Ciencia Latina Revista Científica Multidisciplinar, Ciudad de México, México.
ISSN 2707-2207 / ISSN 2707-2215 (en línea), julio-agosto 2024,
Volumen 8, Número 4.

https://doi.org/10.37811/cl_rcm.v8i4

APORTES MATEMÁTICOS EN ECONOMETRÍA TEÓRICA

MATHEMATICAL CONTRIBUTIONS IN THEORETICAL ECONOMETRICS

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DOI: https://doi.org/10.37811/cl_rcm.v8i4.13556

Aportes Matemáticos en Econometría Teórica

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RESUMEN

En sentido amplio *econometría* equivale a "*métrica económica*". La econometría, consiste, por tanto, en la aplicación de la estadística matemática a los datos económicos con la finalidad de proporcionar soporte empírico a los modelos construidos por la economía matemática y obtener resultados numéricos (Gerhard Tintner, *Methodology of Mathematical Economics and Econometrics*, The University of Chicago Press, Chicago, 1968, p. 74). La econometría también puede definirse como el análisis cuantitativo de fenómenos económicos reales, basados en el desarrollo simultáneo de la teoría y la observación, relacionados mediante métodos apropiados de inferencia (P.A. Samuelson, T.C. Koopmans y J.R.N. Stone, "Report of the Evaluative Committee for *Econometrica*", *Econometrica*, vol. 22, núm. 2, abril de 1954, pp. 141-146). La econometría se define como la ciencia social en la cual las herramientas de la teoría económica, las matemáticas y la inferencia estadística se aplican al análisis de los fenómenos económicos (Arthur S. Goldberger, *Econometric Theory*, John Wiley & Sons, Nueva York, 1964, p. 1.). Por tanto, el método de la investigación econométrica persigue fundamentalmente conjugar la teoría económica y la medición real, con la teoría y la técnica de la inferencia estadística como puente (T. Haavelmo, "The Probability Approach in Econometrics", suplemento de *Econometrica*, vol. 12, 1944, prefacio, p. iii.). En ese contexto, la econometría teórica refiere a la construcción de modelos matemáticos de carácter estadístico, sin perjuicio de otras visiones conceptuales. El propósito de la economía teórica, es la predicción y anticipación demostrativa de variables que son ignotas en el sistema económico examinado. La econometría teórica, tiene como principal objetivo, pronosticar, la trayectoria de una variable económica que se encuentra en constante indeterminancia o variabilidad. Visto lo anterior, el propósito de este manuscrito, es dotar de herramientas matemáticas que permitan el análisis lógico de la data con soporte en la teoría y la fenomenología económicas, configurando un modelo econométrico de predicción de variables que mutan constantemente.

Palabras clave: econometría, métrica económica, teoría económica, matemática económica

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Mathematical Contributions in Theoretical Econometrics

ABSTRACT

In a broad sense, econometrics is equivalent to "economic metrics". Econometrics consists, therefore, in the application of mathematical statistics to economic data in order to provide empirical support to the models built by mathematical economics and to obtain numerical results (Gerhard Tintner, *Methodology of Mathematical Economics and Econometrics*, The University of Chicago Press, Chicago, 1968, p. 74). Econometrics can also be defined as the quantitative analysis of real economic phenomena, based on the simultaneous development of theory and observation, related by appropriate methods of inference (P.A. Samuelson, T.C. Koopmans and J.R.N. Stone, "Report of the Evaluative Committee for Econometrica", Econometrica, vol. 22, no. 2, April 1954, pp. 141-146). Econometrics is defined as the social science in which the tools of economic theory, mathematics, and statistical inference are applied to the analysis of economic phenomena (Arthur S. Goldberger, Econometric Theory, John Wiley & Sons, New York, 1964, p. 1.). Therefore, the method of econometric research fundamentally seeks to combine economic theory and real measurement, with the theory and technique of statistical inference as a bridge (T. Haavelmo, "The Probability Approach in Econometrics", supplement to Econometrica, vol. 12, 1944, preface, p. iii.). In this context, theoretical econometrics refers to the construction of mathematical models of a statistical nature, without prejudice to other conceptual visions. The purpose of theoretical economics is the demonstrative prediction and anticipation of variables that are unknown in the economic system examined. The main objective of theoretical econometrics is to forecast the trajectory of an economic variable that is in constant indeterminacy or variability. In view of the above, the purpose of this manuscript is to provide mathematical tools that allow the logical analysis of data with support from economic theory and phenomenology.

Keywords: econometrics, economic metrics, economic theory, economic mathematics

Artículo recibido 10 julio 2024
Aceptado para publicación: 15 agosto 2024

INTRODUCCIÓN

El objetivo nuclear de la economía matemática consiste en expresar, la teoría económica a través de ecuaciones, sin perjuicio de la capacidad de medición o de verificación empírica de un cuerpo teórico específico. La econometría, tiene como finalidad, la verificación empírica del componente cualitativo de la economía en sus distintas ramas.

¨En econometría, el que construye el modelo a menudo se enfrenta a datos ***provenientes de la observación*** *más que de la* ***experimentación****. Esto tiene dos implicaciones importantes para la creación empírica de modelos en econometría. Primero, se requiere que quien elabore modelos domine muy distintas habilidades en comparación con las que se necesitan para analizar los datos experimentales…Segundo, la separación de quien recopila los datos y el analista exige que quien elabora modelos se familiarice por completo con la naturaleza y la estructura de los datos en cuestión¨* (Aris Spanos, Probability Theory and Statistical Inference: Econometric Modeling with Observational Data, Cambridge University Press, Reino Unido, 1999, p. 21).

La econometría teórica está íntimamente vinculada con la elaboración de métodos idóneos para calcular las relaciones económicas en examen. En ese contexto, la econometría se apoya, fundamentalmente en estadística, sin perjuicio de otros modelos matemáticos.

Es por ello, que el presente trabajo, propone un modelo econométrico teórico no tradicional, tendiente a proporcionar soluciones lógico – matemáticas a relaciones económicas específicas, sin desprendernos de la evidencia empírica que arroja la realidad en estudio de que se trate.

METODOLOGÍA

La teorización desplegada en el presente manuscrito, resulta de la aplicación de una metodología de investigación integral, esto es, bajo un enfoque híbrido, tanto desde el punto de vista cualitativo como en su dimensión cuantitativa. El tipo de investigación que ha sido desarrollado a lo largo del presente Artículo Científico, es esencialmente predictivo, a la luz de la econometría teórica, más no, acusa carácter empírico o experimental. Por otro lado, las líneas de investigación adoptadas para la formulación del estado del arte, se ajustan al constructivismo. Cabe indicar, que no existe población de estudio en la medida en que el presente artículo científico, no es de carácter sociológico o social, más aun, en mérito a su impacto en la realidad de transformación. Tampoco se han implementado técnicas de recolección de información, tales como encuestas, entrevistas, etc, salvo revisión bibliográfica, a razón del campo de investigación abordado. Adicionalmente a lo antes expuesto, es perciso resaltar, que el material de apoyo es meramente bibliográfico. La técnica metodológica, dada la complejidad de la temática escrutada, es deductiva, pues la teorización en sentido estricto, ha sido desarrollada desde principios y premisas generales que son inherentes a la econometría teórica en sentido amplio.

RESULTADOS Y DISCUSIÓN

En sentido estricto, $y' = 2xy + 1$ se tiene como una ecuación diferencial ordinaria, en la que $y = f(x)$ se define como la variable dependiente, x como la variable independiente y, $y' = \frac{dy}{dx}$ como la derivada de y con respecto a x.

La expresión $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ se tiene como una ecuación con derivadas parciales.

Una ecuación es lineal cuando posee la siguiente forma: $a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \ldots + a_1(x)y' + a_0(x)y = g(x)$, esto es:

1. Que, ninguna de las funciones y sus derivadas se elevan a potencia distinta de uno o cero.
2. En cada coeficiente multiplicado, interviene únicamente la variable independiente.
3. Una combinación lineal, sea cual sea, proporciona una solución.
4. $y' = y$ es una ecuación diferencial ordinaria lineal de primer orden, cuya solución es $y = f(x) = k \cdot e^x$, en la que, *k* es un número real cualquiera.
5. $y'' + y = 0$ es una ecuación diferencial ordinaria lineal de segundo orden, cuya solución es $y = f(x) = a\cos(x) + b\sin(x)$, en la que, *a* y *b* son reales.
6. $y'' - y = 0$ es una ecuación diferencial ordinaria lineal de segundo orden, cuya solución es $a \cdot e^x + b \cdot 1/(e^x)$, en la que, *a* y *b* son reales.

Ahora bien, si

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \begin{cases} \frac{dy}{dx} = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

O en su forma implícita:

$$f\left(x, y, \frac{dy}{dx}\right) = 0 \text{ con } y(x_0) = y_0 \quad M(x)dx = N(y)dy$$

$$\int_{x_0}^{x} M(x)dx = \int_{y_0}^{y} N(y)dy \qquad \frac{dy}{dx} = \frac{x^2 y + y^3 - xy^2}{x^3 - 7xy^2} \quad u(x,y) = \frac{x}{y}$$

$$u(y,x) = \frac{y}{x} \quad \frac{dy}{dx} = F\left(\frac{y}{x}\right) \quad \ln x = \int \frac{du}{F(u) - u} + C \quad \frac{dy}{dx} + \alpha y(x) = f(x)$$

$$y(x) = e^{-\alpha(x - x_0)}(y_0 + \int_{x_0}^{x} f(z) e^{\alpha(z - x_0)} dz)$$

En donde $f(x)=b=\text{cte.}$ y $x_0=0$, cuya forma es:

$$y(x) = y_0 e^{-\alpha x} + \frac{b}{\alpha}(1 - e^{-\alpha x}) \qquad \frac{dy}{dx} + P(x)y = Q(x)y^{\alpha}$$

$$y(x) = \frac{e^{-\int P(x)dx}}{\sqrt[\alpha-1]{(1-\alpha)\int Q(x)\,dx + C}}$$

$$a_n(x)D^n y(x) + a_{n-1}(x)D^{n-1}y(x) + \cdots + a_1(x)Dy(x) + a_0(x)y(x) = f(x)$$

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

$$\mathcal{L}y = f, \qquad \mathcal{L} = \sum_{k=0}^{n} a_k(x)D^k(\cdot) \begin{cases} y' + P(x)y = Q(x) \\ y(x_0) = y_0 \end{cases}$$

Donde $P(x)$ y $Q(x)$ son funciones continuas en un intervalo $[a,b] \subseteq \mathbb{R}$.

La solución es:

$$y(x) = e^{-\int_{x_0}^{x} P(x)dx}\left[y_0 + \int_{x_0}^{x} Q(x)e^{\int P(x)dx}dx\right]$$

$$y^{(n)} + A_1(x)y^{(n-1)} + \ldots + A_n(x)y = R(x) Y_0(x) := y(x), \quad Y_k(x) := \frac{d^k y}{dx^k}$$

$$Y_k(x) = \frac{dY_{k-1}}{dx} \text{ con } k \le n-1, \quad Y_n(x) := -A_1(x)Y_{n-1}(x) - \ldots - A_n(x)Y_0(x) + R(x)$$

$$\begin{bmatrix} Y_0' \\ Y_1' \\ \ldots \\ Y_n' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & & & \ldots & \\ -A_n & -A_{n-1} & -A_{n-2} & \ldots & -A_1 \end{bmatrix} \begin{bmatrix} Y_0 \\ Y_1 \\ \ldots \\ Y_n \end{bmatrix}$$

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y + b = 0$$

Donde $a_k(k = 0, 1, \ldots, n) \in \mathbb{R}$ son coeficientes constantes conocidos.

Siendo así que:

$$r^n + a_1 r^{n-1} + \ldots + a_n = 0$$

$$y(x) = y_1(x) + \ldots + y_n(x)$$

En el que, calculando las raíces de λ_n pueden darse los siguientes resultados:

1. Raíces reales disímiles: La solución equivale a $y(x) = y_1(x) + \ldots + y_n(x)$, donde $y_k(x) = C_k e^{\lambda_k x}$, siendo C_k una constante integral.

2. Raíces reales continuas: No es posible expresar la solución como $y(x) = 2Ce^{\lambda x}$. En estos casos, la solución es $y(x) = C_1 e^{\lambda x} + C_2 x e^{\lambda x}$. En general, en una ecuación de orden *n*, si una raíz λ_0 aparece repetida *q* veces, la solución es:

$$y(x) = \sum_{j=1}^{q} C_j x^{j-1} e^{\lambda_0 x}$$

3. Raíces complejas: Cuando las raíces equivalen a $\lambda_k = a_k + b_k i$, la solución es:

$$y_k(x) = e^{a_k x}[\cos(b_k x) + \sin(b_k x)].$$

Más, si las raíces están uniformes, la ecuación se expresará así:

$$y_k(x) = e^{a_k x}[\sum_{j=1}^{q} C_j x^{j-1} \cos(b_k x) + \sum_{j=1}^{q} C_j x^{j-1} \sin(b_k x)].$$

$$y(x) = y_p(x) + y_h(x) = y_p(x) + \sum_{k=1}^{n} C_k e^{\lambda_k x}.$$

Si tenemos $b = x$ y si se plantea $y_p(x) = Ax + B$, las constantes A y B quedarán determinadas así:

Si tenemos $b = \cos(2x)$ y si se propone $y_p(x) = A\cos(2x) + B\sin(2x)$, las constantes A y B quedarán determinadas así:

$$M(x,y)\,dx + N(x,y)\,dy = 0,$$

En la que las derivadas parciales de M y N: $\frac{\partial M}{\partial y}$ y $\frac{\partial N}{\partial x}$ son iguales. Esto es:

$$dF(x,y) = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$$

Donde $\frac{\partial F}{\partial x} = M(x,y)$ y $\frac{\partial F}{\partial y} = N(x,y)$.

Tomando en consideración que *F(x,y)* es una función diferencial, entonces las derivadas cruzadas quedan $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{\partial^2 F}{\partial x \partial y}$.

$$f(x,y) = \int M\,dx + g(y) = \int N\,dy + g(x)$$

En tratándose del factor integrante, éste siempre existe, pero sólo en ciertos casos, tales como:

$$\mu(x) = e^{\int \frac{M_y - N_x}{N}\,dx} \mu(y) = e^{\int \frac{N_x - M_y}{M}\,dy} \mu(x+y) = e^{\int \frac{N_x - M_y}{M - N}\,dz}$$

$$\mu(xy) = e^{\int \frac{N_x - M_y}{N * y - M * x}\,dz}$$

Cabe mencionar que si:

$$M_y = \frac{\partial M}{\partial y}, N_x = \frac{\partial N}{\partial x}$$

Entonces:

$$(a_1 + b_1 x + c_1 y)(xdy - ydx) - (a_2 + b_2 x + c_2 y)dy + (a_3 + b_3 x + c_3 y)dx = 0$$

$$\alpha_1 x + \alpha_2 y + \alpha_3 = K \ \forall \alpha_i, K \in \mathbb{R}$$

Donde la matriz es:

$$A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

Por lo que, el espectro de A es:

$$\sigma(A) = \{\lambda_1, \lambda_2, \lambda_3\} \subset \mathbb{K}$$

Siendo los autovalores distintos, así:

$$\left.\begin{matrix} \sum_{i=1}^{3} k_i = 0 \\ \sum_{i=i}^{3} \lambda_i k_i = 0 \end{matrix}\right\}$$

Esto es, que los coeficientes son $k_1 = \lambda_2 - \lambda_3;\ k_2 = \lambda_3 - \lambda_1;\ k_3 = \lambda_1 - \lambda_2$

Más, si la función es implícita, entonces:

$$f_i(x,y) = \alpha_1^i x + \alpha_2^i y + \alpha_3^i \begin{pmatrix} a_1 - \lambda_i & a_2 & a_3 \\ b_1 & b_2 - \lambda_i & b_3 \\ c_1 & c_2 & c_3 - \lambda_i \end{pmatrix} \begin{pmatrix} \alpha_1^i \\ \alpha_2^i \\ \alpha_3^i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\sum_{i=1}^{3} f_i(x,y)^{k_i} = \beta \ \forall \beta \in \mathbb{R}\ y(x) = x\frac{dy}{dx} + f\left(\frac{dy}{dx}\right).$$

Quedando así:

$$\frac{dy}{dx} = \frac{dy}{dx} + x\frac{d^2y}{dx^2} + f'\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2},$$

Por lo que:

$$0 = \left(x + f'\left(\frac{dy}{dx}\right)\right)\frac{d^2y}{dx^2}\cdot 0 = \frac{d^2y}{dx^2}0 = x + f'\left(\frac{dy}{dx}\right).$$

Por otro lado, en tratándose de una solución singular, se emplean notaciones paramétricas, así:

$$xy''' + (y''')^2 = y''.$$

Más si:

$$y'' = p,$$

Entonces:

$$xp' + (p')^2 = p,$$

Cuya solución es:

$$p = y'' = Cx + C^2,$$

En espectro integral sería:

$$y = \int\int y''\,dxdx = \int\int(Cx+C^2)\,dxdx = \int(\frac{Cx^2}{2}+C^2x+D)\,dx = \frac{Cx^3}{6}+\frac{C^2x^2}{2}+Dx+E,$$

Siendo la solución específica:

$$y = \frac{Cx^3}{6} + \frac{C^2x^2}{2} + Dx + E.$$

Ahora bien, una función con diversas variables se entenderá diferenciable en $x_0 \in \mathbb{R}^n$ si $f : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$, siendo Ω un conjunto abierto en $\mathbb{R}^n$, en tanto exista una transformación lineal T que cumpla lo que sigue:

$$f(x_0 + h) = f(x_0) + T(h) + \theta(h)$$

Donde θ(*h*) equivale a:

$$\lim_{h \to 0} \frac{\|\Theta(h)\|}{\|h\|} = 0$$

Cuya aplicación lineal es:

$$\lim_{h \to 0} \frac{\|f(a+h) - f(a) - T(h)\|}{\|h\|} = 0$$

Ahora bien, la función *f(x,y)* es diferenciable, si *x,y* son diferentes de *0*, esto es:

$$f(x, y) = e^{x+y}$$

En cambio, cuando la función *g*(*x*,*y*) es continua, se soluciona así:

$$g(x, y) = \frac{x|y|}{\sqrt{x^2 + y^2}}$$

La función $h(x,y)$ no es diferenciable en (0,0) en la medida en que, no es continua:

$$h(x,y) = \frac{xy^2}{x^4 + y^4}$$

Una función continua comporta una regla de correspondencia, es decir, $\lim_{\Delta x \to 0} \Delta y = 0$, por lo que, usando la expresión $\Delta y + y = f(\Delta x + x)$, queda $\lim_{\Delta x \to 0} f(\Delta x + x) - y = 0$ donde en este caso, $f(x) = y$. Esto permite afirmar que $\lim_{x \to a} f_{(x)} = f_{(a)}$, y si este último límite existe, entonces aplica la regla de que, un límite existe si y sólo si los dos límites laterales existen y son iguales, así:

$$\lim_{x \to a+} f(x) = \lim_{x \to a-} f(x) = \lim_{x \to a} f(x) = f(a)$$

Un ejemplo puede ser la función de módulo, en el punto $(0,0)$. Dicha función es equivalente a: $\begin{cases} x, & \text{si } x \geq 0 \\ -x, & \text{si } x < 0 \end{cases}$

Por lo que, para valores infinitamente cercanos a 0, el resultado siempre se aproximará a 0, aunque las derivadas pueden resultar de este modo $\begin{cases} 1, & \text{si } x > 0 \\ -1, & \text{si } x < 0 \end{cases}$.

Cuando x vale 0, las derivadas laterales dan resultados desiguales, de tal suerte que, la derivada de la función en el punto a se define como sigue:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

También puede definirse así:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

Lo cual representa una aproximación de la pendiente de la secante a la pendiente de la tangente.

Por otro lado, existen diversas maneras para nombrar a la derivada.

1. $f'(x)$. Notación de Lagrange.
2. $\mathrm{D}_x f$ o $\partial_x f$. Notaciones de Cauchy y Jacobi.
3. $\dot{x}$. Notación de Newton
4. $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}f}{\mathrm{d}x}$ó $\frac{\mathrm{d}}{\mathrm{d}x}f(x)$. Notación de Leibniz.

Las derivadas de f, se identifican así:

$f'(a)$ para la derivada de primer orden,

$f''(a)$ para la derivada de segundo orden,

$f'''(a)$ para la derivada de tercer orden,

$f^{(n)}(a)$ para la derivada infinita (*n* > 3).

Para la función derivada de f en x, se escribe $f'(x)$. De manera semejante, para la derivada de segundo orden de f en x, se escribe $f''(x)$, y así sucesivamente.

La otra notación común es la de Leibniz, que se escribe así:

$$\frac{\mathrm{d}\,(f(x))}{\mathrm{d}x}.$$

Lo que equivale a:

$$\frac{\mathrm{d}\left(f(x)\right)}{\mathrm{d}x}\bigg|_{x=a} = \left(\frac{\mathrm{d}\left(f(x)\right)}{\mathrm{d}x}\right)(a).$$

Si $y = f(x)$, la derivada equivale a:

$$\frac{\mathrm{d}y}{\mathrm{d}x}$$

En el que, las derivadas continuadas se expresan así:

$$\frac{\mathrm{d}^n\left(f(x)\right)}{\mathrm{d}x^n}$$ o $$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$$

$$\frac{\mathrm{d}\left(\frac{\mathrm{d}\left(\frac{\mathrm{d}(f(x))}{\mathrm{d}x}\right)}{\mathrm{d}x}\right)}{\mathrm{d}x}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^3 \left(f(x)\right) = \frac{\mathrm{d}^3}{\left(\mathrm{d}x\right)^3}\left(f(x)\right).$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}.$$

En análisis no-estándar, se pueden derivar así:

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} = x'(t)$$

$$\ddot{x} = x''(t)$$

y así sucesivamente.

La derivada de una función f es la pendiente geométrica de la línea tangente:

$$\frac{f(x+h)-f(x)}{h}$$

Cuyo coeficiente diferencial es:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$

La función derivada se puede calcular, así también:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Por ejemplo, sea:

$$f\,(x)=x^2$$

Entonces:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{(x+h)^2-x^2}{h}\\
&= \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}\\
&= \lim_{h\to 0}\frac{2xh+h^2}{h}\\
&= \lim_{h\to 0}(2x+h)\\
&= 2x
\end{aligned}$$

En las ecuaciones siguientes se considera que $x, a, b, k \in \mathbb{R}, n \in \mathbb{N}$:

$$f\left(x\right) = af'\left(x\right) = 0f\left(x\right) = xf\left(x\right) = axf'\left(x\right) = 1f'\left(x\right) = a$$
$$f\left(x\right) = ax + bf'\left(x\right) = af\left(x\right) = x^n f'\left(x\right) = nx^{n-1}$$
$$f\left(x\right) = \sqrt{x}^{f'\left(x\right) = \frac{1}{2\sqrt{x}f\left(x\right) = e^x f'\left(x\right) = e^x f\left(x\right) = \ln(x)^{f'\left(x\right) = \frac{1}{x}}}}$$
$$f\left(x\right) = a^x(a > 0)f'\left(x\right) = a^x \ln(a)f\left(x\right) = \log_b(x)^{f'\left(x\right) = \frac{1}{x\ln(b)}}$$
$$f\left(x\right) = \frac{1}{x^n} = x^{-n}f'\left(x\right) = -nx^{-n-1} = \frac{-n}{x^{n+1}f\left(x\right) = \operatorname{sen}(x)}$$
$$f'\left(x\right) = \cos(x)f\left(x\right) = \cos(x)f'\left(x\right) = -\operatorname{sen}(x)f\left(x\right) = \tan(x)$$
$$f'\left(x\right) = \sec^2(x) = \frac{1}{cos^2(x)} = 1 + \tan^2(x)_{f'\left(x\right) = -\csc(x)\cot(x)}$$
$$f\left(x\right) = \csc(x)f\left(x\right) = \sec(x)f'\left(x\right) = \sec(x)\tan(x)f\left(x\right) = \cot(x)$$
$$f'\left(x\right) = -\csc^2(x)^{f'\left(x\right) = \frac{1}{\sqrt{1-x^2}f\left(x\right) = \arccos(x)f\left(x\right) = \operatorname{arcsen}(x)}}$$
$$f'\left(x\right) = \frac{-1}{\sqrt{1-x^2}f'\left(x\right) = \frac{1}{1+x^2 f\left(x\right) = \arctan(x)f\left(x\right) = g(x) \pm h(x)}}$$
$$f'\left(x\right) = g'(x) \pm h'(x)f\left(x\right) = g(x) \cdot h(x)$$
$$f'\left(x\right) = g'(x) \cdot h(x) + g(x) \cdot h'(x)f\left(x\right) = k \cdot g(x)f'\left(x\right) = k \cdot g'(x)$$
$$f\left(x\right) = \frac{g(x)}{h(x)^{f'\left(x\right) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h^2(x)}}} f\left(x\right) = g\left(x\right)^{h(x)}$$

$$f'\left(x\right) = h\left(x\right) \cdot g'\left(x\right) \cdot g\left(x\right)^{(h(x)-1)} + g\left(x\right)^{h(x)} \cdot h'\left(x\right) \cdot ln\left(g\left(x\right)\right)$$

$$f\left(x\right) = g \circ h = g(h(x))f'\left(x\right) = (g' \circ h) \cdot h' = g'(h(x)) \cdot h'(x)$$
$$f'(c) = f''(c) = f'''(c) = \cdots = f^{(n-1)}(c) = 0$$

Por tanto, $f^{(n)}(x) < 0 \implies x = c$ es un punto máximo local, por lo que, $f^{(n)}(x) > 0 \implies x = c$ es un punto mínimo local.

