Ciencia Latina Revista Científica Multidisciplinar, Ciudad de México, México.
ISSN 2707-2207 / ISSN 2707-2215 (en línea), julio-agosto 2025,
Volumen 9, Número 4.

https://doi.org/10.37811/cl_rcm.v9i2

SUPERGRAVEDAD CUÁNTICA RELATIVISTA. DIMENSIONES EN R^H

RELATIVISTIC QUANTUM SUPERGRAVITY. DIMENSIONS IN R^H

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DOI: https://doi.org/10.37811/cl_rcm.v9i4.18475

Supergravedad cuántica relativista. Dimensiones en $\mathbb{R}^{\eta}$.

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RESUMEN

En este trabajo nos propondremos esbozar algunas características inherentes a la supergravedad cuántica, a propósito de la existencia de supermembranas y superespacios. Para efectos de este trabajo, entiéndase por supermembranas, a las dimensiones múltiples creadas por una partícula estrella u oscura, según sea el caso, en tanto que los superespacios, son distintos niveles de realidad cuántica, en los que las supermembranas, alteran la materia y la energía, transformándola en distintos estados morfológicos y de interacción. Este trabajo, contempla en sí, un modelamiento matemático de supergravedad cuántica en espacios superplanckianos y modificados por gravedad extrema, y por ende, el sistema de referencia de toda partícula interactuante (momentum, energía, masa, momento angular, espín, etc) y los respectivos sistemas de supersimetría de gauge fijos o corregidos. En este trabajo también, se destaca la dualidad holográfica provocada por la supergravedad cuántica, es decir, dimensiones originadas de un mismo plano pero distintas, dimensionalmente.

Palabras clave: supergravedad cuántica, supermembranas, supersimetría de gauge, superespacios, dualidad holográfica

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Relativistic quantum supergravity. Dimensions in $\mathbb{R}^{\eta}$.

ABSTRACT

In this paper we will outline some characteristics inherent to quantum supergravity, regarding the existence of supermembranes and superspaces. For the purposes of this work, supermembranes are understood to be the multiple dimensions created by a star or dark particle, as the case may be, while superspaces are different levels of quantum reality, in which supermembranes alter matter and energy, transforming it into different morphological and interaction states. This work contemplates in itself, a mathematical modeling of quantum supergravity in superplanckian spaces and modified by extreme gravity, and therefore, the reference system of all interacting particles (momentum, energy, mass, angular momentum, spin, etc.) and the respective fixed or corrected gauge supersymmetry systems. In this work, the holographic duality caused by quantum supergravity is also highlighted, that is, dimensions originating from the same plane but different, dimensionally.

Keywords: quantum supergravity, supermembranes, gauge supersymmetry, superspaces, holographic duality.

Artículo recibido 07 mayo 2025

Aceptado para publicación: 11 junio 2025

INTRODUCCIÓN.

Una de las características esenciales de la supergravedad cuántica, es la formación de dimensiones múltiples. Y no es un fenómeno menor, pues, a diferencia de la gravedad cuántica, en la que, el espacio – tiempo cuántico no se despoja de su estado cuatridimensional, pues, cuando el espacio – tiempo es afectado por supergravedad cuántica, no solamente se desdobla, sino que además, construye dimensiones distintas a la de origen, en las que, las leyes de la física, no son las mismas para cada sistema de referencia en el superespacio. En este punto, es importante precisar que el tiempo, pasa a convertirse en una dimensión física perfectamente maleable debido a la gravedad, de tal suerte que cuanta más gravedad, más contraído se vuelve en tanto que, a mayor gravedad, más se dilata. Es importante destacar, que ante un escenario de supergravedad cuántica, la materia y la energía, se diluyen, transformándose y desplazándose de manera simultánea, entre dimensiones disímiles. Las dimensiones entre sí, se tienen por opuestas, en la medida en que no corresponden al mismo espacio – tiempo cuántico, de ahí que, una partícula supermasiva por ejemplo, no necesariamente desempeña las mismas propiedades fenomenológicas que en su espacio – tiempo cuántico de origen, tal es así, que en otra dimensión, la partícula supermasiva, se propaga sin deformar el tejido espacio – temporal en el que interactúa, repercutiendo o no, en las trayectorias orbitales de las partículas circundantes, o incluso, coexistiendo con partículas de similares características pero de distinta masa o energía, momentum, spin o momento angular, etc, según sea el caso o convirtiéndose en partícula ligera o en antimateria. Las dimensiones múltiples se producen infinitas veces, lo que supone un proceso inflacionario del espacio – tiempo cuántico sin límite externo, a diferencia de la gravedad cuántica, en la que la inflación temporo – espacial, crece sin límite interno.

RESULTADOS Y DISCUSIÓN.

Multidimensiones – Supermembranas – Superespacios y Supersimetrías a propósito de la existencia de supergravedad cuántica. Modelo Nambu-Goto – Weyl – Neumann – Dirichlet – Witten – Kaluza – Klein – Hodge - Calabi - Yau – Kähler - Eguchi-Hanson.

$$l_{\mathrm{p}} = \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.6 \times 10^{-33} \text{ cm}$$

$$m_{\mathrm{p}} = \left(\frac{\hbar c}{G}\right)^{1/2} = 1.2 \times 10^{19} \mathrm{GeV}/c^2$$

$$S_\sigma = -\frac{T}{2}\int \sqrt{-h} h^{\alpha\beta} \eta_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu d\sigma d\tau$$

$$T(\alpha) = T_0 + \alpha T_1 + \alpha^2 T_2 + \cdots$$

$$S_0 = -\alpha \int ds$$

$$S_0 = -m \int ds$$

$$ds^2 = -g_{\mu\nu}(X) dX^\mu dX^\nu$$

$$S_0 = -m \int \sqrt{-g_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu} d\tau$$

$$\tilde{S}_0 = \frac{1}{2}\int d\tau (e^{-1}\dot{X}^2 - m^2 e)$$

$$S_p = -T_p \int d\mu_p$$

$$d\mu_p = \sqrt{-\det G_{\alpha\beta}} d^{p+1}\sigma,$$

$$G_{\alpha\beta} = g_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu \ \alpha, \beta = 0, \ldots, p$$

$$[T_p] = (\text{ length })^{-p-1} = \frac{\text{mass}}{(\text{ length })^p}$$

$$S_0 = -\alpha \int \sqrt{dt^2 - d\vec{x}^2} = -\alpha \int dt \sqrt{1 - \vec{v}^2} \approx -\alpha \int dt \left(1 - \frac{1}{2}\vec{v}^2 + \cdots\right)$$

$$S_{\mathrm{nr}} = \int dt \frac{1}{2} m \vec{v}^2$$

$$S_0 = -m \int \sqrt{-\frac{dX^\mu}{d\tau}\frac{dX_\mu}{d\tau}} d\tau$$

$$d\tau' = \frac{df(\tau)}{d\tau} d\tau = \dot{f}(\tau) d\tau \text{ and } \frac{dX^\mu}{d\tau} = \frac{dX^\mu}{d\tau'} \frac{d\tau'}{d\tau} = \frac{dX^\mu}{d\tau'} \cdot \dot{f}(\tau)$$

$$S_0' = -m \int \sqrt{-\frac{dX^\mu}{d\tau'} \frac{dX_\mu}{d\tau'} \dot{f}(\tau)} \cdot \frac{d\tau'}{\dot{f}(\tau)} = -m \int \sqrt{-\frac{dX^\mu}{d\tau'} \frac{dX_\mu}{d\tau'}} \cdot d\tau'$$

$$\tau \to \tau' = f(\tau) = \tau - \xi(\tau)$$

$$\delta X^\mu = X^{\mu\prime}(\tau) - X^\mu(\tau) = \xi(\tau) \dot{X}^\mu$$

$$e'(\tau') d\tau' = e(\tau) d\tau$$

$$\delta e = e'(\tau) - e(\tau) = \frac{d}{d\tau}(\xi e)$$

$$\delta \tilde{S}_0 = \frac{1}{2} \int d\tau \left(\frac{2\dot{X}^\mu \delta \dot{X}_\mu}{e} - \frac{\dot{X}^\mu \dot{X}_\mu}{e^2} \delta e - m^2 \delta e \right)$$

$$\delta \dot{X}_\mu = \frac{d}{d\tau} \delta X_\mu = \dot{\xi} \dot{X}_\mu + \xi \ddot{X}_\mu$$

$$\delta \tilde{S}_0 = \frac{1}{2} \int d\tau \left[\frac{2\dot{X}^\mu}{e} (\dot{\xi} \dot{X}_\mu + \xi \ddot{X}_\mu) - \frac{\dot{X}^\mu \dot{X}_\mu}{e^2} (\dot{\xi} e + \xi \dot{e}) - m^2 \frac{d(\xi e)}{d\tau} \right]$$

$$\delta \tilde{S}_0 = \frac{1}{2} \int d\tau \cdot \frac{d}{d\tau} \left(\frac{\xi}{e} \dot{X}^\mu \dot{X}_\mu \right)$$

$$\frac{\delta \tilde{S}_0}{\delta e} = -\frac{1}{2} \left(e^{-2} \dot{X}^\mu \dot{X}_\mu + m^2 \right) = 0$$

$$\dot{X}^\mu \dot{X}_\mu + m^2 = 0$$

$$-\frac{d}{d\tau}(g_{\mu\nu} \dot{X}^\nu) + \frac{1}{2} \partial_\mu g_{\rho\lambda} \dot{X}^\rho \dot{X}^\lambda$$
$$= -(\partial_\rho g_{\mu\nu}) \dot{X}^\rho \dot{X}^\nu - g_{\mu\nu} \ddot{X}^\nu + \frac{1}{2} \partial_\mu g_{\rho\lambda} \dot{X}^\rho \dot{X}^\lambda = 0.$$

$$\ddot{X}^\mu + \Gamma^\mu_{\rho\lambda} \dot{X}^\rho \dot{X}^\lambda = 0$$

$$\Gamma^\mu_{\rho\lambda} = \frac{1}{2} g^{\mu\nu} (\partial_\rho g_{\lambda\nu} + \partial_\lambda g_{\rho\nu} - \partial_\nu g_{\rho\lambda})$$

$$G_{\alpha\beta} = \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta} g_{\mu\nu} = (f^{-1})^\gamma_\alpha \frac{\partial X^\mu}{\partial \tilde{\sigma}^\gamma} (f^{-1})^\delta_\beta \frac{\partial X^\nu}{\partial \tilde{\sigma}^\delta} g_{\mu\nu}$$

$$f^\alpha_\beta(\tilde{\sigma}) = \frac{\partial \sigma^\alpha}{\partial \tilde{\sigma}^\beta}$$

$$\det\left(g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta} \right) = J^{-2} \det\left(g_{\mu\nu} \frac{\partial X^\mu}{\partial \tilde{\sigma}^\gamma} \frac{\partial X^\nu}{\partial \tilde{\sigma}^\delta} \right).$$

$$d^{p+1}\sigma = J d^{p+1}\tilde{\sigma}$$

$$\tilde{S}_p = -T_p \int d^{p+1}\tilde{\sigma} \sqrt{-\det\left(g_{\mu\nu}\frac{\partial X^\mu}{\partial \tilde{\sigma}^\gamma}\frac{\partial X^\nu}{\partial \tilde{\sigma}^\delta}\right)}$$

$$S_{\text{NG}} = -T \int d\sigma d\tau \sqrt{(\dot{X}\cdot X')^2 - \dot{X}^2 X'^2}$$

$$\dot{X}^\mu = \frac{\partial X^\mu}{\partial \tau} \text{ and } X^{\mu\prime} = \frac{\partial X^\mu}{\partial \sigma}$$

$$h = \det h_{\alpha\beta} \text{ and } h^{\alpha\beta} = (h^{-1})_{\alpha\beta},$$

$$S_\sigma = -\frac{1}{2}T \int d^2\sigma \sqrt{-h} h^{\alpha\beta}\partial_\alpha X \cdot \partial_\beta X$$

$$T_{\alpha\beta} = -\frac{2}{T}\frac{1}{\sqrt{-h}}\frac{\delta S_\sigma}{\delta h^{\alpha\beta}} = 0$$

$$\delta h = -h h_{\alpha\beta}\delta h^{\alpha\beta},$$

$$\delta\sqrt{-h} = -\frac{1}{2}\sqrt{-h} h_{\alpha\beta}\delta h^{\alpha\beta}$$

$$T_{\alpha\beta} = \partial_\alpha X \cdot \partial_\beta X - \frac{1}{2}h_{\alpha\beta}h^{\gamma\delta}\partial_\gamma X \cdot \partial_\delta X = 0$$

$$\partial_\alpha X \cdot \partial_\beta X = \frac{1}{2}h_{\alpha\beta}h^{\gamma\delta}\partial_\gamma X \cdot \partial_\delta X$$

$$\sqrt{-\det(\partial_\alpha X \cdot \partial_\beta X)} = \frac{1}{2}\sqrt{-h} h^{\gamma\delta}\partial_\gamma X \cdot \partial_\delta X.$$

$$\Delta X^\mu = -\frac{1}{\sqrt{-h}}\partial_\alpha\left(\sqrt{-h} h^{\alpha\beta}\partial_\beta X^\mu\right) = 0$$

$$S_{\text{NG}} = -T \int d\tau d\sigma \sqrt{-\det G_{\alpha\beta}}, G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$$

$$\begin{aligned} \det G_{\alpha\beta} &= \det\begin{pmatrix} \partial_\tau X^\mu \partial_\tau X_\mu & \partial_\tau X^\mu \partial_\sigma X_\mu \\ \partial_\sigma X^\mu \partial_\tau X_\mu & \partial_\sigma X^\mu \partial_\sigma X_\mu \end{pmatrix} \\ &= \det\begin{pmatrix} -1 + \partial_\tau X^i \partial_\tau X_i & \partial_\tau X^i \partial_\sigma X_i \\ \partial_\sigma X^i \partial_\tau X_i & 1 + \partial_\sigma X^i \partial_\sigma X_i \end{pmatrix} \end{aligned}$$

$$\det G_{\alpha\beta} \approx -1 + \partial_\tau X^i \partial_\tau X_i - \partial_\sigma X^i \partial_\sigma X_i + \cdots$$

$$S_{\mathrm{NG}} = -T\int d\tau d\sigma\sqrt{|-1+\partial_\tau X^i\partial_\tau X_i - \partial_\sigma X^i\partial_\sigma X_i|}$$
$$\approx T\int d\tau d\sigma\left(-1+\frac{1}{2}\partial_\tau X^i\partial_\tau X_i - \frac{1}{2}\partial_\sigma X^i\partial_\sigma X_i\right)$$

$$S_\sigma = -\frac{T}{2}\int d^2\sigma\sqrt{-h}h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu + \Lambda\int d^2\sigma\sqrt{-h}$$

$$\frac{2}{\sqrt{-h}}\frac{\delta S_\sigma}{\delta h^{\gamma\delta}} = -T\left[\partial_\gamma X^\mu\partial_\delta X_\mu - \frac{1}{2}h_{\gamma\delta}\left(h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu\right)\right] - \Lambda h_{\gamma\delta} = 0$$

$$h_{\gamma\delta}h^{\gamma\delta}\Lambda = T\left(\frac{1}{2}h_{\gamma\delta}h^{\gamma\delta} - 1\right)h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu.$$

$$S_\sigma = -\frac{T_p}{2}\int d^{p+1}\sigma\sqrt{-h}h^{\alpha\beta}\partial_\alpha X\cdot\partial_\beta X + \Lambda_p\int d^{p+1}\sigma\sqrt{-h}$$

$$T_p\left[\partial_\gamma X\cdot\partial_\delta X - \frac{1}{2}h_{\gamma\delta}\left(h^{\alpha\beta}\partial_\alpha X\cdot\partial_\beta X\right)\right] + \Lambda_p h_{\gamma\delta} = 0.$$

$$h_{\alpha\beta} = \partial_\alpha X\cdot\partial_\beta X$$

$$T_p\left(1-\frac{1}{2}h^{\alpha\beta}h_{\alpha\beta}\right) + \Lambda_p = 0$$

$$\Lambda_p = \frac{1}{2}(p-1)T_p$$

$$\delta X^\mu = a^\mu{}_\nu X^\nu + b^\mu \text{ and } \delta h^{\alpha\beta} = 0.$$

$$\sigma^\alpha \to f^\alpha(\sigma) = \sigma'^\alpha \text{ and } h_{\alpha\beta}(\sigma) = \frac{\partial f^\gamma}{\partial\sigma^\alpha}\frac{\partial f^\delta}{\partial\sigma^\beta}h_{\gamma\delta}(\sigma')$$

$$h_{\alpha\beta} \to e^{\phi(\sigma,\tau)}h_{\alpha\beta} \text{ and } \delta X^\mu = 0$$

$$h_{\alpha\beta} = \begin{pmatrix} h_{00} & h_{01} \\ h_{10} & h_{11}\end{pmatrix},$$

$$h_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1\end{pmatrix}$$

$$S = \frac{T}{2}\int d^2\sigma(\dot{X}^2 - X'^2)$$

$$S_1 = \lambda_1\int d^2\sigma\sqrt{-h} \text{ and } S_2 = \lambda_2\int d^2\sigma\sqrt{-h}R^{(2)}(h)$$

$$\partial_\alpha\partial^\alpha X^\mu = 0 \text{ or } \left(\frac{\partial^2}{\partial\sigma^2} - \frac{\partial^2}{\partial\tau^2}\right)X^\mu = 0.$$

$$T_{01} = T_{10} = \dot{X}\cdot X' \text{ and } T_{00} = T_{11} = \frac{1}{2}(\dot{X}^2 + X'^2).$$

$$X^\mu \to X^\mu + \delta X^\mu$$

$$-T \int d\tau \left[X'_\mu \delta X^\mu \big|_{\sigma=\pi} - X'_\mu \delta X^\mu \big|_{\sigma=0} \right]$$

$$X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau).$$

$$X'_\mu = 0 \ \text{ at } \ \sigma = 0, \pi$$

$$X^\mu|_{\sigma=0} = X_0^\mu \ \text{ and } \ X^\mu|_{\sigma=\pi} = X_\pi^\mu,$$

$$\sigma^\pm = \tau \pm \sigma.$$

$$\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma) \ \text{ and } \ \begin{pmatrix} \eta_{++} & \eta_{+-} \\ \eta_{-+} & \eta_{--} \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\partial_+ \partial_- X^\mu = 0$$

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu = 0$$
$$T_{--} = \partial_- X^\mu \partial_- X_\mu = 0$$

$$X^\mu(\sigma,\tau) = X_{\mathrm{R}}^\mu(\tau-\sigma) + X_{\mathrm{L}}^\mu(\tau+\sigma),$$

$$(\partial_- X_{\mathrm{R}})^2 = (\partial_+ X_{\mathrm{L}})^2 = 0$$

$$X_{\mathrm{R}}^\mu = \frac{1}{2}x^\mu + \frac{1}{2}l_s^2 p^\mu(\tau-\sigma) + \frac{i}{2}l_s \sum_{n\neq 0} \frac{1}{n}\alpha_n^\mu e^{-2in(\tau-\sigma)}$$
$$X_{\mathrm{L}}^\mu = \frac{1}{2}x^\mu + \frac{1}{2}l_s^2 p^\mu(\tau+\sigma) + \frac{i}{2}l_s \sum_{n\neq 0} \frac{1}{n}\tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)}$$

$$T = \frac{1}{2\pi\alpha'} \ \text{ and } \ \frac{1}{2}l_s^2 = \alpha'$$

$$\alpha_{-n}^\mu = \left(\alpha_n^\mu\right)^\star \ \text{ and } \ \tilde{\alpha}_{-n}^\mu = \left(\tilde{\alpha}_n^\mu\right)^\star$$

$$\partial_- X_{\mathrm{R}}^\mu = l_s \sum_{m=-\infty}^{+\infty} \alpha_m^\mu e^{-2im(\tau-\sigma)}$$
$$\partial_+ X_{\mathrm{L}}^\mu = l_s \sum_{m=-\infty}^{+\infty} \tilde{\alpha}_m^\mu e^{-2im(\tau+\sigma)}$$

$$\alpha_0^\mu = \tilde{\alpha}_0^\mu = \frac{1}{2}l_s p^\mu$$

$$P^\mu(\sigma,\tau) = \frac{\delta S}{\delta \dot{X}_\mu} = T\dot{X}^\mu$$

$$[P^\mu(\sigma,\tau), P^\nu(\sigma',\tau)]_{\text{P.B.}} = [X^\mu(\sigma,\tau), X^\nu(\sigma',\tau)]_{\text{P.B.}} = 0$$
$$[P^\mu(\sigma,\tau), X^\nu(\sigma',\tau)]_{\text{P.B.}} = \eta^{\mu\nu}\delta(\sigma-\sigma')$$

$$[\dot{X}^\mu(\sigma,\tau), X^\nu(\sigma',\tau)]_{\text{P.B.}} = T^{-1}\eta^{\mu\nu}\delta(\sigma-\sigma').$$

$$[\alpha_m^\mu, \alpha_n^\nu]_{\text{P.B.}} = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu]_{\text{P.B.}} = im\eta^{\mu\nu}\delta_{m+n,0}$$

$$[\alpha_m^\mu, \tilde{\alpha}_n^\nu]_{\text{P.B.}} = 0.$$

$$\delta(\sigma - \sigma') = \frac{1}{\pi}\sum_{n=-\infty}^{+\infty} e^{2in(\sigma-\sigma')}$$

$$[\dots]_{\text{P.B.}} \to i[\dots]$$

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\eta^{\mu\nu}\delta_{m+n,0}, [\alpha_m^\mu, \tilde{\alpha}_n^\nu] = 0$$

$$a_m^\mu = \frac{1}{\sqrt{m}}\alpha_m^\mu \text{ and } a_m^{\mu\dagger} = \frac{1}{\sqrt{m}}\alpha_{-m}^\mu \text{ for } m > 0$$

$$[a_m^\mu, a_n^{\nu\dagger}] = [\tilde{a}_m^\mu, \tilde{a}_n^{\nu\dagger}] = \eta^{\mu\nu}\delta_{m,n} \text{ for } m,n > 0$$

$$[a_m^0, a_m^{0\dagger}] = -1$$

$$a_m^\mu|0\rangle = 0 \text{ for } m > 0$$

$$|\phi\rangle = a_{m_1}^{\mu_1\dagger}a_{m_2}^{\mu_2\dagger}\cdots a_{m_n}^{\mu_n\dagger}|0;k\rangle$$

$$p^\mu|\phi\rangle = k^\mu|\phi\rangle$$

$$a_m^{0\dagger}|0\rangle \text{ with norm } \langle 0|a_m^0 a_m^{0\dagger}|0\rangle = -1,$$

$$X^\mu(\tau,\sigma) = x^\mu + l_s^2 p^\mu \tau + il_s \sum_{m\neq 0}\frac{1}{m}\alpha_m^\mu e^{-im\tau}\cos(m\sigma).$$

$$2\partial_\pm X^\mu = \dot{X}^\mu \pm X'^\mu = l_s \sum_{m=-\infty}^{\infty} \alpha_m^\mu e^{-im(\tau\pm\sigma)}$$

$$\phi \to \phi + \delta_\varepsilon \phi,$$

$$\mathcal{L} \to \mathcal{L} + \varepsilon\partial_\alpha \mathcal{J}^\alpha.$$

$$\delta X^\mu = a^\mu{}_\nu X^\nu + b^\mu,$$

$$P_\alpha^\mu = T\partial_\alpha X^\mu$$
$$J_\alpha^{\mu\nu} = T(X^\mu\partial_\alpha X^\nu - X^\nu\partial_\alpha X^\mu)$$

$$H = \int_0^\pi (\dot{X}_\mu P_0^\mu - \mathcal{L})d\sigma = \frac{T}{2}\int_0^\pi (\dot{X}^2 + X'^2)d\sigma$$

$$P_0^\mu = \frac{\delta S}{\delta \dot{X}_\mu} = T\dot{X}^\mu$$

$$H = \sum_{n=-\infty}^{+\infty} (\alpha_{-n} \cdot \alpha_n + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n)$$

$$H = \frac{1}{2} \sum_{n=-\infty}^{+\infty} \alpha_{-n} \cdot \alpha_n$$

$$T_{--} = 2l_s^2 \sum_{m=-\infty}^{+\infty} L_m e^{-2im(\tau-\sigma)} \text{ and } T_{++} = 2l_s^2 \sum_{m=-\infty}^{+\infty} \tilde{L}_m e^{-2im(\tau+\sigma)},$$

$$L_m = \frac{1}{2} \sum_{n=-\infty}^{+\infty} \alpha_{m-n} \cdot \alpha_n \text{ and } \tilde{L}_m = \frac{1}{2} \sum_{n=-\infty}^{+\infty} \tilde{\alpha}_{m-n} \cdot \tilde{\alpha}_n.$$

$$\frac{1}{2} H = L_0 + \tilde{L}_0 = \frac{1}{2} \sum_{n=-\infty}^{+\infty} (\alpha_{-n} \cdot \alpha_n + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n)$$

$$H = L_0 = \frac{1}{2} \sum_{n=-\infty}^{+\infty} \alpha_{-n} \cdot \alpha_n.$$

$$L_m = 0 \text{ for } m = 0, \pm 1, \pm 2, \ldots$$

$$L_0 = \tilde{L}_0 = 0$$

$$M^2 = -p_\mu p^\mu$$

$$p^\mu = T \int_0^\pi d\sigma \dot{X}^\mu(\sigma)$$

$$L_0 = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n + \frac{1}{2} \alpha_0^2 = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n + \alpha' p^2 = 0$$

$$M^2 = \frac{1}{\alpha'} \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n$$

$$M^2 = \frac{2}{\alpha'} \sum_{n=1}^{\infty} (\alpha_{-n} \cdot \alpha_n + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n)$$

$$[L_m, L_n]_{\text{P.B.}} = i(m-n) L_{m+n}$$

$$\partial^\alpha \xi^\beta + \partial^\beta \xi^\alpha = \Lambda \eta^{\alpha\beta}$$

$$\xi^+ = \xi^+(\sigma^+) \text{ and } \xi^- = \xi^-(\sigma^-).$$

$$V^\pm = \frac{1}{2} \xi^\pm(\sigma^\pm) \frac{\partial}{\partial \sigma^\pm}$$

$$\xi_n^\pm(\sigma^\pm) = e^{2in\sigma^\pm} \; n \in \mathbb{Z}$$

$$V_n = e^{in\sigma^+}\frac{\partial}{\partial\sigma^+} + e^{in\sigma^-}\frac{\partial}{\partial\sigma^-}\ n \in \mathbb{Z}$$

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{+\infty} \alpha_{m-n}\cdot\alpha_n = 0 \ \text{ for } \ m \in \mathbb{Z}$$

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\cdot\alpha_n:.$$

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

$$(L_0 - a)|\phi\rangle = 0$$

$$(L_0 - a)|\phi\rangle = \left(\tilde{L}_0 - a\right)|\phi\rangle = 0$$

$$\alpha' M^2 = \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n - a = N - a$$

$$N = \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n = \sum_{n=1}^{\infty} n a_n^{\dagger}\cdot a_n,$$

$$\frac{1}{4}\alpha' M^2 = \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n - a = \sum_{n=1}^{\infty} \tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n - a = N - a = \tilde{N} - a.$$

$$\left(L_0 - \tilde{L}_0\right)|\phi\rangle = 0,$$

$$L_m|\phi\rangle = 0\ m > 0.$$

$$(L_0 - a)|\phi\rangle = 0,$$

$$L_{-m} = L_m^{\dagger},$$

$$\langle\phi|L_m = 0\ m < 0.$$

$$J^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu - i\sum_{n=1}^{\infty}\frac{1}{n}\left(\alpha_{-n}^{\mu}\alpha_n^{\nu} - \alpha_{-n}^{\nu}\alpha_n^{\mu}\right)$$

$$[L_m, J^{\mu\nu}] = 0$$

$$(L_0 - a)|\psi\rangle = 0 \ \text{ and } \ \langle\phi \mid \psi\rangle = 0$$

$$|\psi\rangle = \sum_{n=1}^{\infty} L_{-n}|\chi_n\rangle \ \text{ with } \ (L_0 - a + n)|\chi_n\rangle = 0$$

$$|\psi\rangle = L_{-1}|\chi_1\rangle + L_{-2}|\chi_2\rangle$$

$$\langle\phi \mid \psi\rangle = \sum_{n=1}^{\infty} \langle\phi|L_{-n}|\chi_n\rangle = \sum_{n=1}^{\infty} \langle\chi_n|L_n|\phi\rangle^{\star} = 0.$$

$$\langle\psi \mid \psi\rangle = \sum_{n=1}^{\infty} \langle\chi_n|L_n|\psi\rangle = 0$$

$$|\psi\rangle = L_{-1}|\chi_1\rangle$$

$$(L_0 - a + 1)|\chi_1\rangle = 0 \text{ and } L_m|\chi_1\rangle = 0\, m > 0.$$

$$L_m|\psi\rangle = (L_0 - a)|\psi\rangle = 0 \text{ for } m = 1,2,\ldots$$

$$L_1 L_{-1} = 2L_0 + L_{-1}L_1$$

$$L_1|\psi\rangle = L_1 L_{-1}|\chi_1\rangle = (2L_0 + L_{-1}L_1)|\chi_1\rangle = 2(a-1)|\chi_1\rangle = 0$$

$$|\psi\rangle = (L_{-2} + \gamma L_{-1}^2)|\tilde{\chi}\rangle.$$

$$(L_0 + 1)|\tilde{\chi}\rangle = L_m|\tilde{\chi}\rangle = 0 \text{ for } m = 1,2,\ldots$$

$$\begin{aligned}[L_1, L_{-2} + \gamma L_{-1}^2] &= 3L_{-1} + 2\gamma L_0 L_{-1} + 2\gamma L_{-1} L_0 \\ &= (3 - 2\gamma)L_{-1} + 4\gamma L_0 L_{-1}\end{aligned}$$

$$L_1|\psi\rangle = L_1(L_{-2} + \gamma L_{-1}^2)|\tilde{\chi}\rangle = [(3 - 2\gamma)L_{-1} + 4\gamma L_0 L_{-1}]|\tilde{\chi}\rangle$$

$$L_0 L_{-1}|\tilde{\chi}\rangle = L_{-1}(L_0 + 1)|\tilde{\chi}\rangle = 0.$$

$$\left[L_2, L_{-2} + \frac{3}{2}L_{-1}^2\right] = 13L_0 + 9L_{-1}L_1 + \frac{D}{2}$$

$$L_2|\psi\rangle = L_2\left(L_{-2} + \frac{3}{2}L_{-1}^2\right)|\tilde{\chi}\rangle = \left(-13 + \frac{D}{2}\right)|\tilde{\chi}\rangle.$$

$$J^{\mu\nu} = \int_0^{\pi} J_0^{\mu\nu} d\sigma = T\int_0^{\pi} (X^{\mu}\dot{X}^{\nu} - X^{\nu}\dot{X}^{\mu}) d\sigma$$

$$\begin{aligned}X^{\mu}(\tau,\sigma) &= x^{\mu} + l_s^2 p^{\mu}\tau + i l_s \sum_{m\neq 0} \frac{1}{m}\alpha_m^{\mu} e^{-im\tau}\cos(m\sigma), \\ \dot{X}^{\mu}(\tau,\sigma) &= l_s^2 p^{\mu} + l_s \sum_{m\neq 0} \alpha_m^{\mu} e^{-im\tau}\cos(m\sigma),\end{aligned}$$

$$J^{\mu\nu} = x^{\mu}p^{\nu} - x^{\nu}p^{\mu} - i\sum_{m=1}^{\infty} \frac{1}{m}\left(\alpha_{-m}^{\mu}\alpha_m^{\nu} - \alpha_{-m}^{\nu}\alpha_m^{\mu}\right).$$

$$X^{\pm} = \frac{1}{\sqrt{2}}(X^0 \pm X^{D-1})$$

$$v \cdot w = v_\mu w^\mu = -v^+ w^- - v^- w^+ + \sum_i v^i w^i$$

$$v^- = -v_+, v^+ = -v_-, \text{ and } v^i = v_i.$$

$$\sigma^\pm \to \xi^\pm(\sigma^\pm)$$

$$\tilde{\tau} = \frac{1}{2}[\xi^+(\sigma^+) + \xi^-(\sigma^-)],$$
$$\tilde{\sigma} = \frac{1}{2}[\xi^+(\sigma^+) - \xi^-(\sigma^-)].$$

$$\left(\frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2}\right)\tilde{\tau} = 0$$

$$X^+(\tilde{\sigma}, \tilde{\tau}) = x^+ + l_s^2 p^+ \tilde{\tau}$$

$$\alpha_n^+ = 0 \text{ for } n \neq 0$$

$$\dot{X}^- \pm X^{-\prime} = \frac{1}{2p^+ l_s^2}\left(\dot{X}^i \pm X^{i\prime}\right)^2.$$

$$X^- = x^- + l_s^2 p^- \tau + i l_s \sum_{n \neq 0} \frac{1}{n} \alpha_n^- e^{-in\tau} \cos n\sigma$$

$$\alpha_n^- = \frac{1}{p^+ l_s}\left(\frac{1}{2}\sum_{i=1}^{D-2} \sum_{m=-\infty}^{+\infty} :\alpha_{n-m}^i \alpha_m^i: - a\delta_{n,0}\right)$$

$$M^2 = -p_\mu p^\mu = 2p^+ p^- - \sum_{i=1}^{D-2} p_i^2 = 2(N - a)/l_s^2$$

$$N = \sum_{i=1}^{D-2} \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i$$

$$\frac{1}{2}\sum_{i=1}^{D-2} \sum_{n=-\infty}^{+\infty} \alpha_{-n}^i \alpha_n^i = \frac{1}{2}\sum_{i=1}^{D-2} \sum_{n=-\infty}^{+\infty} :\alpha_{-n}^i \alpha_n^i: + \frac{1}{2}(D-2)\sum_{n=1}^{\infty} n.$$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$$

$$\frac{1}{2}(D-2)\sum_{n=1}^{\infty} n = -\frac{D-2}{24}$$

$$\frac{D-2}{24} = 1$$

$$[J^{i-}, J^{j-}] = 0$$

$$\alpha_{-2}^{i}|0;k\rangle \text{ and } \alpha_{-1}^{i}\alpha_{-1}^{j}|0;k\rangle,$$

$$\operatorname{tr} w^{N} = \prod_{n=1}^{\infty}\prod_{i=1}^{24}\operatorname{tr} w^{\alpha_{-n}^{i}\alpha_{n}^{i}} = \prod_{n=1}^{\infty}(1-w^{n})^{-24}$$

$$d_{n} = \frac{1}{2\pi i}\oint \frac{\operatorname{tr} w^{N}}{w^{n+1}}dw$$

$$\operatorname{tr} w^{N} = \prod_{n=1}^{\infty}(1-w^{n})^{-24} \sim \exp\left(\frac{4\pi^{2}}{1-w}\right)$$

$$\eta(-1/\tau) = (-i\tau)^{1/2}\eta(\tau)$$

$$\eta(\tau) = e^{i\pi\tau/12}\prod_{n=1}^{\infty}\left(1-e^{2\pi i n\tau}\right)$$

$$d_{n} \sim \text{ const. } n^{-27/4}\exp\left(4\pi\sqrt{n}\right)$$

$$M_{0} = \left(4\pi\sqrt{\alpha'}\right)^{-1}$$

$$\alpha' M^{2} = 4(N-1) = 4(\tilde{N}-1)$$

$$\alpha' M^{2} = -4$$

$$\left|\Omega^{ij}\right\rangle = \alpha_{-1}^{i}\tilde{\alpha}_{-1}^{j}|0;k\rangle,$$

$$\begin{aligned}
X^{0} &= B\tau,\\
X^{1} &= B\cos(\tau)\cos(\sigma),\\
X^{2} &= B\sin(\tau)\cos(\sigma),\\
X^{i} &= 0, i > 2,
\end{aligned}$$

$$\frac{E^{2}}{|J|} = 2\pi T = \frac{1}{\alpha'}.$$

$$(\partial_{\tau}X)^{2} + (\partial_{\sigma}X)^{2} = 0, \partial_{\tau}X^{\mu}\partial_{\sigma}X_{\mu} = 0$$

$$\begin{aligned}
X^{0} &= 3A\tau\\
X^{1} &= A\cos(3\tau)\cos(3\sigma)\\
X^{2} &= A\sin(a\tau)\cos(b\sigma)
\end{aligned}$$

$$X^{\mu} = X_{L}^{\mu}(\sigma^{-}) + X_{R}^{\mu}(\sigma^{+}).$$

$$X^{25}(0,\tau) = X_{0}^{25} \text{ and } X^{25}(\pi,\tau) = X_{\pi}^{25}.$$

$$X^{25}(0,\tau) = X_{0}^{25} \text{ and } \partial_{\sigma}X^{25}(\pi,\tau) = 0$$

$$|\phi_{1}\rangle = \alpha_{-1}^{i}|0;k\rangle, \qquad |\phi_{2}\rangle = \alpha_{-1}^{i}\alpha_{-1}^{j}|0;k\rangle$$
$$|\phi_{3}\rangle = \alpha_{-3}^{i}|0;k\rangle, \qquad |\phi_{4}\rangle = \alpha_{-1}^{i}\alpha_{-1}^{j}\alpha_{-2}^{k}|0;k\rangle$$

$$|\phi_1\rangle = \alpha_{-1}^{i}\tilde{\alpha}_{-1}^{j}|0;k\rangle, |\phi_2\rangle = \alpha_{-1}^{i}\alpha_{-1}^{j}\tilde{\alpha}_{-2}^{k}|0;k\rangle.$$

$$|\phi_3\rangle = \alpha_{-1}^{i}\tilde{\alpha}_{-2}^{j}|0;k\rangle$$

$$[X^\mu(\sigma,\tau), X^\nu(\sigma',\tau)] = [P^\mu(\sigma,\tau), P^\nu(\sigma',\tau)] = 0$$

$$[X^\mu(\sigma,\tau), P^\nu(\sigma',\tau)] = i\eta^{\mu\nu}\delta(\sigma-\sigma')$$

$$J^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu - i\sum_{n=1}^{\infty}\frac{1}{n}\left(\alpha_{-n}^{\mu}\alpha_n^{\nu} - \alpha_{-n}^{\nu}\alpha_n^{\mu}\right)$$

$$[p^\mu, p^\nu] = 0$$
$$[p^\mu, J^{\nu\sigma}] = -i\eta^{\mu\nu}p^\sigma + i\eta^{\mu\sigma}p^\nu$$
$$[J^{\mu\nu}, J^{\sigma\lambda}] = -i\eta^{\nu\sigma}J^{\mu\lambda} + i\eta^{\mu\sigma}J^{\nu\lambda} + i\eta^{\nu\lambda}J^{\mu\sigma} - i\eta^{\mu\lambda}J^{\nu\sigma}$$

$$[L_m, J^{\mu\nu}] = 0.$$

$$|\phi\rangle = (A\alpha_{-1}\cdot\alpha_{-1} + B\alpha_0\cdot\alpha_{-2} + C(\alpha_0\cdot\alpha_{-1})^2)|0;k\rangle,$$

$$[L_m, L_n] = (m-n)L_{m+n} + A(m)\delta_{m+n,0}$$

$$\left[[L_m, L_n], L_p\right] + \left[[L_p, L_m], L_n\right] + \left[[L_n, L_p], L_m\right] = 0$$

$$z = e^{2(\tau - i\sigma)} \text{ and } \bar{z} = e^{2(\tau + i\sigma)}$$

$$ds^2 = e^{\omega(x)}dx\cdot dx$$

$$x^\mu \to \frac{x^\mu}{x^2}$$

$$dx\cdot dx \to \frac{dx\cdot dx}{(x^2)^2}$$

$$x^\mu \to \frac{x^\mu + b^\mu x^2}{1 + 2b\cdot x + b^2x^2}$$

$$\delta x^\mu = b^\mu x^2 - 2x^\mu b\cdot x$$

$$\delta x^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2x^\mu(b\cdot x).$$

$$D + \frac{1}{2}D(D-1) + 1 + D = \frac{1}{2}(D+2)(D+1)$$

$$z \to f(z) \text{ and } \bar{z} \to \bar{f}(\bar{z})$$

$$z \to z' = z - \varepsilon_n z^{n+1} \text{ and } \bar{z} \to \bar{z}' = \bar{z} - \bar{\varepsilon}_n\bar{z}^{n+1}, n \in \mathbb{Z}$$

$$\ell_n = -z^{n+1}\partial \text{ and } \bar{\ell}_n = -\bar{z}^{n+1}\bar{\partial}$$

$$[\ell_m, \ell_n] = (m-n)\ell_{m+n} \text{ and } [\bar{\ell}_m, \bar{\ell}_n] = (m-n)\bar{\ell}_{m+n}$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

$$\begin{aligned} \ell_{-1}&: \quad z \to z - \varepsilon, \\ \ell_0&: \quad z \to z - \varepsilon z, \\ \ell_1&: \quad z \to z - \varepsilon z^2. \end{aligned}$$

$$z \to \frac{az+b}{cz+d} \text{ with } a,b,c,d \in \mathbb{C}, ad-bc=1$$

$$\partial^\alpha T_{\alpha\beta} = 0$$

$$\bar{\partial} T_{zz} = 0 \text{ and } \partial T_{\bar{z}\bar{z}} = 0.$$

$$T_{zz} = T(z) \text{ and } T_{\bar{z}\bar{z}} = \tilde{T}(\bar{z}).$$

$$X_{\mathrm{R}}^\mu(\sigma,\tau) \to X_{\mathrm{R}}^\mu(z) = \frac{1}{2}x^\mu - \frac{i}{4}p^\mu \ln z + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu z^{-n}$$

$$X_{\mathrm{L}}^\mu(\sigma,\tau) \to X_{\mathrm{L}}^\mu(\bar{z}) = \frac{1}{2}x^\mu - \frac{i}{4}p^\mu \ln \bar{z} + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^\mu \bar{z}^{-n}.$$

$$\partial X^\mu(z,\bar{z}) = -\frac{i}{2}\sum_{n=-\infty}^{\infty} \alpha_n^\mu z^{-n-1}$$

$$\bar{\partial} X^\mu(z,\bar{z}) = -\frac{i}{2}\sum_{n=-\infty}^{\infty} \tilde{\alpha}_n^\mu \bar{z}^{-n-1}$$

$$T_X(z) = -2{:}\partial X \cdot \partial X{:} = \sum_{n=-\infty}^{+\infty} \frac{L_n}{z^{n+2}}$$

$$\tilde{T}_X(\bar{z}) = -2{:}\bar{\partial} X \cdot \bar{\partial} X{:} = \sum_{n=-\infty}^{+\infty} \frac{\tilde{L}_n}{\bar{z}^{n+2}}$$

$$\delta z = \varepsilon(z) \text{ and } \delta\bar{z} = \tilde{\varepsilon}(\bar{z}),$$

$$Q = Q_\varepsilon + Q_{\tilde{\varepsilon}} = \frac{1}{2\pi i}\oint [T(z)\varepsilon(z)dz + \tilde{T}(\bar{z})\tilde{\varepsilon}(\bar{z})d\bar{z}]$$

$$\delta_\varepsilon \Phi(z,\bar{z}) = [Q_\varepsilon, \Phi(z,\bar{z})] \text{ and } \delta_{\tilde{\varepsilon}}\Phi(z,\bar{z}) = [Q_{\tilde{\varepsilon}}, \Phi(z,\bar{z})]$$

$$\Phi(z,\bar{z}) \to \left(\frac{\partial w}{\partial z}\right)^h \left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^{\tilde{h}} \Phi(w,\bar{w})$$

$$\Phi(z,\bar{z})(dz)^h(d\bar{z})^{\tilde{h}}$$

$$\delta_\varepsilon \Phi(w,\bar{w}) = \frac{1}{2\pi i}\oint dz\varepsilon(z)[T(z),\Phi(w,\bar{w})]$$

$$\delta_\varepsilon \Phi(w,\bar{w}) = h\partial\varepsilon(w)\Phi(w,\bar{w}) + \varepsilon(w)\partial\Phi(w,\bar{w}),$$
$$\delta_{\tilde{\varepsilon}} \Phi(w,\bar{w}) = \tilde{h}\bar{\partial}\tilde{\varepsilon}(\bar{w})\Phi(w,\bar{w}) + \tilde{\varepsilon}(\bar{w})\bar{\partial}\Phi(w,\bar{w}).$$

$$T(z)\Phi(w,\bar{w}) = \frac{h}{(z-w)^2}\Phi(w,\bar{w}) + \frac{1}{z-w}\partial\Phi(w,\bar{w}) + \cdots$$
$$\tilde{T}(\bar{z})\Phi(w,\bar{w}) = \frac{\tilde{h}}{(\bar{z}-\bar{w})^2}\Phi(w,\bar{w}) + \frac{1}{\bar{z}-\bar{w}}\bar{\partial}\Phi(w,\bar{w}) + \cdots$$

$$X^\mu(z)X^\nu(w) = -\frac{1}{4}\eta^{\mu\nu}\ln(z-w) + \cdots$$

$$:\partial X^\mu(z)\partial X^\nu(z): = \lim_{w\to z}\left(\partial_z X^\mu(z)\partial_w X^\nu(w) + \frac{\eta^{\mu\nu}}{4(z-w)^2}\right)$$

$$T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{2}{(z-w)^2}T(w) + \frac{1}{z-w}\partial T(w) + \cdots$$

$$(\partial w)^2 T'(w) = T(z) - \frac{c}{12}S(w,z)$$

$$S(w,z) = \frac{2(\partial w)(\partial^3 w) - 3(\partial^2 w)^2}{2(\partial w)^2}$$

$$\psi(z)\psi(w) = \frac{1}{z-w}$$

$$T(z) = -\frac{1}{2}:\psi(z)\partial\psi(z):$$

$$\psi_\pm =: \exp(\pm i\phi):.$$

$$L_0|\Phi\rangle = h|\Phi\rangle \text{ and } L_n|\Phi\rangle = 0, n > 0.$$

$$|\Phi\rangle = \lim_{z\to 0}\Phi(z)|0\rangle$$

$$\Phi(z) = \sum_{n=-\infty}^{\infty}\frac{\Phi_n}{z^{n+h}}$$

$$\Phi_n|0\rangle = 0 \text{ for } n > -h \text{ and } \Phi_{-h}|0\rangle = |\Phi\rangle.$$

$$L_{-n_1}L_{-n_2}\ldots L_{-n_k}|\Phi\rangle$$

$$(L_0 - 1)|\phi\rangle = 0$$

$$L_n|\phi\rangle = 0 \text{ with } n > 0.$$

$$(L_0 - h)|\Phi\rangle = (\tilde{L}_0 - \tilde{h})|\Phi\rangle = 0$$

$$L_n|\Phi\rangle = \tilde{L}_n|\Phi\rangle = 0 \text{ with } n > 0$$

$$[J_0^A, J_0^B] = if^{AB}{}_C J_0^C$$

$$J^A(z) = \sum_{n=-\infty}^{\infty} \frac{J_n^A}{z^{n+1}} \quad A = 1,2,\dots,\mathrm{dim}G$$

$$J^A(z)J^B(w) \sim \frac{k\delta^{AB}}{2(z-w)^2} + \frac{if_C^{AB}J^C(w)}{z-w} + \cdots$$

$$[J_m^A, J_n^B] = \frac{1}{2}km\delta^{AB}\delta_{m+n,0} + if^{AB}{}_C J_{m+n}^C$$

$$T(z) = \frac{1}{k+\tilde{h}_G}\sum_{A=1}^{\mathrm{dim}G} :J^A(z)J^A(z):,$$

$$f^{BC}{}_D f^{B'D}{}_C = c_A\delta^{BB'}$$

$$c = \frac{k\mathrm{dim}G}{k+\tilde{h}_G}.$$

$$T(z) = T_G(z) - T_H(z)$$

$$T_G(z)J^a(w) \sim \frac{J^a(w)}{(z-w)^2} + \frac{\partial J^a(w)}{z-w} + \cdots$$

$$T_H(z)J^a(w) \sim \frac{J^a(w)}{(z-w)^2} + \frac{\partial J^a(w)}{z-w} + \cdots$$

$$T(z)J^a(w) \sim O(1)$$

$$T(z)T_H(w) \sim O(1)$$

$$[L_m, L_n^H] = [L_m^G - L_m^H, L_n^H] = 0$$

$$[L_m, L_n] = [L_m^G, L_n^G] - [L_m^H, L_n^H] - [L_m^H, L_n] - [L_m, L_n^H],$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3-m)\delta_{m+n,0}$$

$$c = c_G - c_H.$$

$$(\hat{G}_1)_{k_1} \times (\hat{G}_2)_{k_2} \times \ldots \times (\hat{G}_n)_{k_n}$$

$$\frac{\widehat{SU}(2)_k \times \widehat{SU}(2)_l}{\widehat{SU}(2)_{k+l}}$$

$$c = \frac{3k}{k+2} + \frac{3l}{l+2} - \frac{3(k+l)}{k+l+2}.$$

$$c = 1 - \frac{6(p'-p)^2}{pp'}$$

$$c = 1 - \frac{6}{(m+2)(m+3)} \quad m = 1,2,\ldots$$

$$\frac{\widehat{SU}(2)_1 \otimes \widehat{SU}(2)_m}{\widehat{SU}(2)_{m+1}}$$

$$c = 1 + \frac{3m}{m+2} - \frac{3(m+1)}{m+3} = 1 - \frac{6}{(m+2)(m+3)},$$

$$h_{pq} = \frac{[(m+3)p - (m+2)q]^2 - 1}{4(m+2)(m+3)}, 1 \leq p \leq m+1 \text{ and } 1 \leq q \leq p.$$

$$\partial X^\mu(z)\partial X^\nu(w) = -\frac{1}{4}\frac{\eta^{\mu\nu}}{(z-w)^2} + \cdots$$

$$\langle \partial X^\mu(z)\partial X^\nu(w)\rangle = -\frac{1}{4}\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\langle 0|\alpha_m^\mu \alpha_n^\nu|0\rangle z^{-m-1}w^{-n-1}$$

$$-\frac{1}{4}\sum_{m,n=1}^{+\infty}\langle 0|\alpha_m^\mu \alpha_{-n}^\nu|0\rangle z^{-m-1}w^{n-1} = -\frac{\eta^{\mu\nu}}{4}\sum_{m,n=1}^{+\infty} m\delta_{m,n} z^{-m-1}w^{n-1}$$
$$= -\frac{1}{4}\frac{\eta^{\mu\nu}}{(z-w)^2}$$

$$T(z) = \sum_{n=-\infty}^{+\infty}\frac{L_n}{z^{n+2}} \text{ or } L_n = \oint \frac{dz}{2\pi i} z^{n+1}T(z)$$

$$[L_m, L_n] = \left[\oint \frac{dz}{2\pi i} z^{m+1}T(z), \oint \frac{dw}{2\pi i} w^{n+1}T(w)\right].$$

$$\oint \frac{dw}{2\pi i} w^{n+1} \oint_C \frac{dz}{2\pi i} z^{m+1}\left[\frac{c/2}{(z-w)^4} + \frac{2}{(z-w)^2}T(w) + \frac{1}{z-w}\partial T(w) + \cdots\right]$$

$$= \oint \frac{dw}{2\pi i}\left[\frac{c}{12}(m^3 - m)w^{m+n-1} + 2(m+1)w^{n+m+1}T(w) + w^{m+n+2}\partial T(w)\right]$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$

$$\delta_\varepsilon T(z) = -2\partial\varepsilon(z)T(z) - \varepsilon(z)\partial T(z) - \frac{c}{12}\partial^3\varepsilon(z).$$

$$\delta_\varepsilon T(w) = \oint \frac{dz}{2\pi i}\varepsilon(z)[T(z), T(w)] = \oint_C \frac{dz}{2\pi i}\varepsilon(z)T(z)T(w),$$

$$\oint_C \frac{dz}{2\pi i}\varepsilon(z)\left[\frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}\right]$$
$$= 2\partial\varepsilon(w)T(w) + \varepsilon(w)\partial T(w) + \frac{c}{12}\partial^3\varepsilon(w)$$

$$(\partial w)^2 T(w) = T(z) - \frac{c}{12}S(u,z) - \frac{c}{12}(\partial u)^2 S(w,u),$$

$$S(w,z) = S(u,z) + (\partial u)^2 S(w,u)$$

$$\frac{dw}{du} = \left(\frac{du}{dz}\right)^{-1} \frac{dw}{dz} = \frac{w'}{u'}$$

$$\frac{d^2w}{du^2} = \frac{w''u' - w'u''}{(u')^3}$$

$$\frac{d^3w}{du^3} = \frac{w'''(u')^2 - 3w''u''u' - w'u'''u' + 3w'(u'')^2}{(u')^5}$$

$$c(z)b(w) = \frac{1}{z-w} + \cdots \quad \text{and} \quad b(z)c(w) = \frac{\varepsilon}{z-w} + \cdots$$

$$T_{bc}(z) = -\lambda\!: b(z)\partial c(z) + \varepsilon(\lambda - 1)\!: c(z)\partial b(z)\!:.$$

$$c(\varepsilon,\lambda) = -2\varepsilon(6\lambda^2 - 6\lambda + 1)$$

$$S_{\mathrm{q}} = \frac{1}{2\pi}\int \left(2\partial X^\mu \bar{\partial} X_\mu + b\bar{\partial}c + \tilde{b}\partial\tilde{c}\right) d^2z$$

$$T(z) = T_X(z) + T_{bc}(z)$$

$$T_{bc}(z) = -2\!: b(z)\partial c(z)\!: + \!: c(z)\partial b(z)\!:.$$

$$\begin{aligned} \delta X^\mu &= \eta c \partial X^\mu \\ \delta c &= \eta c \partial c \\ \delta b &= \eta T \end{aligned}$$

$$Q_{\mathrm{B}} = \frac{1}{2\pi i}\oint (cT_X + \!: bc\partial c\!:) dz$$

$$\{Q_{\mathrm{B}}, b(z)\} = T(z)$$

$$Q_{\mathrm{B}} = \sum_{m=-\infty}^{\infty} \left(L_{-m}^{(X)} - \delta_{m,0}\right) c_m - \frac{1}{2}\sum_{m,n=-\infty}^{\infty} (m-n)\!: c_{-m}c_{-n}b_{m+n}\!:.$$

$$Q_{\mathrm{B}}^2 = 0$$

$$\{[Q_{\mathrm{B}}, L_m], b_n\} = \{[L_m, b_n], Q_{\mathrm{B}}\} + [\{b_n, Q_{\mathrm{B}}\}, L_m].$$

$$\left[Q_{\mathrm{B}}^2, b_n\right] = [Q_{\mathrm{B}}, \{Q_{\mathrm{B}}, b_n\}] = [Q_{\mathrm{B}}, L_n].$$

$$U = \frac{1}{2\pi i}\oint \,: c(z)b(z)\!: dz = \frac{1}{2}(c_0b_0 - b_0c_0) + \sum_{n=1}^{\infty} (c_{-n}b_n - b_{-n}c_n)$$

$$-\frac{1}{2}(\partial\phi)^2 + \frac{3i}{2}\partial^2\phi = c(z)\partial b(z) - 2b(z)\partial c(z),$$

$$\frac{1}{2}\phi_0^2 + \sum_{n=1}^{\infty} \phi_{-n}\phi_n - 1/8 = \sum_{n=1}^{\infty} n(b_{-n}c_n + c_{-n}b_n).$$

$$\Psi(\phi(\sigma)+2\pi)=-\Psi(\phi(\sigma))$$

$$\delta\mathcal{L}=2\partial\delta X\cdot\bar{\partial}X+2\partial X\cdot\bar{\partial}\delta X+\delta b\bar{\partial}c+b\bar{\partial}\delta c=\delta\mathcal{L}_1+\delta\mathcal{L}_3$$

$$\delta\mathcal{L}_1=2\eta\partial(c\partial X)\cdot\bar{\partial}X+2\eta\partial X\cdot\bar{\partial}(c\partial X)+\eta T_X\bar{\partial}c=2\eta\partial\left(c\partial X^\mu\bar{\partial}X_\mu\right)$$

$$\delta\mathcal{L}_3=\eta T_{bc}\bar{\partial}c-\eta b\bar{\partial}(c\partial c)=-\eta\partial(bc\bar{\partial}c)$$

$$S_g=\frac{1}{4\pi\alpha'}\int_M\sqrt{h}h^{\alpha\beta}g_{\mu\nu}(X)\partial_\alpha X^\mu\partial_\beta X^\nu d^2z$$

$$S_B=\frac{1}{4\pi\alpha'}\int_M\varepsilon^{\alpha\beta}B_{\mu\nu}(X)\partial_\alpha X^\mu\partial_\beta X^\nu d^2z$$

$$S_A=q\int A_\mu\dot{x}^\mu d\tau$$

$$S_\Phi=\frac{1}{4\pi}\int_M\sqrt{h}\Phi(X)R^{(2)}(h)d^2z$$

$$\chi(M)=\frac{1}{4\pi}\int_M\sqrt{h}R^{(2)}(h)d^2z$$

$$\chi(M)=2-2n_{\mathrm{h}}-n_{\mathrm{b}}-n_{\mathrm{c}}$$

$$|\phi\rangle=\prod_i\alpha_{-m_i}^{\mu_i}\prod_j\tilde{\alpha}_{-n_j}^{\nu_j}|0;k\rangle,$$

$$\alpha_{-m}^\mu=\frac{1}{\pi}\oint z^{-m}\partial X^\mu dz$$

$$\alpha_{-m}^\mu\rightarrow\frac{2i}{(m-1)!}\partial^m X^\mu, m>0.$$

$$V_\phi(z,\bar{z})=:\prod_i\partial^{m_i}X^{\mu_i}(z)\prod_j\bar{\partial}^{n_j}X^{\nu_j}(\bar{z})e^{ik\cdot X(z,\bar{z})}:$$

$$\frac{k^2}{8}=1-\sum_i m_i=1-\sum_j n_j.$$

$$b_{-m}\rightarrow\frac{1}{(m-2)!}\partial^{m-1}b, m\geq 2$$

$$c_{-m}\rightarrow\frac{1}{(m+1)!}\partial^{m+1}c, m\geq -1$$

$$V_{\mathrm{t}}(z,\bar{z})=:c(z)\tilde{c}(\bar{z})e^{ik\cdot X(z,\bar{z})}:.$$

$$T(z):e^{ik\cdot X(w,\bar{w})}:=-2:\partial X^\mu(z)\partial X_\mu(z)::e^{ik\cdot X(w,\bar{w})}:.$$

$$\langle \partial X^{\mu}(z) X^{\nu}(w)\rangle = -\frac{1}{4}\frac{\eta^{\mu\nu}}{z-w}$$

$$\partial X^{\mu}(z) \colon e^{ik\cdot X(w,\bar{w})}\colon \sim \langle \partial X^{\mu}(z) ik\cdot X(w)\rangle \colon e^{ik\cdot X(w,\bar{w})}\colon$$
$$\sim -\frac{i}{4}\frac{k^{\mu}}{z-w}\colon e^{ik\cdot X(w,\bar{w})}\colon$$

$$T(z)\colon e^{ik\cdot X(w,\bar{w})}\colon \sim \frac{k^2/8}{(z-w)^2}\colon e^{ik\cdot X(w,\bar{w})}\colon + \cdots$$

$$V = f_{\mu\nu}\colon \partial X^{\mu}(w)\bar{\partial} X^{\nu}(\bar{w}) e^{ik\cdot X(w,\bar{w})}\colon.$$

$$-2 f_{\mu\nu}\colon \partial X^{\rho}(z)\partial X_{\rho}(z)\colon\colon \partial X^{\mu}(w)\bar{\partial} X^{\nu}(\bar{w}) e^{ik\cdot X(w,\bar{w})}\colon.$$

$$\mathcal{K}_3 = -\frac{i}{4} k^{\mu} f_{\mu\nu}\frac{\bar{\partial} X^{\nu}(\bar{w})}{(z-w)^3}$$

$$k^{\mu} f_{\mu\nu} = 0$$

$$\mathcal{K}_2 = \frac{1+k^2/8}{(z-w)^2} V$$

$$S_{\mathrm{WS}} = \int_M \mathcal{L}\left(h_{\alpha\beta}; X^{\mu};\ \text{background fields}\ \right) d^2 z$$

$$Z \sim \int D h_{\alpha\beta} \int DX^{\mu} \cdots e^{-S[h,X,\ldots]}$$

$$h_{\alpha\beta} = e^{\psi}\delta_{\alpha\beta}.$$

$$\psi = 0 \Rightarrow R(h) = 0 \Rightarrow \chi(M) = 0.$$

$$Z = \sum_{n_{\mathrm{h}}=0}^{\infty} Z_{n_{\mathrm{h}}}$$

$$S_{\mathrm{dil}} = \Phi_0 \chi(M) = \Phi_0 (2 - 2n_{\mathrm{h}})$$

$$\exp\left(-S_{\mathrm{dil}}\right) = \exp\left(\Phi_0(2n_{\mathrm{h}} - 2)\right) = g_s^{2n_{\mathrm{h}}-2}$$

$$g_s = e^{\Phi_0}$$

$$\dim_{\mathbb{C}} \mathcal{M}_{n_{\mathrm{h}},N} = 3n_{\mathrm{h}} - 3 + N$$

$$A_N(k_1, k_2, \ldots, k_N) = g_s^{N-2} \int d\mu_N(z) \prod_{i<j} \left|z_i - z_j\right|^{k_i\cdot k_j/2}$$

$$d\mu_N(z) = |(z_A - z_B)(z_B - z_C)(z_C - z_A)|^2$$

$$\times \delta^2(z_A - z_A^0)\delta^2(z_B - z_B^0)\delta^2(z_C - z_C^0)\prod_{i=1}^{N} d^2 z_i$$

$$z \sim z + w_1, z \sim z + w_2$$

$$T^2 = \mathbb{C}/\Lambda_{(w_1,w_2)}$$

$$w_1' = aw_1 + bw_2 \text{ and } w_2' = cw_1 + dw_2$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

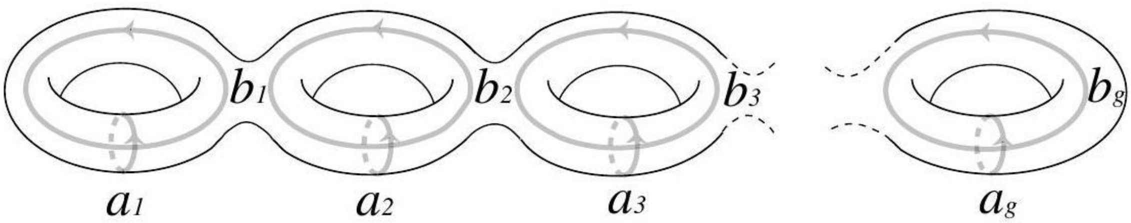
$$\tau' = \frac{\omega_1'}{\omega_2'} = \frac{a\tau + b}{c\tau + d}$$

$$\mathcal{M}_{n_\mathrm{h}=1} = \mathcal{H}/PSL(2,\mathbb{Z})$$

$$|\mathrm{Re}\tau| \leq 1/2, \mathrm{Im}\tau > 0, |\tau| \geq 1,$$

$$\int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2} \int_{T^2} \mu(\tau, z)\langle V_1(0)V_2(z_2) \dots V_N(z_N)\rangle d^2z_2 \dots d^2z_N$$

$$\tau \to \frac{a\tau + b}{c\tau + d}, z_i \to \frac{z_i}{c\tau + d}$$



$$\oint_{a_i} \omega_j = \delta_{ij}$$

$$\oint_{b_i} \omega_j = \Omega_{ij}$$

$$\Omega \to \Omega' = (A\Omega + B)(C\Omega + D)^{-1}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(n_\mathrm{h}, \mathbb{Z})$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} A^T & C^T \\ B^T & D^T \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$d^2\tau \to |c\tau + d|^{-4} d^2\tau \text{ and } \mathrm{Im}\tau \to |c\tau + d|^{-2}\mathrm{Im}\tau,$$

$$\mathcal{I} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2},$$

$$\mathcal{I} = \int_{-1/2}^{+1/2} dx \int_{\sqrt{1-x^2}}^{\infty} \frac{dy}{y^2} = \int_{-1/2}^{+1/2} \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{3}$$

$$\Phi(X^\mu, Y) = kY(z, \bar{z}),$$

$$T(z) = -2\left(\partial X^\mu \partial X_\mu + \partial Y \partial Y\right) + k\partial^2 Y.$$

$$c = \tilde{c} = D + 3k^2.$$

$$k = \sqrt{\frac{26-D}{3}}.$$

$$t''(y) - 2kt'(y) + 8t(y) = 0$$

$$q = q_\pm = k \pm \sqrt{(2-D)/3}$$

$$T(z) = T_0(z) + a_\mu \partial^2 X^\mu(z),$$

$$T(z)T(w) = T_0(z)T_0(w) + a_\mu a_\nu \partial^2 X^\mu(z) \partial^2 X^\nu(w) + \cdots$$

$$T_0(z)T_0(w) = \frac{D/2}{(z-w)^4} \quad \text{and} \quad \partial^2 X^\mu(z)\partial^2 X^\nu(w) = \frac{3}{2}\frac{\eta^{\mu\nu}}{(z-w)^4}$$

$$T(z)T(w) = \frac{(D+3a^2)/2}{(z-w)^4} + \cdots$$

$$c = D + 3k^2.$$

$$A_{ij}(x^\rho) = \sum_{a,\mu} (\lambda^a)_{ij} A_\mu^a(x^\rho) dx^\mu$$

$$A[x^\rho(\sigma), c(\sigma)].$$

- Yang – Mills.

$$\sum_k A_{ik} \wedge B_{kj} = C_{ij}$$

$$A * B = C$$

$$dA = \frac{1}{2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) dx^\mu \wedge dx^\nu$$

$$F = dA + A \wedge A$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \left[A_\mu, A_\nu\right]$$

$$F = Q_{\mathrm{B}} A + A * A$$

$$\delta A = d\Lambda + [A, \Lambda]$$

$$\delta F = [F, \Lambda].$$

$$\delta A = Q_{\mathrm{B}}\Lambda + [A, \Lambda]$$

$$\delta F = [F, \Lambda].$$

$$S \sim \int \ g^{\mu\rho} g^{\nu\lambda} F_{\mu\nu} F_{\rho\lambda}$$

$$\int \ Y = \int \ D^{26} X^\mu(\sigma) D\phi(\sigma) \exp\left(-\frac{3i}{2}\phi(\pi/2)\right) Y[X^\mu(\sigma), \phi(\sigma)] \times \prod_{\sigma<\pi/2} \ \delta^{26}(X^\mu(\sigma) - X^\mu(\pi-\sigma))\delta(\phi(\sigma) - \phi(\pi-\sigma))$$

$$\int \ Q_{\mathrm{B}} Y = 0 \ \text{ and } \int \ [Y_1, Y_2] = 0$$

$$S \sim \int \ \left(A * Q_{\mathrm{B}} A + \frac{2}{3} A * A * A\right)$$

$$T(z) X^\mu(w, \bar{w}) \sim \frac{1}{z-w} \partial X^\mu(w, \bar{w}) + \cdots$$

$$\partial X^\mu(w, \bar{w}) \ \bar{\partial} X^\mu(w, \bar{w}), \partial^2 X^\mu(w, \bar{w}).$$

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\eta^{\mu\nu}\delta_{m+n,0}, [\alpha_m^\mu, \tilde{\alpha}_n^\nu] = 0$$

$$\Phi(z) = \sum_{n=-\infty}^{+\infty} \frac{\Phi_n}{z^{n+h}}$$

$$(L_n - \delta_{n,0})|\phi\rangle = 0, (\tilde{L}_n - \delta_{n,0})|\phi\rangle = 0 \ n \geq 0.$$

- Ramond-Neveu-Schwarz:

$$S = -\frac{1}{2\pi}\int \ d^2\sigma \partial_\alpha X_\mu \partial^\alpha X^\mu$$

$$S = -\frac{1}{2\pi}\int \ d^2\sigma\left(\partial_\alpha X_\mu \partial^\alpha X^\mu + \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu\right)$$

$$\{\rho^\alpha, \rho^\beta\} = 2\eta^{\alpha\beta}$$

$$\rho^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \ \text{ and } \ \rho^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\{\psi^\mu, \psi^\nu\} = 0$$

$$\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix}$$

$$\bar{\psi} = \psi^\dagger \beta, \beta = i\rho^0$$

$$\psi_+^\star = \psi_+ \text{ and } \psi_-^\star = \psi_-$$

$$S_f = \frac{i}{\pi}\int d^2\sigma(\psi_-\partial_+\psi_- + \psi_+\partial_-\psi_+)$$

$$\partial_+\psi_- = 0 \text{ and } \partial_-\psi_+ = 0$$

$$\rho^\alpha\partial_\alpha = \begin{pmatrix} 0 & \partial_1 - \partial_0 \\ \partial_1 + \partial_0 & 0 \end{pmatrix} = 2\begin{pmatrix} 0 & -\partial_- \\ \partial_+ & 0 \end{pmatrix}$$

$$\delta X^\mu = \bar{\varepsilon}\psi^\mu$$
$$\delta\psi^\mu = \rho^\alpha\partial_\alpha X^\mu\varepsilon$$

$$\varepsilon = \begin{pmatrix} \varepsilon_- \\ \varepsilon_+ \end{pmatrix}$$

$$\delta X^\mu = i(\varepsilon_+\psi_-^\mu - \varepsilon_-\psi_+^\mu)$$
$$\delta\psi_-^\mu = -2\partial_- X^\mu\varepsilon_+$$
$$\delta\psi_+^\mu = 2\partial_+ X^\mu\varepsilon_-$$

$$\theta_A = \begin{pmatrix} \theta_- \\ \theta_+ \end{pmatrix}$$

- Grassmann:

$$\{\theta_A, \theta_B\} = 0$$

$$Y^\mu(\sigma^\alpha, \theta) = X^\mu(\sigma^\alpha) + \bar{\theta}\psi^\mu(\sigma^\alpha) + \frac{1}{2}\bar{\theta}\theta B^\mu(\sigma^\alpha)$$

$$Q_A = \frac{\partial}{\partial\bar{\theta}^A} - (\rho^\alpha\theta)_A\partial_\alpha$$

$$\delta\theta^A = [\bar{\varepsilon}Q, \theta^A] = \varepsilon^A,$$
$$\delta\sigma^\alpha = [\bar{\varepsilon}Q, \sigma^\alpha] = -\bar{\varepsilon}\rho^\alpha\theta = \bar{\theta}\rho^\alpha\varepsilon,$$

$$\delta Y^\mu = [\bar{\varepsilon}Q, Y^\mu] = \bar{\varepsilon}QY^\mu$$

$$\theta_A\bar{\theta}_B = -\frac{1}{2}\delta_{AB}\bar{\theta}_C\theta_C$$

$$\delta X^\mu = \bar{\varepsilon}\psi^\mu$$
$$\delta\psi^\mu = \rho^\alpha\partial_\alpha X^\mu\varepsilon + B^\mu\varepsilon$$
$$\delta B^\mu = \bar{\varepsilon}\rho^\alpha\partial_\alpha\psi^\mu$$

$$D_A = \frac{\partial}{\partial\bar{\theta}^A} + (\rho^\alpha\theta)_A\partial_\alpha$$

$$S = \frac{i}{4\pi}\int d^2\sigma d^2\theta\bar{D}Y^\mu DY_\mu$$

$$\delta S = \frac{i}{4\pi}\int d^2\sigma d^2\theta\bar{\varepsilon}Q(\bar{D}Y^\mu DY_\mu)$$

$$\int d\theta(a+\theta b)=b$$

$$\int d^2\theta\bar{\theta}\theta=-2i$$

$$DY^\mu=\psi^\mu+\theta B^\mu+\rho^\alpha\theta\partial_\alpha X^\mu-\frac{1}{2}\bar{\theta}\theta\rho^\alpha\partial_\alpha\psi^\mu,$$
$$\bar{D}Y^\mu=\bar{\psi}^\mu+B^\mu\bar{\theta}-\bar{\theta}\partial_\alpha X^\mu\rho^\alpha+\frac{1}{2}\bar{\theta}\theta\partial_\alpha\bar{\psi}^\mu\rho^\alpha.$$

$$S=-\frac{1}{2\pi}\int d^2\sigma\left(\partial_\alpha X_\mu\partial^\alpha X^\mu+\bar{\psi}^\mu\rho^\alpha\partial_\alpha\psi_\mu-B_\mu B^\mu\right)$$

$$S=\frac{1}{\pi}\int d^2\sigma(2\partial_+X\partial_-X+i\psi_-\partial_+\psi_-+i\psi_+\partial_-\psi_+)$$

$$\delta X^\mu=\bar{\varepsilon}\psi^\mu,\delta\psi^\mu=\rho^\alpha\partial_\alpha X^\mu\varepsilon$$

$$[\delta_{\varepsilon_1},\delta_{\varepsilon_2}]\psi^\mu=\delta_{\varepsilon_1}\left(\delta_{\varepsilon_2}\psi^\mu\right)-\delta_{\varepsilon_2}\left(\delta_{\varepsilon_1}\psi^\mu\right)=\delta_{\varepsilon_1}(\rho^\alpha\partial_\alpha X^\mu\varepsilon_2)-\delta_{\varepsilon_2}(\rho^\alpha\partial_\alpha X^\mu\varepsilon_1)$$
$$=\rho^\alpha\varepsilon_2\partial_\alpha\delta_{\varepsilon_1}X^\mu-\rho^\alpha\varepsilon_1\partial_\alpha\delta_{\varepsilon_2}X^\mu=\rho^\alpha(\varepsilon_2\bar{\varepsilon}_1-\varepsilon_1\bar{\varepsilon}_2)\partial_\alpha\psi^\mu$$

$$-\bar{\varepsilon}_1\rho_\beta\varepsilon_2\rho^\alpha\rho^\beta\partial_\alpha\psi^\mu=-2\bar{\varepsilon}_1\rho^\alpha\varepsilon_2\partial_\alpha\psi^\mu+\bar{\varepsilon}_1\rho_\beta\varepsilon_2\rho^\beta\rho^\alpha\partial_\alpha\psi^\mu$$

$$a^\alpha=-2\bar{\varepsilon}_1\rho^\alpha\varepsilon_2$$

$$[\delta_{\varepsilon_1},\delta_{\varepsilon_2}]X^\mu=\bar{\varepsilon}_2\delta_{\varepsilon_1}\psi^\mu-\bar{\varepsilon}_1\delta_{\varepsilon_2}\psi^\mu=-2\bar{\varepsilon}_1\rho^\alpha\varepsilon_2\partial_\alpha X^\mu$$

$$\delta Y^\mu=[\bar{\varepsilon}Q,Y^\mu(\sigma,\theta)]=\bar{\varepsilon}QY^\mu(\sigma,\theta),$$

$$Q_A=\frac{\partial}{\partial\bar{\theta}^A}-(\rho^\alpha\theta)_A\partial_\alpha$$

$$\delta Y^\mu(\sigma,\theta)=\bar{\varepsilon}^AQ_A\left(X^\mu(\sigma)+\bar{\theta}\psi^\mu(\sigma)+\frac{1}{2}\bar{\theta}\theta B^\mu(\sigma)\right)$$
$$=\bar{\varepsilon}^A\psi^\mu_A(\sigma)-\bar{\varepsilon}^A(\rho^\alpha\theta)_A\partial_\alpha X^\mu(\sigma)+\bar{\varepsilon}^A\theta_AB^\mu(\sigma)-\bar{\varepsilon}^A(\rho^\alpha\theta)_A\bar{\theta}^B\partial_\alpha\psi^\mu_B(\sigma)$$
$$=\bar{\varepsilon}\psi^\mu(\sigma)+\bar{\theta}\rho^\alpha\varepsilon\partial_\alpha X^\mu(\sigma)+\bar{\theta}\varepsilon B^\mu(\sigma)+\frac{1}{2}\bar{\theta}\theta\bar{\varepsilon}\rho^\alpha\partial_\alpha\psi^\mu(\sigma)$$

$$S=\frac{i}{4\pi}\int d^2\sigma d^2\theta\left(-\bar{\psi}^\mu\rho^\alpha\partial_\alpha\psi_\mu\bar{\theta}\theta+B^\mu B_\mu\bar{\theta}\theta-\bar{\theta}\rho^\alpha\partial_\alpha X^\mu\rho^\beta\theta\partial_\beta X_\mu\right)$$

$$\bar{\theta}\rho^\alpha\partial_\alpha X^\mu\rho^\beta\theta\partial_\beta X_\mu=\partial^\alpha X^\mu\partial_\alpha X_\mu\bar{\theta}\theta$$

$$T_{\alpha\beta}=\partial_\alpha X^\mu\partial_\beta X_\mu+\frac{1}{4}\bar{\psi}^\mu\rho_\alpha\partial_\beta\psi_\mu+\frac{1}{4}\bar{\psi}^\mu\rho_\beta\partial_\alpha\psi_\mu-(\text{ trace })$$

$$\delta S\sim\int d^2\sigma(\partial_\alpha\bar{\varepsilon})J^\alpha$$

$$J^\alpha_A=-\frac{1}{2}\left(\rho^\beta\rho^\alpha\psi_\mu\right)_A\partial_\beta X^\mu$$

$$(\rho_\alpha)_{AB} J_B^\alpha = 0$$

$$T_{++} = \partial_+ X_\mu \partial_+ X^\mu + \frac{i}{2}\psi_+^\mu \partial_+ \psi_{+\mu}$$
$$T_{--} = \partial_- X_\mu \partial_- X^\mu + \frac{i}{2}\psi_-^\mu \partial_- \psi_{-\mu}$$

$$J_+ = \psi_+^\mu \partial_+ X_\mu \text{ and } J_- = \psi_-^\mu \partial_- X_\mu$$

$$\partial_- J_+ = \partial_+ J_- = 0$$

$$\partial_- T_{++} = \partial_+ T_{--} = 0$$

$$\{\psi_A^\mu(\sigma,\tau), \psi_B^\nu(\sigma',\tau)\} = \pi \eta^{\mu\nu} \delta_{AB} \delta(\sigma - \sigma')$$

$$J_+ = J_- = T_{++} = T_{--} = 0$$

$$S = \frac{1}{\pi} \int d^2\sigma (2\partial_+ X \cdot \partial_- X + i\psi_- \cdot \partial_+ \psi_- + i\psi_+ \cdot \partial_- \psi_+)$$

$$\delta_+(2\partial_+ X \cdot \partial_- X + i\psi_- \cdot \partial_+ \psi_- + i\psi_+ \cdot \partial_- \psi_+)$$
$$= a^+\big(-2\partial_-(\partial_+ X \cdot \partial_+ X) + i\partial_+(\psi_+ \cdot \partial_- \psi_+) - i\partial_-(\psi_+ \cdot \partial_+ \psi_+)\big)$$

$$-2a^+(\partial_- T_{++} + \partial_+ T_{-+})$$

$$T_{++} = \partial_+ X \cdot \partial_+ X + \frac{i}{2}\psi_+ \cdot \partial_+ \psi_+.$$

$$T_{--} = \partial_- X \cdot \partial_- X + \frac{i}{2}\psi_- \cdot \partial_- \psi_-.$$

$$\delta_- X^\mu = i\varepsilon_- \psi_+^\mu$$
$$\delta_- \psi_+^\mu = -2\partial_+ X^\mu \varepsilon_- \text{ and } \delta_- \psi_-^\mu = 0.$$

$$\delta_-(2\partial_+ X \cdot \partial_- X + i\psi_- \cdot \partial_+ \psi_- + i\psi_+ \cdot \partial_- \psi_+) = -4i\varepsilon_- \partial_-(\psi_+ \cdot \partial_+ X)$$

$$S_f \sim \int d^2\sigma (\psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+)$$

$$\delta S \sim \int d\tau (\psi_+ \delta\psi_+ - \psi_- \delta\psi_-)\Big|_{\sigma=\pi} - (\psi_+ \delta\psi_+ - \psi_- \delta\psi_-)|_{\sigma=0}$$

$$\psi_+^\mu = \pm \psi_-^\mu$$

$$\psi_+^\mu\big|_{\sigma=0} = \psi_-^\mu|_{\sigma=0}.$$

$$\psi_+^\mu\big|_{\sigma=\pi} = \psi_-^\mu|_{\sigma=\pi}$$

$$\psi_-^\mu(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{n\in\mathbb{Z}} d_n^\mu e^{-in(\tau-\sigma)},$$
$$\psi_+^\mu(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{n\in\mathbb{Z}} d_n^\mu e^{-in(\tau+\sigma)}.$$

- Majorana:

$$\psi_+^\mu\big|_{\sigma=\pi} = -\psi_-^\mu|_{\sigma=\pi}$$

$$\psi_-^\mu(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{r\in\mathbb{Z}+1/2} b_r^\mu e^{-ir(\tau-\sigma)}$$
$$\psi_+^\mu(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{r\in\mathbb{Z}+1/2} b_r^\mu e^{-ir(\tau+\sigma)}$$

$$m,n\in\mathbb{Z} \text{ while } r,s\in\mathbb{Z}+\frac{1}{2}$$

$$\psi_\pm(\sigma) = \pm\psi_\pm(\sigma+\pi)$$

$$\psi_-^\mu(\sigma,\tau) = \sum_{n\in\mathbb{Z}} d_n^\mu e^{-2in(\tau-\sigma)} \text{ or } \psi_-^\mu(\sigma,\tau) = \sum_{r\in\mathbb{Z}+1/2} b_r^\mu e^{-2ir(\tau-\sigma)}$$

$$\psi_+^\mu(\sigma,\tau) = \sum_{n\in\mathbb{Z}} \tilde{d}_n^\mu e^{-2in(\tau+\sigma)} \text{ or } \psi_+^\mu(\sigma,\tau) = \sum_{r\in\mathbb{Z}+1/2} \tilde{b}_r^\mu e^{-2ir(\tau+\sigma)}$$

$$[\alpha_m^\mu,\alpha_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu}$$

$$\{b_r^\mu,b_s^\nu\} = \eta^{\mu\nu}\delta_{r+s,0} \text{ and } \{d_m^\mu,d_n^\nu\} = \eta^{\mu\nu}\delta_{m+n,0}$$

$$\alpha_m^\mu|0\rangle_{\mathrm{R}} = d_m^\mu|0\rangle_{\mathrm{R}} = 0 \text{ for } m>0$$

$$\alpha_m^\mu|0\rangle_{\mathrm{NS}} = b_r^\mu|0\rangle_{\mathrm{NS}} = 0 \text{ for } m,r>0.$$

$$\{d_0^\mu,d_0^\nu\} = \eta^{\mu\nu}$$

$$\{\Gamma^\mu,\Gamma^\nu\} = 2\eta^{\mu\nu}.$$

$$d_0^\mu|a\rangle = \frac{1}{\sqrt{2}}\Gamma_{ba}^\mu|b\rangle.$$

$$L_m = \frac{1}{\pi}\int_{-\pi}^{\pi} d\sigma e^{im\sigma}T_{++} = L_m^{(\mathrm{b})} + L_m^{(\mathrm{f})}$$

$$L_m^{(\mathrm{b})} = \frac{1}{2}\sum_{n\in\mathbb{Z}} :\alpha_{-n}\cdot\alpha_{m+n}: m\in\mathbb{Z}$$

$$L_m^{(\mathrm{f})} = \frac{1}{2}\sum_{r\in\mathbb{Z}+1/2}\left(r+\frac{m}{2}\right):b_{-r}\cdot b_{m+r}: m\in\mathbb{Z}$$

$$G_r = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} d\sigma e^{ir\sigma} J_+ = \sum_{n\in\mathbb{Z}} \alpha_{-n}\cdot b_{r+n}\, r\in\mathbb{Z}+\frac{1}{2}$$

$$L_0 = \frac{1}{2}\alpha_0^2 + N$$

$$N = \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n + \sum_{r=1/2}^{\infty} rb_{-r}\cdot b_r$$

$$L_m^{(f)} = \frac{1}{2}\sum_{n\in\mathbb{Z}} \left(n+\frac{m}{2}\right): d_{-n}\cdot d_{m+n}: m\in\mathbb{Z}$$

$$F_m = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} d\sigma e^{im\sigma} J_+ = \sum_{n\in\mathbb{Z}} \alpha_{-n}\cdot d_{m+n}\, m\in\mathbb{Z}$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{8}m^3\delta_{m+n,0}$$
$$[L_m, F_n] = \left(\frac{m}{2}-n\right)F_{m+n}$$
$$\{F_m, F_n\} = 2L_{m+n} + \frac{D}{2}m^2\delta_{m+n,0}$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{8}m(m^2-1)\delta_{m+n,0}$$
$$[L_m, G_r] = \left(\frac{m}{2}-r\right)G_{m+r}$$
$$\{G_r, G_s\} = 2L_{r+s} + \frac{D}{2}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0}$$

$$G_r|\phi\rangle = 0\ r>0,$$
$$L_m|\phi\rangle = 0\ m>0,$$
$$(L_0 - a_{\mathrm{NS}})|\phi\rangle = 0.$$

$$F_n|\phi\rangle = 0\ n\geq 0,$$
$$L_m|\phi\rangle = 0\ m>0,$$
$$(L_0 - a_{\mathrm{R}})|\phi\rangle = 0.$$

$$\left(p\cdot\Gamma + \frac{2\sqrt{2}}{l_s}\sum_{n=1}^{\infty}(\alpha_{-n}\cdot d_n + d_{-n}\cdot\alpha_n)\right)|\phi\rangle = 0$$

$$\alpha_0^\mu = \frac{1}{2}l_s p^\mu, d_0^\mu = \frac{1}{\sqrt{2}}\Gamma^\mu$$

$$F_0 = \sum_{n=-\infty}^{\infty} \alpha_{-n}\cdot d_n = \alpha_0\cdot d_0 + \sum_{n=1}^{\infty}(\alpha_{-n}\cdot d_n + d_{-n}\cdot\alpha_n)$$

$$a_{\mathrm{NS}} = \frac{1}{2}\ \text{ and }\ a_{\mathrm{R}} = 0.$$

$$|\psi\rangle = G_{-1/2}|\chi\rangle,$$

$$G_{1/2}|\chi\rangle = G_{3/2}|\chi\rangle = \left(L_0 - a_{\mathrm{NS}} + \frac{1}{2}\right)|\chi\rangle = 0.$$

$$G_{1/2}|\psi\rangle = G_{1/2}G_{-1/2}|\chi\rangle = \left(2L_0 - G_{-1/2}G_{1/2}\right)|\chi\rangle = (2a_{\mathrm{NS}} - 1)|\chi\rangle.$$

$$\langle\alpha \mid \psi\rangle = \langle\alpha|G_{-1/2}|\chi\rangle = \langle\chi|G_{1/2}|\alpha\rangle^\star = 0,$$

$$|\psi\rangle = \left(G_{-3/2} + \lambda G_{-1/2}L_{-1}\right)|\chi\rangle.$$

$$G_{1/2}|\chi\rangle = G_{3/2}|\chi\rangle = (L_0 + 1)|\chi\rangle = 0.$$

$$\begin{aligned} G_{1/2}|\psi\rangle &= (2-\lambda)L_{-1}|\chi\rangle \\ G_{3/2}|\psi\rangle &= (D-2-4\lambda)|\chi\rangle \end{aligned}$$

$$|\psi\rangle = F_0 F_{-1}|\chi\rangle,$$

$$F_1|\chi\rangle = (L_0 + 1)|\chi\rangle = 0$$

$$L_1|\psi\rangle = \left(\frac{1}{2}F_1 + F_0 L_1\right)F_{-1}|\chi\rangle = \frac{1}{4}(D-10)|\chi\rangle.$$

$$X^+(\sigma,\tau) = x^+ + p^+\tau$$

$$\psi^+(\sigma,\tau) = 0$$

$$\alpha' M^2 = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{r=1/2}^{\infty} r b_{-r}^i b_r^i - \frac{1}{2}.$$

$$\alpha_n^i|0;k\rangle_{\mathrm{NS}} = b_r^i|0;k\rangle_{\mathrm{NS}} = 0 \ \text{ for } \ n,r>0$$

$$\alpha_0^\mu|0;k\rangle_{\mathrm{NS}} = \sqrt{2\alpha'}k^\mu|0;k\rangle_{\mathrm{NS}}.$$

$$\alpha' M^2 = -\frac{1}{2}.$$

$$b_{-1/2}^i|0;k\rangle_{\mathrm{NS}}.$$

$$\alpha' M^2 = \frac{1}{2} - a_{\mathrm{NS}}$$

$$\alpha' M^2 = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{n=1}^{\infty} n d_{-n}^i d_n^i$$

$$\alpha_n^i|0;k\rangle_{\mathrm{R}} = d_n^i|0;k\rangle_{\mathrm{R}} = 0 \ \text{ for } \ n>0$$

$$G = (-1)^{F+1} = (-1)^{\sum_{r=1/2}^{\infty} b_{-r}^i b_r^i + 1} \ (\mathrm{NS})$$

$$G = \Gamma_{11}(-1)^{\sum_{n=1}^{\infty} d_{-n}^i d_n^i} \ (\mathrm{R})$$

$$\Gamma_{11} = \Gamma_0\Gamma_1 \dots \Gamma_9$$

$$(\Gamma_{11})^2 = 1 \text{ and } \{\Gamma_{11}, \Gamma^\mu\} = 0$$

$$\Gamma_{11}\psi = \pm\psi$$

$$P_\pm = \frac{1}{2}(1 \pm \Gamma_{11})$$

$$(-1)^{F_{\mathrm{NS}}} = -1,$$

$$G|0\rangle_{\mathrm{NS}} = -|0\rangle_{\mathrm{NS}}.$$

$$\alpha_{-1}^i b_{-1/2}^j|0\rangle, b_{-1/2}^i b_{-1/2}^j b_{-1/2}^k|0\rangle, b_{-3/2}^i|0\rangle,$$

$$\alpha_{-1}^i|\psi_0\rangle, d_{-1}^i|\psi_0'\rangle$$

$$f_{\mathrm{NS}}(w) = \sum_{n=0}^{\infty} d_{\mathrm{NS}}(n)w^n \text{ and } f_{\mathrm{R}}(w) = \sum_{n=0}^{\infty} d_{\mathrm{R}}(n)w^n.$$

$$\mathrm{tr} w^{a^\dagger a} = 1 + w + w^2 + \cdots = \frac{1}{1-w}$$

$$\mathrm{tr} w^{b^\dagger b} = 1 + w.$$

$$\mathrm{tr} w^{N-1/2} = \frac{1}{\sqrt{w}} \prod_{m=1}^{\infty} \left(\frac{1 + w^{m-1/2}}{1 - w^m}\right)^8.$$

$$f_{\mathrm{NS}}(w) = \frac{1}{2\sqrt{w}} \left[\prod_{m=1}^{\infty} \left(\frac{1 + w^{m-1/2}}{1 - w^m}\right)^8 - \prod_{m=1}^{\infty} \left(\frac{1 - w^{m-1/2}}{1 - w^m}\right)^8\right].$$

$$f_{\mathrm{R}}(w) = 8 \prod_{m=1}^{\infty} \left(\frac{1 + w^m}{1 - w^m}\right)^8$$

$$S = \frac{1}{\pi} \int \partial X^\mu \bar{\partial} X_\mu d^2 z$$

$$T = -2{:}\partial X^\mu \partial X_\mu{:} = \sum_{n=-\infty}^{\infty} \frac{L_n}{z^{n+2}}$$

$$T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{2}{(z-w)^2} T(w) + \frac{1}{z-w} \partial T(w)$$

$$S_{\mathrm{matter}} = \frac{1}{2\pi} \int \left(2\partial X^\mu \bar{\partial} X_\mu + \frac{1}{2}\psi^\mu \bar{\partial}\psi_\mu + \frac{1}{2}\tilde{\psi}^\mu \partial \tilde{\psi}_\mu\right) d^2 z$$

$$T_{\mathrm{B}}(z) = -2\partial X^\mu(z)\partial X_\mu(z) - \frac{1}{2}\psi^\mu(z)\partial\psi_\mu(z) = \sum_{n=-\infty}^{\infty} \frac{L_n}{z^{n+2}}$$

$$\psi^{\mu}(z)\psi^{\nu}(w) \sim \frac{\eta^{\mu\nu}}{z-w}$$

$$T_{\mathrm{F}}(z) = 2i\psi^{\mu}(z)\partial X_{\mu}(z) = \sum_{r=-\infty}^{\infty} \frac{G_r}{z^{r+3/2}}$$

$$T_{\mathrm{F}}(z)T_{\mathrm{F}}(w) \sim \frac{\hat{c}}{4(z-w)^3} + \frac{T_{\mathrm{B}}(w)}{2(z-w)} + \cdots$$

$$T(z,\theta) = T_{\mathrm{F}}(z) + \theta T_{\mathrm{B}}(z)$$

$$T(z_1,\theta_1)T(z_2,\theta_2) \sim \frac{\hat{c}}{4z_{12}^3} + \frac{3\theta_{12}}{2z_{12}^2}T(z_2,\theta_2) + \frac{D_2T(z_2,\theta_2)}{2z_{12}} + \frac{\theta_{12}}{z_{12}}\partial_2 T(z_2,\theta_2) + \cdots$$

$$D = \frac{\partial}{\partial\theta} + \theta\frac{\partial}{\partial_z}$$

$$T(z_1,\theta_1)\Phi(z_2,\theta_2) \sim h\frac{\theta_{12}}{z_{12}^2}\Phi(z_2,\theta_2) + \frac{1}{2z_{12}}D_2\Phi + \frac{\theta_{12}}{z_{12}}\partial_2\Phi + \cdots$$

$$\gamma(z)\beta(w) \sim \frac{1}{z-w}$$

$$\beta(z)\gamma(w) \sim -\frac{1}{z-w}$$

$$S_{\text{ghost}} = \frac{1}{2\pi}\int (b\bar{\partial}c + \bar{b}\partial\bar{c} + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma})d^2z$$

$$T_{\mathrm{B}}^{\text{ghost}} = -2b\partial c + c\partial b - \frac{3}{2}\beta\partial\gamma - \frac{1}{2}\gamma\partial\beta$$
$$T_{\mathrm{F}}^{\text{ghost}} = -2b\gamma + c\partial\beta + \frac{3}{2}\beta\partial c$$

$$\delta X^{\mu} = \eta\left(c\partial X^{\mu} - \frac{i}{2}\gamma\psi^{\mu}\right)$$
$$\delta\psi^{\mu} = \eta\left(c\partial\psi^{\mu} - \frac{1}{2}\psi^{\mu}\partial c + 2i\gamma\partial X^{\mu}\right)$$
$$\delta c = \eta(c\partial c - \gamma^2)$$
$$\delta b = \eta T_{\mathrm{B}}$$
$$\delta\gamma = \eta\left(c\partial\gamma - \frac{1}{2}\gamma\partial c\right)$$
$$\delta\beta = \eta T_{\mathrm{F}}$$

$$Q_{\mathrm{B}} = \frac{1}{2\pi i}\oint \left(cT_{\mathrm{B}}^{\text{matter}} + \gamma T_{\mathrm{F}}^{\text{matter}} + bc\partial c - \frac{1}{2}c\gamma\partial\beta - \frac{3}{2}c\beta\partial\gamma - b\gamma^2\right)dz$$

$$\{Q_{\mathrm{B}}, b(z)\} = T_{\mathrm{B}}(z)$$

$$[Q_{\mathrm{B}}, \beta(z)] = T_{\mathrm{F}}(z).$$

$$S_0 = \int d\tau \left(\frac{1}{2}\dot{X}^\mu \dot{X}_\mu - i\psi^\mu \dot{\psi}_\mu\right)$$

$$\delta X^\mu = i\varepsilon\psi^\mu, \delta\psi^\mu = \frac{1}{2}\varepsilon\dot{X}^\mu,$$

$$\tilde{S}_0 = \int d\tau \left(\frac{\dot{X}^\mu \dot{X}_\mu}{2e} + \frac{i\dot{X}^\mu \psi_\mu \chi}{e} - i\psi^\mu \dot{\psi}_\mu\right)$$

$$\delta X^\mu = \xi\dot{X}^\mu, \quad \delta\psi^\mu = \xi\dot{\psi}^\mu$$
$$\delta e = \frac{d}{d\tau}(\xi e), \quad \delta\chi = \frac{d}{d\tau}(\xi\chi)$$

$$\delta X^\mu = i\varepsilon\psi^\mu, \delta\psi^\mu = \frac{1}{2e}(\dot{X}^\mu - i\chi\psi^\mu)\varepsilon,$$
$$\delta\chi = \dot{\varepsilon}, \delta e = -i\chi\varepsilon.$$

$$[X^\mu, \dot{X}^\nu] = i\eta^{\mu\nu} \text{ and } \{\psi^\mu, \psi^\nu\} = \eta^{\mu\nu}$$

$$\psi_A \bar{\chi}_B = -\frac{1}{2}(\bar{\chi}\psi\delta_{AB} + \bar{\chi}\rho_\alpha\psi\rho^\alpha_{AB} + \bar{\chi}\rho_3\psi(\rho_3)_{AB}),$$

$$X^\mu(\sigma,\tau) \text{ and } \Theta^a(\sigma,\tau).$$

$$S = -m\int \sqrt{-\dot{X}_\mu \dot{X}^\mu}\, d\tau.$$

$$\delta\Theta^{Aa} = \varepsilon^{Aa},$$
$$\delta X^\mu = \bar{\varepsilon}^A \Gamma^\mu \Theta^A.$$

$$[\delta_1, \delta_2]\Theta^A = 0 \text{ and } [\delta_1, \delta_2]X^\mu = -2\bar{\varepsilon}_1^A \Gamma^\mu \varepsilon_2^A = a^\mu.$$

$$\Pi_0^\mu = \dot{X}^\mu - \bar{\Theta}^A \Gamma^\mu \dot{\Theta}^A$$

$$\Pi_\alpha^\mu = \partial_\alpha X^\mu - \bar{\Theta}^A \Gamma^\mu \partial_\alpha \Theta^A, \alpha = 0,1,\ldots,p$$

$$\dot{X}^\mu \to \Pi_0^\mu$$

$$S_1 = -m\int \sqrt{-\Pi_0 \cdot \Pi_0}\, d\tau$$

$$\Theta = \Theta^1 + \Theta^2$$

$$\Theta^1 = \frac{1}{2}(1 + \Gamma_{11})\Theta \text{ and } \Theta^2 = \frac{1}{2}(1 - \Gamma_{11})\Theta$$

$$\Gamma_{11} = \Gamma_0 \Gamma_1 \ldots \Gamma_9$$

$$\Pi_0^\mu = \dot{X}^\mu - \bar{\Theta}\Gamma^\mu \dot{\Theta}$$

$$P_\mu = \frac{\delta S_1}{\delta \dot{X}^\mu} = \frac{m}{\sqrt{-\Pi_0 \cdot \Pi_0}}(\dot{X}_\mu - \bar{\Theta}\Gamma_\mu \dot{\Theta}).$$

$$\dot{P}_\mu = 0.$$

$$P^2 = -m^2.$$

$$P \cdot \Gamma\dot{\Theta} = 0$$

$$(P \cdot \Gamma + m\Gamma_{11})\dot{\Theta} = 0$$

$$(P \cdot \Gamma + m\Gamma_{11})^2 = (P \cdot \Gamma)^2 + m\{P \cdot \Gamma, \Gamma_{11}\} + (m\Gamma_{11})^2 = P^2 + m^2 = 0.$$

$$S_2 = -m\int \bar{\Theta}\Gamma_{11}\dot{\Theta}d\tau$$

$$S = S_1 + S_2 = -m\int \sqrt{-\Pi_0 \cdot \Pi_0}d\tau - m\int \bar{\Theta}\Gamma_{11}\dot{\Theta}d\tau$$

$$\delta X^\mu = \bar{\Theta}\Gamma^\mu\delta\Theta = -\delta\bar{\Theta}\Gamma^\mu\Theta$$

$$\delta\Pi_0^\mu = -2\delta\bar{\Theta}\Gamma^\mu\dot{\Theta}$$

$$\delta S_1 = m\int \frac{\Pi_0 \cdot \delta\Pi_0}{\sqrt{-\Pi_0^2}}d\tau$$

$$\delta S_1 = -2m\int \frac{\Pi_0^\mu\delta\bar{\Theta}\Gamma_\mu\dot{\Theta}}{\sqrt{-\Pi_0^2}}d\tau = -2m\int \delta\bar{\Theta}\gamma\Gamma_{11}\dot{\Theta}d\tau$$

$$\gamma = \frac{\Gamma \cdot \Pi_0}{\sqrt{-\Pi_0^2}}\Gamma_{11}$$

$$\gamma^2 = \frac{(\Gamma \cdot \Pi_0)^2}{\Pi_0^2} = 1$$

$$P_\pm = \frac{1}{2}(1 \pm \gamma)$$

$$\delta S_2 = -2m\int \delta\bar{\Theta}\Gamma_{11}\dot{\Theta}d\tau$$

$$\delta(S_1 + S_2) = -2m\int \delta\bar{\Theta}(1+\gamma)\Gamma_{11}\dot{\Theta}d\tau = -4m\int \delta\bar{\Theta}P_+\Gamma_{11}\dot{\Theta}d\tau$$

$$\delta\bar{\Theta} = \bar{\kappa}P_-,$$

$$\delta\bar{\Theta} = \bar{\kappa}P_- \text{ and } \delta X^\mu = -\bar{\kappa}P_-\Gamma^\mu\Theta.$$

$$\bar{\Theta}_1\Gamma_\mu\Theta_2 = -\bar{\Theta}_2\Gamma_\mu\Theta_1$$

$$\mathcal{C}\Gamma_\mu\mathcal{C}^{-1} = -\Gamma_\mu^T$$

$$\bar{\Theta}_1\Gamma_\mu\Theta_2 = \Theta_1^\dagger\Gamma_0\Gamma_\mu\Theta_2 = \Theta_1^T\mathcal{C}\Gamma_\mu\Theta_2$$

$$-\Theta_2^T \Gamma_\mu^T \mathcal{C}^T \Theta_1 = -\Theta_2^T \mathcal{C} \Gamma_\mu \Theta_1 = -\bar{\Theta}_2 \Gamma_\mu \Theta_1$$

$$\bar{\Theta}_1 \Gamma_{\mu_1 \cdots \mu_n} \Theta_2 = (-1)^{n(n+1)/2} \bar{\Theta}_2 \Gamma_{\mu_1 \cdots \mu_n} \Theta_1,$$

$$\Gamma_{\mu_1 \mu_2 \cdots \mu_n} = \Gamma_{[\mu_1} \Gamma_{\mu_2} \cdots \Gamma_{\mu_n]}$$

$$\delta_1 \delta_2 X^\mu = \delta_1 \left(\bar{\varepsilon}_2^A \Gamma^\mu \Theta^A \right) = \bar{\varepsilon}_2^A \Gamma^\mu \varepsilon_1^A.$$

$$[\delta_1, \delta_2] X^\mu = \bar{\varepsilon}_2^A \Gamma^\mu \varepsilon_1^A - \bar{\varepsilon}_1^A \Gamma^\mu \varepsilon_2^A = -2 \bar{\varepsilon}_1^A \Gamma^\mu \varepsilon_2^A,$$

$$\delta\left(\dot{X}^\mu - \bar{\Theta}^A \Gamma^\mu \dot{\Theta}^A\right) = \frac{d}{d\tau}(\bar{\varepsilon}^A \Gamma^\mu \Theta^A) - \bar{\varepsilon}^A \Gamma^\mu \dot{\Theta}^A - \bar{\Theta}^A \Gamma^\mu \dot{\varepsilon}^A$$
$$= \bar{\varepsilon}^A \Gamma^\mu \dot{\Theta}^A - \bar{\varepsilon}^A \Gamma^\mu \dot{\Theta}^A = 0$$

$$P^\mu = \frac{\delta \mathcal{L}}{\delta \dot{X}_\mu} = m \frac{\Pi^\mu}{\sqrt{-\Pi^2}}$$

$$\dot{P}^\mu = 0$$

$$\frac{d}{d\tau} \frac{\delta \mathcal{L}}{\delta \dot{\bar{\Theta}}^A} - \frac{\delta \mathcal{L}}{\delta \bar{\Theta}^A} = 0$$

$$\frac{d}{d\tau}\left(P_\mu \Gamma^\mu \Theta^A\right) + P_\mu \Gamma^\mu \dot{\Theta}^A = 0$$

$$P \cdot \Gamma \dot{\Theta}^A = 0$$

$$\Gamma_{11} \Theta^A = (-1)^{A+1} \Theta^A$$
$$\Gamma_{11} \Theta^A = \Theta^A$$

$$S_{\mathrm{NG}} = -\frac{1}{\pi} \int d^2\sigma \sqrt{-\det\left(\partial_\alpha X^\mu \partial_\beta X_\mu\right)}$$

$$S_1 = -\frac{1}{\pi} \int d^2\sigma \sqrt{-G}$$

$$\Pi_\alpha^\mu = \partial_\alpha X^\mu - \bar{\Theta}^A \Gamma^\mu \partial_\alpha \Theta^A$$

$$\delta X^\mu = \bar{\Theta}^A \Gamma^\mu \delta \Theta^A = -\delta \bar{\Theta}^A \Gamma^\mu \Theta^A$$

$$\delta \Pi_\alpha^\mu = -2 \delta \bar{\Theta}^A \Gamma^\mu \partial_\alpha \Theta^A$$

$$\delta S_1 = \frac{2}{\pi} \int d^2\sigma \sqrt{-G} G^{\alpha\beta} \Pi_\alpha^\mu \delta \bar{\Theta}^A \Gamma_\mu \partial_\beta \Theta^A$$

$$S_2 = \int \Omega_2 = \frac{1}{2} \int d^2\sigma \epsilon^{\alpha\beta} \Omega_{\alpha\beta}$$

