Ciencia Latina Revista Científica Multidisciplinar, Ciudad de México, México.
ISSN 2707-2207 / ISSN 2707-2215 (en línea), septiembre-octubre 2025,
Volumen 9, Número 5.

https://doi.org/10.37811/cl_rcm.v9i5

SUPERGRAVEDAD CUÁNTICA RELATIVISTA. SUPERSIMETRÍAS ADS/SCFT EN SUPERESPACIOS DE WEYL –DE SITTER Y ANTI – DE SITTER. VOLUMEN I

RELATIVISTIC QUANTUM SUPERGRAVITY. ADS/SCFT SUPERSYMMETRIES IN WEYL–DE SITTER AND ANTI–DE SITTER SUPERSPACES. VOLUME I

Manuel Ignacio Albuja Bustamante
Investigador Independiente

DOI: https://doi.org/10.37811/cl_rcm.v9i5.19763

SUPERGRAVEDAD CUÁNTICA RELATIVISTA. SUPERSIMETRÍAS ADS/SCFT EN SUPERESPACIOS DE WEYL –DE SITTER Y ANTI – DE SITTER. VOLUMEN I

Manuel Ignacio Albuja Bustamante[1]

ignaciomanuelalbujabustamante@gmail.com

https://orcid.org/0009-0005-0115-767X

Investigador Independiente

RESUMEN

En recientes trabajos, este autor ha esbozado planteamientos generales a propósito de la supergravedad cuántica relativista, la misma que, en sentido estricto, se tiene como una teoría del todo. La supergravedad cuántica relativista, es un pilar esencial en la Teoría Cuántica de Campos Relativistas o Curvos, planteada por este investigador, en la medida en que, unifica la relatividad general y la relatividad especial con la mecánica cuántica. Sin embargo, a diferencia de los postulados que son propios de la supergravedad, la supergravedad cuántica relativista, usa la supersimetría en grupos de gauge infinitos (supersimetrías de gauge) con la finalidad de integrar la gravedad a escala subatómica, a partir de la existencia de superpartículas, capaces de deformar el espacio – tiempo cuántico, en dimensiones más altas (supersimetrías de gauge en dimensiones D), lo que ocurre, por interacción de la propia superpartícula a razón de su masa y energía extremadamente densas, o por interacción de la superpartícula con el supergravitón, incluyendo, aquellas interacciones con regiones de antimateria, lo que supone, la permeabilización del campo cuántico repercutido por el campo supergravitónico. En este trabajo, se robustecerá este concepto esencial que sostiene la Teoría Cuántica de Campos Relativistas o Curvos.

Palabras clave: Supersimetría, Supergravedad en D dimensiones, relatividad general, grupos de gauge, superpartículas, supergravitón.

[1] Autor principal
Correspondencia: ignaciomanuelalbujabustamante@gmail.com

RELATIVISTIC QUANTUM SUPERGRAVITY. ADS/SCFT SUPERSYMMETRIES IN WEYL–DE SITTER AND ANTI–DE SITTER SUPERSPACES. VOLUME I

ABSTRACT

In recent works, this author has outlined general approaches to relativistic quantum supergravity, which, strictly speaking, is considered a theory of everything. Relativistic quantum supergravity is an essential pillar in the Quantum Theory of Relativistic or Curved Fields, proposed by this researcher, insofar as it unifies general relativity and special relativity with quantum mechanics. However, unlike the postulates that are typical of supergravity, relativistic quantum supergravity uses supersymmetry in infinite gauge groups (gauge supersymmetries) in order to integrate gravity at the subatomic scale, based on the existence of superparticles, capable of deforming quantum space-time, in higher dimensions (gauge supersymmetries in D dimensions). which occurs by interaction of the superparticle itself due to its extremely dense mass and energy, or by interaction of the superparticle with the supergraviton, including those interactions with regions of antimatter, which supposes the permeabilization of the supergravitonic field, which supposes the permeabilization of the quantum field impacted by the supergravitonic field. In this work, this essential concept that explains the Quantum Theory of Relativistic or Curved Fields will be strengthened.

Keywords: Supersymmetry, Supergravity in D dimensions, general relativity, gauge groups, superparticles, supergraviton.

Artículo recibido 11 agosto 2025

Aceptado para publicación: 13 setiembre 2025

INTRODUCCIÓN.

A diferencia de otras líneas de investigación relacionadas a la supergravedad, en este trabajo, abordaré la supergravedad a escala cuántica y en dimensión relativista, en D dimensiones, sin regulación por campos de gauge finitos, sino en contrario, suponiendo que, a escala cuántica, la supersimetría de gauge, está dada en infinitas dimensiones, a propósito de la distorsión del espacio – tiempo cuántico, en dimensiones más altas, a causa de la interacción de la superpartícula, por su propio centro de masa o energía (en el caso de las partículas estrella o blancas), o por interacción de la superpartícula con el supergravitón, incluyendo, aquellas interacciones con regiones de antimateria.

En este trabajo, esbozaré un modelo matemático unitario, a propósito de la supergravedad cuántica relativista, concebida así, en la Teoría Cuántica de Campos Relativistas o Curvos, esto es, la deformación del espacio – tiempo cuántico, y su desdoblamiento en dimensiones infinitas, por efectos de la gravedad causada por la interacción de la superpartícula, con el supergravitón, lo que supone, la permeabilización del campo cuántico repercutido por el campo supergravitónico, o en su propio campo cuántico, en condiciones relativistas, a razón de su masa y energía, extremadamente densas, como ocurre con las partículas blancas.

RESULTADOS Y DISCUSIÓN.

Este trabajo, se encuentra dividido en dos volúmenes. En consecuencia, los cálculos aquí desarrollados se complementan con el volumen II de este manuscrito.

DESARROLLO MATEMÁTICO: En un escenario inflacionario oscilatorio, la supergravedad cuántica relativista en un campo de gauge perturbativo, se calcula así:

Métrica EMDA.

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - e^{a\phi}F^2 - \frac{1}{2}e^{c\phi}(\partial\chi)^2 + b\chi\tilde{F}^{\mu\nu}F_{\mu\nu}\right]$$

$$\phi \longrightarrow \phi + \lambda, A_\mu \longrightarrow e^{-\frac{1}{2}a\lambda}A_\mu$$

$$\chi \longrightarrow e^{-\frac{1}{2}c\lambda}\chi$$

$$c = -2a.$$

$$V^{\pm} = \pm\partial_{r_*}f + f^2 + 2\beta f = \pm\partial_{r_*}W + W^2 - \beta^2$$

$$R_{\mu\nu}=\frac{1}{2}\partial_\mu\phi\partial_\nu\phi+\frac{1}{2}e^{c\phi}\partial_\mu\chi\partial_\nu\chi+2e^{a\phi}\left(F_{\mu\rho}F_\nu^\rho-\frac{1}{4}F^{\rho\sigma}F_{\rho\sigma}g_{\mu\nu}\right)$$
$$\nabla_\nu\left(e^{a\phi}F^{\mu\nu}-b\chi\tilde{F}^{\mu\nu}\right)=0$$
$$\square\,\phi=ae^{a\phi}F^2+\frac{1}{2}ce^{c\phi}(\partial\chi)^2$$
$$\nabla_\mu\left(e^{c\phi}\nabla^\mu\chi\right)+b\tilde{F}^{\mu\nu}F_{\mu\nu}\;=0$$

$$ds^2=-\Delta dt^2+\frac{dr^2}{\Delta}+R^2(d\theta^2+\sin^2\,\theta d\varphi^2)$$
$$e^{-a\phi}=f_-^{\frac{2a^2}{1+a^2}},\chi=0,A=\frac{Q}{r}dt$$
$$\Delta=f_+f_-^{\frac{1-a^2}{1+a^2}},R=rf_-^{\frac{a^2}{1+a^2}},f_\pm=1-\frac{r_\pm}{r}$$

$$Q^2=\frac{r_+r_-}{1+a^2}$$

$$ds^2=-e^{2\nu}dt^2+e^{2\mu_1}dr^2+e^{2\mu_2}d\theta^2+e^{2\psi}(d\varphi-\mathcal{B}_\alpha dx^\alpha)^2$$

$$e^{\underline{0}}=e^\nu dt,e^{\underline{1}}=e^{\mu_1}dr,e^{\underline{2}}=e^{\mu_2}d\theta,e^{\underline{3}}=e^\psi(d\varphi-\mathcal{B}_\alpha dx^\alpha)$$

$$E_{\underline{0}}^\mu\partial_\mu=e^{-\nu}\left(\frac{\partial}{\partial t}+\mathcal{B}_0\frac{\partial}{\partial\varphi}\right),E_{\underline{1}}^\mu\partial_\mu=e^{-\mu_1}\left(\frac{\partial}{\partial r}+\mathcal{B}_1\frac{\partial}{\partial\varphi}\right)$$
$$E_{\underline{2}}^\mu\partial_\mu=e^{-\mu_2}\left(\frac{\partial}{\partial\theta}+\mathcal{B}_2\frac{\partial}{\partial\varphi}\right),E_{\underline{3}}^\mu\partial_\mu=e^{-\psi}\frac{\partial}{\partial\varphi}$$

$$\gimel_{axial}:\quad\left(\delta\mathcal{B}_0,\delta\mathcal{B}_1,\delta\mathcal{B}_2,\delta F_{\underline{0}}\underline{3},\delta F_{\underline{1}}\underline{3},\delta F_{\underline{2}}\underline{3},\delta\chi\right)$$
$$\lambda_{polar}:\quad\left(\delta\nu,\delta\mu_1,\delta\mu_2,\delta\psi,\delta F_{\underline{0}},\delta F_{\underline{0}},\delta F_{\underline{1}}\underline{2},\delta\phi\right)$$

$$\nu=\bar{\nu}+\epsilon\delta\nu,\mu_1=\bar{\mu}_1+\epsilon\delta\mu_1,\mu_2=\bar{\mu}_2+\epsilon\delta\mu_2,\psi=\bar{\psi}+\epsilon\delta\psi$$
$$\mathcal{B}_\alpha=\epsilon\delta\mathcal{B}_\alpha,\phi=\bar{\phi}+\epsilon\delta\phi,\chi=\epsilon\delta\chi$$

$$F_{ab}=\bar{F}_{ab}+\epsilon\delta F_{ab}$$

$$\bar{\nu}=\frac{1}{2}\log\,\Delta,\bar{\mu}_1=-\bar{\nu},\bar{\mu}_2=\log\,R,\bar{\psi}=\log\,R+\log\,\sin\,\theta$$
$$\overline{\mathcal{B}}_\alpha=0,a\bar{\phi}=2\log\,r-2\log\,R,\bar{F}_{\underline{0}}\underline{1}=\frac{Q}{r^2}$$

$$(\mathfrak{E}_{einstein})_{ab}\equiv R_{ab}-\frac{1}{2}R\eta_{ab}-\frac{1}{2}\phi_a\phi_b+\frac{1}{4}\phi^c\phi_c\eta_{ab}-\frac{1}{2}e^{c\phi}\chi_a\chi_b+\frac{1}{4}e^{c\phi}\chi^c\chi_c\eta_{ab}$$
$$-2e^{a\phi}\left(F_{ac}F_b^c-\frac{1}{4}\eta_{ab}F^{cd}F_{cd}\right)=0$$
$$(\mathfrak{M}_{maxwell})^a\equiv e_\mu^a\partial_\nu\left(\sqrt{g}E^{b\mu}E^{c\nu}\left(e^{a\phi}F_{bc}-b\chi\tilde{F}_{bc}\right)\right)=0$$
$$E_\phi\equiv\square\,\phi-ae^{a\phi}F^{ab}F_{ab}-\frac{1}{2}ce^{c\phi}\chi^a\chi_a=0$$
$$E_\chi\equiv\partial_\mu\left(\sqrt{g}e^{c\phi}E^{a\mu}\chi_a\right)+b\sqrt{g}\tilde{F}^{ab}F_{ab}=0$$

$$(\,\beth_{\text{Bianchi}}\,)^a\equiv e_\mu^a\varepsilon^{\mu\nu\rho\sigma}\partial_\nu\left(e_\rho^be_\sigma^cF_{bc}\right)=0$$

$$(\mathfrak{E}_{einstein})_1\underline{3}: \quad e^{\bar{\nu}-\bar{\mu}_2}\frac{1}{\sin^2\,\theta}\partial_\theta(\sin^3\,\theta\delta\mathcal{G}_{12})+e^{-\bar{\nu}+\bar{\mu}_2}\sin\,\theta\partial_t\delta\mathcal{G}_{01}-\frac{4Q}{r^2}e^{a\bar{\phi}}\delta F_{\underline{0}\underline{3}}=0$$
$$(\mathfrak{E}_{einstein})_{\underline{23}}: \quad e^{-2\bar{\mu}_2}\partial_r(e^{2\bar{\nu}+2\bar{\mu}_2}\delta\mathcal{G}_{12})-e^{-2\bar{\nu}}\partial_t\delta\mathcal{G}_{02}=0$$

$$(\mathfrak{M}_{maxwell})^{\underline{3}}: \quad e^{-\bar{\nu}-\bar{\mu}_2}\partial_t\delta F_{\underline{0}\underline{3}}-e^{-2\bar{\mu}_2-a\bar{\phi}}\partial_r\left(e^{\bar{\nu}+\bar{\mu}_2+a\bar{\phi}}\delta F_{\underline{1}\underline{3}}\right)-$$
$$e^{-2\bar{\mu}_2}\partial_\theta\left(\delta F_{\underline{2}\underline{3}}+\frac{bQ}{r^2}\delta\chi\right)+\frac{Q\sin\,\theta}{r^2}\delta\mathcal{G}_{01}=0$$

$$E_\chi: -e^{-2\bar{\nu}+c\bar{\phi}}\partial_t^2\delta\chi+e^{-2\bar{\mu}_2}\partial_r\left(e^{2\bar{\nu}+2\bar{\mu}_2+c\bar{\phi}}\partial_r\delta\chi\right)$$
$$+e^{-2\bar{\mu}_2+c\bar{\phi}}\frac{1}{\sin\,\theta}\partial_\theta(\sin\,\theta\partial_\theta\delta\chi)-\frac{4bQ}{r^2}\delta F_{\underline{2}\underline{3}}=0$$

$$(\mathfrak{Z}_{\text{Bianchi}})^{\underline{1}}: \quad \partial_t\delta F_{\underline{2}\underline{3}}=e^{\bar{\nu}-\bar{\mu}_2}\frac{1}{\sin\,\theta}\partial_\theta\left(\sin\,\theta\delta F_{\underline{0}\underline{3}}\right)$$
$$(\mathfrak{Z}_{\text{Bianchi}})^{\underline{2}}: \quad \partial_t\delta F_{\underline{13}}=e^{\bar{\nu}-\bar{\mu}_2}\partial_r\left(e^{\bar{\nu}+\bar{\mu}_2}\delta F_{\underline{0}\underline{3}}\right)$$

$$\delta F_{\underline{03}}(t,r,\theta)=\frac{\mathrm{i}\omega\ell(\ell+1)e^{-\bar{\nu}}}{r\sin\,\theta}e^{-\mathrm{i}\omega t}B_{\underline{03}}(r)C_{\ell+1}^{-\frac{1}{2}}(\cos\,\theta)=\frac{-\mathrm{i}\omega e^{-\bar{\nu}}}{r}e^{-\mathrm{i}\omega t}B_{\underline{03}}(r)\partial_\theta P_\ell(\cos\,\theta)$$

$$\delta\mathcal{G}_{12}(t,r,\theta)=\frac{2\mathrm{i}\omega\mu\ell(\ell+1)e^{-2\bar{\nu}-\bar{\mu}_2}}{3\sin^3\,\theta}e^{-\mathrm{i}\omega t}\mathcal{G}_{12}(r)C_{\ell+2}^{-\frac{3}{2}}(\cos\,\theta)$$
$$=\frac{-2\mathrm{i}\omega e^{-2\bar{\nu}-\bar{\mu}_2}}{\mu}e^{-\mathrm{i}\omega t}\mathcal{G}_{12}(r)\partial_\theta(\csc\,\theta\partial_\theta P_\ell(\cos\,\theta)),$$
$$\delta F_{\underline{2}\underline{3}}(t,r,\theta)=\frac{e^{-\bar{\mu}_2}}{r}e^{-\mathrm{i}\omega t}B_{\underline{2}\underline{3}}(r)P_\ell(\cos\,\theta),$$
$$\delta\chi(t,r,\theta)=-2\sqrt{\ell(\ell+1)}e^{-\bar{\mu}_2-\frac{1}{2}c\bar{\phi}}e^{-\mathrm{i}\omega t}\chi(r)P_\ell(\cos\,\theta),$$

$$\mu=\sqrt{\ell(\ell+1)-2}$$

$$\left(\partial_{r_*}^2+\omega^2\right)B_{\underline{0}\underline{+}}+T_1B_{\underline{03}}+S_1\mathcal{G}_{12}+P_1\chi=0,$$
$$\left(\partial_{r_*}^2+\omega^2\right)\mathcal{G}_{12}+T_2\mathcal{G}_{12}+S_1B_{\underline{03}}=0,$$
$$\left(\partial_{r_*}^2+\omega^2\right)\chi+T_3\chi+P_1B_{\underline{03}}=0,$$

$$T_1=-e^{4\bar{\nu}}\left(a\bar{\nu}'\bar{\phi}'+\frac{1}{4}a^2\bar{\phi}'^2+\frac{1}{2}a\bar{\phi}''\right)-\frac{1}{r^2}e^{2\bar{\nu}+a\bar{\phi}}\left(\frac{4Q^2}{r^2}+\ell(\ell+1)\right)$$
$$T_2=-e^{4\bar{\nu}}\left(a\bar{\nu}'\bar{\phi}'+\frac{1}{4}a^2\bar{\phi}'^2+\frac{1}{2}a\bar{\phi}''+\frac{2}{r^2}-\frac{2}{r}\bar{\nu}'-\frac{a}{r}\bar{\phi}'\right)-\frac{1}{r^2}e^{2\bar{\nu}+a\bar{\phi}}(\ell(\ell+1)-2)$$
$$T_3=-e^{4\bar{\nu}}\left(-(a-c)\bar{\nu}'\bar{\phi}'+\frac{1}{4}(a-c)^2\bar{\phi}'^2-\frac{1}{2}(a-c)\bar{\phi}''+\frac{2}{r}\bar{\nu}'-\frac{(a-c)}{r}\bar{\phi}'\right)$$

$$S_1=-\frac{1}{r^2}e^{2\bar{\nu}+a\bar{\phi}}\ell(\ell+1)-\frac{2\mu Q}{r^3}e^{2\bar{\nu}+a\bar{\phi}}$$
$$P_1=-\frac{2b\sqrt{\ell(\ell+1)}Q}{r^3}e^{2\bar{\nu}-\frac{c}{2}\bar{\phi}}$$

$$\partial_{r_*}=e^{2\bar{\nu}}\partial_r$$

$$e^{\bar{\mu}_2} = re^{-\frac{1}{2}a\bar{\phi}}$$

$$B_{\underline{2}}\underline{3} = -\ell(\ell+1)B_{\underline{0}}\underline{3}.$$

$$a = 1, c = -2$$

$$\delta\mathcal{G}_{01} = 2e^{-\bar{\mu}_2}e^{-\mathrm{i}\omega t}\mathcal{G}_{01}(r)\csc\,\theta\partial_\theta P_\ell$$
$$\delta\mathcal{G}_{02} = \frac{2e^{-\bar{\mu}_2}}{\mu}e^{-\mathrm{i}\omega t}\mathcal{G}_{02}(r)\partial_\theta(\csc\,\theta\partial_\theta P_\ell)$$
$$\delta\mathcal{G}_{12} = -2\mathrm{i}\omega\frac{e^{-2\bar{\nu}-\bar{\mu}_2}}{\mu}e^{-\mathrm{i}\omega t}\mathcal{G}_{12}(r)\partial_\theta(\csc\,\theta\partial_\theta P_\ell)$$
$$\delta F_{\underline{0}}\underline{3} = -\mathrm{i}\omega\frac{e^{-\bar{\nu}}}{r}e^{-\mathrm{i}\omega t}B_{\underline{0}}\underline{3}(r)\partial_\theta P_\ell$$
$$\delta F_{\underline{1}}\underline{3} = \frac{e^{-\bar{\nu}}}{r}e^{-\mathrm{i}\omega t}B_{\underline{1}}\underline{3}(r)\partial_\theta P_\ell$$
$$\delta F_{\underline{2}}\underline{3} = \frac{e^{-\bar{\mu}_2}}{r}e^{-\mathrm{i}\omega t}B_{\underline{2}}\underline{3}(r)P_\ell$$
$$\delta\chi = -2\sqrt{\ell(\ell+1)}e^{-\bar{\mu}_2-\frac{1}{2}c\bar{\phi}}e^{-\mathrm{i}\omega t}\chi(r)P_\ell$$

$$(\mathfrak{E}_{einstein})_{\underline{13}}: \frac{2Q}{r}B_{\underline{03}} - e^{2\bar{\mu}_2}\mathcal{G}_{01} + \sqrt{(\ell+2)(\ell-1)}\mathcal{G}_{12} = 0$$
$$(\mathfrak{E}_{einstein})_{\underline{23}}: e^{-\bar{\mu}_2}\partial_r(e^{\bar{\mu}_2}\mathcal{G}_{12}) - e^{-2\bar{\nu}}\mathcal{G}_{02} = 0$$
$$(\mathfrak{Bianchi})^{\underline{1}}: \ell(\ell+1)B_{\underline{03}} + B_{\underline{2}}\underline{3} = 0$$
$$(\mathfrak{Bianchi})^{\underline{2}}: re^{-\bar{\mu}_2}\partial_r\left(\frac{1}{r}e^{\bar{\mu}_2}B_{\underline{03}}\right) - e^{-2\bar{\nu}}B_{\underline{13}} = 0$$
$$(\mathfrak{M}_{maxwell})^{\underline{3}}: \frac{1}{r}e^{\bar{\mu}_2}\partial_r\left(re^{-\bar{\mu}_2}B_{\underline{13}}\right) + \omega^2e^{-2\bar{\nu}}B_{\underline{03}} + e^{-2\bar{\mu}_2}B_{\underline{23}}$$
$$-\frac{2Q}{r}\mathcal{G}_{01} - \frac{2b\sqrt{\ell(\ell+1)}Q}{r^3}e^{-\frac{1}{2}c\bar{\phi}}\chi = 0$$
$$E_\chi: \chi'' + 2\bar{\nu}'\chi' + \left[\omega^2e^{-4\bar{\nu}} - \bar{\mu}_2'(\bar{\mu}_2' + 2\bar{\nu}') - c\bar{\phi}'\left(\bar{\mu}_2' + \bar{\nu}' + \frac{1}{4}c\bar{\phi}'\right) - \bar{\mu}_2''\right.$$
$$\left. -\frac{1}{2}c\bar{\phi}'' - \ell(\ell+1)e^{-2\bar{\mu}_2-2\bar{\nu}}\right]\chi + \frac{2bQ}{\sqrt{\ell(\ell+1)}}\frac{e^{-2\bar{\nu}-\frac{1}{2}c\bar{\phi}}}{r^3}B_{\underline{2}} = 0$$
$$d\mathcal{G}: e^{\bar{\mu}_2}\partial_r(e^{-\bar{\mu}_2}\mathcal{G}_{02}) + \omega^2e^{-2\bar{\nu}}\mathcal{G}_{12} - \sqrt{(\ell+2)(\ell-1)}\mathcal{G}_{01} = 0$$

$$\{\mathcal{G}_{02}, \mathcal{G}_{12}, B_{\underline{0}}\underline{3}, B_{\underline{1}}\underline{3}, \chi, \xi\}$$

$$n = \frac{1}{2}\ell(\ell+1) - 1 = \frac{1}{2}\mu^2$$

$$\mathcal{G}_{02}' = \bar{\mu}_2' \mathcal{G}_{02} + (2ne^{-2\bar{\mu}_2} - \omega^2 e^{-2\bar{\nu}})\mathcal{G}_{12} + \frac{2\sqrt{2n}Q}{r} e^{-2\bar{\mu}_2} B_{\underline{0}\underline{3}}$$

$$\mathcal{G}_{12}' = e^{-2\bar{\nu}} \mathcal{G}_{02} - \bar{\mu}_2' \mathcal{G}_{12}$$

$$B_{\underline{03}}' = \left(\frac{1}{r} - \bar{\mu}_2'\right) B_{\underline{03}} + e^{-2\bar{\nu}} B_{\underline{13}}$$

$$B_{\underline{13}}' = \left(2(n+1)e^{-2\bar{\mu}_2} - \omega^2 e^{-2\bar{\nu}} + \frac{4Q^2}{r^2} e^{-2\bar{\mu}_2}\right) B_{\underline{03}} - \left(\frac{1}{r} - \bar{\mu}_2'\right) B_{\underline{13}} + \frac{2\sqrt{2n}Q}{r} e^{-2\bar{\mu}_2} \mathcal{G}_{12} + \frac{2b\sqrt{2(n+1)}Q}{r^3} e^{-\frac{1}{2}c\bar{\phi}} \chi$$

$$\chi' = \xi$$

$$\xi' = \left[2(n+1)e^{-2\bar{\mu}_2 - 2\bar{\nu}} + \bar{\mu}_2'(\bar{\mu}_2' + 2\bar{\nu}') + c\bar{\phi}'\left(\bar{\mu}_2' + \bar{\nu}' + \frac{1}{4}c\bar{\phi}'\right) + \bar{\mu}_2'' + \frac{1}{2}c\bar{\phi}'' - \omega^2 e^{-4\bar{\nu}}\right]\chi + \frac{2b\sqrt{2(n+1)}Q}{r^3} e^{-2\bar{\nu} - \frac{1}{2}c\bar{\phi}} B_{\underline{03}} - 2\bar{\nu}'\xi$$

$$\left(\partial_{r_*}^2 + \omega^2\right) Z^- - V^- Z^- = 0$$

$$Z^- = f_1 \mathcal{G}_{12} + f_2 B_{\underline{0}\underline{3}} + f_3 \chi$$

$$e_1 \mathcal{G}_{02} + e_2 \mathcal{G}_{12} + e_3 B_{\underline{0}\underline{3}} + e_4 B_{\underline{1}\underline{3}} + e_5 \chi + e_6 \xi = 0$$

$$e_i = 0, 1 \le i \le 6$$

$$e_1 = 2f_+ f_-^{\frac{1-a^2}{1+a^2}} f_1' = 0 \Longrightarrow f_1 = c_1$$

$$e_4 = 2f_+ f_-^{\frac{1-a^2}{1+a^2}} f_2' = 0 \Longrightarrow f_2 = c_2$$

$$e_6 = 2(f_+ f_-)^2 f_-^{\frac{-4a^2}{1+a^2}} f_3' = 0 \Longrightarrow f_3 = c_3$$

$$V^- = \frac{1}{(1+a^2)^2 c_1 r^5} f_+ f_-^{\frac{-4a^2}{1+a^2}} [2(n+1)(1+a^2)^2 c_1 r^3$$

$$- (1+a^2)\left[\left[\left(2n(1+a^2) + (5-a^2)\right)r_- + 3(1+a^2)r_+\right]c_1\right.$$

$$\left. - 2\sqrt{2n}Q(1+a^2)c_2\right] r^2 + \left[(3r_- + 7(1+a^2)r_+)r_- c_1 - 2\sqrt{2n}Q(1+a^2)^2 c_2 r_-\right] r$$

$$- (4+a^2)c_1 r_-^2 r_+]$$

$$\left(\frac{3-a^2}{1+a^2} r_- + 3r_+\right) c_1 c_2 + 2\sqrt{2n}Q(c_1^2 - c_2^2) + 2b\sqrt{2(n+1)}Q f_-^{\frac{a(2a+c)}{1+a^2}} c_1 c_3 = 0$$

$$\sqrt{2n}Qr(r - r_-)c_2 c_3 - b\sqrt{2(n+1)}Qr(r - r_-) f_-^{\frac{a(2a+c)}{1+a^2}} c_1 c_2 - B(r) c_1 c_3 = 0$$

$$B(r) = \frac{1}{2(1+a^2)^2} [4(1+a^2)[(1+a^2)r_+ + (1-a^2)r_-]r^2$$
$$+ r_-[(a^2c^2 - 4)r_- - (1+a^2)(ac+10)r_+]r + (ac+2)(3+a^2-ac)r_-^2 r_+]$$

$$B(r) = \frac{(r - r_-)}{1+a^2} [2[(1+a^2)r_+ + (1-a^2)r_-]r - 3(1-a^2)r_- r_+]$$

$$a = 1, c = -2$$

$$(r_- + 3r_+)c_1c_2 + 2\sqrt{2n}Q(c_1^2 - c_2^2) + 2b\sqrt{2(n+1)}Qc_1c_3 = 0$$
$$\left(\sqrt{2n}c_3 - b\sqrt{2(n+1)}c_1\right)Qc_2 - 2r_+c_1c_3 = 0$$

$$c_1 = -\frac{pc_2r_-}{4\sqrt{2n}Q},$$

$$c_3 = -\frac{b\sqrt{2(n+1)}Qc_2p}{2(p+2n)r_+}$$

$$\tilde{\Lambda}(p) \equiv p^3r_- + p^2[2n(1+b^2)r_- - 2(1-b^2)r_- - 6r_+] - 4np(r_- + 7r_+) - 32n^2r_+ = 0.$$

$$Z^-_{(p)} = c_1\mathcal{G}_{12} + c_2B_{\underline{03}} + c_3\chi$$

$$\left(\partial^2_{r_*} + \omega^2\right)Z^-_{(p)} - V^-_{(p)}Z^-_{(p)} = 0$$

$$V^-_{(p)} = \frac{(r-r_+)[8(n+1)r^3 - 4(2(n+1)r_- + 3r_+)r^2 + (3r_- + 14r_+)r_-r - 5r_-^2r_+]}{4(r-r_-)^2r^4}$$
$$-\frac{8n(r-r_+)r_+}{p(r-r_-)r^3}$$

$$D_p = 64n^2[16(n+1)^3r_-^3r_+b^6 + (n+1)^2(r_-^2 + (48n-34)r_-r_+ - 95r_+^2)r_-^2b^4$$
$$+2(n+1)[(n-1)r_-^3 + (24n^2+2n+7)r_-^2r_+ + (157n+53)r_-r_+^2 + 69r_+^3]\,r_-b^2$$
$$+[(n+1)r_- - r_+]^2[r_-^2 + 2(8n+3)r_-r_+ + 9r_+^2]]$$

$$D_p = 64n^2[16(n+1)b^2r_-r_+[3r_+ - (n+1)b^2r_-]^2 + r_+^2[3r_+ - (n+1)(3+b^2)r_-]^2$$

$$+4(n+1)(77n+25)b^2r_-^2r_+^2 + r_-[r_- + 2(8n+3)r_+][r_+ - (n+1)r_-]^2$$
$$+2(n+1)b^2r_-^3[(n-1)r_- + (24n^2+2n+7)r_+]$$
$$+(n+1)^2b^4r_-^3[r_- + (48n-34)r_+]]$$

$$p_1p_2p_3 = \frac{32n^2r_+}{r_-} > 0 \text{ and } p_1p_2 + p_2p_3 + p_1p_3 = -\frac{4n(r_- + 7r_+)}{r_-} < 0$$

$$p_1 \le p_2 < 0, p_3 > 0$$

$$b^2 = 1$$

$$\tilde{\Lambda}_1(p) \equiv p^3r_- + p^2[4nr_- - 6r_+] - 4np(r_- + 7r_+) - 32n^2r_+ = 0$$

$$V^{\pm} = \pm\partial_{r_*}f + \alpha f^2 + 2\beta f$$

$$V^+ - V^- = 2\partial_{r_*}f$$

$$r_+ = \frac{p[p^2 + 4np - 4n]r_-}{2(2n+p)(8n+3p)}$$

$$f_{(p)} = \frac{r_-(r-r_+)[(4n+2p)r - pr_-]}{2r^2(r-r_-)(4nr+pr_-)}$$

$$\alpha_{(p)} = 1, \beta_{(p)} = \frac{4n(n+1)}{(2n+p)r_-}$$

$$V^{\pm}_{(p)} = \pm\partial_{r*} f_{(p)} + f^2_{(p)} + \frac{8n(n+1)}{(2n+p)r_-} f_{(p)}.$$

$$[\partial^2_{r*} + \omega^2 - V^{\pm}]Z^{\pm} = 0$$

$$D_- D_+ Z^- + (\omega^2 + \beta^2)Z^- = 0$$
$$D_+ D_- Z^+ + (\omega^2 + \beta^2)Z^+ = 0$$

$$D_{\pm} = \partial_{r*} \pm (f + \beta),$$

$$D_- Z^+ = (\mathrm{i}\omega - \beta)Z^-, D_+ Z^- = (\mathrm{i}\omega + \beta)Z^+$$

$$\omega = \pm \mathrm{i}\beta$$

$$D_+ Z^- = 0, D_- Z^+ = 0$$

$$Z^{\pm} = \exp\left[\pm \int^r (f(r') + \beta)\,(f(r') + \beta) e^{-2\bar{\nu}(r')} dr'\right]$$

$$Z^+_{(p)} = e^{\beta_{(p)} r_*} \left(1 - \frac{r_-}{r}\right)^{\frac{1}{2}} \left(4n + \frac{pr_-}{r}\right)^{-1}$$
$$Z^-_{(p)} = e^{-\beta_{(p)} r_*} \left(1 - \frac{r_-}{r}\right)^{-\frac{1}{2}} \left(4n + \frac{pr_-}{r}\right)$$

$$e^{-\mathrm{i}\omega t} Z^+_{(p)} = e^{\beta_{(p)}(r_* \pm t)} \left(1 - \frac{r_-}{r}\right)^{\frac{1}{2}} \left(4n + \frac{pr_-}{r}\right)^{-1}$$
$$e^{-\mathrm{i}\omega t} Z^-_{(p)} = e^{-\beta_{(p)}(r_* \mp t)} \left(1 - \frac{r_-}{r}\right)^{-\frac{1}{2}} \left(4n + \frac{pr_-}{r}\right)$$

$$\tilde{Z}^+_{(p)}(r) = Z^+_{(p)}(r) \int^r \frac{e^{-2\bar{\nu}(r')}}{\left(Z^+_{(p)}(r')\right)^2} \frac{e^{-2\bar{\nu}(r')}}{\left(Z^+_{(p)}(r')\right)^2} dr', \tilde{Z}^-_{(p)}(r) = Z^-_{(p)}(r) \int^r \frac{e^{-2\bar{\nu}(r')}}{\left(Z^-_{(p)}(r')\right)^2} \frac{e^{-2\bar{\nu}(r')}}{\left(Z^-_{(p)}(r')\right)^2} dr'$$

$$r_+ - r_- = \frac{[p + 2(\ell - 1)][p - 2(\ell + 2)][p + 2(\ell + 2)(\ell - 1)]r_-}{2[p + (\ell + 2)(\ell - 1)][3p + 4(\ell + 2)(\ell - 1)]}$$

$(\mathbf{1})$:$p > 2(\ell + 2)$
$(\mathbf{2})$:$-(\ell + 2)(\ell - 1) < p < -2(\ell - 1)$
$(\mathbf{3})$: $-2(\ell + 2)(\ell - 1) < p < -\frac{4}{3}(\ell + 2)(\ell - 1)$

$$\ell = 2 + y, r = r_+(1 + z)$$

$$p = 2(\ell + 2)(1 + x)$$

$$p = -(\ell+2)(\ell-1) + \frac{\ell(\ell-1)}{1+x}$$
$$p = -2(\ell+2)(\ell-1) + \frac{2(\ell+2)(\ell-1)}{3(1+x)}$$

$$V_i^- = R_i P_i, i = 1,2,3$$

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - e^{\phi}F^2 - \frac{1}{2}e^{-2\phi}(\partial\chi)^2 + \chi\tilde{F}^{\mu\nu}F_{\mu\nu} - \frac{e^{-2\phi}}{1+\chi^2 e^{2\phi}}\left(e^{\phi}H^2 + \chi\tilde{H}^{\mu\nu}H_{\mu\nu}\right)\right]$$

$$\delta H_{\underline{03}}(t,r,\theta) = -\frac{\mathrm{i}\omega e^{-\bar{\nu}+\bar{\phi}}}{r}e^{-\mathrm{i}\omega t}Z_H^-(r)\partial_\theta P_\ell(\cos\,\theta),$$

$$\delta H_{\underline{12}}(t,r,\theta) = -\frac{\mathrm{i}\omega e^{-\bar{\nu}+\bar{\phi}}}{r^2}e^{-\mathrm{i}\omega t}Z_H^+(r)\partial_\theta P_\ell(\cos\,\theta)$$

$$\delta H_{\underline{1}\underline{3}} = e^{\bar{\nu}-\bar{\mu}_2}\partial_r(re^{-\bar{\mu}_2}Z_H^-(r))e^{-\mathrm{i}\omega t}\partial_\theta P_\ell(\cos\,\theta)$$
$$\delta H_{\underline{2}\underline{3}} = -\frac{\ell(\ell+1)e^{-\bar{\mu}_2+\bar{\phi}}}{r}Z_H^-(r)e^{-\mathrm{i}\omega t}P_\ell(\cos\,\theta)$$

$$\delta H_{\underline{12}} = -\frac{\mathrm{i}\omega e^{-\bar{\nu}+\bar{\phi}}}{r^2}Z_H^+(r)e^{-\mathrm{i}\omega t}\partial_\theta P_\ell(\cos\,\theta)$$
$$\delta H_{\underline{01}} = \frac{\ell(\ell+1)e^{-\bar{\mu}_2+\bar{\phi}}}{r^2}Z_H^+(r)e^{-\mathrm{i}\omega t}P_\ell(\cos\,\theta)$$

$$\left(\partial_{r_*}^2 + \omega^2 - V_H^{\pm}\right)Z_H^{\pm} = 0$$

$$V_H^{\pm} = \frac{2(n+1)e^{2\bar{\nu}+\bar{\phi}}}{r^2} + e^{4\bar{\nu}}\left[\frac{1}{4}\bar{\phi}'^2 \pm \left(\bar{\nu}'\bar{\phi}' + \frac{1}{2}\bar{\phi}''\right)\right]$$

$$V_H^{\pm} = \pm\partial_{r_*}f_H + f_H^2 + 2\beta_H f_H$$

$$f_H = \frac{1}{2}e^{2\bar{\nu}}\bar{\phi}', \beta_H = \frac{2(n+1)e^{\bar{\phi}}}{r^2\bar{\phi}'} = -\frac{2(n+1)}{r_-}$$

$$V_H^- = \frac{(r-r_+)[8(n+1)r^3 - 4(2n+3)r_-r^2 + 3(r_- + 2r_+)r_-r - 5r_-^2r_+]}{4r^4(r-r_-)^2}$$
$$V_H^+ = \frac{(r-r_+)[8(n+1)r^3 - 4(2n+1)r_-r^2 - (r_- + 6r_+)r_-r + 3r_-^2r_+]}{4r^4(r-r_-)^2}$$

$$f_H = -\frac{r_-(r-r_+)}{2r^2(r-r_-)}, \beta_H = -\frac{2(n+1)}{r_-}$$

$$\omega = \pm\mathrm{i}\beta_H = \mp\frac{2\mathrm{i}(n+1)}{r_-}$$
$$Z_H^+ = e^{\beta_H r_*}\left(1-\frac{r_-}{r}\right)^{-\frac{1}{2}}, Z_H^- = e^{-\beta_H r_*}\left(1-\frac{r_-}{r}\right)^{\frac{1}{2}}$$

$$e^{-i\omega t}Z_H^+ = e^{\beta_H(r_*\pm t)}\left(1-\frac{r_-}{r}\right)^{-\frac{1}{2}}$$

$$e^{-i\omega t}Z_H^- = e^{-\beta_H(r_*\mp t)}\left(1-\frac{r_-}{r}\right)^{\frac{1}{2}}$$

$$\tilde{Z}_H^+ = \frac{\left(2\beta_H r_- e^{\beta_H(r-2r_+)}\mathrm{Ei}[1+2\beta_H r_+, 2\beta_H(r-r_+)] - e^{-\beta_H r}\right)\sqrt{r}}{2\beta_H\sqrt{r-r_-}(r-r_+)^{\beta_H r_+}}$$

$$r_+ - r_- = \frac{r_-(p^3+[2n(1+b^2)-2(1-b^2)-6]p^2-32np-32n^2)}{2(p+2n)(3p+8n)}$$

$$b^2 = 0 : \tilde{\Lambda}(p) = (p+2n)[r_- p^2 - 2(r_- + 3r_+)p - 16nr_+]$$

$$b^2 = \frac{4n+3}{12(n+1)} : \tilde{\Lambda}(p) = \frac{1}{6}(3p+8n)[2r_- p^2 - 3(r_- + 4r_+)p - 24nr_+]$$

Fixed : $p = -2n$

Range (i) : $-\frac{8n}{3} < p < 4(1-\sqrt{n+1}) < 0$

Range (ii) : $0 < 4(1+\sqrt{n+1}) < p < \infty$

Fixed : $p = -\frac{8n}{3}$

Range (i) : $-2n < p < \frac{15}{4}\left(1-\sqrt{1+\frac{64n}{75}}\right) < 0$

Range (ii) : $0 < \frac{15}{4}\left(1+\sqrt{1+\frac{64n}{75}}\right) < p < \infty.$

$$b^2 = \frac{(p_z+2n)[2(\ell+2)-p_z][p_z+2(\ell-1)]}{2p_z^2(n+1)} > 0 \;\forall p_z < -2n$$

$$b^2 = \frac{4(4n+3)x^3+12(2n+1)x^2+9nx}{3(n+1)(3+4x)^2(1+x)}, \text{ with } 0 < x < \infty$$

Range (i) : $-\frac{8n}{3} < p < p_a < -2n$ where $p_a = -\frac{2n(3+4x)}{3(1+x)}$,

Range (ii) : $-2n < p < p_{-,a} < 0$ where $p_{-,a} = \frac{4\left(3(1+x)(3+5x)-\sqrt{P_a}\right)}{(3+4x)^2}$,

Range (iii) : $0 < p_{+,a} < p < \infty$ where $p_{+,a} = \frac{4\left(3(1+x)(3+5x)+\sqrt{P_a}\right)}{(3+4x)^2}$,

where $P_a = 3n(1+x)(3+4x)^3 + [3(1+x)(3+5x)]^2$.

$$b^2 = \frac{(1+4x)(4nx^2+8nx+4n+12x+3)}{12(1+x)^2(n+1)} < \infty, \text{ with } 0 < x < \infty$$

Range (i): $-\infty < p_b < p < -\frac{8n}{3}$ where $p_b = -\frac{8n}{3}(1+x)$

Range (ii) : $-2n < p < p_{-,b} < 0$ where $p_{-,b} = \frac{3(5+8x) - \sqrt{P_b}}{4(1+x)^2}$

Range (iii) : $0 < p_{+,b} < p < \infty$ where $p_{+,b} = \frac{3(5+8x) + \sqrt{P_b}}{4(1+x)^2}$

where $P_b = 3n[4(1+x)]^3 + [3(5+8x)]^2$

$$V_{(p)}^{-} = -\frac{A}{p} + B$$

$$A = \frac{8n(r-r_+)r_+}{(r-r_-)r^3}$$

$$B = \frac{(r-r_+)[8(n+1)r^3 - 4(2(n+1)r_- + 3r_+)r^2 + (3r_- + 14r_+)r_- r - 5r_-^2 r_+]}{4(r-r_-)^2 r^4}$$

$$r = r_+(1+z), r_+ = r_-(1+w)$$

$$B = \frac{(r-r_+)r_-^2 r_+}{4r^4(r-r_-)^2}[8(n+1)(1+w)^2 z^3 + 4(1+w)[4n+1+3(2n+1)w]z^2 + [8n+1+2(16n-1)w + 24nw^2]z + 2w[4n-1+2(2n-1)w]]$$

$$r_+ = 2r_-, \ell = 2, b = 5$$

$$V_{(p)}^{-} = \frac{(r-r_+)}{4r^4(r-r_-)^2}U(r)$$

$$U(r) = 8(n+1)r^3 - 4[(2(n+1)r_- + 3r_+]r^2 + (3r_- + 14r_+)r_- r - 5r_-^2 r_+ - \frac{32n(r-r_-)r_+ r}{p}$$

$$U(r) \longrightarrow -\infty \quad \text{when } r \longrightarrow -\infty$$
$$U(r) \longrightarrow +\infty \quad \text{when } r \longrightarrow +\infty$$

$$U(0) = -5r_+ r_-^2 < 0, U(r_-) = -3(r_+ - r_-)r_-^2 < 0$$

$$r_1 r_2 r_3 = \frac{5}{8(n+1)} r_+ r_-^2 > 0$$

$$\frac{r_1}{r_+}\frac{r_2}{r_+}\frac{r_3}{r_+} = \frac{5}{8(n+1)}\frac{r_-^2}{r_+^2}$$

$$\frac{r_1}{r_-} = \frac{5}{8(n+1)}\frac{r_+}{r_3}\frac{r_-}{r_2}$$

$$U(r) = \frac{(r-r_3)}{r_3(r_3-r_-)}P_2(r)$$

$$P_2(r) = 8(n+1)r_3(r_3-r_-)r^2 - r_-[8(n+1)r_3^2 - (8n+5)r_3 r_- + 2r_3 r_+ - 5r_- r_+]r + 5(r_3-r_-)r_-^2 r_+$$

$$U(r_+) = \frac{2(r_+ - r_-)r_+}{p}[pr_- + (4np - 2p - 16n)r_+],$$

$$p = \frac{ur_- + 16nr_+}{2(2n-1)r_+ + r_-},$$

$$U'(r_+) = \frac{r_-^3}{[2(2n-1)r_+ + r_-]}[48n + (8n+1)u + 2[64n^2 + 72n + (16n-1)u]w + 8n(32n + 28 + 3u)w^2 + 128n(n+1)w^3]$$

$$U(r_+) = 2(r_+ - r_-)r_+W(r_+), W(r_+) = \frac{pr_- - 2(p + 8n - 2np)r_+}{p}$$

$$U(r_+) = 0 \implies p = \frac{16nr_+}{2(2n-1)r_+ + r_-}$$

$$b^2 = \frac{[r_- + 2(2n+3)r_+][3r_-^2 - 2r_-r_+ + (n+1)(2n-1)r_+^2]}{16(n+1)r_-r_+[r_- + 2(2n-1)r_+]}$$

$$r_+ = r_-(1+w), n = 2 + y$$

$$\frac{\partial b^2}{\partial w} > 0, \forall w \geq 0$$

$$r_+ \longrightarrow r_-$$

$$p = \frac{16n}{4n-1}, b^2 = \frac{(4n+7)(16n^2 + 8n - 7)}{16(n+1)(4n-1)}$$

$$b_{\min} = \{1.80525, 2.53171, 3.23273, 3.93299, \cdots\}$$

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - e^{\phi}F^2 - \frac{1}{2}e^{-2\phi}(\partial\chi)^2 + b\chi\tilde{F}^{\mu\nu}F_{\mu\nu}\right]$$

$$V^{\pm} = \pm\partial_{r_*}W + W^2 - \beta^2$$

$$\mathcal{Q} = (\mathbf{p} + \mathrm{i}W)\mathbf{b}$$

$$\{\mathbf{b}^{\dagger}, \mathbf{b}\} = 1, \{\mathbf{b}, \mathbf{b}\} = 0, \{\mathbf{b}^{\dagger}, \mathbf{b}^{\dagger}\} = 0, [\mathbf{p}, \mathbf{b}] = 0, [\mathbf{p}, \mathbf{b}^{\dagger}] = 0$$

$$H = \frac{1}{2}\left(-\partial_{r_*}^2 + V^+ + \beta^2\right)\mathbf{b}\mathbf{b}^{\dagger} + \frac{1}{2}\left(-\partial_{r_*}^2 + V^- + \beta^2\right)\mathbf{b}^{\dagger}\mathbf{b}$$

$$H = \frac{1}{2}\begin{pmatrix} -\partial_{r_*}^2 + V^+ + \beta^2 & 0 \\ 0 & -\partial_{r_*}^2 + V^- + \beta^2 \end{pmatrix}$$

$$\mathbf{Z} = \begin{pmatrix} Z^+ \\ Z^- \end{pmatrix}$$

$$H\mathbf{Z} = \frac{1}{2}(\omega^2 + \beta^2)\mathbf{Z}$$

$$a = 1, c = -2$$

$$\left(\partial_{r_*}^2 + \omega^2 + T_1\right)\Psi + \frac{e^{2\bar{v}+\bar{\phi}}}{r^3} W\Psi = 0, \Psi = \begin{pmatrix} \mathcal{G}_{12} \\ B_0 \underline{3} \\ \underline{\chi} \end{pmatrix}$$

$$W = \begin{pmatrix} r_- + 3r_+ & -2\sqrt{2n}Q & 0 \\ -2\sqrt{2n}Q & 0 & -2\sqrt{2(n+1)}bQ \\ 0 & -2\sqrt{2(n+1)}bQ & r_- - r_+ \end{pmatrix}$$

$$\frac{S_{i1}}{S_{i2}} = \frac{2\sqrt{n}\sqrt{r_- r_+}}{(r_- + 3r_+ - s_i)}, \frac{S_{i3}}{S_{i2}} = -\frac{4nr_- r_+ + (r_- + 3r_+ - s_i)s_i}{2b\sqrt{n+1}\sqrt{r_- r_+}(r_- + 3r_+ - s_i)},$$

$$s^3 - 2(r_- + r_+)s^2 + (r_-^2 - 3r_+^2 + 2r_- r_+[1 - 2n - 2(n+1)b^2])s$$
$$+4r_- r_+[n(r_- - r_+) + (n+1)b^2(r_- + 3r_+)] = 0$$

$$\frac{S_{i1}}{S_{i2}} = \frac{c_1}{c_2}, \frac{S_{i3}}{S_{i2}} = \frac{c_3}{c_2},$$

$$s = r_- + \left(3 + \frac{8n}{p}\right) r_+$$

Cuyos superespacios de Sitter y anti – de Sitter, se calculan así:

$$ds_{I,st}^2 = f(\vec{v})\mathrm{AdS}_{d+1} + g_{ij,(9-d)}(\vec{v})dv^i dv^j, \Phi(\vec{v})$$
$$\mathrm{AdS}_{d+1} = \frac{u^2}{l^2}\left(\lambda dt^2 + d\vec{x}_{d-2}^2 + dy^2\right) + \frac{l^2 du^2}{u^2}$$

$$ds_{II,st}^2 = f(\vec{v},u)\left[s_1(u)\left(\lambda dt^2 + d\vec{x}_{d-3}^2 + dy^2\right) + s_2(u)du^2 + s_3(u)d\phi^2\right] +$$
$$+g_{ij,(9-d)}(\vec{v},u)\left(dv^i - A^i\right)\left(dv^j - A^j\right), \Phi(\vec{v},u)$$
$$A^i = s_{4,i}(u)d\phi$$

$$ds_{III,st}^2 = \hat{h}(u)^{-\frac{1}{2}}\left(\lambda dt^2 + dy^2 + d\vec{x}_{d-2}^2\right) + \hat{h}(u)^{\frac{1}{2}} e^{2k(u)} du^2 + g_{ij,(9-d)}(u,\vec{v})dv^i dv^j, \Phi(u)$$

$$\Sigma_{8,I} = \Sigma_{8,III} = [u, \vec{x}_{d-2}, \vec{v}_{9-d}], \Sigma_{8,II} = [u, \vec{x}_{d-3}, \phi, \vec{v}_{9-d}]$$
$$t(u), y(u)$$

$$ds_{\Sigma_{8,I}}^2 = f(\vec{v})\left(du^2\left[\frac{l^2}{u^2} + \frac{u^2}{l^2}(\lambda t'^2 + y'^2)\right] + \frac{u^2}{l^2}d\vec{x}_{d-2}^2\right) + g_{ij,(9-d)}(\vec{v})dv^i dv^j$$
$$ds_{\Sigma_{8,II}}^2 = f(\vec{v},u)(du^2[s_2(u) + s_1(u)(\lambda t'^2 + y'^2)] + s_1(u)d\vec{x}_{d-3}^2 + s_3(u)d\phi^2) + g_{ij,(9-d)}(\vec{v},u)(dv^i - A^i)(dv^j - A^j)$$
$$ds_{\Sigma_{8,III}}^2 = du^2\left[\hat{h}(u)^{\frac{1}{2}} e^{2k} + \hat{h}(u)^{-\frac{1}{2}}(\lambda t'^2 + y'^2)\right] + \hat{h}(u)^{-\frac{1}{2}}d\vec{x}_{d-2}^2 + g_{ij,(9-d)}(\vec{v},u)dv^i dv^j$$

$$4G_{10}S_{EE} = \int_{\Sigma_8} \sqrt{e^{-4\Phi}\det\left[g_{ind,\Sigma_8}\right]}$$

$$e^{-4\Phi}\det\left[g_{\Sigma_{8,I}}\right] = e^{-4\Phi(\vec{v})}f(\vec{v})^{d-1}\det[g_{9-d}]\left[\left(\frac{u}{l}\right)^{2(d-3)} + \left(\frac{u}{l}\right)^{2(d-1)}(y'^2 + \lambda t'^2)\right]$$

$$4G_{10}S_{EE,I} = \mathcal{N}_I\int du\sqrt{F^2(u)(y'^2 + \lambda t'^2) + G^2(u)},$$

$$F(u) = \left(\frac{u}{l}\right)^{(d-1)}, G(u) = \left(\frac{u}{l}\right)^{(d-3)}, \mathcal{N}_I = \int d^{9-d}vd^{d-2}x\sqrt{e^{-4\Phi(\vec{v})}f(\vec{v})^{d-1}\det[g_{9-d}]}$$

$$e^{-4\Phi}\det\left[g_{\Sigma_{8,II}}\right] = e^{-4\Phi(\vec{v},u)}f(\vec{v},u)^{d-1}\det[g_{9-d}(\vec{v},u)]s_3(u)s_1(u)^{d-3}[s_1(u)(\lambda t'^2 + y'^2) + s_2(u)]$$

$$e^{-4\Phi}\det\left[g_{\Sigma_{8,III}}\right] = e^{-4\Phi(u)}\det[g_{9-d}(\vec{v},u)]\hat{h}(u)^{\frac{(1-d)}{2}}\left[\lambda t'^2 + y'^2 + e^{2k}\hat{h}(u)\right]$$

$$4G_{10}S_{EE} = \mathcal{N}\int_{u_0}^{\infty} du\sqrt{F^2(u)(y'^2 + \lambda t'^2) + G^2(u)}$$

$$\frac{F^2t'}{\sqrt{G^2 + F^2(\lambda t'^2 + y'^2)}} = \lambda c_t, \frac{F^2y'}{\sqrt{G^2 + F^2(\lambda t'^2 + y'^2)}} = c_y$$

$$t'^2(u) = \frac{c_t^2G^2(u)}{F^2(u)\left(F^2(u) - F^2(u_0)\right)}, y'^2(u) = \frac{c_y^2G^2(u)}{F^2(u)\left(F^2(u) - F^2(u_0)\right)}$$

$$F^2(u_0) = c_y^2 + \lambda c_t^2$$

$$T = 2c_t\int_{u_0}^{\infty} du\frac{G(u)}{F(u)\sqrt{F^2(u) - F^2(u_0)}}, Y = 2c_y\int_{u_0}^{\infty} du\frac{G(u)}{F(u)\sqrt{F^2(u) - F^2(u_0)}}$$

$$\frac{4G_{10}}{\mathcal{N}}S_{tEE} = 2\int_{u_0}^{\infty} du\frac{G(u)F(u)}{\sqrt{F^2(u) - F^2(u_0)}} - 2\int_{u_*}^{\infty} G(u)du$$

$$T_{app}(u_0) = \pi\frac{G(u_0)}{F'(u_0)}$$

$$\frac{dS(u_0)}{dY(u_0)} = F(u_0).$$

$$\frac{dS(u_0)}{dT(u_0)} \approx \frac{dS_{\text{approx}}(u_0)}{dT_{\text{app}}(u_0)} \approx F(u_0),$$

$$S_{\text{approx}}(u_0) = \int F(u_0)dT_{\text{app}}(u_0) = \pi\int F(u_0)\frac{d}{du_0}\left(\frac{G(u_0)}{F'(u_0)}\right)du_0$$

$$ds_{I,st}^2 = f(\vec{v})\text{AdS}_{d+1} + g_{ij,(9-d)}(\vec{v})dv^idv^j, \Phi(\vec{v})$$

$$\text{AdS}_{d+1} = \frac{u^2}{l^2}\left(\lambda dt^2 + d\vec{x}_{d-2}^2 + dy^2\right) + \frac{l^2du^2}{u^2}$$

or,

$$AdS_{d+1} = u^2\left(\lambda dt^2 + t^2d\Omega_{d-2}^{(\lambda)} + dy^2\right) + \frac{du^2}{u^2}$$

$$T = 2c_t l^{d+1}\int_{u_0}^{\infty}\frac{du}{u^2}\frac{1}{\sqrt{u^{2(d-1)}-u_0^{2(d-1)}}}, Y = 2c_y l^{d+1}\int_{u_0}^{\infty}\frac{du}{u^2}\frac{1}{\sqrt{u^{2(d-1)}-u_0^{2(d-1)}}}$$

$$\frac{4G_{10}}{\mathcal{N}}S_{EE} = \frac{2}{l^{d-3}}\left[\int_{u_0}^{\infty}du\frac{u^{2d-4}}{\sqrt{u^{2d-2}-u_0^{2d-2}}} - \int_0^{\infty}u^{d-3}du\right]$$

$$u_0 = \left(c_y^2 - c_t^2\right)^{\frac{1}{2(d-1)}}$$

$$u_0 = e^{\frac{i\pi}{2(d-1)}}\left[\sqrt{|c_y^2 - c_t^2|}\right]^{\frac{1}{d-1}} = (i\tilde{u}_0)^{\frac{1}{d-1}}$$

$$T = \frac{2c_t l^{d+1}}{u_0^d}I_1, Y = \frac{2c_y l^{d+1}}{u_0^d}I_1$$

$$I_1 = \int_0^1 dr\frac{r^{d-1}}{\sqrt{1-r^{2(d-1)}}} = \frac{\sqrt{\pi}\Gamma\left(\frac{d}{2d-2}\right)}{\Gamma\left(\frac{1}{2(d-1)}\right)}$$

$$\frac{2G_{10}l^{d-3}}{\mathcal{N}u_0^{d-2}}S_{EE} = I_2 - I_3,$$

$$I_2 = \int_{\epsilon}^1\frac{dr}{r^{d-1}\sqrt{1-r^{2d-2}}} = -\frac{1}{(d-2)r^{d-2}}\,{}_2F_1\left(\frac{1}{2},\frac{2-d}{2d-2};\frac{d}{2(d-1)};r^{2d-2}\right)\Big|_{\epsilon}^1$$

$$I_3 = \int_{\epsilon}^{\infty}\frac{dr}{r^{d-1}} = \frac{1}{(d-2)\epsilon^{d-2}}$$

$$\frac{2G_{N,10}l^{d-3}}{\mathcal{N}u_0^{d-2}}S_{EE} = \frac{1}{(2-d)}\,{}_2F_1\left(\frac{1}{2},\frac{2-d}{2d-2};\frac{d}{2d-2};1\right)$$

$$c_t = \frac{T}{2l^{d+1}I_1}u_0^d, c_y = \frac{Y}{2l^{d+1}I_1}u_0^d, u_0 = \frac{2l^{d+1}I_1}{\sqrt{Y^2-T^2}}$$

$$S_{EE} = \frac{\mathcal{N}(I_2-I_3)2^{d-3}l^{(d-1)^2}I_1^{d-2}}{G_{10}}\times\frac{1}{(Y^2-T^2)^{\frac{(d-2)}{2}}}$$

$$S_{EE} = \frac{\mathcal{N}(I_2-I_3)2^{d-3}l^{(d-1)^2}I_1^{d-2}}{G_{10}}\times\frac{e^{-i\pi\frac{(d-2)}{2}}}{|Y^2-T^2|^{\frac{(d-2)}{2}}}$$

$$T_{app} = \pi\frac{G(u_0)}{F'(u_0)} = \frac{\pi}{(d-1)\sqrt{\lambda}u_0}$$

$$S_{EE,\text{ approx}} = \int^{u_0}d\,dzF(z)T'_{app}(z) = -\frac{\pi}{(d^2-3d+2)}u_0^{d-2}$$

$$S_{EE,\,\text{approx}} = -\frac{\pi^{d-1}}{\sqrt{\lambda^{d-2}}(d-1)^{d-2}(d^2-3d+2)}\frac{1}{\left|T_{app}\right|^{d-2}}$$

$$ds_8^2\big|_{\Sigma_8} = f(\vec{v})[1+\lambda u^4 t'^2(u)]\frac{du^2}{u^2} + f(\vec{v})u^2 t^2 d\Omega_{d-2}^{(\lambda)} + g_{ij}(\vec{v})dv^i dv^j$$

$$S_{EE}^{(\lambda)}[\Sigma_8] = \frac{1}{4G_{10}}\int d^8x\sqrt{e^{-4\Phi}\det g_8} = \frac{\hat{\mathcal{N}}}{4G_{10}}\int duu^{d-3}t^{d-2}\sqrt{1+\lambda u^4 t'^2(u)}$$

$$\hat{\mathcal{N}} = \text{Vol}\left(\Omega_{d-2}^{(\lambda)}\right)\int d^{9-d}v\sqrt{e^{-4\Phi}\det g_{ij}}f^{\frac{d-1}{2}}(\vec{v})$$

$$\lambda u^3 t(u)((d-1)\lambda u^4 t'(u)^3 + (d+1)t'(u) + ut''(u)) - (d-2)(\lambda u^4 t'(u)^2 + 1) = 0$$

$$t(u) = \frac{\sqrt{R^2u^2-\lambda}}{u}, \lambda = \pm 1$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}\sqrt{\lambda^{d-2}}}S_{EE}^{(\lambda)}[\Sigma_8] = \int_1^{\frac{R}{\sqrt{\lambda}\epsilon}} dx(x^2-1)^{\frac{d-3}{2}}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}\sqrt{\lambda^{d-2}}}S_{EE}^{(\lambda)}[\Sigma_8] = \sum_{j=0}^{\left[\frac{d-3}{2}\right]}\frac{\left(\frac{3-d}{2}\right)_j}{j!\,(d-2j-2)}\left(\frac{R}{\sqrt{\lambda}\epsilon}\right)^{d-2j-2} - (-1)^{\frac{d+1}{2}}\frac{\sqrt{\pi}\Gamma\left(\frac{(d-1)}{2}\right)}{2\Gamma\left(\frac{d}{2}\right)}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}\sqrt{\lambda^{d-2}}}S_{EE}^{(\lambda)}[\Sigma_8] = \sum_{j=0}^{\left[\frac{d-3}{2}\right]}\frac{\left(\frac{3-d}{2}\right)_j}{j!\,(d-2j-2)}\left(\frac{R}{\sqrt{\lambda}\epsilon}\right)^{d-2j-2} - \frac{\Gamma\left(\frac{d-1}{2}\right)}{\Gamma\left(\frac{d}{2}\right)}\frac{(-1)^{d/2}}{\sqrt{\pi}}\left(\log\left(\frac{2R}{\epsilon\sqrt{\lambda}}\right) + \frac{1}{2}\mathcal{H}_{\frac{d-2}{2}}\right)$$

$$(d-2)!!\,c_{LM,odd} = (R\partial_R - 1)\cdots(R\partial_R - d + 2)S_{EE}^{(\lambda)}[\Sigma_8].$$

$$(d-2)!!\,c_{LM,\,\text{even}} = R\partial_R\cdots(R\partial_R - d + 2)S_{EE}^{(\lambda)}[\Sigma_8]$$

$$c_{slab} = \kappa\frac{T^{d-2}}{L^{d-2}}T\partial_T S_{EE}^{(\lambda)}[\Sigma_8]$$

$$c_{slab} = \frac{\mathcal{N}}{4G_{10}}\frac{\kappa}{L^{d-2}}\left(\frac{2\sqrt{\pi}\Gamma\left(\frac{d}{2d-2}\right)}{\Gamma\left(\frac{1}{2(d-1)}\right)}\right)^{d-1}\frac{1}{\sqrt{\lambda^{d-2}}}$$

$$F_i = 2N_i - N_{i-1} - N_{i+1},$$

$$G_{\text{global}} = SU(F_1)\times\ldots\times SU(F_P), G_{\text{local}} = SU(N_1)\times\ldots\times SU(N_P).$$

$$\mathcal{R}(\eta)=\begin{cases}N_1\eta & 0\le\eta\le 1\\ N_l+(N_{l+1}-N_l)(\eta-l) & l\le\eta\le l+1, l:=1,\dots.,P-2\\ N_{P-1}(P-\eta) & (P-1)\le\eta\le P.\end{cases}$$

$$\mathcal{R}''(\eta)=\sum_{i=1}^{P}F_i\delta(\eta-i).$$

$$R_k=\frac{2}{P}\int_0^P\mathcal{R}(\eta)\sin\left(\frac{k\pi\eta}{P}\right)d\eta=\frac{2P}{\pi^2k^2}\sum_{j=1}^{P-1}F_j\sin\left(\frac{k\pi j}{P}\right).$$

$$ds_{10}^2=f_1\left[ds_{AdS_4}^2+f_2d\Omega_2(\theta,\phi)+f_3d\tilde{\Omega}_2(\tilde{\theta},\tilde{\phi})+f_4(d\sigma^2+d\eta^2)\right]$$

$$ds_{AdS_4}^2=u^2(\lambda dt^2+dx^2+dy^2)+\frac{du^2}{u^2},e^{-4\Phi}=f_5^2$$

$$f_1=\frac{\pi}{2}\sqrt{\frac{\sigma^3\partial_{\sigma\eta}^2V}{\partial_\sigma(\sigma\partial_\eta V)}},f_2=-\frac{\partial_\eta V\partial_\sigma(\sigma\partial_\eta V)}{\sigma\Lambda},f_3=\frac{\partial_\sigma(\sigma\partial_\eta V)}{\sigma\partial_{\sigma\eta}^2V}$$

$$f_4=-\frac{\partial_\sigma(\sigma\partial_\eta V)}{\sigma^2\partial_\eta V},f_5=-16\Lambda\frac{\partial_\eta V}{\partial_{\sigma\eta}^2V},\Lambda=\partial_\eta V\partial_{\sigma\eta}^2V+\sigma\left(\partial_{\sigma\eta}^2V\right)^2+\sigma\left(\partial_\eta^2V\right)^2$$

$$V(\sigma,\eta)=\frac{1}{\sigma}\sum_{k=1}^{\infty}R_k\cos\left(\frac{k\pi\eta}{P}\right)e^{-\frac{k\pi|\sigma|}{P}}$$

$$ds_8^2=\frac{f_1}{u^2}(1-\lambda u^4t'^2(u))du^2+f_1u^2dx_1^2+f_1f_2d\Omega_2(\theta,\phi)+f_1f_3d\tilde{\Omega}_2(\tilde{\theta},\tilde{\phi})+f_1f_4(d\sigma^2+d\eta^2)$$

$$\frac{4G_{10}}{\mathcal{N}}S_{EE}[\Sigma_8]=\begin{cases}-\dfrac{4\pi\Gamma\left(\frac{3}{4}\right)^2}{\Gamma\left(\frac{1}{4}\right)^2}\dfrac{1}{|T|} & (\lambda=+1)\\ \dfrac{4\pi i\Gamma\left(\frac{3}{4}\right)^2}{\Gamma\left(\frac{1}{4}\right)^2}\dfrac{1}{|T|} & (\lambda=-1)\end{cases}$$

$$\mathcal{N}=-16\pi^6L_{x_1}\int_0^\infty d\sigma\int_0^P d\eta\sigma^2\partial_\eta V\partial_\sigma(\sigma\partial_\eta V)$$

$$T_{app}=\begin{cases}\dfrac{\pi}{2u_0} & (\lambda=+1)\\ \dfrac{-i\pi}{2u_0} & (\lambda=-1)\end{cases}$$

$$S_{EE,\text{ approx}}=\begin{cases}-\dfrac{\pi^2}{4}\dfrac{1}{|T_{app}|} & (\lambda=+1)\\ \dfrac{i\pi^2}{4}\dfrac{1}{|T_{app}|} & (\lambda=-1)\end{cases}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}}S_{EE}^{(\lambda)}[\Sigma_8] = \begin{cases} \frac{R}{\epsilon} - 1 & (\lambda = +1) \\ \frac{R}{\epsilon} - i & (\lambda = -1) \end{cases}$$

$$\hat{\mathcal{N}} = -16\pi^6 \mathrm{Vol}\left(\Omega_1^{(\lambda)}\right) \int_0^\infty d\sigma \int_0^P d\eta \sigma^2 \partial_\eta V \partial_\sigma \left(\sigma \partial_\eta V\right)$$

$$c_{LM} = \frac{\hat{\mathcal{N}}}{4G_{10}}, c_{\text{slab}} = \frac{\mathcal{N}}{4G_{10}} \frac{\kappa}{L} \frac{4\pi\Gamma\left(\frac{3}{4}\right)^2}{\Gamma\left(\frac{1}{4}\right)^2}$$

$$c_{LM} \propto c_{slab} \propto c_{hol} \sim \sum_{k=1}^{\infty} k R_k^2$$

$$ds_{10}^2 = \sqrt{f_1^3 f_5}\left[4ds_{AdS_5}^2 + f_2 d\Omega_2(\theta, \phi) + f_3 d\chi^2 + f_4(d\sigma^2 + d\eta^2)\right]$$

$$ds_{AdS_5}^2 = u^2(\lambda dt^2 + dx_1^2 + dx_2^2 + dy^2) + \frac{du^2}{u^2}$$

$$e^{-4\Phi} = (f_1 f_5)^{-3}$$

$$f_1^3 = \frac{\dot{V}\Delta}{2V''}, f_2 = \frac{2V''\dot{V}}{\Delta}, f_3 = \frac{4\sigma^2 V''}{2\dot{V} - \ddot{V}}, f_4 = \frac{2V''}{\dot{V}}, f_5 = \frac{2(2\dot{V} - \ddot{V})}{\dot{V}\Delta}$$

$$\Delta = (2\dot{V} - \ddot{V})V'' + (\dot{V}')^2, \dot{V} = \sigma\partial_\sigma V, V'' = \partial_\eta^2 V$$

$$V(\sigma, \eta) = -\sum_{k=1}^{\infty} R_k \sin\left(\frac{k\pi\eta}{P}\right) K_0\left(\frac{k\pi\sigma}{P}\right)$$

$$\frac{4G_{10}}{\mathcal{N}} S_{EE}\left[\Sigma_8^{(\lambda)}\right] = \begin{cases} -\frac{4\pi^{3/2}\Gamma\left(\frac{2}{3}\right)^3}{\Gamma\left(\frac{1}{6}\right)^3} \frac{1}{|T|^2} & (\lambda = +1) \\ \frac{4\pi^{3/2}\Gamma\left(\frac{2}{3}\right)^3}{\Gamma\left(\frac{1}{6}\right)^3} \frac{1}{|T|^2} & (\lambda = -1) \end{cases}$$

$$\mathcal{N} = 256\pi^2 L_{x_1} L_{x_2} \int_0^\infty d\sigma \int_0^P d\eta \sigma \dot{V} V''$$

$$T_{app} = \begin{cases} \frac{\pi}{3u_0} & (\lambda = +1) \\ \frac{-i\pi}{3u_0} & (\lambda = -1) \end{cases}$$

$$S_{EE,\,\text{approx}} = \begin{cases} -\dfrac{\pi^3}{54}\dfrac{1}{|T_{app}|^2} & (\lambda = +1) \\ \dfrac{\pi^3}{54}\dfrac{1}{|T_{app}|^2} & (\lambda = -1) \end{cases}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}} S_{EE}\left[\hat{\Sigma}_8^{(\lambda)}\right] = \begin{cases} \dfrac{R^2}{2\epsilon^2} - \dfrac{1}{2}\log\left(\dfrac{2R}{\epsilon}\right) - \dfrac{1}{4} & (\lambda = +1) \\ \dfrac{R^2}{2\epsilon^2} + \dfrac{1}{2}\log\left(\dfrac{2R}{\epsilon}\right) + \dfrac{1}{4}(1 - i\pi) & (\lambda = -1) \end{cases}$$

$$\hat{\mathcal{N}} = 256\pi^2 \text{Vol}\left(\Omega_2^{(\lambda)}\right) \int_0^\infty d\sigma \int_0^P d\eta \sigma \dot{V} V''$$

$$c_{LM} = \frac{\hat{\mathcal{N}}}{8G_{10}}, c_{slab} = \frac{\mathcal{N}}{4G_{10}} \frac{\kappa}{L_{x_1} L_{x_2}} \frac{8\pi^{3/2}\Gamma\left(\frac{2}{3}\right)^3}{\Gamma\left(\frac{1}{6}\right)^3}$$

$$c_{LM} \propto c_{slab} \propto c_{hol} \sim P \sum_{k=1}^{\infty} R_k^2$$

$$ds_{10}^2 = f_1\left[ds_{AdS_6}^2 + f_2 d\Omega_2(\theta, \phi) + f_3(d\sigma^2 + d\eta^2)\right]$$

$$ds_{AdS_6}^2 = u^2(\lambda dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dy^2) + \frac{du^2}{u^2}$$

$$e^{-4\Phi} = f_6^2$$

$$f_1 = \frac{2}{3}\left(\frac{\sigma\left(\sigma\partial_\eta^2 V + 3\partial_\sigma V\right)}{\partial_\eta^2 V}\right)^{1/2}, f_2 = \frac{\partial_\sigma V \partial_\eta^2 V}{3\tilde{\Lambda}}, f_3 = \frac{\partial_\eta^2 V}{3\sigma\partial_\sigma V}$$

$$f_6 = 18^2 \frac{3\sigma^2 \partial_\sigma V \partial_\eta^2 V \tilde{\Lambda}}{\left(3\partial_\sigma V + \sigma\partial_\eta^2 V\right)^2}, \tilde{\Lambda} = \sigma\left(\partial_{\sigma\eta}^2 V\right)^2 + \partial_\eta^2 V(\partial_\sigma V - \sigma\partial_\sigma^2 V)$$

$$V(\sigma, \eta) = -\frac{1}{\sigma}\sum_{k=1}^{\infty} \frac{P}{2\pi k} R_k \sin\left(\frac{k\pi\eta}{P}\right) e^{-\frac{k\pi|\sigma|}{P}}$$

$$\frac{4G_{10}}{\mathcal{N}} S_{EE}\left[\Sigma_8^{(\lambda)}\right] = \begin{cases} -\dfrac{16\pi^2\Gamma\left(\frac{5}{8}\right)^4}{3\Gamma\left(\frac{1}{8}\right)^4}\dfrac{1}{|T|^3} & (\lambda = +1) \\ -\dfrac{16i\pi^2\Gamma\left(\frac{5}{8}\right)^4}{3\Gamma\left(\frac{1}{8}\right)^4}\dfrac{1}{|T|^3} & (\lambda = -1) \end{cases}$$

$$\mathcal{N} = \frac{2^8\pi}{3} L_{x_1} L_{x_2} L_{x_3} \int_0^\infty d\sigma \int_0^P d\eta \sigma^3 \partial_\sigma V \partial_\eta^2 V$$

$$T_{app} = \begin{cases} \dfrac{\pi}{4u_0} & (\lambda = +1) \\ \dfrac{-i\pi}{4u_0} & (\lambda = -1) \end{cases}$$

$$S_{EE,\,\text{approx}} = \begin{cases} -\dfrac{\pi^4}{768}\dfrac{1}{|T_{app}|^3} & (\lambda = +1) \\ -\dfrac{i\pi^4}{768}\dfrac{1}{|T_{app}|^3} & (\lambda = -1) \end{cases}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}} S_{EE}\left[\hat{\Sigma}_8^{(\lambda)}\right] = \begin{cases} \dfrac{R^3}{3\epsilon^3} - \dfrac{R}{\epsilon} + \dfrac{2}{3} & (\lambda = +1) \\ \dfrac{R^3}{3\epsilon^3} + \dfrac{R}{\epsilon} - i\dfrac{2}{3} & (\lambda = -1) \end{cases}$$

$$\hat{\mathcal{N}} = \frac{2^8\pi}{3}\mathrm{Vol}\left(\Omega_3^{(\lambda)}\right)\int_0^\infty d\sigma \int_0^P d\eta \sigma^3 \partial_\sigma V \partial_\eta^2 V$$

$$c_{LM} = \frac{\hat{\mathcal{N}}}{4G_{10}}\frac{2}{3}, c_{\text{slab}} = \frac{\mathcal{N}}{4G_{10}}\frac{\kappa}{L_{x_1}L_{x_2}L_{x_3}}\frac{16\pi^2\Gamma\left(\frac{5}{8}\right)^4}{\Gamma\left(\frac{1}{8}\right)^4}$$

$$c_{LM} \propto c_{slab} \propto c_{hol} \sim P^2 \sum_{k=1}^{\infty} \frac{R_k^2}{k}$$

$$ds_{10}^2 = f_1 ds_{AdS_7}^2 + f_2 d\eta^2 + f_3 d\Omega_2(\theta, \phi)$$

$$ds_{AdS_6}^2 = u^2(\lambda dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dy^2) + \frac{du^2}{u^2}$$

$$e^{-4\Phi} = f_6^{-4}$$

$$f_1 = 8\sqrt{2}\pi\sqrt{-\frac{V}{V''}}, f_2 = \sqrt{2}\pi\sqrt{-\frac{V''}{V}}$$

$$f_3 = f_2\frac{V^2}{(V'^2 - 2VV'')}, f_6^4 = \frac{2^5\pi^{10}3^{16}\left(-\frac{V}{V''}\right)^3}{(V'^2 - 2VV'')^2}$$

$$V''' = -162\pi^3 F_0$$

$$\frac{4G_{10}}{\mathcal{N}} S_{EE}\left[\Sigma_8^{(\lambda)}\right] = \begin{cases} -\dfrac{8\pi^{5/2}\Gamma\left(\frac{3}{5}\right)^5}{\Gamma\left(\frac{1}{10}\right)^5}\dfrac{1}{|T|^4} & (\lambda = +1) \\ -\dfrac{8\pi^{5/2}\Gamma\left(\frac{3}{5}\right)^5}{\Gamma\left(\frac{1}{10}\right)^5}\dfrac{1}{|T|^4} & (\lambda = -1) \end{cases}$$

$$\mathcal{N} = \frac{4}{3}\left(\frac{2}{3}\right)^7 L_{x_1} \ldots . L_{x_4} \int_0^P (-V''V) d\eta$$

$$T_{app} = \begin{cases} \dfrac{\pi}{5u_0} & (\lambda = +1) \\ \dfrac{-i\pi}{5u_0} & (\lambda = -1) \end{cases}$$

$$S_{EE,\,\text{approx}} = \begin{cases} -\dfrac{\pi^5}{12500}\dfrac{1}{|T_{app}|^4} & (\lambda = +1) \\ -\dfrac{\pi^5}{12500}\dfrac{1}{|T_{app}|^4} & (\lambda = -1) \end{cases}$$

$$\frac{4G_{10}}{\hat{\mathcal{N}}} S_{EE}^{(\lambda)}\left[\hat{\Sigma}_8^{(\lambda)}\right] = \begin{cases} \dfrac{R^4}{4\epsilon^4} - \dfrac{3}{4}\dfrac{R^2}{\epsilon^2} + \dfrac{3}{8}\left(\log\left(\dfrac{2R}{\epsilon}\right) + \dfrac{3}{4}\right) & (\lambda = +1) \\ \dfrac{R^4}{4\epsilon^4} + \dfrac{3}{4}\dfrac{R^2}{\epsilon^2} + \dfrac{3}{8}\left(\log\left(\dfrac{2R}{\epsilon}\right) - \dfrac{i\pi}{2} + \dfrac{3}{4}\right) & (\lambda = -1) \end{cases}$$

$$\hat{\mathcal{N}} = \frac{4}{3}\left(\frac{2}{3}\right)^7 \text{Vol}\left(\Omega_4^{(\lambda)}\right)\int_0^P (-V''V)d\eta$$

$$c_{LM} = \frac{\hat{\mathcal{N}}}{4G_{10}}\frac{3}{8}, c_{slab} = \frac{\mathcal{N}}{4G_{10}}\frac{\kappa}{L_{x_1}\ldots L_{x_4}}\frac{32\pi^{5/2}\Gamma\left(\frac{3}{5}\right)^5}{\Gamma\left(\frac{1}{10}\right)^5}$$

$$c_{LM} \propto c_{slab} \propto c_{hol} \sim P^3 \sum_{k=1}^{\infty} \frac{R_k^2}{k^2}$$

$$ds^2 = \frac{\hat{G}}{\sqrt{\hat{h}_4 h_8}}\left(ds^2(\text{AdS}_3) + \frac{h_8\hat{h}_4}{4h_8\hat{h}_4 + \left(\hat{G}'\right)^2} ds^2(\text{S}^2)\right) + \sqrt{\frac{\hat{h}_4}{h_8}} ds^2(\text{CY}_2) + \frac{\sqrt{\hat{h}_4 h_8}}{\hat{G}} d\rho^2$$

$$e^{-\Phi} = \frac{h_8^{\frac{3}{4}}}{2\hat{h}_4^{\frac{1}{4}}\sqrt{\hat{G}}}\sqrt{4h_8\hat{h}_4 + \left(\hat{G}'\right)^2} \cdot\ ds^2(\text{AdS}_3) = \frac{u^2}{l^2}(-dt^2 + dy^2) + \frac{l^2 du^2}{u^2}$$

$$ds_{\Sigma_8}^2 = \frac{\hat{G}}{\sqrt{\hat{h}_4 h_8}}\left(du^2\left[\frac{l^2}{u^2} + \frac{u^2}{l^2}(y'^2 - t'^2)\right] + \frac{h_8\hat{h}_4}{4h_8\hat{h}_4 + \left(\hat{G}'\right)^2} ds^2(\text{S}^2)\right) + \sqrt{\frac{\hat{h}_4}{h_8}} ds^2(\text{CY}_2) + \frac{\sqrt{\hat{h}_4 h_8}}{\hat{G}} d\rho^2$$

$$e^{-4\Phi}\det\left[g_{\Sigma_8}\right] = \frac{\hat{h}_4^2 h_8^2}{16}\text{Vol}_{S^2}\text{Vol}_{CY_2}\left[\frac{l^2}{u^2} + \frac{u^2}{l^2}(y'^2 - t'^2)\right]$$

$$\frac{4G_{10}}{\mathcal{N}} S_{EE} = \int_{u_0}^{\infty} du \sqrt{\frac{l^2}{u^2} + \frac{u^2}{l^2}(y'^2 - t'^2)}$$

$$\mathcal{N} = \pi\text{Vol}_{CY_2}\int_0^P \hat{h}_4 h_8 d\rho$$

$$\frac{T}{2c_t l^3}=\frac{Y}{2c_y l^3}=\int_{u_0}^{\infty}\frac{du}{u^2\sqrt{u^2-u_0^2}}=\frac{1}{u_0^2}$$

$$\frac{4G_{10}}{\mathcal{N}}S_{EE}=\lim_{\Lambda\to\infty}\int_{u_0}^{\Lambda}\frac{du}{\sqrt{u^2-u_0^2}}-\int_{\frac{1}{\Lambda}}^{\Lambda}\frac{du}{u}=\log\left[\frac{\sqrt{Y^2-T^2}}{\Lambda l^3}\right]$$

$$ds_{10}^2=\frac{u^2}{l^2}[\lambda dt^2+dy^2+dx^2+f(u)d\phi^2]+\frac{l^2du^2}{f(u)u^2}+l^2d\tilde{\Omega}_5^2$$

$$d\tilde{\Omega}_5^2=d\theta^2+\sin^2\theta d\psi^2+\sin^2\theta\sin^2\psi(d\varphi_1-A_1)^2+\sin^2\theta\cos^2\psi(d\varphi_2-A_1)^2+\cos^2\theta(d\varphi_3-A_1)^2$$

$$A_1=Q\left(1-\frac{l^2Q^2}{u^2}\right)d\phi, f(u)=1-\left(\frac{Ql}{u}\right)^6$$

$$ds_8^2=\left[\frac{u^2}{l^2}(\lambda t'^2+y'^2)+\frac{l^2}{u^2f(u)}\right]du^2+\frac{u^2}{l^2}dx^2+\frac{u^2}{l^2}f(u)d\phi^2+l^2d\tilde{\Omega}_5^2$$

$$S_{EE}^{(\lambda)}=\frac{\hat{\mathcal{N}}}{4G_{10}}\int_{u_0}^{\infty}\sqrt{G^2(u)+F^2(u)(\lambda t'^2(u)+y'^2(u))}$$

$$G(u)=\frac{u}{l}, F(u)=\frac{u^3}{l^3}\sqrt{f(u)}, \hat{\mathcal{N}}=l^5L_xL_\phi\mathrm{Vol}(\tilde{S}^5), L_\phi=\frac{1}{3Q}$$

$$\frac{T}{2l^5c_t}=\frac{Y}{2l^5c_y}=\int_{u_0}^{\infty}\frac{udu}{\sqrt{u^6-u_\Lambda^6}\sqrt{u^6-u_0^6}}$$

$$\frac{2lG_{10}}{\hat{\mathcal{N}}}S_{EE}=\int_{u_0}^{\infty}du\frac{u\sqrt{u^6-u_\Lambda^6}}{\sqrt{u^6-u_0^6}}-\int_{u_\Lambda}^{\infty}udu$$

$$u_0=l\left(Q^6+c_y^2-c_t^2\right)^{1/6}$$

$$u_0=le^{i\pi/6}\left|Q^6+c_y^2-c_t^2\right|^{1/6}$$

$$r=\frac{u_0}{u}, \gamma=\frac{u_\Lambda}{u_0}$$

$$J_1=\int_0^1 dr\frac{r^3}{\sqrt{(1-r^6)(1-\gamma^6r^6)}}=\frac{\sqrt{\pi}\Gamma\left(\frac{5}{3}\right)}{4\Gamma\left(\frac{7}{6}\right)}{}_2F_1\left[\frac{1}{2},\frac{2}{3},\frac{7}{6},\gamma^6\right]$$

$$J_2=\int_\epsilon^1\frac{dr}{r^3}\sqrt{\frac{1-\gamma^6r^6}{1-r^6}}=\mathcal{J}_2+\frac{1}{2\epsilon^2}+O(\epsilon^4)$$

$$\mathcal{J}_2=-\frac{\sqrt{\pi}\Gamma\left(\frac{2}{3}\right)}{2\Gamma\left(\frac{1}{6}\right)}{}_2F_1\left[-\frac{1}{2},-\frac{1}{3},\frac{1}{6},\gamma^6\right], J_3=\int_\epsilon^{1/\gamma}\frac{dr}{r^3}=\frac{1}{2\epsilon^2}-\frac{\gamma^2}{2}$$

$$c_t=\frac{Tu_0^4}{2J_1l^5}, c_y=\frac{Yu_0^4}{2J_1l^5}, \frac{2lG_{10}}{\hat{\mathcal{N}}u_0^2}S_{EE}=J_2-J_3$$

$$\Delta^2 = (1-\gamma^6)\frac{4l^4 J_1^2}{u_0^2}, S_{EE} = \frac{\hat{\mathcal{N}}}{2lG_N}(J_2 - J_3)u_0^2$$

$$\mathrm{d}s^2 = \frac{\zeta(u,\theta)}{L^2}\left[u^2\left(-\mathrm{d}t^2 + \mathrm{d}y^2 + \mathrm{d}x^2 + L^2\tilde{f}(u)\mathrm{d}\phi^2\right) + \frac{L^2\,\mathrm{d}u^2}{\tilde{f}(u)u^2\lambda^6(u)} + L^4\,\mathrm{d}\theta^2\right]$$
$$+\frac{L^2}{\zeta(u,\theta)}[\cos^2\,\theta\,\mathrm{d}\psi^2 + \cos^2\,\theta\sin^2\,\psi\mathrm{D}\phi_1^2 + \cos^2\,\theta\cos^2\,\psi\mathrm{D}\phi_2^2 + \lambda^6(u)\sin^2\,\theta\mathrm{D}\phi_3^2]$$

$$A^1 = A^2 = q_1[\lambda^6(u) - \lambda^6(u_\star)]L\,\mathrm{d}\phi, A^3 = q_2\left[\frac{1}{\lambda^6(u)} - \frac{1}{\lambda^6(u_\star)}\right]L\,\mathrm{d}\phi.$$

$$\zeta(u,\theta) = \sqrt{1+\varepsilon\frac{\ell^2}{u^2}\cos^2\,\theta}, \lambda^6(u) = \frac{u^2+\varepsilon\ell^2}{u^2}$$
$$\tilde{f}(u) = \frac{1}{L^2} - \frac{\varepsilon\ell^2 L^2}{u^4}\left(q_1^2 - \frac{q_2^2}{\lambda^6(u)}\right)$$

$$ds_{\Sigma_8}^2 = \frac{\zeta(u,\theta)}{L^2}\left[u^2\left(\,\mathrm{d}x^2 + L^2\tilde{f}(u)\mathrm{d}\phi^2\right) + \frac{L^2\,\mathrm{d}u^2}{\tilde{f}(u)u^2\lambda^6(u)}\left[1 + \frac{\tilde{f}(u)\lambda^6(u)u^4}{L^2}(-t'^2 + y'^2)\right] + L^4\,\mathrm{d}\theta^2\right]$$
$$+\frac{L^2}{\zeta(u,\theta)}[\cos^2\,\theta\,\mathrm{d}\psi^2 + \cos^2\,\theta\sin^2\,\psi\mathrm{D}\phi_1^2 + \cos^2\,\theta\cos^2\,\psi\mathrm{D}\phi_2^2 + \lambda^6(u)\sin^2\,\theta\mathrm{D}\phi_3^2]$$
$$e^{-4\Phi}\det\left[g_{\Sigma_8}\right] = L^8\cos^6\,\theta\sin^2\,\theta\cos^2\,\psi\sin^2\,\psi\left[u^2 + \frac{\tilde{f}(u)\lambda^6(u)u^6}{L^2}(y'^2 - t'^2)\right]$$

$$\frac{4G_{10}}{\mathcal{N}_{II}}S_{EE} = \int_{u_0}^{\infty} du\sqrt{G^2(u) + F^2(u)(y'^2 - t'^2)}$$

$$G(u) = u, F(u) = \frac{u^3\lambda^3(u)\sqrt{\tilde{f}(u)}}{L}, \text{ and}$$
$$\mathcal{N}_{II} = L^4 L_x L_\phi \int d\theta d\psi d\phi_1 d\phi_2 d\phi_3 \cos^3\,\theta\sin\,\theta\cos\,\psi\sin\,\psi$$

$$G(u) = u, \tilde{f}(u) = \frac{[u^6 + \epsilon\ell^2 u^4 - L^6 q^6]}{L^2 u^4(u^2+\epsilon\ell^2)}$$
$$F(u) = \frac{1}{L^2}[u^6 + \epsilon\ell^2 u^4 - L^6 q^6]^{\frac{1}{2}}$$

$$\frac{T}{2c_t L^4} = \frac{Y}{2c_y L^4} = \int_{u_0}^{\infty} du\frac{u}{\sqrt{(u^6 + \epsilon\ell^2 u^4 - L^6 q^6)[(u^6 - u_0^6) + \epsilon\ell^2(u^4 - u_0^4)]}}$$

$$\frac{2G_{10}}{\mathcal{N}_{II}}S_{EE} = \int_{u_0}^{\infty} duu\sqrt{\frac{(u^6 + \epsilon\ell^2 u^4 - L^6 q^6)}{[(u^6 - u_0^6) + \epsilon\ell^2(u^4 - u_0^4)]}} - \int_{u_*}^{\infty} udu$$

$$u = \frac{u_0}{r}, \qquad \mu = \frac{\epsilon\ell^2}{u_0^2}, \nu = \frac{L^6 q^6}{u_0^6}$$

$$\frac{Tu_0^4}{2c_tL^4}=\frac{Yu_0^4}{2c_yL^4}=\int_0^1 dr\frac{r^3}{\sqrt{(1+\mu r^2-\nu r^6)[(1-r^6)+\mu r^2(1-r^4)]}}$$

$$\frac{2G_{10}}{\mathcal{N}_{II}u_0^2}S_{EE}=\int_0^1\frac{dr}{r^3}\sqrt{\frac{(1+\mu r^2-\nu r^6)}{[(1-r^6)+\mu r^2(1-r^4)]}}-\int_0^{\frac{u_0}{u_*}}\frac{dr}{r^3}$$

$$\Delta^2=Y^2-T^2=\frac{4L^2}{u_0^2}(1+\mu-\nu)\mathcal{J}_1^2.$$

$$F(u_0)^2=c_y^2-c_t^2,\longrightarrow u_0^6+\epsilon\ell^2u_0^4=L^6\left(q^6+c_y^2-c_t^2\right).$$

$$27u_0^6=\left[-\epsilon\ell^2+\frac{\ell^4}{\mathcal{Z}}+\mathcal{Z}\right]^3,$$

$$\mathcal{Z}^3=\frac{27m}{2}\left(1-\frac{2\epsilon\ell^6}{27m}+\sqrt{1-\frac{4\epsilon\ell^6}{27m}}\right),m=L^6\left(q^6+c_y^2-c_t^2\right).$$

$$\mathrm{d}s^2=f_1^{\frac{3}{2}}f_5^{\frac{1}{2}}[4\,\mathrm{d}s_5^2+f_2D\mu_iD\mu_i+f_4(\,\mathrm{d}\sigma^2+\mathrm{d}\eta^2)+f_3(\,\mathrm{d}\chi+\mathcal{A})^2],e^{\frac{4}{3}\Phi}=f_1f_5$$

$$\mathrm{d}s_5^2=\frac{u^2}{l^2}(\lambda\,\mathrm{d}t^2+\mathrm{d}y^2+\mathrm{d}x^2+f(u)\mathrm{d}\phi^2)+\frac{l^2\,\mathrm{d}u^2}{u^2f(u)}$$

$$f(u)=1-\frac{Q^2l^2}{u^6},\mathcal{A}=Q\left(\frac{1}{u^2}-\frac{1}{u_*^2}\right)\mathrm{d}\phi,u_*=(Ql)^{1/3}$$

$$\mu_1=\sin\,\theta\sin\,\varphi,\mu_2=\sin\,\theta\cos\,\varphi,\mu_3=\cos\,\theta$$

$$D\mu_1=\mathrm{d}\mu_1+2\mu_2\mathcal{A},D\mu_2=\mathrm{d}\mu_2-2\mu_1\mathcal{A},D\mu_3=\mathrm{d}\mu_3$$

$$f_1=\left(\frac{\dot{V}\tilde{\Delta}}{2V''}\right)^{\frac{1}{3}},f_2=\frac{2V''\dot{V}}{\tilde{\Delta}},f_3=\frac{4\sigma^2}{\Lambda},f_4=\frac{2V''}{\dot{V}}$$

$$f_5=\frac{2\Lambda V''}{\dot{V}\tilde{\Delta}},\tilde{\Delta}=\Lambda(V'')^2+(\dot{V}')^2,\Lambda=\frac{2\dot{V}-\ddot{V}}{V''}$$

$$ds_8^2=(f_1^3f_5)^{\frac{1}{2}}\left(4\left[\frac{u^2}{l^2}(\lambda t'^2+y'^2)+\frac{l^2}{u^2f(u)}\right]du^2+\frac{4u^2}{l^2}dx^2+\frac{4u^2}{l^2}f(u)d\phi^2+\right.$$
$$\left.f_2D\mu_iD\mu_i+f_4(\,\mathrm{d}\sigma^2+\mathrm{d}\eta^2)+f_3(\,\mathrm{d}\chi+\mathcal{A})^2\right)$$

$$e^{-4\Phi}\det\left[g_{\Sigma_8}\right]=f_1^9f_2^2f_4^2f_3f_5\mathrm{Vol}_{S^2(\theta,\varphi)}\mathrm{Vol}_{S^1(\chi)}\left[\frac{u^2}{l^2}+\frac{u^6}{l^6}f(u)(y'^2+\lambda t'^2)\right]$$

$$S_{EE}^{(\lambda)}=\frac{\tilde{\mathcal{N}}}{4G_{10}}\int_{u_0}^{\infty}\sqrt{G^2(u)+F^2(u)(\lambda t'^2(u)+y'^2(u))}$$

$$G(u)=\frac{u}{l},F(u)=\frac{u^3}{l^3}\sqrt{f(u)},\tilde{\mathcal{N}}=256\pi^2L_xL_\phi\int_0^\infty d\sigma\int_0^P d\eta\sigma\dot{V}V'',L_\phi=\frac{1}{3Q}$$

$$\mathrm{d}s_{10}^2=\tilde{f}_1^{\frac{3}{2}}\tilde{f}_5^{\frac{1}{2}}\left[4\tilde{\gamma}\,\mathrm{d}s_5^2+\tilde{f}_2\mathrm{D}\mu_i\mathrm{D}\mu^i+\tilde{f}_3(\,\mathrm{d}\chi+B)^2+\tilde{f}_4(\,\mathrm{d}\sigma^2+\mathrm{d}\eta^2)\right]$$

$$e^{\frac{4}{3}\Phi}=\tilde{f}_1\tilde{f}_5$$

$$\tilde{f}_1 = \left(\frac{\dot{V}\tilde{\Delta}}{2V''}\right)^{1/3}, \tilde{Y} = \frac{Z}{X(u)} = \frac{Z}{\lambda^2(u)}, \tilde{f}_2 = \frac{2\dot{V}V''}{Z^2\tilde{\Delta}}, \tilde{f}_3 = \frac{4X^3\sigma^2V''}{2X^3\dot{V} - \ddot{V}}Z$$
$$\tilde{f}_4 = \frac{2V''}{\dot{V}}Z, \tilde{f}_5 = \frac{2(2X^3\dot{V} - \ddot{V})}{Z^2\dot{V}\tilde{\Delta}}$$
$$\tilde{\Delta} = (\dot{V}')^2 + V''(2\dot{V} - \ddot{V}), Z = \left[\frac{(\dot{V}')^2 + V''(2X^3\dot{V} - \ddot{V})}{(\dot{V}')^2 + V''(2\dot{V} - \ddot{V})}\right]^{1/3}$$

$$ds_5^2 = \frac{u^2\lambda(u)^2}{L^2}\left(-dt^2 + dy^2 + dx^2 + L^2\tilde{f}(u)d\phi^2\right) + \frac{du^2}{u^2\lambda(u)^4\tilde{f}(u)}$$
$$\mu_1 = \sin\theta\cos\varphi, \mu_2 = \sin\theta\sin\varphi, \mu_3 = \cos\theta$$
$$D\mu_i = \left(d\mu_1 + 2\mu_2 A_{1\phi}d\phi\right)\delta_{i,1} + \left(d\mu_2 - 2\mu_1 A_{1\phi}d\phi\right)\delta_{i,2} + d\mu_3\delta_{i,3}$$

$$e^{-4\Phi}\det\left[g_{\Sigma_8}\right] = 4^3\tilde{f}_1^9\tilde{f}_2^2\tilde{f}_4^2\tilde{f}_3\tilde{f}_5\tilde{\gamma}^3\left[\frac{u^2}{L^2} + \frac{u^6\lambda^6(u)\tilde{f}(u)}{L^4}(y'^2 - t'^2)\right]\mathrm{Vol}_{S^2(\theta,\varphi)}\mathrm{Vol}_{S^1(\chi)}$$

$$\tilde{\mathcal{N}}_{II} = 256\pi^2 L_x L_\phi \int_0^\infty d\sigma \int_0^P d\eta\sigma\dot{V}V''$$

$$ds_{st}^2 = e^{\Phi}\hat{h}^{-\frac{1}{2}}(u)\left[\lambda dt^2 + dy^2 + dx_1^2 + dx_2^2 + \hat{h}(u)e^{2k(u)}du^2 + \hat{h}(u)e^{2h(u)}(d\theta^2 + \sin^2\theta d\varphi^2)\right.$$
$$\left. + \frac{\hat{h}(u)e^{2g(u)}}{4}\left[\left(\omega_1 - A^{(1)}\right)^2 + \left(\omega_2 - A^{(2)}\right)^2\right] + \frac{\hat{h}(u)e^{2k(u)}}{4}(\omega_3 -\right.$$
$$\Phi = \Phi(u)$$

$$\omega_1 = \cos\tilde{\psi}d\tilde{\theta} + \sin\tilde{\psi}\sin\tilde{\theta}d\tilde{\varphi}, \omega_2 = -\sin\tilde{\psi}d\tilde{\theta} + \cos\tilde{\psi}\sin\tilde{\theta}d\tilde{\varphi}, \omega_3 = d\tilde{\psi} + \cos\tilde{\theta}d\tilde{\varphi}$$
$$A^{(1)} = -a(u)d\theta, A^{(2)} = a(u)\sin\theta d\varphi, A^{(3)} = -\sin\theta d\varphi$$

$$e^{2h} = \frac{(P^2 - Q^2)}{4(P\coth(2u) - Q)}, e^{2g} = (P\coth(2u) - Q), e^{2k} = \frac{P'}{2}, e^{4\Phi} = 2e^{4\Phi_0}\frac{\sinh^2(2u)}{(P^2 - Q^2)P'}$$
$$a = \frac{P}{P\cosh(2u) - Q\sinh(2u)}, \hat{h} = 1 - \kappa^2 e^{2\Phi}, Q = N_c(2u\coth(2u) - 1)$$

$$P'' + P'\left(\frac{P' - Q'}{P + Q} + \frac{P' + Q'}{P - Q} - 4\coth(2u)\right) = 0$$

$$P = 2N_c u$$

$$P = e^{\frac{4u}{3}}\left[c_+ + \frac{e^{-\frac{8u}{3}}N_c^2}{c_+}\left(4u^2 - 4u + \frac{13}{4}\right) + e^{-4u}\left(c_- - \frac{8c_+}{3}u\right)\right.$$
$$\left. + \frac{N_c^4 e^{-\frac{16u}{3}}}{c_+^3}\left(\frac{18567}{512} + \frac{2781}{32}u + \frac{27}{4}u^2 + 36u^3\right) + O\left(e^{-\frac{20u}{3}}\right)\right]$$

$$P = h_1 u + \frac{4h_1}{15}\left(1 - \frac{4N_c^2}{h_1^2}\right)u^3 + \frac{16h_1}{525}\left(1 - \frac{4N_c^2}{3h_1^2} - \frac{32N_c^4}{3h_1^4}\right)u^5 + O(u^7).$$

$$ds^2_{\Sigma_8} = e^{\Phi}\hat{h}^{-\frac{1}{2}}(u)[dx_1^2 + dx_2^2 + (\hat{h}(u)e^{2k(u)} + \lambda t'^2 + y'^2)du^2 + \hat{h}(u)e^{2h(u)}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{\hat{h}(u)e^{2g(u)}}{4}\left[(\omega_1 - A^{(1)})^2 + (\omega_2 - A^{(2)})^2\right] + \frac{\hat{h}(u)e^{2k(u)}}{4}(\omega_3 - A^{(3)})^2\Big]$$

$$e^{-4\Phi}\det[g_{\Sigma_8}] = \frac{\hat{h}^2}{64}e^{4\Phi+4h+4g+4k}\left(\frac{e^{-2k}}{\hat{h}}(\lambda t'^2 + y'^2) + 1\right)\mathrm{Vol}_{S^2}\mathrm{Vol}_{S^3}$$

$$4G_{10}S_{EE} = \mathcal{N}_{III}\int du\sqrt{G^2(u) + F^2(u)(y'^2 + \lambda t'^2)}$$

$$G^2(u) = e^{4\Phi+4h+4g+4k}\hat{h}^2 = \frac{e^{4\Phi_0}}{32}\sinh^2(2u)(P^2 - Q^2)P'\hat{h}^2$$

$$F^2(u) = \frac{e^{-2k}G^2(u)}{\hat{h}} = \frac{e^{4\Phi_0}}{16}\sinh^2(2u)(P^2 - Q^2)\hat{h}$$