Ahora bien, si $f(x) : A \subset \mathbb{R} \longrightarrow \mathbb{R}$, sea $x_0 \in A$ y sea $P\left(x_0, f(x_0)\right)$ un punto concerniente a la función, entonces:

Se dirá que P es un máximo local de f cuando existe un entorno reducido de centro x_0, donde para todo elemento x de $E'(x_0)$ se cumple $f(x) \leq f(x_0)$. Para esto, se debe cumplir $f(x) < f(x_0)$.

Se dirá que el punto P es un mínimo local de f cuando existe un entorno reducido de centro x_0, donde para todo elemento x de $E'(x_0)$ se cumple $f(x) \geq f(x_0)$.

En tanto que, si $f(x) : A \subset \mathbb{R} \longmapsto \mathbb{R}$, sea $x_0 \in A$ y sea $P(x_0, f(x_0))$ un punto concerniente a la función, entonces:

Se dirá que P es un máximo absoluto de f si, para todo x distinto de x_0 su valor es menor o igual que la de x_0.

$P(x_0, f(x_0))$ es máximo absoluto de $f \iff \forall x \neq x_0, x \in A, f(x_0) \geq f(x)$.

Se dirá que P es un mínimo absoluto de f si, para todo x distinto de x_0 su valor, es mayor o igual que la de x_0.

$P(x_0, f(x_0))$ es mínimo absoluto de $f \iff \forall x \neq x_0, x \in A, f(x_0) \leq f(x)$.

Si la función es $f(x) : A \subset \mathbb{R} \longrightarrow \mathbb{R}$, las derivadas son:

1. $f \rightarrow f'(x)$
2. $f \rightarrow f''(x)$
3. $f'(x) = 0$
4. $x = \{x_1, x_2, ..., x_n / f'(x_i) = 0 \quad \forall i = 1, 2, ..., n\}$.
5. $f''(x_i) < 0$, se tiene un máximo en el punto $M(x_i, f(x_i))$.
6. $f''(x_i) > 0$, se tiene un mínimo en el punto $m(x_i, f(x_i))$.
7. $f''(x_i) = 0$,

Si la derivada es par, se trata de un extremo local; en un máximo si $f^n\left(x_i\right) < 0$ y en un mínimo si $f^n\left(x_i\right) > 0$. Si la derivada no es par, se trata de un punto de inflexión.

Más concretamente:

$$f \rightarrow f'(x)$$
$$f \rightarrow f''(x)$$
$$f \rightarrow f'''(x)$$
$$f''(x) = 0$$
$$x = \{x_1, x_2, ..., x_n / f''(x_i) = 0 \quad \forall i = 1, 2, ..., n\}.$$

Si $f'''\left(x_i\right) \neq 0$, entonces se tiene un punto de inflexión en $P\left(x_i, f(x_i)\right)$.

Si $f'''\left(x_i\right) = 0$, entonces, debemos sustituir x_i en las sucesivas derivadas hasta obtener un valor distinto a cero.

En el punto crítico se verifica:

$$\left(\frac{\partial P}{\partial v}\right)_{T=T_c} = 0$$
$$\left(\frac{\partial^2 P}{\partial v^2}\right)_{T=T_c} = 0$$

En cuanto a relaciones definitorias, tenemos:

$$[J_{ij}, J_{k\ell}] = \delta_{jk} J_{i\ell} - \delta_{j\ell} J_{ik} - \delta_{ik} J_{j\ell} + \delta_{i\ell} J_{jk}$$

Por lo que:

$$[J_{ij}, Q_a] = \frac{1}{4}\left(\gamma_i\gamma_j - \gamma_j\gamma_i\right)_{ab} Q_b$$,

De lo que se obtiene:

$$[Q_a, Q_b] = \gamma^{[i}_{ac}\gamma^{j]}_{cb} J_{ij}.$$

A partir de lo anterior, la identidad de Jacobi es igual a

$$E_8 \leftarrow \mathrm{SO}(10) \leftarrow \mathrm{SU}(5) \leftarrow \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$$

$$(\pm 1, \pm 1, 0, 0, 0, 0, 0, 0)$$

En la siguiente ecuación se describe:

$$m = \frac{\Delta y}{\Delta x}$$

Dados dos puntos (x_1,y_1) y (x_2,y_2), la diferencia en X es $x_2 - x_1$, mientras que el cambio en Y se calcula como $y_2 - y_1$. Por tanto:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Si la pendiente m de una recta y el punto (x_0,y_0) de la misma, están identificados, entonces la ecuación es:

$$y - y_0 = m(x - x_0)$$

La pendiente de la recta es:

$$Ax + By + C = 0$$

La misma que está dada por:

$$m = -\frac{A}{B}$$

Definiéndose así:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

Por tanto:

$$\frac{\mathrm{d}\left(f(x)\right)}{\mathrm{d}x}.$$

$$\left.\frac{\mathrm{d}\left(f(x)\right)}{\mathrm{d}x}\right|_{x=a} = \left(\frac{\mathrm{d}\left(f(x)\right)}{\mathrm{d}x}\right)(a).$$

$\frac{\mathrm{d}^n\left(f(x)\right)}{\mathrm{d}x^n}$ o $\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$

$$\frac{\mathrm{d}\left(\frac{\mathrm{d}\left(\frac{\mathrm{d}(f(x))}{\mathrm{d}x}\right)}{\mathrm{d}x}\right)}{\mathrm{d}x}$$

Lo cual se puede explicar de esta manera:

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^3 (f(x)) = \frac{\mathrm{d}^3}{(\mathrm{d}x)^3} (f(x)).$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}.$$

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} = x'(t)$$

$$\ddot{x} = x''(t)$$

Y así sucesivamente.

Ahora bien, si la función *f* tiene límite, entonces el límite de una función *f(x)*, se calculará de la siguiente manera:

$$\lim_{x \to c} f(x) = L \iff \forall \varepsilon > 0 \;\; \exists \delta > 0 / \forall x \in \mathrm{Dom}(f), 0 < |x - c| < \delta \longrightarrow |f(x) - L| < \epsilon$$

No obstante, hay casos como la función de Dirichlet $D : \mathbb{R} \to \mathbb{R}$ en la que:

$$D(x) = \begin{cases} c \\ d \end{cases}$$

Donde no existe un número *c* para el cual exista $\lim_{x \to c} f(x)$.

En cuyo caso, los límites se calculan así:

$$\lim_{x \to c^+} f(x) = L$$

o

$$\lim_{x \to c^-} f(x) = L$$

O así, según el siguiente cuadro:

Límite de	Expresión.
Una constante.	$\lim_{x \to c} k = k$
La función identidad.	$\lim_{x \to c} x = c$
El producto de una función y una constante.	$\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x)$
Una suma.	$\lim_{x \to c}(f(x) + g(x)) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$
Una resta.	$\lim_{x \to c}(f(x) - g(x)) = \lim_{x \to c} f(x) - \lim_{x \to c} g(x)$
Un producto.	$\lim_{x \to c}(f(x)g(x)) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$
Un cociente.	$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$ si $\lim_{x \to c} g(x) \neq 0,$
Una potencia.	$\lim_{x \to c} f(x)^{g(x)} = \lim_{x \to c} f(x)^{\lim_{x \to c} g(x)}$ si $f(x) > 0$
Un logaritmo.	$\lim_{x \to c} \log f(x) = \log \lim_{x \to c} f(x)$
El número e.	$\lim_{x \to 0} (1+x)^{\frac{1}{x}} = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$
Función f(x) acotada y g(x) infinitesimal.	$\lim_{x \to c} (f(x) \cdot g(x)) = 0$.

O así (desigualdades):

$$\lim_{t \to 0} \frac{t}{t^2} = \infty \qquad \lim_{t \to 0} \frac{t}{t} = 1 \qquad \lim_{t \to 0} \frac{t^2}{t} = 0$$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)} \qquad \lim_{x \to \infty} x \sin\left(\frac{2\pi}{x}\right) \cos\left(\frac{2\pi}{x}\right) = 2\pi$$

$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{x}{\sin x} = 1 \qquad \lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{x}{\tan x} = 1$$

$$\lim_{x \to 0} \frac{\sin x}{\tan x} = \lim_{x \to 0} \frac{\tan x}{\sin x} = 1 \lim_{x \to 0} \frac{1 - \cos x}{x^2} = 1/21 < \frac{x}{\sin x} < \frac{1}{\cos x}$$

$$\cos x < \frac{\sin x}{x} < 1$$

Calculando el límite cuando *x* tiende a 0:

$$\lim_{x\to 0} \cos x < \lim_{x\to 0} \frac{\sin x}{x} < \lim_{x\to 0} 1$$

Es igual a:

$$1 < \lim_{x\to 0} \frac{\sin x}{x} < 1$$

De tal suerte, que, aplicando el teorema de estricción, el límite es 1:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Cuyo tercer componente, se calcula así:

$$\lim_{x\to 0} \left(\frac{\tan x}{x}\right) = \lim_{x\to 0} \left(\frac{\sin x}{x}\right) \cdot \lim_{x\to 0} \frac{1}{\cos x} = 1 \cdot 1 = 1$$

Formalmente, la sucesión a_n se aproxima hasta su límite *L*, siendo que converge o que es convergente a *L*, es decir:

$$\lim_{n\to\infty} a_n = L$$

De manera formal, se diría:

$$a_n \to L \Leftrightarrow \forall \varepsilon > 0, \exists N > 0 : \forall n > N, |a_n - L| < \varepsilon$$

Cuyo límite, si se puede demostrar, se tendría por único.

$$\lim_{x \to c} f(x) = L$$
$$\lim_{x \to c} f(x) = L \iff \forall \varepsilon > 0 \;\; \exists\, \delta > 0 : 0 < |x - c| < \delta \longrightarrow |f(x) - L| < \varepsilon$$

$$\lim_{n \to \infty} A_n = \liminf_{n \to \infty} A_n = \limsup_{n \to \infty} A_n = \bigcup_{n=1}^{\infty} \left(\bigcap_{m=n}^{\infty} A_m \right) = \bigcap_{n=1}^{\infty} \left(\bigcup_{m=n}^{\infty} A_m \right)$$

Así, si la función *f* continúa en el punto x_1, por lo que:

$$\exists \lim_{x \to x_1^+} f(x) \in \mathbb{R} \exists \lim_{x \to x_1^-} f(x) \in \mathbb{R} \exists \lim_{x \to x_1^+} f(x) \in \mathbb{R} \quad \wedge \quad \exists \lim_{x \to x_1^-} f(x) \in \mathbb{R}$$

$$\lim_{x \to x_1^-} f(x) = \lim_{x \to x_1^+} f(x)$$

$$\lim_{x \to x1} f(x) = \lim_{x \to x_1^-} f(x) = \lim_{x \to x_1^+} f(x)_{\exists f(x_1)} \lim_{x \to x1} f(x) = f(x)$$

Una función *f* es continua en un intervalo *I*, si y solo si la función es continua en todos los puntos del intervalo, es decir:

$$\forall a \in I, \lim_{x \to a} f(x) = f(a)$$

Por tanto:

$$f(x) = \frac{1}{x} \qquad L^- = \underset{x \to a^-}{Lím}\ f(x) \qquad L^+ = \underset{x \to a^-}{Lím}\ f(x)$$

$$\left.\begin{array}{l} \underset{x \to a^-}{Lím}\ f(x) = L^- \\ \underset{x \to a^+}{Lím}\ f(x) = L^+ \\ L^- = L^+ = L \end{array}\right\} \quad \left.\begin{array}{r} \underset{x \to a}{Lím}\ f(x) = L\ \underset{x \to a}{Lím}\ f(x) = L \\ f(a) = L \end{array}\right\}$$

La discontinuidad de una función se cataloga en:

$$\left.\begin{array}{r} \underset{x \to a}{Lím}\ f(x) = L \\ f(a) \neq L \end{array}\right\}$$

$$\left.\begin{array}{r} \underset{x \to a}{Lím}\ f(x) = L \\ \nexists\ f(a) \end{array}\right\}$$

$$\left.\begin{array}{r} \underset{x \to a}{Lím}\ f(x) = L \\ f(a) = L \end{array}\right\}$$

$$\left.\begin{array}{c} \underset{x \to a^-}{Lím}\ f(x) = L^- \\ \underset{x \to a^+}{Lím}\ f(x) = L^+ \\ L^- \neq L^+ \end{array}\right\}$$

$$Salto = |\lim_{x \to a^-} f(x) - \lim_{x \to a^+} f(x)|$$

$$\left.\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = L \\ \\ \underset{x\to a^+}{Lím}\ f(x) = \pm\infty \end{matrix}\right\}$$

$$\left.\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = \pm\infty \\ \\ \underset{x\to a^+}{Lím}\ f(x) = L \end{matrix}\right\}$$

$$\left.\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = \pm\infty \\ \\ \underset{x\to a^+}{Lím}\ f(x) = \pm\infty \end{matrix}\right\}$$

$$\left\{\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = L1 \\ \\ \underset{x\to a^+}{Lím}\ f(x) = L2 \\ \\ \nexists\ f(a) \end{matrix}\right. \left\{\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = L1 \\ \\ \underset{x\to a^+}{Lím}\ f(x) = L2 \\ \\ f(a) = L1 \end{matrix}\right. \left\{\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = L1 \\ \\ \underset{x\to a^+}{Lím}\ f(x) = L2 \\ \\ f(a) = L2 \end{matrix}\right.$$

$$\left\{\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = L1 \\ \\ \underset{x\to a^+}{Lím}\ f(x) = L2 \\ \\ f(a) \neq L1 \neq L2 \end{matrix}\right. \left\{\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = L \\ \\ \underset{x\to a^+}{Lím}\ f(x) = \infty \\ \\ \nexists\ f(a) \end{matrix}\right. \left\{\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = L \\ \\ \underset{x\to a^+}{Lím}\ f(x) = \infty \\ \\ f(a) = L \end{matrix}\right.$$

$$\left\{\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = L \\ \\ \underset{x\to a^+}{Lím}\ f(x) = -\infty \\ \\ \nexists\ f(a) \end{matrix}\right. \left\{\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = L \\ \\ \underset{x\to a^+}{Lím}\ f(x) = -\infty \\ \\ f(a) = L \end{matrix}\right. \left\{\begin{matrix} \underset{x\to a^-}{Lím}\ f(x) = \infty \\ \\ \underset{x\to a^+}{Lím}\ f(x) = L \\ \\ \nexists\ f(a) \end{matrix}\right.$$

$$\begin{cases} \underset{x\to a^-}{Lím}\ f(x)=\infty \\ \underset{x\to a^+}{Lím}\ f(x)=L \\ f(a)=L \end{cases} \begin{cases} \underset{x\to a^-}{Lím}\ f(x)=-\infty \\ \underset{x\to a^+}{Lím}\ f(x)=L \\ \nexists\, f(a) \end{cases} \begin{cases} \underset{x\to a^-}{Lím}\ f(x)=-\infty \\ \underset{x\to a^+}{Lím}\ f(x)=L \\ f(a)=L \end{cases}$$

$$\begin{cases} \underset{x\to a^-}{Lím}\ f(x)=\infty \\ \underset{x\to a^+}{Lím}\ f(x)=\infty \end{cases} \begin{cases} \underset{x\to a^-}{Lím}\ f(x)=-\infty \\ \underset{x\to a^+}{Lím}\ f(x)=\infty \end{cases} \begin{cases} \underset{x\to a^-}{Lím}\ f(x)=\infty \\ \underset{x\to a^+}{Lím}\ f(x)=-\infty \end{cases}$$

$$\begin{cases} \underset{x\to a^-}{Lím}\ f(x)=-\infty \\ \underset{x\to a^+}{Lím}\ f(x)=-\infty \end{cases} \begin{cases} \underset{x\to a^-}{Lím}\ f(x)=\infty \\ \nexists\, \underset{x\to a^+}{Lím}\ f(x) \end{cases} \begin{cases} \underset{x\to a^-}{Lím}\ f(x)=-\infty \\ \nexists\, \underset{x\to a^+}{Lím}\ f(x) \end{cases}$$

$$\begin{cases} \underset{x\to a^-}{Lím}\ f(x)=L \\ \nexists\, \underset{x\to a^+}{Lím}\ f(x) \\ \nexists\, f(a) \end{cases} \begin{cases} \underset{x\to a^-}{Lím}\ f(x)=L \\ \nexists\, \underset{x\to a^+}{Lím}\ f(x) \\ f(a)=L \end{cases} \begin{cases} \nexists\, \underset{x\to a^-}{Lím}\ f(x) \\ \underset{x\to a^+}{Lím}\ f(x)=\infty \end{cases}$$

$$\begin{cases} \nexists\, \underset{x\to a^-}{Lím}\ f(x) \\ \underset{x\to a^+}{Lím}\ f(x)=-\infty \end{cases} \begin{cases} \nexists\, \underset{x\to a^-}{Lím}\ f(x) \\ \underset{x\to a^+}{Lím}\ f(x)=L \\ \nexists\, f(a) \end{cases} \begin{cases} \nexists\, \underset{x\to a^-}{Lím}\ f(x) \\ \underset{x\to a^+}{Lím}\ f(x)=L \\ f(a)=L \end{cases}$$

$$\begin{cases} \underset{x\to a^-}{Lím}\ f(x)=L \\ \underset{x\to a^+}{Lím}\ f(x)=L \\ \nexists\, f(a) \end{cases} \begin{cases} \underset{x\to a^-}{Lím}\ f(x)=L \\ \underset{x\to a^+}{Lím}\ f(x)=L \\ f(a)\neq L \end{cases}$$

$$\left.\begin{array}{l}\left.\begin{array}{l}\underset{x\to a^-}{Lím}\ f(x)=L^-\\ \underset{x\to a^+}{Lím}\ f(x)=L^+\\ L^-=L^+=L\end{array}\right\}\underset{x\to a}{Lím}\ f(x)=L\\ \qquad\qquad\qquad\qquad f(a)=L\end{array}\right\}\underset{x\to a}{Lím}\ f(x)=f(a)$$

El dominio de una función se calcula así:

$$\begin{array}{l}D\subset X\mid\ \ \forall x\in D:\ \exists y\in Y\quad\wedge\quad y=f(x)\\ D=\{x\in\mathbb{R}\quad\wedge\quad a<x<b\mid\exists y\in\mathbb{R}\quad\wedge\quad y=f(x)\}\\ D=\{x\in\mathbb{R}\quad\wedge\quad x\in(a,b)\mid\exists y\in\mathbb{R}\quad\wedge\quad y=f(x)\}\\ D=\{x\in\mathbb{R}\quad\wedge\quad a<x\leq b\mid\exists y\in\mathbb{R}\quad\wedge\quad y=f(x)\}\\ D=\{x\in\mathbb{R}\quad\wedge\quad x\in(a,b]\mid\exists y\in\mathbb{R}\quad\wedge\quad y=f(x)\}\end{array}$$

$$\begin{array}{l}D=\{x\in\mathbb{R}\quad\wedge\quad a\leq x<b\mid\exists y\in\mathbb{R}\quad\wedge\quad y=f(x)\}\\ D=\{x\in\mathbb{R}\quad\wedge\quad x\in[a,b)\mid\exists y\in\mathbb{R}\quad\wedge\quad y=f(x)\}\\ D=\{x\in\mathbb{R}\quad\wedge\quad a\leq x\leq b\mid\exists y\in\mathbb{R}\quad\wedge\quad y=f(x)\}\\ D=\{x\in\mathbb{R}\quad\wedge\quad x\in[a,b]\mid\exists y\in\mathbb{R}\quad\wedge\quad y=f(x)\}\\ \mathrm{epi}f=\{(x,\mu)\ :\ x\in\mathbb{R}^n,\ \mu\in\mathbb{R},\ f(x)\leq\mu\}\subseteq\mathbb{R}^{n+1}.\end{array}$$

Una ecuación de primer grado, equivale también a:

$$y=mx+n\qquad \begin{array}{rccc}f: & X & \longrightarrow & Y\\ & x & \mapsto & y\end{array}\qquad \mathbf{1}_A:X\to\{0,1\}\qquad \mathbf{1}_A(x)=\begin{cases}1 & \text{if } x\in A,\\ 0 & \text{if } x\notin A.\end{cases}$$

$$\mathbf{1}_A(x)=\begin{cases}1 & \text{si } x\in A,\\ 0 & \text{si } x\notin A.\end{cases}$$

Siguiendo este mismo orden de ideas, la función de Heaviside, es una función discontinua cuyo valor es 0 para cualquier argumento negativo, y 1 para cualquier argumento positivo:

Se calcula así:

$$H(x) = u(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases} \quad H(-x) = 1 - H(x) \quad H'(x-a) = \delta(x-a)$$

$$\mathcal{L}\{H(x-a)\}(s) = \frac{e^{-as}}{s} \qquad H(x) = \lim_{n\to\infty} \frac{1}{e^{-nx}+1}$$

$$H(x) = \lim_{t\to 0}\left(\frac{1}{2} + \frac{1}{\pi}\arctan\frac{x}{t}\right) \quad H(x) = \int_{-\infty}^{x} \delta(t)dt \quad H(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}, & x = 0 \\ 1, & x > 0 \end{cases}$$

$$H(x) = \frac{1}{2}(1 + \operatorname{sgn}(x)) \qquad H_n(x) = \begin{cases} 0, & x < 0 \\ n, & x = 0 \\ 1, & x > 0 \end{cases}$$

$$H(x) = \lim_{\epsilon\to 0} -\frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{1}{\tau + i\epsilon} e^{-ix\tau} d\tau \quad f : \mathbb{R} \to \mathbb{Z}$$

$$\lceil x \rceil = \min\{k \in \mathbb{Z} \mid x \le k\}$$

$$y = \lceil x \rceil : \quad y = \{y : \quad y \in \mathbb{Z} \quad \wedge \quad x \in \mathbb{R} \quad \wedge \quad y - 1 < x \le y\}$$

$$\int_{\epsilon}^{x+\epsilon} \delta(1 - \chi_{\mathbb{Z}}(y))dy = \lceil x \rceil, \qquad 0 < \epsilon < 1 \quad \lfloor x \rfloor = \max\{k \in \mathbb{Z} \mid k \le x\}$$

$$y = \lfloor x \rfloor : \quad y = \{y : \quad y \in \mathbb{Z} \quad \wedge \quad x \in \mathbb{R} \quad \wedge \quad y \le x < y + 1\}$$

$$\{x\} = \frac{1}{2} - \frac{1}{\pi}\sum_{k=1}^{\infty} \frac{\sin(2\pi k x)}{k}.$$

$$x \in \mathbb{Z} \Leftrightarrow \lfloor x \rfloor = x$$

$$\lfloor x \rfloor = x - \frac{1}{2} + \frac{1}{\pi}\sum_{k=1}^{\infty} \frac{\sin(2\pi k x)}{k}. \qquad \lceil x \rceil = x + \frac{1}{2} + \frac{1}{\pi}\sum_{k=1}^{\infty} \frac{\sin(2\pi k x)}{k}.$$

$$\operatorname{int}(x) = x - \frac{\sin(x)}{2} + \frac{1}{\pi}\sum_{k=1}^{\infty} \frac{\sin(2\pi k x)}{k}.$$

En tanto que, la función entera es aquella que está compuesta por la función piso y la función techo, definiéndose así:

$$\text{int}(x) = [x] = \begin{cases} \text{si} \quad x \geq 1 & [x] = \lfloor x \rfloor \\ \text{si} \quad -1 < x < 1 & [x] = 0 \\ \text{si} \quad x \leq -1 & [x] = \lceil x \rceil \end{cases}$$

$$\text{sgn}(x) = \begin{cases} 1, & \text{si } x > 0 \\ 0, & \text{si } x = 0 \\ -1, & \text{si } x < 0 \end{cases} \qquad \text{sgn}(x) = \frac{d|x|}{dx} = \begin{cases} 1, & \text{si } x > 0 \\ -1, & \text{si } x < 0 \end{cases}$$

$$u(x) = \begin{cases} 1, & \text{si } x > 0 \\ \frac{1}{2}, & \text{si } x = 0 \\ 0, & \text{si } x < 0 \end{cases} \quad \text{sgn}(-x) = -\text{sgn}(x) \quad x = \text{sgn}(x) \cdot |x| \qquad x \in \mathbb{R}$$

$$\frac{d|x|}{dx} = \text{sgn}(x) \, . \frac{d \;\; \text{sgn}(x)}{dx} = 2\delta(x) \, . |a| = \begin{cases} a, & \text{si } a \geq 0 \\ -a, & \text{si } a < 0 \end{cases}$$

$\lvert a\rvert \geq 0$	No negatividad.
$\lvert a\rvert = 0 \iff a = 0$	Definición positiva.
$\lvert ab\rvert = \lvert a\rvert\lvert b\rvert$	Propiedad multiplicativa.
$\lvert a+b\rvert \leq \lvert a\rvert + \lvert b\rvert$	Desigualdad triangular.

