Ciencia Latina Revista Científica Multidisciplinar, Ciudad de México, México.
ISSN 2707-2207 / ISSN 2707-2215 (en línea), julio-agosto 2025,
Volumen 9, Número 4.

https://doi.org/10.37811/cl_rcm.v9i2

SUPERGRAVEDAD CUÁNTICA RELATIVISTA. TEORIZACIÓN INICIAL

RELATIVISTIC QUANTUM SUPERGRAVITY. INITIAL THEORIZATION

Manuel Ignacio Albuja Bustamante
Investigador independiente

DOI: https://doi.org/10.37811/cl_rcm.v9i4.18474

Supergravedad cuántica relativista. Teorización inicial

Manuel Ignacio Albuja Bustamante[1]
ignaciomanuelalbujabustamante@gmail.com
Investigador independiente

RESUMEN

La supergravedad cuántica, en contraposición a la gravedad cuántica, es un estado del espacio – tiempo cuántico, en el que, una partícula oscura o una partícula estrella, deforman hipergeométricamente el referido espacio, provocando agujeros negros cuánticos, a razón de su aniquilación o colapso gravitacional, o en su defecto o simultáneamente, provocando D – dimensiones, en los que es perfectamente posible la transformación y desplazamiento de materia y energía, de un punto a otro, en dimensiones distintas, es decir, por fuera de la dimensión en $\mathbb{R}^4$. En este punto, la dimensión del tiempo es maleable a razón de la supergravedad. La dualidad holográfica, es una de las principales características de este fenómeno, en la medida en que, si bien se tratan de desdoblamientos espaciales, las leyes de la física no son las mismas, lo que al contrario ocurre con la gravedad cuántica. En este artículo, me propongo sentar bases formales de la supergravedad cuántica, la misma que, puede ser endógena o exógena, al tenor de las premisas fundamentales contenidas en la Teoría Cuántica de Campos Relativistas o Curvos, formulada por este autor en trabajos anteriores. Este trabajo, al igual que los anteriores y paralelos, se conciben como un intento de unificación.

Palabras clave: supergravedad cuántica, partícula estrella, partícula oscura, supermembranas, supersimetría de gauge, supersimetría de Yang – Mills

[1] Autor principal
Correspondencia: ignaciomanuelalbujabustamante@gmail.com

Relativistic quantum supergravity. Initial theorization

ABSTRACT

Quantum supergravity, as opposed to quantum gravity, is a state of quantum space-time, in which a dark particle or a star particle hypergeometrically deforms the aforementioned space, causing quantum black holes, due to their annihilation or gravitational collapse, or failing that or simultaneously, causing D-dimensions, in which the transformation and displacement of matter and energy is perfectly possible. from one point to another, in different dimensions, that is, outside the dimension in $\mathbb{R}^4$. At this point, the dimension of time is malleable due to supergravity. Holographic duality is one of the main characteristics of this phenomenon, insofar as, although they are spatial splittings, the laws of physics are not the same, which is the opposite of quantum gravity. In this article, I propose to lay the formal foundations of quantum supergravity, which can be endogenous or exogenous, according to the fundamental premises contained in the Quantum Theory of Relativistic or Curved Fields, formulated by this author in previous works. This work, like the previous and parallel ones, is conceived as an attempt at unification.

Keywords: quantum supergravity, star particle, dark particle, supermembranes, gauge supersymmetry, yang-mills supersymmetry.

Artículo recibido 07 mayo 2025
Aceptado para publicación: 11 junio 2025

INTRODUCCIÓN.

Existe una diferencia sustancial entre gravedad cuántica y supergravedad cuántica, pues la primera (GC) consiste esencialmente en la deformación del espacio – tiempo cuántico, a propósito de las interacciones y simetría fija o corregida de una partícula supermasiva que a razón de su masa y energía, esto es, de su centro de materia – energía, curva geométricamente el plano cuántico en el que se propaga vectorialmente y lo torsiona o tuerce, cuya matematización es puramente tensorial y geométrica, esto, a raíz de un espacio de Hilbert – Einstein y una superficie de Riemann, sin que por esto, se produzcan pluridimensiones de gauge. Asimismo, la gravedad cuántica (GC), se desarrolla o cumple los parámetros anteriores, cuando una hiperpartícula, con o sin masa, se aproxima, iguala o supera la velocidad de la luz (momentum), por lo que, en cualquiera de los casos anteriores, el resultado es la formación de la curvatura de Dirac del espacio – tiempo cuántico en el que interactúa, lo que sería el caso del taquión, partícula hipotética que cumple esta descripción. En cuanto a la gravedad cuántica se refiere, la dualidad holográfica se tiene por inexistente, lo que implica, que para el espacio – tiempo cuántico deformado geométricamente, operan las mismas leyes de la física cuántica, pues se trata de la misma dimensión en $\mathbb{R}^4$. Ahora bien, la supergravedad cuántica, en contrario a la gravedad cuántica, comporta las siguientes características: 1) el espacio – tiempo cuántico se deforma hipergeométricamente; 2) comporta la existencia de D – dimensiones, esto es, supermembranas y por ende, superespacios; 3) comporta la existencia de agujeros negros cuánticos masivos o supermasivos, según sea el caso; provocados por aniquilación entre dos partículas supermasivas, una partícula supermasiva y otra partícula repercutida o por colisión, según las categorías antes referidas; 4) comporta la existencia de supersimetrías de gauge en invariancia o covariancia; 5) comporta la existencia de dualidad holográfica; 6) la deformación del espacio – tiempo cuántico opera por una partícula oscura o una partícula estrella, siendo la primera, aquella que es extremadamente densa pero su energía – momentum es inferior, en tanto que la segunda, es aquella cuya masa es extremadamente densa al igual que su energía – momentum, por lo que, su colapso o aniquilación, genera radiación. Sin perjuicio de lo anterior, ambos tipos de gravedad, surgen de forma endógena o exógena, según corresponda. En este artículo, abordaremos específicamente la supergravedad cuántica.

RESULTADOS Y DISCUSIÓN.

Supergravedad cuántica. Modelo Matemático.

Cálculos preliminares para campos superplanckianos.

$$m^2 = \frac{J}{\alpha'}$$

$$s = -(p_1 + p_2)^2\, , t = -(p_2 + p_3)^2\, , u = -(p_1 + p_3)^2$$

$$A_J(s,t) \sim \frac{(-s)^J}{t - M^2}$$

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))}$$

$$\alpha(s) = \alpha(0) + \alpha' s$$

$$A(s,t) = -\sum_{n=0}^{\infty} \frac{(\alpha(s)+1)\ldots(\alpha(s)+n)}{n!} \frac{1}{\alpha(t) - n}$$

$$\frac{1}{M_{\text{Planck}}^4} \int_0^{\Lambda} dE E^3 \sim \frac{\Lambda^4}{M_{\text{Planck}}^4}$$

Partícula Supermasiva. Comportamiento de campo.

$$S = m \int_{s_i}^{s_f} ds = m \int_{\tau_0}^{\tau_1} d\tau \sqrt{-\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$$

$$p_\mu = -\frac{\delta L}{\delta \dot{x}^\mu} = \frac{m\dot{x}_\mu}{\sqrt{-\dot{x}^2}}$$

$$\partial_\tau \left(\frac{m\dot{x}_\mu}{\sqrt{-\dot{x}^2}} \right) = 0$$

$$p^2 + m^2 = 0$$

$$H_{can} = \frac{\partial L}{\partial \dot{x}^\mu} \dot{x}^\mu - L$$

$$H = \frac{N}{2m}(p^2 + m^2)$$

$$\dot{x}^\mu = \{x^\mu, H\} = \frac{N}{m} p^\mu = \frac{N\dot{x}^\mu}{\sqrt{-\dot{x}^2}}$$

$$\dot{x}^2 = -N^2$$

$$S=-\frac{1}{2}\int d\tau e(\tau)(e^{-2}(\tau)(\dot{x}^{\mu})^{2}-m^{2})$$

$$S=-\frac{1}{2}\int d\tau\sqrt{\det g_{\tau\tau}}(g^{\tau\tau}\partial_{\tau}x\cdot\partial_{\tau}x-m^{2})$$

$$\delta x^{\mu}(\tau)=x^{\mu}(\tau+\xi(\tau))-x^{\mu}(\tau)=\xi(\tau)\dot{x}^{\mu}+\mathcal{O}(\xi^{2})$$

$$\delta S=\frac{1}{2}\int d\tau\left(\frac{1}{e^{2}(\tau)}(\dot{x}^{\mu})^{2}+m^{2}\right)\delta e(\tau)$$

$$e^{-2}x^{2}+m^{2}=0\ \rightarrow\ e=\frac{1}{m}\sqrt{-\dot{x}^{2}}$$

$$\delta S=\frac{1}{2}\int d\tau e(\tau)(e^{-2}(\tau)2\dot{x}^{\mu})\partial_{\tau}\delta x^{\mu}$$

$$\partial_{\tau}(e^{-1}\dot{x}^{\mu})=0$$

$$\langle x\mid x'\rangle=N\int_{x(0)=x}^{x(1)=x'}DeDx^{\mu}\exp\left(\frac{1}{2}\int_{0}^{1}\left(\frac{1}{e}(\dot{x}^{\mu})^{2}-em^{2}\right)d\tau\right)$$

$$\delta e=\partial_{\tau}(\xi e)$$

$$L=\int_{0}^{1}d\tau\sqrt{\det g_{\tau\tau}}=\int_{0}^{1}d\tau e$$

$$\langle x\mid x'\rangle=N\int_{0}^{\infty}dL\int_{x(0)=x}^{x(1)=x'}Dx^{\mu}\exp\left(-\frac{1}{2}\int_{0}^{1}\left(\frac{1}{L}\dot{x}^{2}+Lm^{2}\right)d\tau\right)$$

$$x^{\mu}(\tau)=x^{\mu}+(x'^{\mu}-x^{\mu})\tau+\delta x^{\mu}(\tau),$$

$$\|\delta x\|^{2}=\int_{0}^{1}d\tau e(\delta x^{\mu})^{2}=L\int_{0}^{1}d\tau(\delta x^{\mu})^{2}$$

$$Dx^{\mu}\sim\prod_{\tau}\sqrt{L}d\delta x^{\mu}(\tau)$$

$$\langle x\mid x'\rangle=N\int_{0}^{\infty}dL\int\prod_{\tau}\sqrt{L}d\delta x^{\mu}(\tau)e^{-\frac{(x'-x)^{2}}{2L}-m^{2}L/2}e^{-\frac{1}{2L}\int_{0}^{1}(\delta x^{\mu})^{2}}$$

$$\int\prod_{\tau}\sqrt{L}d\delta x^{\mu}(\tau)e^{-\frac{1}{L}\int_{0}^{1}(\delta\dot{x}^{\mu})^{2}}\sim\left(\det\left(-\frac{1}{L}\partial_{\tau}^{2}\right)\right)^{-\frac{D}{2}}$$

$$-\frac{1}{L}\partial_{\tau}^{2}\psi(\tau)=\lambda\psi(\tau)$$

$$\psi_n(\tau) = C_n \sin(n\pi\tau), \lambda_n = \frac{n^2}{L}, n = 1,2, \ldots$$

$$\det\left(-\frac{1}{L}\partial_\tau^2\right) = \prod_{n=1}^{\infty} \frac{n^2}{L}$$

$$\prod_{n=1}^{\infty} L^{-1} = L^{-\zeta(0)} = L^{1/2}, \prod_{n=1}^{\infty} n^a = e^{-a\zeta'(0)} = (2\pi)^{a/2}$$

$$\langle x \mid x' \rangle = \frac{1}{2(2\pi)^{D/2}} \int_0^\infty dL L^{-\frac{D}{2}} e^{-\frac{(x'-x)^2}{2L} - m^2 L/2} =$$

$$= \frac{1}{(2\pi)^{D/2}} \left(\frac{|x-x'|}{m}\right)^{(2-D)/2} K_{(D-2)/2}(m|x-x'|)$$

$$|p\rangle = \int d^D x e^{ip\cdot x} |x\rangle$$

$$\langle p \mid p' \rangle = \int d^D x e^{-ip\cdot x} \int d^D x' e^{ip'\cdot x'} \langle x \mid x' \rangle$$

$$= \frac{1}{2} \int d^D x' e^{i(p'-p)\cdot x'} \int_0^\infty dL e^{-\frac{L}{2}(p^2+m^2)}$$

$$= (2\pi)^D \delta(p-p') \frac{1}{p^2+m^2}$$

Campos cuánticos relativistas o curvos. Propagadores, operadores, números fantasma, supersimetrías, antisimetrías, gravedad supermembranas y superespacios y pluridimensiones en un campo de Hilbert – Einstein. Métrica de Nambu – Goto.

$$S_{NG} = -T \int dA$$

$$ds^2 = G_{\mu\nu}(X) dX^\mu dX^\nu = G_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^i} \frac{\partial X^\nu}{\partial \xi^j} d\xi^i d\xi^j = G_{ij} d\xi^i d\xi^j$$

$$G_{ij} = G_{\mu\nu} \partial_i X^\mu \partial_j X^\nu$$

$$S_{NG} = -T \int \sqrt{-\det G_{ij}} d^2\xi = -T \int \sqrt{(\dot{X} \cdot X')^2 - (\dot{X}^2)(X'^2)} d^2\xi$$

$$\partial_\tau \left(\frac{\delta L}{\delta \dot{X}^\mu}\right) + \partial_\sigma \left(\frac{\delta L}{\delta X^\mu}\right) = 0$$

$$X^\mu(\sigma + \bar{\sigma}) = X^\mu(\sigma)$$

- Neumann :

$$\left.\frac{\delta L}{\delta X'^\mu}\right|_{\sigma=0,\bar{\sigma}} = 0$$

- Dirichlet :

$$\left.\frac{\delta L}{\delta \dot{X}^{\mu}}\right|_{\sigma=0,\bar{\sigma}} = 0$$

$$\Pi^{\mu} = \frac{\delta L}{\delta \dot{X}^{\mu}} = -T\frac{(\dot{X}\cdot X')X'^{\mu} - (X')^{2}\dot{X}^{\mu}}{[(X'\cdot\dot{X})^{2} - (\dot{X})^{2}(X')^{2}]^{1/2}}$$

$$\Pi\cdot X' = 0\,,\Pi^{2} + T^{2}X'^{2} = 0$$

$$H = \int_{0}^{\bar{\sigma}} d\sigma(\dot{X}\cdot\Pi - L)$$

$$S_{P} = -\frac{T}{2}\int d^{2}\xi\sqrt{-\mathrm{det}g}\,g^{\alpha\beta}\partial_{\alpha}X^{\mu}\partial_{\beta}X^{\nu}\eta_{\mu\nu}$$

$$T_{\alpha\beta} \equiv -\frac{2}{T}\frac{1}{\sqrt{-\mathrm{det}g}}\frac{\delta S_{P}}{\delta g^{\alpha\beta}} = \partial_{\alpha}X\cdot\partial_{\beta}X - \frac{1}{2}g_{\alpha\beta}g^{\gamma\delta}\partial_{\gamma}X\cdot\partial_{\delta}X$$

$$g_{\alpha\beta} = \partial_{\alpha}X\cdot\partial_{\beta}X$$

$$\frac{1}{\sqrt{-\mathrm{det}g}}\partial_{\alpha}\left(\sqrt{-\mathrm{det}g}\,g^{\alpha\beta}\partial_{\beta}X^{\mu}\right) = 0$$

$$\lambda_{1}\int\sqrt{-\mathrm{det}g}$$

$$\lambda_{2}\int\sqrt{-\mathrm{det}g}R^{(2)}$$

- Poincaré:

$$\delta X^{\mu} = \omega^{\mu}_{\nu}X^{\nu} + \alpha^{\mu}\,,\delta g_{\alpha\beta} = 0$$

$$\delta g_{\alpha\beta} = \xi^{\gamma}\partial_{\gamma}g_{\alpha\beta} + \partial_{\alpha}\xi^{\gamma}g_{\beta\gamma} + \partial_{\beta}\xi^{\gamma}g_{\alpha\gamma} = \nabla_{\alpha}\xi_{\beta} + \nabla_{\beta}\xi_{\alpha}$$

$$\delta X^{\mu} = \xi^{\alpha}\partial_{\alpha}X^{\mu}$$

$$\delta(\sqrt{-\mathrm{det}g}) = \partial_{\alpha}\left(\xi^{\alpha}\sqrt{-\mathrm{det}g}\right)$$

- Weyl:

$$\delta X^{\mu} = 0\,,\delta g_{\alpha\beta} = 2\Lambda g_{\alpha\beta}$$

$$\delta g_{\alpha\beta} = 2\Lambda(x)g_{\alpha\beta}\,,\delta\phi^{i} = d_{i}\Lambda(x)\phi^{i}$$

$$0 = \delta S = \int d^{2}\xi\left[2\frac{\delta S}{\delta g^{\alpha\beta}}g^{\alpha\beta} + \sum_{i} d_{i}\frac{\delta S}{\delta\phi_{i}}\phi_{i}\right]\Lambda$$

$$T^{\alpha}_{\alpha} \sim \frac{\delta S}{\delta g^{\alpha\beta}} g^{\alpha\beta} = 0$$

$$g_{\alpha\beta} = e^{2\Lambda(\xi)}\eta_{\alpha\beta}$$

$$\xi_{+} = \tau + \sigma\,, \xi_{-} = \tau - \sigma$$

$$ds^2 = -d\xi_{+}d\xi_{-}$$

$$g_{++} = g_{--} = 0\,, g_{+-} = g_{-+} = -\frac{1}{2}$$

$$\partial_{\pm} = \frac{1}{2}(\partial_{\tau} \pm \partial_{\sigma})$$

- Polyakov:

$$S_P \sim T\int d^2\xi \partial_{+}X^{\mu}\partial_{-}X^{\nu}\eta_{\mu\nu}$$

$$\xi_{+} \longrightarrow f(\xi_{+})\,, \xi_{-} \longrightarrow g(\xi_{-})$$

$$\frac{d(d+1)}{2} - d - 1$$

$$\delta S = T\int d^2\xi\big(\delta X^{\mu}\partial_{+}\partial_{-}X_{\mu}\big) - T\int_{\tau_0}^{\tau_1} d\tau X'_{\mu}\delta X^{\mu}$$

$$X'^{\mu}|_{\sigma=0,\bar{\sigma}} = 0$$

$$\partial_{+}\partial_{-}X^{\mu} = 0$$

$$T_{\alpha\beta} = 0$$

$$T_{10} = T_{01} = \frac{1}{2}\dot{X}\cdot X' = 0\,, T_{00} = T_{11} = \frac{1}{4}(\dot{X}^2 + X'^2) = 0$$

$$(\dot{X} \pm X')^2 = 0$$

$$T_{++} = \frac{1}{2}\partial_{+}X\cdot\partial_{+}X\,, T_{--} = \frac{1}{2}\partial_{-}X\cdot\partial_{-}X\,, T_{+-} = T_{-+} = 0$$

$$\partial_{-}T_{++} + \partial_{+}T_{-+} = \partial_{+}T_{--} + \partial_{-}T_{+-} = 0$$

$$\partial_{-}T_{++} = \partial_{+}T_{--} = 0$$

$$Q_f = \int_0^{\bar{\sigma}} f(\xi^{+})T_{++}(\xi^{+})$$

$$0 = \int d\sigma\partial_{-}(f(\xi^{+})T_{++}) = \partial_{\tau}Q_f + f(\xi^{+})T_{++}|_0^{\bar{\sigma}}$$

$$P_{\mu}^{\alpha} = -T\sqrt{\det g}\, g^{\alpha\beta}\partial_{\beta}X_{\mu}$$
$$J_{\mu\nu}^{\alpha} = -T\sqrt{\det g}\, g^{\alpha\beta}\left(X_{\mu}\partial_{\beta}X_{\nu} - X_{\nu}\partial_{\beta}X_{\mu}\right)$$

$$P_{\mu} = \int_0^{\bar{\sigma}} d\sigma P_{\mu}^{\tau}\,, J_{\mu\nu} = \int_0^{\bar{\sigma}} d\sigma J_{\mu\nu}^{\tau}$$

$$\frac{\partial P_{\mu}}{\partial \tau} = T\int_0^{\bar{\sigma}} d\sigma \partial_{\tau}^2 X_{\mu} = T\int_0^{\bar{\sigma}} d\sigma \partial_{\sigma}^2 X_{\mu} = T\left(\partial_{\sigma}X_{\mu}(\sigma = \bar{\sigma}) - \partial_{\sigma}X_{\mu}(\sigma = 0)\right)$$

Expansiones oscilatorias.

$$\partial_+\partial_- X^{\mu} = 0$$

$$X^{\mu}(\tau, \sigma + 2\pi) = X^{\mu}(\tau, \sigma)$$

$$X^{\mu}(\tau, \sigma) = X_L^{\mu}(\tau + \sigma) + X_R^{\mu}(\tau - \sigma)$$

$$X_L^{\mu}(\tau + \sigma) = \frac{x^{\mu}}{2} + \frac{p^{\mu}}{4\pi T}(\tau + \sigma) + \frac{i}{\sqrt{4\pi T}}\sum_{k\neq 0}\frac{\bar{\alpha}_k^{\mu}}{k}e^{-ik(\tau+\sigma)}$$

$$X_R^{\mu}(\tau - \sigma) = \frac{x^{\mu}}{2} + \frac{p^{\mu}}{4\pi T}(\tau - \sigma) + \frac{i}{\sqrt{4\pi T}}\sum_{k\neq 0}\frac{\alpha_k^{\mu}}{k}e^{-ik(\tau-\sigma)}$$

$$\left(\alpha_k^{\mu}\right)^* = \alpha_{-k}^{\mu} \text{ and } \left(\bar{\alpha}_k^{\mu}\right)^* = \bar{\alpha}_{-k}^{\mu}$$

$$\partial_- X_R^{\mu} = \frac{1}{\sqrt{4\pi T}}\sum_{k\in\mathbb{Z}} \alpha_k^{\mu} e^{-ik(\tau-\sigma)},$$
$$\partial_+ X_L^{\mu} = \frac{1}{\sqrt{4\pi T}}\sum_{k\in\mathbb{Z}} \bar{\alpha}_k^{\mu} e^{-ik(\tau+\sigma)}.$$

$$X'^{\mu}(\tau, \sigma)|_{\sigma=0,\pi} = 0$$

$$X'^{\mu}|_{\sigma=0} = \frac{p^{\mu} - \bar{p}^{\mu}}{\sqrt{4\pi T}} + \frac{1}{\sqrt{4\pi T}}\sum_{k\neq 0} e^{ik\tau}\left(\bar{\alpha}_k^{\mu} - \alpha_k^{\mu}\right)$$

$$p^{\mu} = \bar{p}^{\mu} \text{ and } \alpha_k^{\mu} = \bar{\alpha}_k^{\mu}$$

$$X^{\mu}(\tau, \sigma) = x^{\mu} + \frac{p^{\mu}\tau}{\pi T} + \frac{i}{\sqrt{\pi T}}\sum_{k\neq 0}\frac{\alpha_k^{\mu}}{k}e^{-ik\tau}\cos(k\sigma)$$

$$\partial_{\pm} X^{\mu} = \frac{1}{\sqrt{4\pi T}}\sum_{k\in\mathbb{Z}} \alpha_k^{\mu} e^{-ik(\tau\pm\sigma)}$$

$$X_{CM}^{\mu} \equiv \frac{1}{\bar{\sigma}}\int_0^{\bar{\sigma}} d\sigma X^{\mu}(\tau, \sigma) = x^{\mu} + \frac{p^{\mu}\tau}{\pi T}$$

$$p^{\mu}_{CM} = T\int_0^{\bar{\sigma}} d\sigma \dot{X}^{\mu} = \frac{T}{\sqrt{4\pi T}}\int d\sigma \sum_k (\alpha^{\mu}_k + \bar{\alpha}^{\mu}_k)e^{-ik(\tau\pm\sigma)} = \frac{2\pi T}{\sqrt{4\pi T}}(\alpha^{\mu}_0 + \bar{\alpha}^{\mu}_0) = p^{\mu}$$

$$J^{\mu\nu} = T\int_0^{\bar{\sigma}} d\sigma(X^{\mu}\dot{X}^{\nu} - X^{\nu}\dot{X}^{\mu}) = l^{\mu\nu} + E^{\mu\nu} + \bar{E}^{\mu\nu}$$

$$l^{\mu\nu} = x^{\mu}p^{\nu} - x^{\nu}p^{\mu}$$
$$E^{\mu\nu} = -i\sum_{n=1}^{\infty}\frac{1}{n}(\alpha^{\mu}_{-n}\alpha^{\nu}_n - \alpha^{\nu}_{-n}\alpha^{\mu}_n)$$
$$\bar{E}^{\mu\nu} = -i\sum_{n=1}^{\infty}\frac{1}{n}(\bar{\alpha}^{\mu}_{-n}\bar{\alpha}^{\nu}_n - \bar{\alpha}^{\nu}_{-n}\bar{\alpha}^{\mu}_n)$$

$$\{X^{\mu}(\sigma,\tau),\dot{X}^{\nu}(\sigma',\tau)\}_{PB} = \frac{1}{T}\delta(\sigma-\sigma')\eta^{\mu\nu}$$

$$\{\alpha^{\mu}_m, \alpha^{\nu}_n\} = \{\bar{\alpha}^{\mu}_m, \bar{\alpha}^{\nu}_n\} = -im\delta_{m+n,0}\eta^{\mu\nu}$$
$$\{\bar{\alpha}^{\mu}_m, \alpha^{\nu}_n\} = 0, \{x^{\mu}, p^{\nu}\} = \eta^{\mu\nu}.$$

$$H = \int d\sigma(\dot{X}\Pi - L) = \frac{T}{2}\int d\sigma(\dot{X}^2 + X'^2)$$

$$H = \frac{1}{2}\sum_{n\in Z}(\alpha_{-n}\alpha_n + \bar{\alpha}_{-n}\bar{\alpha}_n)$$

$$H = \frac{1}{2}\sum_{n\in Z}\alpha_{-n}\alpha_n$$

$$L_m = 2T\int_0^{2\pi} d\sigma T_{--}e^{im(\tau-\sigma)}, \bar{L}_m = 2T\int_0^{2\pi} d\sigma T_{++}e^{im(\sigma+\tau)}$$

$$L_m = \frac{1}{2}\sum_n \alpha_{m-n}\alpha_n, \bar{L}_m = \frac{1}{2}\sum_n \bar{\alpha}_{m-n}\bar{\alpha}_n$$

$$L^*_m = L_{-m} \text{ and } \bar{L}^*_m = \bar{L}_{-m}$$

$$H = L_0 + \bar{L}_0.$$

$$L_m = 2T\int_0^{\pi} d\sigma\{T_{--}e^{im(\tau-\sigma)} + T_{++}e^{im(\sigma+\tau)}\}$$

$$L_m = \frac{1}{2}\sum_n \alpha_{m-n}\alpha_n$$

$$H = L_0$$

$$\{L_m, L_n\}_{PB} = -i(m-n)L_{m+n}$$
$$\{\bar{L}_m, \bar{L}_n\}_{PB} = -i(m-n)\bar{L}_{m+n}$$
$$\{L_m, \bar{L}_n\}_{PB} = 0$$

$$\{\,,\}_{PB} \longrightarrow -i[,].$$

Cuantización canonical- Bosonificación.

$$[x^\mu, p^\nu] = i\eta^{\mu\nu}$$
$$[\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu}$$

$$[a_m^\mu, a_n^{\nu\dagger}] = \delta_{m,n}\eta^{\mu\nu}$$

$$\alpha_m|p\rangle = 0 \ \forall m > 0.$$

$$|p\rangle, \alpha_{-1}^\mu|p^\mu\rangle, \alpha_{-1}^\mu\alpha_{-1}^\nu\alpha_{-2}^\nu|p^\mu\rangle, \text{ etc.}$$

$$\left|\alpha_{-1}^0|p\rangle\square\right|^2 = \langle p|\alpha_1^0\alpha_{-1}^0|p\rangle = -1,$$

$$L_m = \frac{1}{2}\sum_{n\in\mathbb{Z}} :\alpha_{m-n}\cdot\alpha_n:.$$

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

$$0 = \langle\phi|[L_m, L_{-m}]|\phi\rangle = 2m\langle\phi|L_0|\phi\rangle + \frac{d}{12}m(m^2-1)\langle\phi \mid \phi\rangle \neq 0$$

$$L_{m>0} \mid \text{ phys } \rangle = 0, (L_0 - a) \mid \text{ phys } \rangle = 0$$

$$\alpha' m^2 = 4(N-a)$$

$$N = \sum_{m=1}^{\infty} \alpha_{-m}\cdot\alpha_m$$

$$\xi_+' = f(\xi_+), \xi_-' = g(\xi_-)$$

$$X^+ = x^+ + \alpha' p^+ \tau$$

$$X^\pm = X^0 \pm X^1$$

$$\alpha_n^+ = \bar{\alpha}_n^+ = \sqrt{\frac{\alpha'}{2}}p^+\delta_{n,0}$$
$$\alpha_n^- = \frac{1}{\sqrt{2\alpha' p^+}}\left\{\sum_{m\in\mathbb{Z}} :\alpha_{n-m}^i\alpha_m^i: -2a\delta_{n,0}\right\}$$

$$\alpha_{-1}^{i}\bar{\alpha}_{-1}^{j}|p\rangle$$

$$\alpha_{-1}^{i}\bar{\alpha}_{-1}^{j}|p\rangle = \alpha_{-1}^{[i}\bar{\alpha}_{-1}^{j]}|p\rangle + \left[\alpha_{-1}^{\{i}\bar{\alpha}_{-1}^{j\}} - \frac{1}{d-2}\delta^{ij}\alpha_{-1}^{k}\bar{\alpha}_{-1}^{k}\right]|p\rangle + \\ + \frac{1}{d-2}\delta^{ij}\alpha_{-1}^{k}\bar{\alpha}_{-1}^{k}|p\rangle$$

$$\alpha' m^2 = 4(1-a)$$

$$\alpha_{-1}^{i}|p\rangle,$$

$$\alpha_{-2}^{i}|p\rangle, \alpha_{-1}^{i}\alpha_{-1}^{j}|p\rangle,$$

$$j^{\max} = \alpha' m^2 + 1$$

$$\Omega|p,i,j\rangle = \epsilon|p,j,i\rangle$$

$$Z = \int \frac{\mathcal{D}g\mathcal{D}X^{\mu}}{V_{\text{gauge}}} e^{iS_p(g,X^{\mu})}$$

$$\|\delta g\| = \int d^2\sigma\sqrt{g}g^{\alpha\beta}g^{\delta\gamma}\delta g_{\alpha\gamma}\delta g_{\beta\delta}$$
$$\|\delta X^{\mu}\| = \int d^{\sigma}\sqrt{g}\delta X^{\mu}\delta X^{\nu}\eta_{\mu\nu}$$

$$g_{\alpha\beta} = e^{2\phi}h_{\alpha\beta}$$

$$\delta g_{\alpha\beta} = \nabla_{\alpha}\xi_{\beta} + \nabla_{\beta}\xi_{\alpha} + 2\Lambda g_{\alpha\beta} = (\hat{P}\xi)_{\alpha\beta} + 2\tilde{\Lambda}g_{\alpha\beta}$$

$$\mathcal{D}g = \mathcal{D}(\hat{P}\xi)\mathcal{D}(\tilde{\Lambda}) = \mathcal{D}\xi\mathcal{D}\Lambda\left|\frac{\partial(P\xi,\tilde{\Lambda})}{\partial(\xi,\Lambda)}\right|,$$

$$\left|\frac{\partial(P\xi,\tilde{\Lambda})}{\partial(\xi,\Lambda)}\right| = \left|\det\begin{pmatrix}\hat{P} & 0\\ * & 1\end{pmatrix}\right| = |\det P| = \sqrt{\det\hat{P}\hat{P}^{\dagger}}$$

$$Z = \int \mathcal{D}X^{\mu}\sqrt{\det PP^{\dagger}}e^{iS_p(\hat{h}_{\alpha\beta},X^{\mu})}$$

- Faddeev-Popov:

$$\sqrt{\det PP^{\dagger}} = \int \mathcal{D}c\mathcal{D}b e^{i\int d^2\sigma\sqrt{g}g^{\alpha\beta}b_{\alpha\gamma}\nabla_{\beta}c^{\alpha}}$$

$$Z = \int \mathcal{D}X\mathcal{D}c\mathcal{D}b e^{i(S_p[X]+S_{gh}[c,b])}$$

$$S_p[X] = T\int d^2\sigma\partial_+X^{\mu}\partial_-X_{\mu}$$
$$S_{gh}[b,c] = \int b_{++}\partial_-c^+ + b_{--}\partial_+c^-$$

$$\delta g_{\alpha\beta} = \nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha + 2\Lambda g_{\alpha\beta} = (\hat{P}\xi)_{\alpha\beta} + 2\tilde{\Lambda} g_{\alpha\beta}$$

$$\hat{P}\xi^* = 0$$

$$(V_\alpha, W_\alpha) = \int d^2\xi \sqrt{\det g}\, g^{\alpha\beta} V_\alpha W_\beta$$

$$(T_{\alpha\beta}, S_{\alpha\beta}) = \int d^2\xi \sqrt{\det g}\, g^{\alpha\gamma} g^{\beta\delta} T_{\alpha\beta} S_{\gamma\delta}$$

$$\left(\hat{P}^\dagger t\right)_\alpha = -2\nabla^\beta t_{\alpha\beta}.$$

$$\hat{P}^\dagger t^* = 0$$

$$\left[\delta_\alpha, \delta_\beta\right] = f_{\alpha\beta}^\gamma \delta_\gamma$$

$$F^A(\phi_i) = 0$$

$$\int \frac{\mathcal{D}\phi}{V_{\text{gauge}}} e^{-S_0} \sim \int \mathcal{D}\phi \delta(F^A(\phi) = 0) \mathcal{D}b_A \mathcal{D}c^\alpha e^{-S_0 - \int b_A (\delta_\alpha F^A) c^\alpha}$$

$$\sim \int \mathcal{D}\phi \mathcal{D}B_A \mathcal{D}b_A \mathcal{D}c^\alpha e^{-S_0 - i\int B_A F^A(\phi) - \int b_A (\delta_\alpha F^A) c^\alpha} = \int \mathcal{D}\phi \mathcal{D}B_A \mathcal{D}b_A \mathcal{D}c^\alpha e^{-S}$$

$$S = S_0 + S_1 + S_2\,, S_1 = i\int B_A F^A(\phi)\,, S_2 = \int b_A (\delta_\alpha F^A) c^\alpha$$

$$\begin{aligned} \delta_{BRST}\phi_i &= -i\epsilon c^\alpha \delta_\alpha \phi_i \\ \delta_{BRST} b_A &= -\epsilon B_A \\ \delta_{BRST} c^\alpha &= -\frac{1}{2}\epsilon c^\beta c^\gamma f_{\beta\gamma}^\alpha \\ \delta_{BRST} B_A &= 0 \end{aligned}$$

$$\delta_{BRST}(b_A F^A) = \epsilon[B_A F^A(\phi) + b_A c^\alpha \delta_\alpha F^A(\phi)].$$

$$\epsilon\delta_F\langle\psi \mid \psi'\rangle = -i\langle\psi|\delta_{BRST}(b_A \delta F^A)|\psi'\rangle = \langle\psi|\{Q_B, b_A \delta F^A\}|\psi'\rangle,$$

$$Q_B \mid \text{phys}\,\rangle = 0$$

$$\begin{aligned} 0 = [Q_B, \delta H] &= [Q_B, \delta_B(b_A \delta F^A)] \\ &= [Q_B, \{Q_B, b_A \delta F^A\}] = [Q_B^2, b_A \delta F^A] \end{aligned}$$

$$Q_B^2 = 0$$

$$|\psi'\rangle = |\psi\rangle + Q_B|\chi\rangle$$

$$Q_B \mid \text{phys}\,\rangle = 0$$
$$\text{and} \mid \text{phys}\,\rangle \neq Q_B \mid \text{something}\,\rangle.$$

$$\delta_B X^\mu = i\epsilon(c^+\partial_+ + c^-\partial_-)X^\mu$$
$$\delta_B c^\pm = \pm i\epsilon(c^+\partial_+ + c^-\partial_-)c^\pm$$
$$\delta_B b_\pm = \pm i\epsilon\left(T_\pm^X + T_\pm^{gh}\right)$$

$$S_{gh} = \int d^2\sigma(b_{++}\partial_- c^+ + b_{--}\partial_+ c^-)$$

$$T_{++}^{gh} = i(2b_{++}\partial_+ c^+ + \partial_+ b_{++} c^+)$$
$$T_{--}^{gh} = i(2b_{--}\partial_- c^- + \partial_- b_{--} c^-)$$

$$\partial_- T_{++}^{gh} = \partial_+ T_{--}^{gh} = 0$$

$$\partial_- b_{++} = \partial_+ b_{--} = \partial_- c^+ = \partial_+ c^- = 0$$

$$c^+ = \sum \bar{c}_n e^{-in(\tau+\sigma)}, c^- = \sum c_n e^{-in(\tau-\sigma)}$$
$$b_{++} = \sum \bar{b}_n e^{-in(\tau+\sigma)}, b_{--} = \sum c_n e^{-in(\tau-\sigma)}$$

$$\{b_m, c_n\} = \delta_{m+n,0}\, , \{b_m, b_n\} = \{c_m, c_n\} = 0$$

$$L_m^{gh} = \sum_n (m-n){:}\, b_{m+n}c_{-n}{:}\, , \bar{L}_m^{gh} = \sum_n (m-n){:}\, \bar{b}_{m+n}\bar{c}_{-n}{:}$$

- Virasoro: Campos Fantasma.

$$\left[L_m^{gh}, L_n^{gh}\right] = (m-n)L_{m+n}^{gh} + \frac{1}{6}(m-13m^3)\delta_{m+n,0}$$

$$L_m = L_m^X + L_m^{gh} - a\delta_m$$

$$[L_m, L_n] = (m-n)L_{m+n} + A(m)\delta_{m+n}$$

$$A(m) = \frac{d}{12}m(m^2-1) + \frac{1}{6}(m-13m^3) + 2am$$

$$j_B = cT^X + \frac{1}{2}{:}\, cT^{gh}{:} = cT^X + {:}\, bc\partial c{:}\, ,$$

$$Q_B = \int d\sigma j_B$$

$$Q_B = \sum_n c_n L_{-n}^X + \sum_{m,n} \frac{m-n}{2}{:}\, c_m c_n b_{-m-n}{:} - c_0$$

$$b_0|\text{phys}\rangle = 0$$

$$b_{n>0} \mid \text{ghost vacuum} \rangle = c_{n>0} \mid \text{ghost vacuum} \rangle = 0.$$

$$b_0|\downarrow\rangle = 0, \quad b_0|\uparrow\rangle = |\downarrow\rangle,$$
$$c_0|\uparrow\rangle = 0, \quad c_0|\downarrow\rangle = |\uparrow\rangle.$$

$$0 = Q_B | \downarrow, p\rangle = (L_0^X - 1)c_0 | \downarrow, p\rangle.$$

$$|\psi\rangle = (\zeta \cdot \alpha_{-1} + \xi_1 c_{-1} + \xi_2 b_{-1}) | \downarrow, p\rangle,$$

$$0 = Q_B |\psi\rangle = 2(p^2 c_0 + (p \cdot \zeta) c_{-1} + \xi_1 p \cdot \alpha_{-1}) | \downarrow, p\rangle.$$

$$Q_B |\chi\rangle = 2(p \cdot \zeta' c_{-1} + \xi_1' p \cdot \alpha_{-1}) | \downarrow, p\rangle.$$

Interacciones y amplitudes.

$$g_{\mu\nu} \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \Omega(x) g_{\mu\nu}(x)$$

$$ds'^2 = ds^2 - \left(\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu\right) dx^\mu dx^\nu$$

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d} (\partial \cdot \epsilon) \eta_{\mu\nu}$$

$$\Box\, \epsilon_\nu + \left(1 - \frac{2}{d}\right) \partial_\nu (\partial \cdot \epsilon) = 0$$

$$\partial_\mu \Box\, \epsilon_\nu + \partial_\nu \Box\, \epsilon_\mu = \frac{2}{d} \eta_{\mu\nu} \Box\, (\partial \cdot \epsilon)$$

$$\left[\eta_{\mu\nu} \Box + (d-2) \partial_\mu \partial_\nu\right] \partial \cdot \epsilon = 0$$

$\epsilon^\mu = a^\mu$translationes

$\epsilon^\mu = \omega^\mu_\nu x^\nu$ rotationes $\left(\omega_{\mu\nu} = -\omega_{\nu\mu}\right)$,

$\epsilon^\mu = \lambda x^\mu$ transformationes escalares

$$\epsilon^\mu = b^\mu x^2 - 2x^\mu (b \cdot x)$$

$$d + \frac{1}{2} d(d-1) + 1 + d = \frac{1}{2}(d+2)(d+1)$$

$$\partial_1 \epsilon_1 = \partial_2 \epsilon_2, \partial_1 \epsilon_2 = -\partial_2 \epsilon_1$$

$$\partial \bar{\epsilon} = 0, \bar{\partial} \epsilon = 0$$

$$z \to f(z) \text{ and } \bar{z} \to \bar{f}(\bar{z})$$

$$\epsilon(z) = -\sum a_n z^{n+1}$$

$$\ell_n = -z^{n+1} \partial_z$$

$$[\ell_m, \ell_n] = (m-n)\ell_{m+n}, \left[\bar{\ell}_m, \bar{\ell}_n\right] = (m-n)\bar{\ell}_{m+n}$$

$$z \to \frac{az + b}{cz + d}$$

Invariancia y covariancia de gauge.

$$z \to f(z) = \frac{az+b}{cz+d}\ , \bar{z} \to \bar{f}(\bar{z}) = \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}$$

$$\Phi(z,\bar{z}) \to \left(\frac{\partial f}{\partial z}\right)^{h} \left(\frac{\partial \bar{f}}{\partial \bar{z}}\right)^{\bar{h}} \Phi(f(z), \bar{f}(\bar{z}))$$

$$\Phi(z,\bar{z}) dz^{h} d\bar{z}^{\bar{h}}$$

$$\left\langle \prod_{i=1}^{N} \Phi_i(z_i,\bar{z}_i) \right\rangle = \prod_{i=1}^{N} \left(\frac{\partial f}{\partial z}\right)_{z\to z_i}^{h_i} \left(\frac{\partial \bar{f}}{\partial \bar{z}}\right)_{\bar{z}\to\bar{z}_i}^{\bar{h}_i} \left\langle \prod_{j=1}^{N} \Phi_j\left(f(z_j), \bar{f}(\bar{z}_j)\right) \right\rangle.$$

$$\delta_{\epsilon,\bar{\epsilon}} \Phi(z,\bar{z}) = [(h\partial\epsilon + \epsilon\partial) + (\bar{h}\bar{\partial}\bar{\epsilon} + \bar{\epsilon}\bar{\partial})]\Phi(z,\bar{z})$$

$$\delta_{\epsilon,\bar{\epsilon}} G^{(2)}(z_i,\bar{z}_i) = \langle \delta_{\epsilon,\bar{\epsilon}}\Phi_1, \Phi_2 \rangle + \langle \Phi_1, \delta_{\epsilon,\bar{\epsilon}}\Phi_2 \rangle = 0$$

$$\left[\left(\epsilon(z_1)\partial_{z_1} + h_1\partial\epsilon(z_1) + \epsilon(z_2)\partial_{z_2} + h_2\partial\epsilon(z_2)\right) + (\ \mathfrak{B}_{\text{barred terms}}\ )\right] G^{(2)}(z_i,\bar{z}_i) = 0$$

$$G^{(2)}(z_i,\bar{z}_i) = \frac{C_{12}}{z_{12}^{2h} \bar{z}_{12}^{2\bar{h}}}$$

$$G^{(3)}(z_i,\bar{z}_i) = \frac{C_{123}}{z_{12}^{\Delta_{12}} z_{23}^{\Delta_{23}} z_{31}^{\Delta_{31}} \bar{z}_{12}^{\bar{\Delta}_{12}} \bar{z}_{12}^{\bar{\Delta}_{12}} \bar{z}_{12}^{\bar{\Delta}_{12}}}$$

$$G^{(4)}(z_i,\bar{z}_i) = f(x,\bar{x}) \prod_{i<j} z_{ij}^{-(h_i+h_j)+h/3} \prod_{i<j} \bar{z}_{ij}^{-(\bar{h}_i+\bar{h}_j)+\bar{h}/3}$$

$$G^{N}(z_1,\bar{z}_1, \dots z_N, \bar{z}_N) = \left\langle \prod_{i=1}^{N} \Phi_i(z_i,\bar{z}_i) \right\rangle$$

$$\sum_{i=1}^{N} \partial_i G^{N} = 0$$

$$\sum_{i=1}^{N} (z_i\partial_i + h_i) G^{N} = 0$$

$$\sum_{i=1}^{N} \left(z_i^2\partial_i + 2z_i h_i\right) G^{N} = 0$$

Cuantización radial.

$$z = e^{\tau + i\sigma}, \bar{z} = e^{\tau - i\sigma}$$

$$H = \ell_0 + \bar{\ell}_0$$

$$T_\mu{}^\mu = 0$$

$$T_{z\bar{z}} = T_{\bar{z}z} = \frac{1}{4}(T_{00} + T_{11}) = \frac{1}{4}T_\mu{}^\mu$$

$$\partial_z T_{\bar{z}\bar{z}} = 0 \text{ and } \partial_{\bar{z}} T_{zz} = 0$$

$$T(z) \equiv T_{zz} \text{ and } \bar{T}(\bar{z}) \equiv T_{\bar{z}\bar{z}}$$

$$Q_\epsilon = \frac{1}{2\pi i}\oint dz\epsilon(z)T(z)\,, Q_{\bar{\epsilon}} = \frac{1}{2\pi i}\oint d\bar{z}\bar{\epsilon}(\bar{z})\bar{T}(\bar{z}).$$

$$z \to z + \epsilon(z), \bar{z} \to \bar{z} + \bar{\epsilon}(\bar{z})$$

$$\delta_{\epsilon,\bar{\epsilon}}\Phi(z,\bar{z}) = [Q_\epsilon + Q_{\bar{\epsilon}}, \Phi(z,\bar{z})]$$

$$R(A(z)B(w)) = \begin{cases} A(z)B(w) & |z| > |w| \\ (-1)^F B(w)A(z) & |z| < |w| \end{cases}.$$

$$\left[\int d\sigma B, A\right] = \oint dz R(B(z)A(w))$$

$$\begin{aligned}\delta_{\epsilon,\bar{\epsilon}}\Phi(z,\bar{z}) &= \frac{1}{2\pi i}\oint \left(dz\epsilon(z)R(T(z)\Phi(w,\bar{w})) + d\bar{z}\bar{\epsilon}(\bar{z})R(\bar{T}(\bar{z})\Phi(w,\bar{w}))\right) \\ &= [(h\partial\epsilon(w) + \epsilon(w)\partial) + (\bar{h}\bar{\partial}\bar{\epsilon}(\bar{w}) + \bar{\epsilon}(\bar{w})\bar{\partial})]\Phi(w,\bar{w})\end{aligned}$$

$$R(T(z)\Phi(w,\bar{w})) = \frac{h}{(z-w)^2}\Phi(w,\bar{w}) + \frac{1}{z-w}\partial_w\Phi(w,\bar{w}) + \cdots$$
$$R(\bar{T}(\bar{z})\Phi(w,\bar{w})) = \frac{\bar{h}}{(\bar{z}-\bar{w})^2}\Phi(w,\bar{w}) + \frac{1}{\bar{z}-\bar{w}}\partial_{\bar{w}}\Phi(w,\bar{w}) + \cdots$$

$$F^N(z, z_i, \bar{z}_i) = \left\langle T(z)\prod_{i=1}^{N} \Phi_i(z_i, \bar{z}_i)\right\rangle,$$

$$F^N(z, z_i, \bar{z}_i) = \sum_{i=1}^{N}\left(\frac{h_i}{(z-z_i)^2} + \frac{\partial_{z_i}}{z-z_i}\right)\left\langle\prod_{i=1}^{N} \Phi_i(z_i, \bar{z}_i)\right\rangle.$$

$$\Phi_i(z,\bar{z})\Phi_j(w,\bar{w}) = \sum_k C_{ijk}(z-w)^{h_k-h_i-h_j}(\bar{z}-\bar{w})^{\bar{h}_k-\bar{h}_i-\bar{h}_j}\Phi_k(w,\bar{w})$$

Bosón libre.

$$S=\frac{1}{4\pi}\int d^2z\partial X\bar{\partial}X$$

$$\langle X(z,\bar{z})X(w,\bar{w})\rangle=-\log\left(|z-w|^2\mu^2\right)$$

$$\partial_z X(z)\partial_w X(w)=\partial_z\partial_w\langle XX\rangle+:\partial_z X\partial_w X:=-\frac{1}{(z-w)^2}+:\partial_z X\partial_w X:$$

$$T(z)=-\frac{1}{2}:\partial X\partial X:=-\frac{1}{2}\lim_{z\to w}\left[\partial_z X\partial_w X+\frac{1}{(z-w)^2}\right]$$
$$\bar{T}(\bar{z})=-\frac{1}{2}:\bar{\partial}X\bar{\partial}X:=-\frac{1}{2}\lim_{\bar{z}\to\bar{w}}\left[\partial_{\bar{z}}X\partial_{\bar{w}}X+\frac{1}{(\bar{z}-\bar{w})^2}\right]$$

- Wick:

$$T(z)\partial X(w)=-\frac{1}{2}:\partial X(z)\partial X(z):\partial X(w)\ =-\partial X(z)\langle\partial X(z)\partial X(w)\rangle+\cdots=\partial X(z)\frac{1}{(z-w)^2}+\cdots$$

$$=\frac{\partial X(w)}{(z-w)^2}+\frac{1}{z-w}\partial^2 X(w)+\cdots$$

$$T(z)V_a(w,\bar{w})=-\frac{1}{2}:\partial X(z)\partial X(z):\sum_{n=0}^{\infty}\frac{i^n a^n}{n!}:X^n(w,\bar{w}):$$

$$T(z)V_a(w)=-\frac{1}{2}[ia\partial\langle XX\rangle]^2e^{iaX(w)}-\frac{1}{2}2ia:\partial X(z)\partial\langle XX\rangle e^{iaX(w)}:+\cdots$$

$$=\frac{a^2/2}{(z-w)^2}e^{iaX(w)}+\frac{ia\partial X(z)}{z-w}e^{iaX(w)}+\cdots=\frac{a^2/2}{(z-w)^2}V_a(w)+\frac{1}{z-w}\partial V_a(w)+\cdots$$

$$G^N=\left\langle\prod_{i=1}^{N}V_{a_i}(z_i,\bar{z}_i)\right\rangle=\exp\left[\frac{1}{2}\sum_{i,j=1;i\neq j}^{N}a_i a_j\langle X(z_i,\bar{z}_i)X(z_j,\bar{z}_j)\rangle\right]$$

$$\sum_i a_i=0$$

$$\langle V_a(z)V_{-a}(w)\rangle=\langle:e^{iaX(z)}::e^{-iaX(w)}:\rangle=e^{-a^2\log|z-w|^2}=\frac{1}{|z-w|^{2a^2}}$$

$$i\partial_z XV_a(w,\bar{w})=a\frac{V_a(w,\bar{w})}{(z-w)}+\text{ finito}$$

Carga central – OPE.

$$T(z)T(w) = \frac{c/2}{(z-w)^4} + 2\frac{T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \cdots$$

$$T(z)\bar{T}(\bar{w}) = \text{ regular}$$

$$T(z)T(w) = \frac{1}{4}\{2(\partial\partial\langle XX\rangle)^2 + 4{:}\,\partial X(z)\partial X(w){:}\,\partial\partial\langle XX\rangle + \cdots\}$$

$$= \frac{1/2}{(z-w)^4} + \frac{2}{(z-w)^2}T(w) + \frac{1}{z-w}\partial T(w) + \cdots$$

$$\tilde{T} = -\frac{1}{2}{:}\,\partial X\partial X{:} + iQ\partial^2 X$$

Fermión libre.

$$c = 1 - 12Q^2$$

$$\partial\!\!\!/ = \sigma^1\partial_1 + \sigma^2\partial_2 = \begin{pmatrix} 0 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & \partial \\ \bar{\partial} & 0 \end{pmatrix}.$$

- Majorana espinor $\binom{\psi}{\bar{\psi}}$:

$$S = -\frac{1}{8\pi}\int d^2z(\psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi})$$

$$\bar{\partial}\psi = \partial\bar{\psi} = 0$$

$$\psi(z)\psi(w) = \frac{1}{z-w}\,, \bar{\psi}(\bar{z})\bar{\psi}(\bar{w}) = \frac{1}{\bar{z}-\bar{w}}$$

$$T(z) = -\frac{1}{2}{:}\,\psi(z)\partial\psi(z){:}\,, \bar{T}(\bar{z}) = -\frac{1}{2}{:}\,\bar{\psi}(\bar{z})\bar{\partial}\bar{\psi}(\bar{z}){:}\,.$$

$$T(z)T(w) = \frac{1/4}{(z-w)^4} + \frac{2}{(z-w)^2}T(w) + \frac{1}{z-w}\partial T(w)$$

$$T(z) = \sum_{n\in\mathbb{Z}} z^{-n-2}L_n\,, \bar{T}(\bar{z}) = \sum_{n\in\mathbb{Z}} \bar{z}^{-n-2}\bar{L}_n$$

$$w = \tau + i\sigma \to z = e^w$$

$$\Phi_{\text{cyl}}(w) = \sum_{n\in\mathbb{Z}} \phi_n e^{-nw} = \sum_{n\in\mathbb{Z}} \phi_n e^{in(i\tau-\sigma)} = \sum_{n\in\mathbb{Z}} \phi_n z^{-n}$$

$$\Phi(z) = \sum_{n\in\mathbb{Z}} \phi_n z^{-n-h}$$

$$T(z) \to (f')^2 T(f(z)) + \frac{c}{12}\left[\frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2\right]$$

$$L_n = \oint \frac{dz}{2\pi i} z^{n+1} T(z)\,, \bar{L}_n = \oint \frac{d\bar{z}}{2\pi i} \bar{z}^{n+1} \bar{T}(\bar{z})$$

$$[L_n, L_m] = \left(\oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} - \oint \frac{dw}{2\pi i} \oint \frac{dz}{2\pi i}\right) z^{n+1} T(z) w^{m+1} T(w)$$

$$= \oint \frac{dw}{2\pi i} \oint_{C_w} \frac{dz}{2\pi i} z^{n+1} w^{m+1} \left(\frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \cdots\right)$$

$$= \oint \frac{dw}{2\pi i} \left(\frac{c}{12}(n+1)n(n-1) w^{n-2} w^{m+1}\right.$$

$$\left. +\ 2(n+1) w^n w^{m+1} T(w) + w^{n+1} w^{m+1} \partial T(w)\right)$$

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}$$

$$[\bar{L}_n, \bar{L}_m] = (n-m) \bar{L}_{n+m} + \frac{\bar{c}}{12}(n^3 - n)\delta_{n+m,0}$$

$$[L_n, \bar{L}_m] = 0$$

$$T_\alpha^\alpha = \frac{c}{96\pi^3} \sqrt{g} R^{(2)}$$

$$\int [DX]_{\hat{g}} e^{-S[\hat{g}_{\alpha\beta}, X]} = e^{-cS_L[g_{\alpha\beta}, \phi]} \int [DX]_g e^{-S[g_{\alpha\beta}, X]}$$

$$S_L[g_{\alpha\beta}, \phi] = \frac{1}{96\pi} \int \sqrt{\det g}\, g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + \frac{1}{48\pi} \int \sqrt{\det g} R^{(2)} \phi$$