$$\int_D \Omega_3 = \int_M \Omega_2$$

$$\Gamma^\mu d\Theta d\bar{\Theta} \Gamma_\mu d\Theta = 0$$

$$\Omega_3 = c\left(d\bar{\Theta}^1\Gamma_\mu d\Theta^1 - d\bar{\Theta}^2\Gamma_\mu d\Theta^2\right)\Pi^\mu$$

$$d\Omega_3 = c\left(d\bar{\Theta}^1\Gamma_\mu d\Theta^1 - d\bar{\Theta}^2\Gamma_\mu d\Theta^2\right)d\Pi^\mu$$

$$\delta\Omega_3 = 2c\left(d\delta\bar{\Theta}^1\Gamma_\mu d\Theta^1 - d\delta\bar{\Theta}^2\Gamma_\mu d\Theta^2\right)\Pi^\mu$$
$$-2c\left(d\bar{\Theta}^1\Gamma_\mu d\Theta^1 - d\bar{\Theta}^2\Gamma_\mu d\Theta^2\right)\delta\bar{\Theta}^A\Gamma^\mu d\Theta^A$$

$$-2c\left(\delta\bar{\Theta}^1\Gamma_\mu d\Theta^1 - \delta\bar{\Theta}^2\Gamma_\mu d\Theta^2\right)d\Pi^\mu$$

$$\delta\Omega_3 = d\left[2c\left(\delta\bar{\Theta}^1\Gamma_\mu d\Theta^1 - \delta\bar{\Theta}^2\Gamma_\mu d\Theta^2\right)\Pi^\mu\right]$$

$$\delta\Omega_2 = 2c\left(\delta\bar{\Theta}^1\Gamma_\mu d\Theta^1 - \delta\bar{\Theta}^2\Gamma_\mu d\Theta^2\right)\Pi^\mu$$

$$\delta S_2 = \frac{2}{\pi}\int d^2\sigma\varepsilon^{\alpha\beta}\left(\delta\bar{\Theta}^1\Gamma_\mu\partial_\alpha\Theta^1 - \delta\bar{\Theta}^2\Gamma_\mu\partial_\alpha\Theta^2\right)\Pi_\beta^\mu$$

$$\delta S = \frac{4}{\pi}\int d^2\sigma\varepsilon^{\alpha\beta}\left(\delta\bar{\Theta}^1 P_+\Gamma_\mu\partial_\alpha\Theta^1 - \delta\bar{\Theta}^2 P_-\Gamma_\mu\partial_\alpha\Theta^2\right)\Pi_\beta^\mu$$

$$P_\pm = \frac{1}{2}(1\pm\gamma)$$

$$\gamma = -\frac{\varepsilon^{\alpha\beta}\Pi_\alpha^\mu\Pi_\beta^\nu\Gamma_{\mu\nu}}{2\sqrt{-G}}$$

$$\delta\bar{\Theta}^1 = \bar{\kappa}^1 P_- \text{ and } \delta\bar{\Theta}^2 = \bar{\kappa}^2 P_+$$

$$\Omega_2 = c\left(\bar{\Theta}^1\Gamma_\mu d\Theta^1 - \bar{\Theta}^2\Gamma_\mu d\Theta^2\right)dX^\mu - c\bar{\Theta}^1\Gamma_\mu d\Theta^1\bar{\Theta}^2\Gamma^\mu d\Theta^2$$

$$S = S_1 + S_2$$

$$\gamma^2 = -\frac{1}{4G}\left(\varepsilon^{\alpha\beta}\Pi_\alpha^\mu\Pi_\beta^\nu\Gamma_{\mu\nu}\right)^2 = -\frac{1}{8G}\varepsilon^{\alpha_1\beta_1}\varepsilon^{\alpha_2\beta_2}\Pi_{\alpha_1}^{\mu_1}\Pi_{\beta_1}^{\nu_1}\Pi_{\alpha_2}^{\mu_2}\Pi_{\beta_2}^{\nu_2}\{\Gamma_{\mu_1\nu_1},\Gamma_{\mu_2\nu_2}\}$$

$$\{\Gamma_{\mu_1\nu_1},\Gamma_{\mu_2\nu_2}\} = -2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} + 2\eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} + 2\Gamma_{\mu_1\nu_1\mu_2\nu_2}$$

$$\gamma^2 = \frac{1}{4G}\varepsilon^{\alpha_1\beta_1}\varepsilon^{\alpha_2\beta_2}\left(G_{\alpha_1\alpha_2}G_{\beta_1\beta_2} - G_{\alpha_1\beta_2}G_{\beta_1\alpha_2}\right) = 1$$

$$h_{\alpha\beta} = e^\phi\eta_{\alpha\beta}$$

$$X^+ = x^+ + p^+\tau$$

$$\Gamma^+\Theta^A = 0, \text{ where } \Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^9).$$

$$S = S_1 + S_2$$

$$S_1 = -\frac{1}{2\pi}\int d^2\sigma\sqrt{-h}h^{\alpha\beta}\Pi_\alpha\cdot\Pi_\beta$$

$$S_2 = \frac{1}{\pi}\int d^2\sigma \varepsilon^{\alpha\beta}\left[-\partial_\alpha X^\mu\left(\bar{\Theta}^1\Gamma_\mu\partial_\beta\Theta^1 - \bar{\Theta}^2\Gamma_\mu\partial_\beta\Theta^2\right) - \bar{\Theta}^1\Gamma^\mu\partial_\alpha\Theta^1\bar{\Theta}^2\Gamma_\mu\partial_\beta\Theta^2\right]$$

$$\Pi_\alpha \cdot \Pi_\beta = \frac{1}{2}h_{\alpha\beta}h^{\gamma\delta}\Pi_\gamma \cdot \Pi_\delta$$
$$\Gamma \cdot \Pi_\alpha P_-^{\alpha\beta}\partial_\beta\Theta^1 = \Gamma \cdot \Pi_\alpha P_+^{\alpha\beta}\partial_\beta\Theta^2 = 0$$
$$\partial_\alpha\left[\sqrt{-h}\left(h^{\alpha\beta}\partial_\beta X^\mu - 2P_-^{\alpha\beta}\bar{\Theta}^1\Gamma^\mu\partial_\beta\Theta^1 - 2P_+^{\alpha\beta}\bar{\Theta}^2\Gamma^\mu\partial_\beta\Theta^2\right)\right] = 0$$

$$P_\pm^{\alpha\beta} = \frac{1}{2}\left(h^{\alpha\beta} \pm \frac{\varepsilon^{\alpha\beta}}{\sqrt{-h}}\right).$$

$$\bar{\Theta}^A\Gamma^\mu\partial_\alpha\Theta^A$$

$$\left(\Gamma_\mu\Pi_\alpha^\mu\right)P_-^{\alpha\beta}\partial_\beta\Theta^1 = \left(\Gamma_+\Pi_\alpha^+ + \Gamma_i\Pi_\alpha^i\right)P_-^{\alpha\beta}\partial_\beta\Theta^1 = 0$$

$$\Gamma^+\left(\Gamma_+\Pi_\alpha^+ + \Gamma_i\Pi_\alpha^i\right)P_-^{\alpha\beta}\partial_\beta\Theta^1 = 2\Pi_\alpha^+P_-^{\alpha\beta}\partial_\beta\Theta^1 = 0$$

$$P_-^{0\beta}\partial_\beta\Theta^1 = 0$$

$$\left(\frac{\partial}{\partial\tau} + \frac{\partial}{\partial\sigma}\right)\Theta^1 = 0$$

$$\text{IIA}: \sqrt{p^+}\Theta^A \to \mathbf{8_s} + \mathbf{8_c} = \left(S_1^a, S_2^{\dot{a}}\right)$$
$$\text{IIB}: \sqrt{p^+}\Theta^A \to \mathbf{8_s} + \mathbf{8_s} = \left(S_1^a, S_2^a\right)$$

$$\partial_+\partial_- X^i = 0, \partial_+ S_1^a = 0 \text{ and } \partial_- S_2^a \text{ or } \dot{a} = 0.$$

$$S = -\frac{1}{2\pi}\int d^2\sigma\partial_\alpha X_i\partial^\alpha X^i + \frac{i}{\pi}\int d^2\sigma\left(S_1^a\partial_+S_1^a + S_2^a\partial_-S_2^a\right)$$

$$S = -\frac{1}{2\pi}\int d^2\sigma\left(\partial_\alpha X_i\partial^\alpha X^i + \bar{S}^a\rho^\alpha\partial_\alpha S^a\right)$$

$$\{S^{Aa}(\sigma,\tau), S^{Bb}(\sigma',\tau)\} = \pi\delta^{ab}\delta^{AB}\delta(\sigma - \sigma')$$

$$S^{1a}|_{\sigma=0} = S^{2a}|_{\sigma=0} \text{ and } S^{1a}|_{\sigma=\pi} = S^{2a}|_{\sigma=\pi}$$

$$S^{1a} = \frac{1}{\sqrt{2}}\sum_{n=-\infty}^{\infty} S_n^a e^{-in(\tau-\sigma)}$$
$$S^{2a} = \frac{1}{\sqrt{2}}\sum_{n=-\infty}^{\infty} S_n^a e^{-in(\tau+\sigma)}$$

$$\{S_m^a, S_n^b\} = \delta_{m+n,0}\delta^{ab}$$

$$S^{Aa}(\sigma,\tau) = S^{Aa}(\sigma+\pi,\tau)$$

$$S^{1a} = \sum_{-\infty}^{\infty} S_n^a e^{-2in(\tau-\sigma)},$$
$$S^{2a} = \sum_{-\infty}^{\infty} \tilde{S}_n^a e^{-2in(\tau+\sigma)}.$$

$$\alpha' M^2 = \sum_{n=1}^{\infty} \left(\alpha_{-n}^i \alpha_n^i + n S_{-n}^a S_n^a\right)$$

$$\{S_0^a, S_0^b\} = \delta^{ab} \; a, b = 1, \dots, 8$$

$$|\dot{a}\rangle = \Gamma_{\dot{a}b}^i S_0^b |i\rangle \text{ and } |i\rangle = \Gamma_{\dot{a}b}^i S_0^b |\dot{a}\rangle.$$

$$(8_v + 8_c) \otimes (8_v + 8_s)$$

$$8_\mathbf{v} \otimes 8_\mathbf{v} = 1 + 28 + 35 \text{ and } 8_\mathbf{s} \otimes 8_\mathbf{c} = 8_\mathbf{v} + 56_\mathbf{t}$$

$$\bar{\zeta}\Gamma_i \chi \text{ and } \bar{\zeta}\Gamma_{ijk}\chi$$

$$(8_v + 8_c) \otimes (8_v + 8_c)$$

$$8_\mathrm{v} \otimes 8_\mathrm{v} = 1 + 28 + 35 \text{ and } 8_\mathrm{c} \otimes 8_\mathrm{c} = 1 + 28 + 35_+$$

$$\mathbf{8_v} \otimes \mathbf{8_v} = \phi \oplus B_{\mu\nu} \oplus G_{\mu\nu}$$

$$(\partial_\tau + \partial_\sigma)\Theta^1 = (\partial_\tau - \partial_\sigma)\Theta^2 = 0$$

$$8_v \otimes 8_v = 1 + 28 + 35$$

$$8_\mathrm{s} \otimes 8_\mathrm{s} = 1 + 28 + 35_-$$

$$t_{ijkl} = \pm \frac{1}{4!} \varepsilon^{ijkli'j'k'l'} t_{i'j'k'l'}$$

$$\partial_\mu J^\mu = a\epsilon^{\mu_1\mu_2\dots\mu_{2n}} F_{\mu_1\mu_2} \cdots F_{\mu_{2n-1}\mu_{2n}}$$

$$(\langle B| + \langle C|) \times (|B\rangle + |C\rangle).$$

$$A = \sum_{\mu,a} A_\mu^a(x) \lambda^a dx^\mu$$

$$F = \frac{1}{2} \sum_{\mu\nu} F_{\mu\nu} dx^\mu \wedge dx^\nu = dA + A \wedge A$$

$$\delta_\Lambda A = d\Lambda + [A, \Lambda] \text{ and } \delta_\Lambda F = [F, \Lambda]$$

$$\omega = \sum_{\mu,a} \omega_\mu^a(x) \lambda^a dx^\mu$$

$$R = d\omega + \omega \wedge \omega$$

$$\delta_{\Theta}\omega = d\Theta + [\omega, \Theta] \text{ and } \delta_{\Theta}R = [R, \Theta].$$

$$dX(R,F) = 0 \text{ and } \delta_{\Lambda}X(R,F) = \delta_{\Theta}X(R,F) = 0.$$

$$\mathrm{tr}(F \wedge \ldots \wedge F) \equiv \mathrm{tr}(F^k)$$
$$\mathrm{tr}(R \wedge \ldots \wedge R) \equiv \mathrm{tr}(R^k)$$

$$\delta S_{\mathrm{eff}} = \int G_{2n}$$

$$I_{2n+2} = d\omega_{2n+1}$$

$$\delta\omega_{2n+1} = dG_{2n}$$

$$I_{2n+2} = \sum_{\alpha} I_{2n+2}^{(\alpha)}$$

$$I_{1/2}(R,F) = \left[\hat{A}(R)\mathrm{tr}_{\rho}e^{iF}\right]_{2n+2}$$

$$\hat{A}(R) = \prod_{i=1}^{n} \frac{\lambda_i/2}{\sinh \lambda_i/2},$$

$$R \sim \begin{pmatrix} 0 & \lambda_1 & & & & & \\ -\lambda_1 & 0 & & & & & \\ & & 0 & \lambda_2 & & & \\ & & -\lambda_2 & 0 & & & \\ & & & & & 0 & \lambda_n \\ & & & & & & -\lambda_n \\ & & & & & & 0 \end{pmatrix}.$$

$$\hat{A}(R) = 1 + \frac{1}{48}\mathrm{tr}R^2 + \frac{1}{16}\left[\frac{1}{288}(\mathrm{tr}R^2)^2 + \frac{1}{360}\mathrm{tr}R^4\right] + \cdots$$

$$I_{3/2}(R) = \left(\sum_j 2\cosh \lambda_j - 1\right)\prod_i \frac{\lambda_i/2}{\sinh \lambda_i/2}.$$

$$I_A(R) = -\frac{1}{8}L(R)$$

$$L(R) = \prod_{i=1}^{n} \frac{\lambda_i}{\tanh \lambda_i}$$

$$\hat{A}(R/2) = \sqrt{L(R/4)\hat{A}(R)},$$

$$I(R) = I_{3/2}(R) - I_{1/2}(R) + I_A(R)$$

$$\left(2\sum_{j=1}^{5}\cosh\lambda_j-2\right)\prod_{i=1}^{5}\frac{\lambda_i/2}{\sinh\lambda_i/2}-\frac{1}{8}\prod_{i=1}^{5}\frac{\lambda_i}{\tanh\lambda_i}$$

$$I_{\text{sugra}}=\frac{1}{2}\left[I_{3/2}(R)-I_{1/2}(R)\right]_{12}=-\frac{1}{2}[I_A(R)]_{12}=\frac{1}{16}[L(R)]_{12}.$$

$$I=\left[\frac{1}{2}\hat{A}(R)\mathrm{Tr}e^{iF}+\frac{1}{16}L(R)\right]_{12}$$

$$\mathrm{tr}_{\rho_1\times\rho_2}e^{iF}=\left(\mathrm{tr}_{\rho_1}e^{iF}\right)\left(\mathrm{tr}_{\rho_2}e^{iF}\right)$$

$$\mathrm{Tr}e^{iF}=\frac{1}{2}\left(\mathrm{tr}e^{iF}\right)^2-\frac{1}{2}\mathrm{tr}e^{2iF}=\frac{1}{2}(\mathrm{trcos}\ F)^2-\frac{1}{2}\mathrm{trcos}\ 2F.$$

$$I=\frac{1}{4}\hat{A}(R)(\mathrm{trcos}\ F)^2-\frac{1}{4}\hat{A}(R)\mathrm{trcos}\ 2F+\frac{1}{16}L(R)$$

$$I'=\frac{1}{4}\hat{A}(R)(\mathrm{trcos}\ F)^2-16\hat{A}(R/2)\mathrm{trcos}\ F+256L(R/4)$$

$$I'=Y^2\ \text{ where }\ Y=\frac{1}{2}\sqrt{\hat{A}(R)}\mathrm{trcos}\ F-16\sqrt{L(R/4)}$$

$$[I]_{12}=[(Y_4+Y_8+\cdots)^2]_{12}=2Y_4Y_8$$

$$Y_0=\frac{N-32}{2}$$

$$Y_{\mathrm{B}}=\frac{1}{2}\sqrt{\hat{A}(R)}\mathrm{trcos}\ F$$

$$Y_{\mathrm{C}}=-16\sqrt{L(R/4)}$$

$$I'=Y^2=Y_{\mathrm{B}}^2+2Y_{\mathrm{B}}Y_{\mathrm{C}}+Y_{\mathrm{C}}^2$$

$$\delta S_{\mathrm{ct}}=-\int G_{10}$$

$$G_{10}=2G_2Y_8$$

$$Y_4=\frac{1}{4}(\mathrm{tr}R^2-\mathrm{tr}F^2)$$

$$d\omega_{3\,\mathrm{L}}=\mathrm{tr}R^2\ \text{ and }\ d\omega_{3\mathrm{Y}}=\mathrm{tr}F^2$$

$$S_{\mathrm{ct}}=\mu\int C_2Y_8$$

$$\mu\delta C_2=-2G_2$$

$$\tilde{F}_3=dC_2+2\mu^{-1}\omega_3$$

The case of $E_8 \times E_8$

$$\mathrm{Tr}F^6 = \frac{1}{48}\mathrm{Tr}F^2\mathrm{Tr}F^4 - \frac{1}{14{,}400}(\mathrm{Tr}F^2)^3$$

$$\mathrm{Tr}F^2 = 30\mathrm{tr}F^2$$
$$\mathrm{Tr}F^4 = 24\mathrm{tr}F^4 + 3(\mathrm{tr}F^2)^2$$
$$\mathrm{Tr}F^6 = 15\mathrm{tr}F^2\mathrm{tr}F^4$$

$$X_4 = \mathrm{tr}R^2 - \frac{1}{30}\mathrm{Tr}F^2$$

$$X_8 = \frac{1}{8}\mathrm{tr}R^4 + \frac{1}{32}(\mathrm{tr}R^2)^2 - \frac{1}{240}\mathrm{tr}R^2\mathrm{Tr}F^2 + \frac{1}{24}\mathrm{Tr}F^4 - \frac{1}{7200}(\mathrm{Tr}F^2)^2.$$

$$\mathrm{Tr}F^{2n} = \mathrm{Tr}F_1^{2n} + \mathrm{Tr}F_2^{2n},$$

$$F = \begin{pmatrix} F_1 & 0 \\ 0 & F_2 \end{pmatrix}$$

$$\mathrm{Tr}F_i^4 = \frac{1}{100}\left(\mathrm{Tr}F_i^2\right)^2 \text{ and } \mathrm{Tr}F_i^6 = \frac{1}{7200}\left(\mathrm{Tr}F_i^2\right)^3 \; i = 1{,}2.$$

$$X_4X_8 = X_4^{(1)}X_8^{(1)} + X_4^{(2)}X_8^{(2)}$$

$$X_4^{(i)} = \frac{1}{2}\mathrm{tr}R^2 - \frac{1}{30}\mathrm{Tr}F_i^2 \; i = 1{,}2$$

$$X_8^{(i)} = \frac{1}{8}\mathrm{tr}R^4 + \frac{1}{32}(\mathrm{tr}R^2)^2 - \frac{1}{120}\mathrm{tr}R^2\mathrm{Tr}F_i^2 + \frac{1}{3600}\left(\mathrm{Tr}F_i^2\right)^2 \; i = 1{,}2.$$

$$n_H - n_V = 244.$$

$$I_{3/2}(R) + (n_V - n_H - 1)I_{1/2}(R)$$

$$I_{1/2}^{(8)}(R) = \frac{1}{128 \cdot 180}(4\mathrm{tr}R^4 + 5(\mathrm{tr}R^2)^2)$$
$$I_{3/2}^{(8)}(R) = \frac{1}{128 \cdot 180}(980\mathrm{tr}R^4 - 215(\mathrm{tr}R^2)^2)$$

$$p(R) = \det\left(1 + \frac{R}{2\pi}\right) = \prod_{i=1}^{n} (1 + (\lambda_i/2\pi)^2).$$

$$p_1 = -\frac{1}{2}\frac{1}{(2\pi)^2}\mathrm{tr}R^2 \text{ and } p_2 = \frac{1}{8}\frac{1}{(2\pi)^4}((\mathrm{tr}R^2)^2 - 2\mathrm{tr}R^4)$$

$$I_{1/2}^{(8)} = \frac{1}{5760}(7p_1^2 - 4p_2) \text{ and } I_A^{(8)} = \frac{1}{5760}(16p_1^2 - 112p_2)$$

$$I_8 = 2I_{1/2}^{(8)} + I_A^{(8)} = \frac{1}{192}(p_1^2 - 4p_2) = \frac{1}{192}\frac{1}{(2\pi)^4}\left(\mathrm{tr}R^4 - \frac{1}{4}(\mathrm{tr}R^2)^2\right).$$

$$\int H_3 \wedge \omega_7$$

$$\delta \int H_3 \wedge \omega_7 = \int H_3 \wedge dG_6 = -\int dH_3 \wedge G_6$$

$$dH_3 = \delta_W$$

$$\int F_4 \wedge \omega_7$$

$$\bar{\Theta}_1 \Gamma_{\mu_1 \cdots \mu_n} \Theta_2 = (-1)^{n(n+1)/2} \bar{\Theta}_2 \Gamma_{\mu_1 \cdots \mu_n} \Theta_1$$

$$\Gamma^\mu d\Theta d\bar{\Theta} \Gamma_\mu d\Theta = 0$$

$$\{\Gamma_{\mu_1 \nu_1}, \Gamma_{\mu_2 \nu_2}\} = -2\eta_{\mu_1 \mu_2} \eta_{\nu_1 \nu_2} + 2\eta_{\mu_1 \nu_2} \eta_{\nu_1 \mu_2} + 2\Gamma_{\mu_1 \nu_1 \mu_2 \nu_2}$$

$$\omega_3 = \mathrm{tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

$$X^{25}(\sigma + \pi, \tau) = X^{25}(\sigma, \tau) + 2\pi R W, W \in \mathbb{Z}$$

$$X^{25}(\sigma, \tau) = x^{25} + 2\alpha' p^{25} \tau + 2RW\sigma + \cdots,$$

$$p^{25} = \frac{K}{R}, K \in \mathbb{Z}$$

$$X^{25}(\sigma, \tau) = X_{\mathrm{L}}^{25}(\tau + \sigma) + X_{\mathrm{R}}^{25}(\tau - \sigma),$$

$$X_{\mathrm{R}}^{25}(\tau - \sigma) = \frac{1}{2}(x^{25} - \tilde{x}^{25}) + \left(\alpha' \frac{K}{R} - WR\right)(\tau - \sigma) + \cdots,$$
$$X_{\mathrm{L}}^{25}(\tau + \sigma) = \frac{1}{2}(x^{25} + \tilde{x}^{25}) + \left(\alpha' \frac{K}{R} + WR\right)(\tau + \sigma) + \cdots,$$

$$X_{\mathrm{R}}^{25}(\tau - \sigma) = \frac{1}{2}(x^{25} - \tilde{x}^{25}) + \sqrt{2\alpha'} \alpha_0^{25} (\tau - \sigma) + \cdots$$
$$X_{\mathrm{L}}^{25}(\tau + \sigma) = \frac{1}{2}(x^{25} + \tilde{x}^{25}) + \sqrt{2\alpha'} \tilde{\alpha}_0^{25} (\tau + \sigma) + \cdots$$

$$\sqrt{2\alpha'} \alpha_0^{25} = \alpha' \frac{K}{R} - WR$$
$$\sqrt{2\alpha'} \tilde{\alpha}_0^{25} = \alpha' \frac{K}{R} + WR$$

$$M^2 = -\sum_{\mu=0}^{24} p_\mu p^\mu$$

$$\frac{1}{2} \alpha' M^2 = \left(\tilde{\alpha}_0^{25}\right)^2 + 2N_{\mathrm{L}} - 2 = \left(\alpha_0^{25}\right)^2 + 2N_{\mathrm{R}} - 2$$

$$N_{\mathrm{R}} - N_{\mathrm{L}} = WK$$

$$\alpha' M^2 = \alpha'\left[\left(\frac{K}{R}\right)^2 + \left(\frac{WR}{\alpha'}\right)^2\right] + 2N_{\mathrm{L}} + 2N_{\mathrm{R}} - 4$$

$$\alpha_0 \to -\alpha_0 \text{ and } \tilde{\alpha}_0 \to \tilde{\alpha}_0$$

$$X_{\mathrm{R}} \to -X_{\mathrm{R}} \text{ and } X_{\mathrm{L}} \to X_{\mathrm{L}}$$

$$\tilde{X}(\sigma,\tau) = X_{\mathrm{L}}(\tau+\sigma) - X_{\mathrm{R}}(\tau-\sigma),$$

$$\tilde{X}(\sigma,\tau) = \tilde{x} + 2\alpha'\frac{K}{R}\sigma + 2RW\tau + \cdots$$

$$\int \left(\frac{1}{2}V^\alpha V_\alpha - \epsilon^{\alpha\beta}X\partial_\beta V_\alpha\right) d^2\sigma$$

$$\frac{1}{2}\int \partial^\alpha \tilde{X}\partial_\alpha \tilde{X} d^2\sigma$$

$$\epsilon^{\alpha\beta}\epsilon_\alpha^{\gamma} = -\eta^{\beta\gamma}$$

$$\frac{1}{2}\int \partial^\alpha X\partial_\alpha X d^2\sigma$$

$$\partial_\alpha \tilde{X} = -\epsilon_\alpha{}^\beta \partial_\beta X$$

$$S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma \eta^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu$$

$$\delta S = -\frac{1}{2\pi\alpha'}\int d\tau \partial_\sigma X_\mu \delta X^\mu \Big|_{\sigma=0}^{\sigma=\pi}$$

$$\frac{\partial}{\partial\sigma}X^\mu(\sigma,\tau) = 0, \text{ for } \sigma = 0,\pi$$

$$X(\tau,\sigma) = x + p\tau + i\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in\tau}\cos(n\sigma)$$

$$X_{\mathrm{R}}(\tau-\sigma) = \frac{x-\tilde{x}}{2} + \frac{1}{2}p(\tau-\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in(\tau-\sigma)}$$
$$X_{\mathrm{L}}(\tau+\sigma) = \frac{x+\tilde{x}}{2} + \frac{1}{2}p(\tau+\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in(\tau+\sigma)}$$

$$X_{\mathrm{R}} \to -X_{\mathrm{R}} \text{ and } X_{\mathrm{L}} \to X_{\mathrm{L}}$$

$$\tilde{X}(\tau,\sigma) = X_{\mathrm{L}} - X_{\mathrm{R}} = \tilde{x} + p\sigma + \sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in\tau}\sin(n\sigma)$$

$$\tilde{X}(\tau,0) = \tilde{x} \text{ and } \tilde{X}(\tau,\pi) = \tilde{x} + \frac{\pi K}{R} = \tilde{x} + 2\pi K\tilde{R}$$

$$\delta^{ii'}\delta^{jj'}\delta^{kk'}\lambda^1_{ij}\lambda^2_{j'k}\lambda^3_{k'i'} = \mathrm{Tr}\lambda^1\lambda^2\lambda^3$$

$$|\phi, k, ij\rangle.$$

$$|\phi, k, \lambda\rangle = \sum_{i,j=1}^{N} |\phi, k, ij\rangle \lambda_{ij}$$

$$U = \exp\, i \int_0^{2\pi R} A dx$$

$$A = -\frac{1}{2\pi R} \mathrm{diag}(\theta_1, \theta_2, \dots, \theta_N)$$

$$e^{ip2\pi R} = e^{-i(\theta_i - \theta_j)}$$

$$p = \frac{K}{R} - \frac{\theta_i - \theta_j}{2\pi R}, K \in \mathbb{Z}$$

$$\tilde{X}_{ij}^{25} = \tilde{x}_0 + \theta_i \tilde{R} + 2\tilde{R}\sigma \left(K + \frac{\theta_j - \theta_i}{2\pi}\right) + \cdots$$

$$M_{ij}^2 = \left(\frac{K}{R} + \frac{\theta_j - \theta_i}{2\pi R}\right)^2 + \frac{1}{\alpha'}(N-1).$$

$$\alpha_{-1}^{25}|0, k\rangle,$$

$$\alpha_{-1}^{\mu}|0, k\rangle \text{ with } \mu = 0, \dots, 24$$

$$X_L^{\mu} = \frac{x^{\mu} + \tilde{x}^{\mu}}{2} + \frac{1}{2} l_s^2 p^{\mu}(\tau + \sigma) + \frac{i}{2} l_s \sum_{m \neq 0} \frac{1}{m} \alpha_m^{\mu} e^{-im(\tau+\sigma)}$$

$$X_R^{\mu} = \frac{x^{\mu} - \tilde{x}^{\mu}}{2} + \frac{1}{2} l_s^2 p^{\mu}(\tau - \sigma) + \frac{i}{2} l_s \sum_{m \neq 0} \frac{1}{m} \alpha_m^{\mu} e^{-im(\tau-\sigma)}.$$

$$X^i = X_L^i + X_R^i \text{ and } X^I = X_L^I - X_R^I,$$

$$X^I(0, \tau) = X^I(\pi, \tau) = \tilde{x}^I$$

$$T_{++} = \partial_+ X^i \partial_+ X_i + \partial_+ X^I \partial_+ X_I = \partial_+ X_L^{\mu} \partial_+ X_{L\mu}$$

$$M^2 = 2(N-1)/l_s^2$$

$$M^2 = -2/l_s^2 = -1/\alpha'$$

$$S = \int \left(-m\sqrt{-\dot{X}^{\mu}\dot{X}_{\mu}} - e\dot{X}^{\mu}A_{\mu}\right) d\tau$$

$$A = -\frac{\theta}{2\pi R}$$

$$p = \frac{K}{R} + \frac{e\theta}{2\pi R'}$$

$$S = \int \left(-m\sqrt{1-v^2} - e\left(A_0 + \vec{A}\cdot\vec{v}\right)\right) dt$$

$$P = \frac{\delta S}{\delta \dot{X}} = p - eA = p + \frac{e\theta}{2\pi R}$$

$$p = \frac{m\dot{X}}{\sqrt{1-v^2}}$$

$$\Psi(x) \sim e^{iPX}$$

$$p = \frac{K}{R} - \frac{e\theta}{2\pi R}$$

$$A_n = \frac{1}{n!} A_{\mu_1\mu_2\cdots\mu_n} dx^{\mu_1} \wedge dx^{\mu_2} \wedge \cdots \wedge dx^{\mu_n}$$

$$F_{n+1} = \frac{1}{(n+1)!} F_{\mu_1\mu_2\cdots\mu_{n+1}} dx^{\mu_1} \wedge dx^{\mu_2} \wedge \cdots \wedge dx^{\mu_{n+1}}$$

$$dF = 0 \text{ and } d \star F = 0$$

$$dF = \star J_m \text{ and } d \star F = \star J_e.$$

$$J_\mu = (\rho, \vec{j}),$$

$$e = \int_{S^2} \star F \text{ and } g = \int_{S^2} F,$$

$$e \cdot g \in 2\pi\mathbb{Z}$$

$$S_{\text{int}} = e\int A = e\int d\tau A_\mu \frac{dx^\mu}{d\tau}$$

$$(\star F)^{\mu_1\mu_2\cdots\mu_{D-2}} = \frac{\varepsilon^{\mu_1\mu_2\cdots\mu_D}}{2\sqrt{-g}} F_{\mu_{D-1}\mu_D}$$

$$S_{\text{int}} = \mu_p \int A_{p+1}$$

$$\int A_{p+1} = \frac{1}{(p+1)!}\int A_{\mu_1\cdots\mu_{p+1}} \frac{\partial x^{\mu_1}}{\partial \sigma^0} \cdots \frac{\partial x^{\mu_{p+1}}}{\partial \sigma^p} d^{p+1}\sigma.$$

$$\mu_p \mu_{6-p} \in 2\pi\mathbb{Z}$$

$$Q = Q_1 + \Gamma^{01\cdots p} Q_2$$

$$X_{\mathrm{L}}^9 \to X_{\mathrm{L}}^9 \text{ and } X_{\mathrm{R}}^9 \to -X_{\mathrm{R}}^9$$

$$\psi_{\mathrm{L}}^9 \to \psi_{\mathrm{L}}^9 \text{ and } \psi_{\mathrm{R}}^9 \to -\psi_{\mathrm{R}}^9$$

$$\partial_\sigma X^\mu|_{\sigma=0} = \partial_\sigma X^\mu|_{\sigma=\pi} = 0, \mu = 0, \dots, p,$$

$$X^i\big|_{\sigma=0} = d_1^i \text{ and } X^i\big|_{\sigma=\pi} = d_2^i, i = p+1, \dots, 9,$$

$$X^\mu(\tau,\sigma) = x^\mu + p^\mu\tau + i\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu \cos n\sigma e^{-in\tau},$$

$$X^i(\tau,\sigma) = d_1^i + \left(d_2^i - d_1^i\right)\frac{\sigma}{\pi} + \sum_{n\neq 0}\frac{1}{n}\alpha_n^i \sin n\sigma e^{-in\tau}.$$

$$\frac{1}{g_s^2}\int d^{10}x\mathcal{L}_{\text{NS}}$$

$$\frac{2\pi R}{g_s^2}\int d^9x\mathcal{L}_{\text{NS}}$$

$$\frac{2\pi\tilde{R}}{\tilde{g}_s^2}\int d^9x\mathcal{L}_{\text{NS}}$$

$$\tilde{g}_s = \frac{\sqrt{\alpha'}}{R}g_s$$

$$\mathcal{A} = \begin{pmatrix} A & T \\ \bar{T} & A' \end{pmatrix},$$

$$V(T_0) + 2NT_{\text{D}p} = 0.$$

$$(E, E') \sim 0 \Leftrightarrow E \sim E'$$

$$(E \oplus H, E' \oplus H) \sim (E, E')$$

$$\mathbb{R}^{9,1} = \mathbb{R}^{p,1} \times \mathbb{R}^{9-p}$$

$$\tilde{K}(S^{9-p}) = \begin{cases} \mathbb{Z} & p = \text{ odd} \\ 0 & p = \text{ even} \end{cases}$$

$$K(X \times S^1, S^1) = K^{-1}(X) \oplus \tilde{K}(X)$$

$$K^{-1}(S^{8-p}) \cong \tilde{K}(S^{9-p})$$

$$K^{-1}(S^{9-p}) = \begin{cases} \mathbb{Z} & \text{for } p = \text{even} \\ 0 & \text{for } p = \text{odd} \end{cases}$$

$$K^{-1}(X \times S^1, S^1) = \tilde{K}(X) \oplus K^{-1}(X).$$

$$\psi(x) = \exp\left(ie\int_{x_0}^{x} A_1\right)\psi_0(x),$$

$$\psi(x) \to U(\gamma)\psi(x), U(\gamma) = e^{ie\oint_\gamma A_1},$$

$$\oint_\gamma A_1 = \int_D F_2$$

$$\int_D F_2 - \int_{D'} F_2 = \int_{D-D'} F_2 = g$$

$$eg \in 2\pi\mathbb{Z}$$

$$\tilde{F}_{D-p-2} =\star F_{p+2}$$

$$\mu_p = \int_{S^{D-p-2}} \tilde{F}_{D-p-2}$$

$$\mu_{D-p-4} = \int_{S^{p+2}} F_{p+2}$$

$$\psi(\beta) = \exp\left(i\mu_p \int_{\beta_0}^{\beta} A_{p+1}\right)\psi_0(\beta)$$

$$\psi(\beta) \rightarrow U(\gamma)\psi(\beta), U(\gamma) = \exp\left(i\mu_p \int_\gamma A_{p+1}\right)$$

$$\oint\ _\gamma A_{p+1} = \int_D F_{p+2}.$$

$$\int_D F_{p+2} - \int_{D'} F_{p+2} = \int_{D-D'} F_{p+2} = \mu_{D-p-4}$$

$$\mu_p \mu_{D-p-4} \in 2\pi\mathbb{Z}$$

$$X_R^9 \rightarrow -X_R^9$$

$$\psi_R^9 \rightarrow -\psi_R^9$$

$$d_0^9 \rightarrow -d_0^9$$

$$\Gamma^\mu = \sqrt{2} d_0^\mu$$

$$\Gamma_\mu \rightarrow \Gamma_\mu (\text{ for } \mu \neq 9) \text{ and } \Gamma_9 \rightarrow -\Gamma_9$$

$$\Gamma_{11} = \Gamma_0 \Gamma_1 \cdots \Gamma_9 \rightarrow -\Gamma_{11}$$

$$S_2^{\dot{a}} \rightarrow \Gamma_{b\dot{a}}^j S_2^b (\text{ for IIA }) \text{ and } S_2^a \rightarrow \Gamma_{a\dot{b}}^j S_2^{\dot{b}}$$

$$\Omega: \sigma \rightarrow -\sigma$$

$$P = \frac{1}{2}(1 + \Omega)$$

$$b_{-1/2}^\mu |0; a\rangle \text{ and } \tilde{b}_{-1/2}^\mu |a; 0\rangle.$$

$$\Omega b_{-1/2}^\mu |0, ij\rangle = \pm b_{-1/2}^\mu |0, ji\rangle,$$

$$X_R \rightarrow -X_R, \psi_R \rightarrow -\psi_R$$

$$X = X_L + X_R \to \tilde{X} = X_L - X_R$$

$$\tilde{X} \to -\tilde{X}$$

$$(\mathbb{R}^{8,1} \times S^1)/\Omega$$

$$(\mathbb{R}^{8,1} \times S^1)/\Omega \cdot \mathcal{I},$$

$$\tilde{X}_I = \theta_I \tilde{R}, I = 1,2,\dots,16$$

$$\theta_I = 0 \text{ for } I = 1,\dots,8+N \text{ and } \theta_I = \pi \text{ for } I = 9+N,\dots,16.$$

$$SO(16+2N) \times SO(16-2N) \times U(1)^2.$$

$$SO(16-2N) \times U(1) \to E_{9-N}$$

$$T_9 \Omega T_9 = I_9 \Omega$$

$$T_9 \Omega T_9 \colon (X_L^9, X_R^9) \to (X_L^9, -X_R^9) \to (-X_R^9, X_L^9) \to (-X_R^9, -X_L^9)$$

$$I_9 \Omega \colon (X_L^9, X_R^9) \to (X_R^9, X_L^9) \to (-X_R^9, -X_L^9)$$

$$S = S_g + S_B + S_\Phi,$$

$$S_g = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{\alpha\beta} g_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu$$
$$S_B = \frac{1}{4\pi\alpha'} \int d^2\sigma \varepsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu$$
$$S_\Phi = \frac{1}{4\pi} \int d^2\sigma \sqrt{-h} \Phi R^{(2)}$$

$$4\pi\alpha' S = \int d^2\sigma \big[\sqrt{-h} h^{\alpha\beta} \big(-g_{99} V_\alpha V_\beta - 2 g_{9\mu} V_\alpha \partial_\beta X^\mu - g_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu\big) +$$
$$\varepsilon^{\alpha\beta} \big(B_{9\mu} V_\alpha \partial_\beta X^\mu + B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu\big) + \tilde{X}^9 \varepsilon^{\alpha\beta} \partial_\alpha V_\beta + \alpha' \sqrt{-h} R^{(2)} \Phi(X)\big]$$

$$\varepsilon^{\alpha\beta} \partial_\alpha V_\beta = 0$$

$$\tilde{S} = S_{\tilde{g}} + S_{\tilde{B}} + S_{\tilde{\Phi}}$$

$$\tilde{g}_{99} = \frac{1}{g_{99}}, \tilde{g}_{9\mu} = \frac{B_{9\mu}}{g_{99}}, \tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{B_{9\mu} B_{9\nu} - g_{9\mu} g_{9\nu}}{g_{99}}$$
$$\tilde{B}_{9\mu} = -\tilde{B}_{\mu 9} = \frac{g_{9\mu}}{g_{99}}, \tilde{B}_{\mu\nu} = B_{\mu\nu} + \frac{g_{9\mu} B_{9\nu} - B_{9\mu} g_{9\nu}}{g_{99}}$$

$$\tilde{\Phi} = \Phi - \frac{1}{2} \log\, g_{99}$$

$$\tilde{C}_9 = C, \tilde{C}_\mu = C_{\mu 9}, \tilde{C}_{\mu\nu 9} = C_{\mu\nu}, \tilde{C}_{\mu\nu\lambda} = C_{\mu\nu\lambda 9}$$

$$X^\mu(\sigma)$$

$$\Theta^{1a}(\sigma) \text{ and } \Theta^{2a}(\sigma)$$

$$S_{\text{BI}} \sim \int \sqrt{-\det(\eta_{\alpha\beta} + kF_{\alpha\beta})}d^4\sigma$$

$$\int \sqrt{1 - k^2F_{01}^2}d^2\sigma$$

$$A_1 = -\frac{1}{2\pi\alpha'}\tilde{X}^1$$

$$F_{01} = -\frac{1}{2\pi\alpha'}v \text{ where } v = \dot{\tilde{X}}^1$$

$$-m\int \sqrt{1 - v^2}dt$$

$$k = 2\pi\alpha'$$

$$S_1 = -T_{\text{D}p}\int d^{p+1}\sigma\sqrt{-\det(G_{\alpha\beta} + k\mathcal{F}_{\alpha\beta})}$$

$$G_{\alpha\beta} = \eta_{\mu\nu}\Pi_\alpha^\mu\Pi_\beta^\nu$$

$$\Pi_\alpha^\mu = \partial_\alpha X^\mu - \bar{\Theta}^A\Gamma^\mu\partial_\alpha\Theta^A$$

$$\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} + b_{\alpha\beta}$$

$$b = (\bar{\Theta}^1\Gamma_\mu d\Theta^1 - \bar{\Theta}^2\Gamma_\mu d\Theta^2)\left(dX^\mu - \frac{1}{2}\bar{\Theta}^A\Gamma^\mu d\Theta^A\right)$$

$$S_{\text{Maxwell}} = -\frac{1}{4g^2}\int F_{\alpha\beta}F^{\alpha\beta}d^{p+1}\sigma$$

$$T_{\text{D}p} = \frac{c_p}{g_s}$$

$$\frac{2\pi R c_p}{g_s} = \frac{c_{p-1}}{\tilde{g}_s}$$

$$c_p = \frac{1}{2\pi\sqrt{\alpha'}}c_{p-1}$$

$$T_{\text{D}p} = \frac{1}{g_s(2\pi)^p(\alpha')^{(p+1)/2}}$$

$$S_2 = \int \Omega_{p+1}$$

$$d\Omega_{p+1} = d\bar{\Theta}^A\mathcal{T}_p^{AB}d\Theta^B$$

$$\Omega_1 = -m\bar{\Theta}\Gamma_{11}d\Theta = m(\bar{\Theta}^1d\Theta^2 - \bar{\Theta}^2d\Theta^1)$$

$$\mathcal{T}_0 = m\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$m = T_{\mathrm{D0}} = \frac{1}{g_s\sqrt{\alpha'}}$$

$$\mathcal{T}^{AB} = \sum_{p=0}^{\infty} \mathcal{T}_p^{AB}$$

$$\mathcal{T}^{AB} = me^{2\pi\alpha'\mathcal{F}} f^{AB}(\psi)$$

$$\psi = \frac{1}{\sqrt{2\pi\alpha'}} \Gamma_\mu \Pi_\alpha^\mu d\sigma^\alpha$$

$$f(\psi) = \begin{pmatrix} 0 & \cos\,\psi \\ -\cosh\,\psi & 0 \end{pmatrix}$$

$$f(\psi) = \begin{pmatrix} 0 & \sin\,\psi \\ \sinh\,\psi & 0 \end{pmatrix}$$

$$S_{\mathrm{DBI}} = -T_{\mathrm{D}p} \int d^{p+1}\sigma \sqrt{-\det(\eta_{\alpha\beta} + k^2 \partial_\alpha \Phi^i \partial_\beta \Phi^i + kF_{\alpha\beta})}$$

$$T_{\mathrm{D9}} \int d^{10}\sigma \sqrt{-\det(\eta_{\alpha\beta} + kF_{\alpha\beta} - 2k^2 \bar{\lambda}\Gamma_\alpha \partial_\beta \lambda + k^3 \bar{\lambda}\Gamma^\gamma \partial_\alpha \lambda \bar{\lambda}\Gamma_\gamma \partial_\beta \lambda)}$$

$$P[g + B]_{\alpha\beta} = (g_{\mu\nu} + B_{\mu\nu}) \partial_\alpha X^\mu \partial_\beta X^\nu$$

$$S_{\mathrm{D}p} = -T_{\mathrm{D}p} \int d^{p+1}\sigma e^{-\Phi_0} \sqrt{-\det(g_{\alpha\beta} + B_{\alpha\beta} + k^2 \partial_\alpha \Phi^i \partial_\beta \Phi^i + kF_{\alpha\beta})}$$

$$\delta B = d\Lambda$$

$$\star F_{n+1} = F_{9-n}.$$

$$\mu_p \int C_{p+1}$$

$$C = \sum_{n=0}^{8} C_n.$$

$$S_{\mathrm{CS}} = \mu_p \int \left(Ce^{B+kF}\right)_{p+1}$$

$$[D_\alpha, D_\beta] \sim F_{\alpha\beta}$$

$$A_\alpha = \sum_n A_\alpha^{(n)} T_n \ \text{ and } \ \Phi^i = \sum_n \Phi^{i(n)} T_n$$

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + i[A_\alpha, A_\beta]$$
$$D_\alpha \Phi^i = \partial_\alpha \Phi^i + i[A_\alpha, \Phi^i]$$

$$S_1 = -T_{\mathrm{D9}} \int d^{10}\sigma e^{-\Phi_0} \mathrm{Tr}\left(\sqrt{-\det\left(g_{\alpha\beta} + B_{\alpha\beta} + kF_{\alpha\beta}\right)}\right)$$

- Chern-Simons - Meyers:

$$S_2 = \mu_9 \int \mathrm{Tr}\left(Ce^{B+kF}\right)_{10}$$

$$V(\Phi) \sim -\frac{1}{4}\mathrm{Tr}\left([\Phi^i,\Phi^j][\Phi^i,\Phi^j]\right) - \frac{i}{3} f\epsilon_{ijk}\mathrm{Tr}\left(\Phi^i\Phi^j\Phi^k\right)$$

$$\left[[\Phi^i,\Phi^j],\Phi^j\right] + if\epsilon_{ijk}\left[\Phi^j,\Phi^k\right] = 0$$

$$\left[\alpha^i,\alpha^j\right] = 2i\epsilon_{ijk}\alpha^k$$

$$\mathrm{Tr}\left(\alpha^i\alpha^j\right) = \frac{1}{3}N(N^2-1)\delta_{ij}$$

$$\left\langle \left(X^i\right)^2 \right\rangle = \frac{1}{N}(2\pi\alpha')^2 \mathrm{Tr}\left[\left(\Phi^i\right)^2\right]$$

$$R^2 = (\pi\alpha' f)^2(N^2-1)$$

$$\det\left(G_{\alpha\beta} + kF_{\alpha\beta}\right) = \det\left(G_{\alpha\beta} + kF_{\alpha\beta}\right)^T = \det\left(G_{\alpha\beta} - kF_{\alpha\beta}\right)$$

$$M = kG^{-1}F$$

$$\begin{aligned} \sqrt{-\det(G+kF)} &= \sqrt{-\det G}\sqrt{\det(1+M)} \\ &= \sqrt{-\det G}[\det(1-M^2)]^{1/4} \end{aligned}$$

$$\log\, \det(1-M^2) = \mathrm{tr}\log\,(1-M^2) = -\mathrm{tr}\left(M^2 + \frac{1}{2}M^4 + \cdots\right)$$

$$\begin{aligned} [\det(1-M^2)]^{1/4} &= \exp\,\left(-\frac{1}{4}\mathrm{tr}M^2 - \frac{1}{8}\mathrm{tr}M^4 + \cdots\right) \\ &= 1 - \frac{1}{4}\mathrm{tr}M^2 - \frac{1}{8}\mathrm{tr}M^4 + \frac{1}{32}(\mathrm{tr}M^2)^2 + \cdots \end{aligned}$$

$$\begin{aligned} S_1 &= -T_{\mathrm{D}p} \int d^{p+1}\sigma \sqrt{-\det\left(G_{\alpha\beta} + kF_{\alpha\beta}\right)} \\ &= -T_{\mathrm{D}p} \int d^{p+1}\sigma \sqrt{-\det G}\left(1 + \frac{k^2}{4}F_{\alpha\beta}F^{\alpha\beta}\right. \\ &\left. - \frac{k^4}{8}\left(F_{\alpha\beta}F^{\alpha\beta}\right)^2 + \frac{k^4}{32}F_{\alpha\beta}F^{\beta\gamma}F_{\gamma\delta}F^{\delta\alpha} + \cdots\right) \end{aligned}$$

$$S_{\mathrm{DBI}} = -T_{\mathrm{D}p} \int d^{p+1}\sigma \sqrt{-\det\left(\eta_{\alpha\beta} + k^2\partial_\alpha\Phi^i\partial_\beta\Phi^i + kF_{\alpha\beta}\right)}$$

$$A = -\frac{1}{2\pi R}\mathrm{diag}(\theta_1, \theta_2, \ldots, \theta_N)$$

$$F_{\mu_1\ldots\mu_n} = \bar{\psi}_{\mathrm{L}}\Gamma_{\mu_1\ldots\mu_n}\psi_{\mathrm{R}}.$$

$$\partial_{[\mu}F_{\mu_1\ldots\mu_n]} = 0, \partial^{\mu}F_{\mu\mu_2\ldots\mu_n} = 0$$

$$E_r = F_{rt} = \frac{e}{\sqrt{(r^4 + r_0^4)}}, r_0^2 = 2\pi\alpha' e$$

$$S = -\frac{1}{2\pi}\int d^2\sigma \partial_\alpha X_\mu \partial^\alpha X^\mu$$

$$S = -\frac{1}{2\pi}\int d^2\sigma\left(\partial_\alpha X_\mu \partial^\alpha X^\mu + \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu\right)$$

$$S = -\frac{1}{2\pi}\int d^2\sigma\left(\partial_\alpha X_\mu \partial^\alpha X^\mu + \bar{\lambda}^A \rho^\alpha \partial_\alpha \lambda^A\right)$$

$$S = \frac{1}{\pi}\int d^2\sigma\left(2\partial_+ X_\mu \partial_- X^\mu + i\lambda_-^A \partial_+ \lambda_-^A + i\lambda_+^A \partial_- \lambda_+^A\right)$$

$$S = \frac{1}{\pi}\int d^2\sigma\left(2\partial_+ X_\mu \partial_- X^\mu + i\psi^\mu \partial_+ \psi_\mu + i\sum_{A=1}^{32} \lambda^A \partial_- \lambda^A\right)$$

$$\delta X^\mu = i\varepsilon\psi^\mu \text{ and } \delta\psi^\mu = -2\varepsilon\partial_- X^\mu$$

$$G_r|\phi\rangle = L_m|\phi\rangle = \left(L_0 - \frac{1}{2}\right)|\phi\rangle = 0, r, m > 0$$

$$\left(L_0 - \frac{1}{2}\right)|\phi\rangle = \left(\frac{p^2}{8} + N_{\mathrm{R}} - \frac{1}{2}\right)|\phi\rangle = 0$$

$$N_{\mathrm{R}} = \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n + \sum_{r=1/2}^{\infty} r b_{-r}\cdot b_r$$

$$F_m|\phi\rangle = L_m|\phi\rangle = 0, m \geq 0$$

$$L_0|\phi\rangle = \left(\frac{p^2}{8} + N_{\mathrm{R}}\right)|\phi\rangle = 0$$

$$N_{\mathrm{R}} = \sum_{n=1}^{\infty} (\alpha_{-n}\cdot\alpha_n + n d_{-n}\cdot d_n)$$

$$L_0|\phi\rangle = \left(\frac{p^2}{8} + N_{\mathrm{R}}\right)|\phi\rangle = 0$$

$$N_{\mathrm{R}} = \sum_{n=1}^{\infty} \left(\alpha_{-n}^i \alpha_n^i + n S_{-n}^a S_n^a\right)$$

$$M^2 = 8N_{\mathrm{R}}$$

$$\lambda^A(\tau+\sigma) = \sum_{n\in\mathbb{Z}} \lambda_n^A e^{-2in(\tau+\sigma)}$$

$$\{\lambda_m^A, \lambda_n^B\} = \delta^{AB}\delta_{m+n,0}$$

$$\lambda^A(\tau+\sigma) = \sum_{r\in\mathbb{Z}+1/2} \lambda_r^A e^{-2ir(\tau+\sigma)}$$

$$\{\lambda_r^A, \lambda_s^B\} = \delta^{AB}\delta_{r+s,0}$$

$$\tilde{L}_m|\phi\rangle = (\tilde{L}_0 - \tilde{a})|\phi\rangle = 0, m > 0$$

$$(\tilde{L}_0 - \tilde{a}_\text{P})|\phi\rangle = \left(\frac{p^2}{8} + N_\text{L} - \tilde{a}_\text{P}\right)|\phi\rangle = 0,$$

$$N_\text{L} = \sum_{n=1}^{\infty} (\tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n + n\lambda_{-n}^A\lambda_n^A)$$

$$(\tilde{L}_0 - \tilde{a}_\text{A})|\phi\rangle = \left(\frac{p^2}{8} + N_\text{L} - \tilde{a}_\text{A}\right)|\phi\rangle = 0$$

$$N_\text{L} = \sum_{n=1}^{\infty} \tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n + \sum_{r=1/2}^{\infty} r\lambda_{-r}^A\lambda_r^A$$

$$\tilde{a}_\text{A} = \frac{8}{24} + \frac{32}{48} = 1,$$
$$\tilde{a}_\text{P} = \frac{8}{24} - \frac{32}{24} = -1.$$

$$\frac{1}{8}M^2 = N_\text{R} = N_\text{L} - 1$$

$$\frac{1}{8}M^2 = N_\text{R} = N_\text{L} + 1$$

$$|i\rangle_\text{R} \text{ and } |\dot{a}\rangle_\text{R},$$

$$\tilde{\alpha}_{-1}^i|0\rangle_\text{L}$$

$$\lambda_{-1/2}^A\lambda_{-1/2}^B|0\rangle_\text{L}$$

$$|i\rangle_\text{R} \otimes \tilde{\alpha}_{-1}^j|0\rangle_\text{L}$$

$$|\dot{a}\rangle_\text{R} \otimes \tilde{\alpha}_{-1}^j|0\rangle_\text{L}$$

$$(-1)^F = \bar{\lambda}_0(-1)^{\sum_1^\infty \lambda_{-n}^A\lambda_n^A}$$

$$\bar{\lambda}_0 = \lambda_0^1\lambda_0^2 \dots \lambda_0^{32}$$

$$SU(3)\times SU(2)\times U(1) \subset SU(5) \subset SO(10) \subset E_6 \subset E_7 \subset E_8$$

$$\tilde{a}_{\mathrm{AA}} = \frac{8}{24} + \frac{n}{48} + \frac{32-n}{48} = 1,$$
$$\tilde{a}_{\mathrm{AP}} = \frac{8}{24} + \frac{n}{48} - \frac{32-n}{24} = \frac{n}{16} - 1,$$
$$\tilde{a}_{\mathrm{PA}} = \frac{8}{24} - \frac{n}{24} + \frac{32-n}{48} = 1 - \frac{n}{16},$$
$$\tilde{a}_{\mathrm{PP}} = \frac{8}{24} - \frac{n}{24} - \frac{32-n}{24} = -1.$$

$$\tilde{\alpha}^{i}_{-1}|0\rangle_{\mathrm{L}}$$

$$\lambda^{A}_{-1/2}\lambda^{B}_{-1/2}|0\rangle_{\mathrm{L}}$$

$$\begin{array}{lll} (\mathbf{120},\mathbf{1}) & \text{if} & A,B = 1,\dots,16 \\ (\mathbf{1},\mathbf{120}) & \text{if} & A,B = 17,\dots,32 \\ (\mathbf{16},\mathbf{16}) & \text{if} & A = 1,\dots,16, B = 17,\dots,32 \end{array}$$

$$\text{PA}: (\mathbf{128},\mathbf{1}) \oplus (\mathbf{128}',\mathbf{1}),$$
$$\text{AP}: (\mathbf{1},\mathbf{128}) \oplus (\mathbf{1},\mathbf{128})'.$$

$$(-1)^{F_1} = \bar{\lambda}_0^{(1)}(-1)^{\sum_{n=1}^{\infty}\sum_{A=1}^{16}\lambda^{A}_{-n}\lambda^{A}_{n}}$$

$$\bar{\lambda}_0^{(1)} = \lambda_0^1\lambda_0^2 \dots \lambda_0^{16}$$

$$(-1)^{F_2} = \bar{\lambda}_0^{(2)}(-1)^{\sum_{n=1}^{\infty}\sum_{A=17}^{32}\lambda^{A}_{-n}\lambda^{A}_{n}}$$

$$\bar{\lambda}_0^{(2)} = \lambda_0^{17}\lambda_0^{18} \dots \lambda_0^{32}$$

$$(128,1) \oplus (1,128)$$

$$J^a(z) = \frac{1}{2}T^a_{AB}\lambda^A(z)\lambda^B(z)$$

$$[T^a, T^b] = if^{abc}T^c$$

$$J^a(z)J^b(w) = \frac{k\delta^{ab}}{2(z-w)^2} + i\frac{f^{abc}}{z-w}J^c(w) + \cdots$$

$$\mathrm{tr}\left(T^aT^b\right) = k\delta^{ab}$$

$$\lambda^A(z)\lambda^B(w) = \frac{\delta^{AB}}{z-w}$$

$$\left\langle \frac{1}{2}T^a_{AB}\lambda^A(z)\lambda^B(z)\frac{1}{2}T^b_{CD}\lambda^C(w)\lambda^D(w)\right\rangle = \frac{1}{2}\frac{\mathrm{tr}\left(T^aT^b\right)}{(z-w)^2} = \frac{k\delta^{ab}}{2(z-w)^2}$$

$$c = \frac{k\dim G}{k + \tilde{h}_G}$$

$$\frac{1}{8}M^2 = N_{\mathrm{R}}.$$

$$\tilde{L}_0 - 1 = 0 \rightarrow \frac{1}{8}M^2 = N_{\mathrm{L}} - 1.$$

$$\frac{1}{8}M^2 = N_{\mathrm{R}} = N_{\mathrm{L}} - 1.$$

$$\frac{1}{8}M^2 = N_{\mathrm{R}} = N_{\mathrm{L}} + 1.$$

$$ds^2 = \sum_{\mu,\nu=0}^{d-1} \eta_{\mu\nu} dX^\mu dX^\nu + \sum_{I,J=1}^{n} G_{IJ} dY^I dY^J$$

$$G_{IJ} = R_I^2 \delta_{IJ}$$

$$\begin{aligned} &X^\mu(\sigma + \pi, \tau) = X^\mu(\sigma, \tau) \\ &Y^I(\sigma + \pi, \tau) = Y^I(\sigma, \tau) + 2\pi W^I \ \text{ with } \ W^I \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} X^\mu(\sigma,\tau) &= X_{\mathrm{L}}^\mu(\tau+\sigma) + X_{\mathrm{R}}^\mu(\tau-\sigma) \\ X_{\mathrm{L}}^\mu(\tau+\sigma) &= \frac{1}{2}x^\mu + p_{\mathrm{L}}^\mu(\tau+\sigma) + \frac{i}{2}\sum_{n\neq 0} \frac{1}{n}\tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)} \\ X_{\mathrm{R}}^\mu(\tau-\sigma) &= \frac{1}{2}x^\mu + p_{\mathrm{R}}^\mu(\tau-\sigma) + \frac{i}{2}\sum_{n\neq 0} \frac{1}{n}\alpha_n^\mu e^{-2in(\tau-\sigma)} \end{aligned}$$

$$p_{\mathrm{L}}^\mu = p_{\mathrm{R}}^\mu = \frac{1}{2}p^\mu$$

$$\begin{aligned} Y^I(\sigma,\tau) &= Y_{\mathrm{L}}^I(\tau+\sigma) + Y_{\mathrm{R}}^I(\tau-\sigma), \\ Y_{\mathrm{L}}^I(\tau+\sigma) &= \frac{1}{2}y^I + p_{\mathrm{L}}^I(\tau+\sigma) + \frac{i}{2}\sum_{n\neq 0} \frac{1}{n}\tilde{\alpha}_n^I e^{-2in(\tau+\sigma)}, \\ Y_{\mathrm{R}}^I(\tau-\sigma) &= \frac{1}{2}y^I + p_{\mathrm{R}}^I(\tau-\sigma) + \frac{i}{2}\sum_{n\neq 0} \frac{1}{n}\alpha_n^I e^{-2in(\tau-\sigma)}. \end{aligned}$$

$$Y^I(\sigma,\tau) = Y_{\mathrm{L}}^I(\tau+\sigma) + Y_{\mathrm{R}}^I(\tau-\sigma) = y^I + \left(p_{\mathrm{L}}^I + p_{\mathrm{R}}^I\right)\tau + \left(p_{\mathrm{L}}^I - p_{\mathrm{R}}^I\right)\sigma + \cdots,$$

$$p_{\mathrm{L}}^I - p_{\mathrm{R}}^I = 2W^I \ \text{ with } \ W^I \in \mathbb{Z}.$$

$$p_{\mathrm{L}}^I + p_{\mathrm{R}}^I = K_I \ \text{ with } \ K_I \in \mathbb{Z}$$

$$S = -\frac{1}{2\pi}\int d^2\sigma \left(G_{IJ}\eta^{\alpha\beta} - B_{IJ}\varepsilon^{\alpha\beta}\right)\partial_\alpha Y^I \partial_\beta Y^J$$

$$p_I = \frac{\delta S}{\delta \dot{Y}^I} = \frac{1}{\pi}\left(G_{IJ}\dot{Y}^J + B_{IJ}Y'^J\right).$$

$$K_I = \int_0^\pi p_I d\sigma = G_{IJ}\left(p_{\mathrm{L}}^J + p_{\mathrm{R}}^J\right) + B_{IJ}\left(p_{\mathrm{L}}^J - p_{\mathrm{R}}^J\right) \ \text{ with } \ K_I \in \mathbb{Z}$$

$$p_{\mathrm{L}}^{I} = W^{I} + G^{IJ}\left(\frac{1}{2}K_{J} - B_{JK}W^{K}\right)$$
$$p_{\mathrm{R}}^{I} = -W^{I} + G^{IJ}\left(\frac{1}{2}K_{J} - B_{JK}W^{K}\right)$$

$$(L_0 - 1)|\Phi\rangle = (\tilde{L}_0 - 1)|\Phi\rangle = 0$$

$$\frac{1}{8}M^2 = \frac{1}{2}G_{IJ}p_{\mathrm{L}}^{I}p_{\mathrm{L}}^{J} + N_{\mathrm{L}} - 1 = \frac{1}{2}G_{IJ}p_{\mathrm{R}}^{I}p_{\mathrm{R}}^{J} + N_{\mathrm{R}} - 1$$

$$N_{\mathrm{R}} = \sum_{m=1}^{\infty} \alpha_{-m}\cdot\alpha_{m} \text{ and } N_{\mathrm{L}} = \sum_{m=1}^{\infty} \tilde{\alpha}_{-m}\cdot\tilde{\alpha}_{m}$$

$$N_{\mathrm{R}} - N_{\mathrm{L}} = \frac{1}{2}G_{IJ}\left(p_{\mathrm{L}}^{I}p_{\mathrm{L}}^{J} - p_{\mathrm{R}}^{I}p_{\mathrm{R}}^{J}\right) = W^{I}K_{I}$$

$$M^2 = M_0^2 + 4(N_{\mathrm{R}} + N_{\mathrm{L}} - 2) \text{ with } M_0^2 = 2G_{IJ}\left(p_{\mathrm{L}}^{I}p_{\mathrm{L}}^{J} + p_{\mathrm{R}}^{I}p_{\mathrm{R}}^{J}\right)$$

$$\frac{1}{2}M_0^2 = (W \quad K)\mathcal{G}^{-1}\begin{pmatrix} W \\ K \end{pmatrix}$$

$$\mathcal{G}^{-1} = \begin{pmatrix} 2(G - BG^{-1}B) & BG^{-1} \\ -G^{-1}B & \frac{1}{2}G^{-1} \end{pmatrix}$$

$$\mathcal{G} = \begin{pmatrix} \frac{1}{2}G^{-1} & -G^{-1}B \\ BG^{-1} & 2(G - BG^{-1}B) \end{pmatrix}$$

$$W^{I} \leftrightarrow K_{I}, \mathcal{G} \leftrightarrow \mathcal{G}^{-1}.$$

$$B_{IJ} \to B_{IJ} + \frac{1}{2}N_{IJ} \text{ with } W^{I} \to W^{I}, K_{I} \to K_{I} + N_{IJ}W^{J}$$

$$A^{T}\begin{pmatrix} 0 & 1_n \\ 1_n & 0 \end{pmatrix}A = \begin{pmatrix} 0 & 1_n \\ 1_n & 0 \end{pmatrix}$$

$$\mathcal{G} \to A\mathcal{G}A^{T} \text{ and } \begin{pmatrix} W \\ K \end{pmatrix} \to \begin{pmatrix} W' \\ K' \end{pmatrix} = A\begin{pmatrix} W \\ K \end{pmatrix}.$$

$$W^{I}K_{I} = \frac{1}{2}(W \quad K)\begin{pmatrix} 0 & 1_n \\ 1_n & 0 \end{pmatrix}\begin{pmatrix} W \\ K \end{pmatrix}$$

$$\text{inversion}: \; A = \begin{pmatrix} 0 & 1_n \\ 1_n & 0 \end{pmatrix},$$

$$\text{shift}: \; A = \begin{pmatrix} 1_n & 0 \\ N_{IJ} & 1_n \end{pmatrix}$$

$$\mathcal{M}_{n,n}^{0} = O(n,n;\mathbb{R})/[O(n;\mathbb{R}) \times O(n;\mathbb{R})]$$

$$\mathcal{M}_{n,n} = \mathcal{M}_{n,n}^{0}/O(n,n;\mathbb{Z})$$

$$M^2 = \frac{K^2}{R^2} + 4R^2W^2 + 4(N_\mathrm{L} + N_\mathrm{R} - 2)$$

$$N_\mathrm{R} - N_\mathrm{L} = KW$$

$$\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0\rangle$$

$$\left|V_1^{\mu}\right\rangle = \alpha_{-1}^{\mu}\tilde{\alpha}_{-1}|0\rangle,$$
$$\left|V_2^{\mu}\right\rangle = \alpha_{-1}\tilde{\alpha}_{-1}^{\mu}|0\rangle,$$

$$|\phi\rangle = \alpha_{-1}\tilde{\alpha}_{-1}|0\rangle$$

$$\left|V_{++}^{\mu}\right\rangle = \alpha_{-1}^{\mu}|+1,+1\rangle \text{ and } \left|V_{--}^{\mu}\right\rangle = \alpha_{-1}^{\mu}|-1,-1\rangle,$$

$$|\phi_{++}\rangle = \alpha_{-1}|+1,+1\rangle \text{ and } |\phi_{--}\rangle = \alpha_{-1}|-1,-1\rangle.$$

$$M^2 = \frac{1}{R^2} + 4R^2 - 4 = \left(\frac{1}{R} - 2R\right)^2$$

$$\left|V_{+-}^{\mu}\right\rangle = \tilde{\alpha}_{-1}^{\mu}|+1,-1\rangle \text{ and } \left|V_{-+}^{\mu}\right\rangle = \tilde{\alpha}_{-1}^{\mu}|-1,+1\rangle,$$

$$|\phi_{+-}\rangle = \tilde{\alpha}_{-1}|+1,-1\rangle \text{ and } |\phi_{-+}\rangle = \tilde{\alpha}_{-1}|-1,+1\rangle.$$

$$\int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2} I(\tau, \ldots)$$

$$\tau \to \tau' = \frac{a\tau + b}{c\tau + d}$$

$$\mathrm{Tr}\left(q^{L_0}\bar{q}^{\tilde{L}_0}\right),$$

$$q = e^{2\pi i\tau}$$

$$\mathrm{Tr}\left(q^{\frac{1}{2}p_\mathrm{R}^2}\bar{q}^{\frac{1}{2}p_\mathrm{L}^2}\right) = \sum_{W^I, K_I} e^{\pi i\tau_1(p_\mathrm{R}^2 - p_\mathrm{L}^2)}e^{-\pi\tau_2(p_\mathrm{L}^2 + p_\mathrm{R}^2)},$$

$$\int \exp\left(-\pi\tau_2 p^2\right)d^n p = (\tau_2)^{-n/2}$$

$$F(\tau; G, B) = (\tau_2)^{n/2}\mathrm{Tr}\left(q^{\frac{1}{2}p_\mathrm{R}^2}\bar{q}^{\frac{1}{2}p_\mathrm{L}^2}\right)$$

$$p_\mathrm{R}^2 - p_\mathrm{L}^2 = 2(N_\mathrm{L} - N_\mathrm{R}) = -2W^I K_I$$

$$f(A) = \sum_{\{M\}} \exp\left(-\pi M^T A M\right),$$

$$f(A) = \frac{1}{\sqrt{\det A}} f(A^{-1}).$$

$$F(\tau, G, B) = (\tau_2)^{n/2} f(A)$$

$$A=\tau_2\begin{pmatrix}2(G-BG^{-1}B) & BG^{-1}\\ -G^{-1}B & \frac{1}{2}G^{-1}\end{pmatrix}+i\tau_1\begin{pmatrix}0 & 1_n\\ 1_n & 0\end{pmatrix}.$$

$$\det A=|\tau|^{2n}$$

$$A^{-1}=\tilde{\tau}_2\begin{pmatrix}\frac{1}{2}G^{-1} & -G^{-1}B\\ BG^{-1} & 2(G-BG^{-1}B)\end{pmatrix}+i\tilde{\tau}_1\begin{pmatrix}0 & 1_n\\ 1_n & 0\end{pmatrix},$$

$$\tilde{\tau}=-\frac{1}{\tau}=\frac{-\tau_1+i\tau_2}{|\tau|^2}.$$

$$F(\tau;G,B)=F\left(-\frac{1}{\tau};G,B\right),$$

$$\Lambda=\left\{\sum_{i=1}^{m}n_ie_i,n_i\in\mathbb{Z}\right\},$$

$$g_{ij}=e_i\cdot e_j$$

$$\Lambda^\star=\{w\in V\text{ such that }w\cdot v\in\mathbb{Z},\text{ for all }v\in\Lambda\}.$$

$$\Lambda^\star=\left\{\sum_{i=1}^{m}n_ie_i^\star,n_i\in\mathbb{Z}\right\}$$

$$e_i^\star\cdot e_j=\delta_{ij}.$$

$$g_{ij}^\star=e_i^\star\cdot e_j^\star$$

$$p^2=p_{\mathrm{L}}^2-p_{\mathrm{R}}^2$$

$$p^2=2W^IK_I\in 2\mathbb{Z}$$

$$M^2=2(W^I\quad K_I)\mathcal{G}^{-1}\begin{pmatrix}W^I\\ K_I\end{pmatrix}+4(N_{\mathrm{R}}+N_{\mathrm{L}}-2).$$

$$W^I\leftrightarrow K_I,\mathcal{G}\leftrightarrow\mathcal{G}^{-1}.$$

- Kaluza – Klein:

$$J^{\pm}(z)=e^{\pm 2iX^{25}(z)/\sqrt{\alpha'}}$$

$$J^3(z)=i\sqrt{2/\alpha'}\partial X^{25}(z)$$

$$J^i(z)J^j(w)\sim\frac{\delta^{ij}}{(z-w)^2}+i\varepsilon^{ijk}\frac{J^k(w)}{z-w}+\cdots$$

$$J^i(z)=\sum_{n\in\mathbb{Z}}J_n^iz^{-n-1}\quad\text{or}\quad J_n^i=\oint\frac{dz}{2\pi i}z^nJ^i(z)$$

$$\left[J_m^i, J_n^j\right] = i\varepsilon^{ijk} J_{m+n}^k + m\delta^{ij}\delta_{m+n,0}$$

$$\tilde{G} + \tilde{B} = \frac{1}{4}(G+B)^{-1}$$

$$\tilde{G} = \frac{1}{8}[(G+B)^{-1} + (G-B)^{-1}]$$

$$\tilde{B} = \frac{1}{8}[(G+B)^{-1} - (G-B)^{-1}]$$

$$\tilde{G} = \frac{1}{4}(G - BG^{-1}B)^{-1} \text{ and } \tilde{B} = -G^{-1}B\tilde{G}$$

$$\tilde{\mathcal{G}} = \mathcal{G}^{-1}$$

$$\mathrm{Tr}\left(q^{\frac{1}{2}p_{\mathrm{R}}^2}\bar{q}^{\frac{1}{2}p_{\mathrm{L}}^2}\right) = \sum_{\{W^I, K_I\}} e^{\pi i \tau_1 (p_{\mathrm{R}}^2 - p_{\mathrm{L}}^2)} e^{-\pi\tau_2(p_{\mathrm{L}}^2 + p_{\mathrm{R}}^2)}$$

$$p_{\mathrm{R}}^2 - p_{\mathrm{L}}^2 = -2W^I K_I \text{ and } p_{\mathrm{R}}^2 + p_{\mathrm{L}}^2 = (W \quad K)\mathcal{G}^{-1}\begin{pmatrix} W \\ K \end{pmatrix}$$

$$\sum_{\{M\}} \exp\left(-\pi M^T A M\right),$$

$$A = \begin{pmatrix} 2\tau_2(G - BG^{-1}B) & i\tau_1 1_n + \tau_2 BG^{-1} \\ i\tau_1 1_n - \tau_2 G^{-1}B & \frac{1}{2}\tau_2 G^{-1} \end{pmatrix} \text{ and } M = \begin{pmatrix} W \\ K \end{pmatrix}$$

$$\begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} = \begin{pmatrix} 1_n & M_2 M_4^{-1} \\ 0 & 1_n \end{pmatrix}\begin{pmatrix} M_1 - M_2 M_4^{-1} M_3 & 0 \\ M_3 & 1_n \end{pmatrix}\begin{pmatrix} 1_n & 0 \\ 0 & M_4 \end{pmatrix}$$

$$\det(M_1 - M_2 M_4^{-1} M_3)\det M_4$$

$$\det A = |\tau|^{2n}$$

$$p_{\mathrm{L}}^2 - p_{\mathrm{R}}^2 \in 2\mathbb{Z}$$

$$M^2 = 2\left(p_{\mathrm{L}}^2 + p_{\mathrm{R}}^2\right) - 8 + \text{ oscillators}$$

$$O(n,n;\mathbb{R})/O(n;\mathbb{R}) \times O(n,\mathbb{R}).$$

$$\mathcal{G} = \begin{pmatrix} 1/(2R^2) & 0 \\ 0 & 2R^2 \end{pmatrix},$$

$$M_0^2 = (2WR)^2 + (K/R)^2,$$

$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{pmatrix} \text{ and } B = B_{12}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

$$\tau = \tau_1 + i\tau_2 = \frac{G_{12}}{G_{22}} + i\frac{\sqrt{\det G}}{G_{22}}$$

$$\rho = \rho_1 + i\rho_2 = B_{12} + i\sqrt{\det G}$$

$$SO(2,2;\mathbb{Z}) = SL(2,\mathbb{Z}) \times SL(2,\mathbb{Z})$$

$$G + B = \frac{\rho_2}{\tau_2}\begin{pmatrix} \tau_1^2 + \tau_2^2 & \tau_1 \\ \tau_1 & 1 \end{pmatrix} + \rho_1 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

$$G = \begin{pmatrix} \rho_2\tau_2 & 0 \\ 0 & \rho_2/\tau_2 \end{pmatrix}.$$

$$(\tau, \rho) = (i, i)$$

$$(\tau, \rho) = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}, i\right)$$

$$G = \frac{1}{\sqrt{3}}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\mathbf{p}_{\mathrm{L}} \in \Gamma_{16}, p_{\mathrm{L}}^I = \sum_i n_i e_i^I, n_i \in \mathbb{Z}$$

$$\Theta_\Gamma(\tau) = \sum_{p\in\Gamma} e^{i\pi\tau p^2}$$

$$\Theta_\Gamma(\tau') = (c\tau + d)^8 \Theta_\Gamma(\tau)$$

$$\mathcal{M}_{16+n,n} = \mathcal{M}_{16+n,n}^0 / O(16+n, n; \mathbb{Z}),$$

$$\mathcal{M}_{16+n,n}^0 = \frac{O(16+n, n; \mathbb{R})}{O(16+n, \mathbb{R}) \times O(n, \mathbb{R})}$$

$$\tau \to \tau + 1 \text{ and } \tau \to -\frac{1}{\tau}$$

$$\Theta_{\Gamma_{16}}(\tau) = \sum_{p\in\Gamma_{16}} \exp\,(i\pi\tau p^2)$$

$$A_{ij} = -i\tau G_{ij},$$

$$G_{ij} = e^I{}_i e_{Ij}$$

$$\Theta_{\Gamma_{16}}(\tau) = \sum_{p\in\Gamma_{16}} \exp\,(-\pi N^T A N).$$

$$\frac{1}{\sqrt{\det A}} \sum_{p\in\Gamma_{16}} \exp\,(-\pi N^T A^{-1} N).$$

$$\tau^{-8} \sum_{p \in \Gamma_{16}} \exp\left(-\pi N^T A^{-1} N\right) = \tau^{-8} \sum_{p \in \Gamma_{16}} \exp\left(-\frac{i\pi}{\tau} p^2\right),$$

$$\frac{1}{8} M^2 = N_{\mathrm{R}} = N_{\mathrm{L}} - 1 + \frac{1}{2} \sum_{I=1}^{16} (p^I)^2.$$

$$N_{\mathrm{R}} = 1, N_{\mathrm{L}} = 2, \sum_{I=1}^{16} (p^I)^2 = 0$$

$$\tilde{\alpha}_{-1}^{I} \tilde{\alpha}_{-1}^{J} |0\rangle_{\mathrm{L}}, \tilde{\alpha}_{-2}^{I} |0\rangle_{\mathrm{L}}, \tilde{\alpha}_{-1}^{i} \tilde{\alpha}_{1}^{j} |0\rangle_{\mathrm{L}}, \tilde{\alpha}_{-2}^{i} |0\rangle_{\mathrm{L}}, \tilde{\alpha}_{-1}^{i} \tilde{\alpha}_{-1}^{I} |0\rangle_{\mathrm{L}}$$

$$N_{\mathrm{R}} = 1, N_{\mathrm{L}} = 1, \sum_{I=1}^{16} (p^I)^2 = 2$$

$$\tilde{\alpha}_{-1}^{I} \left| p^J, \sum_{J=1}^{16} (p^J)^2 = 2 \right\rangle_{\mathrm{L}}, \tilde{\alpha}_{-1}^{i} \left| p^I, \sum_{I=1}^{16} (p^J)^2 = 2 \right\rangle_{\mathrm{L}}$$

$$N_{\mathrm{R}} = 1, N_{\mathrm{L}} = 0, \sum_{I=1}^{16} (p^I)^2 = 4$$

$$\left| p^I, \sum_{I=1}^{16} (p^I)^2 = 4 \right\rangle_{\mathrm{L}}$$

$$\alpha_{-1}^{i} |j\rangle_{\mathrm{R}}, \alpha_{-1}^{i} |a\rangle_{\mathrm{R}}, ; S_{-1}^{a} |i\rangle_{\mathrm{R}}, S_{-1}^{a} |b\rangle_{\mathrm{R}}.$$

$$f(A) = \sum_{\{M\}} \exp\left(-\pi M^T A M\right)$$

$$f(A) = \frac{1}{\sqrt{\det A}} f(A^{-1})$$

$$f(A, x) = \sum_{\{M\}} \exp\left(-\pi (M + x)^T A (M + x)\right)$$

$$f(A, x) = \sum_{\{N\}} C_N(A) \exp\left(2\pi i N^T x\right)$$

$$C_N(A) = \int_0^1 f(A, x) e^{-2\pi i N^T x} d^m x$$

$$C_N(A) = \int_{-\infty}^{\infty} \exp\left(-\pi x^T A x - 2\pi i N^T x\right) d^m x = \frac{\exp\left(-\pi N^T A^{-1} N\right)}{\sqrt{\det A}}$$

$$f(A)=\sum_{\{N\}} C_N(A)=\frac{1}{\sqrt{\det A}}f(A^{-1})$$

$$e_1=(1,1) \text{ and } e_2=(1,-1)$$

$$S[X]=\frac{1}{\pi}\int_M \partial X\bar{\partial}X d^2z$$

$$X(z+1,\bar{z}+1)=X(z,\bar{z})+2\pi RW_1$$
$$X(z+\tau,\bar{z}+\bar{\tau})=X(z,\bar{z})+2\pi RW_2$$

$$Z_{\mathrm{cl}}=\sum_{W_1,W_2} e^{-S_{\mathrm{cl}}(W_1,W_2)}$$

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 2 \end{pmatrix}$$

$$\tau\to\tau+1 \quad \rho\to\rho+1$$
$$\tau\to -1/\tau \quad \rho\to -1/\rho$$

$$U:(\tau,\rho)\to(\rho,\tau) \text{ and } V:(\sigma,\tau)\to(-\bar{\sigma},-\bar{\tau}).$$

$$ds^2=R^2(dx^2+dy^2+dz^2)$$

$$\left\{Q_\alpha^I,Q_\beta^{\dagger J}\right\}=2M\delta^{IJ}\delta_{\alpha\beta}+2iZ^{IJ}\Gamma^0_{\alpha\beta}$$

$$Z^{IJ}=\begin{pmatrix} 0 & Z_1 & 0 & 0 & \\ -Z_1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & Z_2 & \\ 0 & 0 & -Z_2 & 0 & \\ & \vdots & & & \ddots \end{pmatrix}$$

$$\begin{pmatrix} M & Z \\ Z^\dagger & M \end{pmatrix}$$

$$M\geq |Z_1|.$$

$$\{Q_\alpha,\bar{Q}_{\dot{\beta}}\}=2\sigma^\mu_{\alpha\dot{\beta}}P_\mu, \text{ and } \{Q_\alpha,Q_\beta\}=\{\bar{Q}_{\dot{\alpha}},\bar{Q}_{\dot{\beta}}\}=0.$$

$$\{Q_\alpha,\bar{Q}_{\dot{\beta}}\}=2M\delta_{\alpha\dot{\beta}}=2M\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

$$b_\alpha=\frac{1}{\sqrt{2M}}Q_\alpha \text{ and } b_\alpha^\dagger=\frac{1}{\sqrt{2M}}\bar{Q}_{\dot{\alpha}}$$

$$\left\{b_\alpha,b_\beta^\dagger\right\}=\delta_{\alpha\beta},\{b_\alpha,b_\beta\}=\left\{b_\alpha^\dagger,b_\beta^\dagger\right\}=0$$

$$\left\{Q_{\alpha}^{I}, \bar{Q}_{\dot{\beta}}^{J}\right\} = 2M\delta_{\alpha\dot{\beta}}\delta^{IJ}$$
$$\left\{Q_{\alpha}^{I}, Q_{\beta}^{J}\right\} = 2Z\varepsilon_{\alpha\beta}\varepsilon^{IJ}$$
$$\left\{\bar{Q}_{\dot{\alpha}}^{I}, \bar{Q}_{\dot{\beta}}^{J}\right\} = 2Z\varepsilon_{\dot{\alpha}\dot{\beta}^{\varepsilon}}\varepsilon^{IJ}$$

$$b_{\alpha}^{\pm} = Q_{\alpha}^{1} \pm \varepsilon_{\alpha\beta}\bar{Q}_{\beta}^{2} \text{ and } \left(b_{\alpha}^{\pm}\right)^{\dagger} = \bar{Q}_{\alpha}^{1} \pm \varepsilon_{\alpha\beta}Q_{\beta}^{2}.$$

$$\left\{b_{\alpha}^{+}, \left(b_{\beta}^{+}\right)^{\dagger}\right\} = 4\delta_{\alpha\beta}(M+Z) \text{ and } \left\{b_{\alpha}^{-}, \left(b_{\beta}^{-}\right)^{\dagger}\right\} = 4\delta_{\alpha\beta}(M-Z)$$

$$M \geq Z$$

$$S = \frac{1}{16\pi G_D}\int \sqrt{-g}R d^D x$$

$$\frac{1}{2}(D-1)(D-2) - 1 = \frac{1}{2}D(D-3) = 44$$

$$S_{\Psi} \sim \int \bar{\Psi}_M \Gamma^{MNP}\partial_N \Psi_P d^D x$$

$$A_3 \rightarrow A_3 + d\Lambda_2$$

$$2\kappa_{11}^2 S = \int d^{11}x\sqrt{-G}\left(R - \frac{1}{2}|F_4|^2\right) - \frac{1}{6}\int A_3 \wedge F_4 \wedge F_4$$

$$16\pi G_{11} = 2\kappa_{11}^2 = \frac{1}{2\pi}\left(2\pi\ell_{\mathrm{p}}\right)^9$$

$$G_{MN} = \eta_{AB}E_M^A E_N^B$$

$$|F_n|^2 = \frac{1}{n!}G^{M_1N_1}G^{M_2N_2}\dots G^{M_nN_n}F_{M_1M_2\cdots M_n}F_{N_1N_2\cdots N_n}$$

$$\begin{aligned} \delta E_M^A &= \bar{\varepsilon}\Gamma^A\Psi_M \\ \delta A_{MNP} &= -3\bar{\varepsilon}\Gamma_{[MN}\Psi_{P]} \\ \delta\Psi_M &= \nabla_M\varepsilon + \frac{1}{12}\left(\Gamma_M\mathbf{F}^{(4)} - 3\mathbf{F}_M^{(4)}\right)\varepsilon \end{aligned}$$

$$\mathbf{F}^{(4)} = \frac{1}{4!}F_{MNPQ}\Gamma^{MNPQ}$$

$$\mathbf{F}_M^{(4)} = \frac{1}{2}\left[\Gamma_M, \mathbf{F}^{(4)}\right] = \frac{1}{3!}F_{MNPQ}\Gamma^{NPQ}$$

$$\Gamma_M = E_M^A\Gamma_A,$$

$$\Gamma_{[MN}\Psi_{P]} = \frac{1}{3}(\Gamma_{MN}\Psi_P + \Gamma_{NP}\Psi_M + \Gamma_{PM}\Psi_N)$$

$$\Gamma^{M_1M_2\cdots M_n} = \Gamma^{[M_1}\Gamma^{M_2}\dots\Gamma^{M_n]}$$

$$\nabla_M \varepsilon = \partial_M \varepsilon + \frac{1}{4}\omega_{MAB}\Gamma^{AB}\varepsilon$$

$$\omega_{MAB} = \frac{1}{2}(-\Omega_{MAB} + \Omega_{ABM} - \Omega_{BMA}),$$

$$\Omega_{MN}{}^{A} = 2\partial_{[N}E^{A}_{M]}$$

$$\delta\Psi_M = \nabla_M \varepsilon + \frac{1}{12}\left(\Gamma_M \mathbf{F}^{(4)} - 3\mathbf{F}^{(4)}_M\right)\varepsilon = 0$$

$$T_{\mathrm{M2}} = 2\pi\left(2\pi\ell_{\mathrm{p}}\right)^{-3} \text{ and } T_{\mathrm{M5}} = 2\pi\left(2\pi\ell_{\mathrm{p}}\right)^{-6}$$

$$G_{MN} = e^{-2\Phi/3}\begin{pmatrix} g_{\mu\nu} + e^{2\Phi}A_\mu A_\nu & e^{2\Phi}A_\mu \\ e^{2\Phi}A_\nu & e^{2\Phi} \end{pmatrix}$$

$$ds^2 = G_{MN}dx^M dx^N = e^{-2\Phi/3}g_{\mu\nu}dx^\mu dx^\nu + e^{4\Phi/3}\left(dx^{11} + A_\mu dx^\mu\right)^2.$$

$$E^A_M = \begin{pmatrix} e^{-\Phi/3}e^a_\mu & 0 \\ e^{2\Phi/3}A_\mu & e^{2\Phi/3} \end{pmatrix}$$

$$E^M_A = \begin{pmatrix} e^{\Phi/3}e^\mu_a & 0 \\ -e^{\Phi/3}A_a & e^{-2\Phi/3} \end{pmatrix}$$

$$A^{(11)}_{\mu\nu\rho} = A_{\mu\nu\rho} \text{ and } A^{(11)}_{\mu\nu 11} = B_{\mu\nu}$$

$$F^{(11)}_{\mu\nu\rho\lambda} = F_{\mu\nu\rho\lambda} \text{ and } F^{(11)}_{\mu\nu\rho 11} = H_{\mu\nu\rho}$$

$$F^{(11)}_{ABCD} = E^M_A E^N_B E^P_C E^Q_D F^{(11)}_{MNPQ}$$

$$F^{(11)}_{abcd} = e^{4\Phi/3}\left(F_{abcd} + 4A_{[a}H_{bcd]}\right) = e^{4\Phi/3}\tilde{F}_{abcd}$$
$$F^{(11)}_{abc11} = e^{\Phi/3}H_{abc}$$

$$\mathbf{F}^{(4)} = e^{4\Phi/3}\tilde{\mathbf{F}}^{(4)} + e^{\Phi/3}\mathbf{H}^{(3)}\Gamma_{11}$$

$$\tilde{F}_4 = dA_3 + A_1 \wedge H_3$$

$$\delta\tilde{F}_4 = d(d\Lambda \wedge B) + d\Lambda \wedge H_3 = 0$$

$$\ell_{\mathrm{p}} = g_{\mathrm{s}}^{1/3}\ell_{\mathrm{s}} \text{ with } \ell_s = \sqrt{\alpha'}$$

$$16\pi G_{10} = 2\kappa^2_{10} = \frac{1}{2\pi}(2\pi\ell_{\mathrm{s}})^8 g_s^2$$

$$G_{11} = 2\pi R_{11}G_{10}$$

$$R_{11} = g_s^{2/3}\ell_{\mathrm{p}} = g_{\mathrm{s}}\ell_{\mathrm{s}}$$

$$2\kappa^2 = \frac{1}{2\pi}(2\pi\ell_s)^8$$

$$S = S_{\mathrm{NS}} + S_{\mathrm{R}} + S_{\mathrm{CS}}$$

$$S_{\mathrm{NS}} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H_3|^2\right)$$

$$S_{\mathrm{R}} = -\frac{1}{4\kappa^2}\int d^{10}x\sqrt{-g}\left(|F_2|^2 + \left|\tilde{F}_4\right|^2\right)$$

$$S_{\mathrm{CS}} = -\frac{1}{4\kappa^2}\int B_2 \wedge F_4 \wedge F_4$$

$$\delta\Psi_A = E_A^\mu\partial_\mu\varepsilon + \frac{1}{4}\omega_{ABC}\Gamma^{BC}\varepsilon + \frac{1}{24}(3\mathbf{F}^{(4)}\Gamma_A - \Gamma_A\mathbf{F}^{(4)})\varepsilon$$

$$\omega_{aBC}^{(11)}\Gamma^{BC} = e^{\Phi/3}\left(\omega_{abc}\Gamma^{bc} - \frac{2}{3}\Gamma_a^\mu\partial_\mu\Phi\right) + e^{4\Phi/3}F_{ab}\Gamma^b\Gamma_{11}$$

$$\omega_{11BC}^{(11)}\Gamma^{BC} = -\frac{1}{2}e^{4\Phi/3}F_{bc}\Gamma^{bc} - \frac{4}{3}e^{\Phi/3}\Gamma^\mu\Gamma_{11}\partial_\mu\Phi$$

$$e^{-\Phi/3}\delta\Psi_{11} = -\frac{1}{4}e^\Phi\mathbf{F}^{(2)}\varepsilon - \frac{1}{3}\partial_\mu\Phi\Gamma^\mu\Gamma_{11}\varepsilon + \frac{1}{12}e^\Phi\tilde{\mathbf{F}}^{(4)}\Gamma_{11}\varepsilon + \frac{1}{6}\mathbf{H}^{(3)}\varepsilon$$

$$e^{-\Phi/3}\delta\Psi_a = e_a^\mu\nabla_\mu\varepsilon - \frac{1}{6}\Gamma_a^\mu\partial_\mu\Phi\varepsilon + \frac{1}{4}e^\Phi F_{ab}\Gamma^b\Gamma_{11}\varepsilon$$
$$+\frac{1}{24}e^\Phi(3\tilde{\mathbf{F}}^{(4)}\Gamma_a - \Gamma_a\tilde{\mathbf{F}}^{(4)})\varepsilon - \frac{1}{24}(3\mathbf{H}^{(3)}\Gamma_a + \Gamma_a\mathbf{H}^{(3)})\Gamma_{11}\varepsilon.$$

$$\tilde{\lambda} = e^{-\Phi/6}\Psi_{11}$$
$$\tilde{\Psi}_\mu = e^{-\Phi/6}\left(\Psi_\mu + \frac{1}{2}\Gamma_\mu\Gamma_{11}\Psi_{11}\right)$$

$$\delta\lambda = \left(-\frac{1}{3}\Gamma^\mu\partial_\mu\Phi\Gamma_{11} + \frac{1}{6}\mathbf{H}^{(3)} - \frac{1}{4}e^\Phi\mathbf{F}^{(2)} + \frac{1}{12}e^\Phi\tilde{\mathbf{F}}^{(4)}\Gamma_{11}\right)\varepsilon$$

$$\delta\Psi_\mu = \left(\nabla_\mu - \frac{1}{4}\mathbf{H}_\mu^{(3)}\Gamma_{11} - \frac{1}{8}e^\Phi F_{\nu\rho}\Gamma_\mu{}^{\nu\rho}\Gamma_{11} + \frac{1}{8}e^\Phi\mathbf{F}^{(4)}\Gamma_\mu\right)\varepsilon.$$

$$\int |F_5|^2 d^{10}x$$

$$S = S_{\mathrm{NS}} + S_{\mathrm{R}} + S_{\mathrm{CS}}$$

$$S_{\mathrm{R}} = -\frac{1}{4\kappa^2}\int d^{10}x\sqrt{-g}\left(|F_1|^2 + \left|\tilde{F}_3\right|^2 + \frac{1}{2}\left|\tilde{F}_5\right|^2\right)$$

$$S_{\mathrm{CS}} = -\frac{1}{4\kappa^2}\int C_4 \wedge H_3 \wedge F_3$$

$$\tilde{F}_3 = F_3 - C_0 H_3,$$
$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3.$$

$$\tilde{F}_5 = \star\, \tilde{F}_5$$

$$\delta\lambda = \frac{1}{2}\left(\partial_\mu \Phi - ie^{\Phi}\partial_\mu C_0\right)\Gamma^\mu \varepsilon + \frac{1}{4}\left(ie^{\Phi}\tilde{\mathbf{F}}^{(3)} - \mathbf{H}^{(3)}\right)\varepsilon^\star$$

$$\delta\Psi_\mu = \left(\nabla_\mu + \frac{i}{8}e^{\Phi}\mathbf{F}^{(1)}\Gamma_\mu + \frac{i}{16}e^{\Phi}\tilde{\mathbf{F}}^{(5)}\Gamma_\mu\right)\varepsilon - \frac{1}{8}\left(2\mathbf{H}_\mu^{(3)} + ie^{\Phi}\tilde{\mathbf{F}}^{(3)}\Gamma_\mu\right)\varepsilon^\star$$

$$B_2 = \begin{pmatrix} B_2^{(1)} \\ B_2^{(2)} \end{pmatrix}.$$

$$\Lambda = \begin{pmatrix} d & c \\ b & a \end{pmatrix} \in SL(2,\mathbb{R})$$

$$B_2 \to \Lambda B_2$$

$$\tau = C_0 + ie^{-\Phi}$$

$$\tau \to \frac{a\tau + b}{c\tau + d}.$$

$$\mathcal{M} = e^{\Phi}\begin{pmatrix} |\tau|^2 & -C_0 \\ -C_0 & 1 \end{pmatrix}$$

$$\mathcal{M} \to (\Lambda^{-1})^T \mathcal{M}\Lambda^{-1}$$

$$g_{\mu\nu} = e^{\Phi/2} g_{\mu\nu}^{\mathrm{E}}$$

$$\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}e^{-2\Phi}R \to \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left(R - \frac{9}{2}\partial^\mu\Phi\partial_\mu\Phi\right)$$

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left(R - \frac{1}{12}H_{\mu\nu\rho}^T\mathcal{M}H^{\mu\nu\rho} + \frac{1}{4}\mathrm{tr}\left(\partial^\mu\mathcal{M}\partial_\mu\mathcal{M}^{-1}\right)\right) - \frac{1}{8\kappa^2}\left(\int d^{10}x\sqrt{-g}\left|\tilde{F}_5\right|^2 + \int \varepsilon_{ij}C_4 \wedge H_3^{(i)} \wedge H_3^{(j)}\right)$$

$$\tilde{F}_5 = F_5 + \frac{1}{2}\varepsilon_{ij}B_2^{(i)} \wedge H_3^{(j)}$$

$g_{\mu\nu}, \Phi, C_2$ and A_μ

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi\right) - \frac{1}{2}\left|\tilde{F}_3\right|^2 - \frac{\kappa^2}{g^2}e^{-\Phi}\mathrm{tr}(|F_2|^2)\right]$$

$$\tilde{F}_3 = dC_2 + \frac{\ell_s^2}{4}\omega_3$$

$$\omega_3 = \omega_{\mathrm{L}} - \omega_{\mathrm{YM}}$$

$$\omega_{\mathrm{L}} = \mathrm{tr}\left(\omega \wedge d\omega + \frac{2}{3}\omega \wedge \omega \wedge \omega\right)$$

$$\omega_{\rm YM} = {\rm tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

$$\frac{g_{\rm YM}^2}{4\pi} = \frac{g^2}{4\pi} g_{\rm s} = (2\pi \ell_{\rm s})^6 g_{\rm s}$$

$$\int C_2 \wedge Y_8$$

$$\begin{aligned} \delta\Psi_\mu &= \nabla_\mu \varepsilon - \frac{1}{8} e^{\Phi} \tilde{\mathbf{F}}^{(3)} \Gamma_\mu \varepsilon \\ \delta\lambda &= \frac{1}{2} \partial\!\!\!/ \Phi \varepsilon + \frac{1}{4} e^{\Phi} \tilde{\mathbf{F}}^{(3)} \varepsilon \\ \delta\chi &= -\frac{1}{2} \mathbf{F}^{(2)} \varepsilon \end{aligned}$$