$\lvert -a\rvert = \lvert a\rvert$	Simetría.
$\lvert a-b\rvert = 0 \iff a = b$	Identidad de indiscernibles.
$\lvert a-b\rvert \leq \lvert a-c\rvert + \lvert c-b\rvert$	Desigualdad triangular.
$\lvert a-b\rvert \geq \lvert\lvert a\rvert - \lvert b\rvert\rvert$	Equivalente a la propiedad aditiva.
$\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a\rvert}{\lvert b\rvert} (si \; b \neq 0)$	Preservación de la división.

Es válido también:

$$|a| \leq b \iff -b \leq a \leq b$$
$$|a| \geq b \iff a \geq b \vee a \leq -b$$

En otro orden de ideas, el valor absoluto de un número complejo z es la distancia r desde z al origen. Se calcula así:

$$|a| = \sqrt{a^2} \; z = x + iy |z| = \sqrt{x^2 + y^2}$$
$$|x + i0| = \sqrt{x^2 + 0^2} = \sqrt{x^2} = |x| \; z = x + iy = r(\cos\phi + i\sin\phi) \; \bar{z} = x - iy \; |z| = r$$
$$|z| = |\bar{z}||z| = \sqrt{z\bar{z}}$$

La mantisa del logaritmo de un número *x* mayor que cero, se calcula así:

$$m(x) = \log_{10}(x) - C = \log_{10}(x) - \lfloor \log_{10}(x) \rfloor$$

$$D(x) = \begin{cases} c \\ d \end{cases} \; D(x) = \lim_{k\to\infty}\left(\lim_{j\to\infty}\left(\cos(k!\pi x)^{2j}\right)\right)$$
$$A(m,n) = \begin{cases} n+1, & \text{si } m = 0; \\ A(m-1,1), & \text{si } m > 0 \text{ y } n = 0; \\ A(m-1, A(m, n-1)), & \text{si } m > 0 \text{ y } n > 0 \end{cases}$$
$$f_0(x) = s(x) \; f_1(x) = f_0^{x+2}(x) = s^{x+2}(x) \; f_2(x) = f_1^{x+2}(x) = (s^{x+2})^{x+2}(x)$$
$$f_{k+1}(x) = f_k^{x+2}(x)$$
$$k \in \mathbb{N}, \; f_k \in FRP \qquad a > b, \; f_k(a) > f_k(b) \qquad x, k \in \mathbb{N}, \; f_k(x) > x$$
$$k \in \mathbb{N}, \; f_{k+1}(x) > f_k(x) \qquad A(m,n) = 2 \to (n+3) \to (m-2) - 3$$

$$A(m,n) = \text{hyper}(2, n+3, m-2) - 3 \qquad F_0(x,y) = x + y,$$

$$F_{n+1}(x,0) = x,\ \ n \geq 0\, F_{n+1}(x,y+1) = F_n(F_{n+1}(x,y), F_{n+1}(x,y) + y + 1),\ n \geq 0.$$

$$\sigma_x(n) = \sum_{d|n} d^x.$$

Las notaciones *d*(*n*) y τ(*n*) son usadas para expresar $\sigma_0(n)$, o el número de denominadores de *n*. Esto es:

$$d(p) = 2\, d(p^n) = n + 1\, \sigma(p) = p + 1\ n = \prod_{i=1}^{r} p_i^{a_i} \sigma_x(n) = \prod_{i=1}^{r} \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$$

$$\sigma_x(n) = \prod_{i=1}^{r} \sum_{j=0}^{a_i} p_i^{jx} = \prod_{i=1}^{r} (1 + p_i^x + p_i^{2x} + ... + p_i^{a_i x}).\, \tau(n) = \prod_{i=1}^{r} (a_i + 1).$$

$$\sigma_x(n) = \sum_{\mu=1}^{n} \mu^{x-1} \sum_{\nu=1}^{\mu} \cos \frac{2\pi\nu n}{\mu} \qquad \sum_{n=1}^{\infty} \frac{\sigma_a(n)}{n^s} = \zeta(s)\zeta(s-a)$$

$$\sum_{n=1}^{\infty} \frac{\sigma_a(n)\sigma_b(n)}{n^s} = \frac{\zeta(s)\zeta(s-a)\zeta(s-b)\zeta(s-a-b)}{\zeta(2s-a-b)}.$$

$$\sum_{n=1}^{\infty} q^n \sigma_a(n) = \sum_{n=1}^{\infty} \frac{n^a q^n}{1-q^n} \qquad \forall \epsilon > 0, d(n) = o(n^\epsilon).$$

$$\forall x \geq 1, \sum_{n \leq x} d(n) = x \log x + (2\gamma - 1)x + O(\sqrt{x}),$$

$$\limsup_{n\to\infty} \frac{\sigma(n)}{n\ \log\log n} = e^\gamma, \sigma(n) < e^\gamma n \log\log n \qquad \forall n > 5040$$

$$\sigma(n) \leq H_n + \ln(H_n)e^{H_n} \qquad \varphi(m) = |\{n \in \mathbb{N} | n \leq m \wedge \text{mcd}(m,n) = 1\}|$$

$$\varphi(mn) = \varphi(m)\varphi(n). \qquad n = p_1^{k_1} \cdots p_r^{k_r}$$

$$\varphi(n) = (p_1 - 1)p_1^{k_1 - 1} \cdots (p_r - 1)p_r^{k_r - 1}.\ \varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$$

$$\sum_{d|n} \varphi(d) = n\ \ \varphi(n) = \sum_{d|n} d\mu(n/d)\ \sum_{n=0}^{\infty} \frac{\varphi(n)}{n^s} = \frac{\zeta(s-1)}{\zeta(s)}$$

A partir de este punto, se incorporan modelos matemáticos aplicables a la econometría teórica en sentido lato:

$$\mathrm{li}(x) = \int_0^x \frac{dt}{\ln(t)} \qquad \mathrm{li}(x) = \int_0^x \frac{dt}{\ln(t)}.$$

$$\mathrm{li}(x) = \lim_{\varepsilon \to 0+} \left(\int_0^{1-\varepsilon} \frac{dt}{\ln(t)} + \int_{1+\varepsilon}^x \frac{dt}{\ln(t)} \right). \qquad \mathrm{Li}(x) = \mathrm{li}(x) - \mathrm{li}(2)$$

$$\mathrm{Li}(x) = \int_2^x \frac{dt}{\ln t} \qquad \mathrm{li}(x) = \mathrm{Ei}(\ln(x)),$$

$$\mathrm{li}(e^u) = \mathrm{Ei}(u) = \gamma + \ln u + \sum_{n=1}^{\infty} \frac{u^n}{n \cdot n!} \quad \text{para } u \neq 0 \ ,$$

$$\mathrm{li}(x) = \gamma + \ln \ln x + \sqrt{x} \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (\ln x)^n}{n! \, 2^{n-1}} \sum_{k=0}^{\lfloor (n-1)/2 \rfloor} \frac{1}{2k+1}.$$

$$\mathrm{li}(x) = \mathcal{O}\left(\frac{x}{\ln x} \right) . \qquad \mathrm{li}(x) \sim \frac{x}{\ln x} \sum_{k=0}^{\infty} \frac{k!}{(\ln x)^k}$$

$$\frac{\mathrm{li}(x)}{x/\ln x} \sim 1 + \frac{1}{\ln x} + \frac{2}{(\ln x)^2} + \frac{6}{(\ln x)^3} + \cdots \cdot \mathrm{Ei}(x) = \int_{-\infty}^x \frac{e^t}{t} \, dt.$$

$$\mathrm{E}_1(z) = \int_z^{\infty} \frac{e^{-t}}{t} dt, \qquad |\mathrm{Arg}(z)| < \pi$$

$$\mathrm{E}_1(z) = \int_1^{\infty} \frac{e^{-tz}}{t} \, dt, \qquad \Re(z) \geq 0.$$

$$\lim_{\delta \to 0\pm} \mathrm{E}_1(-x + i\delta) = -\mathrm{Ei}(x) \mp i\pi, \qquad x > 0,$$

$$\mathrm{Ei}(x) = \gamma + \ln x + \sum_{k=1}^{\infty} \frac{x^k}{k \, k!} \qquad x > 0$$

$$\mathrm{E}_1(z) = -\gamma - \ln z + \sum_{k=1}^{\infty} \frac{(-1)^{k+1} z^k}{k \, k!} \qquad (|\mathrm{Arg}(z)| < \pi)$$

$$\mathrm{E}_1(z) = \frac{\exp(-z)}{z} \sum_{n=0}^{N-1} \frac{n!}{(-z)^n}$$

$$\frac{1}{2} e^{-x} \ln\left(1 + \frac{2}{x}\right) < \mathrm{E}_1(x) < e^{-x} \ln\left(1 + \frac{1}{x}\right) \qquad x > 0$$

$$\mathrm{Ein}(z) = \int_0^z (1-e^{-t})\frac{dt}{t} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}z^k}{k\,k!}$$

$$\mathrm{E}_1(z) = -\gamma - \ln z + \mathrm{Ein}(z) \qquad |\mathrm{Arg}(z)| < \pi$$

$$\mathrm{Ei}(x) = \gamma + \ln x - \mathrm{Ein}(-x) \qquad x > 0 \qquad \mathrm{li}(x) = \mathrm{Ei}(\ln x)$$

$$\mathrm{E}_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n}\,dt, \mathrm{E}_n(x) = x^{n-1}\Gamma(1-n,x).\varphi_m(x) = \mathrm{E}_{-m}(x).$$

$$\mathrm{E_n}'(z) = -\mathrm{E_{n-1}}(z) \qquad (n = 1,2,3,\ldots)$$

$$\mathrm{E}_1(z) = \int_1^\infty \frac{e^{-tz}}{t}dt\, \mathrm{E}_1(ix) = i\left(-\frac{\pi}{2} + \mathrm{Si}(x)\right) - \mathrm{Ci}(x) \qquad (x > 0)$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt.\mathrm{erfc}(x) = 1 - \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt.$$

$$w(x) = e^{-x^2}\mathrm{erfc}(-ix). \qquad \mathrm{erf}(-x) = -\mathrm{erf}(x). \qquad \mathrm{erf}(x^*) = \mathrm{erf}(x)^*$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{n!(2n+1)} = \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216} - \cdots\right)$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{n!}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\left(x\prod_{i=1}^{n}\frac{-(2i-1)x^2}{i(2i+1)}\right) = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{x}{2n+1}\prod_{i=1}^{n}\frac{-x^2}{i}$$

$$\frac{-(2i-1)x^2}{i(2i+1)}\frac{d}{dx}\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}e^{-x^2}.\mathrm{erf}^{-1}(x) = \sum_{k=0}^{\infty}\frac{c_k}{2k+1}\left(\frac{\sqrt{\pi}}{2}x\right)^{2k+1},$$

$$c_k = \sum_{m=0}^{k-1}\frac{c_m c_{k-1-m}}{(m+1)(2m+1)} = \left\{1,1,\frac{7}{6},\frac{127}{90},\ldots\right\}.$$

$$\mathrm{erf}^{-1}(x) = \frac{1}{2}\sqrt{\pi}\left(x + \frac{\pi}{12}x^3 + \frac{7\pi^2}{480}x^5 + \frac{127\pi^3}{40320}x^7 + \frac{4369\pi^4}{5806080}x^9 + \frac{34807\pi^5}{182476800}x^{11} + \cdots\right).$$

$$\mathrm{erfc}(x) = \frac{e^{-x^2}}{x\sqrt{\pi}}\left[1 + \sum_{n=1}^{\infty}(-1)^n\frac{1\cdot 3\cdot 5\cdots(2n-1)}{(2x^2)^n}\right] = \frac{e^{-x^2}}{x\sqrt{\pi}}\sum_{n=0}^{\infty}(-1)^n\frac{(2n)!}{n!(2x)^{2n}}.$$

$$\mathrm{erf}^2(x) \approx 1 - \exp\left(-x^2\frac{4/\pi + ax^2}{1+ax^2}\right) \qquad a = -\frac{8(\pi-3)}{3\pi(\pi-4)}.$$

$$\Phi(x)=\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right]=\frac{1}{2}\operatorname{erfc}\left(-\frac{x}{\sqrt{2}}\right).$$

$$\operatorname{probit}(p)=\Phi^{-1}(p)=\sqrt{2}\operatorname{erf}^{-1}(2p-1)=-\sqrt{2}\operatorname{erfc}^{-1}(2p).$$

$$\operatorname{erf}(x)=\frac{2x}{\sqrt{\pi}}\,{}_1F_1\left(\frac{1}{2},\frac{3}{2},-x^2\right).$$

$$\operatorname{erf}(x)=\operatorname{sgn}(x)P\left(\frac{1}{2},x^2\right)=\frac{\operatorname{sgn}(x)}{\sqrt{\pi}}\gamma\left(\frac{1}{2},x^2\right).\operatorname{sgn}(x)$$

$$E_n(x)=\frac{n!}{\sqrt{\pi}}\int_0^x e^{-t^n}\,dt=\frac{n!}{\sqrt{\pi}}\sum_{p=0}^{\infty}(-1)^p\frac{x^{np+1}}{(np+1)p!}\cdot E_0(x)=\frac{x}{e\sqrt{\pi}}$$

$$E_n(x)=\frac{x\left(x^n\right)^{-1/n}\Gamma(n)\left(\Gamma\left(\frac{1}{n}\right)-\Gamma\left(\frac{1}{n},x^n\right)\right)}{\sqrt{\pi}},\qquad x>0$$

$$\operatorname{erf}(x)=1-\frac{\Gamma\left(\frac{1}{2},x^2\right)}{\sqrt{\pi}}\qquad\qquad \mathrm{i}^n\operatorname{erfc}(z)=\int_z^{\infty}\mathrm{i}^{n-1}\operatorname{erfc}(\zeta)\,\mathrm{d}\zeta.$$

$$\mathrm{i}^n\operatorname{erfc}(z)=\sum_{j=0}^{\infty}\frac{(-z)^j}{2^{n-j}j!\Gamma\left(1+\frac{n-j}{2}\right)},$$

$$\mathrm{i}^{2m}\operatorname{erfc}(-z)=-\mathrm{i}^{2m}\operatorname{erfc}(z)+\sum_{q=0}^{m}\frac{z^{2q}}{2^{2(m-q)-1}(2q)!(m-q)!}$$

$$\mathrm{i}^{2m+1}\operatorname{erfc}(-z)=\mathrm{i}^{2m+1}\operatorname{erfc}(z)+\sum_{q=0}^{m}\frac{z^{2q+1}}{2^{2(m-q)-1}(2q+1)!(m-q)!}.$$

$${}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)=\sum_{n=0}^{\infty}\frac{(a_1)_n(a_2)_n\ldots(a_p)_n}{(b_1)_n(b_2)_n\ldots(b_q)_n}\frac{z^n}{n!}$$

$$(a)_n=a(a+1)(a+2)...(a+n-1)\qquad\qquad \Re\left(\sum b_k-\sum a_j\right)>0.$$

$$J_\alpha(x)=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\Gamma(k+\alpha+1)}\left(\frac{x}{2}\right)^{2k+\alpha}=\frac{x^\alpha}{2^\alpha\Gamma(\alpha+1)}\left[1-\frac{x^2}{2(2\alpha+2)}+\frac{x^4}{2\cdot4(2\alpha+2)(2\alpha+4)}-\ldots\right]$$

$$J_0(x)=1-\frac{x^2}{2^2}+\frac{x^4}{2^24^2}-\frac{x^6}{2^24^26^2}\ldots$$

$$J_1(x)=\frac{x}{2}-\frac{x^3}{2^24}+\frac{x^5}{2^24^26}-\frac{x^7}{2^24^26^28}\ldots\qquad J_0'(x)=\frac{dJ_0(x)}{dx}=-J_1(x)$$

$$J_n(x)=\frac{1}{\pi}\int_0^{\pi}\cos(n\tau-x\sin\tau)\,\mathrm{d}\tau.J_n(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{-\mathrm{i}(n\tau-x\sin\tau)}\,\mathrm{d}\tau.$$

$$J_\alpha(x)=\frac{1}{\pi}\int_0^\pi \cos(\alpha\tau - x\sin\tau)\,d\tau - \frac{\sin(\alpha\pi)}{\pi}\int_0^\infty e^{-x\sinh(t)-\alpha t}\,dt.\alpha > -\frac{1}{2}$$

$$J_\alpha(x)=\frac{1}{\Gamma(\alpha+\frac{1}{2})\sqrt{\pi}2^{\alpha-1}}\int_0^x (x^2-\tau^2)^{\alpha-\frac{1}{2}}\cos\tau\,d\tau.$$

$$J_\alpha(x)=\frac{(x/2)^\alpha}{\Gamma(\alpha+1)}\,{}_0F_1(\alpha+1;-x^2/4).\frac{J_\alpha(x)}{\left(\frac{x}{2}\right)^\alpha}=\frac{e^{-t}}{\Gamma(\alpha+1)}\sum_{k=0}\frac{L_k^{(\alpha)}\left(\frac{x^2}{4t}\right)}{\binom{k+\alpha}{k}}\frac{t^k}{k!}.$$

$$Y_\alpha(x)=\frac{J_\alpha(x)\cos(\alpha\pi)-J_{-\alpha}(x)}{\sin(\alpha\pi)},\quad \forall\alpha\notin\mathbb{Z}$$

$$Y_n(x)=\lim_{\alpha\to n}Y_\alpha(x),\quad \forall n\in\mathbb{Z}$$

$$Y_n(x)=\frac{1}{\pi}\int_0^\pi \sin(x\sin\theta-n\theta)d\theta-\frac{1}{\pi}\int_0^\infty\left[e^{nt}+(-1)^n e^{-nt}\right]e^{-x\sinh t}dt$$

$$H_\alpha^{(1)}(x)=J_\alpha(x)+iY_\alpha(x)$$

$$Y_{-n}(x)=(-1)^nY_n(x)\forall n\in\mathbb{Z}\,H_\alpha^{(2)}(x)=J_\alpha(x)-iY_\alpha(x)$$

$$H_\alpha^{(1)}(x)=\frac{J_{-\alpha}(x)-e^{-\alpha\pi i}J_\alpha(x)}{i\sin(\alpha\pi)}$$

$$H_\alpha^{(2)}(x)=\frac{J_{-\alpha}(x)-e^{\alpha\pi i}J_\alpha(x)}{-i\sin(\alpha\pi)}\qquad H_n^{(1)}(x)=\lim_{\alpha\to n}H_\alpha^{(1)}(x)\forall n\in\mathbb{Z},\quad H_n^{(2)}(x)=\lim_{\alpha\to n}H_\alpha^{(2)}(x)\forall n\in\mathbb{Z},$$

$$H_{-\alpha}^{(1)}(x)=e^{\alpha\pi i}H_\alpha^{(1)}(x)$$

$$H_{-\alpha}^{(2)}(x)=e^{-\alpha\pi i}H_\alpha^{(2)}(x)\,H_\alpha^{(1)}(x)=\frac{e^{-\frac{1}{2}\alpha\pi i}}{\pi i}\int_{-\infty}^{+\infty}e^{ix\cosh t-\alpha t}\,dt.$$

$$H_\alpha^{(2)}(x)=-\frac{e^{-\frac{1}{2}\alpha\pi i}}{\pi i}\int_{-\infty}^{+\infty}e^{-ix\cosh t-\alpha t}\,dt.$$

$$\begin{cases} y(x)=AJ_\alpha(x)+BJ_{-\alpha}(x) & \forall\alpha\notin\mathbb{Z}\\ y(x)=AJ_\alpha(x)+BY_\alpha(x) & \forall\alpha\in\mathbb{R}\\ y(x)=AJ_\alpha(x)+BJ_\alpha(x)\int\frac{dx}{xJ_\alpha^2(x)} & \forall\alpha\in\mathbb{R}\\ y(x)=AH_\alpha^{(1)}(x)+BH_\alpha^{(2)}(x) & \forall\alpha\in\mathbb{R}\end{cases}$$