Espacio de Hilbert – Einstein.

$$|A_{\text{in}}\rangle = \lim_{\tau \to -\infty} A(\tau, \sigma)|0\rangle = \lim_{z \to 0} A(z, \bar{z})|0\rangle.$$

$$\tilde{A}(w, \bar{w}) = A(f(w), \bar{f}(\bar{w}))(\partial f(w))^h (\bar{\partial} \bar{f}(\bar{w}))^{\bar{h}}$$

$$\tilde{A}(w, \tilde{w}) = A\left(\frac{1}{w}, \frac{1}{\bar{w}}\right)(-w^{-2})^h (-\bar{w}^{-2})^{\bar{h}}$$

$$\langle A_{\text{out}}| = \lim_{w, \bar{w} \to 0} \langle 0|\tilde{A}(w, \bar{w})$$

$$[A(z, \bar{z})]^\dagger = A\left(\frac{1}{\bar{z}}, \frac{1}{z}\right) \bar{z}^{-2h} z^{-2\bar{h}}$$

$$\langle A_{\text{out}}| = \lim_{w \to 0} \langle 0|\tilde{A}(w, \bar{w}) = \lim_{z \to 0} \langle 0|A\left(\frac{1}{z}, \frac{1}{\bar{z}}\right) \bar{z}^{-2h} z^{-2\bar{h}} = \lim_{z \to 0} \langle 0|[A(z, \bar{z})]^\dagger = |A_{\text{in}}\rangle^\dagger$$

$$T^\dagger(z) = \sum_m \frac{L_m^\dagger}{\bar{z}^{m+2}} \equiv \sum_m \frac{L_m}{\bar{z}^{-m-2}} \frac{1}{\bar{z}^4}$$

$$L_m^\dagger = L_{-m}$$

$$T(z)|0\rangle = \sum_{m\in\mathbb{Z}} L_m z^{-m-2}|0\rangle$$

$$L_m|0\rangle = 0, m \geq -1$$

$$\langle 0|L_m = 0, m \leq 1$$

$$\Phi_{n>-h}|0\rangle = 0$$

$$[L_n, \Phi(w)] = \oint \frac{dz}{2\pi i} z^{n+1} T(z)\Phi(w) = h(n+1)w^n\Phi(w) + w^{n+1}\partial\Phi(w)$$

$$|h\rangle \equiv \Phi(0)|0\rangle.$$

$$L_{m>0}|h\rangle = L_{m>0}\Phi(0)|0\rangle = [L_m, \Phi(0)]|0\rangle + \Phi(0)L_{m>0}|0\rangle = 0$$

$$|\chi\rangle = L_{-n_1}L_{-n_2}\dots L_{-n_k}|h\rangle,$$

$$(L_{-1}\Phi)(z) \equiv \oint_{C_z} \frac{dw}{2\pi i} T(w)\Phi_h(z)$$

$$\Phi_\chi(z) = \prod_{i=1}^{k} \oint \frac{dw_i}{2\pi i}(w_i - z)^{-n_i+1} T(w_i)\Phi_h(z)$$

$$[L_{-1}, O(z,\bar{z})] = \partial_z O(z,\bar{z})$$

$$\chi_h(q) \equiv \mathrm{Tr}\left[q^{L_0 - \frac{c}{24}}\right]$$

$$\chi_h(q) = \frac{q^{h-c/24}}{\prod_{n=1}^{\infty}(1-q^n)}$$

$$\chi_0(q) = \frac{q^{-c/24}}{\prod_{n=2}^{\infty}(1-q^n)}$$

$$\|L_{-n}|0\rangle\|^2 = \langle 0|L_{-n}^{\dagger}L_{-n}|0\rangle = \langle 0|\left[\frac{c}{12}(n^3-n) + 2nL_0\right]|0\rangle = \frac{c}{12}(n^3-n)$$

$$c = 1 - \frac{6}{m(m+1)}$$

$$|\chi\rangle = \left(L_{-2} - \frac{3}{4}L_{-1}^2\right)|1/2\rangle.$$

$$J^a(z)J^b(w) = \frac{G^{ab}}{(z-w)^2} + \frac{if^{ab}{}_c J^c(w)}{z-w} + \text{ finito}$$

$$[J_m^a, J_n^b] = mG^{ab}\delta_{m+n,0} + if^{ab}{}_c J_{m+n}^c.$$

$$T(z)J^a(w) = \frac{J^a(w)}{(z-w)^2} + \frac{\partial J^a(w)}{z-w}$$

$$S=\frac{1}{4\lambda^2}\int_{M_2} d^2\xi \mathrm{Tr}\left(\partial_\mu g\partial^\mu g^{-1}\right)+\frac{ik}{8\pi}\int_{B;\partial B=M_2} d^3\xi \mathrm{Tr}\left(\epsilon_{\alpha\beta\gamma}U^\alpha U^\beta U^\gamma\right)$$

$$T_G(z)=\frac{1}{2(k+\tilde{h})}:J^a(z)J^a(z):$$

$$c_G=\frac{kD_G}{k+\tilde{h}}$$

$$J^a_{m>0}|R_i\rangle=0\,,J^a_0|R_i\rangle=i(T^a_R)_{ij}\left|R_j\right\rangle$$

$$J^a(z)R_i(w,\bar{w})=i\frac{(T^a_R)_{ij}}{(z-w)}R_j(w,\bar{w})+\cdots$$

$$h_R=\frac{C_R}{k+\tilde{h}},$$

$$T_{\mathrm{G/H}}(z)J^{\mathrm{H}}(w)=\text{ regular }\,,T_{\mathrm{G/H}}(z)T_{\mathrm{H}}(w)=\text{ regular}$$

Supersimetrías. Métrica de Ramond – Clifford.

$$S=-\frac{1}{8\pi}\int d^2z\psi^i\bar{\partial}\psi^i$$

$$J^{ij}(z)=i{:}\psi^i(z)\psi^j(z){:}\,,i<j$$

$$\psi^i(z)\psi^j(w)=\frac{\delta^{ij}}{z-w}$$

$$J^{ij}(z)J^{kl}(w)=\frac{G^{ij,kl}}{(z-w)^2}+if^{ij,kl}_{mn}\frac{J^{mn}(w)}{(z-w)}+\cdots$$

$$2f^{ij,kl}{}_{mn}=\left(\delta^{ik}\delta^{ln}-\delta^{il}\delta^{kn}\right)\delta^{jm}+\left(\delta^{jl}\delta^{kn}-\delta^{jk}\delta^{ln}\right)\delta^{im}-(m\leftrightarrow n)$$

$$T(z)=\frac{1}{2(N-1)}\sum_{i<j}^{N}:J^{ij}(z)J^{ij}(z):.$$

$$T(z)T(w)=\frac{c_G/2}{(z-w)^4}+\frac{2T(w)}{(z-w)^2}+\frac{\partial T(w)}{z-w}$$

$$c_G=\frac{kD}{k+\tilde{h}}$$

$$c_G=\frac{N(N-1)/2}{1+N-2}=\frac{N}{2},$$

$$T(z)=-\frac{1}{2}\sum_{i=1}^{N}:\psi^i\partial\psi^i:$$

$$h_V = \frac{(N-1)/2}{1+N-2} = \frac{1}{2}$$

$$J^{ij}(z)\psi^k(w) = i\frac{T^{ij}_{kl}}{z-w}\psi^l(w) + \cdots,$$

$$\psi^i \to -\psi^i.$$

$$\psi(\tau + i\sigma) = \sum_n \psi_n e^{-n(\tau+i\sigma)}$$

$$\psi^i(z) = \sum_n \psi^i_n z^{-n-h} = \sum_n \psi^i_n z^{-n-1/2}$$

$$\{\psi^i_m, \psi^j_n\} = \delta^{ij}\delta_{m+n,0}$$

$$\psi^i_{n>0}|0\rangle = 0$$

$$|i\rangle = \psi^i_{-\frac{1}{2}}|0\rangle$$

$$\{(-1)^F, \psi^i_n\} = 0$$

$$J^{ij}_{-1}|0\rangle = i\psi^i_{-\frac{1}{2}}\psi^j_{-\frac{1}{2}}|0\rangle$$

$$J^{ij}_{-1}|k\rangle = i\left[\delta^{jk}\psi^i_{-\frac{3}{2}} - \delta^{ik}\psi^j_{-\frac{3}{2}} + \psi^i_{-\frac{1}{2}}\psi^j_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}\right]|0\rangle.$$

$$\mathrm{Tr}_{NS}\left[q^{L_0-c/24}\right] = q^{-\frac{N}{48}}\prod_{n=1}^{\infty}\left(1+q^{n-\frac{1}{2}}\right)^N$$

$$\mathrm{Tr}_{NS}\left[q^{L_0-c/24}\right] = \left[\frac{\vartheta_3}{\eta}\right]^{N/2}$$

$$\mathrm{Tr}_{NS}\left[(-1)^F q^{L_0-c/24}\right] = q^{-\frac{N}{48}}\prod_{n=1}^{\infty}\left(1-q^{n-\frac{1}{2}}\right)^N = \left[\frac{\vartheta_4}{\eta}\right]^{N/2}$$

$$\chi_0 = \mathrm{Tr}_{NS}\left[\frac{(1+(-1)^F)}{2}q^{L_0-c/24}\right] = \frac{1}{2}\left(\left[\frac{\vartheta_3}{\eta}\right]^{N/2} + \left[\frac{\vartheta_4}{\eta}\right]^{N/2}\right)$$
$$\chi_V = \mathrm{Tr}_{NS}\left[\frac{(1-(-1)^F)}{2}q^{L_0-c/24}\right] = \frac{1}{2}\left(\left[\frac{\vartheta_3}{\eta}\right]^{N/2} - \left[\frac{\vartheta_4}{\eta}\right]^{N/2}\right).$$

$$\chi_R(v_i) = \mathrm{Tr}_R\left[q^{L_0-c/24}e^{2\pi i\sum_i v_i J^i_0}\right]$$

$$\chi_0(v_i) = \frac{1}{2}\left[\prod_{i=1}^{N/2} \frac{\vartheta_3(v_i)}{\eta} + \prod_{i=1}^{N/2} \frac{\vartheta_4(v_i)}{\eta}\right]$$

$$\chi_V(v_i) = \frac{1}{2}\left[\prod_{i=1}^{N/2} \frac{\vartheta_3(v_i)}{\eta} - \prod_{i=1}^{N/2} \frac{\vartheta_4(v_i)}{\eta}\right]$$

$$\{\psi_0^i, \psi_0^j\} = \delta^{ij}$$

$$\psi_{m>0}^i|\hat{S}_\alpha\rangle = 0\,, \psi_0^i|\hat{S}_\alpha\rangle = \gamma_{\alpha\beta}^i|S_\beta\rangle$$

$$\gamma^{N+1} = \prod_{i=1}^{N} (\psi_0^i/\sqrt{2})\,, \{\gamma^{N+1}, \psi_0^i\} = 0\,, [\gamma^{N+1}]^2 = 1$$

$$(-1)^F = \gamma^{N+1}(-1)^{\sum_{n=1}^{\infty} \psi_{-n}^i \psi_n^i}$$

$$(-1)^F|S\rangle = |S\rangle\,, (-1)^F|C\rangle = -|C\rangle.$$

$$G_R^{ij}(z,w) = \langle\hat{S}|\psi^i(z)\psi^j(w)|\hat{S}\rangle$$

$$G_R^{ij}(z,w) = \delta^{ij}\frac{z+w}{2\sqrt{zw}}\frac{1}{z-w}$$

$$\langle X|T(z)|X\rangle = \frac{h}{z^2}$$

$$T(w) = \lim_{z\to w}\left[-\frac{1}{2}\sum_{i=1}^{N} \psi^i(z)\partial_w\psi^i(w) + \frac{N}{2(z-w)^2}\right]$$

$$\langle\hat{S}|T(z)|\hat{S}\rangle = \frac{N}{16z^2}$$

$$\mathrm{Tr}_R\left[q^{L_0-c/24}\right] = 2^{N/2}q^{\frac{N}{16}-\frac{N}{48}}\prod_{n=1}^{\infty} (1+q^n)^N = \left[\frac{\vartheta_2}{\eta}\right]^{N/2}$$

$$\chi_S = \chi_C = \frac{1}{2}\left[\frac{\vartheta_2}{\eta}\right]^{N/2}.$$

$$\chi_S(v_i) = \frac{1}{2}\left[\prod_{i=1}^{N/2} \frac{\vartheta_2(v_i)}{\eta} + \prod_{i=1}^{N/2} \frac{\vartheta_1(v_i)}{\eta}\right],$$

$$\chi_C(v_i) = \frac{1}{2}\left[\prod_{i=1}^{N/2} \frac{\vartheta_2(v_i)}{\eta} - \prod_{i=1}^{N/2} \frac{\vartheta_1(v_i)}{\eta}\right]$$

$$|\hat{S}_\alpha\rangle = \lim_{z\to 0}\hat{S}_\alpha(z)|0\rangle$$

$$\psi^i(z)\hat{S}_\alpha(w) = \gamma^i_{\alpha\beta}\frac{\hat{S}_\beta(w)}{\sqrt{z-w}} + \cdots$$

$$J^{ij}(z)\hat{S}_\alpha(w) = \frac{i}{2}\left[\gamma^i,\gamma^j\right]_{\alpha\beta}\frac{\hat{S}_\beta(w)}{(z-w)} + \cdots$$

$$\hat{S}_\alpha(z)\hat{S}_\beta(w) = \frac{\delta_{\alpha\beta}}{(z-w)^{N/8}} + \gamma^i_{\alpha\beta}\frac{\psi^i(w)}{(z-w)^{N/8-1/2}} + \frac{i}{2}\left[\gamma^i,\gamma^j\right]_{\alpha\beta}\frac{J^{ij}(w)}{(z-w)^{N/8-1}} + \cdots$$

Supersimetría isotrópica. Métrica de Majorana.

$$S = \frac{1}{2\pi}\int d^2z\partial X\bar{\partial}X + \frac{1}{2\pi}\int d^2z(\psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi})$$

$$\delta X = \epsilon(z)\psi\,, \delta\psi = -\epsilon(z)\partial X\,, \delta\bar{\psi} = 0$$

$$\delta X = \bar{\epsilon}(\bar{z})\bar{\psi}\,, \delta\bar{\psi} = -\bar{\epsilon}(\bar{z})\bar{\partial}X\,, \delta\psi = 0$$

$$G(z) = i\psi\partial X\,, \bar{G}(\bar{z}) = i\bar{\psi}\bar{\partial}X$$

$$G(z)G(w) = \frac{1}{(z-w)^3} + 2\frac{T(w)}{z-w} + \cdots$$

$$T(z)G(w) = \frac{3}{2}\frac{G(w)}{(z-w)^2} + \frac{\partial G(w)}{z-w} + \cdots$$

$$T(z) = -\frac{1}{2}:\partial X\partial X: -\frac{1}{2}:\psi\partial\psi:.$$

$$G(z)G(w) = \frac{\hat{c}}{(z-w)^3} + 2\frac{T(w)}{z-w} + \cdots$$

$$T(z)G(w) = \frac{3}{2}\frac{G(w)}{(z-w)^2} + \frac{\partial G(w)}{z-w} + \cdots$$

$$\{G_r, G_s\} = \frac{\hat{c}}{2}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0} + 2L_{r+s}$$

$$[L_m, G_r] = \left(\frac{m}{2} - r\right)G_{m+r}$$

$$\chi^{NS}_{N=1} = \mathrm{Tr}\left[q^{L_0-c/24}\right] = q^{h-c/24}\prod_{n=1}^{\infty}\frac{1+q^{n-\frac{1}{2}}}{1-q^n}.$$

$$\{G_0, G_0\} = 2L_0 - \frac{\hat{c}}{8}$$

$$\chi^{R}_{N=1} = \mathrm{Tr}\left[q^{L_0-c/24}\right] = q^{h-c/24}\prod_{n=1}^{\infty}\frac{1+q^{n}}{1-q^n}.$$

$$D_\theta = \frac{\partial}{\partial\theta} + \theta\partial_z\,, \bar{D}_{\bar{\theta}} = \frac{\partial}{\partial\bar{\theta}} + \bar{\theta}\partial_{\bar{z}}$$

$$\hat{X}(z,\bar{z},\theta,\bar{\theta}) = X + \theta\psi + \bar{\theta}\bar{\psi} + \theta\bar{\theta}F$$

$$S=\frac{1}{2\pi}\int d^2z\int d\theta d\bar{\theta}D_\theta\hat{X}\bar{D}_{\bar{\theta}}\hat{X}$$

$$G^+(z)G^-(w)=\frac{2c}{3}\frac{1}{(z-w)^3}+\left(\frac{2J(w)}{(z-w)^2}+\frac{\partial J(w)}{z-w}\right)+\frac{2}{z-w}T(w)+\cdots,$$

$$G^+(z)G^+(w)=\text{ regular },G^-(z)G^-(w)=\text{ regular },$$
$$T(z)G^\pm(w)=\frac{3}{2}\frac{G^\pm(w)}{(z-w)^2}+\frac{\partial G^\pm(w)}{z-w}+\cdots,$$
$$J(z)G^\pm(w)=\pm\frac{G^\pm(w)}{z-w}+\cdots,$$
$$T(z)J(w)=\frac{J(w)}{(z-w)^2}+\frac{\partial J(w)}{z-w}+\cdots,$$
$$J(z)J(w)=\frac{c/3}{(z-w)^2}+\cdots$$

$$G^\pm\left(e^{2\pi i}z\right)=e^{\mp 2\pi i\alpha}G^\pm(z)$$

$$J_n^\alpha=J_n-\alpha\frac{c}{3}\delta_{n,0}\ ,L_n^\alpha=L_n-\alpha J_n+\alpha^2\frac{c}{6}\delta_{n,0}$$
$$G_{r+\alpha}^{\alpha,+}=G_r^+\ ,G_{r-\alpha}^{\alpha,-}=G_r^-$$

$$G_0^\pm G_0^\pm=0,\{G_0^+,G_0^-\}=2\left(L_0-\frac{c}{24}\right)$$

$$J_0^{R^\pm}=J_0^{NS}\mp\frac{c}{6},L_0^{R^\pm}-\frac{c}{24}=L_0^{NS}-\frac{1}{2}J_0^{NS}$$

$$O_{q_1}(z)O_{q_2}(z)=O_{q_1+q_2}(z)$$

$$J^a(z)J^b(w)=\frac{k}{2}\frac{\delta^{ab}}{(z-w)^2}+i\epsilon^{abc}\frac{J^c(w)}{(z-w)}+\cdots$$
$$J^a(z)G^\alpha(w)=\frac{1}{2}\sigma_{\beta\alpha}^a\frac{G^\beta(w)}{(z-w)}+\cdots,J^a(z)\bar{G}^\alpha(w)=-\frac{1}{2}\sigma_{\alpha\beta}^a\frac{\bar{G}^\beta(w)}{(z-w)}+\cdots$$
$$G^\alpha(z)\bar{G}^\beta(w)=\frac{4k\delta^{\alpha\beta}}{(z-w)^3}+2\sigma_{\beta\alpha}^a\left[\frac{2J^a(w)}{(z-w)^2}+\frac{\delta J^a(w)}{(z-w)}\right]+2\delta^{\alpha\beta}\frac{T(w)}{(z-w)}+\cdots$$
$$G^\alpha(z)G^\beta(w)=\text{regular},\bar{G}^\alpha(z)\bar{G}^\beta(w)=\text{regular}$$

Campos fantasmas.

$$S_\lambda=\frac{1}{\pi}\int d^2zb\bar{\partial}c$$

$$c(z)b(w)=\frac{1}{z-w},b(z)c(w)=\frac{\epsilon}{z-w}$$

$$c(z)=\sum_{n\in Z}z^{-n-(1-\lambda)}c_n\,,c_n^\dagger=c_{-n}$$
$$b(z)=\sum_{n\in Z}z^{-n-\lambda}b_n\,,b_n^\dagger=\epsilon b_{-n}$$

$$c_m b_n + \epsilon b_n c_m = \delta_{m+n,0}\, , c_m c_n + \epsilon c_n c_m = b_m b_n + \epsilon b_n b_m = 0$$

$$\text{NS: } b_n, n \in \mathbb{Z} - \lambda, c_n, n \in \mathbb{Z} + \lambda$$
$$\text{R: } b_n, n \in \frac{1}{2} + \mathbb{Z} - \lambda, c_n, n \in \frac{1}{2} + \mathbb{Z} + \lambda$$

$$T = -\lambda b \partial c + (1-\lambda)(\partial b)c$$

$$c = -2\epsilon(6\lambda^2 - 6\lambda + 1) = \epsilon(1 - 3Q^2)\, , Q = \epsilon(1 - 2\lambda)$$

$$J(z) = -{:}\, b(z)c(z){:} = \sum_{n\in\mathbb{Z}} z^{-n-1} J_n$$

$$J(z)J(w) = \frac{\epsilon}{(z-w)^2} + \cdots$$

$$J(z)b(w) = -\frac{b(w)}{z-w} + \cdots\, , J(z)c(w) = \frac{c(w)}{z-w} \dots$$

$$T(z)J(w) = \frac{Q}{(z-w)^3} + \frac{J(w)}{(z-w)^2} + \frac{\partial_w J(w)}{z-w} + \cdots$$

$$[L_m, J_n] = -nJ_{m+n} + \frac{Q}{2} m(m+1)\delta_{m+n,0}$$

$$[L_1, J_{-1}] = J_0 + Q\, , Q + J_0^\dagger = [L_1, J_{-1}]^\dagger = -J_0$$

$$\mathbb{Z}_{\text{zero modes of } c} - \mathbb{Z}_{\#\text{ zero modes of } b} = -\frac{\epsilon}{2} Q \chi$$

$$b_{n>-\lambda}|0\rangle = c_{n>\lambda-1}|0\rangle = 0$$

$$J(z) = -\partial\phi\, , \langle \phi(z)\phi(w)\rangle = -\log(z-w)$$

$$\hat{T} = \frac{1}{2}{:}J^2{:} + \frac{1}{2} Q \partial J = \frac{1}{2}(\partial\phi)^2 - \frac{Q}{2}\partial^2\phi.$$

$$S_Q = \frac{1}{2\pi}\int d^2z \left[\partial\phi\bar{\partial}\phi - \frac{Q}{4}\sqrt{g}R^{(2)}\phi\right]$$

$$T = \hat{T} + T_{\eta\xi}$$

$$T(z){:}\,e^{q\phi(w)}{:} = \left[-\frac{q(q+Q)}{(z-w)^2} + \frac{1}{z-w}\partial_w\right]{:}\,e^{q\phi(w)}{:} + \cdots$$
$$J(z){:}\,e^{q\phi(w)}{:} = \frac{q}{z-w}{:}\,e^{q\phi(w)}{:} \dots \to \left[J_0, {:}\,e^{q\phi(w)}{:}\right] = q{:}\,e^{q\phi(w)}{:}.$$

$$c(z) = e^{\phi(z)}\eta(z)\, , b(z) = e^{-\phi(z)}\partial\xi(z)$$

$$g_{ij} = \frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}$$

$$ds^2 = g_{ij}d\sigma_i d\sigma_j = \frac{1}{\tau_2}|d\sigma_1 + \tau d\sigma_2|^2 = \frac{dwd\bar{w}}{\tau_2}$$

$$w = \sigma_1 + \tau\sigma_2\,, \bar{w} = \sigma_1 + \bar{\tau}\sigma_2$$

$$w \to w + 1\,, w \to w + \tau$$

$$T{:}\,\tau \to \tau + 1$$

$$TST{:}\,\tau \to \frac{\tau}{\tau + 1}.$$

$$S{:}\,\tau \to -\frac{1}{\tau}\,, S^2 = 1\,, (ST)^3 = 1$$

$$\tau' = \frac{a\tau + b}{c\tau + d} \leftrightarrow A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$L_0^{cyl} = L_0 - \frac{c}{24}\,, \bar{L}_0^{cyl} = \bar{L}_0 - \frac{\bar{c}}{24},$$

$$\begin{aligned} \int e^{-S} &= \mathrm{Tr}\left[e^{-2\pi\tau_2 H} e^{2\pi i\tau_1 P}\right] = \mathrm{Tr}\left[e^{2\pi i\tau L_0^{cyl}} e^{-2\pi i\bar{\tau}\bar{L}_0^{cyl}}\right] \\ &= \mathrm{Tr}\left[q^{L_0 - c/24}\bar{q}^{\bar{L}_0 - \bar{c}/24}\right] \end{aligned}$$

Escalares compactos.

$$S = \frac{1}{4\pi}\int d^2\sigma\sqrt{g}g^{ij}\partial_i X\partial_j X = \frac{1}{4\pi}\int_0^1 d\sigma_1 \int_0^1 d\sigma_2 \frac{1}{\tau_2}|\tau\partial_1 X - \partial_2 X|^2 = -\frac{1}{4\pi}\int d^2\sigma X \,\square\, X$$

$$\square = \frac{1}{\tau_2}|\tau\partial_1 - \partial_2|^2.$$

$$Z(R) = \int DXe^{-S}$$

$$X_{\mathrm{class}} = 2\pi R(n\sigma_1 + m\sigma_2)\,, m, n \in \mathbb{Z}$$

$$X_{class}(\sigma_1 + 1, \sigma_2) = X(\sigma_1, \sigma_2) + 2\pi nR, X_{class}(\sigma_1, \sigma_2 + 1) = X(\sigma_1, \sigma_2) + 2\pi mR$$

$$S_{m,n} = \frac{\pi R^2}{\tau_2}|m - n\tau|^2$$

$$\begin{aligned} Z(R) &= \sum_{m,n\in\mathbb{Z}} \int D\chi e^{-S_{m,n} - S(\chi)} \\ &= \sum_{m,n\in\mathbb{Z}} e^{-S_{m,n}} \int D\chi e^{-S(\chi)} \end{aligned}$$

$$\square\, \psi_i = -\lambda_i \psi_i$$

$$\psi_{m_1,m_2} = e^{2\pi i(m_1\sigma_1+m_2\sigma_2)}\,,\lambda_{m_1,m_2} = \frac{4\pi^2}{\tau_2}|m_1\tau - m_2|^2$$

$$\int d^2\sigma\psi_{m_1,m_2}\psi_{n_1,n_2} = \delta_{m_1+n_1,0}\delta_{m_2+n_2,0}$$

$$\delta\chi = \sum_{m_1,m_2\in\mathbb{Z}}{}' A_{m_1,m_2}\psi_{m_1,m_2}$$

$$S(\chi) = \frac{1}{4\pi}\sum_{m_1,m_2}^{\prime}\lambda_{m_1,m_2}\left|A_{m_1,m_2}\right|^2$$

$$\|\delta X\| = \int d^2\sigma\sqrt{\det G}(d\chi)^2 = \sum_{m_1,m_2}{}'\left|dA_{m_1,m_2}\right|^2$$

$$\int D\chi = \int_0^{2\pi R} d\chi_0\prod_{m_1,m_2}\frac{dA_{m_1,m_2}}{2\pi}$$

$$\int D\chi e^{-S(\chi)} = \frac{2\pi R}{\prod'_{m_1,m_2}\lambda^{1/2}_{m_1,m_2}} = \frac{2\pi R}{\sqrt{\det'\square}}$$

$$\det'\square = 4\pi^2\tau_2\eta^2(\tau)\bar{\eta}^2(\bar{\tau})$$

$$Z(R) = \frac{R}{\sqrt{\tau_2}|\eta|^2}\sum_{m,n\in Z} e^{-\frac{\pi R^2}{\tau_2}|m-n\tau|^2}$$

$$Z(R) = \sum_{m,n\in\mathbb{Z}}\frac{q^{\frac{P_L^2}{2}}\bar{q}^{\frac{P_R^2}{2}}}{\eta\bar{\eta}}$$

$$P_L = \frac{1}{\sqrt{2}}\left(\frac{m}{R}+nR\right),P_R = \frac{1}{\sqrt{2}}\left(\frac{m}{R}-nR\right)$$

$$J(z) = i\partial X\,,\bar{J}(\bar{z}) = i\bar{\partial}X$$

$$J(z)J(w) = \frac{1}{(z-w)^2} + \text{ finite },\bar{J}(\bar{z})\bar{J}(\bar{w}) = \frac{1}{(\bar{z}-\bar{w})^2} + \text{ finito}$$

- Sugawara:

$$T(z) = -\frac{1}{2}(\partial X)^2 = \frac{1}{2}{:}J^2{:}\,,\bar{T}(\bar{z}) = -\frac{1}{2}(\bar{\partial}X)^2 = \frac{1}{2}{:}\bar{J}^2{:}.$$

$$\Delta = \frac{1}{2}Q_L^2\,,\bar{\Delta} = \frac{1}{2}Q_R^2$$

$$\chi_{Q_L,Q_R}(q,\bar{q}) = \mathrm{Tr}\left[q^{L_0-1/24}\bar{q}^{\bar{L}_0-1/24}\right] = \frac{q^{Q_L^2/2}\bar{q}^{Q_R^2/2}}{\eta\bar{\eta}}$$

$$J_0|m,n\rangle = P_L|m,n\rangle\,, \bar{J}_0|m,n\rangle = P_R|m,n\rangle$$

$$V_{m,n} =: \exp\,[ip_L X + ip_R\bar{X}]:,$$

$$J(z)V_{m,n}(w,\bar{w}) = p_L\frac{V_{m,n}(w,\bar{w})}{z-w} + \cdots$$
$$\bar{J}(\bar{z})V_{m,n}(w,\bar{w}) = p_R\frac{V_{m,n}(w,\bar{w})}{\bar{z}-\bar{w}} + \cdots$$

$$\left\langle \prod_{i=1}^{N} V_{m_i,n_i}(z_i,\bar{z}_i) \right\rangle = \prod_{i<j}^{N} z_{ij}^{p_L^i p_L^j} \bar{z}_{ij}^{p_R^i p_R^j},$$

$$:e^{ia\phi(z)}::e^{ib\phi(w)}:=(z-w)^{ab}:e^{ia\phi(z)+ib\phi(w)}:=(z-w)^{ab}\left[:e^{i(a+b)\phi(w)}:+\mathcal{O}(z-w)\right]$$

$$\left[V_{m_1,n_1}\right]\cdot\left[V_{m_2,n_2}\right] \sim \left[V_{m_1+m_2,n_1+n_2}\right]$$

$$\Delta - \bar{\Delta} - \frac{c-\bar{c}}{24} = \mathfrak{I}_{\text{integer}}$$

$$\lim_{R\to\infty}\frac{Z(R)}{R} = \frac{1}{\sqrt{\tau_2}\eta\bar{\eta}}$$

$$\Delta(\sigma_1,\sigma_2) \equiv \langle\delta\chi(\sigma_1,\sigma_2)\delta\chi(0,0)\rangle = -\sum_{m,n}{}' \frac{1}{|m\tau-n|^2} e^{2\pi i(m\sigma_1+n\sigma_2)}$$

$$\Box\ \Delta(\sigma_1,\sigma_2) = \frac{4\pi^2}{\tau_2}[\delta(\sigma_1)\delta(\sigma_2)-1]$$

$$\Delta(\sigma_1,\sigma_2) = -\log\ G(z,\bar{z})\,, G = e^{-2\pi\frac{Imz^2}{\tau_2}}\left|\frac{\vartheta_1(z)}{\vartheta_1'(0)}\right|^2$$

$$S = \frac{1}{4\pi}\int\ d^2\sigma\sqrt{\det g}\,g^{ab}G_{ij}\partial_a X^i\partial_b X^j + \frac{1}{4\pi}\int\ d^2\sigma\epsilon^{ab}B_{ij}\partial_a X^i\partial_b X^j,$$

$$\mathbb{Z}_{\text{d, d}}(G,B) = \frac{\sqrt{\det G}}{(\sqrt{\tau_2}\eta\bar{\eta})^N}\sum_{\vec{m},\vec{n}} e^{-\frac{\pi(G_{ij}+B_{ij})}{\tau_2}(m_i+n_i\tau)(m_j+n_j\bar{\tau})}$$

$$\mathbb{Z}_{\text{d, d}}(G,B) = \frac{\Gamma_{d,d}(G,B)}{\eta^d\bar{\eta}^d} = \sum_{\vec{m},\vec{n}\in\mathbb{Z}^N}\frac{q^{\frac{1}{2}P_L^2}\bar{q}^{\frac{1}{2}P_R^2}}{\eta^N\bar{\eta}^N}$$

$$P_{L,R}^2 \equiv P_{L,R}^i G_{ij} P_{L,R}^j$$
$$P_L^i = \frac{G^{ij}}{\sqrt{2}}\left(m_j + \left(B_{jk}+G_{jk}\right)n_k\right), P_R^i = \frac{G^{ij}}{\sqrt{2}}\left(m_j + \left(B_{jk}-G_{jk}\right)n_k\right)$$

$$J^i(z) = i\partial X^i \,, \bar{J}^i = i\bar{\partial} X^i$$
$$J^i(z)J^j(w) = \frac{G^{ij}}{(z-w)^2} + \cdots$$

$$T(z) = -\frac{1}{2}G_{ij}\partial X^i \partial X^j = \frac{1}{2}G_{ij}{:}J^iJ^j{:}.$$

$$\Delta = \frac{1}{2}G_{ij}Q_L^iQ_L^j \,, \bar{\Delta} = \frac{1}{2}G_{ij}Q_R^iQ_R^j$$

Supersimetría de Higgs.

$$\Delta = \frac{1}{4}(m+n)^2 \,, \bar{\Delta} = \frac{1}{4}(m-n)^2$$

$$J^{\pm}(z) = \frac{1}{\sqrt{2}}{:}\,e^{\pm i\sqrt{2}X(z)}{:}$$

$$J^3(z) = \frac{1}{\sqrt{2}}J(z) = \frac{i}{\sqrt{2}}\partial X(z)$$

$$J^3(z)J^{\pm}(w) = \pm\frac{J^{\pm}(w)}{z-w} + \cdots \,, J^+(z)J^+(w) = \cdots$$
$$J^-(z)J^-(w) = \cdots,$$
$$J^+(z)J^-(w) = \frac{1/2}{(z-w)^2} + \frac{J^3(w)}{z-w} + \cdots,$$
$$J^3(z)J^3(w) = \frac{1/2}{(z-w)^2} + \cdots$$

$$a_{-1}^{\mu}\bar{a}_{-1}^{\nu}|0\rangle \,, a_{-1}^{\mu}\bar{a}_{-1}^{25}|0\rangle \,, a_{-1}^{25}\bar{a}_{-1}^{\mu}|0\rangle \,, a_{-1}^{25}\bar{a}_{-1}^{25}|0\rangle,$$

$$\left|A_{\mu}^{\pm}\right\rangle = \bar{a}^{\mu}|m = \pm 1, n = \pm 1\rangle$$

$$\left|\bar{A}_{\mu}^{\pm}\right\rangle = a^{\mu}|m = \pm 1, n = \mp 1\rangle$$

Dualidad T.

$$H = L_0 + \bar{L}_0 = \frac{1}{2}\left(\frac{m^2}{R^2} + n^2R^2\right) \,, P = L_0 - \bar{L}_0 = mn$$

$$R \to \frac{1}{R} \,, m \leftrightarrow n$$

$$P_L \to P_L \,, P_R \to -P_R$$

$$J(z) \to J(z) \,, \bar{J}(\bar{z}) \to -\bar{J}(\bar{z})$$

$$\Omega = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & \mathbf{1}_N \\ \mathbf{1}_N & 0 \end{pmatrix}$$

$$\Omega^T L \Omega = L$$

$$E \to (AE+B)(CE+D)^{-1}, \begin{pmatrix} \vec{m} \\ \vec{n} \end{pmatrix} \to \Omega \begin{pmatrix} \vec{m} \\ \vec{n} \end{pmatrix}$$

$$G_{ij} = \frac{T_2}{U_2}\begin{pmatrix} 1 & U_1 \\ U_1 & U_1^2 + U_2^2 \end{pmatrix}, B_{ij} = \begin{pmatrix} 0 & T_1 \\ -T_1 & 0 \end{pmatrix},$$

$$\Gamma_{2,2}(T,U) = \sum_{\vec{m},\vec{n}} \exp\left[-\frac{\pi\tau_2}{T_2U_2}\left| -m_1U + m_2 + T(n_1 + Un_2)\right|^2 + 2\pi i\tau(m_1n_1 + m_2n_2)\right]$$

$$\int e^{-S} = (\det\partial)^{N/2}$$

$$\lambda_{AA} \sim \left(\left(m_1 + \frac{1}{2}\right)\tau + \left(m_2 + \frac{1}{2}\right)\right), m_{1,2} \in \mathbb{Z}$$

$$(\det\partial)_{AA} = \frac{\vartheta_3(\tau)}{\eta(\tau)}$$

$$\lambda_{AP} \sim \left(\left(m_1 + \frac{1}{2}\right)\tau + m_2\right), m_{1,2} \in \mathbb{Z}$$

$$(\det\partial)_{AP} = \frac{\vartheta_4(\tau)}{\eta(\tau)}$$

$$\lambda_{PA} \sim \left(m_1\tau + \left(m_2 + \frac{1}{2}\right)\right), m_{1,2} \in \mathbb{Z}$$

$$(\det\partial)_{PA} = \frac{\vartheta_2(\tau)}{\eta(\tau)}$$

$$(\det\partial)\begin{bmatrix} a \\ b \end{bmatrix} = \frac{\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(\tau)}{\eta(\tau)}$$

$$Z_N^{\text{fermionic}} = \frac{1}{2}\sum_{a,b=0}^{1}\left|\frac{\vartheta\begin{bmatrix} a \\ b \end{bmatrix}}{\eta}\right|^N$$

$$Z_N^{\text{fermionic}} = |\chi_0|^2 + |\chi_V|^2 + |\chi_S|^2 + |\chi_C|^2$$

- Szegö:

$$\langle \psi^i(z)\psi^j(0)\rangle = \delta^{ij} S\begin{bmatrix} a \\ b \end{bmatrix}(z), S\begin{bmatrix} a \\ b \end{bmatrix}(z) = \frac{\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(z)\vartheta_1'(0)}{\vartheta_1(z)\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(0)}$$

$$q^{-N/24}\prod_{n=1}^{\infty}(1-q^n)^N = \eta^N = \left[\frac{1}{2\pi}\frac{\partial_v\vartheta_1(v)|_{v=0}}{\eta}\right]^{N/2}$$

$$\left\langle \prod_{k=1}^{N} \psi^{i_k}(z_k) \right\rangle_{\text{odd}} = \epsilon^{i_1,\dots,i_N} \eta^N$$

Bosonización Majorana-Weyl $\boldsymbol{\psi^i(z)}$:

$$\psi^i(z)\psi^j(w) = \frac{\delta^{ij}}{z-w} + \cdots$$

$$\psi = \frac{1}{\sqrt{2}}(\psi^1 + i\psi^2)\,, \bar{\psi} = \frac{1}{\sqrt{2}}(\psi^1 - i\psi^2)$$

$$J(z) =: \psi\bar{\psi}:\,, J(z)J(w) = \frac{1}{(z-w)^2} + \cdots$$

$$J(z)\psi(w) = \frac{\psi(w)}{z-w} + \cdots\,, J(z)\bar{\psi}(w) = -\frac{\bar{\psi}(w)}{z-w} + \cdots$$

$$T(z) = -\frac{1}{2}:\psi^i \partial \psi^i: = \frac{1}{2}:J^2:.$$

$$J(z) = i\partial X\,, \psi =: e^{iX}:\,, \bar{\psi} =: e^{-iX}:$$

- Poisson:

$$\begin{aligned}\left|\vartheta\begin{bmatrix}a\\b\end{bmatrix}\right|^2 &= \frac{1}{\sqrt{2\tau_2}} \sum_{m,n\in Z} \exp\left[-\frac{\pi}{2\tau_2}|n - b + \tau(m-a)|^2 + i\pi mn\right] \\ &= \frac{1}{\sqrt{2\tau_2}} \sum_{m,n\in Z} \exp\left[-\frac{\pi}{2\tau_2}|n + \tau m|^2 + i\pi(m+a)(n+b)\right]\end{aligned}$$

$$Z = \frac{1}{2}\sum_{a,b=0}^{1} \left|\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}}{\eta}\right|^2 = \frac{1}{2\sqrt{2\tau_2}} \sum_{a,b=0}^{1} \sum_{m,n\in\mathbb{Z}} \exp\left[-\frac{\pi}{2\tau_2}|n+\tau m|^2 + i\pi(m+a)(n+b)\right]$$

$$Z_{\text{Dirac}} = \frac{1}{\sqrt{2\tau_2}} \sum_{m,n\in\mathbb{Z}} \exp\left[-\frac{\pi}{2\tau_2}|n+\tau m|^2\right]$$

Orbifolds.

$$g\left[\prod_{i=1}^{N} a_{-n_i} \prod_{j=1}^{\bar{N}} \bar{a}_{\bar{n}_j} |m,n\rangle\right] = (-1)^{N+\bar{N}} \prod_{i=1}^{N} a_{-n_i} \prod_{j=1}^{\bar{N}} \bar{a}_{\bar{n}_j} |-m,-n\rangle.$$

$$Z(R)^{\text{invariant}} = \frac{1}{2}Z(R) + \frac{1}{2}\mathrm{Tr}\left[g q^{L_0 - 1/24} \bar{q}^{\bar{L}_0 - 1/24}\right].$$

$$\frac{1}{2}\mathrm{Tr}\left[g q^{L_0-1/24} \bar{q}^{\bar{L}_0-1/24}\right] = \frac{1}{2}(q\bar{q})^{-1/24} \prod_{n=1}^{\infty} \frac{1}{(1+q^n)(1+\bar{q}^n)} = \left|\frac{\eta}{\vartheta_2}\right|$$

$$Z(R)^{\text{invariant}} = \frac{1}{2}Z(R) + \left|\frac{\eta}{\vartheta_2}\right|.$$

$$X(\sigma,\tau) = x_0 + \frac{i}{\sqrt{4\pi T}}\sum_{n\in Z}\left(\frac{a_{n+1/2}}{n+1/2}e^{i(n+1/2)(\sigma+\tau)} + \frac{\bar{a}_{n+1/2}}{n+1/2}e^{-i(n+1/2)(\sigma-\tau)}\right).$$

$$a_{n+1/2}|H^{0,\pi}\rangle = \bar{a}_{n+1/2}|H^{0,\pi}\rangle = 0\; n \geq 0$$

$$Z^{\text{twisted}} = \frac{1}{2}\mathrm{Tr}\left[(1+g)q^{L_0-1/24}\bar{q}^{\bar{L}_0-1/24}\right]$$

$$= \frac{1}{2}\frac{1}{(q\bar{q})^{48}}\left[\prod_{n=1}^{\infty}\frac{1}{\left(1-q^{n-\frac{1}{2}}\right)\left(1-\bar{q}^{n-\frac{1}{2}}\right)} + \prod_{n=1}^{\infty}\frac{1}{\left(1+q^{n-\frac{1}{2}}\right)\left(1+\bar{q}^{n-\frac{1}{2}}\right)}\right]$$

$$= \left|\frac{\eta}{\vartheta_4}\right| + \left|\frac{\eta}{\vartheta_3}\right|$$

$$Z^{\text{orb}}(R) = Z^{\text{untwisted}} + Z^{\text{twisted}} = \frac{1}{2}Z(R) + \left|\frac{\eta}{\vartheta_2}\right| + \left|\frac{\eta}{\vartheta_4}\right| + \left|\frac{\eta}{\vartheta_3}\right|$$

$$Z^{\text{orb}} = \frac{1}{2}\sum_{h,g=0}^{1} Z\begin{bmatrix}h\\g\end{bmatrix}$$

$$Z\begin{bmatrix}h\\g\end{bmatrix} = 2\left|\frac{\eta}{\vartheta\begin{bmatrix}1-h\\1-g\end{bmatrix}}\right|, (h,g) \neq (0,0)$$

$$\tau \to \tau+1: Z\begin{bmatrix}h\\g\end{bmatrix} \to Z\begin{bmatrix}h\\h+g\end{bmatrix}$$
$$\tau \to -\frac{1}{\tau}: Z\begin{bmatrix}h\\g\end{bmatrix} \to Z\begin{bmatrix}g\\h\end{bmatrix}$$

$$[H^0]\cdot[H^0] \sim \sum_{n,m} C^{2m,2n}\left[V^{+}_{2m,2n}\right] + C^{2m,2n+1}\left[V^{+}_{2m,2n+1}\right]$$

$$[H^\pi]\cdot[H^\pi] \sim \sum_{n,m} C^{2m,2n}\left[V^{+}_{2m,2n}\right] - C^{2m,2n+1}\left[V^{+}_{2m,2n+1}\right]$$
$$[H^0]\cdot[H^\pi] \sim \sum_{n,m} C^{2m+1,2n}\left[V^{+}_{2m+1,2n}\right]$$

$$C^{m,n} = \sqrt{2}2^{-2(h_{m,n}+\bar{h}_{m,n})}, C_{0,0} = 1$$

$$h_{m,n} = (m/R+nR)^2/4, \bar{h}_{m,n} = (m/R-nR)^2/4$$

$$\left(H^0, H^\pi, V^{+}_{m,n}\right) \to \left(-H^0, H^\pi, (-1)^m V^{+}_{m,n}\right)$$
$$\left(H^0, H^\pi, V^{+}_{m,n}\right) \to \left(H^\pi, H^0, (-1)^n V^{+}_{m,n}\right)$$

$$\left(H^0, H^\pi, V^+_{m,n}\right) \to \left(-H^0, -H^\pi, V^+_{m,n}\right)$$

$$Z^{\text{orb}}(R) = Z^{\text{orb}}(1/R)$$

$$\begin{pmatrix} H^0 \\ H^\pi \end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} H^0 \\ H^\pi \end{pmatrix}$$

$$Z^{\text{Ising}} = \frac{1}{2}\left[\left|\frac{\vartheta_2}{\eta}\right| + \left|\frac{\vartheta_3}{\eta}\right| + \left|\frac{\vartheta_4}{\eta}\right|\right],$$

$$Z\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \sum_{m,n\in\mathbb{Z}} (-1)^m \frac{\exp\left[\frac{i\pi\tau}{2}\left(\frac{m}{R} + nR\right)^2 - \frac{i\pi\bar{\tau}}{2}\left(\frac{m}{R} - nR\right)^2\right]}{\eta\bar{\eta}}$$

$$Z\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \sum_{m,n\in\mathbb{Z}} \frac{\exp\left[\frac{i\pi\tau}{2}\left(\frac{m}{R} + \left(n+\frac{1}{2}\right)R\right)^2 - \frac{i\pi\bar{\tau}}{2}\left(\frac{m}{R} - \left(n+\frac{1}{2}\right)R\right)^2\right]}{\eta\bar{\eta}}.$$

$$Z\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \sum_{m,n\in\mathbb{Z}} (-1)^m \frac{\exp\left[\frac{i\pi\tau}{2}\left(\frac{m}{R} + \left(n+\frac{1}{2}\right)R\right)^2 - \frac{i\pi\bar{\tau}}{2}\left(\frac{m}{R} - \left(n+\frac{1}{2}\right)R\right)^2\right]}{\eta\bar{\eta}}.$$

$$Z\begin{bmatrix} h \\ g \end{bmatrix} = \sum_{m,n\in\mathbb{Z}} (-1)^{gm} \frac{\exp\left[\frac{i\pi\tau}{2}\left(\frac{m}{R} + \left(n+\frac{h}{2}\right)R\right)^2 - \frac{i\pi\bar{\tau}}{2}\left(\frac{m}{R} - \left(n+\frac{h}{2}\right)R\right)^2\right]}{\eta\bar{\eta}}$$

$$Z\begin{bmatrix} h \\ g \end{bmatrix} = \frac{R}{\sqrt{\tau_2}\eta\bar{\eta}} \sum_{m,n,\in\mathbb{Z}} \exp\left[-\frac{\pi R^2}{\tau_2}\left|m + \frac{g}{2} + \left(n + \frac{h}{2}\right)\tau\right|^2\right]$$

$$g_{\text{translation}} = \exp\left[2\pi i \sum_{i=1}^{d} (m_i\theta_i + n_i\phi_i)\right]$$

CFT en superficies de Riemann. Amplitudes escalares y operadores de vértice.

$$\langle 1\rangle_{g=2} = \sum_i q^{h_i - c/24}\bar{q}^{\bar{h}_i - \bar{c}/24}\langle\phi_i\rangle_{g=1}\langle\phi_i\rangle_{g=1}$$

$$S_P = \frac{1}{4\pi\alpha'}\int d^2\sigma\partial X^\mu\bar{\partial}X^\nu\eta_{\mu\nu}$$

$$\langle X^\mu(z,\bar{z})X^\nu(w,\bar{w})\rangle = -\frac{\alpha'}{4}\eta^{\mu\nu}\log|z-w|^2$$

$$T = -\frac{2}{\alpha'}\eta_{\mu\nu}\partial X^\mu\partial X^\nu$$

$$T(z)O(w,\bar{w}) = -ip^\mu\epsilon_{\mu\nu}\frac{\alpha'}{4}\frac{\bar{\partial}x^\nu V_p}{(z-w)^3} + \left(1 + \frac{\alpha' p^2}{4}\right)\frac{O(w,\bar{w})}{(z-w)^2} + \frac{\partial_w O(w,\bar{w})}{z-w} + \cdots$$

$$p^{\mu}\epsilon_{\mu\nu} = p^{\nu}\epsilon_{\mu\nu} = 0$$

$$\Lambda^4 = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z_{\text{bosonic}}\,(\tau,\bar{\tau}) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^{24}}$$

$$\chi = 2(1-g) - B - C.$$

Acción stress – tensor – energía - momentum. Centro de masa. Partícula Supermasiva u Oscura.

$$S_P = \frac{1}{4\pi\alpha'}\int\; d^2\xi\left[\sqrt{g}g^{ab}G_{\mu\nu}(X) + \epsilon^{ab}B_{\mu\nu}(X)\right]\partial_a X^{\mu}\partial_b X_{\nu} + +\frac{1}{8\pi}\int\; d^2\xi\sqrt{g}R^{(2)}\Phi(X)$$

$$\chi = \frac{1}{4\pi}\int\; \sqrt{g}R^{(2)}$$

$$g_{\text{spacetime dimension}} = e^{\Phi_0/2}$$

$$\frac{T_a^a}{\sqrt{g}} = \frac{\beta^{\Phi}}{96\pi^3}R^{(2)} + \frac{1}{2\pi}\left(\beta_{\mu\nu}^G g^{ab} + \beta_{\mu\nu}^B\epsilon^{ab}\right)\partial_a X^{\mu}\partial_b X_{\nu}$$

$$\frac{\beta_{\mu\nu}^G}{\alpha'} = R_{\mu\nu} - \frac{1}{4}H_{\mu\rho\sigma}H_{\nu}^{\rho\sigma} + \nabla_{\mu}\nabla_{\nu}\Phi + \mathcal{O}(\alpha')$$

$$\frac{\beta_{\mu\nu}^B}{\alpha'} = \nabla^{\mu}\left[e^{-\Phi}H_{\mu\nu\rho}\right] + \mathcal{O}(\alpha')$$

$$\beta^{\Phi} = D - 26 + 3\alpha'\left[(\nabla\Phi)^2 - 2\,\square\,\Phi - R + \frac{1}{12}H^2\right] + \mathcal{O}(\alpha'^2)$$

$$H_{\mu\nu\rho} = \partial_{\mu}B_{\nu\rho} + \partial_{\nu}B_{\rho\mu} + \partial_{\rho}B_{\mu\nu}$$

$$\beta^{\Phi} = \beta_{\mu\nu}^G = \beta_{\mu\nu}^B = 0$$

$$\alpha'^{D-2}S^{\text{tree}} \sim \int\; d^D x\sqrt{-\text{det}G}e^{-\Phi}\left[R + (\nabla\Phi)^2 - \frac{1}{12}H^2 + \frac{D-26}{3}\right] + \mathcal{O}(\alpha')$$

$$G_{\mu\nu}^E = e^{-\frac{2\Phi}{D-2}}G_{\mu\nu}$$

$$S_E^{\text{tree}} \sim \frac{1}{\kappa^2}\int\; d^D x\sqrt{G^E}\left[R - \frac{1}{D-2}(\nabla\Phi)^2 - \frac{e^{-4\Phi/(D-2)}}{12}H^2 + e^{2\Phi/(D-2)}\frac{D-26}{3}\right] + \mathcal{O}(\alpha')$$

$$\kappa = g_{\text{spacetime dimension}}\,\alpha'^{(D-2)/2}$$

Supersimetría de Einstein – Polyakov – Virasoro – Majorana - Weyl. Superconformidad de gauge.