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\Phi} \left[R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} \left|\tilde{H}_3\right|^2 - \frac{\kappa^2}{30 g^2} {\rm Tr}(|F_2|^2) \right]$$

$$\tilde{H}_3 = dB_2 + \frac{\ell_{\rm s}^2}{4} \omega_3$$

$$d\tilde{H}_3 = \frac{\ell_{\rm s}^2}{4} \left({\rm tr} R \wedge R - \frac{1}{30} {\rm Tr} F \wedge F \right)$$

$$ {\rm tr} F \wedge F = \frac{1}{30} {\rm Tr} F \wedge F$$

$$\begin{aligned} \delta\Psi_\mu &= \nabla_\mu \varepsilon - \frac{1}{4} \tilde{\mathbf{H}}_\mu^{(3)} \varepsilon \\ \delta\lambda &= -\frac{1}{2} \Gamma^\mu \partial_\mu \Phi \varepsilon + \frac{1}{4} \tilde{\mathbf{H}}^{(3)} \varepsilon, \\ \delta\chi &= -\frac{1}{2} \mathbf{F}^{(2)} \varepsilon. \end{aligned}$$

$$\frac{1}{4} {\rm tr}(\partial^\mu \mathcal{M} \partial_\mu \mathcal{M}^{-1}) = -\frac{\partial^\mu \tau \partial_\mu \bar{\tau}}{2({\rm Im}\tau)^2} = -\frac{1}{2} (\partial^\mu \Phi \partial_\mu \Phi + e^{2\Phi} \partial^\mu C_0 \partial_\mu C_0)$$

$$\mathcal{M} = e^{\Phi} \begin{pmatrix} |\tau|^2 & -C_0 \\ -C_0 & 1 \end{pmatrix}$$

$$\mathcal{M}^{-1} = e^{\Phi} \begin{pmatrix} 1 & C_0 \\ C_0 & |\tau|^2 \end{pmatrix}.$$

$$\begin{aligned} \frac{1}{4} {\rm tr}(\partial^\mu \mathcal{M} \partial_\mu \mathcal{M}^{-1}) &= \frac{1}{2} \partial_\mu (e^{\Phi} |\tau|^2) \partial^\mu (e^{\Phi}) - \frac{1}{2} \partial_\mu (C_0 e^{\Phi}) \partial^\mu (C_0 e^{\Phi}) \\ &= -\frac{1}{2} (\partial^\mu \Phi \partial_\mu \Phi + e^{2\Phi} \partial^\mu C_0 \partial_\mu C_0) \end{aligned}$$

$$\begin{aligned} -\frac{\partial^\mu \tau \partial_\mu \bar{\tau}}{2({\rm Im}\tau)^2} &= -\frac{1}{2} e^{2\Phi} \partial^\mu (C_0 + i e^{-\Phi}) \partial_\mu (C_0 - i e^{-\Phi}) \\ &= -\frac{1}{2} (\partial^\mu \Phi \partial_\mu \Phi + e^{2\Phi} \partial^\mu C_0 \partial_\mu C_0) \end{aligned}$$

$$S_{\rm NS}=\frac{1}{2\kappa^2}\int\ d^{10}x\sqrt{-g}\left(R-\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}e^{-\Phi}|H_3|^3\right)$$
$$S_{\rm R}=-\frac{1}{4\kappa^2}\int\ d^{10}x\sqrt{-g}\left(e^{2\Phi}|F_1|^2+e^{\Phi}\left|\tilde{F}_3\right|^2+\frac{1}{2}\left|\tilde{F}_5\right|^2\right)$$
$$S_{\rm CS}=-\frac{1}{4\kappa^2}\int\ C_4\wedge H_3\wedge F_3$$

$$\begin{aligned}&-\frac{1}{12}H^T_{\mu\nu\rho}\mathcal{M}H^{\mu\nu\rho}+\frac{1}{4}\mathrm{tr}\left(\partial^\mu\mathcal{M}\partial_\mu\mathcal{M}^{-1}\right)\\&=-\frac{1}{2}e^{\Phi}(|\tau|^2|H_3|^2+|F_3|^2-2C_0F\cdot H)-\frac{1}{2}\left(\partial^\mu\Phi\partial_\mu\Phi+e^{2\Phi}\partial^\mu C_0\partial_\mu C_0\right)\\&=-\frac{1}{2}\left(e^{-\Phi}|H_3|^2+e^{\Phi}(F_3-C_0H_3)^2\right)-\frac{1}{2}\left(\partial^\mu\Phi\partial_\mu\Phi+e^{2\Phi}\partial^\mu C_0\partial_\mu C_0\right)\end{aligned}$$

$$S_\theta=\frac{\theta}{16\pi^2}\int\ F^a\wedge F^a$$

$$\tau=\frac{\theta}{2\pi}+i\frac{4\pi}{g^2_{\rm YM}}$$

$$\Phi\rightarrow-\Phi$$

$$g_{\mu\nu}\rightarrow e^{-\Phi}g_{\mu\nu}$$

$$g^{\rm I}_{\rm s}g^{\rm H}_{\rm s}=1.$$

$$T_{\rm D1}=\frac{1}{g_{\rm s}}\frac{1}{2\pi\ell^2_{\rm s}}$$

$$T_{\rm F1}=\frac{1}{2\pi\ell^2_{\rm s}},$$

$$\ell_{\rm s}\rightarrow\ell_{\rm s}\sqrt{g_{\rm s}}$$

$$T_{\rm D5}=\frac{1}{g_{\rm s}(2\pi)^5\ell^6_{\rm s}}$$

$$T_{\rm NS5}=\frac{1}{(g_{\rm s})^2(2\pi)^5\ell^6_{\rm s}}$$

$$\tau=C_0+ie^{-\Phi}$$

$$T_{(p,q)}=|p-q\tau_{\rm B}|T_{\rm F1}=T_{\rm F1}\sqrt{\left(p-q\frac{\theta_0}{2\pi}\right)^2+\frac{q^2}{g^2_s}}$$

$$\tau_{\rm B}=\langle\tau\rangle=\frac{\theta_0}{2\pi}+\frac{i}{g_{\rm s}}$$

$$T_{\rm F1}=T_{(1,0)}=\frac{1}{2\pi\ell^2_{\rm s}}$$

$$T_{\mathrm{D1}} = T_{(0,1)} = \frac{T_{\mathrm{F1}}}{g_{\mathrm{s}}}$$

$$T_{(p_1+p_2,q_1+q_2)} \leq T_{(p_1,q_1)} + T_{(p_2,q_2)}$$

$$\sum_i p^{(i)} = \sum_i q^{(i)} = 0$$

$$M_{11}^2 = -p_M p^M = 0, M = 0,1, \dots, 9,11$$

$$M_{10}^2 = -p_\mu p^\mu = p_{11}^2, \mu = 0,1, \dots, 9$$

$$(M_N)^2 = (N/R_{11})^2 \text{ with } N \in \mathbb{Z}$$

$$R_{11} = \ell_{\mathrm{s}} g_{\mathrm{s}}$$

$$T_{\mathrm{F1}} = 2\pi R_{11} T_{\mathrm{M2}}$$

$$T_{\mathrm{F1}} = \frac{1}{2\pi \ell_{\mathrm{s}}^2} \text{ and } T_{\mathrm{M2}} = \frac{2\pi}{\left(2\pi \ell_{\mathrm{p}}\right)^3}$$

$$ds_5^2 = -dt^2 + ds_{\mathrm{TN}}^2$$

$$ds_{\mathrm{TN}}^2 = V(r)(dr^2 + r^2 d\Omega_2^2) + \frac{1}{V(r)}(dy + R\sin^2(\theta/2)d\phi)^2$$

$$V(r) = 1 + \frac{R}{2r}$$

$$\vec{B} = -\vec{\nabla} V = \vec{\nabla} \times \vec{A} \text{ with } A_\phi = R\sin^2(\theta/2),$$

$$\tilde{R}(r) = V(r)^{-1/2} R,$$

$$T_{\mathrm{D6}} = \frac{2\pi R}{16\pi G_{11}} \int d^3x \nabla^2 V$$

$$T_{\mathrm{D6}} = \frac{(2\pi R)^2}{16\pi G_{11}} = \frac{2\pi R}{16\pi G_{10}} = \frac{2\pi}{(2\pi \ell_{\mathrm{s}})^7 g_{\mathrm{s}}}$$

$$ds^2 = V(\vec{x}) d\vec{x} \cdot d\vec{x} + \frac{1}{V(\vec{x})}(dy + \vec{A} \cdot d\vec{x})^2$$

$$\vec{B} = -\vec{\nabla} V = \vec{\nabla} \times \vec{A} \text{ and } V(\vec{x}) = 1 + \frac{R}{2} \sum_{\alpha=1}^{N} \frac{1}{|\vec{x} - \vec{x}_\alpha|}$$

$$x^{11} \to -x^{11} \text{ and } A_3 \to -A_3.$$

$$B_{\mu\nu} = A_{\mu\nu 11}$$

$$g_{\mathrm{s}} = R_{11}/\ell_{\mathrm{s}}$$

$$T_{\mathrm{H}} = 2\pi R_{11} T_{\mathrm{M2}} = (2\pi \ell_{\mathrm{s}}^{2})^{-1}$$

$$\frac{T_{\mathrm{NS5}}}{T_{\mathrm{D2}}^{2}} = \frac{1}{2\pi}$$

$$T_{\mathrm{NS5}} \to T_{\mathrm{KK5}} \text{ and } T_{\mathrm{D2}} \to 2\pi R_9 T_{\mathrm{D3}}.$$

$$T_{\mathrm{KK5}} = \frac{1}{2\pi}(2\pi R_9 T_{\mathrm{D3}})^2 = \frac{R_9^2}{g_{\mathrm{s}}^2 (2\pi)^5 \ell_{\mathrm{s}}^8}$$

$$R_9' = \ell_{\mathrm{s}}^2 / R_9$$

$$g_{\mathrm{s}}' = \ell_{\mathrm{s}} g_{\mathrm{s}} / R_9$$

$$g_{\mathrm{s}}'' = \frac{1}{g_{\mathrm{s}}'} = \frac{R_9}{\ell_{\mathrm{s}} g_{\mathrm{s}}}$$

$$R_9'' = R_9' \sqrt{g_{\mathrm{s}}''} = R_9' / \sqrt{g_{\mathrm{s}}'} = (\ell_{\mathrm{s}})^{3/2} / \sqrt{R_9 g_{\mathrm{s}}}.$$

$$R_9''' = \ell_{\mathrm{s}}^2 / R_9'' = \sqrt{R_9 \ell_{\mathrm{s}} g_{\mathrm{s}}}$$

$$g_{\mathrm{s}}''' = \ell_{\mathrm{s}} g_{\mathrm{s}}'' / R_9'' = (R_9/\ell_{\mathrm{s}})^{3/2} g_{\mathrm{s}}^{-1/2}$$

$$R_9''' / \ell_{\mathrm{p}} = (g_{\mathrm{s}})^{2/3}$$

$$R_{11} = \frac{R_9^2}{R_9'''}$$

$$M_{\mathrm{B}}^2 = \left(\frac{K}{R_{\mathrm{B}}}\right)^2 + \left(2\pi R_{\mathrm{B}} W T_{(p,q)}\right)^2 + 4\pi T_{(p,q)} (N_{\mathrm{L}} + N_{\mathrm{R}})$$

$$N_{\mathrm{R}} - N_{\mathrm{L}} = KW.$$

$$|W| T_{(p,q)} = |n_1 - n_2 \tau_{\mathrm{B}}| T_{\mathrm{F1}}$$

$$A_{\mathrm{M}} = (2\pi R_{11})^2 \mathrm{Im}\tau_{\mathrm{M}}$$

$$\psi_{n_1,n_2} \sim \exp\left\{\frac{i}{R_{11}}\left[n_2 x - \frac{n_2 \mathrm{Re}\tau_{\mathrm{M}} - n_1}{\mathrm{Im}\tau_{\mathrm{M}}} y\right]\right\}$$

$$M_{\mathrm{KK}}^2 = \frac{1}{R_{11}^2}\left[n_2^2 + \frac{(n_2 \mathrm{Re}\tau_{\mathrm{M}} - n_1)^2}{(\mathrm{Im}\tau_{\mathrm{M}})^2}\right] = \frac{|n_1 - n_2 \tau_{\mathrm{M}}|^2}{(R_{11} \mathrm{Im}\tau_{\mathrm{M}})^2}$$

$$\tau_{\mathrm{M}} = \tau_{\mathrm{B}}$$

$$g^{(\mathrm{M})} = \beta^2 g^{(\mathrm{B})}$$

$$\beta^2 = \frac{2\pi R_{11} T_{\mathrm{M2}}}{T_{\mathrm{F1}}},$$

$$\frac{g_s^2}{T_{F1}R_B^2} = T_{M2}(2\pi R_{11})^3 = T_{M2}\left(\frac{A_M}{\mathrm{Im}\tau_M}\right)^{3/2}.$$

$$L_{(p,q)} = 2\pi R_{11}|p - q\tau_M|.$$

$$T_{(p,q)}^{(11)} = L_{(p,q)}T_{M2}$$

$$T_{(p,q)} = \beta^{-2}T_{(p,q)}^{(11)}$$

$$T_{M2} = 2\pi R_B\beta^3 T_{D3}$$

$$T_{D3} = \frac{(T_{F1})^2}{2\pi g_s},$$

$$T_{M5}A_M = \beta^4 T_{D3}$$

$$T_{M5} = \frac{1}{2\pi}(T_{M2})^2.$$

$$g_s^{(HO)} = \frac{L_1}{L_2}$$

$$L_1 L_2^2 T_{M2} = \left(\frac{T_1^{(HO)}L_O^2}{2\pi}\right)^{-1}$$

$$T_5^{(HO)} = \frac{1}{(2\pi)^2}\left(\frac{L_2}{L_1}\right)^2\left(T_1^{(HO)}\right)^3$$

$$T_5^{(HO)} \sim \left(g_s^{(HO)}\right)^{-2}$$

$$T_{D1} \sim 1/g_s^{(I)} \text{ and } T_{D5} \sim 1/g_s^{(I)},$$

$$E_{5,5} = SO(5,5), E_{4,4} = SL(5,\mathbb{R}), E_{3,3} = SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$$

$$E_3(\mathbb{Z}) = SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z})$$

$$SO(2,2;\mathbb{Z}) = SL(2,\mathbb{Z}) \times SL(2,\mathbb{Z})$$

$$\mathcal{M}_n^0 = E_{n,n}/H_n$$

$$d_3 = \dim E_8 - \dim \mathrm{Spin}(16) = 248 - 120 = 128$$
$$d_4 = \dim E_7 - \dim SU(8) = 133 - 63 = 70$$

$$d_5 = \dim E_6 - \dim USp(8) = 78 - 36 = 42.$$

$$\mathcal{M}_n = \mathcal{M}_n^0/E_n(\mathbb{Z})$$

$$R_i \rightarrow R_i' = \frac{\ell_s^2}{R_i} = \frac{\ell_p^3}{R_3 R_i} \quad i = 1,2$$

$$\frac{1}{g_8^2} = 4\pi^2 \frac{R_1 R_2}{g_s^2} = 4\pi^2 \frac{R_1' R_2'}{(g_s')^2}$$

$$g_s' = \frac{g_s \ell_s^2}{R_1 R_2}$$

$$R_3' = \frac{g_s \ell_s^3}{R_1 R_2} = \frac{\ell_p^3}{R_1 R_2}$$

$$\ell_p^3 = g_s \ell_s^3 \rightarrow \left(\ell_p'\right)^3 = g_s' \ell_s^3$$

$$\left(\ell_p'\right)^3 = \frac{g_s \ell_s^5}{R_1 R_2} = \frac{\ell_p^6}{R_1 R_2 R_3}$$

$$R_1 \rightarrow \frac{\ell_p^3}{R_2 R_3}$$

$$\ell_p^3 \rightarrow \frac{\ell_p^6}{R_1 R_2 R_3}$$

$$g^{(\mathrm{M})} = \beta^2 g^{(\mathrm{B})},$$

$$M_{11} = \beta M_{\mathrm{B}}.$$

$$A_{\mathrm{M}} T_{\mathrm{M2}} = \beta \frac{1}{R_{\mathrm{B}}}.$$

$$\frac{1}{R_{11} \mathrm{Im}\tau_{\mathrm{M}}} = \beta(2\pi R_{\mathrm{B}} T_{\mathrm{F1}}).$$

$$\beta^2 = \frac{R_{\mathrm{B}} A_{\mathrm{M}} T_{\mathrm{M2}}}{2\pi R_{\mathrm{B}} T_{\mathrm{F1}} R_{11} \mathrm{Im}\tau_{\mathrm{M}}} = \frac{2\pi R_{11} T_{\mathrm{M2}}}{T_{\mathrm{F1}}},$$

$$\frac{g_s^2}{R_{\mathrm{B}}^2 T_{\mathrm{F1}}} = T_{\mathrm{M2}} (2\pi R_{11})^3,$$

$$2\pi R_{\mathrm{B}} \beta^5 T_{(p,q)} = L_{(p,q)} T_{\mathrm{M5}},$$

$$T_{\mathrm{D5}} = \frac{R_{11}}{R_{\mathrm{B}}} \beta^{-5} T_{\mathrm{M5}} = \frac{T_{\mathrm{F1}}^3}{(2\pi)^2 g_s} = \frac{1}{(2\pi)^5 \ell_s^6 g_s},$$

$$T_{(p,q)} = |p - q\tau_{\mathrm{M}}| T_{\mathrm{D5}}$$

$$T_{\mathrm{NS5}} = |\tau_{\mathrm{M}}| T_{\mathrm{D5}}$$

$$\frac{1}{R_1} \rightarrow (2\pi R_2)(2\pi R_3) T_{\mathrm{M2}}$$

$$T_{\mathrm{M2}} \rightarrow (2\pi R_1)(2\pi R_2)(2\pi R_3) T_{\mathrm{M5}}$$

$$F_4 = M\varepsilon_4,$$

$$R_{\mu\nu} = -(M_4)^2 g_{\mu\nu}\ \mu, \nu = 0,1,2,3.$$

$$R_{ij} = (M_7)^2 g_{ij}\ i, j = 4,5, \dots, 10$$

$$ds^2 = d\rho^2 + \frac{\rho^2}{4}(d\theta^2 + d\phi^2 + d\psi^2 - 2\cos\ \theta d\phi d\psi)$$

$$ds = -dt^2 + \sum_{i=1}^{5} dx_i^2 + ds_{\mathrm{TN}}^2$$

$$dF_4 = \delta_W$$

$$X^\mu(\sigma + 2\pi) = gX^\mu(\sigma)$$

$$g: z^a \to e^{i\phi^a} z^a, a = 1, \dots, n$$

$$g: Q_\alpha \to \exp\left(i \sum_{a=1}^{n} \varepsilon_\alpha^a \phi^a\right) Q_\alpha$$

$$\frac{1}{2\pi} \sum_{a=1}^{n} \phi^a = 0 \bmod N$$

- Calabi - Yau – Kähler:

$$c_1 = \frac{1}{2\pi}[\mathcal{R}] = 0$$

$$\Omega(z^1, z^2, \dots, z^n) = f(z^1, z^2, \dots, z^n) dz^1 \wedge dz^2 \cdots \wedge dz^n$$

$$b_k = \sum_{p=0}^{k} h^{p,k-p}$$

$$h^{p,0} = h^{n-p,0}$$

$$h^{p,q} = h^{q,p}$$

$$h^{p,q} = h^{n-q,n-p}$$

$$h^{1,0} = h^{0,1} = 0$$

$$\begin{matrix} h^{3,3} \\ h^{3,2} h^{2,3} \\ h^{3,0} h^{2,2} h^{2,1} h^{1,3} h^{1,2} h^{0,3} \\ h^{2,0} h^{1,1} h^{0,2} \\ h^{1,0} h^{0,1} \\ h^{0,0} \end{matrix} = \begin{matrix} & 1 & \\ 0 & 0 & \\ 1 & 0 & h^{2,1} h^{1,1} \\ 0 & h^{1,1} & 0 \\ 0 & 0 & \\ & 1 & \end{matrix}$$

$$\chi = \sum_{p=0}^{6} (-1)^p b_p = 2(h^{1,1} - h^{2,1}).$$

$$1$$

$$0 \quad 0 \quad 0$$

$$0 \quad h^{1,1} \quad 0$$

$$1 \quad h^{2,1}h^{3,1}h^{2,1} \quad 0$$

$$0 \quad h^{2,1}h^{3,1} \quad 1$$

$$0 \quad h^{2,1} \quad 0$$

$$0 \quad 0$$

$$h^{1,1} \quad 0$$

$$1$$

$$h^{2,2} = 2(22 + 2h^{1,1} + 2h^{1,3} - h^{1,2}).$$

$$\chi = \sum_{p=0}^{8} (-1)^p b_p = 6(8 + h^{1,1} + h^{3,1} - h^{2,1}).$$

$$ds^2 = |dz|^2$$

$$\Omega = dz$$

$$z^a \sim z^a + 1 \; z^a \sim z^a + i, a = 1,2$$

$$\mathcal{I}: (z^1, z^2) \to (-z^1, -z^2)$$

$$0, \frac{1}{2}, \frac{i}{2}, \frac{1+i}{2}$$

$$ds^2 = \Delta^{-1} dr^2 + \frac{1}{4} r^2 \Delta (d\psi + \cos\,\theta d\phi)^2 + \frac{1}{4} r^2 d\Omega_2^2$$

- Eguchi-Hanson:

$$J = \frac{1}{2} r dr \wedge (d\psi + \cos\,\theta d\phi) - \frac{1}{4} r^2 \sin\,\theta d\theta \wedge d\phi$$

$$z_1 = r\cos\,(\theta/2)\exp\left[\frac{i}{2}(\psi + \phi)\right] \text{ and } z_2 = r\sin\,(\theta/2)\exp\left[\frac{i}{2}(\psi - \phi)\right]$$

$$\mathcal{K} = \log\left[\frac{r^2 \exp\,(r^4 + a^4)^{1/2}}{a^2 + (r^4 + a^4)^{1/2}}\right]$$

$$dz^1 \wedge d\bar{z}^1, dz^2 \wedge d\bar{z}^2, dz^1 \wedge d\bar{z}^2, dz^2 \wedge d\bar{z}^1$$

$$(z^1, z^2, \ldots, z^{n+1}) \sim (\lambda z^1, \lambda z^2, \ldots, \lambda z^{n+1})$$

$$K = \log\left(1 + \sum_{a=1}^{n} |w^a|^2\right)$$

$$G(\lambda z^1, \ldots, \lambda z^{n+2}) = \lambda^k G(z^1, \ldots, z^{n+2}).$$

$$G(z^1, \ldots, z^{n+2}) = 0$$

$$c_1(X) \sim [k - (n+2)] c_1(\mathbb{C}P^{n+1}).$$

$$\sum_{a=1}^{4} (z^a)^4 = 0$$

$$\sum_{a=1}^{5} (z^a)^5 = 0$$

$$h^{1,1} = 1 \text{ and } h^{2,1} = 101$$

$$\sum_{a=2}^{5} (w^a)^4 dw^a = 0$$

$$\Omega = \frac{dw^2 \wedge dw^3 \wedge dw^4}{(w^5)^4}$$

$$(\lambda^{k_1} z^1, \lambda^{k_2} z^2, \dots, \lambda^{k_{n+1}} z^{n+1}) \sim \lambda^N (z^1, z^2, \dots, z^{n+1})$$

$$g_{xx} = g_{yy} = 2g_{z\bar{z}}, g_{xy} = 0$$

$$J = i g_{z\bar{z}} dz \wedge d\bar{z} = 2g_{z\bar{z}} dx \wedge dy = \sqrt{g} dx \wedge dy$$

$$\frac{1}{6} J \wedge J \wedge J = \sqrt{g} dx^1 \wedge \cdots \wedge dy^3$$

$$V = \frac{1}{6} \int J \wedge J \wedge J$$

$$z^a \sim z^a + 1 \sim z^a + e^{\pi i/3} \; a = 1,2$$

$$0, \frac{1}{\sqrt{3}} e^{\pi i/6}, \frac{2}{\sqrt{3}} e^{\pi i/6}$$

$$1,\ dz^1 \wedge dz^2, d\bar{z}^1 \wedge d\bar{z}^2, dz^1 \wedge d\bar{z}^1, dz^2 \wedge d\bar{z}^2, dz^1 \wedge dz^2 \wedge d\bar{z}^1 \wedge d\bar{z}^2.$$

$$h^{2,2} = h^{0,0} = h^{2,0} = h^{0,2} = 1, h^{1,1} = 2 + 9 \times 2 = 20,$$

$$x^{n+1} + xy^2 + z^2 = 0.$$

$$x = (z^1 z^2)^2, y = \frac{i}{2}((z^1)^{2n} + (z^2)^{2n}), z = \frac{1}{2}((z^1)^{2n} - (z^2)^{2n}) z^1 z^2$$

$$x^{n+1} = (z^1 z^2)^{2n+2}$$
$$xy^2 = -\frac{1}{4}((z^1)^{4n} + (z^2)^{4n} + 2(z^1 z^2)^{2n})(z^1 z^2)^2$$
$$z^2 = \frac{1}{4}((z^1)^{4n} + (z^2)^{4n} - 2(z^1 z^2)^{2n})(z^1 z^2)^2$$

$$x^{n+1} + xy^2 + z^2 = 0$$

$$M_{10} = M_4 \times M$$

$$R_{\mu\nu\rho\lambda} = \frac{R}{12}\left(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\rho}\right)$$

$$H_{\mu\nu\rho} = H_{\mu\nu p} = H_{\mu np} = 0 \text{ and } F_{\mu\nu} = F_{\mu n} = 0.$$

$$\delta\Psi_M = \nabla_M\varepsilon - \frac{1}{4}\mathbf{H}_M\varepsilon$$
$$\delta\lambda = -\frac{1}{2}\partial\Phi\varepsilon + \frac{1}{4}\mathbf{H}\varepsilon$$
$$\delta\chi = -\frac{1}{2}\mathbf{F}\varepsilon$$

$$dH = \frac{\alpha'}{4}\left[\mathrm{tr}(R\wedge R) - \mathrm{tr}(F\wedge F)\right]$$

$$\delta\Psi_M = \nabla_M\varepsilon$$

$$\nabla_M\varepsilon = 0$$

$$\varepsilon(x,y) = \zeta(x)\otimes\eta(y)$$

- Grassmann:

$$\nabla_\mu\zeta = 0$$

$$[\nabla_\mu, \nabla_\nu]\zeta = \frac{1}{4}R_{\mu\nu\rho\sigma}\Gamma^{\rho\sigma}\zeta = 0$$

$$\nabla_m\eta = 0$$

$$[\nabla_m, \nabla_n]\eta = \frac{1}{4}R_{mnpq}\Gamma^{pq}\eta = 0$$

$$R_{mn} = 0$$

$$8 = 4\oplus\overline{4}$$

$$\mathbf{4} = \mathbf{3}\oplus\mathbf{1}$$

$$\mathbf{16} = (\mathbf{2},\mathbf{4})\oplus(\overline{\mathbf{2}},\overline{\mathbf{4}})$$

$$\varepsilon(x,y) = \zeta_+\otimes\eta_+(y) + \zeta_-\otimes\eta_-(y),$$

$$\eta_- = \eta_+^* \text{ and } \zeta_- = \zeta_+^*$$

$$\Gamma_\mu = \gamma_\mu\otimes 1 \text{ and } \Gamma_m = \gamma_5\otimes\gamma_m$$

$$\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$$

- Dirac – Fierz - Nijenhuis.

$$\sigma_2\otimes 1\otimes\sigma_{1,3}\ \sigma_{1,3}\otimes\sigma_2\otimes 1\ 1\otimes\sigma_{1,3}\otimes\sigma_2$$

$$\gamma_7 = \sigma_2\otimes\sigma_2\otimes\sigma_2$$

$$P_{\pm} = (1 \pm \gamma_7)/2.$$

$$\eta_+^\dagger \eta_+ = \eta_-^\dagger \eta_- = 1$$

$$J_m^n = i\eta_+^\dagger \gamma_m^n \eta_+ = -i\eta_-^\dagger \gamma_m^n \eta_-$$

$$J_m{}^n J_n{}^p = -\delta_m{}^p$$

$$\nabla_m J_n^p = 0$$

$$N^p{}_{mn} = 0$$

$$J_a^b = i\delta_a^b, J_{\bar{a}}^{\bar{b}} = -i\delta_{\bar{a}}^{\bar{b}} \text{ and } J_a^{\bar{b}} = J_{\bar{a}}^b = 0$$

$$g_{mn} = J_m^k J_n^l g_{kn}$$

$$J_{mn} = J_m{}^k g_{kn}$$

$$J = \frac{1}{2} J_{mn} dx^m \wedge dx^n$$

$$J_{a\bar{b}} = i g_{a\bar{b}}$$

$$dJ = \partial J + \bar{\partial} J = i\partial_a g_{b\bar{c}} dz^a \wedge dz^b \wedge dz^{\bar{c}} + i\partial_{\bar{a}} g_{b\bar{c}} dz^{\bar{a}} \wedge dz^b \wedge dz^{\bar{c}} = 0$$

$$\Omega_{abc} = \eta_-^T \gamma_{abc} \eta_-$$

$$\Omega = \frac{1}{6} \Omega_{abc} dz^a \wedge dz^b \wedge dz^c$$

$$\Omega_{abc} = f(z) \varepsilon_{abc}$$

$$\|\Omega\|^2 = \frac{1}{3!} \Omega_{abc} \bar{\Omega}^{abc},$$

$$\sqrt{g} = \frac{|f|^2}{\|\Omega\|^2}$$

$$\mathcal{R} = i\partial\bar{\partial} \log \sqrt{g} = -i\partial\bar{\partial} \log \|\Omega\|^2$$

$$Q|\Psi\rangle = 0$$

$$H = Q^\dagger Q$$

$$H|\Psi\rangle = 0$$

$$\gamma_7 = (1 - \gamma_{\bar{1}}\gamma_1)(1 - \gamma_{\bar{2}}\gamma_2)(1 - \gamma_{\bar{3}}\gamma_3) = -(1 - \gamma_1\gamma_{\bar{1}})(1 - \gamma_2\gamma_{\bar{2}})(1 - \gamma_3\gamma_{\bar{3}})$$

$$[\nabla_m, \nabla_n]\eta = \frac{1}{4} R_{mnpq} \Gamma^{pq} \eta$$

$$\nabla_n \eta = \partial_n \eta + \frac{1}{4} \omega_{npq} \gamma^{pq} \eta$$

$$[\nabla_m, \nabla_n]\eta = \left[\partial_m + \frac{1}{4}\omega_{mpq}\gamma^{pq}, \partial_n + \frac{1}{4}\omega_{nrs}\gamma^{rs}\right]\eta.$$

$$\frac{1}{4}(\partial_m\omega_{nrs} - \partial_n\omega_{mrs})\gamma^{rs}\eta + \frac{1}{16}\omega_{mpq}\omega_{nrs}[\gamma^{pq}, \gamma^{rs}]\eta$$

$$\frac{1}{4}\left(\partial_m\omega_{nrs} - \partial_n\omega_{mrs} + \omega_{mrp}\omega_n{}^p{}_s - \omega_{nrp}\omega_m{}^p{}_s\right)\gamma^{rs}\eta = \frac{1}{4}R_{mnrs}\gamma^{rs}\eta$$

$$[\gamma_{rs}, \gamma^{pq}] = -8\delta^{[p}_{[r}\gamma^{q]}_{s]}$$

$$\gamma^n\gamma^{pq}R_{mnpq}\eta = 0$$

$$\gamma^n\gamma^{pq} = \gamma^{npq} + g^{np}\gamma^q - g^{nq}\gamma^p$$

$$\gamma^{npq}R_{mnpq} = \gamma^{npq}R_{m[npq]} = 0$$

$$2g^{nq}\gamma^pR_{mnpq}\eta = 0$$

$$i\eta_-^T\gamma_q\gamma^p\eta_+R_{mp} = J_q^pR_{mp} = 0$$

$$\Omega = \frac{1}{6}\Omega_{a_1a_2a_3}dz^{a_1} \wedge dz^{a_2} \wedge dz^{a_3}$$

$$\frac{1}{36}dz^{a_1} \wedge dz^{a_2} \wedge dz^{a_3} \wedge d\bar{z}^{\bar{b}_1} \wedge d\bar{z}^{\bar{b}_2} \wedge d\bar{z}^{\bar{b}_3}\Omega_{a_1a_2a_3}\bar{\Omega}_{\bar{b}_1\bar{b}_2\bar{b}_3}$$
$$= -\frac{i}{36}J \wedge J \wedge J\left(\Omega_{a_1a_2a_3}\bar{\Omega}_{\bar{b}_1\bar{b}_2\bar{b}_3}g^{a_1\bar{b}_1}g^{a_2\bar{b}_2}g^{a_3\bar{b}_3}\right)$$

$$\nabla_m\|\Omega\|^2 = 0$$

$$\|\Omega\|^2 = \frac{1}{6}g^{a_1\bar{b}_1}g^{a_2\bar{b}_2}g^{a_3\bar{b}_3}\Omega_{a_1a_2a_3}\bar{\Omega}_{\bar{b}_1\bar{b}_2\bar{b}_3}.$$

$$\int d^{10}x\sqrt{-g}\left|F_p\right|^2$$

$$\Delta B_{p-1} = d \star dB_{p-1} = 0$$

$$\Delta = \Delta_4 + \Delta_6$$

$$g_{mn} \to g_{mn} + \delta g_{mn}$$

$$R_{mn}(g + \delta g) = 0.$$

$$\tau = i\frac{R_2}{R_1} \text{ and } \rho = iR_1R_2$$

$$\Omega = dz$$

$$\tau = \frac{\int_A \Omega}{\int_B \Omega},$$

$$R_{mn}(g) = 0 \text{ and } R_{mn}(g + \delta g) = 0.$$

$$\nabla^m \delta g_{mn} = \frac{1}{2} \nabla_n \delta g^m_m$$

$$\nabla^k \nabla_k \delta g_{mn} + 2R_m{}^p{}_n{}^q \delta g_{pq} = 0$$

$$\delta g_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$$

$$\int_{M_r} \underbrace{J \wedge \cdots \wedge J}_{r-\text{ times}} > 0, r = 1,2,3$$

$$\mathcal{J} = B + iJ.$$

$$\Omega_{abc} g^{c\bar{d}} \delta g_{\bar{d}\bar{e}} dz^a \wedge dz^b \wedge d\bar{z}^{\bar{e}}$$

$$ds^2 = \frac{1}{2V} \int g^{a\bar{b}} g^{c\bar{d}} [\delta g_{ac} \delta g_{\bar{b}\bar{d}} + (\delta g_{a\bar{d}} \delta g_{c\bar{b}} - \delta B_{a\bar{d}} \delta B_{c\bar{b}})] \sqrt{g} d^6 x$$

$$\mathcal{M}(M) = \mathcal{M}^{2,1}(M) \times \mathcal{M}^{1,1}(M)$$

$$\chi_\alpha = \frac{1}{2} (\chi_\alpha)_{ab\bar{c}} dz^a \wedge dz^b \wedge d\bar{z}^{\bar{c}} \text{ with } (\chi_\alpha)_{ab\bar{c}} = -\frac{1}{2} \Omega_{ab} \frac{\bar{d}}{\partial t^\alpha}$$

$$\delta g_{\bar{a}\bar{b}} = -\frac{1}{\|\Omega\|^2} \bar{\Omega}^{cd}_{\bar{a}} (\chi_\alpha)_{cd\bar{b}} \delta t^\alpha, \text{ where } \|\Omega\|^2 = \frac{1}{6} \Omega_{abc} \bar{\Omega}^{abc}$$

$$ds^2 = 2G_{\alpha\bar{\beta}} \delta t^\alpha \delta \bar{t}^{\bar{\beta}}$$

$$G_{\alpha\bar{\beta}} \delta t^\alpha \delta \bar{t}^{\bar{\beta}} = -\left(\frac{i \int \chi_\alpha \wedge \bar{\chi}_{\bar{\beta}}}{i \int \Omega \wedge \bar{\Omega}} \right) \delta t^\alpha \delta \bar{t}^{\bar{\beta}}$$

$$\partial_\alpha \Omega = K_\alpha \Omega + \chi_\alpha$$

$$\mathcal{K}^{2,1} = -\log \left(i \int \Omega \wedge \bar{\Omega} \right)$$

$$A^I \cap B_J = -B_J \cap A^I = \delta^I_J \text{ and } A^I \cap A^J = B_I \cap B_J = 0.$$

$$\int_{A^J} \alpha_I = \int \alpha_I \wedge \beta^J = \delta^J_I \text{ and } \int_{B_J} \beta^I = \int \beta^I \wedge \alpha_J = -\delta^I_J$$

$$X^I = \int_{A^I} \Omega \text{ with } I = 0, \dots, h^{2,1}$$

$$t^\alpha = \frac{X^\alpha}{X^0} \text{ with } \alpha = 1, \dots, h^{2,1}$$

$$F_I = \int_{B_I} \Omega$$

$$\Omega = X^I \alpha_I - F_I(X)\beta^I$$

$$\int \Omega \wedge \partial_I \Omega = 0$$

$$F_I = X^J \frac{\partial F_J}{\partial X^I} = \frac{1}{2}\frac{\partial}{\partial X^I}\left(X^J F_J\right)$$

$$F_I = \frac{\partial F}{\partial X^I} \quad \text{where} \quad F = \frac{1}{2} X^I F_I$$

$$2F = X^I \frac{\partial F}{\partial X^I}$$

$$F(\lambda X) = \lambda^2 F(X)$$

$$\int_M \alpha \wedge \beta = -\sum_I \left(\int_{A^I} \alpha \int_{B_I} \beta - \int_{A^I} \beta \int_{B_I} \alpha\right)$$

$$e^{-\mathcal{K}^{2,1}} = -i \sum_{I=0}^{h^{2,1}} \left(X^I \bar{F}_I - \bar{X}^I F_I\right)$$

$$\Omega \to e^{f(X)}\Omega$$

$$\mathcal{K}^{2,1} \to \mathcal{K}^{2,1} - f(X) - \bar{f}(\bar{X})$$

$$K_\alpha = -\partial_\alpha \mathcal{K}^{2,1}$$

$$\mathcal{D}_\alpha = \partial_\alpha + \partial_\alpha \mathcal{K}^{2,1}$$

$$\chi_\alpha = \mathcal{D}_\alpha \Omega$$

$$G(\rho,\sigma) = \frac{1}{2V}\int_M \rho_{a\bar{d}}\sigma_{\bar{b}c} g^{a\bar{b}} g^{c\bar{d}} \sqrt{g}\, d^6x = \frac{1}{2V}\int_M \rho \wedge\star\, \sigma,$$

$$\kappa(\rho,\sigma,\tau) = \int_M \rho \wedge \sigma \wedge \tau,$$

$$\star\, \sigma = -J \wedge \sigma + \frac{1}{4V}\kappa(\sigma,J,J) J \wedge J$$

$$G(\rho,\sigma) = -\frac{1}{2V}\kappa(\rho,\sigma,J) + \frac{1}{8V^2}\kappa(\rho,J,J)\kappa(\sigma,J,J).$$

$$\mathcal{J} = B + iJ = w^\alpha e_\alpha \quad \text{with} \quad \alpha = 1,\dots,h^{1,1}$$

$$G_{\alpha\bar{\beta}} = \frac{1}{2} G\left(e_\alpha, e_\beta\right) = \frac{\partial}{\partial w^\alpha}\frac{\partial}{\partial \bar{w}^{\bar{\beta}}} \mathcal{K}^{1,1},$$

$$e^{-\mathcal{K}^{1,1}} = \frac{4}{3}\int J \wedge J \wedge J = 8V$$

$$\kappa_{\alpha\beta\gamma} = \kappa(e_\alpha, e_\beta, e_\gamma) = \int e_\alpha \wedge e_\beta \wedge e_\gamma$$

$$G(w) = \frac{1}{6}\frac{\kappa_{\alpha\beta\gamma} w^\alpha w^\beta w^\gamma}{w^0} = \frac{1}{6w^0}\int \mathcal{J} \wedge \mathcal{J} \wedge \mathcal{J},$$

$$e^{-\mathcal{K}^{1,1}} = i\sum_{A=0}^{h^{1,1}} \left(w^A \frac{\partial \bar{G}}{\partial \bar{w}^A} - \bar{w}^A \frac{\partial G}{\partial w^A} \right)$$

$$\delta w^\alpha = \varepsilon^\alpha.$$

$$G(w) = \frac{\kappa_{ABC} w^A w^B w^C}{w^0} + i\mathcal{Y}(w^0)^2$$

$$\mathcal{Y} = \frac{\zeta(3)}{2(2\pi)^3}\chi(M)$$

- Peccei-Quinn:

$$\exp\left(-\frac{c_\alpha w^\alpha}{\alpha' w^0}\right),$$

$$\mathcal{K} = -\log\left(i\int \Omega \wedge \bar{\Omega}\right) \text{ and } \Omega = dz$$

$$g = \frac{1}{\tau_2}\begin{pmatrix} \tau_1^2 + \tau_2^2 & \tau_1 \\ \tau_1 & 1 \end{pmatrix}$$

$$ds^2 = \frac{1}{\tau_2}[(\tau_1^2 + \tau_2^2)dx^2 + 2\tau_1 dxdy + dy^2] = 2g_{z\bar{z}}dzd\bar{z}$$

$$dz = dy + \tau dx \text{ and } g_{z\bar{z}} = \frac{1}{2\tau_2}$$

$$\mathcal{K} = -\log\left(i\int dz \wedge d\bar{z}\right) = -\log(2\tau_2)$$

$$G_{\tau\bar{\tau}} = \partial_\tau \partial_{\bar{\tau}}\mathcal{K} = \frac{1}{4\tau_2^2}$$

$$\delta g_{zz} = \frac{d\tau}{2\tau_2^2} \text{ and } \delta g_{\bar{z}\bar{z}} = \frac{d\bar{\tau}}{2\tau_2^2}$$

$$ds^2 = 2G_{\tau\bar{\tau}}d\tau d\bar{\tau} = \frac{1}{2V}\int (g^{z\bar{z}})^2 \delta g_{zz}\delta g_{\bar{z}\bar{z}}\sqrt{g}d^2x = \frac{d\tau d\bar{\tau}}{2\tau_2^2}$$

$$\Omega = \frac{1}{6}\Omega_{abc}dz^a \wedge dz^b \wedge dz^c$$

$$\partial_a\Omega = \frac{1}{6}\frac{\partial\Omega_{abc}}{\partial t^\alpha}dz^a\wedge dz^b\wedge dz^c + \frac{1}{2}\Omega_{abc}dz^a\wedge dz^b\wedge\frac{\partial(dz^c)}{\partial t^\alpha}$$

$$\partial\Omega/\partial t^\alpha \in H^{(3,0)} \bigoplus H^{(2,1)}$$

$$z^c(t^\alpha+\delta t^\alpha) = z^c(t^\alpha) + M^c_\alpha\delta t^\alpha$$

$$\frac{\partial(dz^c)}{\partial t^\alpha} = dM^c_\alpha = \frac{\partial M^c_\alpha}{\partial z^d}dz^d + \frac{\partial M^c_\alpha}{\partial\bar{z}^{\bar{d}}}d\bar{z}^{\bar{d}}$$

$$\frac{1}{2}\Omega_{abc}\frac{\partial M^c_\alpha}{\partial\bar{z}^{\bar{d}}}dz^a\wedge dz^b\wedge d\bar{z}^{\bar{d}}$$

$$\chi_\alpha = -\frac{1}{4}\Omega_{abc}g^{c\bar{e}}\left(\frac{\partial g_{\bar{d}\bar{e}}}{\partial t^\alpha}\right)dz^a\wedge dz^b\wedge d\bar{z}^{\bar{d}}$$

$$\frac{\partial M^c_\alpha}{\partial\bar{z}^{\bar{d}}} = -\frac{1}{2}g^{c\bar{e}}\left(\frac{\partial g_{\bar{d}\bar{e}}}{\partial t^\alpha}\right).$$

$$\frac{\partial g_{\bar{d}\bar{e}}}{\partial t^\alpha} = -2g_{c\bar{e}}\frac{\partial M^c_\alpha}{\partial\bar{z}^{\bar{d}}}$$

$$\mathcal{M}^{1,1}(M)\times\mathcal{M}^{2,1}(M).$$

$$\mathcal{M}^{1,1}(W)\times\mathcal{M}^{2,1}(W)$$

$$\{G_{MN}, A_{MNP}, \Psi_M\}$$

gravity multiplet : $G_{\mu\nu}, A_{ijk}, A_{\mu\nu\rho}$, fermions
$h^{1,1}$ vector multiplets : $A_{\mu j\bar{k}}, G_{j\bar{k}}$, fermions
$h^{2,1}$ hypermultiplets : $A_{ij\bar{k}}, G_{jk}$, fermions.

$$4h^{2,1} + h^{1,1} + 3$$

$$\left\{G_{MN}, B_{MN}, \Phi, C_M, C_{MNP}, \Psi^{(+)}_M, \Psi^{(-)}_M, \Psi^{(+)}, \Psi^{(-)}\right\}$$

gravity multiplet : $G_{\mu\nu}, \Psi_\mu, \widetilde{\Psi}_\mu, C_\mu$
$h^{1,1}$ vector multiplets : $C_{\mu i\bar{j}}, G_{i\bar{j}}, B_{i\bar{j}}$, fermions
$h^{2,1}$ hypermultiplets : $C_{ij\bar{k}}, G_{ij}$, fermions
universal hypermultiplet : $C_{ijk}, \Phi, B_{\mu\nu}$, fermions.

$$\left\{G_{MN}, B_{MN}, \Phi, C, C_{MN}, C_{MNPQ}, \Psi^{(+)}_M, \widetilde{\Psi}^{(+)}_M, \Psi^{(-)}, \widetilde{\Psi}^{(-)}\right\}$$

gravity multiplet : $G_{\mu\nu}, \Psi_\mu, \widetilde{\Psi}_\mu, C_{\mu ijk}$
$h^{2,1}$ vector supermultiplets : $C_{\mu ij\bar{k}}, G_{ij}$, fermions
$h^{1,1}$ hypermultiplets : $C_{\mu\nu i\bar{j}}, G_{i\bar{j}}, B_{i\bar{j}}, C_{i\bar{j}}$, fermions
universal hypermultiplet : $\Phi, C, B_{\mu\nu}, C_{\mu\nu}$, fermions.

$$X^1 = \int_{A^1}\Omega$$

$$X^1 \to X^1 \text{ and } F_1 \to F_1 + X^1.$$

$$F_1(X^1) = \text{ const } + \frac{1}{2\pi i} X^1 \log X^1$$

$$\mathcal{K}^{2,1} \sim \log\left(|X^1|^2 \log|X^1|^2\right)$$

$$\delta_\varepsilon \Theta = \varepsilon \text{ and } \delta_\varepsilon X^M = i\bar{\varepsilon}\Gamma^M \Theta$$

$$\delta_\kappa \Theta = 2P_+\kappa(\sigma) \text{ and } \delta_\kappa X^M = 2i\bar{\Theta}\Gamma^M P_+\kappa(\sigma)$$

$$P_\pm = \frac{1}{2}\left(1 \pm \frac{i}{6}\varepsilon^{\alpha\beta\gamma}\partial_\alpha X^M \partial_\beta X^N \partial_\gamma X^P \Gamma_{MNP}\right)$$

$$\delta_\varepsilon \Theta + \delta_\kappa \Theta = \varepsilon + 2P_+\kappa(\sigma) = 0$$

$$P_- \varepsilon = 0$$

$$\partial_{[\alpha} X^a \partial_{\beta]} X^{\bar{b}} J_{a\bar{b}} = 0$$

$$\partial_\alpha X^a \partial_\beta X^b \partial_\gamma X^c \Omega_{abc} = e^{-i\varphi} e^{\mathcal{K}} \varepsilon_{\alpha\beta\gamma}$$

$$\mathcal{K} = \frac{1}{2}(\mathcal{K}^{1,1} - \mathcal{K}^{2,1}),$$

$$\int_\Sigma \varepsilon^\dagger P_-^\dagger P_- \varepsilon d^3\sigma \geq 0$$

$$P_-^\dagger P_- = P_- P_- = P_-$$

$$2\mathcal{V} \geq e^{-\mathcal{K}}\left(e^{i\varphi}\int_\Sigma \Omega + e^{-i\varphi}\int_\Sigma \bar{\Omega}\right),$$

$$\mathcal{V} \geq e^{-\mathcal{K}}\left|\int_\Sigma \Omega\right|.$$

$$\mathcal{V} = e^{-\mathcal{K}}\left|\int_\mathcal{C} \Omega\right|$$

$$M \geq e^{\mathcal{K}^{2,1}/2}\left|\int_\mathcal{C} \Omega\right| = e^{\mathcal{K}^{2,1}/2}\left|\int_M \Omega \wedge \Gamma\right|,$$

$$\Gamma = q^I \alpha_I - p_I \beta^I$$

$$M \geq e^{\mathcal{K}^{2,1}/2}|p_I X^I - q^I F_I|.$$

$$\bar{\partial} X^a = 0 \text{ and } \partial X^{\bar{a}} = 0$$

$$\sum_{m=1}^{5} (X^m)^5 = 0$$

$$\Omega = \frac{dY^1 \wedge dY^2 \wedge dY^3}{(Y^4)^4}$$

$$\|\Omega\|^2 = \frac{1}{6}\Omega_{abc}\bar{\Omega}^{abc} = \frac{1}{\hat{g}|Y^4|^8},$$

$$e^{-\mathcal{K}^{2,1}} = i\int \Omega \wedge \bar{\Omega} = V\|\Omega\|^2 = \frac{1}{8}e^{-\mathcal{K}^{1,1}}\|\Omega\|^2$$

$$\|\Omega\|^2 = 8e^{2\mathcal{K}},$$

$$\hat{g} = \frac{e^{-2\mathcal{K}}}{8|Y^4|^8}$$

$$h_{\alpha\beta} = 2\partial_\alpha Y^a g_{a\bar{b}}\partial_\beta Y^{\bar{b}}$$

$$\sqrt{h} = \sqrt{8\hat{g}}|\det(\partial Y)| = |\det(\partial Y)|\frac{e^{-\mathcal{K}}}{|Y^4|^4}$$

$$\partial_\alpha Y^a \partial_\beta Y^b \partial_\gamma Y^c \Omega_{abc} = \frac{\varepsilon_{abc}\partial_\alpha Y^a \partial_\beta Y^b \partial_\gamma Y^c}{(Y^4)^4} = e^{-i\phi}e^{\mathcal{K}}\sqrt{h}\varepsilon_{\alpha\beta\gamma}$$

$$\varepsilon = \lambda \otimes \eta_+ + \lambda^* \otimes \eta_-$$

$$\left(1 - \frac{i}{6}\varepsilon^{\alpha\beta\gamma}\partial_\alpha X^m \partial_\beta X^n \partial_\gamma X^p \gamma_{mnp}\right)\left(e^{-i\theta}\eta_+ + \text{c.c.}\right) = 0$$

$$\gamma_{a\bar{b}\bar{c}}\eta_+ = -2iJ_{a[\bar{b}}\gamma_{\bar{c}]}\eta_+$$

$$\gamma_{\bar{a}\bar{b}\bar{c}}\eta_+ = e^{-\mathcal{K}}\bar{\Omega}_{\bar{a}\bar{b}\bar{c}}\eta_-$$

$$\begin{aligned} &e^{-i\theta}\eta_+ + \frac{i}{6}e^{i\theta}\varepsilon^{\alpha\beta\gamma}\partial_\alpha X^a \partial_\beta X^b \partial_\gamma X^c e^{-\mathcal{K}}\Omega_{abc}\eta_+ \\ &-e^{-i\theta}\varepsilon^{\alpha\beta\gamma}\partial_\alpha X^a \partial_\beta X^{\bar{b}} \partial_\gamma X^{\bar{c}} J_{a\bar{b}}\gamma_{\bar{c}}\eta_+ + \text{c.c.} \;= 0 \end{aligned}$$

$$\varepsilon^{\alpha\beta\gamma}\partial_\alpha X^a \partial_\beta X^{\bar{b}} \partial_\gamma X^{\bar{c}} J_{a\bar{b}} = 0$$

$$e^{-i\theta} + \frac{i}{6}e^{i\theta}\varepsilon^{\alpha\beta\gamma}\partial_\alpha X^a \partial_\beta X^b \partial_\gamma X^c e^{-\mathcal{K}}\Omega_{abc} = 0$$

$$\partial_{[\alpha}X^a \partial_{\beta]}X^{\bar{b}} J_{a\bar{b}} = 0$$

$$\partial_\alpha X^a \partial_\beta X^b \partial_\gamma X^c \Omega_{abc} = -ie^{-2i\theta}e^{\mathcal{K}}\varepsilon_{\alpha\beta\gamma}$$

$$H^{p,q}(M) = H^{3-p,q}(W)$$

$$\mathcal{M}^{1,1}(M) = \mathcal{M}^{2,1}(W) \text{ and } \mathcal{M}^{1,1}(W) = \mathcal{M}^{2,1}(M).$$

$$\chi(M) = -\chi(W).$$

$$\alpha' M^2 = \alpha' \left[\left(\frac{K}{R} \right)^2 + \left(\frac{WR}{\alpha'} \right)^2 \right] + 2N_\mathrm{L} + 2N_\mathrm{R} - 4,$$

$$N_\mathrm{R} - N_\mathrm{L} = WK.$$

$$\tau = i\frac{R_2}{R_1} \text{ and } \rho = iR_1R_2$$

$$\tilde{\tau} = iR_1R_2 \text{ and } \tilde{\rho} = i\frac{R_2}{R_1}$$

$$\mathbf{248} = (\mathbf{78}, \mathbf{1}) + (\mathbf{1}, \mathbf{8}) + (\mathbf{27}, \mathbf{3}) + (\overline{\mathbf{27}}, \overline{\mathbf{3}})$$

$$A_M = \left(A_\mu, A_a, A_{\bar{a}} \right)$$

$$A_{a\bar{d}\bar{e}s} = A_{a,sb} g^{b\bar{c}} \bar{\Omega}_{\bar{c}\bar{d}\bar{e}}$$

$$248 = (45,1) + (1,15) + (10,6) + (16,4) + (\overline{16}, \overline{4}).$$

$$\delta g_{ab} \sim \Omega_{ac} g^{c\bar{d}} \omega_{b\bar{d}} + (a \leftrightarrow b),$$

$$\mathcal{M}_{19,3} = \mathcal{M}_{19,3}^0 / O(19,3;\mathbb{Z})$$

$$\mathcal{M}_{19,3}^0 = \frac{O(19,3;\mathbb{R})}{O(19,\mathbb{R}) \times O(3,\mathbb{R})}$$

$$\mathcal{M}_{16+n,n} = \mathcal{M}_{16+n,n}^0 / O(16+n,n;\mathbb{Z})$$

$$\mathcal{M}_{16+n,n}^0 = \frac{O(16+n,n;\mathbb{R})}{O(16+n,\mathbb{R}) \times O(n,\mathbb{R})}$$

$$F_3 = \sum_{i=1}^{3} \partial_- X^i \omega_+^i + \sum_{i=1}^{19} \partial_+ X^i \omega_-^i$$

$$p + p' = D - 4$$

$$T_\mathrm{F1} = T_\mathrm{M5} V_\mathrm{K3} \text{ and } T_\mathrm{NS5} V_{T^3} = T_\mathrm{M2}$$

$$\frac{1}{\ell_\mathrm{s}^2} \sim \frac{V_\mathrm{K3}}{\ell_\mathrm{p}^6} \text{ and } \frac{V_{T^3}}{g_\mathrm{s}^2 \ell_\mathrm{s}^6} \sim \frac{1}{\ell_\mathrm{p}^3},$$

$$g_\mathrm{s} (V_{T^3}/\ell_\mathrm{s}^3)^{-1/2} \sim \left(V_\mathrm{K3}/\ell_\mathrm{p}^4 \right)^{3/4}$$

$$\mathbb{R}^+ \times \mathcal{M}_{20,4}$$

$$\frac{1}{\ell_\mathrm{H}^2} \sim \frac{V_\mathrm{K3}}{g_A^2 \ell_\mathrm{A}^6} \text{ and } \frac{V_{T^4}}{g_\mathrm{H}^2 \ell_\mathrm{H}^6} \sim \frac{1}{\ell_\mathrm{A}^2}.$$

$$g_{6\mathrm{H}}^2 = g_\mathrm{H}^2 \left(V_{T^4}/\ell_\mathrm{H}^4 \right)^{-1} \text{ and } g_{6\,\mathrm{A}}^2 = g_\mathrm{A}^2 \left(V_\mathrm{K3}/\ell_\mathrm{A}^4 \right)^{-1}$$

$$g_{6\mathrm{H}}^{2} \sim g_{6\,\mathrm{A}}^{-2}.$$

$$\mathbb{R}^{+} \times \mathcal{M}_{21,5}$$

$$\tau = C_0 + ie^{-\Phi}$$

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$\frac{1}{2}\int \sqrt{-g}\left(R - g^{\mu\nu}\frac{\partial_\mu \tau \partial_\nu \bar{\tau}}{(\mathrm{Im}\tau)^2}\right) d^{10}x$$

$$ds^2 = e^{A(r,\theta)}(dr^2 + r^2 d\theta^2) - (dx^0)^2 + (dx^1)^2 + \cdots + (dx^7)^2.$$

$$j(\tau) = \sum_{n=-1}^{\infty} c_n e^{2\pi i n \tau}$$

$$j(\tau) \sim e^{-2\pi i \tau}$$

$$j(\tau(z)) = Cz,$$

$$\tau(z) \sim -\frac{1}{2\pi i}\log z$$

$$T_7 = \frac{1}{2}\int d^2x \frac{\vec{\partial}\tau \cdot \vec{\partial}\bar{\tau}}{(\mathrm{Im}\tau)^2} = \frac{1}{2}\int d^2x \frac{\partial\tau\bar{\partial}\bar{\tau} + \bar{\partial}\tau\partial\bar{\tau}}{(\mathrm{Im}\tau)^2}$$

$$T_7 = \frac{1}{2}\int d^2x \frac{\partial\tau\bar{\partial}\bar{\tau}}{(\mathrm{Im}\tau)^2} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2} = \frac{\pi}{6}$$

$$R_{00} - \frac{1}{2}g_{00}R = -\frac{1}{2}g_{00}e^{-A}\frac{\partial\tau\bar{\partial}\bar{\tau}}{(\mathrm{Im}\tau)^2}$$

$$\partial\bar{\partial}A = -\frac{1}{2}\frac{\partial\tau\bar{\partial}\bar{\tau}}{(\tau - \bar{\tau})^2} = \partial\bar{\partial}\log\,\mathrm{Im}\tau$$

$$A \sim -\frac{1}{6}\log r.$$

$$ds^2 \sim d\rho^2 + \rho^2\left(\frac{11}{12}d\theta\right)^2$$

$$e^A = |f(z)|^2\mathrm{Im}\tau$$

$$f(z) = [\eta(\tau)]^2 \prod_{i=1}^{N} (z - z_i)^{-1/12}$$

- Dedekind:

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)$$

$$q = e^{2\pi i\tau}$$

$$\eta(-1/\tau) = \sqrt{-i\tau}\eta(\tau)$$

$$\sum \phi_i = 4\pi$$

$$y^2 = x^3 + ax + b$$

$$27a^3 - 4b^2 = 0$$

$$\mathbb{R}^+ \times \mathcal{M}_{18,2}$$

$$M_{11} = M_4 \times M_7$$

$$\delta\psi_M = \nabla_M \varepsilon = 0$$

$$\varphi = dy^{123} + dy^{145} + dy^{167} + dy^{246} - dy^{257} - dy^{347} - dy^{356},$$

$$dy^{ijk} = dy^i \wedge dy^j \wedge dy^k$$

$$\Phi = \frac{1}{6}\Phi_{abc} e^a \wedge e^b \wedge e^c$$

$$\alpha: (y^1, \dots, y^7) \to (y^1, y^2, y^3, -y^4, -y^5, -y^6, -y^7),$$
$$\beta: (y^1, \dots, y^7) \to (y^1, -y^2, -y^3, y^4, y^5, 1/2 - y^6, -y^7),$$
$$\gamma: (y^1, \dots, y^7) \to (-y^1, y^2, -y^3, y^4, 1/2 - y^5, y^6, 1/2 - y^7).$$

$$P\_\epsilon = \frac{1}{2}\left(1 - \frac{i}{6}\varepsilon^{\alpha\beta\gamma}\partial_\alpha X^M \partial_\beta X^N \partial_\gamma X^P \Gamma_{MNP}\right)\epsilon = 0$$

$$\partial_{[\alpha} X^a \partial_\beta X^b \partial_{\gamma]} X^c \Phi_{abc} = \varepsilon_{\alpha\beta\gamma}$$

$$P\_\epsilon = \frac{1}{2}\left(1 - \frac{i}{4!}\varepsilon^{\alpha\beta\gamma\sigma}\partial_\alpha X^M \partial_\beta X^N \partial_\gamma X^Q \partial_\sigma X^P \Gamma_{MNPQ}\right)\epsilon = 0.$$

$$ds^2 = dr^2 + r^2 d\Omega_{n-1}^2$$

$$\begin{pmatrix} e^{2\pi i/n} & 0 \\ 0 & e^{-2\pi i/n} \end{pmatrix}$$

$$\begin{pmatrix} e^{\pi i/(k-2)} & 0 \\ 0 & e^{-\pi i/(k-2)} \end{pmatrix} \text{ and } \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$\Omega = dy^{1234} + dy^{1256} + dy^{1278} + dy^{1357} - dy^{1368} - dy^{1458} - dy^{1467} - dy^{2358} - dy^{2367} - dy^{2457} + dy^{2468} + dy^{3456} + dy^{3478} + dy^{5678}$$

$$dy^{ijkl} = dy^i \wedge dy^j \wedge dy^k \wedge dy^l$$

$$a_{14}+a_{36}+a_{27}=0, a_{15}+a_{73}+a_{26}=0,$$
$$a_{16}+a_{43}+a_{52}=0, a_{17}+a_{35}+a_{42}=0,$$
$$a_{76}+a_{54}+a_{32}=0, a_{12}+a_{74}+a_{65}=0,$$
$$a_{13}+a_{57}+a_{64}=0.$$

$$A_p=\frac{1}{p!}A_{\mu_1\cdots\mu_p}dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_p}$$

$$dA_p=\frac{1}{p!}\partial_{\mu_1}A_{\mu_2\cdots\mu_{p+1}}dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_{p+1}}$$

$$ddA_0=d\left(\frac{\partial A_0}{\partial x^\mu}dx^\mu\right)=\frac{\partial^2 A_0}{\partial x^\mu\partial x^\nu}dx^\mu\wedge dx^\nu$$

$$dA_p=0$$

$$A_p=dA_{p-1}$$

$$H^p(M)=C^p(M)/Z^p(M)$$

$$\chi(M)=\sum_{i=0}^{d}(-1)^i b_i(M)$$

$$\delta z_p=0.$$

- Poincaré – Stokes – Laplace – Ricci – Lorentz – Riemann – Hodge – Levi – Civita – Nijenhuis – Dolbeault – Kähler.

$$\int_N dA=\int_{\delta N}A.$$

$$\int_M A\wedge B=\int_N B,$$

$$ds^2=g_{\mu\nu}(x)dx^\mu dx^\nu$$

$$e^\alpha=e_\mu^\alpha dx^\mu$$

$$g=\eta_{\alpha\beta}e^\alpha\otimes e^\beta$$

$$g_{\mu\nu}=\eta_{\alpha\beta}e_\mu^\alpha e_\nu^\beta$$

$$\Delta_p=d^\dagger d+dd^\dagger=\left(d+d^\dagger\right)^2$$

$$d^\dagger=(-1)^{dp+d+1}\star d\star$$

$$\star\left(dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_p}\right)=\frac{\varepsilon^{\mu_1\cdots\mu_p\mu_{p+1}\cdots\mu_d}}{(d-p)!\,|g|^{1/2}}g_{\mu_{p+1}\nu_{p+1}}\cdots g_{\mu_d\nu_d}dx^{\nu_{p+1}}\wedge\cdots\wedge dx^{\nu_d}$$

$$\Delta_p A=0$$

$$(dd^\dagger + d^\dagger d)A_p = 0$$

$$(A_p, (dd^\dagger + d^\dagger d)A_p) = 0 \Rightarrow (d^\dagger A_p, d^\dagger A_p) + (dA_p, dA_p) = 0$$

$$A_p = A_p^{\mathrm{h}} + dA_{p-1}^{\mathrm{e.}} + d^\dagger A_{p-1}^{\mathrm{c.e.}}$$

$$A_p = A_p^{\mathrm{h}} + dA_{p-1}^{\mathrm{e}}$$

$$b_p = b_{d-p}$$

$$\nabla_\mu e_\nu^\alpha = \partial_\mu e_\nu^\alpha - \Gamma_{\mu\nu}^\rho e_\rho^\alpha + \omega_\mu^\alpha{}_\beta e_\nu^\beta = 0$$

$$\Gamma_{\mu\nu}^\rho = \left\{ {\rho \atop \mu\nu} \right\} + K_{\mu\nu}^\rho$$

$$\left\{ {\rho \atop \mu\nu} \right\} = \frac{1}{2} g^{\rho\lambda}(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu})$$

$$\omega_\mu^\alpha{}_\beta = -e_\beta^\nu(\partial_\mu e_\nu^\alpha - \Gamma_{\mu\nu}^\lambda e_\lambda^\alpha)$$

$$R_\beta^\alpha = d\omega_\beta^\alpha + \omega_\gamma^\alpha \wedge \omega_\beta^\gamma,$$

$$R = d\omega + \omega \wedge \omega$$

$$R_{\nu\lambda} = R_{\nu\mu\lambda}^\mu$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$\varepsilon \to U\varepsilon$$

$$\varepsilon \to U_1 U_2 \varepsilon = U_3 \varepsilon.$$

$$J_a^b = i\delta_a^b, J_{\bar{a}}^{\bar{b}} = -i\delta_{\bar{a}}^{\bar{b}}, J_a^{\bar{b}} = J_{\bar{a}}^b = 0$$

$$J_m{}^n J_n{}^p = -\delta_m{}^p$$

$$N^p{}_{mn} = J_m{}^q \partial_{[q} J_{n]}{}^p - J_n{}^q \partial_{[q} J_{m]}{}^p$$

$$A_{p,q} = \frac{1}{p!\,q!} A_{a_1 \cdots a_p \bar{b}_1 \cdots \bar{b}_q} dz^{a_1} \wedge \cdots \wedge dz^{a_p} \wedge d\bar{z}^{\bar{b}_1} \wedge \cdots \wedge d\bar{z}^{\bar{b}_q}$$

$$d = \partial + \bar{\partial}$$

$$\partial = dz^a \frac{\partial}{\partial z^a} \text{ and } \bar{\partial} = d\bar{z}^{\bar{a}} \frac{\partial}{\partial \bar{z}^{\bar{a}}}$$

$$\partial^2 = \bar{\partial}^2 = 0$$

$$\partial\bar{\partial} + \bar{\partial}\partial = 0.$$

$$ds^2 = g_{ab} dz^a dz^b + g_{a\bar{b}} dz^a d\bar{z}^{\bar{b}} + g_{\bar{a}b} d\bar{z}^{\bar{a}} dz^b + g_{\bar{a}\bar{b}} d\bar{z}^{\bar{a}} d\bar{z}^{\bar{b}}.$$

$$g_{ab} = g_{\bar{a}\bar{b}} = 0$$

$$\Delta_\partial = \partial\partial^\dagger + \partial^\dagger\partial \text{ and } \Delta_{\bar{\partial}} = \bar{\partial}\bar{\partial}^\dagger + \bar{\partial}^\dagger\bar{\partial}$$

$$J = i g_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$$

$$dJ = 0$$

$$g_{a\bar{b}} = \frac{\partial}{\partial z^a}\frac{\partial}{\partial \bar{z}^{\bar{b}}}\mathcal{K}(z,\bar{z}),$$

$$J = i\partial\bar{\partial}K$$

$$\tilde{\mathcal{K}}(z,\bar{z}) = \mathcal{K}(z,\bar{z}) + f(z) + \bar{f}(\bar{z})$$

$$\Delta_d = 2\Delta_{\bar{\partial}} = 2\Delta_\partial.$$

$$H^{p,q}_{\bar{\partial}}(M) = H^{p,q}_{\partial}(M) = H^{p,q}(M).$$

$$b_k = \sum_{p=0}^{k} h^{p,k-p}$$

$$h^{p,q} = h^{q,p}.$$

$$h^{n-p,n-q} = h^{p,q}.$$

$$\mathcal{R} = i R_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$$

$$c_1 = \frac{1}{2\pi}[\mathcal{R}].$$

$$\int_M A \wedge B = \alpha_i \beta_j \int_M w^i \wedge v^j \equiv \alpha_i \beta_j m^{ij}.$$

$$\int_{Z_j} v^i = \delta^i_j$$

$$\int_N B = \int_{\alpha_i m^{ij} Z_j} \beta_\gamma v^\gamma = \alpha_i \beta_\gamma m^{ij} \delta^\gamma_j = \alpha_i \beta_j m^{ij} = \int_M A \wedge B$$

$$H^p(M) \approx H_{d-p}(M)$$

$$\star\, dx = dy \text{ and } \star\, dy = -dx$$

$$\star\, dz = -i dz \text{ and } \star\, d\bar{z} = i d\bar{z}$$

$$N^p{}_{mn} = J^q{}_m \nabla_q J^p{}_n - J^q{}_n \nabla_q J^p{}_m - J^p{}_q \nabla_m J^q{}_n + J^p{}_q \nabla_n J^q{}_m.$$

$$J^q{}_m \nabla_q J^p{}_n + J^p{}_q \nabla_n J^q{}_m - J^q{}_n \nabla_q J^p{}_m - J^p{}_q \nabla_m J^q{}_n$$
$$= J^q{}_m \left(\partial_q J^p{}_n + \Gamma_{q\lambda}{}^p J^\lambda{}_n - \Gamma_{qn}{}^\lambda J^p{}_\lambda \right)$$
$$+ J^p{}_q \left(\partial_n J^q{}_m + \Gamma_{n\lambda}{}^q J^\lambda{}_m - \Gamma_{nm}{}^\lambda J^q{}_\lambda \right) - (n \leftrightarrow m).$$

$$J^p{}_q \Gamma_{n\lambda}{}^q J^\lambda{}_m - J^q{}_m \Gamma_{qn}{}^\lambda J^p{}_\lambda$$

$$N^p{}_{mn} = J^q{}_m \partial_q J^p{}_n + J^p{}_q \partial_n J^q{}_m - (n \leftrightarrow m),$$

$$R = -2 \frac{dz \wedge d\bar{z}}{(1 + z\bar{z})^2}$$

- Lichnerowicz:

$$\nabla^k \nabla_k \delta g_{mn} + 2 R_m{}^p{}_n{}^q \delta g_{pq} = 0.$$

$$\frac{T^2 \times T^2 \times T^2}{\mathbb{Z}_4},$$

$$y^2 = x^3 + f(z_1, z_2)x + g(z_1, z_2),$$

$$\int_{\gamma_{n+1}} F$$

$$\int_{\gamma_{D-n-1}} \star F$$

$$2\kappa_{11}^2 S = \int d^{11}x \sqrt{-G} \left(R - \frac{1}{2} |F_4|^2 \right) - \frac{1}{6} \int A_3 \wedge F_4 \wedge F_4$$

$$\delta \Psi_M = \nabla_M \varepsilon + \frac{1}{12} \left(\Gamma_M \mathbf{F}^{(4)} - 3 \mathbf{F}_M^{(4)} \right) \varepsilon = 0$$

$$ds^2 = \Delta(y)^{-1} \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{\substack{\text{3D flat} \\ \text{space-time}}} + \Delta(y)^{1/2} \underbrace{g_{mn}(y) dy^m dy^n}_{\substack{\text{8D internal} \\ \text{manifold}}}$$

$$ds^2 = d\theta^2 + d\varphi^2 \text{ with } 0 \le \theta \le \pi, 0 \le \varphi \le 2\pi$$

$$ds^2 = d\theta^2 + \sin^2 \theta d\varphi^2$$

$$\Gamma_\mu = \Delta^{-1/2} \left(\gamma_\mu \otimes \gamma_9 \right) \text{ and } \Gamma_m = \Delta^{1/4} (1 \otimes \gamma_m)$$

$$\gamma_0 = i\sigma_1, \gamma_1 = \sigma_2 \text{ and } \gamma_2 = \sigma_3$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ and } \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\sigma_2 \otimes \sigma_2 \otimes 1 \otimes \sigma_{1,3}, \quad \sigma_2 \otimes \sigma_{1,3} \otimes \sigma_2 \otimes 1,$$
$$\sigma_2 \otimes 1 \otimes \sigma_{1,3} \otimes \sigma_2, \quad \sigma_{1,3} \otimes 1 \otimes 1 \otimes 1.$$

$$\gamma_9 = \gamma_1 \dots \gamma_8 = \sigma_2 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_2$$

$$P_{\pm} = (1 \pm \gamma_9)/2$$

$$\varepsilon(x,y) = \zeta(x) \otimes \eta(y) + \zeta^*(x) \otimes \eta^*(y)$$

$$\eta = \eta_1 + i\eta_2 \ \text{ with } \ (\gamma_9 - 1)\eta = 0$$

$$\mathbf{8}_c \to \mathbf{6} \oplus \mathbf{1} \oplus \mathbf{1}.$$

$$v_a = \eta_1^\dagger \gamma_a \eta_2$$

$$\nabla_\mu \varepsilon \ \to \nabla_\mu \varepsilon - \frac{1}{4}\Delta^{-7/4}(\gamma_\mu \otimes \gamma_9 \gamma^m)\partial_m \Delta \varepsilon,$$
$$\nabla_m \varepsilon \ \to \nabla_m \varepsilon + \frac{1}{8}\Delta^{-1}\partial_n \Delta(1 \otimes \gamma_m^n)\varepsilon.$$

$$F_{mnpq}(y) \ \text{ and } \ F_{\mu\nu\rho m} = \varepsilon_{\mu\nu\rho} f_m(y)$$

$$\begin{aligned}
\mathbf{F}^{(4)} &= \Delta^{-1}(1 \otimes \mathbf{F}) + \Delta^{5/4}(1 \otimes \gamma_9 \mathbf{f}) \\
\mathbf{F}_\mu^{(4)} &= \Delta^{3/4}(\gamma_\mu \otimes \mathbf{f}) \\
\mathbf{F}_m^{(4)} &= -\Delta^{3/2} f_m(y)(1 \otimes \gamma_9) + \Delta^{-3/4}(1 \otimes \mathbf{F}_m)
\end{aligned}$$

$$\mathbf{F} = \frac{1}{24} F_{mnpq}\gamma^{mnpq}, \mathbf{F}_m = \frac{1}{6} F_{mnpq}\gamma^{npq} \ \text{ and } \ \mathbf{f} = \gamma^m f_m$$

$$\nabla_\mu \zeta = 0$$

$$\partial\!\!\!/ \Delta^{-3/2}\eta + \mathbf{f}\eta + \frac{1}{2}\Delta^{-9/4}\mathbf{F}\eta = 0$$

$$\mathbf{F}\eta = 0$$

$$f_m(y) = -\partial_m \Delta^{-3/2}$$

$$\nabla_m \eta + \frac{1}{4}\Delta^{-1}\partial_m \Delta \eta - \frac{1}{4}\Delta^{-3/4}\mathbf{F}_m \eta = 0$$

$$\mathbf{F}_m \xi = 0 \ \text{ and } \ \nabla_m \xi = 0$$

$$\xi = \Delta^{1/4}\eta$$

$$J_a{}^b = -i\xi^\dagger \gamma_a{}^b \xi$$

$$\{\gamma^a, \gamma^b\} = 0, \{\gamma^{\bar{a}}, \gamma^{\bar{b}}\} = 0, \{\gamma^a, \gamma^{\bar{b}}\} = 2g^{a\bar{b}},$$

$$J_{a\bar{b}} = ig_{a\bar{b}} = -i\xi^\dagger \gamma_{a\bar{b}} \xi = -i\xi^\dagger(\gamma_a \gamma_{\bar{b}} - g_{a\bar{b}})\xi$$

$$0 = \xi^\dagger \gamma_a \gamma_{\bar{b}} \xi = (\gamma_{\bar{a}} \xi)^\dagger (\gamma_{\bar{b}} \xi)$$

$$\gamma_{\bar{a}} \xi = \gamma^a \xi = 0$$

$$F^{4,0} = F^{0,4} = F^{1,3} = F^{3,1} = 0$$

$$F_{a\bar{b}c\bar{d}}g^{c\bar{d}} = 0$$

$$F^{2,2} \wedge J = 0$$

$$F \in H^{(2,2)}_{\text{primitive}}(M)$$

$$\begin{aligned} J_3 &: G \to \frac{1}{2}(d-n)G \\ J_- &: G \to J \wedge G \\ J_+ &: G \to \frac{1}{2(n-2)!} J^{p_1 p_2} G_{p_1 p_2 \dots p_n} dx^{p_3} \wedge \dots \wedge dx^{p_n} \end{aligned}$$

$$|j,m\rangle \text{ with } m = -j, -j+1, \dots, j-1, j.$$

$$J_+ G_{\text{primitive}} = 0,$$

$$J_-^{2j+1} G_{\text{primitive}} = 0 \text{ where } j = \frac{d-n}{2}.$$

$$J_+ G = 0 \text{ or } J_- G = 0.$$

$$H^n(M) = \bigoplus_k J_-^k H^{n-2k}_{\text{primitive}}(M)$$

$$H^{(p,q)}(M) = \bigoplus_k J_-^k H^{(p-k,q-k)}_{\text{primitive}}(M)$$

$$\underbrace{J \wedge \dots \wedge J}_{5-p-q \text{ times}} \wedge F^{p,q}_{\text{primitive}} = 0$$

$$\delta S = -T_{\text{M2}} \int_M A_3 \wedge X_8$$

$$X_8 = \frac{1}{(2\pi)^4}\left[\frac{1}{192}\text{tr}R^4 - \frac{1}{768}(\text{tr}R^2)^2\right]$$

$$dF = 0$$

$$d \star F_4 = -\frac{1}{2} F_4 \wedge F_4 - 2\kappa_{11}^2 T_{\text{M2}} X_8$$

$$d \star_8 d\log \Delta = \frac{1}{3} F \wedge F + \frac{4}{3}\kappa_{11}^2 T_{\text{M2}} X_8$$

$$P_1 = \frac{1}{(2\pi)^2}\left(-\frac{1}{2}\text{tr}R^2\right) \text{ and } P_2 = \frac{1}{(2\pi)^4}\left[-\frac{1}{4}\text{tr}R^4 + \frac{1}{8}(\text{tr}R^2)^2\right]$$

$$X_8 = \frac{1}{192}(P_1^2 - 4P_2)$$

$$P_1 = c_1^2 - 2c_2 \text{ and } P_2 = c_2^2 - 2c_1c_3 + 2c_4$$

$$X_8 = \frac{1}{192}(c_1^4 - 4c_1^2c_2 + 8c_1c_3 - 8c_4)$$

$$\int_M X_8 = -\frac{1}{24}\int_M c_4 = -\frac{\chi}{24}$$

$$\frac{1}{4\kappa_{11}^2 T_{\mathrm{M2}}}\int_M F \wedge F = \frac{\chi}{24}$$

$$4\kappa_{11}^2 T_{\mathrm{M2}} = 2(2\pi\ell_{\mathrm{p}})^6$$

$$F_{mnpq} \simeq O\left(\frac{\ell_{\mathrm{p}}^3}{\sqrt{v}}\right)$$

$$\Delta \simeq 1 + O\left(\frac{\ell_{\mathrm{p}}^6}{v^{3/4}}\right)$$

$$N + \frac{1}{4\kappa_{11}^2 T_{\mathrm{M2}}}\int_M F \wedge F = \frac{\chi}{24}$$

$$J = \sum_{A=1}^{h^{1,1}} K^A e_A$$

$$W^{3,1}(T) = \frac{1}{2\pi}\int_M \Omega \wedge F$$

$$W^{3,1} = \mathcal{D}_I W^{3,1} = 0 \text{ with } I = 1, \dots, h^{3,1}$$

$$\mathcal{K}^{3,1} = -\log\left(\int_M \Omega \wedge \bar{\Omega}\right)$$

$$F^{4,0} = F^{0,4} = 0$$

$$F^{2,2} \wedge J = 0$$

$$W^{1,1}(K) = \int_M J \wedge J \wedge F$$

$$W^{1,1} = \partial_A W^{1,1} = 0 \text{ with } A = 1, \dots, h^{1,1}$$

$$F_4 = \frac{1}{\tau - \bar{\tau}}(G_3^\star \wedge dz - G_3 \wedge d\bar{z})$$

$$dz = d\sigma_1 + \tau d\sigma_2.$$

$$F^{1,3} = \frac{1}{\tau - \bar{\tau}}[(G_3^\star)^{0,3} \wedge dz - (G_3)^{1,2} \wedge d\bar{z}]$$

$$F^{0,4} = -\frac{1}{\tau - \bar{\tau}}(G_3)^{0,3} \wedge d\bar{z}$$

$$G_3 \in H^{(2,1)},$$

$$\mathbf{F}_m \mathbf{F}^m \xi = -2\mathbf{F}^2 \xi - \frac{1}{12} F_{mnpq}(F^{mnpq} \mp \star F^{mnpq})\xi$$

$$(F \mp \star F)^2 = 0$$

$$F = \pm \star F \text{ for } \gamma_9 \xi = \pm \xi$$

$$F_{m\bar{a}\bar{b}\bar{c}} \gamma^{\bar{a}\bar{b}\bar{c}} \xi + 3F_{m\bar{a}\bar{b}} \gamma^{\bar{a}\bar{b}c} \xi = 0$$

$$F_{m\bar{a}\bar{b}\bar{c}} \{\gamma_{\bar{d}}, \gamma^{\bar{a}\bar{b}\bar{c}}\} \xi = 6F_{m\bar{d}\bar{b}\bar{c}} \gamma^{\bar{b}\bar{c}} \xi = 0$$

$$F_{m\bar{d}b\bar{c}} [\gamma_{\bar{e}}, \gamma^{\bar{b}\bar{c}}] \xi = 4F_{m\bar{d}\bar{e}\bar{c}} \gamma^{\bar{c}} \xi = 0$$

$$F_{m\bar{d}\bar{e}\bar{c}} \{\gamma_{\bar{f}}, \gamma^{\bar{c}}\} \xi = 2F_{m\bar{d}\bar{e}\bar{f}} \xi = 0$$

$$F^{4,0} = F^{3,1} = F^{1,3} = F^{0,4} = 0$$

$$F_{a\bar{b}c\bar{d}} g^{c\bar{d}} = 0$$

$$F \wedge J = 0$$

$$F^{3,1} = \frac{1}{6} F_{abc\bar{d}} dz^a \wedge dz^b \wedge dz^c \wedge dz^{\bar{d}}$$

$$F_{abc\bar{d}} J^{c\bar{d}} = 0$$

$$\varepsilon_{abcd\bar{p}\bar{q}\bar{r}\bar{s}} = (g_{a\bar{p}} g_{b\bar{q}} g_{c\bar{r}} g_{d\bar{s}} \pm \text{ permutations })$$

$$\star F^{(p,4-p)} = (-1)^p F^{(p,4-p)}$$

$$\partial_I \Omega = K_I \Omega + \chi_I, I = 1, \dots, h^{3,1}$$

$$J = K^A e_A, A = 1, \dots, h^{1,1}$$

$$\int_M \Omega \wedge F^{0,4} = 0 \text{ and } \int_M \chi_I \wedge F^{1,3} = 0$$

$$\int_M \star F^{3,1} \wedge F^{1,3} = \int_M \star (F^{1,3})^* \wedge F^{1,3} = \int_M |F^{1,3}|^2 \sqrt{g} d^8 x = 0$$

$$F^{1,3} = F^{3,1} = F^{0,4} = F^{4,0} = 0.$$

$$\int e_A \wedge J \wedge F^{2,2} = 0$$

$$\star(J \wedge F^{2,2}) = \sum_{A=1}^{h^{1,1}} U^A e_A$$

$$\int_M \star(J \wedge F^{2,2}) \wedge (J \wedge F^{2,2}) = \int_M |J \wedge F^{2,2}|^2 \sqrt{g} d^8x = 0$$

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} \left[R - \frac{|\partial\tau|^2}{2(\mathrm{Im}\tau)^2} - \frac{|G_3|^2}{2\mathrm{Im}\tau} - \frac{|\tilde{F}_5|^2}{4} \right] + \frac{1}{8i\kappa^2} \int \frac{C_4 \wedge G_3 \wedge G_3^\star}{\mathrm{Im}\tau}$$

$$G_3 = F_3 - \tau H_3$$

$$\tau = C_0 + ie^{-\Phi}$$

$$\tilde{F}_5 = \star_{10} \tilde{F}_5$$

$$|G_3|^2 = \frac{1}{3!} G^{M_1N_1} G^{M_2N_2} G^{M_3N_3} G_{M_1M_2M_3} G^\star_{N_1N_2N_3}$$

$$ds_{10}^2 = \sum_{M,N=0}^{9} G_{MN} dx^M dx^N = e^{2A(y)} \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{4\mathrm{D}} + e^{-2A(y)} \underbrace{g_{mn}(y) dy^m dy^n}_{6\mathrm{D}}$$

$$\tilde{F}_5 = (1 + \star_{10}) d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

$$R_{MN} = \kappa^2 \left(T_{MN} - \frac{1}{8} G_{MN} T \right)$$

$$T_{MN} = -\frac{2}{\sqrt{-G}} \frac{\delta S}{\delta G^{MN}}$$

$$R_{MN} = -\frac{1}{4} G_{MN} \left(\frac{1}{2\mathrm{Im}\tau} |G_3|^2 + e^{-8A} |\partial\alpha|^2 \right) \quad M, N = 0,1,2,3$$

$$\Delta A = \frac{e^{4A}}{8\mathrm{Im}\tau} |G_3|^2 + \frac{1}{4} e^{-8A} |\partial\alpha|^2$$

$$\Delta e^{4A} = \frac{e^{8A}}{2\mathrm{Im}\tau} |G_3|^2 + e^{-4A} (|\partial\alpha|^2 + |\partial e^{4A}|^2)$$

$$2\kappa^2 e^{2A} \mathcal{J}_{\mathrm{loc}}$$

$$\mathcal{J}_{\mathrm{loc}} = \frac{1}{4} \left(\sum_{M=5}^{9} T^M_M - \sum_{M=0}^{3} T^M_M \right)_{\mathrm{loc}}$$

$$T^{\mathrm{loc}}_{MN} = -\frac{2}{\sqrt{-G}} \frac{\delta S_{\mathrm{loc}}}{\delta G^{MN}}$$

$$S_{\text{loc}} = -\int_{\mathbb{R}^4\times\Sigma} d^{p+1}\xi T_p\sqrt{-g} + \mu_p \int_{\mathbb{R}^4\times\Sigma} C_{p+1}$$

$$-\mu_3 \int_{\mathbb{R}^4\times\Sigma} C_4 \wedge \frac{p_1(R)}{48}$$

$$d\tilde{F}_5 = H_3 \wedge F_3 + 2\kappa^2 T_3 \rho_3.$$

$$\frac{1}{2\kappa^2 T_3}\int_M H_3 \wedge F_3 + Q_3 = 0$$

$$\frac{\chi(X)}{24} = N_{D3} + \frac{1}{2\kappa^2 T_3}\int_M H_3 \wedge F_3,$$

$$\Delta(e^{4A} - \alpha) = \frac{1}{6\text{Im}\tau} e^{8A}|iG_3 - \star G_3|^2 + e^{-4A}|\partial(e^{4A} - \alpha)|^2$$
$$+2\kappa^2 e^{2A}\left(\mathcal{J}_{\text{loc}} - T_3\rho_3^{\text{loc}}\right)$$

$$\mathcal{J}_{\text{loc}} \geq T_3\rho_3^{\text{loc}}$$

$$\star G_3 = iG_3$$

$$e^{4A} = \alpha$$

$$\mathcal{J}_{\text{loc}} = T_3\rho_3^{\text{loc}}$$

$$T_0^0 = T_1^1 = T_2^2 = T_3^3 = -T_3\rho_3 \text{ and } T_m^m = 0.$$

$$W = \int_M \Omega \wedge G_3$$

$$\mathcal{K}(\rho) = -3\log[-i(\rho - \bar{\rho})].$$

$$\mathcal{K}(\tau) = -\log[-i(\tau - \bar{\tau})] \text{ and } \mathcal{K}(z^\alpha) = -\log\left(i\int_M \Omega \wedge \bar{\Omega}\right)$$

$$\mathcal{K} = \mathcal{K}(\rho) + \mathcal{K}(\tau) + \mathcal{K}(z^\alpha).$$

$$\mathcal{D}_a W = \partial_a W + \partial_a \mathcal{K} W = 0$$

$$\mathcal{D}_\rho W = \partial_\rho \mathcal{K} W = -\left(\frac{3}{\rho - \bar{\rho}}\right) W = 0$$

$$W = 0$$

$$\mathcal{D}_\tau W = \frac{1}{\tau - \bar{\tau}}\int_M \Omega \wedge \bar{G}_3 = 0$$

$$\mathcal{D}_\alpha W = \int_M \chi_\alpha \wedge G_3 = 0$$

$$G_3 \in H^{(2,1)}(M).$$

$$\chi = \frac{1}{2}\chi_{ab\bar{c}}dz^a \wedge dz^b \wedge d\bar{z}^{\bar{c}}$$

$$\chi = v \wedge J + (\chi - v \wedge J) = \chi_{\|} + \chi_{\perp}$$

$$v = \frac{3}{2}\chi_{ap\bar{q}}J^{p\bar{q}}dz^a$$

$$\chi_{\perp} \wedge J = 0$$

$$G_3 \in H^{(2,1)}_{\text{primitive}}$$

$$\varepsilon_{abc\bar{p}\bar{q}\bar{r}} = -i\left(g_{a\bar{p}}g_{b\bar{q}}g_{c\bar{r}} \pm \text{ permutations }\right)$$

$$\sum_{A=1}^{4} (w^A)^2 = 0 \text{ for } w^A \in \mathbb{C}^4$$

$$\vec{x} \cdot \vec{x} - \frac{1}{2}\rho^2 = 0, \vec{y} \cdot \vec{y} - \frac{1}{2}\rho^2 = 0, \vec{x} \cdot \vec{y} = 0$$

$$ds^2 = dr^2 + r^2 d\Sigma^2,$$

$$d\Sigma^2 = \frac{1}{9}\left(2d\beta + \sum_{i=1}^{2} \cos\theta_i d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2} \left(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right).$$

$$0 \leq \beta \leq 2\pi, 0 \leq \theta_i \leq \pi \text{ and } 0 \leq \phi_i \leq 2\pi,$$

$$\begin{aligned} g^1 &= \frac{1}{\sqrt{2}}(e^1 - e^3), & g^2 &= \frac{1}{\sqrt{2}}(e^2 - e^4), \\ g^3 &= \frac{1}{\sqrt{2}}(e^1 + e^3) \quad , & g^4 &= \frac{1}{\sqrt{2}}(e^2 + e^4), \\ g^5 &= e^5, \end{aligned}$$

$$\begin{aligned} e^1 &= -\sin\theta_1 d\phi_1, e^2 = d\theta_1 \\ e^3 &= \cos 2\beta \sin\theta_2 d\phi_2 - \sin 2\beta d\theta_2 \\ e^4 &= \sin 2\beta \sin\theta_2 d\phi_2 + \cos 2\beta d\theta_2 \\ e^5 &= 2d\beta + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 \end{aligned}$$

$$d\Sigma^2 = \frac{1}{9}(g^5)^2 + \frac{1}{6}\sum_{i=1}^{4} \left(g^i\right)^2$$

$$\sum_{A=1}^{4} (w^A)^2 = z$$

$$z = \vec{x} \cdot \vec{x} - \vec{y} \cdot \vec{y}$$

$$\rho^2 = \vec{x} \cdot \vec{x} + \vec{y} \cdot \vec{y}$$

$$z \leq \rho^2 < \infty$$

$$\det\begin{pmatrix} X & U \\ V & Y \end{pmatrix} = 0$$

$$\begin{pmatrix} X & U \\ V & Y \end{pmatrix}\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0$$

$$(\lambda_1, \lambda_2) \simeq \lambda(\lambda_1, \lambda_2) \;\text{ with }\; \lambda \in \mathbb{C}^\star$$

$$\int_{S^3} F_3 = 4\pi^2\alpha' M$$

$$F_3 = \frac{M\alpha'}{2}\omega_3 \;\text{ where }\; \omega_3 = g^5 \wedge \omega_2$$

$$\omega_2 = \frac{1}{2}(\sin\,\theta_1 d\theta_1 \wedge d\phi_1 - \sin\,\theta_2 d\theta_2 \wedge d\phi_2)$$

$$H_3 = \frac{3}{2r} g_s M\alpha' dr \wedge \omega_2$$

$$d\tilde{F}_5 = H_3 \wedge F_3 + 2\kappa^2 T_3 \rho_3$$

$$\tilde{F}_5 = (1 + \star_{10})\mathcal{F},$$

$$\mathcal{F} = \frac{1}{2}\pi(\alpha')^2 N_{\text{eff}}(r)\omega_2 \wedge \omega_3$$

$$N_{\text{eff}}(r) = N + \frac{3}{2\pi} g_s M^2 \log\left(\frac{r}{r_0}\right).$$

$$\int_\Sigma \tilde{F}_5 = \frac{1}{2}(\alpha')^2 \pi N_{\text{eff}}(r)$$

$$ds_{10}^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(r)}(dr^2 + r^2 d\Sigma^2)$$

$$\sqrt{-g} = \frac{1}{54} e^{-2A} r^5 \sin\,\theta_1 \sin\,\theta_2$$

$$\omega_2 \wedge \omega_3 = -d\beta \wedge \sin\,\theta_1 d\theta_1 \wedge d\phi_1 \wedge \sin\,\theta_2 d\theta_2 \wedge d\phi_2$$

$$\star\,(\omega_2 \wedge \omega_3) = 54 r^{-5} e^{8A} dr \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

$$\star_{10}\,\mathcal{F} = d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

$$d\alpha = 27\pi(\alpha')^2 \alpha^2 r^{-5} N_{\text{eff}}(r) dr$$

$$e^{-4A(r)} = \frac{27\pi(\alpha')^2}{4r^4}\left[g_s N + \frac{3}{2\pi}(g_s M)^2 \log\left(\frac{r}{r_0}\right) + \frac{3}{8\pi}(g_s M)^2\right],$$

$$S \sim \int d^5x\sqrt{-G}(R - 12\Lambda) - T\int d^4x\sqrt{-g}$$

$$ds^2 = e^{-2A(x_5)}\eta_{\mu\nu}dx^\mu dx^\nu + dx_5^2$$

$$A(x_5) = \sqrt{-\Lambda}|x_5|.$$

$$T = 12\sqrt{-\Lambda},$$

$$M_4^2 = M_5^3 \int dx_5 e^{-2\sqrt{-\Lambda}|x_5|}$$

$$G_4 = G_5 \left(\int dx_5 e^{-2\sqrt{-\Lambda}|x_5|}\right)^{-1}$$

$$S = \int d^5x\sqrt{-G}(R - 12\Lambda) - T_{\text{SM}} \int d^4x\sqrt{-g^{\text{SM}}} - T_{\text{P}} \int d^4x\sqrt{-g^P}$$

$$ds^2 = e^{-2A(x_5)}\eta_{\mu\nu}dx^\mu dx^\nu + dx_5^2$$

$$A(x_5) = \sqrt{-\Lambda}|x_5|,$$

$$T_{\text{P}} = -T_{\text{SM}} = 12\sqrt{-\Lambda}$$

$$g_{\mu\nu}^{\text{P}} = \eta_{\mu\nu}$$

$$g_{\mu\nu}^{\text{SM}} = e^{-2\pi r\sqrt{-\Lambda}}\eta_{\mu\nu}$$

$$\frac{1}{2\pi\alpha'}\int_A F_3 = 2\pi M \text{ and } \frac{1}{2\pi\alpha'}\int_B H_3 = -2\pi K.$$

$$W = \int G_3 \wedge \Omega = (2\pi)^2\alpha'\left(M\int_B \Omega - K\tau\int_A \Omega\right),$$

$$z = \int_A \Omega$$

$$\int_B \Omega = \frac{z}{2\pi i}\log z + \mathbb{H}_{\text{holomorphic}}$$

$$\mathcal{D}_z W \simeq (2\pi)^2\alpha'\left(\frac{M}{2\pi i}\log z - i\frac{K}{g_s} + \cdots\right)$$

$$z \simeq e^{-2\pi K/Mg_s}.$$

$$ds^2 = \left(\frac{r}{R}\right)^2 |d\vec{x}|^2 + \left(\frac{R}{r}\right)^2 \left(dr^2 + r^2 d\Omega_5^2\right) \text{ with } R^4 = 4\pi g_s N(\alpha')^2$$

$$r_{\min} \simeq \rho_{\min}^{2/3} = z^{1/3} \simeq e^{-2\pi K/3Mg_s},$$

$$\mathcal{K} = -3\log[-i(\rho - \bar{\rho})] - \log[-i(\tau - \bar{\tau})] - \log\left(i\int_M \Omega \wedge \bar{\Omega}\right)$$

$$ds^2 = e^{-6u(x)} \underbrace{g_{\mu\nu}dx^\mu dx^\nu}_{4D} + e^{2u(x)} \underbrace{g_{mn}dy^m dy^n}_{\text{CY}_3}$$

$$C_{\mu\nu pq} = a_{\mu\nu} J_{pq},$$

$$da = e^{-8u(x)} \star db.$$

$$\rho = \frac{b}{\sqrt{2}} + ie^{4u},$$

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left(R - \frac{1}{2} \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{(\mathrm{Im}\tau)^2} - \frac{3}{2} \frac{\partial_\mu \rho \partial^\mu \bar{\rho}}{(\mathrm{Im}\rho)^2} \right)$$

$$V(T,K) = \frac{1}{4\mathcal{V}^3} \left(\int_M F \wedge \star F - \frac{1}{6} T_{\mathrm{M2}} \chi \right)$$

$$F^{2,2} = F_o^{2,2} + J \wedge F_o^{1,1} + J \wedge J F_o^{0,0}$$

$$\star F^{2,2} = F^{2,2} - 2J \wedge F_o^{1,1},$$

$$\star F^{4,0} = F^{4,0} \text{ and } \star F^{3,1} = -F^{3,1},$$

$$\star F = F - 2F^{3,1} - 2F^{1,3} - 2J \wedge F_o^{1,1}$$

$$\int_M F \wedge \star F = \int_M F \wedge F - 4 \int_M F^{3,1} \wedge F^{1,3} - 2 \int_M J \wedge F_o^{1,1} \wedge J \wedge F_o^{1,1}$$

$$F = \star F \text{ and } F \notin H^{(2,2)}_{\text{primitive}}$$

$$F \sim \Omega \text{ or } F \sim J \wedge J$$

$$\int_M F^{3,1} \wedge F^{1,3} = -e^{\mathcal{K}^{3,1}} G^{I\bar{J}} \mathcal{D}_I W^{3,1} \mathcal{D}_{\bar{J}} \bar{W}^{3,1}$$

$$\int_M J \wedge F_o^{1,1} \wedge J \wedge F_o^{1,1} = -\mathcal{V}^{-1} G^{AB} \mathcal{D}_A W^{1,1} \mathcal{D}_B W^{1,1}$$

$$\mathcal{D}_A = \partial_A - \frac{1}{2} \partial_A \mathcal{K}^{1,1} \text{ with } \mathcal{K}^{1,1} = -3\log \mathcal{V},$$

$$G_{AB} = -\frac{1}{2} \partial_A \partial_B \log \mathcal{V}$$

$$V(T,K) = e^{\mathcal{K}} G^{I\bar{J}} \mathcal{D}_I W^{3,1} \mathcal{D}_{\bar{J}} \bar{W}^{3,1} + \frac{1}{2} \mathcal{V}^{-4} G^{AB} \mathcal{D}_A W^{1,1} \mathcal{D}_B W^{1,1},$$

$$V = e^{\mathcal{K}} \left(G^{a\bar{b}} \mathcal{D}_a W \mathcal{D}_{\bar{b}} \bar{W} - 3|W|^2 \right)$$

$$\mathcal{K}(z,\bar{z}) \to \mathcal{K}(z,\bar{z}) - f(z) - \bar{f}(\bar{z})$$

$$W(z) \to e^{f(z)} W(z).$$

$$G^{\rho\bar{\rho}} \mathcal{D}_\rho W \mathcal{D}_{\bar{\rho}} \bar{W} - 3|W|^2 = 0.$$

$$V = e^{\mathcal{K}} \sum_{i,j \neq \rho} G^{i\bar{j}} \mathcal{D}_i W \mathcal{D}_{\bar{j}} \bar{W},$$

$$\mathcal{D}_i W = 0,$$

$$\mathcal{D}_\rho W \neq 0.$$

$$W = W_0 + Ae^{ia\rho}$$

$$W_{\text{inst}} = T(z^\alpha) e^{2\pi i \rho}$$

$$V = e^{\mathcal{K}} \left(G^{\rho\bar{\rho}} \mathcal{D}_\rho W \mathcal{D}_{\bar{\rho}} \bar{W} - 3|W|^2 \right)$$

$$V_0 = -\frac{a^2 A^2}{6\sigma_0} e^{-2a\sigma_0}$$

$$\delta V = \frac{D}{\sigma^2},$$

$$V(\sigma) = \frac{aAe^{-a\sigma}}{2\sigma^2} \left(\frac{1}{3} \sigma a A e^{-a\sigma} + W_0 + Ae^{-a\sigma} \right) + \frac{D}{\sigma^2}.$$

$$W = W_0 + Ae^{ia\rho}$$

$$\mathcal{D}_\rho W = \partial_\rho W + \partial_\rho \mathcal{K} W = 0 \text{ with } \mathcal{K} = -3\log\left[-i(\rho - \bar{\rho})\right]$$

$$W_0 = -A\left(\frac{2}{3} a\sigma_0 + 1 \right) e^{-a\sigma_0}$$

$$W = -\frac{2}{3} A a \sigma_0 e^{-a\sigma_0}$$

$$V = e^{\mathcal{K}} \left(G^{\rho\bar{\rho}} \mathcal{D}_\rho W \mathcal{D}_{\bar{\rho}} \bar{W} - 3|W|^2 \right)$$

$$V_0 = -\frac{a^2 A^2}{6\sigma_0} e^{-2a\sigma_0},$$

$$\tilde{\mathcal{K}} = \mathcal{K} + \log |W|^2.$$

$$V = e^{\tilde{\mathcal{K}}} \left(G^{a\bar{b}} \frac{\mathcal{D}_a W}{W} \frac{\mathcal{D}_{\bar{b}} \bar{W}}{\bar{W}} - 3 \right).$$

$$G_{a\bar{b}} = \partial_a \partial_{\bar{b}} \mathcal{K} = \partial_a \partial_{\bar{b}} \tilde{\mathcal{K}},$$

$$\frac{\mathcal{D}_a W}{W} = \partial_a \log W + \partial_a \mathcal{K} = \partial_a \tilde{\mathcal{K}}$$

$$V = e^{\tilde{\mathcal{K}}} \left(G^{a\bar{b}} \partial_a \tilde{\mathcal{K}} \partial_{\bar{b}} \tilde{\mathcal{K}} - 3 \right)$$

$$\kappa_4^4 V = e^{\kappa_4^2 \mathcal{K}} \left(\kappa_4^4 G^{a\bar{b}} \mathcal{D}_a W \mathcal{D}_{\bar{b}} \bar{W} - 3\kappa_4^6 |W|^2 \right)$$

$$V = e^{\kappa_4^2 \mathcal{K}} \left(G^{a\bar{b}} \mathcal{D}_a W \mathcal{D}_{\bar{b}} \bar{W} - 3\kappa_4^2 |W|^2 \right)$$

$$V = G^{a\bar{b}} \partial_a W \partial_{\bar{b}} \bar{W} + \mathcal{O}(\kappa_4^2)$$

$$ds^2 = e^{2D(y)} (\underbrace{g_{\mu\nu}(x) dx^\mu dx^\nu}_{4\text{D}} + \underbrace{g_{mn}(y) dy^m dy^n}_{6\text{D}})$$

$$D(y) = \Phi(y)$$

$$T^\alpha = de^\alpha + \omega^\alpha{}_\gamma \wedge e^\gamma$$

$$T^\alpha = \Gamma^r_{mn} e^\alpha_r dx^m \wedge dx^n$$

$$\omega_{\alpha\beta} = h\varepsilon_{\alpha\beta},$$

$$\delta\Psi_M = \nabla_M \varepsilon - \frac{1}{4} \mathbf{H}_M \varepsilon = 0$$

$$\delta\lambda = -\frac{1}{2} \partial\Phi\varepsilon + \frac{1}{4} \mathbf{H}\varepsilon = 0$$

$$\delta\chi = -\frac{1}{2} \mathbf{F}\varepsilon = 0$$

$$dH = \frac{\alpha'}{4} [\mathrm{tr}(R \wedge R) - \mathrm{tr}(F \wedge F)]$$

$$H_{\mu\nu\rho} = H_{\mu\nu p} = H_{\mu np} = 0 \text{ and } F_{\mu\nu} = F_{\mu n} = 0.$$

$$H_{mnp} \neq 0 \text{ and } \partial_m \Phi \neq 0$$

$$\nabla_M \varepsilon = \partial_M \varepsilon + \frac{1}{4} \omega_{MAB} \Gamma^{AB} \varepsilon$$

$$\widetilde{\nabla}_M \varepsilon = \left(\nabla_M - \frac{1}{8} H_{MAB} \Gamma^{AB} \right) \varepsilon$$

$$\tilde{\omega}^A_B = \omega^A{}_B - \frac{1}{2} H_M{}^A_B dx^M$$

$$\tilde{T}^A = T^A + \frac{1}{2} H^A_{MN} dx^M \wedge dx^N$$

$$\varepsilon(x, y) = \zeta_+(x) \otimes \eta_+(y) + \zeta_-(x) \otimes \eta_-(y),$$

$$\zeta_- = \zeta_+^\star \text{ and } \eta_- = \eta_+^\star$$

$$\Gamma_\mu = \gamma_\mu \otimes 1 \text{ and } \Gamma_m = \gamma_5 \otimes \gamma_m$$

$$\delta\psi_\mu = \nabla_\mu \zeta_+ = 0$$

$$\widetilde{\nabla}_m \eta_\pm = \left(\nabla_m - \frac{1}{8} H_{mnp} \gamma^{np} \right) \eta_\pm = 0$$

$$J_m{}^n = i\eta_+^\dagger \gamma_m{}^n \eta_+ = -i\eta_-^\dagger \gamma_m{}^n \eta_-$$

$$J_m{}^n J_n{}^p = -\delta_m{}^p$$

$$g_{mn} = J_m{}^k J_n{}^l g_{kl},$$

$$J_{mn} = J_m{}^k g_{kn}$$

$$J = \frac{1}{2} J_{mn} dx^m \wedge dx^n$$

$$\tilde{\nabla}_m J_n^p = \nabla_m J_n^p - \frac{1}{2} H_{sm}^p J_n^s - \frac{1}{2} H_{mn}^s J_s^p = 0.$$

$$J_a^b = i\delta_a^b, J_{\bar{a}}^{\bar{b}} = -i\delta_{\bar{a}}^{\bar{b}} \text{ and } J_a^{\bar{b}} = J_{\bar{a}}^b = 0.$$

$$J_{a\bar{b}} = i g_{a\bar{b}}$$

$$H = i(\partial - \bar{\partial})J$$

$$\partial_a \Phi = -\frac{1}{2} H_{ab\bar{c}} g^{b\bar{c}}$$

$$\Omega = e^{-2\Phi} \eta_-^T \gamma_{abc} \eta_- dz^a \wedge dz^b \wedge dz^c$$

$$\bar{\partial}\Omega = 0$$

$$\|\Omega\|^2 = \Omega_{a_1 a_2 a_3} \bar{\Omega}_{\bar{b}_1 \bar{b}_2 \bar{b}_3} g^{a_1 \bar{b}_1} g^{a_2 \bar{b}_2} g^{a_3 \bar{b}_3}$$

$$\|\Omega\|^2 = e^{-4(\Phi + \Phi_0)}$$

$$d\left(e^{-2\Phi} J \wedge J\right) = 0$$

$$\left(F_{ab}\gamma^{ab} + F_{\bar{a}\bar{b}}\gamma^{\bar{a}\bar{b}} + 2F_{a\bar{b}}\gamma^{a\bar{b}}\right)\eta = 0$$

$$g^{a\bar{b}} F_{a\bar{b}} = F_{ab} = F_{\bar{a}\bar{b}} = 0$$

$$i\partial\bar{\partial}J = \frac{\alpha'}{8}[\mathrm{tr}(R \wedge R) - \mathrm{tr}(F \wedge F)],$$

$$d(\|\Omega\| J \wedge J) = 0$$

$$H = i(\partial - \bar{\partial})J \text{ and } \Phi = \Phi_0 - \frac{1}{2}\log \|\Omega\|$$

$$g_{mn}(y) = e^{2D(y)} g_{mn}^{\mathrm{K3}}(y)$$

$$\left(\partial_m \Phi + \frac{1}{2}\partial_m h\right)\gamma^m \eta = 0$$

$$\nabla_m \eta + \frac{1}{4}\partial_n h \gamma_m{}^n \eta = 0$$

$$\Phi(y) = -\frac{1}{2}h(y) + \text{ const.}$$

$$\widetilde{\nabla}_m \eta + \frac{1}{2}\partial_n D \gamma_m{}^n \eta + \frac{1}{4}\partial_n h \gamma_m{}^n \eta = 0$$

$$D(y) = \Phi(y)$$

$$d \star d\Phi = -\frac{\alpha'}{8}[\text{tr}(R \wedge R) - \text{tr}(F \wedge F)]$$

$$F_{\bar{a}\bar{b}} = F_{ab} = g^{a\bar{b}}F_{a\bar{b}} = 0$$

$$N_{mn}{}^p = J_m{}^q J_{[n}{}^p{}_{,q]} - J_n{}^q J_{[m}{}^p{}_{,q]}.$$

$$N_{mnp} = \frac{1}{2}\left(H_{mnp} - 3J_{[m}{}^q J_n{}^s H_{p]qs}\right)$$

$$\begin{aligned} J_{[m}{}^p J_{n]}{}^q &= \frac{1}{4}g^{pr}g^{qs}(J \wedge J)_{mrns} + \frac{1}{2}J_{mn}J^{pq} \\ &= \frac{1}{2}\eta^\dagger \gamma^{pq}{}_{mn}\eta - \frac{1}{2}\eta^\dagger \gamma^{pq}\eta\eta^\dagger \gamma_{mn}\eta \end{aligned}$$

$$\frac{1}{2}(J \wedge J) = * J$$

$$\begin{aligned} N_{mnp} &= -\frac{1}{12}\eta_+^\dagger\{H, \gamma_{mnp} + 3i\gamma_{[m}J_{np]}\}\eta_+ \\ &= -\frac{1}{12}\eta_+^\dagger\left[\partial\Phi, \gamma_{mnp} + 3i\gamma_{[m}J_{np]}\right]\eta_+ \\ &= 0. \end{aligned}$$

$$\nabla_{\bar{k}}\Omega_{abc} = \partial_{\bar{k}}\Omega_{abc} - 3\Gamma^p_{\bar{k}[a}\Omega_{bc]p} = \partial_{\bar{k}}\Omega_{abc} - \Gamma^p_{\bar{k}p}\Omega_{abc}$$

$$\Gamma^p_{\bar{k}p} = g^{p\bar{q}}\partial_{[\bar{k}}g_{\bar{q}]p} = \frac{1}{2}H_{\bar{k}p\bar{q}}g^{p\bar{q}} = \partial_{\bar{k}}\Phi$$

$$\nabla_{\bar{k}}\Omega_{abc} = \partial_{\bar{k}}\Omega_{abc} - \partial_{\bar{k}}\Phi\Omega_{abc}$$

$$\nabla_{\bar{k}}\Omega_{abc} = -\partial_{\bar{k}}\Phi\Omega_{abc}$$

$$\nabla_{\bar{k}}\Omega_{abc} = \nabla_{\bar{k}}\left(e^{-2\Phi}\eta_-^T\gamma_{abc}\eta_-\right) = -2\partial_{\bar{k}}\Phi\Omega_{abc} + 2e^{-2\Phi}\eta_-^T\gamma_{abc}\nabla_{\bar{k}}\eta_-$$

$$-2\partial_{\bar{k}}\Phi\Omega_{abc} + \frac{1}{2}H_{\bar{k}n\bar{p}}g^{n\bar{p}}\Omega_{abc} = -\partial_{\bar{k}}\Phi\Omega_{abc}$$

$$L_{\text{eff}} = \int d^{10}x\sqrt{-G}e^{-2\Phi}\left(\frac{4}{\alpha'^4}R - \frac{1}{\alpha'^3}\text{tr}|F|^2\right) + \cdots$$

$$L_{\text{eff}} = \int d^4x\mathcal{V}\sqrt{-G}e^{-2\Phi}\left(\frac{4}{\alpha'^4}R - \frac{1}{\alpha'^3}\text{tr}|F|^2\right) + \cdots$$

$$G_4 = \frac{e^{2\Phi}\alpha'^4}{64\pi\mathcal{V}}$$

$$\alpha_{\mathrm{U}} = \frac{e^{2\Phi}\alpha'^3}{16\pi\mathcal{V}}$$

$$G_4 = \frac{1}{4}\alpha_{\mathrm{U}}\alpha'$$

$$\mathcal{V} \ll \frac{\alpha'^3}{16\pi\alpha_{\mathrm{U}}}$$

$$G_4 > \frac{\alpha_{\mathrm{U}}^{4/3}}{M_{\mathrm{U}}^2}$$

$$L = \frac{1}{2\kappa_{11}^2}\int_{M_{11}} d^{11}x\sqrt{g}R - \sum_i \frac{1}{8\pi(4\pi\kappa_{11}^2)^{2/3}}\int_{M_i^{10}} d^{10}x\sqrt{g}|F_i|^2$$

$$G_4 = \frac{\kappa_{11}^2}{8\pi^2\mathcal{V}d} \text{ and } \alpha_{\mathrm{U}} = \frac{(4\pi\kappa_{11}^2)^{2/3}}{2\mathcal{V}}$$

$$(dF)_{11IJKL} = -\frac{3\sqrt{2}}{2\pi}\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}\left[\mathrm{tr}F_{[IJ}F_{KL]} - \frac{1}{2}\mathrm{tr}R_{[IJ}R_{KL]}\right]\delta(x^{11})$$

$$[F_4/2\pi] = \lambda(F) - \lambda(R)/2$$

$$G_4 \geq \frac{\alpha_{\mathrm{U}}^2}{M_{\mathrm{U}}^2}$$

$$N_{\mathrm{RR}}^{\alpha} = \eta^{\alpha\beta}\int_{\Sigma_\beta} F_3 \text{ and } N_{\mathrm{NS}}^{\alpha} = \eta^{\alpha\beta}\int_{\Sigma_\beta} H_3$$

$$0 \leq \eta_{\alpha\beta}N_{\mathrm{RR}}^{\alpha}N_{\mathrm{NS}}^{\beta} \leq L$$

$$L = \chi/24 - N_{D3}.$$

$$\Pi_\alpha = \int_{\Sigma_\alpha} \Omega$$

$$W = (N_{\mathrm{RR}}^{\alpha} - \tau N_{\mathrm{NS}}^{\alpha})\Pi_\alpha = N \cdot \Pi.$$

$$\mathcal{D}_i W = 0$$

$$W = N \cdot \Pi = A\tau + B$$

$$A = -\left(N_{\mathrm{NS}}^1 + iN_{\mathrm{NS}}^2\right) = a_1 + ia_2$$
$$B = N_{\mathrm{RR}}^1 + iN_{\mathrm{RR}}^2 = b_1 + ib_2$$

$$\mathcal{D}_\tau W = \partial_\tau W + \partial_\tau K W = \partial_\tau W - \frac{1}{\tau - \bar{\tau}}W = -\frac{A\bar{\tau} + B}{\tau - \bar{\tau}} = 0$$

$$\tau = -\bar{B}/\bar{A}$$

$$T_{00} = \rho, T_{ij} = pg_{ij}$$

$$p = w\rho$$

- Friedmann-Robertson-Walker:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\,\theta d\phi^2)\right)$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$$R_{\text{curv}} = a|k|^{-1/2}$$

$$H^2 = \frac{1}{3M_{\text{P}}^2}\rho_{\text{tot}} - \frac{k}{a^2} + \frac{\Lambda}{3},$$
$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\text{P}}^2}(\rho_{\text{tot}} + 3p_{\text{tot}}) + \frac{\Lambda}{3},$$

$$H(t) = \dot{a}(t)/a(t)$$

$$\rho_{\text{tot}} = \sum_i \rho_i, p_{\text{tot}} = \sum_i p_i$$

$$\rho_{\text{c}} = 3H^2M_{\text{P}}^2$$

$$\Omega - 1 = \frac{k}{a^2H^2} - \frac{\Lambda}{3H^2}.$$

$$\dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + p_{\text{tot}}) = 0.$$

$$\rho \sim \frac{1}{a^{3(w+1)}}$$

$$\ddot{a}(t) > 0.$$

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0$$

$$\rho_{\text{tot}} + 3p_{\text{tot}} < 0$$

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi),$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$H^2 = \frac{1}{3M_{\text{P}}^2}\left[V(\phi) + \frac{1}{2}\dot{\phi}^2\right]$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}.$$