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-(x^2+\alpha^2)y=0,$$

$$I_\alpha(x)=\sum_{k=0}^\infty\frac{1}{k!\Gamma(k+\alpha+1)}\left(\frac{x}{2}\right)^{2k+\alpha}=\frac{x^\alpha}{2^\alpha\Gamma(\alpha+1)}\left[1+\frac{x^2}{2(2\alpha+2)}+\frac{x^4}{2\cdot4(2\alpha+2)(2\alpha+4)}+\ldots\right]$$

$$I_\alpha(x)=i^{-\alpha}J_\alpha(ix)=e^{-\alpha\pi i/2}J_\alpha(ix).\,I_0(x)=1+\frac{x^2}{2^2}+\frac{x^4}{2^24^2}+\frac{x^6}{2^24^26^2}\cdots$$

$$I_1(x) = \frac{x}{2} + \frac{x^3}{2^2 4} + \frac{x^5}{2^2 4^2 6} + \frac{x^7}{2^2 4^2 6^2 8} \dots \qquad I_0'(x) = \frac{dI_0(x)}{dx} = I_1(x)$$

$$K_\alpha(x) = \frac{\pi}{2} \frac{I_{-\alpha}(x) - I_\alpha(x)}{\sin(\alpha\pi)} \forall \alpha \notin \mathbb{Z}$$

$$K_n(x) = \lim_{p \to n} K_p(x) = \lim_{p \to n} \frac{\pi}{2} \frac{I_{-p}(x) - I_p(x)}{\sin(p\pi)} \forall n \in \mathbb{Z}$$

$$K_\alpha(x) = \frac{\pi}{2} i^{\alpha+1} H_\alpha^{(1)}(ix) = -\frac{\pi}{2} i^{\alpha+1} e^{-i\pi\alpha} H_\alpha^{(2)}(-ix).$$

$$K_\alpha(x) = \frac{1}{2} e^{-\frac{1}{2}\alpha\pi i} \int_{-\infty}^{+\infty} e^{-ix\sinh t - \alpha t} dt$$

$$\begin{cases} y(x) = AI_\alpha(x) + BI_{-\alpha}(x) & \alpha \notin \mathbb{Z} \\ y(x) = AI_\alpha(x) + BK_\alpha(x) & \forall \alpha \in \mathbb{R} \\ y(x) = AI_\alpha(x) + BI_\alpha(x) \int \dfrac{dx}{xI_\alpha^2(x)} & \forall \alpha \in \mathbb{R} \end{cases}$$

$$x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} + [x^2 - n(n+1)]y = 0. \qquad j_n(x) = \sqrt{\frac{\pi}{2x}} J_{n+1/2}(x),$$

$$y_n(x) = \sqrt{\frac{\pi}{2x}} Y_{n+1/2}(x) = (-1)^{n+1} \sqrt{\frac{\pi}{2x}} J_{-n-1/2}(x).$$

$$j_n(x) = (-x)^n \left(\frac{1}{x}\frac{d}{dx}\right)^n \frac{\sin x}{x}, \qquad y_n(x) = -(-x)^n \left(\frac{1}{x}\frac{d}{dx}\right)^n \frac{\cos x}{x}.$$

$$j_0(x) = \frac{\sin x}{x} \; j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x} \; j_2(x) = \left(\frac{3}{x^2} - 1\right) \frac{\sin x}{x} - \frac{3\cos x}{x^2}$$

$$j_3(x) = \left(\frac{15}{x^3} - \frac{6}{x}\right) \frac{\sin x}{x} - \left(\frac{15}{x^2} - 1\right) \frac{\cos x}{x}, \quad y_0(x) = -j_{-1}(x) = -\frac{\cos x}{x}$$

$$y_1(x) = j_{-2}(x) = -\frac{\cos x}{x^2} - \frac{\sin x}{x}$$

$$y_2(x) = -j_{-3}(x) = \left(-\frac{3}{x^2} + 1\right) \frac{\cos x}{x} - \frac{3\sin x}{x^2};$$

$$J_{n+\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sum_{i=0}^{\frac{n+1}{2}} (-1)^{n-i} \left[\sin(x) \left(\frac{2}{x}\right)^{n-2i} \frac{(n-i)!}{i!} \binom{-\frac{1}{2} - i}{n-2i} - \cos(x) \left(\frac{2}{x}\right)^{n+1-2i} \frac{(n-i)!}{i!} i \binom{-\frac{1}{2} - i}{n-2i+1}\right].$$

$$h_n^{(2)}(x) = j_n(x) - iy_n(x). \; h_n^{(1)}(x) = (-i)^{n+1} \frac{e^{ix}}{x} \sum_{m=0}^{n} \frac{i^m}{m!(2x)^m} \frac{(n+m)!}{(n-m)!}$$

$$i_n(x) = \sqrt{\frac{\pi}{2x}} I_{n+1/2}(x) = i^{1-n} j_n(ix)$$
$$k_n(x) = \sqrt{\frac{\pi}{2x}} K_{n+1/2}(x) = \frac{\pi}{2} i^n h_n^{(1)}(ix)$$
$$k_n(x) = \frac{\pi}{2} \frac{e^{-x}}{x} \sum_{m=0}^{n} \frac{(n+m)!}{m!(n-m)!} \frac{1}{(2x)^m} \frac{1}{z} \cos\sqrt{z^2 - 2zt} = \sum_{n=0}^{\infty} \frac{t^n}{n!} j_{n-1}(z),$$

$$\frac{1}{z} \sin\sqrt{z^2 + 2zt} = \sum_{n=0}^{\infty} \frac{(-t)^n}{n!} y_{n-1}(z).$$

$$f_n(z) = \{j_n(z), y_n(z), h_n^{(1)}(z), h_n^{(2)}(z)\} \, \forall n \in \mathbb{Z}$$

$$\left(\frac{1}{z}\frac{d}{dz}\right)^m \left(z^{n+1} f_n(z)\right) = z^{(n-m)+1} f_{(n-m)}(z).$$

$$S_n(x) = x j_n(x) = \sqrt{\pi x/2} J_{n+1/2}(x)$$

$$C_n(x) = -x y_n(x) = -\sqrt{\pi x/2} Y_{n+1/2}(x)$$

$$\xi_n(x) = x h_n^{(1)}(x) = \sqrt{\pi x/2}\, H_{n+1/2}^{(1)}(x) = S_n(x) - iC_n(x)$$

$$\zeta_n(x) = x h_n^{(2)}(x) = \sqrt{\pi x/2} H_{n+1/2}^{(2)}(x) = S_n(x) + iC_n(x)$$

$$x^2 \frac{d^2 y}{dx^2} + [x^2 - n(n+1)]y = 0 \qquad J_\alpha(x) \approx \frac{1}{\Gamma(\alpha+1)} \left(\frac{x}{2}\right)^\alpha$$

$$Y_\alpha(x) \approx \begin{cases} \frac{2}{\pi}\left[\ln(x/2) + \gamma\right] & \text{si } \alpha = 0 \\ -\frac{\Gamma(\alpha)}{\pi}\left(\frac{2}{x}\right)^\alpha & \text{si } \alpha > 0 \end{cases} J_\alpha(x) \approx \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right)$$

$$Y_\alpha(x) \approx \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right).$$
$$I_\alpha(x) \approx \frac{e^x}{\sqrt{2\pi x}} \left(1 + \frac{(1-2\alpha)(1+2\alpha)}{8x} + \cdots\right), K_\alpha(x) \approx \sqrt{\frac{\pi}{2x}} e^{-x}.$$

$$I_\alpha(x) \approx \frac{1}{\Gamma(\alpha+1)} \left(\frac{x}{2}\right)^\alpha \qquad K_\alpha(x) \approx \begin{cases} -\ln(x/2) - \gamma & \text{si } \alpha = 0 \\ \frac{\Gamma(\alpha)}{2}\left(\frac{2}{x}\right)^\alpha & \text{si } \alpha > 0 \end{cases}$$

$$e^{(x/2)(t-1/t)} = \sum_{n=-\infty}^{\infty} J_n(x) t^n, \qquad e^{iz\cos\phi} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{in\phi},$$

$$f(z) = a_0^\nu J_\nu(z) + 2 \cdot \sum_{k=1} a_k^\nu J_{\nu+k}(z), \qquad a_k^0 = \frac{1}{2\pi i} \int_{|z|=c} f(z) O_k(z)\,\mathrm{d}z,$$

$$f(z) = \sum_{k=0} a_k^\nu J_{\nu+2k}(z) \qquad a_k^\nu = 2(\nu+2k) \int_0^\infty f(z) \frac{J_{\nu+2k}(z)}{z} \mathrm{d}z$$

$$\int_0^\infty J_\alpha(z) J_\beta(z) \frac{\mathrm{d}z}{z} = \frac{2}{\pi} \frac{\sin\left(\frac{\pi}{2}(\alpha-\beta)\right)}{\alpha^2-\beta^2}.$$

$$\mathcal{L}\left\{\sum_{k=0} a_k J_{\nu+k}\right\}(s) = \frac{1}{\sqrt{1+s^2}} \sum_{k=0} \frac{a_k}{(s+\sqrt{1+s^2})^{\nu+k}}$$

$$\sum_{k=0} a_k \xi^{\nu+k} = \frac{1+\xi^2}{2\xi} \mathcal{L}\{f\} \left(\frac{1-\xi^2}{2\xi}\right),$$

$$J_\nu(z) = \frac{\left(\frac{z}{2}\right)^\nu}{\Gamma(\nu+\frac{1}{2})\sqrt{\pi}} \int_{-1}^1 e^{izs}(1-s^2)^{\nu-\frac{1}{2}} ds,$$

$$= \frac{2}{\left(\frac{z}{2}\right)^\nu \cdot \sqrt{\pi} \cdot \Gamma\left(\frac{1}{2}-\nu\right)} \int_1^\infty \frac{\sin(zu)}{(u^2-1)^{\nu+\frac{1}{2}}} du,$$

$$\frac{2\alpha}{x} Z_\alpha(x) = Z_{\alpha-1}(x) + Z_{\alpha+1}(x) \qquad 2\frac{dZ_\alpha}{dx} = Z_{\alpha-1}(x) - Z_{\alpha+1}(x)$$

$$\left(\frac{d}{xdx}\right)^m [x^\alpha Z_\alpha(x)] = x^{\alpha-m} Z_{\alpha-m}(x) \left(\frac{d}{xdx}\right)^m \left[\frac{Z_\alpha(x)}{x^\alpha}\right] = (-1)^m \frac{Z_{\alpha+m}(x)}{x^{\alpha+m}}$$

$$e^{(x/2)(t+1/t)} = \sum_{n=-\infty}^\infty I_n(x) t^n, \qquad e^{z\cos\theta} = I_0(z) + 2\sum_{n=1}^\infty I_n(z)\cos(n\theta),$$

$$C_{\alpha-1}(x) - C_{\alpha+1}(x) = \frac{2\alpha}{x} C_\alpha(x)$$

$$C_{\alpha-1}(x) + C_{\alpha+1}(x) = 2\frac{dC_\alpha}{dx}$$

$$\int_0^1 x J_\alpha(xu_{\alpha,m}) J_\alpha(xu_{\alpha,n}) dx = \frac{\delta_{m,n}}{2}[J_{\alpha+1}(u_{\alpha,m})]^2 = \frac{\delta_{m,n}}{2}[J'_\alpha(u_{\alpha,m})]^2,$$

$$\int_0^1 x^2 j_\alpha(xu_{\alpha,m}) j_\alpha(xu_{\alpha,n}) dx = \frac{\delta_{m,n}}{2}[j_{\alpha+1}(u_{\alpha,m})]^2$$

$$\int_0^\infty xJ_\alpha(ux)J_\alpha(vx)dx = \frac{1}{u}\delta(u-v)\int_0^\infty x^2 j_\alpha(ux)j_\alpha(vx)dx = \frac{\pi}{2u^2}\delta(u-v)$$

$$A_\alpha(x)\frac{dB_\alpha}{dx} - \frac{dA_\alpha}{dx}B_\alpha(x) = \frac{C_\alpha}{x},$$

$$J_\alpha(x)\frac{dY_\alpha}{dx} - \frac{dJ_\alpha}{dx}Y_\alpha(x) = \frac{2}{\pi x}, I_\alpha(x)\frac{dK_\alpha}{dx} - \frac{dI_\alpha}{dx}K_\alpha(x) = \frac{-1}{x},$$

$$\lambda^{-\nu}J_\nu(\lambda z) = \sum_{n=0}^\infty \frac{1}{n!}\left(\frac{(1-\lambda^2)z}{2}\right)^n J_{\nu+n}(z)$$

$$y_p(\alpha x) = \{J_p(\alpha x), Y_p(\alpha x), I_p(\alpha x), H_p^{(1)}(\alpha x), H_p^{(2)}(\alpha x)\}$$

$$\frac{d}{dx}y_p(\alpha x) = \alpha y_{p-1}(\alpha x) - \frac{p}{x}y_p(\alpha x) \quad \frac{d}{dx}y_p(\alpha x) = -\alpha y_{p-1}(\alpha x) - \frac{p}{x}y_p(\alpha x)$$

$$y_p(\alpha x) = \{J_p(\alpha x), Y_p(\alpha x), K_p(\alpha x), H_p^{(1)}(\alpha x), H_p^{(2)}(\alpha x)\}$$

$$\frac{d}{dx}y_p(\alpha x) = -\alpha y_{p+1}(\alpha x) + \frac{p}{x}y_p(\alpha x)$$

$$\frac{d}{dx}y_p(\alpha x) = \alpha y_{p+1}(\alpha x) + \frac{p}{x}y_p(\alpha x)$$

$$y_p(\alpha x) = \{J_p(\alpha x), Y_p(\alpha x), H_p^{(1)}(\alpha x), H_p^{(2)}(\alpha x)\},$$

$$\frac{d}{dx}y_p(\alpha x) = \frac{\alpha}{2}[y_{p-1}(\alpha x) - y_{p+1}(\alpha x)]\, y_{p-1}(\alpha x) + y_{p+1}(\alpha x) = \frac{2p}{\alpha x}y_p(\alpha x)$$

$$I_{-\frac{1}{2}}\left(\frac{z}{2}\right) + I_{\frac{1}{2}}\left(\frac{z}{2}\right) = \frac{2e^{\frac{z}{2}}}{\sqrt{\pi z}}; \qquad I_\nu(z) = \sum_{k=0} \frac{z^k}{k!}J_{\nu+k}(z);$$

$$J_\nu(z) = \sum_{k=0}(-1)^k\frac{z^k}{k!}I_{\nu+k}(z); I_\nu(\lambda z) = \lambda^\nu \sum_{k=0}\frac{\left((\lambda^2-1)\frac{z}{2}\right)^k}{k!}I_{\nu+k}(z);$$

$$I_\nu(z_1+z_2) = \sum_{k=-\infty}^\infty I_{\nu-k}(z_1)I_k(z_2);$$

$$J_\nu(z) = \frac{z}{2\nu}(J_{\nu-1}(z) + J_{\nu+1}(z)), \quad I_\nu(z) = \frac{z}{2\nu}(I_{\nu-1}(z) - I_{\nu+1}(z));$$

$$J'_\nu(z) = \frac{1}{2}(J_{\nu-1}(z) - J_{\nu+1}(z)), \quad I'_\nu(z) = \frac{1}{2}(I_{\nu-1}(z) + I_{\nu+1}(z));$$

$$\left(\frac{x}{2}\right)^\nu = \sum_{k=0}(-1)^k \frac{\Gamma(k+\nu)}{k!}(2k+\nu)I_{2k+\nu}(x).$$

$$z(1+W)\frac{dW}{dz} = W \quad \text{para } z \neq -1/e\,,$$

$$\frac{dW}{dz} = \frac{W(z)}{z(1+W(z))} \quad \text{para } z \neq -1/e\,.$$

$$\int W(x)\,dx = x\left(W(x) - 1 + \frac{1}{W(x)}\right) + C$$

$$W_0(x) = \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!}\ x^n = x - x^2 + \frac{3}{2}x^3 - \frac{8}{3}x^4 + \frac{125}{24}x^5 - \cdots$$

$$X = Ye^Y \iff Y = W(X) \qquad p^{ax+b} = cx + d$$

$$p > 0 \land c, d \neq 0 -t = ax + \frac{ad}{c} \qquad tp^t = R = -\frac{a}{c}p^{b-\frac{ad}{c}} \qquad t = \frac{W(R\ln p)}{\ln p}$$

$$x = -\frac{W(-\frac{a\ln p}{c}\,p^{b-\frac{ad}{c}})}{a\ln p} - \frac{d}{c}\ H(\omega) = \mathcal{F}\{h\}(\omega) = \begin{cases} +j & \text{si } \omega < 0 \\ -j & \text{si } \omega > 0 \end{cases}$$

$$H(\omega) = \mathcal{F}\{h\}(\omega) = -j \cdot \operatorname{sgn}(\omega) \qquad \mathcal{F}\{\widehat{s}\}(\omega) = H(\omega) \cdot \mathcal{F}\{s\}(\omega),$$

$$\mathcal{F}\{s\}(\omega) = -H(\omega) \cdot \mathcal{F}\{\widehat{s}\}(\omega) s(t) = -(h * \widehat{s})(t) = -\mathcal{H}\{\widehat{s}\}(t).$$

Notación. $s(t)$	**Transformación de Hilbert.** $\mathcal{H}\{s\}(t)$
$\sin(t)$	$-\cos(t)$
$\cos(t)$	$\sin(t)$
$\frac{1}{t^2+1}$	$\frac{t}{t^2+1}$
$\frac{\sin(t)}{t}$ Función sinc.	$\frac{1-\cos(t)}{t}$
$\sqcap(t)$ Función rectángulo.	$\frac{1}{\pi}\ln\left\lvert\frac{t+\frac{1}{2}}{t-\frac{1}{2}}\right\rvert$
δ(*t*) Función delta de Dirac.	$\frac{1}{\pi t}$

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt.$$

$$F(s) = \mathcal{L}\{f(t)\} = \lim_{\epsilon\to 0} \int_{-\epsilon}^\infty e^{-st} f(t)\, dt.$$

$$F_B(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^\infty e^{-st} f(t)\, dt.$$

$$z = \int X(x) e^{ax}\, dx \; z = \int X(x) x^A\, dx \int X(x) e^{-ax} a^x\, dx, \int x^s \phi(s)\, dx,$$

$$y = e^{at} \int e^{-at} f(t) dt + c_1 e^{at} \qquad y = \frac{1}{D-a} f(t) = e^{at} \int e^{-at} f(t) dt + c_1 e^{at}$$

$$y = \frac{e^t}{D^2 - 3D + 2} = \frac{e^t}{(D-1)(D-2)} = \frac{1}{D-2} e^t - \frac{1}{D-1} e^t$$

$$y = (e^{2t} \int e^{-2t} f(t) dt + c_1 e^{2t}) - (e^t \int e^{-t} f(t) dt + c_2 e^t) = e^{2t}(-e^{-t}) + c_1 e^{2t}) - (e^t(t) + c_2 e^t)$$

$$\left(y = c_1 e^{2t} - (c_2 + 1) e^t - t e^t\right) \mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$$

$$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)\, \mathcal{L}\{f''(t)\} = s^2 \mathcal{L}\{f(t)\} - sf(0) - f'(0)$$

$$\mathcal{L}\left\{f^{(n)}(t)\right\} = s^n \mathcal{L}\{f(t)\} - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$$

$$\mathcal{L}\left\{f^{(n)}(t)\right\} = s^n \mathcal{L}\{f(t)\} - \sum_{i=1}^{n} s^{n-i} f^{(i-1)}(0) \mathcal{L}\left\{\int_0^t f(\tau)\, d\tau\right\} = \frac{1}{s}\mathcal{L}\{f\}$$

$$\mathcal{L}\{tf(t)\} = -F'(s)\, \mathcal{L}\left\{e^{at} f(t)\right\} = F(s-a)$$

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as} F(s) \qquad \mathcal{L}^{-1}\left\{e^{-as} F(s)\right\} = f(t-a)u(t-a)$$

$$\mathcal{L}\{\, t^n f(t)\} = (-1)^n D_s^n [F(s)] \qquad \mathcal{L}\{f * g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$$

$$\mathcal{L}\{f\} = \frac{1}{1 - e^{-ps}} \int_0^p e^{-st} f(t)\, dt \qquad \mathcal{L}\{(e^{t^2})\}$$

$$\mathcal{L}\{f(t) + g(t)\} = \mathcal{L}\{f(t)\} + \mathcal{L}\{g(t)\}\; \mathcal{L}\{af(t)\} = a\mathcal{L}\{f(t)\}$$

Función	**Dominio (tiempo)** $x(t) = \mathcal{L}^{-1}\{X(s)\}$	**Dominio (frecuencia)** $X(s) = \mathcal{L}\{x(t)\}$	**Región de Convergencia**
Retraso ideal	$\delta(t-\tau)$	$e^{-\tau s}$	
Impulso unitario	$\delta(t)$	1	todo s

Enésima potencia retrasada y con desplazamiento en la frecuencia	$\frac{(t-\tau)^n}{n!}e^{-\alpha(t-\tau)}\cdot u(t-\tau)$	$\frac{e^{-\tau s}}{(s+\alpha)^{n+1}}$	$s>-\alpha$
n-ésima potencia	$\frac{t^n}{n!}\cdot u(t)$	$\frac{1}{s^{n+1}}$	$s>0$
q-ésima potencia	$\frac{t^q}{\Gamma(q+1)}\cdot u(t)$	$\frac{1}{s^{q+1}}$	$s>0$
Escalón unitario	$u(t)$	$\frac{1}{s}$	$s>0$
Escalón unitario con retraso	$u(t-\tau)$	$\frac{e^{-\tau s}}{s}$	$s>0$
Rampa	$t\cdot u(t)$	$\frac{1}{s^2}$	$s>0$
Potencia n-ésima con cambio de frecuencia	$\frac{t^n}{n!}e^{-\alpha t}\cdot u(t)$	$\frac{1}{(s+\alpha)^{n+1}}$	$s>-\alpha$
Amortiguación exponencial	$e^{-\alpha t}\cdot u(t)$	$\frac{1}{s+\alpha}$	$s>-\alpha$
Convergencia exponencial	$(1-e^{-\alpha t})\cdot u(t)$	$\frac{\alpha}{s(s+\alpha)}$	$s>0$
Exponencial doble	$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$	$\frac{1}{(s+a)(s+b)}$	$s>-a$ y $s>-b$
Seno	$\sin(\omega t)\cdot u(t)$	$\frac{\omega}{s^2+\omega^2}$	$s>0$
Coseno	$\cos(\omega t)\cdot u(t)$	$\frac{s}{s^2+\omega^2}$	$s>0$
Seno con fase	$\sin(\omega t+\varphi)\cdot u(t)$	$\frac{s\sin(\varphi)+\omega\cos\varphi}{s^2+\omega^2}$	$s>0$
Seno hiperbólico	$\sinh(\alpha t)\cdot u(t)$	$\frac{\alpha}{s^2-\alpha^2}$	$s>\lvert\alpha\rvert$
Coseno hiperbólico	$\cosh(\alpha t)\cdot u(t)$	$\frac{s}{s^2-\alpha^2}$	$s>\lvert\alpha\rvert$
Onda senoidal con	$e^{-\alpha t}\sin(\omega t)\cdot u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$	$s>-\alpha$

amortiguamiento exponencial			
Onda cosenoidal con amortiguamiento exponencial	$e^{-\alpha t}\cos(\omega t)\cdot u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$	$s > -\alpha$
Raíz n-ésima	$\sqrt[n]{t}\cdot u(t)$	$s^{-(n+1)/n}\cdot\Gamma\left(1+\frac{1}{t}\right)$	$s > 0$
Logaritmo natural	$\ln\left(\frac{t}{t_0}\right)\cdot u(t)$	$-\frac{t_0}{s}\ [\ \ln(t_0 s)+\gamma\ ]$	$s > 0$
Función de Bessel de primer tipo, de orden n	$J_n(\omega t)\cdot u(t)$	$\frac{\omega^n\left(s+\sqrt{s^2+\omega^2}\right)^{-n}}{\sqrt{s^2+\omega^2}}$	$s > 0$ $(n > -1)$
Función de Bessel modificada de primer tipo, de orden n	$I_n(\omega t)\cdot u(t)$	$\frac{\omega^n\left(s+\sqrt{s^2-\omega^2}\right)^{-n}}{\sqrt{s^2-\omega^2}}$	$s > \lvert\omega\rvert$
Función de Bessel de segundo tipo, de orden 0	$Y_0(\alpha t)\cdot u(t)$		
Función de Bessel modificada de segundo tipo, de orden 0	$K_0(\alpha t)\cdot u(t)$		
Función de error	$\mathrm{erf}(t)\cdot u(t)$	$\frac{e^{s^2/4}\mathrm{erfc}\,(s/2)}{s}$	$s > 0$