$$S_P^{II} = \frac{1}{4\pi\alpha'}\int\; \sqrt{g}\left[g^{ab}\partial_a X^{\mu}\partial_b X^{\mu} + \frac{i}{2}\psi^{\mu}\,\partial\psi^{\mu} + \frac{i}{2}\left(\chi_a\gamma^b\gamma^a\psi^{\mu}\right)\left(\partial_b X^{\mu} - \frac{i}{4}\chi_b\psi^{\mu}\right)\right]$$

$$\delta g_{ab} = i\epsilon(\gamma_a\chi_b + \gamma_b\chi_a)\,,\delta\chi_a = 2\nabla_a\epsilon,$$
$$\delta X^\mu = i\epsilon\psi^\mu\,,\delta\psi^\mu = \gamma^a\left(\partial_a X^\mu - \frac{i}{2}\chi_a\psi^\mu\right)\epsilon\,,\delta\bar{\psi}^\mu = 0,$$

$$g_{ab} = e^\phi\delta_{ab}\,,\chi_a = \gamma_a\zeta$$

$$G_{\text{matter}} = i\psi^\mu\partial X^\mu\,,\bar{G}_{\text{matter}} = i\bar{\psi}^\mu\bar{\partial}X^\mu$$

$$j_{BRST} = \gamma G_{\text{matter}} + cT_{\text{matter}} + \frac{1}{2}(cT_{\text{ghost}} + \gamma G_{\text{ghost}})$$

$$G_{\text{matter}} = i\psi^\mu\partial X^\mu\,,T_{\text{matter}} = -\frac{1}{2}\partial X^\mu\partial X^\mu - \frac{1}{2}\psi^\mu\partial\psi^\mu,$$

$$G_{\text{ghost}} = -i\left(c\partial\beta - \frac{1}{2}\gamma b + \frac{3}{2}\partial c\beta\right)\,,T_{\text{ghost}} = T_{bc} - \frac{1}{2}\gamma\partial\beta - \frac{3}{2}\partial\gamma\beta.$$

$$Q = \frac{1}{2\pi i}\left[\oint \; dz j_{BRST} + \oint \; d\bar{z}\bar{j}_{BRST}\right]$$

$$X^+ = x^+ + p^+\tau\,,\psi^+ = \bar{\psi}^+ = 0$$

$$0 = \{G_0, G_0\} = 2\left(L_0 - \frac{D-2}{16}\right)$$

$$L_0 = \bar{L}_0\,,L_0 - \frac{1}{2} = 0$$

$$(-1)^F = \exp\left[i\pi\sum_{r\in Z+1/2}\psi_r^i\psi_{-r}^i\right].$$

$$(-1)^F = \prod_{\mu=0}^{9}\psi_0^\mu\exp\left[i\pi\sum_{n=1}^{\infty}\psi_n^i\psi_{-n}^i\right] = \Gamma^{11}\exp\left[i\pi\sum_{n=1}^{\infty}\psi_n^i\psi_{-n}^i\right]$$

$$\{(-1)^{F_L}, G_0\} = 0\,,\{(-1)^{F_R}, \bar{G}_0\} = 0$$

$$\{\Gamma^{11}, \partial\!\!\!/\} = 0$$

$$Z^{IIB} = \frac{(\chi_V - \chi_S)(\bar{\chi}_V - \bar{\chi}_S)}{(\sqrt{\tau_2}\eta\bar{\eta})^8}$$

$$Z^{IIB} = \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8}\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\frac{1}{2}\sum_{\bar{a},\bar{b}=0}^{1}(-1)^{\bar{a}+\bar{b}+\bar{a}\bar{b}}\frac{\vartheta^4\begin{bmatrix}a\\b\end{bmatrix}\bar{\vartheta}^4\begin{bmatrix}\bar{a}\\\bar{b}\end{bmatrix}}{\eta^4\bar{\eta}^4},$$

$$Z^{IIA} = \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8}\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b}\frac{1}{2}\sum_{\bar{a},\bar{b}=0}^{1}(-1)^{\bar{a}+\bar{b}+\bar{a}\bar{b}}\frac{\vartheta^4\begin{bmatrix}a\\b\end{bmatrix}\bar{\vartheta}^4\begin{bmatrix}\bar{a}\\\bar{b}\end{bmatrix}}{\eta^4\bar{\eta}^4}.$$

Estados supermasivos y pluridimensiones por deformación de superficie. Métrica de Majorana.

$$\{\Gamma^{\mu},\Gamma^{\nu}\} = -2\eta^{\mu\nu}\,,\eta^{\mu\nu} = (-++\cdots+)$$

$$\Gamma_{\mu} = \eta_{\mu\nu}\Gamma^{\nu}\;\Gamma^{\mu} = \eta^{\mu\nu}\Gamma_{\nu}$$

$$\Gamma^{0}\Gamma_{\mu}^{\dagger}\Gamma^{0} = \Gamma_{\mu}\,,\Gamma^{0}\Gamma_{\mu}\Gamma^{0} = -\Gamma_{\mu}^{T}$$

$$\Gamma_{11} = \Gamma_{0}\ldots\Gamma_{9}\,,(\Gamma_{11})^{2} = 1\,,\{\Gamma_{11},\Gamma^{\mu}\} = 0;$$

$$\Gamma^{\mu_1\cdots\mu_k} = \frac{1}{k!}\Gamma^{[\mu_1}\ldots\Gamma^{\mu_k]} = \frac{1}{k!}(\Gamma^{\mu_1}\ldots\Gamma^{\mu_k}\pm\text{ permutations }).$$

$$\Gamma_{11}\Gamma^{\mu_1\cdots\mu_k} = \frac{(-1)^{\left[\frac{k}{2}\right]}}{(10-k)!}\epsilon^{\mu_1\cdots\mu_{10}}\Gamma_{\mu_{k+1}\cdots\mu_{10}}$$

$$\Gamma^{\mu_1\cdots\mu_k}\Gamma_{11} = \frac{(-1)^{\left[\frac{k+1}{2}\right]}}{(10-k)!}\epsilon^{\mu_1\cdots\mu_{10}}\Gamma_{\mu_{k+1}\cdots\mu_{10}}$$

$$\Gamma^{\mu}\Gamma^{\nu_1\cdots\nu_k} = \Gamma^{\mu\nu_1\cdots\nu_k} - \frac{1}{(k-1)!}\eta^{\mu[\nu_1}\Gamma^{\nu_2\cdots\nu_k]}$$

$$\Gamma^{\nu_1\cdots\nu_k}\Gamma^{\mu} = \Gamma^{\nu_1\cdots\nu_k\mu} - \frac{1}{(k-1)!}\eta^{\mu[\nu_k}\Gamma^{\nu_1\cdots\nu_{k-1}]}$$

$$F_{\alpha\beta} = S_{\alpha}(i\Gamma^{0})_{\beta\gamma}\tilde{S}_{\gamma}$$

$$\Gamma_{11}F = F\,,F\Gamma_{11} = -\xi F$$

$$F_{\alpha\beta} = \sum_{k=0}^{10}\frac{i^{k}}{k!}F_{\mu_1\cdots\mu_k}(\Gamma^{\mu_1\cdots\mu_k})_{\alpha\beta}$$

$$F^{\mu_1\cdots\mu_k} = \frac{(-1)^{\left[\frac{k+1}{2}\right]}}{(10-k)!}\epsilon^{\mu_1\cdots\mu_{10}}F_{\mu_{k+1}\cdots\mu_{10}}$$

$$F^{\mu_1\cdots\mu_k} = \xi\frac{(-1)^{\left[\frac{k}{2}\right]+1}}{(10-k)!}\epsilon^{\mu_1\cdots\mu_{10}}F_{\mu_{k+1}\cdots\mu_{10}}$$

$$(p_{\mu}\Gamma^{\mu})F = F(p_{\mu}\Gamma^{\mu}) = 0$$

$$p^{[\mu}F^{\nu_1\cdots\nu_k]} = p_{\mu}F^{\mu\nu_2\cdots\nu_k} = 0$$

$$dF = d^{*}F = 0$$

$$F_{\mu_1\cdots\mu_k} = \frac{1}{(k-1)!}\partial_{[\mu_1}C_{\mu_2\cdots\mu_k]}$$

$$F_{(k)} = dC_{(k-1)}$$

$$X(\sigma+2\pi) = X(2\pi-\sigma)$$

$$Z_{\text{compact}}(\bar{q}) = \sum_{L_{16}} \frac{\bar{q}^{\frac{\bar{p}_R^2}{2}}}{\bar{\eta}^{16}} = \frac{\bar{\Gamma}_{16}(\bar{q})}{\bar{\eta}^{16}}$$

$$Z^{\text{heterotic}} = \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8} \frac{\bar{\Gamma}_{16}}{\bar{\eta}^{16}} \frac{1}{2} \sum_{a,b=0}^{1} (-1)^{a+b+ab} \frac{\vartheta \begin{bmatrix} a \\ b \end{bmatrix}^4}{\eta^4}$$

$$\tau \to -\frac{1}{\tau}: \bar{\Gamma}_{16} \to \bar{\tau}^8 \bar{\Gamma}_{16}$$

$$\bar{\Gamma}_{\mathrm{E}_8 \times \mathrm{E}_8} = (\bar{\Gamma}_8)^2 = \left[\frac{1}{2} \sum_{a,b=0,1} \bar{\vartheta} \begin{bmatrix} a \\ b \end{bmatrix}^8\right] = 1 + 2 \cdot 240\bar{q} + \mathcal{O}(\bar{q}^2).$$

$$\bar{\Gamma}_{\mathrm{O}(32)/\mathrm{Z}_2} = \frac{1}{2} \sum_{a,b=0,1} \bar{\vartheta} \begin{bmatrix} a \\ b \end{bmatrix}^{16} = 1 + 480\bar{q} + \mathcal{O}(\bar{q}^2)$$

$$\bar{J}^{ij} = i\bar{\psi}^i \bar{\psi}^j$$

$$L_0 = \frac{1}{2}, \bar{L}_0 = 1$$

$$G_0 = 0, \bar{L}_0 = 1$$

$$Z_{\text{fermions}} \begin{bmatrix} h \\ g \end{bmatrix} = \frac{1}{2} \sum_{a,b=0}^{1} (-1)^{a+b+ab+ag+bh+gh} \frac{\vartheta^4 \begin{bmatrix} a \\ b \end{bmatrix}}{\eta^4}$$

$$\bar{Z}_{E_8} \begin{bmatrix} h \\ g \end{bmatrix} = \frac{1}{2} \sum_{\gamma,\delta=0}^{1} (-1)^{\gamma g + \delta h} \frac{\bar{\vartheta}^8 \begin{bmatrix} \gamma \\ \delta \end{bmatrix}}{\bar{\eta}^8}$$

$$Z^{\text{heterotic}}_{\mathrm{O}(16)\times\mathrm{O}(16)} = \frac{1}{2} \sum_{h,g=0}^{1} \frac{\bar{Z}_{E_8} \begin{bmatrix} h \\ g \end{bmatrix}^2}{(\sqrt{\tau_2}\eta\bar{\eta})^8} \frac{1}{2} \sum_{a,b=0}^{1} (-1)^{a+b+ab+ag+bh+gh} \frac{\vartheta^4 \begin{bmatrix} a \\ b \end{bmatrix}}{\eta^4}.$$

$$\hat{X}^\mu(z,\theta) = X^\mu(z) + \theta\psi^\mu(z)$$

$$\int dz \int d\theta V(z,\theta) = \int dz \int d\theta (V_0(z) + \theta V_{-1}(z)) = \int dz V_{-1}$$

$$V^{\text{boson}}(\epsilon, p, z, \theta) = \epsilon_\mu : D\hat{X}^\mu e^{ip\cdot\hat{X}}$$

$$V_0^{\text{boson}} = \epsilon_\mu \psi^\mu e^{ip\cdot X}, V_{-1}^{\text{boson}}(\epsilon, p, z) = \epsilon_\mu : (\partial X^\mu + ip \cdot \psi\psi^\mu) e^{ip\cdot X} :,$$

$$V_{-1}^{\text{boson}}(\epsilon, p, z) = \left[Q_{\mathrm{BRST}}, \xi(z) e^{-\phi(z)} \epsilon \cdot \psi e^{ip\cdot X}\right]$$

$$V_{-1/2}^{\text{fermion}}(u, p, z) = u^\alpha(p) : e^{-\phi(z)/2} S_\alpha(z) e^{ip\cdot X} :,$$

$$V_{1/2}^{\text{fermion}}(u,p) = \left[Q_{\text{BRST}}, \xi(z) V_{-1/2}^{\text{fermion}}(u,p,z)\right] = u^{\alpha}(p) e^{\phi/2} S_{\alpha} e^{ip\cdot X} + \cdots$$

$$Q_{\alpha} = \frac{1}{2\pi i}\oint \ dz{:}\, e^{-\phi(z)/2} S_{\alpha}(z)$$

$$\left[Q_{\alpha}, V_{-1/2}^{\text{fermion}}(u,p,z)\right] = V_{-1}^{\text{boson}}\left(\epsilon^{\mu} = u^{\beta}\gamma_{\beta\alpha}^{\mu}, p, z\right),$$

$$\left[Q_{\alpha}, V_{0}^{\text{boson}}(\epsilon,p,z)\right] = V_{-1/2}^{\text{fermion}}\left(u^{\beta} = ip^{\mu}\epsilon^{\nu}\left(\gamma_{\mu\nu}\right)_{\alpha}^{\beta}, p, z\right)$$

Acciones supersimétricas y de supergravedad. Métrica de Yang – Mills - Chern-Simons. Superinvariancias y supercovariancias.

$$\delta_{\epsilon}\phi \sim \phi^{m}\psi\epsilon\,, \delta_{\epsilon}\psi \sim \partial\phi^{m}\epsilon + \phi^{m}\psi^{2}\epsilon$$

$$L_{\text{YM}} = -\frac{1}{4}F_{\mu\nu}^{a}F^{a,\mu\nu} - \bar{\chi}^{a}\Gamma^{\mu}D_{\mu}\chi^{a}$$

$$F_{\mu\nu}^{a} = \partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a} + gf_{bc}^{a}A_{\mu}^{b}A_{\nu}^{c}$$
$$D_{\mu}\chi^{a} = \partial_{\mu}\chi^{a} + gf_{bc}^{a}A_{\mu}^{b}\chi^{c}$$

$$L_{\text{SUGRA}}^{N=1} = -\frac{1}{2\kappa^{2}}R - \frac{3}{4}\phi^{-\frac{3}{2}}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{9}{16\kappa^{2}}\frac{\partial_{\mu}\phi\partial^{\mu}\phi}{\phi^{2}} - \frac{1}{2}\bar{\psi}^{\mu}\Gamma^{\mu\nu\rho}\nabla_{\nu}\psi_{\rho} - \frac{1}{2}\bar{\lambda}\Gamma^{\mu}\nabla_{\mu}\lambda$$

$$-\frac{3\sqrt{2}}{8}\frac{\partial_{\nu}}{\phi}\bar{\psi}^{\mu}\Gamma^{\nu}\Gamma^{\mu}\lambda$$

$$+\frac{\sqrt{2}\kappa}{16}\phi^{-3/4}H_{\nu\rho\sigma}\left[\bar{\psi}_{\mu}\Gamma^{\mu\nu\rho\sigma\tau}\psi_{\tau} + +6\bar{\psi}^{\nu}\Gamma^{\rho}\psi^{\sigma} - \sqrt{2}\bar{\psi}_{\mu}\Gamma^{\nu\rho\sigma}\Gamma^{\mu}\lambda\right] + (\ \mathbb{F}_{fermion}\ )^{4}$$

$$L_{\text{SUGRA+YM}}^{N=1} = L_{\text{SUGRA}}^{N=1}{}' + \phi^{-3/4}L_{\text{YM}}'$$

$$\hat{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - \frac{\kappa}{\sqrt{2}}\omega_{\mu\nu\rho}^{CS}$$

$$\omega_{\mu\nu\rho}^{CS} = A_{\mu}^{a}F_{\nu\rho}^{a} - \frac{g}{3}f_{abc}A_{\mu}^{a}A_{\nu}^{b}A_{\rho}^{c} + \text{ cyclic}$$

$$\delta B = \frac{\kappa}{\sqrt{2}}\text{Tr}[\Lambda dA]$$

$$L^{D=11} = \frac{1}{2\kappa^{2}}\left[R - \frac{1}{2\cdot 4!}G_{4}^{2}\right] - i\bar{\psi}_{\mu}\Gamma^{\mu\nu\rho}\tilde{\nabla}_{\nu}\psi_{\rho} + \frac{1}{2\kappa^{2}(144)^{2}}G_{4}\wedge G_{4}\wedge\hat{C}$$

$$+\frac{1}{192}\left[\bar{\psi}_{\mu}\Gamma^{\mu\nu\rho\sigma\tau\upsilon}\psi_{\upsilon} + 12\bar{\psi}^{\nu}\Gamma^{\rho\sigma}\psi^{\tau}\right](G+\hat{G})_{\nu\rho\sigma\tau}$$

$$\tilde{\omega}_{\mu,ab} = \omega_{\mu,ab} + \frac{i\kappa^{2}}{4}\left[-\bar{\psi}^{\nu}\Gamma_{\nu\mu ab\rho}\psi^{\rho} + 2\left(\bar{\psi}_{\mu}\Gamma_{b}\psi_{a} - \bar{\psi}_{\mu}\Gamma_{a}\psi_{b} + \bar{\psi}_{b}\Gamma_{\mu}\psi_{a}\right)\right]$$

$$G_{\mu\nu\rho\sigma} = \partial_{\mu}\hat{C}_{\nu\rho\sigma} - \partial_{\nu}\hat{C}_{\rho\sigma\mu} + \partial_{\rho}\hat{C}_{\sigma\mu\nu} - \partial_{\sigma}\hat{C}_{\mu\nu\rho}$$

$$\tilde{G}_{\mu\nu\rho\sigma} = G_{\mu\nu\rho\sigma} - 6\kappa^2 \bar{\psi}_{[\mu}\Gamma_{\nu\rho}\psi_{\sigma]}$$

$$G_{\mu\nu} = \begin{pmatrix} g_{\mu\nu} + e^{2\sigma}A_\mu A_\nu & e^{2\sigma}A_\mu \\ e^{2\sigma}A_\mu & e^{2\sigma} \end{pmatrix}$$

$$C_{\mu\nu\rho} = \hat{C}_{\mu\nu\rho} - \left(\hat{C}_{\nu\rho,11}A_\mu + \text{ cyclic }\right), B_{\mu\nu} = \hat{C}_{\mu\nu,11}$$

$$S^{IIA} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{g}e^{\sigma}\left[R - \frac{1}{2\cdot 4!}\hat{G}^2 - \frac{1}{2\cdot 3!}e^{-2\sigma}H^2 - \frac{1}{4}e^{2\sigma}F^2\right] + \frac{1}{2\kappa^2(48)^2}\int B\wedge G\wedge G$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \text{ cíclico}$$

$$\hat{G}_{\mu\nu\rho\sigma} = G_{\mu\nu\rho\sigma} + \left(F_{\mu\nu}B_{\rho\sigma} + 5 \text{ permutaciones }\right).$$

$$\tilde{S}_{10} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{g}e^{-\Phi}\left[\left(R + (\nabla\Phi)^2 - \frac{1}{12}H^2\right) - \frac{1}{2\cdot 4!}\hat{G}^2 - \frac{1}{4}F^2\right] + \frac{1}{2\kappa^2(48)^2}\int B\wedge G\wedge G$$

$$S = \chi + ie^{-\phi/2}$$

$$S \to \frac{aS+b}{cS+d}, \begin{pmatrix} B^N_{\mu\nu} \\ B^R_{\mu\nu} \end{pmatrix} \to \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}\begin{pmatrix} B^N_{\mu\nu} \\ B^R_{\mu\nu} \end{pmatrix}$$

$$S^{IIB} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-\det g}\left[R - \frac{1}{2}\frac{\partial S\partial\bar{S}}{S_2^2} - \frac{1}{12}\frac{|H^R + SH^N|^2}{S_2}\right]$$

$$J^\mu = \frac{\delta\Gamma^{\text{eff}}}{\delta A_\mu}, T^{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta\Gamma^{\text{eff}}}{\delta g_{\mu\nu}}$$

$$\delta_\Lambda\Gamma^{\text{eff}} = \text{Tr}\int D_\mu\Lambda\frac{\delta\Gamma^{\text{eff}}}{\delta A_\mu} = \text{Tr}\int \Lambda D_\mu\frac{\delta\Gamma^{\text{eff}}}{\delta A_\mu} = \int \text{Tr}\left[\Lambda D_\mu J^\mu\right]$$

$$\delta_{diff}\Gamma^{\text{eff}} = \int \left(\nabla^\mu\epsilon^\nu + \nabla^\nu\epsilon^\mu\right)\frac{\delta\Gamma^{\text{eff}}}{\delta g_{\mu\nu}} = \int \epsilon^\mu\nabla_\nu T^{\mu\nu}$$

$$\delta\Gamma|_{\text{gauge}} \sim \int d^{10}x\left[c_1\text{Tr}\left[\Lambda F_0^5\right] + c_2\text{Tr}[\Lambda F_0]\text{Tr}[F_0^4] + c_3\text{Tr}[\Lambda F_0](\text{Tr}[F_0^2])^2\right]$$

$$\delta\Gamma|_{\text{grav}} \sim \int d^{10}x\left[d_1\text{Tr}\left[\Theta R_0^5\right] + d_2\text{Tr}[\Theta R_0]\text{Tr}[R_0^4] + d_3\text{Tr}[\Theta R_0](\text{Tr}[R_0^2])^2\right]$$

$$\delta\Gamma|_{\text{mixed}} \sim \int d^{10}x[e_1\text{Tr}[\Lambda F_0]\text{Tr}[R_0^4] + e_2\text{Tr}[\Theta R_0]\text{Tr}[F_0^4]$$

$$+ + e_3\text{Tr}[\Theta R_0](\text{Tr}[F_0^2])^2 + e_4\text{Tr}[\Lambda F_0](\text{Tr}[R_0])^2]$$

- Wess-Zumino:

$$\delta_{\Lambda_1}G(\Lambda_2) - \delta_{\Lambda_2}G(\Lambda_1) = G([\Lambda_1, \Lambda_2])$$

$$\delta F = [F,\Lambda]\,, \delta R = [R,\Theta]$$

$$d\mathrm{Tr}[R^m] = d\mathrm{Tr}[F^m] = 0$$

$$I^{D+2}(R,F) = d\Omega^{D+1}(\omega,A)$$

$$\delta_\Lambda \Omega^{D+1}(\omega,A) = d\Omega^{D}(\omega,A,\Lambda)$$

- Mecanismo de Green-Schwarz:

$$\delta\Gamma|_{\mathrm{reduc}} \sim \int d^{10}x(\mathrm{Tr}[\Lambda F_0] + \mathrm{Tr}[\Theta R_0])(a_1 \mathrm{Tr}[F_0^4]$$

$$+ a_2\mathrm{Tr}[R_0^4] + a_3(\mathrm{Tr}[F_0^2])^2 + a_4(\mathrm{Tr}[R_0^2])^2 + a_5\mathrm{Tr}[F_0^2]\mathrm{Tr}[R_0^2])$$

$$\hat{H} = dB + \Omega^{CS}(A) + \Omega^{CS}(\omega)$$

$$\delta_\Lambda \Omega^{CS}(A) = d\mathrm{Tr}[\Lambda dA]\,, \delta_\Theta \Omega^{CS}(\omega) = d\mathrm{Tr}[\Theta d\omega]$$

$$\delta B = -\mathrm{Tr}[\Lambda F_0 + \Theta R_0]$$

$$\Gamma_{\mathrm{counter}} \sim \int d^{10}xB(a_1\mathrm{Tr}[F_0^4] + a_2\mathrm{Tr}[R_0^4] + a_3(\mathrm{Tr}[F_0^2])^2 +$$

$$+a_4(\mathrm{Tr}[R_0^2])^2 + a_5\mathrm{Tr}[F_0^2]\mathrm{Tr}[R_0^2])$$

$$F_{\mu_1\dots\mu_{D/2}} = \pm\frac{i}{(D/2)!}\epsilon_{\mu_1\dots\mu_D}F^{\mu_{D/2+1}\dots\mu_D}$$

$$R_0 = \begin{pmatrix} 0 & x_1 & 0 & 0 & & \cdots & \\ -x_1 & 0 & 0 & 0 & & \cdots & \\ 0 & 0 & 0 & x_2 & & \cdots & \\ 0 & 0 & -x_2 & 0 & & \cdots & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & \cdots & & & & 0 & x_{D/2} \\ & \cdots & & & & -x_{D/2} & 0 \end{pmatrix}$$

$$\hat{I}_{1/2}(R) = \prod_{i=1}^{D/2}\left(\frac{x_i/2}{\sinh{(x_i/2)}}\right)$$

$$I_{3/2}(R) = \hat{I}_{1/2}(R)\left(-1 + 2\sum_{i=1}^{D/2}\cosh{(x_i)}\right)$$

$$I_A(R) = -\frac{1}{8}\prod_{i=1}^{D/2}\left(\frac{x_i}{\tanh{(x_i)}}\right)$$

$$I_{1/2}(R,F) = \mathrm{Tr}\left[e^{iF}\right]\hat{I}_{1/2}(R)$$

$$I_{1/2}(R,F)\big|_{12-\text{ form}} = -\frac{\mathrm{Tr}[F^6]}{720} + \frac{\mathrm{Tr}[F^4]\mathrm{Tr}[R^2]}{24\cdot 48} + \\ -\frac{\mathrm{Tr}[F^2]}{256}\left(\frac{\mathrm{Tr}[R^4]}{45} + \frac{(\mathrm{Tr}[R^2])^2}{36}\right) + \\ +\frac{n}{64}\left(\frac{\mathrm{Tr}[R^6]}{5670} + \frac{\mathrm{Tr}[R^2]\mathrm{Tr}[R^4]}{4320} + \frac{(\mathrm{Tr}[R^2])^3}{10368}\right)$$

$$I_{3/2}(R)\big|_{12-\text{ form}} = -\frac{495}{64}\left(\frac{\mathrm{Tr}[R^6]}{5670} + \frac{\mathrm{Tr}[R^2]\mathrm{Tr}[R^4]}{4320} + \frac{(\mathrm{Tr}[R^2])^3}{10368}\right) + \\ +\frac{\mathrm{Tr}[R^2]}{384}\left(\mathrm{Tr}[R^4] + \frac{(\mathrm{Tr}[R^2])^2}{4}\right)$$

$$I_A(R)|_{12-\text{ form}} = \hat{I}_{1/2}(R)\big|_{12-\text{ form}} - I_{3/2}(R)\big|_{12-\text{ form}}$$

$$2I^{N=1} = I_{3/2}(R) - I_{1/2}(R) + I_{1/2}(R,F)$$

$$96I^{\text{total}} = -\frac{\mathrm{Tr}[F^6]}{15} + \frac{\mathrm{Tr}[R^2]\mathrm{Tr}[F^4]}{24} + \frac{\mathrm{Tr}[R^2]\mathrm{Tr}[R^4]}{8} + \frac{(\mathrm{Tr}[R^2])^3}{32} - \\ -\frac{\mathrm{Tr}[F^2]}{960}(4\mathrm{Tr}[R^4] + 5(\mathrm{Tr}[R^2])^2)$$

$$\mathrm{Tr}[F^6] = \frac{1}{48}\mathrm{Tr}[F^2]\mathrm{Tr}[F^4] - \frac{1}{14400}(\mathrm{Tr}[F^2])^3.$$

$$96I^{\text{total}} = \left(\mathrm{Tr}[R^2] - \frac{1}{30}\mathrm{Tr}[F^2]\right)X_8$$

$$X_8 = \frac{\mathrm{Tr}[F^4]}{24} - \frac{(\mathrm{Tr}[F^2])^2}{720} - \frac{\mathrm{Tr}[F^2]\mathrm{Tr}[R^2]}{240} + \frac{\mathrm{Tr}[R^4]}{8} + \frac{(\mathrm{Tr}[R^2])^2}{32},$$

$$\mathrm{Tr}[F^6] = (N-32)\mathrm{tr}[F^6] + 15\mathrm{tr}[F^2]\mathrm{tr}[F^4]$$
$$\mathrm{Tr}[F^4] = (N-8)\mathrm{tr}[F^4] + 3(\mathrm{tr}[F^2])^2\,, \mathrm{Tr}[F^2] = (N-2)\mathrm{tr}[F^2]$$

$$\mathrm{Tr}[F^6] = \frac{1}{7200}(\mathrm{Tr}[F^2])^3\,, \mathrm{Tr}[F^4] = \frac{1}{100}(\mathrm{Tr}[F^2])^2$$

$$d\hat{H} = \mathrm{tr}[R^2] - \frac{1}{30}\mathrm{Tr}[F^2]$$

$$\int \mathrm{tr}[R^2] = \frac{1}{30}\int \mathrm{Tr}[F^2]$$

$$\mathrm{tr}_S[F^6] = 16\mathrm{tr}[F^6] - 15\mathrm{tr}[F^2]\mathrm{tr}[F^4] + \frac{15}{4}(\mathrm{tr}[F^2])^3$$
$$\mathrm{tr}_S[F^4] = -8\mathrm{tr}[F^4] + 6(\mathrm{tr}[F^2])^2\,, \mathrm{tr}_S[F^2] = 16\mathrm{tr}[F^2]$$

Compactificación y rompimiento de supersimetría. Métrica de Kaluza-Klein.

$$Z_D^{\text{heterotic}} = \frac{\Gamma_{10-D,10-D}(G,B)\bar{\Gamma}_H}{\tau_2^{\frac{D-2}{2}}\,\eta^8\bar{\eta}^8}\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\frac{\vartheta^4\begin{bmatrix}a\\b\end{bmatrix}}{\eta^4},$$

$$V_{\text{Higgs}} \sim f^{a}{}_{bc} f^{a}{}_{b'c'} G^{\alpha\gamma} G^{\beta\delta} Y_{\alpha}^{b} Y_{\beta}^{c} Y_{\gamma}^{b'} Y_{\delta}^{c'}$$

$$[m^2]^{ab} \sim G^{\alpha\beta} f_{d}^{ca} f_{d}^{cb'} Y_{\alpha}^{d} Y_{\beta}^{d'}$$

$$Z_D^{\text{heterotic}} = \frac{\Gamma_{10-D,26-D}(G,B,Y)}{\tau_2^{\frac{D-2}{2}} \eta^8 \bar{\eta}^8} \frac{1}{2} \sum_{a,b=0}^{1} (-1)^{a+b+ab} \frac{\vartheta^4 \begin{bmatrix} a \\ b \end{bmatrix}}{\eta^4},$$

$$\alpha'^8 S_{10-d}^{\text{heterotic}} = \int d^{10}x \sqrt{-\det G_{10}}\, e^{-\Phi} \left[R + (\nabla\Phi)^2 - \frac{1}{12} \hat{H}^2 - \frac{1}{4} \mathrm{Tr}[F^2] \right] + \mathcal{O}(\alpha')$$

$$F_{\mu\nu}^{I} = \partial_\mu A_\nu^I - \partial_\nu A_\mu^I$$

$$\hat{H}_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \frac{1}{2} \sum_I A_\mu^I F_{\nu\rho}^I + \text{ cyclic}$$

$$S_D^{\text{heterotic}} = \int d^D x \sqrt{-\det G}\, e^{-\Phi} \Big[R + \partial^\mu \Phi \partial_\mu \Phi - \frac{1}{12} \hat{H}^{\mu\nu\rho} \hat{H}_{\mu\nu\rho} - \\ - \frac{1}{4} \left(\hat{M}^{-1}\right)_{ij} F_{\mu\nu}^{i} F^{j\mu\nu} + \frac{1}{8} \mathrm{Tr}\left(\partial_\mu \hat{M} \partial^\mu \hat{M}^{-1}\right) \Big]$$

$$\hat{H}_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \frac{1}{2} L_{ij} A_\mu^i F_{\nu\rho}^j + \text{ cyclic}$$

$$\hat{M} \to \Omega \hat{M} \Omega^T, A_\mu \to \Omega \cdot A_\mu$$

$$S_D^{\text{heterotic}} = \int d^D x \sqrt{-\det G_E} \Big[R - \frac{1}{D-2} \partial^\mu \Phi \partial_\mu \Phi$$

$$- \frac{e^{-\frac{4\Phi}{D-2}}}{12} \hat{H}^{\mu\nu\rho} \hat{H}_{\mu\nu\rho} - \frac{e^{-\frac{2\Phi}{D-2}}}{4} \left(\hat{M}^{-1}\right)_{ij} F_{\mu\nu}^{i} F^{j\mu\nu} + \frac{1}{8} \mathrm{Tr}\left(\partial_\mu \hat{M} \partial^\mu \hat{M}^{-1}\right) \Big]$$

$$e^{-2\phi} \hat{H}_{\mu\nu\rho} = \frac{\epsilon_{\mu\nu\rho}^{\sigma}}{\sqrt{-\det g_E}} \nabla_\sigma a$$

$$\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-\det g_E}} \partial_\mu \hat{H}_{\nu\rho\sigma} = -L_{ij} F_{\mu\nu}^{i} \tilde{F}^{j,\mu\nu},$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-\det g_E}} F_{\rho\sigma},$$

$$\nabla^\mu e^{2\phi} \nabla_\mu a = -\frac{1}{4} F_{\mu\nu}^{i} \tilde{F}^{j,\mu\nu}$$

$$\tilde{S}_{D=4}^{\text{heterotic}} = \int d^4x\sqrt{-\det g_E}\Big[R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}e^{2\phi}\partial^\mu a\partial_\mu a \pm \frac{1}{4}e^{-\phi}(M^{-1})_{ij}F_{\mu\nu}^{i}F^{j,\mu\nu}$$

$$+\frac{1}{4}aL_{ij}F_{\mu\nu}^{i}\tilde{F}^{j,\mu\nu} + \frac{1}{8}\mathrm{Tr}\big(\partial_\mu M\partial^\mu M^{-1}\big)\Big]$$

$$S = a + ie^{-\phi}$$

$$\tilde{S}_{D=4}^{\text{heterotic}} = \int d^4x\sqrt{-\det g_E}\Big[R - \frac{1}{2}\frac{\partial^\mu S\partial_\mu\bar{S}}{\mathrm{Im}S^2} - \frac{1}{4}\mathrm{Im}S(M^{-1})_{ij}F_{\mu\nu}^{i}F^{j,\mu\nu} + \frac{1}{4}\mathrm{Re}SL_{ij}F_{\mu\nu}^{i}\tilde{F}^{j,\mu\nu}$$

$$+\frac{1}{8}\mathrm{Tr}\big(\partial_\mu M\partial^\mu M^{-1}\big)\Big]$$

$$\delta\psi_\mu = \nabla_\mu\epsilon + \frac{\sqrt{2}}{32}e^{2\Phi}\big(\gamma_\mu\gamma_5\otimes H\big)\epsilon,$$
$$\delta\psi_m = \nabla_m\epsilon + \frac{\sqrt{2}}{32}e^{2\Phi}(\gamma_m H - 12H_m)\epsilon,$$
$$\delta\lambda = \sqrt{2}(\gamma^m\nabla_m\Phi)\epsilon + \frac{1}{8}e^{2\Phi}H\epsilon,$$
$$\delta\chi^a = -\frac{1}{4}e^{\Phi}F_{m,n}^{a}\gamma^{mn}\epsilon,$$

$$H = H_{mnr}\gamma^{mnr}\,, H_m = H_{mnr}\gamma^{nr}$$

$$\Gamma^\mu = \gamma^\mu\otimes\mathbf{1}_6\,, \Gamma^m = \gamma^5\otimes\gamma^m$$
$$\gamma^5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^{\mu\nu\rho\sigma}\,, \gamma = \frac{i}{6!}\sqrt{\det g}\epsilon_{mnrpqs}\gamma^{mnrpqs}$$

$$\nabla_m\xi = 0$$

$$F_{mn}^{a}\gamma^{mn}\xi = 0$$

$$R_{[mn}^{rs}R_{pq]rs} = \frac{1}{30}F_{[mn}^{a}F_{pq]}^{a}$$

$$G_{MN} \sim h_{\mu\nu}(x)\otimes\phi(y) + A_\mu(x)\otimes f_m(y) + \Phi(x)\otimes h_{mn}(y)$$

$$\Box_y\,\phi(y) = 0$$

- Lichnerowicz:

$$-\Box\, h_{mn} + 2R_{mnrs}h^{rs} = 0\,, \nabla^m h_{mn} = g^{mn}h_{mn} = 0.$$

$$\Box\, f_{mn} - R_{mnrs}f^{rs} = \Box\, f_{mn} + 2R_{mrsn}f^{rs} = 0,$$
$$\nabla_m A_{np} + \nabla_p A_{mn} + \nabla_n A_{pm} = 0, \nabla^m A_{mn} = 0$$

$$h_{mn} = A_m^p S_{pm} + A_n^p S_{pm}$$

$$B_{MN} \sim B_{\mu\nu}(x)\otimes\phi(y) + B_\mu(x)\otimes f_m(y) + \Phi(x)\otimes B_{mn}(y)$$

$$S_{\mathrm{K3}}^{IIA} = \int d^6x\sqrt{-\det G_6}\,e^{-\Phi}\left[R + \nabla^{\mu}\Phi\nabla_{\mu}\Phi - \frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho}\right.$$

$$\left. + \frac{1}{8}\mathrm{Tr}\left(\partial_{\mu}\hat{M}\partial^{\mu}\hat{M}^{-1}\right) + \frac{1}{4}\int d^6x\sqrt{-\det G}\left(\hat{M}^{-1}\right)_{IJ}F_{\mu\nu}^{I}F^{J\mu\nu}\right]$$

$$+ \frac{1}{16}\int d^6x\,\epsilon^{\mu\nu\rho\sigma\tau\upsilon}B_{\mu\nu}F_{\rho\sigma}^{I}\hat{L}_{IJ}F_{\tau\upsilon}^{J}$$

Espacio – tiempo cuántico supersimétrico.

$$\left\{Q_{\alpha}^{I}, Q_{\beta}^{J}\right\} = \epsilon_{\alpha\beta}Z^{IJ}$$
$$\left\{\bar{Q}_{\dot{\alpha}}^{I}, Q_{\beta}^{J}\right\} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{Z}^{IJ}$$
$$\left\{Q_{\alpha}^{I}, \bar{Q}_{\dot{\alpha}}^{J}\right\} = \delta^{IJ}\sigma_{\alpha\dot{\alpha}}^{\mu}P_{\mu}$$

$$Q_{\alpha}^{I} = \frac{1}{2\pi i}\oint dz e^{-\phi/2}S_{\alpha}\Sigma^{I}\,, \bar{Q}_{\dot{\alpha}}^{I} = \frac{1}{2\pi i}\oint dz e^{-\phi/2}C_{\dot{\alpha}}\bar{\Sigma}^{I}$$

$$:e^{q_1\phi(z)}::e^{q_2\phi(w)}:= (z-w)^{-q_1q_2}:e^{(q_1+q_2)\phi(w)}:+\cdots,$$
$$S_{\alpha}(z)C_{\dot{\alpha}}(w) = \sigma_{\alpha\dot{\alpha}}^{\mu}\psi^{\mu}(w) + \mathcal{O}(z-w),$$
$$S_{\alpha}(z)S_{\beta}(w) = \frac{\epsilon_{\alpha\beta}}{\sqrt{z-w}} + \mathcal{O}(\sqrt{z-w}),$$
$$C_{\dot{\alpha}}(z)C_{\dot{\beta}}(w) = \frac{\epsilon_{\dot{\alpha}\dot{\beta}}}{\sqrt{z-w}} + \mathcal{O}(\sqrt{z-w}),$$

$$\Sigma^{I}(z)\bar{\Sigma}^{J}(w) = \frac{\delta^{IJ}}{(z-w)^{3/4}} + (z-w)^{1/4}J^{IJ}(w) + \cdots$$
$$\Sigma^{I}(z)\Sigma^{J}(w) = (z-w)^{-1/4}\Psi^{IJ}(w) + \cdots$$
$$\bar{\Sigma}^{I}(z)\bar{\Sigma}^{J}(w) = (z-w)^{-1/4}\bar{\Psi}^{IJ}(w) + \cdots$$

$$G^{\mathrm{int}}(z)\Sigma^{I}(w) \sim (z-w)^{-1/2}\,, G^{\mathrm{int}}(z)\bar{\Sigma}^{I}(w) \sim (z-w)^{-1/2}$$

$$\lambda^{I}(z) = \Sigma^{I}(z)e^{iX/2}\,, \bar{\lambda}(z) = \bar{\Sigma}^{I}(z)e^{-iX/2}$$

$$\lambda^{I}(z)\bar{\lambda}^{J}(w) = \frac{\delta^{IJ}}{z-w} + \hat{J}^{IJ} + \mathcal{O}(z-w),$$
$$\lambda^{I}(z)\lambda^{J}(w) = e^{iX}\Psi^{IJ} + \mathcal{O}(z-w),$$
$$\bar{\lambda}^{I}(z)\bar{\lambda}^{J}(w) = e^{-iX}\bar{\Psi}^{IJ} + \mathcal{O}(z-w),$$

$$\hat{J}^{II}(z)\hat{J}^{JJ}(w) = \frac{\delta^{IJ}}{(z-w)^2} + \text{ regular}$$

$$J^{II}(z)J^{JJ}(w) = \frac{\delta^{IJ}-3/4}{(z-w)^2} + \text{ regular}$$

$$J = 2J^{11}\,, J(z)J(w) = \frac{3}{(z-w)^2} + \text{ regular}$$

$$\langle J(z_1)\Sigma(z_2)\bar{\Sigma}(z_3)\rangle = \frac{3}{2}\frac{z_{23}^{1/4}}{z_{12}z_{13}}$$

$$J = i\sqrt{3}\partial\Phi\,, \Sigma = e^{i\sqrt{3}\Phi/2}W^{+}\,, \bar{\Sigma} = e^{-i\sqrt{3}\Phi/2}W^{-}$$

$$G^{int} = \sum_{q\geq 0} e^{iq\Phi}T^{(q)} + e^{-q\Phi}T^{(-q)}$$

$$J(z)G^{\pm}(w) = \pm\frac{G^{\pm}(w)}{(z-w)} + \cdots$$

$$|h,q;\bar{h}\rangle:\ |0,0;0\rangle, |0,0;1\rangle^{I}, |1/2,\pm 1;1\rangle^{i}$$

$$J^{s}(z)J^{s}(w) = \frac{1}{(z-w)^2} + \cdots$$
$$J^{3}(z)J^{3}(w) = \frac{1/2}{(z-w)^2} + \cdots$$
$$J^{s}(z)J^{3}(w) = \cdots$$

$$J^{s} = i\partial\phi\,, J^{3} = \frac{i}{\sqrt{2}}\partial\chi$$
$$\Sigma^{1} = \exp\left[\frac{i}{2}\phi + \frac{i}{\sqrt{2}}\chi\right]\,, \Sigma^{2} = \exp\left[\frac{i}{2}\phi - \frac{i}{\sqrt{2}}\chi\right]$$
$$\bar{\Sigma}^{1} = \exp\left[-\frac{i}{2}\phi - \frac{i}{\sqrt{2}}\chi\right], \bar{\Sigma}^{2} = \exp\left[-\frac{i}{2}\phi + \frac{i}{\sqrt{2}}\chi\right].$$

$$G^{\text{int}} = G_{(2)} + G_{(4)}, G_{(2)} = G_{(2)}^{+} + G_{(2)}^{-}$$
$$G_{(4)} = G_{(4)}^{+} + G_{(4)}^{-},$$
$$J^{s}(z)G_{(2)}^{\pm}(w) = \pm\frac{G_{(2)}^{\pm}(w)}{z-w} + \cdots,$$
$$J^{3}(z)G_{(4)}^{\pm}(w) = \pm\frac{1}{2}\frac{G_{(4)}^{\pm}(w)}{z-w} + \cdots,$$
$$J^{s}(z)G_{(4)}^{\pm}(w) = \text{ finite }, J^{3}(z)G_{(2)}^{\pm}(w) = \text{ finite}$$
$$G_{(2)}^{\pm} = e^{\pm i\phi}Z^{\pm}.$$

$$\psi^{0} = \frac{1}{\sqrt{2}}(\psi^{3} + i\psi^{4}), \psi^{1} = \frac{1}{\sqrt{2}}(\psi^{5} + i\psi^{6})$$
$$\psi^{2} = \frac{1}{\sqrt{2}}(\psi^{7} + i\psi^{8}), \psi^{3} = \frac{1}{\sqrt{2}}(\psi^{9} + i\psi^{10})$$

$$\langle\psi^{I}(z)\bar{\psi}^{J}(w)\rangle = \frac{\delta^{IJ}}{z-w}\,, \langle\psi^{I}(z)\psi^{J}(w)\rangle = \langle\bar{\psi}^{I}(z)\bar{\psi}^{J}(w)\rangle = 0$$

$$J^{I}(z) = i\partial_{z}\phi^{I}(z)\,, \langle\phi^{I}(z)\phi^{J}(w)\rangle = -\delta^{IJ}\log(z-w)$$

$$\psi^{I} =: e^{i\phi^{I}}:, \bar{\psi}^{I} =: e^{-i\phi^{I}}:$$

$$V(\epsilon_I) =: \exp\left[\frac{i}{2}\sum_{I=0}^{3}\epsilon_I\phi^I\right]:$$

$$\psi^I \to e^{2\pi i\theta^I}\psi^I\,, \bar{\psi}^I \to e^{-2\pi i\theta^I}\bar{\psi}^I$$

$$\phi^I \to \phi^I + 2\pi\theta^I$$

$$V^{\pm,\epsilon} = \bar{\partial}X^\pm V_S(\epsilon)e^{ip\cdot X}\,, X^\pm = \frac{1}{\sqrt{2}}(X^3 \pm iX^4)$$

$$G^{\text{int}} = \sum_{i=5}^{10}\psi^i\partial X^i$$

$$\phi^2 \to \phi^2 + \pi\,, \phi^3 \to \phi^3 - \pi$$

$$[\mathbf{120}] \to [\mathbf{3},\mathbf{1},\mathbf{1}] \oplus [\mathbf{1},\mathbf{3},\mathbf{1}] \oplus [\mathbf{1},\mathbf{1},\mathbf{66}] \oplus [\mathbf{2},\mathbf{1},\mathbf{12}] \oplus [\mathbf{1},\mathbf{2},\mathbf{12}]$$

$$\in \mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{O}(12)[\mathbf{128}] \to [\mathbf{2},\mathbf{1},\mathbf{32}] \oplus [\mathbf{1},\bar{\mathbf{2}},\mathbf{32}]$$

$$\in \mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{O}(12)$$

$$\mathrm{E}_8 \ni [\mathbf{248}] \to [\mathbf{1},\mathbf{133}] \oplus [\mathbf{3},\mathbf{1}] \oplus [\mathbf{2},\mathbf{56}] \in \mathrm{SU}(2)\times\mathrm{E}_7$$

$$\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\frac{\vartheta^2\begin{bmatrix}a\\b\end{bmatrix}\vartheta\begin{bmatrix}a+h\\b+g\end{bmatrix}\vartheta\begin{bmatrix}a-h\\b-g\end{bmatrix}}{\eta^4}$$

$$Z_{(4,4)}\begin{bmatrix}0\\0\end{bmatrix} = \frac{\Gamma_{4,4}}{\eta^4\bar{\eta}^4}\,, Z_{(4,4)}\begin{bmatrix}h\\g\end{bmatrix} = 2^4\frac{\eta^2\bar{\eta}^2}{\vartheta^2\begin{bmatrix}1-h\\1-g\end{bmatrix}\bar{\vartheta}^2\begin{bmatrix}1-h\\1-g\end{bmatrix}}\,, (h,g) \neq (0,0)$$

$$\frac{1}{2}\sum_{\gamma,\delta=0}^{1}\frac{\bar{\vartheta}\begin{bmatrix}\gamma+h\\\delta+g\end{bmatrix}\bar{\vartheta}\begin{bmatrix}\gamma-h\\\delta-g\end{bmatrix}\bar{\vartheta}^6\begin{bmatrix}\gamma\\\delta\end{bmatrix}}{\bar{\eta}^8}$$

$$Z_{N=2}^{\text{heterotic}} = \frac{1}{2}\sum_{h,g=0}^{1}\frac{\Gamma_{2,2}\bar{\Gamma}_{E_8}Z_{(4,4)}\begin{bmatrix}h\\g\end{bmatrix}}{\tau_2\eta^4\bar{\eta}^{12}}\frac{1}{2}\sum_{\gamma,\delta=0}^{1}\frac{\bar{\vartheta}\begin{bmatrix}\gamma+h\\\delta+g\end{bmatrix}\bar{\vartheta}\begin{bmatrix}\gamma-h\\\delta-g\end{bmatrix}\bar{\vartheta}^6\begin{bmatrix}\gamma\\\delta\end{bmatrix}}{\bar{\eta}^8}$$
$$\times\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\frac{\vartheta^2\begin{bmatrix}a\\b\end{bmatrix}\vartheta\begin{bmatrix}a+h\\b+g\end{bmatrix}\vartheta\begin{bmatrix}a-h\\b-g\end{bmatrix}}{\eta^4}$$

$$J^a = -\frac{i}{2}\left[\psi^0\psi^a + \frac{1}{2}\epsilon^{abc}\psi^b\psi^c\right]\,, \tilde{J}^a = -\frac{i}{2}\left[\psi^0\psi^a - \frac{1}{2}\epsilon^{abc}\psi^b\psi^c\right].$$

$$V_{\alpha\beta}^{\pm} = \pm i\left(\delta_{\alpha\beta}\psi^0 \pm i\sigma_{\alpha\beta}^a\psi^a\right)$$

$$V_{\alpha\gamma}^{+}(z)V_{\gamma\beta}^{+}(w)=V_{\alpha\gamma}^{-}(z)V_{\gamma\beta}^{-}(w)\;=\frac{\delta_{\alpha\beta}}{z-w}-2\sigma_{\alpha\beta}^{a}\left(J^{a}(w)-\tilde{J}^{a}(w)\right)+\mathcal{O}(z-w)$$
$$V_{\alpha\gamma}^{+}(z)V_{\gamma\beta}^{-}(w)\;=\frac{3\delta_{\alpha\beta}}{z-w}+4\sigma_{\alpha\beta}^{a}\tilde{J}^{a}(w)+\mathcal{O}(z-w)$$
$$V_{\alpha\gamma}^{-}(z)V_{\gamma\beta}^{+}(w)\;=\frac{3\delta_{\alpha\beta}}{z-w}-4\sigma_{\alpha\beta}^{a}J^{a}(w)+\mathcal{O}(z-w)$$

$$Z_{N=2}^{\text{heterotic}}=\frac{1}{2}\sum_{h,g=0}^{1}\frac{\Gamma_{2,18}(\epsilon)\begin{bmatrix}h\\g\end{bmatrix}Z_{(4,4)}\begin{bmatrix}h\\g\end{bmatrix}}{\tau_2\eta^4\bar{\eta}^{20}}\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\frac{\vartheta^2\begin{bmatrix}a\\b\end{bmatrix}\vartheta\begin{bmatrix}a+h\\b+g\end{bmatrix}\vartheta\begin{bmatrix}a-h\\b-g\end{bmatrix}}{\eta^4}$$

$$G=\frac{T_2-\frac{W_2^I W_2^I}{2U_2}}{U_2}\begin{pmatrix}1 & U_1\\ U_1 & |U|^2\end{pmatrix}$$
$$B=\left(T_1-\frac{W_1^I W_2^I}{2U_2}\right)\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