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{2}{p}}\frac{\phi}{M_{\mathrm{P}}}\right),$$

$$a(t) = a_0 t^p$$
$$\phi(t) = \sqrt{2p}M_{\mathrm{P}}\log\left(\sqrt{\frac{V_0}{p(3p-1)}}\frac{t}{M_{\mathrm{P}}}\right).$$

$$H^2 \approx \frac{V(\phi)}{3M_{\mathrm{P}}^2},$$
$$3H\dot{\phi} \approx -V'(\phi),$$

$$\varepsilon(\phi) = \frac{1}{2}M_{\mathrm{P}}^2(V'/V)^2 \ll 1,$$
$$|\eta(\phi)| = M_{\mathrm{P}}^2|V''/V| \ll 1.$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0,$$

$$-\frac{\dot{H}}{H^2} < 1$$

$$-\frac{\dot{H}}{H^2} \approx \frac{M_{\mathrm{P}}^2}{2}\left(\frac{V'}{V}\right)^2 = \varepsilon.$$

$$\varepsilon = \eta/2 = 1/p$$

$$V(\phi,\psi) = a(\psi^2 - 1)\phi^2 + b\phi^4 + c$$

$$N(t) = \log\left(\frac{a(t_{\mathrm{end}})}{a(t)}\right),$$

$$N(t) = \int_t^{t_{\mathrm{end}}} \frac{\dot{a}}{a}dt = \int_t^{t_{\mathrm{end}}} Hdt \approx \frac{1}{M_{\mathrm{P}}^2}\int_{\phi_{\mathrm{end}}}^{\phi} \frac{V}{V'}d\phi$$

$$\delta_H(k) = \sqrt{\frac{512\pi}{75}\frac{V^{2/3}}{M_{\mathrm{P}}^3 V'}}\Bigg|_{k=aH},$$
$$A_G(k) = \sqrt{\frac{32}{75}\frac{V^{1/2}}{M_{\mathrm{P}}^2}}\Bigg|_{k=aH}.$$

$$\delta_H(k) \approx k^{n-1}, A_G^2(k) \approx k^{n_G}$$

$$n - 1 = \frac{d\ln\ \delta_H^2}{d\ln\ k}, n_G = \frac{d\ln\ A_G^2}{d\ln\ k}$$

$$n = 1 - 6\varepsilon + 2\eta$$
$$n_G = -2\varepsilon$$

$$V(r) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3}{M_{10}^8 r^4}\right),$$

$$V(\phi) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3^3}{M_{10}^8 \phi^4}\right)$$

$$\epsilon = \frac{1}{2}M_{\text{P}}^2(V'/V)^2 \sim \frac{L^6}{r^{10}}$$
$$\eta = M_{\text{P}}^2(V''/V) \sim \frac{L^6}{r^6}$$

$$V(\phi, L) \approx \frac{2T_3}{L^{12}}$$

$$\mathcal{K}(\rho,\bar{\rho},\phi,\bar{\phi}) = -3\log\left[\rho + \bar{\rho} - k(\phi,\bar{\phi})\right]$$

$$2L = \rho + \bar{\rho} - k(\phi,\bar{\phi})$$

$$W(\rho) = W_0 + Ae^{-a\rho}.$$

$$W_0 = \int G_3 \wedge \Omega$$

$$V = e^{\mathcal{K}}\left(G^{a\bar{b}}D_a W D_{\bar{b}}\bar{W} - 3|W|^2\right)$$

$$V = \frac{1}{6L}\left(\left|\partial_\rho W\right|^2 - \frac{3}{2L}\left(\bar{W}\partial_\rho W + W\partial_{\bar{\rho}}\bar{W}\right)\right) + \left(\frac{\left|\partial_\rho W\right|^2}{12L^2}\right)\phi\bar{\phi}$$

$$V = \frac{1}{6L}\left(\left|\partial_\rho W\right|^2 - \frac{3}{2L}\left(\bar{W}\partial_\rho W + W\partial_{\bar{\rho}}\bar{W}\right)\right) + \left(\frac{\left|\partial_\rho W\right|^2}{12L^2}\right)\phi\bar{\phi} + \frac{D}{(2L)^2}$$

$$V = \frac{V_0(\rho_c)}{(1 - \varphi\bar{\varphi}/3)^2} \approx V_0(\rho_c)\left(1 + \frac{2}{3}\varphi\bar{\varphi}\right)$$

$$V(\phi, T) = a((\phi/\ell_s)^2 - b)T^2 + cT^4 + V(\phi)$$

$$\begin{aligned}
&[\gamma_m, \gamma^r] = 2\gamma_m{}^r && \{\gamma_m, \gamma^r\} = 2\delta_m{}^r \\
&[\gamma_{mn}, \gamma^r] = -4\delta_{[m}{}^r\gamma_{n]} && \{\gamma_{mn}, \gamma^r\} = 2\gamma_{mn}{}^r \\
&[\gamma_{mnp}, \gamma^r] = 2\gamma_{mnp}{}^r && \{\gamma_{mnp}, \gamma^r\} = 6\delta_{[m}{}^r\gamma_{np]} \\
&[\gamma_{mnpq}, \gamma^r] = -8\delta_{[m}{}^r\gamma_{npq]} && \{\gamma_{mnpq}, \gamma^r\} = 2\gamma_{mnpq}{}^r \\
&[\gamma_{mnpqk}, \gamma^r] = 2\gamma_{mnpqk}{}^r && \{\gamma_{mnpqk}, \gamma^r\} = 10\delta_{[m}{}^r\gamma_{npqk]}
\end{aligned}$$

$$[\gamma_{mn},\gamma^{rs}] = -8\delta_{[m}{}^{[r}\gamma_{n]}{}^{s]} \quad \{\gamma_{mn},\gamma^{rs}\} = 2\gamma_{mn}{}^{rs} - 4\delta_{[mn]}{}^{rs}$$
$$[\gamma_{mnp},\gamma^{rs}] = 12\delta_{[m}{}^{[r}\gamma_{np]}{}^{s]} \quad \{\gamma_{mnp},\gamma^{rs}\} = 2\gamma_{mnp}{}^{rs} - 12\delta_{[mn}{}^{rs}\gamma_{p]}$$
$$[\gamma_{mnpq},\gamma^{rs}] = -16\delta_{[m}{}^{[r}\gamma_{npq]}{}^{s]} \quad \{\gamma_{mnpq},\gamma^{rs}\} = 2\gamma_{mnpq}{}^{rs} - 24\delta_{[mn}{}^{rs}\gamma_{pq]}$$
$$[\gamma_{mnpqk},\gamma^{rs}] = 20\delta_{[m}{}^{[r}\gamma_{npqk]}{}^{s]} \quad \{\gamma_{mnpqk},\gamma^{rs}\} = 2\gamma_{mnpqk}{}^{rs} - 40\delta_{[mn}{}^{rs}\gamma_{pqk]}$$
$$[\gamma_{mnp},\gamma^{rst}] = 2\gamma_{mnp}{}^{rst} - 36\delta_{[mn}{}^{[rs}\gamma_{p]}{}^{t]}$$
$$[\gamma_{mnpq},\gamma^{rst}] = -24\delta_{[m}{}^{[r}\gamma_{npq]}{}^{st]} + 48\delta_{[mnp}{}^{rst}\gamma_{q]}$$
$$[\gamma_{mnpqk},\gamma^{rst}] = 2\gamma_{mnpqk}{}^{rst} - 120\delta_{[mn}{}^{[rs}\gamma_{pqk]}{}^{t]}$$
$$\{\gamma_{mnp},\gamma^{rst}\} = 18\delta_{[m}{}^{[r}\gamma_{np]}{}^{st]} - 12\delta_{[mnp]}{}^{rst}$$
$$\{\gamma_{mnpq},\gamma^{rst}\} = 2\gamma_{mnpq}{}^{rst} - 72\delta_{[mn}{}^{[rs}\gamma_{pq]}{}^{t]}$$
$$\{\gamma_{mnpqk},\gamma^{rst}\} = 30\delta_{[m}{}^{[r}\gamma_{npqk]}{}^{st]} - 120\delta_{[mnp}{}^{rst}\gamma_{qk]}$$
$$[\gamma_{mnpq},\gamma^{rstu}] = -32\delta_{[m}{}^{[r}\gamma_{npq]}{}^{stu]} + 192\delta_{[mnp}{}^{[rst}\gamma_{q]}{}^{u]}$$
$$[\gamma_{mnpqk},\gamma^{rstu}] = 40\delta_{[m}{}^{[r}\gamma_{npqk]}{}^{stu]} - 480\delta_{[mnp}{}^{[rst}\gamma_{qk]}u]$$
$$\{\gamma_{mnpq},\gamma^{rstu}\} = 2\gamma_{mnpq}{}^{rstu} - 144\delta_{[mn}{}^{[rs}\gamma_{pq]}{}^{tu]} + 48\delta_{[mnpq]}{}^{rstu}$$
$$\{\gamma_{mnpqk},\gamma^{rstu}\} = 2\gamma_{mnpqk}{}^{rstu} - 240\delta_{[mn}{}^{[rs}\gamma_{pqk]}{}^{tu]} + 240\delta_{[mnpq}{}^{rstu}\gamma_{k]}$$
$$[\gamma_{mnpqk},\gamma^{rstuv}] = 2\gamma_{mnpqk}{}^{rstuv} - 400\delta_{[mn}{}^{[rs}\gamma_{pqk]}{}^{tuv]} + 1200\delta_{[mnpq}{}^{[rstu}\gamma_{k]}{}^{v]}$$
$$\{\gamma_{mnpqk},\gamma^{rstuv}\} = 50\delta_{[m}{}^{[r}\gamma_{npqk]}{}^{stuv]} - 1200\delta_{[mnp}{}^{[rst}\gamma_{qk]}{}^{uv]} + 240\delta_{[mnpqk]}{}^{rstuv}$$

$$\left.\begin{matrix}[\gamma_{m_1\ldots m_p},\gamma^{n_1\ldots n_q}] \\ \{\gamma_{m_1\ldots m_p},\gamma^{n_1\ldots n_q}\}\end{matrix} pq \text{ odd } pq \text{ even}\right\} = 2\gamma_{m_1\ldots m_p}{}^{n_1\ldots n_q} - \frac{2p!\,q!}{2!\,(p-2)!\,(q-2)!}\delta_{[m_1 m_2}{}^{[n_1 n_2}\gamma_{m_3\ldots m_p]}{}^{n_3\ldots n_q]} + \frac{2p!\,q!}{4!\,(p-4)!\,(q-4)!}\delta_{[m_1\ldots m_4}{}^{[n_1\ldots n_4}\gamma_{m_5\ldots m_p]}{}^{n_5\ldots n_q]} - \cdots$$

$$\left.\begin{matrix}[\gamma_{m_1\ldots m_p},\gamma^{n_1\ldots n_q}]\, pq \text{ even} \\ \{\gamma_{m_1\ldots m_p},\gamma^{n_1\ldots n_q}\}\, pq \text{ odd}\end{matrix}\right\} = \frac{(-1)^{p-1}2p!\,q!}{1!\,(p-1)!\,(q-1)!}\delta_{[m_1}{}^{[n_1}\gamma_{m_2\ldots m_p]}{}^{n_2\ldots n_q]} - \frac{(-1)^{p-1}2p!\,q!}{3!\,(p-3)!\,(q-3)!}\delta_{[m_1 m_2 m_3}{}^{[n_1 n_2 n_3}\gamma_{m_4\ldots m_p]}{}^{n_4\ldots n_q]} + \cdots$$

$$\chi\bar{\psi} = \frac{1}{2^{[d/2]}}\sum_{p=0}^{d}\frac{1}{p!}\gamma^{m_p\ldots m_1}\bar{\psi}\gamma_{m_1\ldots m_p}\chi$$

$$\hat{\nabla}_M\eta = \nabla_M\eta + \frac{1}{2}\Omega^{-1}\nabla_N\Omega\Gamma_M^N\eta$$

$$\nabla_p J_m{}^n = 0$$

$$P_\pm\xi = \frac{1}{2}(1 \pm \gamma^9)\xi = \xi_\pm$$

$$\nabla_m\xi_+ - \frac{1}{4}\Delta^{-3/4}\mathbf{F}_m\xi_- = 0, \nabla_m\xi_- = 0$$

$$\eta_1 = v^a \gamma_a \eta_2.$$

$$F \wedge J + \star\, dv = 0$$

$$V = e^{\mathcal{K}}\left(G^{a\bar{b}} \mathcal{D}_a W \mathcal{D}_{\bar{b}} \bar{W} - 3|W|^2\right),$$

$$W = \int_M \Omega \wedge G_3$$

$$|Z(\gamma)|^2 = \frac{\left|\int_\gamma \Omega\right|^2}{\int \ \Omega \wedge \bar{\Omega}}$$

$$J = \frac{2}{3} dr \wedge g_5 + \frac{1}{3}(e^2 \wedge e^1 + e^3 \wedge e^4)$$

$$\star_6 H = -e^{-a\Phi} d\left(e^{a\Phi} J\right).$$

- Agujeros negros cuánticos.

$$S = \frac{1}{16\pi G_D} \int \ d^D x \sqrt{-g} R$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

$$R_{\mu\nu} = 0$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\left(1 - \frac{r_{\mathrm{H}}}{r}\right) dt^2 + \left(1 - \frac{r_{\mathrm{H}}}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

$$r_{\mathrm{H}} = 2G_4 M$$

$$d\Omega_2^2 = d\theta^2 + \sin^2\ \theta d\phi^2$$

$$g_{tt} \sim -(1 + 2\Phi)$$

$$\Phi = -\frac{MG_4}{r}$$

$$ds^2 = -h dt^2 + h^{-1} dr^2 + r^2 d\Omega_{D-2}^2$$

$$h = 1 - \left(\frac{r_{\mathrm{H}}}{r}\right)^{D-3}$$

$$r_{\mathrm{H}}^{D-3} = \frac{16\pi M G_D}{(D-2)\Omega_{D-2}}$$

$$\Omega_n = \frac{2\pi^{(n+1)/2}}{\Gamma\left(\frac{n+1}{2}\right)}$$

$$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{12r_{\mathrm{H}}^2}{r^6}$$

$$u = \left(\frac{r}{r_{\mathrm{H}}} - 1\right)^{1/2} e^{r/2r_{\mathrm{H}}} \cosh\left(\frac{t}{2r_{\mathrm{H}}}\right)$$
$$v = \left(\frac{r}{r_{\mathrm{H}}} - 1\right)^{1/2} e^{r/2r_{\mathrm{H}}} \sinh\left(\frac{t}{2r_{\mathrm{H}}}\right)$$

$$ds^2 = \frac{4r_{\mathrm{H}}^3}{r} e^{-r/r_{\mathrm{H}}}(-dv^2 + du^2) + r^2 d\Omega_2^2$$

$$u^2 - v^2 = \left(\frac{r}{r_{\mathrm{H}}} - 1\right) e^{r/r_{\mathrm{H}}}$$

$$-\infty < u < +\infty \text{ and } v^2 < u^2 + 1.$$

$$t = r_{\mathrm{H}} \log\left(\frac{u+v}{u-v}\right)$$

$$A = 4\pi r_{\mathrm{H}}^2 = 16\pi (MG_4)^2$$

$$ds^2 = -\Delta dt^2 + \Delta^{-1} dr^2 + r^2 d\Omega_2^2$$

$$\Delta = 1 - \frac{2MG_4}{r} + \frac{Q^2 G_4}{r^2}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_4 T_{\mu\nu}$$

$$T_{\mu\nu} = F_{\mu\rho} F_{\nu}^{\rho} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}$$

$$F_{tr} = E_r = \frac{Q}{r^2}$$

$$r = r_{\pm} = MG_4 \pm \sqrt{(MG_4)^2 - Q^2 G_4},$$

$$\frac{r_-}{G_4 M} \; 1 \; \frac{r_+}{G_4 M}$$

$$Q > G_4 M^2$$

$$Q = G_4 M^2$$

$$Q < G_4 M^2$$

$$Q^2 = 0$$

$$M\sqrt{G_4} \geq |Q|$$

$$r_{\pm} = MG_4 \text{ or } M\sqrt{G_4} = |Q|$$

$$ds^2 = -\left(1-\frac{r_0}{r}\right)^2 dt^2 + \left(1-\frac{r_0}{r}\right)^{-2} dr^2 + r^2 d\Omega_2^2$$

$$ds^2 = -\left(1+\frac{r_0}{r}\right)^{-2} dt^2 + \left(1+\frac{r_0}{r}\right)^2 (dr^2 + r^2 d\Omega_2^2)$$

$$ds^2 = -\left[1-\left(\frac{r_0}{r}\right)^2\right]^2 dt^2 + \left[1-\left(\frac{r_0}{r}\right)^2\right]^{-2} dr^2 + r^2 d\Omega_3^2$$

$$ds_5^2 = -\left[1+\left(\frac{r_0}{r}\right)^2\right]^{-2} dt^2 + \left[1+\left(\frac{r_0}{r}\right)^2\right] (dr^2 + r^2 d\Omega_3^2)$$

$$A = \Omega_3 r_0^3 = 2\pi^2 r_0^3$$

$$M = \frac{Q}{\sqrt{G_5}} = \frac{3\pi r_0^2}{4G_5}$$

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

$$\nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = 0$$
$$\epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = 0$$

$$ds^2 = -e^{2A(r)}dt^2 + e^{2B(r)}dr^2 + r^2 d\Omega_2^2$$

$$\sqrt{-g} = e^{A+B}r^2 \sin\,\theta$$

$$\partial_r\left(\sqrt{-g}F^{rt}\right) = \partial_r\left(e^{A+B}r^2 \sin\,\theta \cdot (-e^{-2A-2B}F_{rt})\right)$$
$$= \partial_r(e^{-A-B}r^2 \sin\,\theta F_{tr}) = 0$$

$$F_{tr} = e^{A+B}\frac{q(\theta,\phi)}{r^2}$$

$$F_{tr} = e^{A+B}\frac{q}{r^2}$$

$$\partial_\theta\left(e^{A+B}r^2 \sin\,\theta F^{\theta\phi}\right) = 0$$
$$\partial_\phi\left(e^{A+B}r^2 \sin\,\theta F^{\phi\theta}\right) = 0$$

$$F_{\theta\phi} = p(r,t)\sin\,\theta$$

$$F_{\theta\phi} = \mathrm{psin}\,\theta$$

$$Q_{\mathrm{mag}} = \frac{1}{4\pi}\int F = \frac{1}{4\pi}\int_0^\pi d\theta \int_0^{2\pi} d\phi F_{\theta\phi}$$

$$Q_{\mathrm{el}} = \frac{1}{4\pi}\int \star F = \frac{1}{4\pi}\int_0^\pi d\theta \int_0^{2\pi} d\phi (\star F)_{\theta\phi}$$

$$(\star F)_{\theta\phi} = \sqrt{-g}F^{rt} = e^{A+B}r^2\sin\,\theta e^{-2(A+B)}F_{tr} = q\sin\,\theta.$$

$$ds^2 = -\left(\frac{r_0}{r}\right)^{-2} dt^2 + \left(\frac{r_0}{r}\right)^2 dr^2 + r_0^2 d\Omega_2^2$$

$$ds^2 = \left(\frac{r_0}{r}\right)^2 (-dt^2 + dr^2) + r_0^2 d\Omega_2^2$$

$$Z = \mathrm{Tr}\left(e^{-\beta H}\right)$$

$$r = r_{\mathrm{H}}(1 + \rho^2)$$

$$ds^2 \approx 4r_{\mathrm{H}}^2 \left(d\rho^2 + \rho^2 \left(\frac{d\tau}{2r_{\mathrm{H}}}\right)^2 + \frac{1}{4} d\Omega_2^2\right)$$

$$\beta = 4\pi r_{\mathrm{H}} = 8\pi M G_4.$$

$$dM = TdS$$

$$dS/dt \geq 0$$

$$S = 4\pi M^2 G_4$$

$$A = 4\pi r_{\mathrm{H}}^2 = 16\pi (M G_4)^2$$

$$S = \frac{A}{4G_4}$$

$$S = \frac{A}{4G_D}$$

$$S = \pi r_+^2 / G_4$$

$$T = \frac{\sqrt{(MG_4)^2 - Q^2 G_4}}{2\pi r_+^2}$$

$$ds^2 = \frac{4r_+^3}{r_+ - r_-}\left[d\rho^2 + \rho^2 \left(\frac{(r_+ - r_-)d\tau}{2r_+^2}\right)^2 + \frac{r_+ - r_-}{4r_+} d\Omega^2\right]$$

$$\beta = \frac{4\pi r_+^2}{r_+ - r_-}$$

$$T = \frac{r_+ - r_-}{4\pi r_+^2} = \frac{\sqrt{(MG_4)^2 - Q^2 G_4}}{2\pi r_+^2}$$

$$r_H = \frac{2G_4 M}{c^2} \sim 3.0 \times 10^3\ \mathrm{m}, T = \frac{\hbar c^3}{8\pi M G_4 k_{\mathrm{B}}} \sim 6.0 \times 10^{-8}\ \mathrm{K}$$
$$S = \frac{A}{4G_4} = \frac{\pi R^2 c^3}{G_4 \hbar} \sim 1.0 \times 10^{77}, \Delta t \sim \frac{G_4^2 M^3}{\alpha \hbar c^4} \sim 10^{66}\ \text{years.}$$

$$g_{rr} = (1 + (r_0/r)^{D-3})^{\frac{2}{D-3}}$$

$$ds^2 = -\lambda^{-2/3} dt^2 + \lambda^{1/3}(dr^2 + r^2 d\Omega_3^2)$$

$$\lambda = \prod_{i=1}^{3} \left[1 + \left(\frac{r_i}{r}\right)^2\right]$$

$$A = 2\pi^2 r_1 r_2 r_3$$

$$M = M_1 + M_2 + M_3 \text{ where, } M_i = \frac{\pi r_i^2}{4G_5}$$

$$G_5 = \frac{G_{10}}{(2\pi)^5 RV}$$

$$r_i^2 = \frac{g_s^2 \ell_s^8}{RV} M_i$$

$$M_1 = 2\pi R T_{\mathrm{D1}} Q_1 = \frac{Q_1 R}{g_s \ell_s^2}$$

$$M_2 = (2\pi)^5 RV T_{\mathrm{D5}} Q_5 = \frac{Q_5 RV}{g_s \ell_s^6},$$

$$M_3 = \frac{n}{R}.$$

$$g_s Q_1 \gg \frac{V}{\ell_s^4}, g_s Q_5 \gg 1, g_s^2 n \gg \frac{R^2 V}{\ell_s^6}.$$

$$S = \frac{A}{4G_5} = \frac{2\pi g_s \ell_s^4}{\sqrt{RV}} \sqrt{M_1 M_2 M_3}$$

$$S = 2\pi\sqrt{Q_1 Q_5 n}$$

$$S = 2\pi\sqrt{\Delta}$$

$$\Delta = -\frac{1}{48}\mathrm{tr}(\Omega Z \Omega Z \Omega Z),$$

$$x_1 = Q_1 - Q_2 - Q_3, \quad x_2 = -Q_1 + Q_2 - Q_3$$
$$x_3 = -Q_1 - Q_2 + Q_3, \quad x_4 = Q_1 + Q_2 + Q_3$$

$$\Delta = \frac{1}{24}\sum x_i^3 = Q_1 Q_2 Q_3$$

$$ds^2 = -h\lambda^{-2/3} dt^2 + \lambda^{1/3}\left(\frac{dr^2}{h} + r^2 d\Omega_3^2\right)$$

$$h = 1 - \frac{r_0^2}{r^2}$$

$$\lambda=\prod_{i=1}^{3}\left[1+\left(\frac{r_i}{r}\right)^2\right] \text{ with } r_i^2=r_0^2\sinh^2\alpha_i, i=1,2,3$$

$$M=\frac{\pi r_0^2}{8G_5}(\cosh 2\alpha_1+\cosh 2\alpha_2+\cosh 2\alpha_3)$$

$$\lambda(r_0)=\prod_{i=1}^{3}\cosh^2\alpha_i$$

$$r_{\mathrm{H}}=r_0[\lambda(r_0)]^{1/6}$$

$$A=2\pi^2 r_{\mathrm{H}}^3=2\pi^2 r_0^3\cosh\alpha_1\cosh\alpha_2\cosh\alpha_3.$$

$$S=\frac{A}{4G_5}=\frac{2\pi r_0^3 V_6}{\ell_{\mathrm{p}}^9}\cosh\alpha_1\cosh\alpha_2\cosh\alpha_3$$

$$\hat{Q}_i=Q_i-\bar{Q}_i\ i=1,2,3.$$

$$S=\frac{A}{4G_5}=2\pi\prod_{i=1}^{3}\left(\sqrt{Q_i}+\sqrt{\bar{Q}_i}\right)$$

$$x^1=r\cos\theta\cos\psi,\quad x^2=r\cos\theta\sin\psi$$
$$x^3=r\sin\theta\cos\phi,\quad x^4=r\sin\theta\sin\phi$$

$$dx^i dx^i=dr^2+r^2 d\Omega_3^2$$

$$d\Omega_3^2=d\theta^2+\sin^2\theta d\phi^2+\cos^2\theta d\psi^2, 0\le\theta\le\pi/2, 0\le\phi,\psi\le 2\pi$$

$$ds^2=-\lambda^{-2/3}\left(dt-\frac{a}{r^2}\sin^2\theta d\phi+\frac{a}{r^2}\cos^2\theta d\psi\right)^2+\lambda^{1/3}(dr^2+r^2 d\Omega_3^2)$$

$$J=\frac{\pi a}{4G_5}$$

$$S=\frac{A}{4G_5}=2\pi\sqrt{Q_1 Q_5 n-J^2}$$

$$ds^2=-\lambda^{-1/2}dt^2+\lambda^{1/2}(dr^2+r^2 d\Omega_2^2),$$

$$\lambda=\prod_{i=1}^{4}\left(1+\frac{r_i}{r}\right).$$

$$M=\sum_{i=1}^{4}M_i \text{ with } M_i=\frac{r_i}{4G_4}.$$

$$A=4\pi\sqrt{r_1 r_2 r_3 r_4}.$$

$$S=\frac{A}{4G_4}=16\pi G_4\sqrt{M_1M_2M_3M_4}.$$

$$M_1=(2\pi R_1)(2\pi R_6)T_{\mathrm{D}2}Q_1=\frac{1}{g_s\ell_s^3}(R_1R_6)Q_1$$
$$M_2=(2\pi R_1)\cdots(2\pi R_6)T_{\mathrm{D}6}Q_2=\frac{1}{g_s\ell_s^7}(R_1\cdots R_6)Q_2$$
$$M_3=(2\pi R_1)\cdots(2\pi R_5)T_{\mathrm{NS}5}Q_3=\frac{1}{g_s^2\ell_s^6}(R_1\cdots R_5)Q_3$$
$$M_4=\frac{1}{R_1}Q_4$$

$$G_4=\frac{G_{10}}{(2\pi R_1)\cdots(2\pi R_6)}=\frac{g_s^2\ell_s^8}{8R_1\cdots R_6}$$

$$S=2\pi\sqrt{Q_1Q_2Q_3Q_4}$$

$$Z_{AB}=q_{AB}+ip_{AB}$$

$$\Delta=\mathrm{tr}(Z\bar{Z}Z\bar{Z})-\frac{1}{4}(\mathrm{tr}Z\bar{Z})^2+4(\mathrm{Pf}Z+\mathrm{Pf}\bar{Z}),$$

$$\mathrm{Pf}Z=\frac{1}{2^4\cdot 4!}\varepsilon^{ABCDEFGH}Z_{AB}Z_{CD}Z_{EF}Z_{GH}$$

$$\Delta=2\sum|z_i|^4-\left(\sum|z_i|^2\right)^2+8\mathrm{Re}(z_1z_2z_3z_4).$$

$$z_1=\frac{1}{4}(Q_1+Q_2+Q_3+Q_4),z_2=\frac{1}{4}(Q_1+Q_2-Q_3-Q_4),$$
$$z_3=\frac{1}{4}(Q_1-Q_2+Q_3-Q_4),z_4=\frac{1}{4}(Q_1-Q_2-Q_3+Q_4),$$

$$\Delta=Q_1Q_2Q_3Q_4-\frac{1}{4}P_1^2Q_1^2$$

$$S=2\pi\sqrt{Q_1Q_2Q_3Q_4-\frac{1}{4}P_1^2Q_1^2}$$

$$S=2\pi\sqrt{P_0Q_2Q_3Q_4-\frac{1}{4}P_0^2Q_0^2}$$

$$ds_{\mathrm{TN}}^2=\left(1+\frac{R}{2r}\right)(dr^2+r^2d\Omega_2^2)$$
$$+\left(1+\frac{R}{2r}\right)^{-1}(dy+R\sin^2(\theta/2)d\phi)^2$$

$$M_1=\frac{Q_0}{g_s\ell_s},M_2=\frac{Q_4}{(2\pi)^4g_s\ell_s^5}(2\pi)^4V,M_3=\frac{Q_1}{2\pi\ell_s^2}2\pi R$$

$$S=\frac{A}{4G_5}=\frac{2\pi g_s\ell_s^4}{\sqrt{RV}}\sqrt{M_1M_2M_3}=2\pi\sqrt{Q_0Q_4Q_1}$$

$$ds^2=R^2d\Omega_3^2-(a/R^2)^2(\cos^2\theta d\psi-\sin^2\theta d\phi)^2$$
$$=R^2d\theta^2+R^2(\cos\theta\sin\theta)^2(d\phi+d\psi)^2+(R^2-(a/R^2)^2)(\cos^2\theta d\psi-\sin^2\theta d\phi)^2$$

$$R^2=(r_1r_2r_3)^{2/3}$$

$$e_1=Rd\theta$$
$$e_2=R\cos\theta\sin\theta(d\phi+d\psi)$$
$$e_3=\sqrt{R^2-(a/R^2)^2}(\cos^2\theta d\psi-\sin^2\theta d\phi)$$

$$A=\int e_1\wedge e_2\wedge e_3=R^2\sqrt{R^2-(a/R^2)^2}\int\cos\theta\sin\theta d\theta\wedge d\phi\wedge d\psi$$
$$=2\pi^2\sqrt{(r_1r_2r_3)^2-a^2}$$

$$S=\frac{A}{4G_5}=2\pi\sqrt{Q_1Q_5n-J^2}$$

$$J=\frac{\pi a}{4G_5}$$

$$M_1=(2\pi R_2)(2\pi R_3)T_{D2}=\frac{R_2R_3}{g_s\ell_s^3}Q_1,M_2=\frac{R_4R_5}{g_s\ell_s^3}Q_2$$
$$M_3=\frac{R_1R_6}{g_s\ell_s^3}Q_3,M_4=(2\pi)^6(R_1\cdots R_6)T_{D6}P_0=\frac{R_1\cdots R_6}{g_s\ell_s^7}P_0$$

$$S=\frac{A}{4G_4}=16\pi\frac{g_s^2\ell_s^8}{8R_1\cdots R_6}\sqrt{M_1M_2M_3M_4}=2\pi\sqrt{Q_1Q_2Q_3P_0}$$

$$ds^2=-\lambda^{-1/2}\left(1-\frac{r_0}{r}\right)dt^2+\lambda^{1/2}\left(\left(1-\frac{r_0}{r}\right)^{-1}dr^2+r^2d\Omega_2^2\right)$$
$$\lambda=\prod_{i=1}^{4}\left(1+\frac{r_0\sinh^2\alpha_i}{r}\right)$$

$$M=\frac{r_0}{4G_4}\sum_{i=1}^{4}\sinh^2\alpha_i+\frac{r_0}{2G_4}=\frac{r_0}{8G_4}\sum_{i=1}^{4}\cosh 2\alpha_i$$

$$M_i=\frac{r_0\cosh 2\alpha_i}{8G_4}$$

$$A=4\pi r_0^2\prod_{i=1}^{4}\cosh\alpha_i$$

$$\hat{Q}_i=Q_i-\bar{Q}_i$$

$$S=\frac{A}{4G_4}=2\pi\prod_{i=1}^{4}\left(\sqrt{Q_i}+\sqrt{\bar{Q}_i}\right)$$

$$W = Q_1 Q_5$$

$$|nW| = \sum_{i=1}^{4} \sum_{m=1}^{\infty} m\left(N_m^i + n_m^i\right)$$

$$G(w) = N_0 \prod_{m=1}^{\infty} \left(\frac{1+w^m}{1-w^m}\right)^4$$

$$\theta_4(0 \mid \tau) = \frac{1}{\sqrt{-i\tau}} \theta_2(0 \mid -1/\tau)$$

$$\theta_4(0 \mid \tau) = \prod_{m=1}^{\infty} \left(\frac{1-w^m}{1+w^m}\right)$$

$$\theta_2(0 \mid \tau) = \sum_{n=-\infty}^{\infty} w^{(n-1/2)^2}$$

$$G(w) \to \left(-\frac{\log\, w}{\pi}\right)^2 \exp\left(-\frac{\pi^2}{\log\, w}\right)$$

$$d(Q_1, Q_5, n) = \frac{1}{2\pi i} \oint \frac{G(w)dw}{w^{N+1}}$$

$$d(Q_1, Q_5, n) \sim (Q_1 Q_5 n)^{-7/4} \exp\left(2\pi\sqrt{Q_1 Q_5 n}\right)$$

$$S = \log\, d \sim 2\pi\sqrt{Q_1 Q_5 n} - \frac{7}{4} \log\,(Q_1 Q_5 n) + \cdots$$

$$S = 2\pi\left(\sqrt{Q_1 Q_5 n} + \sqrt{Q_1 Q_5 \bar{n}}\right)$$

$$N_L = n Q_1 Q_5 \text{ and } N_R = \bar{n} Q_1 Q_5$$

$$d \sim \exp\left(2\pi\sqrt{N_L} + 2\pi\sqrt{N_R}\right)$$

$$S = 2\pi\left(\sqrt{N_L} + \sqrt{N_R}\right)$$

$$d\Gamma(\omega) = \frac{A}{e^{\omega/T} - 1} \frac{d^4 k}{(2\pi)^4}$$

$$T = \frac{2\sqrt{\bar{n}}}{\pi R}$$

$$\mathcal{K} = -\log\left(i \int_M \Omega \wedge \bar{\Omega}\right),$$

$$A^I \cap B_J = -B_J \cap A^I = \delta_J^I \text{ and } A^I \cap A^J = B_I \cap B_J = 0.$$

$$X^I = e^{\mathcal{K}/2} \int_{A^I} \Omega \text{ and } F_I = e^{\mathcal{K}/2} \int_{B_I} \Omega$$

$$\int_{A^I} \Gamma = \int_M \Gamma \wedge \beta^I = p^I \text{ and } \int_{B_I} \Gamma = \int_M \Gamma \wedge \alpha_I = q_I$$

$$Z(\Gamma) = e^{i\alpha}|Z| = e^{\mathcal{K}/2} \int_M \Gamma \wedge \Omega = e^{\mathcal{K}/2} \int_{\mathcal{C}} \Omega$$

$$Z(\Gamma) = e^{\mathcal{K}/2} \sum_I \left(\int_{A^I} \Gamma \int_{B_I} \Omega - \int_{B_I} \Gamma \int_{A^I} \Omega \right) = p^I F_I - q_I X^I$$

$$ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} d\vec{x} \cdot d\vec{x}$$

$$\delta\psi_\mu = \delta\lambda^\alpha = 0$$

$$\frac{dU(\tau)}{d\tau} = -e^{U(\tau)}|Z|$$
$$\frac{dt^\alpha(\tau)}{d\tau} = -2e^{U(\tau)} G^{\alpha\bar{\beta}} \partial_{\bar{\beta}} |Z|$$

$$2\frac{d}{d\tau}\left[e^{-U(\tau)+\mathcal{K}/2} \mathrm{Im}\left(e^{-i\alpha}\Omega\right)\right] \sim -\Gamma$$

$$2\frac{d}{d\tau}\left[e^{-U(\tau)} \mathrm{Im}\left(e^{-i\alpha} X^I\right)\right] = -p^I$$

$$2\frac{d}{d\tau}\left[e^{-U(\tau)} \mathrm{Im}\left(e^{-i\alpha} F_I\right)\right] = -q_I$$

$$2\frac{d}{d\tau}\left[e^{-U(\tau)} \mathrm{Im}\left(e^{-i\alpha}(q_I X^I - p^I F_I)\right)\right] = 0$$

$$e^{-i\alpha}(q_I X^I - p^I F_I) = -|Z|$$

$$2e^{-U(\tau)+\mathcal{K}/2} \mathrm{Im}\left(e^{-i\alpha}\Omega\right) \sim -\Gamma\tau + 2\left[e^{-U(\tau)+\mathcal{K}/2} \mathrm{Im}\left(e^{-i\alpha}\Omega\right)\right]_{\tau=0}$$

$$\frac{d|Z|}{d\tau} = \frac{dt^\alpha(\tau)}{d\tau} \partial_\alpha |Z| + \frac{d\bar{t}^{\bar{\alpha}}(\tau)}{d\tau} \partial_{\bar{\alpha}} |Z| = -4e^U G^{\alpha\bar{\beta}} \partial_\alpha |Z| \partial_{\bar{\beta}} |Z| \leq 0.$$

$$\frac{d|Z|}{d\tau} \to 0 \text{ as } \tau \to \infty.$$

$$\tau^{-1} e^{-U(\tau)} \to |Z_\star|.$$

$$ds^2 \to -\frac{r^2}{|Z_\star|^2} dt^2 + |Z_\star|^2 \frac{dr^2}{r^2} + |Z_\star|^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$A = 4\pi |Z_\star|^2.$$

$$2e^{\mathcal{K}/2} \mathrm{Im}(\bar{Z}_\star \Omega) \sim -\Gamma$$

$$\Gamma = \Gamma_{(3,0)} + \Gamma_{(0,3)}$$

$$p^I = -2\mathrm{Im}(\bar{Z} X^I) \text{ and } q_I = -2\mathrm{Im}(\bar{Z} F_I)$$

$$\tau = \sum_{p} \frac{1}{|\vec{x} - \vec{x}_p|}.$$

$$ds^2 = -e^{2U}\left(dt + \omega_i dx^i\right)^2 + e^{-2U} d\vec{x} \cdot d\vec{x}$$

$$H = 2e^{-U}\mathrm{Im}\left(e^{-i\alpha} e^{\mathcal{K}/2}\Omega\right)$$

$$\star\, d\omega = \int_M dH \wedge H$$

$$H = -\sum_{p=1}^{N} \Gamma_p \frac{1}{|\vec{x} - \vec{x}_p|} + 2\mathrm{Im}\left(e^{-i\alpha} e^{\mathcal{K}/2}\Omega\right)_{r=\infty}$$

$$\int_M \Delta H \wedge H = 0$$

$$\Delta \frac{1}{|\vec{x} - \vec{x}_p|} = -4\pi\delta^{(3)}\left(\vec{x} - \vec{x}_p\right)$$

$$\sum_{q=1}^{N} \frac{1}{|\vec{x}_p - \vec{x}_q|} \int_M \Gamma_p \wedge \Gamma_q = 2\mathrm{Im}\left[e^{-i\alpha} Z\left(\Gamma_p\right)\right]_{r=\infty}$$

$$|\vec{x}_1 - \vec{x}_2| = \frac{\int_M \Gamma_1 \wedge \Gamma_2}{2\mathrm{Im}[e^{-i\alpha} Z(\Gamma_1)]_{r=\infty}}$$

$$ds_5^2 = -f^{-2}(dt + \omega)^2 + f ds_X^2$$

$$ds_X^2 = \sum_{m,n=1}^{4} h_{mn} dx^m dx^n$$

$$F^A = d[f^{-1} Y^A (dt + \omega)] + \Theta^A$$

$$\Theta^A = \star_4 \Theta^A.$$

$$d\omega + \star_4 d\omega = -f Y_A \Theta^A$$

$$\nabla^2 (f Y_A) = 3 D_{ABC} \Theta^B \Theta^C$$

$$ds_X^2 = H^0 d\vec{x} \cdot d\vec{x} + \frac{1}{H^0}(dx^5 + \omega^0)^2 \text{ and } d\omega^0 = \star_3 dH^0$$

$$H^0 = \frac{4}{R_{\mathrm{TN}}^2} + \frac{1}{|\vec{x}|} \quad H_0 = -\frac{q_0}{L} + \frac{q_0}{|\vec{x} - \vec{x}_0|},$$

$$H^1 = \frac{p^1}{|\vec{x} - \vec{x}_0|}, \qquad H_1 = 1 + \frac{q_1}{|\vec{x} - \vec{x}_0|}$$

$$\Theta^1 = d\left[\frac{H^1}{H^0}(dx^5 + \omega^0)\right] + \star_3 dH^1$$

$$f = H_1 + 3\frac{(H^1)^2}{H^0}$$

$$\omega = -\left[H_0 + 2\frac{(H^1)^3}{(H^0)^2} + \frac{H_1 H^1}{H^0}\right](dx^5 + \omega^0) + \omega^{(4)}$$

$$d\omega^{(4)} = H_I \star_3 dH^I - H^I \star_3 dH^I.$$

$$ds_5^2 = G_{\mu\nu}^{(4)} dx^\mu dx^\nu + \lambda\left(dx^5 - A_\mu dx^\mu\right)^2,$$

$$\frac{d}{d\tau}\left[e^{-U}\left(e^{-i\alpha+\mathcal{K}/2}\Omega - e^{i\alpha+\mathcal{K}/2}\bar{\Omega}\right)\right] \sim -i\Gamma$$

$$-\frac{d}{d\tau}(e^{-U})e^{\mathcal{K}}\Omega \wedge \bar{\Omega} - e^{-U}e^{-i\alpha+\mathcal{K}/2}\Omega \wedge \frac{d}{d\tau}\left(e^{i\alpha+\mathcal{K}/2}\bar{\Omega}\right)$$
$$\sim -ie^{-i\alpha+\mathcal{K}/2}\Omega \wedge \Gamma$$

$$\int e^{i\alpha+\mathcal{K}/2}\bar{\Omega} \wedge \frac{d}{d\tau}\left(e^{-i\alpha+\mathcal{K}/2}\Omega\right) = \int e^{-i\alpha+\mathcal{K}/2}\Omega \wedge \frac{d}{d\tau}\left(e^{i\alpha+\mathcal{K}/2}\bar{\Omega}\right)$$

$$\int \left(e^{-i\alpha+\mathcal{K}/2}\Omega\right) \wedge \left(e^{i\alpha+\mathcal{K}/2}\bar{\Omega}\right) = -i$$

$$i\frac{d}{d\tau}(e^{-U})e^{\mathcal{K}}\int \Omega \wedge \bar{\Omega} = -\frac{1}{2}e^{\mathcal{K}/2}\int \left(e^{-i\alpha}\Omega + e^{i\alpha}\bar{\Omega}\right) \wedge \Gamma$$

$$\frac{d}{d\tau}(e^{-U}) = |Z|$$

$$p_{\mathrm{R}}^2 - p_{\mathrm{L}}^2 = 2N$$

$$\frac{1}{4}\alpha' M^2 = \frac{1}{2}p_{\mathrm{R}}^2 + N_{\mathrm{R}} = \frac{1}{2}p_{\mathrm{L}}^2 + N_{\mathrm{L}} - 1$$

$$\alpha' M^2 = 2p_{\mathrm{R}}^2$$

$$N_{\mathrm{L}} - 1 = N$$

$$d_N \approx \exp\left(4\pi\sqrt{N}\right)$$

$$S = \log\, d_N \approx 4\pi\sqrt{N}$$

$$Z(\beta) = \sum d_N e^{-\beta N} = \frac{16}{\Delta(q)}$$

$$q = e^{-\beta} = e^{2\pi i\tau}$$

$$\Delta(q) = \eta(\tau)^{24} = q\prod_{n=1}^{\infty}(1-q^n)^{24}$$

$$\eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau)$$

$$\Delta\left(e^{-\beta}\right)=\left(\frac{\beta}{2\pi}\right)^{-12}\Delta\left(e^{-4\pi^2/\beta}\right)$$

$$\Delta\left(e^{-\beta}\right)\approx\left(\frac{\beta}{2\pi}\right)^{-12}e^{-4\pi^2/\beta}$$

$$d_N=\frac{1}{2\pi i}\oint\ Z(\beta)\frac{dq}{q^{N+1}}=\frac{1}{2\pi i}\oint\ \frac{16}{\Delta(q)}\frac{dq}{q^{N+1}}$$

$$d_N\approx 16\hat{I}_{13}(4\pi\sqrt{N})$$

$$\hat{I}_\nu(z)=\frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty}(t/2\pi)^{-\nu-1}e^{t+z^2/4t}dt$$

$$S=\log\ d_N\approx 4\pi\sqrt{N}-\frac{27}{2}\log\ \sqrt{N}+\frac{15}{2}\log\ 2+\cdots$$

$$F(X^I,W^2)=\sum_{h=0}^{\infty}F_h(X^I)W^{2h}$$

$$X^I\partial_I F(X^I,W^2)+W\partial_W F(X^I,W^2)=2F(X^I,W^2)$$

$$\int\ d^4xd^4\theta W^{2h}F_h(X^I)$$

$$p^I=\mathrm{Re}(CX^I)$$
$$q_I=\mathrm{Re}(CF_I)$$

$$C^2W^2=256$$

$$S=\frac{\pi i}{2}\left(q_I\overline{CX}^I-p^I\overline{CF}_I\right)+\frac{\pi}{2}\mathrm{Im}(C^3\partial_C F)$$

$$CX^I=p^I+\frac{i}{\pi}\phi^I$$

$$\mathcal{F}(\phi,p)=-\pi\mathrm{Im}F\left(p^I+\frac{i}{\pi}\phi^I,256\right)$$

$$q_I=\frac{1}{2}\left(CF_I+\overline{CF}_I\right)=-\frac{\partial}{\partial\phi^I}\mathcal{F}(\phi,p)$$

$$\frac{\partial}{\partial\phi^I}=\frac{i}{\pi C}\frac{\partial}{\partial X^I}-\frac{i}{\pi\bar{C}}\frac{\partial}{\partial\bar{X}^I}$$

$$C\partial_C F\left(X^I,\frac{256}{C^2}\right)=X^I\frac{\partial}{\partial X^I}F-2F$$

$$S(p,q)=\mathcal{F}(\phi,p)-\phi^I\frac{\partial}{\partial\phi^I}\mathcal{F}(\phi,p)$$

$$\mathcal{Z}(\phi^I, p^I) = e^{\mathcal{F}(\phi^I, p^I)} = \sum_{q^I} \Omega(q_I, p^I) e^{-\phi^I q_I}$$

$$S(q,p) = \log\, \Omega(q,p)$$

$$\Omega(q_I, p^I) = \int\ e^{\mathcal{F}(\phi^I, p^I) + \phi^I q_I} \prod\ d\phi^I$$

$$F_0 = -\frac{1}{2} C_{ab} X^a X^b \left(\frac{X^1}{X^0}\right), a, b = 2, \dots, 23$$

$$\tau = \tau_1 + i\tau_2 = X^1/X^0$$

$$\frac{1}{8\pi}\int\ \tau_1(\mathrm{tr}R \wedge R - \mathrm{tr}F \wedge F)$$

$$\tau = \frac{1}{2\pi i}\log\, q \rightarrow \frac{24}{2\pi i}\log\, \eta(\tau) = \frac{1}{2\pi i}\log\, \Delta(q)$$

$$\frac{1}{16\pi^2}\mathrm{Im}\int\ \log\, \Delta(q)\mathrm{tr}[(R - iR^\star) \wedge (R - iR^\star)]$$

$$F_1 = \frac{i}{128\pi}\log\, \Delta(q)$$

$$F(X, W^2) = -\frac{1}{2} C_{ab} X^a X^b \left(\frac{X^1}{X^0}\right) - \frac{W^2}{128\pi i}\log\, \Delta(q).$$

$$\phi_0 = -2\pi\sqrt{\frac{p^1}{q_0}}$$

$$S \sim \log\left(16\hat{I}_{13}\left(4\pi\sqrt{p^1 q_0}\right)\right).$$

$$\mathcal{K} = -\log\,[2\mathrm{Im}(\bar{X}^I F_I)].$$

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1 - q^n) = \sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{3}{2}(n-1/6)^2}$$

$$S = 2\pi\int_{S^2}\varepsilon_{\mu\nu}\varepsilon_{\rho\lambda}\frac{\partial\mathcal{L}}{\partial R_{\mu\nu\rho\lambda}}d^2\Omega$$

$$SO(d,1) \rightarrow SO(d-p) \times SO(p,1)$$

$$2\kappa_{11}^2 S = \int\ d^{11}x\sqrt{-G}\left(R - \frac{1}{2}|F_4|^2\right) - \frac{1}{6}\int\ A_3 \wedge F_4 \wedge F_4$$

$$\delta\Psi_M = \nabla_M \varepsilon + \frac{1}{12}\left(\Gamma_M \mathbf{F}^{(4)} - 3\mathbf{F}_M^{(4)}\right)\varepsilon = 0$$

$$ds^2 = H^{-2/3}dx \cdot dx + H^{1/3}dy \cdot dy$$

$$F_4 = dx^0 \wedge dx^1 \wedge dx^2 \wedge dH^{-1}$$

$$H = 1 + \frac{r_2^6}{r^6},$$

$$r_2^6 = 32\pi^2 N_2 \ell_p^6$$

$$ds^2 = H^{-1/3} dx \cdot dx + H^{2/3} dy \cdot dy$$

$$H = 1 + \frac{r_5^3}{r^3}$$

$$r_5^3 = \pi N_5 \ell_p^3.$$

$$F_4 = \star\,(dx^0 \wedge dx^1 \wedge \ldots \wedge dx^5 \wedge dH^{-1})$$

$$dy \cdot dy = dr^2 + r^2 d\Omega_7^2$$

$$r^2 H^{1/3} \to r_2^2$$

$$ds^2 \sim (r/r_2)^4 dx \cdot dx + (r_2/r)^2 dr^2 + r_2^2 d\Omega_7^2$$

$$ds^2 = R^2 \frac{dx \cdot dx + dz^2}{z^2}$$

$$z = \frac{r_2^3}{2r^2}$$

$$ds^2 \sim (r/r_5) dx \cdot dx + (r_5/r)^2 dr^2 + r_5^2 d\Omega_4^2$$

$$S^{(p)} = \frac{1}{2\kappa^2} \int \sqrt{-g} \left[e^{-2\Phi} (R + 4(\partial\Phi)^2) - \frac{1}{2} \left| F_{p+2} \right|^2 \right] d^{10}x$$

$$ds^2 = H_p^{-1/2} dx \cdot dx + H_p^{1/2} dy \cdot dy$$

$$H_p(r) = 1 + \left(\frac{r_p}{r}\right)^{7-p}$$

$$e^{\Phi} = g_s H_p^{(3-p)/4}$$

$$F_{p+2} = dH_p^{-1} \wedge dx^0 \wedge dx^1 \wedge \ldots \wedge dx^p$$

$$C_{01\ldots p} = H_p(r)^{-1} - 1$$

$$F_{p+2} = Q \star \omega_{8-p}$$

$$F_5 = Q(\omega_5 + \star\,\omega_5)$$

$$T_{Dp} = (2\pi)^{-p} \ell_s^{-(p+1)} g_s^{-1},$$

$$(r_p/\ell_S)^{7-p} = (2\sqrt{\pi})^{5-p} \Gamma\left(\frac{7-p}{2}\right) g_s N$$

$$R^4 = 4\pi g_s N\alpha'^2$$

$$ds^2 \sim (r/R)^2 dx \cdot dx + (R/r)^2 dr^2 + R^2 d\Omega_5^2$$

$$ds^2 \sim R^2 \frac{dx \cdot dx + dz^2}{z^2} + R^2 d\Omega_5^2$$

$$ds^2 = -\Delta_+(r)\Delta_-(r)^{-1/2} dt^2 + \Delta_-(r)^{1/2} dx^i dx^i$$
$$+\Delta_+(r)^{-1}\Delta_-(r)^{\gamma} dr^2 + r^2 \Delta_-(r)^{\gamma+1} d\Omega_{8-p}^2$$

$$\gamma = -\frac{1}{2} - \frac{5-p}{7-p}$$

$$\Delta_\pm(r) = 1 - \left(\frac{r_\pm}{r}\right)^{7-p}$$

$$e^{\Phi} = g_s \Delta_-(r)^{(p-3)/4}$$

$$\star F_{p+2} = Q\omega_{8-p}$$

$$F_5 = Q(\omega_5 + \star\, \omega_5)$$

$$\tilde{r}^{7-p} = r^{7-p} - r_+^{7-p}$$

$$T = \frac{\Omega_{8-p}}{2\kappa_{10}^2}\left[(8-p) r_+^{7-p} - r_-^{7-p}\right]$$

$$Q = \frac{(7-p)}{2}(r_+ r_-)^{(7-p)/2}$$

$$T \geq N T_{\mathrm{D}p}$$

$$\frac{S_1}{S_0} = k\frac{r_0}{R}$$

$$V(\phi) = (\phi^2 - a^2)^2,$$

$$\phi = \phi_\pm = \pm a.$$

$$\phi(\vec{x}, 0) = 0 \text{ and } \dot{\phi}(\vec{x}, 0) = v.$$

$$dF = 0 \text{ and } d^\dagger F = dz\delta(t)\delta(x)\delta(y)$$

$$F = \mathrm{Re}\left(dz \wedge d\frac{1}{\sqrt{t^2 - x^2 - y^2 - i\epsilon}}\right)$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(\sin^2\theta d\phi^2 + d\theta^2)$$

$$ds^2 = -\left(1 - \frac{2P}{t}\right)^{-1} dt^2 + \left(1 - \frac{2P}{t}\right) dr^2 + t^2 d\Sigma^2$$

$$d\Sigma^2 = \sinh^2\ \theta d\phi^2 + d\theta^2$$

$$t = 2P\cosh^2\ (\eta/2)$$

$$ds^2 = C^2(\eta)(-d\eta^2 + d\Sigma^2) + D^2(\eta)dr^2$$

$$C(\eta) = t(\eta)\ \text{ and }\ D(\eta) = \tanh\left(\frac{\eta}{2}\right)$$

$$g_{00} \sim -1 + \frac{2}{3}(r_2/r)^6 = -1 + \frac{16\pi G_{11} N_2 T_{M2}}{9\Omega_7 r^6}$$

$$r_2^6 = 32\pi^2 N_2 \ell_p^6.$$

$$ds^2 = -\Delta_+(r)dt^2 + dx^i dx^i + \Delta_+(r)^{-1}dr^2 + r^2 d\Omega_{8-p}^2$$

$$p_{11} = \frac{N}{R}$$

$$\mathcal{L} = \frac{1}{2R}\mathrm{Tr}\left(-\left(D_\tau X^i\right)^2 + \frac{1}{2}\left[X^i, X^j\right]^2\right),$$

$$\mathcal{L} = \mathrm{Tr}\left(\frac{1}{2g}F_{\mu\nu}^2 - i\bar{\psi}D\psi + \frac{1}{g}\left(\bar{D}^\mu A_\mu\right)^2\right) + \mathcal{L}_\mathcal{G}$$

$$\bar{D}^\mu A_\mu = \partial^\mu A_\mu + \left[B^\mu, A_\mu\right]$$

$$\begin{aligned} F_{0i} &= \partial_\tau X_i + [A, X_i] \\ F_{ij} &= \left[X_i, X_j\right] \\ D_\tau \psi &= \partial_\tau \psi + [A, \psi] \\ D_i \psi &= [X_i, \psi] \end{aligned}$$

$$X^i = B^i + \sqrt{g}Y^i,$$

$$B^1 = i\frac{v\tau}{2}\sigma^3\ \text{ and }\ B^2 = i\frac{b}{2}\sigma^3$$

$$A = \frac{i}{2}(A_0\mathbb{1} + A_a\sigma^a)$$

$$\mathcal{L} = \mathcal{L}_Y + \mathcal{L}_A + \mathcal{L}_\mathcal{G} + \mathcal{L}_{\text{fermi}}$$

$$r^2 = b^2 + (v\tau)^2$$

$$(-\partial_\tau^2 + \mu^2 + (v\tau)^2)\Delta_\mathcal{B}(\tau, \tau' \mid \mu^2 + (v\tau)^2) = \delta(\tau - \tau')$$

$$\Delta_\mathcal{B}(\tau, \tau' \mid \mu^2 + (v\tau)^2) = \int_0^\infty ds e^{-\mu^2 s}\sqrt{\frac{v}{2\pi\sinh\ 2sv}}\exp\left[-\frac{v}{2}\left(\frac{(\tau^2 + \tau'^2)\cosh\ 2sv - 2\tau\tau'}{\sinh\ 2sv}\right)\right]$$

$$(-\partial_\tau + m_\mathcal{F})\Delta_\mathcal{F}(\tau, \tau' \mid m_\mathcal{F}) = \delta(\tau - \tau')$$

$$\Delta_\mathcal{F}(\tau, \tau' \mid m_\mathcal{F}) = (\partial_\tau + m_\mathcal{F})\Delta_\mathcal{B}(\tau, \tau' \mid r^2 - v\gamma_1).$$

$$\delta(b,v) = -\int d\tau V(b^2 + v^2\tau^2)$$

$$\frac{\det^4(-\partial_\tau^2 + r^2 + v)\det^4(-\partial_\tau^2 + r^2 - v)}{\det^{-1}(-\partial_\tau^2 + r^2 + 2v)\det^{-1}(-\partial_\tau^2 + r^2 - 2v)\det^{-6}(-\partial_\tau^2 + r^2)}$$

$$\delta = \int_0^\infty \frac{ds}{s}\frac{e^{-sb^2}}{\sinh\ sv}(3 - 4\cosh\ sv + \cosh\ 2sv)$$

$$V(r) = \frac{15}{16}\frac{v^4}{r^7}$$

$$\int d\tau\lambda_4\Delta_1(\tau,\tau \mid m_1)\Delta_2(\tau,\tau \mid m_2)$$

$$\int d\tau d\tau' \lambda_3^{(1)}\lambda_3^{(2)}\Delta_1(\tau,\tau' \mid m_1)\Delta_2(\tau,\tau' \mid m_2)\Delta_3(\tau,\tau' \mid m_3)$$

$$\int \frac{dp}{p^2}$$

$$\mathcal{L}_2 = \alpha_0\frac{1}{r^2} + \alpha_2\frac{v^2}{r^6} + \alpha_4\frac{v^4}{r^{10}} + \cdots$$

$$g\mathcal{L} = \sum_{m=0}^{\infty} g^m\mathcal{L}_m = c_{00}v^2 + \sum_{m,n=1}^{\infty} c_{mn}g^m\frac{v^{2n+2}}{r^{3m+4n}}$$

$$\begin{aligned}
\mathcal{L}_0 &= c_{00}v^2 \\
\mathcal{L}_1 &= c_{11}\frac{v^4}{r^7} + c_{12}\frac{v^6}{r^{11}} + c_{13}\frac{v^8}{r^{15}} + \cdots \\
\mathcal{L}_2 &= c_{21}\frac{v^4}{r^{10}} + c_{22}\frac{v^6}{r^{14}} + c_{23}\frac{v^8}{r^{18}} + \cdots \\
\mathcal{L}_3 &= c_{31}\frac{v^4}{r^{13}} + c_{32}\frac{v^6}{r^{17}} + c_{33}\frac{v^8}{r^{21}} + \cdots
\end{aligned}$$

$$G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{--} = \frac{2\kappa_{11}^2 p_-}{7\Omega_8 r^7}\delta(x^-) = \frac{15\pi N_1}{RM_{\mathrm{p}}^9 r^7}\delta(x^-)$$

$$h_{--} = \frac{15N_1}{2R^2M_{\mathrm{p}}^9 r^7}$$

$$\begin{aligned}
S &= -m\int d\tau\left(-G_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\right)^{1/2} \\
&= -m\int d\tau(-2\dot{x}^- - v^2 - h_{--}\dot{x}^-\dot{x}^-)^{1/2}
\end{aligned}$$

$$p_- = m\frac{1 + h_{--}\dot{x}^-}{(-2\dot{x}^- - v^2 - h_{--}\dot{x}^-\dot{x}^-)^{1/2}}.$$

$$\mathcal{L}'(p_-) = -\mathcal{R}(p_-) = \mathcal{L} - p_-\dot{x}^-(p_-)$$

$$\dot{x}^- = \frac{\sqrt{1 - h_{--}v^2} - 1}{h_{--}}$$

$$-p_-\dot{x}^-(p_-) = p_-\left\{\frac{v^2}{2} + \frac{h_{--}v^4}{8} + \frac{h_{--}^2v^6}{16} + \cdots\right\}$$
$$= \frac{N_2}{2R}v^2 + \frac{15}{16}\frac{N_1N_2}{R^3M_\mathrm{p}^9}\frac{v^4}{r^7} + \frac{225}{64}\frac{N_1^2N_2}{R^5M_\mathrm{p}^{18}}\frac{v^6}{r^{14}} + \cdots$$

$$\frac{N_1N_2^2 + N_1^2N_2}{2},$$

$$E_2(\mathbb{Z}) = SL(2,\mathbb{Z}), E_3(\mathbb{Z}) = SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z}), E_4(\mathbb{Z}) = SL(5,\mathbb{Z})$$

$$(\partial_\tau - v\tau\gamma_1 - b\gamma_2)(\partial_\tau + v\tau\gamma_1 + b\gamma_2) = \partial_\tau^2 - r^2 + v\gamma_1$$

$$g_\mathrm{eff}^2(E) \sim g_\mathrm{YM}^2 N E^{p-3}$$

$$\lambda = g_\mathrm{YM}^2 N$$

$$g_\mathrm{YM}^2 = 4\pi g_\mathrm{s}$$

$$R = \lambda^{1/4}\ell_\mathrm{s}$$

$$\int_{S^5} F_5 = N$$

$$\chi = V - E + F$$

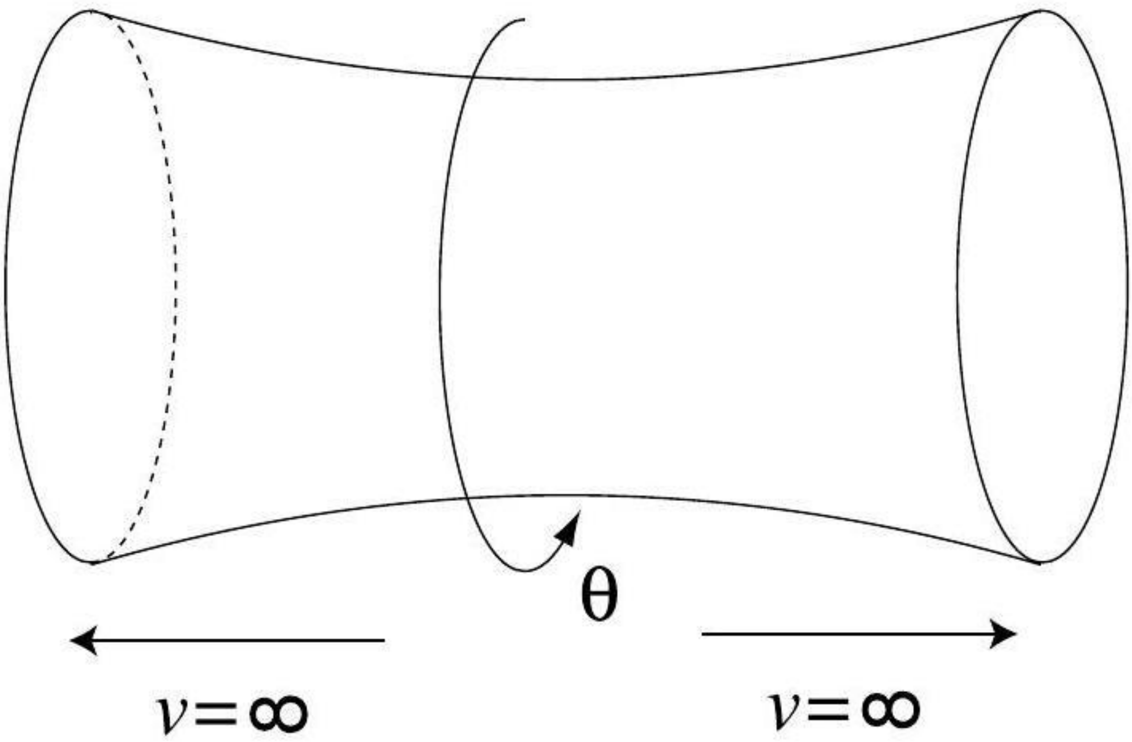
$$\ell_\mathrm{p} = g_\mathrm{s}^{1/4}\ell_\mathrm{s}$$

$$N = \frac{1}{4\pi}\left(R/\ell_\mathrm{p}\right)^4.$$

$$SO(3,2) \times SO(8) \approx Sp(4) \times Spin(8).$$

$$SO(6,2) \times SO(5) \approx \mathrm{Spin}(6,2) \times U\mathrm{Sp}(4)$$

$$ds^2 = \frac{R^2}{z^2}\left((dx^2)_{d+1} + dz^2\right), z \geq 0$$



$$y_1^2 + \cdots + y_d^2 - t_1^2 - t_2^2 = -R^2 = -1$$

$$\left(z, x^0, x^i\right) = ((t_1 + y_d)^{-1}, t_2 z, y_i (t_1 + y_d)^{-1}).$$

$$(y_1, \ldots, y_d) \to \left(v, \Omega_p\right) \text{ and } (t_1, t_2) \to (\tau, \theta)$$

$$ds^2 = \sum dy_i^2 - \sum dt_j^2 = \frac{dv^2}{1 + v^2} + v^2 d\Omega_p^2 - (1 + v^2) d\theta^2$$

$$ds^2 = \frac{1}{\cos^2 \rho}\left(d\rho^2 + \sin^2 \rho d\Omega_p^2 - dt^2\right)$$

$$H = \frac{1}{2}(P_0 + K_0),$$

$$y_1^2 + \cdots + y_d^2 - t_1^2 + t_2^2 = -1$$

$$ds^2 = \frac{1}{z^2}(dz^2 + (dx^2)_d)$$

$$ds^2 = d\rho^2 + \sinh^2 \rho d\Omega_d^2$$

$$ds^2 = \frac{4 \sum du_i^2}{\left(1 - \sum u_i^2\right)^2}$$

$$E \sim 1/z \sim r$$

$$E = kr\ell_s^{-2},$$

$$\tau_{IIB} = C_0 + ie^{-\Phi}$$

$$\tau_{\mathrm{YM}} = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\mathrm{YM}}^2}$$

$$\tau_{\mathrm{YM}} = \tau_{IIB},$$

$$Z_{\text{spacetime dimensions}}(\phi_0) = \int_{\phi_0} D\phi e^{-S_{\text{spacetime dimensions}}}$$

$$\left\langle \exp \int_{S^d} \phi_0 \mathcal{O} \right\rangle_{CFT}$$

$$S \sim \int \ d^4ydz\left[z^2\left(\partial_y\phi\right)^2 + z^2(\partial_z\phi)^2 + m^2R^2\phi^2\right]/z^5$$

$$\alpha(\alpha-4) = m^2R^2,$$

$$\alpha_{\pm} = 2 \pm \sqrt{4+m^2R^2}.$$

$$\phi_0(x) = \lim_{z\to 0} z^{-\alpha_-}\phi(x,z).$$

$$\Delta = 4-\alpha_- = 2+\sqrt{4+m^2R^2}.$$

$$m^2R^2 > -d^2/4$$

$$1-d^2/4 > m^2R^2 > -d^2/4$$

$$\mathcal{O}^{I_1I_2\cdots I_n} = \mathrm{Tr}(\phi^{I_1}\phi^{I_2}\cdots\phi^{I_n})$$

$$\left(\nabla^\mu J_\mu\right)^a = \frac{N^2-1}{384\pi^2} i d^{abc}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}^bF_{\rho\lambda}^c$$

$$S_{\mathrm{CS}} = \frac{iN^2}{96\pi^2}\int_{\mathrm{AdS}_5} d^5x\left(d^{abc}\epsilon^{\mu\nu\rho\lambda\sigma}A_\mu^a\partial_\nu A_\rho^b\partial_\lambda A_\sigma^c + \cdots\right)$$

$$-\frac{iN^2}{384\pi^2}\int \ d^4x d^{abc}\epsilon^{\mu\nu\rho\lambda}\Lambda^a F_{\mu\nu}^b F_{\rho\lambda}^c$$

$$ds^2 = \frac{R^2}{z^2}(-f(z)dt^2 + d\vec{x}\cdot d\vec{x} + f^{-1}(z)dz^2) + R^2d\Omega_5^2$$

$$f(z) = 1-(z/z_0)^4.$$

$$ds^2 = \frac{R^2}{z_0^2}\left(\frac{4\varepsilon}{z_0}d\tau^2 + dx^idx^i + \frac{z_0}{4\epsilon}d\varepsilon^2\right) + R^2d\Omega_5^2$$

$$\frac{S}{V} = \frac{1}{4G_{10}}\left(\frac{R}{z_0}\right)^3\cdot R^5\Omega_5 = \frac{\pi^2}{2}N^2T^3$$

$$\frac{S}{V} = \frac{2\pi^2}{3}N^2T^3$$

$$\frac{S}{V} = c(\lambda)\frac{\pi^2}{2}N^2T^3$$

$$c(\lambda) = \frac{4}{3} - \frac{2\lambda}{\pi^2} + \cdots \quad \text{for small } \lambda$$
$$c(\lambda) = 1 + \frac{15\zeta(3)}{8\lambda^{3/2}} + \cdots \quad \text{for large } \lambda$$

$$W(\mathcal{C}) = \mathrm{Tr}\left(P\exp \oint_{\mathcal{C}} A\right)$$

$$\langle W \rangle \sim \exp(-T \cdot \text{ Area })$$

$$ds^2 = \frac{R^2}{z^2}\left(-dt^2 + f(z)dy^2 + dx_1^2 + dx_2^2 + f^{-1}(z)dz^2\right)$$

$$f(z) = 1 - (z/z_0)^4$$

$$Z_{\text{spacetime dimensions}} \sim e^{-S_1} + e^{-S_2},$$

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_V + \mathcal{L}_W,$$

$$\mathcal{L}_W = \int d^2\theta W + \int d^2\bar{\theta}\bar{W}$$

$$[R, Q_\alpha] = Q_\alpha \text{ and } [R, \bar{Q}_{\dot{\alpha}}] = -\bar{Q}_{\dot{\alpha}}$$

$$\mathcal{L}_K = \int d^4\theta \sum_{i=1}^{3} \mathrm{tr}\left(\Phi_i^\dagger e^V \Phi_i\right)$$

$$\mathcal{L}_V = \frac{1}{4g^2}\int d^2\theta \mathrm{tr}(W^\alpha W_\alpha) + \text{ h.c.}$$

$$\mathcal{L}_W = \int d^2\theta W + \text{ h.c..}$$

$$W \sim \epsilon^{ijk}\mathrm{tr}\left(\Phi_i\Phi_j\Phi_k\right) \sim \mathrm{tr}(\Phi_1[\Phi_2, \Phi_3])$$

$$W = \sqrt{2}\mathrm{tr}(\Phi_1[\Phi_2, \Phi_3]).$$

$$(g^2)^{E-V}N^F = (\lambda/N)^{E-V}N^F = N^\chi\lambda^{E-V}.$$

$$[D, M_{\mu\nu}] = 0, [D, P_\mu] = -iP_\mu, [D, K_\mu] = iK_\mu$$

$$[P_\mu, K_\nu] = 2iM_{\mu\nu} - 2i\eta_{\mu\nu}D$$

$$\{Q_\alpha^A, S_{\beta B}\} = c_1\sigma_{\alpha\beta}^{\mu\nu}\delta_B^A M_{\mu\nu} + c_2\varepsilon_{\alpha\beta}R_B^A + c_3\varepsilon_{\alpha\beta}\delta_B^A D$$

$$X = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\mathrm{Str}X = \mathrm{tr}A - \mathrm{tr}D = 0$$

$$X=\begin{pmatrix} \frac{1}{2}M_{\mu\nu}(\sigma^{\mu\nu})^{\alpha}{}_{\beta}+D\delta^{\alpha}{}_{\beta} & K_{\mu}(\sigma^{\mu})^{\alpha}{}_{\dot\beta} & S^{\alpha}{}_{B} \\ P_{\mu}(\sigma^{\mu})^{\dot\alpha}{}_{\beta} & \frac{1}{2}M_{\mu\nu}(\sigma^{\mu\nu})^{\dot\alpha}{}_{\dot\beta}-D\delta^{\dot\alpha}{}_{\dot\beta} & Q^{\dot\alpha}{}_{B} \\ Q^{A}{}_{\beta} & S^{A}{}_{\dot\beta} & R^{A}{}_{B} \end{pmatrix}.$$

$$\mathcal{O}^{I_1 I_2 \cdots I_n}=\operatorname{Tr}(\phi^{I_1}\phi^{I_2}\cdots\phi^{I_n})+\cdots$$

$$\mathcal{O}^{IJ}=\operatorname{Tr}(\phi^{I}\phi^{J})-\frac{1}{6}\delta^{IJ}(\phi^{K}\phi^{K})$$

$$ds^2=H_3^{-1/2}dx\cdot dx+H_3^{1/2}\left(dr^2+r^2d\Omega_5^2\right)$$

$$R^4=\lambda\ell_s^4$$

$$ds^2=H_3^{-1/2}dx\cdot dx+H_3^{1/2}\left(dr^2+r^2ds_5^2\right)$$

$$R^4=4\pi\lambda\ell_s^4\frac{\operatorname{Vol}(S^5)}{\operatorname{Vol}(X_5)}$$

$$\Delta\geq 3R/2$$

$$W\sim\operatorname{tr}(A_1B_1A_2B_2-A_1B_2A_2B_1).$$

$$\mathcal{O}_k=\operatorname{tr}\left(A_{i_1}B_{j_1}\ldots A_{i_k}B_{j_k}\right)$$

$$SU(M+N)\times SU(N).$$

$$e^{-4A(r)}=\frac{L^4}{r^4}\log(r/r_s),L^2=\frac{9g_sM\alpha'}{2\sqrt{2}}$$

$$\partial_\mu J^\mu=\frac{M}{16\pi^2}\left(F_{\mu\nu}^{a}\tilde{F}^{a\mu\nu}-G_{\mu\nu}^{b}\tilde{G}^{b\mu\nu}\right)$$

$$\frac{1}{\alpha_1(\mu)}+\frac{1}{\alpha_2(\mu)}=\frac{1}{g_s}$$

$$\frac{1}{\alpha_1(\mu)}-\frac{1}{\alpha_2(\mu)}=\frac{3M}{\pi}\log(\mu/\Lambda)+\text{ const.}$$

$$\partial_\mu J^\mu=\frac{K}{32\pi^2}F_{\mu\nu}^{a}\tilde{F}^{a\mu\nu}$$

$$K=4N\cdot(-1/2)+2(M+N)\cdot 1=2M$$

$$K=4(M+N)\cdot(-1/2)+2N\cdot 1=-2M$$

$$\begin{aligned} ds^2(AdS_5)&=R^2(-\cosh^2\rho dt^2+d\rho^2+\sinh^2\rho d\Omega_3^2)\\ ds^2(S^5)&=R^2\left(\cos^2\theta d\phi^2+d\theta^2+\sin^2\theta d\tilde{\Omega}_3^2\right)\end{aligned}$$

$$\begin{gathered} r=R\sinh\rho, y=R\sin\theta,\\ x^+=t/\mu, x^-=\mu R^2(\phi-t).\end{gathered}$$

$$ds^2 = 2dx^+dx^- + g_{++}(x^I)(dx^+)^2 + \sum_{I=1}^{8} dx^I dx^I$$

$$g_{++}(x^I) = -\mu^2(r^2 + y^2)$$

$$r^2 = \sum_{1}^{4} (x^I)^2 \text{ and } y^2 = \sum_{5}^{8} (x^I)^2.$$

$$F_5 \sim \mu dx^+ \wedge (dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 + dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8)$$

$$S = \frac{1}{2\pi\alpha'} \int d^2\sigma \left(\frac{1}{2}(\dot{x}^2 - x'^2 - \mu^2 x^2) + i\bar{S}(\rho \cdot \partial + \mu\Gamma_*)S\right)$$

$$\left[a_m^I, a_n^{J\dagger}\right] = \delta^{IJ}\delta_{mn}\ m, n \in \mathbb{Z}, I, J = 1,2, \ldots ,8.$$

$$\omega_n = \sqrt{1 + (n/\mu\alpha)^2}$$

$$H_{\ell c} = \mu \sum_{n=-\infty}^{\infty} \sum_{I=1}^{8} \omega_n a_n^{I\dagger} a_n^I + \text{ fermions.}$$

$$\sum_{n=-\infty}^{\infty} \sum_{I=1}^{8} n a_n^{I\dagger} a_n^I + \text{ fermions } = 0$$

$$\lambda' = g_{\mathrm{YM}}^2 N/J^2,$$

$$\Delta_{\mathrm{a}} = \Delta - J \leftrightarrow P_+ = H_{\ell c}$$

$$\lambda' \leftrightarrow 1/(\mu\alpha' P_-)^2$$
$$g_2 = J^2/N \leftrightarrow 4\pi g_s(\mu\alpha' P_-)^2$$
$$1/J \leftrightarrow 1/(\mu R^2 P_-)$$

$$\Delta_{\mathrm{a}} = 2\sqrt{1 + \lambda' n^2}$$

$$ds^2 = 2dx^+dx^- + g_{++}(x^I)(dx^+)^2 + \sum_{I=1}^{9} dx^I dx^I$$

$$g_{++}(x^I) = -\mu^2((r_3/3)^2 + (r_6/6)^2).$$

$$F_4 \sim \mu dx^+ \wedge dx^1 \wedge dx^2 \wedge dx^3$$

$$(-\partial_\tau^2 + \partial_\sigma^2)X^I - X^I = 0$$

$$X^I(\sigma, \tau) = i \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{2\omega_n P_-}} \left(e^{-i\omega_n \tau} a_n^I - e^{i\omega_n \tau} a_n^{\dagger I}\right) e^{-ik_n \sigma},$$

$$\omega_n = \sqrt{k_n^2 + 1} \text{ and } k_n = \frac{n}{\alpha' P_-}.$$

$$[X^I(\sigma), P^J(\sigma')] = i\delta^{IJ}\delta(\sigma - \sigma'),$$

$$P^I(\sigma, \tau) = \dot{X}^I(\sigma, \tau)/(2\pi\alpha'),$$

$$i(\dot{S} + S'^{\dagger}) + \Gamma_* S = 0$$

$$S^a(\sigma, \tau) = \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{4\omega_n P_-}} \big([\Gamma_* + \omega_n - k_n] S_n^a e^{-i\omega_n \tau} \\ + [1 - (\omega_n - k_n)\Gamma_*] S_n^{\dagger a} e^{i\omega_n \tau}\big) e^{-ik_n \sigma}$$

$$\{S^a(\sigma, \tau), S^{b\dagger}(\sigma', \tau)\} = 2\pi\alpha'\delta(\sigma - \sigma')$$

$$\{S_m^a, S_n^{b\dagger}\} = \delta_{m,n}\delta^{ab} \; m, n \in \mathbb{Z}$$

$$X = \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

$$(z_1, z_2) \to (\omega z_1, \omega^{-1} z_2)$$

$$\mathcal{L} = \mathcal{L}_Y + \mathcal{L}_A + \mathcal{L}_{\mathcal{G}} + \mathcal{L}_{\text{fermi}},$$

$$\mathcal{S}_Y = i \int d\tau \Big(\frac{1}{2} Y_1^i(\partial_\tau^2 - r^2) Y_1^i + \frac{1}{2} Y_2^i(\partial_\tau^2 - r^2) Y_2^i + \frac{1}{2} Y_3^i \partial_\tau^2 Y_3^i \\ - \sqrt{g}\epsilon^{a3d}\epsilon^{cbd} B_3^i Y_a^j Y_b^i Y_c^j - \frac{g}{4}\epsilon^{abe}\epsilon^{cde} Y_a^i Y_b^j Y_c^i Y_d^j\Big)$$

$$\mathcal{S}_A = i \int d\tau \Big(\frac{1}{2} A_1(\partial_\tau^2 - r^2) A_1 + \frac{1}{2} A_2(\partial_\tau^2 - r^2) A_2 + \frac{1}{2} A_3 \partial_\tau^2 A_3 \\ + 2\epsilon^{ab3}\partial_\tau B_3^i A_a Y_b^i + \sqrt{g}\epsilon^{abc}\partial_\tau Y_a^i A_b Y_c^i \\ - \sqrt{g}\epsilon^{a3d}\epsilon^{bcd} B_3^i A_a A_b Y_c^i - \frac{g}{2}\epsilon^{abe}\epsilon^{cde} A_a Y_b^i A_c Y_d^i\Big)$$

$$\mathcal{S}_{\mathcal{G}} = i \int d\tau (C_1^*(-\partial_\tau^2 + r^2) C_1 + C_2^*(-\partial_\tau^2 + r^2) C_2 - C_3^* \partial_\tau^2 C_3 \\ + \sqrt{g}\epsilon^{abc}\partial_\tau C_a^* C_b A_c - \sqrt{g}\epsilon^{a3d}\epsilon^{cbd} B_3^i C_a^* C_b Y_c^i)$$

$$\mathcal{S}_{\text{fermi}} = i \int d\tau \Big(\psi_+^T(\partial_\tau - v\tau\gamma_1 - b\gamma_2)\psi_- + \sqrt{\frac{g}{2}}(Y_1^i - iY_2^i)\psi_+^T\gamma^i\psi_3 \\ + \frac{1}{2}\psi_3^T\partial_\tau\psi_3 + \sqrt{\frac{g}{2}}(Y_1^i + iY_2^i)\psi_3^T\gamma^i\psi_- - i\sqrt{\frac{g}{2}}(A_1 - iA_2)\psi_+^T\psi_3 \\ + i\sqrt{\frac{g}{2}}(A_1 + iA_2)\psi_-^T\psi_3 - \sqrt{g} Y_3^i \psi_+^T\gamma^i\psi_- + i\sqrt{g} A_3 \psi_+^T\psi_-\Big)$$

$$\psi_+ = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2) \; \psi_- = \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2).$$

$$\Gamma^0 = \sigma^3 \otimes \mathbb{1}_{16\times16}, \Gamma^i = i\sigma^1 \otimes \gamma^i$$

Apéndice A.

Multidimensiones – Supermembranas – Superespacios y Supersimetrías a propósito de la existencia de supergravedad cuántica. Modelo Sugawara Landau-Ginzburg.

$$d^2z \equiv 2dxdy, \delta^2(z,\bar{z}) \equiv \frac{1}{2}\delta(x)\delta(y)$$

$$p_\mu p^\mu + m^2 = 0$$

$$L_0|\psi\rangle = 0$$

$$ip_\mu\Gamma^\mu + m = 0$$

$$\{\Gamma^\mu, \Gamma^v\} = 2\eta^{\mu v},$$

$$S = \frac{1}{4\pi}\int d^2z\left(\frac{2}{\alpha'}\partial X^\mu\bar{\partial}X_\mu + \psi^\mu\bar{\partial}\psi_\mu + \tilde{\psi}^\mu\partial\tilde{\psi}_\mu\right)$$

$$X^\mu(z,\bar{z})X^v(0,0) \sim -\frac{\alpha'}{2}\eta^{\mu v}\ln\,|z|^2.$$

$$\psi^\mu(z)\psi^v(0) \sim \frac{\eta^{\mu v}}{z}, \tilde{\psi}^\mu(\bar{z})\tilde{\psi}^v(0) \sim \frac{\eta^{\mu v}}{\bar{z}}.$$

$$T_F(z) = i(2/\alpha')^{1/2}\psi^\mu(z)\partial X_\mu(z), \tilde{T}_F(\bar{z}) = i(2/\alpha')^{1/2}\tilde{\psi}^\mu(\bar{z})\bar{\partial}X_\mu(\bar{z})$$

$$j^\eta(z) = \eta(z)T_F(z), \tilde{j}^\eta(\bar{z}) = \eta(z)^*\tilde{T}_F(\bar{z})$$

$$\begin{aligned}\epsilon^{-1}(2/\alpha')^{1/2}\delta X^\mu(z,\bar{z}) &= +\eta(z)\psi^\mu(z) + \eta(z)^*\tilde{\psi}^\mu(\bar{z})\\ \epsilon^{-1}(\alpha'/2)^{1/2}\delta\psi^\mu(z) &= -\eta(z)\partial X^\mu(z)\\ \epsilon^{-1}(\alpha'/2)^{1/2}\delta\tilde{\psi}^\mu(\bar{z}) &= -\eta(z)^*\bar{\partial}X^\mu(\bar{z})\end{aligned}$$

$$\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1} = \delta_v, v(z) = -2\eta_1(z)\eta_2(z)$$

$$T_B = -\frac{1}{\alpha'}\partial X^\mu\partial X_\mu - \frac{1}{2}\psi^\mu\partial\psi_\mu$$

$$\begin{aligned}T_B(z)T_B(0) &\sim \frac{3D}{4z^4} + \frac{2}{z^2}T_B(0) + \frac{1}{z}\partial T_B(0),\\ T_B(z)T_F(0) &\sim \frac{3}{2z^2}T_F(0) + \frac{1}{z}\partial T_F(0),\\ T_F(z)T_F(0) &\sim \frac{D}{z^3} + \frac{2}{z}T_B(0),\end{aligned}$$

$$c = \left(1 + \frac{1}{2}\right)D = \frac{3}{2}D.$$

$$
\begin{aligned}
T_B(z)T_B(0) &\sim \frac{c}{2z^4} + \frac{2}{z^2}T_B(0) + \frac{1}{z}\partial T_B(0) \\
T_B(z)T_F(0) &\sim \frac{3}{2z^2}T_F(0) + \frac{1}{z}\partial T_F(0) \\
T_F(z)T_F(0) &\sim \frac{2c}{3z^3} + \frac{2}{z}T_B(0)
\end{aligned}
$$

$$
\begin{aligned}
h_b &= \lambda, h_c = 1 - \lambda \\
h_\beta &= \lambda - \frac{1}{2}, h_\gamma = \frac{3}{2} - \lambda
\end{aligned}
$$

$$
S_{BC} = \frac{1}{2\pi}\int d^2z(b\bar{\partial}c + \beta\bar{\partial}\gamma)
$$

$$
\begin{aligned}
T_B &= (\partial b)c - \lambda\partial(bc) + (\partial\beta)\gamma - \frac{1}{2}(2\lambda - 1)\partial(\beta\gamma) \\
T_F &= -\frac{1}{2}(\partial\beta)c + \frac{2\lambda - 1}{2}\partial(\beta c) - 2b\gamma
\end{aligned}
$$

$$
[-3(2\lambda - 1)^2 + 1] + [3(2\lambda - 2)^2 - 1] = 9 - 12\lambda
$$

$$
0 = \frac{3}{2}D - 15 \Rightarrow D = 10
$$

$$
\begin{aligned}
T_B &= -(\partial b)c - 2b\partial c - \frac{1}{2}(\partial\beta)\gamma - \frac{3}{2}\beta\partial\gamma \\
T_F &= (\partial\beta)c + \frac{3}{2}\beta\partial c - 2b\gamma
\end{aligned}
$$

$$
\begin{aligned}
T_B(z) &= -\frac{1}{\alpha'}\partial X^\mu\partial X_\mu + V_\mu\partial^2X^\mu - \frac{1}{2}\psi^\mu\partial\psi_\mu \\
T_F(z) &= i(2/\alpha')^{1/2}\psi^\mu\partial X_\mu - i(2\alpha')^{1/2}V_\mu\partial\psi^\mu
\end{aligned}
$$

$$
c = \frac{3}{2}D + 6\alpha' V^\mu V_\mu .
$$

$$
\frac{1}{4\pi}\int d^2w\left(\psi^\mu\partial_{\bar{w}}\psi_\mu + \tilde{\psi}^\mu\partial_w\tilde{\psi}_\mu\right)
$$

$$
\begin{aligned}
\psi^\mu(w + 2\pi) &= +\psi^\mu(w), \\
\psi^\mu(w + 2\pi) &= -\psi^\mu(w),
\end{aligned}
$$

$$
\begin{aligned}
\psi^\mu(w + 2\pi) &= \exp(2\pi i\nu)\psi^\mu(w) \\
\tilde{\psi}^\mu(\bar{w} + 2\pi) &= \exp(-2\pi i\tilde{\nu})\tilde{\psi}^\mu(\bar{w})
\end{aligned}
$$

$$
\begin{aligned}
T_F(w + 2\pi) &= \exp(2\pi i\nu)T_F(w) \\
\tilde{T}_F(\bar{w} + 2\pi) &= \exp(-2\pi i\tilde{\nu})\tilde{T}_F(\bar{w})
\end{aligned}
$$

$$
\psi^\mu(w) = i^{-1/2}\sum_{r\in\mathbf{Z}+\nu}\psi_r^\mu\exp(irw), \tilde{\psi}^\mu(\bar{w}) = i^{1/2}\sum_{r\in\mathbf{Z}+\tilde{\nu}}\tilde{\psi}_r^\mu\exp(-ir\bar{w}),
$$

$$
\psi^\mu_{z^{1/2}}(z) = (\partial_z w)^{1/2}\psi^\mu_{w^{1/2}}(w) = i^{1/2}z^{-1/2}\psi^\mu_{w^{1/2}}(w)
$$

$$\psi^\mu(z) = \sum_{r\in\mathbf{Z}+v} \frac{\psi_r^\mu}{z^{r+1/2}}, \tilde{\psi}^\mu(\bar{z}) = \sum_{r\in\mathbf{Z}+\tilde{v}} \frac{\tilde{\psi}_r^\mu}{\bar{z}^{r+1/2}}$$

$$\partial X^\mu(z) = -i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{\infty} \frac{\alpha_m^\mu}{z^{m+1}}, \bar{\partial} X^\mu(\bar{z}) = -i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{\infty} \frac{\tilde{\alpha}_m^\mu}{\bar{z}^{m+1}},$$

$$\begin{aligned} \{\psi_r^\mu, \psi_s^v\} &= \{\tilde{\psi}_r^\mu, \tilde{\psi}_s^v\} = \eta^{\mu v}\delta_{r,-s} \\ [\alpha_m^\mu, \alpha_n^v] &= [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^v] = m\eta^{\mu v}\delta_{m,-n} \end{aligned}$$

$$\begin{aligned} T_F(z) &= \sum_{r\in\mathbf{Z}+v} \frac{G_r}{z^{r+3/2}}, \tilde{T}_F(\bar{z}) = \sum_{r\in\mathbf{Z}+\tilde{v}} \frac{\tilde{G}_r}{\bar{z}^{r+3/2}} \\ T_B(z) &= \sum_{m=-\infty}^{\infty} \frac{L_m}{z^{m+2}}, \tilde{T}_B(\bar{z}) = \sum_{m=-\infty}^{\infty} \frac{\tilde{L}_m}{\bar{z}^{m+2}} \end{aligned}$$