Función.	**Transformación.**
$\delta(x)$	1
$u(x)$	$1/2(\delta(f)+1/(i\pi f))$

$\sin(w_0 x)$	$\frac{\pi}{i}[\delta(w-w_0)-\delta(w+w_0)]$
$\cos(w_0 x)$	$\pi[\delta(w-w_0)+\delta(w+w_0)]$
1	$2\pi\delta(f)=\delta(w)$
$e^{-at}u(t), \mathrm{Re}(a)>0$	$\frac{1}{a+iw}$
$e^{-a\|t\|},$	$\frac{2a}{a^2+w^2}$
$te^{-at}u(t), \mathrm{Re}(a)>0$	$\frac{1}{(a+iw)^2}$
$x(t)=\begin{cases}1, & \text{si } \|t\|<T_1 \\ 0, & \text{si } \|t\|>T_1\end{cases}$	$\operatorname{sinc}\left(\frac{wT_1}{\pi}\right)=2\frac{\sin(wT_1)}{w}$
$x(t)=\operatorname{tri}\left(\frac{t}{2T_1}\right)=\begin{cases}1-\frac{\|t\|}{T_1}, & \text{si } \|t\|<T_1 \\ 0, & \text{si } \|t\|>T_1\end{cases}$	$\operatorname{sinc}^2\left(\frac{wT_1}{\pi}\right)$

$$(1)\quad \check{\hat{f}}=f \qquad \sup_{x\in\mathbb{R}}|x^m\varphi^{(n)}(x)|<\infty, \qquad \check{\hat{f}}=f,\quad f\in\mathcal{S}.$$

$$[T\varphi](x)=\sum_{k=0}^{m}P_k(x)\left(\frac{d}{dx}\right)^n\varphi(x). \qquad \mathcal{F}\{\frac{d\varphi}{dx}\}(\xi)=i\xi\cdot\mathcal{F}\{\varphi\}(\xi)$$

$$\mathcal{F}\{-ix\cdot\varphi(x)\}(\xi)=\frac{d}{d\xi}\Big(\mathcal{F}\{\varphi\}(\xi)\Big), \qquad \hat{g}(k)=e^{-iky}\hat{f}(k)$$

$$(\widehat{f*g})(k)=\hat{f}(k)\cdot\hat{g}(k) \qquad \langle f(x),g(x)\rangle=\int_{-\infty}^{+\infty}f(x)*g(x)dx$$

$$\forall\delta_p:\mathcal{F}(M)\longrightarrow\mathbb{R}\quad \forall f,g\in\mathcal{F}(M),\forall\lambda\in\mathbb{R},\quad \delta_p(g+f)=\delta_p(g)+\delta_p(f),$$

$$\delta_p(\lambda f)=\lambda\delta_p(f). \qquad \begin{array}{rccc}\frac{\partial\,\cdot}{\partial x_i}_{|p}: & \mathcal{F}(M) & \longrightarrow & \mathbb{R} \\ & f & \mapsto & \frac{\partial f}{\partial x_i}_{|p}\end{array}. \quad \frac{\partial(f+g)}{\partial x_i}_{|p}=\frac{\partial f}{\partial x_i}_{|p}+\frac{\partial g}{\partial x_i}_{|p},$$

$$\frac{\partial(\lambda g)}{\partial x_i}_{|p}=\lambda\frac{\partial g}{\partial x_i}_{|p}. \qquad \frac{\partial(f\cdot g)}{\partial x_i}_{|p}=\frac{\partial f}{\partial x_i}_{|p}g_{|p}+f_{|p}\frac{\partial g}{\partial x_i}_{|p}.$$

$$\begin{array}{rccc}\frac{\partial\cdot}{\partial v}_{|p}: & \mathcal{F}(M) & \longrightarrow & \mathbb{R} \\ & f & \mapsto & \frac{\partial f}{\partial v}_{|p}\end{array}.$$

$\delta_p(f) = \delta_p(\lambda) = \delta_p(\lambda \cdot 1) =_{\lambda\delta_p(1),}$

$\delta_p(1) = \delta_p(1 \cdot 1) = \delta_p(1)1 + 1\delta_p(1) = \delta_p(1) + \delta_p(1),\quad p \in M,\ \forall \delta_p \in \mathcal{T}_pM,$

$f \in \mathcal{F}(M)$ $\delta_p(\rho \cdot f) = \delta_p(f).$

$\delta_p(\rho \cdot f) = \delta_p(\rho)f(p) + \rho(p)\delta_p(f), delta_p(\rho \cdot f) = 0 \cdot f(p) + 1 \cdot \delta_p(f) =$

$\delta_p(f).\quad p \in M,\ \forall \delta_p \in \mathcal{T}_pM$ y $f, g \in \mathcal{F}(M)$ $f_{|V} = g_{|V},$ $\delta_p(f) = \delta_p(g).$

$\rho \cdot f = \rho \cdot g$ en $\rho \cdot f, \rho \cdot g \in \mathcal{F}(M)$ $\delta_p(\rho \cdot f) = \delta_p(\rho \cdot g)$ $\delta_p(f) = \delta_p(g).$

$\forall \varepsilon > 0, \exists n_0 > 0 : \forall n > n_0, d(x_n, l) < \varepsilon.$ $\lim_{n\to\infty} x_n = l$ $x_n \xrightarrow[n\to\infty]{} l$

$\sup_{n\geq 0} \inf_{k\geq n} x_k = \sup\{\inf\{x_k : k \geq n\} : n \geq 0\}$ $\lim_{n\to\infty}\left(\inf_{k\geq n} x_k\right)$ $\liminf_{n\to\infty} x_n$

$\underline{\lim}_{n\to\infty} x_n.$ $\limsup_{n\to\infty} x_n = \overline{\lim}_{n\to\infty} x_n = \inf_{n\geq 0} \sup_{k\geq n} x_k = \lim_{n\to\infty}\left(\sup_{k\geq n} x_k\right).$

$\liminf_{n\to\infty} x_n \leq \limsup_{n\to\infty} x_n\ .$

$(f+g)' = f' + g'$ $(cf)' = cf'$ $(fg)' = f'.g + f.g'$ $\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$

$\left(\frac{f}{g}\right)' = \frac{f'.g - f.g'}{g^2},$ $g \neq 0$ $(f \circ g)' = g'(f)f'$ $\frac{d}{dx}c = 0$ $\frac{d}{dx}x = 1$

$\frac{d}{dx}(cx) = c$ $\frac{d}{dx}x^c = cx^{c-1}$ $\frac{d}{dx}(cx^n) = cnx^{n-1}$

$\frac{d}{dx}|x| = \frac{x}{|x|} = \operatorname{sgn} x,$ $x \neq 0$ $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}\left(x^{-1}\right) = -x^{-2} = -\frac{1}{x^2}$

$\frac{d}{dx}\left(\frac{1}{x^c}\right) = \frac{d}{dx}\left(x^{-c}\right) = -cx^{-c-1} = -\frac{c}{x^{c+1}}$ $\frac{d}{dx}(\sqrt[n]{x}) = \frac{1}{n\sqrt[n]{x^{n-1}}}$ sea $x > 0$

$\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}},$ $x > 0$

$\frac{d}{dx}f(x)^n = nf(x)^{n-1} \cdot \frac{d}{dx}f(x)$ $(f^{-1})' = \frac{1}{f' \circ f^{-1}},$

$$\frac{d}{dx}c^x = c^x \ln c, \qquad c > 0 \qquad\qquad \frac{d}{dx}e^x = e^x \frac{d}{dx}(x)$$

$$\frac{d}{dx}\log_c x = \frac{1}{x \ln c} \quad , c > 0, c \neq 1 \quad \frac{d}{dx}\ln x = \frac{1}{x} \quad , x > 0 \quad \frac{d}{dx}\ln|x| = \frac{1}{x}$$

$$\frac{d}{dx}x^x = x^x(1 + \ln x) \qquad\qquad (f^g)' = f^g\left(g' \ln f + \frac{g}{f}f'\right)$$

$$\frac{d}{dx}f(x)^{g(x)} = f(x)^{g(x)}\left(\frac{d}{dx}f(x) \cdot \frac{g(x)}{f(x)} + \frac{d}{dx}g(x) \cdot \ln f(x)\right), \qquad f(x) > 0$$

$$\frac{d}{dx}\operatorname{senh} x = \cosh x \qquad \frac{d}{dx}\operatorname{argsenh} x = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}\cosh x = \operatorname{senh} x \qquad \frac{d}{dx}\operatorname{argcosh} x = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x \qquad \frac{d}{dx}\operatorname{argtanh} x = \frac{1}{1-x^2}$$

$$\frac{d}{dx}\operatorname{sech} x = -\tanh x \operatorname{sech} x \qquad \frac{d}{dx}\operatorname{argsech} x = \frac{-1}{|x|\sqrt{1-x^2}}$$

$$\frac{d}{dx}\operatorname{csch} x = -\coth x \operatorname{csch} x \qquad \frac{d}{dx}\operatorname{argcsch} x = \frac{-1}{|x|\sqrt{1+x^2}}$$

$$\frac{d}{dx}\coth x = -\operatorname{csch}^2 x \qquad \frac{d}{dx}\operatorname{argcoth} x = -\frac{1}{x^2-1}$$

$$\zeta(x) = -\sum_{n=1}^{\infty}\frac{\ln n}{n^x} = -\frac{\ln 2}{2^x} - \frac{\ln 3}{3^x} - \frac{\ln 4}{4^x} - \cdots$$

$$(\zeta(x))' = -\sum_{p \text{ primo}} \frac{p^{-x}\ln p}{(1-p^{-x})^2} \prod_{q \text{ primo},\ q \neq p} \frac{1}{1-q^{-x}}$$

$$H'(x-a)=\delta(x-a)\begin{cases}\frac{d}{dx}\text{sn } x=\text{cn } x \text{ dn } x\\ \frac{d}{dx}\text{dn } x=-k^2\text{sn } x \text{ cn } x\\ \frac{d}{dx}\text{cn } x=-\text{sn } x \text{ dn } x\\ \frac{d}{dx}\text{sc } x=\text{dc } x \text{ nc } x\end{cases}$$

$$\frac{d}{dx}\int_{g_1(x)}^{g_2(x)} f(x,s)\,ds=\int_{g_1(x)}^{g_2(x)}\frac{\partial f(x,s)}{\partial x}\,ds+f(x,g_2(x))\frac{dg_2}{dx}-f(x,g_1(x))\frac{dg_1}{dx}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \qquad f'(x_0)\approx\frac{f(x_0+h)-f(x_0)}{h}$$

$$f'(x_0)\approx\frac{f(x_0)-f(x_0-h)}{h} \quad f'(x_0)\approx\frac{f(x_0+h)-f(x_0-h)}{2h}$$

$$f''(x_0)\approx\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2} \quad f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} \quad f'(x)=\lim_{h\to 0}\frac{\sin(x)\cos(h)+\cos(x)\sin(h)-\sin(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x)\sin(h)-\sin(x)(1-\cos(h))}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x)\sin(h)}{h}-\lim_{h\to 0}\frac{\sin(x)(1-\cos(h))}{h}$$

$$f'(x)=cos(x)\lim_{h\to 0}\frac{\sin(h)}{h}-\sin(x)\lim_{h\to 0}\frac{(1-\cos(h))}{h}$$

$$\lim_{h\to 0}\frac{\sin(h)}{h} \quad \text{y} \quad \lim_{h\to 0}\frac{(1-\cos(h))}{h} \qquad f'(x)=\cos(x)$$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x)(\cos(h)-1)-\sin(x)\sin(h)}{h}$$

$$f'(x) = \cos(x)\lim_{h\to 0}\frac{\cos(h)-1}{h} - \sin(x)\lim_{h\to 0}\frac{\sin(h)}{h}$$

$$\lim_{h\to 0}\frac{\sin(h)}{h} \quad \text{y} \quad \lim_{h\to 0}\frac{(\cos(h)-1)}{h}\; f'(x) = -\sin(x)$$

$f(x)$, $f(x) = \dfrac{g(x)}{h(x)}$ $h(x) \neq 0$, $\dfrac{d}{dx}f(x) = f'(x) = \dfrac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$

$$\tan(x) = \frac{sin(x)}{\cos(x)} \qquad g(x) = \sin(x) \qquad g'(x) = \cos(x) \qquad h(x) = \cos(x)$$

$$h'(x) = -\sin(x) \qquad f'(x) = \frac{\cos(x)\cos(x) - \sin(x)[-\sin(x)]}{\cos^2(x)}$$

$$f'(x) = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} \quad \cos^2(x) + \sin^2(x) = 1 \quad \sec^2(x) = \frac{1}{cos^2(x)}$$

$$f'(x) = \sec^2(x) \quad \cos y \cdot \frac{dy}{dx} = 1 \quad \frac{dy}{dx} = \frac{1}{\cos y} \quad \cos y = \sqrt{1-\sin^2 y} \quad x = \sin y$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \int_0^1 \sqrt{x}\cdot dx = \int_0^1 x^{\frac{1}{2}}\cdot dx = \int_0^1 d\left(\tfrac{2}{3x^{\frac{3}{2}}}\right) = \tfrac{2}{3}.$$

$$a = x_0 \leq t_1 \leq x_1 \leq t_2 \leq x_2 \leq \cdots \leq x_{n-1} \leq t_n \leq x_n = b.$$

$$\sum_{i=1}^{n} f(t_i)\Delta_i; \qquad \left|S - \sum_{i=1}^{n} f(t_i)\Delta_i\right| < \varepsilon \int 1_A d\mu = \mu(A).$$

$$\begin{aligned}\int s\,d\mu &= \int\left(\sum_{i=1}^{n} a_i 1_{A_i}\right)d\mu \\ &= \sum_{i=1}^{n} a_i \int 1_{A_i}\,d\mu \\ &= \sum_{i=1}^{n} a_i\,\mu(A_i) \qquad \int_E s\,d\mu = \sum_{i=1}^{n} a_i\,\mu(A_i\cap E).\end{aligned}$$

$$\int_E f\,d\mu = \sup\left\{\int_E s\,d\mu : 0 \leq s \leq f\right.$$

$$f^+(x) = \begin{cases} f(x), \\ 0, \end{cases} \quad f^-(x) = \begin{cases} -f(x), \\ 0, \end{cases} \int_E |f| \, d\mu < \infty,$$
$$\int_E f \, d\mu = \int_E f^+ \, d\mu - \int_E f^- \, d\mu.$$

$$f \mapsto \int_a^b f \, dx \int_a^b (\alpha f + \beta g)(x) \, dx = \alpha \int_a^b f(x) \, dx + \beta \int_a^b g(x) \, dx.$$

$$f \mapsto \int_E f d\mu \int_E (\alpha f + \beta g) \, d\mu = \alpha \int_E f \, d\mu + \beta \int_E g \, d\mu. f \mapsto \int_E f d\mu,$$

$$m(b-a) \leq \int_a^b f(x) \, dx \leq M(b-a). \int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx.$$

$$\int_c^d f(x) \, dx \leq \int_a^b f(x) \, dx.$$
$$(fg)(x) = f(x)g(x), \; f^2(x) = (f(x))^2, \; |f|(x) = |f(x)|.$$
$$\left| \int_a^b f(x) \, dx \right| \leq \int_a^b |f(x)| \, dx.$$
$$\left(\int_a^b (fg)(x) \, dx \right)^2 \leq \left(\int_a^b f(x)^2 \, dx \right) \left(\int_a^b g(x)^2 \, dx \right).$$
$$\left| \int f(x)g(x) \, dx \right| \leq \left(\int |f(x)|^p \, dx \right)^{1/p} \left(\int |g(x)|^q \, dx \right)^{1/q}.$$
$$\left(\int |f(x) + g(x)|^p \, dx \right)^{1/p} \leq \left(\int |f(x)|^p \, dx \right)^{1/p} + \left(\int |g(x)|^p \, dx \right)^{1/p}.$$
$$\int_a^b f(x) \, dx \int_a^b f(x) \, dx = -\int_b^a f(x) \, dx. \int_a^a f(x) \, dx = 0.$$
$$\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx.$$
$$\begin{aligned} \int_a^c f(x) \, dx &= \int_a^b f(x) \, dx - \int_c^b f(x) \, dx \\ &= \int_a^b f(x) \, dx + \int_b^c f(x) \, dx \end{aligned} \qquad \int_M \omega = -\int_{M'} \omega \, .$$

$$F(x) = \int_a^x f(t) \, dt.$$

Entonces F es continua en $[a, b]$. Si f es continua en x de $[a, b]$, entonces F es derivable en x, y $F'(x) = f(x)$, lo que se calculará así:

$$\int_a^b f(t)\,dt = F(b) - F(a).\ \ F(x) = \int_a^x f(t)\,dt\ \ \int_a^b f(t)\,dt = F(b) - F(a).$$

$$\int_0^\infty \frac{dx}{(x+1)\sqrt{x}} = \pi \qquad \int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$$

$$\int_a^b f(x)dx = \lim_{\epsilon\to 0}\int_{a+\epsilon}^b f(x)dx$$

$$\begin{aligned}\int_0^\infty \frac{dx}{(x+1)\sqrt{x}} &= \lim_{s\to 0}\int_s^1 \frac{dx}{(x+1)\sqrt{x}} + \lim_{t\to\infty}\int_1^t \frac{dx}{(x+1)\sqrt{x}}\\ &= \lim_{s\to 0}\left(-\frac{\pi}{2} + 2\arctan\frac{1}{\sqrt{s}}\right) + \lim_{t\to\infty}\left(\frac{\pi}{2} - 2\arctan\frac{1}{\sqrt{t}}\right)\\ &= \frac{\pi}{2} + \frac{\pi}{2}\\ &= \pi.\end{aligned}$$

$$\begin{aligned}\int_{-1}^1 \frac{dx}{\sqrt[3]{x^2}} &= \lim_{s\to 0}\int_{-1}^{-s} \frac{dx}{\sqrt[3]{x^2}} + \lim_{t\to 0}\int_t^1 \frac{dx}{\sqrt[3]{x^2}}\\ &= \lim_{s\to 0} 3(1-\sqrt[3]{s}) + \lim_{t\to 0} 3(1-\sqrt[3]{t})\\ &= 3+3\\ &= 6.\end{aligned}$$

$$\int_{-1}^1 \frac{dx}{x_n}\int_E f(x)\,dx\ W = \vec{F}\cdot\vec{d}$$

$$W = \int_C \vec{F}\cdot d\vec{s}\int_S \mathbf{v}\cdot d\mathbf{S}\int_S f\,dx^1\cdots dx^m.\ \mathbf{d}\omega = \sum_{i=1}^n \frac{\partial f}{\partial x_i}dx^i \wedge dx^a.$$

$$\int_\Omega \mathbf{d}\omega = \int_{\partial\Omega}\omega\int (ax^k+b)^n dx = \frac{1}{a}\int (ax^k+b)^n a dx = \frac{1}{a}\frac{(ax^k+b)^{n+1}}{(n+1)kx}$$

$$\int (ax+b)^n dx = \frac{1}{a}\int (ax+b)^n a dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1}$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\int (ax+b)^{-1} a dx = \frac{1}{a}\ln|ax+b|$$

$$\int x(ax+b)^n dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)}(ax+b)^{n+1}$$

$$\int \frac{x\ dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b|$$

$$\int \frac{x\ dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b|$$

$$\int \frac{x\ dx}{(ax+b)^n} = \frac{a(1-n)x-b}{a^2(n-1)(n-2)(ax+b)^{n-1}}$$

$$\int \frac{x^2\,dx}{ax+b} = \frac{1}{a^3}\left(\frac{(ax+b)^2}{2} - 2b(ax+b) + b^2\ln|ax+b|\right)$$
$$\int \frac{x^2\,dx}{(ax+b)^2} = \frac{1}{a^3}\left(ax+b-2b\ln|ax+b| - \frac{b^2}{ax+b}\right)$$
$$\int \frac{x^2\,dx}{(ax+b)^3} = \frac{1}{a^3}\left(\ln|ax+b| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}\right)$$

$$\int \frac{x^2\,dx}{(ax+b)^n} = \frac{1}{a^3}\left(-\frac{1}{(n-3)(ax+b)^{n-3}} + \frac{2b}{(n-2)(a+b)^{n-2}} - \frac{b^2}{(n-1)(ax+b)^{n-1}}\right)$$

$$\int \frac{dx}{x(ax+b)} = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right| \qquad \int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{dx}{x^2(ax+b)^2} = -a\left(\frac{1}{b^2(ax+b)} + \frac{1}{ab^2x} - \frac{2}{b^3}\ln\left|\frac{ax+b}{x}\right|\right)$$
$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} \int \frac{dx}{x^2-a^2} = -\frac{1}{a}\operatorname{artanh}\frac{x}{a} = \frac{1}{2a}\ln\frac{a-x}{a+x}$$
$$\int \frac{dx}{x^2-a^2} = -\frac{1}{a}\operatorname{arcoth}\frac{x}{a} = \frac{1}{2a}\ln\frac{x-a}{x+a}$$
$$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}}$$

$$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{b^2-4ac}}\operatorname{artanh}\frac{2ax+b}{\sqrt{b^2-4ac}} = \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| \qquad (\text{for } 4ac-b^2<0)$$

$$\int \frac{x\,dx}{ax^2+bx+c} = \frac{1}{2a}\ln\left|ax^2+bx+c\right| - \frac{b}{2a}\int \frac{dx}{ax^2+bx+c}$$
$$\int \frac{mx+n}{ax^2+bx+c}dx = \frac{m}{2a}\ln\left|ax^2+bx+c\right| + \frac{2an-bm}{a\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} \qquad (\text{para } 4ac-b^2>0)$$

$$\int \frac{mx+n}{ax^2+bx+c}dx = \frac{m}{2a}\ln\left|ax^2+bx+c\right| + \frac{2an-bm}{a\sqrt{b^2-4ac}}\operatorname{artanh}\frac{2ax+b}{\sqrt{b^2-4ac}} \qquad (\text{para } 4ac-b^2<0)$$

$$\int \frac{dx}{(ax^2+bx+c)^n} = \frac{2ax+b}{(n-1)(4ac-b^2)(ax^2+bx+c)^{n-1}} + \frac{(2n-3)2a}{(n-1)(4ac-b^2)}\int \frac{dx}{(ax^2+bx+c)^{n-1}}$$

$$\int \frac{x\,dx}{(ax^2+bx+c)^n} = \frac{bx+2c}{(n-1)(4ac-b^2)(ax^2+bx+c)^{n-1}} - \frac{b(2n-3)}{(n-1)(4ac-b^2)}\int \frac{dx}{(ax^2+bx+c)^{n-1}}$$

$$\int \frac{dx}{x(ax^2+bx+c)} = \frac{1}{2c}\ln\left|\frac{x^2}{ax^2+bx+c}\right| - \frac{b}{2c}\int \frac{dx}{ax^2+bx+c}$$

$$r = \sqrt{x^2+a^2}$$

$$\int r\,dx = \frac{1}{2}\left(xr + a^2\,\ln(x+r)\right)$$

$$\int r^3\,dx = \frac{1}{4}xr^3 + \frac{1}{8}3a^2xr + \frac{3}{8}a^4\ln(x+r)$$

$$\int r^5\,dx = \frac{1}{6}xr^5 + \frac{5}{24}a^2xr^3 + \frac{5}{16}a^4xr + \frac{5}{16}a^6\ln(x+r)$$

$$\int xr\,dx = \frac{r^3}{3}$$

$$\int xr^3\,dx = \frac{r^5}{5}$$

$$\int xr^{2n+1}\,dx = \frac{r^{2n+3}}{2n+3}$$

$$\int x^2r\,dx = \frac{xr^3}{4} - \frac{a^2xr}{8} - \frac{a^4}{8}\ln(x+r)$$

$$\int x^2r^3\,dx = \frac{xr^5}{6} - \frac{a^2xr^3}{24} - \frac{a^4xr}{16} - \frac{a^6}{16}\ln(x+r)$$

$$\int x^3r\,dx = \frac{r^5}{5} - \frac{a^2r^3}{3}$$

$$\int x^3r^3\,dx = \frac{r^7}{7} - \frac{a^2r^5}{5}$$

$$\int x^3r^{2n+1}\,dx = \frac{r^{2n+5}}{2n+5} - \frac{a^3r^{2n+3}}{2n+3}$$

$$\int x^4r\,dx = \frac{x^3r^3}{6} - \frac{a^2xr^3}{8} + \frac{a^4xr}{16} + \frac{a^6}{16}\ln(x+r)$$

$$\int x^4r^3\,dx = \frac{x^3r^5}{8} - \frac{a^2xr^5}{16} + \frac{a^4xr^3}{64} + \frac{3a^6xr}{128} + \frac{3a^8}{128}\ln(x+r)$$

$$\int x^5r\,dx = \frac{r^7}{7} - \frac{2a^2r^5}{5} + \frac{a^4r^3}{3}$$

$$\int x^5r^3\,dx = \frac{r^9}{9} - \frac{2a^2r^7}{7} + \frac{a^4r^5}{5}$$

$$\int x^5r^{2n+1}\,dx = \frac{r^{2n+7}}{2n+7} - \frac{2a^2r^{2n+5}}{2n+5} + \frac{a^4r^{2n+3}}{2n+3}$$

$$\int \frac{r\,dx}{x} = r - a\ln\left|\frac{a+r}{x}\right| = r - a\sinh^{-1}\frac{a}{x}$$