- Kähler:

$$f=S\left(TU-\frac{1}{2}W^I W^I\right),K=-\log\left(S_2\right)-\log\left[U_2T_2-\frac{1}{2}W_2^I W_2^I\right]$$

$$\tau_2B_2=\tau_2\langle\lambda^2\rangle=\Gamma_{2,18}\begin{bmatrix}0\\1\end{bmatrix}\frac{\bar{\vartheta}_3^2\bar{\vartheta}_4^2}{\bar{\eta}^{24}}-\Gamma_{2,18}\begin{bmatrix}1\\0\end{bmatrix}\frac{\bar{\vartheta}_2^2\bar{\vartheta}_3^2}{\bar{\eta}^{24}}-\Gamma_{2,18}\begin{bmatrix}1\\1\end{bmatrix}\frac{\bar{\vartheta}_2^2\bar{\vartheta}_4^2}{\bar{\eta}^{24}}$$

$$=\frac{\Gamma_{2,18}\begin{bmatrix}0\\0\end{bmatrix}+\Gamma_{2,18}\begin{bmatrix}0\\1\end{bmatrix}}{2}\bar{F}_1-\frac{\Gamma_{2,18}\begin{bmatrix}0\\0\end{bmatrix}-\Gamma_{2,18}\begin{bmatrix}0\\1\end{bmatrix}}{2}\bar{F}_1+\frac{\Gamma_{2,18}\begin{bmatrix}1\\0\end{bmatrix}+\Gamma_{2,18}\begin{bmatrix}1\\0\end{bmatrix}}{2}\bar{F}_+$$
$$-\frac{\Gamma_{2,18}\begin{bmatrix}1\\0\end{bmatrix}-\Gamma_{2,18}\begin{bmatrix}1\\0\end{bmatrix}}{2}\bar{F}_-$$

$$\bar{F}_1=\frac{\bar{\vartheta}_3^2\bar{\vartheta}_4^2}{\bar{\eta}^{24}},\bar{F}_\pm=\frac{\bar{\vartheta}_2^2(\bar{\vartheta}_3^2\pm\bar{\vartheta}_4^2)}{\bar{\eta}^{24}}$$

$$\tau\to\tau+1:\;B_2\to B_2,\tau\to-\frac{1}{\tau}:\;B_2\to\tau^2B_2$$

$$F_1=\frac{1}{q}+\sum_{n=0}^{\infty}d_1(n)q^n=\frac{1}{q}+16+156q+\mathcal{O}(q^2)$$
$$F_+=\frac{8}{q^{3/4}}+q^{1/4}\sum_{n=0}^{\infty}d_+(n)q^n=\frac{8}{q^{3/4}}+8q^{1/4}\left(30+481q+\mathcal{O}(q^2)\right)$$
$$F_-=\frac{32}{q^{1/4}}+q^{3/4}\sum_{n=0}^{\infty}d_-(n)q^n=\frac{32}{q^{1/4}}+32q^{3/4}\left(26+375q+\mathcal{O}(q^2)\right)$$

$$M^2=\frac{\left|-m_1U+m_2+Tn_1+\left(TU-\frac{1}{2}\vec{W}^2\right)n_2+\vec{W}\cdot\vec{Q}\right|^2}{4S_2\left(T_2U_2-\frac{1}{2}\mathrm{Im}\vec{W}^2\right)}$$

$$\rho = \vec{m} \cdot \vec{\epsilon}_R + \vec{n} \cdot \epsilon_L - \vec{Q} \cdot \vec{\zeta}$$

$$s = \vec{m} \cdot \vec{n} - \frac{1}{2} \vec{Q} \cdot \vec{Q}$$

$$M^2 = \left| \left(m_1 + \frac{1}{2} \epsilon_L^1 \right) U - \left(m_2 + \frac{1}{2} \epsilon_L^2 \right) - T \left(n_1 + \frac{1}{2} \epsilon_R^1 \right) - \left(TU - \frac{1}{2} \vec{W}^2 \right) \left(n_2 + \frac{1}{2} \epsilon_R^2 \right) \right.$$

$$+$$

$$\left. - \vec{W} \cdot \left(\vec{Q} + \frac{1}{2} \vec{\zeta} \right) \right|^2 / 4 S_2 \left(T_2 U_2 - \frac{1}{2} \mathrm{Im} \vec{W}^2 \right) (12.4.36)$$

$$s' = \left(\vec{m} + \frac{\vec{\epsilon}_L}{2} \right) \cdot \left(\vec{n} + \frac{\vec{\epsilon}_R}{2} \right) - \frac{1}{2} \left(\vec{Q} + \frac{\vec{\zeta}}{2} \right) \cdot \left(\vec{Q} + \frac{\vec{\zeta}}{2} \right),$$

$$m_L^2 = \frac{1}{4} \left(\frac{1}{R} + nR \right)^2, m_R^2 = \frac{1}{4} \left(\frac{1}{R} - nR \right)^2$$

$$Z_{Z_2 \times Z_2}^{N=1} = \frac{1}{\tau_2 \eta^2 \bar{\eta}^2} \frac{1}{4} \sum_{h_1, g_1 = 0, h_2, g_2 = 0}^{1} \frac{1}{2} \sum_{\alpha, \beta = 0}^{1} (-)^{\alpha + \beta + \alpha\beta}$$

$$\times \frac{\vartheta \begin{bmatrix} \alpha \\ \beta \end{bmatrix}}{\eta} \frac{\vartheta \begin{bmatrix} \alpha + h_1 \\ \beta + g_1 \end{bmatrix}}{\eta} \frac{\vartheta \begin{bmatrix} \alpha + h_2 \\ \beta + g_2 \end{bmatrix}}{\eta} \frac{\vartheta \begin{bmatrix} \alpha - h_1 - h_2 \\ \beta - g_1 - g_2 \end{bmatrix}}{\eta} \frac{\bar{\Gamma}_8}{\bar{\eta}^8} Z_{2,2}^1 \begin{bmatrix} h_1 \\ g_1 \end{bmatrix} \times Z_{2,2}^2 \begin{bmatrix} h_2 \\ g_2 \end{bmatrix} Z_{2,2}^3 \begin{bmatrix} h_1 + h_2 \\ g_1 + g_2 \end{bmatrix}$$

$$\times \frac{1}{2} \sum_{\bar{\alpha}, \bar{\beta} = 0}^{1} \frac{\vartheta \begin{bmatrix} \bar{\alpha} \\ \bar{\beta} \end{bmatrix}^5}{\bar{\eta}^5} \frac{\vartheta \begin{bmatrix} \bar{\alpha} + h_1 \\ \bar{\beta} + g_1 \end{bmatrix}}{\bar{\eta}} \frac{\vartheta \begin{bmatrix} \bar{\alpha} + h_2 \\ \bar{\beta} + g_2 \end{bmatrix}}{\bar{\eta}} \frac{\vartheta \begin{bmatrix} \bar{\alpha} - h_1 - h_2 \\ \bar{\beta} - g_1 - g_2 \end{bmatrix}}{\bar{\eta}}$$

$$Z_{6-d}^{II-\lambda} = \frac{1}{2} \sum_{h,g=0}^{1} \frac{Z_{(4,4)} \begin{bmatrix} h \\ g \end{bmatrix}}{\tau_2^2 \eta^4 \bar{\eta}^4} \times \frac{1}{2} \sum_{a,b=0}^{1} (-1)^{a+b+ab} \frac{\vartheta^2 \begin{bmatrix} a \\ b \end{bmatrix} \vartheta \begin{bmatrix} a+h \\ b+g \end{bmatrix} \vartheta \begin{bmatrix} a-h \\ b-g \end{bmatrix}}{\eta^4} \times$$
$$\frac{1}{2} \sum_{\bar{a}, \bar{b} = 0}^{1} (-1)^{\bar{a} + \bar{b} + \lambda \bar{a} \bar{b}} \frac{\bar{\vartheta}^2 \begin{bmatrix} \bar{a} \\ \bar{b} \end{bmatrix} \bar{\vartheta} \begin{bmatrix} \bar{a} + h \\ \bar{b} + g \end{bmatrix} \bar{\vartheta} \begin{bmatrix} \bar{a} - h \\ \bar{b} - g \end{bmatrix}}{\bar{\eta}^4}$$

$$\chi = \frac{1}{g} [\chi(M) - \chi(F)] + \chi(N).$$

$$\left. \frac{1}{g_i^2} \right|_{\text{tree}} = \frac{k_i}{g_{\text{spacetime dimensions}}^2} = k_i S_2$$

$$V_G^{\mu,a} \sim (\partial X^\mu + i(p \cdot \psi) \psi^\mu) \bar{J}^a e^{ip \cdot X}$$

$$\int d^4x \frac{1}{g^2(T_i)} F^a_{\mu\nu} F^{a,\mu\nu}$$

$$V^{IJ}_{\text{modulus}} = (\partial X^I + i(p\cdot\psi)\psi^I)\bar{\partial} X^J e^{ip\cdot X}$$

$$I_{1-\text{ loop}} = \int \langle V^{a,\mu}(p_1,z) V^{b,\nu}(p_2,w) V^{IJ}_{\text{modulus}}(p_3,0)\rangle \sim \delta^{ab}(p_1\cdot p_2 \eta^{\mu\nu} - p_1^\mu p_2^\nu) F^{IJ}(T,U) + \mathcal{O}(p^4)$$

$$F^{IJ} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \int \frac{d^2z}{\tau_2} \int d^2w \langle\psi(z)\psi(w)\rangle^2 \langle \bar{J}^a(\bar{z}) \bar{J}^b(\bar{w})\rangle \langle \partial X^I(0) \bar{\partial} X^J(0)\rangle$$

$$S\begin{bmatrix} a \\ b \end{bmatrix}(z) = \langle \psi(z)\psi(0)\rangle|^a_b = \frac{\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(z)\vartheta_1'(0)}{\vartheta_1(z)\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(0)} = \frac{1}{z} + \cdots$$

$$S^2\begin{bmatrix} a \\ b \end{bmatrix}(z) = \mathcal{P}(z) + 4\pi i \partial_\tau \log \frac{\theta\begin{bmatrix} a \\ b \end{bmatrix}(\tau)}{\eta(\tau)}$$

$$\langle\langle \psi(z)\psi(0)\rangle\rangle = \frac{1}{2} \sum_{(a,b)\neq(1,1)} (-1)^{a+b+ab} \frac{\vartheta^2\begin{bmatrix} a \\ b \end{bmatrix} \vartheta\begin{bmatrix} a+h \\ b+g \end{bmatrix} \vartheta\begin{bmatrix} a-h \\ b-g \end{bmatrix}}{\eta^4} S^2\begin{bmatrix} a \\ b \end{bmatrix}(z)$$

$$= 4\pi^2\eta^2\vartheta\begin{bmatrix} 1+h \\ 1+g \end{bmatrix}\vartheta\begin{bmatrix} 1-h \\ 1-g \end{bmatrix}$$

$$\langle \bar{J}^a(\bar{z})\bar{J}^b(0)\rangle = \frac{k\delta^{ab}}{4\pi^2} \bar{\partial}_{\bar{z}}^2 \log \bar{\vartheta}_1(\bar{z}) + \text{Tr}[J_0^a J_0^b] = \delta^{ab}\left(\frac{k}{4\pi^2}\bar{\partial}_{\bar{z}}^2 \log \bar{\vartheta}_1(\bar{z}) + \text{Tr}[Q^2]\right)$$

$$\langle \partial X^I(0)\bar{\partial} X^J(0)\rangle = \frac{\sqrt{\det G}}{(\sqrt{\tau_2}\eta\bar{\eta})^2} \sum_{\vec{m},\vec{n}} (m^I + n^I\tau)(m^J + n^J\bar{\tau}) \times$$

$$\times \exp\left[-\frac{\pi(G_{KL}+B_{KL})}{\tau_2}(m_K + n_K\tau)(m_L + n_L\bar{\tau})\right]$$

$$V_{T_i} = v_{IJ}(T_i)\partial X^I \bar{\partial} X^J$$

$$v(T) = -\frac{i}{2U_2}\begin{pmatrix} 1 & U \\ \bar{U} & |U|^2 \end{pmatrix}, v(U) = \frac{iT_2}{U_2^2}\begin{pmatrix} 1 & \bar{U} \\ \bar{U} & \bar{U}^2 \end{pmatrix}$$

$$v(\bar{T}) = \overline{v(T)}, v(\bar{U}) = \overline{v(U)}$$

$$\langle V_{T_i}\rangle = -\frac{\tau_2}{2\pi}\partial_{T_i}\frac{\Gamma_{2,2}}{\eta^2\bar{\eta}^2}$$

$$\frac{\partial}{\partial T_i}\frac{16\pi^2}{g_i^2}\bigg|_{1-\text{loop}} \sim \frac{\partial}{\partial T_i}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}\frac{\tau_2\Gamma_{2,2}}{\bar{\eta}^4}\text{Tr}_R^{int}\left[(-1)^F\left(Q_i^2 - \frac{k_i}{4\pi\tau_2}\right)\right] + \text{ constant}$$

$$\langle X(z,\bar{z})X(0)\rangle = -\log|\vartheta_1(z)|^2 + 2\pi\frac{\text{Im}z^2}{\tau_2}$$

$$\int \frac{d^2z}{\tau_2}(S^2[{}^a_b](z) - \langle X\partial X\rangle^2)\left(\frac{k}{4\pi^2}\bar{\partial}^2\log\,\bar{\vartheta}_1(\bar{z}) + \mathrm{Tr}[Q^2]\right)$$

$$= 4\pi i\partial_\tau \log\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}}{\eta}\left(\mathrm{Tr}[Q^2] - \frac{k}{4\pi\tau_2}\right)$$

$$Z_2^I = \left.\frac{16\pi^2}{g_I^2}\right|_{1-\text{ loop}} = \frac{1}{4\pi^2}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\frac{1}{\eta^2\bar{\eta}^2}\sum_{\text{even}} 4\pi i\partial_\tau\left(\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}}{\eta}\right)\mathrm{Tr}_{\text{int}}\left[Q_I^2 - \frac{k_I}{4\pi\tau_2}\right]\begin{bmatrix}a\\b\end{bmatrix}$$

$$\left.\frac{16\pi^2}{g_I^2}\right|_{1-\text{ loop}}^{\text{IR}} = \int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\mathrm{Str}Q_I^2\left(\frac{1}{12} - s^2\right)$$

$$\left.\frac{16\pi^2}{g_I^2}\right|_{1-\text{ loop}}^{\text{IR}} = b_I\log(\mu^2\alpha') + \text{ finite}$$

$$b_I = \mathrm{Str}Q_I^2\left.\left(\frac{1}{12} - s^2\right)\right|_{\text{massless}}$$

$$V_{\text{grav}} = \epsilon_{\mu\nu}(\partial X^\mu + ip\cdot\psi\psi^\mu)\bar{\partial}X^\nu$$

$$\int \frac{d^2z}{\tau_2}\langle X\bar{\partial}_{\bar{z}}^2X\rangle = \int \frac{d^2z}{\tau_2}\left(\bar{\partial}_{\bar{z}}^2\log\,\bar{\vartheta}_1(\bar{z}) + \frac{\pi}{\tau_2}\right) = 0.$$

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = (dX^0)^2 + \frac{N}{4}\left(d\alpha^2 + d\beta^2 + d\gamma^2 + 2\sin\left(\beta/\sqrt{\alpha'}\right)d\alpha d\gamma\right)$$

$$B_{\mu\nu}dX^\mu \wedge dX^\nu = \frac{N}{2}\cos\left(\beta/\sqrt{\alpha'}\right)d\alpha\wedge d\gamma\,,\Phi = \frac{X^0\sqrt{\alpha'}}{\sqrt{N+2}}$$

$$L_0 = -\frac{1}{2\alpha'} + E^2 + \frac{1}{4\alpha'(N+2)} + \frac{j(j+1)}{\alpha'(N+2)} + \cdots$$

$$\mu^2 = \frac{M_{\text{spacetime dimensions}}^2}{2(N+2)}\,,M_{\text{spacetime dimensions}} = \frac{1}{\sqrt{\alpha'}}.$$

$$Z(\mu) = \Gamma\left(\mu/M_{\text{spacetime dimensions}}\right)Z(0),$$

$$\Gamma\left(\mu/M_{\text{spacetime dimensions}}\right) = 4\sqrt{x}\frac{\partial}{\partial x}\left.[\rho(x) - \rho(x/4)]\right|_{x=N+2}$$

$$\rho(x) = \sqrt{x}\sum_{m,n\in Z}\exp\left[-\frac{\pi x}{\tau_2}|m+n\tau|^2\right]$$

$$\delta I = \int d^2z\left(A_\mu^a(X)\partial X^\mu + F_{\mu\nu}^a\psi^\mu\psi^\nu\right)\bar{J}^a$$

$$\delta I = \int d^2zB^a(J^3 + i\psi^1\psi^2)\bar{J}^a$$

$$k_I \frac{16\pi^2}{g^2_{\text{spacetime dimensions}}} + Z_2^I(\mu/M_{\text{spacetime dimensions}})$$

$$\frac{16\pi^2}{g^2_{I\,\text{bare}}} + b_I(4\pi)^\epsilon \int_0^\infty \frac{dt}{t^{1-\epsilon}} \Gamma_{\text{EFT}}\left(\frac{\mu}{\sqrt{\pi} M_{\text{spacetime dimensions}}}, t\right)$$

$$\frac{16\pi^2}{g^2_{I\,\text{bare}}} = \frac{16\pi^2}{g_I^2(\mu)} - b_I(4\pi)^\epsilon \int_0^\infty \frac{dt}{t^{1-\epsilon}} e^{-t\mu^2/M^2}$$

$$\left.\frac{16\pi^2}{g_I^2(\mu)}\right|_{\overline{DR}} = k_I \frac{16\pi^2}{g^2_{\text{spacetime dimensions}}} + Z_2^I(\mu/M_{\text{spacetime dimensions}}) - b_I(2\gamma+2),$$

$$\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \Gamma(\mu/M_{\text{spacetime dimensions}}) = \log \frac{M^2_{\text{spacetime dimensions}}}{\mu^2} + \log \frac{2e^{\gamma+3}}{\pi\sqrt{27}} + \mathcal{O}\left(\frac{\mu}{M_{\text{spacetime dimensions}}}\right)$$

$$\left.\frac{16\pi^2}{g_I^2(\mu)}\right|_{\overline{DR}} = k_I \frac{16\pi^2}{g^2_{\text{spacetime dimensions}}} + b_I \log \frac{M^2_{\text{spacetime dimensions}}}{\mu^2} + b_I \log \frac{2e^{1-\gamma}}{\pi\sqrt{27}} + \Delta_I,$$

$$\Delta_I = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \left[\frac{1}{|\eta|^4} \sum_{\text{even}} \frac{i}{\pi} \partial_\tau \left(\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}}{\eta}\right) \text{Tr}_{\text{int}} \left[Q_I^2 - \frac{k_I}{4\pi\tau_2}\right] \begin{bmatrix}a\\b\end{bmatrix} - b_I\right].$$

Umbrales gravitacionales.

$$\Delta_{\text{grav}} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \left[\frac{1}{|\eta|^4} \sum_{\text{even}} \frac{i}{\pi} \partial_\tau \left(\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}}{\eta}\right) \frac{\hat{\bar{E}}_2}{12} C^{\text{int}} \begin{bmatrix}a\\b\end{bmatrix} - b_{\text{grav}}\right]$$

$$\zeta_{U(1)} = \epsilon^1_{\mu\nu} \epsilon^2_\rho \int \frac{\delta^2 z}{\tau_2} \Big\langle (\partial x^\mu + ip_1 \cdot \psi\psi^\mu) \bar{\partial} X^\nu e^{ip_1 \cdot X}\Big|_z \times$$

$$\times\, \psi^\rho \bar{J} e^{ip_2 \cdot X}\Big|_0 \oint dw \left(\psi^\sigma \partial X^\sigma + G^{\text{int}}\right)\Big|_w \Big\rangle$$

$$\zeta_{U(1)} = \epsilon^1_{\mu\nu} \epsilon^2_\rho \epsilon^{a\mu\rho\sigma} p_1^a \langle \partial X^\sigma \bar{\partial} X^\nu(\bar{z})\rangle \langle \bar{J}\rangle + \mathcal{O}(p^2)$$

$$\zeta_{U(1)} \sim \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \frac{1}{\bar{\eta}^2} \text{Tr}[(-1)^F Q]_R$$

$$\zeta_{U(1)} \sim \sum_{i,\,\text{massless}} q^i$$

$$S = \int \sqrt{\det G} \left[-\frac{1}{12} e^{-2\phi} H_{\mu\nu\rho} H^{\mu\nu\rho} + \zeta B \wedge F\right]$$

$$\tilde{S} = \int \sqrt{\det G}\, e^{2\phi} \left(\partial_\mu a + \zeta A_\mu\right)^2$$

$$V_D \sim \zeta e^\phi \left(e^{-\phi} + \sum_i q^i h_i |c_i|^2\right)^2$$

$$\frac{i}{2\pi}\frac{1}{2}\sum_{\text{even}}(-1)^{a+b+ab}\partial_\tau\left(\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}}{\eta}\right)\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}\vartheta\begin{bmatrix}a+h\\b+g\end{bmatrix}\vartheta\begin{bmatrix}a-h\\b-g\end{bmatrix}}{\eta^3}\frac{Z_{4,4}\begin{bmatrix}h\\g\end{bmatrix}}{|\eta|^4}=4\frac{\eta^2}{\bar\vartheta\begin{bmatrix}1+h\\1+g\end{bmatrix}\bar\vartheta\begin{bmatrix}1-h\\1-g\end{bmatrix}},$$

$$\bar\chi_0^{\mathrm{E}_8}(v_i)=\frac{1}{2}\sum_{a,b=0}^{1}\frac{\prod_{i=1}^{8}\bar\vartheta\begin{bmatrix}a\\b\end{bmatrix}(v_i)}{\bar\eta^8}$$

$$\left[\frac{1}{(2\pi i)^2}\partial_{v_1}^2-\frac{1}{4\pi\tau_2}\right]\bar\chi_0^{E_8}(v_i)\Big|_{v_i=0}=\frac{1}{12}\left(\hat{\bar E}_2\bar E_4-\bar E_6\right)$$

$$\frac{1}{2}\sum_{(h,g)\neq(0,0)}\sum_{a,b=0}^{1}\frac{\bar\vartheta\begin{bmatrix}a\\b\end{bmatrix}\sqrt{6}\begin{bmatrix}a+h\\b+g\end{bmatrix}\bar\vartheta\begin{bmatrix}a-h\\b-g\end{bmatrix}}{\bar\vartheta\begin{bmatrix}1+h\\1+g\end{bmatrix}\bar\vartheta\begin{bmatrix}1-h\\1-g\end{bmatrix}}=-\frac{1}{4}\frac{\bar E_6}{\bar\eta^6}$$

$$\Delta_{\mathrm{E}_8}=\int_{\mathcal F}\frac{d^2\tau}{\tau_2}\left[-\frac{1}{12}\Gamma_{2,2}\frac{\hat{\bar E}_2\bar E_4\bar E_6-\bar E_6^2}{\bar\eta^{24}}+60\right]$$

$$\left[\mathrm{Tr}Q_{\mathrm{E}_7}^2-\frac{1}{4\pi\tau_2}\right]=\left[\frac{1}{(2\pi i)^2}\partial_v^2-\frac{1}{4\pi\tau_2}\right]\frac{1}{2}\sum_{a,b}\frac{\bar\vartheta\begin{bmatrix}a\\b\end{bmatrix}(v)\bar\vartheta^5\begin{bmatrix}a\\b\end{bmatrix}\bar\vartheta\begin{bmatrix}a+h\\b+g\end{bmatrix}\bar\vartheta\begin{bmatrix}a-h\\b-g\end{bmatrix}}{\bar\eta^8}\Bigg|_{v=0}$$

$$\Delta_{\mathrm{E}_7}=\int_{\mathcal F}\frac{d^2\tau}{\tau_2}\left[-\frac{1}{12}\Gamma_{2,2}\frac{\hat{\bar E}_2\bar E_4\bar E_6-\bar E_4^3}{\bar\eta^{24}}-84\right]$$

$$\Delta_{\mathrm{E}_8}-\Delta_{\mathrm{E}_7}=-144\Delta\,,\Delta=\int_{\mathcal F}\frac{d^2\tau}{\tau_2}(\Gamma_{2,2}-1)$$

$$\Delta=-\log\left[4\pi^2T_2U_2|\eta(T)\eta(u)|^4\ \right]$$

$$\lim_{T_2\to\infty}\Delta=\frac{\pi}{3}T_2+\mathcal{O}(\log\,T_2)$$

$$\Delta_{\mathrm{E}_8}^{N=1}=\int_{\mathcal F}\frac{d^2\tau}{\tau_2}\left[-\frac{1}{12}\sum_{i=1}^{3}\Gamma_{2,2}(T_i,U_i)\frac{\hat{\bar E}_2\bar E_4\bar E_6-\bar E_6^2}{\bar\eta^{24}}+\frac{3}{2}60\right]$$
$$\Delta_{\mathrm{E}_6}^{N=1}=\int_{\mathcal F}\frac{d^2\tau}{\tau_2}\left[-\frac{1}{12}\sum_{i=1}^{3}\Gamma_{2,2}(T_i,U_i)\frac{\hat{\bar E}_2\bar E_4\bar E_6-\bar E_4^3}{\bar\eta^{24}}-\frac{3}{2}84\right].$$

$$Z_{N=4\to N=2}=\frac{1}{2}\sum_{h,g=0}^{1}\frac{1}{\tau_2|\eta|^4}\frac{\Gamma_{2,2}\begin{bmatrix}h\\g\end{bmatrix}}{|\eta|^4}\frac{\bar\Gamma_{\mathrm{E}_8}}{\bar\eta^8}Z_{(4,4)}\begin{bmatrix}h\\g\end{bmatrix}\frac{1}{2}\sum_{\gamma,\delta=0}^{1}\frac{\bar\vartheta\begin{bmatrix}\gamma+h\\\delta+g\end{bmatrix}\bar\vartheta\begin{bmatrix}\gamma-h\\\delta-g\end{bmatrix}\bar\vartheta^6\begin{bmatrix}\gamma\\\delta\end{bmatrix}}{\bar\eta^8}$$

$$\times\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\frac{\vartheta^2\begin{bmatrix}a\\b\end{bmatrix}\vartheta\begin{bmatrix}a+h\\b+g\end{bmatrix}\vartheta\begin{bmatrix}a-h\\b-g\end{bmatrix}}{\eta^4}$$

$$m_{3/2}^2=\frac{|U|^2}{T_2U_2}$$

$$b_{\mathrm{E}_8}=-60\,,b_{\mathrm{E}_7}=-12\,,b_{\mathrm{SU}(2)}=52$$

$$\Delta_I=b_I\Delta+\left(\frac{\tilde{b}_I}{3}-b_I\right)\delta-k_IY$$

$$\Delta=\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\left[\sum_{h,g}{}' \ \Gamma_{2,2}\begin{bmatrix}h\\g\end{bmatrix}-1\right]=-\log\left[\frac{\pi^2}{4}|\vartheta_4(T)|^4|\vartheta_2(U)|^4T_2U_2\right]$$

$$\delta=\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\sum_{h,g}{}' \ \Gamma_{2,2}\begin{bmatrix}h\\g\end{bmatrix}\bar{\sigma}\begin{bmatrix}h\\g\end{bmatrix}$$

$$Y=\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\sum_{h,g}{}' \ \Gamma_{2,2}\left[\frac{1}{12}\frac{\hat{\bar{E}}_2}{\bar{\eta}^{24}}\bar{\Omega}\begin{bmatrix}h\\g\end{bmatrix}+\bar{\rho}\begin{bmatrix}h\\g\end{bmatrix}+40\bar{\sigma}\begin{bmatrix}h\\g\end{bmatrix}\right]$$

$$\Omega\begin{bmatrix}0\\1\end{bmatrix}=\frac{1}{2}E_4\vartheta_3^4\vartheta_4^4(\vartheta_3^4+\vartheta_4^4),$$

$$\sigma\begin{bmatrix}h\\g\end{bmatrix}=-\frac{1}{4}\frac{\vartheta^{12}\begin{bmatrix}h\\g\end{bmatrix}}{\eta^{12}},$$

$$\rho\begin{bmatrix}0\\1\end{bmatrix}=f(1-x)\,,\rho\begin{bmatrix}1\\0\end{bmatrix}=f(x)\,,\rho\begin{bmatrix}1\\1\end{bmatrix}=f(x/(x-1)),$$

$$f(x)=\frac{4(8-49x+66x^2-49x^3+8x^4)}{3x(1-x)^2}$$

$$\Delta_I=\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\left[\frac{\Gamma_{2,2}}{\bar{\eta}^{24}}\left(\mathrm{Tr}[Q_I^2]-\frac{k_I}{4\pi\tau_2}\right)\bar{\Omega}-b_I\right]$$

$$\Delta_{\mathrm{grav}}=\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\left[\frac{\Gamma_{2,2}}{\bar{\eta}^{24}}\frac{\hat{\bar{E}}_2}{12}\bar{\Omega}-b_{\mathrm{grav}}\right]$$

$$\bar{\Omega}=\xi\bar{E}_4\bar{E}_6$$

$$\Delta_I=\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\left[\Gamma_{2,2}\left(\frac{\xi k_I}{12}\frac{\hat{\bar{E}}_2\bar{E}_4\bar{E}_6}{\bar{\eta}^{24}}+A_I\bar{j}+B_I\right)-b_I\right]$$

$$\Delta_{\mathrm{grav}}=\xi\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\left[\Gamma_{2,2}\frac{\hat{\bar{E}}_2\bar{E}_4\bar{E}_6}{12\bar{\eta}^{24}}-b_{\mathrm{grav}}\right]$$

$$A_I=-\frac{\xi k_I}{12}$$

$$744A_I+B_I-b_I+k_ib_{\mathrm{grav}}=0$$

$$b_{\text{grav}} = \frac{22 - N_V + N_H}{12}$$

$$\Delta_I = b_I \Delta - k_I Y$$

$$\begin{aligned}\Delta &= \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \left[\Gamma_{2,2}(T,U) - 1\right] \\ &= -\log\left(4\pi^2 |\eta(T)|^4 |\eta(U)|^4 \operatorname{Im}T \operatorname{Im}U\right)\end{aligned}$$

$$Y = \frac{1}{12} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \Gamma_{2,2}(T,U) \left[\frac{\hat{\bar{E}}_2 \bar{E}_4 \bar{E}_6}{\bar{\eta}^{24}} - \bar{j} + 1008\right]$$

$$\Delta_{\text{grav}} = -\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \left[\Gamma_{2,2} \frac{\hat{\bar{E}}_2 \bar{E}_4 \bar{E}_6}{12\bar{\eta}^{24}} - 22\right]$$

$$\frac{16\pi^2}{g_I^2(\mu)} = k_I \frac{16\pi^2}{g_{\text{renorm}}^2} + b_I \log \frac{M_s^2}{\mu^2} + \hat{\Delta}_I$$

$$g_{\text{renorm}}^2 = \frac{g_{\text{string}}^2}{1 - \frac{Y}{16\pi^2} g_{\text{string}}^2}$$

$$M_{\text{string}} = \frac{M_P g_{\text{renorm}}}{\sqrt{1 + \frac{Y}{16\pi^2} g_{\text{renorm}}^2}}$$

$$\frac{1}{g_I^2} = \frac{k_I}{g_U^2}$$

$$\frac{16\pi^2}{g_I^2(\mu)} = k_I \frac{16\pi^2}{g_U^2} + b_I \log \frac{M_U^2}{\mu^2}$$

$$g_U = g_{\text{renorm}} = \frac{g_{\text{spacetime dimensions}}}{\sqrt{1 - \frac{g_{\text{spacetime dimensions}}^2 Y}{16\pi^2}}}$$

$$M_U^2 = \frac{2e^{1-\gamma}}{\pi\sqrt{27}} e^{\Delta} M_P^2 g_{\text{spacetime dimensions}}^2 = \frac{2e^{1-\gamma}}{\pi\sqrt{27}} e^{\Delta} M_P^2 \frac{g_U}{\sqrt{1 + \frac{g_U^2 Y}{16\pi^2}}}$$

$$Z_{D=9}^{O(32)} = \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^7} \frac{\Gamma_{1,17}(R, Y^I)}{\eta\bar{\eta}^{17}} \frac{1}{2} \sum_{a,b=0}^{1} (-1)^{a+b+ab} \frac{\vartheta^4 \begin{bmatrix} a \\ b \end{bmatrix}}{\eta^4},$$

$$\Gamma_{1,17}(R) = R \sum_{m,n\in Z} \exp\left[-\frac{\pi R^2}{\tau_2}|m+\tau n|^2\right]\frac{1}{2}\sum_{a,b} \bar{\vartheta}^8\begin{bmatrix}a\\b\end{bmatrix}\bar{\vartheta}^8\begin{bmatrix}a+n\\b+m\end{bmatrix}$$

$$= \frac{1}{2}\sum_{h,g=0}^{1} \Gamma_{1,1}(2R)\begin{bmatrix}h\\g\end{bmatrix}\frac{1}{2}\sum_{a,b} \bar{\vartheta}^8\begin{bmatrix}a\\b\end{bmatrix}\bar{\vartheta}^8\begin{bmatrix}a+h\\b+g\end{bmatrix}$$

$$\Gamma_{1,1}(R)\begin{bmatrix}h\\g\end{bmatrix} = R\sum_{m,n\in Z} \exp\left[-\frac{\pi R^2}{\tau_2}\left|\left(m+\frac{g}{2}\right)+\tau\left(n+\frac{h}{2}\right)\right|^2\right]$$
$$= \frac{1}{R}\sum_{m,n\in Z} (-1)^{mh+ng}\exp\left[-\frac{\pi}{\tau_2 R^2}|m+\tau n|^2\right].$$

$$\partial X^9 \to \partial X^9\,, \psi^9 \to \psi^9\,, \bar{\partial} X^9 \to -\bar{\partial} X^9\,, \bar{\psi}^9 \to -\bar{\psi}^9.$$

Tensores antisimétricos y supermembranas.

$$A_p \equiv A_{\mu_1\mu_2\ldots\mu_p} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p}$$

$$A_p \to A_p + d\Lambda_{p-1},$$

$$F_{p+1} = dA_p$$

$$d^{*}F_{p+1} = 0$$

$$\exp\left[iQ_p \int_{\text{world-volume}} A_{p+1}\right] = \exp\left[iQ_p \int A_{\mu_0\ldots\mu_p} dx^{\mu_0} \wedge \ldots \wedge dx^{\mu_p}\right]$$

$$d\tilde{A}_{D-p-3} = \tilde{F}_{D-p-2} = {}^{*}F_{p+2} = {}^{*}dA_{p+1}$$

$$\Phi = \int_{S^{D-p-2}} {}^{*}F_{p+2} = \int_{S^{D-p-3}} \tilde{A}_{D-p-3}$$

$$\Phi\tilde{Q}_{D-p-4} = Q_p\tilde{Q}_{D-p-4} = 2\pi N\,, n \in Z$$

$$M_{m,n}^2 = \frac{|m+n\tau|^2}{\tau_2}$$

$$M_{m_0,n_0} = \sum_{i=1}^{N} \sqrt{M_i^2 + \vec{p}_i^{\,2}} \geq \sum_{i=1}^{N} M_i$$

$$M_{m_0,n_0} \geq \sum_{i=1}^{N} M_{m_i,n_i}$$

$$\|v_0\| \leq \sum_{i=1}^{N} \|v_i\|$$

N - Dimensiones y p - supermembranas.

$$S^{\text{het}} = \int d^{10}x\sqrt{G}e^{-\Phi}\left[R + (\nabla\Phi)^2 - \frac{1}{12}\hat{H}^2 - \frac{1}{4}F^2\right]$$

$$S^{I} = \int d^{10}x\sqrt{G}\left[e^{-\Phi}(R + (\nabla\Phi)^2) - \frac{1}{4}e^{-\Phi/2}F^2 - \frac{1}{12}\hat{H}^2\right]$$

$$S_E^{\text{het}} = \int d^{10}x\sqrt{g}\left[R - \frac{1}{8}(\nabla\Phi)^2 - \frac{1}{4}e^{-\Phi/4}F^2 - \frac{1}{12}e^{-\Phi/2}\hat{H}^2\right]$$
$$S_E^{I} = \int d^{10}x\sqrt{g}\left[R - \frac{1}{8}(\nabla\Phi)^2 - \frac{1}{4}e^{\Phi/4}F^2 - \frac{1}{12}e^{\Phi/2}\hat{H}^2\right]$$

$$\text{NN NS sector}\ \ \psi + \bar{\psi}\big|_{\sigma=0} = \psi - \bar{\psi}\big|_{\sigma=\pi} = 0$$

$$\text{NN R sector}\ \ \psi - \bar{\psi}\big|_{\sigma=0} = \psi - \bar{\psi}\big|_{\sigma=\pi} = 0.$$

$$\text{DDNS sector}\quad \psi - \bar{\psi}\big|_{\sigma=0} = \psi + \bar{\psi}\big|_{\sigma=\pi} = 0$$
$$\text{DDR sector}\quad \psi + \bar{\psi}\big|_{\sigma=0} = \psi + \bar{\psi}\big|_{\sigma=\pi} = 0$$

$$X^I(\sigma,\tau) = x^I + w^I\sigma + 2\sum_{n\neq 0}\frac{a_n^I}{n}e^{in\tau}\sin(n\sigma),$$
$$X^\mu(\sigma,\tau) = x^\mu + p^\mu\tau - 2i\sum_{n\neq 0}\frac{a_n^\mu}{n}e^{in\tau}\cos(n\sigma).$$

$$\psi^I(\sigma,\tau) = \sum_{n\in Z} b_{n+1/2}^I e^{i(n+1/2)(\sigma+\tau)}\,, \psi^\mu(\sigma,\tau) = \sum_{n\in Z} b_{n+1/2}^\mu e^{i(n+1/2)(\sigma+\tau)},$$

$$\psi^I(\sigma,\tau) = \sum_{n\in Z} b_n^I e^{in(\sigma+\tau)}\,, \psi^\mu(\sigma,\tau) = \sum_{n\in Z} b_n^\mu e^{in(\sigma+\tau)}.$$

$$\bar{b}_{n+1/2}^I = b_{n+1/2}^I\,, \bar{b}_n^I = -b_n^I$$
$$\bar{b}_{n+1/2}^\mu = -b_{n+1/2}^\mu\,, \bar{b}_n^\mu = b_n^\mu.$$

$$\Omega b_{-1/2}^\mu|0\rangle = \bar{b}_{-1/2}^\mu|0\rangle = -b_{-1/2}^\mu|0\rangle,$$

$$\Omega b_{-1/2}^I|0\rangle = \bar{b}_{-1/2}^I|0\rangle = b_{-1/2}^I|0\rangle.$$

$$\Gamma^{11}|R\rangle = |R\rangle$$

$$\Omega|R\rangle = -\Gamma^2 \ldots \Gamma^9|R\rangle = |R\rangle$$

$$\Gamma^0\Gamma^1|R\rangle = -|R\rangle$$

$$\partial_\tau X^I|_{\sigma=0} = 0,\ \partial_\sigma X^I|_{\sigma=\pi} = 0,$$
$$\text{DNNS sector}\ \ \psi + \bar{\psi}\big|_{\sigma=0} = \psi + \bar{\psi}\big|_{\sigma=\pi} = 0,$$
$$\text{DNR sector}\ \ \psi - \bar{\psi}\big|_{\sigma=0} = \psi + \bar{\psi}\big|_{\sigma=\pi} = 0,$$

$$b^{\mu}_{-1/2}|p;i,j\rangle \,, b^{I}_{-1/2}|p;i,j\rangle$$

$$\tilde{S}^{IIA} = \frac{1}{2\kappa_{10}^2}\left[\int\ d^{10}x\sqrt{g}e^{-\Phi}\left[\left(R + (\nabla\Phi)^2 - \frac{1}{12}H^2\right) - \frac{1}{2\cdot 4!}\hat{G}^2 - \frac{1}{4}F^2\right] + \frac{1}{(48)^2}\int\ B\wedge G\wedge G\right]$$

$$\{Q^1_{\alpha}, Q^2_{\dot{\alpha}}\} = \delta_{\alpha\dot{\alpha}}W$$

$$M \geq \frac{c_0}{\lambda}|W|$$

$$M = \frac{c}{\lambda}|n| \,, n \in Z.$$

$$R = \lambda^{2/3}$$

$$e^{\phi/2} = \lambda + \frac{c}{r^8} \,, \chi = \chi_0 + i\frac{c}{\lambda(\lambda r^8 + c)},$$

$$ds^2 = A(r)^{-3/4}[-(dx^0)^2 + (dx^1)^2] + A(r)^{1/4}dy\cdot dy$$

$$S = \chi_0 + i\frac{e^{-\phi_0/2}}{\sqrt{A(r)}}$$

$$B^1 = 0 \,, B^2_{01} = \frac{1}{\sqrt{\Delta}A(r)}$$

$$A(r) = 1 + \frac{Q\sqrt{\Delta}}{3r^6} \,, Q = \frac{3\kappa^2 T}{\pi^4} \,, \Delta = e^{\phi_0/2}\left[\chi_0^2 + e^{-\phi_0}\right].$$

$$\tilde{T} = T\sqrt{\Delta}.$$

$$T_{p,q} = T\frac{|p + qS|}{\sqrt{S_2}}$$

$$M_B^2 = \frac{m^2}{R_B^2} + \left(2\pi R_B n T_{p,q}\right)^2 + 4\pi T_{p,q}(N_L + N_R)$$

$$M_B^2\big|_{\mathrm{BPS}} = \left(\frac{m}{R_B} + 2\pi R_B n T_{p,q}\right)^2$$

$$M_B^2\big|_{BPS} = \left(\frac{m}{R_B} + 2\pi R_B T\frac{|n_1 + n_2 S|}{\sqrt{S_2}}\right)^2$$

$$M_{11}^2 = (m(2\pi R_{11})^2\tau_2 T_{11})^2 + \frac{|n_1 + n_2\tau|^2}{R_{11}^2\tau_2^2} + \cdots$$

$$S = \tau \,, \frac{1}{R_B^2} = TT_{11}A_{11}^{3/2} \,, \beta = 2\pi R_{11}\frac{\sqrt{\tau_2}T_{11}}{T}$$

$$\mathcal{A} = 2V_{p+1}\int \frac{d^{p+1}k}{(2\pi)^{p+1}}\int_0^\infty \frac{dt}{2t} e^{-2\pi\alpha' t k^2 - t\frac{|Y|^2}{2\pi\alpha'}} \frac{1}{\eta^{12}(it)}\frac{1}{2}\sum_{a,b} (-1)^{a+b+ab}\vartheta^4 \begin{bmatrix} a \\ b \end{bmatrix}(it)$$

$$= 2V_{p+1}\int_0^\infty \frac{dt}{2t}(8\pi^2\alpha' t)^{-\frac{p+1}{2}} e^{-t\frac{|Y|^2}{2\pi\alpha'}} \frac{1}{\eta^{12}(it)}\frac{1}{2}\sum_{a,b=0}^{1} (-1)^{a+b+ab}\vartheta^4 \begin{bmatrix} a \\ b \end{bmatrix}(it)$$

$$\mathcal{A}|^{\text{spacetime dimensions}}_{\text{supermassive.}} = 8(1-1)V_{p+1}\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-\frac{p+1}{2}} t^4 e^{-\frac{t|Y|^2}{2\pi\alpha'}}$$

$$= 2\pi(1-1)V_{p+1}(4\pi^2\alpha')^{3-p} G_{9-p}(|Y|)$$

$$G_d(|Y|) = \frac{1}{4\pi^{d/2}}\int_0^\infty \frac{dt}{t^{(4-d)/2}} e^{-t|Y|^2}$$

$$S = \frac{\alpha_p}{2}\int F^*_{p+2}F_{p+2} + iT_p\int_{\text{branes}} A_{p+1}$$

$$\mathcal{A}|_{\text{field theory}} = \frac{(iT_p)^2}{\alpha_p} V_{p+1}G_{9-p}(|Y|)$$

$$\frac{T_p^2}{\alpha_p} = 2\pi(4\pi^2\alpha')^{3-p}$$

$$\frac{T_pT_{6-p}}{\alpha_p} = 2\pi n$$

$$2\kappa_{10}^2 = (2\pi)^7\alpha'^4$$

$$S_p = -T_p\int_{W_{p+1}} d^{p+1}\xi e^{-\Phi/2}\sqrt{\det\hat{G}} - iT_p\int A_{p+1}$$

$$\hat{G}_{\alpha\beta} = G_{\mu\nu}\frac{\partial X^\mu}{\partial\xi^\alpha}\frac{\partial X^\nu}{\partial\xi^\beta}$$

$$\int A_{p+1} = \frac{1}{(p+1)!}\int d^{p+1}\xi A_{\mu_1\cdots\mu_{p+1}}\frac{\partial X^{\mu_1}}{\partial\xi^{\alpha_1}}\cdots\frac{\partial X^{\mu_{p+1}}}{\partial\xi^{\alpha_{p+1}}}\epsilon^{\alpha_1\cdots\alpha_{p+1}}$$

$$2\kappa_{10}^2 T_pT_{6-p} = 2\pi n$$

$$T_p = \frac{1}{(2\pi)^p(\alpha')^{(p+1)/2}}$$

$$2\kappa_{11}^2 = (2\pi)^8(\alpha')^{9/2}$$

$$T_2^M = T_2 = \frac{1}{(2\pi)^2(\alpha')^{3/2}}$$

$$2\kappa_{11}^2 T_2^M T_5^M = 2\pi \rightarrow T_5^M = \frac{1}{(2\pi)^5(\alpha')^3}$$

$$2\pi\sqrt{\alpha'}T_5^M = T_4$$

$$S_{BI} = \int d^{10}x e^{-\Phi/2}\sqrt{\det(\delta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}$$

$$S_B = \frac{i}{2\pi\alpha'}\int_{M_2} d^2\xi \epsilon^{\alpha\beta} B_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu - \frac{i}{2}\int_{B_1} ds A_\mu \partial_s x^\mu$$

$$\delta S_B = \frac{i}{\pi\alpha'}\int_{B_1} ds \Lambda_\mu \partial_s x^\mu$$

$$\mathcal{F}_{\mu\nu} = 2\pi\alpha' F_{\mu\nu} - B_{\mu\nu} = 2\pi\alpha'(\partial_\mu A_\nu - \partial_\nu A_\mu) - B_{\mu\nu}$$

$$S_p = -T_p \int_{W_{p+1}} d^{p+1}\xi e^{-\Phi/2}\sqrt{\det(\hat{G} + \mathcal{F})} - iT_p \int A_{p+1}$$

$$S_p = -T_p \int_{W_{p+1}} d^{p+1}\xi e^{-\frac{\Phi}{2}}\sqrt{\det(\hat{G} + \mathcal{F})} \pm iT_p \int A$$

$$S_1 = -\frac{1}{2\pi\alpha'}\left[\int d^2\xi \frac{|S|}{\sqrt{S_2}}\sqrt{\det(\hat{G} + \mathcal{F})} + i\int \left(B^N + \frac{iS_1}{2\pi}\mathcal{F}\right)\right]$$

$$e^{-\Phi/2} \to \frac{\sqrt{\alpha'}}{R} e^{-\Phi/2}$$

$$T_{p+1} = \frac{T_p}{2\pi\sqrt{\alpha'}}$$

$$S_p^N = -T_p \mathrm{Str}\int_{W_{p+1}} d^{p+1}\xi e^{-\Phi/2}(F_{\mu\nu}^2 + 2F_{\mu I}^2 + F_{IJ}^2)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$
$$F_{\mu I} = \partial_\mu X^I + [A_\mu, X^I], F_{IJ} = [X^I, X^J]$$

$$S_{D=6}^{\mathrm{het}} = \int d^6x\sqrt{-G}\left[R - \frac{1}{4}\partial^\mu\Phi\partial_\mu\Phi - \frac{e^{-\Phi}}{12}\hat{H}^{\mu\nu\rho}\hat{H}_{\mu\nu\rho} - \frac{e^{-\frac{\Phi}{2}}}{4}(\hat{M}^{-1})_{ij}F_{\mu\nu}^i F^{j\mu\nu} + \frac{1}{8}\mathrm{Tr}(\partial_\mu\hat{M}\partial^\mu\hat{M}^{-1})\right]$$

$$S_{D=6}^{IIA} = \int d^6x\sqrt{-G}\left[R - \frac{1}{4}\partial^\mu\Phi\partial_\mu\Phi\right.$$
$$\left. - \frac{1}{12}e^{-\Phi}H^{\mu\nu\rho}H_{\mu\nu\rho} - \frac{1}{4}e^{\Phi/2}(\hat{M}^{-1})_{ij}F_{\mu\nu}^i F^{j\mu\nu} + \frac{1}{8}\mathrm{Tr}(\partial_\mu\hat{M}\partial^\mu\hat{M}^{-1})\right]$$
$$+ \frac{1}{16}\int d^6x \epsilon^{\mu\nu\rho\sigma\tau\varepsilon} B_{\mu\nu}F_{\rho\sigma}^i \hat{L}_{ij} F_{\tau\varepsilon}^j$$

$$\Phi' = -\Phi, G'_{\mu\nu} = G_{\mu\nu}, \hat{M}' = \hat{M}, A'^i_\mu = A^i_\mu$$
$$e^{-\Phi}\hat{H}_{\mu\nu\rho} = \frac{1}{6}\frac{\epsilon_{\mu\nu\rho}{}^{\sigma\tau\varepsilon}}{\sqrt{-G}}H'_{\sigma\tau\varepsilon}$$

$$\phi = \Phi - \frac{1}{2}\log\left[\det G_{\alpha\beta}\right]$$

$$S_{D=4}^{\text{het}} = \int d^4x\sqrt{-g}e^{-\phi}\left[R + L_B + L_{\text{gauge}} + L_{\text{scalar}}\right]$$

$$\mathcal{L}_{g+\phi} = R + \partial^\mu\phi\partial_\mu\phi,$$
$$\mathcal{L}_B = -\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho},$$