En tanto que la supersimetría de gauge respecto de un campo cuántico relativista en supergravedad, queda expresada así:

$$S_{\mathrm{SYM}} = \int d^{10}x\mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\chi\gamma^{\mu}\mathcal{D}_{\mu}\chi\right]$$

$$(\bar{\gamma}^{\mu})^{\alpha\lambda}(\gamma^{\nu})_{\lambda\beta} + (\bar{\gamma}^{\nu})^{\alpha\lambda}(\gamma^{\mu})_{\lambda\beta} = 2\eta^{\mu\nu}\delta^{\alpha}_{\beta}$$

$$\delta_{\Lambda}A_{\mu} = \mathcal{D}_{\mu}\Lambda, \delta_{\Lambda}\chi = [\chi, \Lambda]$$

$$\delta_{\xi}A_{\mu} = \xi^{\rho}\partial_{\rho}A_{\mu} + \partial_{\mu}\xi^{\rho}A_{\rho}, \delta_{\xi}\chi = \xi^{\mu}\partial_{\mu}\chi + \frac{1}{4}\omega^{\mu\nu}\gamma_{\mu\nu}\chi$$

$$\delta_{\epsilon}A_{\mu} = i\epsilon\gamma_{\mu}\chi, \delta_{\epsilon}\chi = \frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon$$

$$(\gamma^{\mu\nu})^{\alpha}_{\beta} := \frac{1}{2}\left[(\bar{\gamma}^{\mu})^{\alpha\lambda}(\gamma^{\nu})_{\lambda\beta} - (\bar{\gamma}^{\nu})^{\alpha\lambda}(\gamma^{\mu})_{\lambda\beta}\right].$$

$$\mathcal{D}_{\nu}F^{\nu\mu} - \frac{i}{2}[\chi, \gamma^{\mu}\chi] = 0, i\gamma^{\mu}\mathcal{D}_{\mu}\chi = 0$$

$$\varphi = \partial \cdot A$$

$$S_{\mathrm{SYM}} = \int d^{10}x\mathrm{Tr}\Big[\frac{1}{2}A_{\mu}\,\square\,A^{\mu} + (\partial \cdot A)\varphi - \frac{1}{2}\varphi^2 + \frac{i}{2}\chi\gamma^{\mu}\partial_{\mu}\chi - \partial_{\mu}A_{\nu}[A^{\mu}, A^{\nu}] + \frac{i}{2}\chi\gamma^{\mu}[A_{\mu}, \chi] - \frac{1}{4}[A_{\mu}, A_{\nu}][A^{\mu}, A^{\nu}]\Big].$$

$$\mathcal{X} = \bigoplus_{i\in\mathbb{Z}} X_i$$

$$B_n: \mathcal{X}^{\otimes n} \longrightarrow \mathcal{X}, |B_n| = 1, n \geq 1,$$

$$S = \sum_{n=1}^{\infty} \frac{1}{(n+1)!} \langle \mathcal{A}, B_n(\underbrace{\mathcal{A}, \dots, \mathcal{A}}_{n \text{ times}}) \rangle$$

$$\frac{\delta S}{\delta \mathcal{A}} = \sum_{n=1}^{\infty} \frac{1}{n!} B_n(\underbrace{\mathcal{A}, \dots, \mathcal{A}}_{n \text{ times}})$$

$$\xrightarrow{B_1} \underset{\Lambda}{X_{-1}} \xrightarrow{B_1} \underset{\mathcal{A}}{X_0} \xrightarrow{B_1} \underset{\mathcal{E}}{X_1} \xrightarrow{B_1} \underset{\mathcal{N}}{X_2} \xrightarrow{B_1}$$

$$\mathcal{X}_{\text{SYM}} = \mathcal{X}^{\text{B}} \oplus \mathcal{X}^{\text{F}}$$

$$\underset{\Lambda}{X^{\text{B}}_{-1}} \xrightarrow{B_1} \underset{\substack{A_\mu \\ \varphi}}{X^{\text{B}}_0} \xrightarrow{B_1} \underset{\substack{E \\ E_\mu}}{X^{\text{B}}_1} \xrightarrow{B_1} \underset{N}{X^{\text{B}}_2}$$

$$B_1(\Lambda) = \begin{pmatrix} \partial_\mu \Lambda \\ \Box\, \Lambda \end{pmatrix} \in X^{\text{B}}_0$$

$$B_1 \begin{pmatrix} A_\mu \\ \varphi \end{pmatrix} = \begin{pmatrix} \partial \cdot A - \varphi \\ \Box\, A_\mu - \partial_\mu \varphi \end{pmatrix} \in X^{\text{B}}_1$$

$$B_1 \begin{pmatrix} E \\ E^\mu \end{pmatrix} = \begin{pmatrix} \emptyset \\ \Box\, E - \partial_\mu E^\mu \end{pmatrix} \in X^{\text{B}}_2$$

$$b^2 = 0, bB_1 + B_1 b = \Box.$$

$$\begin{array}{ccccccc} X^{\text{B}}_{-1} & \xrightarrow{B_1} & X^{\text{B}}_0 & \xrightarrow{B_1} & X^{\text{B}}_1 & \xrightarrow{B_1} & X^{\text{B}}_2 \\ \Lambda & \xleftarrow{b} & A_\mu & \xleftarrow{b} & E & \xleftarrow{b} & \\ & & \varphi & & E_\mu & & N \end{array} .$$

$$b \begin{pmatrix} \emptyset \\ N \end{pmatrix} = \begin{pmatrix} N \\ 0 \end{pmatrix} \in X^{\text{B}}_1,$$
$$b \begin{pmatrix} E \\ E_\mu \end{pmatrix} = \begin{pmatrix} E_\mu \\ 0 \end{pmatrix} \in X^{\text{B}}_0,$$
$$b \begin{pmatrix} A_\mu \\ \varphi \end{pmatrix} = \begin{pmatrix} \varphi \\ \emptyset \end{pmatrix} \in X^{\text{B}}_{-1}.$$

$$\begin{array}{ccccccccc} 0 & \longrightarrow & X_0^{\mathrm{F}} & \xrightarrow{B_1} & X_1^{\mathrm{F}} & \longrightarrow & 0 \\ & & \chi & & \mathcal{E}_\chi & & \end{array}$$

$$b(\mathcal{E}_\chi) = -i\gamma^\mu \partial_\mu \mathcal{E}_\chi \in X_0^{\mathrm{F}}, b(\chi) = 0,$$

$$B_1(\chi) = \begin{pmatrix} i\gamma^\mu \partial_\mu \chi \\ \square\, \chi \end{pmatrix} \in X_1^{\mathrm{F}}$$

$$b\begin{pmatrix} \mathcal{E}_\chi \\ E_\chi \end{pmatrix} = E_\chi \in X_0^{\mathrm{F}}, b(\chi) = 0.$$

$$\begin{array}{ccccccccccc} 0 & \longrightarrow & X_0^{\mathrm{F}} & \xrightarrow{B_1} & X_1^{\mathrm{F}} & \xrightarrow{B_1} & X_2^{\mathrm{F}} & \xrightarrow{B_1} & \cdots \\ & & \chi & \xrightarrow{i\gamma^\mu \partial_\mu} & \mathcal{E}_\chi & \xrightarrow{i\gamma^\nu \partial_\nu} & \mathcal{N}_\chi & \xrightarrow{i\gamma^\rho \partial_\rho} & \cdots \\ & & & \searrow^{\square} & & \searrow^{1} & & \searrow^{\square} & \\ & & & & E_\chi & \xrightarrow{-i\gamma^\mu \partial_\mu} & N_\chi & \xrightarrow{-i\gamma^\nu \partial_\nu} & \cdots \end{array}.$$

$$\begin{array}{ccccccccc} X_{-1} & \xrightarrow{B_1} & X_0 & \xrightarrow{B_1} & X_1 & \xrightarrow{B_1} & X_2 & \xrightarrow{B_1} & \cdots \\ \Lambda & \xleftarrow{b} & A_\mu, \chi & \xleftarrow{b} & E, \mathcal{E}_\chi & \xleftarrow{b} & \mathcal{N}_\chi & & \cdots \\ & & \varphi & & E_\mu, E_\chi & & N, N_\chi & & \cdots \end{array}$$

$$\mathcal{K}_{\mathrm{SYM}} = \bigoplus_{i=0}^{\infty} K_i$$

$$m_n : \mathcal{K}^{\otimes n} \longrightarrow \mathcal{K}, |m_n| = 2 - n, n \geq 1$$

$$\mathcal{X}_{\mathrm{SYM}} = \mathcal{K}_{\mathrm{SYM}} \otimes \mathfrak{g}, \mathfrak{g}$$

$$\begin{aligned} B_1(x) &= m_1(u^a) \otimes t_a \\ B_2(x_1, x_2) &= (-1)^{x_1} f^a{}_{bc} m_2(u_1^b, u_2^c) \otimes t_a \\ B_3(x_1, x_2, x_3) &= f^a{}_{be} f^e{}_{cd} \left[(-1)^{x_2} m_3(u_1^b, u_2^c, u_3^d) + (-1)^{x_1(x_2+1)} m_3(u_2^b, u_1^c, u_3^d)\right] \otimes t_a \end{aligned}$$

$$\begin{array}{ccccccccc} K_0 & \xrightarrow{m_1} & K_1 & \xrightarrow{m_1} & K_2 & \xrightarrow{m_1} & K_3 & \xrightarrow{m_1} & \cdots \\ \Lambda & & A_\mu, \chi & & E, \mathcal{E}_\chi & & \mathcal{N}_\chi & & \cdots \\ & & \varphi & & E_\mu, E_\chi & & N, N_\chi & & \cdots \end{array}$$

$$b^2 = 0, bm_1 + m_1 b = \square, |b| = -1.$$

$$H_{m,n}(\Xi_1, \ldots, \Xi_m): \mathcal{X}_{\mathrm{SYM}}^{\otimes n} \longrightarrow \mathcal{X}_{\mathrm{SYM}},$$

$$\Sigma_n(\Xi): \mathcal{X}_{\mathrm{SYM}}^{\otimes n} \longrightarrow \mathcal{X}_{\mathrm{SYM}}, |\Sigma_n(\Xi)| = 0, n \geq 1$$

$$\delta_\Xi \mathcal{A} =: \Sigma_1(\Xi \mid \mathcal{A}) + \frac{1}{2}\Sigma_2(\Xi \mid \mathcal{A}, \mathcal{A}) + \cdots$$

$$\Sigma_1(\xi \mid \Lambda) = \begin{pmatrix} \xi \cdot \partial\Lambda \\ \emptyset \end{pmatrix} \in X_{-1}$$

$$\Sigma_1(\xi \mid \mathcal{A}) = \begin{pmatrix} \xi \cdot \partial A_\mu + \partial_\mu \xi \cdot A, \xi \cdot \partial\chi + \frac{1}{4}\omega_{\mu\nu}\gamma^{\mu\nu}\chi \\ \xi \cdot \partial\varphi \end{pmatrix} \in X_0$$

$$\Sigma_1(\xi \mid \mathcal{E}) = \begin{pmatrix} \xi \cdot \partial E, \xi \cdot \partial\mathcal{E}_\chi + \frac{1}{4}\omega_{\mu\nu}\gamma^{\mu\nu}\mathcal{E}_\chi \\ \xi \cdot \partial E_\mu + \partial_\mu \xi^\nu E_\nu, \xi \cdot \partial E_\chi + \frac{1}{4}\omega_{\mu\nu}\gamma^{\mu\nu}E_\chi \end{pmatrix} \in X_1$$

$$\Sigma_1(\xi \mid \mathcal{N}) = \begin{pmatrix} \xi \cdot \partial\mathcal{N}_\chi + \frac{1}{4}\omega_{\mu\nu}\gamma^{\mu\nu}\mathcal{N}_\chi \\ \xi \cdot \partial N, \xi \cdot \partial N_\chi + \frac{1}{4}\omega_{\mu\nu}\gamma^{\mu\nu}N_\chi \end{pmatrix} \in X_2$$

$$\Sigma_1(\epsilon \mid \Lambda) = 0$$

$$\Sigma_1(\epsilon \mid \mathcal{A}_B) = \begin{pmatrix} \partial_\mu A_\nu \gamma^{\mu\nu}\epsilon \\ \emptyset \end{pmatrix} \in X_0^{\mathrm{F}}, \Sigma_1(\epsilon \mid \mathcal{A}_F) = \begin{pmatrix} i\epsilon\gamma_\mu\chi \\ 0 \end{pmatrix} \in X_0^{\mathrm{B}}$$

$$\Sigma_1(\epsilon \mid \mathcal{E}_B) = \begin{pmatrix} i\big(E_\mu - \partial_\mu E\big)\gamma^\mu\epsilon \\ \partial_\mu E_\nu \gamma^{\mu\nu}\epsilon \end{pmatrix} \in X_1^{\mathrm{F}}, \Sigma_1(\epsilon \mid \mathcal{E}_F) = \begin{pmatrix} \epsilon\mathcal{E}_\chi \\ i\epsilon\gamma_\mu E_\chi \end{pmatrix} \in X_1^{\mathrm{B}}$$

$$\Sigma_1(\epsilon \mid \mathcal{N}_B) = \begin{pmatrix} N\epsilon \\ -i\,\partial N\epsilon \end{pmatrix} \in X_2^{\mathrm{F}}, \Sigma_1(\epsilon \mid \mathcal{N}_F) = \begin{pmatrix} \emptyset \\ \epsilon N_\chi \end{pmatrix} \in X_2^{\mathrm{B}}$$

$$B_1\Sigma_1(\Xi \mid x) = \Sigma_1(\Xi \mid B_1(x))$$
$$B_1\Sigma_2(\Xi \mid x_1, x_2) + B_2(\Sigma_1(\Xi \mid x_1), x_2) + B_2\big(x_1, \Sigma_1(\Xi \mid x_2)\big)$$
$$= \Sigma_1\big(\Xi \mid B_2(x_1, x_2)\big) + \Sigma_2(\Xi \mid B_1(x_1), x_2) + (-1)^{x_1}\Sigma_2\big(\Xi \mid x_1, B_1(x_2)\big)$$
$$B_1\Sigma_3(\Xi \mid x_1, x_2, x_3) + B_2(\Sigma_2(\Xi \mid x_1, x_2), x_3)$$
$$+B_3(\Sigma_1(\Xi \mid x_1), x_2, x_3)$$
$$= \Sigma_1\big(\Xi \mid B_3(x_1, x_2, x_3)\big) + \Sigma_2(\Xi \mid B_2(x_1, x_2), x_3)$$
$$\vdots$$

$$\sum_{i+j=n} \big[B_i, \Sigma_j(\Xi)\big] = 0, \forall n \geq 2$$

$$[\Sigma(\Xi_1), \Sigma(\Xi_2)] = \Sigma\big([\Xi_1, \Xi_2]_{\mathcal{G}}\big)$$

$$[\Sigma_1(\Xi_1), \Sigma_1(\Xi_2)] = \Sigma_1(\Xi_{12}) + \big[B_1, H_{2,1}(\Xi_1, \Xi_2)\big],$$
$$2\big[\Sigma_1\big(\Xi_{[1}\big), \Sigma_2\big(\Xi_{2]}\big)\big] = \Sigma_2(\Xi_{12}) + \big[B_1, H_{2,2}(\Xi_1, \Xi_2)\big] + \big[H_{2,1}(\Xi_1, \Xi_2), B_2\big],$$

$$H_{m,n}(\Xi_1, \ldots, \Xi_m): \mathcal{X}_{\mathrm{SYM}}^{\otimes n} \longrightarrow \mathcal{X}_{\mathrm{SYM}}, m, n \geq 1$$

$$[\Sigma_1(\epsilon_1),\Sigma_1(\epsilon_2)] = \Sigma_1(a_{12}) + \left[B_1, H_{2,1}(\epsilon_1,\epsilon_2)\right]$$
$$[\Sigma_1(\epsilon_1),\Sigma_2(\epsilon_2)] - [\Sigma_1(\epsilon_2),\Sigma_2(\epsilon_1)] = \left[B_1, H_{2,2}(\epsilon_1,\epsilon_2)\right] + \left[H_{2,1}(\epsilon_1,\epsilon_2), B_2\right]$$

$$\epsilon_{12}^{\mu[n]} = \epsilon_1 \gamma^{\mu[n]} \epsilon_2 \equiv \epsilon_1 \gamma^{\mu_1 \cdots \mu_n} \epsilon_2$$
$$\chi_{12} := \frac{7}{8}\epsilon_{12}^{\mu}\gamma_\mu - \frac{1}{16}\frac{1}{5!}\epsilon_{12}^{\mu[5]}\gamma_{\mu[5]}$$

$$[\Sigma_1(\epsilon_1),\Sigma_1(\epsilon_2)]\Lambda = \Sigma_1(a_{12} \mid \Lambda) + H_{2,1}(\epsilon_1,\epsilon_2 \mid B_1\Lambda)$$
$$[\Sigma_1(\epsilon_1),\Sigma_1(\epsilon_2)]\mathcal{A}_{\mathrm{B}} = \Sigma_1(a_{12} \mid \mathcal{A}_{\mathrm{B}})\big) + B_1 H_{2,1}(\epsilon_1,\epsilon_2 \mid \mathcal{A}_{\mathrm{B}}) + H_{2,1}(\epsilon_1,\epsilon_2 \mid B_1\mathcal{A}_{\mathrm{B}})$$

$$[\Sigma_1(\epsilon_1),\Sigma_1(\epsilon_2)]\mathcal{A}_{\mathrm{F}} = \Sigma_1(a_{12} \mid \mathcal{A}_{\mathrm{F}})\big) + H_{2,1}(\epsilon_1,\epsilon_2 \mid B_1\mathcal{A}_{\mathrm{F}})$$
$$[\Sigma_1(\epsilon_1),\Sigma_1(\epsilon_2)]\mathcal{E}_{\mathrm{B}} = \Sigma_1(a_{12} \mid \mathcal{E}_{\mathrm{B}})\big) + B_1 H_{2,1}(\epsilon_1,\epsilon_2 \mid \mathcal{E}_{\mathrm{B}})$$
$$[\Sigma_1(\epsilon_1),\Sigma_1(\epsilon_2)]\mathcal{E}_{\mathrm{F}} = \Sigma_1(a_{12} \mid \mathcal{E}_{\mathrm{F}})\big) + B_1 H_{2,1}(\epsilon_1,\epsilon_2 \mid \mathcal{E}_{\mathrm{F}}) + H_{2,1}(\epsilon_1,\epsilon_2 \mid B_1\mathcal{E}_{\mathrm{F}})$$
$$[\Sigma_1(\epsilon_1),\Sigma_1(\epsilon_2)]\mathcal{N}_{\mathrm{B}} = \Sigma_1(a_{12} \mid \mathcal{N}_{\mathrm{B}})\big)$$
$$[\Sigma_1(\epsilon_1),\Sigma_1(\epsilon_2)]\mathcal{N}_{\mathrm{F}} = \Sigma_1(a_{12} \mid \mathcal{N}_{\mathrm{F}})\big) + B_1 H_{2,1}(\epsilon_1,\epsilon_2 \mid \mathcal{N}_{\mathrm{F}}) + H_{2,1}(\epsilon_1,\epsilon_2 \mid B_1\mathcal{N}_{\mathrm{F}})$$

$$\Sigma_1(a_{12} \mid -) := a_{12}^{\mu}\partial_\mu(-), \qquad \Lambda_{12} := 2i\epsilon_{12}^{\mu}A_\mu$$
$$H_{2,1}(\epsilon_1,\epsilon_2 \mid \mathcal{A}_{\mathrm{B}}) := \begin{pmatrix}\Lambda_{12}\\ \emptyset\end{pmatrix} \in X_{-1}^{\mathrm{B}}, \qquad \varphi_{12} := -2i\epsilon_{12}^{\mu}E_\mu$$
$$H_{2,1}(\epsilon_1,\epsilon_2 \mid \mathcal{E}_{\mathrm{B}}) := \begin{pmatrix}0\\ \varphi_{12}\end{pmatrix} \in X_0^{\mathrm{B}}, \qquad \chi_{12} := \chi_{12}\mathcal{E}_\chi$$
$$H_{2,1}(\epsilon_1,\epsilon_2 \mid \mathcal{E}_{\mathrm{F}}) := \begin{pmatrix}\chi_{12}\\ \emptyset\end{pmatrix} \in X_0^{\mathrm{F}}, \qquad \mathcal{E}_\chi^{12} := \epsilon_{12}^{\mu}\gamma_\mu\mathcal{N}_\chi$$
$$H_{2,1}(\epsilon_1,\epsilon_2 \mid \mathcal{N}_{\mathrm{F}}) := \begin{pmatrix}\mathcal{E}_\chi^{12}\\ E_\chi^{12}\end{pmatrix} \in X_1^{\mathrm{F}}, \qquad E_\chi^{12} := -\chi_{12}N_\chi$$
$$\mathcal{N}_\chi^{12} := \epsilon_{12}^{\mu}\gamma_\mu\mathcal{R}_\chi$$
$$H_{2,1}(\epsilon_1,\epsilon_2 \mid \mathcal{R}_{\mathrm{F}}) := \begin{pmatrix}\mathcal{N}_\chi^{12}\\ N_\chi^{12}\end{pmatrix} \in X_2^{\mathrm{F}}, \qquad N_\chi^{12} := -\epsilon_{12}^{\mu}\gamma_\mu R_\chi$$

$$2\left[\Sigma_1\left(\epsilon_{[1}\right),\Sigma_2\left(\epsilon_{2]}\right)\right](\Lambda,\Lambda) = 0$$
$$2\left[\Sigma_1\left(\epsilon_{[1}\right),\Sigma_2\left(\epsilon_{2]}\right)\right](\mathcal{A}_{\mathrm{B}},\mathcal{A}_{\mathrm{B}}) = H_{2,1}(\epsilon_1,\epsilon_2 \mid B_2(A,A)) + 2B_2\left(A, H_{2,1}(\epsilon_1,\epsilon_2 \mid A)\right)$$
$$2\left[\Sigma_1\left(\epsilon_{[1}\right),\Sigma_2\left(\epsilon_{2]}\right)\right](\mathcal{A}_{\mathrm{B}},\mathcal{A}_{\mathrm{F}}) = H_{2,1}(\epsilon_1,\epsilon_2 \mid B_2(A,\chi)) + B_2\left(\chi, H_{2,1}(\epsilon_1,\epsilon_2 \mid A)\right)$$
$$2\left[\Sigma_1\left(\epsilon_{[1}\right),\Sigma_2\left(\epsilon_{2]}\right)\right](\mathcal{A}_{\mathrm{F}},\mathcal{A}_{\mathrm{F}}) = H_{2,1}(\epsilon_1,\epsilon_2 \mid B_2(\chi,\chi))$$

$$[\Sigma_1(\xi_1),\Sigma_1(\xi_2)] = \Sigma_1(\xi_{12}), \xi_{12} = \mathcal{L}_{\xi_1}\xi_2^{\mu}$$

$$[\Sigma_1(\xi),\Sigma_1(\epsilon)] = \Sigma_1(\epsilon_{\mathrm{eff}})$$

$$\epsilon_{\mathrm{eff}} := \frac{1}{4}\omega_{\mu\nu}\gamma^{\mu\nu}\epsilon$$

$$\rho_n(\Xi): \mathcal{K}_{\mathrm{SYM}}^{\otimes n} \longrightarrow \mathcal{K}_{\mathrm{SYM}}, |\rho_n(\Xi)| = 1 - n$$
$$h_{m,n}(\Xi_1,\ldots,\Xi_m): \mathcal{K}_{\mathrm{SYM}}^{\otimes n} \longrightarrow \mathcal{K}_{\mathrm{SYM}}, \left|h_{m,n}(\Xi_1,\ldots,\Xi_m)\right| = 2 - m - n$$

$$\Sigma_1(\Xi \mid x) = \rho_1(\Xi \mid u^a) \otimes t_a$$
$$\Sigma_2(\Xi \mid x_1, x_2) = (-1)^{x_1} f_{bc}^{a} \rho_2\left(\Xi \mid u_1^b, u_2^c\right) \otimes t_a$$

$$H_{2,1}(\Xi_1,\Xi_2 \mid x) = h_{2,1}(\Xi_1,\Xi_2 \mid u^a) \otimes t_a$$
$$H_{2,2}(\Xi_1,\Xi_2 \mid x_1,x_2) = (-1)^{x_1} f^a{}_{bc} h_{2,2}(\Xi_1,\Xi_2 \mid u_1^b,u_2^c) \otimes t_a.$$

$$[b,\rho_1(\Xi)] = \left[b,h_{2,1}(\Xi_1,\Xi_2)\right] = 0$$

$$\square \equiv \tilde{\square},$$

$$\left.\begin{array}{l} \mathcal{X}_{\mathrm{SYM}} = \mathcal{K}_{\mathrm{SYM}} \otimes \mathfrak{g} \overset{\text{color-strip}}{\rightarrow} \mathcal{K}_{\mathrm{SYM}} \\ \tilde{\mathcal{X}}_{\mathrm{SYM}} = \tilde{\mathcal{K}}_{\mathrm{SYM}} \otimes \tilde{\mathfrak{g}} \overset{\text{color-strip}}{\rightarrow} \tilde{\mathcal{K}}_{\mathrm{SYM}} \end{array}\right\} \overset{\text{tensor}}{\underset{\text{product}}{\rightarrow}} \mathcal{X}_{\mathrm{SDFT}} := \left(\mathcal{K}_{\mathrm{SYM}} \otimes \tilde{\mathcal{K}}_{\mathrm{SYM}}\right)\big|_{\text{section}}$$

$$b^- := \frac{1}{2}(b \otimes \tilde{\mathbb{1}} - \mathbb{1} \otimes \tilde{b}), (b^-)^2 = 0.$$

$$\begin{array}{ccccccccccccc} X_{-2} & \xrightarrow{\mathbb{B}_1} & X_{-1} & \xrightarrow{\mathbb{B}_1} & X_0 & \xrightarrow{\mathbb{B}_1} & X_1 & \xrightarrow{\mathbb{B}_1} & X_2 & \xrightarrow{\mathbb{B}_1} & X_3 & \xrightarrow{\mathbb{B}_1} & \cdots \\ \mathbb{t} & & \mathbb{\Lambda} & & \mathbb{H} & & \mathbb{E} & & \mathbb{N} & & \mathbb{R} & & \cdots \end{array},$$

$$\mathbb{B}_1 := m_1 \otimes \tilde{\mathbb{1}} + \mathbb{1} \otimes \tilde{m}_1,$$

$$\mathbb{B}_2 := -\frac{1}{2} b^- (m_2 \otimes \tilde{m}_2)$$
$$\vdots$$

$$\mathbb{B}_1(\mathbb{H}) + \frac{1}{2}\mathbb{B}_2(\mathbb{H},\mathbb{H}) + \frac{1}{6}\mathbb{B}_3(\mathbb{H},\mathbb{H},\mathbb{H}) + \cdots = 0$$

$$\delta_\wedge \mathbb{H} = \mathbb{B}_1(\wedge) + \mathbb{B}_2(\wedge,\mathbb{H}) + \cdots.$$

$$\Lambda = \begin{pmatrix} \Lambda_\mu, \tilde{\Lambda}_{\tilde{\mu}} \\ \eta \end{pmatrix} \oplus \begin{pmatrix} \varepsilon^{\tilde{\alpha}}, \tilde{\varepsilon}^{\alpha} \\ \emptyset \end{pmatrix}$$

$$\Lambda_\mu^{\mathrm{triv}} = \partial_\mu \tau, \tilde{\Lambda}_{\tilde{\mu}}^{\mathrm{triv}} = \tilde{\partial}_{\tilde{\mu}} \tau, \eta^{\mathrm{triv}} = \square\, \tau.$$

$$\mathbb{H} = \mathbb{H}_{\mathrm{NS-NS}} \oplus \mathbb{H}_{\mathrm{NS-R}} \oplus \mathbb{H}_{\mathrm{R-R}}$$

$$\mathbb{H}_{\mathrm{NS-NS}} = \begin{pmatrix} e_{\mu\tilde{\nu}}, e, \tilde{e} \\ f_\mu, \tilde{f}_{\tilde{\mu}} \end{pmatrix}$$

$$\begin{aligned} \square\, e_{\mu\tilde{\nu}} + \tilde{\partial}_{\tilde{\nu}} f_\mu - \partial_\mu \tilde{f}_{\tilde{\nu}} &= 0, \\ \partial_\mu \tilde{e} - \tilde{\partial}^{\tilde{\nu}} e_{\mu\tilde{\nu}} - f_\mu &= 0, \quad \tilde{\partial}_{\tilde{\nu}} e + \partial^\mu e_{\mu\tilde{\nu}} - \tilde{f}_{\tilde{\nu}} = 0 \\ \square\, e - \partial^\mu f_\mu &= 0, \qquad \square\, \tilde{e} - \tilde{\partial}^{\tilde{\mu}} \tilde{f}_{\tilde{\mu}} = 0 \end{aligned}$$

$$\mathbb{B}_1(\Lambda) = \delta_\wedge^{\mathrm{lin}} \mathbb{H}_{\mathrm{NS-NS}}$$

$$\begin{aligned} \delta_\wedge e_{\mu\tilde{\nu}} &= \partial_\mu \tilde{\Lambda}_{\tilde{\nu}} - \tilde{\partial}_{\tilde{\nu}} \Lambda_\mu, \\ \delta_\wedge e &= \partial^\mu \Lambda_\mu - \eta, \qquad \delta_\wedge \tilde{e} = \tilde{\partial}^{\tilde{\mu}} \tilde{\Lambda}_{\tilde{\mu}} - \eta \\ \delta_\wedge f_\mu &= \square\, \Lambda_\mu - \partial_\mu \eta, \qquad \delta_\wedge \tilde{f}_{\tilde{\mu}} = \square\, \tilde{\Lambda}_{\tilde{\mu}} - \tilde{\partial}_{\tilde{\mu}} \eta \end{aligned}$$

$$\mathbb{H}_{\mathrm{NS-R}}=\begin{pmatrix}\psi_{\mu}^{\tilde{\alpha}},\rho_{\tilde{\alpha}};\tilde{\psi}_{\tilde{\mu}}^{\alpha},\tilde{\rho}_{\alpha}\\ \varphi^{\tilde{\alpha}};\tilde{\varphi}^{\alpha}\end{pmatrix}$$

$$\begin{gathered}\gamma^{\tilde{\nu}}\tilde{\partial}_{\tilde{\nu}}\psi_{\mu}-\partial_{\mu}\rho\ =0,\gamma^{\nu}\partial_{\nu}\tilde{\psi}_{\tilde{\mu}}-\tilde{\partial}_{\tilde{\mu}}\tilde{\rho}\ =0,\\ \partial^{\mu}\psi_{\mu}-\varphi\ =0,\tilde{\partial}^{\tilde{\mu}}\tilde{\psi}_{\tilde{\mu}}-\tilde{\varphi}\ =0,\\ \gamma^{\tilde{\mu}}\tilde{\partial}_{\tilde{\mu}}\rho-\varphi\ =0,\gamma^{\mu}\partial_{\mu}\tilde{\rho}-\tilde{\varphi}\ =0,\end{gathered}$$

$$\begin{gathered}\delta_{\wedge}\psi_{\mu}\ =\partial_{\mu}\varepsilon,\delta_{\wedge}\tilde{\psi}_{\tilde{\mu}}\ =\tilde{\partial}_{\tilde{\mu}}\tilde{\varepsilon}\\ \delta_{\wedge}\rho\ =\gamma^{\tilde{\mu}}\tilde{\partial}_{\tilde{\mu}}\varepsilon,\delta_{\wedge}\tilde{\rho}\ =\gamma^{\mu}\partial_{\mu}\tilde{\varepsilon}\\ \delta_{\wedge}\varphi\ =\square\ \varepsilon,\delta_{\wedge}\tilde{\varphi}\ =\square\ \tilde{\varepsilon}\end{gathered}$$

$$\mathbb{H}_{\mathrm{R-R}}=\begin{pmatrix}F^{\alpha\tilde{\beta}}\\ \emptyset\end{pmatrix}$$

$$\gamma^{\tilde{\mu}}\tilde{\partial}_{\tilde{\mu}}F=0,\gamma^{\mu}\partial_{\mu}F=0.$$

$$\Xi:=(\Xi,\tilde{\Xi})\equiv(\xi,\epsilon),$$

$$\begin{gathered}\mathbb{Z}_1(\boldsymbol{\Xi}):=\rho_1(\Xi)\otimes\tilde{\mathbb{1}}+\mathbb{1}\otimes\tilde{\rho}_1(\tilde{\Xi})\\ \Sigma_2(\boldsymbol{\Xi}):=\frac{1}{2}b^{-}\big(\rho_2(\Xi)\otimes\tilde{m}_2+m_2\otimes\tilde{\rho}_2(\tilde{\Xi})\big)\end{gathered}$$

$$[\mathbb{B}_1,\Sigma_1(\boldsymbol{\Xi})]=0,[\mathbb{B}_1,\Sigma_2(\boldsymbol{\Xi})]=[\mathbb{Z}_1(\boldsymbol{\Xi}),\mathbb{B}_2]$$

$$\begin{gathered}[\mathbb{Z}_1(\boldsymbol{\Xi}_1),\Sigma_1(\boldsymbol{\Xi}_2)]\ =\mathbb{Z}_1(\boldsymbol{\Xi}_{12})+\big[\mathbb{B}_1,\mathbb{H}_{2,1}(\boldsymbol{\Xi}_1,\boldsymbol{\Xi}_2)\big]\\ 2\big[\mathbb{Z}_1\big(\boldsymbol{\Xi}_{[1]}\big),\Sigma_2\big(\boldsymbol{\Xi}_{2]}\big)\big]\ =\Sigma_2(\boldsymbol{\Xi}_{12})+\big[\mathbb{B}_1,\mathbb{H}_{2,2}(\boldsymbol{\Xi}_1,\boldsymbol{\Xi}_2)\big]+\big[\mathbb{H}_{2,1}(\boldsymbol{\Xi}_1,\boldsymbol{\Xi}_2),\mathbb{B}_2\big]\end{gathered}$$

$$\begin{gathered}\mathbb{H}_{2,1}(\boldsymbol{\Xi}_1,\boldsymbol{\Xi}_2):=h_{2,1}(\Xi_1,\Xi_2)\otimes\tilde{\mathbb{1}}+\mathbb{1}\otimes\tilde{h}_{2,1}\big(\tilde{\Xi}_1,\tilde{\Xi}_2\big)\\ \mathbb{H}_{2,2}(\boldsymbol{\Xi}_1,\boldsymbol{\Xi}_2):=-\frac{1}{2}b^{-}\big(h_{2,2}(\Xi_1,\Xi_2)\otimes\tilde{m}_2+m_2\otimes\tilde{h}_{2,2}\big(\tilde{\Xi}_1,\tilde{\Xi}_2\big)\\ +\rho_2(\Xi_1)\otimes\tilde{\rho}_2\big(\tilde{\Xi}_2\big)-\rho_2(\Xi_2)\otimes\tilde{\rho}_2\big(\tilde{\Xi}_1\big)\big)\end{gathered}$$

$$\begin{aligned}[\Sigma_1(\boldsymbol{\Xi}_1),\Sigma_1(\boldsymbol{\Xi}_2)]=&\big[\rho_1(\Xi_1)\otimes\tilde{\mathbb{1}}+\mathbb{1}\otimes\tilde{\rho}_1\big(\tilde{\Xi}_1\big),\rho_1(\Xi_2)\otimes\tilde{\mathbb{1}}+\mathbb{1}\otimes\tilde{\rho}_1\big(\tilde{\Xi}_2\big)\big]\\ =&\big\{\rho_1(\Xi_{12})+\big[m_1,h_{2,1}(\Xi_1,\Xi_2)\big]\big\}\otimes\tilde{\mathbb{1}}\\ &+\mathbb{1}\otimes\big\{\tilde{\rho}_1\big(\tilde{\Xi}_{12}\big)+\big[\tilde{m}_1,\tilde{h}_{2,1}\big(\tilde{\Xi}_1,\tilde{\Xi}_2\big)\big]\big\}\\ =&\big\{\rho_1(\Xi_{12})\otimes\tilde{\mathbb{1}}+\big[m_1\otimes\tilde{\mathbb{1}}+\mathbb{1}\otimes\tilde{m}_1,h_{2,1}(\Xi_1,\Xi_2)\otimes\tilde{\mathbb{1}}\big]\big\}\\ &+\big\{\mathbb{1}\otimes\tilde{\rho}_1\big(\tilde{\Xi}_{12}\big)+\big[m_1\otimes\tilde{\mathbb{1}}+\mathbb{1}\otimes\tilde{m}_1,\mathbb{1}\otimes\tilde{h}_{2,1}\big(\tilde{\Xi}_1,\tilde{\Xi}_2\big)\big]\big\}\\ =&\Sigma_1(\boldsymbol{\Xi}_{12})+\big[\mathbb{B}_1,\mathbb{H}_{2,1}(\boldsymbol{\Xi}_1,\boldsymbol{\Xi}_2)\big],\end{aligned}$$

$$\begin{aligned}&[\mathbb{Z}_1(\boldsymbol{\Xi}_1),\mathbb{Z}_2(\boldsymbol{\Xi}_2)]-(1\leftrightarrow2)\\ &=\left[\rho_1(\Xi_1)\otimes\tilde{\mathbb{1}}+\mathbb{1}\otimes\tilde{\rho}_1\big(\tilde{\Xi}_1\big),\frac{1}{2}b^{-}\left(\rho_2(\Xi_2)\otimes\tilde{m}_2+m_2\otimes\tilde{\rho}_2\big(\tilde{\Xi}_2\big)\right)\right]-(1\leftrightarrow2)\\ &=\frac{1}{2}b^{-}\big\{\big(\rho_2(\Xi_{12})+\big[m_1,h_{2,2}(\Xi_1,\Xi_2)\big]+\big[h_{2,1}(\Xi_1,\Xi_2),m_2\big]\big)\otimes\tilde{m}_2\\ &-\rho_2(\Xi_1)\otimes\big[\tilde{m}_1,\tilde{\rho}_2\big(\tilde{\Xi}_2\big)\big]-(1\leftrightarrow2)+[m_1,\rho_2(\Xi_1)]\otimes\tilde{\rho}_2\big(\tilde{\Xi}_2\big)-(1\leftrightarrow2)\\ &+m_2\otimes\big(\tilde{\rho}_2\big(\tilde{\Xi}_{12}\big)+\big[\tilde{m}_1,\tilde{h}_{2,2}\big(\tilde{\Xi}_1,\tilde{\Xi}_2\big)\big]+\big[\tilde{h}_{2,1}\big(\tilde{\Xi}_1,\tilde{\Xi}_2\big),\tilde{m}_2\big]\big)\big\}\\ &=\mathbb{Z}_2(\boldsymbol{\Xi}_{12})+\big[\mathbb{B}_1,\mathbb{H}_{2,2}(\boldsymbol{\Xi}_1,\boldsymbol{\Xi}_2)\big]+\big[\mathbb{H}_{2,1}(\boldsymbol{\Xi}_1,\boldsymbol{\Xi}_2),\mathbb{B}_2\big]\end{aligned}$$

$$[b,\rho_1(\Xi)]=\left[b,h_{2,1}(\Xi_1,\Xi_2)\right]=0$$

$$\delta_{\xi}\mathbb{H}=\mathcal{L}_{\xi}\mathbb{H}$$

$$\begin{aligned}
\delta_\epsilon e_{\mu\tilde{\nu}} &= i\tilde{\epsilon}^{\tilde{\alpha}}\gamma_{\tilde{\nu}}\tilde{\alpha}^{\tilde{\beta}}\psi_{\mu}^{\tilde{\beta}}-i\epsilon^{\alpha}\gamma_{\mu\alpha\beta}\tilde{\psi}_{\tilde{\nu}}^{\beta},\\
\delta_\epsilon e &= i\epsilon^{\alpha}\tilde{\rho}_{\alpha}, \qquad \delta_\epsilon\tilde{e}=i\tilde{\epsilon}^{\tilde{\alpha}}\rho_{\tilde{\alpha}}\\
\delta_\epsilon f_\mu &= i\epsilon^{\alpha}\gamma_{\mu\alpha\beta}\tilde{\varphi}^{\beta}, \qquad \delta_\epsilon\tilde{f}_{\tilde{\mu}}=i\tilde{\epsilon}^{\tilde{\alpha}}\gamma_{\tilde{\mu}}\tilde{\alpha}\tilde{\beta}\varphi^{\tilde{\beta}};\\
\delta_\epsilon\psi_{\mu}^{\tilde{\alpha}} &= \tilde{\partial}_{\tilde{\nu}}e_{\mu\tilde{\rho}}\left(\gamma^{\tilde{\nu}\tilde{\rho}}\tilde{\epsilon}\right)^{\tilde{\alpha}}-\epsilon^{\beta}\gamma_{\mu\beta\lambda}F^{\lambda\tilde{\alpha}}, \qquad \delta_\epsilon\tilde{\psi}_{\tilde{\mu}}^{\alpha}=-\partial_{\nu}e_{\rho\tilde{\mu}}(\gamma^{\nu\rho}\epsilon)^{\alpha}-\tilde{\epsilon}^{\tilde{\beta}}\gamma_{\tilde{\mu}\tilde{\beta}\tilde{\lambda}}F^{\alpha\tilde{\lambda}},\\
\delta_\epsilon\rho_{\tilde{\alpha}} &= \tilde{f}_{\tilde{\mu}}\gamma^{\tilde{\mu}}{}_{\tilde{\alpha}\tilde{\beta}}\tilde{\epsilon}^{\tilde{\beta}}-\tilde{\partial}_{\tilde{\mu}}\tilde{e}^{\tilde{\mu}}{}_{\tilde{\alpha}\tilde{\beta}}\tilde{\epsilon}^{\tilde{\beta}}, \qquad \delta_\epsilon\tilde{\rho}_{\alpha}=f_{\mu}\gamma^{\mu}{}_{\alpha\beta}\epsilon^{\beta}-\partial_{\mu}e\gamma^{\mu}{}_{\alpha\beta}\epsilon^{\beta},\\
\delta_\epsilon\varphi^{\tilde{\alpha}} &= \tilde{\partial}_{\tilde{\mu}}\tilde{f}_{\tilde{\nu}}\gamma^{\tilde{\mu}\tilde{\nu}}\tilde{\alpha}_{\tilde{\beta}}\tilde{\epsilon}^{\tilde{\beta}}, \qquad \delta_\epsilon\tilde{\varphi}^{\alpha}=\partial_{\mu}f_{\nu}\gamma^{\mu\nu\alpha}{}_{\beta}\epsilon^{\beta};\\
\delta_\epsilon F^{\alpha\tilde{\beta}} &= i\partial_{\mu}\psi_{\nu}{}^{\tilde{\beta}}\gamma^{\mu\nu\alpha}{}_{\lambda}\epsilon^{\lambda}+i\tilde{\partial}_{\tilde{\mu}}\tilde{\psi}_{\tilde{\nu}}{}^{\alpha}\gamma^{\tilde{\mu}\tilde{\nu}}\tilde{\beta}_{\tilde{\lambda}}\tilde{\epsilon}^{\tilde{\lambda}}
\end{aligned}$$

$$e_{\mu\nu}=h_{\mu\nu}+B_{\mu\nu},\ h_{\mu\nu}:=e_{(\mu\nu)},B_{\mu\nu}:=e_{[\mu\nu]}$$

$$\delta_{\wedge}h_{\mu\nu}=\partial_{\mu}\lambda_{\nu}+\partial_{\nu}\lambda_{\mu},\delta_{\wedge}B_{\mu\nu}=\partial_{\mu}\zeta_{\nu}-\partial_{\nu}\zeta_{\mu}$$

$$\lambda_{\mu}:=\frac{1}{2}\left(\tilde{\Lambda}_{\mu}-\Lambda_{\mu}\right),\zeta_{\mu}:=\frac{1}{2}\left(\tilde{\Lambda}_{\mu}+\Lambda_{\mu}\right)$$

$$\phi:=\frac{1}{2}(\tilde{e}-e-h).$$

$$\lambda_{\tilde{\alpha}}:=\rho_{\tilde{\alpha}}-(\gamma^{\mu})_{\tilde{\alpha}\tilde{\beta}}\psi_{\mu}^{\tilde{\beta}},\tilde{\lambda}_{\alpha}:=\tilde{\rho}_{\alpha}-(\gamma^{\mu})_{\alpha\beta}\tilde{\psi}_{\mu}^{\beta},$$

$$\psi_{\mu}^{\mathrm{RS}}:=\psi_{\mu}+\frac{1}{8}\gamma_{\mu}\lambda,\tilde{\psi}_{\mu}^{\mathrm{RS}}:=\tilde{\psi}_{\mu}+\frac{1}{8}\gamma_{\mu}\tilde{\lambda}$$

$$h_{\mu\nu}^{\mathrm{E}}:=h_{\mu\nu}+\frac{1}{4}\eta_{\mu\nu}\phi.$$

$$\mathrm{Spin}(1,9)\equiv\widetilde{\mathrm{Spin}}(1,9)$$

$$F^{\alpha\beta}=(\gamma^{\mu})^{\alpha\beta}F_{\mu}+\frac{1}{3!}(\gamma^{\mu\nu\rho})^{\alpha\beta}F_{\mu\nu\rho}+\frac{1}{2\cdot 5!}(\gamma^{\mu\nu\rho\sigma\tau})^{\alpha\beta}F_{\mu\nu\rho\sigma\tau},$$

$$\begin{aligned}
\partial_{[\mu}F_{\nu]} &= 0,\partial^{\mu}F_{\mu}=0\\
\partial_{[\mu}F_{\nu\rho\sigma]} &= 0,\partial^{\mu}F_{\mu\nu\rho}=0\\
\partial_{[\mu}F_{\nu\rho\sigma\tau\omega]} &= 0,\partial^{\mu}F_{\mu\nu\rho\sigma\tau}=0
\end{aligned}$$

$$\begin{aligned}
\psi_{\mu} &:= \psi_{\mu}^{\mathrm{RS}}+i\tilde{\psi}_{\mu}^{\mathrm{RS}},\mathrm{R}=1/2\\
\lambda &:= \lambda-i\tilde{\lambda},\mathrm{R}=3/2\\
A &:= \phi-4iC,\mathrm{R}=2\\
A_{\mu\nu} &:= B_{\mu\nu}+2iC_{\mu\nu},\mathrm{R}=1
\end{aligned}$$

$$\delta_\epsilon h^{\mathrm{E}}_{\mu\nu} = \frac{1}{2}\left(\epsilon\gamma_{(\mu}\psi^*_{\nu)} - \epsilon^*\gamma_{(\mu}\psi_{\nu)}\right)$$

$$\delta_\epsilon A = \frac{1}{2}\epsilon\lambda$$

$$\delta_\epsilon A_{\mu\nu} = -\epsilon\gamma_{[\mu}\psi_{\nu]} - \frac{1}{8}\epsilon^*\gamma_{\mu\nu}\lambda$$

$$\delta_\epsilon C_{\mu\nu\rho\sigma} = \frac{i}{2}\left(\epsilon^*\gamma_{[\mu\nu\rho}\psi_{\sigma]} + \epsilon\gamma_{[\mu\nu\rho}\psi^*_{\sigma]}\right)$$

$$\delta_\epsilon\lambda = -2i\gamma^\mu\partial_\mu A\epsilon^* - \frac{i}{3}\gamma^{\mu\nu\rho}\mathbf{F}_{\mu\nu\rho}\epsilon$$

$$\delta_\epsilon\psi_\mu = -i\partial_\nu h^{\mathrm{E}}_{\mu\rho}\gamma^{\nu\rho}\epsilon + \frac{i}{24}\left(\gamma_\mu^{\ \nu\rho\sigma}\mathbf{F}_{\nu\rho\sigma} - 9\gamma^{\nu\rho}\mathbf{F}_{\mu\nu\rho}\right)\epsilon^* - \frac{1}{2\cdot 5!}\gamma^{\nu[5]}\gamma_\mu F_{\nu[5]}\epsilon$$

$$\tilde{X}^{\tilde{\alpha}} \equiv \tilde{X}_\alpha$$

$$\Psi_\mu := \left(\psi^{\mathrm{RS}\alpha}_\mu, \tilde{\psi}^{\mathrm{RS}}_{\mu\dot{\alpha}}\right), \lambda := \left(\lambda_\alpha, \tilde{\lambda}^{\dot{\alpha}}\right)$$

$$F^\alpha{}_\beta = \delta^\alpha_\beta F + \frac{1}{2!}(\gamma^{\mu\nu})^\alpha{}_\beta F_{\mu\nu} + \frac{1}{4!}(\gamma^{\mu\nu\rho\sigma})^\alpha{}_\beta F_{\mu\nu\rho\sigma},$$

$$\partial_\mu F = 0,$$

$$\partial_{[\mu}F_{\nu\rho]} = 0, \partial^\mu F_{\mu\nu} = 0$$
$$\partial_{[\mu}F_{\nu\rho\sigma\tau]} = 0, \partial^\mu F_{\mu\nu\rho\sigma} = 0$$

$$\eta_{\mu\nu} := (-1,1,\ldots,1)$$

$$\epsilon^{0123456789} = -\epsilon_{0123456789} = 1, \epsilon^{\mu_1,\ldots,\mu_{10}}\epsilon_{\mu_1,\ldots,\mu_{10}} = -10!$$

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$$

$$\Psi^C := \Psi^T C = \Psi^\dagger\Gamma_0$$

$$C := \begin{pmatrix} 0 & \delta^{\dot{\beta}}_\alpha \\ -\delta^{\dot{\beta}}_\gamma & 0 \end{pmatrix}$$

$$\Psi =: \begin{pmatrix} \chi^\alpha \\ \bar{\chi}_{\dot{\beta}} \end{pmatrix}$$

$$C\Gamma^\mu =: \begin{pmatrix} (\gamma^\mu)_{\alpha\beta} & 0 \\ 0 & (\bar{\gamma}^\mu)^{\dot{\beta}\dot{\alpha}} \end{pmatrix}$$

$$(\gamma^\mu)_{\alpha\lambda}(\bar{\gamma}^\nu)^{\lambda\beta} + (\gamma^\nu)_{\alpha\lambda}(\bar{\gamma}^\mu)^{\lambda\beta} = 2\eta^{\mu\nu}\delta^\beta_\alpha$$

$$\gamma^{\mu[n]} = \gamma^{\mu_1\ldots\mu_n} := \gamma^{[\mu_1}\ldots\gamma^{\mu_n]}, n \le D,$$

$$(\gamma^\mu)_{\alpha(\beta}(\gamma_\mu)_{\lambda\delta)} = 0$$

$$\left(\gamma^{[10]}\right)_{\alpha}{}^{\beta} = -\epsilon^{[10]}\delta_{\alpha}{}^{\beta}, \qquad \left(\gamma^{[10]}\right)^{\alpha}{}_{\beta} = +\epsilon^{\mu[10]}\delta^{\alpha}{}_{\beta}$$
$$\left(\gamma^{\mu[9]}\right)_{\alpha\beta} = -\epsilon^{\mu[9]\nu[1]}\left(\gamma_{\nu[1]}\right)_{\alpha\beta}, \qquad \left(\gamma^{\mu[9]}\right)^{\alpha\beta} = +\epsilon^{\mu[9]\nu[1]}\left(\gamma_{\nu[1]}\right)^{\alpha\beta}$$
$$\left(\gamma^{\mu[8]}\right)_{\alpha}{}^{\beta} = +\frac{1}{2}\epsilon^{\mu[8]\nu[2]}\left(\gamma_{\nu[2]}\right)_{\alpha}{}^{\beta}, \quad \left(\gamma^{\mu[8]}\right)^{\alpha}{}_{\beta} = -\frac{1}{2}\epsilon^{\mu[8]\nu[2]}\left(\gamma_{\nu[2]}\right)^{\alpha}{}_{\beta},$$

$$\left(\gamma^{\mu[7]}\right)_{\alpha\beta} = +\frac{1}{3!}\epsilon^{\mu[7]\nu[3]}\left(\gamma_{\nu[3]}\right)_{\alpha\beta}, \qquad \left(\gamma^{\mu[7]}\right)^{\alpha\beta} = -\frac{1}{3!}\epsilon^{\mu[7]\nu[3]}\left(\gamma_{\nu[3]}\right)^{\alpha\beta}$$
$$\left(\gamma^{\mu[6]}\right)_{\alpha}{}^{\beta} = -\frac{1}{4!}\epsilon^{\mu[6]\nu[4]}\left(\gamma_{\nu[4]}\right)_{\alpha}{}^{\beta}, \quad \left(\gamma^{\mu[6]}\right)^{\alpha}{}_{\beta} = +\frac{1}{4!}\epsilon^{\mu[6]\nu[4]}\left(\gamma_{\nu[4]}\right)^{\alpha}{}_{\beta}$$
$$\left(\gamma^{\mu[5]}\right)_{\alpha\beta} = -\frac{1}{5!}\epsilon^{\mu[5]\nu[5]}\left(\gamma_{\nu[5]}\right)_{\alpha\beta}, \qquad \left(\gamma^{\mu[5]}\right)^{\alpha\beta} = +\frac{1}{5!}\epsilon^{\mu[5]\nu[5]}\left(\gamma_{\nu[5]}\right)^{\alpha\beta}$$

$$\xi_{\alpha}\chi^{\beta} = \frac{1}{16}\delta_{\alpha}^{\beta}(\xi\chi) + \frac{1}{2!}\frac{1}{16}(\gamma^{\mu\nu})_{\alpha}^{\beta}\left(\xi\gamma_{\mu\nu}\chi\right) + \frac{1}{4!}\frac{1}{16}(\gamma^{\mu\nu\rho\sigma})_{\alpha}^{\beta}\left(\xi\gamma_{\mu\nu\rho\sigma}\chi\right)$$
$$\xi^{\alpha}\chi^{\beta} = \frac{1}{16}(\gamma^{\mu})^{\alpha\beta}\left(\xi\gamma_{\mu}\chi\right) + \frac{1}{3!}\frac{1}{16}(\gamma^{\mu\nu\rho})^{\alpha\beta}\left(\xi\gamma_{\mu\nu\rho}\chi\right) + \frac{1}{5!}\frac{1}{32}(\gamma^{\mu\nu\rho\sigma\tau})^{\alpha\beta}\left(\xi\gamma_{\mu\nu\rho\sigma\tau}\chi\right)$$

$$\begin{aligned}
\gamma^{\mu}\gamma^{\nu} &= \gamma^{\mu\nu} + \eta^{\mu\nu}, & \gamma^{\mu}\gamma_{\mu} &= D \\
\gamma^{\mu\nu}\gamma^{\rho} &= \gamma^{\mu\nu\rho} + 2\gamma^{[\mu}\eta^{\nu]\rho}, & \gamma^{\mu\nu}\gamma_{\nu} &= (D-1)\gamma^{\mu} \\
\gamma^{\mu\nu\rho}\gamma^{\sigma} &= \gamma^{\mu\nu\rho\sigma} + 3\gamma^{[\mu\nu}\eta^{\rho]\sigma}, & \gamma^{\mu\nu\rho}\gamma_{\rho} &= (D-2)\gamma^{\mu\nu} \\
\gamma^{\mu\nu\rho\sigma}\gamma^{\tau} &= \gamma^{\mu\nu\rho\sigma\tau} + 4\gamma^{[\mu\nu\rho}\eta^{\sigma]\tau}, & \gamma^{\mu\nu\rho\sigma}\gamma_{\sigma} &= (D-3)\gamma^{\mu\nu\rho} \\
\gamma^{\mu\nu\rho\sigma\tau}\gamma^{\lambda} &= \gamma^{\mu\nu\rho\sigma\tau\lambda} + 5\gamma^{[\mu\nu\rho\sigma}\eta^{\tau]\lambda}, & \gamma^{\mu\nu\rho\sigma\tau}\gamma_{\tau} &= (D-4)\gamma^{\mu\nu\rho\sigma} \\
&\vdots & &\vdots \\
\gamma^{\mu_1\ldots\mu_p}\gamma^{\nu} &= \gamma^{\mu_1\ldots\mu_p\nu} + p\gamma^{[\mu_1\ldots\mu_{p-1}}\eta^{\mu_p]\nu}, & \gamma^{\mu_1\ldots\mu_p\nu}\gamma_{\nu} &= (D-p)\gamma^{\mu_1\ldots\mu_p}
\end{aligned}$$

$$\gamma_{\mu\nu}\gamma^{\rho\sigma} = \gamma_{\mu\nu}^{\rho\sigma} - 4\gamma_{[\mu}^{[\rho}\delta_{\nu]}^{\sigma]} - 2\delta_{\mu\nu}^{\rho\sigma}$$
$$\gamma_{\mu\nu\rho}\gamma^{\sigma\tau} = \gamma_{\mu\nu\rho}^{\sigma\tau} - 6\gamma_{[\mu\nu}^{[\sigma}\delta_{\rho]}^{\tau]} - 6\gamma_{[\mu}\delta_{\nu\rho]}^{\sigma\tau}$$
$$\gamma_{\mu\nu\rho\sigma}\gamma^{\tau\omega} = \gamma_{\mu\nu\rho\sigma}{}^{\tau\omega} - 8\gamma_{[\mu\nu\rho}^{[\tau}\delta_{\sigma]}^{\omega]} - 12\gamma_{[\mu\nu}\delta_{\rho\sigma]}^{\tau\omega}$$
$$\gamma_{\mu\nu\rho\sigma\tau}\gamma^{\omega\lambda} = \gamma_{\mu\nu\rho\sigma\tau}{}^{\omega\lambda} - 10\gamma_{[\mu\nu\rho\sigma}{}^{[\omega}\delta_{\tau]}^{\lambda]} - 20\gamma_{[\mu\nu\rho}\delta_{\sigma\tau]}^{\omega\lambda}$$

$$\gamma_{\mu_1\ldots\mu_p}\gamma^{\nu\rho} = \gamma_{\mu_1\ldots\mu_p}^{\nu\rho} - 2p\gamma_{[\mu_1\ldots\mu_{p-1}}^{[\nu}\delta_{\mu_p]}^{\rho]} - p(p-1)\gamma_{[\mu_1\ldots\mu_{p-2}}\delta_{\mu_{p-1}\mu_p]}^{\nu\rho}$$

$$\gamma_{\mu\rho}\gamma^{\rho\nu} = (D-1)\delta_{\mu}^{\nu} + (D-2)\gamma_{\mu}^{\nu}$$
$$\gamma_{\mu\nu}\gamma^{\mu\nu} = (1-D)D$$
$$\gamma^{\mu}\gamma^{\nu}\gamma_{\mu} = (2-D)\gamma^{\nu}$$
$$\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma_{\mu} = (D-4)\gamma^{\nu\rho} + D\eta^{\nu\rho}$$
$$\gamma_{\mu\nu}\gamma_{\rho}\gamma^{\mu\nu} = -(D-1)(D-4)\gamma_{\rho}$$
$$\gamma_{\rho\sigma}\gamma^{\mu\nu}\gamma^{\rho\sigma} = -[(D-7)(D-2)+2]\gamma^{\mu\nu}$$
$$\gamma^{\nu}\gamma_{\mu[p]}\gamma_{\nu} = -(D+(-1)^{p}2p)\gamma_{\mu[p]}$$
$$\gamma^{\nu}\gamma_{\mu[5]}\gamma_{\nu} = 0 \text{ for } D = 10, p = 5$$

$$\begin{aligned}
\left[P_{\mu}, P_{\nu}\right] &= 0 \\
\left[P_{\mu}, M_{\rho\sigma}\right] &= -i\left(\eta_{\mu\rho}P_{\sigma} - \eta_{\mu\sigma}P_{\rho}\right) \\
\left[M_{\mu\nu}, M_{\rho\sigma}\right] &= i\left(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}\right)
\end{aligned}$$

$$L = \oplus_{i=0}^{i=n} L_i,$$

$$[,\}: L \times L \to L,$$

$$\begin{aligned}
&\left[L_i, L_j\right\} \in L_{i+j} \bmod n + 1 \\
&\left[L_i, L_j\right\} = -(-1)^{\eta_i \eta_j}\left[L_j, L_i\right\} \\
&\left[L_i, \left\{L_j, L_k\right\}\right\}(-1)^{\eta_i \eta_k} + \left[L_j, \left[L_k, L_i\right\}\right\}(-1)^{\eta_i \eta_j} + \left[L_k, \left[L_i, L_j\right\}\right\}(-1)^{\eta_j \eta_k}
\end{aligned}$$

$$L_{\mathrm{SP}} = L_0 \oplus L_1,$$

$$\begin{aligned}
&\left[P_\mu, Q_\alpha^I\right] = \left[P_\mu, \bar{Q}_{\dot{\alpha}}^I\right] = 0, && \left\{Q_\alpha^I, \bar{Q}_{\dot{\alpha}}^J\right\} = 2\delta^{IJ}\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \\
&\left[M_{\mu\nu}, Q_\alpha^I\right] = -i\left(\sigma_{\mu\nu}\right)_\alpha{}^\beta Q_\beta^I, && \left\{Q_\alpha^I, Q_\beta^J\right\} = \epsilon_{\alpha\beta} Z^{IJ} \\
&\left[M_{\mu\nu}, \bar{Q}_{\dot{\alpha}}^I\right] = -i\left(\bar{\sigma}_{\mu\nu}\right)_{\dot{\alpha}\dot{\beta}} \bar{Q}^{I\dot{\beta}}, && \left\{\bar{Q}_{\dot{\alpha}}^I, \bar{Q}_{\dot{\beta}}^J\right\} = \epsilon_{\dot{\alpha}\dot{\beta}} (Z^{IJ})^*
\end{aligned}$$

$$J^2|j,m\rangle = j(j+1)|j,m\rangle, J_3|j,m\rangle = m|j,m\rangle$$

$$J_3(Q_1^I|j,m\rangle) = \left(m - \frac{1}{2}\right) Q_1^I|j,m\rangle,$$

$$\begin{aligned}
\langle\psi|\left\{Q_\alpha^I, \bar{Q}_{\dot{\alpha}}^I\right\}|\psi\rangle &= 2\mathcal{N}\sigma_{\alpha\dot{\alpha}}^\mu \langle\psi|P_\mu|\psi\rangle \\
&| \\
&= \langle\psi|\left[Q_\alpha^I (Q_\alpha^I)^\dagger + (Q_\alpha^I)^\dagger Q_\alpha^I\right]|\psi\rangle = \|Q_\alpha^I|\psi\rangle\|^2 + \|(Q_\alpha^I)^\dagger|\psi\rangle\|^2
\end{aligned}$$

$$\langle\psi|P_0|\psi\rangle \geq 0$$

$$\mathrm{Tr}(-1)^{N_\mathrm{F}} = n_\mathrm{B} - n_\mathrm{F}.$$

$$\{(-1)^{N_\mathrm{F}}, Q_\alpha^I\} = 0$$

$$\begin{aligned}
0 &= \mathrm{Tr}\left[(-1)^{N_\mathrm{F}} Q_\alpha^I \bar{Q}_{\dot{\alpha}}^I - \bar{Q}_{\dot{\alpha}}^I (-1)^{N_\mathrm{F}} Q_\alpha^I\right] = \mathrm{Tr}\left[(-1)^{N_\mathrm{F}}\left\{Q_\alpha^I, \bar{Q}_{\dot{\alpha}}^I\right\}\right] = \\
&= 2\mathcal{N}[\mathrm{Tr}(-1)^{N_\mathrm{F}}]\sigma_{\alpha\dot{\alpha}}^\mu P_\mu = 2\mathcal{N}(n_\mathrm{B} - n_\mathrm{F})\sigma_{\alpha\dot{\alpha}}^\mu P_\mu
\end{aligned}$$

$$\left\{Q_\alpha^I, \bar{Q}_{\dot{\alpha}}^J\right\} = 2\delta^{IJ}\sigma_{\alpha\dot{\alpha}}^\mu P_\mu = \begin{pmatrix} 4E & 0 \\ 0 & 0 \end{pmatrix}\delta^{IJ}$$

$$a_I = \frac{1}{2\sqrt{E}} Q_{1I}, a_I^\dagger = \frac{1}{2\sqrt{E}} \bar{Q}_{\dot{1}I} = (a_I)^\dagger$$

$$\left\{a_I, a_J^\dagger\right\} = \delta_{IJ}, \left\{a_I, a_J\right\} = \left\{a_I^\dagger, a_J^\dagger\right\} = 0$$

$$a_I|\lambda_0\rangle = 0 \; \forall I$$

$$|\lambda_0\rangle, a_I^\dagger|\lambda_0\rangle = \left|\lambda_0 + \frac{1}{2}\right\rangle_I, \ldots \; a_{\mathcal{N}^\dagger}^\dagger \ldots a_1^\dagger|\lambda_0\rangle = \left|\lambda_0 + \frac{\mathcal{N}}{2}\right\rangle,$$

$$\left\{\lambda_0, \lambda_0 + \frac{1}{2}\right\} \oplus \left\{-\left(\lambda_0 + \frac{1}{2}\right), -\lambda_0\right\},$$

$$\left\{0, \frac{1}{2}\right\} \oplus \left\{-\frac{1}{2}, 0\right\}$$

$$\left\{\frac{1}{2},1\right\}\oplus\left\{-1,-\frac{1}{2}\right\}$$

$$\left\{-\frac{1}{2},0,0,\frac{1}{2}\right\}\oplus\left\{-\frac{1}{2},0,0,\frac{1}{2}\right\}$$

$$\{Q_\alpha,\bar{Q}_{\dot{\alpha}}\}=2M\delta_{\alpha\dot{\alpha}}$$

$$b_\alpha=\frac{1}{\sqrt{2M}}Q_\alpha, b_{\dot{\alpha}}^\dagger=\frac{1}{\sqrt{2M}}\bar{Q}_{\dot{\alpha}}$$

$$\left\{b_\alpha,b_{\dot{\alpha}}^\dagger\right\}=\delta_{\alpha\dot{\alpha}},\left\{b_\alpha,b_\beta\right\}=\left\{b_{\dot{\alpha}}^\dagger,b_{\dot{\beta}}^\dagger\right\}=0$$

$$|j_0=0\rangle \quad\rightarrow\left\{-\frac{1}{2},0,0,\frac{1}{2}\right\}$$
$$\left|j_0=\frac{1}{2}\right\rangle \quad\rightarrow\left\{0,2\times\frac{1}{2},1\right\}\oplus\left\{-1,2\times-\frac{1}{2},0\right\}$$

$$\left\{c_\alpha^r,\left(c_\beta^s\right)^\dagger\right\}=(2m+Z_r)\delta^{rs}\delta_{\alpha\beta},\left\{d_\alpha^r,\left(d_\beta^s\right)^\dagger\right\}=(2m-Z_r)\delta^{rs}\delta_{\alpha\beta}$$

$$|Z_r|\leq 2m\ \forall r,$$

$$[\bar{Q}_{\dot{\alpha}},\phi]=0$$

$$[Q_\alpha,\phi]\equiv\psi_\alpha$$

$$\{Q_\alpha,\psi_\beta\}=F_{\alpha\beta},\{\bar{Q}_{\dot{\alpha}},\psi_\beta\}=H_{\dot{\alpha}\beta}$$

$$F_{\alpha\beta}+F_{\beta\alpha}=0\ \leftrightarrow F_{\alpha\beta}=F\epsilon_{\alpha\beta},H_{\dot{\alpha}\beta}=2\sigma_{\beta\dot{\alpha}}^\mu[P_\mu,\phi]\sim\partial_\mu\phi$$

$$[Q_\alpha,F]=0,[\bar{Q}_{\dot{\alpha}},F]=-2\left(\partial_\mu\psi\sigma^\mu\right)_{\dot{\alpha}}$$

$$\{\phi,\psi,F\}.$$

$$\delta_\epsilon=i(\epsilon Q+\bar{\epsilon}\bar{Q})$$

$$\delta\phi=\sqrt{2}\epsilon\psi$$
$$\delta\psi=i\sqrt{2}(\sigma^\mu\bar{\epsilon})\partial_\mu\phi+\sqrt{2}F\epsilon$$
$$\delta F=i\sqrt{2}\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\psi$$

$$\left[\delta_{\epsilon_1},\delta_{\epsilon_2}\right]=-2i(\epsilon_1\sigma^\mu\bar{\epsilon}_2-\epsilon_2\sigma^\mu\bar{\epsilon}_1)\partial_\mu$$

$$\theta\rightarrow\theta+\epsilon,\bar{\theta}\rightarrow\bar{\theta}+\bar{\epsilon},\delta_\epsilon x^\mu=i(\epsilon\sigma^\mu\bar{\theta}-\theta\sigma\bar{\epsilon})$$

$$G(x,\theta,\bar{\theta})=e^{i\left(-x^\mu P_\mu+\epsilon Q+\bar{\epsilon}\bar{Q}\right)}$$

$$F(x^\mu,\theta_\alpha,\bar{\theta}_{\dot{\alpha}})=f(x)+\theta\psi(x)+\bar{\theta}\bar{\chi}(x)+\theta^2M(x)+\bar{\theta}^2\bar{N}(x)+\theta\sigma^\mu\bar{\theta}A_\mu(x)$$
$$+\theta^2\bar{\theta}\bar{\lambda}(x)+\bar{\theta}^2\theta\zeta(x)+\theta^2\bar{\theta}^2D(x)$$

$$e^{i(\epsilon Q+\bar{\epsilon}\bar{Q})}F(x,\theta,\bar{\theta})e^{-i(\epsilon Q+\bar{\epsilon}\bar{Q})}$$

$$Q_\alpha = -i\partial_\alpha - (\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \bar{Q}_{\dot{\alpha}} = i\bar{\partial}_{\dot{\alpha}} + (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = -2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$

$$D_\alpha = \partial_\alpha + i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

$$\{D, D\} = \{\bar{D}, \bar{D}\} = \{D, Q\} = \{D, \bar{Q}\} = \{\bar{D}, Q\} = \{\bar{D}, \bar{Q}\} = 0$$
$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$

$$\int d^4x d^2\theta d^2\bar{\theta} F(x, \theta, \bar{\theta})$$

$$\begin{aligned}\delta_\epsilon \int d^4x d^2\theta d^2\bar{\theta} F(x, \theta, \bar{\theta}) &= \int d^4x d^2\theta d^2\bar{\theta}\delta_\epsilon F(x, \theta, \bar{\theta}) \\ &= \int d^4x d^2\theta d^2\bar{\theta}(i\epsilon Q + i\bar{\epsilon}\bar{Q}) F(x, \theta, \bar{\theta}) \\ &= \int d^4x d^2\theta d^2\bar{\theta}\left[\epsilon^\alpha\partial_\alpha + \bar{\epsilon}_{\dot{\alpha}}\partial^{\dot{\alpha}} + i(\epsilon\sigma^\mu\bar{\theta} - \theta\sigma^\mu\bar{\epsilon})\partial_\mu\right] F(x, \theta, \bar{\theta})\end{aligned}$$

$$\bar{D}_{\dot{\alpha}}\Phi = 0$$

$$x^\mu \longrightarrow y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

$$D_\alpha = \partial_\alpha + 2i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}}$$

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\begin{aligned}\Phi(x, \theta, \bar{\theta}) =& \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) \\ &-\frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \frac{1}{4}\theta^2\bar{\theta}^2\,\square\,\phi(x)\end{aligned}$$

$$S = \int d^4x d^2\theta d^2\bar{\theta}\bar{\Phi}\Phi$$

$$\mathcal{L} = -\partial_\mu\bar{\phi}\partial^\mu\phi - i\psi\sigma^\mu\partial_\mu\bar{\psi} + \bar{F}F,$$

$$\int d^4x d^2\theta d^2\bar{\theta} K(\bar{\Phi}, \Phi)$$

$$K(\bar{\Phi}, \Phi) \longrightarrow K(\bar{\Phi}, \Phi) + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}),$$

$$K_i \equiv \partial_i K \equiv \frac{\partial K}{\partial\phi^i},$$

$$\begin{aligned}\int d^2\theta d^2\bar{\theta} K(\Phi, \bar{\Phi}) =& K_{i\bar{\jmath}}\left[-\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{\jmath}} - \left(\frac{i}{2}\bar{\psi}^{\bar{\jmath}}\bar{\sigma}^\mu\partial_\mu\psi^i + h.c.\right) + F^i\bar{F}^{\bar{\jmath}}\right] \\ &-\left[\frac{i}{2}K_{ij\bar{k}}\left(\psi^i\sigma^\mu\bar{\psi}^{\bar{k}}\partial_\mu\phi^j - i\psi^i\psi^j\bar{F}^{\bar{k}}\right) + h.c.\right] \\ &+\frac{1}{4}K_{ij\bar{k}\bar{l}}\psi^i\psi^j\bar{\psi}^{\bar{k}}\bar{\psi}^{\bar{l}}\end{aligned}$$

$$K_{i\bar{\jmath}} = \partial_i\partial_{\bar{\jmath}}K$$

$$\Gamma^i_{jk} = K^{i\bar{l}}K_{jk\bar{l}}, \Gamma^{\bar{l}}_{\bar{j}\bar{k}} = K^{l\bar{l}}K_{\bar{j}\bar{k}l}, R_{i\bar{j}k\bar{l}} = K_{m\bar{l}}\partial_{\bar{j}}\Gamma^m_{ik}$$

$$D_\mu\psi^i = \partial_\mu\psi^i + \Gamma^i_{jk}(\sigma^\mu\psi^j)\partial_\mu\phi^k, D_iV_j = \partial_iV_j - \Gamma^l_{ij}V_l$$

$$\int d^2\theta W(\Phi) + h.c.$$

$$\int d^2\theta W(\Phi) = W_iF^i - \frac{1}{2}W_{ij}\psi^i\psi^j$$

$$S_{\text{chiral}} = \int d^4x\left\{\int d^2\theta d^2\bar{\theta}K(\Phi,\bar{\Phi}) + \left(\int d^2\theta W(\Phi) + \text{ h.c. }\right)\right\}$$

$$\begin{aligned}\mathcal{L}_{\text{chiral}} =& K_{i\bar{j}}\left[-\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}} - \left(\frac{i}{2}\bar{\psi}^{\bar{j}}\bar{\sigma}^\mu\partial_\mu\psi^i + \text{ h.c. }\right) + F^i\bar{F}^{\bar{j}}\right]\\ &-\left[\frac{i}{2}K_{ij\bar{k}}\left(\psi^i\sigma^\mu\bar{\psi}^{\bar{k}}\partial_\mu\phi^j - i\psi^i\psi^jF^{\bar{k}}\right) + \text{ h.c. }\right]\\ &+\frac{1}{4}K_{ij\bar{k}\bar{l}}\psi^i\psi^j\bar{\psi}^{\bar{k}}\bar{\psi}^{\bar{l}} + \left(W_iF^i - \frac{1}{2}W_{ij}\psi^i\psi^j + \text{ h.c. }\right)\end{aligned}$$

$$F^i = -K^{i\bar{j}}\bar{W}_{\bar{j}} + \Gamma^i_{jk}\psi^j\psi^k,$$

$$\begin{aligned}\mathcal{L}_{\text{chiral}} =& -K_{i\bar{j}}\left[\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}} + \left(\frac{i}{2}\bar{\psi}^{\bar{j}}\bar{\sigma}^\mu D_\mu\psi^i + \text{ h.c. }\right)\right] - K^{i\bar{j}}D_iW\bar{D}_{\bar{j}}\bar{W}\\ &-\frac{1}{2}\left(D_iD_jW\psi^i\psi^j + \text{ h.c. }\right) + \frac{1}{4}R_{i\bar{j}k\bar{l}}\psi^i\psi^k\bar{\psi}^{\bar{j}}\bar{\psi}^{\bar{l}}\end{aligned}$$

$$V(\phi,\bar{\phi}) = K^{i\bar{j}}D_iW\bar{D}_{\bar{j}}\bar{W},$$

$$K_{\text{WZ}}(\Phi) = \bar{\Phi}\Phi, W_{\text{WZ}}(\Phi) = \frac{m}{2}\Phi^2 + \frac{\lambda}{3}\Phi^3$$

$$\mathcal{L}_{\text{WZ}} = -\partial_\mu\phi\partial^\mu\bar{\phi} - \left(\frac{i}{2}\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + \text{ h.c. }\right) - \frac{m}{2}(\psi\psi + \bar{\psi}\bar{\psi}) - |m\phi + \lambda\phi^2|^2 - \lambda(\phi\psi\psi + \text{ h.c. }),$$

$$\delta\phi = \sqrt{2}\epsilon\psi, \delta\psi = i\sqrt{2}(\sigma^\mu\bar{\epsilon})\partial_\mu\phi - \sqrt{2}(m\bar{\phi} + \lambda\bar{\phi}^2)\epsilon,$$

$$V = \bar{V}.$$

$$\begin{aligned}V(x,\theta,\bar{\theta}) =& C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) - \theta\sigma^\mu\bar{\theta}A_\mu(x)\\ &+\frac{i}{2}\theta^2[M(x) + iN(x)] - \frac{i}{2}\bar{\theta}^2[M(x) - iN(x)]\\ &+i\theta^2\bar{\theta}\left[\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right] - i\bar{\theta}^2\theta\left[\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\right]\\ &+\frac{1}{2}\theta^2\bar{\theta}^2[D(x) + \Box\, C(x)].\end{aligned}$$

$$V \longrightarrow V + i\Lambda - i\bar{\Lambda},$$

$$A_\mu \longrightarrow A_\mu + \partial_\mu(\alpha + \bar{\alpha})$$

$$V(x,\theta,\bar{\theta}) = -\theta\sigma^{\mu}\bar{\theta}A_{\mu}(x) + i\theta^{2}\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}^{2}\theta\lambda(x) + \frac{1}{2}\theta^{2}\bar{\theta}^{2}D(x)$$

$$\begin{aligned}\delta A_{\mu} &= i\left(\bar{\epsilon}\bar{\sigma}_{\mu}\lambda - \bar{\lambda}\bar{\sigma}_{\mu}\epsilon\right)\\ \delta\lambda &= iD\epsilon + F_{\mu\nu}\sigma^{\mu\nu}\epsilon\\ \delta D &= -\epsilon\sigma^{\mu}\partial_{\mu}\bar{\lambda} - \partial_{\mu}\lambda\sigma^{\mu}\bar{\epsilon}\end{aligned}$$

$$W_{\alpha} = -\frac{1}{4}\bar{D}^{2}D_{\alpha}V$$

$$W_{\alpha}(y,\theta) = -i\lambda_{\alpha}(y) + \left[D(y)\delta_{\alpha}^{\beta} - iF_{\mu\nu}(y)(\sigma^{\mu\nu})_{\alpha}^{\beta}\right] + \theta^{2}\left(\sigma^{\mu}\partial_{\mu}\bar{\lambda}\right)_{\alpha}$$

$$\mathcal{L} = \frac{1}{4}\int d^{2}\theta W^{\alpha}W_{\alpha} + \frac{1}{4}\int d^{2}\bar{\theta}\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \left(\frac{i}{2}\lambda\sigma^{\mu}\partial_{\mu}\bar{\lambda} + h.c.\right) + \frac{1}{2}D^{2},$$

$$\mathcal{L}_{\mathrm{FI}} = 2\xi\int d^{2}\theta d^{2}\bar{\theta}V = \xi D$$

$$V = V_{a}T^{a}, \Lambda = \Lambda_{a}T^{a}, [T^{a},T^{b}] = f^{a}{}_{bc}T^{c}, \mathrm{Tr}\left(T^{a}T^{b}\right) = \delta^{ab}$$

$$W_{\alpha} = -\frac{1}{8g}\bar{D}^{2}(e^{-2gV}D_{\alpha}e^{2gV})$$

$$W_{\alpha} = -\frac{1}{4}\bar{D}^{2}D_{\alpha}V + \frac{g}{4}\bar{D}^{2}[V, D_{\alpha}V]$$

$$W_{\alpha}^{a}(y,\theta) = -i\lambda_{\alpha}^{a}(y) + \left[D^{a}(y)\delta_{\alpha}^{\beta} - iF_{\mu\nu}^{a}(y)(\sigma^{\mu\nu})_{\alpha}^{\beta}\right]\theta_{\beta} + \theta^{2}\left(\sigma^{\mu}\mathcal{D}_{\mu}\bar{\lambda}^{a}\right)_{\alpha}$$

$$F_{\mu\nu}^{a} = \partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a} - gf^{a}{}_{bc}A_{\mu}^{b}A_{\nu}^{c}, \mathcal{D}_{\mu}\lambda^{a} = \partial_{\mu}\lambda^{a} - gf^{a}{}_{bc}A_{\mu}^{b}\lambda^{c}$$

$$\Phi \longrightarrow e^{-i\Lambda}\Phi$$

$$\bar{\Phi}e^{2gV}\Phi$$

$$\mathcal{L}_{\mathrm{SYM}} = \int d^{2}\theta d^{2}\bar{\theta}\bar{\Phi}e^{2gV}\Phi + \left(\int d^{2}\theta\left[\frac{1}{4}\mathrm{Tr}(W^{\alpha}W_{\alpha}) + W(\Phi)\right] + h.c.\right)$$

$$\begin{aligned}\delta\phi^{i} &= \sqrt{2}\bar{\epsilon}\psi^{i}, & \delta A_{\mu}^{a} &= i\left(\bar{\epsilon}\bar{\sigma}_{\mu}\lambda^{a} - \bar{\lambda}^{a}\bar{\sigma}_{\mu}\epsilon\right)\\ \delta\psi^{i} &= i\sqrt{2}(\sigma^{\mu}\bar{\epsilon})\mathcal{D}_{\mu}\phi^{i} + \sqrt{2}F^{i}\epsilon, & \delta\lambda^{a} &= iD^{a}\epsilon + F_{\mu\nu}^{a}\sigma^{\mu\nu}\epsilon\\ \delta F^{i} &= \sqrt{2}\bar{\epsilon}\bar{\sigma}^{\mu}\mathcal{D}_{\mu}\psi + 2ig(T_{a})^{i}{}_{j}\phi^{j}\bar{\epsilon}\bar{\lambda}^{a} & \delta D^{a} &= -\epsilon\sigma^{\mu}\mathcal{D}_{\mu}\bar{\lambda}^{a} - \mathcal{D}_{\mu}\lambda^{a}\sigma^{\mu}\bar{\epsilon}\end{aligned}$$

$$\mathcal{D}_{\mu}\phi^{i} = \partial_{\mu}\phi^{i} + igA_{\mu}^{a}(T_{a})^{i}{}_{j}\phi^{j}, \mathcal{D}_{\mu}\psi^{i} = \partial_{\mu}\psi^{i} + igA_{\mu}^{a}(T_{a})^{i}{}_{j}\psi^{j}$$

$$V(\phi,\bar{\phi}) = \sum_{i}\left|F^{i}\right|^{2} + \frac{1}{2}\sum_{a}D^{a}D_{a},$$

$$F^{i} = -\bar{W}_{\bar{\imath}}, D^{a} = -g(T^{a})_{ij}\phi^{i}\bar{\phi}^{\bar{\jmath}}$$

$$F^{i} = -K^{i\bar{\jmath}}\bar{W}_{\bar{\jmath}} = 0, D^{a} = -g(T^{a})_{ij}\phi^{i}\bar{\phi}^{\bar{\jmath}} - \xi^{a} = 0$$

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}(\bar{\Phi}_1 e^{2gV}\Phi_1 + \bar{\Phi}_2 e^{-2gV}\Phi_2 + 2\xi V) + \left[\int d^2\theta \left(\frac{1}{4}W^\alpha W_\alpha + m\Phi_1\Phi_2\right) + h.c.\right]$$

$$V = m^2(|\phi_1|^2 + |\phi_2|^2) + \frac{1}{2}[g(|\phi_1|^2 - |\phi_2|^2) + 2\xi],$$

$$F_1 = -m\bar{\phi}_2 = 0, F_2 = -m\bar{\phi}_1 = 0, D = -\xi - g(|\phi_1|^2 - |\phi_2|^2) = 0.$$

$$W(\Phi) = a_i\Phi_i + \frac{1}{2}b_{ij}\Phi^i\Phi^j + \frac{1}{3}c_{ijk}\Phi^i\Phi^j\Phi^k + \cdots$$

$$a_i + b_{ij}\phi^j + c_{ijk}\phi^i\phi^j + \cdots = 0.$$

$$K(\Phi, \bar{\Phi}) = \bar{\Phi}\Phi, W(\Phi) = f\Phi$$

$$W_{\mathrm{O,R}} = \frac{h}{2}X\Phi_1^2 + m\Phi_1\Phi_2 + fX$$

$$\begin{aligned}\bar{F}_1 &= -hX\phi_1 - m\phi_2 = 0\\ \bar{F}_2 &= -m\phi_1 = 0\\ \bar{F}_X &= -\frac{h}{2}\phi_1^2 - f = 0\end{aligned}$$

$$y \equiv \left|\frac{hf}{m^2}\right|$$

$$\phi_2 = -\frac{h}{m}X\phi, \phi_1 = \pm i\sqrt{\frac{2f}{h}\left(\frac{y-1}{y}\right)},$$

$$\langle\delta\psi^i\rangle \sim \langle F^i\rangle\epsilon, \langle\delta\lambda^a\rangle \sim \langle D^a\rangle\epsilon$$

$$G = \frac{1}{f}\left(\langle F_i\rangle\psi^i + \langle D_a\rangle\lambda^a\right)$$

$$-\frac{1}{2}(\psi^i \quad \lambda^a)\begin{pmatrix} W_{ij} & -i\sqrt{2}g\bar{\phi}^{\bar{j}}(T_b)_{i\bar{j}} \\ -i\sqrt{2}g\bar{\phi}^j(T_a)_{ij} & 0\end{pmatrix}\begin{pmatrix}\psi^j \\ \lambda^b\end{pmatrix}$$

$$\chi'(x') = \chi(x) + f\epsilon$$

$$\delta_\epsilon\chi(x) = \chi'(x) - \chi(x) = f\epsilon + \frac{i}{f}[\chi(x)\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\chi}(x)]\partial_\mu\chi(x)$$

$$[\delta_1, \delta_2]\chi = -2i(\epsilon_1\sigma^\mu\bar{\epsilon}_2 - \epsilon_2\sigma^\mu\bar{\epsilon}_1)\partial_\mu\chi$$

$$a_\mu^\alpha = \delta_\mu^\alpha + \frac{i}{f^2}\left(\chi\sigma^\alpha\partial_\mu\bar{\chi} - \partial_\mu\chi\sigma^\alpha\bar{\chi}\right)$$

$$\mathcal{L}_{\mathrm{VA}} = -\frac{f^2}{2}\mathrm{det}a = -\frac{f^2}{2} - \frac{i}{2}\left(\chi\sigma^\mu\partial_\mu\bar{\chi} - \partial_\mu\chi\sigma^\mu\bar{\chi}\right) + \cdots$$

$$S^2 = 0$$

$$S = \frac{G^2}{2F_S} + \sqrt{2}\theta G + \theta^2 F_S$$

$$K(S,\bar{S}) = \bar{S}S, W(S) = fS$$

$$F_S = -f\left(1 + \frac{\bar{G}^2}{4f^4}\,\square\, G^2 - \frac{3}{16f^8}G^2\bar{G}^2\,\square\, G^2\,\square\,\bar{G}^2\right),$$

$$\mathcal{L}_S = -f^2 - iG\sigma^\mu\partial_\mu\bar{G} + \frac{1}{4f^2}\bar{G}^2\,\square\, G^2 - \frac{1}{16f^6}G^2\bar{G}^2\,\square\, G^2\,\square\,\bar{G}^2,$$

$$S(\Phi - \bar{\Phi}) = 0$$

$$\Phi = \phi + \sqrt{2}\theta\chi_\phi + \theta^2 F_\phi, \phi = A + iB$$

$$B = -\frac{G\sigma^\mu\bar{G}}{2|F_S|^2}\partial_\mu A$$

$$\chi_\phi = -i\sigma^\mu\frac{\bar{G}}{\bar{F}_S}\partial_\mu\phi$$

$$F_\phi = -\partial_\nu\left(\frac{\bar{G}}{\bar{F}_S}\right)\bar{\sigma}^\mu\sigma^\nu\frac{\bar{G}}{\bar{F}_S}\partial_\mu A + \frac{1}{2}\left(\frac{\bar{G}}{\bar{F}_S}\right)^2\square A$$

$$K = h(\mathcal{A})\mathcal{B}^2 + \bar{S}S, W = f(\Phi)S + g(\Phi)$$

$$\Phi = \mathcal{A} + i\mathcal{B} \;\leftrightarrow\; \mathcal{A} = \frac{1}{2}(\Phi + \bar{\Phi}), \mathcal{B} = \frac{1}{2i}(\Phi - \bar{\Phi})$$

$$\begin{aligned}\mathcal{L} =& |F_S|^2 + \frac{h}{2}\partial_\mu A\partial^\mu A - \left(\frac{i}{2}\partial_\mu G\sigma^\mu\bar{G} + \text{ h.c. }\right) + \frac{1}{4}\frac{\bar{G}^2}{\bar{F}_S}\square\left(\frac{G^2}{F_S}\right)\\ &+ \frac{h}{2}\left[i\partial^\mu\left(\frac{G}{F_S}\right)\sigma^\nu\left(\frac{\bar{G}}{\bar{F}_S}\right)\partial_\mu A\partial_\nu A - \frac{i}{2}\partial_\mu\left(\frac{G}{F_S}\right)\sigma^\mu\left(\frac{\bar{G}}{\bar{F}_S}\right)(\partial A)^2 + h.c.\right]\\ &+ \left[fF_S + \frac{i}{2}f'\frac{G\sigma^\mu\bar{G}}{\bar{F}_S}\partial_\mu A + \frac{g''}{2}\left(\frac{\bar{G}}{\bar{F}_S}\right)^2(\partial A)^2 + \frac{g'}{2}\left(\frac{\bar{G}}{\bar{F}_S}\right)^2\square A\right.\\ &\left. - g'\partial_\nu\left(\frac{\bar{G}}{\bar{F}_S}\right)\bar{\sigma}^\mu\sigma^\nu\frac{\bar{G}}{\bar{F}_S}\partial_\mu A + h.c.\right].\end{aligned}$$

$$\begin{aligned}\mathcal{L} =& -|f|^2 - \frac{h}{2}\partial_\mu A\partial^\mu A + \left(\frac{i}{2}\partial_\mu G\sigma^\mu\bar{G} + \text{ h.c. }\right) - \left(i\frac{f'}{2f} + \text{ h.c. }\right)G\sigma^\mu\bar{G}\partial_\mu A\\ &+ \frac{h}{2|f|^2}\left[i\partial^\mu G\sigma^\nu\bar{G}\partial_\mu A\partial_\nu A - \frac{i}{2}\partial_\mu G\sigma^\mu\bar{G}\partial_\nu A\partial^\nu A + \text{ h.c. }\right]\\ &- \left[\frac{\bar{g}'}{\bar{f}^2}\left(\frac{1}{2}G^2\,\square\, A - \partial_\mu G\sigma^\mu\bar{\sigma}^\nu G\partial_\nu A\right) + \xi G^2\partial_\mu A\partial^\mu A + \text{ h.c. }\right]\\ &+ \frac{1}{4|f|^2}\bar{G}^2\,\square\, G^2,\end{aligned}$$

$$\xi \equiv \frac{\bar{g}''}{2\bar{f}^2} - \frac{\bar{g}'\bar{f}'}{\bar{f}^3}$$

$$\delta G^{\alpha}=\frac{h}{4|f|^2}(\partial A)^2 G^{\alpha}, \delta G^{\alpha}=-i\frac{g'}{f^2}(\bar{G}\bar{\sigma}^{\mu})^{\alpha}\partial_{\mu}A, \delta A=\frac{\bar{g}'}{2h\bar{f}^2}G^2+h.c,$$

$$\begin{aligned}\mathcal{L}=&-|f|^2-\frac{h}{2}\partial_{\mu}A\partial^{\mu}A+\left(\frac{i}{2}\partial_{\mu}G\sigma^{\mu}\bar{G}+\text{ h.c.}\right)+(mG^2+\text{ h.c.})+\lambda G^2\bar{G}^2\\&-\left(i\frac{f'}{2f}G\sigma^{\mu}\bar{G}\partial_{\mu}A+\tilde{\xi}G^2\partial_{\mu}A\partial^{\mu}A+\text{ h.c.}\right)-\frac{1}{8h}\left(\frac{\bar{g}'^2}{\bar{f}^4}G^2\,\square\,G^2+\text{ h.c.}\right)\\&+\frac{1}{2|f|^2}\left(h-\frac{2|g'|^2}{|f|^2}\right)\left(i\partial^{\mu}G\sigma^{\nu}\bar{G}\partial_{\mu}A\partial_{\nu}A+\text{ h.c.}\right)\\&+\frac{1}{4|f|^2}\left(1-\frac{|g'|^2}{h|f|^2}\right)\bar{G}^2\,\square\,G^2,\end{aligned}$$

$$m=\frac{g'f'}{hf}, \lambda=\frac{(f'^2+ff'')g'^2}{2h^2f^4}, \tilde{\xi}=\xi-\frac{g'h'}{4hf^2}$$

$$\mathcal{A}(G,\bar{G}\to G,\bar{G})=\frac{h|f|^2-|g'|^2}{h|f|^4}s^2$$

$$\mathcal{A}(G,G\to\bar{G},\bar{G})=\frac{2\bar{g}'^2}{h\bar{f}^4}s^2$$

$$\mathcal{A}(G,A\to G,A)=\frac{h|f|^2-2|g'|^2}{2|f|^4}s^2$$

$$\left.\frac{d^2}{ds^2}\mathcal{A}(G,\bar{G}\to G,\bar{G})\right|_{s=t=0}\geq 0\Leftrightarrow h|f|^2\geq|g'|^2$$

$$\left.\frac{d^2}{ds^2}\mathcal{A}(G,A\to G,A)\right|_{s=t=0}\geq 0\Leftrightarrow h|f|^2\geq 2|g'|^2$$

$$\left.\frac{d^2}{ds^2}\left[\mathcal{A}(G,\bar{G}\to G,\bar{G})-\frac{1}{2}|\mathcal{A}(G,G\to\bar{G},\bar{G})|\right|_{\blacksquare\aleph|_{s=t=0}}\geq 0\Leftrightarrow h|f|^2\geq 2|g'|^2\right.$$

$$h(0)|f(0)|^2\geq 2|g'(0)|^2$$

$$\left\{\left(1+\frac{h\dot{A}^2}{2|f|^2}\right)^2k_0^2-\left[\left(1-\frac{h\dot{A}^2}{2|f|^2}\right)^2+4\dot{A}^2\frac{|g'|^2}{|f|^4}\right]k_3^3\right\}G_{\alpha}=0$$

$$c_s^2=1-\frac{4\dot{A}^2}{\left(|f(A)|^2+\frac{h(A)\dot{A}^2}{2}\right)^2}\left(\frac{h(A)}{2}|f(A)|^2-|g'(A)|^2\right)$$

$$h(A)|f(A)|^2\geq 2|g'(A)|^2,$$

$$\mathcal{H}=H+\sqrt{2}\theta\chi_H+\theta^2F_H,$$

$$S\overline{\mathcal{H}}=\text{ chiral .}$$

$$\chi_H = -i\sigma^\mu \frac{\bar{G}}{\bar{F}_S}\partial_\mu H$$

$$F_H = -\partial_\nu\left(\frac{\bar{G}}{\bar{F}_S}\right)\bar{\sigma}^\mu\sigma^\nu\frac{\bar{G}}{\bar{F}_S}\partial_\mu H + \frac{1}{2}\left(\frac{\bar{G}}{\bar{F}_S}\right)^2 \square H,$$

$$K = \kappa(\mathcal{H},\overline{\mathcal{H}}) + \bar{S}S, W(\mathcal{H}) = f(\mathcal{H})S + g(\mathcal{H}),$$

$$\begin{aligned}\mathcal{L} =& -|f|^2 + \left(\frac{i}{2}\partial_\mu G\sigma^\mu\bar{G} + \text{ h.c. }\right) - \kappa_{H\bar{H}}\partial_\mu H\partial^\mu\bar{H}\\ &+\frac{1}{4|f|^2}\bar{G}^2\,\square\, G^2 + \frac{\kappa_H\bar{H}}{2|f|^2}(i\partial^\mu G\sigma^\nu\bar{G} + \text{ h.c. })\left(\partial_\mu H\partial_\nu\bar{H} + \partial_\nu H\partial_\mu\bar{H}\right)\\ &-\frac{\kappa_{H\bar{H}}}{2|f|^2}(i\partial_\mu G\sigma^\mu\bar{G} + \text{ h.c. })\partial_\nu H\partial^\nu\bar{H} - \left[i\frac{f_H}{f}G\sigma^\mu\bar{G}\partial_\mu H + \text{ h.c. }\right]\\ &+\frac{\kappa_{H\bar{H}}}{|f|^2}\left[\frac{1}{2}G\sigma^\mu\bar{G}(i\partial_\mu H\,\square\,\bar{H} + \text{ h.c. }) - \epsilon^{\mu\nu\alpha\beta}\partial_\mu G\sigma_\nu\bar{G}\partial_\alpha H\partial_\beta\bar{H}\right]\\ &+\left[\frac{g'}{f^2}\bar{G}\bar{\sigma}^\mu\sigma^\nu\partial_\nu\bar{G}\partial_\mu H - \frac{g''}{2f^2}\bar{G}^2(\partial H)^2 - \frac{g'}{2f^2}\bar{G}^2\,\square\, H + \text{ h.c. }\right].\end{aligned}$$

$$\delta G^\alpha = -\frac{\kappa_H\bar{H}}{2|f|^2}\partial_\mu H\partial^\mu\bar{H}G^\alpha, \qquad \delta H = -\frac{i}{2|f|^2}G\sigma^\mu\bar{G}\partial_\mu H,$$

$$\delta G^\alpha = i\frac{g'}{f^2}(\bar{G}\bar{\sigma}^\mu)^\alpha\partial_\mu H, \qquad \delta H = \frac{\bar{g}'}{2\kappa_{H\bar{H}}\bar{f}^2}G^2,$$

$$\begin{aligned}\mathcal{L} =& -|f|^2 - \kappa_{H\bar{H}}\partial_\mu H\partial^\mu\bar{H} + \left(\frac{i}{2}\partial_\mu G\sigma^\mu\bar{G} + \text{ h.c. }\right) - (m_1G^2 + \text{ h.c. })\\ &+2\left[\bar{m}_1(\bar{k}\bar{H} - kH)\bar{G}^2 + h.c.\right] + \frac{1}{4|f|^2}\left(1 - \frac{|g'|^2}{|f|^2\kappa_{H\bar{H}}}\right)\bar{G}^2\,\square\, G^2\\ &+\frac{1}{2}\left(k^2H^2 + \bar{k}^2\bar{H}^2 - 2|k|^2|H|^2\right)(m_1G^2 + \bar{m}_1\bar{G}^2) - m_2G^2\bar{G}^2\\ &+\frac{1}{2|f|^2}\left(\kappa_{H\bar{H}} + \frac{|g'|^2}{|f|^2}\right)(i\partial^\mu G\sigma^\nu\bar{G}\partial_\mu H\partial_\nu\bar{H} + \text{ h.c. })\\ &+\frac{1}{2|f|^2}\left(\kappa_{H\bar{H}} - 3\frac{|g'|^2}{|f|^2}\right)(i\partial^\mu G\sigma^\nu\bar{G}\partial_\mu\bar{H}\partial_\nu H + \text{ h.c. })\\ &-\left[\lambda_1G^2\partial_\mu H\partial^\mu\bar{H} + \lambda_2\bar{G}^2(\partial H)^2 + h.c.\right]\\ &-\left[\frac{i\left(2f_H\bar{f} - \partial_H|f|^2\right)}{2|f|^2}G\sigma^\mu\bar{G}\partial_\mu H + h.c.\right]\\ &-\frac{1}{|f|^2}\left(\kappa_{H\bar{H}} + \frac{|g'|^2}{|f|^2}\right)\epsilon^{\mu\nu\alpha\beta}\partial_\mu G\sigma_\nu\bar{G}\partial_\alpha H\partial_\beta\bar{H},\end{aligned}$$