$$\int \frac{r^3\,dx}{x} = \frac{r^3}{3} + a^2r - a^3\ln\left|\frac{a+r}{x}\right|$$

$$\int \frac{r^5\,dx}{x} = \frac{r^5}{5} + \frac{a^2r^3}{3} + a^4r - a^5\ln\left|\frac{a+r}{x}\right|$$

$$\int \frac{r^7\,dx}{x} = \frac{r^7}{7} + \frac{a^2r^5}{5} + \frac{a^4r^3}{3} + a^6r - a^7\ln\left|\frac{a+r}{x}\right|$$

$$\int \frac{dx}{r} = \sinh^{-1}\frac{x}{a} = \ln|x+r|$$

$$\int \frac{dx}{r^3} = \frac{x}{a^2r}$$

$$\int \frac{x\,dx}{r} = r$$

$$\int \frac{x\,dx}{r^3} = -\frac{1}{r}$$

$$\int \frac{x^2\,dx}{r} = \frac{x}{2}r - \frac{a^2}{2}\sinh^{-1}\frac{x}{a} = \frac{x}{2}r - \frac{a^2}{2}\ln|x+r|$$

$$\int \frac{dx}{xr} = -\frac{1}{a}\sinh^{-1}\frac{a}{x} = -\frac{1}{a}\ln\left|\frac{a+r}{x}\right|$$

$$s = \sqrt{x^2-a^2} \quad \int xs\,dx = \frac{1}{3}s^3$$

$$\int \frac{s\,dx}{x} = s - a\cos^{-1}\left|\frac{a}{x}\right| \quad \int \frac{dx}{s} = \int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|\frac{x+s}{a}\right| \quad \int \frac{x\,dx}{s} = s$$

$$\int \frac{x\,dx}{s^3} = -\frac{1}{s} \qquad \int \frac{x\,dx}{s^5} = -\frac{1}{3s^3} \qquad \int \frac{x\,dx}{s^7} = -\frac{1}{5s^5}$$

$$\int \frac{x\,dx}{s^{2n+1}} = -\frac{1}{(2n-1)s^{2n-1}} \ldots$$

$$\int \frac{x^{2m}\,dx}{s^{2n+1}} = -\frac{1}{2n-1}\frac{x^{2m-1}}{s^{2n-1}} + \frac{2m-1}{2n-1}\int \frac{x^{2m-2}\,dx}{s^{2n-1}}$$

$$\int \frac{x^2\,dx}{s} = \frac{xs}{2} + \frac{a^2}{2}\ln\left|\frac{x+s}{a}\right| \quad \int \frac{x^2\,dx}{s^3} = -\frac{x}{s} + \ln\left|\frac{x+s}{a}\right|$$

$$\int \frac{x^4\,dx}{s} = \frac{x^3 s}{4} + \frac{3}{8}a^2 xs + \frac{3}{8}a^4\ln\left|\frac{x+s}{a}\right|$$

$$\int \frac{x^4\,dx}{s^3} = \frac{xs}{2} - \frac{a^2 x}{s} + \frac{3}{2}a^2\ln\left|\frac{x+s}{a}\right| \quad \int \frac{x^4\,dx}{s^5} = -\frac{x}{s} - \frac{1}{3}\frac{x^3}{s^3} + \ln\left|\frac{x+s}{a}\right|$$

$$\int \frac{x^{2m}\,dx}{s^{2n+1}} = (-1)^{n-m}\frac{1}{a^{2(n-m)}}\sum_{i=0}^{n-m-1}\frac{1}{2(m+i)+1}\binom{n-m-1}{i}\frac{x^{2(m+i)+1}}{s^{2(m+i)+1}} \qquad (n > m \geq 0)$$

$$\int \frac{dx}{s^3} = -\frac{1}{a^2}\frac{x}{s} \quad \int \frac{dx}{s^5} = \frac{1}{a^4}\left[\frac{x}{s} - \frac{1}{3}\frac{x^3}{s^3}\right] \quad \int \frac{dx}{s^7} = -\frac{1}{a^6}\left[\frac{x}{s} - \frac{2}{3}\frac{x^3}{s^3} + \frac{1}{5}\frac{x^5}{s^5}\right]$$

$$\int \frac{dx}{s^9} = \frac{1}{a^8}\left[\frac{x}{s} - \frac{3}{3}\frac{x^3}{s^3} + \frac{3}{5}\frac{x^5}{s^5} - \frac{1}{7}\frac{x^7}{s^7}\right] \qquad \int \frac{x^2\,dx}{s^5} = -\frac{1}{a^2}\frac{x^3}{3s^3}$$

$$\int \frac{x^2\,dx}{s^7} = \frac{1}{a^4}\left[\frac{1}{3}\frac{x^3}{s^3} - \frac{1}{5}\frac{x^5}{s^5}\right] \quad \int \frac{x^2\,dx}{s^9} = -\frac{1}{a^6}\left[\frac{1}{3}\frac{x^3}{s^3} - \frac{2}{5}\frac{x^5}{s^5} + \frac{1}{7}\frac{x^7}{s^7}\right]$$

$$t = \sqrt{a^2-x^2} \quad \int t\,dx = \frac{1}{2}\left(xt + a^2\sin^{-1}\frac{x}{a}\right) \qquad (|x| \leq |a|)$$

$$\int xt\,dx = -\frac{1}{3}t^3 \qquad (|x| \leq |a|) \quad \int \frac{t\,dx}{x} = t - a\ln\left|\frac{a+t}{x}\right| \qquad (|x| \leq |a|)$$

$$\int \frac{dx}{t} = \sin^{-1}\frac{x}{a} \qquad (|x| \leq |a|)$$

$$\int \frac{x^2\,dx}{t} = -\frac{x}{2}t + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \qquad (|x| \leq |a|)$$

$$\int t\,dx = \frac{1}{2}\left(xt - \operatorname{sgn} x\,\cosh^{-1}\left|\frac{x}{a}\right|\right) \qquad (|x| \geq |a|)$$

$$R = \sqrt{ax^2+bx+c} \qquad \int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln\left|2\sqrt{a}R + 2ax + b\right|$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\,\sinh^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln|2ax+b|$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = -\frac{1}{\sqrt{-a}}\arcsin\frac{2ax+b}{\sqrt{b^2-4ac}}$$

$$\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{4ax+2b}{(4ac-b^2)\sqrt{R}}$$

$$\int \frac{dx}{\sqrt{(ax^2+bx+c)^5}} = \frac{4ax+2b}{3(4ac-b^2)\sqrt{R}}\left(\frac{1}{R} + \frac{8a}{4ac-b^2}\right)$$

$$\int \frac{dx}{\sqrt{(ax^2+bx+c)^{2n+1}}} = \frac{4ax+2b}{(2n-1)(4ac-b^2)R^{(2n-1)/2}} + \frac{8a(n-1)}{(2n-1)(4ac-b^2)}\int \frac{dx}{R^{(2n-1)/2}}$$

$$\int \frac{x\,dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{R}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{R}}$$

$$\int \frac{x\,dx}{\sqrt{(ax^2+bx+c)^3}} = -\frac{2bx+4c}{(4ac-b^2)\sqrt{R}}$$

$$\int \frac{x\,dx}{\sqrt{(ax^2+bx+c)^{2n+1}}} = -\frac{1}{(2n-1)aR^{(2n-1)/2}} - \frac{b}{2a}\int \frac{dx}{R^{(2n+1)/2}}$$

$$\int \frac{dx}{x\sqrt{ax^2+bx+c}} = -\frac{1}{\sqrt{c}}\ln\left(\frac{2\sqrt{c}R + bx + 2c}{x}\right)$$

$$\int \frac{dx}{x\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{-c}} \sin^{-1}\left(\frac{bx+2c}{|x|\sqrt{b^2-4ac}}\right)$$

$$\int \frac{dx}{x\sqrt{ax^2+bx+c}} = -\frac{1}{\sqrt{c}} \sinh^{-1}\left(\frac{bx+2c}{|x|\sqrt{4ac-b^2}}\right)\sqrt{ax+b}$$

$$\int \frac{dx}{x\sqrt{ax+b}} = \frac{-2}{\sqrt{b}} \tanh^{-1}\sqrt{\frac{ax+b}{b}}$$

$$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\left(\sqrt{ax+b} - \sqrt{b}\tanh^{-1}\sqrt{\frac{ax+b}{b}}\right)$$

$$\int \frac{x^n}{\sqrt{ax+b}}\,dx = \frac{2}{a(2n+1)}\left(x^n\sqrt{ax+b} - bn\int \frac{x^{n-1}}{\sqrt{ax+b}}\,dx\right)$$

$$\int x^n\sqrt{ax+b}\,dx = \frac{2}{2n+1}\left(x^{n+1}\sqrt{ax+b} + bx^n\sqrt{ax+b} - nb\int x^{n-1}\sqrt{ax+b}\,dx\right)$$

$$\int \sin cx\;dx = -\frac{1}{c}\cos cx$$

$$\int \sin^n cx\;dx = -\frac{\sin^{n-1} cx\cos cx}{nc} + \frac{n-1}{n}\int \sin^{n-2} cx\;dx \qquad (\text{para } n>0)$$

$$\int x\sin cx\;dx = \frac{\sin cx}{c^2} - \frac{x\cos cx}{c}$$

$$\int x^n\sin cx\;dx = -\frac{x^n}{c}\cos cx + \frac{n}{c}\int x^{n-1}\cos cx\;dx \qquad (\text{para } n>0)$$

$$\int \frac{\sin cx}{x}dx = \sum_{i=0}^{\infty}(-1)^i\frac{(cx)^{2i+1}}{(2i+1)\cdot(2i+1)!}$$

$$\int \frac{\sin cx}{x^n}dx = -\frac{\sin cx}{(n-1)x^{n-1}} + \frac{c}{n-1}\int \frac{\cos cx}{x^{n-1}}dx$$

$$\int \frac{dx}{\sin cx} = \frac{1}{c}\ln\left|\tan\frac{cx}{2}\right|$$

$$\int \frac{dx}{\sin^n cx} = \frac{\cos cx}{c(n-1)\sin^{n-1} cx} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} cx}$$

$$\int \frac{dx}{1\pm\sin cx} = \frac{1}{c}\tan\left(\frac{cx}{2}\mp\frac{\pi}{4}\right)$$

$$\int \frac{x\;dx}{1-\sin cx} = \frac{x}{c}\cot\left(\frac{\pi}{4}-\frac{cx}{2}\right) + \frac{2}{c^2}\ln\left|\sin\left(\frac{\pi}{4}-\frac{cx}{2}\right)\right|$$

$$\int \frac{\sin cx\;dx}{1\pm\sin cx} = \pm x + \frac{1}{c}\tan\left(\frac{\pi}{4}\mp\frac{cx}{2}\right)$$

$$\int \sin c_1x\sin c_2x\;dx = \frac{\sin(c_1-c_2)x}{2(c_1-c_2)} - \frac{\sin(c_1+c_2)x}{2(c_1+c_2)}$$

$$\int \cos cx\ dx = \frac{1}{c}\sin cx$$
$$\int \cos^n cx\ dx = \frac{\cos^{n-1} cx \sin cx}{nc} + \frac{n-1}{n}\int \cos^{n-2} cx\ dx$$
$$\int x\cos cx\ dx = \frac{\cos cx}{c^2} + \frac{x\sin cx}{c}$$
$$\int x^n \cos cx\ dx = \frac{x^n \sin cx}{c} - \frac{n}{c}\int x^{n-1}\sin cx\ dx$$
$$\int \frac{\cos cx}{x}dx = \ln|cx| + \sum_{i=1}^{\infty}(-1)^i \frac{(cx)^{2i}}{2i\cdot(2i)!}$$
$$\int \frac{\cos cx}{x^n}dx = -\frac{\cos cx}{(n-1)x^{n-1}} - \frac{c}{n-1}\int \frac{\sin cx}{x^{n-1}}dx$$
$$\int \frac{dx}{\cos cx} = \frac{1}{c}\ln\left|\tan\left(\frac{cx}{2}+\frac{\pi}{4}\right)\right|$$
$$\int \frac{dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)cos^{n-1}cx} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} cx}$$
$$\int \frac{dx}{1+\cos cx} = \frac{1}{c}\tan\frac{cx}{2}\int \frac{dx}{1-\cos cx} = -\frac{1}{c}\cot\frac{cx}{2}$$
$$\int \frac{x\ dx}{1+\cos cx} = \frac{x}{c}\tan cx2 + \frac{2}{c^2}\ln\left|\cos\frac{cx}{2}\right|$$
$$\int \frac{x\ dx}{1-\cos cx} = -\frac{x}{x}\cot cx2 + \frac{2}{c^2}\ln\left|\sin\frac{cx}{2}\right| \int \frac{\cos cx\ dx}{1+\cos cx} = x - \frac{1}{c}\tan\frac{cx}{2}$$
$$\int \frac{\cos cx\ dx}{1-\cos cx} = -x - \frac{1}{c}\cot\frac{cx}{2}$$
$$\int \cos c_1 x \cos c_2 x\ dx = \frac{\sin(c_1-c_2)x}{2(c_1-c_2)} + \frac{\sin(c_1+c_2)x}{2(c_1+c_2)}$$
$$\int \tan cx\ dx = -\frac{1}{c}\ln|\cos cx|$$
$$\int \tan^n cx\ dx = \frac{1}{c(n-1)}\tan^{n-1} cx - \int \tan^{n-2} cx\ dx$$
$$\int \frac{dx}{\tan cx + 1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx + \cos cx|$$
$$\int \frac{dx}{\tan cx - 1} = -\frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx|$$
$$\int \frac{\tan cx\ dx}{\tan cx + 1} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx + \cos cx|$$
$$\int \frac{\tan cx\ dx}{\tan cx - 1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx| \int \cot cx\ dx = \frac{1}{c}\ln|\sin cx|$$

$$\int \cot^n cx\; dx = -\frac{1}{c(n-1)}\cot^{n-1} cx - \int \cot^{n-2} cx\; dx$$

$$\int \frac{dx}{1+\cot cx} = \int \frac{\tan cx\; dx}{\tan cx + 1}\int \frac{dx}{1-\cot cx} = \int \frac{\tan cx\; dx}{\tan cx - 1}$$

$$\int \frac{dx}{\cos cx \pm \sin cx} = \frac{1}{c\sqrt{2}} \ln\left|\tan\left(\frac{cx}{2} \pm \frac{\pi}{8}\right)\right|$$

$$\int \frac{dx}{(\cos cx \pm \sin cx)^2} = \frac{1}{2c}\tan\left(cx \mp \frac{\pi}{4}\right)$$

$$\int \frac{\cos cx\; dx}{\cos cx + \sin cx} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{\cos cx\; dx}{\cos cx - \sin cx} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx - \cos cx|$$

$$\int \frac{\sin cx\; dx}{\cos cx + \sin cx} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{\sin cx\; dx}{\cos cx - \sin cx} = -\frac{x}{2} - \frac{1}{2c}\ln|\sin cx - \cos cx|$$

$$\int \frac{\cos cx\; dx}{\sin cx(1+\cos cx)} = -\frac{1}{4c}\tan^2\frac{cx}{2} + \frac{1}{2c}\ln\left|\tan\frac{cx}{2}\right|$$

$$\int \frac{\cos cx\; dx}{\sin cx(1+-\cos cx)} = -\frac{1}{4c}\cot^2\frac{cx}{2} - \frac{1}{2c}\ln\left|\tan\frac{cx}{2}\right|$$

$$\int \frac{\sin cx\; dx}{\cos cx(1+\sin cx)} = \frac{1}{4c}\cot^2\left(\frac{cx}{2}+\frac{\pi}{4}\right) + \frac{1}{2c}\ln\left|\tan\left(\frac{cx}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \frac{\sin cx\; dx}{\cos cx(1-\sin cx)} = \frac{1}{4c}\tan^2\left(\frac{cx}{2}+\frac{\pi}{4}\right) - \frac{1}{2c}\ln\left|\tan\left(\frac{cx}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \sin cx \cos cx\; dx = \frac{1}{2c}\sin^2 cx$$

$$\int \sin c_1 x \cos c_2 x\; dx = -\frac{\cos(c_1+c_2)x}{2(c_1+c_2)} - \frac{\cos(c_1-c_2)x}{2(c_1-c_2)}$$

$$\int \sin^n cx \cos cx\; dx = \frac{1}{c(n+1)}\sin^{n+1} cx \qquad (\text{para } n \neq 1)$$

$$\int \sin cx \cos^n cx\; dx = -\frac{1}{c(n+1)}\cos^{n+1} cx \qquad (\text{para } n \neq 1)$$

$$\int \sin^n cx \cos^m cx\; dx = -\frac{\sin^{n-1} cx \cos^{m+1} cx}{c(n+m)} + \frac{n-1}{n+m}\int \sin^{n-2} cx \cos^m cx\; dx$$

$$\int \sin^n cx \cos^m cx \, dx = \frac{\sin^{n+1} cx \cos^{m-1} cx}{c(n+m)} + \frac{m-1}{n+m} \int \sin^n cx \cos^{m-2} cx \, dx$$

$$\int \frac{dx}{\sin cx \cos cx} = \frac{1}{c} \ln |\tan cx|$$

$$\int \frac{dx}{\sin cx \cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx} + \int \frac{dx}{\sin cx \cos^{n-2} cx}$$

$$\int \frac{dx}{\sin^n cx \cos cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} + \int \frac{dx}{\sin^{n-2} cx \cos cx}$$

$$\int \frac{\sin cx \, dx}{\cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx}$$

$$\int \frac{\sin^2 cx \, dx}{\cos cx} = -\frac{1}{c} \sin cx + \frac{1}{c} \ln \left| \tan \left(\frac{\pi}{4} + \frac{cx}{2} \right) \right|$$

$$\int \frac{\sin^2 cx \, dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)\cos^{n-1} cx} - \frac{1}{n-1} \int \frac{dx}{\cos^{n-2} cx}$$

$$\int \frac{\sin^n cx \, dx}{\cos cx} = -\frac{\sin^{n-1} cx}{c(n-1)} + \int \frac{\sin^{n-2} cx \, dx}{\cos cx} \qquad (\text{for } n \neq 1)$$

$$\int \frac{\sin^n cx \, dx}{\cos^m cx} = \frac{\sin^{n+1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-m+2}{m-1} \int \frac{\sin^n cx \, dx}{\cos^{m-2} cx}$$

$$\int \frac{\sin^n cx \, dx}{\cos^m cx} = -\frac{\sin^{n-1} cx}{c(n-m)\cos^{m-1} cx} + \frac{n-1}{n-m} \int \frac{\sin^{n-2} cx \, dx}{\cos^m cx}$$

$$\int \frac{\sin^n cx \, dx}{\cos^m cx} = \frac{\sin^{n-1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-1}{n-1} \int \frac{\sin^{n-1} cx \, dx}{\cos^{m-2} cx}$$

$$\int \frac{\cos cx \, dx}{\sin^n cx} = -\frac{1}{c(n-1)\sin^{n-1} cx}$$

$$\int \frac{\cos^2 cx \, dx}{\sin cx} = \frac{1}{c} \left(\cos cx + \ln \left| \tan \frac{cx}{2} \right| \right)$$

$$\int \frac{\cos^2 cx \, dx}{\sin^n cx} = -\frac{1}{n-1} \left(\frac{\cos cx}{c \sin^{n-1} cx} + \int \frac{dx}{\sin^{n-2} cx} \right)$$

$$\int \frac{\cos^n cx\ dx}{\sin^m cx} = -\frac{\cos^{n+1} cx}{c(m-1)\sin^{m-1} cx} - \frac{n-m-2}{m-1}\int \frac{\cos^n cx\ dx}{\sin^{m-2} cx}$$

$$\int \frac{\cos^n cx\ dx}{\sin^m cx} = \frac{\cos^{n-1} cx}{c(n-m)\sin^{m-1} cx} + \frac{n-1}{n-m}\int \frac{\cos^{n-2} cx\ dx}{\sin^m cx}$$

$$\int \frac{\cos^n cx\ dx}{\sin^m cx} = -\frac{\cos^{n-1} cx}{c(m-1)\sin^{m-1} cx} - \frac{n-1}{m-1}\int \frac{\cos^{n-2} cx\ dx}{\sin^{m-2} cx}$$

$$\int \sin cx \tan cx\ dx = \frac{1}{c}(\ln|\sec cx + \tan cx| - \sin cx)$$

$$\int \frac{\tan^n cx\ dx}{\sin^2 cx} = \frac{1}{c(n-1)}\tan^{n-1}(cx)$$

$$\int \frac{\tan^n cx\ dx}{\cos^2 cx} = \frac{1}{c(n+1)}\tan^{n+1} cx$$

$$\int \frac{\cot^n cx\ dx}{sin^2 cx} = \frac{1}{c(n+1)}\cot^{n+1} cx$$

$$\int \frac{\cot^n cx\ dx}{\cos^2 cx} = \frac{1}{c(1-n)}\tan^{1-n} cx \qquad \int \cot cx \tan cx\ dx = x$$

$$\int \sec^2 u\ du = \tan u + c \int \sec u\ du = \ln|\sec u + \tan u| + c$$

$$\int \sinh cx\ dx = \frac{1}{c}\cosh cx \int \cosh cx\ dx = \frac{1}{c}\sinh cx$$

$$\int \sinh^2 cx\ dx = \frac{1}{4c}\sinh 2cx - \frac{x}{2}\int \cosh^2 cx\ dx = \frac{1}{4c}\sinh 2cx + \frac{x}{2}$$

$$\int \sinh^n cx\ dx = \frac{1}{cn}\sinh^{n-1} cx \cosh cx - \frac{n-1}{n}\int \sinh^{n-2} cx\ dx$$

$$\int \sinh^n cx\ dx = \frac{1}{c(n+1)}\sinh^{n+1} cx \cosh cx - \frac{n+2}{n+1}\int \sinh^{n+2} cx\ dx$$

$$\int \cosh^n cx\ dx = \frac{1}{cn}\sinh cx \cosh^{n-1} cx + \frac{n-1}{n}\int \cosh^{n-2} cx\ dx$$

$$\int \cosh^n cx\ dx = -\frac{1}{c(n+1)}\sinh cx \cosh^{n+1} cx - \frac{n+2}{n+1}\int \cosh^{n+2} cx\ dx$$

$$\int \frac{dx}{\sinh cx} = \frac{1}{c} \ln \left|\tanh \frac{cx}{2}\right| \int \frac{dx}{\sinh cx} = \frac{1}{c} \ln \left|\frac{\cosh cx - 1}{\sinh cx}\right|$$

$$\int \frac{dx}{\sinh cx} = \frac{1}{c} \ln \left|\frac{\sinh cx}{\cosh cx + 1}\right| \int \frac{dx}{\sinh cx} = \frac{1}{c} \ln \left|\frac{\cosh cx - 1}{\cosh cx + 1}\right|$$

$$\int \frac{dx}{\cosh cx} = \frac{2}{c} \arctan e^{cx}$$

$$\int \frac{dx}{\sinh^n cx} = \frac{\cosh cx}{c(n-1)\sinh^{n-1} cx} - \frac{n-2}{n-1} \int \frac{dx}{\sinh^{n-2} cx}$$

$$\int \frac{dx}{\cosh^n cx} = \frac{\sinh cx}{c(n-1)\cosh^{n-1} cx} + \frac{n-2}{n-1} \int \frac{dx}{\cosh^{n-2} cx}$$

$$\int \frac{\cosh^n cx}{\sinh^m cx} dx = \frac{\cosh^{n-1} cx}{c(n-m)\sinh^{m-1} cx} + \frac{n-1}{n-m} \int \frac{\cosh^{n-2} cx}{\sinh^m cx} dx$$

$$\int \frac{\cosh^n cx}{\sinh^m cx} dx = -\frac{\cosh^{n+1} cx}{c(m-1)\sinh^{m-1} cx} + \frac{n-m+2}{m-1} \int \frac{\cosh^n cx}{\sinh^{m-2} cx} dx$$

$$\int \frac{\cosh^n cx}{\sinh^m cx} dx = -\frac{\cosh^{n-1} cx}{c(m-1)\sinh^{m-1} cx} + \frac{n-1}{m-1} \int \frac{\cosh^{n-2} cx}{\sinh^{m-2} cx} dx$$