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \frac{1}{2}\left[B_{\mu\alpha}F_{\nu\rho}^{A,\alpha} + A_\mu^\alpha F_{\alpha,\nu\rho}^B + \hat{L}_{ij}A_\mu^i F_{\nu\rho}^j\right] + \text{ cyclic}$$
$$\equiv \partial_\mu B_{\nu\rho} - \frac{1}{2}L_{IJ}A_\mu^I F_{\nu\rho}^J + \text{ cyclic}$$

$$L = \begin{pmatrix} 0 & 0 & 1 & 0 & \vec{0} \\ 0 & 0 & 0 & 1 & \vec{0} \\ 1 & 0 & 0 & 0 & \vec{0} \\ 0 & 1 & 0 & 0 & \vec{0} \\ \vec{0} & \vec{0} & \vec{0} & \vec{0} & \hat{L} \end{pmatrix}$$

$$C_{\alpha\beta} = \epsilon_{\alpha\beta}B - \frac{1}{2}\hat{L}_{ij}Y_\alpha^i Y_\beta^j$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\Big\{\left[\left(\hat{M}^{-1}\right)_{ij} + \hat{L}_{ki}\hat{L}_{lj}Y_\alpha^k G^{\alpha\beta}Y_\beta^l\right]F_{\mu\nu}^i F^{j,\mu\nu} + G^{\alpha\beta}F_{\alpha,\mu\nu}^B F_{B,\beta}^{\mu\nu}$$

$$+\left[G_{\alpha\beta} + C_{\gamma\alpha}G^{\gamma\delta}C_{\delta\beta} + Y_\alpha^i\left(\hat{M}^{-1}\right)_{ij}Y_\beta^j\right]F_{\mu\nu}^{A,\alpha}F_A^{\beta,\mu\nu} - 2G^{\alpha\gamma}C_{\gamma\beta}F_{\alpha,\mu\nu}^B F^{A,\beta,\mu\nu}$$

$$-2\hat{L}_{ij}Y_\alpha^i G^{\alpha\beta}F_{\mu\nu}^j F_\beta^{B,\mu\nu} + 2\left(Y_\alpha^i\left(\hat{M}^{-1}\right)_{ij} + C_{\gamma\alpha}G^{\gamma\beta}\hat{L}_{ij}Y_\beta^i\right)F_{\mu\nu}^{\alpha,A}F^{j,\mu\nu}\Big\}$$

$$\equiv -\frac{1}{4}(M^{-1})_{IJ}F_{\mu\nu}^I F^{J,\mu\nu}$$

$$\mathcal{L}_{\text{scalar}} = \partial_\mu\phi\partial^\mu\phi + \frac{1}{8}\text{Tr}\left[\partial_\mu\hat{M}\partial^\mu\hat{M}^{-1}\right] + \frac{1}{2}G^{\alpha\beta}\left(\hat{M}^{-1}\right)_{ij}\partial_\mu Y_\alpha^i\partial^\mu Y_\beta^j + \frac{1}{4}\partial_\mu G_{\alpha\beta}\partial^\mu G^{\alpha\beta}$$

$$+\frac{1}{2\det G}\left[\partial_\mu B + \epsilon^{\alpha\beta}\hat{L}_{ij}Y_\alpha^i\partial_\mu Y_\beta^j\right]\left[\partial^\mu B + \epsilon^{\alpha\beta}\hat{L}_{ij}Y_\alpha^i\partial^\mu Y_\beta^j\right]$$

$$= \partial_\mu\phi\partial^\mu\phi + \frac{1}{8}\text{Tr}\left[\partial_\mu M\partial^\mu M^{-1}\right]$$

$$g_{\mu\nu} \to e^{-\phi}g_{\mu\nu}$$

$$S_{D=4}^{\text{het,E}} = \int d^4x\sqrt{-g}\Big[R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi$$

$$-\frac{1}{12}e^{-2\phi}H^{\mu\nu\rho}H_{\mu\nu\rho} - \frac{1}{4}e^{-\phi}(M^{-1})_{IJ}F_{\mu\nu}^I F^{J,\mu\nu} + \frac{1}{8}\text{Tr}\left(\partial_\mu M\partial^\mu M^{-1}\right)\Big]$$

$$e^{-2\phi}H_{\mu\nu\rho}=\frac{\epsilon^{\sigma}_{\mu\nu\rho}}{\sqrt{-g}}\partial_\sigma a$$

$$\tilde{S}^{\text{het}}_{D=4}=\int d^4x\sqrt{-g}\Big[R-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi-\frac{1}{2}e^{2\phi}\partial^\mu a\partial_\mu a-\frac{1}{4}e^{-\phi}(M^{-1})_{IJ}F^I_{\mu\nu}F^{J,\mu\nu}$$

$$+\frac{1}{4}aL_{IJ}F^I_{\mu\nu}\tilde{F}^{J,\mu\nu}+\frac{1}{8}\text{Tr}\big(\partial_\mu M\partial^\mu M^{-1}\big)\Big]$$

$$\tilde{F}^{\mu\nu}=\frac{1}{2}\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}F_{\rho\sigma}$$

$$S=a+ie^{-\phi}$$

$$\tilde{S}^{\text{het}}_{D=4}=\int d^4x\sqrt{-g}\left[R-\frac{1}{2}\frac{\partial^\mu S\partial_\mu\bar{S}}{\text{Im}S^2}-\frac{1}{4}\text{Im}S(M^{-1})_{IJ}F^I_{\mu\nu}F^{J,\mu\nu}++\frac{1}{4}\text{Re}SL_{IJ}F^I_{\mu\nu}\tilde{F}^{J,\mu\nu}+\frac{1}{8}\text{Tr}\big(\partial_\mu M\partial^\mu M^{-1}\big)\right]$$

$$e^{-2\phi}H_{\mu\nu\rho}=\frac{\epsilon^{\sigma}_{\mu\nu\rho}}{\sqrt{-g}}\left[\partial_\sigma a+\frac{1}{2}\hat{L}_{ij}Y^i_\alpha\delta_\sigma Y^j_\beta\epsilon^{\alpha\beta}\right]$$

$$\tilde{S}^{IIA}_{D=4}=\int d^4x\sqrt{-g}\left[R+\mathcal{L}^{\text{even}}_{\text{gauge}}+\mathcal{L}^{\text{odd}}_{\text{gauge}}+\mathcal{L}_{\text{scalar}}\right]$$

$$\mathcal{L}^{\text{even}}_{\text{gauge}}=-\frac{1}{4}\int d^4x\sqrt{-g}\Big[e^{-\phi}G^{\alpha\beta}\big(F^B_{\alpha,\mu\nu}-B_{\alpha\gamma}F^{A,\gamma}_{\mu\nu}\big)\left(F^{B,\mu\nu}_\beta-B_{\alpha\delta}F^{\delta,\mu\nu}_A\right)$$

$$+e^{-\phi}G_{\alpha\beta}F^{A,\alpha}_{\mu\nu}F^{\beta,\mu\nu}_A\sqrt{\text{det}G_{\alpha\beta}}\big(\hat{M}^{-1}\big)_{ij}\big(F^i_{\mu\nu}+Y^i_\alpha F^{A,\alpha}_{\mu\nu}\big)\left(F^{j,\mu\nu}+Y^j_\beta F^{\beta,\mu\nu}_A\right)\Big]$$

$$\mathcal{L}^{\text{odd}}_{\text{gauge}}=\frac{1}{2}\int d^4x\epsilon^{\mu\nu\rho\sigma}\Big[\frac{1}{4}aF^B_{\alpha,\mu\nu}F^{A,\alpha}_{\rho\sigma}$$

$$+\frac{1}{2}\epsilon^{\alpha\beta}\hat{L}_{ij}Y^i_\beta F^B_{\alpha,\mu\nu}\left(F^j_{\rho\sigma}+\frac{1}{2}Y^j_\gamma F^{A,\gamma}_{\rho\sigma}\right)-\frac{1}{8}\epsilon^{\alpha\beta}\hat{L}_{ij}B_{\alpha\beta}\big(F^i_{\mu\nu}+Y^i_\gamma F^{A,\gamma}_{\mu\nu}\big)\big(F^j_{\rho\sigma}+Y^j_\delta F^{A,\delta}_{\rho\sigma}\big)\Big]$$

$$\mathcal{L}_{\text{scalar}}=-\frac{1}{2}(\partial\phi)^2+\frac{1}{4}\partial^\mu G_{\alpha\beta}\partial_\mu G^{\alpha\beta}-\frac{1}{2\text{det}G}\partial_\mu B\partial^\mu B+\frac{1}{8}\text{Tr}\big[\partial_\mu\hat{M}\partial^\mu\hat{M}^{-1}\big]$$

$$-\frac{1}{2}e^{2\phi}\left(\partial_\mu a+\frac{1}{2}\hat{L}_{ij}\epsilon^{\alpha\beta}Y^i_\alpha\partial^\mu Y^j_\beta\right)^2-\frac{1}{2}e^{\phi}\left(\sqrt{\text{det}G_{\alpha\beta}}\hat{M}^{-1}\right)_{ij}G^{\alpha\beta}\partial_\mu Y^i_\alpha\partial^\mu Y^j_\beta$$

$$e^{-\phi}=\sqrt{\text{det}G'_{\alpha\beta}},\,e^{-\phi'}=\sqrt{\text{det}G_{\alpha\beta}}$$

$$\frac{G_{\alpha\beta}}{\sqrt{\text{det}G_{\alpha\beta}}}=\frac{G'_{\alpha\beta}}{\sqrt{\text{det}G'_{\alpha\beta}}},\,A'^{\alpha}_\mu=A^\alpha_\mu$$

$$g_{\mu\nu}=g'_{\mu\nu}\ \text{Einstein frame}$$

$$\hat{M}'=\hat{M}\,,A^i_\mu=A'^i_\mu\,,Y^i_\alpha=Y'^i_\alpha$$

$$A = B' \,, A' = B$$

$$\frac{1}{2}\frac{\epsilon_{\mu\nu}^{\rho\sigma}}{\sqrt{-g}}\epsilon^{\alpha\beta}F_{\beta,\rho\sigma}^{B'} = e^{-\phi}G^{\alpha\beta}\left[F_{\beta,\mu\nu}^{B} - C_{\beta\gamma}F_{\mu\nu}^{A,\gamma} - \hat{L}_{ij}Y_{\beta}^{i}F_{\mu\nu}^{j}\right] - \frac{1}{2}a\frac{\epsilon_{\mu\nu}^{\rho\sigma}}{\sqrt{-g}}F_{\rho\sigma}^{A,\alpha}$$

$$W^{i} = W_{1}^{i} + iW_{2}^{i} = -Y_{2}^{i} + UY_{1}^{i}$$

$$G_{\alpha\beta} = \frac{T_{2} - \frac{\sum_{i}\left(W_{2}^{i}\right)^{2}}{2U_{2}}}{U_{2}}\begin{pmatrix} 1 & U_{1} \\ U_{1} & |U|^{2} \end{pmatrix} , B = T_{1} - \frac{\sum_{i} W_{1}^{i}W_{2}^{i}}{2U_{2}}$$

$$\mathcal{L}_{\text{scalar}}^{\text{het}} = -\frac{1}{2}\partial_{z^{i}}\partial_{\bar{z}^{j}}K(z_{k},\bar{z}_{k})\partial_{\mu}z^{i}\partial^{\mu}\bar{z}^{j}$$

$$K = \log\left[S_{2}\left(T_{2}U_{2} - \frac{1}{2}\sum_{i}\left(W_{2}^{i}\right)^{2}\right)\right]$$

$$G_{\alpha\beta} = \frac{T_{2}}{U_{2}}\begin{pmatrix} 1 & U_{1} \\ U_{1} & |U|^{2} \end{pmatrix} , B = T_{1}$$

$$S = a - \frac{\sum_{i} W_{1}^{i}W_{2}^{i}}{2U_{2}} + i\left(e^{-\phi} - \frac{\sum_{i}\left(W_{2}^{i}\right)^{2}}{2U_{2}}\right)$$

$$S' = T \,, T' = S \,, U = U' \,, W^{i} = W'^{i}$$

$$\begin{pmatrix} m_{1} \\ m_{2} \\ n_{1} \\ n_{2} \\ q^{i} \end{pmatrix} \rightarrow \begin{pmatrix} m_{1} \\ m_{2} \\ \tilde{n}_{2} \\ -\tilde{n}_{1} \\ q^{i} \end{pmatrix}, \begin{pmatrix} \tilde{m}_{1} \\ \tilde{m}_{2} \\ \tilde{n}_{1} \\ \tilde{n}_{2} \\ \tilde{q}^{i} \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{m}_{1} \\ \tilde{m}_{2} \\ -n_{2} \\ n_{1} \\ \tilde{q}^{i} \end{pmatrix}$$

$$\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(v \mid \tau) = \sum_{n \in Z} q^{\frac{1}{2}\left(n - \frac{a}{2}\right)^{2}} e^{2\pi i\left(v - \frac{b}{2}\right)\left(n - \frac{a}{2}\right)}$$

$$\vartheta\begin{bmatrix} a+2 \\ b \end{bmatrix}(v \mid \tau) = \vartheta\begin{bmatrix} a \\ b \end{bmatrix}(v \mid \tau) \,, \vartheta\begin{bmatrix} a \\ b+2 \end{bmatrix}(v \mid \tau) = e^{i\pi a}\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(v \mid \tau),$$
$$\vartheta\begin{bmatrix} -a \\ -b \end{bmatrix}(v \mid \tau) = \vartheta\begin{bmatrix} a \\ b \end{bmatrix}(-v \mid \tau) \,, \vartheta\begin{bmatrix} a \\ b \end{bmatrix}(-v \mid \tau) = e^{i\pi ab}\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(v \mid \tau) \; (a, b \in Z).$$

$$\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(v \mid \tau + 1) = e^{-\frac{i\pi}{4}a(a-2)}\vartheta\begin{bmatrix} a \\ a+b-1 \end{bmatrix}(v \mid \tau),$$
$$\vartheta\begin{bmatrix} a \\ b \end{bmatrix}\left(\frac{v}{\tau}\middle| -\frac{1}{\tau}\right) = \sqrt{-i\tau}e^{\frac{i\pi}{2}ab + i\pi\frac{v^{2}}{\tau}}\vartheta\begin{bmatrix} b \\ -a \end{bmatrix}(v \mid \tau).$$

$$\vartheta_1(v \mid \tau) = 2q^{\frac{1}{8}}\sin[\pi v]\prod_{n=1}^{\infty}(1-q^n)(1-q^n e^{2\pi i v})(1-q^n e^{-2\pi i v})$$
$$\vartheta_2(v \mid \tau) = 2q^{\frac{1}{8}}\cos[\pi v]\prod_{n=1}^{\infty}(1-q^n)(1+q^n e^{2\pi i v})(1+q^n e^{-2\pi i v})$$
$$\vartheta_3(v \mid \tau) = \prod_{n=1}^{\infty}(1-q^n)(1+q^{n-1/2} e^{2\pi i v})(1+q^{n-1/2} e^{-2\pi i v})$$
$$\vartheta_4(v \mid \tau) = \prod_{n=1}^{\infty}(1-q^n)(1-q^{n-1/2} e^{2\pi i v})(1-q^{n-1/2} e^{-2\pi i v})$$

$$\eta(\tau) = q^{\frac{1}{24}}\prod_{n=1}^{\infty}(1-q^n).$$

$$\frac{\partial}{\partial v}\vartheta_1(v)\Big|_{v=0} \equiv \vartheta_1' = 2\pi\eta^3(\tau)$$

$$\eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\,\eta(\tau)$$

$$\vartheta\begin{bmatrix} a \\ b \end{bmatrix}\left(v + \frac{\epsilon_1}{2}\tau + \frac{\epsilon_2}{2}\,\Big|\, \tau\right) = e^{-\frac{i\pi\tau}{4}\epsilon_1^2 - \frac{i\pi\epsilon_1}{2}(2v-b) - \frac{i\pi}{2}\epsilon_1\epsilon_2}\vartheta\begin{bmatrix} a-\epsilon_1 \\ b-\epsilon_2 \end{bmatrix}(v \mid \tau)$$

$$\vartheta_2(0 \mid \tau)\vartheta_3(0 \mid \tau)\vartheta_4(0 \mid \tau) = 2\eta^3$$
$$\vartheta_2^4(v \mid \tau) - \vartheta_1^4(v \mid \tau) = \vartheta_3^4(v \mid \tau) - \vartheta_4^4(v \mid \tau)$$

$$\vartheta_2(2\tau) = \frac{1}{\sqrt{2}}\sqrt{\vartheta_3^2(\tau) - \vartheta_4^2(\tau)}\,,\vartheta_3(2\tau) = \frac{1}{\sqrt{2}}\sqrt{\vartheta_3^2(\tau) + \vartheta_4^2(\tau)}$$
$$\vartheta_4(2\tau) = \sqrt{\vartheta_3(\tau)\vartheta_4(\tau)}\,,\eta(2\tau) = \sqrt{\frac{\vartheta_2(\tau)\eta(\tau)}{2}}$$

$$\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\prod_{i=1}^{4}\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(v_i) = -\prod_{i=1}^{4}\vartheta_1(v_i'),$$

$$v_1' = \frac{1}{2}(-v_1+v_2+v_3+v_4)\,,\quad v_2' = \frac{1}{2}(v_1-v_2+v_3+v_4)$$
$$v_3' = \frac{1}{2}(v_1+v_2-v_3+v_4)\,,\quad v_4' = \frac{1}{2}(v_1+v_2+v_3-v_4)$$

$$\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\prod_{i=1}^{4}\vartheta\begin{bmatrix} a+h_i \\ b+g_i \end{bmatrix}(v_i) = -\prod_{i=1}^{4}\vartheta\begin{bmatrix} 1-h_i \\ 1-g_i \end{bmatrix}(v_i')$$

$$\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b}\prod_{i=1}^{4}\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(v_i) = -\prod_{i=1}^{4}\vartheta_1(v_i') + \prod_{i=1}^{4}\vartheta_1(v_i)$$

$$\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b}\prod_{i=1}^{4}\vartheta\begin{bmatrix}a+h_i\\b+g_i\end{bmatrix}(v_i)=-\prod_{i=1}^{4}\vartheta\begin{bmatrix}1-h_i\\1-g_i\end{bmatrix}(v_i')+\prod_{i=1}^{4}\vartheta\begin{bmatrix}1+h_i\\1+g_i\end{bmatrix}(v_i)$$

$$\left[\frac{1}{(2\pi i)^2}\frac{\partial^2}{\partial v^2}-\frac{1}{i\pi}\frac{\partial}{\partial\tau}\right]\vartheta\begin{bmatrix}a\\b\end{bmatrix}(v\mid\tau)=0$$

$$\frac{1}{4\pi i}\frac{\vartheta_2''}{\vartheta_2}=\partial_\tau\log\,\vartheta_2=\frac{i\pi}{12}(E_2+\vartheta_3^4+\vartheta_4^4)$$
$$\frac{1}{4\pi i}\frac{\vartheta_3''}{\vartheta_3}=\partial_\tau\log\,\vartheta_3=\frac{i\pi}{12}(E_2+\vartheta_2^4-\vartheta_4^4)$$
$$\frac{1}{4\pi i}\frac{\vartheta_4''}{\vartheta_4}=\partial_\tau\log\,\vartheta_4=\frac{i\pi}{12}(E_2-\vartheta_2^4-\vartheta_3^4)$$

$$\mathcal{P}(z)=4\pi i\partial_\tau\log\,\eta(\tau)-\partial_z^2\log\,\vartheta_1(z)=\frac{1}{z^2}+\mathcal{O}(z^2)$$

$$\mathcal{P}(-z)=\mathcal{P}(z)\,,\mathcal{P}(z+1)=\mathcal{P}(z+\tau)=\mathcal{P}(z)$$
$$\mathcal{P}(z,\tau+1)=\mathcal{P}(z,\tau)\,,\mathcal{P}\left(\frac{z}{\tau},-\frac{1}{\tau}\right)=\tau^2\mathcal{P}(z,\tau)$$

$$\int\frac{d^2z}{\tau_2}\mathcal{P}(z,\tau)=4\pi i\partial_\tau\log\left(\sqrt{\tau_2}\eta\right)$$
$$\int\frac{d^2z}{\tau_2}|\mathcal{P}(z,\tau)|^2=\left|4\pi i\partial_\tau\log\left(\sqrt{\tau_2}\eta\right)\right|^2$$
$$\int\frac{d^2z}{\tau_2}\overline{\mathcal{P}}(\bar z,\bar\tau)\left[\partial_z\log\,\vartheta_1(z)+2\pi i\frac{Imz}{\tau_2}\right]^2=4\pi i\partial_\tau\log\left(\eta\sqrt{\tau_2}\right)$$
$$\int\frac{d^2z}{\tau_2}\partial_z^2\log\,\vartheta_1(z)=-\frac{\pi}{\tau_2}$$

$$\tilde f(k)\equiv\frac{1}{2\pi}\int_{-\infty}^{+\infty}f(x)e^{ikx}dx$$

$$\sum_{n\in Z}f(2\pi n)=\sum_{n\in Z}\tilde f(n).$$

$$\sum_{n\in Z}e^{-\pi an^2+\pi bn}=\frac{1}{\sqrt a}\sum_{n\in Z}e^{-\frac{\pi}{a}\left(n+i\frac{b}{2}\right)^2},$$
$$\sum_{n\in Z}ne^{-\pi an^2+\pi bn}=-\frac{i}{\sqrt a}\sum_{n\in Z}\frac{\left(n+i\frac{b}{2}\right)}{a}e^{-\frac{\pi}{a}\left(n+i\frac{b}{2}\right)^2},$$
$$\sum_{n\in Z}n^2e^{-\pi an^2+\pi bn}=\frac{1}{\sqrt a}\sum_{n\in Z}\left[\frac{1}{2\pi a}-\frac{\left(n+i\frac{b}{2}\right)^2}{a^2}\right]e^{-\frac{\pi}{a}\left(n+i\frac{b}{2}\right)^2}.$$

$$\sum_{m_i\in Z}e^{-\pi m_im_jA_{ij}+\pi B_im_i}=(\det A)^{-\frac{1}{2}}\sum_{m_i\in Z}e^{-\pi(m_k+iB_k/2)(A^{-1})_{kl}(m_l+iB_l/2)}$$

$$S_{p,q}=\frac{1}{4\pi}\int d^2\sigma\sqrt{\det g}\,g^{ab}G_{\alpha\beta}\partial_a X^\alpha\partial_b X^\beta+\frac{1}{4\pi}\int d^2\sigma\epsilon^{ab}B_{\alpha\beta}\partial_a X^\alpha\partial_b X^\beta$$
$$+\frac{1}{4\pi}\int d^2\sigma\sqrt{\det g}\sum_I \psi^I(\bar\nabla+Y_\alpha^I(\bar\nabla X^\alpha)\bar\psi^I$$

$$Z_{p,p+16}(G,B,Y)=\frac{\sqrt{\det G}}{\tau_2^{p/2}\eta^p\bar\eta^{p+16}}$$
$$\times\sum_{m^\alpha,n^\alpha\in Z}\exp\left[-\frac{\pi}{\tau_2}(G+B)_{\alpha\beta}(m^\alpha+\tau n^\alpha)(m^\beta+\bar\tau n^\beta)\right]$$
$$\times\frac{1}{2}\sum_{a,b=0}^{1}\prod_{I=1}^{16}e^{i\pi\left(m^\alpha Y_\alpha^I Y_\beta^I n^\beta-bn^\alpha Y_\alpha^I\right)}\bar\vartheta\begin{bmatrix}a-2n^\alpha Y_\alpha^I\\ b-2m^\beta Y_\beta^I\end{bmatrix}$$

$$=\frac{\sqrt{\det G}}{\tau_2^{p/2}\eta^p\bar\eta^{p+16}}\sum_{m^\alpha,n^\alpha\in Z}\exp\left[-\frac{\pi}{\tau_2}(G+B)_{\alpha\beta}(m^\alpha+\tau n^\alpha)(m^\beta+\bar\tau n^\beta)\right]$$
$$\times\exp\left[-i\pi\sum_I n^\alpha(m^\beta+\bar\tau n^\beta)Y_\alpha^I Y_\beta^I\right]\frac{1}{2}\sum_{a,b=0}^{1}\prod_{I=1}^{16}\bar\vartheta\begin{bmatrix}a\\ b\end{bmatrix}\left(Y_\gamma^I(m^\gamma+\bar\tau n^\gamma)\mid\bar\tau\right)$$

$$\tau\to\tau+1\,,Z_{p,p+16}\to e^{4\pi i/3}Z_{p,p+16}$$

$$\Gamma_{p,p+16}(G,B,Y)=\sum_{m_\alpha,n_\alpha,Q_I}q^{P_L^2/2}\bar q^{P_R^2/2}$$

$$M=\begin{pmatrix}G^{-1} & G^{-1}C & G^{-1}Y^t\\ C^tG^{-1} & G+C^tG^{-1}C+Y^tY & C^tG^{-1}Y^t+Y^t\\ YG^{-1} & YG^{-1}C+Y & \mathbf{1}_{16}+YG^{-1}Y^t\end{pmatrix}$$

$$C_{\alpha\beta}=B_{\alpha\beta}-\frac{1}{2}Y_\alpha^I Y_\beta^I$$

$$L=\begin{pmatrix}0 & \mathbf{1}_p & 0\\ \mathbf{1}_p & 0 & 0\\ 0 & 0 & \mathbf{1}_{16}\end{pmatrix}$$

$$M^TLM=MLM=L\,,M^{-1}=LML$$

$$\frac{1}{2}P_L^2=\frac{1}{4}(m^\alpha,n_\alpha,Q_I)\cdot(M-L)\cdot\begin{pmatrix}m^\alpha\\ n_\alpha\\ Q_I\end{pmatrix}$$
$$\frac{1}{2}P_R^2=\frac{1}{4}(m^\alpha,n_\alpha,Q_I)\cdot(M+L)\cdot\begin{pmatrix}m^\alpha\\ n_\alpha\\ Q_I\end{pmatrix}.$$

$$\frac{1}{2}P_R^2-\frac{1}{2}P_L^2=m^\alpha n_\alpha-\frac{1}{2}Q_IQ_I$$

$$\Gamma_{p,p+16}(G,B,Y=0)=\Gamma_{p,p}(G,B)\bar\Gamma_{O(32)/Z_2}$$

$$\Gamma_{p,p+16}(G,B,Y=\tilde Y)=\Gamma_{p,p}(G',B')\bar\Gamma_{E_8\times E_8}$$

$$\begin{pmatrix} m^\alpha \\ n_\alpha \\ Q_I \end{pmatrix} \to \Omega \cdot \begin{pmatrix} m^\alpha \\ n_\alpha \\ Q_I \end{pmatrix}, M \to \Omega M \Omega^T$$

$$Z^N_{d,d+16}(\epsilon)\begin{bmatrix} h \\ g \end{bmatrix} = \frac{\Gamma_{p,p+16}(\epsilon)\begin{bmatrix} h \\ g \end{bmatrix}}{\eta^p \bar{\eta}^{p+16}} = \frac{\sum_{\lambda \in L+\epsilon\frac{h}{N}} e^{\frac{2\pi i g \epsilon \cdot \lambda}{N}} q^{p_L^2/2} \bar{q}^{p_R^2/2}}{\eta^p \bar{\eta}^{p+16}}$$

$$Z^N(-\epsilon)\begin{bmatrix} h \\ g \end{bmatrix} = Z^N(\epsilon)\begin{bmatrix} h \\ g \end{bmatrix}, Z^N(\epsilon)\begin{bmatrix} -h \\ -g \end{bmatrix} = Z^N(\epsilon)\begin{bmatrix} h \\ g \end{bmatrix}$$

$$Z^N(\epsilon)\begin{bmatrix} h+1 \\ g \end{bmatrix} = \exp\left[-\frac{i\pi g \epsilon^2}{N}\right] Z^N(\epsilon)\begin{bmatrix} h \\ g \end{bmatrix}, Z^N(\epsilon)\begin{bmatrix} h \\ g+1 \end{bmatrix} = Z^N(\epsilon)\begin{bmatrix} h \\ g \end{bmatrix}$$

$$Z^N(\epsilon + N\epsilon')\begin{bmatrix} h \\ g \end{bmatrix} = \exp\left[\frac{2\pi i g h \epsilon \cdot \epsilon'}{N}\right] Z^N(\epsilon)\begin{bmatrix} h \\ g \end{bmatrix}$$

$$\tau \to \tau + 1: \; Z^N(\epsilon)\begin{bmatrix} h \\ g \end{bmatrix} \to \exp\left[\frac{4\pi i}{3} + \frac{i\pi h^2 \epsilon^2}{N^2}\right] Z^N(\epsilon)\begin{bmatrix} h \\ h+g \end{bmatrix}$$

$$\tau \to -\frac{1}{\tau}: \; Z^N(\epsilon)\begin{bmatrix} h \\ g \end{bmatrix} \to \exp\left[-\frac{2\pi i h g \epsilon^2}{N^2}\right] Z^N(\epsilon)\begin{bmatrix} g \\ -h \end{bmatrix}$$

$$Z^N(\epsilon, \Omega M \Omega^T)\begin{bmatrix} h \\ g \end{bmatrix} = Z^N(\Omega \cdot \epsilon, M)\begin{bmatrix} h \\ g \end{bmatrix}$$

$$\hat{e}^{\hat{r}}_{\hat{\mu}} = \begin{pmatrix} e^r_\mu & A^\beta_\mu E^a_\beta \\ 0 & E^a_\alpha \end{pmatrix}, \hat{e}^{\hat{\mu}}_{\hat{r}} = \begin{pmatrix} e^\mu_r & -e^\nu_r A^\alpha_\nu \\ 0 & E^\alpha_a \end{pmatrix}$$

$$\hat{G}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu} + A^\alpha_\mu G_{\alpha\beta} A^\beta_\nu & G_{\alpha\beta} A^\beta_\mu \\ G_{\alpha\beta} A^\beta_\nu & G_{\alpha\beta} \end{pmatrix}, \hat{G}^{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g^{\mu\nu} & -A^{\mu\alpha} \\ -A^{\nu\alpha} & G^{\alpha\beta} + A^\alpha_\rho A^{\beta,\rho} \end{pmatrix}.$$

$$\alpha'^{D-2} S^{\text{heterotic}}_D = \int d^D x \sqrt{-\det g}\, e^{-\phi} \Big[R + \partial_\mu \phi \partial^\mu \phi + \frac{1}{4}\partial_\mu G_{\alpha\beta} \partial^\mu G^{\alpha\beta} + \\ -\frac{1}{4} G_{\alpha\beta} F^{A^\alpha}_{\mu\nu} F^{\beta,\mu\nu}_A\Big]$$

$$\phi = \hat{\Phi} - \frac{1}{2}\log\left(\det G_{\alpha\beta}\right)$$
$$F^{A^\alpha}_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu$$

$$-\frac{1}{12}\int d^{10}x \sqrt{-\det \hat{G}}\, e^{-\hat{\Phi}} \hat{H}^{\hat{\mu}\hat{\nu}\hat{\rho}} \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}}$$

$$= -\int d^D x \sqrt{-\det g}\, e^{-\phi}\left[\frac{1}{4} H_{\mu\alpha\beta} H^{\mu\alpha\beta} + \frac{1}{4} H_{\mu\nu\alpha} H^{\mu\nu\alpha} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho}\right]$$

$$H_{\mu\alpha\beta} = e^r_\mu \hat{e}^{\hat{\mu}}_{\hat{r}} \hat{H}_{\hat{\mu}\alpha\beta} = \hat{H}_{\mu\alpha\beta}$$

$$H_{\mu\nu\alpha} = e^r_\mu e^s_\nu \hat{e}^{\hat{\mu}}_r \hat{e}^{\hat{\nu}}_s \hat{H}_{\hat{\mu}\hat{\nu}\alpha} = \hat{H}_{\mu\nu\alpha} - A^\beta_\mu \hat{H}_{\nu\alpha\beta} + A^\beta_\nu \hat{H}_{\mu\alpha\beta}$$
$$H_{\mu\nu\rho} = e^r_\mu e^s_\nu e^t \hat{e}^{\hat{\mu}}_r \hat{e}^{\hat{\nu}}_s \hat{e}^{\hat{\rho}}_t \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \hat{H}_{\mu\nu\rho} + \left[-A^\alpha_\mu \hat{H}_{\alpha\nu\rho} + A^\alpha_\mu A^\beta_\nu \hat{H}_{\alpha\beta\rho} + \text{ cyclic}\right]$$

$$\int d^{10}x\sqrt{-\det\hat{G}}e^{-\hat{\Phi}}\sum_{I=1}^{16}\hat{F}^{I}_{\hat{\mu}\hat{\nu}}F^{I,\hat{\mu}\hat{\nu}}=\int d^{D}x\sqrt{-\det g}e^{-\phi}\sum_{I=1}^{16}\left[\tilde{F}^{I}_{\mu\nu}\tilde{F}^{I,\mu\nu}+2\tilde{F}^{I}_{\mu\alpha}\tilde{F}^{I,\mu\alpha}\right]$$

$$Y^{I}_{\alpha}=\hat{A}^{I}_{\alpha}\,,A^{I}_{\mu}=\hat{A}^{I}_{\mu}-Y^{I}_{\alpha}A^{\alpha}_{\mu}\,,\tilde{F}^{I}_{\mu\nu}=F^{I}_{\mu\nu}+Y^{I}_{\alpha}F^{A,\alpha}_{\mu\nu}$$
$$\tilde{F}^{I}_{\mu\alpha}=\partial_{\mu}Y^{I}_{\alpha}\,,F^{I}_{\mu\nu}=\partial_{\mu}A^{I}_{\nu}-\partial_{\nu}A^{I}_{\mu}$$

$$\hat{H}_{\mu\alpha\beta}=\partial_{\mu}\hat{B}_{\alpha\beta}+\frac{1}{2}\sum_{I}\left[Y^{I}_{\alpha}\partial_{\mu}Y^{I}_{\beta}-Y^{I}_{\beta}\partial_{\mu}Y^{I}_{\alpha}\right]$$

$$C_{\alpha\beta}\equiv\hat{B}_{\alpha\beta}-\frac{1}{2}\sum_{I}Y^{I}_{\alpha}Y^{J}_{\beta}$$

$$H_{\mu\alpha\beta}=\partial_{\mu}C_{\alpha\beta}+\sum_{I}Y^{I}_{\alpha}\partial_{\mu}Y^{I}_{\beta}$$

$$\hat{H}_{\mu\nu\alpha}=\partial_{\mu}\hat{B}_{\nu\alpha}-\partial_{\nu}\hat{B}_{\mu\alpha}+\frac{1}{2}\sum_{I}\left[\hat{A}^{I}_{\nu}\partial_{\mu}Y^{I}_{\alpha}-\hat{A}^{I}_{\mu}\partial_{\nu}Y^{I}_{\alpha}-Y^{I}_{\alpha}\hat{F}^{I}_{\mu\nu}\right]$$

$$B_{\mu,\alpha}\equiv\hat{B}_{\mu\alpha}+B_{\alpha\beta}A^{\beta}_{\mu}+\frac{1}{2}\sum_{I}Y^{I}_{\alpha}A^{I}_{\mu}$$
$$F^{B}_{\alpha,\mu\nu}=\partial_{\mu}B_{\alpha,\nu}-\partial_{\nu}B_{\alpha,\mu}$$

$$H_{\mu\nu\alpha}=F^{B}_{\alpha\mu\nu}-C_{\alpha\beta}F^{A,\beta}_{\mu\nu}-\sum_{I}Y^{I}_{\alpha}F^{I}_{\mu\nu}$$

$$B_{\mu\nu}=\hat{B}_{\mu\nu}+\frac{1}{2}\left[A^{\alpha}_{\mu}B_{\nu\alpha}+\sum_{I}A^{I}_{\mu}A^{\alpha}_{\nu}Y^{I}_{\alpha}-(\mu\leftrightarrow\nu)\right]-A^{\alpha}_{\mu}A^{\beta}_{\nu}B_{\alpha\beta}$$

$$\begin{aligned}H_{\mu\nu\rho}&=\partial_{\mu}B_{\nu\rho}-\frac{1}{2}\left[B_{\mu\alpha}F^{A,\alpha}_{\nu\rho}+A^{\alpha}_{\mu}F^{B}_{a,\nu\rho}+\sum_{I}A^{I}_{\mu}F^{I}_{\nu\rho}\right]+\text{ cyclic}\\&\equiv\partial_{\mu}B_{\nu\rho}-\frac{1}{2}L_{ij}A^{i}_{\mu}F^{j}_{\nu\rho}+\text{ cyclic}\end{aligned}$$

$$\begin{aligned}S_{D}^{\text{heterotic}}=\int d^{D}x\sqrt{-\det g}e^{-\phi}\Big[&R+\partial^{\mu}\phi\partial_{\mu}\phi-\frac{1}{12}\tilde{H}^{\mu\nu\rho}\tilde{H}_{\mu\nu\rho}-\\&-\frac{1}{4}(M^{-1})_{ij}F^{i}_{\mu\nu}F^{j\mu\nu}+\frac{1}{8}\mathrm{Tr}\left(\partial_{\mu}M\partial^{\mu}M^{-1}\right)\Big]\end{aligned}$$

$$S_{C}=-\frac{1}{2\cdot 4!}\int d^{d}x\sqrt{-G}\hat{F}^{2}$$

$$\hat{F}_{\mu\nu\rho\sigma}=\partial_{\mu}\hat{C}_{\nu\rho\sigma}-\partial_{\sigma}\hat{C}_{\mu\nu\rho}+\partial_{\rho}\hat{C}_{\sigma\mu\nu}-\partial_{\nu}\hat{C}_{\rho\sigma\mu}$$

$$C_{\alpha\beta\gamma}=\hat{C}_{\alpha\beta\gamma},C_{\mu\alpha\beta}=\hat{C}_{\mu\alpha\beta}-C_{\alpha\beta\gamma}A^{\gamma}_{\mu}$$
$$C_{\mu\nu\alpha}=\hat{C}_{\mu\nu\alpha}+\hat{C}_{\mu\alpha\beta}A^{\beta}_{\nu}-\hat{C}_{\nu\alpha\beta}A^{\beta}_{\mu}+C_{\alpha\beta\gamma}A^{\beta}_{\mu}A^{\gamma}_{\nu}$$
$$C_{\mu\nu\rho}=\hat{C}_{\mu\nu\rho}+\left(-\hat{C}_{\nu\rho\alpha}A^{\alpha}_{\mu}+\hat{C}_{\alpha\beta\rho}A^{\alpha}_{\mu}A^{\beta}_{\nu}+\text{ cyclic}\right)-C_{\alpha\beta\gamma}A^{\alpha}_{\mu}A^{\beta}_{\nu}A^{\gamma}_{\rho}$$

$$S_C = -\frac{1}{2\cdot 4!}\int d^D x\sqrt{-g}\sqrt{\det G_{\alpha\beta}}\left[F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} + 4F_{\mu\nu\rho\alpha}F^{\mu\nu\rho\alpha} + 6F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} + 4F_{\mu\alpha\beta\gamma}F^{\mu\alpha\beta\gamma}\right]$$

$$F_{\mu\alpha\beta\gamma} = \partial_\mu C_{\alpha\beta\gamma}, F_{\mu\nu\alpha\beta} = \partial_\mu C_{\nu\alpha\beta} - \partial_\nu C_{\mu\alpha\beta} + C_{\alpha\beta\gamma}F^{\gamma}_{\mu\nu}$$
$$F_{\mu\nu\rho\alpha} = \partial_\mu C_{\nu\rho\alpha} + C_{\mu\alpha\beta}F^{\beta}_{\nu\rho} + \text{ cyclic}$$
$$F_{\mu\nu\rho\sigma} = \left(\partial_\mu C_{\nu\rho\sigma} + 3\text{ perm.}\right) + \left(C_{\rho\sigma\alpha}F^{\alpha}_{\mu\nu} + 5\text{ perm.}\right)$$

$$\mathcal{L}_{N=1} = -\frac{1}{2\kappa^2}R + G_{i\bar{j}}D_\mu\phi^i D^\mu\bar{\phi}^{\bar{j}} + V(\phi,\bar{\phi}) + \sum_a \frac{1}{4g_a^2}\left[F_{\mu\nu}F^{\mu\nu}\right]_a + \frac{\theta_a}{4}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right]_a$$

$$G_{i\bar{j}} = \partial_i\partial_{\bar{j}}K(\phi,\bar{\phi})$$

$$\frac{1}{g_a^2} = \mathrm{Re}f_a(\phi)\,, \theta_a = -\mathrm{Im}f_a(\phi)$$

$$V(\phi,\bar{\phi}) = e^{\kappa^2 K}\left(D_i W G^{i\bar{\imath}}\bar{D}_{\bar{\imath}}\bar{W} - 3\kappa^2|W|^2\right)$$

$$D_i W = \frac{\partial W}{\partial\phi^i} + \kappa^2\frac{\partial K}{\partial\phi^i}W$$

$$K \to K + \Lambda(\phi) + \bar{\Lambda}(\bar{\phi})\,, W \to W e^{-\Lambda}\,, f_a \to f_a$$

$$K = -\log\left[i(\bar{Z}^I F_I - Z^I\bar{F}_I)\right]$$

$$K = -\log\left[2\left(f(T^i) + \bar{f}(\bar{T}^i)\right) - (T^i - \bar{T}^i)(f_i - \bar{f}_i)\right]$$

$$R_{i\bar{j}k\bar{l}} = G_{i\bar{j}}G_{k\bar{l}} + G_{i\bar{l}}G_{k\bar{j}} - e^{-2K}W_{ikm}G^{m\bar{m}}\bar{W}_{\bar{m}\bar{j}\bar{l}}$$

$$\mathcal{L}^{\text{vectors}} = -\frac{1}{4}\Xi_{IJ}F^I_{\mu\nu}F^{J,\mu\nu} - \frac{\theta_{IJ}}{4}F^I_{\mu\nu}\tilde{F}^{J,\mu\nu}$$

$$\Xi_{IJ} = \frac{i}{4}\left[N_{IJ} - \bar{N}_{IJ}\right]\,, \theta_{IJ} = \frac{1}{4}\left[N_{IJ} + \bar{N}_{IJ}\right]$$
$$N_{IJ} = \bar{F}_{IJ} + 2i\frac{\mathrm{Im}F_{IK}\mathrm{Im}F_{JL}Z^K Z^L}{\mathrm{Im}F_{MN}Z^M Z^N}$$

$$M^2_{BPS} = \frac{|e_I Z^I + q^I F_I|^2}{\mathrm{Im}(Z^I\bar{F}_I)}$$

$$M^2_{BPS} = \frac{1}{4\mathrm{Im}S}(\alpha^t + S\beta^t)M_+(\alpha + \bar{S}\beta) + \frac{1}{2}\sqrt{(\alpha^t M_+\alpha)(\beta^t M_+\beta) - (\alpha^t M_+\beta)^2},$$

$$\left\{Q^I_\alpha, Q^J_\beta\right\} = \epsilon_{\alpha\beta}Z^{IJ}\,, \left\{\bar{Q}^I_{\dot{\alpha}}, Q^J_{\dot{\beta}}\right\} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{Z}^{IJ}\,, \left\{Q^I_\alpha, \bar{Q}^J_{\dot{\alpha}}\right\} = \delta^{IJ}2\sigma^\mu_{\alpha\dot{\alpha}}P_\mu$$

$$\left\{Q^I_\alpha, \bar{Q}^J_{\dot{\alpha}}\right\} = 2M\delta_{\alpha\dot{\alpha}}\delta^{IJ}\,, \left\{Q^I_\alpha, Q^J_\beta\right\} = \left\{\bar{Q}^I_{\dot{\alpha}}, \bar{Q}^J_{\dot{\beta}}\right\} = 0$$

$$A_\alpha^I = \frac{1}{\sqrt{2M}} Q_\alpha^I \, , A_\alpha^{\dagger I} = \frac{1}{\sqrt{2M}} \bar{Q}_{\dot\alpha}^I$$

$$\{Q_\alpha^I, \bar{Q}_{\dot\alpha}^J\} = 2\begin{pmatrix} 2E & 0 \\ 0 & 0 \end{pmatrix} \delta^{IJ}$$

$$\{Q_\alpha^{am}, \bar{Q}_{\dot\alpha}^{bn}\} = 2M\delta^{\alpha\dot\alpha}\delta^{ab}\delta^{mn} \, , \{Q_\alpha^{am}, Q_\beta^{bn}\} = Z_n \epsilon^{\alpha\beta}\epsilon^{ab}\delta^{mn}$$

$$A_\alpha^m = \frac{1}{\sqrt{2}}\left[Q_\alpha^{1m} + \epsilon_{\alpha\beta} Q_\beta^{2m}\right] , B_\alpha^m = \frac{1}{\sqrt{2}}\left[Q_\alpha^{1m} - \epsilon_{\alpha\beta} Q_\beta^{2m}\right],$$

$$\{A_\alpha^m, A_\beta^n\} = \{A_\alpha^m, B_\beta^n\} = \{B_\alpha^m, B_\beta^n\} = 0,$$
$$\left\{A_\alpha^m, A_\beta^{\dagger n}\right\} = \delta_{\alpha\beta}\delta^{mn}(2M + Z_n), \left\{B_\alpha^m, B_\beta^{\dagger n}\right\} = \delta_{\alpha\beta}\delta^{mn}(2M - Z_n)$$

$$M \geq \max\left[\frac{Z_n}{2}\right]$$

$$B_{2n}(R) = \mathrm{Tr}_R\left[(-1)^{2\lambda}\lambda^{2n}\right]$$

$$Z_R(y) = \mathrm{str} y^{2\lambda}$$

$$Z_{[j]} = \begin{cases} (-)^{2j}\left(\dfrac{y^{2j+1} - y^{-2j-1}}{y - 1/y}\right) & \text{massive} \\ (-)^{2j}\left(y^{2j} + y^{-2j}\right) & \text{supermassive} \end{cases} .$$

$$Z_{r\otimes\tilde r} = Z_r Z_{\tilde r}$$

$$B_n(R) = \left(y^2 \frac{d}{dy^2}\right)^n Z_R(y)\bigg|_{y=1}$$

$$Z_m(y) = Z_{[j]}(y)(1 - y)^m(1 - 1/y)^m$$

$$L_j : \ [j] \otimes ([1] + 4[1/2] + 5[0])$$

$$S_j : \ [j] \otimes (2[1/2] + 4[0])$$

$$M_\lambda^0 : \pm(\lambda + 1/2) + 2(\pm\lambda) + \pm(\lambda - 1/2)$$

$$B_0(\text{ any rep }) = 0$$
$$B_2\left(M_\lambda^0\right) = (-1)^{2\lambda+1} , B_2\left(S_j\right) = (-1)^{2j+1} D_j , B_2\left(L_j\right) = 0.$$

$$\mathrm{L_j} : \ [j] \otimes (42[0] + 48[1/2] + 27[1] + 8[3/2] + [2])$$

$$s = 2 \text{ supermassive long} : 42[0] + 48[1/2] + 27[1] + 8[3/2] + [2]$$

$$I_j : [j] \otimes (14[0] + 14[1/2] + 6[1] + [3/2])$$

$$I_{3/2} : 14[0] + 14[1/2] + 6[1] + [3/2]$$

$$S_j : \ [j] \otimes (5[0] + 4[1/2] + [1]),$$

$$S_1 : 5[0] + 4[1/2] + [1]$$

$$M_\lambda^0 : [\pm(\lambda+1)] + 4[\pm(\lambda+1/2)] + 6[\pm(\lambda)] + 4[\pm(\lambda-1/2)] + [\pm(\lambda-1)],$$

$$M_1^0 : \ [\pm 2] + 4[\pm 3/2] + 6[\pm 1] + 4[\pm 1/2] + 2[0]$$

$$L_j \to 2I_j + I_{j+1/2} + I_{j-1/2}$$

$$I_j \to 2S_j + S_{j+1/2} + S_{j-1/2}$$

$$S_j \to \sum_{\lambda=0}^{j} M_\lambda^0 \, , j - \lambda \in \mathbb{Z}$$

$$B_n(\text{ any rep }) = 0 \text{ for } n = 0{,}2.$$

$$B_4(L_j) = B_4(I_j) = 0, B_4(S_j) = (-1)^{2j}\frac{3}{2}D_j$$

$$B_4(M_\lambda^0) = (-1)^{2\lambda}3, B_4(V^0) = \frac{3}{2}$$

$$B_6(L_j) = 0\, , B_6(I_j) = (-1)^{2j+1}\frac{45}{4}D_j\, , B_6(S_j) = (-1)^{2j}\frac{15}{8}D_j^3$$

$$B_6(M_\lambda^0) = (-1)^{2\lambda}\frac{15}{4}(1+12\lambda^2)\, , B_6(V^0) = \frac{15}{8}$$

$$B_8(L_j) = (-1)^{2j}\frac{315}{4}D_j\, , B_8(I_j) = (-1)^{2j+1}\frac{105}{16}D_j(1+D_j^2)$$

$$B_8(S_j) = (-1)^{2j}\frac{21}{64}D_j(1+2D_j^4)$$

$$B_8(M_\lambda^0) = (-1)^{2\lambda}\frac{21}{16}(1+80\lambda^2+160\lambda^4)\, , B_8(V^0) = \frac{63}{32}$$

$$(\lambda \pm 2) + 8\left(\lambda \pm \frac{3}{2}\right) + 28(\lambda \pm 1) + 56\left(\lambda \pm \frac{1}{2}\right) + 70(\lambda).$$

$$(\pm 2) + 8\left(\pm \frac{3}{2}\right) + 28(\pm 1) + 56\left(\pm \frac{1}{2}\right) + 70(0)$$

$$[j] \otimes ([2] + 8[3/2] + 27[1] + 48[1/2] + 42[0])$$

$$S^j \to \sum_{\lambda=0}^{j} M_0^\lambda$$

$$I_1^j : [j] \otimes ([5/2] + 10[2] + 44[3/2] + 110[1] + 165[1/2] + 132[0])$$

$$I_2^j : [j] \otimes ([3] + 12[5/2] + 65[2] + 208[3/2] + 429[1] +$$
$$+572[1/2] + 429[0])$$

$$I_3^j : [j] \otimes ([7/2] + 14[3] + 90[5/2] + 350[2] + 910[3/2] +$$
$$+1638[1] + 2002[1/2] + 1430[0])$$

$$[j] \otimes ([4] + 16[7/2] + 119[3] + 544[5/2] + 1700[2] + 3808[3/2] + 6188[1] + 7072[1/2] + 4862[0])$$

$$L^j \to I_3^{j+\frac{1}{2}} + 2I_3^j + I_3^{j-\frac{1}{2}}$$
$$I_3^j \to I_2^{j+\frac{1}{2}} + 2I_2^j + I_2^{j-\frac{1}{2}}$$
$$I_2^j \to I_1^{j+\frac{1}{2}} + 2I_1^j + I_1^{j-\frac{1}{2}}$$
$$I_1^j \to S^{j+\frac{1}{2}} + 2S^j + S^{j-\frac{1}{2}}$$

$$B_8(M_0^\lambda) = (-1)^{2\lambda}315$$

$$B_{10}(M_0^\lambda) = (-1)^{2\lambda}\frac{4725}{2}(6\lambda^2 + 1)$$
$$B_{12}(M_0^\lambda) = (-1)^{2\lambda}\frac{10395}{16}(240\lambda^4 + 240\lambda^2 + 19)$$
$$B_{14}(M_0^\lambda) = (-1)^{2\lambda}\frac{45045}{16}(336\lambda^6 + 840\lambda^4 + 399\lambda^2 + 20)$$
$$B_{16}(M_0^\lambda) = (-1)^{2\lambda}\frac{135135}{256}(7680\lambda^8 + 35840\lambda^6 + 42560\lambda^4 + 12800\lambda^2 + 457)$$

$$B_8(S^j) = (-1)^{2j} \cdot \frac{315}{2} D_j,$$
$$B_{10}(S^j) = (-1)^{2j} \cdot \frac{4725}{8} D_j(D_j^2 + 1),$$
$$B_{12}(S^j) = (-1)^{2j} \cdot \frac{10395}{32} D_j(3D_j^4 + 10D_j^2 + 6),$$
$$B_{14}(S^j) = (-1)^{2j} \cdot \frac{45045}{128} D_j(3D_j^6 + 21D_j^4 + 42D_j^2 + 14),$$
$$B_{16}(S^j) = (-1)^{2j} \cdot \frac{45045}{512} D_j(10D_j^8 + 120D_j^6 + 504D_j^4 + 560D_j^2 + 177),$$

$$B_8(I_1^j) = 0$$
$$B_{10}(I_1^j) = (-1)^{2j+1} \cdot \frac{14175}{4} D_j$$
$$B_{12}(I_1^j) = (-1)^{2j+1} \cdot \frac{155925}{16} D_j(2D_j^2 + 3)$$
$$B_{14}(I_1^j) = (-1)^{2j+1} \cdot \frac{2837835}{64} D_j(D_j^2 + 1)(D_j^2 + 4)$$
$$B_{16}(I_1^j) = (-1)^{2j+1} \cdot \frac{2027025}{128} D_j(4D_j^6 + 42D_j^4 + 112D_j^2 + 57)$$

$$B_8(I_3^j) = B_{10}(I_3^j) = B_{12}(I_3^j) = 0$$

$$B_{14}\left(I_3^j\right) = (-1)^{2j+1} \cdot \frac{42567525}{8} D_j,$$
$$B_{16}\left(I_3^j\right) = (-1)^{2j+1} \cdot \frac{212837625}{8} D_j\left(2D_j^2 + 5\right),$$
$$B_8\left(L^j\right) = B_{10}\left(L^j\right) = B_{12}\left(L^j\right) = B_{14}\left(L^j\right) = 0,$$
$$B_{16}\left(L^j\right) = (-1)^{2j} \cdot \frac{638512875}{2} D_j.$$

$$F_d(-1/\tau) = \tau^d F_d(\tau)\; F_d(\tau+1) = F_d(\tau)$$

$$E_2 = \frac{12}{i\pi}\partial_\tau \log\eta = 1 - 24\sum_{n=1}^{\infty}\frac{nq^n}{1-q^n}$$
$$E_4 = \frac{1}{2}(\vartheta_2^8 + \vartheta_3^8 + \vartheta_4^8) = 1 + 240\sum_{n=1}^{\infty}\frac{n^3q^n}{1-q^n}$$
$$E_6 = \frac{1}{2}(\vartheta_2^4 + \vartheta_3^4)(\vartheta_3^4 + \vartheta_4^4)(\vartheta_4^4 - \vartheta_2^4) = 1 - 504\sum_{n=1}^{\infty}\frac{n^5q^n}{1-q^n}.$$

$$H_2 \equiv \frac{1-E_2}{24} = \sum_{n=1}^{\infty}\frac{nq^n}{1-q^n} \equiv \sum_{n=1}^{\infty} d_2(n)q^n$$
$$H_4 \equiv \frac{E_4-1}{240} = \sum_{n=1}^{\infty}\frac{n^3q^n}{1-q^n} \equiv \sum_{n=1}^{\infty} d_4(n)q^n$$
$$H_6 \equiv \frac{1-E_6}{504} = \sum_{n=1}^{\infty}\frac{n^5q^n}{1-q^n} \equiv \sum_{n=1}^{\infty} d_6(n)q^n$$

$$d_{2k}(N) = \sum_{n|N} n^{2k-1}\,, k = 1,2,3$$

$$\hat{E}_2 = E_2 - \frac{3}{\pi\tau_2}$$

$$j = \frac{E_4^3}{\eta^{24}} = \frac{1}{q} + 744 + \cdots\,, \eta^{24} = \frac{1}{2^6\cdot 3^3}[E_4^3 - E_6^2].$$

$$F_{d+2} = \left(\frac{i}{\pi}\partial_\tau + \frac{d/2}{\pi\tau_2}\right)F_d \equiv D_dF_d$$

$$D_{d_1+d_2}\left(F_{d_1}F_{d_2}\right) = F_{d_2}\left(D_{d_1}F_{d_1}\right) + F_{d_1}\left(D_{d_2}F_{d_2}\right)$$

$$D_2\hat{E}_2 = \frac{1}{6}E_4 - \frac{1}{6}\hat{E}_2^2\,, D_4E_4 = \frac{2}{3}E_6 - \frac{2}{3}\hat{E}_2E_4\,, D_6E_6 = E_4^2 - \hat{E}_2E_6$$

$$\frac{\vartheta_1'''}{\vartheta_1'} = -\pi^2 E_2, \frac{\vartheta_1^{(5)}}{\vartheta_1'} = -\pi^2 E_2 (4\pi i \partial_\tau \log\, E_2 - \pi^2 E_2)$$

$$-3\frac{\vartheta_1^{(5)}}{\vartheta_1'} + 5\left(\frac{\vartheta_1'''}{\vartheta_1'}\right)^2 = 2\pi^4 E_4$$

$$-15\frac{\vartheta_1^{(7)}}{\vartheta_1'} - \frac{350}{3}\left(\frac{\vartheta_1'''}{\vartheta_1'}\right)^3 + 105\frac{\vartheta_1^{(5)}\vartheta_1'''}{\vartheta_1'^2} = \frac{80\pi^6}{3}E_6$$

$$\frac{1}{2}\sum_{i=2}^{4}\frac{\vartheta_i''\vartheta_i^7}{(2\pi i)^2} = \frac{1}{12}(E_2E_4 - E_6)$$

$$\xi(v) = \prod_{n=1}^{\infty}\frac{(1-q^n)^2}{(1-q^n e^{2\pi i v})(1-q^n e^{-2\pi i v})} = \frac{\sin\,\pi v}{\pi}\frac{\vartheta_1'}{\vartheta_1(v)}\;\xi(v) = \xi(-v)$$

$$\xi(0) = 1\,, \xi^{(2)}(0) = -\frac{1}{3}\left(\pi^2 + \frac{\vartheta_1'''}{\vartheta_1'}\right) = -\frac{\pi^2}{3}(1 - E_2)$$

$$\xi^{(4)}(0) = \frac{\pi^4}{5} + \frac{2\pi^2}{3}\frac{\vartheta_1'''}{\vartheta_1'} + \frac{2}{3}\left(\frac{\vartheta_1'''}{\vartheta_1'}\right)^2 - \frac{1}{5}\frac{\vartheta_1^{(5)}}{\vartheta_1'} = \frac{\pi^4}{15}(3 - 10E_2 + 2E_4 + 5E_2^2)$$

$$\xi^{(6)}(0) = -\frac{\pi^6}{7} - \pi^4\frac{\vartheta_1'''}{\vartheta_1'} - \frac{10\pi^2}{3}\left(\frac{\vartheta_1'''}{\vartheta_1'}\right)^2 + \pi^2\frac{\vartheta_1^{(5)}}{\vartheta_1'} +$$

$$-\frac{10}{3}\left(\frac{\vartheta_1'''}{\vartheta_1'}\right)^3 + 2\frac{\vartheta_1^{(5)}\vartheta_1'''}{\vartheta_1'^2} - \frac{1}{7}\frac{\vartheta_1^{(7)}}{\vartheta_1'}$$

$$= \frac{\pi^6}{63}(-9 + 63E_2 - 105E_2^2 - 42E_4 + 16E_6 +$$

$$+42E_2E_4 + 35E_2^3)$$

$$Z(v,\bar{v}) = \mathrm{Str}\left[q^{L_0}\bar{q}^{\bar{L}_0}e^{2\pi i v\lambda_R - 2\pi i\bar{v}\lambda_L}\right]$$

$$Z_{D=4}^{\text{heterotic}} = \frac{1}{\tau_2\eta^2\bar{\eta}^2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}}{\eta}C^{\text{int}}\begin{bmatrix}a\\b\end{bmatrix}$$

$$Z_{D=4}^{\text{heterotic}}(v,\bar{v}) = \frac{\xi(v)\bar{\xi}(\bar{v})}{\tau_2\eta^2\bar{\eta}^2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}(v)}{\eta}C^{\text{int}}\begin{bmatrix}a\\b\end{bmatrix}$$

$$Z_{D=4}^{\text{heterotic}}(v,\bar{v}) = \frac{\xi(v)\bar{\xi}(\bar{v})}{\tau_2\eta^2\bar{\eta}^2}\frac{\vartheta\begin{bmatrix}1\\1\end{bmatrix}(v/2)}{\eta}C^{\text{int}}\begin{bmatrix}1\\1\end{bmatrix}(v/2)$$

$$C^{\text{int}}\begin{bmatrix}1\\1\end{bmatrix}(v) = \mathrm{Tr}_R\left[(-1)^{F^{\text{int}}}e^{2\pi i v J_0}q^{L_0^{\text{int}}-3/8}\bar{q}^{\bar{L}_0^{\text{int}}-11/12}\right]$$

$$Q = \frac{1}{2\pi i}\frac{\partial}{\partial v}\,, \bar{Q} = -\frac{1}{2\pi i}\frac{\partial}{\partial\bar{v}}$$

$$B_{2n} \equiv \mathrm{Str}[\lambda^{2n}] = (Q+\bar{Q})^{2n}Z_{D=4}^{\text{heterotic}}(v,\bar{v})\Big|_{v=\bar{v}=0}$$

$$Z_{N=4}^{\text{heterotic}}(v,\bar{v}) = \frac{\vartheta_1^4(v/2)}{\eta^{12}\bar{\eta}^{24}}\xi(v)\bar{\xi}(\bar{v})\frac{\Gamma_{6,22}}{\tau_2}$$

$$B_4 = \langle (Q+\bar{Q})^4\rangle = \langle Q^4\rangle = \frac{3}{2}\frac{1}{\bar{\eta}^{24}}$$

$$B_6 = \langle (Q+\bar{Q})^6\rangle = \langle Q^6 + 15Q^4\bar{Q}^2\rangle = \frac{15}{8}\frac{2-\bar{E}_2}{\bar{\eta}^{24}}$$

$$Z_{N=8}^{II}(v,\bar{v}) = \mathrm{Str}\left[q^{L_0}\bar{q}^{\bar{L}_0}e^{2\pi i v\lambda_R - 2\pi i\bar{v}\lambda_L}\right]$$

$$= \frac{1}{4}\sum_{\alpha,\beta=0}^{1}\sum_{\bar{\alpha},\bar{\beta}=0}^{1}(-1)^{\alpha+\beta+\alpha\beta}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(v)\vartheta^3\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0)}{\eta^4}$$