$$\kappa_{H,\bar{H}}(0)|f(0)|^2 \geq |g'(0)|$$

$$c_s^2 = 1 - \frac{4|\dot{H}|^2}{(|f|^2 + \kappa_{H\bar{H}}|\dot{H}|^2)^2}[\kappa_{H\bar{H}}|f|^2 - |g'|^2],$$

$$\kappa_{H,\bar{H}}(H,\bar{H})|f(H)|^2 \geq |g'(H)|,$$

$$SQ = 0,$$

$$Q = Q + \sqrt{2}\theta\chi_Q + \theta^2 F_Q.$$

$$Q = \frac{\chi_Q G}{F_S} - \frac{F_Q}{2F_S^2}G^2.$$

$$K = S\bar{S} + Q\overline{Q} + \frac{1}{2}Q\overline{Q}\left(h_1 Q + \bar{h}_1\overline{Q}\right) + \frac{h_2}{4}(Q\overline{Q})^2, W = f(Q)S + g(Q),$$

$$\mathcal{L} = \left(\frac{i}{2}\partial_\mu G\sigma^\mu\bar{G} + \text{ h.c. }\right) + \left(\frac{i}{2}\partial_\mu\chi_q\sigma^\mu\bar{\chi}_q + \text{ h.c. }\right) + \frac{1}{2|f|^2}\left[\frac{g'}{f}\partial_\mu(\chi_q G)\partial^\mu\bar{G}^2 + \text{ h.c. }\right]$$
$$+ \frac{1}{|f|^2}(\bar{\chi}_q\bar{G}) \square (\chi_q G) + \frac{1}{4}|f|^2\left(1 + \frac{|g'|^2}{|f|^2}\right)\bar{G}^2 \square G^2 + \cdots,$$

$$\bar{S}SQ_L = 0,$$

$$Q_{\mathrm{L}} = -2\frac{\bar{D}_{\dot{\alpha}}\bar{S}\bar{D}^{\dot{\alpha}}Q_{\mathrm{L}}}{\bar{D}^2\bar{S}} - \frac{\bar{S}\bar{D}^2 Q_{\mathrm{L}}}{\bar{D}^2\bar{S}} - 2\frac{D^\alpha S D_\alpha\bar{D}^2(\bar{S}Q_{\mathrm{L}})}{D^2 S\bar{D}^2\bar{S}} - S\frac{D^2\bar{D}^2(\bar{S}Q_{\mathrm{L}})}{D^2 S\bar{D}^2\bar{S}},$$

$$\bar{S}S(\Phi - \bar{\Phi}) = 0$$
$$\bar{S}SD_\alpha\Phi = 0$$
$$\bar{S}SD^2\Phi = 0$$

$$\bar{S}S(\Phi - \bar{\Phi}) = 0, \bar{S}SD_\alpha\Phi = 0$$

$$B = \frac{i}{4}\left[\left(\frac{\bar{G}}{\bar{F}_S}\right)^2\bar{F}_\phi - \left(\frac{G}{F_S}\right)^2 F_\phi\right] - \frac{G\sigma^\mu\bar{G}}{2|F_S|^2}\partial_\mu A + \cdots$$
$$\psi_\alpha = \frac{F_\phi}{F_S}G_\alpha - i\frac{(\sigma^\mu\bar{G})_\alpha}{\bar{F}_S}\partial_\mu A + \frac{i}{4}\frac{(\sigma^\mu\bar{G})_\alpha}{\bar{F}_S}\partial_\mu\left[\left(\frac{G}{F_S}\right)^2 F_\phi\right] + \cdots$$

$$K = \bar{S}S + \kappa(\Phi,\bar{\Phi}), W(\Phi) = f(\Phi)S + g(\Phi)$$

$$\mathcal{L} = -\left[|f|^2 + \frac{|g'|^2}{\kappa_{\phi\bar{\phi}}}\right] - \kappa_{\phi\bar{\phi}}\partial_\mu A\partial^\mu A + Z\left(\frac{i}{2}\partial_\mu G\sigma^\mu\bar{G} + \text{ h.c. }\right) + (mG^2 + \text{ h.c. })$$
$$+ cG^2\bar{G}^2 - Z\left(i\frac{f'}{2f} + \text{ h.c. }\right)G\sigma^\mu\bar{G}\partial_\mu A + (dG^2 + \text{ h.c. })(\partial A)^2$$
$$+ \left[\frac{\bar{g}'}{\bar{f}^2}\partial_\mu G\sigma^\mu\bar{\sigma}^\nu G\partial_\nu A + \frac{\kappa_{\phi\bar{\phi}}}{|f|^2}i\partial^\mu G\sigma^\nu\bar{G}\partial_\mu A\partial_\nu A - \frac{\kappa_{\phi\bar{\phi}}}{2|f|^2}i\partial_\mu G\sigma^\mu\bar{G}(\partial A)^2 + \text{ h.c. }\right]$$
$$+ \frac{1}{4|f|^2}\left(1 + \frac{|g'|^2}{2\kappa_{\phi\bar{\phi}}|f|^2}\right)\bar{G}^2 \square G^2 - \frac{1}{16\kappa_{\phi\bar{\phi}}}\left(\frac{\bar{g}'^2}{\bar{f}^4}G^2 \square G^2 + \text{ h.c. }\right),$$

$$Z \equiv 1 + \frac{|g'|^2}{\kappa_{\phi\bar{\phi}}|f|^2}$$

$$\delta G^{\alpha}=\frac{\kappa_{\phi\bar{\phi}}}{2Z|f|^{2}}G^{\alpha}(\partial A)^{2},\delta G^{\alpha}=-\frac{i}{Z}\frac{g'}{f^{2}}(\bar{G}\bar{\sigma}^{\mu})^{\alpha}\partial_{\mu}A$$

$$\begin{aligned}\mathcal{L}=&-\left[|f|^{2}+\frac{|g'|^{2}}{\kappa_{\phi\bar{\phi}}}\right]-\kappa_{\phi\bar{\phi}}\partial_{\mu}A\partial^{\mu}A+Z\left(\frac{i}{2}\partial_{\mu}G\sigma^{\mu}\bar{G}+\text{ h.c. }\right)\\&+(mG^{2}+\text{ h.c. })+\lambda G\sigma^{\mu}\bar{G}\partial_{\mu}A+cG^{2}\bar{G}^{2}+\left(\tilde{d}G^{2}+\text{ h.c. }\right)\partial_{\mu}A\partial^{\mu}A\\&+\left(\frac{\kappa_{\phi\bar{\phi}}}{|f|^{2}+\frac{|g'|^{2}}{\kappa_{\phi\bar{\phi}}}}\right)\left(i\partial^{\mu}G\sigma^{\nu}\bar{G}\partial_{\mu}A\partial_{\nu}A+\text{ h.c. }\right)+\frac{1}{4|f|^{2}}\left(1+\frac{|g'|^{2}}{2\kappa_{\phi\bar{\phi}}|f|^{2}}\right)\bar{G}^{2}\,\square\,G^{2}\\&-\frac{1}{16\kappa_{\phi\bar{\phi}}}\left(\frac{\bar{g}'^{2}}{\bar{f}^{4}}G^{2}\,\square\,G^{2}+\text{ h.c. }\right),\end{aligned}$$

$$c_{s}^{2}=1-\frac{4\dot{A}^{2}|f|^{2}\kappa_{\phi\bar{\phi}}}{\left(|f|^{2}+\frac{|g'|^{2}}{\kappa_{\phi\bar{\phi}}}+\kappa_{\phi\bar{\phi}}\dot{A}^{2}\right)^{2}},$$

$$\epsilon\longrightarrow\epsilon(x).$$

$$\delta\psi_{\mu}\sim\partial_{\mu}\epsilon,$$

$$\frac{1}{M_{\mathrm{P}}}\bar{\psi}_{\mu}\Xi^{\mu},$$

$$\delta\Xi^{\mu}\sim T^{\mu\nu}\gamma_{\nu}\epsilon.$$

$$g_{\mu\nu}T^{\mu\nu}.$$

$$\delta\psi_{\mu}\sim M_{\mathrm{P}}\partial_{\mu}\epsilon,$$

$$\delta g_{\mu\nu}\sim\frac{1}{M_{\mathrm{P}}}\bar{\epsilon}\gamma_{(\mu}\psi_{\nu)}.$$

$$T_{A}=\{P_{a},M_{ab},Q_{\alpha}\},$$

$$A_{\mu}^{A}T_{A}=e_{\mu}{}^{a}P_{a}+\frac{1}{2}\omega_{\mu}^{ab}M_{ab}+\bar{\psi}_{\mu}Q.$$

$$\delta A_{\mu}^{A}=\partial_{\mu}\epsilon^{A}+f^{A}{}_{BC}A_{\mu}^{B}\epsilon^{C},$$

$$\delta e_{\mu}^{a}=\frac{1}{2}\bar{\epsilon}\gamma^{a}\psi_{\mu},\delta\psi_{\mu}=D_{\mu}\epsilon,$$

$$D_{\mu}\epsilon=\partial_{\mu}\epsilon+\frac{1}{4}\omega_{\mu}^{ab}\gamma_{ab}\epsilon.$$

$$T_{\mu\nu}^{a}=2\left(\partial_{[\mu}e_{\nu]}^{a}+\omega_{[\mu b}^{a}e_{\nu]}^{b}\right)-\frac{1}{2}\bar{\psi}_{[\mu}\gamma^{a}\psi_{\nu]},$$

$$\hat{\omega}_\mu^{ab} \equiv \omega_\mu^{ab}(e,\psi) = \omega_\mu^{ab}(e) - \frac{1}{4}\left(\bar{\psi}^{[a}\gamma_\mu\psi^{b]} - \bar{\psi}_\mu\gamma^{[a}\psi^{b]} - \bar{\psi}^{[a}\gamma^{b]}\psi_\mu\right).$$

$$e^{-1}\mathcal{L}_{\text{SUGRA}} = \frac{1}{2}R(\hat{\omega}) - \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu(\hat{\omega})\psi_\rho,$$

$$\begin{aligned}\delta e_\mu{}^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu,\\ \delta\psi_\mu &= D_\mu(\hat{\omega})\epsilon.\end{aligned}$$

$$e^{-1}\mathcal{L}_{\text{SUGRA}} = \frac{1}{2}R - \frac{1}{2}\psi_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho + \mathcal{L}_{\text{torsion}},$$

$$e^{-1}\mathcal{L}_{\text{torsion}} = -\frac{1}{32}\left[(\bar{\psi}^\mu\gamma^\nu\psi^\rho)\left(\bar{\psi}_\mu\gamma_\nu\psi_\rho + \bar{\psi}_\mu\gamma_\rho\psi_\nu\right) - 4\left(\bar{\psi}_\mu\gamma^\rho\psi_\rho\right)\left(\bar{\psi}^\mu\gamma^\rho\psi_\rho\right)\right].$$

$$[\delta_1,\delta_2] = \delta_\xi - \delta_{\hat{\Lambda}} - \delta_{\hat{\epsilon}},$$

$$\xi^a = -\frac{1}{2}\bar{\epsilon}_1\gamma^a\epsilon_2, \hat{\Lambda}^{ab} = \xi^\rho\omega_\rho^{ab}, \hat{\epsilon} = \xi^\rho\psi_\rho$$

$$K(\phi,\bar{\phi}) \longrightarrow K(\phi,\bar{\phi}) + \Lambda(\phi) + \bar{\Lambda}(\bar{\phi}),$$

$$\psi_\mu \to e^{-\frac{i}{2}\Im\mathfrak{m}(\Lambda)\gamma_5}\psi_\mu$$

$$\mathcal{D}_\mu\psi_\nu = D_\mu\psi_\nu + \frac{i}{2}Q_\mu\gamma_5\psi_\nu,$$

$$Q_\mu = \frac{i}{2}\left[K_{\bar{\imath}}\partial_\mu\bar{\phi}^{\bar{\imath}} - K_i\partial_\mu\phi^i\right].$$

$$\chi_{\mu L}^i \to e^{\frac{i}{2}\Im\mathfrak{m}(\Lambda)\gamma_5}\chi_{\mu L}^i, \mathcal{D}_\mu\chi_{\nu L}^i = D_\mu\chi_\nu^i + \Gamma_{jk}^i\partial_\mu\phi^j\chi_L^k - \frac{i}{2}Q_\mu\gamma_5\chi_L^i,$$

$$e^{\frac{K}{2}}W$$

$$W(\phi) \to e^{-\Lambda(\phi)}W(\phi)$$

$$e^{\frac{K}{2}}\mathcal{D}_iW = e^{\frac{K}{2}}(\partial_i + K_i)W.$$

$$\mathcal{G} \equiv K + \log|W|^2,$$

$$\begin{aligned}e^{-1}\mathcal{L} =& \frac{1}{2}R - \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\mathcal{D}_\mu\psi_\nu - K_{i\bar{\jmath}}\left(\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{\jmath}} + \bar{\chi}_L^i\Big/\mathcal{D}_R\bar{\jmath} + \bar{\chi}_R^{\bar{\jmath}}\Big/\mathcal{D}_L^i\right)\\ &+\left[\frac{1}{2}e^{\frac{K}{2}}W\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R} - e^{\frac{K}{2}}\mathcal{D}_i\mathcal{D}_jW\bar{\chi}_L^i\chi_L^j + \text{ h.c.}\right]\\ &+\left[K_{i\bar{\jmath}}\bar{\psi}_{\mu L}\gamma^\nu\gamma^\mu\chi_L^i\partial_\nu\bar{\phi}^{\bar{\jmath}} + \bar{\psi}_{\mu R}\gamma^\mu\chi_L^ie^{\frac{K}{2}}\mathcal{D}_iW + \text{ h.c.}\right] - V(\phi,\bar{\phi})\\ &+\mathcal{L}_{4\text{ F}}.\end{aligned}$$

$$V(\phi,\bar{\phi}) = e^K\left(K^{i\bar{\jmath}}\mathcal{D}_iW\overline{\mathcal{D}}_{\bar{\jmath}}\bar{W} - 3|W|^2\right),$$

$$V = e^{\mathcal{G}}\left(\mathcal{G}^{i\bar{j}}\mathcal{G}_i\mathcal{G}_{\bar{j}} - 3\right)$$

$$\mathcal{L}_{4\,\mathrm{F}} = \mathcal{L}_{\mathrm{torsion}} - \frac{1}{2}K_{i\bar{j}}\bar{\psi}_\mu\chi^i\bar{\psi}^\mu\chi^{\bar{j}} + \frac{1}{4}\left(R_{i\bar{j}k\bar{l}} - K_{i\bar{j}}K_{k\bar{l}}\right)\bar{\chi}^i\chi^k\bar{\chi}^{\bar{j}}\chi^{\bar{l}}$$

$$\delta e_\mu^a = \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \delta\psi_{\mu L} = \mathcal{D}_\mu\epsilon_L + \frac{1}{2}e^{\frac{K}{2}}W\gamma_\mu\epsilon_R$$
$$\delta\phi^i = \bar{\epsilon}_L\chi_L^i, \delta\chi_L^i = \frac{1}{2}\partial\phi^i\epsilon_R - \frac{1}{2}e^{\frac{K}{2}}K^{i\bar{j}}\overline{\mathcal{D}}_{\bar{j}}\bar{W}\epsilon_L$$

$$V(\phi,\bar{\phi}) = e^K\left(K^{i\bar{j}}\mathcal{D}_iW\overline{\mathcal{D}}_{\bar{j}}\bar{W} - 3|W|^2\right) + \frac{1}{2}\left(\Re\mathfrak{e}f_{IJ}\right)D^ID^J,$$

$$\langle\delta\psi_{\mu L}\rangle = \langle\mathcal{D}_\mu\epsilon_L\rangle + \frac{1}{2}\left\langle e^{\frac{K}{2}}W\right\rangle\gamma_\mu\epsilon_R = 0$$
$$\langle\delta\chi_L^i\rangle = -\frac{1}{2}\left\langle K^{i\bar{j}}e^{\frac{K}{2}}\overline{\mathcal{D}}_{\bar{j}}\bar{W}\right\rangle\epsilon_L = 0$$
$$\langle\delta\lambda_L^I\rangle = \frac{i}{2}\langle D^I\rangle\epsilon_L = 0$$

$$\langle\mathcal{D}_iW\rangle = 0, \langle D^I\rangle = 0$$

$$V_0 = -3\langle e^K|W|^2\rangle$$

$$\Lambda^4_{\mathrm{supy}} = \left\langle e^KK^{i\bar{j}}\mathcal{D}_iW\overline{\mathcal{D}}_{\bar{j}}\bar{W}\right\rangle + \frac{1}{2}\left\langle\left(\Re\mathfrak{e}f_{IJ}\right)D^ID^J\right\rangle,$$

$$V_0 = \Lambda^4_{\mathrm{supy}} - 3\langle e^K|W|^2\rangle.$$

$$G_L = \left\langle\frac{\mathcal{D}_iW}{W}\right\rangle\chi_L^i$$

$$\delta G_L = -\sqrt{3}\left\langle e^{\frac{K}{2}}\bar{W}\right\rangle\epsilon_L$$

$$\mathcal{L} = -\frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho - \left(\bar{G}_L\,\partial G_R + \bar{G}_R\,\partial G_L\right)$$
$$+\frac{1}{2}\left[\left\langle e^{\frac{K}{2}}W\right\rangle\left(\bar{G}_LG_L + 2\sqrt{3}\bar{\psi}_{\mu R}\gamma^\mu G_L + \bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R}\right) + h.c.\right].$$

$$\psi_{\mu L} \to \psi_{\mu L} + \frac{2}{\sqrt{3}}\left\langle e^{\frac{K}{2}}\bar{W}\right\rangle^{-1}\partial_\mu G_L + \frac{1}{\sqrt{3}}\gamma_\mu G_R$$

$$\mathcal{L} = -\frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho + \frac{1}{2}\left(m_{3/2}\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R} + \text{ h.c. }\right)$$

$$\delta\psi_{\mu L} = \partial_\mu\epsilon_L + \left\langle e^{\frac{K}{2}}\middle| W\right|\rangle\gamma_\mu\epsilon_R$$

$$\epsilon = \frac{2}{\sqrt{3}}\left\langle e^{\frac{K}{2}}\middle| W\right|\rangle^{-1}G$$

$$\left|m_{3/2}\right| = \frac{1}{M_{\mathrm{P}}^2}\left\langle e^{\frac{K}{2}}\right| W|\rangle \sim \frac{\Lambda_{\mathrm{susy}}^2}{M_{\mathrm{P}}},$$

$$\psi_\mu \to \frac{1}{m_{3/2}}\partial_\mu G$$

$$\frac{1}{M_{\mathrm{P}}}\bar{\Xi}^\mu\psi_\mu \longrightarrow \frac{1}{m_{3/2}M_{\mathrm{P}}}\bar{\Xi}^\mu\partial_\mu G \sim \frac{1}{\Lambda_{\mathrm{su/y}}^2}\partial_\mu\bar{\Xi}^\mu G$$

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\bar{\Psi}_\mu\gamma^{\mu\nu\rho}D_\nu\Psi_\rho + \frac{m(\phi)}{2}\bar{\Psi}_\mu\gamma^{\mu\nu}\Psi_\nu - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + e^{-1}\mathcal{L}_{\mathrm{torsion}},$$

$$m(\phi) = g(\phi)$$

$$V(\phi) = f^2(\phi) - 3g^2(\phi).$$

$$g_{\mu\nu} = a(\tau)^2\eta_{\mu\nu}$$

$$G_{\mu\nu} = T_{\mu\nu}^{(\phi)},$$

$$G_{\mu\nu} = a^2\left[\left(4H^2 - \frac{R}{3}\right)\delta_\mu^0\delta_\nu^0 + \left(H^2 - \frac{R}{3}\right)\eta_{\mu\nu}\right], R = 6\frac{a''}{a^3},$$

$$H = \frac{a'}{a^2},$$

$$T_{\mu\nu}^{(\phi)} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right) =$$
$$= a^2\delta_\mu^0\delta_\nu^0\left(\frac{\phi'^2}{2a^2} + V(\phi)\right) + a^2\delta_{ij}\delta_\mu^i\delta_\nu^j\left(\frac{\phi'^2}{2a^2} - V(\phi)\right).$$

$$3(H^2 + m^2) = \frac{\phi'^2}{2a^2} + f^2,$$
$$\left(H^2 - 3m^2 - \frac{R}{3}\right) = \frac{\phi'^2}{2a^2} - f^2,$$

$$e^{-1}\mathcal{L}_\Psi = -\frac{1}{2}\bar{\Psi}_\mu\gamma^{\mu\nu\rho}\nabla_\nu\Psi_\rho + \frac{m}{2}\bar{\Psi}_\mu\gamma^{\mu\nu}\Psi_\nu$$

$$\gamma^{\mu\nu\rho}\nabla_\nu\Psi_\rho = m\gamma^{\mu\nu}\Psi_\nu$$

$$\gamma^{\mu\nu}\nabla_\mu\Psi_\nu = \frac{3}{2}m\gamma^\mu\Psi_\mu$$
$$G_{\mu\nu}\gamma^\mu\Psi^\nu = 3m^2\gamma^\mu\Psi_\mu + 2(\partial_\mu m)\gamma^{\mu\nu}\Psi_\nu$$

$$\triangle\!\!\!/\Psi^\mu - \nabla^\mu(\gamma^\rho\Psi_\rho) = -\frac{m}{2}(\gamma^{\mu\rho}\Psi_\rho + 3\Psi^\mu)$$

$$\Psi_\mu = \frac{1}{\sqrt{a}}\psi_\mu$$

$$\mathcal{L}_\psi = -\frac{1}{2}\bar{\psi}_\mu \bar{\gamma}^{\mu\nu\rho}\partial_\nu \psi_\rho + \frac{am}{2}\bar{\psi}_\mu \bar{\gamma}^{\mu\nu}\psi_\nu - aH\bar{\psi}_\rho \bar{\gamma}^\rho \psi_0$$

$$\bar{\gamma}^{\mu\nu\rho}\partial_\nu \psi_\rho - am\bar{\gamma}^{\mu\rho}\psi_\rho + aH\left(\bar{\gamma}^\mu \psi_0 - \delta_0^\mu \bar{\gamma}^\rho \psi_\rho\right) = 0$$

$$\gamma^\mu = e^\mu{}_a \gamma^a = a(\tau)^{-1}\delta^\mu{}_a \gamma^a \equiv a(\tau)^{-1}\bar{\gamma}^\mu$$

$$\partial_\mu \psi^\mu - \partial\left(\bar{\gamma}^\rho \psi_\rho\right) - 2aH\psi_0 + \frac{1}{2}(3am - aH\bar{\gamma}^0)\left(\bar{\gamma}^\rho \psi_\rho\right) = 0$$

$$\bar{\gamma}^0 \psi_0 = C(\tau)\bar{\gamma}^i \psi_i$$

$$C(\tau) = C_{\mathrm{R}}(\tau) + C_{\mathrm{I}}(\tau)\bar{\gamma}_0 = \frac{H^2 - 3m^2 - R/3}{3(H^2 + m^2)} + \frac{2m'}{3a(H^2 + m^2)}\bar{\gamma}_0$$

$$\psi_\mu(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^3}\psi_{\mu,k}(\tau)e^{i\vec{k}\cdot\vec{x}}$$

$$\psi_{i,k} = \psi_{i,k}^{\mathrm{t}} + \left(P_\gamma\right)_i \vec{\bar{\gamma}} \cdot \vec{\psi}_k + (P_k)_i \vec{k} \cdot \vec{\psi}_k$$

$$\left(P_\gamma\right)_i = \frac{1}{2}\left[\frac{k_i}{\vec{k}^2}(\vec{k} \cdot \vec{\bar{\gamma}}) - \bar{\gamma}_i\right], (P_k)_i = \frac{1}{2\vec{k}^2}\left[\bar{\gamma}_i(\vec{k} \cdot \vec{\bar{\gamma}}) - 3k_i\right].$$

$$\left(P_\gamma\right)^i \psi_{i,k}^{\mathrm{t}} = (P_k)^i \psi_{i,k}^{\mathrm{t}} = 0$$

$$\vec{k} \cdot \vec{\psi}_k = \left[\vec{k} \cdot \vec{\bar{\gamma}} + ia(m - H\bar{\gamma}^0)\right]\vec{\bar{\gamma}} \cdot \vec{\psi}_k, \bar{\gamma}^0 \psi_{0,k} = C(\tau)\left(\vec{\bar{\gamma}} \cdot \vec{\psi}_k\right)$$

$$\psi_{i,k} = \psi_{i,k}^{\mathrm{t}} + \hat{O}_i \theta_k$$

$$\hat{O}_i = \frac{1}{\vec{k}^2}\left\{-k_i(\vec{k} \cdot \vec{\gamma}) + \frac{ia}{2}\left[\bar{\gamma}_i(\vec{k} \cdot \vec{\gamma}) - 3k_i\right](m - H\bar{\gamma}^0)\right\}.$$

$$\theta_k = \sqrt{\frac{3(m^2 + H^2)a^2}{2\vec{k}^2}}\bar{\gamma}^0 \vartheta_k$$

$$\mathcal{L}_\psi = \mathcal{L}_{\mathrm{t}} + \mathcal{L}_\vartheta,$$

$$\mathcal{L}_{\mathrm{t}} = -\frac{1}{2}\sum_{i=1}^{3} \bar{\psi}_{i,k}^{\mathrm{t}}\left[\bar{\gamma}^0 \partial_0 + i(\vec{k} \cdot \vec{\bar{\gamma}}) + am\right]\psi_{i,k}^{\mathrm{t}}$$

$$\mathcal{L}_\vartheta = -\frac{1}{2}\bar{\vartheta}_k\left[\bar{\gamma}^0 \partial_0 - i(\vec{k} \cdot \vec{\bar{\gamma}})C + aM\right]\vartheta_k$$

$$c_s^2 = |C^2(\tau)| = C_{\mathrm{R}}^2(\tau) + C_{\mathrm{I}}^2(\tau),$$

$$M \equiv -\left[\frac{m}{2} + \frac{3}{2}(mC_{\mathrm{R}} + HC_{\mathrm{I}})\right]$$

$$c_s^2 = 1 - \frac{2\dot{\phi}^2}{\left(\frac{\dot{\phi}^2}{2} + 3m^2 + V\right)^2}[V + 3m^2 - 2(m')^2]$$

$$c_s^2 = 1 - \frac{2\dot{\phi}^2}{\left(\frac{\dot{\phi}^2}{2} + f(\phi)^2\right)^2}[f(\phi)^2 - 2g'(\phi)^2]$$

$$c_s \leq 1 \leftrightarrow f(\phi)^2 \leq 2g'(\phi)^2,$$

$$V_{\text{improved}} = f(\phi)^2 + 2g'(\phi)^2 - 3g(\phi)^2,$$

$$c_s^2 = 1 - \frac{2\dot{\phi}^2 f(\phi)^2}{\left(\frac{\dot{\phi}^2}{2} + f(\phi)^2 + g'(\phi)^2\right)^2},$$

$$\begin{pmatrix} a_{\text{out}} \\ a_{\text{out}}^\dagger \end{pmatrix} = \begin{pmatrix} \alpha & \beta^\dagger \\ \beta & \alpha^\dagger \end{pmatrix} \begin{pmatrix} a_{\text{in}} \\ a_{\text{in}}^\dagger \end{pmatrix}.$$

$$|\alpha|^2 - |\beta|^2 = 1,$$

$$\langle N_{\text{out}} \rangle_{\text{in}} = |\beta|^2.$$

$$\partial_\tau^2 \varphi_k + \omega_k^2 \varphi_k = 0$$

$$A_k(\tau) = \frac{\partial_\tau \omega_k}{\omega_k^2}$$

$$\omega_k^2 = c_s^2 k^2 + a^2 M^2,$$

$$A_k(\tau) = \frac{c_s(\partial_\tau c_s)\vec{k}^2 + a^3 H M^2 + a^2 M \partial_\tau M}{(c_s^2 k^2 + a^2 M^2)^{\frac{3}{2}}}$$

$$S_\Phi = \int d^4x\sqrt{-g}\mathcal{L}_\Phi =$$

$$= -\frac{1}{2}\int d^4x\sqrt{-g}\left(\partial_\mu \Phi \partial^\mu \Phi + m^2\Phi^2\right) = \int d^4x \frac{a^2}{2}\left[(\Phi')^2 - (\vec{\nabla}\Phi)^2 - a^2 m^2 \Phi^2\right]$$

$$\Phi \equiv \frac{\varphi}{a},$$

$$S_\varphi = \int d^4x \mathcal{L}_\varphi = \frac{1}{2}\int d^4x \left\{(\varphi')^2 - \varphi\left(-\vec{\nabla}^2 + a^2 m^2 - \frac{a''}{a}\right)\varphi\right\}$$

$$\varphi(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left\{\varphi_{\boldsymbol{k}}(\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \bar{\varphi}_{\boldsymbol{k}}(\tau)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right\}$$

$$\varphi_{\boldsymbol{k}}'' + \omega^2 \varphi_{\boldsymbol{k}} = 0$$

$$\omega^2 \equiv \vec{k}^2 + a^2 m^2 - \frac{a''}{a} \equiv \omega_0^2 - \frac{a''}{a}$$

$$\Pi_\varphi \equiv \frac{\partial \mathcal{L}_\varphi}{\partial \varphi'} = \varphi'$$

$$\mathcal{H}_\varphi = \frac{1}{2}\left[\Pi_\varphi^2 + \varphi\left(-\vec{\nabla}^2 + a^2 m^2 - \frac{a''}{a}\right)\varphi\right].$$

$$\left[\varphi(\tau, \vec{x}), \Pi_\varphi(\tau, \vec{y})\right] = i\delta^{(3)}(\vec{x} - \vec{y})$$

$$\varphi(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left\{\varphi_k(\tau) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} a_{\boldsymbol{k}} + \bar{\varphi}_k(\tau) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} a_{\boldsymbol{k}}^\dagger\right\}$$

$$\Pi_\varphi(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left\{\Pi_k(\tau) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} a_{\boldsymbol{k}} + \bar{\Pi}_k(\tau) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} a_{\boldsymbol{k}}^\dagger\right\}$$

$$\left[a_{\boldsymbol{k}}, a_{\boldsymbol{k}'}^\dagger\right] = \delta^{(3)}\left(\vec{k} - \vec{k}'\right), \left[a_{\boldsymbol{k}}, a_{\boldsymbol{k}'}\right] = 0 = \left[a_{\boldsymbol{k}}^\dagger, a_{\boldsymbol{k}'}^\dagger\right]$$

$$\varphi_k \bar{\Pi}_k - \bar{\varphi}_k \Pi_k = i$$

$$H_\varphi = \int d^3k \mathcal{H}_\varphi = \frac{1}{2}\int d^3k \begin{pmatrix} a_{\boldsymbol{k}}^\dagger & a_{-\boldsymbol{k}} \end{pmatrix} \begin{pmatrix} \mathcal{E}_k & \mathcal{F}_k^\dagger \\ \mathcal{F}_k & \mathcal{E}_k \end{pmatrix} \begin{pmatrix} a_{\boldsymbol{k}} \\ a_{-\boldsymbol{k}}^\dagger \end{pmatrix}$$

$$\mathcal{E}_k = |\Pi_k|^2 + \omega^2 |\varphi_k|^2, \mathcal{F}_k = \Pi_k^2 + \omega^2 \varphi_k^2$$

$$\varphi_k = \frac{1}{\sqrt{2\omega}}(\alpha_k + \beta_k), \qquad \Pi_k = -\frac{i\omega}{\sqrt{2\omega}}(-\alpha_k + \beta_k)$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1,$$

$$\alpha_k' = -i\omega\alpha_k + \frac{\omega'}{2\omega}\beta_k, \qquad \beta_k' = i\omega\beta_k + \frac{\omega'}{2\omega}\alpha_k$$

$$\begin{pmatrix} \tilde{a}_{\boldsymbol{k}} \\ \tilde{a}_{-\boldsymbol{k}}^\dagger \end{pmatrix} = \begin{pmatrix} \alpha_k & \beta_k^\dagger \\ \beta_k & \alpha_k^\dagger \end{pmatrix} \begin{pmatrix} a_{\boldsymbol{k}} \\ a_{-\boldsymbol{k}}^\dagger \end{pmatrix},$$

$$H_\varphi = \frac{1}{2}\int d^3k \begin{pmatrix} a_{\boldsymbol{k}}^\dagger & a_{-\boldsymbol{k}} \end{pmatrix} \begin{pmatrix} \alpha_k^\dagger & \beta_k^\dagger \\ \beta_k & \alpha_k \end{pmatrix} \begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix} \begin{pmatrix} \alpha_k & \beta_k^\dagger \\ \beta_k & \alpha_k^\dagger \end{pmatrix} \begin{pmatrix} a_{\boldsymbol{k}} \\ a_{-\boldsymbol{k}}^\dagger \end{pmatrix} =$$
$$= \int d^3k \frac{\omega}{2}\left(\tilde{a}_{\boldsymbol{k}}^\dagger \tilde{a}_{\boldsymbol{k}} + \tilde{a}_{\boldsymbol{k}} \tilde{a}_{\boldsymbol{k}}^\dagger\right)$$

$$\alpha_l(\tau_0) = 1, \beta_k(\tau_0) = 0$$

$$a_{\boldsymbol{k}}|0\rangle_{\mathrm{BD}} \equiv \tilde{a}_{\boldsymbol{k}}(\tau_0)|0\rangle_{\mathrm{BD}} \equiv 0$$

$$\left\langle N_\varphi(\tau)\right\rangle_{\mathrm{BD}} = |\beta_k(\tau)|^2$$

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu}[(\partial\Phi)^2 + m^2\Phi^2]$$

$$E_\varphi = \int d^3x\sqrt{-g}T_0^0 = \int d^3x\frac{a^2}{2}\{(\Phi')^2 + (\vec{\nabla}\Phi)^2 + a^2m^2\Phi^2\} =$$
$$= \frac{1}{2}\int d^3x\{(\varphi' - aH\varphi)^2 + \varphi(-\vec{\nabla}^2 + a^2m^2)\varphi\}$$

$$E_\varphi = \frac{1}{2}\int d^3k\begin{pmatrix}\tilde{a}_{\boldsymbol{k}}^\dagger & \tilde{a}_{-\boldsymbol{k}}\end{pmatrix}\begin{pmatrix}\mathcal{E}_k & \mathcal{F}_k^\dagger \\ \mathcal{F}_k & \mathcal{E}_k\end{pmatrix}\begin{pmatrix}\tilde{a}_{\boldsymbol{k}} \\ \tilde{a}_{-\boldsymbol{k}}^\dagger\end{pmatrix}$$

$$\mathcal{E}_k = \omega_0^2 + \omega^2 + a^2H^2$$
$$\mathcal{F}_k = \omega_0^2 - \omega^2 + a^2H^2 - 2iaH\omega$$

$$\begin{pmatrix}\tilde{a}_{\boldsymbol{k}} \\ \tilde{a}_{-\boldsymbol{k}}^\dagger\end{pmatrix} \equiv \begin{pmatrix}\hat{\alpha}_k & \hat{\beta}_k^\dagger \\ \hat{\beta}_k & \hat{\alpha}_k^\dagger\end{pmatrix}\begin{pmatrix}\hat{a}_{\boldsymbol{k}} \\ \hat{a}_{-\boldsymbol{k}}^\dagger\end{pmatrix}$$

$$\hat{\alpha}_k = \frac{\omega + \omega_0 + iaH}{2\sqrt{\omega\omega_0}}, \qquad \hat{\beta}_k = \frac{\omega - \omega_0 - iaH}{2\sqrt{\omega\omega_0}}$$

$$E_\varphi = \int d^3k\frac{\omega_0}{2}(\hat{a}_{\boldsymbol{k}}^\dagger\hat{a}_{\boldsymbol{k}} + \hat{a}_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}^\dagger)$$

$$\langle E_\varphi(\tau_0)\rangle_{\mathrm{BD}} = V_3\left[\omega_0\left(\frac{1}{2} + |\hat{\beta}_k|^2\right)\right](\tau_0) = V_3\left[\frac{\omega^2 + \omega_0^2 + a^2H^2}{4\omega}\right](\tau_0),$$

$$\lim_{k\to\infty}\hat{\beta}_k = 0$$

$$\mathcal{H}_{\mathrm{t}} = \frac{1}{2}\sum_{i=1}^{3}\bar{\psi}_{i,k}^{\mathrm{t}}[i(\vec{k}\cdot\vec{\gamma}) + am]\psi_{i,k}^{\mathrm{t}}, \mathcal{H}_\vartheta = \frac{1}{2}\bar{\vartheta}_k[-i(\vec{k}\cdot\vec{\gamma})C + aM]\vartheta_k.$$

$$\Pi_\vartheta \equiv \frac{\partial\mathcal{L}_\vartheta}{\partial\partial_0\vartheta} = \frac{i}{2}\vartheta^\dagger,$$

$$\{\vartheta(\tau,\vec{x}), \Pi_\vartheta(\tau,\vec{y})\} = \delta^{(3)}(\vec{x} - \vec{y}), \{\vartheta(\tau,\vec{x}), \vartheta(\tau,\vec{y})\} = \{\Pi_\vartheta(\tau,\vec{x}), \Pi_\vartheta(\tau,\vec{y})\} = 0$$

$$\vartheta(\tau,\vec{x}) = \int\frac{d^3k}{(2\pi)^{3/2}}\sum_r e^{i\vec{k}\cdot\vec{x}}[\vartheta_{r,\boldsymbol{k}}a_{r,\boldsymbol{k}} + v_{r,\boldsymbol{k}}a_{r,-\boldsymbol{k}}^\dagger]$$

$$\vartheta_{r,\boldsymbol{k}}^\dagger\vartheta_{s,\boldsymbol{k}} = \delta_{rs} = v_{r,\boldsymbol{k}}^\dagger v_{s,\boldsymbol{k}}$$

$$\bar{v}_{r,\boldsymbol{k}} = \vartheta_{r,-\boldsymbol{k}}^{\mathrm{T}}C_4,$$

$$\vartheta_{r,\boldsymbol{k}} = \begin{pmatrix}\vartheta_A(r,\boldsymbol{k})\\ \vartheta_B(r,\boldsymbol{k})\end{pmatrix} = \frac{1}{2}\begin{pmatrix}(\vartheta_+ - r\vartheta_-)\psi_r\\ (\vartheta_+ + r\vartheta_-)\psi_r\end{pmatrix},$$

$$\psi_+ = \begin{pmatrix}1\\0\end{pmatrix}, \psi_- = \begin{pmatrix}0\\1\end{pmatrix}, \qquad \sigma^3\psi_r = r\psi_r$$

$$\{a_{r,\boldsymbol{k}}, a_{s,\boldsymbol{q}}^\dagger\} = \delta_{rs}\delta^{(3)}(\vec{k} - \vec{q}), \{a_{r,\boldsymbol{k}}, a_{s,\boldsymbol{q}}\} = 0 = \{a_{r,\boldsymbol{k}}^\dagger, a_{s,\boldsymbol{q}}^\dagger\}$$

$$|\vartheta_+|^2 + |\vartheta_-|^2 = 2$$

$$\vartheta'_{\pm} = -i(C_{\mathrm{R}} \mp iC_{\mathrm{I}})k\vartheta_{\mp} \mp iaM\vartheta_{\pm}.$$

$$\mathcal{H}_{\vartheta} = \frac{1}{2}\sum_{r}\left[\begin{pmatrix} a^{\dagger}_{r,\boldsymbol{k}} & a_{-r,-\boldsymbol{k}}\end{pmatrix}\begin{pmatrix}\mathcal{E}_k & \mathcal{F}^{\dagger}_k \\ \mathcal{F}_k & -\mathcal{E}_k\end{pmatrix}\begin{pmatrix} a_{r,\boldsymbol{k}} \\ a^{\dagger}_{-r,-\boldsymbol{k}}\end{pmatrix}\right],$$

$$\begin{aligned}\varepsilon_k &= -\frac{C_{\mathrm{R}}k}{2}(\vartheta^*_+\vartheta_- + \vartheta_+\vartheta^*_-) + \frac{i}{2}C_{\mathrm{I}}k(\vartheta^*_+\vartheta_- - \vartheta_+\vartheta^*_-) + \frac{aM}{2}(|\vartheta_+|^2 - |\vartheta_-|^2)\\ \mathcal{F}_k &= -\frac{C_{\mathrm{R}}k}{2}(\vartheta^2_+ - \vartheta^2_-) - \frac{i}{2}C_{\mathrm{I}}k(\vartheta^2_+ + \vartheta^2_-) - aM\vartheta_+\vartheta_-\end{aligned}$$

$$\begin{aligned}\vartheta_+ &= c_+(k)\alpha_{\boldsymbol{k}} - c_-(k)\beta_{\boldsymbol{k}}\\ \vartheta_- &= c_-(k)\alpha_{\boldsymbol{k}} + c_+(k)\beta_{\boldsymbol{k}}\end{aligned}$$

$$\alpha_{-\boldsymbol{k}} = \alpha_{\boldsymbol{k}}, \beta_{-\boldsymbol{k}} = -\beta_{\boldsymbol{k}}$$

$$|\alpha_{\boldsymbol{k}}|^2 + |\beta_{\boldsymbol{k}}|^2 = 1,$$

$$c^2_+(k) + c^2_-(k) = 2, c_+(-k) = c_+(k), c_-(-k) = -c_-(k)$$

$$\begin{pmatrix}\tilde{a}_{r,\boldsymbol{k}} \\ \tilde{a}^{\dagger}_{-r,-\boldsymbol{k}}\end{pmatrix} = \begin{pmatrix}\alpha_{\boldsymbol{k}} & -\beta^{\dagger}_{\boldsymbol{k}} \\ \beta_{\boldsymbol{k}} & \alpha^{\dagger}_{\boldsymbol{k}}\end{pmatrix}\begin{pmatrix} a_{r,\boldsymbol{k}} \\ a^{\dagger}_{-r,-\boldsymbol{k}}\end{pmatrix}$$

$$\mathcal{H}_{\vartheta} = \frac{1}{2}\sum_{r}\begin{pmatrix}\tilde{a}^{\dagger}_{r,\boldsymbol{k}} & \tilde{a}_{-r,-\boldsymbol{k}}\end{pmatrix}\begin{pmatrix}A_k & B_k \\ B^{\dagger}_k & -A_k\end{pmatrix}\begin{pmatrix}\tilde{a}_{r,\boldsymbol{k}} \\ \tilde{a}^{\dagger}_{-r,-\boldsymbol{k}}\end{pmatrix}$$

$$\begin{aligned}A_k &= -[c_+(k)c_-(k)]C_{\mathrm{R}}k + \left[\frac{c^2_+(k) - c^2_-(k)}{2}\right]aM\\ B_k &= -\left[\frac{c^2_+(k) - c^2_-(k)}{2}\right]C_{\mathrm{R}}k + iC_{\mathrm{I}}k - [c_+(k)c_-(k)]aM\end{aligned}$$

$$\begin{pmatrix}\hat{b}_{r,\boldsymbol{k}} \\ \hat{b}^{\dagger}_{-r,-\boldsymbol{k}}\end{pmatrix} = \begin{pmatrix}\hat{\alpha}_{\boldsymbol{k}} & -\hat{\beta}^{\dagger}_{\boldsymbol{k}} \\ \hat{\beta}_{\boldsymbol{k}} & \hat{\alpha}^{\dagger}_{\boldsymbol{k}}\end{pmatrix}\begin{pmatrix}\tilde{a}_{r,\boldsymbol{k}} \\ \tilde{a}^{\dagger}_{-r,-\boldsymbol{k}}\end{pmatrix}$$

$$\hat{\alpha}^{\dagger}_{\boldsymbol{k}} = -\frac{B_k}{\omega - A_k}\hat{\beta}_{\boldsymbol{k}}, |\hat{\alpha}_{\boldsymbol{k}}|^2 = \frac{\omega + A_k}{2\omega}, \left|\hat{\beta}_{\boldsymbol{k}}\right|^2 = \frac{\omega - A_k}{2\omega}$$

$$\omega^2 = c_s^2k^2 + a^2M^2, c_s^2 = C^2_{\mathrm{R}} + C^2_{\mathrm{I}}$$

$$\mathcal{H}_{\vartheta} = \frac{\omega}{2}\sum_{r}\left(\hat{b}^{\dagger}_{r,\boldsymbol{k}}\hat{b}_{r,\boldsymbol{k}} - \hat{b}_{r,\boldsymbol{k}}\hat{b}^{\dagger}_{r,\boldsymbol{k}}\right).$$

$$\begin{aligned}T^{\mu\nu} =& \frac{1}{2}\bar{\Psi}_{\rho}\gamma^{\rho\sigma(\mu}\left(\nabla_{\sigma}\Psi^{\nu)} - \nabla^{\nu)}\Psi_{\sigma}\right) - \frac{m}{2}\bar{\Psi}_{\rho}\gamma^{\rho(\mu}\Psi^{\nu)}\\ &-\frac{1}{2}\nabla_{\rho}\left(\bar{\Psi}^{\rho}\gamma^{(\mu}\Psi^{\nu)}\right) - \frac{1}{2}\nabla^{(\mu}\left(\bar{\Psi}^{\nu)}\gamma^{\rho}\Psi_{\rho}\right) - \frac{1}{2}g^{\mu\nu}\nabla_{\rho}(\bar{\Psi}_{\sigma}\gamma^{\sigma}\Psi^{\rho}).\end{aligned}$$

$$E_{\psi} = \int d^3x\sqrt{-g}T^0_0 = \frac{1}{2}\int d^3x\{\bar{\psi}_i\bar{\gamma}^{ijl}\partial_j\psi_l - am\bar{\psi}_i\bar{\gamma}^{ij}\psi_j\}$$

$$E_\psi = \int d^3k[\mathcal{E}_{\mathrm{t}} + \mathcal{E}_\theta]$$

$$\varepsilon_{\mathrm{t}} = \frac{1}{2}\sum_{j=1}^{3}\left[\bar{\psi}^{\mathrm{t}}_{j,k}(\vec{k}\cdot\vec{\gamma})\psi^{\mathrm{t}}_{j,k} + am\bar{\psi}^{\mathrm{t}}_{j,k}\psi^{\mathrm{t}}_{j,k}\right]$$

$$\varepsilon_\vartheta = \frac{1}{2}\bar{\vartheta}_k[(\vec{k}\cdot\vec{\gamma})D + am]\vartheta_k$$

$$D = d_{\mathrm{R}} + id_{\mathrm{I}}\bar{\gamma}^0 \equiv \frac{3m^2 + H^2}{3(m^2 + H^2)} + \frac{2mH}{3(m^2 + H^2)}\bar{\gamma}_0$$

$$\mathcal{E}_\vartheta = \frac{\omega_d}{2}\sum_r \left(\hat{a}^\dagger_{r,\boldsymbol{k}}\hat{a}_{r,\boldsymbol{k}} - \hat{a}_{r,\boldsymbol{k}}\hat{a}^\dagger_{r,\boldsymbol{k}}\right)$$

$$\omega_d^2 = d_s^2k^2 + a^2m^2, d_s^2 = d_{\mathrm{R}}^2 + d_{\mathrm{I}}^2 = \frac{m^2 + H^2/9}{m^2 + H^2}$$

$$\begin{pmatrix}\hat{a}_{r,\boldsymbol{k}}\\ \hat{a}^\dagger_{-r,-\boldsymbol{k}}\end{pmatrix} = \begin{pmatrix}x_{\boldsymbol{k}} & -y^\dagger_{\boldsymbol{k}}\\ y_{\boldsymbol{k}} & x^\dagger_{\boldsymbol{k}}\end{pmatrix}\begin{pmatrix}\tilde{a}_{r,\boldsymbol{k}}\\ \tilde{a}^\dagger_{-r,-\boldsymbol{k}}\end{pmatrix} = \begin{pmatrix}x_{\boldsymbol{k}} & -y^\dagger_{\boldsymbol{k}}\\ y_{\boldsymbol{k}} & x^\dagger_{\boldsymbol{k}}\end{pmatrix}\begin{pmatrix}\alpha_{\boldsymbol{k}} & -\beta^\dagger_{\boldsymbol{k}}\\ \beta_{\boldsymbol{k}} & \alpha^\dagger_{\boldsymbol{k}}\end{pmatrix}\begin{pmatrix}a_{r,\boldsymbol{k}}\\ a^\dagger_{-r,-\boldsymbol{k}}\end{pmatrix}$$

$$x^\dagger_{\boldsymbol{k}} = -\frac{B^{\mathrm{D}}_k}{\omega_d - A^{\mathrm{D}}_k}y_{\boldsymbol{k}}, |x_{\boldsymbol{k}}|^2 = \frac{\omega_d + A^{\mathrm{D}}_k}{2\omega_d}, |y_{\boldsymbol{k}}|^2 = \frac{\omega_d - A^{\mathrm{D}}_k}{2\omega_d},$$

$$A^{\mathrm{D}}_k = [c_+(k)c_-(k)]d_{\mathrm{R}}k + \left(\frac{c_+^2(k) - c_-^2(k)}{2}\right)am$$

$$B^{\mathrm{D}}_k = \left(\frac{c_+^2(k) - c_-^2(k)}{2}\right)d_{\mathrm{R}}k - [c_+(k)c_-(k)]am - id_{\mathrm{I}}k$$

$$\begin{pmatrix}\hat{a}_{r,\boldsymbol{k}}\\ \hat{a}_{-r,-\boldsymbol{k}},^\dagger\end{pmatrix} = \begin{pmatrix}\mathcal{A}_{\boldsymbol{k}} & -\mathcal{B}^\dagger_{\boldsymbol{k}}\\ \mathcal{B}_{\boldsymbol{k}} & \mathcal{A}^\dagger_{\boldsymbol{k}}\end{pmatrix}\begin{pmatrix}\hat{b}_{r,\boldsymbol{k}}\\ \hat{b}_{-r,-\boldsymbol{k}},^\dagger\end{pmatrix},$$

$$\mathcal{A}_{\boldsymbol{k}} = x_{\boldsymbol{k}}\hat{\alpha}^\dagger_{\boldsymbol{k}} + y^\dagger_{\boldsymbol{k}}\hat{\beta}_{\boldsymbol{k}}, \mathcal{B}_{\boldsymbol{k}} = y_{\boldsymbol{k}}\hat{\alpha}^\dagger_{\boldsymbol{k}} - x^\dagger_{\boldsymbol{k}}\hat{\beta}_{\boldsymbol{k}}$$

$$|\mathcal{A}_{\boldsymbol{k}}|^2 = \frac{1}{2}\left[1 - \frac{(C_{\mathrm{R}}d_{\mathrm{R}} + C_{\mathrm{I}}d_{\mathrm{I}})}{\omega\omega_d}k^2 + \frac{a^2mM}{\omega\omega_d}\right]$$

$$|\mathcal{B}_{\boldsymbol{k}}|^2 = \frac{1}{2}\left[1 + \frac{(C_{\mathrm{R}}d_{\mathrm{R}} + C_{\mathrm{I}}d_{\mathrm{I}})}{\omega\omega_d}k^2 - \frac{a^2mM}{\omega\omega_d}\right]$$

$$\lim_{k\to\infty}|\mathcal{B}_{\boldsymbol{k}}|^2 = \frac{1}{2}\left(1 + \frac{C_{\mathrm{R}}d_{\mathrm{R}} + C_{\mathrm{I}}d_{\mathrm{I}}}{c_sd_s}\right)$$

$$\theta(t,\vec{k}) = \begin{pmatrix}\chi_{\boldsymbol{k}}(t)\\ \bar{\chi}_{-\boldsymbol{k}}(t)\end{pmatrix}$$

$$\mathcal{L}_\theta = \frac{a^2f^2}{2\vec{k}^2}\left\{\left(1 + \frac{\dot{\phi}^2}{2f^2}\right)\frac{i}{2}\partial_0\chi_{\boldsymbol{k}}\sigma^0\bar{\chi}_{\boldsymbol{k}} + \left(1 - \frac{\dot{\phi}^2}{2f^2}\right)\frac{1}{2}\chi_{\boldsymbol{k}}(\vec{k}\cdot\vec{\sigma})\bar{\chi}_{\boldsymbol{k}} + \right.$$
$$\left. + \frac{g'\dot{\phi}}{f^2}\chi_{\boldsymbol{k}}(\vec{k}\cdot\vec{\sigma})\bar{\sigma}^0\chi_{-\boldsymbol{k}} + \frac{1}{2}\left[\left(\frac{\dot{\phi}^2}{f^2} - 1\right)am + \left(\frac{3g'\dot{\phi}}{f^2}\right)aH\right]\chi_{\boldsymbol{k}}\chi_{-\boldsymbol{k}} + \text{ h.c.}\right\}$$

$$\mathcal{L}_G = \left(1+\frac{\dot{\phi}^2}{2f^2}\right)\frac{i}{2}\partial_0 G_{\boldsymbol{k}}\sigma^0\bar{G}_{\boldsymbol{k}} + \left(1-\frac{\dot{\phi}^2}{2f^2}\right)\frac{1}{2}G_{\boldsymbol{k}}(\vec{k}\cdot\vec{\sigma})\bar{G}_{\boldsymbol{k}} +$$
$$+i\frac{g'\dot{\phi}}{f^2}G_{\boldsymbol{k}}(\vec{k}\cdot\vec{\sigma})\bar{\sigma}^0 G_{-\boldsymbol{k}} + \text{ h.c.}$$

$$\chi_{\boldsymbol{k}} = \frac{2k}{af}G_{\boldsymbol{k}}$$

$$\psi_\mu \longrightarrow \frac{M_{\mathrm{P}}}{f}\partial_\mu G$$

$$D^2\mathcal{S}^\mu = 0,$$

$$\left(\sigma_\mu\right)_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}\mathcal{S}^\mu_{\mathrm{FZ}} = D_\alpha X,$$

$$\mathcal{S}^\mu_{\mathrm{FZ}} = \{J^\mu, \Xi^\mu, T^{\mu\nu}, x\},$$

$$\delta J^\mu = -i\bar{\epsilon}\gamma_5\Xi^\mu$$
$$\delta\Xi^\mu = T^{\mu\nu}\gamma_\nu\epsilon + \frac{1}{8}\varepsilon^{\mu\nu\rho\sigma}\partial_\rho J_\sigma\gamma_\nu\epsilon + \frac{i}{4}\partial_\rho J^\mu\gamma_5\gamma^\rho\epsilon + \frac{i}{4}\partial^\mu(A - iB\gamma_5)\gamma_5\epsilon$$
$$\delta T^{\mu\nu} = -\frac{1}{4}\bar{\epsilon}\gamma_{(\mu}{}^\lambda\partial_\lambda\Xi_{\nu)}$$
$$\delta x = 2i\bar{\epsilon}_R\gamma^\rho\Xi_{\rho L}$$

$$\delta g_{\mu\nu} = \bar{\epsilon}\gamma_{(\mu}\psi_{\nu)}$$
$$\delta\psi_\mu = \partial_\mu\epsilon - \frac{1}{4}\partial_\nu g_{\rho\mu}\gamma^{\nu\rho}\epsilon - \frac{1}{6}(M + iN\gamma_5)\gamma_\mu\epsilon + \frac{i}{2}\left(V_\mu - \frac{1}{3}\gamma_\mu V\right)\gamma_5\epsilon$$
$$\delta V_\mu = \frac{i}{4}\bar{\epsilon}\gamma_5\left(\gamma_\mu^{\rho\sigma} - 4\delta_\mu^{[\rho}\gamma^{\sigma]}\right)\partial_\rho\psi_\sigma$$
$$\delta Z = \bar{\epsilon}_R\gamma^{\mu\nu}\partial_\mu\psi_{\nu R}$$

$$e^{-1}\mathcal{L}_{\mathrm{OM}} = \mathcal{L}_{\mathrm{SUGRA}} + \frac{1}{3}\left(|Z|^2 + V_\mu V^\mu\right)$$

$$\left(\sigma_\mu\right)_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}\mathcal{S}^\mu_{\mathrm{R}} = \chi_\alpha$$

$$\bar{D}_{\dot{\alpha}}\chi_\alpha = 0, D^\alpha\chi_\alpha = \bar{D}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}$$

$$\mathcal{S}^\mu_{\mathrm{R}} = \left\{J^\mu_{\mathrm{R}}, \Xi^\mu, T^{\mu\nu}, X^{\mu\nu}\right\}$$

$$X^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}\partial_\rho X_\sigma$$

$$\delta J_\mu = -i\bar{\epsilon}\gamma_5\Xi_\mu$$
$$\delta\Xi_\mu = T_{\mu\nu}\gamma^\nu\epsilon + X_{\mu\nu}\gamma^\nu\epsilon + \frac{1}{8}\varepsilon_{\mu\nu\rho\sigma}\partial^\rho J^\sigma\gamma^\nu\epsilon + \frac{i}{4}\partial_\rho J_\mu\gamma_5\gamma^\rho\epsilon$$
$$\delta T_{\mu\nu} = -\frac{1}{4}\bar{\epsilon}\gamma_{(\mu}{}^\lambda\partial_\lambda\Xi_{\nu)}$$
$$\delta X_{\mu\nu} = -\frac{i}{8}\varepsilon_{\mu\nu\rho\sigma}\bar{\epsilon}\gamma_5\gamma^\rho\gamma^\lambda\partial^\sigma\Xi_\lambda$$

$$\delta g_{\mu\nu} = \bar{\epsilon}\gamma_{(\mu}\psi_{\nu)}$$
$$\delta\psi_\mu = \partial_\mu\epsilon - \frac{1}{4}\partial_\nu g_{\rho\mu}\gamma^{\nu\rho}\epsilon + i\gamma_5 V_\mu\epsilon - \frac{i}{4}\gamma_\mu G_\nu\gamma_5\gamma^\nu\epsilon$$
$$\delta V_\mu = -\frac{i}{4}\bar{\epsilon}\gamma_5\gamma_\mu\gamma^{\nu\rho}\partial_\nu\psi_\rho + \frac{3}{8}\varepsilon^{\rho\sigma}_{\mu\nu}\bar{\epsilon}\gamma^\nu\partial_\rho\psi_\sigma$$
$$\delta C_{\mu\nu} = \bar{\epsilon}\gamma_{[\mu}\psi_{\nu]}$$

$$e^{-1}\mathcal{L}_{\mathrm{NM}} = \mathcal{L}_{\mathrm{SUGRA}} - 3V_\mu V^\mu + 2V^\mu C_\mu$$

$$\left(\sigma_\mu\right)_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}\mathcal{S}^\mu = D_\alpha X + \chi_\alpha$$
$$\bar{D}_{\dot{\alpha}}X = \bar{D}_{\dot{\alpha}}\chi_\alpha = 0$$
$$D^\alpha\chi_\alpha = \bar{D}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}$$

$$\mathcal{L}_{\mathrm{FP}} = \partial_\mu h_{\nu\rho}\partial^\nu h^{\mu\rho} - \frac{1}{2}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\mu h\partial^\mu h - \frac{m^2}{2}\left(h^{\mu\nu}h_{\mu\nu} - h^2\right)$$

$$\Lambda_\lambda = \left(m^{\lambda-1}M_{\mathrm{P}}\right)^{\frac{1}{\lambda}}$$

$$\left\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\right\}$$

$$\delta A_\mu = \frac{1}{2}\bar{\epsilon}\lambda_\mu$$
$$\delta\lambda_\mu = -\frac{i}{4}\left(mh_{\mu\rho} + \tilde{F}_{\mu\rho} + 2i\partial_\rho A_\mu\gamma_5\right)\gamma_5\gamma^\rho\epsilon$$
$$\delta h_{\mu\nu} = \frac{i}{m}\bar{\epsilon}\gamma_5\left(\partial_{(\mu}\lambda_{\nu)} - m\gamma_{(\mu}\chi_{\nu)}\right)$$
$$\delta\chi_\mu = \frac{i}{4}\partial_\rho h_{\mu\sigma}\gamma^{\rho\sigma}\gamma_5\epsilon + \frac{1}{8m}\partial_\mu F_{\rho\sigma}\gamma^{\rho\sigma}\epsilon + \frac{m}{4}A_\rho\left(2\delta^\rho_\mu + \gamma^\rho{}_\mu\right)\epsilon$$

$$\mathcal{L}_{\mathrm{SFP}} = -\frac{3}{8}F^{\mu\nu}F_{\mu\nu} - \frac{3}{4}m^2A^\mu A_\mu - \frac{1}{2}\bar{\lambda}_\mu\gamma^{\mu\nu\rho}\partial_\nu\lambda_\rho - \frac{1}{2}\bar{\chi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\chi_\rho - m\bar{\lambda}_\mu\gamma^{\mu\nu}\chi_\nu$$
$$+\frac{1}{4}\partial_\mu h_{\nu\rho}\partial^\nu h^{\mu\rho} - \frac{1}{8}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} - \frac{1}{4}\partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{8}\partial_\mu h\partial^\mu h - \frac{m^2}{8}\left(h^{\mu\nu}h_{\mu\nu} - h^2\right)$$

$$\partial_\mu F^{\mu\nu} - m^2A^\nu = 0, \quad \partial_{[\mu}F_{\nu\rho]} = 0,$$
$$\gamma^{\mu\nu\rho}\partial_\nu\lambda_\rho + m\gamma^{\mu\nu}\chi_\nu = 0, \quad \gamma^{\mu\nu\rho}\partial_\nu\chi_\rho + m\gamma^{\mu\nu}\lambda_\nu = 0,$$
$$\Box\, h_{\mu\nu} - 2\partial_\rho\partial_{(\mu}h^\rho_{\nu)} + \eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h - \eta_{\mu\nu}\Box\, h - m^2\left(h_{\mu\nu} - \eta_{\mu\nu}h\right) = 0$$

$$\Box\, A_\mu = m^2A_\mu, \qquad \partial^\mu A_\mu = 0,$$
$$\Box\, h_{\mu\nu} = m^2h_{\mu\nu}, \qquad \partial^\rho h_{\rho\mu} = 0, h = 0,$$
$$\partial\lambda_\mu = m\chi_\mu, \Box\,\lambda_\mu = m^2\lambda_\mu, \qquad \partial^\mu\lambda_\mu = 0, \gamma^\mu\lambda_\mu = 0,$$
$$\partial\chi_\mu = m\lambda_\mu, \Box\,\chi_\mu = m^2\chi_\mu, \qquad \partial^\mu\chi_\mu = 0, \gamma^\mu\chi_\mu = 0.$$

$$[\delta_1, \delta_2] = -\frac{1}{2}(\bar{\epsilon}_1\gamma^\rho\epsilon_2)\partial_\rho$$

$$\lambda_\mu \to e^{i\alpha\gamma_5}\lambda_\mu, \chi_\mu \to e^{-i\alpha\gamma_5}\chi_\mu, \epsilon \to e^{-i\alpha\gamma_5}\epsilon$$

$$J^\mu_0 = \varepsilon^{\mu\nu\rho\sigma}\left(\bar{\lambda}_\rho\gamma_\nu\lambda_\sigma - \bar{\chi}_\rho\gamma_\nu\chi_\sigma\right)$$

$$\Xi_0^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\{\big(mh_{\rho\tau} + \tilde{F}_{\rho\tau} - 2i\partial_\tau A_\rho\gamma_5\big)\gamma^\tau\gamma_\nu\lambda_\sigma$$
$$+\left[\partial_\lambda h_{\tau\rho}\gamma^{\lambda\tau}\gamma_5 - \frac{i}{2m}\partial_\rho F_{\lambda\tau}\gamma^{\lambda\tau} + imA_\tau\big(2\delta_\rho^\tau + \gamma_\rho^\tau\big)\right]\gamma_\nu\gamma_5\chi_\sigma\Big\}$$

$$X_0^\mu = \frac{1}{2}A_\nu\big(mh^{\nu\mu} + \tilde{F}^{\nu\mu}\big) + \frac{i}{4}\bar{\lambda}^\nu\gamma_5\gamma^\mu\lambda_\nu$$

$$T_0^{\mu\nu} = T_{\mathrm{K}}^{\mu\nu} + \partial_\rho B_0^{\rho\mu\nu} + \partial_\rho\partial_\sigma G_0^{\mu\sigma\nu\rho}$$

$$T_{\mathrm{K}}^{\mu\nu} = \frac{3}{2}T_{\mathrm{P}}^{\mu\nu} + T_{\mathrm{RS}}^{\mu\nu} + \frac{1}{4}T_{\mathrm{FP}}^{\mu\nu}$$

$$\begin{aligned}
T_{\mathrm{P}}^{\mu\nu} =& F^{\mu\rho}F_\rho^\nu - \frac{1}{4}\eta^{\mu\nu}F^2 + m^2A^\mu A^\nu - \frac{m^2}{2}\eta^{\mu\nu}A^2,\\
T_{\mathrm{RS}}^{\mu\nu} =& \frac{1}{2}\big(\bar{\lambda}^\sigma\gamma^{(\mu}\partial^{\nu)}\lambda_\sigma + \bar{\chi}^\sigma\gamma^{(\mu}\partial^{\nu)}\chi_\sigma\big) - \big(\bar{\lambda}^\sigma\gamma^{(\mu}\partial_\sigma\psi^{\nu)} + \bar{\chi}^\sigma\gamma^{(\mu}\partial_\sigma\chi^{\nu)}\big),\\
T_{\mathrm{FP}}^{\mu\nu} =& \partial^\mu h^{\rho\sigma}\partial^\nu h_{\rho\sigma} + \partial^\rho h^{\mu\sigma}\partial_\rho h_\sigma^\nu - 2\partial^{(\mu|}h^{\rho\sigma}\partial_\rho h^{|\nu)}{}_\sigma + m^2h^{\mu\rho}h_\rho^\nu\\
&+\frac{\eta^{\mu\nu}}{2}\big(-\partial^\lambda h^{\rho\sigma}\partial_\lambda h_{\rho\sigma} + \partial^\lambda h^{\rho\sigma}\partial_\rho h_{\lambda\sigma} - m^2h^{\rho\sigma}h_{\rho\sigma}\big).
\end{aligned}$$

$$\begin{aligned}
B_0^{\rho\mu\nu} =& \frac{1}{4}\big(h^{(\nu}{}_\sigma\partial^{\mu)}h^{\rho\sigma} - \partial^{(\mu}h^{\nu)}{}_\sigma h^{\rho\sigma}\big) + \frac{5}{4}\big(A^{(\nu}\partial^{\mu)}A^\rho - A^\rho\partial^{(\mu}A^{\nu)}\big)\\
&+\frac{1}{4m}\big(\partial^{(\mu}\tilde{F}^{\rho\sigma}h^{\nu)}{}_\sigma - \tilde{F}^{\rho\sigma}\partial^{(\mu}h^{\nu)}{}_\sigma + \tilde{F}^{(\nu}{}_\sigma\partial^{\mu)}h^{\rho\sigma} - \partial^{(\mu}\tilde{F}^{\nu)}{}_\sigma h^{\rho\sigma}\big)\\
&+\frac{1}{2m}\big(\partial^{(\mu}\bar{\chi}^{\nu)}\lambda^\rho - \bar{\chi}^{(\nu}\partial^{\mu)}\lambda^\rho + \partial^{(\mu}\bar{\lambda}^{\nu)}\chi^\rho - \bar{\lambda}^{(\nu}\partial^{\mu)}\chi^\rho\big),
\end{aligned}$$

$$\begin{aligned}
G_0^{\mu\sigma\nu\rho} =& \frac{1}{2m^2}\eta^{\nu][\sigma}F^{\mu]\lambda}F_\lambda^{[\rho} + \frac{1}{8m^2}\eta^{\nu[\sigma}\eta^{\mu]\rho}F^2 + 3\eta^{\nu][\sigma}A^{\mu]}A^{[\rho} + \frac{1}{4}\eta^{\nu[\mu}\eta^{\sigma]\rho}A^2\\
&+\frac{1}{2m}\eta^{\rho][\mu}\big(\tilde{F}^{\sigma]}{}_\lambda h^{\lambda[\nu} - h^{\sigma]}{}_\lambda\tilde{F}^{\lambda[\nu}\big) - h^{\mu[\rho}h^{\nu]\sigma} + \frac{1}{2}\eta^{\rho][\mu}h^{\sigma]}{}_\lambda h^{\lambda[\nu}\\
&+\frac{1}{m}\big(\bar{\lambda}^{[\sigma}\gamma^{\mu][\rho}\chi^{\nu]} + \bar{\lambda}^{[\rho}\gamma^{\nu][\sigma}\chi^{\mu]}\big)
\end{aligned}$$

$$\partial_\rho B_0^{\rho\mu\nu} = \partial_\rho B_0^{\rho(\mu\nu)} = \partial_\rho B_0^{[\rho\mu]\nu} = \partial_\rho B_0^{[\rho|\mu|\nu]}$$
$$\partial_\rho\partial_\sigma G_0^{\mu\sigma\nu\rho} = \partial_\rho\partial_\sigma G_0^{(\mu|\sigma|\nu)\rho} = \partial_\rho\partial_\sigma G_0^{[\mu\sigma]\nu\rho} = \partial_\rho\partial_\sigma G_0^{\mu\sigma[\nu\rho]}$$

$$S_0^\mu = \{J_0^\mu, \Xi_0^\mu, T_0^{\mu\nu}, X_0^{\mu\nu}\}$$

$$\begin{aligned}
\frac{4}{5}\partial_\rho B_A^{\rho\mu\nu} &= \partial_\rho\big(A^{(\nu}\partial^{\mu)}A^\rho - A^\rho\partial^{(\mu}A^{\nu)}\big) =\\
&= \partial_\rho\big(A^\nu\partial^{[\mu}A^{\rho]} + A^{[\mu}\partial^{\rho]}A^\nu + A^{[\mu|}\partial^\nu A^{|\rho]}\big) + \underbrace{\partial_\rho\big(A^{[\nu|}\partial^\rho A^{|\mu]}\big)}_{=0} = \frac{4}{5}\partial_\rho B_A^{[\rho\mu]\nu}
\end{aligned}$$

$$\mathcal{L} \sim -\frac{1}{2}\big(g_{\mu\nu} - \eta_{\mu\nu}\big)T^{\mu\nu} - \frac{1}{2}\bar{\psi}_\mu\Xi^\mu - \frac{1}{2}V_\mu J^\mu + C_\mu X^\mu,$$

$$g_{\mu\nu}\partial_\rho\partial_\sigma G_0^{\mu\sigma\nu\rho} \approx \big(\partial_\rho\partial_{[\sigma}g_{\mu][\nu}\big)G_0^{[\mu\sigma][\nu\rho]} \sim -\frac{1}{2}R_{\mu\sigma\nu\rho}G_0^{\mu\sigma\nu\rho}$$

$$R_{\mu\sigma\nu\rho} \sim -2\partial_\rho\partial_{[\sigma}\partial_{\mu][\nu}$$

$$g_{\mu\nu}\partial_\rho B_0^{\rho\mu\nu} \approx \left(\partial_{[\rho} g_{\mu]\nu} + \frac{1}{2}\partial_\nu g_{\mu\rho}\right) B_0^{[\rho\mu]\nu} \sim \Gamma_{\rho\mu\nu} B_0^{\rho\mu\nu},$$

$$g_{\rho\sigma}\Gamma^\sigma_{\mu\nu} \sim \partial_{[\mu} g_{\rho]\nu} + \frac{1}{2}\partial_\nu g_{\mu\rho}$$

$$\mathcal{S}_0^\mu \longrightarrow \mathcal{S}^\mu = \mathcal{S}_0^\mu + \Delta\mathcal{S}^\mu,$$

$$\Delta\mathcal{S}^\mu = \{\Delta J^\mu, \Delta\Xi^\mu, \Delta T^{\mu\nu}, \Delta X^{\mu\nu}\}$$

$$\Delta\mathcal{S}^\mu = \partial_\nu \mathcal{L}^{\mu\nu} + \sigma^{\mu\alpha\dot\beta}\left[D_\alpha, \bar D_{\dot\beta}\right]\mathcal{U}$$

$$D^2\mathcal{L}^{\mu\nu} = 0$$

$$\Delta J^\mu = \frac{1}{m}\partial_\nu L^{\mu\nu},$$

$$L^{\mu\nu} = c_{\mathrm{B}1} h_\lambda^{[\mu}\partial^\lambda A^{\nu]} + c_{\mathrm{B}2} h_\lambda^{[\mu}\partial^{\nu]} A^\lambda + \frac{c_{\mathrm{B}3}}{m}\varepsilon^{\mu\nu\rho\sigma}\partial_\lambda A_\rho \partial_\sigma A^\lambda$$
$$+ c_{\mathrm{F}1}\varepsilon^{\mu\nu\rho\sigma}\bar\chi_\rho\lambda_\sigma + i c_{\mathrm{F}2}\bar\chi^{[\mu}\gamma_5\lambda^{\nu]}$$

$$\delta_{\mathrm{L}}^2 L^{\mu\nu} = 0$$

$$\epsilon_{\mathrm{L}}\bar\epsilon_{\mathrm{L}} = -\frac{1}{2}(\bar\epsilon_{\mathrm{L}}\epsilon_{\mathrm{L}})P_{\mathrm{L}}$$

$$\delta_{\mathrm{L}}^2\left(h^{[\mu}{}_\sigma\partial^\sigma A^{\nu]}\right) = \frac{i}{4}(\bar\epsilon_{\mathrm{L}}\epsilon_{\mathrm{L}})\left[-\frac{1}{m}\partial_\sigma\bar\lambda_{\mathrm{L}}^{[\mu}\partial^{\nu]}\lambda_{\mathrm{L}}^\sigma + \bar\chi_{\mathrm{R}}^\sigma\gamma^{[\mu}\partial^{\nu]}\lambda_{\sigma\mathrm{L}}\right],$$

$$\delta_{\mathrm{L}}^2\left(h^{[\mu}{}_\sigma\partial^{\nu]} A^\sigma\right) = \frac{i}{4}(\bar\epsilon_{\mathrm{L}}\epsilon_{\mathrm{L}})\left[\frac{1}{m}\partial_\sigma\bar\lambda_{\mathrm{L}}^{[\mu}\partial^{\nu]}\lambda_{\mathrm{L}}^\sigma + \bar\chi_{\mathrm{R}}^\sigma\gamma^{[\mu}\partial^{\nu]}\lambda_{\sigma\mathrm{L}}\right],$$

$$\delta_{\mathrm{L}}^2\left(\frac{1}{m}\varepsilon^{\mu\nu\rho\sigma}\partial_\lambda A_\rho\partial_\sigma A^\lambda\right) = \frac{i}{4}(\bar\epsilon_{\mathrm{L}}\epsilon_{\mathrm{L}})\left[\frac{2}{m}\partial_\sigma\bar\lambda_{\mathrm{L}}^{[\mu}\partial^{\nu]}\lambda_{\mathrm{L}}^\sigma - 2\bar\chi_{\mathrm{R}}^\sigma\gamma^{[\mu}\partial^{\nu]}\lambda_{\sigma\mathrm{L}}\right],$$

$$\delta_{\mathrm{L}}^2\left(\varepsilon^{\mu\nu\rho\sigma}\bar\chi_\rho\lambda_\sigma\right) = 0,$$

$$\delta_{\mathrm{L}}^2\left(\bar\chi^{[\mu}\gamma_5\lambda^{\nu]}\right) = 0.$$

$$c_{\mathrm{B}2} = 0, c_{\mathrm{B}1} = 2c_{\mathrm{B}3} \equiv 2c_{\mathrm{B}}$$

$$L^{\mu\nu} = c_{\mathrm{B}}\left(2h_\lambda^{[\mu|}\partial^\lambda A^{|\nu]} + \frac{1}{m}\varepsilon^{\mu\nu\rho\sigma}\partial_\lambda A_\rho\partial_\sigma A^\lambda\right) + c_{\mathrm{F}1}\varepsilon^{\mu\nu\rho\sigma}\bar\chi_\rho\lambda_\sigma + i c_{\mathrm{F}2}\bar\chi^{[\mu}\gamma_5\lambda^{\nu]}$$

$$\partial_\rho\Delta B_{\mathrm{B}}^{\rho\mu\nu} = \frac{1}{8}\partial_\rho\left(A^{(\mu}\partial^{\nu)}A^\rho - A^\rho\partial^{(\mu}A^{\nu)}\right)$$
$$+\frac{1}{8m^2}\partial_\rho\left(\partial^\sigma A^{(\mu}\partial_\sigma\partial^{\nu)}A^\rho - \partial_\sigma A^\rho\partial^\sigma\partial^{(\mu}A^{\nu)}\right),$$
$$\partial_\rho\Delta B_{\mathrm{F}1}^{\rho\mu\nu} = \frac{1}{2m}\partial_\rho\left(\partial^{(\mu}\bar\chi^{\nu)}\lambda^\rho - \bar\chi^{(\nu}\partial^{\mu)}\lambda^\rho + \partial^{(\mu}\bar\lambda^{\nu)}\chi^\rho - \bar\lambda^{(\nu}\partial^{\mu)}\chi^\rho\right)$$
$$+\frac{1}{2m}\partial_\rho\left(\partial^{(\mu|}\bar\lambda_\sigma\gamma^\rho\gamma^{|\nu)}\chi^\sigma - \partial^\rho\bar\lambda_\sigma\gamma^{(\mu}\gamma^{\nu)}\chi^\sigma\right),$$
$$\partial_\rho\Delta B_{\mathrm{F}2}^{\rho\mu\nu} = -\frac{1}{4}\partial_\rho B_0^{\rho\mu\nu}.$$

$$c_{\mathrm{B}} = c_{\mathrm{F1}} = 0, c_{\mathrm{F2}} = 4$$

$$\mathcal{S}^{\mu} = \{J^{\mu}, \Xi^{\mu}, T^{\mu\nu}, X^{\mu\nu}\}$$

$$J^{\mu} = \varepsilon^{\mu\nu\rho\sigma}\left(\bar{\lambda}_{\rho}\gamma_{\nu}\lambda_{\sigma} - \bar{\chi}_{\rho}\gamma_{\nu}\chi_{\sigma}\right) + \frac{4i}{m}\partial_{\nu}\left(\bar{\chi}^{[\mu}\gamma_{5}\lambda^{\nu]}\right)$$

$$\Xi^{\mu} = \Xi_{0}^{\mu} - \frac{i}{m}\partial_{\nu}\left[\left(mh_{\lambda}^{[\nu} + \tilde{F}_{\lambda}^{[\nu} - 2i\partial_{\lambda}A^{[\nu}\gamma_{5}\right)\gamma^{\lambda}\gamma_{5}\chi^{\mu]}\right.$$
$$\left. + \left(\partial_{\rho}h^{[\mu}{}_{\sigma}\gamma^{\rho\sigma}\gamma_{5} - \frac{i}{2m}\partial^{[\mu}F_{\rho\sigma}\gamma^{\rho\sigma} + 2imA^{[\mu} - imA_{\rho}\gamma^{\rho[\mu}\right)\lambda^{\nu]}\right]$$

$$T^{\mu\nu} = T_{\mathrm{K}}^{\mu\nu} + \partial_{\rho}\partial_{\sigma}\mathcal{G}^{\mu\sigma\nu\rho}$$

$$X^{\mu} = \frac{i}{4}\bar{\lambda}^{\nu}\gamma_{5}\gamma^{\mu}\lambda_{\nu} + \frac{3m}{4}A_{\nu}h^{\nu\mu} + \frac{9}{8}A_{\nu}\tilde{F}^{\nu\mu} + \frac{1}{4m^{2}}\partial_{\rho}\partial_{\sigma}\left(\tilde{F}^{\mu\rho}A^{\sigma}\right)$$
$$+ \frac{1}{8m}\partial_{\nu}\left(F^{\nu\rho}h_{\rho}^{\mu} - F^{\mu\rho}h_{\rho}^{\nu}\right) - \frac{3}{4m}\partial_{\nu}\left(\partial^{\mu}A^{\rho}h_{\rho}^{\nu} + \partial_{\rho}A^{\nu}h^{\rho\mu}\right)$$
$$+ \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}\partial_{\nu}h_{\rho}^{\lambda}h_{\sigma\lambda} - \frac{1}{4m}\varepsilon^{\mu\nu\rho\sigma}\left(\bar{\chi}_{\nu}\partial_{\rho}\lambda_{\sigma} + \bar{\lambda}_{\nu}\partial_{\rho}\chi_{\sigma}\right)$$

$$\mathcal{G}^{\mu\sigma\nu\rho} = -\frac{1}{2}h^{\rho[\mu}h^{\sigma]\nu} + \frac{1}{2}\eta^{\rho][\mu}h^{\sigma]}{}_{\lambda}h^{\lambda[\nu} + 2\eta^{\rho][\mu}A^{\sigma]}A^{[\nu} - \frac{3}{4m^{2}}F^{\mu\sigma}F^{\nu\rho} + \frac{1}{2m^{2}}\eta^{\rho][\mu}F^{\sigma]}{}_{\lambda}F^{\lambda[\nu}$$
$$+ \frac{1}{m^{2}}\left(\eta^{\sigma][\nu}F^{\rho]}\partial^{\lambda}A^{[\mu} + \eta^{\rho][\mu}F^{\sigma]}{}_{\lambda}\partial^{\lambda}A^{[\nu}\right) + \frac{1}{2m}\eta^{\rho][\mu}\left(\tilde{F}^{\sigma]}h^{\lambda[\nu} - h^{\sigma]}{}_{\lambda}\tilde{F}^{\lambda[\nu}\right)$$
$$+ \frac{1}{2m}\left(\varepsilon^{\mu\sigma\lambda\tau}A_{\lambda}\partial^{[\nu}h^{\rho]}{}_{\tau} + \varepsilon^{\nu\rho\lambda\tau}A_{\lambda}\partial^{[\mu}h^{\sigma]}{}_{\tau}\right) + \frac{1}{m}\left(\bar{\lambda}^{[\sigma}\gamma^{\mu][\rho}\chi^{\nu]} + \bar{\lambda}^{[\rho}\gamma^{\nu][\sigma}\chi^{\mu]}\right)$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SUGRA}} + \mathcal{L}_{\mathrm{SFP}} + \mathcal{L}_{\psi} + \mathcal{L}_{\text{non-min}} + \mathcal{L}_{\text{quartic}} + \cdots,$$

$$e^{-1}\mathcal{L}_{\mathrm{SFP}} = -\frac{3}{8}F^{\mu\nu}F_{\mu\nu} - \frac{3}{4}m^{2}A^{\mu}A_{\mu} - \frac{1}{2}\bar{\lambda}_{\mu}\gamma^{\mu\nu\rho}D_{\nu}\lambda_{\rho} - \frac{1}{2}\bar{\chi}_{\mu}\gamma^{\mu\nu\rho}D_{\nu}\chi_{\rho} - m\bar{\lambda}_{\mu}\gamma^{\mu\nu}\chi_{\nu}$$
$$+ \frac{1}{4}\nabla_{\mu}h_{\nu\rho}\nabla^{\nu}h^{\mu\rho} - \frac{1}{8}\nabla_{\rho}h_{\mu\nu}\nabla^{\rho}h^{\mu\nu} - \frac{1}{4}\nabla_{\mu}h^{\mu\nu}\nabla_{\nu}h + \frac{1}{8}\nabla_{\mu}h\nabla^{\mu}h - \frac{m^{2}}{8}\left(h^{\mu\nu}h_{\mu\nu} - h^{2}\right)$$

$$e^{-1}\mathcal{L}_{\psi} = -\frac{e^{-1}}{4}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\left[\left(mh_{\rho\tau} + \tilde{F}_{\rho\tau} - 2i\nabla_{\tau}A_{\rho}\gamma_{5}\right)\gamma^{\tau}\gamma_{\nu}\lambda_{\sigma}\right.$$
$$\left. + \left(\nabla_{\lambda}h_{\tau\rho}\gamma^{\lambda\tau}\gamma_{5} - \frac{i}{2m}\nabla_{\rho}F_{\lambda\tau}\gamma^{\lambda\tau} + imA_{\tau}\left(2\delta_{\rho}^{\tau} + \gamma_{\rho}^{\tau}\right)\right)\gamma_{\nu}\gamma_{5}\chi_{\sigma}\right]$$
$$+ \frac{i}{2m}\bar{\rho}_{\mu\nu}\left[\left(mh_{\lambda}^{\nu} + \tilde{F}_{\lambda}^{\nu} - 2i\nabla_{\lambda}A^{\nu}\gamma_{5}\right)\gamma^{\lambda}\gamma_{5}\chi^{\mu}\right.$$
$$\left. + \left(\nabla_{\rho}h^{\mu}{}_{\sigma}\gamma^{\rho\sigma}\gamma_{5} - \frac{i}{2m}\nabla^{\mu}F_{\rho\sigma}\gamma^{\rho\sigma} + 2imA^{\mu} - imA_{\rho}\gamma^{\rho\mu}\right)\lambda^{\nu}\right]$$

$$e^{-1}\mathcal{L}_{\text{non-min}} = \frac{1}{4}R_{\mu\sigma\nu\rho}\left[-\frac{1}{2}h^{\rho\mu}h^{\sigma\nu} + \frac{1}{2}g^{\rho\mu}h^{\sigma}{}_{\lambda}h^{\lambda\nu} + 2g^{\rho\mu}A^{\sigma}A^{\nu} - \frac{3}{4m^{2}}F^{\mu\sigma}F^{\nu\rho} + \frac{1}{2m^{2}}g^{\rho\mu}F^{\sigma}{}_{\lambda}F^{\lambda\nu}\right.$$
$$\left. + \frac{2}{m^{2}}g^{\sigma\nu}F^{\rho}{}_{\lambda}\nabla^{\lambda}A^{\mu} + \frac{e^{-1}}{m}\varepsilon^{\mu\sigma\lambda\tau}A_{\lambda}\nabla^{\nu}h^{\rho}{}_{\tau} + \frac{1}{m}g^{\rho\mu}\tilde{F}^{\sigma}{}_{\lambda}h^{\lambda\nu} + \frac{2}{m}\bar{\lambda}^{\sigma}\gamma^{\mu\rho}\chi^{\nu}\right]$$

$$e^{-1}\mathcal{L}_{\text{quartic}} = -\frac{1}{4}\left(e^{-1}\varepsilon_{\mu}{}^{\nu\rho\sigma}\left(\bar{\lambda}_{\rho}\gamma_{\nu}\lambda_{\sigma} - \bar{\chi}_{\rho}\gamma_{\nu}\chi_{\sigma}\right) + \frac{4i}{m}\nabla^{\nu}\left(\bar{\chi}_{[\mu}\gamma_{5}\lambda_{\nu]}\right)\right)\left[\frac{i}{4}\bar{\lambda}^{\tau}\gamma_{5}\gamma^{\mu}\lambda_{\tau} + \frac{3m}{4}A_{\tau}h^{\tau\mu}\right.$$
$$+\frac{9}{8}A_{\tau}\tilde{F}^{\tau\mu} + \frac{1}{8m}\nabla_{\tau}\left(F^{\tau\lambda}h^{\mu}{}_{\lambda} - F^{\mu\lambda}h^{\tau}{}_{\lambda}\right) - \frac{3}{4m}\nabla_{\tau}\left(\nabla^{\mu}A^{\lambda}h^{\tau}{}_{\lambda} + \nabla_{\lambda}A^{\tau}h^{\lambda\mu}\right)$$
$$\left.+\frac{1}{4m^{2}}\nabla_{\lambda}\nabla_{\pi}\left(\tilde{F}^{\mu\lambda}A^{\pi}\right) + \frac{e^{-1}}{4}\varepsilon^{\mu\tau\lambda\pi}\nabla_{\tau}h^{\alpha}{}_{\lambda}h_{\pi\alpha} - \frac{e^{-1}}{4m}\varepsilon^{\mu\tau\lambda\pi}\left(\bar{\chi}_{\tau}D_{\lambda}\lambda_{\pi} + \bar{\lambda}_{\tau}D_{\lambda}\chi_{\pi}\right)\right]$$

$$T^{\mu\nu} \to T^{\mu\nu} + (\partial^{\mu}\partial^{\nu} - \eta^{\mu\nu}\,\Box)U, \Xi^{\mu} \to \Xi^{\mu} - 2\gamma^{\mu\nu}\partial_{\nu}\Upsilon$$

$$U = \mathcal{U}|_{\vartheta=0}, \delta U = \bar{\epsilon}\Upsilon$$

$$e^{-1}\mathcal{L}_{u} = -\frac{1}{2}RU + \bar{\rho}_{\mu\nu}\gamma^{\mu\nu}\Upsilon$$

$$E^{a} = \phi^{-\frac{1}{2}}e^{a}, \qquad \mathcal{A} = a(dy + B) + A$$
$$E^{4} = \phi(dy + B), \quad \Psi = \phi^{\frac{5}{4}}\zeta(dy + B) + \phi^{-\frac{1}{4}}\left(\psi - \frac{1}{2}e^{a}\gamma_{a}\gamma_{5}\zeta\right)$$

$$\psi_{\mu}(-y) = \sigma_{3}\gamma_{5}\psi_{\mu}(y), \zeta(-y) = -\sigma_{3}\gamma_{5}\zeta(y)$$

$$B_{\mu}(x,y) = 0, \phi(x,y) = \phi(x), a(x,y) = a(x), \zeta(x,y) = \frac{1}{2}(1 - \sigma_{3}\gamma_{5})\zeta(x)$$

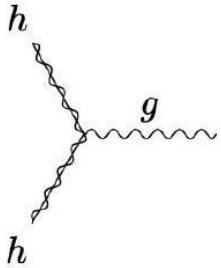
$$\delta B_{\mu}(x,y) = \frac{\phi(x)^{-\frac{3}{2}}}{2}\bar{\epsilon}(x)\gamma_{5}\psi_{\mu}(x,y) + \phi(x)^{-3}g_{\mu\nu}(x,y)\partial_{y}\xi^{\nu}(x,y) + \partial_{\mu}\xi^{5}(x,y)$$
$$\delta\phi(x,y) = \frac{\phi(x)}{2}\bar{\epsilon}(x)\gamma_{5}\zeta(x) + \xi^{\mu}(x,y)\partial_{\mu}\phi(x) + \partial_{y}\xi^{5}(x,y)\phi(x)$$

$$\partial_{y}\xi^{5}(x,y) = -\xi^{\mu}(x,y)\partial_{\mu}\log\,\phi(x)$$

$$\partial_{y}\xi^{\mu}(x,y) + \phi(x)^{3}g^{\mu\nu}(x,y)\partial_{\nu}\xi^{5}(x,y) = -\frac{\phi(x)^{\frac{3}{2}}}{2}g^{\mu\nu}(x,y)\bar{\epsilon}(x)\gamma_{5}\psi_{\nu}(x,y)$$

$$\delta e_{\mu}{}^{a}(x,y) = \frac{1}{2}\bar{\epsilon}(x)\gamma^{a}\psi_{\mu}(x,y) + D_{\mu}(\xi^{\nu}(x,y)e_{\nu}{}^{a}(x,y)) + \xi^{5}(x,y)\partial_{y}e_{\mu}{}^{a}(x,y),$$

