Ciencia Latina Revista Científica Multidisciplinar, Ciudad de México, México.
ISSN 2707-2207 / ISSN 2707-2215 (en línea), julio-agosto 2025,
Volumen 9, Número 4.

https://doi.org/10.37811/cl_rcm.v9i2

GRAVEDAD CUÁNTICA RELATIVISTA

RELATIVISTIC QUANTUM GRAVITY

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DOI: https://doi.org/10.37811/cl_rcm.v9i4.18477

Gravedad cuántica relativista

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RESUMEN

La teoría cuántica de campos relativistas o curvos, esbozada por este autor en trabajos anteriores, define la gravedad, como una característica fenomenológica de las llamadas “partículas supermasivas”. Este efecto gravitacional cuántico, puede ser ordinario o extraordinario, según sea el caso. Cabe precisar, que el efecto gravitacional ordinario, es el resultado de la deformación del espacio – tiempo cuántico, sin causar pliegues de campo, es decir, distintas capas geométricas y en D – dimensiones, conservándose la misma dimensión en $\mathbb{R}^4$, en tanto que, el efecto gravitacional extraordinario, es el resultado de la deformación con ruptura de simetría del espacio – tiempo cuántico, a razón de la existencia de un agujero negro cuántico en dimensión D, es decir, en multidimensiones, por lo que, el tejido del espacio – tiempo se hipergeometriza, en una superficie de Riemann y más concretamente, en un espacio superplanckiano. Sin embargo, en este artículo, nos ocuparemos específicamente del efecto gravitacional ordinario, el mismo que puede ser endógeno o exógeno, según sea el caso, es decir, causado por la propia partícula supermasiva a propósito de su masa o por la interacción de una partícula con el gravitón, esto es, por la permeabilización de un espacio – tiempo cuántico por un campo gravitónico de gauge y local.

Palabras clave: gravedad cuántica, espacio de Hilbert – Einstein, Dimensión en $\mathbb{R}^4$, efecto gravitacional ordinario.

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Relativistic quantum gravity

ABSTRACT

The quantum theory of relativistic or curved fields, outlined by this author in previous works, defines gravity as a phenomenological characteristic of the so-called "supermassive particles". This quantum gravitational effect can be ordinary or extraordinary, as the case may be. It should be noted that the ordinary gravitational effect is the result of the deformation of quantum space-time, without causing field folds, that is, different geometric and D-dimensional layers, preserving the same dimension in $\mathbb{R}^4$, while the extraordinary gravitational effect is the result of the deformation with symmetry breaking of quantum space-time, due to the existence of a quantum black hole in dimension D, that is, in multidimensions, so that the fabric of space-time is hypergeometrized, on a Riemann surface and more specifically, in a superplanckian space. However, in this article, we will deal specifically with the ordinary gravitational effect, which can be endogenous or exogenous, as the case may be, that is, caused by the supermassive particle itself in terms of its mass or by the interaction of a particle with the graviton, that is, by the permeabilization of a quantum space-time by a gauge and local gravitonic field.

Keywords: quantum gravity, Hilbert-Einstein space, Dimension in, ordinary gravitational effect.$\mathbb{R}^4$

Artículo recibido 07 mayo 2025

Aceptado para publicación: 11 junio 2025

INTRODUCCIÓN

En este trabajo, nos proponemos demostrar el comportamiento de un espacio – tiempo cuántico por efecto gravitacional ordinario. Para estos efectos, consideramos el propagador de una partícula supermasiva, el mismo que se desplaza en una superficie de Riemann y por ende, en un espacio de Hilbert – Einstein, en el que, la partícula supermasiva, a propósito de la gravedad causada, deforma el espacio – tiempo cuántico, provocando un rompimiento de simetría de gauge susceptible de reparación, afectando por ende, las trayectorias inicial y final de las partículas repercutidas, su momento angular y las líneas geométricas de colisión y aniquilación. Asimismo, se considera la deformación del espacio – tiempo cuántico, cuando la energía – stress – momentum de una partícula supermasiva, curva el espacio – tiempo cuántico, esto a propósito de la brecha de masa, que supera el estado de vacío, es decir, cuando el salto de energía es superior a cero. Nos ocuparemos también de las hiperpartículas, esto es, de aquellas partículas que aunque tengan o no tengan masa, deforman el espacio – tiempo cuántico, específicamente cuando superan la velocidad de la luz, liberando cargas de energía infinitas. En este trabajo, no nos ocuparemos de la partícula oscura ni de la partícula estrella respectivamente, pues aquellas, son propias de la supergravedad cuántica.

En este trabajo, se intenta extrapolar las ecuaciones de campo de Einstein a un espacio – tiempo cuántico específico, en dimensión $\mathbb{R}^4$.

RESULTADOS Y DISCUSIÓN.

En este apartado, pasamos a diseñar el modelo matemático de gravedad cuántica relativo a un espacio – tiempo cuántico relativista o curvo.

Cálculos preliminares de un campo de gauge y sus transformaciones e invariancias, respecto de un espacio planckiano.

$$X^{\mu}(\tau,\sigma) \sim x^{\mu} + p^{\mu}\tau + \frac{\mathrm{i}}{\sqrt{2}} \sum_{n\in\mathbb{Z}^*} \frac{1}{n}\left(\alpha_n^{\mu}\mathrm{e}^{-\mathrm{i}n(\tau-\sigma)} + \bar{\alpha}_n^{\mu}\mathrm{e}^{-\mathrm{i}n(\tau+\sigma)}\right),$$

$$[x^{\mu}, p^{\nu}] = \mathrm{i}\eta^{\mu\nu}$$

$$\left[\alpha_m^{\mu}, \alpha_n^{\nu}\right] = m\eta^{\mu\nu}\delta_{m+n,0}$$

$$\begin{aligned} H_{\text{closed}} &= -\frac{m^2}{2} + N + \bar{N} - 2 \\ H_{\text{open}} &= -m^2 + N - 1 \end{aligned}$$

$$\begin{aligned} N &= \sum_{n\in\mathbb{N}} nN_n, \quad N_n = \frac{1}{n}\alpha_{-n}\cdot\alpha_n, \\ \bar{N} &= \sum_{n\in\mathbb{N}} n\bar{N}_n, \quad \bar{N}_n = \frac{1}{n}\bar{\alpha}_{-n}\cdot\bar{\alpha}_n. \end{aligned}$$

$$H|\psi\rangle = 0$$

$$(N-\bar{N})|\psi\rangle = 0$$

$$p^{\mu}|k\rangle = k^{\mu}|k\rangle, \forall n > 0\colon \alpha_n^{\mu}|k\rangle = 0$$

$$|\psi\rangle = \prod_{n>0}\prod_{\mu=0}^{D-1}\left(\alpha_{-n}^{\mu}\right)^{N_{n,\mu}}|k\rangle,$$

$$\text{closed}: m^2 = -4, \ \text{open}: m^2 = -1$$

$$\alpha_{-1}^{\mu}|k\rangle$$

$$m^2 = 0$$

$$|A\rangle = \int \mathrm{d}^D k A_{\mu}(k)\alpha_{-1}^{\mu}|k\rangle$$

$$k^2 A_{\mu} = 0$$

$$k^{\mu}A_{\mu} = 0$$

$$A_{\mu} \longrightarrow A_{\mu} + k_{\mu}\lambda$$

$$\alpha_{-1}^{\mu}\bar{\alpha}_{-1}^{\nu}|k\rangle$$

$$m^2 = 0$$

$$\left(\alpha_{-1}^{\mu}\bar{\alpha}_{-1}^{\nu}+\alpha_{-1}^{\nu}\bar{\alpha}_{-1}^{\mu}-\frac{1}{D}\eta^{\mu\nu}\alpha_{-1}\cdot\bar{\alpha}_{-1}\right)|p\rangle$$

$$\left(\alpha_{-1}^{\mu}\bar{\alpha}_{-1}^{\nu}-\alpha_{-1}^{\nu}\bar{\alpha}_{-1}^{\mu}\right)|p\rangle, \frac{1}{D}\eta_{\mu\nu}\alpha_{-1}^{\mu}\bar{\alpha}_{-1}^{\nu}|p\rangle$$

$$|\text{ boson }\rangle = Q\,|\text{ fermion }\rangle.$$

$$D(N=0)=26, D(N=1)=10$$

Superficies de Riemann y amplitudes en base a la función de Green.

$$\chi = 2-2g-b$$

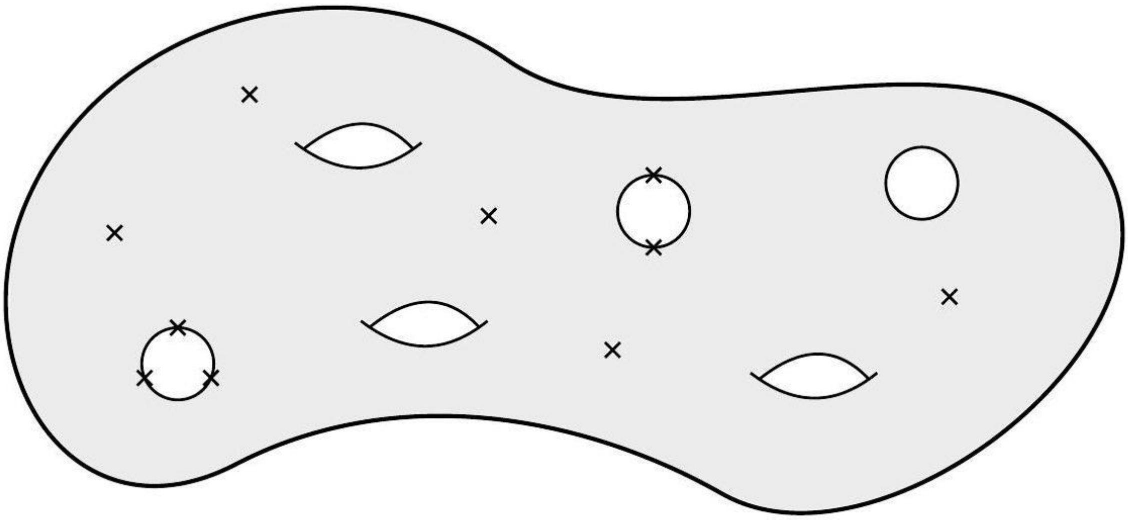


Figura 1. Modelamiento de curvaturas locales.

$$A_n(k_1,\ldots,k_n)=\sum_{g\geq 0} g_s^{n-2+2g}A_{g,n}$$

$$A_{g,n}=\int\prod_{i=1}^{n}\mathrm{d}^2 z_i\int\mathrm{d}g_{ab}\,\mathrm{d}\Psi\mathrm{e}^{-S_{\mathrm{cft}}[g_{ab},\Psi]}\prod_{i=1}^{n}V_i(k_i,z_i)$$

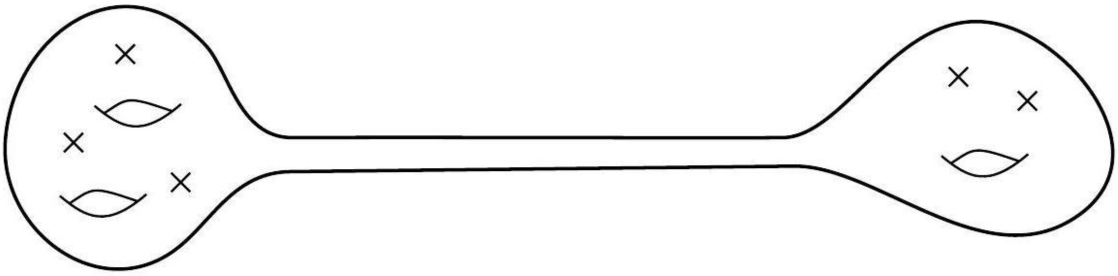
$$\dim_{\mathbb{R}}\mathcal{M}_{g,n}=6g-6+2n.$$

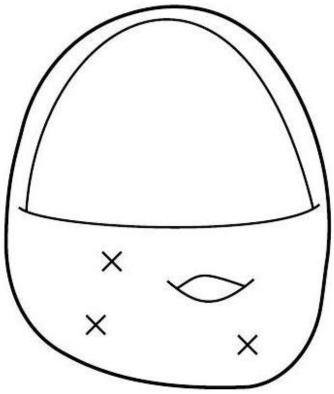
$$A_{g,n}=\int_{\mathcal{M}_{g,n}}\prod_{\lambda=1}^{6g-6+2n}\mathrm{d}t_\lambda F(t)$$

$$F(t)=\left\langle\prod_{i=1}^{n}V_i\times\text{ ghosts }\times\text{ super-ghosts}\right\rangle_{\Sigma_{g,n}}$$

Divergencias de Feynman – acción de los osciladores y propagadores de campo.

$$\frac{1}{k^2+m^2}=\int_0^\infty \mathrm{d}s\mathrm{e}^{-s\left(k^2+m^2\right)}$$





Figuras 2 y 3. Curvatura bilateral y curvatura local. La gravedad cuántica ocupa la segunda demostración, en tanto que la supergravedad cuántica, ocupa la primera demostración.

$$S_0=\frac{1}{2}K_\Psi(\Psi,\Psi)+\frac{1}{2}K_\Phi(\Phi,\Phi)$$

$$S_{\text{int}}=\sum_{m,n}\mathcal{V}_{m,n}(\Phi^m,\Psi^n)$$

$$|\Psi\rangle=\sum_n\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\psi_\alpha(k)|k,\alpha\rangle$$

$$\psi_\alpha=\{T,G_{\mu\nu},B_{\mu\nu},\Phi,\ldots\}.$$

Simetrías difeomorfistas y tensores de simetría – Acción Nambu – Goto – Polyakov – Regge – Weyl – DeWitt.

$$\sigma \in [0,2\pi), \sigma \sim \sigma + 2\pi.$$

$$S_{\mathrm{NG}}[X^\mu] = \frac{1}{2\pi\alpha'}\int \mathrm{d}^2\sigma \sqrt{\det G_{\mu\nu}(X)\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}}$$

$$S_{\mathrm{P}}[g, X^\mu] = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{g}g^{ab}G_{\mu\nu}(X)\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}$$

$$\chi_g := \chi(\Sigma_g) := 2 - 2g = \frac{1}{4\pi}\int_{\Sigma_g} \mathrm{d}^2\sigma\sqrt{g}R$$

$$\sigma'^a = f^a(\sigma^b), g'(\sigma') = f^*g(\sigma), \Psi'(\sigma') = f^*\Psi(\sigma)$$

$$g'_{ab}(\sigma') = \frac{\partial\sigma^c}{\partial\sigma'^a}\frac{\partial\sigma^d}{\partial\sigma'^b}g_{cd}(\sigma), X'^\mu(\sigma') = X^\mu(\sigma)$$

$$\delta_\xi\sigma^a = \xi^a, \delta_\xi\Psi = \mathcal{L}_\xi\Psi, \delta_\xi g_{ab} = \mathcal{L}_\xi g_{ab}$$

$$\mathcal{L}_\xi g_{ab} = \xi^c\partial_c g_{ab} + g_{ac}\partial_b\xi^c + g_{bc}\partial_a\xi^c = \nabla_a\xi_b + \nabla_b\xi_a.$$

$$\Gamma_g := \pi_0\left(\mathrm{Diff}(\Sigma_g)\right) = \frac{\mathrm{Diff}(\Sigma_g)}{\mathrm{Diff}_0(\Sigma_g)}$$

$$g'_{ab}(\sigma) = \mathrm{e}^{2\omega(\sigma)}g_{ab}(\sigma), \Psi'(\sigma) = \Psi(\sigma)$$

$$\delta_\omega g_{ab} = 2\omega g_{ab}, \delta_\omega\Psi = 0$$

$$\mathrm{Conf}(\Sigma_g) := \frac{\mathrm{Met}(\Sigma_g)}{\mathrm{Weyl}(\Sigma_g)}$$

$$G := \mathrm{Diff}(\Sigma_g) \ltimes \mathrm{Weyl}(\Sigma_g).$$

$$G_0 := \mathrm{Diff}_0(\Sigma_g) \ltimes \mathrm{Weyl}(\Sigma_g).$$

$$g' = f^*(\mathrm{e}^{2\omega}g) = \mathrm{e}^{2f^*\omega}f^*g$$

$$g_{ab}(\sigma) = \mathrm{e}^{2\phi(\sigma)}\hat{g}_{ab}(\sigma)$$

$$\hat{g}_{ab} = \delta_{ab}, \phi$$

$$\hat{g}_{ab} = \delta_{ab}, \phi = 0$$

$$S_m[f^*g, f^*\Psi] = S_m[g, \Psi], S_m[\mathrm{e}^{2\omega}g, \Psi] = S_m[g, \Psi]$$

$$X^\mu(\tau, \sigma) \sim X^\mu(\tau, \sigma + 2\pi)$$

$$c_m = D + c_\perp.$$

$$\mathbb{M}_{\text{matter CFT parameters}}: D, c_m.$$

$$T_{m,ab} := -\frac{4\pi}{\sqrt{g}}\frac{\delta S_m}{\delta g^{ab}}$$

$$\nabla^a T_{m,ab} = 0 \ \text{(on-shell)}.$$

$$g^{ab}T_{m,ab} = 0 \ \text{(off-shell)}$$

$$P^a := \int \mathrm{d}\sigma T_m^{0a}$$

$$g^{ab}T_{m,ab} \propto R$$

$$Z_g := \int \frac{\mathrm{d}_g g_{ab}}{\Omega_{\text{gauge}}[g]} Z_m[g], Z_m[g] := \int \mathrm{d}_g \Psi \mathrm{e}^{-S_m[g,\Psi]}$$

$$(\delta\Phi_1, \delta\Phi_2)_g := \int \mathrm{d}^2\sigma\sqrt{g}\gamma_g(\delta\Phi_1, \delta\Phi_2), |\delta\Phi|_g^2 := (\delta\Phi, \delta\Phi)_g$$

$$\int \mathrm{d}_g\delta\Phi \mathrm{e}^{-\frac{1}{2}(\delta\Phi,\delta\Phi)_g} = \frac{1}{\sqrt{\det\gamma_g}}$$

$$\int \mathrm{d}\Phi\sqrt{\det\gamma_g}$$

$$\int \mathrm{d}_g\delta\Phi \mathrm{e}^{-\frac{1}{2}(\delta\Phi,\delta\Phi)_g} = 1$$

$$\Phi(\sigma) \longrightarrow \Phi'(\sigma) = \Phi(\sigma) + \varepsilon(\sigma)$$

$$(\delta f, \delta f)_g := \int \mathrm{d}^2\sigma\sqrt{g}\delta f^2$$

$$(\delta V^a, \delta V^a)_g := \int \mathrm{d}^2\sigma\sqrt{g}g_{ab}\delta V^a\delta V^b$$

$$(\delta T_{ab}, \delta T_{ab})_g := \int \mathrm{d}^2\sigma\sqrt{g}G^{abcd}\delta T_{ab}\delta T_{cd}$$

$$G^{abcd} := G_\perp^{abcd} + ug^{ab}g^{cd}, G_\perp^{abcd} := g^{ac}g^{bd} + g^{ad}g^{bc} - g^{ab}g^{cd}$$

$$G^{abcd}T_{cd} = G_\perp^{abcd}T_{cd} = 2T_{ab}, G^{abcd}(\Lambda g_{cd}) = 2u(\Lambda g_{ab})$$

$$\delta g_{ab} = g_{ab}\delta\Lambda + \delta g_{ab}^\perp, \delta\Lambda = \frac{1}{2}g^{ab}\delta g_{ab}, g^{ab}\delta g_{ab}^\perp = 0$$

$$|\delta g_{ab}|_g^2 = 4u|\delta\Lambda|_g^2 + \left|\delta g_{\mu\nu}^\perp\right|_g^2$$

$$u > 0$$

$$\mathrm{d}_g g_{ab} = \mathrm{d}_g\Lambda\, \mathrm{d}_g g_{ab}^\perp$$

$$\begin{aligned} G^{abcd}\delta g_{ab}\delta g_{cd} &= \left(G_{\perp}^{abcd} + u g^{ab} g^{cd}\right)\left(g_{ab}\delta\Lambda + \delta g_{ab}^{\perp}\right)\left(g_{cd}\delta\Lambda + \delta g_{cd}^{\perp}\right) \\ &= \left(2u g^{cd}\delta\Lambda + G_{\perp}^{abcd}\delta g_{ab}^{\perp}\right)\left(g_{cd}\delta\Lambda + \delta g_{cd}^{\perp}\right) \\ &= 4u(\delta\Lambda)^2 + G_{\perp}^{abcd}\delta g_{ab}^{\perp}\delta g_{cd}^{\perp} \\ &= 4u\delta\Lambda^2 + 2g^{ac}g^{bd}\delta g_{ab}^{\perp}\delta g_{cd}^{\perp} \end{aligned}$$

$$G^{abcd} = g^{ac}g^{bd} + c g^{ab}g^{cd}$$

$$(\delta X^{\mu}, \delta X^{\mu})_g = \int \mathrm{d}^2\sigma\sqrt{g}G_{\mu\nu}(X)\delta X^{\mu}\delta X^{\nu}$$

Reparación de gauge Faddeev-Popov en superficies de Riemann – Roch. Acción de Moduli. Métrica de David-Distler-Kawai y acción de Liouville - Faddeev-Popov.

$$(f^*g, f^*\Psi) \sim (g, \Psi), (\mathrm{e}^{2\omega}g, \Psi) \sim (g, \Psi)$$

$$\mathrm{d}(\text{ fields }) = \text{ Jacobian } \times \mathrm{d}(\text{ gauge }) \times \mathrm{d}(\text{ physical }).$$

$$Z = \int_{\mathbb{R}^2} \mathrm{d}x\, \mathrm{d}y \mathrm{e}^{-(x-y)^2}$$

$$r = x - y, y = a$$

$$Z = \int_{\mathbb{R}} \mathrm{d}a \int_0^{\infty} \mathrm{e}^{-r^2} = \frac{\sqrt{\pi}}{2}\mathrm{Vol}(\mathbb{R})$$

$$\mathcal{M}_g := \frac{\mathrm{Met}\left(\Sigma_g\right)}{G}$$

$$\mathcal{T}_g := \frac{\mathrm{Met}\left(\Sigma_g\right)}{G_0}$$

$$\mathcal{M}_g = \frac{\mathcal{T}_g}{\Gamma_g}$$

$$M_g := \dim_{\mathbb{R}}\mathcal{M}_g = \dim_{\mathbb{R}}\mathcal{T}_g = \begin{cases} 0 & g = 0 \\ 2 & g = 1 \\ 6g - 6 & g \geq 2 \end{cases}$$

$$\int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g}t = \frac{1}{\Omega_{\Gamma_g}}\int_{\mathcal{T}_g} \mathrm{d}^{\mathrm{M}_g}t$$

$$\Omega_G := \int_G \mathrm{d}g = \int_G \mathrm{d}(hg),$$

$$\alpha = \alpha^i T_i.$$

$$\Omega_G = \int \mathrm{d}\alpha := \int \prod_i \mathrm{d}\alpha^i$$

$$g_{ab} = \hat{g}_{ab}^{(f,\phi)}(t) := \mathrm{e}^{2f^*\phi}f^*\hat{g}_{ab}(t) = f^*\left(\mathrm{e}^{2\phi}\hat{g}_{ab}(t)\right)$$

$$g_{ab}(\sigma) = \hat{g}_{ab}^{(f,\phi)}(\sigma;t) := \mathrm{e}^{2\phi(\sigma)}\hat{g}'_{ab}(\sigma;t), \hat{g}'_{ab}(\sigma;t) = \frac{\partial\sigma'^c}{\partial\sigma^a}\frac{\partial\sigma'^d}{\partial\sigma^b}\hat{g}_{cd}(\sigma';t)$$

$$\delta g_{ab} = 2\phi g_{ab} + \nabla_a\xi_b + \nabla_b\xi_a + \delta t_i\partial_i g_{ab}$$

$$\partial_i := \frac{\partial}{\partial t_i}.$$

$$\Omega_{\mathrm{Diff}_0}[g] := \int \mathrm{d}_g\xi$$
$$\Omega_{\mathrm{Weyl}}[g] := \int \mathrm{d}_g\phi$$

$$\Omega_{\mathrm{Diff}}[g] = \Omega_{\mathrm{Diff}_0}[g]\Omega_{\Gamma_g}$$

$$\Omega_{\mathrm{Diff}_0}[g] := \Omega_{\mathrm{Diff}_0}[\mathrm{e}^{2\phi}\hat{g}] = \Omega_{\mathrm{Diff}_0}[\hat{g}], \Omega_{\mathrm{Weyl}}[g] := \Omega_{\mathrm{Weyl}}[\mathcal{L}_\xi\hat{g}] = \Omega_{\mathrm{Weyl}}[\hat{g}]$$

$$\Omega_{\mathrm{gauge}}[g] := \Omega_{\mathrm{gauge}}[\hat{g}]$$

$$\Omega_{\mathrm{Diff}_0}[\mathrm{e}^{2\phi}\hat{g}] = \int \mathrm{d}_{\mathrm{e}^{2\phi}\mathcal{L}_\xi\hat{g}}\xi = \int \mathrm{d}_{\mathrm{e}^{2\phi}\hat{g}}\xi = \int \mathrm{d}_{\hat{g}}\xi = \Omega_{\mathrm{Diff}_0}[\hat{g}]$$
$$\Omega_{\mathrm{Weyl}}[\mathcal{L}_\xi\hat{g}] = \int \mathrm{d}_{\mathrm{e}^{2\phi}\mathcal{L}_\xi\hat{g}}\phi = \int \mathrm{d}_{\mathrm{e}^{2\phi}\hat{g}}\phi = \Omega_{\mathrm{Weyl}}[\hat{g}]$$

$$|\delta\phi|^2 = \int \mathrm{d}^2\sigma\sqrt{g}\delta\phi^2 = \int \mathrm{d}^2\sigma\sqrt{\hat{g}}\mathrm{e}^{2\phi}\delta\phi^2$$

$$F_{ab} := g_{ab} - \hat{g}_{ab}^{(f,\phi)}(t),$$

$$\delta g_{ab} = 2\tilde{\Lambda}g_{ab} + (P_1\xi)_{ab} + \delta t_i\mu_{iab}$$

$$(P_1\xi)_{ab} = \nabla_a\xi_b + \nabla_b\xi_a - g_{ab}\nabla_c\xi^c,$$
$$\mu_{iab} = \partial_i g_{ab} - \frac{1}{2}g_{ab}g^{cd}\partial_i g_{cd},$$
$$\tilde{\Lambda} = \Lambda + \frac{1}{2}\delta t_i g^{ab}\partial_i g_{ab}, \Lambda = \phi + \frac{1}{2}\nabla_c\xi^c.$$

$$Z_g = \int \mathrm{d}^{\mathrm{M}_g}t\, \mathrm{d}_g\tilde{\Lambda}\, \mathrm{d}_g(P_1\xi)\Omega_{\mathrm{gauge}}[g]^{-1}Z_m[g]$$

$$(P_1\xi, \tilde{\Lambda}) \longrightarrow (\xi, \phi)$$

$$\mathrm{d}_g(P_1\xi)\mathrm{d}_g\tilde{\Lambda} \stackrel{?}{=} \mathrm{d}_g\xi\, \mathrm{d}_g\phi\Delta_{\mathrm{FP}}[g]$$

$$\Delta_{\mathrm{FP}}[g] = \det\frac{\partial(P_1\xi, \tilde{\Lambda})}{\partial(\xi,\phi)} = \det\begin{pmatrix} P_1 & 0 \\ \star & 1 \end{pmatrix} = \det P_1$$

$$(T, P_1 v)_g = (P_1^\dagger T, v)_g,$$

$$(P_1^\dagger T)_a = -2\nabla^b T_{ab}$$

$$\mathrm{dimker}P_1^\dagger - \mathrm{dimker}P_1 = -3\chi_g = 6g - 6$$

Deformaciones de Teichmüller.

$$(\delta g, P_1\xi)_g = 0 \implies \left(P_1^\dagger \delta g, \xi\right)_g = 0$$

$$P_1^\dagger \delta g = 0$$

$$\delta g \in \mathrm{ker}P_1^\dagger$$

$$\mathrm{ker}P_1^\dagger = \mathrm{Span}\{\phi_i\}, i = 1, \dots, \mathrm{dimker}P_1^\dagger$$

$$\dim_{\mathbb{R}}\mathrm{ker}P_1^\dagger = \mathrm{M}_g = \begin{cases} 0 & g = 0 \\ 2 & g = 1 \\ 6g - 6 & g > 1 \end{cases}$$

$$\Pi := P_1 \frac{1}{P_1^\dagger P_1} P_1^\dagger$$

$$\delta t_i \mu_i = \delta t_i (1 - \Pi)\mu_i + \delta t_i \Pi \mu_i = \delta t_i (1 - \Pi)\mu_i + \delta t_i P_1 \zeta_i.$$

$$\zeta_i := \frac{1}{P_1^\dagger P_1} P_1^\dagger \mu_i.$$

$$(1 - \Pi)\mu_i = \phi_j (M^{-1})_{jk} (\phi_k, \mu_i)_g$$

$$M_{ij} := \left(\phi_i, \phi_j\right)_g.$$

$$\delta g_{ab} = \left(P_1 \tilde{\xi}\right)_{ab} + 2\tilde{\Lambda} g_{ab} + Q_{iab}\delta t_i.$$

$$\tilde{\xi} = \xi + \zeta_i \delta t_i, Q_{iab} = \phi_{jab}(M^{-1})_{jk}(\phi_k, \mu_i)_g.$$

$$|\delta g|_g^2 = |\delta\tilde{\Lambda}|_g^2 + \left|P_1\tilde{\xi}\right|_g^2 + |Q_i \delta t_i|_g^2$$

$$\mathrm{d}_g g_{ab} = \mathrm{d}_g \tilde{\Lambda}\, \mathrm{d}_g\left(P_1 \tilde{\xi}\right) \mathrm{d}_g (Q_i \delta t_i).$$

$$\left(\tilde{\xi}, \tilde{\Lambda}, Q_i \delta t_i\right) \longrightarrow (\xi, \Lambda, \delta t_i),$$

$$\mathrm{d}_g \tilde{\Lambda}\, \mathrm{d}_g\left(P_1 \tilde{\xi}\right) \mathrm{d}_g (Q_i \delta t_i) = \mathrm{d}^{\mathrm{M}_g} t\, \mathrm{d}_g \Lambda\, \mathrm{d}_g (P_1 \xi) \frac{\det\left(\phi_i, \mu_j\right)_g}{\sqrt{\det\left(\phi_i, \phi_j\right)_g}},$$

$$Z_g = \int_{\mathcal{T}_g} \mathrm{d}^{\mathrm{M}_g} t \frac{1}{\Omega_{\text{gauge}}[\hat{g}]} \int \mathrm{d}_g \Lambda\, \mathrm{d}_g (P_1 \xi) \frac{\det\left(\phi_i, \mu_j\right)_g}{\sqrt{\det\left(\phi_i, \phi_j\right)_g}} Z_m[g]$$

Vectores Conformal Killing.

$$\int \mathrm{d}_g\xi = \Omega_{\mathrm{Diff}_0}[\hat{g}]$$

$$\xi^{(0)} \in \mathcal{K}_g := \ker P_1$$

$$\left(P_1\xi^{(0)}\right)_{ab} = \nabla_a\xi_b^{(0)} + \nabla_b\xi_a^{(0)} - g_{ab}\nabla_c\xi^{(0)c} = 0$$

$$\mathrm{K}_g := \dim_{\mathbb{R}}\mathcal{K}_g = \dim_{\mathbb{R}}\ker P_1 = \begin{cases} 6 & g = 0 \\ 2 & g = 1 \\ 0 & g > 1 \end{cases}$$

$$g = 0: \mathcal{K}_0 = \mathrm{SL}(2,\mathbb{C}), g = 1: \mathcal{K}_1 = \mathrm{U}(1) \times \mathrm{U}(1).$$

$$\xi = \xi^{(0)} + \xi'$$

$$\left(\xi^{(0)}, \xi'\right)_g = 0$$

$$(P_1\xi, \Lambda) \longrightarrow (\xi', \phi).$$

$$\mathrm{d}_g\Lambda\ \mathrm{d}_g(P_1\xi) = \mathrm{d}_g\phi\ \mathrm{d}_g\xi'\Delta_{\mathrm{FP}}[g]$$

$$\Delta_{\mathrm{FP}}[g] = \det{}'\frac{\partial(P_1\xi, \Lambda)}{\partial(\xi', \phi)} = \det{}'P_1 = \sqrt{\det{}'P_1P_1^{\dagger}}$$

$$Z_g = \int_{\mathcal{T}_g} \mathrm{d}^{\mathrm{M}_g}t\Omega_{\mathrm{gauge}}[\hat{g}]^{-1}\int \mathrm{d}_g\phi\ \mathrm{d}_g\xi'\frac{\det\left(\phi_i, \mu_j\right)_g}{\sqrt{\det\left(\phi_i, \phi_j\right)_g}}\Delta_{\mathrm{FP}}[g]Z_m[g]$$

$$\Delta_{\mathrm{FP}}[g] = \det{}'\frac{\partial(P_1\xi, \Lambda)}{\partial(\xi', \phi)} = \det{}'\begin{pmatrix} P_1 & 0 \\ \frac{1}{2}\nabla & 1 \end{pmatrix} = \det{}'P_1$$

$$\sqrt{\det{}'P_1^{\dagger}P_1} = \det{}'P_1$$

$$\begin{aligned} 1 &= \int \mathrm{d}_g\delta\Lambda\ \mathrm{d}_g(P_1\delta\xi)\mathrm{e}^{-|\delta\Lambda|_g^2-|P_1\delta\xi'|_g^2} \\ &= \Delta_{\mathrm{FP}}[g]\int \mathrm{d}_g\delta\phi\ \mathrm{d}_g\delta\xi'\mathrm{e}^{-\left|\delta\phi+\frac{1}{2}\nabla_c\delta\xi^c\right|_g^2-|P_1\delta\xi'|_g^2} \\ &= \Delta_{\mathrm{FP}}[g]\int \mathrm{d}_g\delta\phi\ \mathrm{d}_g\delta\xi'\mathrm{e}^{-|\delta\phi|_g^2-\left(\delta\xi', P_1^{\dagger}P_1\delta\xi'\right)_g} \\ &= \Delta_{\mathrm{FP}}[g]\left(\det{}'P_1^{\dagger}P_1\right)^{-1/2} \end{aligned}$$

$$\Omega'_{\mathrm{Diff}_0}[g] := \Omega'_{\mathrm{Diff}_0}[\hat{g}] = \int \mathrm{d}_g\xi'$$

$$\ker P_1 = \mathrm{Span}\{\psi_i\}, i = 1, \dots, \mathrm{K}_g$$

$$\xi' \longrightarrow \xi$$

$$\mathrm{d}_g\xi' = \frac{1}{\sqrt{\det(\psi_i,\psi_j)_g}}\frac{\mathrm{d}_g\xi}{\Omega_{\mathrm{ckv}}[g]},$$

$$\Omega_{\mathrm{Diff}_0}[g] = \sqrt{\det(\psi_i,\psi_j)_g}\,\Omega_{\mathrm{ckv}}[g]\Omega'_{\mathrm{Diff}_0}[g]$$

$$\xi^{(0)} = \alpha_i\psi_i$$

$$\xi \longrightarrow (\xi',\alpha_i).$$

$$\begin{aligned}
1 &= \int \mathrm{d}\xi \mathrm{e}^{-|\xi|_g^2} = J\int \mathrm{d}\xi^{(0)}\mathrm{d}\xi'\mathrm{e}^{-|\xi'|_g^2-|\xi^{(0)}|_g^2} \\
&= J\int\prod_i \mathrm{d}\alpha_i \mathrm{e}^{-\alpha_i\alpha_j(\psi_i,\psi_j)_g}\int \mathrm{d}\xi'\mathrm{e}^{-|\xi'|_g^2} \\
&= J\left(\det(\psi_i,\psi_j)_g\right)^{-1/2}
\end{aligned}$$

$$\mathrm{d}\xi = \sqrt{\det(\psi_i,\psi_j)_g}\,\mathrm{d}\xi'\prod_i \mathrm{d}\alpha_i$$

$$\Omega_{\mathrm{ckv}}[g] = \int\prod_i \mathrm{d}\alpha_i$$

$$Z_g = \int_{\mathcal{T}_g} \mathrm{d}^{\mathrm{M}_g}t\,\Omega_{\mathrm{gauge}}[\hat{g}]^{-1}\int \mathrm{d}_g\phi\,\mathrm{d}_g\xi\frac{\det(\phi_i,\mu_j)_g}{\sqrt{\det(\phi_i,\phi_j)_g}}\frac{\Omega_{\mathrm{ckv}}[g]^{-1}}{\sqrt{\det(\psi_i,\psi_j)_g}}\Delta_{\mathrm{FP}}[g]Z_m[g]$$

$$(f^*\hat{g},f^*\phi,f^*\Psi) \longrightarrow (\hat{g},\phi,\Psi)$$

$$Z_g = \int_{\mathcal{T}_g} \mathrm{d}^{\mathrm{M}_g}t\frac{\Omega_{\mathrm{Diff}_0}[\hat{g}]}{\Omega_{\mathrm{gauge}}[\hat{g}]}\int \mathrm{d}_g\phi\frac{\det(\phi_i,\mu_j)_g}{\sqrt{\det(\phi_i,\phi_j)_g}}\frac{\Omega_{\mathrm{ckv}}[g]^{-1}}{\sqrt{\det(\psi_i,\psi_j)_g}}\Delta_{\mathrm{FP}}[g]Z_m[g]$$

$$g_{ab} := g_{ab}^{(\phi)} = \mathrm{e}^{2\phi}\hat{g}_{ab}$$

$$Z_g = \frac{1}{\Omega_{\Gamma_g}}\int_{\mathcal{T}_g} \mathrm{d}^{\mathrm{M}_g}t\frac{\Omega_{\mathrm{Diff}}[\hat{g}]}{\Omega_{\mathrm{gauge}}[\hat{g}]}\int \mathrm{d}_g\phi\frac{\det(\phi_i,\mu_j)_g}{\sqrt{\det(\phi_i,\phi_j)_g}}\frac{\Omega_{\mathrm{ckv}}[g]^{-1}}{\sqrt{\det(\psi_i,\psi_j)_g}}\Delta_{\mathrm{FP}}[g]Z_m[g]$$

$$Z_g = \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g}t\frac{\Omega_{\mathrm{Diff}}[\hat{g}]}{\Omega_{\mathrm{gauge}}[\hat{g}]}\int \mathrm{d}_g\phi\frac{\det(\phi_i,\mu_j)_g}{\sqrt{\det(\phi_i,\phi_j)_g}}\frac{\Omega_{\mathrm{ckv}}[g]^{-1}}{\sqrt{\det(\psi_i,\psi_j)_g}}\Delta_{\mathrm{FP}}[g]Z_m[g]$$

Transformaciones de Weyl – difeoformismos gravitacionales y anomalías cuánticas – nodos tensoriales de campo – parámetros de Ricci.

$$\frac{\Delta_{\mathrm{FP}}\left[\mathrm{e}^{2\phi}\hat{g}\right]}{\sqrt{\det\left(\phi_i,\phi_j\right)_{\mathrm{e}^{2\phi}\hat{g}}}}=\mathrm{e}^{\frac{c_{\mathrm{gh}}}{6}S_L[\hat{g},\phi]}\frac{\Delta_{\mathrm{FP}}[\hat{g}]}{\sqrt{\det\left(\hat{\phi}_i,\hat{\phi}_j\right)_{\hat{g}}}}$$

$$Z_m\left[\mathrm{e}^{2\phi}\hat{g}\right]=\mathrm{e}^{\frac{c_m}{6}S_L[\hat{g},\phi]}Z_m[\hat{g}]$$

$$S_L[\hat{g},\phi]:=\frac{1}{4\pi}\int\mathrm{d}^2\sigma\sqrt{\hat{g}}\left(\hat{g}^{ab}\partial_a\phi\partial_b\phi+\hat{R}\phi\right)$$

$$c_{\mathrm{gh}}=-26$$

$$\det\left(\phi_i,\mu_j\right)_{\mathrm{e}^{2\phi}\hat{g}}=\det\left(\hat{\phi}_i,\hat{\mu}_j\right)_{\hat{g}},\det\left(\psi_i,\psi_j\right)_{\mathrm{e}^{2\phi}\hat{g}}=\det\left(\psi_i,\psi_j\right)_{\hat{g}},$$
$$\Omega_{\mathrm{ckv}}\left[\mathrm{e}^{2\phi}\hat{g}\right]=\Omega_{\mathrm{ckv}}[\hat{g}].$$

$$\left\langle g^{\mu\nu}T_{\mu\nu}\right\rangle=\frac{c}{12}R$$

$$Z_g=\int_{\mathcal{M}_g}\mathrm{d}^{\mathrm{M}_g}t\frac{\Omega_{\mathrm{Diff}}[\hat{g}]}{\Omega_{\mathrm{gauge}}[\hat{g}]}\frac{\det\left(\phi_i,\hat{\mu}_j\right)_{\hat{g}}}{\sqrt{\det\left(\phi_i,\phi_j\right)_{\hat{g}}}}\frac{\Omega_{\mathrm{ckv}}[\hat{g}]^{-1}}{\sqrt{\det\left(\psi_i,\psi_j\right)_{\hat{g}}}}\Delta_{\mathrm{FP}}[\hat{g}]Z_m[\hat{g}]\int\mathrm{d}_g\phi\mathrm{e}^{-\frac{c_L}{6}S_L[\hat{g},\phi]}$$

$$c_L:=26-c_m$$

$$c_L=0\implies c_m=26$$

$$c_\perp=26-D$$

$$\int\mathrm{d}_g\phi=\Omega_{\mathrm{Weyl}}[\hat{g}]$$

$$\Omega_{\mathrm{gauge}}[\hat{g}]=\Omega_{\mathrm{Diff}}[\hat{g}]\times\Omega_{\mathrm{Weyl}}[\hat{g}]$$

$$Z_g=\int_{\mathcal{M}_g}\mathrm{d}^{\mathrm{M}_g}t\frac{\det\left(\phi_i,\hat{\mu}_j\right)_{\hat{g}}}{\sqrt{\det\left(\phi_i,\phi_j\right)_{\hat{g}}}}\frac{\Omega_{\mathrm{ckv}}[\hat{g}]^{-1}}{\sqrt{\det\left(\psi_i,\psi_j\right)_{\hat{g}}}}\Delta_{\mathrm{FP}}[\hat{g}]Z_m[\hat{g}]$$

Ambiguedades, ultralocalidad y constante cosmológica – Métrica Weil-Petersson.

$$S_\mu[g]=\int\mathrm{d}^2\sigma\sqrt{g}$$

$$\sqrt{\det\gamma_g}=\mathrm{e}^{-\mu_\gamma S_\mu[g]},$$

$$\Omega_\Phi=\lim_{\lambda\to0}\int\mathrm{d}_g\Phi\mathrm{e}^{-\lambda(\Phi,\Phi)_g}$$

$$\int \mathrm{d}_g\Phi \mathrm{e}^{-\lambda(\Phi,\Phi)_g} = \mathrm{e}^{-\mu(\lambda)S_\mu[g]}$$

$$\Omega_\Phi = \int \mathrm{d}_g\Phi = \mathrm{e}^{-\mu(0)S_\mu[g]}$$

$$\lim_{\epsilon\to 0}\frac{1}{\epsilon}\int \mathrm{d}^2\sigma\sqrt{g}$$

$$\begin{aligned} Z_g &= \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g}t \frac{\det(\phi_i,\hat{\mu}_j)_{\hat{g}}}{\sqrt{\det(\phi_i,\phi_j)_{\hat{g}}}} \frac{\Omega_{\mathrm{ckv}}[\hat{g}]^{-1}}{\sqrt{\det(\psi_i,\psi_j)_{\hat{g}}}} \Delta_{\mathrm{FP}}[\hat{g}]Z_m[\hat{g}] \\ &= \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g}t \sqrt{\frac{\det(\phi_i,\hat{\mu}_j)_{\hat{g}}^2}{\det(\phi_i,\phi_j)_{\hat{g}}} \frac{\det{}'\hat{P}_1^\dagger\hat{P}_1}{\det(\psi_i,\psi_j)_{\hat{g}}} \frac{Z_m[\hat{g}]}{\Omega_{\mathrm{ckv}}[\hat{g}]}} \end{aligned}$$

$$\mathrm{d(WP)} = \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g}t \frac{\det(\phi_i,\hat{\mu}_j)_{\hat{g}}}{\sqrt{\det(\phi_i,\phi_j)_{\hat{g}}}}$$

$$\sigma'^a = \hat{f}^a(\sigma^b), \hat{g}'(\sigma') = f^*\hat{g}(\sigma), \phi'(\sigma') = f^*\phi(\sigma), \Psi'(\sigma') = f^*\Psi(\sigma)$$

$$g_{ab} = f^*(\mathrm{e}^{2\phi}\hat{g}_{ab}(t))$$

$$g'_{ab}(\sigma) = \mathrm{e}^{2\omega(\sigma)}g_{ab}(\sigma), \phi'(\sigma) = \phi(\sigma) - \omega(\sigma), \Psi'(\sigma) = \Psi(\sigma)$$

$$g'_{ab} = F^*g_{ab}, g'_{ab} = f'^*(\mathrm{e}^{2\phi'}\hat{g}'_{ab}), g_{ab} = f^*(\mathrm{e}^{2\phi}\hat{g}_{ab})$$

$$\hat{F} = f'^{-1}\circ F\circ f, \phi' = \hat{F}^*(\phi-\omega), \hat{g}'_{ab} = \hat{F}^*(\mathrm{e}^{2\omega}\hat{g}_{ab})$$

$$g'_{ab} = \tilde{f}^*(\mathrm{e}^{2\phi}\hat{g}_{ab})$$

$$\begin{aligned} g'_{ab} &= F^*g_{ab} = F^*\left(f^*(\mathrm{e}^{2\phi}\hat{g}_{ab})\right) = F^*\left(f^*(\mathrm{e}^{2(\phi-\omega)}\mathrm{e}^{2\omega}\hat{g}_{ab})\right) \\ &= f'^*(\mathrm{e}^{2\phi'}\hat{g}'_{ab}) \end{aligned}$$

Campos fantasmas. Métrica de Grassmann y Métrica de Faddeev-Popov.

$$\Delta_{\mathrm{FP}}[g] = \int \mathrm{d}'_g b\, \mathrm{d}'_g c \mathrm{e}^{-S_{\mathrm{gh}}[g,b,c]}$$

$$\begin{aligned} S_{\mathrm{gh}}[g,b,c] &:= \frac{1}{4\pi}\int \mathrm{d}^2\sigma\sqrt{g}g^{ab}g^{cd}b_{ac}(P_1c)_{bd} \\ &= \frac{1}{4\pi}\int \mathrm{d}^2\sigma\sqrt{g}g^{ab}(b_{ac}\nabla_b c^c + b_{bc}\nabla_a c^c - b_{ab}\nabla_c c^c) \end{aligned}$$

$$(P_1c)_{ab} = \nabla_a c_b + \nabla_b c_a - g_{ab}\nabla_c c^c = 0, \left(P_1^\dagger b\right)_a = -2\nabla^b b_{ab} = 0$$

$$T_{ab}^{\mathrm{gh}} = -b_{ac}\nabla_b c^c - b_{bc}\nabla_a c^c + c^c\nabla_c b_{ab} + g_{ab}b_{cd}\nabla^c c^d$$

$$g^{ab}T^{\mathrm{gh}}_{ab} = 0$$

$$S_{\mathrm{gh}}[\mathrm{e}^{2\omega}g, b, c] = S_{\mathrm{gh}}[g, b, c]$$

$$N_{\mathrm{gh}}(b) = -1, N_{\mathrm{gh}}(c) = 1.$$

$$N_{\mathrm{gh}}(\Psi) = 0$$

$$Z_g = \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g}t \frac{\det(\phi_i, \hat{\mu}_j)_{\hat{g}}}{\sqrt{\det(\phi_i, \phi_j)_{\hat{g}}}} \frac{\Omega_{\mathrm{ckv}}[\hat{g}]^{-1}}{\sqrt{\det(\psi_i, \psi_j)_{\hat{g}}}} \int \mathrm{d}_{\hat{g}}\Psi\ \mathrm{d}'_{\hat{g}}b\ \mathrm{d}'_{\hat{g}}c\,\mathrm{e}^{-S_m[\hat{g},\Psi]-S_{\mathrm{gh}}[\hat{g},b,c]}$$

$$\langle 0|c_{-1}\bar{c}_{-1}c_0\bar{c}_0c_1\bar{c}_1|0\rangle = 1$$

$$A_{0,2}(k,k') = \frac{C_{S^2}}{\mathrm{Vol}\mathcal{K}_{0,2}} \langle \mathcal{V}_k(\infty,\infty)c_0\bar{c}_0\mathcal{V}_{k'}(0{,}0)\rangle_{S^2}$$

Weyl ghost.

$$F^{\perp}_{ab} = \sqrt{g}g_{ab} - \sqrt{\hat{g}}\hat{g}_{ab} = 0$$

$$S'_{\mathrm{gh}}[g, b, c, c_w] = \frac{1}{4\pi}\int \mathrm{d}^2\sigma\sqrt{g}g^{ab}(b_{ac}\nabla_b c^c + b_{bc}\nabla_a c^c + 2b_{ab}c_w)$$

$$\nabla_a c_b + \nabla_b c_a + 2g_{ab}c_w = 0, \nabla^a b_{ab} = 0, g^{ab}b_{ab} = 0$$

$$c_w = -\frac{1}{2}\nabla_a c^a$$

$$T'^{\mathrm{gh}}_{ab} = -(b_{ac}\nabla_b c^c + b_{bc}\nabla_a c^c + 2b_{ab}c_w) - \nabla_c(b_{ab}c^c) + \frac{1}{2}g_{ab}g^{cd}(b_{ce}\nabla_d c^e + b_{de}\nabla_c c^e + 2b_{cd}c_w)$$

$$g^{ab}T'^{\mathrm{gh}}_{ab} = -g^{ab}\nabla_c(b_{ab}c^c)$$

$$\begin{aligned} g^{ab}(b_{ac}\delta\nabla_b c^c + b_{bc}\delta\nabla_a c^c) &= 2g^{ab}b_{ac}\delta\nabla_b c^c = 2g^{ab}b_{ac}\delta\Gamma^c_{bd}c^d \\ &= g^{ab}b_{ac}g^{ce}(\nabla_b\delta g_{de} + \nabla_d\delta g_{be} - \nabla_e\delta g_{bd})c^d \\ &= b^{ab}(\nabla_a\delta g_{bc} + \nabla_c\delta g_{ab} - \nabla_b\delta g_{ac})c^c \\ &= b^{ab}\nabla_c\delta g_{ab}c^c \end{aligned}$$

$$\int \mathrm{d}_g c_w \mathrm{e}^{-(c_w, g^{ab}b_{ab})_g} = \delta(g^{ab}b_{ab})$$

Modo Cero de los campos fantasmas.

$$Z_g = \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g}t \frac{\Omega_{\mathrm{ckv}}[\hat{g}]^{-1}}{\sqrt{\det(\psi_i, \psi_j)_{\hat{g}}}} \int \mathrm{d}_{\hat{g}}\Psi\ \mathrm{d}_{\hat{g}}b\ \mathrm{d}'_{\hat{g}}c \prod_{i=1}^{\mathrm{M}_g} (b, \hat{\mu}_i)_{\hat{g}}\mathrm{e}^{-S_m[\hat{g},\Psi]-S_{\mathrm{gh}}[\hat{g},b,c]}$$

$$(b, \hat{\mu}_i)_{\hat{g}} = \int \mathrm{d}^2\sigma\sqrt{\hat{g}}G^{abcd}_{\perp}b_{ab}\hat{\mu}_{i,cd} = \int \mathrm{d}^2\sigma\sqrt{\hat{g}}g^{ac}g^{bd}b_{ab}\hat{\mu}_{i,cd}.$$

$$b = b_0 + b', b_0 = b_{0i}\phi_i$$

$$1 = \int \mathrm{d}_{\hat{g}} b \mathrm{e}^{-|b|_{\hat{g}}^2} = J \int \mathrm{d}_{\hat{g}} b' \prod_i \mathrm{d}b_{0i} \mathrm{e}^{-|b'|_{\hat{g}}^2 - |b_{0i}\phi_i|^2} = J\sqrt{\det(\phi_i, \phi_j)}.$$

$$\int \mathrm{d}^{\mathrm{M}_g} b_{0i} \prod_j b_0(\sigma_j^0) = \int \mathrm{d}^{\mathrm{M}_g} b_{0i} \prod_j [b_{0i}\phi_i(\sigma_j^0)] = \det\phi_i(\sigma_j^0).$$

$$\frac{\mathrm{d}_{\hat{g}} b'}{\sqrt{\det(\phi_i, \phi_j)_{\hat{g}}}} = \frac{\mathrm{d}_{\hat{g}} b}{\det\phi_i(\sigma_j^0)} \prod_{j=1}^{\mathrm{M}_g} b(\sigma_j^0).$$

$$\mathrm{d}_{\hat{g}} b' \frac{\det(\phi_i, \hat{\mu}_j)_{\hat{g}}}{\sqrt{\det(\phi_i, \phi_j)_{\hat{g}}}} = \mathrm{d}_{\hat{g}} b \prod_{j=1}^{\mathrm{M}_g} (b, \hat{\mu}_j)_{\hat{g}}$$

$$\prod_{j=1}^{\mathrm{M}_g} b(\sigma_j^0) = \prod_{j=1}^{\mathrm{M}_g} [b_{0i}\phi_i(\sigma_j^0)] = \det\phi_i(\sigma_j^0) \prod_{j=1}^{\mathrm{M}_g} b_{0i}$$

$$\det(\phi_i, \hat{\mu}_j)_{\hat{g}} \prod_{j=1}^{\mathrm{M}_g} b_{0i} = \prod_{j=1}^{\mathrm{M}_g} \left[b_{0i}(\phi_i, \hat{\mu}_j)_{\hat{g}}\right] = \prod_{j=1}^{\mathrm{M}_g} (b_{0i}\phi_i, \hat{\mu}_j)_{\hat{g}} = \prod_{j=1}^{\mathrm{M}_g} (b, \hat{\mu}_j)_{\hat{g}}$$

$$Z_g = \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g} t \frac{\Omega_{\mathrm{ckv}}[\hat{g}]^{-1}}{\det\psi_i(\sigma_j^0)} \int \mathrm{d}_{\hat{g}}\Psi \, \mathrm{d}_{\hat{g}} b \, \mathrm{d}_{\hat{g}} c \prod_{j=1}^{\mathrm{K}_g^c} \frac{\epsilon_{ab}}{2} c^a(\sigma_j^0) c^b(\sigma_j^0)$$
$$\times \prod_{i=1}^{\mathrm{M}_g} (\hat{\mu}_i, b)_{\hat{g}} \mathrm{e}^{-S_m[\hat{g},\Psi] - S_{\mathrm{gh}}[\hat{g},b,c]}$$

$$c \longrightarrow c + c_0, P_1 c_0 = 0.$$

$$b \longrightarrow b + b_0, P_1^\dagger b_0 = 0.$$

$$\Phi_0 = \ln \, g_s$$

$$g_s^{2g-2} = \mathrm{e}^{-\Phi_0 \chi_g} = \exp\left(-\frac{\Phi_0}{4\pi} \int \mathrm{d}^2\sigma \sqrt{g} R\right) = \mathrm{e}^{-\Phi_0 S_{\mathrm{EH}}[g]}$$

Amplitudes en un espacio de Moduli – Operadores de vértice. Métrica perturbativa de Green.

$$Z_g = \int \frac{\mathrm{d}_g g_{ab}}{\Omega_{\mathrm{gauge}}[g]} Z_m[g], Z_m[g] = \int \mathrm{d}_g \Psi \mathrm{e}^{-S_m[g,\Psi]}$$

$$V_\alpha(k_i) := \int \mathrm{d}^2\sigma \sqrt{g(\sigma)} V_\alpha(k;\sigma).$$

$$S_{\mathrm{EH}}[g] := \frac{1}{4\pi} \int \mathrm{d}^2\sigma \sqrt{g} R + \frac{1}{2\pi} \oint \mathrm{d}s k = \chi_{g,n}$$

$$\chi_{g,n} := \chi(\Sigma_{g,n}) = 2 - 2g - n$$

$$g_s^{-\chi_{g,n}} = \mathrm{e}^{-\Phi_0 S_{\mathrm{EH}}[g]}, \Phi_0 := \ln\ g_s.$$

$$A_{g,n}(\{k_i\})_{\{\alpha_i\}} := \int \frac{\mathrm{d}_g g_{ab}}{\Omega_{\mathrm{gauge}}[g]} \mathrm{d}_g \Psi \mathrm{e}^{-S_m[g,\Psi] - \Phi_0 S_{\mathrm{EH}}[g]} \prod_{i=1}^{n} \left(\int \mathrm{d}^2\sigma_i \sqrt{g(\sigma_i)} V_{\alpha_i}(k_i;\sigma_i) \right)$$

$$A_{g,n}(\{k_i\})_{\{\alpha_i\}} := A_{g,n}(k_1,\ldots,k_n)_{\alpha_1,\ldots,\alpha_n} := A_{g,n}\left(V_{\alpha_1}(k_1),\ldots,V_{\alpha_n}(k_n)\right).$$

$$A_n(k_1,\ldots,k_n)_{\alpha_1,\ldots,\alpha_n} = \sum_{g=0}^{\infty} A_{g,n}(k_1,\ldots,k_n)_{\alpha_1,\ldots,\alpha_n}$$

$$A_{g,n}(\{k_i\})_{\{\alpha_i\}} = \int \frac{\mathrm{d}_g g_{ab}}{\Omega_{\mathrm{gauge}}[g]} \mathrm{e}^{-\Phi_0 S_{\mathrm{EH}}[g]} \int \prod_{i=1}^{n} \mathrm{d}^2\sigma_i \sqrt{g} \left\langle \prod_{i=1}^{n} V_{\alpha_i}(k_i;\sigma_i) \right\rangle_{m,g}$$

$$H = 0.$$

$$S = 1 + \mathrm{i}T,$$

$$A_n(k_1,\ldots,k_n) = G_n(k_1,\ldots,k_n) \prod_{i=1}^{n} (k_i^2 + m_i^2).$$

$$G_n(k_1,\ldots,k_n) \propto \delta^{(D)}(k_1 + \cdots + k_n).$$

$$T_2 = G_2(k,k')(k^2+m^2)^2 \sim (k^2+m^2)\delta^{(D)}(k+k') \underset{k^2\to -m^2}{\rightarrow} 0$$

$$G_2(k,k') = \frac{\delta^{(D)}(k+k')}{k^2+m^2}$$

$$A_2(k,k') = 2k^0(2\pi)^{D-1}\delta^{(D-1)}(\boldsymbol{k}-\boldsymbol{k}').$$

$$\left[a(\boldsymbol{k}), a^\dagger(\boldsymbol{k}')\right] = 2k^0(2\pi)^{D-1}\delta^{(D-1)}(\boldsymbol{k}-\boldsymbol{k}').$$

$$\phi(\alpha) = \int \mathrm{d}^{D-1}\boldsymbol{k}\,\alpha(\boldsymbol{k})^* a^\dagger(\boldsymbol{k}).$$

Reparaciones de Gauge. Métrica de Faddeev-Popov.

$$\delta_\xi V_{\alpha_i}(k_i) = \delta_\xi \int \mathrm{d}^2\sigma \sqrt{g} V_{\alpha_i}(k_i;\sigma) = 0$$
$$\delta_\omega V_{\alpha_i}(k_i) = \delta_\omega \int \mathrm{d}^2\sigma \sqrt{g} V_{\alpha_i}(k_i;\sigma) = 0$$

$$A_{g,n}(\{k_i\})_{\{\alpha_i\}} = g_s^{-\chi_{g,n}} \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g} t \frac{\det(\phi_i, \hat{\mu}_j)_{\hat{g}}}{\sqrt{\det(\phi_i, \phi_j)_{\hat{g}}}} \frac{\Omega_{\mathrm{ckv}}[\hat{g}]^{-1}}{\sqrt{\det(\psi_i, \psi_j)_{\hat{g}}}}$$

$$\times \int \prod_{i=1}^{n} \mathrm{d}^2\sigma_i \sqrt{\hat{g}} \left\langle \prod_{i=1}^{n} \hat{V}_{\alpha_i}(k_i; \sigma_i) \right\rangle_{m,\hat{g}}$$

$$A_{g,n}(\{k_i\})_{\{\alpha_i\}} = g_s^{-\chi_{g,n}} \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g} t \frac{\Omega_{\mathrm{ckv}}[\hat{g}]^{-1}}{\sqrt{\det(\psi_i, \psi_j)_{\hat{g}}}} \int \mathrm{d}_{\hat{g}} b\, \mathrm{d}'_{\hat{g}} c \prod_{i=1}^{\mathrm{M}_g} (b, \hat{\mu}_i)_{\hat{g}} \mathrm{e}^{-S_{\mathrm{gh}}[\hat{g}, b, c]}$$

$$\times \int \prod_{i=1}^{n} \mathrm{d}^2\sigma_i \sqrt{\hat{g}} \left\langle \prod_{i=1}^{n} \hat{V}_{\alpha_i}(k_i; \sigma_i) \right\rangle_{m,\hat{g}}$$

$$A_{g,n} = g_s^{-\chi_{g,n}} \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g} t \frac{\Omega_{\mathrm{ckv}}[\hat{g}]^{-1}}{\det\psi_i(\sigma_j^0)} \int \mathrm{d}_{\hat{g}} b\, \mathrm{d}_{\hat{g}} c \prod_{j=1}^{\mathrm{K}_g^c} \frac{\epsilon_{ab}}{2} c^a(\sigma_j^0) c^b(\sigma_j^0) \prod_{i=1}^{\mathrm{M}_g} (\hat{\mu}_i, b)_{\hat{g}} \mathrm{e}^{-S_{\mathrm{gh}}[\hat{g}, b, c]}$$

$$\times \int \prod_{i=1}^{n} \mathrm{d}^2\sigma_i \sqrt{\hat{g}} \left\langle \prod_{i=1}^{n} \hat{V}_{\alpha_i}(k_i; \sigma_i) \right\rangle$$

$$1 = \Delta(\sigma_j^0) \int \mathrm{d}\xi \prod_{j=1}^{\mathrm{K}_S^c} \delta^{(2)}\left(\sigma_j - \sigma_j^{0(\xi)}\right), \sigma_j^{0(\xi)} = \sigma_j^0 + \delta_\xi \sigma_j^0, \delta_\xi \sigma_j^0 = \xi(\sigma_j^0)$$

$$\Delta(\sigma_j^0) = \det\psi_i(\sigma_j^0)$$

$$\xi(\sigma_j^0) = \alpha_i \psi_i(\sigma_j^0)$$

$$1 = \int \prod_{j=1}^{\mathrm{K}_g^c} \mathrm{d}^2\delta\sigma_j \mathrm{e}^{-\sum_j (\delta\sigma_j, \delta\sigma_j)} = \Delta \int \prod_{j=1}^{\mathrm{K}_g} \mathrm{d}\alpha_i \mathrm{e}^{-\sum_{j,i,i'} \left(\alpha_i \psi_i(\sigma_j), \alpha_{i'} \psi_{i'}(\sigma_j)\right)}$$

$$= \Delta \left(\det\psi_i(\sigma_j)\right)^{-1}$$

$$\chi_{g,n} = 2 - 2g - n < 0.$$

$$A_{g,n}(\{k_i\})_{\{\alpha_i\}} = g_s^{-\chi_{g,n}} \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g} \quad t \int \mathrm{d}_{\hat{g}} b\, \mathrm{d}_{\hat{g}} c \prod_{j=1}^{\mathrm{K}_g^c} \frac{\epsilon_{ab}}{2} c^a(\sigma_j^0) c^b(\sigma_j^0) \prod_{i=1}^{\mathrm{M}_g} (\hat{\mu}_i, b)_{\hat{g}} \mathrm{e}^{-S_{\mathrm{gh}}[\hat{g}, b, c]}$$

$$\times \int \prod_{i=\mathrm{K}_g^c+1}^{n} \mathrm{d}^2\sigma_i \sqrt{\hat{g}} \left\langle \prod_{j=1}^{\mathrm{K}_g^c} \hat{V}_{\alpha_j}(k_j; \sigma_j^0) \prod_{i=\mathrm{K}_g^c+1}^{n} \hat{V}_{\alpha_i}(k_i; \sigma_i) \right\rangle$$

$$A_{g,n}(\{k_i\})_{\{\alpha_i\}} = g_s^{-\chi_{g,n}} \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g} t \int_{i=\mathrm{K}_g^c+1}^{n} \mathrm{d}^2\sigma_i \sqrt{\hat{g}} \left\langle \prod_{j=1}^{\mathrm{K}_g^c} \frac{\epsilon_{ab}}{2} c^a(\sigma_j^0) c^b(\sigma_j^0) \prod_{i=1}^{\mathrm{M}_g} (\hat{\mu}_i, b)_{\hat{g}} \right\rangle_{\mathrm{gh},\hat{g}}$$

$$\times \left\langle \prod_{j=1}^{\mathrm{K}_g^c} \hat{V}_{\alpha_j}(k_j;\sigma_j^0) \prod_{i=\mathrm{K}_g^c+1}^{n} \hat{V}_{\alpha_i}(k_i;\sigma_i) \right\rangle_{m,\hat{g}}$$

$$A_{g,n}(\{k_i\})_{\{\alpha_i\}} = g_s^{-\chi_{g,n}} \int_{\mathcal{M}_g} \mathrm{d}^{\mathrm{M}_g} t \int_{i=\mathrm{K}_g^c+1}^{n} \mathrm{d}^2\sigma_i \sqrt{\hat{g}} \left\langle \prod_{i=1}^{\mathrm{M}_g} \hat{B}_i \prod_{j=1}^{\mathrm{K}_g^c} \hat{\mathcal{V}}_{\alpha_j}(k_j;\sigma_j^0) \prod_{i=\mathrm{K}_g^c+1}^{n} \hat{V}_{\alpha_i}(k_i;\sigma_i) \right\rangle$$

$$\hat{\mathcal{V}}_{\alpha_j}(k_j;\sigma_j^0) := \frac{\epsilon_{ab}}{2} c^a(\sigma_j^0) c^b(\sigma_j^0) \hat{V}_{\alpha_j}(k_j;\sigma_j^0), \hat{B}_i := (\hat{\mu}_i, b)_{\hat{g}}$$

$$A_{g,n}(\{k_i\})_{\{\alpha_i\}} = g_s^{-\chi_{g,n}} \int_{\mathcal{M}_g \times \mathbb{C}^{n-K_g^c}} \left\langle \bigwedge_{i=1}^{\mathrm{M}_g} \hat{B}_i \, \mathrm{d}t_i \prod_{j=1}^{\mathrm{K}_g^c} \hat{\mathcal{V}}_{\alpha_i}(k_i;\sigma_j^0) \prod_{i=K_g^c+1}^{n} \hat{V}_{\alpha_i}(k_i;\sigma_i) \mathrm{d}^2\sigma_i \sqrt{\hat{g}} \right\rangle.$$

$$\langle V_k(z,\bar{z}) V_{k'}(z',\bar{z}')\rangle_{S^2} = \frac{\mathrm{i}(2\pi)^D \delta^{(D)}(k+k')}{|z-z'|^4}$$

$$A_{0,2}(k,k') = \frac{C_{S_2}}{\mathrm{Vol}\mathcal{K}_{0,0}} \int \mathrm{d}^2 z \, \mathrm{d}^2 z' \langle V_k(z,\bar{z}) V_{k'}(z',\bar{z}')\rangle_{S^2}$$

$$A_{0,2}(k,k') = \frac{C_{S^2}}{\mathrm{Vol}\mathcal{K}_{0,2}} \langle V_k(\infty,\infty) V_{k'}(0,0)\rangle_{S^2}$$

$$A_{0,2}(k,k') = (2\pi)^{D-1} \delta^{(D-1)}(\boldsymbol{k}+\boldsymbol{k}') \frac{C_{S_2} 2\pi \mathrm{i} \delta(0)}{\mathrm{Vol}\mathcal{K}_{0,2}}$$

$$A_{0,2}(k,k') = \frac{C_{S^2}}{\mathrm{Vol}\mathcal{K}_{0,2}} \langle V_k(\infty,\infty) V_{k'}(0,0)\rangle_{S^2}$$

$$\mathrm{Vol}\mathcal{K}_{0,2} = \int \frac{\mathrm{d}^2 z}{|z|^2} = 2\int_0^{2\pi} \mathrm{d}\sigma \int_0^{\infty} \frac{\mathrm{d}r}{r}$$

$$\mathrm{Vol}\mathcal{K}_{0,2} = 4\pi \int_0^{\infty} \frac{\mathrm{d}r}{r} = 4\pi \int_{-\infty}^{\infty} \mathrm{d}\tau = 4\pi \lim_{\varepsilon\to 0} \int_{-\infty}^{\infty} \mathrm{d}\tau \mathrm{e}^{\mathrm{i}\varepsilon\tau} = 4\pi \times 2\pi \lim_{\varepsilon\to 0} \delta(\varepsilon)$$

$$\mathrm{Vol}_{\varepsilon}\mathcal{K}_{0,2} = 8\pi^2 \delta(\varepsilon)$$

$$\mathrm{Vol}_{M,E}\mathcal{K}_{0,2} = 8\pi^2 \mathrm{i}\delta(E)$$

$$X^0(z,\bar{z}) = x^0 + \frac{\mathrm{i}}{2} \alpha' k^0 \ln |z|^2 = x^0 + \mathrm{i}\alpha' k^0 \tau,$$

$$X_M^0 = x_M^0 + \alpha' k_M^0 t$$

$$\mathrm{Vol}_M\mathcal{K}_{0,2} \longrightarrow \frac{8\pi^2\mathrm{i}\delta(0)}{\alpha' k_M^0} = \frac{C_{S_2}2\pi\mathrm{i}\delta(0)}{2k_M^0}$$

$$A_{0,2}(k,k') = 2k^0(2\pi)^{D-1}\delta^{(D-1)}(\boldsymbol{k}+\boldsymbol{k}')$$

$$Z_0 \sim \frac{\delta^{(D)}(0)}{\mathrm{Vol}\mathcal{K}_0}$$

Cuántización y simetría BRST – Transformaciones BRST. Parámetros de Grassmann.

$$Z_g = \int_{\mathcal{M}_g}\frac{\mathrm{d}^{\mathrm{M}_g}t}{\Omega_{\mathrm{ckv}}[g]}\int \mathrm{d}_g g_{ab}\,\mathrm{d}_g\Psi\,\mathrm{d}_g b\,\mathrm{d}_g' c\,\delta\left(\sqrt{g}g_{ab} - \sqrt{\hat{g}}\hat{g}_{ab}\right)\prod_{i=1}^{\mathrm{M}_g}(\phi_i,b)_g \mathrm{e}^{-S_m[g,\Psi]-S_{\mathrm{gh}}[g,b,c]}$$

$$Z_g = \int_{\mathcal{M}_g}\frac{\mathrm{d}^{\mathrm{M}_g}t}{\Omega_{\mathrm{ckv}}[g]}\int \mathrm{d}_g g_{ab}\,\mathrm{d}_g B^{ab}\,\mathrm{d}_g\Psi\,\mathrm{d}_g b\,\mathrm{d}_g' c\prod_{i=1}^{\mathrm{M}_g}(\phi_i,b)_g \mathrm{e}^{-S_m[g,\Psi]-S_{\mathrm{gf}}[g,\hat{g},B]-S_{\mathrm{gh}}[g,b,c]}$$

$$S_{\mathrm{gf}}[g,\hat{g},B] = -\frac{\mathrm{i}}{4\pi}\int \mathrm{d}^2\sigma B^{ab}\left(\sqrt{g}g_{ab} - \sqrt{\hat{g}}\hat{g}_{ab}\right)$$

$$\delta_\epsilon g_{ab} = \mathrm{i}\epsilon\mathcal{L}_c g_{ab}, \delta_\epsilon\Psi = \mathrm{i}\epsilon\mathcal{L}_c\Psi,$$
$$\delta_\epsilon c^a = \mathrm{i}\epsilon\mathcal{L}_c c^a, \delta_\epsilon b_{ab} = \epsilon B_{ab}, \delta_\epsilon B_{ab} = 0,$$

$$\delta_\epsilon g_{ab} = \mathrm{i}\epsilon\mathcal{L}_c g_{ab} + \mathrm{i}\epsilon g_{ab}c_w, \delta_\epsilon c_w = \mathrm{i}\epsilon\mathcal{L}_c c_w$$

$$B_{ab} = \mathrm{i}T_{ab} := \mathrm{i}\left(T_{ab}^m + T_{ab}^{\mathrm{gh}}\right),$$

$$\delta_\epsilon\Psi = \mathrm{i}\epsilon\mathcal{L}_c\Psi, \delta_\epsilon c^a = \mathrm{i}\epsilon\mathcal{L}_c c^a, \delta_\epsilon b_{ab} = \mathrm{i}\epsilon T_{ab}$$

$$\delta_\epsilon c^a = \epsilon c^b\partial_b c^a$$

$$Q_B = \int \mathrm{d}\sigma j_B^0$$

$$Q_B^2 = 0$$

$$N_{\mathrm{gh}}(Q_B) = 1$$

$$\delta_\epsilon\Psi = \mathrm{i}[\epsilon Q_B,\Psi]_\pm$$

$$T_{ab} = [Q_B, b_{ab}]$$

$$|\psi\rangle \in \mathcal{H}(Q_B) := \frac{\ker Q_B}{\mathrm{Im}Q_B},$$

$$Q_B|\psi\rangle = 0, \nexists|\chi\rangle: |\psi\rangle = Q_B|\chi\rangle.$$

$$|\psi\rangle \sim |\psi\rangle + Q_B|\Lambda\rangle.$$

$$\int \mathrm{d}\sigma b_{ab}|\psi\rangle = 0$$

$$b^+ := \int \mathrm{d}\sigma b_{00}, b^- := \int \mathrm{d}\sigma b_{01}$$

$$\mathcal{H}^-(Q_B) = \mathcal{H}(Q_B) \cap \ker b^-, \mathcal{H}^0(Q_B) = \mathcal{H}^-(Q_B) \cap \ker b^+.$$

$$P_\sigma|\psi\rangle = 0$$

$$P_\sigma|\psi\rangle = \int \mathrm{d}\sigma T_{01}|\psi\rangle = \int \mathrm{d}\sigma\{Q_B, b_{01}\}|\psi\rangle = Q_B \int \mathrm{d}\sigma b_{01}|\psi\rangle,$$

$$b^-|\psi\rangle = 0$$

$$\mathcal{H}^- := \mathcal{H}_\downarrow \oplus \mathcal{H}_\uparrow, \mathcal{H}_\downarrow := \mathcal{H}^0 := \mathcal{H}^- \cap \ker b^+.$$

$$b^+|\downarrow\rangle = 0, b^+|\uparrow\rangle = |\downarrow\rangle$$

$$Q_B|\psi_\downarrow\rangle = H|\psi_\uparrow\rangle, Q_B|\psi_\uparrow\rangle = 0$$

$$\begin{gathered} \mathrm{Im} Q_B = \{|\psi_\uparrow\rangle \in \mathcal{H}_\uparrow | H|\psi_\uparrow\rangle \neq 0\} \\ \ker Q_B = \{|\psi_\uparrow\rangle \in \mathcal{H}_\uparrow\} \cup \{|\psi_\downarrow\rangle \in \mathcal{H}_\downarrow | H|\psi_\downarrow\rangle = 0\} \end{gathered}$$

$$H|\psi\rangle = 0$$

$$b^+|\psi\rangle = 0$$

$$Q_B \int \mathrm{d}\sigma b_{00}|\psi\rangle = 0$$

Coordenadas geométricas complejas de un campo de gauge. Métrica de Liouville y simetrías tensoriales.

$$\mathrm{d}s^2 = g_{ab}\,\mathrm{d}\sigma^a\,\mathrm{d}\sigma^b = \mathrm{e}^{2\phi(\tau,\sigma)}(\mathrm{d}\tau^2 + \mathrm{d}\sigma^2),$$

$$g_{ab} = \mathrm{e}^{2\phi}\delta_{ab}, \hat{g}_{ab} = \delta_{ab}.$$

$$\begin{gathered} z = \tau + \mathrm{i}\sigma, \quad \bar{z} = \tau - \mathrm{i}\sigma \\ \tau = \frac{z + \bar{z}}{2}, \quad \sigma = \frac{z - \bar{z}}{2\mathrm{i}} \end{gathered}$$

$$\mathrm{d}s^2 = 2g_{z\bar{z}}\,\mathrm{d}z\,\mathrm{d}\bar{z} = \mathrm{e}^{2\phi(z,\bar{z})}|\mathrm{d}z|^2.$$

$$\begin{gathered} g_{z\bar{z}} = \frac{\mathrm{e}^{2\phi}}{2}, g_{zz} = g_{\bar{z}\bar{z}} = 0 \\ g^{z\bar{z}} = 2\mathrm{e}^{-2\phi}, g^{zz} = g^{\bar{z}\bar{z}} = 0 \end{gathered}$$

$$\hat{g}_{z\bar{z}} = \frac{1}{2}, \hat{g}^{z\bar{z}} = 2$$

$$w = w(z), \bar{w} = \bar{w}(\bar{z}).$$

$$\mathrm{e}^{2\phi(z,\bar{z})} = \left|\frac{\partial w}{\partial z}\right|^2 \mathrm{e}^{2\phi(w,\bar{w})}$$

$$\mathrm{d}s^2 = \mathrm{e}^{2\phi(w,\bar{w})}|\mathrm{d}w|^2$$

$$\mathrm{d}^2\sigma := \mathrm{d}\tau\,\mathrm{d}\sigma = \frac{1}{2}\,\mathrm{d}^2 z,\ \mathrm{d}^2 z := \mathrm{d}z\,\mathrm{d}\bar{z}$$

$$\delta^{(2)}(z) = \frac{1}{2}\delta^{(2)}(\sigma)$$

$$\int \mathrm{d}^2 z\,\delta^{(2)}(z) = \int \mathrm{d}^2\sigma\,\delta^{(2)}(\sigma) = 1$$

$$\partial_z = \frac{1}{2}(\partial_\tau - \mathrm{i}\partial_\sigma), \partial_{\bar{z}} = \frac{1}{2}(\partial_\tau + \mathrm{i}\partial_\sigma)$$
$$\mathrm{d}z = \mathrm{d}\tau + \mathrm{i}\,\mathrm{d}\sigma, \mathrm{d}\bar{z} = \mathrm{d}\tau - \mathrm{i}\,\mathrm{d}\sigma$$

$$\epsilon_{01} = \epsilon^{01} = 1$$
$$\epsilon_{z\bar{z}} = \frac{\mathrm{i}}{2}, \epsilon^{z\bar{z}} = -2\mathrm{i}$$

$$V^z = V^0 + \mathrm{i}V^1, V^{\bar{z}} = V^0 - \mathrm{i}V^1$$

$$V = V^0\partial_0 + V^1\partial_1 = V^z\partial_z + V^{\bar{z}}\partial_{\bar{z}}$$

$$V^w = \frac{\partial w}{\partial z}V^z, V^{\bar{w}} = \frac{\partial \bar{w}}{\partial \bar{z}}V^{\bar{z}}$$

$$T\Sigma_g \simeq T\Sigma_g^+ \oplus T\Sigma_g^-$$
$$V^z\partial_z \in T\Sigma_g^+, V^{\bar{z}}\partial_{\bar{z}} \in T\Sigma_g^-$$

$$\omega_z = \frac{1}{2}(\omega_0 - \mathrm{i}\omega_1), \omega_{\bar{z}} = \frac{1}{2}(\omega_0 + \mathrm{i}\omega_1)$$

$$\omega = \omega_0\,\mathrm{d}\sigma^0 + \omega_1\,\mathrm{d}\sigma^1 = \omega_z\,\mathrm{d}z + \omega_{\bar{z}}\,\mathrm{d}\bar{z}$$

$$T^*\Sigma_g \simeq \Omega^{1,0}(\Sigma_g) \oplus \Omega^{0,1}(\Sigma_g)$$
$$\omega_z\,\mathrm{d}z \in \Omega^{1,0}(\Sigma_g), \omega_{\bar{z}}\,\mathrm{d}\bar{z} \in \Omega^{0,1}(\Sigma_g)$$

$$V_z = g_{z\bar{z}}V^{\bar{z}}, V_{\bar{z}} = g_{z\bar{z}}V^z$$

$$T^{\overbrace{z'\cdots z}^{q_+ + p_-}}{}_{\underbrace{z\cdots z}_{p_+ + q_-}} = (g^{z\bar{z}})^{p_-}(g_{z\bar{z}})^{q_-}T^{\overbrace{z''z}^{q_+}\overbrace{\bar{z}\cdots\bar{z}}^{q_-}}{}_{\underbrace{z\cdots z}_{p_+}\underbrace{\bar{z}\cdots\bar{z}}_{p_-}}.$$

$$T^{\overbrace{w\cdots w}^{q}}{}_{\underbrace{w\cdots w}_{p}} = \left(\frac{\partial w}{\partial z}\right)^n T^{\overbrace{z\cdots z}^{q}}{}_{\underbrace{z\cdots z}_{p}}, n := q - p.$$

$$T^{zz} = 2(T^{00} + \mathrm{i}T^{01}) \in \mathcal{J}^2, T^{\bar{z}\bar{z}} = 2(T^{00} - \mathrm{i}T^{01}) \in \mathcal{J}^{-2}, T^z{}_z = 0$$

$$T_{zz} = g_{z\bar{z}}g_{z\bar{z}}T^{\bar{z}\bar{z}} = \frac{1}{2}(T^{00} - \mathrm{i}T^{01})$$

$$T^{zz} = \left(\frac{\partial z}{\partial \tau}\right)^2 T^{00} + \left(\frac{\partial z}{\partial \sigma}\right)^2 T^{11} + 2\frac{\partial z}{\partial \tau}\frac{\partial z}{\partial \sigma}T^{01} = T^{00} - T^{11} + 2\mathrm{i}T^{01}$$

$$\int \mathrm{d}^2 z(\partial_z v^z + \partial_{\bar{z}} v^{\bar{z}}) = -\mathrm{i}\oint (\mathrm{d}z v^{\bar{z}} - \mathrm{d}\bar{z} v^z) = -2\mathrm{i}\oint_{\partial R}(v_z \,\mathrm{d}z - v_{\bar{z}} \,\mathrm{d}\bar{z})$$

$$\psi_i(z,\bar{z}) = \psi_i^z \partial_z + \psi_i^{\bar{z}} \partial_{\bar{z}}, \phi_i(z,\bar{z}) = \phi_{i,zz}(\,\mathrm{d}z)^2 + \phi_{i,\bar{z}}(\,\mathrm{d}\bar{z})^2$$

$$(P_1\xi)_{zz} = 2\nabla_z \xi_z = \partial_z \xi^{\bar{z}}, (P_1\xi)_{\bar{z}\bar{z}} = 2\nabla_{\bar{z}}\xi_{\bar{z}} = \partial_{\bar{z}}\xi^z$$
$$\left(P_1^\dagger T\right)_z = -2\nabla^z T_{zz} = -4\partial_{\bar{z}} T_{zz}, \left(P_1^\dagger T\right)_{\bar{z}} = -2\nabla^{\bar{z}} T_{\bar{z}\bar{z}} = -4\partial_z T_{\bar{z}\bar{z}}$$

$$\psi^z = \psi^z(z), \psi^{\bar{z}} = \psi^{\bar{z}}(\bar{z}), \phi_{zz} = \phi_{zz}(z), \phi_{\bar{z}\bar{z}} = \phi_{\bar{z}\bar{z}}(\bar{z})$$

$$\ker P_1 = \mathrm{Span}\{\psi_K(z)\} \oplus \mathrm{Span}\{\bar{\psi}_K(\bar{z})\}, K = 1, \ldots, \mathrm{K}_g^c$$
$$\ker P_1^\dagger = \mathrm{Span}\{\phi_I(z)\} \oplus \mathrm{Span}\{\bar{\phi}_I(\bar{z})\}, I = 1, \ldots, \mathrm{M}_g^c$$

$$m_I = t_{2I-1} + \mathrm{i}t_{2I}, \bar{m}_I = t_{2I-1} - \mathrm{i}t_{2I}, I = 1, \ldots, \mathrm{M}_g^c$$

$$\mathrm{d}^{\mathrm{M}_g} t = \mathrm{d}^{2\mathrm{M}_g^c} m$$

$$(T_1, T_2) = 2\int \mathrm{d}^2\sigma\sqrt{\hat{g}}\hat{g}^{ac} g^{bd} T_{1,ab} T_{2,cd} = 4\int \mathrm{d}^2 z\left(T_{1,zz}T_{2,\bar{z}\bar{z}} + T_{1,\bar{z}\bar{z}}T_{2,zz}\right)$$
$$(\xi_1, \xi_2) = \int \mathrm{d}^2\sigma\sqrt{\hat{g}}\hat{g}_{ab}\xi^a\xi^b = \frac{1}{4}\int \mathrm{d}^2 z\left(\xi_1^z \xi_2^{\bar{z}} + \xi_1^{\bar{z}}\xi_2^z\right)$$

$$\mu_{izz} = \partial_i \bar{g}_{zz}, \mu_{i\bar{z}\bar{z}} = \partial_i \bar{g}_{\bar{z}\bar{z}}$$

$$Z_g = \int_{\mathcal{M}_g} \mathrm{d}^{2\mathrm{M}_g^c} m \frac{\left|\det\left(\phi_I, \mu_J\right)\right|^2}{\left|\det\left(\phi_I, \bar{\phi}_J\right)\right|} \frac{\det' P_1^\dagger P_1}{\left|\det\left(\psi_I, \bar{\psi}_J\right)\right|} \frac{Z_m[\delta]}{\Omega_{\mathrm{ckv}}[\delta]}$$

$$c := c^z, \bar{c} := c^{\bar{z}}, b := b_{zz}, \bar{b} := b_{\bar{z}\bar{z}}$$

$$S_{\mathrm{gh}}[g, b, c] = \frac{1}{2\pi}\int \mathrm{d}^2 z\left(b\partial_{\bar{z}} c + \bar{b}\partial_z \bar{c}\right)$$

$$\partial_z c = 0, \partial_z b = 0, \partial_{\bar{z}}\bar{c} = 0, \partial_{\bar{z}}\bar{b} = 0$$

$$\bigwedge_{i=1}^{\mathrm{M}_g} B_i \,\mathrm{d}t_i = \bigwedge_{I=1}^{\mathrm{M}_g^c} B_I \bar{B}_I \,\mathrm{d}m_I \wedge \bar{m}_I, B_I := (\mu_I, b)$$

$$Z_g = \int_{\mathcal{M}_g} \mathrm{d}^{2\mathrm{M}_g^c} m \frac{\Omega_{\mathrm{ckv}}[\delta]^{-1}}{\left|\det\psi_I\left(z_j^0\right)\right|^2}\int \mathrm{d}(b,\bar{b})\mathrm{d}(c,\bar{c})\prod_{j=1}^{\mathrm{K}_g^c} c\left(z_j^0\right)\bar{c}\left(\bar{z}_j^0\right)\prod_{I=1}^{\mathrm{M}_g^c} |(\mu_I, b)|^2 \mathrm{e}^{-S_{\mathrm{gh}}[b,c]} Z_m[\delta].$$

Simetría conforme en dimensión $\mathbb{R}^4$. Ecuaciones de Killing y métrica Beltrami-Laplace.

$$x^\mu \longrightarrow x'^\mu = x'^\mu(x)$$

$$g_{\mu\nu}(x) \longrightarrow g'_{\mu\nu}(x') = \frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} g_{\rho\sigma}(x) = \Omega(x')^2 g_{\mu\nu}(x')$$

$$\frac{u \cdot v}{|u||v|} = \frac{u' \cdot v'}{|u'||v'|}$$

$$\Omega := \mathrm{e}^{\omega}.$$

$$\delta x^{\mu} = \xi^{\mu}$$

$$\delta g_{\mu\nu} = \mathcal{L}_{\xi} g_{\mu\nu} = \nabla_{\mu}\xi_{\nu} + \nabla_{\nu}\xi_{\mu} = \frac{2}{d} g_{\mu\nu}\nabla_{\rho}\xi^{\rho}$$

$$\Omega^2 = 1 + \frac{2}{d}\nabla_{\rho}\xi^{\rho}$$

$$\mathrm{ISO}(\mathcal{M}) \subset \mathrm{CISO}(\mathcal{M})$$

$$\delta g_{\mu\nu} = \mathcal{L}_{\xi} g_{\mu\nu} = \nabla_{\mu}\xi_{\nu} + \nabla_{\nu}\xi_{\mu} = 0$$

$$\eta = \mathrm{diag}(\underbrace{-1, \dots, -1}_{q}, \underbrace{1, \dots, 1}_{p})$$

$$\left(\eta_{\mu\nu}\Delta + (D-2)\partial_{\mu}\partial_{\nu}\right)\partial \cdot \epsilon = 0$$

$$\begin{gathered} \xi^{\mu} = a^{\mu} \\ \xi^{\mu} = \omega^{\mu}{}_{\nu} x^{\nu} \\ \xi^{\mu} = \lambda x^{\mu} \\ \xi^{\mu} = b^{\mu} x^2 - 2b \cdot x x^{\mu} \end{gathered}$$

$$\mathrm{ISO}(\mathbb{R}^{p,q}) = \mathrm{SO}(p,q), \mathrm{CISO}(\mathbb{R}^{p,q}) = \mathrm{SO}(p+1, q+1)$$

$$\dim \mathrm{SO}(p+1, q+1) = \frac{1}{2}(p+q+2)(p+q+1)$$

Planos complejos.

Métrica de Riemann.

$$\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}.$$

$$\lim_{r \to \infty} r\mathrm{e}^{\mathrm{i}\theta} := \infty$$

$$z = \mathrm{e}^{\mathrm{i}\phi} \cot \frac{\theta}{2}$$

$$\begin{gathered} z = x + \mathrm{i}y, \quad \bar{z} = x - \mathrm{i}y \\ x = \frac{z + \bar{z}}{2}, \quad y = \frac{z - \bar{z}}{2\mathrm{i}} \end{gathered}$$

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 = \mathrm{d}z\, \mathrm{d}\bar{z}$$

$$\partial := \partial_z = \frac{1}{2}\left(\partial_x - \mathrm{i}\partial_y\right), \bar{\partial} := \partial_{\bar{z}} = \frac{1}{2}\left(\partial_x + \mathrm{i}\partial_y\right)$$

$$\partial\phi(z_1)\partial\phi(z_2) := \partial_{z_1}\partial_{z_2}\phi(z_1)\phi(z_2).$$

$$w = \frac{1}{z}$$

Métrica de Lorentz.

$$t \in \mathbb{R}, \sigma \in [0, L), \sigma \sim \sigma + L,$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\sigma^2 = -\mathrm{d}\sigma^+ \mathrm{d}\sigma^-,$$

$$\mathrm{d}\sigma^\pm = \mathrm{d}t \pm \mathrm{d}\sigma$$

$$\tau = \mathrm{i}t,$$

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + \mathrm{d}\sigma^2.$$

$$w = \tau + \mathrm{i}\sigma, \bar{w} = \tau - \mathrm{i}\sigma$$

$$\mathrm{d}s^2 = \mathrm{d}w\, \mathrm{d}\bar{w}.$$

$$w = \mathrm{i}(t + \sigma) = \mathrm{i}\sigma^+, \bar{w} = \mathrm{i}(t - \sigma) = \mathrm{i}\sigma^-.$$

$$z = \mathrm{e}^{2\pi w/L}, \bar{z} = \mathrm{e}^{2\pi \bar{w}/L},$$

$$\mathrm{d}s^2 = \left(\frac{L}{2\pi}\right)^2 \frac{\mathrm{d}z\, \mathrm{d}\bar{z}}{|z|^2}.$$

Álgebra de Witt – Métrica de Cauchy-Riemann- Killing – Laurent – Möbius. Definiciones cuánticas de un CFT y sus operadores y propagadores. Métrica de Virasoro. Funciones de Correlación.

$$\mathrm{d}s^2 = \mathrm{d}z\, \mathrm{d}\bar{z},$$

$$z \longrightarrow z' = f(z), \bar{z} \longrightarrow \bar{z}' = \bar{f}(\bar{z})$$

$$\mathrm{d}s^2 = \mathrm{d}z' \mathrm{d}\bar{z}' = \left|\frac{\mathrm{d}f}{\mathrm{d}z}\right|^2 \mathrm{d}z\, \mathrm{d}\bar{z}.$$

$$\delta z = v(z), \delta\bar{z} = \bar{v}(\bar{z}),$$

$$\bar{\partial} v = 0, \partial \bar{v} = 0.$$

$$v(z) = \sum_{n \in \mathbb{Z}} v_n z^{n+1}, \bar{v}(\bar{z}) = \sum_{n \in \mathbb{Z}} \bar{v}_n \bar{z}^{n+1},$$

$$\ell_n = -z^{n+1}\partial_z, \bar{\ell}_n = -\bar{z}^{n+1}\partial_{\bar{z}}, n \in \mathbb{Z}.$$

$$[\ell_m, \ell_n] = (m - n)\ell_{m+n}, [\bar{\ell}_m, \bar{\ell}_n] = (m - n)\bar{\ell}_{m+n}, [\ell_m, \bar{\ell}_n] = 0.$$

$$\delta g_{z\bar{z}} = \partial v + \bar{\partial}\bar{v}, \delta g_{zz} = \delta g_{\bar{z}\bar{z}} = 0$$

$$\lim_{|z| \to 0} v(z) < \infty \implies \forall n < -1: v_n = 0$$

$$v(1/w) = \frac{\mathrm{d}z}{\mathrm{d}w} \sum_n v_n w^{-n-1}$$

$$\lim_{|z|\to\infty} v(z) = \lim_{|w|\to 0} \frac{\mathrm{d}z}{\mathrm{d}w} v(1/w) = -\lim_{|w|\to 0} \frac{v(1/w)}{w^2} < \infty \implies \forall n > 1: v_n = 0$$

$$\{\ell_{-1}, \ell_0, \ell_1\} \cup \{\bar{\ell}_{-1}, \bar{\ell}_0, \bar{\ell}_1\}$$

$$\ell_{-1} = -\partial_z, \ell_0 = -z\partial_z, \ell_1 = -z^2\partial_z$$

$$[\ell_0, \ell_{\pm 1}] = \mp \ell_{\pm 1}, [\ell_1, \ell_{-1}] = 2\ell_0$$

$$\mathrm{PSL}(2,\mathbb{C}) := \mathrm{SL}(2,\mathbb{C})/\mathbb{Z}_2 \sim \mathrm{SO}(3,1)$$

$$\mathcal{K}_0 = \mathrm{PSL}(2,\mathbb{C})$$

$$g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, a, b, c, d \in \mathbb{C}, \det g = ad - bc = 1$$

$$\mathrm{K}_0 := \dim \mathrm{SL}(2,\mathbb{C}) = 6.$$

$$f_g(z) = \frac{az + b}{cz + d}$$

$$v(z) = \beta + 2\alpha z + \gamma z^2, \bar{v}(\bar{z}) = \bar{\beta} + 2\bar{\alpha}\bar{z} + \bar{\gamma}\bar{z}^2$$

$$a = 1 + \alpha, b = \beta, c = -\gamma, d = 1 - \alpha$$

$$\text{translation:} \quad f_g(z) = z + a, \qquad a \in \mathbb{C},$$

$$\text{rotation:} \quad f_g(z) = \zeta z, \qquad |\zeta| = 1,$$

$$\text{dilatation:} \quad f_g(z) = \lambda z, \qquad \lambda \in \mathbb{R},$$

$$\text{SCT:} \quad f_g(z) = \frac{z}{cz+1}, \qquad c \in \mathbb{C}.$$

$$\text{inversion:} \; I^+(z) := I(z) := \frac{1}{z}$$

$$I^-(z) := -I(z) = I(-z) = -\frac{1}{z}$$

$$g_{\infty,0,1}(z) = \frac{1}{1-z}$$

$$\forall f \text{ meromorphic}: \; \mathcal{O}(z,\bar{z}) = \left(\frac{\mathrm{d}f}{\mathrm{d}z}\right)^h \left(\frac{\mathrm{d}\bar{f}}{\mathrm{d}\bar{z}}\right)^{\bar{h}} \mathcal{O}'(f(z), \bar{f}(\bar{z})),$$

$$\forall f \in \mathrm{PSL}(2,\mathbb{C}): \mathcal{O}(z,\bar{z}) = \left(\frac{\mathrm{d}f}{\mathrm{d}z}\right)^{h} \left(\frac{\mathrm{d}\bar{f}}{\mathrm{d}\bar{z}}\right)^{\bar{h}} \mathcal{O}'(f(z),\bar{f}(\bar{z}))$$

$$\Delta := h + \bar{h}, s := h - \bar{h}.$$

$$\mathcal{O}(z,\bar{z})\mathrm{d}z^{h}\,\mathrm{d}\bar{z}^{\bar{h}}$$

$$f \circ \mathcal{O}(z,\bar{z}) := f'(z)^{h}\bar{f}'(\bar{z})^{\bar{h}}\mathcal{O}'(f(z),\bar{f}(\bar{z}))$$

$$\delta z = v(z), \delta\bar{z} = \bar{v}(\bar{z})$$

$$\delta\mathcal{O}(z,\bar{z}) = (h\partial v + v\partial)\mathcal{O}(z,\bar{z}) + (\bar{h}\bar{\partial}\bar{v} + \bar{v}\bar{\partial})\mathcal{O}(z,\bar{z})$$

$$\nabla^{\nu}T_{\mu\nu} = 0, g^{\mu\nu}T_{\mu\nu} = 0$$

$$g^{\mu\nu}T_{\mu\nu} = 4T_{z\bar{z}} = T_{xx} + T_{yy} = 0$$

$$T_{z\bar{z}} = 0$$

$$\partial_z T_{\bar{z}\bar{z}} = 0, \partial_{\bar{z}} T_{zz} = 0$$

$$T(z) := T_{zz}(z), \bar{T}(\bar{z}) := T_{\bar{z}\bar{z}}(\bar{z})$$

$$J_v(z) := J_v^{\bar{z}}(z) = -T(z)v(z), \bar{J}_v(\bar{z}) := J_v^{z}(\bar{z}) = -\bar{T}(\bar{z})\bar{v}(\bar{z})$$

$$Z = \int \mathrm{d}\Psi \mathrm{e}^{-S[\Psi]}$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m-1)(m+1)\delta_{m+n}$$
$$[\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n} + \frac{\bar{c}}{12}m(m-1)(m+1)\delta_{m+n}$$
$$[L_m, \bar{L}_n] = 0, [c, L_m] = 0, [\bar{c}, \bar{L}_m] = 0$$

$$\left\langle \prod_{i=1}^{n} \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle = \int \mathrm{d}\Psi \mathrm{e}^{-S[\Psi]} \prod_{i=1}^{n} \mathcal{O}_i(z_i,\bar{z}_i),$$

$$\left\langle \prod_{i=1}^{n} \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle = \prod_{i=1}^{n} \left(\frac{\mathrm{d}f}{\mathrm{d}z}(z_i)\right)^{h_i} \left(\frac{\mathrm{d}f}{\mathrm{d}\bar{z}}(\bar{z}_i)\right)^{\bar{h}_i} \times \left\langle \prod_{i=1}^{n} \mathcal{O}_i\left(f(z_i),\bar{f}(\bar{z}_i)\right) \right\rangle.$$

$$\delta\left\langle \prod_{i=1}^{n} \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle = \sum_{i=1}^{n} (h_i\partial_i v(z_i) + v(z_i)\partial_i + \text{ c.c. }) \left\langle \prod_{i=1}^{n} \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle = 0,$$

$$\langle \mathcal{O}_i(z_i, \bar{z}_i) \rangle = \delta_{h_i,0}\delta_{\bar{h}_i,0},$$
$$\langle \mathcal{O}_i(z_i, \bar{z}_i)\mathcal{O}_j(z_j, \bar{z}_j) \rangle = \delta_{h_i,h_j}\delta_{\bar{h}_i,\bar{h}_j}\frac{g_{ij}}{z_{ij}^{2h_i}\bar{z}_{ij}^{2\bar{h}_i}},$$
$$\langle \mathcal{O}_i(z_i, \bar{z}_i)\mathcal{O}_j(z_j, \bar{z}_j)\mathcal{O}_k(z_k, \bar{z}_k) \rangle = \frac{C_{ijk}}{z_{ij}^{h_i+h_j-h_k} z_{jk}^{h_j+h_k-h_i} z_{ki}^{h_i+h_k-h_j}}$$
$$\times \frac{1}{\bar{z}_{ij}^{\bar{h}_i+\bar{h}_j-\bar{h}_k} \bar{z}_{jk}^{\bar{h}_j+\bar{h}_k-\bar{h}_i} \bar{z}_{ki}^{\bar{h}_i+\bar{h}_k-\bar{h}_j}},$$

$$z_{ij} = z_i - z_j.$$

$$\left\langle \prod_{i=1}^{4} \mathcal{O}_i(z_i, \bar{z}_i) \right\rangle = f(x, \bar{x}) \prod_{i<j} \frac{1}{z_{ij}^{(h_i+h_j)-h/3}} \times \text{ c.c.}$$

$$h := \sum_{i=1}^{4} h_i, \bar{h} := \sum_{i=1}^{4} \bar{h}_i.$$

$$x := \frac{z_{12}z_{34}}{z_{13}z_{24}}.$$

Operadores formales y cuantización radial. Ordenadores radiales y conmutadores. Productos de expansión.

$$z = \mathrm{e}^{\tau+\mathrm{i}\sigma} = x + \mathrm{i}y.$$

$$\tau \longrightarrow \tau + T$$

$$z \longrightarrow \mathrm{e}^{T} z.$$

$$H = \frac{2\pi}{L}(L_0 + \bar{L}_0)$$

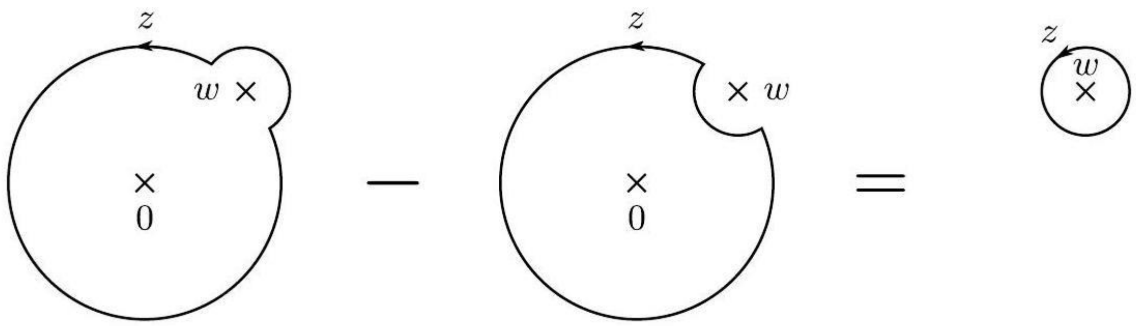
$$\text{on-shell state: } h + \bar{h} = 0.$$

$$R(A(z)B(w)) = \begin{cases} A(z)B(w) & |z| > |w| \\ (-1)^F B(w)A(z) & |w| > |z| \end{cases}$$

$$[A(z), B(w)]_{\pm,|z|=|w|} = \lim_{\delta\to 0}\left(A(z)B(w)|_{|z|=|w|+\delta} \pm B(w)A(z)|_{|z|=|w|-\delta} \right)$$

$$A = \oint_{C_0} \frac{\mathrm{d}z}{2\pi\mathrm{i}} a(z), B = \oint_{C_0} \frac{\mathrm{d}z}{2\pi\mathrm{i}} b(z)$$

$$[A, B]_{\pm} = \oint_{C_0} \frac{\mathrm{d}w}{2\pi\mathrm{i}} \oint_{C_\mathrm{w}} \frac{\mathrm{d}z}{2\pi\mathrm{i}} a(z)b(w)$$
$$[A, b(w)]_{\pm} = \oint_{C_\mathrm{w}} \frac{\mathrm{d}z}{2\pi\mathrm{i}} a(z)b(w)$$



$$\partial_\mu j^\mu = \partial j^z + \bar{\partial} j^{\bar{z}} = 2\left(\partial j_{\bar{z}} + \bar{\partial} j_z\right) = 0$$

$$Q = \frac{1}{2\pi\mathrm{i}} \oint_{C_0} (j_z\, \mathrm{d}z - j_{\bar{z}}\, \mathrm{d}\bar{z})$$

$$j(z) := j_z(z), \bar{j}(\bar{z}) := j_{\bar{z}}(\bar{z})$$

$$Q = Q_L + Q_R, Q_L := \frac{1}{2\pi\mathrm{i}} \oint_{C_0} j(z)\mathrm{d}z, Q_R := -\frac{1}{2\pi\mathrm{i}} \oint_{C_0} \bar{j}(\bar{z})\mathrm{d}\bar{z}$$

$$\delta_\epsilon \mathcal{O}(z,\bar{z}) = -[\epsilon Q, \mathcal{O}(z,\bar{z})] = -\epsilon \oint_{C_z} \frac{\mathrm{d}w}{2\pi\mathrm{i}} j(w)\mathcal{O}(z,\bar{z}) + \epsilon \oint_{C_z} \frac{\mathrm{d}\bar{w}}{2\pi\mathrm{i}} \bar{j}(\bar{w})\mathcal{O}(z,\bar{z})$$

$$Q = \frac{1}{2\pi} \int \mathrm{d}\sigma j^0$$

$$Q = \frac{1}{2\pi} \int (\mathrm{d}\sigma j^0 - \mathrm{d}\tau j^1) = -\frac{1}{2\pi} \int \epsilon_{\mu\nu} j^\mu \mathrm{d}x^\nu$$

$$Q = -\frac{1}{2\pi} \oint \epsilon_{z\bar{z}} (j^z\, \mathrm{d}\bar{z} - j^{\bar{z}}\, \mathrm{d}z) = -\frac{\mathrm{i}}{4\pi} \oint (j^z\, \mathrm{d}\bar{z} - j^{\bar{z}}\, \mathrm{d}z) = -\frac{1}{2\pi\mathrm{i}} \oint (j_z\, \mathrm{d}z - j_{\bar{z}}\, \mathrm{d}\bar{z})$$

$$\mathcal{O}_i(z_i,\bar{z}_i)\mathcal{O}_j(z_j,\bar{z}_j) = \sum_k \frac{c_{ij}^k}{z_{ij}^{h_i+h_j-h_k} \bar{z}_{ij}^{\bar{h}_i+\bar{h}_j-\bar{h}_k}} \mathcal{O}_k(z_j,\bar{z}_j),$$

$$C_{ijk} = g_{k\ell} c_{ij}^\ell.$$

Identidad OPE.

$$\phi(z)1 = \sum_{n\in\mathbb{N}} \frac{(z-w)^n}{n!} \partial^n \phi(w)$$

$$A(z)B(w) := \sum_{n=-\infty}^{N} \frac{\{AB\}_n(z)}{(z-w)^n}$$

$$A(z)B(w) \sim \sum_{n=1}^{N} \frac{\{AB\}_n(z)}{(z-w)^n} =: A(z)B(w)$$

$$\phi_i(z_i)\phi_j(z_j) \sim \sum_k \theta(h_i + h_j - h_k) \frac{c_{ij}^k}{(z-w)^{h_i+h_j-h_k}} \phi_k(w)$$

$$T(z)\phi(w) \sim \frac{h\phi(w)}{(z-w)^2} + \frac{\partial\phi(w)}{z-w}$$

$$\begin{aligned} \delta\phi(z) &= \oint_{C_z} \frac{\mathrm{d}w}{2\pi\mathrm{i}} v(w)T(w)\phi(z) \sim \oint_{C_z} \frac{\mathrm{d}w}{2\pi\mathrm{i}} v(w)\left(\frac{h\phi(z)}{(w-z)^2} + \frac{\partial\phi(z)}{w-z}\right) \\ &= h\partial v(z)\phi(z) + v(z)\partial\phi(z) \end{aligned}$$

$$T(z)T(w) \sim \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

$$\delta T = 2\partial v T + v\partial T + \frac{c}{12}\partial^3 v$$

$$T'(w) = \left(\frac{\mathrm{d}z}{\mathrm{d}w}\right)^{-2}\left(T(z) - \frac{c}{12}S(w,z)\right) = \left(\frac{\mathrm{d}z}{\mathrm{d}w}\right)^{-2} T(z) + \frac{c}{12}S(z,w)$$

$$S(w,z) = \frac{w^{(3)}}{w'} - \frac{3}{2}\left(\frac{w''}{w'}\right)^2$$

$$S(u,z) = S(w,z) + \left(\frac{\mathrm{d}w}{\mathrm{d}z}\right)^2 S(u,w)$$

$$\begin{aligned} \delta T(z) &= \oint_{C_z} \frac{\mathrm{d}w}{2\pi\mathrm{i}} v(w)T(w)T(z) \sim \oint_{C_z} \frac{\mathrm{d}w}{2\pi\mathrm{i}} v(w)\left(\frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}\right) \\ &= \frac{c}{2\times 3!}\partial^3 v(z) + 2\partial v(z)T(z) + v(z)\partial T(z) \end{aligned}$$

Conjugaciones BPZ y Modo de Expansión.

$$t^\dagger = -\mathrm{i}\tau^\dagger = t \implies \tau^\dagger = -\tau.$$

$$z \overset{\tau\to-\tau}{\to} \mathrm{e}^{-\tau+\mathrm{i}\sigma} = \frac{1}{z^*} = I(\bar{z}),$$

$$\mathcal{O}(z,\bar{z})^\ddagger := (\bar{I} \circ \mathcal{O}(z,\bar{z}))^\dagger,$$

$$\mathcal{O}(z,\bar{z})^\ddagger = \left[\frac{1}{\bar{z}^{2h} z^{2\bar{h}}} \mathcal{O}\left(\frac{1}{\bar{z}}, \frac{1}{z}\right)\right]^\dagger = \frac{1}{z^{2h}\bar{z}^{2\bar{h}}} \mathcal{O}^\dagger\left(\frac{1}{z}, \frac{1}{\bar{z}}\right)$$

$$(\lambda\mathcal{O}_1 \cdots \mathcal{O}_n)^\ddagger = \lambda^* \mathcal{O}_n^\ddagger \cdots \mathcal{O}_1^\ddagger, \lambda \in \mathbb{C},$$

$$\mathcal{O}(z,\bar{z})^t := I^\pm \circ \mathcal{O}(z,\bar{z}) = \frac{(\mp 1)^{h+\bar{h}}}{z^{2h}\bar{z}^{2\bar{h}}} \mathcal{O}\left(\pm\frac{1}{z}, \pm\frac{1}{\bar{z}}\right)$$

$$(\lambda\mathcal{O}_1 \cdots \mathcal{O}_n)^t = \lambda \mathcal{O}_1^t \cdots \mathcal{O}_n^t, \lambda \in \mathbb{C}$$

$$1^\ddagger = 1^t = 1$$

$$\mathcal{O}(z,\bar{z}) = \sum_{m,n} \frac{\mathcal{O}_{m,n}}{z^{m+h}\bar{z}^{n+\bar{h}}}$$

$$m+h \in \mathbb{Z}+\nu, n+\bar{h} \in \mathbb{Z}+\bar{\nu}, \nu,\bar{\nu} = \begin{cases} 0 & \text{periodic} \\ 1/2 & \text{anti-periodic} \end{cases}$$

$$\mathcal{O}\left(\mathrm{e}^{2\pi\mathrm{i}}z,\bar{z}\right) = \mathrm{e}^{2\pi\mathrm{i}\nu}\mathcal{O}(z,\bar{z}), \mathcal{O}\left(z,\mathrm{e}^{2\pi\mathrm{i}}\bar{z}\right) = \mathrm{e}^{2\pi\mathrm{i}\bar{\nu}}\mathcal{O}(z,\bar{z})$$

$$\nu,\bar{\nu} = \begin{cases} 0 & \text{NS} \\ 1/2 & \text{R} \end{cases}$$

$$\nu,\bar{\nu} = \begin{cases} 0 & \text{untwisted} \\ 1/2 & \text{twisted} \end{cases}$$

$$\left(\mathcal{O}^{\ddagger}\right)_{-m,-n} = \left(\mathcal{O}_{m,n}\right)^{\dagger}.$$

$$\mathcal{O}^{\ddagger} = \mathcal{O} \implies \left(\mathcal{O}_{m,n}\right)^{\dagger} = \mathcal{O}_{-m,-n}.$$

$$\phi(z) = \sum_{n\in\mathbb{Z}+h+\nu} \frac{\phi_n}{z^{n+h}}.$$

$$\phi_n = \oint_{C_0} \frac{\mathrm{d}z}{2\pi\mathrm{i}} z^{n+h-1}\phi(z)$$

$$\phi^{\ddagger} = \phi \implies (\phi_n)^{\dagger} = \phi_{-n}.$$

$$\phi_n^t = (I^{\pm}\circ\phi)_n = (-1)^h(\pm 1)^n\phi_{-n}.$$

$$\begin{aligned}
\phi_n^t = (I^{\pm}\circ\phi)_n &= \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} z^{n+h-1} I^{\pm}\circ\phi(z) \\
&= \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} z^{n+h-1}\left(\mp\frac{1}{z^2}\right)^h \phi\left(\pm\frac{1}{z}\right) \\
&= (\mp 1)^h \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} z^{n-h-1}\phi\left(\pm\frac{1}{z}\right) \\
&= (\mp 1)^h \oint \frac{\mathrm{d}w}{2\pi\mathrm{i}} \left(\pm\frac{1}{w}\right)^{n-h} w^{-1}\phi(w) \\
&= (\mp 1)^h(\pm 1)^{n-h} \oint \frac{\mathrm{d}w}{2\pi\mathrm{i}} w^{-n+h-1}\phi(w)
\end{aligned}$$

$$\frac{\mathrm{d}z}{z} = \mp\frac{\mathrm{d}w}{w^2 z} = -\frac{\mathrm{d}w}{w}$$

$$T(z) = \sum_{n\in\mathbb{Z}} \frac{L_n}{z^{n+2}}, L_n = \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} T(z) z^{n+1}$$

$$L_n^{\dagger} = L_{-n}.$$

$$[L_m, \phi_n] = (m(h-1)-n)\phi_{m+n}$$

$$[L_m, \phi(z)] = z^m(z\partial + (n+1)h)\phi(z).$$

$$[L_0, L_{-n}] = nL_{-n}, [L_0, \phi_{-n}] = n\phi_{-n}$$

$$L_n^{\pm} = L_n \pm \bar{L}_n$$

$$j(z) = \sum_n \frac{j_n}{z^{n+1}}$$

$$Q_L = j_0$$

Espacios de Hilbert y correspondencia operador – estado.

$$L_0|0\rangle = 0, L_{\pm 1}|0\rangle = 0$$

$$\langle \mathcal{O} \rangle := \langle 0|\mathcal{O}|0\rangle.$$

$$|\mathcal{O}\rangle = \lim_{z,\bar{z}\to 0} \mathcal{O}(z,\bar{z})|0\rangle = \mathcal{O}(0,0)|0\rangle$$

$$|\phi\rangle = \lim_{z\to 0} \phi(z)|0\rangle = \phi(0)|0\rangle.$$

$$\forall n \geq -h+1 : \phi_n|0\rangle = 0$$

$$|\phi\rangle = \phi_{-h}|0\rangle = \oint \frac{\mathrm{d}z}{2\pi \mathrm{i}} \frac{\phi(z)}{z}|0\rangle.$$

$$\phi(z)|0\rangle = \mathrm{e}^{zL_{-1}}\phi(0)\mathrm{e}^{-zL_{-1}}|0\rangle = \mathrm{e}^{zL_{-1}}|\phi\rangle.$$

$$\forall n \geq -1 : L_n|0\rangle = 0$$

$$\forall |\phi\rangle \in \mathcal{H} : \langle \Omega|L_0|\Omega\rangle \leq \langle \phi|L_0|\phi\rangle.$$

$$L_0|\Omega\rangle := a_\Omega|\Omega\rangle,$$

$$\langle 0| = |0\rangle^{\ddagger} = |0\rangle^{t}$$

$$\langle 0|L_0 = 0, \langle 0|L_{\pm 1} = 0.$$

$$\begin{aligned}\langle \mathcal{O}^{\ddagger}| &= \lim_{w,\bar{w}\to 0} \langle 0|\mathcal{O}(w,\bar{w})^{\ddagger} = \lim_{w,\bar{w}\to 0} \frac{1}{w^{2h}\bar{w}^{2\bar{h}}} \langle 0|\mathcal{O}\left(\frac{1}{\bar{w}}, \frac{1}{w}\right)^{\dagger} \\ &= \lim_{z,\bar{z}\to\infty} z^{2h}\bar{z}^{2\bar{h}} \langle 0|\mathcal{O}^{\dagger}(z,\bar{z}) \\ &= \langle 0|I \circ \mathcal{O}^{\dagger}(0,0)\end{aligned}$$

$$\begin{aligned}\langle \phi^{\ddagger}| &= \lim_{\bar{w}\to 0} \langle 0|\phi(w)^{\ddagger} = \lim_{\bar{w}\to 0} \frac{1}{w^{2h}} \langle 0|\phi^{\dagger}\left(\frac{1}{w}\right) \\ &= \lim_{z\to\infty} z^{2h} \langle 0|\phi^{\dagger}(z) \\ &= \langle 0|I \circ \phi^{\dagger}(0).\end{aligned}$$

$$\langle \phi^{\ddagger}| = \langle 0|\left(\phi^{\dagger}\right)_h.$$

$$\langle\phi| := \lim_{w\to 0}\langle 0|\phi(w)^t$$
$$= (\pm 1)^h \lim_{z\to\infty} z^{2h}\langle 0|\phi(z)$$
$$= \langle 0|I^{\pm}\circ\phi(0)$$

$$\langle\phi| = (\pm 1)^h\langle 0|\phi_h.$$

$$\langle\phi^{\ddagger}| = (\pm 1)^h\langle\phi|.$$

$$\forall n \le h-1{:}\ \langle 0|\phi_n = 0,$$

$$\forall n \le 1{:}\ \langle 0|L_n = 0.$$

$$\langle 0|T(z)|0\rangle = 0.$$

$$\langle\phi_i^{\ddagger} \mid \phi_j\rangle = \langle 0|\bar{I}\circ\phi_j(0)\phi_i(0)|0\rangle = \lim_{\substack{z\to\infty\\ w\to 0}} z^{2h_i}\langle 0|\phi_i^{\dagger}(z)\phi_j(w)|0\rangle,$$
$$\langle\phi_i \mid \phi_j\rangle = \langle 0|I\circ\phi_j(0)\phi_i(0)|0\rangle = (\pm 1)^{h_i}\lim_{\substack{z\to\infty\\ w\to 0}} z^{2h_i}\langle 0|\phi_i(z)\phi_j(w)|0\rangle.$$

$$\langle\phi_i \mid \phi_j\rangle = \langle I\circ\phi_i(0)\phi_j(0)\rangle, \langle\phi_i^{\ddagger} \mid \phi_j\rangle = \langle I\circ\phi_i^{\dagger}(0)\phi_j(0)\rangle.$$

$$\langle\phi_i|\phi_j(z)|\phi_k\rangle = (\pm 1)^{h_i}\lim_{w\to\infty} w^{2h_i}\langle\phi_i(w)\phi_j(z)\phi_k(0)\rangle.$$

$$\langle\phi_i^c \mid \phi_j\rangle = \delta_{ij}$$

Módulos Verma.

$$L_0|\phi\rangle = h|\phi\rangle, \forall n \ge 1{:}\ L_n|\phi\rangle = 0.$$

$$|\phi_{\{n_i\}}\rangle := \prod_i L_{-n_i}|\phi\rangle,$$

$$L_0 = h + \sum_i n_i.$$

$$\langle 0|{:}\mathcal{O}{:}|0\rangle = 0.$$

$$\langle\Omega|\,{}^{\star}_{\star}\mathcal{O}^{\star}_{\star}|\Omega\rangle = 0.$$

$$:A(z)B(w): \stackrel{?}{=} A(z)B(w) - \langle A(z)B(w)\rangle.$$

$$:A(z)B(z): \stackrel{?}{=} \lim_{w\to z}(A(z)B(w) - \langle A(z)B(w)\rangle).$$

$$:A(z)B(w): := A(z)B(w) - A(z)B(w) = \sum_{n\in\mathbb{N}} (z-w)^n\{AB\}_{-n}(z)$$

$$:AB(z): =: A(z)B(z): = \lim_{w\to z} :A(z)B(w): = \{AB\}_0(z)$$

$$:AB(z): = \oint_{C_z}\frac{\mathrm{d}w}{2\pi\mathrm{i}}\frac{A(z)B(w)}{z-w}$$

$$AB(z):=:AB(z):.$$

$$:AB(z):=\sum_{m}\frac{:AB:_m}{z^{m+h_A+h_B}}$$

$$:AB:_m=\sum_{n\leq -h_A}A_nB_{m-n}+\sum_{n>-h_A}B_{m-n}A_n$$

$$:AB(z):\neq:BA(z):,:A(BC)(z):\neq:(AB)C(z):.$$

$$A_1(z):A_2A_3(w):=:A_1(z)A_2A_3(w):+\sqrt{A_1(z):A_2}A_3(w):$$
$$A_1(z):A_2A_3(w):=A_1(z)A_2(w):A_3(w):+A_1(z)A_3(w):A_2(w):.$$

$$A(z):B(w)^n:=nA(z)B(w):B(w)^{n-1}:$$
$$A(z):\mathrm{e}^{B(w)}:=A(z)B(w):\mathrm{e}^{B(w)}:$$
$$:\mathrm{e}^{A(z)}:\mathrm{e}^{B(w)}:=\exp\left(A(z)B(w)\right):\mathrm{e}^{A(z)}\mathrm{e}^{B(w)}:.$$

$$\prod_{i=1}^{n}:\mathrm{e}^{A_i}:=:\exp\left(\sum_{i=1}^{n}A_i\right):\exp\sum_{i<j}\langle A_iA_j\rangle,$$
$$\left\langle\prod_{i=1}^{n}:\mathrm{e}^{A_i}:\right\rangle=\exp\sum_{i<j}\langle A_iA_j\rangle.$$

$$A(z):\mathrm{e}^{B(w)}:=A(z)\sum_{n}\frac{1}{n!}:B(w)^n:=A(z)B(w)\sum_{n}\frac{1}{(n-1)!}:B(w)^{n-1}:.$$

$$\begin{aligned}:\mathrm{e}^{A(z)}:\mathrm{e}^{B(w)}:&=\sum_{m,n}\frac{1}{m!\,n!}:A(z)^m::B(w)^n:\\&=\sum_{m,n,k}\frac{k!}{m!\,n!}\binom{m}{k}\binom{n}{k}\left(A(z)B(w)\right)^k:A(z)^{m-k}::B(w)^{n-k}:\\&=\sum_{m,n,k}\frac{1}{k!\,(m-k)!\,(n-k)!}\left(A(z)B(w)\right)^k:A(z)^{m-k}::B(w)^{n-k}:.\end{aligned}$$

$$\overset{\star}{\star}AB(z)\overset{\star}{\star}=\sum_{m}\frac{\overset{\star}{\star}AB(z)^{\star}_{\star n}}{z^{m+h_A+h_B}},$$
$$\overset{\star}{\star}AB^{\star}_{\star m}=\sum_{n\leq 0}A_nB_{m-n}+\sum_{n>0}B_{m-n}A_n.$$

$$:AB:_m=\overset{\star}{\star}AB^{\star}_{\star m}+\sum_{n=0}^{h_A-1}[B_{m+n},A_{-n}].$$

$$
\begin{aligned}
:AB:_m &= \sum_{n\leq -h_A} A_n B_{m-n} + \sum_{n>-h_A} B_{m-n} A_n \\
&= \sum_{n\geq h_A} A_{-n} B_{m+n} + \sum_{n>0} B_{m-n} A_n + \sum_{n=0}^{h_A-1} B_{m+n} A_{-n} \\
&= \sum_{n\geq 0} A_{-n} B_{m+n} + \sum_{n>0} B_{m-n} A_n + \sum_{n=0}^{h_A-1} [B_{m+n}, A_{-n}] \\
&= {}^\star_\star AB^\star_{\star m} + \sum_{n=0}^{h_A-1} [B_{m+n}, A_{-n}].
\end{aligned}
$$

$$\phi(z) = \left(\frac{L}{2\pi}\right)^h z^{-h} \phi_{\text{cyl}}(w)$$

$$\phi_{\text{cyl}} = \left(\frac{2\pi}{L}\right)^h \sum_{n\in\mathbb{Z}} \phi_n \mathrm{e}^{-\frac{2\pi}{L}w} = \left(\frac{2\pi}{L}\right)^h \sum_{n\in\mathbb{Z}} \frac{\phi_n}{z^n}.$$

$$T_{\text{cyl}}(w) = \left(\frac{2\pi}{L}\right)^2 \left(T(z)z^2 - \frac{c}{24}\right)$$

$$(L_0)_{\text{cyl}} = L_0 - \frac{c}{24}$$

$$H = (L_0)_{\text{cyl}} + (\bar{L}_0)_{\text{cyl}} = L_0 + \bar{L}_0 - \frac{c+\bar{c}}{24}$$

Sistemas CFT – Escalar libre – Acción covariante.

$$S = \frac{\epsilon}{4\pi\ell^2} \int \mathrm{d}^2 x \sqrt{g} g^{\mu\nu} \partial_\mu X \partial_\nu X$$

$$\epsilon := \begin{cases} +1 & \text{spacelike} \\ -1 & \text{timelike} \end{cases}, \sqrt{\epsilon} := \begin{cases} +1 & \text{spacelike} \\ \mathrm{i} & \text{timelike} \end{cases}$$

$$X(\tau,\sigma) \sim X(\tau, \sigma + 2\pi).$$

$$T_{\mu\nu} = -\frac{\epsilon}{\ell^2}\left[\partial_\mu X \partial_\nu X - \frac{1}{2} g_{\mu\nu} (\partial X)^2\right],$$

$$T^\mu_\mu = 0.$$

$$\Delta X = 0,$$

$$0 = \int \mathrm{d}X \frac{\delta}{\delta X(\sigma)} \left(\mathrm{e}^{-S[X]} X(\sigma')\right)$$

$$\langle \partial^2 X(\sigma) X(\sigma') \rangle = -2\pi\epsilon\ell^2 \delta^{(2)}(\sigma - \sigma').$$

$$\langle X(\sigma) X(\sigma') \rangle = -\frac{\epsilon\ell^2}{2} \ln|\sigma - \sigma'|^2.$$

$$\langle X(\sigma)X(\sigma')\rangle = G(r), r = |\sigma - \sigma'|.$$

$$\Delta G(r) = \frac{1}{r}\partial_r(rG'(r)).$$

$$-2\pi\epsilon\ell^2 = 2\pi\int_0^r \mathrm{d}r'r' \times \frac{1}{r'}\partial_{r'}\big(r'G'(r')\big) = 2\pi rG'(r).$$

$$G'(r) = -\epsilon\ell^2 \ln\, r$$

$$\ln\, r = \frac{1}{2}\ln\, r^2 = \frac{1}{2}\ln\, |\sigma - \sigma'|^2.$$

$$X \longrightarrow X + a, a \in \mathbb{R}.$$

$$J^\mu := 2\pi\mathrm{i}\epsilon\frac{\partial\mathcal{L}}{\partial\big(\partial_\mu X\big)} = \frac{\mathrm{i}}{\ell^2}g^{\mu\nu}\partial_\nu X, \nabla_\mu J^\mu = 0.$$

$$p = \frac{1}{2\pi}\int \mathrm{d}\sigma J^0 = \frac{\mathrm{i}}{2\pi\ell^2}\int \mathrm{d}\sigma\partial^0 X.$$

$$\tilde{J}^\mu := -\mathrm{i}\epsilon^{\mu\nu}J_\nu = \frac{1}{\ell^2}\epsilon^{\mu\nu}\partial_\nu X$$

$$\nabla_\mu\tilde{J}^\mu \propto \epsilon^{\mu\nu}\big[\nabla_\mu, \nabla_\nu\big]X = 0$$

$$w = \frac{1}{2\pi}\int \mathrm{d}\sigma\tilde{J}^0 = \frac{1}{2\pi\ell^2}\int_0^{2\pi} \mathrm{d}\sigma\partial_1 X = \frac{1}{2\pi\ell^2}(X(\tau, 2\pi) - X(\tau, 0))$$

$$J_a^\mu = \frac{\mathrm{i}}{2\pi\ell^2}\eta_{ab}\partial^\mu X^b$$

Acción en un plano complejo.

$$S = \frac{\epsilon}{2\pi\ell^2}\int \mathrm{d}z\, \mathrm{d}\bar{z}\partial_z X\partial_{\bar{z}}X$$

$$\partial_z\partial_{\bar{z}}X = 0.$$

$$X(z, \bar{z}) = X_L(z) + X_R(\bar{z})$$

$$X(z) := X_L(z), X(\bar{z}) := X_R(\bar{z})$$

$$J := J_z = \frac{\mathrm{i}}{\ell^2}\partial_z X, \bar{J} := J_{\bar{z}} = \frac{\mathrm{i}}{\ell^2}\partial_{\bar{z}}X$$

$$\bar{\partial}J = 0, \partial\bar{J} = 0$$

$$p = p_L + p_R, p_L = \frac{1}{2\pi\mathrm{i}}\oint \mathrm{d}zJ, p_R = -\frac{1}{2\pi\mathrm{i}}\oint \mathrm{d}\bar{z}\bar{J}$$

$$\tilde{J}_z = \frac{\mathrm{i}}{\ell^2}\partial_z X = J, \tilde{J}_{\bar{z}} = -\frac{\mathrm{i}}{\ell^2}\partial_{\bar{z}}X = -\bar{J}$$

$$w = p_L - p_R$$

$$p_L \; = \frac{p+w}{2}, p_R = \frac{p-w}{2}$$
$$p^2 + w^2 \; = p_L^2 + p_R^2, 2pw = p_L^2 - p_R^2$$

$$T := T_{zz} = -\frac{\epsilon}{\ell^2}\partial_z X \partial_z X, \bar{T} := T_{\bar{z}\bar{z}} = -\frac{\epsilon}{\ell^2}\partial_{\bar{z}} X \partial_{\bar{z}} X, T_{z\bar{z}} = 0$$

$$V_k(z,\bar{z}) :=: \mathrm{e}^{\mathrm{i}\epsilon k X(z,\bar{z})}:.$$

$$V_{k_L,k_R}(z,\bar{z}) :=: \mathrm{e}^{2\mathrm{i}\epsilon(k_L X(z) + k_R X(\bar{z}))}:$$

OPE y Modos de Expansión.

$$X(z)X(w) \sim -\frac{\epsilon\ell^2}{2}\ln(z-w).$$

$$\partial X(z)X(w) \sim -\frac{\epsilon\ell^2}{2}\frac{1}{z-w},$$

$$\partial X(z)\partial X(w) \sim -\frac{\epsilon\ell^2}{2}\frac{1}{(z-w)^2}$$

$$T(z)\partial X(w) \sim \frac{\partial X(w)}{(z-w)^2} + \frac{\partial(\partial X(w))}{z-w}.$$

$$T(z)T(w) \sim \frac{1}{2}\frac{1}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

$$c = 1.$$

$$h = n$$

$$T(z)\partial^n X(w) \sim \cdots + \frac{n\partial^n X(w)}{(z-w)^2} + \frac{\partial(\partial^n X(w))}{z-w}$$

$$T(z)\partial^2 X(w) \sim \frac{2\partial X(w)}{(z-w)^3} + \frac{2\partial^2 X}{(z-w)^2} + \frac{\partial(\partial^2 X(w))}{z-w}.$$

$$J(z)V_k(w,\bar{w}) \sim \frac{\ell^2 k}{2}\frac{V_k(w,\bar{w})}{z-w}.$$

$$T(z)V_k(w,\bar{w}) \sim \frac{h_k V_k(w,\bar{w})}{(z-w)^2} + \frac{\partial V_k(w,\bar{w})}{z-w}$$

$$(h_k, \bar{h}_k) = \left(\frac{\epsilon\ell^2 k^2}{4}, \frac{\epsilon\ell^2 k^2}{4}\right), \Delta_k = \frac{\epsilon\ell^2 k^2}{2}, s_k = 0$$

$$V_k(z,\bar{z})V_{k'}(w,,\bar{w}) \sim \frac{V_{k+k'}(w,\bar{w})}{(z-w)^{-\epsilon k k' \ell^2/2}}$$

$$T(z)\partial X(w) = -\frac{\epsilon}{\ell^2}:\partial X(z)\partial X(z):\partial X(w) \sim -\frac{2\epsilon}{\ell^2}:\partial X(z)\partial X(z):\partial X(w) \sim \frac{\partial X(z)}{(z-w)^2}$$

$$\begin{aligned}
T(z)\partial X(w) =& \frac{1}{\ell^4}:\partial X(z)\partial X(z)::\partial X(w)\partial X(w): \\
&\sim \frac{1}{\ell^4}[:\partial X(z)\partial X(z)::\partial X(w)\partial X(w):+:\partial X(z)\partial X(z)::\partial X(w)\partial X(w): \\
&\quad +:\partial X(z)\partial X(z)::\partial X(w)\partial X(w):+\text{perms}] \\
&\sim 2\times\frac{1}{4}\frac{1}{(z-w)^4} - 4\times\frac{1}{2\ell^2}\frac{1}{(z-w)^2}:\partial X(z)\partial X(w): \\
&\sim \frac{1}{2}\frac{1}{(z-w)^4} - \frac{2}{\ell^2}\frac{1}{(z-w)^2}(:\partial X(w)\partial X(w):+(z-w):\partial^2 X(w)\partial X(w):).
\end{aligned}$$

$$\begin{aligned}
T(z)\partial^n X(w) &\sim \partial_w^{n-1}\frac{\partial X(z)}{(z-w)^2} \\
&\sim n!\frac{\partial X(z)}{(z-w)^{n+1}} \\
\sim \frac{n!}{(z-w)^{n+1}}&\Big(\cdots+\frac{1}{(n-1)!}(z-w)^{n-1}\partial^{n-1}(\partial X(w)) \\
&+\frac{1}{n!}(z-w)^n\partial^n(\partial X(w))\Big)
\end{aligned}$$

$$\partial X(z)V_k(w,\bar{w}) \sim \mathrm{i}\epsilon k\partial X(z)X(w)V_k(w,\bar{w}) \sim \mathrm{i}\epsilon k\left(-\frac{\epsilon\ell^2}{2}\frac{1}{z-w}\right)V_k(w,\bar{w})$$

$$\begin{aligned}
T(z)V_k(w,\bar{w}) \sim& -\frac{\epsilon}{\ell^2}:\partial X(z)\partial X(z)::\mathrm{e}^{\mathrm{i}\epsilon kX(w,\bar{w})}: \\
&\sim \frac{\mathrm{i}\epsilon k}{2}\frac{1}{z-w}\partial X(z):\mathrm{e}^{\mathrm{i}\epsilon kX(w,\bar{w})}:-\frac{\epsilon}{\ell^2}\partial X(z):\partial X(z)\mathrm{e}^{\mathrm{i}\epsilon kX(w,\bar{w})}: \\
&\sim \frac{\mathrm{i}\epsilon k}{2}\frac{1}{z-w}\left(:\partial X(z)\mathrm{e}^{\mathrm{i}\epsilon kX(w,\bar{w})}:+\partial X(z):\mathrm{e}^{\mathrm{i}\epsilon kX}(w,\bar{w}):\right) \\
&+\frac{\mathrm{i}\epsilon k}{2}\frac{\partial X(z)\mathrm{e}^{\mathrm{i}\epsilon kX(w,\bar{w})}:}{z-w} \\
&\sim \frac{\epsilon k^2\ell^2}{4}\frac{V_k(w,\bar{w})}{(z-w)^2}+\mathrm{i}\epsilon k\frac{:\partial X(w)\mathrm{e}^{\mathrm{i}\epsilon kX(w,\bar{w})}:}{z-w}.
\end{aligned}$$

$$\begin{aligned}
V_k(z,\bar{z})V_{k'}(w,\bar{w}) &\sim \exp\left(-kk'X(z,\bar{z})X(w,\bar{w})\right):\mathrm{e}^{\mathrm{i}\epsilon kX(z,\bar{z})}\mathrm{e}^{\mathrm{i}\epsilon k'X(w,\bar{w})}: \\
&\sim (z-w)^{\epsilon kk'\ell^2/2}V_{k+k'}(w,\bar{w})
\end{aligned}$$

$$\partial X = -\mathrm{i}\sqrt{\frac{\ell^2}{2}}\sum_{n\in\mathbb{Z}}\alpha_n z^{-n-1}, \bar{\partial}X = -\mathrm{i}\sqrt{\frac{\ell^2}{2}}\sum_{n\in\mathbb{Z}}\bar{\alpha}_n\bar{z}^{-n-1}$$

$$\alpha_n = \mathrm{i}\oint\frac{\mathrm{d}z}{2\pi\mathrm{i}}z^{n-1}\partial X(z), \bar{\alpha}_n = \mathrm{i}\oint\frac{\mathrm{d}z}{2\pi\mathrm{i}}z^{n-1}\bar{\partial}X(z).$$

$$X(z) = \frac{x_L}{2} - \mathrm{i}\sqrt{\frac{\ell^2}{2}}\alpha_0\ln z + \mathrm{i}\sqrt{\frac{\ell^2}{2}}\sum_{n\neq 0}\frac{\alpha_n}{n}z^{-n}$$

$$X(\bar{z}) = \frac{x_R}{2} - \mathrm{i}\sqrt{\frac{\ell^2}{2}}\bar{\alpha}_0 \ln\, \bar{z} + \mathrm{i}\sqrt{\frac{\ell^2}{2}}\sum_{n\neq 0} \frac{\bar{\alpha}_n}{n}\bar{z}^{-n}$$

$$p_L = \frac{\alpha_0}{\sqrt{2\ell^2}}, p_R = \frac{\bar{\alpha}_0}{\sqrt{2\ell^2}}$$

$$X(z) = \frac{x_L}{2} - \mathrm{i}\ell^2 p_L \ln\, z + \mathrm{i}\sqrt{\frac{\ell^2}{2}}\sum_{n\neq 0} \frac{\alpha_n}{n} z^{-n},$$

$$p = \frac{1}{\sqrt{2\ell^2}}(\alpha_0 + \bar{\alpha}_0), w = \frac{1}{\sqrt{2\ell^2}}(\alpha_0 - \bar{\alpha}_0)$$

$$\alpha_0 = \sqrt{\frac{\ell^2}{2}}(p + w), \bar{\alpha}_0 = \sqrt{\frac{\ell^2}{2}}(p - w)$$

$$x_L = x + q, x_R = x - q$$

$$x = \frac{1}{2}(x_L + x_R), q = \frac{1}{2}(x_L - x_R)$$

$$X(z, \bar{z}) = x - \mathrm{i}\frac{\ell^2}{2}\left(p\ln\, |z|^2 + w\ln\, \frac{z}{\bar{z}}\right) + \mathrm{i}\sqrt{\frac{\ell^2}{2}}\sum_{n\neq 0} \frac{1}{n}(\alpha_n z^{-n} + \bar{\alpha}_n \bar{z}^{-n})$$

$$X(\tau, \sigma) = x - \mathrm{i}\ell^2 p\tau + \ell^2 w\sigma + \cdots$$

$$\begin{aligned} p_L &= \frac{1}{2\pi\mathrm{i}}\oint \,\mathrm{d}z J = \frac{\mathrm{i}}{\ell^2}\frac{1}{2\pi\mathrm{i}}\oint \,\mathrm{d}z\partial X = \frac{\mathrm{i}}{\ell^2}\frac{1}{2\pi\mathrm{i}}\oint \,\mathrm{d}z\partial X \\ &= \frac{1}{\sqrt{2\ell^2}}\frac{1}{2\pi\mathrm{i}}\oint \,\mathrm{d}z\sum_n \alpha_n z^{-n-1} = \frac{1}{\sqrt{2\ell^2}}\alpha_0. \end{aligned}$$

$$X(\tau, \sigma + 2\pi) \sim X(\tau, \sigma)$$

$$X\left(\mathrm{e}^{2\pi\mathrm{i}}z, \mathrm{e}^{-2\pi\mathrm{i}}\bar{z}\right) \sim X(z, \bar{z}).$$

$$X\left(\mathrm{e}^{2\pi\mathrm{i}}z, \mathrm{e}^{-2\pi\mathrm{i}}\bar{z}\right) = X(z, \bar{z}) - \mathrm{i}\sqrt{\frac{\ell^2}{2}}(\alpha_0 - \bar{\alpha}_0)$$

$$\alpha_0 = \bar{\alpha}_0 \implies p_L = p_R = \frac{p}{2}, w = 0.$$

$$N_n = \frac{\epsilon}{n}\alpha_{-n}\alpha_n, \bar{N}_n = \frac{\epsilon}{n}\bar{\alpha}_{-n}\bar{\alpha}_n.$$

$$N = \sum_{n>0} nN_n.$$

$$L_m = \frac{\epsilon}{2}\sum_n :\alpha_n\alpha_{m-n}$$

$$m \neq 0: L_m = \frac{\epsilon}{2}\sum_{n\neq 0,m} :\alpha_n\alpha_{m-n}: +\epsilon\alpha_0\alpha_m,$$

$$L_0 = \frac{\epsilon}{2}\sum_n :\alpha_n\alpha_{-n}: = N + \frac{\epsilon}{2}\alpha_0^2 = N + \epsilon\ell^2 p_L^2$$

$$\hat{L}_0 := N$$

$$\bar{L}_0 = \bar{N} + \epsilon\ell^2 p_R^2, \hat{\bar{L}}_0 := \bar{N}$$

$$L_0^+ = N + \bar{N} + \epsilon\ell^2(p_L^2 + p_R^2) = N + \bar{N} + \frac{\epsilon\ell^2}{2}(p^2 + w^2)$$
$$L_0^- = N - \bar{N} + \epsilon\ell^2(p_L^2 - p_R^2) = N - \bar{N} + \epsilon\ell^2 wp$$

Conmutadores.

$$[\alpha_m, \alpha_n] = \epsilon m\delta_{m+n,0}, [\bar{\alpha}_m, \bar{\alpha}_n] = \epsilon m\delta_{m+n,0}, [\alpha_m, \bar{\alpha}_n] = 0$$

$$[p, w] = [p, p] = [w, w] = 0, [p, \alpha_n] = [p, \bar{\alpha}_n] = [w, \alpha_n] = [w, \bar{\alpha}_n] = 0$$

$$[x_L, p_L] = \mathrm{i}\epsilon, [x_R, p_R] = \mathrm{i}\epsilon$$

$$[x, p] = [q, w] = \mathrm{i}\epsilon, [x, w] = [q, p] = 0$$

$$[L_m, \alpha_n] = -n\alpha_{m+n}$$

$$[L_0, \alpha_{-n}] = n\alpha_{-n}$$

$$[N_m, \alpha_{-n}] = \alpha_{-m}\delta_{m,n}$$

$$|k\rangle := \lim_{z,\bar{z}\to 0} V_k(z, \bar{z})|0\rangle = \mathrm{e}^{\mathrm{i}\epsilon kx}|0\rangle,$$

$$p|k\rangle = k|k\rangle.$$

$$\forall n > 0: \alpha_n|k\rangle = 0,$$

$$N_n|k\rangle = 0.$$

$$L_0^+|k\rangle = 2\epsilon\ell^2 k^2|k\rangle, L_0^-|k\rangle = 0,$$

$$\mathcal{F}(k) = \mathrm{Span}\{|k; \{N_n\}\rangle\},$$
$$|k; \{N_n\}\rangle := \prod_{n\geq 1} \frac{(\alpha_{-n})^{N_n}}{\sqrt{n^{N_n N_n!}}}|k\rangle, N_n \in \mathbb{N}^*$$

$$\mathcal{H} = \int_{\mathbb{R}} \mathrm{d}k\mathcal{F}(k)$$

$$\lim_{z,\bar{z}\to 0} \mathrm{e}^{\mathrm{i}\epsilon k X(z,\bar{z})}|0\rangle = \lim_{z,\bar{z}\to 0} \exp\ \mathrm{i}\epsilon k\left[x - \mathrm{i}\frac{\ell^2}{2}p\ln\ |z|^2 + \mathrm{i}\sqrt{\frac{\ell^2}{2}}\sum_{n\neq 0}\frac{1}{n}(\alpha_n z^{-n} + \bar{\alpha}_n\bar{z}^{-n})\right]|0\rangle$$

$$= \lim_{z,\bar{z}\to 0} \exp\left[\mathrm{i}\epsilon k x - \epsilon k\sqrt{\frac{\ell^2}{2}}\sum_{n\neq 0}\frac{1}{n}(\alpha_n z^{-n} + \bar{\alpha}_n\bar{z}^{-n})\right]|0\rangle.$$

$$\begin{aligned} p|k\rangle &= \frac{1}{\ell^2}\frac{1}{2\pi\mathrm{i}}\oint\ (\,\mathrm{d}z\mathrm{i}\partial X(z) + \mathrm{d}\bar{z}\mathrm{i}\bar{\partial}X(\bar{z}))V_k(0{,}0)|0\rangle \\ &= \frac{1}{\ell^2}\frac{1}{2\pi\mathrm{i}}\oint\ \left(\frac{\mathrm{d}z}{z}\frac{\ell^2 k}{2} + \frac{\mathrm{d}\bar{z}}{\bar{z}}\frac{\ell^2 k}{2}\right)V_k(0{,}0)|0\rangle \\ &= kV_k(0{,}0)|0\rangle \end{aligned}$$

$$x^\dagger = x\ p^\dagger = p, \alpha_n^\dagger = \alpha_{-n}.$$

$$L_n^\dagger = L_{-n}$$

$$\langle k| = |k\rangle^\ddagger = \langle 0|\mathrm{e}^{-\mathrm{i}\epsilon k x}, \langle k|p = \langle k|k.$$

$$\alpha_n^t = -(\pm 1)^n\alpha_{-n}$$

$$p^t = -p, \langle -k| = |k\rangle^t.$$

$$\langle k \mid k'\rangle = 2\pi\delta(k-k')$$

$$\langle k^c| = \frac{1}{2\pi}\langle k|.$$

$$|k\rangle^\ddagger = -|k\rangle^t$$

Sistema Ghost.