$$\int \frac{\sinh^m cx}{\cosh^n cx} dx = \frac{\sinh^{m-1} cx}{c(m-n)\cosh^{n-1} cx} + \frac{m-1}{m-n} \int \frac{\sinh^{m-2} cx}{\cosh^n cx} dx$$

$$\int \frac{\sinh^m cx}{\cosh^n cx} dx = \frac{\sinh^{m+1} cx}{c(n-1)\cosh^{n-1} cx} + \frac{m-n+2}{n-1} \int \frac{\sinh^m cx}{\cosh^{n-2} cx} dx$$

$$\int \frac{\sinh^m cx}{\cosh^n cx} dx = -\frac{\sinh^{m-1} cx}{c(n-1)\cosh^{n-1} cx} + \frac{m-1}{n-1} \int \frac{\sinh^{m-2} cx}{\cosh^{n-2} cx} dx$$

$$\int x \sinh cx \, dx = \frac{1}{c} x \cosh cx - \frac{1}{c^2} \sinh cx$$

$$\int x \cosh cx \, dx = \frac{1}{c} x \sinh cx - \frac{1}{c^2} \cosh cx \int \tanh cx \, dx = \frac{1}{c} \ln |\cosh cx|$$

$$\int \coth cx \, dx = \frac{1}{c} \ln |\sinh cx|$$

$$\int \tanh^n cx \, dx = -\frac{1}{c(n-1)} \tanh^{n-1} cx + \int \tanh^{n-2} cx \, dx$$

$$\int \coth^n cx\,dx = -\frac{1}{c(n-1)}\coth^{n-1} cx + \int \coth^{n-2} cx\,dx$$

$$\int \sinh bx \sinh cx\,dx = \frac{1}{b^2-c^2}(b \sinh cx \cosh bx - c \cosh cx \sinh bx)$$

$$\int \cosh bx \cosh cx\,dx = \frac{1}{b^2-c^2}(b \sinh bx \cosh cx - c \sinh cx \cosh bx)$$

$$\int \cosh bx \sinh cx\,dx = \frac{1}{b^2-c^2}(b \sinh bx \sinh cx - c \cosh bx \cosh cx)$$

$$\int \sinh(ax+b)\sin(cx+d)\,dx = \frac{a}{a^2+c^2}\cosh(ax+b)\sin(cx+d) - \frac{c}{a^2+c^2}\sinh(ax+b)\cos(cx+d)$$

$$\int \sinh(ax+b)\cos(cx+d)\,dx = \frac{a}{a^2+c^2}\cosh(ax+b)\cos(cx+d) + \frac{c}{a^2+c^2}\sinh(ax+b)\sin(cx+d)$$

$$\int \cosh(ax+b)\sin(cx+d)\,dx = \frac{a}{a^2+c^2}\sinh(ax+b)\sin(cx+d) - \frac{c}{a^2+c^2}\cosh(ax+b)\cos(cx+d)$$

$$\int \cosh(ax+b)\cos(cx+d)\,dx = \frac{a}{a^2+c^2}\sinh(ax+b)\cos(cx+d) + \frac{c}{a^2+c^2}\cosh(ax+b)\sin(cx+d)$$

$$\int e^{cx}\,dx = \frac{1}{c}e^{cx}$$

$$\int xe^{cx}\,dx = \frac{e^{cx}}{c^2}(cx-1)$$

$$\int x^2 e^{cx}\,dx = e^{cx}\left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right)$$

$$\int x^n e^{cx}\,dx = \frac{1}{c}x^n e^{cx} - \frac{n}{c}\int x^{n-1}e^{cx}dx$$

$$\int \frac{e^{cx}\,dx}{x} = \ln|x| + \sum_{i=1}^{\infty}\frac{(cx)^i}{i \cdot i!}$$

$$\int \frac{e^{cx}\,dx}{x^n} = \frac{1}{n-1}\left(-\frac{e^{cx}}{x^{n-1}} + c\int \frac{e^{cx}dx}{x^{n-1}}\right)$$

$$\int e^{cx}\ln x\,dx = \frac{1}{c}\left(e^{cx}\ln|x| - \int \frac{e^{cx}dx}{x}\right)$$

$$\int e^{cx}\sin bx\,dx = \frac{e^{cx}}{c^2+b^2}(c \sin bx - b \cos bx)$$

$$\int e^{cx}\cos bx\;dx = \frac{e^{cx}}{c^2+b^2}(c\cos bx + b\sin bx)$$

$$\int e^{cx}\sin^n x\;dx = \frac{e^{cx}\sin^{n-1}x}{c^2+n^2}(c\sin x - n\cos x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\sin^{n-2}x\;dx$$

$$\int e^{cx}\cos^n x\;dx = \frac{e^{cx}\cos^{n-1}x}{c^2+n^2}(c\cos x + n\sin x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\cos^{n-2}x\;dx$$

$$\int \frac{1}{\sigma\sqrt{2\pi}}\,e^{-(x-\mu)^2/2\sigma^2} \;\; = \frac{1}{2\sigma}(1+\operatorname{erf}\frac{x-\mu}{\sigma\sqrt{(2)}})\int_{-\infty}^{\infty} e^{-\alpha x^2}dx = \sqrt{\frac{\pi}{\alpha}} = I(\alpha)$$

$$\int_{-\infty}^{\infty} x^{2n}e^{-\alpha x^2}dx = (-1)^n\frac{d^n}{d\alpha^n}\,I(\alpha)\;\xrightarrow{\text{si n = 1}}\; = \frac{\sqrt{\pi}}{2\alpha^{3/2}}$$

$$\int \ln x\,dx = x\ln x - x\int (\ln x)^2\;dx = x(\ln x)^2 - 2x\ln x + 2x$$

$$\int (\ln x)^n\;dx = x(\ln x)^n - n\int (\ln x)^{n-1}dx$$

$$\int \frac{dx}{\ln x} = \ln|\ln x| + \ln x + \sum_{i=2}^{\infty}\frac{(\ln x)^i}{i\cdot i!}$$

$$\int \frac{dx}{(\ln x)^n} = -\frac{x}{(n-1)(\ln x)^{n-1}} + \frac{1}{n-1}\int \frac{dx}{(\ln x)^{n-1}}$$

$$\int x^m\ln x\;dx = x^{m+1}\left(\frac{\ln x}{m+1} - \frac{1}{(m+1)^2}\right)$$

$$\int x^m(\ln x)^n\;dx = \frac{x^{m+1}(\ln x)^n}{m+1} - \frac{n}{m+1}\int x^m(\ln x)^{n-1}dx$$

$$\int \frac{(\ln x)^n\;dx}{x} = \frac{(\ln x)^{n+1}}{n+1}$$

$$\int \frac{\ln x\,dx}{x^m} = -\frac{\ln x}{(m-1)x^{m-1}} - \frac{1}{(m-1)^2x^{m-1}}$$

$$\int \frac{(\ln x)^n\;dx}{x^m} = -\frac{(\ln x)^n}{(m-1)x^{m-1}} + \frac{n}{m-1}\int \frac{(\ln x)^{n-1}dx}{x^m}$$

$$\int \frac{x^m\;dx}{(\ln x)^n} = -\frac{x^{m+1}}{(n-1)(\ln x)^{n-1}} + \frac{m+1}{n-1}\int \frac{x^m\,dx}{(\ln x)^{n-1}}$$

$$\int \frac{dx}{x\ln x} = \ln|\ln x| \int \frac{dx}{x^n\ln x} = \ln|\ln x| + \sum_{i=1}^{\infty}(-1)^i\frac{(n-1)^i(\ln x)^i}{i\cdot i!}$$

$$\int \frac{dx}{x(\ln x)^n} = -\frac{1}{(n-1)(\ln x)^{n-1}}$$
$$\int \sin(\ln x)\ dx = \frac{x}{2}(\sin(\ln x) - \cos(\ln x))$$
$$\int \cos(\ln x)\ dx = \frac{x}{2}(\sin(\ln x) + \cos(\ln x))$$
$$\int \arcsin \frac{x}{c}\, dx = x \arcsin \frac{x}{c} + \sqrt{c^2 - x^2}$$
$$\int x \arcsin \frac{x}{c}\, dx = \left(\frac{x^2}{2} - \frac{c^2}{4} \right) \arcsin \frac{x}{c} + \frac{x}{4}\sqrt{c^2 - x^2}$$
$$\int x^2 \arcsin \frac{x}{c}\, dx = \frac{x^3}{3} \arcsin \frac{x}{c} + \frac{x^2 + 2c^2}{9}\sqrt{c^2 - x^2}$$
$$\int \arccos \frac{x}{c}\, dx = x \arccos \frac{x}{c} - \sqrt{c^2 - x^2}$$
$$\int x \arccos \frac{x}{c}\, dx = \left(\frac{x^2}{2} - \frac{c^2}{4} \right) \arccos \frac{x}{c} - \frac{x}{4}\sqrt{c^2 - x^2}$$
$$\int x^2 \arccos \frac{x}{c}\, dx = \frac{x^3}{3} \arccos \frac{x}{c} - \frac{x^2 + 2c^2}{9}\sqrt{c^2 - x^2}$$
$$\int \arctan \frac{x}{c}\, dx = x \arctan \frac{x}{c} - \frac{c}{2} \ln(c^2 + x^2)$$
$$\int x \arctan \frac{x}{c}\, dx = \frac{c^2 + x^2}{2} \arctan \frac{x}{c} - \frac{cx}{2}$$
$$\int x^2 \arctan \frac{x}{c}\, dx = \frac{x^3}{3} \arctan \frac{x}{c} - \frac{cx^2}{6} + \frac{c^3}{6} \ln c^2 + x^2$$
$$\int x^n \arctan \frac{x}{c}\, dx = \frac{x^{n+1}}{n+1} \arctan \frac{x}{c} - \frac{c}{n+1} \int \frac{x^{n+1} dx}{c^2 + x^2}$$
$$\int \operatorname{arccot} \frac{x}{c}\, dx = x \operatorname{arccot} \frac{x}{c} + \frac{c}{2} \ln(c^2 + x^2)$$
$$\int x \operatorname{arccot} \frac{x}{c}\, dx = \frac{c^2 + x^2}{2} \operatorname{arccot} \frac{x}{c} + \frac{cx}{2}$$
$$\int x^2 \operatorname{arccot} \frac{x}{c}\, dx = \frac{x^3}{3} \operatorname{arccot} \frac{x}{c} + \frac{cx^2}{6} - \frac{c^3}{6} \ln(c^2 + x^2)$$
$$\int x^n \operatorname{arccot} \frac{x}{c}\, dx = \frac{x^{n+1}}{n+1} \operatorname{arccot} \frac{x}{c} + \frac{c}{n+1} \int \frac{x^{n+1} dx}{c^2 + x^2}$$
$$\int \operatorname{arsinh} \frac{x}{c}\, dx = x \operatorname{arsinh} \frac{x}{c} - \sqrt{x^2 + c^2}$$

$$\int \operatorname{arcosh}\frac{x}{c}\,dx = x\operatorname{arcosh}\frac{x}{c} - \sqrt{x^2 - c^2}$$

$$\int \operatorname{artanh}\frac{x}{c}\,dx = x\operatorname{artanh}\frac{x}{c} + \frac{c}{2}\ln|c^2 - x^2|$$

$$\int \operatorname{arcoth}\frac{x}{c}\,dx = x\operatorname{arcoth}\frac{x}{c} + \frac{c}{2}\ln|x^2 - c^2|$$

$$\int \operatorname{arsech}\frac{x}{c}\,dx = x\operatorname{arsech}\frac{x}{c} - c\arctan\frac{x\sqrt{\frac{c-x}{c+x}}}{x-c}$$

$$\int \operatorname{arcsch}\frac{x}{c}\,dx = x\operatorname{arcsch}\frac{x}{c} + c\,\ln\frac{x+\sqrt{x^2+c^2}}{c}$$

$$\int f(u,n)\cdot du = g(u,n) + a\int f(u,n-b)\cdot du$$

$$I_n = \int \left(\frac{u-a}{u-b}\right)^n + 1du = -\frac{(u-a)^n}{(n-1)(u-b)^{n-1}} + \frac{n}{n-1}I_{n-1}$$

$$I_{m,n} = \int \frac{(u-a)^m}{(u-b)^n}\cdot du = -\frac{(u-a)^m}{(n-1)(u-b)^{n-1}} + \frac{m}{n-1}I_{m-1,n-1}$$

$$I_{m,n} = \int (u-a)^m(u-b)^n du = \frac{1}{m+n+1}(u-a)^{m+1}(u-b)^n + \frac{n(a-b)}{m+n+1}I_{m,n-1}$$

$$I_{m,n} = \int \frac{du}{(u-a)^m(u-b)^n} = \frac{1}{(n-1)(a-b)(x-a)^{m-1}(x-b)^{n-1}} + \frac{m+n+2}{(n-1)(a-b)}I_{m,n-1}$$

$$I_{m,n} = \int \frac{u^n du}{(a+bu)^m} = -\frac{u^n}{b(m-1)(a+bu)^{m-1}} + \frac{n}{b(m-1)}I_{m-1,n-1}$$

$$I_n = \int \frac{u^n du}{\sqrt{a+bu}} = \frac{2}{b(2n+1)} u^n \sqrt{a+bu} - \frac{2na}{(2n+1)b} I_{n-1}$$

$$I_{m,n} = \int \frac{(r+su)^n}{(a+bu)^m} \cdot du = -\frac{(r+su)^n}{(m-1)b(a+bu)^{m-1}} + \frac{ns}{(m-1)b} I_{m-1,n-1}$$

$$I_{m,n} = \int \frac{1}{u^n} (a+bu)^m du = -\frac{(a+bu)^m}{(n-1)u^{n-1}} + \frac{mb}{n-1} I_{m-1,n-1}$$

$$I_n = \int \frac{1}{u^n} \sqrt{a+bu} \cdot du = -\frac{1}{(n-1)au^{n-1}} (a+bu)^{3/2} - \frac{(2n-5)b}{2(n-1)a} I_{n-1}$$

$$I_{m,n} = \int u^n (a+bu)^m du = \frac{1}{(m+n+1)b} u^n (a+bu)^{m+1} - \frac{na}{(m+n+1)b} I_{m,n-1}$$

$$I_{m,n} = \int u^n (a+bu)^m du = \frac{1}{m+n+1} u^{n+1} (a+bu)^m + \frac{ma}{m+n+1} I_{m-1,n}$$

$$I_n = \int u^n \sqrt{a+bu} \cdot du = \frac{2}{(2n+3)b} u^n (a+bu)^{3/2} - \frac{2na}{(2n+3)b} I_{n-1}$$

$$I_{m,n} = \int (r+su)^n (a+bu)^m du = \frac{1}{(m+n+1)b} (r+su)^n (a+bu)^{m+1} - \frac{n(as-br)}{(m+n+1)b} I_{m,n-1}$$

$$I_{m,n} = \int \frac{du}{u^n (a+bu)^m} = -\frac{1}{(n-1)u^{n-1}(a+bu)^m} - \frac{mb}{n-1} I_{m+1,n-1}$$

$$I_n = \int \frac{du}{u^n\sqrt{a+bu}} = -\frac{1}{(n-1)au^{n-1}}\sqrt{a+bu} - \frac{(2n-3)b}{2(n-1)a} I_{n-1}$$

$$I_{m,n} = \int \frac{du}{u^n(a+bu)^m} = -\frac{1}{(m-1)bu^n(a+bu)^{m-1}} - \frac{n}{(m-1)b} I_{m-1,n+1}$$

$$I_{m,n} = \int \frac{du}{u^n(a+bu)^m} = -\frac{1}{(n-1)au^{n-1}(a+bu)^{m-1}} - \frac{(m+n-2)b}{(n-1)a} I_{m,n-1}$$

$$I_{m,n} = \int \frac{du}{u^n(a+bu)^m} = \frac{1}{(m-1)au^{n-1}(a+bu)^{m-1}} + \frac{m+n-2}{(m-1)a} I_{m-1,n}$$

$$I_{m,n} = \int \frac{du}{(r+su)^n(a+bu)^m} = -\frac{1}{(n-1)(as-br)(r+su)^{n-1}(a+bu)^{m-1}} - \frac{(m+n-2)b}{(n-1)(as-br)} I_{m,n-1}$$

$$I_n = \int \frac{du}{(a^2 \pm u^2)^n} = \frac{u}{2a^2(n-1)(a^2 \pm u^2)^{n-1}} + \frac{2n-3}{2a^2(n-1)} I_{n-1}$$

$$I_n = \int \frac{du}{(u^2 \pm a^2)^n} = \pm\frac{u}{2a^2(n-1)(u^2 \pm a^2)^{n-1}} \pm \frac{2n-3}{2a^2(n-1)} I_{n-1}$$

$$I_n = \int (a^2 \pm u^2)^n \, du = \frac{u(a^2 \pm u^2)^n}{2n+1} + \frac{2a^2 n}{2n+1} I_{n-1}$$

$$I_n = \int (u^2 - a^2)^n \, du = \frac{u(u^2 - a^2)^n}{2n+1} - \frac{2a^2 n}{2n+1} I_{n-2}$$

$$I_{m,n} = \int \frac{u^m du}{(a^2 \pm u^2)^n} = \mp\frac{u^{m-1}}{2(n-1)(a^2 \pm u^2)^{n-1}} \pm \frac{m-1}{2(n-1)} I_{m-2,n-1}$$

$$I_n = \int \frac{u^n du}{\sqrt{a^2 - u^2}} = -\frac{1}{n}u^{n-1}\sqrt{a^2 - u^2} + \frac{n-1}{n}a^2 I_{n-2}$$

$$I_{m,n} = \int \frac{u^m du}{(u^2 \pm a^2)^n} = -\frac{u^{m-1}}{2(n-1)(u^2 \pm a^2)^{n-1}} + \frac{m-1}{2(n-1)} I_{m-2,n-1}$$

$$I_n=\int\frac{u^n du}{\sqrt{u^2\pm a^2}}=\frac{1}{n}u^{n-1}\sqrt{u^2\pm a^2}\mp\frac{n-1}{n}I_{n-2}$$

$$I_{m,n}=\int\frac{1}{u^n}\left(a^2\pm u^2\right)^m du=-\frac{1}{(n-1)u^{n-1}}\left(a^2\pm u^2\right)^m\pm\frac{2m}{n-1}I_{m-1,n-2}$$

$$I_{m,n}=\int\frac{1}{u^n}\left(u^2\pm a^2\right)^m du=-\frac{1}{(n-1)u^{n-1}}\left(u^2\pm a^2\right)^m+\frac{2m}{n-1}I_{m-1,n-2}$$

$$I_{m,n}=\int\frac{1}{u^n}\left(a^2\pm u^2\right)^m du=\frac{1}{(2m-n+1)u^{n-1}}\left(a^2\pm u^2\right)^m+\frac{(n-1)a^2}{2m-n+1}I_{m-1,n}$$

$$I_{m,n}=\int\frac{1}{u^n}\left(u^2\pm a^2\right)^m du=\frac{1}{(2m-n+1)u^{n-1}}\left(u^2\pm a^2\right)^m\pm\frac{(n-1)a^2}{2m-n+1}I_{m-1,n}$$

$$I_n=\int\frac{1}{u^n}\sqrt{u^2\pm a^2}\cdot du=\mp\frac{1}{(n-1)a^2u^{n-1}}\left(u^2\pm a^2\right)^{3/2}\mp\frac{n-4}{(n-1)a^2}I_{n-2}$$

$$I_{m,n}=\int\frac{1}{u^n}\left(a^2\pm u^2\right)^m du=-\frac{2(m+1)\left(a^2\pm u^2\right)^{m+1}}{(n-1)^2a^2u^{n-1}}\pm\frac{2(m+1)(2m-n+3)}{(n-1)^2a^2}I_{m,n-2}$$

$$I_n=\int\frac{1}{u^n}\sqrt{a^2-u^2}\cdot du=\frac{1}{(n-1)a^2u^{n-1}}\left(a^2-u^2\right)^{3/2}-\frac{n-4}{(n-1)a^2}I_{n-2}$$

$$I_{m,n}=\int u^n\left(u^2\pm a^2\right)^m du=\frac{1}{2m+n+1}u^{n-1}\left(u^2\pm a^2\right)^{m+1}\mp\frac{(n-1)a^2}{2m+n+1}I_{m,n-2}$$

$$I_n=\int u^n\sqrt{u^2\pm a^2}\cdot du=\frac{1}{n+2}u^{n-1}\left(u^2\pm a^2\right)^{3/2}\mp\frac{n-1}{n+2}a^2I_{n-2}$$

$$I_{m,n}=\int u^n\left(a^2\pm u^2\right)^m du=\pm\frac{1}{2m+n+1}u^{n-1}\left(a^2\pm u^2\right)^{m+1}\mp\frac{(n-1)a^2}{2m+n+1}I_{m,n-2}$$

$$I_n=\int u^n\sqrt{a^2-u^2}\cdot du=\frac{1}{n+2}u^{n-1}\left(a^2-u^2\right)^{3/2}-\frac{n-1}{n+1}a^2I_{n-2}$$

$$I_{m,n}=\int u^n\left(u^2\pm a^2\right)^m du=\frac{1}{2m+n+1}u^{n+1}\left(a^2\pm u^2\right)^m\pm\frac{2ma^2}{2m+n+1}I_{m-1,n}$$

$$I_{m,n}=\int u^n\left(a^2\pm u^2\right)^m du=\frac{1}{2m+n+1}u^{n+1}\left(u^2\pm a^2\right)^m+\frac{2ma^2}{2m+n+1}I_{m-1,n}$$

$$I_{m,n}=\int\frac{du}{u^n\left(a^2\pm u^2\right)^m}=\frac{1}{2(m-1)a^2u^{n-1}\left(a^2\pm u^2\right)^{m-1}}+\frac{2m+n-3}{2(m-1)a^2}I_{m-1,n}$$

$$I_n=\int\frac{du}{u^n\sqrt{a^2-u^2}}=-\frac{1}{(n-1)a^2u^{n-1}}\sqrt{a^2-u^2}+\frac{n-2}{(n-1)a^2}I_{n-2}$$

$$I_{m,n}=\int\frac{du}{u^n\left(u^2\pm a^2\right)^m}=\pm\frac{1}{2(m-1)a^2u^{n-1}\left(u^2\pm a^2\right)^{m-1}}\pm\frac{2m+n-3}{2(m-1)a^2}I_{m-1,n}$$

$$I_n=\int\frac{du}{u^n\sqrt{u^2\pm a^2}}=\mp\frac{1}{(n-1)a^2u^{n-1}}\sqrt{u^2\pm a^2}\mp\frac{n-2}{(n-1)a^2}I_{n-2}$$

$$I_{m,n}=\int\frac{du}{u^n\left(a^2\pm u^2\right)^m}=-\frac{1}{(n-1)a^2u^{n-1}\left(a^2\pm u^2\right)^{m-1}}\mp\frac{2m+n-3}{(n-1)a^2}I_{m,n-2}$$

$$I_{m,n}=\int\frac{du}{u^n\left(u^2\pm a^2\right)^m}=\mp\frac{1}{(n-1)a^2u^{n-1}\left(u^2\pm a^2\right)^{m-1}}\mp\frac{2m+n-3}{(n-1)a^2}I_{m,n-2}$$

$$I_n=\int\frac{dx}{(ax^2+bx+c)^n}=\frac{2ax+b}{(n-1)\left(4ac-b^2\right)\left(ax^2+bx+c\right)^{n-1}}+\frac{2(2n-3)a}{(n-1)\left(4ac-b^2\right)}I_{n-1}$$

$$I_m=\int\frac{du}{\left(u^n\pm a^n\right)^m}=\pm\frac{u}{n(m-1)a^n\left(u^n\pm a^n\right)^{m-1}}\pm\frac{n(m-1)-1}{n(m-1)a^n}I_{m-1}$$

$$I_m = \int \frac{du}{(a^n \pm u^n)^m} = \frac{u}{n(m-1)a^n (a^n \pm u^n)^{m-1}} + \frac{n(m-1)-1}{n(m-1)a^n} I_{m-1}$$