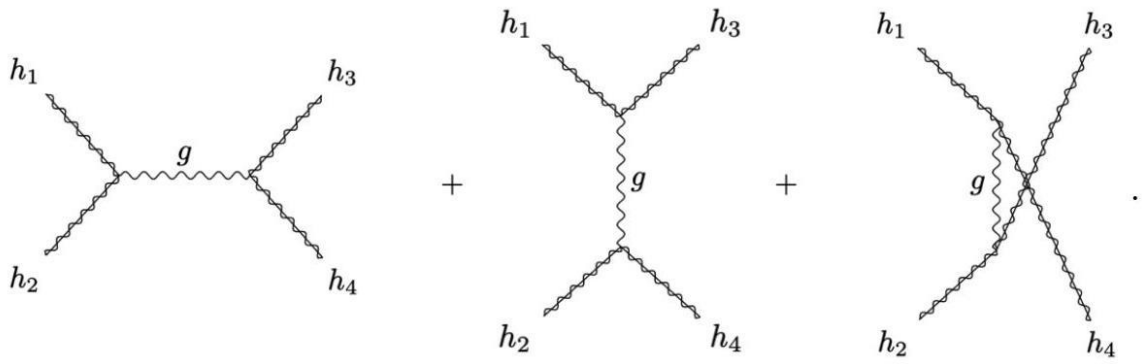
$$\xi^{\mu}(\epsilon_{1},\epsilon_{2}) \sim -\frac{1}{2}\bar{\epsilon}_{1}\gamma^{\mu}\epsilon_{2} + \frac{1}{4m^{2}}\bar{\epsilon}_{1}h^{\nu\rho}\psi_{\rho}\partial_{\nu}(\bar{\epsilon}_{2}h^{\mu\sigma}\psi_{\sigma}) - \frac{1}{4m^{2}}\bar{\epsilon}_{2}h^{\nu\rho}\psi_{\rho}\partial_{\nu}(\bar{\epsilon}_{1}h^{\mu\sigma}\psi_{\sigma})$$



$$= -i\left[\frac{1}{4}T_{\text{FP}}^{\mu\nu} + \partial_{\rho}\partial_{\sigma}\left(-\frac{1}{2}h^{\rho[\mu}h^{\sigma]\nu} + \frac{1}{2}\eta^{\rho][\mu}h^{\sigma]}{}_{\lambda}h^{\lambda[\nu}\right)\right],$$

$$i(2\pi)^{4}\delta^{(4)}(p_{1} + p_{2} - p_{1}' - p_{2}')\mathcal{A} = -\frac{\kappa^{2}}{2}\langle\,\text{out}\,|\int d^{4}x d^{4}y\colon T_{h}^{\mu\nu}(x)D_{\mu\nu\rho\sigma}(x-y)T_{h}^{\rho\sigma}(y)\colon|\,\text{in}\,\rangle$$

$$D_{\mu\nu\rho\sigma}(p) = -\frac{i}{2p^2}\left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}\right)$$



$$E_{\boldsymbol{p}} = \sqrt{\boldsymbol{p}^2 + m^2}, \boldsymbol{p} = |\boldsymbol{p}|(\sin\,\theta\cos\,\phi, \sin\,\theta\sin\,\phi, \cos\,\theta) \equiv |\boldsymbol{p}|\hat{\boldsymbol{p}},$$

$$h_{\mu\nu}(x) = 2\int \frac{d^3p}{(2\pi)^{\frac{3}{2}}\sqrt{2E_{\boldsymbol{p}}}}\sum_{\sigma}\left(\epsilon_{\mu\nu}(\sigma,\boldsymbol{p})e^{ip\cdot x}a_{\sigma,\boldsymbol{p}} + \bar{\epsilon}_{\mu\nu}(\sigma,\boldsymbol{p})e^{-ip\cdot x}a^{\dagger}_{\sigma,\boldsymbol{p}}\right)$$

$$\epsilon^{\mu\nu}(\sigma,\boldsymbol{p})\bar{\epsilon}_{\mu\nu}(\sigma',\boldsymbol{p}) = \delta_{\sigma\sigma'}$$

$$\sum_{\sigma}\epsilon^{\mu\nu}(\sigma,\boldsymbol{p})\bar{\epsilon}^{\rho\sigma}(\sigma,\boldsymbol{p}) = \frac{1}{2}(P^{\mu\rho}P^{\nu\sigma} + P^{\nu\rho}P^{\nu\sigma}) - \frac{1}{3}P^{\mu\nu}P^{\rho\sigma}$$

$$p^{\mu}\epsilon_{\mu\nu}(\sigma,\boldsymbol{p}) = 0, \epsilon^{\mu}_{\mu}(\sigma,\boldsymbol{p}) = 0$$

$$\epsilon^{\mu\nu}(\pm 2,\boldsymbol{p}) = \epsilon^{\mu}_{\pm 1}(\boldsymbol{p})\epsilon^{\nu}_{\pm 1}(\boldsymbol{p})$$

$$\epsilon^{\mu\nu}(\pm 1,\boldsymbol{p}) = \frac{1}{\sqrt{2}}\left(\epsilon^{\mu}_{\pm 1}(\boldsymbol{p})\epsilon^{\nu}_{0}(\boldsymbol{p}) + \epsilon^{\nu}_{\pm 1}(\boldsymbol{p})\epsilon^{\mu}_{0}(\boldsymbol{p})\right)$$

$$\epsilon^{\mu\nu}(0,\boldsymbol{p}) = \frac{1}{\sqrt{6}}\left(\epsilon^{\mu}_{1}(\boldsymbol{p})\epsilon^{\nu}_{-1}(\boldsymbol{p}) + \epsilon^{\nu}_{1}(\boldsymbol{p})\epsilon^{\mu}_{-1}(\boldsymbol{p}) + 2\epsilon^{\mu}_{0}(\boldsymbol{p})\epsilon^{\nu}_{0}(\boldsymbol{p})\right)$$

$$\epsilon^{\mu}_{\pm 1}(\boldsymbol{p}) = \pm\frac{e^{\pm i\phi}}{\sqrt{2}}(0, -\cos\,\theta\cos\,\phi \pm i\sin\,\phi, -\cos\,\theta\sin\,\phi \mp i\cos\,\phi, \sin\,\theta)$$

$$\epsilon^{\mu}_{0}(\boldsymbol{p}) = \frac{E_{\boldsymbol{p}}}{m}\left(\sqrt{1 - \frac{m^2}{E^2_{\boldsymbol{p}}}}, \hat{\boldsymbol{p}}\right)$$

$$\mathcal{A}(0,0,0,0) = \frac{\kappa^2}{16m^4 s(s} - 4m^2)\left[s^5 \sin^2\,\theta(3+\cos^2\,\theta) - \frac{4m^2 s^4}{3}(21 - 29\cos^2\,\theta)\right.$$
$$+2m^4 s^3\left(\frac{32}{\sin^2\,\theta} + \frac{37 - 231\cos^2\,\theta + 18\cos^4\,\theta}{3}\right)$$
$$-16m^6 s^2\left(\frac{16}{\sin^2\,\theta} - 7 + 2\cos^2\,\theta - \cos^4\,\theta\right)$$
$$\left.+32m^8 s\left(\frac{4}{\sin^2\,\theta} + \frac{11 + 26\cos^2\,\theta}{3}\right) + 128m^{10}\cos\,(2\theta)\right]$$

$$\mathcal{A}(1,1,0,0) = \frac{\kappa^2}{384m^6 s(s - 4m^2)} \times$$
$$\times [s^6(5 - \cos^2\,\theta) - m^2 s^5(49 - 9\cos^4\,\theta) + 2m^4 s^4(217 - 113\cos^2\,\theta + 26\cos^4\,\theta)$$
$$-4m^6 s^3(480 - 265\cos^2\,\theta + 69\cos^4\,\theta) + 32m^8 s^2(103 + 96\cos^2\,\theta - 9\cos^4\,\theta)$$
$$-64m^{10} s(56 + 119\cos^2\,\theta + \cos^4\,\theta) - 3072m^{12}\cos\,(2\theta)],$$

$$T_U^{\mu\nu} = (\partial^\mu\partial^\nu - \eta^{\mu\nu}\,\Box)\left(c_1 h^{\rho\sigma}h_{\rho\sigma} + c_2\partial_\lambda h^{\rho\sigma}\partial^\lambda h_{\rho\sigma} + c_3\partial_\rho\partial_\sigma\left(h^{\rho\lambda}h_\lambda^\sigma\right)\right)$$

$$-\frac{3}{16m^2}R_{\mu\sigma\nu\rho}F^{\mu\sigma}F^{\nu\rho}$$

$$\frac{\kappa}{2}\left[g_{\mu\nu}^{(1)}\left(\partial^\mu h^{\rho\sigma}\partial^\nu h_{\rho\sigma} - 4\partial^\mu h^{\rho\sigma}\partial_\rho h_\sigma^\nu\right) + h^{\mu\nu}\left(\partial_\mu g_{\rho\sigma}^{(1)}\partial_\nu h^{\rho\sigma} - \partial_\rho g_{\mu\sigma}^{(1)}\partial_\nu h^{\rho\sigma}\right)\right.$$
$$\left.+\partial_\mu\partial^\rho g_{\nu\rho}^{(1)}\left(2h_\sigma^\mu h^{\nu\sigma} - \frac{1}{4}\eta^{\mu\nu}h_{\sigma\lambda}h^{\sigma\lambda}\right)\right]$$

$$S = -T\int\,dA = -T\int\,d^2\xi\sqrt{-\gamma}$$

$$\gamma_{ab} = \frac{\partial X^\mu}{\partial\xi^a}\frac{\partial X^\nu}{\partial\xi^b}G_{\mu\nu} = \begin{pmatrix}\dot{X}^\mu\dot{X}_\mu & \dot{X}^\mu X'_\mu \\ X'^\mu\dot{X}_\mu & X'^\mu X'_\mu\end{pmatrix}$$

$$S_{\mathrm{NG}} = -T\int\,d^2\xi\sqrt{(\dot{X}\cdot X')^2 - (\dot{X}\cdot\dot{X})(X'\cdot X')}$$

$$S_{\mathrm{P}} = -\frac{T}{2}\int\,d^2\xi\sqrt{-g}g^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}$$

$$T_{ab} = \partial_a X^\mu\partial_b X_\mu - \frac{1}{2}g_{ab}\left(g^{cd}\partial_c X^\mu\partial_d X_\mu\right) = 0$$

$$\xi^\pm = \tau \pm \sigma, ds^2 = -d\xi^+ d\xi^-, \eta = \begin{pmatrix}0 & -\frac{1}{2} \\ -\frac{1}{2} & 0\end{pmatrix}$$

$$S_{\mathrm{P}} = 2T\int\,d\xi^+ d\xi^-\partial_+ X^\mu\partial_- X_\mu$$

$$\delta S_{\mathrm{P}} = 2T\int d\xi^{+}d\xi^{-}\left(\partial_{+}\delta X^{\mu}\partial_{-}X_{\mu} + \partial_{+}X^{\mu}\partial_{-}\delta X_{\mu}\right)$$
$$= 2T\int d\xi^{+}d\xi^{-}\left[\partial_{+}\left(\delta X^{\mu}\partial_{-}X_{\mu}\right) + \partial_{-}\left(\partial_{+}X^{\mu}\delta X_{\mu}\right) - 2(\partial_{+}\partial_{-}X^{\mu})\delta X_{\mu}\right]$$
$$= -4T\int d\xi^{+}d\xi^{-}(\partial_{+}\partial_{-}X^{\mu})\delta X_{\mu} - T\int d^{2}\xi\partial_{\sigma}\left(\delta X^{\mu}\partial_{\sigma}X_{\mu}\right)$$

$$\int_{0}^{\pi} d\sigma\partial_{\sigma}\left(\delta X^{\mu}\partial_{\sigma}X_{\mu}\right) = \left(\delta X^{\mu}\partial_{\sigma}X_{\mu}\right)\big|_{\sigma=\pi} - \left(\delta X^{\mu}\partial_{\sigma}X_{\mu}\right)\big|_{\sigma=0}$$

$$\text{NN: } \partial_{\sigma}X^{\mu}|_{\sigma=0} = \partial_{\sigma}X^{\mu}|_{\sigma=\pi} = 0, \text{DD: } \delta X^{\mu}|_{\sigma=0} = \delta X^{\mu}|_{\sigma=\pi} = 0$$

$$\partial_{+}\partial_{-}X^{\mu} = 0$$

$$X^{\mu}(\tau,\sigma) = X_{\mathrm{L}}^{\mu}(\tau+\sigma) + X_{\mathrm{R}}^{\mu}(\tau-\sigma)$$

$$X_{\mathrm{L}}^{\mu}(\tau+\sigma) = x_{\mathrm{L}}^{\mu} + \frac{\alpha'}{2}p_{\mathrm{L}}^{\mu}(\tau+\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\alpha_{n}^{\mu}}{n}e^{-in(\tau+\sigma)}$$
$$X_{\mathrm{R}}^{\mu}(\tau-\sigma) = x_{\mathrm{R}}^{\mu} + \frac{\alpha'}{2}p_{\mathrm{R}}^{\mu}(\tau-\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\bar{\alpha}_{n}^{\mu}}{n}e^{-in(\tau-\sigma)}$$

$$T_{++} = \partial_{+}X^{\mu}\partial_{+}X_{\mu}, T_{--} = \partial_{-}X^{\mu}\partial_{-}X_{\mu}$$

$$X^{\mu}(\tau,\sigma) = x^{\mu} + \alpha' p^{\mu}\tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\in\mathbb{Z}\setminus\{0\}}\frac{1}{n}\left[\alpha_{n}^{\mu}e^{-in(\tau+\sigma)} + \bar{\alpha}_{n}^{\mu}e^{-in(\tau-\sigma)}\right]$$

$$\alpha_{0}^{\mu} \equiv \sqrt{\frac{\alpha'}{2}}p^{\mu}$$

$$T_{++} = \alpha'\sum_{n} L_{n}e^{-in(\tau+\sigma)}, T_{--} = \alpha'\sum_{n}\bar{L}_{n}e^{-in(\tau-\sigma)}$$

$$L_{n} = \frac{1}{2}\sum_{m}\alpha_{n-m}^{\mu}\alpha_{m\mu}, \bar{L}_{n} = \frac{1}{2}\sum_{m}\bar{\alpha}_{n-m}^{\mu}\bar{\alpha}_{m\mu}$$

$$\{L_{n}, L_{m}\} = -i(n-m)L_{n+m}, \{\bar{L}_{n}, \bar{L}_{m}\} = -i(n-m)\bar{L}_{n+m}$$

$$L_{n} = \bar{L}_{n} = 0\ \forall n \in \mathbb{Z}.$$

$$M^{2} = \frac{2}{\alpha'}\sum_{m\neq 0}\alpha_{-m}^{\mu}\alpha_{m\mu}$$

$$\sum_{m\neq 0}\alpha_{-m}^{\mu}\alpha_{m\mu} = \sum_{m\neq 0}\bar{\alpha}_{-m}^{\mu}\bar{\alpha}_{m\mu}$$

$$\alpha_{n}^{\mu} = \bar{\alpha}_{n}^{\mu}$$

$$X^{\mu}(\tau,\sigma) = x^{\mu} + 2\alpha' p^{\mu}\tau + i\sqrt{2\alpha'}\sum_{n\in\mathbb{Z}\setminus\{0\}}\frac{\alpha_n^{\mu}}{n}\cos(n\sigma)e^{-in\tau}$$

$$M^2 = \frac{1}{2\alpha'}\sum_{m\neq 0}\alpha_{-m}^{\mu}\alpha_{m\mu}$$

$$X^{\mu}(\tau,\sigma) = x^{\mu} + d^{\mu}\sigma + \sqrt{2\alpha'}\sum_{n\in\mathbb{Z}\setminus\{0\}}\frac{\alpha_n^{\mu}}{n}\sin(n\sigma)e^{-in\tau}$$

$$[X^{\mu}(\tau,\sigma),\Pi^{\nu}(\tau,\sigma')] = i\eta^{\mu\nu}\delta(\sigma-\sigma')$$

$$X^{\pm} \equiv \frac{1}{\sqrt{2}}(X^0 \pm X^{D-1}), ds^2 = -2dX^+dX^- + \sum_{i=1}^{D-2} dX^i dX_i$$

$$X^{+}(\tau,\sigma) = x^{+} + 2\alpha' p^{+}\tau$$

$$X^{-}(\tau,\sigma) = x^{-} + 2\alpha' p^{-}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^{-}}{n}\cos(n\sigma)e^{-in\tau}$$

$$X^{i}(\tau,\sigma) = x^{i} + 2\alpha' p^{i}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^{i}}{n}\cos(n\sigma)e^{-in\tau}$$

$$T_{++} = T_{--} = 0 \implies \partial_{\pm}X^{-} = \frac{1}{2\alpha' p^{+}}\partial_{\pm}X^{i}\partial_{\pm}X_i$$

$$\alpha_n^{-} = \sqrt{\frac{2}{\alpha'}}\frac{1}{4p^{+}}\sum_{m}\alpha_{n-m}^{i}\alpha_{mi}.$$

$$M^2 = \frac{1}{2\alpha'}\sum_{m\neq 0}\alpha_{-m}^{i}\alpha_{mi},$$

$$\left[\alpha_n^{i},\alpha_m^{j}\right] = n\delta_{n+m,0}\delta^{ij}$$

$$M^2 = \frac{1}{2\alpha'}\sum_{m\neq 0} :\alpha_{-m}^{i}\alpha_{mi}: = \frac{1}{\alpha'}\sum_{m>0}\alpha_{-m}^{i}\alpha_{mi} - \frac{D-2}{24\alpha'} \equiv \frac{1}{\alpha'}\left(N - \frac{D-2}{24}\right).$$

$$M^2|0\rangle = -\frac{D-2}{24\alpha'}|0\rangle,$$

$$M^2\left(\alpha_{-1}^{i}|0\rangle\right) = -\frac{D-26}{24\alpha'}\left(\alpha_{-1}^{i}|0\rangle\right)$$

$$\alpha_{-2}^{i}|0\rangle, \alpha_{-1}^{i}\alpha_{-1}^{j}|0\rangle$$

$$(D-2) + \frac{(D-2)(D-1)}{2} = \frac{D(D-1)}{2} - 1$$
$$= \left[\frac{(D-2)(D-1)}{2} - 1\right] + (D-2) + 1.$$

$$M^2 = \frac{2}{\alpha'}\left[\sum_{n>0}\left(\alpha^i_{-m}\alpha_{mi} + \bar{\alpha}^i_{-m}\bar{\alpha}_{mi}\right) - \frac{D-2}{12}\right] \equiv \frac{2}{\alpha'}\left(N + \bar{N} - \frac{D-2}{12}\right),$$

$$\alpha^i_{-1}\bar{\alpha}^j_{-1}|0,\overline{0}\rangle.$$

$$(D-2)(D-2) = \left[\frac{(D-2)(D-1)}{2} - 1\right] + \frac{(D-2)(D-3)}{2} + 1$$

$$S_{\mathrm{P}} = -\frac{1}{4\pi\alpha'}\int d^2\xi\left\{g^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + \varepsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X) - \alpha' R(g)\phi(X)\right\}$$

$$\begin{aligned}
\beta^{(G)}_{\mu\nu} &= R_{\mu\nu} + \frac{1}{4}H_\mu{}^{\lambda\rho}H_{\nu\lambda\rho} - 2\nabla_\mu\nabla_\nu\phi + \mathcal{O}(\alpha') = 0\\
\beta^{(B)}_{\mu\nu} &= \nabla_\lambda H^\lambda{}_{\mu\nu} - 2\nabla_\lambda\phi H^\lambda{}_{\mu\nu} + \mathcal{O}(\alpha') = 0\\
\beta^{(\phi)} &= 4\nabla_\lambda\phi\nabla^\lambda\phi - 4\nabla_\lambda\nabla^\lambda\phi + R + \frac{1}{12}H_{\lambda\rho\sigma}H^{\lambda\rho\sigma} + \mathcal{O}(\alpha') = 0
\end{aligned}$$

$$S = \frac{1}{2}\int d^{26}x\sqrt{-G}e^{-2\phi}\left(R - \frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho} + 4\partial_\mu\phi\partial^\mu\phi\right)$$

$$z = \int DX(\tau,\sigma)e^{-S_{\mathrm{P}}}$$

$$\langle q_{\mathrm{F}}, t_{\mathrm{F}} \mid q_{\mathrm{I}}, t_{\mathrm{I}}\rangle = \int_{q_{\mathrm{I}}}^{q_{\mathrm{F}}} Dq(t)e^{-\frac{1}{\hbar}S(q,\dot{q})}$$

$$\frac{\phi}{4\pi}\int d^2\xi\sqrt{-g}R(g) = \phi\chi$$

$$\chi = 2 - 2h - b - c$$

$$z \sim \frac{1}{g_s^2}[\chi = 2] + \frac{1}{g_s}[\chi = 1] + [\chi = 0] + \sum_{n>0} g_s^n[\chi = n]$$

$$\Gamma = -\log z = -\frac{V_D}{2(4\pi)^{\frac{D}{2}}}\int_\epsilon^\infty \frac{dt}{t^{\frac{D}{2}+1}}\mathrm{Str}\left(e^{-tM^2}\right)$$

$$M^2 = \frac{2}{\alpha'}(N + \bar{N} - 2)$$

$$\delta(N - \bar{N}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} ds e^{2\pi i(N-\bar{N})s}$$

$$\Gamma = -\frac{V_{26}}{2(4\pi)^{13}}\int_{-\frac{1}{2}}^{\frac{1}{2}} ds\int_\epsilon^\infty \frac{dt}{t^{14}}\mathrm{Tr}\left[e^{-\frac{2t}{\alpha'}(N+\bar{N}-2)}e^{2\pi i(N-\bar{N})s}\right]$$

$$\tau = \tau_1 + i\tau_2 \equiv s + \frac{i}{\pi\alpha'}t, q = e^{2\pi i\tau}$$

$$\Gamma = -\frac{V_{26}}{2(4\pi^2\alpha')^{13}}\int_{\mathcal{D}}\frac{d^2\tau}{\tau_2^{14}}\mathrm{Tr}\left(q^{N-1}\bar{q}^{\bar{N}-1}\right)$$

$$\mathrm{Tr}(q^{N-1}) = \frac{1}{q}\mathrm{Tr}\left(q^{\sum_{n>0}\alpha_{-n}^i\alpha_{ni}}\right) = \frac{1}{q}\prod_{i=1}^{24}\prod_{n=1}^{\infty}\mathrm{Tr}\left(q^{\alpha_{-n}^i\alpha_n^i}\right)$$

$$= \frac{1}{q}\prod_{i=1}^{24}\prod_{n=1}^{\infty}\sum_{k_n}\langle k_n|q^{\alpha_{-n}^i\alpha_n^i}|k_n\rangle = \frac{1}{q}\prod_{i=1}^{24}\prod_{n=1}^{\infty}\sum_{k_n}q^{k_n n}$$

$$= \frac{1}{q}\prod_{i=1}^{24}\prod_{n=1}^{\infty}\frac{1}{1-q^n} = \left[\frac{1}{q^{\frac{1}{24}}\prod_{n=1}^{\infty}(1-q^n)}\right]^{24} = \frac{1}{\eta(\tau)^{24}}$$

$$\Gamma = -\frac{V_{26}}{2(4\pi^2\alpha')^{13}}\int_{\mathcal{D}}\frac{d^2\tau}{\tau_2^{14}}\frac{1}{|\eta(\tau)|^{48}}$$

$$\tau \to \frac{a\tau+b}{c\tau+d}$$

$$T = \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}, S = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}, (S)^2 = (ST)^3 = -\mathbb{1}$$

$$\mathcal{F} = \mathbb{H}^+ \setminus \mathrm{PSL}(2,\mathbb{Z}) = \left\{-\frac{1}{2} < \tau_1 \le \frac{1}{2}, |\tau| \ge 1, \tau_2 > 0\right\}$$

$$\mathcal{T} = \frac{1}{(4\pi^2\alpha')^{13}}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^{14}}\frac{1}{|\eta(\tau)|^{48}}$$

$$T\colon \eta(\tau+1) = e^{\frac{i\pi}{12}}\eta(\tau), S\colon \eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\eta(\tau)$$

$$\frac{1}{\eta(q)^{24}\bar{\eta}(\bar{q})^{24}} = \sum_{n,m}c(n,m)q^n\bar{q}^m$$

$$\left.\frac{1}{\eta(q)^{24}\bar{\eta}(\bar{q})^{24}}\right|_{\text{physical}} = \sum_n c(n,n)(q\bar{q})^n = \frac{1}{q\bar{q}} + 576 + 104976q\bar{q} + \cdots$$

$$\mathcal{A} = \frac{1}{2(4\pi^2\alpha')^{24}}\int_0^{\infty}\frac{d\tau_2}{\tau_2^{14}}\frac{1}{\eta\left(\frac{i\tau_2}{2}\right)^{24}}$$

$$\frac{1}{\eta(q)^{24}} = \frac{1}{q} + 24 + 324q \ldots$$

$$\tilde{\mathcal{A}} = \frac{2^{-13}}{2(4\pi^2\alpha')^{24}}\int_0^{\infty}d\ell\frac{1}{\eta(i\ell)^{24}}$$

$$\rho^0 = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}, \qquad \rho^1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \{\rho^a, \rho^b\} = 2\eta^{ab}$$

$$\psi^{\mu}=\begin{pmatrix}\psi_{-}^{\mu}\\ \psi_{+}^{\mu}\end{pmatrix},\psi_{\pm}^{\mu*}=\psi_{\pm}^{\mu},\bar{\psi}^{\mu}=i\psi^{\mu\dagger}\rho^{0}$$

$$S_{\mathrm{P}}=-\frac{1}{4\pi\alpha'}\int d^{2}\xi\left(\partial_{a}X^{\mu}\partial^{a}X_{\mu}+\bar{\psi}^{\mu}\rho^{a}\partial_{a}\psi_{\mu}\right)$$

$$\delta X^{\mu}=\bar{\epsilon}\psi^{\mu},\delta\psi^{\mu}=\rho^{a}\partial_{a}X^{\mu}\epsilon$$

$$T_{\pm\pm}=\partial_{\pm}X^{\mu}\partial_{\pm}X_{\mu}+\frac{i}{2}\psi_{\pm}^{\mu}\partial_{\pm}\psi_{\pm\mu},\Xi_{\pm}=\partial_{\pm}X^{\mu}\psi_{\pm\mu}$$

$$\int d\tau\left(\psi_{-}^{\mu}\delta\psi_{-\mu}-\psi_{+}^{\mu}\delta\psi_{+\mu}\right)\Big|_{\sigma=0}^{\sigma=\begin{Bmatrix}2\pi\\ \pi\end{Bmatrix}}=0$$

Bosonic		$X^{\mu}(\tau,\sigma+2\pi)=X^{\mu}(\tau,\sigma)$	NN: DD:	$\partial_{\sigma}X^{\mu}(\tau,0)=\partial_{\sigma}X^{\mu}(\tau,\pi)=0$ $\delta X^{\mu}(\tau,0)=\delta X^{\mu}(\tau,\pi)=0$
Fermionic	R-R: NS-NS:	$\psi_{\pm}^{\mu}(\tau,\sigma+2\pi)=+\psi_{\pm}^{\mu}(\tau,\sigma)$ $\psi_{\pm}^{\mu}(\tau,\sigma+2\pi)=-\psi_{\pm}^{\mu}(\tau,\sigma)$		$\psi_{+}^{\mu}(\tau,0)=\psi_{-}^{\mu}(\tau,0)$
	R-NS:	$\psi_{+}^{\mu}(\tau,\sigma+2\pi)=+\psi_{+}^{\mu}(\tau,\sigma)$ $\psi_{-}^{\mu}(\tau,\sigma+2\pi)=-\psi_{+}^{\mu}(\tau,\sigma)$	R:	$\psi_{+}^{\mu}(\tau,\pi)=+\psi_{-}^{\mu}(\tau,\pi)$
	NS-R:	$\psi_{+}^{\mu}(\tau,\sigma+2\pi)=-\psi_{+}^{\mu}(\tau,\sigma)$ $\psi_{-}^{\mu}(\tau,\sigma+2\pi)=+\psi_{-}^{\mu}(\tau,\sigma)$	NS:	$\psi_{+}^{\mu}(\tau,\pi)=-\psi_{-}^{\mu}(\tau,\pi)$

$$\mathrm{R}:\psi_{+}^{\mu}=\sqrt{2\pi\alpha'}\sum_{n\in\mathbb{Z}}d_{n}^{\mu}e^{-2i\pi n(\tau+\sigma)},\psi_{-}^{\mu}=\sqrt{2\pi\alpha'}\sum_{n\in\mathbb{Z}}\tilde{d}_{n}^{\mu}e^{-2i\pi n(\tau-\sigma)}$$
$$\mathrm{NS}:\psi_{+}^{\mu}=\sqrt{2\pi\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}b_{r}^{\mu}e^{-2i\pi r(\tau+\sigma)},\psi_{-}^{\mu}=\sqrt{2\pi\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\tilde{b}_{r}^{\mu}e^{-2i\pi r(\tau-\sigma)}$$

$$\mathrm{R}:\psi_{\pm}^{\mu}=\sqrt{\pi\alpha'}\sum_{n\in\mathbb{Z}}d_{n}^{\mu}e^{-i\pi n(\tau\pm\sigma)},\mathrm{NS}:\psi_{\pm}^{\mu}=\sqrt{\pi\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\tilde{b}_{r}^{\mu}e^{-i\pi r(\tau\pm\sigma)}$$

$$X^{+}=x^{+}+2\pi\alpha'p^{+}\tau,\psi_{\pm}^{+}=0$$

$$\partial_{\pm}X^{-}=\frac{1}{2\pi\alpha'p^{+}}\left(\partial_{\pm}X^{i}\partial_{\pm}X_{i}+\frac{i}{2}\psi_{\pm}^{i}\partial_{\pm}\psi_{\pm i}\right),\psi_{\pm}^{-}=\frac{1}{\pi\alpha'p^{+}}\psi_{\pm}^{i}\partial_{\pm}X_{i}$$

$$\mathrm{R}:N_{\psi}=\sum_{n>0}nd_{-n}^{i}d_{ni},\qquad\mathrm{NS}:N_{\psi}=\sum_{r\geq\frac{1}{2}}rb_{-r}^{i}b_{ri}$$

$$\mathrm{R}:M^{2}=\frac{1}{\alpha'}\left(N_{X}+N_{\psi}\right),\mathrm{NS}:M^{2}=\frac{1}{\alpha'}\left(N_{X}+N_{\psi}-\frac{D-2}{16}\right).$$

$$\mathrm{R}:M_{L}^{2}=\frac{4}{\alpha'}\left(N_{X}+N_{\psi}\right),\mathrm{NS}:M_{L}^{2}=\frac{4}{\alpha'}\left(N_{X}+N_{\psi}-\frac{D-2}{16}\right),$$

$$M^{2}|0\rangle_{\mathrm{NS}}=-\frac{(D-2)}{16\alpha'}|0\rangle_{\mathrm{NS}}$$

$$M^2\left(b^i_{-\frac{1}{2}}|0\rangle_{\text{NS}}\right)=\frac{10-D}{2\alpha'}\left(b^i_{-\frac{1}{2}}|0\rangle_{\text{NS}}\right).$$

$$M^2|0,\overline{0}\rangle_{\text{NSNS}}=-\frac{(D-2)}{4\alpha'}|0,\overline{0}\rangle_{\text{NSNS}}$$

$$b^i_{-\frac{1}{2}}\bar{b}^j_{-\frac{1}{2}}|0,\overline{0}\rangle_{\text{NSNS}}, M^2\left(b^i_{-\frac{1}{2}}\bar{b}^j_{-\frac{1}{2}}|0,\overline{0}\rangle_{\text{NSNS}}\right)=\frac{10-D}{2\alpha'}\left(b^i_{-\frac{1}{2}}\bar{b}^j_{-\frac{1}{2}}|0,\overline{0}\rangle_{\text{NSNS}}\right).$$

$$\begin{aligned}|0,\overline{0}\rangle_{\text{RR}}&\simeq(\mathbf{8}_s\oplus\mathbf{8}_c)\otimes(\mathbf{8}_s\oplus\mathbf{8}_c)=\\&=(\mathbf{8}_s\otimes\mathbf{8}_s)\oplus(\mathbf{8}_c\otimes\mathbf{8}_c)\oplus[(\mathbf{8}_s\otimes\mathbf{8}_c)\oplus(\mathbf{8}_c\otimes\mathbf{8}_s)]=\\&=(\mathbf{1}\oplus\mathbf{28}\oplus\mathbf{35}+)\oplus(\mathbf{1}\oplus\mathbf{28}\oplus\mathbf{35}_)\oplus[(\mathbf{8}_{\text{v}}\oplus\mathbf{56})\oplus(\mathbf{8}'_{\text{v}}\oplus\mathbf{56}')].\end{aligned}$$

$$\begin{aligned}|0,\overline{0}\rangle_{\text{NSR}}&\simeq\mathbf{8}_{\text{v}}\otimes(\mathbf{8}_s\oplus\mathbf{8}_c)=(\mathbf{8}_{\text{v}}\otimes\mathbf{8}_s)\oplus(\mathbf{8}_{\text{v}}\otimes\mathbf{8}_c)\\&=(\mathbf{56}_s\oplus\mathbf{8}_s)\oplus(\mathbf{56}_c\oplus\mathbf{8}_c)\end{aligned}$$

$$\text{NS: }(-1)^{G_{\text{NS}}}=(-1)^{\sum_{r\geq\frac{1}{2}}b^i_{-r}b_{ri}},\ \text{R: }(-1)^{G_{\text{R}}}=\gamma_9(-1)^{\sum_{n\geq1}a^i_{-n}d_{ni}},$$

$$(-1)^{G_{\text{NS}}}|0\rangle_{\text{NS}}=|0\rangle_{\text{NS}}$$

$$\mathcal{P}^{\text{NS}}_{\text{GSO}}=\frac{1-(-1)^{G_{\text{NS}}}}{2}$$

$$\mathcal{P}^{\text{R}}_{\text{GSO}}=\frac{1\pm(-1)^{G_{\text{R}}}}{2}$$

$$b^i_{-\frac{1}{2}}\bar{b}^j_{-\frac{1}{2}}|0,\overline{0}\rangle_{\text{NSNS}}=b^i_{-\frac{1}{2}}|0\rangle_{\text{NS}}\otimes\bar{b}^j_{-\frac{1}{2}}|\overline{0}\rangle_{\text{NS}}$$

type IIA: $|+\rangle_{\text{R}}\otimes|-\rangle_{\text{R}}$, type IIB: $|+\rangle_{\text{R}}\otimes|+\rangle_{\text{R}}$,

		Type IIA	Type IIB
Bosons	graviton	$G_{\mu\nu}$	$G_{\mu\nu}$
	dilaton	ϕ	ϕ
	Kalb–Ramond	$B_{\mu\nu}$	$B_{\mu\nu}$
	R-R forms	$C^{(1)}$, $C^{(3)}$	$C^{(0)}$, $C^{(2)}$, $C^{(4)}_+$
		$C^{(5)}$, $C^{(7)}$	$C^{(6)}$, $C^{(8)}$
Fermions	gravitini	ψ_{1R}, ψ_{2L}	ψ_{1L}, ψ_{2L}
	dilatini	λ_{1R}, λ_{2L}	λ_{1L}, λ_{2L}

$$\text{type IIA:}\quad b^i_{-\frac{1}{2}}|0\rangle_{\text{NS}} \otimes |-\rangle_{\text{R}} \in \mathbf{56}_s \oplus \mathbf{8}_s,$$
$$\text{type IIB:}\quad b^i_{-\frac{1}{2}}|0\rangle_{\text{NS}} \otimes |+\rangle_{\text{R}} \in \mathbf{56}_s \oplus \mathbf{8}_s,$$

$$\text{type IIA:}\quad |+\rangle_{\text{R}} \otimes \bar{b}^i_{-\frac{1}{2}}|0\rangle_{\text{NS}} \in \mathbf{56}_c \oplus \mathbf{8}_c$$
$$\text{type IIB:}\quad |+\rangle_{\text{R}} \otimes \bar{b}^i_{-\frac{1}{2}}|0\rangle_{\text{NS}} \in \mathbf{56}_s \oplus \mathbf{8}_s$$

$$\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(z\mid\tau)=\sum_n q^{\frac{1}{2}(n+\alpha)^2}e^{2i\pi(n+\alpha)(z+\beta)}$$
$$=e^{2\pi i\alpha(z+\beta)}q^{\frac{\alpha^2}{2}}\prod_{n=1}^{\infty}(1-q^n)\left(1+q^{n+\alpha-\frac{1}{2}}e^{2i\pi(z+\beta)}\right)\left(1+q^{n+\alpha-\frac{1}{2}}e^{-2i\pi(z+\beta)}\right)$$

$$\vartheta_1(z\mid\tau)=\vartheta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(z\mid\tau)=2\sin(\pi z)q^{\frac{1}{8}}\prod_{n=1}^{\infty}(1-q^n)\left(1-q^ne^{2\pi iz}\right)\left(1-q^ne^{-2\pi iz}\right)$$
$$\vartheta_2(z\mid\tau)=\vartheta\begin{bmatrix}\frac{1}{2}\\ 0\end{bmatrix}(z\mid\tau)=2\cos(\pi z)q^{\frac{1}{8}}\prod_{n=1}^{\infty}(1-q^n)\left(1+q^ne^{2\pi iz}\right)\left(1+q^ne^{-2\pi iz}\right)$$
$$\vartheta_3(z\mid\tau)=\vartheta\begin{bmatrix}0\\ 0\end{bmatrix}(z\mid\tau)=\prod_{n=1}^{\infty}(1-q^n)\left(1+q^{n-\frac{1}{2}}e^{2\pi iz}\right)\left(1+q^{n-\frac{1}{2}}e^{-2\pi iz}\right)$$
$$\vartheta_4(z\mid\tau)=\vartheta\begin{bmatrix}0\\ \frac{1}{2}\end{bmatrix}(z\mid\tau)=\prod_{n=1}^{\infty}(1-q^n)\left(1-q^{n-\frac{1}{2}}e^{2\pi iz}\right)\left(1-q^{n-\frac{1}{2}}e^{-2\pi iz}\right)$$

$$T:\quad \vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(z\mid\tau+1)=\mathrm{e}^{-\mathrm{i}\pi\alpha(\alpha-1)}\vartheta\begin{bmatrix}\alpha\\ \beta+\alpha-\frac{1}{2}\end{bmatrix}(z\mid\tau)$$
$$S:\quad \vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\left(\frac{z}{\tau}\middle|-\frac{1}{\tau}\right)=(-\mathrm{i}\tau)^{\frac{1}{2}}\mathrm{e}^{2\mathrm{i}\pi\alpha\beta+\mathrm{i}\pi z^2/\tau}\vartheta\begin{bmatrix}\beta\\ \alpha\end{bmatrix}(z\mid\tau)$$

$$T:\quad \vartheta_{1,2}\to e^{\frac{i\pi}{4}}\vartheta_{1,2},\qquad \vartheta_{3,4}\to\vartheta_{4,3}$$
$$S:\quad \vartheta_{1,3}\to\sqrt{-i\tau}\vartheta_{1,3},\quad \vartheta_{2,4}\to\sqrt{-i\tau}\vartheta_{4,2}$$

$$O_{2n}(q)=\frac{\vartheta_3^n+\vartheta_4^n}{2\eta(q)^n},V_{2n}(q)=\frac{\vartheta_3^n-\vartheta_4^n}{2\eta(q)^n}$$
$$S_{2n}(q)=\frac{\vartheta_2^n+i^{-n}\vartheta_1^n}{2\eta(q)^n},C_{2n}(q)=\frac{\vartheta_2^n-i^{-n}\vartheta_1^n}{2\eta(q)^n}$$

$$T=e^{-\frac{in\pi}{12}}\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&e^{\frac{in\pi}{4}}&0\\0&0&0&e^{\frac{in\pi}{4}}\end{pmatrix},S=\frac{1}{2}\begin{pmatrix}1&1&1&1\\1&1&-1&-1\\1&-1&i^{-n}&-i^{-n}\\1&-1&-i^{-n}&i^{-n}\end{pmatrix}$$

$$\begin{aligned} O_{2n} &\sim q^{-\frac{n}{24}}, & V_{2n} \\ S_{2n} &\sim 2^{n-1} q^{\frac{n}{12}}, & C_{2n} q^{\frac{1}{2}-\frac{n}{24}}, \\ & & \sim 2^{n-1} q^{\frac{n}{12}} \end{aligned}$$

$$\frac{O_8}{\eta^8}, \frac{V_8}{\eta^8}, \frac{S_8}{\eta^8}, \frac{C_8}{\eta^8},$$

$$\begin{aligned} O_{2n} &= O_{p-1}O_{2n+1-p} + V_{p-1}V_{2n+1-p}, V_{2n} = O_{p-1}V_{2n+1-p} + V_{p-1}O_{2n+1-p}, \\ S_{2n} &= S_{p-1}S_{2n+1-p} + C_{p-1}C_{2n+1-p}, C_{2n} = S_{p-1}C_{2n+1-p} + C_{p-1}S_{2n+1-p}, \end{aligned}$$

$$\begin{aligned} \mathrm{Str}_{\mathrm{NS}}\left[\frac{1+(-1)^{G_{\mathrm{NS}}}}{2} q^{N_X+N_\psi-\frac{1}{2}}\right] &= O_8(q), \mathrm{Str}_{\mathrm{R}}\left[\frac{1+(-1)^{G_{\mathrm{R}}}}{2} q^{N_X+N_\psi}\right] = S_8(q), \\ \mathrm{Str}_{\mathrm{NS}}\left[\frac{1-(-1)^{G_{\mathrm{NS}}}}{2} q^{N_X+N_\psi-\frac{1}{2}}\right] &= V_8(q), \mathrm{Str}_{\mathrm{R}}\left[\frac{1-(-1)^{G_{\mathrm{R}}}}{2} q^{N_X+N_\psi}\right] = C_8(q). \end{aligned}$$

$$\begin{aligned} \mathcal{T}_{\mathrm{IIA}} &= \frac{1}{(4\pi\alpha')^5} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6} \frac{1}{\eta(q)^8 \bar{\eta}(\bar{q})^8} (V_8 - S_8)(\bar{V}_8 - \bar{C}_8), \\ \mathcal{T}_{\mathrm{IIB}} &= \frac{1}{(4\pi\alpha')^5} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6} \frac{1}{\eta(q)^8 \bar{\eta}(\bar{q})^8} (V_8 - S_8)(\bar{V}_8 - \bar{S}_8). \end{aligned}$$

$$\vartheta_3^4 - \vartheta_4^4 - \vartheta_2^4 = 0$$

$$\mathcal{T} = \bar{\chi}^i \mathcal{N}_{ij} \chi^j,$$

$$\begin{aligned} \mathcal{T}_{0\,\mathrm{A}} &= |V_8|^2 + |O_8|^2 + S_8\bar{C}_8 + \bar{S}_8 C_8, \\ \mathcal{T}_{0\,\mathrm{B}} &= |V_8|^2 + |O_8|^2 + |S_8|^2 + |C_8|^2. \end{aligned}$$

$$\mathcal{A}_{pp} = \frac{1}{(8\pi\alpha')^{\frac{p+1}{2}}} \int_0^\infty \frac{d\tau_2}{\tau_2^{\frac{p+3}{2}}} \frac{V_8 - S_8}{\eta^8} \left(\frac{i\tau_2}{2}\right)$$

$$\tilde{\mathcal{A}}_{pp} = \frac{2^{-\frac{p+1}{2}}}{(8\pi\alpha')^{\frac{p+1}{2}}} \int_0^\infty \frac{d\ell}{\ell^{\frac{9-p}{2}}} \frac{V_8 - S_8}{\eta^8} (i\ell),$$

$$\mathcal{A}_{pp} = O_{p-1}V_{9-p} + V_{p-1}O_{9-p} - S_{p-1}S_{9-p} - C_{p-1}C_{9-p}.$$

$$\mathcal{A}_{pp} = \frac{N\bar{N}}{(8\pi\alpha')^{\frac{p+1}{2}}} \int_0^\infty \frac{d\tau_2}{\tau_2^{\frac{p+3}{2}}} \frac{V_8 - S_8}{\eta^8} \left(\frac{i\tau_2}{2}\right)$$

$$\Omega_{\mathrm{closed}} : \sigma \to -\sigma$$

$$\alpha_m^\mu \leftrightarrow \bar{\alpha}_m^\mu, b_r^\mu \leftrightarrow \bar{b}_r^\mu, d_n^\mu \leftrightarrow \bar{d}_m^\mu$$

$$\Omega_{\mathrm{open}} : \sigma \to \pi - \sigma$$

$$\alpha_m^\mu \to (-1)^m \alpha_m^\mu, b_r^\mu \to (-1)^{r-\frac{1}{2}} b_r^\mu, d_n^\mu \to (-1)^n d_n^\mu$$

$$\mathrm{Str}\left[\frac{1+\Omega}{2}\mathcal{P}_{\mathrm{GSO}}q^{N_X+N_\psi-\Delta}\bar{q}^{\bar{N}_X+\bar{N}_\psi-\Delta}\right]$$

$$\mathcal{K}=\frac{1}{(4\pi\alpha')^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{V_8-S_8}{\eta^8}(2i\tau_2)$$

$$\frac{1}{2}\mathcal{T}+\mathcal{K}\sim\frac{1}{2}\left|\frac{V_8-S_8}{\eta^8}\right|^2+\frac{V_8-S_8}{2\eta^8}=$$
$$=\frac{|V_8|^2+|S_8|^2+V_8-S_8}{2\eta^8}-\frac{V_8\bar{S}_8+\bar{V}_8S_8}{2\eta^8}\sim(35+1+28)-(56+8),$$

$$\tilde{\mathcal{K}}=\frac{2^5}{(4\pi\alpha')^5}\int_0^\infty d\ell\frac{V_8-S_8}{\eta^8}(i\ell)$$

$$\mathrm{Str}\left[\frac{1+\Omega}{2}\mathcal{P}_{\mathrm{GSO}}q^{N_X+N_\psi+\Delta}\right]$$

$$\mathcal{A}=\frac{N^2}{(8\pi\alpha')^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{V_8-S_8}{\eta^8}\left(\frac{i\tau_2}{2}\right)$$
$$\tilde{\mathcal{A}}=\frac{2^{-5}N^2}{(8\pi\alpha')^5}\int_0^\infty d\ell\frac{V_8-S_8}{\eta^8}(i\ell)$$

$$\hat{\chi}_i(q)=e^{-i\pi\left(h_i-\frac{c}{24}\right)}\chi_i(q)$$

$$\mathcal{M}=\frac{\epsilon N}{(8\pi\alpha')^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{\hat{V}_8-\hat{S}_8}{\hat{\eta}^8}\left(\frac{1}{2}+\frac{i\tau_2}{2}\right)$$

$$\mathcal{A}+\mathcal{M}\sim\frac{N(N+\epsilon)}{2}(8-8)$$

$$P=TST^2S\colon\ \frac{1}{2}+\frac{i\tau_2}{2}\overset{P}{\to}\frac{1}{2}+\frac{i}{2\tau_2}\equiv\frac{1}{2}+i\ell$$

$$\tilde{\mathcal{M}}=\frac{2\epsilon N}{(8\pi\alpha')^5}\int_0^\infty d\ell\frac{\hat{V}_8-\hat{S}_8}{\hat{\eta}^8}\left(\frac{1}{2}+i\ell\right)$$

$$\tilde{\mathcal{K}}+\tilde{\mathcal{A}}+\tilde{\mathcal{M}}\sim2^5+2^{-5}N^2+2\epsilon N=2^{-5}(N+2^5\epsilon)^2,$$

$$g=(-1)^F\delta$$

$$\delta\colon X^9\to X^9+\pi R,$$

$$X^9=x^9+2\alpha'\frac{m}{R}\tau+2nR\sigma+[\ \mathfrak{O}_{\text{oscillators}}\ ]$$

$$X^9_{\mathrm{L,R}}=x^9_{\mathrm{L,R}}+\alpha' p^9_{\mathrm{L,R}}(\tau\mp\sigma)+[\ \mathfrak{O}_{\text{oscillators}}\ ],p^9_{\mathrm{L,R}}=\frac{m}{R}\pm\frac{nR}{\alpha'}$$

$$M^2 = \frac{m^2}{R^2} + \frac{n^2R^2}{\alpha'^2} + \frac{2}{\alpha'}\left(N_X + N_\psi + \bar{N}_X + \bar{N}_\psi - 1\right)$$

$$P_m \equiv q^{\frac{\alpha' m^2}{R^2}}, W_n \equiv q^{\frac{n^2R^2}{4\alpha'}}, \Lambda_{m,n} = q^{\frac{\alpha'}{4}\left(\frac{m}{R}+\frac{nR}{\alpha'}\right)^2}\bar{q}^{\frac{\alpha'}{4}\left(\frac{m}{R}-\frac{nR}{\alpha'}\right)^2}$$

$$\mathcal{T} = \frac{1}{2|\eta|^{16}}\sum_{m,n}\left(|V_8 - S_8|^2\Lambda_{m,n} + |V_8 + S_8|^2(-1)^m\Lambda_{m,n}\right.$$
$$\left.+|O_8 - C_8|^2\Lambda_{m,n+1/2} + |O_8 + C_8|^2(-1)^m\Lambda_{m,n+1/2}\right)$$

$$g = (-1)^F\delta^2, \qquad \delta^2: X^9 \to X^9 + 2\pi R$$

$$\mathcal{T} = \frac{1}{|\eta|^{16}}\sum_{m,n}\left[(|V_8|^2 + |S_8|^2)\Lambda_{m,2n} - (V_8\bar{S}_8 + \bar{V}_8S_8)\Lambda_{m+1/2,2n}\right.$$
$$\left.+(|O_8|^2 + |C_8|^2)\Lambda_{m,2n+1} - (O_8\bar{C}_8 + \bar{O}_8C_8)\Lambda_{m+1/2,2n+1}\right]$$

$$\mathcal{K}_1 = \frac{1}{2}\frac{V_8 - S_8}{\eta^8}\sum_m P_m, \tilde{\mathcal{K}}_1 = \frac{2^5v}{2}\frac{V_8 - S_8}{\eta^8}\sum_n W_{2n},$$

$$\Omega_2 = \Omega\Pi, \Pi: X^9 \to 2\pi - X^9$$

$$\mathcal{K}_2 = \frac{1}{2\eta^8}\sum_n [(V_8 - S_8)W_{2n} + (O_8 - C_8)W_{2n+1}].$$

$$\tilde{\mathcal{K}}_2 = \frac{2^5}{2v}\frac{1}{\eta^8}\sum_m (V_8P_{2m} - S_8P_{2m+1})$$

$$\Omega_3 = \Omega\Pi(-1)^{G_L}$$

$$\mathcal{K}_3 = \frac{1}{2\eta^8}\sum_n [(V_8 - S_8)W_{2n} - (O_8 - C_8)W_{2n+1}]$$
$$\tilde{\mathcal{K}}_3 = \frac{2^5}{2v}\frac{1}{\eta^8}\sum_m (S_8P_{2m} - V_8P_{2m+1})$$

$$\Omega_{\mathrm{DP}} = \Omega\delta$$

$$\mathcal{T}_{\mathrm{DP}} = \left|\frac{V_8 - S_8}{\eta^8}\right|^2\sum_{m,n}\Lambda_{m,n}$$

$$\mathcal{K}_{\mathrm{DP}} = \frac{1}{2}\frac{V_8 - S_8}{\eta^8}\sum_m (-1)^m P_m, \tilde{\mathcal{K}}_{\mathrm{DP}} = \frac{2^5v}{2}\frac{V_8 - S_8}{\eta^8}\sum_n W_{2n+1}.$$

$$\Omega_{\mathrm{DP}}\left|G_{\mu\nu}, \phi, C^{(2)}, C^{(6)}, C^{(10)}\right\rangle^{(m)} = (-1)^m\left|G_{\mu\nu}, \phi, C^{(2)}, C^{(6)}, C^{(10)}\right\rangle^{(m)}$$
$$\Omega_{\mathrm{DP}}\left|B_{\mu\nu}, \phi, C^{(0)}, C_+^{(4)}, C^{(8)}\right\rangle^{(m)} = -(-1)^m\left|B_{\mu\nu}, \phi, C^{(0)}, C_+^{(4)}, C^{(8)}\right\rangle^{(m)}$$

$$\Omega_{\mathrm{DP}}\left|\psi_{1\mu}, \lambda_1\right\rangle^{(m)} = (-1)^m\left|\psi_{2\mu}, \lambda_2\right\rangle^{(m)}, \Omega_{\mathrm{DP}}\left|\psi_{2\mu}, \lambda_2\right\rangle^{(m)} = (-1)^m\left|\psi_{1\mu}, \lambda_1\right\rangle^{(m)}$$

$$\psi_{\mu}^{\prime(m)} \equiv \frac{\psi_{1\mu}^{(m)} + (-1)^m \psi_{2\mu}^{(m)}}{\sqrt{2}}, \lambda^{\prime(m)} \equiv \frac{\lambda_1^{(m)} + (-1)^m \lambda_2^{(m)}}{\sqrt{2}}$$

$$\left\{G_{\mu\nu}, \phi, C^{(2)}, C^{(6)}, C^{(10)}, \psi_1^{\prime\mu}, \lambda_1^{\prime}\right\}^{(2m)}, \quad M^2 = \frac{(2m)^2}{R^2},$$
$$\left\{B_{\mu\nu}, C^{(0)}, C_+^{(4)}, C^{(8)}, \psi_2^{\prime\mu}, \lambda_2^{\prime}\right\}^{(2m+1)}, \quad M^2 = \frac{(2m+1)^2}{R^2}.$$

$$\tilde{\mathcal{A}}_{55} = \frac{2^{-3} v N^2}{2} \frac{V_4 O_4 + O_4 V_4 - S_4 S_4 - C_4 C_4}{\eta^8} \sum_n W_n,$$
$$\tilde{\mathcal{M}}_5 = -N \left(\frac{4\epsilon v}{\hat{\eta}^2 \hat{\vartheta}_2^2}\right) \left(\hat{V}_4 \hat{O}_4 - \hat{O}_4 \hat{V}_4 + \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4\right) \sum_n W_{2n+1},$$

$$\mathcal{A}_{55} = \frac{N^2}{2} \frac{V_4 O_4 + O_4 V_4 - S_4 S_4 - C_4 C_4}{\eta^8} \sum_m P_m$$
$$\mathcal{M}_5 = \frac{N}{2} \left(\frac{4\epsilon}{\hat{\eta}^2 \hat{\vartheta}_2^2}\right) \left(\hat{V}_4 \hat{O}_4 - \hat{O}_4 \hat{V}_4 + \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4\right) \sum_m (-1)^m P_m$$

$$\mathcal{A}_{55} + \mathcal{M}_5 \sim q^{-\frac{1}{3}} \left[\frac{N(N+\epsilon)}{2} (V_4 O_4 - C_4 C_4) + \frac{N(N-\epsilon)}{2} (O_4 V_4 - S_4 S_4)\right]$$

$$\tilde{\mathcal{A}}_{55} = \frac{2^{-3}}{2v} \frac{V_8 - S_8}{\eta^8} \sum_m \left[e^{\frac{i\pi m}{2}} N + e^{-\frac{i\pi m}{2}} \bar{N}\right]^2 P_m$$
$$= \frac{2^{-3}}{2v} \frac{V_8 - S_8}{\eta^8} \sum_m \left[(N+\bar{N})^2 P_{2m} - (N-\bar{N})^2 P_{2m+1}\right]$$

$$\mathcal{A}_{55} = \frac{V_8 - S_8}{\eta^8} \sum_n \left(N\bar{N} W_n + \frac{N^2 + \bar{N}^2}{2} W_{n+1/2}\right)$$

$$\tilde{\mathcal{A}}_{77} = \frac{2^{-4}}{2v} \frac{1}{\eta^8} \sum_m \left[(N+\bar{N})^2 (V_8 P_{2m} - S_8 P_{2m+1}) - (N-\bar{N})^2 (V_8 P_{2m+1} - S_8 P_{2m})\right]$$
$$= \frac{2^{-4}}{v} \frac{1}{\eta^8} \sum_m \left[N\bar{N}(V_8 - S_8) + \frac{N^2 + \bar{N}^2}{2} (-1)^m (V_8 + S_8)\right] P_m$$

$$\mathcal{A}_{77} = \frac{1}{\eta^8} \left[N\bar{N}(V_8 - S_8) \sum_n W_n + \frac{N^2 + \bar{N}^2}{2} (O_8 - C_8) \sum_n W_{n+1/2}\right].$$

$$\tilde{\mathcal{A}}_{77}^{\mathrm{IIB}} = \frac{2^{-4}}{v} \frac{1}{\eta^8} \sum_m \left[(M\bar{M} + N\bar{N})(V_8 - S_8) + (M\bar{N} + N\bar{M})(V_8 + S_8)\right] P_m$$

$$M = N_1 + e^{2\pi i a m} N_2, N = (-1)^m \left(\bar{N}_1 + e^{2\pi i a m} \bar{N}_2\right)$$

$$\tilde{\mathcal{A}}_{77} = \frac{2^{-4}}{2v}\frac{1}{\eta^8}\sum_m \Big[\left|N_1+\bar{N}_1+e^{2\pi i a m}(N_2+\bar{N}_2)\right|^2\left(V_8P_m^{\text{even}}-S_8P_m^{\text{odd}}\right)$$
$$+\left|N_1-\bar{N}_1+e^{2\pi i a m}(N_2-\bar{N}_2)\right|^2\left(V_8P_m^{\text{odd}}-S_8P_m^{\text{even}}\right)\Big]$$
$$=\frac{2^{-3}}{v}\frac{1}{\eta^8}\sum_m P_m\Big[\left(N_1\bar{N}_1+N_2\bar{N}_2+\cos(2\pi a m)(N_1\bar{N}_2+N_2\bar{N}_1)\right)(V_8-S_8)$$
$$+\left(\frac{N_1^2+\bar{N}_1^2+N_2^2+\bar{N}_2^2}{2}+\cos(2\pi a m)(N_1N_2+\bar{N}_1\bar{N}_2)\right)(-1)^m(V_8+S_8)\Big]$$

$$\langle \mathrm{D}p, X^9=x \mid \mathrm{D}p, X^9=y\rangle = \sum_m e^{i\frac{m}{R}(y-x)}\left\langle \mathrm{D}p, p^9=\frac{m}{R}\right| \mathrm{D}p, p^9=\frac{m}{R}\Big\rangle$$

$$\mathcal{A}_{77}=\sum_n\left[N_1\bar{N}_1W_n+N_2\bar{N}_2W_n+\frac{N_1\bar{N}_2+N_2\bar{N}_1}{2}(W_{n-a}+W_{n+a})\right]\frac{V_8-S_8}{\eta^8}$$
$$+\sum_n\left[\frac{N_1^2+\bar{N}_1^2+N_2^2+\bar{N}_2^2}{2}W_{n+\frac{1}{2}}+\frac{N_1N_2+\bar{N}_1\bar{N}_2}{2}\left(W_{n+\frac{1}{2}+a}+W_{n+\frac{1}{2}-a}\right)\right]\frac{O_8-C_8}{\eta^8},$$

$$\tilde{\mathcal{A}}_{88}=\frac{2^{-\frac{9}{2}}v}{2}\frac{N^2}{\eta^8}(V_7O_1+O_7V_1)\sum_n W_n,$$
$$\tilde{\mathcal{M}}_8=-\frac{2vN\epsilon}{\hat{\eta}^{\frac{13}{2}}\hat{\vartheta}_2^{\frac{1}{2}}}(\hat{V}_7\hat{O}_1-\hat{O}_7\hat{V}_1)\sum_n W_{2n+1},$$

$$\mathcal{A}_{88}=\frac{N^2}{2}\frac{1}{\eta^8}[(O_7+V_7)(O_1+V_1)-2S_7'S_1']\sum_m P_m$$
$$\mathcal{M}_8=-\epsilon\frac{N}{2}\frac{1}{\hat{\eta}^7}\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^{\frac{1}{2}}[\hat{O}_7\hat{O}_1+\hat{V}_7\hat{V}_1-\hat{O}_7\hat{V}_1+\hat{V}_7\hat{O}_1]\sum_m(-1)^mP_m$$

$$\tilde{\mathcal{A}}_{33}=\frac{2^{-2}}{2v}\frac{1}{\eta^8}\sum_m[(N+\bar{N})^2(V_8P_{2m}-S_8P_{2m+1})-(N-\bar{N})^2(V_8P_{2m+1}-S_8P_{2m})]$$
$$=\frac{2^{-4}}{v}\frac{1}{\eta^8}\sum_m\left[N\bar{N}(V_8-S_8)+\frac{N^2+\bar{N}^2}{2}(-1)^m(V_8+S_8)\right]P_m$$
$$\mathcal{A}_{33}=N\bar{N}\left(\frac{V_8-S_8}{\eta^8}\right)\sum_n W_n+\frac{N^2+\bar{N}^2}{2}\left(\frac{O_8-C_8}{\eta^8}\right)\sum_n W_{n+1/2}$$

$$\mathcal{A}_{44}=\frac{N^2}{2}\frac{1}{\eta^8}[(O_3+V_3)(O_5+V_5)-2S_5'S_5']\sum_m P_m$$
$$\mathcal{M}_4=\epsilon\frac{N}{2}\frac{1}{\hat{\eta}^3}\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^{\frac{5}{2}}[\hat{O}_3\hat{O}_5+\hat{V}_3\hat{V}_5-\hat{O}_3\hat{V}_5+\hat{V}_3\hat{O}_5]\sum_m(-1)^mP_m$$

$$Q_o=V_4O_4-C_4C_4,\quad Q_v=O_4V_4-S_4S_4$$
$$Q_s=O_4C_4-S_4O_4,\quad Q_c=V_4S_4-C_4V_4$$

	tensor/ gravity	hyper/gravitino
$\mathcal{Q}_o$	$g_{\mu\nu}$, $C_{\mu\nu+}$, ψ^i_μ C_+ , $B_{\mu\nu-}$, λ^i_+	$B_{\mu\nu+}$, $C_{\mu\nu+I}$, $\psi^{\hat{i}}_\mu$ $e^{-\phi}$, C_I , $\lambda^{\hat{i}}$
$\mathcal{Q}_v$	v_4 , $C_{\mu\nu-}$, λ^i $B_{\hat{I}}$, $C_{\mu\nu-\hat{I}}$, $\lambda^i_{\hat{I}}$	C_- , B_I , $\lambda^{\hat{i}}_-$ $C_{\hat{I}}$, $g_{\hat{I}I}$, $\lambda^{\hat{i}}_{\hat{I}}$
$\mathcal{Q}_s$	B_A , $B_{\mu\nu-A}$, λ^i_A	C_A , g_{AI} , $\lambda^{\hat{i}}_A$

	tensor/ gravity	hyper
$\mathcal{Q}_o$	$g_{\mu\nu}$, $C_{\mu\nu+}$, ψ^i_μ	$e^{-\phi}$, C_I, $\lambda^{\hat{i}}$
$\mathcal{Q}_v$	v_4, $C_{\mu\nu-}$, λ^i	$C_{\hat{I}}$, $g_{\hat{I}I}$, $\lambda^{\hat{i}}_{\hat{I}}$
$\mathcal{Q}_s$		C_A , g_{AI} , $\lambda^{\hat{i}}_A$

$$\mathcal{T} = \frac{1}{2}\left[\left|\frac{Q_o+Q_v}{\eta^8}\right|^2 \sum_{m,n} q^{\frac{\alpha'}{4}p_{\mathrm{L}}^{\mathsf{T}} g^{-1} p_{\mathrm{L}}} q^{\frac{\alpha'}{4}p_{\mathrm{R}}^{\mathsf{T}} g^{-1} p_{\mathrm{R}}} + \frac{16}{|\eta|^4}\left(\left|\frac{Q_o-Q_v}{\vartheta_2^2}\right|^2 + \left|\frac{Q_s-Q_c}{\vartheta_3^2}\right|^2 + \left|\frac{Q_s+Q_c}{\vartheta_4^2}\right|^2\right)\right],$$

$$\mathcal{K} = \frac{1}{4}\left[\frac{Q_o+Q_v}{\eta^8}\left(\sum_m q^{\frac{\alpha'}{2}m^{\mathsf{T}} g^{-1} m} + \sum_n q^{\frac{1}{2\alpha'}n^{\mathsf{T}} g n}\right) + 2\times 16\frac{Q_s+Q_c}{\eta^2\vartheta_4^2}\right],$$

$$\tilde{\mathcal{K}} = \frac{2^5}{4}\left[\frac{Q_o+Q_v}{\eta^8}\left(v_4\sum_n q^{\frac{1}{\alpha'}n^{\mathsf{T}} g n} + \frac{1}{v_4}\sum_m q^{\alpha' m^{\mathsf{T}} g^{-1} m}\right) + 8\frac{Q_o-Q_v}{\eta^2\vartheta_2^2}\right]$$
$$\overset{q=0}{\sim} 8\left[\frac{Q_o}{\eta^8}\Big|_{q=0}\left(\sqrt{v_4}+\frac{1}{\sqrt{v_4}}\right)^2 + \frac{Q_v}{\eta^8}\Big|_{q=0}\left(\sqrt{v_4}-\frac{1}{\sqrt{v_4}}\right)^2\right].$$

$$\omega^0 = \frac{i}{2}(dz_1\wedge d\bar{z}_1 + dz_2\wedge d\bar{z}_2),\quad \omega^+ = dz_1\wedge dz_2, \omega^- = d\bar{z}_1\wedge d\bar{z}_2, \omega^{\hat{+}} = dz_1\wedge d\bar{z}_2, \omega^{\hat{-}} = d\bar{z}_1\wedge dz_2$$

$$C^{(0)} = C_+ + C_-$$
$$C^{(2)} = C_I\omega^I + C_{\hat{I}}\omega^{\hat{I}} + C_A\omega^A + \frac{1}{2}C_{\mu\nu}dx^\mu\wedge dx^\nu$$
$$C^{(4)} = \frac{1}{4}(C_+ - C_-)\omega^+\wedge\bar{\omega}^- + \frac{1}{2}\left(C_{\mu\nu+I}\omega^I + C_{\mu\nu-\hat{I}}\omega^{\hat{I}} + C_{\mu\nu-A}\omega^A\right)dx^\mu\wedge dx^\nu$$

$$H_2(\mathrm{K3},\mathbb{Z}) = \left\langle \frac{1}{2}e_{ij} + \frac{1}{2}\sum_{\substack{A\in(\mathbb{Z}_2)^4\\ a_i=a_j=0}} e_A\Big|_{i<j}, \frac{1}{2}\sum_{\substack{A\in(\mathbb{Z}_2)^4\\ a_i=0}} e_A\Big|_{i=1}^4, \frac{1}{2}\sum_{A\in(\mathbb{Z}_2)^4} e_A, e_A|_{\sum_i a_i\le 2}\right\rangle,$$

$$(e_{ij}, e_{kl}) = 2\varepsilon_{ijkl}, (e_A, e_B) = -2\delta_{AB} = -2\prod_{i=1}^4 \delta_{a_i b_i}$$

$$\delta X_i = X_i + \pi R_i.$$

$$\mathcal{K} = \frac{1}{4}\left[\frac{Q_o+Q_v}{\eta^8}\left(\sum_m (-1)^m q^{\frac{\alpha'}{2}m^{\mathsf{T}} g^{-1} m} + \sum_n q^{\frac{1}{\alpha'}n^{\mathsf{T}} g n}\right) + 2\times(8-8)\frac{Q_s+Q_c}{\eta^2\vartheta_4^2}\right].$$

$$\tilde{\mathcal{K}} = \frac{2^5}{4}\left[\frac{Q_o+Q_v}{\eta^8}\left(v_4\sum_{n\ \text{odd}} q^{\frac{1}{\alpha'}n^{\mathrm{T}}gn} + \frac{1}{v_4}\sum_m q^{\alpha' m^{\mathrm{T}}g^{-1}m}\right) + (1-1)\frac{Q_o-Q_v}{\eta^2\vartheta_2^2}\right].$$

	tensor/ gravity	hyper
$\mathcal{Q}_o$	$g_{\mu\nu},\ C_{\mu\nu+}\ ,\ \psi^i_\mu$	$e^{-\phi},\ C_I,\ \lambda^{\hat{i}}$
$\mathcal{Q}_v$	$v_4, C_{\mu\nu-},\ \lambda^i$ $B_{\hat{\pm}},\ C_{\mu\nu-\hat{\pm}}\ ,\ \lambda^i_{\hat{\pm}}$	$C_{\hat{0}},\ g_{\hat{0}I}\ ,\ \lambda^{\hat{i}}_{\hat{0}}$
$\mathcal{Q}_s$	$B_{\hat{a}},\ C_{\mu\nu-\hat{a}}\ ,\ \lambda^i_{\hat{a}}$	$C_a\ ,\ g_{aI}\ ,\ \lambda^{\hat{i}}_a$

$$\Omega' = \Omega Z_4, (\Omega')^2 = Z_2$$

$$Z_4(z_1,z_2) = (iz_1,-iz_2) \leftrightarrow Z_4(X_1,X_2,X_3,X_4) = (-X_2,X_1,X_4,-X_3)$$

$$\mathcal{K} = \frac{1}{4\eta^2}\left[8\frac{Q_o-Q_v}{\vartheta_2^2} + 2\times(4+6-6)\frac{Q_s-Q_c}{\vartheta_3^2}\right].$$

$$\tilde{\mathcal{K}} = \frac{16}{\eta^2}\left[\left(\frac{1}{\vartheta_4^2}-\frac{1}{\vartheta_3^2}\right)Q_s + \left(\frac{1}{\vartheta_4^2}+\frac{1}{\vartheta_3^2}\right)Q_c\right].$$

$$\frac{1}{36(4\pi)^4}\left(\frac{n_H+29n_T-273}{5}\mathrm{Tr}R^4 + \frac{n_H-7n_T+51}{4}\mathrm{Tr}R^2\mathrm{Tr}R^2\right) \overset{\tilde{\tilde{\mathfrak{H}}}}{\underset{\substack{n_H=12\\ n_T=9}}{\triangleq}} 0.$$

$$\Omega' = \Omega(-1)^{F_L}\delta,$$

$$(\Omega')^2 = g,$$

$$\mathcal{K}_4 = \frac{1}{2}\frac{V_8+S_8}{\eta^8}\sum_m (-1)^m P_m$$

$$\tilde{\mathcal{K}}_4 = \frac{2^5 v}{2}\frac{O_8-C_8}{\eta^8}\sum_n W_{2n+1}$$

$$\tau \to \frac{a\tau+b}{c\tau+d}$$

$$\begin{pmatrix} B^{(2)} \\ C^{(2)} \end{pmatrix} = \begin{pmatrix} dB^{(2)} - cC^{(2)} \\ aC^{(2)} - bB^{(2)} \end{pmatrix}$$

$$\{1,(-1)^F\}\to \mathrm{Mp}(2,\mathbb{Z})\to \mathrm{GL}(2,\mathbb{Z})$$

$$\Omega(-1)^{F_L}=S^2$$

$$S(-1)^{F_L}=\Omega S, S^4=(-1)^F$$

$$\Omega'\left|C^{(0)},C_+^{(4)},C^{(8)}\right\rangle^{(m)}=(-1)^m\left|C^{(0)},C_+^{(4)},C^{(8)}\right\rangle^{(m)}$$
$$\Omega'\left|B^{(2)},C^{(2)},C^{(6)},C^{(10)}\right\rangle^{(m)}=-(-1)^m\left|B^{(2)},C^{(2)},C^{(6)},C^{(10)}\right\rangle^{(m)}$$
$$\Omega'\left|\psi_1^\mu,\lambda_2\right\rangle^{\left(m+\frac{1}{2}\right)}=-i(-1)^m\left|\psi_2^\mu,\lambda_1\right\rangle^{\left(m+\frac{1}{2}\right)}$$
$$\Omega'\left|\psi_2^\mu,\lambda_1\right\rangle^{\left(m+\frac{1}{2}\right)}=i(-1)^m\left|\psi_1^\mu,\lambda_2\right\rangle^{\left(m+\frac{1}{2}\right)}$$

$$|\psi'^\mu,\lambda'\rangle^{\left(m+\frac{1}{2}\right)}\equiv\frac{\left|\psi_1^\mu,\lambda_2\right\rangle^{\left(m+\frac{1}{2}\right)}-i(-1)^m\left|\psi_2^\mu,\lambda_1\right\rangle^{\left(m+\frac{1}{2}\right)}}{\sqrt{2}}$$

$$\left(G_{\mu\nu},\phi,C^{(0)},C_+^{(4)}\right)^{(2m)},M^2=\frac{(2m)^2}{R^2}$$
$$\left(B^{(2)},C^{(2)}\right)^{(2m+1)},M^2=\frac{(2m+1)^2}{R^2}$$
$$\left(\psi'_\mu,\lambda'\right)^{\left(m+\frac{1}{2}\right)},M^2=\frac{\left(m+\frac{1}{2}\right)^2}{R^2}$$

$$\left(g_{\mu\nu},C^{(0)},C_+^{(4)}\right)(y+\pi R)=\left(g_{\mu\nu},C^{(0)},C_+^{(4)}\right)(y)$$
$$\left(B^{(2)},C^{(2)}\right)(y+\pi R)=-\left(B^{(2)},C^{(2)}\right)(y)$$
$$\psi_\pm(y+\pi R)=\pm i\psi_\pm(y)\ \rightarrow\ \psi_\pm(y+2\pi R)=-\psi_\pm(y)$$

$$\left(g_{\mu\nu},\tau,C_+^{(4)}\right)(y,x)=\sum_m e^{2mi\frac{y}{R}}\left(g_{\mu\nu},\tau,C_+^{(4)}\right)^{(m)}(x)$$
$$\left(B^{(2)},C^{(2)}\right)(y,x)=\sum_m e^{(2m+1)i\frac{y}{R}}\left(B^{(2)},C^{(2)}\right)^{(m)}(x)$$
$$\psi_+(y,x)=\sum_m e^{\left(2m+\frac{1}{2}\right)i\frac{y}{R}}\psi_+^{(m)}(x)$$
$$\psi_-(y,x)=\sum_m e^{\left(2m-\frac{1}{2}\right)i\frac{y}{R}}\psi_-^{(m)}(x)$$

$$T^2: z\overset{\mathbb{Z}_4}{\rightarrow}-z,\qquad S^1: X^9\overset{\mathbb{Z}_4}{\rightarrow}X^9+\frac{\pi}{2}R_9$$

$$\left(G_{\mu\nu},G_{IJ},C_{\mu IJ},C_{\mu\nu\rho}\right)^{(2m)},M^2=\frac{(2m)^2}{R^2}$$
$$\left(C_{\mu\nu I},g_{\mu I}\right)^{(2m+1)},M^2=\frac{(2m+1)^2}{R^2}$$
$$\left(\Psi_{\mu L},\Psi_{IR}\right)^{(2m+1/2)},M^2=\frac{(2m+1/2)^2}{R^2}$$
$$\left(\Psi_{\mu R},\Psi_{IL}\right)^{(2m+3/2)},M^2=\frac{(2m+3/2)^2}{R^2}$$

$$V = V_{\mathrm{SS}} + V_{\mathrm{K}} = -\left(\frac{1}{2}\mathcal{T} + \mathcal{K}_4\right)$$

$$V_{\mathrm{SS}} = -\frac{1}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^{11/2}}\frac{1}{|\eta|^{16}}\sum_{m,n}\Big[\big[|V_8 - S_8|^2\Lambda_{m,n} + |V_8 + S_8|^2(-1)^m\Lambda_{m,n}$$
$$+|O_8 - C_8|^2\Lambda_{m,n+1/2} + |O_8 + C_8|^2(-1)^m\Lambda_{m,n+1/2}\Big]$$
$$V_{\mathrm{K}} = -\frac{1}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_0^\infty \frac{d\tau_2}{\tau_2^{\frac{11}{2}}}\frac{V_8 + S_8}{\eta^8}(2i\tau_2)\sum_m (-1)^m P_m$$

$$V_{\mathrm{SS}} \sim -\frac{1}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^{11/2}}\left|\frac{V_8 + S_8}{\eta^8}\right|^2\sum_{m\in\mathbb{Z}}(-1)^m\Lambda_{m,0}$$
$$\sim -\frac{16^2}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_0^\infty\frac{d\tau_2}{\tau_2^{11/2}}\sum_{m\in\mathbb{Z}}\left[e^{-\pi\tau_2\frac{\alpha' m^2}{R^2}} - e^{-\pi\tau_2\frac{\alpha'\left(m+\frac{1}{2}\right)^2}{R^2}}\right]$$
$$\sim -2\frac{16^2 R}{(2\pi)^9\alpha'^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\sum_{k=0}^\infty e^{-\frac{4\pi R^2}{\alpha'\tau_2}\left(k+\frac{1}{2}\right)^2} =$$
$$= -\frac{24}{\pi^{14}R^9}\sum_{k=0}^\infty\frac{1}{(2k+1)^{10}} = -\frac{31}{7560(2\pi)^4R^9}$$

$$V_{\mathrm{K}} = -\frac{1}{2(4\pi^2\alpha')^{\frac{9}{2}}}\int_0^\infty \frac{d\tau_2}{\tau_2^{\frac{11}{2}}}\frac{V_8 + S_8}{\eta^8}(2i\tau_2)\sum_m (-1)^m e^{-\pi\tau_2\alpha'\frac{m^2}{R^2}}$$
$$= -\frac{R}{(2\pi)^9\alpha'^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{V_8 + S_8}{\eta^8}(2i\tau_2)\sum_{k=0}^\infty e^{-\frac{\pi R^2}{\alpha'\tau_2}\left(k+\frac{1}{2}\right)^2}$$
$$\sim -\frac{24}{\pi^{14}R^9}\sum_{k=0}^\infty\frac{2^5}{(2k+1)^{10}} = -\frac{31}{3780\pi^4R^9}$$

$$V \sim -\frac{1}{(4\pi)^{9/2}}\int_0^\infty\frac{dt}{t^{\frac{11}{2}}}\sum_{m=-\infty}^\infty\left(2\times 36e^{-\left(\frac{2m}{R}\right)^2 t} + 2\times 28e^{-\left(\frac{2m+1}{R}\right)^2 t} - 64e^{-\left(\frac{m+\frac{1}{2}}{R}\right)^2 t}\right)$$

$$\sum_{n\in\mathbb{Z}} e^{-\pi nAn + 2i\pi bn} = \frac{1}{\sqrt{A}}\sum_{m\in Z} e^{-\pi(m-b)A^{-1}(m-b)}$$

$$\frac{V_8 + S_8}{\eta^8} = \frac{\vartheta_2(0)^4}{\eta^{12}} = 16\prod_{n\geq 1}\frac{(1+q^n)^8}{(1-q^n)^8} = 16\sum_{n=0}^\infty c(n)q^n$$

$$V_{\mathrm{SS}} = \frac{2R}{(2\pi)^9\alpha'^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\int_{-\frac{1}{2}}^{\frac{1}{2}}d\tau_1\left|\frac{V_8 + S_8}{\eta^8}\right|^2\sum_{k=0}^\infty e^{-\frac{4\pi R^2}{\alpha'\tau_2}\left(k+\frac{1}{2}\right)^2}$$
$$V_{\mathrm{K}} = -\frac{R}{(2\pi)^9\alpha'^5}\int_0^\infty\frac{d\tau_2}{\tau_2^6}\frac{V_8 + S_8}{\eta^8}(2i\tau_2)\sum_{k=0}^\infty e^{-\frac{\pi R^2}{\alpha'\tau_2}\left(k+\frac{1}{2}\right)^2}$$

$$V = -\frac{403}{189(4\pi)^4}\frac{1}{R^9}$$

$$-\frac{1}{R^4}\sum_{n\geq 1}\sum_{k\geq 0}\frac{\left(\frac{n}{\alpha'}\right)^{\frac{5}{2}}}{\left(k+\frac{1}{2}\right)^5}\left[c(n)^2K_5\left(8\pi R\sqrt{\frac{n}{\alpha'}}\left(k+\frac{1}{2}\right)\right)+2c(n)K_5\left(4\pi R\sqrt{\frac{n}{\alpha'}}\left(k+\frac{1}{2}\right)\right)\right],$$

$$\hat{V} = -\frac{403}{189(4\pi)^4}\frac{1}{R^9}$$

$$-\frac{1}{2\ell_{10}^5 R_{(m,n)}^4}\sum_{-\mathbb{Z}^2\setminus(0,0)}\sum_{k\geq 0}\frac{\left(e^{\frac{\phi}{2}}|n+\tau m|\right)^{\frac{5}{2}}}{\left(k+\frac{1}{2}\right)^5}\left[+2c(\gcd(m,n))K_5\left(4\pi\frac{R}{\ell_{10}}\sqrt{e^{\frac{\phi}{2}}|n+\tau m|}\left(k+\frac{1}{2}\right)\right)\right.$$

$$\left.+c(\gcd(m,n))^2K_5\left(8\pi\frac{R}{\ell_{10}}\sqrt{e^{\frac{\phi}{2}}|n+\tau m|}\left(k+\frac{1}{2}\right)\right)\right]$$

$$R\times\min_{m,n}\sqrt{e^{\frac{\phi}{2}}|n+\tau m|}\leq\sqrt{2}\ell_{10}$$

$$M_{\mathrm{F1-D1}}^2 = -2\frac{|n+\tau m|}{\alpha'}+\frac{R^2|n+\tau m|^2}{\alpha'^2}.$$

$$M_{\mathrm{D1}}^2 = -2\frac{\sqrt{\frac{1}{g_s^2}+C_0^2}}{\alpha'}+\frac{R^2}{\alpha'^2}\left(\frac{1}{g_s^2}+C_0^2\right)\approx\frac{R^2}{\alpha'^2 g_s^2}-\frac{2}{\alpha' g_s}+\mathcal{O}(g_s^0)$$

$$4e^{-2\phi}\partial_\tau\partial_{\bar{\tau}}\hat{V} = \frac{1}{16}\left(\partial_{\log R}^2+22\partial_{\log R}+117\right)\hat{V}$$

$$\int ds f(s)\frac{\ell_{10}^{4s}}{R^{9+4s}}e^{-\frac{\phi}{2}}K_{s-\frac{1}{2}}\left(2\pi|N|e^{-\Phi}\right)e^{2\pi iNC_0}$$

$$S_{\mathrm{D}(-1)} = 2\pi|N|e^{-\Phi}+2\pi iNC_0$$

$$V\sim\sum_{h=1}^{\infty}d_h\frac{1}{R^9}\frac{\ell_{10}^{8(h-1)}}{R^{8(h-1)}}$$

$$\sigma(X^8,X^9,X^{10}) = (-X^8,X^9+\pi R,-X^{10})$$

$$\mathcal{M}_3 = \mathbb{R}^3/\pi_1(\mathcal{M}_3)$$

$$T_{a_1,a_2,a_3}(X^8,X^9,X^{10}) = \left(X^8+2\pi a_1\frac{\alpha'}{R_{\mathrm{B}}},X^9+2\pi a_3 R,X^{10}+2\pi a_2 e^{\frac{2}{3}\Phi_{\mathrm{A}}}\sqrt{\alpha'}\right)$$

$$H_1(\mathcal{M}_3,\mathbb{Z}) = \pi_1(\mathcal{M}_3)/[\pi_1(\mathcal{M}_3),\pi_1(\mathcal{M}_3)] = \mathbb{Z}\oplus\mathbb{Z}_2\oplus\mathbb{Z}_2,$$

$$H_0(\mathcal{M}_3,\mathbb{Z})=\mathbb{Z},\quad H_1(\mathcal{M}_3,\mathbb{Z})=\mathbb{Z}\oplus\mathbb{Z}_2\oplus\mathbb{Z}_2,$$
$$H_2(\mathcal{M}_3,\mathbb{Z})=\mathbb{Z},\quad H_3(\mathcal{M}_3,\mathbb{Z})=\mathbb{Z}.$$

$$S^1(X^9)\in H_1(\mathcal{M}_3,\mathbb{Z}),T^2(X^8,X^{10})\in H_2(\mathcal{M}_3,\mathbb{Z})$$

$$S^1(X^8)\in H_1(\mathcal{M}_3,\mathbb{Z}),S^1(X^{10})\in H_1(\mathcal{M}_3,\mathbb{Z})$$

$$\begin{aligned}\mathcal{A}=&\frac{1}{8\eta^8}\sum_m\big[(\alpha^2+\beta^2+\gamma^2+\delta^2)\big(V_8P_m-S_8P_{m+1/2}\big)\\&+(\alpha^2+\beta^2-\gamma^2-\delta^2)\big(V_8P_{m+1/2}-S_8P_m\big)\\&+(\alpha^2-\beta^2+\gamma^2-\delta^2)\big(O_8P_m-C_8P_{m+1/2}\big)\\&+(\alpha^2-\beta^2-\gamma^2+\delta^2)\big(O_8P_{m+1/2}-C_8P_m\big)\big]\end{aligned}$$
$$\tilde{\mathcal{A}}=\frac{2^{-5}v}{2}\frac{1}{\eta^8}\sum_n\left[(\alpha^2V_8-\beta^2S_8)W_{2n}+(\gamma^2O_8-\delta^2C_8)W_{2n+1}\right]$$

$$\mathcal{M}_9=-\frac{1}{2\eta^8}\sum_m\left[\epsilon_1\gamma(-1)^mP_m\hat{O}_8-i\epsilon_2\delta(-1)^mP_{m+1/2}\hat{C}_8\right]$$
$$\tilde{\mathcal{M}}_9=-\frac{v}{\eta^8}\sum_n\left[\epsilon_1\gamma\hat{O}_8+(-1)^n\epsilon_2\delta\hat{C}_8\right]W_{2n+1}$$

$$\alpha=N_1+\overline{N_1}+N_2+\overline{N_2},\quad \beta=i\big(N_1-\overline{N_1}+N_2-\overline{N_2}\big),$$
$$\gamma=N_1+\overline{N_1}-N_2-\overline{N_2},\quad \delta=i\big(N_1-\overline{N_1}-N_2+\overline{N_2}\big).$$

$$A_{99}=\frac{1}{\eta^8}\left[N\bar{N}\sum_m\big(V_8P_m-S_8P_{m+1/2}\big)+\frac{N^2+\bar{N}^2}{2}\sum_m\big(O_8P_m-C_8P_{m+1/2}\big)\right]$$
$$M_9=-\frac{1}{2\hat{\eta}^8}\sum_m(-1)^m\left[\epsilon_1(N+\bar{N})O_8P_m-\epsilon_2(N-\bar{N})C_8P_{m+1/2}\right]$$

$$\tilde{\mathcal{M}}_7=-\frac{2v}{\hat{\eta}^5\hat{\vartheta}_2}\sum_n\left[\epsilon_1\gamma\big(\hat{O}_6\hat{O}_2+\hat{V}_6\hat{V}_2\big)(-1)^n-\epsilon_2\delta\big(\hat{S}_6\hat{C}_2-\hat{C}_6\hat{S}_2\big)\right]W_{2n+1}$$

$$\alpha=N_1+\overline{N_1}+N_2+\overline{N_2},\quad \gamma=i\big(N_1-\overline{N_1}+N_2-\overline{N_2}\big),$$
$$\beta=N_1+\overline{N_1}-N_2-\overline{N_2},\quad \delta=i\big(N_1-\overline{N_1}-N_2+\overline{N_2}\big).$$

$$\mathcal{A}_{77}=\frac{1}{\eta^8}\left[N\bar{N}\sum_m\big(V_8P_m-S_8P_{m+1/2}\big)+\frac{N^2+\bar{N}^2}{2}\sum_m\big(V_8P_{m+1/2}-S_8P_m\big)\right]$$
$$\mathcal{M}_7=\left(\frac{2}{\hat{\eta}^5\hat{\vartheta}_2}\right)\frac{N-\bar{N}}{2}\sum_m(-1)^m\left[\epsilon_1\big(\hat{V}_6\hat{O}_2-\hat{O}_6\hat{V}_2\big)P_{m+1/2}+\epsilon_2\big(\hat{S}_6\hat{S}_2-\hat{C}_6\hat{C}_2\big)P_m\right]$$

$$\mathrm{D7}_{01234589}\ \xrightarrow[\text{T- duality}]{}\ \mathrm{KK6}_{0123459}.$$

$$A_{77} = \frac{V_8 - S_8}{\eta^8} \sum_n \left(N\bar{N} W_n + \frac{N^2 + \bar{N}^2}{2} W_{n+1/2} \right)$$

$$\tilde{\mathcal{A}}_{77} = \frac{2^{-4}}{2v} \left(\frac{V_8 - S_8}{\eta^8} \right) \sum_m \left[e^{\frac{i\pi m}{2}} N + e^{-\frac{i\pi m}{2}} \bar{N} \right]^2 P_m$$

$$= \frac{2^{-4}}{2v} \left(\frac{V_8 - S_8}{\eta^8} \right) \sum_m \left[(N + \bar{N})^2 P_{2m} - (N - \bar{N})^2 P_{2m+1} \right]$$

$$= \frac{2^{-4}}{v} \left(\frac{V_8 - S_8}{\eta^8} \right) \sum_m \left[N\bar{N} + (-1)^m \frac{N^2 + \bar{N}^2}{2} \right] P_m$$

$$\mathrm{D7}_{01234568} \xrightarrow[\text{T- duality}]{} \mathrm{KK6}_{0123456}$$

$$\tilde{\mathcal{A}}_{55} = \frac{2^{-3}}{2v} \frac{1}{\eta^8} \sum_m \left[\left(e^{\frac{i\pi m}{2}} N + e^{-\frac{i\pi m}{2}} \bar{N} \right)^2 V_8 + \left(e^{\frac{i\pi m}{2}} N - e^{-\frac{i\pi m}{2}} \bar{N} \right)^2 S_8 \right] P_m$$

$$= \frac{2^{-3}}{2v} \frac{1}{\eta^8} \sum_m \left[(N + \bar{N})^2 (V_8 P_{2m} - S_8 P_{2m+1}) - (N - \bar{N})^2 (V_8 P_{2m+1} - S_8 P_{2m}) \right]$$

$$= \frac{2^{-4}}{v} \frac{1}{\eta^8} \sum_m \left[N\bar{N} (V_8 - S_8) + \frac{N^2 + \bar{N}^2}{2} (-1)^m (V_8 + S_8) \right] P_m$$

$$\mathcal{A}_{55} = \frac{1}{\eta^8} \left[N\bar{N} (V_8 - S_8) \sum_n W_n + \frac{N^2 + \bar{N}^2}{2} (O_8 - C_8) \sum_n W_{n+1/2} \right]$$

KK number	Fields	Representation
$2m$	vectors $A_\mu^{(m)}$ +6 scalars	$N\overline{N}$
	4 fermions	$\frac{N(N-1)}{2} + \overline{\frac{N(N+1)}{2}}$
$2m+1$	vectors $A_\mu^{(m)}$ +6 scalars	$N\overline{N}$
	4 fermions	$\frac{N(N+1)}{2} + \overline{\frac{N(N-1)}{2}}$
$2m+\frac{1}{2}$	vectors $A_\mu^{(m)}$	$\frac{N(N-1)}{2} + \overline{\frac{N(N+1)}{2}}$
	4 fermions	$N\overline{N}$
	6 complex scalars	$\frac{N(N+1)}{2} + \overline{\frac{N(N-1)}{2}}$
$2m+\frac{3}{2}$	vectors $A_\mu^{(m)}$	$\frac{N(N+1)}{2} + \overline{\frac{N(N-1)}{2}}$
	4 fermions	$N\overline{N}$
	6 complex scalars	$\frac{N(N-1)}{2} + \overline{\frac{N(N+1)}{2}}$

$$\mathrm{D5}_{012348} \xrightarrow[\text{T- duality}]{} \mathrm{M5}_{0123410}.$$

$$\mathrm{D5}_{012345} \xrightarrow[\text{T- duality}]{} \mathrm{KK6}_{0123458},$$

$$\mathcal{A}_{33}=\frac{1}{\eta^8}\left[N\bar{N}\sum_m\left(V_8P_m-S_8P_{m+1/2}\right)+\frac{N^2+\bar{N}^2}{2}\sum_m\left(V_8P_{m+1/2}-S_8P_m\right)\right]$$

$$\mathcal{M}_3=\left(\frac{8\hat{\eta}}{\hat{\vartheta}_2^3}\right)\frac{N-\bar{N}}{2}\sum_m(-1)^m\left[\epsilon_1\left(\hat{O}_2\hat{V}_6-\hat{V}_2\hat{O}_6\right)P_{m+1/2}+\epsilon_2\left(\hat{S}_2\hat{S}_6-\hat{C}_2\hat{C}_6\right)P_m\right]$$

$$\varsigma=\begin{pmatrix}0 & i]\,]_{N\times N}\\ ]\,]_{N\times N} & 0\end{pmatrix}$$

$$Z_4A_\mu(X)=-\varsigma A_\mu(X+\pi R)^\top\varsigma^\dagger,\quad Z_4\phi_{ij}(X)=\varsigma\phi_{ij}(X+\pi R)^\top\varsigma^\dagger$$
$$Z_4\lambda_{\alpha i}(X)=i\varsigma\lambda_{\alpha i}(X+\pi R)^\top\varsigma^\dagger,\quad Z_4\lambda^i_{\dot{\alpha}}(X)=-i\varsigma\lambda^i_{\dot{\alpha}}(X+\pi R)^\top\varsigma^\dagger$$

$$\mathrm{D3}_{0189}\ \underset{\text{T- duality}}{\rightarrow}\ \mathrm{M2}_{019}$$

$$\mathrm{D3}_{0129}\ \underset{\text{T- duality}}{\rightarrow}\ \mathrm{M5}_{0128910},$$

$$\tilde{\mathcal{A}}_{11}=\frac{2^{-1}v}{2\eta^8}\sum_n\left[(N+\bar{N})^2(V_8W_{2n}+O_8W_{2n+1})+(N-\bar{N})^2(S_8W_{2n}+C_8W_{2n+1})\right]$$

$$\tilde{\mathcal{M}}_1=-v\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^4\sum_n\left[\epsilon_1(N+\bar{N})\left(\hat{O}_0\hat{O}_8-\hat{V}_0\hat{V}_8\right)+i\epsilon_2(-1)^n(N-\bar{N})\left(\hat{S}_0\hat{C}_8-\hat{C}_0\hat{S}_8\right)\right]W_{2n+1}.$$

$$\mathcal{A}_{11}=\frac{1}{\eta^8}\left[N\bar{N}\sum_m\left(V_8P_m-S_8P_{m+1/2}\right)+\frac{N^2+\bar{N}^2}{2}\sum_m\left(O_8P_m-C_8P_{m+1/2}\right)\right]$$

$$\mathcal{M}_1=\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^4\sum_m(-1)^m\left[\epsilon_1\frac{N+\bar{N}}{2}\left(\hat{O}_0\hat{O}_8-\hat{V}_0\hat{V}_8\right)P_m-\epsilon_2\frac{N-\bar{N}}{2}\left(\hat{S}_0\hat{C}_8-\hat{C}_0\hat{S}_8\right)P_{m+1/2}\right]$$

$$O_8=O_0O_8+V_0V_8,V_8=V_0O_8+O_0V_8$$
$$S_8=S_0S_8+C_0C_8,C_8=S_0C_8+C_0S_8$$

$$\mathrm{D1}_{09}\ \underset{\text{T- duality}}{\rightarrow}\ \mathrm{M2}_{089},$$

$$Z_2(X^6,X^7,X^8,X^9)=(-X^6,-X^7,-X^8,-X^9)$$

$$\delta_5X^5=X^5+\pi R_5$$

$$\Omega'=\Omega Z_4\delta_5,$$

$$(\Omega')^2=Z_2\delta_5^2$$

$$\mathcal{T}=\frac{1}{2}\Bigg\{\left|\frac{Q_o+Q_v}{\eta^8}\right|^2\sum_{m,n}\left(q^{\frac{\alpha'}{4}}p_{\mathrm{L}}^{\mathrm{T}}g^{-1}p_{\mathrm{L}}q^{\frac{\alpha'}{4}}p_{\mathrm{R}}^{\mathrm{T}}g^{-1}p_{\mathrm{R}}\right)\sum_{m_5,n_5}\left(\Lambda^{(1,1)}_{m_5,2n_5}+\Lambda^{(1,1)}_{m_5+\frac{1}{2},2n_5}\right)$$
$$+\frac{16}{|\eta|^4}\Bigg[\left|\frac{Q_o-Q_v}{\vartheta_2^2}\right|^2\sum_{m_5,n_5}\left(\Lambda^{(1,1)}_{m_5,2n_5}-\Lambda^{(1,1)}_{m_5+\frac{1}{2},2n_5}\right)$$
$$+\left|\frac{Q_s-Q_c}{\vartheta_3^2}\right|^2\sum_{m_5,n_5}\left(\Lambda^{(1,1)}_{m_5,2n_5+1}-\Lambda^{(1,1)}_{m_5+\frac{1}{2},2n_5+1}\right)$$
$$+\left|\frac{Q_s+Q_c}{\vartheta_4^2}\right|^2\sum_{m_5,n_5}\left(\Lambda^{(1,1)}_{m_5,2n_5+1}+\Lambda^{(1,1)}_{m_5+\frac{1}{2},2n_5+1}\right)\Bigg]\Bigg\}$$

$$\mathcal{K}=\frac{2^2}{2}\frac{Q_o-Q_v}{\eta^2\vartheta_2^2}\sum_{m_5}(-1)^{m_5}P_{m_5}$$
$$\tilde{\mathcal{K}}=\frac{2^3}{2}\frac{R_5}{\sqrt{\alpha'}}\frac{Q_s+Q_c}{\eta^2\vartheta_4^2}\sum_{n_5}W_{2n_5+1}$$

$$T_{(\mu\nu)}\equiv\frac{1}{2}\left(T_{\mu\nu}+T_{\nu\mu}\right),T_{[\mu\nu]}\equiv\frac{1}{2}\left(T_{\mu\nu}-T_{\nu\mu}\right)$$

$$T_{\mu]\dots[\nu}\equiv T_{[\mu|\dots|\nu]}$$

$$[O_1,O_2]\equiv O_1O_2-O_1O_2=2O_{[1}O_{2]},\{O_1,O_2\}\equiv O_1O_2+O_1O_2=2O_{(1}O_{2)}$$

$$\Gamma^{\mu}_{\nu\rho}=\frac{1}{2}g^{\mu\sigma}\left(\partial_\rho g_{\nu\sigma}+\partial_\nu g_{\rho\sigma}-\partial_\sigma g_{\nu\rho}\right)$$
$$\omega_\mu^{ab}=2e^{\nu[a}\partial_{[\mu}e_{\nu]}^{b]}-e_c^\mu e^{\nu[a}e^{b]\sigma}\partial_\nu e_\sigma^c$$

$$R^{\mu}_{\nu\rho\sigma}=2\partial_{[\rho}\Gamma^{\mu}_{\sigma]\nu}+2\Gamma^{\mu}_{\lambda[\rho}\Gamma^{\lambda}_{\sigma]\nu},R_{\mu\nu}=R^{\rho}_{\mu\rho\nu},G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$$

$$T_{\mu\nu}=-\frac{2}{e}\frac{\delta(e\mathcal{L})}{\delta g^{\mu\nu}}=-2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}}+\frac{1}{2}g_{\mu\nu}\mathcal{L}$$

$$\varepsilon^{\mu_1\dots\mu_p\rho_1\dots\rho_{D-p}}\varepsilon_{\nu_1\dots\nu_p\rho_1\dots\rho_{D-p}}=-\frac{1}{p!\,(D-p)!}\delta^{\mu_1\dots\mu_p}_{\nu_1\dots\nu_p}$$

$$\delta^{\mu_1\dots\mu_p}_{\nu_1\dots\nu_p}\equiv\delta^{\mu_1}_{[\nu_1}\dots\delta^{\mu_p}_{\nu_p]}$$

$$\bar{\psi}_{\dot{\alpha}}=(\psi_\alpha)^\dagger=(\psi^\alpha)^*$$

$$\epsilon_{\alpha\beta}=\epsilon_{\dot{\alpha}\dot{\beta}}=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\epsilon^{\alpha\beta}=\epsilon^{\dot{\alpha}\dot{\beta}}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$$

$$\psi\chi=\psi^\alpha\chi_\alpha,\bar{\psi}\bar{\chi}=\bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}$$

$$\sigma^1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\sigma^2=\begin{pmatrix}0&-1\\i&0\end{pmatrix},\sigma^3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\sigma^m = \left(-\mathbb{1}, \sigma^i\right), \bar{\sigma}^m = \left(-\mathbb{1}, -\sigma^i\right), \bar{\sigma}^{m\dot{\alpha}\alpha} = \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}\sigma^m_{\beta\dot{\beta}}$$

$$(\sigma^{mn})^{\beta}_{\alpha} = \frac{1}{4}(\sigma^m\bar{\sigma}^n - \sigma^n\bar{\sigma}^m)^{\beta}_{\alpha}, (\bar{\sigma}^{mn})^{\dot{\alpha}}_{\dot{\beta}} = \frac{1}{4}(\bar{\sigma}^m\sigma^n - \bar{\sigma}^n\sigma^m)^{\dot{\alpha}}_{\dot{\beta}},$$

$$\mathrm{Tr}(\sigma^m\bar{\sigma}^n) = -2\eta^{mn}$$
$$\sigma^m_{\alpha\dot{\alpha}}\bar{\sigma}^{\dot{\beta}\beta}_m = -2\delta^{\beta}_{\alpha}\delta^{\dot{b}}_{\dot{\alpha}}$$
$$\sigma^m\bar{\sigma}^n + \sigma^n\bar{\sigma}^m = -2\eta^{mn}\mathbb{1}$$
$$\bar{\sigma}^m\sigma^n + \bar{\sigma}^n\sigma^m = -2\eta^{mn}\mathbb{1}$$
$$\sigma^m\bar{\sigma}^n\sigma^p = (-\eta^{mn}\eta^{rs} + \eta^{mr}\eta^{ns} - \eta^{nr}\eta^{ms} + i\epsilon^{mnrs})\sigma_s$$
$$\bar{\sigma}^m\sigma^n\bar{\sigma}^p = (-\eta^{mn}\eta^{rs} + \eta^{mr}\eta^{ns} - \eta^{nr}\eta^{ms} - i\epsilon^{mnrs})\bar{\sigma}_s$$

$$\theta^{\alpha}\theta^{\beta} = -\frac{1}{2}\theta^2\epsilon^{\alpha\beta}, \qquad \psi\chi = \psi^{\alpha}\chi_{\alpha} = -\chi_{\alpha}\psi^{\alpha} = \chi\psi$$

$$\bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \frac{1}{2}\bar{\theta}^2\epsilon^{\dot{\alpha}\dot{\beta}}, \qquad \bar{\psi}\bar{\chi} = \bar{\psi}\dot{\alpha}\bar{\chi}^{\dot{\alpha}} = -\bar{\chi}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}} = \bar{\chi}\bar{\psi}$$

$$\theta_{\alpha}\theta_{\beta} = \frac{1}{2}\theta^2\epsilon^{\alpha\beta}, \qquad (\theta\sigma^m\bar{\psi})(\theta\sigma^n\bar{\chi}) = \frac{1}{2}\theta^2(\bar{\psi}\bar{\sigma}^m\sigma^n\bar{\chi})$$

$$\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}\bar{\theta}^2\epsilon_{\dot{\alpha}\dot{\beta}}, \qquad (\psi\sigma^m\bar{\theta})(\chi\sigma^n\bar{\theta}) = \frac{1}{2}\bar{\theta}^2(\psi\sigma^m\bar{\sigma}^n\chi)$$

$$\psi\sigma^m\bar{\chi} = -\bar{\chi}\bar{\sigma}^m\psi, \qquad (\theta\sigma^m\bar{\theta})(\theta\sigma^n\bar{\theta}) = \frac{1}{2}\theta^2\bar{\theta}^2\eta^{mn}$$

$$\theta\sigma^m\bar{\sigma}^n\theta = \theta^2\eta^{mn}, \qquad (\theta\psi)(\theta\chi) = -\frac{1}{2}\theta^2\psi\chi$$

$$\psi\sigma^m\bar{\sigma}^n\chi = \chi\sigma^n\bar{\sigma}^m\psi, \qquad (\bar{\theta}\bar{\psi})(\bar{\theta}\bar{\chi}) = -\frac{1}{2}\bar{\theta}^2\bar{\psi}\bar{\chi}$$

$$\theta_1^2 = \theta_2^2 = \bar{\theta}_{\dot{1}}^2 = \bar{\theta}_{\dot{2}}^2 = 0 \leftrightarrow \theta^3 = \bar{\theta}^3 = 0.$$

$$F(\vartheta) = F_0 + F_1\vartheta.$$

$$\int d\vartheta = 0, \int d\vartheta\vartheta = 1$$

$$\int d\vartheta F(\vartheta) = F_1 = \partial_{\vartheta}F(\vartheta), \int d\vartheta\vartheta F(\vartheta) = F_0, \int d\vartheta\partial_{\vartheta}F(\vartheta) = 0$$

$$d^2\theta = -\frac{1}{4}\epsilon_{\alpha\beta}d\theta^{\alpha}d\theta^{\beta} = \frac{1}{2}d\theta^1d\theta^2$$
$$d^2\bar{\theta} = -\frac{1}{4}\epsilon^{\dot{\alpha}\dot{\beta}}d\bar{\theta}_{\dot{\alpha}}d\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}d\bar{\theta}_1d\bar{\theta}_2$$
$$d^4\theta = d^2\theta d^2\bar{\theta}$$

$$\int \theta^2d^2\theta = \int \bar{\theta}^2d^2\bar{\theta} = 1$$

$$D_{\alpha} = \partial_{\alpha} + i(\sigma^m\bar{\theta})_{\alpha}\partial_m, \qquad \bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i(\theta\sigma^m)_{\dot{\alpha}}\partial_m$$

$$D_\alpha D_\beta = -\frac{1}{2}\varepsilon_{\alpha\beta}D^2, \qquad \bar{D}_{\dot\alpha}\bar{D}_{\dot\beta} = \frac{1}{2}\varepsilon_{\dot\alpha\dot\beta}\bar{D}^2$$

$$\{D_\alpha, \bar{D}_{\dot\alpha}\} = -2i\sigma^m_{\alpha\dot\alpha}\partial_m, \qquad \{D_\alpha, D_\beta\} = \{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = 0$$

$$D^2\theta^2 = -4 + 4i\theta\sigma^m\bar{\theta}\partial_m + \theta^2\bar{\theta}^2\,\Box, \qquad [D_\alpha, \bar{D}^2] = -4i\sigma^m_{\alpha\dot\alpha}\bar{D}^{\dot\alpha}\partial_m$$

$$\bar{D}^2\bar{\theta}^2 = -4 - 4i\theta\sigma^m\bar{\theta}\partial_m + \theta^2\bar{\theta}^2\,\Box, \qquad [D^2, \bar{D}^2] = -8iD^\alpha\sigma^m_{\alpha\dot\alpha}\bar{D}^{\dot\alpha}\partial_m + 16\,\Box$$

$$\Psi = \begin{pmatrix}\psi_\alpha \\ \bar{\chi}^{\dot\alpha}\end{pmatrix}$$

$$\{\gamma^m, \gamma^n\} = 2\eta^{mn}$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \;\leftrightarrow\; (\gamma^0)^\dagger = -\gamma^0, \left(\gamma^i\right)^\dagger = \gamma^i$$

$$\bar{\Psi} = i\Psi^\dagger\gamma^0$$

$$\gamma^m = \begin{pmatrix}0 & -i\sigma^m \\ -i\bar{\sigma}^m & 0\end{pmatrix}, \qquad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix}\mathbb{1} & 0 \\ 0 & -\mathbb{1}\end{pmatrix}$$

$$P_L = \frac{1-\gamma_5}{2} = \begin{pmatrix}0 & 0 \\ 0 & \mathbb{1}\end{pmatrix}, P_R = \frac{1+\gamma_5}{2} = \begin{pmatrix}\mathbb{1} & 0 \\ 0 & 0\end{pmatrix}$$

$$\Psi_{L,R} \equiv P_{L,R}\Psi, \Psi_L = \begin{pmatrix}0 \\ \bar{\chi}^{\dot\alpha}\end{pmatrix}, \Psi_R = \begin{pmatrix}\psi_\alpha \\ 0\end{pmatrix}, \bar{\Psi}_{L,R} = \bar{\Psi}P_{L,R} = \overline{\Psi_{R,L}}$$

$$\{\mathbb{1}, \gamma^m, \gamma^{mn}, \gamma^m\gamma_5, \gamma_5\}$$

$$\gamma^{m_1\ldots m_r} = -\frac{i}{(4-r)!}\varepsilon^{m_r\ldots m_1 r_1\ldots r_{4-r}}\gamma_{r_1\ldots r_{4-r}}\gamma_5$$

$$C_4 = \begin{pmatrix}\sigma^2 & 0 \\ 0 & -\sigma^2\end{pmatrix}, C_4^{\mathrm{T}} = C_4^\dagger = C_4^{-1} = -C_4$$

$$(C_4\gamma^{m_1\ldots m_r})^{\mathrm{T}} = -t_r C_4\gamma^{m_1\ldots m_r}$$

$$M = \frac{1}{4}\mathrm{Tr}(M)\mathbb{1} + \frac{1}{4}\mathrm{Tr}(\gamma^m M)\gamma_m - \frac{1}{8}\mathrm{Tr}(\gamma^{mn}M)\gamma_{mn} - \frac{1}{4}\mathrm{Tr}(\gamma^m\gamma_5 M)\gamma_m\gamma_5 + \frac{1}{4}\mathrm{Tr}(\gamma_5 M)\gamma_5$$

$$\lambda = \begin{pmatrix}\psi_\alpha \\ \bar{\psi}^{\dot\alpha}\end{pmatrix}$$

$$\bar{\lambda} = \lambda^{\mathrm{T}}C_4.$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \gamma^\mu = e^\mu{}_m\gamma^m$$

$$\gamma^\mu = e^\mu{}_a\gamma^a = a(\tau)^{-1}\gamma^a \equiv a(\tau)^{-1}\bar{\gamma}^\mu$$

$$\vec{k}\cdot\vec{x}=\sum_{i=1}^{3}k^{i}x^{i}=k^{i}x_{i},\vec{\gamma}\cdot\vec{\psi}_{k}=\sum_{i=1}^{3}\bar{\gamma}^{i}\psi_{i,k}=\bar{\gamma}^{i}\psi_{i,k}$$

$$\vec{k}\cdot\vec{\psi}_{k}=\sum_{i=1}^{3}k^{i}\psi_{i,k}=k^{i}\psi_{i,k},\vec{k}\cdot\vec{\bar{\gamma}}=\sum_{i=1}^{3}k^{i}\bar{\gamma}^{i}=k_{i}\bar{\gamma}^{i}$$

$$(\vec{k}\cdot\vec{\gamma})(\vec{k}\cdot\vec{\gamma})=\vec{k}^{2},(\vec{k}\cdot\vec{\gamma})\bar{\gamma}^{i}(\vec{k}\cdot\vec{\gamma})=2k^{i}(\vec{k}\cdot\vec{\gamma})-\vec{k}^{2}\bar{\gamma}^{i}$$

$$\bar{\gamma}^{i}\left(P_{\gamma}\right)_{i}=-1,\qquad \bar{\gamma}^{i}(P_{k})_{i}=0$$

$$k^{i}\left(P_{\gamma}\right)_{i}=0,\qquad k^{i}(P_{k})_{i}=-1$$

$$\left\{\left(P_{\gamma}\right)_{i},\bar{\gamma}_{0}\right\}=0,\qquad \left\{\left(P_{\gamma}\right)_{i},\bar{\gamma}_{j}\right\}=-\eta_{ij}+\frac{k_{i}k_{j}}{\vec{k}^{2}},$$