Acción covariante y holomórfica o antiholomórfica.

$$S = \frac{1}{4\pi}\int\ \mathrm{d}^2x\sqrt{g}g^{\mu\nu}b_{\mu\mu_1\cdots\mu_{\lambda-1}}\nabla_\nu c^{\mu_1\cdots\mu_{\lambda-1}}$$

$$b_{\mu_1\cdots\mu_n}\longrightarrow \mathrm{e}^{-\mathrm{i}\theta}b_{\mu_1\cdots\mu_n}, c^{\mu_1\cdots\mu_{n-1}}\longrightarrow \mathrm{e}^{\mathrm{i}\theta}c^{\mu_1\cdots\mu_{n-1}}.$$

$$b(z) := b_{z\cdots z}(z), \bar{b}(\bar{z}) := b_{\bar{z}\cdots\bar{z}}(\bar{z}), c(z) := c^{z\cdots z}(\bar{z}), \bar{c}(\bar{z}) := c^{\bar{z}\cdots\bar{z}}(z)$$

$$S = \frac{1}{2\pi}\int\ \mathrm{d}^2 z(b\bar{\partial}c + \bar{b}\partial\bar{c})$$

$$\partial\bar{b} = 0, \bar{\partial}b = 0, \partial\bar{c} = 0, \bar{\partial}c = 0$$

$$h(b) = \lambda, h(c) = 1-\lambda, h(\bar{b}) = \lambda, h(\bar{c}) = 1-\lambda,$$

$$\begin{aligned} T &= -\lambda : b\partial c : + (1-\lambda) : \partial b c : \\ &= -\lambda : \partial(bc) : + : \partial b c : \\ &= (1-\lambda) : \partial(bc) : - : b\partial c : . \end{aligned}$$

$$b(z)c(w) = -\epsilon c(w)b(z), b(z)b(w) = -\epsilon b(w)b(z), c(z)c(w) = -\epsilon c(w)c(z)$$

$$\epsilon = \begin{cases} +1 & \text{anticommuting} \\ -1 & \text{commuting} \end{cases}$$

$$\delta b = -\mathrm{i}b, \delta c = \mathrm{i}c, \delta \bar{b} = -\mathrm{i}\bar{b}, \delta \bar{c} = \mathrm{i}\bar{c}$$

$$j(z) = -: b(z)c(z):, \bar{j}(\bar{z}) = -: \bar{b}(\bar{z})\bar{c}(\bar{z}):$$

$$N_{\mathrm{gh}} = N_{\mathrm{gh},L} + N_{\mathrm{gh},R}, N_{\mathrm{gh},L} = \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} j(z), N_{\mathrm{gh},R} = -\oint \frac{\mathrm{d}\bar{z}}{2\pi\mathrm{i}} \bar{j}(\bar{z})$$

$$N_{\mathrm{gh}}(c) = 1, N_{\mathrm{gh}}(b) = -1, N_{\mathrm{gh}}(\bar{c}) = 1, N_{\mathrm{gh}}(\bar{b}) = -1$$

$$\int \mathrm{d}'b\,\mathrm{d}'c \frac{\delta}{\delta b(z)} \left[b(w) \mathrm{e}^{-S[b,c]} \right] = 0$$

$$\delta^{(2)}(z-w) + \frac{1}{2\pi} \langle b(w) \bar{\partial} c(z) \rangle = 0$$

$$\langle c(z)b(w) \rangle = \frac{1}{z-w}$$

$$\begin{aligned} T(z)b(w) &\sim \lambda \frac{b(w)}{(z-w)^2} + \frac{\partial b(w)}{z-w} \\ T(z)c(w) &\sim (1-\lambda) \frac{c(w)}{(z-w)^2} + \frac{\partial c(w)}{z-w} \end{aligned}$$

$$T(z)T(w) \sim \frac{c_\lambda/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

$$c_\lambda = 2\epsilon(-1 + 6\lambda - 6\lambda^2) = -2\epsilon(1 + 6\lambda(\lambda - 1))$$

$$q_\lambda = \epsilon(1 - 2\lambda)$$

$$c_\lambda = \epsilon\left(1 - 3q_\lambda^2\right)$$

$$\begin{aligned} j(z)b(w) &\sim -\frac{b(w)}{z-w} \\ j(z)c(w) &\sim \frac{c(w)}{z-w} \end{aligned}$$

$$j(z)\mathcal{O}(w) \sim N_{\mathrm{gh}}(\mathcal{O}) \frac{\mathcal{O}(w)}{z-w}$$

$$j(z)j(w) \sim \frac{\epsilon}{(z-w)^2}$$

$$T(z)j(w) \sim \frac{q_\lambda}{(z-w)^3} + \frac{j(w)}{(z-w)^2} + \frac{\partial j(w)}{z-w}.$$

$$j(z) = \frac{\mathrm{d}w}{\mathrm{d}z} j'(w) + \frac{q_\lambda}{2} \frac{\mathrm{d}}{\mathrm{d}z} \ln \frac{\mathrm{d}w}{\mathrm{d}z} = \frac{\mathrm{d}w}{\mathrm{d}z} j'(w) + \frac{q_\lambda}{2} \frac{\partial_z^2 w}{\partial_z w}.$$

$$j(z) = \frac{\mathrm{d}w}{\mathrm{d}z}\left(j^{\mathrm{cyl}}(w) - \frac{q_\lambda}{2}\right),$$

$$N_{\mathrm{gh}} = N_{\mathrm{gh}}^{\mathrm{cyl}} - q_\lambda, N_{\mathrm{gh},L} = N_{\mathrm{gh},L}^{\mathrm{cyl}} - \frac{q_\lambda}{2}, N_{\mathrm{gh},R} = N_{\mathrm{gh},R}^{\mathrm{cyl}} - \frac{q_\lambda}{2}.$$

$$N^c - N^b = -\frac{\epsilon q_\lambda}{2} \chi_g = (1-2\lambda)(g-1)$$

$$\begin{aligned} T(z)b(w) &= (-\lambda{:}\, b(z)\partial c(z){:} + (1-\lambda){:}\, \partial b(z) c(z){:}\,) b(w) \\ &\sim -\lambda{:}\, b(z)\partial c(z){:}\, b(w) + (1-\lambda){:}\, \partial b(z) c(z){:}\, b(w) \\ &\sim -\lambda b(z) \partial_z \frac{1}{z-w} + (1-\lambda)\partial b(z) \frac{1}{z-w} \\ &\sim \lambda(b(w) + (z-w)\partial b(w)) \frac{1}{(z-w)^2} + (1-\chi) \frac{\partial b(w)}{z-w} \end{aligned}$$

$$\begin{aligned} T(z)c(w) &= (-\lambda{:}\, b(z)\partial c(z){:} + (1-\lambda){:}\, \partial b(z) c(z){:}\,) c(w) \\ &\sim \epsilon\lambda{:}\, \partial c(z) b(z){:}\, c(w) - \epsilon(1-\lambda){:}\, c(z)\partial b(z){:}\, c(w) \\ &\sim \lambda \frac{\partial c(z)}{z-w} - (1-\lambda) c(z) \partial_z \frac{1}{z-w} \\ &\sim \lambda \frac{\partial c(w)}{z-w} + (1-\lambda)(c(w) + (z-w)\partial c(w)) \frac{1}{(z-w)^2} \\ &\sim (1-\lambda) \frac{c(w)}{(z-w)^2} + \frac{\partial c(w)}{(z-w)^2} \end{aligned}$$

$$j(z)b(w) = -{:}\, b(z)c(z){:}\, b(w) \sim -{:}\, b(z)c(z){:}\, b(w) \sim -\frac{b(z)}{z-w} \sim -\frac{b(w)}{z-w}.$$

$$j(z)c(w) = -{:}\, b(z)c(z){:}\, c(w) \sim \epsilon{:}\, c(z)b(z){:}\, c(w) \sim \frac{c(z)}{z-w} \sim \frac{c(w)}{z-w}.$$

$$\begin{aligned} j(z)j(w) &= {:}\, b(z)c(z){:}{:}\, b(w)c(w){:} \\ &\sim {:}\, b(z)c(z){:}{:}\, b(w)c(w){:} + {:}\, b(z)c(z){:}{:}\, b(w)c(w){:} + {:}\, b(z)c(z){:}{:}\, b(w)c(w){:} \\ &\sim \frac{\epsilon}{(z-w)^2} + \frac{\epsilon{:}\, c(z)b(w){:}}{z-w} + \frac{{:}\, b(z)c(w){:}}{z-w} \sim \frac{\epsilon}{(z-w)^2}. \end{aligned}$$

$$b(z) = \sum_{n \in \mathbb{Z}+\lambda+\nu} \frac{b_n}{z^{n+\lambda}}, c(z) = \sum_{n \in \mathbb{Z}+\lambda+\nu} \frac{c_n}{z^{n+1-\lambda}},$$

$$b_n = \oint \frac{\mathrm{d}z}{2\pi \mathrm{i}} z^{n+\lambda-1} b(z), c_n = \oint \frac{\mathrm{d}z}{2\pi \mathrm{i}} z^{n-\lambda} c(z).$$

$$b \longrightarrow -b, c \longrightarrow -c.$$

$$N_n^b = {:}\, b_{-n} c_n{:}\,, N_n^c = \epsilon{:}\, c_{-n} b_n{:}$$

$$N^b = \sum_{n>0} nN_n^b, N^c = \sum_{n>0} nN_n^c$$

$$L_m = \sum_n (n-(1-\lambda)m): b_{m-n}c_n: = \sum_n (\lambda m - n): b_n c_{m-n}:.$$

$$L_0 = -\sum_n n: b_n c_{-n}: = \sum_n n: b_{-n}c_n:.$$

$$j_m = -\sum_n : b_{m-n}c_n: = -\sum_n : b_n c_{m-n}:.$$

$$N_{\mathrm{gh},L} = j_0 = -\sum_n : b_{-n}c_n:.$$

$$b_n^{\pm} = b_n \pm \bar{b}_n, c_n^{\pm} = \frac{1}{2}(c_n \pm \bar{c}_n).$$

$$b_n^- b_n^+ = 2b_n\bar{b}_n, c_n^- c_n^+ = \frac{1}{2}c_n\bar{c}_n$$

$$\begin{aligned}
T &= -\lambda: b\partial c: + (1-\lambda): \partial bc: \\
&= \sum_{m,n} \left(\lambda: b_m c_n: \frac{n+1-\lambda}{z^{m+\lambda}z^{m+2-\lambda}} - (1-\lambda): b_m c_n: \frac{m+\lambda}{z^{m+\lambda+1}z^{m+1-\lambda}}\right) \\
&= \sum_{m,n} (\lambda(n+1-\lambda) - (1-\lambda)(m+\lambda)) \frac{: b_m c_n:}{z^{m-n+2}} \\
&= \sum_{m,n} (\lambda(n+1-\lambda) - (1-\lambda)(m-n+\lambda)) \frac{: b_{m-n}c_n:}{z^{m+2}} \\
&= \sum_{m,n} (n-m+\lambda m) \frac{: b_{m-n}c_n:}{z^{m+2}} = \sum_m \frac{L_m}{z^{m+2}}.
\end{aligned}$$

$$j = -: bc: = \sum_{m,n} \frac{: b_m c_n:}{z^{m+\lambda}z^{n+1-\lambda}} = \sum_{m,n} \frac{: b_{m-n}c_n:}{z^{m+1}} = \sum_m \frac{j_m}{z^{m+1}}.$$

$$[b_m, c_n]_\epsilon = \delta_{m+n,0}, [b_m, b_n]_\epsilon = 0, [c_m, c_n]_\epsilon = 0$$

$$[b_m^+, c_n^+]_\epsilon = \delta_{m+n}, [b_m^-, c_n^-]_\epsilon = \delta_{m+n}$$

$$[N_m^b, b_{-n}] = b_{-n}\delta_{m,n}, [N_m^c, c_{-n}] = c_{-n}\delta_{m,n}$$

$$[L_m, b_n] = (m(\lambda-1)-n)b_{m+n}, [L_m, c_n] = -(m\lambda+n)c_{m+n}$$

$$[L_0, b_0] = 0, [L_0, c_0] = 0$$

$$[j_m, j_n] = m\delta_{m+n,0}$$

$$[L_m, j_n] = -nj_{m+n} + \frac{q_\lambda}{2}m(m+1)\delta_{m+n,0}$$

$$[N_{\mathrm{gh}}, b(w)] = -b(w), [N_{\mathrm{gh}}, c(w)] = c(w)$$

$$\begin{aligned}[b_m, c_n]_\epsilon &= \epsilon \oint_{C_0} \frac{\mathrm{d}w}{2\pi \mathrm{i}} w^{-1} \oint_{C_\mathrm{w}} \frac{\mathrm{d}z}{2\pi \mathrm{i}} z^{-1} w^{n+\lambda} z^{m-\lambda+1} b(z) c(w) \\ &\sim \epsilon \oint_{C_0} \frac{\mathrm{d}w}{2\pi \mathrm{i}} w^{-1} \oint_{C_\mathrm{w}} \frac{\mathrm{d}z}{2\pi \mathrm{i}} z^{-1} w^{n+\lambda} z^{m-\lambda+1} \frac{\epsilon}{z-w} \\ &= \oint_{C_0} \frac{\mathrm{d}w}{2\pi \mathrm{i}} w^{m+n-1} = \delta_{m+n,0}\end{aligned}$$

$$[N_{\mathrm{gh}}, b(w)] = \oint \frac{\mathrm{d}z}{2\pi \mathrm{i}} j(z) b(w) \sim -\oint \frac{\mathrm{d}z}{2\pi \mathrm{i}} \frac{b(w)}{z-w} = -b(w)$$

$$\forall n > -\lambda : b_n |0\rangle = 0, \forall n > \lambda - 1 : c_n |0\rangle = 0$$

$$|n_1, \ldots, n_{\lambda-1}\rangle = c_1^{n_1} \cdots c_{\lambda-1}^{n_{\lambda-1}} |0\rangle$$

$$L_0 |n_1, \ldots, n_{\lambda-1}\rangle = -\left(\sum_{j=1}^{\lambda-1} j n_j \right) |n_1, \ldots, n_{\lambda-1}\rangle$$

Estado de energía – momentum. Métrica Grassmann.

$$\{|\downarrow\rangle, |\uparrow\rangle\},$$

$$|\downarrow\rangle := c_1 \cdots c_{\lambda-1} |0\rangle, |\uparrow\rangle := c_0 c_1 \cdots c_{\lambda-1} |0\rangle.$$

$$|\Omega\rangle = \omega_\downarrow |\downarrow\rangle + \omega_\uparrow |\uparrow\rangle, \omega_\downarrow, \omega_\uparrow \in \mathbb{C}.$$

$$b_0 |\uparrow\rangle = |\downarrow\rangle, c_0 |\downarrow\rangle = |\uparrow\rangle, b_0 |\downarrow\rangle = 0, c_0 |\uparrow\rangle = 0.$$

$$\forall n > 0 : b_n |\downarrow\rangle = b_n |\uparrow\rangle = 0, c_n |\downarrow\rangle = b_n |\downarrow\rangle = 0.$$

$$|0\rangle = b_{1-\lambda} \cdots b_{-1} |\downarrow\rangle = b_{1-\lambda} \cdots b_{-1} b_0 |\uparrow\rangle.$$

$$L_0 |\downarrow\rangle = a_\lambda |\downarrow\rangle, L_0 |\uparrow\rangle = a_\lambda |\uparrow\rangle,$$

$$a_\lambda = -\sum_{n=1}^{\lambda-1} n = -\frac{\lambda(\lambda-1)}{2} = \frac{c_\lambda}{24} + \frac{2}{24}.$$

$$\{|\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\uparrow\rangle\},$$

$$|\downarrow\downarrow\rangle := c_1 \bar{c}_1 \cdots c_{\lambda-1} \bar{c}_{\lambda-1} |0\rangle$$

$$|\uparrow\downarrow\rangle := c_0 |\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle := \bar{c}_0 |\downarrow\downarrow\rangle, |\uparrow\uparrow\rangle := c_0 \bar{c}_0 |\downarrow\downarrow\rangle.$$

$$c_0 |\downarrow\downarrow\rangle = |\uparrow\downarrow\rangle, \bar{c}_0 |\downarrow\downarrow\rangle = |\downarrow\uparrow\rangle, c_0 |\downarrow\uparrow\rangle = -\bar{c}_0 |\uparrow\downarrow\rangle = |\uparrow\uparrow\rangle$$

$$b_0 |\uparrow\uparrow\rangle = |\downarrow\uparrow\rangle, \bar{b}_0 |\uparrow\uparrow\rangle = -|\uparrow\downarrow\rangle, b_0 |\uparrow\downarrow\rangle = \bar{b}_0 |\downarrow\uparrow\rangle = |\downarrow\downarrow\rangle,$$

$$
\begin{aligned}
b_0|\downarrow\downarrow\rangle &= \bar{b}_0|\downarrow\downarrow\rangle = 0, &\qquad c_0|\uparrow\downarrow\rangle &= \bar{b}_0|\uparrow\downarrow\rangle = 0,\\
b_0|\downarrow\uparrow\rangle &= \bar{c}_0|\downarrow\uparrow\rangle = 0, &\qquad c_0|\uparrow\uparrow\rangle &= \bar{c}_0|\uparrow\uparrow\rangle = 0.
\end{aligned}
$$

$$
\{|\downarrow\downarrow\rangle, |+\rangle, |-\rangle, |\uparrow\uparrow\rangle\},
$$

$$
\begin{aligned}
c_0^{\pm}|\downarrow\downarrow\rangle &= \frac{1}{2}|\pm\rangle, &\quad c_0^{\mp}|\pm\rangle &= \pm|\uparrow\uparrow\rangle,\\
b_0^{\pm}|\pm\rangle &= \pm 2|\downarrow\downarrow\rangle, &\quad b_0^{\mp}|\uparrow\uparrow\rangle &= \pm|\pm\rangle.
\end{aligned}
$$

$$
\begin{aligned}
b_0^{+}|\downarrow\downarrow\rangle &= b_0^{-}|\downarrow\downarrow\rangle = 0, &\quad c_0^{-}|-\rangle &= b_0^{+}|-\rangle = 0,\\
c_0^{+}|+\rangle &= b_0^{-}|+\rangle = 0 &\quad c_0^{+}|\uparrow\uparrow\rangle &= c_0^{-}|\uparrow\uparrow\rangle = 0,
\end{aligned}
$$

$$
c_0^{-}c_0^{+}|\downarrow\downarrow\rangle = \frac{1}{2}|\uparrow\uparrow\rangle, b_0^{+}b_0^{-}|\uparrow\uparrow\rangle = 2|\downarrow\downarrow\rangle.
$$

$$
\begin{gathered}
2c_0^{+}|\pm\rangle = (c_0 + \bar{c}_0)|\uparrow\downarrow\rangle \pm (c_0 + \bar{c}_0)|\downarrow\uparrow\rangle = \bar{c}_0|\uparrow\downarrow\rangle \pm c_0|\downarrow\uparrow\rangle = (-1 \pm 1)|\uparrow\uparrow\rangle\\
b_0^{+}|\pm\rangle = \left(b_0 + \bar{b}_0\right)|\uparrow\downarrow\rangle \pm \left(b_0 + \bar{b}_0\right)|\downarrow\uparrow\rangle = b_0|\uparrow\downarrow\rangle \pm \bar{b}_0|\downarrow\uparrow\rangle = (1 \pm 1)|\downarrow\downarrow\rangle\\
2c_0^{\pm}|\downarrow\downarrow\rangle = (c_0 \pm \bar{c}_0)|\downarrow\downarrow\rangle = c_0|\downarrow\downarrow\rangle \pm \bar{c}_0|\downarrow\downarrow\rangle = |\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle = |\pm\rangle\\
b_0^{\pm}|\uparrow\uparrow\rangle = \left(b_0 \pm \bar{b}_0\right)|\uparrow\uparrow\rangle = b_0|\uparrow\uparrow\rangle \pm \bar{b}_0|\uparrow\uparrow\rangle = |\downarrow\uparrow\rangle \mp |\uparrow\downarrow\rangle = \mp|\mp\rangle
\end{gathered}
$$

$$
L_0 = \sum_n n{}^{\star}_{\star}b_{-n}c_n{}^{\star}_{\star} + a_\lambda = N^b + N^c + a_\lambda
$$

$$
\hat{L}_0 = N^b + N^c.
$$

$$
L_m = \sum_n (n - (1-\lambda)m){}^{\star}_{\star}b_{m-n}c_n^{\star} + a_\lambda\delta_{m,0}
$$

$$
\begin{aligned}
N_{\mathrm{gh},L} &= j_0 = \sum_n {}^{\star}_{\star}b_{-n}c_{n\star}^{\star} - \left(\frac{q_\lambda}{2} + \frac{1}{2}\right)\\
&= \sum_{n>0} \left(N_n^c - N_n^b\right) + \frac{1}{2}\left(N_0^c - N_0^b\right) - \frac{q_\lambda}{2}
\end{aligned}
$$

$$
j_m = \sum_n {}^{\star}b_{m-n}c_n{}^{\star} - \left(\frac{q_\lambda}{2} + \frac{1}{2}\right)\delta_{m,0}.
$$

$$
\hat{N}_{\mathrm{gh},L} := \sum_{n>0} \left(N_n^c - N_n^b\right)
$$

$$
\begin{aligned}
j_0|\downarrow\rangle &= (\lambda - 1)|\downarrow\rangle = \left(-\frac{q_\lambda}{2} - \frac{1}{2}\right)|\downarrow\rangle,\\
j_0|\uparrow\rangle &= \lambda|\uparrow\rangle = \left(-\frac{q_\lambda}{2} + \frac{1}{2}\right)|\uparrow\rangle.
\end{aligned}
$$

$$
j_0|0\rangle = 0
$$

$$
j_0^{\mathrm{cyl}}|\downarrow\rangle = -\frac{1}{2}|\downarrow\rangle, j_0^{\mathrm{cyl}}|\uparrow\rangle = \frac{1}{2}|\uparrow\rangle.
$$

$$\begin{aligned}L_0 &= -\sum_n n{:}b_n c_{-n}{:} = -\sum_{n\leq -\lambda} nb_n c_{-n} + \epsilon \sum_{n>-\lambda} nc_{-n}b_n \\ &= \sum_{n\geq\lambda} nb_{-n}c_n + \epsilon \sum_{n>-\lambda} nc_{-n}b_n \\ &= \sum_{n\geq\lambda} nb_{-n}c_n + \epsilon \sum_{n>0} nc_{-n}b_n + \epsilon \sum_{n=-\lambda+1}^{0} nc_{-n}b_n \\ &= \sum_{n\geq\lambda} nb_{-n}c_n + \epsilon \sum_{n>0} nc_{-n}b_n + \epsilon \sum_{n=0}^{\lambda-1} nb_{-n}c_n + a_\lambda \\ &= \sum_{n>0} nb_{-n}c_n + \epsilon \sum_{n>0} nc_{-n}b_n + a_\lambda \\ &= \sum_n n \,{}^\star_\star b_{-n}c_n^\star + a_\lambda\end{aligned}$$

$$\sum_{n=-\lambda+1}^{0} c_{-n}b_n = -\sum_{n=0}^{\lambda-1} nc_n b_{-n} = -\sum_{n=0}^{\lambda-1} n(-\epsilon b_{-n}c_n + 1) = \epsilon \sum_{n=0}^{\lambda-1} nb_{-n}c_n + a_\lambda.$$

$$\begin{aligned}j_0 &= -\sum_n {:}b_{-n}c_n{:} = -\sum_{n\geq\lambda} b_{-n}c_n + \epsilon \sum_{n>-\lambda} c_{-n}b_n \\ &= -\sum_{n\geq\lambda} b_{-n}c_n + \epsilon \sum_{n>0} c_{-n}b_n + \epsilon \sum_{n=1}^{\lambda-1} c_n b_{-n} + \epsilon c_0 b_0 \\ &= -\sum_{n\geq\lambda} b_{-n}c_n + \epsilon \sum_{n>0} c_{-n}b_n - \sum_{n=1}^{\lambda-1} b_{-n}c_n + \epsilon(\lambda-1) + \epsilon c_0 b_0 \\ &= -\sum_{n>0} b_{-n}c_n + \epsilon \sum_{n>0} c_{-n}b_n + \epsilon(\lambda-1) + \epsilon c_0 b_0.\end{aligned}$$

$$\epsilon(\lambda-1) = -\frac{q_\lambda}{2} - \frac{\epsilon}{2}.$$

$$\begin{aligned}\epsilon c_0 b_0 + \epsilon(\lambda-1) &= \frac{\epsilon}{2} c_0 b_0 + \frac{1}{2}(-b_0 c_0 + \epsilon) + \epsilon(\lambda-1) \\ &= \frac{1}{2}(\epsilon c_0 b_0 - b_0 c_0) + \epsilon\left(\lambda - \frac{1}{2}\right).\end{aligned}$$

Estructura del espacio de Hilbert. Métrica Grassmann odd.

$$\mathcal{H}_{\mathrm{gh}} = \mathcal{H}_{\mathrm{gh},0} \oplus c_0\mathcal{H}_{\mathrm{gh},0}, \mathcal{H}_{\mathrm{gh},0} := \mathcal{H}_{\mathrm{gh}} \cap \ker b_0$$

$$\mathcal{H}_{\mathrm{gh},0} = \mathrm{Span}\{|\downarrow; \{N_n^b\}; \{N_n^c\}\rangle\},$$
$$|\downarrow; \{N_n^b\}; \{N_n^c\}\rangle = \prod_{n\geq 1} (b_{-n})^{N_n^b}(c_{-n})^{N_n^c}|\downarrow\rangle, N_n^b, N_n^c \in \mathbb{N}^*$$

$$\psi = \psi_\downarrow + \psi_\uparrow, \psi_\downarrow \in \mathcal{H}_{\mathrm{gh},0}, \psi_\uparrow \in c_0\mathcal{H}_{\mathrm{gh},0}$$

$$\mathcal{H}_{\mathrm{gh}} = \mathcal{H}_{\mathrm{gh},0} \oplus c_0\mathcal{H}_{\mathrm{gh},0} \oplus \bar{c}_0\mathcal{H}_{\mathrm{gh},0} \oplus c_0\bar{c}_0\mathcal{H}_{\mathrm{gh},0}$$

$$\mathcal{H}_{\text{gh},0} := \mathcal{H}_{\text{gh}} \cap \ker b_0 \cap \ker \bar{b}_0.$$

$$\left|\downarrow\downarrow; \{N_n^b\}; \{N_n^c\}; \{\bar{N}_n^b\}; \{\bar{N}_n^c\}\right\rangle = \prod_{n\geq 1} (b_{-n})^{N_n^b} \left(\bar{b}_{-n}\right)^{\bar{N}_n^b} (c_{-n})^{N_n^c} (\bar{c}_{-n})^{\bar{N}_n^c} |\downarrow\downarrow\rangle,$$
$$N_n^b, \bar{N}_n^b, N_n^c, \bar{N}_n^c \in \mathbb{N}^*.$$

$$\psi = \psi_{\downarrow\downarrow} + \psi_{\uparrow\downarrow} + \psi_{\downarrow\uparrow} + \psi_{\uparrow\uparrow},$$

$$\mathcal{H}_{\text{gh}} = \mathcal{H}_{\text{gh},0} \oplus c_0^+ \mathcal{H}_{\text{gh},0} \oplus c_0^- \mathcal{H}_{\text{gh},0} \oplus c_0^- c_0^+ \mathcal{H}_{\text{gh},0}$$

$$\mathcal{H}_{\text{gh},0} := \mathcal{H}_{\text{gh}} \cap \ker b_0^- \cap \ker b_0^+$$

$$\mathcal{H}_{\text{gh},\pm} := \mathcal{H}_{\text{gh}} \cap \ker b_0^\pm = \mathcal{H}_{\text{gh},0} \oplus c_0^\mp \mathcal{H}_{\text{gh},0},$$

$$\mathcal{H}_{\text{gh}} = \mathcal{H}_{\text{gh},\pm} \oplus c_0^\pm \mathcal{H}_{\text{gh},\pm}.$$

$$b_n^\dagger = \epsilon b_{-n}, c_n^\dagger = c_{-n}.$$

$$b_n^t = (-1)^\lambda b_{-n}, c_n^t = (-1)^{1-\lambda} c_{-n},$$

$$|\downarrow\rangle^\ddagger = \langle 0| c_{1-\lambda} \cdots c_{-1}, |\uparrow\rangle^\ddagger = \langle 0| c_{1-\lambda} \cdots c_{-1} c_0.$$

$$\langle\downarrow| := |\downarrow\rangle^t = (-1)^{(1-\lambda)^2} \langle 0| c_{-1} \cdots c_{1-\lambda}$$
$$\langle\uparrow| := |\uparrow\rangle^t = (-1)^{\lambda(1-\lambda)} \langle 0| c_0 c_{-1} \cdots c_{1-\lambda}.$$

$$\langle\downarrow| = (-1)^{a_\lambda + (1-\lambda)(2-\lambda)} |\downarrow\rangle^\ddagger, \langle\uparrow| = (-1)^{a_\lambda} |\uparrow\rangle^\ddagger,$$

$$\langle\downarrow| = (-1)^{(1-\lambda)^2 + \frac{1}{2}(2-\lambda)(1-\lambda)} |\downarrow\rangle^\ddagger = (-1)^{-a_\lambda + (1-\lambda)(2-\lambda)} |\downarrow\rangle^\ddagger$$

$$\sum_{i=1}^{\lambda-2} i = \frac{1}{2}(2-\lambda)(1-\lambda) = -a_\lambda + 1 - \lambda.$$

$$\langle\uparrow| = (-1)^{\lambda(1-\lambda) - \frac{1}{2}\lambda(1-\lambda)} |\uparrow\rangle^\ddagger = (-1)^{\frac{1}{2}\lambda(1-\lambda)} |\uparrow\rangle^\ddagger.$$

$$\langle\uparrow| b_0 = \langle\downarrow|, \langle\downarrow| c_0 = \langle\uparrow|, \langle\downarrow| b_0 = 0, \langle\uparrow| c_0 = 0.$$

$$|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

$$b_0 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, c_0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\langle\downarrow|\downarrow\rangle = \langle\uparrow|\uparrow\rangle = 0$$

$$\langle\uparrow|\downarrow\rangle = \langle\downarrow| c_0 |\downarrow\rangle = \langle 0| c_{1-\lambda} \cdots c_{-1} c_0 c_1 \cdots c_{\lambda-1} |0\rangle = 1.$$

$$\langle 0^c| = \langle 0| c_{1-\lambda} \cdots c_{-1} c_0 c_1 \cdots c_{\lambda-1}, \langle\downarrow^c| = \langle\uparrow|.$$

Cuantización BRST. Reparametrización e invariancia.

$$c = c_m + c_{\mathrm{gh}} = c_m - 26, T(z) = T^m(z) + T^{\mathrm{gh}}(z), \mathcal{H} = \mathcal{H}_m \otimes \mathcal{H}_{\mathrm{gh}}.$$

$$Q_B|\psi\rangle = 0$$

$$|\psi\rangle \sim |\psi\rangle + Q_B|\Lambda\rangle.$$

$$\begin{aligned} j_B(z) &=: c(z)\left(T^m(z) + \frac{1}{2}T^{\mathrm{gh}}(z)\right): + \kappa\partial^2 c(z) \\ &= c(z)T^m(z) + : b(z)c(z)\partial c(z): + \kappa\partial^2 c(z) \end{aligned}$$

$$Q_B = Q_{B,L} + Q_{B,R}, Q_{B,L} = \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} j_B(z), Q_{B,R} = \oint \frac{\mathrm{d}\bar{z}}{2\pi\mathrm{i}} \bar{j}_B(\bar{z}).$$

$$T(z)j_B(w) \sim \left(\frac{c_m}{2} - 4 - 6\kappa\right)\frac{c(w)}{(z-w)^4} + (3-2\kappa)\frac{\partial c(w)}{(z-w)^3} + \frac{j_B(w)}{(z-w)^2} + \frac{\partial j_B(w)}{z-w}.$$

$$c_m = 26, \kappa = \frac{3}{2}.$$

$$T(z)j_B(w) \sim \frac{j_B(w)}{(z-w)^2} + \frac{\partial j_B(w)}{z-w}.$$

$$\begin{aligned} j_B(z)b(w) &\sim \frac{2\kappa}{(z-w)^3} + \frac{j(w)}{(z-w)^2} + \frac{T(w)}{z-w} \\ j_B(z)c(w) &\sim \frac{:c(w)\partial c(w)}{z-w} \end{aligned}$$

$$j_B(z)\phi(w) \sim h\frac{c(w)\phi(w)}{(z-w)^2} + \frac{:h\partial c(w)\phi(w) + c(w)\partial\phi(w):}{z-w}$$

$$j_B(z)j(w) \sim \frac{2\kappa+1}{(z-w)^3} - \frac{2\partial c(w)}{(z-w)^2} - \frac{j_B(w)}{z-w}$$

$$j_B(z)j_B(w) \sim -\frac{c_m-18}{2}\frac{:c(w)\partial c(w):}{(z-w)^3} - \frac{c_m-18}{4}\frac{:c(w)\partial^2 c(w):}{(z-w)^2} - \frac{c_m-26}{12}\frac{:c(w)\partial^3 c(w):}{z-w}.$$

$$\begin{aligned} Q_B &= \sum_m :c_m\left(L_{-m}^m + \frac{1}{2}L_{-m}^{\mathrm{gh}}\right): \\ &= \sum_m c_{-m}L_m^m + \frac{1}{2}\sum_{m,n}(n-m): c_{-m}c_{-n}b_{m+n}: \end{aligned}$$

$$\begin{aligned} Q_B &= \sum_m {}^\star c_m\left(L_{-m}^m + \frac{1}{2}L_{-m}^{\mathrm{gh}}\right){}^\star - \frac{c_0}{2} \\ &= \sum_n c_m L_{-m}^m + \frac{1}{2}\sum_{m,n}(n-m)^\star c_{-m}c_{-n}b_{m+n}{}^\star - c_0, \end{aligned}$$

$$Q_B = c_0 L_0 - b_0 M + \hat{Q}_B$$

$$\hat{Q}_B = \sum_{m\neq 0} c_{-m} L_m^m - \frac{1}{2} \sum_{\substack{m,n\neq 0 \\ m+n\neq 0}} (m-n)^{\star}_{\star} c_{-m} c_{-n} b^{\star}_{m+n\star},$$

$$M = \sum_{m\neq 0} m c_{-m} c_m$$

$$[L_0, M] = [\hat{Q}_B, M] = [\hat{Q}_B, L_0] = 0, \hat{Q}_B^2 = L_0 M$$

$$\{Q_B, b(z)\} = T(z)$$
$$\{Q_B, c(z)\} = c(z)\partial c(z)$$
$$[Q_B, \phi(z)] = h\partial c(z)\phi(z) + c(z)\partial\phi(z)$$

$$[Q_B, \phi(z)] = \partial(c(z)\phi(z))$$

$$\{Q_B, c(z)\phi(z)\} = (1-h)c(z)\partial c(z)\phi(z)$$

$$[Q_B, j(z)] = -j_B(z)$$

$$[N_{\mathrm{gh}}, Q_B] = Q_B$$

$$\{Q_B, Q_B\} = 0$$

$$[Q_B, T(z)] = 0$$

$$c_m = 26.$$

$$L_n = \{Q_B, b_n\}$$

$$[Q_B, L_n] = 0$$

Cohomología BRST para sectores holomórficos y antiholomórficos.

$$k_{\|}^2 = \epsilon_0 (k^0)^2 + (k^1)^2.$$

$$\mathcal{H} := \mathcal{H}_{\|} \otimes \mathcal{H}_{\perp}, \mathcal{H}_{\|} := \int \mathrm{d}k^0 \mathcal{F}_0(k^0) \otimes \int \mathrm{d}k^1 \mathcal{F}_1(k^1) \otimes \mathcal{H}_{\mathrm{gh}}$$

$$|\psi\rangle = |\psi_{\|}\rangle \otimes |\psi_{\perp}\rangle,$$

$$|\psi_{\|}\rangle = c_0^{N_0^c} \prod_{m>0} (\alpha_{-m}^0)^{N_m^0} (\alpha_{-m}^1)^{N_m^1} (b_{-m})^{N_m^b} (c_{-m})^{N_m^c} |k^0, k^1, \downarrow\rangle$$
$$|k^0, k^1, \downarrow\rangle := |k^0\rangle \otimes |k^1\rangle \otimes |\downarrow\rangle, N_m^0, N_m^1 \in \mathbb{N}, N_m^b, N_m^c = 0{,}1.$$

$$\mathcal{H}_0 = \mathcal{H} \cap \ker b_0$$

$$\mathcal{H} = \mathcal{H}_0 \oplus c_0 \mathcal{H}_0.$$

$$L_0 = L_0^m + L_0^{\mathrm{gh}} = (L_0^m - 1) + N^b + N^c,$$

$$L_0 = \left(L_0^{\perp} - m_{\|,L}^2 \ell^2 - 1\right) + \hat{L}_0^{\|}$$

$$m_{\|,L}^2 = -p_{\|,L}^2, \hat{L}_0^{\|} = N^0 + N^1 + N^b + N^c \in \mathbb{N}.$$

$$\text{on-shell: } L_0|\psi\rangle = 0$$

$$\mathcal{H}_{\text{abs}}(Q_B) := \{|\psi\rangle \in \mathcal{H} | Q_B|\psi\rangle = 0, \nexists|\chi\rangle \in \mathcal{H} | |\psi\rangle = Q_B|\chi\rangle\}.$$

$$\{Q, \Delta\} = 1$$

$$|\psi\rangle = \{Q_B, \Delta\}|\psi\rangle = Q_B(\Delta|\psi\rangle)$$

$$\Delta := \frac{b_0}{L_0}$$

$$L_0 = \{Q_B, b_0\}$$

$$Q_B|\psi\rangle = 0, L_0|\psi\rangle \neq 0$$

$$|\psi\rangle = Q_B\left(\frac{b_0}{L_0}|\psi\rangle\right)$$

$$\mathcal{H}_{\text{abs}}(Q_B) \subset \ker L_0$$

$$|\psi\rangle = \frac{L_0}{L_0}|\psi\rangle = \frac{1}{L_0}\{Q_B, b_0\}|\psi\rangle = \frac{1}{L_0}Q_B(b_0|\psi\rangle)$$

$$|\psi\rangle \in \ker L_0: P_0|\psi\rangle = |\psi\rangle, |\psi\rangle \in (\ker L_0)^{\perp}: P_0|\psi\rangle = 0.$$

$$\{Q_B, \Delta(1 - P_0)\} = (1 - P_0).$$

$$\mathcal{H}_0 := \mathcal{H} \cap \ker b_0 = \mathcal{H}_m \otimes \mathcal{H}_{\text{gh},0}$$

$$|\psi\rangle \in \mathcal{H}_0 \implies b_0|\psi\rangle = 0.$$

$$b_0|\psi\rangle = Q_B|\psi\rangle = 0, L_0|\psi\rangle \neq 0 \implies |\psi\rangle = 0.$$

$$\mathcal{H}_{\text{rel}}(Q_B) := \mathcal{H}_0(Q_B) = \{|\psi\rangle \in \mathcal{H}_0 | Q_B|\psi\rangle = 0, \nexists|\chi\rangle \in \mathcal{H} | |\psi\rangle = Q_B|\chi\rangle\}.$$

$$Q_B|\psi\rangle = 0, L_0|\psi\rangle = 0, b_0|\psi\rangle = 0$$

$$Q_B = c_0 L_0 - b_0 M + \hat{Q}_B, \hat{Q}_B^2 = L_0 M$$

$$|\psi\rangle \in \mathcal{H}_0 \cap \ker L_0 \implies Q_B|\psi\rangle = \hat{Q}_B|\psi\rangle, \hat{Q}_B^2|\psi\rangle = 0$$

$$\mathcal{H}_0(Q_B) = \mathcal{H}_0(\hat{Q}_B).$$

$$X_L^{\pm} = \frac{1}{\sqrt{2}}\left(X_L^0 \pm \frac{\mathrm{i}}{\sqrt{\epsilon_0}} X_L^1\right)$$

$$\alpha_n^{\pm} = \frac{1}{\sqrt{2}}\left(\alpha_n^0 \pm \frac{\mathrm{i}}{\sqrt{\epsilon_0}}\alpha_n^1\right), n \neq 0,$$

$$x_L^{\pm} = \frac{1}{\sqrt{2}}\left(x_L^0 \pm \frac{\mathrm{i}}{\sqrt{\epsilon_0}}x_L^1\right), p_L^{\pm} = \frac{1}{\sqrt{2}}\left(p_L^0 \pm \frac{\mathrm{i}}{\sqrt{\epsilon_0}}p_L^1\right),$$

$$[\alpha_m^+, \alpha_n^-] = \epsilon_0 m \delta_{m+n,0}, [x_L^{\pm}, p_L^{\mp}] = \mathrm{i}\epsilon_0.$$

$$2p_L^+ p_L^- = (p_L^0)^2 + \epsilon_0 (p_L^1)^2 = \epsilon_0 p_{\parallel,L}^2,$$
$$x^+ p^- + x^- p^+ = x^0 p^0 + \epsilon_0 x^1 p^1,$$
$$\sum_n \alpha_n^+ \alpha_{m-n}^- = \frac{1}{2}\sum_n (\alpha_n^0 \alpha_{m-n}^0 + \epsilon_0 \alpha_n^1 \alpha_{m-n}^1).$$

$$N_n^{\pm} = \frac{\epsilon_0}{n}\alpha_{-n}^{\pm}\alpha_n^{\mp}, N^{\pm} = \sum_{n>0} n N_n^{\pm}.$$

$$N^+ + N^- = N^0 + N^1.$$

$$L_0 = \left(L_0^{\perp} - m_{\parallel,L}^2 \ell^2 - 1\right) + \hat{L}_0^{\parallel}$$

$$m_{\parallel,L}^2 = -2\epsilon_0 p_L^+ p_L^-, \hat{L}_0^{\parallel} = N^+ + N^- + N^b + N^c$$

$$L_m^0 + L_m^1 = \epsilon_0 \sum_n :\alpha_n^+ \alpha_{m-n}^-: = \epsilon_0 \sum_{n\neq 0,m} :\alpha_n^+ \alpha_{m-n}^-: + \epsilon_0(\alpha_0^- \alpha_m^+ + \alpha_m^+ \alpha_m^-).$$

$$\begin{aligned}[\alpha_m^+, \alpha_n^{\pm}] &= \frac{1}{2}\left[\left(\alpha_m^0 + \frac{\mathrm{i}}{\sqrt{\epsilon_0}}\alpha_m^1\right), \left(\alpha_n^0 \pm \frac{\mathrm{i}}{\sqrt{\epsilon_0}}\alpha_n^1\right)\right] \\ &= \frac{1}{2}\left([\alpha_m^0, \alpha_n^0] \mp \frac{1}{\epsilon_0}[\alpha_m^1, \alpha_n^1]\right) = \frac{\epsilon_0}{2} m \delta_{m+n,0}(1 \mp 1),\end{aligned}$$

$$\begin{aligned}[x_L^-, p_L^{\pm}] &= \frac{1}{2}\left[\left(x_L^0 - \frac{\mathrm{i}}{\sqrt{\epsilon_0}}x_L^1\right), \left(p_L^0 \pm \frac{\mathrm{i}}{\sqrt{\epsilon_0}}p_L^1\right)\right] \\ &= \frac{1}{2}([x_L^0, p_L^0] \pm \epsilon_0 [x_L^1, p_L^1]) = \frac{\epsilon_0}{2}(1 \pm 1)\end{aligned}$$

$$\begin{aligned}\sum_n \alpha_n^+ \alpha_{m-n}^- &= \frac{1}{2}\sum_n \left(\alpha_n^0 + \frac{\mathrm{i}}{\sqrt{\epsilon_0}}\alpha_n^1\right)\left(\alpha_{m-n}^0 - \frac{\mathrm{i}}{\sqrt{\epsilon_0}}\alpha_{m-n}^1\right) \\ &= \frac{1}{2}\sum_n \left(\alpha_n^0 \alpha_{m-n}^0 + \epsilon_0 \alpha_n^1 \alpha_{m-n}^1 + \frac{\mathrm{i}}{\sqrt{\epsilon_0}}(\alpha_{m-n}^0 \alpha_n^1 - \alpha_n^0 \alpha_{m-n}^1)\right)\end{aligned}$$

$$N^0 + N^1 = \sum_n n(N_n^0 + N_n^1) = \sum_n n(N_n^+ + N_n^-) = N^+ + N^-.$$

$$\hat{Q}_B = \sum_{m\neq 0} c_{-m}\left(L_m^{\perp} + \epsilon_0 \sum_n \alpha_n^+ \alpha_{m-n}^-\right) + \frac{1}{2}\sum_{m,n} (n-m):c_{-m}c_{-n}b_{m+n}:$$

$$\deg := N^+ - N^- + \hat{N}^c - \hat{N}^b$$

$$\forall m \neq 0: \deg(\alpha_m^+) = \deg(c_m) = 1, \deg(\alpha_m^-) = \deg(b_m) = -1$$

$$\hat{Q}_B = Q_0 + Q_1 + Q_2, \deg(Q_j) = j$$

$$Q_1 = \sum_{m\neq 0} c_{-m} L_m^{\perp} + \sum_{\substack{m,n\neq 0 \\ m+n\neq 0}} {}^{\star}c_{-m}\left(\epsilon_0 \alpha_n^+ \alpha_{m-n}^- + \frac{1}{2}(m-n)c_{-m}b_{m+n}\right){}^{\star}_{\star}$$

$$Q_0 = \sum_{n\neq 0} \alpha_0^+ c_{-n}\alpha_n^-, Q_2 = \sum_{n\neq 0} \alpha_0^- c_{-n}\alpha_n^+$$

$$Q_0^2 = Q_2^2 = 0, \{Q_0, Q_1\} = \{Q_1, Q_2\} = 0, Q_1^2 + \{Q_0, Q_2\} = 0$$

$$\mathcal{H}_0(\hat{Q}_B) \simeq \mathcal{H}_0(Q_0)$$

$$\hat{\Delta} := \frac{B}{\hat{L}_0^{\parallel}}, B := \epsilon_0 \sum_{n\neq 0} \frac{1}{\alpha_0^+}\alpha_{-n}^+ b_n.$$

$$\hat{L}_0^{\parallel} = \{Q_0, B\} \implies \{Q_0, \hat{\Delta}\} = 1.$$

$$\hat{L}_0^{\parallel}|\psi\rangle = 0, \implies N^{\pm}|\psi\rangle = N^c|\psi\rangle = N^b|\psi\rangle = 0,$$

$$0 = Q_0\hat{L}_0^{\parallel}|\psi\rangle = \hat{L}_0^{\parallel}Q_0|\psi\rangle.$$

$$Q_0|\psi\rangle = 0$$

$$L_0 = L_0^{\perp} - m_{\parallel,L}^2\ell^2 - 1 = 0$$

$$\{Q_0, Q_1\}|\psi_0\rangle = 0 \implies Q_0(Q_1|\psi_0\rangle) = 0.$$

$$Q_1|\psi_0\rangle =: -Q_0|\psi_1\rangle \implies |\psi_1\rangle = -\frac{B}{\hat{L}_0^{\parallel}}Q_1|\psi_0\rangle.$$

$$Q_1|\psi_0\rangle = \left\{Q_0, \frac{B}{\hat{L}_0^{\parallel}}\right\}Q_1|\psi_0\rangle = Q_0\left(\frac{B}{\hat{L}_0^{\parallel}}Q_1|\psi_0\rangle\right).$$

$$\{Q_0, Q_1\}|\psi_1\rangle = Q_0(Q_1|\psi_1\rangle + Q_2|\psi_0\rangle).$$

$$Q_1|\psi_1\rangle + Q_2|\psi_0\rangle = Q_0|\psi_2\rangle, |\psi_2\rangle = -\frac{B}{\hat{L}_0^{\parallel}}(Q_1|\psi_1\rangle + Q_2|\psi_0\rangle)$$

$$\{Q_0, Q_1\}|\psi_1\rangle = Q_0Q_1|\psi_1\rangle - Q_1^2|\psi_0\rangle = Q_0Q_1|\psi_1\rangle + \{Q_0, Q_2\}|\psi_0\rangle.$$

$$|\psi_{k+1}\rangle = -\frac{B}{\hat{L}_0^{\parallel}}(Q_1|\psi_k\rangle + Q_2|\psi_{k-1}\rangle)$$

$$|\psi\rangle = \sum_{k\in\mathbb{N}} |\psi_k\rangle.$$

$$\hat{Q}_B|\psi\rangle = 0.$$

$$N_{\mathrm{gh}}(\psi) = N_{\mathrm{gh}}(\psi_0) = 1$$

$$Q_1|\psi_0\rangle = Q_2|\psi_0\rangle = 0$$

$$\begin{aligned}\hat{Q}_B|\psi\rangle &= \sum_{k\in\mathbb{N}} \hat{Q}_B|\psi_k\rangle \\ &= Q_0|\psi_0\rangle + \underbrace{Q_1|\psi_0\rangle + Q_0|\psi_1\rangle}_{=0} + \underbrace{Q_2|\psi_0\rangle + Q_1|\psi_1\rangle + Q_0|\psi_2\rangle}_{=0} + \cdots \\ &= 0\end{aligned}$$

$$\mathcal{H}_{\mathrm{abs}}(Q_B) = \mathcal{H}_{\mathrm{rel}}(Q_B) \oplus c_0\mathcal{H}_{\mathrm{rel}}(Q_B)$$

$$|\psi\rangle = |k^0, k^1, \downarrow\rangle \otimes |\psi_\perp\rangle, \qquad |\psi_\perp\rangle \in \mathcal{H}_\perp$$
$$\left(L_0^\perp - m_{\|,L}^2\ell^2 - 1\right)|\psi\rangle = 0, \quad p_{L,\|}^2 = -m_{\|,L}^2\ell^2$$

$$(L_0^m - 1)|\psi\rangle = 0, \forall n > 0: L_n^m|\psi\rangle = 0.$$

$$Q_B = c_0L_0 - b_0M + \hat{Q}_B + \bar{c}_0\bar{L}_0 - \bar{b}_0\bar{M} + \hat{\bar{Q}}_B$$

$$Q_B = c_0^+L_0^+ - b_0^+M^+ + c_0^-L_0^- - b_0^-M^- + \hat{Q}_B^+,$$

$$L_0^+ = \left(L_0^{\perp+} - \frac{m_\|^2\ell^2}{2} - 2\right) + \hat{L}_0^{\|+}, L_0^- = L_0^{\perp-} + \hat{L}_0^{\|-}$$

$$M^\pm := \frac{1}{2}(M \pm \bar{M}).$$

$$L_0^+|\psi\rangle = L_0^-|\psi\rangle = 0.$$

$$\hat{L}_0^{\|\pm} = N^0 \pm \bar{N}^0 + N^1 \pm \bar{N}^1 + N^b \pm \bar{N}^b + N^c \pm \bar{N}^c = 0.$$

$$(L_0^m + \bar{L}_0^m - 2)|\psi\rangle = 0, (L_0^m - \bar{L}_0^m)|\psi\rangle = 0$$

$$\forall n > 0: L_n^m|\psi\rangle = \bar{L}_n^m|\psi\rangle = 0$$

$$|\downarrow\downarrow\rangle = c(0)\bar{c}(0)|0\rangle = c_1\bar{c}_1|0\rangle.$$

$$\hat{L}_0|\psi_{\ell,\bar{\ell}}\rangle = \ell|\psi_{\ell,\bar{\ell}}\rangle, \hat{\bar{L}}_0|\psi_{\ell,\bar{\ell}}\rangle = \bar{\ell}|\psi_{\ell,\bar{\ell}}\rangle.$$

$$k^2 = -m^2, m^2 := \frac{2}{\ell^2}(N + \bar{N} - 2),$$

$$m^2\ell^2 = -4 < 0.$$

$$\mathcal{V}(k, z, \bar{z}) = c(z)\bar{c}(\bar{z})\mathrm{e}^{\mathrm{i}k\cdot X(z,\bar{z})}.$$

Campo gravitónico de gauge y campos cuánticos gravitacionales e interacciones locales y no locales de las partículas supermasivas en simetrías axiales.

Expansión de campo.

$$\Psi[X(\sigma), c(\sigma)] := \langle X(\sigma), c(\sigma) \mid \Psi\rangle$$

$$|\Psi\rangle = \sum_{\alpha} \int \frac{\mathrm{d}^D k}{(2\pi)^D} \psi_\alpha(k) |\phi_\alpha(k)\rangle$$

$$\psi_\alpha(x) = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \mathrm{e}^{\mathrm{i}k\cdot x} \psi_\alpha(k)$$

$$|\Psi\rangle = \sum_r \psi_r |\phi_r\rangle$$

Campo escalar.

$$\phi(x) = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \phi(k) \mathrm{e}^{\mathrm{i}k\cdot x}$$

$$|\phi\rangle = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \phi(k)|k\rangle, \phi(k) = \langle k \mid \phi\rangle$$

$$\phi(x) = \langle x \mid \phi\rangle = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \langle x \mid k\rangle\langle k \mid \phi\rangle, \langle x \mid k\rangle = \mathrm{e}^{\mathrm{i}k\cdot x}$$

Warm-up.

$$(-\Delta + m^2)\phi(x) = 0.$$

$$k^2 = -m^2.$$

$$\phi(x) = \int \mathrm{d}k \phi(k) \mathrm{e}^{\mathrm{i}kx}$$

$$\phi(x) = \langle x \mid \phi\rangle, \phi(k) = \langle k \mid \phi\rangle,$$

$$|\phi\rangle = \int \mathrm{d}x \phi(x)|x\rangle = \int \mathrm{d}k \phi(k)|k\rangle$$

$$K(x, x') := \langle x|K|x'\rangle = \delta(x - x')(-\Delta_x + m^2),$$

$$\int \mathrm{d}x' K(x, x')\phi(x') = 0 \iff K|\phi\rangle = 0.$$

$$S = \frac{1}{2}\int \mathrm{d}x \phi(x)(-\Delta + m^2)\phi(x) = \frac{1}{2}\int \mathrm{d}x\, \mathrm{d}x' \phi(x) K(x, x')\phi(x')$$

$$S = \frac{1}{2}\langle\phi|K|\phi\rangle$$

$$Q_B|\psi\rangle = 0$$

$$\Phi \in \mathcal{H}$$

$$Q_B|\Phi\rangle = 0$$

$$\langle A, B\rangle := \langle A \mid B\rangle,$$

$$S = \frac{1}{2}\langle \Phi, Q_B\Phi\rangle = \frac{1}{2}\langle \Phi|Q_B|\Phi\rangle.$$

$$\langle A, B\rangle = (-1)^{|A||B|}\langle B, A\rangle, \langle Q_B A, B\rangle = -(-1)^{|A|}\langle A, Q_B B\rangle,$$

$$N_{\mathrm{gh}}(\Phi) = 1$$

$$|\Phi| = 1$$

$$|\Phi\rangle^{\ddagger} = |\Phi\rangle^{t}$$

$$\begin{aligned}\langle \Phi, Q_B\Phi\rangle &= (-1)^{|\Phi|(|Q_B\Phi|)}\langle Q_B\Phi, \Phi\rangle = (-1)^{|\Phi|(1+|\Phi|)}\langle Q_B\Phi, \Phi\rangle \\ &= \langle Q_B\Phi, \Phi\rangle = -(-1)^{|\Phi|}\langle \Phi, Q_B\Phi\rangle\end{aligned}$$

$$|\Phi\rangle = |\Phi_{\downarrow}\rangle + c_0|\widetilde{\Phi}_{\downarrow}\rangle,$$

$$\Phi_{\downarrow}, \widetilde{\Phi}_{\downarrow} \in \mathcal{H}_0 \implies b_0|\Phi_{\downarrow}\rangle = b_0|\widetilde{\Phi}_{\downarrow}\rangle = 0.$$

$$N_{\mathrm{gh}}(\Phi_{\downarrow}) = 1, N_{\mathrm{gh}}(\widetilde{\Phi}_{\downarrow}) = 0.$$

$$Q_B = c_0 L_0 - b_0 M + \hat{Q}_B$$

$$S = \frac{1}{2}\langle \Phi_{\downarrow}|c_0 L_0|\Phi_{\downarrow}\rangle + \frac{1}{2}\langle \widetilde{\Phi}_{\downarrow}|c_0 M|\widetilde{\Phi}_{\downarrow}\rangle + \langle \widetilde{\Phi}_{\downarrow}|c_0\hat{Q}_B|\Phi_{\downarrow}\rangle.$$

$$0 = -M|\widetilde{\Phi}_{\downarrow}\rangle + \hat{Q}_B|\Phi_{\downarrow}\rangle, 0 = c_0 L_0|\Phi_{\downarrow}\rangle + c_0\hat{Q}_B|\widetilde{\Phi}_{\downarrow}\rangle.$$

$$|\Phi\rangle = |\Phi_{\downarrow}\rangle + |\Phi_{\uparrow}\rangle, |\Phi_{\downarrow}\rangle = \Pi_s|\Phi\rangle, |\Phi_{\uparrow}\rangle = \bar{\Pi}_s|\Phi\rangle.$$

$$\Pi_s Q_B|\Phi\rangle = -b_0 M|\Phi_{\uparrow}\rangle + \hat{Q}_B|\Phi_{\downarrow}\rangle, \bar{\Pi}_s Q_B|\Phi\rangle = c_0 L_0|\Phi_{\downarrow}\rangle + \hat{Q}_B|\Phi_{\uparrow}\rangle,$$

$$[\Pi_s, \hat{Q}_B] = [\Pi_s, M] = [\Pi_s, L_0] = 0$$

$$\begin{aligned}S &= \frac{1}{2}\langle \Phi, Q_B\Phi\rangle \\ &= \frac{1}{2}\langle \Pi_s\Phi + \bar{\Pi}_s\Phi, Q_B\Phi\rangle \\ &= \frac{1}{2}\langle \Pi_s\Phi, \bar{\Pi}_s Q_B\Phi\rangle + \frac{1}{2}\langle \bar{\Pi}_s\Phi, \Pi_s Q_B\Phi\rangle \\ &= \frac{1}{2}\langle \Phi_{\downarrow}, c_0 L_0\Phi_{\downarrow} + \hat{Q}_B\Phi_{\uparrow}\rangle + \frac{1}{2}\langle \Phi_{\uparrow}, -b_0 M\Phi_{\uparrow} + \hat{Q}_B\Phi_{\downarrow}\rangle \\ &= \frac{1}{2}\langle \Phi_{\downarrow}, c_0 L_0\Phi_{\downarrow}\rangle + \frac{1}{2}\langle \Phi_{\downarrow}, \hat{Q}_B\Phi_{\uparrow}\rangle - \frac{1}{2}\langle \Phi_{\uparrow}, b_0 M\Phi_{\uparrow}\rangle + \frac{1}{2}\langle \Phi_{\uparrow}, \hat{Q}_B\Phi_{\downarrow}\rangle.\end{aligned}$$

Invariancia de Gauge.

$$|\phi\rangle \sim |\psi\rangle + Q_B|\lambda\rangle.$$

$$|\Phi\rangle \longrightarrow |\Phi'\rangle = |\Phi\rangle + \delta_\Lambda|\Phi\rangle, \delta_\Lambda|\Phi\rangle = Q_B|\Lambda\rangle\, N_{\mathrm{gh}}(\Lambda) = 0$$

Gauge de Siegel y singularidad.

$$b_0|\Phi\rangle = 0$$

$$|\Lambda\rangle = -\Delta|\Phi\rangle, \Delta = \frac{b_0}{L_0}$$

$$0 = \{Q_B, b_0\}|\psi\rangle = L_0|\psi\rangle$$

$$b_0|\Phi'\rangle = b_0|\Phi\rangle + b_0 Q_B|\Lambda\rangle = 0$$

$$b_0|\Phi\rangle = b_0\{Q_B, \Delta\}|\Phi\rangle = b_0 Q_B \Delta|\Phi\rangle,$$

$$b_0 Q_B(\Delta|\Phi\rangle + |\Lambda\rangle) = 0$$

$$A'_\mu = A_\mu + \partial_\mu \lambda$$

$$\partial^\mu A'_\mu = 0 \implies \Delta\lambda = -\partial^\mu A_\mu$$

$$\lambda = -\frac{k^\mu}{k^2} A_\mu$$

$$|\tilde{\Phi}_\downarrow\rangle = 0 \implies |\Phi\rangle = |\Phi_\downarrow\rangle.$$

$$S = \frac{1}{2}\langle\Phi|c_0 L_0|\Phi\rangle,$$

$$L_0|\Phi\rangle = 0$$

$$\hat{Q}_B|\Phi\rangle = 0$$

$$\begin{aligned} S &= \frac{1}{2}\langle\Phi|Q_B\{c_0, b_0\}|\Phi\rangle = \frac{1}{2}\langle\Phi|Q_B b_0 c_0|\Phi\rangle \\ &= \frac{1}{2}\langle\Phi|\{b_0, Q_B\}c_0|\Phi\rangle - \frac{1}{2}\langle\Phi|b_0 Q_B c_0|\Phi\rangle \\ &= \frac{1}{2}\langle\Phi|c_0 L_0|\Phi\rangle \end{aligned}$$

Campo expansivo, paridad y número fantasma.

$$n_r := N_{\mathrm{gh}}(\phi_r), |\phi_r| = n_r \bmod 2.$$

$$\langle\phi_r^c \mid \phi_s\rangle = \delta_{rs}$$

$$|\phi_r\rangle = |\phi_{\downarrow,r}\rangle + |\phi_{\uparrow,r}\rangle, b_0|\phi_{\downarrow,r}\rangle = c_0|\phi_{\uparrow,r}\rangle = 0$$

$$|\psi_\uparrow\rangle = c_0|\tilde{\psi}_\downarrow\rangle, b_0|\tilde{\psi}_\downarrow\rangle = 0, N_{\mathrm{gh}}(\psi_\uparrow) = N_{\mathrm{gh}}(\tilde{\psi}_\downarrow) + 1.$$

$$|\Phi\rangle = \sum_r \psi_r|\phi_r\rangle$$

$$\forall r\colon |\Phi| = |\psi_r||\phi_r|.$$

$$G(\psi_r) = 1 - n_r$$

$$\Phi = \sum_{n\in\mathbb{Z}} \Phi_n, N_{\mathrm{gh}}(\Phi_n) = n$$

$$\Phi = \Phi_+ + \Phi_-, \Phi_+ = \sum_{n>1} \Phi_n, \Phi_- = \sum_{n\leq 1} \Phi_n$$

$$\forall n_r \neq n\colon \psi_r = 0$$

$$|\Phi_n\rangle = \delta\big(N_{\mathrm{gh}} - n\big)|\Psi\rangle = \sum_r \delta(n_r - n)\psi_r|\phi_r\rangle$$

$$|\Phi\rangle = |\Phi_\downarrow\rangle + |\Phi_\uparrow\rangle = |\Phi_\downarrow\rangle + c_0\big|\widetilde{\Phi}_\downarrow\big\rangle,$$
$$|\Phi_\uparrow\rangle = c_0\big|\widetilde{\Phi}_\downarrow\big\rangle, \big|\widetilde{\Phi}_\downarrow\big\rangle = b_0|\Phi_\uparrow\rangle,$$

$$b_0|\Phi_\downarrow\rangle = 0, c_0|\Phi_\uparrow\rangle = 0, b_0\big|\widetilde{\Phi}_\downarrow\big\rangle = 0$$

$$|\Phi_\downarrow\rangle = \sum_r \psi_{\downarrow,r}\big|\phi_{\downarrow,r}\big\rangle, |\Phi_\uparrow\rangle = \sum_r \psi_{\uparrow,r}\big|\phi_{\uparrow,r}\big\rangle.$$

$$Z = \int \mathrm{d}\Phi_{\mathrm{cl}}\mathrm{e}^{-S[\Phi_{\mathrm{cl}}]} = \int \mathrm{d}\Phi_{\mathrm{cl}}\mathrm{e}^{-\frac{1}{2}\langle\Phi_{\mathrm{cl}}|Q_B|\Phi_{\mathrm{cl}}\rangle}$$

$$Z = \int \prod_s \mathrm{d}\psi_s\mathrm{e}^{-S[\{\psi_r\}]}$$

Transformaciones de gauge bajo la métrica de Faddeev-Popov.

$$F(\Phi_{\mathrm{cl}}) := b_0|\Phi_{\mathrm{cl}}\rangle = 0.$$

$$\delta F = b_0 Q_B|\Lambda_{\mathrm{cl}}\rangle,$$

$$\det\frac{\delta F}{\delta\Lambda_{\mathrm{cl}}} = \det b_0 Q_B.$$

$$\det b_0 Q_B = \int \mathrm{d}B'\mathrm{d}C\mathrm{e}^{-S_{\mathrm{FP}}}, S_{\mathrm{FP}} = -\langle B'|b_0 Q_B|C\rangle.$$

$$N_{\mathrm{gh}}(B') = 3, N_{\mathrm{gh}}(C) = 0.$$

$$|B'\rangle = \delta\big(N_{\mathrm{gh}} - 3\big)\sum_r b'_r|\phi_r\rangle, |C\rangle = \delta\big(N_{\mathrm{gh}}\big)\sum_r c_r|\phi_r\rangle,$$

$$|b_r| = |c_r| = 1.$$

$$|B'| = 0, |C| = 1.$$

$$\delta|C\rangle = Q_B|\Lambda_{-1}\rangle, \quad N_{\text{gh}}(\Lambda_{-1}) = -1$$
$$\delta|B'\rangle = b_0|\Lambda'\rangle, \qquad N_{\text{gh}}(\Lambda') = 4$$

$$|\Lambda\rangle = |\Lambda_0\rangle + Q_B|\Lambda_{-1}\rangle,$$

$$|\Phi'_{\text{cl}}\rangle \longrightarrow |\Phi_{\text{cl}}\rangle + Q_B|\Lambda_0\rangle$$

$$|B'\rangle \ = |B'_{\downarrow}\rangle + c_0|B\rangle, |B\rangle := \left|\tilde{B}'_{\downarrow}\right\rangle,$$
$$|\Lambda'\rangle \ = |\Lambda'_{\downarrow}\rangle + c_0\left|\tilde{\Lambda}'_{\downarrow}\right\rangle.$$

$$\delta|B'_{\downarrow}\rangle = \left|\tilde{\Lambda}'_{\downarrow}\right\rangle, \delta|B\rangle = 0$$

$$|F'\rangle = c_0|B'\rangle = 0 \implies |B'_{\downarrow}\rangle = 0.$$

$$\det\frac{\delta F'}{\delta\Lambda'} = \det c_0 b_0 = \det c_0 \det b_0 = \frac{1}{2}\det\{b_0, c_0\} = \frac{1}{2}$$

$$\Delta_{\text{FP}} = \int \ \mathrm{d}B\ \mathrm{d}C\mathrm{e}^{-S_{\text{FP}}[B,C]}, S_{\text{FP}} = \langle B|Q_B|C\rangle$$

$$b_0|B\rangle = 0, N_{\text{gh}}(B) = 2, |B| = 1$$

$$S_{\text{FP}} = \frac{1}{2}(\langle B|Q_B|C\rangle + \langle C|Q_B|B\rangle).$$

$$|B\rangle = |B_{\downarrow}\rangle + c_0\left|\tilde{B}_{\downarrow}\right\rangle$$

$$\delta|B\rangle = Q_B|\Lambda_1\rangle$$

$$|\Phi\rangle = \sum_n \ |\Phi_n\rangle$$

$$b_0|\Phi\rangle = 0 \implies b_0|\Phi_n\rangle = 0.$$

$$|\beta\rangle = \sum_{n\in\mathbb{Z}} \ |\beta_n\rangle.$$

$$Z = \int \ \mathrm{d}\Phi\ \mathrm{d}\beta\mathrm{e}^{-S[\Phi,\beta]}$$

$$S[\Phi,\beta] \ = \frac{1}{2}\langle\Phi|Q_B|\Phi\rangle + \langle\beta|b_0|\Phi\rangle$$
$$= \sum_{n\in\mathbb{Z}} \left(\frac{1}{2}\langle\Phi_{2-n}|Q_B|\Phi_n\rangle + \langle\beta_{4-n}|b_0|\Phi_n\rangle\right)$$

$$\delta|\Phi\rangle = Q_B|\Lambda\rangle$$

$$|\Lambda\rangle = \sum_{n\in\mathbb{Z}} \ |\Lambda_n\rangle.$$

Acción de espacio – tiempo curvo.

$$|\Phi\rangle = \frac{1}{\sqrt{\alpha'}}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\left(T(k) + A_\mu(k)\alpha_{-1}^{\mu} + \mathrm{i}\sqrt{\frac{\alpha'}{2}}B(k)b_{-1}c_0 + \cdots\right)|k,\downarrow\rangle$$

$$(\alpha' k^2 - 1)T(k) = 0, k^2 A_\mu(k) + \mathrm{i}k_\mu B(k) = 0,$$
$$k^\mu A_\mu(k) + \mathrm{i}B(k) = 0.$$

$$k^2 A_\mu(k) - k_\mu k \cdot A(k) = 0$$

$$(\alpha'\Delta + 1)T = 0, B = \partial^\mu A_\mu, \Delta A_\mu = \partial_\mu B$$

$$Q_B = c_0 L_0 - b_0 M + \hat{Q}_B$$
$$M \sim 2c_{-1}c_1, \hat{Q}_B \sim c_1 L_{-1}^m + c_{-1}L_1^m,$$
$$L_1^m \sim \alpha_0 \cdot \alpha_1, L_{-1}^m \sim \alpha_0 \cdot \alpha_{-1}.$$

$$\begin{aligned}
Q_B|\Phi\rangle = \frac{1}{\sqrt{\alpha'}}\int \frac{\mathrm{d}^D k}{(2\pi)^D} & \Big(T(k)c_0 L_0|k,\downarrow\rangle + A_\mu(k)\big(c_0 L_0 + \eta_{\nu\rho}c_{-1}\alpha_1^\nu \alpha_0^\rho\big)\alpha_{-1}^\mu|k,\downarrow\rangle \\
& + \mathrm{i}\sqrt{\frac{\alpha'}{2}}B(k)\big(-2b_0 c_{-1}c_1 + \eta_{\nu\rho}c_1\alpha_{-1}^\nu\alpha_0^\rho\big)b_{-1}c_0|k,\downarrow\rangle\Big) \\
= \frac{1}{\sqrt{\alpha'}}\int \frac{\mathrm{d}^D k}{(2\pi)^D} & \Big(T(k)(\alpha' k^2 - 1)c_0|k,\downarrow\rangle \\
& + A_\mu(k)\big(\alpha' k^2 c_0 \alpha_{-1}^\mu + \sqrt{2\alpha'}\eta_{\nu\rho}\eta^{\mu\nu}k^\rho c_{-1}\big)|k,\downarrow\rangle \\
& + \mathrm{i}\sqrt{\frac{\alpha'}{2}}B(k)\big(2c_{-1} + \sqrt{2\alpha'}\eta_{\nu\rho}k^\rho\alpha_{-1}^\nu c_0\big)|k,\downarrow\rangle\Big) \\
= \frac{1}{\sqrt{\alpha'}}\int \frac{\mathrm{d}^D k}{(2\pi)^D} & \Big(T(k)(\alpha' k^2 - 1)c_0|k,\downarrow\rangle \\
& + \alpha'\big(A_\mu(k)k^2 + \mathrm{i}k^\mu B(k)c_0\alpha_{-1}^\mu|k,\downarrow\rangle \\
& + \sqrt{2\alpha'}\big(k^\mu A_\mu(k) + \mathrm{i}B(k)\big)c_{-1}|k,\downarrow\rangle\Big)
\end{aligned}$$

$$|\Lambda\rangle = \frac{\mathrm{i}}{\sqrt{2\alpha'}}\int \frac{\mathrm{d}^D k}{(2\pi)^D}(\lambda(k)b_{-1}|k,\downarrow\rangle + \cdots).$$

$$Q_B|\Lambda\rangle = \frac{\mathrm{i}}{\sqrt{\alpha'}}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\lambda(k)\left(-\sqrt{\frac{\alpha'}{2}}k^2 b_{-1}c_0 + k_\mu\alpha_{-1}^\mu\right)|k,\downarrow\rangle,$$

$$\delta A_\mu = -\mathrm{i}k_\mu\lambda, \delta B = k^2\lambda.$$

$$|T\rangle = \int \frac{\mathrm{d}^D k}{(2\pi)^D}T(k)c_1|k,0\rangle,$$

$$\langle T| = \int \frac{\mathrm{d}^D k}{(2\pi)^D} T(k)\langle -k,0|c_{-1}$$
$$\langle T^{\ddagger}| = \int \frac{\mathrm{d}^D k}{(2\pi)^D} T(k)^*\langle k,0|c_{-1}$$

$$T(k)^* = T(-k)$$

$$S[T] = \frac{1}{2}\int \frac{\mathrm{d}^D k}{(2\pi)^D} T(-k)\left(k^2 - \frac{1}{\alpha'}\right)T(k)$$

$$S[A] = \frac{1}{2}\int \frac{\mathrm{d}^D k}{(2\pi)^D} A_\mu(-k)k^2A^\mu(k)$$

$$\frac{L_0^+}{2} = \frac{1}{2}(k^2 + m^2)$$

$$\begin{aligned}\langle T|c_0L_0|T\rangle &= \frac{1}{\alpha'}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{\mathrm{d}^D k'}{(2\pi)^D} T(k)T(k')\langle -k',0|c_{-1}c_0L_0c_1|k,0\rangle \\ &= \frac{1}{\alpha'}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{\mathrm{d}^D k'}{(2\pi)^D} T(k)T(k')(\alpha'k^2-1)\langle -k',0|c_{-1}c_0c_1|k,0\rangle \\ &= \frac{1}{\alpha'}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\, \mathrm{d}^D k' T(k)T(k')(\alpha'k^2-1)\delta^{(D)}(k+k')\end{aligned}$$

$$Q_B|\Psi\rangle = 0$$

$$S = \frac{1}{2}\langle \Psi, Q_B\Psi\rangle.$$

$$N_{\mathrm{gh}}(\Psi) = 2$$

$$N_{\mathrm{gh}}(\langle\cdot,\cdot\rangle) = 1.$$

$$\langle A,B\rangle = \langle A|c_0^-|B\rangle.$$

$$S = \frac{1}{2}\langle \Psi|c_0^- Q_B|\Psi\rangle.$$

$$\mathcal{H} = \mathcal{H}^- \oplus c_0^-\mathcal{H}^-, \mathcal{H}^- := \mathcal{H} \cap \mathrm{ker} b_0^-,$$

$$|\Psi\rangle = |\Psi_-\rangle + c_0^-|\tilde{\Psi}_-\rangle, \Psi_-, \tilde{\Psi}_- \in \mathcal{H}^-$$

$$c_0^-|\Psi\rangle = c_0^-|\Psi_-\rangle.$$

$$b_0^-|\Psi\rangle = 0$$

$$L_0^-|\Psi\rangle = 0,$$

$$\Psi \in \mathcal{H}^- \cap \mathrm{ker} L_0^-.$$

$$|\Psi\rangle \longrightarrow |\Psi'\rangle = |\Psi\rangle + \delta_\Lambda|\Psi\rangle, \delta_\Lambda|\Psi\rangle = Q_B|\Lambda\rangle,$$

$$N_{\mathrm{gh}}(\Lambda) = 1, L_0^-|\Lambda\rangle = 0, b_0^-|\Lambda\rangle = 0$$

$$b_0^+|\Psi\rangle = 0.$$

$$S = \frac{1}{2}\langle\Psi|c_0^- c_0^+ L_0^+|\Psi\rangle = \frac{1}{4}\langle\Psi|c_0\bar{c}_0 L_0^+|\Psi\rangle.$$

$$L_0^+|\Psi\rangle = 0.$$

$$c_0^- Q_B = (c_0 - \bar{c}_0)(c_0 L_0 + \bar{c}_0 \bar{L}_0) = c_0\bar{c}_0(L_0 + \bar{L}_0).$$

Off-shell.

$$A_{0,3} = \left\langle \prod_{i=1}^{3} \mathcal{V}_i(z_i) \right\rangle_{S^2} \propto (z_1 - z_2)^{h_3 - h_1 - h_2} \times \text{ perms } \times \text{ c.c.}$$

$$z \longrightarrow f_g(z) = \frac{az+b}{cz+d} \in \mathrm{SL}(2,\mathbb{C})$$

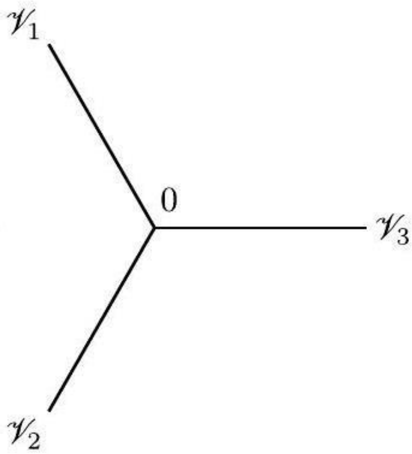
$$z = f_i(w_i), z_i = f_i(0)$$

$$f \circ \mathcal{V}(w) = f'(w)^h \overline{f'(w)}^{\bar{h}} \mathcal{V}(f(w))$$

$$A_{0,3} = \left\langle \prod_{i=1}^{3} f_i \circ \mathcal{V}_i(0) \right\rangle_{S^2} = \left(\prod_{i=1}^{3} f_i'(0)^{h_i} \overline{f_i'(0)}^{\bar{h}_i} \right) \left\langle \prod_{i=1}^{3} \mathcal{V}_i(f_i(0)) \right\rangle_{S^2}$$
$$\propto \left(\prod_{i=1}^{3} f_i'(0)^{h_i} \overline{f_i'(0)}^{\bar{h}_i} \right) (f_1(0) - f_2(0))^{h_3 - h_1 - h_2} \times \text{ perms } \times \text{ c.c.}$$

$$f_i \longrightarrow \frac{af_i + b}{cf_i + d}$$

$$f_i' \longrightarrow \frac{f_i'}{(cf_i + d)^2}, f_i - f_j \longrightarrow \frac{f_i - f_j}{(cf_i + d)(cf_j + d)}$$

$\mathcal{V}_{0,3}(\mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3) :=$  $= A_{0,3}(\mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3).$

$$A_{0,4} = \int \mathrm{d}^2 z_4 \left\langle \prod_{i=1}^{3} c\bar{c}V_i(z_i)V_4(z_4) \right\rangle_{S^2}$$

$$z_4 \longrightarrow z_1, z_2, z_3$$

$$A_{0,4} \propto \prod_{\substack{i,j=1 \\ i<j}}^{3} |z_i - z_j|^{2+k_i \cdot k_j} \int \mathrm{d}^2 z_4 \prod_{i=1}^{3} |z_4 - z_i|^{k_i \cdot k_4}$$

$$\mathcal{F}_{0,4}^{(s)} = \int \frac{\mathrm{d}^2 q}{|q|^2} \left\langle c\bar{c}V_1(z_1)c\bar{c}V_2(z_2)c\bar{c}V_3(0)|qy_4\right|^2 V_4(qy_4)\Big\rangle$$

$$\mathcal{F}_{0,4}^{(s)} = -\int \frac{\mathrm{d}^2 q}{|q|^2} \Big\langle c\bar{c}V_1(z_1)c\bar{c}V_2(z_2)\oint_{|w|=|q|^{1/2}} \mathrm{d}w w(w) \oint_{|w|=|q|^{1/2}} \mathrm{d}\bar{w}\bar{w}\overline{b(w)}c\bar{c}V_4(qy_4)c\bar{c}V_3(0)\Big\rangle$$

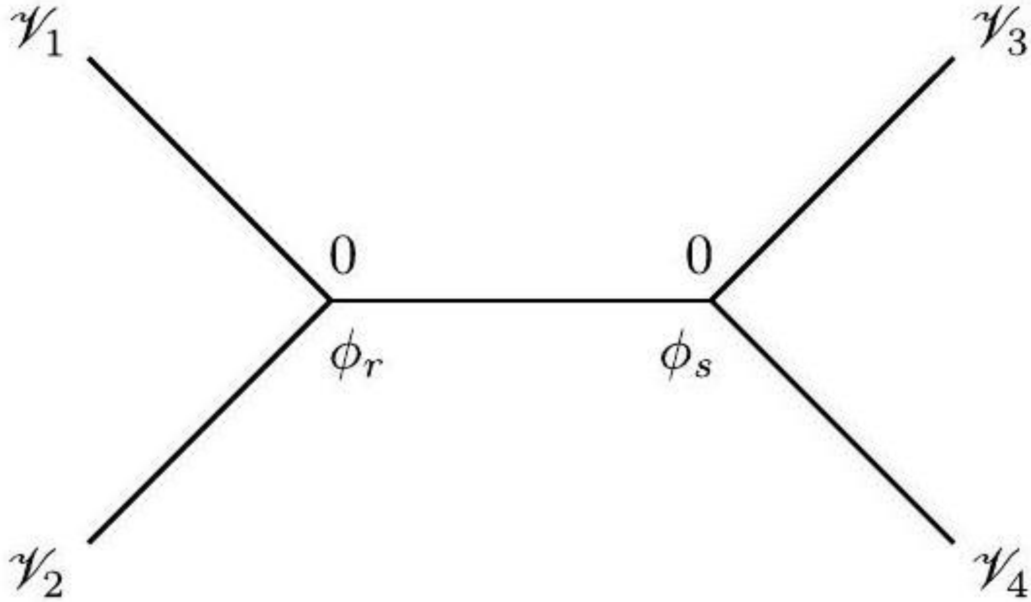
$$\mathcal{F}_{0,4}^{(s)} = -\int \frac{\mathrm{d}^2 q}{|q|^2} \langle c\bar{c}V_1(z_1)c\bar{c}V_2(z_2)\oint \mathrm{d}w w b(w)\oint \mathrm{d}\bar{w}\bar{w}\overline{b(w)}q^{L_0}\bar{q}^{\bar{L}_0}c\bar{c}V_4(y_4)c\bar{c}V_3(0)\rangle$$

$$\mathcal{F}_{0,4}^{(s)} = \langle c\bar{c}V_1(z_1)c\bar{c}V_2(z_2)\phi_r(0)\rangle\langle c\bar{c}V_3(z_3)c\bar{c}V_4(y_4)\phi_s(0)\rangle \int \frac{\mathrm{d}^2 q}{|q|^2} \langle \phi_r^c q^{L_0}\bar{q}^{\bar{L}_0} b_0 \bar{b}_0 \phi_s^c\rangle$$

$$\Delta(\phi_r^c, \phi_s^c) := \langle \phi_r^c|\Delta|\phi_s^c\rangle := -\int \frac{\mathrm{d}^2 q}{|q|^2} \langle \phi_r^c q^{L_0}\bar{q}^{\bar{L}_0} b_0 \bar{b}_0 \phi_s^c\rangle$$

$$\mathcal{F}_{0,4}^{(s)} = \mathcal{V}_{0,3}(c\bar{c}V_1(z_1), c\bar{c}V_2(z_2), \phi_r(0)) \times \Delta(\phi_r^c, \phi_s^c) \times \mathcal{V}_{0,3}(c\bar{c}V_3(z_3), c\bar{c}V_4(y_4), \phi_s(0))$$

$=$



$$q = \mathrm{e}^{-s+\mathrm{i}\theta}, s \in \mathbb{R}_+, \theta \in [0,2\pi)$$

$$\int \frac{\mathrm{d}^2 q}{|q|^2} q^{L_0} \bar{q}^{\bar{L}_0} = 2\int_0^{\infty} \mathrm{d}s \int_0^{2\pi} \mathrm{d}\theta \mathrm{e}^{-s(L_0+\bar{L}_0)} \mathrm{e}^{\mathrm{i}\theta(L_0-\bar{L}_0)} = \frac{2}{L_0+\bar{L}_0}\delta_{L_0,\bar{L}_0}$$

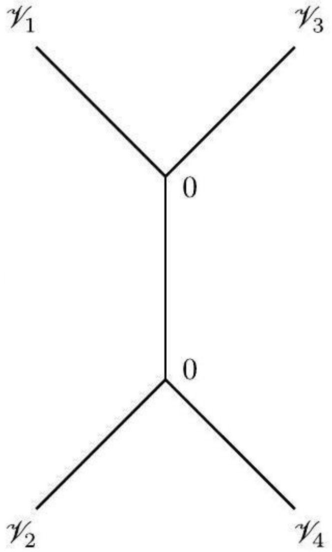
$$\Delta = -\frac{2b_0\bar{b}_0}{L_0+\bar{L}_0}\delta_{L_0,\bar{L}_0} = \frac{b_0^+}{L_0^+} b_0^- \delta_{L_0^-,0}$$

$$L_0|\phi_\alpha(k)\rangle = \bar{L}_0|\phi_\alpha(k)\rangle = \frac{\alpha'}{4}(k^2+m_\alpha^2)|\phi_\alpha(k)\rangle$$

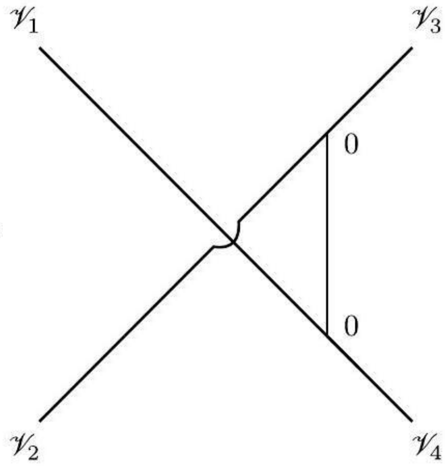
$$\Delta_{\alpha\beta}(k) = \int \frac{\mathrm{d}^2 q}{|q|^2}\langle \phi_\alpha^c(k) q^{L_0}\bar{q}^{\bar{L}_0} b_0 \bar{b}_0 \phi_\beta^c(-k)\rangle = \frac{M_{\alpha\beta}(k)}{k^2+m_\alpha^2}$$

$$M_{\alpha\beta}(k) := \frac{2}{\alpha'}\langle \phi_\alpha^c(k)|b_0^+ b_0^-|\phi_\beta^c(-k)\rangle.$$

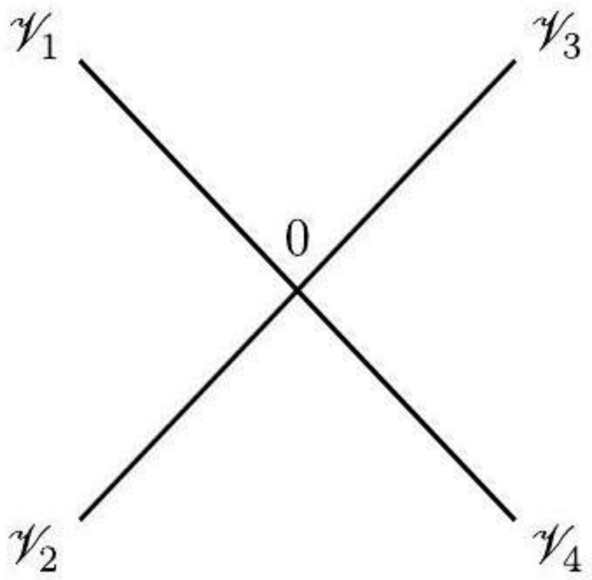
$\mathcal{F}^{(t)}_{0,4} =$



$\mathcal{F}^{(u)}_{0,4} =$



$\mathcal{V}_{0,4} =$



$$A_{0,4} = \mathcal{F}_{0,4}^{(s)} + \mathcal{F}_{0,4}^{(t)} + \mathcal{F}_{0,4}^{(u)} + \mathcal{V}_{0,4}.$$

Estados Off-shell.

$$\mathcal{H} = \mathcal{H}_m \otimes \mathcal{H}_{\text{gh}}.$$

$$\mathcal{H} = \text{Span}\{|\phi_r\rangle\}.$$

$$n_r := N_{\text{gh}}(\phi_r) \in \mathbb{Z}$$

$$|\phi_r| := N_{\text{gh}}(\phi_r) \bmod 2 = \begin{cases} 0 & N_{\text{gh}}(\phi_r) \text{ even} \\ 1 & N_{\text{gh}}(\phi_r) \text{ odd} \end{cases}$$

$$\langle \phi_r^c \mid \phi_s \rangle = \delta_{rs}$$

$$n_r^c := N_{\text{gh}}(\phi_r^c)$$

$$n_r^c + n_r = 6$$

$$\langle \phi_r \mid \phi_s \rangle = 0$$

$$\langle \phi_r \mid \phi_s^c \rangle = (-1)^{|\phi_r|} \delta_{rs}.$$

$$1 = \sum_r |\phi_r\rangle\langle\phi_r^c| = \sum_r (-1)^{|\phi_r|} |\phi_r^c\rangle\langle\phi_r|.$$

$$\mathcal{H} = \mathcal{H}_\pm \oplus c_0^\pm \mathcal{H}_\pm$$

$$\mathcal{H}_\pm := \mathcal{H} \cap \ker b_0^\pm = \mathcal{H}_0 \oplus c_0^\mp \mathcal{H}_0, \mathcal{H}_0 := \mathcal{H} \cap \ker b_0^- \cap \ker b_0^+.$$

$$L_0^- |\phi\rangle = 0, b_0^- |\phi\rangle = 0$$

$$\mathcal{H} \sim \mathcal{H}_{\downarrow\downarrow} \oplus \mathcal{H}_{\downarrow\uparrow} \oplus \mathcal{H}_{\uparrow\downarrow} \oplus \mathcal{H}_{\uparrow\uparrow},$$

$$\mathcal{H}_{\downarrow\uparrow} \sim \mathcal{H}_0, \mathcal{H}_{\downarrow\uparrow} \sim \bar{c}_0 \mathcal{H}_{\downarrow\downarrow}\ \mathcal{H}_{\uparrow\downarrow} \sim c_0 \mathcal{H}_{\downarrow\downarrow}\ \mathcal{H}_{\uparrow\uparrow} \sim c_0 \bar{c}_0 \mathcal{H}_{\downarrow\downarrow}$$

$$\phi_r = \phi_{\downarrow\downarrow,r} + \phi_{\downarrow\uparrow,r} + \phi_{\uparrow\downarrow,r} + \phi_{\uparrow\uparrow,r}$$

$$\begin{aligned} b_0|\phi_{\downarrow\downarrow,r}\rangle = \bar{b}_0|\phi_{\downarrow\downarrow,r}\rangle = 0, \quad & b_0|\phi_{\downarrow\uparrow,r}\rangle = \bar{c}_0|\phi_{\downarrow\uparrow,r}\rangle = 0, \\ c_0|\phi_{\uparrow\downarrow,r}\rangle = \bar{b}_0|\phi_{\uparrow\downarrow,r}\rangle = 0, \quad & c_0|\phi_{\uparrow\uparrow,r}\rangle = \bar{c}_0|\phi_{\uparrow\uparrow,r}\rangle = 0. \end{aligned}$$

$$|\phi_{\downarrow\uparrow,r}\rangle = \bar{c}_0|\phi_{\downarrow\downarrow,r}\rangle\ |\phi_{\uparrow\downarrow,r}\rangle = c_0|\phi_{\downarrow\downarrow,r}\rangle\ |\phi_{\uparrow\uparrow,r}\rangle = c_0\bar{c}_0|\phi_{\downarrow\downarrow,r}\rangle.$$

$$\phi_r^c = \phi_{\downarrow\downarrow,r}^c + \phi_{\downarrow\uparrow,r}^c + \phi_{\uparrow\downarrow,r}^c + \phi_{\uparrow\uparrow,r}^c$$

$$\begin{aligned} \langle\phi_{\downarrow\downarrow,r}^c|c_0 = \langle\phi_{\downarrow\downarrow,r}^c|\bar{c}_0 = 0, \quad & \langle\phi_{\downarrow\uparrow,r}^c|c_0 = \langle\phi_{\downarrow\uparrow,r}^c|\bar{b}_0 = 0, \\ \langle\phi_{\uparrow\downarrow,r}^c|b_0 = \langle\phi_{\uparrow\downarrow,r}^c|\bar{c}_0 = 0, \quad & \langle\phi_{\uparrow\uparrow,r}^c|b_0 = \langle\phi_{\uparrow\uparrow,r}^c|\bar{b}_0 = 0. \end{aligned}$$

$$\langle\phi_{\downarrow\uparrow,r}^c| = \langle\phi_{\downarrow\downarrow,r}^c|\bar{b}_0\ \langle\phi_{\uparrow\downarrow,r}^c| = \langle\phi_{\downarrow\downarrow,r}^c|b_0\ \langle\phi_{\uparrow\uparrow,r}^c| = \langle\phi_{\downarrow\downarrow,r}^c|\bar{b}_0 b_0.$$

$$\langle \phi_{x,r}^c \mid \phi_{y,s} \rangle = \delta_{xy}\delta_{rs}$$

$$b_0^- |\phi_r\rangle = 0 \implies b_0|\phi_r\rangle = \bar{b}_0|\phi_r\rangle$$

$$\phi_{\uparrow\downarrow,r} + \phi_{\uparrow\uparrow,r} = \phi_{\downarrow\downarrow,r} + \phi_{\downarrow\uparrow,r}$$

$$\phi_r = 2\left(\phi_{\downarrow\downarrow,r} + \phi_{\downarrow\uparrow,r}\right)$$

Amplitudes Off-shell.

$$\chi_{g,n} := \chi\left(\Sigma_{g,n}\right) = 2 - 2g - n.$$

$$\mathrm{M}_{g,n} := \dim_{\mathbb{R}} \mathcal{M}_{g,n} = 6g - 6 + 2n, \text{ for } \begin{cases} g \geq 2, \\ g = 1, n \geq 1, \\ g = 0, n \geq 3. \end{cases}$$

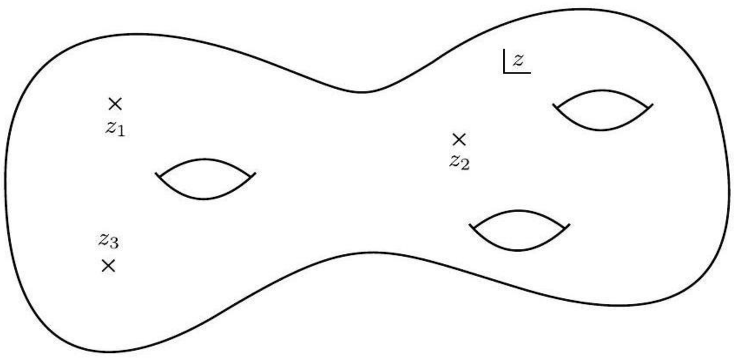
$$A_{g,n}(\mathcal{V}_1, \ldots, \mathcal{V}_n) = \int_{\mathcal{M}_{g,n}} \omega^{g,n}_{\mathbb{M}_{g,n}}(\mathcal{V}_1, \ldots, \mathcal{V}_n)$$

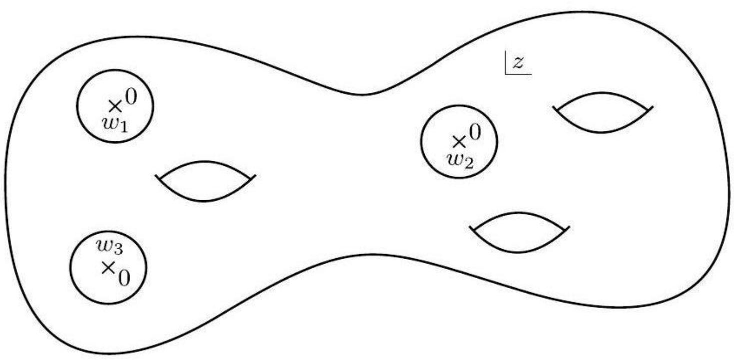
$$\omega^{g,n}_{\mathrm{M}_{g,n}}(\mathcal{V}_1, \ldots, \mathcal{V}_n) = \left\langle \text{ghosts } \times \prod_{i=1}^{n} \mathcal{V}_i \right\rangle_{\Sigma_{g,n}} \times \bigwedge_{\lambda=1}^{\mathrm{M}_{g,n}} \mathrm{d}t_\lambda$$

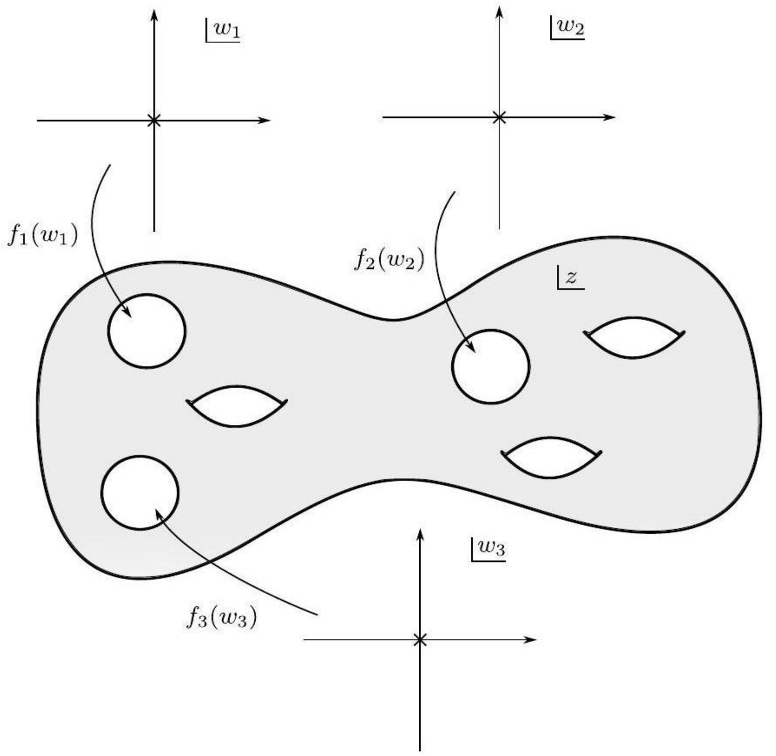
Coordenadas locales.

$$z = f_i(w_i), z_i = f_i(0)$$

$$A_{g,n}(\mathcal{V}_1, \ldots, \mathcal{V}_n)_{\mathcal{S}_{g,n}} = \int_{\mathcal{S}_{g,n}} \omega^{g,n}_{\mathrm{M}_{g,n}}(\mathcal{V}_1, \ldots, \mathcal{V}_n)\bigg|_{\mathcal{S}_{g,n}}$$







Figuras 4, 5 y 6. Curvatura local provocada por una partícula supermasiva.

$$\forall \mathcal{S}_{g,n}\colon A_{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n)_{\mathcal{S}_{g,n}} = A_{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n) \text{ (on-shell).}$$

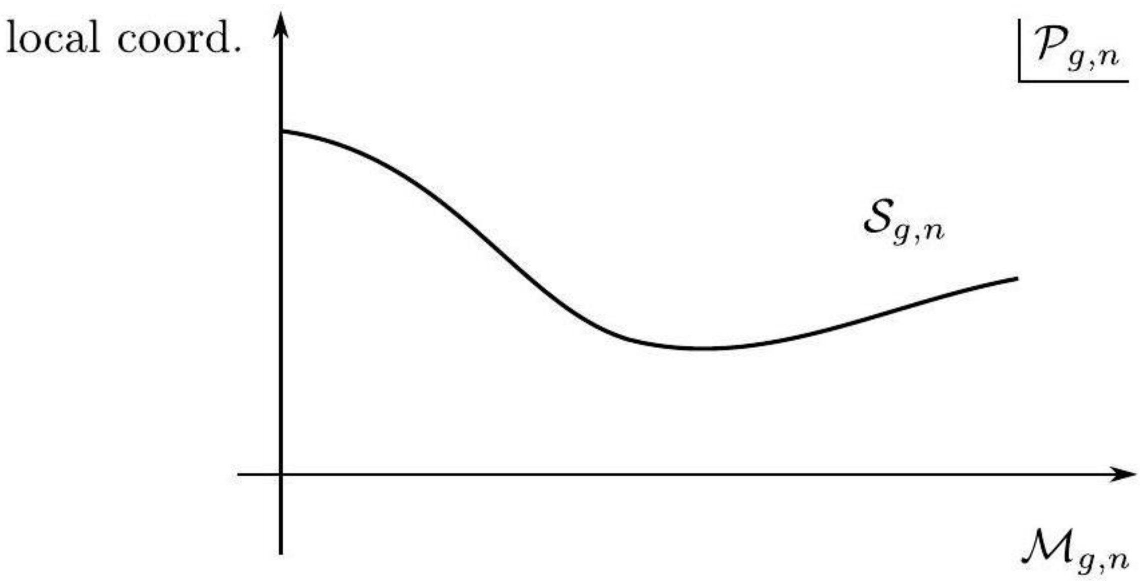


Figura 7. Fluctuaciones en el sistema de coordenadas de una partícula supermasiva al deformar o curvar el espacio – tiempo cuántico.

Geometría en espacios de Moduli y superficies de Riemann.

$$\text{disks } = n.$$

$$\backslash\#\text{spheres } = 2g - 2 + n.$$

$$\text{circles } = \frac{n + 3(2g - 2 + n)}{2} = 3g - 3 + 2n$$

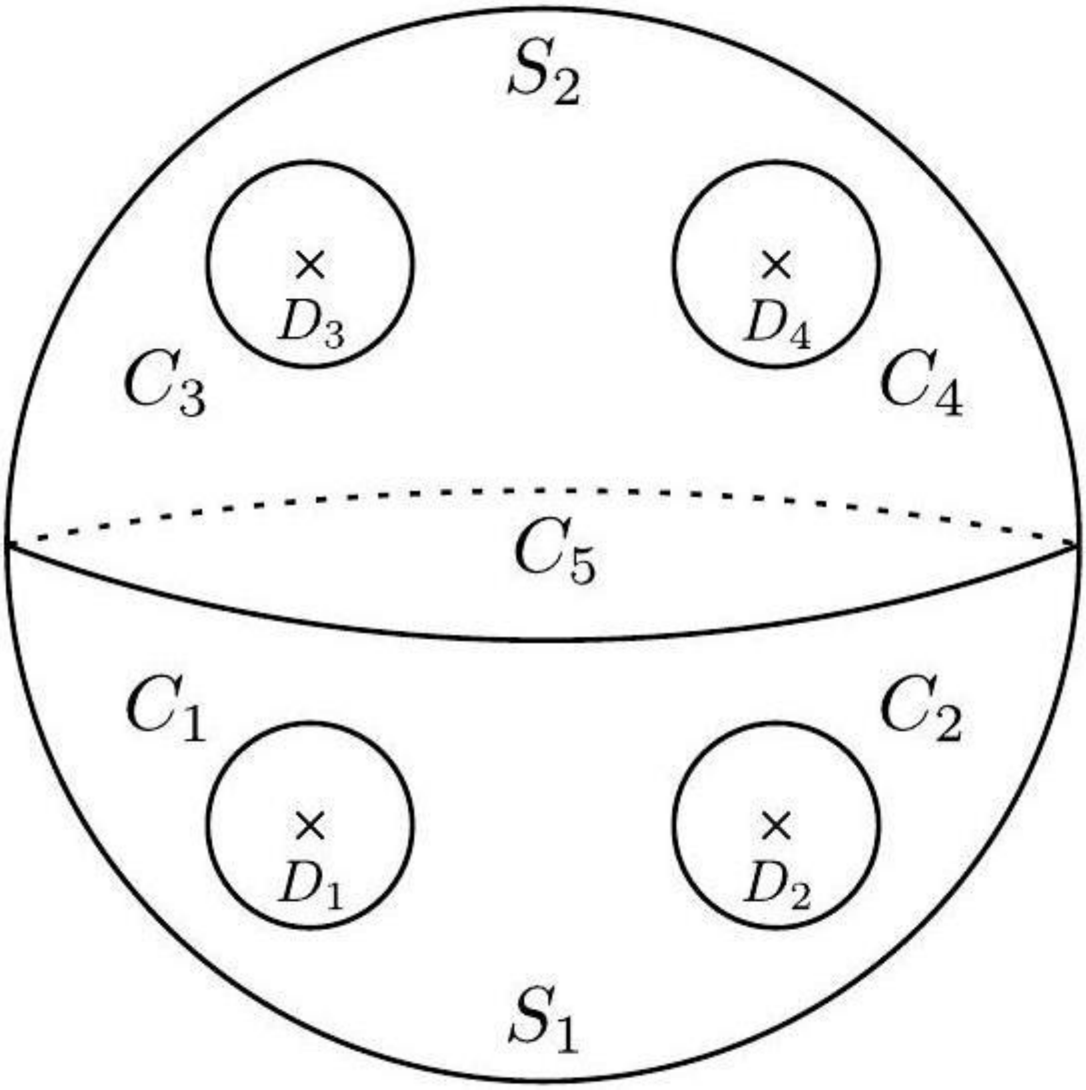
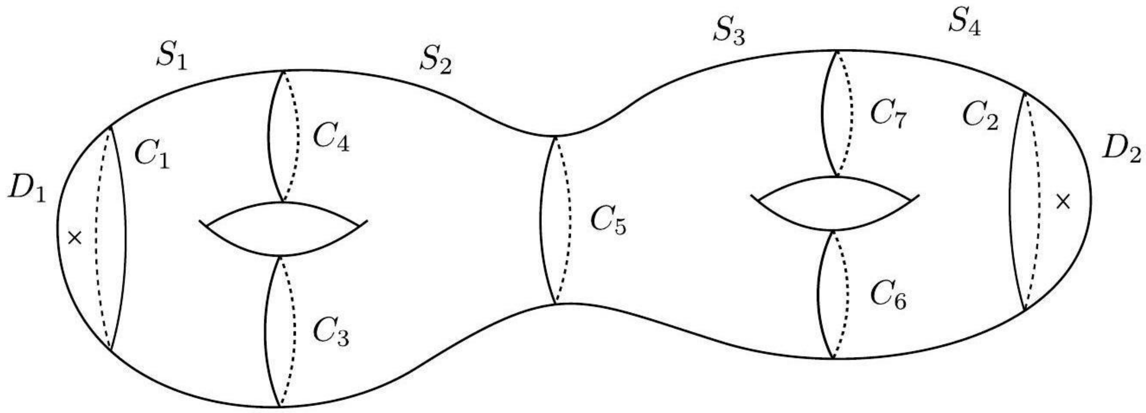
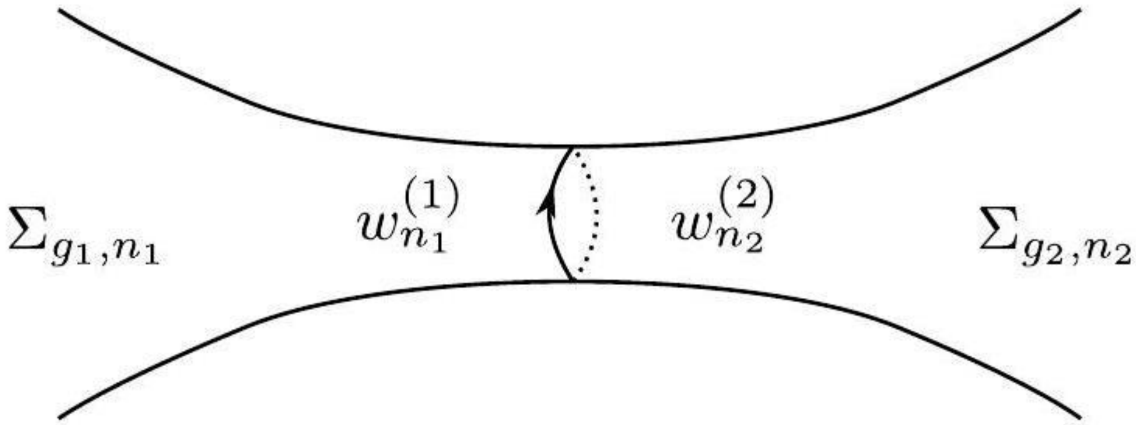


Figura 8. Propagadores de una partícula repercutida en relación al propagador de la partícula supermasiva.