$$I_{m,n} = \int \frac{u^m du}{a^n \pm u^n} = \frac{1}{m-n+1} u^{m-n+1} \mp a^n I_{m-n,n}$$

$$I_{m,n} = \int \frac{du}{u^m (u^n \pm a^n)} = \mp \frac{1}{(m-1)u^{m-1}} \mp I_{m-n,n}$$

$$I_m = \int \frac{du}{u (u^n \pm a^n)^m} = \pm \frac{1}{n(m-1)a^n (u^n \pm a^n)^{m-1}} \pm \frac{1}{a^n} I_{m-1}$$

$$I_{r,m} = \int \frac{du}{u^r (u^n \pm a^n)^m} = \pm \frac{1}{n(m-1)a^n u^{r-1} (u^n \pm a^n)^{m-1}} \pm \frac{1}{a^n} \left(1 + \frac{r-1}{n(m-1)}\right) I_{r,m-1}$$

$$I_{r,m} = \int \frac{du}{u^r (a^n \pm u^n)^m} = \frac{1}{n(m-1)a^n u^{r-1} (a^n \pm u^n)^{m-1}} + \frac{1}{a^n} \left(1 + \frac{r-1}{n(m-1)}\right) I_{r,m-1}$$

$$I_{r,m} = \int \frac{du}{u^r (u^n \pm a^n)^m} = \mp \frac{1}{(r-1)a^n u^{r-1} (u^n \pm a^n)^{m-1}} \mp \frac{1}{a^n} \left(1 + n\frac{m-1}{r-1}\right) I_{r-n,m}$$

$$I_{r,m} = \int \frac{du}{u^r (a^n \pm u^n)^m} = -\frac{1}{(r-1)a^n u^{r-1} (a^n \pm u^n)^{m-1}} \mp \frac{1}{a^n} \left(1 + n\frac{m-1}{r-1}\right) I_{r-n,m}$$

$$I_{r,m} = \int \frac{u^r du}{(u^n \pm a^n)^m} = -\frac{u^{r-n+1}}{n(m-1)(u^n \pm a^n)^{m-1}} \mp \frac{r-n+1}{n(m-1)} I_{r-n,m-1}$$

$$I_{r,m}=\int\frac{u^r du}{(a^n\pm u^n)^m}=\mp\frac{u^{r-n+1}}{n(m-1)\,(a^n\pm u^n)^{m-1}}\pm\frac{r-n+1}{n(m-1)}I_{r-n,m-1}$$

$$I_{r,m}=\int\frac{u^r du}{(u^n\pm a^n)^m}=\pm\frac{u^{r+1}}{n(m-1)a^n\,(u^n\pm a^n)^{m-1}}\pm\frac{1}{a^n}\left(1-\frac{r+1}{n(m-1)}\right)I_{r,m-1}$$

$$I_{r,m}=\int\frac{u^r du}{(a^n\pm u^n)^m}=\frac{u^{r+1}}{n(m-1)a^n\,(a^n\pm u^n)^{m-1}}+\frac{1}{a^n}\left(1-\frac{r+1}{n(m-1)}\right)I_{r,m-1}$$

$$I_{r,m}=\int\frac{1}{u^r}\,(u^n\pm a^n)^m\,du=-\frac{1}{(r-1)x^{r-1}}\,(u^n\pm a^n)^m+\frac{mn}{r-1}I_{r-n,m-1}$$

$$I_{r,m}=\int\frac{1}{u^r}\,(a^n\pm u^n)^m\,du=-\frac{1}{(r-1)x^{r-1}}\,(a^n\pm u^n)^m\pm\frac{mn}{r-1}I_{r-n,m-1}$$

$$I_{r,m}=\int u^r\,(u^n\pm a^n)^m\,du=\frac{1}{mn+r+1}x^{r+1}\,(u^n\pm a^n)^m\pm\frac{mna^n}{mn+r+1}I_{r,m-1}$$

$$I_{r,m}=\int u^r\,(a^n\pm u^n)^m\,du=\frac{1}{mn+r+1}x^{r+1}\,(a^n\pm x^n)^m+\frac{mna^n}{mn+r+1}I_{r,m-1}$$

$$I_{r,m}=\int u^r\,(u^n\pm a^n)^m\,du=\frac{1}{mn+r+1}x^{r-n+1}\,(u^n\pm a^n)^{m+1}\mp\frac{r-n+1}{mn+r+1}a^nI_{r-n,m}$$

$$I_{r,m}=\int u^r\,(a^n\pm u^n)^m\,du=\pm\frac{1}{mn+r+1}x^{r-n+1}\,(a^n\pm x^n)^{m+1}\mp\frac{r-n+1}{mn+r+1}a^nI_{r-n,m}$$

$$I_n=\int\sin^n u\cdot du=-\frac{1}{n}\sin^{n-1}u\cdot\cos u+\frac{n-1}{n}I_{n-2}$$

$$I_n = \int \cos^n u \cdot du = \frac{1}{n} \cos^{n-1} u \cdot \sin u + \frac{n-1}{n} I_{n-2}$$

$$I_n = \int \sec^n u \cdot du = \frac{1}{n-1} \sec^{n-2} u \cdot \tan u + \frac{n-2}{n-1} I_{n-2} \quad \text{if "} n \neq 1 \text{"}$$

$$I_n = \int \csc^n u \cdot du = -\frac{1}{n-1} \csc^{n-2} u \cdot \cot u + \frac{n-2}{n-1} I_{n-2}$$

$$I_n = \int \tan^n u \cdot du = \frac{1}{n-1} \tan^{n-1} u - I_{n-2}$$

$$I_n = \int \cot^n u \cdot du = -\frac{1}{n-1} \cot^{n-1} u - I_{n-2}$$

$$I_{m,n} = \int \sin^n u \cdot \cos^n u \cdot du = \frac{1}{m+n} \sin^{m+1} u \cdot \cos^{n-1} u + \frac{n-1}{m+n} I_{m,n-2}$$

$$I_{m,n} = \int \sin^n u \cdot \cos^n u \cdot du = -\frac{1}{m+n} \sin^{m-1} u \cdot \cos^{n+1} u + \frac{m-1}{m+n} I_{m-2,n}$$

$$I_{m,n} = \int \frac{\sin^m u}{\cos^n u} \cdot du = \frac{\sin^{m-1} u}{(n-1)\cos^{n-1} u} - \frac{m-1}{n-1} I_{m-2,n-2}$$

$$I_{m,n} = \int \frac{\sin^m u}{\cos^n u} \cdot du = \frac{\sin^{m+1} u}{(n-1)\cos^{n-1} u} - \frac{m-n+2}{n-1} I_{m,n-2}$$

$$I_{m,n} = \int \frac{\sin^m u}{\cos^n u} \cdot du = -\frac{\sin^{m-1} u}{(m-n)\cos^{n-1} u} + \frac{m-1}{m-2} I_{m-2,n}$$

$$I_{m,n} = \int \frac{\cos^m u}{\sin^n u} \cdot du = -\frac{\cos^{m-1} u}{(n-1)\sin^{n-1} u} - \frac{m-1}{n-1} I_{m-2,n-2}$$

$$I_{m,n} = \int \frac{\cos^m u}{\sin^n u} \cdot du = -\frac{\cos^{m+1} u}{(n-1)\sin^{n-1} u} - \frac{m-n+2}{n-1} I_{m,n-2}$$

$$I_{m,n} = \int \frac{\cos^m u}{\sin^n u} \cdot du = \frac{\cos^{m-1} u}{(m-n)\sin^{n-1} u} + \frac{m-1}{m-2} I_{m-2,n}$$

$$I_n = \int \frac{\sin^n u}{\cos u} \cdot du = -\frac{1}{(n-1)} \sin^{n-1} u + I_{n-2}$$

$$I_n = \int \frac{\cos^n u}{\sin u} \cdot du = \frac{1}{(n-1)} \cos^{n-1} u + I_{n-2}$$

$$I_{m,n} = \int \frac{du}{\sin^m u \cdot \cos^n u} = \frac{1}{(n-1)\sin^{m-1} u \cdot \cos^{n-1} u} + \frac{m+n-2}{n-1} I_{m,n-2}$$

$$I_{m,n} = \int \frac{du}{\sin^m u \cdot \cos^n u} = -\frac{1}{(m-1)\sin^{m-1} u \cdot \cos^{n-1} u} + \frac{m+n-2}{m-1} I_{m-2,n}$$

$$I_n = \int \frac{du}{\sin^n u \cdot \cos u} = -\frac{1}{(n-1)\sin^{n-1} u} + I_{n-2}$$

$$I_n = \int \frac{du}{\sin u \cdot \cos^n u} = \frac{1}{(n-1)\cos^{n-1} u} + I_{n-2}$$

$$I_n = \int \cos nu \cdot \cos^n u \cdot du = \frac{1}{2n} \sin nu \cdot \cos^n u + \frac{1}{2} I_{n-1}$$

$$I_n = \int \sin nu \cdot cos^n u \cdot du = -\frac{1}{2n} \cos nu \cdot \cos^n u + \frac{1}{2} I_{n-1}$$

$$I_n = \int \cos nu \cdot sin^n u \cdot du = \frac{1}{2n} \sin nu \cdot \sin^n u - \frac{1}{2} \int \sin(n-1)u \cdot \sin^{n-1} u \cdot du$$

$$I_n = \int \sin nu \cdot sin^n u \cdot du = -\frac{1}{2n} \cos nu \cdot \sin^n u + \frac{1}{2} \int \cos(n-1)u \cdot \sin^{n-1} u \cdot du$$

$$I_{m,n} = \int \cos mu \cdot \cos^n u \cdot du = \frac{1}{m+n} \sin mu \cdot \cos^n u + \frac{n}{m+n} I_{m-1,n-1}$$

$$I_{m,n} = \int \sin mu \cdot \cos^n u \cdot du = -\frac{1}{m+n} \cos mu \cdot \cos^n u + \frac{n}{m+n} I_{m-1,n-1}$$

$$I_{m,n} = \int \cos mu \cdot \sin^n u \cdot du = \frac{1}{m+n} \sin mu \cdot \sin^n u - \frac{n}{m+n} \int \sin(m-1)u \cdot \sin^{n-1} u \cdot du$$

$$I_{m,n} = \int \sin mu \cdot \sin^n u \cdot du = -\frac{1}{m+n} \cos mu \cdot \sin^n u + \frac{n}{m+n} \int \cos(m-1)u \cdot \sin^{n-1} u \cdot du$$

$$I_n = \int \frac{\cos nu}{\cos^n u} \cdot du = -\frac{\sin(n-1)u}{(n-1)\cos^{n-1} u} + 2I_{n-1}$$

$$I_n = \int \frac{\sin nu}{\cos^n u} \cdot du = \frac{\cos(n-1)u}{(n-1)\cos^{n-1} u} + 2I_{n-1}$$

$$I_n = \int \frac{\sin nu}{\sin^n u} \cdot du = -\frac{\sin(n-1)u}{(n-1)\sin^{n-1} u} + 2\int \frac{\cos(n-1)u}{\sin^{n-1} u} \cdot du$$

$$I_n = \int \frac{\cos nu}{\sin^n u} \cdot du = -\frac{\cos(n-1)u}{(n-1)\sin^{n-1} u} - 2\int \frac{\sin(n-1)u}{\sin^{n-1} u} \cdot du$$

$$I_{m,n} = \int \frac{\cos mu}{\cos^n u} \cdot du = -\frac{\sin(m-1)u}{(n-1)\cos^{n-1} u} + \frac{m+n-2}{n-1} I_{m-1,n-1}$$

$$I_{m,n} = \int \frac{\sin mu}{\cos^n u} \cdot du = \frac{\cos(m-1)u}{(n-1)\cos^{n-1} u} + \frac{m+n-2}{n-1} I_{m-1,n-1}$$

$$I_{m,n} = \int \frac{\sin mu}{\sin^n u} \cdot du = -\frac{\sin(m-1)u}{(n-1)\sin^{n-1} u} + \frac{m+n-2}{n-1}\int \frac{\cos(m-1)u}{\sin^{n-1} u} \cdot du$$

$$I_{m,n} = \int \frac{\cos mu}{\sin^n u} \cdot du = -\frac{\cos(m-1)u}{(n-1)\sin^{n-1} u} - \frac{m+n-2}{n-1}\int \frac{\sin(m-1)u}{\sin^{n-1} u} \cdot du$$

$$I_n = \int (\sin u \pm \cos u)^n du = -\frac{1}{n}(\cos u \mp \sin u)(\sin u \pm \cos u)^{n-1} + 2\frac{n-1}{n} I_{n-2}$$

$$I_n = \int (\cos u \pm \sin u)^n du = \frac{1}{n}(\sin u \mp \cos u)(\cos u \pm \sin u)^{n-1} + 2\frac{n-1}{n} I_{n-2}$$

$$I_n = \int \frac{du}{(\sin u \pm \cos u)^n} = -\frac{\cos u \mp \sin u}{2(n-1)(\sin u \pm \cos u)^{n-1}} + \frac{n-2}{2(n-1)} I_{n-2}$$

$$I_n = \int \frac{du}{(\cos u \pm \sin u)^n} = \frac{\sin u \mp \cos u}{2(n-1)(\cos u \pm \sin u)^{n-1}} + \frac{n-2}{2(n-1)} I_{n-2}$$

$$I_n = \int (a \sin u + b \cos u)^n du = -\frac{1}{n}(a \cos u - b \sin u)(a \sin u + b \cos u)^{n-1} + \frac{n-1}{n}(a^2+b^2) I_{n-2}$$

$$I_n = \int \frac{du}{(a \sin u + b \cos u)^n} = -\frac{a \cos u - b \sin u}{(n-1)(a^2+b^2)(a \sin u + b \cos u)^{n-1}} + \frac{n-2}{(n-1)(a^2+b^2)} I_{n-2}$$

$$I_n = \int u^n \sin au \cdot du = -\frac{1}{a} u^n \cos au + \frac{n}{a} \int u^{n-1} \cos au \cdot du$$

$$I_n = \int u^n \cos au \cdot du = \frac{1}{a} u^n \sin au - \frac{n}{a} \int u^{n-1} \sin au \cdot du$$

$$I_n = \int \frac{1}{u^n} \sin au \cdot du = -\frac{1}{(n-1)u^{n-1}} \sin au + \frac{a}{n-1} \int \frac{1}{u^{n-1}} \cos au \cdot du$$

$$I_n = \int \frac{1}{u^n} \cos au \cdot du = -\frac{1}{(n-1)u^{n-1}} \cos au - \frac{a}{n-1} \int \frac{1}{u^{n-1}} \sin au \cdot du$$

$$I_n = \int u \sin^n au \cdot du = \frac{1}{a^2 n^2}(\sin au - nau \cos au) \sin^{n-1} au + \frac{n-1}{n} I_{n-2}$$

$$I_n = \int u \cos^n au \cdot du = \frac{1}{a^2 n^2}(\cos au - nau \sin au) \cos^{n-1} au + \frac{n-1}{n} I_{n-2}$$

$$I_n = \int u \sec^n au \cdot du = \frac{u}{(n-1)a} \sec^{n-2} au \cdot \tan au - \frac{1}{(n-1)(n-2)a^2} \sec^{n-2} au + \frac{n-2}{n-1} I_{n-2}$$

$$I_n = \int u \csc^n au \cdot du = -\frac{u}{(n-1)a} \csc^{n-2} au \cdot \cot au - \frac{1}{(n-1)(n-2)a^2} \csc^{n-2} au + \frac{n-2}{n-1} I_{n-2}$$

$$I_n = \int (\arcsin u)^n du = \left(u \arcsin u + n\sqrt{1-u^2}\right) (\arcsin u)^{n-1} - n(n-1) I_{n-2}$$

$$I_n = \int (\arccos u)^n du = \left(u \arccos u - n\sqrt{1-u^2}\right) (\arccos u)^{n-1} - n(n-1) I_{n-2}$$

$$I_n = \int \frac{du}{(\arcsin u)^n} = \frac{u \arcsin u - (n-2)\sqrt{1-x^2}}{(n-1)(n-2)(\arcsin u)^{n-1}} - \frac{1}{(n-1)(n-2)} I_{n-2}$$

$$I_n = \int \frac{du}{(\arccos u)^n} = \frac{u \arccos u + (n-2)\sqrt{1-x^2}}{(n-1)(n-2)(\arccos u)^{n-1}} - \frac{1}{(n-1)(n-2)} I_{n-2}$$

$$I_n = \int u^n \arcsin u \cdot du = \frac{1}{n+1} u^{n+1} \arcsin u - \frac{1}{n+1} \int \frac{u^{n+1} du}{\sqrt{1-u^2}}$$

$$I_n = \int u^n \arccos u \cdot du = \frac{1}{n+1} u^{n+1} \arccos u + \frac{1}{n+1} \int \frac{u^{n+1} du}{\sqrt{1-u^2}}$$

$$I_n = \int u^n \arcsin u \cdot du = \frac{1}{n+1} u^n \left(u \arcsin u + \sqrt{1-u^2}\right) - \frac{n}{n+1} \int u^{n-1} \sqrt{1-u^2} \cdot du$$

$$I_n=\int u^n \arccos u\cdot du=\frac{1}{n+1}u^n\left(u\arccos u-\sqrt{1-u^2}\right)+\frac{n}{n+1}\int u^{n-1}\sqrt{1-u^2}\cdot du$$

$$I_n=\int \frac{1}{u^n}\arcsin u\cdot du=-\frac{1}{(n-1)u^{n-1}}\arcsin u+\frac{1}{n-1}\int\frac{du}{u^{n-1}\sqrt{1-u^2}}$$

$$I_n=\int \frac{1}{u^n}\arccos u\cdot du=-\frac{1}{(n-1)u^{n-1}}\arccos u-\frac{1}{n-1}\int\frac{du}{u^{n-1}\sqrt{1-u^2}}$$

$$I_n=\int u^n\arctan u\cdot du=\frac{1}{n+1}u^{n+1}\arctan u-\frac{1}{n+1}\int\frac{u^{n+1}du}{1+u^2}$$

$$I_n=\int u^n\operatorname{arccot} u\cdot du2=\frac{1}{n+1}u^{n+1}\operatorname{arccot} u+\frac{1}{n+1}\int\frac{u^{n+1}du}{1+u^2}$$

$$I_n=\int \frac{1}{u^n}\arctan u\cdot du=-\frac{1}{(n-1)u^{n-1}}\arctan u+\frac{1}{n-1}\int\frac{du}{u^{n-1}(1+u^2)}$$

$$I_n=\int \frac{1}{u^n}\operatorname{arccot} u\cdot du=-\frac{1}{(n-1)u^{n-1}}\operatorname{arccot} u-\frac{1}{n-1}\int\frac{du}{u^{n-1}(1+u^2)}$$

$$I_n=\int u^n e^{au}du=\frac{1}{a}u^n e^{au}-\frac{n}{a}I_{n-1}$$

$$I_n=\int \frac{1}{u^n}e^{au}du=-\frac{1}{(n-1)u^{n-1}}e^{au}-\frac{a}{n-1}I_{n-1}$$

$$I_n=\int u^n e^{au^2}du=\frac{1}{2a}u^{n-1}e^{au^2}-\frac{n-1}{2a}I_{n-2}$$

$$I_n = \int e^{au} \sin^n bu \cdot du = \frac{1}{a^2+b^2n^2} e^{au}(a \sin bu - nb \cos bu) \sin^{n-1} bu + \frac{n(n-1)b^2}{a^2+b^2n^2} I_{n-2}$$

$$I_n = \int e^{au} \cos^n bu \cdot du = \frac{1}{a^2+b^2n^2} e^{au}(a \cos bu - nb \sin bu) \cos^{n-1} bu + \frac{n(n-1)b^2}{a^2+b^2n^2} I_{n-2}$$

$$I_n = \int x^n e^{-x} dx = -e^{-x} \left[\sum_{k=0}^{n} \frac{d^k (x^n)}{dx^k} \right]; \frac{d^0 (x^n)}{dx^0} = x^n; \frac{d^n (x^n)}{dx^n} = n!$$

$$I_n = \int \ln^n u \cdot du = u \ln^n u - n I_{n-1}$$

$$I_n = \int \frac{du}{\ln^n u} = -\frac{u}{(n-1)\ln^{n-1} u} + \frac{1}{n-1} I_{n-1}$$

$$I_n = \int u^m \ln^n u \cdot du = \frac{1}{m+1} u^{m+1} \ln^n u - \frac{n}{m+1} I_{n-1}$$

$$I_n = \int \frac{1}{u^m} \ln^n u \cdot du = -\frac{1}{(m-1)u^{m-1}} \ln^n u + \frac{n}{m-1} I_{n-1}$$

$$I_n = \int \frac{u^m}{\ln^n u} \cdot du = -\frac{u^{m+1}}{(n-1)\ln^{n-1} u} + \frac{m+1}{n-1} I_{n-1}$$

$$I_n = \int \frac{du}{u^m \ln^n u} = -\frac{1}{(n-1)u^{m-1}\ln^{n-1} u} - \frac{m-1}{n-1} I_{n-1}$$

$$I_n = \int \sinh^n u \cdot du = \frac{1}{n} \sinh^{n-1} u \cdot \cosh u - \frac{n-1}{n} I_{n-2}$$

$$I_n = \int \cosh^n u \cdot du = \frac{1}{n} \cosh^{n-1} u \cdot \sinh u + \frac{n-1}{n} I_{n-2}$$

$$I_n = \int \mathrm{sech}^n \mathrm{u} \cdot du = \frac{1}{n-1} \mathrm{sech}^{n-2} \mathrm{u} \cdot \tanh u + \frac{n-2}{n-1} I_{n-2}$$

$$I_n = \int \mathrm{csch}^n \mathrm{u} \cdot du = -\frac{1}{n-1} \mathrm{csch}^{n-2} \mathrm{u} \cdot \coth u - \frac{n-2}{n-1} I_{n-2}$$

$$I_n = \int \tanh^n u \cdot du = -\frac{1}{n-1} \tanh^{n-1} u + I_{n-2}$$

$$I_n = \int \coth^n u \cdot du = -\frac{1}{n-1} \coth^{n-1} u + I_{n-2}$$

$$I_{m,n} = \int \sinh^m u \cdot \cosh^n u \cdot du = \frac{1}{m+n} \sinh^{m-1} u \cdot \cosh^{n+1} u - \frac{m-1}{m+n} I_{m-2,n}$$

$$I_{m,n} = \int \sinh^m u \cdot \cosh^n u \cdot du = \frac{1}{m+n} \sinh^{m+1} u \cdot \cosh^{n-1} u + \frac{n-1}{m+n} I_{m,n-2}$$

$$I_{m,n} = \int \frac{\sinh^m u}{\cosh^n u} \cdot du = -\frac{\sinh^{m-1} u}{(n-1)\cosh^{n-1} u} + \frac{m-1}{n-1} I_{m-2,n-2}$$

$$I_{m,n} = \int \frac{\cosh^m u}{\sinh^n u} \cdot du = -\frac{\cosh^{m-1} u}{(n-1)\sinh^{n-1} u} + \frac{m-1}{n-1} I_{m-2,n-2}$$

$$I_n = \int u^n \sinh au \cdot du = \frac{1}{a} u^n \cosh au - \frac{n}{a} \int u^{n-1} \cosh au \cdot du$$

$$I_n = \int u^n \cosh au \cdot du = \frac{1}{a} u^n \sinh au - \frac{n}{a} \int u^{n-1} \sinh au \cdot du$$

CONCLUSIONES

Si bien es cierto, la econometría, en amplio espectro, se erige a propósito de la aplicación de la estadística y la probabilidad, como normas lógicas generales, no es menos cierto y como ha quedado reflejado en el presente trabajo, que existen otros modelos matemáticos, que permiten, predecir y anticipar variables mutables y cambiantes, propias de la fenomenología económica, que arrojan resultados deterministas o que permiten, en aproximación, describir y sostener el componente empírico de la ciencia econométrica.

Considero que la econometría, sea teórica o aplicada, o de la naturaleza que fuere, debe incorporar todo modelo matemático posible que coadyuve a su aplicación, esto es, dotar de soporte empírico, a la data económica, pues el fin en sí mismo, es obtener resultados numéricos que desgranen, en órbita cuantitativa, la teoría económica en su dimensión cualitativa.

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