$$\left[\left(P_{\gamma}\right)_{k},\bar{\gamma}_{0}\right]=0,\qquad \left[(P_{k})_{i},\bar{\gamma}_{j}\right]=\frac{1}{\vec{k}^{2}}\left[\bar{\gamma}_{i}k_{j}+\bar{\gamma}_{j}k_{i}+\eta_{ij}(\vec{k}\cdot\vec{\gamma})\right],$$

$$\left(P_{\gamma}\right)_{i}^{\dagger}=\left(P_{\gamma}\right)_{i},\qquad (P_{k})_{i}^{\dagger}=\frac{1}{2\vec{k}^{2}}\left[(\vec{k}\cdot\vec{\gamma})\bar{\gamma}_{i}-3k_{i}\right],$$

$$\left(P_{\gamma}\right)^{i}\left(P_{\gamma}\right)_{i}=\frac{1}{2},\qquad \left(P_{\gamma}\right)^{i}(P_{k})_{i}=-\frac{1}{2\vec{k}^{2}}(\vec{k}\cdot\vec{\gamma}),$$

$$(P_{k})^{i}(P_{k})_{i}=\frac{1}{4\vec{k}^{4}},\qquad (P_{k})^{i}\left(P_{\gamma}\right)_{i}=\frac{1}{2\vec{k}^{2}}(\vec{k}\cdot\vec{\gamma}),$$

$$(P_{k})^{i\dagger}(P_{k})_{i}=\frac{3}{4\vec{k}^{4}},\qquad (P_{k})^{i\dagger}\left(P_{\gamma}\right)_{i}=-\frac{1}{2\vec{k}^{2}}(\vec{k}\cdot\vec{\gamma}).$$

$$\bar{\gamma}^{0}\hat{O}_{i}^{\dagger}\bar{\gamma}^{0}=\frac{1}{\vec{k}^{2}}\left\{-k_{i}(\vec{k}\cdot\vec{\bar{\gamma}})+\frac{ia}{2}(m+H\bar{\gamma}^{0})\left[(\vec{k}\cdot\vec{\bar{\gamma}})\bar{\gamma}_{i}-3k_{i}\right]\right\}$$

$$(\vec{k}\cdot\vec{\bar{\gamma}})\hat{O}_{i}=-\left\{k_{i}+\frac{ia}{2}\left[\frac{k_{i}}{\vec{k}^{2}}(\vec{k}\cdot\vec{\bar{\gamma}})+\bar{\gamma}_{i}\right](m-H\bar{\gamma}^{0})\right\}$$

$$\begin{aligned}T^{\mu\nu}=&\frac{1}{2}\bar{\Psi}_{\rho}\gamma^{\rho\sigma\mu}(\nabla_{\sigma}\Psi^{\nu}-\nabla^{\nu}\Psi_{\sigma})-\frac{m}{2}\bar{\Psi}_{\rho}\gamma^{\rho\mu}\Psi^{\nu}\\&-\frac{1}{4}\nabla_{\rho}(\bar{\Psi}^{\rho}\gamma^{\mu}\Psi^{\nu}+\bar{\Psi}^{\rho}\gamma^{\nu}\Psi^{\mu}-\bar{\Psi}^{\mu}\gamma^{\rho}\Psi^{\nu})\\&-\frac{1}{2}\nabla^{\nu}\left(\bar{\Psi}^{\mu}\gamma^{\rho}\Psi_{\rho}\right)-\frac{1}{2}g^{\mu\nu}\nabla_{\rho}(\bar{\Psi}_{\sigma}\gamma^{\sigma}\Psi^{\rho}).\end{aligned}$$

$$
\begin{aligned}
T^{[\mu\nu]} =& \frac{1}{2}\bar{\Psi}_\rho \gamma^{\rho\sigma[\mu|}\left(\nabla_\sigma \Psi^{|\nu]} - \nabla^{|\nu]}\Psi_\sigma\right) - \frac{m}{2}\bar{\Psi}_\rho \gamma^{\rho[\mu|}\Psi^{|\nu]} \\
& -\frac{1}{2}\nabla^{[\nu}\left(\bar{\Psi}^{\mu]}\gamma^\rho \Psi_\rho\right) + \frac{1}{4}\nabla_\rho\left(\bar{\Psi}^{[\mu|}\gamma^\rho \Psi^{|\nu]}\right) = \\
=& -\frac{1}{2}\left(\bar{\Psi}_\rho \gamma^{\rho[\mu} - \bar{\Psi}^{[\mu}\right)\left[\nabla^{\nu]} - \nabla^{\nu]}\left(\gamma^\rho \Psi_\rho\right)\right] - \frac{m}{2}\bar{\Psi}_\rho \gamma^{\rho[\mu|}\Psi^{|\nu]} \\
& -\frac{1}{2}\bar{\Psi}^\sigma \gamma^{[\mu}\left(\nabla_\sigma \Psi^{\nu]} - \nabla^{\nu]}\Psi_\sigma\right) + \frac{1}{2}\bar{\Psi}_\rho \gamma^\rho \nabla^{[\mu}\Psi^{\nu]} = \\
=& \frac{m}{2}\bar{\Psi}_\rho \gamma^{\rho[\mu}\Psi^{\nu]} - \frac{3}{2}\bar{\Psi}_\rho \gamma^{[\rho}\nabla^\nu \Psi^{\mu]} \\
=& \frac{m}{2}\bar{\Psi}_\rho \gamma^{\rho[\mu}\Psi^{\nu]} - \frac{3}{2}\delta^{\rho\nu\mu}_{\alpha\beta\gamma}\bar{\Psi}_\rho \gamma^\alpha \nabla^\beta \Psi^\gamma = \\
=& \frac{m}{2}\bar{\Psi}_\rho \gamma^{\rho[\mu}\Psi^{\nu]} + \frac{i}{4}\varepsilon^{\rho\nu\mu}{}_\sigma \bar{\Psi}_\rho \gamma_5\left[\gamma^{\sigma\lambda\tau}\nabla_\lambda \Psi_\tau\right] = \\
=& \frac{m}{2}\bar{\Psi}_\rho \gamma^{\rho[\mu}\Psi^{\nu]} + \frac{m}{4}\bar{\Psi}_\rho \gamma^{\rho\nu\mu}\gamma^\sigma \Psi_\sigma \\
=& \frac{m}{2}\bar{\Psi}_\rho \gamma^{\rho[\mu}\Psi^{\nu]} - \frac{m}{2}\bar{\Psi}_\rho \gamma^{\rho[\mu}\Psi^{\nu]} = 0
\end{aligned}
$$

$$
\begin{aligned}
& R^\mu_{[\nu\rho\sigma]} = 0 \\
& R_{\mu\nu\rho\sigma}\gamma^{\mu\nu\lambda}\gamma^{\rho\sigma} = 4G^\lambda_\tau \gamma^\tau \\
& \left[\nabla_\mu, \nabla_\nu\right]\Psi^\rho = \frac{1}{4}R_{\mu\nu\lambda\tau}\gamma^{\lambda\tau}\Psi^\rho + R^\rho_{\sigma\mu\nu}\Psi^\sigma \\
& \left[\nabla_\mu, \nabla_\nu\right]V^{\rho_1\ldots\rho_r} = R^{\rho_1}_{\sigma\mu\nu}V^{\sigma\rho_2\ldots\rho_r} + \cdots + R^{\rho_r}_{\sigma\mu\nu}V^{\rho_1\rho_2\ldots\sigma}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2}\nabla_\mu[\bar{\Psi}_\rho \gamma^{\rho\sigma\mu}(\nabla_\sigma \Psi^\nu - \nabla^\nu \Psi_\sigma) - m\bar{\Psi}_\rho \gamma^{\rho\mu}\Psi^\nu] = \\
& =\frac{1}{2}\bar{\Psi}_\rho \overleftarrow{\nabla}_\mu \gamma^{\rho\sigma\mu}(\nabla_\sigma \Psi^\nu - \nabla^\nu \Psi_\sigma) + \frac{1}{2}\bar{\Psi}_\rho \gamma^{\rho\sigma\mu}\nabla_\mu \nabla_\sigma \Psi^\nu - \frac{1}{2}\bar{\Psi}_\rho \gamma^{\rho\mu\sigma}\nabla_\mu \nabla^\nu \Psi_\sigma \\
& -\frac{m}{2}\bar{\Psi}_\rho \overleftarrow{\nabla}_\mu \gamma^{\rho\mu}\Psi^\nu - \frac{m}{2}\bar{\Psi}_\rho \gamma^{\rho\mu}\nabla_\mu \Psi^\nu - \frac{1}{2}(\partial_\mu m)\bar{\Psi}_\rho \gamma^{\rho\mu}\Psi^\nu = \\
& = -\frac{1}{4}R^\nu_{\rho\alpha\beta}\bar{\Psi}^\alpha \gamma^\rho \Psi^\beta - \frac{1}{2}R^\nu_\rho \bar{\Psi}_\alpha \gamma^\alpha \Psi^\rho
\end{aligned}
$$

$$
\square^{\prime\prime\odot}_{\boxtimes} - \frac{1}{4}\nabla_\mu \nabla_\rho[\bar{\Psi}^\rho \gamma^\nu \Psi^\mu] = -\frac{i}{8}\left[\nabla_\mu, \nabla_\rho\right]V^{\rho\nu\mu} =
$$

$$
\square^{\prime\prime\odot}_{\boxtimes} = -\frac{1}{8}\left(R^\rho_{\sigma\mu\rho}V^{\sigma\nu\mu} + R^\mu_{\sigma\mu\rho}V^{\rho\nu\sigma} + R^\nu_{\sigma\mu\rho}V^{\rho\sigma\mu}\right) = \frac{1}{8}R^\nu_{\rho\alpha\beta}\bar{\Psi}^\alpha \gamma^\rho \Psi^\beta
$$

$$
\begin{aligned}
& \square^{\prime\prime\odot}_{\boxtimes} - \frac{1}{4}\nabla_\mu \nabla_\rho[\bar{\Psi}^\rho \gamma^\mu \Psi^\nu - \bar{\Psi}^\mu \gamma^\rho \Psi^\nu] = -\frac{1}{4}\left[\nabla_\mu, \nabla_\rho\right]V^{\rho\mu\nu} \\
& \quad = -\frac{1}{4}\left(R^\rho_{\sigma\mu\rho}V^{\sigma\mu\nu} + R^\mu_{\sigma\mu\rho}V^{\rho\sigma\nu} + R^\nu_{\sigma\mu\rho}V^{\rho\mu\sigma}\right) = -\frac{1}{4}R^\nu_{\beta\rho\alpha}\bar{\Psi}^\alpha \gamma^\rho \Psi^\beta,
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{2}\nabla_\mu\left[g^{\mu\nu}\nabla_\rho(\bar{\Psi}_\alpha \gamma^\alpha \Psi^\rho) + \nabla^\nu\left(\bar{\Psi}^\mu \gamma^\rho \Psi_\rho\right)\right] \\
& = -\frac{1}{2}\left[\nabla^\nu, \nabla_\mu\right](\bar{\Psi}_\rho \gamma^\rho \Psi^\mu)V^\mu = \frac{1}{2}R^\nu_\rho \bar{\Psi}_\alpha \gamma^\alpha \Psi^\rho.
\end{aligned}
$$

$$
\nabla_\mu T^{\mu\nu} = 0
$$

$$
\gamma^0 = \mathbb{1} \otimes (i\sigma_1 \otimes \mathbb{1}), \gamma^i = \mathbb{1} \otimes \left(\sigma_2 \otimes \sigma^i\right), \gamma^4 = \mathbb{1} \otimes (\sigma_3 \otimes \mathbb{1}),
$$

$$\mathcal{C}_5 = \sigma_2 \otimes (\mathbb{1} \otimes \sigma_2) \equiv \sigma_2 \otimes (\gamma_5 C_4),$$

$$\mathcal{C}_5^{-1} = \mathcal{C}_5^{\mathrm{T}} = \mathcal{C}_5^{\dagger} = \mathcal{C}_5.$$

$$(\mathfrak{C}_5 \gamma^{M_1 \ldots M_r})^{\mathrm{T}} = -t_r \mathfrak{C}_5 \gamma^{M_1 \ldots M_r},$$

$$\bar{\lambda} = \lambda^{\mathrm{T}} \mathcal{C}_5.$$

$$\begin{aligned}
\delta E^A &= \frac{1}{2}\bar{\varepsilon}\gamma^A \Psi \\
\delta \mathcal{A} &= -\frac{i}{2}\bar{\varepsilon}\Psi \\
\delta \Psi &= \mathcal{D}\varepsilon + iE^A\left(\frac{1}{8}\gamma_A^{BC}\mathcal{F}_{BC} - \frac{1}{2}\mathcal{F}_{AB}\gamma^B\right)\varepsilon
\end{aligned}$$

$$\begin{aligned}
E^a &= \phi^{-\frac{1}{2}} e^a \\
E^4 &= \phi(dy + B) \\
\mathcal{A} &= a(dy + B) + A \\
\Psi &= \phi^{\frac{5}{4}}\zeta(dy + B) + \phi^{-\frac{1}{4}}\left(\psi - \frac{1}{2}e^a\gamma_a\gamma_5\zeta\right)
\end{aligned}$$

$$\varepsilon = \phi^{-\frac{1}{4}}\epsilon.$$

$$\mathcal{F}_{ab} = \phi e_a{}^{\mu} e_b{}^{\nu} F_{\mu\nu}, \mathcal{F}_{a4} = \phi^{-\frac{1}{2}} e_a{}^{\mu}\left(\partial_\mu a - \partial_y A_\mu\right)$$

$$dE^A = \frac{1}{2}\Omega_{BC}{}^A E^B \wedge E^C, (\omega_C)_{AB} = \frac{1}{2}(\Omega_{CAB} - \Omega_{CBA} + \Omega_{BAC}),$$

$$\begin{aligned}
\Omega_{bc}^a &= \phi^{\frac{1}{2}}\Omega_{bc}^{(4)a} - \phi^{-\frac{1}{2}}\partial_\mu\phi e_{[b}^{\mu}\delta_{c]}^a, \qquad \Omega_{ab}^4 = \phi^2 e_{[a}^{\mu} e_{b]}^{\nu} G_{\mu\nu} \\
\Omega_{b4}^a &= \frac{1}{2}\phi^{-2}\partial_y\phi\delta_b^a - \phi^{-1}e_b^{\mu}\partial_y e_\mu^a, \quad \Omega_{a4}^4 = \phi^{-1/2}e_a^{\mu}\left(\partial_\mu\phi - \phi\partial_y B_\mu\right).
\end{aligned}$$

$$\delta_{\mathrm{LL}} E^A = \Lambda^A{}_B E^B$$

$$\Lambda_b^a = \frac{1}{4}\phi^{\frac{1}{4}}(\bar{\varepsilon}\gamma^a{}_b\gamma_5\zeta), \Lambda^a{}_4 = \frac{1}{4}\phi^{\frac{1}{4}}(\bar{\varepsilon}\gamma^a\zeta)$$

$$\delta e_\mu^a = \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu$$
$$\delta\phi = \frac{\phi}{2}\bar{\epsilon}\gamma_5\zeta$$
$$\delta a = -\frac{i}{2}\phi\bar{\epsilon}\zeta$$
$$\delta A_\mu = -\frac{i}{2}\phi^{-\frac{1}{2}}\bar{\epsilon}\left(\psi_\mu - \frac{1}{2}\gamma_\mu\gamma_5\zeta\right)$$
$$\delta B_\mu = \frac{\phi^{-\frac{3}{2}}}{2}\bar{\epsilon}\left(\gamma_5\psi_\mu + \frac{3}{2}\gamma_\mu\zeta\right)$$
$$\delta\zeta = -\frac{\phi^{-\frac{5}{2}}}{4}\partial_y\phi\epsilon + \frac{\phi^{-\frac{3}{2}}}{4}Q_{ab}\gamma^{ab}\epsilon + \frac{i}{8}\phi^{\frac{1}{2}}(F_{\rho\sigma} + i\phi G_{\rho\sigma}\gamma_5)\gamma^{\rho\sigma}\gamma_5\epsilon$$
$$+\frac{i}{2}\phi^{-1}[\partial_\rho a - \partial_y A_\rho + i(\phi\partial_y B_\rho - \partial_\rho\phi)\gamma_5]\gamma^\rho\epsilon$$
$$\delta\psi_\mu = D_\mu\epsilon - \frac{3}{8}\phi^{-\frac{5}{2}}\partial_y\phi\gamma_\mu\gamma_5\epsilon + \frac{\phi^{-\frac{3}{2}}}{2}e_\mu^a\left(P_{ab}\gamma^b + \frac{1}{4}Q_{bc}\gamma_a\gamma^{bc}\right)\gamma_5\epsilon$$
$$-\frac{1}{4}\partial_y B_\rho\gamma_\mu\gamma^\rho\epsilon + \frac{3i}{4}\phi^{-1}(\partial_y A_\mu - \partial_\mu a)\gamma_5\epsilon - \frac{i}{8}\phi^{\frac{1}{2}}(3F_{\mu\rho}^{+,5} - i\phi G_{\mu\rho}^{+,5}\gamma_5)\gamma^\rho\epsilon$$

$$P_{ab} \equiv e_{(a}{}^\lambda\partial_y e_{b)\lambda}, Q_{ab} \equiv e_{[a}{}^\lambda\partial_y e_{b]\lambda}$$

$$F_{\mu\nu}^{\pm,5} \equiv F_{\mu\nu} \pm i\tilde{F}_{\mu\nu}\gamma_5, \tilde{F}^{\mu\rho} \equiv \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$$e_\mu{}^a(y) = e_\mu^{(0)a}(x) + \sum_{n=1}^{\infty} e_\mu^{(n)a}(x)\cos\left(\frac{ny}{R}\right), \quad A_\mu(y) = \sum_{n=1}^{\infty} A_\mu^{(n)}(x)\sin\left(\frac{ny}{R}\right)$$
$$\phi(y) = \phi^{(0)}(x)\left[1 + \sum_{n=1}^{\infty}\phi^{(n)}(x)\cos\left(\frac{ny}{R}\right)\right] \quad B_\mu(y) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x)\sin\left(\frac{ny}{R}\right)$$
$$a(y) = a^{(0)}(x) + \sum_{n=1}^{\infty} a^{(n)}(x)\cos\left(\frac{ny}{R}\right)$$

$$\Psi_{i\mu}(-y) = \sigma_{ij}^3\gamma_5\Psi_{j\mu}(y), \Psi_{i5}(-y) = -\sigma_{ij}^3\gamma_5\Psi_{j5}(y)$$

$$\psi_{1\mu}(-y) = \gamma_5\psi_{1\mu}(y), \quad \zeta_{1\mu}(-y) = -\gamma_5\zeta_{1\mu}(y)$$
$$\psi_{2\mu}(-y) = -\gamma_5\psi_{2\mu}(y), \quad \zeta_{2\mu}(-y) = \gamma_5\zeta_{2\mu}(y)$$

$$\psi_{1\mu}=\psi_{1\mu\mathrm{R}}+\psi_{1\mu\,\mathrm{L}}=\psi_{1\mu\mathrm{R}}^{(0)}+\sum_{n=1}^{\infty}\left[\psi_{1\mu\mathrm{R}}^{(n)}\cos\left(\frac{ny}{R}\right)+i\psi_{1\mu\,\mathrm{L}}^{(n)}\sin\left(\frac{ny}{R}\right)\right]$$

$$\psi_{2\mu}=\psi_{1\mu\,\mathrm{L}}+\psi_{1\mu\mathrm{R}}=\psi_{2\mu\,\mathrm{L}}^{(0)}+\sum_{n=1}^{\infty}\left[\psi_{2\mu\,\mathrm{L}}^{(n)}\cos\left(\frac{ny}{R}\right)+i\psi_{2\mu\mathrm{R}}^{(n)}\sin\left(\frac{ny}{R}\right)\right]$$

$$\zeta_1=\zeta_{1\,\mathrm{L}}+\zeta_{1\mathrm{R}}=\zeta_{1\,\mathrm{L}}^{(0)}+\sum_{n=1}^{\infty}\left[\zeta_{1\,\mathrm{L}}^{(n)}\cos\left(\frac{ny}{R}\right)+i\zeta_{1\mathrm{R}}^{(n)}\sin\left(\frac{ny}{R}\right)\right]$$

$$\zeta_2=\zeta_{2\mathrm{R}}+\zeta_{2\,\mathrm{L}}=\zeta_{2\mathrm{R}}^{(0)}+\sum_{n=1}^{\infty}\left[\zeta_{2\mathrm{R}}^{(n)}\cos\left(\frac{ny}{R}\right)+i\zeta_{2\,\mathrm{L}}^{(n)}\sin\left(\frac{ny}{R}\right)\right]$$

$$\zeta_{2\mathrm{R}}^{(n)}=-\left(\zeta_{1\,\mathrm{L}}^{(n)}\right)^{*},\zeta_{2\,\mathrm{L}}^{(n)}=-\left(\zeta_{1\mathrm{R}}^{(n)}\right)^{*},\psi_{2\mu\mathrm{R}}^{(n)}=\left(\psi_{1\mu\,\mathrm{L}}^{(n)}\right)^{*},\psi_{2\mu\,\mathrm{L}}^{(n)}=\left(\psi_{1\mu\mathrm{R}}^{(n)}\right)^{*}$$

$$\epsilon_i(-y)=\sigma_{ij}^{3}\gamma_5\epsilon_j(y)$$

$$\epsilon_1=\epsilon_{1\mathrm{R}},\epsilon_2=\epsilon_{2\,\mathrm{L}}$$

$$\epsilon\equiv\epsilon_{1\mathrm{R}}+\epsilon_{2\,\mathrm{L}}$$

$$\delta e_{\mu}^{(0)a}=\frac{1}{2}\left(\bar{\epsilon}_1\gamma^{a}\psi_{1\mu}^{(0)}+\bar{\epsilon}_2\gamma^{a}\psi_{2\mu}^{(0)}\right)$$

$$\delta\phi^{(0)}=\frac{\phi^{(0)}}{2}\left(\bar{\epsilon}_2\zeta_2^{(0)}-\bar{\epsilon}_1\zeta_1^{(0)}\right)$$

$$\delta a^{(0)}=-\frac{i}{2}\phi^{(0)}\left(\bar{\epsilon}_1\zeta_1^{(0)}+\bar{\epsilon}_2\zeta_2^{(0)}\right)$$

$$\delta\psi_{1\mu R}^{(0)}=D_\mu\epsilon_1-\frac{3i}{4}\phi^{(0)-1}\partial_\mu a^{(0)}\epsilon_1$$

$$\delta\psi_{2\mu L}^{(0)}=D_\mu\epsilon_2+\frac{3i}{4}\phi^{(0)^{-1}}\partial_\mu a^{(0)}\epsilon_2$$

$$\delta\zeta_{2R}^{(0)}=\frac{\phi^{(0)-1}}{2}\partial_\rho\left(\phi^{(0)}+ia^{(0)}\right)\gamma^{\rho}\epsilon_2$$

$$\delta\zeta_{1L}^{(0)}=-\frac{\phi^{(0)-1}}{2}\partial_\rho\left(\phi^{(0)}-ia^{(0)}\right)\gamma^{\rho}\epsilon_1$$

$$\psi_\mu\equiv\psi_{1\mu R}^{(0)}+\psi_{2\mu L}^{(0)},\zeta\equiv\phi^{(0)}\left(\zeta_{2R}^{(0)}-\zeta_{1L}^{(0)}\right)$$

$$T\equiv\phi^{(0)}+ia^{(0)}$$

$$\delta e_{\mu}^{a}=\frac{1}{2}\bar{\epsilon}\gamma^{a}\psi_\mu$$

$$\delta\psi_\mu=D_\mu\epsilon+\frac{i}{2}Q_\mu\gamma_5\epsilon$$

$$\delta T=\bar{\epsilon}_{\mathrm{R}}\zeta_{\mathrm{R}}$$

$$\delta\zeta_{\mathrm{R}}=\frac{1}{2}\partial T\epsilon_{\mathrm{L}}$$

$$K=-3\log\left(T+\bar{T}\right),Q_\mu=-\frac{3}{2i(T+\bar{T})}\left(\partial_\mu T-\partial_\mu\bar{T}\right)$$

$$m_n = \phi^{(0)-\frac{3}{2}}\frac{n}{R}$$

$$\partial_y\phi = -\phi^{(0)\frac{5}{2}}\sum_n m_n\phi^{(n)}\sin\left(\frac{ny}{R}\right), \qquad P_{ab} = -\phi^{(0)\frac{3}{2}}\sum_n m_n e_{(a}^{(0)\rho}e_{b)}^{(n)}\rho\sin\left(\frac{ny}{R}\right),$$

$$\partial_y B_\mu = \phi^{(0)\frac{3}{2}}\sum_n m_n B_\mu^{(n)}\cos\left(\frac{ny}{R}\right), \qquad Q_{ab} = -\phi^{(0)\frac{3}{2}}\sum_n m_n e_{[a}^{(0)\rho}e_{b]\rho}^{(n)}\rho\sin\left(\frac{ny}{R}\right),$$

$$\partial_y A_\mu = \phi^{(0)\frac{3}{2}}\sum_n m_n A_\mu^{(n)}\cos\left(\frac{ny}{R}\right), \qquad \omega_\mu^{(n)ab} = 2e^{(0)[a|\alpha}\partial_{[\mu}e_{\alpha]}^{(n)|b]} - e_\alpha^{(0)[a}e_\beta^{(0)b]}e_\mu^{(0)c}\partial^\alpha e_c^{(n)\beta},$$

$$\begin{aligned}
\delta e_n^a &= \frac{1}{2}\left(\bar\epsilon_1\gamma^a\psi_{1\mu}^{(n)} + \bar\epsilon_2\gamma^a\psi_{2\mu}^{(n)}\right)\\
\delta\phi^{(n)} &= \frac{1}{2}\left(\bar\epsilon_2\zeta_2^{(n)} - \bar\epsilon_1\zeta_1^{(n)}\right)\\
\delta a^{(n)} &= -\frac{i}{2}\phi^{(0)}\left(\bar\epsilon_1\zeta_1^{(n)} + \bar\epsilon_2\zeta_2^{(n)}\right)\\
\delta A_\mu^{(n)} &= \frac{\phi^{(0)-\frac{1}{2}}}{2}\left[\bar\epsilon_1\left(\psi_{1\mu}^{(n)} - \frac{1}{2}\gamma_\mu\zeta_1^{(n)}\right) + \bar\epsilon_2\left(\psi_{2\mu}^{(n)} - \frac{1}{2}\gamma_\mu\zeta_2^{(n)}\right)\right]\\
\delta B_\mu^{(n)} &= \frac{i}{2}\phi^{(0)-\frac{3}{2}}\left[\bar\epsilon_2\left(\psi_{2\mu}^{(n)} + \frac{3}{2}\gamma_\mu\zeta_2^{(n)}\right) - \bar\epsilon_1\left(\psi_{1\mu}^{(n)} - \frac{3}{2}\gamma_\mu\zeta_1^{(n)}\right)\right]
\end{aligned}$$

$$\delta\zeta_{1\,\mathrm{L}}^{(n)} = \frac{m_n}{2}\left[-i\left(\phi^{(0)\frac{1}{2}}A_\rho^{(n)} - \frac{\phi^{(0)-1}}{m_n}\partial_\rho a^{(n)}\right) + \left(\phi^{(0)\frac{3}{2}}B_\rho^{(n)} - \frac{1}{m_n}\partial_\rho\phi^{(n)}\right)\right]\gamma^\rho\epsilon_1,$$

$$\delta\zeta_{1\mathrm{R}}^{(n)} = \frac{\phi^{(0)\frac{1}{2}}}{8}\left(F_{\rho\sigma}^{(n)} + i\phi^{(0)}G_{\rho\sigma}^{(n)}\right)\gamma^{\rho\sigma}\epsilon_1 + \frac{i}{4}m_n\left(e_a^{(0)\rho}e_b^{(n)}{}_\rho\gamma^{ab} - \phi^{(n)}\right)\epsilon_1,$$

$$\delta\zeta_{2\,\mathrm{L}}^{(n)} = -\frac{\phi^{(0)\frac{1}{2}}}{8}\left(F_{\rho\sigma}^{(n)} - i\phi^{(0)}G_{\rho\sigma}^{(n)}\right)\gamma^{\rho\sigma}\epsilon_2 + \frac{i}{4}m_n\left(e_a^{(0)\rho}e_b^{(n)}{}_\rho\gamma^{ab} - \phi^{(n)}\right)\epsilon_2,$$

$$\delta\zeta_{2\mathrm{R}}^{(n)} = \frac{m_n}{2}\left[-i\left(\phi^{(0)\frac{1}{2}}A_\rho^{(n)} - \frac{\phi^{(0)-1}}{m_n}\partial_\rho a^{(n)}\right) - \left(\phi^{(0)\frac{3}{2}}B_\rho^{(n)} - \frac{1}{m_n}\partial_\rho\phi^{(n)}\right)\right]\gamma^\rho\epsilon_2,$$

$$\begin{aligned}
\delta\psi_{1\mu\mathrm{R}}^{(n)} =& \frac{1}{4}\left(2e^{(0)a\alpha}\partial_{[\mu}e_{\alpha]}^{(n)b} - e_\alpha^{(0)a}e_\beta^{(0)b}e_\mu^{(0)c}\partial^\alpha e_c^{(n)\beta}\right)\gamma_{ab}\epsilon_1 - \frac{m_n}{4}\phi^{(0)\frac{3}{2}}B_\rho^{(n)}\gamma_\mu\gamma^\rho\epsilon_1\\
&+\frac{3i}{4}m_n\left(\phi^{(0)\frac{1}{2}}A_\mu^{(n)} - \frac{\phi^{(0)^{-1}}}{m_n}\partial_\mu a^{(n)}\right)\epsilon_1\\
\delta\psi_{1\mu\,\mathrm{L}}^{(n)} =& -\frac{\phi^{(0)\frac{1}{2}}}{8}\left(3F_{\mu\rho}^{(n)+,5} + i\phi^{(0)}G_{\mu\rho}^{(n)+,5}\right)\gamma^\rho\epsilon_1 + \frac{i}{4}m_n g_{\mu\rho}^{(n)}\gamma^\rho\epsilon_1\\
&+\frac{i}{4}m_n\left(e_a^{(0)\rho}e_b^{(n)}{}_\rho\gamma_\mu\gamma^{ab} - \phi^{(n)}\gamma_\mu\right)\epsilon_1\\
\delta\psi_{2\mu\mathrm{R}}^{(n)} =& -\frac{\phi^{(0)\frac{1}{2}}}{8}\left(3F_{\mu\rho}^{(n)+,5} - i\phi^{(0)}G_{\mu\rho}^{(n)+,5}\right)\gamma^\rho\epsilon_2 - \frac{i}{4}m_n g_{\mu\rho}^{(n)}\gamma^\rho\epsilon_2\\
&-\frac{i}{4}m_n\left(e_a^{(0)\rho}e_b^{(n)}\gamma_\mu\gamma^{ab} - \phi^{(n)}\gamma_\mu\right)\epsilon_2,\\
\delta\psi_{2\mu\,\mathrm{L}}^{(n)} =& \frac{1}{4}\left(2e^{(0)a\alpha}\partial_{[\mu}e_{\alpha]}^{(n)b} - e_\alpha^{(0)a}e_\beta^{(0)b}e_\mu^{(0)c}\partial^\alpha e_c^{(n)\beta}\right)\gamma_{ab}\epsilon_2 - \frac{m_n}{4}\phi^{(0)\frac{3}{2}}B_\rho^{(n)}\gamma_\mu\gamma^\rho\epsilon_2\\
&-\frac{3i}{4}m_n\left(\phi^{(0)\frac{1}{2}}A_\mu^{(n)} - \frac{\phi^{(0)-1}}{m_n}\partial_\mu a^{(n)}\right)\epsilon_2.
\end{aligned}$$

$$A_\mu^{(n)\mathrm{S}} = \phi^{(0)\frac{1}{2}} A_\mu^{(n)} - \frac{\phi^{(0)-1}}{m_n}\partial_\mu a^{(n)},$$

$$\delta A_\mu^{(n)\mathrm{S}} = \frac{1}{2}\left[\bar{\epsilon}_1\left(\psi_{1\mu\,\mathrm{L}}^{(n)} - \frac{1}{2}\gamma_\mu\zeta_{1\mathrm{R}}^{(n)} + \frac{i}{m_n}\partial_\mu\zeta_{1\,\mathrm{L}}^{(n)}\right) + \bar{\epsilon}_2\left(\psi_{2\mu\mathrm{R}}^{(n)} + \frac{1}{2}\gamma_\mu\zeta_{2\,\mathrm{L}}^{(n)} + \frac{i}{m_n}\partial_\mu\zeta_{2\mathrm{R}}^{(n)}\right)\right]$$

$$\psi_{1\mu\,\mathrm{L}}^{(n)}\mathrm{S} = \psi_{1\mu\,\mathrm{L}}^{(n)} - \frac{1}{2}\gamma_\mu\zeta_{1\mathrm{R}}^{(n)} + \frac{i}{m_n}\partial_\mu\zeta_{1\,\mathrm{L}}^{(n)}$$
$$\psi_{2\mu\mathrm{R}}^{(n)}\mathrm{S} = \psi_{2\mu\mathrm{R}}^{(n)} + \frac{1}{2}\gamma_\mu\zeta_{2\,\mathrm{L}}^{(n)} + \frac{i}{m_n}\partial_\mu\zeta_{2\mathrm{R}}^{(n)}$$

$$\lambda_\mu^{(n)\mathrm{S}} \equiv \psi_{1\mu\,\mathrm{L}}^{(n)\mathrm{S}} + \psi_{2\mu\mathrm{R}}^{(n)\mathrm{S}} = \left(\psi_{1\mu\,\mathrm{L}}^{(n)} + \psi_{2\mu\mathrm{R}}^{(n)}\right) - \frac{1}{2}\gamma_\mu\left(\zeta_{1\mathrm{R}}^{(n)} - \zeta_{2\,\mathrm{L}}^{(n)}\right) + \frac{i}{m_n}\partial_\mu\left(\zeta_{1\,\mathrm{L}}^{(n)} + \zeta_{2\mathrm{R}}^{(n)}\right),$$

$$\chi_\mu^{(n)\mathrm{S}} = -i\left[\left(\psi_{1\mu\mathrm{R}}^{(n)} - \psi_{2\mu\,\mathrm{L}}^{(n)}\right) - \frac{1}{2}\gamma_\mu\left(\zeta_{1\,\mathrm{L}}^{(n)} + \zeta_{2\mathrm{R}}^{(n)}\right) + \frac{i}{m_n}\partial_\mu\left(\zeta_{1\mathrm{R}}^{(n)} - \zeta_{2\,\mathrm{L}}^{(n)}\right)\right]$$

$$g_{\mu\nu}^{(n)\mathrm{S}} = 2e_{(\mu}^{(0)}e_{\nu)}^{(n)\,a} - \phi^{(n)}g_{\mu\nu}^{(0)} + \frac{\phi^{(0)\frac{3}{2}}}{m_n}\left(\partial_\mu B_\nu^{(n)} + \partial_\nu B_\mu^{(n)}\right) - \frac{2}{m_n^2}\partial_\mu\partial_\nu\phi^{(n)}$$

$$\tilde{\mathcal{A}}_{77} = \frac{2^{-4}v}{2}\frac{1}{\eta^8}[(N+\bar{N})^2V_8 + (N-\bar{N})^2S_8]\sum_n W_n$$
$$\tilde{\mathcal{M}}_7 = \left(\frac{2\epsilon v}{\hat{\eta}^5\hat{\vartheta}_2}\right)\left[(N+\bar{N})\left(\hat{O}_6\hat{V}_2 - \hat{V}_6\hat{O}_2\right) - (N-\bar{N})\left(\hat{S}_6\hat{S}_2 - \hat{C}_6\hat{C}_2\right)\right]\sum_n W_{2n+1}$$

$$\mathcal{A}_{77} = \frac{1}{\eta^8}\left[N\bar{N}(V_8 - S_8) + \frac{N^2+\bar{N}^2}{2}(O_8 - C_8)\right]\sum_m P_m$$
$$\mathcal{M}_7 = -\left(\frac{2\epsilon}{\hat{\eta}^5\hat{\vartheta}_2}\right)\left[\frac{N+\bar{N}}{2}\left(\hat{O}_6\hat{O}_2 + \hat{V}_6\hat{V}_2\right) + \frac{N-\bar{N}}{2}\left(\hat{S}_6\hat{C}_2 - \hat{C}_6\hat{S}_2\right)\right]\sum_m (-1)^m P_m$$

$$\tilde{\mathcal{A}}_{33} = \frac{2^{-2}v}{2}\frac{1}{\eta^8}[(N+\bar{N})^2V_8 + (N-\bar{N})^2S_8]\sum_n W_n,$$
$$\tilde{\mathcal{M}}_3 = \left(\frac{8\epsilon v\hat{\eta}}{\hat{\vartheta}_2^3}\right)\left[(N+\bar{N})\left(\hat{O}_2\hat{V}_6 - \hat{V}_2\hat{O}_6\right) - (N-\bar{N})\left(\hat{S}_2\hat{S}_6 - \hat{C}_2\hat{C}_6\right)\right]\sum_n W_{2n+1},$$

$$\mathcal{A}_{33} = \left[N\bar{N}\left(\frac{V_8 - S_8}{\eta^8}\right) + \frac{N^2+\bar{N}^2}{2}\left(\frac{O_8 - C_8}{\eta^8}\right)\right]\sum_m P_m$$
$$\mathcal{M}_3 = \left(\frac{8\epsilon\hat{\eta}}{\hat{\vartheta}_2^3}\right)\left[\frac{N+\bar{N}}{2}\left(\hat{O}_2\hat{O}_6 + \hat{V}_2\hat{V}_6\right) + \frac{N-\bar{N}}{2}\left(\hat{S}_2\hat{C}_6 - \hat{C}_2\hat{S}_6\right)\right]\sum_m (-1)^m P_m$$

$$\tilde{\mathcal{A}}_{pp} = \frac{2^{-\frac{p+1}{2}}v}{2}\frac{N^2}{\eta^8}\left(V_{p-1}O_{9-p} + O_{p-1}V_{9-p}\right)\sum_n W_n,$$
$$\tilde{\mathcal{M}}_p = -\frac{\sqrt{2}vN\epsilon}{\hat{\eta}^{p-1}}\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^{\frac{9-p}{2}}\left(\hat{V}_{p-1}\hat{O}_{9-p} - \hat{O}_{p-1}\hat{V}_{9-p}\right)\sum_n W_{2n+1},$$

$$\mathcal{A}_{pp} = \frac{N^2}{2}\frac{1}{\eta^8}\left[(O_{p-1}+V_{p-1})(O_{9-p}+V_{9-p}) - 2S'_{p-1}S'_{9-p}\right]\sum_m P_m$$

$$\mathcal{M}_p = -\frac{\epsilon N}{\sqrt{2}}\frac{1}{\hat{\eta}^{p-1}}\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^{\frac{9-p}{2}}\left[\sin\frac{(p-5)\pi}{4}\left(\hat{O}_{p-1}\hat{O}_{9-p}+\hat{V}_{p-1}\hat{V}_{9-p}\right)\right.$$
$$\left.+\cos\frac{(p-5)\pi}{4}\left(\hat{O}_{p-1}\hat{V}_{9-p}-\hat{V}_{p-1}\hat{O}_{9-p}\right)\right]\sum_m (-1)^m P_m$$

$$\tilde{\mathcal{A}}_{pp} = \frac{2^{-\frac{p+1}{2}}}{2v\eta^8}\left(e^{\frac{i\pi m}{2}}N + e^{-\frac{i\pi m}{2}}\bar{N}\right)^2\left(V_{p-1}O_{9-p}+O_{p-1}V_{9-p}\right)\sum_m P_m$$

$$\mathcal{A}_{pp} = \frac{1}{\eta^8}\left(N\bar{N}\sum_n W_n + \frac{N^2+\bar{N}^2}{2}\sum_n W_{n+\frac{1}{2}}\right)\left[(O_{p-1}+V_{p-1})(O_{9-p}+V_{9-p}) - 2S'_{p-1}S'_{9-p}\right]$$

$$\tilde{\mathcal{A}}_{55} = \frac{2^{-3}v}{2\eta^8}\sum_n \left[(N+\bar{N})^2(V_8W_{2n}+O_8W_{2n+1}) + (N-\bar{N})^2(S_8W_{2n}+C_8W_{2n+1})\right]$$

$$\tilde{\mathcal{M}}_5 = -\left(\frac{4v}{\hat{\eta}^2\hat{\vartheta}_2^2}\right)\sum_n \left[\epsilon_1(N+\bar{N})\left(\hat{O}_4\hat{O}_4-\hat{V}_4\hat{V}_4\right) + i(-1)^n\epsilon_2(N-\bar{N})\left(\hat{S}_4\hat{C}_4-\hat{C}_4\hat{S}_4\right)\right]W_{2n+1}$$

$$\mathcal{A}_{55} = \frac{1}{\eta^8}\left[N\bar{N}\sum_m \left(V_8P_m - S_8P_{m+1/2}\right) + \frac{N^2+\bar{N}^2}{2}\sum_m \left(O_8P_m - C_8P_{m+1/2}\right)\right],$$

$$\mathcal{M}_5 = \left(\frac{2}{\hat{\eta}^2\hat{\vartheta}_2^2}\right)\sum_m (-1)^m\left[\epsilon_1(N+\bar{N})\left(\hat{O}_4\hat{O}_4-\hat{V}_4\hat{V}_4\right)P_m + \epsilon_2(N-\bar{N})\left(\hat{S}_4\hat{C}_4-\hat{C}_4\hat{S}_4\right)P_{m+1/2}\right].$$

$$\mathrm{D5}_{012389} \xrightarrow[\text{T- duality}]{} \mathrm{M5}_{0123910}$$

$$\mathrm{D5}_{012349} \xrightarrow[\text{T- duality}]{} \mathrm{KK6}_{0123489},$$

$$A_{33} = \sum_n \left(N\bar{N}W_n + \frac{N^2+\bar{N}^2}{2}W_{n+1/2}\right)\frac{V_8-S_8}{\eta^8}$$

$$\tilde{\mathcal{A}}_{33} = \frac{2^{-2}}{2v}\left(\frac{V_8-S_8}{\eta^8}\right)\sum_m \left[e^{\frac{i\pi m}{2}}N + e^{-\frac{i\pi m}{2}}\bar{N}\right]^2 P_m$$
$$= \frac{2^{-2}}{v}\left(\frac{V_8-S_8}{\eta^8}\right)\sum_m \left[N\bar{N} + (-1)^m\frac{N^2+\bar{N}^2}{2}\right]P_m$$

$$\mathrm{D3}_{0128} \xrightarrow[\text{T- duality}]{} \mathrm{M2}_{012}, \mathrm{D3}_{0123} \xrightarrow[\text{T- duality}]{} \mathrm{M5}_{0123810},$$

$$\mathrm{D1}_{01} \xrightarrow[\text{T- duality}]{} \mathrm{M2}_{018}, \mathrm{D1}_{08} \xrightarrow[\text{T- duality}]{} \mathrm{KK}(X^{10}),$$

$$\tilde{\mathcal{A}}_{pp} = \frac{2^{-\frac{p+1}{2}}v}{2\eta^8}\left[(N_1+N_2)^2V_8\sum_n W_{2n} + (N_1-N_2)^2O_8\sum_n W_{2n+1}\right]$$

$$\tilde{\mathcal{M}}_p = -\sqrt{2}v(N_1-N_2)\frac{1}{\hat{\eta}^{p-1}}\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^{\frac{9-p}{2}}\left(\hat{O}_{p-1}\hat{O}_{9-p}+\hat{V}_{p-1}\hat{V}_{9-p}\right)\sum_n W_{2n+1},$$

$$\mathcal{A}_{pp} = \frac{N_1^2+N_2^2}{2\eta^8}\left[\left(O_{p-1}+V_{p-1}\right)\left(O_{9-p}+V_{9-p}\right)\sum_m P_m - 2S'_{p-1}S'_{9-p}\sum_m P_{m+\frac{1}{2}}\right]$$
$$+\frac{N_1N_2}{\eta^8}\left[\left(O_{p-1}+V_{p-1}\right)\left(O_{9-p}+V_{9-p}\right)\sum_m P_{m+\frac{1}{2}} - 2S'_{p-1}S'_{9-p}\sum_m P_m\right]$$

$$\mathcal{M}_p = -\frac{N_1-N_2}{\sqrt{2}}\frac{1}{\hat{\eta}^{p-1}}\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2}\right)^{\frac{9-p}{2}}\left[\sin\frac{(p-5)\pi}{4}\left(\hat{V}_{p-1}\hat{O}_{9-p}-\hat{O}_{p-1}\hat{V}_{9-p}\right)\right.$$
$$\left.+\cos\frac{(p-5)\pi}{4}\left(\hat{O}_{p-1}\hat{O}_{9-p}+\hat{V}_{p-1}\hat{V}_{9-p}\right)\right]\sum_m (-1)^m P_m$$

En supergravedad cuántica relativista, un agujero negro cuántico, se forma por la interacción de la superpartícula, en la medida en que deforma el espacio – tiempo cuántico, en dimensiones más altas, lo que provoca una suerte de desdoblamiento en D dimensiones del espacio – tiempo cuántico en el que interactúa, esto es, por la gravedad intensa generada por la propia superpartícula, a propósito de su centro de masa y energía extremadamente denso o por interacción con el supergravitón, lo que equivale a que, el campo supergravitónico, permea y por ende, distorsiona el espacio – tiempo cuántico, por lo que, el modelamiento matemático de este fenómeno, se expresa así, utilizando la métrica de Kerr – Hawking:

$$ds^2 = -\frac{\Delta_r}{W}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + W\left(\frac{dr^2}{\Delta_r}+\frac{d\theta^2}{\Delta_\theta}\right) + \frac{\Delta_\theta\sin^2\theta}{W}\left[adt - \frac{r_1r_2+a^2}{\Xi}d\phi\right]^2$$

$$A_1 = \frac{2\sqrt{2}m\sinh(\delta_1)\cosh(\delta_1)r_2}{W}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right) + \alpha_{41}dt$$

$$A_2 = \frac{2\sqrt{2}m\sinh(\delta_2)\cosh(\delta_2)r_1}{W}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right) + \alpha_{42}dt$$

$$e^{\varphi_1} = \frac{r_1^2+a^2\cos^2\theta}{W} = 1+\frac{r_1(r_1-r_2)}{W}$$

$$\chi_1 = \frac{a(r_2-r_1)\cos\theta}{r_1^2+a^2\cos^2\theta}$$

$$r_i \equiv r + 2m\sinh^2(\delta_i), (i=1,2)$$
$$\Delta_r \equiv r^2 + a^2 - 2mr + g^2r_1r_2(r_1r_2+a^2)$$
$$\Delta_\theta \equiv 1 - g^2a^2\cos^2\theta$$
$$W \equiv r_1r_2 + a^2\cos^2\theta$$
$$\Xi \equiv 1 - a^2g^2$$

$$T_{\mathrm{H}}=\frac{\Delta_r'(r_+)}{4\pi(r_1(r_+)r_2(r_+)+a^2)},\Omega_{\mathrm{H}}=\frac{a\left(1+g^2r_1(r_+)r_2(r_+)\right)}{r_1(r_+)r_2(r_+)+a^2}$$
$$\Phi_1=\Phi_2=\frac{2m\sinh(\delta_1)\cosh(\delta_1)r_2(r_+)}{r_1(r_+)r_2(r_+)+a^2},\Phi_3=\Phi_4=\frac{2m\sinh(\delta_2)\cosh(\delta_2)r_1(r_+)}{r_1(r_+)r_2(r_+)+a^2}$$

$$S_{\mathrm{BH}}=\frac{\pi\hat{r}_+^2}{\Xi},$$

$$\hat{r}_\pm^2\equiv r_1(r_\pm)r_2(r_\pm)+a^2$$

$$E=\frac{m}{\Xi^2}\left(1+\sinh^2(\delta_1)+\sinh^2(\delta_2)\right),\quad J=\frac{ma}{\Xi^2}\left(1+\sinh^2(\delta_1)+\sinh^2(\delta_2)\right)$$
$$Q_1=Q_2=\frac{m\sinh(\delta_1)\cosh(\delta_1)}{2\Xi},\qquad Q_3=Q_4=\frac{m\sinh(\delta_2)\cosh(\delta_2)}{2\Xi}$$

$$m=\frac{g^2r_+^3}{2}(1+k_1)^2(1+k_2)^2$$

$$T_{\mathrm{H}}=\frac{g^2r_+}{4\pi}(3+k_1+k_2-k_1k_2),\Omega_{\mathrm{H}}=ag^2$$
$$\Phi_1=\Phi_2=gr_+\sqrt{k_1}(1+k_2),\Phi_3=\Phi_4=gr_+\sqrt{k_2}(1+k_1)$$
$$E=\frac{1}{\Xi^2}\frac{g^2r_+^3}{2}(1+k_1)^2(1+k_2)^2,J=\frac{g^2r_+^3a}{2\Xi^2}(1+k_1)^2(1+k_2)^2$$
$$Q_1=Q_2=\frac{gr_+^2(1+k_1)(1+k_2)\sqrt{k_1}}{4\Xi},Q_3=Q_4=\frac{gr_+^2(1+k_1)(1+k_2)\sqrt{k_2}}{4\Xi}$$
$$S_{\mathrm{BH}}=\frac{\pi r_+^2(1+k_1)(1+k_2)}{\Xi}$$

$$ds^2=-\frac{\Delta}{\rho^2}(dt-a\sin^2\theta d\phi)^2+\frac{\sin^2\theta}{\rho^2}\left((r^2+a^2)d\phi-adt\right)^2+\frac{\rho^2}{\Delta}dr^2+\rho^2d\theta^2$$
$$\Delta\equiv r^2-2Mr+a^2,\rho^2\equiv r^2+a^2\cos^2\theta$$

$$r_\pm=M\pm\sqrt{M^2-a^2}.$$

$$T_{\mathrm{H}}=\frac{r_+-M}{4\pi Mr_+},\Omega_{\mathrm{H}}=\frac{a}{2Mr_+}$$

$$\hat{t}=\frac{\lambda t}{2M},x=\frac{r-M}{\lambda M},\hat{\phi}=\phi-\frac{t}{2M},$$

$$ds^2=2J\Omega^2\left(-x^2d\hat{t}^2+\frac{dx^2}{x^2}+d\theta^2+\Lambda^2(d\hat{\phi}+xd\hat{t})^2\right)$$

$$\Omega^2=\frac{1+\cos^2\theta}{2},\Lambda=\frac{2\sin\theta}{1+\cos^2\theta}$$

$$\zeta_\epsilon=\epsilon_n(\phi)\partial_\phi-r\epsilon_n'(\phi)\partial_r,\ \text{with}\ \epsilon_n(\phi)=-e^{-in\phi}.$$

$$T_{\mathrm{L}}=\frac{1}{2\pi},T_{\mathrm{R}}=0$$

$$S = \frac{\pi^2}{3} c_{\mathrm{L}} T_{\mathrm{L}} = 2\pi J$$

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi(t, r, \theta, \phi) \right) = 0$$

$$\Phi(t, r, \theta, \phi) = e^{-i\omega t + im\phi} R(r) S(\theta)$$

$$\left[\frac{1}{\sin\ \theta} \partial_\theta (\sin\ \theta \partial_\theta) - \frac{m^2}{\sin^2\ \theta} + \omega^2 a^2 \cos^2\ \theta \right] S(\theta) = -K_\ell S(\theta),$$

$$\left[\partial_r \Delta \partial_r + \frac{(2Mr_+\omega - ma)^2}{(r - r_+)(r_+ - r_-)} - \frac{(2Mr_-\omega - ma)^2}{(r - r_-)(r_+ - r_-)} + (r^2 + 2M(r + 2M))\omega^2 \right] R(r) = K_\ell R(r)$$

$$\nabla^2_{S^2} S(\theta) = \left[\frac{1}{\sin\ \theta} \partial_\theta (\sin\ \theta \partial_\theta) - \frac{m^2}{\sin^2\ \theta} \right] S(\theta) = -\ell(\ell + 1) S(\theta), m = -\ell, \cdots, \ell$$

$$\left[\partial_r \Delta \partial_r + \frac{(2Mr_+\omega - ma)^2}{(r - r_+)(r_+ - r_-)} - \frac{(2Mr_-\omega - ma)^2}{(r - r_-)(r_+ - r_-)} \right] R(r) = \ell(\ell + 1) R(r).$$

$$\begin{aligned} R(r) =& C_1 \left(\frac{r - r_+}{r - r_-} \right)^{-\frac{i(\omega - m\Omega_{\mathrm{H}})}{4\pi T_{\mathrm{H}}}} (r - r_-)^{-1-\ell} \\ & \times\ {}_2F_1 \left[1 + \ell - i\frac{4M}{r_+ - r_-}(M\omega - r_+ m\Omega_{\mathrm{H}}), 1 + \ell - i2M\omega; 1 - \frac{i(\omega - m\Omega_{\mathrm{H}})}{2\pi T_{\mathrm{H}}}; \frac{r - r_+}{r - r_-} \right] \\ & + C_2 \left(\frac{r - r_+}{r - r_-} \right)^{\frac{i(\omega - m\Omega_{\mathrm{H}})}{4\pi T_{\mathrm{H}}}} (r - r_-)^{-1-\ell} \\ & \times\ {}_2F_1 \left[1 + \ell + i\frac{4M}{r_+ - r_-}(M\omega - r_+ m\Omega_{\mathrm{H}}), 1 + \ell + i2M\omega; 1 + \frac{i(\omega - m\Omega_{\mathrm{H}})}{2\pi T_{\mathrm{H}}}; \frac{r - r_+}{r - r_-} \right] \end{aligned}$$

$$\omega^+ = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_{\mathrm{R}} \phi}$$

$$\omega^- = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_{\mathrm{L}} \phi - \frac{t}{2M}}$$

$$y = \sqrt{\frac{r_+ - r_-}{r - r_-}} e^{\pi(T_{\mathrm{L}} + T_{\mathrm{R}})\phi - \frac{t}{4M}}$$

$$T_{\mathrm{L}} \equiv \frac{r_+ + r_-}{4\pi a}, T_{\mathrm{R}} \equiv \frac{r_+ - r_-}{4\pi a},$$

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i\left(\omega^+ \partial_+ + \frac{1}{2} y \partial_y \right) \\ H_{-1} &= i\left((\omega^+)^2 \partial_+ + \omega^+ y \partial_y - y^2 \partial_- \right) \end{aligned}$$

$$\bar{H}_1 = i\partial_-$$
$$\bar{H}_0 = i\left(\omega^-\partial_- + \frac{1}{2}y\partial_y\right)$$
$$\bar{H}_{-1} = i\left((\omega^-)^2\partial_- + \omega^- y\partial_y - y^2\partial_+\right)$$

$$[H_0, H_{\pm 1}] = \mp iH_{\pm 1}, \quad [H_{-1}, H_1] = -2iH_0$$
$$[\bar{H}_0, \bar{H}_{\pm 1}] = \mp i\bar{H}_{\pm 1}, \quad [\bar{H}_{-1}, \bar{H}_1] = -2i\bar{H}_0$$

$$S = \frac{\pi^2}{3}(c_{\mathrm{L}}T_{\mathrm{L}} + c_{\mathrm{R}}T_{\mathrm{R}}) = 2\pi M r_+,$$

$$\omega^+ = R(r)e^{2\pi T_{\mathrm{R}}\phi}$$
$$\omega^- = R(r)e^{2\pi T_{\mathrm{L}}\phi - \Delta_r'(r_+)t/r_*^2}$$
$$y = Q(r)e^{\pi(T_{\mathrm{L}}+T_{\mathrm{R}})\phi - \Delta_r'(r_+)t/(2r_*^2)}$$

$$T_{\mathrm{L}} \equiv \frac{\Delta_r'(r_+)}{4\pi a\Xi}\frac{\hat{r}_+^2 + \hat{r}_-^2}{r_*^2}, T_{\mathrm{R}} \equiv \frac{\Delta_r'(r_+)}{4\pi a\Xi}$$

$$r_*^2 \equiv \hat{r}_+^2 - \hat{r}_-^2$$

$$t = \frac{1}{\Delta_r'(r_+)}\left(\hat{r}_+^2 \ln\frac{\omega^+}{\omega^-} + \hat{r}_-^2 \ln(\omega^+\omega^- + y^2)\right),$$
$$r = r_+ + \frac{\omega^+\omega^-\Delta_r'(r_+)}{\omega^+\omega^- + y^2},$$
$$\phi = \frac{a\Xi}{\Delta_r'(r_+)}\ln\frac{\omega^+(\omega^+\omega^- + y^2)}{\omega^-}.$$

$$\begin{aligned} ds^2 =& \frac{4W_+}{y^2}d\omega^+ d\omega^- + \frac{4\Delta_\theta r_*^4 a^2\sin^2\theta}{\Delta_r'(r_+)^2 y^2 W_+}dy^2 + \frac{W_+}{\Delta_\theta}d\theta^2 \\ & -\frac{4\omega^+}{y^3}\left(r_*^2 - \frac{\Delta_\theta a^2\sin^2\theta}{W_+\Delta_r'(r_+)^2}(r_*^4 - \Delta_r'(r_+)r_*^2\tilde{r}_+)\right)d\omega^- dy \\ & -\frac{4\omega^-}{y^3}\left(W_+ + W_- - \frac{\Delta_\theta a^2\sin^2\theta}{W_+\Delta_r'(r_+)^2}(r_*^4 + \Delta_r'(r_+)r_*^2\tilde{r}_+)\right)d\omega^+ dy + \cdots \end{aligned}$$

$$W_\pm = r_1(r_\pm)r_2(r_\pm) + a^2\cos^2\theta$$
$$\tilde{r}_+ = r_1(r_+) + r_2(r_+)$$

$$\epsilon_{\theta+-y} = \frac{4W_+ r_*^2 a\sin\theta}{\Delta_r'(r_+)y^3} + \cdots$$

$$\Gamma_{+y}^+ + \Gamma_{-y}^- = -\frac{2}{y}$$

$$\zeta(\varepsilon) = \varepsilon\partial_+ + \frac{1}{2}(\partial_+\varepsilon)y\partial_y, \bar{\zeta}(\bar{\varepsilon}) = \bar{\varepsilon}\partial_- + \frac{1}{2}(\partial_-\bar{\varepsilon})y\partial_y$$

$$\varepsilon_n = 2\pi T_{\mathrm{R}}(\omega^+)^{1+\frac{in}{2\pi T_{\mathrm{R}}}}, \bar{\varepsilon}_n = 2\pi T_{\mathrm{L}}(\omega^-)^{1+\frac{in}{2\pi T_{\mathrm{L}}}}$$

$$[\zeta_m, \zeta_n] = i(n-m)\zeta_{n+m}, [\bar{\zeta}_m, \bar{\zeta}_n] = i(n-m)\bar{\zeta}_{n+m}$$

$$\{\mathcal{Q}_n, \mathcal{Q}_m\} = (m-n)\mathcal{Q}_{m+n} + K_{m,n}$$

$$K_{m,n} = \delta\mathcal{Q}\left(\zeta_n, \mathcal{L}_{\zeta_m} g; g\right) = \frac{c_{\mathrm{R}} m^3}{12}\delta_{m+n}$$

$$\delta\mathcal{Q} = \delta\mathcal{Q}_{\mathrm{IW}} + \delta\mathcal{Q}_{\mathrm{ct}}$$

$$\delta\mathcal{Q}_{\mathrm{IW}} = \frac{1}{16\pi}\int_{\partial\Sigma} * F_{\mathrm{IW}}$$

$$F_{\mathrm{IW}}^{ab} = \frac{1}{2}\nabla^a\zeta^b h + \nabla^a h^{cb}\zeta_c + \nabla_c\zeta^a h^{cb} + \nabla_c h^{ca}\zeta^b - \nabla^a h\zeta^b - (a \leftrightarrow b)$$

$$F_{\mathrm{IW}}^{-y} = 4h_m^{y-}\zeta_n^y\Gamma_{-y}^-$$

$$h_m^{-y} = g^{+-}\partial_+\zeta_m^y = \frac{y^3\varepsilon_m''}{4W_+}, \text{ with } \varepsilon_m'' \equiv \partial_+^2\varepsilon$$

$$\delta\mathcal{Q}_{\mathrm{ct}} = \frac{1}{16\pi}\int_{\partial\Sigma} F_{\mathrm{ct}}^{ab} d\Sigma_{ab}$$

$$F_{\mathrm{ct}}^{ab} = -2N_d\ {}^c\nabla_c\left(\zeta^a h^{db}\right) - (a \leftrightarrow b),$$

$$N_+^+ = 1, N_-^- = -1$$

$$F_{\mathrm{ct}}^{-y} = 2h_m^{-y}\zeta_n^y\left(\Gamma_{+y}^+ - \Gamma_{-y}^-\right)$$

$$\delta\mathcal{Q} = \frac{1}{16\pi}\int d\theta d\omega^+ \epsilon_{\theta+-y}\left(F_{\mathrm{IW}}^{-y} + F_{\mathrm{ct}}^{-y}\right)$$
$$= i\frac{a r_*^2}{2\Delta_r'(r_+)} m^3\delta_{m+n,0}$$

$$c_{\mathrm{R}} = \frac{6a r_*^2}{\Delta_r'(r_+)} = \frac{6a}{\Delta_r'(r_+)}\left(r_1(r_+)r_2(r_+) - r_1(r_-)r_2(r_-)\right)$$

$$S_{\mathrm{Cardy}} = \frac{\pi^2}{3}(c_{\mathrm{L}}T_{\mathrm{L}} + c_{\mathrm{R}}T_{\mathrm{R}})$$

$$T_{\mathrm{L}} = \frac{\Delta_r'(r_+)}{4\pi a\Xi}\frac{\hat{r}_+^2 + \hat{r}_-^2}{r_*^2}$$
$$= \frac{\Delta_r'(r_+)\left[\left(r_+ + 2m\sinh^2(\delta_1)\right)\left(r_+ + 2\sinh^2(\delta_2)\right) + \left(r_- + 2m\sinh^2(\delta_1)\right)\left(r_- + 2m\sinh^2(\delta_2)\right) + 2a^2\right]}{4\pi a(1-a^2g^2)\left[\left(r_+ + 2m\sinh^2(\delta_1)\right)\left(r_+ + 2m\sinh^2(\delta_2)\right) - \left(r_- + 2m\sinh^2(\delta_1)\right)\left(r_- + 2m\sinh^2(\delta_2)\right)\right]}$$

$$T_{\mathrm{R}} = \frac{\Delta_r'(r_+)}{4\pi a\Xi} = \frac{\Delta_r'(r_+)}{4\pi a(1-a^2g^2)}$$

$$c_{\mathrm{L}} = c_{\mathrm{R}}$$
$$= \frac{6ar_*^2}{\Delta_r'(r_+)} = \frac{6a}{\Delta_r'(r_+)}\left(r_1(r_+)r_2(r_+) - r_1(r_-)r_2(r_-)\right)$$
$$= \frac{6a}{\Delta_r'(r_+)}\Big(\big(r_+ + 2m\sinh^2(\delta_1)\big)\big(r_+ + 2m\sinh^2(\delta_2)\big) - \big(r_- + 2m\sinh^2(\delta_1)\big)\big(r_- + 2m\sinh^2(\delta_2)\big)\Big)$$

$$S_{\mathrm{Cardy}} = \frac{\pi \hat{r}_+^2}{\Xi} = \frac{\pi\left[\big(r_1 + 2m\sinh^2(\delta_1)\big)\big(r_+ + 2m\sinh^2(\delta_2)\big) + a^2\right]}{1 - a^2 g^2},$$

$$T_0^{\mu\nu} = \rho u^\mu u^\nu + \mathcal{P}P^{\mu\nu}$$
$$J_{i,0}^\mu = r_i u^\mu$$
$$J_{S,0}^\mu = s u^\mu$$

$$T_1^{\mu\nu} = -\zeta\vartheta P^{\mu\nu} - 2\eta\sigma^{\mu\nu} + q^\mu u^\nu + u^\mu q^\nu$$
$$J_{i,1}^\mu = q_i^\mu$$
$$J_{S,1}^\mu = \frac{1}{\mathcal{T}}\left(q^\mu - \mu_i q_i^\mu\right)$$

$$\vartheta = \nabla_\mu u^\mu,$$
$$\sigma^{\mu\nu} = \frac{1}{2}\left(P^{\mu\lambda}\nabla_\lambda u^\nu + P^{\nu\lambda}\nabla_\lambda u^\mu\right) - \frac{1}{2}\vartheta P^{\mu\nu},$$
$$q^\mu = -\kappa P^{\mu\nu}(\partial_\nu \mathcal{T} + a_\nu \mathcal{T}),$$
$$q_i^\mu = -D_{ij}P^{\mu\nu}\partial_\nu\left(\frac{\mu_j}{\mathcal{T}}\right),$$

$$\Phi \equiv \mathcal{E} - \mathcal{T}\mathcal{S} - \mu_i\mathcal{R}_i, i = 1, \cdots, 4,$$

$$d\Phi = -\mathcal{S}d\mathcal{T} - \mathcal{P}dV - \mathcal{R}_i d\mu_i.$$

$$\Phi = -\mathrm{vol}(\mathrm{S}^2)\mathcal{T}^d h(v_i),$$

$$\rho = 2\mathcal{P} = 2\mathcal{T}^3 h(v)$$
$$r_i = \mathcal{T}^2 h_i(v)$$
$$s = \mathcal{T}^2(3h - v_i h_i)$$

$$E = \frac{2\mathrm{vol}(\mathrm{S}^2)\tau^3 h(v)}{(1-\omega_1^2)^2}$$
$$L_1 = \frac{2\mathrm{vol}(\mathrm{S}^2)\tau^3 h(v)\omega_1}{(1-\omega_1^2)^2}$$
$$R_i = \frac{\mathrm{vol}(\mathrm{S}^2)\tau^2 h_i(v)}{1-\omega_1^2}$$
$$S = \frac{\mathrm{vol}(\mathrm{S}^2)\tau^2[3h(v) - v_i h_i(v)]}{1-\omega_1^2}$$

$$T = \left(\frac{\partial E}{\partial S}\right)_{L_1,R_i} = \tau$$

$$\Omega_1 = \left(\frac{\partial E}{\partial L_1}\right)_{S,R_i} = \omega_1$$

$$\zeta_i = \left(\frac{\partial E}{\partial R_i}\right)_{S,L_1,R_j} = \tau\nu_i$$

$$\mathcal{Z} = \exp\left[-\frac{E - TS - \Omega_1 L_1 - \zeta_i R_i}{T}\right] = \exp\left[\frac{\mathrm{vol}(\mathrm{S}^2)T^2 h(\zeta/T)}{1-\Omega_1^2}\right]$$

$$h = \frac{\mathcal{P}}{\mathcal{T}^3} = \frac{4\pi^2(1+k_1)^2(1+k_2)^2}{\Xi g^4(3+k_1+k_2-k_1k_2)^3}$$

$$h_1 \equiv \frac{\partial h}{\partial \nu_1} = \frac{r_1}{\mathcal{T}^2} = \frac{\pi(1+k_1)(1+k_2)\sqrt{k_1}}{\Xi g^3(3+k_1+k_2-k_1k_2)^2}, h_2 \equiv \frac{\partial h}{\partial \nu_2} = \frac{r_2}{\mathcal{T}^2} = h_1$$

$$h_3 \equiv \frac{\partial h}{\partial \nu_3} = \frac{r_3}{\mathcal{T}^2} = \frac{\pi(1+k_1)(1+k_2)\sqrt{k_2}}{\Xi g^3(3+k_1+k_2-k_1k_2)^2}, h_4 \equiv \frac{\partial h}{\partial \nu_4} = \frac{r_4}{\mathcal{T}^2} = h_3$$

$$\nu_1 = \nu_2 = \frac{4\pi(1+k_2)\sqrt{k_1}}{3+k_1+k_2-k_1k_2}, \nu_3 = \nu_4 = \frac{4\pi(1+k_1)\sqrt{k_2}}{3+k_1+k_2-k_1k_2}$$

$$\mathcal{S} = \mathrm{vol}(S^2)\cdot s = \frac{\pi r_+^2(1+k_1)(1+k_2)}{\Xi},$$

$$\log \mathcal{Z} = \frac{16\pi^3(2N)^{3/2}T^2(1+k_1)^2(1+k_2)^2}{3(1-\Omega_1^2)(3+k_1+k_2-k_1k_2)^3}.$$

$$Z(q,q';y_1,y_2) = \mathrm{Tr}_{\mathcal{H}}\left[e^{-\beta'\{Q,S\}}e^{-\beta(\epsilon+j_3)}e^{-\gamma_1 h_1-\gamma_2 h_2}\right]$$

$$q \equiv e^{-\beta}, q' \equiv e^{-\beta'}, y_1 \equiv e^{-\gamma_1}, y_2 \equiv e^{-\gamma_2}$$

$$Z(q,q';y_1,y_2) = \frac{1}{(N!)^2}\sum_{\mathfrak{m},\tilde{\mathfrak{m}}\in\mathbb{Z}^N}\int\left[\prod_{i=1}^{N}\frac{d\alpha_i}{2\pi}\frac{d\tilde{\alpha}_i}{2\pi}\right]\prod_{\substack{i<j\\ \mathfrak{m}_i=\mathfrak{m}_j}}\left[2\sin\left(\frac{\alpha_i-\alpha_j}{2}\right)\right]^2\prod_{\substack{i<j\\ \tilde{\mathfrak{m}}_i=\tilde{\mathfrak{m}}_j}}\left[2\sin\left(\frac{\tilde{\alpha}_i-\tilde{\alpha}_j}{2}\right)\right]^2\times Z_0 Z_{\mathrm{cl}} Z_{\mathrm{ch}} Z_{\mathrm{g}}$$

$$Z_{\mathrm{cl}} = e^{ik\sum_{i=1}^{N}(\mathfrak{m}_i\alpha_i-\tilde{\mathfrak{m}}_i\tilde{\alpha}_i)}$$

$$Z_{\mathrm{ch}} = \prod_{i,j=1}^{N}\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(f_{ij}^{+}(q^n,q'^n;y_1^n,y_2^n)e^{in(\tilde{\alpha}_j-\alpha_i)}+f_{ij}^{-}(q^n,q'^n;y_1^n,y_2^n)e^{in(\alpha_i-\tilde{\alpha}_j)}\right)\right]$$

$$Z_{\mathrm{g}} = \prod_{i,j=1}^{N}\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(f_{ij}^{\mathrm{adj}}(q^n,q'^n)e^{-in(\alpha_i-\alpha_j)}+\tilde{f}_{ij}^{\mathrm{adj}}(q^n,q'^n)e^{-in(\tilde{\alpha}_i-\tilde{\alpha}_j)}\right)\right]$$

$$Z_0 = \prod_{i,j=1}^{N}\exp\left[-\beta\left(|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|-\sum_{i<j}|\mathfrak{m}_{ij}|-\sum_{i<j}|\tilde{\mathfrak{m}}_{ij}|\right)\right]$$

$$f_{ij}^{\pm\mathrm{B}}(q,q';y_1,y_2) \equiv \sum_{\substack{4\text{ scalars}\\ s=\pm1}} \sum_{J=\frac{|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|}{2}}^{\infty} \sum_{j_3=-J}^{J} \left(q^{\epsilon_J+j_3} q'^{\epsilon_J-h_3-j_3} y_1^{h_1} y_2^{h_2}\right)$$

$$f_{ij}^{\pm\mathrm{F}}(q,q';y_1,y_2) \equiv \sum_{4\text{ fermions}} \sum_{J=\frac{|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|\pm1}{2}}^{\infty} \sum_{j_3=-J}^{J} \left(q^{\epsilon_J+j_3} q'^{\epsilon_J-h_3-j_3} y_1^{h_1} y_2^{h_2}\right)$$

$$f_{ij}^{+\mathrm{B}}(q,q';y_1,y_2) = \left(\sqrt{\frac{y_1}{y_2}}+\sqrt{\frac{y_2}{y_1}}\right) \sum_{J=\frac{|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|}{2}}^{\infty} q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2J-l} q^l + \left(\sqrt{y_1y_2}+\frac{1}{\sqrt{y_1y_2}}\right) \sum_{J=\frac{|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|}{2}}^{\infty} q' q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l,$$

$$f_{ij}^{-\mathrm{B}}(q,q';y_1,y_2) = \left(\sqrt{y_1y_2}+\frac{1}{\sqrt{y_1y_2}}\right) \sum_{J=\frac{|\tilde{\mathfrak{m}}_i-\mathfrak{m}_j|}{2}}^{\infty} q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l + \left(\sqrt{\frac{y_1}{y_2}}+\sqrt{\frac{y_2}{y_1}}\right) \sum_{J=\frac{|\tilde{\mathfrak{m}}_i-\mathfrak{m}_j|}{2}}^{\infty} q' q^{\frac{1}{2}\sum_{l=0}^{2j}} q'^{2j-l} q^l$$

$$f_{ij}^{+\mathrm{F}}(q,q';y_1,y_2) = -\left(\sqrt{y_1y_2}+\frac{1}{\sqrt{y_1y_2}}\right) \sum_{J=\frac{|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|+1}{2}}^{\infty} q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l - \left(\sqrt{\frac{y_1}{y_2}}+\sqrt{\frac{y_2}{y_1}}\right) \sum_{J=\frac{|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|-1}{2}}^{\infty} q' q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l$$

$$f_{ij}^{-\mathrm{F}}(q,q';y_1,y_2) = -\left(\sqrt{\frac{y_1}{y_2}}+\sqrt{\frac{y_2}{y_1}}\right) \sum_{J=\frac{|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|+1}{2}}^{\infty} q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l - \left(\sqrt{y_1y_2}+\frac{1}{\sqrt{y_1y_2}}\right) \sum_{J=\frac{|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|-1}{2}}^{\infty} q' q^{\frac{1}{2}} \sum_{l=0}^{2j} q'^{2j-l} q^l.$$

$$f_{ij}^{\mathrm{adj,\,B}}(q,q') = \sum_{J=\frac{|\mathfrak{m}_{ij}|}{2}+1}^{} \frac{q^{2J+1}-q^{2J+1}}{q-q'} + \sum_{J=\frac{|\mathfrak{m}_{ij}|}{2}}^{} \frac{qq'(q^{2J+1}-q^{2J+1})}{q-q'} + \left(1-\delta_{\mathfrak{m}_{ij}}\right)\frac{q'q^{|\mathfrak{m}_{ij}|}-qq'^{|\mathfrak{m}_{ij}|}}{q-q'},$$

$$f_{ij}^{\mathrm{adj,\,F}}(q,q') = \sum_{J=\frac{|\mathfrak{m}_{ij}|+1}{2}}^{} \frac{q'(q^{2J+1}-q^{2J+1})}{q-q'} - \sum_{J=\frac{|\mathfrak{m}_{ij}|+1}{2}}^{} q\frac{q^{2J+1}q'^{2J+1}}{q-q'} - \left(1-\delta_{\mathfrak{m}_{ij}}\right)\frac{q(q^{\mathfrak{m}_{ij}}-q'^{\mathfrak{m}_{ij}})}{q-q'},$$

$$t_1 = \frac{1}{\sqrt{y_1y_2}}, t_2 = \sqrt{y_1y_2}, t_3 = \sqrt{\frac{y_2}{y_1}}, t_4 = \sqrt{\frac{y_1}{y_2}}$$

$$
\begin{aligned}
&f_{ij}^{+}(q^n,q'^n;t_i^n)\\
=&f_{ij}^{+\mathrm{B}}(q^n,q'^n;t_i^n)+(-1)^{n-1}f_{ij}^{+\mathrm{F}}(q^n,q'^n;t_i^n)\\
=&\frac{t_{3,4}^n}{q^n-q'^n}\left(\frac{q^{n(1/2+|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|)}(q^n+(-1)^nq'^n)}{1-q^{2n}}-\frac{q^{n/2}q'^{n(1+|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|)}(1+(-1)^n)}{1-q'^{2n}}\right)\\
&+\frac{t_{1,2}^n}{q^n-q'^n}\left(\frac{q^{n(3/2+|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|)}(q'^n+(-1)^nq^n)}{1-q^{2n}}-\frac{q^{n/2}q'^{n(2+|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|)}(1+(-1)^n)}{1-q'^{2n}}\right),\\
&f_{ij}^{-}(q^n,q'^n;t_i^n)\\
=&f_{ij}^{-\mathrm{B}}(q^n,q'^n;t_i^n)+(-1)^{n-1}f_{ij}^{-\mathrm{F}}(q^n,q'^n;t_i^n)\\
=&\frac{t_{1,2}^{-n}}{q^n-q'^n}\left(\frac{q^{n(1/2+|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|)}(q^n+(-1)^nq'^n)}{1-q^{2n}}-\frac{q^{n/2}q'^{n(1+|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|)}(1+(-1)^n)}{1-q'^{2n}}\right)\\
&+\frac{t_{3,4}^{-n}}{q^n-q'^n}\left(\frac{q^{n(3/2+|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|)}(q'^n+(-1)^nq^n)}{1-q^{2n}}-\frac{q^{n/2}q'^{n(2+|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|)}(1+(-1)^n)}{1-q'^{2n}}\right),
\end{aligned}
$$

$$
\begin{aligned}
&f_{ij}^{\mathrm{adj}}(q^n,q'^n)=f_{ij}^{\mathrm{adj,\,B}}(q^n,q'^n)+(-1)^{n-1}f_{ij}^{\mathrm{adj,\,F}}(q^n,q'^n)\\
=&\left(\frac{q^{n(|\mathfrak{m}_{ij}|+2)}}{1-q^{2n}}-\frac{q'^{n(|\mathfrak{m}_{ij}|+2)}}{1-q'^{2n}}\right)\frac{q^n+q'^n}{q^n-q'^n}(1+(-1)^n)\\
&+\frac{1-\delta_{\mathfrak{m}_{ij}}}{q^n-q'^n}\left(q^{n|\mathfrak{m}_{ij}|}(q'^n+(-1)^nq^n)-q^nq'^{n|\mathfrak{m}_{ij}|}(1+(-1)^n)\right)\\
&\tilde{f}_{ij}^{\mathrm{adj}}(q^n,q'^n)=\tilde{f}_{ij}^{\mathrm{adj,\,B}}(q^n,q'^n)+(-1)^{n-1}\tilde{f}_{ij}^{\mathrm{adj,\,F}}(q^n,q'^n)\\
=&\left(\frac{q^{n(|\tilde{\mathfrak{m}}_{ij}|+2)}}{1-q^{2n}}-\frac{q'^{n(|\tilde{\mathfrak{m}}_{ij}|+2)}}{1-q'^{2n}}\right)\frac{q^n+q'^n}{q^n-q'^n}(1+(-1)^n)\\
&+\frac{1-\delta_{\tilde{\mathfrak{m}}_{ij}}}{q^n-q'^n}\left(q^{n|\tilde{\mathfrak{m}}_{ij}|}(q'^n+(-1)^nq^n)-q^nq'^{n|\tilde{\mathfrak{m}}_{ij}|}(1+(-1)^n)\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{m}_i&\to\frac{N^\alpha}{\beta}x_i,\alpha_i\to y_i\\
\tilde{\mathfrak{m}}_i&\to\frac{N^\alpha}{\beta}x_i,\tilde{\alpha}_i\to\tilde{y}_i,
\end{aligned}
$$

$$
\sum_{i,j}e^{-nt\beta|\mathfrak{m}_i-\tilde{\mathfrak{m}}_j|\pm in(\tilde{\alpha}_j-\alpha_i)}\to\frac{2N^{2-\alpha}}{nt}\int dx\rho^2(x)e^{\pm in\delta y}
$$

$$\log Z_{\mathrm{ch}} = 2N^{3/2}\int dx\rho^2(x)\sum_{n=1}^{\infty}\left[\left(\frac{t_{3,4}^n q^{3n/2}e^{in\delta y}}{n^2(q^n-q'^n)(1-q^{2n})}+\frac{\left(-t_{3,4}\right)^n q^{3n/2}q'^n e^{in\delta y}}{n^2(q^n-q'^n)(1-q^{2n})}\right.\right.$$
$$+\frac{t_{1,2}^n q^{3n/2}q'^n e^{in\delta y}}{n^2(q^n-q'^n)(1-q^{2n})}+\frac{\left(-t_{1,2}\right)^n q^{5n/2}e^{in\delta y}}{n^2(q^n-q'^n)(1-q^{2n})}$$
$$+\frac{t_{1,2}^{-n} q^{3n/2}e^{-in\delta y}}{n^2(q^n-q'^n)(1-q^{2n})}+\frac{\left(-t_{1,2}\right)^{-n} q^{3n/2}q'^n e^{-in\delta y}}{n^2(q^n-q'^n)(1-q^{2n})}$$
$$\left.+\frac{t_{3,4}^{-n} q^{3n/2}q'^n e^{-in\delta y}}{n^2(q^n-q'^n)(1-q^{2n})}+\frac{\left(-t_{3,4}\right)^{-n} q^{5n/2}e^{-in\delta y}}{n^2(q^n-q'^n)(1-q^{2n})}\right)$$
$$-(1+(-1)^n)\frac{1+\lambda}{1-\lambda}\left(\frac{t_{3,4}^n q'^n q^{n/2}e^{in\delta y}}{n^2(q^n-q'^n)(1-q'^{2n})}+\frac{t_{1,2}^n q'^{2n}q^{n/2}e^{in\delta y}}{n^2(q^n-q'^n)(1-q^{2n})}\right.$$
$$\left.\left.+\frac{t_{1,2}^{-n} q'^n q^{n/2}e^{-in\delta y}}{n^2(q^n-q'^n)(1-q'^{2n})}+\frac{t_{3,4}^{-n} q'^{2n}q^{n/2}e^{-in\delta y}}{n^2(q^n-q'^n)(1-q'^{2n})}\right)\right],$$

$$\log Z_{\mathrm{g}} = 4N^{3/2}\int dx\rho^2(x)\sum_{n=1}^{\infty}\frac{1+(-1)^n}{n^2}\left[\frac{q^{3n}+q^{2n}q'^n}{(q^n-q'^n)(1-q^{2n})}-\frac{1+\lambda}{1-\lambda}\left(\frac{q^n q'^{2n}+q'^{3n}}{(q^n-q'^n)(1-q'^{2n})}\right)\right]$$

$$\bar{\omega} = \lambda\bar{\beta} \ll \lambda_i$$

$$\beta = \frac{1}{2}(\bar{\beta}+\bar{\omega}) = \frac{1}{2}(1+\lambda)\bar{\beta}$$
$$\beta' = \frac{1}{2}(\bar{\beta}-\bar{\omega}) = \frac{1}{2}(1-\lambda)\bar{\beta}$$

$$\frac{q^{l_1 n}q'^{l_2 n}}{(q^n-q'^n)(1-q^{2n})} = -\left(\frac{1}{n^2\lambda(1+\lambda)\bar{\beta}^2}+\frac{(2+\lambda)-l_1(1+\lambda)-l_2(1-\lambda)}{2n\lambda(1+\lambda)\bar{\beta}}\right)+\mathcal{O}(\bar{\beta}^0)$$
$$\frac{q^{l_1 n}q'^{l_2 n}}{(q^n-q'^n)(1-q'^{2n})} = -\left(\frac{1}{n^2\lambda(1-\lambda)\bar{\beta}^2}+\frac{(2-\lambda)-l_1(1+\lambda)-l_2(1-\lambda)}{2n\lambda(1-\lambda)\bar{\beta}}\right)+\mathcal{O}(\bar{\beta}^0)$$

$$Z = \int \mathcal{D}\rho\mathcal{D}\delta y \exp\left(-W_{\mathrm{eff}}[x;\rho,\delta y]\right)$$

$$W_{\text{eff}}[x;\rho,\delta y] = 2N^{3/2}\int dx\left(\frac{ik\rho(x\delta y-\mu)}{\bar{\beta}(1+\lambda)}\right.$$
$$+\rho^2(x)\left[\frac{1}{\lambda(1+\lambda)\bar{\beta}^2}\left(\text{Li}_4\left[e^{i(\lambda_i+\delta y)}\right]+\text{Li}_4\left[e^{-i(\lambda_i+\delta y)}\right]+\text{Li}_4\left[e^{i(\pi+\lambda_i+\delta y)}\right]+\text{Li}_4\left[e^{-i(\pi+\lambda_i+\delta y)}\right]\right)\right.$$
$$-\frac{1+\lambda}{8\lambda(1-\lambda)^2\bar{\beta}^2}\left(\text{Li}_4\left[e^{2i(\lambda_i+\delta y)}\right]+\text{Li}_4\left[e^{-2i(\lambda_i+\delta y)}\right]\right)+\frac{4\text{Li}_4[1]}{8\lambda(1+\lambda)\bar{\beta}^2}-\frac{4(1+\lambda)\text{Li}_4[1]}{8\lambda(1-\lambda)^2\bar{\beta}^2}$$
$$+\frac{2+\lambda-\frac{3}{2}(1+\lambda)}{2\lambda(1+\lambda)\bar{\beta}}\left(\text{Li}_3\left[e^{i(\lambda_{3,4}+\delta y)}\right]+\text{Li}_3\left[e^{-i(\lambda_{1,2}+\delta y)}\right]\right)$$
$$+\frac{2+\lambda-\frac{1}{2}(1+\lambda)-(1-\lambda)}{2\lambda(1+\lambda)\bar{\beta}}\left(\text{Li}_3\left[e^{i(\pi+\lambda_{3,4}+\delta y)}\right]+\text{Li}_3\left[e^{-i(\pi+\lambda_{1,2}+\delta y)}\right]\right)$$
$$+\frac{2+\lambda-\frac{3}{2}(1+\lambda)-(1-\lambda)}{2\lambda(1+\lambda)\bar{\beta}}\left(\text{Li}_3\left[e^{i(\lambda_{1,2}+\delta y)}\right]+\text{Li}_3\left[e^{-i(\lambda_{3,4}+\delta y)}\right]\right)$$
$$+\frac{2+\lambda-\frac{5}{2}(1+\lambda)}{2\lambda(1+\lambda)\bar{\beta}}\left(\text{Li}_3\left[e^{i(\pi+\lambda_{1,2}+\delta y)}\right]+\text{Li}_3\left[e^{-i(\pi+\lambda_{3,4}+\delta y)}\right]\right)$$
$$-\frac{2-\lambda-\frac{1}{2}(1+\lambda)-(1-\lambda)}{4\cdot 2\lambda(1-\lambda)\bar{\beta}}\frac{1+\lambda}{1-\lambda}\left(\text{Li}_3\left[e^{2i(\lambda_{3,4}+\delta y)}\right]+\text{Li}_3\left[e^{-2i(\lambda_{1,2}+\delta y)}\right]\right)$$
$$-\frac{2-\lambda-\frac{1}{2}(1+\lambda)-2(1-\lambda)}{4\cdot 2\lambda(1-\lambda)\bar{\beta}}\frac{1+\lambda}{1-\lambda}\left(\text{Li}_3\left[e^{2i(\lambda_{1,2}+\delta y)}\right]+\text{Li}_3\left[e^{-2i(\lambda_{3,4}+i\delta y)}\right]\right)$$
$$+\left(\frac{2(2+\lambda-3(1+\lambda))}{2\lambda(1+\lambda)\bar{\beta}}+\frac{2(2+\lambda-2(1+\lambda)-(1-\lambda))}{2\lambda(1+\lambda)\bar{\beta}}\right)\frac{\text{Li}_3[1]}{4}$$
$$\left.\left.-\left(\frac{2(2+\lambda-(1+\lambda)-2(1-\lambda))}{2\lambda(1-\lambda)\bar{\beta}}\frac{1+\lambda}{1-\lambda}+\frac{2(1+\lambda-3(1-\lambda))}{2\lambda(1-\lambda)\bar{\beta}}\frac{1+\lambda}{1-\lambda}\frac{\text{Li}_3[1]}{4}\right]+\mathcal{O}(\bar{\beta}^0)\right)\right.$$

$$\text{Li}_n[z]=\sum_{k=1}^{\infty}\frac{z^k}{k^n}$$

$$\text{Li}_n[z]+\text{Li}_n[-z]=2^{1-n}\text{Li}_n[z^2]$$

$$Q_i=\frac{\partial\log Z}{\partial\lambda_i}.$$

$$e^{2\delta_1+2\delta_2}=1+\frac{2}{ag},$$

$$S_{BH}=\frac{\pi^2}{3}c_LT_L+\frac{\pi^2}{3}c_RT_R$$
$$=2\pi\sqrt{\frac{c_LN_L}{6}}+2\pi\sqrt{\frac{c_RN_R}{6}}$$

$$T_L=\frac{1}{\pi}\sqrt{\frac{6N_L}{c_L}},T_R=\frac{1}{\pi}\sqrt{\frac{6N_R}{c_R}}.$$

$$Z_R = \sum_{N_R} q^{N_R} d(N_R) = \sum_{N_R} q^{N_R} e^{S_R} = \sum_{N_R} q^{N_R} e^{2\pi\sqrt{c_R N_R/6}}$$

$$N_R = q\frac{\partial}{\partial q}\log\, Z_R \approx \frac{\pi^2 c_R}{6(\log\,(q))^2}$$

$$N_L = q\frac{\partial}{\partial q}\log\, Z_L \approx \frac{\pi^2 c_L}{6(\log\,(q))^2}$$

$$\rho_L(k_0) = \frac{q^n}{1-q^n} = \frac{e^{-k_0/T_L}}{1-e^{-k_0/T_L}},$$
$$\rho_R(k_0) = \frac{q^n}{1-q^n} = \frac{e^{-k_0/T_R}}{1-e^{-k_0/T_R}},$$

$$\rho_L(k_0) = \frac{e^{-k_0/T_L}}{1-e^{-k_0/T_L}} \approx \frac{T_L}{k_0}, \rho_R(k_0) = \frac{e^{-k_0/T_R}}{1-e^{-k_0/T_R}} \approx \frac{T_R}{k_0}.$$

$$\rho_L(k_0) \approx \rho_R(k_0) \approx \frac{e^{-2k_0/T_H}}{1-e^{-2k_0/T_H}},$$

$$d\Gamma \sim \frac{d^4k}{2k_0}\frac{1}{p_0^L p_0^R}|\mathcal{A}|^2 c_L c_R \rho_L(k_0)\rho_R(k_0)$$

$$d\Gamma \sim (\text{ horizon area }) \cdot \frac{e^{-2k_0/T_H}}{1-e^{-2k_0/T_H}} d^4k$$