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m,-n} \\ \{G_r, G_s\} &= 2L_{r+s} + \frac{c}{12}(4r^2 - 1)\delta_{r,-s} \\ [L_m, G_r] &= \frac{m-2r}{2} G_{m+r} \end{aligned}$$

$$\begin{aligned} L_m &= \frac{1}{2}\sum_{n\in\mathbf{Z}} {}_\circ\alpha_{m-n}^\mu \alpha_{\mu n}^{\circ}{}_\circ + \frac{1}{4}\sum_{r\in\mathbf{Z}+v} (2r-m) \mathring{\circ}\, \psi_{m-r}^\mu \psi_{\mu r}^{\circ}{}_\circ + a^{\mathrm{m}}\delta_{m,0} \\ G_r &= \sum_{n\in\mathbf{Z}} \alpha_n^\mu \psi_{\mu r-n} \end{aligned}$$

$$\mathrm{R}: a^{\mathrm{m}} = \frac{1}{16}D, \mathrm{NS}: a^{\mathrm{m}} = 0.$$

$$\psi^\mu(0,\sigma^2) = \exp(2\pi i v)\tilde{\psi}^\mu(0,\sigma^2), \psi^\mu(\pi,\sigma^2) = \exp(2\pi i v')\tilde{\psi}^\mu(\pi,\sigma^2).$$

$$\psi^\mu(\sigma^1,\sigma^2) = \tilde{\psi}^\mu(2\pi - \sigma^1,\sigma^2)$$

$$\psi_r^\mu|0\rangle_{\mathrm{NS}} = 0, r > 0.$$

$$\Gamma^\mu \cong 2^{1/2}\psi_0^\mu$$

$$|s_0, s_1, \ldots, s_4\rangle_{\mathrm{R}} \equiv |\mathbf{s}\rangle_{\mathrm{R}}, s_a = \pm\frac{1}{2}$$

$$\exp(\pi i F)$$

$$\Sigma^{\mu\lambda} = -\frac{i}{2}\sum_{r\in\mathbf{Z}+v} [\psi_r^\mu, \psi_{-r}^\lambda]$$

$$S_a = i^{\delta_{a,0}}\Sigma^{2a,2a+1}$$

$$F = \sum_{a=0}^{4} S_a$$

$$S_1(\psi_r^2 \pm i\psi_r^3) = (\psi_r^2 \pm i\psi_r^3)(S_1 \pm 1)$$

$$\begin{aligned} \exp(\pi i F)|0\rangle_{\text{NS}} &= -|0\rangle_{\text{NS}} \\ \exp(\pi i F)|\mathbf{s}\rangle_{\text{R}} &= |\mathbf{s}'\rangle_{\text{R}}\Gamma_{\mathbf{s}'\mathbf{s}} \end{aligned}$$

$$\begin{aligned} \mathbf{32}_{\text{Dirac}} \times \mathbf{32}_{\text{Dirac}} &= [0] + [1] + [2] + \cdots + [10] \\ &= [0]^2 + [1]^2 + \cdots + [4]^2 + [5] \end{aligned}$$

$$\psi_r^\mu|1\rangle = 0, r = \frac{1}{2}, \frac{3}{2}, \ldots,$$

$$|1\rangle = |0\rangle$$

$$\psi_{-r}^\mu \to \frac{1}{(r-1/2)!}\partial^{r-1/2}\psi^\mu(0)$$

$$\delta_\eta \mathcal{A}(z,\bar{z}) = -\epsilon \sum_{n=0}^{\infty} \frac{1}{n!}\left[\partial^n \eta(z) G_{n-1/2} + (\partial^n \eta(z))^* \tilde{G}_{n-1/2}\right] \cdot \mathcal{A}(z,\bar{z})$$

$$H(z)H(0) \sim -\ln z.$$

$$\begin{aligned} e^{iH(z)}e^{-iH(0)} &\sim \frac{1}{z} \\ e^{iH(z)}e^{iH(0)} &= O(z) \\ e^{-iH(z)}e^{-iH(0)} &= O(z) \end{aligned}$$

$$\left\langle \prod_i e^{i\epsilon_i H(z_i)} \right\rangle_{S_2} = \prod_{i<j} z_{ij}^{\epsilon_i \epsilon_j}, \sum_i \epsilon_i = 0.$$

$$\psi = 2^{-1/2}(\psi^1 + i\psi^2), \bar{\psi} = 2^{-1/2}(\psi^1 - i\psi^2).$$

$$\begin{aligned} \psi(z)\bar{\psi}(0) &\sim \frac{1}{z}, \\ \psi(z)\psi(0) &= O(z), \\ \bar{\psi}(z)\bar{\psi}(0) &= O(z). \end{aligned}$$

$$\psi(z) \cong e^{iH(z)}, \bar{\psi}(z) \cong e^{-iH(z)}$$

$$\tilde{\psi}(\bar{z}) \cong e^{i\tilde{H}(\bar{z})}, \bar{\tilde{\psi}}(\bar{z}) \cong e^{-i\tilde{H}(\bar{z})}$$

$$\begin{aligned} e^{iH(z)}e^{-iH(-z)} &= \frac{1}{2z} + i\partial H(0) + 2zT_B^H(0) + O(z^2) \\ \psi(z)\bar{\psi}(-z) &= \frac{1}{2z} + \psi\bar{\psi}(0) + 2zT_B^\psi(0) + O(z^2) \end{aligned}$$

$$\psi\bar{\psi} \cong i\partial H, T_B^\psi \cong T_B^H$$

$$\psi(z) \cong {}^{\circ}_{\circ} e^{iH(z)} {}^{\circ}_{\circ}.$$

$$\overset{\circ}{\circ} e^{iH(z)\circ} \circ e^{iH(z')\circ} = \exp\{-[H(z), H(z')]\} \circ e^{iH(z')\circ} e^{iH(z)\circ} = -e^{iH(z')\circ} e^{iH(z)\circ}$$

$$H(z) \circ e^{iH(z')\circ} = \circ e^{iH(z')\circ}(H(z) + i[H(z), H(z')]) = \circ e^{iH(z')\circ}\big(H(z) - \pi \mathrm{sign}(\sigma_1 - \sigma_1')\big)$$

$$\mathrm{Tr}(q^{L_0}) = \left(\sum_{k_L \in \mathbf{Z}} q^{k_L^2/2}\right) \prod_{n=1}^{\infty} (1 - q^n)^{-1}.$$

$$\mathrm{Tr}(q^{L_0}) = \prod_{n=1}^{\infty} \left(1 + q^{n-1/2}\right)^2$$

$$1 + 2q^{1/2} + q + 2q^{3/2} + 4q^2 + 4q^{5/2} + \cdots$$

$$\psi(w + 2\pi) = \exp(2\pi i v)\psi(w)$$

$$\psi(z) = \sum_{r \in \mathbf{Z}+v} \frac{\psi_r}{z^{r+1/2}}, \bar{\psi}(z) = \sum_{s \in \mathbf{Z}-v} \frac{\bar{\psi}_s}{z^{s+1/2}},$$

$$\{\psi_r, \bar{\psi}_s\} = \delta_{r,-s}$$

$$\psi_{n+v}|0\rangle_v = \bar{\psi}_{n+1-v}|0\rangle_v = 0, n = 0,1, \ldots$$

$$\psi(z)\mathcal{A}_v(0) = O\left(z^{-v+1/2}\right), \bar{\psi}(z)\mathcal{A}_v(0) = O\left(z^{v-1/2}\right)$$

$$\exp[i(-v + 1/2)H] \cong \mathcal{A}_v$$

$$|0\rangle_{v+1} = \bar{\psi}_{-v}|0\rangle_v$$

$$|s\rangle \cong e^{isH}, s = \pm\frac{1}{2}$$

$$2^{-1/2}(\pm\psi^0 + \psi^1) \cong e^{\pm iH^0}$$
$$2^{-1/2}(\psi^{2a} \pm i\psi^{2a+1}) \cong e^{\pm iH^a}, a = 1, \ldots, 4$$

$$\Theta_{\mathbf{s}} \cong \exp\left[i \sum_a s_a H^a\right]$$

$$T_B^{(\lambda)} = T_B^{(1/2)} - \left(\lambda - \frac{1}{2}\right)\partial(bc)$$

$$T_B^{(\lambda)} \cong T_B^H - i\left(\lambda - \frac{1}{2}\right)\partial^2 H$$

$$b \cong e^{iH}, c \cong e^{-iH}$$

$$c = 1 - 3(2\lambda - 1)^2 = 1 + 12V^2$$

$$H \to i\rho;\ c \cong e^{\rho}, b \cong e^{-\rho}.$$

$$\beta(z) = \sum_{r\in \mathbf{Z}+v} \frac{\beta_r}{z^{r+3/2}}, \gamma(z) = \sum_{r\in \mathbf{Z}+v} \frac{\gamma_r}{z^{r-1/2}}$$
$$b(z) = \sum_{m=-\infty}^{\infty} \frac{b_m}{z^{m+2}}, c(z) = \sum_{m=-\infty}^{\infty} \frac{c_m}{z^{m-1}}$$

$$[\gamma_r, \beta_s] = \delta_{r,-s}, \{b_m, c_n\} = \delta_{n,-m}$$

$$\beta_r|0\rangle_{\mathrm{NS}} = 0, r \geq \frac{1}{2}, \gamma_r|0\rangle_{\mathrm{NS}} = 0, r \geq \frac{1}{2}$$
$$\beta_r|0\rangle_{\mathrm{R}} = 0, r \geq 0, \gamma_r|0\rangle_{\mathrm{R}} = 0, r \geq 1,$$
$$b_m|0\rangle_{\mathrm{NS,R}} = 0, m \geq 0, c_m|0\rangle_{\mathrm{NS,R}} = 0, m \geq 1.$$

$$L_m^{\mathrm{g}} = \sum_{n\in \mathbf{Z}} (m+n)^{\circ}_{\circ} b_{m-n} c_n^{\circ} + \sum_{r\in \mathbf{Z}+v} \frac{1}{2}(m+2r)^{\circ}_{\circ}\beta_{m-r}\gamma_r{}^{\circ}_{\circ} + a^{\mathrm{g}}\delta_{m,0}$$
$$G_r^{\mathrm{g}} = -\sum_{n\in \mathbf{Z}} \left[\frac{1}{2}(2r+n)\beta_{r-n}c_n + 2b_n\gamma_{r-n}\right]$$

$$\mathrm{R}: a^{\mathrm{g}} = -\frac{5}{8}, \mathrm{NS}: a^{\mathrm{g}} = -\frac{1}{2}$$

$$\beta_r|1\rangle = 0, r \geq -\frac{1}{2}, \gamma_r|1\rangle = 0, r \geq \frac{3}{2}.$$

$$\gamma(z)\delta(\gamma(0)) = O(z), \beta(z)\delta(\gamma(0)) = O(z^{-1})$$

$$\beta\gamma(z)\beta\gamma(0) \sim -\frac{1}{z^2}$$

$$\beta\gamma(z) \cong \partial\phi(z)$$

$$\beta(z) \overset{?}{\cong} e^{-\phi(z)}, \gamma(z) \overset{?}{\cong} e^{\phi(z)}.$$

$$\beta(z)\beta(0) \overset{?}{=} O(z^{-1}), \beta(z)\gamma(0) \overset{?}{=} O(z^1), \gamma(z)\gamma(0) \overset{?}{=} O(z^{-1})$$

$$\beta(z)\beta(0) = O(z^0), \beta(z)\gamma(0) = O(z^{-1}), \gamma(z)\gamma(0) = O(z^0)$$

$$\beta(z) \cong e^{-\phi(z)}\partial\xi(z), \gamma \cong e^{\phi(z)}\eta(z)$$

$$\eta(z)\xi(0) \sim \frac{1}{z}, \eta(z)\eta(0) = O(z), \partial\xi(z)\partial\xi(0) = O(z).$$

$$T(z)\beta\gamma(0) = \frac{1-2\lambda'}{z^3} + \cdots$$

$$T_B^{\phi} = -\frac{1}{2}\partial\phi\partial\phi + \frac{1}{2}(1-2\lambda')\partial^2\phi.$$

$$T_B^{\eta\xi} = -\eta\partial\xi$$

$$T_B^{\beta\gamma} \cong T_B^{\phi} + T_B^{\eta\xi}$$

$$\eta \cong e^{-\chi}, \xi \cong e^{\chi},$$
$$\beta \cong e^{-\phi+\chi}\partial\chi, \gamma \cong e^{\phi-\chi}.$$

$$T_B = -\frac{1}{2}\partial\phi\partial\phi + \frac{1}{2}\partial\chi\partial\chi + \frac{1}{2}(1-2\lambda')\partial^2\phi + \frac{1}{2}\partial^2\chi$$

$$\delta(\gamma) \cong e^{-\phi}, h = \frac{1}{2}$$

$$e^{-\phi}, e^{-\phi}e^{\pm iH^a}$$

$$\beta(z)\Sigma(0) = O\left(z^{-1/2}\right), \gamma(z)\Sigma(0) = O\left(z^{1/2}\right)$$

$$\Sigma = e^{-\phi/2}, h = \frac{3}{8}$$

$$\mathcal{V}_{\mathbf{s}} = e^{-\phi/2}\Theta_{\mathbf{s}}$$

$$\exp\,(ik_L \cdot H_L + ik_R \cdot H_R),$$

$$C_k(\alpha_0)\,{}^{\circ}_{\circ}\exp\,(ik_L \cdot H_L + ik_R \cdot H_R)^{\circ}$$

$$C_k(\alpha_0) = \exp\left(\pi i \sum_{\alpha>\beta} n_\alpha \alpha_{0\beta} k_\alpha \circ k_\beta\right)$$

$$L_n^{\mathrm{m}}|\psi\rangle = 0, n > 0, G_r^{\mathrm{m}}|\psi\rangle = 0, r \geq 0$$

$$L_n^{\mathrm{m}}|\chi\rangle \cong 0, n < 0, G_r^{\mathrm{m}}|\chi\rangle \cong 0, r < 0.$$

$$L_0|\psi\rangle = H|\psi\rangle = 0$$

$$H = \begin{cases} \alpha' p^2 + N - \frac{1}{2} \text{ (NS)} \\ \alpha' p^2 + N \text{ (R)} \end{cases}$$

$$\text{NS: } 8\left(-\frac{1}{24} - \frac{1}{48}\right) = -\frac{1}{2}, R\text{: } 8\left(-\frac{1}{24} + \frac{1}{24}\right) = 0.$$

$$G_0^{\mathrm{m}} = (2\alpha')^{1/2} p_\mu \psi_0^\mu + \cdots$$
$$G_{\pm 1/2}^{\mathrm{m}} = (2\alpha')^{1/2} p_\mu \psi_{\pm 1/2}^\mu + \cdots$$

$$m^2 = -k^2 = -\frac{1}{2\alpha'}.$$

$$|e; k\rangle_{\mathrm{NS}} = e \cdot \psi_{-1/2}|0; k\rangle_{\mathrm{NS}}.$$

$$0 = L_0|e; k\rangle_{\mathrm{NS}} = \alpha' k^2 |e; k\rangle_{\mathrm{NS}}$$
$$0 = G_{1/2}^{\mathrm{m}}|e; k\rangle_{\mathrm{NS}} = (2\alpha')^{1/2} k \cdot e|0; k\rangle_{\mathrm{NS}}$$

$$G_{-1/2}^{\mathrm{m}}|0;k\rangle_{\mathrm{NS}} = (2\alpha')^{1/2}k\cdot\psi_{-1/2}|0;k\rangle_{\mathrm{NS}}$$

$$k^2 = 0, e\cdot k = 0, e^{\mu} \cong e^{\mu} + \lambda k^{\mu}.$$

$$|u;k\rangle_{\mathrm{R}} = |\mathbf{s};k\rangle_{\mathrm{R}}u_{\mathbf{s}}.$$

$$0 = L_0|u;k\rangle_{\mathrm{R}} = \alpha' k^2|u;k\rangle_{\mathrm{R}}$$
$$0 = G_0^{\mathrm{m}}|u;k\rangle_{\mathrm{R}} = \alpha'^{1/2}|\mathbf{s}';k\rangle_{\mathrm{R}}k\cdot\Gamma_{\mathbf{s}'\mathbf{s}}u_{\mathbf{s}}.$$

$$k\cdot\Gamma_{\mathbf{s}'\mathbf{s}}u_{\mathbf{s}} = 0$$

$$k_0\Gamma^0 + k_1\Gamma^1 = -k_1\Gamma^0(\Gamma^0\Gamma^1 - 1) = -2k_1\Gamma^0\left(S_0 - \frac{1}{2}\right)$$

$$\left(S_0 - \frac{1}{2}\right)|\mathbf{s},0;k\rangle_{\mathrm{R}}u_{\mathbf{s}} = 0$$

$$\mathbf{16} \rightarrow \left(+\frac{1}{2},\mathbf{8}\right) + \left(-\frac{1}{2},\mathbf{8}'\right)$$
$$\mathbf{16} \rightarrow \left(+\frac{1}{2},\mathbf{8}'\right) + \left(-\frac{1}{2},\mathbf{8}\right)$$

$$\frac{\alpha'}{4}m^2 = N - v = \tilde{N} - \tilde{v}$$

$$|i,\mathbf{s}\rangle\Gamma_{\mathbf{ss}'}^{i}$$

$$Q_{\mathrm{B}} = \frac{1}{2\pi i}\oint\ (dzj_{\mathrm{B}} - d\bar{z}\tilde{j}_{\mathrm{B}})$$

$$\begin{aligned} j_{\mathrm{B}} &= cT_B^{\mathrm{m}} + \gamma T_F^{\mathrm{m}} + \frac{1}{2}\left(cT_B^{\mathrm{g}} + \gamma T_F^{\mathrm{g}}\right) \\ &= cT_B^{\mathrm{m}} + \gamma T_F^{\mathrm{m}} + bc\partial c + \frac{3}{4}(\partial c)\beta\gamma + \frac{1}{4}c(\partial\beta)\gamma - \frac{3}{4}c\beta\partial\gamma - b\gamma^2 \end{aligned}$$

$$j_{\mathrm{B}}(z)b(0) \sim \cdots + \frac{1}{z}T_B(0), j_{\mathrm{B}}(z)\beta(0) \sim \cdots + \frac{1}{z}T_F(0),$$

$$\{Q_{\mathrm{B}}, b_n\} = L_n, [Q_{\mathrm{B}}, \beta_r] = G_r$$

$$\begin{aligned} Q_{\mathrm{B}} = \sum_m c_{-m}L_m^{\mathrm{m}} + \sum_r \gamma_{-r}G_r^{\mathrm{m}} - \sum_{m,n}\frac{1}{2}(n-m)\,{}^{\circ}_{\circ}b_{-m-n}c_m c_n{}^{\circ}_{\circ} \\ + \sum_{m,r}\left[\frac{1}{2}(2r-m)\,{}^{\circ}_{\circ}\beta_{-m-r}c_m\gamma_r{}^{\circ}_{\circ} - {}^{\circ}_{\circ}b_{-m}\gamma_{m-r}\gamma_r{}^{\circ}_{\circ}\right] + a^{\mathrm{g}}c_0 \end{aligned}$$

$$b_0|\psi\rangle = L_0|\psi\rangle = 0$$

$$\beta_0|\psi\rangle = G_0|\psi\rangle = 0$$

$$(\alpha, F),$$

$$\alpha = 1 - 2v$$

$$(\alpha, F, \tilde{\alpha}, \tilde{F}).$$

$$\exp\ \pi i\left(F_1\alpha_2 - F_2\alpha_1 - \tilde{F}_1\tilde{\alpha}_2 + \tilde{F}_2\tilde{\alpha}_1\right)$$

$$F_1\alpha_2 - F_2\alpha_1 - \tilde{F}_1\tilde{\alpha}_2 + \tilde{F}_2\tilde{\alpha}_1 \in 2\mathbf{Z}$$

$$\left(\alpha_1 + \alpha_2, F_1 + F_2, \tilde{\alpha}_1 + \tilde{\alpha}_2, \tilde{F}_1 + \tilde{F}_2\right).$$

$$X^2 \to -X^2, \psi^2 \to -\psi^2, \tilde{\psi}^2 \to -\tilde{\psi}^2,$$

$$\begin{array}{lllll} 0\text{ A:} & (\mathrm{NS}+, \mathrm{NS}+) & (\mathrm{NS}-, \mathrm{NS}-) & (\mathrm{R}+, \mathrm{R}-) & (\mathrm{R}-, \mathrm{R}+), \\ 0\text{ B:} & (\mathrm{NS}+, \mathrm{NS}+) & (\mathrm{NS}-, \mathrm{NS}-) & (\mathrm{R}+, \mathrm{R}+) & (\mathrm{R}-, \mathrm{R}-). \end{array}$$

$$\begin{array}{ll} \text{IIA}: & [0] + [1] + [2] + [3] + (2) + \mathbf{8} + \mathbf{8}' + \mathbf{56} + \mathbf{56}', \\ \text{IIB}: & [0]^2 + [2]^2 + [4]_+ + (2) + \mathbf{8}'^2 + \mathbf{56}^2. \end{array}$$

$$\exp\,(\pi i F) = \exp\,(\pi i \tilde{F}) = +1,$$

$$\exp\,(\pi i F) = +1, \exp\,(\pi i \tilde{F}) = (-1)^{\tilde{\alpha}}.$$

$$\alpha = \tilde{\alpha}, \exp\,(\pi i F) = \exp\,(\pi i \tilde{F})$$

$$\psi^{\mu}_{-1/2}|0;\mathbf{s};k\rangle_{\text{NS-R}}\, u_{\mu\mathbf{s}}.$$

$$k^2 = k^{\mu}u_{\mu\mathbf{s}} = k\cdot\Gamma_{\mathbf{s}\mathbf{s}'}u_{\mu\mathbf{s}'} = 0$$

$$u_{\mu\mathbf{s}} \cong u_{\mu\mathbf{s}} + k_{\mu}\zeta_{\mathbf{s}}$$

$$\mathcal{V}_{\mathbf{s}}e^{-\tilde{\phi}}\tilde{\psi}^{\mu}e^{ik\cdot X}, e^{-\phi}\psi^{\mu}\tilde{\mathcal{V}}_{\mathbf{s}}e^{ik\cdot X}.$$

$$\mathcal{V}_{\mathbf{s}}\tilde{\mathcal{V}}_{\mathbf{s}'}$$

$$\tilde{\mathcal{V}}_{\mathbf{s}}\mathcal{V}_{\mathbf{s}'} = -\mathcal{V}_{\mathbf{s}'}\tilde{\mathcal{V}}_{\mathbf{s}}$$

$$[0] + [2] + (2) + \mathbf{8}' + \mathbf{56} = \mathbf{1} + \mathbf{28} + \mathbf{35} + \mathbf{8}' + \mathbf{56}.$$

$$\begin{array}{ll} \text{I:} & \mathrm{NS}+, \mathrm{R}+ = \mathbf{8}_v + \mathbf{8}, \\ \text{I:} & \mathrm{NS}+, \mathrm{R}- = \mathbf{8}_v + \mathbf{8}'. \end{array}$$

$$[0] + [2] + (2) + \mathbf{8}' + \mathbf{56} + (\mathbf{8}_v + \mathbf{8})_{SO(n)} \text{ or } Sp(k).$$

$$\boldsymbol{Z}_{T_2} = V_{10}\int_F \frac{d^2\tau}{4\tau_2}\int \frac{d^{10}k}{(2\pi)^{10}} \sum_{i\in\mathcal{H}^{\perp}} (-1)^{\mathbf{F}_i} q^{\alpha'(k^2+m_i^2)/4}\bar{q}^{\alpha'(k^2+\tilde{m}_i^2)/4},$$

$$m^2 = 4H^{\perp}/\alpha', \tilde{m}^2 = 4\tilde{H}^{\perp}/\alpha'.$$

$$Z_X(\tau) = (4\pi^2\alpha'\tau_2)^{-1/2}(q\bar{q})^{-1/24}\prod_{n=1}^{\infty}\left(\sum_{N_n,\tilde{N}_n=1}^{\infty} q^{nN_n}\bar{q}^{n\tilde{N}_n}\right) = (4\pi^2\alpha'\tau_2)^{-1/2}|\eta(\tau)|^{-2}$$

$$\psi(w+2\pi) = \exp\left[\pi i(1-\alpha)\right]\psi(w)$$

$$\psi_{-m+(1-\alpha)/2}, \bar{\psi}_{-m+(1+\alpha)/2}, m = 1,2,\ldots$$

$$\mathrm{Tr}_{\alpha}(q^{H}) = q^{(3\alpha^2-1)/24} \prod_{m=1}^{\infty} \left[1+q^{m-(1-\alpha)/2}\right]\left[1+q^{m-(1+\alpha)/2}\right]$$

$$\begin{aligned} Z_{\beta}^{\alpha}(\tau) &= \mathrm{Tr}_{\alpha}\left[q^{H}\exp\left(\pi i\beta Q\right)\right] \\ &= q^{(3\alpha^2-1)/24}\exp\left(\pi i\alpha\beta/2\right) \\ &\times \prod_{m=1}^{\infty} \left[1+\exp\left(\pi i\beta\right)q^{m-(1-\alpha)/2}\right]\left[1+\exp\left(-\pi i\beta\right)q^{m-(1+\alpha)/2}\right] \\ &= \frac{1}{\eta(\tau)}\vartheta\begin{bmatrix}\alpha/2\\ \beta/2\end{bmatrix}(0,\tau) \end{aligned}$$

$$\begin{aligned} Z_{0}^{0}(\tau) &= \mathrm{Tr}_{\mathrm{NS}}[q^{H}], \\ Z_{1}^{0}(\tau) &= \mathrm{Tr}_{\mathrm{NS}}[\exp\left(\pi iF\right)q^{H}], \\ Z_{0}^{1}(\tau) &= \mathrm{Tr}_{\mathrm{R}}[q^{H}], \\ Z_{1}^{1}(\tau) &= \mathrm{Tr}_{\mathrm{R}}[\exp\left(\pi iF\right)q^{H}]. \end{aligned}$$

$$Z_{\psi}^{\pm}(\tau) = \frac{1}{2}\left[Z_{0}^{0}(\tau)^{4} - Z_{1}^{0}(\tau)^{4} - Z_{0}^{1}(\tau)^{4} \mp Z_{1}^{1}(\tau)^{4}\right].$$

$$Z_{T_2} = iV_{10}\int_{F}\frac{d^2\tau}{16\pi^2\alpha'\tau_2^2}Z_X^8 Z_{\psi}^{\pm}(\tau)Z_{\psi}^{\pm}(\tau)^{*}$$

$$\begin{aligned} \psi(w+2\pi) &= -\exp\left(-\pi i\alpha\right)\psi(w) \\ \psi(w+2\pi\tau) &= -\exp\left(-\pi i\beta\right)\psi(w) \end{aligned}$$

$$\psi[w+2\pi(\tau+1)] = \exp\left[-\pi i(\alpha+\beta)\right]\psi(w).$$

$$\begin{aligned} \psi'(w'+2\pi) &= -\exp\left(-\pi i\beta\right)\psi'(w') \\ \psi'(w'-2\pi/\tau) &= -\exp\left(\pi i\alpha\right)\psi'(w') \end{aligned}$$

$$Z_{\beta}^{\alpha}(\tau) = Z_{-\alpha}^{\beta}(-1/\tau) = \exp\left[-\pi i(3\alpha^2-1)/12\right]Z_{\alpha+\beta-1}^{\alpha}(\tau+1)$$

$$\frac{1}{2}\left[|Z^{0}{}_{0}(\tau)|^{N} + |Z^{0}{}_{1}(\tau)|^{N} + |Z^{1}{}_{0}(\tau)|^{N} \mp |Z^{1}{}_{1}(\tau)|^{N}\right]$$

$$Z^{0}{}_{0}(\tau)^{4} - Z^{0}{}_{1}(\tau)^{4} - Z^{1}{}_{0}(\tau)^{4} = 0.$$

$$\alpha = \tilde{\alpha}, \exp\left(\pi iF\right) = \exp\left(\pi i\tilde{F}\right)$$

$$(k_R, k_L) = (n_1, n_2) \text{ or } \left(n_1+\frac{1}{2}, n_2+\frac{1}{2}\right)$$

$$\partial H\bar{\partial}H \cong -\bar{\psi}\psi\tilde{\bar{\psi}}\tilde{\psi}$$

$$k_R = m/3^{1/2}, k_L = n/3^{1/2}, m-n \in 3\mathbf{Z}$$

$$\exp\left[\pm i3^{1/2}H(z)\right], \exp\left[\pm i3^{1/2}\tilde{H}(\bar{z})\right]$$

$$(H,\tilde{H}) \to (H,\tilde{H}) + \frac{2\pi}{2\times 3^{1/2}}(1,-1).$$

$$Z_{C_2} = Z_{C_2,0} + Z_{C_2,1},$$

$$Z_{C_2,0} = iV_{10}n^2\int_0^\infty \frac{dt}{8t}(8\pi^2\alpha' t)^{-5}\eta(it)^{-8}[Z_0^0(it)^4 - Z_0^1(it)^4]$$
$$Z_{C_2,1} = iV_{10}n^2\int_0^\infty \frac{dt}{8t}(8\pi^2\alpha' t)^{-5}\eta(it)^{-8}[-Z_1^0{}_1(it)^4 - Z_1^1(it)^4]$$

$$\eta(it) = t^{-1/2}\eta(i/t), Z_\beta^\alpha(it) = Z_{-\alpha}^\beta(i/t)$$

$$Z_{C_2,0} = i\frac{V_{10}n^2}{8\pi(8\pi^2\alpha')^5}\int_0^\infty ds\eta(is/\pi)^{-8}[Z_0^0(is/\pi)^4 - Z_1^0(is/\pi)^4]$$
$$= i\frac{V_{10}n^2}{8\pi(8\pi^2\alpha')^5}\int_0^\infty ds[16 + O(\exp(-2s))]$$

$$\mu_{10}\int C_{10}$$

$$\frac{\mu_{10}^2}{0}$$

$$\frac{1+\exp(\pi i F)}{2}\cdot\frac{1+\exp(\pi i\tilde{F})}{2}$$

$$\psi(w+2\pi it) = -R\psi(w)R^{-1} = -\exp(\pi i\beta)\tilde{\psi}(\bar{w})$$
$$\tilde{\psi}(\bar{w}+2\pi it) = -R\tilde{\psi}(\bar{w})R^{-1} = -\exp(\pi i\tilde{\beta})\psi(w)$$

$$\psi(w+4\pi it) = \exp[\pi i(\beta+\tilde{\beta})]\psi(w).$$

$$\text{NS-NS: } q^{-1/3}\prod_{m=1}^{\infty}(1+q^{2m-1})^8 = Z_0^0(2it)^4$$
$$\text{R}-\text{R:} -16q^{2/3}\prod_{m=1}^{\infty}(1+q^{2m})^8 = -Z_0^1(2it)^4$$

$$Z_{K_2,0} = iV_{10}\int_0^\infty \frac{dt}{8t}(4\pi^2\alpha' t)^{-5}\eta(2it)^{-8}[Z_0^0(2it)^4 - Z_0^1(2it)^4]$$

$$= i\frac{2^{10}V_{10}}{8\pi(8\pi^2\alpha')^5}\int_0^\infty ds\eta(is/\pi)^{-8}[Z_0^0(is/\pi)^4 - Z_1^0(is/\pi)^4]$$

$$= i\frac{2^{10}V_{10}}{8\pi(8\pi^2\alpha')^5}\int_0^\infty ds[16 + O(\exp(-2s))]$$

$$\Omega\psi^\mu(w)\Omega^{-1} = \tilde{\psi}^\mu(\pi-\bar{w}) = \psi^\mu(w-\pi),$$

$$\Omega\psi_r^\mu\Omega^{-1} = \exp\left(-\pi i r\right)\psi_r^\mu.$$

$$\Omega^2 = \exp\left(\pi i F\right).$$

$$\frac{1+\Omega+\Omega^2+\Omega^3}{4}.$$

$$\psi^\mu(w+4\pi i t) = -\exp\left(\pi i\beta\right)\psi^\mu(w+2\pi i t-\pi) = \psi^\mu(w-2\pi).$$

$$-16q^{1/3}\prod_{m=1}^{\infty}\left[1+(-1)^m q^m\right]^8 - (1-1)^4 q^{1/3}\prod_{m=1}^{\infty}\left[1-(-1)^m q^m\right]^8$$
$$= Z^0{}_1(2it)^4 Z^1{}_0(2it)^4$$

$$Z_{M_2,1} = \pm inV_{10}\int_0^\infty \frac{dt}{8t}(8\pi^2\alpha' t)^{-5}\frac{Z_1^0(2it)^4 Z_1^1(2it)^4}{\eta(2it)^8 Z_0^0(2it)^4}$$

$$= \pm 2in\frac{2^5 V_{10}}{8\pi(8\pi^2\alpha')^5}\int_0^\infty ds\frac{Z_1^0(2is/\pi)^4 Z_0^1(2is/\pi)^4}{\eta(2is/\pi)^8 Z_0^0(2is/\pi)^4}$$

$$= \pm 2in\frac{2^5 V_{10}}{8\pi(8\pi^2\alpha')^5}\int_0^\infty ds\left[16+O(\exp\left(-2s\right))\right]$$

$$Z_1 = -i(n\mp 32)^2\frac{V_{10}}{8\pi(8\pi^2\alpha')^5}\int_0^\infty ds\left[16+O(\exp\left(-2s\right))\right]$$

$$j(z)j(0)\sim z^{-2h}$$

$$c_h = (-1)^{2h+1}\left[3(2h-1)^2-1\right]$$
$$c_2 = -26, c_{3/2} = +11, c_1 = -2, c_{1/2} = -1, c_0 = -2$$

$$T_F^\pm = 2^{-1/2}(T_{F1}\pm iT_{F2})$$

$$\begin{aligned}
T_B(z)T_F^\pm(0) &\sim \frac{3}{2z^2}T_F^\pm(0)+\frac{1}{z}\partial T_F^\pm(0),\\
T_B(z)j(0) &\sim \frac{1}{z^2}j(0)+\frac{1}{z}\partial j(0),\\
T_F^+(z)T_F^-(0) &\sim \frac{2c}{3z^3}+\frac{2}{z^2}j(0)+\frac{2}{z}T_B(0)+\frac{1}{z}\partial j(0),\\
T_F^+(z)T_F^+(0) &\sim T_F^-(z)T_F^-(0)\sim 0,\\
j(z)T_F^\pm(0) &\sim \pm\frac{1}{z}T_F^\pm(0),\\
j(z)j(0) &\sim \frac{c}{3z^2}.
\end{aligned}$$

$$S = \frac{1}{2\pi}\int d^2z(\partial\bar{Z}\bar{\partial}Z+\bar{\psi}\bar{\partial}\psi+\bar{\tilde{\psi}}\partial\tilde{\psi})$$

$$T_B = -\partial\bar{Z}\partial Z - \frac{1}{2}(\bar{\psi}\partial\psi + \psi\partial\bar{\psi}), j = -\bar{\psi}\psi$$
$$T_F^+ = 2^{1/2} i\psi\partial\bar{Z}, T_F^- = 2^{1/2} i\bar{\psi}\partial Z$$

$$X^\mu(z,\bar{z}), \tilde{\psi}^\mu(\bar{z}), \mu = 0, \ldots, 9,$$

$$\lambda^A(z), A = 1, \ldots, 32.$$

$$S = \frac{1}{4\pi}\int d^2z\left(\frac{2}{\alpha'}\partial X^\mu\bar{\partial}X_\mu + \lambda^A\bar{\partial}\lambda^A + \tilde{\psi}^\mu\partial\tilde{\psi}_\mu\right)$$

$$X^\mu(z,\bar{z})X^\nu(0,0) \sim -\eta^{\mu\nu}\frac{\alpha'}{2}\ln |z|^2$$
$$\lambda^A(z)\lambda^B(0) \sim \delta^{AB}\frac{1}{z}$$
$$\tilde{\psi}^\mu(\bar{z})\tilde{\psi}^\nu(0) \sim \eta^{\mu\nu}\frac{1}{\bar{z}}$$

$$T_B = -\frac{1}{\alpha'}\partial X^\mu\partial X_\mu - \frac{1}{2}\lambda^A\partial\lambda^A$$
$$\tilde{T}_B = -\frac{1}{\alpha'}\bar{\partial}X^\mu\bar{\partial}X_\mu - \frac{1}{2}\tilde{\psi}^\mu\bar{\partial}\tilde{\psi}_\mu$$
$$\tilde{T}_F = i(2/\alpha')^{1/2}\tilde{\psi}^\mu\bar{\partial}X_\mu$$

$$\lambda^A(w + 2\pi) = O^{AB}\lambda^B(w).$$

$$\mathbf{s}\cdot\mathbf{s}' + \frac{l}{2} \in \mathbf{Z}$$

$$\exp(\pi i\tilde{F}) = 1;$$

$$\lambda^A(w + 2\pi) = \pm\lambda^A(w)$$

$$\exp(\pi i F) = 1$$

$$\lambda^{K\pm} = 2^{-1/2}(\lambda^{2K-1} \pm i\lambda^{2K}), K = 1, \ldots, 16.$$

$$F = \sum_{K=1}^{16} q_K,$$

$$Z_{16}(\tau) = \frac{1}{2}[Z_0^0(\tau)^{16} + Z_1^0(\tau)^{16} + Z_0^1(\tau)^{16} + Z_1^1(\tau)^{16}].$$

$$\mathrm{NS}: -\frac{8}{24} - \frac{32}{48} = -1, \mathrm{R}: -\frac{8}{24} + \frac{32}{24} = +1.$$

$$\lambda_{-1/2}^A|0\rangle_{\mathrm{NS}}$$

$$\alpha_{-1}^i|0\rangle_{\mathrm{NS}}, \lambda_{-1/2}^A\lambda_{-1/2}^B|0\rangle_{\mathrm{NS}}.$$

$$(\mathbf{8}_v, \mathbf{1}) \times (\mathbf{8}_v + \mathbf{8}) = (\mathbf{1}, \mathbf{1}) + (\mathbf{28}, \mathbf{1}) + (\mathbf{35}, \mathbf{1}) + (\mathbf{56}, \mathbf{1}) + (\mathbf{8}', \mathbf{1})$$

$$(1{,}496)\times(8_v+8)=(8_v,496)+(8{,}496)$$

$$\lambda^A(w+2\pi)=\begin{cases}\eta\lambda^A(w), & A=1,\dots,16\\ \eta'\lambda^A(w), & A=17,\dots,32\end{cases}$$

$$\exp(\pi i F_1), \exp(\pi i F_1')$$

$$\exp(\pi i F_1)=\exp(\pi i F_1')=\exp(\pi i\tilde{F})=1.$$

$$Z_8(\tau)^2=\frac{1}{4}[Z_0^0(\tau)^8+Z_1^0(\tau)^8+Z_0^1(\tau)^8+Z_1^1(\tau)^8]^2$$

$$\alpha_{-1}^i|0\rangle_{\mathrm{NS}'_{\mathrm{NS}}}\lambda_{-1/2}^A\lambda_{-1/2}^B|0\rangle_{\mathrm{NS}_{,\mathrm{NS}'}}$$
$$1\le A,B\le 16 \text{ or } 17\le A,B\le 32$$

$$-\frac{8}{24}+\frac{16}{24}-\frac{16}{48}=0$$

$$(8_v,1{,}1)+(1{,}120{,}1)+(1{,}1{,}120)+(1{,}128{,}1)+(1{,}1{,}128).$$

$$\exp\left[i\sum_{K=1}^{16}q_K H^K(z)\right]$$

$$q_K=\begin{cases}\pm\frac{1}{2}, & K=1,\dots 8\\ 0, & K=9,\dots 16\end{cases},\sum_{K=1}^{16}q_K\in 2\mathbf{Z}$$

$$(1{,}1{,}1)+(28{,}1{,}1)+(35{,}1{,}1)+(56{,}1{,}1)+(8',1{,}1)$$
$$+(8_v,248{,}1)+(8{,}248{,}1)+(8_v,1{,}248)+(8{,}1{,}248)$$

$$\frac{1}{2}[Z^0{}_0(\tau)^{16}Z^0{}_0(\tau)^{*4}-Z^0{}_1(\tau)^{16}Z^0{}_1(\tau)^{*4}$$
$$-Z^1{}_0(\tau)^{16}Z^1{}_0(\tau)^{*4}-Z^1{}_1(\tau)^{16}Z^1{}_1(\tau)^{*4}]$$

$$\exp[\pi i(F+\tilde{F})]=1.$$

$$\lambda_{-1/2}^A|0\rangle_{\mathrm{NS,NS}}, m^2=-\frac{2}{\alpha'}, \exp(\pi i F)=\exp(\pi i\tilde{F})=-1$$

$$\alpha_{-1}^i\tilde{\psi}_{-1/2}^j|0\rangle_{\mathrm{NS,NS}}, \lambda_{-1/2}^A\lambda_{-1/2}^B\tilde{\psi}_{-1/2}^j|0\rangle_{\mathrm{NS,NS}}$$

$$\frac{1+\exp[\pi i(F+\tilde{F})]}{2}\cdot\frac{1+\exp(\pi i\tilde{F})}{2}=\frac{1+\exp(\pi i F)}{2}\cdot\frac{1+\exp(\pi i\tilde{F})}{2}$$

$$-\frac{8}{24}+\frac{k}{24}-\frac{(32-k)}{48}=-1+\frac{k}{16}.$$

$$Z=\frac{1}{\mathrm{order}(H)}\sum_{\substack{h_1,h_2\in H\\ [h_1,h_2]=0}}Z_{h_1,h_2},$$

$$Z = \frac{1}{\text{order}(H)} \sum_{\substack{h_1,h_2 \in H \\ [h_1,h_2]=0}} \epsilon(h_1,h_2) Z_{h_1,h_2}$$

$$\begin{aligned} \epsilon(h_1,h_2) &= \epsilon(h_2,h_1)^{-1} \\ \epsilon(h_1,h_2)\epsilon(h_1,h_3) &= \epsilon(h_1,h_2h_3) \\ \epsilon(h,h) &= 1 \end{aligned}$$

$$\hat{h}_2|\psi\rangle_{h_1} = \epsilon(h_1,h_2)^{-1}|\psi\rangle_{h_1}$$

$$\hat{h} \to \epsilon(h_1,h)\hat{h}.$$

$$(h_1,h_2) = \left(\exp\left[\pi i\left(k_1F_1 + l_1\tilde{F}\right)\right], \exp\left[\pi i\left(k_2F_1 + l_2\tilde{F}\right)\right]\right)$$

$$\epsilon(h_1,h_2) = (-1)^{k_1l_2+k_2l_1}$$

$$\exp(\pi i F_1) = \exp(\pi i F_1') = \exp(\pi i \tilde{F}) = 1,$$

$$\exp[\pi i(F_1 + \alpha_1' + \tilde{\alpha})] = \exp[\pi i(F_1' + \alpha_1 + \tilde{\alpha})] = \exp\left[\pi i\left(\tilde{F} + \alpha_1 + \alpha_1'\right)\right] = 1$$

$$\begin{aligned} &(\text{NS}+,\text{NS}+,\text{NS}+) \\ &(\text{NS}-,\text{NS}-,\text{R}+),(\text{NS}-,\text{R}+,\text{NS}-),(\text{NS}+,\text{R}-,\text{R}-) \\ &(\text{R}+,\text{NS}-,\text{NS}-),(\text{R}-,\text{R}-,\text{NS}+),(\text{R}-,\text{NS}+,\text{R}-) \\ &(\text{R}+,\text{R}+,\text{R}+) \end{aligned}$$

$$\begin{aligned} &(\text{NS}+,\text{NS}+,\text{NS}+): (\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{28},\mathbf{1},\mathbf{1}) + (\mathbf{35},\mathbf{1},\mathbf{1}) \\ &+(\mathbf{8}_v,\mathbf{120},\mathbf{1}) + (\mathbf{8}_v,\mathbf{1},\mathbf{120}), \\ &(\text{R}+,\text{NS}-,\text{NS}-): (\mathbf{8},\mathbf{16},\mathbf{16}), \\ &(\text{R}-,\text{R}-,\text{NS}+): (\mathbf{8}',\mathbf{128}',\mathbf{1}), \\ &(\text{R}-,\text{NS}+,\text{R}-): (\mathbf{8}',\mathbf{1},\mathbf{128}'). \end{aligned}$$

$$[T^a,T^b] = if^{ab}{}_cT^c$$

$$\left[T^a,[T^b,T^c]\right] + \left[T^b,[T^c,T^a]\right] + \left[T^c,[T^a,T^b]\right] = 0.$$

$$\exp(i\theta_aT^a),$$

$$\left(T^a,T^b\right) = d^{ab}$$

$$([T,T'],T'') + (T',[T,T'']) = 0.$$

$$\text{Tr}\left(t_r^at_r^b\right) = T_rd^{ab}$$

$$t_r^at_r^bd_{ab} = Q_r$$

$$MTM^{-1} = -T^T.$$

$$M = i\begin{bmatrix} 0 & I_k \\ -I_k & 0 \end{bmatrix}$$

$$MUM^{-1} = (U^T)^{-1}$$

$$[H^i, E^\alpha] = \alpha^i E^\alpha.$$

$$[E^\alpha, E^\beta] = \begin{cases} \epsilon(\alpha,\beta) E^{\alpha+\beta} & \text{if } \alpha+\beta \text{ is a root}\,, \\ 2\alpha \cdot H/\alpha^2 & \text{if } \alpha+\beta = 0, \\ 0 & \text{otherwise}\,. \end{cases}$$

$$(w^1, \dots, w^{\mathrm{rank}(g)}),$$

$$\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$$

$$(\pm 1, 0^{k-1})$$

$$(+1,+1,0^{k-2}), (+1,-1,0^{k-2}), (-1,-1,0^{k-2})$$

$$(\pm 1, 0^{k-1})$$

$$(\pm 2, 0^{k-1})$$

$$(+1,-1,0^{n-2}).$$

$$\left(+\frac{1}{2},+\frac{1}{2},+\frac{1}{2},+\frac{1}{2},+\frac{1}{2},+\frac{1}{2},+\frac{1}{2},+\frac{1}{2}\right)$$

$$\alpha^i \in \mathbf{Z} \text{ for all } i, \text{ or } \alpha^i \in \mathbf{Z} + \frac{1}{2} \text{ for all } i,$$

$$\sum_{i=1}^{8} \alpha^i \in 2\mathbf{Z}, \sum_{i=1}^{8} (\alpha^i)^2 = 2.$$

$$-\sum_{c,d} f^{ac}{}_d f^{bd}{}_c = h(g)\psi^2 d^{ab}.$$

$$E_8 \to SU(3) \times E_6.$$

$$248 \to (8,1) + (1,78) + (3,27) + (\overline{3}, \overline{27}).$$

$$\mathbf{27} \to (\mathbf{3},\mathbf{2})_1 + (\overline{\mathbf{3}},\mathbf{1})_{-4} + (\mathbf{1},\mathbf{1})_6 + (\overline{\mathbf{3}},\mathbf{1})_2 + (\mathbf{1},\mathbf{2})_{-3} + [\mathbf{1}_0]$$

$$+[(\overline{\mathbf{3}},\mathbf{1})_2 + (\mathbf{3},\mathbf{1})_{-2}] + [(\mathbf{1},\mathbf{2})_{-3} + (\mathbf{1},\mathbf{2})_3] + [\mathbf{1}_0]$$

$$j^a(z) j^b(0) \sim \frac{k^{ab}}{z^2} + \frac{ic_c^{ab}}{z} j^c(0)$$

$$j^a(z) = \sum_{m=-\infty}^{\infty} \frac{j_m^a}{z^{m+1}}$$

$$[j_m^a, j_n^b] = mk^{ab}\delta_{m,-n} + ic^{ab}{}_c j_{m+n}^c$$

$$[j_0^a, j_0^b] = ic^{ab}{}_c j_0^c$$

$$c^{ab}{}_c = f^{ab}{}_c.$$

$$f^{bc}{}_d k^{ad} + f^{ba}{}_d k^{dc} = 0.$$

$$k^{ab} = \hat{k} d^{ab}$$

$$[L_m, j_n^a] = -n j_{m+n}^a$$

$$\hat{k} d^{aa} = \langle 1 | [j_1^a, j_{-1}^a] | 1 \rangle = \| j_{-1}^a | 1 \rangle \|^2$$

$$J^3 = \frac{\alpha \cdot H}{\alpha^2}, J^{\pm} = E^{\pm\alpha}$$

$$[J^3, J^{\pm}] = \pm J^{\pm}, [J^+, J^-] = 2J^3$$

$$\frac{\alpha \cdot H_0}{\alpha^2}, E_0^{\alpha}, E_0^{-\alpha},$$
$$\frac{\alpha \cdot H_0 + \hat{k}}{\alpha^2}, E_1^{\alpha}, E_{-1}^{-\alpha}$$

$$k = \frac{2\hat{k}}{\psi^2}$$

$$j^a(z) j^b(0) \sim \frac{\delta^{ab}}{z^2}$$

$$j^a = i\partial H^a$$

$$i\lambda^A \lambda^B$$

$$\frac{1}{2} \lambda^A \lambda^B t_{r,AB}^a$$

$$:jj(z_1) := \lim_{z_2 \to z_1} \left(j^a(z_1) j^a(z_2) - \frac{\hat{k} \dim(g)}{z_{12}^2} \right),$$

$$j^a(z_1) j^a(z_2) j^c(z_3)$$
$$= \frac{\hat{k}}{z_{31}^2} j^c(z_2) + \frac{i f^{cad}}{z_{31}} j^d(z_1) j^a(z_2) + \frac{\hat{k}}{z_{32}^2} j^c(z_1) + \frac{i f^{cad}}{z_{32}} j^a(z_1) j^d(z_2)$$
$$+ \mathfrak{H}_{\text{terms holomorphic in } z_3}$$

$$:jj(z_1): j^c(z_3) \sim \frac{2\hat{k}}{z_{13}^2} j^c(z_1) + \frac{f^{cad} f^{ead}}{z_{13}^2} j^e(z_1) = \frac{2\hat{k} + h(g)\psi^2}{z_{13}^2} j^c(z_1)$$

$$= (k + h(g))\psi^2 \left[\frac{1}{z_{13}^2} j^c(z_3) + \frac{1}{z_{13}} \partial j^c(z_3) \right]$$

$$T_B^s(z) = \frac{1}{(k + h(g))\psi^2} :jj(z):$$

$$T_B^s(z)j^c(0) \sim T_B(z)j^c(0)$$

$$j^a(z_1)j^a(z_2)T_B^s(z_3)$$

$$= \frac{1}{z_{31}^2}j^a(z_1)j^a(z_2) + \frac{1}{z_{31}}\partial j^a(z_1)j^a(z_2) + \frac{1}{z_{32}^2}j^a(z_1)j^a(z_2) + \frac{1}{z_{32}}j^a(z_1)\partial j^a(z_2)$$

$$+ \mathfrak{H}\text{terms holomorphic in } z_3$$

$$T_B^s(z_1)T_B^s(z_3) \sim \frac{c^{g,k}}{2z_{13}^4} + \frac{2}{z_{13}^2}T_B^s(z_3) + \frac{1}{z_{13}}\partial T_B^s(z_3)$$

$$c^{g,k} = \frac{k\dim(g)}{k + h(g)}$$

$$L_0^s = \frac{1}{(k + h(g))\psi^2}\left(j_0^a j_0^a + 2\sum_{n=1}^{\infty} j_{-n}^a j_n^a\right)$$

$$L_m^s = \frac{1}{(k + h(g))\psi^2}\sum_{n=-\infty}^{\infty} j_n^a j_{m-n}^a, m \neq 0$$

$$T_B^s = \frac{1}{2}:jj:$$

$$T_B' = T_B - T_B^s$$

$$T_B'(z_1)j^a(z_2) \sim 0$$

$$T_B'(z)T_B'(0) = T_B(z)T_B(0) - T_B^s(z)T_B^s(0) - T_B'(z)T_B^s(0) - T_B^s(z)T_B'(0) \sim \frac{c'}{2z^4} + \frac{2}{z^2}T_B'(0) + \frac{1}{z}\partial T_B'(0)$$

$$c' = c - c^{g,k}.$$

$$c^{g,k} \leq c,$$

$$c^{g,k} = c,$$

$$c^{g,k} = \frac{k\dim(g)\text{rank}(g)}{\dim(g) + (k-1)\text{rank}(g)}.$$

$$c^{g,1} = \text{rank}(g)$$

$$c^{g,k} = \frac{3k}{2 + k} = 1, \frac{3}{2}, \frac{9}{5}, 2, \frac{15}{7}, \ldots \to 3.$$

$$\text{rank}(g) \leq c^{g,k} \leq \dim(g)$$

$$j_0^a|r, i\rangle = |r, j\rangle t_{r,ji}^a$$

$$L_0^s|r,i\rangle = \frac{1}{(k+h(g))\psi^2}|r,k\rangle t^a_{r,kj}t^a_{r,ji} = \frac{Q_r}{(k+h(g))\psi^2}|r,i\rangle$$

$$h_r = \frac{Q_r}{(k+h(g))\psi^2} = \frac{Q_r}{2\hat{k}+Q_g},$$

$$h_j = \frac{j(j+1)}{k+2}$$

$$\langle r,\lambda|[E_1^{\alpha},E_{-1}^{-\alpha}]|r,\lambda\rangle = 2\langle r,\lambda|\left(\alpha\cdot H_0+\hat{k}\right)|r,\lambda\rangle/\alpha^2 = 2(\alpha\cdot\lambda+\hat{k})/\alpha^2$$

$$\hat{k} \geq |\alpha\cdot\lambda|$$

$$k \geq \frac{2|\psi\cdot\lambda|}{\psi^2} = 2|J^3|,$$

$$\dot{X}^{\mu}\lambda^a e^{ik\cdot X}$$

$$\lambda^a(y_1)\lambda^b(y_2) = [\theta(y_1-y_2)d_c^{ab}+\theta(y_2-y_1)d_c^{ba}]\lambda^c(y_2)$$

$$l_{L,R} = (\alpha'/2)^{1/2}k_{L,R}$$

$$(l_L^m, l_R^n), d \leq m \leq 25, d \leq n \leq 9,$$

$$l\circ l' = l_L\cdot l_L' - l_R\cdot l_R',$$

$$l\circ l \in 2\mathbf{Z} \text{ for all } l\in\Gamma,$$
$$\Gamma = \Gamma^*.$$

$$(n_1,\ldots,n_{16}) \text{ or } \left(n_1+\frac{1}{2},\ldots,n_{16}+\frac{1}{2}\right)$$
$$\sum_i n_i \in 2\mathbf{Z}$$

$$(n_1,\ldots,n_8) \text{ or } \left(n_1+\frac{1}{2},\ldots,n_8+\frac{1}{2}\right)$$
$$\sum_i n_i \in 2\mathbf{Z}$$

$$\Gamma_r \subset \Gamma_g^*.$$

$$\Gamma_w = \Gamma_g^*.$$

$$(0): 0+\mathbb{A}_{\text{any root}};$$
$$(v): (1,0,0,\ldots,0)+\mathbb{A}_{\text{any root}};$$
$$(s): \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\ldots,\frac{1}{2}\right)+\mathbb{A}_{\text{any root}};$$
$$(c): \left(-\frac{1}{2},\frac{1}{2},\frac{1}{2},\ldots,\frac{1}{2}\right)+\mathbb{A}_{\text{any root}}.$$

$$\Gamma = \sum_r \Gamma_r \times \tilde{\Gamma}_r.$$

$$\Gamma = \Lambda\Gamma_0, \Lambda \in O(26-d, 10-d, \mathbf{R})$$

$$\Lambda_1\Lambda\Lambda_2\Gamma_0 \cong \Lambda\Gamma_0$$

$$\Lambda_1 \in O(26-d, \mathbf{R}) \times O(10-d, \mathbf{R}), \Lambda_2 \in O(26-d, 10-d, \mathbf{Z})$$

$$\frac{O(26-d, 10-d, \mathbf{R})}{O(26-d, \mathbf{R}) \times O(10-d, \mathbf{R}) \times O(26-d, 10-d, \mathbf{Z})}.$$

$$l_L^2 = 2, l_R = 0$$

$$\frac{1}{2}[(36-2d)(35-2d) - (26-d)(25-d) - (10-d)(9-d)] = (26-d)(10-d)$$

$$\begin{aligned} k_{Lm} &= \frac{n_m}{R} + \frac{w^n R}{\alpha'}(G_{mn} + B_{mn}) - q^I A_m^I - \frac{w^n R}{2} A_n^I A_m^I, \\ k_L^I &= (q^I + w^m R A_m^I)(2/\alpha')^{1/2}, \\ k_{Rm} &= \frac{n_m}{R} + \frac{w^n R}{\alpha'}(-G_{mn} + B_{mn}) - q^I A_m^I - \frac{w^n R}{2} A_n^I A_m^I, \end{aligned}$$

$$RA_9^I = \mathrm{diag}\left(\frac{1}{2}^8, 0^8\right)$$

$$R' A_9^I = \mathrm{diag}(1, 0^7, 1, 0^7)$$

$$k_{L,R} = \frac{\tilde{n}}{R} \pm \frac{2mR}{\alpha'}, k'_{L,R} = \frac{\tilde{n}'}{R'} \pm \frac{2m'R'}{\alpha'},$$

$$\begin{aligned} \mathbf{8}_v &\to +1, 0^6, -1, \\ \mathbf{8} &\to +\frac{1}{2}^4, -\frac{1}{2}^4, \end{aligned}$$

$$\begin{aligned} \mathbf{8}_v \times \mathbf{8}_v &\to +2, +1^{12}, 0^{38}, -1^{12}, -2, \\ \mathbf{8} \times \mathbf{8}_v &\to \frac{3}{2}^4, \frac{1}{2}^{28}, -\frac{1}{2}^{28}, -\frac{3}{2}^4. \end{aligned}$$

$$\left\{Q_\alpha, Q_\beta^\dagger\right\} = 2P_\mu(\Gamma^\mu\Gamma^0)_{\alpha\beta} + 2P_{Rm}(\Gamma^m\Gamma^0)_{\alpha\beta}.$$

$$2(M + k_{Rm}\Gamma^m\Gamma^0)_{\alpha\beta}$$

$$2(M \pm |k_R|),$$

$$M^2 = \begin{cases} k_R^2 + 4\left(\tilde{N} - \frac{1}{2}\right)/\alpha' & \text{(NS)} \\ k_R^2 + 4\tilde{N}/\alpha' & \text{(R)} \end{cases}$$

$$M^2 = k_L^2 + 4(N-1)/\alpha'$$

$$N = 1 + \alpha'(k_R^2 - k_L^2)/4 = 1 - n_m w^m - q^I q^I/2.$$

$$\left\{Q_\alpha, Q_\beta^\dagger\right\} = 2P_M(\Gamma^M\Gamma^0)_{\alpha\beta} - 2\frac{\Delta X_m}{2\pi\alpha'}(\Gamma^m\Gamma^0)_{\alpha\beta},$$

$$\frac{1}{2\pi\alpha'}\int_M B = \frac{1}{2}\int d^{10}x j^{MN}(x) B_{MN}(x)$$

$$j^{MN}(x) = \frac{1}{2\pi\alpha'}\int_M d^2\sigma(\partial_1 X^M \partial_2 X^N - \partial_1 X^N \partial_2 X^M)\delta^{10}(x - X(\sigma))$$

$$Q^M = \int d^9 x j^{M0} = \frac{1}{2\pi\alpha'}\int dX^M$$

$$\left\{Q_\alpha, Q_\beta^\dagger\right\} = 2(P_M - Q_M)(\Gamma^M\Gamma^0)_{\alpha\beta}$$

$$\Gamma = \sum_r \Gamma_r \times \tilde{\Gamma}_r$$

$$k \circ k = k_L^I k_L' + G^{mn}(k_{Lm}k_{Ln}' - k_{Rm}k_{Rn}')$$

$$2\kappa_{11}^2 \boldsymbol{S}_{11} = \int d^{11}x(-G)^{1/2}\left(R - \frac{1}{2}|F_4|^2\right) - \frac{1}{6}\int A_3 \wedge F_4 \wedge F_4.$$

$$\int d^d x(-G)^{1/2}\left|F_p\right|^2 = \int d^d x \frac{(-G)^{1/2}}{p!} G^{M_1N_1} \dots G^{M_pN_p} F_{M_1\dots M_p} F_{N_1\dots N_p}$$

$$ds^2 = G_{MN}^{11}(x^\mu)dx^M dx^N = G_{\mu\nu}^{10}(x^\mu)dx^\mu dx^\nu + \exp\left(2\sigma(x^\mu)\right)[dx^{10} + A_\nu(x^\mu)dx^\nu]^2$$

$$\boldsymbol{S}_1 = \frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left(e^\sigma R - \frac{1}{2}e^{3\sigma}|F_2|^2\right)$$

$$\boldsymbol{S}_2 = -\frac{1}{4\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left(e^{-\sigma}|F_3|^2 + e^\sigma\left|\tilde{F}_4\right|^2\right)$$

$$\boldsymbol{S}_3 = -\frac{1}{4\kappa_{10}^2}\int A_2 \wedge F_4 \wedge F_4 = -\frac{1}{4\kappa_{10}^2}\int A_3 \wedge F_3 \wedge F_4$$

$$\tilde{F}_4 = dA_3 - A_1 \wedge F_3$$

$$-d\lambda_0 \wedge F_3 = -d(\lambda_0 \wedge F_3)$$

$$\delta' A_3 = \lambda_0 \wedge F_3$$

$$d\tilde{F}_4 = -F_2 \wedge F_3$$

$$G_{\mu\nu} = e^{-\sigma}G_{\mu\nu}(\text{ new }), \sigma = \frac{2\Phi}{3}.$$

$$\boldsymbol{S}_{\mathrm{IIA}} = \boldsymbol{S}_{\mathrm{NS}} + \boldsymbol{S}_{\mathrm{R}} + \boldsymbol{S}_{\mathrm{CS}}$$
$$\boldsymbol{S}_{\mathrm{NS}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2}e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H_3|^2\right)$$
$$\boldsymbol{S}_{\mathrm{R}} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left(|F_2|^2 + |\tilde{F}_4|^2\right)$$
$$\boldsymbol{S}_{\mathrm{CS}} = -\frac{1}{4\kappa_{10}^2}\int B_2 \wedge F_4 \wedge F_4$$

$$C_1 = e^{-\Phi}C_1',$$

$$\int d^{10}x(-G)^{1/2}|F_2|^2 = \int d^{10}x(-G)^{1/2}e^{-2\Phi}|F_2'|^2$$
$$F_2' \equiv dC_1' - d\Phi \wedge C_1'$$

$$dF_2' = d\Phi \wedge F_2', \delta C_1' = d\lambda_0' - \lambda_0' d\Phi$$

$$\mathcal{V}_\alpha \tilde{\mathcal{V}}_\beta (C\Gamma^{\mu_1\ldots\mu_p})_{\alpha\beta} e_{\mu_1\ldots\mu_p}(X).$$

$$\Gamma^\nu\Gamma^{\mu_1\ldots\mu_p}\partial_\nu e_{\mu_1\ldots\mu_p}(X) = \Gamma^{\mu_1\ldots\mu_p}\Gamma^\nu\partial_\nu e_{\mu_1\ldots\mu_p}(X) = 0$$

$$\Gamma^\nu\Gamma^{\mu_1\ldots\mu_p} = \Gamma^{\nu\mu_1\ldots\mu_p} + p\eta^{\nu[\mu_1}\Gamma^{\mu_2\ldots\mu_p]}$$
$$\Gamma^{\mu_1\ldots\mu_p}\Gamma^\nu = (-1)^p\Gamma^{\nu\mu_1\ldots\mu_p} + (-1)^{p+1}p\eta^{\nu[\mu_1}\Gamma^{\mu_2\ldots\mu_p]}$$

$$de_p = d * e_p = 0$$

$$T_F = i(2/\alpha')^{1/2}\psi^\mu\partial X_\mu - 2i(\alpha'/2)^{1/2}\Phi_{,\mu}\partial\psi^\mu$$
$$G_0 \sim (\alpha'/2)^{1/2}\psi_0^\mu(p_\mu + i\Phi_{,\mu})$$

$$(d - d\Phi\wedge)e_p = (d - d\Phi\wedge) * e_p = 0.$$

$$\frac{1}{2\pi\alpha'}\int_M B_2$$

$$\tilde{F}_6 =* \tilde{F}_4, \tilde{F}_8 =* F_2;$$

$$d * F_{10} = 0$$

$$* F_{10} = \text{constant}$$

$$\boldsymbol{S}_{\mathrm{IIA}}' = \tilde{\boldsymbol{S}}_{\mathrm{IIA}} - \frac{1}{4\kappa_{10}^2}\int d^{10}x(-G)^{1/2}M^2 + \frac{1}{2\kappa_{10}^2}\int MF_{10}$$

$$\boldsymbol{S}_{\mathrm{IIB}} = \boldsymbol{S}_{\mathrm{NS}} + \boldsymbol{S}_{\mathrm{R}} + \boldsymbol{S}_{\mathrm{CS}}$$
$$\boldsymbol{S}_{\mathrm{NS}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2}e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H_3|^2\right)$$
$$\boldsymbol{S}_{\mathrm{R}} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right)$$
$$\boldsymbol{S}_{\mathrm{CS}} = -\frac{1}{4\kappa_{10}^2}\int C_4 \wedge H_3 \wedge F_3$$

$$\tilde{F}_3 = F_3 - C_0 \wedge H_3,$$
$$\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3.$$

$$d * \tilde{F}_5 = d\tilde{F}_5 = H_3 \wedge F_3$$

$$* \tilde{F}_5 = \tilde{F}_5$$

$$G_{\mathrm{E}\mu\nu} = e^{-\Phi/2} G_{\mu\nu}, \tau = C_0 + ie^{-\Phi}$$
$$\mathcal{M}_{ij} = \frac{1}{\mathrm{Im}\tau} \begin{bmatrix} |\tau|^2 & -\mathrm{Re}\tau \\ -\mathrm{Re}\tau & 1 \end{bmatrix}, F_3^i = \begin{bmatrix} H_3 \\ F_3 \end{bmatrix}.$$

$$\boldsymbol{S}_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10} x (-G_{\mathrm{E}})^{1/2} \left(R_{\mathrm{E}} - \frac{\partial_\mu \bar{\tau} \partial^\mu \tau}{2(\mathrm{Im}\tau)^2} - \frac{\mathcal{M}_{ij}}{2} F_3^i \cdot F_3^j - \frac{1}{4} \left| \tilde{F}_5 \right|^2 \right) - \frac{\epsilon_{ij}}{8\kappa_{10}^2} \int C_4 \wedge F_3^i \wedge F_3^j$$

$$\tau' = \frac{a\tau + b}{c\tau + d}$$
$$F_3^{i\prime} = \Lambda_j^i F_3^j, \Lambda_j^i = \begin{bmatrix} d & c \\ b & a \end{bmatrix},$$
$$\tilde{F}_5' = \tilde{F}_5, G_{\mathrm{E}\mu\nu}' = G_{\mathrm{E}\mu\nu},$$

$$\mathcal{M}' = (\Lambda^{-1})^T \mathcal{M} \Lambda^{-1}$$

$$\boldsymbol{S}_{\mathrm{I}} = \boldsymbol{S}_{\mathrm{c}} + \boldsymbol{S}_{\mathrm{o}}$$
$$\boldsymbol{S}_{\mathrm{c}} = \frac{1}{2\kappa_{10}^2} \int d^{10} x (-G)^{1/2} \left[e^{-2\Phi} \left(R + 4\partial_\mu \Phi \partial^\mu \Phi \right) - \frac{1}{2} \left| \tilde{F}_3 \right|^2 \right]$$
$$\boldsymbol{S}_{\mathrm{o}} = -\frac{1}{2g_{10}^2} \int d^{10} x (-G)^{1/2} e^{-\Phi} \mathrm{Tr}_{\mathrm{v}} (|F_2|^2)$$

$$\tilde{F}_3 = dC_2 - \frac{\kappa_{10}^2}{g_{10}^2} \omega_3,$$

$$\omega_3 = \mathrm{Tr}_{\mathrm{v}} \left(A_1 \wedge dA_1 - \frac{2i}{3} A_1 \wedge A_1 \wedge A_1 \right).$$

$$\delta\omega_3 = d\mathrm{Tr}_{\mathrm{v}} (\lambda dA_1).$$

$$\delta C_2 = \frac{\kappa_{10}^2}{g_{10}^2} \mathrm{Tr}_{\mathrm{v}} (\lambda dA_1)$$

$$S_{\mathrm{het}} = \frac{1}{2\kappa_{10}^2} \int d^{10} x (-G)^{1/2} e^{-2\Phi} \left[R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} \left| \tilde{H}_3 \right|^2 - \frac{\kappa_{10}^2}{g_{10}^2} \mathrm{Tr}_{\mathrm{v}} (|F_2|^2) \right]$$

$$\tilde{H}_3 = dB_2 - \frac{\kappa_{10}^2}{g_{10}^2} \omega_3, \delta B_2 = \frac{\kappa_{10}^2}{g_{10}^2} \mathrm{Tr}_{\mathrm{v}} (\lambda dA_1)$$

$$G_{\mathrm{I}\mu\nu} = e^{-\Phi_{\mathrm{h}}} G_{\mathrm{h}\mu\nu}, \Phi_{\mathrm{I}} = -\Phi_{\mathrm{h}}$$
$$\tilde{F}_{\mathrm{I}3} = \tilde{H}_{\mathrm{h}3}, A_{\mathrm{I}1} = A_{\mathrm{h}1}$$

$$S_{\text{int}} = \int \ d^2z(j_z^a A_{\bar{z}}^a + j_{\bar{z}}^a A_z^a)$$

$$Z[A] = \frac{1}{2}\int \ d^2z_1 d^2z_2 \left[\frac{\hat{k}_L}{z_{12}^2} A_{\bar{z}}^a(z_1,\bar{z}_1) A_{\bar{z}}^a(z_2,\bar{z}_2) + \frac{\hat{k}_R}{\bar{z}_{12}^2} A_z^a(z_1,\bar{z}_1) A_z^a(z_2,\bar{z}_2)\right]$$

$$\delta Z[A] = 2\pi \int \ d^2z \lambda^a(z,\bar{z}) [\hat{k}_L \partial_z A_{\bar{z}}^a(z,\bar{z}) + \hat{k}_R \partial_{\bar{z}} A_z^a(z,\bar{z})]$$

$$\delta Z[A] = -2\pi\hat{k}\delta \int \ d^2z A_z^a(z,\bar{z}) A_{\bar{z}}^a(z,\bar{z})$$

$$Z'[A] = Z[A] + 2\pi\hat{k} \int \ d^2z A_z^a(z,\bar{z}) A_{\bar{z}}^a(z,\bar{z})$$
$$= \frac{\hat{k}}{2}\int \ d^2z_1 d^2z_2 \ln |z_{12}^2| F_{z\bar{z}}^a(z_1,\bar{z}_1) F_{z\bar{z}}^a(z_2,\bar{z}_2)$$

$$\text{gauge anomaly:} \ \sum_L q^2 - \sum_R q^2 = 0,$$
$$\text{gravitational anomaly:} \ \sum_L 1 - \sum_R 1 = 0,$$
$$\text{mixed anomaly:} \ \sum_L q - \sum_R q = 0.$$

$$\text{gauge anomaly:} \ \sum_L q^3 = 0$$
$$\text{mixed anomaly:} \ \sum_L q = 0$$

$$e_\mu{}^p(x)' = e_\mu{}^q(x)\Theta_q{}^p(x)$$

$$\hat{I}_{d+2} = d\hat{I}_{d+1}, \delta\hat{I}_{d+1} = d\hat{I}_d$$

$$\delta \ln Z = \frac{-i}{(2\pi)^5}\int \ \hat{I}_d(F_2,R_2)$$

$$\hat{I}_{\mathbf{8}}(F_2,R_2) = -\frac{\mathrm{Tr}(F_2^6)}{1440} + \frac{\mathrm{Tr}(F_2^4)\mathrm{tr}(R_2^2)}{2304} - \frac{\mathrm{Tr}(F_2^2)\mathrm{tr}(R_2^4)}{23040} - \frac{\mathrm{Tr}(F_2^2)[\mathrm{tr}(R_2^2)]^2}{18432} + \frac{n\mathrm{tr}(R_2^6)}{725760}$$
$$+ \frac{n\mathrm{tr}(R_2^4)\mathrm{tr}(R_2^2)}{552960} + \frac{n[\mathrm{tr}(R_2^2)]^3}{1327104}$$

$$\hat{I}_{56}(F_2,R_2) = -495\frac{\mathrm{tr}(R_2^6)}{725760} + 225\frac{\mathrm{tr}(R_2^4)\mathrm{tr}(R_2^2)}{552960} - 63\frac{[\mathrm{tr}(R_2^2)]^3}{1327104}.$$

$$\hat{I}_{\mathrm{SD}}(R_2) = 992\frac{\mathrm{tr}(R_2^6)}{725760} - 448\frac{\mathrm{tr}(R_2^4)\mathrm{tr}(R_2^2)}{552960} + 128\frac{[\mathrm{tr}(R_2^2)]^3}{1327104}.$$

$$\hat{I}_{\mathrm{IIB}}(R_2) = -2\hat{I}_{\mathbf{8}}(R_2) + 2\hat{I}_{\mathbf{56}}(R_2) + \hat{I}_{\mathrm{SD}}(R_2) = 0.$$

$$\boldsymbol{S}' = \int B_2 \mathrm{Tr}(F_2^4)$$

$$\delta \boldsymbol{S}' \propto \int \mathrm{Tr}(\lambda d A_1)\mathrm{Tr}(F_2^4)$$

$$\hat{I}_d \propto \mathrm{Tr}(\lambda d A_1)\mathrm{Tr}(F_2^4), \hat{I}_{d+1} \propto \mathrm{Tr}(A_1 F_2)\mathrm{Tr}(F_2^4),$$
$$\hat{I}_{d+2} \propto \mathrm{Tr}(F_2^2)\mathrm{Tr}(F_2^4).$$

$$\boldsymbol{S}'' = \int B_2[\mathrm{Tr}(F_2^2)]^2$$

$$\mathrm{Tr_a}(t^2) = (n-2)\mathrm{Tr_v}(t^2)$$
$$\mathrm{Tr_a}(t^4) = (n-8)\mathrm{Tr_v}(t^4) + 3\mathrm{Tr_v}(t^2)\mathrm{Tr_v}(t^2)$$
$$\mathrm{Tr_a}(t^6) = (n-32)\mathrm{Tr_v}(t^6) + 15\mathrm{Tr_v}(t^2)\mathrm{Tr_v}(t^4)$$

$$\mathrm{Tr_a}(t^4) = \frac{1}{100}[\mathrm{Tr_a}(t^2)]^2, \mathrm{Tr_a}(t^6) = \frac{1}{7200}[\mathrm{Tr_a}(t^2)]^3$$

$$\int B_2 X_8(F_2, R_2)$$

$$\widetilde{H}_3 = dB_2 - c\omega_{3Y} - c'\omega_{3L}$$

$$\omega_{3L} = \omega_1 d\omega_1 + \frac{2}{3}\omega_1^3$$

$$\delta\omega_{3L} = d\mathrm{tr}(\Theta d\omega_1)$$

$$\delta A_1 = d\lambda$$
$$\delta\omega_1 = d\Theta$$
$$\delta B_2 = c\mathrm{Tr}(\lambda d A_1) + c'\mathrm{tr}(\Theta d\omega_1)$$

$$[c\mathrm{Tr}(F_2^2) + c'\mathrm{Tr}(R_2^2)]X_8(F_2, R_2).$$

$$\hat{I}_{\mathrm{I}} = \hat{I}_{\mathbf{56}}(R_2) - \hat{I}_{\mathbf{8}}(R_2) + \hat{I}_{\mathbf{8}}(F_2, R_2)$$
$$= \frac{1}{1440}\left\{-\mathrm{Tr_a}(F_2^6) + \frac{1}{48}\mathrm{Tr_a}(F_2^2)\mathrm{Tr_a}(F_2^4) - \frac{[\mathrm{Tr_a}(F_2^2)]^3}{14400}\right\} + (n$$
$$- 496)\left\{\frac{\mathrm{tr}(R_2^6)}{725760} + \frac{\mathrm{tr}(R_2^4)\mathrm{tr}(R_2^2)}{552960} + \frac{[\mathrm{tr}(R_2^2)]^3}{1327104}\right\} + \frac{Y_4 X_8}{768}$$

$$Y_4 = \mathrm{tr}(R_2^2) - \frac{1}{30}\mathrm{Tr_a}(F_2^2)$$
$$X_8 = \mathrm{tr}(R_2^4) + \frac{[\mathrm{tr}(R_2^2)]^2}{4} - \frac{\mathrm{Tr_a}(F_2^2)\mathrm{tr}(R_2^2)}{30} + \frac{\mathrm{Tr_a}(F_2^4)}{3} - \frac{[\mathrm{Tr_a}(F_2^2)]^2}{900}$$

$$\frac{Y_4 X_8}{768}.$$

$$\theta^2 = \bar{\theta}^2 = \{\theta, \bar{\theta}\} = 0.$$

$$\partial_z = \frac{\partial z'}{\partial z}\partial_{z'} + \frac{\partial \bar{z}'}{\partial z}\partial_{\bar{z}'}.$$

$$D_\theta = \partial_\theta + \theta\partial_z, D_{\bar{\theta}} = \partial_{\bar{\theta}} + \bar{\theta}\partial_{\bar{z}},$$

$$D_\theta^2 = \partial_z, D_{\bar{\theta}}^2 = \partial_{\bar{z}}, \{D_\theta, D_{\bar{\theta}}\} = 0.$$

$$D_\theta = D_\theta\theta'\partial_{\theta'} + D_\theta z'\partial_{z'} + D_\theta\bar{\theta}'\partial_{\bar{\theta}'} + D_\theta\bar{z}'\partial_{\bar{z}'},$$

$$D_\theta\bar{\theta}' = D_\theta\bar{z}' = 0, D_\theta z' = \theta' D_\theta\theta'$$

$$D_\theta = (D_\theta\theta')D_{\theta'}$$

$$\partial_{\bar{z}}z' = \partial_{\bar{\theta}}z' = \partial_{\bar{z}}\theta' = \partial_{\bar{\theta}}\theta' = 0$$

$$\begin{aligned} z'(z,\theta) &= f(z) + \theta g(z)h(z), \theta'(z,\theta) = g(z) + \theta h(z) \\ h(z) &= \pm[\partial_z f(z) + g(z)\partial_z g(z)]^{1/2} \end{aligned}$$

$$\delta z = \epsilon[v(z) - i\theta\eta(z)], \delta\theta = \epsilon\left[-i\eta(z) + \frac{1}{2}\theta\partial v(z)\right]$$

$$(D_\theta\theta')^{2h}(D_{\bar{\theta}}\bar{\theta}')^{2\tilde{h}}\boldsymbol{\phi}'(\mathbf{z}',\bar{\mathbf{z}}') = \boldsymbol{\phi}(\mathbf{z},\bar{\mathbf{z}}),$$

$$\delta\boldsymbol{\phi}(\mathbf{z},\bar{\mathbf{z}}) = -\epsilon\left[2h\theta\partial\eta(z) + \eta(z)Q_\theta + 2\tilde{h}\bar{\theta}\bar{\partial}\bar{\eta}(\bar{z}) + \bar{\eta}(\bar{z})Q_{\bar{\theta}}\right]\boldsymbol{\phi}(\mathbf{z},\bar{\mathbf{z}}),$$

$$\phi(z) = \mathcal{O}(z) + \theta\Psi(z)$$

$$\delta\mathcal{O} = -\epsilon\eta\Psi, \delta\Psi = -\epsilon[2h\partial\eta\mathcal{O} + \eta\partial\mathcal{O}].$$

$$\begin{aligned} G_{-1/2}\cdot\mathcal{O} &= \Psi, G_r\cdot\mathcal{O} = 0, r \geq \frac{1}{2}, \\ G_{-1/2}\cdot\Psi &= \partial\mathcal{O}, G_{1/2}\cdot\Psi = 2h\mathcal{O}, G_r\cdot\Psi = 0, r \geq \frac{3}{2}. \end{aligned}$$

$$G_{-1/2} * -iQ_\theta = -i(\partial_\theta - \theta\partial_z)$$

$$dz'd\theta' = dzd\theta D_\theta\theta'.$$

$$S = \frac{1}{4\pi}\int d^2zd^2\theta D_{\bar{\theta}}\boldsymbol{X}^\mu D_\theta\boldsymbol{X}_\mu.$$

$$\boldsymbol{X}^\mu(\mathbf{z},\bar{\mathbf{z}}) = X^\mu + i\theta\psi^\mu + i\bar{\theta}\tilde{\psi}^\mu + \theta\bar{\theta}F^\mu.$$

$$S = \frac{1}{4\pi}\int d^2z\left(\partial_{\bar{z}}X^\mu\partial_z X_\mu + \psi^\mu\partial_{\bar{z}}\psi_\mu + \tilde{\psi}^\mu\partial_z\tilde{\psi}_\mu + F^\mu F_\mu\right).$$

$$D_\theta D_{\bar{\theta}}\boldsymbol{X}^\mu(\mathbf{z},\bar{\mathbf{z}}) = 0.$$

$$\boldsymbol{X}^\mu(\mathbf{z}_1,\bar{z}_1)\boldsymbol{X}^v(\mathbf{z}_2,\bar{z}_2) \sim -\eta^{\mu v}\ln|z_1 - z_2 - \theta_1\theta_2|^2$$

$$S_{BC} = \frac{1}{2\pi}\int d^2zd^2\theta BD_{\bar{\theta}}C$$

$$D_{\bar{\theta}}B = D_{\bar{\theta}}C = 0$$

$$B(z) = \beta(z) + \theta b(z), C(z) = c(z) + \theta\gamma(z)$$

$$B(z_1)C(z_2) \sim \frac{\theta_1 - \theta_2}{z_1 - z_2 - \theta_1\theta_2} = \frac{\theta_1 - \theta_2}{z_1 - z_2}.$$

$$S = \frac{1}{4\pi}\int d^2zd^2\theta\left[G_{\mu\nu}(\boldsymbol{X}) + B_{\mu\nu}(\boldsymbol{X})\right]D_{\bar{\theta}}\boldsymbol{X}^{\nu}D_{\theta}\boldsymbol{X}^{\mu}$$

$$= \frac{1}{4\pi}\int d^2z\{\left[G_{\mu\nu}(X) + B_{\mu\nu}(X)\right]\partial_z X^{\mu}\partial_{\bar{z}}X^{\nu} + G_{\mu\nu}(X)\left(\psi^{\mu}\mathcal{D}_{\bar{z}}\psi^{\nu} + \tilde{\psi}^{\mu}\mathcal{D}_z\tilde{\psi}^{\nu}\right)$$

$$+ \frac{1}{2}R_{\mu\nu\rho\sigma}(X)\psi^{\mu}\psi^{\nu}\tilde{\psi}^{\rho}\tilde{\psi}^{\sigma}\Big\}$$

$$\mathcal{D}_{\bar{z}}\psi^{\nu} = \partial_{\bar{z}}\psi^{\nu} + \left[\Gamma^{\nu}{}_{\rho\sigma}(X) + \frac{1}{2}H^{\nu}{}_{\rho\sigma}(X)\right]\partial_{\bar{z}}X^{\rho}\psi^{\sigma},$$
$$\mathcal{D}_z\tilde{\psi}^{\nu} = \partial_z\tilde{\psi}^{\nu} + \left[\Gamma^{\nu}{}_{\rho\sigma}(X) - \frac{1}{2}H^{\nu}{}_{\rho\sigma}(X)\right]\partial_z X^{\rho}\tilde{\psi}^{\sigma}.$$

$$X^{\mu} = X^{\mu} + i\bar{\theta}\tilde{\psi}^{\mu}$$
$$\lambda^A = \lambda^A + \bar{\theta}G^A$$

$$S = \frac{1}{4\pi}\int d^2zd\bar{\theta}\{\left[G_{\mu\nu}(\boldsymbol{X}) + B_{\mu\nu}(\boldsymbol{X})\right]\partial_z\boldsymbol{X}^{\mu}D_{\bar{\theta}}\boldsymbol{X}^{\nu} - \lambda^A\mathcal{D}_{\bar{\theta}}\lambda^A\}$$

$$= \frac{1}{4\pi}\int d^2z\{\left[G_{\mu\nu}(X) + B_{\mu\nu}(X)\right]\partial_z X^{\mu}\partial_{\bar{z}}X^{\nu} + G_{\mu\nu}(X)\tilde{\psi}^{\mu}\mathcal{D}_z\tilde{\psi}^{\nu} + \lambda^A\mathcal{D}_{\bar{z}}\lambda^A$$

$$+ \frac{i}{2}F_{\rho\sigma}^{AB}(X)\lambda^A\lambda^B\tilde{\psi}^{\rho}\tilde{\psi}^{\sigma}\Big\}$$

$$\mathcal{D}_{\bar{\theta}}\lambda^A = D_{\bar{\theta}}\lambda^A - iA_{\mu}^{AB}(\boldsymbol{X})D_{\bar{\theta}}\boldsymbol{X}^{\mu}\lambda^B$$
$$\mathcal{D}_{\bar{z}}\lambda^A = \partial_{\bar{z}}\lambda^A - iA_{\mu}^{AB}(X)\partial_{\bar{z}}X^{\mu}\lambda^B$$

$$\delta A_{\mu}^{AB} = D_{\mu}\chi^{AB}, \delta\lambda^A = i\chi^{AB}\lambda^B$$

$$A_{\bar{z}}^{AB}(z,\bar{z}) = \frac{1}{2\pi}A_{\mu}^{AB}(X)\partial_{\bar{z}}X^{\mu}$$

$$\delta Z[A] = \frac{1}{8\pi}\int d^2z\mathrm{Tr}_v\left[\chi(X)F_{\mu\nu}(X)\right]\partial_z X^{\mu}\partial_{\bar{z}}X^{\nu}$$

$$\delta B_{\mu\nu} = \frac{1}{2}\mathrm{Tr}_v\left(\chi F_{\mu\nu}\right)$$

$$\frac{\kappa_{10}^2}{g_{10}^2} = \frac{1}{2} \to \frac{\alpha'}{4}$$

$$\frac{\kappa^2}{g_{\mathrm{YM}}^2} \equiv \frac{e^{2\Phi}\kappa_{10}^2}{e^{2\Phi}g_{10}^2} = \frac{\alpha'}{4}.$$

$$\delta(\gamma)\delta(\tilde{\gamma}) = e^{-\phi-\tilde{\phi}}$$

$$\mathcal{V}^{0,0} = G_{-1/2}\tilde{G}_{-1/2} \cdot \mathcal{O}$$

$$\mathcal{V}^{-1,-1} = e^{-\phi-\tilde{\phi}}\mathbb{O}$$

$$\psi^{\mu}_{-1/2}\tilde{\psi}^{v}_{-1/2}|0;k\rangle_{\mathrm{NS}}$$

$$\mathcal{V}^{-1,-1} = g_c e^{-\phi-\tilde{\phi}}\psi^{\mu}\tilde{\psi}^{v}e^{ik\cdot X}$$

$$G_{-1/2}\tilde{G}_{-1/2}\psi^{\mu}_{-1/2}\tilde{\psi}^{v}_{-1/2}|0;k\rangle_{\mathrm{NS}} = -\left(\alpha^{\mu}_{-1} + \alpha_0 \cdot \psi_{-1/2}\psi^{\mu}_{-1/2}\right)\left(\tilde{\alpha}^{v}_{-1} + \tilde{\alpha}_0 \cdot \tilde{\psi}_{-1/2}\tilde{\psi}^{v}_{-1/2}\right)|0;k\rangle_{\mathrm{NS}}$$

$$\mathcal{V}^{0,0} = -\frac{2g_c}{\alpha'}\left(i\partial_z X^{\mu} + \frac{1}{2}\alpha' k \cdot \psi\psi^{\mu}\right)\left(i\partial_{\bar{z}} X^{v} + \frac{1}{2}\alpha' k \cdot \tilde{\psi}\tilde{\psi}^{v}\right)e^{ik\cdot X},$$

$$\mathcal{V}^{-1} = g_o e^{-\phi} t^{a}\psi^{\mu}e^{ik\cdot X}$$
$$\mathcal{V}^{0} = g_o(2\alpha')^{-1/2}t^{a}(i\dot{X}^{\mu} + 2\alpha' k \cdot \psi\psi^{\mu})e^{ik\cdot X}$$

$$\mathcal{V}^{-1} = g_c\hat{k}^{-1/2}e^{-\tilde{\phi}}j^{a}\tilde{\psi}^{\mu}e^{ik\cdot X}$$
$$\mathcal{V}^{0} = g_c(2/\alpha')^{1/2}\hat{k}^{-1/2}j^{a}\left(i\bar{\partial}X^{\mu} + \frac{1}{2}\alpha' k \cdot \tilde{\psi}\tilde{\psi}^{\mu}\right)e^{ik\cdot X}$$

type I: $g_o = g_{\mathrm{YM}}(2\alpha')^{1/2};\ g_{\mathrm{YM}} \equiv g_{10}e^{\Phi/2}$

heterotic: $g_c = \dfrac{\kappa}{2\pi} = \dfrac{\alpha'^{1/2}g_{\mathrm{YM}}}{4\pi};\ \kappa \equiv \kappa_{10}e^{\Phi}, g_{\mathrm{YM}} \equiv g_{10}e^{\Phi}$

type I /II: $g_c = \dfrac{\kappa}{2\pi};\ \kappa \equiv \kappa_{10}e^{\Phi}$

$$\frac{1}{\alpha' g_0^2}\left\langle c\mathcal{V}_1^{-1}(x_1)c\mathcal{V}_2^{-1}(x_2)c\mathcal{V}_3^{0}(x_3)\right\rangle + (\mathcal{V}_1 \leftrightarrow \mathcal{V}_2),$$

$$\langle c(x_1)c(x_2)c(x_3)\rangle = x_{12}x_{13}x_{23}$$
$$\left\langle e^{-\phi}(x_1)e^{-\phi}(x_2)\right\rangle = x_{12}^{-1}$$
$$\langle\psi^{\mu}(x_1)\psi^{v}(x_2)\rangle = \eta^{\mu v}x_{12}^{-1}$$

$$\left\langle\psi^{\mu}e^{ik_1\cdot X(x_1)}\psi^{v}e^{ik_2\cdot X(x_2)}(i\dot{X}^{\rho}+2\alpha' k_3\cdot\psi\psi^{\rho})e^{ik_3\cdot X(x_3)}\right\rangle$$

$$= 2i\alpha'(2\pi)^{10}\delta^{10}\left(\sum_i k_i\right)\left(-\frac{\eta^{\mu v}k_1^{\rho}}{x_{12}x_{13}} - \frac{\eta^{\mu v}k_2^{\rho}}{x_{12}x_{23}} + \frac{\eta^{\mu\rho}k_3^{v} - \eta^{v\rho}k_3^{\mu}}{x_{13}x_{23}}\right)$$

$$ig_{\mathrm{YM}}(2\pi)^{10}\delta^{10}\left(\sum_i k_i\right)e_{1\mu}e_{2v}e_{3\rho}V^{\mu v\rho}\mathrm{Tr_v}([t^{a_1},t^{a_2}]t^{a_3})$$

$$V^{\mu v\rho} = \eta^{\mu v}k_{12}^{\rho} + \eta^{v\rho}k_{23}^{\mu} + \eta^{\rho\mu}k_{31}^{v},$$

$$\langle e^{-\phi/2}(x_1)e^{-\phi/2}(x_2)e^{-\phi}(x_3)\rangle = x_{12}^{-1/4}x_{13}^{-1/2}x_{23}^{-1/2},$$
$$\langle \Theta_\alpha(x_1)\Theta_\beta(x_2)\rangle = x_{12}^{-5/4}C_{\alpha\beta},$$
$$\langle \Theta_\alpha(x_1)\Theta_\beta(x_2)\psi^\mu(x_3)\rangle = 2^{-1/2}(C\Gamma^\mu)_{\alpha\beta}x_{12}^{-3/4}x_{13}^{-1/2}x_{23}^{-1/2}.$$

$$\psi^\mu(x)\Theta_\alpha(0) = (2x)^{-1/2}\Theta_\beta(0)\Gamma^\mu_{\beta\alpha} + O\left(x^{1/2}\right)$$

$$ig_{\mathrm{YM}}(2\pi)^{10}\delta^{10}\left(\sum_i k_i\right)e_\mu\bar{u}_1\Gamma^\mu u_2\mathrm{Tr}_\mathrm{v}([t^{a_1},t^{a_2}]t^{a_3}).$$

$$\langle j^a(z_1)j^b(z_2)\rangle = \frac{\hat{k}\delta^{ab}}{z_{12}^2}$$
$$\langle j^a(z_1)j^b(z_2)j^c(z_3)\rangle = \frac{i\hat{k}f^{abc}}{z_{12}z_{13}z_{23}}$$

$$i\hat{k}^{-1/2}f^{abc} = 2^{1/2}\mathrm{Tr}_\mathrm{v}\left([t^a,t^b]t^c\right)$$

$$\left\langle \prod_{i=1}^{3} ie_i\cdot\partial X e^{ik_i\cdot X}(z_i,\bar{z}_i)\right\rangle = \frac{\alpha'^2 e_{1\mu}e_{2\nu}e_{3\rho}T^{\mu\nu\rho}}{8iz_{12}z_{13}z_{23}}$$

$$T^{\mu\nu\rho} = k_{23}^\mu\eta^{\nu\rho} + k_{31}^\nu\eta^{\rho\mu} + k_{12}^\rho\eta^{\mu\nu} + \frac{\alpha'}{8}k_{23}^\mu k_{31}^\nu k_{12}^\rho$$

$$4\pi g_\mathrm{c}\alpha'^{-1/2}(2\pi)^{10}\delta^{10}\left(\sum_i k_i\right)e_{1\mu}e_{2\nu}e_{3\rho}V^{\mu\nu\rho}\mathrm{Tr}_\mathrm{v}\left([t^a,t^b]t^c\right)$$

$$\pi ig_\mathrm{c}(2\pi)^{10}\delta^{10}\left(\sum_i k_i\right)e_{1\mu\sigma}e_{2\nu\omega}e_{3\rho\lambda}T^{\mu\nu\rho}V^{\sigma\omega\lambda}.$$

$$\pi ig_\mathrm{c}(2\pi)^{10}\delta^{10}\left(\sum_i k_i\right)e_{1\mu\nu}e_{2\rho}e_{3\sigma}k_{23}^\nu V^{\mu\rho\sigma}\delta^{ab}$$

$$\pi ig_\mathrm{c}(2\pi)^{10}\delta^{10}\left(\sum_i k_i\right)e_{1\mu\sigma}e_{2\nu\omega}e_{3\rho\lambda}V^{\mu\nu\rho}V^{\sigma\omega\lambda}.$$

$$(-G_\mathrm{h})^{1/2}e^{-2\Phi_\mathrm{h}}R_\mathrm{h}^2 \to (-G_\mathrm{I})^{1/2}e^{-\Phi_\mathrm{I}}R_\mathrm{I}^2$$

$$\mathcal{V}_\alpha(z)\mathcal{V}_\beta(0) \sim \frac{(C\Gamma^\mu)_{\alpha\beta}}{2^{1/2}z}e^{-\phi}\psi_\mu$$

$$\langle \mathcal{V}_\alpha(z_1)\mathcal{V}_\beta(z_2)\mathcal{V}_\gamma(z_3)\mathcal{V}_\delta(z_4)\rangle = \frac{(C\Gamma^\mu)_{\alpha\beta}\left(C\Gamma_\mu\right)_{\gamma\delta}}{2z_{12}z_{23}z_{24}z_{34}} + \frac{(C\Gamma^\mu)_{\alpha\gamma}\left(C\Gamma_\mu\right)_{\delta\beta}}{2z_{13}z_{34}z_{32}z_{42}} + \frac{(C\Gamma^\mu)_{\alpha\delta}\left(C\Gamma_\mu\right)_{\beta\gamma}}{2z_{14}z_{42}z_{43}z_{23}}$$

$$\Gamma^{\mu}_{\alpha\beta}\Gamma_{\mu\gamma\delta} + \Gamma^{\mu}_{\alpha\gamma}\Gamma_{\mu\delta\beta} + \Gamma^{\mu}_{\alpha\delta}\Gamma_{\mu\beta\gamma} = 0$$

$$\frac{i}{2} g_{\mathrm{o}}^2 (2\pi)^{10} \delta^{10}\left(\sum_i k_i\right) \mathrm{Tr}_{\mathrm{v}}(t^{a_1} t^{a_2} t^{a_3} t^{a_4}) \int_0^1 dx x^{-\alpha' s - 1} (1 - x)^{-\alpha' u - 1} \times (\bar{u}_1 \Gamma^{\mu} u_2 \bar{u}_3 \Gamma^{\mu} u_4 + x \bar{u}_1 \Gamma^{\mu} u_3 \bar{u}_2 \Gamma^{\mu} u_4)$$

$$-16 i g_{\mathrm{YM}}^2 \alpha'^2 (2\pi)^{10} \delta^{10}\left(\sum_i k_i\right) K(u_1, u_2, u_3, u_4) \times \left[\mathrm{Tr}_{\mathrm{v}}(t^{a_1} t^{a_2} t^{a_3} t^{a_4}) \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' u)}{\Gamma(1 - \alpha' s - \alpha' u)} + 2 \text{ permutations}\right]$$

$$K(u_1, u_2, u_3, u_4) = \frac{1}{8}\left(u \bar{u}_1 \Gamma^{\mu} u_2 \bar{u}_3 \Gamma_{\mu} u_4 - s \bar{u}_1 \Gamma^{\mu} u_4 \bar{u}_3 \Gamma_{\mu} u_2\right)$$

$$s = -(k_1 + k_2)^2, t = -(k_1 + k_3)^2, u = -(k_1 + k_4)^2.$$

$$K(e_1, e_2, e_3, e_4) = \frac{1}{8}\left(4 M^1_{\mu\nu} M^2_{\nu\sigma} M^3_{\sigma\rho} M^4_{\rho\mu} - M^1_{\mu\nu} M^2_{\nu\mu} M^3_{\sigma\rho} M^4_{\rho\sigma}\right) + 2 \text{ permutations}$$

$$\equiv t^{\mu\nu\sigma\rho\alpha\beta\gamma\delta} k_{1\mu} e_{1\nu} k_{2\sigma} e_{2\rho} k_{3\alpha} e_{3\beta} k_{4\gamma} e_{4\delta}$$

$$K(e_1, e_2, e_3, e_4) = -\frac{1}{4}(s t e_1 \cdot e_4 e_2 \cdot e_3 + 2 \text{ permutations }) + \frac{1}{2}(s e_1 \cdot k_4 e_3 \cdot k_2 e_2 \cdot e_4 + 11 \text{ permutations })$$

$$\frac{1}{\alpha'^2 s u} - \frac{\pi^2}{6} + O(\alpha')$$

$$\frac{\pi^2 \alpha'^2}{2 \times 4!\, g_{\mathrm{YM}}^2} t^{\mu\nu\sigma\rho\alpha\beta\gamma\delta} \mathrm{Tr}_{\mathrm{v}}\left(F_{\mu\nu} F_{\sigma\rho} F_{\alpha\beta} F_{\gamma\delta}\right)$$

$$A_{\mathrm{c}}(s, t, u, \alpha', g_{\mathrm{c}}) = -\frac{\pi i g_{\mathrm{c}}^2 \alpha'}{g_{\mathrm{o}}^4} A_{\mathrm{o}}\left(s, t, \frac{1}{4}\alpha', g_{\mathrm{o}}\right) A_{\mathrm{o}}\left(t, u, \frac{1}{4}\alpha', g_{\mathrm{o}}\right)^* \sin\frac{\pi \alpha' t}{4}$$

$$-\frac{i\kappa^2 \alpha'^3}{4} \frac{\Gamma\left(-\frac{1}{4}\alpha' s\right)\Gamma\left(-\frac{1}{4}\alpha' t\right)\Gamma\left(-\frac{1}{4}\alpha' u\right)}{\Gamma\left(1 + \frac{1}{4}\alpha' s\right)\Gamma\left(1 + \frac{1}{4}\alpha' t\right)\Gamma\left(1 + \frac{1}{4}\alpha' u\right)} K_{\mathrm{c}}(e_1, e_2, e_3, e_4)$$

$$K_{\mathrm{c}}(e_1, e_2, e_3, e_4) = t^{\mu_1 \nu_1 \dots \mu_4 \nu_4} t^{\rho_1 \sigma_1 \dots \rho_4 \sigma_4} \prod_{j=1}^{4} e_{j\mu_j \rho_j} k_{j\nu_j} k_{j\sigma_j}$$

$$-\frac{64}{\alpha'^3 s t u} - 2\zeta(3) + O(\alpha')$$

$$\zeta(k) = \sum_{m=1}^{\infty} \frac{1}{m^k}.$$

$$\frac{\zeta(3)\alpha'^3}{2^9 \times 4!\,\kappa^2} t^{\mu_1\nu_1\ldots\mu_4\nu_4} t^{\rho_1\sigma_1\ldots\rho_4\sigma_4} R_{\mu_1\nu_1\rho_1\sigma_1} R_{\mu_2\nu_2\rho_2\sigma_2} R_{\mu_3\nu_3\rho_3\sigma_3} R_{\mu_4\nu_4\rho_4\sigma_4}$$

$$\hat{k}^{-2}\langle j^{a_1}(z_1) j^{a_2}(z_2) j^{a_3}(z_3) j^{a_4}(z_4)\rangle = \frac{\delta^{a_1a_2}\delta^{a_3a_4}}{z_{12}^2 z_{34}^2} - \frac{f^{a_1a_2b} f^{ba_3a_4}}{\hat{k} z_{12} z_{23} z_{24} z_{34}} + (2 \leftrightarrow 3) + (2 \leftrightarrow 4)$$

$$-\hat{k}^{-1} f^{a_1a_2b} f^{ba_3a_4} = 2\mathrm{Tr_v}\left([t^{a_1}, t^{a_2}] t^b\right) \mathrm{Tr_v}\left(t^b [t^{a_3}, t^{a_4}]\right) = 2\mathrm{Tr_v}\left([t^{a_1}, t^{a_2}][t^{a_3}, t^{a_4}]\right)$$

$$\int d^2 z_4 \ldots d^2 z_n \left\langle \langle \mathcal{V}_1^{-1} \middle| \mathrm{T}[\mathcal{V}_3^0 \mathcal{V}_4^0 \ldots \mathcal{V}_n^0] \mid \mathcal{V}_2^{-1} \right\rangle_{\text{matter}}$$

$$\left|\mathcal{V}_2^{-1}\right\rangle = 2L_0^{\mathrm{m}} \left|\mathcal{V}_2^{-1}\right\rangle = \left\{G_{1/2}^{\mathrm{m}}, G_{-1/2}^{\mathrm{m}}\right\} \left|\mathcal{V}_2^{-1}\right\rangle = G_{1/2}^{\mathrm{m}} G_{-1/2}^{\mathrm{m}} \left|\mathcal{V}_2^{-1}\right\rangle,$$

$$\int d^2 z_4 \ldots d^2 z_n \left\langle \mathcal{V}_1^0 \middle| \mathrm{T}[\mathcal{V}_3^0 \mathcal{V}_4^0 \ldots \mathcal{V}_n^0] \middle| \mathcal{V}_2^0 \right\rangle_{\text{matter}}$$

$$X(z) \equiv Q_{\mathrm{B}} \cdot \xi(z) = T_F(z)\delta(\beta(z)) - \partial b(z)\delta'(\beta(z))$$

$$\delta(\beta) \cong e^{\phi}.$$

$$\gamma(z) f(\beta(0), \gamma(0)) \sim \frac{1}{z} \partial_\beta f(\beta(0), \gamma(0)),$$

$$\theta(\beta) \cong \xi$$

$$j_{\mathrm{B}}(z)\theta(\beta(0)) \sim -\frac{1}{z^2} b(0)\delta'(\beta(0)) + \frac{1}{z} T_F(0)\delta(\beta(0)).$$

$$\langle \xi(z) \rangle = 1$$

$$X(z_1)\xi(z_2) = Q_{\mathrm{B}} \cdot \xi(z_1)\xi(z_2) = \xi(z_1) Q_{\mathrm{B}} \cdot \xi(z_2) = \xi(z_1) X(z_2)$$

$$\lim_{z \to 0} X(z)\mathcal{V}^{-1}(0),$$

$$e^{\phi} T_F^{\mathrm{m}}(z) e^{-\phi} \mathcal{O}(0) = z T_F^{\mathrm{m}}(z) \mathcal{O}(0) + O(z^2).$$

$$\lim_{z \to 0} X(z)\mathcal{V}^{-1}(0) = \mathcal{V}^0(0).$$

$$n_X = 2g - 2 + n_B + \frac{n_F}{2},$$

$$\begin{aligned} z_m &= f_{mn}(z_n) + \theta_n g_{mn}(z_n) h_{mn}(z_n) \\ \theta_m &= g_{mn}(z_n) + \theta_n h_{mn}(z_n) \\ h_{mn}^2(z_n) &= \partial_z f_{mn}(z_n) + g_{mn}(z_n)\partial_z g_{mn}(z_n) \end{aligned}$$

$$z_m = f_{mn}(z_n)$$
$$\theta_m = \theta_n h_{mn}(z_n), h_{mn}^2(z_n) = \partial_z f_{mn}(z_n).$$

$$h_{mn} h_{np} h_{pm} = 1$$

$$u = 1/z, \phi = i\theta/z$$

$$f(z) = \frac{\alpha z + \beta}{\gamma z + \delta}, g(z) = \epsilon_1 + \epsilon_2 z$$

$$(z, \theta) \cong (z + 2\pi, \eta_1 \theta) \cong (z + 2\pi\tau, \eta_2 \theta).$$

$$(z, \theta) \cong (z + 2\pi, \theta) \cong (z + 2\pi\tau + \theta v, \theta + v)$$

$$(z, \theta) \to (z + \theta\epsilon, \theta + \epsilon).$$

$$n_v = 2g - 2 + n_B + \frac{n_F}{2}.$$

$$S(1; \dots; n) = \sum_{\chi,\gamma} \frac{e^{-\lambda\chi}}{n_R} \int_{\chi,\gamma} d^{n_e}t d^{n_o}v \left\langle \prod_{j=1}^{n_e} B_j \prod_{a=1}^{n_o} \delta(B_a) \prod_{i=1}^{n} \hat{\mathcal{V}}_i \right\rangle.$$

$$B_j = \sum_{(mn)} \int_{C_{mn}} \frac{dz_m d\theta_m}{2\pi i} B(z_m, \theta_m) \left[\frac{\partial z_m}{\partial t_j} - \frac{\partial \theta_m}{\partial t_j} \theta_m\right]_{z_n, \theta_n}$$

$$Q_B \cdot B(z) = T(z) = T_F(z) + \theta T_B(z)$$

$$f_{12}(z_2) = z_2, g_{12}(z_2) = v\alpha(z_2)$$

$$B[\alpha] = \oint \frac{dz_1}{2\pi i} \alpha(z_1) \beta(z_1, \theta)$$

$$vT[\alpha],$$

$$T[\alpha]\delta(B[\alpha]) = Q_B \cdot \theta(B[\alpha]).$$

$$\alpha(z_1) = \frac{1}{z_1 - z_0}$$

$$\int B_2 \mathrm{Tr}_v(F_2^4).$$

$$\left(\frac{2}{\alpha'}\right)^{5/2} g_c^5 \int_F \frac{d\tau d\bar{\tau}}{8\tau_2} \left[\prod_{i=1}^{5} \int d^2 w_i\right] \langle b(0)\tilde{b}(0)\tilde{c}(0)c(0)X(0)$$

$$\times \left[\prod_{i=1}^{4} \hat{k}^{-1/2} j^{a_i} \left(ie_i \cdot \bar{\partial}X + \frac{1}{2}\alpha' k_i \cdot \tilde{\psi} e_i\right.\right.$$

$$\left.\left.\cdot \tilde{\psi}\right) e^{ik_i \cdot X}(w_i, \bar{w}_i)\right] \times ie_{5\mu\nu} \partial X^\mu \delta(\tilde{\gamma}) \tilde{\psi}^\nu e^{ik_5 \cdot X}(w_5, \bar{w}_5)\rangle_{(P,P)}$$

$$\delta(\tilde{\beta}) i (2/\alpha')^{1/2} \tilde{\psi}^\rho \bar{\partial} X_\rho$$

$$\left\langle \prod_{i=1}^{10} \tilde{\psi}^{\mu_i} \right\rangle_{\tilde{\psi}(\mathrm{P,P})} = \epsilon^{\mu_1 \dots \mu_{10}} \bar{q}^{10/24} \prod_{n=1}^{\infty} (1-\bar{q}^n)^{10} = \epsilon^{\mu_1 \dots \mu_{10}} [\eta(\tau)^{10}]^*$$

$$\left\langle \partial X^\mu(w_5) \bar{\partial} X^\rho(0) \prod_{i=1}^{5} e^{ik_i \cdot X(w_i, \bar{w}_i)} \right\rangle_X$$

$$-i(2\pi)^{10} \delta^{10}\left(\sum_i k_i\right) \frac{\eta^{\mu\rho}\alpha'}{8\pi\tau_2(4\pi^2\alpha'\tau_2)^5 |\eta(\tau)|^{20}}.$$

$$\langle b(0)\tilde{b}(0)\tilde{c}(0)c(0)\rangle_{bc} = |\eta(\tau)|^4,$$

$$\langle \delta(\tilde{\beta}(0))\delta(\tilde{\gamma}(\bar{w}_5))\rangle_{\beta\gamma} = [\eta(\tau)^{-2}]^*.$$

$$\hat{k}^{-2} \langle j^{a_1}(w_1) j^{a_2}(w_2) j^{a_3}(w_3) j^{a_4}(w_4)\rangle_g$$

$$j^a(w) j^b(0) \to \mathrm{T}\left[\hat{Q}^a(w)\hat{Q}^b(0)\right] - \pi\delta^2(w,\bar{w})\delta^{ab}$$

$$\frac{\delta^{ab}}{2} \int_{|\sigma^2|<\delta} d^2w \frac{1}{w^2} (w\bar{w})^{k\cdot k'}$$

$$(-1 + k\cdot k')^{-1} \partial_w \left(w^{-1+k\cdot k'} \bar{w}^{k\cdot k'}\right)$$

$$\langle j^{a_1}(w_1) j^{a_2}(w_2)\rangle \to \mathrm{Tr}\left\{\exp(2\pi i\tau H) \mathrm{T}\left[\hat{Q}^{a_1}(w_1)\hat{Q}^{a_2}(w_2)\right]\right\} - \delta^{ab}\pi\delta^2(w_{12}, \bar{w}_{12})$$

$$\to \mathrm{Tr}\left\{\exp(2\pi i\tau H) \hat{Q}^{(a_1}\hat{Q}^{a_2)}\right\} - \frac{\delta^{ab}}{8\pi\tau_2}$$

$$f(q,z) \equiv \langle \exp(z\cdot \bar{J})\rangle = \exp\left(-\frac{z\cdot z}{16\pi\tau_2}\right) \mathrm{Tr}[\exp(2\pi i\tau H)\exp(z\cdot\hat{Q})]$$

$$f(q,z) = \eta(\tau)^{-16} \exp\left(-\frac{z\cdot z}{16\pi\tau_2}\right) \sum_{l\in\Gamma} q^{l^2/2} \exp\left(2^{-1/2} z\cdot l\right)$$

$$-\frac{i g_c^5}{\pi\alpha'^3} (2\pi)^{10}\delta^{10}\left(\sum_i k_i\right) \epsilon^{\mu_1\dots\mu_{10}} k_{1\mu_1} e_{1\mu_2} \dots k_{4\mu_7} e_{4\mu_8} e_{5\mu_9\mu_{10}} \times \int_F \frac{d^2\tau}{\tau_2^2} \frac{\partial^4 \hat{f}(q,z)}{\partial z^{a_1} \cdots \partial z^{a_4}}\bigg|_{z=0}$$

$$\hat{f}(q,z) = \eta(\tau)^{-8} f(q,z).$$

$$-\frac{1}{2^9\pi^6\alpha'} \int B_2 \int_F \frac{d^2\tau}{\tau_2^2} \hat{f}(q,F_2)$$

$$\frac{\hat{f}(q,F_2)}{\tau_2^2} = -\frac{32\pi i}{F_2\cdot F_2} \frac{\partial \hat{f}(q,F_2)}{\partial\bar{\tau}}$$

$$-\frac{1}{2^4\pi^5\alpha'}\int \frac{B_2}{F_2\cdot F_2}\hat{f}(q,F_2)\Big|_{q^0\text{ term}}$$

$$-\frac{1}{2^4\pi^5 6!\,\alpha'}\int \frac{B_2\mathrm{Tr_a}(F_2^6)}{F_2\cdot F_2} = -\frac{1}{2^4\pi^5 4!\,\alpha'}\int B_2\frac{\mathrm{Tr_a}(F_2^6)}{\mathrm{Tr_a}(F_2^2)}.$$

$$\frac{1}{7200}\{[\mathrm{Tr_{a1}}(F_2^2)]^2+[\mathrm{Tr_{a2}}(F_2^2)]^2-\mathrm{Tr_{a1}}(F_2^2)\mathrm{Tr_{a2}}(F_2^2)\}.$$

$$\frac{4g_c^4}{\alpha'^2}\int_F\frac{d\tau d\bar{\tau}}{8\tau_2}\left[\prod_{i=1}^4\int d^2w_i\right]\sum_{\gamma\neq(\mathrm{P,P})}\langle b(0)\tilde{b}(0)\tilde{c}(0)c(0)$$

$$\times\left[\prod_{i=1}^4 \hat{k}^{-1/2}j^{a_i}\left(ie_i\cdot\bar{\partial}X+\frac{1}{2}\alpha' k_i\cdot\tilde{\psi}e_i\cdot\tilde{\psi}\right)e^{ik_i\cdot X}(w_i,\bar{w}_i)\right]\Big\rangle_\gamma$$

$$\tilde{\Theta}_\alpha\to\begin{Bmatrix}\exp\left[\frac{1}{2}i(\tilde{H}_1+\tilde{H}_2+\tilde{H}_3+\tilde{H}_4)\right]=\exp\left(i\tilde{H}_1'\right)\\ \exp\left[\frac{1}{2}i(\tilde{H}_1+\tilde{H}_2-\tilde{H}_3-\tilde{H}_4)\right]=\exp\left(i\tilde{H}_2'\right)\\ \exp\left[\frac{1}{2}i(\tilde{H}_1-\tilde{H}_2+\tilde{H}_3-\tilde{H}_4)\right]=\exp\left(i\tilde{H}_3'\right)\\ \exp\left[\frac{1}{2}i(\tilde{H}_1-\tilde{H}_2-\tilde{H}_3+\tilde{H}_4)\right]=\exp\left(i\tilde{H}_4'\right)\end{Bmatrix}\to\tilde{\theta}_\alpha.$$

$$\tilde{H}_i'(\bar{z})\tilde{H}_j'(0)\sim-\delta_{ij}\ln\,\bar{z}.$$

$$\frac{1}{2}\sum_\gamma\langle\rangle_{\tilde{\psi},\gamma}=\langle\rangle_{\tilde{\theta}(\mathrm{P,P})}$$

$$k_ie_j\tilde{\psi}^{[i}\tilde{\psi}^{j]}$$

$$\tilde{\psi}^{[i}\tilde{\psi}^{j]}\to\frac{1}{4}\tilde{\theta}^T\Gamma^{ij}\tilde{\theta}$$

$$\left\langle\prod_{a=1}^4\frac{1}{4}\tilde{\theta}^T\Gamma^{i_aj_a}\tilde{\theta}\right\rangle_{\tilde{\theta}(\mathrm{P,P})}=\frac{1}{2^8}\epsilon^{\alpha_1\ldots\alpha_8}\Gamma^{i_1j_1}_{\alpha_1\alpha_2}\cdots\Gamma^{i_4j_4}_{\alpha_7\alpha_8}=t^{i_1j_1\ldots i_4j_4}+\epsilon^{i_1j_1\ldots i_4j_4}$$