$$\times(-1)^{\bar{\alpha}+\bar{\beta}+\bar{\alpha}\bar{\beta}}\frac{\bar{\vartheta}\begin{bmatrix}\bar{\alpha}\\ \bar{\beta}\end{bmatrix}\bar{\vartheta}^3\begin{bmatrix}\bar{\alpha}\\ \bar{\beta}\end{bmatrix}(0)}{\bar{\eta}^4}\frac{\xi(v)\bar{\xi}(\bar{v})}{\mathrm{Im}\tau|\eta|^4}\frac{\Gamma_{6,6}}{|\eta|^{12}} = \frac{\Gamma_{6,6}}{\mathrm{Im}\tau}\frac{\vartheta_1^4(v/2)}{\eta^{12}}\frac{\bar{\vartheta}_1^4(\bar{v}/2)}{\bar{\eta}^{12}}\xi(v)\bar{\xi}(\bar{v})$$

$$B_8 = \langle (Q+\bar{Q})^8\rangle = 70\langle Q^4\bar{Q}^4\rangle = \frac{315}{2}\frac{\Gamma_{6,6}}{\mathrm{Im}\tau}$$

$$M^2 = \frac{1}{4}p_L^2\,,\vec{m}\cdot\vec{n} = 0$$

$$B_{10} = \langle (Q+\bar{Q})^{10}\rangle = 210\langle Q^6\bar{Q}^4 + Q^4\bar{Q}^6\rangle$$
$$= -\frac{4725}{8\pi^2}\frac{\Gamma_{6,6}}{\mathrm{Im}\tau}\left(\frac{\vartheta_1'''}{\vartheta_1'} + 3\xi'' + cc\right) = \frac{4725}{4}\frac{\Gamma_{6,6}}{\mathrm{Im}\tau}$$

$$B_{12} = \langle 495(Q^4\bar{Q}^8 + Q^8\bar{Q}^4) + 924Q^6\bar{Q}^6\rangle = \left[\frac{10395}{2} + \frac{31185}{64}(E_4+\bar{E}_4)\right]\frac{\Gamma_{6,6}}{\mathrm{Im}\tau}$$

$$= \left[\frac{10395\cdot 19}{32} + \frac{10395\cdot 45}{4}\left(\frac{E_4-1}{240} + cc\right)\right]\frac{\Gamma_{6,6}}{\mathrm{Im}\tau}$$

$$I_2^j\colon \sum_j (-1)^{2j}D_j = d_4(N)$$

$$B_{14} = \langle (Q+\bar{Q})^{14}\rangle =$$
$$= \left[\frac{45045}{32}20 + \frac{14189175}{16}\left(2\frac{E_4-1}{240} + \frac{1-E_6}{504} + cc\right)\right]\frac{\Gamma_{6,6}}{\mathrm{Im}\tau}$$

$$I_2^j\colon \sum_j (-1)^{2j}D_j^3 = d_6(N)$$

$$Z^{II} = \frac{1}{8}\sum_{g,h=0}^{1}\sum_{\alpha,\beta=0}^{1}\sum_{\bar{\alpha},\bar{\beta}=0}^{1}(-1)^{\alpha+\beta+\alpha\beta}\frac{\vartheta^2\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}{\eta^2}\frac{\vartheta\begin{bmatrix}\alpha+h\\ \beta+g\end{bmatrix}}{\eta}\frac{\vartheta\begin{bmatrix}\alpha-h\\ \beta-g\end{bmatrix}}{\eta}\times$$
$$(-1)^{\bar{\alpha}+\bar{\beta}+\bar{\alpha}\bar{\beta}}\frac{\bar{\vartheta}}{\bar{\eta}^2}\begin{bmatrix}\bar{\alpha}\bar{\beta}\\ \bar{\eta}^2\end{bmatrix}\frac{\bar{\vartheta}\begin{bmatrix}\bar{\alpha}+h\\ \bar{\beta}+g\end{bmatrix}}{\bar{\eta}}\frac{\bar{\vartheta}\begin{bmatrix}\bar{\alpha}-h\\ \bar{\beta}-g\end{bmatrix}}{\bar{\eta}}\frac{1}{\mathrm{Im}\tau|\eta|^4}\frac{\Gamma_{2,2}}{|\eta|^4}Z_{4,4}\begin{bmatrix}h\\ g\end{bmatrix}$$

$$Z_{4,4}\begin{bmatrix}0\\0\end{bmatrix} = \frac{\Gamma_{4,4}}{|\eta|^8}\,, Z_{4,4}\begin{bmatrix}0\\1\end{bmatrix} = 16\frac{|\eta|^4}{|\vartheta_2|^4} = \frac{|\vartheta_3\vartheta_4|^4}{|\eta|^8}$$

$$Z_{4,4}\begin{bmatrix}1\\0\end{bmatrix} = 16\frac{|\eta|^4}{|\vartheta_4|^4} = \frac{|\vartheta_2\vartheta_3|^4}{|\eta|^8}\,, Z_{4,4}\begin{bmatrix}1\\1\end{bmatrix} = 16\frac{|\eta|^4}{|\vartheta_3|^4} = \frac{|\vartheta_2\vartheta_4|^4}{|\eta|^8}$$

$$Z^{II}(v,\bar{v}) = \frac{1}{4}\sum_{\alpha\beta\bar{\alpha}\bar{\beta}} (-1)^{\alpha+\beta+\alpha\beta+\bar{\alpha}+\bar{\beta}+\bar{\alpha}\bar{\beta}}\frac{\vartheta\begin{bmatrix}\alpha\\\beta\end{bmatrix}(v)\vartheta\begin{bmatrix}\alpha\\\beta\end{bmatrix}(0)}{\eta^6}$$

$$\times\frac{\bar{\vartheta}\begin{bmatrix}\bar{\alpha}\\\bar{\beta}\end{bmatrix}(\bar{v})\bar{\vartheta}\begin{bmatrix}\bar{\alpha}\\\bar{\beta}\end{bmatrix}(0)}{\bar{\eta}^6}\xi(v)\bar{\xi}(\bar{v})C\begin{bmatrix}\alpha & \bar{\alpha}\\ & \bar{\beta}\end{bmatrix}\frac{\Gamma_{2,2}}{\tau_2}$$

$$= \frac{\vartheta_1^2(v/2)\bar{\vartheta}_1^2(\bar{v}/2)}{\eta^6\bar{\eta}^6}\xi(v)\bar{\xi}(\bar{v})C\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}(v/2,\bar{v}/2)\frac{\Gamma_{2,2}}{\tau_2}$$

$$C\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}(v,0) = 8\sum_{i=2}^{4}\frac{\vartheta_i^2(v)}{\vartheta_i^2(0)}$$

$$\langle\lambda^4\rangle = \rangle(Q+\bar{Q})^4\rangle = 6\langle Q^2\bar{Q}^2 + Q^2\bar{Q}^4\rangle = 36\frac{\Gamma_{2,2}}{\tau_2}.$$

$$\langle\lambda^6\rangle = \rangle(Q+\bar{Q})^6\rangle = 15\langle Q^4\bar{Q}^2 + Q^2\bar{Q}^4\rangle = 90\frac{\Gamma_{2,2}}{\tau_2},$$

$$\partial_v^2 C\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}(v,0)\Big|_{v=0} = -16\pi^2 E_2$$

$$L_{\text{gauge}} = -\frac{1}{8}\mathrm{Im}\int d^4x\sqrt{-\det g}\,\mathbf{F}_{\mu\nu}^i N_{ij}\mathbf{F}^{j,\mu\nu}$$

$$\mathbf{F}_{\mu\nu} = F_{\mu\nu} + i^\star F_{\mu\nu}\,, {}^\star F_{\mu\nu} = \frac{1}{2}\frac{\epsilon_{\mu\nu}{}^{\rho\sigma}}{\sqrt{-g}}F_{\rho\sigma}$$

$$L_{\text{gauge}} = -\frac{1}{4}\int d^4x\left[\sqrt{-g}F_{\mu\nu}^i N_2^{ij}F^{j,\mu\nu} + F_{\mu\nu}^i N_1^{ij\star}F^{j,\mu\nu}\right]$$

$$\mathbf{G}_{\mu\nu}^i = N_{ij}\mathbf{F}_{\mu\nu}^j = N_1F - N_2\,{}^\star F + i(N_2F + N_1\,{}^\star F)$$

$$\mathrm{Im}\nabla^\mu\begin{pmatrix}\mathbf{G}_{\mu\nu}^i\\\mathbf{F}_{\mu\nu}^i\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix}\mathbf{G}'_{\mu\nu}\\\mathbf{F}'_{\mu\nu}\end{pmatrix} = \begin{pmatrix}A & B\\C & D\end{pmatrix}\begin{pmatrix}\mathbf{G}_{\mu\nu}\\\mathbf{F}_{\mu\nu}\end{pmatrix}$$

$$N' = (AN+B)(CN+D)^{-1}$$

$$F' = C(N_1F - N_2\,{}^\star F) + DF\,, {}^\star F' = C(N_2F + N_1\,{}^\star F) + D^\star F$$

$$F' = N_1 F - N_2^{\star} F\ ,\ {}^{\star}F' = N_2 F + N_1^{\star} F\ , N' = -\frac{1}{N}.$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \mathbf{1} - e & -e \\ e & \mathbf{1} - e \end{pmatrix}, e = \begin{pmatrix} 1 & 0 & \dots \\ 0 & 0 & \dots \\ . & & . \end{pmatrix},.$$

$$N'_{00} = -\frac{1}{N_{00}}, N'_{0i} = \frac{N_{0i}}{N_{00}}, N'_{i0} = \frac{N_{i0}}{N_{00}}, N'_{ij} = N_{ij} - \frac{N_{i0} N_{0j}}{N_{00}}.$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \mathbf{1} - e_1 & e_2 \\ -e_2 & \mathbf{1} - e_1 \end{pmatrix},$$

$$e_1 = \begin{pmatrix} 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & . \\ . & . & . & . \end{pmatrix}, e_2 = \begin{pmatrix} 0 & 1 & 0 & \dots \\ -1 & 0 & 0 & \dots \\ 0 & 0 & 0 & . \\ . & . & . & . \end{pmatrix}.$$

$$N'_{\alpha\beta} = -\frac{N_{\alpha\beta}}{\det N_{\alpha\beta}}\ N'_{\alpha i} = -\frac{N_{\alpha\beta} \epsilon^{\beta\gamma} N_{\gamma i}}{\det N_{\alpha\beta}}\ , N'_{i\alpha} = \frac{N_{i\beta} \epsilon^{\beta\gamma} N_{\alpha\gamma}}{\det N_{\alpha\beta}}$$

$$N'_{ij} = N_{ij} + \frac{N_{i\alpha} \epsilon^{\alpha\beta} N_{\beta\gamma} \epsilon^{\gamma\delta} N_{\delta j}}{\det N_{\alpha\beta}}$$

$$N = S_1 L + i S_2 M^{-1}\ , S = S_1 + i S_2$$

$$N' = -N^{-1} = -\frac{S_1}{|S|^2} L + i \frac{S_2}{|S|^2} M = -\frac{S_1}{|S|^2} L + i \frac{S_2}{|S|^2} L M^{-1} L$$

$$N' = -L N^{-1} L = -\frac{S_1}{|S|^2} L + i \frac{S_2}{|S|^2} M^{-1}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} a \mathbf{1}_{28} & bL \\ cL & d \mathbf{1}_{28} \end{pmatrix}\ , ad - bc = 1$$

Supergravedad cuántica perturbativa en campos cuánticos relativistas. Modelo Matemático. Espacio de Moduli, amplitudes y gauge fixing. Invariante BRST y tensores. Invariancia difeomorfista. Invariancia de gauge. Operadores de Vértice.

$$I = I_X + I_{\mathrm{gh}}$$

$$I_{\mathrm{gh}} = \frac{1}{2\pi} \int \mathrm{d}^2 \sigma \sqrt{g} b_{ij} D^i c^j$$

$$\{Q_B, b_{ij}\} = T_{ij}$$

$$\delta I = \frac{1}{4\pi} \int \mathrm{d}^2 \sigma \sqrt{g} \delta g_{ij} T^{ij}$$

$$[Q_B, g_{ij}] = \delta g_{ij}, \{Q_B, \delta g_{ij}\} = 0$$

$$I \to \hat{I} = I + \frac{1}{4\pi} \int \mathrm{d}^2 \sigma \sqrt{g} \delta g_{ij} b^{ij}$$

$$F(g \mid \delta g)=\int \mathcal{D}(X,b,c)\exp\left(-\hat{I}(X,b,c;g,\delta g)\right)=\int \mathcal{D}(X,b,c)\exp(-I)\exp\left(-\frac{1}{4\pi}\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\delta g_{ij}b^{ij}\right)$$

$$[Q_B, F(g \mid \delta g)\}=0$$

$$\exp\left(-\frac{1}{4\pi}\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\delta g_{ij}b^{ij}\right)=\sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{1}{4\pi}\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\delta g_{ij}b^{ij}\right)^n$$

$$Q_B=\int_\Sigma \mathrm{d}^2\sigma\sum_{i,j=1,2}\sqrt{g}\delta g_{ij}\frac{\delta}{\delta g_{ij}}$$

$$\mathrm{d}=\sum_{i=1}^{s}\mathrm{d}x^i\frac{\partial}{\partial x^i},$$

$$F(x^1\ldots \mid \cdots \mathrm{d}x^s)=\sum_{i_1<\cdots<i_p}F_{i_1\ldots i_p}(x^1\ldots x^s)\mathrm{d}x^{i_1}\ldots\ \mathrm{d}x^{i_p}$$

$$g_{ij}\to g_{ij}+\epsilon\left(D_iv_j+D_jv_i\right)$$

$$\delta g_{ij}\to \delta g_{ij}+\epsilon\left(D_iv_j+D_jv_i\right)$$

$$\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\left(D_iv_j+D_jv_i\right)\frac{\delta}{\delta\left(\delta g_{ij}\right)}F(g \mid \delta g)=0$$

$$\int \mathcal{D}(X,b,c)\exp(-\hat{I})\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\left(D_iv_j+D_jv_i\right)b^{ij}=0$$

$$D_ib^{ij}=0.$$

$$c^i\to c^i+\epsilon v^i,$$

$$\int \mathcal{D}(X,b,c)\exp(-\hat{I})\to\int \mathcal{D}(X,b,c)\exp(-\hat{I})\left(1+\frac{\epsilon}{2\pi}\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}b_{ij}D^iv^j\right)$$

$$b_{ij}=\sum_{\alpha=1}^{6g-6}\mathrm{u}_\alpha \mathrm{b}_{\alpha,ij}+\sum_\lambda w_\lambda \mathrm{b}'_{\lambda ij}.$$

$$\hat{I}=\cdots+\frac{1}{4\pi}\sum_{\alpha=1}^{6g-6}\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\delta g_{ij}\mathrm{u}_\alpha \mathrm{b}_\alpha^{ij}.$$

$$\prod_{\alpha=1}^{6g-6}\int \mathrm{d}u_\alpha\exp\left(\frac{\mathrm{u}_\alpha}{4\pi}\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\delta g_{ij}\ \mathrm{b}_\alpha^{ij}\right)=\prod_{\alpha=1}^{6g-6}\frac{1}{4\pi}\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\delta g_{ij}\ \mathrm{b}_\alpha^{ij}.$$

$$v_\alpha=\frac{1}{4\pi}\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\delta g_{ij}\ \mathrm{b}_\alpha^{ij}$$

$$\prod_{\alpha=1}^{6g-6} \delta(v_\alpha)$$

$$c^i = \sum_\lambda \gamma_\lambda c_\lambda^i$$

$$\prod_\lambda \int D\gamma_\lambda \exp\left(m_\lambda \gamma_\lambda w_\lambda\right) = \prod_\lambda m_\lambda \cdot \prod_\lambda \delta(w_\lambda)$$

$$Z_{\mathrm{g}} = \int_{\mathcal{M}_{\mathrm{g}}} F(g \mid \delta g)$$

$$\delta g_{ij} = \sum_{s=1}^{\mathrm{p}} \frac{\partial g_{ij}}{\partial m_s}\, \mathrm{d}m_s.$$

$$\frac{1}{4\pi}\int_\Sigma \mathrm{d}^2\sigma\sqrt{g}\,\delta g_{ij} b^{ij} = \frac{1}{4\pi}\sum_{s=1}^{\mathrm{p}} \mathrm{d}m_s \int_\Sigma \mathrm{d}^2\sigma \frac{\partial\left(\sqrt{g}g_{ij}\right)}{\partial m_s} b^{ij}$$

$$F(g \mid \delta g) = \int \mathcal{D}(X,b,c)\exp\left(-\hat{I}\right) = \int \mathcal{D}(X,b,c)\exp\left(-I - \frac{1}{4\pi}\sum_{s=1}^{\mathrm{p}} \mathrm{d}m_s \int_\Sigma \mathrm{d}^2\sigma \frac{\partial\left(\sqrt{g}g_{ij}\right)}{\partial m_s} b^{ij}\right)$$

$$\mathrm{d}m_1, \dots, \mathrm{d}m_{\mathrm{p}}$$

$$F_{\mathrm{top}}\left(m_1 \dots \mid \cdots \mathrm{d}m_{\mathfrak{p}}\right) = (-1)^{\mathfrak{p}(\mathfrak{p}+1)/2}\, \mathrm{d}m_1 \dots \mathrm{d}m_{\mathfrak{p}} \int \mathcal{D}(X,b,c) e^{-I} \prod_{s=1}^{\mathfrak{p}} \frac{1}{4\pi}\int_\Sigma \mathrm{d}^2\sigma \frac{\partial\left(\sqrt{g}g_{ij}\right)}{\partial m_s} b^{ij}$$

$$\Psi_s = \frac{1}{4\pi}\int_\Sigma \mathrm{d}^2\sigma \frac{\partial\left(\sqrt{g}g_{ij}\right)}{\partial m_s} b^{ij}.$$

$$(-1)^{\mathfrak{p}(\mathfrak{p}+1)/2} \int \mathcal{D}\left(m_1 \dots \mid \cdots \mathrm{d}m_{\mathfrak{p}}\right) \mathrm{d}m_1 \dots \mathrm{d}m_{\mathfrak{p}} \int \mathcal{D}(X,b,c)\exp(-I)\Psi_1 \dots \Psi_{\mathfrak{p}}.$$

$$(-1)^{\mathfrak{p}(\mathfrak{p}+1)/2} \int \mathcal{D}\left(m_1 \dots \mid \cdots \mathrm{d}m_{\mathfrak{p}}\right) \mathrm{d}m_1 \dots \mathrm{d}m_{\mathfrak{p}} \int \mathcal{D}(X,b,c)\exp(-I)\delta(\Psi_1) \dots \delta\left(\Psi_{\mathfrak{p}}\right).$$

$$\int \mathcal{D}\left(m_1, \dots, m_{\mathfrak{p}}\right) \int \mathcal{D}(X,b,c)\exp(-I)\delta(\Psi_1) \dots \delta\left(\Psi_{\mathfrak{p}}\right)$$

$$F_\Omega(g \mid \delta g) = \int \mathcal{D}(X,b,c)\exp\left(-\hat{I}(X,b,c,g,\delta g)\right)\Omega.$$

$$\mathrm{d}F_\Omega + F_{Q_B\Omega} = 0$$

$$\Omega = \prod_{s=1}^{n} \mathcal{V}_s(X, b, c; p_s)$$

$$b_n \mathcal{V}_s = \tilde{b}_n \mathcal{V}_s = 0, n \geq 0$$

$$b_n = \frac{1}{2\pi i} \oint \mathrm{d}z z^{n+1} b_{zz}, \tilde{b}_n = \frac{1}{2\pi i} \oint \mathrm{d}\tilde{z} \tilde{z}^{n+1} b_{\tilde{z}\tilde{z}}$$

$$b_n \mathcal{W}_s = \tilde{b}_n \mathcal{W}_s = 0, n \geq 0.$$

$$\delta c^i = c^j \partial_j c^i$$
$$\delta X = c^j \partial_j X$$
$$\delta g_{ij} = D_i c_j + D_j c_i - g_{ij} D_k c^k$$

$$v^i(p_1) = \cdots = v^i(p_\mathrm{n}) = 0$$

$$\mathcal{M}_{\mathrm{g,n}} = \mathcal{J}/\mathcal{D}_{p_1,\ldots,p_\mathrm{n}}$$

$$\int \mathcal{D}(X, b, c) \exp(-\hat{I}) \prod_{s=1}^{\mathrm{n}} \mathcal{V}_s(X, b, c; p_s) \int_\Sigma \mathrm{d}^2\sigma \sqrt{g} \left(D_i v_j + D_j v_i \right) b^{ij} = 0$$

$$\langle \mathcal{V}_1 \ldots \mathcal{V}_\mathrm{n} \rangle_\mathrm{g} = \int_{\mathcal{M}_{\mathrm{g,n}}} F_{\mathcal{V}_1,\ldots,\mathcal{V}_\mathrm{n}}$$

$$\int_{\mathcal{M}_{\mathrm{g,n}}} F_{\{Q_B, \mathcal{W}_1\}, \mathcal{V}_2,\ldots,\mathcal{V}_\mathrm{n}} = 0$$

$$\mathrm{d}F_{\mathcal{W}_1, \mathcal{V}_2,\ldots,\mathcal{V}_\mathrm{n}} + F_{\{Q_B, \mathcal{W}_1\}, \mathcal{V}_2,\ldots,\mathcal{V}_\mathrm{n}} = 0$$

$$\int_{\mathcal{M}_{\mathrm{g,n}}} F_{\{Q_B, \mathcal{W}_1\}, \mathcal{V}_2,\ldots,\mathcal{V}_\mathrm{n}} = -\int_{\mathcal{M}_{\mathrm{g,n}}} \mathrm{d}F_{\mathcal{W}_1, \mathcal{V}_2,\ldots,\mathcal{V}_\mathrm{n}}$$

- Deligne-Mumford – Riemann.

$$z = \mathbf{z}, \tilde{z} = \tilde{\mathbf{z}}$$

$$z = \begin{cases} w + \mathbf{z} & \text{if } |w| < \epsilon \\ w & \text{if } |w| > 1 - \epsilon \end{cases}$$

$$z = w + f(|w|)\mathbf{z}$$

$$\tilde{z} = \widetilde{w} + f(|w|)\tilde{\mathbf{z}}$$

$$\partial_{\mathbf{z}} g_{IJ} = D_I v_J + D_J v_I$$
$$\partial_{\tilde{\mathbf{z}}} g_{IJ} = D_I \tilde{v}_J + D_J \tilde{v}_I,$$

$$\left(v^w, v^{\widetilde{w}} \right) = -\left(\partial_{\mathbf{z}}|_{z,\tilde{z}} w, \partial_{\mathbf{z}}|_{z,\tilde{z}} \widetilde{w} \right)$$

$$\left(v^{w}, v^{\widetilde{w}}\right) = (1,0)$$

$$\left(\tilde{v}^{w}, \tilde{v}^{\widetilde{w}}\right) = (0,1)$$

$$\mathrm{d}\mathbf{z}\ \mathrm{d}\tilde{\mathbf{z}}\Psi_{\mathbf{z}}\Psi_{\tilde{\mathbf{z}}}$$

$$\Psi_{\mathbf{z}} = \frac{1}{2\pi}\int_{\Sigma} \mathrm{d}w\ \mathrm{d}\widetilde{w}\sqrt{g}b^{ij}D_i v_j, \Psi_{\tilde{\mathbf{z}}} = \frac{1}{2\pi}\int_{\Sigma} \mathrm{d}w\ \mathrm{d}\widetilde{w}\sqrt{g}b^{ij}D_i \tilde{v}_j$$

$$\Psi_{\mathbf{z}}\Psi_{\tilde{\mathbf{z}}}\tilde{c}cV(0)$$

$$\int_{\Sigma} \mathrm{d}\mathbf{z}\ \mathrm{d}\tilde{\mathbf{z}}V(\mathbf{z}, \tilde{\mathbf{z}})$$

$$\begin{array}{ll} \Sigma \rightarrow & \mathcal{M}_{\mathrm{g,n}} \\ & \downarrow \pi \\ & \mathcal{M}_{\mathrm{g,n-1}} \end{array}$$

$$\mathcal{V} = \delta(\tilde{c})\delta(c)V$$

$$\delta I = \frac{-i}{4\pi}\int_{\Sigma} \mathrm{d}\tilde{z}\ \mathrm{d}z\left(\delta J_{\tilde{z}}^{z} T_{zz} - \delta J_{z}^{\tilde{z}} T_{\tilde{z}\tilde{z}}\right)$$

$$\hat{I} = I + \frac{-i}{4\pi}\int_{\Sigma} \mathrm{d}\tilde{z}\ \mathrm{d}z\left(\delta J_{\tilde{z}}^{z} b_{zz} - \delta J_{z}^{\tilde{z}} b_{\tilde{z}\tilde{z}}\right)$$

Espacios de Supermoduli - Riemann. Variables holomórficas y antiholomórficas y deformaciones.

$$D_\theta = \frac{\partial}{\partial\theta} + \theta\frac{\partial}{\partial z}.$$

$$\Phi^{[n]} = u + \theta v,$$

$$v_f = f(z)(\partial_\theta - \theta\partial_z)$$

$$V_g = g(z)\partial_z + \frac{g'(z)}{2}\theta\partial_\theta$$

$$\begin{aligned} G_r &= z^{r+1/2}(\partial_\theta - \theta\partial_z), && r \in \mathbb{Z} + 1/2 \\ L_n &= -z^{n+1}\partial_z - \frac{1}{2}(n+1)z^n\theta\partial_\theta, && n \in \mathbb{Z} \end{aligned}$$

$$r \geq -1/2, n \geq -1$$

$$[L_m, L_n] = (m-n)L_{m+n}$$
$$\{G_r, G_s\} = 2L_{r+s}$$
$$[L_m, G_r] = \left(\frac{m}{2} - r\right)G_{m+r}$$

$$\delta\mathcal{J}_{\tilde{z}}^{z} = h_{\tilde{z}}^{z} + \theta\chi_{\tilde{z}}^{\theta}$$
$$\delta\mathcal{J}_{\theta}^{\tilde{z}} = e_{\theta}^{\tilde{z}} + \theta h_{z}^{\tilde{z}}$$

$$q^{\tilde{z}}\partial_{\tilde{z}}+\left(q^{z}\partial_{z}+\frac{1}{2}D_{\theta}q^{z}D_{\theta}\right)+q^{\theta}D_{\theta}$$

$$\delta D_{\theta}=\left[D_{\theta},q^{\theta}D_{\theta}\right]=\left(D_{\theta}q^{\theta}\right)D_{\theta}-2q^{\theta}\partial_{z}$$

$$\delta\mathcal{J}_{\tilde{z}}^{z}=\partial_{\tilde{z}}q^{z}$$

$$\delta\mathcal{J}_{\theta}^{\tilde{z}}=D_{\theta}q^{\tilde{z}}(3.10)$$

Propagadores, indentidades y números fantasma.

$$\delta I=\frac{1}{2\pi}\int_{\Sigma}\mathcal{D}(\tilde{z},z\mid\theta)\left(\delta\mathcal{J}_{\tilde{z}}^{z}\mathcal{S}_{z\theta}+\delta\mathcal{J}_{\theta}^{\tilde{z}}T_{\tilde{z}\tilde{z}}\right)$$

$$\mathcal{D}(\tilde{z},z\mid\theta)=-i[\,\mathrm{d}\tilde{z};\mathrm{d}z\mid\mathrm{d}\theta]$$

$$\mathcal{S}_{z\theta}=S_{z\theta}+\theta T_{zz}$$

$$\partial_{\tilde{z}}\mathcal{S}_{z\theta}=0=D_{\theta}T_{\tilde{z}\tilde{z}}=\partial_{z}T_{\tilde{z}\tilde{z}},$$

$$\mathcal{S}_{z\theta}(z\mid\theta)=\frac{1}{2}\sum_{r\in\mathbb{Z}+1/2}z^{-r-3/2}G_{r}+\theta\sum_{n\in\mathbb{Z}}z^{-n-2}L_{n}$$

$$T_{\tilde{z}\tilde{z}}=\sum_{n\in\mathbb{Z}}\tilde{z}^{-n-2}\tilde{L}_{n}.$$

$$C^{z}=c^{z}+\theta\gamma^{\theta}$$

$$B_{z\theta}=\beta_{z\theta}+\theta b_{zz}$$

$$\tilde{B}_{\tilde{z}z}=\tilde{b}_{\tilde{z}\tilde{z}}+\theta\tilde{f}_{\tilde{z}\tilde{z}\theta},\tilde{C}^{\tilde{z}}=\tilde{c}^{\tilde{z}}+\theta\tilde{g}_{\theta}^{\tilde{z}}$$

$$I_{\mathrm{gh}}=\frac{1}{2\pi}\int\mathcal{D}(\tilde{z},z\mid\theta)\left(B_{z\theta}\partial_{\tilde{z}}C^{z}+\tilde{B}_{\tilde{z}\tilde{z}}D_{\theta}\tilde{C}^{\tilde{z}}\right)$$

$$\partial_{\tilde{z}}B_{z\theta}=0=D_{\theta}\tilde{B}_{\tilde{z}\tilde{z}}$$

$$[Q_{B},B_{z\theta}]=\mathcal{S}_{z\theta}$$

$$\{Q_{B},\tilde{B}_{\tilde{z}z}\}=T_{\tilde{z}\tilde{z}}$$

$$\left\{Q_{B},\delta\mathcal{J}_{\tilde{z}}^{\tilde{z}}\right\}=\left[Q_{B},\delta\mathcal{J}_{\theta}^{\tilde{z}}\right]=0$$

$$I\rightarrow\hat{I}=I+\frac{1}{2\pi}\int_{\Sigma}\mathcal{D}(\tilde{z},z\mid\theta)\left(\delta\mathcal{J}_{\tilde{z}}^{z}B_{z\theta}-\delta\mathcal{J}_{\theta}^{\tilde{z}}\tilde{B}_{\tilde{z}\tilde{z}}\right)$$

$$F(\mathcal{J},\delta\mathcal{J})=\int\mathcal{D}(X,B,C,\tilde{B},\tilde{C})\exp\left(-\hat{I}\right)$$

$$[Q_{B},F(\mathcal{J},\delta\mathcal{J})\}=0$$

$$\mathrm{d}F(\mathcal{J},\delta\mathcal{J})=0$$

Supermanifolds e integración bosónica.

$$F(x,\lambda\, \mathrm{d}x) = \lambda^s F(x, \mathrm{d}x)$$

$$\mathrm{d} = \sum_I \mathrm{d}x^I \frac{\partial}{\partial x^I},\ \mathrm{d}^2 = 0$$

$$\int \mathcal{D}(x, \mathrm{d}x) F(x, \mathrm{d}x)$$

$$F(x, \mathrm{d}x) = f(t^1 \dots | \cdots \theta^n)\mathrm{d}t^1 \dots\ \mathrm{d}t^m \delta(\mathrm{d}\theta^1) \dots \delta(\mathrm{d}\theta^n)$$

$$\begin{aligned} F(x, \mathrm{d}x) &= f(t^1 \dots | \cdots \theta^n)\delta(\mathrm{d}t^1) \dots \delta(\mathrm{d}\theta^n) \\ &= f(t^1 \dots | \cdots \theta^n)\delta^{m|n}(\mathrm{d}t^1 \dots | \cdots \mathrm{d}\theta^n) \end{aligned}$$

$$\frac{\partial^r}{\partial(\mathrm{d}\theta)^r}\delta(\mathrm{d}\theta)$$

$$F(x, \mathrm{d}x) = \theta^2\ \mathrm{d}t\delta'(\mathrm{d}\theta^1)$$

$$I_{\beta\gamma} = \frac{1}{\pi}\int_\Sigma \mathrm{d}^2 z \beta \partial_{\tilde{z}} \gamma$$

$$\int \mathcal{D}(\mathrm{d}\theta) \frac{\partial^r}{\partial(\mathrm{d}\theta)^r}\delta(\mathrm{d}\theta) = \delta_{r,0}$$

$$\int \mathcal{D}(u,v)\exp\left(-\sum_{i,j} u_i m_{ij} v_j\right) = \frac{1}{\det m}$$

$$\int \mathcal{D}(u,v)\exp\left(-\sum_{i,j} u_i m_{ij} v_j - \sum_k (r_k u_k + s_k v_k)\right) = \frac{\exp\left(\sum_{i,j} s_i (m^{-1})_{ij} r_j\right)}{\det m}$$

$$\int \mathcal{D}(u,v)\exp\left(-\sum_{i,j} u_i m_{ij} v_j - \sum_k s_k v_k\right) = \frac{1}{\det m}$$

$$G(v) = \int \mathcal{D}(u)\exp\left(-(u, mv)\right)$$

$$\int \mathcal{D}(v) A(v) G(v) = \int \mathcal{D}(u,v) A(v) \exp\left(-(u, mv)\right)$$

$$\int \mathcal{D}(v)\exp\left(-(s,v)\right) G(v) = \frac{1}{\det m}$$

$$\int \mathcal{D}(u)\exp\left(-(u, mv)\right) = \delta(mv)$$

$$\int \mathcal{D}(v)\delta(mv)=\frac{1}{\det m}$$

$$\tilde{I}_{\delta\mathcal{J}B}=\frac{1}{2\pi}\int \mathcal{D}(\tilde{z},z\mid\theta)\delta\mathcal{J}_{\tilde{z}}^{z}B_{z\theta}$$

$$B=\sum_{\alpha=1\ldots|\cdots 2g-2}u_\alpha \mathrm{B}_\alpha+\sum_\lambda w_\lambda \mathrm{B}'_\lambda.$$

$$C=\sum_\lambda \gamma_\lambda \mathrm{C}_\lambda,$$

$$\int \mathrm{d}\gamma\exp\left(-mw\gamma\right)=\delta(mw)$$

$$\tilde{I}_{\delta\mathcal{J}B}=\sum_{\alpha=1\ldots|\cdots 2g-2}u_\alpha\Psi_\alpha,$$

$$\Psi_\alpha=\frac{1}{2\pi}\int_\Sigma \mathcal{D}(\tilde{z},z\mid\theta)\delta\mathcal{J}_{\tilde{z}}^{z}\,\mathrm{B}_{z\theta\alpha}$$

$$\prod_{\alpha=1\ldots|\cdots 2g-2}\int \mathrm{d}u_\alpha\exp\left(u_\alpha\Psi_\alpha\right)=\prod_{\alpha=1\ldots|\cdots 2g-2}\delta(\Psi_\alpha)=\delta^{3g-3|2g-2}\left(\Psi'_1\ldots|\cdots\Psi''_{2g-2}\right)$$

$$\tilde{I}_{\delta\mathcal{J}\tilde{B}}=\frac{1}{2\pi}\int_\Sigma \mathcal{D}(\tilde{z},z\mid\theta)\delta\mathcal{J}_{\theta}^{\tilde{z}}\tilde{B}_{\tilde{z}\tilde{z}}$$

$$\tilde{I}_{\delta\mathcal{J}\tilde{B}}=\sum_{\alpha=1}^{3g-3}\tilde{\mathrm{u}}_\alpha\widetilde{\Psi}_\alpha$$

$$\widetilde{\Psi}_\alpha=\frac{1}{2\pi}\int_\Sigma[\mathrm{d}\tilde{z};\mathrm{d}z\mid\mathrm{d}\theta]\delta\mathcal{J}_{\theta}^{\tilde{z}}\,\widetilde{\mathrm{B}}_{\tilde{z}\tilde{z}}$$

$$\prod_{\alpha=1}^{3g-3}\int \mathrm{d}\tilde{\mathrm{u}}_\alpha\exp\left(\tilde{\mathrm{u}}_\alpha\widetilde{\Psi}_\alpha\right)=\delta^{3g-3}\left(\widetilde{\Psi}_1,\ldots,\widetilde{\Psi}_{3g-3}\right)$$

$$\delta^{3g-3}\left(\widetilde{\Psi}_1\ldots\widetilde{\Psi}_{3g-3}\right)\delta^{3g-3|2g-2}\left(\Psi'_1\ldots|\cdots\Psi''_{2g-2}\right)$$

$$\delta\mathcal{J}_{\tilde{z}}^{z}\to\delta\mathcal{J}_{\tilde{z}}^{z}+\partial_{\tilde{z}}q^z,\delta\mathcal{J}_{\theta}^{\tilde{z}}\to\delta\mathcal{J}_{\theta}^{\tilde{z}}+D_\theta q^{\tilde{z}}$$

$$m'=f(m\mid\eta^1,\eta^2)$$
$$\eta'^i=\psi^i(m\mid\eta^1,\eta^2),i=1{,}2$$

$$m'=f_0(m)+\eta^1\eta^2 f_2(m)$$

$$\widetilde{m}'=\tilde{f}(\widetilde{m})$$

$$Z_{\mathrm{g}}=\int_\Gamma \mathcal{D}(\mathcal{J},\delta\mathcal{J})F(\mathcal{J},\delta\mathcal{J})$$

$$\delta \mathcal{J}_{\tilde{z}}^{z} = \sum_{\alpha=1\ldots|\cdots 2g-2} \frac{\partial \mathcal{J}_{\tilde{z}}^{z}}{\partial \boldsymbol{m}_{\alpha}} \mathrm{d}\boldsymbol{m}_{\alpha}$$

$$\delta \mathcal{J}_{\theta}^{\tilde{z}} = \sum_{\beta=1\ldots 3\mathbf{g}-3} \frac{\partial \mathcal{J}_{\theta}^{\tilde{z}}}{\partial \widetilde{m}_{\beta}} \mathrm{d}\widetilde{m}_{\beta}$$

$$I_{\mathrm{d}\boldsymbol{m},\,\mathrm{d}\widetilde{m}} = \sum_{\alpha=1\ldots|\cdots 2g-2} \mathrm{d}\boldsymbol{m}_{\alpha} B^{(\alpha)} + \sum_{\beta=1\ldots 3g-3} \mathrm{d}\widetilde{m}_{\beta} \widetilde{B}^{(\beta)}$$

$$B^{(\alpha)} = \frac{1}{2\pi}\int_{\Sigma} \mathcal{D}(\tilde{z}, z \mid \theta) \frac{\partial \mathcal{J}_{\tilde{z}}^{z}}{\partial \boldsymbol{m}_{\alpha}} B_{z\theta}$$

$$\widetilde{B}^{(\beta)} = \frac{1}{2\pi}\int_{\Sigma} \mathcal{D}(\tilde{z}, z \mid \theta) \frac{\partial \mathcal{J}_{\theta}^{\tilde{z}}}{\partial \widetilde{m}_{\beta}} \widetilde{B}_{\tilde{z}\tilde{z}}$$

$$\prod_{\beta=1\ldots 3g-3} \int \mathcal{D}(\,\mathrm{d}\widetilde{m}_{\beta}) \exp\left(-\mathrm{d}\widetilde{m}_{\beta}\widetilde{B}^{(\beta)}\right) \cdot \prod_{\alpha=1\ldots|\cdots 2g-2} \mathcal{D}(\,\mathrm{d}\boldsymbol{m}_{\alpha}) \exp\left(-\mathrm{d}\boldsymbol{m}_{\alpha} B^{(\alpha)}\right)$$

$$= \prod_{\beta=1\ldots 3g-3} \delta\left(\widetilde{B}^{(\beta)}\right) \cdot \prod_{\alpha=1\ldots|\cdots 2g-2} \delta\left(B^{(\alpha)}\right)$$

$$\delta^{3g-3}\left(\widetilde{B}^{(\beta)}\right)\delta^{3g-3|2g-2}\left(B^{(\alpha)}\right)$$

$$\Lambda(\widetilde{m}, \boldsymbol{m}) = \int \mathcal{D}(X, B, C, \widetilde{B}, \widetilde{C}) \exp\,(-I)\delta^{3g-3}\left(\widetilde{B}^{(\beta)}\right)\delta^{3g-3|2g-2}\left(B^{(\alpha)}\right)$$

$$\mathcal{D}(x) = \left[\mathrm{d}\widetilde{m}_{1} \ldots \mathrm{d}\widetilde{m}_{3g-3}; \mathrm{d}m_{1} \ldots \mathrm{d}m_{3g-3} \mid \mathrm{d}\eta_{1} \ldots \mathrm{d}\eta_{2g-2}\right]$$

$$\Xi(x) = \mathcal{D}(x)\Lambda(x)$$

$$\Xi(x) = \int \mathcal{D}(\mathrm{d}x) F(\mathcal{J}, \delta\mathcal{J})$$

$$Z_{\mathrm{g}} = \int_{\Gamma} \Xi(x)$$

Sistema de Coordenadas – Moduli y transformaciones de gauge.

$$\Xi = [\mathrm{d}\widetilde{m}; \mathrm{d}m \mid \mathrm{d}\eta^{1}, \mathrm{d}\eta^{2}](\Upsilon_{0}(\widetilde{m}, m) + \Upsilon_{2}(\widetilde{m}, m)\eta^{1}\eta^{2})$$

$$m' = m + a(m)\eta^{1}\eta^{2}$$
$$\eta'^{1} = \eta^{1}$$
$$\eta'^{2} = \eta^{2}$$

$$\Xi = [\mathrm{d}\widetilde{m}; \mathrm{d}m' \mid \mathrm{d}\eta'^{1}, \mathrm{d}\eta'^{2}](\Upsilon_{0}'(\widetilde{m}, m') + \Upsilon_{2}'(\widetilde{m}, m')\eta'^{1}\eta'^{2})$$

$$\Upsilon_{2}'(\widetilde{m}, m') = \Upsilon_{2}(\widetilde{m}, m') - \partial_{m'}\left(a(m')\Upsilon_{0}(\widetilde{m}; m')\right)$$

$$\delta\mathcal{J}_{\tilde{z}}^{z} = h_{\tilde{z}}^{z} + \theta\chi_{\tilde{z}}^{\theta}$$

$$\chi_{\tilde{z}}^{\theta} \rightarrow \chi_{\tilde{z}}^{\theta} + \partial_{\tilde{z}} y^{\theta}$$

$$\chi_{\tilde{z}}^{\theta}=\sum_{\sigma=1}^{2g-2}\eta_{\sigma}\chi_{\tilde{z}}^{(\sigma)\theta}$$

$$\partial_{\tilde{z}}y^{\theta}=\sum_{\sigma}e_{\sigma}\chi_{\tilde{z}}^{(\sigma)\theta}$$

$$\chi_{\tilde{z}}^{(\sigma)\theta}\rightarrow\chi_{\tilde{z}}^{(\sigma)\theta}+\partial_{\tilde{z}}y^{(\sigma)\theta}$$

$$\chi\rightarrow\chi+\partial_{\tilde{z}}y,$$

$$y=\sum_{\sigma=1}^{2g-2}\eta_{\sigma}y^{(\sigma)}$$

$$h_{\tilde{z}}^{z}\rightarrow h_{\tilde{z}}^{z}+y^{\theta}\chi_{\tilde{z}}^{\theta}.$$

$$h_{\tilde{z}}^{z}\rightarrow h_{\tilde{z}}^{z}+\sum_{\sigma,\sigma'}\eta_{\sigma}\eta_{\sigma'}y^{(\sigma)\theta}\chi_{\tilde{z}}^{(\sigma')\theta}$$

$$I_{\eta}=\sum_{\sigma=1}^{2\,g-2}\frac{\eta_{\sigma}}{2\pi}\int_{\Sigma_{\mathrm{red}}}\mathrm{d}^{2}z\chi_{\tilde{z}}^{(\sigma)\theta}S_{z\theta}$$

$$I_{\mathrm{d}\eta}=\sum_{\sigma=1}^{2\,g-2}\frac{\mathrm{d}\eta_{\sigma}}{2\pi}\int_{\Sigma_{\mathrm{red}}}\mathrm{d}^{2}z\chi_{\tilde{z}}^{(\sigma)\theta}\beta_{z\theta}$$

$$\exp\left(-\frac{1}{2\pi}\int_{\Sigma_{\mathrm{red}}}\mathrm{d}^{2}z\chi_{\tilde{z}}^{(\sigma)\theta}(\eta_{\sigma}S_{z\theta}+\mathrm{d}\eta_{\sigma}\beta_{z\theta})\right)$$

$$\delta\left(\int_{\Sigma_{\mathrm{red}}}\mathrm{d}^{2}z\chi_{\tilde{z}}^{(\sigma)\theta}\beta_{z\theta}\right)\cdot\int_{\Sigma_{\mathrm{red}}}\mathrm{d}^{2}z\chi_{\tilde{z}}^{(\sigma)\theta}S_{z\theta}$$

$$\delta\left(\int_{\Sigma_{\mathrm{red}}}\mathrm{d}^{2}z\chi_{\tilde{z}}^{(\sigma)\theta}\beta_{z\theta}\right)\cdot\delta\left(\int_{\Sigma_{\mathrm{red}}}\mathrm{d}^{2}z\chi_{\tilde{z}}^{(\sigma)\theta}S_{z\theta}\right)$$

$$\chi_{\tilde{z}}^{(\sigma)\theta}\rightarrow\lambda\chi_{\tilde{z}}^{(\sigma)\theta},\eta_{\sigma}\rightarrow\lambda^{-1}\eta_{\sigma},\lambda\in\mathbb{C}^{*}$$

$$\begin{pmatrix}\chi_{\tilde{z}}^{(1)}\\ \vdots\\ \chi_{\tilde{z}}^{(s)\theta}\end{pmatrix}\rightarrow M\begin{pmatrix}\chi_{\tilde{z}}^{(1)}\\ \vdots\\ \chi_{\tilde{z}}^{(s)\theta}\end{pmatrix}$$

$$(\eta_{1}\ldots\eta_{s})\rightarrow(\eta_{1}\ldots\eta_{s})M^{-1},(\,\mathrm{d}\eta_{1}\ldots\,\mathrm{d}\eta_{s})\rightarrow(\mathrm{d}\eta_{1}\ldots\,\mathrm{d}\eta_{s})M^{-1}$$

$$\prod_{\sigma=1}^{s}\left(\delta\left(\int_{\Sigma_{\mathrm{red}}}\mathrm{d}^{2}z\chi_{\tilde{z}}^{(\sigma)\theta}\beta_{z\theta}\right)\cdot\int_{\Sigma_{\mathrm{red}}}\mathrm{d}^{2}z\chi_{\tilde{z}}^{(\sigma)\theta}S_{z\theta}\right)$$

Operador Picture-Changing.

$$\chi_{\tilde{z}}^{(\sigma)\theta} = \delta_{p_\sigma}.$$

$$y(p) = \delta(\beta(p))S_{z\theta}(p)$$

$$\chi_{\tilde{z}}^{(\sigma)\theta} = \delta_{p_\sigma}.$$

$$\prod_{\sigma=1}^{2g-2} y(p_\sigma)$$

$$\partial_{\tilde{z}} y^\theta = \sum_{\sigma=1}^{2g-2} e_\sigma \delta_{p_\sigma}$$

$$H^0\left(\Sigma_{\text{red}}, T^{1/2}(\Sigma_\sigma p_\sigma)\right) \neq 0$$

$$\chi_{\tilde{z}}^{(1)\theta} = \delta^2(z), \chi_{\tilde{z}}^{(2)\theta} = \delta^2(z-\epsilon)$$

$$\hat{\chi}_{\tilde{z}}^{(2)\theta} = -\frac{1}{\epsilon}\left(\chi_{\tilde{z}}^{(2)\theta} - \chi_{\tilde{z}}^{(1)\theta}\right) = -\frac{1}{\epsilon}(\delta^2(z-\epsilon) - \delta^2(z)).$$

$$\hat{\chi}_{\tilde{z}}^{(2)\theta} = \partial_z \delta^2(z).$$

$$\delta\left(\partial_z \beta(z_0)\right)\partial_z S_{z\theta}(z_0).$$

$$\delta\left(\int_{\Sigma_{\text{red}}} \mathrm{d}^2 z \chi_{\tilde{z}}^{(\sigma)\theta} \beta_{z\theta}\right)$$

$$\partial_z \delta(\beta(z)) = \partial_z \beta(z) \cdot \delta'(\beta(z))$$

$$\delta^{(k)}\left(\int_{\Sigma_{\text{red}}} \mathrm{d}^2 z \chi_{\tilde{z}}^{(\sigma)\theta} \beta_{z\theta}\right)$$

$$r - s = 2g - 2$$

$$\delta^{3g-3|2g-2}\left(B^{(\alpha)}\right) = \prod_{\alpha'=1}^{3g-3} \delta\left(b^{(\alpha')}\right) \prod_{\alpha''=1}^{2g-2} \delta\left(\beta^{(\alpha'')}\right)$$

Métrica Neveu-Schwarz.

$$b_n \mathcal{V} = \tilde{b}_n \mathcal{V} = \beta_r \mathcal{V} = 0, n, r \geq 0$$

$$\beta_r = \frac{1}{2\pi i} \oint \mathrm{d}z z^{r+1/2} \beta_{z\theta}$$

$$\begin{aligned} L_n \mathcal{V} &= 0, \quad n \geq 0 \\ G_r \mathcal{V} &= 0, \quad r \geq 1/2, \end{aligned}$$

$$\tilde{L}_n\mathcal{V} = 0, n \geq 0.$$

$$\delta(\tilde{b})\delta(b)\delta(\beta)$$

$$\mathcal{V} = \tilde{c}c\delta(\gamma)V,$$

$$L_n^X V = \frac{1}{2}\delta_{n,0}V, n \geq 0$$
$$G_r^X V = 0, r > 0$$
$$\tilde{L}_n^X V = \delta_{n,0}V, n \geq 0$$

$$Q_B^* = \sum_{r\in\mathbb{Z}+1/2} \gamma_r G_{-r}^X$$

$$\gamma(z) = \sum_{r\in\mathbb{Z}+1/2} z^{-r+1/2}\gamma_r$$

$$F_{\mathcal{V}_1,\ldots,\mathcal{V}_n}(\mathcal{J},\delta\mathcal{J}) = \int \mathcal{D}(X,B,C,\tilde{B},\tilde{C})\exp\left(-\hat{I}\right)\prod_{i=1}^{n} \mathcal{V}_i(p_i)$$

$$F_{\{Q_B,\mathcal{W}_1\},\mathcal{V}_2,\ldots,\mathcal{V}_\mathrm{n}} + \mathrm{d}F_{\mathcal{W}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n} = 0$$

$$\tilde{b}_n\mathcal{V} = b_n\mathcal{V} = \beta_r\mathcal{V} = 0, n, r \geq 0.$$

$$\tilde{b}_n\mathcal{W} = b_n\mathcal{W} = \beta_r\mathcal{W} = 0, n, r \geq 0.$$

$$\hat{I} \to \hat{I} + \frac{1}{2\pi}\int_\Sigma \mathcal{D}(\tilde{z}, z \mid \theta)\partial_{\tilde{z}}q^z B_{z\theta}$$

$$\langle\mathcal{V}_1 \ldots \mathcal{V}_\mathrm{n}\rangle = \int_\Gamma F_{\mathcal{V}_1,\ldots,\mathcal{V}_\mathrm{n}}(\mathcal{J},\delta\mathcal{J})$$

$$\int_\Gamma F_{\{Q_B,\mathcal{W}_1\},\mathcal{V}_2,\ldots,\mathcal{V}_\mathrm{n}} = -\int_\Gamma \mathrm{d}F_{\mathcal{W}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n} = 0$$

$$K_\mathcal{V} = \int_\Sigma \mathcal{D}(\tilde{z}, z \mid \theta)V(\tilde{z}; z \mid \theta)$$

$$\left(\frac{1}{2\pi}\int_\Sigma \mathrm{d}^2 z g^z \partial_{\tilde{z}} b_{zz}\right) \cdot c^z(0) = g^z(0)$$

$$\delta\left(\frac{1}{2\pi}\int_\Sigma \mathrm{d}^2 z g^z \partial_{\tilde{z}} b_{zz}\right) \cdot \delta(c^z(0)) = g^z(0)$$

$$\delta\left(\frac{1}{2\pi}\int_\Sigma \mathrm{d}^2 z f^\theta \partial_{\tilde{z}} \beta_{z\theta}\right) \cdot \delta\left(\gamma^\theta(0)\right) = \frac{1}{f^\theta(0)}$$

$$\delta\left(\tilde{B}^{(\tilde{\mathbf{z}})}\right)\delta\left(B^{(\mathbf{z})}\right)\delta\left(B^{(\boldsymbol{\theta})}\right)$$

$$[\mathrm{d}\tilde{\mathbf{z}}; \mathrm{d}\mathbf{z} \mid \mathrm{d}\boldsymbol{\theta}]\delta\left(\tilde{B}^{(\tilde{\mathbf{z}})}\right)\delta\left(B^{(\mathbf{z})}\right)\delta\left(B^{(\boldsymbol{\theta})}\right)\tilde{c}c\delta(\gamma)V$$

$$[\mathrm{d}\tilde{\mathbf{z}};\mathrm{d}\mathbf{z}\mid\mathrm{d}\boldsymbol{\theta}]V(\tilde{\mathbf{z}};\mathbf{z}\mid\boldsymbol{\theta}).$$

$$\int_\Sigma [\mathrm{d}\tilde{z};\mathrm{d}z\mid\mathrm{d}\theta]V = \begin{cases}\int_\Sigma [\mathrm{d}\tilde{z};\mathrm{d}z\mid\mathrm{d}\theta]D_\theta W\\ \int_\Sigma [\mathrm{d}\tilde{z};\mathrm{d}z\mid\mathrm{d}\theta]\partial_{\tilde{z}}W'\end{cases}$$

$$\left\langle \prod_{i=1}^{3}\tilde{c}cV_i(\tilde{z}_i,z_i)\prod_{j=4}^{n}\int \mathrm{d}^2z_jV(\tilde{z}_j,z_j)\right\rangle$$

$$\int \mathrm{d}\theta_3\tilde{c}cV_3(\tilde{z}_3;z_3\mid\theta_3)=\tilde{c}cD_\theta V_3(\tilde{z}_3;z_3\mid 0)$$

$$\langle\tilde{c}c\delta(\gamma)V_1(\tilde{z}_1,z_1\mid 0)\tilde{c}c\delta(\gamma)V_2(\tilde{z}_2,z_2\mid 0)\tilde{c}cD_\theta V_3(\tilde{z}_3,z_3\mid 0)\rangle$$

$$\left\langle\tilde{c}c\delta(\gamma)V_1(\tilde{z}_1,z_1\mid 0)\tilde{c}c\delta(\gamma)V_2(\tilde{z}_2,z_2\mid 0)\tilde{c}cD_\theta V_3(\tilde{z}_3,z_3\mid 0)\prod_{j=4}^{n}\int [\mathrm{d}\tilde{z}_j;\mathrm{d}z_j\mid\mathrm{d}\theta_j]V_j(\tilde{z}_j;z_j\mid\theta_j)\right\rangle$$

$$\chi_{\tilde{z}}^{\theta}=\sum_{\sigma=1}^{2g-2+n}\eta_\sigma\chi_{\tilde{z}}^{(\sigma)\theta}$$

$$H^0\left(\Sigma_{\mathrm{red}},T^{1/2}\otimes\mathcal{O}\left(\sum_{\sigma=1}^{2\,\mathrm{g}-2+\mathrm{n}}p_\sigma-\sum_{i=1}^{\mathrm{n}}q_i\right)\right)\neq 0$$

$$\left\langle\prod_{i=1}^{3}\tilde{c}c\mathcal{V}^{(s_i)}(\tilde{z}_i,z_i)\prod_{j=4}^{n}\int \mathrm{d}^2z_j\hat{V}_j^{(s_j)}(\tilde{z}_j,z_j)\right\rangle$$

$$\sum_i s_i=-2$$

Métrica Ramond.