Un campo cuántico relativista, bajo supergravedad, supone la existencia de supermembranas en dimensiones altas, lo que se calcula así:

$$\frac{\mathcal{G}}{\mathcal{H}} = \frac{SL(2,\mathbb{R}) \times SU(2) \times \mathbb{R}_t \times \mathbb{S}^1_y}{U(1)_L \times U(1)_R} \times \mathcal{M}$$

$$u = t - y, v = t + y$$

$$\begin{aligned} ds^2 =& n_5(d\theta^2 + d\rho^2) + \frac{1}{\Sigma}\Big[-f_0 du dv + \frac{s(s+1)}{kR_y}\Delta dv^2 \\ &+\big(n_5\sinh^2\,\rho + n_5(s+1)^2 + \mathrm{k}^2R_y^2\big)\sin^2\,\theta d\phi^2 + \big(n_5\sinh^2\,\rho + n_5 s^2 + \mathrm{k}^2R_y^2\big)\cos^2\,\theta d\psi^2 \\ &+2\Big(\big(\mathrm{k}R_y - (s+1)\Delta\big)dy - (s+1)\Delta dt\Big)\sin^2\,\theta d\phi + 2\Big(\big(\mathrm{k}R_y + s\Delta\big)dy + s\Delta dt\Big)\cos^2\,\theta d\psi\Big] \\ &+d\mathbf{z}\cdot d\mathbf{z} \end{aligned}$$

$$\begin{aligned} B =& \frac{1}{\Sigma}\Big[\frac{\mathrm{k}R_y}{n_5}\Delta dt \wedge dy + \big(n_5\sinh^2\,\rho + n_5(s+1)^2 + \mathrm{k}^2R_y^2\big)\cos^2\,\theta d\phi \wedge d\psi \\ &+\Big(\big(\mathrm{k}R_y - (s+1)\Delta\big)dy - (s+1)\Delta dt\Big)\wedge\cos^2\,\theta d\psi + \Big(\big(\mathrm{k}R_y + s\Delta\big)dy + s\Delta dt\Big)\wedge\sin^2\,\theta d\phi\Big] \end{aligned}$$

$$e^{2\Phi} = \frac{\mathrm{k}R_y V_4}{n_1}\frac{\Delta}{\Sigma}$$

$$\Delta = \mathrm{k}R_y + \frac{n_5 s(s+1)}{\mathrm{k}R_y}, \Sigma = f_0 + \frac{\mathrm{k}R_y}{n_5}\Delta, f_0 = \sinh^2\,\rho - s\sin^2\,\theta + (s+1)\cos^2\,\theta$$

$$\frac{s(s+1)}{\mathrm{k}} \in \mathbb{Z}.$$

$$\tilde{t} = \frac{t}{R_y}, \tilde{y} = \frac{y}{R_y},$$

$$\begin{aligned}\frac{1}{n_5} ds^2 =& -\frac{1}{\mathrm{k}^2}\cosh^2\,\rho d\tilde{t}^2 + d\rho^2 + \frac{1}{\mathrm{k}^2}\sinh^2\rho d\tilde{y}^2 \\ &+ d\theta^2 + \cos^2\,\theta\left(d\psi + \frac{s}{\mathrm{k}}d\tilde{t} + \frac{s+1}{\mathrm{k}}d\tilde{y}\right)^2 + \sin^2\,\theta\left(d\phi - \frac{s+1}{\mathrm{k}}d\tilde{t} - \frac{s}{\mathrm{k}}d\tilde{y}\right)^2\end{aligned}$$

$$\psi_{\mathrm{NS}} = \psi + \frac{s}{\mathrm{k}}\tilde{t} + \frac{s+1}{\mathrm{k}}\tilde{y}, \phi_{\mathrm{NS}} = \phi - \frac{s+1}{\mathrm{k}}\tilde{t} - \frac{s}{\mathrm{k}}\tilde{y}, \tilde{t}_{\mathrm{NS}} = t, \tilde{y}_{\mathrm{NS}} = \tilde{y}$$

$$(\tilde{y}, \psi_{\mathrm{NS}}, \phi_{\mathrm{NS}}) \sim (\tilde{y}, \psi_{\mathrm{NS}}, \phi_{\mathrm{NS}}) + 2\pi\left(1, \frac{s+1}{\mathrm{k}}, -\frac{s}{\mathrm{k}}\right).$$

$$g_{tt} = -\frac{f_0 - \Delta_p}{\Sigma}, \Delta_p \equiv s(s+1) + n_5\left(\frac{s(s+1)}{kR_y}\right)^2.$$

$$ds^2 = d\theta^2 + \cos^2\,\theta d\psi^2 + \sin^2\,\theta d\phi^2$$

$$g_{\mathrm{su}} = e^{-i(\phi-\psi)\sigma_3/2} e^{i(\pi/2-\theta)\sigma_2} e^{-i(\phi+\psi)\sigma_3/2} = \begin{pmatrix} e^{-i\phi}\sin\,\theta & e^{i\psi}\cos\,\theta \\ -e^{-i\psi}\cos\,\theta & e^{i\phi}\sin\,\theta \end{pmatrix}$$

$$g(z, \bar{z}) = g_\ell(z) g_r(\bar{z})$$

$$g_\ell(z) = e^{-iv'z\sigma_3/2}, g_r(\bar{z}) = e^{-iv'\bar{z}\sigma_3/2}$$

$$g_\ell(z) = e^{-i\alpha'_\ell\sigma_1/2} e^{-iv'z\sigma_3/2}, g_r(\bar{z}) = e^{-iv'\bar{z}\sigma_3/2} e^{-i\alpha'_r\sigma_1/2}$$

$$g(\xi_0) = \begin{pmatrix} \cos\,v'\xi_0\cos\frac{\alpha'_\ell + \alpha'_r}{2} - i\sin\,v'\xi_0\cos\frac{\alpha'_\ell - \alpha'_r}{2} & -i\cos\,v'\xi_0\sin\frac{\alpha'_\ell + \alpha'_r}{2} + \sin\,v'\xi_0\sin\frac{\alpha'_\ell - \alpha'_r}{2} \\ -i\cos\,v'\xi_0\sin\frac{\alpha'_\ell + \alpha'_r}{2} - \sin\,v'\xi_0\sin\frac{\alpha'_\ell - \alpha'_r}{2} & \cos\,v'\xi_0\cos\frac{\alpha'_\ell + \alpha'_r}{2} + i\sin\,v'\xi_0\cos\frac{\alpha'_\ell - \alpha'_r}{2} \end{pmatrix}$$

$$-\cos\,2\theta = \cos\,(\alpha'_\ell + \alpha'_r)\cos^2\,(v'\xi_0) + \cos\,(\alpha'_\ell - \alpha'_r)\sin^2\,(v'\xi_0).$$

$$\theta_\pm = \frac{1}{2}(\pi - |\alpha'_\ell \mp \alpha'_r|)$$

$$\begin{aligned}\mathcal{E} &= \frac{n_5}{2}\mathrm{tr}[\partial g \partial g^{-1}] + \frac{n_5}{2}\mathrm{tr}\left[\bar{\partial} g \bar{\partial} g^{-1}\right] = \frac{n_5}{2}(v')^2, \\ J^3_{\mathrm{su}} &= -\frac{in_5}{2}\mathrm{tr}[(\partial g)g^{-1}\sigma_3] = n_5(\cos^2\,\theta\partial\psi - \sin^2\,\theta\partial\phi) = -\frac{n_5}{2}v'\cos\,(\alpha'_\ell), \\ \bar{J}^3_{\mathrm{su}} &= -\frac{in_5}{2}\mathrm{tr}\left[g^{-1}(\bar{\partial} g)\sigma_3\right] = -n_5\left(\cos^2\,\theta\bar{\partial}\psi + \sin^2\,\theta\bar{\partial}\phi\right) = -\frac{n_5}{2}v'\cos\,(\alpha'_r);\end{aligned}$$

$$j' = \frac{n_5}{2}v', \frac{m'}{j'} = -\cos\,(\alpha'_\ell), \frac{\bar{m}'}{j'} = -\cos\,(\alpha'_r)$$

$$ds^2 = -\cosh^2\,\rho d\tau^2 + d\rho^2 + \sinh^2\,\rho d\sigma^2$$

$$g_{\mathrm{sl}} = e^{i(\tau+\sigma)\sigma_3/2} e^{\rho\sigma_1} e^{i(\tau-\sigma)\sigma_3/2} = \begin{pmatrix} e^{i\tau}\cosh\,\rho & e^{i\sigma}\sinh\,\rho \\ e^{-i\sigma}\sinh\,\rho & e^{-i\tau}\cosh\,\rho \end{pmatrix}$$

$$g_\ell(z) = e^{i\nu z\sigma_3/2}, g_r(\bar{z}) = e^{i\nu\bar{z}\sigma_3/2}$$

$$g_\ell(z) = e^{\alpha_\ell\sigma_1/2} e^{i\nu z\sigma_3/2}, g_r(\bar{z}) = e^{i\nu\bar{z}\sigma_3/2} e^{\alpha_r\sigma_1/2}$$

$$g(\xi_0) = \begin{pmatrix} \cos\,\nu\xi_0\cosh\,\dfrac{\alpha_\ell+\alpha_r}{2} + i\sin\,\nu\xi_0\cosh\,\dfrac{\alpha_\ell-\alpha_r}{2} & \cos\,\nu\xi_0\sinh\,\dfrac{\alpha_\ell+\alpha_r}{2} - i\sin\,\nu\xi_0\sinh\,\dfrac{\alpha_\ell-\alpha_r}{2} \\ \cos\,\nu\xi_0\sinh\,\dfrac{\alpha_\ell+\alpha_r}{2} + i\sin\,\nu\xi_0\sinh\,\dfrac{\alpha_\ell-\alpha_r}{2} & \cos\,\nu\xi_0\cosh\,\dfrac{\alpha_\ell+\alpha_r}{2} - i\sin\,\nu\xi_0\cosh\,\dfrac{\alpha_\ell-\alpha_r}{2} \end{pmatrix}$$

$$\cosh\,2\rho = \cos^2\,(\nu\xi_0)\cosh\,(\alpha_\ell+\alpha_r) + \sin^2\,(\nu\xi_0)\cosh\,(\alpha_\ell-\alpha_r),$$

$$\rho_\pm = \frac{1}{2}|\alpha_\ell \pm \alpha_r|.$$

$$\begin{aligned} \mathcal{E} &= -\frac{n_5}{2}\mathrm{tr}[\partial g\partial g^{-1}] - \frac{n_5}{2}\mathrm{tr}\left[\bar{\partial} g\bar{\partial} g^{-1}\right] = -\frac{n_5}{2}\nu^2 \\ J^3_{\mathrm{sl}} &= -\frac{in_5}{2}\mathrm{tr}[(\partial g)g^{-1}\sigma_3] = n_5(\cosh^2\,\rho\partial\tau - \sinh^2\,\rho\partial\sigma) = \frac{n_5}{2}\nu\cosh\,(\alpha_\ell) \\ \bar{J}^3_{\mathrm{sl}} &= -\frac{in_5}{2}\mathrm{tr}\left[g^{-1}(\bar{\partial} g)\sigma_3\right] = n_5\left(\cosh^2\,\rho\bar{\partial}\tau + \sinh^2\,\rho\bar{\partial}\sigma\right) = \frac{n_5}{2}\nu\cosh\,(\alpha_r); \end{aligned}$$

$$j = \frac{n_5}{2}\nu, \frac{m}{j} = \cosh\,\alpha_\ell, \frac{\bar{m}}{j} = \cosh\,\alpha_r.$$

$$\mathrm{m} = s+1, \mathrm{n} = s,$$

$$\mathcal{S} = \int d\xi\sqrt{\mathrm{g}}\left(\mathrm{g}^{\xi\xi}\partial_\xi X^\mu\partial_\xi X^\nu G_{\mu\nu}(X) + \mathrm{M}^2\right)$$

$$\mathrm{g}^{\xi\xi}\partial_\xi X^\mu\partial_\xi X^\nu G_{\mu\nu}(X) = \mathrm{M}^2.$$

$$\mathcal{L}_g = \mathcal{L}_{\mathrm{sl}} + \mathcal{L}_{\mathrm{su}} = \frac{n_5}{2}\left[(-\cosh^2\,\rho\dot{\tau}^2 + \dot{\rho}^2 + \sinh^2\,\rho\dot{\sigma}^2) + \left(\dot{\theta}^2 + \cos^2\,\theta\dot{\psi}^2_{\mathrm{NS}} + \sin^2\,\theta\dot{\phi}^2_{\mathrm{NS}}\right)\right]$$

$$\dot{\psi}_{\mathrm{NS}} = \dot{\psi} + \mathrm{n}\dot{\tau} + \mathrm{m}\dot{\sigma}, \dot{\phi}_{\mathrm{NS}} = \dot{\phi} - \mathrm{m}\dot{\tau} - \mathrm{n}\dot{\sigma}$$

$$\begin{aligned} -p_\tau &= n_5\left(\cosh^2\,\rho\dot{\tau} - \mathrm{n}\cos^2\,\theta\dot{\psi}_{\mathrm{NS}} + \mathrm{m}\sin^2\,\theta\dot{\phi}_{\mathrm{NS}}\right) \\ p_\sigma &= n_5\left(\sinh^2\,\rho\dot{\sigma} + \mathrm{m}\cos^2\,\theta\dot{\psi}_{\mathrm{NS}} - \mathrm{n}\sin^2\,\theta\dot{\phi}_{\mathrm{NS}}\right) \\ m_\psi &= n_5\left(\cos^2\,\theta\dot{\psi}_{\mathrm{NS}}\right) \\ m_\phi &= n_5\left(\sin^2\,\theta\dot{\phi}_{\mathrm{NS}}\right) \end{aligned}$$

$$\begin{aligned} \mathrm{k}\varepsilon &= n_5\cosh^2\,\rho\dot{\tau} - \mathrm{n}m_\psi + \mathrm{m}m_\phi \\ \mathrm{k}n_y &= n_5\sinh^2\,\rho\dot{\sigma} + \mathrm{m}m_\psi - \mathrm{n}m_\phi \end{aligned}$$

$$\begin{aligned} n_5\cosh^2\,\rho\dot{\tau} &= m + \bar{m}, m_\phi = m' + \bar{m}' \\ -n_5\sinh^2\,\rho\dot{\sigma} &= m - \bar{m}, m_\psi = \bar{m}' - m' \end{aligned}$$

$$\mathrm{k}\varepsilon = m + \bar{m} + (2s+1)m' + \bar{m}'$$
$$-\mathrm{k}n_y = m - \bar{m} + (2s+1)m' - \bar{m}'$$

$$\mathcal{H}_{\mathrm{sl}} = n_5^2\dot{\rho}^2 + \frac{(m-\bar{m})^2}{\sinh^2\,\rho} - \frac{(m+\bar{m})^2}{\cosh^2\,\rho} = -4j^2$$
$$\mathcal{H}_{\mathrm{su}} = n_5^2\dot{\theta}^2 + \frac{(m'-\bar{m}')^2}{\cos^2\,\theta} + \frac{(m'+\bar{m}')^2}{\sin^2\,\theta} = +4(j')^2$$

$$\mathcal{H}_{\mathrm{tot}} = \mathcal{H}_{\mathrm{sl}} + \mathcal{H}_{\mathrm{su}} = \mathrm{M}^2$$

$$m' = -j + \hat{\mathfrak{m}}, m = j + \hat{\mathfrak{n}}$$

$$\sinh^2\,\rho = \frac{\hat{\mathfrak{n}}}{2j}\,, \cos^2\,\theta = \frac{\hat{\mathfrak{m}}}{2j}$$

$$D_{j'm'\bar{m}'}(\theta,\phi,\psi) = e^{-im'(\phi+\psi)}e^{-i\bar{m}'(\phi-\psi)}d_{j'm'\bar{m}'}(\theta)$$
$$d_{j'm'\bar{m}'}(\theta) = (\cos\,\theta)^a(\sin\,\theta)^b P_q^{(a,b)}(\theta)$$
$$P_q^{(a,b)}(\theta) = \sum_{p=0}^{q}\binom{q+a}{q-p}\binom{q+b}{p}(\sin\,\theta)^{2p}(\cos\,\theta)^{2q-2p}$$

$$j' \to -j\,, m' \to m\,, \bar{m}' \to \bar{m}\,, \hat{\mathfrak{m}} \to \hat{\mathfrak{n}}$$

$$\left(\frac{\pi}{2} - \theta\right) \to -i\rho\,, \phi \to -\tau\,, \psi \to \sigma + \frac{\pi}{2}$$

$$e^{-2ij'\phi}(\sin\,\theta)^{2j'} \longrightarrow e^{-2ij\tau}(\cosh\,\rho)^{-2j}$$

$$\mathrm{k}\varepsilon = -\mathrm{k}n_y = \hat{\mathfrak{n}} + (2s+1)\hat{\mathfrak{m}} - 2sj$$

$$\Delta_{\ell\hat{\mathfrak{m}}\hat{\mathfrak{n}}}e^{iv_{\ell\hat{\mathfrak{m}}\hat{\mathfrak{n}}}} = \left(\frac{r_{\mathrm{b}}}{\sqrt{r_{\mathrm{b}}^2+a^2}}\right)^{\hat{\mathfrak{n}}}\left(\frac{a}{\sqrt{r_{\mathrm{b}}^2+a^2}}\right)^{\ell}\sin^{\ell-\hat{\mathfrak{m}}}\,\theta\cos^{\hat{\mathfrak{m}}}\,\theta e^{i[(\hat{\mathfrak{n}}+(2s+1)\hat{\mathfrak{m}}-2sj)v/\mathbf{k}+(\ell-\hat{\mathfrak{m}})\phi-\hat{\mathfrak{m}}\psi]}$$

$$\frac{r_b^2}{a^2} = \sinh^2\,\rho$$

$$\mathrm{k}\varepsilon = -\mathrm{k}n_y = 2j(\sinh^2\,\rho + (2s+1)\cos^2\,\theta - s) = n_5 v f_0$$

$$\mathrm{k}(\varepsilon - n_y) \approx n_5 v[\cosh\,(\rho_+ + \rho_-) + (\mathrm{m}+\mathrm{n})\cos\,(\theta_+ + \theta_-)]$$
$$\mathrm{k}(\varepsilon + n_y) \approx n_5 v[\cosh\,(\rho_+ - \rho_-) - (\mathrm{m}-\mathrm{n})\cos\,(\theta_+ - \theta_-)]$$

$$\mathrm{k}\varepsilon \approx n_5 v[\cosh\,\rho_+\cosh\,\rho_- - \mathrm{m}\sin\,\theta_+\sin\,\theta_- + \mathrm{n}\cos\,\theta_+\cos\,\theta_-]$$

$$\mathrm{k}(\varepsilon + n_y) \approx \frac{n_5 v}{2}(\alpha_r^2 + (\alpha_r')^2)$$

$$f_0 = \cosh^2\,\rho - (s+1)\sin^2\,\theta + s\cos^2\,\theta$$

$$\frac{\mathcal{G}}{\mathcal{H}} = \frac{SL(2,\mathbb{R}) \times SU(2) \times \mathbb{R}_{\mathrm{t}} \times \mathbb{S}^1_{\mathrm{y}}}{U(1)_L \times U(1)_R}$$

$$ds^2 = n_5(-\cosh^2\,\rho d\tau^2 + d\rho^2 + \sinh^2\,\rho d\sigma^2) + n_5(d\theta^2 + \cos^2\,\theta d\psi^2 + \sin^2\,\theta d\phi^2) - d\mathrm{t}^2 + d\mathrm{y}^2.$$

$$\begin{aligned} \mathcal{J} &= J^3_{\mathrm{sl}} + l_2 J^3_{\mathrm{su}} + l_3 i\partial\mathrm{t} + l_4 i\partial\mathrm{y}, \quad \boldsymbol{\lambda} = \psi^3_{\mathrm{sl}} + l_2\psi^3_{\mathrm{su}} + l_3\psi^{\mathrm{t}} + l_4\psi^{\mathrm{y}} \\ \overline{\mathcal{J}} &= \bar{J}^3_{\mathrm{sl}} + r_2 \bar{J}^3_{\mathrm{su}} + r_3 i\bar{\partial}\mathrm{t} + r_4 i\bar{\partial}\mathrm{y}, \quad \overline{\boldsymbol{\lambda}} = \bar{\psi}^3_{\mathrm{sl}} + r_2\bar{\psi}^3_{\mathrm{su}} + r_3\bar{\psi}^{\mathrm{t}} + r_4\bar{\psi}^{\mathrm{y}} \end{aligned}$$

$$n_5(-1 + l_2^2) - l_3^2 + l_4^2 = n_5(-1 + r_2^2) - r_3^2 + r_4^2 = 0$$

$$\begin{aligned} &l_2 = 2s + 1, r_2 = 1, \qquad l_3 = r_3 = -\Delta = -\left(\mathrm{k}R_y + \frac{n_5 s(s+1)}{\mathrm{k}R_y}\right), \\ &l_4 = \mathrm{k}R_y - \frac{n_5 s(s+1)}{\mathrm{k}R_y}, \quad r_4 = -\mathrm{k}R_y - \frac{n_5 s(s+1)}{\mathrm{k}R_y}. \end{aligned}$$

$$\Sigma = \sinh^2\,\rho + l_2 r_2 \cos^2\,\theta + \frac{1 - l_2 r_2}{2} + \frac{l_3 r_3 - l_4 r_4}{2n_5}.$$

$$\begin{aligned} &\delta\tau = l_1\alpha + r_1\beta = (\alpha + \beta), \qquad \delta\phi = -l_2\alpha - r_2\beta = -(\alpha + \beta)(s+1) - (\alpha - \beta)s, \\ &\delta\sigma = l_1\alpha + r_1\beta = (\alpha - \beta), \qquad \delta\psi = l_2\alpha - r_2\beta = (\alpha + \beta)s + (\alpha - \beta)(s+1), \\ &\delta\mathrm{t} = l_3\alpha + r_3\beta = -(\alpha + \beta)\Delta, \quad \delta\mathrm{y} = -l_4\alpha - r_4\beta = -\mathrm{k}R_y(\alpha - \beta) + \frac{n_5 s(s+1)}{\mathrm{k}R_y}(\alpha + \beta). \end{aligned}$$

$$\tilde{t}_{\mathrm{gi}} = \tilde{\mathrm{t}} + \mathrm{k}\tau, \tilde{y}_{\mathrm{gi}} = \tilde{\mathrm{y}} + \mathrm{k}\sigma,$$

$$\phi_{\mathrm{gi},1} = \phi - \frac{s+1}{\mathrm{k}}\tilde{\mathrm{t}} - \frac{s}{\mathrm{k}}\tilde{\mathrm{y}}, \psi_{\mathrm{gi},1} = \psi + \frac{s^2}{\mathrm{k}}\tilde{\mathrm{t}} + \frac{s+1}{\mathrm{k}}\tilde{\mathrm{y}},$$

$$\phi_{\mathrm{gi},2} = \phi + (s+1)\tau + s\sigma, \psi_{\mathrm{gi},2} = \psi - s\tau - (s+1)\sigma.$$

$$\delta(\sigma, \psi, \phi) = \frac{2\pi}{\mathrm{k}}(1, s+1, -s)$$

$$\begin{aligned} &|++\rangle_{\mathrm{k},s}: h = \frac{\mathrm{k}}{4} + \frac{s(s+1)}{\mathrm{k}}, M' = s + \frac{1}{2} \\ &\bar{h} = \frac{\mathrm{k}}{4}, \bar{M}' = \frac{1}{2}. \end{aligned}$$

$$\begin{aligned} |--\rangle_{\mathrm{k},s} = |++\rangle_{\mathrm{k},s-1}: h &= \frac{\mathrm{k}}{4} + \frac{s(s-1)}{\mathrm{k}}, M' = s - \frac{1}{2} \\ \bar{h} &= \frac{\mathrm{k}}{4}, \bar{M}' = \frac{1}{2}. \end{aligned}$$

$$\mathcal{Q}_{\mathrm{BRST}} = \oint dz[(cT + \gamma G + \text{ ghosts }) + (\tilde{c}\mathcal{J} + \tilde{\gamma}\boldsymbol{\lambda})]$$

$$\Phi^{(w)}_{j;m,\bar{m}} \Psi^{(w',\bar{w}')}_{j';m',\bar{m}'} e^{-iE\mathrm{t} + iP_y\mathrm{y} + i\bar{P}_y\bar{\mathrm{y}}}$$

$$P_y = \frac{n_y}{R_y} + w_y R_y, \bar{P}_y = \frac{n_y}{R_y} - w_y R_y$$

$$J^a_{\mathrm{sl}} = j^a_{\mathrm{sl}} - \frac{i}{2}(\epsilon_{\mathrm{sl}})^a_{bc}\psi^b_{\mathrm{sl}}\psi^c_{\mathrm{sl}}\,, J^a_{\mathrm{su}} = j^a_{\mathrm{su}} - \frac{i}{2}(\epsilon_{\mathrm{su}})^a_{bc}\psi^b_{\mathrm{su}}\psi^c_{\mathrm{su}}$$

$$L_0 - \frac{1}{2} = -\frac{j(j-1)}{n_5} + \frac{j'(j'+1)}{n_5} - Mw - \frac{n_5}{4}w^2 + M'w' + \frac{n_5}{4}w'^2 - \frac{1}{4}E^2 + \frac{1}{4}P_y^2 + h_L = 0$$
$$\bar{L}_0 - \frac{1}{2} = -\frac{j(j-1)}{n_5} + \frac{j'(j'+1)}{n_5} - \bar{M}w - \frac{n_5}{4}w^2 + \bar{M}'\bar{w}' + \frac{n_5}{4}\bar{w}'^2 - \frac{1}{4}E^2 + \frac{1}{4}\bar{P}_y^2 + h_R = 0$$

$$\Psi^{(w',\bar{w}')}_{j';m',\bar{m}'} = \Lambda^{(\alpha_{\mathrm{su}},\bar{\alpha}_{\mathrm{su}})}_{j';m',\bar{m}'}\exp\left[i\frac{2}{\sqrt{n_5}}\left(\left(M' + \frac{n_5}{2}w'\right)Y' + \left(\bar{M}' + \frac{n_5}{2}\bar{w}'\right)\bar{Y}'\right)\right]$$

$$J^3_{\mathrm{su}} = i\sqrt{n_5}\partial Y'\,, \bar{J}^3_{\mathrm{su}} = i\sqrt{n_5}\bar{\partial}\bar{Y}'$$

$$h_\Lambda = \frac{j'(j'+1)}{n_5} - \frac{M'^2}{n_5} + \frac{\alpha^2_{\mathrm{su}}}{2},$$

$$h_\Psi = h_\Lambda + \frac{1}{n_5}\left(M' + \frac{n_5}{2}w'\right)^2$$

$$(2M + n_5 w) + l_2(2M' + n_5 w') + l_3 E + l_4 P_y = 0$$
$$(2\bar{M} + n_5 w) + r_2(2\bar{M}' + n_5\bar{w}') + r_3 E + r_4\bar{P}_y = 0$$

$$\mathcal{V}^{\alpha\dot{\alpha}}, \mathcal{S}^{AB},$$

$$(|++\rangle_{\mathrm{k}})^{2j'+1} \longrightarrow |I\rangle_{(2j'+1)\mathrm{k}}$$

$$n_y w_y = 0.$$

$$E = \frac{\varepsilon}{R_y}$$

$$j = j' + 1$$

$$\mathrm{k}(\varepsilon - n_y) = 2M + 2(2s+1)M',$$
$$\mathrm{k}(\varepsilon + n_y) = 2\bar{M} + 2\bar{M}'.$$

$$\mathrm{k}\varepsilon = M + \bar{M} + (2s+1)M' + \bar{M}'$$
$$-\mathrm{k}n_y = M - \bar{M} + (2s+1)M' - \bar{M}'$$

$$\bar{M} = \bar{J} = j + \bar{\epsilon}, \bar{M}' = -\bar{J}' = -(j' + \bar{\epsilon}')$$

$$\varepsilon = -n_y.$$

$$M + (2s+1)M' + \mathrm{k}n_y = 0$$

$$M = J + \hat{\mathfrak{n}} = (j + \epsilon) + \hat{\mathfrak{n}}, M' = -J' + \hat{\mathfrak{m}} = -(j' + \epsilon') + \hat{\mathfrak{m}},$$

$$\hat{\mathrm{n}} + (2s+1)\hat{\mathrm{m}} - 2sj' + \epsilon - (2s+1)\epsilon' + 1 = -\mathrm{k}n_y.$$

$$-2sj' + \epsilon - (2s+1)\epsilon' + 1 = -\mathrm{k}n_y$$

$$-2sj' - 2s\epsilon' = -\mathrm{k}n_y$$

$$\left(|++\rangle_{\mathrm{k},s}\right)^{2j'+1} \longrightarrow |++\rangle_{(2j'+1)\mathrm{k},(2j'+1)s}$$

$$\left(|++\rangle_{\mathrm{k},s}\right)^{2j'+1} \longrightarrow |--\rangle_{(2j'+1)\mathrm{k},(2j'+1)s} = |++\rangle_{(2j'+1)\mathrm{k},(2j'+1)(s-1)}$$

$$|\hat{\mathrm{m}},\hat{\mathrm{n}};I\rangle_{(2j'+1)\mathrm{k},(2j'+1)s} = \left(\mathrm{J}^{+}_{-\frac{(2s+1)}{\mathrm{k}}}\right)^{\hat{\mathrm{m}}}\left(\mathrm{L}_{-\frac{1}{\mathrm{k}}} - \mathrm{J}^{3}_{-\frac{1}{\mathrm{k}}}\right)^{\hat{\mathrm{n}}}|I\rangle_{(2j'+1)\mathrm{k},(2j'+1)s}$$

$$\hat{\mathrm{n}} + (2s+1)\hat{\mathrm{m}} - 2sj' + (2s+2) = -\mathrm{k}n_y$$

$$\left(\mathrm{G}^{+1}_{-\frac{s+1}{\mathrm{k}}}\mathrm{G}^{+2}_{-\frac{s+1}{\mathrm{k}}} + \frac{1}{\mathrm{k}}\,\mathrm{J}^{+}_{-\frac{2s+1}{\mathrm{k}}}\left(\mathrm{L}_{-\frac{1}{\mathrm{k}}} - \mathrm{J}^{3}_{-\frac{1}{\mathrm{k}}}\right)\right)|\hat{\mathrm{m}},\hat{\mathrm{n}};++\rangle_{(2j'+1)\mathrm{k},(2j'+1)s}.$$

$$\hat{\mathrm{n}} + (2s+1)\hat{\mathrm{m}} - 2sj' + 2 = -\mathrm{k}n_y$$

$$\left(\mathrm{G}^{+1}_{-\frac{s+1}{\mathrm{k}}}\mathrm{G}^{+2}_{-\frac{s+1}{\mathrm{k}}} + \frac{1}{\mathrm{k}}\,\mathrm{J}^{+}_{-\frac{2s+1}{\mathrm{k}}}\left(\mathrm{L}_{-\frac{1}{\mathrm{k}}} - \mathrm{J}^{3}_{-\frac{1}{\mathrm{k}}}\right)\right)|\hat{\mathrm{m}},\hat{\mathrm{n}};--\rangle_{(2j'+1)\mathrm{k},(2j'+1)s}.$$

$$\mathrm{L}_{\frac{n}{k}} \rightarrow \mathrm{L}_{\frac{n}{k}},\ \mathrm{J}^{\pm}_{\frac{n}{k}} \rightarrow \mathrm{J}^{\pm}_{\frac{n\mp 2s}{k}}, J^{3}_{\frac{n}{k}} \rightarrow \mathrm{J}^{3}_{\frac{n}{k}}, \mathrm{G}^{\pm,A}_{\frac{n}{k}} \rightarrow \mathrm{G}^{\pm,A}_{\frac{n\mp s}{k}},$$

$$E = w_y R_y + \frac{\varepsilon}{R_y}$$

$$\mathrm{k}(\varepsilon - n_y) = 2\left(M_{\mathrm{tot}} + \frac{n_5}{2}w\right) + (2s+1)2\left(M'_{\mathrm{tot}} + \frac{n_5}{2}w'\right) - \frac{2s(s+1)}{\mathrm{k}}n_5 w_y$$
$$\mathrm{k}(\varepsilon + n_y) = 2\left(\bar{M}_{\mathrm{tot}} + \frac{n_5}{2}w\right) + 2\left(\bar{M}'_{\mathrm{tot}} + \frac{n_5}{2}\bar{w}'\right)$$

$$M_{\mathrm{tot}} = M + M_{\mathrm{osc}}, M'_{\mathrm{tot}} = M' + M'_{\mathrm{osc}}$$

$$0 = \frac{-j(j-1) + j'(j'+1)}{n_5} - M_{\mathrm{tot}}w + M'_{\mathrm{tot}}w' - \frac{n_5}{4}(w^2 - (w')^2) - \frac{w_y}{2}(\varepsilon - n_y) + h_L,$$
$$0 = \frac{-j(j-1) + j'(j'+1)}{n_5} - \bar{M}_{\mathrm{tot}}w + \bar{M}'_{\mathrm{tot}}\bar{w}' - \frac{n_5}{4}(w^2 - (\bar{w}')^2) - \frac{w_y}{2}(\varepsilon + n_y) + h_R,$$

$$h_L = N_L + \frac{\alpha_{\mathrm{sl}}^2 + \alpha_{\mathrm{su}}^2}{2} - \frac{1}{2}, h_R = N_R + \frac{\bar{\alpha}_{\mathrm{sl}}^2 + \bar{\alpha}_{\mathrm{su}}^2}{2} - \frac{1}{2}.$$

$$J^3_{\mathrm{sl}}, J^3_{\mathrm{su}}, i\partial\mathrm{t} \equiv J_t, i\partial\mathrm{y} \equiv J_y\ ;\ \bar{J}^3_{\mathrm{sl}}, \bar{J}^3_{\mathrm{su}}, i\bar{\partial}\mathrm{t} \equiv \bar{J}_t, i\bar{\partial}\mathrm{y} \equiv \bar{J}_y,$$

$$M_{\mathrm{tot}}+\frac{n_5}{2}w,\quad M'_{\mathrm{tot}}+\frac{n_5}{2}w',\quad \frac{E}{2},\quad \frac{1}{2}\left(\frac{n_y}{R_y}+w_yR_y\right)$$
$$\bar{M}_{\mathrm{tot}}+\frac{n_5}{2}w,\quad \bar{M}'_{\mathrm{tot}}+\frac{n_5}{2}\bar{w}',\quad \frac{E}{2},\quad \frac{1}{2}\left(\frac{n_y}{R_y}-w_yR_y\right)$$

$$\delta w=q,\delta w'=-l_2q,\delta\bar{w}'=-r_2q,$$
$$\delta E=l_3q,\delta P_y=-l_4q,\delta\bar{P}_y=-r_4q,q\in\mathbb{Z},$$
$$\Longrightarrow\ \delta n_y=-n_5\frac{s(s+1)}{\mathrm{k}}q,\delta w_y=-\mathrm{k}q.$$

$$\begin{aligned}\mathcal{M}&=J^3_{\mathrm{sl}}-\frac{n_5}{l_3}J_t,&\quad\overline{\mathcal{M}}&=\bar{J}^3_{\mathrm{sl}}-\frac{n_5}{l_3}\bar{J}_t\\ \mathcal{M}'&=J^3_{\mathrm{su}}-\frac{n_5l_2}{l_4}J_y,&\quad\overline{\mathcal{M}}'&=\bar{J}^3_{\mathrm{su}}-\frac{n_5r_2}{r_4}\bar{J}_y\\ \mathcal{L}&=J_t+\frac{l_3}{l_4}J_y,&\quad\overline{\mathcal{L}}&=\bar{J}_t+\frac{l_3}{r_4}\bar{J}_y\end{aligned}$$

$$\mathcal{M}+l_2\mathcal{M}'+\frac{(l_3^2+n_5)}{l_3}\mathcal{L}=\mathcal{J}$$

$$\mathcal{M}-\overline{\mathcal{M}}=M_{\mathrm{tot}}-\bar{M}_{\mathrm{tot}},$$

$$\mathcal{M}+\overline{\mathcal{M}}=M_{\mathrm{tot}}+\bar{M}_{\mathrm{tot}}+n_5\left(w+\frac{w_y}{\mathrm{k}}\right)$$

$$\mathcal{E}=\varepsilon-n_5\frac{s(s+1)}{\mathrm{k}}\frac{w_y}{\mathrm{k}}$$

$$\mathcal{N}_y=n_y+n_5\frac{s(s+1)}{\mathrm{k}}\frac{w_y}{\mathrm{k}},$$

$$\mathcal{M}'=M'_{\mathrm{tot}}+\frac{n_5}{2}\left(w'-(2s+1)\frac{w_y}{\mathrm{k}}\right),\overline{\mathcal{M}}'=\bar{M}'_{\mathrm{tot}}+\frac{n_5}{2}\left(\bar{w}'-\frac{w_y}{\mathrm{k}}\right)$$

$$w_y\in\{0,1,\dots,\mathrm{k}-1\}$$

$$\begin{aligned}\mathcal{E}-\mathcal{N}_y=\frac{2}{\mathrm{k}w+w_y}\Big[&\frac{-j(j-1)+j'(j'+1)}{n_5}+h_L\\&+M'_{\mathrm{tot}}\left(w'-(2s+1)\frac{w_y}{\mathrm{k}}\right)+(2s+1)M'_{\mathrm{tot}}\left(w+\frac{w_y}{\mathrm{k}}\right)\\&+\frac{n_5}{4}\left(\left(w+\frac{w_y}{\mathrm{k}}\right)^2+\left(w'-(2s+1)\frac{w_y}{\mathrm{k}}\right)^2\right.\\&\left.+2(2s+1)\left(w+\frac{w_y}{\mathrm{k}}\right)\left(w'-(2s+1)\frac{w_y}{\mathrm{k}}\right)\right)\Big]\end{aligned}$$

$$\begin{aligned}\hat{\mathrm{n}}=\frac{\mathrm{k}}{\mathrm{k}w+w_y}\Big[&\frac{-j(j-1)+j'(j'+1)}{n_5}+h_L\\&-(J+M_{\mathrm{osc}})\left(w+\frac{w_y}{\mathrm{k}}\right)+M'_{\mathrm{tot}}\left(w'-(2s+1)\frac{w_y}{\mathrm{k}}\right)\\&+\frac{n_5}{4}\left(-\left(w+\frac{w_y}{\mathrm{k}}\right)^2+\left(w'-(2s+1)\frac{w_y}{\mathrm{k}}\right)^2\right)\Big]\end{aligned}$$

$$\mathcal{E}+\mathcal{N}_y=\frac{2}{\mathrm{k}w+w_y}\Big[\frac{-j(j-1)+j'(j'+1)}{n_5}+h_R+\bar{M}'_{\mathrm{tot}}\left(\bar{w}'-\frac{w_y}{\mathrm{k}}\right)+\bar{M}'_{\mathrm{tot}}\left(w+\frac{w_y}{\mathrm{k}}\right)$$
$$+\frac{n_5}{4}\left(\left(w+\frac{w_y}{\mathrm{k}}\right)^2+\left(\bar{w}'-\frac{w_y}{\mathrm{k}}\right)^2+2\left(w+\frac{w_y}{\mathrm{k}}\right)\left(\bar{w}'-\frac{w_y}{\mathrm{k}}\right)\right)\Big]$$

$$\hat{\bar{\mathrm{n}}}=\frac{\mathrm{k}}{\mathrm{k}w+w_y}\left[\frac{-j(j-1)+j'(j'+1)}{n_5}+h_R-(\bar{J}+\bar{M}_{\mathrm{osc}})\left(w+\frac{w_y}{\mathrm{k}}\right)+\bar{M}'_{\mathrm{tot}}\left(\bar{w}'-\frac{w_y}{\mathrm{k}}\right)\right.$$
$$\left.+\frac{n_5}{4}\left(-\left(w+\frac{w_y}{\mathrm{k}}\right)^2+\left(\bar{w}'-\frac{w_y}{\mathrm{k}}\right)^2\right)\right]$$

$$\left(|++\rangle_{\mathrm{k},s}\right)^{2j'+1+n_5\mathrm{w}}\longrightarrow|++\rangle_{(2j'+1+n_5\mathrm{w})\mathrm{k},(2j'+1+n_5\mathrm{w})s}$$

$$\Delta J^3_{\mathrm{su}}=\Delta\bar{J}^3_{\mathrm{su}}=-\left(j'+\frac{n_5}{2}\mathrm{w}\right)$$
$$\Delta L_0=-(2j'+n_5\mathrm{w})\frac{s}{\mathrm{k}},\Delta\bar{L}_0=0$$

$$w'=2s\frac{w_y}{\mathrm{k}}-w,\bar{w}'=-w$$

$$\left.\frac{\mathcal{E}-\mathcal{N}_y}{2}\right|_{\propto n_5}=-n_5\frac{s}{\mathrm{k}}\left(w+\frac{w_y}{\mathrm{k}}\right),\left.\frac{\mathcal{E}+\mathcal{N}_y}{2}\right|_{\propto n_5}=0$$

$$\hat{\mathrm{n}}=\frac{\mathrm{k}}{\mathrm{k}w+w_y}\left[\frac{-j(j-1)+j'(j'+1)}{\mathrm{k}}+h_L\right]-(J+M_{\mathrm{osc}}+M'_{\mathrm{tot}})$$
$$\hat{\bar{\mathrm{n}}}=\frac{\mathrm{k}}{\mathrm{k}w+w_y}\left[\frac{-j(j-1)+j'(j'+1)}{\mathrm{k}}+h_R\right]-(J+M_{\mathrm{osc}}+M'_{\mathrm{tot}}).$$

$$\mathcal{E}=2\left(\frac{-j(j-1)+j'(j'+1)}{n_5(\mathrm{k}w+w_y)}\right)+\frac{h_L+h_R}{\mathrm{k}w+w_y}+\left(\frac{s}{\mathrm{k}}\right)\left[2M'_{\mathrm{tot}}-n_5\left(w+\frac{w_y}{\mathrm{k}}\right)\right]$$
$$\mathcal{N}_y=-\frac{h_L-h_R}{\mathrm{k}w+w_y}-\left(\frac{s}{\mathrm{k}}\right)\left[2M'_{\mathrm{tot}}-n_5\left(w+\frac{w_y}{\mathrm{k}}\right)\right]$$

$$s+1=\ell_1m_1,s=\ell_2m_2,\mathrm{k}=\ell_1\ell_2,$$

$$j=j'+1,\bar{M}=J=J'=-\bar{M}',w=-\bar{w}',\bar{M}_{\mathrm{osc}}=\bar{M}'_{\mathrm{osc}}=0$$

$$w_y=p\ell_1,w=w'=pm_2,M'=-J',\varepsilon=-n_y=-\frac{2J'm_2}{\ell_1},p=1,\dots,\ell_2-1$$

$$w_y=-p\ell_2,w=-w'=pm_1,M'=J',\varepsilon=-n_y=\frac{2J'm_1}{\ell_2},p=1,\dots,\ell_1-1$$

$$\left(|++\rangle_{\mathrm{k},s}\right)^{2j'+n_5pm_2+[n_5p/\ell_2]}\longrightarrow|\Upsilon_1\rangle|00\rangle_{(2j'+n_5pm_2)\mathrm{k}+n_5p\ell_1,2j's+n_5pm_2(s+1)}$$

$$\left(|++\rangle_{\mathrm{k},s}\right)^{2j'+n_5pm_1-[n_5p/\ell_1]}\longrightarrow|\Upsilon_2\rangle|00\rangle_{(2j'+n_5pm_1)\mathbf{k}-n_5p\ell_2,2j'(s+1)+n_5pm_1s}$$

$$j=\frac{1}{2}\left(1+\sqrt{(2j'+1)^2+2n_5(h_L+h_R)}\right).$$

$$\frac{1}{2} < j < \frac{n_5+1}{2}.$$

$$ds^2 = \frac{1}{H_1}\left(-dt_b^2 + dy_b^2\right) + H_5\left(dr_b^2 + r_b^2 d\Omega_3^2\right) + d\mathbf{z}\cdot d\mathbf{z}$$

$$e^{2\Phi} = g_s^2 \frac{H_5}{H_1}\,, H_{1,5} = 1 + \frac{Q_{1,5}}{r_b^2}$$

$$Q_5 = n_5\alpha'\,, Q_1 = \frac{g_s^2 n_1 (\alpha')^3}{V_4}$$

$$ds^2 = \frac{\tilde{r}^2}{\tilde{r}^2 + n_1/V_4}\left(-dt_b^2 + dy_b^2\right) + \left(g_s^2 + \frac{n_5}{\tilde{r}^2}\right)\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2\right) + d\mathbf{z}\cdot d\mathbf{z}$$

$$e^{2\Phi} = \frac{g_s^2\tilde{r}^2 + n_5}{\tilde{r}^2 + n_1/V_4}$$

$$g_s \ll 1, \frac{n_5}{n_1} V_4 \ll 1,$$

$$r_b = g_s\tilde{r} = ar\,, a^2 \equiv \frac{g_s^2 n_1 n_5}{V_4 R_y^2}\,, t = t_b/R_y\,, y = y_b/R_y$$

$$ds^2 = \frac{r^2 R_y^2}{r^2 + R_y^2/n_5}\left(-dt^2 + dy^2\right) + \left(\frac{g_s^2 n_1 n_5}{V_4 R_y^2} + \frac{n_5}{r^2}\right)\left(dr^2 + r^2\Omega_3^2\right) + d\mathbf{z}\cdot d\mathbf{z}$$

$$e^{2\Phi} = \frac{g_s^2 r^2 + V_4 R_y^2/n_1}{r^2 + R_y^2/n_5}$$

$$\tilde{Q}_1 = \frac{n_1}{V_4}, Q_5 = n_5$$

$$n_1 \gg n_5 V_4 \;\Rightarrow\; \tilde{Q}_1 \gg Q_5$$

$$(s+1)^2 + \frac{\mathrm{k}^2 R_y^2}{n_5}\,,\; s^2 + \frac{\mathrm{k}^2 R_y^2}{n_5}.$$

$$e^{2\Phi} = \frac{n_5 V_4}{n_1},$$

Los superestratos y simetrías superconformes, en un campo cuántico relativista bajo supergravedad, se calculan así:

$$ds_{10}^2 = \ell_{\mathrm{AdS}}^2\left(d\rho^2 + \cosh^2\rho\, dt_{\mathrm{E}}^2 + \sinh^2\rho\, d\varphi^2\right) + R_+^2 d\Omega_+^2 + R_-^2 d\Omega_-^2 + L^2 dy^2$$

$$H = \frac{2}{\ell_{\mathrm{AdS}}}\omega_{\mathrm{AdS}} + \frac{2}{R_+}\omega_+ + \frac{2}{R_-}\omega_-$$

$$\frac{R_\pm}{\ell_s} = \sqrt{k_\pm}, \frac{L}{\ell_s} = g_s^2 \frac{N_1}{k_+ k_- \sqrt{k_+ + k_-}}$$

$$\alpha = \frac{k_-}{k_+}$$

$$\gamma = \frac{\mathrm{k}_-}{\mathrm{k}_+ + \mathrm{k}_-} = \frac{\alpha}{1+\alpha}$$

$$\frac{\ell_{\mathrm{AdS}}}{\ell_s} = \sqrt{k}$$

$$c = 6kN_1 \equiv 6\mathrm{k}, k \equiv \frac{k_+ k_-}{k_+ + k_-},$$

$$h \geq \gamma \ell_+ + (1-\gamma)\ell_-.$$

$$\begin{aligned}\chi_{\mathrm{L}}^{\mathrm{g}}(h,\ell_+,\ell_-) =& \left(q^h + q^{h+2}\right)\chi_{\ell_+}(r_+)\chi_{\ell_-}(r_-) \\ &+ \left(q^{h+\frac{1}{2}} + q^{h+\frac{3}{2}}\right)\left(\chi_{\ell_++\frac{1}{2}}(r_+) + \chi_{\ell_--\frac{1}{2}}(r_+)\right)\left(\chi_{\ell_-+\frac{1}{2}}(r_-) + \chi_{\ell_--\frac{1}{2}}(r_-)\right) \\ &+ q^{h+1}\left(2\chi_{\ell_+}(r_+)\chi_{\ell_-}(r_-) + \left(\chi_{\ell_++1}(r_+) + \chi_{\ell_+-1}(r_+)\right)\chi_{\ell_-}(r_-)\right. \\ &\left. + \chi_{\ell_+}(r_+)\left(\chi_{\ell_-+1}(r_-) + \chi_{\ell_--1}(r_-)\right)\right)\end{aligned}$$

$$\chi_\ell(r) = \frac{r^{\ell+\frac{1}{2}} - r^{-\ell-\frac{1}{2}}}{r^{\frac{1}{2}} - r^{-\frac{1}{2}}}, \ell \geq 0 \ (\chi_\ell(r) = 0, \ell < 0)$$

$$q = e^{2\pi i \tau}, r_\pm = e^{2\pi i \rho_\pm}$$

$$\begin{aligned}\chi_{\mathrm{S}}^{\mathrm{g}}(\ell_+,\ell_-) =& q^{h_s}\chi_{\ell_+}(r_+)\chi_{\ell_-}(r_-) + q^{\frac{3}{2}}\chi_{\ell_+-\frac{1}{2}}(r_+)\chi_{\ell_--\frac{1}{2}}(r_-) \\ &+ q^{h_s+\frac{1}{2}}\left(\chi_{\ell_+-\frac{1}{2}}(r_+)\chi_{\ell_--\frac{1}{2}}(r_-) + \chi_{\ell_++\frac{1}{2}}(r_+)\chi_{\ell_--\frac{1}{2}}(r_-) + \chi_{\ell_+-\frac{1}{2}}(r_+)\chi_{\ell_--\frac{1}{2}}(r_-)\right) \\ &+ q^{h_s+1}\left(\chi_{\ell_+}(r_+)\chi_{\ell_--1}(r_-) + \chi_{\ell_+-1}(r_+)\chi_{\ell_-}(r_-) + \chi_{\ell_+}(r_+)\chi_{\ell_-}(r_-)\right),\end{aligned}$$

$$h_s = h_s(\ell_+,\ell_-) \equiv \gamma\ell_+ + (1-\gamma)\ell_-.$$

$$\chi_{\mathrm{L}}^{\mathrm{g}}(h,\ell_+,\ell_-) = q^{h-h_s(\ell_+,\ell_-)}\left[\chi_{\mathrm{S}}^{\mathrm{g}}(\ell_+,\ell_-) + \chi_{\mathrm{S}}^{\mathrm{g}}\left(\ell_+ + \frac{1}{2}, \ell_- + \frac{1}{2}\right)\right].$$

$$\chi_{\mathrm{S}}^{\mathrm{g}}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right) = -q^{\frac{1}{2}}\chi_{\frac{1}{2}}(r_+)\chi_{\frac{1}{2}}(r_-) + q\left(1 + \chi_1(r_+) + \chi_1(r_-)\right) - q^{\frac{3}{2}}\chi_{\frac{1}{2}}(r_+)\chi_{\frac{1}{2}}(r_-) + q^2.$$

$$\sum_{\ell=0}^{\infty}\left|\chi_{\mathrm{S}}^{\mathrm{g}}(\ell,\ell) + \chi_{\mathrm{S}}^{\mathrm{g}}\left(\ell+\frac{1}{2},\ell+\frac{1}{2}\right)\right|^2 + \sum_{\ell_+\neq\ell_-}\left|\chi_{\mathrm{L}}^{\mathrm{g}}(h,\ell_+,\ell_-)\right|^2.$$

$$h(\ell_+,\ell_-) = \sqrt{\frac{k}{k_+}\left(\ell_+ + \frac{1}{2}\right)^2 + \frac{k}{k_-}\left(\ell_- + \frac{1}{2}\right)^2} - \frac{1}{2}$$

$$\sum_{\ell_+,\ell_-} \left|\chi_{\mathrm{L}}^{\mathrm{g}}(h(\ell_+,\ell_-),\ell_+,\ell_-)\right|^2$$

$$h \geq \frac{1}{\mathrm{k}_+ + \mathrm{k}_-}[\mathrm{k}_+\ell_- + \mathrm{k}_-\ell_+ + (\ell_+ - \ell_-)^2 + u^2]$$
$$= \gamma\ell_+ + (1-\gamma)\ell_- + \frac{(\ell_+ - \ell_-)^2 + u^2}{(k_+ + k_-)N_1}$$

$$h - \frac{c}{24} \geq \frac{1}{\mathrm{k}_+ + \mathrm{k}_-}\left[\left(\ell_+ + \ell_- - \frac{1}{2}\right)^2 + u^2\right]$$
$$= \frac{1}{N_1(k_+ + k_-)}\left[\left(\ell_+ + \ell_- - \frac{1}{2}\right)^2 + u^2\right]$$

$$h - \frac{c}{24} \geq \frac{6}{c}\frac{\alpha}{(1+\alpha)^2}\left(\ell_+ + \ell_- - \frac{1}{2}\right)^2$$

$$\chi_{\mathrm{NS}}^{\gamma}(q, r_+, r_-; \delta) = \mathrm{Tr}_{\mathrm{NS}}\left(e^{i\pi\delta F} q^{L_0 - \frac{c}{24}} r_+^{J_0^{3,+}} r_-^{J_0^{3,-}}\right)$$

$$\boldsymbol{\chi}_{\mathrm{NS}}^{\gamma}(q, r_+, r_-; \delta) = q^{h-\frac{c}{24}} \frac{\vartheta_{3+\delta}^2\left(\frac{\rho_+ + \rho_-}{2}, \tau\right)\vartheta_{3+\delta}^2\left(\frac{\rho_+ - \rho_-}{2}, \tau\right)}{\eta(\tau)^6\vartheta_1(\rho_+, \tau)\vartheta_1(\rho_-, \tau)}\mu_{\mathrm{NS}}(q, r_+, r_-; \delta)$$

$$\mu_{\mathrm{NS}}(q, r_+, r_-; \delta) = \sum_{n,m\in\mathbb{Z}} q^{n^2\mathrm{k}_+ + m^2\mathrm{k}_- + n(2\ell_+ + 1) + m(2\ell_- + 1)} \sum_{\epsilon_\pm = \pm} \epsilon_+\epsilon_-\hat{\mu}_{\mathrm{NS}}\left(q, r_+^{\epsilon_+}, r_-^{\epsilon_-}\right)$$

$$\hat{\mu}_{\mathrm{NS}}(q, r_+, r_-) = r_+^{n\mathrm{k}_+ + \ell_+ + \frac{1}{2}} r_-^{m\mathrm{k}_- + \ell_- + \frac{1}{2}} \times \begin{cases} 1, & \text{long} \\ \dfrac{1}{1 + e^{i\pi\delta} r_+^{\frac{1}{2}} r_-^{\frac{1}{2}} q^{m+n+\frac{1}{2}}}, & \text{short} \end{cases}$$

$$\boldsymbol{\chi}_{\mathrm{R}}^{\gamma}(q, r_+, r_-; \delta) = q^{h-\frac{c}{24}} \frac{\vartheta_{2-\delta}^2\left(\frac{\rho_+ + \rho_-}{2}, \tau\right)\vartheta_{2-\delta}^2\left(\frac{\rho_+ - \rho_-}{2}, \tau\right)}{\eta(\tau)^6\vartheta_1(\rho_+, \tau)\vartheta_1(\rho_-, \tau)}\mu_{\mathrm{R}}(q, r_+, r_-; \delta)$$

$$\mu_{\mathrm{R}}(q, r_+, r_-; \delta) = \sum_{n,m\in\mathbb{Z}} q^{n^2\mathrm{k}_+ + m^2\mathrm{k}_- + 2\ell_+ n + 2\ell_- m} \sum_{\epsilon_\pm = \pm} \epsilon_+\epsilon_-\hat{\mu}_{\mathrm{R}}\left(q, r_+^{\epsilon_+}, r_-^{\epsilon_-}\right)$$

$$\hat{\mu}_{\mathrm{R}}(q, r_+, r_-) = r_+^{n\mathrm{k}_+ + \ell_+} r_-^{m\mathrm{k}_- + \ell_-} \times \begin{cases} 1, & \text{long} \\ \dfrac{1}{r_+^{-1} q^{-n} + e^{i\pi\delta} r_-^{-1} q^{-m}}, & \text{short} \end{cases}$$

$$L_n, \mathcal{G}_r^+ \equiv G_r^3 + iG_r^0, \mathcal{G}_r^- \equiv G_r^3 - iG_r^0, J_n \equiv 2\left(\gamma J_n^{+,3} - (1-\gamma)J_n^{-,3}\right),$$

$$\{\mathcal{G}_r^+, \mathcal{G}_s^-\} = 4L_{r+s} + 2(r-s)J_{r+s} + \frac{c}{3}\delta_{r+s,0}\left(r^2 - \frac{1}{4}\right)$$
$$\left[J_n, \mathcal{G}_r^\pm\right] = \pm\mathcal{G}_{r+n}^\pm$$

$$\left\{\mathcal{G}_{-\frac{1}{2}}^+, \mathcal{G}_{\frac{1}{2}}^-\right\} = 4L_0 - 2J_0$$

$$\mathrm{Tr}_{\mathrm{NS}}\left((-1)^{F_s} q^{L_0 - J_0/2}\right)$$

$$\left(\frac{d}{d\rho_+}-\frac{d}{d\rho_-}\right)\mathrm{Tr}_{\mathrm{NS}}\left((-1)^{F_s}q^{L_0-\frac{c}{24}}r_+^{J_0^{3,+}}r_-^{J_0^{3,-}}\right)\Big|_{\rho_+=\gamma\tau,\rho_-=-(1-\gamma)\tau}$$

$$\mathcal{S}_+:(h(\ell_+,\ell_-),\ell_+,\ell_-)_{\mathrm{NS}}\mapsto\left(h(\ell_+,\ell_-)-\ell_++\frac{\mathrm{k}_+}{4},\frac{\mathrm{k}_+}{2}+\ell_+,\ell_-+\frac{1}{2}\right)_{\mathrm{R}}$$
$$\mathcal{S}_-:(h(\ell_+,\ell_-),\ell_+,\ell_-)_{\mathrm{NS}}\mapsto\left(h(\ell_+,\ell_-)-\ell_-+\frac{\mathrm{k}_-}{4},\ell_++\frac{1}{2},\frac{\mathrm{k}_-}{2}-\ell_-+\right)_{\mathrm{R}}$$

$$(t_{\mathrm{E}},\varphi,\phi_1^+,\phi_2^+,\phi_1^-,\phi_2^-,y)$$
$$\sim(t_{\mathrm{E}}+\beta,\varphi+i\beta\mu,\phi_1^++i\beta\nu_1^+,\phi_2^++i\beta\nu_2^+,\phi_1^-+i\beta\nu_1^-,\phi_2^-+i\beta\nu_2^-,y)$$

$$\tau=\frac{\beta\mu+i\beta}{2\pi},q=e^{2\pi i\tau}$$

$$\rho_\pm=\beta\left(\nu_2^\pm-\nu_1^\pm\right),\bar{\rho}_\pm=\beta\left(\bar{\nu}_2^\pm+\bar{\nu}_1^\pm\right),r_\pm=e^{2\pi i\rho_\pm},$$

$$\mathrm{t}=\mathrm{t}_1+i\mathrm{t}_2,\mathfrak{z}=e^{2\pi i\mathrm{t}}$$

$$\mathcal{Z}_{\mathfrak{sl}(2)}(\mathrm{t})=\frac{\beta\sqrt{k}}{2\pi\sqrt{\mathrm{t}_2}}\sum_{n,m\in\mathbb{Z}}\mathcal{Z}_{\mathfrak{sl}(2)}^{(n,m)}(\mathrm{t})$$
$$\mathcal{Z}_{\mathfrak{sl}(2)}^{(n,m)}(\mathrm{t})=\frac{\exp\left(-\frac{k\beta^2}{4\pi\mathrm{t}_2}|m-n\mathrm{t}|^2+\frac{2\pi}{\mathrm{t}_2}\left(\mathrm{Im}\left(\mathrm{u}_{n,m}\right)\right)^2\right)}{\left|\vartheta_1\left(\mathrm{u}_{n,m},\mathrm{t}\right)\right|^2}$$

$$\mathrm{u}_{n,m}=\tau(n\mathrm{t}-m)$$

$$\mathcal{Z}_{\mathfrak{su}(2)}^{(n,m)}(\mathrm{t})=e^{\frac{\pi}{\mathrm{t}_2}\left(\mathrm{Im}(\rho_{n,m})\right)^2[k|n\mathrm{t}-m|^2-(k-2)]}\sum_{\ell=0}^{\frac{k}{2}-1}\left|\frac{\Theta_{2\ell+1}^{(k)}\left(\rho_{n,m},\mathrm{t}\right)-\Theta_{-2\ell-1}^{(k)}\left(\rho_{n,m},\mathrm{t}\right)}{\vartheta_1\left(\rho_{n,m},\mathrm{t}\right)}\right|^2$$

$$\Theta_l^{(k)}(\rho,\tau)=\sum_{n\in\mathbb{Z}+\frac{l}{2k}}q^{kn^2}r^{nk},q=e^{2\pi i\tau},r=e^{2\pi i\rho}$$

$$\rho_{n,m}=\rho(n\mathrm{t}-m)$$

$$\mathcal{Z}_{\mathrm{u}(1)}^{(n,m)}(\mathrm{t})=\frac{1}{|\eta(\mathrm{t})|^2}\sum_{p_L,p_R}\mathfrak{z}^{p_L^2}\bar{\mathfrak{z}}^{-2}$$

$$\mathcal{Z}_{bc}(\mathrm{t})=|\eta(\mathrm{t})|^4$$

$$\left(\mathcal{Z}_{\mathrm{PF}}^{\pm}\right)^{(n,m)}=\mathrm{Tr}_{\mathrm{NS}}\left(\frac{1-e^{i\pi F}}{2}e^{2\pi i\mathrm{t}L_0}\right)-\mathrm{Tr}_{\mathrm{R}}\left(\frac{1\pm e^{i\pi F}}{2}e^{2\pi i\mathrm{t}L_0}\right)$$
$$=\frac{1}{2}\left(\mathcal{Z}_3^{(n,m)}-\mathcal{Z}_4^{(n,m)}-\mathcal{Z}_2^{(n,m)}\mp\mathcal{Z}_1^{(n,m)}\right)$$

$$\mathcal{Z}_a^{(n,m)} = \underbrace{\frac{e^{-\frac{\pi}{\mathrm{t}_2}\left(\mathrm{Im}(\mathrm{u}_{n,m})\right)^2}\vartheta_a\left(\mathrm{u}_{n,m},\mathrm{t}\right)\sqrt{\vartheta_a(0,\mathrm{t})}}{\eta(\mathrm{t})^{\frac{3}{2}}}}_{\mathfrak{sl}(2)} \times \underbrace{e^{-\frac{\pi}{\mathrm{t}_2}\left(\mathrm{Im}(\rho^+_{n,m})\right)^2}\frac{\vartheta_a\left(\rho^+_{n,m},\mathrm{t}\right)\sqrt{\vartheta_a(0,\mathrm{t})}}{\eta(\mathrm{t})^{\frac{3}{2}}}}_{\mathfrak{su}(2)_+}$$
$$\times \underbrace{e^{-\frac{\pi}{\mathrm{t}_2}\left(\mathrm{Im}(\rho^-_{n,m})\right)^2}\frac{\vartheta_a\left(\rho^-_{n,m},\mathrm{t}\right)\sqrt{\vartheta_a(0,\mathrm{t})}}{\eta(\mathrm{t})^{\frac{3}{2}}}}_{\mathfrak{su}(2)_-} \times \underbrace{\sqrt{\frac{\vartheta_a(0,\mathrm{t})}{\eta(\mathrm{t})}} \times \underbrace{\frac{\eta(\mathrm{t})}{\vartheta_a(0,\mathrm{t})}}_{\beta\gamma}}_{\mathfrak{u}(1)}$$

$$\mathcal{Z}_{\mathrm{PF}}^{(n,m)}(\mathrm{t}) = \frac{e^{-\frac{\pi}{\mathrm{t}_2}\left[\left(\mathrm{Im}\left((\mathrm{u}_{n,m})\right)^2+\left(\mathrm{Im}\left((\rho^+_{n,m})\right)^2+\left(\mathrm{Im}\left((\rho^-_{n,m})\right)^2\right]\right.}}{\eta(\mathrm{t})^4} \times \prod_{\eta_\pm=\pm 1}\vartheta_1\left(\frac{-\mathrm{u}_{n,m}+\eta_+\rho^+_{n,m}+\eta_-\rho^-_{n,m}}{2},\mathrm{t}\right)$$

$$\left(\mathcal{Z}^{\pm}_{\mathrm{AF}}\right)^{(n,m)} = \mathrm{Tr}_{\mathrm{NS}}\left(\left[\frac{1-e^{i\pi F}}{2}\frac{1+e^{i\pi n}}{2}+\frac{1+e^{i\pi F}}{2}e^{i\pi m}\frac{1-e^{i\pi n}}{2}\right]e^{2\pi i\mathrm{t}L_0}\right)$$
$$-\mathrm{Tr}_{\mathrm{R}}\left(\left[\frac{1\pm e^{i\pi F}}{2}\frac{1-e^{i\pi n}}{2}+\frac{1+e^{i\pi F}}{2}e^{i\pi m}\frac{1-e^{i\pi n}}{2}\right]e^{2\pi i\mathrm{t}L_0}\right)$$

$$\mathcal{Z}_{\mathrm{IIB}} = \int_{\mathcal{F}}\frac{d^2\mathrm{t}}{\mathrm{t}_2}\sum_{n,m\in\mathbb{Z}}\mathcal{Z}^{(n,m)}_{\mathfrak{su}(2)}(\mathrm{t})\mathcal{Z}^{(n,m)}_{\mathfrak{su}(2)_+}(\mathrm{t})\mathcal{Z}^{(n,m)}_{\mathfrak{su}(2)_-}(\mathrm{t})\mathcal{Z}_{bc}(\mathrm{t})\left|\mathcal{Z}^{(n,m)}_{\mathrm{PF}}(\mathrm{t})\right|^2$$

$$\mathcal{Z}_{\mathrm{IIB}}(\tau,\rho_+,\rho_-) = -\beta\sum_{m=1}^{\infty} f_{\mathrm{PF}}(m\tau,\rho_+,\rho_-),$$

$$z_{\mathrm{CFT}}(\tau,\rho_+,\rho_-)|_{\text{thermal AdS}_3} = \exp\left(S_{\text{tree}} + \mathcal{Z}_{\mathrm{IIB}} + \cdots\right)$$

$$f_{\mathrm{PF}}(\tau,\rho_+,\rho_-) = f_{\mathrm{PF,disc}}(\tau,\rho_+,\rho_-) + f_{\mathrm{PF,cont}}(\tau,\rho_+,\rho_-)$$

$$f_{\mathrm{PF}} = \frac{\sqrt{k}\beta}{2\pi}\int_0^\infty\frac{d\mathrm{t}_2}{\mathrm{t}_2^{\frac{3}{2}}}\int_{-\frac{1}{2}}^{\frac{1}{2}}d\mathrm{t}_1 e^{-\frac{k\beta^2}{4\pi\mathrm{t}_2}}\mathcal{I}_{\mathrm{PF}}$$
$$= \frac{4}{ik}\int_{-\infty}^{\infty}\zeta d\zeta\int_0^\infty d\mathrm{t}_2\int_{-\frac{1}{2}}^{\frac{1}{2}}d\mathrm{t}_1 e^{2i\beta\zeta-\frac{4\pi}{k}\mathrm{t}_2\zeta^2}\mathcal{I}_{\mathrm{PF}}$$

$$\mathcal{I}_{\mathrm{PF}} = \left|\frac{\prod_{\epsilon_\pm=\pm}\vartheta_1\left(\frac{\tau+\epsilon_+\rho_++\epsilon_-\rho_-}{2},\mathrm{t}\right)}{\vartheta_1(\tau,\mathrm{t})\vartheta_1(\rho_+,\mathrm{t})\vartheta_1(\rho_-,\mathrm{t})\eta(\mathrm{t})^3}\right|^2 \times\left(\sum_{p_L,p_R}\mathfrak{z}^{p_L^2}\bar{\mathfrak{z}}_R^2\right)$$
$$\times\prod_{\epsilon\in\{\pm\}}\left[\sum_{\ell_\epsilon=0}^{\frac{1}{2}k_\epsilon-1}\left|\sum_{n_\epsilon\in\mathbb{Z}}\mathfrak{z}^{\frac{\left(n_\epsilon k_\epsilon+\ell_\epsilon+\frac{1}{2}\right)^2}{k_\epsilon}}\left(r_\epsilon^{nk_\epsilon+\ell_\epsilon+\frac{1}{2}}-r_\epsilon^{-n_\epsilon k_\epsilon-\ell_\epsilon-\frac{1}{2}}\right)\right|^2\right]$$

$$\mathcal{I}_{\mathrm{PF}} = (q\bar{q})^{-\frac{1}{2}}I_0\times I_{\text{primaries}}\times I_{\text{oscillators}}$$

$$I_0 = \left| 1 + \frac{\chi_{\mathrm{S}}^{\mathrm{g}}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)}{1-q} \right|^2$$

$$I_{\text{primaries}} = \left(\sum_{p_L,p_R} \mathfrak{z}^{p_L^2} \bar{\mathfrak{z}}^{p_R^2} \right) \times \prod_{\epsilon\in\{\pm\}} \left(\sum_{\ell_\epsilon=0}^{\frac{k_\epsilon}{2}-1} \left| \sum_{n_\epsilon\in\mathbb{Z}} \mathfrak{z}^{\frac{\left(n_\epsilon k_\epsilon+\ell_\epsilon+\frac{1}{2}\right)^2}{k_\epsilon}} \chi_{n_\epsilon k_\epsilon+\ell_\epsilon}(r_\epsilon) \right|^2 \right)$$

$$I_{\text{oscillators}} = \left| \prod_{n=1}^{\infty} \frac{\prod_{\pm=\pm1}\left(1-\left(qr_+^{\epsilon_+}r_-^{\epsilon_-}\right)^{\frac{1}{2}}\mathfrak{z}^n\right)\left(1-\left(qr_+^{\epsilon_+}r_-^{\epsilon_-}\right)^{-\frac{1}{2}}\mathfrak{z}^n\right)}{(1-\mathfrak{z}^n)^2\prod_{\epsilon=\pm1}(1-q^\epsilon\mathfrak{z}^n)(1-r_+^\epsilon\mathfrak{z}^n)(1-r_-^\epsilon\mathfrak{z}^n)} \right|^2$$

$$\prod_{n=1}^{\infty} \frac{1-\mathfrak{z}^n}{(1-q\mathfrak{z}^n)(1-q^{-1}\mathfrak{z}^n)} = \sum_{w=0}^{\infty} q^w \mathfrak{z}^{-\frac{1}{2}w(w+1)} \sum_{N\in\mathbb{Z}} \mathbf{P}_N^w(q)\mathfrak{z}^N$$

$$I_{\text{oscillators}} = \left| \sum_{w=0}^{\infty} \sum_{N\in\mathbb{Z}} \mathbf{Q}_N^w(q,r_+,r_-) q^w \mathfrak{z}^{-\frac{1}{2}w(w+1)+N} \right|^2$$

$$\mathcal{I}_{\text{PF}} = (q\bar{q})^{-\frac{1}{2}} I_0 \hat{\sum} (q\bar{q})^w \mathrm{Q}_{N,\bar{N}}^w \times (\mathfrak{z}\bar{\mathfrak{z}})^{-\frac{1}{2}w(w+1)} \mathfrak{z}^{p_L^2+h_{\mathfrak{su}(2)}+N} \bar{\mathfrak{z}}_R^{2} + \bar{h}_{\mathfrak{su}(2)} + \bar{N}$$

$$h_{\mathfrak{su}(2)} = \frac{\left(n_+k_+ + \ell_+ + \frac{1}{2}\right)^2}{k_+} + \frac{\left(n_-k_- + \ell_- + \frac{1}{2}\right)^2}{k_-}$$

$$\bar{h}_{\mathfrak{su}(2)} = \frac{\left(\bar{n}_+k_+ + \ell_+ + \frac{1}{2}\right)^2}{k_+} + \frac{\left(\bar{n}_-k_- + \ell_- + \frac{1}{2}\right)^2}{k_-}$$

$$\mathbf{Q}_{N,\bar{N}}^w \equiv \mathbf{Q}_N^w(q,r_+,r_-)\overline{\mathbf{Q}}_{\bar{N}}^w(\bar{q},\bar{r}_+,\bar{r}_-)\left[\prod_{\epsilon=\pm1} \chi_{n_\epsilon k_\epsilon+\ell_\epsilon}(r_\epsilon)\chi_{\bar{n}_\epsilon k_\epsilon+\ell_\epsilon}(\bar{r}_\epsilon)\right];$$

$$\hat{\sum} \equiv \sum_{w=0}^{\infty} \sum_{N,\bar{N}\in\mathbb{Z}} \sum_{n_\pm,\bar{n}_\pm\in\mathbb{Z}} \sum_{p_L,p_R} \sum_{\ell_+=0}^{\frac{k_+}{2}-1} \sum_{\ell_-=0}^{\frac{k_-}{2}-1}$$

$$\delta_{\text{LM}}\colon p_L^2 + h_{\mathfrak{su}(2)} + N = p_R^2 + \bar{h}_{\mathfrak{su}(2)} + \bar{N}$$

$$f_{\text{PF}} = \frac{k}{4\pi} I_0 \hat{\sum} \delta_{\text{LM}} (q\bar{q})^{w-\frac{1}{2}} \mathrm{Q}_{N,\bar{N}}^w \times \int_{-\infty}^{\infty} \frac{d\zeta}{i\pi} \frac{\zeta e^{2i\beta\zeta}}{\zeta^2+\Delta_w^2} \left(e^{-\frac{2\beta}{w+1}\frac{\zeta^2+\Delta_w^2}{k}} - e^{-\frac{2\beta}{w}\frac{\zeta^2+\Delta_w^2}{k}} \right)$$

$$\Delta_w^2 = k\left[p_L^2 + h_{\mathfrak{su}(2)} + N - \frac{1}{2}w(w+1)\right]$$

$$\zeta_* = i\Delta_w$$

$$f_{\mathrm{PF,disc}}(\tau,\rho_\pm)=I_0\hat{\sum}\,\delta_{\mathrm{LM}}\mathrm{Q}^{w}_{N,\bar{N}}(q\bar{q})^{w-\frac{1}{2}+\Delta_w}$$

$$\frac{k}{2}w\le\Delta_w\le\frac{k}{2}(w+1)$$

$$f_{\mathrm{PF,cont}}(\tau,\rho_\pm)=I_0\sum_{w=1}^{\infty}\int_{-\infty}^{\infty}\frac{ds}{i\pi}\boldsymbol{\varrho}(s;q,\bar{q},r_\pm,\bar{r}_\pm)(q\bar{q})^{\frac{kw}{4}+\frac{1}{wk}(s^2+\Delta_w^2)+w-\frac{1}{2}}$$

$$\Delta_{w=0}-\frac{1}{2}=\sqrt{kp_L^2+k\left(\frac{\left(n_+k_++\ell_++\frac{1}{2}\right)^2}{k_+}+\frac{\left(n_-k_-+\ell_-+\frac{1}{2}\right)^2}{k_-}\right)}-\frac{1}{2}$$

$$h_{\mathrm{CFT}}=\sqrt{\frac{k}{k_+}\left(\ell_++\frac{1}{2}\right)^2+\frac{k}{k_-}\left(\ell_-+\frac{1}{2}\right)^2}-\frac{1}{2}.$$

$$f_{\mathrm{PF,disc}}(\tau,\rho_\pm)\big|_{\mathrm{sugra}}=I_0\sum_{\ell_+=0}^{\frac{k_+}{2}-1}\sum_{\ell_-=0}^{\frac{k_-}{2}-1}(q\bar{q})^{h_{\mathrm{CFT}}}\left|\chi_{\ell_+}(r_+)\chi_{\ell_-}(r_-)\right|^2$$

$$\chi^{\mathrm{g}}_{\mathrm{S}}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)q^h\chi_{\ell_+}(r_+)\chi_{\ell_-}(r_-)=\chi^{\mathrm{g}}_{\mathrm{L}}(h,\ell_+,\ell_-)-q^h(1-q)\chi_{\ell_+}(r_+)\chi_{\ell_-}(r_-).$$

$$f_{\mathrm{PF,disc}}(\tau,\rho_\pm)\big|_{\mathrm{sugra}}=\sum_{\ell_+=0}^{\frac{k_+}{2}-1}\sum_{\ell_-=0}^{\frac{k_-}{2}-1}\left|\chi^{\mathrm{g}}_{\mathrm{L}}(h_{\mathrm{CFT}},\ell_+,\ell_-)\right|^2$$

$$I_0=1+\frac{\chi^{\mathrm{g}}_{\mathrm{S}}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)}{1-q}+\frac{\bar{\chi}^{\mathrm{g}}_{\mathrm{S}}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)}{1-\bar{q}}+\frac{\chi^{\mathrm{g}}_{\mathrm{S}}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\bar{\chi}^{\mathrm{g}}_{\mathrm{S}}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)}{|1-q|^2}$$

$$\exp\left(\sum_{j=1}^{\infty}\frac{1}{j}I_{0,\mathrm{hol}}(j\tau,j\rho_\pm)\right)=\frac{\vartheta_4^2\left(\frac{\rho_++\rho_-}{2},\tau\right)\vartheta_4^2\left(\frac{\rho_+-\rho_-}{2},\tau\right)}{\eta(\tau)^6\vartheta_1(\rho_+,\tau)\vartheta_1(\rho_-,\tau)}\frac{\left(r_+^{\frac{1}{2}}-r_+^{-\frac{1}{2}}\right)\left(r_-^{\frac{1}{2}}-r_-^{-\frac{1}{2}}\right)(1-q)}{\prod_{\epsilon_\pm=\pm1}\left(1-r_+^{\frac{1}{2}\epsilon_+}r_-^{\frac{1}{2}\epsilon_-}q^{\frac{1}{2}}\right)}$$

$$\exp\left(\sum_{j=1}^{\infty}\frac{1}{j}I_{0,\mathrm{hol}}(j\tau,j\rho_\pm)\right)=\chi^{\gamma}_{\mathrm{NS}}(q,r_+,r_-,u)\big|_{n=m=0}$$

$$\mu_{\mathrm{NS}}(q,r_+,r_-)|_{n=m=0}=\sum_{\epsilon_\pm=\pm1}\frac{\epsilon_+\epsilon_-}{1-r_+^{\frac{1}{2}\epsilon_+}r_-^{\frac{1}{2}\epsilon_-}q^{\frac{1}{2}}}=\frac{\left(r_+^{\frac{1}{2}}-r_+^{-\frac{1}{2}}\right)\left(r_-^{\frac{1}{2}}-r_-^{-\frac{1}{2}}\right)(1-q)}{\prod_{\epsilon_\pm=\pm1}\left(1-r_+^{\frac{1}{2}\epsilon_+}r_-^{\frac{1}{2}\epsilon_-}q^{\frac{1}{2}}\right)}$$

$$f(r_+,r_-)\mapsto\sum_{n,m\in\mathbb{Z}}q^{k_+n^2+k_-m^2}r_+^{k_+n}r_-^{k_-m}f(q^2r_+,q^{2m}r_-)$$

$$f_{\mathrm{PF}}=-4i\int_{-\infty}^{\infty}\zeta d\zeta\int_0^{\infty}d\mathrm{t}_2\int_{-\frac{1}{2}}^{\frac{1}{2}}d\mathrm{t}_1e^{2i\beta\zeta-4\pi\mathrm{t}_2\zeta^2}\mathcal{I}_{\mathrm{PF}}$$

$$\mathcal{I}_{\mathrm{PF}}=\left|\frac{\prod_{\pm}\vartheta_1\left(\frac{\tau+\epsilon_+\rho_++\epsilon_-\rho_-}{2},\mathrm{t}\right)}{\eta(\mathrm{t})^6}\times\frac{\eta(\mathrm{t})^3}{\vartheta_1(\tau,\mathrm{t})}\right|^2\times\left(\sum_{p_L,p_R}\mathfrak{z}^{p_L^2}\bar{\mathfrak{z}}^{p_R^2}\right)$$

$$f_{\mathrm{PF}}(\tau,\rho_+,\rho_-)\ =I_0\sum_{N,\bar{N}\in\mathbb{Z}}\sum_{p_L,p_R}\sum_{w=0}^{\infty}\delta_{N+p_L^2-p_R^2,\bar{N}}\widetilde{\mathrm{Q}}_{N,\bar{N}}^{w}(q\bar{q})^{w-\frac{1}{2}}\mathfrak{I}_{k=1}$$

$$\mathfrak{I}_{k=1}\ =\int_{-\infty}^{\infty}d\zeta\frac{\zeta e^{2i\beta\zeta}}{\zeta^2+\Delta_w^2}\left[(q\bar{q})^{\frac{\zeta^2+\Delta_w^2}{w+1}}-(q\bar{q})^{\frac{\zeta^2+\Delta_w^2}{w}}\right]$$

$$\Delta_w^2=p_L^2+N+\frac{1}{4}-\frac{1}{2}w(w+1)$$

$$\frac{w}{2}<\sqrt{p_L^2+N+\frac{1}{4}-\frac{1}{2}w(w+1)}<\frac{w+1}{2}$$

$$\frac{(3w-1)(w+1)}{4}<p_L^2+N<\frac{w(3w+4)}{4}$$

$$f_{\mathrm{PF,cont}}\supset I_0\sum_{w=1}^{\infty}\sum_{N,p_L,p_R}\delta_{\mathrm{LM}}\int_{-\infty}^{\infty}\frac{ds}{i\pi}(q\bar{q})^{E_w(s)}\left[\frac{\left(s+\frac{iw}{2}\right)\widetilde{\mathrm{Q}}_N^{w-1}}{\left(s+\frac{iw}{2}\right)^2+\Delta_{w-1}^2}-\frac{\left(s+\frac{iw}{2}\right)\widetilde{\mathrm{Q}}_N^{w}}{\left(s+\frac{iw}{2}\right)^2+\Delta_w^2}\right]$$

$$E_w(s)=\frac{3}{4}w-1+\frac{1}{w}\left(s^2+\frac{1}{4}+N+p_L^2\right)$$

$$\begin{aligned}z_{\mathrm{CFT}}^{\mathrm{NS}}(\tau,\rho_+,\rho_-,\bar{\tau},\bar{\rho}_+,\bar{\rho}_-)\big|_{\mathrm{BTZ}}&=\mathrm{Tr}_{\mathrm{NS}}\left(q^{L_0}r_+^{J_0^{3,+}}r_-^{J_0^{3,-}}\bar{q}^{\bar{L}_0}\bar{r}_+^{\bar{J}_0^{3,+}}\bar{r}_-^{\bar{J}_0^{3,-}}\right)\Big|_{\mathrm{BTZ}}\\&=\mathrm{Tr}_{\mathrm{NS}}\left(q_{\mathrm{AdS}}^{L_0}r_{+,\mathrm{AdS}}^{J_0^{3,+}}r_{-,\mathrm{AdS}}^{J_0^{3,-}}\bar{q}_{\mathrm{AdS}}^{\bar{L}_0}\bar{r}_{+,\mathrm{AdS}}^{\bar{J}_0^{3,+}}\bar{r}_{-,\mathrm{AdS}}^{\bar{J}_0^{3,-}}\right)\Big|_{\mathrm{AdS}}\\z_{\mathrm{CFT}}^{\mathrm{R}}(\tau,\rho_+,\rho_-,\bar{\tau},\bar{\rho}_+,\bar{\rho}_-)\big|_{\mathrm{BTZ}}&=\mathrm{Tr}_{\mathrm{R}}\left(q^{L_0}r_+^{J_0^{3,+}}r_-^{J_0^{3,}}\bar{q}^{\bar{L}_0}\bar{r}_+^{\bar{J}_0^{3,+}}\bar{r}_-^{J_0^{3,-}}\right)\Big|_{\mathrm{BTZ}}\\&=\mathrm{Tr}_{\mathrm{NS}}\left((-1)^{F_s}q_{\mathrm{AdS}}^{L_0}r_{+,\mathrm{AdS}}^{J_0^{3,+}}r_{-,\mathrm{AdS}}^{J_0^{3,-}}\bar{q}_{\mathrm{AdS}}^{\bar{L}_0}\bar{r}_{+,\mathrm{AdS}}^{\bar{J}_0^{3,+}}\bar{r}_{-,\mathrm{AdS}}^{\bar{J}_0^{3,-}}\right)\Big|_{\mathrm{AdS}}\end{aligned}$$

$$q_{\mathrm{AdS}}=e^{-\frac{2\pi i}{\tau}},(r_{\pm})_{\mathrm{AdS}}=e^{-2\pi i\frac{\rho}{\tau}}$$

$$T=\frac{1}{i\pi}\frac{1}{\bar{\tau}-\tau},\Omega=\pi(\tau+\bar{\tau})$$

$$\tau\to i\infty,\bar{\tau}\to-i0,|\tau|<1.$$

$$\mathrm{Tr}_{\mathrm{NS}}\left(q_{\mathrm{AdS}}^{L_0}r_{+,\mathrm{AdS}}^{J_0^{3,+}}r_{-,\mathrm{AdS}}^{J_0^{3,-}}\bar{q}_{\mathrm{AdS}}^{\bar{L}_0}\bar{r}_{+,\mathrm{AdS}}^{\bar{J}_0^{3,+}}\bar{r}_{-,\mathrm{AdS}}^{\bar{J}_0^{3,-}}\right)\Big|_{\mathrm{AdS}}=e^{S_{\mathrm{tree}}}\times\left|\boldsymbol{\chi}_{\mathrm{NS}}^{\gamma}(q,r_+,r_-;0)\right|^2\times\cdots$$

$$z^{\mathrm{NS}}_{\mathrm{CFT}}(\tau,\rho_{\pm},\bar{\tau},\bar{\rho}_{\pm})\big|_{\mathrm{BTZ}} \sim e^{S_{\mathrm{tree}}+\beta+\frac{T}{\gamma T_{\mathrm{gap}}}(\rho_+^2+\alpha\rho_-^2)}\left(\frac{T}{T_{\mathrm{gap}}}\right)^5 \mathfrak{F}_+(\rho_+)\mathfrak{F}_-(\rho_-),$$

$$\mathfrak{F}_{\pm}(\rho) \equiv \sum_{m\in\mathbb{Z}} \frac{\left(m+\frac{\rho}{2}\right)}{\sin(\pi\rho)} e^{-4\frac{\mathrm{k}_{\pm}}{\mathrm{k}}\frac{T}{T_{\mathrm{gap}}}\left(m+\frac{\rho}{2}\right)^2}$$

$$\mathfrak{F}_{\pm}(\rho) = \frac{\pi^{\frac{3}{2}}}{4}\left(\frac{\mathrm{k}}{\mathrm{k}_{\pm}}\frac{T_{\mathrm{gap}}}{T}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} n\chi_{\frac{n-1}{2}}(r) e^{-\frac{\pi^2}{4}\frac{\mathrm{k}}{\mathrm{k}_{\pm}}\frac{T_{\mathrm{gap}}}{T}n^2}$$

$$\tilde{Z}^{\mathrm{NS}}_{\mathrm{CFT}}\left(T,\rho_+,\rho_-,Q_{\mathrm{U}(1)}=0\right)\Big|_{\mathrm{BTZ}} \sim \frac{\alpha^{\frac{3}{2}}}{(1+\alpha)^3}\left(\frac{T}{T_{\mathrm{gap}}}\right)^{\frac{3}{2}} e^{S_{\mathrm{tree}}+\beta+\frac{\rho_+^2+\alpha\rho_-^2}{\gamma}\frac{T}{T_{\mathrm{gap}}}}$$
$$\times \sum_{m,n=1}^{\infty} mn\chi_{\frac{m-1}{2}}(r_+)\chi_{\frac{n-1}{2}}(r_-) e^{-\beta E_{\mathrm{gap}}\frac{\alpha m^2+n^2}{1+\alpha}}$$

$$\tilde{z}^{\mathrm{NS}}_{\mathrm{CFT}}\left(T,j_+,j_-,Q_{\mathrm{U}(1)}=0\right)\Big|_{\mathrm{BTZ}} \sim \left(\frac{T}{T_{\mathrm{gap}}}\right)^{\frac{3}{2}} e^{S_{\mathrm{tree}}+S_1 T-\beta\frac{E_{\mathrm{gap}}}{1+\alpha}(\alpha(2j_++1)^2+(2j_-+1)^2+E_1)}$$

$$\mathrm{Tr}_{\mathrm{NS}}\left((-1)^F_s q^{L_0}_{\mathrm{AdS}} r^{J_0^{3,+}}_{+,\mathrm{AdS}} r^{J_0^{3,-}}_{-,\mathrm{AdS}} \bar{q}^{\bar{L}_0}_{\mathrm{AdS}} \bar{r}^{\bar{J}_0^{3,+}}_{+,\mathrm{AdS}} \bar{r}^{\bar{J}_0^{3,-}}_{-,\mathrm{AdS}}\right)\Big|_{\mathrm{AdS}} = e^{S_{\mathrm{tree}}} \times \left|\chi^{\gamma}_{\mathrm{NS}}(q,r_+,r_-;1)\right|^2 \times \cdots$$

$$z^{\mathrm{R}}_{\mathrm{CFT}}(\tau,\rho_{\pm},\bar{\tau},\bar{\rho}_{\pm})\big|_{\mathrm{BTZ}} \sim 64\frac{T}{T_{\mathrm{gap}}}\exp\left(S_{\mathrm{tree}}+2y(\rho_+^2+\alpha\rho_-^2)\right)$$
$$\times \frac{\cos^2\left(\frac{\pi(\rho_++\rho_-)}{2}\right)\cos^2\left(\frac{\pi(\rho_+-\rho_-)}{2}\right)}{\sin(\pi\rho_+)\sin(\pi\rho_-)}\mathfrak{B}_{\alpha}(y,\rho_+,\rho_-)$$

$$y = \frac{1}{2}\frac{T}{T_{\mathrm{gap}}}\frac{\mathrm{k}_+}{\mathrm{k}} = \frac{T}{2\gamma T_{\mathrm{gap}}}$$

$$\mathfrak{B}_{\alpha}(y,\rho_+,\rho_-) \equiv \sum_{n_{\pm}\in\mathbb{Z}} \frac{\left(n_+ + \frac{1}{2}\rho_+\right)\left(n_- + \frac{1}{2}\rho_-\right) e^{-8y\left(\left(n_+ + \frac{1}{2}\rho_+\right)^2 + \alpha\left(n_- + \frac{1}{2}\rho_-\right)^2\right)}}{\left[1-4\left(n_+ + n_- + \frac{\rho_+ + \rho_-}{2}\right)^2\right]\left[1-4\left(n_+ - n_- + \frac{\rho_+ - \rho_-}{2}\right)^2\right]}$$

$$m_1 = n_+ + n_-, m_2 = n_+ - n_-, \rho_1 = \rho_+ + \rho_-, \rho_2 = \rho_+ - \rho_-$$

$$\mathfrak{B}_{\alpha}(y,\rho_1,\rho_2) = \frac{1}{16}\sum_{m_1,m_2\in\mathbb{Z}}\left[\frac{1}{1-4\left(m_1+\frac{\rho_1}{2}\right)^2} - \frac{1}{1-4\left(m_2+\frac{\rho_2}{2}\right)^2}\right]$$
$$\times e^{-2y\left(\left(m_1+m_2+\frac{\rho_1+\rho_2}{2}\right)^2+\alpha\left(m_1-m_2+\frac{\rho_1-\rho_2}{2}\right)^2\right)}$$

$$z^{\mathrm{R}}_{\mathrm{CFT}}(\tau,\rho_{\pm},\bar{\tau},\bar{\rho}_{\pm})\big|_{\mathrm{BTZ}} \sim \frac{4\sqrt{\alpha}}{1+\alpha}\left(\frac{T}{T_{\mathrm{gap}}}\right)^{\frac{1}{2}}\exp\left(S_{\mathrm{tree}}+2y(\rho_+^2+\alpha\rho_-^2-\gamma)\right)$$
$$\times \cos^2\left(\frac{\pi(\rho_++\rho_-)}{2}\right)\cos^2\left(\frac{\pi(\rho_+-\rho_-)}{2}\right)\int_0^{2\gamma y} du\,\frac{e^u}{\sqrt{u}}\mathfrak{f}_{\alpha}(u)$$

$$\mathfrak{f}_\alpha(u)=\sum_{m_1,m_2=0}^{\infty}\left(1-\frac{\delta_{m_1,0}}{2}\right)\left(1-\frac{\delta_{m_2,0}}{2}\right)\mathrm{sign}(m_1-m_2)e^{-\frac{\pi^2 m_2^2}{2y(1+\alpha)}}$$
$$\times\left[e^{-\frac{\pi^2(m_1+\eta m_2)^2}{4u}}\chi_{\frac{m_1+m_2-1}{2}}(\rho_+)\chi_{\frac{|m_1-m_2|-1}{2}}(\rho_-)+(m_2\leftrightarrow -m_2)\right].$$

$$\eta=1-2\gamma=\frac{1-\alpha}{1+\alpha}$$

$$\tilde{z}^{\mathrm{R}}_{\mathrm{CFT}}\big(T,\rho_+,\rho_-,Q_{\mathrm{U}(1)}=0\big)\big|_{\mathrm{BTZ}}\sim\cos^2\left(\frac{\pi(\rho_++\rho_-)}{2}\right)\cos^2\left(\frac{\pi(\rho_+-\rho_-)}{2}\right)$$
$$\times\left(\frac{T}{T_{\mathrm{gap}}}\right)^{\frac{3}{2}}e^{S_{\mathrm{tree}}+\frac{T}{\gamma T_{\mathrm{gap}}}(\rho_+^2+\alpha\rho_-^2-\gamma)-\beta E_{\mathrm{gap}}}$$

$$z^{\mathrm{R}}_{\mathrm{CFT}}(\tau,\rho_+=1+\rho_-)\big|_{\mathrm{BTZ}}=\mathrm{Tr}_{\mathrm{R}}\left(q^{L_0}e^{i\pi(J_0^{3,+}-J_0^{3,-})}e^{i\pi\rho_1(J_0^{3,+}+J_0^{3,-})}\right)\Big|_{\mathrm{BTZ}}$$

$$\frac{\cos^2\left(\frac{\pi}{2}\rho_1\right)\cos^2\left(\frac{\pi}{2}\rho_2\right)}{\sin\left(\frac{\pi}{2}(\rho_1+\rho_2)\right)\sin\left(\frac{\pi}{2}(\rho_1-\rho_2)\right)}\to-\frac{\pi^2}{4}(1-\rho_2)^2+\mathrm{O}((\rho_2-1)^3)$$

$$\mathfrak{C}_\alpha(y,\rho_1)=\lim_{\rho_2\to 1}\cos\left(\frac{\pi}{2}\rho_2\right)\mathfrak{B}_\alpha(y,\rho_1,\rho_2).$$

$$\mathfrak{C}_\alpha(y,\rho_1)=-\frac{\pi}{64}e^{-\frac{2\alpha}{1+\alpha}y}\sum_{m\in\mathbb{Z}}\left[e^{-2y(1+\alpha)\left(m+\frac{1}{2}\rho_1+\frac{1}{2}\eta\right)^2}-e^{-2y(1+\alpha)\left(m+\frac{1}{2}\rho_1-\frac{1}{2}\eta\right)^2}\right]$$
$$=-\frac{\pi^{\frac{3}{2}}e^{-2\gamma y}}{64\sqrt{2y(1+\alpha)}}\left[\vartheta_3\left(e^{i\pi(\rho_1+\eta)},e^{-\frac{\pi^2}{(1+\alpha)y}}\right)-\vartheta_3\left(e^{i\pi(\rho_1-\eta)},e^{-\frac{\pi^2}{(1+\alpha)y}}\right)\right]$$
$$=\frac{\pi^{\frac{3}{2}}e^{-2\gamma y}}{16\sqrt{2y(1+\alpha)}}\sum_{n=1}^{\infty}\sin(n\pi\rho_1)\sin(n\pi\eta)e^{-\frac{\pi^2}{2y(1+\alpha)}n^2}$$

$$z^{\mathrm{R}}_{\mathrm{CFT}}(T,\rho_+=1+\rho_-)\big|_{\mathrm{BTZ}}\sim 2\pi^2\frac{\sqrt{\pi\alpha}}{1+\alpha}(\rho_2-1)\sqrt{\frac{T}{T_{\mathrm{gap}}}}\sin(\pi\rho_1)e^{S_{\mathrm{tree}}+S_1'\frac{T}{T_{\mathrm{gap}}}}$$
$$\times\sum_{n=1}^{\infty}\chi_{\frac{n-1}{2}}(\rho_1)\sin\left(n\pi\frac{1-\alpha}{1+\alpha}\right)e^{-\frac{\pi^2\alpha}{(1+\alpha)^2}\beta T_{\mathrm{gap}}n^2}$$

$$S_1'=-1+\frac{(1+\alpha)^2}{4\alpha}(1+\rho_1^2+2\rho_1\eta)$$

$$E_{\mathrm{R}}(\mathrm{j})=\frac{16\alpha}{(1+\alpha)^2}\left(\mathrm{j}+\frac{1}{2}\right)^2E_{\mathrm{gap}}$$

$$\tilde{z}^{\mathrm{R}}_{\mathrm{CFT}}\big(T,\rho_+=1+\rho_-,Q_{\mathrm{U}(1)=0}\big)\big|_{\mathrm{BTZ}}\sim 2\pi^2\frac{\sqrt{\pi\alpha}}{1+\alpha}\sin(\pi\rho_1)e^{S_{\mathrm{tree}}+S_1'\frac{T}{T_{\mathrm{gap}}}}$$
$$\times\sum_{\mathrm{j}=0}^{\infty}\chi_{\mathrm{j}}(\rho_1)\sin((2\mathrm{j}+1)\pi\eta)e^{-\beta E_{\mathrm{R}}(\mathrm{j})}$$

$$\tilde{z}^{\mathrm{R}}_{\mathrm{CFT}}\left(T,\rho_+=\gamma,\rho_-=\gamma-1,Q_{\mathrm{U}(1)=0}\right)\big|_{\mathrm{BTZ}}\sim\frac{2\pi^{\frac{5}{2}}\sqrt{\alpha}e^{S_{\mathrm{tree}}}}{(1+\alpha)}\sum_{\mathrm{j}=0}^{\infty}\sin^2\left((2\mathrm{j}+1)\pi\eta\right)e^{-\beta E_{\mathrm{R}}(\mathrm{j})}.$$

$$A^{\pm i}_{ab}=\frac{1}{2}(\pm\delta_{ia}\delta_{b0}\mp\delta_{ib}\delta_{a0}+\epsilon_{iab})$$

$$c=\frac{6\mathrm{k}_+\mathrm{k}_-}{\mathrm{k}_++\mathrm{k}_-}.$$

$$\begin{aligned}
\{G^a_r,G^b_s\}=&\frac{c}{3}\delta^{ab}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0}+2\delta^{ab}L_{r+s}\\
&+4i(r-s)\left(\gamma A^{+i}_{ab}J^{+,i}_{r+s}+(1-\gamma)A^{-i}_{ab}J^{-,i}_{r+s}\right)\\
\{Q^a_r,G^b_s\}=&2A^{+i}_{ab}J^{+,i}_{r+s}-2A^{-i}_{ab}J^{-,i}_{r+s}+\delta^{ab}U_{r+s}\\
\{Q^a_r,Q^b_s\}=&\frac{\mathrm{k}_++\mathrm{k}_-}{2}\delta^{ab}\delta_{r+s,0}\\
\left[J^{\pm,i}_m,G^a_r\right]=&iA^{\pm i}_{ab}G^b_{m+r}\mp\frac{2\mathrm{k}_\pm}{\mathrm{k}_++\mathrm{k}_-}mA^{pmi}_{ab}Q^b_{m+r}\\
[U_m,G^a_r]=&mQ^a_{m+r}\\
\left[J^{\pm,i}_m,Q^a_r\right]=&iA^{\pm i}_{ab}Q^b_{m+r}\\
\left[J^{\pm,i}_m,J^{\pm,j}_n\right]=&i\epsilon^{ijl}J^{\pm,l}_{m+n}+\frac{\mathrm{k}_\pm}{2}m\delta^{ij}\delta_{m+n,0}\\
[U_m,U_n]=&\frac{\mathrm{k}_++\mathrm{k}_-}{2}m\delta_{m+n,0}
\end{aligned}$$

$$\mathcal{A}_\gamma=\tilde{\mathcal{A}}_\gamma\oplus\mathcal{A}_{\mathrm{QU}}$$

$$\tilde{\mathrm{k}}_\pm=\mathrm{k}_\pm-1$$

$$\tilde{c}=c-3=3\frac{2\tilde{\mathrm{k}}_+\tilde{\mathrm{k}}_-+\tilde{\mathrm{k}}_++\tilde{\mathrm{k}}_-}{\tilde{\mathrm{k}}_++\tilde{\mathrm{k}}_-+2}$$

$$\gamma\to\frac{\mathrm{k}_--1}{\mathrm{k}_++\mathrm{k}_-},1-\gamma\to\frac{\mathrm{k}_+-1}{\mathrm{k}_++\mathrm{k}_-}.$$

$$\tilde{c}'=6\frac{(\mathrm{k}_+-1)(\mathrm{k}_--1)}{\mathrm{k}_++\mathrm{k}_-}$$

$$h\geq\frac{1}{\tilde{\mathrm{k}}_++\tilde{\mathrm{k}}_-+2}\left[\left(\tilde{\mathrm{k}}_++1\right)\ell_-+\left(\tilde{\mathrm{k}}_-+1\right)\ell_++(\ell_+-\ell_-)^2\right]$$

$$\frac{\prod_{\pm=\pm}\vartheta_1\left(\frac{\tau+\epsilon_+\rho_++\epsilon_-\rho_-}{2},\mathrm{t}\right)}{\eta(\mathrm{t})^6}=\vartheta^{\frac{1}{4}}q^{-1}\left[1-q+\chi^{\mathrm{g}}_{\mathrm{S}}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right]\sum_{h_{\mathrm{osc}}\geq0}D_{h_{\mathrm{osc}}}(q,r_+,r_-)\vartheta^{h_{\mathrm{osc}}}$$

$$\frac{\prod_{\epsilon_\pm=\pm}\vartheta_1\left(\frac{\tau+\epsilon_+\rho_++\epsilon_-\rho_-}{2},\mathrm{t}\right)}{\eta(\mathrm{t})^4\vartheta^{\frac{1}{4}}q^{-1}}=\left[1-q+\chi^{\mathrm{g}}_{\mathrm{S}}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right]\left(\prod_{n=1}^{\infty}\sum_{m=0}^{8}A_m(q,r_\pm)\vartheta^{mn}\right)$$

$$A_0 = A_8 = 1$$

$$A_1 = A_7 = -q^{-\frac{1}{2}}(1+q)\chi_{\frac{1}{2}}(r_+)\chi_{\frac{1}{2}}(r_-)$$

$$A_2 = A_6 = q^{-1}\left[(1+q^2)\left(\chi_1(r_+)+\chi_1(r_-)\right)+q\left(1+\chi_1(r_+)\right)\left(1+\chi_1(r_-)\right)\right]$$

$$A_3 = A_5 = -q^{-\frac{3}{2}}\chi_{\frac{1}{2}}(r_+)\chi_{\frac{1}{2}}(r_-)\left[1+q^3+(q+q^2)\left(\chi_1(r_+)+\chi_1(r_-)\right)\right]$$

$$A_4 = q^{-2}\left[1+q^4+(q+q^3)\left(1+\chi_1(r_+)\right)\left(1+\chi_1(r_-)\right)+q^2\left(\chi_1(r_+)+\chi_1(r_-)\right)^2\right]$$

$$\frac{\eta(\mathfrak{t})^3}{\vartheta_1(\tau,\mathfrak{t})} = \sum_{n\in\mathbb{Z}}(-1)^{n+1}\left[\sum_{r\geq n+w+1}(-1)^r\mathfrak{z}^{\frac{r(r+1)}{2}}\right]\mathfrak{z}^{-\frac{n(n+1)}{2}}q^{n+\frac{1}{2}}$$

$$\mathcal{I}_{\mathrm{PF}} = |1-q|^2 I_0\left|\tilde{\sum}(-1)^{n+r+1}D_{h_{\mathrm{osc}}}\mathfrak{z}^{\frac{1}{4}+\frac{r(r+1)}{2}-\frac{n(n+1)}{2}+h_{\mathrm{osc}}+p_L^2}q^{n-\frac{1}{2}}\right|^2$$

$$f_{\mathrm{PF}} = |1-q|^2 I_0\sum_{w=0}^{\infty}\hat{\sum}q^{n-\frac{1}{2}}\bar{q}^{\bar{n}-\frac{1}{2}}D_{h_{\mathrm{osc}}}\bar{D}_{\bar{h}_{\mathrm{osc}}}\mathfrak{I}_{k=1}$$

$$\mathfrak{I}_{k=1} \equiv \int_{-\infty}^{\infty}\frac{\zeta d\zeta}{i\pi}\frac{e^{2i\beta\zeta}}{\zeta^2+\Delta_w^2}\left[e^{-\frac{2\beta}{w+1}(\zeta^2+\Delta_w^2)}-e^{-\frac{2\beta}{w}(\zeta^2+\Delta_w^2)}\right]$$

$$\Delta_w^2 = \frac{1}{4}+\frac{r(r+1)}{2}-\frac{n(n+1)}{2}+h_{\mathrm{osc}}+p_L^2$$

$$\hat{\sum} = \sum_{n,\bar{n}\in\mathbb{Z}}\sum_{r\geq n+w}\sum_{\bar{r}\geq\bar{n}+w}(-1)^{n+r+\bar{n}+\bar{r}}\sum_{h_{\mathrm{osc}},\bar{h}_{\mathrm{osc}}}\sum_{p_L,p_R}\delta_{\mathrm{LM}},$$

$$\delta_{\mathrm{LM}}: \frac{r(r+1)}{2}-\frac{n(n+1)}{2}+h_{\mathrm{osc}}+p_L^2 = \frac{\bar{r}(\bar{r}+1)}{2}-\frac{\bar{n}(\bar{n}+1)}{2}+\bar{h}_{\mathrm{osc}}+p_R^2,$$

$$\frac{r(r+1)}{2}-\frac{n(n+1)}{2}+h_{\mathrm{osc}} \to N-\frac{w(w+1)}{2}$$

$$\mathfrak{B}_1(y,\rho_1,\rho_2) = \mathfrak{b}(y,\rho_2)\hat{\mathfrak{b}}(y,\rho_1)-\mathfrak{b}(y,\rho_1)\hat{\mathfrak{b}}(y,\rho_2),$$

$$\mathfrak{b}(y,\rho) \equiv \sum_{m\in\mathbb{Z}}e^{-4y\left(m+\frac{\rho}{2}\right)^2}$$

$$\hat{\mathfrak{b}}(y,\rho) \equiv \sum_{m\in\mathbb{Z}}\frac{e^{-4y\left(m+\frac{\rho}{2}\right)^2}}{1-4\left(m+\frac{\rho}{2}\right)^2}$$

$$\mathfrak{b}(y,\rho) = \sqrt{\frac{\pi}{4y}}\sum_{n\in\mathbb{Z}}e^{i\pi n\rho}$$

$$e^{-y}\frac{\mathrm{d}}{\mathrm{d}y}\left(e^{y}\hat{\mathfrak{b}}(y,\rho)\right) = \mathfrak{b}(y,\rho)$$

$$\mathfrak{B}_1(y,\rho_+,\rho_-) \sim \frac{\pi}{8}\sin(\pi\rho_+)\sin(\pi\rho_-)e^{-\frac{\pi^2}{4y}} + \cdots$$

$$\frac{1}{x} = \int_C dt e^{-xt}$$

$$\mathfrak{B}_\alpha(y,\rho_1,\rho_2) = \mathfrak{F}_\alpha(y,\rho_1,\rho_2) - \mathfrak{F}_\alpha(y,\rho_2,\rho_1),$$

$$\mathfrak{F}_\alpha(y,\rho_1,\rho_2) = e^{-2y(\alpha\rho_-^2+\rho_+^2)}\int_C dt e^{-t(1-\rho_1^2)}\sum_{m_1,m_2\in\mathbb{Z}} e^{-\mathbf{m}^T\cdot\mathbf{A}\cdot\mathbf{m}+2\pi i\mathbf{z}^T\cdot\mathbf{m}}$$

$$\mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, \mathbf{z} = \frac{i(1+\alpha)}{\pi} y \begin{pmatrix} \left(1-\frac{2t}{y(1+\alpha)}\right)\rho_1 + \eta\rho_2 \\ \rho_2 + \eta\rho_1 \end{pmatrix}$$

$$\mathbf{A} = 2y(1+\alpha)\begin{pmatrix} 1-\frac{2t}{y(1+\alpha)} & \eta \\ \eta & 1 \end{pmatrix}$$

$$\mathfrak{F}_\alpha(y,\rho_1,\rho_2) = \frac{e^{-2\gamma y}}{64\sqrt{2}\sqrt{y(1+\alpha)}}\int_{C_u} du\frac{e^u}{\sqrt{u}}\tilde{\mathfrak{f}}_\alpha, u = 2\gamma y - t$$

$$\tilde{\mathfrak{f}}_\alpha = \sum_{m_1,m_2\in\mathbb{Z}} e^{-\frac{\pi^2}{4u}(m_1+\eta m_2)^2-\frac{\pi^2}{2y(1+\alpha)}m_2^2}\left[e^{i\pi(m_1\rho_1+m_2\rho_2)} - e^{i\pi(m_2\rho_1+m_1\rho_2)}\right]$$

$$\tilde{\mathfrak{f}}_\alpha = -4\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\left(1-\frac{\delta_{m_1,0}}{2}\right)\left(1-\frac{\delta_{m_2,0}}{2}\right)e^{-\frac{\pi^2 m_2^2}{2y(1+\alpha)}}\tilde{F}_\alpha(m_1,m_2)$$

$$\tilde{F}_\alpha(m_1,m_2) = \left[e^{-\frac{\pi^2(m_1+\eta m_2)^2}{4u}}\sin(\pi(m_1+m_2)\rho_+)\sin(\pi(m_1-m_2)\rho_-)\right.$$
$$\left. + e^{-\frac{\pi^2(m_1-\eta m_2)^2}{4u}}\sin(\pi(m_1-m_2)\rho_+)\sin(\pi(m_1+m_2)\rho_-)\right]$$

$$\mathfrak{B}_\alpha = -\frac{1}{16\sqrt{2}}\frac{e^{-2\gamma y}}{\sqrt{y(1+\alpha)}}\sin(\pi\rho_+)\sin(\pi\rho_-)\int_0^{2\gamma y} du\frac{e^u}{\sqrt{u}}\mathfrak{f}_\alpha$$

$$\mathfrak{f}_\alpha = \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\left(1-\frac{\delta_{m_1,0}}{2}\right)\left(1-\frac{\delta_{m_2,0}}{2}\right)e^{-\frac{\pi^2 m_2^2}{2y(1+\alpha)}}F_\alpha(m_1,m_2)$$

$$F_\alpha(m_1,m_2) = \text{sign}(m_1-m_2)\left[e^{-\frac{\pi^2(m_1+\eta m_2)^2}{4u}}\chi_{\frac{m_1+m_2-1}{2}}(\rho_+)\chi_{\frac{|m_1-m_2|-1}{2}}(\rho_-)\right.$$
$$\left. + e^{-\frac{\pi^2(m_1-\eta m_2)^2}{4u}}\chi_{\frac{|m_1-m_2|-1}{2}}(\rho_+)\chi_{\frac{m_1+m_2-1}{2}}(\rho_-)\right]$$