Figuras 9 y 10. Entrelazamiento cuántico por curvatura local en dimensión $\mathbb{R}^4$, es decir, en la misma dimensión.

$$C_{\Lambda(ab)} := S_a \cap S_b, C_{i(a)} := S_a \cap D_i, \{C_\alpha\} = \{C_\Lambda, C_i\}$$

$$\Lambda = 1, \dots, \mathrm{M}_{g,n}^c, \mathrm{M}_{g,n}^c = 3g - 3 + n$$

$$\text{on } C_{\Lambda(ab)}: z_a = F_{ab}(z_b)$$

$$\text{on } C_{i(a)}: z_a = f_{ai}(w_i)$$

$$w_i \longrightarrow \tilde{w}_i = \mathrm{e}^{\mathrm{i}\alpha_i} w_i$$

$$z = f_i(w_i), z = \tilde{f}_i(\tilde{w}_i)$$

$$f_i(w_i) = \tilde{f}_i\left(\mathrm{e}^{\mathrm{i}\alpha_i} w_i\right)$$

$$\hat{\mathcal{P}}_{g,n} = \mathcal{P}_{g,n}/\mathrm{U}(1)^n$$

$$\text{on } C_{\Lambda(ab)}: z_a - z_{a,m} = \frac{q_\Lambda}{z_b - z_{b,n}}$$

$$\text{on } C_{i(a)}: z_a - z_{a,m} = w_i + \sum_{N=1}^{\infty} p_{i,N} w_i^N$$

$$\{x_s\} = \{q_\Lambda, p_{i,N}\}$$

$$z_a = F_{ab}(z_b; x_s), z_a = f_{ai}(w_i; x_s)$$

$$\text{on } C_\alpha: \sigma_\alpha = F_\alpha(\tau_\alpha; x_s)$$

Espacio tangencial.

$$\delta x_s = \epsilon V_s$$

$$\epsilon V_s \partial_s f = f(x_s + \epsilon V_s) - f(x_s)$$

$$F_\alpha \longrightarrow F_\alpha + \epsilon \delta F_\alpha$$

$$\sigma_\alpha = F_\alpha(\tau_\alpha)$$

$$\sigma'_\alpha = F_\alpha(\tau_\alpha) + \epsilon\delta F_\alpha(\tau_\alpha) = \sigma_\alpha + \epsilon\delta F_\alpha(\tau_\alpha) = \sigma_\alpha + \epsilon\delta F_\alpha\left(F_\alpha^{-1}(\sigma_\alpha)\right)$$

$$\sigma'_\alpha = \sigma_\alpha + \epsilon v^{(\alpha)}(\sigma_\alpha), v^{(\alpha)} = \delta F_\alpha \circ F_\alpha^{-1}$$

$$V^{(\alpha)} \sim \left(v^{(\alpha)}, C_{(\alpha)}\right).$$

$$V \sim (v, C), C \subseteq \bigcup_\alpha C_\alpha$$

$$v|_{C_\alpha} = v^{(\alpha)}$$

$$x_s \longrightarrow x_s + \epsilon \delta x_s$$

$$C_\alpha: F_\alpha \longrightarrow F_\alpha + \epsilon\delta F_\alpha, \delta F_\alpha = \frac{\partial F_\alpha}{\partial x_s}\delta x_s$$

$$\sigma'_\alpha = \sigma_\alpha + \epsilon v_s^{(\alpha)}(\sigma_\alpha)\delta x_s, v_s^{(\alpha)}(\sigma_\alpha) = \frac{\partial F_\alpha}{\partial x_s}\left(F_\alpha^{-1}(\sigma_\alpha)\right)$$

Plumbing fixture.

$$D_q^{(1)} = \left\{\left|w_{n_1}^{(1)}\right| \leq |q|^{1/2}\right\}, D_q^{(2)} = \left\{\left|w_{n_2}^{(2)}\right| \leq |q|^{1/2}\right\}$$

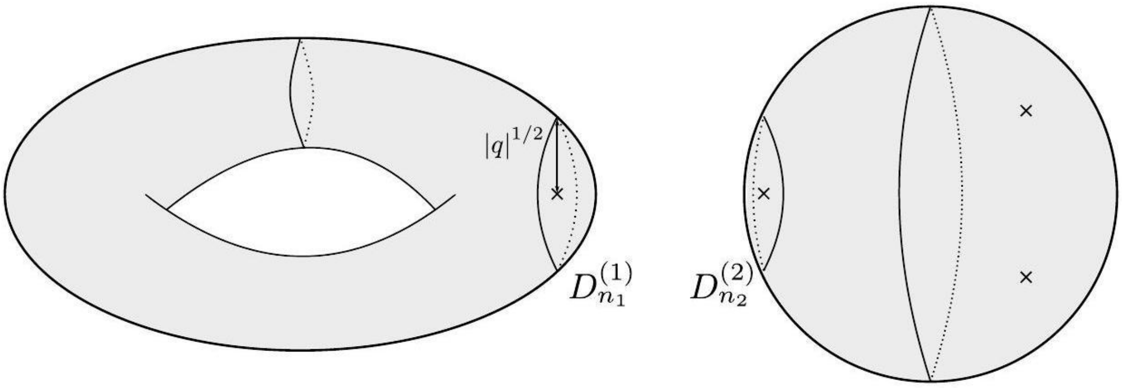
$$\Sigma_{g,n} = \Sigma_{g_1,n_1} \qquad \Sigma_{g_2,n_2}, \begin{cases} g = g_1 + g_2 \\ n = n_1 + n_2 - 2 \end{cases}$$

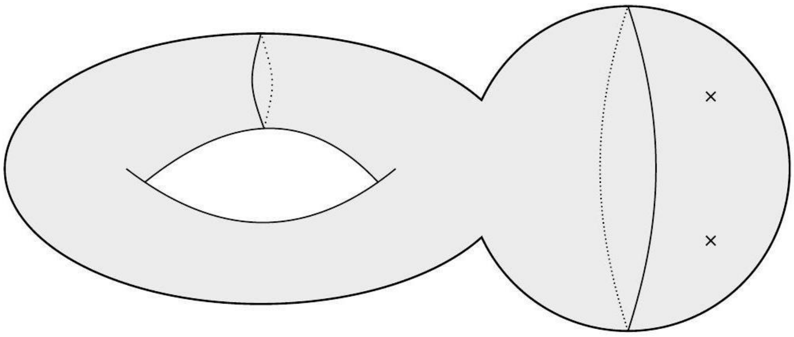
$$w_{n_1}^{(1)} w_{n_2}^{(2)} = q, |q| \leq 1,$$

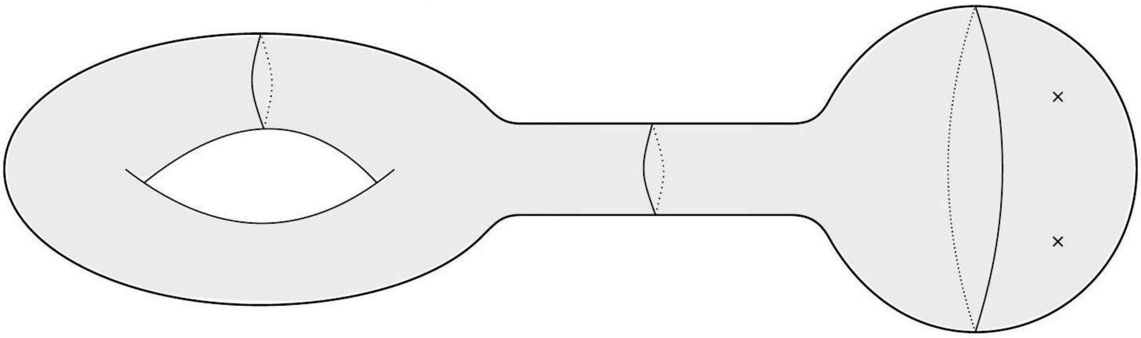
$$q = \mathrm{e}^{-s+\mathrm{i}\theta}, s \in \mathbb{R}_+, \theta \in [0,2\pi).$$

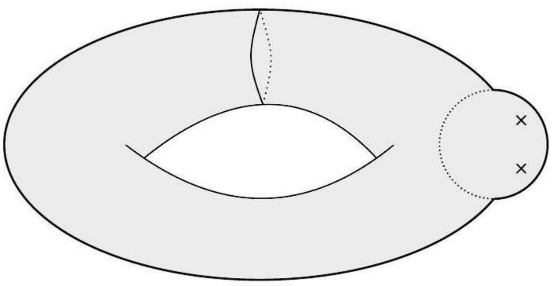
$$\left|w_{n_2}^{(2)}\right| = \frac{|q|}{\left|w_{n_1}^{(1)}\right|} > |q|^{1/2}$$

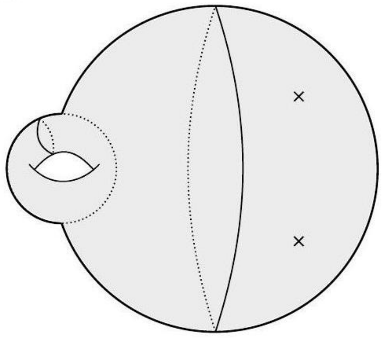
$$z_a^{(1)} = f_{an_1}^{(1)}\left(w_{n_1}^{(1)}\right), z_b^{(2)} = f_{bn_2}^{(2)}\left(w_{n_2}^{(2)}\right)$$

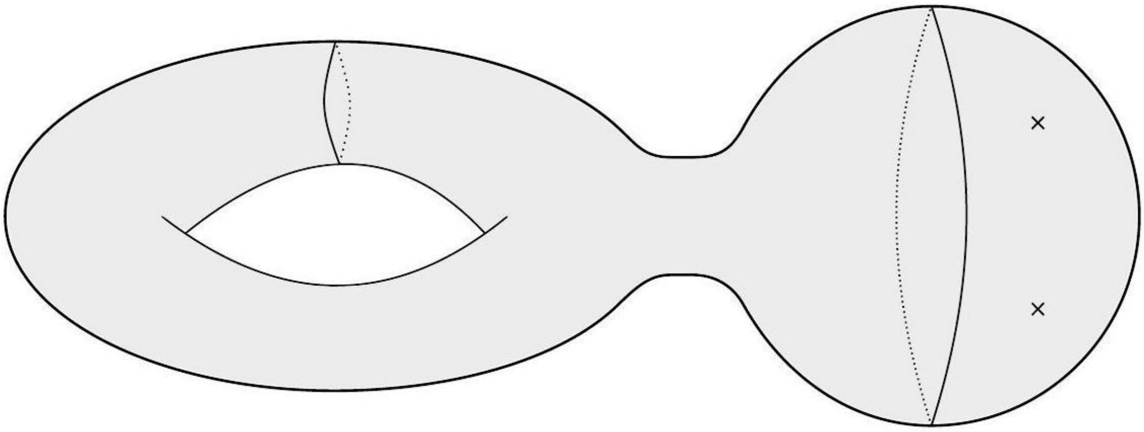


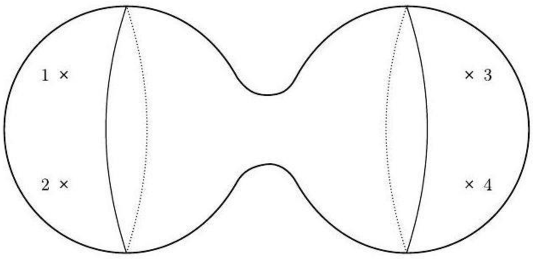








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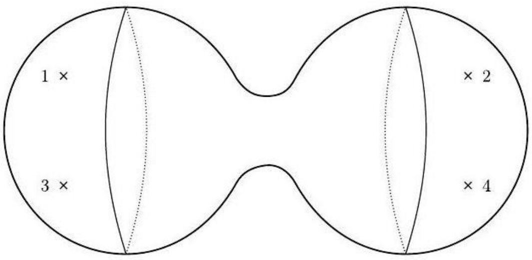
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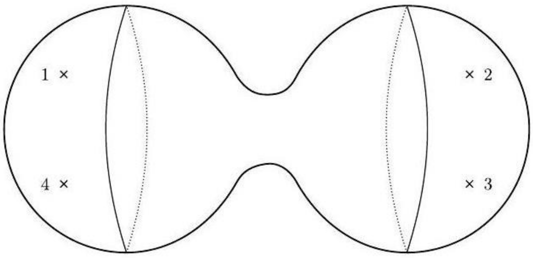
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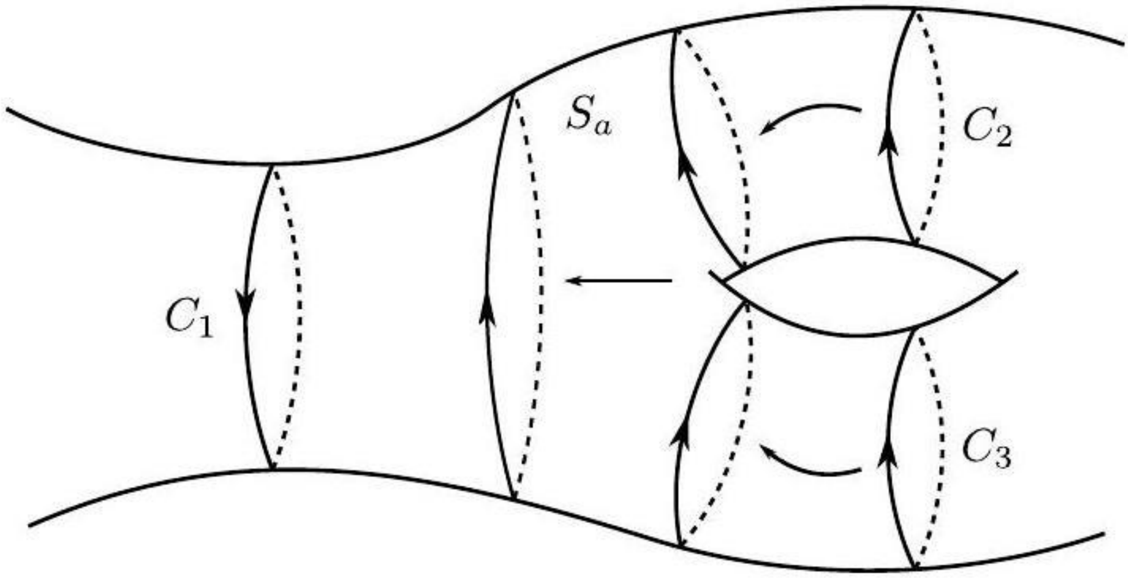


Figuras 11, 12, 13, 14, 15, 16, 17, 18, 19 y 20. Puente de Einstein – Rosen a escala cuántica por curvatura local en dimensión $\mathbb{R}^4$, es decir, en la misma dimensión.

$$z_a^{(1)} = f_{an_1}^{(1)}\left(w_{n_1}^{(1)}\right) = f_{an_1}^{(1)}\left(\frac{q}{w_{n_2}^{(2)}}\right) = f_{an_1}^{(1)}\left(\frac{q}{f_{bn_2}^{(2)-1}\left(z_b^{(2)}\right)}\right)$$

$$z_a = F_{ab}(z_b), F_{ab} = f_{an_1}^{(1)} \circ (q \cdot I) \circ f_{bn_2}^{(2)-1},$$

$$\mathrm{M}_{g_1,n_1} + \mathrm{M}_{g_2,n_2} = 6g_1 - 6 + 2n_1 + 6g_2 - 6 + 2n_2 = \mathrm{M}_{g,n} - 2.$$

$$\Sigma_{g,n} = \Sigma_{g_1,n_1}, \begin{cases} g = g_1 + 1 \\ n = n_1 - 2 \end{cases}$$

$$w_{n_1-1}^{(1)} w_{n_1}^{(1)} = q$$

$$\mathrm{M}_{g_1,n_1} = \mathrm{M}_{g,n} - 2.$$

$$\mathcal{M}_{g_1,n_1}\mathcal{M}_{g_2,n_2} \subset \mathcal{M}_{g,n}$$

$$\mathcal{M}_{g_1,n_1} \subset \mathcal{M}_{g,n}$$

$$\mathcal{F}_{g,n} := \mathcal{M}_{g-1,n+2} \bigcup \left(\bigcup_{\substack{n_1+n_2=n+2 \\ g_1+g_2=g}} \mathcal{M}_{g_1,n_1} \# \mathcal{M}_{g_2,n_2} \right)$$

$$\mathcal{V}_{g,n} := \mathcal{M}_{g,n} - \mathcal{F}_{g,n}$$

$$\mathcal{F}_{g,n}^{1\mathrm{PR}} := \bigcup_{\substack{n_1+n_2=n+2 \\ g_1+g_2=g}} \mathcal{M}_{g_1,n_1}\mathcal{M}_{g_2,n_2}$$

$$\mathcal{V}_{g,n}^{1\mathrm{PI}} := \mathcal{M}_{g,n} - \mathcal{F}_{g,n}^{1\mathrm{PR}}$$

$$\mathcal{V}_{g,n}^{1\mathrm{PI}} = \mathcal{V}_{g,n}\, U\left(\bigcup_{g'} \#\mathcal{M}_{g-g',n+g'}\right)$$

$$\mathcal{V}_{0,3} = \mathcal{M}_{0,3}, \mathcal{F}_{0,3} = \emptyset.$$

$$\mathcal{F}_{0,4} = \mathcal{V}_{0,3}\mathcal{V}_{0,3}$$

$$\mathcal{F}_{0,5} = \mathcal{M}_{0,4}\mathcal{M}_{0,3}$$
$$\mathcal{F}_{1,1} = \mathcal{V}_{0,3}$$

$$\mathcal{V}_{0,3} = \mathcal{V}_{0,3}\mathcal{V}_{0,3}\mathcal{V}_{0,3} + \mathcal{V}_{0,4}\mathcal{V}_{0,3}$$
$$\mathcal{F}_{0,4} = \mathcal{V}_{0,3} + \mathcal{V}_{0,4}$$

$$r(\Sigma_{g_1,n_1}\#\Sigma_{0,3}) = r(\Sigma_{g_1,n_1}) + 1, r(\#\Sigma_{g_1,n_1}) = r(\Sigma_{g_1,n_1}) + 1.$$

$$r(\Sigma_{g,n}) = 3g + n - 2 \in \mathbb{N}^*$$

$$r(\Sigma_{0,3}) = 1$$

$$r(\Sigma_{g_1,n_1}\#\Sigma_{g_2,n_2}) = r(\Sigma_{g_1,n_1}) + r(\Sigma_{g_2,n_2}).$$

Stubs.

$$q = \mathrm{e}^{-s+\mathrm{i}\theta}, s \in [s_0,\infty), \theta \in [0,2\pi), s_0 \geq 0.$$

$$s_0 < s_0': \mathcal{F}_{g,n}(s_0') \subset \mathcal{F}_{g,n}(s_0)\ \mathcal{V}_{g,n}(s_0) \subset \mathcal{V}_{g,n}(s_0')$$

$$w_1 = \lambda\tilde{w}_1, w_2 = \lambda\tilde{w}_2$$

$$\tilde{w}_1\tilde{w}_2 = \mathrm{e}^{-\tilde{s}+\mathrm{i}\tilde{\theta}}$$

$$\tilde{s} = s + 2\ln\ |\lambda|, \tilde{\theta} = \theta + \mathrm{i}\ln\ \frac{\lambda}{\bar{\lambda}}$$

$$\tilde{s} \in [s_0,\infty), s_0 := 2\ln\ |\lambda|.$$

Amplitudes y estados de superficie Off-shell.

$$\omega_{i_1\cdots i_p} := \omega_p\left(\partial_{s_1},\dots,\partial_{s_p}\right),$$

$$\omega_{i_1 i_2\cdots i_p} = -\omega_{i_2 i_1\cdots i_p},$$

$$\omega_p\left(V^{(1)},\dots,V^{(p)}\right) = \omega_p\left(V_{s_1}^{(1)}\partial_{s_1},\dots,V_{s_p}^{(p)}\partial_{s_p}\right) = \omega_{i_1\cdots i_p}V_{s_1}^{(1)}\cdots V_{s_p}^{(p)},$$

$$\omega_p(\mathcal{V}_1,\dots,\mathcal{V}_n) := \omega_p(\otimes_i \mathcal{V}_i).$$

$$\omega_0 = (2\pi\mathrm{i})^{-\mathrm{M}_{g,n}^c}\left\langle\prod_{i=1}^{n} f_i\circ\mathcal{V}_i(0)\right\rangle_{\Sigma_{g,n}}$$

$$B(V) := \oint_C \frac{\mathrm{d}z}{2\pi\mathrm{i}} b(z)v(z) + \oint_C \frac{\mathrm{d}\bar{z}}{2\pi\mathrm{i}}\bar{b}(\bar{z})\bar{v}(\bar{z})$$

$$B(V) := \sum_{\alpha} \oint_{C_\alpha} \frac{\mathrm{d}z}{2\pi\mathrm{i}} b(z) v(z) + \text{ c.c.}$$

$$T(V) := \oint_{C} \frac{\mathrm{d}z}{2\pi\mathrm{i}} T(z) v(z) + \oint_{C} \frac{\mathrm{d}\bar{z}}{2\pi\mathrm{i}} \bar{T}(\bar{z}) \bar{v}(\bar{z})$$

$$T(V) = \{Q_B, B(V)\}$$

$$B = B_s \, \mathrm{d}x_s, B_s := B(\partial_s)$$

$$B_s = \sum_{\alpha} \oint_{C_\alpha} \frac{\mathrm{d}\sigma_\alpha}{2\pi\mathrm{i}} b(\sigma_\alpha) \frac{\partial F_\alpha}{\partial x_s} \left(F_\alpha^{-1}(\sigma_\alpha)\right) + \sum_{\alpha} \oint_{C_\alpha} \frac{\mathrm{d}\bar{\sigma}_\alpha}{2\pi\mathrm{i}} \bar{b}(\bar{\sigma}_\alpha) \frac{\partial \bar{F}_\alpha}{\partial x_s} \left(\bar{F}_\alpha^{-1}(\bar{\sigma}_\alpha)\right)$$

$$\omega_p\left(V^{(1)}, \dots, V^{(p)}\right)(\mathcal{V}_1, \dots, \mathcal{V}_n) := (2\pi\mathrm{i})^{-\mathrm{M}_{g,n}^c} \left\langle B\left(V^{(1)}\right) \cdots B\left(V^{(p)}\right) \prod_{i=1}^{n} \mathcal{V}_i \right\rangle_{\Sigma_{g,n}}$$

$$\begin{aligned} \omega_p &= \omega_{p,s_1\cdots s_p} \, \mathrm{d}x_{s_1} \wedge \cdots \wedge \, \mathrm{d}x_{s_p} \\ &= (2\pi\mathrm{i})^{-\mathrm{M}_{g,n}^c} \left\langle B_{s_1} \, \mathrm{d}x_{s_1} \wedge \cdots \wedge B_{s_p} \, \mathrm{d}x_{s_p} \prod_{i=1}^{n} \mathcal{V}_i \right\rangle_{\Sigma_{g,n}} \end{aligned}$$

$$x_s = x_s\left(t_1, \dots, t_q\right)$$

$$\forall p \leq q \colon \; \omega_p\big|_{\mathcal{S}} = (2\pi\mathrm{i})^{-\mathrm{M}_{g,n}^c \left\langle B_{r_1} \frac{\partial x_{s_1}}{\partial t_{r_1}} \mathrm{d}t_{r_1} \wedge \cdots \wedge B_{r_p} \frac{\partial x_{s_p}}{\partial t_{r_p}} \mathrm{d}t_{r_p} \prod_{i=1}^{n} \mathcal{V}_i \right\rangle_{\Sigma_{g,n}}},$$
$$\forall p > q \colon \omega_p \mid \mathcal{S} = 0.$$

$$B_r := \frac{\partial x_s}{\partial t_r} B_s.$$

$$A_{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n)_{\mathcal{S}_{g,n}} := \int_{\mathcal{S}_{g,n}} \omega_{\mathrm{M}_{g,n}}^{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n)\Big|_{\mathcal{S}_{g,n}}$$

$$\omega_{\mathrm{M}_{g,n}}^{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n)\Big|_{\mathcal{S}_{g,n}} = (2\pi\mathrm{i})^{-\mathrm{M}_{g,n}^c} \left\langle \bigwedge_{\lambda=1}^{\mathrm{M}_{g,n}} B_s \frac{\partial x_s}{\partial t_\lambda} \mathrm{d}t_\lambda \prod_{i=1}^{n} f_i \circ \mathcal{V}_i(0) \right\rangle$$

$$A_n(\mathcal{V}_1, \dots, \mathcal{V}_n) := \sum_{g \geq 0} A_{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n).$$

$$\mathcal{R}_{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n) := \int_{\mathcal{R}_{g,n}} \omega_{\mathbb{M}_{g,n}}^{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n),$$

$$\mathcal{R}_n := \sum_{g \geq 0} \mathcal{R}_{g,n},$$

$$\mathcal{R}_n(\mathcal{V}_1, \dots, \mathcal{V}_n) := \sum_{g \geq 0} \mathcal{R}_{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n) = \sum_{g \geq 0} \int_{\mathcal{R}_{g,n}} \omega_{\mathbb{M}_{g,n}}^{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n).$$

$$\langle \Sigma_{g,n}|B_{s_1}\cdots B_{s_p}|\otimes_i \mathcal{V}_i\rangle := \omega_{s_1\cdots s_p}(\mathcal{V}_1,\dots,\mathcal{V}_n),$$
$$\langle \omega_p^{g,n}\ |\otimes_i \mathcal{V}_i\rangle := \omega_p(\mathcal{V}_1,\dots,\mathcal{V}_n),$$
$$\langle A_{g,n}\ |\otimes_i \mathcal{V}_i\rangle := A_{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n).$$

$$\langle \mathcal{R}_{g,n}\ |\otimes_i \mathcal{V}_i\rangle := \mathcal{R}_{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n).$$

$$\langle \omega_p^{g,n}| = \langle \Sigma_{g,n}|B_{s_1}\,\mathrm{d}x^{s_1}\cdots B_{s_p}\,\mathrm{d}x^{s_p}, \langle A_{g,n}| = \int_{\mathcal{M}_{g,n}} \langle \omega_p^{g,n}|.$$

$$C_1: w_1 = z_1 - y_1,\quad C_3: w_3 = z_2 - y_3,\quad C_5: z_1 = z_2$$
$$C_2: w_2 = z_1 - y_2,\quad C_4: w_4 = z_2 - y_4.$$

$$y_4 \longrightarrow y_4 + \delta y_4, \bar{y}_4 \longrightarrow \bar{y}_4 + \delta\bar{y}_4$$

$$z_2' = z_2 + \delta y_4, \bar{z}_2' = \bar{z}_2 + \delta\bar{y}_4.$$

$$v = 1, \bar{v} = 0,$$

$$v = 0, \bar{v} = 1.$$

$$B(\partial_{y_4}) = \oint\ _{C_4} \mathrm{d}z_2 b(z_2)(+1), B(\partial_{\bar{y}_4}) = \oint\ _{C_4} \mathrm{d}\bar{z}_2\bar{b}(\bar{z}_2)(+1)$$

$$\omega_2(\partial_{y_4},\partial_{\bar{y}_4}) = \frac{1}{2\pi\mathrm{i}}\left\langle B(\partial_{y_4})B(\partial_{\bar{y}_4})\prod_{i=1}^{4}\ \mathcal{V}_i\right\rangle_{\Sigma_{0,4}}$$
$$= \frac{1}{2\pi\mathrm{i}}\left\langle \oint\ _{C_4} \mathrm{d}z_2 b(z_2)\oint\ _{C_4} \mathrm{d}\bar{z}_2\bar{b}(\bar{z}_2)\prod_{i=1}^{4}\ \mathcal{V}_i\right\rangle_{\Sigma_{0,4}}$$

$$\omega_2(\partial_{y_4},\partial_{\bar{y}_4}) = \frac{1}{2\pi\mathrm{i}}\left\langle \prod_{i=1}^{3}\ c\bar{c}V_i(y_i,\bar{y}_i)\oint\ _{C_4} \mathrm{d}z_2 b(z_2)\oint\ _{C_4} \mathrm{d}\bar{z}_2\bar{b}(\bar{z}_2)\bar{c}(\bar{y}_4)c(y_4)V_4(y_4,\bar{y}_4)\right\rangle_{\Sigma_{0,4}}$$

$$\oint\ _{C_4} \mathrm{d}z_2 b(z_2)c(y_4) \sim \oint\ _{C_4} \mathrm{d}z_2\frac{1}{z_2 - y_4}$$

$$A_{0,4} = \frac{1}{2\pi\mathrm{i}}\int\ \mathrm{d}y_4 \wedge \mathrm{d}\bar{y}_4\left\langle \prod_{i=1}^{3}\ c\bar{c}V_i(y_i)V_4(y_4)\right\rangle_{\Sigma_{0,4}}$$

$$z_a \longrightarrow z_a + \phi(z_a)$$

$$C_i: v^{(i)} = \phi|_{C_i}.$$

$$B(V) = \sum_{i=1}^{3}\ \oint\ _{C_i} \mathrm{d}z_a b(z_a)\phi(z_a) +\ \text{c.c.}$$

$$w_i \longrightarrow (1 + \mathrm{i}\alpha_i)w_i, \bar{w}_i \longrightarrow (1 - \mathrm{i}\alpha_i)\bar{w}_i,$$

$$v = \mathrm{i}w_i, \bar{v} = -\mathrm{i}\bar{w}_i.$$

$$B(V) = \mathrm{i}\oint_{C_i} \mathrm{d}w_i w_i b(w_i) - \mathrm{i}\oint_{C_i} \mathrm{d}\bar{w}_i \bar{w}_i \bar{b}(\bar{w}_i),$$

$$B(V)\mathcal{V}_i(0) = \mathrm{i}\oint_{C_i} \mathrm{d}w_i w_i b(w_i)\mathcal{V}_i(0) - \mathrm{i}\oint_{C_i} \mathrm{d}\bar{w}_i \bar{w}_i \bar{b}(\bar{w}_i)\mathcal{V}_i(0),$$

$$\mathrm{i}\left(b_0 - \bar{b}_0\right)|\mathcal{V}_i\rangle,$$

$$b_0^- |\mathcal{V}_i\rangle = 0$$

$$\mathcal{V}_i \in \mathcal{H}^-,$$

$$w_i \longrightarrow f(w_i), f(0) = 0$$

$$f(w_i) = \sum_{m \geq 0} p_m w_i^{m+1}$$

$$v_m = w_i^{m+1}, \bar{v}_m = 0$$

$$B\left(\partial_{p_m}\right) = \oint_{C_i} \mathrm{d}w_i b(w_i) w_i^{m+1}$$

$$\forall m \geq 0 \colon b_m|\mathcal{V}_i\rangle = 0, \bar{b}_m|\mathcal{V}_i\rangle = 0$$

$$\omega_p\left(\sum_i Q_B^{(i)} \otimes_i \mathcal{V}_i\right) = (-1)^p \, \mathrm{d}\omega_{p-1}(\otimes \mathcal{V}_i)$$

$$Q_B^{(i)} = 1_{i-1} \otimes Q_B \otimes 1_{n-i}$$

$$Q_B \mathcal{V}_i(z, \bar{z}) = \frac{1}{2\pi \mathrm{i}} \oint \mathrm{d}w j_B(w) \mathcal{V}_i(z, \bar{z}) + \text{ c.c.}$$

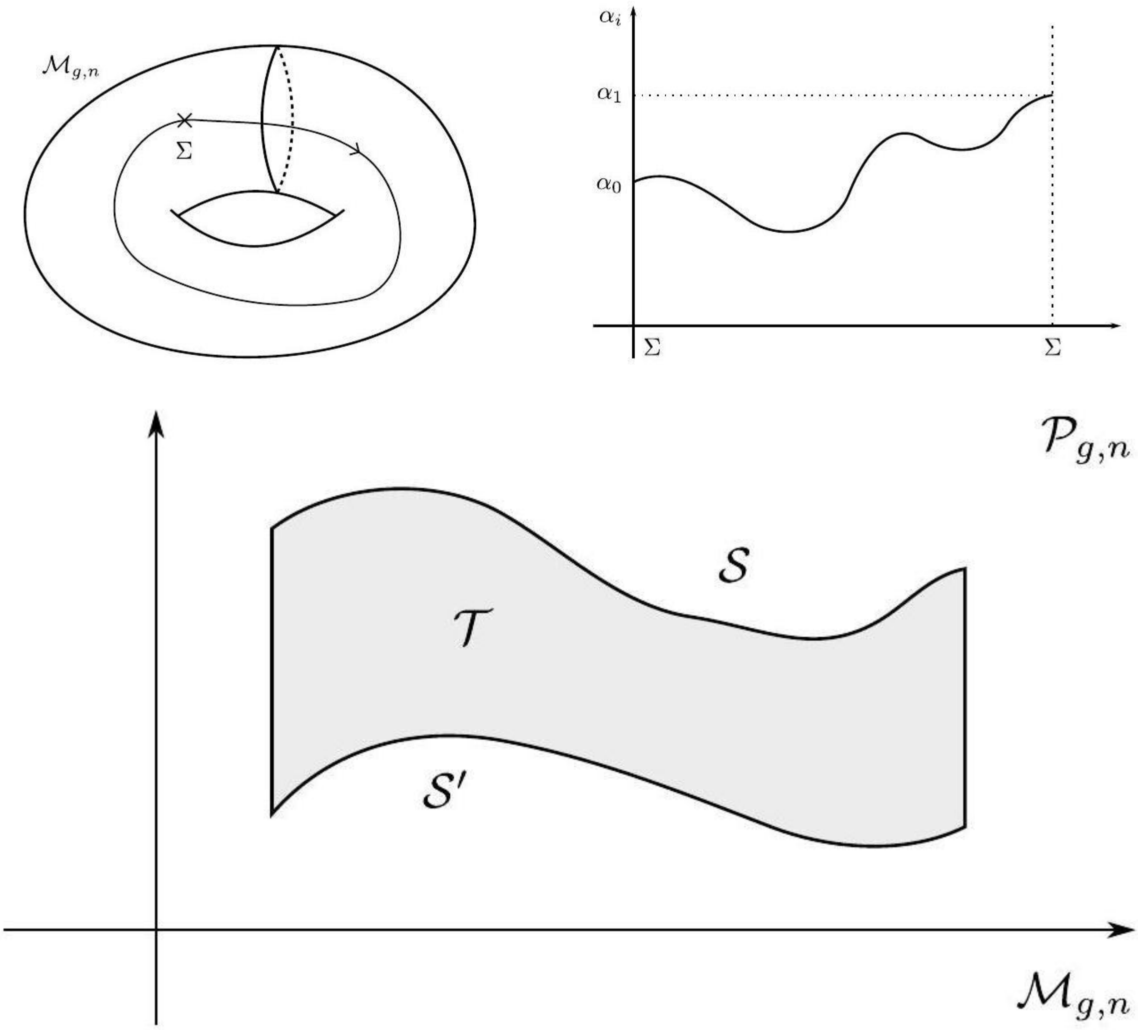
$$\omega_p\left(\sum_i Q_B^{(i)} \otimes_i \mathcal{V}_i\right) = \omega_p(Q_B \mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_n) + (-1)^{|\mathcal{V}_1|} \omega_p(\mathcal{V}_1, Q_B \mathcal{V}_2, \dots, \mathcal{V}_n)$$
$$+ \cdots + (-1)^{|\mathcal{V}_1| + \cdots + |\mathcal{V}_{n-1}|} \omega_p(\mathcal{V}_1, \mathcal{V}_2, \dots, Q_B \mathcal{V}_n)$$

$$T_s = \{Q_B, B_s\}$$

$$\mathrm{d}x_s\{Q_B, B_s\} = \mathrm{d}x_s T_s = \mathrm{d}x_s \partial_s = \mathrm{d}$$

$$N_{\text{gh}}\left(\omega_p(\mathcal{V}_1, \dots, \mathcal{V}_n)\right) = \sum_{i=1}^{n} N_{\text{gh}}(\mathcal{V}_i) - p = 6 - 6g$$

$$N_{\text{gh}}\left(\omega_{\mathrm{M}_{g,n}}\right) = 6 - 6g \implies \sum_{i=1}^{n} N_{\text{gh}}(\mathcal{V}_i) = 2n$$



Figuras 21 y 22. Deformación local del espacio – tiempo cuántico y fluctuaciones de materia y energía de la partícula supermasiva.

$$L_0^-|\mathcal{V}_i\rangle = 0$$

$$b_0^-|\mathcal{V}_i\rangle = 0$$

$$\mathcal{V}_i \in \mathcal{H}^- \cap \ker L_0^-,$$

$$\mathcal{V}_i(0) \longrightarrow \left(\mathrm{e}^{\mathrm{i}\alpha_i}\right)^h \left(\mathrm{e}^{-\mathrm{i}\alpha_i}\right)^{\bar{h}} \mathcal{V}_i(0)$$

$$|\mathcal{V}_i\rangle \longrightarrow \mathrm{e}^{\mathrm{i}\alpha_i(L_0-\bar{L}_0)}|\mathcal{V}_i\rangle$$

$$\forall i\colon Q_B|\mathcal{V}_i\rangle = 0 \implies \mathrm{d}\omega_{p-1}(\mathcal{V}_1, \dots, \mathcal{V}_n) = 0.$$

$$\int_{\mathcal{S}} \omega_{\mathrm{M}_{g,n}} - \int_{\mathcal{S}'} \omega_{\mathrm{M}_{g,n}} = \int_{\partial\mathcal{T}} \omega_{\mathrm{M}_{g,n}-1} = \int_{\mathcal{T}} \mathrm{d}\omega_{\mathrm{M}_{g,n}-1} = 0$$

$$|\mathcal{V}_1\rangle = Q_B|\Lambda\rangle, Q_B|\mathcal{V}_i\rangle = 0$$

$$\omega_{\mathrm{M}_{g,n}}(Q_B\Lambda,\mathcal{V}_2,\dots,\mathcal{V}_n)=\mathrm{d}\omega_{\mathrm{M}_{g,n}-1}(\Lambda,\mathcal{V}_2,\dots,\mathcal{V}_n),$$

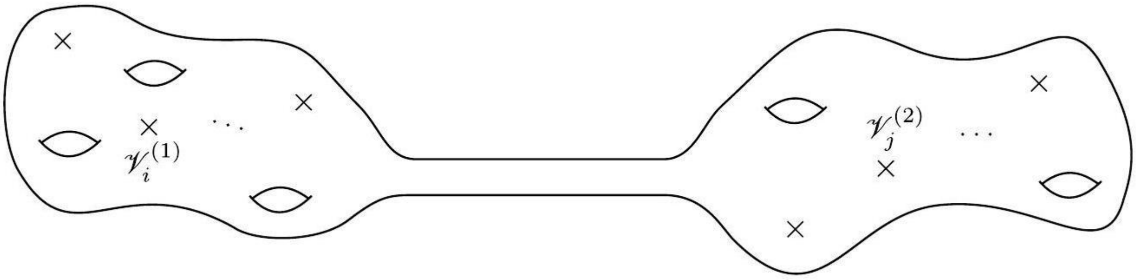
$$\int_{\mathcal{S}}\omega_{\mathrm{M}_{g,n}}(Q_B\Lambda,\mathcal{V}_2,\dots,\mathcal{V}_n)=\int_{\mathcal{S}}\mathrm{d}\omega_{\mathrm{M}_{g,n}-1}(\Lambda,\mathcal{V}_2,\dots,\mathcal{V}_n)=\int_{\partial\mathcal{S}}\omega_{\mathrm{M}_{g,n}-1}(\Lambda,\mathcal{V}_2,\dots,\mathcal{V}_n)$$

$$\int_{\mathcal{S}}\omega_{\mathrm{M}_{g,n}}(Q_B\Lambda,\mathcal{V}_2,\dots,\mathcal{V}_n)=0$$

Factorización de amplitudes y diagramas de Feynman.

$$w_{n_1}^{(1)}w_{n_2}^{(2)}=q.$$

$$A_{g,n}=\int_{\mathcal{S}_{g,n}}\omega_{\mathrm{M}_{g,n}}^{g,n}\left(\mathcal{V}_1^{(1)},\dots,\mathcal{V}_{n_1-1}^{(1)},\mathcal{V}_1^{(2)},\dots,\mathcal{V}_{n_2-1}^{(2)}\right).$$



$$w_{n_1}'^{(1)}=w_{n_1}^{(1)}+\frac{w_{n_1}^{(1)}}{q}\delta q$$

$$v_q=\frac{w_{n_1}^{(1)}}{q},B_q=\frac{1}{q}\oint_{C_q}\mathrm{d}w_{n_1}^{(1)}b\left(w_{n_1}^{(1)}\right)w_{n_1}^{(1)}$$

$$w_{n_1}'^{(1)}w_{n_2}^{(2)}=q+\delta q$$

$$w_{n_1}'^{(1)}=\frac{w_{n_1}^{(1)}}{q}(q+\delta q)=w_{n_1}^{(1)}+\frac{q}{w_{n_1}^{(1)}}\delta q$$

$$\omega_{\mathrm{M}_{g,n}}\left(V_1^{(1)},\dots,V_{\mathrm{M}_{g_1,n_1}^{(1)}}^{(2)},\partial_q,\partial_{\bar{q}},V_1^{(2)},\dots,V_{\mathrm{M}_{g_2,n_2}}^{(2)}\right.$$
$$=(2\pi\mathrm{i})^{-\mathrm{M}_{g,n}^c}\left\langle\prod_{\lambda=1}^{\mathrm{M}_{g_1,n_1}}B\left(V_\lambda^{(1)}\right)B(\partial_q)B(\partial_{\bar{q}})\prod_{\kappa=1}^{\mathrm{M}_{g_2,n_2}}B\left(V_\kappa^{(2)}\right)\prod_{i=1}^{n_1-1}\mathcal{V}_i^{(1)}\prod_{j=1}^{n_2-1}\mathcal{V}_j^{(2)}\right\rangle_{\Sigma_{g,n}}$$

$$\left\langle\Sigma_{n_1}\mid\mathcal{V}_{n_1}^{(1)}\right\rangle:=\omega_{\mathrm{M}_{g_1,n_1}}\left(\mathcal{V}_1^{(1)},\dots,\mathcal{V}_{n_1}^{(1)}\right)=(2\pi\mathrm{i})^{-\mathrm{M}_{g_1,n_1}^c}\left\langle\prod_{\lambda=1}^{\mathrm{M}_{g_1,n_1}}B\left(V_\lambda^{(1)}\right)\prod_{i=1}^{n_1-1}\mathcal{V}_i^{(1)}\right\rangle_{\Sigma_{g_1,n_1}}$$

$$\left\langle\Sigma_{n_2}\mid\mathcal{V}_{n_2}^{(2)}\right\rangle:=\omega_{\mathrm{M}_{g_2,n_2}}\left(\mathcal{V}_1^{(2)},\dots,\mathcal{V}_{n_2}^{(2)}\right)=(2\pi\mathrm{i})^{-\mathrm{M}_{g_2,n_2}^c}\left\langle\prod_{\lambda=1}^{\mathrm{M}_{g_1,n_1}}B\left(V_\lambda^{(2)}\right)\prod_{j=1}^{n_2-1}\mathcal{V}_j^{(2)}\right\rangle_{\Sigma_{g_2,n_2}}$$

$$\langle\Sigma_{n_1}| = \langle 0|I\circ\Sigma_{n_1}(0), \langle\Sigma_{n_2}| = \langle 0|I\circ\Sigma_{n_2}(0),$$

$$\left\langle\Sigma_{n_1}\mid\mathcal{V}_{n_1}^{(1)}\right\rangle = \left\langle I\circ\Sigma_{n_1}(0)\mathcal{V}_{n_1}(0)\right\rangle_{w_{n_1}^{(1)}}, \left\langle\Sigma_{n_2}\mid\mathcal{V}_{n_2}^{(2)}\right\rangle = \left\langle I\circ\Sigma_{n_2}(0)\mathcal{V}_{n_2}(0)\right\rangle_{w_{n_2}^{(2)}}$$

$$w_1 = I\left(w_{n_1}^{(1)}\right), w_2 = I\left(w_{n_2}^{(2)}\right)$$

$$w_{n_1}^{(1)} = \frac{q}{w_{n_2}^{(2)}} = \frac{q}{I(w_2)} = qw_2 := f(w_2)$$

$$\omega_{\mathrm{M}_{g,n}} = \frac{1}{2\pi\mathrm{i}}\left\langle I\circ\Sigma_{n_1}(0)B_qB_{\bar{q}}f\circ\Sigma_{n_2}(0)\right\rangle_{w_{n_1}^{(1)}} = \frac{1}{2\pi\mathrm{i}}\langle\Sigma_{n_1}|B_qB_{\bar{q}}q^{L_0}\bar{q}^{\bar{L}_0}|\Sigma_2\rangle,$$

$$\langle\Sigma_{n_1}|B_qB_{\bar{q}}\left|\mathcal{V}_{n_1}^{(1)}\right\rangle = \frac{1}{q\bar{q}}\langle\Sigma_{n_1}|b_0\bar{b}_0\left|\mathcal{V}_{n_1}^{(1)}\right\rangle$$

$$B_q\mathcal{V}_{n_1}^{(1)}(z,\bar{z}) = \frac{1}{q}\oint_{C_q}\mathrm{d}w_{n_1}^{(1)}b\left(w_{n_1}^{(1)}\right)w_{n_1}^{(1)}\mathcal{V}_{n_1}^{(1)}(z,\bar{z}) \longrightarrow \frac{1}{q}b_0\left|\mathcal{V}_{n_1}^{(1)}\right\rangle$$

$$\omega_{\mathrm{M}_{g,n}} = \frac{1}{2\pi\mathrm{i}}\frac{1}{q\bar{q}}\langle\Sigma_{n_1}|b_0\bar{b}_0q^{L_0}\bar{q}^{\bar{L}_0}|\Sigma_{n_2}\rangle.$$

$$\mathcal{F}_{g,n}\left(\mathcal{V}_i^{(1)}\mid\mathcal{V}_j^{(2)}\right) := \frac{1}{2\pi\mathrm{i}}\int\bigwedge_{\lambda=1}^{\mathrm{M}_{g_1,n_1}}\mathrm{d}t_\lambda^{(1)}\bigwedge_{\kappa=1}^{\mathrm{M}_{g_2,n_2}}\mathrm{d}t_\kappa^{(2)}\wedge\frac{\mathrm{d}q}{q}\wedge\frac{\mathrm{d}\bar{q}}{\bar{q}}\langle\Sigma_{n_1}|b_0\bar{b}_0q^{L_0}\bar{q}^{\bar{L}_0}|\Sigma_{n_2}\rangle.$$

$$\begin{aligned}\mathcal{F}_{g,n}&\left(\mathcal{V}_i^{(1)}\mid\mathcal{V}_j^{(2)}\right)\\ &= \frac{1}{2\pi\mathrm{i}}\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\frac{\mathrm{d}^Dk'}{(2\pi)^D}(-1)^{|\phi_\alpha|}\times\int\frac{\mathrm{d}q}{q}\\ &\wedge\frac{\mathrm{d}\bar{q}}{\bar{q}}\langle\phi_\alpha(k)^c|b_0\bar{b}_0q^{L_0}\bar{q}^{\bar{L}_0}|\phi_\beta(k')^c\rangle\\ &\times\int\bigwedge_{\lambda=1}^{\mathrm{M}_{g_1,n_1}}\mathrm{d}t_\lambda^{(1)}\langle\Sigma_{n_1}\mid\phi_\alpha(k)\rangle\bigwedge_{\kappa=1}^{\mathrm{M}_{g_2,n_2}}\mathrm{d}t_\kappa^{(2)}\langle\phi_\beta(k')\mid\Sigma_{n_2}\rangle\end{aligned}$$

$$\begin{aligned}A_{g_1,n_1}\left(\mathcal{V}_1^{(1)},\ldots,\mathcal{V}_{n_1-1}^{(1)},\phi_\alpha(k)\right) &= \int_{\mathcal{S}_{g_1,n_1}}\omega_{\mathrm{M}_{g_1,n_1}}\left(\mathcal{V}_1^{(1)},\ldots,\mathcal{V}_{n_1-1}^{(1)},\phi_\alpha(k)\right)\\ &= \int_{\mathcal{S}_{g_1,n_1}}\bigwedge_{\lambda=1}^{\mathrm{M}_{g_1,n_1}}\mathrm{d}t_\lambda^{(1)}\langle\Sigma_{n_1}\mid\phi_\alpha(k)\rangle\end{aligned}$$

$$A_{g_2,n_2}\left(\mathcal{V}_1^{(2)},\ldots,\mathcal{V}_{n_2-1}^{(2)},\phi_\beta(k')\right)=\int_{\mathcal{S}_{g_2,n_2}}\omega_{\mathrm{M}_{g_2,n_2}}\left(\mathcal{V}_1^{(2)},\ldots,\mathcal{V}_{n_2-2}^{(2)},\phi_\beta(k')\right)$$
$$=\int_{\mathcal{S}_{g_2,n_2}}\bigwedge_{\lambda=1}^{\mathrm{M}_{g_2,n_2}}\mathrm{d}t_\lambda^{(2)}\langle\Sigma_{n_2}\mid\phi_\beta(k')\rangle$$

$$\Delta_{\alpha\beta}(k,k'):=\Delta\left(\phi_\alpha(k)^c,\phi_\beta(k')^c\right):=\frac{1}{2\pi\mathrm{i}}\int\frac{\mathrm{d}q}{q}\wedge\frac{\mathrm{d}\bar{q}}{\bar{q}}\langle\phi_\alpha(k)^c|b_0\bar{b}_0q^{L_0}\bar{q}^{\bar{L}_0}|\phi_\beta(k')^c\rangle$$

$$\mathcal{F}_{g,n}\left(\mathcal{V}_i^{(1)}\mid\mathcal{V}_j^{(2)}\right)=\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\frac{\mathrm{d}^Dk'}{(2\pi)^D}A_{g_1,n_1}\left(\mathcal{V}_1^{(1)},\ldots,\mathcal{V}_{n_1-1}^{(1)},\phi_\alpha(k)\right)\Delta_{\alpha\beta}(k,k')$$
$$\times A_{g_2,n_2}\left(\mathcal{V}_1^{(2)},\ldots,\mathcal{V}_{n_2-1}^{(2)},\phi_\beta(k')\right)$$

$$A_{g_1,n_1}\sim\delta^{(D)}\left(k_1^{(1)}+\cdots+k_{n_1-1}^{(1)}+k\right),A_{g_2,n_2}\sim\delta^{(D)}\left(k_1^{(2)}+\cdots+k_{n_2-1}^{(2)}+k'\right).$$

$$\mathcal{F}_{g,n}\sim\delta^{(D)}\left(k_1^{(1)}+\cdots+k_{n_1-1}^{(1)}+k_1^{(2)}+\cdots+k_{n_2-1}^{(2)}\right)$$

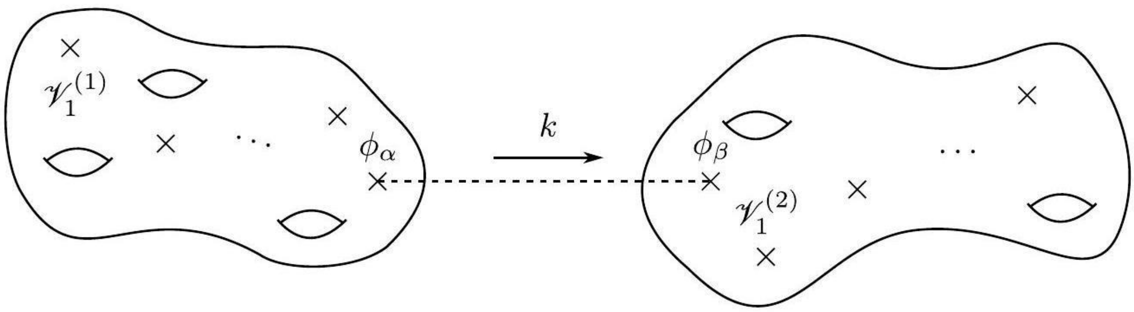
$$N_{\mathrm{gh}}(\phi_\alpha)=2n_1-\sum_{i=1}^{n_1-1}N_{\mathrm{gh}}\left(\mathcal{V}_i^{(1)}\right),N_{\mathrm{gh}}(\phi_\beta)=2n_2-\sum_{j=1}^{n_2-1}N_{\mathrm{gh}}\left(\mathcal{V}_j^{(2)}\right)$$

$$N_{\mathrm{gh}}(\phi_\alpha)+N_{\mathrm{gh}}(\phi_\beta)=4$$

$$N_{\mathrm{gh}}(\phi_\alpha)=N_{\mathrm{gh}}(\phi_\beta)=2$$

$$w_{n_1-1}w_{n_1}=q$$

$$A_{g,n}=\int_{\mathcal{S}_{g,n}}\omega_{\mathrm{M}_{g,n}}^{g,n}\left(\mathcal{V}_1^{(1)},\ldots,\mathcal{V}_{n_1-2}^{(1)}\right)$$



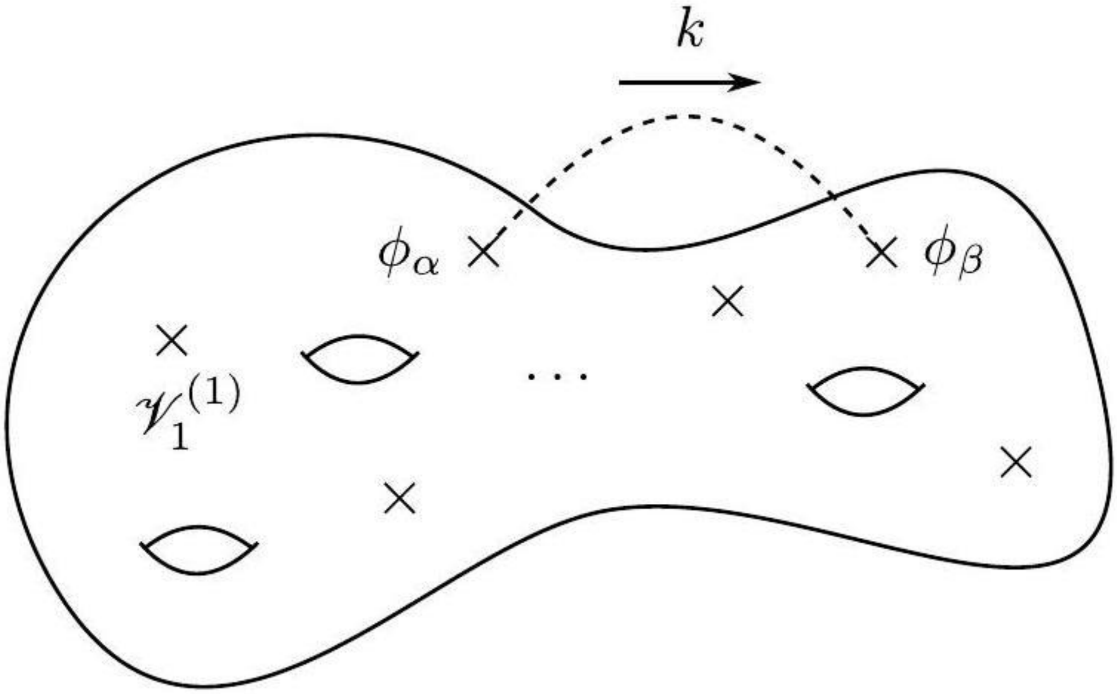
$$\omega_{\mathrm{M}_{g,n}}\left(V_1^{(1)},\ldots,V_{\mathrm{M}_{g_1,n_1}}^{(1)},\partial_q,\partial_{\bar{q}}\right)=(2\pi\mathrm{i})^{-\mathrm{M}_{g,n}^c}\left\langle\prod_{\lambda=1}^{\mathrm{M}_{g_1,n_1}}B\left(V_\lambda^{(1)}\right)B(\partial_q)B(\partial_{\bar{q}})\prod_{i=1}^{n_1-2}\mathcal{V}_i^{(1)}\right\rangle_{\Sigma_{g,n}}$$

$$\left\langle\Sigma_{n_1-1,n_1}\mid\mathcal{V}_{n_1-1}^{(1)}\otimes\mathcal{V}_{n_1}^{(1)}\right\rangle:=\omega_{\mathrm{M}_{g_1,n_1}}\left(\mathcal{V}_1^{(1)},\ldots,\mathcal{V}_{n_1}^{(1)}\right)$$

$$\mathcal{F}_{g,n}\left(\mathcal{V}_i^{(1)}\,|\right) = \int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{\mathrm{d}^D k'}{(2\pi)^D} A_{g_1,n_1}\left(\mathcal{V}_1^{(1)},\dots,\mathcal{V}_{n_1-2}^{(1)},\phi_\alpha(k),\phi_\beta(k')\right)\Delta_{\alpha\beta}(k,k')$$

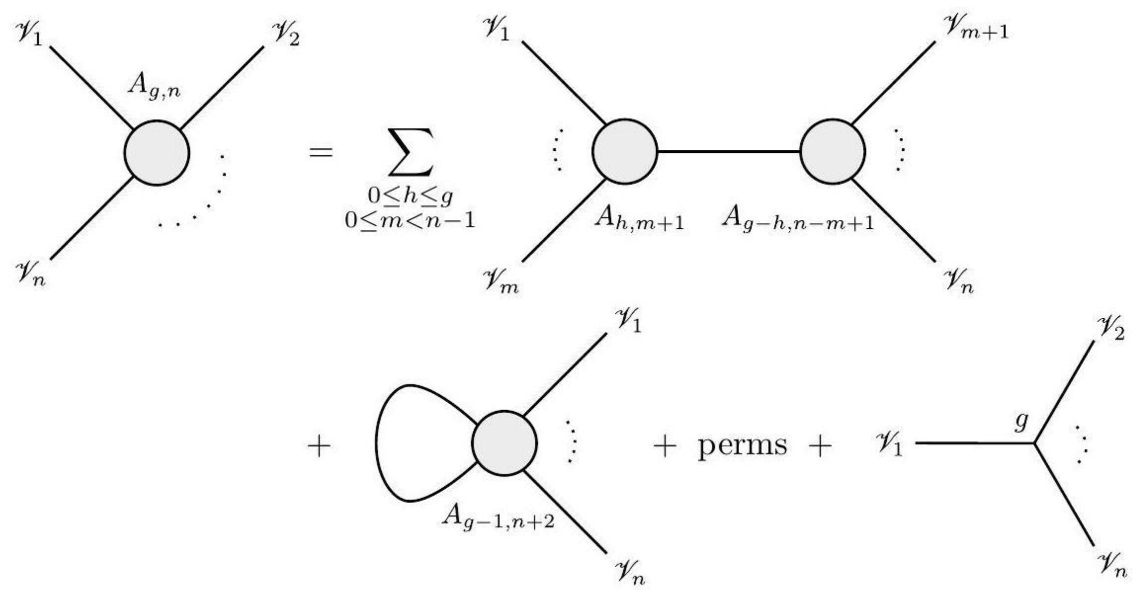
$$N_{\mathrm{gh}}(\phi_\alpha) + N_{\mathrm{gh}}(\phi_\beta) = 2n_1 - \sum_{i=1}^{n_1-2} N_{\mathrm{gh}}\left(\mathcal{V}_i^{(1)}\right) = 4$$

$$A_{g_1,n_1}\left(\mathcal{V}_1^{(1)},\dots,\mathcal{V}_{n_1-2}^{(1)},\phi_\alpha(k),\phi_\beta(-k)\right) \sim \delta^{(D)}\left(k_1^{(1)} + \cdots + k_{n_1-2}^{(1)}\right)$$



$$r(A_{g,n}) := r(\Sigma_{g,n}) = 3g + n - 2$$

$$\mathcal{V}_{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n) := \mathcal{V}_1 \overset{g}{\rightarrow}/^{\mathcal{V}_2}_{\mathcal{V}_n} := \int_{\mathcal{R}_{g,n}} \omega^{g,n}_{\mathrm{M}_{g,n}}(\mathcal{V}_1,\dots,\mathcal{V}_n),$$



$$\mathcal{V}_{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n) := \mathcal{V}_{g,n}(\otimes_i \mathcal{V}_i) := \{\mathcal{V}_1,\dots,\mathcal{V}_n\}_g.$$

Escalar QFT.

Propagador.

$$\Delta = \frac{1}{2\pi\mathrm{i}}\int \frac{\mathrm{d}q}{q}\wedge\frac{\mathrm{d}\bar{q}}{\bar{q}}\, b_0\bar{b}_0 q^{L_0}\bar{q}^{\bar{L}_0}$$

$$q = \mathrm{e}^{-s+\mathrm{i}\theta}, s\in\mathbb{R}_+, \theta\in[0,2\pi)$$

$$\frac{\mathrm{d}q}{q}\wedge\frac{\mathrm{d}\bar{q}}{\bar{q}} = -2\mathrm{i}\mathrm{d}s\wedge\,\mathrm{d}\theta$$

$$\Delta = \frac{1}{2\pi} b_0^+ b_0^- \int_0^\infty \mathrm{d}s\mathrm{e}^{-sL_0^+}\int_0^{2\pi}\,\mathrm{d}\theta\mathrm{e}^{\mathrm{i}\theta L_0^-}$$

$$\int_0^\infty \mathrm{d}s\mathrm{e}^{-sL_0^+} = \frac{1}{L_0^+}, \int_0^{2\pi}\,\mathrm{d}\theta\mathrm{e}^{\mathrm{i}\theta L_0^-} = 2\pi\delta_{L_0^-,0}$$

$$\Delta = \frac{b_0^+}{L_0^+} b_0^- \delta_{L_0^-,0}$$

$$\Delta = -2b_0\bar{b}_0\frac{1}{L_0+\bar{L}_0}\delta_{L_0,\bar{L}_0}$$

$$L_0^+|\phi_\alpha(k)\rangle = \frac{\alpha'}{2}(k^2+m_\alpha^2)|\phi_\alpha(k)\rangle, L_0^-|\phi_\alpha(k)\rangle = 0$$

$$\Delta_{\alpha\beta}(k,k') := \langle \phi_\alpha(k)^c | \Delta \big| \phi_\beta(k')^c \big\rangle := (2\pi)^D \delta^{(D)}(k+k')\Delta_{\alpha\beta}(k),$$
$$\Delta_{\alpha\beta}(k) := \frac{M_{\alpha\beta}(k)}{k^2+m_\alpha^2}, M_{\alpha\beta}(k) := \frac{2}{\alpha'}\langle \phi_\alpha^c(k) | b_0^+ b_0^- \big| \phi_\beta^c(-k) \big\rangle,$$

$$b_0^+ |\phi_\alpha^c\rangle \neq 0, b_0^- |\phi_\alpha^c\rangle \neq 0$$

$$c_0^+ |\phi_\alpha^c\rangle = 0, c_0^- |\phi_\alpha^c\rangle = 0$$

$$|\phi_\alpha^c\rangle = |\phi_1\rangle + b_0^\pm |\phi_2\rangle, c_0^\pm |\phi_1\rangle = c_0^\pm |\phi_2\rangle = 0$$

$$c_0^\pm |\phi_\alpha^c\rangle = 0 \implies |\phi_2\rangle = 0$$

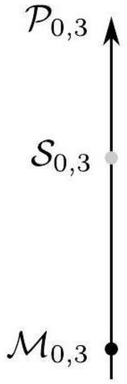
$$b_0^+ |\phi_\alpha\rangle = 0, b_0^- |\phi_\alpha\rangle = 0$$

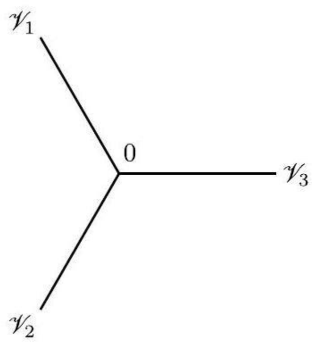
$$b_0^+ |\mathcal{V}_i\rangle = 0$$

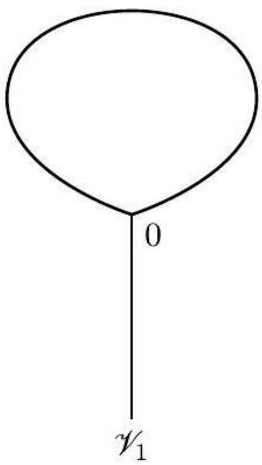
$$\Delta^{-1} = c_0^+ c_0^- L_0^+ \delta_{L_0^-,0}$$

Vértices fundamentales.

$$\mathcal{V}_{0,3}(\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3) := A_{0,3}(\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3) = \omega_0^{0,3}(\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3).$$







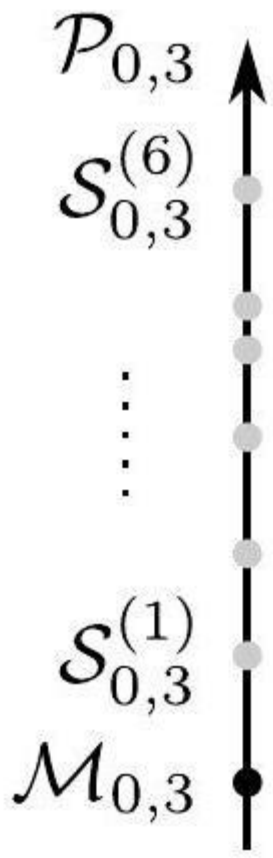
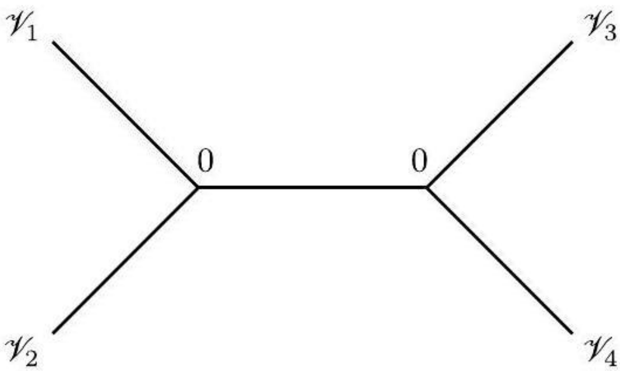


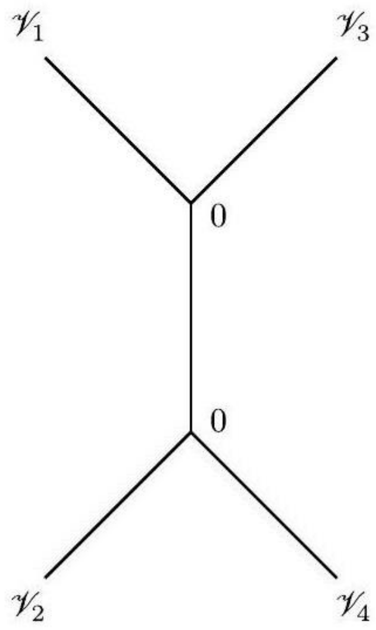
Figura 23, 23A y 23B. Operadores de vértice.

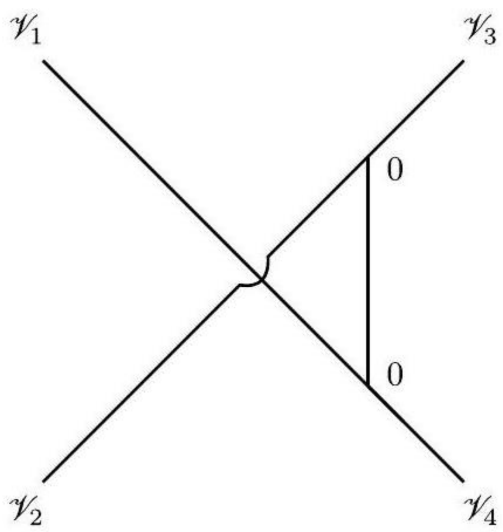
$$\mathcal{V}_{0,4}(\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3,\mathcal{V}_4) := \int_{\mathcal{R}_{0,4}} \omega_2^{0,4}(\mathcal{V}_1,\ldots,\mathcal{V}_4)$$

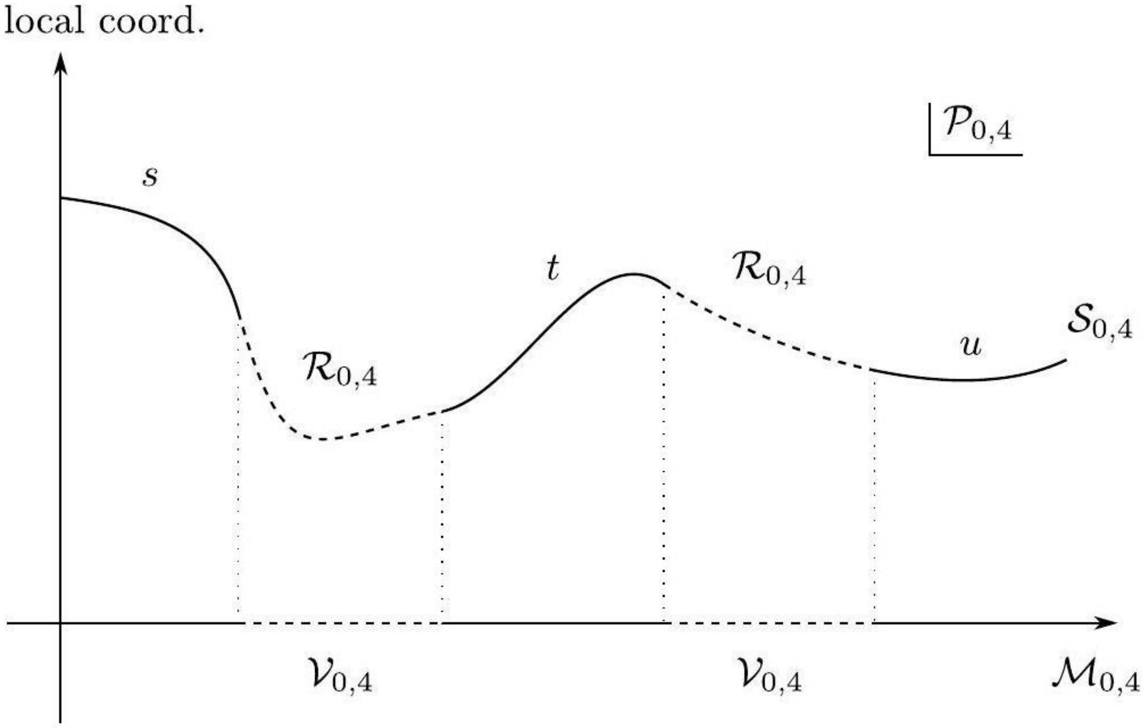
$$A_{0,4} = \mathcal{F}_{0,4}^{(s)} + \mathcal{F}_{0,4}^{(t)} + \mathcal{F}_{0,4}^{(u)} + \mathcal{V}_{0,4}$$

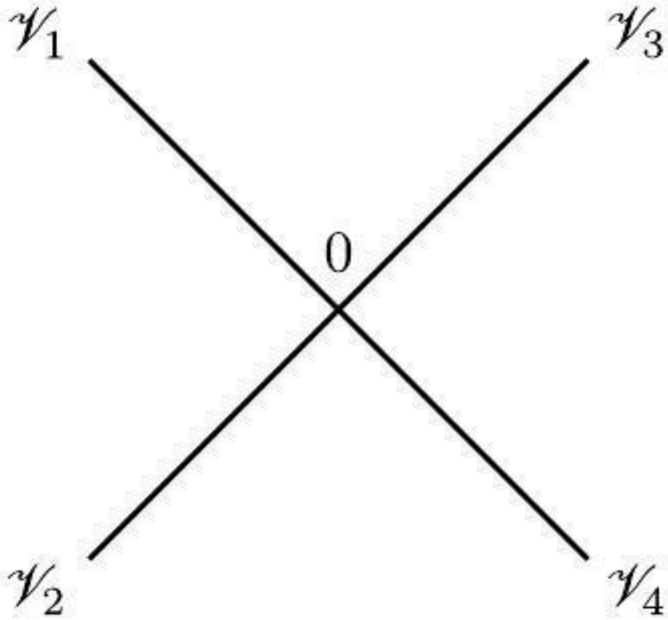
$$\mathcal{V}_{0,5}(\mathcal{V}_1,\ldots,\mathcal{V}_5) := \int_{\mathcal{R}_{0,5}} \omega_4^{0,5}(\mathcal{V}_1,\ldots,\mathcal{V}_5)$$

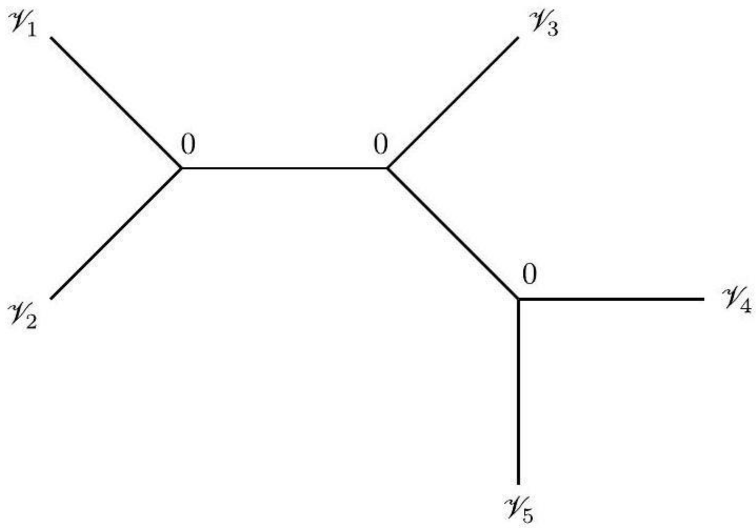


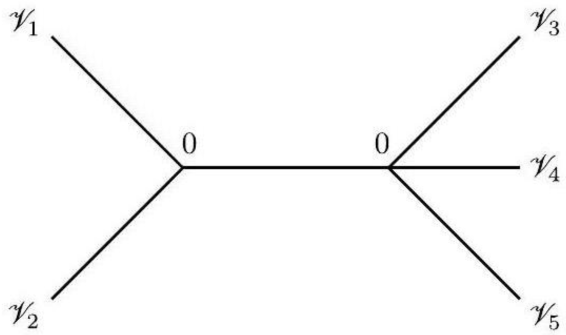

$\mathscr{V}_1$
$\mathscr{V}_3$
0
0
$\mathscr{V}_2$
$\mathscr{V}_4$

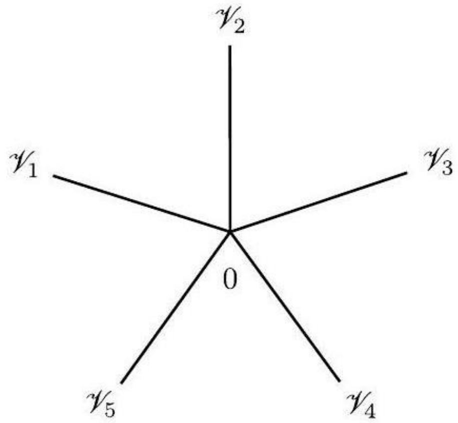

$\mathscr{V}_1$
$\mathscr{V}_3$
0
0
$\mathscr{V}_2$
$\mathscr{V}_4$


local coord.
$\mathcal{P}_{0,4}$
s
t
$\mathcal{R}_{0,4}$
$\mathcal{R}_{0,4}$
u
$\mathcal{S}_{0,4}$
$\mathcal{V}_{0,4}$
$\mathcal{V}_{0,4}$
$\mathcal{M}_{0,4}$


$\mathscr{V}_1$
$\mathscr{V}_3$
0
$\mathscr{V}_2$
$\mathscr{V}_4$







Figuras 24, 25, 26, 27, 28, 29 y 30. Diagramas de Feynman de una partícula supermasiva.

$$\mathcal{V}_{0,2} := \Delta^{-1}\Delta\Delta^{-1} = \Delta^{-1}$$

$$\mathcal{V}_{0,2}(\mathcal{V}_1, \mathcal{V}_2) := \langle \mathcal{V}_1 | c_0^+ c_0^- L_0^+ \delta_{L_0^-,0} | \mathcal{V}_2 \rangle.$$

$$\mathcal{V}_n(\mathcal{V}_1, \dots, \mathcal{V}_n) := \sum_{g \geq 0} (\hbar g_s^2)^g \mathcal{V}_{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n)$$

$$q = \mathrm{e}^{-s+\mathrm{i}\theta}, s \in [s_0, \infty), \theta \in [0,2\pi).$$

$$\int_{s_0}^{\infty} \mathrm{d}s \mathrm{e}^{-sL_0^+} = \frac{\mathrm{e}^{-s_0 L_0^+}}{L_0^+}$$

$$\Delta(s_0) = b_0^+ \frac{\mathrm{e}^{-s_0 L_0^+}}{L_0^+} b_0^- \delta_{L_0^-,0}.$$

$$\Delta_{\alpha\beta}(k) := \frac{\mathrm{e}^{-\frac{\alpha' s_0}{2}(k^2+m_\alpha^2)}}{k^2+m_\alpha^2} M_{\alpha\beta}(k)$$

$$\mathcal{V}_{g,n}^{1\mathrm{PI}}(\mathcal{V}_1,\dots,\mathcal{V}_n) := \mathcal{V}_1 \;\overset{g/1\mathrm{PI}}{\widetilde{\mathcal{V}_n}} \; := \int_{\mathcal{R}_{g,n}^{1\mathrm{PI}}} \omega_{\mathrm{M}_{g,n}}^{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n),$$

$$\langle \mathcal{V}^{g,n} \,| \otimes_i \mathcal{V}_i \rangle := \mathcal{V}_{g,n}(\otimes_i \mathcal{V}_i).$$

$$\mathcal{V}_{g,n+1}(\mathcal{V}_0,\mathcal{V}_1,\dots,\mathcal{V}_n) := \langle \mathcal{V}_0 | c_0^- \big| \ell_{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n) \big\rangle.$$

$$\ell_{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n) := [\mathcal{V}_1,\dots,\mathcal{V}_n]_g$$

$$\ell_{g,0} := [\cdot]_g \in \mathcal{H}.$$

$$N_{\mathrm{gh}}\left(\ell_{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n)\right) = 3 - 2n + \sum_{i=1}^{n} N_{\mathrm{gh}}(\mathcal{V}_i) = 3 + \sum_{i=1}^{n} \left(N_{\mathrm{gh}}(\mathcal{V}_i) - 2\right),$$

$$\left|\ell_{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n)\right| = 1 + \sum_{i=1}^{n} |\mathcal{V}_i| \bmod 2,$$

$$0 = \sum_{\substack{g_1,g_2\geq 0\\ g_1+g_2=g}} \sum_{\substack{n_1,n_2\geq 0\\ n_1+n_2=n}} \frac{n!}{n_1!\,n_2!} \mathcal{V}_{g_1,n_1+1}\left(\Psi^{n_1}, \ell_{g_2,n_2}(\Psi^{n_2})\right) + (-1)^{|\phi_s|} \mathcal{V}_{g-1,n+2}(\phi_s, b_0^- \phi_s^c, \Psi^n)$$

$$\mathcal{V}_{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n) = \frac{1}{N} \sum_{a=1}^{N} \int_{\mathcal{R}_{g,n}^{(a)}} \omega_{\mathrm{M}_{g,n}}^{g,n}(\mathcal{V}_1,\dots,\mathcal{V}_n)$$

$$\mathcal{V}_{0,3}(\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3) = \mathcal{V}_{0,3}(\mathcal{V}_3,\mathcal{V}_1,\mathcal{V}_2) + \cdots$$

$$\mathcal{V}_{0,3}(\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3) = \omega_0^{0,3}(\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3)\big|_{\mathcal{S}_{0,3}} = \langle f_1 \circ \mathcal{V}_1(0) f_2 \circ \mathcal{V}_2(0) f_3 \circ \mathcal{V}_3(0) \rangle,$$

$$\mathcal{V}_{0,3}(\mathcal{V}_3,\mathcal{V}_1,\mathcal{V}_2) = \langle f_1 \circ \mathcal{V}_3(0) f_2 \circ \mathcal{V}_1(0) f_3 \circ \mathcal{V}_2(0) \rangle \neq \mathcal{V}_{0,3}(\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),$$

$$g(z_1) = z_2, g(z_2) = z_3, g(z_3) = z_1$$

$$\mathcal{V}_{0,3}(\mathcal{V}_3,\mathcal{V}_1,\mathcal{V}_2) = \langle g \circ f_1 \circ \mathcal{V}_3(0) g \circ f_2 \circ \mathcal{V}_1(0) g \circ f_3 \circ \mathcal{V}_2(0) \rangle.$$

$$g \circ f_1 = f_2, g \circ f_2 = f_3, g \circ f_3 = f_1$$

$$\mathcal{V}_{0,3}(\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3) = \frac{1}{6} \sum_{a=1}^{6} \omega_0^{0,3}(\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3)\Big|_{\mathcal{S}_{0,3}^{(a)}}$$

Espacio de Hilbert en CFT.

$$|\Psi\rangle = \sum_r \psi_r |\phi_r\rangle.$$

$$|\Psi\rangle = \sum_r \left(\psi_{\downarrow\downarrow,r}\left|\phi_{\downarrow\downarrow,r}\right\rangle + \psi_{\downarrow\uparrow,r}\left|\phi_{\downarrow\uparrow,r}\right\rangle + \psi_{\uparrow\downarrow,r}\left|\phi_{\uparrow\downarrow,r}\right\rangle + \psi_{\uparrow\uparrow,r}\left|\phi_{\uparrow\uparrow,r}\right\rangle\right),$$

$$\begin{aligned} b_0\left|\phi_{\downarrow\downarrow,r}\right\rangle &= \bar{b}_0\left|\phi_{\downarrow\downarrow,r}\right\rangle = 0, \quad b_0\left|\phi_{\downarrow\uparrow,r}\right\rangle = \bar{c}_0\left|\phi_{\downarrow\uparrow,r}\right\rangle = 0, \\ c_0\left|\phi_{\uparrow\downarrow,r}\right\rangle &= \bar{b}_0\left|\phi_{\uparrow\downarrow,r}\right\rangle = 0, \quad c_0\left|\phi_{\uparrow\uparrow,r}\right\rangle = \bar{c}_0\left|\phi_{\uparrow\uparrow,r}\right\rangle = 0. \end{aligned}$$

$$\langle \phi_r^c \mid \phi_s \rangle = \delta_{rs}$$

$$\begin{gathered} \left\langle\phi_{\downarrow\downarrow,r}^c\right|c_0 = \left\langle\phi_{\downarrow\downarrow,r}^c\right|\bar{c}_0 = 0, \left\langle\phi_{\downarrow\uparrow,r}^c\right|c_0 = \left\langle\phi_{\downarrow\uparrow,r}^c\right|\bar{b}_0 = 0 \\ \left\langle\phi_{\uparrow\downarrow,r}^c\right|b_0 = \left\langle\phi_{\uparrow\downarrow,r}^c\right|\bar{c}_0 = 0, \left\langle\phi_{\uparrow\uparrow,r}^c\right|b_0 = \left\langle\phi_{\uparrow\uparrow,r}^c\right|\bar{b}_0 = 0, \\ \left\langle\phi_{x,r}^c \mid \phi_{y,s}\right\rangle = \delta_{xy}\delta_{rs}, \end{gathered}$$

$$G(\psi_r) = 2 - n_r$$

$$n_r = N_{\text{gh}}(\phi_r), n_r^c = N_{\text{gh}}(\phi_r^c) = 6 - n_r$$

Gauge fixed y transformaciones de gauge bajo propagador.

$$\Delta = b_0^+ b_0^- \frac{1}{L_0^+}\delta_{L_0^-,0}, \Delta_{rs} = \langle \phi_r^c | b_0^+ b_0^- \frac{1}{L_0^+}\delta_{L_0^-,0} | \phi_s^c \rangle.$$

$$S_{0,2} = \frac{1}{2}\langle \Psi | K | \Psi \rangle = \frac{1}{2}\psi_r K_{rs} \psi_s$$

$$K = c_0^- c_0^+ L_0^+ \delta_{L_0^-,0} \; K_{rs} = \langle \phi_r | c_0^- c_0^+ L_0^+ \delta_{L_0^-,0} | \phi_s \rangle$$

$$K = \frac{1}{2} c_0 \bar{c}_0 L_0^+ \delta_{L_0^-,0}$$

$$\ker K|_{\mathcal{H}} \neq \emptyset$$

$$K_{rs} = \frac{1}{2}\begin{pmatrix} \left\langle\phi_{\downarrow\downarrow,r}\right| \\ \left\langle\phi_{\downarrow\uparrow,r}\right| \\ \left\langle\phi_{\uparrow\downarrow,r}\right| \\ \left\langle\phi_{\uparrow\uparrow,r}\right| \end{pmatrix}^t \begin{pmatrix} c_0\bar{c}_0 L_0^+ & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \left|\phi_{\downarrow\downarrow,s}\right\rangle \\ \left|\phi_{\downarrow\uparrow,s}\right\rangle \\ \left|\phi_{\uparrow\downarrow,s}\right\rangle \\ \left|\phi_{\uparrow\uparrow,s}\right\rangle \end{pmatrix}.$$

$$L_0^- |\Psi\rangle = 0, b_0^- |\Psi\rangle = 0, b_0^+ |\Psi\rangle = 0$$

$$|\Psi\rangle = \sum_r \psi_{\downarrow\downarrow,r}\left|\phi_{\downarrow\downarrow,r}\right\rangle.$$

$$
\begin{aligned}
K_{rs}\Delta_{st} &= \langle \phi_r | c_0^- c_0^+ L_0^+ \delta_{L_0^-,0} | \phi_s \rangle \langle \phi_s^c | b_0^+ b_0^- \frac{1}{L_0^+} \delta_{L_0^-,0} | \phi_t^c \rangle \\
&= \langle \phi_r | c_0^- c_0^+ L_0^+ \delta_{L_0^-,0} b_0^+ b_0^- \frac{1}{L_0^+} \delta_{L_0^-,0} | \phi_t^c \rangle \\
&= \langle \phi_r | \{c_0^-, b_0^-\}\{c_0^+, b_0^+\} | \phi_t^c \rangle \\
&= \langle \phi_r \mid \phi_t^c \rangle = \delta_{rt}.
\end{aligned}
$$

$$
1 = \sum_r |\phi_r\rangle\langle\phi_r^c| = \sum_r |\phi_{\downarrow\downarrow,r}\rangle\langle\phi_{\downarrow\downarrow,r}^c|.
$$

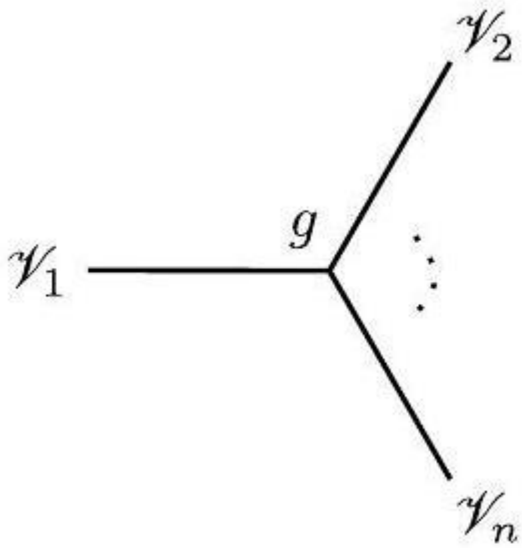
$$
S_{0,2} = \frac{1}{2}\mathcal{V}_{0,2}(\Psi^2) = \frac{1}{2}\langle \Psi | c_0^- c_0^+ L_0^+ \delta_{L_0^-,0} | \Psi \rangle.
$$

$$
S = \int \mathrm{d}^D x \left(\frac{1}{2}\phi(x)(-\partial^2 + m^2)\phi(x) + \frac{\lambda}{n!}\phi(x)^n \right)
$$

$$
\phi_k(x) = \mathrm{e}^{\mathrm{i}k\cdot x}
$$

$$
\begin{aligned}
V_n(k_1, \dots, k_n) &= \frac{\lambda}{n!}\int \mathrm{d}^D x n! \prod_{i=1}^{n} \phi_{k_i}(x) = \lambda \int \mathrm{d}^D x \mathrm{e}^{\mathrm{i}(k_1+\cdots+k_n)x} \\
&= \lambda(2\pi)^D \delta^{(D)}(k_1 + \cdots + k_n)
\end{aligned}
$$

$$
\mathcal{V}_{g,n}(\mathcal{V}_1, \dots, \mathcal{V}_n) = \int_{\mathcal{R}_{g,n}} \omega^{g,n}_{\mathrm{M}_{g,n}}(\mathcal{V}_1, \dots, \mathcal{V}_n) =
$$



$$
S_{g,n} = \hbar^g \frac{g_s^{2g-2+n}}{n!}\mathcal{V}_{g,n}(\Psi^n)
$$

$$
\begin{aligned}
&\mathcal{V}_{g,m}(\mathcal{V}_1, \dots, \mathcal{V}_{m-1}, \phi_r)\langle \phi_r^c | b_0^+ b_0^- \frac{1}{L_0^+} | \phi_s^c \rangle \mathcal{V}_{g',n}(\mathcal{W}_1, \dots, \mathcal{W}_{n-1}, \phi_s) \\
&= \mathcal{V}_{g,m}(\mathcal{V}_1, \dots, \mathcal{V}_{m-1}, \phi_{\downarrow\downarrow,r})\langle \phi_{\downarrow\downarrow,r}^c | b_0^+ b_0^- \frac{1}{L_0^+} | \phi_{\downarrow\downarrow,s}^c \rangle \mathcal{V}_{g',n}(\mathcal{W}_1, \dots, \mathcal{W}_{n-1}, \phi_{\downarrow\downarrow,s})
\end{aligned}
$$

$$S=\sum_{g,n\geq 0}\hbar^g\frac{g_s^{2g-2+n}}{n!}\mathcal{V}_{g,n}(\Psi^n):=\frac{1}{2}\langle\Psi|c_0^-c_0^+L_0^+\delta_{L_0^-,0}|\Psi\rangle+\sum_{g,n\geq 0}{}^{'}\hbar^g\frac{g_s^{2g-2+n}}{n!}\mathcal{V}_{g,n}(\Psi^n)$$

$$\mathcal{V}_{g,n}(\Psi^n):=\langle\Psi|c_0^-\big|\ell_{g,n-1}(\Psi^{n-1})\big\rangle$$

$$S=\sum_{g,n\geq 0}\hbar^g\frac{g_s^{2g-2+n}}{n!}\langle\Psi|c_0^-\big|\ell_{g,n-1}(\Psi^{n-1})\big\rangle$$

$$\ell_{0,1}(\Psi_{\text{cl}})=c_0^+L_0^+|\Psi_{\text{cl}}\rangle$$

$$S=\sum_{\substack{g,n\geq 0\\ \chi_{g,n}\leq 0}}\hbar^g\frac{g_s^{2g-2+n}}{n!}\mathcal{V}_{g,n}(\Psi^n)$$

$$S_{\text{cl}}=\frac{1}{2}\langle\Psi_{\text{cl}}|c_0^-c_0^+L_0^+|\Psi_{\text{cl}}\rangle+\sum_{n\geq 3}\frac{g_s^n}{n!}\mathcal{V}_{0,n}(\Psi_{\text{cl}}^n)$$

$$S=\sum_{g,n\geq 0}\hbar^g g_s^{2g-2}\frac{1}{n!}\mathcal{V}_{g,n}(\Psi^n):=\frac{1}{2g_s^2}\langle\Psi|c_0^-c_0^+L_0^+\delta_{L_0^-,0}|\Psi\rangle+\frac{1}{g_s^2}\sum_{g,n\geq 0}{}^{'}\frac{(\hbar g_s^2)^g}{n!}\mathcal{V}_{g,n}(\Psi^n)$$

$$\frac{S}{\hbar}=\sum_{g,n\geq 0}(\hbar g_s^2)^{g-1}\frac{1}{n!}\mathcal{V}_{g,n}(\Psi^n)$$

$$S\to\alpha S\implies g_s^2\to\frac{g_s^2}{\alpha}$$

$$L_0^+|\Psi\rangle=0$$

$$\Psi_{\text{cl}}\in\mathcal{H}^-\cap\ker L_0^-,N_{\text{gh}}(\Psi_{\text{cl}})=2$$

$$S_{0,2}=\frac{1}{2}\langle\Psi|c_0^-Q_B|\Psi\rangle.$$

$$S_{\text{cl}}=\frac{1}{2}\langle\Psi_{\text{cl}}|c_0^-Q_B|\Psi_{\text{cl}}\rangle+\frac{1}{g_s^2}\sum_{n\geq 3}\frac{g_s^n}{n!}\mathcal{V}_{0,n}(\Psi_{\text{cl}}^n)$$

$$\mathcal{V}_{0,2}\big(\Psi_{\text{cl}}^2\big):=\langle\Psi_{\text{cl}}|c_0^-Q_B|\Psi_{\text{cl}}\rangle$$

$$S_{\text{cl}}=\frac{1}{g_s^2}\sum_{n\geq 2}\frac{g_s^n}{n!}\mathcal{V}_{0,n}(\Psi_{\text{cl}}^n)=\frac{1}{g_s^2}\sum_{n\geq 2}\frac{g_s^n}{n!}\langle\Psi_{\text{cl}}|c_0^-\big|\ell_{0,n-1}\big(\Psi_{\text{cl}}^{n-1}\big)\big\rangle,$$

$$\ell_{0,1}(\Psi_{\text{cl}})=Q_B|\Psi_{\text{cl}}\rangle.$$

$$\mathcal{F}_{\mathrm{cl}}(\Psi_{\mathrm{cl}}) := \sum_{n\geq 1}\frac{g_s^{n-1}}{n!}\ell_{0,n}(\Psi_{\mathrm{cl}}^n) = Q_B|\Psi_{\mathrm{cl}}\rangle + \sum_{n\geq 2}\frac{g_s^{n-1}}{n!}\ell_{0,n}(\Psi_{\mathrm{cl}}^n) = 0$$

$$\delta S_{\mathrm{cl}} = \frac{1}{g_s^2}\sum_{n\geq 2}\frac{g_s^n}{n!}n\{\delta\Psi_{\mathrm{cl}},\Psi_{\mathrm{cl}}^{n-1}\}_0 = \frac{1}{g_s^2}\sum_{n\geq 2}\frac{g_s^n}{(n-1)!}\langle\delta\Psi_{\mathrm{cl}}|c_0^-\Big|\ \ell_{0,n-1}(\Psi_{\mathrm{cl}}^{n-1})\rangle.$$

$$\delta_\Lambda S_{\mathrm{cl}} = 0$$

$$\delta_\Lambda\Psi_{\mathrm{cl}} = \sum_{n\geq 0}\frac{g_s^n}{n!}\ell_{0,n+1}(\Psi_{\mathrm{cl}}^n,\Lambda) = Q_B|\Lambda\rangle + \sum_{n\geq 1}\frac{g_s^n}{n!}\ell_{0,n+1}(\Psi_{\mathrm{cl}}^n,\Lambda)$$

$$[\delta_{\Lambda_2},\delta_{\Lambda_1}]\Psi_{\mathrm{cl}} = \delta_{\Lambda(\Lambda_1,\Lambda_2,\Psi_{\mathrm{cl}})}|\Psi_{\mathrm{cl}}\rangle + \sum_{n\geq 0}\frac{g_s^{n+2}}{n!}\ell_{0,n+3}\big(\Psi_{\mathrm{cl}}^n,\Lambda_2,\Lambda_1,\mathcal{F}_{\mathrm{cl}}(\Psi_{\mathrm{cl}})\big),$$

$$\Lambda(\Lambda_1,\Lambda_2,\Psi_{\mathrm{cl}}) = \sum_{n\geq 0}\frac{g_s^{n+1}}{n!}\ell_{0,n+2}(\Lambda_1,\Lambda_2,\Psi_{\mathrm{cl}}^n) = g_s\ell_{0,2}(\Lambda_1,\Lambda_2) + \sum_{n\geq 1}\frac{g_s^{n+1}}{n!}\ell_{0,n+2}(\Lambda_1,\Lambda_2,\Psi_{\mathrm{cl}}^n)$$

$$\forall n\geq 4: \mathcal{V}_{0,4}(\mathcal{V}_1,\ldots,\mathcal{V}_n) = 0, \ell_{g,n-1}(\mathcal{V}_1,\ldots,\mathcal{V}_{n-1}) = 0, (\text{ cubic theory }),$$

$$\begin{aligned}\delta_\Lambda S_{\mathrm{cl}} &= \sum_{n\geq 2}\frac{g_s^{n-2}}{n!}n\mathcal{V}_{0,n}\big(\delta\Psi_{\mathrm{cl}},\Psi_{\mathrm{cl}}^{n-1}\big) = \sum_{m,n\geq 0}\frac{g_s^{m+n-1}}{m!\,n!}\mathcal{V}_{0,n+1}\big(\ell_{0,m+1}(\Psi_{\mathrm{cl}}^m,\Lambda),\Psi_{\mathrm{cl}}^n\big)\\ &= \sum_{m\geq 0}\sum_{n=0}^{m}\frac{g_s^{m-1}}{(m-n)!\,n!}\langle\ell_{m-n+1}(\Psi_{\mathrm{cl}}^{m-n},\Lambda)|c_0^-|\ell_{0,n}(\Psi_{\mathrm{cl}}^n)\rangle\end{aligned}$$

$$\begin{aligned}&= \big\langle\ell_{0,n}(\Psi_{\mathrm{cl}}^n)\big|c_0^-\big|\ell_{0,m-n+1}(\Psi_{\mathrm{cl}}^{m-n},\Lambda)\big\rangle\\ &= \mathcal{V}_{0,m-n+2}\big(\ell_{0,n}(\Psi_{\mathrm{cl}}^n),\Psi_{\mathrm{cl}}^{m-n},\Lambda\big)\\ &= -\mathcal{V}_{0,m-n+2}\big(\Lambda,\ell_{0,n}(\Psi_{\mathrm{cl}}^n),\Psi_{\mathrm{cl}}^{m-n}\big)\\ &= \langle\Lambda|c_0^-\big|\ell_{0,m-n+1}\big(\ell_{0,n}(\Psi_{\mathrm{cl}}^n),\Psi_{\mathrm{cl}}^{m-n}\big)\big\rangle.\end{aligned}$$

$$0 = \sum_{n=0}^{m}\frac{m!}{(m-n)!\,n!}\ell_{0,m-n+1}\big(\ell_{0,n}(\Psi_{\mathrm{cl}}^n),\Psi_{\mathrm{cl}}^{m-n}\big)$$

Formalismo BV.

$$\begin{aligned}S &= \frac{1}{g_s^2}\sum_{g\geq 0}\hbar^g g_s^{2g}\sum_{n\geq 0}\frac{g_s^n}{n!}\mathcal{V}_{g,n}(\Psi^n)\\ &= \frac{1}{2}\langle\Psi|c_0^-Q_B|\Psi\rangle + {\sum_{g,n\geq 0}}'\frac{\hbar^g g_s^{2g-2+n}}{n!}\mathcal{V}_{g,n}(\Psi^n)\\ &= \frac{1}{g_s^2}\sum_{g,n\geq 0}\frac{\hbar^g g_s^{2g-2+n}}{n!}\langle\Psi|c_0^-\big|\ell_{g,n-1}(\Psi^{n-1})\big\rangle\end{aligned}$$

$$\Psi\in\mathcal{H}^-\cap\ker L_0^-.$$

$$(S,S) - 2\hbar\Delta S = 0$$

$$|\Psi\rangle=\sum_r \psi_r|\phi_r\rangle$$

$$\Psi=\Psi_+ + \Psi_-$$

$$\Psi_- =\sum_r \sum_{n_r\leq 2}|\phi_r\rangle\psi^r, \Psi_+ =\sum_r \sum_{n_r^c>2} b_0^-|\phi_r^c\rangle\psi_r^*$$

$$G(\psi^r)\geq 0, G(\psi_r^*)<0$$

$$G(\psi_r^*)=-1-G(\psi^r)$$

$$\begin{aligned}G(\psi_r^*)&=2-N_{\mathrm{gh}}(b_0^-\phi_r^c)=2+1-n_r^c\\&=3-(6-n_r)=-3+\big(2-G(\psi^r)\big)=-1-G(\psi^r)\end{aligned}$$

$$\frac{\partial_R S}{\partial\psi^r}\frac{\partial_L S}{\partial\psi_r^*}+\hbar\frac{\partial_R\partial_L S}{\partial\psi^r\partial\psi_r^*}=0$$

$$\sum_{\substack{g_1,g_2\geq 0\\ g_1+g_2=g}}\sum_{\substack{n_1,n_2\geq 0\\ n_1+n_2=n}}\frac{\partial_R S_{g_1,n_1}}{\partial\psi^r}\frac{\partial_L S_{g_2,n_2}}{\partial\psi_r^*}+\hbar\frac{\partial_R\partial_L S_{g-1,n}}{\partial\psi^r\partial\psi_r^*}=0$$

$$\begin{aligned}\mathcal{V}_n^{1\mathrm{PI}}(\mathcal{V}_1,\ldots,\mathcal{V}_n)&:=\mathcal{V}_1\longrightarrow\sum_{\mathcal{V}_n}^{\mathcal{V}_2}:=\sum_{g\geq 0}(\hbar g_s^2)^g\mathcal{V}_{g,n}^{1\mathrm{PI}}(\mathcal{V}_1,\ldots,\mathcal{V}_n),\\ \mathcal{V}_{g,n}^{1\mathrm{PI}}(\mathcal{V}_1,\ldots,\mathcal{V}_n)&:=\int_{\mathcal{R}_{g,n}^{1\mathrm{PI}}}\omega_{\mathrm{M}_{g,n}}^{g,n}(\mathcal{V}_1,\ldots,\mathcal{V}_n),\end{aligned}$$

$$S_{1\mathrm{PI}}=\frac{1}{g_s^2}\sum_{n\geq 0}\frac{g_s^n}{n!}\mathcal{V}_n^{1\mathrm{PI}}(\Psi^n):=\frac{1}{2}\langle\Psi|c_0^-c_0^+L_0^+|\Psi\rangle+\frac{1}{g_s^2}\sum_{n\geq 0}{}'\frac{g_s^n}{n!}\mathcal{V}_n^{1\mathrm{PI}}(\Psi^n)$$

$$S_{1\mathrm{PI}}=\frac{1}{2}\langle\Psi|c_0^-Q_B|\Psi\rangle+\frac{1}{g_s^2}\sum_{n\geq 0}{}'\frac{g_s^n}{n!}\mathcal{V}_n^{1\mathrm{PI}}(\Psi^n)$$

Independencia background.

$$\delta S_{\mathrm{cft}}=\frac{\lambda}{2\pi}\int \mathrm{d}^2z\varphi(z,\bar z)$$

$$S_1[\Psi_1]=\frac{1}{g_s^2}\left(\frac{1}{2}\langle\Psi_1|c_0^-Q_B|\Psi_1\rangle+\sum_{n\geq 0}{}'\frac{1}{n!}\mathcal{V}_n^{1\mathrm{PI}}(\Psi_1^n)\right)$$

$$\mathcal{F}_1(\Psi_1)=Q_B|\Psi_1\rangle+\sum_n\frac{1}{n!}\ell_n(\Psi_1^n)=0$$

Deformación de CFT.

$$S_{\text{cft},2}[\psi_1] = S_{\text{cft},1}[\psi_1] + \frac{\lambda}{2\pi}\int \mathrm{d}^2 z \varphi(z,\bar{z})$$

$$\left\langle \prod_i \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle_2 = \left\langle \exp\left(-\frac{\lambda}{2\pi}\int \mathrm{d}^2 z\varphi(z,\bar{z})\right) \prod_i \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle_1$$
$$\approx \left\langle \prod_i \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle_1 - \frac{\lambda}{2\pi}\int_\Sigma \mathrm{d}^2 z \left\langle \varphi(z,\bar{z}) \prod_i \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle_1$$

$$\delta \left\langle \prod_i \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle_1 = -\frac{\lambda}{2\pi}\int_{\Sigma-\cup_i D_i} \mathrm{d}^2 z \left\langle \varphi(z,\bar{z}) \prod_i \mathcal{O}_i(z_i,\bar{z}_i) \right\rangle_1 .$$

$$\delta L_n = \lambda \oint_{|z|=1} \frac{\mathrm{d}\bar{z}}{2\pi \mathrm{i}} z^{n+1}\varphi(z,\bar{z}), \delta \bar{L}_n = \lambda \oint_{|z|=1} \frac{\mathrm{d}z}{2\pi \mathrm{i}} \bar{z}^{n+1}\varphi(z,\bar{z}).$$

$$\delta Q_B = \lambda \oint_{|z|=1} \frac{\mathrm{d}\bar{z}}{2\pi \mathrm{i}} c(z)\varphi(z,\bar{z}) + \lambda \oint_{|z|=1} \frac{\mathrm{d}\bar{z}}{2\pi \mathrm{i}} \bar{c}(\bar{z})\varphi(z,\bar{z}).$$

$$\{Q_B, \delta Q_B\} = O(\lambda^2)$$

$$\delta L_0^- |\mathcal{O}\rangle = \lambda \oint_{|z|=1} \frac{\mathrm{d}\bar{z}}{2\pi \mathrm{i}} z \sum_{p,q} z^{p-1}\bar{z}^{q-1} |\mathcal{O}_{p,q}\rangle - \lambda \oint_{|z|=1} \frac{\mathrm{d}\bar{z}}{2\pi \mathrm{i}} \sum_{p,q} z^{p-1}\bar{z}^{q-1} |\mathcal{O}_{p,q}\rangle,$$

$$\varphi(z,\bar{z})\mathcal{O}(0,0) = \sum_{p,q} z^{p-1}\bar{z}^{q-1}\mathcal{O}_{p,q}(0,0)$$

$$S_2[\Psi_1] = S_1[\Psi_1] + \delta S_1[\Psi_1]$$

$$\delta S_1[\Psi_1] = \frac{1}{g_s^2}\left(\frac{1}{2}\langle \Psi_1 | c_0^- \delta Q_B | \Psi_1\rangle + \sum_{n\geq 0} \frac{1}{n!}\delta \mathcal{V}_n(\Psi_1^n)\right)$$

$$\mathcal{F}_2(\Psi_1) = \mathcal{F}_1(\Psi_1) + \lambda\delta\mathcal{F}_1(\Psi_1) = 0$$

$$\lambda\delta\mathcal{F}_1(\Psi_1) = \delta Q_B|\Psi_1\rangle + \sum_n \frac{1}{n!}\delta \ell_n(\Psi_1^n)$$

$$|\Psi_1\rangle = \lambda|\Psi_0\rangle, |\Psi_0\rangle = c_1\bar{c}_1(0)|\varphi\rangle$$

$$|\Psi_1\rangle = \lambda|\Psi_0\rangle + |\Psi'\rangle,$$

$$S_1[\Psi_1] = S_1[\Psi_0] + S'[\Psi']$$

$$S'[\Psi'] = \frac{1}{g_s^2}\left(\frac{1}{2}\langle \Psi' | c_0^- Q_B | \Psi'\rangle + \sum_n \frac{1}{n!}\left(\mathcal{V}_n(\Psi'^n) + \lambda\mathcal{V}_{n+1}(\Psi_0, \Psi'^n)\right)\right)$$

$$\mathcal{F}'(\Psi') := \mathcal{F}_1(\Psi') + \lambda\delta\mathcal{F}'(\Psi') = 0$$

$$\delta\mathcal{F}'(\Psi') = \sum_n \frac{1}{n!}\ell_{n+1}(\Psi_0, \Psi'^n)$$

$$\mathcal{F}_1(\Psi_1) + \lambda\delta\mathcal{F}_1(\Psi_1) = \big(1 + \lambda\mathcal{M}(\Psi')\big)\big(\mathcal{F}_1(\Psi') + \lambda\delta\mathcal{F}'(\Psi')\big)$$
$$|\Psi_1\rangle = |\Psi'\rangle + \lambda|\delta\Psi'\rangle$$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\mathcal{F}_1(\Psi' + \lambda\delta\Psi')\Big|_{\lambda=0} + \delta\mathcal{F}_1(\Psi_1) - \delta\mathcal{F}'(\Psi') = \mathcal{M}(\Psi')\mathcal{F}_1(\Psi')$$

$$\lambda Q_B|\delta\Psi'\rangle + \lambda\sum_n \frac{1}{n!}\ell_{n+1}(\delta\Psi', \Psi'^n) + \delta Q_B|\Psi'\rangle$$
$$+ \sum_n \frac{1}{n!}\delta\ell_n(\Psi'^n) - \lambda\sum_n \frac{1}{n!}\ell_{n+1}(\Psi_0, \Psi'^n) = 0$$

$$\Delta := \lambda\langle A|c_0^- Q_B|\delta\Psi'\rangle + \lambda\sum_n \frac{1}{n!}\mathcal{V}_{n+2}(A, \delta\Psi', \Psi'^n) + \langle A|c_0^-\delta Q_B|\Psi'\rangle + \sum_n \frac{1}{n!}\delta\mathcal{V}_{n+1}(A, \Psi'^n)$$

$$- \lambda\sum_n \frac{1}{n!}\mathcal{V}_{n+2}(A, \Psi_0, \Psi'^n)$$

$$\langle A|c_0^-\delta Q_B|B\rangle = \lambda\mathcal{V}'_{0,3}(\Psi_0, B, A), \delta\mathcal{V}_n(\Psi'^n) = \lambda\mathcal{V}'_{n+1}(\Psi_0, \Psi'^n),$$

$$\langle A|c_0^-|\delta\Psi'\rangle = \sum_n \frac{1}{n!}\mathcal{B}_{n+2}(\Psi_0, \Psi'^n, A)$$

$$\mathcal{V}'_n = \sum_{g\geq 0}\mathcal{V}'_{g,n}, \mathcal{B}_n = \sum_{g\geq 0}\mathcal{B}_{g,n}$$

$$\Delta = -\sum_n \frac{1}{n!}\mathcal{B}_{n+2}\Big(\Psi_0, \Psi'^n, Q_B A) + \sum_{m,n}\frac{1}{m!\,n!}\mathcal{B}_{n+2}\big(\Psi_0, \Psi'^m, \ell_{n+1}(A, \Psi'^n)\big)$$

$$+ \sum_n \frac{1}{n!}\mathcal{V}'_{n+2}(A, \Psi_0, \Psi'^n) - \sum_n \frac{1}{n!}\mathcal{V}_{n+2}(A, \Psi_0, \Psi'^n)$$

$$\mathcal{B}_{n+2}(\Psi_0, \Psi'^n, Q_B A) = \partial\mathcal{B}_{n+2}(\Psi_0, \Psi'^n, A) + n\mathcal{B}_{n+2}(\Psi_0, \Psi'^{n-1}, Q_B\Psi', A)$$
$$= \partial\mathcal{B}_{n+2}(\Psi_0, \Psi'^n, A) - \sum_m \frac{n}{m!}\mathcal{B}_{n+2}(\Psi_0, \Psi'^{n-1}, \ell_m(\Psi'^m), A)$$

$$\Delta = \sum_{n} \frac{1}{n!} \partial \mathcal{B}_{n+2}(\Psi_0, \Psi'^n, A) - \sum_{m,n} \frac{1}{m!\,n!} \mathcal{B}_{n+3}(\Psi_0, \Psi'^n, \ell_m(\Psi'^m), A)$$

$$+ \sum_{m,n} \frac{1}{m!\,n!} \mathcal{B}_{n+2}\big(\Psi_0, \Psi'^m, \ell_{n+1}(A, \Psi'^n)\big) + \sum_{n} \frac{1}{n!} \mathcal{V}'_{n+2}(A, \Psi_0, \Psi'^n)$$

$$- \sum_{n} \frac{1}{n!} \mathcal{V}_{n+2}(A, \Psi_0, \Psi'^n)$$

$$\partial \mathcal{B}_{n+2}(\Psi_0, \Psi'^n, A)$$

$$= -\mathcal{V}'_{n+2}(A, \Psi_0, \Psi'^n) + \mathcal{V}_{n+2}(A, \Psi_0, \Psi'^n)$$

$$+ \sum_{\substack{m_1, m_2 \\ m_1 + m_2 = n}} \frac{n!}{m_1!\, m_2!} \mathcal{B}_{m_1+3}\big(\Psi_0, \Psi'^{m_1}, \ell_{m_2}(\Psi'^{m_2}), A\big)$$

$$- \sum_{\substack{m_1, m_2 \\ m_1 + m_2 = n}} \frac{n!}{m_1!\, m_2!} \mathcal{B}_{m_1+2}\left(\Psi_0, \Psi'^{m_1}, \ell_{m_2+1}(A, \Psi'^{m_2})\right)$$

Membranas, Simetrías y espacios planckianos en dimensión $\mathbb{R}^4$. Singularidad.

$$\partial \mathcal{B}_{0,3} = \mathcal{V}_{0,3} - \mathcal{V}'_{0,3}$$

$$T(z)T(w) \sim \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

$$G(z)G(w) \sim \frac{2c/3}{(z-w)^3} + \frac{2T(w)}{(z-w)}$$

$$T(z)G(w) \sim \frac{3}{2} \frac{G(w)}{(z-w)^2} + \frac{\partial G(w)}{(z-w)}$$

$$h(\beta) = \left(\frac{3}{2}, 0\right), h(\gamma) = \left(-\frac{1}{2}, 0\right)$$

$$\gamma(z)\beta(w) \sim \frac{1}{z-w}, \beta(z)\gamma(w) \sim -\frac{1}{z-w}$$

$$T^{\mathrm{gh}} = -2b\partial c + c\partial b, T^{\beta\gamma} = \frac{3}{2}\beta\partial\gamma + \frac{1}{2}\gamma\partial\beta$$

$$N_{\mathrm{gh}}(b) = N_{\mathrm{gh}}(\beta) = -1, N_{\mathrm{gh}}(c) = N_{\mathrm{gh}}(\gamma) = 1$$

$$\gamma = \eta \mathrm{e}^{\phi}, \beta = \partial \xi \mathrm{e}^{-\phi}$$

$$\delta(\gamma) = \mathrm{e}^{-\phi}, \delta(\beta) = \mathrm{e}^{\phi}$$

$$\int \mathrm{d}c_0 = 0 \Rightarrow \int \mathrm{d}c_0 c_0 = 1$$

$$\int d\gamma_0=\infty \Longrightarrow \int d\gamma_0\delta(\gamma_0)=1$$

$$T^{\beta\gamma}=T^{\eta\xi}+T^{\phi},$$

$$T^{\eta\xi}=-\eta\partial\xi, T^{\phi}=-\frac{1}{2}(\partial\phi)^2-\partial^2\phi.$$

$$\xi(z)\eta(w)\sim\frac{1}{z-w}, \mathrm{e}^{q_1\phi(z)}\mathrm{e}^{q_2\phi(w)}\sim\frac{\mathrm{e}^{(q_1+q_2)\phi(w)}}{(z-w)^{q_1q_2}}, \partial\phi(z)\partial\phi(w)\sim-\frac{1}{(z-w)^2}.$$

$$N_{\mathrm{gh}}(\eta)=1, N_{\mathrm{gh}}(\xi)=-1, N_{\mathrm{gh}}(\phi)=0.$$

$$N_{\mathrm{pic}}\left(\mathrm{e}^{q\phi}\right)=q, N_{\mathrm{pic}}(\xi)=1, N_{\mathrm{pic}}(\eta)=-1.$$

$$N_{\mathrm{pic}}=2(g-1)=-\chi_g.$$

$$h\left(\mathrm{e}^{q\phi}\right)=-\frac{q}{2}(q+2)$$

$$h\left(\mathrm{e}^{\phi}\right)=\frac{3}{2}, h\left(\mathrm{e}^{-\phi}\right)=\frac{1}{2}.$$

$$j_B=c\left(T^{\mathrm{m}}+T^{\beta\gamma}\right)+\gamma G+bc\partial c-\frac{1}{4}\gamma^2 b$$
$$\bar{j}_B=\bar{c}\bar{T}^{\mathrm{m}}+\bar{b}\bar{c}\bar{\partial}\bar{c}$$

$$\mathcal{X}(z)=\{Q_B,\xi(z)\}=c\partial\xi+\mathrm{e}^{\phi}G-\frac{1}{4}\partial\eta\mathrm{e}^{2\phi}b-\frac{1}{4}\partial\left(\eta\mathrm{e}^{2\phi}b\right),$$

$$\mathcal{X}_0=\frac{1}{2\pi\mathrm{i}}\oint\frac{\mathrm{d}z}{z}\mathcal{X}(z)$$

$$\mathcal{H}_{\text{small}}=\{|\psi\rangle|\eta_0|\psi\rangle=0\}.$$

$$\mathcal{H}_{\text{small}}=\mathcal{H}_{\text{large}}\cap\ker\eta_0$$

$$\langle k|c_{-1}\bar{c}_{-1}c_0\bar{c}_0c_1\bar{c}_1\mathrm{e}^{-2\phi(z)}|k'\rangle=(2\pi)^D\delta^{(D)}(k+k')$$

$$\langle k|c_{-1}\bar{c}_{-1}c_0\bar{c}_0c_1\bar{c}_1\mathrm{e}^{-2\phi(z)}\mathrm{e}^{-\bar{\phi}(\bar{w})}|k'\rangle=-(2\pi)^D\delta^{(D)}(k+k')$$

$$\mathcal{H}_T=\mathcal{H}_{\mathrm{NS}}\oplus\mathcal{H}_{\mathrm{R}}$$

$$\widehat{\mathcal{H}}_T=\mathcal{H}_{-1}\oplus\mathcal{H}_{-1/2}, \widetilde{\mathcal{H}}_T=\mathcal{H}_{-1}\oplus\mathcal{H}_{-3/2}$$

$$|p\rangle=\mathrm{e}^{p\phi}(0)|0\rangle.$$

$$\forall n\geq-p-\frac{1}{2}:\beta_n|p\rangle=0$$

$$\forall n\geq p+\frac{3}{2}:\gamma_n|p\rangle=0$$

$$\widehat{\mathcal{H}}_T=\operatorname{Span}\{|\phi_r\rangle\}, \widetilde{\mathcal{H}}_T=\operatorname{Span}\{|\phi_r^c\rangle\}$$

$$\langle \phi_r^c \mid \phi_s \rangle = \delta_{rs}.$$

$$1 = \sum_r |\phi_r\rangle\langle\phi_r^c|$$

$$1 = \sum_r (-1)^{|\phi_r|}|\phi_r^c\rangle\langle\phi_r|$$

$$\mathcal{G} = \begin{cases} 1 & \text{NS sector}\,, \\ \mathcal{X}_0 & \text{R sector}\,. \end{cases}$$

$$[\mathcal{G}, L_0^\pm] = [\mathcal{G}, b_0^\pm] = [\mathcal{G}, Q_B] = 0.$$

$$\mathrm{M}_{g,m,n} := \dim\mathcal{M}_{g,m,n} = 6g - 6 + 2m + 2n$$

$$N_{\mathrm{gh}} = 6 - 6g, N_{\mathrm{pic}} = 2g - 2$$

$$n_{\mathrm{pco}} := 2g - 2 + m + \frac{n}{2}$$

$$A_{g,m,n}(\mathcal{V}_i^{\mathrm{NS}}, \mathcal{V}_j^{\mathrm{R}}) = \int_{\mathcal{S}_{g,m,n}} \Omega_{\mathrm{M}_{g,m,n}}(\mathcal{V}_i^{\mathrm{NS}}, \mathcal{V}_j^{\mathrm{R}}),$$

$$\Omega_{\mathrm{M}_{g,m,n}} = (-2\pi\mathrm{i})^{-\mathrm{M}_{g,m,n}^c} \left\langle \bigwedge_{\lambda=1}^{\mathrm{M}_{g,m,n}} \mathcal{B}_\lambda \mathrm{d}t_\lambda \prod_{A=1}^{n_{\mathrm{pco}}} \mathcal{X}(y_A) \prod_{i=1}^{m} \mathcal{V}_i^{\mathrm{NS}} \prod_{j=1}^{n} \mathcal{V}_j^{\mathrm{R}} \right\rangle_{\Sigma_{g,n}}$$

$$\mathcal{B}_\lambda = \sum_\alpha \oint_{C_\alpha} \frac{\mathrm{d}\sigma_\alpha}{2\pi\mathrm{i}} b(\sigma_\alpha) \frac{\partial F_\alpha}{\partial t_\lambda}\left(F_\alpha^{-1}(\sigma_\alpha)\right) + \sum_\alpha \oint_{C_\alpha} \frac{\mathrm{d}\bar\sigma_\alpha}{2\pi\mathrm{i}} \bar b(\bar\sigma_\alpha) \frac{\partial \bar F_\alpha}{\partial t_\lambda}\left(\bar F_\alpha^{-1}(\bar\sigma_\alpha)\right)$$
$$- \sum_A \frac{1}{\mathcal{X}(y_A)} \frac{\partial y_A}{\partial t_\lambda} \partial\xi(y_A)$$

$$\mathcal{X}(y_A) - \partial\xi(y_A)\mathrm{d}y_A$$

$$n_{\mathrm{pco}}^{(1)} + n_{\mathrm{pco}}^{(2)} = 2(g_1 + g_2) - 2 + (m_1 + m_2 - 2) + \frac{n_1 + n_2}{2} = n_{\mathrm{pco}}^{(\mathrm{NS})}$$

$$\Delta_{\mathrm{NS}} = b_0^+ b_0^- \frac{1}{L_0 + \bar L_0} \delta(L_0^-)$$

$$n_{\mathrm{pco}}^{(1)} + n_{\mathrm{pco}}^{(2)} = 2(g_1 + g_2) - 2 + (m_1 + m_2) + \frac{n_1 + n_2 - 2}{2} - 1 = n_{\mathrm{pco}}^{(\mathrm{R})} - 1$$

$$\Delta_{\mathrm{R}} = b_0^+ b_0^- \frac{\mathcal{X}_0}{L_0 + \bar L_0} \delta(L_0^-)$$

$$\mathcal{X}_0 = \frac{1}{2\pi\mathrm{i}} \oint \frac{\mathrm{d}w_n^{(1)}}{w_n^{(1)}} \mathcal{X}\left(w_n^{(1)}\right) = \frac{1}{2\pi\mathrm{i}} \oint \frac{\mathrm{d}w_n^{(2)}}{w_n^{(2)}} \mathcal{X}\left(w_n^{(2)}\right)$$

$$\Delta = b_0^+ b_0^- \frac{\mathcal{G}}{L_0 + \bar{L}_0} \delta(L_0^-)$$

$$\Delta_{\text{NS}} \sim \frac{1}{k^2 + m^2}, \Delta_{\text{R}} \sim \frac{\mathrm{i}\,\partial\!\!\!/ + m}{k^2 + m^2}$$

$$\mathcal{C}(x_i, y_j, z_q) = \left\langle \prod_{i=1}^{n+1} \xi(x_i) \prod_{j=1}^{n} \eta(y_j) \prod_{k=1}^{m} \mathrm{e}^{q_k \phi(z_k)} \right\rangle$$

$$= \frac{\prod_{j'=1}^{n} \vartheta_\delta\left(-y_{j'} + \sum_i x_i - \sum_j y_j + \sum_k q_k z_k\right)}{\prod_{i'=1}^{n+1} \vartheta_\delta\left(-x_{i'} + \sum_i x_i - \sum_j y_j + \sum_k q_k z_k\right)} \times \frac{\prod_{i<i'} E(x_i, x_{i'}) \prod_{j<j'} E\left(y_j, y_{j'}\right)}{\prod_{i,j} E\left(x_i, y_j\right) \prod_{k,\ell} E(z_k, z_\ell)^{q_k q_\ell}}$$

$$\sum_k q_k = 0.$$

$$E(x, y) = \frac{\vartheta_1(x - y)}{\vartheta_1'(0)} \sim_{x \to y} x - y$$

$$\vartheta_\delta\left(\sum_{i=2}^{n+1} x_i - \sum_{j=1}^{n} y_j + \sum_{k=1}^{m} q_k z_k\right) = 0$$

$$\langle A^{(p)} | Q = 0,$$

$$Q = Q_B \otimes 1^{\otimes n-1} + \cdots + 1^{\otimes n-1} \otimes Q_B$$

$$\langle A^{(p)} | \eta = 0$$

$$\eta = \eta_0 \otimes 1^{\otimes n-1} + \cdots + 1^{\otimes n-1} \otimes \eta_0.$$

$$\langle A^{(p)} | = \langle \alpha^{(p)} | Q,$$

$$\langle \alpha^{(p)} | \eta Q = 0$$

$$\langle A^{(p-1)} | = \langle \alpha^{(p)} | \eta$$

$$\Psi = \Psi_{-1} + \Psi_{-1/2}$$

$$\eta_0 | \Psi \rangle = 0$$

$$\Delta = b_0^+ b_0^- \frac{\mathcal{G}}{L_0 + \bar{L}_0} \delta(L_0^-), \mathcal{G} = \begin{cases} 1 & \text{NS} \\ \mathcal{X}_0 & \text{R} \end{cases}$$

$$b_0^- | \Psi \rangle = L_0^- | \Psi \rangle = 0, b_0^+ | \Psi \rangle = 0$$

$$X = G_0 \delta(\beta_0) + b_0 \delta'(\beta_0), Y = -c_0 \delta'(\gamma_0)$$

$$[Q_B, X] = 0$$

$$XYX = X$$

$$XY|\Psi_{-1/2}\rangle = |\Psi_{-1/2}\rangle$$

$$Bc_0^-|\Psi\rangle = |\Psi\rangle,$$

$$B = b_0^- \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \mathrm{e}^{\mathrm{i}\theta L_0^-} = \delta(b_0^-)\delta(L_0^-)$$

$$S_{0,2} = -\frac{1}{2}\langle\Psi_{-1}|c_0^- Q_B|\Psi_{-1}\rangle - \frac{1}{2}\langle\Psi_{-1/2}|c_0^- Y Q_B|\Psi_{-1/2}\rangle.$$

$$\delta|\Psi\rangle = Q_B|\Lambda\rangle$$

$$\Lambda = \Lambda_{-1} + \Lambda_{-1/2}$$

$$\widetilde{\Psi} = \widetilde{\Psi}_{-1} + \widetilde{\Psi}_{-3/2}$$

$$b_0^-|\widetilde{\Psi}\rangle = L_0^-|\widetilde{\Psi}\rangle = 0, b_0^+|\widetilde{\Psi}\rangle = 0$$

$$S_{0,2} = \frac{1}{2}\langle\widetilde{\Psi}|c_0^- c_0^+ L_0^+ \mathcal{G}|\widetilde{\Psi}\rangle - \langle\widetilde{\Psi}|c_0^- c_0^+ L_0^+|\Psi\rangle.$$

$$K = c_0^- c_0^+ L_0^+ \begin{pmatrix} -\mathcal{G} & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Delta = b_0^- b_0^+ \frac{1}{L_0^+} \begin{pmatrix} 0 & 1 \\ 1 & \mathcal{G} \end{pmatrix}$$

$$\delta|\Psi\rangle = Q_B|\Lambda\rangle, \delta|\widetilde{\Psi}\rangle = Q_B|\widetilde{\Lambda}\rangle,$$

$$Q_B|\Psi\rangle = 0, Q_B|\widetilde{\Psi}\rangle = 0$$

$$Q_B(|\Psi\rangle - \mathcal{G}|\widetilde{\Psi}\rangle) = 0, Q_B|\widetilde{\Psi}\rangle = |J(\Psi)\rangle,$$

$$Q_B|\Psi\rangle = \mathcal{G}|J(\Psi)\rangle$$

$$S_{0,2} = -\frac{1}{2}\langle\langle\Psi_0, \eta_0 Q_B \Psi_0\rangle\rangle,$$

$$\delta|\Psi_0\rangle = Q_B|\Lambda_0\rangle + \eta_0|\Omega_1\rangle,$$

$$Q_B\eta_0|\Psi_0\rangle = 0$$

$$\xi_0|\Psi_0\rangle = 0$$

$$|\Psi_0\rangle = \xi_0|\Psi_{-1}\rangle$$

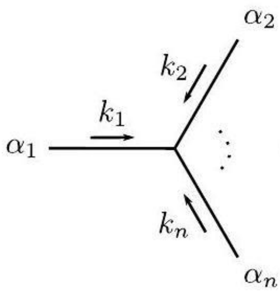
$$Q_B|\Psi_{-1}\rangle = 0$$

Momentum en dimensión $\mathbb{R}^4$– espacio SFT.

$$|\Psi\rangle = \sum_j \int \frac{\mathrm{d}^D k}{(2\pi)^D} \psi_\alpha(k)|\phi_\alpha(k)\rangle$$

$$S = -\int \mathrm{d}^D k \psi_\alpha(k) K_{\alpha\beta}(k) \psi_\beta(-k) - \sum_{n\geq 0} \int \mathrm{d}^D k_1 \cdots \mathrm{d}^D k_n V^{(n)}_{\alpha_1 \cdots \alpha_n}(k_1, \dots, k_n) \psi_{\alpha_1}(k_1) \cdots \psi_{\alpha_n}(k_n)$$

$$\alpha - \vec{\vec{k}}\beta = K_{\alpha\beta}(k)^{-1} = \frac{-\mathrm{i}M_{\alpha\beta}}{k^2 + m_\alpha^2} Q_\alpha(k),$$



$$= \mathrm{i}\, V^{(n)}_{\alpha_1 \cdots \alpha_n}(k_1, \dots, k_n) := \mathrm{i}\, \mathcal{V}_n\big(\phi_{\alpha_1}(k_1), \dots, \phi_{\alpha_n}(k_n)\big)$$

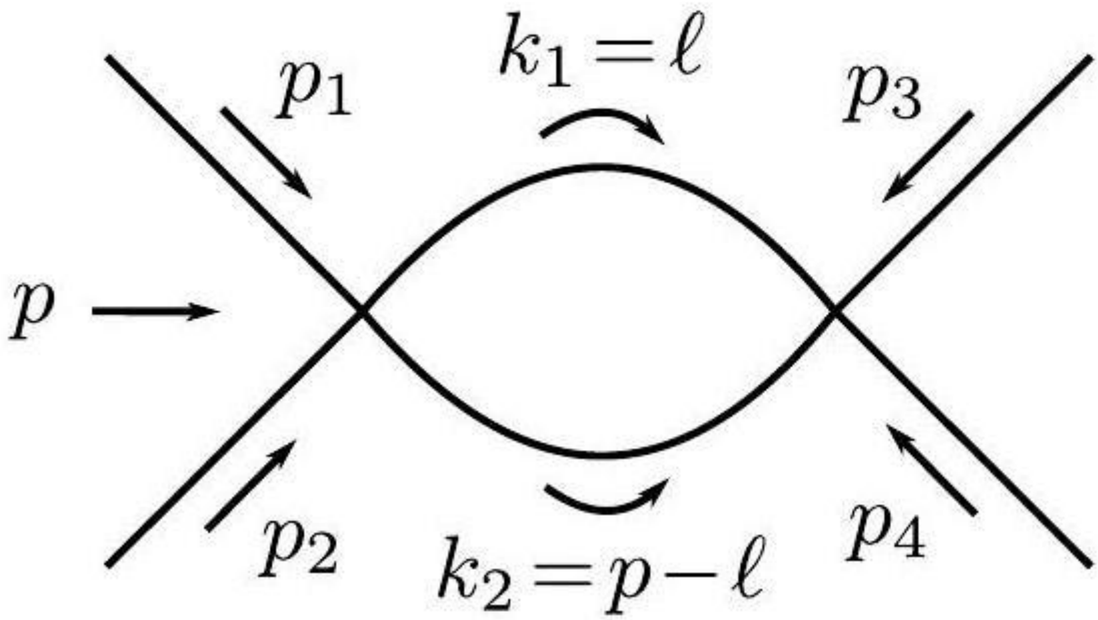
$$= \mathrm{i} \int \mathrm{d}t\, \mathrm{e}^{-g_{ij}^{\{\alpha_k\}}(t)\, k_i \cdot k_j - \lambda \sum_\alpha m_\alpha^2} P_{\alpha_1, \dots, \alpha_n}(k_1, \dots, k_n; t),$$

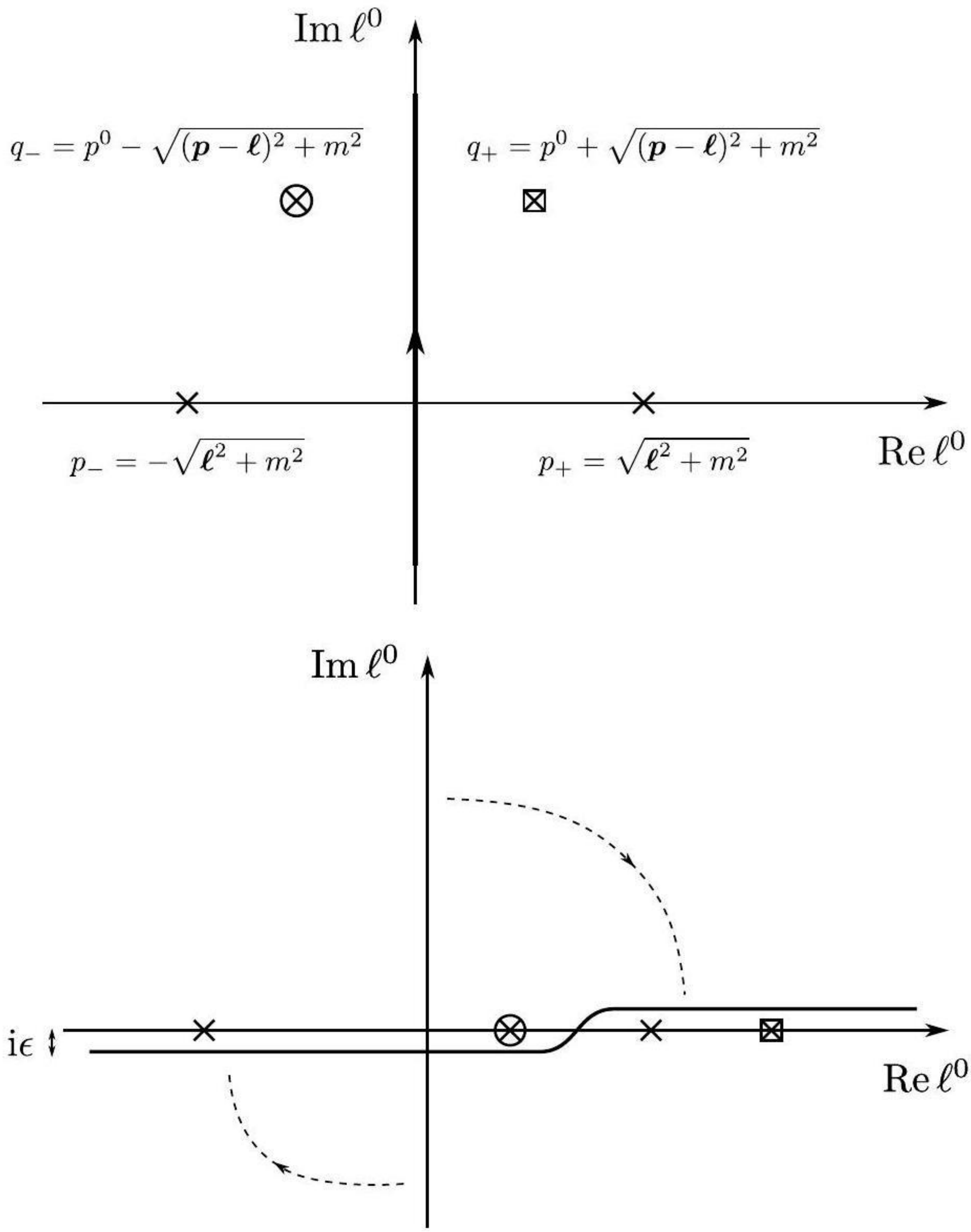
$$\lim_{k^0 \to \pm \mathrm{i}\infty} V^{(n)} = 0, \lim_{k^0 \to \pm\infty} V^{(n)} = \infty$$

$$F_{g,n}(p_1, \dots, p_n) \sim \int \mathrm{d}T \prod_s \mathrm{d}^D \ell_s \mathrm{e}^{-G_{rs}(T)\ell_r \cdot \ell_s} - 2H_{ri}(T)\ell_r \cdot p_i - F_{ij}(T) p_i \cdot p_j$$
$$\times \prod_a \frac{1}{k_a^2 + m_a^2} \mathcal{P}(p_i, \ell_r; T)$$

Rotación de Wick.

$$p_\pm = \pm\sqrt{\ell^2 + m^2}, q_\pm = p^0 \pm \sqrt{(\boldsymbol{p} - \ell)^2 + m^2}.$$





Figuras 31, 32 y 33. Coordenadas de una partícula supermasiva en aniquilación.

Sistema de Coordenadas.

$$x^\mu = \left(x^0, x^i\right), \mu = 0, \dots, D-1 = d \; i = 1, \dots, d$$

$$\sigma^a = (\sigma^0, \sigma^\alpha), a = 0, \dots, p-1, \alpha = 1, \dots, p.$$

$$\eta_{\mu\nu} = \mathrm{diag}(-1, \underbrace{1, \dots, 1}_{d}).$$

$$\delta_{\mu\nu} = \mathrm{diag}(\underbrace{1,\ldots,1}_{D}).$$

$$\epsilon_{01} = -\epsilon^{01} = 1$$

$$t = -\mathrm{i}\tau.$$

$$V_M^0 = -\mathrm{i}V_E^0, V_{M,0} = \mathrm{i}V_{E,0}.$$

$$x^{\pm} = x^0 \pm x^1.$$

$$\tau \in \mathbb{R}, \sigma \in [0, L), \sigma \sim \sigma + L,$$

$$\mathcal{L} = \frac{1}{2\pi}\int_0^L \mathrm{d}\sigma = \frac{L}{2\pi}$$

$$w = \tau + \mathrm{i}\sigma, \bar{w} = \tau - \mathrm{i}\sigma$$

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + \mathrm{d}\sigma^2 = \mathrm{d}w\,\mathrm{d}\bar{w}$$

$$w = \mathrm{i}\sigma^+, \bar{w} = \mathrm{i}\sigma^-.$$

$$z = \mathrm{e}^{2\pi w/L}, \bar{z} = \mathrm{e}^{2\pi\bar{w}/L}.$$

$$\epsilon_{z\bar{z}} = \frac{\mathrm{i}}{2}, \epsilon^{z\bar{z}} = -2\mathrm{i}$$

Sistema de Operadores.

$$[A, B] := [A, B]_- = AB - BA, \{A, B\} := [A, B]_+ = AB + BA.$$

$$|A| = \begin{cases} +1 & \text{Grassmann odd} \\ 0 & \text{Grassmann even} \end{cases}$$

$$AB = (-1)^{|A||B|}BA.$$

Sistema QFT.

$$p^{\mu} := \left(E, p^i\right)$$

$$(N_L, N_R), N = N_L + N_R$$

$$\delta\phi(x) = \phi'(x) - \phi(x).$$

$$J_a^{\mu} = \lambda\frac{\partial\mathcal{L}}{\partial\left(\partial_{\mu}\phi\right)}\frac{\delta\phi}{\delta\alpha^a}, \nabla_{\mu}J_a^{\mu} = 0$$

$$Q_a = \frac{1}{\lambda}\oint_{\Sigma}\mathrm{d}^{D-1}x\sqrt{h}J_a^0$$

$$\delta_{\alpha^a}\phi(x) = \mathrm{i}\alpha^a[Q_a, \phi(x)]$$

$$J_a^\mu = \mathrm{i}\lambda \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \frac{\delta\phi}{\delta\alpha^a}$$

$$\delta_{\alpha^a}\phi(x) = -\alpha^a [Q_a, \phi(x)]$$

Espacio curvo y gravedad.

$$\nabla_\mu = \partial_\mu + \Gamma_\mu$$

$$\nabla_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\nu_{\mu\rho} A^\rho$$

$$\Delta = g^{\mu\nu}\nabla_\mu \nabla_\nu = \frac{1}{\sqrt{g}} \nabla_\mu \left(\sqrt{g} g^{\mu\nu} \nabla_\nu\right)$$

$$T_{\mu\nu} = -\frac{2\lambda}{\sqrt{g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

Análisis Complejo - Cauchy-Riemann.

$$\oint_{C_z} \frac{\mathrm{d}w}{2\pi \mathrm{i}} \frac{f(w)}{(w-z)^n} = \frac{f^{(n-1)}(z)}{(n-1)!},$$

$$\bar{\partial} \frac{1}{z} = 2\pi \delta^{(2)}(z)$$

QFT, espacios curvos y gravedad. Función de Green.

$$D_x G(x,y) = \frac{\delta(x-y)}{\sqrt{g}} - P(x,y),$$

$$\nabla_\mu v^\mu = \frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g} v^\mu\right)$$

$$\delta x^\mu = \xi^\mu,$$

$$\delta g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

$$\int_V \mathrm{d}^D x \nabla_\mu v^\mu = \oint_{\partial V} \mathrm{d}\Sigma_\mu v^\mu, \mathrm{d}\Sigma_\mu := \epsilon n_\mu \mathrm{d}^{D-1}\Sigma,$$

$$\mathrm{d}^{D-1}\Sigma = \sqrt{g}\, \mathrm{d}^{D-1}x, n_\mu = \delta^0_\mu$$

$$Q_S = \frac{1}{\lambda} \int_S \mathrm{d}\Sigma_\mu J_a^\mu$$

Dimensiones D. Teorema de Stokes.

$$\int \mathrm{d}^2 x \partial_\mu v^\mu = \oint \epsilon_{\mu\nu} \mathrm{d}x^\nu v^\mu = \oint (v^0\, \mathrm{d}\sigma - v^1\, \mathrm{d}\tau)$$

$$\chi_{g;b} := \frac{1}{4\pi}\int \mathrm{d}^2\sigma\sqrt{g}R + \frac{1}{2\pi}\oint \mathrm{d}sk$$
$$= 2 - 2g - b$$

$$T_{ab} = -\frac{4\pi}{\sqrt{g}}\frac{\delta S}{\delta g^{ab}}$$

Plano Complejo.

$$z = x + \mathrm{i}y, z = x - \mathrm{i}y$$

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 = \mathrm{d}z\,\mathrm{d}\bar{z}, g_{z\bar{z}} = \frac{1}{2}, g_{zz} = g_{\bar{z}\bar{z}} = 0$$
$$\epsilon_{z\bar{z}} = \frac{\mathrm{i}}{2}, \epsilon^{z\bar{z}} = -2\mathrm{i}$$
$$\partial := \partial_z = \frac{1}{2}(\partial_x - \mathrm{i}\partial_y), \bar{\partial} := \partial_{\bar{z}} = \frac{1}{2}(\partial_x + \mathrm{i}\partial_y)$$
$$V^z = V^x + \mathrm{i}V^y, V^{\bar{z}} = V^x - \mathrm{i}V^y$$
$$\mathrm{d}^2x = \mathrm{d}x\,\mathrm{d}y = \frac{1}{2}\,\mathrm{d}^2z,\ \mathrm{d}^2z = \mathrm{d}z\,\mathrm{d}\bar{z}$$
$$\delta(z) = \frac{1}{2}\delta^{(2)}(x), 1 = \int \mathrm{d}^2z\delta^{(2)}(z) = \int \mathrm{d}^2x\delta^{(2)}(x)$$
$$\int_R \mathrm{d}^2z(\partial_z v^z + \partial_{\bar{z}}v^{\bar{z}}) = -\mathrm{i}\oint_{\partial R}(\mathrm{d}zv^{\bar{z}} - \mathrm{d}\bar{z}v^z) = -2\mathrm{i}\oint_{\partial R}(v_z\,\mathrm{d}z - v_{\bar{z}}\,\mathrm{d}\bar{z})$$

Propiedades generales.

$$f \circ \phi(z) = \left(\frac{\mathrm{d}f}{\mathrm{d}z}\right)^h \phi(f(z))$$

$$\phi(z) = \sum_n \frac{\phi_n}{z^{n+h}}, \phi_n = \oint_{C_0}\frac{\mathrm{d}z}{2\pi\mathrm{i}}z^{n+h-1}\phi(z),$$

$$\forall n \geq -h + 1: \phi_n|0\rangle = 0$$

$$\forall n \leq h - 1: \langle 0|\phi_n = 0.$$

$$|\phi\rangle := \phi(0)|0\rangle = \phi_{-h}|0\rangle.$$

$$\langle\phi^{\ddagger}| := \langle 0|I \circ \phi^{\dagger}(0) = \lim_{z\to\infty} z^{2h}\langle 0|\phi^{\dagger}(z), \langle\phi| := \langle 0|I^{\pm} \circ \phi(0) = (\pm 1)^h \lim_{z\to\infty} z^{2h}\langle 0|\phi(z).$$

$$T(z) = \sum_n \frac{L_n}{z^{n+2}}.$$

$$T(z)\phi(w) \sim \frac{h\phi(w)}{(z-w)^2} + \frac{\partial\phi(w)}{z-w}$$

$$T(z)T(w) \sim \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

Conjugaciones Hermitianas y BPZ.