$$\frac{1}{2^8\pi^5 4!\,\alpha'}t^{\mu\nu\sigma\rho\alpha\beta\gamma\delta}\mathrm{Tr_v}\left(F_{\mu\nu}F_{\sigma\rho}F_{\alpha\beta}F_{\gamma\delta}\right)$$

$$G_{\mu\nu}(x)=\eta_{\mu\nu},\Phi(x)=\Phi_0$$

$$\int d^{10}x(-G)^{1/2}V(\Phi)$$

$$X_R''(\bar{z})=-X_R^9(\bar{z})$$

$$\tilde{\psi}'^9(\bar{z})=-\tilde{\psi}^9(\bar{z}).$$

$$\mathcal{V}'_\alpha(z) = \mathcal{V}_\alpha(z), \tilde{\mathcal{V}}'_\alpha(\bar{z}) = \beta^9_{\alpha\beta}\tilde{\mathcal{V}}_\beta(\bar{z}),$$

$$\overline{\mathcal{V}}\Gamma^{\mu_1\dots\mu_p}\tilde{\mathcal{V}}.$$

$$\begin{aligned} C_9 &\to C, \\ C_\mu, C_{\mu\nu 9} &\to C_{\mu 9}, C_{\mu\nu}, \\ C_{\mu\nu\lambda} &\to C_{\mu\nu\lambda 9}, \end{aligned}$$

$$\prod_m \beta^m,$$

$$\beta^m\beta^n = \exp\,(\pi i\tilde{\mathbf{F}})\beta^n\beta^m$$

$$\psi^\mu_{-1/2}|k\rangle_{\mathrm{NS}}, \psi^9_{-1/2}|k\rangle_{\mathrm{NS}}, |\alpha; k\rangle_{\mathrm{R}}.$$

$$Q'_\alpha + \left(\beta^9\tilde{Q}'\right)_\alpha.$$

$$\frac{1}{2\pi\alpha'}\int_{\partial M} ds\partial_n X'^9$$

$$\int_{\partial M} ds\mathcal{V}'_\alpha = -\int_{\partial M} ds\left(\beta^9\tilde{\mathcal{V}}'\right)_\alpha$$

$$Q'_\alpha + \left(\beta^\perp\tilde{Q}'\right)_\alpha$$

$$\beta^\perp = \prod_m \beta^m$$

$$\int C_{p+1}$$

$$\begin{aligned} F_2 &= dC_1, d\wedge\!* dC_1 = 0 \\ *F_2 &= (*F)_8 = dC_7, d\wedge\!* dC_7 = 0. \end{aligned}$$

$$\begin{aligned} \left\{Q_\alpha, \bar{Q}_\beta\right\} &= -2\left[P_M + (2\pi\alpha')^{-1}Q^{\mathrm{NS}}_M\right]\Gamma^M_{\alpha\beta} \\ \left\{\tilde{Q}_\alpha, \bar{\tilde{Q}}_\beta\right\} &= -2\left[P_M - (2\pi\alpha')^{-1}Q^{\mathrm{NS}}_M\right]\Gamma^M_{\alpha\beta} \\ \left\{Q_\alpha, \bar{\tilde{Q}}_\beta\right\} &= -2\sum_p \frac{\tau_p}{p!}Q^{\mathrm{R}}_{M_1\dots M_p}\left(\beta^{M_1}\cdots\beta^{M_p}\right)_{\alpha\beta} \end{aligned}$$

$$\beta^{\mu_1}\cdots\beta^{\mu_p} = \beta^\perp\Gamma^0,$$

$$\mathcal{A}_{\mathrm{NS-NS}} \approx \frac{iV_{p+1}4\times 16}{8\pi(8\pi^2\alpha')^5}\int_0^\infty \frac{\pi dt}{t^2}(8\pi^2\alpha' t)^{(9-p)/2}\exp\left(-\frac{ty^2}{2\pi\alpha'}\right) = iV_{p+1}2\pi(4\pi^2\alpha')^{3-p}G_{9-p}(y)$$

$$G_d(y) = 2^{-2}\pi^{-d/2}\Gamma\left(\frac{1}{2}d - 1\right)y^{2-d}$$

$$\mathcal{A}_{\mathrm{R-R}} = -\mathcal{A}_{\mathrm{NS-NS}}$$

$$2i\kappa^2\tau_p^2 G_{9-p}(y)$$

$$\tau_p^2 = \frac{\pi}{\kappa^2}(4\pi^2\alpha')^{3-p}$$

$$-\frac{1}{4\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left|F_{p+2}\right|^2 + \mu_p \int C_{p+1}$$

$$-2\kappa_{10}^2 i\mu_p^2 G_{9-p}(y).$$

$$\mu_p^2 = \frac{\pi}{\kappa_{10}^2}(4\pi^2\alpha')^{3-p} = e^{2\Phi_0}\tau_p^2 = T_p^2$$

$$\int_{S_2} F_2 = \mu_{\mathrm{m}}$$

$$\exp\left(i\mu_{\mathrm{e}}\oint_P A_1\right) = \exp\left(i\mu_{\mathrm{e}}\int_D F_2\right)$$

$$\exp\left(i\mu_{\mathrm{e}}\int_{S_2} F_2\right) = \exp\left(i\mu_{\mathrm{e}}\mu_{\mathrm{m}}\right)$$

$$\mu_{\mathrm{e}}\mu_{\mathrm{m}} = 2\pi n$$

$$\int_{S_{p+2}} F_{p+2} = \mu_{6-p}/2\kappa_{10}^2$$

$$\mu_p\mu_{6-p} = \pi n/\kappa_{10}^2$$

$$\boldsymbol{S}_{\mathrm{D}p} = -\mu_p\int d^{p+1}\xi \mathrm{Tr}\{e^{-\Phi}[-\det(G_{ab}+B_{ab}+2\pi\alpha' F_{ab})]^{1/2}\},$$

$$O([X^m, X^n]^2)$$

$$\int C_2 = \int dx^0(dx^1 C_{01} + dx^2 C_{02}) = \int dx^0 dx^1 (C_{01} + \partial_1 X^2 C_{02})$$

$$\int dx^0 dx^1 dx^2 (C_{012} + 2\pi\alpha' F_{12} C_0)$$

$$i\mu_p\int_{p+1}\mathrm{Tr}\left[\exp(2\pi\alpha' F_2 + B_2)\wedge\sum_q C_q\right]$$

$$-i\int d^{p+1}\xi \mathrm{Tr}\left(\bar{\lambda}\Gamma^a D_a\lambda\right)$$

$$\frac{\tau_{\mathrm{F1}}}{\tau_{\mathrm{D1}}} = \frac{1}{2\pi\alpha'}\frac{\kappa}{4\pi^{5/2}\alpha'} = \frac{\kappa}{8\pi^{7/2}\alpha'^2}.$$

$$g = \frac{\tau_{\mathrm{F1}}}{\tau_{\mathrm{D1}}}$$

$$\kappa^2 = \frac{1}{2}(2\pi)^7 g^2 \alpha'^4$$

$$\tau_p = \frac{1}{g(2\pi)^p \alpha'^{(p+1)/2}} = (2\kappa^2)^{-1/2}(2\pi)^{(7-2p)/2}\alpha'^{(3-p)/2}.$$

$$\kappa_{10}^2 = \frac{1}{2}(2\pi)^7 \alpha'^4$$

$$g_{\mathrm{D}p}^2 = \frac{1}{(2\pi\alpha')^2\tau_p} = (2\pi)^{p-2} g\alpha^{(p-3)/2}$$

$$\frac{1}{4g_{\mathrm{D}p}^2}\mathrm{Tr}_{\mathrm{f}}$$

$$\tilde{t} = \begin{bmatrix} t & 0 \\ 0 & -t^T \end{bmatrix}$$

$$\frac{1}{4g_{\mathrm{D}p}^2}\mathrm{Tr}_{\mathrm{f}}(t^2) = \frac{1}{4g_{\mathrm{D}p,SO(2n)}^2}\mathrm{Tr}_{\mathrm{v}}(\tilde{t}^2)$$

$$\kappa_{10-k}^2 = (2\pi R)^{-k}\kappa^2(\text{ type I }), g_{10-k,\mathrm{YM}}^2 = (2\pi R)^{-k} g_{\mathrm{YM}}^2.$$

$$\kappa_{10-k}^2 = 2(2\pi R')^{-k}\kappa^2, g_{10-k,\mathrm{YM}}^2 = g_{\mathrm{D}(9-k),\mathrm{SO}(32)}^2$$

$$\frac{g_{\mathrm{YM}}^2}{\kappa} = 2(2\pi)^{7/2}\alpha'$$

$$\boldsymbol{S} = -\frac{1}{(2\pi\alpha')^2 g_{\mathrm{YM}}^2}\int d^{10}x\mathrm{Tr}\left\{\left[-\det\left(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)\right]^{1/2}\right\}$$

$$\frac{(2\pi\alpha')^2}{32 g_{\mathrm{YM}}^2}\mathrm{Tr}_{\mathrm{v}}\left(4F_{\mu\nu}F^{\nu\sigma}F_{\sigma\rho}F^{\rho\mu} - F_{\mu\nu}F^{\nu\mu}F_{\sigma\rho}F^{\rho\sigma}\right).$$

$$\det{}^{1/2}(1+M) = \exp\left[\frac{1}{2}\mathrm{tr}\left(M - \frac{1}{2}M^2 + \frac{1}{3}M^3 - \frac{1}{4}M^4 + \cdots\right)\right]$$

$$Q_\alpha + \left(\beta^\perp \tilde{Q}\right)_{\alpha}, \beta^\perp = \prod_{m\in S_{\mathrm{D}}} \beta^m$$

$$Q_\alpha + \left(\beta^{\perp\prime}\tilde{Q}\right)_\alpha = Q_\alpha + \left[\beta^\perp\left(\beta^{\perp-1}\beta^{\perp\prime}\right)\tilde{Q}\right]_{\alpha}, \beta^{\perp\prime} = \prod_{m\in S_{\mathrm{D}}'} \beta^m$$

$$\beta \equiv (\beta^\perp)^{-1}\beta^{\perp\prime} = \exp\left[\pi i\left(J_1 + \cdots + J_j\right)\right]$$

$$X^\mu(w,\bar{w}) = X^\mu(w) + \tilde{X}^\mu(\bar{w})$$

$$X^{\mu}(w) = x^{\mu} + \left(\frac{\alpha'}{2}\right)^{1/2}\left[-\alpha_0^{\mu} w + i \sum_{\substack{m\in\mathbf{Z}\\ m\neq 0}} \frac{\alpha_m^{\mu}}{m} \exp\,(imw)\right],$$

$$X^{\mu}(w) = i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{r\in\mathbf{Z}+1/2} \frac{\alpha_r^{\mu}}{r} \exp\,(irw).$$

$$\tilde{X}^{\mu}(\bar{w}) = X^{\mu}(2\pi - \bar{w})$$

$$\tilde{X}^{\mu}(\bar{w}) = -X^{\mu}(2\pi - \bar{w})$$

$$(8 - \#_{\mathrm{ND}})\left(-\frac{1}{24} - \frac{1}{48}\right) + \#_{\mathrm{ND}}\left(\frac{1}{48} + \frac{1}{24}\right) = -\frac{1}{2} + \frac{\#_{\mathrm{ND}}}{8}.$$

$$Q_{\alpha} + \left(\rho^{-1}\beta^{\perp}\rho\tilde{Q}\right)_{\alpha}$$

$$(\beta^{\perp})^{-1}\rho^{-1}\beta^{\perp}\rho = (\beta^{\perp})^{-1}\beta^{\perp}\rho^{2} = \rho^{2}$$

$$\exp\left(2i\sum_{a=1}^{4} s_a\phi_a\right)$$

$$\mathrm{diag}[\exp\,(i\phi_1), \exp\,(i\phi_2), \exp\,(i\phi_3), \exp\,(i\phi_4)].$$

$$\sigma^1 = 0{:}\ \partial_1 \mathrm{Re}(Z^a) = \mathrm{Im}(Z^a) = 0$$
$$\sigma^1 = \pi{:}\ \partial_1 \mathrm{Re}[\exp\,(-i\phi_a)Z^a] = \mathrm{Im}[\exp\,(-i\phi_a)Z^a] = 0$$

$$Z^a(w,\bar{w}) = \mathcal{Z}^a(w) + \overline{\mathcal{Z}^a}(-\bar{w}) = \exp\,(-2i\phi_a)\mathcal{Z}^a(w + 2\pi) + \overline{\mathcal{Z}^a}(-\bar{w})$$

$$\mathcal{Z}^a(w) = i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{r\in\mathbf{Z}+v_a} \frac{\alpha_r^a}{r} \exp\,(irw),$$

$$q^{E_0}\prod_{m=0}^{\infty}\left[1 - q^{m+(\phi/\pi)}\right]^{-1}\left[1 - q^{m+1-(\phi/\pi)}\right]^{-1} = -i\frac{\exp\,(\phi^2 t/\pi)\eta(it)}{\vartheta_{11}(i\phi t/\pi, it)}$$

$$E_0 = \frac{1}{24} - \frac{1}{2}\left(\frac{\phi}{\pi} - \frac{1}{2}\right)^2.$$

$$\vartheta_{11}(v,it) = -2q^{1/8}\sin\,\pi v\prod_{m=1}^{\infty}(1 - q^m)(1 - zq^m)(1 - z^{-1}q^m)$$
$$\vartheta_{11}(-iv/t, i/t) = -it^{1/2}\exp\,(\pi v^2/t)\vartheta_{11}(v,it)$$

$$Z^{\alpha}{}_{\beta}(\phi,it) = \frac{\vartheta_{\alpha\beta}(i\phi t/\pi, it)}{\exp\,(\phi^2 t/\pi)\eta(it)}$$

$$\frac{1}{2}\left[\prod_{a=1}^{4} Z_0^0(\phi_a,it) - \prod_{a=1}^{4} Z_1^0(\phi_a,it) - \prod_{a=1}^{4} Z_0^1(\phi_a,it) - \prod_{a=1}^{4} Z_1^1(\phi_a,it)\right]$$

$$\prod_{a=1}^{4} Z^1{}_1(\phi'_a, it)$$

$$\phi'_1 = \frac{1}{2}(\phi_1 + \phi_2 + \phi_3 + \phi_4), \quad \phi'_2 = \frac{1}{2}(\phi_1 + \phi_2 - \phi_3 - \phi_4)$$
$$\phi'_3 = \frac{1}{2}(\phi_1 - \phi_2 + \phi_3 - \phi_4), \quad \phi'_4 = \frac{1}{2}(\phi_1 - \phi_2 - \phi_3 + \phi_4)$$

$$V = -\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-1/2}\exp\left(-\frac{ty_1^2}{2\pi\alpha'}\right)\prod_{a=1}^{4}\frac{\vartheta_{11}(i\phi'_a t/\pi, it)}{\vartheta_{11}(i\phi_a t/\pi, it)}.$$

$$\vartheta_{11}(i\phi_a t/\pi, it)^{-1} \to iL\eta(it)^{-3}(8\pi^2\alpha' t)^{-1/2}$$

$$i\eta(it)^{-3}\exp\left[-\frac{t(y_8^2 + y_9^2)}{2\pi\alpha'}\right],$$

$$\prod_{a=1}^{4}\frac{\vartheta_{11}(i\phi'_a t/\pi, it)}{\vartheta_{11}(i\phi_a t/\pi, it)} \to \prod_{a=1}^{4}\frac{\sin\ \phi'_a}{\sin\ \phi_a}$$

$$m^2 = \frac{y_1^2}{4\pi^2\alpha'^2} - \frac{\phi_1}{2\pi\alpha'}, 0 \le \phi_1 \le \pi$$

$$X^3 = X'^0\tanh\ u,$$

$$\mathcal{A} = -iV_p\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-p/2}\exp\left(-\frac{ty^2}{2\pi\alpha'}\right)\frac{\vartheta_{11}(ut/2\pi, it)^4}{\eta(it)^9\vartheta_{11}(ut/\pi, it)},$$

$$\mathcal{A} = \frac{V_p}{(8\pi^2\alpha')^{p/2}}\int_0^\infty \frac{dt}{t}t^{(6-p)/2}\exp\left(-\frac{ty^2}{2\pi\alpha'}\right)\frac{\vartheta_{11}(iu/2\pi, i/t)^4}{\eta(i/t)^9\vartheta_{11}(iu/\pi, i/t)}.$$

$$\mathcal{A} = -i\int_{-\infty}^{\infty} d\tau V(r(\tau), v)$$

$$r(\tau)^2 = y^2 + v^2\tau^2, v = \tanh\ u,$$

$$V(r,v) = i\frac{2V_p}{(8\pi^2\alpha')^{(p+1)/2}}\int_0^\infty dt t^{(5-p)/2} \times \exp\left(-\frac{tr^2}{2\pi\alpha'}\right)\frac{(\tanh\ u)\vartheta_{11}(iu/2\pi, i/t)^4}{\eta(i/t)^9\vartheta_{11}(iu/\pi, i/t)}$$

$$V(r,v) = -v^4\frac{V_p}{(8\pi^2\alpha')^{(p+1)/2}}\int_0^\infty dt t^{(5-p)/2}\exp\left(-\frac{tr^2}{2\pi\alpha'}\right) + O(v^6)$$
$$= -\frac{v^4}{r^{7-p}}\frac{V_p}{\alpha'^{p-3}}2^{2-2p}\pi^{(5-3p)/2}\Gamma\left(\frac{7-p}{2}\right) + O(v^6)$$

$$V(r,v) \approx -2V_p\int_0^\infty \frac{dt}{(8\pi^2\alpha' t)^{(p+1)/2}}\exp\left(-\frac{tr^2}{2\pi\alpha'}\right)\frac{\tanh\ u\sin^4\ ut/2}{\sin\ ut}.$$

$$r \approx \alpha^{1/2}v^{1/2}$$

$$\delta x\delta t \gtrsim \alpha'$$

$$\delta x \gtrsim \frac{1}{m\delta v} = \frac{g\alpha'^{1/2}}{\delta v}$$

$$\delta x \gtrsim g^{1/3}\alpha^{1/2}$$

$$L = \mathrm{Tr}\left\{\frac{1}{2g\alpha'^{1/2}} D_0 X^i D_0 X^i + \frac{1}{4g\alpha'^{1/2}(2\pi\alpha')^2}\left[X^i, X^j\right]^2 - \frac{i}{2}\lambda D_0 \lambda + \frac{1}{4\pi\alpha'}\lambda\Gamma^0\Gamma^i\left[X^i, \lambda\right]\right\}.$$

$$H = \mathrm{Tr}\left\{\frac{g\alpha'^{1/2}}{2} p_i p_i - \frac{1}{16\pi^2 g\alpha'^{5/2}}\left[X^i, X^j\right]^2 - \frac{1}{4\pi\alpha'}\lambda\Gamma^0\Gamma^i\left[X^i, \lambda\right]\right\}$$

$$\left[p_{iab}, X^j_{cd}\right] = -i\delta^j_l \delta_{ad}\delta_{bc}$$

$$X^i = g^{1/3}\alpha^{1/2} Y^i$$

$$H = \frac{g^{1/3}}{\alpha'^{1/2}}\mathrm{Tr}\left\{\frac{1}{2} p_{Yi} p_{Yi} - \frac{1}{16\pi^2}\left[Y^i, Y^j\right]^2 - \frac{1}{4\pi}\lambda\Gamma^0\Gamma^i\left[Y^i, \lambda\right]\right\}$$

$$|s_3, s_4\rangle_{\mathrm{NS}}$$

$$-\exp\left[\pi i(s_3 + s_4)\right] = +1 \Rightarrow s_3 = s_4$$

$$|s_1, s_2\rangle_{\mathrm{R}}.$$

$$\boldsymbol{S} = -\frac{1}{4g_{\mathrm{D9}}^2}\int d^{10}x F_{MN}F^{MN} - \frac{1}{4g_{\mathrm{D5}}^2}\int d^6x F'_{MN}F'^{MN} - \int d^6x \left[D_\mu \chi^\dagger D^\mu \chi + \frac{g_{\mathrm{D5}}^2}{2}\sum_{A=1}^{3}\left(\chi_i^\dagger \sigma_{ij}^A \chi_j\right)^2\right]$$

$$A'_M \to A'_\mu, X'_m/2\pi\alpha'$$

$$A_i \to X_i/2\pi\alpha', A'_i \to X'_i/2\pi\alpha'.$$

$$\left(\frac{X_i - X'_i}{2\pi\alpha'}\right)^2 \chi^\dagger \chi$$

$$\frac{1}{2}\left\{\begin{bmatrix} Q_\alpha \\ \tilde{Q}_\alpha \end{bmatrix}, \left[Q_\beta^\dagger \tilde{Q}_\beta^\dagger\right]\right\} = M\delta_{\alpha\beta}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{L_1}{2\pi\alpha'}(\Gamma^0\Gamma^1)_{\alpha\beta}\begin{bmatrix} p & q/g \\ q/g & -p \end{bmatrix}$$

$$M \pm L_1 \frac{(p^2 + q^2/g^2)^{1/2}}{2\pi\alpha'}$$

$$\frac{M}{L_1} \geq \frac{(p^2 + q^2/g^2)^{1/2}}{2\pi\alpha'}$$

$$\tau_{(0,1)} + \tau_{(1,0)} = \frac{g^{-1} + 1}{2\pi\alpha'}$$

$$\tau_{(1,1)} = \frac{(g^{-2}+1)^{1/2}}{2\pi\alpha'}$$

left-moving: $\Gamma^0\Gamma^1 Q = Q$, right-moving: $\Gamma^0\Gamma^1\tilde{Q} = -\tilde{Q}$,

$$\boldsymbol{S}_1 = -T_1 \int d^2\xi e^{-\Phi}[-\det(G_{ab}+B_{ab}+2\pi\alpha' F_{ab})]^{1/2}$$

$$\tau_{(1,0)} + \tau_{(0,1)} - \tau_{(1,1)} = \frac{1 - O(g)}{2\pi\alpha'}$$

$$\oint_{\partial M} ds \delta X^{\mu} \left(\frac{1}{2\pi\alpha'}\partial_n X_{\mu} + iF_{\mu v}\partial_t X^{v}\right)$$

$$\partial_n X_{\mu} + 2\pi\alpha' i F_{\mu v}\partial_t X^{v} = 0$$

$$\tau_{(1,1)} + \tau_{(0,1)} = \frac{(g^{-2}+1)^{1/2} + g^{-1}}{2\pi\alpha'} = \frac{2g^{-1} + g/2 + O(g^3)}{2\pi\alpha'}$$

$$\tau_{(1,2)} = \frac{(4g^{-2}+1)^{1/2}}{2\pi\alpha'} = \frac{2g^{-1} + g/4 + O(g^3)}{2\pi\alpha'}.$$

$$\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}.$$

$$W(\Phi) = \mathrm{Tr}(\Phi_1[\Phi_2,\Phi_3]) + m\mathrm{Tr}(\Phi_i\Phi_i).$$

$$\frac{1}{2}\left\{\begin{bmatrix}Q_{\alpha}\\ \tilde{Q}_{\alpha}\end{bmatrix}, \begin{bmatrix}Q_{\beta}^{\dagger}\tilde{Q}_{\beta}^{\dagger}\end{bmatrix}\right\} = M\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\delta_{\alpha\beta} + \begin{bmatrix}0 & Z_{\alpha\gamma}\\ -Z_{\alpha\gamma}^{\dagger} & 0\end{bmatrix}\Gamma_{\gamma\beta}^{0}$$

$$Z = \tau_0 + \tau_p V_p \beta, \beta = \beta^1 \cdots \beta^p$$

$$M^2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \geq \begin{bmatrix}0 & Z\\ -Z^{\dagger} & 0\end{bmatrix}\Gamma^0\begin{bmatrix}0 & Z\\ -Z^{\dagger} & 0\end{bmatrix}\Gamma^0 = \begin{bmatrix}ZZ^{\dagger} & 0\\ 0 & Z^{\dagger}Z\end{bmatrix}$$

$$ZZ^{\dagger} = \tau_0^2 + \tau_0\tau_p V_p(\beta+\beta^{\dagger}) + \tau_p^2 V_p^2 \beta\beta^{\dagger}$$

$$M \geq \tau_0 + \tau_p V_p$$

$$M \geq \left(\tau_0^2 + \tau_p^2 V_p^2\right)^{1/2}$$

$$\tau_0 + \left(\tau_0 + \frac{p_9^2}{2\tau_0}\right)$$

$$2\tau_0 + \frac{p_9^2}{4\tau_0}$$

$$\int_{\mathrm{D}2} F_2 = 2\pi$$

$$i\mu_2 \int (C_3 + 2\pi\alpha' F_2 \wedge C_1)$$

$$\sum_{n=0}^{\infty} q^n D_n = 2^8 \prod_{k=1}^{\infty} \left(\frac{1+q^k}{1-q^k}\right)^8 .$$

$$\frac{g_{\mathrm{D0}}^2}{4} \sum_{A=1}^{3} \left(\chi_i^{\dagger} \sigma_{ij}^{A} \chi_j\right)^2 + \sum_{i=1}^{5} \frac{(X_i - X_i')^2}{(2\pi\alpha')^2} \chi^{\dagger}\chi$$

$$X_i - X_i' = 0, \chi \neq 0$$

$$X_i - X_i' \neq 0, \chi = 0$$

$$D^A \equiv \chi_i^{\dagger} \sigma_{ij}^{A} \chi_j = 0$$

$$\chi_{ia}^{\dagger} \sigma_{ij}^{A} \chi_{ja} = 0$$

$$\chi_{ia} = v\delta_{ia}$$

$$\chi_{ia} = vU_{ia}$$

$$* F_2 = \pm F_2,$$

$$\delta\lambda \propto F_{MN}\Gamma^{MN}\zeta.$$

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}', \mathbf{1}, \mathbf{2})$$

$$\frac{1}{2}(2\pi\alpha')^2 \mu_4 \int C_1 \wedge \mathrm{Tr}(F_2 \wedge F_2)$$

$$\int_{\mathrm{D4}} \mathrm{Tr}(F_2 \wedge F_2) = 8\pi^2$$

$$(\Gamma^0 \partial_0 + \Gamma^1 \partial_1) u = 0$$

$$\mathbf{16} \rightarrow \left(\frac{1}{2}, \mathbf{8}\right) + \left(-\frac{1}{2}, \mathbf{8}'\right)$$

$$\frac{\tau_{\mathrm{F1}}}{\tau_{\mathrm{D1}}} = g = e^{\Phi}$$

$$l_0 = (4\pi^3)^{-1/8} \kappa^{1/4}$$

$$\tau_{\mathrm{F1}}^{-1/2} : l_0 : \tau_{\mathrm{D1}}^{-1/2} = g^{-1/4} : 1 : g^{1/4}.$$

$$\begin{aligned} \Phi' &= -\Phi, G_{\mu\nu}' = e^{-\Phi} G_{\mu\nu} \\ B_2' &= C_2, C_2' = -B_2 \\ C_4' &= C_4 \end{aligned}$$

$$G_{\mathrm{E}\mu\nu} = e^{-\Phi/2} G_{\mu\nu} = e^{-\Phi'/2} G_{\mu\nu}'$$

$$\int_M B_2' = \int_M (B_2 d + C_2 c)$$

$$\tau_{(p,q)}^2 = l_0^{-4}(\mathcal{M}^{-1})^{ij} q_i q_j = l_0^{-4}[e^{\Phi}(p + C_0 q)^2 + e^{-\Phi} q^2]$$

$$C_0 \rightarrow C_0 + b$$

$$\int d^{10}x(-G)^{1/2} e^{-2\Phi}(R + 4\partial_\mu \Phi \partial^\mu \Phi) - \frac{1}{2}\int e^{2\alpha\Phi}|F_q|^2,$$

$$\int_{S_q} F_q = Q$$

$$d * (e^{2\alpha\Phi} F_q) = 0$$

$$\int_{S_{10-q}} * e^{2\alpha\Phi} F_q = Q'$$

$$\begin{aligned} G_{mn} &= e^{2\Phi}\delta_{mn}, G_{\mu v} = \eta_{\mu v} \\ H_{mnp} &= -\epsilon_{mnp}{}^{q}\partial_q \Phi \\ e^{2\Phi} &= e^{2\Phi(\infty)} + \frac{Q}{2\pi^2 r^2} \end{aligned}$$

$$\tau_{\mathrm{NS5}} = \frac{2\pi^2 \alpha'}{\kappa^2} = \frac{1}{(2\pi)^5 g^2 \alpha'^3}$$

$$e^{2\Phi} = e^{2\Phi(\infty)} + \frac{1}{2\pi^2}\sum_{i=1}^{N} \frac{Q_i}{(x - x_i)^2}$$

$$g_{\mathrm{D3}}^2 = 2\pi g$$

$$g_{\mathrm{D3}}^2 \rightarrow \frac{4\pi^2}{g_{\mathrm{D3}}^2}$$

$$\frac{1}{4\pi}\int C_0 \mathrm{Tr}(F_2 \wedge F_2)$$

$$-\frac{1}{2g_{\mathrm{D3}}^2}\int d^4x \mathrm{Tr}(|F_2|^2) + \frac{\theta}{8\pi^2}\int \mathrm{Tr}(F_2 \wedge F_2)$$

$$\mathbf{27} \rightarrow \mathbf{10} + \mathbf{16} + \mathbf{1}.$$

$$(\mathrm{D}_9,\ \mathrm{F}_9) \xrightarrow{T_{78}} (\mathrm{D}_{789},\ \mathrm{F}_9) \xrightarrow{S} (\mathrm{D}_{789}, \mathrm{D}_9) \xrightarrow{T_9} (\mathrm{D}_{78}, \mathrm{D}_{\emptyset}).$$

$$(\mathrm{D}_{6789}, \mathrm{D}_{\emptyset}) \xrightarrow{T_6} (\mathrm{D}_{789}, \mathrm{D}_6) \xrightarrow{S} (\mathrm{D}_{789},\ \mathrm{F}_6) \xrightarrow{T_{6789}} (\mathrm{D}_6, p_6) \xrightarrow{S} (\ \mathrm{F}_6, p_6).$$

$$(N, \tilde{N}) = \begin{cases} (nm, 0), nm > 0 \\ (0, -nm), nm < 0 \end{cases}$$

$$\mathrm{Tr} q^N = 2^8 \prod_{k=1}^{\infty} \left(\frac{1 + q^k}{1 - q^k}\right)^8$$

$$-\beta = -\exp\left[\pi i(s_1 + s_2 + s_3 + s_4)\right],$$

$$|s_0; i\rangle,$$

$$\exp(\pi i F) = -i\exp\left[\pi i(s_0 + \cdots + s_4)\right]$$

$$\begin{aligned} G_{\mathrm{I}\mu\nu} &= e^{-\Phi_{\mathrm{h}}} G_{\mathrm{h}\mu\nu}, \Phi_{\mathrm{I}} = -\Phi_{\mathrm{h}} \\ \tilde{F}_{\mathrm{I}3} &= \tilde{H}_{\mathrm{h}3}, A_{\mathrm{I}1} = A_{\mathrm{h}1} \end{aligned}$$

$$\Lambda_0^i = L^{-1/2} \int_0^L dx^1 \Lambda^i(x^1)$$

$$\left\{\Lambda_0^i, \Lambda_0^j\right\} = \delta^{ij}$$

$$\tau_{\mathrm{D}1}(\text{ type I}) = \frac{\pi^{1/2}}{2^{1/2}\kappa}(4\pi^2\alpha') = \frac{g_{\mathrm{YM}}^2}{8\pi\kappa^2}$$

$$\frac{\pi^2\alpha'^2}{2 \times 4!\, g_{\mathrm{YM}}^2}(tF^4) = \frac{g_{\mathrm{YM}}^2}{2^{10}\pi^5 4!\, \kappa^2}(tF^4),$$

$$\frac{1}{2^8\pi^5 4!\, \alpha'}(tF^4) = \frac{g_{\mathrm{YM}}^2}{2^{10}\pi^5 4!\, \kappa^2}(tF^4)$$

$$\psi_{-1/2}^{\mu}|0, k; ij\rangle\lambda_{ij}, \psi_{-1/2}^{m}|0, k; ij\rangle\lambda'_{ij},$$

$$M\lambda M^{-1} = -\lambda^T, M\lambda' M^{-1} = \lambda'^T,$$

$$\lambda = \sigma^a, \lambda' = I.$$

$$\hat{\Omega}|\psi; ij\rangle = \gamma_{jj'}|\Omega\psi; j'i'\rangle\gamma_{i'i}^{-1}.$$

$$V = e^{i(H_3 + H_4)/2},$$

$$(\psi^6 + i\psi^7)_{-1/2}(\psi^8 + i\psi^9)_{-1/2}|0\rangle.$$

$$\gamma_9^T\gamma_9^{-1} = \Omega_{5-9}^2\gamma_5^T\gamma_5^{-1}.$$

$$\left(\tau_p\right)^{1/(p+1)} \approx g^{-1/(p+1)}\alpha'^{-1/2}$$

$$\tau_0 = \frac{1}{g\alpha'^{1/2}}$$

$$n\tau_0 = \frac{n}{g\alpha'^{1/2}}$$

$$R_{10} = g\alpha'^{1/2}$$

$$\kappa_{11}^2 = 2\pi R_{10}\kappa^2 = \frac{1}{2}(2\pi)^8 g^3\alpha'^{9/2}$$

$$M_{11} = g^{-1/3}\alpha'^{-1/2}$$

$$g=(M_{11}R_{10})^{3/2}, \alpha'=M_{11}^{-3}R_{10}^{-1}$$

$$d=9: U=SL(2,\mathbf{Z})$$

$$d=8: U=SL(3,\mathbf{Z})\times SL(2,\mathbf{Z})$$

$$d=6: U=SO(5,5,\mathbf{Z})$$

$$\tau_{\mathrm{D2}}=\frac{1}{(2\pi)^2 g\alpha'^{3/2}}=\frac{M_{11}^3}{(2\pi)^2}$$

$$\tau_{\mathrm{F1}}=\frac{1}{2\pi\alpha'}=2\pi R_{10}\tau_{\mathrm{D2}}$$

$$S[F,\lambda,X]=-\tau_2\int d^3x\left\{\left[-\det\left(\eta_{\mu\nu}+\partial_\mu X^m\partial_\nu X^m+2\pi\alpha' F_{\mu\nu}\right)\right]^{1/2}+\frac{\epsilon^{\mu\nu\rho}}{2}\lambda\partial_\mu F_{\nu\rho}\right\}$$

$$S[\lambda,X]=-\tau_2\int d^3x\left\{-\det\left[\eta_{\mu\nu}+\partial_\mu X^m\partial_\nu X^m+(2\pi\alpha')^{-2}\partial_\mu\lambda\partial_\nu\lambda\right]\right\}^{1/2}$$

$$\delta\psi_M=D_M^-\zeta, \delta\tilde{\psi}_M=D_M^+\zeta.$$

$$SO(9,1)\to SO(5,1)\times SO(4)$$

$$\begin{aligned} 16 &\to (4,2)+(4',2') \\ 16' &\to (4,2')+(4',2) \end{aligned}$$

$$\tau_{\mathrm{NS5}}=\frac{1}{(2\pi)^5 g^2\alpha'^3}=\frac{\tau_{\mathrm{D2}}^2}{2\pi}=\frac{M_{11}^6}{(2\pi)^5}.$$

$$\tau_{\mathrm{D2}}=\tau_{\mathrm{M2}}, \tau_{\mathrm{NS5}}=\tau_{\mathrm{M5}}$$

$$\tau_{\mathrm{D4}}=2\pi R_{10}\tau_{\mathrm{M5}}$$

$$\tau_1=r\tau_{\mathrm{M2}}.$$

$$R_9'\propto R_9^{-1}, g'\propto gR_9^{-1}.$$

$$\frac{1}{g^2}\int d^{10}x=\frac{2\pi R_9}{g^2}\int d^9x$$

$$g_{\mathrm{I}}\propto g'^{-1}\propto g^{-1}R_9, R_{9\mathrm{I}}\propto g'^{-1/2}R_9'\propto g^{-1/2}R_9^{-1/2}.$$

$$g_{I'}\propto g_I R_{9I}^{-1}\propto g^{-1/2}R_9^{3/2}, R_{9I'}\propto R_{9I}^{-1}\propto g^{1/2}R_9^{1/2}.$$

$$R_{10\mathrm{M}}\propto g_{\mathrm{I}'}^{2/3}\propto g^{-1/3}R_9, R_{9\mathrm{M}}\propto g_{\mathrm{I}'}^{-1/3}R_{9\mathrm{I}'}\propto g^{2/3}.$$

$$\mathrm{F}_8\overset{T_9}{\rightarrow}\mathrm{F}_8\overset{S}{\rightarrow}\mathrm{D}_8\overset{T_9}{\rightarrow}\mathrm{D}_{89}\overset{S}{\rightarrow}\mathrm{M}_{8,10'}.$$

$$\mathrm{p}_9\overset{T_9}{\rightarrow}\mathrm{F}_9\overset{S}{\rightarrow}\mathrm{D}_9\overset{T_9}{\rightarrow}\mathrm{D}_\emptyset\overset{S}{\rightarrow}\mathrm{p}_{10}=\mathrm{p}_{9'}.$$

$$E \gtrsim \frac{1}{g^2 \alpha'^{1/2}}$$

$$E = (p_{10}^2 + q^2 + m^2)^{1/2} \approx p_{10} + \frac{q^2 + m^2}{2p_{10}} = \frac{n}{R_{10}} + \frac{R_{10}}{2n}(q^2 + m^2).$$

$$E - n/R_{10} = O(R_{10}/n)$$

$$H = R_{10}\mathrm{Tr}\left\{\frac{1}{2}p_i p_i - \frac{M_{11}^6}{16\pi^2}\left[X^i, X^j\right]^2 - \frac{M_{11}^3}{4\pi}\lambda\Gamma^0\Gamma^i\left[X^i, \lambda\right]\right\}.$$

$$E = \frac{R_{10}}{2}\mathrm{Tr}(p_i p_i) = \frac{q^2}{2p_{10}},$$

$$X^i = X_0^i + x^i$$
$$X_0^i = Y_1^i I_1 + Y_2^i I_2, x^i = x_{11}^i + x_{22}^i + x_{12}^i + x_{21}^i.$$

$$\psi(x_{11}, x_{22}) = \psi_0(x_{11})\psi_0(x_{22}).$$

$$\left[X_0^i, x_{12}^j\right] = \left(Y_1^i - Y_2^i\right)x_{12}^j$$

$$L_{\mathrm{eff}} = -V(r, v) = 4\pi^{5/2}\Gamma(7/2)\alpha'^3 n_1 n_2 \frac{v^4}{r^7} = \frac{15\pi^3}{2}\frac{p_{10}p'_{10}}{M_{11}^9 R_{10}}\frac{v^4}{r^7}$$

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 & \\ 0 & \alpha & 0 & 0 & \cdots \\ 0 & 0 & \alpha^2 & 0 & \\ 0 & 0 & 0 & \alpha^3 & \\ & \vdots & & & \ddots \end{bmatrix}, V = \begin{bmatrix} 0 & 0 & 0 & & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & & 0 \\ 0 & 0 & 1 & & 0 \\ & \vdots & & \ddots & \end{bmatrix}$$

$$U^n = V^n = 1, UV = \alpha VU,$$

$$X^i = \sum_{r,s=[1-n/2]}^{[n/2]} X_{rs}^i U^r V^s,$$

$$X^i \to X^i(p, q) = \sum_{r,s=[1-n/2]}^{[n/2]} X_{rs}^i \exp\,(ipr + iqs)$$

$$\left[X^i, X^j\right] \to \frac{2\pi i}{n}\left(\partial_q X^i \partial_p X^j - \partial_p X^i \partial_q X^j\right) + O(n^{-2})$$
$$\equiv \frac{2\pi i}{n}\left\{X^i, X^j\right\}_{\mathrm{PB}} + O(n^{-2})$$

$$\mathrm{Tr} = n\int \frac{dqdp}{(2\pi)^2}$$

$$R_{10}\int dqdp\left(\frac{n}{8\pi^2}\Pi_i\Pi_i + \frac{M_{11}^6}{16\pi^2 n}\left\{X^i, X^j\right\}_{\mathrm{PB}}^2 - i\frac{M_{11}^3}{8\pi^2}\lambda\Gamma^0\Gamma^i\left\{X^i, \lambda\right\}_{\mathrm{PB}}\right)$$

$$X^1 = aq, X^2 = bp;$$

$$\frac{M_{11}^6 R_{10} a^2 b^2}{2n} = \frac{M_{11}^6 A^2}{2(2\pi)^4 p_{10}} = \frac{\tau_{\mathrm{M2}}^2 A^2}{2p_{10}}.$$

$$X^i = Y_1^i I_1 + Y_2^i I_2$$

$$(x^0, x^{10}) \cong \left(x^0 - \pi\tilde{R}_{10}, x^{10} + \pi\tilde{R}_{10}\right)$$

$$(x^0, x^{10}) \cong \left(x^0 - \pi\tilde{R}_{10}, x^{10} + \pi\tilde{R}_{10} + 2\pi\epsilon^2\tilde{R}_{10}\right)$$

$$(x'^0, x'^{10}) \cong \left(x'^0, x'^{10} + 2\pi\epsilon\tilde{R}_{10}\right),$$

$$x'^0 \pm x'^{10} = \epsilon^{\mp 1}(x^0 \pm x^{10})$$

$$E', p'_{10} \propto O(\epsilon^{-1}), E' - p'_{10} \propto O(\epsilon)$$

$$R_{10} = \epsilon\tilde{R}_{10}$$

$$\frac{R_{10}}{\alpha'^{1/2}} \prod_m \frac{\alpha'^{1/2}}{R_m} = R_{10}^{(3-k)/2} (M_{11}^3)^{(1-k)/2} \prod_m R_m^{-1},$$

- Agujeros negros cuánticos.

$$ds^2 = Z(r)^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + Z(r)^{1/2} dx^m dx^m$$
$$e^{2\Phi} = g^2 Z(r)^{(3-p)/2}$$

$$Z(r) = 1 + \frac{\rho^{7-p}}{r^{7-p}}, r^2 = x^m x^m$$
$$\rho^{7-p} = \alpha^{(7-p)/2} g Q (4\pi)^{(5-p)/2} \Gamma\left(\frac{7-p}{2}\right)$$

$$\begin{aligned} ds^2 =& Z_1^{-1/2} Z_5^{-1/2} \left[\eta_{\mu\nu} dx^\mu dx^\nu + (Z_n - 1)(dt + dx_5)^2\right] \\ & + Z_1^{1/2} Z_5^{1/2} dx^i dx^i + Z_1^{1/2} Z_5^{-1/2} dx^m dx^m \\ e^{-2\Phi} =& Z_5 / Z_1 \end{aligned}$$

$$Z_1 = 1 + \frac{r_1^2}{r^2}, \quad r_1^2 = \frac{(2\pi)^4 g Q_1 \alpha'^3}{V_4}$$
$$Z_5 = 1 + \frac{r_5^2}{r^2}, \quad r_5^2 = g Q_5 \alpha'$$
$$Z_n = 1 + \frac{r_n^2}{r^2}, \quad r_n^2 = \frac{(2\pi)^5 g^2 p_5 \alpha'^4}{L V_4}$$

$$\begin{aligned} ds_{\mathrm{E}}^2 = & Z_1^{-3/4} Z_5^{-1/4} \left[\eta_{\mu\nu} dx^\mu dx^\nu + (Z_n - 1)(dt + dx_5)^2\right] \\ & + Z_1^{1/4} Z_5^{3/4} dx^i dx^i + Z_1^{1/4} Z_5^{-1/4} dx^m dx^m \end{aligned}$$

$$\left(\frac{r_1^2}{r^2}\right)^{1/4} \left(\frac{r_5^2}{r^2}\right)^{3/4} r^2 d\Omega^2 = r_1^{1/2} r_5^{3/2} d\Omega^2$$

$$A = 2\pi^2 L V_4 r_1 r_5 r_n = 2^6 \pi^7 g^2 \alpha'^4 (Q_1 Q_5 n_5)^{1/2} = \kappa^2 (Q_1 Q_5 n_5)^{1/2}.$$

$$S = \frac{2\pi A}{\kappa^2} = 2\pi (Q_1 Q_5 n_5)^{1/2}$$

$$V = \frac{1}{(2\pi\alpha')^2} |X_i \chi - \chi Y_i|^2 + \frac{g_1^2}{4} D_1^A D_1^A + \frac{g_5^2}{4V_4} D_5^A D_5^A$$

$$X^i = x^i I_{Q_1}, Y^i = x^i I_{Q_5}$$

$$\mathrm{Tr}[\exp(-\beta H)] \approx \exp(\pi c L / 12\beta)$$

$$\int_0^\infty dE n(E) \exp(-\beta E) = \mathrm{Tr}[\exp(-\beta H)]$$

$$n(E) \approx \exp\left[(\pi c E L/3)^{1/2}\right]$$

$$n(E) \approx \exp\left[2\pi (Q_1 Q_5 n_5)^{1/2}\right]$$

$$ds^2 = \frac{r^2}{r_1 r_5} \eta_{\mu v} dx^\mu dx^v + \frac{r_1 r_5}{r^2} dr^2 + r_1 r_5 d\Omega^2 + \frac{r_1}{r_5} dx^m dx^m$$

$$AdS_3 \times S^3 \times T^4$$

$$AdS_5 \times S^5$$

$$R \approx G_{\mathrm{N}} M$$
$$S_{\mathrm{bh}} \approx \frac{R^2}{G_{\mathrm{N}}} \approx G_{\mathrm{N}} M^2.$$

$$G_{\mathrm{N}} \approx g^2 \alpha'.$$

$$\frac{R}{\alpha'^{1/2}} \approx g S_{\mathrm{bh}}^{1/2}.$$

$$\exp\left\{\pi M [(c + \tilde{c})\alpha'/3]^{1/2}\right\}.$$

$$S_{\mathrm{s}} \approx M \alpha^{1/2} \approx g^{-1} M G_{\mathrm{N}}^{1/2}$$

$$g S_{\mathrm{bh}}^{1/2} \approx 1,$$

Multidimensiones – Supermembranas – Superespacios y Supersimetrías a propósito de la existencia de supergravedad cuántica. Modelo Sugawara Landau-Ginzburg.

$$L_0 |h\rangle = h |h\rangle$$
$$L_m |h\rangle = 0, m > 0$$

$$[L_m, L_n] = (m - n) L_{m+n} + \frac{c}{12} (m^3 - m) \delta_{m,-n}$$

$$L_{-k_1} L_{-k_2} \dots L_{-k_l} |h\rangle.$$

$$|\phi\rangle \to |\phi\rangle - |i\rangle\langle i \mid \phi\rangle$$

$$\langle\phi|L_{-n}L_n|\phi\rangle > 0.$$

$$\mathcal{M}^1 \equiv \langle h|L_1L_{-1}|h\rangle = 2h,$$

$$\mathcal{M}^2 = \begin{bmatrix}\langle h|L_1^2\\ \langle h|L_2\end{bmatrix}[L_{-1}^2|h\rangle L_{-2}|h\rangle]$$

$$\mathcal{M}^2 = \begin{pmatrix}8h^2+4h & 6h\\ 6h & 4h+c/2\end{pmatrix}$$

$$\begin{aligned}\det(\mathcal{M}^2) &= 32h(h-h_+)(h-h_-)\\ 16h_\pm &= (5-c)\pm[(1-c)(25-c)]^{1/2}\end{aligned}$$

$$\mathcal{M}^N_{\{k\},\{k'\}}(c,h) = \langle h,\{k\} \mid h,\{k'\}\rangle, \sum_i k_i = N.$$

$$\det[\mathcal{M}^N(c,h)] = K_N \prod_{1\leq rs\leq N} \left(h-h_{r,s}\right)^{P(N-rs)}$$

$$h_{r,s} = \frac{c-1}{24} + \frac{1}{4}(r\alpha_+ + s\alpha_-)^2,$$

$$\alpha_\pm = (24)^{-1/2}\left[(1-c)^{1/2} \pm (25-c)^{1/2}\right].$$

$$\prod_{n=1}^{\infty}\frac{1}{1-q^n} = \sum_{k=0}^{\infty} P(k)q^k.$$

$$\begin{aligned}c &= 1-\frac{6}{m(m+1)}, m = 2,3,\ldots\\ &= 0,\frac{1}{2},\frac{7}{10},\frac{4}{5},\frac{6}{7},\ldots\\ h = h_{r,s} &= \frac{[r(m+1)-sm]^2-1}{4m(m+1)}\end{aligned}$$

$$h_{r,s} = \frac{25-(3r+2s)^2}{24}$$

$$h = h_{r,s} + rs = \frac{25-(3r-2s)^2}{24}$$

$$\langle T(z)\mathcal{O}_1(z_1)\ldots\mathcal{O}_n(z_n)\rangle_{S_2} = \sum_{i=1}^{n}\left[\frac{h_i}{(z-z_i)^2} + \frac{1}{(z-z_i)}\frac{\partial}{\partial z_i}\right]\langle\mathcal{O}_1(z_1)\ldots\mathcal{O}_n(z_n)\rangle_{S_2}$$

$$T(z)\mathcal{O}_1(z_1) = \sum_{k=-\infty}^{\infty} (z-z_1)^{k-2}L_{-k}\cdot\mathcal{O}_1(z_1)$$

$$\langle[L_{-k}\cdot\mathcal{O}_1(z_1)]\mathcal{O}_2(z_2)\ldots\mathcal{O}_n(z_n)\rangle_{S_2} = \mathcal{L}_{-k}\langle\mathcal{O}_1(z_1)\ldots\mathcal{O}_n(z_n)\rangle_{S_2},$$

$$\mathcal{L}_{-k} = \sum_{i=2}^{n} \left[\frac{h_i(k-1)}{(z_i - z_1)^k} - \frac{1}{(z_i - z_1)^{k-1}}\frac{\partial}{\partial z_i}\right]$$

$$\left\langle\left[L_{-k_1} \dots L_{-k_\ell}\tilde{L}_{-l_1} \dots \tilde{L}_{-l_m} \cdot \mathcal{O}_1(z_1)\right] \dots \mathcal{O}_n(z_n)\right\rangle_{S_2}$$
$$= \mathcal{L}_{-k_\ell} \dots \mathcal{L}_{-k_1}\tilde{\mathcal{L}}_{-l_m} \dots \tilde{\mathcal{L}}_{-l_1}\langle\mathcal{O}_1(z_1) \dots \mathcal{O}_n(z_n)\rangle_{S_2}$$

$$\mathcal{O}_m(z,\bar{z})\mathcal{O}_n(0,0) = \sum_{i,\{k,\tilde{k}\}} z^{-h_m-h_n+h_i+N}\bar{z}^{-\tilde{h}_m-\tilde{h}_n+\tilde{h}_i+\tilde{N}}$$
$$\times\, c^{i\{k,\tilde{k}\}}{}_{mn}L_{-\{k\}}\tilde{L}_{-\{\tilde{k}\}} \cdot \mathcal{O}_i(0,0)$$

$$c^{i\{k,\tilde{k}\}_{mn}} = \sum_{\{k',\tilde{k}'\}} \mathcal{M}^{-1}_{\{k\},\{k'\}}\mathcal{M}^{-1}_{\{\tilde{k}\},\{\tilde{k}'\}} \times \mathcal{L}_{-\{k'\}}\tilde{\mathcal{L}}_{-\{\tilde{k}'\}}\langle\mathcal{O}_m(\infty,\infty)\mathcal{O}_n(1,1)\mathcal{O}_i(z_1,\bar{z}_1)\rangle_{S_2}\Big|_{z_1=0}$$

$$c^{i\{k,\tilde{k}\}_{mn}} = \beta^{i\{k\}}_{mn}\tilde{\beta}^{i\{\tilde{k}\}}_{mn}c^{i}_{mn}.$$

$$\left\langle\mathcal{O}_j(\infty,\infty)\mathcal{O}_l(1,1)\mathcal{O}_m(z,\bar{z})\mathcal{O}_n(0,0)\right\rangle_{S_2} = \sum_i c^i_{jl}c_{imn}\mathcal{F}^{jl}_{mn}(i \mid z)\tilde{\mathcal{F}}^{jl}_{mn}(i \mid \bar{z}),$$

$$\mathcal{F}^{jl}_{mn}(i \mid z) = \sum_{\{k\},\{k'\}} z^{-h_m-h_n+h_i+N}\beta^{i\{k\}}_{jl}\mathcal{M}_{\{k\},\{k'\}}\beta^{i\{k'\}}_{mn}.$$

$$Z(\tau) = \sum_{i,\{k,\tilde{k}\}} q^{-c/24+h_i+N}\bar{q}^{-\tilde{c}/24+\tilde{h}_i+\tilde{N}} = \sum_i \chi_{c,h_i}(q)\chi_{\tilde{c},\tilde{h}_i}(\bar{q})$$

$$\chi_{c,h}(q) = q^{-c/24+h}\sum_{\{k\}} q^N$$

$$\chi_{c,h}(q) = q^{-c/24+h}\prod_{n=1}^{\infty}\frac{1}{1-q^n}$$

$$Z(i\ell) \overset{\ell\to 0}{\sim} \exp(\pi c/6\ell)$$

$$\chi_{c,h}(q) \leq q^{h+(1-c)/24}\eta(i\ell)^{-1} \overset{\ell\to 0}{\sim} \ell^{1/2}\exp(\pi/12\ell).$$

$$Z(i\ell) \leq \mathcal{N}\ell\exp(\pi/6\ell)$$

$$h = h_{1,2} = \frac{c-1}{24} + \frac{(\alpha_+ + 2\alpha_-)^2}{4}.$$

$$\mathcal{N}_{1,2} = \left[L_{-2} - \frac{3}{2(2h_{1,2}+1)}L^2_{-1}\right] \cdot \mathcal{O}_{1,2} = 0$$

$$0=\left\langle \mathcal{N}_{1,2}(z_1)\prod_{i=2}^{n}\mathcal{O}_i(z_i)\right\rangle_{S_2}=\left[\mathcal{L}_{-2}-\frac{3}{2(2h_{1,2}+1)}\mathcal{L}_{-1}^2\right]\mathcal{A}_n$$

$$=\left[\sum_{i=2}^{n}\frac{h_i}{(z_i-z_1)^2}-\sum_{i=2}^{n}\frac{1}{z_i-z_1}\frac{\partial}{\partial z_i}-\frac{3}{2(2h_{1,2}+1)}\frac{\partial^2}{\partial z_1^2}\right]\mathcal{A}_n$$

$$\mathcal{A}_n=\left\langle \mathcal{O}_{1,2}(z_1,\bar{z}_1)\prod_{i=2}^{n}\mathcal{O}_i(z_i,\bar{z}_i)\right\rangle_{S_2}$$

$$\left[\sum_{i=2}^{4}\frac{h_i}{(z_i-z_1)^2}-\sum_{2\le i<j\le 4}\frac{h_{1,2}-h_2-h_3-h_4+2(h_i+h_j)}{(z_i-z_1)(z_j-z_1)}+\sum_{i=2}^{4}\frac{1}{z_i-z_1}\frac{\partial}{\partial z_1}\right.$$

$$\left.-\frac{3}{2(2h_{1,2}+1)}\frac{\partial^2}{\partial z_1^2}\right]\mathcal{A}_4=0$$

$$\left\langle \mathcal{O}_{r,s;\tilde{r},\tilde{s}}(z_1,\bar{z}_1)\prod_{i=2}^{4}\mathcal{O}_i(z_i,\bar{z}_i)\right\rangle_{S_2}=\sum_{i=1}^{rs}\sum_{j=1}^{\tilde{r}\tilde{s}}a_{ij}f_i(z)\tilde{f}_j(\bar{z}),$$

$$\frac{3}{2(2h_{1,2}+1)}\kappa(\kappa-1)+\kappa-h_i=0.$$

$$h_\pm=h_{1,2}+h_i+\kappa_\pm$$

$$h_i=\frac{c-1}{24}+\frac{\gamma^2}{4}$$

$$h_\pm=\frac{c-1}{24}+\frac{(\gamma\pm\alpha_-)^2}{4}$$

$$\mathcal{O}_{1,2}\mathcal{O}_{(\gamma)}=[\mathcal{O}_{(\gamma+\alpha_-)}]+[\mathcal{O}_{(\gamma-\alpha_-)}];$$

$$\mathcal{O}_{2,1}\mathcal{O}_{(\gamma)}=[\mathcal{O}_{(\gamma+\alpha_+)}]+[\mathcal{O}_{(\gamma-\alpha_+)}].$$

$$\mathcal{O}_{1,2}\mathcal{O}_{r,s}=[\mathcal{O}_{r,s+1}]+[\mathcal{O}_{r,s-1}],$$
$$\mathcal{O}_{2,1}\mathcal{O}_{r,s}=[\mathcal{O}_{r+1,s}]+[\mathcal{O}_{r-1,s}].$$

$$c=1-6\frac{(p-q)^2}{pq}$$
$$h_{r,s}=\frac{(rq-sp)^2-(p-q)^2}{4pq}$$

$$h_{p-r,q-s}=h_{r,s}$$

$$1\le r\le p-1, 1\le s\le q-1$$

$$p=m, q=m+1.$$

$$h_{1,1} = 0, h_{2,1} = \frac{1}{2}, h_{1,2} = \frac{1}{16}$$

$$\mathcal{O}_{r_1,s_1}\mathcal{O}_{r_2,s_2} = \sum [\mathcal{O}_{r,s}],$$
$$r = |r_1 - r_2| + 1, |r_1 - r_2| + 3, \dots,$$
$$\min(r_1 + r_2 - 1, 2p - 1 - r_1 - r_2),$$
$$s = |s_1 - s_2| + 1, s_1 + s_2 + 3, \dots,$$
$$\min(s_1 + s_2 - 1, 2q - 1 - s_1 - s_2).$$

$$J(z)\mathcal{O}_i(0) = z^{h_{i'} - h_i - h}[\mathcal{O}_{i'}(0)]$$

$$2\pi(h_{i'} - h_i - h) = 2\pi Q_i$$

$$\mathcal{O}_{p-1,1}\mathcal{O}_{p-1,1} = [\mathcal{O}_{1,1}]$$

$$Q_{r,s} = \frac{p(1-s) + q(1-r)}{2}\mathrm{mod}1$$

$$c = 1 - 24\alpha_0^2$$

$$T = -\frac{1}{2}\partial\phi\partial\phi + 2^{1/2}i\alpha_0\partial^2\phi.$$

$$V_\alpha = \exp\left(2^{1/2}i\alpha\phi\right)$$

$$\alpha = \alpha_0 - \frac{\gamma}{2}$$

$$\frac{c-1}{24} + \frac{\gamma^2}{4}$$

$$\langle V_{\alpha_1}V_{\alpha_2}V_{\alpha_3}V_{\alpha_4}\rangle$$

$$\sum_i \alpha_i = 2\alpha_0$$

$$J_\pm = \exp\left(2^{1/2}i\alpha_\pm\phi\right)$$

$$Q_\pm = \oint dzJ_\pm$$

$$n_+ = \frac{1}{2}\sum_i r_i - 2, n_- = \frac{1}{2}\sum_i s_i - 2.$$

$$L_m|r,i\rangle = j_m^a|r,i\rangle, m > 0,$$
$$j_0^a|r,i\rangle = |r,j\rangle t_{r,ji}^a.$$

$$T(z) = \frac{1}{(k + h(g))\psi^2}:jj(z):$$

$$c^{g,k} = \frac{k\dim(g)}{k + h(g)}$$

$$h_r = \frac{Q_r}{(k + h(g))\psi^2}$$

$$\left\langle \left(j_{-m}^a \cdot \mathcal{O}_1(z_1)\right)\mathcal{O}_2(z_2) \ldots \mathcal{O}_n(z_n)\right\rangle_{S_2} = \mathcal{J}_{-m}^a \langle \mathcal{O}_1(z_1) \ldots \mathcal{O}_n(z_n)\rangle_{S_2},$$

$$\mathcal{J}_{-m}^a = -\sum_{i=2}^{n} \frac{t^{a(i)}}{(z_i - z_1)^m},$$

$$L_m = \frac{1}{(k + h(g))\psi^2} \sum_{n=-\infty}^{\infty} j_n^a j_{m-n}^a$$

$$\mathcal{L}_{-1} - \frac{2}{(k + h(g))\psi^2} \sum_a t^{a(i)} \mathcal{J}_{-1}^a.$$

$$\left[\frac{\partial}{\partial z_1} - \frac{2}{(k + h(g))\psi^2} \sum_a \sum_{i=2}^{n} \frac{t^{a(1)} t^{a(i)}}{z_1 - z_i}\right] \langle \mathcal{O}_1(z_1) \ldots \mathcal{O}_n(z_n)\rangle_{S_2} = 0.$$

$$j_{-1}^+, j_0^3 - \frac{k}{2}, j_1^-.$$

$$|j, m\rangle,$$

$$(j_{-1}^+)^{k-2m+1} |j, m\rangle = 0.$$

$$0 = \left\langle (\mathcal{J}_{-1}^+)^{k-2m_1+1} \cdot \mathcal{O}_1(z_1) \ldots \mathcal{O}_n(z_n)\right\rangle_{S_2} = \left[-\sum_{i=2}^{n} \frac{t^{+(i)}}{z_i - z_1}\right]^{k-2m_1+1} \langle \mathcal{O}_1(z_1) \ldots \mathcal{O}_n(z_n)\rangle_{S_2}$$

$$0 = \sum_{m_2, m_3} \left[\left(t^{+(2)}\right)^{l_2}\right]_{m_2, n_2} \left[\left(t^{+(3)}\right)^{l_3}\right]_{m_3, n_3} \left\langle \mathcal{O}_{j_1, m_1} \mathcal{O}_{j_2, m_2} \mathcal{O}_{j_3, m_3}\right\rangle_{S_2}$$

$$l_2 + l_3 \geq k - 2m_1 + 1$$

$$\left\langle \mathcal{O}_{j_1, j_1} \mathcal{O}_{j_2, m_2} \mathcal{O}_{j_3, m_3}\right\rangle_{S_2} = 0 \text{ if } j_1 + j_2 + j_3 > k$$

$$[j_1] \times [j_2] = [|j_1 - j_2|] + [|j_1 - j_2| + 1] + \cdots + [\min(j_1 + j_2, k - j_1 - j_2)].$$

$$[j_1] \times [k/2] = [k/2 - j_1].$$

$$Z(\tau) = \sum_{r, \tilde{r}} n_{r\tilde{r}} \chi_r(q) \chi_{\tilde{r}}(q)^*$$

$$\chi_r(q') = \sum_{r'} S_{rr'} \chi_{r'}(q)$$

$$S^\dagger n S = n$$

$$S_{jj'} = \left(\frac{2}{k+2}\right)^{1/2} \sin \frac{\pi(2j+1)(2j'+1)}{k+2}$$

$$n_{j\tilde{j}} = \delta_{j\tilde{j}}$$

$$n_{j\tilde{j}} = \delta_{j\tilde{j}}\big|_{j\in\mathbf{Z}} + \delta_{k/2-j,\tilde{j}}\big|_{j\in\mathbf{Z}+k/4}.$$

$$S = \frac{1}{2\pi\alpha'}\int d^2z(G_{mn} + B_{mn})\partial X^m \bar{\partial} X^n$$

$$H_{mnp} = \frac{q}{r^3}\epsilon_{mnp}$$

$$R_{mn} = \frac{2}{r^2}G_{mn}$$

$$\beta^G_{mn} = \alpha' G_{mn}\left(\frac{2}{r^2} - \frac{q^2}{2r^6}\right)$$

$$\beta^\Phi = \frac{1}{2} - \frac{\alpha' q^2}{4r^6}$$

$$r^2 = \frac{|q|}{2} + O(\alpha')$$

$$c = 6\beta^\Phi = 3 - \frac{6\alpha'}{r^2} + O\left(\frac{\alpha'^2}{r^4}\right)$$

$$c = 3 - \frac{6}{k+2}$$

$$\exp\left(\frac{i}{2\pi\alpha'}\int_M B\right) = \exp\left(\frac{i}{2\pi\alpha'}\int_N H\right)$$

$$1 = \exp\left(\frac{i}{2\pi\alpha'}\int_{S_3} H\right) = \exp\left(\frac{\pi i q}{\alpha'}\right)$$

$$q = 2\alpha' n, r^2 = \alpha'|n|$$

$$c = 3 - \frac{6}{|n|} + O\left(\frac{1}{n^2}\right)$$

$$g = x^4 + ix^i\sigma^i, \sum_{i=1}^{4}\left(x^i\right)^2 = 1$$

$$S = \frac{|n|}{4\pi}\int_M d^2z \mathrm{Tr}\left(\partial g^{-1}\bar{\partial} g\right) + \frac{in}{12\pi}\int_N \mathrm{Tr}(\omega^3)$$

$$d(\omega^3) = 0$$

$$\frac{n}{12\pi}\int_M \mathrm{Tr}(\chi)$$

$$\delta S = \frac{|n|}{2\pi}\int d^2z\mathrm{Tr}[\bar{\partial}gg^{-1}\partial(g^{-1}\delta g)] = \frac{|n|}{2\pi}\int d^2z\mathrm{Tr}[g^{-1}\partial g\bar{\partial}(\delta gg^{-1})]$$

$$\delta g(z,\bar{z}) = i\epsilon_L g(z,\bar{z}) - ig(z,\bar{z})\epsilon_R$$

$$\delta g(z,\bar{z}) = i\epsilon_L(z)g(z,\bar{z}) - ig(z,\bar{z})\epsilon_R(\bar{z})$$

$$|n|\mathrm{Tr}(\epsilon_R g^{-1}\partial g), |n|\mathrm{Tr}(\epsilon_L\bar{\partial}gg^{-1})$$

$$g = 1 + i(2|n|)^{-1/2}\phi_a\sigma^a + \cdots$$

$$\begin{aligned}\mathcal{L} &= \frac{1}{4\pi}\partial\phi^a\bar{\partial}\phi^a + O(\phi^3)\\ j_R^a &= |n|^{1/2}\partial\phi^a + O(\phi^2)\\ j_L^a &= |n|^{1/2}\bar{\partial}\phi^a + O(\phi^2)\end{aligned}$$

$$D_{ij}^r(g) = \mathcal{O}_i^r(z)\tilde{\mathcal{O}}_j^r(\bar{z})$$

$$T^G = T^H + T^{G/H}.$$

$$c^{G/H} = c^G - c^H.$$

$$\begin{aligned}G = SU(2)_k \oplus SU(2)_1, c^G &= 4 - \frac{6}{k+2},\\ H = SU(2)_{k+1}, c^H &= 3 - \frac{6}{k+3},\end{aligned}$$

$$c^{G/H} = 1 - \frac{6}{(k+2)(k+3)}$$

$$\chi_r^G(q) = \sum_{r',r''} n_{r'r''}^r \chi_{r'}^H(q)\chi_{r''}^{G/H}(q),$$

$$S_{rs,r's'} = \left[\frac{8}{(p+1)(q+1)}(-1)^{(r+s)(r'+s')}\right]^{1/2}\sin\frac{\pi rr'}{p}\sin\frac{\pi ss'}{q}$$

$$\frac{SU(2)_k}{U(1)}, c = 2 - \frac{6}{k+2}.$$

$$j^3 = i(k/2)^{1/2}\partial H, T^H = -\frac{1}{2}\partial H\partial H.$$

$$j^+ = \exp\left[iH(2/k)^{1/2}\right]\psi_1, j^- = \exp\left[-iH(2/k)^{1/2}\right]\psi_1^\dagger,$$

$$:(j^+)^l: = (j_{-1}^+)^l\cdot 1 \equiv \exp\left(ilH(2/k)^{1/2}\right)\psi_l$$

$$\mathcal{O}_{j,m} = \exp\left[imH(2/k)^{1/2}\right]\psi_m^j$$

$$\psi_l(z)\psi_{l'}(0) \approx z^{-2ll'/k}(\psi_{l+l'} + \cdots)$$

$$:j^a_{(1)}j^a_{(1)}:, :j^a_{(1)}j^a_{(2)}:, :j^a_{(2)}j^a_{(2)}:.$$

$$[j^b_{(1)}(z)+j^b_{(2)}(z)]:j^a_{(i)}j^a_{(j)}(0):=\sum_{k=0}^{\infty}\frac{1}{z^{k+1}}[j^b_{k(1)}+j^b_{k(2)}]j^a_{-1(i)}j^a_{-1(j)}\cdot 1$$

$$G=SU(n)_{k_1}\oplus SU(n)_{k_2}, c^G=(n^2-1)\left[\frac{k_1}{k_1+n}+\frac{k_2}{k_2+n}\right]$$

$$H=SU(n)_{k_1+k_2}, c^H=(n^2-1)\frac{k_1+k_2}{k_1+k_2+n}$$

$$d^{abc}\propto \mathrm{Tr}\left(t^a\{t^b,t^c\}\right),$$

$$W(z)W(0)\sim\frac{c}{3z^6}+\frac{2}{z^4}T(0)+\frac{1}{z^3}\partial T(0)+\frac{3}{10z^2}\partial^2T(0)+\frac{1}{15z}\partial^3T(0)$$
$$+\frac{16}{220+50c}\left(\frac{2}{z^2}+\frac{1}{z}\partial\right)[10:T^2(0):-3\partial^2T(0)]$$

$$c=2-\frac{24}{(k+3)(k+4)}.$$

$$\det(\mathcal{M}^N)_{\mathrm{R,NS}}=(h-\epsilon\hat{c}/16)K_N\prod_{1\le rs\le 2N}\left(h-h_{r,s}\right)^{P_{\mathrm{R,NS}}(N-rs/2)}.$$

$$h_{r,s}=\frac{\hat{c}-1+\epsilon}{16}+\frac{1}{4}(r\hat{\alpha}_+ +s\hat{\alpha}_-)^2,$$

$$\hat{\alpha}_\pm=\frac{1}{4}\left[(1-\hat{c})^{1/2}\pm(9-\hat{c})^{1/2}\right].$$

$$\prod_{n=1}^{\infty}\frac{1+q^{n-1}}{1-q^n}=\sum_{k=0}^{\infty}P_{\mathrm{R}}(k)q^k$$
$$\prod_{n=1}^{\infty}\frac{1+q^{n-1/2}}{1-q^n}=\sum_{k=0}^{\infty}P_{\mathrm{NS}}(k)q^k$$

$$\hat{c}\ge 1, h\ge\epsilon\frac{\hat{c}}{16},$$

$$c=\frac{3}{2}-\frac{12}{m(m+2)}, m=2,3,\ldots$$
$$=0,\frac{7}{10},1,\frac{81}{70},\ldots,$$
$$h=h_{r,s}\equiv\frac{[r(m+2)-sm]^2-4}{8m(m+2)}+\frac{\epsilon}{16}$$

$$G=SU(2)_k\oplus SU(2)_2, H=SU(2)_{k+2}.$$

$$Z(\tau) = \sum_{i,\tilde{j}} n_{i\tilde{j}} \chi_i(q) \chi_{\tilde{j}}(q)^*,$$

$$N_{ijkl} = N_{ij}^r N_{rkl},$$

$$N_{ij}^r N_{rkl} = N_{ik}^r N_{rjl} = N_{il}^r N_{rjk}$$

$$\tau_1 \tau_2 \tau_3 \tau_4 = \tau_{12} \tau_{13} \tau_{23}$$

$$\tau_1 \colon \mathcal{F}_{ij}^{kl}(r \mid z) \rightarrow \exp\,(2\pi i h_i) \mathcal{F}_{ij}^{kl}(r \mid z),$$
$$\tau_{12} \colon \mathcal{F}_{ij}^{kl}(r \mid z) \rightarrow \exp\,(2\pi i h_r) \mathcal{F}_{ij}^{kl}(r \mid z).$$

$$N_{ijkl}\left(h_i + h_j + h_k + h_l\right) - \sum_r \left(N_{ij}^r N_{rkl} + N_{ik}^r N_{rjl} + N_{il}^r N_{rjk}\right) h_r \in \mathbf{Z}.$$

$$\sum_r N_{ii}^r N_{rii} (4h_i - 3h_r) \in \mathbf{Z}.$$

$$N_{00}^0 = N_{1/2,1/2}^0 = N_{1/2,1/2}^1 = N_{11}^0 = N_{11}^1 = N_{3/2,3/2}^0 = 1.$$

$$8h_{1/2} - 3h_1, 5h_1, 4h_{3/2}$$

$$S^4 = (ST)^3 = 1$$

$$1 = [(\det S)^4]^{-3} [(\det S \det T)^3]^4 = (\det T)^{12}.$$

$$T \colon \chi_i(q) \rightarrow \exp\,[2\pi i (h_i - c/24)] \chi_i(q).$$

$$\frac{\mathcal{N} c}{2} - 12 \sum_i h_i \in \mathbf{Z},$$

$$N_{jk}^i = \sum_r \frac{S_j^r S_k^r S_r^{\dagger i}}{S_0^r}$$

$$T' = L_{ab} \colon j^a j^b \colon$$

$$L_{ab} = 2 L_{ac} k^{cd} L_{db} - L_{cd} L_{ef} f_a^{ce} f_b^{df} - L_{cd} f_f^{ce} f_{(a}^{df} L_{b)e}$$

$$\delta_s z = \epsilon z$$

$$\delta_s g_{ab} = 2 \epsilon g_{ab}$$

$$-\frac{\epsilon}{2\pi} \int d^2 \sigma T_a^a(\sigma)$$

$$T_a^a = \partial_a \mathcal{K}^a$$

$$\epsilon^{-1} \delta_s \left\langle \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle = -\frac{1}{2\pi} \int d^2 \sigma \left\langle T_a^a(\sigma) \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle - \sum_n \Delta_{i_n}{}^j \left\langle \mathcal{A}_j(\sigma_n) \prod_{m \neq n} \mathcal{A}_{i_m}(\sigma_m) \right\rangle$$

$$\epsilon^{-1}\delta_s\mathcal{A}_i(\sigma) = -\Delta_i{}^j\mathcal{A}_j(\sigma)$$

$$\int d^d\sigma T^a_a = -2\pi\sum_i{}' \int d^d\sigma\beta^i(g)\mathcal{A}_i$$

$$S = \sum_i{}' g^i \int d^d\sigma\mathcal{A}_i(\sigma)$$

$$\epsilon^{-1}\delta_s\left\langle\prod_m \mathcal{A}_{i_m}(\sigma_m)\right\rangle = -\sum_i{}' \beta^i(g)\frac{\partial}{\partial g^i}\left\langle\prod_m \mathcal{A}_{i_m}(\sigma_m)\right\rangle - \sum_n \Delta_{i_n}{}^j\left\langle\mathcal{A}_j(\sigma_n)\prod_{m\neq n} \mathcal{A}_{i_m}(\sigma_m)\right\rangle$$

.

$$F(r^2) = z^4\langle T_{zz}(z,\bar{z})T_{zz}(0,0)\rangle$$
$$G(r^2) = 4z^3\bar{z}\langle T_{zz}(z,\bar{z})T_{z\bar{z}}(0,0)\rangle$$
$$H(r^2) = 16z^2\bar{z}^2\langle T_{z\bar{z}}(z,\bar{z})T_{z\bar{z}}(0,0)\rangle$$

$$4\dot{F} + \dot{G} - 3G = 0, 4\dot{G} - 4G + \dot{H} - 2H = 0$$

$$C = 2F - G - \frac{3}{8}H$$

$$\dot{C} = -\frac{3}{4}H$$

$$S = S_0 + \lambda^i \int d^2z\mathcal{O}_i$$

$$-T_{zz}(z,\bar{z})\lambda^i \int d^2w\mathcal{O}_i(w,\bar{w})$$

$$\partial_{\bar{z}}T_{zz}(z)\mathcal{O}_i(w,\bar{w}) = \partial_{\bar{z}}[(z-w)^{-2}h_i + (z-w)^{-1}\partial_w]\mathcal{O}_i(w,\bar{w})$$
$$= -2\pi h_i\partial_z\delta^2(z-w)\mathcal{O}_i(w,\bar{w}) + 2\pi\delta^2(z-w)\partial_w\mathcal{O}_i(w,\bar{w})$$

$$\partial_{\bar{z}}T_{zz}(z,\bar{z}) = 2\pi\lambda_i(h_i - 1)\partial_z\mathcal{O}_i(z,\bar{z}).$$

$$\partial_{\bar{z}}T_{zz} + \partial_z T_{\bar{z}z} = 0.$$

$$T_{\bar{z}z} = 2\pi\lambda^i(1-h_i)O_i(z,\bar{z}).$$

$$\beta^i = 2(h_i - 1)\lambda^i,$$

$$\delta\lambda^i = 2\epsilon(1-h_i)\lambda^i.$$

$$\frac{1}{2}\int d^2z\mathcal{O}_i(z,\bar{z})\int d^2w\mathcal{O}_j(w,\bar{w})$$

$$\mathcal{O}_i(z,\bar{z})\mathcal{O}_j(w,\bar{w}) \sim \frac{1}{|z-w|^2}c^k_{ij}\mathcal{O}_k(w,\bar{w}),$$

$$2\pi\int\frac{dr}{r}c_{ij}^{k}\int d^2w\mathcal{O}_k(w,\bar{w}).$$

$$\delta\lambda^k=-2\pi\epsilon c^k{}_{ij}\lambda^i\lambda^j.$$

$$\beta^k=2\pi c_{ij}^{k}\lambda^i\lambda^j.$$

$$c_{ij}^{k}\lambda^i\lambda^j=0$$

$$\beta^i=2(h_i-1)\lambda^i+2\pi c^k{}_{ij}\lambda^i\lambda^j,$$

$$\dot{C}=-12\pi^2\beta^i\beta^jG_{ij},$$

$$G_{ij}=z^2\bar{z}^2\langle\mathcal{O}_i(z,\bar{z})\mathcal{O}_j(0,0)\rangle$$

$$\beta^i=\frac{\partial}{\partial\lambda_i}U(\lambda),$$

$$U(\lambda)=(h_i-1)\lambda^i\lambda_i+\frac{2\pi}{3}c_{ijk}\lambda^i\lambda^j\lambda^k,$$

$$\dot{C}=24\pi^2\beta_j\dot{\lambda}^j=24\pi^2\dot{U}$$

$$C=c+24\pi^2U$$

$$\dot{\lambda}=(1-h)\lambda-\pi c_{111}\lambda^2,$$

$$\lambda'=\frac{1-h}{\pi c_{111}}.$$

$$c'=c-8\frac{(1-h)^3}{c_{111}^2}.$$

$$Z=\int[dq]\exp(-\beta H)$$

$$Z=\int[d\phi]\exp(-S/\hbar)$$

$$H=-\sum_{\text{links}}\sigma_i\sigma_{i'}$$

$$\langle\sigma_i\sigma_j\rangle\sim\exp[-|i-j|/\xi(\beta)]$$

$$\langle\sigma_i\sigma_j\rangle\sim v^2(\beta)+\exp[-|i-j|/\xi'(\beta)].$$

$$\langle\sigma_i\sigma_j\rangle\sim|i-j|^{-\eta},\beta=\beta_c.$$

$$\mathcal{O}_{1,1}:h=0,\mathcal{O}_{1,2}:h=\frac{1}{16},\mathcal{O}_{1,3}:h=\frac{1}{2}.$$

$$\sigma_i\rightarrow\sigma(z,\bar{z})=\mathcal{O}_{1,2}(z)\tilde{\mathcal{O}}_{1,2}(\bar{z}).$$

$$\langle\sigma(z,\bar{z})\sigma(0,0)\rangle \propto (z\bar{z})^{-2h} = (z\bar{z})^{-1/8}$$

$$\mathcal{O}_{1,3}(z)\tilde{\mathcal{O}}_{1,3}(\bar{z})$$

$$[\mathbb{O}_{1,1}\tilde{\mathcal{O}}_{1,1}] + [\mathbb{O}_{1,2}\tilde{\mathcal{O}}_{1,2}] + [\mathbb{O}_{1,3}\tilde{\mathcal{O}}_{1,3}].$$

$$H = -\sum_{\text{links}} \text{Re}(\sigma_i\sigma_{i'}^*).$$

$$\mathcal{L} = \partial\phi\bar{\partial}\phi + \lambda_1\phi^2 + \lambda_2\phi^4.$$

$$\mathbb{O}_{2,2}\mathbb{O}_{2,2} = [\mathbb{O}_{1,1}] + [\mathbb{O}_{3,1}] + [\mathbb{O}_{3,3}] + [\mathbb{O}_{1,3}].$$

$$\phi^n = \mathcal{O}_{n+1,n+1}, 0 \le n \le m-2.$$

$$\phi^{m-1+n} = \mathcal{O}_{n+1,n+2}, 0 \le n \le m-3.$$

$$m\lambda_m\phi^{2m-3} = \partial\bar{\partial}\phi = L_{-1}\tilde{L}_{-1}\cdot\phi,$$

$$\mathcal{O}_{m-1,m-2} = \mathcal{O}_{1,3}, h = 1 - \frac{2}{m+1}$$

$$\mathbb{O}_{1,3}\mathbb{O}_{1,3} = [\mathbb{O}_{1,1}] + [\mathbb{O}_{1,3}] + [\mathbb{O}_{1,5}],$$

$$c' = c - \frac{12}{m^3}$$

$$J^+ = iw/2^{1/2}, J^3 = iq\partial\phi/2^{1/2} - w\chi,$$
$$J^- = i[w\chi^2 + (2-q^2)\partial\chi]/2^{1/2} + q\chi\partial\phi.$$

$$X^m \to \theta^{mn}X^n + v^m$$

$$\tilde{\psi}^m \to \theta^{mn}\tilde{\psi}^n.$$

$$\lambda^A \to \gamma^{AB}\lambda^B.$$

$$K = R^6/S$$

$$T^6 = R^6/\Lambda$$

$$(\theta,w)\cdot(1,v)\cdot(\theta,w)^{-1} = (1,\theta v)$$

$$\bar{P} \equiv S/\Lambda$$

$$K = T^6/\bar{P}$$

$$\gamma(\theta_1,v_1)\gamma(\theta_2,v_2) = \gamma((\theta_1,v_1)\cdot(\theta_2,v_2)).$$

$$\varphi(\sigma^1 + 2\pi) = h\cdot\varphi(\sigma^1)$$

$$\theta = \exp[2\pi i(\phi_2 J_{45} + \phi_3 J_{67} + \phi_4 J_{89})].$$

$$Z^i = 2^{-1/2}(X^{2i} + iX^{2i+1}), i = 2,3,4,$$

$$Z^{\bar{\imath}} \equiv \overline{Z^i} = 2^{-1/2}(X^{2i} - iX^{2i+1})$$

$$Z^i(\sigma + 2\pi) = \exp(2\pi i\phi_i)Z^i(\sigma).$$

$$\tilde{\psi}^i(\sigma + 2\pi) = \exp[2\pi i(\phi_i + v)]\tilde{\psi}^i(\sigma)$$

$$\begin{gathered} \alpha^i: n + \phi_i, \alpha^{\bar{\imath}}: n - \phi_i, \\ \tilde{\alpha}^i: n - \phi_i, \tilde{\alpha}^{\bar{\imath}}: n + \phi_i, \\ \tilde{\psi}^i: n - \phi_i(\mathrm{R}), n - \phi_i + \frac{1}{2}(\mathrm{NS}), \\ \tilde{\psi}^{\bar{\imath}}: n + \phi_i(\mathrm{R}), n + \phi_i + \frac{1}{2}(\mathrm{NS}). \end{gathered}$$

$$\gamma = \mathrm{diag}[\exp(2\pi i\beta_1), \ldots, \exp(2\pi i\beta_{16})].$$

$$\lambda^{K\pm} \to \exp(\pm 2\pi i\beta_K)\lambda^{K\pm}.$$

$$\phi_i = \frac{r_i}{N}, \beta_K = \frac{s_K}{N},$$

$$\frac{1}{2}\sum_{i=2}^{4} \phi_i, \frac{1}{2}\sum_{K=1}^{8} \beta_K, \frac{1}{2}\sum_{K=9}^{16} \beta_K.$$

$$\sum_{i=2}^{4} r_i = \sum_{K=1}^{8} s_K = \sum_{K=9}^{16} s_K = 0 \mathrm{mod} 2$$

$$\frac{1}{24} - \frac{1}{8}(2\theta - 1)^2$$

$$L_0 - \tilde{L}_0 = -\sum_{i=2}^{4} (N^i + \tilde{N}^i + \tilde{N}_\psi^i)\phi_i - \sum_{K=1}^{16} N^K\beta_K - \frac{1}{2}\sum_{i=2}^{4} \phi_i(1 - \phi_i) + \frac{1}{2}\sum_{K=1}^{16} \beta_K(1 - \beta_K) \mathrm{mod} 1$$

$$-\frac{1}{2}\sum_{i=2}^{4} \phi_i(1 - \phi_i) + \frac{1}{2}\sum_{K=1}^{16} \beta_K(1 - \beta_K) = \frac{m}{N}$$

$$\sum_{i=2}^{4} (N^i + \tilde{N}^i + \tilde{N}_\psi^i)\phi_i + \sum_{K=1}^{16} N^K\beta_K = \frac{m}{N} \mathrm{mod} 1$$

$$\delta = \frac{1}{2}\left(-\tilde{N}_\psi - 1 + \sum_{i=2}^{4} \phi_i\right),$$

$$\delta = k/N$$

$$\sum_{i=2}^{4} r_i^2 - \sum_{K=1}^{16} s_K^2 = 0 \mathrm{mod} 2N.$$

$$\tilde{T}_F = i \sum_{I,J,K=1}^{18} \tilde{\chi}_I \tilde{\chi}_J \tilde{\chi}_K c_{IJK}$$

$$c_{IJM} c_{KLM} + c_{JKM} c_{ILM} + c_{KIM} c_{JLM} = 0$$

$$18 c_{IKL} c_{JKL} = \delta_{IJ}$$

$$e^{ik \cdot X_R}, k^2 = 6/\alpha'; \; e^{il \cdot X_R} \bar{\partial} X_R, l^2 = 2/\alpha'.$$

$$\delta m \approx \frac{\alpha}{l}$$

$$\delta m \approx \alpha m \ln \frac{1}{ml}$$

$$\frac{m_b}{m_\tau} \approx 3.$$

$$Q_\alpha \to D(\phi)_{\alpha\beta} Q_\beta$$

$$Q_{\mathbf{s}} \to \exp\left(2\pi i \mathbf{s} \cdot \phi\right) Q_{\mathbf{s}}$$

$$\phi_2 + \phi_3 + \phi_4 = 0$$

$$SO(9,1) \to SO(3,1) \times SO(6) \to SO(3,1) \times SU(3),$$

$$16 \to (2,4) + (\overline{2}, \overline{4}) \to (2,3) + (2,1) + (\overline{2}, \overline{3}) + (\overline{2}, 1).$$

$$\phi_2 + \phi_3 = \phi_4 = 0$$

$$P \subset SU(2) \subset SU(3) \subset SO(6).$$

$$t_i : Z^i \to Z^i + R_i,$$
$$u_i : Z^i \to Z^i + \alpha R_i, \alpha = \exp\left(2\pi i/3\right)$$

$$r : Z^2 \to \alpha Z^2, Z^3 \to \alpha Z^3, Z^4 \to \alpha^{-2} Z^4.$$

$$\phi_i = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right),$$

$$\beta_K = (\phi_2, \phi_3, \phi_4, 0^5; 0^8) = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, 0^5; 0^8\right).$$

$$\begin{aligned}
&\alpha^0: \quad \alpha_{-1}^{\mu} |a\rangle, \quad |a\rangle \in (\mathbf{8}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{78}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{248}), \\
&\alpha^1: \quad \alpha_{-1}^{i} |a\rangle, \quad |a\rangle \in (\mathbf{3}, \mathbf{27}, \mathbf{1}), \\
&\alpha^2: \quad \alpha_{-1}^{\bar{\imath}} |a\rangle, \quad |a\rangle \in (\overline{\mathbf{3}}, \overline{\mathbf{27}}, \mathbf{1}).
\end{aligned}$$

$$SU(3) \times E_6 \times E_8 \subset E_8 \times E_8.$$

$$\exp\left[2\pi i(q_1+q_2-2q_3)/3\right]$$

$$\begin{aligned}
&\alpha^0: \quad \tilde{\psi}^{\mu}_{-1/2}|0\rangle_{\mathrm{NS}}, \left|\frac{1}{2},\mathbf{1}\right\rangle_{\mathrm{R}} \ \left|-\frac{1}{2},\overline{\mathbf{1}}\right\rangle_{\mathrm{R}}\\
&\alpha^1: \quad \tilde{\psi}^{i}_{-1/2}|0\rangle_{\mathrm{NS}}, \left|\frac{1}{2},\mathbf{3}\right\rangle_{\mathrm{R}}\\
&\alpha^2: \quad \tilde{\psi}^{\bar{\imath}}_{-1/2}|0\rangle_{\mathrm{NS}}, \left|-\frac{1}{2},\overline{\mathbf{3}}\right\rangle_{\mathrm{R}}
\end{aligned}$$

$$\alpha^{\mu}_{-1}\tilde{\psi}^{\nu}_{-1/2}|0\rangle_{\mathrm{NS}},$$

$$\tilde{\psi}^{\mu}_{-1/2}|a\rangle_{\mathrm{NS}}, a \in (\mathbf{8},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{78},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{248}).$$

$$\alpha^{i}_{-1}\tilde{\psi}^{\bar{J}}_{-1/2}|0,0\rangle_{\mathrm{NS}}$$

$$\tilde{\psi}^{\bar{J}}_{-1/2}|a\rangle_{\mathrm{NS}}, a \in (\mathbf{3},\mathbf{27},\mathbf{1}).$$

$$G_{i\bar{J}}dZ^{i}dZ^{\bar{J}}$$

$$\alpha^{2\pm i3}_{-1}|s_1,\mathbf{1}\rangle_{\mathrm{R}}$$

$$\left|a,\frac{1}{2},\mathbf{3}\right\rangle_{\mathrm{R}}, a \in (\overline{\mathbf{3}},\overline{\mathbf{27}},\mathbf{1})$$

$$\alpha^{\bar{\imath}}_{-1}\left|\frac{1}{2},\mathbf{3}\right\rangle_{\mathrm{R}}.$$

$$h = r t_2^{n_2} t_3^{n_3} t_4^{n_4}, n_i \in \{0,1,2\}.$$

$$Z^{i} = \frac{\exp\left(i\pi/6\right)}{3^{1/2}}(n_2R_2, n_3R_3, n_4R_4).$$

$$e^{isH}, s = \frac{1}{2} - \zeta \mathrm{mod} 1, e^{i\tilde{S}\tilde{H}}, \tilde{s} = -\frac{1}{2} + \zeta \mathrm{mod} 1$$

$$\exp\left(i\tilde{S}_a\tilde{H}_a\right), \tilde{\mathbf{s}} = \left(\pm\frac{1}{2},-\frac{1}{6},-\frac{1}{6},-\frac{1}{6}\right).$$

$$\exp\left[\pi i(\tilde{s}_1+\tilde{s}_2+\tilde{s}_3+\tilde{s}_4)\right] = 1$$

$$\left|+\frac{1}{2}\right\rangle_{h,\mathrm{R}}$$

$$-\frac{2}{24}-\frac{2}{48}+\frac{3}{36}+\frac{3}{72}=0.$$

$$|0\rangle_{h,\mathrm{NS}}$$

$$-\frac{2}{24}+\frac{3}{36}-\frac{3}{36}+\frac{10}{24}-\frac{16}{48}=0.$$

$$\sum_{K=1}^{8} q_K \in 2\mathbf{Z}$$

$$-\frac{2}{24}+\frac{3}{36}+\frac{3}{72}-\frac{10}{48}-\frac{16}{48}=-\frac{1}{2}.$$

$$\lambda_{-1/6}^{1+}\lambda_{-1/6}^{2+}\lambda_{-1/6}^{3+}|0\rangle_{\mathrm{NS,NS}}, \lambda_{-1/2}^{I}|0\rangle_{\mathrm{NS,NS}}, 7 \leq I \leq 16$$
$$\lambda_{-1/6}^{K+}\alpha_{-1/3}^{\bar{J}}|0\rangle_{\mathrm{NS,NS}}, K=1,2,3$$

$$(\mathbf{1},\overline{\mathbf{27}},\mathbf{1})+(\mathbf{3},\mathbf{1},\mathbf{1})^3.$$

$$h_1=(r,0;\gamma), h_2=(1,t_2;\gamma_2), h_3=(1,t_3;\gamma_3), h_4=(1,t_4;\gamma_4),$$

$$h_1 h_2^{n_2} h_3^{n_3} h_4^{n_4}$$

$$j^a = j_{(1)}^a + j_{(2)}^a$$

$$\gamma_2=\left(0^7,\frac{2}{3};0,\frac{1}{3},\frac{1}{3},0^6\right)$$
$$\gamma_3=0$$
$$\gamma_4=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{2}{3},\frac{1}{3},0,\frac{1}{3},\frac{1}{3};\frac{1}{3},\frac{1}{3},0^6\right)$$

$$r': Z^2 \to iZ^2, Z^3 \to iZ^3, Z^4 \to i^{-2}Z^4.$$

$$dZ^4 dZ^4$$

$$dZ^i dZ^{\bar{J}}$$

$$i\sum_{i=2}^{4} \bar{\partial} Z^i \tilde{\psi}^{\bar{\imath}}, i\sum_{i=2}^{4} \bar{\partial} Z^{\bar{\imath}} \tilde{\psi}^{i}$$

$$\sum_{i=2}^{4} \tilde{\psi}^i \tilde{\psi}^{\bar{\imath}},$$

$$i\sum_{i=2}^{4} \partial Z^i \lambda^{(i-1)-}, i\sum_{i=2}^{4} \partial Z^{\bar{\imath}} \lambda^{(i-1)+}.$$

$$\boldsymbol{S}_{\mathrm{het}}=\frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}\left|\tilde{H}_3\right|^2-\frac{\alpha'}{4}\mathrm{Tr}_{\mathrm{v}}(|F_2|^2)\right]$$

$$\tilde{H}_3 = dB_2-\frac{\alpha'}{4}\mathrm{Tr}_{\mathrm{v}}(A_1\wedge dA_1-2iA_1\wedge A_1\wedge A_1/3)$$

$$G_{\mu v}, B_{\mu v}, \Phi, G_{i\bar{J}}, B_{i\bar{J}}, A_\mu^a, A_{i\bar{J}\bar{x}}, A_{\bar{\imath}jx}$$

$$\boldsymbol{S}=\frac{1}{2\kappa_4^2}\int d^4x(-G)^{1/2}\left[R-2\partial_\mu\Phi_4\partial^\mu\Phi_4-\frac{1}{2}e^{-4\Phi_4}|H_3|^2-\frac{1}{2}G^{i\bar{J}}G^{k\bar{l}}\left(\partial_\mu G_{i\bar{l}}\partial^\mu G_{\bar{J}k}+\partial_\mu B_{i\bar{l}}\partial^\mu B_{\bar{J}k}\right)\right]$$

$$\Phi_4 = \Phi - \frac{1}{4}\det G_{mn}$$

$$G_{\mu v \text{ Einstein}} = e^{-2\Phi_4} G_{\mu v};$$

$$G_{i\bar{j}} = G_{\bar{j}i} = G^*_{\bar{j}\bar{\imath}} = G^*_{\bar{\imath}j},$$
$$B_{i\bar{j}} = -B_{\bar{j}i} = -B^*_{\bar{j}\bar{\imath}} = B^*_{\bar{\imath}j}.$$

$$-\frac{1}{2}\int \ d^4x(-G)^{1/2} e^{-4\Phi_4} |H_3|^2 + \int \ a dH_3$$
$$\rightarrow -\frac{1}{2}\int \ d^4x(-G)^{1/2} e^{4\Phi_4} \partial_\mu a \partial^\mu a$$

$$\frac{1}{2\kappa_4^2}\int \ d^4x(-G)^{1/2}\left[R - \frac{2\partial_\mu S^* \partial^\mu S}{(S+S^*)^2} - \frac{1}{2} G^{i\bar{j}} G^{k\bar{l}} \partial_\mu T_{i\bar{l}} \partial^\mu T_{\bar{j}k}\right]$$

$$S = e^{-2\Phi_4} + ia, T_{i\bar{j}} = G_{i\bar{j}} + B_{i\bar{j}}$$

$$\kappa_4^2 K = -\ln(S+S^*) - \ln\ \det\left(T_{i\bar{j}} + T^*_{i\bar{j}}\right)$$

$$-\frac{1}{2}\int \ d^4x(-G)^{1/2} e^{-4\Phi_4}\left|\tilde{H}_3\right|^2 + \int \ a\left[d\tilde{H}_3 + \frac{\alpha'}{4}\mathrm{Tr_v}(F_2 \wedge F_2)\right]$$

$$-\frac{1}{4g_4^2}\int \ e^{-2\Phi_4}\mathrm{Tr_v}(|F_2|^2) + \frac{1}{2g_4^2}\int \ a\mathrm{Tr_v}(F_2 \wedge F_2)$$

$$f_{ab} = \frac{\delta_{ab}}{g_4^2} S$$

$$\kappa_4^2 K = -\ln(S+S^*) - \ln\ \det\left[T_{i\bar{j}} + T^*_{i\bar{j}} - \alpha' \mathrm{Tr_v}\left(A_i A_j^*\right)\right]$$

$$W = \epsilon^{ijk}\mathrm{Tr_v}\left(A_i\left[A_j, A_k\right]\right)$$

$$W = \epsilon^{ijk}\epsilon^{\bar{l}\bar{m}\bar{n}} d^{\bar{x}\bar{y}\bar{z}} A_{il\bar{x}} A_{jm\bar{y}} A_{kn\bar{z}}$$

$$T_{i\bar{j}} = T_i \delta_{i\bar{j}},$$

$$\kappa_4^2 K = -\ln(S+S^*) - \sum_i \ln(T_i + T_i^*) + \alpha' \sum_i \frac{\mathrm{Tr_v}(A_i A_i^*)}{T_i + T_i^*}.$$

$$T_i \rightarrow \frac{a_i T_i - i b_i}{i c_i T_i + d_i}, a_i d_i - b_i c_i = 1$$

$$T_i + T_i^* \rightarrow \frac{T_i + T_i^*}{|i c_i T_i + d_i|^2}$$

$$\kappa_4^2 K \rightarrow \kappa_4^2 K + \mathrm{Re}\left[\sum_i \ln(i c_i T_i + d_i)\right].$$

$$A_i \to \frac{A_i}{ic_iT_i + d_i}$$

$$W \to \frac{W}{\prod_{i=2}^{4}\ (ic_iT_i + d_i)}$$

$$\frac{SU(3,3)}{SU(3) \times SU(3) \times SU(3,3,\mathbf{Z})}.$$

$$\frac{SU(1,1)}{U(1) \times PSL(2,\mathbf{Z})}.$$

$$\lambda_{-1/6}^{K+}\alpha_{-1/3}^{\bar{J}}|0\rangle_{\text{NS,NS}}, K = 1,2,3$$

$$D^a \propto M_{K\bar{J}}^* t_{KL}^a M_{L\bar{J}} = \text{Tr}\big(M^\dagger t^a M\big),$$

$$MM^\dagger = \rho^2 I \Rightarrow M = \rho U$$

$$C_\alpha C_\alpha^* \prod_{i=2}^{4}\ (T_i + T_i^*)^{n_\alpha^i}$$

$$C_\alpha \to C_\alpha \prod_{i=2}^{4}\ (ic_iT_i + d_i)^{n_\alpha^i}$$

$$\frac{1}{g_a^2(\mu)} = \frac{Sk_a}{g_4^2} + \frac{b_a}{16\pi^2}\ln\frac{m_{\text{SU}}^2}{\mu^2} + \frac{1}{16\pi^2}\tilde{\Delta}_a,$$

$$\beta_a = \frac{b_a g_a^3}{16\pi^2}$$

$$\tilde{\Delta}_a = \Delta_a + 16\pi^2 k_a Y$$

$$\Delta_a = \int_\Gamma \frac{d^2\tau}{\tau_2}[\mathcal{B}_a(\tau,\bar{\tau}) - b_a]$$

$$m_{\text{SU}} = \frac{2\exp\,[(1-\gamma)/2]}{3^{3/4}(2\pi\alpha')^{1/2}},$$

$$r'^2: Z^2 \to -Z^2, Z^3 \to -Z^3, Z^4 \to +Z^4.$$

$$\Delta_a = c_a - \sum_i \frac{b_a^i|P^i|}{|P|}\{\ln\,[(T_i + T_i^*)|\eta(T_i)|^4] + \ln\,[(U_i + U_i^*)|\eta(U_i)|^4]\},$$

$$\ln\,[(T_i + T_i^*)|\eta(T_i)|^4] = \ln\,(T_i + T_i^*) + 4\text{Re}[\ln\,\eta(T_i)]$$

$$X^m \to -X^m, m = 6,7,8,9$$

$$Z^i \to \exp\,(2\pi i/3)Z^i, i = 3,4$$

$$G_{MN}=\begin{bmatrix} f(y)\eta_{\mu\nu} & 0 \\ 0 & G_{mn}(y)\end{bmatrix}$$

$$\begin{aligned}\delta\psi_\mu &= \nabla_\mu\varepsilon\\ \delta\psi_m &= \left(\partial_m+\frac{1}{4}\Omega^-_{mnp}\Gamma^{np}\right)\varepsilon\\ \delta\chi &= \left(\Gamma^m\partial_m\Phi-\frac{1}{12}\Gamma^{mnp}\widetilde{H}_{mnp}\right)\varepsilon\\ \delta\lambda &= F_{mn}\Gamma^{mn}\varepsilon\end{aligned}$$

$$\Omega^\pm_{MNP}=\omega_{MNP}\pm\frac{1}{2}H_{MNP}$$

$$16\to(2,4)+(\overline{2},\overline{4}).$$

$$\varepsilon(y)\to\varepsilon_{\alpha\beta}(y)+\varepsilon^*_{\alpha\beta}(y)$$

$$\varepsilon_{\alpha\beta}=u_\alpha\zeta_\beta(y)$$

$$\widetilde{H}_{mnp}=0.$$

$$\partial_m\Phi=0$$

$$G_{\mu\nu}=\eta_{\mu\nu}$$

$$\nabla_m\zeta=0$$

$$[\nabla_m,\nabla_n]\zeta=\frac{1}{4}R_{mnpq}\Gamma^{pq}\zeta=0$$

$$F_{ij}=F_{\bar{\imath}\bar{\jmath}}=0, G^{i\bar{\jmath}}F_{i\bar{\jmath}}=0$$

$$d\widetilde{H}_3=\frac{\alpha'}{4}[\mathrm{tr}(R_2\wedge R_2)-\mathrm{Tr}_v(F_2\wedge F_2)]$$

$$R_{mn}=0,\nabla^m F_{mn}=0.$$

$$\chi(K)=\sum_{p=0}^{d}(-1)^p b_p.$$

$$\Delta_d=*d*d+d*d*=(d+*d*)^2$$

$$b_p=b_{d-p}$$

$$\int_K\omega_p\wedge\alpha_{d-p}=\int_{N(\omega)}\alpha_{d-p}$$

$$z'^i(z^j)$$

$$G_{ij}=G_{\bar{\imath}\bar{\jmath}}=0.$$

$$\omega_{i_1\cdots i_p\bar{\jmath}_1\cdots\bar{\jmath}_q}$$

$$\partial = dz^i \partial_i, \bar{\partial} = d\bar{z}^{\bar{\imath}} \partial_{\bar{\imath}}.$$

$$\partial^2 = \bar{\partial}^2 = 0.$$

$$\int d^n z d^n \bar{z} (G)^{1/2} G^{\bar{\imath}'} \cdots G^{j'\bar{\jmath}'} \cdots \left(\omega_{i\cdots\bar{\jmath}} \cdots\right)^* \omega_{i'\cdots\bar{\jmath}'\cdots},$$

$$\Delta_\partial = \partial\partial^\dagger + \partial^\dagger\partial, \Delta_{\bar{\partial}} = \bar{\partial}\bar{\partial}^\dagger + \bar{\partial}^\dagger\bar{\partial}.$$

$$J_{1,1} = iG_{i\bar{\jmath}} dz^i d\bar{z}^{\bar{\jmath}}$$

$$dJ_{1,1} = 0$$

$$G_{i\bar{\jmath}} = \frac{\partial}{\partial z^i} \frac{\partial}{\partial \bar{z}^{\bar{\jmath}}} K(z, \bar{z}).$$

$$K'(z, \bar{z}) = K(z, \bar{z}) + f(z) + f(z)^*$$

$$\Delta_d = 2\Delta_{\bar{\partial}} = 2\Delta_\partial.$$

$$H_{\bar{\partial}}^{p,q}(K) = H_{\partial}^{p,q}(K) \equiv H^{p,q}(K)$$

$$b_k = \sum_{p=0}^{k} h^{p,k-p}$$

$$h^{p,q} = h^{q,p}$$

$$h^{n-p,n-q} = h^{p,q}$$

$$J_{1,1} = \sum_A v^A \omega_{1,1A}$$

$$\mathcal{R}_{1,1} = R_{i\bar{\jmath}} dz^i d\bar{z}^{\bar{\jmath}}$$

$$h^{p,0} = h^{3-p,0}.$$

$$b_1 = h^{1,0} = h^{0,1} = 0.$$

$$h^{3,3} \qquad c^1$$
$$h^{3,2} h^{2,3} \qquad 0$$
$$h^{3,1} h^{2,2} h^{1,3} \qquad 0$$
$$h^{3,0} h^{2,1} h^{1,2} h^{0,3} = 1 h^{1,1} 0$$
$$h^{2,0} h^{1,1} h^{0,2} h^{2,1} 1 .$$
$$h^{1,0} h^{0,1} \qquad 0 h^{1,1} 0$$
$$h^{0,0} \qquad 0$$
$$h^{0,1} \qquad 0^0$$