$$D_\theta^*=\partial_\theta+\theta z\partial_z.$$

$$\tau|\zeta=(\log\, z)|\theta$$

$$(D_\theta^*)^2=z\partial_z,$$

$$v_f=f(z)(\partial_\theta-\theta z\partial_z)$$

$$V_g=z\left(g(z)\partial_z+\frac{g'(z)}{2}\theta\partial_\theta\right)$$

$$G_r = z^r(\partial_\theta - \theta z \partial_z)$$

$$L_n = -z^{n+1}\partial_z - \frac{nz^n}{2}\theta\partial_\theta$$

$$[L_m, L_n] = (m-n)L_{m+n}$$
$$\{G_r, G_s\} = 2L_{r+s}$$
$$[L_m, G_r] = \left(\frac{m}{2} - r\right)G_{m+r}$$

$$G_0|_{z=0} = \partial_\theta$$

$$\theta \to \theta + \alpha$$

$$G_r\mathcal{V} = L_n\mathcal{V} = 0, r, n \geq 0$$

$$\mathcal{V} = \tilde{c}c\Theta V,$$

$$L_n^X V = \frac{5}{8}\delta_{n,0}V, n \geq 0$$
$$G_r^X V = 0, r \geq 0$$

$$\beta(z) = \sum_r z^{-r-3/2}\beta_r, \gamma(z) = \sum_r z^{-r+1/2}\gamma_r$$

$$Q_B^* = \sum_{r\in\mathbb{Z}} \gamma_{-r}G_r^X$$

$$\gamma_r\Theta = 0, r > 0$$

$$\beta_r\Theta = 0, r \geq 0$$

$$\dim\mathfrak{M}_{\mathrm{g},\mathfrak{n}_{\mathrm{NS}},\mathfrak{n}_{\mathrm{R}}} = 3\,\mathrm{g} - 3 + \mathfrak{n}_{\mathrm{NS}} + \mathfrak{n}_{\mathrm{R}} \,\Big|\, 2\,\mathrm{g} - 2 + \mathfrak{n}_{\mathrm{NS}} + \frac{1}{2}\mathfrak{n}_{\mathrm{R}}$$

$$\gamma_r\Theta = 0, \quad r \geq 0$$
$$\beta_r\Theta = 0, \quad r > 0.$$

$$D_\theta^* = \partial_\theta + \theta w(z)\partial_z$$

$$w(z) = \prod_{i=1}^{\mathrm{n_R}} (z - z_i)$$

$$\nu_f = f(z)(\partial_\theta - \theta w(z)\partial_z)$$

$$V_g = w(z)\left(g(z)\partial_z + \frac{g'(z)}{2}\theta\partial_\theta\right)$$

$$D_\theta^* = \partial_\theta + \theta z(z-a)\partial_z$$

$$\nu_s = z^s(\partial_\theta - \theta z(z-a)\partial_z)$$

$$V_n = z(z-a)\left(z^n\partial_z + \frac{nz^{n-1}}{2}\theta\partial_\theta\right)$$

$$c\Theta(a)c\Theta(0)\sim c\partial c\delta(\gamma),a\to 0$$

$$\Theta(a)\Theta(0)\sim\delta(\gamma),a\to 0$$

$$I_X=\frac{1}{2\pi i}\int_\Sigma[\mathrm{d}\tilde{\tau};\mathrm{d}\tau\mid\mathrm{d}\zeta]G_{IJ}(X^K)\partial_{\tilde{\tau}}X^ID_\zeta X^J$$

$$\begin{aligned}\tilde{\tau}&=\log\ \tilde{z}\\ \tau&=\log\ z\\ \zeta&=\theta\end{aligned}$$

$$I_X=\frac{1}{2\pi i}\int_\Sigma[\mathrm{d}\tilde{z};\mathrm{d}z\mid\mathrm{d}\theta]\frac{1}{z}G_{IJ}(X^K)\partial_{\tilde{z}}X^ID_\theta^*X^J$$

$$I_X=\frac{1}{2\pi}\int_{\Sigma_{\text{red}}}\mathrm{d}^2z\left(G_{IJ}\partial_{\tilde{z}}x^I\partial_zx^J+\frac{1}{z}G_{IJ}\psi^I\frac{D}{D\tilde{z}}\psi^J\right)$$

$$\tau\cong\tau+2\pi i$$

$$I_\psi=\frac{1}{2\pi}\int_{\Sigma_{\text{red}}}\mathrm{d}^2\tau G_{IJ}(x^K)\psi^I\frac{D}{D\tilde{\tau}}\psi^J$$

$$I_{BC}=\frac{1}{2\pi i}\int_\Sigma[\mathrm{d}\tilde{z};\mathrm{d}z\mid\mathrm{d}\theta]B\partial_{\tilde{z}}C$$

$$[dz\mid\mathrm{d}\theta]=[\mathrm{d}z^*\mid\mathrm{d}\theta^*]\mathrm{Ber}^{-1}\begin{pmatrix}\partial_zz^* & \partial_z\theta^*\\ \partial_\theta z^* & \partial_\theta\theta^*\end{pmatrix}$$

$$\begin{aligned}C&=C^*\cdot\mathrm{Ber}^{-2}\begin{pmatrix}\partial_zz^* & \partial_z\theta^*\\ \partial_\theta z^* & \partial_\theta\theta^*\end{pmatrix}\\ B&=B^*\cdot\mathrm{Ber}^{3}\begin{pmatrix}\partial_zz^* & \partial_z\theta^*\\ \partial_\theta z^* & \partial_\theta\theta^*\end{pmatrix}\end{aligned}$$

$$\mathcal{R}^2\cong T\otimes\mathcal{O}\left(-q_1-\cdots-q_{n_{\mathrm{R}}}\right)$$

$$\mathcal{R}^2\cong T\otimes\mathcal{O}(-q)$$

$$\mathcal{R}^{-2}\cong K\otimes\mathcal{O}(q)$$

$$C(z\mid\theta)=\hat{c}(z)+\theta\hat{\gamma}(z)$$

$$B=\hat{\beta}+\theta\hat{b}$$

$$I_{BC}=\frac{1}{2\pi}\int_{\Sigma_{\text{red}}}\mathrm{d}^2z\left(\hat{b}\partial_{\tilde{z}}\hat{c}+\hat{\beta}\partial_{\tilde{z}}\hat{\gamma}\right)$$

$$\gamma\sim z^{1/2},\beta\sim z^{-1/2},z\to 0$$

$$\gamma(z)=\sum_{r\in\mathbb{Z}}z^{-r+1/2}\gamma_r,\beta(z)=\sum_{r\in\mathbb{Z}}z^{-r-3/2}\beta_r$$

$$\gamma_r \Theta_{-1/2} = 0, \quad r > 0$$
$$\beta_r \Theta_{-1/2} = 0, \quad r \geq 0$$

$$\beta_{z\theta'}(z)\gamma^{\theta'}(w) \sim -\frac{1}{z-w}$$

$$\left\langle \beta_{z\theta'}(z)\gamma^{\theta'}(w) \right\rangle_{\Theta_{-1/2}} = -\frac{1}{z-w}\sqrt{\frac{w}{z}}.$$

$$J_{\beta\gamma}(w) = \lim_{z\to w}\left(-\beta(z)\gamma(w) - \frac{1}{z-w}\right),$$

$$\left\langle J_{\beta\gamma}(w) \right\rangle_{\Theta_{-1/2}} = -\frac{1}{2w},$$

$$T_{\beta\gamma} =: \partial_z \beta_{z\theta'} \cdot \gamma^{\theta'} : -\frac{3}{2}\partial_z\left(: \beta_{z\theta'}\gamma^{\theta'} :\right)$$

$$\left\langle T_{\beta\gamma}(w) \right\rangle_{\Theta_{-1/2}} = \frac{3}{8w^2},$$

$$\left\langle \beta_{z\theta'}(z)\gamma^{\theta'}(w) \right\rangle_{\Theta_{-t}} = -\frac{1}{z-w}\left(\frac{w}{z}\right)^t$$

$$\left\langle J_{\beta\gamma}(w) \right\rangle_{\Theta_{-t}} = -\frac{t}{w}$$
$$\left\langle T_{\beta\gamma}(w) \right\rangle_{\Theta_{-t}} = -\frac{t(t-2)}{2w^2}$$

$$b_n \mathcal{V}_1 = \beta_r \mathcal{V}_1 = 0, n, r \geq 0,$$

$$F_{\{Q_B, \mathcal{W}_1\}, \mathcal{V}_2, \ldots, \mathcal{V}_s} + \mathrm{d}F_{\mathcal{W}_1, \mathcal{V}_2, \ldots, \mathcal{V}_s} = 0$$

$$\int_\Gamma F_{\{Q_B, \mathcal{W}_1\}, \mathcal{V}_2, \ldots, \mathcal{V}_s} + \int_\Gamma \mathrm{d}F_{\mathcal{W}_1, \mathcal{V}_2, \ldots, \mathcal{V}_s} = 0$$

$$\begin{array}{c} \Sigma \to \mathfrak{M}_{\mathrm{g}, \mathfrak{n}_{\mathrm{NS}}, \mathfrak{n}_{\mathrm{R}}} \\ \downarrow \pi \\ \mathfrak{M}_{\mathrm{g}, \mathfrak{n}_{\mathrm{NS}}-1, \mathfrak{n}_{\mathrm{R}}}. \end{array}$$

$$\mathcal{R}^2 \cong T \otimes \mathcal{O}\left(-q_1 - \cdots - q_{\mathfrak{n}_{\mathrm{R}}}\right)$$

$$\deg \mathcal{R} = 1 - \mathrm{g} - \frac{1}{2}\mathfrak{n}_{\mathrm{R}}$$

$$\deg \widehat{\mathcal{R}} = 1 - \mathrm{g} - \mathrm{n}_{\mathrm{NS}} - \frac{1}{2}\mathrm{n}_{\mathrm{R}}$$

$$\chi_{\tilde{z}}^{\theta} = \sum_{\sigma=1}^{\Delta} \eta_\sigma \delta_{r_\sigma}$$

$$H^0\left(\Sigma, \hat{\mathcal{R}} \otimes \mathcal{O}\left(\sum_{\sigma=1}^{\Delta} r_\sigma\right)\right) \neq 0$$

$$\left\langle \tilde{c}c\delta(\gamma)V_1(\tilde{z}_1; z_1 \mid 0)\tilde{c}c\Theta_{-1/2}V_2'(\tilde{z}_2; z_2)\tilde{c}c\Theta_{-1/2}V_3'(\tilde{z}_3; z_3)\right\rangle.$$

$$\left\langle \prod_{i=1}^{3} \tilde{c}c\Theta_{-1/2}V_i'(\tilde{z}_i; z_i) \int \mathrm{d}^2 z_4 \Theta_{-1/2}V_4'(\tilde{z}_4; z_4)\right\rangle$$

Dualidad isotrópica.

$$\omega(\mathcal{V}, \mathcal{W}) = \langle \mathcal{V}(0)\mathcal{W}(1)\rangle.$$

$$\omega(Q_B\mathcal{U}, \mathcal{V}) + (-1)^{|\mathcal{U}|}\omega(\mathcal{U}, Q_B\mathcal{V}) = 0.$$

$$\langle c\partial c U(0) cV(1)\rangle \neq 0.$$

$$\omega: \mathcal{H}_n^* \cong \mathcal{H}_{3-n}.$$

$$\omega: \mathcal{H}_{n;k}^* \cong \mathcal{H}_{1-n;-2-k}$$

$$\langle c\partial c\delta(\gamma)U(0)c\delta(\gamma)V(1)\rangle \neq 0$$

$$\omega: \mathcal{H}_{n;-3/2}^* \cong \mathcal{H}_{1-n;-1/2}$$

$$D_\theta^* = \partial_\theta + z(z-1)\theta\partial_z$$

$$\nu = \partial_\theta - z(z-1)\theta\partial_z$$

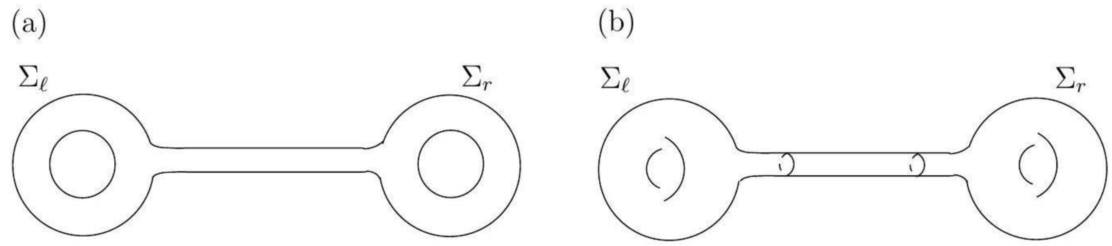
Supermasa y deformaciones dimensionales – GSO – Propagador de una Partícula Supermasiva – NS - Ramond en un espacio de Fock.

$$\left\langle c\partial c\Theta_{-1/2}u^\alpha\Sigma_\alpha(p) \cdot c\Theta_{-3/2}v_\beta\Sigma^\beta(q)\right\rangle = u^\alpha v_\alpha(2\pi)^{10}\delta^{10}(p+q).$$

$$p^I\Gamma_{I\alpha\beta}u^\beta = 0$$

$$v_\beta \cong v_\beta + p^I\Gamma_{I\beta\gamma}w^\gamma$$

$$\frac{1}{p^2+m^2} = \int_0^\infty \mathrm{dsexp}\left(-s(p^2+m^2)\right)$$



$$xy = q$$

$$x = e^{\varrho}, y = qe^{-\varrho}$$

$$\varrho = u + i\varphi, u, \varphi \in \mathbb{R}$$

$$0 \leq u \leq s, 0 \leq \varphi \leq \pi$$

$$\mathrm{d}s^2 = \mathrm{d}u^2 + \mathrm{d}\varphi^2$$

$$L_0 = -\frac{1}{2\pi}\int_0^{\pi} \mathrm{d}\varphi T_{uu}$$

$$b_0 = -\frac{1}{2\pi}\int_0^{\pi} \mathrm{d}\varphi b_{uu}$$

$$z = -i\varrho = \varphi - iu$$

$$T_{zz}(u,\varphi) = T_{\tilde{z}}(u,-\varphi) = T_{zz}(u,\varphi + 2\pi)$$
$$b_{zz}(u,\varphi) = b_{\tilde{z}\tilde{z}}(u,-\varphi) = b_{zz}(u,\varphi + 2\pi)$$

$$L_0 = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi T_{zz}$$

$$b_0 = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi b_{zz}$$

$$0 \leq t \leq 1, 0 \leq \varphi \leq \pi$$

$$\mathrm{d}s^2 = s^2\,\mathrm{d}t^2 + \mathrm{d}\varphi^2$$

$$\Psi_s = \frac{1}{4\pi}\int_S \mathrm{d}t\,\mathrm{d}\varphi \frac{\partial\left(\sqrt{g}g_{ij}\right)}{\mathrm{d}s} b^{ij} = \frac{1}{2\pi}\int_0^1 \mathrm{d}t \int_0^{\pi} \mathrm{d}\varphi s^2 b^{tt}$$
$$= \frac{1}{2\pi}\int_0^1 \mathrm{d}t \int_0^{\pi} \mathrm{d}\varphi b_{uu} = \int_0^1 \mathrm{d}t b_0 = b_0$$

$$\int_0^{\infty} \mathrm{d}s b_0 \exp\left(-sL_0\right) = \frac{b_0}{L_0}$$

$$b_0 \int_0^1 \frac{\mathrm{d}q}{q} q^{L_0}$$

$$L_0 = \frac{\alpha'}{4}p^2 + N,$$

$$\frac{4}{\alpha'}\frac{b_0}{p^2 + M^2}$$

$$Z_{\Sigma;s} = \langle\psi_\ell|\frac{b_0}{L_0}|\psi_r\rangle.$$

$$\tilde{\theta} = \begin{cases} \theta & \text{if } \varphi = 0 \\ -\theta & \text{if } \varphi = \pi \end{cases}$$

$$\varphi \to \varphi + 2\pi, \theta \to -\theta$$

$$\chi_z^{\tilde{\theta}} = \begin{cases} \chi_{\tilde{z}}^{\theta} & \text{if } \varphi = 0 \\ -\chi_{\tilde{z}}^{\theta} & \text{if } \varphi = \pi \end{cases}$$

$$\chi_{\tilde{z}}^{\theta}(u, \varphi + 2\pi) = -\chi_{\tilde{z}}^{\theta}(u, \varphi)$$

$$\chi_{\tilde{z}}^{\theta} \to \chi_{\tilde{z}}^{\theta} + \partial_{\tilde{z}} y^{\theta},$$

$$\frac{b_0}{L_0} \Pi_{\text{GSO}}.$$

$$\int_0^{\infty} \mathrm{d}s \mathrm{Tr} \mathcal{O} b_0 \exp\left(-sL_0\right) \Pi_{\text{GSO}}$$

$$\begin{aligned} \beta_r |\mathrm{q}\rangle &= 0, \quad r > -\mathrm{q} - 3/2 \\ \gamma_r |\mathrm{q}\rangle &= 0, \quad r \geq \mathrm{q} + 3/2 \end{aligned}$$

$$\beta_r |\mathrm{q}\rangle = \gamma_r |\mathrm{q}\rangle = 0, r > 0$$

$$\det \tilde{\partial}_{\beta\gamma} = \mathrm{Tr}(-1)^F \exp\left(-sL_{0;\beta^*\gamma^*}\right) = q^f \prod_{r=\frac{1}{2},\frac{3}{2},\ldots} (1 - q^r)$$

$$\frac{1}{\det \tilde{\partial}_{\beta\gamma}} = q^{-f} \prod_{r=\frac{1}{2},\frac{3}{2},\ldots} \frac{1}{1 - q^r}.$$

$$\mathrm{Tr} \mathcal{O} q^{L_0}$$

$$\tilde{\theta} = \begin{cases} \theta & \text{if } \varphi = 0 \\ \theta & \text{if } \varphi = \pi \end{cases}$$

$$G_0 = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\varphi S_{z\theta}(0, \varphi)$$

$$\beta_0 = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\varphi \beta_{z\theta}(u, \varphi)$$

$$\chi_{\tilde{z}}^{\theta}(u, \varphi + 2\pi) = \chi_{\tilde{z}}^{\theta}(u, \varphi).$$

$$\chi_{\tilde{z}}^{\theta} = \eta,$$

$$\chi_{\tilde{z}}^{\theta} \to \chi_{\tilde{z}}^{\theta} + \partial_{\tilde{z}} y^{\theta}$$

- Deligne-Mumford:

$$\exp\left(-\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi \int_0^{s} \mathrm{d}u(\eta S_{z\theta} + \mathrm{d}\eta\beta_{z\theta})\right) = \exp\left(-s(\eta G_0 + \mathrm{d}\eta\beta_0)\right)$$

$$G_0\delta(\beta_0)$$

$$\frac{b_0\delta(\beta_0)\Pi_{\mathrm{GSO}}G_0}{L_0}$$

$$\frac{b_0\delta(\beta_0)\Pi_{\mathrm{GSO}}}{G_0}$$

$$\mathrm{d}s^2 = \mathrm{d}u^2 + \mathrm{d}\varphi^2, 0 \le u \le s, 0 \le \varphi \le 2\pi.$$

$$L_0 = -\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi T_{zz}$$

$$\tilde{L}_0 = -\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi T_{\tilde{z}\tilde{z}}$$

$$b_0 = -\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi b_{zz}$$

$$\tilde{b}_0 = -\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi b_{\tilde{z}\tilde{z}}$$

$$\hat{\varphi} = \varphi - \alpha f(u),$$

$$\mathrm{d}s^2 = \mathrm{d}u^2 + \mathrm{d}(\hat{\varphi} + \alpha f(u))^2.$$

$$\Psi_\alpha = \frac{1}{4\pi}\int_0^{2\pi} \mathrm{d}\varphi \int_0^{s} \mathrm{d}u \frac{\partial\left(\sqrt{g}g_{ij}\right)}{\partial\alpha} b^{ij}$$

$$\frac{\partial g_{ij}}{\partial\alpha} = D_i v_j + D_j v_i$$

$$v = f(u)\frac{\partial}{\partial\varphi},$$

$$\Psi_\alpha = \frac{1}{4\pi}\int_0^{2\pi} \mathrm{d}\varphi \int_0^{s} \mathrm{d}u\sqrt{g}b^{ij}\left(D_i v_j + D_j v_i\right)$$

$$\Psi_\alpha = -\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi b_{u\varphi}(\varphi, s) = b_0 - \tilde{b}_0$$

$$\exp\left(-s\left(L_0 + \tilde{L}_0\right)\right)\exp\left(-i\alpha\left(L_0 - \tilde{L}_0\right)\right)$$

$$2\Psi_s\Psi_\alpha\int_0^{\infty} \mathrm{d}s \int_0^{2\pi} \mathrm{d}\alpha \exp\left(-s\left(L_0 + \tilde{L}_0\right)\right)\exp\left(-i\alpha\left(L_0 - \tilde{L}_0\right)\right)$$

$$= 4\pi\tilde{b}_0 b_0 \delta_{L_0 - \tilde{L}_0}\int_0^{\infty} \mathrm{d}s \exp\left(-s\left(L_0 + \tilde{L}_0\right)\right) = \frac{2\pi\tilde{b}_0 b_0 \delta_{L_0 - \tilde{L}_0}}{L_0}$$

$$\tilde{b}_0 b_0 \int_{|q|\leq 1} \frac{\mathrm{d}^2 q}{|q|^2} q^{L_0} \tilde{q}^{\tilde{L}_0}$$

$$\frac{2\pi \tilde{b}_0 b_0 \delta_{L_0-\tilde{L}_0} \Pi_{\mathrm{GSO}}}{L_0}$$

$$\frac{2\pi \tilde{b}_0 b_0 \delta(\beta_0) \delta_{L_0-\tilde{L}_0} G_0 \Pi_{\mathrm{GSO}}}{L_0} = \frac{2\pi \tilde{b}_0 b_0 \delta(\beta_0) \delta_{L_0-\tilde{L}_0} \Pi_{\mathrm{GSO}}}{G_0}$$

$$\frac{2\pi \tilde{b}_0 b_0 \delta_{L_0-\tilde{L}_0} \Pi_{\mathrm{GSO}} \tilde{\Pi}_{\mathrm{GSO}}}{L_0}$$

$$\frac{2\pi \tilde{b}_0 b_0 \delta_{L_0-\tilde{L}_0} \delta(\beta_0) G_0 \Pi_{\mathrm{GSO}} \tilde{\Pi}_{\mathrm{GSO}}}{L_0}$$

$$\frac{2\pi \tilde{b}_0 b_0 \delta_{L_0-\tilde{L}_0} \delta(\tilde{\beta}_0) \tilde{G}_0 \Pi_{\mathrm{GSO}} \tilde{\Pi}_{\mathrm{GSO}}}{L_0}$$

$$\frac{2\pi \tilde{b}_0 b_0 \delta(\beta_0) G_0 \delta(\tilde{\beta}_0) \tilde{G}_0 \delta_{L_0-\tilde{L}_0} \Pi_{\mathrm{GSO}} \tilde{\Pi}_{\mathrm{GSO}}}{L_0}$$

$$\langle \phi_{\alpha\beta}(p) \phi_{\alpha'\beta'}(-p) \rangle = \frac{(\Gamma \cdot p)_{\alpha\alpha'} (\Gamma \cdot p)_{\beta\beta'}}{p^2} + \text{ constant.}$$

Métrica de Feynman – singularidad y propagador de Lorentz – BRST – OPE.

$$\frac{1}{p^2 + m^2} = \int_0^\infty \mathrm{dtexp}\left(-\mathrm{t}(p^2 + m^2)\right)$$

$$\frac{1}{p^2 + m^2 - i\epsilon} = i \int_0^\infty \mathrm{d}\tau \mathrm{exp}\left(-i\tau(p^2 + m^2) - \epsilon\tau\right)$$

$$\mathcal{D}_{\text{nonsep}} \cong \widehat{\mathcal{M}}_{g-1,n+2}.$$

$$\mathcal{D}_{\text{sep}} \cong \widehat{\mathcal{M}}_{\mathrm{g}_1,\mathrm{n}_1+1} \times \widehat{\mathcal{M}}_{\mathrm{g}_2,n_2+1},$$

$$\mathrm{g}_1 + \mathrm{g}_2 = \mathrm{g}, \mathrm{n}_1 + \mathrm{n}_2 = \mathrm{n}.$$

$$(x-a)(y-b) = q.$$

$$\tilde{b}_0 b_0 \int_{|q|\leq 1} \frac{\mathrm{d}^2 q}{|q|^2} q^{L_0} \bar{q}^{\tilde{L}_0}$$

$$\int \frac{\mathrm{d}^d p}{(2\pi)^d} \frac{1}{p^2}$$

$$\begin{aligned} xy &= -\varepsilon^2 \\ y\theta &= \varepsilon\psi \\ x\psi &= -\varepsilon\theta \\ \theta\psi &= 0. \end{aligned}$$

$$q_{\mathrm{NS}} = -\varepsilon^2$$

$$D_\theta^* = \frac{\partial}{\partial\theta} + \theta x \frac{\partial}{\partial x}, D_\psi^* = \frac{\partial}{\partial\psi} + \psi y \frac{\partial}{\partial y}.$$

$$xy = q_{\mathrm{R}}$$
$$\theta = \pm\sqrt{-1}\psi.$$

$$\mathcal{D}_{\text{nonsep}} \cong \widehat{\mathfrak{M}}_{\mathrm{g},\mathrm{n}_{\mathrm{NS}}+2,\mathrm{n}_{\mathrm{R}}}$$

$$\mathcal{D}_{\mathrm{sep}} \cong \widehat{\mathfrak{M}}_{\mathrm{g}_1,\mathrm{n}_{\mathrm{NS},1}+1,\mathrm{n}_{\mathrm{R},1}} \times \widehat{\mathfrak{M}}_{\mathrm{g}_2,\mathrm{n}_{\mathrm{NS},2}+1,\mathrm{n}_{\mathrm{R},2}}$$

$$\boldsymbol{\Pi}: \mathcal{D}_{\mathrm{sep}} \to \widehat{\mathfrak{M}}_{\mathrm{g}_1,\mathrm{n}_{\mathrm{NS},1},\mathrm{n}_{\mathrm{R},1}+1} \times \widehat{\mathfrak{M}}_{\mathrm{g}_2,\mathrm{n}_{\mathrm{NS},2},\mathrm{n}_{\mathrm{R},2}+1}$$

$$G_0 = \frac{\partial}{\partial\psi} - \psi y \frac{\partial}{\partial y}$$

$$\mathcal{D}_{\mathrm{sep}} \cong \widehat{\mathfrak{M}}_{\mathrm{g}_1,\mathrm{n}_{\mathrm{NS},1}+1,\mathrm{n}_{\mathrm{R},1}} \times \widehat{\mathfrak{M}}_{\mathrm{g}_2,\mathrm{n}_{\mathrm{NS},2}+1,\mathrm{n}_{\mathrm{R},2}},$$

$$\mathrm{R} = b_0\Pi_0$$

$$\{Q_B, \mathrm{R}\} = \{Q_B, b_0\}\Pi_0 = L_0\Pi_0 = 0$$

$$\bigoplus_{n\in\mathbb{Z}} \mathcal{H}_n \otimes \mathcal{H}_{n+1}^*$$

$$\mathrm{R}' \in \bigoplus_{n\in\mathbb{Z}} \mathcal{H}_n \otimes \mathcal{H}_{2-n}$$

$$\mathrm{R}' \in (\mathcal{H} \otimes \mathcal{H})_2$$

$$\mathrm{R}' = \sum_i cU_i \otimes cU^i + Q_B\mathcal{X}$$

$$\mathcal{X} = \sum_j \mathcal{S}_j \otimes \mathcal{T}_j, \mathcal{S}_j, \mathcal{T}_j \in \mathcal{H}$$

$$\mathrm{d}F_\Omega + F_{Q_B\Omega} = 0.$$

$$\mathrm{d}F_{\mathcal{V}_1\cdots\mathcal{V}_\mathrm{n}\mathcal{X}} + F_{\mathcal{V}_1\cdots\mathcal{V}_\mathrm{n}Q_B\mathcal{X}} = 0$$

$$\int_{\mathcal{M}_{\mathrm{g}-1,\mathrm{n}+2}} F_{\mathcal{V}_1\cdots\mathcal{V}_\mathrm{n}Q_B\mathcal{X}} = -\int_{\mathcal{M}_{\mathrm{g}-1,\mathrm{n}+2}} \mathrm{d}F_{\mathcal{V}_1\cdots\mathcal{V}_\mathrm{n}\mathcal{X}}$$

$$\widetilde{\mathrm{R}}' = \sum_i \left(c\partial cU_i \otimes cU^i + cU_i \otimes c\partial cU^i\right) + Q_B\mathcal{Y}$$

$$(c\partial c\partial^2 c\otimes 1+1\otimes c\partial c\partial^2 c)$$

Cohomología de Ramond.

$$\mathrm{R}'=\sum_i c\delta(\gamma)U_i\otimes c\delta(\gamma)U^i+Q_B\mathcal{X}$$

$$\mathrm{R}=b_0\delta(\beta_0)G_0\Pi_0$$

$$\mathrm{R}'\in(\mathcal{H}\otimes\mathcal{H})_{1;-1/2\otimes-1/2}$$

$$\mathrm{R}'=\sum_i c\Theta_{-1/2}\Phi_i\otimes c\Theta_{-1/2}\Phi_i'+Q_B\mathcal{X}$$

$$\sum_{\alpha\beta}(p\cdot\Gamma)^{\alpha\beta}c\Theta_{-1/2}\Sigma_\alpha(p)\otimes c\Theta_{-1/2}\Sigma_\beta(-p)$$

$$\mathrm{R}'=\sum_i \tilde{c}cU_i\otimes\tilde{c}cU^i+\{Q_B,\mathcal{X}\}$$

$$\widetilde{\mathrm{R}}'=\sum_i\left(\tilde{c}c(\tilde{\partial}\tilde{c}+\partial c)U_i\otimes\tilde{c}cU^i+\tilde{c}cU_i\otimes\tilde{c}c(\tilde{\partial}\tilde{c}+\partial c)U^i\right)+\{Q_B,\mathcal{X}\}$$

$$\sum_I\left(\tilde{c}\tilde{\partial}\tilde{c}\tilde{\partial}^2\tilde{c}\cdot c\partial X^I\otimes c\partial X_I+c\partial X_I\otimes\tilde{c}\tilde{\partial}\tilde{c}\tilde{\partial}^2\tilde{c}\cdot c\partial X^I\right)+z\leftrightarrow\tilde{z}$$

$$(x-a)(y-b)=q$$

$$(x,a)\to(\lambda x,\lambda a),(y,b)\to(\tilde{\lambda}y,\tilde{\lambda}b),q\to\lambda\tilde{\lambda}q.$$

$$\Omega\sim\mathrm{d}a\,\mathrm{d}b\frac{\mathrm{d}q}{q^2}$$

$$\Omega\sim c(a)\otimes c(b)\frac{\mathrm{d}q}{q^2}$$

$$\mathcal{V}_\ell(a)\otimes\mathcal{V}_r(b)\mathrm{d}q q^{L_0-1}$$

$$\begin{aligned}(x-a-\alpha\theta)(y-b-\beta\psi)&=-\varepsilon^2\\(y-b-\beta\psi)(\theta-\alpha)&=\varepsilon(\psi-\beta)\\(x-a-\alpha\theta)(\psi-\beta)&=-\varepsilon(\theta-\alpha)\\(\theta-\alpha)(\psi-\beta)&=0\end{aligned}$$

$$\begin{aligned}(x,a,\alpha)&\to\left(\lambda x,\lambda a,\lambda^{1/2}\alpha\right)\\(y,b,\beta)&\to\left(\tilde{\lambda}y,\tilde{\lambda}b,\tilde{\lambda}^{1/2}\beta\right)\\\varepsilon&\to(\lambda\tilde{\lambda})^{1/2}\varepsilon\end{aligned}$$

$$\Omega\sim[\mathrm{d}a\mid\mathrm{d}\alpha]\otimes[\mathrm{d}b\mid\mathrm{d}\beta]\frac{\mathrm{d}\varepsilon}{\varepsilon^2}$$

$$\Omega \sim c\delta(\gamma) \otimes c\delta(\gamma)\frac{\mathrm{d}\varepsilon}{\varepsilon^2}$$

$$\mathcal{V}_\ell(a \mid \alpha) \otimes \mathcal{V}_r(b \mid \beta)\mathrm{d}\varepsilon\varepsilon^{2L_0-1}$$

$$\mathrm{d}\varepsilon\varepsilon^{2L_0-1} \sim \mathrm{d}q_{\mathrm{NS}}q_{\mathrm{NS}}^{L_0-1}$$

$$(\hat{x}-\hat{a})(\hat{y}-\hat{b}) = \hat{q}$$

$$\hat{q} = q\frac{\partial\hat{a}}{\partial a}\frac{\partial\hat{b}}{\partial b}$$

Compactificación Deligne-Mumford.

$$\bar{\tilde{q}} = q_{\mathrm{NS}}$$

$$\bar{\tilde{q}} = q_{\mathrm{NS}}\left(1+\mathcal{O}(\eta_i\eta_j)\right),$$

$$\bar{\tilde{q}} = q_{\mathrm{NS}} + \mathcal{O}(\eta_i\eta_j).$$

$$\Xi = [\mathrm{d}\tilde{q}; \mathrm{d}q_{\mathrm{NS}} \mid \mathrm{d}\eta_1, \mathrm{d}\eta_2]\tilde{q}^{-1}.$$

$$\int \Xi \to \int \Xi + \int [\mathrm{d}\tilde{q}; \mathrm{d}q_{\mathrm{NS}} \mid \mathrm{d}\eta_1, \mathrm{d}\eta_2]\eta_1\eta_2\frac{\partial}{\partial q_{\mathrm{NS}}}\frac{1}{\tilde{q}}.$$

$$\tilde{q} \to e^{\tilde{\varphi}}\tilde{q}, q_{\mathrm{NS}} \to e^{\varphi}q_{\mathrm{NS}}$$

$$\tilde{q} = \tilde{u}_1 - \tilde{u}_2$$
$$q_{\mathrm{NS}} = u_1 - u_2 - \zeta_1\zeta_2.$$

$$q_{\mathrm{NS}}\left(1+\mathcal{O}(\eta^2)\right) = t^1 + it^2$$
$$\tilde{q}_{\mathrm{NS}}\left(1+\mathcal{O}(\tilde{\eta}^2)\right) = t^1 - it^2$$

$$t^1 = q_{\mathrm{NS}}\left(1+\mathcal{O}(\eta^2)\right)$$

Anomalías BRST para las partículas sin masa. Supersimetrías de gauge.

$$\int_\Gamma F_{\{Q_B,\mathcal{W}_1\},\mathcal{V}_2,\ldots,\mathcal{V}_n} = -\int_{\partial\Gamma} F_{\mathcal{W}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n}$$

$$b_0\int_0^\infty \mathrm{d}s\exp(-sL_0)$$

$$2\pi(b_0-\tilde{b}_0)\delta_{L_0-\tilde{L}_0}\exp\left(-s(L_0+\tilde{L}_0)\right).$$

$$\sum_i \left(c\partial cU_i \otimes cU^i + cU_i \otimes c\partial cU^i\right)$$

$$\mathcal{O} = \sum_i a_i\mathcal{Y}^i$$

$$\sum_i a_i \langle \mathcal{Y}^i \mathcal{V}_{\mathrm{n}} \rangle_{\mathrm{g}_r}.$$

$$\mathcal{Q}_B(\mathcal{W}_1) = \{Q_B, \mathcal{W}_1\} - g_{\mathrm{st}}^{2\,\mathrm{g}_\ell} \mathcal{O}(\mathcal{W}_1),$$

$$\mathcal{Q}_B(\mathcal{V}_{\mathrm{n}}) = \{Q_B, \mathcal{V}_{\mathrm{n}}\} + \sum_i y^i c\partial c U_i,$$

Rompimiento de simetría de gauge - BRST.

$$\sigma \to \sigma - g_{\mathrm{st}}^{2\,\mathrm{g}_\ell} \lambda$$

$$\int \left(F_{IJ}F^{IJ} + (\partial_I \sigma + A_I)^2\right)$$

$$\lambda \to \lambda - g_{\mathrm{st}}^{2g_\ell} \zeta.$$

$$\mathcal{W}_1 = \varepsilon_I \cdot (c^z \partial_z X^I - \tilde{c}\tilde{z}\partial_{\tilde{z}} X^I) \exp(ip \cdot X), p^2 = \varepsilon \cdot p = 0,$$

$$W = \varepsilon_I \, \mathrm{d}X^I \exp(ip \cdot X), p^2 = \varepsilon \cdot p = 0.$$

$$\mathcal{O}(\mathcal{W}) = \mathcal{W}_{\mathrm{open}}, \mathcal{W}_{\mathrm{open}} = cW_{\mathrm{open}}$$

$$\int \mathrm{d}^D x \left(\frac{1}{g_{\mathrm{st}}^2} H_{IJK}^2 + \frac{1}{g_{\mathrm{st}}} \left(\partial_I A_J - \partial_J A_I + B_{IJ}\right)^2 \right), H = \mathrm{d}B$$

$$\int_{\mathbb{R}^4} \mathrm{d}^4 x e^{-2\phi} ((\mathrm{d}A)^2 + (\mathrm{d}B)^2)$$

$$\int_{\mathbb{R}^4} B \wedge \mathrm{d}A$$

$$xy = q,$$

$$\mathcal{D} = \widehat{\mathcal{M}}_\ell \times \widehat{\mathcal{M}}_r$$

$$q \to e^{f_\ell + f_r} q$$

$$\mathcal{N} = \mathcal{L}_\ell \otimes \mathcal{L}_r$$

$$\mathcal{A}_g(p_1, \zeta_1; \ldots; p_n, \zeta_n) = \int_{\widehat{\mathcal{M}}} F_{p_1,\zeta_1;\ldots;p_n,\zeta_n}(g \mid \delta g)$$

$$\mathcal{A}_{g,\,\mathrm{sing}} = \sum_{\alpha=1}^{s} \int \frac{\mathrm{d}^2 q}{\bar{q} q} \int_{\widehat{\mathcal{M}}_\ell} \mathcal{G}_{\ell,\alpha} \int_{\widehat{\mathcal{M}}_r} \mathcal{G}_{r,\alpha}.$$

$$\int_{\widehat{\mathcal{M}}_r} \mathcal{G}_{r,\alpha} = 0, \alpha = 1, \ldots, s.$$

$$\int_{\epsilon<|q|<\Lambda} \frac{\mathrm{d}^2 q}{\bar{q} q}$$

$$\mathcal{A}_{\mathrm{g},\epsilon} = \int_{\widehat{\mathcal{M}}_\epsilon} F(g \mid \delta g),$$

$$\mathcal{A}_{\mathrm{g}} \to \mathcal{A}_{\mathrm{g}} - 4\pi \sum_\alpha \int_{\widehat{\mathcal{M}}_\ell \times \widehat{\mathcal{M}}_r} h \mathcal{G}_{\ell,\alpha} \mathcal{G}_{r,\alpha}$$

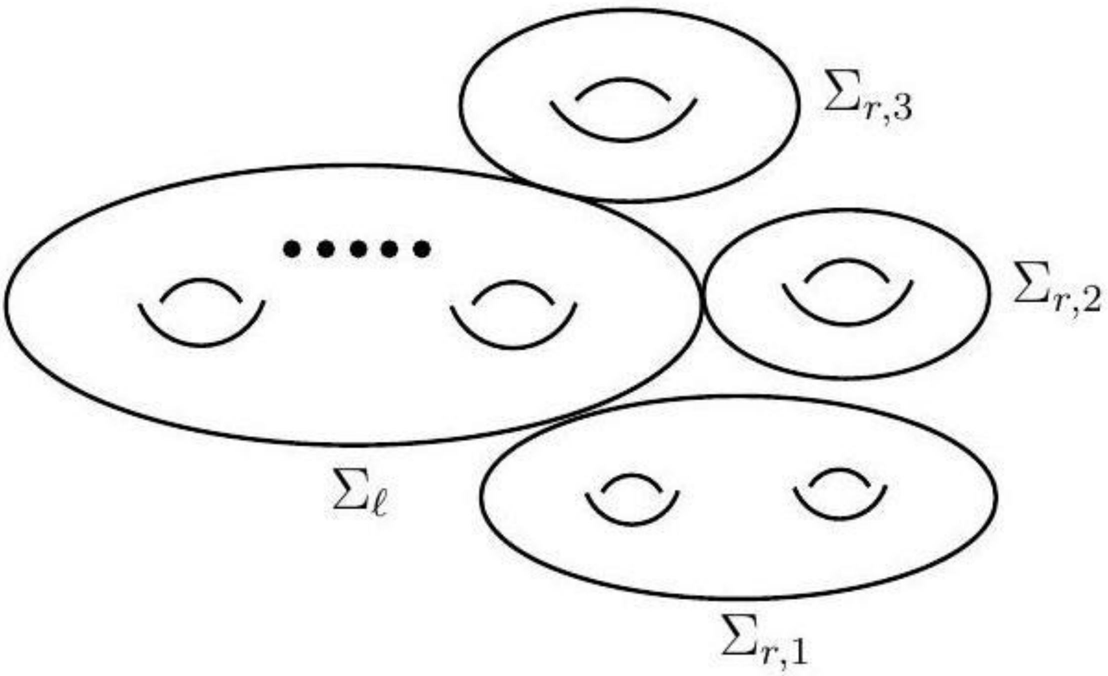
$$\Delta_{g_r} \phi_\alpha = -\frac{1}{4\pi} \int_{\widehat{\mathcal{M}}_r} h_r \mathcal{G}_{r,\alpha}$$

$$\mathcal{A}_{\mathrm{g}} \to \mathcal{A}_{\mathrm{g}} + \sum_\alpha \Delta_{\mathrm{g}_r} \phi_\alpha \int_{\widehat{\mathcal{M}}_\ell} \mathcal{G}_{\ell,\alpha}$$

$$\mathcal{A}_{\mathrm{g}} \to \mathcal{A}_{\mathrm{g}} + \sum_\alpha \Delta_{\mathrm{g}_r} \phi_\alpha \frac{\partial}{\partial \phi_\alpha} \mathcal{A}_{\mathrm{g}_\ell}$$

$$\mathcal{A}_{\mathrm{g}} \to \mathcal{A}_{\mathrm{g}} + \sum_{\mathrm{g}_\ell + \mathrm{g}_r = \mathrm{g}} \sum_\alpha \Delta_{\mathrm{g}_r} \phi_\alpha \frac{\partial}{\partial \phi_\alpha} \mathcal{A}_{\mathrm{g}_\ell}$$

$$\mathcal{A} \to \mathcal{A} + \sum_\alpha \Delta \phi_\alpha \frac{\partial}{\partial \phi_\alpha} \mathcal{A}.$$



$$\mathcal{A} \to \exp\left(\sum_\alpha \Delta \phi_\alpha \frac{\partial}{\partial \phi_\alpha}\right) \mathcal{A}.$$

$$\mathcal{K} = \sum_{\alpha} \Delta\phi_{\alpha} \frac{\partial}{\partial\phi_{\alpha}}$$

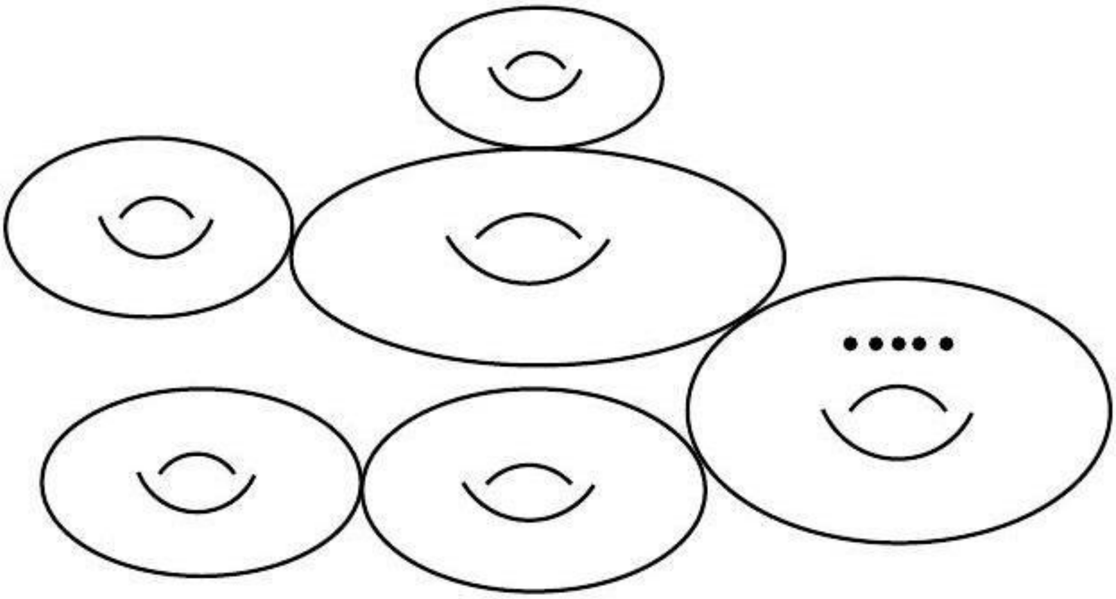


Figura 1. Superespacios producidos por supermembranas.

$$F_0 = \sum_{\alpha} \frac{\mathrm{d}^2 q}{\bar{q} q} \wedge \mathcal{G}_{\ell,\alpha} \wedge \mathcal{G}_{r,\alpha}$$

$$\mathcal{A}_{g,\epsilon} = \int_{\widehat{\mathcal{M}}_{\epsilon}} F - \int_{\partial\widehat{\mathcal{M}}_{\epsilon}} \Lambda$$

$$\chi_{r,\alpha} = \frac{\mathrm{d}^2 q}{\bar{q} q} \mathcal{G}_{r,\alpha}$$

$$\chi_{r,\alpha} = \mathrm{d}\lambda_{r,\alpha}$$

$$\Lambda = \sum_{\alpha} \mathcal{G}_{\ell,\alpha} \wedge \lambda_{r,\alpha}$$

$$\Delta\phi_{\alpha} = -\int_{\mathcal{X}_{\epsilon}} \Delta\lambda_{r,\alpha}$$

$$\Lambda \to \Lambda + \sum_{\alpha} \mathcal{G}_{\ell,\alpha} \wedge \Delta\lambda_{r,\alpha}$$

$$\mathcal{A}_{\mathrm{g}} \to \mathcal{A}_{\mathrm{g}} + \sum_{\alpha} \Delta\phi_{\alpha} \int_{\widehat{\mathcal{M}}_{\ell}} \mathcal{G}_{\ell,\alpha}.$$

$$\lambda_{r,\alpha}^{(0)} = \frac{\mathrm{d}^2 q}{\bar{q} q} \beta_{r,\alpha}.$$

$$\lambda_{r,\alpha} = \frac{\mathrm{d}^2 q}{\bar{q}q}\beta_{r,\alpha} + \frac{\mathrm{d}q}{q}\gamma_{r,\alpha} + \frac{\mathrm{d}\bar{q}}{\bar{q}}\tilde{\gamma}_{r,\alpha},$$

Renormalización de la función de onda.

$$\mathcal{A}_{\mathrm{g,sing}} = \sum_{\alpha=1}^{s} \int \frac{\mathrm{d}^2 q}{\bar{q}q} \int_{\widehat{\mathcal{M}}_\ell} \mathcal{G}_{\ell,\alpha} \int_{\widehat{\mathcal{M}}_r} \mathcal{G}_{r,\alpha}$$

$$\int_{\widehat{\mathcal{M}}_\ell} \mathcal{G}_{\ell,\alpha} = 0$$

$$xy = q$$

$$\mathcal{B} \cong \widehat{\mathcal{M}}_\ell \times \widehat{\mathcal{M}}_r$$

$$\sum_i \left(c\partial c U_i \otimes c U^i + c U_i \otimes c\partial c U^i\right)$$

$$\mathcal{O}(\mathcal{W}_1) = \sum_i a_i \mathcal{V}_i$$

$$\int_{\widehat{\mathcal{M}}_{\mathrm{g,n}}} F_{\{Q_B,\mathcal{W}_1\},\mathcal{V}_2,\ldots,\mathcal{V}_{\mathrm{n}}} = -\int_{\partial\widehat{\mathcal{M}}_{\mathrm{g,n}}} F_{\mathcal{W}_1,\mathcal{V}_2,\ldots,\mathcal{V}_{\mathrm{n}}}$$

$$\frac{\mathrm{d}q}{q}\sum_i cU_i \otimes cU^i$$

$$\frac{\mathrm{d}q}{q}\sum cU_i \otimes cU^i + \sum_i \left(c\partial c U_i \otimes c U^i + c U_i \otimes c\partial c U^i\right)$$

$$v(x)\partial_x + \left.\frac{\partial v(x)}{\partial x}\right|_{x=0} q\partial_q, v(0) = 0$$

$$\mathcal{X}_i = \frac{\mathrm{d}q}{q} cU_i + c\partial c U_i$$

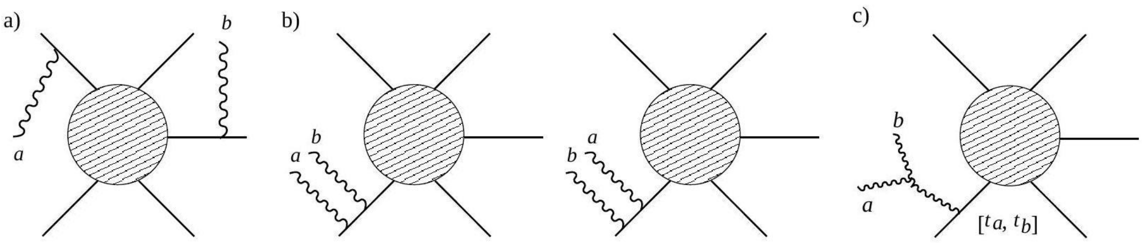
$$a_i = \int_{\mathcal{M}_\ell^*} F_{\mathcal{W}_1,\ldots,\mathcal{X}_i}$$

$$\frac{\mathrm{d}(|q|^2)}{|q|^2}\sum_i \tilde{c}cU_i \otimes \tilde{c}cU^i + \sum_i \left(\tilde{c}c(\tilde{\partial}\tilde{c} + \partial c)U_i \otimes \tilde{c}cU^i + \tilde{c}cU_i \otimes \tilde{c}c(\tilde{\partial}\tilde{c} + \partial c)U^i\right)$$

Supersimetrías de espacio – tiempo.

$$\frac{i}{p_i^2 - m_i^2}$$

$$-ie_i\varepsilon \cdot (p_i + p_i'), p_i' = p_i + k$$



$$-ie_i\varepsilon\cdot(p_i+p_i')\frac{i}{(p_i')^2-m_i^2}=\frac{e_i\varepsilon\cdot p_i}{k\cdot p_i},$$

$$\mathcal{A}\sim\sum_{i=1}^{n}\frac{e_i\varepsilon\cdot p_i}{k\cdot p_i}\mathcal{A}'.$$

$$\sum_i e_i=0,$$

$$-it_{a,i}\varepsilon\cdot(p+p')$$

$$\mathcal{A}\sim\sum_{i=1}^{\mathrm{n}}\frac{t_{a,i}\varepsilon\cdot p_i}{k\cdot p_i}\mathcal{A}'$$

$$\sum_i t_{a,i}\mathcal{A}'=0.$$

$$\sum_{i\neq i'}\frac{t_{a,i}\varepsilon_a\cdot p_i}{k_a\cdot p_i}\frac{t_{b,i'}\varepsilon_b\cdot p_{i'}}{k_b\cdot p_{i'}}\mathcal{A}'$$

$$\sum_{i\neq i'}t_{a,i}\frac{t_{b,i'}\varepsilon_b\cdot p_{i'}}{k_b\cdot p_{i'}}\mathcal{A}'=-\sum_i\frac{t_{b,i}\varepsilon_b\cdot p_i}{k_b\cdot p_i}t_{a,i}\mathcal{A}'$$

$$\left(\sum_i\frac{t_{a,i}\varepsilon_a\cdot p_i}{k_a\cdot p_i}\frac{t_{b,i}\varepsilon_b\cdot p_i}{(k_a+k_b)\cdot p_i}+a\leftrightarrow b\right)\mathcal{A}'.$$

$$\left(\sum_i\frac{t_{b,i}\varepsilon_b\cdot p_i}{k_b\cdot p_i}t_{a,i}+\sum_i\left[t_{a,i},t_{b,i}\right]\frac{\varepsilon_b\cdot p_i}{(k_a+k_b)\cdot p_i}\right)\mathcal{A}'.$$

Gravedad y supergravedad. Métrica de Ward.