Hermitiana.

$$(\lambda A_1 \cdots A_n|0\rangle)^\dagger = \lambda^* \langle 0| A_n^\dagger \cdots A_1^\dagger.$$

BPZ.

$$\phi_n^t = (I^\pm \circ \phi)_n = (-1)^h (\pm 1)^n \phi_{-n}$$

$$(\lambda A_1 \cdots A_n|0\rangle)^t = \lambda \langle 0| (A_1)^t \cdots (A_n)^t.$$

$$\langle A, B\rangle = (-1)^{|A||B|} \langle B, A\rangle.$$

$$\forall A: \langle A \mid B\rangle = 0 \implies |B\rangle = 0.$$

$$\langle \phi_r^c \mid \phi_s \rangle = \delta_{rs}$$

$$\langle \phi_r \mid \phi_s^c \rangle = (-1)^{|\phi_r|} \delta_{rs}.$$

Campo escalar.

$$\mathrm{i}\partial X^\mu = \sum_n \frac{\alpha_n^\mu}{z^{n+1}}, \mathrm{i}\bar{\partial} X^\mu = \sum_n \frac{\bar{\alpha}_n^\mu}{\bar{z}^{n+1}}$$

$$[\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu}, [\bar{\alpha}_m^\mu, \bar{\alpha}_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu}, [\alpha_m^\mu, \bar{\alpha}_n^\nu] = 0.$$

$$\alpha_0^\mu = \bar{\alpha}_0^\mu = \sqrt{\frac{\alpha'}{2}} p^\mu.$$

$$[x^\mu, p^\nu] = \eta^{\mu\nu}.$$

$$V_k(z, \bar{z}) =: \mathrm{e}^{\mathrm{i}k\cdot X(z,\bar{z})}:, h = \bar{h} = \frac{\alpha'^2 k^2}{4}.$$

$$p^\mu |k\rangle = k^\mu |k\rangle, \forall n > 0: \alpha_n^\mu |k\rangle = 0, \bar{\alpha}_n^\mu |k\rangle = 0.$$

$$|k\rangle = V_k(0,0)|0\rangle = \mathrm{e}^{\mathrm{i}k\cdot x}|0\rangle.$$

$$\langle k| p^\mu = \langle k| k^\mu, \langle k| = |k\rangle^\dagger, \langle -k| = |k\rangle^t.$$

Reparametrizaciones fantasmas.

$$\epsilon = 1, \lambda = 2, c_{\mathrm{gh}} = -26, q_{\mathrm{gh}} = -3, a_{\mathrm{gh}} = -1.$$

$$h(b) = 2, h(c) = -1$$

$$b(z)=\sum_{n\in\mathbb{Z}}\frac{b_n}{z^{n+2}},c(z)=\sum_{n\in\mathbb{Z}}\frac{c_n}{z^{n-1}},$$
$$b_n=\oint\frac{\mathrm{d}z}{2\pi\mathrm{i}}z^{n+1}b(z),c_n=\oint\frac{\mathrm{d}z}{2\pi\mathrm{i}}z^{n-2}c(z).$$

$$\{b_m,c_n\}=\delta_{m+n,0},\{b_m,b_n\}=0,\{c_m,c_n\}=0.$$

$$T=-2{:}b\partial c{:}-{:}\partial bc{:}$$
$$L_m=\sum_n(n+m){:}b_{m-n}c_n{:}=\sum_n(2m-n){:}b_nc_{m-n}{:}$$

$$L_0=-\sum_n n{:}b_nc_{-n}{:}=\sum_n n{:}b_{-n}c_n{:}$$

$$[L_m,b_n]=(m-n)b_{m+n},[L_m,c_n]=-(2m+n)c_{m+n}$$

$$[L_0,b_0]=0,[L_0,c_0]=0$$

$$j=-{:}bc{:},N_{\mathrm{gh},L}=\oint\frac{\mathrm{d}z}{2\pi\mathrm{i}}j(z)$$

$$N_{\mathrm{gh}}(c)=1,N_{\mathrm{gh}}(b)=-1.$$

$$j_m=-\sum_n{:}b_{m-n}c_n{:}=-\sum_n{:}b_nc_{m-n}{:},N_{\mathrm{gh},L}=j_0=-\sum_n{:}b_{-n}c_n{:}$$

$$[j_m,j_n]=m\delta_{m+n,0},[L_m,j_n]=-nj_{m+n}-\frac{3}{2}m(m+1)\delta_{m+n,0}$$

$$\left[N_{\mathrm{gh}},b(w)\right]=-b(w),\left[N_{\mathrm{gh}},c(w)\right]=c(w)$$

$$N^b=\sum_{n>0}nN_n^b,N^c=\sum_{n>0}nN_n^c,$$
$$N_n^b={:}b_{-n}c_n{:},N_n^c={:}c_{-n}b_n{:}$$

$$[N_m^b,b_{-n}]=b_{-n}\delta_{m,n},[N_m^c,c_{-n}]=c_{-n}\delta_{m,n}$$

$$c(z)b(w)\sim\frac{1}{z-w},b(z)c(w)\sim\frac{1}{z-w},b(z)b(w)\sim0,c(z)c(w)\sim0,$$
$$T(z)b(w)\sim\frac{2b(w)}{(z-w)^2}+\frac{\partial b(w)}{z-w},T(z)c(w)\sim\frac{-c(w)}{(z-w)^2}+\frac{\partial c(w)}{z-w}.$$
$$j(z)b(w)\sim-\frac{b(w)}{z-w},j(z)c(w)\sim\frac{c(w)}{z-w}.j(z)\mathcal{O}(w)\sim N_{\mathrm{gh}}(\mathcal{O})\frac{\mathcal{O}(w)}{z-w},$$
$$j(z)j(w)\sim\frac{1}{(z-w)^2}.$$
$$T(z)j(w)\sim\frac{-3}{(z-w)^3}+\frac{j(w)}{(z-w)^2}+\frac{\partial j(w)}{z-w}.$$

$$N^c-N^b=3-3g$$

$$N_{\mathrm{gh},L}=N_{\mathrm{gh},L}^{\mathrm{cyl}}+\frac{3}{2}$$

$$\forall n>-2{:}\ b_n|0\rangle=0, \forall n>1{:}\ c_n|0\rangle=0$$

$$|\downarrow\rangle:=c_1|0\rangle, |\uparrow\rangle:=c_0c_1|0\rangle.$$

$$L_0|\downarrow\rangle=a_{\mathrm{gh}}|\downarrow\rangle, L_0|\uparrow\rangle=a_{\mathrm{gh}}|\uparrow\rangle, a_{\mathrm{gh}}=-1.$$

$$L_m=\sum_n\ (n-(1-\lambda)m)^\star b_{m-n}c_n{}^\star+a_{\mathrm{gh}}\delta_{m,0},$$

$$j_m=\sum_n\ {}^\star b_{m-n}c_n{}^\star+\delta_{m,0}.$$

$$L_0=\sum_n\ n_\star^\star b_{-n}c_{n\star}^\star+a_{\mathrm{gh}}=\hat{L}_0-1,$$

$$N_{\mathrm{gh},L}=j_0=\sum_n^\star\ b_{-n}c_n^\star+1=\widehat{N}_{\mathrm{gh},L}+\frac{1}{2}\left(N_0^c-N_0^b\right)-\frac{3}{2},$$

$$\hat{L}_0=N^b+N^c, \widehat{N}_{\mathrm{gh},L}:=\sum_{n>0}\ \left(N_n^c-N_n^b\right).$$

$$N_{\mathrm{gh}}|0\rangle=0, N_{\mathrm{gh}}|\downarrow\rangle=|\downarrow\rangle, N_{\mathrm{gh}}|\uparrow\rangle=2|\uparrow\rangle.$$

$$N_{\mathrm{gh}}^{\mathrm{cyl}}|\downarrow\rangle=-\frac{1}{2}|\downarrow\rangle, N_{\mathrm{gh}}^{\mathrm{cyl}}|\uparrow\rangle=\frac{1}{2}|\uparrow\rangle.$$

$$b_n^\dagger=b_{-n}, c_n^\dagger=c_{-n}$$

$$b_n^t=(\pm 1)^n b_{-n}, c_n^t=-(\pm 1)^n c_{-n}$$

$$|\downarrow\rangle^\ddagger=\langle 0|c_{-1}, |\uparrow\rangle^\ddagger=\langle 0|c_{-1}c_0.$$

$$\langle\downarrow|:=|\downarrow\rangle^t=\mp\langle 0|c_{-1}, \langle\uparrow|:=|\uparrow\rangle^t=\pm\langle 0|c_0c_{-1}$$

$$\langle\downarrow|=\mp|\downarrow\rangle^\ddagger, \langle\uparrow|=\mp|\uparrow\rangle^\ddagger.$$

$$\langle\uparrow|\downarrow\rangle=\langle\downarrow|c_0|\downarrow\rangle=\langle 0|c_{-1}c_0c_1|0\rangle=1,$$

$$\langle 0^c|=\langle 0|c_{-1}c_0c_1$$

$$b_n^\pm=b_n\pm\bar{b}_n, c_n^\pm=\frac{1}{2}(c_n\pm\bar{c}_n)$$

$$\{b_m^+,c_n^+\}=\delta_{m+n}, \{b_m^-,c_n^-\}=\delta_{m+n}$$

$$b_n^-b_n^+=2b_n\bar{b}_n, c_n^-c_n^+=\frac{1}{2}c_n\bar{c}_n$$

Campo gravitónico. Operadores, progagadores, funciones fantasmas, supersimetrías, antisimetría y supermembranas y sistema de coordenadas.

$$(\alpha_n)^t = -(\pm 1)^n \alpha_{-n}, (b_n)^t = (\pm 1)^n b_{-n}, (c_n)^t = -(\pm 1)^n c_{-n}$$

$$L_n^\pm = L_n \pm \bar{L}_n, b_n^\pm = b_n \pm \bar{b}_n, c_n^\pm = \frac{1}{2}(c_n \pm \bar{c}_n)$$

$$\langle A, B\rangle = \langle A|c_0^-|B\rangle,$$

$$\langle A, B\rangle = \langle A \mid B\rangle.$$

$$|k,0\rangle := |k\rangle \otimes |0\rangle, |k,\downarrow\rangle := |k\rangle \otimes |\downarrow\rangle.$$

$$\langle k,\downarrow |c_0|k,\downarrow\rangle = \langle k',0|c_{-1}c_0c_1|k,0\rangle = (2\pi)^D\delta^{(D)}(k+k')$$

$$\langle k,\downarrow\downarrow |c_0\bar{c}_0|k,\downarrow\downarrow\rangle = \langle k',0|c_{-1}\bar{c}_{-1}c_0\bar{c}_0c_1\bar{c}_1|k,0\rangle = (2\pi)^D\delta^{(D)}(k+k')$$

$$|\delta\Phi|^2 = G(\Phi)(\delta\Phi,\delta\Phi),$$

$$|\Phi|^2 = G(\Phi)(\Phi,\Phi),$$

$$\mathrm{d}\Phi\sqrt{\det G(\Phi)}.$$

$$(\delta\Phi_1,\delta\Phi_2) = G(\Phi)(\delta\Phi_1,\delta\Phi_2), (\Phi_1,\Phi_2) = G(\Phi)(\Phi_1,\Phi_2).$$

$$|\delta\Phi_a|^2 = \int \mathrm{d}x\rho(x)\gamma_{ab}(\Phi(x))\delta\Phi_a(x)\delta\Phi_b(x),$$

$$G_{ab}(x,y)(\Phi) = \delta(x-y)\rho(x)\gamma_{ab}(\Phi(x)).$$

$$\int \mathrm{d}\delta\Phi \mathrm{e}^{-G(\Phi)(\delta\Phi,\delta\Phi)} = \frac{1}{\sqrt{\det G(\Phi)}}$$

$$\int \mathrm{d}\Phi \mathrm{e}^{-G(\Phi)(\Phi,\Phi)}$$

$$\int \mathrm{d}\Phi\sqrt{\det G(\Phi)}F(\Phi)$$

$$S = -\frac{1}{2}\mathrm{trln}\ G(\Phi)$$

$$Z = \int \mathrm{d}\Phi \mathrm{e}^{-S_{cl}(\Phi)}$$

$$G(\delta\Phi, D\delta\Phi) = G\big(D^\dagger\delta\Phi,\delta\Phi\big)$$

$$\Phi \to \Phi + \varepsilon$$

$$\int \mathrm{d}\Phi \mathrm{e}^{-\frac{1}{2}|\Phi+\varepsilon|^2} = \int \mathrm{d}\Phi \mathrm{e}^{-\frac{1}{2}|\Phi|^2}$$

$$\int \mathrm{d}\Phi \mathrm{e}^{-\frac{1}{2}|\Phi+\varepsilon|^2} = \int \mathrm{d}\widetilde{\Phi}\mathrm{det}\frac{\delta\Phi}{\delta\widetilde{\Phi}} \mathrm{e}^{-\frac{1}{2}|\widetilde{\Phi}|^2} = \int \mathrm{d}\widetilde{\Phi} \mathrm{e}^{-\frac{1}{2}|\widetilde{\Phi}|^2}$$

$$\mathrm{d}\Phi\sqrt{\mathrm{det}G(\Phi)} = \mathrm{d}\widetilde{\Phi}\sqrt{\mathrm{det}\tilde{G}(\widetilde{\Phi})}, G(\Phi)(\delta\Phi,\delta\Phi) = \tilde{G}(\widetilde{\Phi})(\delta\widetilde{\Phi},\delta\widetilde{\Phi})$$

$$\mathrm{d}\Phi = J(\Phi,\widetilde{\Phi})\mathrm{d}\widetilde{\Phi}, J(\Phi,\widetilde{\Phi}) = \left|\mathrm{det}\frac{\partial\Phi}{\partial\widetilde{\Phi}}\right| = \sqrt{\frac{\mathrm{det}\tilde{G}(\widetilde{\Phi})}{\mathrm{det}G(\Phi)}}$$

$$\int \mathrm{d}\delta\Phi \mathrm{e}^{-|\delta\Phi|^2} = 1$$

$$J(\widetilde{\Phi})^{-1} = \int \mathrm{d}\delta\widetilde{\Phi}\mathrm{e}^{-\tilde{G}(\delta\widetilde{\Phi},\delta\widetilde{\Phi})}$$

$$J = \mathrm{det}\frac{\partial\tilde{x}^\mu}{\partial x^\mu} = \mathrm{det}\frac{\partial\tilde{v}^\mu}{\partial v^\mu}$$

$$\tilde{v}^\mu = v^\nu\frac{\partial\tilde{x}^\mu}{\partial x^\nu}$$

$$D\Phi_0 = 0$$

$$\Phi = \Phi_0 + \Phi', (\Phi_0,\Phi') = 0$$

$$Z[D] = \int \mathrm{d}\Phi\sqrt{\mathrm{det}G}\mathrm{e}^{-\frac{1}{2}(\Phi,D\Phi)} = \left(\int \mathrm{d}\Phi_0\right)\int \mathrm{d}\Phi'\mathrm{e}^{-\frac{1}{2}(\Phi',D\Phi')}$$

$$Z[D,J] = \int \mathrm{d}\Phi\sqrt{\mathrm{det}G}\mathrm{e}^{-\frac{1}{2}(\Phi,D\Phi)-(J,\Phi)}$$

$$Z[D,J] = \int \mathrm{d}\Phi_0\mathrm{e}^{-(J,\Phi_0)}\int \mathrm{d}\Phi'\mathrm{e}^{-\frac{1}{2}(\Phi',D\Phi')-(J,\Phi')}$$

$$\int \mathrm{d}x = \infty$$

$$\int \mathrm{d}\theta = 0$$

$$\int \mathrm{d}\theta\theta = \int \mathrm{d}\theta\delta(\theta) = 1$$

$$\int \mathrm{d}x\delta(x) = 1$$

$$\theta_0(x) = \theta_{0i}\psi_i(x), \mathrm{ker}D = \mathrm{Span}\{\psi_i\}$$

$$\mathrm{d}\theta = \frac{1}{\sqrt{\det(\psi_i, \psi_j)}} \mathrm{d}\theta' \prod_{i=1}^{n} \mathrm{d}\theta_{0i}$$

$$\mathrm{d}\theta \prod_{i=1}^{n} \theta(x_i) = \frac{\det\psi_i(x_j)}{\sqrt{\det(\psi_i, \psi_j)}} \mathrm{d}\theta'$$

$$\begin{aligned} 1 &= \int \mathrm{d}\theta \mathrm{e}^{-|\theta|^2} = \int \mathrm{d}\theta' \mathrm{d}\theta_0 \mathrm{e}^{-|\theta|^2 - |\theta_0|^2} \\ &= J \int \mathrm{d}\theta' \prod_i \mathrm{d}\theta_{0i} \mathrm{e}^{-|\theta'|^2 - |\theta_{0i}\psi_i|^2} = J\sqrt{\det(\psi_i, \psi_j)} \end{aligned}$$

$$\begin{aligned} \int \mathrm{d}\theta_0 \prod_{j=1}^{n} \theta(x_j) &= \int \mathrm{d}\theta_0 \prod_{j=1}^{n} \theta_0(x_j) = \frac{1}{\sqrt{\det(\psi_i, \psi_j)}} \int \mathrm{d}^n\theta_{0i} \prod_{j=1}^{n} [\theta_{0i}\psi_i(x_j)] \\ &= \frac{\det\psi_i(x_j)}{\sqrt{\det(\psi_i, \psi_j)}} \int \prod_i \mathrm{d}\theta_{0i}\theta_{0i} = \frac{\det\psi_i(x_j)}{\sqrt{\det(\psi_i, \psi_j)}} \end{aligned}$$

Cuantización BRST.

$$\delta\phi^i = \epsilon^a \delta_a \phi^i = \epsilon^a R_a^i(\phi)$$

$$\epsilon^a \delta_a S_m = 0$$

$$[\delta_a, \delta_b] = f_{ab}^c \delta_c$$

$$R_a^i(\phi) = \left(T_a^{\mathbf{R}}\right)^i{}_j \phi^j,$$

$$R_{a\mu}^b = \delta_a^b \partial_\mu + f_{ab}^c A_\mu^b$$

$$Z = \Omega_{\text{gauge}}^{-1} \int \mathrm{d}\phi^i \mathrm{e}^{-S_m}$$

$$F^A(\phi^i) = 0$$

$$\begin{aligned} S_{\text{gh}} &= b_A c^a \delta_a F^A(\phi^i), \\ S_{\text{gf}} &= -\mathrm{i} B_A F^A(\phi^i) \end{aligned}$$

$$Z = \int \mathrm{d}\phi^i \, \mathrm{d}b_A \, \mathrm{d}c^a \, \mathrm{d}B_A \mathrm{e}^{-S_{\text{tot}}}$$

$$S_{\text{tot}} = S_m + S_{\text{gf}} + S_{\text{gh}}$$

$$\delta_\epsilon S_{\text{tot}} = 0$$

$$\delta_\epsilon \phi^i = \mathrm{i}\epsilon c^a \delta_a \phi^i, \delta_\epsilon c^a = -\frac{\mathrm{i}}{2} \epsilon f_{bc}^a c^b c^c, \delta_\epsilon b_A = \epsilon B_A, \delta_\epsilon B_A = 0$$

$$\delta_\epsilon \delta_{\epsilon'} = 0$$

$$\delta_\epsilon \phi^i = \mathrm{i}\left[\epsilon Q_B, \phi^i\right]$$

$$S_{\mathrm{gf}} + S_{\mathrm{gh}} = \{Q_B, b_A F^A\}$$

$$\delta S = \{Q_B, b_A \delta F^A\}$$

$$[Q_B, \{Q_B, b_A \delta F^A\}] = 0 \implies Q_B^2 = 0$$

$$|\psi\rangle \text{ closed } \Leftrightarrow |\psi\rangle \in \ker Q_B \Leftrightarrow Q_B|\psi\rangle = 0.$$

$$|\psi\rangle \text{ exact } \Leftrightarrow |\psi\rangle \in \mathrm{Im} Q_B \Leftrightarrow \exists |\chi\rangle : |\psi\rangle = Q_B|\chi\rangle.$$

$$|\psi\rangle \in \mathcal{H}(Q_B) \Leftrightarrow |\psi\rangle \in \ker Q_B, \nexists |\chi\rangle : |\psi\rangle = Q_B|\chi\rangle.$$

$$\mathcal{H}(Q_B) = \frac{\ker Q_B}{\mathrm{Im} Q_B}$$

$$|\psi\rangle \simeq |\psi\rangle + Q_B|\chi\rangle.$$

$$\delta_F \langle \psi_f \mid \psi_i \rangle = \langle \psi_f | \{Q_B, b_A \delta F^A\} | \psi_i \rangle$$

$$Q_B|\psi\rangle = 0$$

$$\langle \psi | Q_B | \chi \rangle = 0$$

$$|\psi\rangle \text{ physical } \Leftrightarrow |\psi\rangle \in \mathcal{H}(Q_B).$$

$$\frac{\delta F^A}{\delta \phi^i} B_A = -\frac{\delta S_m}{\delta \phi^i}$$

$$\delta_\epsilon b_A = -\epsilon \left(\frac{\delta F^A}{\delta \phi^i}\right)^{-1} \frac{\delta S_m}{\delta \phi^i}$$

$$\{Q_B, b_A B_B M^{AB}\} = \mathrm{i} B_A M^{AB} B_B$$

$$S[\phi, b, c, B] = S_0[\phi] + Q_B \Psi[\phi, b, c, B]$$

Formalismo Batalin-Vilkovisky.

$$\mathcal{F}_i(\phi) = \frac{\partial S_0}{\partial \phi^i}$$

$$[T_a, T_b] = F_{ab}^c(\phi) T_c + \lambda_{ab}^i \mathcal{F}_i(\phi)$$

$$m_1 = m_0 - \mathrm{rank} R_a^i$$

$$\delta A_p = \mathrm{d}\lambda_{p-1}$$

$$\delta \lambda_{p-1} = \mathrm{d}\lambda_{p-2}$$

$$\delta\lambda_{p-2} = \mathrm{d}\lambda_{p-3}$$

$$\psi^r = \{\phi^i, B_A, b_A, c^a\}$$

$$S[\psi^r, \psi_r^*] = S_0[\phi] + Q_B\psi^r\psi_r^*$$

$$\psi_r^* = \{\phi_i^*, B^{A*}, b^{A*}, c_a^*\}$$

$$\psi_r^* = \frac{\partial\Psi}{\partial\psi^r}$$

$$\delta\phi^i = \epsilon_0^{a_0} R_{a_0}^i(\phi^i)$$

$$\delta c_0^{a_0} = \epsilon_1^{a_1} R_{a_1}^{a_0}(\phi^i, c^{a_0})$$

$$\delta c_n^{a_n} = \epsilon_{n+1}^{a_{n+1}} R_{a_{n+1}}^{a_n}(\phi^i, c_0^{a_0}, \dots, c_n^{a_n})$$

$$\psi^r = \{c_n^{a_n}\}_{n=-1,\dots,\ell}, c_{-1} := \phi$$

$$N_{\mathrm{gh}}(\phi^i) = 0, N_{\mathrm{gh}}(c_n^{a_n}) = n + 1$$

$$|c_n| = |\epsilon_n^{a_n}| + n + 1$$

$$N_{\mathrm{gh}}(\psi_r^*) = -1 - N_{\mathrm{gh}}(\psi^r), |\psi_r^*| = -|\psi^r|.$$

$$\omega = \sum_r \mathrm{d}\psi^r \wedge \mathrm{d}\psi_r^*$$

$$(\psi^r, \psi_s^*) = \delta_{rs}, (\psi^r, \psi^s) = 0, (\psi_r^*, \psi_s^*) = 0$$

$$(A, B) = \frac{\partial_R A}{\partial\psi^r}\frac{\partial_L B}{\partial\psi_r^*} - \frac{\partial_R A}{\partial\psi_r^*}\frac{\partial_L B}{\partial\psi^r}$$

$$(A, B) = -(-1)^{(|A|+1)(|B|+1)}(B, A)$$

$$N_{\mathrm{gh}}((A, B)) = N_{\mathrm{gh}}(A) + N_{\mathrm{gh}}(B) + 1, |(A, B)| = |A| + |B| + 1 \bmod 2$$

$$(A, BC) = (A, B)C + (-1)^{|B|C}(A, C)B$$

$$N_{\mathrm{gh}}(S) = 0, |S| = 0$$

$$S[\psi^r, \psi_r^* = 0] = S_0[\phi^i], \left.\frac{\partial_L\partial_R S}{\partial c_{n-1,a_{n-1}}^* \partial c_n^{a_n}}\right|_{\psi^*=0} = R_{a_n}^{a_{n-1}}$$

$$\delta_\theta\psi^r = \theta s\psi^r = -\theta(S, \psi_r) = \theta\frac{\partial_R S}{\partial\psi_r^*}$$
$$\delta_\theta\psi_r^* = \theta s\psi_r^* = -\theta(S, \psi_r^*) = -\theta\frac{\partial_R S}{\partial\psi^r}$$

$$\delta_\theta F = \theta sF = -\theta(S, F)$$

$$(S,S)=0$$

$$(S,(S,F))=0$$

$$s^2=0$$

$$s\mathcal{O}=0$$

$$S'=S+(S,\delta F)$$

$$\psi'=\psi-\frac{\delta F}{\delta\psi^*},\psi'^*=\psi^*+\frac{\delta F}{\delta\psi}$$

$$S'[\psi,\psi^*]=S\left[\psi-\frac{\delta F}{\delta\psi^*},\psi^*+\frac{\delta F}{\delta\psi}\right]$$

$$S'[\psi,\psi^*]=S[\psi,\psi^*]+(S,\psi)\frac{\delta F}{\delta\psi}+(S,\psi^*)\frac{\delta F}{\delta\psi^*}=S[\psi,\psi^*]-\frac{\partial_R S}{\partial\psi^*}\frac{\delta F}{\delta\psi}+\frac{\partial_R S}{\partial\psi}\frac{\delta F}{\delta\psi^*}$$

$$(\psi'^r,\psi'^*_s)=\delta_{rs},(S',S')=0$$

$$G'=G+(\delta F,G)$$

$$S_\Psi[\psi^r]=S\left[\psi^r,\frac{\partial\Psi}{\partial\psi^r}\right],\psi_r^*=\frac{\partial\Psi}{\partial\psi^r}$$

$$N_{\mathrm{gh}}(\Psi)=-1,|\Psi|=1$$

$$|B|=-|\bar{c}|,N_{\mathrm{gh}}(B)=N_{\mathrm{gh}}(\bar{c})+1,$$
$$s\bar{c}=B,sB=0$$

$$\bar{S}=S[\psi^r,\psi_r^*]-B\bar{c}^*$$

$$(\bar{S},\bar{S})=(S,S)=0$$

$$\left(B_{0a_0},\bar{c}_{0a_0}\right):=\left(B^0_{0a_0},\bar{c}^0_{0a_0}\right)$$

$$|B_0|=|\epsilon_0|,|\bar{c}_0|=-|\epsilon_0|,N_{\mathrm{gh}}(B_0)=0,N_{\mathrm{gh}}(c_0)=-1.$$

$$\left(B^0_{1a_1},\bar{c}^0_{1a_1}\right),\left(\bar{B}_1^{1a_1},c_1^{1a_1}\right)$$

$$\delta_\theta\psi^r=\theta s\psi^r=\theta\left.\frac{\partial_R S}{\partial\psi_r^*}\right|_{\psi_r^*=\partial_r\Psi}$$

$$s^2\propto \text{ eom.}$$

Cuántica BV.

$$Z=\int \mathrm{d}\psi^r\,\mathrm{d}\psi_r^*\mathrm{e}^{-W[\psi^r,\psi_r^*]/\hbar}$$

$$\delta_\theta F=\theta\sigma F=(W,F)-\hbar\Delta F$$

$$\Delta = \frac{\partial_R}{\partial \psi_r^*}\frac{\partial_L}{\partial \psi^r}$$

$$(W, W) - 2\hbar \Delta W = 0,$$

$$\Delta \mathrm{e}^{-W/\hbar} = 0$$

$$\delta W = \frac{1}{2}(W, W),$$

$$\mathrm{sdet} J \sim 1 + \Delta W$$

$$W = S + \sum_{p\geq 1} \hbar^p W_p$$

$$\sigma \mathcal{O} = 0$$

$$\delta \langle \mathcal{O} \rangle = 0$$

$$Z = \int \mathrm{d}\psi^r \mathrm{e}^{-W_\Psi[\psi^r]}, W_\Psi[\psi^r] = W\left[\psi^r, \frac{\partial \Psi}{\partial \psi^r}\right]$$

$$Z = \int \mathrm{d}\psi^r \mathrm{e}^{-W_\Psi[\psi^r]} \left(\frac{\partial_R S}{\partial \psi_r^*}\right)_{\psi^* = \partial_\psi \Psi} \frac{\partial(\delta \Psi)}{\partial \psi^r}$$

Gravedad cuántica endógena en planos cuánticos relativistas.

Cálculos preliminares.

$$|\dot{H}| \ll H^2,$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\boldsymbol{x}^2.$$

$$M_P \equiv \sqrt{\frac{\hbar c}{G}} = 1.2 \times 10^{19} \mathrm{GeV}/c^2,$$

$$M_{\mathrm{pl}}^{-2} \equiv 8\pi G = (2.4 \times 10^{18} \mathrm{GeV})^{-2}$$

$$\mathcal{R}_{\boldsymbol{k}} = \int \mathrm{d}^3 x \mathcal{R}(\boldsymbol{x}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}.$$

$$\langle \mathcal{R}_{\boldsymbol{k}} \mathcal{R}_{\boldsymbol{k}'} \rangle = (2\pi)^3 P_{\mathcal{R}}(k) \delta(\boldsymbol{k} + \boldsymbol{k}').$$

$$\Delta_{\mathcal{R}}^2(k) \equiv \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k).$$

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \tilde{\eta} \equiv \frac{\dot{\varepsilon}}{H\varepsilon}$$

$$\epsilon \equiv \frac{M_{\mathrm{pl}}^2}{2}\left(\frac{V'}{V}\right)^2, \eta \equiv M_{\mathrm{pl}}^2 \frac{V''}{V}$$

$$\ell_s^2 \equiv \alpha', M_s^2 \equiv \frac{1}{\alpha'}$$

$$2\kappa^2 = (2\pi)^7 (\alpha')^4$$

Métrica de Friedmann-Robertson-Walker (FRW).

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\boldsymbol{x}^2.$$

$$\mathrm{d}s^2 = a^2(\tau)[-\mathrm{d}\tau^2 + \mathrm{d}\boldsymbol{x}^2],$$

$$\Delta\tau = \int_0^t \frac{\mathrm{d}t'}{a(t')} = \int_0^a \frac{\mathrm{d}\ln\ a}{aH}, \text{ where } H \equiv \frac{1}{a}\frac{da}{dt}$$

$$\frac{d}{dt}(aH)^{-1} = -\frac{1}{a}\left[\frac{\dot{H}}{H^2} + 1\right] < 0 \ \Rightarrow \ \varepsilon \equiv -\frac{\dot{H}}{H^2} < 1$$

$$a(t) \propto e^{Ht}$$

$$3M_{\mathrm{pl}}^2 H^2 \ = \rho$$
$$6M_{\mathrm{pl}}^2(\dot{H} + H^2) \ = -(\rho + 3P)$$

$$2M_{\mathrm{pl}}^2 \dot{H} = -(\rho + P)$$

$$\varepsilon = \frac{3}{2}\left(1 + \frac{P}{\rho}\right)$$

Acción de Goldstone.

$$U(t, \boldsymbol{x}) \equiv t + \pi(t, \boldsymbol{x})$$

$$\delta\psi_m(t, \boldsymbol{x}) \equiv \psi_m(t + \pi(t, \boldsymbol{x})) - \psi_m(t).$$

$$g_{ij} = a^2(t)\delta_{ij}$$

$$g_{ij} = a^2(t)e^{2\mathcal{R}(t,x)}\delta_{ij}$$

$$\mathcal{R} = -H\pi + \cdots,$$

$$S = \int \ \mathrm{d}^4x\sqrt{-g}\mathcal{L}\left[U, \left(\partial_\mu U\right)^2, \Box\, U, \cdots\right]$$

$$S_\pi^{(2)} = \int \ \mathrm{d}^4x\sqrt{-g}\frac{M_{\mathrm{pl}}^2|\dot{H}|}{c_s^2}\left[\dot{\pi}^2 - \frac{c_s^2}{a^2}(\partial_i\pi)^2 + 3\varepsilon H^2\pi^2\right]$$

$$S_{\mathcal{R}}^{(2)} = \frac{1}{2}\int \ \mathrm{d}^4x a^3 y^2(t)\left[\dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2}(\partial_i\mathcal{R})^2\right]$$

$$y^2 \equiv 2M_{\mathrm{pl}}^2\frac{\varepsilon}{c_s^2}$$

$$v \equiv y\mathcal{R} = \int \mathrm{d}^3k\left[v_k(t)a_{\boldsymbol{k}}e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + c.c.\right]$$

$$\ddot{v}_k + 3H\dot{v}_k + \frac{c_s^2k^2}{a^2}v_k = 0$$

$$\omega_k(t) \equiv \frac{c_sk}{a(t)}$$

$$\hat{v}_{\boldsymbol{k}} = v_k(t)\hat{a}_{\boldsymbol{k}} + h.c.$$

$$|v_k|^2 = \frac{1}{a^3}\frac{1}{2\omega_k}.$$

$$|v_k|^2 = \frac{1}{2}\frac{1}{a_\star^3}\frac{1}{c_sk/a_\star},$$

$$\frac{c_sk}{a_\star} = H.$$

$$|v_k|^2 = \frac{1}{2}\frac{H^2}{(c_sk)^3},$$

Perturbaciones de Curvatura.

$$P_{\mathcal{R}}(k) \equiv |\mathcal{R}_k|^2 = \frac{1}{4}\frac{H^4}{M_{\mathrm{pl}}^2|\dot{H}|c_s}\frac{1}{k^3}$$

$$\Delta_{\mathcal{R}}^2(k) \equiv \frac{k^3}{2\pi^2}P_{\mathcal{R}}(k) = \frac{1}{8\pi^2}\frac{H^4}{M_{\mathrm{pl}}^2|\dot{H}|c_s}.$$

$$n_s - 1 \equiv \frac{d\ln\ \Delta_{\mathcal{R}}^2}{d\ln\ k} = -2\varepsilon - \tilde{\eta} - \kappa$$

$$\tilde{\eta} \equiv \frac{\dot{\varepsilon}}{H\varepsilon} \text{ and } \kappa \equiv \frac{\dot{c}_s}{Hc_s}$$

Ondas gravitacionales.

$$g_{ij} = a^2(t)\left(\delta_{ij} + 2h_{ij}\right)$$

$$S_h^{(2)} = \frac{1}{2}\int \mathrm{d}^4xa^3y^2\left[\left(\dot{h}_{ij}\right)^2 - \frac{1}{a^2}\left(\partial_kh_{ij}\right)^2\right]$$

$$y^2 \equiv \frac{1}{4}M_{\mathrm{pl}}^2$$

$$\Delta_h^2(k) \equiv \frac{k^3}{2\pi^2}P_h(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\mathrm{pl}}^2},$$

$$n_t \equiv \frac{d\ln\ \Delta_h^2}{d\ln\ k} = -2\varepsilon$$

$$r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2}$$

$$\frac{H}{M_{\mathrm{pl}}} = \pi\Delta_{\mathcal{R}}(k_\star)\sqrt{\frac{r}{2}},$$

$$H = 3 \times 10^{-5}\left(\frac{r}{0.1}\right)^{1/2} M_{\mathrm{pl}}$$

$$E_{\mathrm{inf}} \equiv \left(3H^2M_{\mathrm{pl}}^2\right)^{1/4} = 8 \times 10^{-3}\left(\frac{r}{0.1}\right)^{1/4} M_{\mathrm{pl}}$$

Anisotropías CMB.

$$e + p \rightarrow H + \gamma$$

$$\Delta T(\boldsymbol{n}) \equiv T(\boldsymbol{n}) - \bar{T}.$$

$$C(\theta) \equiv \left\langle \frac{\Delta T}{\bar{T}}(\boldsymbol{n})\frac{\Delta T}{\bar{T}}(\boldsymbol{n}')\right\rangle,$$

$$\frac{\Delta T(\boldsymbol{n})}{\bar{T}} = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{+\ell} a_{\ell m}Y_{\ell m}(\boldsymbol{n})$$

$$\langle a_{\ell m}a_{\ell' m'}^*\rangle = C_\ell\delta_{\ell\ell'}\delta_{mm'}$$

$$C_\ell = 2\pi\int_{-1}^{1} \mathrm{dcos}\ \theta C(\theta)P_\ell(\cos\ \theta)$$

$$\hat{C}_\ell = \frac{1}{2\ell+1}\sum_m |a_{\ell m}|^2$$

$$\mathrm{var}(\hat{C}_\ell) \equiv \langle\hat{C}_\ell\hat{C}_\ell\rangle - \langle\hat{C}_\ell\rangle^2 = \frac{2}{2\ell+1}C_\ell^2$$

$$C_\ell = \int \mathrm{dln}\ k\Delta_{\mathcal{R}}^2(k)T_\ell^2(k)$$

Polarización CMB.

$$(Q \pm iU)'(\boldsymbol{n}) = e^{\mp 2i\psi}(Q \pm iU)(\boldsymbol{n}).$$

$$(Q \pm iU)(\boldsymbol{n}) = \sum_{\ell m} a_{\pm 2,\ell m \pm 2}Y_{\ell m}(\boldsymbol{n})$$

$$E(\boldsymbol{n}) \equiv a_{E,\ell m} Y_{\ell m}(\boldsymbol{n}), \quad a_{E,\ell m} \equiv -\frac{a_{2,\ell m} + a_{-2,\ell m}}{2}$$
$$B(\boldsymbol{n}) \equiv a_{B,\ell m} Y_{\ell m}(\boldsymbol{n}), \quad a_{B,\ell m} \equiv -\frac{a_{2,\ell m} - a_{-2,\ell m}}{2i}$$

$$\left\langle a_{X,\ell m} a^{*}_{Y,\ell' m'} \right\rangle = C_{\ell}^{XY} \delta_{\ell \ell'} \delta_{mm'}, X, Y \equiv \{T, E, B\}.$$

Estructura Escalar.

$$P_{\delta}(z,k) = T_{\delta}^{2}(z,k) P_{\mathcal{R}}(k)$$

$$\delta_{g}(z,\boldsymbol{x}) = b(z)\delta(z,\boldsymbol{x})$$

$$D_{V}(\bar{z}) \equiv \left[(1+\bar{z})^{2} D_{A}^{2}(\bar{z}) \frac{c\bar{z}}{H(\bar{z})}\right]^{1/3}$$

Modelo CDM.

$$\Delta_{\mathcal{R}}^{2}(k) = A_{s}\left(\frac{k}{k_{\star}}\right)^{n_{s}-1}.$$

$$A_{s} = \left(2.196^{+0.051}_{-0.060}\right) \times 10^{-9}.$$

$$n_{s} = 0.9603 \pm 0.0073$$

Fluctuaciones Escalares.

$$\Delta_{\mathcal{R}}^{2}(k) = A_{s}\left(\frac{k}{k_{\star}}\right)^{n_{s}-1+\frac{1}{2}\alpha_{s}\ln(k/k_{\star})}$$

Fluctuaciones Tensoriales.

$$r < 0.12 \text{ (95\% limit).}$$

$$\langle 0|\hat{\mathcal{R}}_{\boldsymbol{k}_1}\hat{\mathcal{R}}_{\boldsymbol{k}_2}|0\rangle = (2\pi)^{3} P_{\mathcal{R}}(k_1)\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2),$$

$$\langle \Omega|\hat{\mathcal{R}}_{\boldsymbol{k}_1}\cdots\hat{\mathcal{R}}_{\boldsymbol{k}_n}|\Omega\rangle \propto \int [\mathcal{D}\mathcal{R}]\mathcal{R}_{\boldsymbol{k}_1}\cdots\mathcal{R}_{\boldsymbol{k}_n} e^{iS[\mathcal{R}]},$$

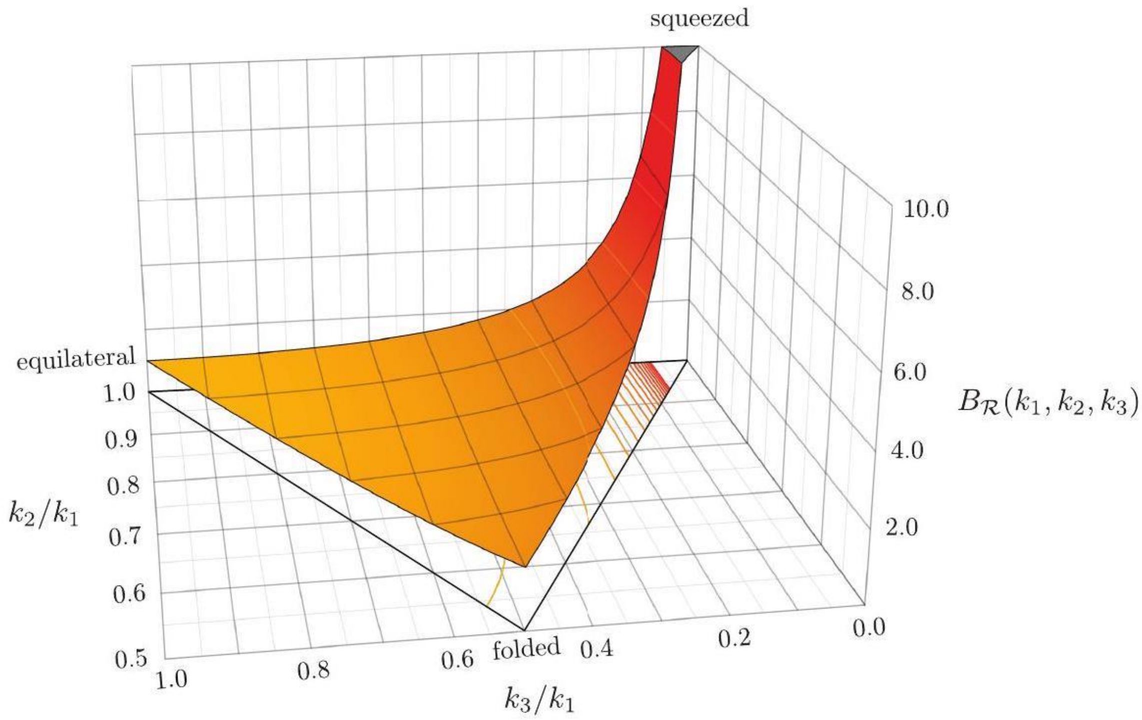
$$\langle \Omega|\hat{\mathcal{R}}_{\boldsymbol{k}_1}\hat{\mathcal{R}}_{\boldsymbol{k}_2}\hat{\mathcal{R}}_{\boldsymbol{k}_3}|\Omega\rangle = (2\pi)^{3} B_{\mathcal{R}}(k_1,k_2,k_3)\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3).$$

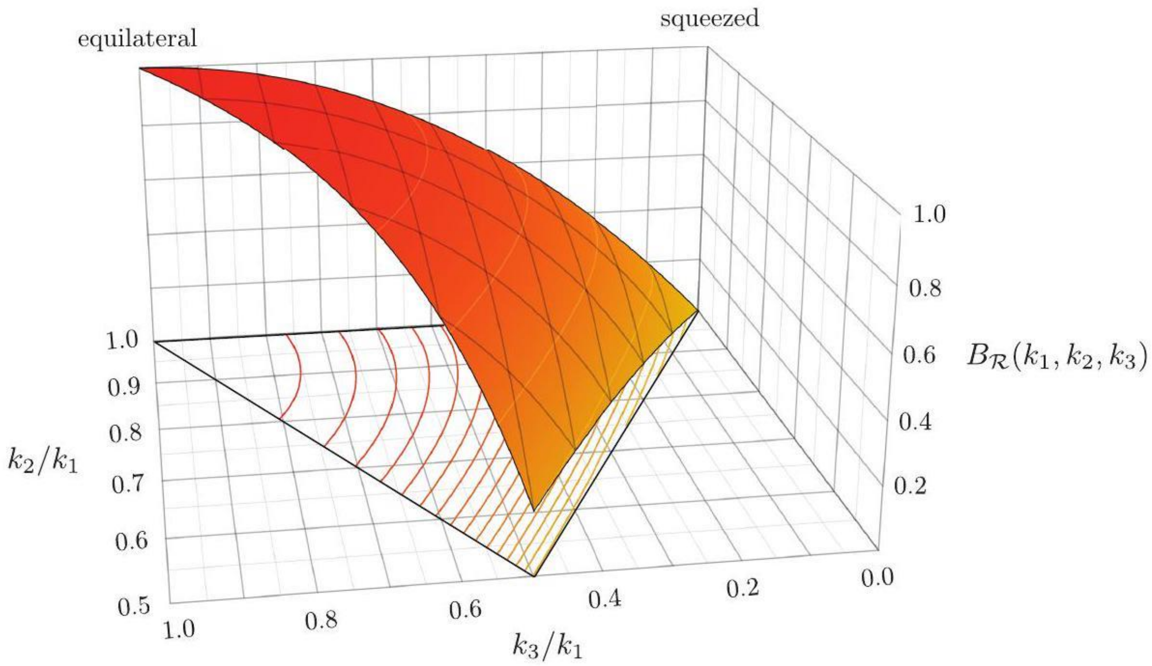
$$f_{\mathrm{NL}} \equiv \frac{5}{18}\frac{B_{\mathcal{R}}(k,k,k)}{P_{\mathcal{R}}^{2}(k)}$$

$$
\begin{aligned}
B_{\text{local}} &\equiv \frac{6}{5}\left(P_1P_2 + \text{ perms.}\right)\\
B_{\text{equil}} &\equiv \frac{3}{5}\left(6(P_1^3P_2^2P_3)^{1/3} - 3P_1P_2 - 2(P_1P_2P_3)^{2/3} + \text{ perms.}\right)\\
B_{\text{ortho}} &\equiv \frac{3}{5}\left(18(P_1^3P_2^2P_3)^{1/3} - 9P_1P_2 - 8(P_1P_2P_3)^{2/3} + \text{ perms.}\right)
\end{aligned}
$$

$$
\mathcal{R}(\boldsymbol{x}) \equiv \mathcal{R}_g(\boldsymbol{x}) + \frac{3}{5}f_{\text{NL}}^{\text{local}}\left[\mathcal{R}_g^2(\boldsymbol{x}) - \langle\mathcal{R}_g^2\rangle\right]
$$

$$
\lim_{k_1\to 0}\frac{B_{\mathcal{R}}(k_1,k_2,k_3)}{P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)} = (1-n_s) \ll 1
$$





Figuras 34 y 35. Curvatura local por deformación del espacio – tiempo cuántico, a propósito de una particular supermasiva, esto en dimension $\mathbb{R}^4$.

$$S_\pi^{(3)} = \int \mathrm{d}^4 x \sqrt{-g}\, \frac{M_{\mathrm{pl}}^2 \dot{H}}{c_s^2} (1 - c_s^2) \left(\frac{\dot{\pi}(\partial_i \pi)^2}{a^2} + \frac{A}{c_s^2} \dot{\pi}^3 \right)$$

$$\begin{aligned} f_{\mathrm{NL}}^{\mathrm{local}} &= 2.7 \pm 5.8, \\ f_{\mathrm{NL}}^{\mathrm{equil}} &= -42 \pm 75, \\ f_{\mathrm{NL}}^{\mathrm{ortho}} &= -25 \pm 39. \end{aligned}$$

Adiabaticity.

$$\delta_I(t, \boldsymbol{x}) \equiv \frac{\bar{\rho}_I(t + \pi(t, \boldsymbol{x})) - \bar{\rho}_I(t)}{\bar{\rho}_I(t)} \approx \frac{\dot{\bar{\rho}}_I}{\bar{\rho}_I} \pi(t, \boldsymbol{x}).$$

$$\mathcal{S} \equiv \delta_c - \frac{3}{4} \delta_\gamma$$

$$\alpha \equiv \frac{P_{\mathcal{S}}}{P_{\mathcal{R}}}$$

$$\alpha_0 < 0.036$$

$$\alpha_{+1} < 0.0025.$$

Curvatura isotrópica.

$$n_t = 2\frac{\dot{H}}{H^2}$$

$$n_t = -\frac{r}{8}$$

$$\alpha_s = 16\varepsilon^2 - 6\varepsilon\tilde{\eta} + \tilde{\eta}\chi$$

$$\mathcal{N}^{\mathrm{CMB}} \sim \left(\frac{\ell_{\max}}{\ell_{\min}}\right)^2 \sim 10^6$$

$$\mathcal{N}_{\mathrm{linear}}^{\mathrm{LSS}} \sim \left(\frac{k_{\max}}{k_{\min}}\right)^3 \sim 10^9$$

$$\mathcal{N}_{\mathrm{linear}}^{\mathrm{Euclid}} \sim \left(\frac{k_{\max}}{k_{\min}}\right)^3 \sim 10^6,$$

$$\left|f_{\mathrm{NL}}^{\mathrm{local}}\right| \gtrsim \mathcal{O}(1)$$

$$f_{\mathrm{NL}}^{\mathrm{equil}} \sim \mathcal{O}(1)$$

$$M_{\mathrm{pl}} \equiv \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}\mathrm{GeV}$$

Acción efectiva.

$$\mathcal{L}[\phi,\Psi] = \mathcal{L}_l[\phi] + \mathcal{L}_h[\Psi] + \mathcal{L}_{lh}[\phi,\Psi]$$

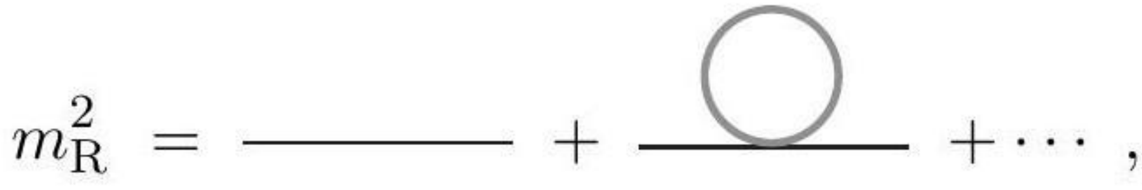
$$e^{iS_{\mathrm{eff}}[\phi]} = \int [\mathcal{D}\Psi] e^{iS[\phi,\Psi]}$$

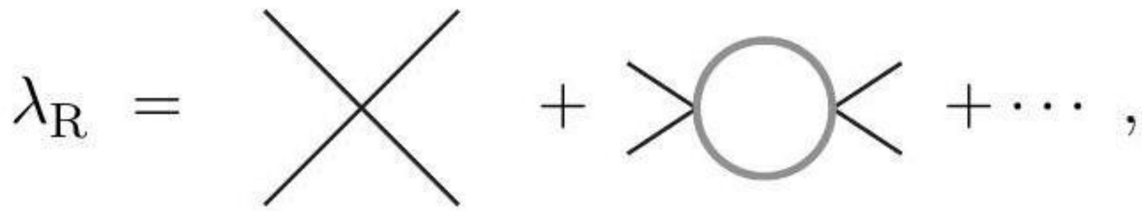
$$\phi(-\Box + M^2)^{-1}\phi = \frac{\phi}{M^2}\left(1 + \frac{\Box}{M^2} + \cdots\right)\phi,$$

$$\mathcal{L}_{\mathrm{eff}}[\phi] = \mathcal{L}_l[\phi] + \sum_i c_i(g)\frac{\mathcal{O}_i[\phi]}{M^{\delta_i - 4}},$$

$$\mathcal{L}[\phi,\Psi] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 - \frac{1}{2}(\partial\Psi)^2 - \frac{1}{2}M^2\Psi^2 - \frac{1}{4}g\phi^2\Psi^2.$$

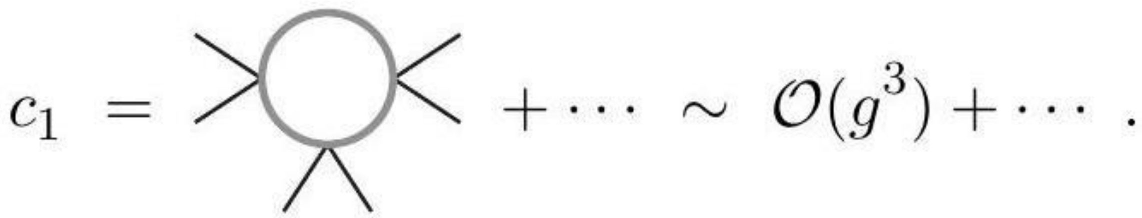
$$\mathcal{L}_{\mathrm{eff}}[\phi] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_{\mathrm{R}}^2\phi^2 - \frac{1}{4!}\lambda_{\mathrm{R}}\phi^4 - \sum_{i=1}^{\infty}\left(\frac{c_i(g)}{M^{2i}}\phi^{4+2i} + \frac{d_i(g)}{M^{2i}}(\partial\phi)^2\phi^{2i} + \cdots\right)$$





$$m_{\mathrm{R}}^2 = m^2 + \frac{g}{32\pi^2}(\Lambda^2 - M^2 L) + \cdots$$
$$\lambda_{\mathrm{R}} = \lambda - \frac{3g^2}{32\pi^2} L + \cdots$$

$$L \to \frac{1}{\epsilon} + \gamma - \ln(4\pi)$$



Parametrizaciones de simetría.

$$\mathcal{L}_{\mathrm{eff}}[\phi] = \mathcal{L}_l[\phi] + \sum_i c_i \frac{\mathcal{O}_i[\phi]}{\Lambda^{\delta_i - 4}},$$

$$\Delta m_e = \alpha\Lambda$$

$$\Delta m_e = \alpha m_e \ln(\Lambda/m_e)$$

$$m_{\pi^+}^2 - m_{\pi^0}^2 = \frac{3\alpha}{4\pi}\Lambda^2$$

$$\frac{m_{K_L^0} - m_{K_S^0}}{m_{K_L^0}} = \frac{G_F^2 f_K^2}{6\pi^2} \sin^2 \theta_c \Lambda^2,$$

$$\Delta m_H^2 \sim \frac{y_t^2}{(4\pi)^2}\Lambda^2$$

$$\langle T_{\mu\nu} \rangle = -\rho_{\mathrm{vac}} g_{\mu\nu}$$

$$\Delta\rho_{\mathrm{vac}} \sim \Lambda^4$$

$$\Delta m^2 \propto \Lambda^2$$

$$\phi \mapsto \phi + \text{ const .}$$

$$\Delta m^2 \propto m^2$$

$$\lim_{g \to 0} c_i(g) = 0$$

$$S_{\mathrm{EH}} = \frac{M_{\mathrm{pl}}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} R$$

$$S_{\mathrm{EH}} = \int \mathrm{d}^4 x \left[(\partial h)^2 + \frac{1}{M_{\mathrm{pl}}} h (\partial h)^2 + \frac{1}{M_{\mathrm{pl}}^2} h^2 (\partial h)^2 + \cdots \right]$$

$$S_{\mathrm{YM}} = \int \mathrm{d}^4 x [(\partial A)^2 + g A^2 \partial A + g^2 A^4]$$

$$S_g = \int \mathrm{d}^4 x \sqrt{-g} \left[M_\Lambda^4 + \frac{M_{\mathrm{pl}}^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{1}{M^2} (d_1 R^3 + \cdots) + \cdots \right]$$

$$S_g = \int \mathrm{d}^{10} X \sqrt{-G} \left[\frac{M_{10}^2}{2} R + \frac{\zeta(3)}{3 \cdot 2^5} \frac{1}{M^6} \mathcal{R}^4 + \cdots \right]$$

$$M^2 = \frac{4}{\alpha'}$$

$$S_{\mathrm{eff}}[\phi, g] = S_g + S_{\mathrm{eff}}[\phi] + S_{g,\phi}$$

$$S_{g,\phi} = \int \mathrm{d}^4 x \sqrt{-g} \left[\sum_i c_i \frac{\mathcal{O}_i[g,\phi]}{\Lambda^{\delta_i - 4}} \right]$$

$$S_{g,\phi}^{(4)} = \int \mathrm{d}^4 x \sqrt{-g} \xi \phi^2 R$$

$$g_{\mu\nu} \mapsto \bar{g}_{\mu\nu} \equiv e^{2\omega(\phi)} g_{\mu\nu},$$

Dinámicas y perturbaciones inflacionarias.

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{M_{\mathrm{pl}}^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right]$$

$$3 M_{\mathrm{pl}}^2 H^2 = \frac{1}{2} \dot{\phi}^2 + V \quad \text{and} \quad \ddot{\phi} + 3 H \dot{\phi} = -V',$$

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2} \dot{\phi}^2}{M_{\mathrm{pl}}^2 H^2}$$

$$\epsilon \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1\,, |\eta| \equiv M_{\rm pl}^2 \frac{|V''|}{V} \ll 1.$$

$$\pi = \frac{\delta\phi}{\dot{\phi}}$$

$$\mathcal{R}(t,\boldsymbol{x}) = -H\pi(t,\boldsymbol{x}) = -\frac{H}{\dot{\phi}}\delta\phi(t,\boldsymbol{x})$$

$$\Delta_{\mathcal{R}}^2 = \frac{1}{24\pi^2}\frac{1}{\epsilon}\frac{V}{M_{\rm pl}^4}\,, \Delta_h^2 = \frac{2}{3\pi^2}\frac{V}{M_{\rm pl}^4}$$

$$\begin{aligned} n_s - 1 &= 2\eta - 6\epsilon \\ r &= 16\epsilon \end{aligned}$$

$$N_\star = \int_{\phi_{\rm end}}^{\phi_\star} \frac{{\rm d}\phi}{M_{\rm pl}} \frac{1}{\sqrt{2\epsilon}}$$

$$V(\phi) = \mu^{4-p}\phi^p$$

$$\epsilon = \frac{p^2}{2}\left(\frac{M_{\rm pl}}{\phi}\right)^2, \eta = p(p-1)\left(\frac{M_{\rm pl}}{\phi}\right)^2.$$

$$N_\star \approx \frac{1}{2p}\left(\frac{\phi_\star}{M_{\rm pl}}\right)^2,$$

$$n_s - 1 = -\frac{(2+p)}{2N_\star}\,, r = \frac{4p}{N_\star}.$$

$$\begin{aligned} p = 1: &\quad n_s \approx 0.975, \quad r \approx 0.07, \quad \phi_\star \approx 11M_{\rm pl}, \\ p = 2: &\quad n_s \approx 0.967, \quad r \approx 0.13, \quad \phi_\star \approx 15M_{\rm pl}, \\ p = 3: &\quad n_s \approx 0.958, \quad r \approx 0.20, \quad \phi_\star \approx 19M_{\rm pl}, \\ p = 4: &\quad n_s \approx 0.950, \quad r \approx 0.27, \quad \phi_\star \approx 22M_{\rm pl}. \end{aligned}$$

$$V(\phi) = \frac{V_0}{2}\left[1 - \cos\left(\frac{\phi}{f}\right)\right],$$

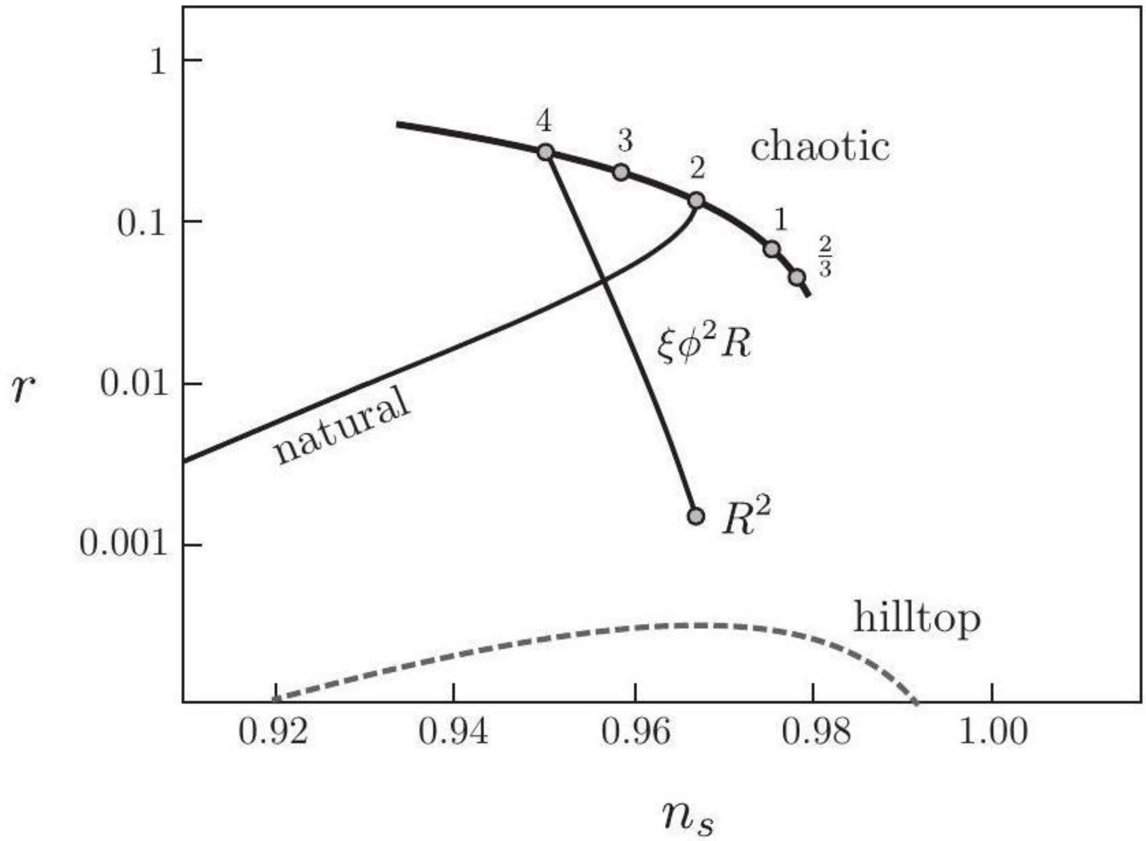
$$\begin{aligned} n_s - 1 &= -\alpha\frac{e^{N_\star\alpha}+1}{e^{N_\star\alpha}-1} \overset{\alpha\ll 1}{\to} -\frac{2}{N_\star}, \\ r &= 8\alpha\frac{1}{e^{N_\star\alpha}-1} \overset{\alpha\ll 1}{\to} +\frac{8}{N_\star}, \end{aligned}$$

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \cdots$$

$$V(\phi) = V_0\left[1 + \frac{1}{2}\eta_0\frac{\phi^2}{M_{\rm pl}^2} + \cdots\right], \text{ where } \eta \approx \eta_0 < 0$$

$$n_s - 1 = 2\eta_0$$

$$r = 2(1-n_s)^2 e^{-N_\star(1-n_s)}\left(\frac{\phi_{\text{end}}}{M_{\text{pl}}}\right)^2 \approx 10^{-3}\left(\frac{\phi_{\text{end}}}{M_{\text{pl}}}\right)^2.$$



$$V(\phi) = V_0\left[1 + \lambda_0\frac{\phi}{M_{\text{pl}}} + \frac{1}{2}\eta_0\frac{\phi^2}{M_{\text{pl}}^2} + \frac{1}{3!}\mu_0\frac{\phi^3}{M_{\text{pl}}^3} + \cdots\right]$$

$$V(\phi) \approx V_0\left[1 + \lambda_0\frac{\phi}{M_{\text{pl}}} + \frac{1}{3!}\mu_0\frac{\phi^3}{M_{\text{pl}}^3} + \cdots\right]$$

$$n_s - 1 = -4\sqrt{\frac{\lambda_0\mu_0}{2}}\cot\left(N_\star\sqrt{\frac{\lambda_0\mu_0}{2}}\right),$$

$$r = 16\lambda_0^2$$

$$V(\phi,\Psi) = V(\phi) + V(\Psi) + \frac{1}{2}g\phi^2\Psi^2,$$

$$V(\Psi) \equiv \frac{1}{4\lambda}(M^2 - \lambda\Psi^2)^2$$

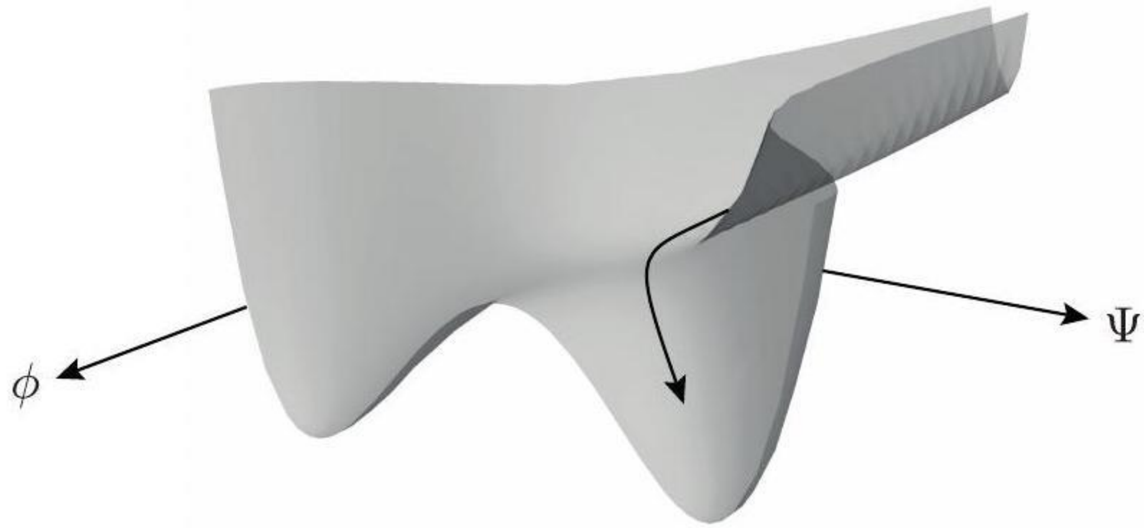


Figura 36. Membranas en dimensión $\mathbb{R}^4$

$$M_{\Psi}^2(\phi) = -M^2 + g\phi^2$$

$$S = \frac{M_{\rm pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{\alpha}{2M_{\rm pl}^2}R^2\right)$$

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left(\frac{M_{\rm pl}^2}{2}\tilde{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

$$V(\phi) = \frac{M_{\rm pl}^4}{4\alpha}\left(1 - \exp\left[-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm pl}}\right]\right)^2.$$

$$\eta = -\frac{4}{3}e^{-\sqrt{2/3}\phi/M_{\rm pl}}, \epsilon = \frac{3}{4}\eta^2$$

$$\alpha = 2.2 \times 10^8$$

$$n_s - 1 \approx -\frac{2}{N_\star}, r \approx \frac{12}{N_\star^2}$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}\left(1 + \xi\frac{\varphi^2}{M_{\rm pl}^2}\right)R - \frac{1}{2}(\partial\varphi)^2 - \frac{\lambda}{4}\varphi^4\right]$$

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\frac{M_{\rm pl}^2}{2}\tilde{R} - \frac{1}{2}k(\varphi)(\partial\varphi)^2 - V(\varphi)\right]$$

$$k(\varphi) = \frac{1 + (6\xi + 1)\psi^2}{(1 + \psi^2)^2}$$

$$V(\varphi) = \frac{\lambda M_{\rm pl}^4}{4\xi^2}\frac{\psi^4}{(1 + \psi^2)^2}, \psi^2 \equiv \frac{\xi\varphi^2}{M_{\rm pl}^2}$$

$$\frac{\phi}{M_{\rm pl}} = \sqrt{\frac{6\xi+1}{\xi}}\sinh^{-1}\left(\sqrt{6\xi+1}\psi\right) - \sqrt{6}\sinh^{-1}\left(\sqrt{6\xi}\frac{\psi}{\sqrt{1+\psi^2}}\right).$$

$$\frac{\phi}{M_{\rm pl}} \approx \sqrt{\frac{3}{2}}\ln\left(1+\psi^2\right),$$

$$V(\phi) = \frac{\lambda M_{\rm pl}^4}{4\xi^2}\left(1-\exp\left[-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm pl}}\right]\right)^2.$$

$$\xi = 47000\sqrt{\lambda}$$

$$\begin{aligned} n_s - 1 &= -\frac{32\xi}{16\xi N_\star - 1} \overset{\xi\gg1}{\rightarrow} -\frac{2}{N_\star}, \\ r &= +\frac{12}{N_\star^2}\frac{6\xi+1}{6\xi} \overset{\xi\gg1}{\rightarrow} +\frac{12}{N_\star^2}. \end{aligned}$$

Inflación K.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + P(X,\phi)\right]$$

$$3M_{\rm pl}^2H^2 = 2P_{,X}X - P \quad \text{and} \quad \frac{d}{dt}\left(a^3P_{,X}\dot{\phi}\right) = a^3P_{,\phi}$$

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{3XP_{,X}}{2XP_{,X} - P}$$

$$c_s^2 = \frac{dP}{d\rho} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}.$$

$$\begin{aligned} n_s - 1 &= -2\varepsilon - \tilde{\eta} - \kappa \\ r &= 16\varepsilon c_s \end{aligned}$$

$$S_{\rm eff}[\phi] = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + \mathcal{L}_l[\phi] + \sum_i c_i\frac{\mathcal{O}_i[\phi]}{\Lambda^{\delta_i-4}}\right]$$

$$\Delta m^2 \sim \Lambda^2.$$

$$\Delta\eta \sim \frac{\Lambda^2}{H^2} \gtrsim 1$$

$$\Delta m^2 \sim H^2$$

$$\Delta\eta \sim 1.$$

$$\phi \mapsto \phi + \text{ const }.$$

Operadores en altas dimensiones.

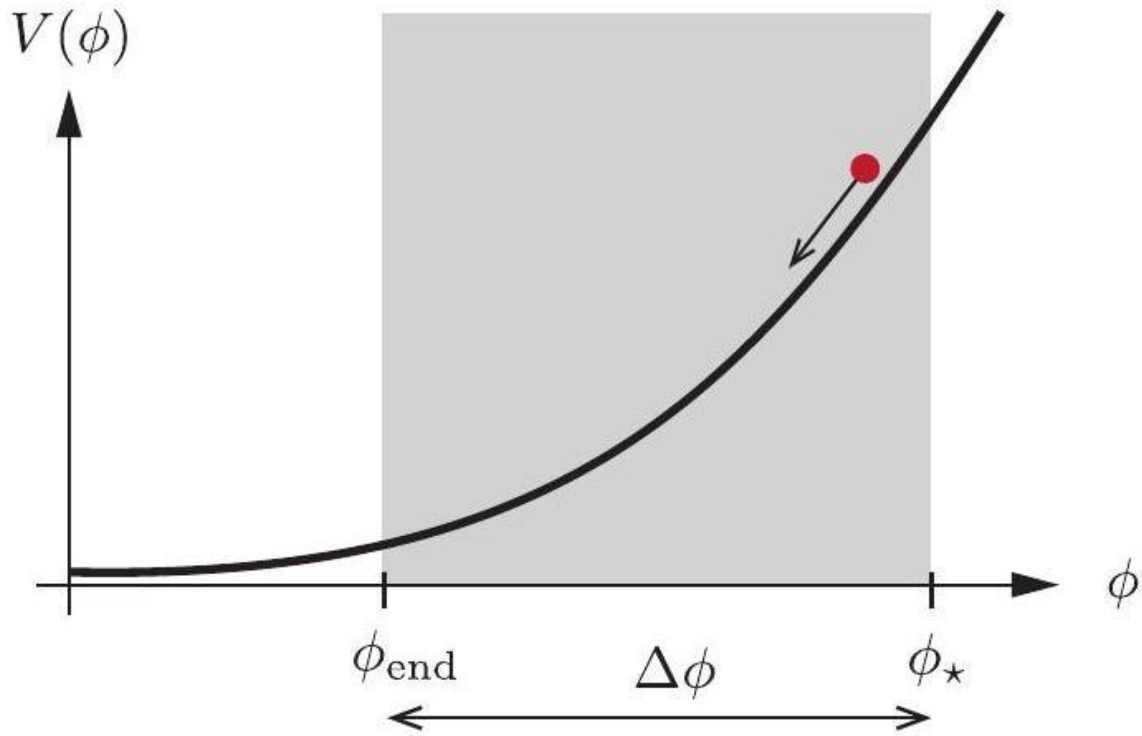
$$\mathcal{O}_6 = cV_l(\phi)\frac{\phi^2}{\Lambda^2}$$

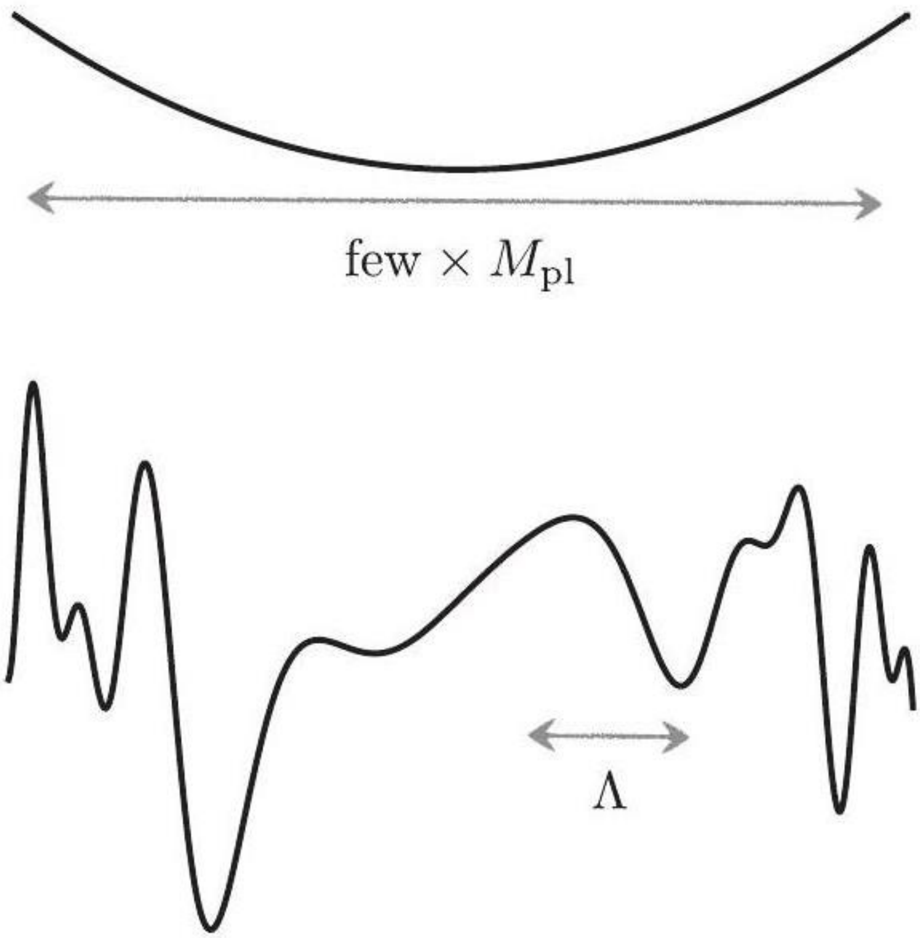
$$\Delta\eta \approx 2c\left(\frac{M_{\mathrm{pl}}}{\Lambda}\right)^2$$

$$\mathcal{O}_\delta = c\langle V\rangle\left(\frac{\phi}{\Lambda}\right)^{\delta-4}$$

$$\Delta\eta \approx c(\delta-4)(\delta-5)\left(\frac{M_{\mathrm{pl}}}{\Lambda}\right)^2\left(\frac{\phi}{\Lambda}\right)^{\delta-6}.$$

Ondas gravitacionales en campos planckianos y campos inflacionarios.





Figuras 37 y 38. Fluctuaciones gravitacionales en la curvatura local.

$$r = 8\left(\frac{1}{M_{\mathrm{pl}}}\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2, \text{ where } \mathrm{d}N \equiv H\,\mathrm{d}t.$$

$$\frac{\Delta\phi}{M_{\mathrm{pl}}} = \int_0^{N_\star} \mathrm{d}N \sqrt{\frac{r(N)}{8}}.$$

$$N_{\mathrm{eff}} \equiv \int_0^{N_\star} \mathrm{d}N \sqrt{\frac{r(N)}{r_\star}}$$

$$\frac{\Delta\phi}{M_{\mathrm{pl}}} = N_{\mathrm{eff}}\sqrt{\frac{r_\star}{8}}.$$

$$\frac{\mathrm{d}\ln\, r}{\mathrm{d}N} = -\left[n_s - 1 + \frac{r}{8}\right],$$

$$\frac{\Delta\phi}{M_{\mathrm{pl}}} \gtrsim \left(\frac{r}{0.01}\right)^{1/2}$$

$$\frac{\Delta\phi}{M_{\text{pl}}} \gtrsim 0.25 \times \left(\frac{r}{0.01}\right)^{1/2}$$

$$\frac{\Delta V}{V} = c_1 \frac{V''}{M_{\text{pl}}^2} + c_2 \frac{V}{M_{\text{pl}}^4},$$

$$\mathcal{L}_{\text{eff}}[\phi] = \mathcal{L}_l[\phi] + \sum_{i=1}^{\infty} \left(\frac{c_i}{\Lambda^{2i}} \phi^{4+2i} + \frac{d_i}{\Lambda^{2i}} (\partial\phi)^2 \phi^{2i} + \frac{e_i}{\Lambda^{4i}} (\partial\phi)^{2(i+1)} + \cdots\right),$$

$$\phi \mapsto \phi + \text{ const .}$$

$$\frac{\Delta\phi}{M_{\text{pl}}} = \left(c_s P_{,X}\right)^{-1/2} \sqrt{\frac{r}{8}} \Delta N.$$

$$\Delta\mathcal{L} = P\left(X - V(\phi)\frac{\phi^2}{M_{\text{pl}}^2}\right) = P(X) - P_{,X} V(\phi) \frac{\phi^2}{M_{\text{pl}}^2} + \cdots$$

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2X P_{,XX}}$$

$$P = X + \frac{1}{2}\frac{X^2}{\Lambda^4} + \cdots$$

$$\left|f_{\text{NL}}^{\text{equil}}\right| \sim \frac{1}{c_s^2} - 1 \approx \frac{X}{\Lambda^4} + \cdots \ll 1.$$

$$\mathcal{O}_5 = \frac{\psi X}{\Lambda}$$

$$\Lambda \gtrsim 10^5 H$$

$$\Lambda \gtrsim \left(\frac{r}{0.01}\right)^{1/2} M_{\text{pl}}$$

$$S_{\text{P}} = -\frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma \sqrt{-h} h^{ab} \partial_a X^M(\sigma) \partial_b X^N(\sigma) \eta_{MN}$$

$$S_{\text{P}} = -\frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma \partial^a X^M \partial_a X_M$$

$$S_\sigma = -\frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma \sqrt{-h} \left([h^{ab} G_{MN}(X) + \epsilon^{ab} B_{MN}(X)] \partial_a X^M \partial_b X^N + \alpha' \Phi(X) R(h) \right)$$

$$S_{\text{B}} = \frac{1}{2\kappa_D^2} \int \mathrm{d}^D X \sqrt{-G} e^{-2\Phi} \left(R + 4(\partial\Phi)^2 - \frac{1}{2}|H_3|^2 - \frac{2(D-26)}{3\alpha'} + \mathcal{O}(\alpha') \right)$$

$$e^{-\Phi\chi} = e^{-\Phi(2-2g)} \equiv g_s^{2g-2}$$

$$S = S_{\mathrm{P}} + S_{\mathrm{F}} = -\frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma(\partial^a X^M \partial_a X_M - i\bar{\psi}^M \rho^a \partial_a \psi_M)$$

$$\{\rho^a, \rho^b\} = -2\eta^{ab}$$

$$\psi^M \equiv \begin{pmatrix} \psi_-^M \\ \psi_+^M \end{pmatrix}$$

$$S_{\mathrm{F}} = \frac{i}{2\pi\alpha'}\int \mathrm{d}^2\sigma(\psi_-^M \partial_+ \psi_-^N + \psi_+^M \partial_- \psi_+^N)\eta_{MN}$$

$$S_{\mathrm{NS}} = \frac{1}{2\kappa^2}\int \mathrm{d}^{10}X\sqrt{-G}e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - \frac{1}{2}|H_3|^2\right)$$

$$2\kappa^2 = (2\pi)^7(\alpha')^4$$

$$S_{\mathrm{IIA}} = S_{\mathrm{NS}} + S_{\mathrm{R}}^{(\mathrm{IIA})} + S_{\mathrm{CS}}^{(\mathrm{IIA})},$$

$$\begin{aligned} S_{\mathrm{R}}^{(\mathrm{IIA})} &= -\frac{1}{4\kappa^2}\int \mathrm{d}^{10}X\sqrt{-G}\left(|F_2|^2 + |\tilde{F}_4|^2\right), \\ S_{\mathrm{CS}}^{(\mathrm{IIA})} &= -\frac{1}{4\kappa^2}\int B_2 \wedge F_4 \wedge F_4, \end{aligned}$$

$$S_{\mathrm{IIB}} = S_{\mathrm{NS}} + S_{\mathrm{R}}^{(\mathrm{IIB})} + S_{\mathrm{CS}}^{(\mathrm{IIB})},$$

$$\begin{aligned} S_{\mathrm{R}}^{(\mathrm{IIB})} &= -\frac{1}{4\kappa^2}\int \mathrm{d}^{10}X\sqrt{-G}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right), \\ S_{\mathrm{CS}}^{(\mathrm{IIB})} &= -\frac{1}{4\kappa^2}\int C_4 \wedge H_3 \wedge F_3, \end{aligned}$$

$$\tilde{F}_5 = \star\, \tilde{F}_5$$

$$G_{E,MN} \equiv e^{-\Phi/2}G_{MN}$$

$$\begin{aligned} G_3 &\equiv F_3 - \tau H_3 \\ \tau &\equiv C_0 + ie^{-\Phi} \end{aligned}$$

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa^2}\int \mathrm{d}^{10}X\sqrt{-G_E}\left[R_E - \frac{|\partial\tau|^2}{2(\mathrm{Im}(\tau))^2} - \frac{|G_3|^2}{2\mathrm{Im}(\tau)} - \frac{|\tilde{F}_5|^2}{4}\right] - \frac{i}{8\kappa^2}\int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\mathrm{Im}(\tau)}$$

Membranas en dimensión $\mathbb{R}^4$.

$$S_{\mathrm{CS}} = \mu_p \int_{\Sigma_{p+1}} C_{p+1}$$

$$S_{\mathrm{D}} = -T_p \int \mathrm{d}^{p+1}\sigma\sqrt{-\det(G_{ab})}$$

$$G_{ab} \equiv \frac{\partial X^M}{\partial \sigma^a}\frac{\partial X^N}{\partial \sigma^b} G_{MN}$$

$$S_{\mathrm{BI}} = -Q_p \int \mathrm{d}^{p+1}\sigma\sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab})} = -Q_p \int \mathrm{d}^{p+1}\sigma\left(1 + \frac{(2\pi\alpha')^2}{4} F_{ab}F^{ab} + \cdots\right)$$

$$S_{\mathrm{DBI}} = -g_s T_p \int \mathrm{d}^{p+1}\sigma e^{-\Phi}\sqrt{-\det(G_{ab} + \mathcal{F}_{ab})}$$

$$\mathcal{F}_{ab} \equiv B_{ab} + 2\pi\alpha' F_{ab}$$

$$T_p \equiv \frac{1}{(2\pi)^p g_s (\alpha')^{(p+1)/2}}$$

$$S_{\mathrm{CS}} = i\mu_p \int_{\Sigma_{p+1}} \sum_n C_n \wedge e^{\mathcal{F}}$$

$$S_{\mathrm{D}p} = S_{\mathrm{DBI}} + S_{\mathrm{CS}}$$

$$r_+^{7-p} = d_p g_s N (\alpha')^{\frac{1}{2}(7-p)}$$

$$e^{\Phi} = g_s\left(1 + \left(\frac{r_+}{r}\right)^{7-p}\right)^{\frac{1}{4}(3-p)}$$

$$1 \ll g_s N \ll N$$

$$\mathcal{M}_{10} = \mathcal{M}_4 \times X_6$$

$$G_{MN}\,\mathrm{d}X^M\,\mathrm{d}X^N = \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}\,\mathrm{d}y^m\,\mathrm{d}y^n$$

$$G_{MN}\,\mathrm{d}X^M\,\mathrm{d}X^N = e^{2A(y)} g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + e^{-2A(y)} g_{mn}\,\mathrm{d}y^m\,\mathrm{d}y^n,$$

$$\mathcal{L}_\Phi = -K_{i\bar{j}}\partial^\mu\phi^i\partial_\mu\bar{\phi}^j - V_F,$$

$$V_F(\phi^i,\bar{\phi}^i) = e^{K/M_{\mathrm{pl}}^2}\left[K^{i\bar{j}} D_i W \overline{D_j W} - \frac{3}{M_{\mathrm{pl}}^2}|W|^2\right]$$

$$G_{MN}\,\mathrm{d}X^M\,\mathrm{d}X^N = e^{-6u(x)} g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + e^{2u(x)} \hat{g}_{mn}\,\mathrm{d}y^m\,\mathrm{d}y^n,$$

$$\int_{X_6} \mathrm{d}^6 y\sqrt{\hat{g}} \equiv \mathcal{V}$$

$$S_{\mathrm{EH}}^{(10)} = \frac{1}{2\kappa^2}\int \mathrm{d}^{10}X\sqrt{-G}e^{-2\Phi}R_{10}$$

$$\bar{g}_{MN} = e^{2\omega(x)}g_{MN}$$

$$e^{2\omega}\bar{R} = R - 2(D-1)\nabla^2\omega - (D-2)(D-1)g^{MN}\nabla_M\omega\nabla_N\omega.$$

$$e^{2\omega}\bar{\nabla}^2 = \nabla^2 + (D-2)g^{MN}\nabla_M\omega\nabla_N.$$

$$S_{\mathrm{EH}}^{(10)} = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\int_{X_6}\mathrm{d}^6y\sqrt{\hat{g}}e^{-2\Phi}\left(R_4 + e^{-8u}\hat{R}_6 + 12\partial_\mu u\partial^\mu u\right)$$

$$S_{\mathrm{EH}}^{(4)} = \frac{M_{\mathrm{pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R_4$$

$$M_{\mathrm{pl}}^2 \equiv \frac{\mathcal{V}}{g_s^2\kappa^2}$$

$$K = -3\ln\left(T + \bar{T}\right)$$

Métrica de Moduli - Kähler.

$$J \equiv i g_{i\bar{j}}\,\mathrm{d}z^i \wedge \mathrm{d}\bar{z}^{\bar{j}}$$

$$J = t^I(x)\omega_I$$

$$\delta g_{ij} = \frac{i}{\|\Omega\|^2}\zeta^A(x)(\chi_A)_{i\bar{\imath}\bar{j}}\Omega^{\bar{\imath}\bar{j}}{}_j$$

$$\begin{aligned} B_2 &= B_2(x) + b^I(x)\omega_I, \\ C_2 &= C_2(x) + c^I(x)\omega_I, \\ C_4 &= \vartheta^I(x)\tilde{\omega}_I. \end{aligned}$$

$$\mathcal{O} = (-1)^{F_L}\Omega_{ws}\sigma$$

$$H^{1,1} = H_+^{1,1} \oplus H_-^{1,1}$$

$$J = t^i(x)\omega_i$$

$$\begin{aligned} B_2 &= b^\alpha(x)\omega_\alpha, \\ C_2 &= c^\alpha(x)\omega_\alpha, \\ C_4 &= \vartheta^i(x)\tilde{\omega}_i. \end{aligned}$$

$$\tau = C_0 + ie^{-\Phi}$$

$$G_\alpha \equiv c_\alpha - \tau b_\alpha$$

$$\mathcal{V} = \frac{1}{6}\int_{X_6} J \wedge J \wedge J = \frac{1}{6}c_{ijk}t^it^jt^k,$$

$$T_i \equiv \frac{1}{2}c_{ijk}t^jt^k + i\vartheta_i + \frac{1}{4}e^{\Phi}c_{i\alpha\beta}G^\alpha(G-\bar{G})^\beta,$$

$$T_i = \frac{1}{2}c_{ijk}t^j t^k + i\vartheta_i$$

$$\tau_i = \frac{\partial \mathcal{V}}{\partial t^i} = \frac{1}{2}c_{ijk}t^j t^k,$$

$$T_i = \tau_i + i\vartheta_i$$

Simetrías de Peccei-Quinn (PQ).

$$a \mapsto a + \text{ const}.$$

$$b_I = \frac{1}{\alpha'}\int_{\Sigma_2^I} B_2, c_I = \frac{1}{\alpha'}\int_{\Sigma_2^I} C_2, \vartheta_I = \frac{1}{(\alpha')^2}\int_{\Sigma_4^I} C_4$$

$$\int_{\Sigma_2^I} \omega^J = \alpha'\delta_I^J, \int_{\Sigma_4^I} \tilde{\omega}^J = (\alpha')^2\delta_I^J$$

$$S_\sigma \supset -\frac{1}{4\pi\alpha'}\int_{\Sigma_2} \mathrm{d}^2\sigma\epsilon^{ab}\partial_a X^M \partial_b X^N B_{MN}(X),$$

$$S_\sigma \supset -\frac{1}{2\pi\alpha'}\int_{\Sigma_2} B_2 \equiv -\frac{b}{2\pi},$$

$$B_{MN}(X) = B_{MN}\left(X_{(0)}\right) + X^P\partial_P B_{MN}\left(X_{(0)}\right) + \cdots$$

$$-\frac{1}{4\pi\alpha'}\int_{\Sigma_2} \mathrm{d}^2\sigma\partial_a\left(\epsilon^{ab}X^M\partial_b X^N B_{MN}\left(X_{(0)}\right)\right)$$

$$\mathcal{S}_{\text{inst}} = \exp\left(-\frac{1}{2\pi\alpha'}\int_{\Sigma_2} (J + iB_2)\right) \propto \exp\left(-i\frac{b}{2\pi}\right)$$

$$S_{\mathrm{ED}p} = \exp\left(-T_p \mathrm{Vol}(\Sigma_p)\right),$$

$$\mathcal{L}(a) = -\frac{1}{2}f^2(\partial a)^2 - \Lambda^4[1 - \cos(a/2\pi)] + \cdots,$$

$$B_2 = b_\alpha(x)\omega^\alpha.$$

$$\frac{1}{2(2\pi)^7 g_s^2(\alpha')^4}\int \mathrm{d}^{10}X|\,\mathrm{d}B_2|^2 \supset \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\gamma^{\alpha\beta}\left(\partial^\mu b_\alpha \partial_\mu b_\beta\right),$$

$$\gamma^{\alpha\beta} \equiv \frac{1}{6(2\pi)^7 g_s^2(\alpha')^4}\int_{X_6} \omega^\alpha \wedge \star_6\, \omega^\beta.$$

$$\frac{f^2}{M_{\mathrm{pl}}^2} \approx \frac{1}{6}\frac{\alpha'^2}{L^4}$$

Compactifiaciones de Calabi-Yau.

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}\,dy^m\,dy^n$$

$$\tilde{F}_5 = (1+\star_{10})d\alpha(y)\wedge dx^0\wedge\ dx^1\wedge\ dx^2\wedge\ dx^3$$

$$\nabla^2 e^{4A} = \frac{e^{8A}}{2\mathrm{Im}(\tau)}|G_3|^2 + e^{-4A}(|\partial\alpha|^2 + |\partial e^{4A}|^2) + 2\kappa^2 e^{2A}\mathcal{J}_{\mathrm{loc}}$$

$$\mathcal{J}_{\mathrm{loc}} \equiv \frac{1}{4}\left(\sum_{M=4}^{9} T^M{}_M - \sum_{M=0}^{3} T^M{}_M\right)_{\mathrm{loc}}$$

$$d\tilde{F}_5 = H_3\wedge F_3 + 2\kappa^2 T_3\rho_3^{\mathrm{loc}}$$

$$\frac{1}{2\kappa^2 T_3}\int_{X_6} H_3\wedge F_3 + Q_3^{\mathrm{loc}} = 0$$

$$\nabla^2(e^{4A}-\alpha) = \frac{e^{8A}}{24\mathrm{Im}(\tau)}|iG_3 - \star_6 G_3|^2 + e^{-4A}|\partial(e^{4A}-\alpha)|^2 + 2\kappa^2 e^{2A}(\mathcal{J}_{\mathrm{loc}} - \mathcal{Q}_{\mathrm{loc}})$$

$$\mathcal{J}_{\mathrm{loc}} \geq \mathcal{Q}_{\mathrm{loc}}$$

$$\star_6\, G_3 = iG_3,$$

$$e^{4A} = \alpha$$

$$V_{\mathrm{flux}} = \frac{1}{2\kappa^2}\int d^{10}X\sqrt{-G_E}\left[-\frac{|G_3|^2}{2\mathrm{Im}(\tau)}\right]$$

$$K_0 = -2\ln(\mathcal{V}) - \ln(-i(\tau-\bar{\tau})) - \ln\left(-i\int \Omega\wedge\bar{\Omega}\right)$$

$$W_0 = \frac{c}{\alpha'}\int G_3\wedge\Omega$$

$$V_F = e^{K_0}\left[K_0^{I\bar{J}}D_I W_0\overline{D_J W_0} - 3|W_0|^2\right]$$

$$D_I W_0 \equiv \partial_I W_0 + (\partial_I K)W_0 = 0$$

$$\sum_{I,J=T_i} K_0^{I\bar{J}}\partial_I K_0\partial_{\bar{J}}K_0 = 3.$$

$$V_F = e^{K_0}\sum_{I,J\neq T_i} K_0^{I\bar{J}}D_I W_0\overline{D_J W_0}$$

$$K(T,\bar{T},z_\alpha,\bar{z}_\alpha) = -3\ln\left[T+\bar{T}-\gamma k(z_\alpha,\bar{z}_\alpha)\right],$$

Correcciones perturbativas Kaluza-Klein (KK) y singularidad.

$$K=-2\ln\left[\mathcal{V}+\frac{\xi}{2g_s^{3/2}}\right], \xi\equiv-\frac{\chi(X_6)\zeta(3)}{2(2\pi)^3}$$

$$\delta K_{(g_s)}=\delta K_{(g_s)}^{\mathrm{KK}}+\delta K_{(g_s)}^{\mathrm{W}}$$

$$\delta K_{(g_s)}^{\mathrm{KK}}=-\frac{1}{128\pi^2}\sum_{i=1}^{3}\frac{\mathcal{E}_i^{\mathrm{KK}}(\zeta,\bar{\zeta})}{\mathrm{Re}(\tau)\tau_i},$$

$$\delta K_{(g_s)}^{\mathrm{W}}=-\frac{1}{128\pi^2}\sum_{i=1}^{3}\frac{\mathcal{E}_i^{\mathrm{W}}(\zeta,\bar{\zeta})}{\tau_j\tau_k}\bigg|_{j\neq k\neq i}$$

$$S=\frac{1}{2g_7^2}\int_{\Sigma_4}\mathrm{d}^4\sigma\sqrt{g_{\mathrm{ind}}}\,e^{-4A(y)}\cdot\int\mathrm{d}^4x\sqrt{-g}\,\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right]$$

$$g_7^2=2(2\pi)^5(\alpha')^2$$

$$\frac{1}{g^2}=\frac{T_3\mathcal{V}_4}{8\pi^2},$$

$$\mathcal{V}_4\equiv\int_{\Sigma_4}\mathrm{d}^4\sigma\sqrt{g_{\mathrm{ind}}}\,e^{-4A(y)}$$

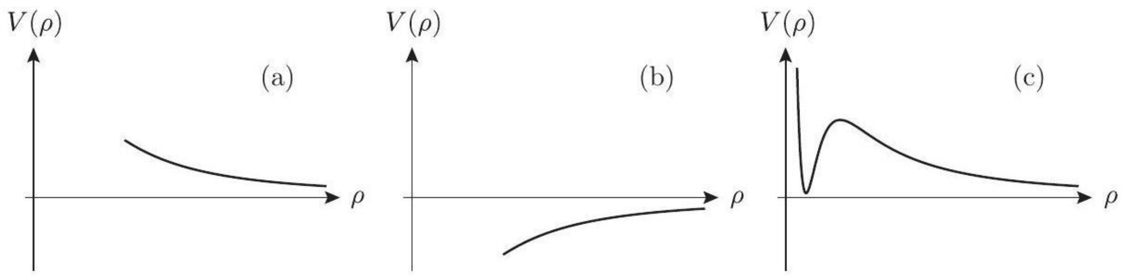
$$|W_{\lambda\lambda}|=16\pi^2M_{\mathrm{UV}}^3\exp\left(-\frac{1}{N_c}\frac{8\pi^2}{g^2}\right)\propto\exp\left(-\frac{T_3\mathcal{V}_4}{N_c}\right).$$

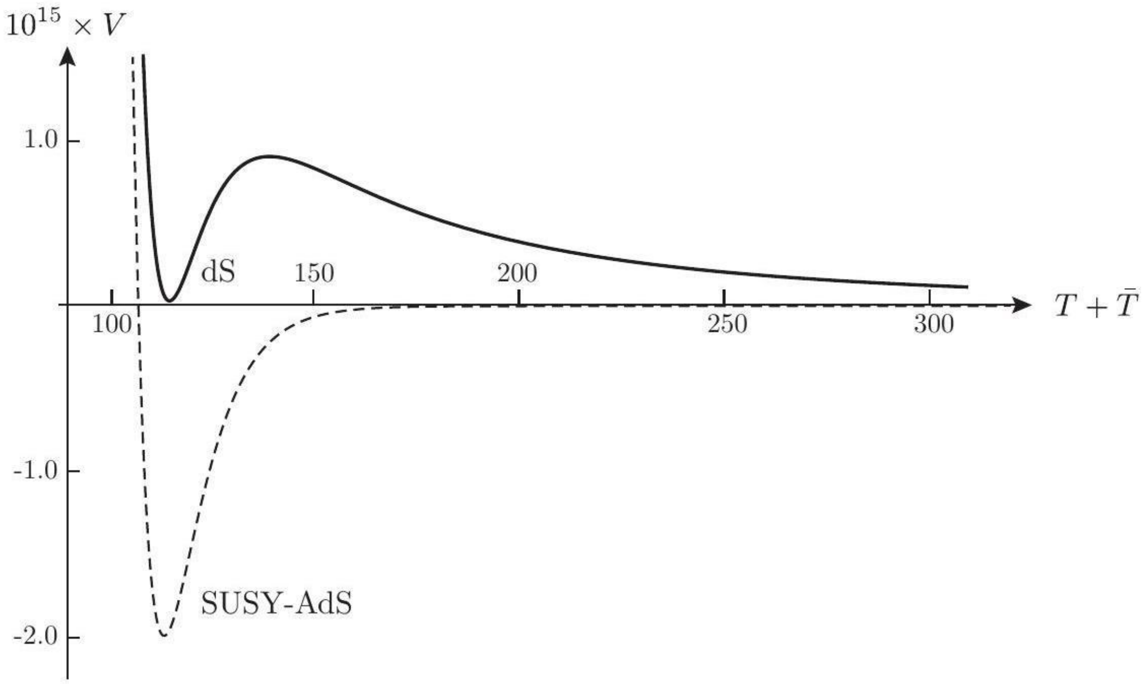
$$W_{\lambda\lambda}=\mathcal{A}e^{-aT}$$

$$W_{\mathrm{ED3}}=\mathcal{A}e^{-aT},$$

$$\chi(\mathcal{O}_D)\equiv\sum_{i=0}^{3}(-1)^i h^{0,i}(D)=1$$

$$h^{0,1}(D)=h^{0,2}(D)=h^{0,3}(D)=0$$





Figuras 39 y 40. Fluctuaciones del centro de masa – energía de la partícula supermasiva.

$$W = W_0 + \sum_{i=1}^{h_+^{1,1}} \mathcal{A}_i e^{-a_i T_i} + \cdots$$

$$V_{(np)} = e^K K^{j\bar{\imath}} \left[a_j \mathcal{A}_j a_{\bar{\imath}} \overline{\mathcal{A}}_{\bar{\imath}} e^{-(a_j T_j + a_i \bar{T}_i)} - \left(a_j \mathcal{A}_j e^{-a_j T_j} \bar{W} \partial_{\bar{\imath}} K + a_{\bar{\imath}} \overline{\mathcal{A}}_{\bar{\imath}} e^{-a_i \bar{T}_i} W \partial_j K \right) \right]$$

$$V_{(np)} = \frac{a\mathcal{A} e^{-a(T+\bar{T})}}{2(T+\bar{T})^2} \left[\left(1 + \frac{T+\bar{T}}{3} \right) a\mathcal{A} e^{-a(T+\bar{T})} + W_0 \right]$$

$$W_0 = -\mathcal{A} e^{-a(T+\bar{T})_\star} \left(1 + \frac{2}{3} a(T+\bar{T})_\star \right)$$

$$W_0 = W_0|_{Z=0} + \ell_A Z^A + m_{AB} Z^A Z^B + \cdots$$

$$\delta V_{(\alpha')} = 3\hat{\xi} e^K \frac{(\hat{\xi}^2 + 7\hat{\xi}\mathcal{V} + \mathcal{V}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} W_0^2 \approx \frac{3}{4} \hat{\xi} W_0^2 \frac{1}{\mathcal{V}^3},$$

$$V_{(np)} + \delta V_{(\alpha')} = e^K \left\{ K^{j\bar{\imath}} \left[a_j \mathcal{A}_j a_{\bar{\imath}} \overline{\mathcal{A}}_{\bar{\imath}} e^{-(a_j T_j + a_i \bar{T}_i)} - \left(a_j \mathcal{A}_j e^{-a_j T_j} \bar{W} \partial_{\bar{\imath}} K + a_{\bar{\imath}} \overline{\mathcal{A}}_{\bar{\imath}} e^{-a_i \bar{T}_i} W \partial_j K \right) \right] + \frac{3}{4} \hat{\xi} W_0^2 \frac{1}{\mathcal{V}} \right\}$$

$$\mathcal{V} \to \infty, \text{ with } a_s \tau_s = \ln \mathcal{V}$$

$$\{T_i\} = \{T_b^\rho\} \cup \{T_s^r\},$$

$$\mathcal{V} \to \infty, \text{ with } a_s^r \tau_s^r = \ln \mathcal{V} \text{ for all } r = 1, \ldots, N_s$$

$$\mathcal{V} = \alpha\tau_b^{3/2} - p^{(3/2)}(\tau_s^r),$$

$$\mathcal{V} = \alpha\left(\tau_b^{3/2} - \sum_{r=1}^{N_s} \lambda_r(\tau_s^r)^{3/2}\right)$$

$$\rho_S = \frac{D}{\mathcal{V}^\alpha}$$

$$\frac{1}{(2\pi)^2\alpha'}\int_{S^3} F_3 \equiv M$$

$$R^4 = 4\pi g_s N(\alpha')^2,$$

$$\nabla^2\Phi = 4\pi e\delta(\boldsymbol{x}),$$

$$\Phi(\boldsymbol{x}') = -\frac{e}{|\boldsymbol{x} - \boldsymbol{x}'|}.$$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\phi}{f}F_{\mu\nu}F_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}$$

$$\boldsymbol{E}\cdot\boldsymbol{B} = -\frac{eB}{r^2}\cos\,\theta$$

$$\nabla^2\phi = \frac{eB}{fr^2}\cos\,\theta$$

$$\mathrm{d}G_- = -\mathrm{d}\left(\frac{\Phi_- G_+}{\Phi_+}\right)$$

$$V = V_{\mathfrak{F}}(\zeta_1, \dots, \zeta_{h^{2,1}})$$

$$\mathrm{d}\mathcal{N}_{\min}(\zeta) \equiv \sum_i \delta(\zeta - \zeta_i)$$

$$\mathrm{d}\mathcal{I}_{\min}(\zeta) \equiv \sum_i \delta(\zeta - \zeta_i)(-1)^{F_i}$$

$$\sum_{L\le L_{\max}} \mathrm{d}\mathcal{I}_{\min} = \frac{(2\pi L_{\max})^{b_3}}{\pi^{b_3/2}b_3!}\det(-\mathcal{R} - \omega)$$

Gravedad cuántica y simetrías de gauge local.

$$\mathcal{N}_{dS} = \mathcal{N}_{\text{c.p.}} \times f_{dS}$$

$$\mathcal{N}_{\min} = \mathcal{N}_{\text{c.p.}} \times f_{\min}$$

$$\mathfrak{C} \equiv \bigcup_{\mathfrak{F}_\star} \left\{p_i^{(\mathfrak{F}_\star)}\right\},$$

$$\mathcal{N}_{\text{c.p.}} \gtrsim \mathcal{N}_{\mathfrak{F}}$$

$$V = e^K (F_a \bar{F}^a - 3|W|^2)$$

$$\mathcal{H} = \begin{pmatrix} \partial^2_{a\bar{b}} V & \partial^2_{ab} V \\ \partial^2_{\bar{a}\bar{b}} V & \partial^2_{\bar{a}b} V \end{pmatrix}$$

$$f_{\min} = P(\lambda_1 > 0)$$

$$F_a \equiv \mathcal{D}_a W, Z_{ab} \equiv \mathcal{D}_a \mathcal{D}_b W, U_{abc} \equiv \mathcal{D}_a \mathcal{D}_b \mathcal{D}_c W$$

$$\mathcal{H} = \begin{pmatrix} Z_a^{\bar{c}} \bar{Z}_{\bar{b}\bar{c}} - F_a \bar{F}_{\bar{b}} - R_{a\bar{b}c\bar{d}} \bar{F}^c F^{\bar{d}} & U_{abc} \bar{F}^c - Z_{ab} \bar{W} \\ \bar{U}_{\bar{a}\bar{b}\bar{c}} F^{\bar{c}} - \bar{Z}_{\bar{a}\bar{b}} W & \bar{Z}_{\bar{a}}^c Z_{bc} - F_b \bar{F}_{\bar{a}} - R_{b\bar{a}c\bar{d}} \bar{F}^c F^{\bar{d}} \end{pmatrix} + \mathbb{1}(F^2 - 2|W|^2)$$

$$F \ll m_{\text{susy}} M_{\text{pl}}$$

$$F \gtrsim m_{\text{susy}} M_{\text{pl}}$$

$$F_{\text{rms}} \sim Z_{\text{rms}} \sim U_{\text{rms}}$$

$$M = A + A^\dagger$$

$$\rho(\lambda) = \frac{1}{2\pi N \sigma^2} \sqrt{4N\sigma^2 - \lambda^2}$$

$$M = AA^\dagger$$

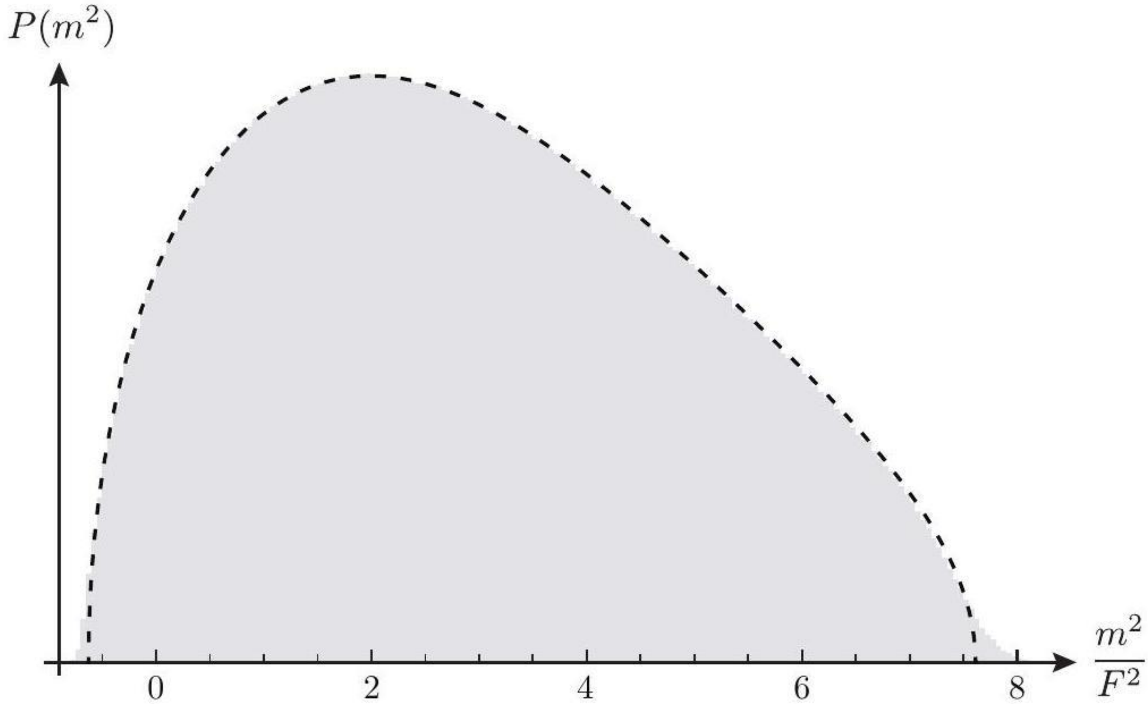
$$\rho(\lambda) = \frac{1}{2\pi N \sigma^2 \lambda} \sqrt{(\eta_+ - \lambda)(\lambda - \eta_-)}$$

$$\mathcal{H}_Z \equiv \begin{pmatrix} Z_a^{\bar{c}} \bar{Z}_{\bar{b}\bar{c}} & 0 \\ 0 & \bar{Z}_{\bar{a}}^c Z_{bc} \end{pmatrix}$$

$$\mathcal{H}_{WWW} \equiv \mathcal{H}_{\text{Wigner}} + \mathcal{H}^{(I)}_{\text{Wishart}} + \mathcal{H}^{(II)}_{\text{Wishart}},$$

$$\rho(\mathcal{H}_{WWW}) = \rho(\mathcal{H}_{\text{Wigner}}) \boxplus \rho\left(\mathcal{H}^{(I)}_{\text{Wishart}}\right) \boxplus \rho\left(\mathcal{H}^{(II)}_{\text{Wishart}}\right).$$

$$f_> \equiv \frac{\int_0^\infty \rho(\lambda)}{\int_{-\infty}^\infty \rho(\lambda)}$$



$$P(\lambda_1 > \zeta) = \exp\left(-N^2\Psi(\zeta)\right)$$

$$f_{\min} \equiv P(\lambda_1 > 0) = \exp\left(-cN^2\right),$$

$$f_{\min} \equiv P(\lambda_1 > 0) = \exp\left(-dN\right)$$

$$\begin{aligned} K &= K_l(\phi, \bar{\phi}) + K_h(\Sigma, \bar{\Sigma}) \\ W &= W_l(\phi) + W_h(\Sigma) \end{aligned}$$

$$f_{\min} \equiv P(\lambda_1 > 0) = \exp\left(-cN_l^2\right),$$

$$S_{10}[\mathcal{C}] \mapsto S_4[\Phi(t)].$$

$$M_{\rm pl} \sim g_s^{-1}(M_{\rm s}/M_{\rm KK})^3 M_{\rm s} \gg M_{\rm s}.$$

$$M_{\rm SUSY} < H < M_{\rm KK} < M_{\rm s} < M_{\rm pl}$$

$$\Delta\mathcal{L} \supset \frac{1}{M_{AB}^{\delta_A+\delta_B-4}} \mathcal{O}_A^{(\delta_A)} \mathcal{O}_B^{(\delta_B)}$$

$$\Delta\mathcal{L} \supset \frac{V_0}{M_{AB}^2}\phi^2$$

$$\frac{M_{AB}}{M_{\rm pl}} \lesssim g_{\rm s}\left(\frac{\ell_{\rm s}}{L}\right)^2$$

$$\frac{M_{AB}}{M_{\rm pl}} \lesssim g_{\rm s}\left(\frac{\ell_{\rm s}}{L}\right)^{\frac{1}{2}p-1}\left(\frac{\ell_{\rm s}}{S}\right)^{\frac{1}{2}(6-p)}$$

$$V(r) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3 g_s^2 \kappa^2}{r^4}\right),$$

$$\eta \approx -\frac{10}{\pi^3}\frac{\mathcal{V}}{r^6},$$

$$\nabla_y^2\left(\delta e^{-4A(y_b;y)}\right) = -\mathcal{C}\left(\frac{\delta(y_b - y)}{\sqrt{g(y)}} - \bar{\rho}(y)\right)$$

$$\delta e^{-4A(y_b;y)} = \mathcal{C}\left(\mathcal{G}(y_b; y) - \int \ \mathrm{d}^6 y' \sqrt{g}\mathcal{G}(y; y')\bar{\rho}(y')\right)$$

$$\nabla_{y'}^2\mathcal{G}(y; y') = \nabla_{y'}^2\mathcal{G}(y; y') = -\frac{\delta(y - y')}{\sqrt{g}} + \frac{1}{\mathcal{V}}.$$

$$\nabla_{y_b}^2\left(\delta e^{-4A(y_b;y)}\right) = -\mathcal{C}\left(\frac{\delta(y_b - y)}{\sqrt{g(y_b)}} - \frac{1}{\mathcal{V}}\right)$$

$$V(y_b) = 2T_3 e^{4A(y_b)} \approx 2T_3\left(1 - \delta e^{-4A(y_b)}\right)$$

$$\mathrm{Tr}(\eta) \approx -\frac{M_{\mathrm{pl}}^2}{T_3}\nabla_{y_b}^2\left(\delta e^{-4A(y_b;y)}\right) = -2$$

$$\mathcal{L} = -K_{\varphi\bar{\varphi}}\partial_\mu\varphi\partial^\mu\bar{\varphi} - e^{K/M_{\mathrm{pl}}^2}\left[K^{\varphi\bar{\varphi}}D_\varphi W\overline{D_\varphi W} - \frac{3}{M_{\mathrm{pl}}^2}|W|^2\right].$$

$$K = K(0) + K_{\varphi\bar{\varphi}}(0)\varphi\bar{\varphi} + \cdots,$$

$$\mathcal{L} \approx -\partial_\mu\phi\partial^\mu\bar{\phi} - V(0)\left(1 + \frac{\phi\bar{\phi}}{M_{\mathrm{pl}}^2} + \cdots\right),$$

$$m_\phi^2 = \frac{V(0)}{M_{\mathrm{pl}}^2} + \cdots = 3H^2 + \cdots \ \Rightarrow \ \eta = 1 + \cdots$$

$$K = -3\ln\left[T + \bar{T} - \gamma k(z_\alpha, \bar{z}_\alpha)\right] \equiv -2\ln\,\mathcal{V},$$

$$K = (\phi - \bar{\phi})^2$$

$$\Delta\phi^2 < \frac{1}{8\pi}\frac{M_s^2}{g_s}\left(\frac{L}{\ell_s}\right)^{p-1}.$$

$$M_{\mathrm{pl}}^2 = \frac{1}{\pi}\frac{M_s^2}{g_s^2}\left(\frac{L}{\ell_s}\right)^6.$$

$$\frac{\Delta\phi^2}{M_{\mathrm{pl}}^2} < \frac{g_s}{8}\left(\frac{\ell_s}{L}\right)^{7-p}$$

$$\frac{\Delta\phi^2}{M_{\rm pl}^2} < \frac{g_{\rm s}}{4\pi}\frac{L}{\ell_{\rm s}}$$

$$\Delta\Phi = \sqrt{N}\Delta\phi$$

$$\delta M_{\rm pl}^2 \sim \frac{N}{16\pi^2}\Lambda_{\rm UV}^2$$

$$V^{1/4} \ll M_{\rm KK} \ll M_{\rm s} \ll M_{\rm pl}$$

$$\mathcal{L}_{\rm eff}[\Phi] = \mathcal{L}_l[\Phi] + \sum_i c_i \frac{\mathcal{O}_i[\Phi]}{\Lambda^{\delta_i - 4}},$$

Campos multiperturbativos y espacios anti – De Sitter.

$$\mathcal{R} = f(\pi_\star, \psi_\star)$$

$$\mu_{p,q} = \frac{1}{2\pi\alpha'}\sqrt{(p - C_0 q)^2 + e^{-2\Phi}q^2}$$

$$\mathrm{d}s^2 = e^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + e^{-2A(r)}\left(\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_{S_5}^2\right)$$

$$e^{-4A(r)} = 1 + \frac{L^4}{r^4}, \text{ with } \frac{L^4}{(\alpha')^2} = 4\pi g_s N$$

$$\alpha(r) \equiv (C_4)_{tx^i} = e^{4A(r)}$$

$$\mathrm{d}s_{AdS_5}^2 = \frac{L^2}{r^2}\,\mathrm{d}r^2 + \frac{r^2}{L^2}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu,$$

$$S_{\rm D3} = -T_3\int \mathrm{d}^4\sigma\sqrt{-\det(G_{ab}^E)} + \mu_3\int C_4$$

$$\mathcal{L} = -T_3 e^{4A(r)}\sqrt{1 + e^{-4A(r)}g^{\mu\nu}\partial_\mu r\partial_\nu r} + T_3\alpha(r)$$

$$\mathcal{L} \approx -\frac{1}{2}(\partial\phi)^2 - T_3\left(e^{4A(\phi)} - \alpha(\phi)\right),$$

$$\sum_{A=1}^{4} z_A^2 = 0$$

$$x\cdot x = \frac{1}{2}\rho^2, y\cdot y = \frac{1}{2}\rho^2, x\cdot y = 0$$

$$T^{1,1} = [SU(2)\times SU(2)]/U(1)$$

$$d\Omega_{T^{1,1}}^2 \equiv \frac{1}{9}\left(\mathrm{d}\psi + \sum_{i=1}^{2}\cos\,\theta_i\,\mathrm{d}\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(\mathrm{d}\theta_i^2 + \sin^2\,\theta_i\,\mathrm{d}\phi_i^2\right)$$

$$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2_{T^{1,1}}$$

$$\mathrm{d}s^2 = k_{\alpha\bar{\beta}}\mathrm{d}z^\alpha \mathrm{d}\overline{z^\beta}$$

$$k(z_\alpha,\bar{z}_\alpha) = \frac{3}{2}\left(\sum_{A=1}^{4} |z^A|^2\right)^{2/3}$$

$$\sum_{A=1}^{4} z_A^2 = \varepsilon^2$$

$$x\cdot x - y\cdot y = \varepsilon^2$$
$$x\cdot x + y\cdot y = \rho^2$$

$$\mathrm{d}s^2 = e^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(r)}\left(\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2_{T^{1,1}}\right),$$

$$e^{-4A(r)} = 1 + \frac{L^4}{r^4} \quad \text{with} \quad L^4 \equiv \frac{27\pi}{4} g_s N(\alpha')^2$$

$$\frac{1}{(2\pi)^2\alpha'}\int_A F_3 = M \quad \text{and} \quad \frac{1}{(2\pi)^2\alpha'}\int_B H_3 = K$$

$$\mathrm{d}s^2 = e^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(r)}\mathrm{d}\tilde{s}^2,$$

$$e^{-4A(r)} = \frac{L^4}{r^4}\left(1 + \frac{3g_sM}{8\pi K} + \frac{3g_sM}{2\pi K}\ln\frac{r}{r_{\mathrm{UV}}}\right)$$

$$L^4 \equiv \frac{27\pi}{4} g_s N(\alpha')^2, N \equiv MK$$

$$e^{A_{\mathrm{IR}}} = \exp\left(-\frac{2\pi K}{3g_sM}\right)$$

$$\mathcal{V}_{\mathcal{T}} \equiv \int \mathrm{d}\Omega^2_{T^{1,1}} \int_{r_{\mathrm{IR}}}^{r_{\mathrm{UV}}} r^5\,\mathrm{d}r e^{-4A(r)} = 2\pi^4 g_s N(\alpha')^2 r_{\mathrm{UV}}^2$$

$$M_{\mathrm{pl}}^2 > \frac{N}{4}\frac{r_{\mathrm{UV}}^2}{(2\pi^3)g_s(\alpha')^2}$$

$$\Delta\phi^2 < T_3 r_{\mathrm{UV}}^2 = \frac{r_{\mathrm{UV}}^2}{(2\pi^3)g_s(\alpha')^2}$$

$$\frac{\Delta\phi}{M_{\mathrm{pl}}} \leq \frac{2}{\sqrt{N}}$$

$$V(\phi) = T_3\left(e^{4A(\phi)} - \alpha(\phi)\right).$$

$$V_{\mathcal{C}}(\phi) = D_0\left(1 - \frac{27}{64\pi^2}\frac{D_0}{\phi^4}\right)$$

$$D_0 \equiv 2T_3 e^{4A(r_{\mathrm{IR}})}$$

$$V_{\mathcal{R}}(\phi) = \frac{1}{12}R\phi^2.$$

$$V(\phi) = V_{\mathcal{C}}(\phi) + V_{\mathcal{R}}(\phi) + \cdots \approx V_0 + H^2\phi^2 + \cdots \;\Rightarrow\; \eta \approx \frac{2}{3} + \cdots$$

$$V_F = e^K\left[K^{I\bar{J}}D_I W\overline{D_J W} - 3|W|^2\right]$$

$$K = -2\ln(\mathcal{V})$$

$$\mathcal{V} = \left(T + \bar{T} - \gamma k(z_\alpha, \bar{z}_\alpha)\right)^{3/2}$$

$$\gamma \equiv \frac{T_3}{6}(T + \bar{T})_{\mathrm{IR}}$$

$$K(Z^I, \bar{Z}^I) = -3\ln\left[T + \bar{T} - \gamma k(z_\alpha, \bar{z}_\alpha)\right] \equiv -3\ln\left[U(Z^I, \bar{Z}^I)\right].$$

$$V_F(T, z_\alpha) = \frac{1}{3U^2}\Big[\left(T + \bar{T} + \gamma\left(k_\gamma k^{\gamma\bar{\delta}}k_{\bar{\delta}} - k\right)\right)\left|W_{,T}\right|^2 - 3\left(\bar{W}W_{,T} + c.c.\right)$$

$$+ \underbrace{\left(k^{\alpha\bar{\delta}}k_{\bar{\delta}}W_{,T}W_{,\alpha} + c.c.\right) + \frac{k^{\alpha\bar{\beta}}}{\gamma}W_{,\alpha}W_{,\beta}\Big]}_{\Delta V_F}$$

$$V_F(r) \approx \frac{V_0}{\left(1 - \frac{1}{6}\phi^2\right)^2} \approx V_0 + \frac{1}{3}\frac{V_0}{M_{\mathrm{pl}}^2}\phi^2,$$

$$|\Delta W| \propto \exp\left(-\frac{2\pi}{N_c}\mathcal{V}_4\right)$$

$$f(z_\alpha) = 0$$

$$W(T, z_\alpha) = W_0 + \mathcal{A}(z_\alpha)e^{-aT}, a \equiv \frac{2\pi}{N_c}$$

$$\mathcal{A}(z_\alpha) = \mathcal{A}_0\left(\frac{f(z_\alpha)}{f(0)}\right)^{1/N_c}$$

$$f(z_1) = \mu - z_1$$

$$V_F(\phi) \approx V_0 + \cdots + \lambda\phi^{3/2} + \frac{V_0}{M_{\mathrm{pl}}^2}\phi^2 + \cdots$$

$$\delta\Phi(r) = \delta\Phi(r_{\mathrm{UV}})\left(\frac{r}{r_{\mathrm{UV}}}\right)^{\Delta}.$$

$$\Phi_- \equiv e^{4A} - \alpha$$

$$\mathrm{d}s^2 = e^{2A(y)} g_{\mu\nu} \mathrm{d}x^{\mu} \mathrm{d}x^{\nu} + e^{-2A(y)} g_{mn}\, \mathrm{d}y^m\, \mathrm{d}y^n,$$

$$\nabla^2 \Phi_- = R_4 + \frac{g_s}{96}|\Lambda|^2 + e^{-4A}|\nabla\Phi_-|^2 + \mathcal{S}_{\mathrm{loc}}$$

$$\Lambda \equiv \Phi_+ G_- + \Phi_- G_+$$

$$G_{\pm} \equiv (\star_6 \pm i) G_3 \ \text{ and } \ \Phi_{\pm} \equiv e^{4A} \pm \alpha$$

$$\mathrm{d}\Lambda + \frac{i}{2}\frac{\mathrm{d}\tau}{\mathrm{Im}\tau} \wedge (\Lambda + \bar{\Lambda}) = 0$$

$$V(x, \Psi) = V_0 + V_{\mathcal{C}}(x) + V_{\mathcal{R}}(x) + V_{\mathcal{B}}(x, \Psi),$$

$$V_{\mathcal{C}}(x) = D_0 \left(1 - \frac{27}{64\pi^2}\frac{D_0}{T_3^2 r_{\mathrm{UV}}^4}\frac{1}{x^4}\right)$$

$$V_{\mathcal{R}}(x) = \frac{1}{3}\mu^4 x^2 + \cdots, \ \text{ where } \ \mu^4 \equiv (V_0 + D_0)\frac{T_3 r_{\mathrm{UV}}^2}{M_{\mathrm{pl}}^2}$$

$$V_{\mathcal{B}}(x, \Psi) = \mu^4 \sum_{LM} c_{LM} x^{\Delta(L)} f_{LM}(\Psi)$$

$$\nabla^2 \Phi_h = 0$$

$$\nabla^2 \Phi_f = \frac{g_s}{96}|\Lambda|^2$$

$$\Delta_h(L) \equiv -2\sqrt{H(j_1, j_2, R) + 4}$$

$$H(j_1, j_2, R) \equiv 6\left[j_1(j_1+1) + j_2(j_2+1) - \frac{1}{8}R^2\right]$$

$$\Delta_h = \frac{3}{2}, 2, 3, \sqrt{28} - 2, \cdots$$

$$\Delta_f(L) = \delta_i(L) + \delta_j(L) - 4$$

$$\begin{gathered}\delta_1(L) \equiv -1 + \sqrt{H(j_1, j_2, R+2) + 4},\\ \delta_2(L) \equiv \sqrt{H(j_1, j_2, R) + 4},\\ \delta_3(L) \equiv 1 + \sqrt{H(j_1, j_2, R-2) + 4}.\end{gathered}$$

$$\Delta_f = 1, 2, \frac{5}{2}, \sqrt{28} - \frac{5}{2}, \cdots$$

$$\Delta = \{\Delta_h, \Delta_f\} = 1, \frac{3}{2}, \sqrt{28} - \frac{5}{2}, 3, \sqrt{28} - 2, \frac{7}{2}, \sqrt{28} - \frac{3}{2}, \cdots$$

$$\mathcal{M} = \begin{pmatrix} A\bar{A} + B\bar{B} & C \\ \bar{C} & \bar{A}A + \bar{B}B \end{pmatrix}$$

$$\tau_{\text{therm}} \ll \tau_{\text{graviton}} \ll \tau_{\text{tunnel}}\,,$$

$$V(\phi) \approx V_0\left[1 + \lambda_0 \frac{\phi}{M_{\text{pl}}} + \frac{1}{3!}\mu_0 \frac{\phi^3}{M_{\text{pl}}^3} + \cdots\right],$$

$$n_s - 1 \approx -\frac{4\pi}{N_{\text{tot}}}\cot\left(\pi \frac{N_\star}{N_{\text{tot}}}\right) \approx -\frac{4}{N_\star}\left(1 + \mathcal{O}\left(\frac{N_\star^2}{N_{\text{tot}}^2}\right)\right),$$

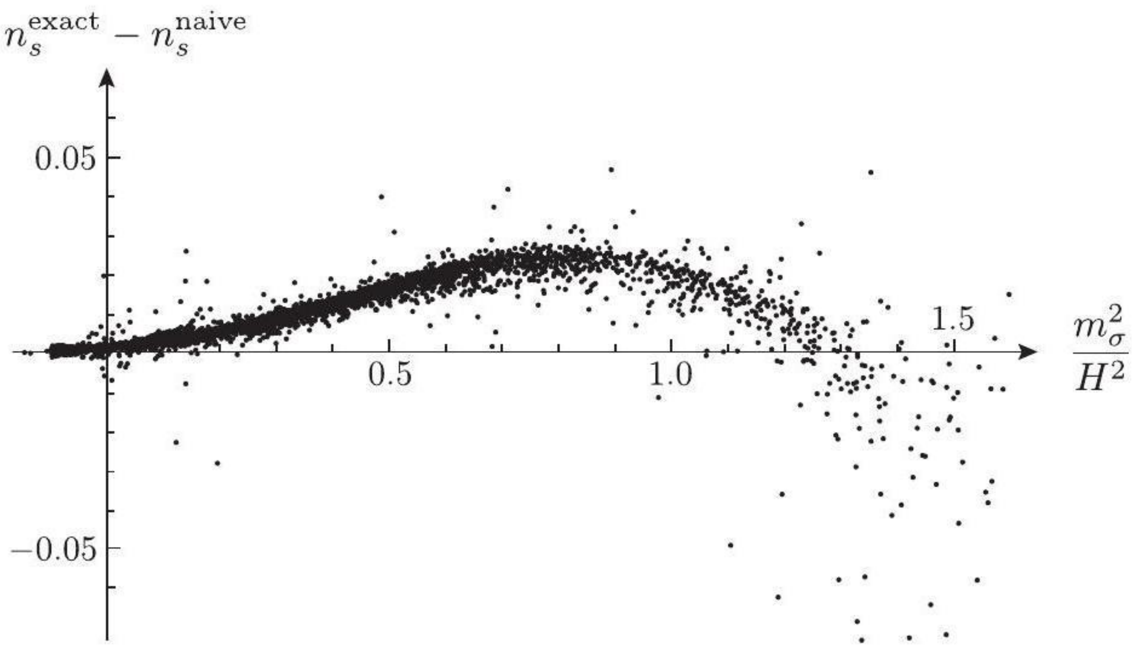
$$N_{\text{tot}} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\epsilon}} \frac{\mathrm{d}\phi}{M_{\text{pl}}} = \pi\sqrt{\frac{2}{\lambda_0\lambda_1}}.$$

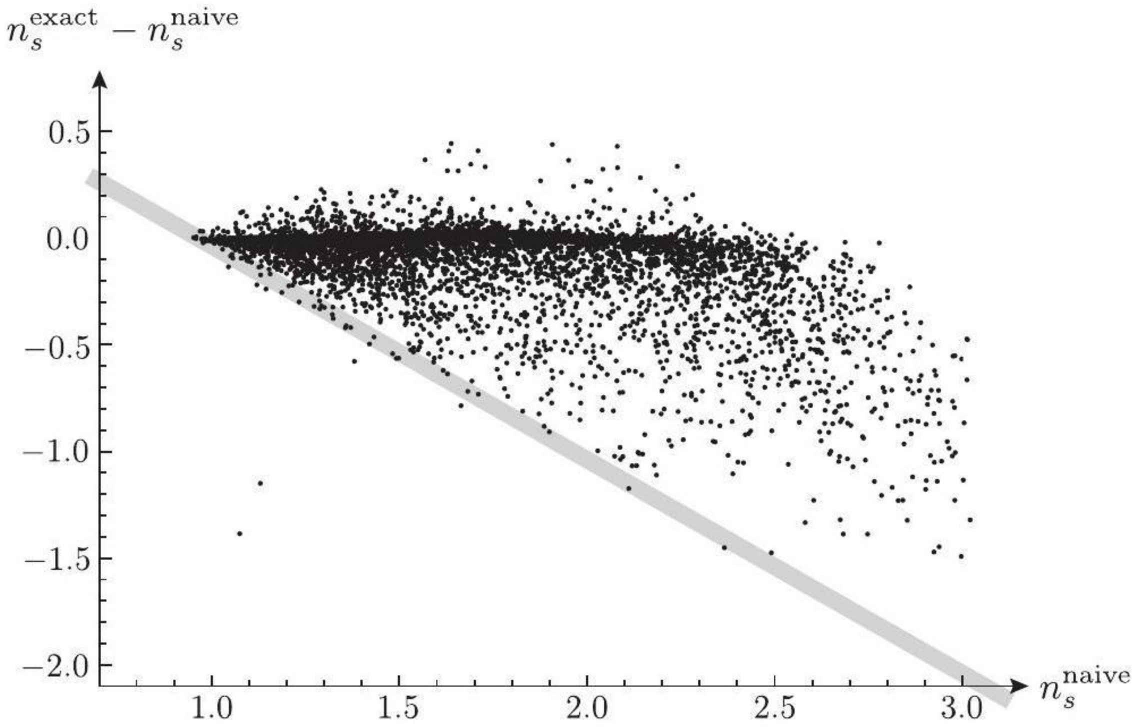
$$N_e(\phi) = \int_{\phi_{\text{end}}}^{\phi} \frac{1}{\sqrt{2\epsilon}} \frac{\mathrm{d}\phi}{M_{\text{pl}}} = \frac{N_{\text{tot}}}{\pi}\arctan\left(\frac{\eta(\phi)N_{\text{tot}}}{2\pi}\right)\Bigg|_{\phi_{\text{end}}}^{\phi}$$

$$\alpha_s = -\frac{4\pi^2}{N_{\text{tot}}^2}\sin^{-2}\left(\pi \frac{N_\star}{N_{\text{tot}}}\right) \approx -\frac{4}{N_\star^2}\left(1 + \mathcal{O}\left(\frac{N_\star^2}{N_{\text{tot}}^2}\right)\right).$$

$$r < \frac{4}{N} \times 0.01 \ll 0.01$$

$$P(N_e) = P(N_\star)\left(\frac{N_\star}{N_e}\right)^3$$





$$T_{(p,q)} \approx \frac{e^{2A_{\mathrm{IR}}}}{2\pi\alpha'}\sqrt{\frac{q^2}{g_s^2} + \left(\frac{bM}{\pi}\right)^2 \sin^2\left(\frac{\pi(p - qC_0)}{M}\right)}$$

$$V_D(\phi) = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{16\pi^2}U(x)\right),$$

$$U(x) \equiv (x^2+1)^2 \ln(x^2+1) + (x^2-1)^2 \ln(x^2-1) - 4x^4 \ln(x) - 4\ln(2),$$

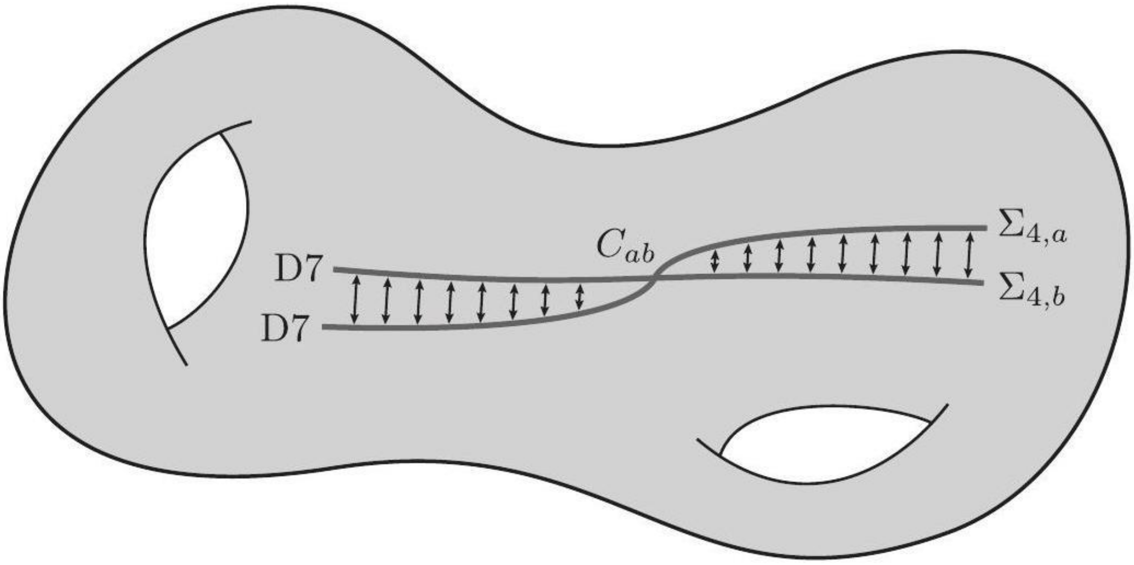
$$K(Z^I, \bar{Z}^I) = -3\ln[T + \bar{T} - \gamma k(z_\alpha, \bar{z}_\alpha)]$$

$$k = \frac{1}{2}(z_1 + \bar{z}_1)^2 = x^2,$$

$$W = W_0 + \left[\vartheta_1\left(\sqrt{2\pi}(z_1+\mu), \zeta\right)\vartheta_1\left(\sqrt{2\pi}(z_1-\mu), \zeta\right)\right]^{-1/N_c} e^{-2\pi T/N_c}$$

$$W = \Lambda^{3-1/N_c} m^{1/N_c},$$

$$V(\phi) = V_D(\phi) - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$



$$V(\phi) = V_D(\phi) + V_F(\phi)$$

$$V_D(\phi) = V_0\big(1 + \alpha \ln{(\phi/\phi_0)}\big)$$

$$K \supset -\ln\left(S + \bar{S} - \frac{(Y + \bar{Y})^2}{T + \bar{T}}\right)$$

$$M_{\mathrm{pl}} \propto \sqrt{\mathrm{Vol}(K3) L_1 L_2},$$

$$\Delta\phi_1 \propto \sqrt{L_1/L_2}$$

$$\mathrm{d}s^2 = \left(\frac{r}{L}\right)^2 \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + \left(\frac{L}{r}\right)^2 \left(\mathrm{d}r^2 + r^2\, \mathrm{d}\Omega_{S^5}^2\right),$$

$$\mathcal{L} = -\frac{\phi^4}{\lambda}\left(\sqrt{1 + \frac{\lambda}{\phi^4}(\partial\phi)^2} - 1\right) - V(\phi),$$

$$\lambda \equiv T_3 L^4 = \frac{1}{2\pi^2} N$$

$$\mathcal{L} = -T(\phi)\left(\sqrt{1 + \frac{(\partial\phi)^2}{T(\phi)}} - 1\right) - V(\phi),$$

$$\mathrm{d}s^2 = e^{2A(r)} \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(r)}\left(\mathrm{d}r^2 + r^2\, \mathrm{d}\Omega_{X_5}^2\right),$$

$$e^{-4A(r)} \approx \frac{L^4}{r^4} \quad \text{with} \quad L^4 \equiv \frac{4\pi^4 g_s}{\mathrm{Vol}(X_5)} N(\alpha')^2$$

$$\frac{\Delta\phi}{M_{\mathrm{pl}}} \leq \frac{2}{\sqrt{N}}$$

$$V(\phi) = V_0 - \frac{1}{2}\beta H^2\phi^2,$$

$$\gamma \equiv \left(1 - \frac{\dot{\phi}^2}{T(\phi)}\right)^{-1/2}$$

$$\dot{\phi}^2 < T(\phi).$$

$$\begin{aligned} \rho &= (\gamma - 1)T + V, \\ P &= (\gamma - 1)\frac{T}{\gamma} - V. \end{aligned}$$

$$3M_{\mathrm{pl}}^2 H^2 = (\gamma - 1)T + V(\phi)$$

$$\dot{\phi} = -\frac{2M_{\mathrm{pl}}^2 H'}{\gamma},$$

$$\gamma = \sqrt{1 + \frac{\left(2M_{\mathrm{pl}}^2 H'\right)^2}{T}}$$

$$\begin{aligned} \varepsilon &= -\frac{\dot{H}}{H^2} = \frac{2M_{\mathrm{pl}}^2}{\gamma}\left(\frac{H'}{H}\right)^2 \\ \tilde{\eta} &= \frac{\dot{\varepsilon}}{H\varepsilon} = \frac{2M_{\mathrm{pl}}^2}{\gamma}\left[2\left(\frac{H'}{H}\right)^2 - 2\frac{H''}{H} + \frac{H'}{H}\frac{\gamma'}{\gamma}\right] \end{aligned}$$

$$\frac{V}{\gamma T} \gg 1$$

$$\gamma^2 = \frac{2}{3}\epsilon\frac{V}{T} \gg 1$$

dilatation :x^μ

$$\phi(x) \mapsto \tilde{x}^\mu \equiv (1 + c)x^\mu$$

SCTs : $\mapsto \phi(\tilde{x}) + c$

$$x^\mu \mapsto \tilde{x}^\mu \equiv x^\mu + (b \cdot x)x^\mu - \frac{1}{2}\left(x^2 + \frac{\lambda}{\phi^2}\right)b^\mu$$

$$\phi(x) \mapsto \phi(\tilde{x})(1 - (b \cdot x))$$

$$S = \int \mathrm{d}^4x\phi^4 f((\partial\phi)^2/\phi^4) + \cdots$$

$$f(z) = \alpha[\sqrt{1 + \lambda z} - \beta]$$

$$V_C(\phi) = D_0\left(1 - \frac{\pi}{4\mathrm{Vol}(X_5)}\frac{D_0}{\phi^4}\right).$$

$$\frac{V(\phi)}{\gamma T(\phi)} = \frac{\lambda}{2\gamma}\frac{m^2}{\phi^2}$$

$$M_{\mathrm{KK}} \sim \frac{1}{L}e^{A_{\mathrm{IR}}} \sim \frac{r_{\mathrm{IR}}}{L^2},$$

$$r_{\mathrm{IR}} \sim mL^2$$

$$\phi^2 \gtrsim \phi_{\mathrm{IR}}^2 \sim T_3(mL^2)^2 = \lambda m^2.$$

$$\frac{V(\phi)}{\gamma T(\phi)} \lesssim \frac{1}{2\gamma} \ll 1$$

$$\frac{V(\phi)}{\gamma T(\phi)} = \frac{1}{\gamma}\frac{e^{4A} - \alpha}{e^{4A}}$$

$$|\alpha| \gg \gamma e^{4A}$$

$$N_e^{\mathrm{EFT}} \approx \frac{N^{1/8}}{\sqrt{\beta}}$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{pl}}^2}{2}R + P(X,\phi)\right]$$

$$P(X,\phi) = -T(\phi)\left(\sqrt{1 - \frac{2X}{T(\phi)}} - 1\right) - V(\phi).$$