$$\chi = 2(h^{1,1} - h^{2,1}).$$

$$G_{i\bar{\jmath}} = \left(1 + \frac{\rho^6}{r^6}\right)^{1/3} \left[\delta_{i\bar{\jmath}} - \frac{\rho^6 w_i \bar{w}_{\bar{\jmath}}}{r^2(\rho^6 + r^6)}\right],$$

$$w_i \cong \exp(2\pi i/3) w_i$$

$$h^{1,1} = 36, h^{2,1} = 0, \chi = 72$$

$$(z_1, z_2, \dots, z_{n+1}) \cong (\lambda z_1, \lambda z_2, \dots, \lambda z_{n+1})$$

$$G(\lambda z_1, \dots, \lambda z_{n+1}) = \lambda^k G(z_1, \dots, z_{n+1})$$

$$G(z_1, \dots, z_{n+1}) = 0$$

$$h^{1,1} = 1, h^{2,1} = 101, \chi = -200$$

$$z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0$$

$$\tilde{\psi}^i \to \exp(i\theta)\tilde{\psi}^i.$$

$$iG_{\bar{\imath}j}\bar{\partial}X^{\bar{\imath}}\tilde{\psi}^j + iG_{i\bar{\jmath}}\bar{\partial}X^i\tilde{\psi}^{\bar{\jmath}},$$

$$\nabla_M \nabla^M = \partial_\mu \partial^\mu + \nabla_m \nabla^m$$
$$\Gamma_M \nabla^M = \Gamma_\mu \partial^\mu + \Gamma_m \nabla^m$$

$$E_8 \times E_8 \to SU(3) \times E_6 \times E_8$$

$$a: (\mathbf{1}, \mathbf{78}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{248})$$
$$ix: (\mathbf{3}, \mathbf{27}, \mathbf{1}), \overline{\imath x}: (\overline{\mathbf{3}}, \overline{\mathbf{27}}, \mathbf{1}), i\bar{\jmath}: (\mathbf{8}, \mathbf{1}, \mathbf{1}).$$

$$a_{i\bar{l}\bar{m}x} = a_{i,jx} G^{j\bar{k}} \Omega_{\bar{k}\bar{l}\bar{m}}$$

$$16 \to (2,1) + (2,3) + (\bar{2}, \bar{1}) + (\bar{2}, \bar{3}).$$

$$|h^{2,1} - h^{1,1}| = \frac{|\chi|}{2}.$$

$$g_{i\bar{l}\bar{m}} = g_{ij} G^{j\bar{k}} \Omega_{\bar{k}\bar{l}\bar{m}}$$

$$\varphi(x, y) = \sum_m \phi_m(x) f_m(y).$$

$$\mathcal{L}_4(\phi) = \int d^6 y \mathcal{L}_{10}(\varphi)$$

$$\varphi = \varphi_1 + \varphi_{\mathrm{h}}$$

$$m\varphi_{\mathrm{h}}^2 + g\varphi_{\mathrm{h}}\varphi_1^2$$

$$-\frac{g^2}{4m}\varphi_1^4$$

$$a_{i,\bar{j}\bar{x}}(x,y)=\sum_A \phi^A\bar{x}(x)\omega_{Ai\bar{j}}(y),$$
$$\lambda_{i,\bar{j}\bar{x}}(x,y)=\sum_A \lambda^A\bar{x}(x)\omega_{Ai\bar{j}}(y),$$

$$\int d^6y\mathrm{Tr}_{\mathrm{V}}\left(\bar{\lambda}\Gamma^m[A_m,\lambda]\right)$$

$$d^{\bar{x}\bar{y}\bar{z}}\bar{\lambda}^A\bar{x}\lambda^B\bar{y}\phi^C\bar{z}\int_K \omega_{1,1A}\wedge\omega_{1,1B}\wedge\omega_{1,1C}$$

$$W(\phi)=d^{\bar{x}\bar{y}\bar{z}}\phi^A{}_{\bar{x}}\phi^B{}_{\bar{y}}\phi^C_{\bar{z}}\int_K \omega_{1,1A}\wedge\omega_{1,1B}\wedge\omega_{1,1C}$$

$$(N_A,N_B,N_C)=\int_K \omega_{1,1A}\wedge\omega_{1,1B}\wedge\omega_{1,1C}$$

$$(N^A,N_B)=\int_{N^A}\omega_{1,1B}=\delta_B^A$$

$$\left(g_{i\bar{j}}+b_{i\bar{j}}\right)(x,y)=\sum_A T^A(x)\omega_{Ai\bar{j}}(y)$$

$$G_{A\bar{B}}=\frac{1}{V}\int d^6y(\det G)^{1/2}G^{i\bar{k}}G^{l\bar{j}}\omega_{Ai\bar{j}}\omega^*_{Bk\bar{l}}$$

$$J_{1,1}+iB_{1,1}=T^A\omega_{1,1A},T^A=v^A+ib^A$$

$$G_{A\bar{B}}=-\frac{\partial^2}{\partial T^A\partial T^{B*}}\ln W(v)$$

$$W(v)=(N_A,N_B,N_C)v^Av^Bv^C=\int_K J_{1,1}\wedge J_{1,1}\wedge J_{1,1}$$

$$K_1(T,T^*)=-\ln W(v)$$

$$G'_{A\bar{B}}=\exp\left[\kappa_4^2(K_2-K_1)/3\right]G_{A\bar{B}}$$

$$a_{i,jx}(x,y)=\frac{1}{2}\sum_a \chi_x^a(x)\omega_{ai\bar{k}\bar{l}}(y)\Omega_j^{\bar{k}\bar{l}}(y),$$
$$\lambda_{i,jx}(x,y)=\frac{1}{2}\sum_a \lambda_x^a(x)\omega_{ai\bar{k}\bar{l}}(y)\Omega_j^{\bar{k}\bar{l}}(y),$$

$$G_{a\bar{b}}=-\frac{\int_K \omega_{1,2a}\wedge\omega^*_{1,2b}}{\int_K \Omega_{3,0}\wedge\Omega^*_{3,0}}=-\frac{\partial}{\partial X^a}\frac{\partial}{\partial X^{\bar{b}}}K_2(X,X^*)$$

$$K_2(X,X^*)=\ln\left(i\int_K \Omega_{3,0}\wedge\Omega^*_{3,0}\right)$$

$$\{A^I, B_J\}, I, J = 0, \dots h^{2,1}$$

$$(A^I, B_J) = \delta_J^I, \#(A^I, A^J) = \#(B_I, B_J) = 0$$

$$Z^I = \int_{A^I} \Omega_{3,0}$$

$$(Z^0, Z^1, \dots, Z^n) \cong (\lambda Z^0, \lambda Z^1, \dots, \lambda Z^n)$$

$$\mathcal{G}_I(Z) = \int_{B_I} \Omega_{3,0}$$

$$\mathcal{G}_I = \frac{\partial \mathcal{G}}{\partial Z^I}, \mathcal{G}(\lambda Z) = \lambda^2 \mathcal{G}(Z)$$

$$K_2(Z, Z^*) = \ln\ \mathrm{Im}(Z^{I*} \partial_I \mathcal{G}(Z))$$

$$W(Z, \chi) = \frac{\chi^a \chi^b \chi^c}{3!} \frac{\partial^3 \mathcal{G}(Z)}{\partial Z^a \partial Z^b \partial Z^c}$$

$$G'_{a\bar{b}} = \exp\,[\kappa_4^2 (K_1 - K_2)/3] G_{a\bar{b}}$$

$$\begin{bmatrix} A'^I \\ B'_J \end{bmatrix} = S \begin{bmatrix} A^I \\ B_J \end{bmatrix}$$

$$\begin{bmatrix} Z'^I \\ \mathcal{G}'_J \end{bmatrix} = S \begin{bmatrix} Z^I \\ \mathcal{G}_J \end{bmatrix}$$

$$\frac{1}{2\pi\alpha'} \int_M B_{1,1} = \frac{n_A b^A}{2\pi\alpha'}$$

$$T^A \to T^A + i\epsilon^A$$

$$T^A = c^A T$$

$$-3\ln\,(T + T^*),$$

$$\frac{1}{2\pi\alpha'} \int\ d^2 z G_{i\bar{j}} \left(\partial_z Z^i \partial_{\bar{z}} Z^{\bar{j}} + \partial_z Z^{\bar{\imath}} \partial_{\bar{z}} Z^j\right)$$

$$\frac{1}{2\pi\alpha'} \int_M J_{1,1} = \frac{1}{2\pi\alpha'} \int\ d^2 z G_{i\bar{j}} \left(\partial_z Z^i \partial_{\bar{z}} Z^{\bar{j}} - \partial_z Z^{\bar{\imath}} \partial_{\bar{z}} Z^j\right) = \frac{n_A v^A}{2\pi\alpha'}$$

$$\exp\,(-n_A T^A / 2\pi\alpha')$$

$$Q = \frac{1}{2\pi i} \oint\ (dz j_z - d\bar{z} j_{\bar{z}})$$

$$j_z \bar{\partial} X^\mu e^{ik\cdot X}, \partial X^\mu j_{\bar{z}} e^{ik\cdot X}.$$

$$Q=\frac{1}{2\pi i}\oint\ (dzd\theta J-d\bar{z}d\bar{\theta}\tilde{J})$$

$$R_8(2\pi R_9)=\alpha'/R_8(0)$$

$$\Phi(2\pi R_9)=-\Phi(0),g(2\pi R_9)=1/g(0)$$

$$N_{+\frac{1}{2},r}-N_{-\frac{1}{2},r}=\mathrm{Tr}_{r,\ker(G_0)}\left[i\exp\left(\pi i\tilde{F}_K\right)\right],$$

$$N_{+\frac{1}{2},r}-N_{-\frac{1}{2},r}=\mathrm{Tr}_{r}\left[i\exp\left(\pi i\tilde{F}_K\right)\right]$$

$$\langle\alpha,\mathrm{out}\mid\beta,\mathrm{in}\rangle=\left\langle\overline{\mathcal{V}}_\alpha\mathcal{V}_\beta\right\rangle,$$

$$\langle\alpha,\text{ out }|\beta,\text{ in }\rangle=\left\langle\theta\cdot\overline{\mathcal{V}}_\alpha\theta\cdot\mathcal{V}_\beta\right\rangle=\langle\theta\bar{\beta},\text{ out }|\theta\bar{\alpha},\text{ in }\rangle.$$

$$\langle CPT\cdot\beta,\text{ out }|CPT\cdot\alpha,\text{ in }\rangle=\langle\alpha,\text{ out }|\beta,\text{ in }\rangle,$$

$$\mathcal{A}_i(z,\bar{z})\mathcal{A}_i(0,0)\sim(z\bar{z})^{-2}\bar{z}^{2(q+q^2-s_0^2)}\exp\left(2q\tilde{\phi}+2is_0\tilde{H}_0\right)$$

$$\boldsymbol{S}_\theta=\frac{\theta}{8\pi^2}\int\ \mathrm{Tr}(F_2\wedge F_2)$$

$$|\theta|<10^{-9}.$$

$$\frac{1}{2g_4^2}\int\ aF_2^a\wedge F_2^a$$

$$a\rightarrow a+\epsilon$$

$$\exp\left(-n_AT^A/2\pi\alpha'\right)=\exp\left[-n_A(v^A+ib^A)/2\pi\alpha'\right].$$

$$g_{10}^2=\frac{4\kappa_{10}^2}{\alpha'}$$

$$g_4^2=g_{10}^2/V,\kappa_4^2=\kappa_{10}^2/V,$$

$$g_4^2=\frac{4\kappa_4^2}{\alpha'}$$

$$g_{\mathrm{YM}}^2=\frac{4\kappa^2}{\alpha'}$$

$$g_{\mathrm{YM}}^2=\frac{2\kappa^2}{\hat{k}\alpha'}$$

$$-\frac{1}{4g_{\mathrm{YM}}^2}F_{\mu\nu}^aF^{a\mu\nu}$$

$$\frac{g_{\mathrm{YM}}^2}{\kappa}=2(2\pi)^{7/2}\alpha'\ (\text{ type I, }d=10).$$

$$\frac{g_{\mathrm{YM}}^2}{\kappa} = \frac{2(2\pi)^{7/2}\alpha'}{V^{1/2}} \text{ (type I, } d < 10).$$

$$T_B = T_B^{\mathrm{s}} + T_B'$$

$$j_{-1}^a \cdot 1 = j^a$$

$$\partial X^\mu \tilde{\psi}^a e^{ik\cdot X}$$

$$\tilde{G}_{-1/2} \cdot \tilde{\psi}^a = \tilde{j}^a$$

$$\tilde{G}_{1/2} \cdot \tilde{j}^a = 2\tilde{L}_0 \cdot \tilde{\psi}^a = \tilde{\psi}^a$$

$$\tilde{\psi}^a(\bar{z})\tilde{\psi}^b(0) \sim \frac{k\delta^{ab}}{\bar{z}}$$

$$\tilde{j}^a(\bar{z})\tilde{\psi}^b(0) \sim \frac{if^{abc}}{\bar{z}}\tilde{\psi}^c(0)$$

$$\tilde{T}_F(\bar{z})\tilde{\psi}^a(0) \sim \frac{1}{\bar{z}}\tilde{j}^a(0)$$

$$\tilde{j}^a(\bar{z})\tilde{j}^b(0) \sim \frac{k\delta^{ab}}{\bar{z}^2} + \frac{if^{abc}}{\bar{z}}\tilde{j}^c(0)$$

$$\tilde{T}_F(\bar{z})\tilde{j}^a(0) \sim \frac{1}{\bar{z}^2}\tilde{\psi}^a(0) + \frac{1}{\bar{z}}\bar{\partial}\tilde{\psi}^a(0)$$

$$\tilde{j}^a = \tilde{j}_\psi^a + \tilde{j}'^a$$

$$\tilde{j}_\psi^a = -\frac{i}{2k} f^{abc}\tilde{\psi}^b\tilde{\psi}^c$$

$$\tilde{T}_F = \tilde{T}_F^{\mathrm{s}} + \tilde{T}_F''$$

$$\tilde{T}_F^{\mathrm{s}} = -\frac{i}{6k^2} f^{abc}\tilde{\psi}^a\tilde{\psi}^b\tilde{\psi}^c + \frac{1}{k}\tilde{\psi}^a\tilde{j}'^a$$

$$\tilde{T}_B = \tilde{T}_B^\psi + \tilde{T}_B' + \tilde{T}_B'',$$

$$\tilde{T}_B^\psi = -\frac{1}{2k}\tilde{\psi}^a\bar{\partial}\tilde{\psi}^a$$

$$\tilde{T}_B' = \frac{1}{2(k' + h(g))} :\tilde{j}'\tilde{j}':$$

$$\tilde{c}^\psi = \frac{\dim(g)}{2}, \tilde{c}' = \frac{k'\dim(g)}{k' + h(g)}, \tilde{c}'' = \tilde{c} - \tilde{c}^\psi - \tilde{c}'$$

$$\tilde{c}^\psi + \tilde{c}' = \frac{(3k' + h(g))\dim(g)}{2(k' + h(g))}.$$

$$\frac{\dim(g)}{2} \leq \tilde{c}^\psi + \tilde{c}' \leq \frac{3\dim(g)}{2}$$

$$\tilde{T}_F^{\mathrm{s}} = i\tilde{\psi}\bar{\partial}H, \tilde{T}_B^\psi + \tilde{T}_B' = -\frac{1}{2}\tilde{\psi}\bar{\partial}\tilde{\psi} - \frac{1}{2}\bar{\partial}H\bar{\partial}H$$

$$S_\alpha \mathcal{V}_K e^{ik_\mu X^\mu}$$

$$\tilde{G}_0^2 = \tilde{L}_0 - \frac{\tilde{c}}{24} \geq 0$$

$$\tilde{L}_0^i = \frac{\tilde{c}^i}{24}$$

$$\tilde{L}_0^\psi + \tilde{L}_0' - \frac{\tilde{c}^\psi + \tilde{c}'}{24} \geq \frac{h(g)\dim(g)}{24(k' + h(g))} > 0$$

$$\psi^a \tilde{\psi}^\mu e^{ik\cdot X}, \psi^\mu \tilde{\psi}^a e^{ik\cdot X}$$

$$\tilde{c} \geq \frac{8}{2} + \tilde{c}^{SU(3),1} + \frac{3}{2} + \tilde{c}^{SU(2),1} + \tilde{c}^{U(1)} = 4 + 2 + \frac{3}{2} + 1 + \frac{3}{2} = 10$$

$$\frac{m_{\mathrm{s}}}{m_{\mathrm{grav}}} = g_{\mathrm{YM}}(\hat{k}/2)^{1/2}.$$

$$\frac{Y}{2} = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right).$$

$$g_3 = g_2 = g_1 = g_{SU(5)}$$

$$\frac{1}{2}g'Y = g_{U(1)}t^{U(1)} \Rightarrow g' = (3/5)^{1/2}g_1.$$

$$(5/3)^{1/2}g' = g_2 = g_3$$

$$\mu\frac{\partial}{\partial\mu}g_i = \frac{b_i}{16\pi^2}g_i^3$$

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(m_{\mathrm{GUT}}) + \frac{b_i}{4\pi}\ln\left(m_{\mathrm{GUT}}^2/\mu^2\right)$$

$$b_i = -\frac{11}{3}T_g + \frac{1}{3}\sum_{\substack{\text{complex}\\ \text{scalars}}} T_r + \frac{2}{3}\sum_{\substack{\text{Weyl}\\ \text{fermions}}} T_r$$

$$\sin^2\theta_{\mathrm{w}}(m_Z) = 0.212 \pm 0.003$$

$$\sin^2\theta_{\mathrm{w}}(m_Z) = 0.234 \pm 0.003$$

$$\sin^2\theta_{\mathrm{w}}(m_Z) = 0.2313 \pm 0.0003$$

$$m_{\mathrm{GUT}} = 10^{16.1\pm0.3}\mathrm{GeV}, \alpha_{\mathrm{GUT}}^{-1} \approx 25.$$

$$m_{\mathrm{SU}} = k^{1/2}g_{\mathrm{YM}} \times 5.27 \times 10^{17}\mathrm{GeV} \to 3.8 \times 10^{17}\mathrm{GeV}$$

$$SU(5)' \times U(1) \subset SO(10)$$
$$SU(4) \times SU(2)_L \times SU(2)_R \subset SO(10)$$
$$SU(3)_C \times SU(3)_L \times SU(3)_R \subset E_6$$

$$\Delta_a = c_a - \sum_i \frac{b_a^i |G^i|}{|G|} \ln\left[(T_i + T_i^*)|\eta(T_i)|^4 (U_i + U_i^*)|\eta(U_i)|^4\right].$$

$$\Delta_a \approx \sum_i \frac{b_a^i |G^i|}{|G|} \frac{\pi(T_i + T_i^*)}{6}$$

$$K \times \frac{S_1}{\mathbf{Z}_2}$$

$$10^2 \mathrm{GeV} \lesssim m_{\mathrm{sp}} \lesssim 10^3 \mathrm{GeV}$$

$$\frac{\alpha_2}{\alpha_3} = \frac{\hat{k}_3}{\hat{k}_2} = \frac{k_3}{k_2}.$$

$$\partial H \tilde{\psi} e^{ik\cdot X}$$

$$Q' = Q_{\mathrm{EM}} + \frac{T}{3},$$

$$Q' = Q_{\mathrm{EM}} + \frac{1}{3} T = \frac{1}{2} Y + I_3 + \frac{1}{3} T$$

$$\mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right) + \mathrm{diag}\left(0,0,0, \frac{1}{2}, -\frac{1}{2}\right) + \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, 0,0\right) = \mathrm{diag}(0,0,-1,1,0)$$

$$j' = \lambda^{6-}\lambda^{6+} - \lambda^{7-}\lambda^{7+} = i\partial(H_7 - H_6).$$

$$\lambda^{K+}(\sigma_1 + 2\pi) = \exp(2\pi i v_K)\lambda^{K+}(\sigma_1)$$

$$\exp\left[i \sum_K (1/2 - v_K) H_K\right]$$

$$Q' = v_6 - v_7$$

$$\begin{aligned} j^3_{SU(3)} &= \frac{i}{2}\partial(H^4 - H^5) \\ j^8_{SU(3)} &= \frac{i}{2\times 3^{1/2}}\partial(H^4 + H^5 - 2H^6) \\ j^3_{SU(2)} &= \frac{i}{2}\partial(H^7 - H^8) \\ j_{Y/2} &= \frac{i}{6}\partial[-2(H^4 + H^5 + H^6) + 3(H^7 + H^8)] \end{aligned}$$

$$j' = j_{Y/2} + j^3_{SU(2)} + \frac{2}{3^{1/2}} j^8_{SU(3)} = i\partial(H^7 - H^6)$$

$$\exp[i(H^6 - H^7)]$$

$$\tau_P \approx \left(\frac{M_X}{10^{15}\mathrm{GeV}}\right)^4 \times 10^{31\pm 1} \text{ years}.$$

$$\mu_1 H_1 L + \eta_1 U^c D^c D^c + \eta_2 Q L D^c + \eta_3 L L E^c + \frac{\lambda_1}{M} QQQL + \frac{\lambda_2}{M} U^c U^c D^c E^c + \frac{\lambda_3}{M} LLH_2 H_2$$

$$\tilde{J}_\alpha = e^{-\tilde{\phi}/2} \tilde{S}_\alpha \tilde{\Sigma}, \tilde{J}_{\dot{\alpha}} = e^{-\tilde{\phi}/2} \tilde{S}_{\dot{\alpha}} \bar{\tilde{\Sigma}}$$

$$\tilde{\Sigma}(\bar{z}) \bar{\tilde{\Sigma}}(0) = \bar{z}^{-3/4}$$
$$\tilde{\Sigma}(\bar{z}) \tilde{\Sigma}(0) = \bar{z}^{3/4}$$

$$\tilde{\Sigma}(\bar{z}) \bar{\tilde{\Sigma}}(0) = \bar{z}^{-3/4} \left(1 + \frac{\bar{z}}{2} \tilde{j} + \cdots \right)$$

$$\tilde{J}_\alpha(\bar{z}) \tilde{J}_{\dot{\beta}}(0) \sim \frac{1}{2^{1/2} \bar{z}} (C\Gamma^\mu)_{\alpha\dot{\beta}} e^{-\tilde{\phi}} \tilde{\psi}_\mu(0).$$

$$\tilde{\Sigma}(\bar{z}) \tilde{\Sigma}(0) = O(\bar{z}^{3/4}).$$

$$\left\langle \tilde{\Sigma}(\bar{z}_1) \bar{\tilde{\Sigma}}(\bar{z}_2) \tilde{\Sigma}(\bar{z}_3) \bar{\tilde{\Sigma}}(\bar{z}_4) \right\rangle = \left(\frac{\bar{z}_{13} \bar{z}_{24}}{\bar{z}_{12} \bar{z}_{14} \bar{z}_{23} \bar{z}_{34}} \right)^{3/4} f(\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4),$$

$$\left\langle \tilde{j}(\bar{z}_2) \tilde{\Sigma}(\bar{z}_3) \bar{\tilde{\Sigma}}(\bar{z}_4) \right\rangle = \frac{3 \bar{z}_{34}^{1/4}}{2 \bar{z}_{23} \bar{z}_{24}}$$

$$\tilde{j}(\bar{z}) \tilde{\Sigma}(0) \sim \frac{3}{2\bar{z}} \tilde{\Sigma}(0)$$
$$\tilde{j}(\bar{z}) \tilde{\Sigma}(0) \sim -\frac{3}{2\bar{z}} \bar{\tilde{\Sigma}}(0)$$
$$\tilde{j}(\bar{z}) \tilde{j}(0) \sim \frac{3}{\bar{z}^2}$$

$$\tilde{j}(\bar{z}) = 3^{1/2} i \bar{\partial} \tilde{H}(\bar{z})$$

$$\tilde{T}_B = -\frac{1}{2} \bar{\partial} \tilde{H} \bar{\partial} \tilde{H} + \tilde{T}_B'$$

$$\tilde{\Sigma} = \exp\left(3^{1/2} i \tilde{H}/2\right) \tilde{\Sigma}',$$

$$\tilde{\Sigma} = \exp\left(3^{1/2} i \tilde{H}/2\right), \bar{\tilde{\Sigma}} = \exp\left(-3^{1/2} i \tilde{H}/2\right)$$

$$\tilde{T}_F(\bar{z}) \tilde{\Sigma}(0) = O(\bar{z}^{-1/2}), \tilde{T}_F(\bar{z}) \bar{\tilde{\Sigma}}(0) = O(\bar{z}^{-1/2})$$

$$\tilde{T}_F = \tilde{T}_F^+ + \tilde{T}_F^-$$
$$\tilde{T}_F^+ \propto \exp\left(i\tilde{H}/3^{1/2}\right), \tilde{T}_F^- \propto \exp\left(-i\tilde{H}/3^{1/2}\right).$$

$$\tilde{j}(\bar{z}) \tilde{T}_F^+(0) \sim \frac{1}{\bar{z}} \tilde{T}_F^+(0), \tilde{j}(\bar{z}) \tilde{T}_F^-(0) \sim -\frac{1}{\bar{z}} \tilde{T}_F^-(0).$$

$$\tilde{J}_{\frac{11}{22}} = \exp\left[\frac{1}{2}\left(-\tilde{\phi} + i\tilde{H}_0 + i\tilde{H}_1 + 3^{1/2} i\tilde{H}\right)\right]$$

$$\tilde{j}_{\mathrm{GSO}} = \frac{1}{2} \bar{\partial}\left(-\tilde{\phi} + i\tilde{H}_0 + i\tilde{H}_1 + 3^{1/2} i\tilde{H}\right)$$

$$f_{ab} = \frac{2\hat{k}_a \delta_{ab}}{g_4^2} S$$

$$\int F_2 \wedge F_2$$

$$S \to S + i\epsilon$$

$$S \to tS, G_{\mu v 4\mathrm{E}} \to tG_{\mu v 4\mathrm{E}}$$

$$\boldsymbol{S} \to t\boldsymbol{S}$$

$$f_{ab} = \delta_{ab} \frac{S}{8\pi^2}.$$

$$\frac{g_{\mathrm{YM}}^2}{8\pi^2} = \frac{1}{\mathrm{Re}\langle S \rangle}$$

$$\int d^4x(-G_{4\mathrm{E}})^{1/2} \exp\left(\kappa^2 K\right)\left(K^{\bar{\imath}j} W_{;i}^* W_{;j} - 3\kappa^2 W^* W\right)$$

$$\boldsymbol{S}_L \to t^{1-L} \boldsymbol{S}_L$$

$$\mathcal{G}_{ij} = \left\langle \left\langle \frac{\partial \mathcal{L}_{\mathrm{ws}}}{\partial \phi^i} \right| \frac{\partial \mathcal{L}_{\mathrm{ws}}}{\partial \phi^j} \right\rangle.$$

$$\frac{1}{2} \mathcal{G}_{ij} \partial_\mu \phi^i \partial^\mu \phi^j$$

$$(\phi_2, \phi_3, \phi_4) = \left(0,0, \frac{1}{2}\pi\right)$$

$$(\partial_t \partial_{\bar{t}} K)^{-1} |\partial_t W + \kappa^2 \partial_t K W|^2 = 3\kappa^2 |W|^2$$

$$W(\phi) = \partial_i W(\phi) = D^a(\phi, \phi^*) = 0.$$

$$V = \mathrm{Re}[(S/8\pi^2) + f_1(T)] \frac{D^2}{2}$$

$$D = \frac{1}{\mathrm{Re}[(S/8\pi^2) + f_1(T)]} \left(2\xi - i\kappa^2 K_{,i} \frac{\delta \phi^i}{\delta \lambda}\right)$$

$$S^{-L-1}$$

$$\delta S = iq\delta\lambda$$

$$V \propto \frac{q^2}{(S + S^*)^3}$$

$$\delta \frac{1}{4\pi^2} \int \mathrm{Im}(S) F_2^a \wedge F_2^a = \frac{q\delta\lambda}{4\pi^2} \int F_2^a \wedge F_2^a.$$

$$D=\frac{q}{(S+S^*)}+\sum_{\phi^i\neq S} q_i\phi^{i*}\phi^i$$

$$\delta T^A=iq^A\delta\lambda$$

$$V=\frac{(q^A\partial_A K)^2}{(S+S^*)}$$

$$\delta T^A\propto i\delta\lambda\int_{N^A}F_2$$

$$\beta_8=\frac{b_8}{16\pi^2}g_{E_8}^3,-b_8\gg 1$$

$$g_{E_8}^2(\mu)=\frac{8\pi^2}{\mathrm{Re}(S)+b_8\ln{(m_s/\mu)}}$$

$$\Lambda_8=m_s\exp{[-\mathrm{Re}(S)/|b_8|]}$$

$$\left|\left\langle(\bar{\lambda}\lambda)_{\text{hidden}}\right\rangle\right|\approx\Lambda_8^3.$$

$$\kappa F_S(\bar{\lambda}\lambda)_{\text{hidden}}.$$

$$\kappa F_S\left\langle(\bar{\lambda}\lambda)_{\text{hidden}}\right\rangle\approx F_S\kappa m_{\mathrm{SU}}^3\exp{(-3S/|b_8|)}.$$

$$W\approx\kappa m_{\mathrm{SU}}^3\exp{(-3S/|b_8|)}$$

$$F_S=\frac{\partial W}{\partial S}\approx\kappa m_{\mathrm{SU}}^3\exp{(-3S/|b_8|)}$$

$$V\approx\kappa^2m_{\mathrm{SU}}^6(S+S^*)^k\exp{[-3(S+S^*)/|b_8|]}$$

$$\kappa\langle F_S\rangle\bar{\lambda}\lambda\approx\kappa^2m_{\mathrm{SU}}^3\exp{(-3\langle S\rangle/|b_8|)}\bar{\lambda}\lambda$$

$$m_\lambda\approx\kappa^2m_{\mathrm{SU}}^3\exp{(-3\langle S\rangle/|b_8|)}\approx\exp{(-3\langle S\rangle/|b_8|)}\times 10^{18}\mathrm{GeV}$$

$$\frac{m_{\text{ew}}}{m_{\text{grav}}}\approx 10^{-16}$$

$$m_{\text{SUSY}}^2\approx m_{\text{sp}}m_{\text{grav}}$$

$$m_{\text{sp}}\approx\kappa F_S$$

$$m_\Phi\approx m_{\text{sp}}$$

$$\begin{aligned}K&=-\ln{(S+S^*)}-3\ln{(T+T^*)}\\ W&=-w+\kappa m_{\mathrm{SU}}^3\exp{(-3S/|b_8|)}\end{aligned}$$

$$V = \frac{(S+S^*)\left|W_{;S}\right|^2}{(T+T^*)^3}$$

$$W_{;S} = \frac{w}{S+S^*} - \kappa m_{\mathrm{SU}}^3 \exp\left(-3S/|b_8|\right)\left(\frac{3}{|b_8|} + \frac{1}{S+S^*}\right)$$

$$W_{;T} = -\frac{3W}{T+T^*} \neq 0$$

$$\exp\left(CS^{1/2}\right), \exp\left[C(S+S^*)^{1/2}\right]$$

$$\begin{aligned}
T_B(z)T_F^{\pm}(0) &\sim \frac{3}{2z^2}T_F^{\pm}(0) + \frac{1}{z}\partial T_F^{\pm}(0),\\
T_B(z)j(0) &\sim \frac{1}{z^2}j(0) + \frac{1}{z}\partial j(0),\\
T_F^{+}(z)T_F^{-}(0) &\sim \frac{2c}{3z^3} + \frac{2}{z^2}j(0) + \frac{2}{z}T_B(0) + \frac{1}{z}\partial j(0),\\
T_F^{+}(z)T_F^{+}(0) &\sim T_F^{-}(z)T_F^{-}(0) \sim 0,\\
j(z)T_F^{\pm}(0) &\sim \pm\frac{1}{z}T_F^{\pm}(0),\\
j(z)j(0) &\sim \frac{c}{3z^2}.
\end{aligned}$$

$$\begin{aligned}
T_B(z) &= \sum_{n\in\mathbf{Z}} \frac{L_n}{z^{n+2}}, j(z) = \sum_{n\in\mathbf{Z}} \frac{J_n}{z^{n+1}},\\
T_F^{+}(z) &= \sum_{r\in\mathbf{Z}+v} \frac{G_r^{+}}{z^{r+3/2}}, T_F^{-}(z) = \sum_{r\in\mathbf{Z}-v} \frac{G_r^{-}}{z^{r+3/2}},
\end{aligned}$$

$$\begin{aligned}
\left[L_m, G_r^{\pm}\right] &= \left(\frac{m}{2} - r\right)G_{m+r}^{\pm}\\
[L_m, J_n] &= -nJ_{m+n}\\
\{G_r^{+}, G_s^{-}\} &= 2L_{r+s} + (r-s)J_{r+s} + \frac{c}{3}\left(r^2 - \frac{1}{4}\right)\delta_{r,-s}\\
\{G_r^{+}, G_s^{+}\} &= \{G_r^{-}, G_s^{-}\} = 0\\
\left[J_n, G_r^{\pm}\right] &= \pm G_{r+n}^{\pm}\\
[J_m, J_n] &= \frac{c}{3}m\delta_{m,-n}
\end{aligned}$$

$$\tilde{T}_F = \tilde{T}_F^{+} + \tilde{T}_F^{-}.$$

$$\exp\left[l\tilde{\phi} + is_0\tilde{H}^0 + is_1\tilde{H}^1 + i\tilde{Q}\left(\tilde{H}/3^{1/2}\right)\right],$$

$$l + s_0 + s_1 + \tilde{Q} \in 2\mathbf{Z}.$$

$$\mathcal{U}\exp\left(i\tilde{H}/3^{1/2}\right), \overline{\mathcal{U}}\exp\left(-i\tilde{H}/3^{1/2}\right),$$

$$j^a\exp\left(3^{1/2}i\tilde{H}/2\right), \mathcal{U}\exp\left[-i\tilde{H}/\left(2\times 3^{1/2}\right)\right].$$

$$\overline{\mathcal{U}}\exp\left[i\tilde{H}/\left(2\times 3^{1/2}\right)\right], j^a\exp\left(-3^{1/2}i\tilde{H}/2\right).$$

$$\{\tilde{G}^{+}_{1/2}, \tilde{G}^{-}_{-1/2}\} = 2\tilde{L}_0 + \tilde{J}_0$$
$$\{\tilde{G}^{+}_{-1/2}, \tilde{G}^{-}_{1/2}\} = 2\tilde{L}_0 - \tilde{J}_0$$

$$\tilde{J} = i(\tilde{c}/3)^{1/2}\bar{\partial}\tilde{H}$$

$$2\tilde{h} \geq |\tilde{Q}|$$

$$\tilde{G}^{\pm}_{r}|c\rangle = 0, r > 0$$
$$\tilde{L}_n|c\rangle = \tilde{J}_n|c\rangle = 0, n > 0$$
$$\tilde{G}^{+}_{-1/2}|c\rangle = 0$$

$$\frac{3\tilde{Q}^2}{2\tilde{c}} \leq \frac{|\tilde{Q}|}{2} \Rightarrow |\tilde{Q}| \leq \frac{\tilde{c}}{3}$$

$$G'_{\mu\nu\ \text{Einstein}} = \exp\left[-2(\Phi_4 - \langle\Phi_4\rangle)\right]G_{\mu\nu}$$

$$\tilde{J}_1 \cdot \mathcal{V}^0 = \tilde{J}_1 G_{-1/2} \cdot \mathcal{V}^{-1} = \left(\tilde{G}_{-1/2}\tilde{J}_1 + \tilde{G}^{+}_{1/2} - \tilde{G}^{-}_{1/2}\right) \cdot \mathcal{V}^{-1} = 0$$

$$\exp\left[i(3/\tilde{c})^{1/2}\tilde{Q}\tilde{H}\right] \rightarrow \exp\left[i(3/\tilde{c})^{1/2}\tilde{Q}\tilde{H} - i\eta(\tilde{c}/3)^{1/2}\tilde{H}\right]$$

$$v \rightarrow v + \eta$$

$$\tilde{G}^{\pm}_{n}|\psi\rangle = \tilde{L}_n|\psi\rangle = \tilde{J}_n|\psi\rangle = 0, n \geq 0,$$

$$K = -3\ln\left(T + T^*\right).$$

$$K = -\ln\ \mathrm{Im}\left(\sum_I X^{I*}\partial_I F(X)\right),$$

$$F(X) = \frac{(X^1)^3}{X^0}, T = \frac{iX^1}{X^0}$$

$$\delta X^1 = \epsilon X^0.$$

$$\delta F = 3\epsilon(X^1)^2.$$

$$\delta F = c_{IJ}X^I X^J$$

$$\Delta F = i\lambda(X^0)^2$$

$$\delta X^I = \omega^{IJ}X^J$$

$$T^A = \frac{iX^A}{X^0}, A = 1, \ldots, n$$

$$F = \frac{d_{ABC}X^A X^B X^C}{X^0} + i\lambda(X^0)^2$$

$$|c, \tilde{c}\rangle, |c, \tilde{a}\rangle, |a, \tilde{c}\rangle, |a, \tilde{a}\rangle,$$

$$\Phi^{++}, \Phi^{+-}, \Phi^{-+}, \Phi^{--}$$

$$h \geq \frac{1}{2}(Q_\Phi + Q_\Psi) = h_\Phi + h_\Psi$$

$$\Phi^{++}(z,\bar{z})\Psi^{++}(0,0) \sim (\Phi\Psi)^{++}(0,0)$$

$$j = \psi^i\psi^{\bar{\iota}}, \tilde{j} = \tilde{\psi}^i\tilde{\psi}^{\bar{\iota}}$$

$$b_{i_1\ldots i_p\bar{j}_1\ldots\bar{j}_q}(X)\psi^{i_1}\ldots\psi^{i_p}\tilde{\psi}^{\bar{j}_1}\ldots\tilde{\psi}^{\bar{j}_q}$$

$$Q = p, h = \frac{p}{2}, \tilde{Q} = -q, \tilde{h} = \frac{q}{2}$$

$$(G_0^+)^2 = 0.$$

$$T_B^{\text{top}} \equiv T_B + \frac{1}{2}\partial j$$

$$T_B^{\text{top}}(z)T_B^{\text{top}}(0) \sim \frac{2}{z^2}T_B^{\text{top}}(z) + \frac{1}{z}\partial T_B^{\text{top}}(z)$$
$$T_B^{\text{top}}(z)T_F^+(0) \sim \frac{1}{z^2}T_F^+ + \frac{1}{z}\partial T_F^+$$

$$T_F^+(z)T_F^-(0) = \cdots + \frac{1}{z}T_B^{\text{top}}(0) + \cdots$$

$$\sum_{K=4}^{8} \lambda^{K+}\lambda^{K-}$$

$$\lambda^A\lambda^B, \Theta_{\mathbf{16}}\exp\left(3^{1/2}iH/2\right), \Theta_{\overline{\mathbf{16}}}\exp\left(-3^{1/2}iH/2\right), i\partial H.$$

$$45 + 16 + \overline{16} + 1$$

$$\mathcal{V} = \lambda^A\Phi^{++}$$

$$\Theta_{16}\Phi^{++}\left(1 \to -\frac{1}{2}\right)$$

$$h = \frac{Q}{2} + \frac{Q'^2 - Q^2}{6}, \tilde{h} = \frac{\tilde{Q}}{2}$$

$$\Phi^{++}(1 \to -2)$$

$$G_{-1/2}^- \cdot \Phi^{++}$$

$$G_{-1/2}^-G_{-1/2}^- \cdot \Phi^{+\pm} = 0$$
$$G_{-1/2}^-G_{-1/2}^+ \cdot \Phi^{-\pm} = \left(2L_{-1} - G_{-1/2}^+G_{-1/2}^-\right) \cdot \Phi^{-\pm} = 2\partial\Phi^{-\pm}$$

$$\left(G_{-1/2}^-\right)^2 = 0, G_{-1/2}^- \cdot \Phi^{-\pm} = 0$$

$$G'_{A\bar{B}} = \exp\left[\kappa^2(K_2 - K_1)/3\right]G_{A\bar{B}}$$
$$W(\phi) = \phi^A{}_{\bar{x}}\phi^B{}_{\bar{y}}\phi^C{}_{\bar{z}}d^{\bar{x}\bar{y}\bar{z}}\partial_A\partial_B\partial_C F_1(T)$$

$$G'_{a\bar{b}} = \exp[\kappa^2(K_1 - K_2)/3]G_{a\bar{b}}$$
$$W(\chi) = \chi^a{}_x\chi^b{}_y\chi^c{}_z d^{xyz}\partial_a\partial_b\partial_c F_2(Z)$$

$$c = 3 - \frac{6}{k+2} = \frac{3k}{k+2}, k = 0,1, \ldots$$

$$\text{NS}: h = \frac{l(l+2) - q^2}{4(k+2)}, Q = \frac{q}{k+2}$$
$$\text{R}: h = \frac{l(l+2) - (q \pm 1)^2}{4(k+2)} + \frac{1}{8}, Q = \frac{q \pm 1}{k+2} \mp \frac{1}{2}$$

$$j^+ = \psi_1 \exp\left[i\left(\frac{2}{k}\right)^{1/2} H\right], j^- = \psi_1^\dagger \exp\left[-i\left(\frac{2}{k}\right)^{1/2} H\right].$$

$$T_F^+ = \psi_1 \exp\left[i\left(\frac{k+2}{k}\right)^{1/2} H\right], T_F^- = \psi_1^\dagger \exp\left[-i\left(\frac{k+2}{k}\right)^{1/2} H\right]$$

$$1 - \frac{1}{2}\left(\frac{2}{k}\right) + \frac{1}{2}\left(\frac{k+2}{k}\right) = \frac{3}{2},$$

$$\mathcal{O}_m^j = \psi_m^j \exp\left[im\left(\frac{2}{k}\right)^{1/2} H\right].$$

$$\mathcal{O}_m'^j = \psi_m^j \exp\left[i\frac{2m}{k^{1/2}(k+2)^{1/2}} H\right]$$

$$h = \frac{j(j+1)}{k+2} - \frac{m^2}{k} + \frac{2m^2}{k(k+2)} = \frac{j(j+1) - m^2}{k+2}.$$

$$j = i[k/(k+2)]^{1/2}\partial H,$$

$$\psi_m^j \exp\left[i\frac{2m \pm k/2}{k^{1/2}(k+2)^{1/2}} H\right]$$

$$W(\Phi) = \Phi^{k+2}$$

$$\sigma \to \lambda\sigma, \phi \to \lambda^\omega \phi$$
$$\psi \to \lambda^{\omega - 1/2}\psi, F \to \lambda^{\omega - 1} F,$$

$$\lambda^{2-1+(k+2)\omega}$$

$$h_\phi = \tilde{h}_\phi = \frac{1}{2(k+2)}.$$

$$Q_\phi = \tilde{Q}_\phi = \frac{1}{k+2}$$

$$\partial_\phi W(\phi) = (k+2)\phi^{k+1} = 0$$

$$Q = \frac{c}{3} = \frac{k}{k+2}$$

$$\Phi \rightarrow \exp\left(\frac{2\pi i}{k+2}\right)\Phi.$$

$$\exp(2\pi i Q);$$

$$\sum_i \frac{k_i}{k_i+2} = 3.$$

$$l + s_0 + s_1 + Q \in 2\mathbf{Z}$$

$$g_q = \exp(\pi i s + 2\pi i Q) = \exp(\pi i s)\prod_i \exp(2\pi i Q_i)$$

$$\frac{Nk}{k+2} = 3$$

$$k^N = 1^9, 2^6, 3^5, 6^4.$$

$$\prod_{i=1}^{N} \psi_{m_i}^{j_i}\tilde{\psi}_{m_i}^{j_i}\exp\left[i\frac{2m\left(H_i+\tilde{H}_i\right)}{k^{1/2}(k+2)^{1/2}}\right].$$

$$\exp\left[il\left(\frac{k+2}{k}\right)^{1/2} H_i\right].$$

$$\exp\left[in\left(\frac{k}{k+2}\right)^{1/2} H_i\right]$$

$$\prod_{i=1}^{N} \psi_{m_i}^{j_i}\tilde{\psi}_{m_i}^{j_i}\exp\left[i\frac{(2m_i+nk)H_i+2m_i\tilde{H}_i}{k^{1/2}(k+2)^{1/2}}\right]$$

$$Q = \frac{1}{k+2}\sum_{i=1}^{N}(2m_i+nk) = 3n + \frac{2}{k+2}\sum_{i=1}^{N} m_i$$

$$L_0 - \tilde{L}_0 = \frac{1}{2k(k+2)}\sum_{i=1}^{N}\{(2m_i+nk)^2-(2m_i)^2\}$$
$$= \frac{3n^2}{2} + \frac{2n}{k+2}\sum_{i=1}^{N} m_i$$

$$\tilde{h} - \frac{\tilde{Q}}{2} = \sum_{i=1}^{N}\frac{j_i(j_i+1)-m_i(m_i+1)}{k+2}.$$

$$\sum_i j_i = \frac{k+2}{2}, |j_i| \leq \frac{k}{2}$$

$$m_i = -\tilde{m}_i = -j_i, \text{ all } i$$

$$1^9:(84,0), 2^6:(90,0), 3^5:(101,1), 6^4:(149,1).$$

$$S_5 \ltimes \mathbf{Z}_5^4,$$

$$\exp(2\pi i Q_i), i = 2,3,4,5.$$

$$G(z) = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5$$

$$z_i \to \exp(2\pi i n_i/5) z_i, n_1 = 0$$

$$\sum_{i=1}^{5} \Phi_i^5$$

$$W = PG(\Phi)$$

$$q_\Phi = 1, q_P = -5$$

$$U = |G(\phi)|^2 + |p|^2 \sum_{i=1}^{5} \left|\frac{\partial G}{\partial \phi_i}\right|^2 + \frac{e^2}{2}\left(r + 5|p|^2 - \sum_{i=1}^{5} |\phi_i|^2\right)^2 + (A_2^2 + A_3^2)\left(25|p|^2 + \sum_{i=1}^{5} |\phi_i|^2\right)$$

$$\frac{\partial G}{\partial \phi_i} = 0$$

$$p = 0, \sum_{i=1}^{5} |\phi_i|^2 = r.$$

$$G(\phi) = 0$$

$$R_c^2 \propto r$$

$$|p|^2 = \frac{r}{5}, \phi_i = 0, A_2 = A_3 = 0$$

$$W = \langle p \rangle G(\Phi);$$

$$p \to p, \phi_i \to \exp(2\pi i/5)\phi_i$$

$$i\frac{\theta}{2\pi}\int F_2,$$

$$F_{12} = \frac{\theta}{2\pi}.$$

$$(h^{1,1}, h^{2,1})_{\mathcal{M}} = (h^{2,1}, h^{1,1})_{\mathcal{W}},$$

$$\exp\{2\pi i[r(Q_2 - Q_3) + s(Q_3 - Q_4) + t(Q_4 - Q_5)]\}$$

$$X \to X + 2\pi(\alpha'/n)^{1/2}$$

$$R' = (\alpha'/n)^{1/2}$$

$$(z_1, z_2, z_3, z_4, z_5) \to (z_1, \alpha^r z_2, \alpha^{s-r} z_3, \alpha^{t-s} z_4, \alpha^{-t} z_5)$$

$$\mathcal{W} = \mathcal{M}/\Gamma.$$

$$G(z) = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 - 5\psi z_1 z_2 z_3 z_4 z_5$$

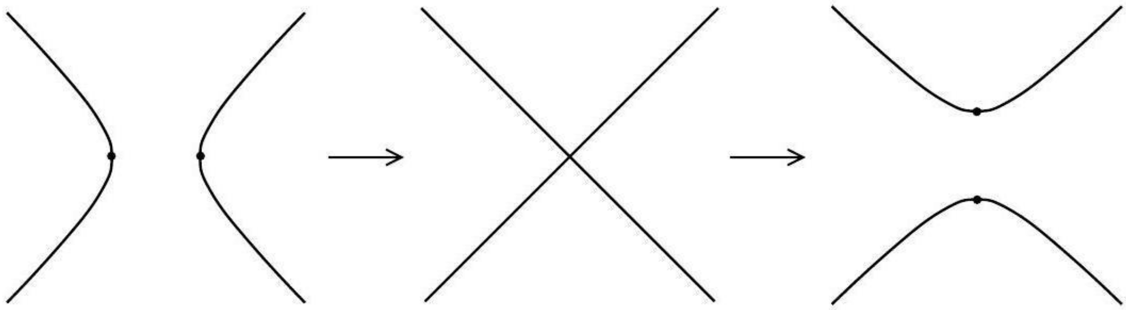
$$Z^I = \int_{A^I} \Omega_{3,0}\,, \mathcal{G}_I(Z) = \int_{B_I} \Omega_{3,0}$$

$$T \approx \frac{5}{2\pi} \ln(5\psi)$$

$$F = (X^0)^2 \left[\frac{5i}{6} T^3 - \frac{25i}{2\pi^3} \zeta(3) + \sum_{k=1}^{\infty} C_k \exp(-2\pi k T) \right]$$

$$n_k = 2875,609250,317206375,242467530000, \ldots$$

$$\mathrm{Re}(T^A) = \int_{N^A} J_{1,1} = \int_{N^A} d^2 w G_{i\bar{j}} \frac{\partial X^i}{\partial w} \frac{\partial X^{\bar{j}}}{\partial \bar{w}} > 0$$



$$\phi^* \cdot \phi - \rho^* \cdot \rho - r = 0$$

$$(\phi_i, \rho_i) \cong \left(e^{i\lambda} \phi_i, e^{-i\lambda} \rho_i \right)$$

$$z_i^5 = \psi z_1 z_2 z_3 z_4 z_5, i = 1, \ldots, 5$$

$$\psi^5 = 1$$

$$\sum_i w_i^2 = 0,$$

$$\sum_i |w_i^2| = 2\rho^2.$$

$$x \cdot x = \rho^2, y \cdot y = \rho^2, x \cdot y = 0.$$

$$S^3 \times S^2 \times \mathbf{R}^+$$

$$\sum_i w_i^2 = \psi - 1.$$

$$x \cdot x = |\psi - 1|, y = 0$$

$$\mathcal{G}_1 = \frac{1}{2\pi i} Z^1 \ln\ Z^1 + \mathbb{H}_{\text{holomorphic terms}}$$

$$\mathcal{G}_1 \to \mathcal{G}_1 + Z^1.$$

$$c_{\mu npq}(x,y) = c_\mu^1(x)\omega_{1npq}(y)$$

$$\int_D c_4 = \int_P c_1^1$$

$$-\frac{1}{32\pi^2}\ln\,(\Lambda^2/M^2)F_{\mu\nu}F^{\mu\nu}$$

$$\frac{1}{8\pi}\text{Re}(i\partial_1\mathcal{G}_1)$$

$$\Phi_\alpha^\dagger \sigma_{\alpha\beta}^A \Phi_\beta = 0, A = 1,2,3$$

$$z^1 H_1(z) + z^2 H_2(z) = 0$$

$$z^1 = z^2 = H_1(z) = H_2(z) = 0$$

$$q^I{}_i = \delta^I{}_i, i = 1, \dots, 15, q^I{}_{16} = -1$$

$$\Phi_{i\alpha}^\dagger \sigma_{\alpha\beta}^A \Phi_{i\beta} - \Phi_{16\alpha}^\dagger \sigma_{\alpha\beta}^A \Phi_{16\beta} = 0, A = 1,2,3, i = 1, \dots, 15.$$

$$\Phi_{i\alpha} = \Phi_{16\alpha}, i = 1, \dots, 15.$$

$$(h^{1,1}, h^{2,1}) = (2,86)$$

$$\begin{aligned} 16\ &\to (4,2) + (4',2') \\ 16'\ &\to (4,2') + (4',2) \end{aligned}$$

$$\begin{matrix} h^{2,2} & & c^1 \\ h^{2,1}h^{1,2} & & 0^{1,1}0 \\ h^{1,0}h^{1,1}h^{0,2} & & h^{0,1} \\ h^{0,0} & & 0^{20}0 \end{matrix}.$$

$$*\,\omega_2 = \pm\omega_2.$$

$$g_{ij} = \Omega_{[i}^{\bar{k}}\omega_{j]\bar{k}}$$

$$H_{\mu\nu\sigma pq} = H_{\mu\nu\sigma}\omega_{pq}$$

$$*_{10} = *_4 *_6$$

$$\frac{SO(20,4,\mathbf{R})}{SO(20,\mathbf{R}) \times SO(4,\mathbf{R})},$$

$$T_F = i\psi_m \partial X^m = i\psi_r e_m^r \partial X^m$$

$$(\mathbf{56},\mathbf{1})^{10}+(\mathbf{1},\mathbf{1})^{65}$$

$$(\mathbf{28},\mathbf{2})^{10}+(\mathbf{1},\mathbf{1})^{65}$$

$$n_H+29n_T-n_V=273.$$

$$F=*F.$$

$$\int_{\mathrm{K3}}\mathrm{tr}(R_2\wedge R_2)=\int_{\mathrm{K3}}\mathrm{Tr}_v(F_2\wedge F_2),$$

$$n_1+n_2+n_5=24$$

$$G_{\mathrm{I}\mu v}=g_{\mathrm{h}}^{-1}G_{\mathrm{h}\mu v}$$
$$g_{\mathrm{I}}=g_{\mathrm{h}}^{-1}$$

$$R_{m\mathrm{I}}\propto g_{\mathrm{h}}^{-1/2}R_{m\,\mathrm{h}}$$

$$g'\propto V_{\mathrm{I}}^{-1}g_{\mathrm{I}}\propto V_{\mathrm{h}}^{-1}g_{\mathrm{h}}^{(k-2)/2}$$
$$R_m'\propto R_{m\mathrm{I}}^{-1}\propto g_{\mathrm{h}}^{1/2}R_{m\,\mathrm{h}}^{-1}$$

$$T^k/\mathbf{Z}_2,\mathbf{Z}_2=\{1,\Omega\hat{\beta}\}$$

$$R_{10\mathrm{M}}\propto g'^{2/3}\propto g_{\mathrm{h}}^{1/3}V_{\mathrm{h}}^{-2/3},$$
$$R_{m\mathrm{M}}\propto g'^{-1/3}R_m'\propto g_{\mathrm{h}}^{1/3}R_{m\,\mathrm{h}}^{-1}V_{\mathrm{h}}^{1/3}.$$

$$g''\propto g'^{-1}\propto g_{\mathrm{h}}^{-1}V_{\mathrm{h}},$$
$$R_m''\propto g'^{-1/2}R_m'\propto R_{m\,\mathrm{h}}^{-1}V_{\mathrm{h}}^{1/2}.$$

$$\Omega\hat{\beta}=\beta_R^{-1}\Omega\beta_R=\Omega\beta_L^{-1}\beta_R$$

$$\hat{\beta}=\beta_L^{-1}\beta_R=\beta_L^{-2}\beta_L\beta_R=\exp(-2\pi iJ_L)\beta=\exp(\pi in\mathbf{F}_L)\beta$$

$$G_{\mu v}+,B_{\mu v}-,\Phi+,C-,C_{\mu v}+,C_{\mu v\rho\sigma}-.$$

$$G_{\mu v}+,B_{\mu v}+,\Phi+,C-,C_{\mu v}-,C_{\mu v\rho\sigma}-.$$

$$T^k/\mathbf{Z}_2,\mathbf{Z}_2=\{1,\exp(\pi i\mathbf{F}_L)\beta\}$$

$$g_{\mathrm{A}}=g''R_9''^{-1}=g_{\mathrm{h}}^{-1}R_{9\,\mathrm{h}}V_{\mathrm{h}}^{1/2}$$
$$R_{9\,\mathrm{A}}=R_9''^{-1}=V_{\mathrm{h}}^{-1/2}R_{9\,\mathrm{h}}$$
$$R_{m\,\mathrm{A}}=R_m''=V_{\mathrm{h}}^{1/2}R_{m\,\mathrm{h}}^{-1},m=6,7,8$$

$$T^4/\mathbf{Z}_2,\mathbf{Z}_2=\{1,\beta\}$$

$$e^{-2\Phi_6}=Ve^{-2\Phi}$$

$$\Phi_6\to-\Phi_6,G_{\mu v}\to e^{-2\Phi_6}G_{\mu v}$$
$$\tilde{H}_3\to e^{-2\Phi_6}*_6\tilde{H}_3,F_2^a\to F_2^a$$

$$\boldsymbol{S}_{\text{het}} = \frac{1}{2\kappa_6^2}\int d^6x(-G_6)^{1/2}e^{-2\Phi_6}\left(R + 4\partial_\mu\Phi_6\partial^\mu\Phi_6 - \frac{1}{2}\left|\tilde{H}_3\right|^2 - \frac{\kappa_6^2}{2g_6^2}|F_2|^2\right)$$

$$S_{\text{IIA}} = \frac{1}{2\kappa_6^2}\int d^6x(-G_6)^{1/2}\left(e^{-2\Phi_6}R + 4e^{-2\Phi_6}\partial_\mu\Phi_6\partial^\mu\Phi_6 - \frac{1}{2}\left|\tilde{H}_3\right|^2 - \frac{\kappa_6^2}{2g_6^2}e^{-2\Phi_6}|F_2|^2\right)$$

$$T^4/\mathbf{Z}_2 = \text{K3}.$$

$$\frac{SO(19,3,\mathbf{R})}{SO(19,\mathbf{R})\times SO(3,\mathbf{R})}\times\mathbf{R}^+.$$

$$e^{-2\Phi_4}\propto R_4R_5e^{-2\Phi_6}$$

$$e^{\Phi_4}\to(R_4R_5)^{-1/2}$$

$$*_4\,\tilde{H}\to e^{-2\Phi_4}dB_{45},$$

$$*_4\,\tilde{H}\propto e^{-2\Phi_4}da$$

$$S\to i\rho^*.$$

$$\frac{SU(1,1)}{U(1)\times SU(1,1,\mathbf{Z})}\times\frac{O(22,6,\mathbf{R})}{O(22,\mathbf{R})\times O(6,\mathbf{R})\times O(22,6,\mathbf{Z})}$$

$$|1,-1\rangle,\left|1,-\frac{1}{2}\right\rangle^2,|1,0\rangle$$

$$\{\Gamma^\mu,\Gamma^v\} = 2\eta^{\mu v}$$

$$\Gamma^{0\pm} = \frac{1}{2}(\pm\Gamma^0 + \Gamma^1)$$
$$\Gamma^{a\pm} = \frac{1}{2}(\Gamma^{2a}\pm i\Gamma^{2a+1}), a = 1,\dots,k$$

$$\{\Gamma^{a+},\Gamma^{b-}\} = \delta^{ab}$$
$$\{\Gamma^{a+},\Gamma^{b+}\} = \{\Gamma^{a-},\Gamma^{b-}\} = 0$$

$$\Gamma^{a-}\zeta = 0 \text{ for all } a$$

$$\zeta^{(\mathbf{s})}\equiv\left(\Gamma^{k+}\right)^{s_k+1/2}\dots(\Gamma^{0+})^{s_0+1/2}\zeta.$$

$$\Gamma^0 = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix},\Gamma^1 = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}.$$

$$\Gamma^\mu = \gamma^\mu\otimes\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix},\mu = 0,\dots,d-3$$
$$\Gamma^{d-2} = I\otimes\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix},\Gamma^{d-1} = I\otimes\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$$

$$\Sigma^{\mu v} = -\frac{i}{4}[\Gamma^\mu,\Gamma^v]$$

$$i[\Sigma^{\mu v},\Sigma^{\sigma\rho}] = \eta^{v\sigma}\Sigma^{\mu\rho} + \eta^{\mu\rho}\Sigma^{v\sigma} - \eta^{v\rho}\Sigma^{\mu\sigma} - \eta^{\mu\sigma}\Sigma^{v\rho}.$$

$$S_a \equiv i^{\delta_{a,0}}\Sigma^{2a,2a+1} = \Gamma^{a+}\Gamma^{a-} - \frac{1}{2}$$

$$\Gamma = i^{-k}\Gamma^0\Gamma^1 \ldots \Gamma^{d-1}$$

$$(\Gamma)^2 = 1, \{\Gamma, \Gamma^\mu\} = 0, [\Gamma, \Sigma^{\mu\nu}] = 0.$$

$$\Gamma = 2^{k+1}S_0S_1 \ldots S_k,$$

$$\mathbf{4}_{\text{Dirac}} = \mathbf{2} + \mathbf{2}'$$

$$32_{\text{Dirac}} = 16 + 16'.$$

$$B_1 = \Gamma^3\Gamma^5 \ldots \Gamma^{d-1}, B_2 = \Gamma B_1$$

$$B_1\Gamma^\mu B_1^{-1} = (-1)^k\Gamma^{\mu*}, B_2\Gamma^\mu B_2^{-1} = (-1)^{k+1}\Gamma^{\mu*}.$$

$$B\Sigma^{\mu\nu}B^{-1} = -\Sigma^{\mu\nu*}$$

$$B_1\Gamma B_1^{-1} = B_2\Gamma B_2^{-1} = (-1)^k\Gamma^*,$$

$$\zeta^* = B\zeta$$

$$B_1^*B_1 = (-1)^{k(k+1)/2}, B_2^*B_2 = (-1)^{k(k-1)/2}.$$

$$\mathrm{P}_\pm = \frac{1 \pm \Gamma}{2}.$$

$$\zeta \to \mathrm{P}_+\zeta, \chi \to \chi + B^*\chi^*$$

$$C\Gamma^\mu C^{-1} = -\Gamma^{\mu T}.$$

$$\Gamma^{\mu\dagger} = \Gamma_\mu = -\Gamma^0\Gamma^\mu(\Gamma^0)^{-1}$$

$$C\Gamma^0\Gamma^\mu(C\Gamma^0)^{-1} = \Gamma^{\mu*}.$$

$$C = B_1\Gamma^0, d = 2\text{mod}4;\ C = B_2\Gamma^0, d = 4\text{mod}4$$

$$C\Sigma^{\mu\nu}C^{-1} = -\Sigma^{\mu\nu T}.$$

$$\bar{\zeta}\chi = \zeta^\dagger\Gamma^0\chi.$$

$$\bar{\zeta}\Gamma^{\mu_1}\Gamma^{\mu_2} \ldots \Gamma^{\mu_m}\chi$$

$$\zeta^T C\Gamma^{\mu_1}\Gamma^{\mu_2} \ldots \Gamma^{\mu_m}\chi$$

$$\zeta^T C\Gamma^{\mu_1\mu_2\cdots\mu_m}\chi$$

$$\Gamma^{\mu_1\mu_2\cdots\mu_m} = \Gamma^{[\mu_1}\Gamma^{\mu_2} \ldots \Gamma^{\mu_m]}$$

$$\Gamma^{\mu_1\cdots\mu_s}\Gamma = -\frac{i^{-k+s(s-1)}}{(d-s)!}\epsilon^{\mu_1\cdots\mu_d}\Gamma_{\mu_{s+1}\cdots\mu_d}.$$

$$\mathbf{2^{k+1}} \times \mathbf{2^{k+1}} = [0] + [1] + \cdots + [k+1],$$

$$\begin{aligned}\mathbf{2^{k+1}_{\text{Dirac}}} \times \mathbf{2^{k+1}_{\text{Dirac}}} &= [0] + [1] + \cdots + [2k+2] \\ &= [0]^2 + [1]^2 + \cdots + [k]^2 + [k+1]\end{aligned}$$

$$\zeta^T C \Gamma^{\mu_1\mu_2\dots\mu_m} \Gamma \chi = (-1)^{k+m+1} (\Gamma\zeta)^T C \Gamma^{\mu_1\mu_2\dots\mu_m} \chi$$

$$\begin{aligned}\mathbf{2^k} \times \mathbf{2^k} &= \begin{cases}[1] + [3] + \cdots + [k+1]_+, & k \text{ even} \\ [0] + [2] + \cdots + [k+1]_+, & k \text{ odd}\end{cases} \\ \mathbf{2^{k\prime}} \times \mathbf{2^{k\prime}} &= \begin{cases}[1] + [3] + \cdots + [k+1]_-, & k \text{ even} \\ [0] + [2] + \cdots + [k+1]_-, & k \text{ odd}\end{cases} \\ \mathbf{2^k} \times \mathbf{2^{k\prime}} &= \begin{cases}[0] + [2] + \cdots + [k], & k \text{ even} \\ [1] + [3] + \cdots + [k], & k \text{ odd}\end{cases}\end{aligned}$$

$$u^*_{ij} = \epsilon_{ii'} \epsilon_{jj'} u_{i'j'}$$

$$\Gamma^0 = i\sigma^2, \Gamma^1 = \sigma^1, \Gamma^2 = \sigma^3$$

$$\begin{aligned}\mathbf{2^{l-1}} \times \mathbf{2^{l-1}} &= \begin{cases}[0] + [2] + \cdots + [l]_+, & l \text{ even}, \\ [1] + [3] + \cdots + [l]_+, & l \text{ odd},\end{cases} \\ \mathbf{2^{l-l\prime}} \times \mathbf{2^{l-l\prime}} &= \begin{cases}[0] + [2] + \cdots + [l]_-, & l \text{ even}, \\ [1] + [3] + \cdots + [l]_-, & l \text{ odd},\end{cases} \\ \mathbf{2^{l-1}} \times \mathbf{2^{l-l\prime}} &= \begin{cases}[1] + [3] + \cdots + [l-1], & l \text{ even}, \\ [0] + [2] + \cdots + [l-1], & l \text{ odd}.\end{cases}\end{aligned}$$

$$SO(9,1) \to SO(3,1) \times SO(6)$$

$$SO(2k+1,1) \to SO(2l+1,1) \times SO(2k-2l)$$

$$\begin{aligned}2^{\mathbf{k}} &\to \left(2^{\mathbf{l}}, 2^{\mathbf{k-l-1}}\right) + \left(2^{\mathbf{l}}, 2^{\mathbf{k-l-1}\prime}\right) \\ 2^{\mathbf{k}\prime} &\to \left(2^{\mathbf{l}}, 2^{\mathbf{k-l-1}}\right) + \left(2^{\mathbf{l}}, 2^{\mathbf{k-l-l}\prime}\right)\end{aligned}$$

$$SO(2n) \to SU(n) \times U(1)$$

$$\zeta \in \mathbf{1}_{-n},$$

$$\mathbf{2^n} \to [0]_{-n} + [1]_{2-n} + [2]_{4-n} + \cdots + [n]_n,$$

$$SO(6) \to SU(3) \times U(1)$$

$$\begin{aligned}\mathbf{4} &\to \mathbf{1}_3 + \mathbf{3}_{-1}, \\ \overline{\mathbf{4}} &\to \mathbf{1}_{-3} + \overline{\mathbf{3}}_1.\end{aligned}$$

$$SO(4) = SU(2) \times SU(2).$$

$$x = x^4 I + i x^i \sigma^i, i = 1,2,3;\ \det x = \sum_{m=1}^{4} (x^m)^2.$$

$$x' = g_1 x g_2^{-1},$$

$$\mathbf{4} = (\mathbf{2}, \mathbf{2}),$$
$$\mathbf{2} = (\mathbf{2}, \mathbf{1}),$$
$$\mathbf{2}' = (\mathbf{1}, \mathbf{2}).$$

$$\{Q_\alpha, \bar{Q}_\beta\} = -2P_\mu \Gamma^\mu_{\alpha\beta}$$
$$[P^\mu, Q_\alpha] = 0$$

$$\left\{Q_\alpha, Q^\dagger_\beta\right\} = 2k^0(1 + \Gamma^0\Gamma^1)_{\alpha\beta} = 2k^0(1 + 2S_0)_{\alpha\beta}$$

$$\left\{Q_{s'_0 s'_1}, Q^\dagger_{s_0 s_1}\right\} = 4k^0 \delta_{s_0,1/2} \delta_{\mathbf{ss}'}$$

$$0 = \langle\psi|\left\{Q_{-1/2,s_1}, Q^\dagger_{-1/2,s_1}\right\}|\psi\rangle$$
$$= \|Q_{-1/2,s_1}|\psi\rangle\|^2 + \|Q^\dagger_{-1/2,s_1}|\psi\rangle\|^2$$

$$b = (4k^0)^{-1/2} Q_{1/2,-1/2}, b^\dagger = (4k^0)^{-1/2} Q_{1/2,1/2}$$

$$\{b, b^\dagger\} = 1, b^2 = b^{\dagger 2} = 0$$

$$S_1|\lambda\rangle = \lambda|\lambda\rangle, b|\lambda\rangle = 0$$

$$b^\dagger|\lambda\rangle = \left|\lambda + \frac{1}{2}\right\rangle, S_1\left|\lambda + \frac{1}{2}\right\rangle = \left(\lambda + \frac{1}{2}\right)\left|\lambda + \frac{1}{2}\right\rangle.$$

$$\left\{Q_{s'_0 s'_1}, Q^\dagger_{s_0 s_1}\right\} = 2m\delta_{\mathbf{ss}'}$$

$$b_1 = (2m)^{-1/2} Q_{1/2,-1/2}, b_2 = (2m)^{-1/2} Q_{-1/2,-1/2}$$
$$\{b_i, b_j^\dagger\} = \delta_{ij}, \{b_i, b_j\} = \{b_i^\dagger, b_j^\dagger\} = 0$$

$$S_1|\lambda\rangle = \lambda|\lambda\rangle, b_i|\lambda\rangle = 0$$

$$b_1^\dagger|\lambda\rangle, b_2^\dagger|\lambda\rangle, b_1^\dagger b_2^\dagger|\lambda\rangle, S_1 = \lambda + \frac{1}{2}, \lambda + \frac{1}{2}, \lambda + 1.$$

$$\Phi^i: \phi^i, \psi^i, F^i,$$
$$V^a: A^a_\mu, \lambda^a, D^a.$$

$$\delta\phi^i/2^{1/2} = i\bar{\zeta}\mathrm{P}_+\psi^i = i\bar{\psi}^i\mathrm{P}_+\zeta$$
$$\delta(\mathrm{P}_+\psi^i)/2^{1/2} = \mathrm{P}_+\zeta F^i + \Gamma^\mu \mathrm{P}_-\zeta D_\mu\phi^i,$$
$$\delta F^i/2^{1/2} = -i\bar{\zeta}\Gamma^\mu D_\mu \mathrm{P}_+\psi^i,$$

$$\delta A^a_\mu = -i\bar{\zeta}\Gamma_\mu\lambda^a$$
$$\delta\lambda^a = \frac{1}{2}\Gamma^{\mu\nu}\zeta F^a_{\mu\nu} + i\Gamma\zeta D^a$$
$$\delta D^a = -\bar{\zeta}\Gamma\Gamma^\mu D_\mu\lambda^a$$

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2,$$

$$\mathcal{L}_1 = -D_\mu \phi^{i*} D^\mu \phi^i - \frac{i}{2}\bar{\psi}^i \Gamma^\mu D_\mu \psi^i - \frac{1}{4g_a^2} F_{\mu\nu}^a F^{a\mu\nu} - \frac{i}{2g_a^2}\bar{\lambda}^a \Gamma^\mu D_\mu \lambda^a$$

$$-\frac{1}{2}\left[iW_{,ij}(\phi)\bar{\psi}^i \mathrm{P}_+ \psi^j + 2^{1/2}\phi^{i*} t_{ij}^a \bar{\lambda}^a \mathrm{P}_+ \psi^j\right] + \text{ c.c.}$$

$$\mathcal{L}_2 = F^{i*}F^i + \frac{1}{2g_a^2} D^{a2} + W_{,i}(\phi)F^i + \text{ c.c. } + \frac{1}{2} D^a\left(2\xi_a + \phi^{i*} t_{ij}^a \phi^j\right)$$

$$-\mathcal{L}_2' = V = \left|F^i(\phi)\right|^2 + \frac{1}{2g_a^2}[D^a(\phi, \phi^*)]^2,$$

$$F^i(\phi) = -W_{,i}(\phi)^*$$
$$D^a(\phi, \phi^*) = -\frac{g_a^2}{2}\left(2\xi_a + \phi^{i*} t_{ij}^a \phi^j\right).$$

$$\phi^i \to \exp(iq_i\alpha)\phi^i, \mathrm{P}_+\psi^i \to \exp[i(q_i - 1)\alpha]\mathrm{P}_+\psi^i$$
$$F^i \to \exp[i(q_i - 2)\alpha]F^i$$
$$A_\mu^a \to A_\mu^a, \mathrm{P}_+\lambda^a \to \exp(i\alpha)\mathrm{P}_+\lambda^a, D^a \to D^a.$$

$$W(\phi) \to \exp(2i\alpha)W(\phi).$$

$$\langle 0|\{Q_\alpha, \chi_\beta\}|0\rangle,$$

$$\delta\psi^i = \delta\lambda^a = 0.$$

$$F^i(\phi) = D^a(\phi) = 0.$$

$$W = f\phi_1,$$

$$W = f\phi_1 + m\phi_2\phi_3 + g\phi_1\phi_2^2,$$

$$\frac{\mathcal{L}_{\text{bos}}}{(-G)^{1/2}} = \frac{1}{2\kappa^2}R - K_{,\bar{\imath}j} D_\mu \phi^{i*} D^\mu \phi^j - \frac{1}{4}\mathrm{Re}(f_{ab}(\phi)) F_{\mu\nu}^a F^{b\mu\nu} - \frac{1}{8}\mathrm{Im}(f_{ab}(\phi))\epsilon^{\mu\nu\sigma\rho} F_{\mu\nu}^a F_{\sigma\rho}^b$$
$$- V(\phi, \phi^*)$$

$$V(\phi, \phi^*) = \exp(\kappa^2 K)\left(K^{\bar{\imath}j} W_{;i}^* W_{;j} - 3\kappa^2 W^* W\right) + \frac{1}{2} f_{ab} D^a D^b.$$

$$W_{;i} = \partial_i W + \kappa^2 \partial_i K W$$
$$\mathrm{Re}(f_{ab}(\phi))D^b = -2\xi_a - K_{,i} t_{ij}^a \phi^j$$

$$K_{,\bar{\imath}j} = \frac{\partial^2 K(\phi, \phi^*)}{\partial\phi^{i*}\partial\phi^j}$$

$$K(\phi, \phi^*) \to K(\phi, \phi^*) + f(\phi) + f(\phi)^*.$$

$$W(\phi) \to \exp[-\kappa^2 f(\phi)]W(\phi).$$

$$\begin{aligned}\delta \mathrm{P}_+\psi^i/2^{1/2} &= -K^{i\bar{j}}W^*_{;j}\mathrm{P}_+\zeta + \Gamma^\mu \mathrm{P}_-\zeta D_\mu \phi^i \\ \delta\lambda^a &= \frac{1}{2}\Gamma^{\mu\nu}\zeta F^a_{\mu\nu} + i\Gamma\zeta D^a \\ \delta\psi_\mu &= D_\mu\zeta + \frac{1}{2}\Gamma_\mu\zeta\exp(\kappa^2K/2)W\end{aligned}$$

$$\{Q^A_\alpha, \bar{Q}^B_\beta\} = -2\delta^{AB}P_\mu\Gamma^\mu_{\alpha\beta}, [P^\mu, Q^A_\alpha] = 0$$

$$\begin{aligned}&\mathscr{h}_{\text{hypermultiplet}}: && \left(-\frac{1}{2}, 0^2, \frac{1}{2}\right) + \left(-\frac{1}{2}, 0^2, \frac{1}{2}\right), \\ &\mathscr{v}_{\text{vector multiplet}}: && \left(-1, -\frac{1}{2}^2, 0\right) + \left(0, \frac{1}{2}^2, 1\right), \\ &\mathfrak{S}_{\text{supergravity multiplet}}: && \left(-2, -\frac{3}{2}^2, -1\right) + \left(1, \frac{3}{2}^2, 2\right).\end{aligned}$$

$$\begin{aligned}&\mathscr{v}_{\text{vector multiplet}}: && \left(-1, -\frac{1}{2}^4, 0^6, \frac{1}{2}^4, 1\right) \\ &\mathfrak{S}_{\text{supergravity multiplet}}: && \left(-2, -\frac{3}{2}^4, -1^6, -\frac{1}{2}^4, 0\right) + \left(0, \frac{1}{2}^4, 1^6, \frac{3}{2}^4, 2\right)\end{aligned}$$

$$\mathfrak{S}_{\text{supergravity multiplet}}: \left(-2, -\frac{3}{2}^8, -1^{28}, -\frac{1}{2}^{56}, 0^{70}, \frac{1}{2}^{56}, 1^{28}, \frac{3}{2}^8, 2\right)$$

$$\begin{aligned}\{Q^A_\alpha, \bar{Q}^B_\beta\} &= -2\delta^{AB}P_\mu\Gamma^\mu_{\alpha\beta} - 2iZ^{AB}\delta_{\alpha\beta} \\ [P^\mu, Q^A_\alpha] &= [Z^{AB}, Q^C_\alpha] = [Z^{AB}, P_\mu] = [Z^{AB}, Z^{CD}] = 0\end{aligned}$$

$$\left\{Q^A_\alpha, Q^{B\dagger}_\beta\right\} = 2m\delta^{AB}\delta_{\alpha\beta} + 2iZ^{AB}\Gamma^0_{\alpha\beta}.$$

$$Z^{AB} = \begin{bmatrix} 0 & q_1 & 0 & 0 & \\ -q_1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & q_2 & \\ 0 & 0 & -q_2 & 0 & \\ & \vdots & & & \ddots \end{bmatrix},$$

$$m \geq q_i$$

$$\begin{gathered}\{Q^A_L, Q^B_L\} = \delta^{AB}(P^0 - P^1), \{Q^A_R, Q^B_R\} = \delta^{AB}(P^0 + P^1) \\ \{Q^A_L, Q^B_R\} = Z^{AB}\end{gathered}$$

$$(A_p \wedge B_q)_{\mu_1\ldots\mu_{p+q}} = \frac{(p+q)!}{p!\,q!}A_{[\mu_1\ldots\mu_p}B_{\mu_{p+1}\ldots\mu_{p+q}]}$$

$$A_p \wedge B_q = (-1)^{pq}B_q \wedge A_p$$

$$(dA_p)_{\mu_1\ldots\mu_{p+1}} = (p+1)\partial_{[\mu_1}A_{\mu_2\ldots\mu_{p+1}]}$$

$$\int d^dx A_{01\ldots d-1} \equiv \int A_d$$

$$\int_{\mathcal{M}} dA_{p-1} = \int_{\partial\mathcal{M}} A_{p-1}$$

$$* A_{\mu_1 \ldots \mu_{d-p}} = \frac{1}{p!} \epsilon_{\mu_1 \ldots \mu_{d-p}}{}^{\nu_1 \ldots \nu_p} A_{\nu_1 \ldots \nu_p}$$

$$** = (-1)^{p(d-p)+1}$$

$$A_p = \frac{1}{p!} A_{\mu_1 \ldots \mu_p} dx^{\mu_1} \ldots dx^{\mu_p}$$

$$F_2 = dA_1, \delta A_1 = d\lambda$$

$$F_2 = dA_1 - iA_1 \wedge A_1 \equiv dA_1 - iA_1^2, \delta A_1 = d\lambda - iA_1\lambda + i\lambda A_1$$

$$F_{p+2} = dA_{p+1}, \delta A_{p+1} = d\lambda_p$$

$$-\frac{1}{2}\int d^d x (-G)^{1/2} \left|F_{p+2}\right|^2 = -\frac{1}{2}\int d^d x \frac{(-G)^{1/2}}{(p+2)!} F_{\mu_1 \ldots \mu_{p+2}} F^{\mu_1 \ldots \mu_{p+2}}$$

$$dF_{p+2} = 0, d * F_{p+2} = 0$$

$$F'_{d-p-2} = * F_{p+2}$$

$$-\frac{1}{2}\int d^d x (-G)^{1/2} \left|dA_{p+1}\right|^2 \to -\frac{1}{2}\int d^d x (-G)^{1/2} \left|F_{p+2}\right|^2 + \int A'_{d-p-3} \wedge dF_{p+2}$$

$$\to -\frac{1}{2}\int d^d x (-G)^{1/2} \left|dA'_{d-p-3}\right|^2$$

$$F_{d/2} = \pm * F_{d/2}$$

$$\left|F_{d/2}\right|^2 = \pm F_{d/2} \wedge F_{d/2} = 0$$

$$\{Q_\alpha, \bar{Q}_\beta\} = -2P_\mu \Gamma^\mu_{\alpha\beta}$$

$$S_{11} = \frac{1}{2\kappa^2}\int d^{11}x (-G)^{1/2} \left(R - \frac{1}{2}|F_4|^2\right) - \frac{1}{12\kappa^2}\int A_3 \wedge F_4 \wedge F_4$$

$$Q^1_\alpha \in \mathbf{16}, Q^2_\alpha \in \mathbf{16}'$$

$$\{Q^1_\alpha, \bar{Q}^1_\beta\} = -2P_\mu (\mathrm{P}_+ \Gamma^\mu)_{\alpha\beta}, \{Q^2_\alpha, \bar{Q}^2_\beta\} = -2P_\mu (\mathrm{P}_- \Gamma^\mu)_{\alpha\beta}$$
$$\{Q^1_\alpha, \bar{Q}^2_\beta\} = -2P_{10} (\mathrm{P}_+ \Gamma)_{\alpha\beta}$$

$$\{Q^A_\alpha, \bar{Q}^B_\beta\} = -2\delta^{AB} P_\mu (\mathrm{P}_+ \Gamma^\mu)_{\alpha\beta}$$

$$\frac{1}{2}k(k+1)$$

$$\frac{1}{3!}k(k-1)(k-2)$$

$$\left\{Q_\alpha^A, \bar{Q}_\beta^B\right\} = -2P_\mu \delta^{AB}\Gamma_{\alpha\beta}^\mu - 2P_m \Gamma_{(7)}^{mAB}\Gamma_{(4)\alpha\beta}.$$

$$\Gamma_{(4)\alpha\beta} = i(\Gamma^0\Gamma^1\Gamma^2\Gamma^3)_{\alpha\beta}$$

$$(P_m\Gamma^m P_n \Gamma^n)^{AB} = \delta^{AB}P_m P^m,$$

$$\left\{Q_\alpha^A, \bar{Q}_\beta^B\right\} = -2P_\mu \delta^{AB}\Gamma_{\alpha\beta}^\mu - 2P^{AB}\delta_{\alpha\beta}.$$

$$\frac{SO(1,1,\mathbf{R}) \times SO(k+r,k,\mathbf{R})}{SO(k+r,\mathbf{R}) \times SO(k,\mathbf{R})}$$

$$\frac{SO(8+r,8,\mathbf{R})}{SO(8+r,\mathbf{R}) \times SO(8,\mathbf{R})}$$

$d = 6, N = 2$ supersymmetry

$$16 \to (4,2) + (4',2').$$

$$\mathbf{4} \to \left(+\frac{1}{2},\mathbf{2}\right) + \left(-\frac{1}{2},\mathbf{2}'\right) = \left(+\frac{1}{2},\frac{1}{2},0\right) + \left(-\frac{1}{2},0,\frac{1}{2}\right)$$
$$\mathbf{4}' \to \left(+\frac{1}{2},\mathbf{2}'\right) + \left(-\frac{1}{2},\mathbf{2}\right) = \left(+\frac{1}{2},0,\frac{1}{2}\right) + \left(-\frac{1}{2},\frac{1}{2},0\right)$$

$$r = \left(\frac{1}{2},\frac{1}{2}\right) + \left(\frac{1}{2},0\right)^2 + \left(0,\frac{1}{2}\right)^2 + (0,0)^4.$$

$$r \times |j_1, j_2\rangle$$

$$|1,1\rangle + |1,0\rangle + |0,1\rangle + |0,0\rangle + \left|\frac{1}{2},\frac{1}{2}\right\rangle^4$$
$$+ \left|1,\frac{1}{2}\right\rangle^2 + \left|\frac{1}{2},1\right\rangle^2 + \left|0,\frac{1}{2}\right\rangle^2 + \left|\frac{1}{2},0\right\rangle^2$$

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle + |0,0\rangle^4 + \left|\frac{1}{2},0\right\rangle^2 + \left|0,\frac{1}{2}\right\rangle^2.$$

$$r' = (1,0) + \left(\frac{1}{2},0\right)^4 + (0,0)^5$$

$$|1,1\rangle + \left|\frac{1}{2},1\right\rangle^4 + |0,1\rangle^5$$

$$|1,0\rangle + \left|\frac{1}{2},0\right\rangle^4 + |0,0\rangle^5,$$

$$\left\{Q_\alpha^A, \bar{Q}_\beta^B\right\} = -2P_\mu \delta^{AB}\Gamma_{\alpha\beta}^\mu - 2P_{Rm}\Gamma^{mAB}\delta_{\alpha\beta}$$

$$-\frac{1}{4g^2}\mathrm{Tr}(F_{MN}F^{MN}) - \frac{i}{2g^2}\mathrm{Tr}\left(\bar{\lambda}\Gamma^M D_M \lambda\right)$$

$$\begin{aligned} \delta A_M &= -i\bar{\zeta}\Gamma_M\lambda \\ \delta\lambda &= \frac{1}{2}F_{MN}\Gamma^{MN}\zeta \end{aligned}$$

$$-\frac{1}{4g^2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu} + 2D_\mu A_m D^\mu A_m - [A_m, A_n]^2\right) - \frac{i}{2g^2}\mathrm{Tr}\left(\bar{\lambda}\Gamma^\mu D_\mu\lambda + i\bar{\lambda}\Gamma_m[A_m, \lambda]\right)$$

$$\begin{aligned} \delta A_\mu &= -i\bar{\zeta}\Gamma_\mu\lambda \\ \delta A_m &= -i\bar{\zeta}\Gamma_m\lambda \\ \delta\lambda &= \left(\frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu} + D_\mu A_n\Gamma^{\mu n} + \frac{i}{2}[A_m, A_n]\Gamma^{mn}\right)\zeta. \end{aligned}$$

$$V = -\frac{1}{4g^2}\mathrm{Tr}([A_m, A_n]^2)$$

$$v = (A_m A_m)^{1/2} \equiv A,$$

$$-\frac{1}{4g^2(A)}F_{\mu\nu}F^{\mu\nu}.$$

$$r'' = \left(\frac{1}{2}, 0\right) + (0,0)^2.$$

$$|1,1\rangle + \left|\frac{1}{2}, 1\right\rangle^2 + |0,1\rangle$$

$$\begin{aligned} \daleth_{\text{half-hypermultiplet}}: &\quad \left|\frac{1}{2}, 0\right\rangle, |0,0\rangle^2, \\ \gimel_{\text{vector multiplet}}: &\quad \left|\frac{1}{2}, \frac{1}{2}\right\rangle, \left|0, \frac{1}{2}\right\rangle^2, \\ \aleph_{\text{tensor multiplet}}: &\quad |1,0\rangle, \left|\frac{1}{2}, 0\right\rangle^2, |0,0\rangle \end{aligned}$$

$$D^{Aa} = \frac{g^2}{2}\Phi_\alpha^{i*}\sigma_{\alpha\beta}^A t_{ij}^a \Phi_\beta^j$$

$$\frac{1}{2g^2}D^{Aa}D^{Aa}$$

$$t\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

$$B_2\mathrm{Tr}(F_2 \wedge F_2)$$

$$G_{rs}(\phi)\partial_\mu\phi^r\partial^\mu\phi^s$$

$$J_j^i = i\delta_j^i, J_{\bar{j}}^{\bar{\imath}} = -i\delta_{\bar{j}}^{\bar{\imath}}, J_{\bar{j}}^i = J_j^{\bar{\imath}} = 0$$

$$J^A J^B = -\delta^{AB} + \epsilon^{ABC}J^C$$

$$-\frac{1}{4g^2}\mathrm{Tr}([A_4, A_5]^2)$$

$$\Phi_{\alpha}^{i\dagger}\left(M_4^2+M_5^2\right)_{ij}\Phi_{\alpha}^{j}$$

$$M_{mij}=A_m^a t_{ij}^a+q_{mij}$$

$$K(A,A^*)=\mathrm{Im}\left(\sum_a A^{a*}\partial_a F(A)\right)$$

$$G_{a\bar{b}}=\mathrm{Im}(\partial_a\partial_b F)$$

$$(X^0,X^1,\dots,X^n)\cong(\lambda X^0,\lambda X^1,\dots,\lambda X^n)$$

$$T^A=\frac{X^A}{X^0},A=1,\dots,n$$

$$F(\lambda X)=\lambda^2 F(X)$$

$$K=-\ln\ \mathrm{Im}\left(\sum_I X^{I*}\partial_I F(X)\right)$$

$$K\to K-\ln\ \lambda-\ln\ \lambda^*$$

$$\begin{bmatrix}X'^I\\ \partial_{I'}F'\end{bmatrix}=S\begin{bmatrix}X^I\\ \partial_I F\end{bmatrix}$$

$$\Gamma^\mu\to U\Gamma^\mu U^{-1}$$

Apéndice B.

Supermembranas, supersimetrías, superespacios y multidimensiones a propósito de la existencia de supergravedad cuántica. Métrica de Yang – Mills.

$$[Q_\alpha,\mathfrak{I}_{\text{internal charges}}]=0.$$

$$\frac{4\pi}{g_i^2(\mu)}=\frac{b_i}{2\pi}\ln\,\frac{\mu}{\Lambda_i}\ i=1,2,3$$

$$g_1(M_Z)\sim 0.46<g_2(M_Z)\sim 0.64<g_3(M_Z)\sim 1.22$$

$$SU(3)_c\times SU(2)_L\times U(1)_Y\subset SU(5)\subset SO(10)\subset E_6\subset E_8$$

$$100\text{GeV}\leq\ \text{susy breaking scale}\ \leq 1000\text{GeV}.$$

$$I=1,\cdots,\mathcal{N}\ \begin{cases}Q_\alpha^I\ \alpha=1,2 & \text{left Weyl spinor}\\ \bar{Q}_{\dot{\alpha}I}=(Q_\alpha^I)^\dagger & \text{right Weyl spinor}\end{cases}$$

$$\{Q_\alpha^I,\bar{Q}_{\dot{\beta}J}\}=2\sigma_{\alpha\dot{\beta}}^\mu P_\mu\delta_J^I$$
$$\{Q_\alpha^I,Q_\beta^J\}=2\epsilon_{\alpha\beta}Z^{IJ}$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \; Q^I = \begin{pmatrix} Q^I_\alpha \\ \bar{Q}^{\dot{\alpha}}_I \end{pmatrix}$$

$$Z^{IJ} = -Z^{JI} \; [Z^{IJ}, \text{anything} \,] = 0$$

$$\left\{Q^I_\alpha, \left(Q^J_\beta\right)^\dagger\right\} = 2\left(\sigma^\mu P_\mu\right)_{\alpha\dot{\beta}} \delta^I_J = \begin{pmatrix} 4E & 0 \\ 0 & 0 \end{pmatrix}_{\alpha\dot{\beta}} \delta^I_J$$

$$\{Q^I_2, (Q^I_2)^\dagger\} = 0 \implies Q^I_2 = 0, Z^{IJ} = 0$$

$$\left\{Q^I_\alpha, \left(Q^J_\beta\right)^\dagger\right\} = 2M\delta^\beta_\alpha \delta^I_J$$

$$Z = \text{diag}(\epsilon Z_1, \cdots, \epsilon Z_r, \#) \; \epsilon^{12} = -\epsilon^{21} = 1$$

$$\left\{Q^i_{\alpha\pm}, \left(Q^j_{\beta\pm}\right)^\dagger\right\} = \delta^i_j \delta^\beta_\alpha (M \pm Z_i)$$

$$M \geq |Z_i| \; i = 1, \cdots, r = \left[\frac{\mathcal{N}}{2}\right]$$

$$\mathcal{L} = -\frac{1}{2g^2}\text{tr}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{8\pi^2}\text{tr}F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{i}{2}\text{tr}\bar{\lambda}\bar{\sigma}^\mu D_\mu \lambda$$

$$\delta_\xi A_\mu = i\bar{\xi}\bar{\sigma}_\mu\lambda - i\bar{\lambda}\bar{\sigma}_\mu\xi$$
$$\delta_\xi \lambda = \sigma^{\mu\nu}F_{\mu\nu}\xi$$

$$\mathcal{L} = -\partial_\mu\phi^*\partial^\mu\phi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi - \left|\frac{\partial U}{\partial\phi}\right|^2 - \text{Re}\left(\psi\psi\frac{\partial^2 U}{\partial\phi^2}\right)$$

$$[x^\mu, \theta_\alpha] = \left[x^\mu, \bar{\theta}^{\dot{\alpha}}\right] = \{\theta_\alpha, \theta_\beta\} = \left\{\theta_\alpha, \bar{\theta}^{\dot{\beta}}\right\} = \left\{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\right\} = 0$$

$$D_\alpha \equiv \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \;\; \bar{D}_{\dot{\alpha}} \equiv -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$

$$\frac{\partial}{\partial\theta^\alpha}\left(1, \theta^\beta, \bar{\theta}^{\dot{\beta}}\right) \equiv \int d\theta^\alpha\left(1, \theta^\beta, \bar{\theta}^{\dot{\beta}}\right) \equiv \left(0, \delta_\alpha{}^\beta, 0\right)$$

$$d^2\theta \equiv \frac{1}{4}d\theta_\alpha d\theta^\alpha, d^2\bar{\theta} \equiv \frac{1}{4}d\bar{\theta}^{\dot{\alpha}}d\bar{\theta}_{\dot{\alpha}}, d^4\theta \equiv d^2\theta d^2\bar{\theta}$$

$$\int d^2\theta\theta\theta = \int d^2\bar{\theta}\bar{\theta}\bar{\theta} = \int d^4\theta\theta\theta\bar{\theta}\bar{\theta} = 1$$

$$S(x, \theta, \bar{\theta}) = \phi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \bar{\theta}\bar{\sigma}^\mu\theta A_\mu(x) + \theta\theta f(x) + \bar{\theta}\bar{\theta}g^*(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\rho(x)$$
$$+\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x)$$

$$\mathfrak{B}_{\text{bosonic superfield}} \; [S, \theta^\alpha] = [S, \bar{\theta}_{\dot{\alpha}}] = 0$$
$$\mathfrak{F}_{\text{fermionic superfield}} \; \{S, \theta^\alpha\} = \{S, \bar{\theta}_{\dot{\alpha}}\} = 0.$$

$$D_\alpha(S_1S_2) = (D_\alpha S_1)S_2 + (-)^{g(S_1)g(S_2)}S_1(D_\alpha S_2)$$
$$D_{\dot\alpha}(S_1S_2) = (D_{\dot\alpha} S_1)S_2 + (-)^{g(S_1)g(S_2)}S_1(D_{\dot\alpha} S_2)$$

$$\delta_\xi S = (\xi Q + \bar\xi\bar Q)S$$

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \ \ \bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$\{Q_\alpha, \bar Q_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu \ \{D_\alpha, \bar D_{\dot\beta}\} = -2\sigma^\mu_{\alpha\dot\beta}P_\mu$$

$$D_\alpha D_\beta D_\gamma = Q_\alpha Q_\beta Q_\gamma = 0$$

$$\bar D_{\dot\alpha}\Phi = 0$$

$$x^\mu_\pm = x^\mu \pm i\theta\sigma^\mu\bar\theta$$

$$\bar D_{\dot\alpha}x^\mu_+ = 0, D_\alpha x^\mu_- = 0$$

$$\Phi(x,\theta,\bar\theta) = \phi(x_+) + \sqrt{2}\theta\psi(x_+) + \theta\theta F(x_+)$$
$$\Phi^\dagger(x,\theta,\bar\theta) = \phi^*(x_-) + \sqrt{2}\bar\theta\bar\psi(x_-) + \bar\theta\bar\theta F^*(x_-)$$

$$V = V^\dagger$$

$$V(x,\theta,\bar\theta) = v(x) + \theta\chi(x) + \bar\theta\bar\chi(x) + \theta\theta f(x) + \bar\theta\bar\theta f^*(x) + \bar\theta\bar\sigma^\mu\theta A_\mu(x)$$
$$+ i\theta\theta\bar\theta\left(\bar\lambda(x) + \frac{1}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right) - i\bar\theta\bar\theta\theta\left(\lambda(x) + \frac{1}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right)$$
$$+\frac{1}{2}\theta\theta\bar\theta\bar\theta\left(D(x) + \frac{1}{2}\partial_\mu\partial^\mu v(x)\right)$$

$$V \longrightarrow V' = V + i\Lambda - i\Lambda^\dagger$$

$$v \longrightarrow v' = v + i\phi - i\phi^*$$
$$\chi \longrightarrow \chi' = \chi + i\sqrt{2}\psi$$
$$f \longrightarrow f' = f + iF$$

$$A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu(\phi + \phi^*).$$

$$e^V \longrightarrow e^{V'} = e^{-i\Lambda^\dagger}e^V e^{i\Lambda}$$

$$V(x,\theta\bar\theta) = \bar\theta\bar\sigma^\mu\theta A_\mu(x) + i\theta\theta\bar\theta\bar\lambda(x) - i\bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x)$$

$$\delta_\xi F \ = i\sqrt{2}\partial_\mu(\bar\xi\bar\sigma^\mu\psi)$$
$$\delta_\xi D \ = \partial_\mu(i\bar\xi\bar\sigma^\mu\lambda - i\bar\lambda\bar\sigma^\mu\xi)$$

$$\text{F} - \ \text{terms} \quad \mathcal{L}_F = F = \int \ d^2\theta\Phi$$
$$\text{D} - \ \text{terms} \quad \mathcal{L}_D = \frac{1}{2}D = \int \ d^4\theta V$$

$$\bar{D}_{\dot{\alpha}}\Phi^i = 0 \implies \bar{D}_{\dot{\alpha}}U(\Phi^i) = 0$$

$$\mathcal{L}_U = \int d^2\theta U(\Phi^i) + \mathfrak{C}_{\text{complex conjugate}}$$

$$\mathcal{L}_U = \sum_i F^i \frac{\partial U}{\partial \phi^i} - \frac{1}{2}\sum_{i,j} \psi^i\psi^j \frac{\partial^2 U}{\partial\phi^i\partial\phi^j} + \mathfrak{C}_{\text{complex conjugate}}$$

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}(e^{-V}D_\alpha e^{+V})$$

$$W_\alpha(x,\theta,\bar{\theta}) = -i\lambda_\alpha(x_+) + \theta_\alpha D(x_+) - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)_\alpha{}^\beta\theta_\beta F_{\mu\nu}(x_+) + \theta\theta\sigma^\mu_{\alpha\dot{\beta}}D_\mu\bar{\lambda}^{\dot{\beta}}(x_+)$$

$$W^{\alpha a}W^b_\alpha = -\lambda^a\lambda^b - i\theta\lambda^a D^b - i\theta\lambda^b D^a - \frac{1}{2}\theta(\sigma^\mu\bar{\sigma}^\nu)(\lambda^a F^b_{\mu\nu} + \lambda^b F^a_{\mu\nu})$$

$$-\theta\theta\left(i\lambda^a\sigma^\mu\partial_\mu\bar{\lambda}^b + i\lambda^b\sigma^\mu\partial_\mu\bar{\lambda}^a + \frac{1}{4}(F^{\mu\nu a} + i\tilde{F}^{\mu\nu a})(F^b_{\mu\nu} + i\tilde{F}^b_{\mu\nu}) - D^a D^b\right)$$

$$\mathcal{L}_G = \int d^2\theta\tau_{ab}(\Phi^i)W^aW^b + \mathfrak{C}_{\text{complex conjugate}}$$

$$\mathcal{L}_G = -\lambda^a\lambda^b\left(F^i\frac{\partial\tau_{ab}}{\partial\phi^i} - \frac{1}{2}\psi^i\psi^j\frac{\partial^2\tau_{ab}}{\partial\phi^i\partial\phi^j}\right)$$

$$-\frac{1}{2\sqrt{2}}\frac{\partial\tau_{ab}}{\partial\phi^i}\psi^i\left(-i\lambda^a D^b - i\lambda^b D^a - \frac{1}{2}(\sigma^\mu\bar{\sigma}^\nu)(\lambda^a F^b_{\mu\nu} + \lambda^b F^a_{\mu\nu})\right)$$

$$-\tau_{ab}\left(i\lambda^a\sigma^\mu\partial_\mu\bar{\lambda}^b + i\lambda^b\sigma^\mu\partial_\mu\bar{\lambda}^a + \frac{1}{4}(F^{\mu\nu a} + i\tilde{F}^{\mu\nu a})(F^b_{\mu\nu} + i\tilde{F}^b_{\mu\nu}) - D^a D^b\right)$$

$$\Phi \longrightarrow \Phi' = e^{-i\Lambda}\Phi$$

$$\mathcal{L}_K = \int d^4\theta K\left(e^V\Phi^i, (\Phi^i)^\dagger\right)$$

$$\mathcal{L}_K \sim -D_\mu\phi^* D^\mu\phi - i\bar{\psi}\bar{\sigma}^\mu D_\mu\psi$$

$$\mathcal{L}_K = -g_{ii^*}D_\mu\phi^i D^\mu\phi^{i^*} - ig_{ii^*}\bar{\psi}^{i^*}\bar{\sigma}^\mu D_\mu\psi^i + \frac{1}{4}R_{ik^*jl^*}\psi^i\psi^j\bar{\psi}^{k^*}\bar{\psi}^{l^*}$$

$$+ g_{ii^*}\left(F^i - \frac{1}{2}\Gamma^i_{jk}\psi^j\psi^k\right)\left(F^{i^*} - \frac{1}{2}\Gamma^{i^*}_{j^*k^*}\psi^{j^*}\psi^{k^*}\right) - \frac{i}{2}D^a(T^a)^j{}_i\phi^i\frac{\partial K}{\partial\phi^j}$$

$$+ \sqrt{2}g_{ii^*}(T^a)^i{}_k\phi^k\bar{\psi}^{i^*} + \mathfrak{C}_{\text{complex conjugate}}$$

$$D_\mu\phi^i = \partial_\mu\phi^i - A^a_\mu(T^a)^i{}_j\phi^j$$
$$D_\mu\psi^i = \partial_\mu\psi^i - A^a_\mu(T^a)^i{}_j\psi^j + \Gamma^i_{jk}D_\mu\phi^j\psi^k$$

$$g_{ii^*} \equiv \frac{\partial^2 K}{\partial\phi^i\partial\phi^{i^*}}$$

$$\mathcal{N} = 2 \text{ gauge } (\text{rep}\mathcal{G}): V \oplus \Phi V \sim \left(A_\mu \lambda_+\right)\Phi \sim (\lambda_- \phi)$$

$$\mathcal{N} = 2 \text{ hyper } (\text{rep}\mathcal{R}): H_1 \oplus H_2 H_1 \sim (H_+ \psi_+) H_2 \sim (H_- \psi_-)$$

$$\mathcal{L} = \text{Re} \int d^2\theta \left(\tau W^a W^a + U\left(\Phi, H_f\right)\right) + \int d^4\theta \left(\Phi^\dagger e^{V_\mathcal{G}} \Phi + \sum_f H_f^\dagger e^{V_\mathcal{R}} H_f\right)$$