$$\sum_{i=1}^{\mathrm{n}}Q_{\alpha,i}\mathcal{A}'=0$$

$$\mathcal{V}=\{Q_B,\mathcal{W}\}=\tilde{c}cik_J\varepsilon_I\tilde{\partial}X^J\partial X^I e^{ik\cdot X}$$

$$\mathcal{V}=\tilde{c}c\tilde{\partial}\left(\varepsilon_I\partial X^I e^{ik\cdot X}\right)$$

$$0=\{Q_B,\mathcal{W}\}$$

$$0 = \tilde{\partial} J, J = \varepsilon_I \partial X^I.$$

$$J^I = \partial X^I.$$

$$\frac{1}{2\pi\alpha'} \oint_{\gamma} J^I \cdot \mathcal{V} = p^I \mathcal{V}$$

$$0 = \left\langle \int_{\Sigma'} \mathrm{d}J \cdot \mathcal{V}_1 \dots \mathcal{V}_n \right\rangle = \sum_{i=1}^{n} \langle \mathcal{V}_1 \dots \mathcal{V}_{i-1} (\oint_{\gamma_i} J \cdot \mathcal{V}_i) \mathcal{V}_{i+1} \dots \mathcal{V}_n \rangle.$$

$$0 = \left(\sum_{i=1}^{n} p_i \right) \langle \mathcal{V}_1 \dots \mathcal{V}_n \rangle.$$

$$\sum_{i=1}^{\mathrm{n}} p_i = 0.$$

Spacetime Supersymmetry. Métrica de Ramond – BRST – Stokes - Ward. Conservación de energía – momentum – stress.

$$\mathcal{W}(k,u) = c\Theta_{-1/2} u^\alpha \Sigma_\alpha e^{ik\cdot X}$$

$$(\gamma \cdot k)_{\alpha\beta} u^\beta = 0$$

$$\mathcal{V}(k,u) = \{Q_B, \mathcal{W}(k,u)\} = i\tilde{c}ck \cdot \tilde{\partial} X \Theta_{-1/2} u^\alpha \Sigma_\alpha e^{ik\cdot X}$$

$$\mathcal{S}_\alpha = c\Theta_{-1/2}\Sigma_\alpha$$

$$\mathrm{d}F_{\mathcal{S}_\alpha} \mathcal{V}_1 \dots \mathcal{V}_\mathrm{n} = 0$$

$$0 = \int_\Gamma \mathrm{d}F_{\mathcal{S}_\alpha \mathcal{V}_1 \dots \mathcal{V}_\mathrm{n}} = \int_{\partial\Gamma} F_{\mathcal{S}_\alpha \mathcal{V}_1 \dots \mathcal{V}_\mathrm{n}}$$

$$\sum_{i=1}^{n} \langle \mathcal{V}_1 \dots \mathcal{V}_{i-1} Q_\alpha(\mathcal{V}_i) \mathcal{V}_{i+1} \dots \mathcal{V}_n \rangle = 0$$

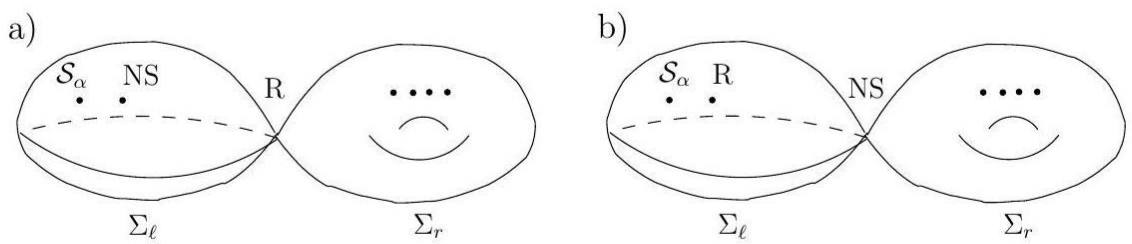
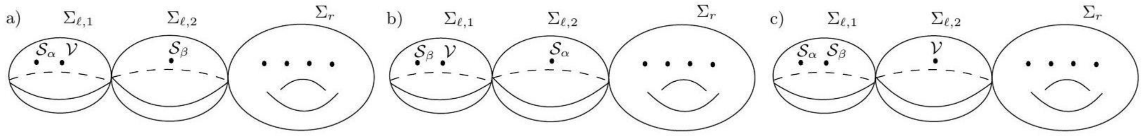


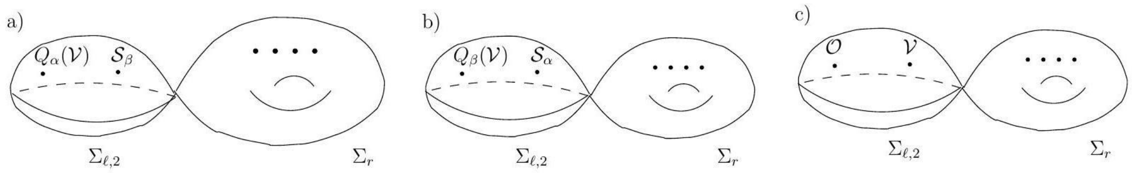
Figura 2. Supermembranas.

$$\mathcal{S}_\alpha = c\hat{\mathcal{S}}_\alpha, \hat{\mathcal{S}}_\alpha = \Theta_{-1/2}\Sigma_\alpha$$

$$\frac{1}{2\pi i}\oint_{|z|=\epsilon} \mathrm{d}z\hat{\mathcal{S}}_\alpha \cdot \mathcal{V}$$

$$0=\int_{\widehat{\mathcal{M}}_{\mathrm{g,n+1}}} \mathrm{d}F_{\mathcal{P}^I\mathcal{V}_1\ldots\mathcal{V}_\mathrm{n}} = \int_{\partial\widehat{\mathcal{M}}_{\mathrm{g,n+1}}} F_{\mathcal{P}^I\mathcal{V}_1\ldots\mathcal{V}_\mathrm{n}}$$





$$0=\int_{\mathcal{B}} \mathrm{d}F(\mathcal{J},\delta\mathcal{J}) = \int_{\partial\mathcal{B}} F(\mathcal{J},\delta\mathcal{J})$$

$$Q_\alpha\big(Q_\beta(\mathcal{V})\big) + Q_\beta(Q_\alpha(\mathcal{V})) + \mathcal{O}(\mathcal{V}) = 0$$

$$\mathcal{O} = -\{Q_\alpha, Q_\beta\}$$

$$\tilde{c}\tilde{\partial}\tilde{c}\tilde{\partial}^2\tilde{c}c\delta(\gamma)D_\theta X^I \otimes c\delta(\gamma)D_\theta X_I + \cdots$$

$$\mathcal{O}_{\mathcal{S}_\alpha,\mathcal{S}_\beta} = \frac{1}{2\pi i}\oint_{|z|=\epsilon} \mathrm{d}z\hat{\mathcal{S}}_\alpha(z)\mathcal{S}_\beta(0)$$

$$\mathcal{V}_\phi = \sum_\alpha Q_\alpha(\mathcal{V}_{\psi_\alpha})$$

$$\mathcal{V}_\phi = \tilde{c}c\delta(\gamma)\tilde{\partial}X_I DX^I$$

$$\mathcal{V}_{\psi_\alpha} = \Gamma_I^{\alpha\beta}\tilde{c}c\tilde{\partial}X^I\Theta_{-1/2}\Sigma_\beta$$

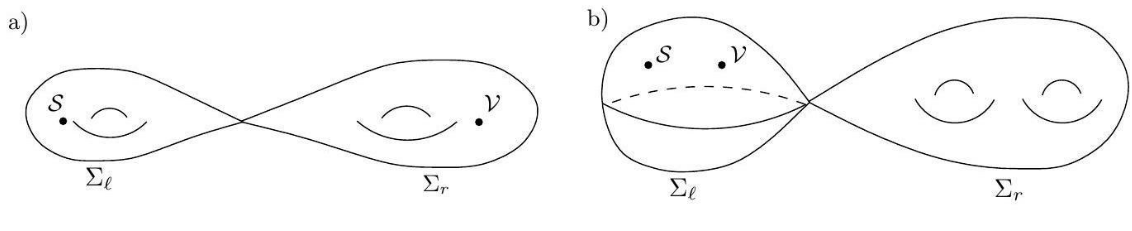


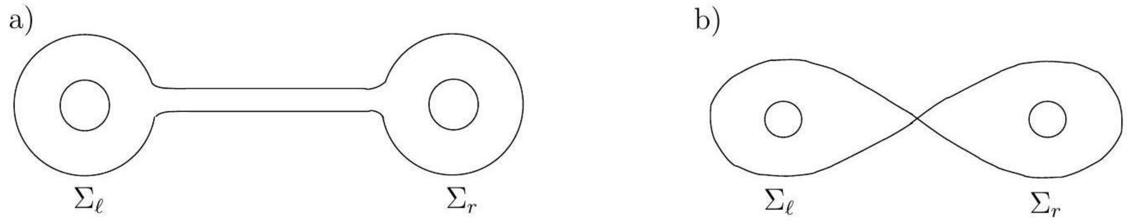
Figura 3. Supermembranas.

$$\sum_\alpha \langle \mathcal{S}_\alpha \mathcal{V}_{\psi_\alpha}\rangle$$

$$0=\int_{\Gamma}\mathrm{d}F_{\mathcal{SV}}=\int_{\partial\Gamma}F_{\mathcal{SV}}$$

Supermembranas. Métrica de Ward – Chan – Paton. Función Delta.

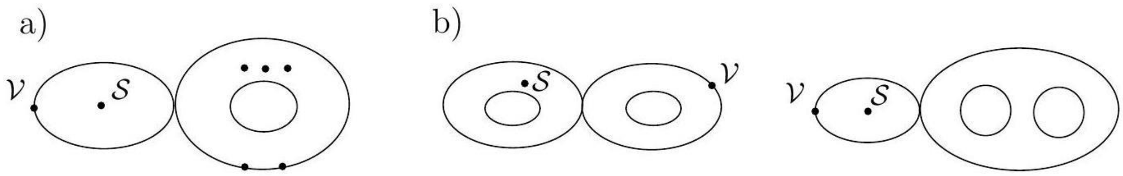
$$\mathcal{V}_{\mathrm{NS,NS}}=\sum_{\alpha}\{Q'_{\alpha},\mathcal{V}^{\alpha}_{\mathrm{R,NS}}\}=\sum_{\beta}\left\{Q''_{\beta},\mathcal{V}^{\beta}_{\mathrm{NS,R}}\right\}$$



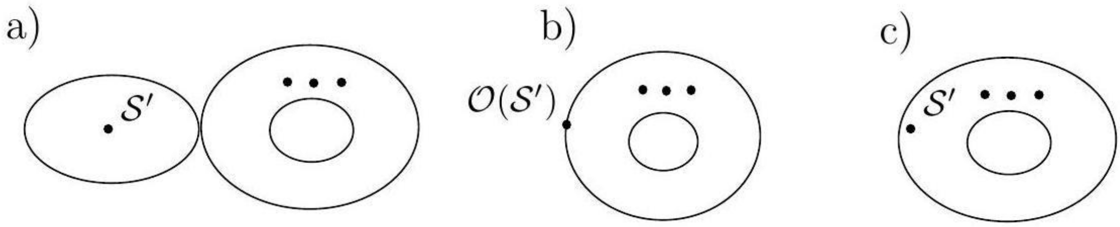
$$q\to e^{f}q$$

$$q\to e^{f_\ell+f_r}q$$

$$\frac{q_{\Sigma_\ell,\mathrm{D}}}{q_{\Sigma_\ell,\mathbb{RP}^2}}\to e^{\kappa}\frac{q_{\Sigma_\ell,\mathrm{D}}}{q_{\Sigma_\ell,\mathbb{RP}^2}},\kappa\in\mathbb{R}$$



$$0=\int_{\partial\hat{\Gamma}}F_{\mathcal{S}_\alpha}\mathcal{V}_1\ldots\mathcal{V}_{\mathrm{n}}$$



$$\mathcal{O}(\mathcal{S}'_\alpha)+\mathcal{O}(\mathcal{S}''_{\tilde{\alpha}})=0$$

$$\mathcal{S}_\alpha=\mathcal{S}'_\alpha+\phi_{\mathrm{D}}(\mathcal{S}'_\alpha)$$

$$\langle\mathcal{V}_{\mathrm{NS-NS}}\rangle+\langle\mathcal{V}_{\mathrm{R-R}}\rangle=0,$$

$$\mathcal{V}_{\mathrm{NS-NS}}=\sum_{\alpha}\{Q'_{\alpha},\mathcal{V}^{\alpha}_{\mathrm{R-NS}}\}$$

$$\mathcal{V}_{\text{NS-NS}} + \mathcal{V}_{\text{R-R}} = \sum_{\alpha} \{Q'_\alpha + Q''_\alpha, \mathcal{V}^{\alpha}_{\text{R-NS}}\}.$$

$$\mathcal{V}_{\text{NS-NS}} + \mathcal{V}_{\text{R-R}} = \sum_{\alpha} \{Q'_\alpha + Q''_\alpha, \mathcal{V}^{\alpha}_{\text{R-NS}} + \mathcal{V}^{\alpha}_{\text{NS-R}}\}$$

$$\begin{aligned} \mathcal{Y}^I &= c\delta(\gamma) D_\theta X^I \exp(ik \cdot X) \\ \mathcal{Z}_\alpha &= c\Theta_{-1/2}\Sigma_\alpha \exp(ik \cdot X) \end{aligned}$$

$$\begin{aligned} \tilde{\mathcal{Y}}^I &= \tilde{c}\delta(\tilde{\gamma}) D_{\tilde{\theta}'} X^I \exp(ik \cdot X) \\ \tilde{\mathcal{Z}}_\alpha &= \tilde{c}\tilde{\Theta}_{-1/2}\tilde{\Sigma}_\alpha \exp(ik \cdot X) \end{aligned}$$

$$\begin{aligned} \mathcal{Z}^\alpha_* &= c\Theta_{-3/2}\Sigma^\alpha \exp(ik \cdot X) \\ \tilde{\mathcal{Z}}^\alpha_* &= \tilde{c}\tilde{\Theta}_{-3/2}\tilde{\Sigma}^\alpha \exp(ik \cdot \tilde{X}) \end{aligned}$$

$$y \cdot \mathcal{Z}^\alpha_* = (\Gamma \cdot k)^{\alpha\beta}\mathcal{Z}_\beta + \{Q_B, \cdot\}$$

$$\hat{\mathcal{S}}_\alpha(z)\mathcal{Z}_\beta(w) \sim \frac{1}{z-w}\Gamma^I_{\alpha\beta}\mathcal{Y}_I$$

$$\{Q_\alpha, \mathcal{Z}_\beta\} = \Gamma^I_{\alpha\beta}\mathcal{Y}_I$$

$$\hat{\mathcal{S}}_\alpha(z)\mathcal{Y}_I(w) \sim \frac{1}{z-w}\Gamma_{I\alpha\beta}\mathcal{Z}^\beta_*$$

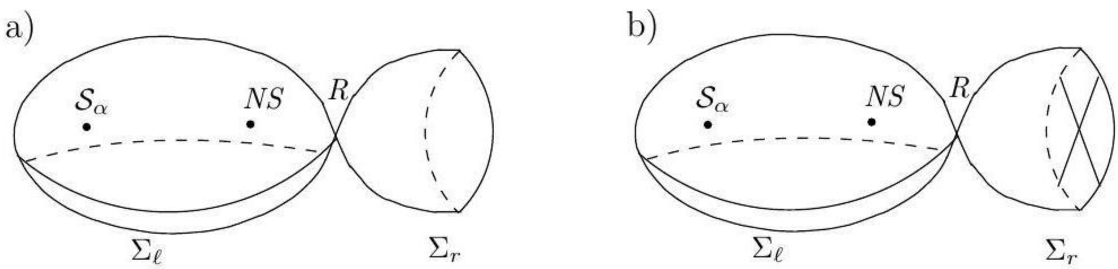
$$\{Q_\alpha, \mathcal{Y}_I\} = (\Gamma_I\Gamma \cdot k)_\alpha{}^\beta\mathcal{Z}_\beta$$

$$\mathcal{S}_\alpha(z)\mathcal{Y}_I(w) \sim \Gamma_{I\alpha\beta}\partial c\mathcal{Z}^\beta_*$$

$$\tilde{v} = \partial_{\tilde{\theta}} - \tilde{\theta}\tilde{z}\partial_{\tilde{z}}$$

$$\tilde{v}^2 = -\tilde{z}\partial_{\tilde{z}}$$

$$\tilde{\theta} \to \tilde{\theta} + \alpha,$$



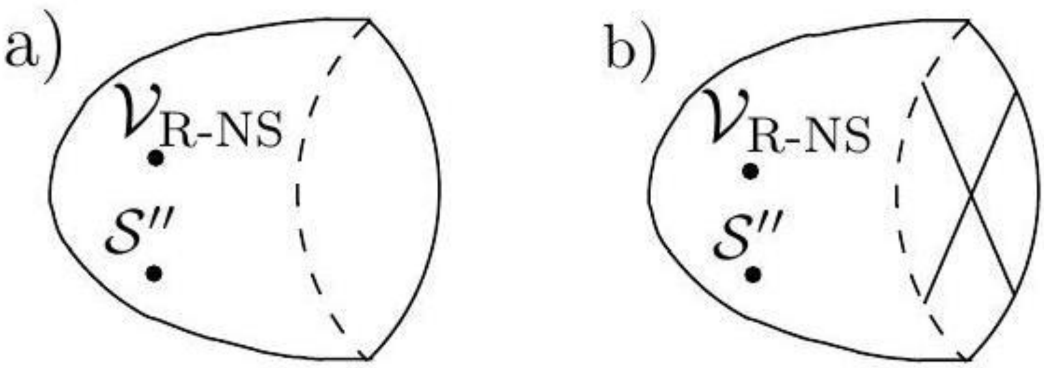


Figura 4. Supermembranas.

$$0 = \int_{\partial\hat{\Gamma}} F_{\mathcal{S}_\alpha} \mathcal{V}^{\alpha}_{\mathrm{NS/R}}$$

$$\langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\Sigma} = 0$$

$$\langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\Sigma} + \langle \mathcal{V}_{\mathrm{R-R}} \rangle_{\Sigma} = 0.$$

$$\mathcal{V}_{\mathrm{R-R}} = \tilde{\partial}\tilde{c}\tilde{\mathcal{Z}}^{\alpha}_{*}\mathcal{Z}_{\alpha} + \tilde{\mathcal{Z}}_{\alpha}\partial c \mathcal{Z}^{\alpha}_{*}$$

$$\langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\mathrm{D}} + \langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\mathbb{RP}^2} = 0.$$

$$I_1 \sim \int \; [\,\mathrm{d}m_1 \ldots | \cdots \mathrm{d}\eta_s]\mathcal{G}_1\big(m_1 \ldots ; q_{\Sigma_\ell,\mathrm{D}} \,|\, \cdots \eta_s\big)\frac{\mathrm{d}q_{\Sigma_\ell,\mathrm{D}}}{q_{\Sigma_\ell,\mathrm{D}}}$$

$$I_2 \sim \int \; [\,\mathrm{d}m_1 \ldots | \cdots \mathrm{d}\eta_s]\mathcal{G}_2\big(m_1 \ldots ; q_{\Sigma_\ell,\mathbb{RP}^2} \,|\, \cdots \eta_s\big)\frac{\mathrm{d}q_{\Sigma_\ell,\mathbb{RP}^2}}{q_{\Sigma_\ell,\mathbb{RP}^2}}$$

$$I_{1,\log} = \int_{\mathcal{M}_\ell} [\mathrm{d}m_1 \ldots | \cdots \mathrm{d}\eta_s]\mathcal{G}_1(m_1 \ldots ; 0 \,|\, \cdots \eta_s)$$
$$I_{2,\log} = \int_{\mathcal{M}_\ell} [\mathrm{d}m_1 \ldots | \cdots \mathrm{d}\eta_s]\mathcal{G}_2(m_1 \ldots ; 0 \,|\, \cdots \eta_s)$$

$$\mathcal{G}_1(m_1 \ldots ; 0 \,|\, \cdots \eta_s) = \mathcal{G}_0(m_1 \ldots | \cdots \eta_s)\langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\mathrm{D}}$$
$$\mathcal{G}_2(m_1 \ldots ; 0 \,|\, \cdots \eta_s) = \mathcal{G}_0(m_1 \ldots | \cdots \eta_s)\langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\mathbb{RP}^2}$$

$$\mathcal{A}_{\mathcal{V}_1 \ldots \mathcal{V}_n ; \mathcal{V}_{\mathrm{NS-NS}}} = \int_{\mathcal{M}_\ell} [\mathrm{d}m_1 \ldots | \cdots \mathrm{d}\eta_s]\mathcal{G}_0$$

$$I_{1,\log} + I_{2,\log} = 0$$

$$I_1 \to I_1 - \int_{\mathcal{M}_\ell} [\mathrm{d}m_1 \ldots | \cdots \mathrm{d}\eta_s] h\mathcal{G}_1(m_1 \ldots ; 0 \,|\, \cdots \eta_s)$$
$$= I_1 - \langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\mathrm{D}} \int_{\mathcal{M}_\ell} [\mathrm{d}m_1 \ldots | \cdots \mathrm{d}\eta_s] h\mathcal{G}_0(m_1, \ldots | \cdots \eta_s)$$

$$I_2 \to I_2 - \langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\mathbb{RP}^2} \int_{\mathcal{M}_\ell} [\mathrm{d}m_1 \ldots | \cdots \mathrm{d}\eta_s] h \mathcal{G}_0(m_1, \ldots | \cdots \eta_s)$$

$$I_2 \to I_2 + \kappa \langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\mathbb{RP}^2} \int_{\mathcal{M}_\ell} [\mathrm{d}m_1 \ldots | \cdots \mathrm{d}\eta_s] \mathcal{G}_0(m_1, \ldots | \cdots \eta_s)$$

$$\mathcal{A}_{\mathcal{V}_1 \ldots \mathcal{V}_\mathrm{n}} \to \mathcal{A}_{\mathcal{V}_1 \ldots \mathcal{V}_\mathrm{n}} + \kappa \langle \mathcal{V}_{\mathrm{NS-NS}} \rangle_{\mathbb{RP}^2} \mathcal{A}_{\mathcal{V}_1 \ldots \mathcal{V}_\mathrm{n}; \mathcal{V}_{\mathrm{NS-NS}}}$$

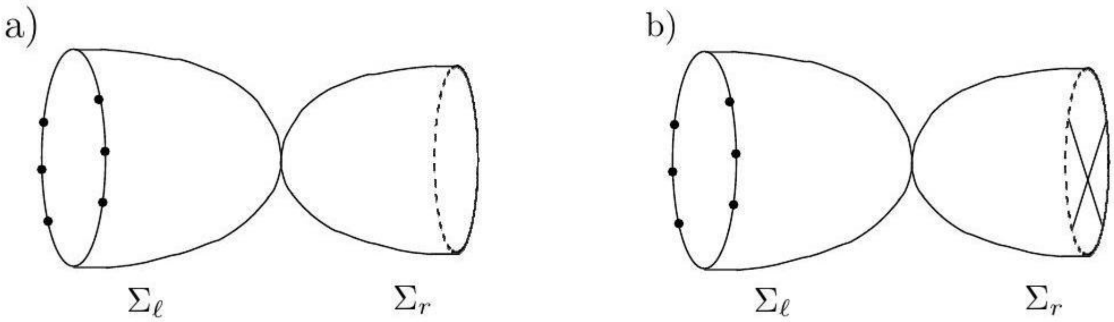


Figura 5. Supermembranas.

$$\langle \delta(\gamma(z_1)) \delta(\beta(z_2)) \rangle \neq 0$$

$$\langle \delta'(\gamma(z_1)) \delta(\beta(z_2)) \rangle = 0$$

$$\langle \delta'(\gamma(z_1)) \delta(\beta(z_2)) \gamma(z_3) \rangle \neq 0$$

$$\langle b(z_1) c(z_2) \rangle \neq 0.$$

$$\langle bc\delta(\beta)\delta(\gamma) \rangle \neq 0$$
$$\langle bc\delta(\beta)\delta'(\gamma)\gamma \rangle \neq 0$$

$$\prod_{i=2}^{n} \oint_{\partial_\ell \Sigma} [\mathrm{d}z \,|\, \mathrm{d}\theta] \varepsilon_I^{(i)} D_\theta X^I \exp\left(ik^{(i)} \cdot X\right)$$

$$\mathcal{V}_1 = c\delta(\gamma)\varepsilon^{(1)} \cdot D_\theta X \exp\left(ik^{(1)} \cdot X\right)$$

$$\left(b_0 + \tilde{b}_0\right) \int_{s_0}^{\infty} \mathrm{ds} \exp\left(-s\left(L_0 + \tilde{L}_0\right)\right) \delta\left(\beta_0 + \tilde{\beta}_0\right)\left(G_0 + \tilde{G}_0\right)$$

$$c\delta(\gamma)\varepsilon^{(1)} \cdot D_\theta X \exp\left(ik^{(1)} \cdot X\right)\left(b_0 + \tilde{b}_0\right) \int_{s_0}^{\infty} \mathrm{ds} \exp\left(-s\left(L_0 + \tilde{L}_0\right)\right) \delta\left(\beta_0 + \tilde{\beta}_0\right)\left(G_0 + \tilde{G}_0\right)$$

$$G_0 = G_0^X + G_0^{\mathrm{gh}}$$

$$\tilde{G}_0 = \tilde{G}_0^X + \tilde{G}_0^{\mathrm{gh}}$$

$$\mathcal{W}_1 = c\delta'(\gamma) \exp\left(ik^{(1)} \cdot X\right)$$

$$c\delta'(\gamma)\exp\left(ik^{(1)}\cdot X\right)\exp\left(-s\left(L_0+\tilde{L}_0\right)\right)\delta\left(\beta_0+\tilde{\beta}_0\right)\left(G_0+\tilde{G}_0\right)$$

$$I_{\beta\gamma}=\frac{1}{2\pi}\int_{\Sigma}\mathrm{d}^2z\beta\partial_{\tilde{z}}\gamma$$

$$I_{\beta^*\gamma^*}=\frac{1}{2\pi}\int_{\Sigma}\mathrm{d}^2z\beta^*\partial_{\tilde{z}}\gamma^*$$

$$\int\mathcal{D}\beta^*\mathcal{D}\gamma^*\exp\left(-I_{\beta*\gamma*}\right)=\det M$$

$$\int\mathcal{D}\beta\mathcal{D}\gamma\exp\left(-I_{\beta\gamma}\right)=\frac{1}{\det M}$$

$$\langle\mathcal{O}\rangle_N=\frac{\langle\mathcal{O}\rangle}{\langle 1\rangle}$$

$$\int\mathrm{d}^nx\exp\left(-\frac{1}{2}(x,Nx)\right)=\frac{1}{\sqrt{\det N}}$$

$$N=\begin{pmatrix}0 & M\\ M^t & 0\end{pmatrix}$$

$$\sqrt{\det N}=\det M,$$

$$\widetilde{N}=\begin{pmatrix}0 & M\\ -M^t & 0\end{pmatrix}$$

$$\langle\gamma(u)\beta(w)\rangle_N=\langle\gamma^*(u)\beta^*(w)\rangle_N=S(u,w),$$

$$\frac{\partial_{\tilde{z}}}{2\pi}S(z,z')=\delta^2(z,z')$$

$$\langle\gamma^*(u_1)\gamma^*(u_2)\ldots\gamma^*(u_s)\beta^*(w_s)\beta^*(w_{s-1})\ldots\beta^*(w_1)\rangle_N=\sum_{\pi}(-1)^{\pi}\prod_{i=1}^{s}S\left(u_i,w_{\pi(i)}\right),$$

$$\left\langle\prod_{i=1}^{s}\gamma(u_i)\prod_{j=1}^{s}\beta(w_j)\right\rangle_N=\sum_{\pi}\prod_{i=1}^{s}S\left(u_i,w_{\pi(i)}\right).$$

$$\langle\delta(\gamma^*(u))\delta(\beta^*(w))\rangle=(\det M)S(u,w)$$

$$\int\mathrm{d}\tau^*\,\mathrm{d}\sigma^*\exp\left(-\sigma^*\gamma^*(u)-\beta^*(w)\tau^*\right)=\gamma^*(u)\beta^*(w)=\delta(\gamma^*(u))\delta(\beta^*(w))$$

$$\langle\delta(\gamma^*(u))\delta(\beta^*(w))\rangle=\int\mathcal{D}\beta^*\mathcal{D}\gamma^*\,\mathrm{d}\tau^*\,\mathrm{d}\sigma^*\exp\left(-I_{\beta^*\gamma^*}-\sigma^*\gamma^*(u)-\beta^*(w)\tau^*\right)$$

$$\hat{I}_{\hat{\beta}^*\hat{\gamma}^*}=I_{\beta^*\gamma^*}+\sigma^*\gamma^*(u)+\beta^*(w)\tau^*$$

$$\hat{\beta}^*=(\beta^*(z)\sigma^*),\hat{\gamma}^*=\begin{pmatrix}\gamma^*(z)\\ \tau^*\end{pmatrix}$$

$$\langle\delta(\gamma^*(u))\delta(\beta^*(w))\rangle = \mathrm{det}\widehat{M}$$

$$\mathrm{det}\widehat{M} = (\mathrm{det}M)S(u,w)$$

$$\delta(\gamma(u))\delta(\beta(w)) = \int\ \mathrm{d}\tau\ \mathrm{d}\sigma \exp\left(-\sigma\gamma(u) - \beta(w)\tau\right)$$

$$\hat{I}_{\hat{\beta}\hat{\gamma}} = I_{\beta\gamma} + \sigma\gamma(u) + \beta(w)\tau,$$

$$\langle\delta(\gamma(u))\delta(\beta(w))\rangle = \int\ \mathcal{D}\hat{\beta}\mathcal{D}\hat{\gamma} \exp\left(-\hat{I}_{\hat{\beta}\hat{\gamma}}\right) = \frac{1}{\mathrm{det}\widehat{M}}$$

$$\langle\delta(\gamma(u))\delta(\beta(w))\rangle_N = \frac{1}{S(u,w)}$$

$$\delta(\gamma(u))\delta(\beta(w)) \sim u - w$$

$$\gamma^*(u) = 0$$

$$\frac{\partial_{\bar{z}}\gamma^*}{2\pi} + \delta^2(z,w)\tau^* = 0$$

$$\int\ \mathcal{D}\beta^*\mathcal{D}\gamma^* \prod_{i=1}^{s}\ \mathrm{d}\tau_i^*\ \mathrm{d}\sigma_i^* \exp\left(-I_{\beta^*\gamma^*} - \sum_i\ \sigma_i^*\gamma^*(u_i) - \sum_j\ \beta^*(w_j)\tau_j^*\right)$$

$$\left\langle \prod_{i=1}^{s}\ \delta(\gamma(u_i)) \prod_{j=1}^{s}\ \delta\left(\beta(w_j)\right)\right\rangle_N = \frac{1}{\sum_\pi\ (-1)^\pi \prod_{i=1}^{s}\ S(u_i, w_{\pi(i)})}$$

$$\delta(\gamma(u_1))\delta(\gamma(u_2)) \sim -\frac{1}{u_1 - u_2}\delta(\gamma)\delta(\partial\gamma)(u_2)$$

$$\Theta_{-t} = \delta(\gamma)\delta(\partial\gamma) \ldots \delta(\partial^{t-1}\gamma)$$

$$\delta(\gamma(u_1))\delta(\beta)\delta(\partial\beta)(u_2) \sim (u_1 - u_2)^2\delta(\beta(u_2))$$

$$\langle\delta(\gamma^*(u))\delta(\beta^*(w))\gamma^*(u')\beta^*(w')\rangle = \mathrm{det}M\left(S(u,w)S(u',w') - S(u,w')S(u',w)\right)$$

$$\langle\delta(\gamma^*(u))\delta(\beta^*(w))\gamma^*(u')\beta^*(w')\rangle = \int\ \mathcal{D}\hat{\beta}^*\mathcal{D}\hat{\gamma}^* \exp\left(-\hat{I}_{\hat{\beta}^*\hat{\gamma}^*}\right)\gamma^*(u')\beta^*(w')$$

$$\langle\delta(\gamma^*(u))\delta(\beta^*(w))\gamma^*(u')\beta^*(w')\rangle = (\mathrm{det}\widehat{M})\hat{S}(u',w'),$$

$$\hat{S}(u',w') = \frac{1}{S(u,w)}\left(S(u,w)S(u',w') - S(u,w')S(u',w)\right).$$

$$\langle\delta(\gamma(u))\delta(\beta(w))\gamma(u')\beta(w')\rangle = \frac{1}{\mathrm{det}\widehat{M}}\hat{S}(u',w').$$

$$\langle\delta(\gamma(u))\delta(\beta(w))\gamma(u')\beta(w')\rangle_N = \frac{1}{S(u,w)^2}\left(S(u,w)S(u',w') - S(u,w')S(u',w)\right).$$

$$\gamma(u')\delta(\gamma(u)) \sim (u'-u)\partial\gamma \cdot \delta(\gamma)(u).$$

$$\beta(w')\delta(\gamma(u)) \sim \frac{1}{w'-u}\delta'(\gamma(u)).$$

$$\gamma(u')\delta'(\gamma(u)) \sim -\delta(\gamma(u))$$

$$\gamma(u')\delta(\beta(w)) \sim \frac{1}{u'-w}\delta'(\beta(w))$$
$$\beta(w')\delta(\beta(w)) \sim (w'-w)\partial\beta \cdot \delta(\beta)(w)$$

$$\langle \gamma(u)\beta(w)\gamma(u')\beta(w')\rangle = S(u,w)S(u',w') + S(u,w')S(u',w)$$

$$\delta(\gamma(u))\delta(\gamma(0)) = \delta\left(\gamma(0) + u\gamma'(0) + \frac{u^2}{2}\gamma''(0) + \cdots\right)\delta(\gamma(0))$$

$$\delta\left(\gamma(0) + u\gamma'(0) + \frac{u^2}{2}\gamma''(0) + \cdots\right)\delta(\gamma(0)) = \delta\left(u\gamma'(0) + \frac{u^2}{2}\gamma''(0) + \cdots\right)\delta(\gamma(0)) = \frac{1}{u}\delta\left(\gamma'(0) + \frac{u}{2}\gamma''(0) + \cdots\right)\delta(\gamma(0))$$

$$\delta\left(\gamma'(0) + \frac{u}{2}\gamma''(0) + \cdots\right) = \delta(\gamma'(0)) + \frac{u}{2}\gamma''(0)\delta'(\gamma'(0)) + \cdots$$

$$\delta(\gamma(u))\delta(\gamma(0)) \sim \frac{1}{u}\delta(\gamma')\delta(\gamma)(0) + \frac{1}{2}\gamma''\delta'(\gamma')\delta(\gamma)(0) + \cdots$$

$$\gamma_f = \int_\Sigma f(z)\gamma(z), \beta_g = \int_\Sigma g(z)\beta(z)$$

$$\left\langle \delta(\gamma_f^*)\delta(\beta_g^*)\right\rangle_N = \left\langle \gamma_f^*\beta_g^*\right\rangle_N = \int_{\Sigma\times\Sigma} f(z)S(z,z')g(z')$$

$$\left\langle \delta(\gamma_f)\delta(\beta_g)\right\rangle_N = \frac{1}{\int_{\Sigma\times\Sigma} f(z)S(z,z')g(z')}$$

$$\langle \delta(\gamma(u))\delta(\partial\beta(w))\rangle_N = \frac{1}{\partial_w S(u,w)}$$

$$H^1(\Sigma, K \otimes \mathcal{L}^{-1}) = 0$$

$$M^{\mathrm{t}}\mathrm{y}_i(z) = 0, i = 1, \ldots, t$$

$$\langle \beta^*(w_1)\ldots\beta^*(w_t)\rangle = \det{}'M\det N.$$

$$\left\langle \delta(\beta^*(w_1))\ldots\delta(\beta^*(w_t))\right\rangle = \det{}'M\det N.$$

$$\left\langle \delta(\beta(w_1))\ldots\delta(\beta(w_t))\right\rangle = \frac{1}{\det{}'M}\frac{1}{\det N}$$

$$\delta(\beta^*(w_1))\ldots\delta(\beta^*(w_t)) = \int \mathrm{d}\tau_1^* \ldots \mathrm{d}\tau_t^* \exp\left(-\sum_i \beta(w_i)\tau_i^*\right)$$

$$\hat{I}_{\beta^*\hat{\gamma}^*} = I_{\beta^*\gamma^*} + \sum_{i=1}^{t} \beta^*(w_i)\tau_i^*$$

$$\left\langle \delta\big(\beta^*(w_1)\big) \dots \delta\big(\beta^*(w_t)\big) \right\rangle = \int \mathcal{D}\beta^* \mathcal{D}\hat{\gamma}^* \exp\left(-\hat{I}_{\beta^*\hat{\gamma}^*}\right)$$

$$\hat{I}_{\beta\hat{\gamma}} = I_{\beta\gamma} + \sum_{i=1}^{t} \beta(w_i)\tau_i,$$

$$\left\langle \delta\big(\beta(w_1)\big) \dots \delta\big(\beta(w_t)\big) \right\rangle = \int \mathcal{D}\beta \mathcal{D}\hat{\gamma} \exp\left(-\hat{I}_{\beta\hat{\gamma}}\right)$$

$$\langle \gamma^*(u)\beta^*(w_1) \dots \beta^*(w_{t+1})\rangle.$$

$$S(u,w) \to S(u,w) + \sum_i h_i(u) \mathrm{y}_i(w)$$

$$N_{(t+1)} = \begin{pmatrix} \mathrm{y}_1(w_1) & \mathrm{y}_1(w_2) & \dots & \mathrm{y}_1(w_{t+1}) \\ \mathrm{y}_2(w_1) & \mathrm{y}_2(w_2) & \dots & \mathrm{y}_2(w_{t+1}) \\ & & \ddots & \\ \mathrm{y}_t(w_1) & \mathrm{y}_t(w_2) & \dots & \mathrm{y}_t(w_{t+1}) \\ S(u,w_1) & S(u,w_2) & \dots & S(u,w_{t+1}) \end{pmatrix}$$

$$\langle \gamma^*(u)\beta^*(w_1) \dots \beta^*(w_{t+1})\rangle = \det{}' M \det N_{(t+1)}$$

$$\left\langle \delta(\gamma^*(u))\delta\big(\beta^*(w_1)\big) \dots \delta\big(\beta^*(w_{t+1})\big) \right\rangle = \det{}' M \det N_{(t+1)}$$

$$\left\langle \delta(\gamma(u))\delta\big(\beta(w_1)\big) \dots \delta\big(\beta(w_{t+1})\big) \right\rangle = \frac{1}{\det{}' M} \frac{1}{\det N_{(t+1)}}$$

$$\frac{\left\langle \gamma^*(u)\delta\big(\beta^*(w_1)\big) \dots \delta\big(\beta^*(w_t)\big)\beta^*(w_{t+1}) \right\rangle}{\left\langle \delta\big(\beta^*(w_1)\big) \dots \delta\big(\beta^*(w_t)\big) \right\rangle} = \frac{\det N_{(t+1)}}{\det N_{(t)}}$$

$$\frac{\left\langle \gamma(u)\delta\big(\beta(w_1)\big) \dots \delta\big(\beta(w_t)\big)\beta(w_{t+1}) \right\rangle}{\left\langle \delta\big(\beta(w_1)\big) \dots \delta\big(\beta(w_t)\big) \right\rangle} = \frac{\det N_{(t+1)}}{\det N_{(t)}}$$

$$\left\langle \gamma(u)\delta\big(\beta(w_1)\big) \dots \delta\big(\beta(w_t)\big)\beta(w_{t+1}) \right\rangle = \frac{\det N_{(t+1)}}{\det{}' M \big(\det N_{(t)}\big)^2}$$

$$N_{(t+s)} = \begin{pmatrix} \mathrm{y}_1(w_1) & \mathrm{y}_1(w_2) & \dots & \mathrm{y}_1(w_{t+s}) \\ \mathrm{y}_2(w_1) & \mathrm{y}_2(w_2) & \dots & \mathrm{y}_2(w_{t+s}) \\ & & \ddots & \\ \mathrm{y}_t(w_1) & \mathrm{y}_t(w_2) & \dots & \mathrm{y}_t(w_{t+s}) \\ S(u_1,w_1) & S(u_1,w_2) & \dots & S(u_1,w_{t+s}) \\ & & \ddots & \\ S(u_s,w_1) & S(u_s,w_2) & \dots & S(u_s,w_{t+s}) \end{pmatrix}$$

$$\langle \gamma^*(u_1) \dots \gamma^*(u_s)\beta^*(u_1) \dots \beta^*(u_{t+s})\rangle = \det{}' M \det N_{(t+s)}$$

$$\langle \delta(\gamma(u_1)) \dots \delta(\gamma^*(u_s))\delta(\beta^*(u_1)) \dots \delta(\beta^*(u_{t+s}))\rangle = \frac{1}{\det' M} \frac{1}{\det N_{(t+s)}}$$

$$\langle \delta(\gamma(u_1))\delta(\gamma(u_2)) \dots \delta(\gamma(u_s))\gamma(u_{s+1})\delta(\beta(w_1)) \dots \delta(\beta(w_{t+s}))\beta(w_{t+s+1})\rangle = \frac{\det N_{(t+s+1)}}{\det' M\left(\det N_{(t+s)}\right)^2}$$

$$\frac{\langle \delta(\gamma(u_1))\delta(\gamma(u_2)) \dots \delta(\gamma(u_s))\gamma(u_{s+1})\delta(\beta(w_1)) \dots \delta(\beta(w_{t+s}))\beta(w_{t+s+1})\rangle}{\langle \delta(\gamma(u_1))\delta(\gamma(u_2)) \dots \delta(\gamma(u_s))\delta(\beta(w_1)) \dots \delta(\beta(w_{t+s}))\rangle} = \frac{\det N_{(t+s+1)}}{\det N_{(t+s)}}$$

$$\mathcal{L}^2 \cong K^{-1} \otimes_{i=1}^{n_{\mathrm{R}}} \mathcal{O}(-p_i)$$

$$\Theta_{-t} = \delta(\gamma)\delta(\partial\gamma) \dots \delta(\partial^{t-1}\gamma)$$

$$\mathcal{L}^2 \cong K^{-1} \otimes \mathcal{O}(-p)$$

$$\hat{\gamma}(z)(z^{-1}\partial_z)^{1/2}$$

$$s = \frac{1}{z^{1/2}} s^\circ$$

$$\gamma^\circ(z)(\partial_z)^{1/2}.$$

$$\langle \gamma(u)\beta(w)\rangle_{N,\delta} = \frac{\det N_{(t+s+1)}}{\det N_{(t+s)}}$$

$$\tilde{\mathcal{L}} = \mathcal{L} \otimes_{i=1}^{s} \mathcal{O}(-u_i) \otimes_{j=1}^{t+s} \mathcal{O}(w_j)$$

$$V_a = \sum_{i=1}^{n-m} v_{a,i} \frac{\partial}{\partial f_i}$$

$$\mathbf{i}_{V_a} = \sum_{i=1}^{n-m} v_{a,i} \frac{\partial}{\partial\, \mathrm{d} f_i}.$$

$$V = \sum_{n=1}^{\infty} L_{-n} U_n$$

$$b_n \mathcal{W} = 0, n \geq 0,$$

$$V = L_{-1}\Phi_0,$$

$$\mathcal{W} = \Phi_0.$$

$$V = \left(L_{-2} + \frac{3}{2} L_{-1}^2\right)\Phi_{-1},$$

$$\mathcal{W} = bc\Phi_{-1} + \frac{3}{2} L_{-1}\Phi_{-1}.$$

$$V = L_{-1}\Phi_0 + \left(L_{-2} + \frac{3}{2} L_{-1}^2\right)\Phi_{-1}$$

$$\mathcal{W} = \Phi_0 + \left(bc + \frac{3}{2}L_{-1}\right)\Phi_{-1}$$

$$Q_B = Q_> + Q_0$$

$$Q_B\mathcal{W} = \mathcal{V},$$

$$N_*^{\mathrm{lc}} = \sum_{m\geq 1} \frac{1}{m}\alpha_{-m}^{+}\alpha_m^{-}$$

$$Q_> = Q_{>,1} + Q_{>,0} + Q_{>,-1}$$

$$Q_{>,1} = -(2\alpha')^{1/2}k^{+}\sum_{m\geq 1}\alpha_m^{-}c_{-m}$$

$$R = \frac{1}{(2\alpha')^{1/2}k^{+}}\sum_{m\geq 1}\alpha_{-m}^{+}b_m$$

$$S = \{Q_{>,1}, R\} = \sum_{m=1}^{\infty}(mc_{-m}b_m - \alpha_{-m}^{+}\alpha_m^{-})$$

$$V = \sum_{n>0} L_{-n}^{X}W_n + \sum_{r>0} G_{-r}^{X}\Lambda_r$$

$$V = G_{-1/2}^{X}\Phi_0$$

$$\mathcal{W} = c\delta'(\gamma)\Phi_0$$

$$\beta_r|-1\rangle = \gamma_r|-1\rangle = 0, r > 0$$

$$\mathcal{W} = -c_1\beta_{-1/2}|-1\rangle \otimes \Phi_0$$

$$V = (G_{-3/2}^{X} + 2G_{-1/2}^{X}L_{-1}^{X})\Phi_{-1}$$

$$\mathcal{W} = (\delta(\gamma)G_{-1/2}^{X} - c\beta\delta(\gamma) + c\delta'(\gamma)L_{-1}^{X})\Phi_{-1}$$

$$\mathcal{W} = (G_{-1/2}^{X} - c_1\beta_{-3/2} - c_1\beta_{-1/2}L_{-1}^{X})|-1\rangle \otimes \Phi_{-1}$$

$$V = \left(L_{-1}^{X} - \frac{1}{2}G_{-1}^{X}G_0^{X}\right)\Phi$$

$$\begin{aligned} \beta_n\Theta_{-1/2} &= 0, \quad n \geq 0 \\ \gamma_n\Theta_{-1/2} &= 0, \quad n > 0 \end{aligned}$$

$$\mathcal{W} = \left(1 - G_0^{\mathrm{gh}}G_0^{X}\right)\Theta_{-1/2}\Phi$$

$$\mathcal{W} = \left(1 + \frac{1}{2}c_1\beta_{-1}G_0^{X}\right)\Theta_{-1/2}\Phi$$

$$Q_B = Q_> + Q_0,$$

$$N_*^{\mathrm{lc}} = \sum_{m\geq 1} \frac{1}{m} \alpha_{-m}^+ \alpha_m^- - \sum_{r\geq 1/2} \psi_{-r}^+ \psi_r^-$$

$$Q_> = Q_{>,1} + Q_{>,0} + Q_{>,-1}$$

$$Q_{>,1} = -(2\alpha')^{1/2} k^+ \sum_{m\geq 1} \alpha_m^- c_{-m} + (2\alpha')^{1/2} k^+ \sum_{r\geq 1/2} \gamma_{-r} \psi_r^-$$

$$\prod_{s\geq 1} \beta_{-s}^{n_s} \prod_{r\geq 0} \gamma_{-r}^{m_r} \Theta_{-1/2}$$

$$N_*^{\mathrm{lc}} = \sum_{m\geq 1} \frac{1}{m} \alpha_{-m}^+ \alpha_m^- - \sum_{r\geq 0} \psi_{-r}^+ \psi_r^-$$

$$Q_{>,1} = -(2\alpha')^{1/2} k^+ \sum_{m\geq 1} \alpha_m^- c_{-m} + (2\alpha')^{1/2} k^+ \sum_{r\geq 0} \gamma_{-r} \psi_r^-$$

$$\begin{aligned} z &\cong z + 1 \\ \theta &\cong -\theta \end{aligned}$$

$$\begin{aligned} z &\cong z + \tau \\ \theta &\cong \theta \end{aligned}$$

$$\tilde{z} \cong \tilde{z} + 1 \cong \tilde{z} + \tilde{\tau}$$

$$q_{\mathrm{NS}} = u_1 - u_2 - \zeta_1 \zeta_2$$

$$\begin{aligned} u &\cong u + 1 \\ \zeta_1 &\cong -\zeta_1 \\ \zeta_2 &\cong \zeta_2 \\ \tilde{u} &\cong \tilde{u} + 1 \end{aligned}$$

$$\begin{aligned} u &\cong u + \tau \\ \zeta_1 &\cong \zeta_1 \\ \zeta_2 &\cong \zeta_2 \\ \tilde{u} &\cong \tilde{u} + \bar{\tau} \end{aligned}$$

$$\tilde{q} = \tilde{u}$$

$$q_{\mathrm{NS}} = u - \zeta_1 \zeta_2$$

$$\bar{\tilde{u}} = u + \zeta_1 \zeta_2 h(\tilde{u}, u)$$

$$h(\tilde{u} + 1, u + 1) = -h(\tilde{u}, u), h(\tilde{u} + \bar{\tau}, u + \tau) = h(\tilde{u}, u)$$

$$h(0,0) = -1$$

$$h_\lambda = \lambda h_1 + (1 - \lambda) h_2, 0 \leq \lambda \leq 1$$

$$\Omega = [\mathrm{d}\tilde{u}; \mathrm{d}u \mid \mathrm{d}\zeta_1\, \mathrm{d}\zeta_2] P(\tilde{u}),$$

$$P(\tilde{u} + 1) = -P(\tilde{u}), P(\tilde{u} + \bar{\tau}) = P(\tilde{u})$$

$$P(\tilde{u}) = \sum_{n,m\in\mathbb{Z}} \frac{(-1)^n}{\tilde{u} + n + m\bar{\tau}}$$

$$I = \int_{\Gamma} \Omega$$

$$u = \overline{\tilde{u}} - \zeta_1\zeta_2 h(\tilde{u}, \overline{\tilde{u}})$$

$$[\mathrm{d}\tilde{u}; \mathrm{d}u \mid \mathrm{d}\zeta_1\ \mathrm{d}\zeta_2] = \left(1 - \zeta_1\zeta_2 \frac{\partial h}{\partial \overline{\tilde{u}}}\right) \left[\mathrm{d}\tilde{u}\ \mathrm{d}\overline{\tilde{u}} \mid \mathrm{d}\zeta_1\ \mathrm{d}\zeta_2\right]$$

$$I = \int_{\Gamma} \left[\mathrm{d}\tilde{u}\ \mathrm{d}\overline{\tilde{u}} \mid \mathrm{d}\zeta_1\ \mathrm{d}\zeta_2\right] \left(1 - \zeta_1\zeta_2 \frac{\partial h}{\partial \overline{\tilde{u}}}\right) P(\tilde{u})$$

$$I = -\int_{\Sigma_{\mathrm{red}}} \mathrm{d}\tilde{u} \wedge\ \mathrm{d}\overline{\tilde{u}} \frac{\partial h}{\partial \overline{\tilde{u}}} P(\tilde{u})$$

$$I = 4\pi i$$

$$\begin{aligned} z &\to -z \\ \theta &\to \pm\sqrt{-1}\theta. \end{aligned}$$

$$\begin{aligned} u &\to -u \\ \zeta_i &\to \pm\sqrt{-1}\zeta_i, i = 1,2 \end{aligned}$$

$$\begin{aligned} u &\to u \\ \zeta_i &\to -\zeta_i, i = 1,2 \end{aligned}$$

CONCLUSIONES.

Se concluye entonces, que la supergravedad cuántica, es un estado de la gravedad propiamente dicha, en la que, el tejido temporo – espacial, no conserva su geometría inicial. La deformación y por ende, la torsión del espacio – tiempo cuántico, no es local, de tal suerte, que la producción de multiespacios es inminente. La supergravedad cuántica, ocurre en circunstancias extremadamente hostiles de la materia, en las que, a escala microscópica, una partícula estrella u oscura, según sea el caso, se aniquila o colapsa, provocando, inicialmente, un agujero negro masivo o supermasivo, según la emisión de radición que se tenga por implícita, y simultáneamente, la formación de dimensiones en $\mathbb{R}^{\eta}$. En este punto, es indispensable anotar, que una partícula, sea cual sea, puede interactuar en distintas dimensiones, al mismo tiempo. En un escenario de supergravedad cuántica, la materia y la energía alcanzan densidades exponenciales, lo que las funde, transformándolas finalmente, en materia y energía oscuras, yuxtapuestas en un tejido espacio – tiempo pluridimensional y no local.

REFERENCIAS BIBLIOGRÁFICAS.

Elias Kiritsis, INTRODUCTION TO SUPERSTRING THEORY, https://arxiv.org/abs/hep-th/9709062v2.

Edward Witten, Perturbative Superstring Theory Revisited, arXiv:1209.5461v4 [hep-th] 5 Nov 2023.

Albuja Bustamante, M. I. (2024). Demostración del Espectro Hamiltoniano para un Campo de Yang-Mills no Abeliano que Poseen una Brecha de Masa Finita con Respecto al Estado de Vacío. *Ciencia Latina Revista Científica Multidisciplinar*, *8*(1). https://doi.org/10.37811/cl_rcm.v8i1.9738.

Manuel Ignacio Albuja Bustamante, Teoría de Campos: Reforzamiento Teórico – Matemático al Modelo Estándar de Partículas, bajo la estructura ecuacional de Yang – Mills, Ciencia Latina Revista Científica Multidisciplinar: Vol. 8 Núm. 2 (2024)

Manuel Ignacio Albuja Bustamante, La brecha de masa y la curvatura de los campos cuánticos, Ciencia Latina Revista Científica Multidisciplinar: Vol. 8 Núm. 4 (2024)

Manuel Ignacio Albuja Bustamante, Aportes Matemáticos en Econometría Teórica, Ciencia Latina Revista Científica Multidisciplinar: Vol. 8 Núm. 4 (2024)

Manuel Ignacio Albuja Bustamante, Formalización Matemática y en Física de Partículas, en Relación a la Brecha de Masa y la Curvatura Geométrica de los Campos Cuánticos, Ciencia Latina Revista Científica Multidisciplinar: Vol. 8 Núm. 5 (2024)

Manuel Ignacio Albuja Bustamante, Campos Cuánticos Relativistas: Aproximaciones Teórico – Matemáticas Relativas a los Espacios Cuánticos Geométricamente Deformados o Perforados por Partículas y Antipartículas Supermasivas y Masivas E Hiperpartículas y Suprapartículas, Ciencia Latina Revista Científica Multidisciplinar: Vol. 9 Núm. 1 (2025)

Manuel Ignacio Albuja Bustamante, Teoría Cuántica de Campos Relativistas. Formalización Teórica, Ciencia Latina Revista Científica Multidisciplinar: Vol. 9 Núm. 3 (2025)

Albuja Bustamante, M. I. (2025). Las partículas supermasivas: naturaleza fenomenológica de la partícula oscura. Ciencia Latina Revista Científica Multidisciplinar, 9(3). https://doi.org/10.37811/cl_rcm.v9i3.18172