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}} = \frac{1}{\gamma^2(\phi)}$$

$$c_s^2 = 1 - \frac{2X}{T(\phi)}.$$

$$r = 16c_s\varepsilon$$

$$\frac{\Delta\phi}{M_{\mathrm{pl}}} = \int_0^{N_\star}\sqrt{\frac{r(N)}{8}\frac{1}{c_sP_{,X}}}\,\mathrm{d}N$$

$$c_sP_{,X} = 1.$$

$$A = c_s^2\left(-1 - \frac{2}{3}\frac{XP_{,XXX}}{P_{,XX}}\right).$$

$$A = -1.$$

$$f_{\mathrm{NL}}^{\mathrm{equil}} = (-0.27 + 0.08A)\frac{1-c_s^2}{c_s^2},$$
$$f_{\mathrm{NL}}^{\mathrm{ortho}} = (+0.02 + 0.02A)\frac{1-c_s^2}{c_s^2}.$$

$$f_{\mathrm{NL}}^{\mathrm{equil}} = -\frac{35}{108}\left(\frac{1}{c_s^2}-1\right) \approx -\frac{35}{108}\gamma^2,$$

$$\gamma \lesssim 24 \ \text{(95\textbackslash\% C.L.)}$$

$$2\left(\frac{M_{\mathrm{pl}}}{\phi}\right)^2 = \varepsilon\gamma = \frac{1}{16}r\gamma^2,$$

$$2\left(\frac{M_{\mathrm{pl}}}{\phi}\right)^2 = \frac{27}{140}r\left|f_{\mathrm{NL}}^{\mathrm{equil}}\right|$$

$$N < \frac{27}{70}r\left|f_{\mathrm{NL}}^{\mathrm{equil}}\right| \lesssim 9 \ \text{(95\textbackslash\% C.L.)}$$

$$\Delta_{\mathcal{R}}^2(k_\star) = \left(\frac{32}{3\pi}\right)^2 \frac{3}{r^4\left(f_{\mathrm{NL}}^{\mathrm{equil}}\right)^2}\frac{\mathrm{Vol}(X_5)}{N} \gtrsim 15\frac{\mathrm{Vol}(X_5)}{N},$$

$$N \gtrsim 7\times 10^9\mathrm{Vol}(X_5)$$

$$f_{\mathrm{NL}}^{\mathrm{equil}} \approx -\frac{35}{108}\gamma^2\cos^2\,\Theta$$

$$\mathcal{L}(\phi) = -\frac{1}{2}(\partial\phi)^2 - \Lambda^4\left[1-\cos\left(\frac{\phi}{f}\right)\right] + \cdots$$

$$\mathcal{L} \supset \sum_{i=1}^{2}\frac{\phi_i}{f_i}\left(\frac{c_{ia}}{32\pi^2}\mathrm{Tr}\left[F^{(a)}\wedge F^{(a)}\right] + \frac{c_{ib}}{32\pi^2}\mathrm{Tr}\left[F^{(b)}\wedge F^{(b)}\right]\right)$$

$$V = \Lambda_a^4\left[1-\cos\left(c_{1a}\frac{\phi_1}{f_1}+c_{2a}\frac{\phi_2}{f_2}\right)\right] + \Lambda_b^4\left[1-\cos\left(c_{1b}\frac{\phi_1}{f_1}+c_{2b}\frac{\phi_2}{f_2}\right)\right].$$

$$\frac{c_{1a}}{c_{2a}} = \frac{c_{1b}}{c_{2b}}$$

$$f_\xi = \frac{(c_{2a}^2 f_1^2 + f_2^2)^{1/2}}{|c_{2b}-c_{2a}|}, \ \text{with} \ \xi = \frac{\phi_2 f_2 - c_{2a}\phi_1 f_1}{c_{2a}^2 f_1^2 + f_2^2}$$

$$\mathcal{L} = \sum_{i=1}^{N}\left[-\frac{1}{2}(\partial\phi_i)^2 - V_i(\phi_i)\right],$$

$$\ddot{\phi}_i + 3H\dot{\phi}_i = -\partial_i V_i, \ \text{where} \ 3M_{\mathrm{pl}}^2 H^2 \approx \sum_{i=1}^{N} V_i.$$

$$V(\Phi) = \frac{1}{2}m^2\Phi^2$$

$$\delta M_{\rm pl}^2 \sim \frac{N}{16\pi^2}\Lambda_{\rm UV}^2$$

$$W = W_0 + \sum_{i=1}^{N} \mathcal{A}_i e^{-a_i T_i}$$

$$K = -2\ln\left[\mathcal{V}(T_i, \bar{T}_i)\right],$$

$$\mathcal{L} = -\frac{1}{2}M_{\rm pl}^2 K_{ij}\partial_\mu\vartheta^i\partial^\mu\vartheta^j - \frac{1}{2}M_{ij}\vartheta^i\vartheta^j + \cdots$$

$$\mathcal{L} = \sum_{i=1}^{N}\left[-\frac{1}{2}(\partial\phi_i)^2 - \frac{1}{2}m_i^2\phi_i^2\right]$$

$$\rho(m^2) = \frac{1}{2\pi N\sigma^2 m^2}\sqrt{(\eta_+ - m^2)(m^2 - \eta_-)}.$$

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}\left(1 + \frac{\zeta(3)\chi(X_6)}{(2\pi)^3}\frac{(\alpha')^3}{\mathcal{V}} + \cdots\right)R_4,$$

$$\chi(X_6) = 2h^{1,1} - 2h^{2,1},$$

$$S_{\rm D5} = \frac{1}{(2\pi)^5 g_s(\alpha')^3}\int_{\mathcal{M}_4\times\Sigma_2} {\rm d}^6\sigma\sqrt{-\det(G_{ab} + B_{ab})}$$

$$V(b) = \frac{\varrho}{(2\pi)^6 g_s(\alpha')^2}\sqrt{(2\pi)^2\ell^4 + b^2}$$

$$V(c) = \frac{\varrho}{(2\pi)^6 g_s^2(\alpha')^2}\sqrt{(2\pi)^2\ell^4 + g_s^2 c^2}$$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \mu^3\phi, \text{ with } \mu^3 \equiv \frac{1}{f}\frac{\varrho}{(2\pi)^6 g_s(\alpha')^2},$$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \mu^{4-p}\phi^p$$

$$S \supset \int {\rm d}^{10}X|C_2 \wedge H_3|^2$$

$$S_{\rm CS} \supset i\mu_5\int_{\mathcal{M}_4\times\Sigma_2} C_4 \wedge \mathcal{F}_2$$

$$S_{\rm CS}^{\rm (D3)} = i\mu_3\int_{\mathcal{M}_4} C_4,$$

$$\int_{\Sigma_2} \mathcal{F}_2 \neq 0$$

$$W = W_0 + \mathcal{A}e^{-2\pi T}$$
$$K = -3\ln\left(T + \bar{T} + \gamma b^2\right)$$

$$S \supset \int_{\Sigma_4} C_2 \wedge \mathcal{F}_2$$

$$W_{\lambda\lambda} = \mathcal{A}e^{-f/N_c},$$

$$f = f_0 + f_1 + f_{np}$$

$$W_{\lambda\lambda} = \mathcal{A}e^{-(f_0+f_1)/N_c}[1 + \mathcal{O}(e^{-S})g(c)],$$

$$V(c) = \mu^3 fc + \hat{V}[T_i(c)]$$

$$\mathrm{Re}(T_i) = \int_{\Sigma_4^i} \mathrm{d}^4 y\sqrt{g}e^{-4A(y;c)}$$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V_0(\phi) - \Lambda^4\cos\left(\frac{\phi}{f}\right)$$

$$b_\star \equiv \frac{\Lambda^4}{V_0'(\phi_\star)f} < 1$$

$$\frac{f^2}{M_{\mathrm{pl}}^2} > \frac{\sqrt{g_s}}{(2\pi)^3\mathcal{V}}$$

$$f \gg (2\Delta_{\mathcal{R}})^{-1/2}\frac{H}{2\pi}$$

$$\frac{f}{M_{\mathrm{pl}}} \gg \sqrt{\frac{r}{16}}\Delta_{\mathcal{R}}^{1/2} \approx 4.5\times 10^{-4}\left(\frac{r}{0.07}\right)^{1/2}$$

$$\Delta_{\mathcal{R}}^2(k) = \Delta_{\mathcal{R}}^2(k_\star)\left(\frac{k}{k_\star}\right)^{n_s-1}\left[1 + \mathcal{A}\cos\left(\frac{\phi_k}{f}\right)\right]$$

$$\mathcal{A} \equiv 3b_\star\left(\frac{2\pi f}{\sqrt{2\epsilon_\star}}\right)^{1/2}$$

$$\mathcal{A}_{p=1} = \frac{6b_\star}{\sqrt{1+(3f\phi_\star)^2}}\sqrt{\frac{\pi}{2}\coth\left(\frac{\pi}{2f\phi_\star}\right)f\phi_\star}$$

$$B_{\mathcal{R}}(k_1,k_2,k_3) = f_{\mathrm{NL}}^{\mathrm{res}} \times \frac{(2\pi\Delta_{\mathcal{R}})^4}{k_1^2k_2^2k_3^2}\left[\sin\left(\frac{\sqrt{2\epsilon_\star}}{f}\ln\frac{K}{k_\star}\right) + \frac{f}{\sqrt{2\epsilon_\star}}\sum_{i\neq j}\cos\left(\frac{\sqrt{2\epsilon_\star}}{f}\ln\frac{K}{k_\star}\right) + \cdots\right]$$

$$f_{\mathrm{NL}}^{\mathrm{res}} \equiv \frac{3\sqrt{2\pi}}{8} b_\star \left(\frac{\sqrt{2\epsilon_\star}}{f}\right)^{3/2}$$

$$f_{\mathrm{NL}}^{\mathrm{res}} \ll \frac{3\sqrt{\pi}}{2}(2\Delta_{\mathcal{R}})^{-3/4} \approx 3\times 10^3$$

$$\mathcal{L} \supset -\frac{\alpha}{4}\frac{\phi}{f}F\tilde{F}$$

$$\left(\frac{\partial^2}{\partial\tau^2} + k^2 \mp 2aHk\xi\right)A_\pm(\tau,k) = 0, \text{ where } \xi \equiv \frac{\alpha\dot{\phi}}{2fH}.$$

$$A_+(\tau,k) \simeq \frac{1}{\sqrt{2k}}\left(\frac{k}{2\xi aH}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}}$$

$$f_{\mathrm{NL}}^{\mathrm{equil}} \simeq \frac{\Delta_{\mathcal{R},0}^6}{\Delta_{\mathcal{R}}^4} f_3(\xi) e^{6\pi\xi}$$

$$f_3(\xi) = 2.8\times 10^{-7}\xi^{-9} \ \text{ for } \ \xi \gg 1$$
$$f_3(\xi) \approx 7.4\times 10^{-8}\xi^{-8.1} \ \text{ for } \ 2 < \xi < 3$$

$$f > \frac{\alpha}{10\pi}\frac{H}{\Delta_{\mathcal{R}}}.$$

$$\frac{f}{M_{\mathrm{pl}}} > \frac{\alpha}{10}\sqrt{\frac{r}{2}} \approx 2\times 10^{-2}\left(\frac{\alpha}{1}\right)\left(\frac{r}{0.07}\right)^{1/2}.$$

$$\Delta_{\mathcal{R}}^2(k) = \Delta_{\mathcal{R},0}^2(k)\left[1 + \Delta_{\mathcal{R},0}^2(k) f_2(\xi) e^{4\pi\xi}\right]$$

$$f_2(\xi) = 7.5\times 10^{-5}\xi^{-6} \ \text{ for } \ \xi \gg 1$$
$$f_2(\xi) \approx 3.0\times 10^{-5}\xi^{-5.4} \ \text{ for } \ 2 < \xi < 3$$

$$W = W_0 + \mathcal{A}e^{-aT} + \mathcal{B}e^{-bT},$$

$$K = -3\ln(T + \bar{T}),$$

$$\delta V = \frac{\varrho}{(T+\bar{T})^2}$$

$$K = K_0 + \delta K_{(\alpha')} = -2\ln(\mathcal{V}) - \frac{\hat{\xi}}{\mathcal{V}}$$

$$K = K_0 + \delta K_{(\alpha')} + \delta K_{(g_s)}$$

$$\delta K_{(g_s)}^{\mathrm{KK}} \sim g_s \sum_{i=1}^{h^{1,1}} \frac{\mathcal{C}_i^{\mathrm{KK}}(\zeta,\bar{\zeta}) M_{\mathrm{KK}}^{-2}}{\mathcal{V}} \sim g_s \sum_{i=1}^{h^{1,1}} \frac{\mathcal{C}_i^{\mathrm{KK}}(\zeta,\bar{\zeta})(a_{ij}t^j)}{\mathcal{V}},$$

$$\delta K_{(g_s)}^{\mathrm{W}} \sim \sum_i \frac{\mathcal{C}_i^{\mathrm{W}}(\zeta,\bar{\zeta}) M_{\mathrm{W}}^{-2}}{\mathcal{V}} \sim \sum_i \frac{\mathcal{C}_i^{\mathrm{W}}(\zeta,\bar{\zeta})}{(b_{ij}t^j)\mathcal{V}},$$

$$\delta K_{(g_s)}^{\mathrm{KK}} \sim \sum_i \frac{t_i}{\mathcal{V}} > \delta K_{(\alpha')} \sim \frac{1}{\mathcal{V}}.$$

$$K = -2\ln\left(\mathcal{V}\right) - \frac{\hat{\xi}}{\mathcal{V}} + \frac{\sqrt{\tau}}{\mathcal{V}}$$

$$V = \frac{W_0^2}{\mathcal{V}^3}\left[0 + \hat{\xi} + 0 \cdot \sqrt{\tau} + \frac{1}{\sqrt{\tau}} + \frac{1}{\tau^{3/2}}\right].$$

$$\delta V_{\mathrm{CW}} \simeq 0 \cdot \Lambda^4 + \Lambda^2\, \mathrm{STr}(M^2) + \mathrm{STr}\left[M^4 \ln\left(\frac{M^2}{\Lambda^2}\right)\right]$$

$$\Lambda = M_{\mathrm{KK}} \simeq \frac{M_{\mathrm{pl}}}{\mathcal{V}^{2/3}}\ ,\ \mathrm{STr}(M^2) \simeq \frac{M_{\mathrm{pl}}^2}{\mathcal{V}^2},$$

$$\delta V_{\mathrm{CW}} \simeq 0 \cdot \frac{1}{\mathcal{V}^{8/3}} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^4}$$

$$\delta V_{\mathrm{CW}} \simeq \frac{1}{\mathcal{V}^3}\left[0 \cdot \sqrt{\tau} + \frac{1}{\sqrt{\tau}} + \frac{1}{\tau^{3/2}}\right]$$

$$\mathcal{V} = \alpha\left(\tau_b^{3/2} - \lambda_\phi \tau_\phi^{3/2} - \lambda_s \tau_s^{3/2}\right)$$

$$W = W_0 + \mathcal{A}_\phi e^{-a_\phi T_\phi} + \mathcal{A}_s e^{-a_s T_s}$$

$$V = W_0^2\left(\mathfrak{a}\frac{\sqrt{\tau_\phi} e^{-2a_\phi \tau_\phi}}{\mathcal{V}} - \mathfrak{b}\frac{\tau_\phi e^{-a_\phi \tau_\phi}}{\mathcal{V}^2} + \mathfrak{c}\frac{\hat{\xi}}{\mathcal{V}^3}\right),$$

$$V(\phi) \simeq V_0\left(1 - c_1 \mathcal{V}^{5/3} \phi^{4/3} \exp\left[-c_2 \mathcal{V}^{2/3} \phi^{4/3}\right]\right),$$

$$\phi \equiv \sqrt{4\lambda_\phi/(3\mathcal{V})}\,\tau_\phi^{3/4},$$

$$a_\phi \langle \tau_\phi \rangle \approx \mathcal{O}(1) \times \ln\left(\mathcal{V}\right)\ \Leftrightarrow\ \langle \phi \rangle \approx \mathcal{O}(1) \times \frac{\ln\left(\mathcal{V}\right)^{3/4}}{\mathcal{V}^{1/2}}$$

$$V(\hat{\phi}) \simeq V_0\left(1 - \kappa_1 e^{-\kappa_2 \hat{\phi}}\right)$$

$$\kappa_1 \equiv c_1 \mathcal{V}^{5/3} \langle \phi \rangle^{4/3} \approx \mathcal{O}(\mathcal{V} \ln\left(\mathcal{V}\right))$$
$$\kappa_2 \equiv \frac{4}{3} c_2 \mathcal{V}^{2/3} \langle \phi \rangle^{1/3} = \mathcal{O}\left(\mathcal{V}^{1/2} \ln\left(\mathcal{V}\right)^{1/4}\right)$$

$$\mathcal{V}^{2/3} \phi^{4/3} \gg 1\ \text{ and }\ \phi \ll 1$$

$$\delta V_{(g_s)} \sim \frac{1}{\sqrt{\tau_\phi} \mathcal{V}^3} \sim \frac{1}{\phi^{2/3} \mathcal{V}^{10/3}}$$

$$\delta \eta \sim \frac{\delta V''_{(g_s)}}{V_0} \sim \frac{1}{\phi^{8/3} \mathcal{V}^{1/3}} \sim \frac{\mathcal{V}}{\tau_\phi^2}$$

$$\delta\eta \approx a_\phi^2 \frac{\mathcal{V}}{\ln(\mathcal{V})^2} \gg 1$$

$$\mathcal{V} = \frac{1}{2}\sqrt{\tau_1}\tau_2,$$

$$\mathcal{V} = \alpha\left(\sqrt{\tau_1}\tau_2 - \lambda_s\tau_s^{3/2}\right)$$

$$W = W_0 + \mathcal{A}_s e^{-a_s T_s}$$

$$V = a_s^2\mathcal{A}_s^2 \frac{\sqrt{\tau_s}}{\mathcal{V}} e^{-2a_s\tau_s} - a_s\mathcal{A}_s W_0 \frac{\tau_s}{\mathcal{V}} e^{-a_s\tau_s} + \hat{\xi} W_0^2 \frac{1}{\mathcal{V}^3}.$$

$$\delta V_{(g_s)} = \frac{W_0^2}{\mathcal{V}^2}\left(\mathfrak{a}\frac{g_s^2}{\tau_1^2} - \mathfrak{b}\frac{1}{\sqrt{\tau_1}\mathcal{V}} + \mathfrak{c}\frac{g_s^2\tau_1}{\mathcal{V}^2}\right)$$

$$V(\phi) = V_0\left(1 - \frac{4}{3}e^{-\phi/\sqrt{3}} + \frac{1}{3}e^{-4\phi/\sqrt{3}} + \frac{\mathfrak{C}}{3}e^{2\phi/\sqrt{3}}\right),$$

$$\phi \equiv \frac{\sqrt{3}}{2}\ln\,\tau_1$$

$$V(\phi) \simeq V_0\left(1 - \frac{4}{3}e^{-\phi/\sqrt{3}}\right)$$

$$W = \mathcal{A}_a \exp\left(-S_a + \mathcal{A}_b e^{-S_b}\right),$$

$$\mathcal{V} = \alpha\left(\sqrt{\tau_1}\tau_2 - \lambda_s\tau_s^{3/2}\right)$$

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2} - (\tau_s + \tau_w)^{3/2}$$

$$W = W_0 + \mathcal{A}\exp[-aT_s] - \mathcal{B}\exp[-bT_s],$$

$$W = W_0 + \mathcal{A}\exp[-a(T_s + c_1e^{-2\pi T_1})] - \mathcal{B}\exp[-b(T_s + c_2e^{-2\pi T_1})],$$

$$V = \frac{F_{\text{poly}}}{\mathcal{V}^{3+p}}\left(1 - (1 + 2\pi\hat{\tau}_1)e^{-2\pi\hat{\tau}_1}\right)$$

$$V(\hat{\phi}) \simeq V_0\left(1 - \kappa_2\hat{\phi}e^{-\kappa_2\hat{\phi}}\right), \hat{\phi} \approx \frac{\sqrt{3}}{2}\frac{\hat{\tau}_1}{\langle\tau_1\rangle},$$

$$V(\phi) \approx V_0\left(1 - \kappa_1 e^{-\kappa_2\phi}\right)$$

$$\eta \simeq -\kappa_1\kappa_2^2 e^{-\kappa_2\phi} \text{ and } \epsilon \simeq \frac{1}{2}\frac{\eta^2}{\kappa_2^2}$$

$$n_s \simeq 1 + 2\eta$$

$$r \simeq \frac{2}{\kappa_2^2}(n_s - 1)^2 \overset{n_s=0.96}{\rightarrow} \frac{3 \times 10^{-3}}{\kappa_2^2}$$

$$r \simeq 6(n_s - 1)^2 \overset{n_s=0.96}{\rightarrow} 0.01.$$

$$r \sim 10^{-5}$$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{O}(\phi, \psi)\right].$$

$$\mathcal{L}_{\rm int} = -\frac{1}{2}g^2(\phi - \phi_0)^2\psi^2$$

$$\phi(t) \approx \phi_0 + \dot{\phi}_0(t - t_0)$$

$$m_\psi^2(t) \equiv g^2(\phi - \phi_0)^2 \approx k_\star^4(t - t_0)^2$$

$$\ddot{\psi}_k + 3H\dot{\psi}_k + \underbrace{\left(\frac{k^2}{a^2} + k_\star^4(t - t_0)^2\right)}_{\equiv\omega_k^2(t)}\psi_k = 0$$

$$|\dot{\omega}_k| > \omega_k^2$$

$$n_k = e^{-\pi k^2/k_\star^2}$$

$$n_\psi(t_0) = \int \frac{d^3k}{(2\pi)^3}n_k \approx \frac{k_\star^3}{(2\pi)^3}$$

$$n_\psi(t) = \frac{k_\star^3}{(2\pi)^3}\frac{a^3(t_0)}{a^3(t)}\Theta(t - t_0)$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = -g^2(\phi - \phi_0)\langle\psi^2\rangle,$$

$$\langle\psi^2\rangle \approx \frac{n_\psi(t)}{g|\phi - \phi_0|}$$

$$\mathcal{L}_{\rm int} = -\frac{1}{2}g^2\sum_{i=1}^{N}(\phi - \phi_i)^2\psi_i^2$$

$$n_{\psi_i}(t_i) \simeq \frac{\left(g\dot{\phi}(t_i)\right)^{3/2}}{(2\pi)^3} \equiv \frac{\dot{\varphi}^{3/2}(t_i)}{(2\pi)^3}$$

$$\ddot{\phi} + 3H\dot{\phi} + V' + \sum_i \frac{g\dot{\varphi}^{3/2}(t_i)}{(2\pi)^3}\frac{a^3(t_i)}{a^3(t)} = 0.$$

$$\sum_i \frac{g\dot{\varphi}^{3/2}(t_i)}{(2\pi)^3}\frac{a^3(t_i)}{a^3(t)} \approx \int^t \frac{\mathrm{d}t'}{\Delta}\frac{\mathrm{d}t'}{\Delta}\frac{\dot{\varphi}^{5/2}(t')}{(2\pi)^3}\frac{a^3(t')}{a^3(t)} \approx \frac{1}{3H\Delta}\frac{\dot{\varphi}^{5/2}(t)}{(2\pi)^3},$$

$$\ddot{\phi} + 3H\dot{\phi} + V' + \frac{1}{24\pi^3}\frac{g^{5/2}}{H\Delta}\dot{\phi}^{5/2} = 0$$

$$\dot{\varphi} = g\dot{\phi} \approx -(24\pi^3 H\Delta V')^{2/5}.$$

$$3M_{\mathrm{pl}}^2 H^2 = \rho_\phi + \rho_\psi \approx V(\phi)$$

$$2M_{\mathrm{pl}}^2\dot{H} \approx -\rho_\psi,$$

$$\varepsilon = -\frac{\dot{H}}{H^2} \approx \frac{3}{2}\frac{\rho_\psi}{V},$$

$$\rho_\psi(t) = \sum_i g|\phi - \phi_i| n_{\psi_i}(t) \approx \int^t \frac{\mathrm{d}t'}{\Delta}\frac{\mathrm{d}t'}{\Delta}|\phi(t) - \phi(t')|\frac{\dot{\varphi}^{5/2}(t')}{(2\pi)^3}\frac{a^3(t')}{a^3(t)}.$$

$$\rho_\psi(t) \approx \frac{1}{(3H)^2}\frac{1}{g\Delta}\frac{\dot{\varphi}^{7/2}(t)}{(2\pi)^3}$$

$$\varepsilon \sim \frac{\epsilon^{7/10}}{g}\left(\frac{H}{M_{\mathrm{pl}}}\frac{\Delta^2}{M_{\mathrm{pl}}^2}\right)^{1/5}$$

$$\begin{aligned} t_x &: (x, u_1, u_2) \mapsto (x+1, u_1, u_2) \\ t_{u_1} &: (x, u_1, u_2) \mapsto (x - Mu_2, u_1 + 1, u_2) \\ t_{u_2} &: (x, u_1, u_2) \mapsto (x, u_1, u_2 + 1), \end{aligned}$$

$$\frac{\mathrm{d}s^2}{\alpha'} = L_{u_1}^2\,\mathrm{d}u_1^2 + \underbrace{L_{u_2}^2\,\mathrm{d}u_2^2 + L_x^2(\,\mathrm{d}x + Mu_1\,\mathrm{d}u_2)^2}_{T^2}$$

$$\frac{\mathrm{d}s_{T^2}^2(u_1)}{\alpha'} = L_{u_2}^2\,\mathrm{d}u_2^2 + L_x^2(\,\mathrm{d}x + Mu_1\,\mathrm{d}u_2)^2$$

$$\frac{\mathrm{d}s_{T^2,\perp}^2}{\alpha'} = L_x^2\,\mathrm{d}y_1^2 + L_{u_2}^2\,\mathrm{d}y_2^2$$

$$S_{\mathrm{D4}} = -\frac{1}{(2\pi)^4 g_s(\alpha')^2}\int \mathrm{d}^4x\sqrt{-g}\sqrt{L_{u_2}^2 + L_x^2 M^2 u_1^2}\left(1 - \frac{1}{2}\alpha' L_{u_1}^2\dot{u}_1^2\right).$$

$$S_{\mathrm{D4}} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}\dot{\phi}^2 - \mu^{10/3}\phi^{2/3}\right)$$

$$\frac{\phi^2}{M_{\mathrm{pl}}^2} = \frac{2}{9}(2\pi)^3 g_s\frac{M}{L^3}\frac{L_{u_1}}{L_{u_2}}u_1^3$$

$$\frac{\mu}{M_{\mathrm{pl}}} = \frac{M_s}{M_{\mathrm{pl}}}\left(\frac{9}{4}\frac{M^2}{(2\pi)^8 g_s^2}\left(\frac{L_x}{L}\right)^3\frac{L_{u_2}}{L_{u_1}}\right)^{1/10}$$

$$\Delta u_1^3 \gg \frac{L^3}{M}$$

$$\int_{\Sigma_2} \mathcal{F}_2 = n \in \mathbb{Z}$$

$$T_{\mathrm{D3}/\Sigma_2} = T_3\sqrt{\ell^2 + (cg_s + n)^2},$$

$$\mathcal{M}_{10} = dS_4 \times X_6,$$

$$\int_{dS_4 \times \Sigma_{p-2}} F_{p+2} = Q_0 \gg 1.$$

$$\mathrm{d}s_{\mathrm{E}}^2 = H^{-2}(\,\mathrm{d}\xi^2 + \sin^2\,\xi\,\mathrm{d}\Omega_3^2) + \mathrm{d}z^2,$$

$$S = \int\;\mathrm{d}z \int\;\;\mathrm{d}\mathcal{H}_3\,\mathrm{d}t \frac{\sinh^3\,(Ht)}{H^3}\left(-2\sigma\delta(z - z_b)\sqrt{1 - (\partial z_b)^2} - \frac{F_5^2}{2 \cdot 5!}\right)$$

$$\frac{F_5^2}{5!} = \mu^5 Q^2 = \mu^5\left(Q_0 + \sum_{j=-\infty}^{\infty}\;[\Theta(z - z_b + j\ell) - \Theta(z + z_b + j\ell)]\right)^2$$

$$S = \int\;\mathrm{d}t\;\mathrm{d}^3 x a^3(t)\left(-2\sigma\sqrt{1 - (\partial z_b)^2} - V(z_b)\right)$$

$$V(z_b) \sim \mu^5\left(Q_0 - \frac{z_b}{\ell}\right)^2.$$

$$\mathcal{L} \supset -\sum_{i=1}^{N} \alpha_i \frac{\phi}{f} F_i \tilde{F}_i$$

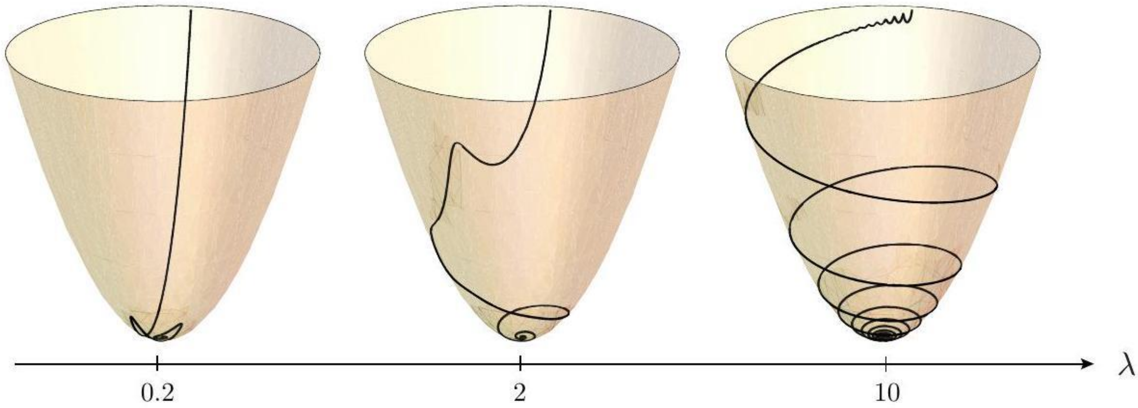
$$S_{\mathrm{CS}} = \frac{i}{2\pi}\int_{\mathcal{M}_4} C_0 \mathrm{Tr}[\mathcal{F}_2 \wedge \mathcal{F}_2]$$

$$S_{\mathrm{eff}} = \int\;\mathrm{d}^4 x \left\{a^3\left[\gamma_C \dot{C}_0^2 - V(C_0) + \gamma_A \frac{\mathrm{Tr}\left(\dot{A}^2\right)}{a^2} + \gamma_A \frac{\mathrm{Tr}([A,A]^2)}{a^4}\right] + \kappa C_0 \mathrm{Tr}(\dot{A}[A,A])\right\}$$

$$C_0(t) \equiv \frac{\phi(t)}{\sqrt{\gamma_C}}$$

$$A_0 \equiv 0, A_i(t) \equiv \frac{a(t)\psi(t)}{\sqrt{\gamma_A}\nu} J_i$$

$$S_{\mathrm{eff}} = \int\;\mathrm{d}^4 x a^3\left[\frac{1}{2}\dot{\phi}^2 - V(\phi) + \frac{3}{2}(\dot{\psi} + H\psi)^2 - \frac{3}{2}g^2\psi^4 - \frac{3g\lambda}{f}\phi\psi^2(\dot{\psi} + H\psi)\right]$$



$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -3\frac{g\lambda}{f}\psi^2(\dot{\psi} + H\psi)$$

$$(N_e)_{\max} \approx \frac{3}{5}\lambda$$

$$S_{\mathrm{CS}} = \frac{i}{(2\pi)^3}\int_{\mathcal{M}_4\times\Sigma_4}\frac{C_0}{24}\left(\mathrm{Tr}[\mathcal{F}_2\wedge\mathcal{F}_2\wedge\mathcal{F}_2\wedge\mathcal{F}_2] + \frac{1}{2}\mathrm{Tr}[\mathcal{F}_2\wedge\mathcal{F}_2]\hat{\chi}(R)\right)$$

$$\lambda = \left[K + \frac{\chi(\Sigma_4)}{24}\right]\times g_s \times \frac{\ell_s^4}{\mathcal{V}_4}$$

$$\delta\ddot{\phi} + \left(M^2 + \frac{k^2}{a^2}\right)\delta\phi + \int^t \mathrm{d}\ \mathrm{d}t' M^2\left(\frac{5}{2}\delta\dot{\phi}(t') - 3H\delta\phi(t')\right)\frac{a^3(t')}{a^3(t)} = -g\Delta n_\psi(k,t)$$

$$M^2 \equiv \frac{g^{5/2}}{(2\pi)^3}\frac{\dot{\phi}^{3/2}}{\Delta}$$

$$\Delta_{\mathcal{R}}^2 \approx g^{8/3}\frac{H}{M}\left(\frac{M}{\Delta}\right)^{2/3}$$

$$\Delta_{\mathcal{R}}^2 \approx g^{9/4}\left(\frac{H}{\Delta}\right)^{1/2}\left(\frac{H^2}{\dot{\phi}}\right)^{1/4},$$

$$n_s - 1 = \frac{\dot{H}}{H^2} - \frac{1}{4}\frac{\ddot{\phi}}{H\dot{\phi}}.$$

$$r = g^{-8/2}\frac{HM}{M_{\mathrm{pl}}^2}\left(\frac{\Delta}{M}\right)^{2/3}.$$

$$f_{\mathrm{NL}}^{\mathrm{equil}} \simeq \frac{M^2}{H^2}$$

Interacciones gravitónicas y transferencia de gravedad al centro de masa de las partículas portadoras, volviéndolas supermasivas. Formalismo de Batalin-Vilkovisky por permeabilidad de un campo gravitónico para inmersión gravitacional de un espacio – tiempo cuántico específico. Propagadores y campos fantasma.

$$Q = \oint_0 j_B(z)dz + \oint_0 \bar{j}_B(\bar{z})d\bar{z}$$

$$j_B = cT_m + bc\partial c, \bar{j}_B = \bar{c}\bar{T}_m + \bar{b}\bar{c}\bar{\partial}\bar{c}$$

$$\oint_0 \frac{dz}{z} = 1, \oint_0 \frac{d\bar{z}}{\bar{z}} = 1$$

$$b(z)c(w) \simeq \frac{1}{z-w}, \bar{b}(\bar{z})\bar{c}(\bar{w}) \simeq \frac{1}{\bar{z}-\bar{w}}$$

$$T_m(z)T_m(w) \simeq \frac{26}{2}\frac{1}{(z-w)^4} + \frac{2}{(z-w)^2}T_m(w) + \frac{1}{z-w}\partial_w T_m(w)$$

$$\bar{T}_m(\bar{z})\bar{T}_m(\bar{w}) \simeq \frac{26}{2}\frac{1}{(\bar{z}-\bar{w})^4} + \frac{2}{(\bar{z}-\bar{w})^2}\bar{T}_m(\bar{w}) + \frac{1}{\bar{z}-\bar{w}}\bar{\partial}_w \bar{T}_m(\bar{w})$$

$$T_{bc}(z) = -\partial bc - 2b\partial c, \bar{T}_{bc}(\bar{z}) = -\bar{\partial}\bar{b}\bar{c} - 2\bar{b}\bar{\partial}\bar{c}$$

$$\partial X^\mu(z)\partial X^\nu(w) = -\frac{\eta^{\mu\nu}}{2(z-w)^2}, \bar{\partial}X^\mu(\bar{z})\bar{\partial}X^\nu(\bar{w}) = -\frac{\eta^{\mu\nu}}{2(\bar{z}-\bar{w})^2}$$

$$T(z) = -\eta_{\mu\nu}\partial X^\mu \partial X^\nu, \bar{T}(\bar{z}) = -\eta_{\mu\nu}\bar{\partial}X^\mu\bar{\partial}X^\nu$$

$$|k\rangle \equiv e^{ik\cdot X}(0)|0\rangle,$$

$$\langle k|c_{-1}\bar{c}_{-1}c_0\bar{c}_0c_1\bar{c}_1|k'\rangle = -(2\pi)^D\delta^{(D)}(k+k')$$

$$\langle k|c_{-1}c_0c_1|k'\rangle' = -(2\pi)^{p+1}\delta^{(p+1)}(k+k')$$

$$\langle k|c_{-1}c_0c_1|k'\rangle = -(2\pi)^{p+1}K\delta^{(p+1)}(k+k').$$

$$|s\rangle \in \mathcal{H}_c \text{ iff } b_0^-|s\rangle = 0, L_0^-|s\rangle = 0$$

$$b_0^\pm = b_0 \pm \bar{b}_0, L_0^\pm = L_0 \pm \bar{L}_0, c_0^\pm = (c_0 \pm \bar{c}_0)/2$$

$$\langle A, B\rangle \equiv \langle A|c_0^-|B\rangle\ A, B \in \mathcal{H}_c$$

$$\langle A, B\rangle' \equiv \langle A \mid B\rangle', \langle A, B\rangle \equiv \langle A \mid B\rangle.$$

$$\gamma = \eta e^\phi, \beta = \partial\xi e^{-\phi}, \delta(\gamma) = e^{-\phi}, \delta(\beta) = e^\phi$$

$$[\gamma] = -\frac{1}{2}, [\beta] = \frac{3}{2}, [\eta] = 1, [\xi] = 0, [\phi] = 0, [e^{q\phi}] = -\frac{1}{2}q(q+2)$$

$$\xi(z)\eta(w) \simeq \frac{1}{z-w}, \partial\phi(z)\partial\phi(w) \simeq -\frac{1}{(z-w)^2}, e^{q_1\phi}(z)e^{q_2\phi}(w) \simeq (z-w)^{-q_1 q_2} e^{(q_1+q_2)\phi}(w)$$

$$T_{\beta\gamma} = \frac{3}{2}\beta\partial\gamma + \frac{1}{2}\gamma\partial\beta = T_{\xi\eta} + T_\phi, T_{\xi\eta} = -\eta\partial\xi, T_\phi = -\frac{1}{2}\partial\phi\partial\phi - \partial^2\phi.$$

$$T_m(z)T_m(w) \simeq \frac{15}{2}\frac{1}{(z-w)^4} + \frac{2}{(z-w)^2}T_m(w) + \frac{1}{z-w}\partial_w T_m(w)$$

$$T_F(z)T_F(w) \simeq \frac{5}{2}\frac{1}{(z-w)^3} + \frac{1}{2}\frac{1}{z-w}T_m(w)$$

$$T_m(z)T_F(w) \simeq \frac{3}{2}\frac{1}{(z-w)^2}T_F(w) + \frac{1}{z-w}T_F(w)$$

$$\psi^\mu(z)\psi^\nu(w) \simeq -\frac{\eta^{\mu\nu}}{2(z-w)}$$

$$j_B = c\left(T_m + T_{\beta\gamma}\right) + bc\partial c + \gamma T_F - \frac{1}{4}\gamma^2 b$$

$$\mathcal{X}(z) = \{Q, \xi(z)\} = c\partial\xi + e^\phi T_F - \frac{1}{4}\partial\eta e^{2\phi} b - \frac{1}{4}\partial\left(\eta e^{2\phi} b\right).$$

$$\begin{gathered}\mathrm{gh}(c) = \mathrm{gh}(\eta) = \mathrm{gh}(\gamma) = 1 \\ \mathrm{gh}(b) = \mathrm{gh}(\xi) = \mathrm{gh}(\beta) = -1\end{gathered}$$

$$\mathrm{pic}(\eta) = -1, \mathrm{pic}(\xi) = 1, \mathrm{pic}\left(e^{q\phi}\right) = q, \mathrm{pic}(\mathcal{X}) = 1$$

GSO odd fields: $\beta, \gamma, \psi^\mu, T_F$

$$|k; m, n\rangle \equiv e^{ik\cdot X}(0)e^{m\phi}(0)e^{n\bar{\phi}}(0)|0\rangle, |k; m\rangle \equiv e^{ik\cdot X}(0)e^{m\phi}(0)|0\rangle$$

$$\langle k; 0,0|c_{-1}\bar{c}_{-1}c_0\bar{c}_0c_1\bar{c}_1 e^{-2\phi}e^{-2\bar{\phi}}|k'; 0,0\rangle = -(2\pi)^D\delta^{(D)}(k+k'),$$

$$\langle k; 0|c_{-1}c_0c_1 e^{-2\phi}|k'; 0\rangle' = (2\pi)^{p+1}\delta^{(p+1)}(k+k'),$$

$$\langle k; 0|c_{-1}c_0c_1 e^{-2\phi}|k'; 0\rangle = (2\pi)^D\delta^{(D)}(k+k')$$

$$\mathcal{C} = \left\langle \prod_i f_i \circ V_i(0) \right\rangle_\Sigma,$$

$$f \circ V(w) = (f'(w))^h \left(\bar{f}'(\bar{w})\right)^{\bar{h}} V(f(w))$$

$$V(w)(dw)^h(d\bar{w})^{\bar{h}} = V(z)(dz)^h(d\bar{z})^{\bar{h}} \rightarrow V(w) = V(z(w))\left(\frac{dz}{dw}\right)^h\left(\frac{d\bar{z}}{d\bar{w}}\right)^{\bar{h}}$$

$$\mathcal{C} = \left\langle \prod_i V_i(w_i = 0) \right\rangle_\Sigma$$

$$\langle \chi_s^c \mid \chi_r \rangle = \delta_{rs} \;\Leftrightarrow\; \sum_r |\chi_r\rangle\langle \chi_r^c| = I$$

$$w_1 w_2 = 1$$

$$w_1 w_2 = -1$$

$$d_{g,n} \equiv \dim_{\mathbb{R}}(\mathcal{M}_{g,n}) = 6g - 6 + 2n$$

$$\chi_{g,n} = 2 - 2g - n$$

$$\mathcal{A}_g(V_1, \cdots, V_n) = (g_s)^{-\chi_{g,n}} \int_{\mathcal{F}_{g,n}} \Omega^{(g,n)}_{6g-6+2n}(V_1, \cdots, V_n)$$

$$\sigma_s = F_s(\tau_s, u)$$

$$\mathcal{B}\left[\frac{\partial}{\partial u^i}\right] \equiv \sum_s \left[\oint_{C_s} \frac{\partial F_s}{\partial u^i} d\sigma_s b(\sigma_s) + \oint_{C_s} \frac{\partial \bar{F}_s}{\partial u^i} d\bar{\sigma}_s \bar{b}(\bar{\sigma}_s)\right]$$

$$\Omega_p^{(g,n)}(A_1, \cdots, A_n)\left[\frac{\partial}{\partial u^{j_1}}, \cdots, \frac{\partial}{\partial u^{j_p}}\right] \equiv \left(-\frac{1}{2\pi i}\right)^{3g-3+n} \left\langle \mathcal{B}\left[\frac{\partial}{\partial u^{j_1}}\right] \cdots \mathcal{B}\left[\frac{\partial}{\partial u^{j_p}}\right] A_1 \cdots A_n \right\rangle_{\Sigma_{g,n}}$$

$$z = F(w, u)$$

$$y(u) = F(0, u)$$

$$\Omega_2(c\bar{c}V) = \left(-\frac{1}{2\pi i}\right) du^1 \wedge du^2 \left\langle \cdots \mathcal{B}\left[\frac{\partial}{\partial u^1}\right] \mathcal{B}\left[\frac{\partial}{\partial u^2}\right] c\bar{c}V(w = 0) \right\rangle$$

$$\mathcal{B}\left[\frac{\partial}{\partial u^1}\right] \mathcal{B}\left[\frac{\partial}{\partial u^2}\right] = \left[\oint b(z) dz \frac{\partial F(w,u)}{\partial u^1} + \oint \bar{b}(\bar{z}) d\bar{z} \frac{\partial \overline{F(w,u)}}{\partial u^1}\right]$$
$$\left[\oint b(z) dz \frac{\partial F(w,u)}{\partial u^2} + \oint \bar{b}(\bar{z}) d\bar{z} \frac{\partial \overline{F(w,u)}}{\partial u^2}\right]$$

$$\Omega_2(c\bar{c}V) = -\left(-\frac{1}{2\pi i}\right) du^1 \wedge du^2 \left(\frac{\partial y}{\partial u^1}\frac{\partial \bar{y}}{\partial u^2} - \frac{\partial y}{\partial u^2}\frac{\partial \bar{y}}{\partial u^1}\right) \langle \cdots V(z = y) \rangle$$

$$du^1 \wedge du^2 \left(\frac{\partial y}{\partial u^1}\frac{\partial \bar{y}}{\partial u^2} - \frac{\partial y}{\partial u^2}\frac{\partial \bar{y}}{\partial u^1}\right) = dy \wedge d\bar{y}$$

$$\Omega_2(c\bar{c}V) = \frac{1}{2\pi i} \langle \cdots (dy \wedge d\bar{y} V(z = y)) \rangle$$

$$\Omega_p^{(g,n)}(QA_1, A_2, \cdots, A_n) + \cdots + (-1)^{A_1 + \cdots + A_{n-1}} \Omega_p^{(g,n)}(A_1, A_2, \cdots, QA_n) = (-1)^p \, \mathrm{d}\Omega_{p-1}^{(g,n)}(A_1, \cdots, A_n)$$

$$\int_{\mathcal{F}'_{g,n}} \Omega^{(g,n)}_{6g-6+2n}(V_1,\cdots,V_n) - \int_{\mathcal{F}_{g,n}} \Omega^{(g,n)}_{6g-6+2n}(V_1,\cdots,V_n) = \int_{\mathcal{R}_{g,n}} \mathrm{d}\Omega^{(g,n)}_{6g-5+2n}(V_1,\cdots,V_n)$$

$$\Omega^{(g,n)}_{6g-6+2n}(QW_1,V_2,\cdots,V_n) = \mathrm{d}\Omega^{(g,n)}_{6g-7+2n}(W_1,V_2,\cdots,V_n)$$

$$n_{-1,-1}, n_{-1,-1/2}, n_{-1/2,-1}, n_{-1/2,-1/2}$$

$$2g-2 = N_L - n_{-1,-1} - n_{-1,-1/2} - \frac{1}{2}n_{-1/2,-1} - \frac{1}{2}n_{-1/2,-1/2}$$

$$2g-2 = N_R - n_{-1,-1} - n_{-1/2,-1} - \frac{1}{2}n_{-1,-1/2} - \frac{1}{2}n_{-1/2,-1/2}$$

$$p = k + \ell + \bar{\ell}$$

$$\Omega^{(g,n)}_p(A_1,\cdots,A_n)\left[\frac{\partial}{\partial u^{j_1}},\cdots,\frac{\partial}{\partial u^{j_k}},\frac{\partial}{\partial y_{\alpha_1}},\cdots,\frac{\partial}{\partial y_{\alpha_\ell}},\frac{\partial}{\partial \bar{y}_{\beta_1}},\cdots,\frac{\partial}{\partial \bar{y}_{\beta_{\bar{\ell}}}}\right]$$

$$\equiv \left(-\frac{1}{2\pi i}\right)^{3g-3+n} \left\langle \mathcal{B}\left[\frac{\partial}{\partial u^{j_1}}\right]\cdots\mathcal{B}\left[\frac{\partial}{\partial u^{j_k}}\right]\left(-\partial\xi(y_{\alpha_1})\right)\cdots\left(-\partial\xi(y_{\alpha_\ell})\right)\left(-\bar{\partial}\bar{\xi}(\bar{y}_{\beta_1})\right)\cdots\left(-\bar{\partial}\bar{\xi}(\bar{y}_{\beta_{\bar{\ell}}})\right)\prod_{\alpha=l+1}^{N_R}\mathcal{X}(y_\alpha)\prod_{\beta=\bar{l}+1}^{N_L}\overline{\mathcal{X}}(\bar{y}_\beta)A_1\cdots A_n\right\rangle_{\Sigma_{g,n}}$$

$$-\int_{y_\alpha}^{y'_\alpha} \partial\xi(y)dy = \xi(y_\alpha) - \xi(y'_\alpha)$$

$$\Omega^{(g,n)}_p(A_1,\cdots,A_n)\left[\frac{\partial}{\partial u^{j_1}},\cdots,\frac{\partial}{\partial u^{j_k}},\frac{\partial}{\partial y_{\alpha_1}},\cdots,\frac{\partial}{\partial y_{\alpha_\ell}}\right]$$

$$= \left(-\frac{1}{2\pi i}\right)^{3g-3+n} \left\langle \mathcal{B}\left[\frac{\partial}{\partial u^{j_1}}\right]\cdots\mathcal{B}\left[\frac{\partial}{\partial u^{j_k}}\right]\left(-\partial\xi(y_{\alpha_1})\right)\cdots\left(-\partial\xi(y_{\alpha_\ell})\right)\prod_{\alpha=l+1}^{N_R}\mathcal{X}(y_\alpha)A_1\cdots A_n\right\rangle_{\Sigma_{g,n}}$$

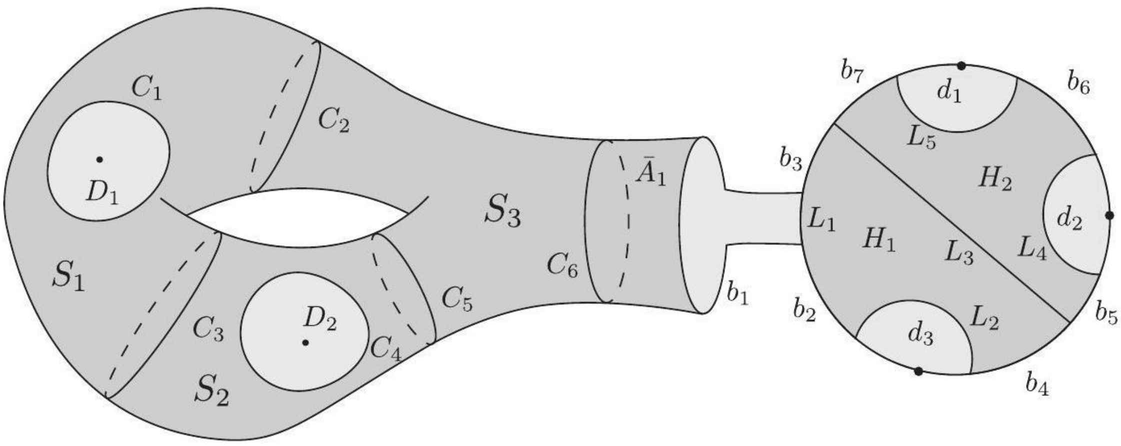


Figura 41. Puente Einstein – Rosen en dimensión $\mathbb{R}^4$ para entrelazamiento cuántico.

$$d_{g,b,n_c,n_o} \equiv \dim_{\mathbb{R}}\left(\mathcal{M}_{g,b,n_c,n_o}\right) = 6g-6+3b+2n_c+n_o$$

$$\chi_{g,b,n_c,n_o} = 2 - 2g - n_c - b - \frac{1}{2}n_o$$

$$S + \frac{1}{2}\#H + \frac{1}{2}\#\bar{A} = -\chi_{g,b,n_c,n_o}$$

$$2\#C = 3S + n_c + A + \bar{A}$$

$$2\#L = 3\#H + n_o + \#\bar{A}$$

$$\mathcal{B}\left[\frac{\partial}{\partial u^i}\right] \equiv \sum_s \left[\oint_{C_s} \frac{\partial F_s}{\partial u^i} d\sigma_s b(\sigma_s) + \oint_{C_s} \frac{\partial \bar{F}_s}{\partial u^i} d\bar{\sigma}_s \bar{b}(\bar{\sigma}_s)\right] + \sum_m \left[\int_{L_m} \frac{\partial G_m}{\partial u^i} d\sigma_m b(\sigma_m) + \int_{L_m} \frac{\partial \bar{G}_m}{\partial u^i} d\bar{\sigma}_m \bar{b}(\bar{\sigma}_m)\right]$$

$$\sigma_m = G_m(\tau_m, u^i)$$

$$\widehat{\Omega}_p^{(g,b,n_c,n_o)}(A_1^c, \cdots, A_{n_c}^c; A_1^o, \cdots, A_{n_o}^o)\left[\frac{\partial}{\partial u^{j_1}}, \cdots, \frac{\partial}{\partial u^{j_p}}\right]$$

$$\sim \left\langle \mathcal{B}\left[\frac{\partial}{\partial u^{j_1}}\right] \cdots \mathcal{B}\left[\frac{\partial}{\partial u^{j_p}}\right] A_1^c \cdots A_{n_c}^c; A_1^o \cdots A_{n_o}^o \right\rangle_{\Sigma_{g,b,n_c,n_o}}$$

$$\Omega_p^{(g,b,n_c,n_o)}(A_1^c, \cdots, A_{n_c}^c; A_1^o, \cdots, A_{n_o}^o) \equiv N_{g,b,n_c,n_o} \hat{\Omega}_p^{(g,b,n_c,n_o)}(A_1^c, \cdots, A_{n_c}^c; A_1^o, \cdots, A_{n_o}^o)$$

$$N_{g,b,n_c,n_o} = \eta_c^{3g-3+n_c+\frac{3}{2}b+\frac{3}{4}n_o}, \eta_c \equiv -\frac{1}{2\pi i} = \frac{i}{2\pi}$$

$$\mathcal{A}_{g,b}(A_1^c, \cdots, A_{n_c}^c; A_1^o, \cdots, A_{n_o}^o) = (g_s)^{-\chi_{g,b,n_c,n_o}} \int_{\mathcal{F}_{g,b,n_c,n_o}} \Omega_{d_{g,b,n_c,n_o}}^{(g,b,n_c,n_o)}(A_1^c, \cdots, A_{n_c}^c; A_1^o, \cdots, A_{n_o}^o)$$

$$\widehat{\Omega}_0^{(0,1,1,0)}(A^c) = \eta_c \langle c_0^- A^c \rangle,$$

$$\Omega_0^{(0,1,1,0)}(A^c) = \eta_c N_{0,1,1,0} \langle c_0^- A^c \rangle.$$

$$c_o \equiv N_{g,b,n_c,n_o+1}/N_{g,b,n_c,n_o}$$

$$z = G(w,u)$$

$$\Omega_1(cV_o) = c_o du \left\langle \cdots \mathcal{B}\left[\frac{\partial}{\partial u}\right] cV_o(w=0) \right\rangle$$

$$\Omega_1(cV_o) = c_o du \left\langle \cdots \oint b(z) dz \frac{\partial G(w,u)}{\partial u} cV_o(z=y) \right\rangle$$

$$\Omega_1(cV_o) = -c_o du \frac{\partial y}{\partial u} \langle \cdots V_o(z=y) \rangle = -c_o dy \langle \cdots V_o(y) \rangle$$

$$\Omega_p^{(g,b,n_c,n_o)}\left(A_1^c,\cdots,A_{n_c}^c;A_1^o,\cdots,A_{n_o}^o\right)\left[\frac{\partial}{\partial u^{j_1}},\cdots,\frac{\partial}{\partial u^{j_k}},\frac{\partial}{\partial y_{\alpha_1}},\cdots,\frac{\partial}{\partial y_{\alpha_\ell}},\frac{\partial}{\partial \bar{y}_{\beta_1}},\cdots,\frac{\partial}{\partial \bar{y}_{\beta_{\bar{\ell}}}}\right]$$

$$\sim N_{g,b,n_c,n_o}\left\langle \mathcal{B}\left[\frac{\partial}{\partial u^{j_1}}\right]\cdots\mathcal{B}\left[\frac{\partial}{\partial u^{j_k}}\right]\left(-\partial\xi(y_{\alpha_1})\right)\cdots\left(-\partial\xi(y_{\alpha_\ell})\right)\left(-\bar{\partial}\bar{\xi}(\bar{y}_{\beta_1})\right)\cdots\left(-\bar{\partial}\bar{\xi}(\bar{y}_{\beta_{\bar{\ell}}})\right)\prod_{\alpha=l+1}^{N_R}\mathcal{X}(y_\alpha)\prod_{\beta=\bar{l}+1}^{N_L}\overline{\mathcal{X}}(\bar{y}_\beta)A_1^c\cdots A_{n_c}^c A_1^o\cdots A_{n_o}^o\right\rangle_{\Sigma_{g,b,n_c,n_o}}$$

$$2(2g-2+b)=N-2n_{-1,-1}-\frac{3}{2}\left(n_{-1,-1/2}+n_{-1/2,-1}\right)-n_{-1/2,-1/2},-n_{-1}-\frac{1}{2}n_{-1/2}$$

$$|\mathbf{b}\rangle=-\frac{dq_r}{q_r}\left(e^{-\Lambda}q_r\right)^{L_0+\bar{L}_0}b_0^+|B\rangle,q_r\in[0,1]$$

$$dyV_o(y)$$

$$\widehat{\Omega}^{(0,1,1,1)}(c\bar{c}V_c;cV_o)=-\langle cV_oc\bar{c}V_c\rangle$$

$$N_{g,b,n_c,n_o}=\eta_c^{3g-3+n_c+\frac{3}{2}b+\frac{3}{4}n_o}$$

$$\eta_c\equiv-\frac{1}{2\pi i}=\frac{i}{2\pi}$$

$$\widehat{\Omega}_p^{(g,b,n_c,n_o)}\left(QA_1^c,\cdots,A_{n_c};A_1^o,\cdots,A_{n_o}^o\right)+\cdots$$

$$+(-1)^{A_1^c+\cdots+A_n^c+A_1^o+\cdots A_{n_o}^o-1}\widehat{\Omega}_p^{(g,b,n_c,n_o)}\left(A_1^c,\cdots,A_{n_c};A_1^o,\cdots,QA_{n_o}^o\right)$$

$$=(-1)^p\,\mathrm{d}\widehat{\Omega}_{p-1}^{(g,b,n_c,n_o)}\left(A_1^c,\cdots,A_{n_c};A_1^o,\cdots,A_{n_o}^o\right)$$

$$\{F,G\}=\frac{\partial_r F}{\partial\psi_m}\frac{\partial_\ell G}{\partial\psi_m^*}-\frac{\partial_r F}{\partial\psi_m^*}\frac{\partial_\ell G}{\partial\psi_m},$$

$$\frac{\partial_r A}{\partial\psi}=(-1)^{\psi(A+1)}\frac{\partial_l A}{\partial\psi}$$

$$\{F,G\}=-(-1)^{(F+1)(G+1)}\{G,F\}$$

$$\Delta F=(-1)^{\psi_m}\frac{\partial_\ell}{\partial\psi_m}\frac{\partial_\ell F}{\partial\psi_m^*}=(-1)^{\psi_m F}\frac{\partial_r}{\partial\psi_m}\left(\frac{\partial_\ell F}{\partial\psi_m^*}\right)$$

$$\{S_{cl},S_{cl}\}=0$$

$$\frac{1}{2}\{S,S\}+\Delta S=0$$

$$\Delta e^S=0$$

$$\psi_m^*=\frac{\partial_\ell\Psi}{\partial\psi_m}$$

$$\tilde{\psi}_m^*=\psi_m^*-\frac{\partial_\ell\Psi}{\partial\psi_m},\tilde{\psi}_m=\psi_m$$

$$\{F,G\} = \frac{\partial_r F}{\partial \psi^i}\omega^{ij}\frac{\partial_l G}{\partial \psi^j}, \omega^{ij} = -\omega^{ji}$$

$$\Delta F = \frac{1}{2}(-1)^{\psi^i}\frac{\partial_\ell}{\partial \psi^i}\left(\omega^{ij}\frac{\partial_\ell F}{\partial \psi^j}\right) = \frac{1}{2}(-1)^{\psi^i F}\frac{\partial_r}{\partial \psi^i}\left(\omega^{ij}\frac{\partial_\ell F}{\partial \psi^j}\right)$$

Campos bosónicos.

$$\Psi = \sum_i \varphi_i \psi^i$$

$$|\Psi\rangle = \sum_i (-1)^{\psi^i \epsilon_0}|\varphi_i\rangle\psi^i$$

$$b_0^-|\Psi\rangle = 0, L_0^-|\Psi\rangle = 0$$

$$\Psi \in \mathcal{H}_c.$$

$$\langle A,B\rangle \equiv \langle A|c_0^-|B\rangle\ |A\rangle, |B\rangle \in \mathcal{H}_c.$$

$$\langle A,B\rangle = (-1)^{(A+1)(B+1)}\langle B,A\rangle$$

$$\langle \lambda A, B\eta\rangle = \lambda\langle A,B\rangle\eta,$$

$$\partial\mathcal{V}_{g,n} = -\Delta_c\mathcal{V}_{g-1,n+2} - \frac{1}{2}\sum_{\substack{g_1+g_2=g \\ n_1+n_2=n+2}} \{\mathcal{V}_{g_1,n_1}, \mathcal{V}_{g_2,n_2}\}_c$$

$$\text{General collection of } \mathcal{V}_{g,n} = \begin{cases} \mathcal{V}_{0,n}, & n \geqslant 3, \\ \mathcal{V}_{1,n}, & n \geqslant 1, \\ \mathcal{V}_{g,n}, & n \geqslant 0, g \geqslant 2. \end{cases}$$

$$w_1 w_2 = e^{i\theta}, 0 \leqslant \theta \leqslant 2\pi$$

$$\{A_1,\cdots,A_n\} \equiv \sum_{g=0}^{\infty} (g_s)^{-\chi_{g,n}}\{A_1,\cdots,A_n\}_g \equiv \sum_{g=0}^{\infty} g_s^{2g+n-2}\int_{\mathcal{V}_{g,n}} \Omega_{d_{g,n}}^{(g,n)}(A_1,\cdots,A_n)$$

$$\langle A_0, [A_1,\cdots,A_n]\rangle = \{A_0,\cdots,A_n\}, \forall A_0 \in \mathcal{H}_c$$

$$[A_1,\cdots,A_n] \equiv \sum_{g=0}^{\infty} g_s^{2g+n-1}[A_1,\cdots,A_n]_g$$

$$\sum_{i=1}^{N}\{A'_1 \dots A'_{i-1}(QA'_i)A'_{i+1}\dots A'_N\}$$

$$= -\frac{1}{2}\sum_{\substack{\ell,k\geqslant 0\\ \ell+k=N}}\sum_{\substack{\{i_a;a=1,\dots\ell\}\\ \{j_b;b=1,\dots k\}\\ \{i_a\}\cup\{j_b\}=\{1,\dots N\}}}\{A'_{i_1}\dots A'_{i_\ell}\varphi_s\}\{\varphi_r A'_{j_1}\dots A'_{j_k}\}\langle\varphi_s^c,\varphi_r^c\rangle$$

$$-\frac{1}{2}\{A'_1\dots A'_N\varphi_s\varphi_r\}\langle\varphi_s^c,\varphi_r^c\rangle$$

$$\langle\varphi_r^c,\varphi_s\rangle = \langle\varphi_s,\varphi_r^c\rangle = \delta_{rs}$$

$$\langle\varphi_i,\varphi_j\rangle = \langle\varphi_j,\varphi_i\rangle, \text{ and } \langle\varphi_i^c,\varphi_j^c\rangle = \langle\varphi_j^c,\varphi_i^c\rangle.$$

$$Q[A'_1\cdots A'_N] = -\sum_{i=1}^{N}[A'_1\dots A'_{i-1}(QA'_i)A'_{i+1}\dots A'_N]$$

$$-\sum_{\substack{\ell,k\geqslant 0\\ \ell+k=N}}\sum_{\substack{\{i_a;a=1,\dots\ell\}\\ \{j_b;b=1,\dots k\}\\ \{i_a\}\cup\{j_b\}=\{1,\dots N\}}}\left[A'_{i_1}\dots A'_{i_\ell}\left[A'_{j_1}\dots A'_{j_k}\right]\right] - \frac{1}{2}[A'_1\dots A'_N\varphi_s\varphi_r]\langle\varphi_s^c,\varphi_r^c\rangle$$

$$S = \frac{1}{2}\langle\Psi,Q\Psi\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\{\Psi^n\}$$

$$\Psi = \sum_r \varphi_r\psi^r \rightarrow \delta\Psi = \sum_r \varphi_r\delta\psi^r$$

$$\delta F = \sum_k \frac{\partial^r F}{\partial\psi^k}\delta\psi^k = \langle F_R,\delta\Psi\rangle \rightarrow \frac{\partial^r F}{\partial\psi^k} = \langle F_R,\varphi_k\rangle$$

$$\delta G = \sum_k \delta\psi^k\frac{\partial^l G}{\partial\psi^k} = \langle\delta\Psi,G_L\rangle \rightarrow \frac{\partial^l G}{\partial\psi^k} = (-1)^{\varphi_k}\langle\varphi_k,G_L\rangle$$

$$\{F,G\} = -\langle F_R,G_L\rangle$$

$$\omega^{rs} = (-1)^{\varphi_s+1}\langle\varphi_r^c,\varphi_s^c\rangle, \omega_{rs} = (-1)^{\varphi_r+1}\langle\varphi_r,\varphi_s\rangle,$$

$$Q\Psi + \sum_{n=1}^{\infty}\frac{1}{n!}[\Psi^n] = 0$$

$$S_{\text{cl}} = \frac{1}{2}\langle\Psi,Q\Psi\rangle + \sum_{n=3}^{\infty}\frac{1}{n!}\{\Psi^n\}_0$$

$$\delta\Psi = Q\Lambda + \sum_{n=1}^{\infty}\frac{1}{n!}[\Lambda,\Psi^n]_0 = Q\Lambda + [\Lambda,\Psi]_0 + \frac{1}{2!}[\Lambda,\Psi,\Psi]_0 + \frac{1}{3!}[\Lambda,\Psi,\Psi,\Psi]_0 + \cdots$$

$$\Psi \in \mathcal{H}_c \equiv \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-1/2} \oplus \mathcal{H}_{-1/2,-1} \oplus \mathcal{H}_{-1/2,-1/2},$$
$$\widetilde{\Psi} \in \widetilde{\mathcal{H}}_c \equiv \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-3/2} \oplus \mathcal{H}_{-3/2,-1} \oplus \mathcal{H}_{-3/2,-3/2}.$$

$$\sum_{i=1}^{N} \{A'_1 \dots A'_{i-1}(QA'_i)A'_{i+1} \dots A'_N\}$$

$$= -\frac{1}{2} \sum_{\substack{\ell,k\geqslant 0\\ \ell+k=N}} \sum_{\substack{\{i_a;a=1,\dots\ell\}\\ \{j_b;b=1,\dots k\}\\ \{i_a\}\cup\{j_b\}=\{1,\dots N\}}} \{A'_{i_1} \dots A'_{i_\ell}\varphi_s\}\{\varphi_r A'_{j_1} \dots A'_{j_k}\}\langle\varphi_s^c, \mathcal{G}\varphi_r^c\rangle$$

$$-\frac{1}{2}\{A'_1 \dots A'_N \varphi_s \varphi_r\}\langle\varphi_s^c, \mathcal{G}\varphi_r^c\rangle$$

$$\mathcal{X}_0 = \oint \frac{dz}{z}\mathcal{X}(z), \overline{\mathcal{X}}_0 = \oint \frac{d\bar{z}}{\bar{z}}\overline{\mathcal{X}}(\bar{z})$$

$$\mathcal{G} = \begin{cases} \mathbf{1} \text{ on } \mathcal{H}_{-1,-1} \\ \mathcal{X}_0 \text{ on } \mathcal{H}_{-1,-3/2} \\ \overline{\mathcal{X}}_0 \text{ on } \mathcal{H}_{-3/2,-1} \\ \mathcal{X}_0\overline{\mathcal{X}}_0 \text{ on } \mathcal{H}_{-3/2,-3/2} \end{cases}$$

$$S = -\frac{1}{2}\langle\widetilde{\Psi}, Q\mathcal{G}\widetilde{\Psi}\rangle + \langle\widetilde{\Psi}, Q\Psi\rangle + \sum_{n=1}^{\infty} \frac{1}{n!}\{\Psi^n\}$$

$$\delta F = \langle F_R, \delta\widetilde{\Psi}\rangle + \langle \tilde{F}_R, \delta\Psi\rangle = \langle \delta\widetilde{\Psi}, F_L\rangle + \langle \delta\Psi, \tilde{F}_L\rangle$$

$$\{F, G\} = -\langle F_R, \tilde{G}_L\rangle - \langle \tilde{F}_R, G_L\rangle - \langle \tilde{F}_R, \mathcal{G}\tilde{G}_L\rangle$$

$$\langle A_1, [A_1, \cdots, A_n]\rangle = \{A_1, \cdots, A_n\}$$

$$\delta|\widetilde{\Psi}\rangle = Q|\widetilde{\Lambda}\rangle + \sum_{n=2}^{\infty} \frac{1}{n!}[\Lambda\Psi^n]_0, \delta|\Psi\rangle = Q|\Lambda\rangle + \sum_{n=2}^{\infty} \frac{1}{n!}\mathcal{G}[\Lambda\Psi^n]_0$$

Campos heteróticos.

$$\Psi \in \mathcal{H}_c \equiv \mathcal{H}_{-1} \oplus \mathcal{H}_{-1/2}$$
$$\widetilde{\Psi} \in \widetilde{\mathcal{H}}_c \equiv \mathcal{H}_{-1} \oplus \mathcal{H}_{-3/2}$$

$$\mathcal{G} = \begin{cases} \mathbf{1} \text{ on } \mathcal{H}_{-1} \\ \mathcal{X}_0 \text{ on } \mathcal{H}_{-3/2} \end{cases}$$

$$S = -\frac{1}{2}\langle\widetilde{\Psi}, Q\mathcal{G}\widetilde{\Psi}\rangle + \langle\widetilde{\Psi}, Q\Psi\rangle + \sum_{n=1}^{\infty} \frac{1}{n!}\{\Psi^n\}$$

$$\Omega_{n-3}^{o(0,n)}(A_1, \cdots, A_n) \equiv K^{-1}\widehat{\Omega}_{n-3}^{(0,1,0,n)}(A_1, \cdots, A_n)$$

$$\partial\mathcal{V}^o_{0,n} = -\frac{1}{2}\sum_{\substack{n_1,n_2\\ n_1+n_2=n+2}} \{\mathcal{V}^o_{0,n_1}, \mathcal{V}^o_{0,n_2}\}_o$$

$$\{A_1,\cdots,A_n\} \equiv g_o^{n-2}\int_{\mathcal{V}^o_{0,n}} \Omega^{o(0,n)}_{n-3}(A_1,\cdots,A_n)$$

$$S_o = \frac{1}{2}\langle\psi_o, Q\psi_o\rangle' + \sum_{n=3}^{\infty}\frac{1}{n!}\{\psi_o^n\}$$

$$\mathcal{T} = \frac{1}{2\pi^2 g_o^2}$$

$$\{A_1\cdots A_iA_{i+1}\cdots A_n\} = (-1)^{A_iA_{i+1}+1}\{A_1\cdots A_{i+1}A_i\cdots A_n\}.$$

$$Q[A_1'\cdots A_N'] = \sum_{i=1}^{N}(-1)^{i-1}[A_1'\dots A_{i-1}'(QA_i')A_{i+1}'\dots A_N']$$
$$+\sum_{\substack{\ell,k\geqslant 0\\ \ell+k=N}}\ \sum_{\substack{\{i_a;a=1,\dots\ell\}\\ \{j_b;b=1,\dots k\}\\ \{i_a\}\cup\{j_b\}=\{1,\dots N\}}} \left[[A_{j_1}'\dots A_{j_k}']A_{i_1}'\dots A_{i_\ell}'\right]$$

$$\langle A_0,[A_1,\cdots,A_n]\rangle' = \{A_0,A_1,\cdots,A_n\}$$

$$\delta F = \langle F_R^o,\delta\psi_o\rangle' = \langle\delta\psi_o, F_L^o\rangle',$$

$$\{F,G\} = -\langle F_R^o, G_L^o\rangle'$$

$$\delta\Phi = Q\Lambda + [\Lambda\Phi] + \frac{1}{2!}[\Lambda,\Phi,\Phi] + \cdots$$

$$[A_1,A_2] = A_1\star A_2 - (-1)^{A_1A_2}A_2\star A_1$$

$$A_1\star(A_2\star A_3) = (A_1\star A_2)\star A_3$$

$$[A_1,A_2,\cdots,A_n] = 0 \ \text{ for } \ n\geqslant 3$$

$$\langle A_1, A_2\star A_3\rangle' = \langle f_1\circ A_1(0) f_2\circ A_2(0) f_3\circ A_3(0)\rangle'_{\text{UHP}}$$

$$\begin{aligned} f_1(w_1) &= h^{-1}\left(e^{2\pi i/3}\big(h(w_1)\big)^{2/3}\right)\\ f_2(w_2) &= h^{-1}\left(\big(h(w_2)\big)^{2/3}\right), h(u)\equiv\frac{1+iu}{1-iu}\\ f_3(w_3) &= h^{-1}\left(e^{-2\pi i/3}\big(h(w_3)\big)^{2/3}\right)\end{aligned}$$

$$\begin{aligned}\psi_o &\in \mathcal{H}_o \equiv \mathcal{H}_{-1}\oplus\mathcal{H}_{-1/2},\\ \tilde{\psi}_o &\in \widetilde{\mathcal{H}}_o \equiv \mathcal{H}_{-1}\oplus\mathcal{H}_{-3/2}.\end{aligned}$$

$$S=-\frac{1}{2}\langle\tilde{\psi}_o,Q\mathcal{G}\tilde{\psi}_o\rangle'+\langle\tilde{\psi}_o,Q\psi_o\rangle'+\sum_{n=1}^{\infty}\frac{1}{n!}\{\psi_o^n\}$$

$$\Psi_c\in\mathcal{H}_c,\widetilde{\Psi}_c\in\widetilde{\mathcal{H}}_c,$$
$$\Psi_o\in\mathcal{H}_o,\widetilde{\Psi}_o\in\widetilde{\mathcal{H}}_o$$

$$\begin{aligned}\partial\mathcal{V}_{g,b,n_c,n_o}=&-\Delta_c\mathcal{V}_{g-1,b,n_c+2,n_o}-\Delta_o'\mathcal{V}_{g,b-1,n_c,n_o+2}-\Delta_o\mathcal{V}_{g-1,b+1,n_c,n_o+2}\\&-\frac{1}{2}\sum_{\substack{g_1+g_2=g,b_1+b_2=b\\n_{c1}+n_{c2}=n_c+2,n_{o1}+n_{o2}=n_o}}\left\{\mathcal{V}_{g_1,b_1,n_{c1},n_{o1}},\mathcal{V}_{g_2,b_2,n_{c2},n_{o2}}\right\}_c\\&-\frac{1}{2}\sum_{\substack{g_1+g_2=g,b_1+b_2=b+1\\n_{c1}+n_{c2}=n_c,n_{o1}+n_{o2}=n_o+2}}\left\{\mathcal{V}_{g_1,b_1,n_{c1},n_{o1}},\mathcal{V}_{g_2,b_2,n_{c2},n_{o2}}\right\}_o\end{aligned}$$

$$\left\{A_1^c,\cdots,A_{n_c}^c;A_1^o,\cdots,A_{n_o}^o\right\}=\sum_{g=0}^{\infty}\sum_{b=0}^{\infty}\left\{A_1^c,\cdots,A_{n_c}^c;A_1^o,\cdots,A_{n_o}^o\right\}_{g,b}$$
$$\left\{A_1^c,\cdots,A_{n_c}^c;A_1^o,\cdots,A_{n_o}^o\right\}_{g,b}=(g_s)^{-\chi_{g,b,n_c,n_o}}\int_{\mathcal{V}_{g,b,n_c,n_o}}\Omega_{d_{g,b,n_c,n_o}}^{(g,b,n_c,n_o)}\left(A_1^c,\cdots,A_{n_c}^c;A_1^o,\cdots,A_{n_o}^o\right)$$

$$\left\{\tilde{A}^c\right\}_D=\Omega_0^{(0,1,1,0)}\left(\tilde{A}^c\right)\equiv-\frac{1}{2\pi i}N_{0,1,1,0}\langle\hat{\mathcal{G}}\tilde{A}^c|c_0^-e^{-\Lambda(L_0+\bar{L}_0)}|B\rangle,$$

$$\hat{\mathcal{G}}\equiv\begin{cases}1\text{ on }\mathcal{H}_{-1,-1}\\\frac{1}{2}\left(\mathcal{X}_0+\overline{\mathcal{X}_0}\right)\text{ on }\mathcal{H}_{-3/2,-3/2}\end{cases}$$

$$\{A^c\}_D=\Omega_0^{(0,1,1,0)}(A^c)\equiv-\frac{1}{2\pi i}N_{0,1,1,0}\langle A^c|c_0^-e^{-\Lambda(L_0+\bar{L}_0)}|B\rangle.$$

$$\langle A_0^c|c_0^-|[A_1^c\cdots A_N^c;A_1^o\cdots A_M^o]_c\rangle=\{A_0^cA_1^c\cdots A_N^c;A_1^o\cdots A_M^o\},\forall|A_0^c\rangle\in\mathcal{H}_c,$$
$$\langle A_0^o\mid[A_1^c\cdots A_N^c;A_1^o\cdots A_M^o]_o\rangle=\{A_1^c\cdots A_N^c;A_0^oA_1^o\cdots A_M^o\},\forall|A_0^o\rangle\in\mathcal{H}_o,$$
$\langle\tilde{A}^c|c_0^-|[]_c\rangle=\left\{\tilde{A}^c\right\}_D$,(4.65)

$$\left\{\left(Q\tilde{A}^c\right)\right\}_D=0$$

$$\sum_{i=1}^{N}\{A_1^c\cdots A_{i-1}^c(QA_i^c)A_{i+1}^c\cdots A_N^c;A_1^o\cdots A_M^o\}+\sum_{j=1}^{M}\{A_1^c\cdots A_N^c;A_1^o\cdots A_{j-1}^o(QA_j^o)A_{j+1}^o\cdots A_M^o\}(-1)^{j-1}$$

$$=-\frac{1}{2}\sum_{k=0}^{N}\sum_{\ell=0}^{M}\sum_{\substack{\{i_1\cdots,i_k\}\subset\{1,\cdots,N\}\\\{j_1,\cdots,j_\ell\}<\{1,\cdots,M\}}}\left(\{A_{i_1}^c\cdots A_{i_k}^c\mathcal{B}^c;A_{j_1}^o\cdots A_{j_\ell}^o\}+\{A_{i_1}^c\cdots A_{i_k}^c;\mathcal{B}^oA_{j_1}^o\cdots A_{j_\ell}^o\}\right)$$

$$-\{[A_1^c\cdots A_N^c;A_1^o\cdots A_M^o]_c\}_D-\frac{1}{2}\{A_1^c\cdots A_N^c\varphi_s\varphi_r;A_1^o\cdots,A_M^o\}\langle\varphi_s^c,\mathcal{G}\varphi_r^c\rangle$$

$$-\frac{1}{2}(-1)^{\hat{\varphi}_s}\{A_1^c\cdots A_N^c;\hat{\varphi}_s\hat{\varphi}_rA_1^o\cdots,A_M^o\}\langle\hat{\varphi}_s^c,\mathcal{G}\hat{\varphi}_r^c\rangle$$

$$\mathcal{B}^c \equiv \mathcal{G}\left[A^c_{\bar{\imath}_1} \cdots A^c_{\bar{\imath}_{N-k}}; A^o_{\bar{\jmath}_1} \cdots A^o_{\bar{\jmath}_{M-\ell}}\right]_c, \mathcal{B}^o \equiv \mathcal{G}\left[A^c_{\bar{\imath}_1} \cdots A^c_{\bar{\imath}_{N-k}}; A^o_{\bar{\jmath}_1} \cdots A^o_{\bar{\jmath}_{M-\ell}}\right]_o$$
$$\{i_1, \cdots, i_k\} \cup \{\bar{\imath}_1, \cdots, \bar{\imath}_{N-k}\} = \{1, \cdots, N\}, \{j_1, \cdots, j_\ell\} \cup \{\bar{\jmath}_1, \cdots, \bar{\jmath}_{M-\ell}\} = \{1, \cdots, M\},$$

$$S = -\frac{1}{2}\left\langle \widetilde{\Psi}_c, Q\mathcal{G}\widetilde{\Psi}_c \right\rangle + \left\langle \widetilde{\Psi}_c, Q\Psi_c \right\rangle - \frac{1}{2}\left\langle \widetilde{\Psi}_o, Q\mathcal{G}\widetilde{\Psi}_o \right\rangle + \left\langle \widetilde{\Psi}_o, Q\Psi_o \right\rangle + \left\{\widetilde{\Psi}_c\right\}_D + \sum_{N=0}^{\infty} \sum_{M=0}^{\infty} \frac{1}{N!\,M!}\{(\Psi_c)^N; (\Psi_o)^M\}$$

$$\delta F = \left\langle F^c_R, \delta\widetilde{\Psi}_c \right\rangle + \left\langle \tilde{F}^c_R, \delta\Psi_c \right\rangle + \left\langle F^o_R, \delta\widetilde{\Psi}_o \right\rangle + \left\langle \tilde{F}^o_R, \delta\Psi_o \right\rangle = \left\langle \delta\widetilde{\Psi}_c, F^c_L \right\rangle + \left\langle \delta\Psi_c, \tilde{F}^c_L \right\rangle + \left\langle \delta\widetilde{\Psi}_o, F^o_L \right\rangle + \left\langle \delta\Psi_o, \tilde{F}^o_L \right\rangle$$

$$\{F, G\} = -\left(\left\langle F^c_R, \tilde{G}^c_L \right\rangle + \left\langle \tilde{F}^c_R, G^c_L \right\rangle + \left\langle \tilde{F}^c_R, \mathcal{G}\tilde{G}^c_L \right\rangle\right) - \left(\left\langle F^o_R, \tilde{G}^o_L \right\rangle + \left\langle \tilde{F}^o_R, G^o_L \right\rangle + \left\langle \tilde{F}^o_R, \mathcal{G}\tilde{G}^o_L \right\rangle\right)$$

$$S = \frac{1}{2}\langle \Psi_c, Q\Psi_c \rangle + \frac{1}{2}\langle \Psi_o, Q\Psi_o \rangle + \{\Psi_c\}_D + \sum_{N=0}^{\infty} \sum_{M=0}^{\infty} \frac{1}{N!\,M!}\{(\Psi_c)^N; (\Psi_o)^M\}$$

$$S_{oc} = \frac{1}{2}\langle \Psi_o, Q\Psi_o \rangle + \sum_{n=3}^{\infty} \frac{1}{n!} g_s^{(n-2)/2} \int \Omega^{(0,1,0,n)}_{n-3}(\Psi_o, \cdots, \Psi_o)$$

$$= \frac{1}{2}K\langle \Psi_o, Q\Psi_o \rangle' + \sum_{n=3}^{\infty} \frac{1}{n!} g_s^{(n-2)/2} N_{0,1,0,n} K \int \Omega^{o(0,n)}_{n-3}(\Psi_o, \cdots, \Psi_o)$$

$$S_o = \frac{1}{2}\langle \psi_o, Q\psi_o \rangle' + \sum_{n=3}^{\infty} \frac{1}{n!} g_o^{n-2} \int \Omega^{o(0,n)}_{n-3}(\psi_o, \cdots, \psi_o)$$

$$\Psi_o = K^{-1/2}\psi_o$$

$$S_{oc} = \frac{1}{2}\langle \psi_o, Q\psi_o \rangle' + \sum_{n=3}^{\infty} \frac{1}{n!} g_s^{(n-2)/2} K^{(2-n)/2} \eta_c^{3(n-2)/4} \int \Omega^{o(0,n)}_{n-3}(\psi_o, \cdots, \psi_o)$$

$$g_o = g_s^{1/2} K^{-1/2} \eta_c^{3/4}$$

$$g_o^2 = (g_s/K)\eta_c^{3/2}$$

Partículas supermasivas – campos de gauge – transformaciones de gauge – campos cuánticos gravitacionales y simetría – osciladores y propagadores.

$$|\psi_o\rangle = (\alpha')^{(p+1)/2} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \left(A_\mu(k) c_1 \alpha^\mu_{-1} - i\sqrt{\frac{1}{2}} B(k) c_0 \right) |k\rangle,$$

$$|\epsilon\rangle = (\alpha')^{(p+1)/2} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \frac{i}{\sqrt{2}} \epsilon(k) |k\rangle.$$

$$\phi(x) = (\alpha')^{(p+1)/2} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \tilde{\phi}(k) e^{ikx}, \text{ so that } ik_\mu \leftrightarrow \partial_\mu$$

$$\mathrm{bpz}(\phi_n) = (-1)^{n+d}\phi_{-n}$$

$$\langle\psi_o| = (\alpha')^{(p+1)/2}\int \frac{d^{p+1}k}{(2\pi)^{p+1}}\langle k|\left(A_\mu(k)c_{-1}\alpha_1^\mu + i\sqrt{\frac{1}{2}}B(k)c_0\right),$$

$$Q = \sum_n c_n\left(L_{-n}^m - \delta_{n,0}\right) + \frac{1}{2}\sum_{m,n}(m-n){:}c_m c_n b_{-n-m}{:}$$

$$L_n^m = \frac{1}{2}\sum_m \alpha_m \cdot \alpha_{n-m}, (n \neq 0), L_0^m = \frac{1}{2}\alpha_0^2 + \sum_{n\geqslant 1}\alpha_{-n}\cdot\alpha_n, \alpha_0^\mu = \sqrt{2\alpha'}k^\mu$$

$$Q = c_0(\alpha' k^2 - 1) + c_0(\alpha_{-1}\cdot\alpha_1 + b_{-1}c_1 + c_{-1}b_1) + \sqrt{2\alpha'}k_\mu\left(\alpha_{-1}^\mu c_1 + c_{-1}\alpha_1^\mu\right) - 2b_0c_{-1}c_1 + \cdots$$

$$Qc_1\alpha_{-1}^\mu|k\rangle = \left(\alpha' k^2 c_0 c_1 \alpha_{-1}^\mu + \sqrt{2\alpha'}k^\mu c_{-1}c_1\right)|k\rangle$$

$$Qc_0|k\rangle = \left(\sqrt{2\alpha'}k_\mu\alpha_{-1}^\mu c_1 c_0 - 2c_{-1}c_1\right)|k\rangle$$
$$Q|k\rangle = \left(\alpha' k^2 c_0 + \sqrt{2\alpha'}k_\mu\alpha_{-1}^\mu c_1\right)|k\rangle$$

$$\langle k|c_{-1}c_0c_1|k'\rangle' = -(2\pi)^{p+1}(\alpha')^{-(p+1)/2}\delta^{(p+1)}(k+k').$$

$$S_2 = \frac{1}{2}\langle\psi_o|Q|\psi_o\rangle'$$

$$= (\alpha')^{(p+1)/2}\int \frac{d^{p+1}k}{(2\pi)^{p+1}}\left(-\frac{1}{2}A^\mu(-k)\alpha' p^2 A_\mu(k) - \sqrt{\alpha'}A^\mu(-k)ik_\mu B(k)\right.$$

$$\left.-\frac{1}{2}B(-k)B(k)\right)$$

$$S_2 = (\alpha')^{-(p+1)/2}\int d^{p+1}x\left(\frac{\alpha'}{2}A^\mu \,\Box\, A_\mu - \sqrt{\alpha'}A^\mu\partial_\mu B - \frac{1}{2}B^2\right)$$

$$\delta A_\mu(k) = i\sqrt{\alpha'}k_\mu\epsilon(k), \delta B(k) = -\alpha' k^2\epsilon(k)$$

$$\delta A_\mu = \sqrt{\alpha'}\partial_\mu\epsilon, \delta B = \alpha' \,\Box\, \epsilon$$

$$S = (\alpha')^{-(p-1)/2}\int d^{p+1}x\left(\frac{1}{2}A_\mu \,\Box\, A^\mu + \frac{1}{2}(\partial\cdot A)^2\right) = (\alpha')^{-(p-1)/2}\int d^{p+1}$$

$$x\left(-\frac{1}{2}\partial_\mu A_\nu\partial^\mu A^\nu + \frac{1}{2}\partial_\mu A_\nu\partial^\nu A^\mu\right) = (\alpha')^{-(p-1)/2}\int d^{p+1}x\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right)$$

$$S_{\mathrm{ws}} = -\frac{1}{4\pi\alpha'}\int dxdy g_{\mu\nu}\left(\partial_x X^\mu\partial_x X^\nu + \partial_y X^\mu\partial_y X^\nu\right) = -\frac{1}{\pi\alpha'}\int dxdy g_{\mu\nu}\partial X^\mu\bar{\partial}X^\nu, z \equiv x + iy$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_{\text{ws}}|_h = -\frac{1}{\pi\alpha'}\int \ dxdy h_{\mu\nu}\partial X^{\mu}\bar{\partial}X^{\nu}$$

$$-g_s\frac{1}{\pi}\int \ dxdyV$$

$$\mathcal{O}_h = \frac{1}{g_s\alpha'}h_{\mu\nu}c\bar{c}\partial X^{\mu}\bar{\partial}X^{\nu} = \frac{1}{g_s}\left(-\frac{1}{2}h_{\mu\nu}\right)c\bar{c}i\sqrt{\frac{2}{\alpha'}}\partial X^{\mu}i\sqrt{\frac{2}{\alpha'}}\bar{\partial}X^{\nu}$$

$$|\Psi\rangle = (\alpha')^{D/2}\frac{1}{g_s}\int \ \frac{d^Dk}{(2\pi)^D}\Bigg(-\frac{1}{2}e_{\mu\nu}(k)\alpha_{-1}^{\mu}\bar{\alpha}_{-1}^{\nu}c_1\bar{c}_1 + e(k)c_1c_{-1} + \bar{e}(k)\bar{c}_1\bar{c}_{-1}$$
$$+i\sqrt{\frac{1}{2}}\left(f_{\mu}(k)c_0^{+}c_1\alpha_{-1}^{\mu} + \bar{f}_{\mu}(k)c_0^{+}\bar{c}_1\bar{\alpha}_{-1}^{\mu}\right)\Bigg)|k\rangle$$

$$e_{\mu\nu} = h_{\mu\nu} + b_{\mu\nu}, \text{ with } h_{\mu\nu} = h_{\nu\mu}, b_{\mu\nu} = -b_{\nu\mu}$$

$$\langle k|c_{-1}\bar{c}_{-1}c_0^{-}c_0^{+}c_1\bar{c}_1|k'\rangle = -\frac{1}{2}(\alpha')^{-D/2}(2\pi)^D\delta^{(D)}(k+k').$$

$$\text{bpz}(\phi_n) = (-1)^d\phi_{-n},$$

$$\langle\Psi| = (\alpha')^{D/2}\frac{1}{g_s}\int \ \frac{d^Dk}{(2\pi)^D}\langle k|\Bigg(-\frac{1}{2}e_{\mu\nu}(k)\alpha_1^{\mu}\bar{\alpha}_1^{\nu}c_{-1}\bar{c}_{-1} + e(k)c_{-1}c_1 + \bar{e}(k)\bar{c}_{-1}\bar{c}_1$$
$$-i\sqrt{\frac{1}{2}}\left(f_{\mu}(k)c_0^{+}c_{-1}\alpha_1^{\mu} + \bar{f}_{\mu}(k)c_0^{+}\bar{c}_{-1}\bar{\alpha}_1^{\mu}\right)$$

$$S^{(2)} = \frac{1}{2}\langle\Psi|c_0^{-}Q|\Psi\rangle$$

$$Q = c_0^{+}\left(\frac{\alpha'}{2}k^2 - 2\right) + c_0^{+}\left(\alpha_{-1}\cdot\alpha_1 + b_{-1}c_1 + c_{-1}b_1 + \bar{\alpha}_{-1}\cdot\bar{\alpha}_1 + \bar{b}_{-1}\bar{c}_1 + \bar{c}_{-1}\bar{b}_1\right) + \sqrt{\frac{\alpha'}{2}}k$$

$$\cdot(\alpha_{-1}c_1 + c_{-1}\alpha_1) + \sqrt{\frac{\alpha'}{2}}k\cdot(\bar{\alpha}_{-1}\bar{c}_1 + \bar{c}_{-1}\bar{\alpha}_1) - b_0^{+}(c_{-1}c_1 + \bar{c}_{-1}\bar{c}_1) + \cdots$$

$$S^{(2)} = (\alpha')^{-D/2}\frac{1}{8g_s^2}\int \ d^Dx\Bigg[\frac{\alpha'}{4}e^{\mu\nu}\,\Box\, e_{\mu\nu} + 2\alpha'\bar{e}\,\Box\, e - f^{\mu}f_{\mu} - \bar{f}^{\mu}\bar{f}_{\mu}$$
$$-\sqrt{\alpha'}f^{\mu}\left(\partial^{\nu}e_{\mu\nu} - 2\partial_{\mu}\bar{e}\right) + \sqrt{\alpha'}\bar{f}^{\nu}\left(\partial^{\mu}e_{\mu\nu} + 2\partial_{\nu}e\right)\Bigg]$$

$$|\Lambda\rangle = (\alpha')^{D/2}\frac{1}{g_s}\int \ \frac{d^Dk}{(2\pi)^D}\left(\frac{i}{\sqrt{2}}\lambda_{\mu}(k)\alpha_{-1}^{\mu}c_1 - \frac{i}{\sqrt{2}}\bar{\lambda}_{\mu}(k)\bar{\alpha}_{-1}^{\mu}\bar{c}_1 + \mu(k)c_0^{+}\right)|k\rangle.$$

$$\delta|\Psi\rangle = Q|\Lambda\rangle$$

$$\delta e_{\mu\nu} = \sqrt{\alpha'}\left(\partial_\mu \bar{\lambda}_\nu + \partial_\nu \lambda_\mu\right)$$

$$\delta f_\mu = -\frac{1}{2}\alpha' \square \lambda_\mu + \sqrt{\alpha'}\partial_\mu \mu$$

$$\delta \bar{f}_\nu = \frac{1}{2}\alpha' \square \bar{\lambda}_\nu + \sqrt{\alpha'}\partial_\nu \mu$$

$$\delta e = -\frac{1}{2}\sqrt{\alpha'}\partial \cdot \lambda + \mu$$

$$\delta \bar{e} = \frac{1}{2}\sqrt{\alpha'}\partial \cdot \bar{\lambda} + \mu$$

$$d = \frac{1}{2}(e - \bar{e}), \text{ and } \chi = \frac{1}{2}(e + \bar{e})$$

$$\delta d = -\frac{1}{4}\sqrt{\alpha'}(\partial \cdot \lambda + \partial \cdot \bar{\lambda})$$

$$\delta \chi = -\frac{1}{4}\sqrt{\alpha'}(\partial \cdot \lambda - \partial \cdot \bar{\lambda}) + \mu$$

$$\chi = 0$$

$$f_\mu = -\frac{1}{2}\sqrt{\alpha'}\left(\partial^\nu e_{\mu\nu} - 2\partial_\mu \bar{e}\right), \bar{f}_\nu = \frac{1}{2}\sqrt{\alpha'}\left(\partial^\mu e_{\mu\nu} + 2\partial_\nu e\right)$$

$$S^{(2)} = (\alpha')^{-(D-2)/2}\frac{1}{8g_s^2}\int d^D x\left[\frac{1}{4}e_{\mu\nu} \square e^{\mu\nu} + \frac{1}{4}\left(\partial^\nu e_{\mu\nu}\right)^2 + \frac{1}{4}\left(\partial^\mu e_{\mu\nu}\right)^2 - 2d\partial^\mu\partial^\nu e_{\mu\nu} - 4d \square d\right]$$

$$\delta e_{\mu\nu} = \sqrt{\alpha'}\left(\partial_\nu \lambda_\mu + \partial_\mu \bar{\lambda}_\nu\right)$$

$$\delta d = -\frac{1}{4}\sqrt{\alpha'}(\partial \cdot \lambda + \partial \cdot \bar{\lambda})$$

$$S^{(2)} = (\alpha')^{-(D-2)/2}\frac{1}{8g_s^2}\int d^D x L[h, b, d]$$

$$L[h, b, d] = \frac{1}{4}h^{\mu\nu}\partial^2 h_{\mu\nu} + \frac{1}{2}\left(\partial^\nu h_{\mu\nu}\right)^2 - 2d\partial^\mu\partial^\nu h_{\mu\nu} - 4d\partial^2 d + \frac{1}{4}b^{\mu\nu}\partial^2 b_{\mu\nu} + \frac{1}{2}\left(\partial^\nu b_{\mu\nu}\right)^2$$

$$S_{\mathrm{st}} = \frac{1}{2\kappa^2}\int d^D x\sqrt{-\mathrm{g}}e^{-2\phi}\left[R - \frac{1}{12}H^2 + 4(\partial\phi)^2\right]$$

$$\mathrm{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \phi = d + \frac{1}{4}\eta^{\mu\nu}h_{\mu\nu}, H_{\mu\nu\rho} = \partial_\mu b_{\nu\rho} + \cdots$$

$$S_{\mathrm{st}}^{(2)} = \frac{1}{2\kappa^2}\int d^D x L[h, b, d]$$

$$\kappa = (\alpha')^{(D-2)/4}(2g_s)$$

$$\delta h_{\mu\nu} \qquad\qquad = \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu$$
$$\delta b_{\mu\nu} = -\partial_\mu \tilde{\epsilon}_\nu + \partial_\nu \tilde{\epsilon}_\mu (4.124)$$
$$\delta d = -\frac{1}{2}\partial \cdot \epsilon \qquad (4.124)$$

$$\epsilon_\mu \equiv \frac{1}{2}\sqrt{\alpha'}(\lambda_\mu + \bar{\lambda}_\mu), \tilde{\epsilon}_\mu \equiv \frac{1}{2}\sqrt{\alpha'}(\lambda_\mu - \bar{\lambda}_\mu)$$

$$b_0^+|\Psi\rangle = 0$$

$$S = \frac{1}{2}\langle\Psi|c_0^- c_0^+ L_0^+|\Psi\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\{\Psi^n\}$$

$$\mathcal{P}_b = -b_0^+ b_0^- (L_0 + \bar{L}_0)^{-1}\delta_{L_0,\bar{L}_0}$$

$$\mathcal{P}_b = -\frac{1}{2\pi} b_0^+ b_0^- \int_0^{2\pi} d\theta \int_0^{\infty} ds e^{-s(L_0+\bar{L}_0)} e^{i\theta(L_0-\bar{L}_0)} = \frac{1}{\pi} b_0 \bar{b}_0 \int_{|q|\leqslant 1} \frac{d^2q}{|q|^2} q^{L_0}\bar{q}^{\bar{L}_0}$$

$$q \equiv e^{-s+i\theta}, d^2q = d\theta ds|q|^2 \equiv \frac{i}{2}dq \wedge d\bar{q}$$

$$w_1 w_2 = q$$

$$(6g_1 - 6 + 2n_1) + (6g_2 - 6 + 2n_2) + 2$$

$$\mathcal{P}_b = \int_{|q|\leqslant 1}\left[\left(-\frac{1}{2\pi i}\right) b_0 \bar{b}_0 \frac{dq \wedge d\bar{q}}{|q|^2}\right] q^{L_0}\bar{q}^{\bar{L}_0}$$

$$\Omega_{\mathcal{P}_b} = -\frac{1}{2\pi i} b_0 \bar{b}_0 \frac{dq \wedge d\bar{q}}{|q|^2}$$

$$\frac{\partial F}{\partial q} = \frac{1}{w_2} = \frac{w_1}{q}, \frac{\partial \bar{F}}{\partial q} = 0; \frac{\partial F}{\partial \bar{q}} = 0, \frac{\partial \bar{F}}{\partial \bar{q}} = \frac{\bar{w}_1}{\bar{q}}.$$

$$\widehat{\Omega}_b = \mathcal{B}\left[\frac{\partial}{\partial q}\right]\mathcal{B}\left[\frac{\partial}{\partial \bar{q}}\right] dq \wedge d\bar{q} = \frac{1}{q}\oint \; w_1 b(w_1) dw_1 \times \frac{1}{\bar{q}}\oint \; \bar{w}_1 \bar{b}(\bar{w}_1) d\bar{w}_1 \times dq \wedge d\bar{q} = \frac{b_0}{q}\frac{\bar{b}_0}{\bar{q}} dq \wedge d\bar{q}$$

$$\Omega_{\mathcal{P}_b} = -\frac{1}{2\pi i}\hat{\Omega}_b$$

$$\left(-\frac{1}{2\pi i}\right)^{3g_1-3+n_1}\left(-\frac{1}{2\pi i}\right)^{3g_2-3+n_2}\left(-\frac{1}{2\pi i}\right)^{1} = \left(-\frac{1}{2\pi i}\right)^{3(g_1+g_2)-3+(n_1+n_2-2)}$$

$$S = -\frac{1}{2}\langle\widetilde{\Psi}|c_0^- c_0^+ L_0^+ \mathcal{G}|\widetilde{\Psi}\rangle + \langle\widetilde{\Psi}|c_0^- c_0^+ L_0^+|\Psi\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\{\Psi^n\}$$

$$K_0 = c_0^- c_0^+ L_0^+ \begin{pmatrix} \mathcal{G} & -1 \\ -1 & 0 \end{pmatrix}$$

$$\mathcal{P}_s = -b_0^+ b_0^- (L_0 + \bar{L}_0)^{-1}\delta_{L_0,\bar{L}_0}\begin{pmatrix} 0 & 1 \\ 1 & \mathcal{G} \end{pmatrix}$$

$$\mathcal{P}_o = -b_0 L_0^{-1} = -b_0 \int_0^1 \frac{dq_o}{q_o} q_o^{L_0} = \int_0^1 \left[(-b_0)\frac{dq_o}{q_o}\right] q_o^{L_0}$$

$$\Omega_{\mathcal{P}_o} = -b_0 \frac{dq_o}{q_o}$$

$$w_1 w_2 = -q_o, q_o \in [0,1]$$

$$\widehat{\Omega}_o = \mathcal{B}\left[\frac{\partial}{\partial q_o}\right] dq_o$$

$$\mathcal{B}\left[\frac{\partial}{\partial q_o}\right] = \int \ dw_1 b(w_1)\frac{w_1}{q_o} + \int \ d\bar{w}_1 \bar{b}(\bar{w}_1)\frac{\bar{w}_1}{q_o} = \frac{1}{q_o}\oint \ _C dw_1 b(w_1) w_1$$

$$\mathcal{B}\left[\frac{\partial}{\partial q_o}\right] = -\frac{1}{q_o} b_0, \widehat{\Omega}_o = -b_0 \frac{dq_o}{q_o}$$

$$\Omega_{\mathcal{P}_o} = \widehat{\Omega}_o$$

$$N_{g_1,b_1,n_{c1},n_{o1}} N_{g_2,b_2,n_{c2},n_{o2}} \sim N_{g_1+g_2,b_1+b_2-1,n_{c1}+n_{c2},n_{o1}+n_{o2}-2}$$

$$N_{g,b,n_c,n_o} \sim N_{g,b+1,n_c,n_o+2}, N_{g,b,n_c,n_o} \sim N_{g+1,b-1,n_c,n_o+2}$$

$$\eta_c N_{g_1,b_1,n_{c1},n_{o1}} N_{g_2,b_2,n_{c2},n_{o2}} \sim N_{g_1+g_2,b_1+b_2,n_{c1}+n_{c2}-2,n_{o1}+n_{o2}}$$
$$\eta_c N_{g-1,b,n_c+2,n_o} \sim N_{g,b,n_c,n_o}$$

Membranas.

$$\mathcal{O}_h = \frac{1}{g_s} h_{\mu\nu} c\bar{c}\partial X^\mu \bar{\partial} X^\nu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \phi = \frac{1}{4}\eta^{\mu\nu} h_{\mu\nu}$$

$$S_p = -\mathcal{T}\int \ d^{p+1}x e^{-\phi}\sqrt{-g}$$

$$S_p\big|_h = -\mathcal{T}\left\{\frac{1}{2}\eta^{\mu\nu} h_{\mu\nu}^{\parallel}(x_\perp = 0) - \phi(x_\perp = 0)\right\}\int \ d^{p+1}x = -\frac{\mathcal{T}}{4}\eta^{\mu\nu}\left(h_{\mu\nu}^{\parallel} - h_{\mu\nu}^{\perp}\right)(2\pi)^{p+1}\delta^{(p+1)}(0)$$

$$\Omega_0^{(0,1,1,0)}(A) = -\frac{1}{2\pi i}\eta_c^{-1/2}\langle c_0^- A\rangle = \eta_c^{1/2}\langle c_0^- A\rangle$$

$$\Omega_0^{(0,1,1,0)}(\mathcal{O}_h) = \eta_c^{1/2}\frac{1}{g_s} h_{\mu\nu}\langle c_0^- c\bar{c}\partial X^\mu \bar{\partial} X^\nu\rangle$$

$$\Omega_0^{(0,1,1,0)}(\mathcal{O}_h) = \eta_c^{1/2}\frac{1}{g_s}\left(h_{\mu\nu}^{\parallel} - h_{\mu\nu}^{\perp}\right)\frac{K}{2}\eta^{\mu\nu}(2\pi)^{p+1}\delta^{(p+1)}(0)$$

$$K = -g_s \frac{\mathcal{T}}{2\sqrt{\eta_c}}$$

$$g_o^2 = -\frac{2}{\mathcal{T}}\eta_c^2 = \frac{1}{2\pi^2\mathcal{T}} \rightarrow \mathcal{T}g_o^2 = \frac{1}{2\pi^2}$$

$$-\frac{g_s}{\pi}\int dy_R dy_I V_c(y)$$

$$g_o\int dx V_o(x)$$

$$-g_s\frac{\mathcal{T}}{2}\langle c_0^- c\bar{c}V_c\rangle'$$

$$-i\pi\mathcal{T}g_s g_o\langle c\bar{c}V_c cV_o\rangle'$$

$$\frac{i}{2}g_s^2\mathcal{T}\int dy\left\langle c\bar{c}V_c^{(1)}(i)(c+\bar{c})V_c^{(2)}(iy)\right\rangle'$$

$$w_1w_2 = q, |q| \leqslant 1$$

$$\mathcal{V}_{0,n}^{1\mathrm{PI}} = \mathcal{V}_{0,n}$$

$$\mathcal{V}_{1,1}^{1\mathrm{PI}} = \mathcal{F}_{1,1}$$

$$\mathcal{V}_{1\mathrm{PI}} \equiv \sum_{g,n}\mathcal{V}_{g,n}^{1\mathrm{PI}}$$

$$\partial\mathcal{V}_{1\mathrm{PI}} + \frac{1}{2}\{\mathcal{V}_{1\mathrm{PI}}, \mathcal{V}_{1\mathrm{PI}}\} = 0$$

$$\{A_1, \cdots, A_n\}_{1\mathrm{PI}} = \sum_{g=0}^{\infty} g_s^{2g+n-2}\int_{\mathcal{V}_{g,n}^{1\mathrm{PI}}}\Omega_{6g-6+2n}^{(g,n)}(A_1, \cdots, A_n)$$

$$S_{1\mathrm{PI}} = -\frac{1}{2}\langle\widetilde{\Psi}, Q\mathcal{G}\widetilde{\Psi}\rangle + \langle\widetilde{\Psi}, Q\Psi\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\{\Psi^n\}_{1\mathrm{PI}}$$

$$\delta|\widetilde{\Psi}\rangle = Q|\widetilde{\Lambda}\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}[\Lambda\Psi^n]_{1\mathrm{PI}}, \delta|\Psi\rangle = Q|\Lambda\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\mathcal{G}[\Lambda\Psi^n]_{1\mathrm{PI}}$$

$$\{S_{1\mathrm{PI}}, S_{1\mathrm{PI}}\} = 0$$

Acción Wilsoniana.

$$[P, L_0^{\pm}] = 0, [P, b_0^{\pm}] = 0, [P, c_0^{\pm}] = 0, [P, Q] = 0, [P, \mathcal{G}] = 0$$

$$\{A_1, \cdots, A_n\}_{\mathrm{eff}}, A_1, \cdots, A_n \in P\mathcal{H}_c.$$

$$\Psi \in P\mathcal{H}_c, \tilde{\Psi} \in P\tilde{\mathcal{H}}_c$$

$$S_{\mathrm{eff}} = -\frac{1}{2}\langle\widetilde{\Psi}, Q\mathcal{G}\widetilde{\Psi}\rangle + \langle\widetilde{\Psi}, Q\Psi\rangle + \sum_{n}\frac{1}{n!}\{\Psi^n\}_{\mathrm{eff}}$$

$$\mathcal{V}^{1\text{PI}} = \sum_{g,n} g_s^{2g-n+2} \mathcal{V}_{g,n}^{1\text{PI}}$$

$$\partial\mathcal{V}^{1\text{PI}} + \frac{1}{2}\{\mathcal{V}^{1\text{PI}}, \mathcal{V}^{1\text{PI}}\} = 0$$

$$S'_{1\text{PI}} - S_{1\text{PI}} = \sum_{g=0}^{\infty}\sum_{n=1}^{\infty} g_s^{2g-n+2}\left(\int_{\mathcal{V}'^{1PI}_{g,n}} - \int_{\mathcal{V}^{1\text{PI}}_{g,n}}\right)\Omega^{(g,n)}_{6g-6+2n}(\Psi^{\otimes n})$$

$$\langle\Phi|c_0^-|\delta\Psi\rangle = -\sum_{g=0}^{\infty}\sum_{n=1}^{\infty} g_s^{2g-n+2}\frac{1}{(n-1)!}\int_{\mathcal{V}^{11\text{I}}_{g,n}} \Omega^{(g,n)}_{6g-5+2n}(\Phi, \Psi^{\otimes(n-1)})[\hat{U}_{g,n}]$$

Teorema del dilatón.

$$D(z,\bar{z}) = \frac{1}{2}(c\partial^2 c - \bar{c}\partial^2\bar{c})$$

$$|D\rangle = (c_1 c_{-1} - \bar{c}_1\bar{c}_{-1})|0\rangle$$

$$b_0^-|D\rangle = 0, L_0^-|D\rangle = 0$$

$$|D\rangle = Qc_0^-|0\rangle = -Q|\chi\rangle, |\chi\rangle = -c_0^-|0\rangle$$

$$L_1|D\rangle = c_0 c_1|0\rangle, \bar{L}_1|D\rangle = -\bar{c}_0\bar{c}_1|0\rangle$$

$$\Omega_2(D) = \left(-\frac{1}{2\pi i}\right) du^1 \wedge du^2 \mathcal{B}\left[\frac{\partial}{\partial u^1}\right]\mathcal{B}\left[\frac{\partial}{\partial u^2}\right] D(w=0)$$

$$\Omega_2(D) = -d\Omega_1(\chi)$$

$$\Omega_1(\chi) = \left(-\frac{1}{2\pi i}\right)\left(du^1\mathcal{B}\left[\frac{\partial}{\partial u^1}\right] + du^2\mathcal{B}\left[\frac{\partial}{\partial u^2}\right]\right)\chi(w=0)$$

$$\int_M \Omega_2(D) + \int_{\partial M}\Omega_1(\chi)$$

$$\chi(M) = \frac{1}{2\pi}\int_M K^{(2)} + \frac{1}{2\pi}\int_{\partial M} k^{(1)}$$

$$K^{(2)} = -2i\partial\bar{\partial}\rho dz \wedge d\bar{z}$$

$$k^{(1)} = d\theta_\gamma - i[dz\partial\log\,\rho - d\bar{z}\bar{\partial}\log\,\rho]$$

$$\Omega_2(D) = -\frac{1}{2\pi}K^{(2)} \text{ and } \Omega_1(\chi) = -\frac{1}{2\pi}k^{(1)}$$

$$[Q, \mathcal{P}_b] = \frac{1}{2\pi}b_0^-\int_0^{2\pi} d\theta \int_0^{\infty} ds\frac{\partial}{\partial s}e^{-s(L_0+\bar{L}_0)}e^{i\theta(L_0-\bar{L}_0)}$$

$$z = F(w; u) = F(w; u^1, u^2)$$

$$y(u) = F(0; u)$$

$$z = F(w; u^1, u^2) = y(u) + a(u)w + \frac{1}{2}b(u)w^2 + \frac{1}{3!}c(u)w^2 + \mathcal{O}(w^4)$$

$$\begin{aligned}\mathcal{B}\left[\frac{\partial}{\partial u^i}\right] &= \oint b(z)dz\frac{\partial F}{\partial u^i} + \oint \bar{b}(\bar{z})d\bar{z}\frac{\partial \bar{F}}{\partial u^i} \\ &= -\oint b(w)dw\left(\frac{\partial F}{\partial w}\right)^{-1}\frac{\partial F}{\partial u^i} - \oint \bar{b}(\bar{w})d\bar{w}\left(\frac{\partial \bar{F}}{\partial \bar{w}}\right)^{-1}\frac{\partial \bar{F}}{\partial u^i}\end{aligned}$$

$$\left(\frac{\partial F}{\partial w}\right)^{-1}\frac{\partial F}{\partial u^i} = \alpha_i + \beta_i w + \gamma_i w^2 + \mathcal{O}(w^3)$$

$$\begin{aligned}\alpha_i &= \frac{1}{a}\frac{\partial y}{\partial u^i} \\ \beta_i &= \frac{1}{a}\frac{\partial a}{\partial u^i} - \frac{b}{a^2}\frac{\partial y}{\partial u^i} \\ \gamma_i &= a\frac{\partial}{\partial u^i}\left(\frac{b}{2a^2}\right) + \left(\frac{b^2}{a^3} - \frac{1}{2}\frac{c}{a^2}\right)\frac{\partial y}{\partial u^i}\end{aligned}$$

$$\mathcal{B}\left[\frac{\partial}{\partial u^i}\right] = -\left(\alpha_i b_{-1} + \beta_i b_0 + \gamma_i b_1 + \bar{\alpha}_i\bar{b}_{-1} + \bar{\beta}_i\bar{b}_0 + \bar{\gamma}_i\bar{b}_1 + \cdots\right)$$

$$\begin{aligned}\Omega_2(D) &= \left(-\frac{1}{2\pi i}\right)du^1 \wedge du^2\mathcal{B}\left[\frac{\partial}{\partial u^1}\right]\mathcal{B}\left[\frac{\partial}{\partial u^2}\right](c_1c_{-1} - \bar{c}_1\bar{c}_{-1})|0\rangle \\ &= \left(\frac{1}{2\pi i}\right)du^1 \wedge du^2[\alpha_1\gamma_2 - \alpha_2\gamma_1 - (\bar{\alpha}_1\bar{\gamma}_2 - \bar{\alpha}_2\bar{\gamma}_1)]|0\rangle\end{aligned}$$

$$\Omega_2(D) = \left(\frac{1}{2\pi i}\right)du^1 \wedge du^2\left[\frac{\partial y}{\partial u^1}\frac{\partial}{\partial u^2}\left(\frac{b}{2a^2}\right) - \frac{\partial y}{\partial u^2}\frac{\partial}{\partial u^1}\left(\frac{b}{2a^2}\right) - (\text{ c.c. })\right]|0\rangle$$

$$\Omega_2(D) = \frac{1}{2\pi i}\left[dy \wedge d\left(\frac{b}{2a^2}\right) - d\bar{y} \wedge d\left(\frac{\bar{b}}{2\bar{a}^2}\right)\right]|0\rangle$$

$$\Omega_2(D) = \frac{1}{2\pi i}dy \wedge d\bar{y}\left[\frac{\partial}{\partial \bar{y}}\left(\frac{b}{2a^2}\right) + \frac{\partial}{\partial y}\left(\frac{\bar{b}}{2\bar{a}^2}\right)\right]$$

$$\begin{aligned}\Omega_1(\chi) &= -\frac{1}{2\pi i}\left(-\frac{1}{2}du^1\mathcal{B}\left[\frac{\partial}{\partial u^1}\right] - \frac{1}{2}du^2\mathcal{B}\left[\frac{\partial}{\partial u^2}\right]\right)(c_0 - \bar{c}_0)|0\rangle \\ &= -\frac{1}{2\pi i}\left(\frac{1}{2}du^1(\beta_1 - \bar{\beta}_1) + \frac{1}{2}du^2(\beta_2 - \bar{\beta}_2)\right)|0\rangle\end{aligned}$$

$$\Omega_1(\chi) = \frac{1}{2\pi i}\left[-\frac{1}{2}d\left(\ln\frac{a}{\bar{a}}\right) + \frac{b}{2a^2}dy - \frac{\bar{b}}{2\bar{a}^2}d\bar{y}\right]$$

$$\rho^w(w) = \rho\left|\frac{dz}{dw}\right| = \left(\rho(y) + (z - y)\left.\frac{\partial\rho}{\partial z}\right|_y + (\bar{z} - \bar{y})\left.\frac{\partial\rho}{\partial\bar{z}}\right|_y + \cdots\right)|a + bw + \cdots|$$

$$\rho^w(w)=\rho\left|\frac{dz}{dw}\right|=\left(\rho(y)+aw\frac{\partial\rho}{\partial y}\bigg|_y+\bar{a}\bar{w}\frac{\partial\rho}{\partial\bar{y}}\bigg|_y+\cdots\right)|a|\left(1+\frac{b}{2a}w+\frac{\bar{b}}{2\bar{a}}\bar{w}+\cdots\right)$$

$$=\rho(y)|a|\left[1+aw\left(\frac{\partial}{\partial y}\log\,\rho+\frac{b}{2a^2}\right)+\bar{a}\bar{w}\left(\frac{\partial}{\partial\bar{y}}\log\,\rho+\frac{\bar{b}}{2\bar{a}^2}\right)+\cdots\right]$$

$$\frac{b}{2a^2}=-\frac{\partial}{\partial y}\log\,\rho,\frac{\bar{b}}{2\bar{a}^2}=-\frac{\partial}{\partial\bar{y}}\log\,\rho$$

$$\Omega_2(D)=\left(\frac{1}{2\pi i}\right)dy\wedge d\bar{y}\left[-2\frac{\partial}{\partial\bar{y}}\frac{\partial}{\partial y}\log\,\rho\right]$$

$$\Omega_1(\chi)=\frac{1}{2\pi i}(-i)\left[d\theta_a-idy\frac{\partial}{\partial y}\log\,\rho+id\bar{y}\frac{\partial}{\partial\bar{y}}\log\,\rho\right]$$

$$\Omega_2(D)=-\frac{1}{2\pi}K^{(2)}\ \text{ and }\ \Omega_1(\chi)=-\frac{1}{2\pi}k^{(1)}$$

$$(2-2g-n)\Omega^{(g,n)}_{6g-6+2n}(A_1,\cdots,A_n)$$

Estructuras algebraicas de campo – álgebra de Lie y álgebra cuántica.

$$\deg\big(b_n(A_1,\cdots,A_n)\big)=-1+\sum_{i=1}^{n}d_{A_i}$$

$$T(V)\equiv V\oplus(V\otimes V)\oplus(V\otimes V\otimes V)\oplus\cdots$$

$$\mathbf{b}=\sum_{i=1}^{n}\sum_{j=0}^{n-i}\mathbb{1}^{\otimes j}\otimes b_i\otimes\mathbb{1}^{n-i-j},\ \text{on}\ V^{\otimes n}$$

$$\mathbf{b}=\sum_{i=1}^{\infty}\mathbf{b}_i=\mathbf{b}_1+\mathbf{b}_2+\cdots$$

$$\mathbf{b}_i=\sum_{j=0}^{n-i}\mathbb{1}^{\otimes j}\otimes b_i\otimes\mathbb{1}^{n-i-j},\ \text{on}\ V^{\otimes n},\text{for } i\leqslant n$$

$$\mathbf{b}=b_1\otimes\mathbb{1}\otimes\mathbb{1}+\mathbb{1}\otimes b_1\otimes\mathbb{1}+\mathbb{1}\otimes\mathbb{1}\otimes b_1+b_2\otimes\mathbb{1}+\mathbb{1}\otimes b_2+b_3,\ \text{on}\ V^{\otimes 3},$$

$$\mathbf{b}(A\otimes B\otimes C)=b_1(A)\otimes B\otimes C+(-1)^{d_A}A\otimes b_1(B)\otimes C+(-1)^{d_A+d_B}A\otimes B\otimes b_1(C)$$

$$+b_2(A,B)\otimes C+(-1)^{d_A}A\otimes b_2(B,C)+b_3(A,B,C)$$

$$\mathbf{b}^2=0$$

$$\sum_{i=1}^{n}\mathbf{b}_i\mathbf{b}_{n+1-i}=0,\ \text{on}\ V^{\otimes n}$$

$$V:0=\mathbf{b}_1\mathbf{b}_1$$
$$V^{\otimes 2}:0=\mathbf{b}_1\mathbf{b}_2+\mathbf{b}_2\mathbf{b}_1$$
$$V^{\otimes 3}:0=\mathbf{b}_1\mathbf{b}_3+\mathbf{b}_2\mathbf{b}_2+\mathbf{b}_3\mathbf{b}_1$$

$$V:0=b_1\circ b_1$$

$$V^{\otimes 2}:0=b_1\circ b_2+b_2\circ(b_1\otimes\mathbb{1}+\mathbb{1}\otimes b_1)$$

$$\begin{aligned}V^{\otimes 3}:0=&b_1\circ b_3+b_2\circ(b_2\otimes\mathbb{1}+\mathbb{1}\otimes b_2),\\&+b_3(b_1\otimes\mathbb{1}\otimes\mathbb{1}+\mathbb{1}\otimes b_1\otimes\mathbb{1}+\mathbb{1}\otimes\mathbb{1}\otimes b_1)\end{aligned}$$

$$0=Q^2A$$

$$0=Q(AB)+(QA)B+(-1)^{d_A}A(QB)$$

$$0=Q(A,B,C)+(AB)C+(-1)^{d_A}A(BC)+(QA,B,C)+(-1)^{d_A}(A,QB,C)+(-1)^{d_A+d_B}(A,B,QC)$$

$$(A,B)=-(-1)^{d_Ad_B}(B,A)$$

$$(QA,B)=-(-1)^{d_A}(A,QB)$$

$$\big(A_1,b_n(A_2,\cdots,A_{n+1})\big)=(-1)^{\#}\big(A_2,b_n(A_3,\cdots,A_{n+1},A_1)\big)=d_{A_2}+d_{A_1}\big(1+d_{A_2}+d_{A_3}+\cdots+d_{A_{n+1}}\big)$$

$$S(\Phi)=\sum_{n=1}^{\infty}\frac{1}{n+1}(\Phi,b_n(\Phi,\cdots,\Phi))$$

$$(A,B)=(-1)^{A+1}\langle A,B\rangle'$$

$$A=d_A+1(\mathrm{mod}2)$$

$$\langle B,A\rangle'=(-1)^{AB}\langle A,B\rangle'$$

$$(B,A)=(-1)^{B+1}\langle B,A\rangle'=(-1)^{AB+B+1}\langle A,B\rangle'=(-1)^{AB+A+B}(A,B)=(-1)^{d_Ad_B+1}(A,B).$$

$$A\star B=(-1)^{A+1}b_2(A,B)=(-1)^{A+1}(AB)$$

$$Q(A\star B)=(-1)^{A+1}Q(AB)=(-1)^{A}((QA)B)-(A(QB))=(QA)\star B+(-1)^{A}A\star QB.$$

$$[A_1\cdots A_n]=(-1)^{n(n+1)/2}\sum_{\sigma}(-1)^{s_\sigma}(-1)^{A_{\sigma(1)}+2A_{\sigma(2)}+\cdots+nA_{\sigma(n)}}b_n\big(A_{\sigma(1)},A_{\sigma(2)},\cdots A_{\sigma(n)}\big)$$

$$[A_1,A_2]=A_1\star A_2-(-1)^{A_1A_2}A_2\star A_1$$

$$[A_1\cdots A_n]=\sum_{\sigma}b_n\big(A_{\sigma(1)},A_{\sigma(2)},\cdots A_{\sigma(n)}\big)$$

$$b_1(B_1),b_2(B_1,B_2),b_3(B_1,B_2,B_3),\cdots$$

$$b_1(B_1)=QB_1,b_1(B_1,B_2)=[B_1,B_2]_0,b_3(B_1,B_2,B_3)=[B_1,B_2,B_3]_0,\cdots,$$

$$b_n(B_1, \cdots, B_k, B_{k+1}, \cdots, B_n) = (-1)^{d_{B_k} d_{B_{k+1}}} b_n(B_1, \cdots, B_{k+1}, B_k, \cdots, B_n)$$

$$\deg\big(b(B_1, \cdots, B_n)\big) = -1 + \sum_{i=1}^{n} d_{B_i}$$

$$T(W) = \sum_{n=1}^{\infty} SW^{\otimes n}$$

$$B_1 \wedge B_2 \wedge \cdots \wedge B_n$$

$$B_1 \wedge \cdots \wedge B_n = \epsilon(\sigma; B) B_{\sigma(1)} \wedge \cdots \wedge B_{\sigma(n)}$$

$$\mathbf{b}(B_1 \wedge \cdots \wedge B_n) = \sum_{l=1}^{n} \sum_{\sigma'} \epsilon(\sigma', B)\big(b_l\big(B_{i_1}, \cdots, B_{i_l}\big) \wedge B_{j_1} \wedge \cdots \wedge B_{j_{n-l}}\big),$$

$$\mathbf{b}(B_1) = b_1(B_1)$$

$$\mathbf{b}(B_1 \wedge B_2) = b_1(B_1) \wedge B_2 + (-1)^{d_{B_1} d_{B_2}} b_1(B_2) \wedge B_1 + b_1(B_1, B_2)$$

$$= b_1(B_1) \wedge B_2 + (-1)^{d_{B_1} B_1 \wedge b_1(B_2) + b_1(B_1, B_2)}$$

$$\mathbf{b}^2 = 0$$

$$\mathbf{b}^2(B_1) = \mathbf{bb}(B_1) = \mathbf{b}(QB_1) = Q(QB_1) = 0$$

$$\mathbf{b}^2(B_1 \wedge B_2) = \mathbf{b}(QB_1 \wedge B_2) + (-1)^{d_{B_1}} \mathbf{b}(B_1 \wedge QB_2) + \mathbf{b}\big(b_2(B_1, B_2)\big) = 0$$

$$\mathbf{b}^2(B_1 \wedge B_2) = \big((-1)^{d_{B_1}+1} QB_1 \wedge QB_2 + b_2(QB_1, B_2)\big)$$
$$+(-1)^{d_{B_1}}\big(QB_1 \wedge QB_2 + b_2(B_1, QB_2)\big) + Qb_2(B_1, B_2) = 0$$

$$Qb_2(B_1, B_2) + b_2(QB_1, B_2) + (-1)^{d_{B_1}} b_2(B_1, QB_2) = 0$$

$$0 = Qb_3(B_1, B_2, B_3) + b_3(QB_1, B_2, B_3) + (-1)^{d_{B_1}} b_3(B_1, QB_2, B_3) + (-1)^{d_{B_1}+d_{B_2}} b_3(B_1, B_2, QB_3)$$

$$+ b_2(b_2(B_1, B_2), B_3) + (-1)^{d_{B_2} d_{B_3}} b_2(b_2(B_1, B_3), B_2)$$

$$+ (-1)^{d_{B_1}(d_{B_2}+d_{B_3})} b_2(b_2(B_2, B_3), B_1)$$

$$0 = \sum_{l=1}^{n} \sum_{\sigma'} \epsilon(\sigma', B) b_{n-l+1}\big(b_l\big(B_{i_1}, \cdots, B_{i_l}\big), B_{j_1}, \cdots, B_{j_{n-l}}\big).$$

$$(B_1, B_2) = (-1)^{(d_{B_1}+1)(d_{B_2}+1)}(B_2, B_1),$$

$$(QB_1, B_2) = (-1)^{d_{B_1}}(B_1, QB_2)$$

$$\{B_1, \cdots, B_n\}_{L_\infty} \equiv \big(B_1, b_{n-1}(B_2, \cdots B_n)\big)$$

$$\{B_1, \cdots, B_k, B_{k+1}, \cdots, B_n\}_{L_\infty} = (-1)^{d_{B_k} d_{B_{k+1}}} \{B_1, \cdots, B_{k+1}, B_k, \cdots, B_n\}_{L_\infty}$$

$$S=\frac{1}{2}(\Psi,Q\Psi)+\sum_{n=3}^{\infty}\frac{1}{n!}\{\Psi^n\}_{L_\infty}$$

$$\bar{b}_1(A_1)\equiv b_1(A_1)$$

$$\bar{b}_2(A_1,A_2)\equiv b_2(A_1,A_2)+(-1)^{d_{A_1}d_{A_2}}b_2(A_2,A_1)$$

$$\bar{b}_1\bar{b}_2(A_1,A_2)+\bar{b}_2(\bar{b}_1(A_1),A_2)+(-1)^{d_{A_1}}\bar{b}_2\left(A_1,\bar{b}_1(A_2)\right)=0$$

$$b_2(b_2(A_1,A_2),A_3)+(-1)^{d_{A_1}}b_2(A_1,b_2(A_2,A_3))=0$$

$$\bar{b}_2(\bar{b}_2(A_1,A_2),A_3)+(-1)^{d_{A_2}d_{A_3}}\bar{b}_2(\bar{b}_2(A_1,A_3),A_2)+(-1)^{d_{A_1}(d_{A_2}+d_{A_3})}\bar{b}_2(\bar{b}_2(A_2,A_3),A_1)=0$$

$$\bar{b}_n(A_1,\cdots,A_n)\equiv\sum_{\sigma}\epsilon(\sigma,A)b_n(A_{\sigma(1)},\cdots,A_{\sigma(n)})$$

$$[A_1\cdots A_n]=(-1)^{d_{A_1}+2d_{A_2}+\cdots+nd_{A_n}}\bar{b}_n(A_1,\cdots,A_n)$$

$$(A_1,A_2)=\langle A_1,A_2\rangle'=(-1)^{d_{A_1}}(A_1,A_2).$$

$$\{A_0A_1\cdots A_n\}_{L_\infty}=\Big(A_0,\bar{b}_n(A_1A_2\cdots A_n)+=(-1)^{d_{A_0}}\left(A_0,\bar{b}_n(A_1A_2\cdots A_n)\right)$$

$$=(-1)^{d_{A_0}}\sum_{\sigma}\epsilon(\sigma,A)\left(A_0,b_n(A_{\sigma(1)},\cdots,A_{\sigma(n)})\right)$$

$$S(\psi_o)=\sum_{n=1}^{\infty}\frac{1}{n+1}(\psi_o,b_n(\psi_o^n))=\frac{1}{2}f\psi_o,Q\psi_o t+\sum_{n=2}^{\infty}\frac{1}{(n+1)!}\{\psi_o^{n+1}\}_{L_\infty}$$

$$[A_1\cdots A_n]=\bar{b}_n(A_1,\cdots,A_n)$$

$$b_n(B_1,\cdots,B_n)=\sum_{g=0}^{\infty}g_s^{2g+n-1}b_n^g(B_1,\cdots,B_n),n=0,1,\cdots$$

$$0=\sum_{l=1}^{n}\sum_{\sigma'}\epsilon(\sigma',B)b_{n-l+1}(b_l(B_{i_1},\cdots,B_{i_l}),B_{j_1},\cdots,B_{j_{n-l}})$$
$$+\frac{1}{2}b_{n+2}(B_1,\cdots,B_n,\varphi_s,\varphi_r)(\varphi_s^c,\varphi_r^c)$$

$$\boldsymbol{\theta}(A_1\wedge\cdots\wedge A_n)=\frac{1}{2}A_1\wedge\cdots\wedge A_n\wedge\varphi_r\wedge\varphi_s(\varphi_r^c,\varphi_s^c)$$

$$\boldsymbol{\pi}_1(\mathbf{b}^2+\mathbf{b}\boldsymbol{\theta})=0$$

$$(\mathbf{b}+\boldsymbol{\theta})^2=0$$

$$\{A_1^c,\cdots,A_{n_c}^c;A_1^o,\cdots,A_{n_o}^o\}_{L_\infty}\equiv(-1)^{d_{A_2^o}+2d_{A_3^o}+\cdots+(n-1)d_{A_n^o}}\{A_1^c,\cdots,A_{n_c}^c;A_1^o,\cdots,A_{n_o}^o\}$$

Transferencia homotópica.

$$\mathbf{M} = \mathbf{Q} + \mathbf{m}, \mathbf{m} = \sum_{n=2}^{\infty} \mathbf{b}_n$$

$$\mathbf{Q}^2 = 0, \text{ and } \mathbf{Qm} + \mathbf{mQ} + \mathbf{m}^2 = 0$$

$$P: V \to \bar{V}, P^2 = P$$

$$(A_1, PA_2) = (PA_1, A_2)$$

$$QP = PQ$$

$$\mathbf{P}(A_1 \otimes A_2 \otimes \cdots \otimes A_n) \equiv PA_1 \otimes PA_2 \otimes \cdots \otimes PA_n$$

$$\mathbf{P}^2 = \mathbf{P}$$

$$\mathbf{QP} = \mathbf{PQ}$$

$$hP = Ph = 0, h^2 = 0, Qh + hQ = 1 - P.$$

$$h(A_1 \otimes A_2 \otimes \cdots \otimes A_n) \equiv (hA_1) \otimes PA_2 \otimes \cdots \otimes PA_n + (-1)^{d_{A_1}} A_1 \otimes hA_2 \otimes \cdots \otimes PA_n \vdots$$

$$\vdots + (-1)^{d_{A_1} + d_{A_2} + \cdots + d_{A_{n-1}}} A_1 \otimes A_2 \otimes \cdots \otimes hA_n$$

$$\mathbf{hP} = \mathbf{Ph} = 0, \mathbf{h}^2 = 0, \mathbf{Qh} + \mathbf{hQ} = \mathbf{1} - \mathbf{P}$$

$$\overline{\mathbf{M}} = \mathbf{PQP} + \mathbf{Pm} \frac{\mathbf{1}}{\mathbf{1} + \mathbf{h\ m}} \mathbf{P}$$

$$\frac{1}{1 + \mathrm{h\ m}} = 1 - \mathrm{h\ m} + \mathrm{hmh\ m} - \cdots$$

$$\overline{\mathbf{M}}^2 = \mathbf{P}\left\{\mathbf{Q}, \mathbf{m} \frac{1}{1 + \mathbf{hm}}\right\} \mathbf{P} + \mathbf{Pm} \frac{1}{1 + \mathbf{hm}} \mathbf{Pm} \frac{1}{1 + \mathbf{hm}} \mathbf{P}$$

$$\begin{aligned} \mathbf{P}\left\{\mathbf{Q}, \mathbf{m} \frac{1}{1 + \mathbf{hm}}\right\} \mathbf{P} &= \mathbf{P}\{\mathbf{Q}, \mathbf{m}\} \frac{1}{1 + \mathbf{hm}} \mathbf{P} + \mathbf{Pm} \frac{1}{1 + \mathbf{hm}} [\mathbf{Q}, \mathbf{1} + \mathbf{hm}] \frac{1}{1 + \mathbf{hm}} \mathbf{P} \\ &= -\mathbf{Pm}^2 \frac{1}{1 + \mathbf{hm}} \mathbf{P} + \mathbf{Pm} \frac{1}{1 + \mathbf{hm}} ((1 - \mathbf{P})\mathbf{m} + \mathbf{h}^2) \frac{1}{1 + \mathbf{hm}} \end{aligned}$$

$$\begin{aligned} \overline{\mathbf{Q}} &= \mathbf{PQP} \\ \overline{\mathbf{m}}_2 &= \mathbf{Pm_2P} \\ \overline{\mathbf{m}}_3 &= \mathbf{P}[\mathbf{m_3} - \mathbf{m_2hm_2}]\mathbf{P}, \\ \overline{\mathbf{m}}_4 &= \mathbf{P}[\mathbf{m_4} - \mathbf{m_2hm_3} - \mathbf{m_3hm_2} + \mathbf{m_2hm_2hm_2}]\mathbf{P}. \end{aligned}$$

$$\begin{aligned} \bar{Q}A_1 &= PQ\bar{A}_1 \\ \bar{m}_2(A_1 \otimes A_2) &= P(\bar{A}_1 \star \bar{A}_2) \\ \bar{m}_3(A_1 \otimes A_2 \otimes A_3) &= Pm_3(\bar{A}_1, \bar{A}_2, \bar{A}_3) - P\big(h(\bar{A}_1 \star \bar{A}_2) \star \bar{A}_3 + \bar{A}_1 \star h(\bar{A}_2 \star \bar{A}_3)\big). \end{aligned}$$

$$\mathbf{L} = \mathbf{Q} + \boldsymbol{\ell}, \boldsymbol{\ell} = \sum_{n=2}^{\infty} \mathbf{b}_n$$

$$\mathbf{Q}^2 = 0, \text{ and } \mathbf{Q}\boldsymbol{\ell} + \boldsymbol{\ell}\mathbf{Q} + \boldsymbol{\ell}^2 = 0$$

$$P^2 = P, PQ = QP, (B_1, PB_2) = (PB_1, B_2)$$

$$\mathbf{P}(B_1 \wedge B_2 \wedge \cdots \wedge B_n) \equiv PB_1 \wedge PB_2 \wedge \cdots \wedge PB_n$$

$$hP = Ph = 0, h^2 = 0, Qh + hQ = 1 - P.$$

$$h(A_1 \wedge \ldots \wedge A_n) \equiv \frac{1}{n!} \sum_{\sigma \in S_n} \epsilon(\sigma; A)\Big(h\big(A_{\sigma(1)} \wedge \ldots \wedge A_{\sigma(n-1)}\big) \wedge A_{\sigma(n)}$$
$$+(-1)^{A_{\sigma(1)}+\cdots+A_{\sigma(n-1)}} \bar{A}_{\sigma(1)} \wedge \cdots \wedge \bar{A}_{\sigma(n-1)} \wedge hA_{\sigma(n)}\Big)$$

$$h(A_1 \wedge A_2) = \frac{1}{2}\Big(hA_1 \wedge A_2 + hA_1 \wedge \bar{A}_2 + (-1)^{A_1}(\bar{A}_1 \wedge hA_2 + A_1 \wedge hA_2)\Big)$$

$$\mathbf{hP} = \mathbf{Ph} = 0, \mathbf{h}^2 = 0, \mathbf{Qh} + \mathbf{hQ} = \mathbf{1} - \mathbf{P}$$

$$\overline{\mathbf{L}} = \mathbf{PQP} + \mathbf{P}\ell \frac{1}{1 + \mathbf{h}\ell}\mathbf{P}$$

$$\bar{Q}A_1 = Q\bar{A}_1 \bar{\ell}_2(A_1 \wedge A_2) = P[\bar{A}_1, \bar{A}_2]\bar{\ell}_3(A_1 \wedge A_2 \wedge A_3)$$
$$= P\ell_3(\bar{A}_1 \wedge \bar{A}_2 \wedge \bar{A}_3)$$
$$- P\big([h[\bar{A}_1, \bar{A}_2], \bar{A}_3] + (-1)^{A_2 A_3}[h[\bar{A}_1, \bar{A}_3], \bar{A}_2] + \big[\bar{A}_1, h[\bar{A}_2, \bar{A}_3]\big]\big)$$

Vórtices – espacios de Moduli.

$$\Delta(\Delta X) = \Delta^2 X = 0, \forall X \in \mathcal{C}$$

$$\Delta(XYZ) = \Delta(XY)Z + (-1)^X X\Delta(YZ) + (-1)^{XY+Y} Y\Delta(XZ)$$
$$-\Delta X(YZ) - (-1)^X X(\Delta Y)Z - (-1)^{X+Y} XY\Delta Z$$

$$\{X, Y\} \equiv (-1)^X \Delta(XY) - (-1)^X (\Delta X)Y - X\Delta Y$$

$$\{X, Y\} = -(-1)^{(X+1)(Y+1)}\{Y, X\}$$
$$0 = (-1)^{(X+1)(Z+1)}\{\{X, Y\}, Z\} + \mathfrak{I}_{\text{cyclic}}$$
$$\{X, YZ\} = \{X, Y\}Z + (-1)^{XY+Y} Y\{X, Z\}$$

$$\mathcal{A}_{g_1,n_1} \times \ldots \times \mathcal{A}_{g_r,n_r}$$

$$\deg\big(\mathcal{A}_{g,n}\big) = \dim\big(\mathcal{A}_{g,n}\big)(\text{mod}2)$$

$$\mathcal{A}_{g_1,n_1} \times \mathcal{A}_{g_2,n_2} = (-1)^{\mathcal{A}_{g_1,n_1} \cdot \mathcal{A}_{g_2,n_2}} \mathcal{A}_{g_2,n_2} \times \mathcal{A}_{g_1,n_1}$$

$$\big[\big[\mathcal{A}_{g_1,n_1}, \ldots, \mathcal{A}_{g_r,n_r}\big]\big] \equiv \frac{1}{n_1! \cdots n_r!} \sum_{\sigma \in S_N} \mathbf{P}_\sigma\big(\mathcal{A}_{g_1,n_1} \times \ldots \times \mathcal{A}_{g_r,n_r}\big) \in \mathcal{C}$$

$$\left[\left[\mathcal{A}_{g_1,n_1},\dots,\mathcal{A}_{g_r,n_r}\right]\right]\cdot\left[\left[\mathcal{B}_{g_1',n_1'},\dots,\mathcal{B}_{g_p',n_p'}\right]\right]=\left[\left[\mathcal{A}_{g_1,n_1},\dots,\mathcal{A}_{g_r,n_r},\mathcal{B}_{g_1',n_1'},\dots,\mathcal{B}_{g_p',n_p'}\right]\right]$$

$$\Delta X\equiv\Delta_{ij}X\equiv\frac{1}{n_1!\cdots n_r!}\sum_{\sigma\in S_N}\Delta_{ij}\mathbf{P}_\sigma\left(\mathcal{A}_{n_1,g_1}\times\cdots\times\mathcal{A}_{g_r,n_r}\right)$$

$$\Delta_{12}(\Delta_{12}\Sigma_X)\left[\frac{\partial}{\partial\theta_{34}},\frac{\partial}{\partial\theta_{12}},\{X\}\right]$$

$$\Delta_{12}(\Delta_{34}\Sigma_X)\left[\frac{\partial}{\partial\theta_{12}},\frac{\partial}{\partial\theta_{34}},\{X\}\right]$$

$$\{X,Y\}\equiv(-1)^X\Delta(XY)-(-1)^X(\Delta X)Y-X\Delta Y$$

$$(-1)^X\Delta(XY)=S_{XX}+S_{YY}+S_{XY}$$

$$(-1)^X(\Delta X)Y=S_{XX},X\Delta Y=S_{YY}$$

$$\partial(XY)=(\partial X)Y+(-1)^X X\partial Y$$

$$\Delta\partial X=-\partial\Delta X$$

$$\partial\{X,Y\}=\{\partial X,Y\}+(-1)^{X+1}\{X,\partial Y\}$$

Estructuras de campo en CFT.

$$\left\langle\Omega_k^{(g,n)}\right|\in(\mathcal{H}^*)^{\otimes n}$$

$$\left\langle\Omega_k^{(g,n)}\mid A_1\right\rangle\otimes\cdots\otimes|A_n\rangle=\Omega_k^{(g,n)}(A_1,\cdots,A_n),$$

$$\int_{\mathcal{A}_{g,n}}\left\langle\Omega_k^{(g,n}\right|\sum_{i=1}^n Q^{(i)}=(-1)^k\int_{\partial\mathcal{A}_{g,n}}\left\langle\Omega_{k-1}^{(g,n)}\right|$$

$$f\left(\mathcal{A}_{g,n}^{(k)}\right)\equiv\frac{1}{n!}\int_{\mathcal{A}_{g,n}^{(k)}}\left\langle\Omega_k^{(g,n)}\mid\Psi\right\rangle_1\cdots|\Psi\rangle_n$$

$$f\left(\left[\left[\mathcal{A}_{g_1,n_1}^{(k_1)}\cdots\mathcal{A}_{g_r,n_r}^{(k_r)}\right]\right]\right)\equiv\prod_{i=1}^r\frac{1}{n_i!}\int_{\mathcal{A}_{g_i,n_i}^{(k_i)}}\left\langle\Omega_{k_i}^{(g_i,n_i)}\mid\Psi\right\rangle_1\cdots|\Psi\rangle_{n_i}$$

$$f(XY)=f(X)f(Y),X,Y\in\mathcal{C}$$

$${}_1\langle A|\,{}_2\langle B\mid R_{12}\rangle=\langle A\mid B\rangle,\langle R_{12}\mid A\rangle_1|B\rangle_2=\langle A\mid B\rangle,$$

$$\langle\omega_{12}|=\langle R_{12}|c_0^{-(2)}$$
$$|S_{12}\rangle=b_0^{-(1)}|R_{12}\rangle$$

$$\{F,G\}=(-1)^{G+1}\frac{\partial F}{\partial|\Psi\rangle}\frac{\partial G}{\partial|\Psi\rangle}|S\rangle$$

$$S_{0,2} = \frac{1}{2}\langle\omega_{12}|Q^{(2)}|\Psi\rangle_1|\Psi\rangle_2$$

$$\Delta F = \frac{1}{2}(-1)^{F+1}\left(\frac{\partial}{\partial|\Psi\rangle}\frac{\partial}{\partial|\Psi\rangle}F\right)|S\rangle$$

$$f(\Delta X) = -\Delta f(X)$$

$$f(\{X,Y\}) = -\{f(X), f(Y)\},$$

$$\{S_{0,2}, f(X)\} = -f(\partial X)$$

$$\mathcal{V} = \sum_{g,n} g_s^{2g+n-2}\mathcal{V}_{g,n}, \text{ with } \begin{cases} n \geqslant 3, \text{ for } g = 0 \\ n \geqslant 1, \text{ for } g = 1 \\ n \geqslant 0, \text{ for } g \geqslant 2 \end{cases}$$