En tanto que, la métrica Super de Yang – Mills, en un campo cuántico relativista, en supergravedad, se calcula así:

$$\mathcal{I}_S(\lambda;\alpha) = 4\sum_{\ell=1}^{\infty}(-1)^{\ell+1}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}\left[\binom{2\ell+1}{\ell}^2 - \alpha^{2(\ell+1)}\binom{2\ell+1}{\ell}\,{}_2F_1(\ell+1,\ell+2;1;1-\alpha)\right],$$

$$\mathcal{I}_{AdS}(\lambda;\beta) = 4\sum_{\ell=1}^{\infty}(-1)^{\ell}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}\left[\binom{2\ell+1}{\ell}^2 - \beta^{2(\ell+1)}\binom{2\ell+1}{\ell}\,{}_2F_1(\ell+1,\ell+2;1;1+\beta)\right],$$

$$\mathcal{I}_S(\lambda;\alpha) \sim 2\alpha - \frac{4\pi^2}{3\lambda} - \sum_{n=1}^{\infty}\frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2\Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)}$$

$$\mathcal{I}_{AdS}(\lambda;\beta) \sim 2\log(1+\beta) - \frac{4\pi^2}{3\lambda} + \sum_{n=1}^{\infty}\frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2\Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)}$$

$$\exp[-2n\sqrt{\lambda}], \exp[-n\sqrt{\lambda}(1-\sqrt{1-\alpha})], \exp[-n\sqrt{\lambda}(1+\sqrt{1-\alpha})],$$

$$\exp[-2n\sqrt{\lambda}], \exp[-n\sqrt{\lambda}(\sqrt{\beta+1}-1)], \exp[-n\sqrt{\lambda}(\sqrt{\beta+1}+1)]$$

$$\mathcal{O}_{\vec{p}}(x,Y) = \mathcal{O}_{(p_1\ldots p_m)}(x,Y) := \frac{p_1\cdots p_m}{p}\mathcal{O}_{p_1}(x,Y)\ldots\mathcal{O}_{p_m}(x,Y)$$

$$\mathcal{O}_q(x,Y) := \frac{1}{q}\mathrm{Tr}(Y\cdot\Phi(x))^q$$

$$\langle\mathcal{O}_{\vec{p}}(x_1,Y_1)\mathcal{O}_{\vec{p}}(x_2,Y_2)\rangle = \mathfrak{R}_{\vec{p}}(N)(d_{12})^p,\ d_{12} := \frac{Y_1\cdot Y_2}{x_{12}^2},$$

$$\frac{\langle\mathcal{O}_{\vec{m}}(x_1,Y_1)\mathcal{O}_{\vec{m}}(x_2,Y_2)\mathcal{O}_{\vec{p}}(x_3,Y_3)\rangle}{\langle\mathcal{O}_{\vec{m}}(x_1,Y_1)\mathcal{O}_{\vec{m}}(x_2,Y_2)\rangle} = \mathfrak{C}_{\vec{m}\vec{m}\vec{p}}(N)\times\left(\frac{d_{23}d_{31}}{d_{12}}\right)^{\frac{p}{2}}$$

$$\mathcal{O}_R^{\text{Schur}}(x,Y) := \frac{1}{m!}\sum_{\sigma\in S_m}\chi_R(\sigma)\mathrm{Tr}(\sigma\varphi)(x,Y),\ \text{with}\ \varphi(x,Y) := Y\cdot\Phi(x)$$

$$\mathrm{Tr}(\sigma\varphi) := \sum_{i_1,\ldots i_m=1}^{N}\varphi^{i_1}_{i_{\sigma(1)}}\ldots\varphi^{i_m}_{i_{\sigma(m)}}$$

$$\mathcal{O}^{\text{Schur}}_{(\overline{1})_m}(x,Y) = \det{}_m\varphi(x,Y)$$

$$\det{}_m X := \frac{1}{m!}\epsilon_{i_1\ldots i_m\ell_1\ldots\ell_{N-m}}\epsilon^{j_1\ldots j_m\ell_1\ldots\ell_{N-m}}X^{i_1}_{j_1}\cdots X^{i_m}_{j_m}$$

$$ds^2 = ds^2_{AdS_5} + ds^2_{S^5}$$

$$ds^2_{AdS_5} = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\tilde{\Omega}_3^2$$
$$ds^2_{S^5} = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\Omega_3^2$$

$$\cos^2\theta_0 = \frac{m}{N} = \alpha$$

$$\sinh^2\ \rho_0 = \beta$$

$$\mathcal{G}_S(x,Y;z) := \sum_{m=0}^{N} z^m \mathcal{O}^{\text{Schur}}_{(\vec{1})_m}(x,Y) = \det(\mathbb{1} + zY\cdot\Phi)(x), z \in \mathbb{C}$$

$$\mathcal{O}^{\text{Schur}}_{(\vec{1})_m}(x,Y) = \oint \frac{dz}{2\pi \mathrm{i}} \frac{1}{z^{m+1}} \mathcal{G}_S(x,Y;z)$$

$$\mathcal{G}_{S;\alpha}(x,Y) := \oint \frac{dz}{2\pi \mathrm{i}} \frac{1}{z^{\alpha N+1}} \mathcal{G}_S(x,Y;z)$$

$$\mathcal{G}_{AdS}(x,Y;z) := \sum_{m=0}^{\infty} z^m \mathcal{O}^{\text{Schur}}_{(m,\vec{0})}(x,Y) = [\det(\mathbb{1} - zY\cdot\Phi)]^{-1}(x),\ z \in \mathbb{C}$$

$$\mathcal{O}^{\text{Schur}}_{(m,\vec{0})}(x,Y) = \oint \frac{dz}{2\pi \mathrm{i}} \frac{1}{z^{m+1}} \mathcal{G}_{AdS}(x,Y;z)$$

$$\mathcal{G}_{AdS;\beta}(x,Y) := \oint \frac{dz}{2\pi \mathrm{i}} \frac{1}{z^{\beta N+1}} \mathcal{G}_{AdS}(x,Y;z)$$

$$\frac{\langle \mathcal{H}(x_1,Y_1)\mathcal{H}(x_2,Y_2)\mathcal{O}_{\vec{p}}(x_3,Y_3)\rangle}{\langle \mathcal{H}(x_1,Y_1)\mathcal{H}(x_2,Y_2)\rangle} = \mathfrak{C}_{\mathcal{H},\mathcal{H},\vec{p}}(N) \times \left(\frac{d_{23}d_{31}}{d_{12}}\right)^{\frac{p}{2}}.$$

$$\mathfrak{C}_{\mathcal{H},\mathcal{H},\vec{p}}(N) = \sum_{g=0}^{\infty} N^{\frac{p}{2}-g} \mathfrak{C}^{(g)}_{\mathcal{H},\mathcal{H},\vec{p}}$$

$$\mathfrak{C}^{(0)}_{\mathcal{G}_{S;\alpha}}\mathcal{G}_{S;\alpha},(2p) = -2^{-p}\oint_{z=0} \frac{dz}{2\pi i} \frac{1-z^2}{z^{2p+1}\sqrt{1+2z^2(2\alpha-1)+z^4}}$$
$$= (-2)^{-p+1}\ {}_2F_1(-p,p;1;1-\alpha)$$

$$\mathfrak{C}_{\mathcal{G}^{(0)}_{AdS;\beta},\mathcal{G}_{AdS;\beta},(2p)} = 2^{-p}\oint_{z=0} \frac{dz}{2\pi i} \frac{1-z^2}{z^{2p+1}\sqrt{1-2z^2(2\beta+1)+z^4}}$$
$$= -(-2)^{-p+1}\ {}_2F_1(-p,p;1;1+\beta)$$

$$\langle \mathcal{O}_2(x_1,Y_1)\mathcal{O}_2(x_2,Y_2)\mathcal{H}(x_3,Y_3)\mathcal{H}(x_4,Y_4)\rangle = \mathcal{C}_{\text{free}}(x_i,Y_i) + \mathcal{R}_4(x_i,Y_i) d_{34}^{\Delta_{\mathcal{H}}-2} \mathcal{C}_{\mathcal{H}}(u,v;\tau)$$

$$\tau := \tau_1 + \mathrm{i}\tau_2 = \frac{\theta}{2\pi} + \mathrm{i}\frac{4\pi}{g^2_{\text{YM}}},$$

$$u := \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, v := \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$N \to \infty, \text{ with } \lambda = g^2_{\text{YM}} N \text{ fixed}$$

$$\mathcal{I}_{\mathcal{H}}(\tau;N) := -\frac{2}{\pi}\int_0^{\infty} dr \int_0^{\pi} d\theta \frac{r^3 \sin^2\ \theta}{u^2} \mathcal{C}_{\mathcal{H}}(u,v;\tau)$$

$$\mathcal{Z}(\tau,\tau_p;m)=\int_{\mathbb{R}^N}d\mu(a_i)\left|\exp\left[\mathrm{i}\pi\tau\left(\sum_{i=1}^{N}a_i^2\right)+\mathrm{i}\sum_{p=3}^{\infty}\pi^{\frac{p}{2}}\tau_p\left(\sum_{i=1}^{N}a_i^p\right)\right]\right|^2 Z_{1-\mathrm{loop}}(a;m)\left|Z_{\mathrm{inst}}(\tau,\tau_p,a;m)\right|^2$$

$$d\mu(a_i)=\prod_{i=1}^{N}da_i\prod_{i<j}a_{ij}^2\delta\left(\sum_{i=1}^{N}a_i\right),$$

$$Z_{1-\mathrm{loop}}(a;m)=\frac{1}{H(m)^N}\prod_{i<j}\frac{H^2(a_{ij})}{H(a_{ij}+m)H(a_{ij}-m)},H(x):=\mathrm{e}^{-(1+\gamma)x^2}G(1+\mathrm{i}x)G(1-\mathrm{i}x)$$

$$a\to\frac{a}{\sqrt{2\pi\tau_2}}$$

$$\left\langle\langle f(a_i)\rangle:=\frac{1}{\mathcal{N}}\int_{\mathbb{R}^N}d\mu(a_i)\exp\left(-\sum_i a_i^2\right)f(a_i)\right.$$

$$\partial_{\tau_{p_1}}\dots\partial_{\tau_{p_\ell}}\mathcal{Z}(\tau,\tau_p;m)\leftrightarrow T_{\vec{p}}(a):=\mathrm{tr}(a^{p_1})\mathrm{tr}(a^{p_2})\cdots\mathrm{tr}(a^{p_\ell})$$

$$\mathcal{O}_{\vec{p}}(a):=T_{\vec{p}}(a)+\sum_{\vec{q}\vdash q<p}u_{\vec{p},\vec{q}}(N)T_{\vec{q}}(a),$$

$$\left\langle\left\langle\mathcal{O}_{\vec{p}}(a)T_{\vec{r}}(a)\right\rangle\right\rangle=0,\forall 0\le r<p\ \text{ and }\ \forall\vec{r}\ \text{partitions of}\ r,$$

$$\left\langle\left\langle\mathcal{O}_{\vec{p}}(a)\mathcal{O}_{\vec{p}}(a)\right\rangle\right\rangle=\Re_{\vec{p}}(N),\frac{\left\langle\left\langle\mathcal{O}_{\vec{m}}(a)\mathcal{O}_{\vec{m}}(a)\mathcal{O}_{\vec{p}}(a)\right\rangle\right\rangle}{\left\langle\left\langle\mathcal{O}_{\vec{m}}(a)\mathcal{O}_{\vec{m}}(a)\right\rangle\right\rangle}=\mathfrak{C}_{\vec{m}\vec{m}\vec{p}}(N)$$

$$T_{\vec{q}}(a)=\mathcal{O}_{\vec{q}}(a)+\sum_{\vec{p}\vdash p<q}v_{\vec{q},\vec{p}}(N)\mathcal{O}_{\vec{p}}(a)$$

$$v_{\vec{q},\vec{p}}(N)=(-1)^{\frac{q-p}{2}}u_{\vec{q},\vec{p}}(N)$$

$$\mathcal{I}_{\mathcal{H}}(\tau;N)=\frac{\partial_{\mathcal{H}}\partial_{\mathcal{H}}\partial_m^2\log\mathcal{Z}(\tau,\tau_p;m)\big|_{\tau_p,m=0}}{\partial_{\mathcal{H}}\partial_{\mathcal{H}}\log\mathcal{Z}(\tau,\tau_p;m)\big|_{\tau_p,m=0}}$$

$$\mathcal{Z}(g_{\mathrm{YM}}^2;m)=\int da\mathrm{e}^{-\mathrm{tr}a^2}Z_{1-\mathrm{loop}}\left(\frac{ag_{\mathrm{YM}}}{2\sqrt{2}\pi};m\right)$$

$$\partial_m^2\log\mathcal{Z}(g_{\mathrm{YM}}^2;m)\big|_{m=0}=\left\langle\left\langle\mathbf{M}_2(a,g_{\mathrm{YM}}^2)\right\rangle\right\rangle=\frac{1}{\mathcal{N}}\int da\mathrm{e}^{-\mathrm{tr}a^2}\mathbf{M}_2(a,g_{\mathrm{YM}}^2)$$

$$\mathbf{M}_2(a,g_{\mathrm{YM}}^2)=-2\sum_{\ell=1}^{\infty}\sum_{j=0}^{2\ell}(-1)^{j+\ell}(2\ell+1)\binom{2\ell}{j}\zeta(2\ell+1)\left(\frac{g_{\mathrm{YM}}^2}{8\pi^2}\right)^{\ell}T_{(2\ell-j,j)}(a)$$

$$\mathbf{M}_2\left(a, g_{\mathrm{YM}}^2\right) = -2\sum_{\ell=1}^{\infty}\sum_{j=0}^{2\ell}(-1)^{j+\ell}(2\ell+1)\binom{2\ell}{j}\zeta(2\ell+1)\left(\frac{g_{\mathrm{YM}}^2}{8\pi^2}\right)^{\ell}\left[\mathcal{O}_{(2\ell-j,j)}(a) + \sum_{\vec{p}\vdash p<2\ell} v_{(2\ell-j,j),\vec{p}}\mathcal{O}_{\vec{p}}(a)\right]$$

$$\mathcal{I}_{\mathcal{H}}\left(g_{\mathrm{YM}}^2; N\right) = \frac{\left\langle\left\langle \mathcal{H}(a)\mathcal{H}(a)\mathbf{M}_2\left(a, g_{\mathrm{YM}}^2\right)\right\rangle\right\rangle}{\langle\langle \mathcal{H}(a)\mathcal{H}(a)\rangle\rangle} - \left\langle\left\langle \mathbf{M}_2\left(a, g_{\mathrm{YM}}^2\right)\right\rangle\right\rangle.$$

$$\mathcal{I}_{\mathcal{H}}\left(g_{\mathrm{YM}}^2; N\right) = -2\sum_{\ell=1}^{\infty}\sum_{j=0}^{2\ell}(-1)^{j+\ell}(2\ell+1)\binom{2\ell}{j}\zeta(2\ell+1)\left(\frac{g_{\mathrm{YM}}^2}{8\pi^2}\right)^{\ell} \times\left(\mathfrak{C}_{\mathcal{H},\mathcal{H},(2\ell-j,j)} + \sum_{\vec{p}\vdash p<2\ell} v_{(2\ell-j,j),\vec{p}}\mathfrak{C}_{\mathcal{H},\mathcal{H},\vec{p}}\right)$$

$$\mathcal{I}_{\mathcal{H}}(\lambda; N) = \sum_{g=0}^{\infty} N^{1-g}\mathcal{I}_{\mathcal{H}}^{(g)}(\lambda).$$

$$\mathcal{I}_{\mathcal{H}}(\lambda; N) = -2N\sum_{\ell=1}^{\infty}(-1)^{\ell}(2\ell+1)\zeta(2\ell+1)\left(\frac{\lambda}{8\pi^2}\right)^{\ell} \times\left[2\mathfrak{C}_{\mathcal{H},\mathcal{H},(2\ell)}^{(0)} + \sum_{j=0}^{\ell}\binom{2\ell}{2j}\sum_{p=1}^{\ell-1} v_{(2\ell-2j,2j),(2p)}^{(0)}\mathfrak{C}_{\mathcal{H},\mathcal{H},(2p)}^{(0)}\right] + O(N^0)$$

$$v_{(2\ell-2j,2j),(2p)}^{(0)} = 2^{p-\ell}\left[\binom{2\ell-2j}{\ell-j+p}C_j + \binom{2j}{j+p}C_{\ell-j}\right],$$

$$\mathcal{I}_{\mathcal{H}}^{(0)}(\lambda) = -4\sum_{\ell=1}^{\infty}(-1)^{\ell}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}\sum_{p=1}^{\ell} 2^p\binom{2\ell+1}{\ell+p+1}\binom{2\ell+1}{\ell+p}\mathfrak{C}_{\mathcal{H},\mathcal{H},(2p)}^{(0)}$$

$$\mathcal{I}_S(\lambda; \alpha) := \mathcal{I}_{\mathcal{G}_S;\alpha}^{(0)}(\lambda), \qquad \mathcal{I}_{AdS}(\lambda; \beta) := \mathcal{I}_{\mathcal{G}_{AdS};\beta}^{(0)}(\lambda)$$

$$\mathcal{I}_S(\lambda; \alpha) = 8\sum_{\ell=1}^{\infty}(-1)^{\ell}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}\sum_{p=1}^{\ell}(-1)^p\binom{2\ell+1}{\ell+p+1}\binom{2\ell+1}{\ell+p}\,{}_2F_1(-p, p; 1; 1-\alpha)$$

$$\mathcal{I}_S(\lambda; \alpha) = \mathcal{I}_{S;0}(\lambda) + \mathcal{I}_{S;1}(\lambda; \alpha)$$

$$\mathcal{I}_{S;0}(\lambda) := 4\sum_{\ell=1}^{\infty}(-1)^{\ell+1}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}\binom{2\ell+1}{\ell}^2,$$

$$\mathcal{I}_{S;1}(\lambda; \alpha) := 4\sum_{\ell=1}^{\infty}(-1)^{\ell}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}\alpha^{2(\ell+1)}\binom{2\ell+1}{\ell}\,{}_2F_1(\ell+1, \ell+2; 1; 1-\alpha).$$

$$\mathcal{I}_S(\lambda; 1) = 4\sum_{\ell=1}^{\infty}(-1)^{\ell+1}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}\left[\binom{2\ell+1}{\ell}^2 - \binom{2\ell+1}{\ell}\right].$$

$$\mathcal{I}_{AdS}(\lambda; \beta) = \mathcal{I}_{AdS;0}(\lambda) + \mathcal{I}_{AdS;1}(\lambda; \beta)$$

$$\mathcal{I}_{AdS;0}(\lambda) := 4\sum_{\ell=1}^{\infty}(-1)^{\ell}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}\binom{2\ell+1}{\ell}^2 = -\mathcal{I}_S^{(0)}(\lambda)$$

$$\mathcal{I}_{AdS;1}(\lambda;\beta) := 4\sum_{\ell=1}^{\infty}(-1)^{\ell+1}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}(1+\beta)^{\ell}\binom{2\ell+1}{\ell}(\ell+1)\,{}_2F_1\left(-\ell,-\ell;2;\frac{1}{1+\beta}\right)$$

$$(1+\beta)^{\ell}(\ell+1)\,{}_2F_1\left(-\ell,-\ell;2;\frac{1}{1+\beta}\right) = \beta^{2(\ell+1)}\,{}_2F_1(\ell+1,\ell+2;1;1+\beta), \forall \ell \in \mathbb{N},$$

$$\mathcal{I}_{AdS;1}(\lambda;\beta) = 4\sum_{\ell=1}^{\infty}(-1)^{\ell+1}\zeta(2\ell+1)\left(\frac{\lambda}{16\pi^2}\right)^{\ell}\beta^{2(\ell+1)}\binom{2\ell+1}{\ell}\,{}_2F_1(\ell+1,\ell+2;1;1+\beta)$$

$$\alpha^{2(\ell+1)}\,{}_2F_1(\ell+1,\ell+2;1;1-\alpha) \sim \frac{(1+\sqrt{1-\alpha})^{2(\ell+1)}}{2\sqrt{\pi}\ell\sqrt[4]{1-\alpha}}, \ell \gg 1$$

$$|\lambda| < R_S(\alpha) := \pi^2\frac{4}{(1+\sqrt{1-\alpha})^2}$$

$$|\lambda| < R_{AdS}(\beta) := \pi^2\frac{4}{(1+\sqrt{1+\beta})^2},$$

$$\mathcal{I}_{S;0}(\lambda) = -\int_{\mathrm{Res}=\frac{1}{2}}\frac{ds}{2\pi\mathrm{i}}\frac{4\pi\zeta(2s+1)}{\sin(\pi s)}\binom{2s+1}{s}^2\left(\frac{\lambda}{16\pi^2}\right)^s$$

$$\mathcal{I}_{S;1}(\lambda;\alpha) = \int_{\mathrm{Res}=\frac{1}{2}}\frac{ds}{2\pi\mathrm{i}}\frac{4\pi\zeta(2s+1)}{\sin(\pi s)}\alpha^{2(s+1)}\binom{2s+1}{s}\,{}_2F_1(s+1,s+2;1;1-\alpha)\left(\frac{\lambda}{16\pi^2}\right)^s$$

$$\mathcal{I}_{AdS;0}(\lambda) = -\mathcal{I}_{S;0}(\lambda)$$

$$\mathcal{I}_{AdS;1}(\lambda;\beta) = -\int_{\mathrm{Res}=\frac{1}{2}}\frac{ds}{2\pi\mathrm{i}}\frac{4\pi\zeta(2s+1)}{\sin(\pi s)}(1+\beta)^s\binom{2s+1}{s}(s+1)\,{}_2F_1\left(-s,-s;2;\frac{1}{1+\beta}\right)\left(\frac{\lambda}{16\pi^2}\right)^s$$

$${}_2F_1(\ell+1,\ell+2;1;1+\beta)\ \ell \in \mathbb{N} \rightarrow {}_2F_1(s+1,s+2;1;1+\beta)\ s \in \mathbb{C},$$

$$\mathcal{I}_S(\lambda;\alpha) = -\mathcal{I}_{AdS}(\lambda;-\alpha), \alpha < 0$$

$$\lim_{\alpha\to 0}\left[\alpha^{2(s+1)}\,{}_2F_1(s+1,s+2;1;1-\alpha)\right] = \lim_{\beta\to 0}\left[(1+\beta)^s(s+1)\,{}_2F_1\left(-s,-s;2;\frac{1}{1+\beta}\right)\right] = \binom{2s+1}{s}$$

$$\lim_{\alpha\to 0}\left[\mathcal{I}_{S;1}(\lambda;\alpha)\right] = -\lim_{\beta\to 0}\left[\mathcal{I}_{AdS;1}(\lambda;\beta)\right] = -\mathcal{I}_{S;0}(\lambda)$$

$$\mathcal{I}_{S;0}(\lambda) = -\mathcal{I}_{AdS;0}(\lambda) = \int_{\mathrm{Res}=\frac{1}{2}}\frac{ds}{2\pi\mathrm{i}}\frac{8\pi^{s-\frac{3}{2}}(2s-3)\xi(2s-2)\tan(\pi s)\Gamma(s-2)^2}{\Gamma\left(s-\frac{3}{2}\right)}\lambda^{1-s}$$

$$\mathcal{I}_{S;1}(\lambda;\alpha) = -\int_{\mathrm{Res}=\frac{1}{2}}\frac{ds}{2\pi\mathrm{i}}4^s\pi^{s-1}(2s-3)\xi(2s-2)\Gamma(s-2)\,{}_2F_1(s-2,s-1;1;1-\alpha)\lambda^{1-s}$$

$$\mathcal{I}_{AdS;1}(\lambda;\beta) = -\int_{\mathrm{Res}=\frac{1}{2}}\frac{ds}{2\pi\mathrm{i}}4^s\pi^{s-1}(2s-3)\xi(2s-2)\Gamma(s-1)\,{}_2F_1\left(s-1,s-1;2;\frac{1}{\beta+1}\right)(\beta+1)^{1-s}\lambda^{1-s}$$

$$\mathcal{I}_{S;0}(\lambda)\big|_{\text{strong}} \sim \mathcal{I}_{S;0}^{(\text{p})}(\lambda)$$

$$= 2[\log(\lambda) + 2\gamma + 2 - 2\log(4\pi)] + \frac{4\pi^2}{3\lambda} - \sum_{n=1}^{\infty} \frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2\Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)}$$

$$\mathcal{I}_{S;1}(\lambda;\alpha)\big|_{\text{strong}} \sim \mathcal{I}_{S;1}^{(\text{p})}(\lambda;\alpha) = 2\alpha - 2[\log(\lambda) + 2\gamma + 2 - 2\log(4\pi)] - \frac{8\pi^2}{3\lambda}$$

$$\mathcal{I}_S(\lambda;\alpha)|_{\text{strong}} = \mathcal{I}_{S;0}(\lambda)\big|_{\text{strong}} + \mathcal{I}_{S;1}(\lambda;\alpha)\big|_{\text{strong}}$$

$$\sim \mathcal{I}_{S;0}^{(\text{p})}(\lambda) + \mathcal{I}_{S;1}^{(\text{p})}(\lambda;\alpha) = 2\alpha - \frac{4\pi^2}{3\lambda} - \sum_{n=1}^{\infty} \frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2\Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)}$$

$$\mathcal{I}_{AdS;1}(\lambda;\beta)\big|_{\text{strong}} \sim \mathcal{I}_{AdS;1}^{(\text{p})}(\lambda;\beta) = 2\log(1+\beta) + 2[\log(\lambda) + 2\gamma + 2 - 2\log(4\pi)]$$

$$\mathcal{I}_{AdS}(\lambda;\beta)|_{\text{strong}} \sim \mathcal{I}_{AdS;0}^{(\text{p})}(\lambda) + \mathcal{I}_{AdS;1}^{(\text{p})}(\lambda;\beta)$$

$$= 2\log(1+\beta) - \frac{4\pi^2}{3\lambda} + \sum_{n=1}^{\infty} \frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2\Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)}$$

$$\mathcal{I}_{AdS}(\lambda;\beta) = \frac{3\beta\lambda\zeta(3)}{2\pi^2} - \frac{15\beta(\beta+4)\lambda^2\zeta(5)}{32\pi^4} + \frac{35\beta(2\beta^2+15\beta+30)\lambda^3\zeta(7)}{512\pi^6} + \cdots$$

$$\mathcal{I}_{AdS}(\lambda;\beta)|_{\text{lead}-\beta} = 4\sum_{\ell=1}^{\infty} (-1)^{\ell+1}\binom{2\ell}{\ell}(\ell+1)\zeta(2\ell+1)\left(\frac{\lambda_\beta}{16\pi^2}\right)^{\ell}$$

$$= 4\int_0^{\infty} dw \frac{w}{\sinh^2(w)}\left[1 - J_0\left(\frac{w\sqrt{\lambda_\beta}}{\pi}\right)\right]$$

$$2^{1-s}\zeta(s)\Gamma(s+1) = \int_0^{\infty} dw \frac{w^s}{\sinh^2(w)}$$

$$\mathcal{I}_{AdS}(\lambda;\beta)|_{\text{lead}-\beta} = 4 + 4\gamma + 2\log\left(\frac{\lambda_\beta}{16\pi^2}\right) + 8\sum_{n=1}^{\infty}\left(n\sqrt{\lambda_\beta}K_1\left(n\sqrt{\lambda_\beta}\right) - K_0\left(n\sqrt{\lambda_\beta}\right)\right)$$

$$= 4 + 4\gamma + 2\log\left(\frac{\lambda_\beta}{16\pi^2}\right)$$

$$-8\sum_{n=1}^{\infty}\left(2\pi n\sqrt{\lambda_\beta}\right)^{\frac{1}{2}} e^{-n\sqrt{\lambda_\beta}} \sum_{k=0}^{\infty} \frac{(4k^2-8k-1)\Gamma\left(k-\frac{1}{2}\right)^2}{\pi\Gamma(k+1)}\left(-2n\sqrt{\lambda_\beta}\right)^{-k}$$

$$\langle\Phi_I(x)\rangle_\theta = \frac{2\sqrt{\lambda_\beta}}{4\pi\sqrt{d_{12}}}\left(e^{i\theta}\Omega_2^{(N)}\frac{(Y_1)_I}{(x-x_1)^2} + e^{-i\theta}\Omega_2^{(N)}\frac{(Y_2)_I}{(x-x_2)^2}\right), \text{ with } I = 1,\ldots,6$$

$$\Omega_2^{(N)} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$\mathcal{I}_{S;0}^{(\mathrm{np})}(\lambda) = \pm \mathrm{i}\frac{64\sqrt{\lambda}}{\pi^2}\sum_{n=1}^{\infty} e^{-2n\sqrt{\lambda}} \int_0^{\infty} dz e^{-2n\sqrt{\lambda}z}\left[(z+1)^3 G_{3,3}^{2,3}\left((z+1)^2 \middle| \begin{matrix} -\frac{1}{2},\frac{1}{2},\frac{1}{2} \\ 0,0,-\frac{3}{2} \end{matrix}\right) - \frac{\pi^3}{4}\right]$$

$$= \pm \mathrm{i}\sum_{n=1}^{\infty} e^{-2n\sqrt{\lambda}}\left[\frac{32}{\sqrt{\lambda}n^2} + \frac{24}{\lambda n^3} + \frac{5}{\lambda^{3/2}n^4} - \frac{9}{4\lambda^2 n^5} + \frac{123}{64\lambda^{5/2}n^6} + O(\lambda^{-2})\right]$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha) = -\sum_{n=1}^{\infty} \int_{\mathrm{Re}s=\frac{3}{2}} \frac{ds}{2\pi \mathrm{i}} 4(2s-1)\Gamma(s)\Gamma(s-1)\,{}_2F_1(s-1,s;1;1-\alpha)\left(\frac{Y_n}{2}\right)^{-2s},$$

$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta) = -\sum_{n=1}^{\infty} \int_{\mathrm{Re}s=\frac{3}{2}} \frac{ds}{2\pi \mathrm{i}} 4(2s-1)\Gamma(s)^2\,{}_2F_1\left(s,s;2;\frac{1}{1+\beta}\right)\left(\frac{Y_n\sqrt{(\beta+1)}}{2}\right)^{-2s},$$

$$\xi(s) = \xi(1-s)$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha) = -16\pi(1-\alpha)^{\frac{1}{4}}\sum_{n=1}^{\infty}\left[e^{-n\sqrt{\lambda}(1-\sqrt{1-\alpha})}\sum_{m=0}^{\infty}\frac{1}{m!}c_m(\sqrt{1-\alpha})(2n\sqrt{\lambda(1-\alpha)})^{-m-1}\right.$$
$$\left. \pm \mathrm{i}e^{-n\sqrt{\lambda}(1+\sqrt{1-\alpha})}\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}c_m(-\sqrt{1-\alpha})(2n\sqrt{\lambda(1-\alpha)})^{-m-1}\right]$$

$$c_m(x) = [m^2+m-1]x\,{}_3\tilde{F}_2\left(-\frac{1}{2},\frac{3}{2},-m;\frac{3}{2}-m,\frac{3}{2}-m;x\right)$$
$$+\,{}_3\tilde{F}_2\left(-\frac{3}{2},\frac{1}{2},-m-1;\frac{1}{2}-m,\frac{1}{2}-m;x\right) - \frac{3}{4}m(m+1)x^2\,{}_3\tilde{F}_2\left(\frac{1}{2},\frac{5}{2},1-m;\frac{5}{2}-m,\frac{5}{2}-m;x\right)$$

$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta) = -16\pi(1+\beta)^{\frac{1}{4}}\sum_{n=1}^{\infty}\left[e^{-n\sqrt{\lambda}(\sqrt{\beta+1}-1)}\sum_{m=0}^{\infty}\frac{1}{m!}c_m(\sqrt{1+\beta})(-2n\sqrt{\lambda(1+\beta)})^{-m-1}\right.$$
$$\left. \mp \mathrm{i}e^{-n\sqrt{\lambda}(\sqrt{\beta+1}+1)}\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}c_m(-\sqrt{1+\beta})(2n\sqrt{\lambda(1+\beta)})^{-m-1}\right]$$

$$\mathcal{I}_{S;0}^{(\mathrm{np})}(\lambda) = -\mathcal{I}_{AdS;0}^{(\mathrm{np})}(\lambda) \propto \exp\left[-2n\sqrt{\lambda}\right]$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)\Big|_{\mathbb{R}} \propto \exp\left[-n\sqrt{\lambda}(1-\sqrt{1-\alpha})\right], \mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)\Big|_{\mathbb{I}} \propto \exp\left[-n\sqrt{\lambda}(1+\sqrt{1-\alpha})\right].$$
$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta)\Big|_{\mathbb{R}} \propto \exp\left[-n\sqrt{\lambda}(\sqrt{\beta+1}-1)\right], \mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta)\Big|_{\mathbb{I}} \propto \exp\left[-n\sqrt{\lambda}(\sqrt{\beta+1}+1)\right].$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)\Big|_{\mathbb{R}} \gg \mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda)\Big|_{\mathbb{I}} \gg \mathcal{I}_{S;0}^{(\mathrm{np})}(\lambda)$$

$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta)\Big|_{\mathbb{R}} \gg \mathcal{I}_{AdS;0}^{(\mathrm{np})}(\lambda) \gg \mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\beta)\Big|_{\mathbb{I}}$$

$$\mathcal{I}^{(\mathrm{np})}_{AdS;0}(\lambda) \gg \mathcal{I}^{(\mathrm{np})}_{AdS;1}(\lambda;\beta)\Big|_{\mathbb{R}} \gg \mathcal{I}^{(\mathrm{np})}_{S;1}(\lambda;\beta)\Big|_{\mathbb{I}}$$

$$\mathcal{I}^{(\mathrm{np})}_{S;1}(\lambda;\alpha)\Big|_{\mathbb{R}/\mathbb{I}} \propto \exp\left[-2\pi L^2\left(1 \mp \sin(\theta_0)\right)nT_F\right]$$
$$\mathcal{I}^{(\mathrm{np})}_{AdS;1}(\lambda;\beta)\Big|_{\mathbb{R}/\mathbb{I}} \propto \exp\left[-2\pi L^2(\cosh(\rho_0) \mp 1)nT_F\right]$$

$$\frac{\mathcal{I}^{(\mathrm{np})}_{S;1}(\lambda;\alpha)\Big|_{\mathbb{I}}}{\mathcal{I}^{(\mathrm{np})}_{S;1}(\lambda;\alpha)\Big|_{\mathbb{R}}} \propto \exp\left[-4\pi L^2 \sin(\theta_0) nT_F\right]$$
$$\frac{\mathcal{I}^{(\mathrm{np})}_{AdS;1}(\lambda;\beta)\Big|_{\mathbb{I}}}{\mathcal{I}^{(\mathrm{np})}_{AdS;1}(\lambda;\beta)\Big|_{\mathbb{R}}} \propto \exp\left[-4\pi L^2 nT_F\right]$$

$$\left[N\mathcal{I}^{(0)}_{\mathcal{G}_{S;\alpha}}(\lambda)\right]_{N\to -N} = -N\mathcal{I}^{(0)}_{\mathcal{G}_{S;-\alpha}}(\lambda) = +N\mathcal{I}^{(0)}_{\mathcal{G}_{AdS;\beta}}\Big|_{\beta=-\alpha}$$

$$\partial_\mu^2\partial_m^2 \log Z(\tau;m,\mu)\big|_{m=\mu=0} \sim NF_1(\lambda) + F_2(\lambda) + N^{-1}F_3(\lambda) + \cdots,$$

$$F_1(\lambda) = 2^6\int_0^\infty \frac{w_1^2 dw_1}{\sinh(w_1)^2}\frac{w_2 dw_2}{\sinh(w_2)^2}J_1(x_2)\frac{w_2J_0(x_2)J_1(x_1) - w_1J_0(x_1)J_1(x_2)}{w_1^2 - w_2^2}$$

$$F_1(\lambda) = \frac{9\zeta(3)^2}{2\pi^4}\lambda^2 - \frac{105\zeta(3)\zeta(5)}{16\pi^6}\lambda^3 + \frac{105(5\zeta(5)^2 + 12\zeta(3)\zeta(7))}{256\pi^8}\lambda^4$$
$$-\frac{21(65\zeta(5)\zeta(7) + 99\zeta(3)\zeta(9))}{512\pi^{10}}\lambda^5 + \frac{105(427\zeta(7)^2 + 1284\zeta(5)\zeta(9) + 2211\zeta(3)\zeta(11))}{65536\pi^{12}}\lambda^6 + \cdots$$

$$F_1(\lambda)|_{\text{strong}} \sim 8\left(2 - \frac{4\pi^2}{\lambda} - \sum_{n=1}^{\infty}\frac{16n\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2\Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\pi^{3/2}\Gamma(n)}\right)$$

$$\frac{x_2J_0(x_2)J_1(x_1) - x_1J_0(x_1)J_1(x_2)}{x_1^2 - x_2^2} = 4\sum_{\ell=1}^{\infty}\frac{\ell J_{2\ell}(x_1)J_{2\ell}(x_2)}{x_1x_2},$$

$$J_\mu(x) = \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty}\frac{ds}{2\pi i}\frac{\Gamma(-s)}{\Gamma(\mu+s+1)}\left(\frac{x}{2}\right)^{\mu+2s}$$
$$J_\mu(x)J_\nu(x) = \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty}\frac{ds}{2\pi i}\frac{\Gamma(-s)\Gamma(2s+\mu+\nu+1)}{\Gamma(s+\mu+1)\Gamma(s+\nu+1)\Gamma(s+\mu+\nu+1)}\left(\frac{x}{2}\right)^{\mu+\nu+2s}$$

$$F_1(\lambda) = \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} dsdt\, 2^{9-2s}\pi^{\frac{5}{2}}\frac{s(2s-1)t(2t-1)\xi(2s)\xi(2t)\,{}_4\tilde{F}_3(2,s,s+1,t+1;2-s,3-s,2-t;-1)}{\sin(2\pi s)\sin(\pi t)\Gamma\left(s-\frac{1}{2}\right)}\left(\frac{\lambda}{4\pi}\right)^{-s-t}$$

$$F_1(\lambda) = -\sum_{n_1,n_2=1}^{\infty}\sum_{\ell=1}^{\infty}\frac{2^6\ell}{\pi}\int_{c'-\mathrm{i}\infty}^{c'+\mathrm{i}\infty} dsdt\Big[\frac{(2s-1)\Gamma(s-\ell-1)\Gamma(s-\ell)\Gamma(s+\ell-1)\Gamma(s+\ell)}{\Gamma(2s-1)\cot(\pi s)}$$
$$\times(2t-1)\Gamma(t-\ell)\Gamma(t+\ell)n_1^{-2s}n_2^{-2t}\left(\frac{\sqrt{\lambda}}{2}\right)^{-2s-2t}\Big]$$

$$\mathrm{i}\exp\left[-2n_1\sqrt{\lambda}\right],\mathrm{i}\exp\left[-(2n_1+n_2)\sqrt{\lambda}\right],\exp\left[-n_2\sqrt{\lambda}\right]$$

$${}_2F_1(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=-\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{4(1-\alpha)^m}{\Gamma(m+1)^2}\int_{\mathrm{Re}s=\frac{3}{2}}\frac{ds}{2\pi i}(2s-1)\Gamma(m+s)\Gamma(m+s-1)\left(\frac{Y_n}{2}\right)^{-2s}$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{8(1-\alpha)^m}{\Gamma(m+1)^2}\left[\left(\frac{Y_n}{2}\right)^{2m-1}K_1(Y_n)+G_{2,3}^{2,1}\left(0;m-1,m,1;\frac{Y_n^2}{4}\right)\right]$$

$$K_1(Y)\sim-\sqrt{\frac{\pi}{2Y}}e^{-Y}\sum_{k=0}^{\infty}\frac{\Gamma\left(k-\frac{1}{2}\right)\Gamma\left(k+\frac{3}{2}\right)}{\pi\Gamma(k+1)}(-2Y)^{-k}$$

$$G_{2,3}^{2,1}\left(0;m-1,m,1;\frac{Y^2}{4}\right)\sim-\sqrt{\frac{\pi}{2Y}}\left(\frac{Y}{2}\right)^{2m}e^{-Y}\sum_{k=0}^{\infty}\frac{\Gamma\left(k-\frac{3}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{2\pi\Gamma(k+1)}$$
$$\times[(2k+1)(2k+3)-16km](-2Y)^{-k}$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=-\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{(1-\alpha)^m}{\Gamma(m+1)^2}\sqrt{\frac{8\pi}{Y_n}}e^{-Y_n}\left(\frac{Y_n}{2}\right)^{2m}$$
$$\times\left[\sum_{k=0}^{\infty}(-2Y_n)^{-k}\frac{\Gamma(k+1/2)\Gamma(k-3/2)}{\pi\Gamma(k+1)}(4k^2+3-16km)\right]$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=-\sum_{n=1}^{\infty}\sqrt{\frac{8\pi}{Y_n}}e^{-Y_n}\sum_{k=0}^{\infty}\frac{\Gamma\left(k-\frac{3}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\pi\Gamma(k+1)}(-2Y_n)^{-k}$$
$$\times\left[(4k^2+3)I_0\left(\sqrt{1-\alpha}Y_n\right)-8kY_n\sqrt{1-\alpha}I_1\left(\sqrt{1-\alpha}Y_n\right)\right]$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;1)=-\sum_{n=1}^{\infty}e^{-Y_n}\sqrt{\frac{8\pi}{Y_n}}\sum_{k=0}^{\infty}\frac{(4k^2+3)\Gamma\left(k-\frac{3}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\pi\Gamma(k+1)}(-2Y_n)^{-k}$$

$$I_\nu(z)=\frac{e^z}{(2\pi z)^{\frac{1}{2}}}\sum_{j=0}^{\infty}\frac{\left(\frac{1}{2}+\nu\right)_j\left(\frac{1}{2}-\nu\right)_j}{j!}(2z)^{-k}$$
$$\pm\mathrm{i}e^{\pm\nu\pi\mathrm{i}}\frac{e^{-z}}{(2\pi z)^{\frac{1}{2}}}\sum_{j=0}^{\infty}\frac{\left(\frac{1}{2}+\nu\right)_j\left(\frac{1}{2}-\nu\right)_j}{j!}(-2z)^{-k}$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=-2\sum_{n=1}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}(-1)^k\left(\frac{1}{2n\sqrt{\lambda}}\right)^{j+k}\frac{\Gamma(j+1/2)^2\Gamma(k+1/2)\Gamma(k-3/2)}{\pi^2\Gamma(j+1)\Gamma(k+1)}\frac{1}{(1-\alpha)^{1/4+j/2}}$$
$$\times\left[e^{-n\sqrt{\lambda}(1-\sqrt{1-\alpha})}\left(\frac{4k^2+3}{n\sqrt{\lambda}}+8k\frac{2j+1}{2j-1}\sqrt{1-\alpha}\right)\pm\mathrm{i}(-1)^je^{-n\sqrt{\lambda}(1+\sqrt{1-\alpha})}\left(\frac{4k^2+3}{n\sqrt{\lambda}}-8k\frac{2j+1}{2j-1}\sqrt{1-\alpha}\right)\right]$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)=-16\pi(1-\alpha)^{\frac{1}{4}}\sum_{n=1}^{\infty}\left[e^{-n\sqrt{\lambda}(1-\sqrt{1-\alpha})}\sum_{m=0}^{\infty}\frac{1}{m!}c_m(\sqrt{1-\alpha})(2n\sqrt{\lambda(1-\alpha)})^{-m-1}\right.$$
$$\left.\pm\mathrm{i}e^{-n\sqrt{\lambda}(1+\sqrt{1-\alpha})}\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}c_m(-\sqrt{1-\alpha})(2n\sqrt{\lambda(1-\alpha)})^{-m-1}\right]$$

$$\mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda;\alpha)\Big|_{\mathbb{R}}\sim\exp\left[-n\sqrt{\lambda}(1-\sqrt{1-\alpha})\right],\ \mathcal{I}_{S;1}^{(\mathrm{np})}(\lambda)\Big|_{\mathbb{I}}\sim\mathrm{i}\exp\left[-n\sqrt{\lambda}(1+\sqrt{1-\alpha})\right]$$

$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta)=-\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{4}{(1+\beta)^m\Gamma(m+1)\Gamma(m+2)}\int_{\mathrm{Res}=\frac{3}{2}}\frac{ds}{2\pi i}(2s-1)\Gamma(m+s)^2\left(\frac{Y_n\sqrt{\beta+1}}{2}\right)^{-2s}$$

$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{8(1+\beta)^{-m}}{\Gamma(m+1)\Gamma(m+2)}\left[\left(\frac{Y_n\sqrt{\beta+1}}{2}\right)^{2m}K_0\left(Y_n\sqrt{\beta+1}\right)+G_{2,3}^{2,1}\left(0;m,m,1;\frac{Y_n^2(\beta+1)}{4}\right)\right]$$

$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta)=\sum_{n=1}^{\infty}e^{-Y_n\sqrt{\beta+1}}\sum_{m=0}^{\infty}\frac{\sqrt{2\pi Y_n}}{\Gamma(m+1)\Gamma(m+2)}\left(\frac{Y_n}{2}\right)^{2m}$$
$$\times\sum_{k=0}^{\infty}(1+\beta)^{1/4-k/2}\frac{\Gamma(k-1/2)^2}{\pi\Gamma(k+1)}(-2Y_n)^{-k}[(4k^2-8k-1)-16km].$$

$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta)=\sum_{n=1}^{\infty}\sqrt{\frac{8\pi}{Y_n}}e^{-Y_n\sqrt{\beta+1}}\sum_{k=0}^{\infty}\frac{\Gamma(k-1/2)^2}{\pi(1+\beta)^{k/2-1/4}\Gamma(k+1)}(-2Y_n)^k$$
$$\times[(4k^2+8k-1)I_1(Y_n)-8kY_nI_0(Y_n)].$$

$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta)=\sum_{n=1}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}(-1)^k\left(\frac{1}{2n\sqrt{\lambda}}\right)^{j+k}\frac{\Gamma(k-1/2)^2\Gamma(j+1/2)\Gamma(j-1/2)}{\pi^2\Gamma(j+1)\Gamma(k+1)}\frac{1}{(\beta+1)^{\frac{k}{2}-\frac{1}{4}}}$$
$$\left[e^{-n\sqrt{\lambda}(\sqrt{\beta+1}-1)}\left(-\frac{(4k^2+8k-1)(2j+1)}{n\sqrt{\lambda}}-8k(2j-1)\right)\right.$$
$$\left.\times\mathrm{i}(-1)^je^{-n\sqrt{\lambda}(\sqrt{\beta+1}+1)}\left(\frac{(4k^2+8k-1)(2j+1)}{n\sqrt{\lambda}}-8k(2j-1)\right)\right]$$

$$\mathcal{I}_{AdS;1}^{(\mathrm{np})}(\lambda;\beta)=-16\pi(1+\beta)^{\frac{1}{4}}\sum_{n=1}^{\infty}\left[e^{-n\sqrt{\lambda}(\sqrt{\beta+1}-1)}\sum_{m=0}^{\infty}\frac{1}{m!}c_m(\sqrt{1+\beta})(-2n\sqrt{\lambda(1+\beta)})^{-m-1}\right.$$
$$\left.\mp\mathrm{i}e^{-n\sqrt{\lambda}(\sqrt{\beta+1}+1)}\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}c_m(-\sqrt{1+\beta})(2n\sqrt{\lambda(1+\beta)})^{-m-1}\right]$$

$$\mathcal{I}_{S;1}(\lambda;1)=\int_{\mathrm{Res}=1/2}\frac{ds}{2\pi\mathrm{i}}\frac{4\pi\zeta(2s+1)}{\sin(\pi s)}\binom{2s+1}{s}\left(\frac{\lambda}{16\pi^2}\right)^s$$

$$\binom{2s+1}{s}=\frac{2^{2s+1}\Gamma\left(s+\frac{3}{2}\right)}{\sqrt{\pi}\Gamma(s+2)}$$

$$\tilde{\mathcal{J}}_{S;1}(\lambda;1;a,b) := \int_{\mathrm{Re}s=1/2} \frac{ds}{2\pi \mathrm{i}} \frac{4\pi\zeta(2s+1)}{\sin(\pi s)} \frac{2^{2s+1}\Gamma\left(s+a+\frac{1}{2}\right)}{\sqrt{\pi}\Gamma(s+b)} \left(\frac{\lambda}{16\pi^2}\right)^s \frac{ds}{2\pi \mathrm{i}}$$

$$\tilde{\mathcal{J}}_{S;1}(\lambda;1;a,b) = 16\sqrt{\pi} \int_{\mathrm{Re}s=1/2} \frac{ds}{2\pi \mathrm{i}} \zeta(2s-2) \frac{\Gamma(2s-2)\Gamma(s-b)}{\Gamma\left(s-a-\frac{1}{2}\right)} \frac{\cos(\pi s)\sin(\pi(s-b))}{\cos(\pi(s-a))\sin(\pi s)} \lambda^{1-s}$$

$$\tilde{\mathcal{J}}_{S;1}(\lambda;1;1,b) = -\int_{\mathrm{Re}s=1/2} \frac{ds}{2\pi \mathrm{i}} 4^s (2s-3)\zeta(2s-2)\Gamma(s-1)\Gamma(s-b) \frac{\sin(\pi(s-b))}{\sin(\pi s)} \lambda^{1-s}$$

$$\tilde{\mathcal{J}}_{S;1}(\lambda;1;1,b) \sim \tilde{\mathcal{J}}^{(\mathrm{p})}_{S;1}(\lambda;1;1,b) = A_1(\lambda;b) + \frac{\sin(\pi b)}{\pi} \sum_{n=1}^{\infty} 4^{n+1}(2n-1)\zeta(2n)\Gamma(n)\Gamma(n+1-b)\lambda^{-n}$$

$$A_1(\lambda;b) := -\frac{2\left[\log\left(\frac{\lambda}{16\pi^2}\right) + \gamma + 2 - \psi(b)\right]}{\Gamma(b)}$$

$$\lim_{b\to 2} A_1(\lambda;b) = -2\left[\log\left(\frac{\lambda}{16\pi^2}\right) + 2\gamma + 1\right],$$

$$\lim_{b\to 2}\left[\frac{\sin(\pi b)}{\pi} \sum_{n=1}^{\infty} 4^{n+1}(2n-1)\zeta(2n)\Gamma(n)\Gamma(n+1-b)\lambda^{-n}\right] = \begin{cases} -\frac{8\pi^2}{3\lambda}, & n=1 \\ 0, & n \geq 2 \end{cases}$$

$$\mathcal{B}: \sum_{n=1}^{\infty} c_n \lambda^{-n} \to \hat{\phi}(w) := \sum_{n=1}^{\infty} \frac{2c_n}{\zeta(2n)\Gamma(2n+1)} (2w)^{2n}$$

$$\hat{\phi}(w;b) = \frac{64w^2}{\Gamma(b-1)}\left[{}_3F_2\left(1,1,2-b;\frac{3}{2},2;4w^2\right) - 2\,{}_2F_1\left(1,2-b;\frac{3}{2};4w^2\right)\right]$$

$$\mathcal{S}_\theta\left[\tilde{\mathcal{J}}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b) = A_1(\lambda;b) + \sqrt{\lambda}\int_0^{e^{i\theta}\infty} \frac{dw}{4\sinh^2(\sqrt{\lambda}w)} \hat{\phi}(w)$$

$$\frac{2^{2s-1}}{\Gamma(2s+1)} \int_0^{\infty} dw \frac{w^{2s}}{\sinh^2(w)} = \zeta(2s)$$

$$(\mathcal{S}_+ - \mathcal{S}_-)\left[\tilde{\mathcal{J}}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b) = \Delta\left[\tilde{\mathcal{J}}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b) = \lambda \int_0^{\infty} dw \frac{1}{4\sinh^2(xw)} \mathrm{Disc}_0 \hat{\phi}(w;b)$$

$$\mathcal{S}_\pm\left[\tilde{\mathcal{J}}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b) = \lim_{\theta\to 0^\pm} \mathcal{S}_\theta\left[\tilde{\mathcal{J}}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b)$$

$$\begin{aligned}\mathrm{Disc}\,{}_2F_1(a,b;c;z) =& \frac{2\pi\mathrm{i}\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-a-b+1)} z^{1-c}(z-1)^{c-a-b} \\ & \times {}_2F_1(1-a,1-b;c-a-b+1;1-z),\end{aligned}$$

$$\mathrm{Disc}_3F_2\left(1,1,2-b;\frac{3}{2},2;z\right) = -\mathrm{i}\sin(\pi b)\sqrt{\pi} z^{-\frac{1}{2}}(z-1)^{b-\frac{1}{2}}\Gamma(b-1)\,{}_2\tilde{F}_1\left(1,b;b+\frac{1}{2};1-z\right)$$

$$\mathrm{Disc}_0\hat{\phi}(w;b) = -2\mathrm{i}\sin(\pi b)\mathrm{disc}(w;b)$$

$$\mathrm{disc}(w;b):=\frac{16\sqrt{\pi}w(4w^2-1)^{b-\frac{3}{2}}\left[(4w^2-1)\,{}_2F_1\left(1,b;b+\frac{1}{2};1-4w^2\right)+1-2b\right]}{\Gamma\left(b+\frac{1}{2}\right)}$$

$$\begin{aligned}\mathcal{S}_{\mathrm{med}}\left[\tilde{\jmath}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b)&=\mathcal{S}_{\pm}\left[\tilde{\jmath}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b)\mp\frac{1}{2}\Delta\left[\tilde{\jmath}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b)\\&=\mathcal{S}_{\pm}\left[\tilde{\jmath}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b)\pm\mathrm{isin}\,(\pi b)\tilde{\jmath}^{(\mathrm{np})}_{S;1}(\lambda;1;1,b)\\&=\mathcal{S}_{\pm}\left[\tilde{\jmath}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b)\pm\mathrm{isin}\,(\pi b)\lambda\int_{\frac{1}{2}}^{\infty}dw\frac{1}{4\sinh^2(\sqrt{\lambda}w)}\mathrm{disc}(w;b)\end{aligned}$$

$$\mathrm{Im}\sigma(b)=\pm\mathrm{isin}\,(\pi b)=\begin{cases}\arg(\lambda)>0\\\arg(\lambda)<0\end{cases}$$

$$\sigma(b)=\epsilon^{\pm\mathrm{i}\pi b}$$

$$\tilde{\jmath}_{S;1}(\lambda;1;1,b)=\mathcal{S}_{\pm}\left[\tilde{\jmath}^{(\mathrm{p})}_{S;1}\right](\lambda;1;1,b)+\sigma(b)\tilde{\jmath}^{(\mathrm{np})}_{S;1}(\lambda;1;1,b)$$

$$\tilde{\jmath}_{S;1}(\lambda;1)=\jmath^{(\mathrm{p})}_{S;1}(\lambda;1)+\tilde{\jmath}^{(\mathrm{np})}_{S;1}(\lambda;1;1,2),$$

$$\begin{aligned}\jmath^{(\mathrm{np})}_{S;1}(\lambda;1)=\tilde{\jmath}^{(\mathrm{np})}_{S;1}(\lambda;1;1,2)&=\lambda\int_{\frac{1}{2}}^{\infty}dw\frac{1}{4\sinh^2(\sqrt{\lambda}w)}\mathrm{disc}(w;2)\\&=-\lambda\int_{\frac{1}{2}}^{\infty}dw\frac{1}{4\sinh^2(\sqrt{\lambda}w)}16\left(2w\sqrt{4w^2-1}+\mathrm{arccosh}(2w)\right)\end{aligned}$$

$$\frac{1}{4\sinh^2(\sqrt{\lambda}w)}=\sum_{n=1}^{\infty}ne^{-2nw\sqrt{\lambda}}$$

$$\jmath^{(\mathrm{np})}_{S;1}(\lambda;1)=-\sum_{n=1}^{\infty}e^{-Y_n}\sqrt{\frac{8\pi}{Y_n}}\sum_{k=0}^{\infty}\frac{(4k^2+3)\Gamma\left(k-\frac{3}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\pi\Gamma(k+1)}(-2Y_n)^{-k}$$

En este punto, el principio de dualidad holográfica, en supersimetrías de gauge, cobra relevancia, calculándose así:

$$\mathcal{W}_{\mathcal{N}=4}=\mathrm{Tr}\Phi_1[\Phi_2,\Phi_3],$$

$$\mathcal{W}^{\mathrm{UV}}_{\mathcal{N}=1}=\mathrm{Tr}\Phi_1[\Phi_2,\Phi_3]+h_1\mathrm{Tr}\Phi_1\{\Phi_2,\Phi_3\}+h_2\mathrm{Tr}(\Phi_1^3+\Phi_2^3+\Phi_3^3)$$

$$\Delta\mathcal{W}=\frac{m}{2}\mathrm{Tr}\Phi_1^2$$

$$\mathrm{Tr}\Phi_2^4,\mathrm{Tr}\Phi_3^4,\mathrm{Tr}\Phi_2^2\Phi_3^2,\mathrm{Tr}\Phi_2\Phi_3\Phi_2\Phi_3,\mathrm{Tr}\Phi_2^3\Phi_3,\mathrm{Tr}\Phi_2\Phi_3^3$$

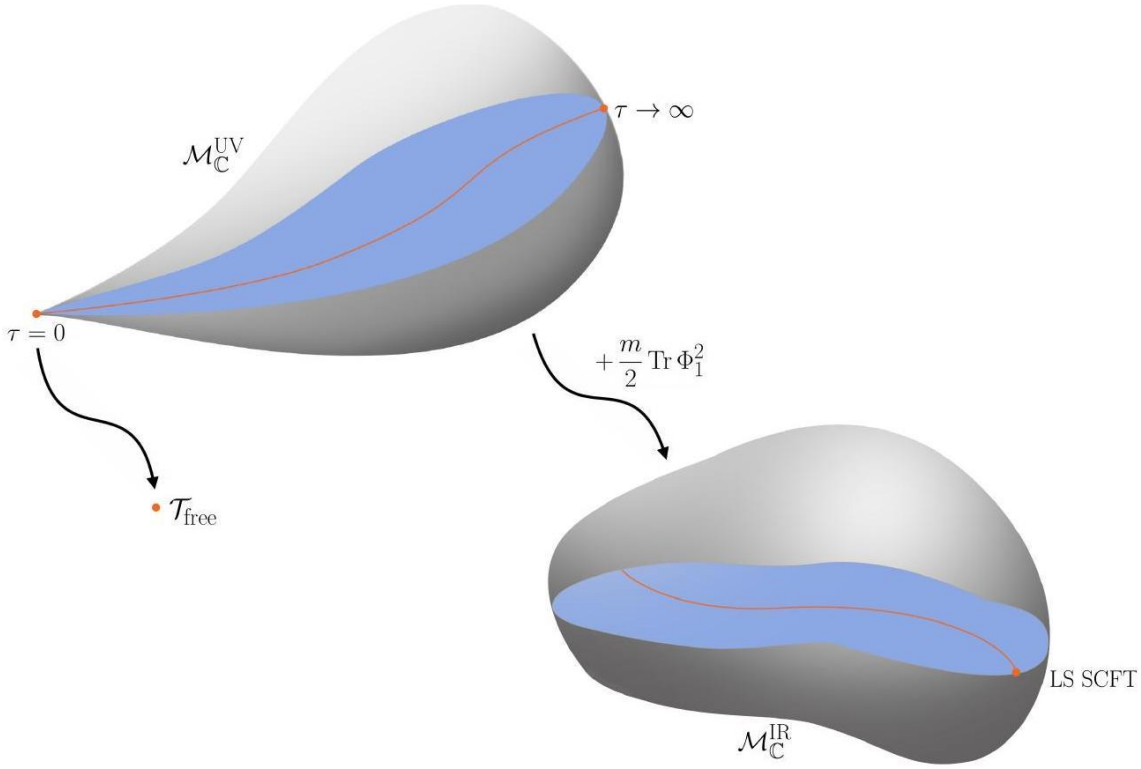


Figura 1. Representación de la dualidad holográfica a escala subatómica.

$$\mathcal{W}_{\mathcal{N}=1}^{\mathrm{IR}} = g_1 \mathrm{Tr}[\Phi_2, \Phi_3]^2 + g_2 \mathrm{Tr}\{\Phi_2, \Phi_3\}^2 + g_3 \mathrm{Tr}(\Phi_2^4 + \Phi_3^4),$$

$$a = \frac{9}{32}\mathrm{Tr}R^3 - \frac{3}{32}\mathrm{Tr}R, c = \frac{9}{32}\mathrm{Tr}R^3 - \frac{5}{32}\mathrm{Tr}R,$$

$$\mathrm{Tr}R_{\mathcal{N}=4}^3 = \frac{8}{9}d_{\mathrm{G}}, \mathrm{Tr}R_{\mathcal{N}=4} = 0$$

$$a_{\mathcal{N}=4} = c_{\mathcal{N}=4} = \frac{1}{4}d_{\mathrm{G}}$$

$$\mathrm{Tr}R_{\mathrm{LS}}^3 = \frac{3}{4}d_{\mathrm{G}}, \mathrm{Tr}R_{\mathrm{LS}} = 0$$

$$\frac{a_{\mathrm{LS}}}{a_{\mathcal{N}=4}} = \frac{c_{\mathrm{LS}}}{c_{\mathcal{N}=4}} = \frac{27}{32}$$

$$\mathcal{K} = -\sum_{a=1}^{4} \log\left(1 - z^a \bar{z}^a\right)$$

$$z^1 = \tanh\left[\frac{1}{2}(\alpha_1 + \alpha_2 + \alpha_3 + \varphi - \mathrm{i}\phi_1 - \mathrm{i}\phi_2 - \mathrm{i}\phi_3 + \mathrm{i}\phi_4)\right],$$
$$z^2 = \tanh\left[\frac{1}{2}(\alpha_1 - \alpha_2 + \alpha_3 - \varphi - \mathrm{i}\phi_1 + \mathrm{i}\phi_2 - \mathrm{i}\phi_3 - \mathrm{i}\phi_4)\right],$$
$$z^3 = \tanh\left[\frac{1}{2}(\alpha_1 + \alpha_2 - \alpha_3 - \varphi - \mathrm{i}\phi_1 - \mathrm{i}\phi_2 + \mathrm{i}\phi_3 - \mathrm{i}\phi_4)\right],$$
$$z^4 = \tanh\left[\frac{1}{2}(\alpha_1 - \alpha_2 - \alpha_3 + \varphi - \mathrm{i}\phi_1 + \mathrm{i}\phi_2 + \mathrm{i}\phi_3 + \mathrm{i}\phi_4)\right],$$
$$\bar{z}^1 = \tanh\left[\frac{1}{2}(\alpha_1 + \alpha_2 + \alpha_3 + \varphi + \mathrm{i}\phi_1 + \mathrm{i}\phi_2 + \mathrm{i}\phi_3 - \mathrm{i}\phi_4)\right],$$
$$\bar{z}^2 = \tanh\left[\frac{1}{2}(\alpha_1 - \alpha_2 + \alpha_3 - \varphi + \mathrm{i}\phi_1 - \mathrm{i}\phi_2 + \mathrm{i}\phi_3 + \mathrm{i}\phi_4)\right],$$
$$\bar{z}^3 = \tanh\left[\frac{1}{2}(\alpha_1 + \alpha_2 - \alpha_3 - \varphi + \mathrm{i}\phi_1 + \mathrm{i}\phi_2 - \mathrm{i}\phi_3 + \mathrm{i}\phi_4)\right],$$
$$\bar{z}^4 = \tanh\left[\frac{1}{2}(\alpha_1 - \alpha_2 - \alpha_3 + \varphi + \mathrm{i}\phi_1 - \mathrm{i}\phi_2 - \mathrm{i}\phi_3 - \mathrm{i}\phi_4)\right].$$

$$\mathcal{L} = -R + 12(\partial\beta_1)^2 + 4(\partial\beta_2)^2 + 2\mathcal{K}_{a\bar{b}}\partial_\mu z^a \partial^\mu \bar{z}^b + \mathcal{P}$$

$$\begin{aligned}\mathcal{W} \equiv & \frac{1}{L}e^{2\beta_1+2\beta_2}(1 + z_1z_2 + z_1z_3 + z_1z_4 + z_2z_3 + z_2z_4 + z_3z_4 + z_1z_2z_3z_4) \\ & + \frac{1}{L}e^{2\beta_1-2\beta_2}(1 - z_1z_2 + z_1z_3 - z_1z_4 - z_2z_3 + z_2z_4 - z_3z_4 + z_1z_2z_3z_4) \\ & + \frac{1}{L}e^{-4\beta_1}(1 + z_1z_2 - z_1z_3 - z_1z_4 - z_2z_3 - z_2z_4 + z_3z_4 + z_1z_2z_3z_4)\end{aligned}$$

$$\mathcal{P} = \frac{1}{2}e^{\mathcal{K}}\left[\frac{1}{6}\partial_{\beta_1}\mathcal{W}\partial_{\beta_1}\overline{\mathcal{W}} + \frac{1}{2}\partial_{\beta_2}\mathcal{W}\partial_{\beta_2}\overline{\mathcal{W}} + \mathcal{K}^{\bar{b}a}\nabla_a\mathcal{W}\nabla_{\bar{b}}\overline{\mathcal{W}} - \frac{8}{3}\mathcal{W}\overline{\mathcal{W}}\right]$$

$$\mathcal{S}_{\text{eff}} = \eta\int d^5x\sqrt{g}\mathcal{L}, \eta = \frac{N^2}{8\pi^2L^3}$$

$$\varphi = \beta_{1,2} = \alpha_{1,2,3} = \phi_{1,2,3,4} = 0, \text{ with } \mathcal{P} = -\frac{12}{L^2},$$

$$\varphi = \alpha_{1,2,3} = \phi_{2,3,4} = 0, \phi_1 = -\frac{\pi}{6}, \beta_1 = \frac{1}{3}\beta_2 = \frac{\log(2)}{12}, \text{ with } \mathcal{P} = -\frac{12}{L_{\text{LS}}^2},$$

$$\frac{a_{\text{LS}}}{a_{\mathcal{N}=4}} = \frac{c_{\text{LS}}}{c_{\mathcal{N}=4}} \rightarrow \frac{L_{\text{LS}}^3}{L^3} = \frac{27}{32}$$

$$\beta_1 \mapsto \frac{1}{2\sqrt{6}}\beta_1, \beta_2 \mapsto \frac{1}{2\sqrt{2}}\beta_2, \Phi_i \mapsto \frac{1}{2}\Phi_i,$$

$$\mathcal{L}_{\text{kin,UV}}=\frac{1}{2}\left[\left(\partial_\mu\beta_1\right)^2+\left(\partial_\mu\beta_2\right)^2+\left(\partial_\mu\varphi\right)^2+\sum_{i=1}^{3}\left(\partial_\mu\alpha_i\right)^2+\sum_{i=1}^{4}\left(\partial_\mu\phi_i\right)^2\right]+\mathcal{L}^{(4)}_{\text{kin,UV}}+\cdots$$

$$\mathcal{P}_{\text{UV}}=\frac{1}{L^2}\left[-12-\frac{1}{2}\left(4\beta_1^2+4\beta_2^2+4\sum_{i=1}^{3}\alpha_i^2+3\sum_{i=1}^{4}\phi_i^2\right)+\mathcal{P}^{(3)}_{\text{UV}}+\mathcal{P}^{(4)}_{\text{UV}}+\cdots\right]$$

$$\begin{aligned}
\alpha_i,\beta_i&:\quad m^2L^2=-4\quad\leftrightarrow\quad\Delta=2,\quad\text{(boson bilinears, part of }\mathbf{20'}\text{)}\\
\phi_i&:\quad m^2L^2=-3\quad\leftrightarrow\quad\Delta=3,\quad\text{(fermion bilinears, part of }\mathbf{10}\oplus\overline{\mathbf{10}}\text{)}\\
\varphi&:\quad m^2L^2=0\quad\leftrightarrow\quad\Delta=4.\quad\left(F_{\mu\nu}F^{\mu\nu},\mathbf{1}\right)
\end{aligned}$$

$$\phi_1\mapsto-\frac{\pi}{6}-\phi_1,\beta_1\mapsto\frac{\log(2)}{12}-\beta_1,\beta_2\mapsto\frac{\log(2)}{4}-\beta_2,$$

$$\beta_1\mapsto\frac{1}{2\sqrt{6}}\beta_1,\beta_2\mapsto\frac{1}{2\sqrt{2}}\beta_2,\Phi_i\mapsto\frac{\sqrt{3}}{4}\Phi_i$$

$$\begin{pmatrix}\hat{\beta}\\ \rho_1\\ \rho_2\end{pmatrix}\equiv M\cdot\begin{pmatrix}\beta_1\\ \beta_2\\ \phi_1\end{pmatrix},M=\frac{1}{2\sqrt{14}}\begin{pmatrix}-\sqrt{42}&\sqrt{14}&0\\ \sqrt{7+\sqrt{7}}&\sqrt{3(7+\sqrt{7})}&2\sqrt{7-\sqrt{7}}\\ -\sqrt{7-\sqrt{7}}&-\sqrt{3(7-\sqrt{7})}&2\sqrt{7+\sqrt{7}}\end{pmatrix}$$

$$\mathcal{L}_{\text{kin,IR}}=\mathcal{L}^{(2)}_{\text{kin,IR}}+\mathcal{L}^{(3)}_{\text{kin,IR}}+\mathcal{L}^{(4)}_{\text{kin,IR}}+\cdots$$

$$\mathcal{P}_{\text{IR}}=\frac{1}{L^2_{\text{LS}}}\left[-12+\mathcal{P}^{(2)}_{\text{IR}}+\mathcal{P}^{(3)}_{\text{IR}}+\mathcal{P}^{(4)}_{\text{IR}}+\cdots\right]$$

$$\mathcal{L}^{(2)}_{\text{kin,IR}}=\frac{1}{2}\left[\left(\partial_\mu\hat{\beta}\right)^2+\left(\partial_\mu\rho_1\right)^2+\left(\partial_\mu\rho_2\right)^2+\left(\partial_\mu\varphi\right)^2+\sum_{i=1}^{3}\left(\partial_\mu\alpha_i\right)^2+\sum_{i=2}^{4}\left(\partial_\mu\phi_i\right)^2\right]$$

$$\mathcal{P}^{(2)}_{\text{IR}}=\frac{1}{2}\left[(4-2\sqrt{7})\rho_1^2+3\alpha_1^2+(4+2\sqrt{7})\rho_2^2-4\hat{\beta}^2-\frac{15}{4}(\alpha_2^2+\alpha_3^2+\phi_2^2+\phi_3^2)-3\phi_4^2\right]$$

	$m^2L^2_{\text{LS}}$	Δ	$\mathcal{N}=1$ multiplet	$SU(2)_{\text{F}}\ U(1)_{\text{R}}$
α_2,α_3	$-\frac{15}{4}$	$\frac{3}{2}$	$L\bar{B}_1[\frac{3}{2};0,0;1]\otimes[1]^{(1)}+\text{c.c.}$	$\mathbf{3}_1\oplus\mathbf{3}_{-1}$
ϕ_2,ϕ_3	$-\frac{15}{4}$	$\frac{5}{2}$	(chiral)	$\mathbf{3}_{-1}\oplus\mathbf{3}_1$
ϕ_4	-3	3	$L\bar{B}_1[3;0,0;2]\otimes[0]^{(2)}+\text{c.c.}$	$\mathbf{1}_2\oplus\mathbf{1}_{-2}$
φ	0	4	(chiral)	$\mathbf{1}_0\oplus\mathbf{1}_0$
$\widehat{\beta}$	-4	2	$A\bar{A}[2;0,0;0]\otimes[1]^{(0)}$ (flavour current)	$\mathbf{3}_0$
ρ_1	$4-2\sqrt{7}$	$1+\sqrt{7}$	$L\bar{L}[1+\sqrt{7};0,0;0]\otimes[0]^{(0)}$	$\mathbf{1}_0$
α_1	3	$2+\sqrt{7}$	(long)	$\mathbf{1}_2\oplus\mathbf{1}_{-2}$
ρ_2	$4+2\sqrt{7}$	$3+\sqrt{7}$		$\mathbf{1}_0$

$$X\bar{Y}[\Delta; j_1, j_2; r] \otimes [k]^{(\hat{y})},$$

bulk cubic couplings CFT 3pt-functions

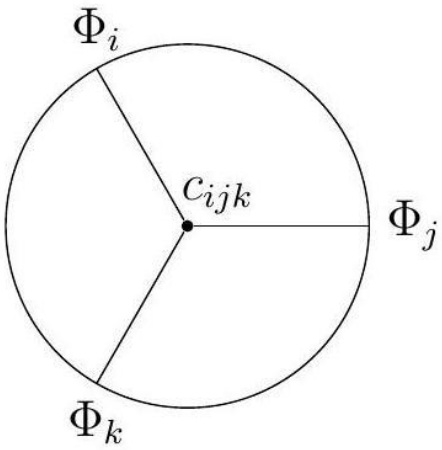
$$c_{ijk}\Phi_i\Phi_j\Phi_k + d_{ijk}\Phi_i\partial_\mu\Phi_j\partial^\mu\Phi_k \subset \mathcal{L}_{\text{eff}} \quad \mapsto \quad \left\langle \mathcal{O}_{\Phi_i}\mathcal{O}_{\Phi_j}\mathcal{O}_{\Phi_k} \right\rangle .$$

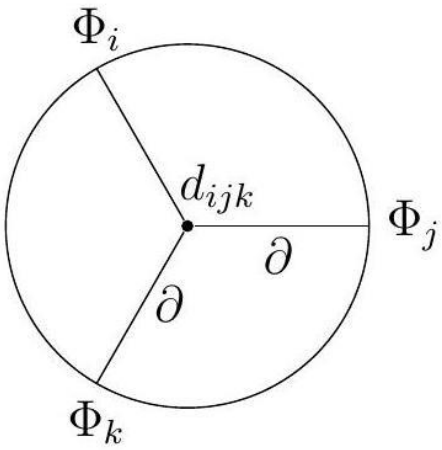
$$\mathcal{S}_{\text{eff}} = \eta \int_{\text{AdS}_{d+1}} d^{d+1}x\sqrt{g}\left(-R + \mathcal{L}_{\text{eff}}(\Phi_i)\right)$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\left(\partial_\mu\Phi_i\right)^2 + d_{ijk}\Phi_i\left(\partial_\mu\Phi_j\right)(\partial^\mu\Phi_k) + \mathcal{L}^{(4)}_{\text{kin}} + \cdots$$
$$+ \frac{1}{L^2}\left[-d(d-1) + m_i^2\Phi_i^2 + c_{ijk}\Phi_i\Phi_j\Phi_k + \mathcal{P}^{(4)} + \cdots\right]$$

$$\left\langle \hat{\mathcal{O}}_\Phi(x_1)\hat{\mathcal{O}}_\Phi(x_2)\right\rangle = \frac{n_\Delta}{|x_1 - x_2|^{2\Delta}}, \text{ with } n_\Delta = \eta\frac{(2\Delta - d)\Gamma(\Delta)}{\pi^{d/2}\Gamma\left(\Delta - \frac{d}{2}\right)},$$

$$m^2L^2 = \Delta(\Delta - d)$$





$$\mathcal{O}_\Phi := \frac{\hat{\mathcal{O}}_\Phi}{\sqrt{n_\Delta}}.$$

$$\left\langle \mathcal{O}_{\Phi_i}(x_1)\mathcal{O}_{\Phi_j}(x_2)\mathcal{O}_{\Phi_k}(x_3)\right\rangle = \frac{C_{\Phi_i\Phi_j\Phi_k}}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1 - x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1}}.$$

$$C_{\Phi_i\Phi_j\Phi_k} = c_{ijk}A_1 + d_{ijk}A_2,$$

$$A_1 = \frac{1}{\sqrt{\eta}}\frac{\Gamma\left(\frac{\Delta_i + \Delta_j - \Delta_k}{2}\right)\Gamma\left(\frac{\Delta_i + \Delta_k - \Delta_j}{2}\right)\Gamma\left(\frac{\Delta_j + \Delta_k - \Delta_i}{2}\right)}{4\pi^{d/4}\sqrt{2\prod_{n\in\{i,j,k\}}^3 \Gamma(\Delta_n)\Gamma\left(\Delta_n + 1 - \frac{d}{2}\right)}}\Gamma\left(\frac{\Delta_i + \Delta_j + \Delta_k - d}{2}\right)$$

$$A_2 = \left[\Delta_j\Delta_k + \frac{1}{2}\left(d - \Delta_i - \Delta_j - \Delta_k\right)\left(\Delta_j + \Delta_k - \Delta_i\right)\right]A_1$$

$$\partial_\mu\left(\Phi_i\Phi_j\partial^\mu\Phi_k + \Phi_i\Phi_k\partial^\mu\Phi_j - \Phi_j\Phi_k\partial^\mu\Phi_i\right) = 2\Phi_i\partial_\mu\Phi_j\partial^\mu\Phi_k - \left(m_i^2 - m_j^2 - m_k^2\right)\Phi_i\Phi_j\Phi_k$$

$$\mathcal{L}'_{\text{eff}} \widehat{=} \mathcal{L}_{\text{eff}} + \gamma_{ijk}\left(-\Phi_i \partial_\mu \Phi_j \partial^\mu \Phi_k + \frac{m_i^2 - m_j^2 - m_k^2}{2}\Phi_i \Phi_j \Phi_k\right)$$

$$\Phi_i \partial_\mu \Phi_j \partial^\mu \Phi_k \mapsto \frac{m_i^2 - m_j^2 - m_k^2}{2}\Phi_i \Phi_j \Phi_k$$

$$\Delta_j \Delta_k + \frac{1}{2}(d - \Delta_i - \Delta_j - \Delta_k)(\Delta_j + \Delta_k - \Delta_i) = 0.$$

$$\begin{aligned} c_{ijk} &= -\Delta_j \Delta_k d_{ijk}, && \text{for } \Delta_i = \Delta_j + \Delta_k, \\ c_{ijk} &= -(d - \Delta_j)\Delta_k d_{ijk}, && \text{for } \Delta_j = \Delta_i + \Delta_k, \\ c_{ijk} &= -(d - \Delta_k)\Delta_j d_{ijk}, && \text{for } \Delta_k = \Delta_i + \Delta_j. \end{aligned}$$

$$\langle \mathcal{O}_i \mathcal{O}_i \rangle \sim \eta^0, \langle \mathcal{O}_i^2 \mathcal{O}_i^2 \rangle \sim \eta^0, \langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle \sim \eta^{-1/2}, \langle \mathcal{O}_i^2 \mathcal{O}_j \mathcal{O}_k \rangle \sim \eta^0.$$

$$\Phi \leftrightarrow \mathcal{O}'_\Phi = \mathcal{O}_\Phi + \mathfrak{c}\mathcal{O}_{\Phi_1,\Phi_2} + \cdots$$

$$\left\langle \mathcal{O}_\Phi^{(s)}(x)\left[\mathcal{O}_{\Phi_1} \cdots \mathcal{O}_{\Phi_k}\right](y)\right\rangle = 0, k \geq 2$$

$$\begin{aligned} \mathcal{P}^{(3)} =& \frac{9}{8}(2\alpha_1\alpha_2\phi_2 + 2\alpha_1\alpha_3\phi_3 - \alpha_2\alpha_3\phi_4) - \frac{21}{8}\phi_2\phi_3\phi_4 \\ &+ \frac{3}{4\sqrt{2}}\hat{\beta}(\alpha_2^2 - \alpha_3^2 - \phi_2^2 + \phi_3^2) \\ &+ \frac{\sqrt{3}}{16\sqrt{14}}(\alpha_2^2 + \alpha_3^2)\left(\sqrt{917 + 29\sqrt{7}}\rho_1 - \sqrt{917 - 29\sqrt{7}}\rho_2\right) \\ &- \frac{\sqrt{3}}{16\sqrt{2}}(\phi_2^2 + \phi_3^2)\left(\sqrt{371 + 107\sqrt{7}}\rho_1 + \sqrt{371 - 107\sqrt{7}}\rho_2\right) \\ &+ \frac{1}{2\sqrt{21}}\hat{\beta}^2\left(\sqrt{217 + 79\sqrt{7}}\rho_1 - \sqrt{217 - 79\sqrt{7}}\rho_2\right) \\ &- \frac{3}{56}\phi_4^2\left(\sqrt{686 + 238\sqrt{7}}\rho_1 - \sqrt{686 - 238\sqrt{7}}\rho_2\right) \\ &+ \frac{\sqrt{3}}{2\sqrt{14}}\rho_1\rho_2\left(\sqrt{35 + 13\sqrt{7}}\rho_1 - \sqrt{35 - 13\sqrt{7}}\rho_2\right) \\ &- \frac{\sqrt{3}}{4\sqrt{14}}\alpha_1^2\left(\sqrt{2891 - 517\sqrt{7}}\rho_1 - \sqrt{2891 + 517\sqrt{7}}\rho_2\right) \\ &+ \frac{1}{6\sqrt{42}}\left(\sqrt{36155 - 13261\sqrt{7}}\rho_1^3 + \sqrt{36155 + 13261\sqrt{7}}\rho_2^3\right) \end{aligned}$$

$$
\begin{aligned}
\mathcal{L}^{(3)}_{\text{kin}} =& -\frac{1}{2}\left(\phi_2\partial_\mu\alpha_1\partial^\mu\alpha_2+\phi_3\partial_\mu\alpha_1\partial^\mu\alpha_3-\phi_4\partial_\mu\alpha_2\partial^\mu\alpha_3\right)\\
&-\frac{1}{2}\left(\phi_2\partial_\mu\alpha_3+\phi_3\partial_\mu\alpha_2-\phi_4\partial_\mu\alpha_1\right)\partial^\mu\varphi\\
&+\frac{1}{2}\left(\phi_2\partial_\mu\phi_3\partial^\mu\phi_4+\phi_3\partial_\mu\phi_2\partial^\mu\phi_4+\phi_4\partial_\mu\phi_2\partial^\mu\phi_3\right)\\
&+\frac{1}{4\sqrt{14}}\left(\sqrt{7-\sqrt{7}}\rho_1+\sqrt{7+\sqrt{7}}\rho_2\right)\\
&\times\left[\left(\partial_\mu\alpha_1\right)^2+\left(\partial_\mu\alpha_2\right)^2+\left(\partial_\mu\alpha_3\right)^2+\left(\partial_\mu\phi_2\right)^2+\left(\partial_\mu\phi_3\right)^2+\left(\partial_\mu\varphi\right)^2+\left(\partial_\mu\phi_4\right)^2\right]\\
&+\frac{1}{2\sqrt{14}}\left(\phi_2\partial_\mu\phi_2+\phi_3\partial_\mu\phi_3+\phi_4\partial_\mu\phi_4\right)\left(\sqrt{7-\sqrt{7}}\partial^\mu\rho_1+\sqrt{7+\sqrt{7}}\partial^\mu\rho_2\right)\\
&+\frac{\sqrt{3}}{28}\left(\sqrt{7-\sqrt{7}}\rho_1+\sqrt{7+\sqrt{7}}\rho_2\right)\partial_\mu\rho_1\partial^\mu\rho_2\\
&+\frac{1}{56}\rho_1\left(\sqrt{35-11\sqrt{7}}\left(\partial_\mu\rho_1\right)^2+\sqrt{21+3\sqrt{7}}\left(\partial_\mu\rho_2\right)^2\right)\\
&+\frac{1}{56}\rho_2\left(\sqrt{21-3\sqrt{7}}\left(\partial_\mu\rho_1\right)^2+\sqrt{35+11\sqrt{7}}\left(\partial_\mu\rho_2\right)^2\right)
\end{aligned}
$$

	frame 1	frame 2
$\alpha_2\alpha_3\phi_4,\ \phi_2\phi_3\phi_4$	$c_{ijk}\neq 0,\ d_{ijk}\neq 0$	$c_{ijk}=d_{ijk}=0$
$\alpha_2\phi_3\varphi,\ \alpha_3\phi_2\varphi$	$c_{ijk}=0,\ d_{ijk}\neq 0$	$c_{ijk}=d_{ijk}=0$

$$
\begin{aligned}
\mathcal{P}^{(3)}\big|_{\text{frame 2}} =& \frac{3}{2}\alpha_1(2\alpha_2\phi_2+2\alpha_3\phi_3-\varphi\phi_4)+\frac{3}{4\sqrt{2}}\hat{\beta}(\alpha_2^2-\alpha_3^2-\phi_2^2+\phi_3^2)\\
&+\frac{\sqrt{3}}{4}(\alpha_2^2+\alpha_3^2-\phi_2^2-\phi_3^2)\left(\sqrt{7+\sqrt{7}}\rho_1+\sqrt{7-\sqrt{7}}\rho_2\right)\\
&+\frac{1}{2\sqrt{21}}\hat{\beta}^2\left(\sqrt{217+79\sqrt{7}}\rho_1-\sqrt{217-79\sqrt{7}}\rho_2\right)\\
&-\frac{\sqrt{3}}{4\sqrt{14}}\phi_4^2\left(\sqrt{77+29\sqrt{7}}\rho_1-\sqrt{77-29\sqrt{7}}\rho_2\right)\\
&-\frac{\sqrt{3}}{4\sqrt{14}}\varphi^2\left(\sqrt{35-13\sqrt{7}}\rho_1-\sqrt{35+13\sqrt{7}}\rho_2\right)\\
&+\frac{\sqrt{3}}{56}\rho_1\rho_2\left(\sqrt{2065+377\sqrt{7}}\rho_1-\sqrt{2065-377\sqrt{7}}\rho_2\right)\\
&-\frac{\sqrt{3}}{4\sqrt{14}}\alpha_1^2\left(\sqrt{3296-523\sqrt{7}}\rho_1-\sqrt{3296+523\sqrt{7}}\rho_2\right)\\
&+\frac{1}{168\sqrt{3}}\left(\sqrt{1659343-602999\sqrt{7}}\rho_1^3+\sqrt{1659343+602999\sqrt{7}}\rho_2^3\right)
\end{aligned}
$$

$$
C_{\phi_2\phi_3\phi_4}=c_{\phi_2\phi_3\phi_4}A_1+\left(d_{\phi_2\phi_3\phi_4}+d_{\phi_3\phi_2\phi_4}+d_{\phi_4\phi_2\phi_3}\right)A_2=0.
$$

$$\frac{1}{L}\left[3+\frac{1}{2}\left(\alpha_{1,2,3}^2+\beta_{1,2}^2\right)-\frac{1}{2\sqrt{6}}\left(\beta_1(\phi_1^2+\phi_2^2-2\phi_3^2)+\sqrt{3}\beta_2(\phi_1^2-\phi_2^2)\right)+(\mathrm{q})\right]$$

$$\frac{1}{L_{\mathrm{LS}}}\left[3+\frac{1}{2}\hat{\beta}^2+\frac{3}{4}\phi_2\phi_3\phi_4+\frac{9}{64}\phi_4^4\right]$$

$$\{\alpha_2,\alpha_3,\phi_2,\phi_3\},\{\phi_4,\varphi\}\in\mathcal{C},\{\hat{\beta}\}\in\mathcal{F},\{\rho_1,\alpha_1,\rho_2\}\in\mathcal{L}.$$

$$C_{\alpha_2\alpha_3\phi_4}=-\frac{2^{5/6}}{9}\frac{\gamma}{N},C_{\alpha_2\phi_3\varphi}=C_{\alpha_3\phi_2\varphi}=-\frac{1}{9\cdot 2^{2/3}}\frac{\gamma}{N},C_{\phi_2\phi_3\phi_4}=0.$$

$$C_{\alpha_2\alpha_2\hat{\beta}}=-C_{\alpha_3\alpha_3\hat{\beta}}=\frac{2\cdot 2^{5/6}}{3}\frac{1}{N},C_{\phi_2\phi_2\hat{\beta}}=-C_{\phi_3\phi_3\hat{\beta}}=-\frac{2\cdot 2^{5/6}}{9}\frac{1}{N}.$$

$$C_{\alpha_2\alpha_2\rho_1}=C_{\alpha_3\alpha_3\rho_1}=C_{\alpha_2\phi_2\alpha_1}=C_{\alpha_3\phi_3\alpha_1}=\frac{\sqrt{1+\sqrt{7}}\Gamma\left(\frac{2-\sqrt{7}}{2}\right)\Gamma\left(\frac{1+\sqrt{7}}{2}\right)}{2^{-\frac{10}{3}+\sqrt{7}}3^{\frac{3}{2}}\pi^{\frac{1}{2}}}\frac{1}{N},$$

$$C_{\phi_2\phi_2\rho_1}=C_{\phi_3\phi_3\rho_1}=\frac{7-2\sqrt{7}}{3}\times C_{\alpha_2\alpha_2\rho_1},$$

$$C_{\alpha_2\alpha_2\rho_2}=C_{\alpha_3\alpha_3\rho_2}=-\frac{\sqrt{7-2\sqrt{7}}}{2\sqrt{14}}\times C_{\alpha_2\alpha_2\rho_1},$$

$$C_{\phi_2\phi_2\rho_2}=C_{\phi_3\phi_3\rho_2}=-\frac{\sqrt{7+2\sqrt{7}}}{2\sqrt{6}}\times C_{\alpha_2\alpha_2\rho_1}.$$

$$C_{\phi_4\phi_4\rho_1}=-\frac{\sqrt{29+11\sqrt{7}}\Gamma\left(\frac{5-\sqrt{7}}{2}\right)\Gamma\left(\frac{1+\sqrt{7}}{2}\right)^2\Gamma\left(\frac{3+\sqrt{7}}{2}\right)}{3\cdot 2^{\frac{1}{6}}\sqrt{21}\Gamma(\sqrt{7})}\frac{1}{N}$$

$$C_{\phi_4\phi_4\rho_2}=-\frac{\sqrt{7-2\sqrt{7}}}{2\sqrt{14}}\times C_{\phi_4\phi_4\rho_1}$$

$$C_{\varphi\varphi\rho_1}=\frac{11-4\sqrt{7}}{12}\times C_{\phi_4\phi_4\rho_1}$$

$$C_{\varphi\varphi\rho_2}=-\frac{\sqrt{133+50\sqrt{7}}}{8\sqrt{42}}\times C_{\phi_4\phi_4\rho_1}$$

$$C_{\phi_4\varphi\alpha_1}=\frac{1}{2\sqrt{2}}\times C_{\phi_4\phi_4\rho_1}.$$

$$C_{\hat{\beta}\hat{\beta}\rho_1}=-\frac{2^{\frac{7}{3}}\sqrt{217+79\sqrt{7}}\Gamma\left(\frac{3-\sqrt{7}}{2}\right)\Gamma\left(\frac{1+\sqrt{7}}{2}\right)^3}{9\sqrt{21\Gamma(\sqrt{7})\Gamma(1+\sqrt{7})}}\frac{1}{N}$$

$$C_{\hat{\beta}\hat{\beta}\rho_2}=\frac{\sqrt{91-34\sqrt{7}}}{2\sqrt{42}}\times C_{\hat{\beta}\hat{\beta}\rho_1}$$

$$C_{\rho_1\rho_1\rho_1}=\frac{2^{\frac{1}{3}}\sqrt{1659343-602999\sqrt{7}}\Gamma\left(\frac{1+\sqrt{7}}{2}\right)^3\Gamma\left(\frac{-1+3\sqrt{7}}{2}\right)}{63\sqrt{3}(\Gamma(\sqrt{7})\Gamma(1+\sqrt{7}))^{3/2}}\frac{1}{N}$$

$$C_{\rho_1\rho_1\rho_2}=\frac{\sqrt{3(2777257+1042598\sqrt{7})}}{1514\sqrt{14}}\times C_{\rho_1\rho_1\rho_1}$$

$$C_{\rho_1\alpha_1\alpha_1}=\frac{\sqrt{1976297+635680\sqrt{7}}}{1514}\times C_{\rho_1\rho_1\rho_1}$$

$$C_{\rho_1\rho_2\rho_2}=-\frac{4963+1156\sqrt{7}}{42392}\times C_{\rho_1\rho_1\rho_1}$$

$$C_{\rho_2\alpha_1\alpha_1}=\frac{\sqrt{325648561+109746134\sqrt{7}}}{3028\sqrt{42}}\times C_{\rho_1\rho_1\rho_1}$$

$$C_{\rho_2\rho_2\rho_2}=\frac{\sqrt{3(7587462463+2704517498\sqrt{7})}}{84784\sqrt{2}}\times C_{\rho_1\rho_1\rho_1}$$

$$\frac{\langle\mathcal{O}_1(y)|\mathcal{C}(x)|\mathcal{O}_2(0)\rangle}{\langle Q^-Q^-\mathcal{O}_1(y)|\mathcal{C}(x)|Q^+Q^+\mathcal{O}_2(0)\rangle},$$

$$\langle\mathcal{O}'_\pm\mid\mathcal{O}'_\pm\rangle=4\left(\Delta\mp\frac{3}{2}q\right)\left(\Delta\mp\frac{3}{2}q-2\right)$$

$$|\rho_1\rangle=|0,0\rangle,|\phi_4\rangle=\frac{1}{\sqrt{2}}(|0,2\rangle+|0,-2\rangle)$$

$$|\alpha_1\rangle=\xi_1\left(\epsilon^{\dot\alpha\dot\beta}Q^-_{\dot\alpha}Q^-_{\dot\beta}|0,0\rangle+\epsilon^{\alpha\beta}Q^+_\alpha Q^+_\beta|0,0\rangle\right)$$
$$|\varphi\rangle=\xi_2\left(\epsilon^{\dot\alpha\dot\beta}Q^-_{\dot\alpha}Q^-_{\dot\beta}|0,2\rangle+\epsilon^{\alpha\beta}Q^+_\alpha Q^+_\beta|0,-2\rangle\right)$$

$$\langle\alpha_1\mid\alpha_1\rangle=8\xi_1^2\Delta_{\rho_1}\left(\Delta_{\rho_1}-2\right)=48\xi_1^2,\langle\varphi\mid\varphi\rangle=64\xi_2^2\Delta_{\phi_4}\left(\Delta_{\phi_4}-2\right)=192\xi_2^2$$

$$\xi_1=\frac{1}{\sqrt{48}},\xi_2=\frac{1}{\sqrt{192}}$$

$$\frac{\langle\rho_1|\phi_4|\phi_4\rangle}{\langle\alpha_1|\phi_4|\varphi\rangle}=\frac{1}{24\sqrt{2}\xi_1\xi_2}=2\sqrt{2}.$$

$$\frac{C_{\rho_1\phi_4\phi_4}}{C_{\alpha_1\phi_4\varphi}}=2\sqrt{2}$$

$$\frac{\langle\rho_1|\alpha_2|\alpha_2\rangle}{\langle\alpha_1|\alpha_2|\phi_2\rangle}=\frac{\langle\rho_1|\alpha_3|\alpha_3\rangle}{\langle\alpha_1|\alpha_3|\phi_3\rangle}=1$$

$$\frac{\langle\alpha_3|\alpha_2|\phi_4\rangle}{\langle\phi_3|\alpha_2|\varphi\rangle}=\frac{\langle\alpha_2|\alpha_3|\phi_4\rangle}{\langle\phi_2|\alpha_3|\varphi\rangle}=2\sqrt{2}.$$

$$\langle\phi_2|\phi_4|\phi_3\rangle=0$$

$$\mathcal{G}_{\Delta,\ell}=g_{\Delta,\ell}+c_1g_{\Delta+1,\ell+1}+c_2g_{\Delta+1,\ell-1}+c_3g_{\Delta+2,\ell}$$

$$c_3 \equiv c_{\Delta,\ell}^{(\mathcal{M}_1\mathcal{M}_2\mathcal{M}_3\mathcal{M}_4)} = \frac{C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}''}C_{\mathcal{O}''\mathcal{O}_3\mathcal{O}_4}}{C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}}C_{\mathcal{O}\mathcal{O}_3\mathcal{O}_4}},$$

$$c_3 = \frac{1}{16}\frac{(\Delta+\ell)(\Delta-\ell-2)}{(\Delta^2-(\ell+1)^2)}.$$

$$c_{\Delta=1+\sqrt{7},\ell=0}^{(\bar{\mathcal{C}}\mathcal{C}\mathcal{C}\mathcal{C})} = \frac{7-2\sqrt{7}}{56} \overset{!}{=} \left(\frac{C_{\alpha_2\alpha_2\rho_2}}{C_{\alpha_2\alpha_2\rho_1}}\right)^2$$

$$\frac{C_{\alpha_2\alpha_2\rho_2}}{C_{\alpha_2\alpha_2\rho_1}} = \frac{C_{\alpha_3\alpha_3\rho_2}}{C_{\alpha_3\alpha_3\rho_1}} = \frac{C_{\phi_4\phi_4\rho_2}}{C_{\phi_4\phi_4\rho_1}}$$

$$c_3 = \frac{1}{16}\frac{(\Delta-2)^2(\Delta+\ell)(\Delta-\ell-2)}{\Delta^2(\Delta^2-(\ell+1)^2)}.$$

$$c_{\Delta=1+\sqrt{7},\ell=0}^{(\mathcal{F}\mathcal{F}\mathcal{F})} = \frac{91-34\sqrt{7}}{168} \overset{!}{=} \left(\frac{C_{\widehat{\beta}\widehat{\beta}\rho_2}}{C_{\widehat{\beta}\widehat{\beta}\rho_1}}\right)^2.$$

$$c_3^{(\mathcal{L}\mathcal{L}\mathcal{L}\mathcal{L})} = \frac{\left[(\Delta+\ell)^2 - 8(\Delta-1)\frac{c_{LLL}^{(2)}}{c_{LLL}}\right]^2}{16\Delta^2(\Delta-\ell-1)(\Delta-\ell-2)(\Delta+\ell)(\Delta+\ell+1)}.$$

$$c_3^{(\bar{\mathcal{C}}\mathcal{C}\mathcal{L}\mathcal{L})} = \frac{(\Delta+\ell)^2 - 8(\Delta-1)\frac{c_{LLL}^{(2)}}{c_{LLL}}}{16\Delta(\Delta^2-(\ell+1)^2)}.$$

$$c_{\Delta=1+\sqrt{7},\ell=0}^{(\bar{\mathcal{C}}\mathcal{C}\mathcal{L}\mathcal{L})} = \frac{C_{\alpha_2\alpha_2\rho_2}C_{\rho_2\rho_1\rho_1}}{C_{\alpha_2\alpha_2\rho_1}C_{\rho_1\rho_1\rho_1}}.$$

$$c_{\Delta=1+\sqrt{7},\ell=0}^{(\mathcal{L}\mathcal{L}\mathcal{L})} = \frac{3(2777257+1042598\sqrt{7})}{32090744} \overset{!}{=} \left(\frac{C_{\rho_1\rho_1\rho_2}}{C_{\rho_1\rho_1\rho_1}}\right)^2.$$

$$C_{\alpha_1\alpha_1\varphi}, C_{\alpha_1\phi_{2,3}\phi_{2,3}}, C_{\varphi\varphi\varphi}, C_{\varphi\phi_{2,3}\phi_{2,3}}$$

$$C_{\widehat{\beta}\widehat{\beta}\varphi}, C_{\widehat{\beta}\alpha_{2,3}\phi_{2,3}}, C_{\rho_{1,2}\alpha_{2,3}\phi_{2,3}}, C_{\rho_{1,2}\rho_{1,2}\varphi}, C_{\rho_{1,2}\alpha_1\phi_4}, C_{\alpha_1\alpha_{2,3}\alpha_{2,3}}, C_{\varphi\phi_4\phi_4}, C_{\varphi\alpha_{2,3}\alpha_{2,3}}, C_{\phi_4\alpha_{2,3}\phi_{2,3}}$$

$-\langle\mathcal{F}\mathcal{F}\mathcal{F}\rangle$: $C_{\widehat{\beta}\widehat{\beta}\widehat{\beta}}$,

$-\langle\mathcal{C}\mathcal{C}\mathcal{F}\rangle$: $C_{\alpha_2\alpha_3\widehat{\beta}}, C_{\phi_2\phi_3\widehat{\beta}}$,

$^{-}\langle\mathcal{C}\mathcal{C}\mathcal{L}\rangle$: $C_{\alpha_2\alpha_3\rho_{1,2}}$,

$-\langle\mathcal{C}\mathcal{C}\mathcal{C}\rangle$: $C_{\alpha_1\alpha_2\phi_3}, C_{\alpha_1\alpha_3\phi_2}, C_{\alpha_2\phi_2\varphi}, C_{\alpha_3\phi_3\varphi}, C_{\alpha_2\alpha_2\phi_4}, C_{\alpha_3\alpha_3\phi_4}, C_{\phi_2\phi_2\phi_4}, C_{\phi_3\phi_3\phi_4}$.