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

$$U(\Phi, H_a) = H_1^T \Phi H_2 + H_1 m H_2$$

$$z_M = \left\{x^\mu, \theta_{\alpha i}, \bar{\theta}^{\dot{\alpha} i}, i = 1{,}2\right\}$$

$$(x^\mu)^\dagger = x^\mu \; (\theta_{\alpha i})^\dagger = \bar{\theta}^{\dot{\alpha} i}$$

$$D_\alpha^i = \frac{\partial}{\partial \theta_i^\alpha} + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha} i} \partial_\mu \;\; \bar{D}_{\dot{\alpha} i} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha} i}} - i\theta_i^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu$$

$$Q_\alpha^i = \frac{\partial}{\partial \theta_i^\alpha} - i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha} i} \partial_\mu \;\; \bar{Q}_{\dot{\alpha} i} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha} i}} + i\theta_i^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu$$

$$S(z_M) = \phi(x) + \theta_i \psi^i(x) + \bar{\theta}^j \bar{\chi}_j(x) + \bar{\theta}^i \bar{\sigma}^\mu \theta_j A_{\mu i}^j(x) + \theta_{\alpha i} \theta_{j\beta} f^{\alpha\beta ij}(x) + \bar{\theta}^{\dot{\alpha} i} \bar{\theta}^{\dot{\beta} j} g_{\dot{\alpha}\dot{\beta} ij}^*(x) + \cdots$$

$$+ \theta_i \theta_j \theta_k \theta_l \bar{\theta}^m \bar{\theta}^n \bar{\theta}^o \bar{\theta}^p D_{mnop}^{ijkl}(x)$$

$$\delta_\xi S = \left(\xi^i Q_i + \bar{\xi}_i \bar{Q}^i\right) S$$

$$\bar{D}_{\dot{\alpha} i} \Phi = 0 \; i = 1{,}2$$

$$x_\pm^\mu = x^\mu \pm i\theta_i \sigma^\mu \bar{\theta}^i$$

$$\bar{D}_{\dot{\alpha} i} x_+^\mu = 0, D^{\alpha i} x_-^\mu = 0$$

$$\Phi(x_+, \theta) = \phi(x_+) + \theta^{\alpha i} \psi_{\alpha i}(x_+) + \theta^{\alpha i} \theta^{\beta j} f_{\alpha\beta ij}(x_+) + \theta^{\alpha i} \theta^j \theta^k \chi_{\alpha ijk}(x_+) + \theta\theta\theta\theta D(x_+)$$

$$\mathcal{L}_D = D = \int d^4\theta \Phi$$

$$D^{\alpha i} D_\alpha^j W = \bar{D}_{\dot{\alpha}}^i \bar{D}^{\dot{\alpha} j} \bar{W}$$

$$W(x_+, \theta) = \Phi(x_+, \theta^1) + \sqrt{2}\theta^{\alpha 2} W_\alpha(x_+, \theta^1) + \theta^2 \theta^2 G(x_+, \theta^1).$$

$$G(x_+, \theta^1) = -\frac{1}{2} \int d^2 \bar{\theta}^1 \Phi(x_+ - i\theta_1 \sigma \bar{\theta}^1, \theta^1, \bar{\theta}^1)^\dagger e^{-2V(x_+ - i\theta_1 \sigma \bar{\theta}^1, \theta^1, \bar{\theta}^1)}$$

$$\mathcal{D}_{\alpha i} = D_{\alpha i} + iA_{\alpha i} \;\; \overline{\mathcal{D}}_{\dot{\alpha} i} = \bar{D}_{\dot{\alpha} i} + i\bar{A}_{\dot{\alpha} i}$$

$$\overline{\mathcal{D}}_{\dot{\alpha} i} W = 0 \; \mathcal{D}^{\alpha i} \mathcal{D}_\alpha^j W = \overline{\mathcal{D}}_{\dot{\alpha}}^i \overline{\mathcal{D}}^{\dot{\alpha} j} \bar{W}$$

$$S_{\mathcal{F}} = \int dz_M \mathrm{tr}\mathcal{F}(W) + \mathfrak{C}_{\text{complex conjugate}}$$

$$S_{\mathcal{F}} = -\frac{1}{2}\int d^2\theta \left(\frac{\partial\mathcal{F}}{\partial\Phi^a}G^a\right) - \int d^2\theta \frac{\partial^2\mathcal{F}}{\partial\Phi^a\partial\Phi^b}W^{\alpha a}W_\alpha^b$$

$$\begin{pmatrix} u_1^- & u_1^+ \\ u_2^- & u_2^+ \end{pmatrix} \in SU(2)_R \begin{cases} u^{+i}u_i^- = 1 \\ u^{\pm i}u_i^{\pm} = 0 \end{cases}$$

$$(x^\mu)^\dagger = x^\mu \ (\theta_{\alpha i})^\dagger = \bar{\theta}^{\dot{\alpha} i} \ \left(u^{\pm i}\right)^\dagger = u_i^{\mp}$$

$$\delta x^\mu = i\xi^i\sigma^\mu\bar{\theta}_i - i\theta^i\sigma^\mu\bar{\xi}_i \qquad \delta\theta_{\alpha i} = \xi_{\alpha i}$$
$$\delta u_i^{\pm} = 0 \qquad \delta\bar{\theta}_{\dot{\alpha}}^i = \bar{\xi}_{\dot{\alpha}}^i$$

$$x_A^\mu \equiv x^\mu - i\theta^+\sigma^\mu\bar{\theta}^- - i\theta^-\sigma^\mu\bar{\theta}^+ \quad \theta_\alpha^{\pm} \equiv \theta_\alpha^i u_i^{\pm}$$
$$\bar{\theta}_{\dot{\alpha}}^{\pm} \equiv \bar{\theta}_{\dot{\alpha}}^i u_i^{\pm}$$

$$\delta x_A^\mu = -2i\left(\xi^i\sigma^\mu\bar{\theta}^+ + \theta^+\sigma^\mu\bar{\xi}_i\right)u_i^- \quad \delta\theta_\alpha^{\pm} = \xi_\alpha^i u_i^{\pm}$$
$$\delta u_i^{\pm} = 0 \quad \delta\bar{\theta}_{\dot{\alpha}}^{\pm} = \bar{\xi}_{\dot{\alpha}}^i u_i^{\pm}$$

$$D_\alpha^+ = \frac{\partial}{\partial\theta^{\alpha-}} \qquad D_\alpha^- = -\frac{\partial}{\partial\theta^{\alpha+}} + 2i\sigma_{\alpha\dot{\alpha}}^\mu\bar{\theta}^{\dot{\alpha}-}D_\mu$$
$$\bar{D}_{\dot{\alpha}}^+ = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}-}} \qquad \bar{D}_{\dot{\alpha}}^- = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}+}} - 2i\theta^{\alpha-}\sigma_{\alpha\dot{\alpha}}^\mu D_\mu$$

$$D^{++} = u^{i+}\frac{\partial}{\partial u^{i-}} - 2i\theta^+\sigma^\mu\bar{\theta}^+D_\mu + \theta^{\alpha+}\frac{\partial}{\partial\theta^{\alpha-}} + \bar{\theta}^{\dot{\alpha}+}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}-}}$$

$$z_A = \left\{x_A^\mu, \theta_\alpha^+, \bar{\theta}_{\dot{\alpha}}^+\right\} \text{ and } u_i^{\pm}$$

$$\left(x_A^\mu\right)^c = x_A^\mu, \left(\theta_\alpha^{\pm}\right)^c = \bar{\theta}_{\dot{\alpha}}^{\pm} \left(\bar{\theta}_{\dot{\alpha}}^{\pm}\right)^c = -\theta_\alpha^{\pm} \left(u^{i\pm}\right)^c = -u_i^{\pm}$$

$$D_\alpha^+\Phi = \bar{D}_{\dot{\alpha}}^+\Phi = 0$$

$$\Phi(z_A,u) = \phi(x_A,u) + \theta^+\psi(x_A,u) + \bar{\theta}^+\bar{\chi}(x_A,u) + \theta^+\theta^+f(x_A,u) + \bar{\theta}^+\bar{\theta}^+g(x_A,u)$$
$$+ \bar{\theta}^+\bar{\sigma}^\mu\theta^+A_\mu(x_A,u) + i\theta^+\theta^+\bar{\theta}^+\bar{\lambda}(x_A,u) - i\bar{\theta}^+\bar{\theta}^+\theta^+\nu(x_A,u)$$
$$+ \frac{1}{2}\theta^+\theta^+\bar{\theta}^+\bar{\theta}^+D(x_A,u)$$

$$\Phi^{(q)}(z_A,u) = \sum_{n=0}^{\infty} \phi^{(i_1\cdots i_{n+q}j_1\cdots j_n)}(z_A)u_{(i_1}^+\cdots u_{i_{n+q}}^+u_{j_1}^-\cdots u_{j_n)}^-$$

$$\int du u_{(i_1}^+\cdots u_{i_m}^+u_{j_1}^-\cdots u_{j_n)}^- = \delta_{m+n,0}$$

$$S[\Phi^+] = \int dz_A du\{(\Phi^+)^c D^{++}\Phi^+ + U(\Phi^+,(\Phi^+)^c)\}$$
$$U(\Phi^+,(\Phi^+)^c) = a(\Phi^+)^4 + b(\Phi^+)^c(\Phi^+)^3 + c((\Phi^+)^c)^2(\Phi^+)^2 + b^*((\Phi^+)^c)^3\Phi^+ + a^*((\Phi^+)^c)^4$$

$$(V^{++})' = e^{i\Lambda}(V^{++} - iD^{++})e^{-i\Lambda} \; (\Phi^{+})' = e^{i\Lambda}\Phi^{+}$$

$$\mathcal{D}^{++} = D^{++} + iV^{++} \; \mathcal{D}_{\alpha}^{-} = D_{\alpha}^{-} + iA_{\alpha}^{-} \; \mathcal{D}_{\dot{\alpha}}^{-} = D_{\dot{\alpha}}^{-} + iA_{\dot{\alpha}}^{-}$$

$$W = -\frac{i}{4} e^{iv}\{\bar{D}_{\dot{\alpha}}^{+}\bar{D}^{\dot{\alpha}+}(e^{-iv}D^{--}e^{iv})\}e^{-iv}$$

$$(D^{++} + iV^{++})e^{iv} = 0$$

$$W(z,u) = \frac{i}{4}\bar{D}_{\dot{\alpha}}^{+}\bar{D}^{\dot{\alpha}+}\sum_{n=1}^{\infty}(-i)^{n}\int du_{1}\cdots du_{n}\frac{V^{++}(z,u_{1})\cdots V^{++}(z,u_{n})}{(uu_{1})(u_{1}u_{2})\cdots(u_{n}u)}$$

$$S = \mathrm{Re}\int dz_{M}du\,\mathrm{tr}\mathcal{F}(W;\tau) + \int dz_{A}du\{(\Phi^{+})^{c}D^{++}\Phi^{+} + U(\Phi^{+},(\Phi^{+})^{c})\}$$

$$\mathcal{L}_{\mathrm{bare}} = \frac{1}{2g(\Lambda)^{2}}\mathrm{tr}F_{(\Lambda)}^{2}$$
$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{2g(\mu)^{2}}\mathrm{tr}F_{(\mu)}^{2} + \frac{1}{\Lambda^{2}}f\left(g,\frac{\mu}{\Lambda}\right)\mathrm{tr}F_{(\mu)}^{3} + \cdots$$

$$m\Phi^{2}\left(\frac{\lambda\Phi}{m}\right)^{n}$$

$$\int d^{2}\theta\tau\mathrm{tr}(WW) \Rightarrow \int d^{2}\theta\tau_{\mathrm{eff}}\mathrm{tr}(WW)$$

$$\beta_{W}(g) = \frac{\partial g(\mu)}{\partial \ln\,\mu}\bigg|_{\Lambda,g(\Lambda)} = (-3C_{2}(\mathcal{G}) + T_{2}(\mathcal{R}))\frac{g^{3}}{16\pi^{2}}$$

$$T_{\mathcal{R}}^{a}T_{\mathcal{R}}^{a} = C_{2}(\mathcal{R})I_{\mathcal{R}} \; \mathrm{tr}(T_{\mathcal{R}}^{a}T_{\mathcal{R}}^{b}) = T_{2}(\mathcal{R})\delta^{ab}$$

$$\tau(\mu) - \tau(\mu') = \frac{1}{2\pi i}(-3C_{2}(\mathcal{G}) + T_{2}(\mathcal{R}))\ln\frac{\mu'}{\mu}$$

$$\tau(\mu) - \tau(\mu') = \frac{1}{\pi i}(-C_{2}(\mathcal{G}) + T_{2}(\mathcal{R}_{H}))\ln\frac{\mu'}{\mu}$$

$$\mathcal{L} = -\frac{1}{2g^{2}}\mathrm{tr}FF - \frac{\theta}{16\pi^{2}}\mathrm{tr}F\tilde{F} - \frac{1}{2g^{2}}\mathrm{tr}D_{\mu}\phi^{\dagger}D^{\mu}\phi + \frac{1}{2g^{2}}\mathrm{tr}[\phi^{\dagger},\phi]^{2} - \frac{i}{2g^{2}}\mathrm{tr}\bar{\lambda}\bar{\sigma}^{\mu}D_{\mu}\lambda$$

$$-\frac{i}{2g^{2}}\mathrm{tr}\bar{\psi}\bar{\sigma}^{\mu}D_{\mu}\psi + i\frac{\sqrt{2}}{g^{2}}\mathrm{tr}([\phi^{\dagger},\psi]\lambda) - i\frac{\sqrt{2}}{g^{2}}\mathrm{tr}([\bar{\lambda},\phi]\bar{\psi})$$

$$F_{\mu\nu} = 0 \; D_{\mu}\phi = 0 \; \mathrm{tr}\left([\phi^{\dagger},\phi]^{2}\right) = 0$$

$$\langle 0|\phi|0\rangle = \sum_{j=1}^{n} a_{j}h_{j}$$

$$\mathcal{G} \longrightarrow U(1)^{n}/\mathrm{Weyl}(\mathcal{G})$$

$$\left(A_{j\mu}, \lambda_{j\pm}, \phi_j\right) \langle 0|\phi_j|0\rangle = a_j \; j = 1, \cdots, n$$

$$\left(V_{\alpha\mu}, \lambda_{\alpha\pm}, \phi_\alpha\right) \alpha = \text{ root of } \mathcal{G}$$

$$M_\alpha^W = |\vec{\alpha} \cdot \vec{a}| \; \vec{a} = (a_1, \cdots, a_n)$$

$$E = \frac{1}{g^2}\int d^3x\left(\mathrm{tr}(D_i\phi)^2 + \mathrm{tr}B_i^2\right) = \frac{1}{g^2}\int d^3x\mathrm{tr}(D_i\phi \pm B_i)^2 \mp \frac{2}{g^2}\int d^3x\partial_i(\mathrm{tr}\phi B_i)$$

$$M_\beta^M = |\vec{\beta} \cdot \tau\vec{a}| \; \tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

$$M_{(\vec{\alpha},\vec{\beta})}^D = \left|\vec{\alpha} \cdot \vec{a} + \vec{\beta} \cdot \vec{a}_D\right|$$

$$W_\alpha^j = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V^j$$

$$\mathcal{L} = \int d^4\theta K\left(\Phi^j, (\Phi^j)^\dagger\right) + \mathrm{Re}\int d^2\theta\left\{U\left(\Phi^j\right) + \tau_{ij}\left(\Phi^k\right)W^iW^j\right\}$$

$$\mathcal{L} = -g_{i\bar{j}}\left(D_\mu\phi^i D^\mu\bar{\phi}^{\bar{j}} + i\bar{\psi}^{\bar{j}}\bar{\sigma}^\mu D_\mu\psi^i\right) - \frac{1}{2}g^{i\bar{j}}\frac{\partial U}{\partial\phi^i}\frac{\partial\bar{U}}{\partial\bar{\phi}^{\bar{j}}} + \mathrm{Re}\left\{\psi^i\psi^j\frac{\partial^2 U}{\phi^i\partial\phi^j}\right\}$$

$$-\mathrm{Re}\left\{\tau_{ij}\left(\frac{i}{2}F_{\mu\nu}^iF^{j\mu\nu} - \frac{1}{2}F_{\mu\nu}^i\tilde{F}^{j\mu\nu} + \bar{\lambda}^i\bar{\sigma}^\mu D_\mu\lambda^j\right)\right\}$$

$$g_{i\bar{j}}(\phi, \bar{\phi}) = \frac{\partial^2 K(\phi, \bar{\phi})}{\partial\phi^i\partial\bar{\phi}^{\bar{j}}}$$

$$D_\mu\phi^i = \partial_\mu\phi^i + \Gamma_{jk}^i\phi^j\partial_\mu\phi^k$$

$$\frac{1}{2i}\left(\tau_{ij}(\phi) - \overline{\tau_{ij}(\phi)}\right) = \frac{\partial^2 K(\phi, \bar{\phi})}{\partial\phi^i\partial\bar{\phi}^j}$$

$$K(\phi, \bar{\phi}) = T_i(\phi)\bar{\phi}^i + \overline{T_i(\phi)}\phi^i$$

$$\frac{1}{2i}\tau_{ij}(\phi) = \frac{\partial T_j(\phi)}{\partial\phi^i} = \frac{\partial T_i(\phi)}{\partial\phi^j}$$

$$T_i(\phi) = \frac{1}{2i}\frac{\partial\mathcal{F}(\phi)}{\partial\phi^i}$$

$$\mathcal{L} = \mathrm{Im}\left(\tau_{ij}\right)F_{\mu\nu}^iF^{j\mu\nu} + \mathrm{Re}\left(\tau_{ij}\right)F_{\mu\nu}^i\tilde{F}^{j\mu\nu} + \mathrm{Im}\left(\partial_\mu\bar{\phi}^j\partial^\mu\phi_{Dj}\right) + \text{ fermions}$$

$$\phi_{Dj} = \frac{\partial\mathcal{F}(\phi)}{\partial\phi^j} \; \tau_{ij}(\phi) = \frac{\partial^2\mathcal{F}(\phi)}{\partial\phi^i\partial\phi^j}$$

$$\tau_{ij} = \frac{\theta_{ij}}{2\pi} + \frac{4\pi i}{g_{ij}^2} \sim \frac{i}{2\pi}\ln\left(a_i - a_j\right)^2/\mu^2$$

$$\mathcal{F} \sim \frac{i}{8\pi}\sum_{i,j} (a_i - a_j)^2 \ln (a_i - a_j)^2/\mu^2$$

$$\mathcal{L} = \mathrm{Im}\{\tau(\phi)(F_{\mu\nu}+i\tilde{F}_{\mu\nu})^2\} + \partial_\mu \begin{pmatrix}\phi_D\\ \phi\end{pmatrix}^\dagger J\partial^\mu \begin{pmatrix}\phi_D\\ \phi\end{pmatrix} + \text{ fermions}$$

$$J = \begin{pmatrix}0 & i\\ -i & 0\end{pmatrix}$$

$$\begin{pmatrix}\phi_D\\ \phi\end{pmatrix} \longrightarrow M\begin{pmatrix}\phi_D\\ \phi\end{pmatrix}\ M^\dagger J M = J$$

$$T_\beta = \begin{pmatrix}1 & \beta\\ 0 & 1\end{pmatrix}\ S = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$

$$\phi_D \longrightarrow \phi_D + \beta\phi$$
$$\phi \longrightarrow \phi$$
$$\tau(\phi) \longrightarrow \tau(\phi) + \beta$$

$$\phi_D \longrightarrow -\phi$$
$$\phi \longrightarrow \phi_D$$
$$\tau(\phi) \longrightarrow -1/\tau(\phi).$$

$$S_G = \int \ \mathrm{Im}\tau(\phi)(F + i\tilde{F})^2 + 2\int \ A_{D\mu}\partial_\nu \tilde{F}^{\mu\nu}$$

$$S_G = \int \ \mathrm{Im}\left[\tau(\phi)\left\{F + i\tilde{F} + \frac{1}{\tau(\phi)}(F_D + i\tilde{F}_D)\right\}^2 - \frac{1}{\tau(\phi)}(F_D + i\tilde{F}_D)^2\right]$$

$$S_G = \int \ \mathrm{Im}\left[-\frac{1}{\tau(\phi)}(F_D + i\tilde{F}_D)^2\right]$$

$$y^2 = (k - \Lambda^2)(k + \Lambda^2)(k - u)$$

$$\begin{pmatrix}B\\ A\end{pmatrix} \longrightarrow M\begin{pmatrix}B\\ A\end{pmatrix}\ M^\dagger J M = J\ M \in SL(2,\mathbf{Z})$$

$$z(P) - z(Q) = \int_Q^P \frac{dk}{y}$$

$$\omega = \oint_A \frac{dk}{y}\ \omega_D = \oint_B \frac{dk}{y}$$

$$\begin{pmatrix}\omega_D\\ \omega\end{pmatrix} \longrightarrow M\begin{pmatrix}\omega_D\\ \omega\end{pmatrix}\ M^\dagger J M = J\ M \in SL(2,\mathbf{Z})$$

$$\tau = \frac{\omega_D}{\omega}\ \mathrm{Im}\tau > 0$$

$$\tau = \frac{\omega_D}{\omega} = \tau(a) = \frac{\partial a_D}{\partial a} = \left(\frac{\partial a_D}{\partial u}\right)\left(\frac{\partial a}{\partial u}\right)^{-1}$$

$$\frac{\partial a}{\partial u} = \omega = \oint_A \frac{dk}{y} \frac{\partial a_D}{\partial u} = \omega_D = \oint_B \frac{dk}{y}.$$

$$a = \oint_A d\lambda \, a_D = \oint_B d\lambda$$

$$\frac{\partial d\lambda}{\partial u} = \frac{dk}{y} = \frac{dk}{\sqrt{(k^2 - \Lambda^4)(k-u)}} \Longrightarrow d\lambda = \frac{(k-u)dk}{y} + \mathfrak{E}_{\text{exact}}$$

$$a_D(u) = \frac{\sqrt{2}}{\pi}\int_{\Lambda^2}^{u} dk \frac{\sqrt{k-u}}{\sqrt{k^2-\Lambda^4}} \; a(u) = \frac{\sqrt{2}}{\pi}\int_{\Lambda^2}^{\Lambda^2} dk \frac{\sqrt{k-u}}{\sqrt{k^2-\Lambda^4}}$$

$$a_D(u) = \frac{\sqrt{2u}}{\pi}\int_{\Lambda^2/u}^{1} dy \frac{\sqrt{y-1}}{\sqrt{y^2 - \frac{\Lambda^4}{u^2}}} \sim -i\frac{\sqrt{2u}\ln u}{\pi}$$

$$a(u) \sim \frac{\sqrt{2u}}{\pi}\int_{-\Lambda^2}^{\Lambda^2} \frac{dk}{\sqrt{\Lambda^4 - k^2}} = \sqrt{2u}$$

$$M = |n_2 a + n_m a_D|$$

$$a(u) \sim \sqrt{u}$$
$$a_D(u) \sim \sqrt{u}\ln \frac{u}{\Lambda^2} \sim a\ln a$$

$$a(u) \sim \frac{i}{\pi} a_D \ln \frac{a_D}{\Lambda}$$
$$a_D(u) \sim (u - \Lambda^2)$$

$$(a - a_D)(u) \sim (u + \Lambda^2)$$
$$a(u) \sim \frac{i}{\pi}(a_D - a)\ln \frac{a_D - a}{\Lambda}$$

$$D_\infty = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} D_{+\Lambda^2} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} D_{-\Lambda^2} = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}$$

$$\left(-\frac{d^2}{dz^2} + V(z)\right)\psi(z) = 0$$

$$V(z) = -\frac{1}{4}\left(\frac{1-\lambda_1^2}{(z+1)^2} + \frac{1-\lambda_2^2}{(z-1)^2} - \frac{1-\lambda_1^2-\lambda_2^2+\lambda_3^2}{(z+1)(z-1)}\right)$$

$$\psi(z) = (z+1)^{(1-\lambda_1)/2}(z-1)^{(1-\lambda_2)/2} f\left(\frac{z+1}{2}\right).$$

$$f_1(x) = (1-x)^{c-a-b}F(c-a, c-b; c+1-a-b; 1-x)$$
$$f_2(x) = (-x)^{-a}F\left(a, a+1-c; a+1-b; \frac{1}{x}\right),$$

$$a_D(u) = i\frac{u-1}{2}F\left(\frac{1}{2},\frac{1}{2};2;\frac{1-u}{2}\right)$$
$$a(u) = \sqrt{2}(u+1)^{\frac{1}{2}}F\left(-\frac{1}{2},\frac{1}{2};1;\frac{2}{u+1}\right).$$

$$F(a,b;c,z)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 dtt^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}$$

$$a_D(u)\ =i\frac{u-1}{2}\frac{\Gamma(2)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right)}\int_0^1 dtt^{-\frac{1}{2}}(1-t)^{\frac{1}{2}}\left(1-t\frac{u-1}{2}\right)^{-\frac{1}{2}}$$

$$a(u)\ =\sqrt{2}(u+1)^{\frac{1}{2}}\frac{\Gamma(1)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}\int_0^1 dtt^{-\frac{1}{2}}(1-t)^{-\frac{1}{2}}\left(1-t\frac{u+1}{2u}\right)^{\frac{1}{2}}$$

$$a=\frac{1}{2\pi i}\oint\ _A d\lambda\ a_D=\frac{1}{2\pi i}\oint\ _B d\lambda\ a_D=\frac{\partial F}{\partial a}$$

$$2\mathrm{Im}\partial_\mu\phi^i\partial^\mu\phi_{Di}=i\partial_\mu\begin{pmatrix}\phi_{Di}\\ \phi_i\end{pmatrix}^\dagger J_{ij}\begin{pmatrix}\phi_{Di}\\ \phi_i\end{pmatrix}\ J=\begin{pmatrix}0 & -I\\ I & 0\end{pmatrix}$$

$$T_\beta:\begin{pmatrix}\phi_{Di}\\ \phi_i\end{pmatrix}\longrightarrow\begin{pmatrix}I & \beta\\ 0 & I\end{pmatrix}\begin{pmatrix}\phi_{Di}\\ \phi_i\end{pmatrix}$$

$$T_\beta:\mathrm{Re}(\tau_{ij})\longrightarrow\mathrm{Re}(\tau_{ij})+\beta_{ij}$$

$$\#(A_i,A_j)=\#(B_i,B_j)=0$$
$$\#(A_i,B_j)=-\#(B_j,A_i)=\delta_{ij}$$

$$\omega_{ij}=\oint\ _{A_i}\Omega_j\ \ \omega_{Dij}=\oint\ _{B_i}\Omega_j$$

$$\tau\equiv\omega_D\omega^{-1}\ \tau_{ij}=\sum_k\ \omega_{Dik}(\omega^{-1})_{kj}$$

$$\mathrm{Im}\tau>0 \text{ for non } - \ \Gamma_{\text{degenerate}}$$
$$\tau_{ij}=\tau_{ji}.$$

$$\begin{pmatrix}B_i\\ A_i\end{pmatrix}\longrightarrow\begin{pmatrix}B_i'\\ A_i'\end{pmatrix}=M\begin{pmatrix}B_i\\ A_i\end{pmatrix}\ M\in\mathrm{Sp}(2g,\mathbf{Z}).$$

$$\omega_{ij}=\frac{\partial a_i}{\partial u_j}=\frac{1}{2\pi i}\oint\ _{A_i}\frac{\partial d\lambda}{\partial u_j}$$
$$\omega_{Dij}=\frac{\partial a_{Di}}{\partial u_j}=\frac{1}{2\pi i}\oint\ _{B_i}\frac{\partial d\lambda}{\partial u_j}$$

$$\tau=\frac{\partial a_D}{\partial u}\left(\frac{\partial a}{\partial u}\right)^{-1}$$

$$a_i=\frac{1}{2\pi i}\oint\ _{A_i}d\lambda\ a_{Di}=\frac{1}{2\pi i}\oint\ _{B_i}d\lambda$$

$$\mathcal{F}^{\text{pert}}(a)=\frac{i}{8\pi}\left[\sum_{\alpha\in\mathcal{R}(\mathcal{G})}(\alpha\cdot a)^2\ln\frac{(\alpha\cdot a)^2}{\mu^2}-\sum_{\lambda\in\mathcal{W}(R)}(\lambda\cdot a+m)^2\ln\frac{(\lambda\cdot a+m)^2}{\mu^2}\right]$$

$$\Gamma(u_i) = \{(k, y)\; y^2 = A(k)^2 - \bar{\Lambda}^2 B(k)\}\; \bar{\Lambda} \equiv \Lambda^{N - \frac{1}{2}N_f}$$

$$A(k) = \prod_{i=1}^{N} (k - u_i)\; B(k) = \prod_{\alpha=1}^{N_f} (k + m_\alpha)$$

$$d\lambda = k d\ln(y + A(k))$$

$$A(x_k^{\pm})^2 - B(x_k^{\pm}) = 0$$

$$d\lambda = \frac{kdk}{y}\left(A'(k) - \frac{1}{2}A(k)\frac{B'(k)}{B(k)}\right).$$

$$d\lambda \sim \frac{1}{2}\frac{m_\alpha dk}{k + m_\alpha}$$

$$d\lambda = k\left(\frac{A'}{A} - \frac{1}{2}\frac{B'}{B}\right)\sum_{m=0}^{\infty} \frac{\Gamma\left(m + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)m!} k\left(\frac{B}{A^2}\right)^m dk$$

$$a_i = u_i + \sum_{m=1}^{\infty} \frac{\bar{\Lambda}^{2m}}{2^{2m}(m!)^2}\left(\frac{\partial}{\partial u_i}\right)^{2m-1} S_i(u_i; u)^m$$

$$S_i(k; u) = \bar{\Lambda}^{-2}(k - u_i)^2 \frac{B(k)}{A(k)^2} = \frac{\prod_\alpha (k + m_\alpha)}{\prod_{l \neq i} (k - u_l)^2}$$

$$d\lambda(\xi) = k\left(\frac{A'}{A} - \frac{1}{2}\frac{B'}{B}\right)\left(1 - \xi^2\frac{B}{A^2}\right)^{-\frac{1}{2}} dk = \sum_{m=0}^{\infty} \frac{\Gamma\left(m + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)m!} k\left(\frac{A'}{A} - \frac{1}{2}\frac{B'}{B}\right)\left(\frac{\xi^2 B}{A^2}\right)^m dk$$

$$d\lambda = \frac{1}{2}\sum_{\alpha=1}^{N_f} \frac{m_\alpha dk}{k + m_\alpha} + \sum_{l=1}^{N} a_l \frac{dk}{k - u_l} + \mathcal{O}(\Lambda)$$

$$\int_{x_1^-}^{x_i^-} d\lambda = \frac{1}{2}\sum_{\alpha=1}^{N_f} m_\alpha \ln(x_i^- + m_\alpha) + \sum_{l=1}^{N} a_l \ln(x_i^- - u_l) + \mathcal{O}(\Lambda)$$

$$\int_{x_1^-}^{x_i^-} d\lambda = a_i \ln\bar{\Lambda} + \frac{1}{2}\sum_{\alpha=1}^{N_f} (a_i + m_\alpha)\ln(x_i^- + m_\alpha) - \sum_{l=1}^{N} (a_i - a_l)\ln(x_i^- - u_l) + \mathcal{O}(\Lambda)$$

$$\prod_{l=1}^{N} (x_i^- - u_l)^2 = \bar{\Lambda}^2 \prod_{j=1}^{N_f} (x_i^- + m_j)$$

$$\int_{k_1^-}^{k_i^-} d\lambda = a_i \ln\bar{\Lambda} + \frac{1}{2}\sum_{\alpha=1}^{N_f} (a_i + m_\alpha)\ln(a_i + m_\alpha) - \sum_{l=1}^{N} (a_i - a_l)\ln(a_i - a_l) + \mathcal{O}(\Lambda)$$

$$\mathcal{F}^{\text{pert}} = -\frac{1}{8\pi i}\left(\sum_{l,m=1}^{N} (a_l - a_m)^2 \ln\frac{(a_l - a_m)^2}{\Lambda^2} - \sum_{l=1}^{N}\sum_{j=1}^{N_f} (a_l + m_j)^2 \ln\frac{(a_l + m_j)^2}{\Lambda^2}\right)$$

$$x_k^{\pm} = u_k \pm \bar{\Lambda} S(x_k^{\pm})^{\frac{1}{2}}$$

$$x_k^{\pm} = u_k + \sum_{m=1}^{\infty} \frac{(\pm)^m \bar{\Lambda}^m}{m!}\left(\frac{\partial}{\partial u_k}\right)^{m-1} S_k(u_k)^{\frac{m}{2}}$$

$$\left.\frac{\partial \mathcal{F}}{\partial \Lambda}\right|_{a_i} = \frac{2N - N_f}{2\pi i}\sum_{i=1}^{N} u_i^2,$$

$$\mathcal{F} = \mathcal{F}^{(0)} + \mathcal{F}^{(1)} + \mathcal{F}^{(2)} + \mathcal{O}(\bar{\Lambda}^6)$$

$$2\pi i \mathcal{F}^{(0)} = -\frac{1}{4}\sum_{j\neq i} (a_i - a_j)^2 \ln\frac{(a_i - a_j)^2}{\Lambda^2} - (\ln 2 - 2N + N_f)\sum_i a_i^2$$
$$+\frac{1}{4}\sum_{i,\alpha} (a_i + m_\alpha)^2 \ln\frac{(a_i + m_\alpha)^2}{\Lambda^2},$$

$$2\pi i \mathcal{F}^{(1)} = \frac{\bar{\Lambda}^2}{4}\sum_i S_i(a_i; a)$$

$$2\pi i \mathcal{F}^{(2)} = \frac{\bar{\Lambda}^4}{16}\left[\sum_{j\neq i}\frac{S_i(a_i;a) S_j(a_j;a)}{(a_i - a_j)^2} + \frac{1}{4}\sum_i S_i(a_i;a)\frac{\partial^2 S_i(a_i;a)}{\partial a_i^2}\right]$$

$$A(k) = k^a \prod_{i=1}^{n} (k^2 - u_i^2)\; B(k) = \Lambda^c k^b \prod_{\alpha=1}^{N_f} (k^2 - m_\alpha^2).$$

$$\mathcal{F}_{SO(2n+1);N_f}\left(a_1, \cdots, a_n; m_1, \cdots, m_{N_f}; \Lambda\right)$$
$$= \mathcal{F}_{SU(2n);2N_f+2}\left(a_1, \cdots, a_n, -a_1, \cdots, -a_n; m_1, \cdots, m_{N_f}, -m_1, \cdots, -m_{N_f}, 0,0; \Lambda\right)$$

$$\mathcal{F}_{Sp(2n);N_f}\left(a_1, \cdots, a_n; m_1, \cdots, m_{N_f-2}, 0,0; \Lambda\right)$$
$$= \mathcal{F}_{SU(2n);2N_f-4}\left(a_1, \cdots, a_n, -a_1, \cdots, -a_n; m_1, \cdots, m_{N_f-2}, -m_1, \cdots, -m_{N_f-2}; \Lambda\right)$$

$$\mathcal{F}_{SO(2n);N_f}\left(a_1, \cdots, a_n; m_1, \cdots, m_{N_f}; \Lambda\right)$$
$$= \mathcal{F}_{SU(2n);2N_f+4}\left(a_1, \cdots, a_n, -a_1, \cdots, -a_n; m_1, \cdots, m_{N_f}, -m_1, \cdots, -m_{N_f}, 0,0,0,0; \Lambda\right)$$

$$H(x_i, p_i)\dot{x}_i = \{x_i, H\} = \frac{\partial H}{\partial p_i}$$
$$\{x_i, p_j\} = \delta_{ij}\dot{p}_i = \{p_i, H\} = -\frac{\partial H}{\partial x_i}$$

$$\dot{I}_i = \{I_i, H\} = 0 \; \{I_i, I_j\} = 0$$

$$I_i(t) = I_i(0) \; \psi_i(t) = \psi_i(0) + c_i I_i t$$

$$\dot{L} = [L, M] \; \Leftrightarrow \begin{pmatrix} \dot{x}_i = \{x_i, H\} \\ \dot{p}_i = \{p_i, H\} \end{pmatrix}$$

$$L^S = S^{-1} L S \; M^S = S^{-1} M S - S^{-1} \dot{S}$$

$$I_i \equiv \mathrm{tr} L^{n_i} \implies \dot{I}_i = n_i \mathrm{tr}(L^{n_i - 1}[L, M]) = 0$$

$$\dot{L}(z) = [L(z), M(z)] \; \Leftrightarrow \begin{pmatrix} \dot{x}_i = \{x_i, H\} \\ \dot{p}_i = \{p_i, H\} \end{pmatrix}$$

$$\Gamma = \{(k, z) \in \mathbf{C} \times \mathbf{C}; \det(kI - L(z)) = 0\}$$

$$d\lambda = kdz$$

$$H = \frac{1}{2} \sum_{i=1}^{n+1} p_i^2 - M^2 \sum_{i=1}^{n} e^{x_{i+1} - x_i}$$

$$H = \frac{1}{2} \sum_{i=1}^{n+1} p_i^2 - M^2 \sum_{i=1}^{n+1} e^{x_{i+1} - x_i}$$

$$\begin{array}{lll} A_n & e_i - e_{i+1} & i = 1, \cdots, n \\ A_n^{(1)} & e_i - e_{i+1} & i = 1, \cdots, n+1, e_{r+2} = e_1. \end{array}$$

$$H = \frac{1}{2} p^2 - M^2 \sum_{\alpha \in \mathcal{R}_*} e^{-\alpha \cdot x}$$

$$H = \frac{1}{2} p^2 - M^2 \sum_{\alpha \in \mathcal{R}_*(\mathcal{G})} e^{-\alpha \cdot x}$$

$$\begin{aligned} L &= p \cdot h + \sum_{\alpha \in \mathcal{R}_*} M e^{-\frac{1}{2}\alpha \cdot x} (E_\alpha - E_{-\alpha}) + \mu^2 e^{\frac{1}{2}\alpha_0 \cdot x} \left(z E_{-\alpha_0} - z^{-1} E_{\alpha_0} \right) \\ M &= -\frac{1}{2} \sum_{\alpha \in \mathcal{R}_*} M e^{-\frac{1}{2}\alpha \cdot x} (E_\alpha + E_{-\alpha}) + \frac{\mu^2}{2} e^{\frac{1}{2}\alpha_0 \cdot x} \left(z E_{-\alpha_0} + z^{-1} E_{\alpha_0} \right) \end{aligned}$$

$$\det(kI - L(z)) = A(k) - \frac{1}{2} \left(z + \frac{\bar{\Lambda}^2}{z} \right).$$

$$A(k) = k^N + k^{N-2} J_2 + k^{N-3} J_3 + \cdots + J_N = \prod_{i=1}^{N} (k - u_i)$$

$$\Gamma = \{(k, y) \in \mathbf{C} \times \mathbf{C}, y^2 = A(k)^2 - \bar{\Lambda}^2\}$$

$$d\lambda = kd\ln z = \frac{A'(k)kdk}{\sqrt{A(k)^2 - \bar{\Lambda}^2}}$$

$$H = \frac{1}{2}\sum_{i=1}^{n+1} p_i^2 - \frac{1}{2}m^2 \sum_{i \neq j}^{n+1} \frac{1}{(x_i - x_j)^2}$$

$$H = \frac{1}{2}\sum_{i=1}^{n+1} p_i^2 - \frac{1}{2}m^2 \sum_{i \neq j}^{n+1} \frac{1}{\sin^2(x_i - x_j)}$$

$$H = \frac{1}{2}\sum_{i=1}^{n+1} p_i^2 - \frac{1}{2}m^2 \sum_{i \neq j}^{n+1} \wp(x_i - x_j; \omega_1, \omega_2)$$

$$\wp(x; \omega_1, \omega_2) \equiv \frac{1}{x^2} + \sum_{(m,n)\neq(0,0)} \left(\frac{1}{(x + 2m\omega_1 + 2n\omega_2)^2} - \frac{1}{(2m\omega_1 + 2n\omega_2)^2}\right).$$

$$\mathcal{R}(A_n) = \{e_i - e_j, i \neq j = 1, \cdots, n+1\}$$

$$H = \frac{1}{2}p^2 - \frac{1}{2}m^2 \sum_{\alpha \in \mathcal{R}} \wp(\alpha \cdot x; \omega_1, \omega_2)$$

$$\begin{aligned} L_{ij}(z) &= p_i \delta_{ij} - m(1 - \delta_{ij})\Phi(x_i - x_j; z) \\ M_{ij}(z) &= d_i(x)\delta_{ij} + m(1 - \delta_{ij})\Phi'(x_i - x_j; z) \end{aligned}$$

$$\Phi(x; z) = \frac{\sigma(z - x)}{\sigma(x)\sigma(z)} e^{x\zeta(z)}$$

$$\left(\frac{d^2}{dz^2} - \wp(x)\right)\Phi(x; z) = 2\wp(z)\Phi(x; z)$$

$$\mathrm{Re}(\omega_2) \to \infty \begin{cases} m = Me^{\delta\omega_2} \\ x_j = X_j + 2\omega_2 \delta j \\ M, X_j, 0 < \delta \leq \frac{1}{N} \text{ fixed} \end{cases}$$

$$\wp(x; -i\pi; \omega_2) = \frac{1}{2}\sum_{k=-\infty}^{\infty} \frac{1}{\mathrm{ch}(x + 2k\omega_2) - 1}$$

$$m^2 \wp(x_i - x_j; -i\pi, 2\omega_2) = \frac{1}{2}\sum_{k=-\infty}^{\infty} \frac{M^2 e^{2\delta\omega_2}}{\mathrm{ch}[X_i - X_j + 2\omega_2(\delta(i-j) + k)] - 1}$$

$$k = 0 \text{ term } \longrightarrow \begin{cases} M^2 \exp(X_j - X_i) & j - i = 1 \\ 0 & j - i \neq 1 \end{cases}$$

$$k = 1 \text{ term } \longrightarrow \begin{cases} M^2 \exp(X_N - X_1) & \delta N = 1 \\ 0 & \delta N < 1 \end{cases}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi \cdot \partial^\mu\phi - M^2 \sum_{\alpha\in\mathcal{R}_*} e^{\alpha\cdot\phi}$$

$$\partial_t u = 6u\partial_x u - \partial_x^3 u$$

$$u(t,x) = 2\sum_j \frac{1}{\left(x - x_j(t)\right)^2}$$

$$\partial_t x_j = \{x_j, \mathrm{tr}L^3\}$$

$$3\partial_{t'}^2 u = \partial_x(\partial_t u - 6u\partial_x u - \partial_x^3 u)$$

$$u(t,t',x) = 2\sum_j \wp\left(x - x_j(t,t')\right)$$

$$\partial_t x_j = \{x_j, \mathrm{tr}L^3\}\, \partial_{t'} x_j = \{x_j, \mathrm{tr}L^2\}$$

$$H(x,p) = \frac{1}{2}\sum_{i=1}^{n} p_i^2 - \frac{1}{2}\sum_{\alpha\in\mathcal{R}(\mathcal{G})} m_{|\alpha|}^2 \wp(\alpha\cdot x),$$

$$H_{\mathcal{G}}^{\text{twisted}} = \frac{1}{2}\sum_{i=1}^{n} p_i^2 - \frac{1}{2}\sum_{\alpha\in\mathcal{R}(\mathcal{G})} m_{|\alpha|}^2 \wp_{\nu(\alpha)}(\alpha\cdot x)$$

$$\wp_\nu(z) = \sum_{\sigma=0}^{\nu-1} \wp\left(z + 2\omega_a \frac{\sigma}{\nu}\right)$$

$$m = Mq^{-\frac{1}{2}\delta}$$
$$x = X - 2\omega_2\delta\rho^\vee$$

$$m^2\wp(\alpha\cdot x) = \frac{1}{2}M^2 \sum_{k=-\infty}^{\infty} \frac{e^{2\delta\omega_2}}{\mathrm{ch}(\alpha\cdot x - 2k\omega_2) - 1}$$

$$\delta \le \delta\alpha\cdot\rho^\vee \le 1 - \delta$$

$$\alpha_i\cdot\rho^\vee = 1, 1 \le i \le n$$

$$h_{\mathcal{G}} = 1 + \max_\alpha(\alpha\cdot\rho^\vee)$$

$$\tau = \frac{i}{2\pi} h_{\mathcal{G}}^\vee \ln\frac{m^2}{M^2} \Leftrightarrow m = Mq^{-\frac{1}{2h_{\mathcal{G}}^\vee}}$$

$$h_{\mathcal{G}}^\vee = 1 + \max_\alpha(\alpha^\vee\cdot\rho)$$

$$\rho = \frac{1}{2}\sum_{\alpha>0} \alpha$$

$$m^2\wp_\nu(x) = \frac{\nu^2}{2}\sum_{n=-\infty}^{\infty}\frac{m^2}{\operatorname{ch}\nu(x-2n\omega_2)-1}$$

$$m^2\wp_\nu(x) = \nu^2 M^2 \begin{cases} e^{-2\omega_2(\delta^\vee\alpha^\vee\cdot\rho-\delta^\vee)-\alpha^\vee\cdot X} + e^{-2\omega_2(1-\delta^\vee\alpha^\vee\cdot\rho-\delta^\vee)+\alpha^\vee\cdot X}, & \text{if } \alpha \text{ is long;} \\ e^{-2\omega_2(\delta^\vee\alpha^\vee\cdot\rho-\delta^\vee)-\alpha^\vee\cdot X}, & \text{if } \alpha \text{ is short.} \end{cases}$$

$$\begin{array}{ll} B_n & m_4 = 0 \\ C_n & m_1 = 0 \\ D_n & m_1 = m_4 = 0 \end{array}$$

$$m_1\left(m_1^2 - 2m_2^2 + \sqrt{2}m_2m_4\right) = 0$$

$$L(z) = P + X, M(z) = D + Y$$

$$P = \operatorname{diag}(p_1, \cdots, p_n; -p_1, \cdots, -p_n; 0)$$

$$D = \operatorname{diag}(d_1, \cdots, d_n; +d_1, \cdots, +d_n; 0)$$

$$X = \begin{pmatrix} A & B_1 & C_1 \\ B_2 & A^T & C_2 \\ C_2^T & C_1^T & 0 \end{pmatrix}, Y = \begin{pmatrix} A' & B_1' & C_1' \\ B_2' & A'^T & C_2' \\ C_2'^T & C_1'^T & 0 \end{pmatrix}$$

$$d_i = -2m_2\sum_k \wp(x_k) + \frac{m_1^2}{m_2}\wp(x_i) + \sqrt{2}m_4\wp(2x_i) + m_2\sum_{k\neq i}[\wp(x_i - x_k) + \wp(x_i + x_k)]$$

$$\begin{aligned} A_{ij} &= m_2\left(1-\delta_{ij}\right)\Phi\left(x_i - x_j, z\right) \\ B_{1ij} &= m_2\left(1-\delta_{ij}\right)\Phi\left(x_i + x_j, z\right) + \sqrt{2}m_4\delta_{ij}\Phi(2x_i, z) \\ B_{2ij} &= m_2\left(1-\delta_{ij}\right)\Phi\left(-x_i - x_j, z\right) + \sqrt{2}m_4\delta_{ij}\Phi(-2x_i, z) \\ C_{1i} &= m_1\Phi(x_i, z) \\ C_{2i} &= m_1\Phi(-x_i, z) \end{aligned}$$

$$\lambda_i = e_i;\ \lambda_{i+n} = -e_i\ 1 \le i \le n;\ \lambda_{2n+1} = 0$$

$$\sqrt{2}u_I = \lambda_I + v_I$$

$$\begin{aligned} \sqrt{2}\left(u_i - u_j\right) &= e_i - e_j + v_i - v_j \\ \sqrt{2}\left(u_{n+j} - u_{n+i}\right) &= e_i - e_j - v_i + v_j \\ \sqrt{2}\left(u_i - u_{n+j}\right) &= e_i + e_j + v_i - v_j \\ \sqrt{2}\left(u_{n+i} - u_j\right) &= -e_i - e_j + v_i - v_j, i \neq j, \end{aligned}$$

$$\begin{aligned} \sqrt{2}(u_i - u_{n+i}) &= 2e_i \\ \sqrt{2}(u_{n+i} - u_i) &= -2e_i \end{aligned}$$

$$\begin{aligned} \sqrt{2}(u_i - u_N) &= e_i + v_i - v_{2n+1} \\ \sqrt{2}(u_N - u_{n+i}) &= e_i - v_i + v_{2n+1} \\ \sqrt{2}(u_{n+i} - u_N) &= -e_i + v_i - v_{2n+1} \\ \sqrt{2}(u_N - u_i) &= -e_i - v_i + v_{2n+1} \end{aligned}$$

$$X=\sum_{\alpha\in\mathcal{R}(BC_n)}\Phi(\alpha\cdot x)\left(\sum_{\lambda_I-\lambda_J=\alpha}C_{IJ}E_{IJ}\right)$$

$$Y=\sum_{\alpha\in\mathcal{R}(BC_n)}\Phi'(\alpha\cdot x)\left(\sum_{\lambda_I-\lambda_J=\alpha}C_{IJ}E_{IJ}\right)$$

$$su_I=\lambda_I+v_I,\lambda_I\perp v_J.$$

$$s^2=\frac{1}{n}\sum_{I=1}^{N}\lambda_I^2=I_2(\Lambda)$$

$$\alpha_{IJ}=\lambda_I-\lambda_J$$

$$[E_{IJ},E_{KL}]=\delta_{JK}E_{IL}-\delta_{IL}E_{KJ}$$
$$[h,E_{IJ}]=(\lambda_I-\lambda_J)E_{IJ},[\tilde{h},E_{IJ}]=(v_I-v_J)E_{IJ}$$

$$E_{II}=\frac{1}{s^2}(\lambda_I\cdot h+v_I\cdot\tilde{h})$$

$$L=P+X,M=D+Y$$

$$X=\sum_{I\neq J}C_{IJ}\Phi_{IJ}(\alpha_{IJ},z)E_{IJ},Y=\sum_{I\neq j}C_{IJ}\Phi'_{IJ}(\alpha_{IJ},z)E_{IJ}$$

$$P=p\cdot h,D=d\cdot(h\oplus\tilde{h})+\Delta$$

$$\Phi_{IJ}=\Phi_{IJ}(\alpha_{IJ}\cdot x),\Phi'_{IJ}=\Phi'_{IJ}(\alpha_{IJ}\cdot x)$$
$$\wp'_{IJ}=\Phi_{IJ}(\alpha_{IJ}\cdot x,z)\Phi'_{JI}(-\alpha_{IJ}\cdot x,z)-\Phi'_{IJ}(\alpha_{IJ}\cdot x,z)\Phi'_{JI}(-\alpha_{IJ}\cdot x,z)$$

$$\sum_{I\neq J}C_{IJ}C_{JI}\wp'_{IJ}\alpha_{IJ}=s^2\sum_{\alpha\in\mathcal{R}(\mathcal{G})}m^2_{|\alpha|}\wp_{v(\alpha)}(\alpha\cdot x)$$
$$\sum_{I\neq J}C_{IJ}C_{JI}\wp'_{IJ}(v_I-v_J)=0$$
$$\sum_{K\neq I,J}C_{IK}C_{KJ}(\Phi_{IK}\Phi'_{KJ}-\Phi'_{IK}\Phi_{KJ})=sC_{IJ}\Phi_{IJ}d\cdot(u_I-u_J)+\sum_{K\neq I,J}\Delta_{IJ}C_{KJ}\Phi_{KJ}$$
$$-\sum_{K\neq I,J}C_{IK}\Phi_{IK}\Delta_{KJ}$$

$$[X,Y]=\sum_{J\notin\{I,L\}}C_{IJ}C_{JL}(\Phi_{IJ}\Phi'_{JL}-\Phi'_{IJ}\Phi_{JL})E_{IL}$$

$$\dot{p}\cdot h=\frac{1}{s^2}\sum_{I\neq J}C_{IJ}C_{JI}\wp'_{IJ}(\lambda_I\cdot h+v_I\cdot\tilde{h})$$

$$\dot{p}=\frac{1}{2}\sum_{\alpha\in\mathcal{R}(\mathcal{G})}m'_{|\alpha|\wp'_{v(\alpha)}}(\alpha\cdot x)$$

$$2m_2^2 = C_{ij}^2 + C_{n+j,n+i}^2$$

$$0 = C_{ij}^2(v_i - v_j) + C_{n+j,n+i}^2(-v_i + v_j)$$

$$m_2^2 = C_{ij}^2 = C_{n+i,n+j}^2 = C_{n+i,j}^2$$

$$2m_4^2 = C_{i,n+i}^2$$

$$m_1^2 = C_{iN}^2 = C_{n+i,N}^2$$

$$m_2 = C_{ij} = C_{n+i,n+j} = C_{n+i,j}$$

$$2m_4 = C_{i,n+i}$$

$$m_1 = C_{iN} = C_{n+i,N}$$

$$m_2 d \cdot (v_i - v_j) = \sum_{k \neq i,j} m_2^2[\wp(x_i - x_j) - \wp(x_k - x_j) + \wp(x_i + x_k) - \wp(x_k + x_j)]$$

$$+ m_1^2[\wp(x_i) - \wp(x_j)] + \sqrt{2} m_2 m_4 [\wp(2x_i) - \wp(2x_j)]$$

$$m_1 d \cdot (v_i - v_N) = \sum_{k \neq i} m_1 m_2 [\wp(x_i - x_j) + \wp(x_i + x_k) - 2\wp(x_k)] + \sqrt{2} m_1 m_4 [\wp(2x_i) - \wp(x_i)]$$

$$d \cdot v_i = d_0 + \frac{m_1^2}{m_2}\wp(x_i) + \sqrt{2} m_4 \wp(2x_i) + \sum_{k \neq i} m_2[\wp(x_i - x_k) + \wp(x_i + x_k)]$$

$$m_1 d \cdot v_N = m_1 d_0 + m_1\left(-2m_2 + \sqrt{2} m_4 + \frac{m_1^2}{m_2}\right)\wp(x_i) + 2m_1 m_2 \sum_k \wp(x_k)$$

$$c(\lambda, \lambda - \delta)c(\lambda - \delta, \mu) = c(\lambda, \mu + \delta)c(\mu + \delta, \mu)$$

when $\delta \cdot \lambda = -\delta \cdot \mu = 1, \lambda \cdot \mu = 0$

$$c(\lambda, \mu)c(\lambda - \delta, \mu) = c(\lambda, \lambda - \delta)$$

when $\delta \cdot \lambda = \lambda \cdot \mu = 1, \delta \cdot \mu = 0$

$$c(\lambda, \mu)c(\lambda, \lambda - \mu) = -c(\lambda - \mu, -\mu)$$

when $\lambda \cdot \mu = 1$

$$\Delta_{ab} = \sum_{\substack{\delta \cdot \beta_a = 1 \\ \delta \cdot \beta_b = 1}} \frac{m_2}{2}\big(c(\beta_a, \delta)c(\delta, \beta_b) + c(\beta_a, \beta_a - \delta)c(\beta_a - \delta, \beta_b)\big)\wp(\delta \cdot x)$$

$$- \sum_{\substack{\delta \cdot \beta_a = 1 \\ \delta \cdot \beta_b = -1}} \frac{m_2}{2}\big(c(\beta_a, \delta)c(\delta, \beta_b) + c(\beta_a, \beta_a - \delta)c(\beta_a - \delta, \beta_b)\big)\wp(\delta \cdot x)$$

$$\Delta_{aa} = \sum_{\beta a \cdot \delta = 1} m_2 \wp(\delta \cdot x) + 2m_2 \wp(\beta_a \cdot x)$$

$$C_{\lambda\mu} = \begin{cases} m_2 c(\lambda,\mu) & \lambda\cdot\mu = 1\ c(\lambda,\mu) = \pm 1 \\ 0 & \mathbb{O}_{\text{otherwise}} \end{cases}$$

$$C_{\lambda,c} = \begin{cases} \sum_{a=1}^{8} \frac{1}{2}(\lambda\cdot\beta_a)c\big(\lambda,\beta_a(\lambda\cdot\beta_a)\big)C_{\beta_a,c} & \lambda \neq \pm\beta_b \\ \pm C_{\beta_b,c} & \lambda = \pm\beta_b \end{cases}$$

$$\sqrt{60}d\cdot u_\lambda = \sum_{\delta\cdot\lambda=1} m_{2\wp}(\delta\cdot x) + 2m_{2\wp}(\lambda\cdot x)$$

$$C_{\lambda\mu} = \begin{cases} \sqrt{\frac{\alpha^2}{2}}m_{|\alpha|} & \text{when } \alpha = \lambda-\mu \text{ is a root} \\ 0 & \mathbb{O}_{\text{otherwise}} \end{cases}$$

$$sd\cdot u_\lambda = \sum_{\lambda\cdot\delta=1;\delta^2=2} m_{|\delta|\wp(\delta\cdot x)}$$

$$C_{\lambda,\mu} = \begin{cases} m_2 & \lambda\cdot\mu = \pm\frac{1}{3} \\ 0 & \mathbb{O}_{\text{otherwise}} \end{cases}$$

$$C_{\lambda,7} = \sqrt{2}m_2$$

$$\sqrt{2}d\cdot u_\lambda = \sum_{\delta^2=1,\lambda\cdot\delta=1} m_2\wp(\delta\cdot x) + m_2\wp(\lambda\cdot x)$$

$$\sqrt{2}d\cdot u_7 = \frac{1}{2}\sum_{\kappa^2=2/3} m_2\wp(\kappa\cdot x).$$

$$C_{\lambda\mu} = \begin{cases} m_2 & \lambda\cdot\mu = 0,\frac{1}{2} \\ 0 & \mathbb{O}_{\text{otherwise}} \end{cases}$$

$$C_{\lambda,a} = m_2\big(1-\delta_{[\lambda],a}\big)$$

$$C_{a,b} = 0$$

$$\sqrt{6}d\cdot u_\lambda = 2m_2\wp(\lambda\cdot x) + m_2\sum_{\delta^2=1,\lambda\cdot\delta}\wp(\delta\cdot x) - \frac{1}{2}m_2\sum_{\kappa\in[\lambda]}\wp(\kappa\cdot x)$$

$$\sqrt{6}d\cdot u_a = -m_2\sum_{[\kappa]=a}\wp(\kappa\cdot x) + \frac{1}{2}m_2\sum_{\kappa}\wp(\kappa\cdot x).$$

$$\pm e_i \pm e_j,\quad 1\leq i<j\leq 5$$

$$\pm\frac{1}{2}\left(\sqrt{3}e_6 + \sum_{i=1}^{5}\epsilon_i e_i\right),\quad \prod_{i=1}^{5}\epsilon_i = 1$$

$$\frac{2}{\sqrt{3}}e_6\,\frac{1}{2\sqrt{3}}e_6 - \frac{1}{2}\sum_{i=1}^{5}\epsilon_i e_i, \prod_{i=1}^{5}\epsilon_i = 1 - \frac{1}{2\sqrt{3}}e_6 \pm e_i, 1\leq i\leq 5$$

$$\lambda\pm\alpha = w_{\text{weight}} \Leftrightarrow \lambda\cdot\alpha = \mp 1$$

$$\sum_{\alpha\in\mathcal{R}(\mathcal{G})} \wp'(\alpha\cdot x) \sum_{\lambda_I-\lambda_J=\alpha} 1 = s^2 \sum_{\alpha\in\mathcal{R}(\mathcal{G})} \wp'(\alpha\cdot x)$$

$$\sum_{\lambda_I-\lambda_J=\alpha} (v_I - v_J) = 0$$

$$\sum_{\lambda_I-\lambda_J=\alpha} (v_I - v_J)\cdot u_K = \sum_{\lambda_I-\lambda_J=\alpha} \left(s(\delta_{IK}-\delta_{JK}) - \frac{1}{s}\alpha\cdot\lambda_K\right)$$

$$\sum_{\lambda_I-\lambda_J=\alpha} (v_I - v_J)\cdot u_K = \sqrt{6}\left\{\left(\sum_{\lambda_I-\lambda_J=\alpha} (\delta_{IK}-\lambda_{JK})\right) - \alpha\cdot\lambda_K\right\}$$

$$\sqrt{6}C_{IJ}d\cdot(u_I - u_J) = m_{\sqrt{2}}^2\left[\sum \left[\wp(\alpha_{IK}\cdot x) - \sum \wp(\alpha_{KJ}\cdot x)\right]\right.,$$

$$\sum_{\beta\cdot\lambda_I=-\beta\cdot\lambda_J=1} \wp(\beta\cdot x) - \sum_{\beta\cdot\lambda_I=-\beta\cdot\lambda_J=-1} \wp(\beta\cdot x),$$

$$\sqrt{6}m_{\sqrt{2}}d\cdot(u_I - u_J) = m_{\sqrt{2}}^2\left(\sum_{\beta\cdot\lambda_I=-1,\beta\cdot\lambda_J=0} \wp(\beta\cdot x) - \sum_{\beta\cdot\lambda_J=-1,\beta\cdot\lambda_I=0} \wp(\beta\cdot x)\right)$$

$$m_{\sqrt{2}}^2\left(\sum_{\beta\cdot\lambda_I=-1} \wp(\beta\cdot x) - \sum_{\beta\cdot\lambda_J=-1} \wp(\beta\cdot x)\right)$$

$$-m_{\sqrt{2}}\left(\sum_{\beta\cdot\lambda_I=-1;\beta\cdot\lambda_J=\pm 1} \wp(\beta\cdot x) - \sum_{\beta\cdot\lambda_I=-1;\beta\cdot\lambda_J=\pm 1} \wp(\beta\cdot x)\right).$$

$$\Phi_{IJ}(x,z) = \begin{cases} \Phi(x,z), & \text{if } I-J \neq 0, \pm n \\ \Phi_2\left(\frac{1}{2}x, z\right), & \text{if } I-J = \pm n \end{cases}$$

$$\Phi_2\left(\frac{1}{2}x, z\right) = \frac{\Phi\left(\frac{1}{2}x, z\right)\Phi\left(\frac{1}{2}x + \omega_1, z\right)}{\Phi(\omega_1, z)}$$

$$C_{IJ} = \begin{cases} m_2 & I-J \neq 0, \pm n \\ m_1 & I-J = \pm n \end{cases}$$

$$d\cdot v_i = m_2 \sum_{J-i\neq 0,n} \wp\left((e_i - \lambda_J)\cdot x\right) + \frac{1}{2}m_1\wp_2(e_i\cdot x)$$

$$\Phi_{IJ}(x,z) = \Phi_2(x + \omega_{IJ}, z)$$

$$\omega_{IJ} = \begin{cases} 0, & \text{if } I \neq J = 1,2,\cdots,2n+1 \\ \omega_2, & \text{if } 1 \leq I \leq 2n, J = 2n+2 \\ -\omega_2, & \text{if } 1 \leq J \leq 2n, I = 2n+2 \end{cases}$$

$$C_{IJ} = \begin{cases} m_2 & I,J = 1,\cdots,2n; I - J \neq \pm n \\ \frac{1}{\sqrt{2}} m_4 = \sqrt{2} m_2 & I = 1,\cdots,2n; J = 2n+1, 2n+2; I \leftrightarrow \mathcal{J} \\ 2m_2 & I = 2n+1, J = 2n+2; I \leftrightarrow J \end{cases}$$

$$\sqrt{2} d \cdot u_I = \sum_{J-I \neq 0, \pm n} m_2 \wp_2 \left((\lambda_I - \lambda_J) \cdot x \right) + 8 m_2 \wp(2\lambda_I \cdot x); I = 1,\cdots,2n$$

$$\sqrt{2} d \cdot u_{2n+1} = \sum_{J=1}^{2n} \wp_2 (\lambda_J \cdot x) + 2 m_2 \wp_2(\omega_2)$$

$$\sqrt{2} d \cdot u_{2n+2} = \sum_{J=1}^{2n} \wp_2 (\lambda_J \cdot x + \omega_2) + 2 m_2 \wp_2(\omega_2)$$

$$\Phi_{\lambda\mu}(x,z) = \begin{cases} \Phi(x,z), & \text{if } \lambda \cdot \mu = 0 \\ \Phi_1(x,z), & \text{if } \lambda \cdot \mu = \frac{1}{2} \\ \Phi_2 \left(\frac{1}{2} x, z \right), & \text{if } \lambda \cdot \mu = -1 \end{cases}$$

$$\Phi_1(x,z) = \Phi(x,z) - e^{\pi i \zeta(z) + \eta_1 z} \Phi(x + \omega_1, z)$$

$$C_{\lambda\mu} = \begin{cases} m_2 & \lambda \cdot \mu = 0 \\ \frac{1}{\sqrt{2}} m_1 & \lambda \cdot \mu = \frac{1}{2} \\ 0 & \lambda \cdot \mu = -\frac{1}{2} \\ \sqrt{2} m_1 & \lambda \cdot \mu = -1 \end{cases}$$

$$\sqrt{6} d \cdot v_\lambda = \sum_{\substack{\delta \text{ long} \\ \delta \cdot \lambda = 1}} m_2 \wp(\delta \cdot x) - \sum_{\kappa \in [\lambda]} \frac{1}{2\sqrt{2}} m_1 \wp_2(\kappa \cdot x) + \frac{1}{\sqrt{2}} m_1 \wp_2(\lambda \cdot x)$$

$$\lambda_i = -\lambda_{i+n} = e_i, 1 \leq i \leq n.$$

$$\sqrt{2} u_i = e_i + v_i, \sqrt{2} u_{n+i} = -e_i + v_i, 1 \leq i \leq n.$$

$$\begin{aligned} \sqrt{2}(u_i - u_j) &= e_i - e_j + v_i - v_j, i \neq j \\ \sqrt{2}(u_{v+i} - u_{v+j}) &= e_i - e_j + v_i - v_j, i \neq j \\ \sqrt{2}(u_i - u_{v+j}) &= e_i + e_j + v_i - v_j, i \neq j \\ \sqrt{2}(u_{v+i} - u_j) &= -e_i - e_j + v_i - v_j, i \neq j \end{aligned}$$

$$\begin{aligned} \sqrt{2}(u_i - u_{v+i} &= 2e_i \\ \sqrt{2}(u_{v+i} - u_i) &= -2e_i \end{aligned}$$

$$C_{IJ} = \begin{cases} m_2, & \text{if } I - J \neq 0, \pm n \\ m_1, & \text{if } I - J = \pm n \end{cases}$$

$$m_I\Lambda(2x_i)d\cdot 2e_i=\sum_{K\neq i,n+i} m_2^2\{\Phi(\alpha_{iK}\cdot x)\Phi'(\alpha_{K(n+i)}\cdot x)-\Phi'(\alpha_{iK}\cdot x)\Phi(\alpha_{K(n+i)}\cdot x)\}$$

$$=m_2^2\Phi(2x_i)\sum_{k\neq i}^{n}\{\wp(\alpha_{ik}\cdot x)-\wp(\alpha_{k(n+i)}\cdot x)$$

$$+\wp(\alpha_{i(n+k)}\cdot x)-\wp(\alpha_{(n+k)(n+i)}\cdot x)\}$$

$$=m_2\Phi(\alpha_{IJ}\cdot x)sd\cdot(u_I-u_J)+\sum_{\substack{I-K\neq 0,\pm n\\ K-J\neq 0,\pm n}} m_2^2\{\Phi(\alpha_{IK}\cdot x)\Phi'(\alpha_{KJ}\cdot x)-\Phi'(\alpha_{IK}\cdot x)\Phi(\alpha_{KJ}\cdot x)\}$$

$$+\sum_{\substack{I-K=\pm n\\ K-J\neq 0,\pm n}} m_1m_2\{\Lambda(\alpha_{IK}\cdot x)\Phi'(\alpha_{KJ}\cdot x)-\Lambda'(\alpha_{IK}\cdot x)\Phi(\alpha_{KJ}\cdot x)\}$$

$$+\sum_{\substack{I-KQ,\pm n\\ K-J=\pm n}} m_1m_2\{\Phi(\alpha_{IK}\cdot x)\Lambda'(\alpha_{KJ}\cdot x)-\Phi'(\alpha_{IK}\cdot x)\Lambda(\alpha_{KJ}\cdot x)\}$$

$$+\sum_{\substack{I-K=\pm n\\ K-J=\pm n}} m_1^2\{\Lambda(\alpha_{IK}\cdot x)\Lambda'(\alpha_{KJ}\cdot x)-\Lambda'(\alpha_{IK}\cdot x)\Lambda(\alpha_{KJ}\cdot x)\}$$

$$m_2\Phi(\alpha_{IJ}\cdot x)sd\cdot(u_I-u_J)$$

$$=\sum_{\substack{I-K\neq 0,\pm n\\ K-J\neq 0,\pm n}} m_2^2\{\Phi(\alpha_{IK}\cdot x)\Phi'(\alpha_{KJ}\cdot x)-\Phi'(\alpha_{IK}\cdot x)\Phi(\alpha_{KJ}\cdot x)\}$$

$$+m_1m_2\{\Lambda(2\lambda_I\cdot x)\Phi'(-(\lambda_I+\lambda_J)\cdot x)-\Lambda'(2\lambda_I\cdot x)\Phi(-(\lambda_I+\lambda_K)\cdot x)\}$$

$$+m_1m_2\left\{\Phi\left((\lambda_I+\lambda_J)\cdot x\right)\Lambda'(-2\lambda_J\cdot x)-\Phi'\left((\lambda_I+\lambda_J)\cdot x\right)\Lambda(-2\lambda_J\cdot x)\right\}$$

$$sd\cdot(u_I-u_J)=\sum_{\substack{I-K\neq 0,\pm n\\ K-J\neq 0,\pm n}}\{\wp(\alpha_{IK}\cdot x)-\wp(\alpha_{KJ}\cdot x)\}+\frac{1}{2}m_1\{\wp_2(\lambda_I\cdot x)-\wp_2(\lambda_J\cdot x)\}$$

$$d\cdot v_i=\sum_{J-i\neq 0,n} m_2\wp\left((e_i-\lambda_j)\cdot x\right)+\frac{1}{2}m_1\wp_2(e_I\cdot x)$$

$$\Phi(u,z)\to\begin{cases}+e^{-\frac{1}{2}u}(1-Z^{-1}e^{u-\omega_2}), & \mathrm{Re}(u)\to+\infty\\ -e^{\frac{1}{2}u}(1-Ze^{-u-\omega_2}), & \mathrm{Re}(u)\to-\infty\end{cases}$$

$$C_{IJ}=\begin{cases}M_{|\alpha|}e^{\delta\omega_2}c_{IJ}, & \text{when } \alpha_{IJ}=\alpha\in\mathcal{R}(\mathcal{G})\\ 0, & \text{when } \alpha_{IJ}\notin\mathcal{R}(\mathcal{G})\end{cases}$$

$$E_\alpha=\sum_{\alpha_{IJ}=\alpha} c_{IJ}E_{IJ}$$

$$C_{IJ} = \begin{cases} M_{|\alpha|} e^{\delta^\vee \omega_2} c_{IJ}, & \text{when } \alpha_{IJ} = \alpha \in \mathcal{R}(\mathcal{G}) \\ 0, & \text{when } \alpha_{IJ} \notin \mathcal{R}(\mathcal{G}) \end{cases}$$

$$C_{IJ}\Phi_{IJ}(\alpha \cdot x, z) \to \begin{cases} \pm\kappa_{\mathcal{G}} M_{|\alpha|} c_{IJ} e^{\mp\frac{1}{2}\alpha^\vee \cdot X}, & \text{if } l^\vee(\alpha^\vee) = \pm 1 \\ \mp\kappa_{\mathcal{G}} M_{|\alpha|} c_{IJ} e^{\pm\frac{1}{2}\alpha_0^\vee \cdot X}, & \text{if } l^\vee(\alpha^\vee) = \pm l_0^\vee \\ 0 & \mathbb{O}_{\text{otherwise}} \end{cases}$$

$$C_{IJ}\Phi'_{IJ}(\alpha \cdot x, z) \to \begin{cases} -\frac{1}{2}\kappa_{\mathcal{G}} M_{|\alpha|} c_{IJ} e^{\mp\frac{1}{2}\alpha^\vee \cdot X}, & \text{if } l^\vee(\alpha^\vee) = \pm 1 \\ -\frac{1}{2}\kappa_{\mathcal{G}} M_{|\alpha|} c_{IJ} e^{\pm\frac{1}{2}\alpha_0^\vee \cdot X}, & \text{if } l^\vee(\alpha^\vee) = \pm l_0^\vee \\ 0 & \mathbb{O}_{\text{otherwise}} \end{cases}$$

$$C_{IJ}\Phi_{IJ}(\alpha \cdot x, z) \to \begin{cases} \pm 2 M_{|\alpha|} c_{IJ} e^{\mp\frac{1}{2}\alpha^\vee \cdot X}, & \text{if } l^\vee(\alpha^\vee) = \pm 1 \\ \mp 2 M_{|\alpha|} c_{IJ} e^{\pm\frac{1}{2}\alpha_0^\vee \cdot X} Z^{-\frac{1}{2}\mp\frac{1}{2}}, & \text{if } l^\vee(\alpha^\vee) = \pm l_0, I < J \\ \mp 2 M_{|\alpha|} c_{IJ} e^{\pm\frac{1}{2}\alpha_0^\vee \cdot X} Z^{\frac{1}{2}\mp\frac{1}{2}}, & \text{if } l^\vee(\alpha^\vee) = \pm l_0, J < I \\ 0 & \mathbb{O}_{\text{otherwise}} \end{cases}$$

$$C_{IJ}\Phi'_{IJ}(\alpha \cdot x, z) \to \begin{cases} -2 M_{|\alpha|} c_{IJ} e^{\mp\frac{1}{2}\alpha^\vee \cdot X}, & \text{if } l^\vee(\alpha^\vee) = \pm 1; \\ -2 M_{|\alpha|} c_{IJ} e^{\pm\frac{1}{2}\alpha_0^\vee \cdot X} Z^{-\frac{1}{2}\mp\frac{1}{2}}, & \text{if } l^\vee(\alpha^\vee) = \pm l_0, I < J; \\ -2 M_{|\alpha|} c_{IJ} e^{\pm\frac{1}{2}\alpha_0^\vee \cdot X} Z^{\frac{1}{2}\mp\frac{1}{2}}, & \text{if } l^\vee(\alpha^\vee) = \pm l_0, J < I; \\ 0 & \mathbb{O}_{\text{otherwise}} \end{cases}$$

$$\omega = \delta\left(\sum_{i=1}^{n} d\lambda(z_i)\right)$$

$$L_{ij}(z) = p_i\delta_{ij} - m(1 - \delta_{ij})\Phi(x_i - x_j, z)$$

$$\tau = \frac{\omega_2}{\omega_1} = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

$$\Gamma = \{(\tilde{k}, z) \in \mathbf{C} \times \Sigma, \det(\tilde{k}I - L(z)) = 0\}$$

$$\det(\tilde{k} - L(z)) = \frac{\vartheta_1\left(\frac{1}{2\omega_1}\left(z - m\frac{\partial}{\partial k}\right)\middle|\, \tau\right)}{\vartheta_1\left(\frac{z}{2\omega_1}\middle|\, \tau\right)} H(k)\Bigg|_{k = \tilde{k} + m\partial_z \ln\, \vartheta_1\left(\frac{z}{2\omega_1}\middle|\tau\right)} = 0$$

$$\lim_{q \to 0} \frac{1}{2\pi i} \oint_{A_i} \tilde{k}\, dz = k_i - \frac{1}{2}m$$

$$\vartheta_1(u \mid \tau) = \sum_{r \in \frac{1}{2} + \mathbf{Z}} q^{\frac{1}{2}r^2} e^{2\pi i r\left(u + \frac{1}{2}\right)}$$

$$k=\tilde{k}+m\partial_z \ln\, \vartheta_1\left(\frac{z}{2\omega_1}\middle|\, \tau\right)+\frac{1}{2}m$$

$$\sum_{n\in\mathbf{Z}}(-)^n q^{\frac{1}{2}n(n-1)}e^{nz}H(k-n\cdot m)=0$$

$$w=e^z$$

$$H(k)-wH(k-m)=0$$

$$d\lambda=\tilde{k}d\ln\, w=kd\ln\frac{H(k)}{H(k-m)}-md\ln\, \vartheta_1\left(\frac{z}{2\pi i}\middle|\, \tau\right)-\frac{1}{2}mdz$$

$$\frac{1}{2\pi i}\oint_{A_i} d\lambda\square\Big|_{q=0}=k_i-\frac{1}{2}m$$

$$w=\frac{H(k)}{H(k-m)}\left[1+\sum_{n=1}^{\infty}\frac{q^n}{n!}\frac{\partial^n}{\partial y^n}F^n(y)\square\Big|_{y=1}\right]$$

$$F(y)=\sum_{n=1}^{\infty}q^{\frac{1}{2}n(n+1)}(-)^n[y^{-n}\eta_n^+(k)-y^{n+1}\eta_n^-(k-m)]$$

$$\eta_n^{\pm}(k)=\frac{H(k\pm mn)H(k\mp m)}{H(k)^{n+1}}$$

$$a_i=k_i+q\bar{S}_i(k_i)+\frac{1}{4}q^2(\bar{S}_i)'''(k_i)+\mathcal{O}(q^3)$$

$$\bar{S}_i(k)\equiv\frac{H(k+m)H(k-m)}{\prod_{j\neq i}\left(k-k_j\right)^2}$$

$$a_j=\frac{1}{2\pi i}\oint_{A_j} d\lambda\, \frac{\partial\mathcal{F}}{\partial\tau}\Big|_{a_j}=H(x,p)=\frac{1}{2}\sum_{i=1}^{N}k_i^2$$

$$\delta a_{Di}=\sum_{j=1}^{N}\oint_{A_j}\tilde{k}\Omega_i$$

$$\Omega_i=\frac{1}{2\pi i}\frac{\partial}{\partial a_i}d\lambda_i$$

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\mathcal{F}}{\partial a_i}\right)=\frac{1}{4\pi i}\frac{\partial}{\partial a_i}\left(\sum_{j=1}^{N}\oint_{A_j}\tilde{k}^2dz\right)$$

$$\mathcal{F}=\mathcal{F}^{(\mathrm{pert})}+\sum_{n=1}^{\infty}q^n\mathcal{F}^{(n)}$$

$$\mathcal{F}^{(\mathrm{pert})}=\frac{\tau}{2}\sum_i a_i^2-\frac{1}{8\pi i}\sum_{i,j}\left[\left(a_i-a_j\right)^2\ln\left(a_i-a_j\right)^2-\left(a_i-a_j-m\right)^2\ln\left(a_i-a_j-m\right)^2\right]$$

$$S_i(a) = \frac{\prod_{j=1}^{N} \left[\left(a_i - a_j\right)^2 - m^2\right]}{\prod_{j\neq i} \left(a - a_j\right)^2}$$

$$\mathcal{F}^{(1)} = \frac{1}{2\pi i}\sum_i S_i(a_i)$$

$$\mathcal{F}^{(2)} = \frac{1}{8\pi i}\left[\sum_i S_i(a_i)\partial_i^2 S_i(a_i) + 4\sum_{i\neq j} \frac{S_i(a_i)S_j(a_j)}{\left(a_i - a_j\right)^2} - \frac{S_i(a_i)S_j(a_j)}{\left(a_i - a_j - m\right)^2}\right]$$

$$\frac{dk}{dz} = 0$$

$$k_i^{\pm} = k_i \pm q^{1/2} k_i^{(1)}, k_i^{(1)} = 2\frac{H^{1/2}(k_i - m)H^{1/2}(k_i + m)}{\prod_{j\neq i} \left(k_i - k_j\right)}$$

$$a_{Di} = \frac{1}{2\pi i}\int_{k_i^+}^{k_i^+ + m} d\lambda = \frac{1}{2\pi i}\int_{k_i^+}^{k_i^+ + m} k d\ln\, w$$

$$w = \frac{H(k)}{H(k-m)} \times \frac{1 + \sqrt{1 - 4q\eta_1^+(k)}}{1 + \sqrt{1 - 4q\eta_1^-(k-m)}}$$

$$\sigma_K(k) = \sigma_K(p) + \sum_{l=1}^{[K/2]} m^{2l} \sum_{\substack{|S_i \cap S_j| = 2\delta_{ij} \\ 1\leq i,j\leq l}} \sigma_{K-2l}\left(p_{\left(\cup_{i=1}^{l} S_i\right)^*}\right)\prod_{i=1}^{l} \left[\wp(S_i) + \frac{\eta_1}{\omega_1}\right]$$

$$k_i = v_1 + x_i, k_{N_1+j} = v_2 + y_j, 1 \leq i,j \leq N_1$$

$$A(x) - t(-)^{N_1}B(x) - 2^{N_1}\Lambda^{N_1}\left(\frac{1}{t} - t^2\right) = 0$$

$$A_i^I = \prod_{\substack{j\neq i \\ j\in I}} \left(x - x_j^{(I)}\right), B^I(x) = \prod_{\substack{j\in J \\ |I-J|=1}} \left(\mu \pm \left(x - x_j^{(J)}\right)\right), S_i^I(x) = \frac{B^I(x)}{A_i^I(x)^2},$$

$$\mathcal{F}_{SU(N_1)\times SU(N_1)}^{(1)} = \frac{(-2\Lambda)^{N_1}}{2\pi i}\sum_{I=1,2}\sum_{i\in I} S_i^I\left(x_i^{(I)}\right)$$

$$\mathcal{F}_{SU(N_1)\times SU(N_1)}^{(2)} = \frac{(-2\Lambda)^{2N_1}}{8\pi i}\sum_{I=1,2}\sum_{i\in I} S_i^I\left(x_i^{(I)}\right)\frac{\partial^2 S_i^I\left(x_i^{(I)}\right)}{\partial x_i^{(I)2}} + \sum_{\substack{i\neq j \\ i,j\in I}} \frac{S_i^I\left(x_i^{(I)}\right)S_j^I\left(x_j^{(I)}\right)}{\left(x_i^{(I)} - x_j^{(I)}\right)^2}$$

$$a_i = \frac{1}{2\pi i}\oint_{A_i} d\lambda = \frac{k_i}{2\pi i}\oint_{A} dz = \frac{2\omega_1}{2\pi i}k_i$$

$$a_{Di} = \frac{1}{2\pi i}\oint_{B_i} d\lambda = \frac{k_i}{2\pi i}\oint_{B} dz = \frac{2\omega_1}{2\pi i}\tau k_i$$

$$x= X + 2\omega_2 \frac{1}{h_{\mathcal{G}}^{V}} \rho$$

$$m = Mq^{-\frac{1}{2h_{\mathcal{G}}^{V}}}$$

$$\frac{\partial \mathcal{F}}{\partial \tau} = H_{\mathcal{G}}^{\text{twisted}}(x, p),$$

$$\frac{1}{Z} = \frac{1}{2}\coth\frac{z}{2} - \frac{1}{z}$$

$$\wp(z) \rightarrow \frac{1}{Z^2} - \frac{1}{6}$$

$$\Phi(x, z) \rightarrow \frac{1}{2}\coth\frac{x}{2} - \frac{1}{Z}$$

$$x = \xi\rho^{\vee}, \alpha \cdot x = \xi l(\alpha), \xi \rightarrow \infty$$

$$\Phi(\alpha \cdot x, z) \rightarrow -\frac{1}{Z} + \begin{cases} +\frac{1}{2}, & \text{if } \alpha > 0 \\ -\frac{1}{2}, & \text{if } \alpha < 0 \end{cases}$$

$$\mu_{ij}^{+} = \begin{cases} 1, & \text{if } i < j \\ 0, & \text{if } i \geq j \end{cases} \mu_{ij}^{-} = \begin{cases} 1, & \text{if } i > j \\ 0, & \text{if } i \leq j \end{cases}$$

$$R(k, z) = \det\begin{pmatrix} kI - P + \frac{m}{Z}\mu - \frac{m}{2}(\mu^{+} - \mu^{-}) & \left(\frac{m}{Z} - \frac{m}{2}\right)\mu \\ \left(\frac{m}{Z} + \frac{m}{2}\right)\mu & kI + P + \frac{m}{Z}\mu + \frac{m}{2}(\mu^{+} - \mu^{-}) \end{pmatrix}$$

$$R(k, z) = \det\left[(kI + P - m\mu^{-})(kI - P - m\mu^{+}) + k\left(m + 2\frac{m}{Z}\right)\mu\right]$$

$$0 = A^2 + mA + 2k\frac{m}{Z} - k^2$$

$$R(k, z) = \det\left[(AI + P - m\mu^{-})(AI - P - m\mu^{-}) + \left(mA + 2k\frac{m}{Z}\right)(\mu + I)\right]$$

$$R(k, z) = \prod_{j=1}^{n}\left(A^2 - p_j^2\right) + \left(mA + 2\frac{m}{Z}\right)\sum_{j=1}^{n}\prod_{i=1}^{j-1}\left((A + m)^2 - p^2\right)\prod_{i=j+1}^{n}\left(A^2 - p_i^2\right)$$

$$H(A) = \prod_{j=1}^{n}\left(A^2 - p_j^2\right) = \sum_{j=0}^{n}(-1)^{n-j}A^{2j}u_{2n-2j}$$

$$R(k, z) = \frac{m^2 + mA - 2k\frac{m}{z}}{m^2 + mA}H(A) + \frac{mA + 2k\frac{m}{Z}}{m^2 + 2mA}H(A + m)$$

$$R(k, z) = \sum_{j=0}^{n}(-1)^{n-j}P_{2j}u_{2n-2j}$$

$$0=P_{2(j+1)}-\left(2k^2+m^2-4k\frac{m}{Z}\right)P_{2j}+k^2\left(k-2\frac{m}{Z}\right)^2 P_{2(j-1)}$$

$$P_4=k^4-4k^2\frac{m^2}{Z^2}+m^2k^2$$
$$P_6=k^6-12k^4\frac{m^2}{Z}+16k^3\frac{m^3}{Z^3}-4k^3\frac{m^3}{Z}-4k^2\frac{m^4}{Z^2}+3k^4m^2+k^2m^4$$
$$P_8=k^8-24k^6\frac{m^2}{Z^2}+6k^6m^2+64k^5\frac{m^3}{Z^3}-16k^5\frac{m^3}{Z}-48k^4\frac{m^4}{Z^4}$$
$$-8k^4\frac{m^4}{Z^2}+5k^4m^4+32k^3\frac{m^5}{Z^3}-8k^3\frac{m^5}{Z}-4k^2\frac{m^6}{Z^2}+k^2m^6$$
$$P_{10}=k^{10}-40k^8\frac{m^2}{Z^2}+160k^7\frac{m^3}{Z^3}-240k^6\frac{m^4}{Z^4}+128k^5\frac{m^5}{Z^5}$$
$$+m^2\left[10k^8-40k^7\frac{m}{Z}+160k^5\frac{m^3}{Z^3}-160k^4\frac{m^4}{Z^4}\right]$$
$$+m^4\left[15k^6-48k^5\frac{m}{Z}+12k^4\frac{m^2}{Z^2}+48k^3\frac{m^3}{Z^3}\right]$$
$$+m^6\left[7k^4-12k^3\frac{m}{Z}-4k^2\frac{m^2}{Z^2}\right]+m^8k^2$$

$$Q_0=1$$
$$Q_2=k^2$$
$$Q_4=k^4-4k^2m^2\wp$$
$$Q_6=k^6-12k^4m^2\wp-8k^3m^3\wp'$$
$$Q_8=k^8-24k^6m^2\wp-32k^5m^3\wp'-48k^4m^4\wp^2+64g_2k^2m^6\wp$$
$$Q_{10}=k^{10}-40k^8m^2\wp-80k^7m^3\wp'-240k^6m^4\wp^2-64k^5m^5\wp\wp'$$
$$+704g_2k^4m^6\wp+512g_2k^3m^7\wp'-768k^2m^8g_3\wp.$$

$$e^u=\frac{k^2-(A+m)^2}{k^2-A^2}$$

$$e^u=\frac{H(A+m)}{H(A)}$$

$$e^z=\frac{\frac{2}{Z}+1}{\frac{2}{Z}-1}=\frac{(k-A)(k+A+m)}{(k+A)(k-A-m)}$$
$$d\lambda=-Adu-md\ln\left(k^2-(A+m)^2\right)$$

$$\mathcal{F}^{1-\text{loop}}=-\frac{1}{8\pi i}\sum_{\alpha\in\mathcal{R}(D_r)}(\alpha\cdot a)^2\ln(\alpha\cdot a)^2-(\alpha\cdot a+m)^2\ln(\alpha\cdot a+m)^2$$

$$\epsilon_{ijk}=\epsilon^{ijk},\epsilon^{123}=1;\ \ \epsilon_{\mu\nu\rho\sigma}=-\epsilon^{\mu\nu\rho\sigma},\epsilon^{0123}=1$$

$$\tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$$

$$\sigma^0=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\ \sigma^1=\begin{pmatrix}0&1\\1&0\end{pmatrix}\ \sigma^2=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\ \sigma^3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = -2\eta^{\mu\nu} I$$

$$\{\gamma_5, \gamma^\mu\} = 0 \; \gamma_5 = \gamma^5 = (\gamma_5)^\dagger = i\gamma^0\gamma^1\gamma^2\gamma^3 \; (\gamma_5)^2 = I$$

$$\Sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \text{ with } [\gamma_5, \Sigma^{\mu\nu}] = 0$$

$$C\gamma_\mu C^{-1} = -(\gamma_\mu)^T$$

$$C\Sigma_{\mu\nu}C^{-1} = -(\Sigma_{\mu\nu})^T, C\gamma_\mu\gamma_5 C^{-1} = (\gamma_\mu\gamma_5)^T, C\gamma_5 C^{-1} = (\gamma_5)^T$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \; C = i\gamma^2\gamma^0$$

$$\gamma^0 = \begin{pmatrix} 0 & i\sigma^2 \\ i\sigma^2 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} -\sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}$$
$$\gamma^5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} \quad C = \begin{pmatrix} 0 & -i\sigma^2 \\ i\sigma^2 & 0 \end{pmatrix}$$

$$\chi^\alpha = \epsilon^{\alpha\beta}\chi_\beta \quad \chi_\alpha = \epsilon_{\alpha\beta}\chi^\beta$$
$$\bar{\psi}^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar{\psi}_{\dot\beta} \quad \bar{\psi}_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar{\psi}^{\dot\beta}$$

$$\epsilon^{12} = \epsilon_{21} = -\epsilon^{21} = -\epsilon_{12} = 1, \quad \epsilon^{11} = \epsilon^{22} = 0; \quad \epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \delta_\alpha^\gamma$$
$$\epsilon^{\dot1\dot2} = \epsilon_{\dot2\dot1} = -\epsilon^{\dot2\dot1} = -\epsilon_{\dot1\dot2} = 1, \quad \epsilon^{\dot1\dot1} = \epsilon^{\dot2\dot2} = 0, \quad \epsilon_{\dot\alpha\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \delta_{\dot\alpha}^{\dot\gamma}$$

$$\psi\chi = \psi^\alpha\chi_\alpha, \bar{\psi}\bar{\chi} = \bar{\psi}_{\dot\alpha}\bar{\chi}^{\dot\alpha}$$
$$\chi\sigma^m\bar{\psi} = \chi^\alpha\sigma^m_{\alpha\dot\alpha}\bar{\psi}^{\dot\alpha}, \bar{\chi}\bar{\sigma}^m\psi = \bar{\chi}_{\dot\alpha}\bar{\sigma}^{m\dot\alpha\alpha}\psi_\alpha$$

$$\psi\chi = \chi\psi(\chi\psi)^\dagger = \bar{\chi}\bar{\psi} = \bar{\psi}\bar{\chi}$$
$$\chi\sigma^m\bar{\psi} = -\bar{\psi}\bar{\sigma}^m\chi, (\chi\sigma^m\bar{\psi})^\dagger = \psi\sigma^m\bar{\chi}$$
$$\chi\sigma^m\bar{\sigma}^n\psi = \psi\sigma^n\bar{\sigma}^m\chi(\chi\sigma^m\bar{\sigma}^n\psi)^\dagger = \bar{\psi}\bar{\sigma}^n\sigma^m\bar{\chi}$$

$$(\psi\varphi)\bar{\chi}_{\dot\beta} = -\frac{1}{2}(\varphi\sigma^m\bar{\chi})(\psi\sigma_m)_{\dot\beta}$$

$$\Psi_{LW} = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix} \; \Psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot\alpha} \end{pmatrix}$$

$$\Psi_{RW} = \begin{pmatrix} 0 \\ \bar{\psi}^{\dot\alpha} \end{pmatrix} \; \Psi_M = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot\alpha} \end{pmatrix}$$

$$\bar{\psi}_1\Gamma\psi_2 = \bar{\psi}_2\tilde{\Gamma}\psi_1 \text{ with } \Gamma = +\tilde{\Gamma} \qquad \Gamma = 1, \gamma_5, \gamma^m\gamma_5$$
$$\Gamma = -\tilde{\Gamma} \qquad \Gamma = \gamma^m, \gamma_{[\mu}\gamma_{\nu]}$$

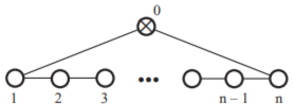
$$\alpha = \sum_{i=1}^{n} l_i\alpha_i \;\; \alpha^\vee = \sum_{i=1}^{n} l_i^\vee\alpha_i^\vee$$

$$l_i^\vee = \frac{\alpha_i^2}{\alpha^2} l_i, i = 1, \cdots, n.$$

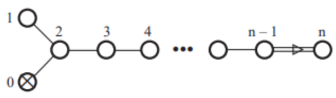
$$\alpha_0 = \sum_{i=1}^{n} a_i \alpha_i \quad \alpha_0^\vee = \sum_{i=1}^{n} a_i^\vee \alpha_i^\vee$$

$$h_{\mathcal{G}} = 1 + \sum_{i=1}^{n} a_i \quad h_{\mathcal{G}}^\vee = 1 + \sum_{i=1}^{n} a_i^\vee$$

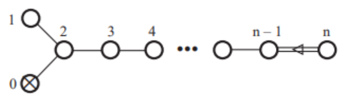
$A_n^{(1)}$



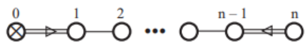
$B_n^{(1)}$



$A_{2n-1}^{(2)} = (B_n^{(1)})^\vee$

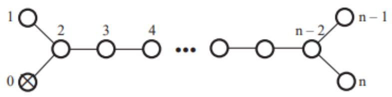


$C_n^{(1)}$



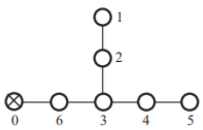
$D_{n+1}^{(2)} = (C_n^{(1)})^\vee$

$D_n^{(1)}$

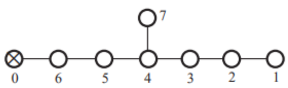


$BC_n = A_{2n}^{(2)}$

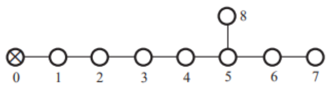
$E_6^{(1)}$



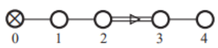
$E_7^{(1)}$



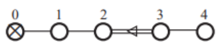
$E_8^{(1)}$



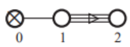
$F_4^{(1)}$



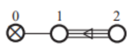
$E_6^{(2)} = (F_4^{(1)})^\vee$



$G_2^{(1)}$



$D_4^{(3)} = (G_2^{(1)})^\vee$



$$\alpha_i^\vee \cdot \lambda_j = \delta_{ij}.$$

$$\Lambda \equiv (q_1, \cdots, q_n) \; \lambda = \sum_{i=1}^{n} q_i \lambda_i$$

$$l(\lambda) = \lambda \cdot \rho^\vee \quad l(\alpha_i) = 1 \qquad i = 1, \cdots, n$$
$$l^\vee(\lambda) = \lambda \cdot \rho \qquad l^\vee(\alpha_i^\vee) = 1 \qquad i = 1, \cdots, n.$$

$$\rho = \sum_{i=1}^{n} \lambda_i = \frac{1}{2} \sum_{\alpha \in \mathcal{R}_+(\mathcal{G})} \alpha$$
$$\rho^\vee = \sum_{i=1}^{n} \lambda_i^\vee = \frac{1}{2} \sum_{\alpha^\vee \in \mathcal{R}_+(\mathcal{G})^\vee} \alpha^\vee$$

$$h_{\mathcal{G}} = 1 + \alpha_0 \cdot \rho^\vee = 1 + l(\alpha_0)$$
$$h_{\mathcal{G}}^\vee = 1 + \alpha_0^\vee \cdot \rho = 1 + l^\vee(\alpha_0^\vee).$$

$$\wp(z; 2\omega_1, 2\omega_2) \equiv \frac{1}{z^2} + \sum_{\substack{(m_1,m_2) \\ \neq (0,0)}} \left\{ \frac{1}{(z + 2\omega_1 m_1 + 2\omega_2 m_2)^2} - \frac{1}{(2\omega_1 m_1 + 2\omega_2 m_2)^2} \right\}$$

$$\wp(z; 2\omega_1, 2\omega_2) = -\frac{d}{dz}\zeta(z; 2\omega_1, 2\omega_2) = -\frac{d^2}{dz^2}\log \sigma(z; 2\omega_1, 2\omega_2)$$

$$\wp(-z) = \wp(z), \quad \wp(z + 2\omega_a) = \wp(z) \qquad a = 1,2,3$$
$$\zeta(-z) = -\zeta(z), \quad \zeta(z + 2\omega_a) = \zeta(z) + 2\eta_a$$
$$\sigma(-z) = \sigma(z), \quad \sigma(z + 2\omega_a) = -\sigma(z) e^{2\eta_a(z + 2\omega_a)},$$

$$\sigma(z) = z + \mathcal{O}(z^5)$$
$$\zeta(z) = \frac{1}{z} + \mathcal{O}(z^3)$$
$$\wp(z) = \frac{1}{z^2} + \mathcal{O}(z^2)$$

$$\sigma(z; 2\omega_1, 2\omega_2) = 2\omega_1 \exp\left(\frac{\eta_1 z^2}{2\omega_1}\right) \frac{\vartheta_1\left(\frac{z}{2\omega_1} \middle| \tau\right)}{\vartheta_1'(0 \mid \tau)}$$

$$q = e^{2\pi i \tau} \; v = \frac{z}{2\omega_1}$$

$$\vartheta_1(u \mid \tau) = 2q^{\frac{1}{4}} \sin \pi u \prod_{n=1}^{\infty} \left(1 - q^n e^{2\pi i u}\right)\left(1 - q^n e^{-2\pi i u}\right)(1 - q^n)$$

$$\vartheta_1'(0 \mid \tau) = \frac{2\pi}{2\omega_1} q^{1/8} \prod_{n=1}^{\infty} (1 - q^n)^3$$

$$\wp'(z)^2 = 4\big(\wp(z) - \wp(\omega_1)\big)\big(\wp(z) - \wp(\omega_2)\big)\big(\wp(z) - \wp(\omega_3)\big).$$

$$\wp(z; 2\omega_1, 2\omega_2) = -\frac{\eta_1}{\omega_1} + \left(\frac{\pi}{2\omega_1}\right)^2 \sum_{n=-\infty}^{\infty} \frac{1}{\sinh^2 \frac{i\pi}{2\omega_1}(z - 2n\omega_2)}$$

$$\frac{\eta_1}{\omega_1} = -\frac{1}{12} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{\sinh^2 \; i\pi n\tau}$$

$$(1 - q^n e^{2\pi i u})(1 - q^n e^{-2\pi i u}) = 4q^{2n} \sin(\pi u + n\pi\tau) \sin(\pi u - n\pi\tau)$$

$$\vartheta_1(u \mid \tau) = 2q^{1/8} \sin(\pi u) \prod_{n=1}^{\infty} (1 - q^n)^3 \prod_{n=1}^{\infty} (4q^2)^n \prod_{n=1}^{\infty} \sin(nu + n\pi\tau) \sin(nu - n\pi\tau)$$

$$\ln \prod_{n=1}^{\infty} (4q^2) = 2\ln\, 2 \sum_{n=1}^{\infty} 1 + \ln\, q \sum_{n=1}^{\infty} n = 2\ln\, 2\zeta(0) + \ln\, q\zeta(-1)$$

$$\frac{\vartheta_1(u \mid \tau)}{\vartheta_1'(0 \mid \tau)} = \frac{\omega_1}{\pi} q^{-\frac{1}{12}} \prod_{n=0}^{\infty} \sin\, \pi(u - n\tau)$$

$$\sigma(z) = \frac{2\omega_1^2}{\pi} \exp\left(\eta_1 \frac{z^2}{2\omega_1} - \frac{i\pi\tau}{6}\right) \prod_{n=-\infty}^{\infty} \sin\, \pi\left(\frac{z}{2\omega_1} - n\tau\right)$$

$$\zeta(z) = \frac{\eta_1}{\omega_1} z + \frac{\pi}{2\omega_1} \sum_{n=-\infty}^{\infty} \operatorname{cotan}\pi\left(\frac{z}{2\omega_1} - n\tau\right)$$

$$\wp(z) = -\frac{\eta_1}{\omega_1} + \frac{\pi^2}{4\omega_1^2} \sum_{n=-\infty}^{\infty} \frac{1}{\sin^2\, \pi\left(\frac{z}{2\omega_1} - n\tau\right)}$$

$$\frac{\eta_1}{\omega_1} = -\frac{1}{12} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{\operatorname{sh}^2 n\omega_1}$$

$$\wp_2(z) = \wp(z; \omega_1, 2\omega_2) = \wp(z) + \wp(z + \omega_1) - \wp(\omega_1)$$

$$= \frac{1}{\wp(\omega_1)} \left[\wp(z)\wp(z + \omega_1) - \big(\wp(\omega_1) - \wp(\omega_2)\big)\big(\wp(\omega_1) - \wp(\omega_3)\big)\right]$$

$$\zeta_2(z) = \zeta(z; \omega_1, 2\omega_2) = \zeta(z) + \zeta(z + \omega_1) + z\wp(\omega_1) - \eta_1$$

$$\sigma_2(z) = \sigma(z; \omega_1, 2\omega_2) = \frac{\sigma(z)\sigma(z + \omega_1)}{\sigma(\omega_1)} e^{\frac{1}{2}z^2\wp(\omega_1) - z\eta_1}$$

$$4\wp(2z) = \wp(z) + \wp(z + \omega_1) + \wp(z + \omega_2) + \wp(z + \omega_1 + \omega_2)$$

$$\wp_3(z) = \wp(z; 2\omega_1/3, 2\omega_2) = \wp(z) + \wp(z + 2\omega_1/3) + \wp(z + 4\omega_1/3)$$

$$\zeta_3(z) = \zeta(z; 2\omega_1/3, 2\omega_2)$$

$$= -\wp(2\omega_1/3) - \wp(4\omega_1/3)\zeta(z) + \zeta(z + 2\omega_1/3) + \zeta(z + 4\omega_1/3) + z\wp(2\omega_1/3)$$

$$+ z\wp(4\omega_1/3) - \eta_1$$

$$\sigma_3(z) = \sigma\left(z; \frac{2\omega_1}{3,2\omega_2}\right) = \frac{\sigma(z)\sigma(z+2\omega_1/3)\sigma(z+4\omega_1/3)}{\sigma(2\omega_1/3)\sigma(4\omega_1/3)} e^{\frac{1}{2}z^2\wp(\omega_1)-z\eta_1}$$

$$9_{\wp}(3z) = \sum_{j,k=0}^{2} \wp\left(z + j\frac{2\omega_1}{3} + k\frac{2\omega_2}{3}\right)$$

$$\wp_\nu(z; 2\omega_1, 2\omega_2) = -\frac{d}{dz}\zeta_\nu(z; 2\omega_1, 2\omega_2) = -\frac{d^2}{dz^2}\log\ \sigma_\nu(z; 2\omega_1, 2\omega_2)$$

$$\Phi(x,z) = \Phi(x,z; 2\omega_1, 2\omega_2) = \frac{\sigma(z-x)}{\sigma(z)\sigma(x)} e^{x\zeta(z)}$$

$$\Phi(x+2\omega_a, z) = \Phi(x,z)e^{2\omega_a\zeta(z)-2\eta_a z}$$

$$\Phi(x,z) = \frac{1}{x} - \frac{1}{2}x\wp(z) + \mathcal{O}(x^2)$$
$$\Phi(x,z) = \left\{-\frac{1}{z} + \zeta(x) + \mathcal{O}(z)\right\} e^{x\zeta(z)}\ z \to 0$$

$$\prod_{\alpha=1}^{n} \Phi(x_\alpha, z) = P_n[\wp(z); x_\alpha] + \wp'(z)Q_n[\wp(z); x_\alpha]$$

$$\Phi(x,z)\Phi(-x,z) = \wp(z) - \wp(x)$$

$$\Phi(x,z)\Phi'(y,z) - \Phi(y,z)\Phi'(x,z) = (\wp(x) - \wp(y))\Phi(x+y,z)$$

$$\Lambda(2x,z) = \Phi_2(x,z) = \frac{\Phi(x,z)\Phi(x+\omega_1,z)}{\Phi(\omega_1,z)}$$

$$\Lambda(2x,z)\Lambda'(2y,z) - \Lambda'(2x,z)\Lambda(2y,z) = \frac{1}{2}(\wp_2(x) - \wp_2(y))\Lambda(2x+2y,z)$$
$$\Phi_2(x,z)\Phi_2'(y,z) - \Phi_2(y,z)\Phi_2'(x,z) = (\wp_2(x) - \wp_2(y))\Phi_2(x+y,z)$$

$$\Lambda(2x,z)\Phi'(-x-y,z) - \Lambda'(2x,z)\Phi(-x-y,z)$$
$$-\Lambda(-2y,z)\Phi'(x+y,z) + \Lambda'(-2y,z)\Phi(x+y,z) = \frac{1}{2}(\wp_2(x) - \wp_2(y))\Phi(x-y,z)$$

$$\Lambda(2x,z)\Lambda(-2x,z) = \wp_2(z) - \wp_2(x)$$

$$\Phi_1(x,z) = \Phi(x,z) + f(z)\Phi(x+\omega_1,z)$$
$$f(z) = -e^{\pi i\zeta(z)+\eta_1 z}$$

$$\Phi_1(x,z)\Phi_1'(y,z) - \Phi_1'(x,z)\Phi_1(y,z) = (\wp_2(x) - \wp_2(y))\Phi_1(x+y,z)$$
$$\Phi_1(x,z)\Phi'(y,z) - \Phi(y,z)\Phi_1'(x,z) = \{\wp(x+\omega_1) - \wp(y)\}\Phi_1(x+y,z)$$
$$+\{\wp(x) - \wp(x+\omega_1)\}\Phi(x+y,z)$$

$$\Phi(2x,z)\Phi_1'(-x-y,z) - \Phi'(2x,z)\Phi_1(-x-y,z) - \Phi(-2y,z)\Phi_1'(x+y,z)$$
$$+\Phi'(-2y,z)\Phi_1(x+y,z) = (\wp(2x) - \wp(2y))\Phi_1(x-y,z)$$
$$\Lambda(2x,z)\Phi_1'(-x-y,z) - \Lambda'(2x,z)\Phi_1(-x-y,z) - \Lambda(-2y,z)\Phi_1'(x+y,z)$$
$$+\Lambda'(-2y,z)\Phi_1(x+y,z) = \frac{1}{2}(\wp_2(x) - \wp_2(y))\Phi_1(x-y,z)$$

CONCLUSIONES.

Expuestos así los resultados, se concluye que, todo espacio – tiempo cuántico deformado por supergravedad cuántica, no solamente provoca la curvatura de Dirac, sino que además, provoca la formación de dimensiones múltiples llamadas supermembranas, yuxtapuestas en superespacios multinivel. Este escenario, ocurre cuando una partícula estrella u oscura, según sea el caso, colapsa en sí misma o en su defecto, sufre aniquilación, necesariamente con una partícula de equivalentes características, sin descartar la supergravedad cuántica residual, esto es, cuando la aniquilación ocurre entre una partícula estrella u oscura y su contraria o una partícula cualquiera. Como ha quedado anotado, estas dimensiones son infinitas y yuxtapuestas en superespacios infinitos, en los que la materia y la energía, se vuelven deformables y por ende, licuables, transformándose y desplazándose entre dimensiones, en condiciones de mutabilidad, siendo capaz, de volver a su estado inicial u originario. Finalmente cabe precisar, que no todas las dimensiones, son por defecto deformables, siendo superplanckianas en sentido estricto aunque susceptibles de corrección hipergeométrica. Es necesario precisar también, que de una dimensión inicial, surge dimensiones distintas, y cada una de ellas, acusa las mismas características de una dimensión originaria, y así, en forma superior a cero.

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