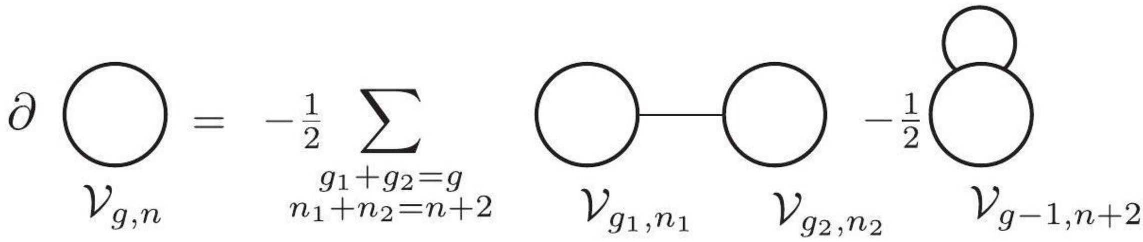
$$\partial\mathcal{V} + \Delta\mathcal{V} + \frac{1}{2}\{\mathcal{V},\mathcal{V}\} = 0$$

$$S = S_{0,2} + f(\mathcal{V})$$

$$\{S_{0,2}, S_{0,2}\} = 0, \Delta S_{0,2} = 0$$

$$0 = \frac{1}{2}\{S,S\} + \Delta S = \{S_{0,2}, f(\mathcal{V})\} + \frac{1}{2}\{f(\mathcal{V}), f(\mathcal{V})\} + \Delta f(\mathcal{V})$$

$$= -f(\partial\mathcal{V}) - \frac{1}{2}f(\{\mathcal{V},\mathcal{V}\}) - f(\Delta\mathcal{V}) - f\left(\partial\mathcal{V} + \frac{1}{2}\{\mathcal{V},\mathcal{V}\} + \Delta\mathcal{V}\right) = 0$$



$$\partial\mathcal{V}_{g,n} = -\Delta\mathcal{V}_{g-1,n+2} - \frac{1}{2}\sum_{\substack{g_1+g_2=g \\ n_1+n_2=n+2}}\{\mathcal{V}_{g_1,n_1}, \mathcal{V}_{g_2,n_2}\} \equiv \mathcal{O}_{g,n}$$

$$\partial\mathcal{V}_{g,n} = -\partial_p R_1,$$

Compactificación de Deligne-Mumford, nodos de superficie y diagramas de Feynman. Transformaciones canonicales.

$$\mathcal{F}_{g,n} = \mathcal{V}_{g,n} \oplus R_1 \oplus \cdots R_{3g-3+n}$$

$$\partial \mathcal{V}_{0,4} = -\frac{1}{2}\{\mathcal{V}_{0,3}, \mathcal{V}_{0,3}\}, \text{ and } \partial \mathcal{V}_{1,1} = -\Delta \mathcal{V}_{0,3}.$$

$$\partial \mathcal{V}_{g,n} = \mathcal{O}_{g,n}$$

$$[\mathcal{O}_{g,n}] \in H_{6g-6+2n-1}(\hat{\mathcal{P}}_{g,n})^{S_n}$$

$$\delta S = \Delta \epsilon + \{S, \epsilon\}$$

$$\delta_{\mathcal{W}} \mathcal{V} = \{\mathcal{V}, \mathcal{W}\} + \Delta \mathcal{W} + \partial \mathcal{W},$$

$$\mathcal{V} + \delta_{\mathcal{W}} \mathcal{V}$$

$$\frac{1}{2}\{\mathcal{V} + \delta_{\mathcal{W}}\mathcal{V}, \mathcal{V} + \delta_{\mathcal{W}}\mathcal{V}\} + \Delta(\mathcal{V} + \delta_{\mathcal{W}}\mathcal{V}) + \partial(\mathcal{V} + \delta_{\mathcal{W}}\mathcal{V})$$
$$= \left\{\partial \mathcal{V} + \Delta \mathcal{V} + \frac{1}{2}\{\mathcal{V}, \mathcal{V}\}, \mathcal{W}\right\} + O((\delta_{\mathcal{W}}\mathcal{V})^2) = 0$$

$$S_{\mathcal{V}+\delta_{\mathcal{W}}\mathcal{V}} = S_{\mathcal{V}} + f(\delta_{\mathcal{W}}\mathcal{V}) = S_{\mathcal{V}} + f(\{\mathcal{V}, \mathcal{W}\} + \Delta \mathcal{W} + \partial \mathcal{W})$$
$$= S_{\mathcal{V}} - \{f(\mathcal{V}), f(\mathcal{W})\} - \Delta f(\mathcal{W}) - \{S_{0,2}, \mathcal{W}\} = S_{\mathcal{V}} - \{S_{\mathcal{V}}, f(\mathcal{W})\} - \Delta f(\mathcal{W})$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{V}(t) = \delta_{\mathcal{W}}\mathcal{V}(t)$$

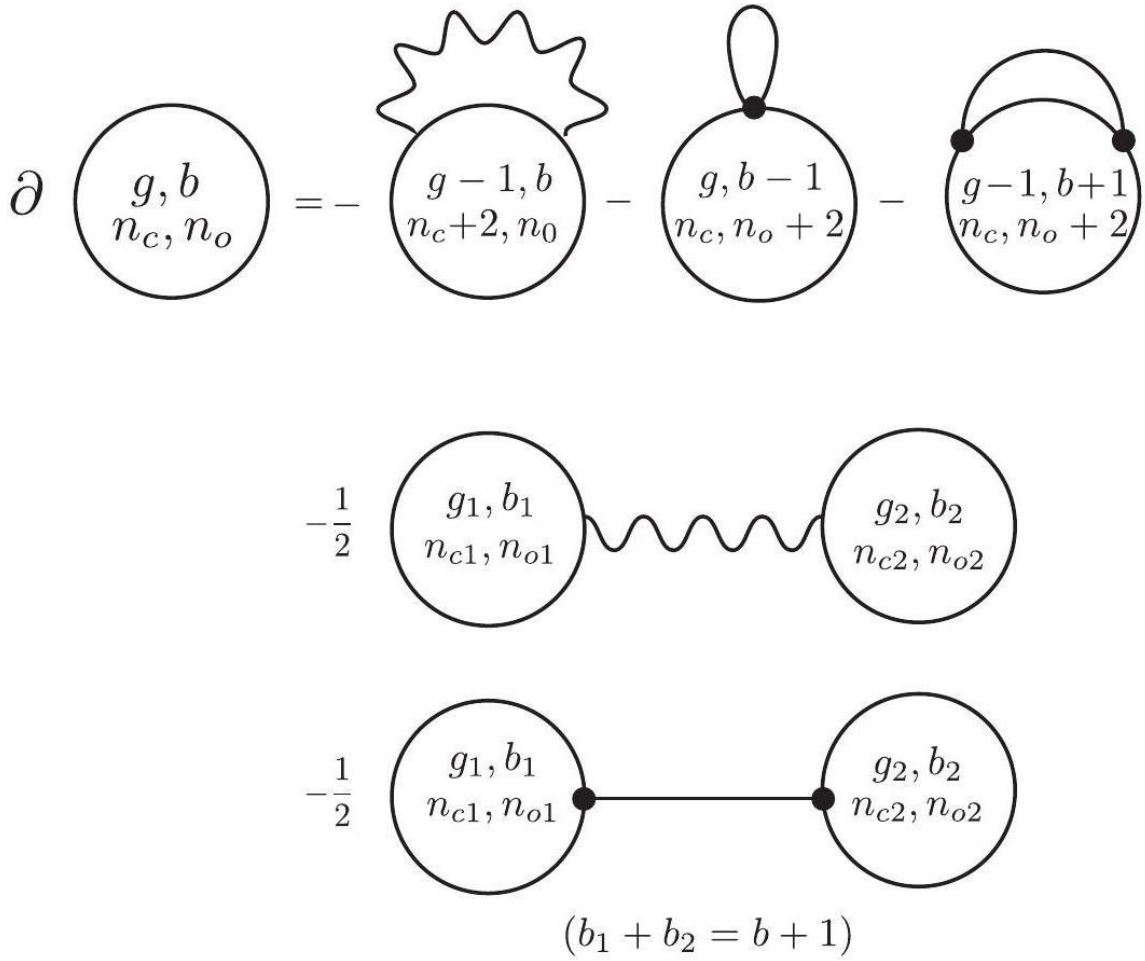
$$M_{\mathcal{V}}(t) \equiv \partial \mathcal{V}(t) + \Delta \mathcal{V}(t) + \frac{1}{2}\{\mathcal{V}(t), \mathcal{V}(t)\}$$

$$\frac{\mathrm{d}M_{\mathcal{V}}}{\mathrm{d}t} = \{M_{\mathcal{V}}(t), \mathcal{W}\}$$

$$\mathcal{V}(t) = \mathcal{V} + t\delta_{\mathcal{W}}\mathcal{V} + \frac{1}{2}t^2\{\delta_{\mathcal{W}}\mathcal{V}, \mathcal{W}\} + \frac{1}{3!}t^3\{\{\delta_{\mathcal{W}}\mathcal{V}, \mathcal{W}\}, \mathcal{W}\} + \cdots$$

$$\exp(\delta_{\mathcal{W}})\mathcal{V} \equiv \mathcal{V} + \delta_{\mathcal{W}}\mathcal{V} + \frac{1}{2}\{\delta_{\mathcal{W}}\mathcal{V}, \mathcal{W}\} + \frac{1}{3!}\{\{\delta_{\mathcal{W}}\mathcal{V}, \mathcal{W}\}, \mathcal{W}\} + \cdots$$

$$\exp(\delta_{\mathcal{W}})\mathcal{V} = \mathcal{V}'$$



$$\mathcal{V} = \sum_{g,b,n_c,n_o} (g_s)^{-\chi_{g,b,n_c,n_o}} \mathcal{V}_{g,b,n_c,n_o},$$

$$\partial\mathcal{V} + \Delta\mathcal{V} + \frac{1}{2}\{\mathcal{V},\mathcal{V}\} = 0$$

$$\partial\mathcal{V}_c + \Delta\mathcal{V}_c + \frac{1}{2}\{\mathcal{V}_c,\mathcal{V}_c\}_c = 0$$

$$\partial\hat{\mathcal{V}}_c + \frac{1}{2}\left\{\hat{\mathcal{V}}_c,\hat{\mathcal{V}}_c\right\}_c = 0$$

$$\partial\mathcal{V}_o + \frac{1}{2}\{\mathcal{V}_o,\mathcal{V}_o\}_o = 0$$

$$\partial\mathcal{V}_1 + \{\mathcal{V}_o,\mathcal{V}_1\}_o = 0$$

$$\partial\mathcal{V}_2 + \{\mathcal{V}_o,\mathcal{V}_2\}_o + \frac{1}{2}\{\mathcal{V}_1,\mathcal{V}_1\}_o + \left\{\mathcal{V}_1,\mathcal{V}_{0,3}\right\}_c = 0$$

$$\overline{\mathcal{V}} \equiv \mathcal{V}_o + \sum_{n=1}^{\infty} \mathcal{V}_n, \mathcal{V}_n \equiv \sum_{n_o=0}^{\infty} (g_s)^{n+\frac{1}{2}n_o-1} \mathcal{V}_{0,1,n,n_o}$$

$$\partial\overline{\mathcal{V}} + \frac{1}{2}\{\overline{\mathcal{V}}, \overline{\mathcal{V}}\}_o + \{\overline{\mathcal{V}}, \hat{\mathcal{V}}_c\}_c = 0$$

$$\tilde{\mathcal{V}} = \sum_{n_c,b,n_o} (g_s)^{n_c+b+\frac{1}{2}n_o-2} \mathcal{V}_{0,b,n_c,n_o}$$

$$\partial\tilde{\mathcal{V}} + \frac{1}{2}\{\tilde{\mathcal{V}}, \tilde{\mathcal{V}}\} + \Delta_o'\tilde{\mathcal{V}} = 0$$

Vórtice de Witten.

$$\begin{aligned} w_1 w_2 &= -1, \quad \text{for } |w_1| = 1, \mathrm{Re} w_1 \leqslant 0 \\ w_2 w_3 &= -1, \quad \text{for } |w_2| = 1, \mathrm{Re} w_2 \leqslant 0 \\ w_3 w_1 &= -1, \quad \text{for } |w_3| = 1, \mathrm{Re} w_3 \leqslant 0 \end{aligned}$$

$$\varphi = \phi(w_i) dw_i^2 = -\frac{1}{w_i^2} dw_i^2$$

$$ds^2 = |\phi(w_i)||dw_i|^2$$

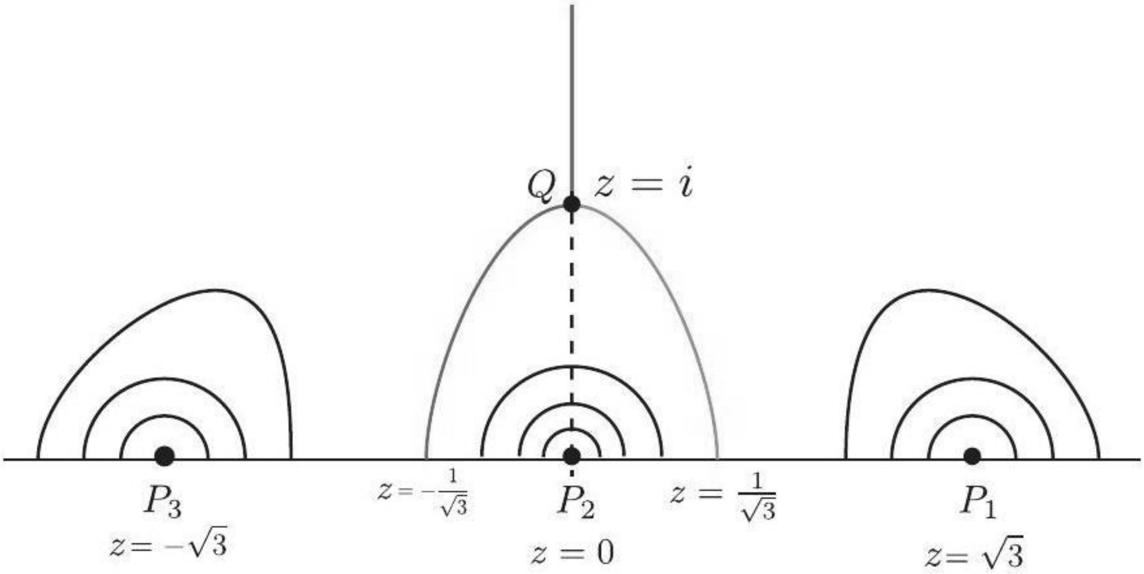
$$h(u) = \frac{1 + iu}{1 - iu}$$

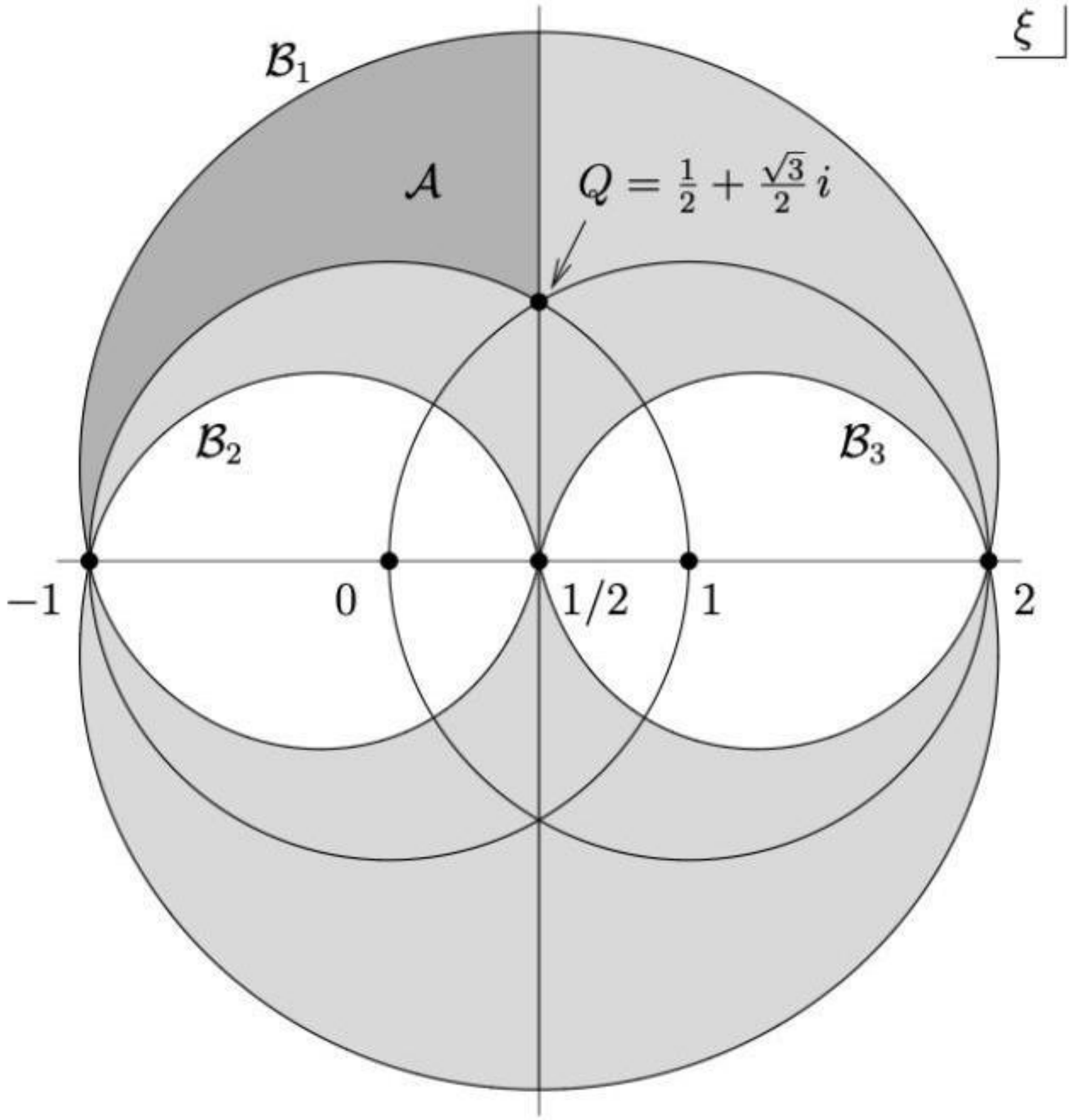
$$w(w_1) = e^{2\pi i/3}\big(h(w_1)\big)^{2/3}, w(w_2) = \big(h(w_2)\big)^{2/3}, w(w_3) = e^{-2\pi i/3}\big(h(w_3)\big)^{2/3},$$

$$z = f_1(w_1) = h^{-1}\big(w(w_1)\big) = \sqrt{3} + \frac{8}{3}w_1 + \frac{16}{9}\sqrt{3}w_1^2 + \frac{248}{81}w_1^3 + \mathcal{O}(w_1^4)$$

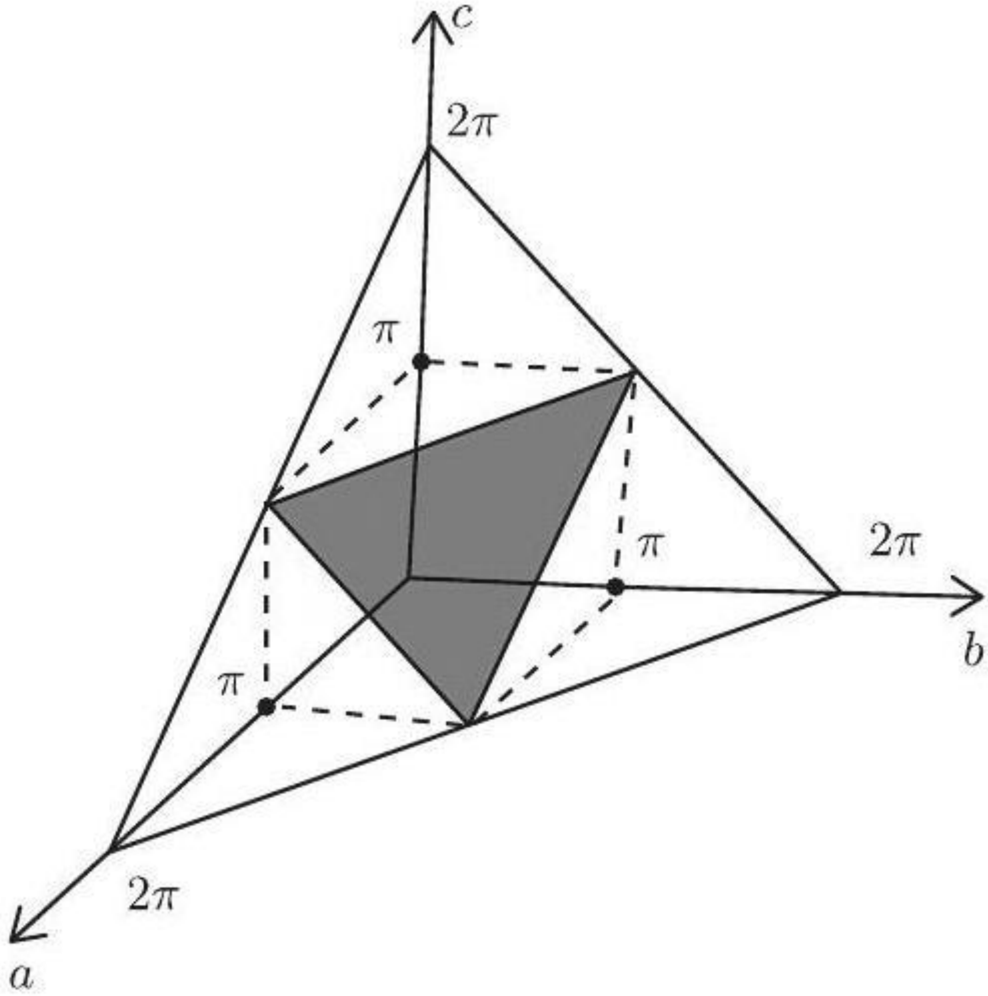
$$z = f_2(w_2) = h^{-1}\big(w(w_2)\big) = \frac{2}{3}w_2 - \frac{10}{81}w_2^3 + \mathcal{O}\big(w_2^5\big)$$

$$z = f_3(w_3) = h^{-1}\big(w(w_3)\big) = -\sqrt{3} + \frac{8}{3}w_3 - \frac{16}{9}\sqrt{3}w_3^2 + \frac{248}{81}w_3^3 + \mathcal{O}(w_3^4)$$


Q
$z = i$
$z = -\frac{1}{\sqrt{3}}$
$z = \frac{1}{\sqrt{3}}$
P_3
$z = -\sqrt{3}$
P_2
$z = 0$
P_1
$z = \sqrt{3}$



$a, b, c \leqslant \pi, a', b', c' \leqslant \pi$



Métricas hiperbólicas y curvatura de Gauss – Riemann – Espacios de Hilbert – Einstein deformados.

$$\mathrm{gr}_\infty\colon \Sigma_{g,n} \to \hat{\mathcal{P}}_{g,n}$$

$$\pi \circ \mathrm{gr}_\infty\colon \mathcal{M}_{g,n,L} \to \mathcal{M}_{g,n}$$

$$\tilde{\mathcal{V}}_{g,n}(L) \equiv \{\Sigma \in \mathcal{M}_{g,n,L} \mid \text{ sys } \Sigma \geqslant L\}$$

$$\mathcal{V}_{g,n}(L) \equiv \mathrm{gr}_\infty\big(\tilde{\mathcal{V}}_{g,n}(L)\big)$$

$$\sinh\left(\frac{1}{2}w_i\right)\sinh\left(\frac{1}{2}\ell_i\right) = 1$$

$$L \leqslant L_* \ \to \ w \geqslant L$$

$$L_c \leqslant L'_*, \text{ and } \sinh\, L_c \sinh\, L_o \leqslant 1$$

$$\mathbf{B}^{[0]}(t) = \sum_{n=0}^{\infty} t^n \mathbf{b}_{n+1}^{(n)}$$

$$\left(\mathbf{B}^{[0]}(t)\right)^2 = 0$$

$$\left\{\boldsymbol{\eta}_0, \mathbf{B}^{[0]}(t)\right\} = 0$$

$$\mathbf{B}^{[m]}(t) = \sum_{n=0}^{\infty} t^n \mathbf{b}_{m+n+1}^{(n)}, m = 0,1, \ldots$$

$$\mathbf{B}(s,t) = \sum_{m=0}^{\infty} s^m \mathbf{B}^{[m]}(t) = \sum_{m,n=0}^{\infty} s^m t^n \mathbf{b}_{m+n+1}^{(n)}$$

$$\boldsymbol{\mu}(s,t) = \sum_{m=0}^{\infty} s^m \boldsymbol{\mu}^{[m]}(t) = \sum_{m,n=0}^{\infty} s^m t^n \boldsymbol{\mu}_{m+n+2}^{(n+1)}$$

$$\{\mathbf{B}(s,t), \mathbf{B}(s,t)\} = 0, \text{ and } \{\boldsymbol{\eta}_0, \mathbf{B}(s,t)\} = 0$$

$$\frac{\partial}{\partial t}\mathbf{B}(s,t) = [\mathbf{B}(s,t), \boldsymbol{\mu}(s,t)], \frac{\partial}{\partial s}\mathbf{B}(s,t) = [\boldsymbol{\eta}_0, \boldsymbol{\mu}(s,t)].$$

$$\mathbf{B}(s,t) = \mathbf{Q} + t\mathbf{b}_2^{(1)} + s\mathbf{b}_2^{(0)} + \cdots, \boldsymbol{\mu}(s,t) = \boldsymbol{\mu}_2^{(1)} + \cdots$$

$$\mathbf{b}_2^{(1)} = \left[\mathbf{Q}, \boldsymbol{\mu}_2^{(1)}\right], \mathbf{b}_2^{(0)} = \left[\boldsymbol{\eta}_0, \boldsymbol{\mu}_2^{(1)}\right].$$

$$\boldsymbol{\mu}_2^{(1)} = \frac{1}{3}\left(\boldsymbol{\xi}_0 \mathbf{b}_2^{(0)} - \mathbf{b}_2^{(0)} \boldsymbol{\xi}_0\right).$$

$$\boldsymbol{\mu}_2^{(1)}(A \otimes B) = \frac{1}{3}\big(\xi_0(A,B) - (\xi_0 A, B) - (-1)^A (A, \xi_0 B)\big).$$

$$\left[\boldsymbol{\eta}_0, \boldsymbol{\mu}_2^{(1)}\right](A \otimes B) = \boldsymbol{\eta}_0 \boldsymbol{\mu}_2^{(1)}(A \otimes B) = \frac{1}{3}\eta_0\big(\xi_0(A,B) - (\xi_0 A, B) - (-1)^A (A, \xi_0 B)\big)$$

$$= \frac{1}{3}\big(\eta_0\xi_0(A,B) + (\eta_0\xi_0 A, B) + (A, \eta_0\xi_0 B)\big) = \frac{1}{3}((A,B) + (A,B) + (A,B))$$

$$= (A,B) = \mathbf{b}_2^{(0)}(A \otimes B)$$

$$(A,B)^* \equiv \mathbf{b}_2^{(1)}(A \otimes B) = \left[\mathbf{Q}, \boldsymbol{\mu}_2^{(1)}\right](A \otimes B)$$

$$= \mathbf{Q}\boldsymbol{\mu}_2^{(1)}(A \otimes B) - \boldsymbol{\mu}_2^{(1)}(QA \otimes B) - (-1)^A \boldsymbol{\mu}_2^{(1)}(A \otimes QB)$$

$$= \frac{1}{3}Q\big(\xi_0(A,B) - (\xi_0 A, B) - (-1)^A (A, \xi_0 B)\big)$$

$$- \frac{1}{3}\big(\xi_0(QA,B) - (\xi_0 QA, B) + (-1)^A (QA, \xi_0 B)\big)$$

$$- \frac{1}{3}(-1)^A\big(\xi_0(A,QB) - (\xi_0 A, QB) - (-1)^A (A, \xi_0 QB)\big)$$

$$(A,B)^* \equiv \frac{1}{3}\big(\mathcal{X}_0(A,B) + (\mathcal{X}_0 A,B) + (A,\mathcal{X}_0 B)\big)$$

$$S(\Phi) = \frac{1}{2}\langle \Phi, Q\Phi\rangle' + \sum_{n=1}^{\infty} \frac{1}{n+2}\left\langle \Phi, b_{n+1}^{(n)}(\Phi,\cdots,\Phi)\right\rangle'$$

$$[A,B]^* \equiv \frac{1}{3}(\mathcal{X}_0[A,B] + [\mathcal{X}_0 A,B] + [A,\mathcal{X}_0 B])$$

Métrica de Berkovits para espacios de Hilbert – Einstein deformados.

$$\{\eta_0,\xi_0\} = 1$$

$$\{Q,\eta_0\} = 0$$

$$S = \frac{1}{2g^2}\left\langle\!\!\left\langle (e^{-\Phi}\eta_0 e^{\Phi})(e^{-\Phi}Qe^{\Phi}) + \int_0^1 dt\Phi\{e^{-t\Phi}Qe^{t\Phi}, e^{-t\Phi}\eta_0 e^{t\Phi}\}\right\rangle\!\!\right\rangle$$

$$\langle\langle A_1 \dots A_n\rangle\rangle = \left\langle h^{-1}\circ f_1^{(n)} \circ A_1(0)\cdots h^{-1}\circ f_n^{(n)} \circ A_n(0)\right\rangle_L', n \geqslant 2,$$

$$f_1^{(n)}(z) = \left(\frac{1+iz}{1-iz}\right)^{2/n}, f_k^{(n)}(z) = e^{2\pi i(k-1)/n} f_1^{(n)}(z), k = 2,\dots n$$

$$\left\langle \xi c\partial c\partial^2 c e^{-2\phi}\right\rangle_L' \neq 0$$

$$\langle\langle A_1 \dots A_n\rangle\rangle = \langle A_1, A_2 \star A_3 \star \cdots \star A_n\rangle_L'$$

$$\eta_0\big(e^{-\Phi}Qe^{\Phi}\big) = 0$$

$$\delta e^{\Phi} = (Q\Lambda)e^{\Phi} + e^{\Phi}(\eta_0\Omega)$$

$$S = \frac{1}{2g^2}\left\langle\!\!\left\langle \frac{1}{2}(Q\Phi)(\eta_0\Phi) + \frac{1}{6}(Q\Phi)(\Phi(\eta_0\Phi) - (\eta_0\Phi)\Phi)\right\rangle\!\!\right\rangle + \mathcal{O}(\Phi^3)$$

$$S_{\text{kin}} \sim \langle\langle (Q\Phi)(\eta_0\Phi)\rangle\rangle = \langle Q\Phi, \eta_0\Phi\rangle_L',$$

$$Q\eta_0|\Phi\rangle = 0$$

$$\delta\Phi = Q|\Lambda\rangle, \delta\Phi = \eta_0|\Omega\rangle$$

$$|\Phi\rangle = |\phi'\rangle + \xi_0|\phi\rangle, \text{ with } \eta_0|\phi'\rangle = \eta_0|\phi\rangle = 0$$

$$\delta|\Phi\rangle = \eta_0|\Omega\rangle = \eta_0(|\omega\rangle + \xi_0|\omega'\rangle) = |\omega'\rangle$$

$$0 = Q\eta_0|\Phi\rangle = Q\eta_0\xi_0|\phi\rangle = Q\{\eta_0,\xi_0\}|\phi\rangle = Q|\phi\rangle$$

Sector holomórfico.

$$\left\langle \xi c\partial c\partial^2 c\bar{c}\partial\bar{c}\partial^2\bar{c}e^{-2\phi}\right\rangle_L \neq 0$$

$$S = 2\left[\frac{1}{2}\langle\eta_0 V, QV\rangle_L + \frac{\kappa}{3!}\langle\eta_0 V, [V, QV]_0\rangle_L\right] + \mathcal{O}(V^3)$$

$$\delta V = Q\Lambda + \eta_0\Omega$$

$$G(\tau V + d\tau V) = G(\tau V) + Q(d\tau V) + \sum_{n=1}^{\infty}\frac{\kappa^n}{n!}[G(\tau V)^n, d\tau V]_0 + \mathcal{O}(d\tau^2)$$

$$\partial_\tau G(\tau V) = QV + \sum_{n=1}^{\infty}\frac{\kappa^n}{n!}[G(\tau V)^n, V]_0$$

$$\partial_\tau G = QV + \kappa[G, V]_0 + \frac{\kappa^2}{2}[G, G, V]_0 + \mathcal{O}(\kappa^3)$$

$$G = G^{(0)} + \kappa G^{(1)} + \kappa^2 G^{(2)} + \mathcal{O}(\kappa^3)$$

$$G(V) = QV + \frac{\kappa}{2}[V, QV]_0 + \frac{\kappa^2}{3!}([V, QV, QV]_0 + [V, [V, QV]_0]_0) + \mathcal{O}(\kappa^3)$$

$$S = 2\int_0^1 dt\langle\eta_0 V, G(tV)\rangle_L$$

$$\left\langle \xi\bar{\xi}c\partial c\partial^2 c\bar{c}\bar{\partial}\bar{c}\bar{\partial}^2\bar{c}e^{-2\phi}e^{-2\bar{\phi}}\right\rangle_L \neq 0$$

$$S_2 = \frac{1}{2}\langle\eta_0\Psi, Q\bar{\eta}_0\Psi\rangle_L$$

$$\delta\Psi = Q\Lambda + \eta_0\Omega + \bar{\eta}_0\bar{\Omega}$$

$$S_3 = \frac{1}{3!}\langle\eta_0\Psi, [Q\bar{\eta}_0\Psi, \bar{\eta}_0\Psi]^*\rangle_L$$

$$[A, B]^* \equiv \frac{1}{3}\left(\overline{\mathcal{X}}_0[A, B]_0 + \left[\overline{\mathcal{X}}_0 A, B\right]_0 + \left[A, \overline{\mathcal{X}}_0 B\right]_0\right)$$

$$S = \int_0^1 dt\langle\eta_0\Psi_t, \mathcal{G}^*(\Psi(t))\rangle_L$$

$$\Psi_t = \Psi + \frac{\kappa}{2}t[\bar{\eta}_0\Psi, \Psi]^* + \mathcal{O}(\kappa^2)$$

$$\mathcal{G}^*(\Psi) = Q\bar{\eta}_0\Psi + \frac{\kappa}{2}t[Q\bar{\eta}_0\Psi, \bar{\eta}_0\Psi]^* + \mathcal{O}(\kappa^2)$$

Sector de Ramond.

$$\beta(z)=\sum_{n\in\mathbb{Z}}\beta_n z^{-n-\frac{3}{2}},\gamma(z)=\sum_{n\in\mathbb{Z}}\gamma_n z^{-n+\frac{1}{2}}$$

$$\beta_n|-1/2\rangle=0 \text{ for } n\geqslant 0,\gamma_n|-1/2\rangle=0 \text{ for } n\geqslant 1$$

$$X=-\delta(\beta_0)G_0^m+\delta'(\beta_0)b_0,Y=-c_0\delta'(\gamma_0)$$

$$\Psi=\phi-(\gamma_0+2c_0b_0\gamma_0+c_0G_0^m)\psi$$

$$b_0\phi=0,\beta_0\phi=0,\eta_0\phi=0,b_0\psi=0,\beta_0\psi=0,\eta_0\psi=0$$

$$XY\Psi=\Psi,\eta_0\Psi=0$$

$$S_R^{(0)}=-\frac{1}{2}\langle\Psi,YQ\Psi\rangle'$$

$$\delta|\Psi\rangle=Q|\lambda\rangle,$$

$$XY\lambda=\lambda,\eta_0\lambda=0$$

$$S=-\frac{1}{2}\langle\Psi,YQ\Psi\rangle'-\int_0^1 dt\left\langle A_t(t),QA_\eta(t)+(F(t)\Psi)^2\right\rangle_L'$$

$$A_\eta(t)=\eta e^{\Phi(t)}e^{-\Phi(t)},A_t(t)=\partial_t e^{\Phi(t)}e^{-\Phi(t)}$$

$$F(t)\Psi=\Theta(\beta_0)\left\{A_\eta(t),\Theta(\beta_0)\left\{A_\eta(t),\cdots\Theta(\beta_0)\left\{A_\eta(t),\Psi\right\}\cdots\right\}\right\},$$

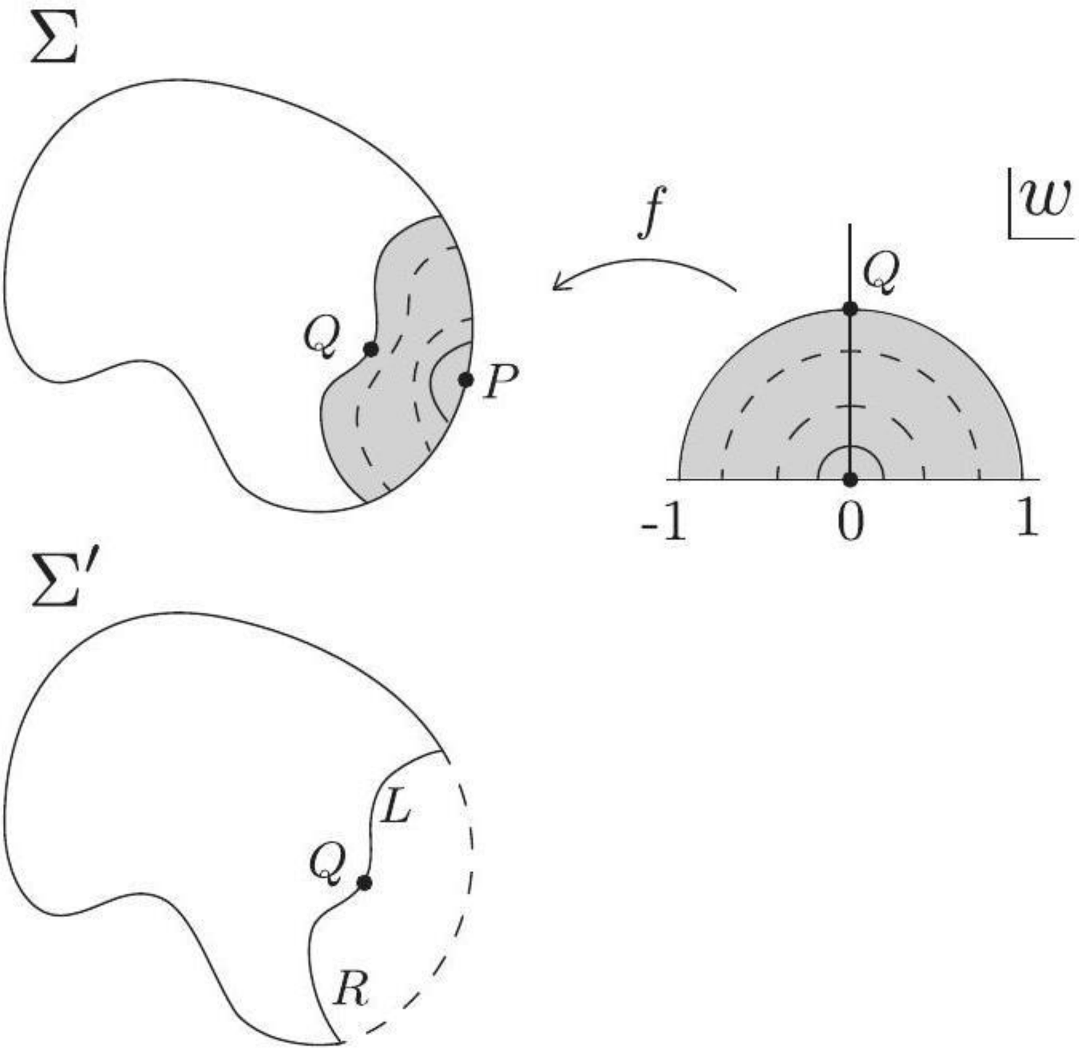
Hiperpartículas – Condensación de campo.

$$-S=Vg_o^{-2}\left[-\frac{1}{2}\phi^2+\frac{1}{3}\left(\frac{3\sqrt{3}}{4}\right)^3\phi^3\right]$$

$$\mathcal{T}=\frac{1}{2\pi^2 g_o^2}$$

$$-S=-Vg_o^{-2}\frac{1}{6}\left(\frac{16}{27}\right)^3=-V\mathcal{T}2\pi^2\frac{1}{6}\left(\frac{16}{27}\right)^3\simeq-0.68V\mathcal{T}$$

$$\langle\Sigma\mid\phi\rangle'=\langle\phi(w=0)\rangle_\Sigma'=\langle f\circ\phi(0)\rangle_\Sigma'$$



$$z = \frac{2}{\pi} \arctan w$$

$$|\Omega_\alpha\rangle \star |\Omega_\beta\rangle = |\Omega_{\alpha+\beta}\rangle,$$

$$\langle \phi, \Omega_\alpha \rangle' = \langle f \circ \phi(0) \rangle'_{C_{\alpha+1}}$$

$$\langle \phi, \Omega_\alpha \rangle' = \langle f_\alpha \circ f \circ \phi(0) \rangle'_{C_1}$$

$$f_{\alpha+\delta\alpha} = f_{\delta\alpha/(1+\alpha)} \circ f_\alpha$$

$$\langle \phi, \Omega_{\alpha+\delta\alpha} \rangle' = \left\langle f_{\delta\alpha/(1+\alpha)} \circ f_\alpha \circ f \circ \phi(0) \right\rangle'_{C_1}$$

$$\langle \phi, \Omega_{\alpha+\delta\alpha} \rangle' = \langle \phi, \Omega_\alpha \rangle' - \frac{\delta\alpha}{1+\alpha} \left\langle \oint_0 \frac{dz}{2\pi i} z T(z) f_\alpha \circ f \circ \phi(0) \right\rangle'_{C_1}$$

$$= \langle \phi, \Omega_\alpha \rangle' - \frac{\delta\alpha}{1+\alpha} \left\langle \oint_0 \frac{dz}{2\pi i} z T(z) f \circ \phi(0) \right\rangle'_{C_{\alpha+1}}$$

$$\left\langle \phi, \frac{d}{d\alpha}\Omega_\alpha \right\rangle' = -\frac{1}{\alpha+1}\left\langle \oint_0 \frac{dz}{2\pi i} zT(z) f\circ\phi(0) \right\rangle'_{C_{\alpha+1}}$$

$$\left\langle \phi, \frac{d}{d\alpha}\Omega_\alpha \right\rangle' = -\left\langle \int_R \frac{dz}{2\pi i} T(z) f\circ\phi(0) \right\rangle'_{C_{\alpha+1}}$$

$$\frac{d}{d\alpha}\Omega_\alpha = -\Omega_\alpha \mathcal{K} = -\mathcal{K}\Omega_\alpha$$

$$\mathcal{K} = \left(\int_L T\right)\mathcal{I}$$

$$\Omega_\alpha = e^{-\alpha\mathcal{K}}$$

$$B = \left(\int_L b\right)\mathcal{I}$$

$$\begin{aligned} &c^2 = 0, B^2 = 0, \{c, B\} = 1 \\ &[\mathcal{K}, B] = 0, [\mathcal{K}, c] = \partial c \\ &Q\mathcal{K} = 0, QB = \mathcal{K}, Qc = c\mathcal{K}c \end{aligned}$$

$$\Psi = Fc\frac{\mathcal{K}}{1-F^2}BcF, \text{ with } F = F(\mathcal{K})$$

$$(1 - FBcF)^{-1} = 1 + \frac{F}{1-F^2}BcF$$

$$\Psi = (1 - FBcF)Q(1 - FBcF)^{-1}$$

$$A = \frac{1-F^2}{\mathcal{K}}B, Q_\Psi A = \mathcal{I}$$

$$F(\mathcal{K}) = e^{-\mathcal{K}/2} = \Omega_{1/2}$$

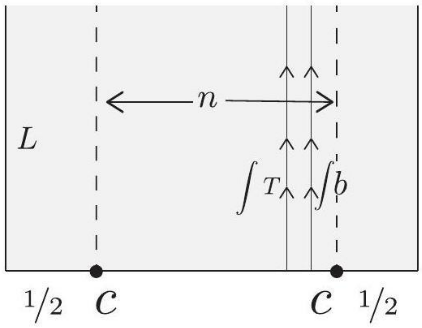
$$\Psi = \Omega_{1/2}c\frac{\mathcal{K}}{1-\Omega_1}Bc\Omega_{1/2} = \Omega_{1/2}c(1 + \Omega_1 + \Omega_2 + \cdots)\mathcal{K}Bc\Omega_{1/2}$$

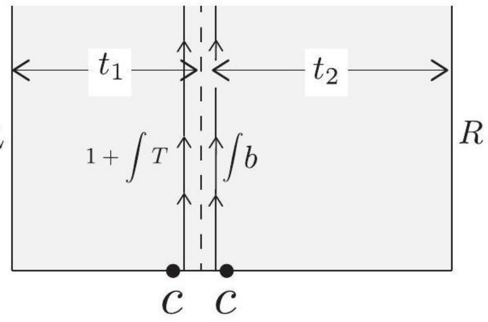
$$= \Omega_{1/2}c\mathcal{K}Bc\Omega_{1/2} + \Omega_{1/2}c\left(\sum_{n=1}^{\infty}\Omega_n\right)\mathcal{K}Bc\Omega_{1/2}$$

$$\Psi = \frac{1}{\sqrt{1+\mathcal{K}}}c(1+\mathcal{K})Bc\frac{1}{\sqrt{1+\mathcal{K}}}$$

$$\frac{1}{\sqrt{1+\mathcal{K}}} = \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{dt}{\sqrt{t}}e^{-t}\Omega_t$$

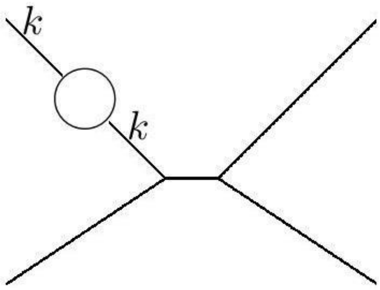
$$\Psi = \frac{1}{\pi}\int_0^\infty\int_0^\infty \frac{dt_1}{\sqrt{t_1}}\frac{dt_2}{\sqrt{t_2}}e^{-(t_1+t_2)}\Omega_{t_1}c(1+\mathcal{K})Bc\Omega_{t_2}$$

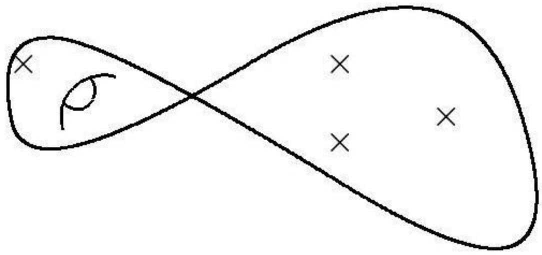




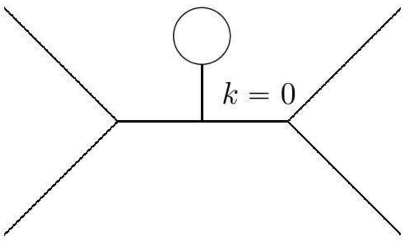
$$-4\pi i\langle E[\mathcal{O}_c] \mid \hat{\Phi}\rangle = \langle \mathcal{O}_c|c_0^-\left|B_{\hat{\Phi}}\right\rangle - \langle \mathcal{O}_c|c_0^-|B_0\rangle.$$

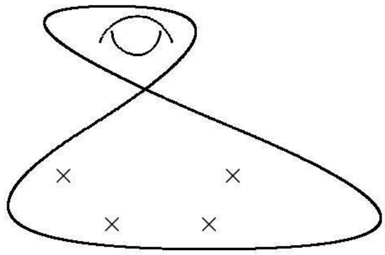
Renormalización de masa.





$$\frac{1}{L_0+\bar{L}_0}\delta_{L_0,\bar{L}_0} = \frac{1}{2\pi}\int_{|q|\leqslant 1}\frac{d^2q}{|q|^2}q^{L_0}\bar{q}^{\bar{L}_0}$$





$$S = -\int d^Dx\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{g_s}{3!}\phi^3\right]$$

$$S = -\int d^Dx\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{g_s}{3!}\phi^3 + cg_s\phi + \mathcal{O}(\phi^2)\right]$$

$$\phi = \pm\sqrt{-2c} + \mathcal{O}(g_s)$$

$$Q(|\Psi\rangle - \mathcal{G}|\widetilde{\Psi}\rangle) = 0$$

$$Q|\widetilde{\Psi}\rangle + \sum_{n=0}^{\infty}\frac{1}{n!}[\Psi^n]_{1\mathrm{PI}} = 0$$

$$|\widetilde{\Psi}\rangle = |\Psi\rangle,$$

$$Q|\Psi\rangle + \sum_{n=0}^{\infty} \frac{1}{n!}[\Psi^n]_{1\mathrm{PI}} = 0$$

$$|\Psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+}\sum_{n=0}^{k+1} \frac{1}{n!}[\Psi_k^n]_{1\mathrm{PI}} + \mathcal{O}\left(g_s^{k+2}\right)$$

$$|\Psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+}(1-\mathbf{P})\sum_{n=0}^{k+1} \frac{1}{n!}[\Psi_k^n]_{1\mathrm{PI}} + |\psi_{k+1}\rangle + \mathcal{O}\left(g_s^{k+2}\right), \mathbf{P}|\psi_{k+1}\rangle = |\psi_{k+1}\rangle$$

$$Q|\psi_{k+1}\rangle = -\mathbf{P}\sum_{n=0}^{k+1} \frac{1}{n!}[\Psi_k^n]_{1\mathrm{PI}} + \mathcal{O}\left(g_s^{k+2}\right)$$

$$|\Psi\rangle = |\Psi_v\rangle + |\Phi\rangle, |\widetilde{\Psi}\rangle = |\Psi_v\rangle + |\widetilde{\Phi}\rangle$$

$$Q(|\Phi\rangle - \mathcal{G}|\widetilde{\Phi}\rangle) = 0, Q|\widetilde{\Phi}\rangle + K|\Phi\rangle = 0$$

$$K|A\rangle \equiv \sum_{n=0}^{\infty} \frac{1}{n!}[\Psi_v^n A]$$

$$(Q + \mathcal{G}K)|\Phi\rangle = 0$$

$$QK + KQ + K\mathcal{G}K = 0$$

$$2L_0^+ = \left(k^2 + \hat{M}^2\right)$$

$$|\Phi_0\rangle = |\phi_n\rangle, |\Phi_{\ell+1}\rangle = -\frac{b_0^+}{L_0^+}(1-P)\mathcal{G}K|\Phi_\ell\rangle + |\phi_n\rangle$$

$$P|\phi_n\rangle = |\phi_n\rangle, Q|\phi_n\rangle = -P\mathcal{G}K|\Phi_{n-1}\rangle + \mathcal{O}(g_s^{n+1})$$

$$(Q + \mathcal{G}K)|\Phi_n\rangle = \mathcal{O}(g_s^{n+1})$$

$$N \equiv \exp\left[\int_0^\infty \frac{dt}{t} F(t)\right]$$

$$F(t) = \frac{1}{2}\mathrm{Tr}\left\{(-1)^f e^{-2\pi t L_0}\right\},$$

$$N = \exp\left[\int_0^\infty \frac{dt}{2t}\left(\sum_b e^{-2\pi t h_b} - \sum_f e^{-2\pi t h_f}\right)\right]$$

$$\int_0^\infty \frac{dt}{2t}\left(e^{-2\pi h_b t} - e^{-2\pi h_f t}\right) = \ln\sqrt{\frac{h_f}{h_b}}$$

$$N=\prod_{b,f}\sqrt{\frac{h_f}{h_b}}=\left(\mathrm{sdet}(L_0)\right)^{-1/2}$$

$$\mathrm{sdet}M=1,\ \text{with}\ M_{rs}\equiv\langle\phi_r|c_0|\phi_s\rangle'.$$

$$\int\frac{d\phi_b}{\sqrt{2\pi}}e^{-\frac{1}{2}h_b\phi_b^2}=h_b^{-1/2},\int\ dp_f dq_f e^{-h_f p_f q_f}=h_f$$

$$|\psi_o\rangle=\sum_r\ |\phi_r\rangle\psi_r$$

$$N=\left(\mathrm{sdet}(L_0)\right)^{-1/2}=\int\ \prod_r\ D\psi_r\exp\left[\frac{1}{2}\langle\psi_o|c_0L_0|\psi_o\rangle'\right],$$

$$\xi^\mu=\frac{1}{\sqrt{2\pi}g_o}\delta y^\mu$$

$$|\psi_o\rangle=p|0\rangle+qc_1c_{-1}|0\rangle+\cdots$$

$$\frac{1}{2}\langle\psi_o|c_0L_0|\psi_o\rangle'=-hpq+\cdots$$

$$|\psi_o\rangle=i\phi c_0|0\rangle+\cdots$$

$$\delta\phi=h\theta$$

$$\int\ dpdq\rightarrow\int\ d\phi e^{S_{\phi,\xi^0}}/\int\ d\theta$$

$$S_{\phi,\xi^0}=\frac{1}{2}\langle\psi_o|Q|\psi_o\rangle'\Big|_{\phi,\xi^0}=-\left(\phi-\sqrt{\frac{h}{2}}\xi^0\right)^2$$

$$\delta\phi=h,\delta\xi^0=\sqrt{2h}$$

$$\int\ d\phi e^{-\phi^2}=\sqrt{\pi}$$

$$\theta=\alpha/g_o$$

$$\int\ d\theta=2\pi/g_o$$

$$\int\ dpdq\prod_\mu\frac{d\xi^\mu}{\sqrt{2\pi}}\rightarrow\left(2\pi\sqrt{\pi}g_o\right)^{-d}\frac{g_o}{2\pi}\sqrt{\pi}\int\ \prod_\mu\ dy^\mu$$

$$N=\left(2\pi\sqrt{\pi}g_o\right)^{-d}\frac{g_o}{2\pi}\sqrt{\pi}\prod_b\ {}'h_b^{-1/2}\prod_f\ {}'h_f^{1/2}(2\pi)^d\delta^{(d)}\left(\sum_i\ p_i\right)$$

$$\prod_b{}' h_b^{-1/2} \prod_f{}' h_f^{1/2} = \prod_b \prod_b{}'' h_b^{-1/2} \prod_f{}'' h_f^{1/2} \exp\left[-\int_0^\infty \frac{dt}{2t}\left(\sum_b{}'''' e^{-2\pi t h_b} - \sum_f{}'''' e^{-2\pi t h_f}\right)\right]$$

$$\mathcal{N}(e^{S_1}-1)$$

$$S = S_0 + \mathcal{N}(e^{S_1}-1) + \mathcal{O}(\mathcal{N}^2)$$

$$\frac{1}{2}\{S_0,S_0\} + \Delta S_0 = 0$$

$$\frac{1}{2}\{S_0+S_1,S_0+S_1\} + \Delta(S_0+S_1) = 0$$

$$\{S_0,S_1\} + \frac{1}{2}\{S_1,S_1\} + \Delta S_1 = 0$$

$$\frac{1}{2}\{S,S\} = \frac{1}{2}\{S_0,S_0\} + \mathcal{N}\{S_0,S_1\}e^{S_1} + \mathcal{O}(\mathcal{N}^2)$$

$$\Delta S = \Delta S_0 + \mathcal{N}\Delta e^{S_1} = \Delta S_0 + \mathcal{N}\left(\frac{1}{2}\{S_1,S_1\} + \Delta S_1\right)e^{S_1}$$

$$\frac{1}{2}\{S,S\} + \Delta S = \mathcal{O}(\mathcal{N}^2)$$

$$\frac{1}{2}\{S_0+2S_1+S_2,S_0+2S_1+S_2\} + \Delta(S_0+2S_1+S_2) = 0$$

$$\{S_0,S_2\} + 2\{S_1,S_2\} + \{S_1,S_1\} + \frac{1}{2}\{S_2,S_2\} = 0$$

$$S = S_0 + \mathcal{N}(e^{S_1}-1) + \frac{1}{2}\mathcal{N}^2(e^{S_2}-1)e^{2S_1} + \mathcal{O}(\mathcal{N}^3)$$

$$\frac{1}{2}\{S,S\} + \Delta S = \mathcal{O}(\mathcal{N}^3)$$

Simetrías.

$$T - T^\dagger = i\sum_n T^\dagger|n\rangle\langle n|T \;\Leftrightarrow\; \sum_n S^\dagger|n\rangle\langle n|S = 1$$

$$\bar{\Delta}(A_1\otimes\cdots\otimes A_n) = \sum_{k=1}^{n-1}(A_1\otimes\cdots A_k)\otimes'(A_{k+1}\cdots A_n), n \geqslant 2, \bar{\Delta}(A_1) = 0$$

Apéndice A.

Gravedad cuántica endógena, es decir, por acción de la partícula supermasiva, a razón de su masa extremadamente densa. Modelo Weyl – Polyakov.

$$d^2z \equiv 2dxdy, \delta^2(z,\bar{z}) \equiv \frac{1}{2}\delta(x)\delta(y)$$

$$M_{\mathrm{P}} = G_{\mathrm{N}}^{-1/2} = 1.22 \times 10^{19}\mathrm{GeV}$$

$$M_{\mathrm{P}}^{-1} = 1.6 \times 10^{-33} \text{ cm.}$$

$$G_{\mathrm{N}}^2 E^2 \int dE'E' = \frac{E^2}{M_{\mathrm{P}}^4} \int dE'E'$$

$$X'^{\mu}(\tau'(\tau)) = X^{\mu}(\tau)$$

$$S_{\mathrm{pp}} = -m \int d\tau \left(-\dot{X}^{\mu}\dot{X}_{\mu}\right)^{1/2}$$

$$\delta S_{\mathrm{pp}} = -m \int d\tau \dot{u}_{\mu} \delta X^{\mu}$$

$$u^{\mu} = \dot{X}^{\mu}(-\dot{X}^{v}\dot{X}_{v})^{-1/2}$$

$$S'_{\mathrm{pp}} = \frac{1}{2} \int d\tau \left(\eta^{-1}\dot{X}^{\mu}\dot{X}_{\mu} - \eta m^2\right)$$

$$\eta'(\tau')d\tau' = \eta(\tau)d\tau$$

$$\eta^2 = -\dot{X}^{\mu}\dot{X}_{\mu}/m^2$$

$$h_{ab} = \partial_a X^{\mu} \partial_b X_{\mu}$$

$$S_{\mathrm{NG}} = \int_M d\tau d\sigma \mathcal{L}_{\mathrm{NG}}$$

$$\mathcal{L}_{\mathrm{NG}} = -\frac{1}{2\pi\alpha'}(-\mathrm{det}h_{ab})^{1/2}$$

$$T = \frac{1}{2\pi\alpha'}$$

$$X'^{\mu}(\tau,\sigma) = \Lambda_v^{\mu} X^{v}(\tau,\sigma) + a^{\mu}$$

$$X'^{\mu}(\tau',\sigma') = X^{\mu}(\tau,\sigma)$$

$$S_{\mathrm{P}}[X,\gamma] = -\frac{1}{4\pi\alpha'} \int_M d\tau d\sigma (-\gamma)^{1/2} \gamma^{ab} \partial_a X^{\mu} \partial_b X_{\mu}$$

$$\delta_\gamma S_{\mathrm{P}}[X,\gamma] = -\frac{1}{4\pi\alpha'}\int_M d\tau d\sigma(-\gamma)^{1/2}\delta\gamma^{ab}\left(h_{ab} - \frac{1}{2}\gamma_{ab}\gamma^{cd}h_{cd}\right)$$

$$\delta\gamma = \gamma\gamma^{ab}\delta\gamma_{ab} = -\gamma\gamma_{ab}\delta\gamma^{ab}$$

$$h_{ab} = \frac{1}{2}\gamma_{ab}\gamma^{cd}h_{cd}$$

$$h_{ab}(-h)^{-1/2} = \gamma_{ab}(-\gamma)^{-1/2}$$

$$S_{\mathrm{P}}[X,\gamma] \to -\frac{1}{2\pi\alpha'}\int \; d\tau d\sigma(-h)^{1/2} = S_{\mathrm{NG}}[X]$$

$$X'^{\mu}(\tau,\sigma) = \Lambda^{\mu}_{\nu}X^{\nu}(\tau,\sigma) + a^{\mu}$$
$$\gamma'_{ab}(\tau,\sigma) = \gamma_{ab}(\tau,\sigma)$$

$$X'^{\mu}(\tau',\sigma') = X^{\mu}(\tau,\sigma)$$
$$\frac{\partial\sigma'^{c}}{\partial\sigma^{a}}\frac{\partial\sigma'^{d}}{\partial\sigma^{b}}\gamma'_{cd}(\tau',\sigma') = \gamma_{ab}(\tau,\sigma)$$

$$X'^{\mu}(\tau,\sigma) = X^{\mu}(\tau,\sigma)$$
$$\gamma'_{ab}(\tau,\sigma) = \exp\,(2\omega(\tau,\sigma))\gamma_{ab}(\tau,\sigma)$$

$$T^{ab}(\tau,\sigma) = -4\pi(-\gamma)^{-1/2}\frac{\delta}{\delta\gamma_{ab}}S_{\mathrm{P}} = -\frac{1}{\alpha'}\left(\partial^{a}X^{\mu}\partial^{b}X_{\mu} - \frac{1}{2}\gamma^{ab}\partial_{c}X^{\mu}\partial^{c}X_{\mu}\right)$$

$$\gamma_{ab}\frac{\delta}{\delta\gamma_{ab}}S_{\mathrm{P}} = 0 \Rightarrow T^{a}_{a} = 0$$

$$T_{ab} = 0$$

$$\partial_a\left[(-\gamma)^{1/2}\gamma^{ab}\partial_b X^{\mu}\right] = (-\gamma)^{1/2}\nabla^2 X^{\mu} = 0$$

$$-\infty < \tau < \infty, 0 \leq \sigma \leq \ell.$$

$$\delta S_{\mathrm{P}} = \frac{1}{2\pi\alpha'}\int_{-\infty}^{\infty} d\tau \int_0^{\ell} d\sigma(-\gamma)^{1/2}\delta X^{\mu}\nabla^2 X_{\mu}$$
$$-\frac{1}{2\pi\alpha'}\int_{-\infty}^{\infty} d\tau(-\gamma)^{1/2}\delta X^{\mu}\partial^{\sigma}X_{\mu}\bigg|_{\sigma=0}^{\sigma=\ell}$$

$$\partial^{\sigma}X^{\mu}(\tau,0) = \partial^{\sigma}X^{\mu}(\tau,\ell) = 0$$

$$n^{a}\partial_{a}X_{\mu} = 0 \;\; \text{on } \partial M$$

$$X^{\mu}(\tau,\ell) = X^{\mu}(\tau,0), \partial^{\sigma}X^{\mu}(\tau,\ell) = \partial^{\sigma}X^{\mu}(\tau,0)$$
$$\gamma_{ab}(\tau,\ell) = \gamma_{ab}(\tau,0)$$

$$\chi = \frac{1}{4\pi}\int_M d\tau d\sigma(-\gamma)^{1/2}R$$

$$(-\gamma')^{1/2}R' = (-\gamma)^{1/2}(R - 2\nabla^2\omega).$$

$$S'_{\mathrm{P}} = S_{\mathrm{P}} - \lambda\chi = -\int_M d\tau d\sigma(-\gamma)^{1/2}\left(\frac{1}{4\pi\alpha'}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu + \frac{\lambda}{4\pi}R\right)$$

$$x^\pm = 2^{-1/2}(x^0 \pm x^1), x^i, i = 2, \dots, D-1$$

$$a^\mu b_\mu = -a^+b^- - a^-b^+ + a^i b^i$$
$$a_- = -a^+, a_+ = -a^-, a_i = a^i$$

$$X^+(\tau) = \tau$$

$$S'_{\mathrm{pp}} = \frac{1}{2}\int d\tau\left(-2\eta^{-1}\dot{X}^- + \eta^{-1}\dot{X}^i\dot{X}^i - \eta m^2\right)$$

$$p_- = -\eta^{-1}, p_i = \eta^{-1}\dot{X}^i$$

$$H = p_-\dot{X}^- + p_i\dot{X}^i - L = \frac{p^i p^i + m^2}{2p^+}$$

$$\left[p_i, X^j\right] = -i\delta_i^j, [p_-, X^-] = -i$$

$$X^+ = \tau$$
$$\partial_\sigma\gamma_{\sigma\sigma} = 0$$
$$\det\gamma_{ab} = -1$$

$$f'd\sigma' = fd\sigma$$

$$\begin{bmatrix}\gamma^{\tau\tau} & \gamma^{\tau\sigma}\\ \gamma^{\sigma\tau} & \gamma^{\sigma\sigma}\end{bmatrix} = \begin{bmatrix}-\gamma_{\sigma\sigma}(\tau) & \gamma_{\tau\sigma}(\tau,\sigma)\\ \gamma_{\tau\sigma}(\tau,\sigma) & \gamma_{\sigma\sigma}^{-1}(\tau)(1-\gamma_{\tau\sigma}^2(\tau,\sigma))\end{bmatrix}.$$

$$L = -\frac{1}{4\pi\alpha'}\int_0^\ell d\sigma\left[\gamma_{\sigma\sigma}\left(2\partial_\tau x^- - \partial_\tau X^i\partial_\tau X^i\right) - 2\gamma_{\sigma\tau}\left(\partial_\sigma Y^- - \partial_\tau X^i\partial_\sigma X^i\right) + \gamma_{\sigma\sigma}^{-1}(1-\gamma_{\tau\sigma}^2)\partial_\sigma X^i\partial_\sigma X^i\right]$$

$$x^-(\tau) = \frac{1}{\ell}\int_0^\ell d\sigma X^-(\tau,\sigma)$$
$$Y^-(\tau,\sigma) = X^-(\tau,\sigma) - x^-(\tau)$$

$$\gamma_{\tau\sigma}\partial_\tau X^\mu - \gamma_{\tau\tau}\partial_\sigma X^\mu = 0 \ \text{ at } \sigma = 0, \ell.$$

$$\gamma_{\tau\sigma} = 0 \ \text{ at } \sigma = 0, \ell$$

$$\partial_\sigma X^i = 0 \ \text{ at } \sigma = 0, \ell$$

$$L = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}\partial_\tau x^- + \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma\left(\gamma_{\sigma\sigma}\partial_\tau X^i\partial_\tau X^i - \gamma_{\sigma\sigma}^{-1}\partial_\sigma X^i\partial_\sigma X^i\right)$$

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau x^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

$$\Pi^i = \frac{\delta L}{\delta(\partial_\tau X^i)} = \frac{1}{2\pi\alpha'}\gamma_{\sigma\sigma}\partial_\tau X^i = \frac{p^+}{\ell}\partial_\tau X^i$$

$$H = p_-\partial_\tau x^- - L + \int_0^\ell d\sigma \Pi_i \partial_\tau X^i = \frac{\ell}{4\pi\alpha' p^+}\int_0^\ell d\sigma \left(2\pi\alpha'\Pi^i\Pi^i + \frac{1}{2\pi\alpha'}\partial_\sigma X^i \partial_\sigma X^i\right)$$

$$\partial_\tau x^- = \frac{\partial H}{\partial p_-} = \frac{H}{p^+}, \partial_\tau p^+ = \frac{\partial H}{\partial x^-} = 0$$

$$\partial_\tau X^i = \frac{\delta H}{\delta \Pi^i} = 2\pi\alpha' c\Pi^i, \partial_\tau \Pi^i = -\frac{\delta H}{\delta X^i} = \frac{c}{2\pi\alpha'}\partial_\sigma^2 X^i$$

$$\partial_\tau^2 X^i = c^2\partial_\sigma^2 X^i$$

$$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i(2\alpha')^{1/2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\frac{1}{n}\alpha_n^i \exp\left(-\frac{\pi i n c\tau}{\ell}\right)\cos\frac{\pi n\sigma}{\ell}$$

$$x^i(\tau) = \frac{1}{\ell}\int_0^\ell d\sigma X^i(\tau,\sigma)$$

$$p^i(\tau) = \int_0^\ell d\sigma \Pi^i(\tau,\sigma) = \frac{p^+}{\ell}\int_0^\ell d\sigma \partial_\tau X^i(\tau,\sigma)$$

$$[x^-, p^+] = i\eta^{-+} = -i$$

$$[X^i(\sigma), \Pi^j(\sigma')] = i\delta^{ij}\delta(\sigma - \sigma')$$

$$[x^i, p^j] = i\delta^{ij}$$

$$[\alpha_m^i, \alpha_n^j] = m\delta^{ij}\delta_{m,-n}$$

$$\alpha_m^i \sim m^{1/2}a, \alpha_{-m}^i \sim m^{1/2}a^\dagger, m > 0$$

$$p^+|0;k\rangle = k^+|0;k\rangle, p^i|0;k\rangle = k^i|0;k\rangle$$

$$\alpha_m^i|0;k\rangle = 0, m > 0$$

$$|N;k\rangle = \left[\prod_{i=2}^{D-1}\prod_{n=1}^{\infty}\frac{(\alpha_{-n}^i)^{N_{in}}}{(n^{N_{in}}N_{in}!)^{1/2}}\right]|0;k\rangle.$$

$$\mathcal{H} = |\text{ vacuum }\rangle \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots$$

$$H = \frac{p^i p^i}{2p^+} + \frac{1}{2p^+\alpha'}\left(\sum_{n=1}^{\infty}\alpha_{-n}^i\alpha_n^i + A\right)$$

$$A = \frac{D-2}{2}\sum_{n=1}^{\infty} n$$

$$\sum_{n=1}^{\infty} n \to -\frac{1}{12}$$

$$\exp\left(-\epsilon\gamma_{\sigma\sigma}^{-1/2}|k_\sigma|\right)$$

$$A \to \frac{D-2}{2}\sum_{n=1}^{\infty} n\exp\left[-\epsilon n(\pi/2p^+\alpha'\ell)^{1/2}\right] = \frac{D-2}{2}\left(\frac{2\ell p^+\alpha'}{\epsilon^2\pi} - \frac{1}{12} + O(\epsilon)\right)$$

$$A = \frac{2-D}{24}$$

$$m^2 = 2p^+H - p^ip^i = \frac{1}{\alpha'}\left(N + \frac{2-D}{24}\right),$$

$$N = \sum_{i=2}^{D-1}\sum_{n=1}^{\infty} nN_{in}$$

$$|0;k\rangle, m^2 = \frac{2-D}{24\alpha'}$$

$$\alpha_{-1}^i|0;k\rangle, m^2 = \frac{26-D}{24\alpha'}$$

$$A = -1, D = 26$$

$$S^{ij} = -i\sum_{n=1}^{\infty}\frac{1}{n}\left(\alpha_{-n}^i\alpha_n^j - \alpha_{-n}^j\alpha_n^i\right)$$

$$\mathbf{v} = (v^1, 0, \ldots, 0) + (0, v^2, \ldots, v^{D-1})$$

$$S^{23} \le 1 + \alpha' m^2$$

$$\sigma' = \sigma + s(\tau)\mathrm{mod}\ell$$

$$\gamma_{\tau\sigma}(\tau, 0) = 0$$

$$\sigma' = \sigma + s\mathrm{mod}\ell.$$

$$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i\left(\frac{\alpha'}{2}\right)^{1/2} \times \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\left\{\frac{\alpha_n^i}{n}\exp\left[-\frac{2\pi in(\sigma + c\tau)}{\ell}\right] + \frac{\tilde{\alpha}_n^i}{n}\exp\left[\frac{2\pi in(\sigma - c\tau)}{\ell}\right]\right\}$$

$$\alpha_n^i, \tilde{\alpha}_n^i, x^i, p^i, x^-, p^+$$

$$
\begin{aligned}
[x^-, p^+] &= -i \\
[x^i, p^j] &= i\delta^{ij} \\
[\alpha_m^i, \alpha_n^j] &= m\delta^{ij}\delta_{m,-n} \\
[\tilde{\alpha}_m^i, \tilde{\alpha}_n^j] &= m\delta^{ij}\delta_{m,-n}
\end{aligned}
$$

$$|N, \tilde{N}; k\rangle = \left[\prod_{i=2}^{D-1}\prod_{n=1}^{\infty}\frac{(\alpha_{-n}^i)^{N_{in}}(\tilde{\alpha}_{-n}^i)^{\tilde{N}_{in}}}{(n^{N_{in}}N_{in}!\,n^{\tilde{N}_{in}}\tilde{N}_{in}!)^{1/2}}\right]|0,0;k\rangle.$$

$$m^2 = 2p^+H - p^ip^i = \frac{2}{\alpha'}\left[\sum_{n=1}^{\infty}(\alpha_{-n}^i\alpha_n^i + \tilde{\alpha}_{-n}^i\tilde{\alpha}_n^i) + A + \tilde{A}\right] = \frac{2}{\alpha'}(N + \tilde{N} + A + \tilde{A})$$

$$A = \tilde{A} = \frac{2-D}{24}$$

$$P = -\int_0^{\ell} d\sigma \Pi^i \partial_\sigma X^i = -\frac{2\pi}{\ell}\left[\sum_{n=1}^{\infty}(\alpha_{-n}^i\alpha_n^i - \tilde{\alpha}_{-n}^i\tilde{\alpha}_n^i) + A - \tilde{A}\right] = -\frac{2\pi}{\ell}(N - \tilde{N})$$

$$N = \tilde{N}$$

$$|0,0;k\rangle, m^2 = \frac{2-D}{6\alpha'}$$

$$\alpha_{-1}^i\tilde{\alpha}_{-1}^j|0,0;k\rangle, m^2 = \frac{26-D}{6\alpha'}$$

$$A = \tilde{A} = -1, D = 26$$

$$e^{ij} = \frac{1}{2}\left(e^{ij} + e^{ji} - \frac{2}{D-2}\delta^{ij}e^{kk}\right) + \frac{1}{2}(e^{ij} - e^{ji}) + \frac{1}{D-2}\delta^{ij}e^{kk},$$

$$[\alpha_m^\mu, \alpha_n^v] = m\eta^{\mu v}\delta_{m,-n}$$

$$\sigma' = \ell - \sigma, \tau' = \tau$$

$$\Omega\alpha_n^i\Omega^{-1} = (-1)^n\alpha_n^i$$

$$
\begin{aligned}
\Omega\alpha_n^i\Omega^{-1} &= \tilde{\alpha}_n^i \\
\Omega\tilde{\alpha}_n^i\Omega^{-1} &= \alpha_n^i
\end{aligned}
$$

$$
\begin{aligned}
\Omega|N;k\rangle &= (-1)^N|N;k\rangle \\
\Omega|N,\tilde{N};k\rangle &= |\tilde{N},N;k\rangle
\end{aligned}
$$

$$\chi = \frac{1}{4\pi}\int_M d\tau d\sigma(-\gamma)^{1/2}R + \frac{1}{2\pi}\int_{\partial M} dsk$$

$$k=\pm t^{a}n_{b}\nabla_{a}t^{b}$$

$$\sum_{n=1}^{\infty}(n-\theta)$$

$$X^{25}(\tau,0)=0,X^{25}(\tau,\ell)=y$$

$$X^{25}(\tau,0)=0,\partial^{\sigma}X^{25}(\tau,\ell)=0$$

$$X^{25}(\tau,\sigma+\ell)=X^{25}(\tau,\sigma)+2\pi R$$

$$X^{25}(\tau,\sigma+\ell)=-X^{25}(\tau,\sigma)$$

$$S=\frac{1}{4\pi\alpha'}\int d^{2}\sigma\left(\partial_{1}X^{\mu}\partial_{1}X_{\mu}+\partial_{2}X^{\mu}\partial_{2}X_{\mu}\right)$$

$$z=\sigma^{1}+i\sigma^{2},\bar{z}=\sigma^{1}-i\sigma^{2}$$

$$\partial_{z}=\frac{1}{2}(\partial_{1}-i\partial_{2}),\partial_{\bar{z}}=\frac{1}{2}(\partial_{1}+i\partial_{2})$$

$$\partial_{z}z=1,\partial_{z}\bar{z}=0,\partial_{\bar{z}}z=0,\partial_{\bar{z}}\bar{z}=1$$

$$v^{z}=v^{1}+iv^{2},v^{\bar{z}}=v^{1}-iv^{2},v_{z}=\frac{1}{2}(v^{1}-iv^{2}),v_{\bar{z}}=\frac{1}{2}(v^{1}+iv^{2})$$

$$g_{z\bar{z}}=g_{\bar{z}z}=\frac{1}{2},g_{zz}=g_{\bar{z}\bar{z}}=0,g^{z\bar{z}}=g^{\bar{z}z}=2,g^{zz}=g^{\bar{z}\bar{z}}=0$$

$$d^{2}z=2d\sigma^{1}d\sigma^{2}$$

$$\int d^{2}z\delta^{2}(z,\bar{z})=1$$

$$\int_{R}d^{2}z(\partial_{z}v^{z}+\partial_{\bar{z}}v^{\bar{z}})=i\oint_{\partial R}(v^{z}d\bar{z}-v^{\bar{z}}dz)$$

$$S=\frac{1}{2\pi\alpha'}\int d^{2}z\partial X^{\mu}\bar{\partial}X_{\mu}$$

$$\partial\bar{\partial}X^{\mu}(z,\bar{z})=0$$

$$\partial\left(\bar{\partial}X^{\mu}\right)=\bar{\partial}(\partial X^{\mu})=0$$

$$\langle\mathcal{F}[X]\rangle=\int[dX]\exp(-S)\mathcal{F}[X]$$

$$0=\int[dX]\frac{\delta}{\delta X_{\mu}(z,\bar{z})}\exp(-S)=-\int[dX]\exp(-S)\frac{\delta S}{\delta X_{\mu}(z,\bar{z})}=-\left\langle\frac{\delta S}{\delta X_{\mu}(z,\bar{z})}\right\rangle=\frac{1}{\pi\alpha'}\left\langle\partial\bar{\partial}X^{\mu}(z,\bar{z})\right\rangle$$

$$\langle \partial\bar{\partial} X^\mu(z,\bar{z}) \ldots \rangle = 0$$

$$\partial\bar{\partial}\hat{X}^\mu(z,\bar{z}) = 0$$

$$0 = \int [dX] \frac{\delta}{\delta X_\mu(z,\bar{z})}[\exp(-S) X^v(z',\bar{z}')]$$

$$= \int [dX] \exp(-S)\left[\eta^{\mu v}\delta^2(z-z',\bar{z}-\bar{z}') + \frac{1}{\pi\alpha'}\partial_z\partial_{\bar{z}}X^\mu(z,\bar{z})X^v(z',\bar{z}')\right]$$

$$= \eta^{\mu v}\langle\delta^2(z-z',\bar{z}-\bar{z}')\rangle + \frac{1}{\pi\alpha'}\partial_z\partial_{\bar{z}}\langle X^\mu(z,\bar{z})X^v(z',\bar{z}')\rangle$$

$$\frac{1}{\pi\alpha'}\partial_z\partial_{\bar{z}}\langle X^\mu(z,\bar{z})X^v(z',\bar{z}') \ldots \rangle = -\eta^{\mu v}\langle\delta^2(z-z',\bar{z}-\bar{z}') \ldots \rangle$$

$$\frac{1}{\pi\alpha'}\partial_z\partial_{\bar{z}}X^\mu(z,\bar{z})X^v(z',\bar{z}') = -\eta^{\mu v}\delta^2(z-z',\bar{z}-\bar{z}')$$

$$:X^\mu(z,\bar{z}) := X^\mu(z,\bar{z})$$

$$:X^\mu(z_1,\bar{z}_1)X^v(z_2,\bar{z}_2) := X^\mu(z_1,\bar{z}_1)X^v(z_2,\bar{z}_2) + \frac{\alpha'}{2}\eta^{\mu v}\ln|z_{12}|^2$$

$$z_{ij} = z_i - z_j$$

$$\partial_1\bar{\partial}_1 : X^\mu(z_1,\bar{z}_1)X^v(z_2,\bar{z}_2) := 0$$

$$\partial\bar{\partial}\ln|z|^2 = 2\pi\delta^2(z,\bar{z}).$$

$$\langle \mathcal{A}_{i_1}(z_1,\bar{z}_1)\mathcal{A}_{i_2}(z_2,\bar{z}_2) \ldots \mathcal{A}_{i_n}(z_n,\bar{z}_n)\rangle,$$

$$\mathcal{A}_i(\sigma_1)\mathcal{A}_j(\sigma_2) = \sum_k c_{ij}^k(\sigma_1-\sigma_2)\mathcal{A}_k(\sigma_2).$$

$$\langle \mathcal{A}_i(\sigma_1)\mathcal{A}_j(\sigma_2) \ldots \rangle = \sum_k c_{ij}^k(\sigma_1-\sigma_2)\langle \mathcal{A}_k(\sigma_2) \ldots \rangle$$

$$X^\mu(z_1,\bar{z}_1)X^v(z_2,\bar{z}_2)$$

$$= -\frac{\alpha'}{2}\eta^{\mu v}\ln|z_{12}|^2 + :X^v X^\mu(z_2,\bar{z}_2)$$

$$+ \sum_{k=1}^{\infty}\frac{1}{k!}\left[(z_{12})^k : X^v\partial^k X^\mu(z_2,\bar{z}_2) : + (\bar{z}_{12})^k : X^v\bar{\partial}^k X^\mu(z_2,\bar{z}_2) :\right]$$

$$:X^{\mu_1}(z_1,\bar{z}_1) \ldots X^{\mu_n}(z_n,\bar{z}_n) := X^{\mu_1}(z_1,\bar{z}_1) \ldots X^{\mu_n}(z_n,\bar{z}_n) + \sum \text{ subtractions}$$

$$:X^{\mu_1}(z_1,\bar{z}_1)X^{\mu_2}(z_2,\bar{z}_2)X^{\mu_3}(z_3,\bar{z}_3):$$

$$= X^{\mu_1}(z_1,\bar{z}_1)X^{\mu_2}(z_2,\bar{z}_2)X^{\mu_3}(z_3,\bar{z}_3)$$

$$+ \left(\frac{\alpha'}{2}\eta^{\mu_1\mu_2}\ln|z_{12}|^2 X^{\mu_3}(z_3,\bar{z}_3) + 2\text{ permutations}\right)$$

$$:\mathcal{F}:= \exp\left(\frac{\alpha'}{4}\int d^2z_1 d^2z_2 \ln|z_{12}|^2 \frac{\delta}{\delta X^\mu(z_1,\bar{z}_1)}\frac{\delta}{\delta X_\mu(z_2,\bar{z}_2)}\right)\mathcal{F}$$

$$\mathcal{F} = \exp\left(-\frac{\alpha'}{4}\int d^2z_1 d^2z_2 \ln|z_{12}|^2 \frac{\delta}{\delta X^\mu(z_1,\bar{z}_1)}\frac{\delta}{\delta X_\mu(z_2,\bar{z}_2)}\right):\mathcal{F}:=:\mathcal{F}:+\sum \text{ contractions}$$

$$:\mathcal{F}::\mathcal{G}:=:\mathcal{F}\mathcal{G}:+\sum \text{ cross-contractions}$$

$$:\mathcal{F}::\mathcal{G}:= \exp\left(-\frac{\alpha'}{2}\int d^2z_1 d^2z_2 \ln|z_{12}|^2 \frac{\delta}{\delta X_F^\mu(z_1,\bar{z}_1)}\frac{\delta}{\delta X_{G\mu}(z_2,\bar{z}_2)}\right):\mathcal{F}\mathcal{G}:,$$

$$:\partial X^\mu(z)\partial X_\mu(z)::\partial' X^\nu(z')\partial' X_\nu(z'):$$

$$=:\partial X^\mu(z)\partial X_\mu(z)\partial' X^\nu(z')\partial' X_\nu(z'):-4\cdot\frac{\alpha'}{2}(\partial\partial'\ln|z-z'|^2):\partial X^\mu(z)\partial' X_\mu(z'):+2$$

$$\cdot\,\eta_\mu^\mu\left(-\frac{\alpha'}{2}\partial\partial'\ln|z-z'|^2\right)^2$$

$$\sim \frac{D\alpha'^2}{2(z-z')^4} - \frac{2\alpha'}{(z-z')^2}:\partial' X^\mu(z')\partial' X_\mu(z'):-\frac{2\alpha'}{z-z'}:\partial'^2 X^\mu(z')\partial' X_\mu(z'):$$

$$\mathcal{F} = e^{ik_1\cdot X(z,\bar{z})}, \mathcal{G} = e^{ik_2\cdot X(0,0)}.$$

$$:e^{ik_1\cdot X(z,\bar{z})}::e^{ik_2\cdot X(0,0)}:= \exp\left(\frac{\alpha'}{2}k_1\cdot k_2\ln|z|^2\right):e^{ik_1\cdot X(z,\bar{z})}e^{ik_2\cdot X(0,0)}=|z|^{\alpha' k_1\cdot k_2}:e^{ik_1\cdot X(z,\bar{z})}e^{ik_2\cdot X(0,0)}:$$

$$:e^{ik_1\cdot X(z,\bar{z})}::e^{ik_2\cdot X(0,0)}:=|z|^{\alpha' k_1\cdot k_2}:e^{i(k_1+k_2)\cdot X(0,0)}[1+O(z,\bar{z})]:.$$

$$c_{ij}^k(\sigma_1-\sigma_2)_{\text{sym}} = \pm c_{ji}^k(\sigma_2-\sigma_1)_{\text{sym}}$$

$$\phi'_\alpha(\sigma) = \phi_\alpha(\sigma) + \delta\phi_\alpha(\sigma),$$

$$[d\phi']\exp(-S[\phi']) = [d\phi]\exp(-S[\phi]).$$

$$\phi'_\alpha(\sigma) = \phi_\alpha(\sigma) + \rho(\sigma)\delta\phi_\alpha(\sigma)$$

$$[d\phi']\exp(-S[\phi']) = [d\phi]\exp(-S[\phi])\left[1+\frac{i\epsilon}{2\pi}\int d^d\sigma g^{1/2} j^a(\sigma)\partial_a\rho(\sigma) + O(\epsilon^2)\right]$$

.

$$0=\int[d\phi']\exp(-S[\phi'])\ldots-\int[d\phi]\exp(-S[\phi])\ldots=\frac{\epsilon}{2\pi i}\int d^d\sigma g^{1/2}\rho(\sigma)\langle\nabla_a j^a(\sigma)\ldots\rangle$$

$$\nabla_a j^a=0$$

$$\delta\mathcal{A}(\sigma_0)+\frac{\epsilon}{2\pi i}\int_R d^d\sigma g^{1/2}\nabla_a j^a(\sigma)\mathcal{A}(\sigma_0)=0$$

$$\nabla_a j^a(\sigma)\mathcal{A}(\sigma_0)=g^{-1/2}\delta^d(\sigma-\sigma_0)\frac{2\pi}{i\epsilon}\delta\mathcal{A}(\sigma_0)+\text{ total }\sigma\text{-derivative}$$

$$\int_{\partial R} dAn_a j^a\mathcal{A}(\sigma_0)=\frac{2\pi}{i\epsilon}\delta\mathcal{A}(\sigma_0)$$

$$\oint_{\partial R}(jdz-\tilde{j}d\bar{z})\mathcal{A}(z_0,\bar{z}_0)=\frac{2\pi}{\epsilon}\delta\mathcal{A}(z_0,\bar{z}_0)$$

$$\mathrm{Res}_{z\to z_0}j(z)\mathcal{A}(z_0,\bar{z}_0)+\overline{\mathrm{Res}}_{\bar{z}\to\bar{z}_0}\tilde{j}(\bar{z})\mathcal{A}(z_0,\bar{z}_0)=\frac{1}{i\epsilon}\delta\mathcal{A}(z_0,\bar{z}_0)$$

$$\delta S=\frac{\epsilon a_\mu}{2\pi\alpha'}\int d^2\sigma\partial^a X^\mu\partial_a\rho$$

$$j_a^\mu=\frac{i}{\alpha'}\partial_a X^\mu$$

$$j^\mu(z):e^{ik\cdot X(0,0)}:\sim\frac{k^\mu}{2z}:e^{ik\cdot X(0,0)}:$$
$$\tilde{j}^\mu(\bar{z}):e^{ik\cdot X(0,0)}:\sim\frac{k^\mu}{2\bar{z}}:e^{ik\cdot X(0,0)}:$$

$$j_a=iv^bT_{ab}$$
$$T_{ab}=-\frac{1}{\alpha'}:\left(\partial_a X^\mu\partial_b X_\mu-\frac{1}{2}\delta_{ab}\partial_c X^\mu\partial^c X_\mu\right):.$$

$$T_{z\bar{z}}=0$$

$$\bar{\partial}T_{zz}=\partial T_{\bar{z}\bar{z}}=0$$

$$T(z)\equiv T_{zz}(z),\tilde{T}(\bar{z})\equiv T_{\bar{z}\bar{z}}(\bar{z})$$

$$T(z)=-\frac{1}{\alpha'}:\partial X^\mu\partial X_\mu:,\tilde{T}(\bar{z})=-\frac{1}{\alpha'}:\bar{\partial}X^\mu\bar{\partial}X_\mu:,$$

$$j(z)=iv(z)T(z),\tilde{j}(\bar{z})=iv(z)^*\tilde{T}(\bar{z})$$

$$T(z)X^\mu(0)\sim\frac{1}{z}\partial X^\mu(0),\tilde{T}(\bar{z})X^\mu(0)\sim\frac{1}{\bar{z}}\bar{\partial}X^\mu(0)$$

$$\delta X^\mu=-\epsilon v(z)\partial X^\mu-\epsilon v(z)^*\bar{\partial}X^\mu$$

$$X'^{\mu}(z',\bar{z}') = X^{\mu}(z,\bar{z}), z' = f(z)$$

$$z' = \zeta z$$

$$ds'^2 = dz'd\bar{z}' = \frac{\partial z'}{\partial z}\frac{\partial \bar{z}'}{\partial \bar{z}}dzd\bar{z}$$

$$T(z)\mathcal{A}(0,0) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}}\mathcal{A}^{(n)}(0,0)$$

$$\delta\mathcal{A}(z,\bar{z}) = -\epsilon \sum_{n=0}^{\infty} \frac{1}{n!}\left[\partial^n v(z)\mathcal{A}^{(n)}(z,\bar{z}) + \bar{\partial}^n v(z)^* \tilde{\mathcal{A}}^{(n)}(z,\bar{z})\right]$$

$$\mathcal{A}'(z',\bar{z}') = \zeta^{-h}\bar{\zeta}^{-\tilde{h}}\mathcal{A}(z,\bar{z})$$

$$T(z)\mathcal{A}(0,0) = \cdots + \frac{h}{z^2}\mathcal{A}(0,0) + \frac{1}{z}\partial\mathcal{A}(0,0) + \cdots,$$

$$\mathcal{O}'(z',\bar{z}') = (\partial_z z')^{-h}(\partial_{\bar{z}}\bar{z}')^{-\tilde{h}}\mathcal{O}(z,\bar{z})$$

$$T(z)\mathcal{O}(0,0) = \frac{h}{z^2}\mathcal{O}(0,0) + \frac{1}{z}\partial\mathcal{O}(0,0) + \cdots$$

$$\left.\begin{array}{lll} X^{\mu} & (0,0), & \partial X^{\mu} \\ \bar{\partial}X^{\mu} & (0,1), & \partial^2 X^{\mu} \\ :e^{ik\cdot X}: & & \left(\frac{\alpha' k^2}{4}, \frac{\alpha' k^2}{4}\right) \end{array}\right\}$$

$$:\left(\prod_i \partial^{m_i} X^{\mu_i}\right)\left(\prod_j \bar{\partial}^{n_j} X^{\nu_j}\right) e^{ik\cdot X}:,$$

$$\left(\frac{\alpha' k^2}{4} + \sum_i m_i, \frac{\alpha' k^2}{4} + \sum_j n_j\right)$$

$$\mathcal{A}_i(z_1,\bar{z}_1)\mathcal{A}_j(z_2,\bar{z}_2) = \sum_k z_{12}^{h_k-h_i-h_j}\bar{z}_{12}^{\tilde{h}_k-\tilde{h}_i-\tilde{h}_j} c_{ij}^k \mathcal{A}_k(z_2,\bar{z}_2)$$

$$\delta e^{ik\cdot X} = -\epsilon v(z)\partial e^{ik\cdot X} - \epsilon v(z)^* \bar{\partial} e^{ik\cdot X}$$

$$T(z)T(0) = \frac{\eta_{\mu}^{\mu}}{2z^4} - \frac{2}{\alpha' z^2}:\partial X^{\mu}(z)\partial X_{\mu}(0): + :T(z)T(0): \sim \frac{D}{2z^4} + \frac{2}{z^2}T(0) + \frac{1}{z}\partial T(0)$$

$$\epsilon^{-1}\delta T(z) = -\frac{D}{12}\partial_z^3 v(z) - 2\partial_z v(z)T(z) - v(z)\partial_z T(z).$$

$$\epsilon^{-1}\delta T(z) = -\frac{c}{12}\partial_z^3 v(z) - 2\partial_z v(z)T(z) - v(z)\partial_z T(z)$$

$$T(z)T(0) \sim \frac{c}{2z^4} + \frac{2}{z^2}T(0) + \frac{1}{z}\partial T(0)$$

$$(\partial_z z')^2 T'(z') = T(z) - \frac{c}{12}\{z', z\}$$

$$\{f, z\} = \frac{2\partial_z^3 f \partial_z f - 3\partial_z^2 f \partial_z^2 f}{2\partial_z f \partial_z f}$$

$$T(z) = -\frac{1}{\alpha'} : \partial X^\mu \partial X_\mu : + V_\mu \partial^2 X^\mu,$$
$$\tilde{T}(\bar{z}) = -\frac{1}{\alpha'} : \bar{\partial} X^\mu \bar{\partial} X_\mu : + V_\mu \bar{\partial}^2 X^\mu,$$

$$c = \tilde{c} = D + 6\alpha' V_\mu V^\mu$$

$$\delta X^\mu = -\epsilon v \partial X^\mu - \epsilon v^* \bar{\partial} X^\mu - \frac{\epsilon}{2}\alpha' V^\mu [\partial v + (\partial v)^*]$$

$$S = \frac{1}{2\pi}\int d^2 z b \bar{\partial} c$$

$$h_b = \lambda, h_c = 1 - \lambda$$

$$\bar{\partial} c(z) = \bar{\partial} b(z) = 0$$
$$\bar{\partial} b(z) c(0) = 2\pi \delta^2(z, \bar{z})$$

$$: b(z_1) c(z_2) := b(z_1) c(z_2) - \frac{1}{z_{12}}$$

$$\bar{\partial}\frac{1}{z} = \partial \frac{1}{\bar{z}} = 2\pi\delta^2(z, \bar{z})$$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}}, c(z_1)b(z_2) \sim \frac{1}{z_{12}}$$

$$b(z_1)b(z_2) = O(z_{12}), c(z_1)c(z_2) = O(z_{12})$$

$$T(z) =: (\partial b) c : -\lambda \partial (: bc :)$$
$$\tilde{T}(\bar{z}) = 0$$

$$c = -3(2\lambda - 1)^2 + 1, \tilde{c} = 0$$

$$S = \frac{1}{2\pi}\int d^2 z \tilde{b} \partial \tilde{c}$$

$$j = -: bc :$$

$$T(z) j(0) \sim \frac{1 - 2\lambda}{z^3} + \frac{1}{z^2} j(0) + \frac{1}{z}\partial j(0).$$

$$\epsilon^{-1}\delta j = -v\partial j - j\partial v + \frac{2\lambda - 1}{2}\partial^2 v,$$

$$(\partial_z z')j_{z'}(z') = j_z(z) + \frac{2\lambda - 1}{2}\frac{\partial_z^2 z'}{\partial_z z'}.$$

$$\psi \ = 2^{-1/2}(\psi_1 + i\psi_2), \bar{\psi} = 2^{-1/2}(\psi_1 - i\psi_2)$$
$$S \ = \frac{1}{4\pi}\int \ d^2z\big(\psi_1\bar{\partial}\psi_1 + \psi_2\bar{\partial}\psi_2\big)$$
$$T \ = -\frac{1}{2}\psi_1\partial\psi_1 - \frac{1}{2}\psi_2\partial\psi_2$$

$$\beta\gamma CFT$$

$$h_\beta = \lambda, h_\gamma = 1 - \lambda.$$

$$S = \frac{1}{2\pi}\int \ d^2z\beta\bar{\partial}\gamma$$

$$\bar{\partial}\gamma(z) = \bar{\partial}\beta(z) = 0.$$

$$\beta(z_1)\gamma(z_2) \sim -\frac{1}{z_{12}}, \gamma(z_1)\beta(z_2) \sim \frac{1}{z_{12}}.$$

$$T =: (\partial\beta)\gamma: -\lambda\partial(:\beta\gamma:)$$
$$\tilde{T} = 0$$

$$c = 3(2\lambda - 1)^2 - 1, \tilde{c} = 0$$

$$\sigma^1 \sim \sigma^1 + 2\pi$$

$$-\infty < \sigma^2 < \infty$$

$$w = \sigma^1 + i\sigma^2$$

$$z = \exp\left(-iw\right) = \exp\left(-i\sigma^1 + \sigma^2\right)$$

$$T_{zz}(z) = \sum_{m=-\infty}^{\infty}\frac{L_m}{z^{m+2}}, \tilde{T}_{\bar{z}\bar{z}}(\bar{z}) = \sum_{m=-\infty}^{\infty}\frac{\tilde{L}_m}{\bar{z}^{m+2}}.$$

$$L_m = \oint \ _C\frac{dz}{2\pi iz}z^{m+2}T_{zz}(z)$$

$$T_{ww}(w) = -\sum_{m=-\infty}^{\infty} \ \exp\left(im\sigma^1 - m\sigma^2\right)T_m$$
$$T_{\bar{w}\bar{w}}(\bar{w}) = -\sum_{m=-\infty}^{\infty} \ \exp\left(-im\sigma^1 - m\sigma^2\right)\tilde{T}_m$$

$$T_m = L_m - \delta_{m,0}\frac{c}{24}, \tilde{T}_m = \tilde{L}_m - \delta_{m,0}\frac{\tilde{c}}{24}$$

$$T_{ww} = (\partial_w z)^2 T_{zz} + \frac{c}{24}$$

$$H = \int_0^{2\pi} \frac{d\sigma^1}{2\pi} T_{22} = L_0 + \tilde{L}_0 - \frac{c+\tilde{c}}{24}$$

$$Q_i\{C\} = \oint_C \frac{dz}{2\pi i} j_i$$

$$Q_1\{C_1\}Q_2\{C_2\} - Q_1\{C_3\}Q_2\{C_2\}$$

$$\hat{Q}_1\hat{Q}_2 - \hat{Q}_2\hat{Q}_1 \equiv [\hat{Q}_1, \hat{Q}_2]$$

$$[Q_1, Q_2]\{C_2\} = \oint_{C_2} \frac{dz_2}{2\pi i} \mathrm{Res}_{z_1\to z_2} j_1(z_1) j_2(z_2)$$

$$[Q, \mathcal{A}(z_2, \bar{z}_2)] = \mathrm{Res}_{z_1\to z_2} j(z_1)\mathcal{A}(z_2, \bar{z}_2) = \frac{1}{i\epsilon}\delta\mathcal{A}(z_2, \bar{z}_2)$$

$$\tilde{Q}\{C\} = -\oint_C \frac{d\bar{z}}{2\pi i} \tilde{j}$$

$$[\tilde{Q}, \mathcal{A}(z_2, \bar{z}_2)] = \overline{\mathrm{Res}}_{\bar{z}_1\to \bar{z}_2} \tilde{j}(\bar{z}_1)\mathcal{A}(z_2, \bar{z}_2) = \frac{1}{i\epsilon}\delta\mathcal{A}(z_2, \bar{z}_2)$$

$$\mathrm{Res}_{z_1\to z_2} z_1^{m+1} T(z_1) z_2^{n+1} T(z_2) = \mathrm{Res}_{z_1\to z_2} z_1^{m+1} z_2^{n+1}\left(\frac{c}{2z_{12}^4} + \frac{2}{z_{12}^2}T(z_2) + \frac{1}{z_{12}}\partial T(z_2)\right)$$
$$= \frac{c}{12}(\partial^3 z_2^{m+1}) z_2^{n+1} + 2(\partial z_2^{m+1}) z_2^{n+1} T(z_2) + z_2^{m+n+2}\partial T(z_2) = \frac{c}{12}(m^3 - m) z_2^{m+n-1} + (m-n) z_2^{m+n+1} T(z_2)$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m,-n}$$

$$[L_0, L_n] = -nL_n$$

$$L_0 L_n|\psi\rangle = L_n(L_0 - n)|\psi\rangle = (h-n)L_n|\psi\rangle$$

$$[L_0, L_1] = -L_1, [L_0, L_{-1}] = L_{-1}, [L_1, L_{-1}] = 2L_0$$

$$\mathcal{O}(z) = \sum_{m=-\infty}^{\infty} \frac{\mathcal{O}_m}{z^{m+h}}$$

$$[L_m, \mathcal{O}_n] = [(h-1)m - n]\mathcal{O}_{m+n}$$

$$0 \le \mathrm{Re} w \le \pi \iff \mathrm{Im} z \ge 0$$

$$T_{ab} n^a t^b = 0$$

$$T_{ww} = T_{\bar{w}\bar{w}}, \mathrm{Re} w = 0, \pi \iff T_{zz} = T_{\bar{z}\bar{z}}, \mathrm{Im} z = 0$$

$$T_{zz}(z) \equiv T_{\bar{z}\bar{z}}(\bar{z}'), \mathrm{Im} z < 0$$

$$L_m = \frac{1}{2\pi i}\int_C (dz z^{m+1} T_{zz} - d\bar{z}\bar{z}^{m+1} T_{\bar{z}\bar{z}}) = \frac{1}{2\pi i}\oint \, dz z^{m+1} T_{zz}(z)$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m,-n}$$

$$\partial X^\mu(z) = -i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{\infty} \frac{\alpha_m^\mu}{z^{m+1}}, \bar{\partial} X^\mu(\bar{z}) = -i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{\infty} \frac{\tilde{\alpha}_m^\mu}{\bar{z}^{m+1}}.$$

$$\alpha_m^\mu = \left(\frac{2}{\alpha'}\right)^{1/2} \oint \, \frac{dz}{2\pi} z^m \partial X^\mu(z)$$
$$\tilde{\alpha}_m^\mu = -\left(\frac{2}{\alpha'}\right)^{1/2} \oint \, \frac{d\bar{z}}{2\pi} \bar{z}^m \bar{\partial} X^\mu(\bar{z})$$

$$p^\mu = \frac{1}{2\pi i}\oint \, _C(dz j^\mu - d\bar{z}\bar{j}^\mu) = \left(\frac{2}{\alpha'}\right)^{1/2} \alpha_0^\mu = \left(\frac{2}{\alpha'}\right)^{1/2} \tilde{\alpha}_0^\mu$$

$$X^\mu(z,\bar{z}) = x^\mu - i\frac{\alpha'}{2} p^\mu \ln\,|z|^2 + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{1}{m}\left(\frac{\alpha_m^\mu}{z^m} + \frac{\tilde{\alpha}_m^\mu}{\bar{z}^m}\right)$$

$$[\alpha_m^\mu, \alpha_n^v] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^v] = m\delta_{m,-n}\eta^{\mu v}$$
$$[x^\mu, p^v] = i\eta^{\mu v}$$

$$L_m \sim \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n}^\mu \alpha_{\mu n}$$

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} (\alpha_{-n}^\mu \alpha_{\mu n}) + a^X$$

$$2L_0|0;0\rangle = (L_1 L_{-1} - L_{-1} L_1)|0;0\rangle = 0$$

$$a^X = 0$$

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n}^\mu \alpha_{\mu n^\circ}^\circ$$

$$X^\mu(z,\bar{z})X^v(z',\bar{z}') = \mathring{\circ}\, X^\mu(z,\bar{z})X^v(z',\bar{z}')\mathring{2} + \frac{\alpha'}{2}\eta^{\mu v}\left[-\ln\,|z|^2 + \sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{z'^m}{z^m} + \frac{\bar{z}'^m}{\bar{z}^m}\right)\right]$$

$$= \circ\, X^\mu(z,\bar{z})X^v(z',\bar{z}') \circ - \frac{\alpha'}{2}\eta^{\mu v} \mathrm{l} = \mathrm{n}\,|z - z'|^2$$

$$\circ\mathring{\circ} X^\mu(z,\bar{z})X^v(z',\bar{z}') \circ =: X^\mu(z,\bar{z})X^v(z',\bar{z}'):$$

$$[X^\mu(z,\bar{z})X^v(z',\bar{z}')]_1 = [X^\mu(z,\bar{z})X^v(z',\bar{z}')]_2 + \eta^{\mu v}\Delta(z,\bar{z},z',\bar{z}'),$$

$$[\mathcal{F}]_1 = \exp\left(\frac{1}{2}\int d^2 z d^2 z' \Delta(z,\bar{z},z',\bar{z}') \frac{\delta}{\delta X^\mu(z,\bar{z})} \frac{\delta}{\delta X_\mu(z',\bar{z}')}\right)[\mathcal{F}]_2$$

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} {}_\circ\alpha^\mu_{m-n}\alpha^\circ_{\mu n}{}^\circ + i\left(\frac{\alpha'}{2}\right)^{1/2}(m+1)V^\mu\alpha_{\mu m}$$

$$b(z) = \sum_{m=-\infty}^{\infty}\frac{b_m}{z^{m+\lambda}}, c(z) = \sum_{m=-\infty}^{\infty}\frac{c_m}{z^{m+1-\lambda}}$$

$$\{b_m, c_n\} = \delta_{m,-n}$$

$$\begin{gathered} b_0|\downarrow\rangle = 0, b_0|\uparrow\rangle = |\downarrow\rangle, \\ c_0|\downarrow\rangle = |\uparrow\rangle, c_0|\uparrow\rangle = 0, \\ b_n|\downarrow\rangle = b_n|\uparrow\rangle = c_n|\downarrow\rangle = c_n|\uparrow\rangle = 0, n > 0. \end{gathered}$$

$$L_m = \sum_{n=-\infty}^{\infty}(m\lambda - n){}^\circ_\circ b_n c^\circ_{m-n}{}^\circ + \delta_{m,0}a^{\mathrm{g}}$$

$$2L_0|\downarrow\rangle = (L_1L_{-1} - L_{-1}L_1)|\downarrow\rangle = (\lambda b_0 c_1)[(1-\lambda)b_{-1}c_0]|\downarrow\rangle = \lambda(1-\lambda)|\downarrow\rangle$$

$$L_m = \sum_{n=-\infty}^{\infty}(m\lambda - n){}^\circ b_n c_{m-n}{}^\circ + \frac{\lambda(1-\lambda)}{2}\delta_{m,0}$$

$$N^{\mathrm{g}} = -\frac{1}{2\pi i}\int_0^{2\pi} dw j_w = \sum_{n=1}^{\infty}(c_{-n}b_n - b_{-n}c_n) + c_0 b_0 - \frac{1}{2}$$

$$[N^{\mathrm{g}}, b_m] = -b_m, [N^{\mathrm{g}}, c_m] = c_m$$

$$N^{\mathrm{g}}|\downarrow\rangle = -\frac{1}{2}|\downarrow\rangle, N^{\mathrm{g}}|\uparrow\rangle = \frac{1}{2}|\uparrow\rangle.$$

$$\alpha_0^\mu = (2\alpha')^{1/2}p^\mu$$

$$X^\mu(z,\bar{z}) = x^\mu - i\alpha' p^\mu \ln|z|^2 + i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty}\frac{\alpha_m^\mu}{m}(z^{-m} + \bar{z}^{-m})$$

$$L_0 = \alpha' p^2 + \sum_{n=1}^{\infty}\alpha^\mu_{-n}\alpha_{\mu n}$$

$$[\alpha^\mu_m, \alpha^v_n] = m\delta_{m,-n}\eta^{\mu v}, [x^\mu, p^v] = i\eta^{\mu v}$$

$$c(z) = \tilde{c}(\bar{z}), b(z) = \tilde{b}(\bar{z}), \mathrm{Im}z = 0$$

$$c(z) \equiv \tilde{c}(\bar{z}'), b(z) \equiv \tilde{b}(\bar{z}'), \mathrm{Im}(z) \leq 0, z' = \bar{z}$$

$$0 \le \mathrm{Re}w \le 2\pi, w \sim w + 2\pi, \mathrm{Im}w \le 0$$

$$|1\rangle = |0;0\rangle$$

$$\alpha^{\mu}_{-m} = \left(\frac{2}{\alpha'}\right)^{1/2} \oint \frac{dz}{2\pi} z^{-m} \partial X^{\mu}(z) \to \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!} \partial^{m} X^{\mu}(0)$$

$$\alpha^{\mu}_{-m}|1\rangle \cong \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!} \partial^{m} X^{\mu}(0), m \ge 1$$

$$\tilde{\alpha}^{\mu}_{-m}|1\rangle \cong \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!} \bar{\partial}^{m} X^{\mu}(0), m \ge 1.$$

$$\alpha^{\mu}_{-m} : \mathcal{A}(0,0) : = : \alpha^{\mu}_{-m} \mathcal{A}(0,0) :$$

$$\alpha^{\mu}_{-m} \to i\left(\frac{2}{\alpha'}\right)^{1/2} \frac{1}{(m-1)!} \partial^{m} X^{\mu}(0), \quad m \ge 1$$
$$\tilde{\alpha}^{\mu}_{-m} \to i\left(\frac{2}{\alpha'}\right)^{1/2} \frac{1}{(m-1)!} \bar{\partial}^{m} X^{\mu}(0), \quad m \ge 1$$

$$x^{\mu}_{0} \to X^{\mu}(0,0)$$

$$|0;k\rangle \cong : e^{ik\cdot X(0,0)} :$$

$$b_m|1\rangle = 0, m \ge -1, c_m|1\rangle = 0, m \ge 2$$

$$|1\rangle = b_{-1}|\downarrow\rangle$$

$$b_{-m} \to \frac{1}{(m-2)!} \partial^{m-2} b(0), m \ge 2$$
$$c_{-m} \to \frac{1}{(m+1)!} \partial^{m+1} c(0), m \ge -1$$

$$(\partial_z w) j_w(w) = j_z(z) + q_0 \frac{\partial_z^2 w}{\partial_z w} = j_z(z) - \frac{q_0}{z}$$

$$Q^{\mathrm{g}} \equiv \frac{1}{2\pi i} \oint dz j_z = N^{\mathrm{g}} + q_0$$

$$0 \le \mathrm{Re}w \le \pi, \mathrm{Im}w \le 0$$

$$\Psi_{\mathcal{A}}[\phi_{\mathrm{b}}] = \int [d\phi_{\mathrm{i}}]_{\phi_{\mathrm{b}}} \exp\left(-S[\phi_{\mathrm{i}}]\right) \mathcal{A}(0)$$

$$\int [d\phi'_{\mathrm{b}}][d\phi_{\mathrm{i}}]_{\phi_{\mathrm{b}},\phi'_{\mathrm{b}}} \exp\left(-S[\phi_{\mathrm{i}}]\right) r^{-L_0-\tilde{L}_0} \Psi[\phi'_{\mathrm{b}}]$$

$$X_{\mathrm{b}}(\theta) = \sum_{n=-\infty}^{\infty} X_n e^{in\theta}, X_n^* = X_{-n}$$

$$\Psi_1[X_\mathrm{b}] = \int [dX_\mathrm{i}]_{X_\mathrm{b}} \exp\left(-\frac{1}{2\pi\alpha'}\int d^2z\partial X\bar{\partial}X\right)$$

$$X_\mathrm{i} = X_\mathrm{cl} + X'_\mathrm{i}$$

$$X_\mathrm{cl}(z,\bar{z}) = X_0 + \sum_{n=1}^{\infty} (z^n X_n + \bar{z}^n X_{-n})$$

$$\Psi_1[X_\mathrm{b}] = \exp(-S_\mathrm{cl}) \int [dX'_\mathrm{i}]_{X_\mathrm{b}=0} \exp\left(-\frac{1}{2\pi\alpha'}\int d^2z\partial X'\bar{\partial}X'\right)$$

$$S_\mathrm{cl} = \frac{1}{\alpha'}\sum_{m=1}^{\infty} mX_m X_{-m} = \frac{1}{\alpha'}\sum_{m=1}^{\infty} mX_m X_{-m}$$

$$\Psi_1[X_\mathrm{b}] \propto \exp\left(-\frac{1}{\alpha'}\sum_{m=1}^{\infty} mX_m X_{-m}\right)$$

$$\alpha_n = -\frac{in}{(2\alpha')^{1/2}}X_{-n} - i\left(\frac{\alpha'}{2}\right)^{1/2}\frac{\partial}{\partial X_n}$$

$$\tilde{\alpha}_n = -\frac{in}{(2\alpha')^{1/2}}X_n - i\left(\frac{\alpha'}{2}\right)^{1/2}\frac{\partial}{\partial X_{-n}}$$

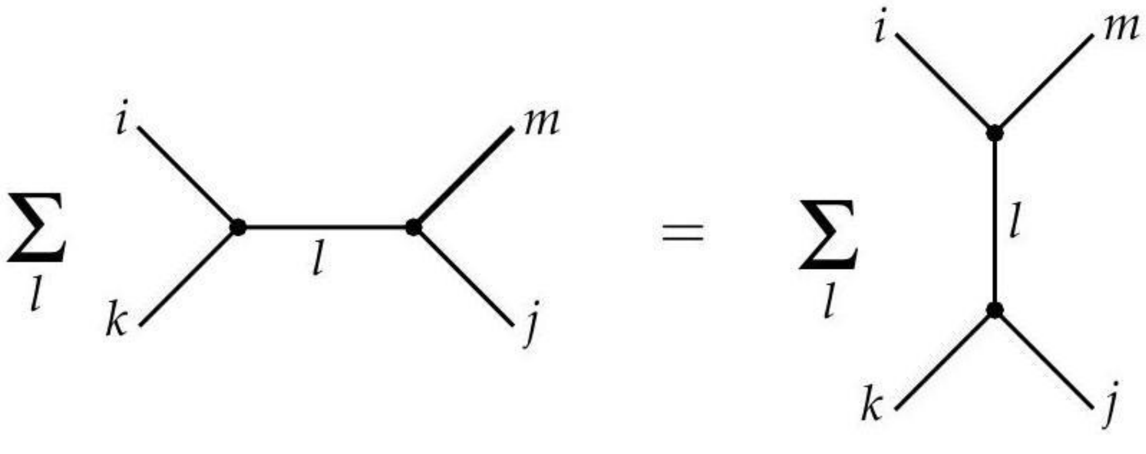
$$\alpha_n\Psi_1[X_\mathrm{b}] = \tilde{\alpha}_n\Psi_1[X_\mathrm{b}] = 0, n \geq 0,$$

$$|1\rangle \propto |0;0\rangle.$$

$$|\partial^k X\rangle = k!\, X_k\Psi_1 = -i\left(\frac{\alpha'}{2}\right)^{1/2}(k-1)!\,\alpha_{-k}|0;0\rangle,$$

$$\mathcal{A}_i(z,\bar{z})\mathcal{A}_j(0,0),$$

$$\mathcal{A}_{ij,z,\bar{z}} = \sum_k z^{h_k-h_i-h_j}\bar{z}^{\tilde{h}_k-\tilde{h}_i-\tilde{h}_j} c_{ij}^k \mathcal{A}_k,$$



$$\mathcal{A}_i(0,0)\mathcal{A}_j(1,1)\mathcal{A}_k(z,\bar{z}),$$

$$L_m|\mathcal{A}\rangle \cong \oint \frac{dz}{2\pi i} z^{m+1}T(z)\mathcal{A}(0,0) \cong L_m \cdot \mathcal{A}(0,0)$$

$$T(z)\mathcal{A}(0,0) = \sum_{m=-\infty}^{\infty} z^{-m-2}L_m \cdot \mathcal{A}(0,0).$$

$$\delta\mathcal{A}(z,\bar{z}) = -\epsilon\sum_{n=0}^{\infty}\frac{1}{n!}\left[\partial^n v(z)L_{n-1} + (\partial^n v(z))^*\tilde{L}_{n-1}\right]\cdot\mathcal{A}(z,\bar{z})$$

$$L_{-1}\cdot\mathcal{A} = \partial\mathcal{A}, \tilde{L}_{-1}\cdot\mathcal{A} = \bar{\partial}\mathcal{A},$$
$$L_0\cdot\mathcal{A} = h\mathcal{A}, \tilde{L}_0\cdot\mathcal{A} = \tilde{h}\mathcal{A}.$$

$$L_0|\mathcal{O}\rangle = h|\mathcal{O}\rangle, \tilde{L}_0|\mathcal{O}\rangle = \tilde{h}|\mathcal{O}\rangle$$
$$L_m|\mathcal{O}\rangle = \tilde{L}_m|\mathcal{O}\rangle = 0, m > 0$$

$$L_m|1\rangle = \tilde{L}_m|1\rangle = 0, m \geq -1$$

$$SL(2,\mathbf{C}).$$

$$L_m^\dagger = L_{-m}, \tilde{L}_m^\dagger = \tilde{L}_{-m}$$

$$\langle 0;k \mid 0;k'\rangle = 2\pi\delta(k-k')$$

$$\alpha_m^\dagger = \alpha_{-m}, \tilde{\alpha}_m^\dagger = \tilde{\alpha}_{-m}$$

$$2h_{\mathcal{O}}\langle\mathcal{O}\mid\mathcal{O}\rangle = 2\langle\mathcal{O}|L_0|\mathcal{O}\rangle = \langle\mathcal{O}|[L_1,L_{-1}]|\mathcal{O}\rangle = \|L_{-1}|\mathcal{O}\rangle\|^2 \geq 0$$

$$L_{-1}\cdot\mathcal{O} = \tilde{L}_{-1}\cdot\mathcal{O} = 0$$

$$\partial\mathcal{A} = 0 \Leftrightarrow h = 0, \bar{\partial}\mathcal{A} = 0 \Leftrightarrow \tilde{h} = 0.$$

$$E = -\frac{c+\tilde{c}}{24}$$

$$E = -\frac{\pi(c+\tilde{c})}{12\ell}$$

$$\sum_{n=1}^{\infty}(n-\theta) = \frac{1}{24} - \frac{1}{8}(2\theta-1)^2$$

$$\partial\bar{\partial}\ln|z|^2 = \partial\frac{1}{\bar{z}} = \bar{\partial}\frac{1}{z} = 2\pi\delta^2(z,\bar{z})$$

$$\left\langle\prod_{i=1}^{n} :e^{ik_i\cdot X(z_i,\bar{z}_i)}:\right\rangle = iC^X(2\pi)^D\delta^D\left(\sum_{i=1}^{n}k_i\right)\prod_{\substack{i,j=1\\i<j}}^{n}|z_{ij}|^{\alpha' k_i\cdot k_j}$$

$$S[\phi]=\int d^{d}\sigma\mathcal{L}(\phi(\sigma),\partial_{a}\phi(\sigma))$$

$$\delta\mathcal{L}=\epsilon\partial_{a}\mathcal{K}^{a}$$

$$j^{a}=2\pi i\left(\frac{\partial\mathcal{L}}{\partial\phi_{\alpha,a}}\epsilon^{-1}\delta\phi_{\alpha}-\mathcal{K}^{a}\right).$$

$$ {}^{\star}_{\star}e^{ik_{1}\cdot X(y_{1})}{}_{\star}^{\star\star}e^{ik_{2}\cdot X(y_{2})}{}_{\star},y_{1}\rightarrow y_{2}(y)$$
$$:e^{ik\cdot X(z,\bar{z})}:,\operatorname{Im}(z)\rightarrow 0$$

$$L_{m}(L_{-m}|0;0\rangle)-L_{-m}(L_{m}|0;0\rangle)$$

$$:b(z)c(z'):-{}^{\circ}_{\circ}b(z)c(z'){}^{\circ}_{\circ}=\frac{(z/z')^{1-\lambda}-1}{z-z'}$$

$$\exp\left(iS_{\mathrm{cl}}/\hbar\right)$$

$$X_{\text{upper}}^{\mu}(\sigma)=X_{\text{lower}}^{\mu}(\pi-\sigma)$$

$$\int[dXdg]\exp\left(-S\right)$$

$$S=S_{X}+\lambda\chi$$

$$S_{X}=\frac{1}{4\pi\alpha'}\int_{M}d^{2}\sigma g^{1/2}g^{ab}\partial_{a}X^{\mu}\partial_{b}X_{\mu}$$
$$\chi=\frac{1}{4\pi}\int_{M}d^{2}\sigma g^{1/2}R+\frac{1}{2\pi}\int_{\partial M}dsk$$

$$\int[d\eta dX]\exp\left[\frac{i}{2}\int d\tau\left(\eta^{-1}\dot{X}^{\mu}\dot{X}_{\mu}-\eta m^{2}\right)\right]$$

$$\eta(\tau)\rightarrow e^{-i\theta}\eta(\tau),X^{0}(\tau)\rightarrow e^{-i\theta}X^{0}(\tau)$$

$$\int[dedX]\exp\left[-\frac{1}{2}\int d\tau\left(e^{-1}\sum_{\mu=1}^{D}\dot{X}^{\mu}\dot{X}^{\mu}+em^{2}\right)\right]$$

$$g_{\mathrm{o}}^{2}\sim g_{\mathrm{c}}\sim e^{\lambda}$$

$$\chi=\tilde{\chi}+\frac{1}{4}n_{\mathrm{c}},$$

$$\exp\left(-\lambda\tilde{\chi}\right)=\exp\left(-\lambda\chi+\lambda n_{\mathrm{c}}/4\right)$$

$$\int\frac{[dXdg]}{V_{\text{diff}\times\text{Weyl}}}\exp\left(-S\right)\equiv Z$$

$$g_{ab}(\sigma)\rightarrow\hat{g}_{ab}(\sigma)$$

$$\hat{g}_{ab}(\sigma)=\delta_{ab}$$

$$\hat{g}_{ab}(\sigma) = \exp[2\omega(\sigma)]\delta_{ab}$$

$$g'^{1/2}R' = g^{1/2}(R - 2\nabla^2\omega).$$

$$R_{abcd} = \frac{1}{2}(g_{ac}g_{bd} - g_{ad}g_{bc})R$$

$$z' \equiv \sigma'^1 + i\sigma'^2 = f(z)$$

$$ds'^2 = \exp(2\omega)|\partial_z f|^{-2}dz'd\bar{z}'$$

$$\omega = \ln|\partial_z f|$$

$$\zeta: g \to g^\zeta, g^\zeta_{ab}(\sigma') = \exp[2\omega(\sigma)]\frac{\partial\sigma^c}{\partial\sigma'^a}\frac{\partial\sigma^d}{\partial\sigma'^b}g_{cd}(\sigma)$$

$$1 = \Delta_{\mathrm{FP}}(g)\int [d\zeta]\delta(g - \hat{g}^\zeta)$$

$$Z[\hat{g}] = \int \frac{[d\zeta dX dg]}{V_{\text{diff} \times \text{Weyl}}}\Delta_{\mathrm{FP}}(g)\delta(g - \hat{g}^\zeta)\exp(-S[X, g]).$$

$$Z[\hat{g}] = \int \frac{[d\zeta dX^\zeta]}{V_{\text{diff} \times \text{Weyl}}}\Delta_{\mathrm{FP}}(\hat{g}^\zeta)\exp(-S[X^\zeta, \hat{g}^\zeta])$$

$$Z[\hat{g}] = \int \frac{[d\zeta dX]}{V_{\text{diff} \times \text{Weyl}}}\Delta_{\mathrm{FP}}(\hat{g})\exp(-S[X, \hat{g}])$$

$$\Delta_{\mathrm{FP}}(g^\zeta)^{-1} = \int [d\zeta']\delta(g^\zeta - \hat{g}^{\zeta'}) = \int [d\zeta']\delta(g - \hat{g}^{\zeta^{-1}\cdot\zeta'})$$
$$= \int [d\zeta'']\delta(g - \hat{g}^{\zeta''}) = \Delta_{\mathrm{FP}}(g)^{-1},$$

$$Z[\hat{g}] = \int [dX]\Delta_{\mathrm{FP}}(\hat{g})\exp(-S[X, \hat{g}])$$

$$\delta g_{ab} = 2\delta\omega g_{ab} - \nabla_a\delta\sigma_b - \nabla_b\delta\sigma_a = (2\delta\omega - \nabla_c\delta\sigma^c)g_{ab} - 2(P_1\delta\sigma)_{ab}$$

$$(P_1\delta\sigma)_{ab} = \frac{1}{2}(\nabla_a\delta\sigma_b + \nabla_b\delta\sigma_a - g_{ab}\nabla_c\delta\sigma^c)$$

$$\Delta_{\mathrm{FP}}(\hat{g})^{-1} = \int [d\delta\omega d\delta\sigma]\delta[-(2\delta\omega - \hat{\nabla}\cdot\delta\sigma)\hat{g} + 2\hat{P}_1\delta\sigma]$$

$$= \int [d\delta\omega d\beta d\delta\sigma]\exp\left\{2\pi i\int d^2\sigma\hat{g}^{1/2}\beta^{ab}[-(2\delta\omega - \hat{\nabla}\cdot\delta\sigma)\hat{g} + 2\hat{P}_1\delta\sigma]ab\right\}$$

$$= \int [d\beta' d\delta\sigma]\exp\left\{4\pi i\int d^2\sigma\hat{g}^{1/2}\beta'^{ab}(\hat{P}_1\delta\sigma)_{ab}\right\}$$

$$\begin{aligned}\delta\sigma^a &\to c^a\\ \beta'_{ab} &\to b_{ab}\end{aligned}$$

$$\Delta_{\mathrm{FP}}(\hat{g}) = \int [dbdc]\exp\left(-S_{\mathrm{g}}\right)$$

$$S_{\mathrm{g}} = \frac{1}{2\pi}\int d^2\sigma\hat{g}^{1/2}b_{ab}\hat{\nabla}^a c^b = \frac{1}{2\pi}\int d^2\sigma\hat{g}^{1/2}b_{ab}\left(\hat{P}_1 c\right)^{ab}$$

$$Z[\hat{g}] = \int [dXdbdc]\exp\left(-S_X - S_{\mathrm{g}}\right)$$

$$Z[\hat{g}] = \left(\det\hat{\nabla}^2\right)^{-D/2}\det\hat{P}_1,$$

$$S_{\mathrm{g}} = \frac{1}{2\pi}\int d^2z(b_{zz}\nabla_{\bar{z}}c^z + b_{\bar{z}\bar{z}}\nabla_z c^{\bar{z}}) = \frac{1}{2\pi}\int d^2z(b_{zz}\partial_{\bar{z}}c^z + b_{\bar{z}\bar{z}}\partial_z c^{\bar{z}})$$

$$n_a\delta\sigma^a = 0$$

$$n_a c^a = 0$$

$$\int_{\partial M} dsn^a b_{ab}\delta c^b = 0$$

$$n_a t_b b^{ab} = 0$$

$$Z\left[g^{\zeta}\right] = Z[g]$$

$$\langle\ldots\rangle_g \equiv \int [dXdbdc]\exp\left(-S[X,b,c,g]\right)\ldots$$

$$\langle\ldots\rangle_{g^{\zeta}} = \langle\ldots\rangle_g$$

$$\delta\langle\ldots\rangle_g = -\frac{1}{4\pi}\int d^2\sigma g(\sigma)^{1/2}\delta g_{ab}(\sigma)\left\langle T^{ab}(\sigma)\ldots\right\rangle_g$$

$$T^{ab}(\sigma) \overset{\text{classical}}{=} \frac{4\pi}{g(\sigma)^{1/2}}\frac{\delta}{\delta g_{ab}(\sigma)}S$$

$$\delta_{\mathrm{W}}\langle\ldots\rangle_g = -\frac{1}{2\pi}\int d^2\sigma g(\sigma)^{1/2}\delta\omega(\sigma)\langle T_a^a(\sigma)\ldots\rangle_g$$

$$T_a^a \overset{?}{=} 0$$

$$T_a^a = a_1 R$$

$$T_{z\bar{z}} = \frac{a_1}{2}g_{z\bar{z}}R$$

$$\nabla^{\bar{z}}T_{\bar{z}z} = \frac{a_1}{2}\nabla^{\bar{z}}(g_{z\bar{z}}R) = \frac{a_1}{2}\partial_z R,$$

$$\nabla^z T_{zz} = -\nabla^{\bar{z}} T_{\bar{z}z} = -\frac{a_1}{2}\partial_z R$$

$$a_1 \partial_z \nabla^2 \delta\omega \approx 4a_1 \partial_z^2 \partial_{\bar{z}} \delta\omega,$$

$$\epsilon^{-1}\delta T_{zz}(z) = -\frac{c}{12}\partial_z^3 v^z(z) - 2\partial_z v^z(z) T_{zz}(z) - v^z(z)\partial_z T_{zz}(z)$$

$$\delta_{\mathrm{W}} T_{zz} = -\frac{c}{6}\partial_z^2 \delta\omega$$

$$c = -12a_1, T_a^a = -\frac{c}{12} R$$

$$R = -2\exp(-2\omega)\partial_a\partial_a\omega$$
$$\nabla^2 = \exp(-2\omega)\partial_a\partial_a$$

$$\delta_{\mathrm{W}} Z[\exp(2\omega)\delta] = \frac{a_1}{\pi} Z[\exp(2\omega)\delta] \int d^2\sigma \delta\omega \partial_a\partial_a\omega$$

$$Z[\exp(2\omega)\delta] = Z[\delta]\exp\left(-\frac{a_1}{2\pi}\int d^2\sigma \partial_a\omega\partial_a\omega\right)$$

$$Z[g] = Z[\delta]\exp\left[\frac{a_1}{8\pi}\int d^2\sigma \int d^2\sigma' g^{1/2}R(\sigma)G(\sigma,\sigma')g^{1/2}R(\sigma')\right]$$

$$g(\sigma)^{1/2}\nabla^2 G(\sigma,\sigma') = \delta^2(\sigma-\sigma')$$

$$\ln\frac{Z[\delta+h]}{Z[\delta]} \approx \frac{a_1}{8\pi^2}\int d^2z \int d^2z' (\partial_z^2 \ln|z-z'|^2) h_{\bar{z}\bar{z}}(z,\bar{z}) \partial_{z'}^2 h_{\bar{z}\bar{z}}(z',\bar{z}')$$

$$= -\frac{3a_1}{4\pi^2}\int d^2z \int d^2z' \frac{h_{\bar{z}\bar{z}}(z,\bar{z})h_{\bar{z}\bar{z}}(z',\bar{z}')}{(z-z')^4}$$

$$\ln\frac{Z[\delta+h]}{Z[\delta]} \approx \frac{1}{8\pi^2}\int d^2z \int d^2z' h_{\bar{z}\bar{z}}(z,\bar{z})h_{\bar{z}\bar{z}}(z',\bar{z}')\langle T_{zz}(z)T_{zz}(z')\rangle_\delta$$

$$c = c^X + c^{\mathrm{g}} = D - 26$$

$$T_a^a = a_1 R + a_2$$

$$S_{\mathrm{ct}} = b\int d^2\sigma g^{1/2}$$

$$T_a^a = a_1 R + (a_2 + 4\pi b)$$

$$\delta_{\mathrm{W}} \ln\langle\ldots\rangle_g = -\frac{1}{2\pi}\int_M d^2\sigma g^{1/2}(a_1 R + a_2)\delta\omega - \frac{1}{2\pi}\int_{\partial M} ds(a_3 + a_4 k + a_5 n^a \partial_a)\delta\omega$$

$$S_{\mathrm{ct}} = \int_M d^2\sigma g^{1/2} b_1 + \int_{\partial M} ds(b_2 + b_3 k)$$

$$\delta_{\mathrm{W}} S_{\mathrm{ct}} = 2\int_M d^2\sigma g^{1/2} b_1 \delta\omega + \int_{\partial M} ds(b_2 + b_3 n^a \partial_a)\delta\omega$$

$$\delta_{\mathrm{W}_1}\left(\delta_{\mathrm{W}_2} \ln \langle \dots \rangle_g\right) = \frac{a_1}{\pi}\int_M d^2\sigma g^{1/2}\delta\omega_2 \nabla^2 \delta\omega_1 - \frac{a_4}{2\pi}\int_{\partial M} ds\delta\omega_2 n^a \partial_a \delta\omega_1$$

$$= -\frac{a_1}{\pi}\int_M d^2\sigma g^{1/2}\partial_a \delta\omega_2 \partial^a \delta\omega_1 + \frac{2a_1 - a_4}{2\pi}\int_{\partial M} ds n^a \delta\omega_2 \partial_a \delta\omega_1$$

$$T_a^a(\sigma) = -\frac{\mathcal{C}(\sigma)}{12} R(\sigma)$$

$$-2\pi t \leq \mathrm{Im} w \leq 0, w \cong w + 2\pi$$

$$z = \exp(-iw), \exp(-2\pi t) \leq |z| \leq 1$$

$$-2\pi t \leq \mathrm{Im} w \leq 0, 0 \leq \mathrm{Re} w \leq \pi$$

$$\exp(-2\pi t) \leq |z| \leq 1, \mathrm{Im} z \geq 0$$

$$S_{j_1 \dots j_n}(k_1, \dots, k_n) = \sum_{\substack{\text{compact} \\ \text{topologics}}} \int \frac{[dXdg]}{V_{\text{diff} \times \text{Weyl}}} \exp(-S_X - \lambda\chi) \prod_{i=1}^{n} \int d^2\sigma_i g(\sigma_i)^{1/2} \mathcal{V}_{j_i}(k_i, \sigma_i)$$

$$V_0 = 2g_{\mathrm{c}} \int d^2\sigma g^{1/2} e^{ik\cdot X} \to g_{\mathrm{c}} \int d^2 z {:} e^{ik\cdot X}{:}$$

$$m^2 = -k^2 = -\frac{4}{\alpha'}$$

$$\frac{2g_c}{\alpha'}\int d^2 z {:} \partial X^\mu \bar{\partial} X^\nu e^{ik\cdot X}{:}.$$

$$h = \tilde{h} = 1 + \frac{\alpha' k^2}{4}$$

$$[\mathcal{F}]_{\mathrm{r}} = \exp\left(\frac{1}{2}\int d^2\sigma d^2\sigma' \Delta(\sigma, \sigma') \frac{\delta}{\delta X^\mu(\sigma)} \frac{\delta}{\delta X_\mu(\sigma')}\right)\mathcal{F}$$

$$\Delta(\sigma, \sigma') = \frac{\alpha'}{2} \ln d^2(\sigma, \sigma')$$

$$\delta_{\mathrm{W}}[\mathcal{F}]_{\mathrm{r}} = [\delta_{\mathrm{W}}\mathcal{F}]_{\mathrm{r}} + \frac{1}{2}\int d^2\sigma d^2\sigma' \delta_{\mathrm{W}}\Delta(\sigma, \sigma') \frac{\delta}{\delta X^\mu(\sigma)} \frac{\delta}{\delta X_\mu(\sigma')} [\mathcal{F}]_{\mathrm{r}}$$

$$\delta_{\mathrm{W}} V_0 = 2g_c \int d^2\sigma g^{1/2}\left(2\delta\omega(\sigma) - \frac{k^2}{2}\delta_{\mathrm{W}}\Delta(\sigma, \sigma)\right)\left[e^{ik\cdot X(\sigma)}\right]_{\mathrm{r}}$$

$$d^2(\sigma, \sigma') \approx (\sigma - \sigma')^2 \exp(2\omega(\sigma))$$

$$\Delta(\sigma,\sigma') \approx \alpha'\omega(\sigma) + \frac{\alpha'}{2}\ln(\sigma-\sigma')^2$$

$$\delta_{\mathrm{W}}\Delta(\sigma,\sigma) = \alpha'\delta\omega(\sigma)$$

$$\delta^D(X(\sigma)-x_0) = \int \frac{d^Dk}{(2\pi)^D}\exp[ik\cdot(X(\sigma)-x_0)]$$

$$\int_M d^2\sigma g(\sigma)^{1/2}\int_M d^2\sigma' g(\sigma')^{1/2}\delta^D\big(X(\sigma)-X(\sigma')\big)$$

$$V_1 = \frac{g_{\mathrm{c}}}{\alpha'}\int d^2\sigma g^{1/2}\left\{\left(g^{ab}s_{\mu\nu}+i\epsilon^{ab}a_{\mu\nu}\right)\left[\partial_aX^\mu\partial_bX^\nu e^{ik\cdot X}\right]_{\mathrm{r}} + \alpha'\phi R\left[e^{ik\cdot X}\right]_{\mathrm{r}}\right\}$$

$$\begin{aligned}
\partial_a\delta_{\mathrm{W}}\Delta(\sigma,\sigma')|_{\sigma'=\sigma} &= \frac{1}{2}\alpha'\partial_a\delta\omega(\sigma)\\
\partial_a\partial_b'\delta_{\mathrm{W}}\Delta(\sigma,\sigma')|_{\sigma'=\sigma} &= \frac{1+\gamma}{2}\alpha'\nabla_a\partial_b\delta\omega(\sigma)\\
\nabla_a\partial_b\delta_{\mathrm{W}}\Delta(\sigma,\sigma')|_{\sigma'=\sigma} &= -\frac{\gamma}{2}\alpha'\nabla_a\partial_b\delta\omega(\sigma)
\end{aligned}$$

$$\delta_{\mathrm{W}}V_1 = \frac{g_{\mathrm{c}}}{2}\int d^2\sigma g^{1/2}\delta\omega\left\{\left(g^{ab}S_{\mu\nu}+i\epsilon^{ab}A_{\mu\nu}\right)\left[\partial_aX^\mu\partial_bX^\nu e^{ik\cdot X}\right]_{\mathrm{r}} + \alpha' FR\left[e^{ik\cdot X}\right]_{\mathrm{r}}\right\}$$

$$\begin{aligned}
S_{\mu\nu} &= -k^2s_{\mu\nu} + k_\nu k^\omega s_{\mu\omega} + k_\mu k^\omega s_{\nu\omega} - (1+\gamma)k_\mu k_\nu s_\omega^\omega + 4k_\mu k_\nu\phi\\
A_{\mu\nu} &= -k^2a_{\mu\nu} + k_\nu k^\omega a_{\mu\omega} - k_\mu k^\omega a_{\nu\omega}\\
F &= (\gamma-1)k^2\phi + \frac{1}{2}\gamma k^\mu k^\nu s_{\mu\nu} - \frac{1}{4}\gamma(1+\gamma)k^2 s_\nu^\nu
\end{aligned}$$

$$\left[\nabla^2X^\mu e^{ik\cdot X}\right]_{\mathrm{r}} = i\frac{\alpha'\gamma}{4}k^\mu R\left[e^{ik\cdot X}\right]_{\mathrm{r}}$$

$$S_{\mu\nu} = A_{\mu\nu} = F = 0$$

$$\begin{aligned}
s_{\mu\nu} &\to s_{\mu\nu} + \xi_\mu k_\nu + k_\mu\xi_\nu\\
a_{\mu\nu} &\to a_{\mu\nu} + \zeta_\mu k_\nu - k_\mu\zeta_\nu\\
\phi &\to \phi + \frac{\gamma}{2}k\cdot\xi
\end{aligned}$$

$$n^\mu s_{\mu\nu} = n^\mu a_{\mu\nu} = 0$$

$$\begin{aligned}
k^2 &= 0\\
k^\nu s_{\mu\nu} &= k^\mu a_{\mu\nu} = 0\\
\phi &= \frac{1+\gamma}{4}s_\mu^\mu
\end{aligned}$$

$$k^\mu = (1,1,0,0,\dots,0),\; n^\mu = \frac{1}{2}(-1,1,0,0,\dots,0)$$

$$
\begin{aligned}
\left[e^{ik\cdot X}\right]_{\mathrm{DR}} &= \left[e^{ik\cdot X}\right]_{\mathrm{r}} \\
\left[\partial_a X^\mu e^{ik\cdot X}\right]_{\mathrm{DR}} &= \left[\partial_a X^\mu e^{ik\cdot X}\right]_{\mathrm{r}} \\
\left[\partial_a X^\mu \partial_b X^\nu e^{ik\cdot X}\right]_{\mathrm{DR}} &= \left[\partial_a X^\mu \partial_b X^\nu e^{ik\cdot X}\right]_{\mathrm{r}} - \frac{\alpha'}{12} g_{ab}\eta^{\mu\nu} R\left[e^{ik\cdot X}\right]_{\mathrm{r}} \\
\left[\nabla_a \partial_b X^\mu e^{ik\cdot X}\right]_{\mathrm{DR}} &= \left[\nabla_a \partial_b X^\mu e^{ik\cdot X}\right]_{\mathrm{r}} + i\frac{\alpha'}{12} g_{ab} k^\mu R\left[e^{ik\cdot X}\right]_{\mathrm{r}}
\end{aligned}
$$

$$
g_{\mathrm{o}} \int_{\partial M} ds \left[e^{ik\cdot X}\right]_{\mathrm{r}}
$$

$$
-i\frac{g_o}{(2\alpha')^{1/2}} e_\mu \int_{\partial M} ds\left[\dot{X}^\mu e^{ik\cdot X}\right]_{\mathrm{r}}
$$

$$
S_{\mathrm{pp}} = \frac{1}{2}\int d\tau\left(\eta^{-1} G_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu - \eta m^2\right)
$$

$$
S_\sigma = \frac{1}{4\pi\alpha'}\int_M d^2\sigma g^{1/2} g^{ab} G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu
$$

$$
G_{\mu\nu}(X) = \eta_{\mu\nu} + \chi_{\mu\nu}(X)
$$

$$
\exp\left(-S_\sigma\right) = \exp\left(-S_{\mathrm{P}}\right)\left(1 - \frac{1}{4\pi\alpha'}\int_M d^2\sigma g^{1/2} g^{ab} \chi_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu + \cdots\right)
$$

$$
\chi_{\mu\nu}(X) = -4\pi g_{\mathrm{c}} e^{ik\cdot X} s_{\mu\nu}.
$$

$$
S_\sigma = \frac{1}{4\pi\alpha'}\int_M d^2\sigma g^{1/2}\left[\left(g^{ab} G_{\mu\nu}(X) + i\epsilon^{ab} B_{\mu\nu}(X)\right)\partial_a X^\mu \partial_b X^\nu + \alpha' R\Phi(X)\right]
$$

$$
\delta B_{\mu\nu}(X) = \partial_\mu \zeta_\nu(X) - \partial_\nu \zeta_\mu(X)
$$

$$
H_{\omega\mu\nu} = \partial_\omega B_{\mu\nu} + \partial_\mu B_{\nu\omega} + \partial_\nu B_{\omega\mu}
$$

$$
G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu = \left[G_{\mu\nu}(x_0) + G_{\mu\nu,\omega}(x_0) Y^\omega + \frac{1}{2} G_{\mu\nu,\omega\rho}(x_0) Y^\omega Y^\rho + \cdots\right]\partial_a Y^\mu \partial_b Y^\nu
$$

$$
\begin{aligned}
G_{\mu\nu}(X) &= \eta_{\mu\nu} - 4\pi g_{\mathrm{c}} s_{\mu\nu} e^{ik\cdot X} \\
B_{\mu\nu}(X) &= -4\pi g_{\mathrm{c}} a_{\mu\nu} e^{ik\cdot X} \\
\Phi(X) &= -4\pi g_{\mathrm{c}} \phi e^{ik\cdot X}
\end{aligned}
$$

$$
T_a^a = -\frac{1}{2\alpha'}\beta_{\mu\nu}^G g^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{i}{2\alpha'}\beta_{\mu\nu}^B \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\beta^\Phi R
$$

$$\beta_{\mu\nu}^{G} \approx -\frac{\alpha'}{2}\left(\partial^2\chi_{\mu\nu} - \partial_\nu\partial^\omega\chi_{\mu\omega} - \partial_\mu\partial^\omega\chi_{\omega\nu} + \partial_\mu\partial_\nu\chi_\omega^\omega\right) + 2\alpha'\partial_\mu\partial_\nu\Phi$$

$$\beta_{\mu\nu}^{B} \approx -\frac{\alpha'}{2}\partial^\omega H_{\omega\mu\nu}$$

$$\beta^{\Phi} \approx \frac{D-26}{6} - \frac{\alpha'}{2}\partial^2\Phi$$

$$\beta_{\mu\nu}^{G} = \alpha'\boldsymbol{R}_{\mu\nu} + 2\alpha'\nabla_\mu\nabla_\nu\Phi - \frac{\alpha'}{4}H_{\mu\lambda\omega}H_\nu{}^{\lambda\omega} + O(\alpha'^2)$$

$$\beta_{\mu\nu}^{B} = -\frac{\alpha'}{2}\nabla^\omega H_{\omega\mu\nu} + \alpha'\nabla^\omega\Phi H_{\omega\mu\nu} + O(\alpha'^2)$$

$$\beta^{\Phi} = \frac{D-26}{6} - \frac{\alpha'}{2}\nabla^2\Phi + \alpha'\nabla_\omega\Phi\nabla^\omega\Phi - \frac{\alpha'}{24}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + O(\alpha'^2)$$

$$\beta_{\mu\nu}^{G} = \beta_{\mu\nu}^{B} = \beta^{\Phi} = 0$$

$$G_{\mu\nu}(X) = \eta_{\mu\nu}, B_{\mu\nu}(X) = 0, \Phi(X) = \Phi_0$$

$$\lambda = \Phi_0$$

$$G_{\mu\nu}(X) = \eta_{\mu\nu}, B_{\mu\nu}(X) = 0, \Phi(X) = V_\mu X^\mu$$

$$V_\mu V^\mu = \frac{26-D}{6\alpha'}.$$

$$\boldsymbol{S} = \frac{1}{2\kappa_0^2}\int d^Dx(-G)^{1/2}e^{-2\Phi}\left[-\frac{2(D-26)}{3\alpha'} + \boldsymbol{R} - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\Phi\partial^\mu\Phi + O(\alpha')\right]$$

$$\delta\boldsymbol{S} = -\frac{1}{2\kappa_0^2\alpha'}\int d^Dx(-G)^{1/2}e^{-2\Phi}\left[\delta G_{\mu\nu}\beta^{G\mu\nu}\right.$$

$$\left. + \delta B_{\mu\nu}\beta^{B\mu\nu} + \left(2\delta\Phi - \frac{1}{2}G^{\mu\nu}\delta G_{\mu\nu}\right)\left(\beta_\omega^{G\omega} - 4\beta^\Phi\right)\right]$$

$$\tilde{G}_{\mu\nu}(x) = \exp\left(2\omega(x)\right)G_{\mu\nu}(x)$$

$$\tilde{\boldsymbol{R}} = \exp(-2\omega)\left[\boldsymbol{R} - 2(D-1)\nabla^2\omega - (D-2)(D-1)\partial_\mu\omega\partial^\mu\omega\right]$$

$$\tilde{\Phi} = \Phi - \Phi_0$$

$$\boldsymbol{S} = \frac{1}{2\kappa^2}\int d^DX(-\tilde{G})^{1/2}\left[-\frac{2(D-26)}{3\alpha'}e^{4\tilde{\Phi}/(D-2)} + \tilde{\boldsymbol{R}} - \frac{1}{12}e^{-8\tilde{\Phi}/(D-2)}H_{\mu\nu\lambda}\tilde{H}^{\mu\nu\lambda} - \frac{4}{D-2}\partial_\mu\tilde{\Phi}\tilde{\partial}^\mu\tilde{\Phi} + O(\alpha')\right]$$

$$\kappa = (8\pi G_{\mathrm{N}})^{1/2} = \frac{(8\pi)^{1/2}}{M_{\mathrm{P}}} = (2.43\times10^{18}\mathrm{GeV})^{-1}$$

$$\boldsymbol{S} = -\frac{1}{4}\int d^Dx\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]$$

$$gA_\mu = A'_\mu, gF_{\mu v} = F'_{\mu v}$$

$$g_{MN} = \begin{bmatrix} \eta_{\mu v} & 0 \\ 0 & g_{mn}(x^p) \end{bmatrix}$$

$$-\frac{1}{4\pi\alpha'}\int_M d^2\sigma g^{1/2} g^{ab}\partial_a X^0 \partial_b X^0$$

$$(t, t') = \int d^2\sigma g^{1/2} t \cdot t'$$

$$\nabla_a j^a = aR$$

$$\delta\langle f \mid i\rangle = -\frac{1}{4\pi}\int d^2\sigma g(\sigma)^{1/2}\delta g_{ab}(\sigma)\langle f|T^{ab}(\sigma)|i\rangle.$$

$$\langle\psi|T^{ab}(\sigma)|\psi'\rangle = 0$$

$$T_{ab} = T^X_{ab} + T^{\mathrm{g}}_{ab}$$

$$T_{ab} = T^{\mathrm{m}}_{ab} + T^{\mathrm{g}}_{ab}$$

$$\left(L^{\mathrm{m}}_n + A\delta_{n,0}\right)|\psi\rangle = 0 \text{ for } n \geq 0$$

$$\langle\psi|L^{\mathrm{m}}_n|\psi'\rangle = \langle L^{\mathrm{m}}_{-n}\psi \mid \psi'\rangle = 0$$

$$L^{\mathrm{m}\dagger}_n = L^{\mathrm{m}}_{-n}$$

$$|\chi\rangle = \sum_{n=1}^{\infty} L^{\mathrm{m}}_{-n}|\chi_n\rangle$$

$$|\psi\rangle \cong |\psi\rangle + |\chi\rangle.$$

$$\mathcal{H}_{\mathrm{OCQ}} = \frac{\mathcal{H}_{\mathrm{phys}}}{\mathcal{H}_{\mathrm{null}}}.$$

$$\begin{aligned} L^{\mathrm{m}}_0 &= \alpha' p^2 + \alpha_{-1}\cdot\alpha_1 + \cdots \\ L^{\mathrm{m}}_{\pm 1} &= (2\alpha')^{1/2} p \cdot \alpha_{\pm 1} + \cdots \end{aligned}$$

$$|e; k\rangle = e \cdot \alpha_{-1}|0; k\rangle.$$

$$\begin{aligned} \langle e; k \mid e; k'\rangle &= \langle 0; k|e^* \cdot \alpha_1 e \cdot \alpha_{-1}|0; k'\rangle = \langle 0; k|(e^* \cdot e + e^* \cdot \alpha_{-1} e \cdot \alpha_1)|0; k'\rangle \\ &= e^{\mu *} e_\mu (2\pi)^D \delta^D(k - k') \end{aligned}$$

$$\alpha^{\mu\dagger}_n = \alpha^\mu_{-n},$$

$$\langle 0; k \mid 0; k'\rangle = (2\pi)^D \delta^D(k - k'),$$

$$m^2 = \frac{1 + A}{\alpha'}$$

$$L_1^{\mathrm{m}}|e;k\rangle \propto p\cdot\alpha_1 e\cdot\alpha_{-1}|0;k\rangle = e\cdot k|0;k\rangle = 0$$

$$L_{-1}^{\mathrm{m}}|0;k\rangle = (2\alpha')^{1/2}k\cdot\alpha_{-1}|0;k\rangle.$$

$$k\cdot e = 0, e_\mu \cong e_\mu + \gamma k_\mu$$

$$\mathcal{H}_{\mathrm{OCQ}} = \mathcal{H}_{\text{light-cone}},$$

$$|0;k\rangle, m^2 = -\frac{4}{\alpha'}$$
$$e_{\mu\nu}\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0;k\rangle, m^2 = 0, k^\mu e_{\mu\nu} = k^\nu e_{\mu\nu} = 0$$
$$e_{\mu\nu} \cong e_{\mu\nu} + a_\mu k_\nu + k_\mu b_\nu, a\cdot k = b\cdot k = 0$$

$$\left(L_0^{\mathrm{m}} + L_0^{\mathrm{g}}\right)|\psi,\downarrow\rangle = 0$$

$$(H^{\mathrm{m}} + H^{\mathrm{g}})|\psi,\downarrow\rangle = 0$$

Cuantización BRST:

$$\left[\delta_\alpha, \delta_\beta\right] = f^{\gamma}{}_{\alpha\beta}\delta_\gamma$$

$$F^A(\phi) = 0$$

$$\int \frac{[d\phi_i]}{V_{\text{gauge}}}\exp\left(-S_1\right) \to \int \ [d\phi_i dB_A db_A dc^\alpha]\exp\left(-S_1 - S_2 - S_3\right)$$

$$S_2 = -iB_A F^A(\phi)$$

$$S_3 = b_A c^\alpha \delta_\alpha F^A(\phi)$$

$$\begin{aligned}
\delta_{\mathrm{B}}\phi_i &= -i\epsilon c^\alpha \delta_\alpha \phi_i \\
\delta_{\mathrm{B}}B_A &= 0 \\
\delta_{\mathrm{B}}b_A &= \epsilon B_A \\
\delta_{\mathrm{B}}c^\alpha &= \frac{i}{2}\epsilon f_{\beta\gamma}^{\alpha}c^\beta c^\gamma
\end{aligned}$$

$$\delta_{\mathrm{B}}(b_A F^A) = i\epsilon(S_2 + S_3).$$

$$\epsilon\delta\langle f \mid i\rangle = i\langle f|\delta_{\mathrm{B}}(b_A\delta F^A)|i\rangle = -\epsilon\langle f|\{Q_{\mathrm{B}}, b_A\delta F^A\}|i\rangle$$

$$\langle\psi|\{Q_{\mathrm{B}}, b_A\delta F^A\}|\psi'\rangle = 0$$

$$Q_{\mathrm{B}}|\psi\rangle = Q_{\mathrm{B}}|\psi'\rangle = 0$$

$$\epsilon^{-1}\delta_{\mathrm{B}}(b_A B_B M^{AB}) = -B_A B_B M^{AB}$$

$$0 = [Q_{\mathrm{B}}, \{Q_{\mathrm{B}}, b_A\delta F^A\}] = Q_{\mathrm{B}}^2 b_A\delta F^A - Q_{\mathrm{B}}b_A\delta F^A Q_{\mathrm{B}} + Q_{\mathrm{B}}b_A\delta F^A Q_{\mathrm{B}} - b_A\delta F^A Q_{\mathrm{B}}^2 = \left[Q_{\mathrm{B}}^2, b_A\delta F^A\right]$$

$$Q_{\mathrm{B}}^2 = 0$$

$$\delta_{\mathrm{B}}(\delta'_{\mathrm{B}}c^{\alpha}) = -\frac{1}{2}\epsilon\epsilon' f^{\alpha}_{\beta\gamma} f^{\gamma}_{\delta\epsilon} c^{\beta} c^{\delta} c^{\epsilon} = 0$$

$$Q_{\mathrm{B}}|\chi\rangle$$

$$\langle\psi|(Q_{\mathrm{B}}|\chi\rangle) = (\langle\psi|Q_{\mathrm{B}})|\chi\rangle = 0$$

$$|\psi'\rangle = |\psi\rangle + Q_{\mathrm{B}}|\chi\rangle$$

$$\mathcal{H}_{\mathrm{BRST}} = \frac{\mathcal{H}_{\mathrm{closed}}}{\mathcal{H}_{\mathrm{exact}}}$$

$$\delta_{\tau_1}X^{\mu}(\tau) = -\delta(\tau-\tau_1)\partial_{\tau}X^{\mu}(\tau), \delta_{\tau_1}e(\tau) = -\partial_{\tau}[\delta(\tau-\tau_1)e(\tau)]$$

$$[\delta_{\tau_1},\delta_{\tau_2}]X^{\mu}(\tau) = -[\delta(\tau-\tau_1)\partial_{\tau}\delta(\tau-\tau_2) - \delta(\tau-\tau_2)\partial_{\tau}\delta(\tau-\tau_1)]\partial_{\tau}X^{\mu}(\tau)$$

$$\equiv \int d\tau_3 f^{\tau_3}{}_{\tau_1\tau_2}\delta_{\tau_3}X^{\mu}(\tau)$$

$$f^{\tau_3}{}_{\tau_1\tau_2} = \delta(\tau_3-\tau_1)\partial_{\tau_3}\delta(\tau_3-\tau_2) - \delta(\tau_3-\tau_2)\partial_{\tau_3}\delta(\tau_3-\tau_1)$$

$$\begin{aligned}
\delta_{\mathrm{B}}X^{\mu} &= i\epsilon c\dot{X}^{\mu} \\
\delta_{\mathrm{B}}e &= i\epsilon(\dot{c e}) \\
\delta_{\mathrm{B}}B &= 0 \\
\delta_{\mathrm{B}}b &= \epsilon B \\
\delta_{\mathrm{B}}c &= i\epsilon c\dot{c}
\end{aligned}$$

$$S = \int d\tau\left(\frac{1}{2}e^{-1}\dot{X}^{\mu}\dot{X}_{\mu} + \frac{1}{2}em^2 + iB(e-1) - e\dot{b}c\right)$$

$$S = \int d\tau\left(\frac{1}{2}\dot{X}^{\mu}\dot{X}_{\mu} + \frac{1}{2}m^2 - \dot{b}c\right)$$

$$\begin{aligned}
\delta_{\mathrm{B}}X^{\mu} &= i\epsilon c\dot{X}^{\mu} \\
\delta_{\mathrm{B}}b &= i\epsilon\left(-\frac{1}{2}\dot{X}^{\mu}\dot{X}_{\mu} + \frac{1}{2}m^2 - \dot{b}c\right) \\
\delta_{\mathrm{B}}c &= i\epsilon c\dot{c}
\end{aligned}$$

$$[p^{\mu}, X^{v}] = -i\eta^{\mu v}, \{b, c\} = 1$$

$$Q_{\mathrm{B}} = cH$$

$$\begin{gathered}
p^{\mu}|k,\downarrow\rangle = k^{\mu}|k,\downarrow\rangle, p^{\mu}|k,\uparrow\rangle = k^{\mu}|k,\uparrow\rangle \\
b|k,\downarrow\rangle = 0, b|k,\uparrow\rangle = |k,\downarrow\rangle \\
c|k,\downarrow\rangle = |k,\uparrow\rangle, c|k,\uparrow\rangle = 0
\end{gathered}$$

$$Q_{\mathrm{B}}|k,\downarrow\rangle = \frac{1}{2}(k^2+m^2)|k,\uparrow\rangle, Q_{\mathrm{B}}|k,\uparrow\rangle = 0$$

$$\begin{aligned}
&|k,\downarrow\rangle, \quad k^2+m^2 = 0 \\
&|k,\uparrow\rangle, \quad \text{all } k^{\mu},
\end{aligned}$$

$$|k,\uparrow\rangle, k^2+m^2 \neq 0$$

$$|k,\downarrow\rangle, k^2+m^2=0;\ |k,\uparrow\rangle, k^2+m^2=0$$

$$b|\psi\rangle=0$$

$$\begin{aligned} \delta_{\mathrm{B}} X^\mu &= i\epsilon(c\partial+\tilde{c}\bar{\partial})X^\mu \\ \delta_{\mathrm{B}} b &= i\epsilon(T^X+T^{\mathrm{g}}), \delta_{\mathrm{B}}\tilde{b}=i\epsilon\left(\tilde{T}^X+\tilde{T}^{\mathrm{g}}\right) \\ \delta_{\mathrm{B}} c &= i\epsilon c\partial c, \delta_{\mathrm{B}}\tilde{c}=i\epsilon\tilde{c}\bar{\partial}\tilde{c} \end{aligned}$$

$$\frac{i}{4\pi}\int d^2\sigma g^{1/2}B^{ab}(\delta_{ab}-g_{ab})$$

$$j_{\mathrm{B}}=cT^{\mathrm{m}}+\frac{1}{2}:cT^{\mathrm{g}}:+\frac{3}{2}\partial^2c=cT^{\mathrm{m}}+:bc\partial c:+\frac{3}{2}\partial^2 c$$

$$\begin{aligned} j_{\mathrm{B}}(z)b(0) &\sim \frac{3}{z^3}+\frac{1}{z^2}j^{\mathrm{g}}(0)+\frac{1}{z}T^{\mathrm{m+g}}(0), \\ j_{\mathrm{B}}(z)c(0) &\sim \frac{1}{z}c\partial c(0), \\ j_{\mathrm{B}}(z)\mathcal{O}^{\mathrm{m}}(0,0) &\sim \frac{h}{z^2}c\theta^{\mathrm{m}}(0,0)+\frac{1}{z}[h(\partial c)\mathcal{O}^{\mathrm{m}}(0,0)+c\partial\theta^{\mathrm{m}}(0,0)]. \end{aligned}$$

$$Q_{\mathrm{B}}=\frac{1}{2\pi i}\oint (dz j_{\mathrm{B}}-d\bar{z}\tilde{j}_{\mathrm{B}})$$

$$\{Q_{\mathrm{B}}, b_m\}=L_m^{\mathrm{m}}+L_m^{\mathrm{g}}$$

$$Q_{\mathrm{B}}=\sum_{n=-\infty}^{\infty}\left(c_n L_{-n}^{\mathrm{m}}+\tilde{c}_n\tilde{L}_{-n}^{\mathrm{m}}\right)+\sum_{m,n=-\infty}^{\infty}\frac{(m-n)}{2}{}^{(c_m c_n b_{-m-n}\overset{}{\underset{\circ}{+}}\tilde{c}_m\tilde{c}_n\tilde{b}_{-m-n})\circ}+a^{\mathrm{B}}(c_0+\tilde{c}_0)$$

$$\{Q_{\mathrm{B}}, b_0\}=L_0^{\mathrm{m}}+L_0^{\mathrm{g}}$$

$$\{Q_{\mathrm{B}}, Q_{\mathrm{B}}\}=0 \text{ only if } c^{\mathrm{m}}=26$$

$$j_{\mathrm{B}}(z)j_{\mathrm{B}}(0)\sim-\frac{c^{\mathrm{m}}-18}{2z^3}c\partial c(0)-\frac{c^{\mathrm{m}}-18}{4z^2}c\partial^2c(0)-\frac{c^{\mathrm{m}}-26}{12z}c\partial^3c(0)$$

$$T(z)j_{\mathrm{B}}(0)\sim\frac{c^{\mathrm{m}}-26}{2z^4}c(0)+\frac{1}{z^2}j_{\mathrm{B}}(0)+\frac{1}{z}\partial j_{\mathrm{B}}(0),$$

$$\left[G_I, G_J\right]=ig^K{}_{IJ}G_K$$

$$\left\{c^I, b_J\right\}=\delta_J^I, \{c^I, c^J\}=\left\{b_I, b_J\right\}=0$$

$$Q_{\mathrm{B}}=c^I G_I^{\mathrm{m}}-\frac{i}{2}g_{IJ}^K c^I c^J b_K=c^I\left(G_I^{\mathrm{m}}+\frac{1}{2}G_I^{\mathrm{g}}\right)$$

$$G_I^{\mathrm{g}}=-ig^K{}_{IJ}c^J b_K$$

$$Q_{\mathrm{B}}^2 = \frac{1}{2}\{Q_{\mathrm{B}}, Q_{\mathrm{B}}\} = -\frac{1}{2} g_{IJ}^{K} g_{KL}^{M} c^I c^J c^L b_M = 0$$

$$\begin{aligned} (\alpha_m^\mu)^\dagger &= \alpha_{-m}^\mu, \quad (\tilde{\alpha}_m^\mu)^\dagger = \tilde{\alpha}_{-m}^\mu \\ (b_m)^\dagger &= b_{-m}, \quad (\tilde{b}_m)^\dagger = \tilde{b}_{-m} \\ (c_m)^\dagger &= c_{-m}, \quad (\tilde{c}_m)^\dagger = \tilde{c}_{-m}. \end{aligned}$$

$$\langle 0; k | c_0 | 0; k' \rangle = (2\pi)^{26} \delta^{26}(k - k')$$
$$\langle 0; k | \tilde{c}_0 c_0 | 0; k' \rangle = i(2\pi)^{26} \delta^{26}(k - k')$$

$$b_0 |\psi\rangle = 0$$

$$L_0 |\psi\rangle = \{Q_{\mathrm{B}}, b_0\} |\psi\rangle = 0$$

$$L_0 = \alpha'(p^\mu p_\mu + m^2)$$

$$\alpha' m^2 = \sum_{n=1}^{\infty} n \left(N_{bn} + N_{cn} + \sum_{\mu=0}^{25} N_{\mu n} \right) - 1.$$

$$|0; \mathbf{k}\rangle, -k^2 = -\frac{1}{\alpha'}.$$

$$Q_{\mathrm{B}} |0; \mathbf{k}\rangle = 0$$

$$|\psi_1\rangle = (e \cdot \alpha_{-1} + \beta b_{-1} + \gamma c_{-1}) |0; \mathbf{k}\rangle, -k^2 = 0$$

$$\langle \psi_1 \| \psi_1 \rangle = \langle 0; \mathbf{k} \| (e^* \cdot \alpha_1 + \beta^* b_1 + \gamma^* c_1)(e \cdot \alpha_{-1} + \beta b_{-1} + \gamma c_{-1}) \mid 0; \mathbf{k}' \rangle$$
$$= (e^* \cdot e + \beta^* \gamma + \gamma^* \beta) \langle 0; \mathbf{k} \| 0; \mathbf{k}' \rangle$$

$$0 = Q_{\mathrm{B}} |\psi_1\rangle = (2\alpha')^{1/2} (c_{-1} k \cdot \alpha_1 + c_1 k \cdot \alpha_{-1}) |\psi_1\rangle = (2\alpha')^{1/2} (k \cdot e c_{-1} + \beta k \cdot \alpha_{-1}) |0; \mathbf{k}\rangle$$

$$Q_{\mathrm{B}} |\chi\rangle = (2\alpha')^{1/2} (k \cdot e' c_{-1} + \beta' k \cdot \alpha_{-1}) |0; \mathbf{k}\rangle.$$

$$b_0 |\psi\rangle = \tilde{b}_0 |\psi\rangle = 0$$

$$L_0 |\psi\rangle = \tilde{L}_0 |\psi\rangle = 0$$

$$L_0 = \frac{\alpha'}{4}(p^2 + m^2), \tilde{L}_0 = \frac{\alpha'}{4}(p^2 + \tilde{m}^2)$$

$$\frac{\alpha'}{4} m^2 = \sum_{n=1}^{\infty} n \left(N_{bn} + N_{cn} + \sum_{\mu=0}^{25} N_{\mu n} \right) - 1,$$
$$\frac{\alpha'}{4} \tilde{m}^2 = \sum_{n=1}^{\infty} n \left(\tilde{N}_{bn} + \tilde{N}_{cn} + \sum_{\mu=0}^{25} \tilde{N}_{\mu n} \right) - 1.$$

$$L_m = L_m^X + L_m^K + L_m^{\mathrm{g}}$$

$$|N,I;k\rangle, |N,\tilde{N},I;k\rangle,$$

$$-\sum_{\mu=0}^{d-1} k_\mu k^\mu \;= m^2$$

$$\alpha' m^2 \;= \sum_{n=1}^{\infty} n\left(N_{bn} + N_{cn} + \sum_{\mu=0}^{d-1} N_{\mu n}\right) + L_0^K - 1$$

$$-\sum_{\mu=0}^{d-1} k_\mu k^\mu \;= m^2 = \tilde{m}^2$$

$$\frac{\alpha'}{4} m^2 \;= \sum_{n=1}^{\infty} n\left(N_{bn} + N_{cn} + \sum_{\mu=0}^{d-1} N_{\mu n}\right) + L_0^K - 1$$

$$\frac{\alpha'}{4} \tilde{m}^2 \;= \sum_{n=1}^{\infty} n\left(\tilde{N}_{bn} + \tilde{N}_{cn} + \sum_{\mu=0}^{d-1} \tilde{N}_{\mu n}\right) + \tilde{L}_0^K - 1$$

$$\langle 0,I;\mathbf{k}\|0,I';\mathbf{k}'\rangle = \langle 0,0,I;\mathbf{k}\|0,0,I';\mathbf{k}'\rangle = 2k^0(2\pi)^{d-1}\delta^{d-1}(\mathbf{k}-\mathbf{k}')\delta_{I,I'}$$

$$\alpha_m^\pm = 2^{-1/2}(\alpha_m^0 \pm \alpha_m^1)$$

$$[\alpha_m^+, \alpha_n^-] = -m\delta_{m,-n}, [\alpha_m^+, \alpha_n^+] = [\alpha_m^-, \alpha_n^-] = 0$$

$$N^{\mathrm{lc}} = \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{1}{m} {:}\alpha_{-m}^+ \alpha_m^-{:}$$

$$Q_{\mathrm{B}} = Q_1 + Q_0 + Q_{-1}$$

$$\left[N^{\mathrm{lc}}, Q_j\right] = jQ_j$$

$$\left[N^{\mathrm{g}}, Q_j\right] = Q_j$$

$$(Q_1^2) + (\{Q_1, Q_0\}) + (\{Q_1, Q_{-1}\} + Q_0^2) + (\{Q_0, Q_{-1}\}) + (Q_{-1}^2) = 0$$

$$Q_1 = -(2\alpha')^{1/2} k^+ \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \alpha_{-m}^- c_m$$

$$R = \frac{1}{(2\alpha')^{1/2} k^+} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \alpha_{-m}^+ b_m$$

$$S \equiv \{Q_1, R\} = \sum_{m=1}^{\infty} (mb_{-m}c_m + mc_{-m}b_m - \alpha_{-m}^+\alpha_m^- - \alpha_{-m}^-\alpha_m^+) = \sum_{m=1}^{\infty} m(N_{bm} + N_{cm} + N_m^+ + N_m^-)$$

$$|\psi\rangle=\frac{1}{s}\{Q_1,R\}|\psi\rangle=\frac{1}{s}Q_1R|\psi\rangle,$$

$$0=Q_1S|\psi\rangle=SQ_1|\psi\rangle$$

$$S+U\equiv\{Q_{\mathrm{B}},R\}$$

$$|\psi\rangle=(1-S^{-1}U+S^{-1}US^{-1}U-\cdots)|\psi_0\rangle$$

$$\langle\psi||\psi'\rangle=\langle\psi_0||\psi_0'\rangle$$

Cuantización BRST-OCQ.

$$\mathcal{H}_{\mathrm{OCQ}}=\mathcal{H}_{\mathrm{BRST}}=\mathcal{H}_{\text{light-cone}}$$

$$|\psi,\downarrow\rangle$$

$$Q_{\mathrm{B}}|\psi,\downarrow\rangle=\sum_{n=0}^{\infty}c_{-n}\left(L_n^{\mathrm{m}}-\delta_{n,0}\right)|\psi,\downarrow\rangle=0$$

$$|\psi,\downarrow\rangle-|\psi',\downarrow\rangle$$

$$|\psi,\downarrow\rangle-|\psi',\downarrow\rangle=Q_{\mathrm{B}}|\chi\rangle,$$

$$|\chi\rangle=\sum_{n=1}^{\infty}b_{-n}|\chi_n,\downarrow\rangle+\cdots;$$

$$|\psi,\downarrow\rangle-|\psi',\downarrow\rangle=\sum_{m,n=1}^{\infty}c_mL_{-m}^{\mathrm{m}}b_{-n}|\chi_n,\downarrow\rangle=\sum_{n=1}^{\infty}L_{-n}^{\mathrm{m}}|\chi_n,\downarrow\rangle$$

$$\{[Q_{\mathrm{B}},L_m],b_n\}-\{[L_m,b_n],Q_{\mathrm{B}}\}-[\{b_n,Q_{\mathrm{B}}\},L_m]=0.$$

$$G_IG_J-(-1)^{F_IF_J}G_JG_I=ig^K{}_{IJ}G_K$$

$$(b_{-1})^{N_b}(c_{-1})^{N_c}(\alpha_{-1}^{+})^{N^+}(\alpha_{-1}^{-})^{N^-}|0\rangle.$$

Simetrías de gauge.

$$\int[dedX]\exp\left[-\frac{1}{2}\int d\tau\left(e^{-1}\dot{X}^{\mu}\dot{X}_{\mu}+em^2\right)\right]$$

$$\frac{\partial\tau'}{\partial\tau}=e(\tau).$$

$$\tau'(\tau)=\int_0^{\tau}d\tau''e(\tau'')$$

$$\tau'(1)=\int_0^1d\tau e(\tau)=l$$

$$e' = l, 0 \leq \tau \leq 1$$

$$e' = 1, 0 \leq \tau \leq l$$

$$\tau \to \tau + v \bmod 1.$$

$$0 \leq \sigma^1 \leq 2\pi, 0 \leq \sigma^2 \leq 2\pi$$

$$(\sigma^1, \sigma^2) \cong (\sigma^1, \sigma^2) + 2\pi(m, n)$$

$$ds^2 = |d\sigma^1 + \tau d\sigma^2|^2$$

$$\tilde{\sigma}^a \cong \tilde{\sigma}^a + 2\pi(mu^a + nv^a)$$

$$w \cong w + 2\pi(m + n\tau)$$

$$T: \tau' = \tau + 1, S: \tau' = -1/\tau$$

$$\tau' = \frac{a\tau + b}{c\tau + d}$$

$$\begin{bmatrix} \sigma^1 \\ \sigma^2 \end{bmatrix} = \begin{bmatrix} d & b \\ c & a \end{bmatrix} \begin{bmatrix} \sigma'^1 \\ \sigma'^2 \end{bmatrix}$$

$$-\frac{1}{2} \leq \mathrm{Re}\tau \leq \frac{1}{2}, |\tau| \geq 1$$

$$\sigma^a \to \sigma^a + v^a$$

Superficies de Moduli y Riemann.

$$\mathcal{M}_r = \frac{\mathcal{G}_r}{(\mathrm{diff} \times \mathrm{Weyl})_r}$$

$$\mathcal{M}_{r,n} = \frac{\mathcal{G}_r \times \mathcal{M}^n}{(\text{ diff } \times \text{ Weyl })_r}$$

$$\frac{\mathrm{diff}_r}{\mathrm{diff}_{r0}}$$

$$\delta g_{ab} = -2(P_1\delta\sigma)_{ab} + (2\delta\omega - \nabla \cdot \delta\sigma)g_{ab}$$

$$0 = \int \ d^2\sigma g^{1/2} \delta' g_{ab}[-2(P_1\delta\sigma)^{ab} + (2\delta\omega - \nabla \cdot \delta\sigma)g^{ab}]$$

$$= \int \ d^2\sigma g^{1/2}[-2(P_1^T\delta' g)_a \delta\sigma^a + \delta' g_{ab} g^{ab}(2\delta\omega - \nabla \cdot \delta\sigma)]$$

$$g^{ab}\delta' g_{ab} = 0$$
$$(P_1^T\delta' g)_a = 0$$

$$(P_1\delta\sigma)_{ab} = 0$$

$$\begin{aligned}\partial_{\bar{z}}\delta' g_{zz} &= \partial_z \delta' g_{\bar{z}\bar{z}} = 0\\ \partial_{\bar{z}}\delta z &= \partial_z \delta \bar{z} = 0\end{aligned}$$

$$\mu - \kappa = -3\chi$$

$$\int d^2\sigma g^{1/2}(P_1\delta\sigma)_{ab}(P_1\delta\sigma)^{ab} = \int d^2\sigma g^{1/2}\delta\sigma_a(P_1^T P_1\delta\sigma)^a = \int d^2\sigma g^{1/2}\left(\frac{1}{2}\nabla_a\delta\sigma_b\nabla^a\delta\sigma^b - \frac{R}{4}\delta\sigma_a\delta\sigma^a\right)$$

$$\begin{aligned}\chi > 0: &\quad \kappa = 3\chi, \mu = 0\\ \chi < 0: &\quad \kappa = 0, \mu = -3\chi\end{aligned}$$

$$\sigma_m^a = f_{mn}^a(\sigma_n)$$

$$z_m^a = f_{mn}^a(z_n)$$

$$ds^2 \propto dz_m d\bar{z}_m$$

$$w \cong w + 2\pi \cong w + 2\pi\tau$$

$$S_{j_1\ldots j_n}(k_1, \ldots, k_n) = \sum_{\substack{\text{compact}\\ \text{topologies}}} \int \frac{[d\phi dg]}{V_{\text{diff}\times\text{Weyl}}} \exp(-S_{\text{m}} - \lambda\chi) \prod_{i=1}^{n} \int d^2\sigma_i g(\sigma_i)^{1/2} \mathcal{V}_{j_i}(k_i, \sigma_i)$$

$$[dg]d^{2n}\sigma \to [d\zeta]d^\mu t d^{2n-\kappa}\sigma$$

$$1 = \Delta_{\text{FP}}(g, \sigma) \int_F d^\mu t \int_{\text{diff}\times\text{Weyl}} [d\zeta]\delta\left(g - \hat{g}(t)^\zeta\right) \prod_{(a,i)\in f} \delta\left(\sigma_i^a - \hat{\sigma}_i^{\zeta a}\right)$$

$$\begin{aligned}S_{j_1\ldots j_n}(k_1, \ldots, k_n) = &\sum_{\substack{\text{compact}\\ \text{topologies}}} \int_F d^\mu t \Delta_{\text{FP}}(\hat{g}(t), \hat{\sigma}) \int [d\phi] \int \prod_{(a,i)\notin f} d\sigma_i^a\\ &\times \exp(-S_{\text{m}}[\phi, \hat{g}(t)] - \lambda\chi) \prod_{i=1}^{n} \left[\hat{g}(\sigma_i)^{1/2}\mathcal{V}_{j_i}(k_i; \sigma_i)\right]\end{aligned}$$

$$\delta g_{ab} = \sum_{k=1}^{\mu} \delta t^k \partial_{t^k}\hat{g}_{ab} - 2\left(\hat{P}_1\delta\sigma\right)_{ab} + (2\delta\omega - \hat{\nabla}\cdot\delta\sigma)\hat{g}_{ab}$$

$$\begin{aligned}\Delta_{\text{FP}}(\hat{g}, \hat{\sigma})^{-1} &= n_R \int d^\mu \delta t [d\delta\omega d\delta\sigma]\delta(\delta g_{ab}) \prod_{(a,i)\in f} \delta\left(\delta\sigma^a(\hat{\sigma}_i)\right)\\ &= n_R \int d^\mu \delta t d^\kappa x [d\beta' d\delta\sigma] \times \exp\left[2\pi i\left(\beta', 2\hat{P}_1\delta\sigma - \delta t^k\partial_k\hat{g}\right) + 2\pi i \sum_{(a,i)\in f} x_{ai}\delta\sigma^a(\hat{\sigma}_i)\right]\end{aligned}$$

$$\begin{aligned}\delta\sigma^a &\to c^a\\ \beta'_{ab} &\to b_{ab}\\ x_{ai} &\to \eta_{ai}\\ \delta t^k &\to \xi^k\end{aligned}$$

$$\Delta_{\mathrm{FP}}(\hat{g},\hat{\sigma})=\frac{1}{n_R}\int\ [dbdc]d^{\mu}\xi d^{\kappa}\eta\times\exp\left[-\frac{1}{4\pi}\left(b,2\hat{P}_1c-\xi^k\partial_k\hat{g}\right)+\sum_{(a,i)\in f}\ \eta_{ai}c^a(\hat{\sigma}_i)\right]$$

$$=\frac{1}{n_R}\int\ [dbdc]\exp\left(-S_{\mathrm{g}}\right)\prod_{k=1}^{\mu}\frac{1}{4\pi}(b,\partial_k\hat{g})\prod_{(a,i)\in f}\ c^a(\hat{\sigma}_i)$$

$$S_{j_1\dots j_n}(k_1,\dots,k_n)$$

$$=\sum_{\substack{\text{compact}\\ \text{topologies}}}\ \int_F\frac{d^{\mu}t}{n_R}\int\ [d\phi dbdc]\exp\left(-S_{\mathrm{m}}-S_{\mathrm{g}}-\lambda\chi\right)$$

$$\times\prod_{(a,i)\notin f}\ \int\ d\sigma_i^a\prod_{k=1}^{\mu}\frac{1}{4\pi}(b,\partial_k\hat{g})\prod_{(a,i)\in f}\ c^a(\hat{\sigma}_i)\prod_{i=1}^{n}\ \hat{g}(\sigma_i)^{1/2}\mathcal{V}j_i(k_i,\sigma_i)$$

$$c^a(\sigma)=\sum_J\ c_J\mathrm{C}_J^a(\sigma),b_{ab}(\sigma)=\sum_K\ b_K\ \mathrm{B}_{Kab}(\sigma).$$

$$S_{\mathrm{g}}=\frac{1}{2\pi}(b,P_1c)=\frac{1}{2\pi}(P_1^Tb,c)$$

$$P_1^TP_1\mathrm{C}_J^a=v_J'^2\mathrm{C}_J^a,P_1P_1^T\ \mathrm{B}_{Kab}=v_K^2\ \mathrm{B}_{Kab}$$

$$\left(\mathrm{C}_J,\mathrm{C}_{J'}\right)\ =\int\ d^2\sigma g^{1/2}\mathrm{C}_J^a\mathrm{C}_{J'a}=\delta_{JJ'}$$

$$(\mathrm{B}_K,\ \mathrm{B}_{K'})\ =\int\ d^2\sigma g^{1/2}\ \mathrm{B}_{Kab}\ \mathrm{B}_{K'}^{ab}=\delta_{KK'}$$

$$(P_1P_1^T)P_1\mathrm{C}_J=P_1(P_1^TP_1)\mathrm{C}_J=v_J'^2P_1\mathrm{C}_J$$

$$\mathrm{B}_{Jab}\ =\frac{1}{v_J}\left(P_1\mathrm{C}_J\right)_{ab},v_J=v_J'\neq 0$$

$$\int\prod_{k=1}^{\mu}\ db_{0k}\prod_{j=1}^{\kappa}\ dc_{0j}\prod_J\ db_Jdc_J\exp\left(-\frac{v_Jb_Jc_J}{2\pi}\right)\prod_{k'=1}^{\mu}\ \frac{1}{4\pi}(b,\partial_{k'}\hat{g})\prod_{(a,i)\in f}\ c^a(\sigma_i)$$

$$\Delta_{\mathrm{FP}}=\int\ \prod_{k=1}^{\mu}\ db_{0k}\prod_{k'=1}^{\mu}\left[\sum_{k''=1}^{\mu}\ \frac{b_{0k''}}{4\pi}(\ \mathrm{B}_{0k''},\partial_{k'}\hat{g})\right]\times\int\ \prod_{j=1}^{\kappa}\ dc_{0j}\prod_{(a,i)\in f}\left[\sum_{j'=1}^{\kappa}\ c_{0j'}\mathrm{C}_{0j'}^a(\sigma_i)\right]$$

$$\times\int\ \prod_J\ db_Jdc_J\exp\left(-\frac{v_Jb_Jc_J}{2\pi}\right)$$

$$\Delta_{\mathrm{FP}}=\det\frac{(\mathrm{B}_{0k},\partial_{k'}\hat{g})}{4\pi}\det\mathrm{C}_{0j}^a(\sigma_i)\det'\left(\frac{P_1^TP_1}{4\pi^2}\right)^{1/2}$$

Teorema de Riemann-Roch.

$$\nabla_a j^a = \frac{1-2\lambda}{4} R$$

$$\frac{\delta([d\phi]\exp(-S))}{[d\phi]\exp(-S)} = \frac{i\epsilon}{2\pi}\int d^2\sigma g^{1/2}\nabla_a j^a \to -i\epsilon\frac{2\lambda-1}{2}\chi$$

$$\frac{1}{2}(\kappa-\mu) = \frac{1}{2}(\mathrm{dimker}P_1 - \mathrm{dimker}P_1^T).$$

$$\mathrm{dimker}P_n - \mathrm{dimker}P_n^T = (2n+1)\chi,$$

$$d^\mu t' = \left|\det\frac{\partial t'}{\partial t}\right| d^\mu t \prod_{k=1}^{\mu}\frac{1}{4\pi}(b,\partial_k'\hat{g}) = \det\left(\frac{\partial t}{\partial t'}\right)\prod_{k=1}^{\mu}\frac{1}{4\pi}(b,\partial_k\hat{g})$$

$$\delta\hat{g}'_{ab}(t;\sigma) = 2\delta\omega(t;\sigma)\hat{g}_{ab}(t;\sigma),$$

$$(b,\partial_k\hat{g}') = \int d^2\sigma\hat{g}'^{1/2}b_{ab}\hat{g}'^{ac}\hat{g}'^{bd}\partial_k\hat{g}'_{cd} = \int d^2\sigma\hat{g}^{1/2}b_{ab}\left(\hat{g}^{ac}\hat{g}^{bd}\partial_k\hat{g}_{cd} + 2\hat{g}^{ab}\partial_k\omega\right)$$

$$= (b,\partial_k\hat{g})$$

$$\delta(b,\partial_k\hat{g}) = -2(b,P_1\partial_k\xi) = -2(P_1^T b,\partial_k\xi) = 0$$

Invariancia BRST.

$$\delta_{\mathrm{B}}\mathcal{V}_{\mathrm{m}} = i\epsilon\partial_a(c^a\mathcal{V}_{\mathrm{m}})$$

$$\delta_{\mathrm{B}}(b,\partial_k\hat{g}) = i\epsilon(T,\partial_k\hat{g})$$

$$\mu_{ka}^{b} = \frac{1}{2}\hat{g}^{bc}\partial_k\hat{g}_{ac}$$

$$\frac{1}{2\pi}(b,\mu_k) = \frac{1}{2\pi}\int d^2z\left(b_{zz}\mu_{k\bar{z}}^{z} + b_{\bar{z}\bar{z}}\mu_{kz}^{\bar{z}}\right)$$

$$z_m' = z_m + \delta t^k v_{km}^{z_m}(z_m,\bar{z}_m)$$

$$dz_m' d\bar{z}_m' \propto dz_m d\bar{z}_m + \delta t^k\left(\mu_{kz_m}^{\bar{z}_m}dz_m dz_m + \mu_{k\bar{z}_m}^{z_m}d\bar{z}_m d\bar{z}_m\right)$$

$$\mu_{kz_m}^{\bar{z}_m} = \partial_{z_m}v_{km}^{\bar{z}_m}, \mu_{k\bar{z}_m}^{z_m} = \partial_{\bar{z}_m}v_{km}^{z_m}$$

$$\frac{1}{2\pi}(b,\mu_k) = \frac{1}{2\pi i}\sum_m \oint_{C_m}\left(dz_m v_{km}^{z_m}b_{z_m z_m} - d\bar{z}_m v_{km}^{\bar{z}_m}b_{\bar{z}_m\bar{z}_m}\right)$$

$$\frac{dz_m}{dt^k} = v_{km}^{z_m}$$

$$\left.\frac{\partial z_m}{\partial t^k}\right|_{z_n} = v_{km}^{z_m} - \left.\frac{\partial z_m}{\partial z_n}\right|_t v_{kn}^{z_n} = v_{km}^{z_m} - v_{kn}^{z_m}$$

$$\frac{1}{2\pi}(b,\mu_k) = \frac{1}{2\pi i}\sum_{(mn)}\int_{C_{mn}}\left(dz_m \left.\frac{\partial z_m}{\partial t^k}\right|_{z_n} b_{z_m z_m} - d\bar{z}_m \left.\frac{\partial \bar{z}_m}{\partial t^k}\right|_{z_n} b_{\bar{z}_m \bar{z}_m}\right)$$

$$z = z' + z_v$$

$$\left.\frac{\partial z'}{\partial z_v}\right|_z = -1.$$

$$\int_C \frac{dz'}{2\pi i} b_{z'z'} \int_C \frac{d\bar{z}'_m}{-2\pi i} b_{\bar{z}'\bar{z}'} = b_{-1}\tilde{b}_{-1}$$

$$S(1;\dots;n) = \sum_{\substack{\text{compact}\\ \text{topologies}}} e^{-\lambda\chi}\int_F \frac{d^m t}{n_R}\left\langle \prod_{k=1}^{m} B_k \prod_{i=1}^{n} \hat{\mathcal{V}}_i \right\rangle$$

$$m = \mu + 2n_{\mathrm{c}} + n_{\mathrm{o}} - \kappa = -3\chi + 2n_{\mathrm{c}} + n_{\mathrm{o}}$$

$$b_{-1}\tilde{b}_{-1} \cdot \tilde{c}c\mathcal{V}_{\mathrm{m}} = \mathcal{V}_{\mathrm{m}}$$

$$u = 1/z.$$

$$ds^2 = \exp\left(2\omega(z,\bar{z})\right) dz d\bar{z}$$

$$ds^2 = \frac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2} = \frac{4r^2 du d\bar{u}}{(1+u\bar{u})^2}$$

$$\delta u = \frac{\partial u}{\partial z}\delta z = -z^{-2}\delta z$$
$$\delta g_{uu} = \left(\frac{\partial u}{\partial z}\right)^{-2} \delta g_{zz} = z^4 \delta g_{zz}$$

$$\delta z = a_0 + a_1 z + a_2 z^2$$
$$\delta\bar{z} = a_0^* + a_1^* \bar{z} + a_2^* \bar{z}^2$$

$$z' = \frac{\alpha z + \beta}{\gamma z + \delta}$$

$$z' = 1/\bar{z}$$

$$z' = \bar{z}$$

$$z' = -1/\bar{z}$$

$$Z[J] = \left\langle \exp\left(i\int d^2\sigma J(\sigma)\cdot X(\sigma)\right)\right\rangle$$

$$X^{\mu}(\sigma) = \sum_{I} x_I^{\mu} \mathrm{X}_I(\sigma)$$

$$\nabla^2 \mathrm{X}_I = -\omega_I^2 \mathrm{X}_I$$

$$\int_M d^2\sigma g^{1/2} \mathrm{X}_I \mathrm{X}_{I'} = \delta_{II'}$$

$$Z[J] = \prod_{I,\mu} \int dx_I^{\mu} \exp\left(-\frac{\omega_I^2 x_I^{\mu} x_{I\mu}}{4\pi\alpha'} + i x_I^{\mu} J_{I\mu}\right)$$

$$J_I^{\mu} = \int d^2\sigma J^{\mu}(\sigma) \mathrm{X}_I(\sigma)$$

$$\mathrm{X}_0 = \left(\int d^2\sigma g^{1/2}\right)^{-1/2}$$

$$Z[J] = i(2\pi)^d \delta^d(J_0) \prod_{I\neq 0} \left(\frac{4\pi^2\alpha'}{\omega_I^2}\right)^{\frac{d}{2}} \exp\left(-\frac{\pi\alpha' J_I \cdot J_I}{\omega_I^2}\right)$$

$$= i(2\pi)^d \delta^d(J_0) \left(\det{}' \frac{-\nabla^2}{4\pi^2\alpha'}\right)^{-d/2} \times \exp\left(-\frac{1}{2}\int d^2\sigma d^2\sigma' J(\sigma) \cdot J(\sigma') G'(\sigma,\sigma')\right)$$

$$G'(\sigma_1,\sigma_2) = \sum_{I\neq 0} \frac{2\pi\alpha'}{\omega_I^2} \mathrm{X}_I(\sigma_1)\mathrm{X}_I(\sigma_2)$$

$$-\frac{1}{2\pi\alpha'}\nabla^2 G'(\sigma_1,\sigma_2) = \sum_{I\neq 0} \mathrm{X}_I(\sigma_1)\mathrm{X}_I(\sigma_2) = g^{-1/2}\delta^2(\sigma_1-\sigma_2) - \mathrm{X}_0^2$$

$$G'(\sigma_1,\sigma_2) = -\frac{\alpha'}{2}\ln|z_{12}|^2 + f(z_1,\bar{z}_1) + f(z_2,\bar{z}_2)$$

$$f(z,\bar{z}) = \frac{\alpha' X_0^2}{4}\int d^2z' \exp[2\omega(z',\bar{z}')]\ln|z-z'|^2 + k$$

$$A_{S_2}^n(k,\sigma) = \left\langle \left[e^{ik_1\cdot X(\sigma_1)}\right]_{\mathrm{r}} \left[e^{ik_2\cdot X(\sigma_2)}\right]_{\mathrm{r}} \cdots \left[e^{ik_n\cdot X(\sigma_n)}\right]_{\mathrm{r}} \right\rangle_{S_2}$$

$$J(\sigma) = \sum_{i=1}^{n} k_i \delta^2(\sigma-\sigma_i)$$

$$A_{S_2}^n(k,\sigma) = iC_{S_2}^X (2\pi)^d \delta^d\left(\sum_i k_i\right) \times \exp\left(-\sum_{\substack{i,j=1\\ i<j}}^{n} k_i \cdot k_j G'(\sigma_i,\sigma_j) - \frac{1}{2}\sum_{i=1}^{n} k_i^2 G'_{\mathrm{r}}(\sigma_i,\sigma_i)\right)$$

$$C_{S_2}^X = \mathrm{X}_0^{-d}\left(\det{}' \frac{-\nabla^2}{4\pi^2\alpha'}\right)_{S_2}^{-d/2}$$

$$G'_{\mathrm{r}}(\sigma,\sigma') = G'(\sigma,\sigma') + \frac{\alpha'}{2}\ln\ d^2(\sigma,\sigma')$$

$$G'_{\mathrm{r}}(\sigma,\sigma) = 2f(z,\bar{z}) + \alpha'\omega(z,\bar{z})$$

$$A^n_{S_2}(k,\sigma) = iC^X_{S_2}(2\pi)^d\delta^d\left(\sum_i k_i\right)\exp\left(-\frac{\alpha'}{2}\sum_i k_i^2\omega(\sigma_i)\right)\prod_{\substack{i,j=1\\ i<j}}^{n} |z_{ij}|^{\alpha' k_i\cdot k_j}$$

$$\left\langle \prod_{i=1}^{n} [e^{ik_i\cdot X(z_i,\bar{z}_i)}]_{\mathrm{r}} \prod_{j=1}^{p} \partial X^{\mu_j}(z'_j) \prod_{k=1}^{q} \bar{\partial} X^{\nu_k}(\bar{z}''_k)\right\rangle_{S_2}$$

$$iC^X_{S_2}(2\pi)^d\delta^d\left(\sum_i k_i\right)\prod_{\substack{i,j=1\\ i<j}}^{n} |z_{ij}|^{\alpha' k_i\cdot k_j} \times \left\langle \prod_{j=1}^{p} [v^{\mu_j}(z'_j) + q^{\mu_j}(z'_j)] \prod_{k=1}^{q} [\tilde{v}^{\nu_k}(\bar{z}''_k) + \tilde{q}^{\nu_k}(\bar{z}''_k)]\right\rangle$$

$$v^\mu(z) = -i\frac{\alpha'}{2}\sum_{i=1}^{n} \frac{k_i^\mu}{z - z_i}, \tilde{v}^\mu(\bar{z}) = -i\frac{\alpha'}{2}\sum_{i=1}^{n} \frac{k_i^\mu}{\bar{z} - \bar{z}_i}$$

$$\langle \partial X^\mu(z_1)\partial X^\nu(z_2)\rangle_{S_2}$$

$$-\frac{\alpha'\eta^{\mu\nu}}{2z_{12}^2}\langle 1\rangle_{S_2} + g(z_1,z_2)$$

$$\partial_u X^\mu = -z^2\partial_z X^\mu$$

$$A^n_{S_2}(k,\sigma) = \left\langle \prod_{i=1}^{n} :e^{ik_i\cdot X(z_i,\bar{z}_i)}:\right\rangle_{S_2}$$

$$\left\langle \partial X^\mu(z) \prod_{i=1}^{n} :e^{ik_i\cdot X(z_i,\bar{z}_i)}:\right\rangle_{S_2} = -\frac{i\alpha'}{2}A^n_{S_2}(k,\sigma)\sum_{i=1}^{n}\frac{k_i^\mu}{z - z_i} + \text{terms holomorphic in } z$$

$$A^n_{S_2}(k,\sigma)\sum_{i=1}^{n} k_i^\mu = 0$$

$$p^\mu = \frac{1}{2\pi i}\oint_C \left(dz j_z^\mu - d\bar{z} j_{\bar{z}}^\mu\right)$$

$$-\frac{i\alpha'}{2}A^n_{S_2}(k,\sigma)\left(\frac{k_1^\mu}{z - z_1} + \sum_{i=2}^{n}\frac{k_i^\mu}{z_1 - z_i} + O(z - z_1)\right).$$

$$ik_1\cdot\partial X(z){:}e^{ik_1\cdot X(z_1,\bar{z}_1)}{:} = \frac{\alpha' k_1^2}{2(z - z_1)}{:}e^{ik_1\cdot X(z_1,\bar{z}_1)}{:} + \partial_{z_1}{:}e^{ik_1\cdot X(z_1,\bar{z}_1)}{:} + O(z - z_1)$$

$$\partial_{z_1} A^n_{S_2}(k,\sigma) = \frac{\alpha'}{2} A^n_{S_2}(k,\sigma) \sum_{i=2}^{n} \frac{k_1 \cdot k_i}{z_{1i}}.$$

$$A^n_{S_2}(k,\sigma) \propto \delta^d\left(\sum_i k_i\right) \prod_{\substack{i,j=1 \\ i<j}}^{n} |z_{ij}|^{\alpha' k_i \cdot k_j},$$

$$G'(\sigma_1,\sigma_2) = -\frac{\alpha'}{2} \ln |z_1 - z_2|^2 - \frac{\alpha'}{2} \ln |z_1 - \bar{z}_2|^2,$$

$$\left\langle \prod_{i=1}^{n} :e^{ik_i \cdot X(z_i,\bar{z}_i)}: \right\rangle_{D_2}$$

$$= iC^X_{D_2} (2\pi)^d \delta^d\left(\sum_i k_i\right) \prod_{i=1}^{n} |z_i - \bar{z}_i|^{\alpha' k_i^2/2} \times \prod_{\substack{i,j=1 \\ i<j}}^{n} |z_i - z_j|^{\alpha' k_i \cdot k_j} |z_i - \bar{z}_j|^{\alpha' k_i \cdot k_j}$$

$${}^{\star}_{\star} X^\mu(y_1) X^\nu(y_2) {}^{\star}_{\star} = X^\mu(y_1) X^\nu(y_2) + 2\alpha' \eta^{\mu\nu} \ln |y_1 - y_2|,$$

$$\left\langle \prod_{i=1}^{n} {}^{\star}_{\star} e^{ik_i \cdot X(y_i)} {}^{\star}_{\star} \right\rangle_{D_2} = iC^X_{D_2} (2\pi)^d \delta^d\left(\sum_i k_i\right) \prod_{\substack{i,j=1 \\ i<j}}^{n} |y_i - y_j|^{2\alpha' k_i \cdot k_j}$$

$$\left\langle \prod_{i=1}^{n} {}_{\star}{}^{\star}\, e^{ik_i \cdot X(y_i)}{}_{\star} \prod_{j=1}^{p} \partial_y X^{\mu_j}(y'_j) \right\rangle_{D_2}$$

$$= iC^X_{D_2} (2\pi)^d \delta^d\left(\sum_i k_i\right) \times \prod_{\substack{i,j=1 \\ i<j}}^{n} |y_{ij}|^{2\alpha' k_i \cdot k_j} \left\langle \prod_{j=1}^{p} \left[v^{\mu_j}(y'_j) + q^{\mu_j}(y'_j) \right] \right\rangle_{D_2}$$

$$v^\mu(y) = -2i\alpha' \sum_{i=1}^{n} \frac{k_i^\mu}{y - y_i}$$

$$G'(\sigma_1,\sigma_2) = -\frac{\alpha'}{2} \ln |z_1 - z_2|^2 - \frac{\alpha'}{2} \ln |1 + z_1 \bar{z}_2|^2.$$

$$
\left\langle \prod_{i=1}^{n} :e^{ik_i\cdot X(z_i,\bar{z}_i)}: \right\rangle_{RP_2}
$$

$$
= iC_{RP_2}^{X}(2\pi)^d\delta^d\left(\sum_i k_i\right)\prod_{i=1}^{n}|1+z_i\bar{z}_i|^{\alpha' k_i^2/2}
$$

$$
\times \prod_{\substack{i,j=1\\ i<j}}^{n} \left|z_i-z_j\right|^{\alpha' k_i\cdot k_j}\left|1+z_i\bar{z}_j\right|^{\alpha' k_i\cdot k_j}
$$

Métrica CFT.

$$
\langle c(z_1)c(z_2)c(z_3)\tilde{c}(\bar{z}_4)\tilde{c}(\bar{z}_5)\tilde{c}(\bar{z}_6)\rangle_{S_2}
$$

$$
\det \mathrm{C}_{0j}^{a}(\sigma_i)
$$

$$
\mathrm{C}^{z} = 1, z, z^2, \mathrm{C}^{\bar{z}} = 0
$$
$$
\mathrm{C}^{z} = 0, \mathrm{C}^{\bar{z}} = 1, \bar{z}, \bar{z}^2
$$

$$
C_{S_2}^{\mathrm{g}} \det\begin{vmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ z_1^2 & z_2^2 & z_3^2 \end{vmatrix} \det\begin{vmatrix} 1 & 1 & 1 \\ \bar{z}_4 & \bar{z}_5 & \bar{z}_6 \\ \bar{z}_4^2 & \bar{z}_5^2 & \bar{z}_6^2 \end{vmatrix} = C_{S_2}^{\mathrm{g}} z_{12}z_{13}z_{23}\bar{z}_{45}\bar{z}_{46}\bar{z}_{56}
$$

$$
\left\langle \prod_{i=1}^{p+3} c(z_i)\prod_{j=1}^{p} b(z_j')\cdot(\text{ anti })\right\rangle_{S_2} = C_{S_2}^{\mathrm{g}}\frac{z_{p+1,p+2}z_{p+1,p+3}z_{p+2,p+3}}{(z_1-z_1')\dots(z_p-z_p')}\cdot(\text{ב}_{\text{anti}}) \pm \boxtimes_{\text{ permutations}}
$$

$$
\oint_C \frac{dz}{2\pi i} j_z = -\oint_C \frac{du}{2\pi i} j_u + 3 \to 3
$$

$$
z_{12}z_{13}z_{23}\bar{z}_{45}\bar{z}_{46}\bar{z}_{56}F(z_1,z_2,z_3)\tilde{F}(\bar{z}_4,\bar{z}_5,\bar{z}_6)
$$

$$
C_{S_2}^{\mathrm{g}}\prod_{\substack{i,i'=1\\ i<i'}}^{p+3} z_{ii'}\prod_{\substack{j,j'=1\\ j<j'}}^{p} z'_{jj'}\prod_{i=1}^{p+3}\prod_{j=1}^{p}\left(z_i-z_j'\right)^{-1}\cdot(\text{ anti }),
$$

$$
\tilde{b}(\bar{z}) = b(z'), \tilde{c}(\bar{z}) = c(z'), z' = \bar{z}, \mathrm{Im}z > 0
$$

$$
\langle c(z_1)c(z_2)c(z_3)\rangle_{D_2} = C_{D_2}^{\mathrm{g}} z_{12}z_{13}z_{23}
$$

$$
\langle c(z_1)c(z_2)\tilde{c}(\bar{z}_3)\rangle_{D_2} = \langle c(z_1)c(z_2)c(z_3')\rangle_{D_2} = C_{D_2}^{\mathrm{g}} z_{12}(z_1-\bar{z}_3)(z_2-\bar{z}_3)
$$

$$
\tilde{b}(\bar{z}) = \left(\frac{\partial z'}{\partial \bar{z}}\right)^2 b(z') = z'^4 b(z'), \tilde{c}(\bar{z}) = \left(\frac{\partial z'}{\partial \bar{z}}\right)^{-1} c(z') = z'^{-2}c(z')
$$

$$
\langle c(z_1)c(z_2)c(z_3)\rangle_{RP_2} = C_{RP_2}^{\mathrm{g}} z_{12}z_{13}z_{23}
$$

$$\langle c(z_1)c(z_2)\tilde{c}(\bar{z}_3)\rangle_{RP_2} = z_3'^{-2}\langle c(z_1)c(z_2)c(z_3')\rangle_{RP_2} = C^{g}_{RP_2} z_{12}(1+z_1\bar{z}_3)(1+z_2\bar{z}_3)$$

$$S_{D_2}(k_1;k_2;k_3) = g_0^3 e^{-\lambda}\left\langle {}^{\star}_{\star}c^1 e^{ik_1\cdot X}(y_1)^{\star}_{\star\star}c^1 e^{ik_2\cdot X}(y_2)^{\star}_{\star\star}c^1 e^{ik_3\cdot X}(y_3)^{\star}_{\star}\right\rangle_{D_2} + (k_2 \leftrightarrow k_3)$$

$$S_{D_2}(k_1;k_2;k_3) = ig_0^3 C_{D_2}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right) \times |y_{12}|^{1+2\alpha' k_1\cdot k_2}|y_{13}|^{1+2\alpha' k_1\cdot k_3}|y_{23}|^{1+2\alpha' k_2\cdot k_3}$$

$$+ (k_2 \leftrightarrow k_3)$$

$$2\alpha' k_1\cdot k_2 = \alpha'(k_3^2 - k_1^2 - k_2^2) = -1$$

$$S_{D_2}(k_1;k_2;k_3) = 2ig_0^3 C_{D_2}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)$$

$$S_{D_2}(k_1;k_2;k_3;k_4) = g_0^4 e^{-\lambda}\int_{-\infty}^{\infty} dy_4 \left\langle \prod_{i=1}^{3} {}^{\star}_{\star}c^1(y_i)e^{ik_i\cdot X(y_i)_{\star}\star} e^{ik_4\cdot X(y_4)_{\star}} \right\rangle + (k_2 \leftrightarrow k_3)$$

$$= ig_0^4 C_{D_2}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)|y_{12}y_{13}y_{23}|\int_{-\infty}^{\infty} dy_4 \prod_{i<j} |y_{ij}|^{2\alpha' k_i\cdot k_j} + (k_2 \leftrightarrow k_3)$$

$$s = -(k_1+k_2)^2, t = -(k_1+k_3)^2, u = -(k_1+k_4)^2.$$

$$s+t+u = \sum_i m_i^2 = -\frac{4}{\alpha'}$$

$$S_{D_2}(k_1;k_2;k_3;k_4) = ig_0^4 C_{D_2}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right) \times \left[\int_{-\infty}^{\infty} dy_4 |y_4|^{-\alpha' u-2}|1-y_4|^{-\alpha' t-2} + (t \to s)\right]$$

$$S_{D_2}(k_1;k_2;k_3;k_4) = 2ig_0^4 C_{D_2}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)[I(s,t)+I(t,u)+I(u,s)]$$

$$I(s,t) = \int_0^1 dy y^{-\alpha' s-2}(1-y)^{-\alpha' t-2}$$

$$I(s,t) = \int_0^r dy y^{-\alpha' s-2} + \varsigma_{\text{terms analytic at } \alpha' s} = -1 = -\frac{r^{-\alpha' s-1}}{\alpha' s+1} + \varsigma_{\text{terms analytic at } \alpha' s} = -1$$

$$= -\frac{1}{\alpha' s+1} + \varsigma_{\text{terms analytic at } \alpha' s} = -1$$

$$\frac{1}{\alpha' s+1} \equiv \frac{1}{\alpha' s+1+i\epsilon} \equiv \mathrm{P}\frac{1}{\alpha' s+1} - i\pi\delta(\alpha' s+1)$$

$$S_{D_2}(k_1;k_2;k_3;k_4) = i\int \frac{d^{26}k}{(2\pi)^{26}} \frac{S_{D_2}(k_1;k_2;k)S_{D_2}(-k;k_3;k_4)}{-k^2+\alpha'^{-1}+i\epsilon} + \mathrm{Q}_{\text{ terms analytic at } k^2=1/\alpha'}$$

$$C_{D_2} = e^{-\lambda} C^X_{D_2} C^{g}_{D_2} = \frac{1}{\alpha' g_o^2}$$

$$S_{D_2}(k_1;k_2;k_3) = \frac{2ig_o}{\alpha'}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)$$

$$I(s,t) = \int_0^r dy\left[y^{-\alpha' s-2} + (\alpha' t+2)y^{-\alpha' s-1} + \cdots\right]$$

$$I(s,t) = \frac{u-t}{2s} + \text{ terms analytic at } \alpha' s = 0$$

$$\alpha' s = -1,0,1,2,\ldots$$

$$B(a,b) = \int_0^1 dy y^{a-1}(1-y)^{b-1}$$

$$I(s,t) = B(-\alpha_0(s), -\alpha_0(t)), \alpha_0(x) = 1+\alpha' x$$

$$w^{a+b-1}B(a,b) = \int_0^w dv v^{a-1}(w-v)^{b-1}$$

$$\Gamma(a+b)B(a,b) = \int_0^\infty dv v^{a-1}e^{-v}\int_0^\infty d(w-v)(w-v)^{b-1}e^{-(w-v)} = \Gamma(a)\Gamma(b)$$

$$S_{D_2}(k_1;k_2;k_3;k_4)$$

$$= \frac{2ig_o^2}{\alpha'}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)$$

$$\times\left[B(-\alpha_0(s),-\alpha_0(t)) + B(-\alpha_0(s),-\alpha_0(u)) + B(-\alpha_0(t),-\alpha_0(u))\right]$$

$$B(-\alpha_o(x), -\alpha_o(y)) = \frac{\Gamma(-\alpha' x-1)\Gamma(-\alpha' y-1)}{\Gamma(-\alpha' x-\alpha' y-2)}$$

$$s\to\infty, t \text{ fixed}$$

$$s\to\infty, t/s \text{ fixed}$$

$$s = E^2, t = (4m^2-E^2)\sin^2\frac{\theta}{2}, u = (4m^2-E^2)\cos^2\frac{\theta}{2}$$

$$S_{D_2}(k_1;k_2;k_3;k_4) \propto s^{\alpha_0(t)}\Gamma(-\alpha_0(t))$$

$$S_{D_2}(k_1;k_2;k_3;k_4) \approx \exp\left[-\alpha'(s\ln s\alpha' + t\ln t\alpha' + u\ln u\alpha')\right] = \exp\left[-\alpha' s f(\theta)\right]$$

$$f(\theta) \approx -\sin^2 \frac{\theta}{2} \ln \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2} \ln \cos^2 \frac{\theta}{2}$$

Factores Chan-Paton e interacciones de gauge.

$$|N; k; ij\rangle,$$

$$\mathrm{Tr}\left(\lambda^a \lambda^b\right) = \delta^{ab},$$

$$|N; k; a\rangle = \sum_{i,j=1}^{n} |N; k; ij\rangle \lambda_{ij}^a$$

$$\mathrm{Tr}(\lambda^{a_1} \lambda^{a_2} \lambda^{a_3} \lambda^{a_4})$$

$$S_{D_2}(k_1, a_1; k_2, a_2; k_3, a_3) = \frac{ig_o}{\alpha'} (2\pi)^{26} \delta^{26} \left(\sum_i k_i \right) \mathrm{Tr}(\lambda^{a_1} \lambda^{a_2} \lambda^{a_3} + \lambda^{a_1} \lambda^{a_3} \lambda^{a_2})$$

$$S_{D_2}(k_1, a_1; k_2, a_2; k_3, a_3; k_4, a_4)$$

$$= \frac{ig_o^2}{\alpha'} (2\pi)^{26} \delta^{26} \left(\sum_i k_i \right)$$

$$\times [\mathrm{Tr}(\lambda^{a_1} \lambda^{a_2} \lambda^{a_4} \lambda^{a_3} + \lambda^{a_1} \lambda^{a_3} \lambda^{a_4} \lambda^{a_2}) B(-\alpha_o(s), -\alpha_o(t))$$

$$+ \mathrm{Tr}(\lambda^{a_1} \lambda^{a_3} \lambda^{a_2} \lambda^{a_4}$$

$$+ \lambda^{a_1} \lambda^{a_4} \lambda^{a_2} \lambda^{a_3}) B(-\alpha_o(t), -\alpha_o(u)) + \mathrm{Tr}(\lambda^{a_1} \lambda^{a_2} \lambda^{a_3} \lambda^{a_4}$$

$$+ \lambda^{a_1} \lambda^{a_4} \lambda^{a_3} \lambda^{a_2}) B(-\alpha_o(s), -\alpha_o(u))].$$

$$\frac{1}{4} \mathrm{Tr}(\{\lambda^{a_1}, \lambda^{a_2}\} \{\lambda^{a_3}, \lambda^{a_4}\})$$

$$\frac{1}{4} \sum_a \mathrm{Tr}(\{\lambda^{a_1}, \lambda^{a_2}\} \lambda^a) \mathrm{Tr}(\{\lambda^{a_3}, \lambda^{a_4}\} \lambda^a)$$

$$\mathrm{Tr}(A\lambda^a) \mathrm{Tr}(B\lambda^a) = \mathrm{Tr}(AB)$$

$$S_{D_2}(k_1, a_1, e_1; k_2, a_2; k_3, a_3)$$

$$= -ig_o' g_o^2 e^{-\lambda} e_{1\mu}$$

$$\times \left\langle {}^{\star} c^1 \dot{X}^\mu e^{ik_1 \cdot X}(y_1)^{\star}_{\star}\, {}^{\star}_{\star} c^1 e^{ik_2 \cdot X}(y_2)^{\star}_{\star}\, {}^{\star}_{\star} c^1 e^{ik_3 \cdot X}(y_3)^{\star}_{\star} \right\rangle_{D_2} \mathrm{Tr}(\lambda^{a_1} \lambda^{a_2} \lambda^{a_3}) + (k_2, a_2)$$

$$\leftrightarrow (k_3, a_3)$$

$$\left\langle {}^{\star}_{\star}\dot{X}^{\mu} e^{ik_1\cdot X}(y_1){}^{\star}_{\star\star} e^{ik_2\cdot X}(y_2){}^{\star}_{\star\star} e^{ik_3\cdot X}(y_3){}^{\star}_{\star} \right\rangle_{D_2}$$

$$= -2i\alpha'\left(\frac{k_2^{\mu}}{y_{12}} + \frac{k_3^{\mu}}{y_{13}}\right)$$

$$\times\, iC_{D_2}^{X}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)|y_{12}|^{2\alpha' k_1\cdot k_2}|y_{13}|^{2\alpha' k_1\cdot k_3}|y_{23}|^{2\alpha' k_2\cdot k_3}$$

$$S_{D_2}(k_1,a_1,e_1;k_2,a_2;k_3,a_3) = -ig_o' e_1\cdot k_{23}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)\mathrm{Tr}(\lambda^{a_1}[\lambda^{a_2},\lambda^{a_3}])$$

$$\mathrm{Tr}([\lambda^{a_1},\lambda^{a_2}][\lambda^{a_3},\lambda^{a_4}])$$

$$g_o' = (2\alpha')^{-1/2}g_o$$

$$S_{D_2}(k_1,a_1,e_1;k_2,a_2,e_2;k_3,a_3,e_3)$$

$$= ig_o'(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)\Big(e_1\cdot k_{23}e_2\cdot e_3 + e_2\cdot k_{31}e_3\cdot e_1 + e_3\cdot k_{12}e_1\cdot e_2 + \frac{\alpha'}{2}e_1$$

$$\cdot\, k_{23}e_2\cdot k_{31}e_3\cdot k_{12}\Big)\mathrm{Tr}(\lambda^{a_1}[\lambda^{a_2},\lambda^{a_3}])$$

$$\boldsymbol{S} = \frac{1}{g_o'^2}\int d^{26}x\left[-\frac{1}{2}\mathrm{Tr}\left(D_\mu\varphi D^\mu\varphi\right) + \frac{1}{2\alpha'}\mathrm{Tr}(\varphi^2) + \frac{2^{1/2}}{3\alpha'^{1/2}}\mathrm{Tr}(\varphi^3) - \frac{1}{4}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)\right]$$

$$-\frac{2i\alpha'}{3g_o'^2}\mathrm{Tr}\left(F_\mu{}^{\nu}F_\nu{}^{\omega}F_\omega{}^{\mu}\right)$$

$$\lambda^a \to U\lambda^a U^\dagger,$$

$$\Omega\alpha_n^\mu\Omega^{-1} = (-1)^n\alpha_n^\mu$$

$$\Omega\alpha_n^\mu\Omega^{-1} = \tilde{\alpha}_n^\mu$$

$$\Omega|N;k\rangle = \omega_N|N;k\rangle, \omega_N = (-1)^{1+\alpha' m^2}$$

$$\Omega|N;k;ij\rangle = \omega_N|N;k;ji\rangle.$$

$$\Omega|N;k;a\rangle = \omega_N s^a|N;k;a\rangle$$

$\alpha' m^2$ even: λ^a antisymmetric
$\alpha' m^2$ odd: λ^a symmetric

$$\Omega_\gamma|N;k;ij\rangle = \omega_N\gamma_{jj'}|N;k;j'i'\rangle\gamma_{i'i}^{-1}$$

$$\Omega_\gamma^2|N;k;ij\rangle = [(\gamma^T)^{-1}\gamma]_{ii'}|N;k;i'j'\rangle(\gamma^{-1}\gamma^T)_{j'j}.$$

$$\gamma^T = \pm\gamma$$

$$|N;k;ij\rangle' = U_{ii'}^{-1}|N;k;i'j'\rangle U_{j'j},$$

$$\gamma' = U^T\gamma U.$$

$$\gamma = M \equiv i\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$$

$$\alpha' m^2 \text{ even: } M(\lambda^a)^T M = -\lambda^a$$
$$\alpha' m^2 \text{ odd: } M(\lambda^a)^T M = +\lambda^a$$

$$\lambda \to (\gamma^T)^{-1}\gamma\lambda\gamma^{-1}\gamma^T = \lambda,$$

$$S_{S_2}(k_1;k_2;k_3) = g_c^3 e^{-2\lambda}\left\langle \prod_{i=1}^{3} :\tilde{c}ce^{ik_i\cdot X}(z_i,\bar{z}_i): \right\rangle_{S_2}$$

$$S_{S_2}(k_1;k_2;k_3) = ig_c^3 C_{S_2}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)$$

$$S_{S_2}(k_1;k_2;k_3;k_4) = g_c^4 e^{-2\lambda}\int_{\mathbf{C}} d^2z_4 \left\langle \prod_{i=1}^{3} :\tilde{c}ce^{ik_i\cdot X}(z_i,\bar{z}_i)::e^{ik_4\cdot X}(z_4,\bar{z}_4): \right\rangle_{S_2}$$

$$S_{S_2}(k_1;k_2;k_3;k_4) = ig_c^4 C_{S_2}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)J(s,t,u)$$

$$J(s,t,u) = \int_{\mathbf{C}} d^2z_4|z_4|^{-\alpha' u/2-4}|1-z_4|^{-\alpha' t/2-4}$$

$$\alpha' s, \alpha' t, \alpha' u = -4,0,4,8,\dots,$$

$$ig_c^4 C_{S_2}\int_{|z_4|>1/\epsilon} d^2z_4|z_4|^{\alpha' s/2} \sim -\frac{8\pi i g_c^4 C_{S_2}}{\alpha' s+4}$$

$$C_{S_2} = \frac{8\pi}{\alpha' g_c^2}$$

$$S_{S_2}(k_1;k_2;k_3) = \frac{8\pi i g_c}{\alpha'}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right).$$

$$S_{S_2}(k_1;k_2;k_3;k_4) = \frac{8\pi i g_c^2}{\alpha'}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)C(-\alpha_c(t),-\alpha_c(u)),$$

$$C(a,b) = \int_{\mathbf{C}} d^2z|z|^{2a-2}|1-z|^{2b-2} = 2\pi\frac{\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(a+b)\Gamma(a+c)\Gamma(b+c)}, a+b+c=1$$

$$S_{S_2}(k_1;k_2;k_3;k_4) \propto s^{2\alpha_c(t)}\frac{\Gamma(-\alpha_c(t))}{\Gamma(1+\alpha_c(t))},$$

$$S_{S_2}(k_1;k_2;k_3;k_4) \propto \exp\left[-\frac{\alpha'}{2}(s\ln\ s\alpha' + t\ln\ t\alpha' + u\ln\ u\alpha')\right]$$

$$S_{S_2}(k_1,e_1;k_2;k_3) = g_c^2 g_c' e^{-2\lambda} e_{1\mu\nu}\left\langle :\tilde{c}c\partial X^\mu \bar{\partial} X^\nu e^{ik_1\cdot X}(z_1,\bar{z}_1):: \tilde{c}ce^{ik_2\cdot X}(z_2,\bar{z}_2):: \tilde{c}ce^{ik_3\cdot X}(z_3,\bar{z}_3):\right\rangle_{S_2}$$

$$= -\frac{\pi i\alpha'}{2} g_c' e_{1\mu\nu} k_{23}^\mu k_{23}^\nu (2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)$$

$$g_c' = \frac{2}{\alpha'}g_c$$

$$\boldsymbol{S}_T = -\frac{1}{2}\int d^{26}x(-G)^{1/2}e^{-2\tilde{\Phi}}\left(G^{\mu\nu}\partial_\mu T\partial_\nu T - \frac{4}{\alpha'}T^2\right)$$

$$\tilde{G}_{\mu\nu} = \eta_{\mu\nu} - 2\kappa e_{\mu\nu}e^{ik\cdot x}$$

$$\kappa = \pi\alpha' g_c' = 2\pi g_c$$

$$S_{S_2}(k_1,e_1;k_2,e_2;k_3,e_3) = \frac{i\kappa}{2}(2\pi)^{26}\delta^{26}\left(\sum_i k_i\right)e_{1\mu\nu}e_{2\alpha\beta}e_{3\gamma\delta}T^{\mu\alpha\gamma}T^{\nu\beta\delta}$$

$$T^{\mu\alpha\gamma} = k_{23}^\mu\eta^{\alpha\gamma} + k_{31}^\alpha\eta^{\gamma\mu} + k_{12}^\gamma\eta^{\mu\alpha} + \frac{\alpha'}{8}k_{23}^\mu k_{31}^\alpha k_{12}^\gamma$$

$$J(s,t,u,\alpha') = -2\sin\ \pi\alpha_c(t)I(s,t,4\alpha')I(t,u,4\alpha')$$

$$\int_{\mathbf{C}} d^2z z^{a-1+m_1}\bar{z}^{a-1+n_1}(1-z)^{b-1+m_2}(1-\bar{z})^{b-1+n_2}$$

$$= 2\sin\left[\pi(b+n_2)\right]B(a+m_1,b+m_2)B(b+n_2,1-a-b-n_1-n_2)m_1-n_1$$

$$\in \mathbf{Z}, m_2-n_2 \in \mathbf{Z}$$

$$A_c(s,t,u,\alpha',g_c) = \frac{\pi i g_c^2\alpha'}{g_o^4}\sin\left[\pi\alpha_c(t)\right]A_o\left(s,t,\frac{1}{4}\alpha',g_o\right)A_o\left(t,u,\frac{1}{4}\alpha',g_o\right)^*$$

$$:e^{ik_1\cdot X(z_1,\bar{z}_1)}::e^{ik_4\cdot X(z_4,\bar{z}_4)}:$$

$$= |z_{14}|^{\alpha' k_1\cdot k_4}:\Big(1 + iz_{14}k_1\cdot\partial X + i\bar{z}_{14}k_1\cdot\bar{\partial}X - z_{14}\bar{z}_{14}k_1\cdot\partial X k_1\cdot\bar{\partial}X$$

$$+\cdots\Big)e^{i(k_1+k_4)\cdot X}(z_4,\bar{z}_4):$$

$$\alpha' k_1\cdot k_4 = \frac{\alpha'}{2}(k_1+k_4)^2 - 4 > -2$$

$$-\frac{1}{4g_o'^2}\int d^{26}x\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

$$-\frac{1}{4g_o'^2}\int d^{26}x(-G)^{1/2}e^{-\tilde{\Phi}}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

$$-\Lambda\int d^{26}x(-G)^{1/2}e^{-\tilde{\Phi}}$$

Invariancia Möbius.

$$z'=\frac{\alpha z+\beta}{\gamma z+\delta}$$

$$\langle\mathcal{A}_i(z_1,\bar{z}_1)\ldots\mathcal{A}_k(z_n,\bar{z}_n)\rangle_{S_2}=\langle\mathcal{A}_i'(z_1,\bar{z}_1)\ldots\mathcal{A}_k'(z_n,\bar{z}_n)\rangle_{S_2}.$$

$$\langle\mathcal{A}_i(0,0)\rangle_{S_2}=\langle\mathcal{A}_i'(0,0)\rangle_{S_2}=\gamma^{-h_i}\bar{\gamma}^{-\tilde{h}_i}\langle\mathcal{A}_i(0,0)\rangle_{S_2}.$$

$$\left\langle\mathcal{A}_i(z_1,\bar{z}_1)\mathcal{A}_j(z_2,\bar{z}_2)\right\rangle_{S_2}=z_{12}^{-h_i-h_j}\bar{z}_{12}^{-\tilde{h}_i-\tilde{h}_j}\left\langle\mathcal{A}_i(1,1)\mathcal{A}_j(0,0)\right\rangle_{S_2},$$

$$\left\langle\mathcal{O}_p(z_1,\bar{z}_1)\mathcal{O}_q(z_2,\bar{z}_2)\right\rangle_{S_2}=0\ \text{ unless } h_p=h_q,\tilde{h}_p=\tilde{h}_q.$$

$$\left\langle\prod_{i=1}^{3}\mathcal{O}_{p_i}(z_i,\bar{z}_i)\right\rangle_{S_2}=C_{p_1p_2p_3}\prod_{\substack{i,j=1\\ i<j}}^{3}z_{ij}^{h-2(h_i+h_j)}z_{ij}^{\tilde{h}-2(\tilde{h}_i+\tilde{h}_j)}$$

$$\left\langle\prod_{i=1}^{4}\mathcal{O}_{p_i}(z_i,\bar{z}_i)\right\rangle_{S_2}=C_{p_1p_2p_3p_4}(z_c,\bar{z}_c)(z_{12}z_{34})^{h}(\bar{z}_{12}\bar{z}_{34})^{\tilde{h}}\times\prod_{\substack{i,j=1\\ i<j}}^{4}z_{ij}^{-h_i-h_j}\bar{z}_{ij}^{-\tilde{h}_i-\tilde{h}_j}$$

$$\left\langle\mathcal{A}_i'(\infty,\infty)\mathcal{A}_j(0,0)\right\rangle_{S_2}.$$

$$\int[d\phi_{\mathrm{b}}]\Psi_{\mathcal{A}_i}[\phi_{\mathrm{b}}^{\Omega}]\Psi_{\mathcal{A}_j}[\phi_{\mathrm{b}}]$$

$$\left\langle\langle i\mid j\right\rangle=\left\langle\mathcal{A}_i'(\infty,\infty)\mathcal{A}_j(0,0)\right\rangle_{S_2}.$$

$$\mathcal{O}_i(z,\bar{z})\mathcal{O}_j(0,0)=\frac{\langle\langle i\mid j\rangle}{z^{h_i+h_j}\tilde{\bar{Z}}_i+\tilde{h}_j\langle 1\rangle_{S_2}}+\cdots$$

$$\langle\langle i\mid j\rangle=\pm\langle\langle j\mid i\rangle,$$

$$\left\langle\mathcal{A}_i'(\infty,\infty)\mathcal{A}_k(1,1)\mathcal{A}_j(0,0)\right\rangle_{S_2}=\left\langle\langle i|\hat{\mathcal{A}}_k(1,1)\mid j\right\rangle,$$

$$\sum_l c_{kj}^{l}\langle\mathcal{A}_i'(\infty,\infty)\mathcal{A}_l(0,0)\rangle_{S_2}=c_{ikj}$$

$$\left\langle\mathcal{A}_i'(\infty,\infty)\mathcal{A}_k(z_1,\bar{z}_1)\mathcal{A}_j(0,0)\right\rangle_{S_2}=z_1^{h_i-h_k-h_j}\bar{z}_1^{\tilde{h}_i-\tilde{h}_k-\tilde{h}_j}c_{ikj}$$

$$\left\langle\mathcal{A}_i'(\infty,\infty)\mathcal{A}_k(z_1,\bar{z}_1)\mathcal{A}_l(z_2,\bar{z}_2)\mathcal{A}_j(0,0)\right\rangle_{S_2}=\left\langle\langle i|\mathrm{T}\left[\hat{\mathcal{A}}_k(z_1,\bar{z}_1)\hat{\mathcal{A}}_l(z_2,\bar{z}_2)\right]\mid j\right\rangle$$

$$1 = |m\rangle\mathcal{G}^{mn}\langle\langle n|.$$

$$\sum_m z_1^{h_i-h_k-h_m}\bar{z}_1^{\tilde{h}_i-\tilde{h}_k-\tilde{h}_m}z_2^{h_m-h_l-h_j}\bar{z}_2^{\tilde{h}_m-\tilde{h}_l-\tilde{h}_j}c_{ikm}c^m{}_{lj}$$

Cálculos de operador.

$$\left\langle :e^{ik_4\cdot X(\infty,\infty)}:' :e^{ik_1\cdot X(z_1,\bar{z}_1)}::e^{ik_2\cdot X(z_2,\bar{z}_2)}::e^{ik_3\cdot X(0,0)}:\right\rangle_{S_2} = \left\langle\langle 0;k_4|\mathrm{T}\left[\,{}^\circ_\circ e^{ik_1\cdot X_1}{}^\circ_\circ\,{}^\circ_\circ e^{ik_2\cdot X_2}{}^\circ_\circ\right] \mid 0;k_3\right\rangle$$

$$e^{ik\cdot X_\circ} = e^{ik\cdot X_C}e^{ik\cdot X_A},$$

$$X_C^\mu(z,\bar{z}) = x^\mu - i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{m=1}^{\infty}\frac{1}{m}\left(\alpha_{-m}^\mu z^m + \tilde{\alpha}_{-m}^\mu \bar{z}^m\right)$$

$$X_A^\mu(z,\bar{z}) = -i\frac{\alpha'}{2}p^\mu \ln|z|^2 + i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\alpha_m^\mu}{z^m}+\frac{\tilde{\alpha}_m^\mu}{\bar{z}^m}\right).$$

$$\langle 0;k_4|e^{ik_1\cdot X_{1C}}e^{ik_1\cdot X_{1A}}e^{ik_2\cdot X_{2C}}e^{ik_2\cdot X_{2A}}|0;k_3\rangle.$$

$$e^{ik_1\cdot X_{1A}}e^{ik_2\cdot X_{2C}} = e^{ik_2\cdot X_{2C}}e^{ik_1\cdot X_{1A}}e^{-[k_1\cdot X_{1A},k_2\cdot X_{2C}]} = e^{ik_2\cdot X_{2C}}e^{ik_1\cdot X_{1A}}|z_{12}|^{\alpha' k_1\cdot k_2}$$

$$|z_{12}|^{\alpha'}k_1\cdot k_2\langle 0;k_4|e^{ik_1\cdot X_{1C}+ik_2\cdot X_{2C}}e^{ik_1\cdot X_{1A}+ik_2\cdot X_{2A}}|0;k_3\rangle$$

$$= |z_{12}|^{\alpha' k_1\cdot k_2}\left\langle\langle 0;k_4|e^{i(k_1+k_2)\cdot x}e^{\alpha'(k_1\ln|z_1|+k_2\ln|z_2|)\cdot p} \mid 0;k_3\right\rangle$$

$$= |z_{12}|^{\alpha' k_1\cdot k_2}|z_1|^{\alpha' k_1\cdot k_3}|z_2|^{\alpha' k_2\cdot k_3}\langle\langle 0;k_1+k_2+k_4 \mid 0;k_3\rangle$$

$$= iC_{S_2}^X(2\pi)^d\delta^d\left(\sum_i k_i\right)|z_{12}|^{\alpha' k_1\cdot k_2}|z_1|^{\alpha' k_1\cdot k_3}|z_2|^{\alpha' k_2\cdot k_3}$$

$$\langle 0;k \mid 0;l\rangle = iC_{S_2}^X(2\pi)^d\delta^d(k+l)$$

$$\langle 0;k \mid 0;l\rangle = (2\pi)^d\delta^d(l-k)$$

$$\langle\langle 0;k| = iC_{S_2}^X\langle 0;-k|.$$

$$\overline{\mathcal{A}(p)} = \mathcal{A}'(p')^\dagger.$$

$$\mathcal{O}(z) = i^h\sum_{n=-\infty}^{\infty}\frac{\mathcal{O}_n}{z^{n+h}}$$

$$\mathcal{O}(z)^\dagger = i^{-h}\sum_{n=-\infty}^{\infty}\frac{\mathcal{O}_n^\dagger}{\bar{z}^{n+h}}.$$

$$\overline{\mathcal{O}(z)} = i^{-h}(-z^{-2})^h\sum_{n=-\infty}^{\infty}\frac{\mathcal{O}_n^\dagger}{z^{-n-h}} = i^h\sum_{n=-\infty}^{\infty}\frac{\mathcal{O}_{-n}^\dagger}{z^{n+h}}.$$

$$\left\langle\langle\overline{\mathcal{A}}_i\right| = K\langle\mathcal{A}_i|$$

$$\langle\bar{\imath} \mid j\rangle \equiv \left\langle\langle 1|\overline{\mathcal{A}_i'(\infty,\infty)} \mid j\right\rangle = K\langle 1|\overline{\mathcal{A}_i'(\infty,\infty)}|j\rangle = K\langle 1|\mathcal{A}_i(0,0)^{\dagger}|j\rangle = K\langle j|\mathcal{A}_i(0,0)|1\rangle^* = K\langle j \mid i\rangle^* = K\langle i \mid j\rangle$$

$$k_1^i k_1^j P_{ij,kl}^J k_3^k k_3^l$$

$$|z|^{-2a} = \frac{1}{\Gamma(a)}\int_0^\infty dt t^{a-1}\exp\left(-tz\bar{z}\right)$$

$$w \cong w + 2\pi \cong w + 2\pi\tau$$

$$(\sigma^1,\sigma^2) \cong (\sigma^1 + 2\pi,\sigma^2) \cong (\sigma^1 + 2\pi\tau_1,\sigma^2 + 2\pi\tau_2)$$

$$z \cong z\exp\left(-2\pi i\tau\right)$$

$$1 \leq |z| \leq \exp\left(2\pi\tau_2\right)$$

$$0 \leq \mathrm{Re}w \leq \pi, w \cong w + 2\pi it$$

$$w' = -\bar{w}$$

$$(\sigma^1, 0) \cong (\sigma^1, 2\pi t)$$

$$w \cong w + 2\pi \cong -\bar{w} + 2\pi it$$

$$(\sigma^1,\sigma^2) \cong (\sigma^1 + 2\pi,\sigma^2) \cong (-\sigma^1,\sigma^2 + 2\pi t)$$

$$w' = -\bar{w} + 2\pi it$$

$$0 \leq \mathrm{Re}w \leq \pi, w \cong -\bar{w} + \pi + 2\pi it$$

$$w' = -\bar{w} \ \text{ and } \ w' = w + \pi(2it + 1)$$

Correlaciones escalares.

$$\frac{2}{\alpha'}\bar{\partial}\partial G'(w,\bar{w};w',\bar{w}') = -2\pi\delta^2(w - w') + \frac{1}{4\pi\tau_2}.$$

$$G'(w,\bar{w};w',\bar{w}') \sim -\frac{\alpha'}{2}\ln\left|\vartheta_1\left(\frac{w - w'}{2\pi}\middle|\,\tau\right)\right|^2.$$

$$G'(w,\bar{w};w',\bar{w}') = -\frac{\alpha'}{2}\ln\left|\vartheta_1\left(\frac{w - w'}{2\pi}\middle|\,\tau\right)\right|^2 + \alpha'\frac{[\mathrm{Im}(w - w')]^2}{4\pi\tau_2} + k(\tau,\bar{\tau}).$$

$$\left\langle\prod_{i=1}^{n} :e^{ik_i\cdot X(z_i,\bar{z}_i)}:\right\rangle_{T^2}$$

$$= iC_{T^2}^{X}(\tau)(2\pi)^d\delta^d\left(\sum_i k_i\right)$$

$$\times\prod_{i<j}\left|\frac{2\pi}{\partial_v\vartheta_1(0\mid\tau)}\vartheta_1\left(\frac{w_{ij}}{2\pi}\middle|\ \tau\right)\exp\left[-\frac{\left(\mathrm{Im}w_{ij}\right)^2}{4\pi\tau_2}\right]\right|^{\alpha' k_i\cdot k_j}$$

$$Z(\tau) = \mathrm{Tr}[\exp(2\pi i\tau_1 P - 2\pi\tau_2 H)] = (q\bar{q})^{-d/24}\mathrm{Tr}\left(q^{L_0}\bar{q}^{\tilde{L}_0}\right)$$

$$V_d(q\bar{q})^{-d/24}\int\frac{d^dk}{(2\pi)^d}\exp(-\pi\tau_2\alpha' k^2)\prod_{\mu,n}\sum_{N_{\mu n},\tilde{N}_{\mu n}=0}^{\infty} q^{nN_{\mu n}}\bar{q}^{n\tilde{N}_{\mu n}}$$

$$\sum_{N=0}^{\infty} q^{nN} = (1-q^n)^{-1}$$

$$Z(\tau) = iV_d Z_X(\tau)^d$$

$$Z_X(\tau) = (4\pi^2\alpha'\tau_2)^{-1/2}|\eta(\tau)|^{-2}$$

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$

$$ds^2 = dwd\bar{w} + \epsilon^* dw^2 + \epsilon d\bar{w}^2 = (1+\epsilon^*+\epsilon)d[w+\epsilon(\bar{w}-w)]d[\bar{w}+\epsilon^*(w-\bar{w})] + O(\epsilon^2)$$

$$w' \cong w' + 2\pi \cong w' + 2\pi(\tau - 2i\tau_2\epsilon)$$

$$\delta\tau = -2i\tau_2\epsilon$$

$$\delta Z(\tau) = -\frac{1}{2\pi}\int d^2w[\delta g_{\bar{w}\bar{w}}\langle T_{ww}(w)\rangle + \delta g_{ww}\langle T_{\bar{w}\bar{w}}(\bar{w})\rangle] = -2\pi i[\delta\tau\langle T_{ww}(0)\rangle - \delta\bar{\tau}\langle T_{\bar{w}\bar{w}}(0)\rangle]$$

$$\partial_w X^\mu(w)\partial_w X_\mu(0) = -\frac{\alpha' d}{2w^2} - \alpha' T_{ww}(0) + O(w)$$

$$Z(\tau)^{-1}\left\langle\partial_w X^\mu(w)\partial_w X_\mu(0)\right\rangle = d\partial_w\partial_{w'}G'(w,\bar{w};w',\bar{w}')|_{w'=0} = \frac{\alpha' d}{2}\frac{\vartheta_1\partial_w^2\vartheta_1 - \partial_w\vartheta_1\partial_w\vartheta_1}{\vartheta_1^2} + \frac{\alpha' d}{8\pi\tau_2}$$

$$\frac{\alpha' d}{6}\frac{\partial_w^3\vartheta_1}{\partial_w\vartheta_1} + \frac{\alpha' d}{8\pi\tau_2}$$

$$\langle T_{ww}(0)\rangle = \left(-\frac{d}{6}\frac{\partial_w^3\vartheta_1(0\mid\tau)}{\partial_w\vartheta_1(0\mid\tau)} - \frac{d}{8\pi\tau_2}\right)Z(\tau)$$

$$\partial_\tau \ln\, Z(\tau) = \frac{\pi i d}{3}\frac{\partial_w^3\vartheta_1(0\mid\tau)}{\partial_w\vartheta_1(0\mid\tau)} + \frac{id}{4\tau_2}$$

$$\partial_w^2\vartheta_1\left(\frac{w}{2\pi}\middle|\ \tau\right) = \frac{i}{\pi}\partial_\tau\vartheta_1\left(\frac{w}{2\pi}\middle|\ \tau\right)$$

$$\partial_\tau \ln\, Z(\tau) = -\frac{d}{3}\partial_\tau \ln\, \partial_w\vartheta_1(0\mid\tau) + \frac{id}{4\tau_2}$$

$$Z(\tau) = |\partial_w\vartheta_1(0\mid\tau)|^{-2d/3}\tau_2^{-d/2}$$

Propagador CFT.

$$\mathrm{Tr}[\exp\,(2\pi i\tau_1 P - 2\pi\tau_2 H)] = (q\bar{q})^{13/12}\mathrm{Tr}\left(q^{L_0}\bar{q}^{\tilde{L}_0}\right) = 4(q\bar{q})^{1/12}\prod_{n=1}^{\infty}\,|1+q^n|^4$$

$$Z(\tau) = \mathrm{Tr}[(-1)^F\exp\,(2\pi i\tau_1 P - 2\pi\tau_2 H)] = 0$$

$$\left\langle c(w_1)b(w_2)\tilde{c}(\bar{w}_3)\tilde{b}(\bar{w}_4)\right\rangle.$$

$$\mathrm{Tr}\left[(-1)^F c_0 b_0\tilde{c}_0\tilde{b}_0\exp\,(2\pi i\tau_1 P - 2\pi\tau_2 H)\right]$$

$$(q\bar{q})^{1/12}\prod_{n=1}^{\infty}\,|1-q^n|^4 = |\eta(\tau)|^4$$

$$Z(\tau) = \sum_i\, q^{h_i - c/24}\bar{q}^{\tilde{h}_i-\tilde{c}/24}(-1)^{F_i}$$

$$h_i - \tilde{h}_i \in \mathbf{Z}$$

$$Z(i\ell) \overset{\ell\to 0}{\approx} \exp\,\left[\frac{\pi(c+\tilde{c})}{12\ell}\right]$$

$$\vartheta(v,\tau) = \sum_{n=-\infty}^{\infty}\,\exp\,(\pi i n^2\tau + 2\pi i n v)$$

$$\vartheta(v+1,\tau) = \vartheta(v,\tau)$$
$$\vartheta(v+\tau,\tau) = \exp\,(-\pi i\tau - 2\pi i v)\vartheta(v,\tau)$$

$$\vartheta(v,\tau+1)\ = \vartheta(v+1/2,\tau)$$
$$\vartheta(v/\tau,-1/\tau)\ = (-i\tau)^{1/2}\exp\,(\pi i v^2/\tau)\vartheta(v,\tau)$$

$$\vartheta(v,\tau) = \prod_{m=1}^{\infty}\,(1-q^m)\left(1+zq^{m-1/2}\right)\left(1+z^{-1}q^{m-1/2}\right)$$

$$q=\exp(2\pi i\tau), z=\exp(2\pi i v)$$

$$\vartheta\begin{bmatrix}a\\b\end{bmatrix}(v,\tau)=\exp[\pi i a^2\tau+2\pi i a(v+b)]\vartheta(v+a\tau+b,\tau)=\sum_{n=-\infty}^{\infty}\exp[\pi i(n+a)^2\tau+2\pi i(n+a)(v+b)]$$

$$\vartheta_{00}(v,\tau)=\vartheta_3(v\mid\tau)=\vartheta\begin{bmatrix}0\\0\end{bmatrix}(v,\tau)=\sum_{n=-\infty}^{\infty}q^{n^2/2}z^n$$
$$\vartheta_{01}(v,\tau)=\vartheta_4(v\mid\tau)=\vartheta\begin{bmatrix}0\\1/2\end{bmatrix}(v,\tau)=\sum_{n=-\infty}^{\infty}(-1)^n q^{n^2/2}z^n$$

$$\vartheta_{10}(v,\tau)=\vartheta_2(v\mid\tau)=\vartheta\begin{bmatrix}1/2\\0\end{bmatrix}(v,\tau)=\sum_{n=-\infty}^{\infty}q^{(n-1/2)^2/2}z^{n-1/2}$$
$$\vartheta_{11}(v,\tau)=-\vartheta_1(v\mid\tau)=\vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}(v,\tau)$$
$$=-i\sum_{n=-\infty}^{\infty}(-1)^n q^{(n-1/2)^2/2}z^{n-1/2}$$

$$\vartheta_{00}(v,\tau)=\prod_{m=1}^{\infty}(1-q^m)\left(1+zq^{m-1/2}\right)\left(1+z^{-1}q^{m-1/2}\right)$$
$$\vartheta_{01}(v,\tau)=\prod_{m=1}^{\infty}(1-q^m)\left(1-zq^{m-1/2}\right)\left(1-z^{-1}q^{m-1/2}\right)$$
$$\vartheta_{10}(v,\tau)=2\exp(\pi i\tau/4)\cos\pi v\prod_{m=1}^{\infty}(1-q^m)(1+zq^m)(1+z^{-1}q^m)$$
$$\vartheta_{11}(v,\tau)=-2\exp(\pi i\tau/4)\sin\pi v\prod_{m=1}^{\infty}(1-q^m)(1-zq^m)(1-z^{-1}q^m)$$

$$\begin{aligned}\vartheta_{00}(v,\tau+1)&=\vartheta_{01}(v,\tau)\\ \vartheta_{01}(v,\tau+1)&=\vartheta_{00}(v,\tau)\\ \vartheta_{10}(v,\tau+1)&=\exp(\pi i/4)\vartheta_{10}(v,\tau)\\ \vartheta_{11}(v,\tau+1)&=\exp(\pi i/4)\vartheta_{11}(v,\tau)\end{aligned}$$

$$\begin{aligned}\vartheta_{00}(v/\tau,-1/\tau)&=(-i\tau)^{1/2}\exp(\pi i v^2/\tau)\vartheta_{00}(v,\tau)\\ \vartheta_{01}(v/\tau,-1/\tau)&=(-i\tau)^{1/2}\exp(\pi i v^2/\tau)\vartheta_{10}(v,\tau)\\ \vartheta_{10}(v/\tau,-1/\tau)&=(-i\tau)^{1/2}\exp(\pi i v^2/\tau)\vartheta_{01}(v,\tau)\\ \vartheta_{11}(v/\tau,-1/\tau)&=-i(-i\tau)^{1/2}\exp(\pi i v^2/\tau)\vartheta_{11}(v,\tau)\end{aligned}$$

$$\vartheta_{00}^4(0,\tau)-\vartheta_{01}^4(0,\tau)-\vartheta_{10}^4(0,\tau)=0$$

$$\vartheta_{11}(0,\tau)=0$$

$$\eta(\tau)=q^{1/24}\prod_{m=1}^{\infty}(1-q^m)=\left[\frac{\partial_v\vartheta_{11}(0,\tau)}{-2\pi}\right]^{1/3}$$

$$\eta(\tau+1) = \exp{(i\pi/12)}\eta(\tau)$$
$$\eta(-1/\tau) = (-i\tau)^{1/2}\eta(\tau)$$

$$S_{T^2}(1;2;\ldots;n) = \frac{1}{2}\int_{F_0} d\tau d\bar{\tau}\left\langle B\tilde{B}\tilde{c}c\mathcal{V}_1(w_1,\bar{w}_1)\prod_{i=2}^{n}\int \; dw_i d\bar{w}_i \mathcal{V}_i(w_i,\bar{w}_i)\right\rangle_{T^2}$$

$$B = \frac{1}{4\pi}(b,\partial_\tau g) = \frac{1}{2\pi}\int \; d^2w b_{ww}(w)\partial_\tau g_{\bar{w}\bar{w}} = \frac{i}{4\pi\tau_2}\int \; d^2w b_{ww}(w) \to 2\pi i b_{ww}$$

$$\int \frac{dwd\bar{w}}{2(2\pi)^2\tau_2}$$

$$S_{T^2}(1;2;\ldots;n) = \int_{F_0} \frac{d\tau d\bar{\tau}}{4\tau_2}\left\langle b(0)\tilde{b}(0)\tilde{c}(0)c(0)\prod_{i=1}^{n}\int \; dw_i d\bar{w}_i \mathcal{V}_i(w_i,\bar{w}_i)\right\rangle_{T^2}$$

$$Z_{T^2} = \int_{F_0} \frac{d\tau d\bar{\tau}}{4\tau_2}\langle b(0)\tilde{b}(0)\tilde{c}(0)c(0)\rangle_{T^2}$$

$$Z_{T^2} = iV_{26}\int_{F_0} \frac{d\tau d\bar{\tau}}{4\tau_2}(4\pi^2\alpha'\tau_2)^{-13}|\eta(\tau)|^{-48}$$

$$\frac{d\tau d\bar{\tau}}{\tau_2^2}$$

$$\begin{aligned} Z_{T^2} &= V_d\int_{F_0} \frac{d\tau d\bar{\tau}}{4\tau_2}\int \frac{d^dk}{(2\pi)^d}\exp{(-\pi\tau_2\alpha' k^2)}\sum_{i\in\mathcal{H}^\perp} q^{h_i-1}\bar{q}^{\tilde{h}_i-1} \\ &= iV_d\int_{F_0} \frac{d\tau d\bar{\tau}}{4\tau_2}(4\pi^2\alpha'\tau_2)^{-d/2}\sum_{i\in\mathcal{H}^\perp} q^{h_i-1}\bar{q}^{\tilde{h}_i-1} \end{aligned}$$

$$Z_{S_1}(m^2) = V_d\int \frac{d^dk}{(2\pi)^d}\int_0^\infty \frac{dl}{2l}\exp{[-(k^2+m^2)l/2]} = iV_d\int_0^\infty \frac{dl}{2l}(2\pi l)^{-d/2}\exp{(-m^2l/2)}$$

$$m^2 = \frac{2}{\alpha'}(h+\tilde{h}-2)$$

$$\delta_{h,\tilde{h}} = \int_{-\pi}^{\pi}\frac{d\theta}{2\pi}\exp{[i(h-\tilde{h})\theta]}$$

$$\begin{aligned} \sum_{i\in\mathcal{H}^\perp} Z_{S_1}(m_i^2) &= iV_d\int_0^\infty \frac{dl}{2l}\int_{-\pi}^{\pi}\frac{d\theta}{2\pi}(2\pi l)^{-\frac{d}{2}} \times \sum_{i\in\mathcal{H}^\perp} \exp\left[-\frac{(h_i+\tilde{h}_i-2)l}{\alpha'} + i(h_i-\tilde{h}_i)\theta\right] \\ &= iV_d\int_R \frac{d\tau d\bar{\tau}}{4\tau_2}(4\pi^2\alpha'\tau_2)^{-d/2}\sum_{i\in\mathcal{H}^\perp} q^{h_i-1}\bar{q}^{\tilde{h}_i-1} \end{aligned}$$

$$R: \tau_2 > 0, |\tau_1| < \frac{1}{2}$$

$$F_0: |\tau| > 1, |\tau_1| < \frac{1}{2}, \tau_2 > 0$$

$$iV_{26} \int^{\infty} \frac{d\tau_2}{2\tau_2} \frac{d\tau_2}{2\tau_2} (4\pi^2\alpha'\tau_2)^{-13} [\exp(4\pi\tau_2) + 24^2 + \cdots]$$

$$iV_d \int^{\infty} \frac{d\tau_2}{2\tau_2} \frac{d\tau_2}{2\tau_2} (4\pi^2\alpha'\tau_2)^{-d/2} \sum_i \exp(-\pi\alpha' m_i^2 \tau_2)$$

$$\boldsymbol{Z}_{\mathrm{vac}}(m^2) = \exp\left[Z_{S_1}(m^2)\right]$$

$$\boldsymbol{Z}_{\mathrm{vac}}(m^2) = \langle 0|\exp(-iHT)|0\rangle = \exp(-i\rho_0 V_d)$$

$$\rho_0 = \frac{i}{V_d} Z_{S_1}(m^2).$$

$$\int_0^{\infty} \frac{dl}{2l} \exp[-(k^2+m^2)l/2] \rightarrow -\frac{1}{2}\ln(k^2+m^2)$$

$$i\int_0^{\infty} \frac{dl}{2l} \int_{-\infty}^{\infty} \frac{dk^0}{2\pi} \exp[-(k^2+m^2)l/2] \rightarrow \frac{\omega_{\mathbf{k}}}{2}$$

$$\rho_0 = \int \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}} \frac{\omega_{\mathbf{k}}}{2}$$

$$\ln \boldsymbol{Z}_{\mathrm{vac}}(m^2) = -\frac{1}{2}\mathrm{Tr}\ln(-\partial^2+m^2) = -\frac{V_d}{2}\int \frac{d^d k}{(2\pi)^d} \ln(k^2+m^2)$$

$$\rho_0 = \frac{i}{V_d}\sum_i (-1)^{\mathbf{F}_i} Z_{S_1}(m_i^2).$$

$$|\rho_0| \lesssim 10^{-44}\mathrm{GeV}^4$$

$$m_{\mathrm{ew}}^4 \approx 10^8 \mathrm{GeV}^4$$

$$Z_{C_2} = \int_0^{\infty} \frac{dt}{2t} \mathrm{Tr}_0'[\exp(-2\pi t L_0)] = iV_d \int_0^{\infty} \frac{dt}{2t} (8\pi^2\alpha' t)^{-d/2} \sum_{i \in \mathcal{H}_{\mathrm{o}}^{1}} \exp[-2\pi t(h_i - 1)]$$

$$\rightarrow iV_{26} n^2 \int_0^{\infty} \frac{dt}{2t} (8\pi^2\alpha' t)^{-13} \eta(it)^{-24}$$

$$\eta(it) = t^{-1/2}\eta(i/t)$$

$$Z_{C_2} = i\frac{V_{26} n^2}{2\pi(8\pi^2\alpha')^{13}} \int_0^{\infty} ds \eta(is/\pi)^{-24}$$

$$\eta(is/\pi)^{-24} = \exp(2s)\prod_{n=1}^{\infty}[1-\exp(-2ns)]^{-24} = \exp(2s) + 24 + O(\exp(-2s))$$

$$\int_0^{\infty} ds \exp(\beta s) \equiv -\frac{1}{\beta}$$

$$\left.\frac{1}{k^2}\right|_{k^{\mu}=0}$$

$$-\frac{1}{g^2}\int d^d x\left(\frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi + g\Lambda\phi\right)$$

$$\partial^2\phi = g\Lambda$$

$$-\Lambda\int d^{26}x(-G)^{1/2}e^{-\tilde{\Phi}}$$

$$\exp(-\alpha' k^2 s/2)$$

$$\langle B|c_0 b_0 \exp\left[-s\left(L_0+\tilde{L}_0\right)\right]|B\rangle.$$

$$\left(\alpha_n^{\mu}+\tilde{\alpha}_{-n}^{\mu}\right)|B\rangle = (c_n+\tilde{c}_{-n})|B\rangle = \left(b_n-\tilde{b}_{-n}\right)|B\rangle = 0,\ \text{all } n$$

$$|B\rangle \propto (c_0+\tilde{c}_0)\exp\left[-\sum_{n=1}^{\infty}\left(n^{-1}\alpha_{-n}\cdot\tilde{\alpha}_{-n}+b_{-n}\tilde{c}_{-n}+\tilde{b}_{-n}c_{-n}\right)\right]|0;0\rangle.$$

$$Z_{K_2} = \int_0^{\infty}\frac{dt}{4t}\mathrm{Tr}_{\mathrm{c}}'\{\Omega\exp\left[-2\pi t\left(L_0+\tilde{L}_0\right)\right]\} = iV_d\int_0^{\infty}\frac{dt}{4t}(4\pi^2\alpha' t)^{-d/2}\sum_{i\in\mathcal{H}_{\mathrm{c}}^{\perp}}\Omega_i\exp\left[-2\pi t\left(h_i+\tilde{h}_i-2\right)\right]$$

$$Z_{K_2} \to iV_{26}\int_0^{\infty}\frac{dt}{4t}(4\pi^2\alpha' t)^{-13}\eta(2it)^{-24}$$

$$0 \le \sigma^1 \le 2\pi, 0 \le \sigma^2 \le 2\pi t$$
$$0 \le \sigma^1 \le \pi, 0 \le \sigma^2 \le 4\pi t$$

$$w \cong w + 2\pi \cong -\bar{w} + 2\pi i t$$

$$w \cong w + 4\pi i t, w + \pi \cong -(\bar{w}+\pi) + 2\pi i t$$

$$Z_{K_2} = i\frac{2^{26}V_{26}}{4\pi(8\pi^2\alpha')^{13}}\int_0^{\infty} ds\,\eta(is/\pi)^{-24}$$

$$Z_{M_2} = iV_d\int_0^{\infty}\frac{dt}{4t}(8\pi^2\alpha' t)^{-d/2}\sum_{i\in\mathcal{H}_{\mathrm{o}}^{\perp}}\Omega_i\exp\left[-2\pi t(h_i-1)\right]$$

$$\exp(2\pi t)\prod_{n=1}^{\infty}[1-(-1)^n\exp(-2\pi n t)]^{-24} = \vartheta_{00}(0,2it)^{-12}\eta(2it)^{-12}$$

$$Z_{M_2} = \pm i n V_{26} \int_0^\infty \frac{dt}{4t} (8\pi^2\alpha' t)^{-13} \vartheta_{00}(0,2it)^{-12} \eta(2it)^{-12}$$

$$Z_{M_2} = \pm 2in \frac{2^{13} V_{26}}{4\pi(8\pi^2\alpha')^{13}} \int_0^\infty ds \vartheta_{00}(0,2is/\pi)^{-12} \eta(2is/\pi)^{-12}$$

$$i \frac{24 V_{26}}{4\pi(8\pi^2\alpha')^{13}} (2^{13} \mp n)^2 \int_0^\infty ds$$

$$\mathrm{Tr}(\lambda^a \lambda^b) \mathrm{Tr}(\lambda^c \lambda^d) = \mathrm{Tr}(\lambda^a \lambda^b \lambda^e \lambda^c \lambda^d \lambda^e).$$

$$x^4 \cong x^4 + 2\pi R$$

$$ds^2 = G_{MN}^D dx^M dx^N = G_{\mu v} dx^\mu dx^v + G_{dd} (dx^d + A_\mu dx^\mu)^2$$

$$x'^d = x^d + \lambda(x^\mu).$$

$$A'_\mu = A_\mu - \partial_\mu \lambda$$

$$\phi(x^M) = \sum_{n=-\infty}^{\infty} \phi_n(x^\mu) \exp\left(inx^d/R\right)$$

$$\partial_\mu \partial^\mu \phi_n(x^\mu) = \frac{n^2}{R^2} \phi_n(x^\mu)$$

$$-p^\mu p_\mu = \frac{n^2}{R^2}$$

$$\boldsymbol{R} = \boldsymbol{R}_d - 2e^{-\sigma} \nabla^2 e^\sigma - \frac{1}{4} e^{2\sigma} F_{\mu v} F^{\mu v}$$

$$\boldsymbol{S}_1 = \frac{1}{2\kappa_0^2} \int d^D x (-G_D)^{1/2} e^{-2\Phi} (\boldsymbol{R} + 4\nabla_\mu \Phi \nabla^\mu \Phi)$$

$$= \frac{\pi R}{\kappa_0^2} \int d^d x (-G_d)^{1/2} e^{-2\Phi+\sigma} \times \left(\boldsymbol{R}_d - 4\partial_\mu \Phi \partial^\mu \sigma + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{4} e^{2\sigma} F_{\mu v} F^{\mu v}\right)$$

$$= \frac{\pi R}{\kappa_0^2} \int d^d x (-G_d)^{1/2} e^{-2\Phi_d} \times \left(\boldsymbol{R}_d - \partial_\mu \sigma \partial^\mu \sigma + 4\partial_\mu \Phi_d \partial^\mu \Phi_d - \frac{1}{4} e^{2\sigma} F_{\mu v} F^{\mu v}\right)$$

$$\partial_\mu + i p_d A_\mu = \partial_\mu + in\tilde{A}_\mu,$$

$$g_d^2 = \frac{\kappa_0^2 e^{2\Phi_d}}{\pi R^3 e^{2\sigma}} = \frac{2\kappa_d^2}{\rho^2}$$

$$\frac{1}{\kappa_d^2} = \frac{2\pi\rho}{\kappa^2}$$

$$\boldsymbol{S}_2 = -\frac{1}{24\kappa_0^2}\int d^D x(-G_D)^{1/2} e^{-2\Phi} H_{MNL}H^{MNL}$$

$$= -\frac{\pi R}{12\kappa_0^2}\int d^d x(-G_d)^{1/2} e^{-2\Phi_d}\left(\tilde{H}_{\mu\nu\lambda}\tilde{H}^{\mu\nu\lambda} + 3e^{-2\sigma} H_{d\mu\nu}H_d^{\mu\nu}\right)$$

$$\tilde{H}_{\mu\nu\lambda} = \left(\partial_\mu B_{\nu\lambda} - A_\mu H_{d\nu\lambda}\right) + \mathfrak{C}\wp_{\text{cyclic permutations}}$$

$$B'_{\nu\lambda} = B_{\nu\lambda} - \lambda H_{d\nu\lambda}$$

$$X \cong X + 2\pi R$$

$$k = \frac{n}{R}, n \in \mathbf{Z}$$

$$X(\sigma + 2\pi) = X(\sigma) + 2\pi R w, w \in \mathbf{Z}$$

$$\partial X(z) = -i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{m=-\infty}^{\infty}\frac{\alpha_m}{z^{m+1}}, \bar{\partial} X(\bar{z}) = -i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{m=-\infty}^{\infty}\frac{\tilde{\alpha}_m}{\bar{z}^{m+1}}.$$

$$2\pi R w = \oint (dz\partial X + d\bar{z}\bar{\partial} X) = 2\pi(\alpha'/2)^{1/2}(\alpha_0 - \tilde{\alpha}_0)$$

$$p = \frac{1}{2\pi\alpha'}\oint (dz\partial X - d\bar{z}\bar{\partial} X) = (2\alpha')^{-1/2}(\alpha_0 + \tilde{\alpha}_0)$$

$$p_L \equiv (2/\alpha')^{1/2}\alpha_0 = \frac{n}{R} + \frac{wR}{\alpha'}$$
$$p_R \equiv (2/\alpha')^{1/2}\tilde{\alpha}_0 = \frac{n}{R} - \frac{wR}{\alpha'}$$

$$L_0 = \frac{\alpha' p_L^2}{4} + \sum_{n=1}^{\infty}\alpha_{-n}\alpha_n,$$
$$\tilde{L}_0 = \frac{\alpha' p_R^2}{4} + \sum_{n=1}^{\infty}\tilde{\alpha}_{-n}\tilde{\alpha}_n.$$

Función de Partición.

$$(q\bar{q})^{-1/24}\mathrm{Tr}\left(q^{L_0}\bar{q}^{\tilde{L}_0}\right) = |\eta(\tau)|^{-2}\sum_{n,w=-\infty}^{\infty} q^{\alpha' p_L^2/4}\bar{q}^{\alpha' p_R^2/4}$$

$$= |\eta(\tau)|^{-2}\sum_{n,w=-\infty}^{\infty}\exp\left[-\pi\tau_2\left(\frac{\alpha' n^2}{R^2} + \frac{w^2R^2}{\alpha'}\right) + 2\pi i\tau_1 n w\right]$$

$$\sum_{n=-\infty}^{\infty}\exp\left(-\pi a n^2 + 2\pi i b n\right) = a^{-1/2}\sum_{m=-\infty}^{\infty}\exp\left[-\frac{\pi(m-b)^2}{a}\right].$$

$$2\pi R Z_X(\tau)\sum_{m,w=-\infty}^{\infty}\exp\left(-\frac{\pi R^2|m-w\tau|^2}{\alpha'\tau_2}\right)$$

$$\begin{aligned} X(\sigma^1+2\pi,\sigma^2) &= X(\sigma^1,\sigma^2)+2\pi wR \\ X(\sigma^1+2\pi\tau_1,\sigma^2+2\pi\tau_2) &= X(\sigma^1,\sigma^2)+2\pi mR \end{aligned}$$

$$X_{\mathrm{cl}}=\sigma^1 wR+\sigma^2(m-w\tau_1)R/\tau_2$$

Operadores de vórtice.

$$[x_L,p_L]=[x_R,p_R]=i$$

$$X(z,\bar{z})=X_L(z)+X_R(\bar{z})$$

$$X_L(z)=x_L-i\frac{\alpha'}{2}p_L\ln\ z+i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{\substack{m=-\infty\\ m\neq 0}}^{\infty}\frac{\alpha_m}{mz^m},$$

$$X_R(\bar{z})=x_R-i\frac{\alpha'}{2}p_R\ln\ \bar{z}+i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{\substack{m=-\infty\\ m\neq 0}}^{\infty}\frac{\tilde{\alpha}_m}{m\bar{z}^m}.$$

$$\begin{gathered} X_L(z_1)X_L(z_2)\sim-\frac{\alpha'}{2}\ln\ z_{12}, X_R(\bar{z}_1)X_R(\bar{z}_2)\sim-\frac{\alpha'}{2}\ln\ \bar{z}_{12} \\ X_L(z_1)X_R(\bar{z}_2)\sim 0 \end{gathered}$$

$$\mathcal{V}_{k_Lk_R}(z,\bar{z})=:e^{ik_LX_L(z)+ik_RX_R(\bar{z})}:$$

$$\mathcal{V}_{k_Lk_R}(z_1,\bar{z}_1)\mathcal{V}_{k'_Lk'_R}(z_2,\bar{z}_2)\sim z_{12}^{\alpha' k_{k'_L}/2}\bar{z}_{12}^{\alpha' k_R k'_R/2}\mathcal{V}_{(k+k')_L(k+k')_R}(z_2,\bar{z}_2).$$

$$\exp\left[\pi i\alpha'(k_Lk'_L-k_Rk'_R)\right]=\exp\left[2\pi i(nw'+wn')\right]=1.$$

$$[X_L(z_1),X_L(z_2)]=\frac{\pi i\alpha'}{2}\operatorname{sign}(\sigma_1^1-\sigma_2^1).$$

$$\mathcal{V}_{k_Lk_R}(z,\bar{z})=\exp\left[\pi i(k_L-k_R)(p_L+p_R)\alpha'/4\right]\circ e^{ik_LX_L(z)+ik_RX_R(\bar{z})}\circ$$

$$\begin{aligned} \exp\{\pi i[(k_L-k_R)(k'_L+k'_R)-(k'_L-k'_R)(k_L+k_R)]\alpha'/4\} \\ =\exp\left[\pi i(nw'-wn')\right] \end{aligned}$$

$$\prod_{\substack{i,j=1\\ i<j}}^{n}|z_{ij}|^{\alpha' k_i k_j}\to\prod_{\substack{i,j=1\\ i<j}}^{n}z_{ij}^{\alpha' k_{Li}k_{Lj}/2}\bar{z}_{ij}^{\alpha' k_{Ri}k_{Rj}/2}$$

$$2\pi R\delta_{\Sigma_i n_i,0}\delta_{\Sigma_i w_i,0}$$

Operadores DDF.

$$X^+(z)X^-(0) \sim \frac{\alpha'}{2}\ln\, z, X^+(z)X^+(0) \sim X^-(z)X^-(0) \sim 0.$$

$$V^i(nk_0, z) = \partial X^i(z)e^{ink_0X^+(z)}(2/\alpha')^{1/2}$$

$$V^i(nk_0, z)V^j(mk_0, 0) \sim -\frac{\delta^{ij}}{z^2}e^{i(n+m)k_0X^+(0)} - \frac{ink_0\delta^{ij}}{z}\partial X^+(0)e^{i(n+m)k_0X^+(0)}$$

$$A_n^i = \oint \frac{dz}{2\pi}V^i(nk_0, z)$$

$$\left[A_m^i, A_n^j\right] = m\delta^{ij}\delta_{m,-n}\frac{\alpha' k_0 p^+}{2}$$

$$m^2 = -k^\mu k_\mu = \left(k_L^{25}\right)^2 + \frac{4}{\alpha'}(N-1) = \left(k_R^{25}\right)^2 + \frac{4}{\alpha'}(\tilde{N}-1)$$

$$\begin{aligned} m^2 &= \frac{n^2}{R^2} + \frac{w^2R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2) \\ 0 &= nw + N - \tilde{N} \end{aligned}$$

$$\alpha_{-1}^\mu\tilde{\alpha}_{-1}^\nu|0;k\rangle, \left(\alpha_{-1}^\mu\tilde{\alpha}_{-1}^{25} + \alpha_{-1}^{25}\tilde{\alpha}_{-1}^\mu\right)|0;k\rangle\left(\alpha_{-1}^\mu\tilde{\alpha}_{-1}^{25} - \alpha_{-1}^{25}\tilde{\alpha}_{-1}^\mu\right)|0;k\rangle, \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{25}|0;k\rangle$$

$$\partial X^\mu\bar{\partial}X^{25} - \partial X^{25}\bar{\partial}X^\mu = \bar{\partial}(X^{25}\partial X^\mu) - \partial\left(X^{25}\bar{\partial}X^\mu\right)$$

$$\frac{2^{1/2}g_{c,25}}{\alpha'}:\left(\partial X^\mu\bar{\partial}X^{25} \pm \partial X^{25}\bar{\partial}X^\mu\right)e^{ik\cdot X}:$$

$$g_{c,25}: e^{ik_L\cdot X_L(z) + ik_R\cdot X_R(\bar{z})}:$$

$$-2^{-1/2}\pi i g_{c,25}(2\pi)^{25}\delta^{25}\left(\sum_i k_i\right)k_{23}^\mu\left(k_{L23}^{25} \pm k_{R23}^{25}\right)$$

$$\rightarrow -2^{3/2}\pi i g_{c,25}(2\pi)^{25}\delta^{25}\left(\sum_i k_i\right)k_2^\mu\left(k_{L2}^{25} \pm k_{R2}^{25}\right)$$

Simetrías de gauge.

$$(n+w)^2 + 4N = (n-w)^2 + 4\tilde{N} = 4$$

$$n = w = \pm 1, N = 0, \tilde{N} = 1, n = -w = \pm 1, N = 1, \tilde{N} = 0$$

$$n = \pm 2, w = N = \tilde{N} = 0, w = \pm 2, n = N = \tilde{N} = 0$$

$$:\bar{\partial}X^\mu e^{ik\cdot X}\exp\left[\pm 2i\alpha'^{-1/2}X_L^{25}\right]:, :\partial X^\mu e^{ik\cdot X}\exp\left[\pm 2i\alpha'^{-1/2}X_R^{25}\right]:.$$

$$j^1(z) \;=: \cos\left[2\alpha'^{-1/2}X_L^{25}(z)\right]:$$
$$j^2(z) \;=: \sin\left[2\alpha'^{-1/2}X_L^{25}(z)\right]:$$
$$j^3(z) \;= i\partial X_L^{25}(z)/\alpha'^{1/2}$$

$$j^i(z)j^j(0) \sim \frac{\delta^{ij}}{2z^2} + i\frac{\epsilon^{ijk}}{z}j^k(0)$$

$$j^i(z) \;= \sum_{m=-\infty}^{\infty} \frac{j_m^i}{z^{m+1}}$$
$$\left[j_m^i, j_n^j\right] \;= \frac{m}{2}\delta_{m,-n}\delta^{ij} + i\epsilon^{ijk}j_{m+n}^k$$

Escalares y acoplamientos.

$$g_{25}^2 = 2\kappa_{25}^2/\alpha'$$

$$g_4^2 = 2\kappa_4^2/\alpha'$$

$$g_{\mathrm{G},4}^2(E) = \kappa_4^2E^2$$

$$g_5^2 = 2\pi\rho_5 g_4^2$$

$$\hat{g}_5^2 = g_5^2E = 2\pi\rho_5 E g_4^2$$

Mecanismo de Higgs.

$$m = \frac{|R^2 - \alpha'|}{R\alpha'} \approx \frac{2}{\alpha'}\left|R - \alpha'^{1/2}\right|,$$

$$:j^i(z)\tilde{j}^j(\bar{z})e^{ik\cdot X(z,\bar{z})}:.$$

$$U(M) \propto \epsilon^{ijk}\epsilon^{i'j'k'}M_{ii'}M_{jj'}M_{kk'} = \det M$$

$$U(M) = \frac{\partial U(M)}{\partial M_{ij}} = 0$$

$$M_{11}M_{22}M_{33} = M_{11}M_{22} = M_{11}M_{33} = M_{22}M_{33} = 0$$

$$m^2 = \frac{n^2}{R^2} + \frac{w^2R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2),$$

$$R \to R' = \frac{\alpha'}{R}, n \leftrightarrow w.$$

$$p_L^{25} \to p_L^{25}, p_R^{25} \to -p_R^{25}$$

$$X'^{25}(z,\bar{z}) = X_L^{25}(z) - X_R^{25}(\bar{z})$$

$$R_{\text{self-dual}} = R_{SU(2)\times SU(2)} = \alpha'^{1/2}$$

$$\rho' = \frac{\alpha'}{\rho}, \kappa' = \frac{\alpha'^{1/2}}{\rho}\kappa$$

$$e^{\Phi'} = \frac{\alpha'^{1/2}}{\rho} e^{\Phi}$$

Dimensiones múltiples.

$$X^m \cong X^m + 2\pi R, 26 - k \leq m \leq 25$$

$$\boldsymbol{S} = \frac{(2\pi R)^k}{2\kappa_0^2} \int d^d x (-G_d)^{1/2} e^{-2\Phi_d} \Big[\boldsymbol{R}_d + 4\partial_\mu \Phi_d \partial^\mu \Phi_d$$

$$-\frac{1}{4} G^{mn} G^{pq} \left(\partial_\mu G_{mp} \partial^\mu G_{nq} + \partial_\mu B_{mp} \partial^\mu B_{nq} \right) - \frac{1}{4} G_{mn} F^m_{\mu v} F^{n\mu v} - \frac{1}{4} G^{mn} H_{m\mu v} H_n^{\mu v}$$

$$-\frac{1}{12} H_{\mu v \lambda} H^{\mu v \lambda} \Big]$$

$$B_{mn} \partial_a \left(g^{1/2} \epsilon^{ab} X^m \partial_b X^n \right)$$

$$X^m(\sigma^1, \sigma^2) = x^m(\sigma^2) + w^m R \sigma^1$$

$$L = \frac{1}{2\alpha'} G_{mn} (\dot{x}^m \dot{x}^n + w^m w^n R^2) - \frac{i}{\alpha'} B_{mn} \dot{x}^m w^n R$$

$$p_m = -\frac{\partial L}{\partial v^m} = \frac{1}{\alpha'} (G_{mn} v^n + B_{mn} w^n R),$$

$$v_m = \alpha' \frac{n_m}{R} - B_{mn} w^n R$$

$$\frac{1}{2\alpha'} G_{mn} (v^m v^n + w^m w^n R^2)$$

$$m^2 = \frac{1}{2\alpha'^2} G_{mn} (v_L^m v_L^n + v_R^m v_R^n) + \frac{2}{\alpha'} (N + \tilde{N} - 2)$$

$$v_{L,R}^m = v^m \pm w^m R$$

$$0 = G_{mn} (v_L^m v_L^n - v_R^m v_R^n) + 4\alpha' (N - \tilde{N}) = 4\alpha' \left(n_m w^m + N - \tilde{N} \right) (8.4.10)$$

$$X^m = (w_1^m \sigma^1 + w_2^m \sigma^2) R$$

$$2\pi i b_{mn} w_1^m w_2^n$$

$$G_{mn} = e_m^r e_n^r$$

$$k_{rL} = e_r^m \frac{v_{mL}}{\alpha'}, k_{rR} = e_r^m \frac{v_{mR}}{\alpha'}$$

$$\begin{aligned} m^2 &= \frac{1}{2}(k_{rL}k_{rL} + k_{rR}k_{rR}) + \frac{2}{\alpha'}(N + \tilde{N} - 2) \\ 0 &= \alpha'(k_{rL}k_{rL} - k_{rR}k_{rR}) + 4(N - \tilde{N}) \end{aligned}$$

Compactación de Narain.

$$:e^{ik_L\cdot X_L(z)+ik_R\cdot X_R(\bar{z})}::e^{ik'_L\cdot X_L(0)+ik'_R\cdot X_R(0)}: \sim z^{l_L\cdot l'_L}\bar{z}_R l_R\cdot l'_R :e^{i(k_L+k'_L)\cdot X_L(0)+i(k_R+k'_R)\cdot X_R(0)}:$$

$$l_L\cdot l'_L - l_R\cdot l'_R \equiv l\circ l' \in \mathbf{Z}$$

$$\Gamma \subset \Gamma^*$$

$$l\circ l \in 2\mathbf{Z} \text{ for all } l\in\Gamma.$$

$$2l\circ l' = (l+l')\circ(l+l') - l\circ l - l'\circ l' \in 2\mathbf{Z}$$

$$Z_\Gamma(\tau) = |\eta(\tau)|^{-2k}\sum_{l\in\Gamma}\exp\left(\pi i\tau l_L^2 - \pi i\bar{\tau}l_R^2\right)$$

$$\sum_{l'\in\Gamma}\delta(l-l') = V_\Gamma^{-1}\sum_{l''\in\Gamma^*}\exp\left(2\pi i l''\circ l\right).$$

$$Z_\Gamma(\tau) = V_\Gamma^{-1}|\eta(\tau)|^{-2k}\sum_{l''\in\Gamma^*}\int d^{2k}l\exp\left(2\pi i l''\circ l + \pi i\tau l_L^2 - \pi i\bar{\tau}l_R^2\right)$$

$$= V_\Gamma^{-1}(\tau\bar{\tau})^{-k/2}|\eta(\tau)|^{-2k}\sum_{l''\in\Gamma^*}\exp\left(-\pi i l_L''^2/\tau + \pi i l_R''^2/\bar{\tau}\right) = V_\Gamma^{-1}Z_{\Gamma^*}(-1/\tau)$$

$$\Gamma = \Gamma^*$$

$$\Gamma' = \Lambda\Gamma$$

$$l_{L,R} = \frac{n}{r} \pm \frac{mr}{2}$$

$$l'_L = l_L\cosh\lambda + l_R\sinh\lambda,\ l'_R = l_L\sinh\lambda + l_R\cosh\lambda$$

$$\frac{O(k,k,\mathbf{R})}{O(k,\mathbf{R})\times O(k,\mathbf{R})}$$

$$\frac{2k(2k-1)}{2} - k(k-1) = k^2$$

$$\begin{gathered} \Lambda\Gamma_0 \cong \Lambda'\Lambda\Lambda''\Gamma_0 \\ \Lambda' \in O(k,\mathbf{R})\times O(k,\mathbf{R}),\Lambda'' \in O(k,k,\mathbf{Z}) \end{gathered}$$

$$\frac{O(k,k,\mathbf{R})}{O(k,\mathbf{R})\times O(k,\mathbf{R})\times O(k,k,\mathbf{Z})}$$

$$x'^m = L^m{}_n x^n$$

$$b_{mn} \to b_{mn} + N_{mn}$$

$$-\frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j$$

$$\frac{16}{d-2}\partial_\mu\Phi\partial^\mu\Phi+G^{mn}G^{pq}\left(\partial_\mu G_{mp}\partial^\mu G_{nq}+\partial_\mu B_{mp}\partial^\mu B_{nq}\right)$$

$$\rho=\frac{R^2}{\alpha'}\left(B_{24,25}+i\det{}^{1/2}G_{mn}\right)=b_{24,25}+i\frac{V}{4\pi^2\alpha'}$$

$$ds^2=\frac{\alpha'\rho_2}{R^2\tau_2}\left|dX^{24}+\tau dX^{25}\right|^2$$

$$\mathrm{PSL}(2,\mathbf{Z})\times\mathrm{PSL}(2,\mathbf{Z})\times\mathbf{Z}_2^2$$

$$\frac{\partial_\mu\tau\partial^\mu\bar{\tau}}{\tau_2^2}+\frac{\partial_\mu\rho\partial^\mu\bar{\rho}}{\rho_2^2}$$

$$\frac{PSL(2,\mathbf{R})\times PSL(2,\mathbf{R})}{U(1)\times U(1)\times PSL(2,\mathbf{Z})\times PSL(2,\mathbf{Z})\times\mathbf{Z}_2^2}.$$

Orbifolds.

$$X^{25}\cong -X^{25}$$

$$X^m\to -X^m, 26-k\le m\le 25.$$

$$t^m: X^{25}\cong X^{25}+2\pi Rm,$$
$$t^m r: X^{25}\cong 2\pi Rm-X^{25},$$

$$X^{25}(\sigma^1+2\pi)=-X^{25}(\sigma^1)$$

$$\left|N,\tilde{N};k^\mu,n,w\right\rangle\to(-1)^{\sum_{m=1}^{\infty}\left(N_m^{25}+\tilde{N}_m^{25}\right)}\left|N,\tilde{N};k^\mu,-n,-w\right\rangle,$$

$$X^{25}(z,\bar{z})=i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{m=-\infty}^{\infty}\frac{1}{m+1/2}\left(\frac{\alpha_{m+1/2}^{25}}{z^{m+1/2}}+\frac{\tilde{\alpha}_{m+1/2}^{25}}{\bar{z}^{m+1/2}}\right).$$

$$X^{25}(\sigma^1+2\pi)=2\pi R-X^{25}(\sigma^1).$$

$$m^2=\frac{4}{\alpha'}\left(N-\frac{15}{16}\right), N=\tilde{N}$$

$$(q\bar{q})^{-1/24}\mathrm{Tr}_U\left(\frac{1+r}{2}q^{L_0}\bar{q}^{\tilde{L}_0}\right)$$

$$\frac{1}{2}Z_{\mathrm{tor}}(R,\tau)+\frac{1}{2}(q\bar{q})^{-1/24}\prod_{m=1}^{\infty}|1+q^m|^{-2}$$

$$(q\bar{q})^{1/48}\mathrm{Tr}_T\left(\frac{1+r}{2}q^{L_0}\bar{q}^{\tilde{L}_0}\right)=(q\bar{q})^{1/48}\left[\prod_{m=1}^{\infty}\left|1-q^{m-1/2}\right|^{-2}+\prod_{m=1}^{\infty}\left|1+q^{m-1/2}\right|^{-2}\right]$$

$$Z_{\text{orb}}\,(R,\tau) = \frac{1}{2} Z_{\text{tor}}\,(R,\tau) + \left|\frac{\eta(\tau)}{\vartheta_{10}(0,\tau)}\right| + \left|\frac{\eta(\tau)}{\vartheta_{01}(0,\tau)}\right| + \left|\frac{\eta(\tau)}{\vartheta_{00}(0,\tau)}\right|$$

$$X^{25}(\sigma^1 + 2\pi, \sigma^2) \;= (-1)^{a+1} X^{25}(\sigma^1, \sigma^2)$$
$$X^{25}(\sigma^1 + 2\pi\tau_1, \sigma^2 + 2\pi\tau_2) \;= (-1)^{b+1} X^{25}(\sigma^1, \sigma^2)$$

Twisting.

$$\phi(\sigma^1 + 2\pi) = h \cdot \phi(\sigma^1)$$

$$P_H = \frac{1}{\text{order}(H)} \sum_{h_2 \in H} \hat{h}_2$$

$$Z = \frac{1}{\text{order}(H)} \sum_{h_1, h_2 \in H} Z_{h_1, h_2}$$

$$\phi'(\sigma^1) = h_2 \cdot \phi(\sigma^1)$$

$$\phi'(\sigma^1 + 2\pi) = h_1' \cdot \phi'(\sigma^1)$$

$$h_1' = h_2 h_1 h_2^{-1}$$

$$c = 1 CFTs$$

$$r' \colon X^{25} \rightarrow X^{25} + \pi\alpha^{1/2}$$

$$Z_{\text{orb}}\left(\alpha'^{1/2}, \tau\right) = Z_{\text{tor}}\left(2\alpha'^{1/2}, \tau\right)$$

$$X^{25} \rightarrow X^{25} + \frac{2\pi\alpha'^{1/2}}{k}$$

$$j^3\tilde{j}^3, j^1\tilde{j}^1 + j^2\tilde{j}^2, j^1\tilde{j}^2 - j^2\tilde{j}^1$$

$$A_{25}(x^M) = -\frac{\theta}{2\pi R} = -i\Lambda^{-1}\frac{\partial\Lambda}{\partial x^{25}}, \Lambda(x^{25}) = \exp\left(-\frac{i\theta x^{25}}{2\pi R}\right)$$

$$W_q = \exp\left(iq\oint \; dx^{25} A_{25}\right) = \exp\left(-iq\theta\right)$$

$$S = \int \; d\tau \left(\frac{1}{2}\dot{X}^M \dot{X}_M + \frac{m^2}{2} - iqA_M\dot{X}^M\right)$$

$$p_{25} = -\frac{\partial L}{\partial v^{25}} = v^{25} - \frac{q\theta}{2\pi R}$$

$$v_{25} = \frac{2\pi l + q\theta}{2\pi R}$$

$$H = \frac{1}{2}\left(p_\mu p^\mu + v_{25}^2 + m^2\right)$$

$$v_{25} = p_{25} = \frac{2\pi l + q\theta}{2\pi R}$$

$$A_{25} = -\frac{1}{2\pi R}\operatorname{diag}(\theta_1, \theta_2, \dots, \theta_n)$$

$$v_{25} = \frac{2\pi l - \theta_j + \theta_i}{2\pi R}$$

$$m^2 = \frac{\left(2\pi l - \theta_j + \theta_i\right)^2}{4\pi^2 R^2} + \frac{1}{\alpha'}(N-1)$$

$$m^2 = \frac{\left(\theta_j - \theta_i\right)^2}{4\pi^2 R^2}$$

$$U(r_1) \times \dots \times U(r_s), \sum_{i=1}^{s} r_i = n$$

$$\operatorname{Tr}([A_m, A_n]^2)$$

Dualidad T.

$$X'^{25}(z,\bar{z}) = X_L^{25}(z) - X_R^{25}(\bar{z}).$$

$$\partial_n X^{25} = -i\partial_t X'^{25}$$

$$X'^{25}(\pi) - X'^{25}(0) = \int_0^\pi d\sigma^1 \partial_1 X'^{25} = -i\int_0^\pi d\sigma^1 \partial_2 X^{25} = -2\pi\alpha' v^{25} = -\frac{2\pi\alpha' l}{R} = -2\pi l R'$$

$$\Delta X'^{25} = X'^{25}(\pi) - X'^{25}(0) = -\left(2\pi l - \theta_j + \theta_i\right)R'.$$

$$X'^{25} = \theta_i R' = -2\pi\alpha' A_{25,ii}.$$

$$X'^{25}(z,\bar{z}) = \theta_i R' - \frac{iR'}{2\pi}\left(2\pi l - \theta_j + \theta_i\right)\ln(z/\bar{z}) + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{\alpha_m^{25}}{m}\left(z^{-m} - \bar{z}^{-m}\right)$$

$$= \theta_i R' + \frac{\sigma^1}{\pi}\Delta X'^{25} - (2\alpha')^{1/2} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{\alpha_m^{25}}{m}\exp(-m\sigma^2)\sin m\sigma^1$$

$$m^2 = \left(\frac{\Delta X'^{25}}{2\pi\alpha'}\right)^2 + \frac{1}{\alpha'}(N-1)$$

Membranas.

$$\alpha_{-1}^{\mu}|k;ii\rangle, \quad \mathcal{V} = i\partial_t X^{\mu}$$
$$\alpha_{-1}^{25}|k;ii\rangle, \quad \mathcal{V} = i\partial_t X^{25} = \partial_n X'^{25}$$

$$\boldsymbol{S}_p = -T_p \int d^{p+1}\xi e^{-\Phi}[-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})]^{1/2}$$

$$G_{ab}(\xi) = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^v}{\partial \xi^b} G_{\mu v}(X(\xi)), B_{ab}(\xi) = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^v}{\partial \xi^b} B_{\mu v}(X(\xi))$$

$$X'^2 = -2\pi\alpha' X^1 F_{12}$$

$$\int dX^1[1 + (\partial_1 X'^2)^2]^{1/2} = \int dX^1[1 + (2\pi\alpha' F_{12})^2]^{1/2}$$

$$\frac{i}{4\pi\alpha'}\int_M d^2\sigma g^{1/2}\epsilon^{ab}\partial_a X^\mu \partial_b X^v B_{\mu v} + i\int_{\partial M} dX^\mu A_\mu$$

$$\delta A_\mu = \partial_\mu \lambda$$

$$\delta B_{\mu v} = \partial_\mu \zeta_v - \partial_v \zeta_\mu$$

$$\delta A_\mu = -\zeta_\mu / 2\pi\alpha'$$

$$B_{\mu v} + 2\pi\alpha' F_{\mu v} \equiv 2\pi\alpha' \mathcal{F}_{\mu v}$$

$$V \propto \mathrm{Tr}([X_m, X_n][X^m, X^n]).$$

$$\boldsymbol{S}_p = -T_p \int d^{p+1}\xi \mathrm{Tr}\{e^{-\Phi}[-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})]^{1/2} + O([X^m, X^n]^2)\}$$

$$T_p e^{-\Phi}\prod_{i=1}^{p}(2\pi R_i)$$

$$2\pi\alpha'^{1/2} T_p e^{-\Phi'}\prod_{i=1}^{p-1}(2\pi R_i)$$

$$T_{p-1} e^{-\Phi'}\prod_{i=1}^{p-1}(2\pi R_i)$$

$$T_p = T_{p-1}/2\pi\alpha'^{1/2},$$

$$\mathcal{A} = iV_{p+1}\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-(p+1)/2}\exp(-ty^2/2\pi\alpha')\eta(it)^{-24}$$

$$= \frac{iV_{p+1}}{(8\pi^2\alpha')^{(p+1)/2}}\int_0^\infty dt t^{(21-p)/2}\exp(-ty^2/2\pi\alpha') \times [\exp(2\pi/t) + 24 + \cdots]$$

$$\mathcal{A} = iV_{p+1}\frac{24}{2^{12}}(4\pi^2\alpha')^{11-p}\pi^{(p-23)/2}\Gamma\left(\frac{23-p}{2}\right)|y|^{p-23} = iV_{p+1}\frac{24\pi}{2^{10}}(4\pi^2\alpha')^{11-p}G_{25-p}(y)$$

$$\boldsymbol{S} = \frac{1}{2\kappa^2}\int d^{26}X(-\tilde{G})^{1/2}\left(\tilde{\boldsymbol{R}} - \frac{1}{6}\nabla_\mu\tilde{\Phi}\tilde{\nabla}^\mu\tilde{\Phi}\right)$$

$$\boldsymbol{S}_p = -\tau_p \int d^{p+1}\xi \exp\left(\frac{p-11}{12}\tilde{\Phi}\right)\left(-\det\tilde{G}_{ab}\right)^{1/2}$$

$$F_\nu \equiv \partial^{\hat{\mu}} h_{\mu\nu} - \frac{1}{2}\partial_\nu h_\mu^{\hat{\mu}} = 0$$

$$\boldsymbol{S} = -\frac{1}{8\kappa^2}\int d^{26}X\left(\partial_\mu h_{\nu\lambda}\partial^{\hat{\mu}} h^{\hat{\nu}\hat{\lambda}} - \frac{1}{2}\partial_\mu h_\nu^{\hat{\nu}}\partial^{\hat{\mu}} h_\lambda^{\hat{\lambda}} + \frac{2}{3}\partial_\mu\tilde{\Phi}\partial^{\hat{\mu}}\tilde{\Phi}\right)$$

$$\langle\tilde{\Phi}\tilde{\Phi}\rangle = -\frac{(D-2)i\kappa^2}{4k^2},$$

$$\langle h_{\mu\nu}h_{\sigma\rho}\rangle = -\frac{2i\kappa^2}{k^2}\left(\eta_{\mu\sigma}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\sigma} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\sigma\rho}\right).$$

$$\boldsymbol{S}_p = -\tau_p \int d^{p+1}\xi\left(\frac{p-11}{12}\tilde{\Phi} - \frac{1}{2}h_{aa}\right)$$

$$\mathcal{A} = \frac{i\kappa^2\tau_p^2}{k_\perp^2}V_{p+1}\left\{6\left[\frac{p-11}{12}\right]^2 + \frac{1}{2}\left[2(p+1) - \frac{1}{12}(p+1)^2\right]\right\} = \frac{6i\kappa^2\tau_p^2}{k_\perp^2}V_{p+1}$$

$$\tau_p^2 = \frac{\pi}{256\kappa^2}(4\pi^2\alpha')^{11-p}.$$

$$\frac{\tau_{25}}{4}(2\pi\alpha')^2\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right).$$

$$\frac{g_o^2}{g_c} = \frac{4\pi\alpha' g_o'^2}{\kappa} = 2^{18}\pi^{25/2}\alpha'^6$$

$$\Omega: X_L^M(z) \leftrightarrow X_R^M(z)$$

$$\begin{aligned}\Omega:\quad & X'^m(z,\bar{z}) \leftrightarrow -X'^m(\bar{z},z)\\ & X^\mu(z,\bar{z}) \leftrightarrow X^\mu(\bar{z},z)\end{aligned}$$

$$\begin{aligned}G_{\mu\nu}(x') &= G_{\mu\nu}(x), B_{\mu\nu}(x') = -B_{\mu\nu}(x)\\ G_{\mu n}(x') &= -G_{\mu n}(x), B_{\mu n}(x') = B_{\mu n}(x)\\ G_{mn}(x') &= G_{mn}(x), B_{mn}(x') = -B_{mn}(x)\end{aligned}$$

$$W = \mathrm{diag}\left(e^{i\theta_1}, e^{-i\theta_1}, e^{i\theta_2}, e^{-i\theta_2}, \cdots, e^{i\theta_{n/2}}, e^{-i\theta_{n/2}}\right)$$

$$2^{12-k}T_p\int d^{p+1}\xi e^{-\Phi}(-\det G_{ab})^{1/2}$$

$$g_{c,d}: e^{ik\cdot X}:$$

$$\mathcal{V}_j(k;z,\bar{z})\overline{\mathcal{V}_{j'}(k;0,0)} = \frac{g_{c,d}^2}{z^2\bar{z}^2} + \cdots$$

$$e^{-2\lambda}\langle\tilde{c}(\bar{z}_1)c(z_1)\tilde{c}(\bar{z}_2)c(z_2)\tilde{c}(\bar{z}_3)c(z_3): e^{ik\cdot X}(z,\bar{z}):\rangle = \frac{8\pi i}{\alpha' g_{c,d}^2}|z_{12}z_{13}z_{23}|^2(2\pi)^d\delta^d(k)$$

$$S_{S_2}(1;2;3) = e^{-2\lambda}\left\langle \hat{\mathcal{V}}_1'(\infty,\infty)\hat{\mathcal{V}}_2(1,1)\hat{\mathcal{V}}_3(0,0)\right\rangle_{S_2}$$

$$\hat{\mathcal{V}}_j = \tilde{c}c\mathcal{V}_j$$

$$S_{S_2}(1;2;3) = e^{-2\lambda}\left\langle\langle\hat{\mathcal{V}}_1\middle|\hat{\mathcal{V}}_2(1,1) \mid \hat{\mathcal{V}}_3\right\rangle = e^{-2\lambda}c_{123}.$$

$$\left|\hat{\mathcal{V}}_3\right\rangle = Q_{\mathrm{B}}|\chi\rangle.$$

$$e^{2\lambda}S_{S_2}(1;2;3) = \left\langle\langle\hat{\mathcal{V}}_1\middle|[\hat{\mathcal{V}}_2(1,1),Q_{\mathrm{B}}] \mid \chi\right\rangle + \left\langle\langle\hat{\mathcal{V}}_1\middle|Q_{\mathrm{B}}\hat{\mathcal{V}}_2(1,1) \mid \chi\right\rangle = 0$$

$$\hat{\mathcal{V}}_3 = Q_{\mathrm{B}}\cdot\mathcal{X}.$$

$$\sum_n S_{mn}S_{pn}^* \stackrel{?}{=} \delta_{mp}$$

$$S_{mn} = \delta_{mn} + iT_{mn}$$

$$T_{mp} - T_{pm}^* = i\sum_n T_{mn}T_{pn}^*$$

$$T_{S_2}(1;2;3) = T_{S_2}(-1;-2;-3)^*$$

$$\hat{\mathcal{V}}_{-i} = -\overline{\hat{\mathcal{V}}}_i.$$

$$\left\langle\hat{\mathcal{V}}_1'(\infty,\infty)\hat{\mathcal{V}}_2(1,1)\hat{\mathcal{V}}_3(0,0)\right\rangle_{S_2} = -\left\langle\hat{\mathcal{V}}_{-1}'(\infty,\infty)\hat{\mathcal{V}}_{-2}(1,1)\hat{\mathcal{V}}_{-3}(0,0)\right\rangle_{S_2}^*,$$

$$S_{S_2}(1;2;3;4) = e^{-2\lambda}\int_{\mathbf{C}} d^2z\left\langle\hat{\mathcal{V}}_1'(\infty,\infty)\hat{\mathcal{V}}_2(1,1)\hat{\mathcal{V}}_3(0,0)\mathcal{V}_4(z,\bar{z})\right\rangle_{S_2}.$$

$$\left\langle\langle\hat{\mathcal{V}}_1\middle|\mathrm{T}[\hat{\mathcal{V}}_2(1,1)\mathcal{V}_4(z,\bar{z})] \mid \hat{\mathcal{V}}_3\right\rangle.$$

$$\mathcal{V}_4(z,\bar{z}) = (z\bar{z})^{-1}\{b_0,[\tilde{b}_0,\hat{\mathcal{V}}_4(z,\bar{z})]\}.$$

$$\theta(1-z|)\left\langle\langle\hat{\mathcal{V}}_1\middle|\hat{\mathcal{V}}_2(1,1)b_0\tilde{b}_0z^{L_0-1}\bar{z}^{\tilde{L}_0-1}\hat{\mathcal{V}}_4(1,1) \mid \hat{\mathcal{V}}_3\right\rangle + \theta(|z|$$
$$-1)\left\langle\langle\hat{\mathcal{V}}_1\middle|\hat{\mathcal{V}}_4(1,1)b_0\tilde{b}_0z^{-L_0-1}\bar{z}^{-\tilde{L}_0-1}\hat{\mathcal{V}}_2(1,1) \mid \hat{\mathcal{V}}_3\right\rangle$$

$$\theta(1-|z|)\sum_{i,i'} z^{\alpha'(k_i^2+m_i^2)/4-1}\bar{z}^{\alpha'(k_i^2+\tilde{m}_i^2)/4-1} \times \left\langle\langle\hat{\mathcal{V}}_1\middle|\hat{\mathcal{V}}_2(1,1)b_0\tilde{b}_0 \mid i\right\rangle \mathcal{G}^{ii'}\left\langle\langle i'|\hat{\mathcal{V}}_4(1,1) \mid \hat{\mathcal{V}}_3\right\rangle + \theta(|z|$$
$$-1)\sum_{i,i'} z^{-\alpha'(k_i^2+m_i^2)/4-1}\bar{z}^{-\alpha'(k_i^2+\tilde{m}_i^2)/4-1}$$
$$\times\left\langle\langle\hat{\mathcal{V}}_1\middle|\hat{\mathcal{V}}_4(1,1)b_0\tilde{b}_0 \mid i\right\rangle \mathcal{G}^{ii'}\left\langle\langle i'|\hat{\mathcal{V}}_2(1,1) \mid \hat{\mathcal{V}}_3\right\rangle$$

$$k_i^2 = (k_1 + k_2)^2 > \frac{4}{\alpha'},$$

$$\int_{|z|<1} d^2 z z^{\alpha'(k^2+m^2)/4-1} \bar{z}^{\alpha'(k^2+\tilde{m}^2)/4-1} = \frac{8\pi}{\alpha'} \frac{\delta_{m^2,\tilde{m}^2}}{k^2+m^2-i\epsilon}$$

$$\left[b_0 \tilde{b}_0 z^{\pm L_0 - 1} \bar{z}^{\pm \tilde{L}_0 - 1}, Q_{\mathrm{B}}\right] = \pm b_0 \partial_{\bar{z}} \left(z^{\pm L_0 - 1} \bar{z}^{\pm \tilde{L}_0}\right) \mp \tilde{b}_0 \partial_z \left(z^{\pm L_0} \bar{z}^{\pm \tilde{L}_0 - 1}\right)$$

$$T_{S_2}(1;2;3;4) - T_{S_2}(-1;-2;-3;-4)^*$$

$$= i \sum_{j_5} \int \frac{d^{d-1}\mathbf{k}_5}{2E_5(2\pi)^{d-1}} T_{S_2}(1;2;5) T_{S_2}(-5;3;4) + 2 \text{ permutations}$$

$$= -ie^{-4\lambda} \sum_{j_5} \int \frac{d^{d-1}\mathbf{k}_5}{2E_5(2\pi)^{d-1}} \left\langle \left\langle \hat{\mathcal{V}}_1 \middle| \hat{\mathcal{V}}_2(1,1) \mid \hat{\mathcal{V}}_5 \right\rangle \left\langle \left\langle \hat{\mathcal{V}}_{-5} \middle| \hat{\mathcal{V}}_3(1,1) \mid \hat{\mathcal{V}}_4 \right\rangle \right.\right.$$

$$+ 2 \text{ permutations}$$

$$\frac{1}{k^2+m^2-i\epsilon} - \frac{1}{k^2+m^2+i\epsilon} = 2\pi i \delta(k^2+m^2)$$

$$T_{S_2}(1;2;3;4) - T_{S_2}(-1;-2;-3;-4)^*$$

$$= \frac{16\pi^2 i}{\alpha' e^{2\lambda}} \sum_{i,i'} \delta_{m_i^2,\tilde{m}_i^2} \delta\left(k_i^2 + m_i^2\right) \langle \hat{\mathcal{V}}_1 | \hat{\mathcal{V}}_2(1,1) b_0 \tilde{b}_0 | i \rangle \mathcal{G}^{ii'} \langle i' | \hat{\mathcal{V}}_3(1,1) | \hat{\mathcal{V}}_4 \rangle$$

$$+ 2 \text{ permutations}$$

$$i \leftrightarrow (j,k)$$

$$\sum_{i,i'} \delta_{m_i^2,\tilde{m}_i^2} \delta\left(k_i^2 + m_i^2\right) b_0 \tilde{b}_0 | i \rangle \mathcal{G}^{ii'} \langle i' | = \delta_{L_0,\tilde{L}_0} \delta(4L_0/\alpha') b_0 \tilde{b}_0$$

$$= -\frac{i\alpha'}{16\pi^2 e^{2\lambda}} \int \frac{d^{d-1}\mathbf{k}}{2E(2\pi)^{d-1}} \sum_{\substack{j \in \hat{\mathcal{H}} \\ k^0 = \pm\omega_k}} \left| \hat{\mathcal{V}}_j(k) \right\rangle \left\langle \hat{\mathcal{V}}_j(k) \right|$$

$$\left\langle \overline{\hat{\mathcal{V}}_j(k)} \right| \tilde{c}_0 c_0 \left| \hat{\mathcal{V}}_{j'}(k') \right\rangle = \frac{8\pi i e^{2\lambda}}{\alpha'} \delta_{jj'} (2\pi)^d \delta^d(k-k')$$

$$\mathcal{P}|b\rangle_{\mathrm{P}} = |b\rangle_{\mathrm{P}}, \mathcal{P}|a\rangle_{\mathrm{U}} = 0, \mathcal{P}|a\rangle_{\mathrm{N}} = 0;$$
$$\mathcal{U}|b\rangle_{\mathrm{P}} = 0, \mathcal{U}|a\rangle_{\mathrm{U}} = 0, \mathcal{U}|a\rangle_{\mathrm{N}} = |a\rangle_{\mathrm{U}}.$$

$$1 = \mathcal{P} + Q_{\mathrm{B}} \mathcal{U} + \mathcal{U} Q_{\mathrm{B}}.$$

$$-i \frac{\eta^{\mu\nu}}{k^2}$$

$$\eta^{\mu\nu} = (\eta^{\mu\nu} - k^\mu n^\nu - n^\mu k^\nu) + k^\mu n^\nu + n^\mu k^\nu.$$

$$\left.\frac{\partial z_1}{\partial z}\right|_{z_2} = \frac{z_1}{z}$$

$$\gamma\colon \frac{z' - U}{z' - V} = K\frac{z - U}{z - V},$$

$$|K|^{-1/2} < \left|\frac{z - U}{z - V}\right| < |K|^{1/2}.$$

$$ds^2 = \frac{dzd\bar{z}}{(\mathrm{Imz})^2}$$

$$(A_1B_1^{-1}A_1^{-1}B_1)(A_2B_2^{-1}A_2^{-1}B_2)\ldots\left(A_gB_g^{-1}A_g^{-1}B_g\right)$$

$$\partial_{\bar{z}}\omega_z = 0, \omega_{\bar{z}} = 0$$

$$\mathrm{dimker}P_0^T - \mathrm{dimker}P_0 = -\chi = 2g - 2.$$

$$\oint_{A_i} dz\omega_{zj} = \delta_{ij}$$

$$\tau_{ij} = \oint_{B_i} dz\omega_{zj}$$

$$y^2 = \prod_{i=1}^{2k}(z - Z_i)$$

$$z_1z_2 = q$$

$$(1 - \epsilon)^{-1}|q|^{1/2} > |z_1| > (1 - \epsilon)|q|^{1/2}$$

$$M = M_1\infty M_2(z_1, z_2, q)$$

$$M = M_0 8(z_1, z_2, q).$$

$$\ln\ |q| < \mathrm{Im}w < 0, w \cong w + 2\pi,$$

$$z^{(0)} = \frac{az}{z - 2}, z^{(1)} = \frac{a(z - 1)}{z + 1}, z^{(\infty)} = \frac{a}{2z - 1}$$

$$z_4 = (q - a^2)^2/4a^2q$$

$$r = 3g + n - 2$$

$$\langle\cdots 1 \cdots 2\rangle_M = \sum_{ij}\left\langle\cdots 1\mathcal{A}_i^{(z_1)}\right\rangle_{M_1}\mathrm{G}^{ij}\left\langle\mathcal{A}_j^{(z_2)}\cdots 2\right\rangle_{M_2}.$$

$$\left\langle \cdots \mathcal{A}_k^{(z_1)} \right\rangle_{M_1} = \sum_{ij} \left\langle \cdots \mathcal{A}_i^{(z_1)} \right\rangle_{M_1} \mathrm{G}^{ij} \left\langle \mathcal{A}_j^{(u)} \mathcal{A}_k^{(z)} \right\rangle_{S_2}.$$

$$\mathrm{G}^{ij}\mathcal{G}_{jk} = \delta_k^i$$

$$\mathrm{G}^{ij} = \mathcal{G}^{ij}$$

$$\mathcal{A}_j^{(z_2')} = q^{h_j}\bar{q}^{\tilde{h}_j}\mathcal{A}_j^{(z_2)}$$

$$\langle \cdots 1 \cdots 2 \rangle_M = \sum_{ij} q^{h_j}\bar{q}^{\tilde{h}_j} \left\langle \cdots 1 \mathcal{A}_i^{(z_1)} \right\rangle_{M_1} \mathcal{G}^{ij} \left\langle \mathcal{A}_j^{(z_2)} \cdots 2 \right\rangle_{M_2}$$

$$\langle \ldots \rangle_M = \sum_{ij} q^{h_j}\bar{q}^{\tilde{h}_j}\mathcal{G}^{ij} \left\langle \ldots \mathcal{A}_i^{(z_1)} \mathcal{A}_j^{(z_2)} \right\rangle_{M_0}$$

$$z_1' = z_1 + \sum_{n=-\infty}^{\infty} \epsilon_n z_1^{n+1}$$

$$z_2' = z_2 - \sum_{n=-\infty}^{\infty} \epsilon_{-n} q^{-n} z_2^{n+1}$$

$$\mathcal{A}_i^{(z_1')}\mathcal{G}^{ij}\mathcal{A}_j^{(z_2')} - \mathcal{A}_i^{(z_1)}\mathcal{G}^{ij}\mathcal{A}_j^{(z_2)} = -\sum_{n=-\infty}^{\infty} \epsilon_n \left[L_n \cdot \mathcal{A}_i^{(z_1)}\mathcal{G}^{ij}\mathcal{A}_j^{(z_2)} - \mathcal{A}_i^{(z_1)}\mathcal{G}^{ij} L_{-n} \cdot \mathcal{A}_j^{(z_2)} \right]$$

$$2\pi(m,n).$$

$$(\sigma^1, \sigma^2) = (\sigma'^1, \sigma'^2)\begin{bmatrix} m & n \\ q & p \end{bmatrix}$$

$$(\sigma'^1, \sigma'^2) = 2\pi(1,0)$$

$$\oint \frac{dz}{2\pi i} b_{z_1 z_1} \frac{\partial z_1}{\partial q}\bigg|_{z_2} = \frac{b_0}{q}$$

$$\int_{|q|<1} \frac{d^2 q}{q\bar{q}} q^{\alpha'(k_i^2+m_i^2)/4} \bar{q}^{\alpha'(k_i^2+\tilde{m}_i^2)/4} \mathcal{G}^{ij} b_0 \tilde{b}_0 = \frac{8\pi\delta_{m_i^2,\tilde{m}_i^2}\mathcal{G}^{ij} b_0 \tilde{b}_0}{\alpha'(k_i^2 + m_i^2 - i\epsilon)}.$$

$$g(i_1, \ldots, i_n) = \sum_g \int_{\mathcal{V}_{g,n}} d^m t \left\langle \prod_{k=1}^{m} B_k \prod_{l=1}^{n} \hat{\mathcal{V}}_{i_l} \right\rangle_g$$

$$\partial X^\mu \bar{\partial} X_\mu \frac{dq d\bar{q}}{q\bar{q}}$$

$$-4\pi\partial X^\mu \bar{\partial} X_\mu \ln(a e^{-\omega})$$

$$\delta \boldsymbol{S} = -\sum_\chi \Lambda_\chi \int d^d x (-G)^{1/2} e^{-\chi\Phi}$$

$$\int [d\Psi]\exp(iS[\Psi])$$

$$\delta\Psi = Q_{\mathrm{B}}\Lambda$$

$$S_0 = \frac{1}{2}\langle\langle\Psi|Q_{\mathrm{B}} \mid \Psi\rangle.$$

$$Q_{\mathrm{B}}\Psi = 0$$

$$\Psi[X, c, \tilde{c}] = \sum_i \Phi_i(x)\Psi_i[X', c, \tilde{c}]$$

$$\int [d\Psi] \to \prod_i \int [d\Phi_i]$$

$$\Psi = [\varphi(x) + A_\mu(x)\alpha_{-1}^{\mu} + B(x)b_{-1} + C(x)c_{-1}$$
$$+ \varphi'(x)c_0 + A'_\mu(x)\alpha_{-1}^{\mu}c_0 + B'(x)b_{-1}c_0 + C'(x)c_{-1}c_0 + \cdots]\Psi_0$$

$$\delta\Psi = Q_{\mathrm{B}}\Lambda + g\Psi * \Lambda - g\Lambda * \Psi.$$

$$Q_{\mathrm{B}}(\Psi_1 * \Psi_2) = (Q_{\mathrm{B}}\Psi_1) * \Psi_2 + \Psi_1 * (Q_{\mathrm{B}}\Psi_2)$$

$$\int \Psi_1 * \Psi_2 = \int \Psi_2 * \Psi_1$$

$$S = \frac{1}{2}\int \Psi * Q_{\mathrm{B}}\Psi + \frac{2g}{3}\int \Psi * \Psi * \Psi$$

$$S = \frac{1}{2}\langle\mathcal{V}_\Psi Q_{\mathrm{B}} \cdot \mathcal{V}_{\ \Psi}\rangle_{D_2} + \frac{2g}{3}\langle\mathcal{V}_\Psi\mathcal{V}_\Psi\mathcal{V}_{\ \Psi}\rangle_{D_2}$$

$$\int_0^\infty dt\exp(-tL_0) = L_0^{-1}$$

$$S_0' = \frac{1}{2}\langle\langle\Psi|(c_0 - \tilde{c}_0)Q_{\mathrm{B}} \mid \Psi\rangle$$

$$\int_{-\infty}^{\infty} dy\exp\left[-\frac{y^2}{2!} - \lambda\frac{y^3}{3!}\right] = \lambda^{-1}\int_{-\infty}^{\infty} dz\exp\left[-\frac{1}{\lambda^2}\left(\frac{z^2}{2!} + \frac{z^3}{3!}\right)\right]$$

$$\sum_{n=0}^{\infty}\frac{(-\lambda)^n}{6^n n!}\int_{-\infty}^{\infty} dy y^{3n}\exp(-y^2/2) = (2\pi)^{1/2}\sum_{k=0}^{\infty}\lambda^{2k}C_{2k}$$

$$C_{2k} = \frac{2^k\Gamma(3k + 1/2)}{\pi^{1/2}3^{2k}(2k)!}$$

$$C_{2k} \approx k^k \approx k!,$$

$$\sum_{k=0}^{\infty} \lambda^{2k} k! \, f_{2k}$$

$$\exp\left[-O(1/\lambda^2)\right]$$

$$\int_0^{\infty} dt \exp(-t) \sum_{k=0}^{\infty} (t\lambda^2)^k f_{2k}$$

$$\sum_{k=0}^{\infty} g_o^{2k} O(k!) = \sum_{k=0}^{\infty} g_c^k O(k!)$$

$$\exp\left[-O(1/g_c)\right]$$

$$\frac{8\pi i g_c^2}{\alpha'} (2\pi)^{26} \delta^{26}\left(\sum_i k_i\right) \int_{\mathrm{C}} d^2 z_4 |z_4|^{-\alpha' u/2-4} |1-z_4|^{-\alpha' t/2-4}$$

$$\frac{\partial}{\partial z_4}(u \ln|z_4|^2 + t \ln|1-z_4|^2) = \frac{u}{z_4} + \frac{t}{z_4-1} = 0$$

$$S \approx \exp\left[-\alpha'(s\ln s + t\ln t + u\ln u)/2\right]$$

$$X_{\mathrm{cl}}^{\mu}(\sigma) = i \sum_i k_i^{\mu} G'(\sigma, \sigma_i),$$

$$\exp\left[-\sum_{i<j} k_i \cdot k_j G'(\sigma_i, \sigma_j)\right]$$

$$z_2^N = \frac{(z_1-a_1)(z_1-a_2)}{(z_1-a_3)(z_1-a_4)}$$

$$S_g \approx \exp\left[-\alpha'(s\ln s + t\ln t + u\ln u)/2(g+1)\right]$$

$$\exp\left[\alpha' t \ln s/2(g+1)\right]$$

$$\langle 0|[X^1(\sigma) - x^1]^2|0\rangle = \sum_{n=1}^{\infty} \frac{\alpha'}{2n^2} \langle 0|(\alpha_n \alpha_{-n} + \tilde{\alpha}_n \tilde{\alpha}_{-n})|0\rangle = \alpha' \sum_{n=1}^{\infty} \frac{1}{n}$$

$$s^{2+\alpha' t/2} \frac{\Gamma(-\alpha' t/4 - 1)}{\Gamma(\alpha' t/4 + 2)} \sim \frac{s^2}{t} \exp\left[-\frac{q^2 \alpha'}{2} \ln(\alpha' s)\right]$$

$$\int_0^{\infty} dm\, n(m) \exp(-\alpha' \pi m^2 \ell) \approx \exp(4\pi/\ell)$$

$$\int_0^{\infty} dm \exp\left(4\pi m \alpha'^{1/2}\right) \exp(-m/T)$$

$$T_{\mathrm{H}}=\frac{1}{4\pi\alpha'^{1/2}}$$

$$F(T,m^2)=T\int\frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}}\ln\left[1-\exp\left(-\omega_k/T\right)\right]=-\int_0^\infty\frac{dt}{t}(2\pi t)^{-d/2}\sum_{r=1}^{\infty}\exp\left(-\frac{m^2t}{2}-\frac{r^2}{2T^2t}\right)$$

$$F(T)=-\int_R\frac{d\tau d\bar{\tau}}{2\tau_2}(4\pi^2\alpha'\tau_2)^{-13}|\eta(\tau)|^{-48}\sum_{r=1}^{\infty}\exp\left(-\frac{r^2}{4\pi T^2\alpha'\tau_2}\right)$$

$$R:\tau_1\leq\frac{1}{2},|\tau_2|>0$$

$$\eta(i\tau_2)=\eta(i/\tau_2)\tau_2^{-1/2}\approx\exp\left(-\pi/12\tau_2\right)\text{ as }\tau_2\to 0$$

$$\tilde{F}(T)=F(T)+\rho_0$$

$$\frac{1}{T}\tilde{F}(T)=\frac{T}{4T_{\mathrm{H}}^2}\tilde{F}\left(4T_{\mathrm{H}}^2/T\right)$$

$$\tilde{F}(T)\to\frac{T^2}{4T_{\mathrm{H}}^2}\rho_0$$

$$G_{\mu v}(x)\ =\eta_{\mu v},B_{\mu v}(x)=0,\Phi(x)=V_\mu x^\mu$$

$$V_\mu\ =\delta_\mu{}^1\left(\frac{26-D}{6\alpha'}\right)^{1/2}$$

$$-\partial^\mu\partial_\mu T(x)+2V^\mu\partial_\mu T(x)-\frac{4}{\alpha'}T(x)=0$$

$$T(x)=\exp\left(q\cdot x\right),(q-V)^2=\frac{2-D}{6\alpha'}$$

$$q_1=\alpha_\pm=\left(\frac{26-D}{6\alpha'}\right)^{1/2}\pm\left(\frac{2-D}{6\alpha'}\right)^{1/2}$$

$$S_\sigma=\frac{1}{4\pi\alpha'}\int_M d^2\sigma g^{1/2}\left[g^{ab}\eta_{\mu v}\partial_a X^\mu\partial_b X^v+\alpha' RV_1X^1+T_0\exp\left(\alpha\_X^1\right)\right]$$

$$S=\frac{1}{4\pi\alpha'}\int_M d^2\sigma g^{1/2}\left(g^{ab}\partial_a X^\mu\partial_b X_\mu+\mu\right)$$

$$g_{ab}(\sigma)=\exp\left(2\phi(\sigma)\right)\hat{g}_{ab}(\sigma)$$

$$g^{1/2}R=\hat{g}^{1/2}\left(\hat{R}-2\hat{\nabla}^2\phi\right)$$

$$S=\frac{1}{4\pi\alpha'}\int_M d^2\sigma\hat{g}^{1/2}\left[\hat{g}^{ab}\partial_a X^\mu\partial_b X_\mu+\mu\exp\ 2\phi+\frac{13\alpha'}{3}\left(\hat{g}^{ab}\partial_a\phi\partial_b\phi+\hat{R}\phi\right)\right]$$

$$\hat{g}_{ab}(\sigma)\ \to \exp\left(2\omega(\sigma)\right)\hat{g}_{ab}(\sigma)$$
$$\phi(\sigma)\ \to \phi(\sigma)-\omega(\sigma)$$

Campos bosónicos.

$$[\hat{q},\hat{p}]=i\hbar$$

$$\langle q_f|\exp\left(-i\hat{H}T/\hbar\right)|q_i\rangle$$

$$\hat{A}(t)=\exp\left(i\hat{H}t/\hbar\right)\hat{A}\exp\left(-i\hat{H}t/\hbar\right)$$

$$\hat{q}(t)|q,t\rangle=|q,t\rangle q$$

$$|q,t\rangle=\exp\left(i\hat{H}t/\hbar\right)|q\rangle$$

$$\langle q_f,T\mid q_i,0\rangle=\int\ dq\langle q_f,T\mid q,t\rangle\langle q,t\mid q_i,0\rangle$$

$$t_m=m\epsilon,\epsilon=T/N$$

$$\langle q_f,T\mid q_i,0\rangle=\int\ dq_{N-1}\ldots dq_1\prod_{m=0}^{N-1}\langle q_{m+1},t_{m+1}\mid q_m,t_m\rangle$$

$$\langle q_{m+1},t_{m+1}\mid q_m,t_m\rangle=\langle q_{m+1}|\exp\left(-i\hat{H}\epsilon/\hbar\right)|q_m\rangle=\int\ dp_m\langle q_{m+1}\mid p_m\rangle\langle p_m|\exp\left(-i\hat{H}\epsilon/\hbar\right)|q_m\rangle$$

$$\langle p_m|\hat{H}(\hat{p},\hat{q})|q_m\rangle=H(p_m,q_m)\langle p_m\mid q_m\rangle.$$

$$\int\ dp_m\exp\left[-\frac{iH(p_m,q_m)\epsilon}{\hbar}\right]\langle q_{m+1}\mid p_m\rangle\langle p_m\mid q_m\rangle$$

$$=\int\ \frac{dp_m}{2\pi\hbar}\exp\left\{-\frac{i}{\hbar}[H(p_m,q_m)\epsilon-p_m(q_{m+1}-q_m)]+O(\epsilon^2)\right\}$$

$$\langle q_f,T\mid q_i,0\rangle\approx\int\ \frac{dp_{N-1}}{2\pi\hbar}dq_{N-1}\ldots\frac{dp_1}{2\pi\hbar}dq_1\frac{dp_0}{2\pi\hbar}\times\exp\left\{-\frac{i}{\hbar}\sum_{m=0}^{N-1}[H(p_m,q_m)\epsilon-p_m(q_{m+1}-q_m)]\right\}$$

$$\to\int\ [dpdq]\exp\left\{\frac{i}{\hbar}\int_0^T dt[p\dot{q}-H(p,q)]\right\}$$

$$0=\frac{\partial}{\partial p}[p\dot{q}-H(p,q)]=\dot{q}-\frac{\partial}{\partial p}H(p,q).$$

$$\langle q_f,T\mid q_i,0\rangle=\int\ [dq]\exp\left[\frac{i}{\hbar}\int_0^T dtL(q,\dot{q})\right]$$

$$\int [dq]_{q_i,0}^{q_f,T} \exp\left(i\int_0^T dtL\right) = \int dq \int [dq]_{q,t}^{q_f,T} \exp\left(i\int_t^T dtL\right)\int [dq]_{q_i,0}^{q,t} \exp\left(i\int_0^t dtL\right)$$

$$\int [dq]_{q_i,0}^{q_f,T} \exp(iS) q(t) = \int dq \langle q_f,T \mid q,t\rangle q\langle q,t \mid q_i,0\rangle = \int dq \langle q_f,T\left|\hat{q}(t)\right|q,t\rangle\langle q,t \mid q_i,0\rangle$$

$$= \langle q_f,T\left|\hat{q}(t)\right|q_i,0\rangle$$

$$\int [dq]_{q_i,0}^{q_f,T} \exp(iS) q(t)q(t') = \langle q_f,T\left|\mathrm{T}[\hat{q}(t)\hat{q}(t')]\right|q_i,0\rangle.$$

$$\mathrm{T}\left[\hat{A}(t)\hat{B}(t')\right] = \theta(t-t')\hat{A}(t)\hat{B}(t') + \theta(t'-t)\hat{B}(t')\hat{A}(t)$$

$$\int [dq]_{q_i,0}^{q_f,T} \exp(iS) \frac{\delta S}{\delta q(t)}\mathcal{F} = -i\int [dq]_{q_i,0}^{q_f,T}\left[\frac{\delta}{\delta q(t)}\exp(iS)\right]\mathcal{F} = i\int [dq]_{q_i,0}^{q_f,T} \exp(iS)\frac{\delta \mathcal{F}}{\delta q(t)}$$

$$\frac{\delta}{\delta q(t)} q(t') = \delta(t-t')$$

$$\frac{1}{\epsilon}\frac{\partial}{\partial q_m}, m = t/\epsilon$$

$$\left\langle q(t')\frac{\delta S}{\delta q(t)} \ldots\right\rangle = i\langle\delta(t-t')\ldots\rangle$$

$$q(t')\frac{\delta S}{\delta q(t)} = i\delta(t-t')$$

$$S = \frac{1}{2}\int dt(\dot{q}^2 - \omega^2 q^2)$$

$$\frac{\delta S}{\delta q(t)} = -\ddot{q}(t) - \omega^2 q(t)$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega^2\right) q(t) = 0$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega^2\right) q(t)q(t') = -i\delta(t-t')$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega^2\right) \hat{q}(t) = 0$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega^2\right) \mathrm{T}[\hat{q}(t)\hat{q}(t')] = -i\delta(t-t')$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega^2\right) \mathrm{T}[\hat{q}(t)\hat{q}(t')] = \mathrm{T}\left\{\left[\ddot{\hat{q}}(t) + \omega^2\hat{q}(t)\right]\hat{q}(t')\right\} + \delta(t-t')[\dot{\hat{q}}(t),\hat{q}(t)] = 0 - i\delta(t-t')$$

$$\langle q_f\left|\exp\left[-i\hat{H}(T-t_>)\right]\hat{q}\exp\left[-i\hat{H}(t_>-t_<)\right]\hat{q}\exp\left[-i\hat{H}t_<\right]\right|q_i\rangle,$$

$$\langle q_f, u|\mathrm{T}\left[\prod_a \hat{q}(u_a)\right]|q_i, 0\rangle_{\mathrm{E}}$$

$$\langle q_f, t|\mathrm{T}\left[\prod_a \hat{q}(t_a)\right]|q_i, 0\rangle$$

$$\langle q_f, U|\mathrm{T}\left[\prod_a \hat{q}(u_a)\right]|q_i, 0\rangle_{\mathrm{E}} = \int [dq]_{q_i,0}^{q_f,U} \exp\left[\int_0^U duL(q, i\partial_u q)\right]\prod_a q(u_a).$$

$$S_{\mathrm{E}} = \int duL_{\mathrm{E}}(q, \partial_u q) = -\int duL(q, i\partial_u q)$$

$$\mathrm{Tr}\exp(-\hat{H}U) = \sum_i \exp(-E_i U)$$

$$\int dq\langle q|\exp(-\hat{H}U)|q\rangle = \int dq\langle q, U \mid q, 0\rangle_{\mathrm{E}} = \int dq \int [dq]_{q,0}^{q,U}\exp(-S_{\mathrm{E}}) = \int [dq]_{\mathrm{P}}\exp(-S_{\mathrm{E}})$$

$$\int [dq]_{\mathrm{A}}\exp(-S_{\mathrm{E}}) = \int dq \int [dq]_{-q,0}^{q,U}\exp(-S_{\mathrm{E}}) = \int dq\langle q, U \mid -q, 0\rangle_{\mathrm{E}} = \int dq\langle q, U|\hat{R}|q, 0\rangle_{\mathrm{E}}$$

$$= \mathrm{Tr}[\exp(-\hat{H}U)\hat{R}]$$

$$\frac{1}{2}(\text{ periodic } + \text{ antiperiodic }) = \sum_{R_i=+1} \exp(-E_i U)$$

$$\hat{A}(u)^\dagger = \hat{A}(-u)$$

$$\overline{\hat{A}(u)} = \hat{A}(-u)^\dagger.$$

$$\frac{1}{2}\int d^d x\phi(x)\Delta\phi(x)$$

$$Z[J] = \left\langle \exp\left[i\int d^d xJ(x)\phi(x)\right]\right\rangle = \int [d\phi]\exp\left[-\frac{1}{2}\int d^d x\phi(x)\Delta\phi(x) + i\int d^d xJ(x)\phi(x)\right]$$

$$Z[J] = Z[0]\exp\left[-\frac{1}{2}\int d^d xd^d yJ(x)\Delta^{-1}(x,y)J(y)\right]$$

$$= Z[0]\exp\left[\frac{1}{2}\int d^d xd^d y\Delta^{-1}(x,y)\frac{\delta^2}{\delta\phi(x)\delta\phi(y)}\right] \times \exp\left[i\int d^d xJ(x)\phi(x)\right]\Bigg|_{\phi=0}$$

$$\mathcal{F}[\phi] = \int [dJ]\exp\left[i\int d^d xJ(x)\phi(x)\right]f[J]$$

$$\langle\mathcal{F}\rangle = Z[0]\exp\left[\frac{1}{2}\int d^d xd^d y\Delta^{-1}(x,y)\frac{\delta^2}{\delta\phi(x)\delta\phi(y)}\right]\mathcal{F}[\phi]\Bigg|_{\phi=0}.$$

$$\frac{\delta}{\delta\phi(x)}=\frac{\delta}{\delta\phi_1(x)}+\frac{\delta}{\delta\phi_2(x)}+\cdots$$

$$\frac{\delta^2}{\delta\phi_i(x)\delta\phi_i(y)} \text{ (no sum on } i\text{)}$$

$$\phi(x)=\sum_i \phi_i\Phi_i(x)$$

$$\begin{aligned}\Delta\Phi_i(x) &= \lambda_i\Phi_i(x)\\ \int d^dx\Phi_i(x)\Phi_j(x) &= \delta_{ij}\end{aligned}$$

$$[d\phi]=\prod_i d\phi_i$$

$$\langle 1\rangle=\prod_i\left[\int d\phi_i\exp\left(-\frac{1}{2}\phi_i^2\lambda_i\right)\right]=\prod_i\left(\frac{2\pi}{\lambda_i}\right)^{1/2}=\left(\det\frac{\Delta}{2\pi}\right)^{-1/2}$$

$$\int [d\phi d\phi^*]\exp\left(-\int d^dx\phi^*\Delta\phi\right)=(\det\Delta)^{-1}$$

$$\int [d\phi^1d\phi^2]\exp\left(i\int d^dx\phi^1\Delta\phi^2\right)=(\det\Delta)^{-1}$$

$$\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\exp(i\lambda xy)=2\pi\int_{-\infty}^{\infty}dx\delta(\lambda x)=\frac{2\pi}{\lambda}$$

$$\begin{aligned}\langle q_f,U\mid q_i,0\rangle_{\mathrm{E}} &= \int [dq]_{q_i,0}^{q_f,U}\exp(-S_{\mathrm{E}})\\ S_{\mathrm{E}} &= \frac{1}{2}\int_0^U du[(\partial_uq)^2+\omega^2q^2]+S_{\mathrm{ct}}\end{aligned}$$

$$q(u)=q_{\mathrm{cl}}(u)+q'(u)$$

$$\begin{gathered}-\ddot{q}_{\mathrm{cl}}(u)+\omega^2q_{\mathrm{cl}}(u)=0\\ q_{\mathrm{cl}}(0)=q_i,q_{\mathrm{cl}}(U)=q_f\end{gathered}$$

$$q'(0)=q'(U)=0$$

$$S_{\mathrm{E}}=S_{\mathrm{cl}}(q_i,q_f)+S'+S_{\mathrm{ct}}$$

$$S_{\mathrm{cl}}(q_i,q_f)=\frac{1}{2}\int_0^U du\left[(\partial_uq_{\mathrm{cl}})^2+\omega^2q_{\mathrm{cl}}^2\right]=\omega\frac{(q_i^2+q_f^2)\cosh\omega U-2q_iq_f}{2\sinh\omega U}$$

$$S'=\frac{1}{2}\int_0^U du[(\partial_uq')^2+\omega^2q'^2]$$

$$\langle q_f, U \mid q_i, 0\rangle = \exp\left[-S_{\mathrm{cl}}(q_i, q_f) - S_{\mathrm{ct}}\right] \int [dq']_{0,0}^{0,U} \exp(-S')$$

$$f_j(u) = \left(\frac{2}{U}\right)^{1/2} \sin \frac{j\pi u}{U}, \lambda_j = \frac{j^2\pi^2}{U^2} + \omega^2$$

$$\det \frac{\Delta}{2\pi} = \prod_{j=1}^{\infty} \frac{j^2\pi^2 + \omega^2 U^2}{2\pi U^2}$$

$$\det \frac{\Delta}{2\pi} \xrightarrow{\text{regulate}} \prod_{j=1}^{\infty} \frac{j^2\pi^2 + \omega^2 U^2}{j^2\pi^2 + \Omega^2 U^2}$$

$$\frac{\Omega \sinh\ \omega U}{\omega \sinh\ \Omega U}$$

$$\langle q_f, U \mid q_i, 0\rangle \rightarrow \left(\frac{\omega}{2\sinh\ \omega U}\right)^{1/2} \exp\left[-S_{\mathrm{cl}}(q_i, q_f) + \frac{1}{2}(\Omega U - \ln\ \Omega) - S_{\mathrm{ct}}\right]$$

$$\langle q_f, U \mid q_i, 0\rangle \rightarrow \langle q_f \mid 0\rangle\langle 0 \mid q_i\rangle \exp(-E_0 U) = \left(\frac{\omega}{\pi}\right)^{1/2} \exp\left[-\frac{\omega}{2}(q_f^2 + q_i^2 + U)\right]$$

$$\langle q_f, U \mid q_i, 0\rangle \rightarrow \omega^{1/2} \exp\left[-\frac{\omega}{2}(q_f^2 + q_i^2 + U) + \frac{1}{2}(\Omega U - \ln\ \Omega) - S_{\mathrm{ct}}\right]$$

$$S_{\mathrm{ct}} = \frac{1}{2}\left[\int_0^U du\Omega\right] - \frac{1}{2}\ln \frac{\Omega}{\pi}$$

$$\langle q_f, U \mid q_i, 0\rangle = \left(\frac{\omega}{2\pi\sinh\ \omega U}\right)^{1/2} \exp(-S_{\mathrm{cl}}),$$

Campos Fermiónicos.

$$\begin{aligned} \hat{\psi}|\downarrow\rangle &= 0, \hat{\chi}|\downarrow\rangle = |\uparrow\rangle, \\ \hat{\psi}|\uparrow\rangle &= |\downarrow\rangle, \hat{\chi}|\uparrow\rangle = 0, \\ \{\hat{\psi}, \hat{\chi}\} &= 1, \hat{\psi}^2 = \hat{\chi}^2 = 0. \end{aligned}$$

$$\{\chi_m, \psi_n\} = \{\chi_m, \chi_n\} = \{\psi_m, \psi_n\} = 0.$$

$$\int d\theta = 0, \int d\theta\theta = -\int \theta d\theta = 1$$

$$\int d\theta \frac{d}{d\theta}(a + \theta b) = \int d\theta b = 0$$

$$\int d\theta_1 \ldots d\theta_n f(\theta) = c$$

$$f(\theta) = \cdots + \theta_n \theta_{n-1} \ldots \theta_1 c.$$

$$\begin{aligned} |\psi\rangle &= |\downarrow\rangle + |\uparrow\rangle\psi, \\ \hat{\psi}|\psi\rangle &= |\psi\rangle\psi. \end{aligned}$$

$$\langle \psi \mid \psi' \rangle = \psi - \psi'.$$

$$(\psi - \psi')f(\psi) = (\psi - \psi')f(\psi')$$

$$\int d\psi(\psi - \psi')f(\psi) = f(\psi')$$

$$\langle \psi | \hat{\psi} = -\psi \langle \psi |,$$

$$\int |\psi\rangle d\psi \langle \psi| = 1$$

$$-\int d\chi \exp [\chi(\psi' - \psi)] = \psi - \psi' = \langle \psi \mid \psi' \rangle$$

$$\langle \psi \mid \downarrow \rangle = \psi, \langle \psi \mid \uparrow \rangle = -1$$

$$\langle \psi | \hat{\chi} | \psi' \rangle = \langle \psi \mid \uparrow \rangle = -\int d\chi \chi \exp [\chi(\psi' - \psi)]$$

$$-\langle \psi | f(\hat{\chi}, \hat{\psi}) | \psi' \rangle = \int d\chi f(\chi, \psi') \exp [\chi(\psi' - \psi)]$$

$$\left\langle \psi_f, T \mid \psi_i, 0 \right\rangle \approx -\int d\chi_0 d\psi_1 d\chi_1 \ldots d\psi_{N-1} d\chi_{N-1}$$

$$\times \exp \sum_{m=0}^{N-1} [-iH(\chi_m, \psi_m)\epsilon - \chi_m(\psi_{m+1} - \psi_m)]$$

$$\rightarrow \int [d\chi d\psi] \exp \left\{ \int_0^T dt[-\chi\dot{\psi} - iH(\chi, \psi)] \right\}$$

$$\begin{aligned} &(\mathrm{A}): \quad \langle \uparrow | \uparrow \rangle = \langle \downarrow | \downarrow \rangle = 1, \langle \uparrow | \downarrow \rangle = \langle \downarrow | \uparrow \rangle = 0, \\ &(\mathrm{B}): \quad \langle \uparrow | \downarrow \rangle = \langle \downarrow | \uparrow \rangle = 1, \langle \uparrow | \uparrow \rangle = \langle \downarrow | \downarrow \rangle = 0. \end{aligned}$$

$$\psi(t)\psi(t') = -\psi(t')\psi(t)$$

$$\mathrm{T}\left[\hat{\psi}(t)\hat{\psi}(t')\right] \equiv \theta(t - t')\hat{\psi}(t)\hat{\psi}(t') - \theta(t' - t)\hat{\psi}(t')\hat{\psi}(t)$$

$$\hat{A}|i\rangle = |j\rangle A_{ji}$$

$$\int d\psi \langle \psi | \hat{A} | \psi \rangle = A_{\downarrow\downarrow} - A_{\uparrow\uparrow} = \mathrm{Tr}\left[(-1)^{\hat{F}} \hat{A}\right]$$

$$\int d\psi \langle \psi, U \mid \psi, 0 \rangle = \mathrm{Tr}\left[(-1)^{\hat{F}} \exp (-\hat{H}U)\right]$$

$$\langle 1 \rangle = \int [d\psi d\chi] \exp \left(\int d^d x \chi \Delta \psi \right)$$

$$\psi(x) = \sum_i \psi_i \Psi_i(x), \chi(x) = \sum_i \chi_i \Upsilon_i(x)$$
$$\Delta\Psi_i(x) = \lambda_i \Psi_i(x), \Delta^T \Upsilon_i(x) = \lambda_i \Upsilon_i(x)$$
$$\int d^d x \Upsilon_i(x)\Psi_j(x) = \delta_{ij}$$

$$\langle 1 \rangle = \prod_i \left[\iint d\psi_i d\chi_i \exp\left(\lambda_i \chi_i \psi_i\right)\right] = \prod_i \lambda_i = \det\Delta$$

$$\int [d\psi] \exp\left(\int d^d x \psi \Delta \psi\right) = (\det\Delta)^{1/2}$$

$$\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \exp\left(2\pi i \lambda x y\right) = \frac{1}{\lambda} = \left[\int d\psi \int d\chi \exp\left(\lambda \chi \psi\right)\right]^{-1}$$

$$\delta(d^m x d^n \theta) = d^m x d^n \theta \left(\sum_i \frac{\partial}{\partial x_i} \delta x_i - \sum_j \frac{\partial}{\partial \theta_j} \delta \theta_j\right)$$

$$\int [dq dq^*]_\theta \exp\left(-S_{\mathrm{E}}\right)$$

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma (\partial_1 X \partial_1 X + \partial_2 X \partial_2 X + m^2 X^2)$$

$$0 \leq \sigma_1 \leq 2\pi, 0 \leq \sigma_2 \leq T$$

$$H = -\frac{1}{2} \frac{\partial^2}{\partial \phi^2}$$

$$\frac{1}{2\kappa^2} \int d^d x (-G)^{1/2} R$$

$$\chi = \sum_{p=0}^{d} (-1)^p B_p \frac{i}{2\pi\alpha'} \int_M B + i \int_{\partial M} \mathcal{A}$$

CONCLUSIONES.

En mérito a lo antes expuesto, concluyese que, las ecuaciones de campo de Einstein engranan, en tratándose de un espacio – tiempo cuántico, en tanto se esté ante un efecto cuántico – gravitacional ordinario, es decir, cuando el tejido del espacio – tiempo, en dimensión $\mathbb{R}^4$, se deforma, sin que por esto, estemos ante pluridimensiones, como sí supone la supergravedad cuántica, lo que no ocupa este estudio. Sin embargo, esta deformación, supone la existencia de otros puntos de realidad en una misma dimensión, en los que impera la simetría de gauge fija o corregida. Por tanto, la curvatura, es puramente local, por lo que, las transformaciones de gauge son locales, sin que existan contrapuntos pluridimensionales, sino pluriespaciales, es decir, interacciones de partículas focalizadas en puntos arbitrarios del espacio de Hilber – Einstein deformado. Adicionalmente y como ha quedado anotado, concebida así, la gravedad cuántica, ésta puede ser endógena o exógena, según la naturaleza fenomenológica de la partícula supermasiva en un campo cuántico específico y según las magnitudes escalares y simetrías tensoriales.

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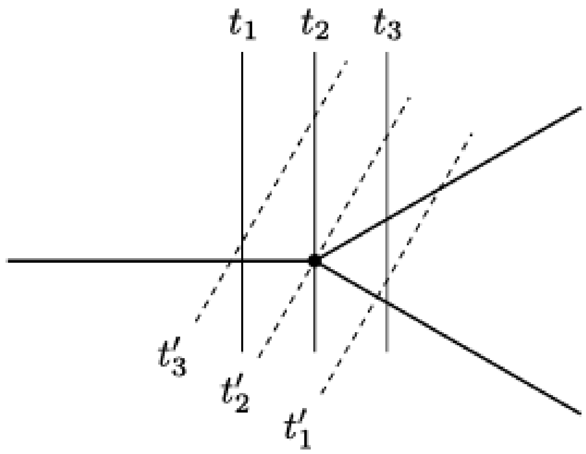
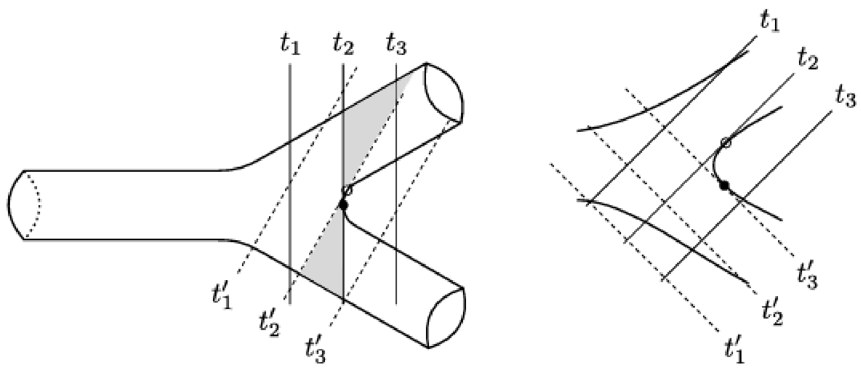
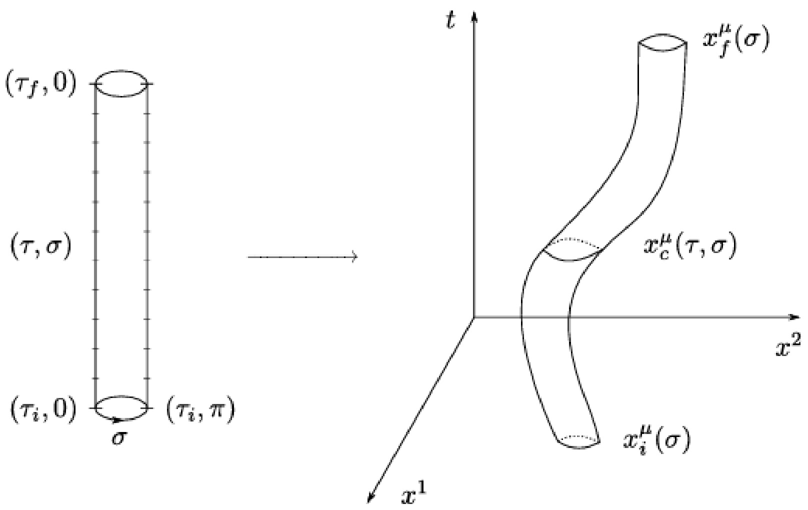
Albuja Bustamante, M. I. (2025). Las partículas supermasivas: naturaleza fenomenológica de la partícula oscura. Ciencia Latina Revista Científica Multidisciplinar, 9(3). https://doi.org/10.37811/cl_rcm.v9i3.18172

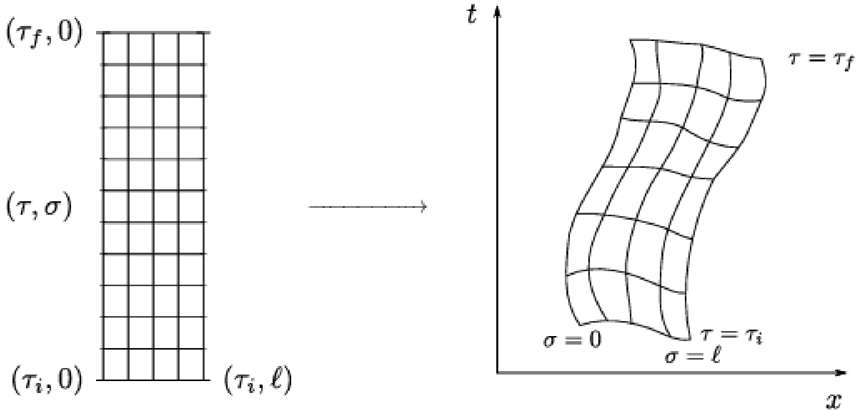
Anexo A.

Acoplamiento de las ecuaciones de campo de Einstein – Jordan - Weyl a un espacio – tiempo cuántico relativista.









$$S = \int_M d^4x\sqrt{-g}\left(\phi^4 R - \frac{\omega}{\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right) \\ +2\int_{\partial M} d^3x\sqrt{h}\phi K$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\omega}{\phi^2}\left[\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)\right] + \\ +\frac{1}{\phi}\left[\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\,\square\,\phi - \frac{1}{2}g_{\mu\nu}U(\phi)\right]$$

$$(3 + 2\omega)\,\square\,\phi = \phi\frac{dU}{d\phi} - 2U(\phi).$$

$$\tilde{g}_{\mu\nu} = (16\pi G\phi)g_{\mu\nu}, \tilde{\phi} = \phi,$$

$$\tilde{S} = \int_M d^4x\sqrt{-\tilde{g}}\left[\frac{1}{16\pi G}\tilde{R} - \frac{2\omega + 3}{(16\pi G)2\phi^2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \right. \\ \left. -V(\phi)\right] + \frac{1}{8\pi G}\int_{\partial M} d^3\sqrt{\tilde{h}}\tilde{K}$$

$$V(\phi) \equiv \frac{U(\phi)}{(16\pi G\phi)^2}$$

$$\left(g_{\mu\nu}(x), \phi(x)\right)$$

$$\left(\tilde{g}_{\mu\nu}(x), \phi(x)\right)$$

$$\left\{\tilde{N}(x), \tilde{\pi}_\phi(x')\right\} = \frac{8\pi G N(x)\delta^{(3)}(x - x')}{\sqrt{16\pi G\phi(x)}} \neq 0.$$

$$g = -N^2(t)dt \otimes dt + a^2(t)dx^i \otimes dx^i$$

$$\mathcal{L}_{FLRW} = -\frac{6a\dot{a}^2}{N}\phi - \frac{6a^2\dot{a}}{N}\dot{\phi} + \frac{\omega a^3}{N\phi}(\dot{\phi})^2 - Na^3U(\phi).$$

$$H_{\mathrm{T}} = N\left[-\frac{\omega\pi_a^2}{12a\phi(2\omega+3)} - \frac{\pi_a\pi_\phi}{2a^2(2\omega+3)} + \frac{\phi\pi_\phi^2}{2a^3(2\omega+3)}\right.$$
$$\left. + a^3U(\phi)\right] + \lambda_N\pi_N \equiv NH + \lambda_N\pi_N$$

$$\dot{N} \approx \{N, H_T\} \approx \lambda_N,$$
$$\dot{\pi}_N \approx \{H, H_T\} \approx -H,$$
$$\dot{a} \approx \{a, H_T\} \approx -\frac{N}{2a(2\omega+3)}\left(\frac{\omega\pi_a}{3\phi} + \frac{\pi_\phi}{a}\right),$$
$$\dot{\pi}_a \approx \{\pi_a, H_T\} \approx -\frac{N}{2a^2(2\omega+3)}\left(\frac{\omega\pi_a^2}{6\phi} + \frac{2\pi_a\pi_\phi}{a} - \frac{3\phi\pi_\phi^2}{a^2}\right)$$
$$-3Na^2U(\phi),$$
$$\dot{\phi} \approx \{\phi, H_T\} \approx \frac{N}{2a^2(2\omega+3)}\left(-\pi_a + \frac{2\phi\pi_\phi}{a}\right),$$
$$\dot{\pi}_\phi \approx \{\pi_\phi, H_T\} \approx -\frac{N}{2a(2\omega+3)}\left(\frac{\omega\pi_a^2}{6\phi^2} + \frac{\pi_\phi^2}{a^2}\right) - Na^3\frac{dU}{d\phi}.$$

$$\tilde{g} = -\tilde{N}^2(t)dt \otimes dt + \tilde{a}^2(t)dx^i \otimes dx^i$$

$$\tilde{N} = N(16\pi G\phi)^{\frac{1}{2}}, \tilde{a} = (16\pi G\phi)^{\frac{1}{2}}a, \tilde{\phi} = \phi.$$

$$\tilde{\mathcal{L}}_{FLRW} = -\frac{1}{\tilde{N}(16\pi G\phi)}\left[6\tilde{a}\dot{\tilde{a}}^2\phi - \frac{(2\omega+3)\tilde{a}^3\dot{\phi}^2}{2\phi}\right]$$
$$-\tilde{N}\tilde{a}^3V(\phi)$$

$$\tilde{H}_T = \tilde{N}\tilde{a}^3\left[-\frac{2\pi G\tilde{\pi}_a^2}{3\tilde{a}^4} + \frac{8\pi G\tilde{\pi}_\phi^2\phi^2}{(2\omega+3)\tilde{a}^6} + V(\phi)\right]$$
$$+\tilde{\lambda}_N\tilde{\pi}_N \equiv \tilde{N}\tilde{H} + \tilde{\lambda}_N\tilde{\pi}_N$$

$$\dot{\tilde{N}} \approx \{\tilde{N}, \tilde{H}_T\} \approx \tilde{\lambda}_N$$
$$\dot{\tilde{\pi}}_N \approx \{\tilde{\pi}_N, \tilde{H}_T\} \approx -\tilde{H}$$
$$\dot{\tilde{a}} \approx \{\tilde{a}, \tilde{H}_T\} \approx -\tilde{N}\frac{4\pi G\tilde{\pi}_a}{3\tilde{a}}$$
$$\dot{\tilde{\pi}}_a \approx \{\tilde{\pi}_a, \tilde{H}_T\}$$
$$\approx \tilde{N}\left[-\frac{(2\pi G)\tilde{\pi}_a^2}{3\tilde{a}^2} + \frac{3(8\pi G)\tilde{\pi}_\phi^2\tilde{\phi}^2}{(2\omega+3)\tilde{a}^4} - 3\tilde{a}^2V(\tilde{\phi})\right]$$
$$\dot{\tilde{\phi}} \approx \{\tilde{\phi}, \tilde{H}_T\} \approx \tilde{N}\frac{(16\pi G)\tilde{\pi}_\phi\tilde{\phi}^2}{(2\omega+3)\tilde{a}^3}$$
$$\dot{\tilde{\pi}}_\phi \approx \{\tilde{\pi}_\phi, \tilde{H}_T\} \approx -\tilde{N}\left[\frac{16\pi G\tilde{\pi}_\phi^2\tilde{\phi}}{(2\omega+3)\tilde{a}^3} + \tilde{a}^3\frac{dV(\tilde{\phi})}{d\tilde{\phi}}\right]$$

$$\tilde{\pi}_a = \frac{\pi_a}{(16\pi G\phi)^{\frac{1}{2}}}, \tilde{\pi}_\phi = \frac{1}{\phi}\left(\phi\pi_\phi - \frac{1}{2}a\pi_a\right)$$
$$\tilde{\pi}_N = \frac{\pi_N}{(16\pi G\phi)^{\frac{1}{2}}}$$

$$\{\tilde{N}, \tilde{\pi}_N\} = 1, \{\tilde{a}, \tilde{\pi}_a\} = 1, \{\phi, \tilde{\pi}_\phi\} = 1$$

$$\{\tilde{N},\tilde{a}\}=0,\{\tilde{N},\tilde{\pi}_a\}=0,\{\tilde{N},\phi\}=0$$
$$\{\tilde{a},\tilde{\pi}_N\}=0,\{\tilde{a},\phi\}=0,\{\tilde{a},\tilde{\pi}_\phi\}=0$$
$$\{\phi,\tilde{\pi}_N\}=0,\{\phi,\tilde{\pi}_a\}=0,\{\tilde{\pi}_N,\tilde{\pi}_a\}=0$$

$$\{\tilde{N},\tilde{\pi}_\phi\}=\left\{N(16\pi G\phi)^{\frac{1}{2}},\frac{1}{\phi}\left(\phi\pi_\phi-\frac{1}{2}a\pi_a\right)\right\}$$
$$=\frac{8\pi GN}{(16\pi G\phi)^{\frac{1}{2}}}\neq 0$$

$$\{\tilde{\pi}_N,\tilde{\pi}_\phi\}=\left\{\frac{\pi_N}{(16\pi G\phi)^{\frac{1}{2}}},\frac{1}{\phi}\left(\phi\pi_\phi-\frac{1}{2}a\pi_a\right)\right\}$$
$$=-\frac{8\pi G\pi_N}{(16\pi G\phi)^{\frac{3}{2}}}\neq 0$$

$$\dot{N}\approx\frac{\tilde{\lambda}_N}{(16\pi G\phi)^{\frac{1}{2}}}-\frac{N^2}{2a^2(2\omega+3)}\left(\frac{\pi_\phi}{a}-\frac{\pi_a}{2\phi}\right),$$
$$\dot{\pi}_N\approx -H+\frac{N\pi_N}{2a^2(2\omega+3)}\left(\frac{\pi_\phi}{a}-\frac{\pi_a}{2\phi}\right),$$
$$\dot{a}\approx -\frac{N}{2a(2\omega+3)}\left(\frac{\omega\pi_a}{3\phi}+\frac{\pi_\phi}{a}\right),$$
$$\dot{\pi}_a\approx -\frac{N}{2a^2(2\omega+3)}\left(\frac{\omega\pi_a^2}{6\phi}+\frac{2\pi_a\pi_\phi}{a}-\frac{3\phi\pi_\phi^2}{a^2}\right)$$
$$-3Na^2U(\phi),$$
$$\dot{\phi}\approx\frac{N}{2a^2(2\omega+3)}\left(-\pi_a+\frac{2\phi\pi_\phi}{a}\right),$$
$$\dot{\pi}_\phi\approx -\frac{N}{2a(2\omega+3)}\left(\frac{\omega\pi_a^2}{4\phi^2}-\frac{7\pi_\phi^2}{2a^2}+\frac{\pi_a\pi_\phi}{2a\phi}\right)$$
$$-Na^3\frac{dU}{d\phi}+\frac{Na^3}{2\phi}U(\phi).$$

$$\dot{\pi}_\phi\approx -\frac{N}{2a(2\omega+3)}\left(\frac{\omega\pi_a^2}{6\phi^2}+\frac{\pi_\phi^2}{a^2}\right)-Na^3\frac{dU}{d\phi}+\frac{H}{2\phi},$$

$$\tilde{\lambda}_N=(16\pi G\phi)^{\frac{1}{2}}\left[\lambda_N+\frac{N^2}{2a^2(2\omega+3)}\left(\frac{\pi_\phi}{a}-\frac{\pi_a}{2\phi}\right)\right].$$

$$\tilde{\pi}_N=\frac{\pi_N}{(16\pi G\phi)^{\frac{1}{2}}}$$

$$N=c_0$$

$$N-c_0\approx 0$$

$$\{N-c_0,H\}\approx 0$$

$$\{N - c_0, \pi_N\} \approx 1$$

$$\chi_0 \equiv N - c_0, \text{ and } \chi_1 \equiv \pi_N$$

$$\dot{\chi}_0 \approx \{N - c_0, H_T\} \approx 0,$$

$$\dot{\chi}_1 \approx \{\pi_N, H_T\} \approx 0,$$

$$C_{\alpha\beta} \equiv \{\chi_\alpha, \chi_\beta\}$$

$$C_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \text{ and } C_{\alpha\beta}^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

$$\{\cdot,\cdot\}_{DB} \equiv \{\cdot,\cdot\} - \{\cdot, \chi_\alpha\} C_{\alpha\beta}^{-1} \{\chi_\beta, \cdot\}$$

$$\begin{aligned}
\dot{a} &\approx \{a, H_T\}_{DB} \\
&\approx -\frac{N}{2a(2\omega+3)}\left(\frac{\omega\pi_a}{3\phi} + \frac{\pi_\phi}{a}\right), \\
\dot{\pi}_a &\approx \{\pi_a, H_T\}_{DB} \\
&\approx -\frac{N}{2a^2(2\omega+3)}\left(\frac{\omega\pi_a^2}{6\phi} + \frac{2\pi_a\pi_\phi}{a} - \frac{3\phi\pi_\phi^2}{a^2}\right) \\
&\quad -3Na^2U(\phi), \\
\dot{\phi} &\approx \{\phi, H_T\}_{DB} \\
&\approx \frac{N}{2a^2(2\omega+3)}\left(-\pi_a + \frac{2\phi\pi_\phi}{a}\right), \\
\dot{\pi}_\phi &\approx \{\pi_\phi, H_T\}_{DB} \\
&\approx -\frac{N}{2a(2\omega+3)}\left(\frac{\omega\pi_a^2}{6\phi^2} + \frac{\pi_\phi^2}{a^2}\right) - Na^3\frac{dU}{d\phi}.
\end{aligned}$$

$$\begin{aligned}
\dot{a} &\approx -\frac{c_0}{2a(2\omega+3)}\left(\frac{\omega\pi_a}{3\phi} + \frac{\pi_\phi}{a}\right), \\
\dot{\pi}_a &\approx -\frac{c_0}{2a^2(2\omega+3)}\left(\frac{\omega\pi_a^2}{6\phi} + \frac{2\pi_a\pi_\phi}{a} - \frac{3\phi\pi_\phi^2}{a^2}\right) \\
&\quad -3c_0a^2U(\phi), \\
\dot{\phi} &\approx \frac{c_0}{2a^2(2\omega+3)}\left(-\pi_a + \frac{2\phi\pi_\phi}{a}\right), \\
\dot{\pi}_\phi &\approx -\frac{c_0}{2a(2\omega+3)}\left(\frac{\omega\pi_a^2}{6\phi^2} + \frac{\pi_\phi^2}{a^2}\right) - c_0a^3\frac{dU}{d\phi}.
\end{aligned}$$

$$\tilde{N} = c_0(16\pi G\phi)^{\frac{1}{2}},$$

$$\tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}} \approx 0.$$

$$\left\{\tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{\pi}_N\right\} \approx 1.$$

$$\left\{\tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{H}\right\} = -\frac{c_0(16\pi G\phi)^{3/2}\tilde{\pi}_\phi}{2(2\omega+3)\tilde{a}^3} \equiv -\eta \neq 0$$

$$\tilde{H}' \equiv \tilde{H} + \eta\tilde{\pi}_N,$$

$$\left\{\tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{H}'\right\} = -\frac{c_0^2(16\pi G)^2\phi\tilde{\pi}_N}{4(2\omega+3)\tilde{a}^3} \approx 0$$

$$\tilde{H}_T' = \tilde{N}\tilde{H}' + \tilde{\lambda}_N\tilde{\pi}_N.$$

$$\tilde{\chi}_0 \equiv \tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \text{ and } \tilde{\chi}_1 \equiv \tilde{\pi}_N.$$

$$\dot{\tilde{\chi}}_0 \approx \left\{\tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{H}_T'\right\} \approx 0,$$

$$\dot{\tilde{\chi}}_1 \approx \{\tilde{\pi}_N, \tilde{H}_T'\} \approx 0,$$

$$C_{\alpha\beta} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \text{ and } C_{\alpha\beta}^{-1} \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

$$\begin{gathered}
\dot{\tilde{a}} \approx \{\tilde{a}, \tilde{H}_T'\}_{DB} \approx -\tilde{N}\frac{4\pi G\tilde{\pi}_a}{3\tilde{a}}, \\
\dot{\tilde{\pi}}_a \approx \{\tilde{\pi}_a, \tilde{H}_T'\}_{DB} \\
\approx \tilde{N}\left[-\frac{(2\pi G)\tilde{\pi}_a^2}{3\tilde{a}^2} + \frac{3(8\pi G)\tilde{\pi}_\phi^2\tilde{\phi}^2}{(2\omega+3)\tilde{a}^4} - 3\tilde{a}^2V(\phi)\right] \\
-\frac{3c_0(16\pi G\phi)^{3/2}\tilde{\pi}_\phi\tilde{\pi}_N}{(2\omega+3)\tilde{a}^4}, \\
\dot{\phi} \approx \{\phi, \tilde{H}_T'\}_{DB} \\
\approx \tilde{N}\frac{(16\pi G)\tilde{\pi}_\phi\phi^2}{(2\omega+3)\tilde{a}^3} - \frac{c_0(16\pi G\phi)^{3/2}\tilde{\pi}_N}{(2\omega+3)\tilde{a}^3} \\
\dot{\tilde{\pi}}_\phi \approx \{\tilde{\pi}_\phi, \tilde{H}_T'\}_{DB} \\
\approx \{\tilde{\pi}_\phi, \tilde{H}_T'\} \\
-\left\{\tilde{\pi}_\phi, \tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}\right\} C_{01}^{-1}\{\tilde{\pi}_N, \tilde{H}_T'\} \\
\approx -\tilde{N}\left[\frac{16\pi G\tilde{\pi}_\phi^2\tilde{\phi}}{(2\omega+3)\tilde{a}^3} + \tilde{a}^3\frac{dV(\phi)}{d\phi}\right] \\
+\frac{c_0\tilde{\pi}_\phi\tilde{\pi}_N}{(2\omega+3)\tilde{a}^3}\frac{3}{2}(16\pi G\phi)^{1/2}16\pi G \\
-\frac{16\pi G c_0}{2(16\pi G\phi)^{1/2}}\tilde{H}'.
\end{gathered}$$

$$\begin{gathered}
\dot{\tilde{a}} \approx -c_0(16\pi G\phi)^{\frac{1}{2}}\frac{4\pi G\tilde{\pi}_a}{3\tilde{a}}, \\
\dot{\tilde{\pi}}_a \approx c_0(16\pi G\phi)^{\frac{1}{2}}\left[-\frac{(2\pi G)\tilde{\pi}_a^2}{3\tilde{a}^2} + \frac{3(8\pi G)\tilde{\pi}_\phi^2\tilde{\phi}^2}{(2\omega+3)\tilde{a}^4}\right. \\
\left.-3\tilde{a}^2V(\phi)\right] \\
\dot{\phi} \approx c_0(16\pi G\phi)^{\frac{1}{2}}\frac{(16\pi G)\tilde{\pi}_\phi\phi^2}{(2\omega+3)\tilde{a}^3} \\
\dot{\tilde{\pi}}_\phi \approx -c_0(16\pi G\phi)^{\frac{1}{2}}\left[\frac{16\pi G\tilde{\pi}_\phi^2\tilde{\phi}}{(2\omega+3)\tilde{a}^3} + \tilde{a}^3\frac{dV(\phi)}{d\phi}\right].
\end{gathered}$$

$$
\begin{aligned}
g =&-\left(N^2 - N_i N^i\right) dt \otimes dt + N_i\left(dx^i \otimes dt + dt \otimes dx^i\right) \\
&+h_{ij} dx^i \otimes dx^j,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{ADM} =&\sqrt{h}\Big[N\phi\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right) \\
&-\frac{\omega}{N\phi}\left(N^2 h^{ij} D_i\phi D_j\phi - \left(\dot{\phi} - N^i D_i\phi\right)^2\right) \\
&+2K\left(\dot{\phi} - N^i D_i\phi\right) - NU(\phi) + 2h^{ij} D_i N D_j \phi\Big]
\end{aligned}
$$

$$
K_{ij} = \frac{1}{2N}\left(-\frac{\partial h_{ij}}{\partial t} + D_i N_j + D_j N_i\right)
$$

$$
H_T = \int d^3x\left(\lambda^N \pi_N + \lambda_i \pi^i + N\mathcal{H} + N_i \mathcal{H}^i\right)
$$

$$
\begin{aligned}
\mathcal{H} =&\sqrt{h}\Bigg\{\left[-\phi^3 R + \frac{1}{\phi h}\left(\pi^{ij}\pi_{ij} - \frac{\pi_h{}^2}{2}\right)\right] \\
&+\frac{\omega}{\phi} D_i\phi D^i\phi + 2D^i D_i \phi + U(\phi) \\
&+\frac{1}{2h\phi}\left(\frac{1}{3+2\omega}\right)\left(\pi_h - \phi\pi_\phi\right)^2\Bigg\}
\end{aligned}
$$

$$
\mathcal{H}^i = -2D_j \pi^{ji} + \left(D^i\phi\right)\pi_\phi.
$$

$$
\begin{aligned}
\tilde{g} =&-\left(\tilde{N}^2 - \tilde{N}_i \tilde{N}^i\right) dt \otimes dt + \tilde{N}_i\left(dx^i \otimes dt + dt \otimes dx^i\right) \\
&+\tilde{h}_{ij} dx^i \otimes dx^j
\end{aligned}
$$

$$
\begin{aligned}
&\tilde{N} = N(16\pi G\phi)^{\frac{1}{2}}, \tilde{N}_i = N_i(16\pi G\phi), \\
&\tilde{h}_{ij} = (16\pi G\phi) h_{ij}, \tilde{\phi} = \phi
\end{aligned}
$$

$$
\tilde{\mathcal{L}}_{ADM} =\frac{\sqrt{\tilde{h}}}{16\pi G}\left[\tilde{N}\left({}^{(3)}\tilde{R} + \tilde{K}_{ij}\tilde{K}^{ij} - \tilde{K}^2\right) - \frac{2\omega+3}{2\tilde{N}\phi^2}\left(\tilde{N}^2 \tilde{h}^{ij}\tilde{D}_i\phi\tilde{D}_j\phi - \left(\dot{\phi} - \tilde{N}^i\tilde{D}_i\phi\right)^2\right) - 16\pi G\tilde{N}V(\phi)\right]
$$

$$
\tilde{H}_T = \int d^3x\left(\tilde{\lambda}^N \tilde{\pi}_N + \tilde{\lambda}_i \tilde{\pi}^i + \tilde{N}\tilde{\mathcal{H}} + \tilde{N}_i \tilde{\mathcal{H}}^i\right)
$$

$$
\begin{aligned}
\tilde{\mathcal{H}} =& \frac{\sqrt{\tilde{h}}}{16\pi G}\Bigg[-{}^3\tilde{R} + \frac{(16\pi G)^2}{\tilde{h}}\left(\tilde{\pi}^{ij}\tilde{\pi}_{ij} - \frac{\tilde{\pi}_h^2}{2}\right) \\
&+\frac{\left(\omega + \frac{3}{2}\right)}{\phi^2}\tilde{D}_i\phi\tilde{D}^i\phi + \frac{64(\pi G)^2\phi^2}{\tilde{h}\left(\omega + \frac{3}{2}\right)}\tilde{\pi}_\phi^2\Bigg] + \sqrt{\tilde{h}}V(\phi)
\end{aligned}
$$

$$
\tilde{\mathcal{H}}^i = -2\tilde{D}_j\tilde{\pi}^{ji} + \tilde{D}^i\phi\tilde{\pi}_\phi,
$$

$$\tilde{\pi}_N = \frac{\pi_N}{(16\pi G\phi)^{\frac{1}{2}}}, \tilde{\pi}^i = \frac{\pi^i}{(16\pi G\phi)},$$
$$\tilde{\pi}^{ij} = \frac{\pi^{ij}}{(16\pi G\phi)^{\frac{1}{2}}}, \tilde{\pi}_\phi = \frac{1}{\phi}(\phi\pi_\phi - \pi_h),$$

$$\begin{aligned}
\{\tilde{N}(x), \tilde{\pi}_N(x')\} &= \delta^{(3)}(x-x') \\
\{\tilde{N}_i(x), \tilde{\pi}^j(x')\} &= \delta_i^j \delta^{(3)}(x-x') \\
\{\tilde{h}_{ij}(x), \tilde{\pi}^{kl}(x')\} &= \delta_i^k \delta_j^l \delta^{(3)}(x-x') \\
\{\phi(x), \tilde{\pi}_\phi(x')\} &= \delta^{(3)}(x-x')
\end{aligned}$$

$$\begin{aligned}
&\{\tilde{N}(x), \tilde{\pi}^j(x')\} = \{\tilde{N}_i(x), \tilde{\pi}_N(x')\} = \{\tilde{\pi}^i(x), \tilde{\pi}_\phi(x')\} \\
&= \{\tilde{N}(x), \tilde{N}_j(x')\} = \{\tilde{\pi}_N(x), \tilde{\pi}^j(x')\} \\
&= \{\tilde{N}(x), \tilde{h}^{ij}(x')\} = \{\tilde{N}(x), \tilde{\pi}^{ij}(x')\} = \{\tilde{\pi}^i(x), \phi(x')\} \\
&= \{\tilde{N}(x), \phi(x')\} = \{\tilde{N}_i(x), \tilde{h}^{ij}(x')\} = \{\tilde{N}_h(x), \tilde{\pi}^{ij}(x')\} \\
&= \{\tilde{N}_i(x), \phi(x')\} = \{\tilde{\pi}_N(x), \tilde{h}_{ij}(x')\} = \{\tilde{\pi}_N(x), \tilde{\pi}^{ij}(x')\} \\
&= \{\tilde{\pi}^i(x), \tilde{\pi}^{ij}(x')\} = \{\tilde{\pi}^i(x), \tilde{h}_{ij}(x')\} = \{\tilde{\pi}_N(x), \phi(x')\} \\
&= \{\tilde{h}_{ij}(x), \phi(x')\} = \{\tilde{h}_{ij}(x), \tilde{\pi}_\phi(x')\} = \{\tilde{\pi}^{ij}(x), \phi(x')\} \\
&= \{\tilde{\pi}^{ij}(x), \tilde{\pi}_\phi(x')\} = 0,
\end{aligned}$$

$$\begin{aligned}
\{\tilde{N}(x), \tilde{\pi}_\phi(x')\} &= \frac{8\pi G N(x)\delta^{(3)}(x-x')}{\sqrt{16\pi G\phi(x)}} \neq 0 \\
\{\tilde{N}_i(x), \tilde{\pi}_\phi(x')\} &= 16\pi G N_i(x)\delta^{(3)}(x-x') \neq 0 \\
\{\tilde{\pi}_N(x), \tilde{\pi}_\phi(x')\} &= -\frac{8\pi G\pi_N(x)\delta^{(3)}(x-x')}{\sqrt{(16\pi G\phi(x))^3}} \neq 0 \\
\{\tilde{\pi}^i(x), \tilde{\pi}_\phi(x')\} &= -\frac{\pi^i(x)}{(16\pi G\phi^2)}\delta^{(3)}(x-x') \neq 0
\end{aligned}$$

$$N = c_0, N_i = c_i$$

$$N - c_0 \approx 0, N_i - c_i \approx 0$$

$$\pi_N \approx 0, \pi^i \approx 0$$

$$\begin{aligned}
\{N(x) - c_0, \pi_N(x')\} &\approx \delta^{(3)}(x-x') \\
\{N_i(x) - c_i, \pi^j(x')\} &= \delta_i^j \delta^{(3)}(x-x')
\end{aligned}$$

$$\begin{aligned}
&\{N - c_0, \mathcal{H}\} \approx 0, \quad \{N_0 - c_0, \mathcal{H}^i\} = 0 \\
&\{N_i - c_i, \mathcal{H}\} \approx 0, \quad \{N_i - c_i, \mathcal{H}^i\} = 0
\end{aligned}$$

$$\begin{aligned}
&\chi_0 \equiv N - c_0, \chi_i \equiv N_i - c_i \\
&\chi_4 \equiv \pi_N, \chi_{i+4} \equiv \pi_i
\end{aligned}$$

$$\dot{\chi}_0 \approx \{N - c_0, H_T\} \approx 0,$$

$$\dot{\chi}_i \approx \{N_i - c_i, H_T\} \approx 0,$$

$$\dot{\chi}_4 \approx \{\pi_N, H_T\} \approx 0,$$
$$\dot{\chi}_{i+4} \approx \{\pi^i, H_T\} \approx 0,$$

$$C_{\alpha\beta}^{-1} \equiv \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -\mathbb{I} \\ 1 & 0 & 0 & 0 \\ 0 & \mathbb{I} & 0 & 0 \end{pmatrix}$$

$$\{\mathcal{H}_i(x), \mathcal{H}_j(x')\}_{DB} = \{\mathcal{H}_i(x), \mathcal{H}_j(x')\}$$
$$\{\mathcal{H}_i(x), \mathcal{H}(x')\}_{DB} = \{\mathcal{H}_i(x), \mathcal{H}(x')\}$$
$$\{\mathcal{H}(x), \mathcal{H}(x')\}_{DB} = \{\mathcal{H}(x), \mathcal{H}(x')\}$$

$$\begin{aligned} \dot{h}_{ij} &\approx \{h_{ij}, H_T\}_{DB} \\ &\approx D_i N_j + D_j N_i + \frac{2N}{\phi\sqrt{h}}\left(\pi_{ij} - \frac{\pi_h}{2} h_{ij}\right) \\ &+ \frac{N}{\phi\sqrt{h}} \frac{(\pi_h - \phi\pi_\phi)}{(2\omega + 3)} h_{ij}. \end{aligned}$$

$$\begin{aligned} \dot{\pi}^{ij} &\approx \{\dot{\pi}_{ij}, H_T\}_{DB} \\ &\approx -N\sqrt{h}\phi\left({}^{(3)}R^{ij} - \frac{h^{ij}}{2}\,{}^{(3)}R\right) \\ &+\sqrt{h}\left(D^i D^j - h^{ij} D^k D_k\right)(N\phi) \\ &+\frac{N h^{ij}}{2\phi\sqrt{h}}\left(\pi^{ij}\pi_{ij} - \frac{\pi_h^2}{2}\right) + \frac{N}{\phi\sqrt{h}}\pi_h \pi^{ij} \\ &-\sqrt{h}\frac{N\omega}{\phi} D^i\phi D^j\phi - \sqrt{h}\frac{N\omega}{2\phi} h^{ij} D_k\phi D^k\phi \\ &+\sqrt{h} h^{ij} D_k N D^k \phi \\ &+\sqrt{h}\left(D^i N D^j\phi + D^j N D^i \phi\right) - \frac{\sqrt{h}}{2} h^{ij} N U(\phi) \\ &+\frac{h^{ij}}{4\sqrt{h}\phi}\left(\frac{N}{2\omega + 3}\right)\left(\pi_h - \phi\pi_\phi\right)^2 \\ &-\frac{\pi^{ij}}{\sqrt{h}\phi}\left(\frac{N}{2\omega + 3}\right)\left(\pi_h - \phi\pi_\phi\right) \\ &+D_k\left(N^k \pi^{ij}\right) - \frac{2N}{\sqrt{h}\phi}\pi^{iq}\pi_q^j \\ &-\left(D_k N^i\right)\pi^{kj} - \left(D_k N^j\right)\pi^{ki} \end{aligned}$$

$$\begin{aligned} \dot{\phi} &\approx \{\phi, H_T\}_{DB} \\ &\approx N^i D_i \phi - \frac{N}{\sqrt{h}(2\omega + 3)}\left(\pi_h - \phi\pi_\phi\right) \end{aligned}$$

$$
\begin{aligned}
\dot{\pi}_\phi \approx & \{\pi_\phi, H_T\}_{DB} \\
& \approx \sqrt{h} N^{(3)} R + \frac{N}{\phi^2\sqrt{h}}\left(\pi^{ij}\pi_{ij} - \frac{\pi_h^2}{2}\right) \\
& + \frac{\sqrt{h} N\omega}{\phi^2} D_i\phi D^i\phi + 2\sqrt{h} D_i\left(\frac{N\omega}{\phi} D^i\phi\right) \\
& - 2\sqrt{h}\left(D^i D_i\right)(N) - N\sqrt{h}\frac{dU}{d\phi} + D_i\left(N^i \pi_\phi\right) \\
& + \frac{N\left(\pi_h - \phi\pi_\phi\right)^2}{2\sqrt{h}\phi^2(2\omega+3)} + \frac{N\left(\pi_h - \phi\pi_\phi\right)\pi_\phi}{\sqrt{h}\phi(2\omega+3)}
\end{aligned}
$$

$$
\dot{h}_{ij} \approx \frac{2c_0}{\phi\sqrt{h}}\left(\pi_{ij} - \frac{\pi_h}{2} h_{ij}\right) + \frac{c_0}{\phi\sqrt{h}} \frac{\left(\pi_h - \phi\pi_\phi\right)}{(2\omega+3)} h_{ij}
$$

$$
\begin{aligned}
\dot{\pi}^{ij} \approx & -c_0\sqrt{h}\phi\left({}^{(3)}R^{ij} - \frac{h^{ij}}{2}\,{}^{(3)}R\right) \\
& + c_0\sqrt{h}\left(D^i D^j - h^{ij} D^k D_k\right)(\phi) \\
& + \frac{c_0 h^{ij}}{2\phi\sqrt{h}}\left(\pi^{ij}\pi_{ij} - \frac{\pi_h^2}{2}\right) + \frac{c_0}{\phi\sqrt{h}}\pi_h\pi^{ij} \\
& - \sqrt{h}\frac{c_0\omega}{\phi} D^i\phi D^j\phi - \sqrt{h}\frac{c_0\omega}{2\phi} h^{ij} D_k\phi D^k\phi \\
& - \frac{\sqrt{h}}{2} h^{ij} c_0 U(\phi) \\
& + \frac{h^{ij}}{4\sqrt{h}\phi}\left(\frac{c_0}{2\omega+3}\right)\left(\pi_h - \phi\pi_\phi\right)^2 \\
& - \frac{\pi^{ij}}{\sqrt{h}\phi}\left(\frac{c_0}{2\omega+3}\right)\left(\pi_h - \phi\pi_\phi\right) \\
& + c^k D_k\left(\pi^{ij}\right) - \frac{2c_0}{\sqrt{h}\phi}\pi^{iq}\pi_q^j
\end{aligned}
$$

$$
\dot{\phi} \approx c^i D_i\phi - \frac{c_0}{\sqrt{h}(2\omega+3)}\left(\pi_h - \phi\pi_\phi\right)
$$

$$
\begin{aligned}
\dot{\pi}_\phi \approx & c_0\sqrt{h}\,{}^{(3)}R + \frac{c_0}{\phi^2\sqrt{h}}\left(\pi^{ij}\pi_{ij} - \frac{\pi_h^2}{2}\right) \\
& + c_0\frac{\sqrt{h}\omega}{\phi^2} D_i\phi D^i\phi + 2c_0\sqrt{h} D_i\left(\frac{\omega}{\phi} D^i\phi\right) \\
& - c_0\sqrt{h}\frac{dU}{d\phi} + c^i D_i\left(\pi_\phi\right) \\
& + \frac{c_0\left(\pi_h - \phi\pi_\phi\right)^2}{2\sqrt{h}\phi^2(2\omega+3)} + \frac{c_0\left(\pi_h - \phi\pi_\phi\right)\pi_\phi}{\sqrt{h}\phi(2\omega+3)}
\end{aligned}
$$

$$
\tilde{N} = c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{N}_i = c_i(16\pi G\phi).
$$

$$
\tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}} \approx 0, \tilde{N}_i - c_i(16\pi G\phi) \approx 0
$$

$$\left\{\widetilde{N}(x) - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{\pi}_N(x')\right\} \approx \delta^{(3)}(x - x')$$
$$\{\widetilde{N}_i(x) - c_i(16\pi G\phi(x)), \tilde{\pi}^j(x')\} \approx \delta_i^j \delta^{(3)}(x - x')$$

$$\left\{\widetilde{N}(x) - c_0(16\pi G\phi(x))^{\frac{1}{2}}, \widetilde{\mathcal{H}}(x')\right\}$$
$$\approx -\frac{(16\pi G\phi(x))^{\frac{3}{2}} c_0 \tilde{\pi}_\phi(x)\delta^{(3)}(x - x')}{4\sqrt{\tilde{h}(x)}\left(\omega + \frac{3}{2}\right)}$$
$$\left\{\widetilde{N}(x) - c_0(16\pi G\phi(x))^{\frac{1}{2}}, \widetilde{\mathcal{H}}^i(x')\right\}$$
$$\approx -\frac{8\pi G\phi(x) c_0 D^i\phi(x)\delta^{(3)}(x - x')}{(16\pi G\phi(x))^{\frac{1}{2}}}$$
$$\{\widetilde{N}_i(x) - c_i(16\pi G\phi(x)), \widetilde{\mathcal{H}}(x')\}$$
$$\approx -\frac{128 c_i \tilde{\pi}_\phi(x)\delta^{(3)}(x - x')(\pi G\phi(x))^2}{\sqrt{\tilde{h}(x)}\left(\omega + \frac{3}{2}\right)}$$
$$\{\widetilde{N}_i(x) - c_i(16\pi G\phi(x)), \widetilde{\mathcal{H}}^j(x')\}$$
$$\approx -(16\pi G)c_i D^j\phi(x)\delta^{(3)}(x - x')$$

$$\widetilde{\mathcal{H}}' \equiv \widetilde{\mathcal{H}} + \eta_N \tilde{\pi}_N + \gamma_i \tilde{\pi}^i$$
$$\widetilde{\mathcal{H}}^{i\prime} \equiv \widetilde{\mathcal{H}}^i + \eta^i \tilde{\pi}_N + \rho_k^i \tilde{\pi}^k$$

$$\eta_N \equiv \frac{(16\pi G\phi(x))^{\frac{3}{2}} c_0 \tilde{\pi}_\phi(x)}{4\sqrt{\tilde{h}(x)}\left(\omega + \frac{3}{2}\right)}$$
$$\eta^i \equiv \frac{8\pi G\phi(x) c_0 D^i\phi(x)}{(16\pi G\phi(x))^{\frac{1}{2}}}$$
$$\gamma_i \equiv \frac{128 c_i \tilde{\pi}_\phi(x)(\pi G\phi(x))^2}{\sqrt{\tilde{h}(x)}\left(\omega + \frac{3}{2}\right)}$$
$$\rho_k^i \equiv (16\pi G)c_k D^i\phi(x)$$

$$\tilde{\chi}_0 \equiv \widetilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{\chi}_i \equiv \widetilde{N}_i - c_i(16\pi G\phi)$$
$$\tilde{\chi}_4 \equiv \tilde{\pi}_N, \tilde{\chi}_{i+4} \equiv \tilde{\pi}^i$$

$$\tilde{C}_{\alpha\beta}^{-1} \equiv \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -\mathbb{I} \\ 1 & 0 & 0 & 0 \\ 0 & \mathbb{I} & 0 & 0 \end{pmatrix}$$

$$\widetilde{H}_T' = \int d^3x\left(\tilde{\lambda}^N \tilde{\pi}_N + \tilde{\lambda}_i \tilde{\pi}^i + \widetilde{N}\widetilde{\mathcal{H}}' + \widetilde{N}_i \widetilde{\mathcal{H}}^{i\prime}\right)$$

$$\dot{\tilde{\chi}}_0(x) \approx \left\{\widetilde{N}(x) - c_0(16\pi G\phi(x))^{\frac{1}{2}}, \widetilde{H}_T'\right\} \approx 0$$

$$\tilde{\lambda}^N(x) \approx 0$$

$$\dot{\tilde{\chi}}_i(x) \approx \{\tilde{N}_i(x) - c_i(16\pi G\phi(x)), \tilde{H}'_T\} \approx 0$$

$$\tilde{\lambda}_i(x) \approx 0.$$

$$\{\tilde{\mathcal{H}}'_i(x), \tilde{\mathcal{H}}'_j(x')\}_{DB} = \{\tilde{\mathcal{H}}'_i(x), \tilde{\mathcal{H}}'_j(x')\},$$
$$\{\tilde{\mathcal{H}}'_i(x), \tilde{\mathcal{H}}'(x')\}_{DB} = \{\tilde{\mathcal{H}}'_i(x), \tilde{\mathcal{H}}'(x')\},$$
$$\{\tilde{\mathcal{H}}'(x), \tilde{\mathcal{H}}'(x')\}_{DB} = \{\tilde{\mathcal{H}}'(x), \tilde{\mathcal{H}}'(x')\}.$$

$$\begin{aligned}\dot{\tilde{h}}_{ij} &\approx \{\tilde{h}_{ij}, \tilde{H}'_{ADM}\}_{DB} \\ &\approx \tilde{D}_i\tilde{N}_j + \tilde{D}_j\tilde{N}_i + \frac{(32\pi G)\tilde{N}}{\sqrt{\tilde{h}}}\left(\tilde{\pi}_{ij} - \frac{\tilde{\pi}_h}{2}\tilde{h}_{ij}\right)\end{aligned}$$

$$\begin{aligned}\dot{\tilde{\pi}}^{ij} &\approx \{\tilde{\pi}^{ij}, \tilde{H}'_{ADM}\}_{DB} \\ &\approx -\frac{\tilde{N}\sqrt{\tilde{h}}}{16\pi G}\left({}^{(3)}\tilde{R}^{ij} - \frac{1}{2}(3)\tilde{R}^{ij}\right) \\ &\quad + \frac{(16\pi G)\tilde{N}}{2\sqrt{\tilde{h}}}\tilde{h}^{ij}\left(\tilde{\pi}^{pk}\tilde{\pi}_{pk} - \frac{\tilde{\pi}_h^2}{2}\right) \\ &\quad -2\frac{(16\pi G)\tilde{N}}{\sqrt{\tilde{h}}}\left(\tilde{\pi}^{ik}\tilde{\pi}_k^j - \frac{1}{2}\tilde{\pi}_h\tilde{\pi}^{ij}\right) - \tilde{h}^{ij}\frac{\sqrt{\tilde{h}}\tilde{N}}{2}V(\phi) \\ &\quad + \frac{\sqrt{\tilde{h}}}{16\pi G}\left(\tilde{D}^i\tilde{D}^j\tilde{N} - \tilde{h}^{ij}\tilde{D}^k\tilde{D}_k\tilde{N}\right) \\ &\quad + \sqrt{\tilde{h}}\tilde{D}_k\left(\frac{1}{\sqrt{\tilde{h}}}\tilde{N}^k\tilde{\pi}^{ij}\right) - \tilde{\pi}^{ki}\tilde{D}_k\tilde{N}^j - \tilde{\pi}^{kj}\tilde{D}_k\tilde{N}^i\end{aligned}$$

$$\begin{aligned}\dot{\phi} &\approx \{\phi, \tilde{H}'_{ADM}\}_{DB} \\ &\approx \frac{8(\pi G)\phi^2}{\sqrt{\tilde{h}}\left(\omega + \frac{3}{2}\right)}\tilde{\pi}_\phi\tilde{N} + \tilde{N}_i\tilde{D}^i\phi\end{aligned}$$

$$\begin{aligned}\dot{\tilde{\pi}}_\phi &\approx \{\tilde{\pi}_\phi, \tilde{H}'_{ADM}\}_{DB} \\ &\approx -\frac{\sqrt{\tilde{h}}}{8\pi G}\left(\omega + \frac{3}{2}\right)\frac{\tilde{N}}{\phi^3}\tilde{D}^i\phi\tilde{D}_i\phi \\ &\quad + \frac{\sqrt{\tilde{h}}}{8\pi G}\frac{\left(\omega + \frac{3}{2}\right)}{\phi^2}\tilde{D}^i\tilde{N}\tilde{D}_i\phi \\ &\quad + \frac{\sqrt{\tilde{h}}}{8\pi G}\left(\omega + \frac{3}{2}\right)\frac{\tilde{N}}{\phi^2}\tilde{D}^i\tilde{D}_i\phi \\ &\quad - \frac{8\tilde{N}(\pi G)\phi\tilde{\pi}_\phi^2}{\sqrt{\tilde{h}}\left(\omega + \frac{3}{2}\right)} + \tilde{D}^i(\tilde{N}_i\tilde{\pi}_\phi) \\ &\quad - \sqrt{\tilde{h}}\tilde{N}\frac{dV(\phi)}{d\phi}\end{aligned}$$

$$\dot{\tilde{h}}_{ij} \approx 16\pi G\left(c_j \widetilde{D}_i\phi + c_i \widetilde{D}_j\phi\right),$$

$$\begin{aligned}
\dot{\tilde{\pi}}^{ij} \approx &-\frac{c_0(16\pi G\phi)^{\frac{1}{2}}\sqrt{\tilde{h}}}{16\pi G}\left({}^{(3)}\tilde{R}^{ij} - \frac{1}{2}(3)\tilde{R}^{ij}\right) \\
&+\frac{(16\pi G)c_0(16\pi G\phi)^{\frac{1}{2}}}{2\sqrt{\tilde{h}}}\tilde{h}^{ij}\left(\tilde{\pi}^{pk}\tilde{\pi}_{pk} - \frac{\tilde{\pi}_h^2}{2}\right) \\
&-2\frac{(16\pi G)c_0(16\pi G\phi)^{\frac{1}{2}}}{\sqrt{\tilde{h}}}\left(\tilde{\pi}^{ik}\tilde{\pi}_k^j - \frac{1}{2}\tilde{\pi}_h\tilde{\pi}^{ij}\right) \\
&-\tilde{h}^{ij}\frac{\sqrt{\tilde{h}}c_0(16\pi G\phi)^{\frac{1}{2}}}{2}V(\phi) \\
&+\frac{c_0\sqrt{\tilde{h}}}{2(16\pi G\phi)^{\frac{1}{2}}}\left[-\frac{\widetilde{D}^i\phi\widetilde{D}^j\phi}{2\phi} + \widetilde{D}^i\widetilde{D}^j\phi\right. \\
&\left.-\tilde{h}^{ij}\left(-\frac{\widetilde{D}^k\phi\widetilde{D}_k\phi}{2\phi} + \widetilde{D}^k\widetilde{D}_k\phi\right)\right] \\
&+(16\pi G)\sqrt{\tilde{h}}c^k\widetilde{D}_k\left(\frac{1}{\sqrt{\tilde{h}}}\phi\tilde{\pi}^{ij}\right) \\
&-(16\pi G)\tilde{\pi}^{ki}c^j\widetilde{D}_k\phi - (16\pi G)\tilde{\pi}^{kj}c^i\widetilde{D}_k\phi
\end{aligned}$$

$$\dot{\phi} \approx \frac{c_0(16\pi G\phi)^{\frac{3}{2}}}{2\sqrt{\tilde{h}}\left(\omega + \frac{3}{2}\right)}\tilde{\pi}_\phi\phi + (16\pi G\phi)c_i\widetilde{D}^i\phi$$

$$\begin{aligned}
\dot{\tilde{\pi}}_\phi \approx &-\frac{c_0\sqrt{\tilde{h}}}{(16\pi G\phi)^{\frac{1}{2}}\phi^2}\left(\omega + \frac{3}{2}\right)\widetilde{D}^i\phi\widetilde{D}_i\phi \\
&+\frac{2c_0\sqrt{\tilde{h}}}{(16\pi G\phi)^{\frac{1}{2}}\phi}\left(\omega + \frac{3}{2}\right)\widetilde{D}^i\widetilde{D}_i\phi \\
&-\frac{c_0(16\pi G\phi)^{\frac{3}{2}}\tilde{\pi}_\phi^2}{2\sqrt{\tilde{h}}\left(\omega + \frac{3}{2}\right)} + 16\pi G c_i\widetilde{D}^i\left(\phi\tilde{\pi}_\phi\right) \\
&-c_0\sqrt{\tilde{h}}(16\pi G\phi)^{\frac{1}{2}}\frac{dV(\phi)}{d\phi}
\end{aligned}$$

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}q^2$$

$$q = \sqrt{\frac{2P}{m\omega}}\sin\ Q$$

$$p = \sqrt{2m\omega P}\cos\ Q$$

$$H = \omega P$$

$$P = \frac{E}{\omega}$$

$$\dot{Q} = \frac{\partial H}{\partial P} = \omega$$

$$Q = \omega t + \alpha$$

$$q(t) = \sqrt{\frac{2E}{m\omega^2}}\sin(\omega t + \alpha)$$

$$\begin{aligned}
&\tilde{N}^* = N, \tilde{\pi}_{N^*} = \pi_N \\
&\tilde{N}_i^* = N_i, \tilde{\pi}^{*i} = \pi^i \\
&\tilde{h}_{ij}^* = (16\pi G\phi)h_{ij}, \tilde{\pi}^{*ij} = \frac{\pi^{ij}}{(16\pi G\phi)^{\frac{1}{2}}} \\
&\tilde{\phi}^* = \phi, \tilde{\pi}_\phi^* = \frac{1}{\phi}\left(\phi\pi_\phi - \pi_h\right)
\end{aligned}$$

$$ds^2 = -dt^2 + \lambda^2 dx^2$$

$$C_\phi \equiv \frac{1}{2}a\pi_a - \phi\pi_\phi$$

$$\begin{aligned}
H_{\mathrm{T}}^{(-3/2)} &= N\left[-\frac{\pi_a^2}{24a\phi} + a^3U(\phi)\right] + \lambda_N\pi_N + \lambda_\phi C_\phi \\
&= NH^{(-3/2)} + \lambda_N\pi_N + \lambda_\phi C_\phi
\end{aligned}$$

$$\dot{\chi}_1 \approx \left\{N - c_0, H_{\mathrm{T}}^{(-3/2)}\right\} \approx 0,$$

$$\dot{\chi}_2 \approx \left\{\pi_N, H_{\mathrm{T}}^{(-3/2)}\right\} \approx 0$$

$$\begin{aligned}
\dot{a} &\approx \left\{a, H_{\mathrm{T}}^{(-3/2)}\right\}_{DB} \approx -\frac{N\pi_a}{12a\phi} + \frac{\lambda_\phi a}{2}, \\
\dot{\pi}_a &\approx \left\{\pi_a, H_{\mathrm{T}}^{(-3/2)}\right\}_{DB} \\
&\approx -\frac{N\pi_a^2}{24\phi a^2} - 3Na^2U(\phi) - \frac{\lambda_\phi\pi_a}{2}, \\
\dot{\phi} &\approx \left\{\phi, H_{\mathrm{T}}^{(-3/2)}\right\}_{DB} \approx -\lambda_\phi\phi, \\
\dot{\pi}_\phi &\approx \left\{\pi_\phi, H_{\mathrm{T}}^{(-3/2)}\right\}_{DB} \\
&\approx -N\frac{\pi_a^2}{24a\phi^2} - Na^3\frac{dU(\phi)}{d\phi} + \lambda_\phi\pi_\phi \\
&\approx -N\frac{\pi_a^2}{24a\phi^2} - \frac{2Na^3U(\phi)}{\phi} + \lambda_\phi\pi_\phi,
\end{aligned}$$

$$\phi\frac{dU(\phi)}{d\phi} = 2U(\phi),$$

$$\dot{a} \approx -\frac{c_0\pi_a}{12a\phi} + \frac{\lambda_\phi a}{2}$$

$$\dot{\pi}_a \approx -\frac{c_0\pi_a^2}{24\phi a^2} - 3c_0a^2U(\phi) - \frac{\lambda_\phi\pi_a}{2}$$

$$\dot{\phi} \approx -\lambda_\phi\phi$$

$$\dot{\pi}_\phi \approx -c_0\frac{\pi_a^2}{24a\phi^2} - \frac{2c_0a^3U(\phi)}{\phi} + \lambda_\phi\pi_\phi$$

$$\begin{aligned}\widetilde{H}_T^{(-3/2)} &= \widetilde{N}\tilde{a}^3\left[-\frac{2\pi G\tilde{\pi}_a^2}{3\tilde{a}^4} + V(\tilde{\phi})\right] + \tilde{\lambda}_N\tilde{\pi}_N + \lambda_\phi\tilde{C}_\phi \\ &= \widetilde{N}\widetilde{H}^{(-3/2)} + \tilde{\lambda}_N\tilde{\pi}_N + \tilde{\lambda}_\phi\tilde{C}_\phi\end{aligned}$$

$$\tilde{C}_\phi = -\phi\tilde{\pi}_\phi.$$

$$\widetilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}} \approx 0,$$

$$\left\{\widetilde{H}^{(-3/2)}, \widetilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}\right\} = 0$$

$$\left\{\widetilde{H}^{(-3/2)}, \tilde{C}_\phi\right\} = -\tilde{a}^3\phi\frac{dV(\phi)}{d\phi} = 0$$

$$\left\{\tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{C}_\phi\right\} = \frac{c_0(16\pi G\phi)^{\frac{1}{2}}}{2}$$

$$\tilde{C}_\phi - \frac{c_0(16\pi G\phi)^{\frac{1}{2}}}{2}\tilde{\pi}_N,$$

$$\left\{\widetilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{C}_\phi - \frac{c_0(16\pi G\phi)^{\frac{1}{2}}}{2}\tilde{\pi}_N\right\} \approx 0$$

$$\begin{aligned}\widetilde{H}_T'^{(-3/2)} &= \widetilde{N}\widetilde{H}^{(-3/2)} + \tilde{\lambda}_N\tilde{\pi}_N \\ &\quad + \tilde{\lambda}_\phi\left[-\phi\tilde{\pi}_\phi - \frac{c_0(16\pi G\phi)^{\frac{1}{2}}}{2}\tilde{\pi}_N\right].\end{aligned}$$

$$\dot{\tilde{\chi}}_0 \approx \left\{\widetilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \widetilde{H}_T'^{(-3/2)}\right\} \approx 0$$

$$\dot{\tilde{\chi}}_1 \approx \left\{\tilde{\pi}_N, \widetilde{H}_T'^{(-3/2)}\right\} \approx 0$$

$$\dot{\tilde{a}} \approx \left\{\tilde{a}, \tilde{H}_T'^{(-3/2)}\right\}_{DB} \approx -\tilde{N}\frac{4\pi G\tilde{\pi}_a}{3\tilde{a}}$$

$$\dot{\tilde{\pi_a}} \approx \left\{\tilde{\pi}_a, \tilde{H}_T'^{(-3/2)}\right\}_{DB}$$

$$\approx \tilde{N}\left[-\frac{(2\pi G)\tilde{\pi}_a^2}{3\tilde{a}^2} - 3\tilde{a}^2 V(\phi)\right]$$

$$\dot{\tilde{\phi}} \approx \left\{\tilde{\phi}, \tilde{H}_T'^{(-3/2)}\right\}_{DB} \approx -\tilde{\lambda}_\phi\tilde{\phi}$$

$$\dot{\tilde{\pi}}_\phi \approx \left\{\tilde{\pi}_\phi, \tilde{H}_T'^{(-3/2)}\right\}_{DB}$$

$$\approx \left\{\tilde{\pi}_\phi, \tilde{H}_T'^{(-3/2)}\right\}$$

$$-\left\{\tilde{\pi}_\phi, \tilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}\right\} C_{01}^{-1}\left\{\tilde{\pi}_N, \tilde{H}_T'^{(-3/2)}\right\}$$

$$\approx -\tilde{N}\tilde{a}^3\frac{dV(\phi)}{d\phi} + \tilde{\lambda}_\phi\tilde{\pi}_\phi - \frac{\tilde{\lambda}_\phi c_0}{2}\frac{16\pi G}{(16\pi G\phi)^{\frac{1}{2}}}\tilde{\pi}_N$$

$$+\frac{c_0}{2}\frac{16\pi G}{(16\pi G\phi)^{\frac{1}{2}}}\tilde{H}^{(-3/2)}$$

$$\dot{\tilde{a}} \approx -c_0(16\pi G\phi)^{\frac{1}{2}}\frac{4\pi G\tilde{\pi}_a}{3\tilde{a}},$$

$$\dot{\tilde{\pi}}_a \approx c_0(16\pi G\phi)^{\frac{1}{2}}\left[-\frac{(2\pi G)\tilde{\pi}_a^2}{3\tilde{a}^2} - 3\tilde{a}^2 V(\phi)\right],$$

$$\dot{\tilde{\phi}} \approx -\tilde{\lambda}_\phi\tilde{\phi},$$

$$\dot{\tilde{\pi}}_\phi \approx \tilde{\lambda}_\phi\tilde{\pi}_\phi.$$

$$H_T^{(-3/2)} = \int d^3x\big(\lambda^N\pi_N + \lambda_i\pi^i + \lambda_\phi C_\phi + N\mathcal{H}^{(-3/2)} + N_i\mathcal{H}^{i(-3/2)}\big)$$

$$C_\phi = \pi^{ij}h_{ij} - \phi\pi_\phi$$

$$\mathcal{H}^{(-3/2)} = \sqrt{h}\left\{\left[-\phi^3 R + \frac{1}{\phi h}\left(\pi^{ij}\pi_{ij} - \frac{\pi_h}{2}^2\right)\right] - \frac{3}{2\phi}D_i\phi D^i\phi + 2D^i D_i\phi + U(\phi)\right\}$$

$$\mathcal{H}^{i(-3/2)} = -2D_j\pi^{ji} + D^i\phi\pi_\phi.$$

$$N = c_0\ N_i = c_i$$

$$\chi_0 \equiv N - c_0 \approx 0, \chi_i \equiv N_i - c_i \approx 0$$

$$\chi_4 \equiv \pi_N \approx 0, \chi_{i+4} \equiv \pi^i \approx 0$$

$$\{N(x) - c_0, \pi_N(x')\} \approx \delta^{(3)}(x - x')$$

$$\left\{N_i(x) - c_i, \pi^j(x')\right\} = \delta_i^j\delta^{(3)}(x - x')$$

$$\dot{\chi}_0 \approx \{N - c_0, H_T\} \approx 0,$$

$$\dot{\chi}_i \approx \{N_i - c_i, H_T\} \approx 0,$$

$$\dot{\chi}_4 \ \approx \{\pi_N, H_T\} \approx 0,$$
$$\dot{\chi}_i + 4 \ \approx \{\pi^i, H_T\} \approx 0,$$

$$\begin{aligned}\dot{h}_{ij} &\approx \{h_{ij}, H_T\}_{DB} \\ &\approx \lambda_\phi h_{ij} + \frac{2c_0}{\phi\sqrt{h}}\left(\pi_{ij} - \frac{\pi_h}{2}h_{ij}\right),\end{aligned}$$

$$\begin{aligned}\dot{\pi}^{ij} &\approx \{\pi^{ij}, H_T\}_{DB} \\ &\approx -\lambda_\phi \pi^{ij} - c_0\sqrt{h}\phi\left({}^{(3)}R^{ij} - \frac{h^{ij}}{2}\,{}^{(3)}R\right) \\ &+ c_0\sqrt{h}\left(D^iD^j - h^{ij}D^kD_k\right)(\phi) \\ &+ \frac{c_0h^{ij}}{2\phi\sqrt{h}}\left(\pi^{ij}\pi_{ij} - \frac{\pi_h^2}{2}\right) + \frac{c_0}{\phi\sqrt{h}}\pi_h\pi^{ij} \\ &- \sqrt{h}\frac{c_0\omega}{\phi}D^i\phi D^j\phi - \sqrt{h}\frac{c_0\omega}{2\phi}h^{ij}D_k\phi D^k\phi \\ &- \frac{\sqrt{h}}{2}h^{ij}c_0U(\phi) \\ &+ c^kD_k\left(\pi^{ij}\right) - \frac{2c_0}{\sqrt{h}\phi}\pi^{iq}\pi_q^j\end{aligned}$$

$$\dot{\phi} \approx \{\phi, H_T\}_{DB} \approx -\lambda_\phi\phi + c^iD_i\phi$$

$$\begin{aligned}\dot{\pi}_\phi &\approx \{\pi_\phi, H_T\}_{DB} \\ &\approx \lambda_\phi\pi_\phi + c_0\sqrt{h}\,{}^{(3)}R + \frac{c_0}{\phi^2\sqrt{h}}\left(\pi^{ij}\pi_{ij} - \frac{\pi_h^2}{2}\right) \\ &+ c_0\frac{\sqrt{h}\omega}{\phi^2}D_i\phi D^i\phi + 2c_0\sqrt{h}D_i\left(\frac{\omega}{\phi}D^i\phi\right) \\ &- c_0\sqrt{h}\frac{dU}{d\phi} + c^iD_i\left(\pi_\phi\right)\end{aligned}$$

$$\begin{aligned}\widetilde{H}_T^{(-3/2)} = &\int d^3x\big(\tilde{\lambda}^N\tilde{\pi}_N + \tilde{\lambda}^i\tilde{\pi}_i + \tilde{\lambda}_\phi\tilde{C}_\phi \\ &+ \widetilde{N}\widetilde{\mathcal{H}}^{(-3/2)} + \widetilde{N}^i\widetilde{\mathcal{H}}_i^{(-3/2)}\big)\end{aligned}$$

$$\tilde{C}_\phi = -\phi\tilde{\pi}_\phi$$

$$\begin{aligned}\widetilde{\mathcal{H}}^{(-3/2)} = &\frac{\sqrt{\tilde{h}}}{16\pi G}\left[-{}^3\tilde{R} + \frac{(16\pi G)^2}{\tilde{h}}\left(\tilde{\pi}^{ij}\tilde{\pi}_{ij} - \frac{\tilde{\pi}_h^2}{2}\right)\right] \\ &+ \sqrt{\tilde{h}}V(\phi)\end{aligned}$$

$$\widetilde{\mathcal{H}}_i^{(-3/2)} = -2\widetilde{D}_j\tilde{\pi}_i^j$$

$$\widetilde{N} = c_0(16\pi G\phi)^{\frac{1}{2}}, \widetilde{N}_i = c_i(16\pi G\phi),$$

$$\widetilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}} \approx 0, \widetilde{N}_i - c_i(16\pi G\phi) \approx 0.$$

$$\left\{\widetilde{N}(x) - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{\pi}_N(x')\right\} \approx \delta^{(3)}(x - x')$$
$$\{\widetilde{N}_i(x) - c_i(16\pi G\phi(x)), \tilde{\pi}^j(x')\} \approx \delta_i^j \delta^{(3)}(x - x')$$

$$\left\{\widetilde{N}(x) - c_0(16\pi G\phi)^{\frac{1}{2}}, \widetilde{\mathcal{H}}^{(-3/2)}(x')\right\} \approx 0,$$
$$\{\widetilde{N}_i(x) - c_i(16\pi G\phi(x)), \widetilde{\mathcal{H}}^{(-3/2)}(x')\} \approx 0$$
$$\left\{\widetilde{N}(x) - c_0(16\pi G\phi)^{\frac{1}{2}}, \widetilde{\mathcal{H}}_i^{(-3/2)}(x')\right\} \approx 0,$$
$$\left\{\widetilde{N}_i(x) - c_i(16\pi G\phi(x)), \widetilde{\mathcal{H}}_i^{(-3/2)}(x')\right\} \approx 0.$$

$$\left\{\widetilde{N}(x) - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{C}_\phi\right\} \approx \frac{c_0(16\pi G\phi)^{\frac{1}{2}}}{2},$$
$$\{\widetilde{N}_i(x) - c_i(16\pi G\phi(x)), \tilde{C}_\phi\} \approx c_i(16\pi G\phi),$$

$$\tilde{C}'_\phi \equiv -\phi\tilde{\pi}_\phi - \frac{c_0(16\pi G\phi)^{\frac{1}{2}}}{2}\tilde{\pi}_N - c_i(16\pi G\phi)\tilde{\pi}^i$$

$$\widetilde{H}_T'^{(-3/2)} = \int d^3x\big(\tilde{\lambda}^N\tilde{\pi}_N + \tilde{\lambda}^i\tilde{\pi}_i + \tilde{\lambda}_\phi\tilde{C}'_\phi + \widetilde{N}\widetilde{\mathcal{H}}^{(-3/2)} + \widetilde{N}^i\widetilde{\mathcal{H}}_i^{(-3/2)}\big)$$

$$\tilde{\chi}_0 \equiv \widetilde{N} - c_0(16\pi G\phi)^{\frac{1}{2}}, \tilde{\chi}_i \equiv \widetilde{N}_i - c_i(16\pi G\phi)$$
$$\tilde{\chi}_4 \equiv \tilde{\pi}_N, \tilde{\chi}_{i+4} \equiv \tilde{\pi}^i$$

$$\dot{\tilde{\chi}}_0 \approx \left\{\widetilde{N}(x) - c_0(16\pi G\phi(x))^{\frac{1}{2}}, \widetilde{H}'_T(-3/2)\right\} \approx 0$$

$$\dot{\tilde{\chi}}_i \approx \{\widetilde{N}_i(x) - c_i(16\pi G\phi(x)), \widetilde{H}'_T(-3/2)\} \approx 0$$

$$\tilde{C}^{-1}_{\alpha\beta} \equiv \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -\mathbb{I} \\ 1 & 0 & 0 & 0 \\ 0 & \mathbb{I} & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}\dot{\tilde{h}}_{ij} &\approx \{\tilde{h}_{ij}, \widetilde{H}'_T(-3/2)\}_{DB} \\ &\approx 16\pi G\big(c_j\widetilde{D}_i\phi + c_i\widetilde{D}_j\phi\big) \\ &\quad + \frac{(32\pi G)c_0(16\pi G\phi)^{\frac{1}{2}}}{\sqrt{\tilde{h}}}\left(\tilde{\pi}_{ij} - \frac{\tilde{\pi}_h}{2}\tilde{h}_{ij}\right)\end{aligned}$$

$$
\begin{aligned}
\dot{\tilde{\pi}}^{ij} \approx & \left\{\tilde{\pi}^{ij}, \widetilde{H}'_T(-3/2)\right\}_{DB} \\
\approx & -\frac{c_0(16\pi G\phi)^{\frac{1}{2}}\sqrt{\tilde{h}}}{16\pi G}\left({}^{(3)}\tilde{R}^{ij} - \frac{1}{2}(3)\tilde{R}^{ij}\right) \\
& +\frac{(16\pi G)c_0(16\pi G\phi)^{\frac{1}{2}}}{2\sqrt{\tilde{h}}}\tilde{h}^{ij}\left(\tilde{\pi}^{pk}\tilde{\pi}_{pk} - \frac{\tilde{\pi}_h^2}{2}\right) \\
& -2\frac{(16\pi G)c_0(16\pi G\phi)^{\frac{1}{2}}}{\sqrt{\tilde{h}}}\left(\tilde{\pi}^{ik}\tilde{\pi}_k^j - \frac{1}{2}\tilde{\pi}_h\tilde{\pi}^{ij}\right) \\
& -\tilde{h}^{ij}\frac{\sqrt{\tilde{h}}c_0(16\pi G\phi)^{\frac{1}{2}}}{2}V(\phi) \\
& +\frac{c_0\sqrt{\tilde{h}}}{2(16\pi G\phi)^{\frac{1}{2}}}\left[-\frac{\widetilde{D}^i\phi\widetilde{D}^j\phi}{2\phi} + \widetilde{D}^i\widetilde{D}^j\phi\right. \\
& \left. -\tilde{h}^{ij}\left(-\frac{\widetilde{D}^k\phi\widetilde{D}_k\phi}{2\phi} + \widetilde{D}^k\widetilde{D}_k\phi\right)\right] \\
& +(16\pi G)\sqrt{\tilde{h}}c^k\widetilde{D}_k\left(\frac{1}{\sqrt{\tilde{h}}}\phi\tilde{\pi}^{ij}\right) \\
& -(16\pi G)\tilde{\pi}^{ki}c^j\widetilde{D}_k\phi - (16\pi G)\tilde{\pi}^{kj}c^i\widetilde{D}_k\phi
\end{aligned}
$$

$$
\dot{\phi} \approx \left\{\phi, \widetilde{H}_T'^{(-3/2)}\right\}_{DB} \approx -\tilde{\lambda}_\phi\phi
$$

$$
\dot{\tilde{\pi}}_\phi \approx \left\{\tilde{\pi}_\phi, \widetilde{H}_T'^{(-3/2)}\right\}_{DB} \approx \tilde{\lambda}_\phi\tilde{\pi}_\phi
$$

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