$$\begin{aligned}\tilde{\mathcal{L}} =& -R-\left[\rho^4\nu^{-4}F_{\mu\nu}^{(1)}F^{(1)\mu\nu}+\rho^4\nu^4F_{\mu\nu}^{(2)}F^{(2)\mu\nu}+\rho^{-8}F_{\mu\nu}^{(3)}F^{(3)\mu\nu}\right]\\&+12\left(\partial_\mu\beta_1\right)^2+4\left(\partial_\mu\beta_2\right)^2+2\sum_{j=1}^{4}\left(\partial_\mu\phi_j\right)^2\\&+\frac{1}{2}\sinh^2(2\phi_1)\left(\partial_\mu\theta_1+\left(A_\mu^{(1)}+A_\mu^{(2)}-A_\mu^{(3)}\right)\right)^2\\&+\frac{1}{2}\sinh^2(2\phi_2)\left(\partial_\mu\theta_1+\left(A_\mu^{(1)}-A_\mu^{(2)}+A_\mu^{(3)}\right)\right)^2\\&+\frac{1}{2}\sinh^2(2\phi_3)\left(\partial_\mu\theta_1+\left(-A_\mu^{(1)}+A_\mu^{(2)}+A_\mu^{(3)}\right)\right)^2\\&+\frac{1}{2}\sinh^2(2\phi_4)\left(\partial_\mu\theta_1-\left(A_\mu^{(1)}+A_\mu^{(2)}+A_\mu^{(3)}\right)\right)^2+\tilde{\mathcal{P}},\end{aligned}$$

$$\tilde{\mathcal{P}}=\frac{1}{2L^2}\left[\sum_{j=1}^{4}\left(\frac{\partial W}{\partial\phi_j}\right)^2+\frac{1}{6}\left(\frac{\partial W}{\partial\beta_1}\right)^2+\frac{1}{2}\left(\frac{\partial W}{\partial\beta_2}\right)^2\right]-\frac{4}{3L^2}W^2,$$

$$\begin{aligned}W=-\frac{1}{2\rho^2\nu^2}[&(1+\nu^4-\nu^2\rho^6)\cosh(2\phi_1)+(-1+\nu^4+\nu^2\rho^6)\cosh(2\phi_2)\\&+(1-\nu^4+\nu^2\rho^6)\cosh(2\phi_3)+(1+\nu^4+\nu^2\rho^6)\cosh(2\phi_4)].\end{aligned}$$

$$\beta_{1,2}=\phi_{1,2,3,4}=0,\ \text{with}\ \tilde{\mathcal{P}}=-\frac{12}{L^2}$$

$$\begin{aligned}\beta_{1,2}:&\quad m^2L^2=-4\quad\leftrightarrow\quad\Delta=2\\\phi_{1,2,3,4}:&\quad m^2L^2=-3\quad\leftrightarrow\quad\Delta=3\end{aligned}$$

$$A_\mu^{(1,2,3)}:m^2L^2=0\ \leftrightarrow\ \Delta=3$$

$$C_{\beta_1\beta_1\beta_1}=-C_{\beta_1\beta_2\beta_2}=-\sqrt{\frac{2}{3}}\frac{1}{N},$$

$$\beta_2=\phi_{2,3,4}=0,\beta_1=-\frac{\log 2}{6},\phi_1=\frac{\log 3}{2},\ \text{with}\ \tilde{\mathcal{P}}=-\frac{12}{L_{\mathrm{LS}}^2},$$

$$\begin{aligned}\hat{\beta}:&m^2L_{\mathrm{LS}}^2=-4&&\leftrightarrow\Delta=2,\\\phi_{2,3}:&m^2L_{\mathrm{LS}}^2=-\frac{15}{4}&&\leftrightarrow\Delta=\frac{5}{2},\\\phi_4:&m^2L_{\mathrm{LS}}^2=-3&&\leftrightarrow\Delta=3,\\\rho_1:&m^2L_{\mathrm{LS}}^2=4-2\sqrt{7}&&\leftrightarrow\Delta=1+\sqrt{7},\\\rho_2:&m^2L_{\mathrm{LS}}^2=4+2\sqrt{7}&&\leftrightarrow\Delta=3+\sqrt{7}.\end{aligned}$$

$$\begin{aligned}\tilde{A}_\mu^{(1,2)}:&\quad m^2L_{\mathrm{LS}}^2=0\quad\leftrightarrow\quad\Delta=3,\\\tilde{A}_\mu^{(3)}:&\quad m^2L_{\mathrm{LS}}^2=6\quad\leftrightarrow\quad\Delta=2+\sqrt{7},\end{aligned}$$

$$\tilde{\mathcal{P}}^{(3)} = \frac{1}{2!}\left(2\sqrt{\frac{2}{3}}\beta_1\beta_2^2\right) + \frac{1}{3!}\left(-2\sqrt{\frac{2}{3}}\beta_1^3\right)$$

$$\tilde{\mathcal{P}}^{(4)} \supset \frac{1}{2!\cdot 2!}\left(-\frac{2}{3}\beta_1^2\beta_2^2\right) + \frac{1}{4!}\left(-2(\beta_1^4+\beta_2^4)\right)$$

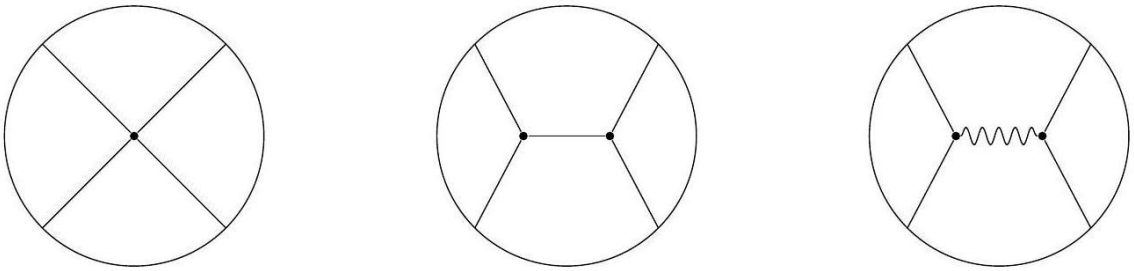


Figura 2. Computación holográfica.

$$\mathcal{P}_{\mathrm{UV}}^{(3)} = \frac{1}{2!}\left(2\sqrt{\frac{2}{3}}\beta_1(\alpha_1^2+\alpha_2^2-2\alpha_3^2+\beta_2^2) + 2\sqrt{2}\beta_2(\alpha_1^2-\alpha_2^2)\right) + \frac{1}{3!}\left(-2\sqrt{\frac{2}{3}}\beta_1^3\right)$$

$$C_{\alpha_1\alpha_1\beta_1} = C_{\alpha_2\alpha_2\beta_1} = -C_{\beta_1\beta_1\beta_1} = C_{\beta_1\beta_2\beta_2} = \sqrt{\frac{2}{3}}\frac{1}{N}$$

$$C_{\alpha_1\alpha_1\beta_2} = -C_{\alpha_2\alpha_2\beta_2} = \sqrt{2}\frac{1}{N}, C_{\alpha_3\alpha_3\beta_1} = -2\sqrt{\frac{2}{3}}\frac{1}{N}$$

$$\mathcal{O}_{\alpha_i} \sim \mathrm{Tr}\left(Z_iZ_i + \tilde{Z}_i\tilde{Z}_i\right), i = 1,2,3$$
$$\mathcal{O}_{\beta_1} \sim \mathrm{Tr}\left(Z_1\tilde{Z}_1 + Z_2\tilde{Z}_2 - 2Z_3\tilde{Z}_3\right)$$
$$\mathcal{O}_{\beta_2} \sim \mathrm{Tr}\left(Z_1\tilde{Z}_1 - Z_2\tilde{Z}_2\right)$$

$$Z_1 = \frac{X_1 + \mathrm{i}X_2}{\sqrt{2}}, Z_2 = \frac{X_3 + \mathrm{i}X_4}{\sqrt{2}}, Z_3 = \frac{X_5 + \mathrm{i}X_6}{\sqrt{2}}$$

$$\left\langle (X_I)_a^b(x_1)\left(X_J\right)_c^d(x_2)\right\rangle = \delta_{IJ}\left(\delta_a^d\delta_c^b - \frac{1}{N}\delta_a^b\delta_c^d\right)g_{12}, g_{12} \equiv \frac{1}{(x_1-x_2)^2}$$

$$\left\langle (Z_i)_a^b(x_1)\left(\tilde{Z}_j\right)_c^d(x_2)\right\rangle = \delta_{ij}\left(\delta_a^d\delta_c^b - \frac{1}{N}\delta_a^b\delta_c^d\right)g_{12}$$

$$\left\langle \mathrm{Tr}\left(Z_i\tilde{Z}_i\right)\mathrm{Tr}\left(Z_j\tilde{Z}_j\right)\right\rangle = \delta_{ij}(N^2-1)g_{12}^2$$
$$\left\langle \mathrm{Tr}(Z_iZ_i)\mathrm{Tr}\left(\tilde{Z}_j\tilde{Z}_j\right)\right\rangle = 2\delta_{ij}(N^2-1)g_{12}^2$$

$$\left\langle \mathrm{Tr}\left(Z_i\tilde{Z}_i\right)\mathrm{Tr}\left(Z_j\tilde{Z}_j\right)\mathrm{Tr}\left(Z_k\tilde{Z}_k\right)\right\rangle = 2\delta_{ij}\delta_{ik}(N^2-1)g_{12}g_{13}g_{23}$$
$$\left\langle \mathrm{Tr}\left(Z_i\tilde{Z}_i\right)\mathrm{Tr}\left(Z_jZ_j\right)\mathrm{Tr}\left(\tilde{Z}_k\tilde{Z}_k\right)\right\rangle = 4\delta_{ij}\delta_{ik}(N^2-1)g_{12}g_{13}g_{23}$$

$$\mathcal{O}_{\alpha_i}=\frac{1}{2\sqrt{N^2-1}}\mathrm{Tr}\left(Z_iZ_i+\tilde{Z}_i\tilde{Z}_i\right),i=1,2,3$$
$$\mathcal{O}_{\beta_1}=\frac{1}{\sqrt{6(N^2-1)}}\mathrm{Tr}\left(Z_1\tilde{Z}_1+Z_2\tilde{Z}_2-2Z_3\tilde{Z}_3\right)$$
$$\mathcal{O}_{\beta_2}=\frac{1}{\sqrt{2(N^2-1)}}\mathrm{Tr}\left(Z_1\tilde{Z}_1-Z_2\tilde{Z}_2\right)$$

$$C^{(\mathcal{N}=4)}_{\alpha_1\alpha_1\beta_1}=C^{(\mathcal{N}=4)}_{\alpha_2\alpha_2\beta_1}=-C^{(\mathcal{N}=4)}_{\beta_1\beta_1\beta_1}=C^{(\mathcal{N}=4)}_{\beta_1\beta_2\beta_2}=\sqrt{\frac{2}{3}}\frac{1}{\sqrt{N^2-1}}$$
$$C^{(\mathcal{N}=4)}_{\alpha_1\alpha_1\beta_2}=-C^{(\mathcal{N}=4)}_{\alpha_2\alpha_2\beta_2}=\sqrt{2}\frac{1}{\sqrt{N^2-1}},C^{(\mathcal{N}=4)}_{\alpha_3\alpha_3\beta_1}=-2\sqrt{\frac{2}{3}}\frac{1}{\sqrt{N^2-1}}.$$

$$\tilde{\mathcal{P}}^{(3)}=\frac{1}{2!}\left(-2\sqrt{\frac{2}{3}}\beta_1\beta_2^2\right)+\frac{1}{3!}\left(2\sqrt{\frac{2}{3}}\beta_1^3\right),$$
$$\tilde{\mathcal{P}}^{(4)}\supset\frac{1}{2!\cdot 2!}\left(-\frac{2}{3}\beta_1^2\beta_2^2\right)+\frac{1}{4!}\left(-2(\beta_1^4+\beta_2^4)\right).$$

$$\mathcal{A}^{(c)}=g_{12}^2g_{34}^2\cdot\frac{\pi^2}{2}u^2\bar{D}_{2222}$$
$$\mathcal{A}_s^{(s)}=g_{12}^2g_{34}^2\cdot\frac{\pi^2}{8}u\bar{D}_{1122}$$
$$\mathcal{A}_s^{(g)}=g_{12}^2g_{34}^2\cdot\frac{2\pi^2}{3}u(-2\bar{D}_{1122}+3\bar{D}_{2123}+3\bar{D}_{2132}-3\bar{D}_{3133})$$

$$\langle\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\rangle=\frac{1}{\eta\left(\sqrt{2\pi^2}\right)^4}\left(\left[\mathcal{A}_s^{(g)}+\mathcal{A}_t^{(g)}+\mathcal{A}_u^{(g)}\right]+\frac{8}{3}\left[\mathcal{A}_s^{(s)}+\mathcal{A}_t^{(s)}+\mathcal{A}_u^{(s)}\right]+2\left[\mathcal{A}^{(c)}\right]\right),$$

$$\langle\mathcal{O}_{\beta_2}\mathcal{O}_{\beta_2}\mathcal{O}_{\beta_2}\mathcal{O}_{\beta_2}\rangle=\langle\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\rangle$$

$$\langle\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_2}\mathcal{O}_{\beta_2}\rangle=\frac{1}{\eta\left(\sqrt{2\pi^2}\right)^4}\left(\left[\mathcal{A}_s^{(g)}\right]-\frac{8}{3}\left[\mathcal{A}_s^{(s)}\right]+\frac{8}{3}\left[\mathcal{A}_t^{(s)}+\mathcal{A}_u^{(s)}\right]+\frac{2}{3}\left[\mathcal{A}^{(c)}\right]\right).$$

$$\mathcal{O}_2(x,y)=y^Iy^J\mathrm{Tr}\left(X_IX_J\right),y\cdot y\equiv y_Iy^I=0$$

$$\langle\mathcal{O}_2(x_1,y_1)\mathcal{O}_2(x_2,y_2)\mathcal{O}_2(x_3,y_3)\mathcal{O}_2(x_4,y_4)\rangle=\frac{y_{12}^4y_{34}^4}{x_{12}^4x_{34}^4}\times\mathcal{G}(u,v;\sigma,\tau),$$

$$u=\frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2},v=\frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2},\sigma=\frac{y_{13}^2y_{24}^2}{y_{12}^2y_{34}^2},\tau=\frac{y_{14}^2y_{23}^2}{y_{12}^2y_{34}^2}.$$

$$\mathcal{G}_{\text{sugra}}\left(u,v;\sigma,\tau\right)=16\left(\sigma u+\tau\frac{u}{v}+\sigma\tau\frac{u^2}{v}\right)+\mathcal{I}\times(-16u^2\bar{D}_{2422}),$$

$$\mathcal{I}=\tau+(1-\sigma-\tau)v+(\tau^2-\tau-\sigma\tau)u+(\sigma^2-\sigma-\sigma\tau)uv+\sigma v^2+\sigma\tau u^2.$$

$$\mathcal{O}_2\left(x,y_{\{k,l\}}\right)+\mathcal{O}_2\left(x,\bar{y}_{\{k,l\}}\right)=2\mathrm{Tr}\left(X_k^2-X_l^2\right).$$

$$\mathcal{O}_{\beta_1}=\frac{1}{4\sqrt{6(N^2-1)}}\sum_{y=y_{\{1,5\}},y_{\{2,5\}},y_{\{3,6\}},y_{\{4,6\}}}(\mathcal{O}_2(x,y)+\mathcal{O}_2(x,\bar{y})),$$

$$\mathcal{O}_{\beta_2}=\frac{1}{4\sqrt{2(N^2-1)}}\sum_{y=y_{\{1,3\}},y_{\{2,4\}}}(\mathcal{O}_2(x,y)+\mathcal{O}_2(x,\bar{y})).$$

$$\langle\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\rangle=\langle\mathcal{O}_{\beta_2}\mathcal{O}_{\beta_2}\mathcal{O}_{\beta_2}\mathcal{O}_{\beta_2}\rangle=\frac{1}{x_{12}^4x_{34}^4}\left[\left(u+\frac{u}{v}+\frac{u^2}{v}\right)-(u+v+1)^2u^2\bar{D}_{2422}\right],$$

$$\langle\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_1}\mathcal{O}_{\beta_2}\mathcal{O}_{\beta_2}\rangle=\frac{1}{x_{12}^4x_{34}^4}\left[\frac{1}{3}\left(u+\frac{u}{v}+\frac{u^2}{v}\right)-\frac{1}{3}(u^2-2uv-2u+v^2+10v+1)u^2\bar{D}_{2422}\right],$$

$$c_{\Delta,\ell}^{(\bar{\mathcal{C}}\mathcal{F}\mathcal{F})}=\sqrt{c_{\Delta,\ell}^{(\bar{\mathcal{C}}\mathcal{C}\mathcal{C})}c_{\Delta,\ell}^{(\mathcal{F}\mathcal{F}\mathcal{F}\mathcal{F})}}$$

$$\langle\mathcal{O}_2(x_1,y_1)\mathcal{O}_2(x_2,y_2)\rangle=2(N^2-1)\frac{(y_1\cdot y_2)^2}{x_{12}^4}$$

$$c_T=\frac{64}{3\pi}\sqrt{2k_{CS}}N^{\frac{3}{2}}+\mathcal{O}\left(N^{1/2}\right)$$

$$\frac{\ell^6}{\ell_{11}^6}=\left(\frac{3\pi c_Tk_{CS}}{2^{11}}\right)^{\frac{2}{3}}+\mathcal{O}(c_T^0)=\frac{Nk_{CS}}{8}+\mathcal{O}(N^0)$$

$$\mathcal{O}_k(x,t)\equiv\mathcal{O}_k^{I_1\dots I_k}t_{I_1}\dots t_{I_k}$$

$$p_j=\frac{1}{\sqrt{2}}\epsilon_j\omega_j\left(1+z_j\bar{z}_j,z_j+\bar{z}_j,-i\left(z_j-\bar{z}_j\right),1-z_j\bar{z}_j\right)$$

$$\mathcal{C}_n^{\Delta_1,\dots,\Delta_n}\left(\left\{u_j,z_j,\bar{z}_j\right\}^{\epsilon_j}\right)=\int_0^{+\infty}\prod_{j=1}^{n}\frac{d\omega_j}{2\pi}\left(-i\epsilon_j\omega_j\right)^{\Delta_j-1}e^{-i\epsilon_j\omega_ju_j}\mathcal{A}_n\left(\left\{\omega_j,z_j,\bar{z}_j\right\}^{\epsilon_j}\right)$$

$$\mathcal{C}_n^{\Delta_1,\dots,\Delta_n}\left(\left\{u_j,z_j,\bar{z}_j\right\}^{\epsilon_j}\right)\equiv\left\langle\prod_{j=1}^{n}\Phi_{\Delta_j}^{\epsilon_j}\left(u_j,z_j,\bar{z}_j\right)\right\rangle$$

$$\mathcal{C}_n^{1,\dots,1}\left(\left\{u_j,z_j,\bar{z}_j\right\}^{\epsilon_j}\right)=\int_0^{+\infty}\prod_{j=1}^{n}\frac{d\omega_j}{2\pi}e^{-i\epsilon_j\omega_ju_j}\mathcal{A}_n\left(\left\{\omega_j,z_j,\bar{z}_j\right\}^{\epsilon_j}\right)$$

$$ds_{AdS_4}^2=-\frac{r^2}{\ell^2}du^2-2dudr+2r^2dzd\bar{z}$$

$$ds_{\mathbb{R}^{3,1}}^2=-2dudr+2r^2dzd\bar{z}$$

$$ds_{\partial AdS_4}^2=-\frac{du^2}{\ell^2}+2dzd\bar{z}$$

$$ds_j^2 = 0du^2 + 2dzd\bar{z}$$

$$c_{\text{boundary}} \equiv \frac{1}{\ell_{\text{bulk}}}$$

$$\langle \mathcal{O}_\Delta(x_1)\mathcal{O}_\Delta(x_2)\rangle = \frac{\mathcal{N}_2}{(x_{12}^2 + i\epsilon)^\Delta},$$

$$\lim_{c\to 0} c^{2\Delta-2}\langle \mathcal{O}_\Delta(x_1)\mathcal{O}_\Delta(x_2)\rangle = \frac{\mathcal{N}_2\delta^2(z_{12})}{2(\Delta-1)(-u_{12}+i\varepsilon)^{2\Delta-2}} \propto \langle \Phi_\Delta^{\epsilon_1}\Phi_\Delta^{\epsilon_2}\rangle,$$

$$\left\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\mathcal{O}_{\Delta_3}(x_3)\right\rangle_K = \frac{\mathcal{N}_3}{c}\frac{1}{(x_{12}^2+i\varepsilon)^{\Delta_{12}}(x_{23}^2+i\varepsilon)^{\Delta_{23}}(x_{13}^2+i\varepsilon)^{\Delta_{13}}}.$$

$$\lim_{c\to 0} c^{3-\sum_{j=1}^3 \Delta_j}\langle \mathcal{O}_{\Delta_1}(u_1,z_1,\bar{z}_1)\mathcal{O}_{\Delta_2}(u_2,z_2,\bar{z}_2)\mathcal{O}_{\Delta_3}(u_3,z_3,\bar{z}_3)\rangle$$
$$= \frac{\tilde{\mathcal{N}}_3\delta(\bar{z}_{12})\delta(\bar{z}_{23})\Theta(z_{12}z_{31})\Theta(z_{13}z_{23})z_{12}^{\Delta_3-2}z_{23}^{\Delta_1}z_{13}^{\Delta_2-2}}{(u_1z_{23}+u_2z_{31}+u_3z_{12}+i\text{sign}z_{23}\varepsilon)^{\sum_{j=1}^3\Delta_j-4}} \propto \langle \Phi_{\Delta_1}\Phi_{\Delta_2}\Phi_{\Delta_3}\rangle$$

$$\left\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\mathcal{O}_{\Delta_3}(x_3)\mathcal{O}_{\Delta_4}(x_4)\right\rangle \propto \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}(U,V)$$

$$U = \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2} = Z\bar{Z}\,, V = \frac{x_{23}^2x_{14}^2}{x_{13}^2x_{24}^2} = (1-Z)(1-\bar{Z})$$

$$\bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}(U,V) \overset{Z\to\bar{Z}}{\to} \hat{\Phi}^{l\cdot s}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4} \equiv \mathcal{K}_\Delta \frac{Z^{\Delta_3+\Delta_4-2}(1-Z)^{\Delta_1+\Delta_4-2}}{(Z-\bar{Z})^{\sum_{i=1}^4\Delta_i-3}}.$$

$$\lim_{c\to 0}\hat{\Phi}^{l\cdot s}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4} = \mathcal{R}(u_i,z_i)\delta(z-\bar{z})$$

$$\lim_{c\to 0} c^{4-\sum_\Delta}\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\mathcal{O}_{\Delta_3}(x_3)\mathcal{O}_{\Delta_4}(x_4)\rangle$$
$$= \mathcal{N}\left(\frac{|z_{23}|^2}{|z_{34}|^2|z_{24}|^2}\right)^{\frac{4-\Sigma_\Delta}{2}}\frac{z^{2-\Delta_1-\Delta_2}(1-z)^{\Delta_1+\Delta_4-2}\delta(z-\bar{z})}{\mathcal{U}^{\sum_{i=1}^4\Delta_i-4}} \propto \left\langle \Phi_{\Delta_1}^{\epsilon_1}\Phi_{\Delta_2}^{\epsilon_2}\Phi_{\Delta_3}^{\epsilon_3}\Phi_{\Delta_4}^{\epsilon_4}\right\rangle,$$

$$\mathcal{U} = u_4 - u_1 z\left|\frac{z_{24}}{z_{12}}\right|^2 + u_2\frac{1-z}{z}\left|\frac{z_{34}}{z_{23}}\right|^2 - u_3\frac{1}{1-z}\left|\frac{z_{14}}{z_{13}}\right|^2$$

$$s = \sum_{k\geq 0} Y_k^{(7)}s_k = \sum_{k\geq 0}\frac{s_k}{\ell^k}\mathcal{C}^{(k)}_{I_1\ldots I_k}Z^{I_1}\ldots Z^{I_k}$$

$$S = \frac{243}{\kappa^2}\int_{\text{AdS}_4} d^4y\sqrt{-\bar{g}_4}\left\{\sum_{k\geq 2}\frac{(2\ell)^7}{2}A_k s_k\left(\Box_{AdS} - m_k^2\right)s_k\langle \mathcal{C}^{(k)}\mathcal{C}^{(k)}\rangle\right.$$
$$\left. + \sum_{k_i\geq 2}\frac{(2\ell)^5}{3}\langle \mathcal{C}^{(k_1)}\mathcal{C}^{(k_2)}\mathcal{C}^{(k_3)}\rangle g_{123}s_{k_1}s_{k_2}s_{k_3}\right\}$$

$$A_k = \frac{4\pi^4 k(k-1)}{3\times 2^k (k+1)(k+2)^2}, m_k^2 = \frac{k(k-6)}{4\ell^2}$$

$$g_{123} = \frac{192\pi^4(\alpha^2-9)(\alpha^2-1)(\alpha+2)}{(2\alpha+6)!!}\prod_{i=1}^{3}\frac{k_i!}{(k_i+2)\Gamma(\alpha_i)}$$

$$\alpha_i = \frac{1}{2}\sum_{j=1}^{3} k_j - k_i, \alpha = \frac{1}{2}\sum_{i=1}^{3} k_i$$

$$S_{int} = \int_{\partial \mathrm{AdS}_4} d^3 y w_k s_k^{(0)} \mathcal{O}_k$$

$$\mathcal{C}^k_{I_1\ldots I_k} = t_{I_1}\ldots t_{I_k}$$

$$\langle \mathcal{C}^{(k_1)}\mathcal{C}^{(k_2)}\rangle \equiv \mathcal{C}^{(k_1)}_{I_1\ldots I_{k_1}}\mathcal{C}^{(k_2)I_1\ldots I_{k_2}} = t_{12}^{k_1}\delta_{k_1,k_2}$$

$$\langle \mathcal{O}_{k_1}(x_1,t_1)\mathcal{O}_{k_2}(x_2,t_2)\rangle = 243 A_k \frac{(2\ell)^7}{\kappa^2}\frac{(k-3)}{\pi^{\frac{3}{2}}}\frac{\Gamma\left(\frac{k}{2}\right)}{\Gamma\left(\frac{k-3}{2}\right)}\frac{w_{k_1}^2 t_{12}^{k_1}\delta_{k_1,k_2}}{(x_{12}^2)^{k_1}}$$

$$\langle \mathcal{O}_{k_1}(x_1,t_1)\mathcal{O}_{k_2}(x_2,t_2)\rangle = \frac{\delta_{k_1,k_2}t_{12}^{k_1}}{(x_{12}^2)^{\frac{k_1}{2}}}$$

$$w_k = \frac{(2\pi)^{\frac{3}{2}}\sqrt{2}\ell}{9}\left(\frac{\ell_{11}}{2\ell}\right)^{\frac{9}{2}}\frac{(k+2)}{(k-3)(k-1)}\frac{\sqrt{\Gamma(2+k)}}{\Gamma\left(\frac{k}{2}+1\right)}$$

$$\langle \mathcal{O}_{k_1}\mathcal{O}_{k_2}\mathcal{O}_{k_3}\rangle = \left(\frac{\ell_{11}}{\ell}\right)^{\frac{9}{2}} R_{k_1,k_2,k_3}\frac{t_{12}^{\alpha_3}t_{23}^{\alpha_1}t_{13}^{\alpha_2}}{x_{12}^{\alpha_3}x_{23}^{\alpha_1}x_{13}^{\alpha_2}}$$

$$\left(\frac{\ell_{11}}{\ell}\right)^{\frac{9}{2}} R_{k_1,k_2,k_3} = \frac{7776(2\ell)^6}{\ell_{11}^9(2\pi)^8}\Gamma\left(\frac{\alpha-3}{2}\right)\prod_{i=1}^{3}\frac{\Gamma\left(\frac{\alpha_i}{2}\right)w_{k_i}}{\Gamma\left(\frac{k_i-3}{2}\right)}g_{123}.$$

$$\left(\frac{\ell_{11}}{\ell}\right)^{\frac{9}{2}} R_{k_1,k_2,k_3} = \frac{\pi}{2^{\frac{5}{2}}}\left(\frac{\ell_{11}}{\ell}\right)^{\frac{9}{2}}\frac{2^{-\alpha}}{\Gamma\left(1+\frac{\alpha}{2}\right)}\prod_{i=1}^{3}\frac{\sqrt{\Gamma(k_i+2)}}{\Gamma\left(\frac{\alpha_i+1}{2}\right)} = \frac{\pi}{N^{\frac{3}{4}}}\frac{2^{-\alpha-\frac{1}{4}}}{\Gamma\left(1+\frac{\alpha}{2}\right)}\prod_{i=1}^{3}\frac{\sqrt{\Gamma(k_i+2)}}{\Gamma\left(\frac{\alpha_i+1}{2}\right)}.$$

$$\langle \mathcal{C}^{k_1}\mathcal{C}^{k_2}\mathcal{C}^{k_3}\rangle = t_{12}^{\alpha_3}t_{23}^{\alpha_1}t_{13}^{\alpha_2}$$

$$\langle \mathcal{O}_{k_1}\ldots\mathcal{O}_{k_4}\rangle = \prod_{i<j}\left(\frac{t_{ij}^2}{x_{ij}^2}\right)^{\frac{\gamma_{ij}^0}{2}}\left(\frac{t_{12}^2 t_{34}^2}{x_{12}^2 x_{34}^2}\right)^{\frac{\mathcal{E}}{2}}\mathcal{G}_{k_1,\ldots,k_4}(U,V,\sigma,\tau).$$

$$U = \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2} = Z\bar{Z}, V = \frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2} = (1-Z)(1-\bar{Z}), \sigma = \frac{t_{13}t_{24}}{t_{12}t_{34}}, \tau = \frac{t_{14}t_{23}}{t_{12}t_{34}}.$$

$$\mathcal{E}=\begin{cases}\dfrac{k_1+k_2+k_3-k_4}{2} & \text{Case I}: k_1+k_4\geq k_2+k_3\\ k_1 & \text{Case II}: k_1+k_4<k_2+k_3\end{cases}$$

$$\gamma_{12}^0=\gamma_{13}^0=0,\qquad \gamma_{34}^0=\frac{\kappa_s}{2},\gamma_{24}^0=\frac{\kappa_u}{2}$$

$$\text{Case I: }\gamma_{14}^0=\frac{\kappa_t}{2},\quad \gamma_{23}^0=0,\ \text{Case II: }\gamma_{14}^0=0,\gamma_{23}^0=\frac{\kappa_t}{2}$$

$$\kappa_s=|k_1+k_2-k_3-k_4|,\kappa_t=|k_1+k_4-k_2-k_3|,\kappa_u=|k_2+k_4-k_1-k_3|.$$

$$\mathcal{G}_{k_1,\ldots,k_4}=\mathcal{G}^0_{k_1,\ldots,k_4}+\frac{1}{c_T}\mathcal{G}^R_{k_1,\ldots,k_4}+\cdots$$

$$\mathcal{G}^R_{k_1,k_2,k_3,k_4}=\mathcal{G}^R_{k_1,k_2,k_3,k_4,s}+\mathcal{G}^R_{k_1,k_2,k_3,k_4,t}+\mathcal{G}^R_{k_1,k_2,k_3,k_4,u}$$

$$\mathcal{G}^c_{k_1,\ldots k_4}(U,V,\sigma,\tau)\equiv\mathcal{G}_{k_1,\ldots k_4}(U,V,\sigma,\tau)-\mathcal{G}^0_{k_1,\ldots k_4}(U,V,\sigma,\tau)$$

$$\mathcal{G}^c_{k_1,\ldots k_4}(U,V,\sigma,\tau)=\int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}-a_s}V^{\frac{t}{2}-a_t}\mathcal{M}_{k_1,\ldots k_4}(s,t;\sigma,\tau)\Gamma_{\{k_i\}}$$

$$\Gamma_{\{k_i\}}=\Gamma\left(\frac{k_1+k_2}{4}-\frac{s}{2}\right)\Gamma\left(\frac{k_3+k_4}{4}-\frac{s}{2}\right)\Gamma\left(\frac{k_1+k_4}{2}-\frac{t}{2}\right)$$
$$\Gamma\left(\frac{k_2+k_3}{4}-\frac{t}{2}\right)\Gamma\left(\frac{k_1+k_3}{4}-\frac{u}{2}\right)\Gamma\left(\frac{k_2+k_4}{4}-\frac{u}{2}\right)$$

$$a_s=\frac{1}{4}(k_1+k_2)-\frac{1}{2}\mathcal{E},a_t=\frac{1}{4}\text{Min}\{k_1+k_4,k_2+k_3\},s+t+u=\frac{1}{2}\sum_{i=1}^{4}k_i.$$

$$\bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}(U,V)=\int_{-i\infty}^{i\infty}\frac{dj_1dj_2}{(2\pi i)^2}U^{j_1}V^{j_2}\Gamma(j_1+j_2+\Delta_2)\Gamma(j_1+j_2+\Delta-\Delta_4)$$
$$\times\Gamma(-j_1)\Gamma(-j_2)\Gamma(-j_1-\Delta+\Delta_3+\Delta_4)\Gamma(-j_2+\Delta-\Delta_2-\Delta_3)$$

$$\mathcal{G}_{2,2,2,2}(U,V,\sigma,\tau)\equiv\int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t}{2}-1}\mathcal{M}_{2,2,2,2}\Gamma^2\left(\frac{2-s}{2}\right)\Gamma^2\left(\frac{2-t}{2}\right)\Gamma^2\left(\frac{s+t-2}{2}\right)$$

$$\mathcal{G}_{2,2,2,2,s}=-\frac{6}{\sqrt{8N^3\pi^3}}[\left(3\sqrt{\pi}U\bar{D}_{3,1,0,0}-\sqrt{\pi}U^2\bar{D}_{4,2,0,0}-2\sqrt{U}\bar{D}_{\frac{5}{2},\frac{1}{2},0,0}\right)$$
$$+\sigma\left(3\sqrt{\pi}U\bar{D}_{2,1,0,1}-\sqrt{\pi}U^2\bar{D}_{3,2,0,1}-2\sqrt{U}\bar{D}_{\frac{3}{2},\frac{1}{2},0,1}\right)$$
$$+\tau\left(3\sqrt{\pi}U\bar{D}_{2,1,1,0}-\sqrt{\pi}U^2\bar{D}_{3,2,1,0}-2\sqrt{U}\bar{D}_{\frac{3}{2},\frac{1}{2},1,0}\right)].$$

$$\mathcal{G}^R_{2,2,2,2,t}(U,V,\sigma,\tau)\ =\tau^2\frac{U}{V}\mathcal{G}^R_{2,2,2,2,s}\left(V,U,\frac{\sigma}{\tau},\frac{1}{\tau}\right)$$
$$\mathcal{G}^R_{2,2,2,2,u}(U,V,\sigma,\tau)\ =\sigma^2U\mathcal{G}_{2,2,2,2,s}\left(\frac{1}{U},\frac{V}{U},\frac{1}{\sigma},\frac{\tau}{\sigma}\right)$$

$$\bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}(V,U)=\bar{D}_{\Delta_3,\Delta_2,\Delta_1,\Delta_4}(U,V)$$
$$\bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}\left(\frac{1}{U},\frac{V}{U}\right)=U^{\Delta_2}\bar{D}_{\Delta_4,\Delta_2,\Delta_3,\Delta_1}(U,V)$$

$$\mathcal{E}=2,\kappa_s=\kappa_t=\kappa_u=0,\gamma_{ij}^0=0$$

$$\langle\mathcal{O}_2(x_1,t_1)\dots\mathcal{O}_2(x_4,t_4)\rangle=\frac{t_{12}^2t_{34}^2}{x_{12}^2x_{34}^2}\mathcal{G}_{2,2,2,2}(U,V,\sigma,\tau)$$

$$\mathcal{G}_{2,2,k,k}(U,V,\sigma,\tau)=\int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t}{2}-\frac{k}{4}-\frac{1}{2}}\mathcal{M}_{2,2,k,k}$$
$$\times\Gamma\left(1-\frac{s}{2}\right)\Gamma\left(\frac{k-s}{2}\right)\Gamma^2\left(\frac{1}{2}+\frac{k}{4}-\frac{t}{2}\right)\Gamma^2\left(\frac{s+t-1}{2}-\frac{k}{4}\right)$$

$$\mathcal{G}_{2,2,k,k,s}=\frac{6}{\sqrt{8\pi^3N^3}}\left[\frac{\sqrt{\pi}}{\Gamma\left(\frac{k}{2}\right)}((1-k)U\partial_U-k)\left(\frac{V}{U}\bar{D}_{-1,1,\frac{k}{2}+1,\frac{k}{2}+1}+\sigma V\bar{D}_{0,1,\frac{k}{2}+1,\frac{k}{2}}+\tau\bar{D}_{0,1,\frac{k}{2},\frac{k}{2}+1}\right)\right.$$
$$\left.+k\left(\frac{V}{U}\bar{D}_{-1,1,\frac{3}{2},\frac{3}{2}}+V\sigma\bar{D}_{0,1,\frac{3}{2},\frac{1}{2}}+\tau\bar{D}_{0,1,\frac{1}{2},\frac{3}{2}}\right)\right]$$

$$\mathcal{G}_{2,2,k,k,t}=(-1)^{\frac{k}{2}}\frac{12k\tau U}{\sqrt{2N^3}}\frac{\Gamma\left(\frac{k}{2}+1\right)}{\Gamma\left(\frac{k}{2}-\frac{1}{2}\right)}\left[\frac{1}{4\pi}\left(\bar{D}_{\frac{1}{2},2-\frac{k}{2},0,\frac{1+k}{2}}+\sigma\bar{D}_{\frac{1}{2},1-\frac{k}{2},1,\frac{1+k}{2}}+\tau\bar{D}_{\frac{1}{2},1-\frac{k}{2},0,\frac{3+k}{2}}\right)\right.$$
$$\left.-\sum_{i=0}^{\left\lceil\frac{k-1}{2}\right\rceil}2^ix_i(k)\left(V\partial_V+\frac{k}{4}+\frac{1}{2}\right)^i\left(\bar{D}_{1,2-\frac{k}{2},0,1+\frac{k}{2}}+\sigma\bar{D}_{1,1-\frac{k}{2},1,1+\frac{k}{2}}+\tau\bar{D}_{1,1-\frac{k}{2},0,2+\frac{k}{2}}\right)\right],$$

$$\mathcal{G}_{2,2,k,k,u}(U,V,\sigma,\tau)=\mathcal{G}_{2,2,k,k,t}\left(\frac{U}{V},\frac{1}{V},\tau,\sigma\right)$$

$$\mathcal{E}=2,\kappa_s=2k-4,\kappa_t=\kappa_u=0,\gamma_{12}^0=\gamma_{13}^0=\gamma_{14}^0=\gamma_{23}^0=\gamma_{24}^0=0,\gamma_{34}^0=k-2$$

$$\langle\mathcal{O}_2\mathcal{O}_2\mathcal{O}_k\mathcal{O}_k\rangle=\left(\frac{t_{34}^2}{x_{34}^2}\right)^{\frac{k-2}{2}}\left(\frac{t_{12}^2t_{34}^2}{x_{12}^2x_{34}^2}\right)\mathcal{G}_{2,2,k,k}(U,V,\sigma,\tau).$$

$$\frac{1}{s}\Gamma\left(\frac{k_1+k_2}{4}-\frac{s}{2}\right)=-\frac{1}{2}\Gamma\left(-\frac{s}{2}\right)\prod_{n=1}^{\frac{k_1+k_2}{4}-1}\left[\frac{k_1+k_2}{4}-\frac{s}{2}-n\right]$$

$$\mathcal{G}_{k_1,k_2,k_3,k_4}^{HE}=-\frac{1}{2}\mathcal{N}_{k_i}P_{k_i}(\sigma,\tau)(-1)^{\frac{k_1+k_2}{2}+\frac{k_1+k_4}{2}+\frac{k_1+k_3}{4}}((1-\alpha)U\partial_U+V\partial_V)^2((1-\bar{\alpha})U\partial_U+V\partial_V)^2$$
$$\sum_r\binom{\frac{k_1+k_3}{4}-1}{r}U^{\frac{k_1+k_2}{4}-1-a_s+r}V^{\frac{2k_1+k_3+k_4}{4}-2-a_t-r}\bar{D}_{a_1+r-1,a_2-3,a_3-r-2,a_4}$$

$$a_1=\frac{k_2-k_4}{4},a_2=\frac{2k_1+k_2+k_4}{2},a_3=\frac{k_1-k_2+k_3+k_4}{4},a_4=\frac{-k_1+k_2+k_3+k_4}{4}.$$

$$\langle \mathcal{O}_{k_1} \dots \mathcal{O}_{k_4} \rangle^{HE} = \prod_{i<j} \left(\frac{t_{ij}^2}{x_{ij}^2}\right)^{\frac{\gamma_{ij}^0}{2}} \left(\frac{t_{12}^2 t_{34}^2}{x_{12}^2 x_{34}^2}\right)^{\frac{\varepsilon}{2}} \mathcal{G}^{HE}_{k_1,\dots,k_4}(U,V,\sigma,\tau).$$

$$ds^2_{AdS_4\times S^7} = ds^2_{AdS_4} + 4\ell^2 ds^2_{S^7}$$

$$\mathcal{O}_k = \sigma_k \ell^{\frac{k}{2}-1}\Phi_k, \sigma_k = \frac{2\pi}{\sqrt{\Gamma(k-1)}}$$

$$\lim_{\ell\to\infty} \frac{\langle \mathcal{O}_{k_1}(x_1)\mathcal{O}_{k_2}(x_2)\rangle}{\ell^{\frac{k_1+k_2}{2}-2}\sigma_{k_1}\sigma_{k_2}} = \langle \Phi_{k_1}\Phi_{k_2}\rangle = \frac{\delta_{k_1,k_2}}{(2\pi)^2}\frac{(-1)^{\frac{k_1}{2}-1}\Gamma(k_1-2)}{(u_{12}-i\epsilon)^{k_1-2}} t_{12}^{k_1}\delta^2(z_{12})$$

$$\frac{\langle \mathcal{O}_{k_1}\mathcal{O}_{k_2}\mathcal{O}_{k_3}\rangle}{\ell^{\frac{k_1+k_2+k_3}{2}}-3} \overset{\ell\to\infty}{\to} \mathcal{O}\left(\frac{1}{\ell^{\frac{11}{2}}}\right)$$

$$\lim_{\ell\to\infty} \ell^{\frac{11}{2}} \frac{\langle \mathcal{O}_{k_1}\mathcal{O}_{k_2}\mathcal{O}_{k_3}\rangle}{\ell^{\frac{k_1+k_2+k_3}{2}}-3} \prod_{i=1}^{3} \sigma_{k_i} = \frac{\Gamma\left(\frac{\alpha}{2}-2\right)\Theta(z_{12}z_{31})\Theta(z_{13}z_{23})}{\Gamma\left(\frac{\alpha_1}{2}\right)\Gamma\left(\frac{\alpha_2}{2}\right)\Gamma\left(\frac{\alpha_3}{2}\right)}\pi^2 R_{k_1,k_2,k_3}\prod_{i=1}^{3}\frac{1}{\sigma_{k_i}}$$
$$\times \delta(\bar{z}_{12})\delta(\bar{z}_{23})\frac{t_{12}^{\alpha_3}t_{23}^{\alpha_1}t_{13}^{\alpha_2} z_{12}^{\frac{k_3}{2}-2} z_{23}^{\frac{k_1}{2}-2} z_{13}^{\frac{k_2}{2}-2}}{(u_1 z_{23}+u_2 z_{31}+u_3 z_{12}+i\varepsilon)^{\frac{k_1+k_2+k_3}{2}-4}}$$

$$U^a V^b \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4} \overset{\ell\to\infty}{\to} \frac{\ell^{-4+\Sigma_\Delta}\mathcal{K}}{\mathcal{U}^{\Sigma_\Delta-4}}\left(\frac{|z_{23}|^2}{|z_{34}|^2|z_{24}|^2}\right)^{\frac{4-\Sigma_\Delta}{2}}\frac{(1-z)^{\Delta_1+\Delta_4-2+2b}}{z^{\Delta_1+\Delta_2-2-2a}}\delta(z-\bar{z})\Theta(z)\Theta(1-z)$$

$$\mathcal{K} = (-1)^{\Delta_1+\Delta_3}2^{\frac{\Sigma_\Delta}{2}}\pi^2\Gamma\left(\frac{\Sigma_\Delta-4}{2}\right)$$

$$\mathcal{G}_{2,2,2,2,s} \overset{\ell\to\infty}{\to} \frac{48\pi\ell^2}{\sqrt{2N^3}\mathcal{U}^2}\left(\frac{|z_{34}|^2|z_{24}|^2}{|z_{23}|^2}\right)(1-z)(1-\alpha z)^2(1-\bar{\alpha}z)^2\delta(z-\bar{z})\Theta(z)\Theta(1-z)$$

$$\langle \mathcal{O}_2(x_1,t_1)\dots\mathcal{O}_2(x_4,t_4)\rangle \overset{\ell\to\infty}{\to} \frac{3\ell_{11}^9\pi}{8\ell^7\mathcal{U}^2}\left(\frac{|z_{24}|^2}{|z_{12}|^2|z_{23}|^2}\right)t_{12}^2t_{34}^2(1-\alpha z)^2(1-\bar{\alpha}z)^2\delta(z-\bar{z})\Theta(z)\Theta(1-z)$$

$$\lim_{\ell\to\infty}\frac{V_7}{(2\pi)^4}\langle \mathcal{O}_1(x_1,t_1)\dots\mathcal{O}_1(x_4,t_4)\rangle = \langle \Phi_2(u_1,z_1,\bar{z}_1)\dots\Phi_2(u_4,z_4,\bar{z}_4)\rangle$$
$$= \frac{\pi\ell_{11}^9}{2\mathcal{U}^2}\left(\frac{|z_{24}|^2}{|z_{12}|^2|z_{23}|^2}\right)t_{12}^2t_{34}^2(1-\alpha z)^2(1-\bar{\alpha}z)^2\delta(z-\bar{z})\Theta(z)\Theta(1-z)$$

$$\mathcal{G}_{2,2,k,k,s} \overset{\ell\to\infty}{\to} \frac{-3U}{\sqrt{2N^3}\pi}\frac{(1-k)}{\Gamma\left(\frac{k}{2}\right)}\left(V\bar{D}_{0,2,\frac{k}{2}+1,\frac{k}{2}+1} + \sigma UV\bar{D}_{1,2,\frac{k}{2}+1,\frac{k}{2}} + \tau U\bar{D}_{1,2,\frac{k}{2},\frac{k}{2}+1}\right)$$

$$\mathcal{G}_{2,2,k,k,t} \overset{\ell\to\infty}{\to} \frac{12k\tau U}{\sqrt{2N^3}}\frac{\Gamma\left(\frac{k}{2}+1\right)}{\Gamma\left(\frac{k}{2}-\frac{1}{2}\right)}2^{\frac{k}{2}}x_{\frac{k}{2}}\left(\bar{D}_{1,2,\frac{k}{2},1+\frac{k}{2}} + \sigma\bar{D}_{1,1,1+\frac{k}{2},1+\frac{k}{2}} + \tau\bar{D}_{1,1,\frac{k}{2},2+\frac{k}{2}}\right),$$

$$\lim_{\ell\to\infty} V_7\left[\ell^{2-k}\frac{\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_k\mathcal{O}_k\rangle}{\sigma_k^2}\right]=\frac{\pi\ell_{11}^9}{2}(1-\alpha z)^2(1-\bar{\alpha}z)^2(-1)^{\frac{k}{2}-1}t_{34}^k t_{12}^2$$
$$\times\frac{|z_{24}|^k}{|z_{12}|^2|z_{23}|^k}\frac{\Gamma(k)(1-z)^{\frac{k}{2}-1}\delta(z-\bar{z})}{\mathcal{U}^k}$$

$$\mathcal{G}^{HE}_{k_1,k_2,k_3,k_4}\overset{\ell\to\infty}{\to}\frac{\mathcal{N}_{k_i}\mathcal{P}_{k_i}}{N^{\frac{3}{2}}\mathcal{U}^{\frac{\sum_i k_i}{2}-2}}(-1)^{\frac{k_1+k_3}{2}}z^{-2a_s}(1-z)^{\frac{\sum_i k_i}{2}-2-2a_t}\pi^{\frac{5}{2}}2^{4-\frac{\sum_i k_i}{4}}\frac{\Gamma\left(\frac{\sum_i k_i}{2}-5\right)\Gamma\left(\frac{\sum_i k_i}{4}-1\right)}{\Gamma\left(\frac{\sum_i k_i}{4}-4\right)\Gamma\left(\frac{\sum_i k_i}{4}-\frac{1}{2}\right)}$$
$$\times(1-\alpha z)^2(1-\bar{\alpha}z)^2\left(\frac{|z_{23}|^2}{|z_{34}|^2|z_{24}|^2}\right)^{1-\frac{\sum_i k_i}{4}}\delta(z-\bar{z})\left(\frac{1}{\ell}\right)^{2-\frac{\sum_i k_i}{2}}$$

$$\lim_{\ell\to\infty} V_7\frac{\langle \mathcal{O}_{k_1}\dots\mathcal{O}_{k_4}\rangle}{\prod_{i=1}^4 \ell^{\frac{k_i}{2}-1}\sigma_k}=\tilde{\mathcal{N}}\left[\prod_{i<j}\left(\frac{t_{ij}}{|z_{ij}|}\right)^{\gamma_{ij}^0}\left(\frac{t_{12}t_{34}}{|z_{12}||z_{34}|}\right)^{\mathcal{E}}\left(\frac{|z_{23}|^2}{|z_{34}|^2|z_{24}|^2}\right)^{1-\frac{\sum_i k_i}{4}}\right]$$
$$\times\left[z^{-2a_s}(1-z)^{\frac{\sum_i k_i}{2}-2-2a_t}\frac{\delta(z-\bar{z})}{\mathcal{U}^{\frac{\sum_i k_i}{2}-2}}\right]\times\left[(1-\alpha z)^2(1-\bar{\alpha}z)^2\mathcal{P}_{k_i}(\sigma,\tau)\right]$$

$$\tilde{\mathcal{N}}=\frac{V_7\ell_{11}^9}{(2\pi)^4}\mathcal{N}_{k_i}\pi^{\frac{5}{2}}2^{-\frac{1+\sum_i k_i}{2}}\frac{\Gamma\left(\frac{\sum_i k_i}{2}-5\right)\Gamma\left(\frac{\sum_i k_i}{4}-1\right)}{\Gamma\left(\frac{\sum_i k_i}{4}-4\right)\Gamma\left(\frac{\sum_i k_i}{4}-\frac{1}{2}\right)}(-1)^{\frac{k_1+k_3}{2}}$$

$$\left(Z\partial_Z-\frac{\alpha}{2}\partial_\alpha\right)\mathcal{G}_{k_1,k_2,k_3,k_4}(Z,\bar{Z},\alpha,\bar{\alpha})\Big|_{\alpha=\frac{1}{\bar{Z}}}=\left(\bar{Z}\partial_{\bar{Z}}-\frac{\alpha}{2}\partial_\alpha\right)\mathcal{G}_{k_1,k_2,k_3,k_4}(Z,\bar{Z},\alpha,\bar{\alpha})\Big|_{\alpha=\frac{1}{\bar{Z}}}=0$$
$$\left(Z\partial_Z-\frac{\bar{\alpha}}{2}\partial_{\bar{\alpha}}\right)\mathcal{G}_{k_1,k_2,k_3,k_4}(Z,\bar{Z},\alpha,\bar{\alpha})\Big|_{\bar{\alpha}=\frac{1}{\bar{Z}}}=\left(\bar{Z}\partial_{\bar{Z}}-\frac{\bar{\alpha}}{2}\partial_{\bar{\alpha}}\right)\mathcal{G}_{k_1,k_2,k_3,k_4}(Z,\bar{Z},\alpha,\bar{\alpha})\Big|_{\bar{\alpha}=\frac{1}{\bar{Z}}}=0$$

$$\mathcal{G}_{k_1,k_2,k_3,k_4}(Z,\bar{Z},\alpha,\bar{\alpha})=\frac{\mathcal{G}^0(Z,\alpha,\bar{\alpha})}{(Z-\bar{Z})^p}+\mathcal{O}\left(\frac{1}{(Z-\bar{Z})^{p-1}}\right)$$

$$\mathcal{G}^0\left(z,\alpha=\frac{1}{z},\bar{\alpha}\right)=\mathcal{G}^0\left(z,\alpha,\bar{\alpha}=\frac{1}{z}\right)=0$$

$$S=\int_{\mathbb{R}^{3,1}}d^4x\left\{\sum_{k=2}^{k_{\max}}\frac{t_{12}^k}{2}s_k\,\square\,s_k+\sum_{k_1,k_2,k_3=2}^{k_{\max}}\frac{t_{12}^{\alpha_3}t_{23}^{\alpha_1}t_{13}^{\alpha_2}}{2\ell}\left(\frac{\ell_{11}}{2\ell}\right)^{\frac{9}{2}}\frac{\tilde{g}_{123}}{3}s_{k_1}s_{k_2}s_{k_3}\right\}$$

$$\tilde{g}_{123}=\frac{144\sqrt{3}2^\alpha(\alpha^2-9)(\alpha^2-1)(\alpha+2)}{(2\alpha+6)!!\,\pi^2}\prod_{i=1}^3\frac{\Gamma(k_i-1)}{\Gamma(\alpha_i)}\sqrt{(k_i+1)k_i(k_i-1)}$$

$$\mathcal{A}_{k_1,k_2}=\frac{t_{12}^{k_1}\delta_{k_1,k_2}}{\omega_1}\delta_{\epsilon_1,-\epsilon_2}\delta(\omega_1-\omega_2)\delta^2(z_{12})$$

$$\mathcal{C}^{\Delta_1,\Delta_2}_{k_1,k_2}=\int\prod_{j=1}^2\frac{d\omega_j}{2\pi}\omega_j^{\Delta_j-1}e^{iu_j\omega_j\epsilon_j}\mathcal{A}_{k_1,k_2}\Big|_{\epsilon_1=-\epsilon_2=-1}=\frac{\delta_{k_1,k_2}}{(2\pi)^2}\frac{(-1)^{\Delta_1-1}\Gamma(\Delta_1+\Delta_2-2)}{(u_{12}-i\epsilon)^{\Delta_1+\Delta_2-2}}t_{12}^{k_1}\delta^2(z_{12})$$

$$\mathcal{C}_{k_1,k_2}^{\frac{k_1}{2},\frac{k_2}{2}} = \frac{\delta_{k_1,k_2}}{(2\pi)^2}\frac{(-1)^{\frac{k_1}{2}-1}\Gamma(k_1-2)}{(u_{12}-i\epsilon)^{k_1-2}}t_{12}^{k_1}\delta^2(z_{12})$$

$$\mathcal{A}_{k_1,k_2,k_3} = \frac{\tilde{g}_{123}}{2\ell}\left(\frac{\ell_{11}}{2\ell}\right)^{\frac{9}{2}}t_{12}^{\alpha_3}t_{23}^{\alpha_1}t_{13}^{\alpha_2}\delta^{(4)}(p_1+p_2+p_3)$$

$$\mathcal{C}_{k_1,k_2,k_3}^{\Delta_1,\Delta_2,\Delta_3} = \int\prod_{j=1}^{3}\frac{d\omega_j}{2\pi}\omega_j^{\Delta_j-1}e^{iu_j\omega_j\epsilon_j}\mathcal{A}_{k_1,k_2,k_3}$$

$$\begin{aligned}C_{k_1,k_2,k_3}^{\Delta_1,\Delta_2,\Delta_3} =& \frac{-i\epsilon_1\epsilon_2\epsilon_3}{(2\pi)^3}\frac{\tilde{g}_{123}}{2\ell}\left(\frac{\ell_{11}}{2\ell}\right)^{\frac{9}{2}}t_{12}^{\alpha_3}t_{23}^{\alpha_1}t_{13}^{\alpha_2}(z_{12})^{\Delta_1-2}(z_{13})^{\Delta_2-2}(z_{23})^{\Delta_3-2}\delta(\bar{z}_{12})\delta(\bar{z}_{23})\\ &\times\Theta\left(-\frac{z_{13}}{z_{23}}\epsilon_1\epsilon_2\right)\Theta\left(\frac{z_{12}}{z_{23}}\epsilon_1\epsilon_3\right)\frac{\Gamma\left(\sum_{i=1}^{3}\Delta_i-4\right)}{\left(z_{23}u_1+z_{31}u_2+z_{12}u_3-i\varepsilon\epsilon_1\mathrm{sign}(z_{23})\right)^{\sum_{i=1}^{3}\Delta_i-4}}\end{aligned}$$

$$\begin{aligned}A_4 =& \frac{-a_4\ell_{11}^9}{stu}\Big(\frac{1}{2}e_2\cdot e_3(se_1\cdot P_3e_4\cdot P_2+te_1\cdot P_2e_4\cdot P_3)+\frac{1}{2}e_1\cdot e_4(se_2\cdot P_4e_3\cdot P_1+te_2\cdot P_1e_3\cdot P_4)\\ &+\frac{1}{2}e_2\cdot e_4(se_1\cdot P_4e_3\cdot P_2+ue_1\cdot P_2e_3\cdot P_4)+\frac{1}{2}e_1\cdot e_3(se_2\cdot P_3e_4\cdot P_1+ue_2\cdot P_1e_4\cdot P_3)\\ &+\frac{1}{2}e_3\cdot e_4(te_1\cdot P_4e_2\cdot P_3+ue_1\cdot P_3e_2\cdot P_4)+\frac{1}{2}e_1\cdot e_2(te_3\cdot P_2e_4\cdot P_1+ue_3\cdot P_1e_4\cdot P_2)\\ &-\frac{1}{4}ste_1\cdot e_4e_2\cdot e_3-\frac{1}{4}sue_1\cdot e_3e_2\cdot e_4-\frac{1}{4}tue_1\cdot e_2e_3\cdot e_4\Big)^2\delta^{(11)}\left(\sum_{i=1}^{4}P_i\right)\end{aligned}$$

$$P_i^\alpha = \left(p_i^\mu,\tilde{p}_i^I\right), p_i\in\mathbb{R}^{1,3},\tilde{p}_i\in\mathbb{R}^7, p_i^\mu\sim\mathcal{O}(1),\tilde{p}_i^I\sim\mathcal{O}\left(\frac{1}{\ell}\right)\approx 0.$$

$$e_i^\alpha = \left(0,0,0,0,\xi_i^I\right)$$

$$t_i = (0,\xi_i)$$

$$e_i\cdot e_j = t_i\cdot t_j\equiv t_{ij}, e_i\cdot p_j\sim\mathcal{O}\left(\frac{1}{\ell}\right)\approx 0$$

$$h_j^{\alpha\beta}(X) = \mathcal{N}_j e_j^\alpha e_j^\beta e^{ip_j\cdot x}\left(\xi_j\cdot\tilde{x}\right)^{k_j-2},$$

$$\Box\,\bar{h}_{\alpha\beta} = 0, \bar{h}_{\alpha\beta} = h_{\alpha\beta}-\frac{1}{2}\eta_{\alpha\beta}h_\gamma^\gamma,$$

$$\begin{aligned}\int_{\mathbb{R}^{1,3}}d^4x\int_{\mathbb{R}^7}d^7\tilde{x}h_1^{\alpha\beta}h_{2,\alpha\beta} &= \mathcal{N}_1\mathcal{N}_2t_{12}^2(2\pi)^4\delta^{(4)}(P_1+P_2)\int_{\mathbb{R}^7}d^7\tilde{x}(\xi_1\cdot\tilde{x})^{k_1-2}(\xi_2\cdot\tilde{x})^{k_2-2}\\ &= \mathcal{N}_1\mathcal{N}_2t_{12}^2(2\pi)^4\delta^{(4)}(P_1+P_2)\times\frac{2\pi^{\frac{7}{2}}t_{12}^{2k_1-2}\delta_{k_1,k_2}}{\Gamma\left(\frac{7}{2}\right)}\int_0^\infty d|\tilde{x}||\tilde{x}|^{k_1+k_2+2}\end{aligned}$$

$$h_j^{\alpha\beta}(X) = \frac{1}{\sqrt{V_7}}e_i^\alpha e_i^\beta e^{ip_j\cdot x}\frac{\left(t_j\cdot Z\right)^{k_j-2}}{(2\ell)^{k_j-2}}.$$

$$h_j^{\alpha\beta}(X) = \frac{1}{\sqrt{V_7}} e_i^{\alpha} e_i^{\beta} e^{ip_j \cdot x}$$

$$\delta^{(11)}\left(\sum_{i=1}^{4} P_i\right) \to \frac{1}{(2\pi)^7 V_7^2} \int_{\mathbb{R}^{3,1}} \frac{d^4x}{(2\pi)^4} e^{i\sum_{j=1}^{4} p_j \cdot x} \int_{S^7} d^8 Z \delta(Z \cdot Z - 4\ell^2) = \frac{\delta^{(4)}\left(\sum_{i=1}^{4} p_i\right)}{(2\pi)^7 V_7}$$

$$\mathcal{A}_4^{2,2,2,2} = \frac{-a_4 \ell_{11}^9 t_{12}^2 t_{34}^2}{(2\pi)^7 V_7 stu} (st\tau + su\sigma + tu)^2 \times \delta^{(4)}\left(\sum_{i=1}^{4} p_i\right)$$

$$V_7 \mathcal{C}_4^{1,\ldots,1}\left(\{u_j, z_j, \bar{z}_j\}^{\epsilon_j}\right) = \frac{a_4 \ell_{11}^9}{2(2\pi)^{11} \mathcal{U}^2} \left(\frac{|z_{24}|^2}{|z_{12}|^2 |z_{23}|^2}\right) t_{12}^2 t_{34}^2 (1-\alpha z)^2 (1-\bar{\alpha} z)^2 \delta(z-\bar{z}) \Theta(z) \Theta(1-z),$$

$$\delta^{(11)}(P_1 + P_2 + P_3 + P_4) \longrightarrow \delta^{(4)}\left(\sum_{k=1}^{4} p_k\right) \frac{1}{(2\pi)^7 V_7^2} \int d^8 Z \delta(Z \cdot Z - 4\ell^2) \prod_{j=1}^{4} \frac{(t_j \cdot Z)^{k_j - 2}}{(2\ell)^{k_j - 2}}$$

$$= \frac{\tilde{N}_{k_i}}{V_7} \prod_{i<j} t_{ij}^{\gamma_{ij}^0} (t_{12} t_{34})^{\mathcal{E}-2} \mathcal{P}_{k_i}(\sigma, \tau) \delta^{(4)}\left(\sum_{j=1}^{4} p_j\right)$$

$$\frac{\epsilon_1 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4}{\epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4} = \frac{t_{13} t_{24}}{t_{12} t_{34}} \equiv \sigma, \frac{\epsilon_2 \cdot \epsilon_3 \epsilon_1 \cdot \epsilon_4}{\epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4} = \frac{t_{23} t_{14}}{t_{12} t_{34}} \equiv \tau.$$

$$\mathcal{A}_4^{k_1,k_2,k_3,k_4} = -\frac{\tilde{N}_{k_i}}{V_7} \prod_{i<j} t_{ij}^{-\gamma_{ij}^0} \frac{(t_{12} t_{34})^{2-\mathcal{E}}}{4\ell_{11}^9 stu} (tu + su\sigma + st\tau)^2 \mathcal{P}_{k_i}(\sigma, \tau)$$

$$\mathcal{C}_{k_1,k_2,k_3,k_4}^{\frac{k_1}{2},\frac{k_2}{2},\frac{k_3}{2},\frac{k_4}{2}} = \int_0^{+\infty} \prod_{j=1}^{4} \frac{d\omega_j}{2\pi} (-i\epsilon_j \omega_j)^{\frac{k_j}{2}-1} e^{-i\epsilon_j \omega_j u_j} \mathcal{A}_4^{k_1,k_2,k_3,k_4}$$

$$= -\frac{\tilde{N}_{k_i} (-1)^{\frac{k_1+k_3}{2}} i^{\sum_{i=1}^{4} \frac{k_i}{2}} \Gamma\left(-2 + \sum_{i=1}^{4} \frac{k_i}{2}\right)}{V_7 4\ell_{11}^9 (2\pi)^4} \left[(t_{12} t_{34})^{2-\mathcal{E}} \prod_{i<j} t_{ij}^{-\gamma_{ij}^0} (1-\alpha z)^2 (1-\bar{\alpha} z)^2\right]$$

$$\times \left[\frac{|z_{14}|^{k_3-2} |z_{24}|^{k_1+2} |z_{34}|^{k_2-4}}{|z_{12}|^{k_1+2} |z_{13}|^{k_3-4} |z_{23}|^{k_2}}\right] \times \left[\frac{\delta(z-\bar{z}) \Theta(z) \Theta(1-z) z^{\frac{k_1-k_2+4}{2}}}{\mathcal{U}^{-2+\sum_{i=1}^{4} \frac{k_i}{2}}} (1-z)^{\frac{k_2-k_3}{2}}\right.$$

$$\left\{Q^{I\alpha}, \bar{Q}_{J\beta}\right\} = 2\delta^I{}_J \gamma^{\mu\alpha}{}_{\beta} P_\mu, \left\{S^{I\alpha}, \bar{S}_{J\beta}\right\} = 2\delta^I{}_J \gamma^{\mu\alpha}{}_{\beta} K_\mu$$
$$\left\{Q^{\alpha I}, \bar{S}_{\beta J}\right\} = -i\delta^I{}_J \left(2\delta^{\alpha}{}_{\beta} D + \left(\gamma^{[\mu} \gamma^{\nu]}\right)^{\alpha}{}_{\beta} M_{\mu\nu}\right) + 2i\delta^{\alpha}{}_{\beta} R^I{}_J$$

$$\bar{Q}_J = (Q^J)^\dagger \gamma^0 = -(Q^J)^T \epsilon, \bar{S}_J = (S^J)^\dagger \gamma^0 = -(S^J)^T \epsilon.$$

$$\gamma^0 = \sigma^3, \gamma^1 = i\sigma^1, \gamma^2 = i\sigma^2$$

$$\left[R_{IJ}, R_{KL}\right] = i(\delta^{IK} R^{JL} + \delta^{JL} R^{IK} - \delta^{IL} R^{JK} - \delta^{JK} R^{IL})$$

$$\left[R_{IJ}, Q^K\right] = i\left(\delta_I{}^K \delta_{JD} - \delta_J{}^K \delta_{ID}\right) Q^D, \left[R_{IJ}, S^K\right] = i\left(\delta_I{}^K \delta_{JD} - \delta_J{}^K \delta_{ID}\right) S^D$$

$$H \to \frac{1}{c}H, B_i \to \frac{1}{c}B_i, K \to \frac{1}{c}K$$

$$Q^{I\alpha} \to \frac{1}{\sqrt{c}}Q^{I\alpha}, S^{I\alpha} \to \frac{1}{\sqrt{c}}S^{I\alpha}$$

$$\left\{Q^{I\alpha}, \bar{Q}_{J\beta}\right\} = -2\delta^I{}_J\gamma^{0\alpha}{}_\beta H, \left\{S^{I\alpha}, \bar{S}_{J\beta}\right\} = -2\delta^I{}_J\gamma^{0\alpha}{}_\beta K,$$
$$\left\{Q^{\alpha I}, \bar{S}_{\beta J}\right\} = -2i\delta^I{}_J\left(\gamma^{[0}\gamma^{i]}\right)^\alpha{}_\beta B_i,$$
$$\left[R_{IJ}, R_{KL}\right] = i(\delta^{IK}R^{JL} + \delta^{JL}R^{IK} - \delta^{IL}R^{JK} - \delta^{JK}R^{IL}),$$
$$\left[R_{IJ}, Q^K\right] = i\left(\delta_I{}^K\delta_{JD} - \delta_J{}^K\delta_{ID}\right)Q^D, \left[R_{IJ}, S^K\right] = i\left(\delta_I{}^K\delta_{JD} - \delta_J{}^K\delta_{ID}\right)S^D.$$

$$\mathfrak{CCarr}_3^{\text{glob}} \simeq \mathfrak{iso}(3,1)$$

$$\left\{\mathcal{Q}_\alpha^I, \overline{\mathcal{Q}}_{\dot{\alpha}}^J\right\} = 2\delta^{IJ}\sigma_{\alpha\dot{\alpha}}^\mu\mathcal{P}_\mu$$

$$\mathcal{P}_0 = -\frac{1}{2}(H + K),, \mathcal{P}_1 = -B_1, \mathcal{P}_2 = -B_2, \mathcal{P}_3 = \frac{1}{2}(H - K)$$

$$\mathcal{Q}_1^I = S^{I1} = -\bar{S}_{I2}, \overline{\mathcal{Q}}_{\dot{1}}^I = \bar{S}^{I1} = S^{I2}, \mathcal{Q}_2^I = \bar{Q}_{I1} = Q^{I2}, \overline{\mathcal{Q}}_{\dot{2}} = Q^{I1} = -\bar{Q}_{I2}$$

$$\left[R_{IJ}, Q^K\right] = i\left(\delta_I{}^K\delta_{JD} - \delta_J{}^K\delta_{ID}\right)Q^D, \left[R_{IJ}, \overline{\mathcal{Q}}^K\right] = i\left(\delta_I{}^K\delta_{JD} - \delta_J{}^K\delta_{ID}\right)\overline{\mathcal{Q}}^D$$

$$\mathfrak{sCCaCrr}_3^{\text{glob},\mathcal{N}} \simeq \mathfrak{spoin}(\mathcal{N}, 4)$$

$$\left[J_{ij}, \phi_\Delta(0)\right] = 0, [B_i, \phi_\Delta(0)] = 0$$
$$[D, \phi_\Delta(0)] = -i\Delta\phi_\Delta(0), [K, \phi_\Delta(0)] = 0, [K_i, \phi_\Delta(0)] = 0$$

$$[S^{I\alpha}, \phi_\Delta(0)] = 0 = [\bar{S}_{I\alpha}, \phi_\Delta(0)]$$

$$\left[R_{IJ}, \phi_\Delta(0)\right] = \mathcal{R}^{(s)} \cdot \phi_\Delta(0)$$

$$\phi_k = \phi^{I_1 \dots I_k} t_{I_1} \dots t_{I_k}$$

$$\phi_\Delta(x, \theta) = U\phi_\Delta(0,0)U^{-1}, U = e^{-i\left(-Hu + P_i x^i + \bar{Q}_{I\alpha}\theta^{I\alpha}\right)}$$

$$\delta\phi_\Delta(x, \theta)$$
$$= i\left[aH + b^j B_j + kK + a^j P_j + \frac{1}{2}r^{jk}J_{jk} + \lambda D + k^j K_j + \epsilon^I\bar{Q}_I + \kappa^I\bar{S}_I + \frac{1}{2}\omega^{IJ}R_{IJ}, \phi_\Delta(x, \theta)\right]$$

$$\sum_{j=1}^n \left\langle \phi_{\Delta_1}(x_1, \theta_1^I) \dots \delta\phi_{\Delta_j}(x_j, \theta_j^I) \dots \phi_{\Delta_n}(x_n, \theta_n^I)\right\rangle = 0$$

$$\phi_\Delta(x^a, \theta^{I\alpha}) = \Phi_\Delta(x^a) + \theta^{I\alpha}\bar{\Psi}_{I\alpha}(x^a) + \cdots$$

$$\left\langle \Phi_{k_1}(x_1) \dots \Phi_{k_n}(x_n)\right\rangle = \int_0^{+\infty} \prod_{j=1}^n \frac{d\omega_j}{2\pi}\left(-i\epsilon_j\omega_k\right)^{\frac{k_j}{2}-1} e^{-i\epsilon_j\omega_j u_j}\mathcal{A}_n^{k_1,\dots,k_n}$$

$$\mathcal{M}_{2,2,2,2}=\frac{1}{c_T}\mathcal{M}^R_{2,2,2,2}+\frac{1}{c_T^{\frac{5}{3}}}B^{R^4}\mathcal{M}^{(4)}_{2,2,2,2}$$
$$+\frac{1}{c_T^{\frac{7}{3}}}\left(B_4^{D^6R^4}\mathcal{M}^{(4)}_{2,2,2,2}+B_6^{D^6R^4}\mathcal{M}^{(6)}_{2,2,2,2}+B_7^{D^6R^4}\mathcal{M}^{(7)}_{2,2,2,2}\right)+\cdots$$

$$\mathcal{M}^R_{2,2,2,2}(s,t;\sigma,\tau)=\mathcal{M}^R_{2,2,2,2,s}+\mathcal{M}^R_{2,2,2,2,t}+\mathcal{M}^R_{2,2,2,2,u}$$
$$\mathcal{M}^R_{2,2,2,2,s}(s,t;\sigma,\tau)=\sum_{m=0}^{\infty}-\frac{3((t-2)(u-2)+(s+2)((t-2)\sigma+(u-2)\tau))}{\sqrt{2\pi}N^{\frac{3}{2}}\Gamma\left(\frac{1}{2}-m\right)^2 m!\,\Gamma\left(m+\frac{5}{2}\right)(s-1-2m)}$$

$$\mathcal{M}^R_{2,2,2,2,s}(s,t;\sigma,\tau)=\frac{3}{\sqrt{8\pi^3N^3}}\frac{1}{s(s+2)}[(t-2)(s+t-2)-\sigma(s+2)(t-2)+\tau(s+2)(s+t-2)]$$
$$\times\left[\sqrt{\pi}(s+4)-4\frac{\Gamma\left(\frac{1-s}{2}\right)}{\Gamma\left(1-\frac{s}{2}\right)}\right]$$

$$\mathcal{M}^R_{2,2,2,2,t}(s,t;\sigma,\tau)=\tau^2\mathcal{M}^R_{2,2,2,2,s}\left(t,s;\frac{\sigma}{\tau},\frac{1}{\tau}\right),\mathcal{M}^R_{2,2,2,2,u}(s,t;\sigma,\tau)=\sigma^2\mathcal{M}^R_{2,2,2,2,s}\left(u,t;\frac{1}{\sigma},\frac{\tau}{\sigma}\right)$$

$$\mathcal{M}_{2,2,k,k}=\frac{1}{c_T}\mathcal{M}^R_{2,2,k,k}$$

$$\mathcal{M}_{2,2,k,k,s}=\frac{3}{8\sqrt{2}\pi^{3/2}N^{3/2}s(s+2)}[(k-2t+2)(k-2u+2)-2(s+2)\sigma(k-2t+2)-2(s+2)\tau(k-2u+2)]$$
$$\times\left(\frac{\sqrt{\pi}(s-k(s+2))}{\Gamma\left(\frac{k}{2}\right)}+\frac{2k\Gamma\left(\frac{1}{2}-\frac{s}{2}\right)}{\Gamma\left(\frac{k-s}{2}\right)}\right)$$

$$\mathcal{M}_{2,2,k,k,t}=\frac{-3k\tau\Gamma\left(\frac{k}{2}+1\right)[(k+2t+2)(k-2u+2)+2\sigma(k-s)(k+2t+2)-2\tau(k-s)(k-2u+2)]}{4\sqrt{2}\pi\sqrt{N^3}(k-2t)\Gamma\left(\frac{k-1}{2}\right)\Gamma\left(\frac{3+k}{2}\right)}$$
$$\times{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{k}{4}-\frac{t}{2};\frac{k}{2}+\frac{3}{2},\frac{k}{4}-\frac{t}{2}+1;1\right)$$

$${}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{k}{4}-\frac{t}{2};\frac{k}{2}+\frac{3}{2},\frac{k}{4}-\frac{t}{2}+1;1\right)=\frac{\pi(k-2t)\Gamma\left(\frac{3+k}{2}\right)}{\left(\frac{2-k+2t}{4}\right)_{\frac{2+k}{2}}}\left[\frac{\Gamma\left(\frac{k}{4}-\frac{t}{2}\right)}{4\pi\Gamma\left(\frac{k}{4}+\frac{1}{2}-\frac{t}{2}\right)}-\sum_{i=0}^{\left\lceil\frac{k-1}{2}\right\rceil}x_i t^i\right]$$

$$x_{\frac{k}{2}}=\frac{\Gamma\left(\frac{1+k}{2}\right)}{4\pi 2^{\frac{k}{2}}\Gamma^2\left(1+\frac{k}{2}\right)}$$

$$\mathcal{M}_{2,2,k,k,t}=\frac{-3k\tau\Gamma\left(\frac{k}{2}+1\right)[-(k+2t+2)(k-2u+2)+2\sigma(k-s)(k+2t+2)-2\tau(k-s)(k-2u+2)]}{4\sqrt{2}\sqrt{N^3}\Gamma\left(\frac{k-1}{2}\right)\left[\prod_{n=0}^{\frac{k}{2}}\left(\frac{t}{2}+\frac{k}{4}+\frac{1}{2}-n\right)\right]}$$

$$\otimes\ \langle\frac{\Gamma\left(\frac{k}{4}-\frac{t}{2}\right)}{4\pi\Gamma\left(\frac{k}{4}+\frac{1}{2}-\frac{t}{2}\right)}-\sum_{i=0}^{\left\lceil\frac{k-1}{2}\right\rceil}x_i t^i\rangle$$

$$\lim_{s,t\to\infty}\mathcal{M}_{k_1,k_2,k_3,k_4}=\frac{\mathcal{N}_{k_i}}{N^{\frac{3}{2}}}\frac{(tu+st\sigma+\tau su)^2}{stu}\mathcal{P}_{k_i}(\sigma,\tau)=\frac{\mathcal{N}_{k_i}}{N^{\frac{3}{2}}}\frac{(s+t-s\alpha)^2(s+t-s\bar{\alpha})^2}{stu}\mathcal{P}_{k_i}(\sigma,\tau),$$

$$\mathcal{P}_{k_i}(\sigma,\tau)=\sum_{\substack{i+j+k=\mathcal{E}-2\\0\le i,j,k\le\mathcal{E}-2}}\frac{(\mathcal{E}-2)!\,\sigma^i\tau^j}{i!\,j!\left(i+\frac{\kappa_u}{2}\right)!\left(j+\frac{\kappa_t}{2}\right)!\left(\frac{\kappa_u}{2}\right)!},$$

$$\sigma=\alpha\bar{\alpha},\tau=(1-\alpha)(1-\bar{\alpha})$$

$$\langle O_1(x_1)\dots O_4(x_4)\rangle=\frac{1}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2}{2}}(x_{34}^2)^{\frac{\Delta_3+\Delta_4}{2}}}\left(\frac{x_{14}^2}{x_{24}^2}\right)^a\left(\frac{x_{14}^2}{x_{13}^2}\right)^b\mathcal{G}(U,V),$$

$$a=\frac{1}{2}(\Delta_2-\Delta_1),b=\frac{1}{2}(\Delta_3-\Delta_4),$$

$$\mathcal{G}(U,V)=\int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t}{2}-\frac{\Delta_2+\Delta_3}{2}}\mathcal{M}(s,t)\Gamma_{\{\Delta_i\}}$$

$$\Gamma_{\{\Delta_i\}}=\Gamma\left(\frac{\Delta_1+\Delta_2-s}{2}\right)\Gamma\left(\frac{\Delta_3+\Delta_4-s}{2}\right)\Gamma\left(\frac{\Delta_1+\Delta_4-t}{2}\right)$$
$$\Gamma\left(\frac{\Delta_2+\Delta_3-t}{2}\right)\Gamma\left(\frac{\Delta_1+\Delta_3-u}{2}\right)\Gamma\left(\frac{\Delta_2+\Delta_4-u}{2}\right)$$

$$\mathcal{M}^c(s,t)=\sum_{a=a_0,b=b_0}^{a_1,b_1}\chi_{a,b}s^a t^b.$$

$$\mathcal{G}^c(U,V)=\sum_{a=a_0,b=b_0}^{a_1,b_1}\chi_{a,b}(2U\partial_U)^a(2V\partial_V+\Delta_2+\Delta_3)^b\left(U^{\frac{\Delta_1+\Delta_2}{2}}\bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}\right)$$

$$\mathcal{G}^{c,l.s}(U,V)=(-1)^{a_1+b_1}\chi_{a_1,b_1}(2U)^{a_1}(2V)^{b_1}U^{\frac{\Delta_1+\Delta_2}{2}}\phi_{\Delta_1+a_1,\Delta_2+a_1+b_1,\Delta_3+b_1,\Delta_4},$$

$$\mathcal{M}^{ex}_{\Delta_E,\ell_E}(s,t)=\sum_{m=0}^{\infty}\frac{f_{m,\ell_E}Q_{\ell_E}(t,u)}{s-\tau_E-2m},$$

$$\mathcal{M}^{ex}_{\Delta_E,\ell_E}(s,t)=\sum_{m=0}^{m_0}\frac{f_{m,\ell_E}Q_{\ell_E}(t,u)}{s-(\Delta_1+\Delta_2)+2k_m},$$

$$\frac{1}{s-(\Delta_1+\Delta_2)+2k_m}\Gamma\left(\frac{\Delta_1+\Delta_2-s}{2}\right)=-\frac{1}{2}\Gamma\left(\frac{\Delta_1+\Delta_2-s}{2}-k_m\right)\prod_{n=1}^{k_m-1}\left[\frac{\Delta_1+\Delta_2-s}{2}-n\right]$$

$$\mathcal{G}^{ex}_{\Delta_E,\ell_E}(U,V)=-\sum_{m=0}^{m_0}\frac{f_{m,\ell_E}}{2}\hat{Q}_{\ell_E}\prod_{n=1}^{k_m-1}\left[\frac{\Delta_1+\Delta_2}{2}-U\partial_U-n\right]\left(U^{\frac{\Delta_1+\Delta_2}{2}-k_m}\bar{D}_{\Delta_1-k_m,\Delta_2-k_m,\Delta_3,\Delta_4}\right)$$

$$t\to\mathcal{D}_t=2V\partial_V+\Delta_2+\Delta_3, u\to\mathcal{D}_u=\Delta_1+\Delta_4-2U\partial_U-2V\partial_V$$

$$\mathcal{Q}_{\ell_E}(s,t)=\tilde{\mathcal{Q}}+\sum_{a+b=\ell_E}\chi_{a,b}t^a u^b$$

$$\mathcal{G}^{ex,l.s}_{\Delta_E,\ell_E}(U,V)=-\frac{1}{2}\left[\sum_{m=0}^{m_0}f_{m,\ell_E}\right]\sum_{a+b=\ell_E}(-1)^a\chi_{a,b}2^{a+b}V^aU^{\frac{\Delta_1+\Delta_2}{2}-1}\Phi^{l.s}_{\Delta_1-1,\Delta_2-1+a+b,\Delta_3+a,\Delta_4+b}$$

$$\mathcal{M}^{ex}_{\Delta_E,\ell_E}(s,t)\overset{s,t\to\infty}{\to}\frac{\sum_{a+b=\ell_E}\chi_{a,b}t^a u^b}{s}\sum_{m=0}^{\infty}f_{m,\ell_E}$$

$$\frac{1}{s}\Gamma\left(\frac{\Delta_1+\Delta_2-s}{2}\right)=-\frac{1}{2}\Gamma\left(-\frac{s}{2}\right)\prod_{n=1}^{k-1}\left[\frac{k-s}{2}-n\right]$$

$$\mathcal{G}^{ex,l.s}_{\Delta_E,\ell_E}(U,V)=-\frac{1}{2}\left[\sum_{m=0}^{\infty}f_{m,\ell_E}\right]\sum_{a+b=\ell_E}(-1)^a\chi_{a,b}2^{a+b}V^aU^{\frac{\Delta_1+\Delta_2}{2}-1}\Phi^{l.s}_{\Delta_1-1,\Delta_2-1+a+b,\Delta_3+a,\Delta_4+b}$$

$$\mathcal{A}_2=(e_1\cdot e_2)^2 2P_1^0(2\pi)^{10}\delta^{(10)}(P_1+P_2)$$

$$\mathcal{A}_2=t_{12}^2 2P_1^0V_7(2\pi)^3\delta^{(3)}(P_1+P_2)$$

$$\mathcal{A}_3=(e_1\cdot e_2e_3\cdot P_1+\text{cyclic})^2\delta^{(11)}\left(\sum_i P_i\right)$$

$$=(e_1\cdot e_2e_3\cdot P_1+e_2\cdot e_3e_1\cdot P_2-e_3\cdot e_1e_2\cdot P_1)^2\delta^{(11)}\left(\sum_i P_i\right),$$

$$\mathcal{A}_3=[(e_1\cdot e_2e_3\cdot P_1)^2+(e_2\cdot e_3e_1\cdot P_2)^2+(e_3\cdot e_1e_2\cdot P_1)^2+2e_1\cdot e_2e_3\cdot P_1e_2\cdot e_3e_1\cdot P_2$$
$$-2e_2\cdot e_3e_1\cdot P_2e_3\cdot e_1e_2\cdot P_1-2e_1\cdot e_2e_3\cdot P_1e_3\cdot e_1e_2\cdot P_1]\delta^{(11)}\left(\sum_i P_i\right)$$

$$P_i^AP_j^B\to\frac{1}{\ell^2}\int d^6\hat{p}_i^A d^6\hat{p}_j^B=\frac{1}{\ell^2}\delta_{ij}\delta^{AB}$$

$$\mathcal{A}_3 \to -\frac{2V_7}{\ell^2} e_1 \cdot e_2 e_2 \cdot e_3 e_3 \cdot e_1 \delta^{(4)}\left(\sum_i P_i\right) = -\frac{2V_7}{\ell^2} t_{12} t_{23} t_{13} \delta^{(4)}\left(\sum_i P_i\right)$$

$$\mathcal{M}_{2,2,2,2} = \frac{1}{c_T} \mathcal{M}^R_{2,2,2,2} + \frac{1}{c_T^{\frac{5}{3}}} B^{R^4} \mathcal{M}^{(4)}_{2,2,2,2}$$
$$+ \frac{1}{c_T^{\frac{7}{3}}} \left(B_4^{D^6 R^4} \mathcal{M}^{(4)}_{2,2,2,2} + B_6^{D^6 R^4} \mathcal{M}^{(6)}_{2,2,2,2} + B_7^{D^6 R^4} \mathcal{M}^{(7)}_{2,2,2,2} \right) + \cdots$$

$$B_4^{R^4} = 1120 \left(\frac{2}{9\pi^8 k_{CS}^2} \right)^{\frac{1}{3}}, B_4^{D^6 R^4} = -\frac{1352960}{9} \left(\frac{36}{\pi^{10} k_{CS}^4} \right)^{\frac{1}{3}}$$
$$B_6^{D^6 R^4} = -220528 \left(\frac{36}{\pi^{10} k_{CS}^4} \right)^{\frac{1}{3}}, B_7^{D^6 R^4} = 16016 \left(\frac{36}{\pi^{10} k_{CS}^4} \right)^{\frac{1}{3}}$$

$$\mathcal{M}^{(4),HE}_{2,2,2,2} = (s + t - s\alpha)^2 (s + t - s\bar{\alpha})^2$$
$$\mathcal{M}^{(6),HE}_{2,2,2,2} = 2(s^2 + t^2 + st)(s + t - s\alpha)^2 (s + t - s\bar{\alpha})^2$$
$$\mathcal{M}^{(7),HE}_{2,2,2,2} = stu(s + t - s\alpha)^2 (s + t - s\bar{\alpha})^2$$

$$\lim_{\ell \to \infty} \frac{V_7}{(2\pi)^4} \langle \mathcal{O}_2 \dots \mathcal{O}_2 \rangle = \left[1 + f_1 \left(\ell_{11} \partial_{u_4} \right)^6 + f_2 \left(\ell_{11} \partial_{u_4} \right)^{10} + f_3 \left(\ell_{11} \partial_{u_4} \right)^{11} \right] \tilde{\mathcal{C}}_4$$

$$f_3 = \mathcal{N}_3 f_1^2, f_2 = \mathcal{N}_2 \left| \frac{z_{24}}{z_{12}} \right|^4 \left((1 - z)^2 \left| \frac{z_{34}}{z_{23}} \right|^4 - z(1 - z) \left| \frac{z_{34}}{z_{23}} \right|^2 + z^2 \right) f_1$$
$$f_1 = \mathcal{N}_1 z^2 |z_{24}|^2 |z_{34}|^2 |z_{14}|^4$$

La supersimetría extendida, en un campo cuántico relativista bajo supergravedad, se calcula así:

$$Z_{\text{SUGRA}} = \sum_{m \in \{ \text{ Configurations } \}} \prod_{i \in \{ \text{ Spectrum } \}} Z_{\text{i-th field in m-th conf.}}$$

$$h \geq j$$

$$\chi^{\text{g}}(h, j) = \text{Tr}\left((-1)^F q^{L_0} z^{J_0^3} \right)$$

$$\chi^{\text{g}}(j) = q^j \chi_j(z) - 2q^{j+\frac{1}{2}} \chi_{j-\frac{1}{2}}(z) + q^{j+1} \chi_{j-1}(z)$$

$$\chi_j(z) = \frac{z^{j+1/2} - z^{-j-1/2}}{z^{1/2} - z^{-1/2}}, j \geq 0$$

$$\chi^{\text{g}}(0) = q^0 \chi_0(z) = 1, \chi^{\text{g}}(1/2) = q^{1/2} \chi_{1/2}(z) + 2q^1 \chi_0(z)$$
$$\chi^{\text{g}}(1) = q^1 \chi_1(z) - 2q^{3/2} \chi_{1/2}(z) + q^2 \chi_0(z)$$

$$\boldsymbol{\xi}^{(2)}(j) = \chi^{\mathrm{g}}(j+1)\bar{\chi}^{\mathrm{g}}(j), \quad \bar{\boldsymbol{\xi}}^{(2)}(j) = \chi^{\mathrm{g}}(j)\bar{\chi}^{\mathrm{g}}(j+1)$$
$$\boldsymbol{\xi}^{\left(\frac{3}{2}\right)}(j) = \chi^{\mathrm{g}}\left(j+\frac{1}{2}\right)\bar{\chi}^{\mathrm{g}}(j), \quad \bar{\boldsymbol{\xi}}^{\left(\frac{3}{2}\right)}(j) = \chi^{\mathrm{g}}(j)\bar{\chi}^{\mathrm{g}}\left(j+\frac{1}{2}\right)$$
$$\boldsymbol{\xi}^{(1)}(j) = \chi^{\mathrm{g}}(j)\bar{\chi}^{\mathrm{g}}(j)$$

$$\boldsymbol{\xi}^{(2)}(0) + \sum_{j\geq\frac{1}{2}}\left[\boldsymbol{\xi}^{(2)}(j) + \mathbf{21}_{21}\boldsymbol{\xi}^{(1)}(j) + \boldsymbol{\xi}^{(1)}\left(j+\frac{1}{2}\right)\right] + \text{cc.}$$

$$\boldsymbol{\xi}^{(2)}(0) = \left[q^1\chi_1(z) - 2q^{3/2}\chi_{1/2}(z) + q^2\chi_0(z)\right] \times \left[(\bar{q}^0\chi_0(\bar{z}))\right]$$

$$1 \times (1,0,1,0) + 2 \times \left(\frac{3}{2}, 0, \frac{1}{2}, 0\right) + 1 \times (2,0,0,0)$$

$$ds^2_{6\mathrm{D}} = d\rho^2 + \cosh^2\,\rho dt^2_{\mathrm{E}} + \sinh^2\,\rho d\varphi^2 + d\theta^2 + \sin^2\,\theta d\phi_1^2 + \cos^2\,\theta d\phi_2^2$$

$$(t_{\mathrm{E}}, \varphi, \phi_1, \phi_2) \sim (t_{\mathrm{E}} + \beta, \varphi + i\beta\mu, \phi_1 + i\beta\nu_1, \phi_2 + i\beta\nu_2)$$

$$\tau = \frac{\beta\mu + i\beta}{2\pi}, q \equiv e^{2\pi i\tau}$$

$$\nu = \beta\nu_2 - \beta\nu_1, z = e^{2\pi i\nu}$$

$$K^{(s)}(x,y;t) = \sum_n \Psi_n^{(s)}(x)\Psi_n^{*(s)}(y)e^{t\lambda_n^{(s)}}$$

$$K_{G/\Gamma}(x,y;t) = \sum_{\gamma\in\Gamma} K_G(x,\gamma(y);t)$$

$$-\frac{1}{2}\ln\,\det\left(-\Delta_{(s)} + M^2\right) = \frac{1}{2}\int_0^\infty \frac{dt}{t}\int_{G/\Gamma} d\mu(x)K^{(s)}(x,x;t)e^{-M^2t}$$

$$\int_{\mathbf{S}^3/\mathbb{Z}_n} d\mu(x)K_{\mathbf{S}^3/\mathbb{Z}_n}(x,x;t) = \sum_{m\in\mathbb{Z}_n}\sum_{\ell,\bar{\ell}} \chi_\ell(z^m)\chi_{\bar{\ell}}(\bar{z}^m)e^{-C(\ell,\bar{\ell})t}$$

$$\frac{\tau_2}{2\pi}\sum_{m\in\mathbb{Z}}\int_0^\infty d\lambda\left[\hat{\chi}_{(j_1,j_2)}\left(e^{i\pi m\tau}\right) + \hat{\chi}_{(j_2,j_1)}\left(e^{i\pi m\tau}\right)\right]e^{-(\lambda^2+s+1)t}$$

$$\hat{\chi}_{j_1,j_2}(\alpha) = \frac{\alpha^{2j_1+1}\bar{\alpha}^{2j_2+1} + \alpha^{-2j_1-1}\bar{\alpha}^{-2j_2-1}}{|\alpha - \alpha^{-1}|^2}$$

$$\frac{\tau_2}{2\pi}\sum_{m\in\mathbb{Z}}\int_0^\infty d\lambda\left[\hat{\chi}_{(j_1,j_2)}\left(e^{i\pi m\tau}\right)\chi_\ell(z^m)\chi_{\bar{\ell}}(\bar{z}^m) + \hat{\chi}_{(j_2,j_1)}\left(e^{i\pi m\tau}\right)\chi_{\bar{\ell}}(z^m)\chi_\ell(\bar{z}^m)\right]e^{-(\lambda^2+s+1+M^2)t}$$

$$\sum_{m=1}^\infty \frac{\pi\tau_2 e^{2\pi i s m\tau_1}\chi_\ell(z^m)\chi_{\bar{\ell}}(\bar{z}^m)}{\sqrt{4\pi t}|\sin\,\pi m\tau|^2} e^{-\frac{4\pi^2m^2\tau_2^2}{4t}} e^{-(s+1+M^2)t}$$

$$Z_{1-\mathrm{loop}} = \det{}^{-1/2}\left(-\Delta_{(s)} + M^2\right)$$

$$\ln Z_{1-\text{ loop}}^{(3/2)} = \frac{1}{2}\ln \det\left(-\Delta_{(3/2)} + \frac{9}{4}\right) + \frac{1}{2}\ln \det\left(-\Delta_{(1/2)} - \frac{3}{4}\right).$$

$$\frac{1}{4\pi^{1/2}}\int_0^\infty \frac{dt}{t^{3/2}} e^{-\frac{\alpha^2}{4t}-\beta^2 t} = \frac{1}{2\alpha} e^{-\alpha\beta}$$

$$\ln Z_{\text{one helicity}}^{(3/2)} = -\sum_{m=1}^{\infty} \frac{(-1)^m}{m} \frac{q^{\frac{3m}{2}}}{1-q^m}\left(z^{+1/2} + z^{-1/2}\right)$$

$$\ln Z_{\text{one helicity}}^{(3/2)} = -\sum_{n=2}^{\infty} \left[\ln\left(1 - z^{+1/2}q^{n-1/2}\right) + \ln\left(1 - z^{-1/2}q^{n-1/2}\right)\right]$$

$$Z^{(3/2)} = \prod_{n=2}^{\infty} \left|1 - z^{+1/2}q^{n-1/2}\right|^2 \left|1 - z^{-1/2}q^{n-1/2}\right|^2$$

$$Z_1(q,z) = \frac{q^{3/2}}{1-q}\chi_{1/2}(z)$$

$$Z_1(q) \to \sum_{m=1}^{\infty} \frac{1}{m} Z_1(q^m)$$

$$L_0 \to L_0 + 2mJ_0^3 + km^2$$
$$J_n^3 \to J_n^3 + km\delta_{n,0}$$
$$J_n^{\pm} \to J_{n\pm w}^{\pm}$$

$$d\theta^2 + \sin^2\theta(d\Phi_1 + A_1)^2 + \cos^2\theta(d\Phi_2 + A_2)^2$$

$$A = A_1 + A_2, \bar{A} = A_2 - A_1.$$

$$S_{\text{gauge}} = -\frac{ik}{4\pi}\int d^3x \text{Tr}\left[AdA + \frac{2}{3}A^3\right] + S_{\text{bnd}}$$

$$L_0^{\text{gauge}} = \frac{1}{8\pi}\oint dw A_w^2, J_0^3 = ik\oint \frac{dw}{2\pi i} A_w$$

$$d\Phi_1 \to d\Phi_1 + mdw + \bar{m}d\bar{w}, d\Phi_2 \to d\Phi_2 + mdw - \bar{m}d\bar{w}$$

$$A \to A + 2mdw, \bar{A} \to \bar{A} + 2\bar{m}d\bar{w}$$

$$L_0 \to L_0 + 2mJ_0^3 + km^2, J_0^3 \to J_0^3 + km,$$

$$A_{\bar{w}} \propto \frac{\nu}{\tau}, \bar{A}_w \propto \frac{\bar{\nu}}{\tau}$$

$$\exp\left[i\int_\gamma A\right] = -1$$

$$A_w \propto -\frac{\nu}{\tau} + 2m, \bar{A}_w \propto -\frac{\bar{\nu}}{\tau} + 2\bar{m},$$

$$\mathrm{Z}_{(0,0)}^{\text{1-loop}} = \underbrace{\frac{1}{\prod_{n=1}^{\infty}|1-z^{+1}q^{n}|^{2}|1-q^{n}|^{2}|1-z^{-1}q^{n}|^{2}}}_{\text{3 vectors}} \times \underbrace{\prod_{n=2}^{\infty}\left|1-z^{1/2}q^{n-1/2}\right|^{4}\left|1-z^{-1/2}q^{n-1/2}\right|^{4}}_{\text{4 gravitini}} \times \underbrace{\frac{1}{\prod_{n=2}^{\infty}|1-q^{n}|^{2}}}_{\text{graviton}}$$

$$S_{\text{on-shell}} = -\frac{i\pi k\tau}{2} + \frac{i\pi\bar{k}\bar{\tau}}{2} - \pi\tau_{2}(A_{\bar{w}}^{2} + \bar{A}_{\bar{w}}^{2})$$

$$\mathrm{Z}_{\text{SUGRA}}^{\text{1- loop}} = \sum_{m,\bar{m}=0}^{\infty} q^{km^{2}+k\bar{m}^{2}} z^{km+k\bar{m}} \times \frac{\prod_{n=2}^{\infty}\left|1-z^{1/2}q^{m+n-1/2}\right|^{4}\left|1-z^{-1/2}q^{-m+n-1/2}\right|^{4}}{\prod_{n=1}^{\infty}|1-zq^{2m+n}|^{2}|1-q^{n}|^{2}|1-z^{-1}q^{-2m+n}|^{2}\prod_{n=2}^{\infty}|1-q^{n}|^{2}}$$

$$\chi(0)|_{m=0} = \prod_{n=1}^{\infty}\frac{\left(1-z^{+1/2}q^{n-1/2}\right)^{2}\left(1-z^{-1/2}q^{n-1/2}\right)^{2}}{(1-zq^{n})(1-q^{n})^{2}(1-z^{-1}q^{n-1})}\left[\frac{1}{(1-z^{+1/2}q^{-1/2})^{2}} - \frac{z^{-1}}{(1-z^{-1/2}q^{-1/2})^{2}}\right]$$

$$\frac{(1-z^{-1})(1-q)}{(1-z^{+1/2}q^{1/2})^{2}(1-z^{-1/2}q^{1/2})^{2}},$$

$$\mathfrak{f}(q,z) \to \sum_{m\in\mathbb{Z}} q^{km^{2}}z^{km}\mathfrak{f}(q,zq^{2m})$$

$$\chi(0)|_{m=0} = q^{\frac{1}{4}}\frac{\vartheta_{4}^{2}\left(z^{1/2},q\right)}{\eta(q)^{3}\vartheta_{1}(z,q)}[\mu(q,z)-\mu(q,z^{-1})]_{m=0}.$$

$$\frac{\vartheta_{4}^{2}\left(z^{1/2},q\right)}{\eta(q)^{3}\vartheta_{1}(z,q)} \to q^{-m^{2}}z^{-m}\frac{\vartheta_{3}^{2}\left(z^{1/2},q\right)}{\eta(q)^{3}\vartheta_{1}(z,q)}.$$

$$[\mu(q,z)-\mu(q,z^{-1})]_{m=0} \to q^{m^{2}}z^{m}[\mu(q,z)-\mu(q,z^{-1})]_{m}.$$

$$\chi(0) = q^{\frac{1}{4}}\frac{\vartheta_{4}^{2}\left(z^{1/2},q\right)}{\eta(q)^{3}\vartheta_{1}(z,q)}[\mu(q,z)-\mu(q,z^{-1})]$$

$$\mathrm{Z}_{\text{SUGRA}}^{\text{1- loop}} = \chi(0)\bar{\chi}(0),$$

$$\boldsymbol{\xi}^{(2)}(0) + 2\boldsymbol{\xi}^{\left(\frac{3}{2}\right)}(0) + \sum_{j\geq\frac{1}{2}}\left[\boldsymbol{\xi}^{(2)}(j) + \mathbf{4}_{5}\boldsymbol{\xi}^{\left(\frac{3}{2}\right)}(j) + \mathbf{5}_{5}\boldsymbol{\xi}^{(1)}(j) + \boldsymbol{\xi}^{(1)}\left(j+\frac{1}{2}\right)\right] + \text{cc}$$

$$\boldsymbol{\xi}^{\left(\frac{3}{2}\right)}(0) = q^{1/2}\chi_{1/2}(z) - 2q$$

$$1\times\left(\frac{1}{2},0,\frac{1}{2},0\right) + 2\times(1,0,0,0)$$

$$\underbrace{\prod_{n=1}^{\infty}\left|1-z^{+1/2}q^{n-1/2}\right|^{4}\left|1-z^{-1/2}q^{n-1/2}\right|^{4}}_{4\text{ spin-}\frac{1}{2}\text{ fermions}}\times\underbrace{\frac{1}{\prod_{n=1}^{\infty}|1-q^{n}|^{8}}}_{4\text{ vectors}}$$

$$\prod_{n=1}^{\infty}\frac{\left(1-z^{+1/2}q^{n-1/2}\right)^{2}\left(1-z^{-1/2}q^{n-1/2}\right)^{2}}{(1-q^{n})^{4}}$$

$$h\geq\left(\frac{1}{1+\alpha}j_{-}+\frac{\alpha}{1+\alpha}j_{+}\right)$$

$$h\geq\frac{1}{k_{+}+k_{-}}[k_{+}\ell_{-}+k_{-}\ell_{+}+(\ell_{+}-\ell_{-})^{2}+u^{2}]$$

$$\chi^{\mathrm{g}}(h,j_{+},j_{-})=\mathrm{Tr}\left((-1)^{F}q^{L_{0}}z_{+}^{J_{0}^{+,3}}z_{-}^{J_{0}^{-,3}}\right)$$

$$\begin{aligned}\chi_{\mathrm{S}}^{\mathrm{g}}(\ell_{+},\ell_{-})=\;&q^{h}\chi_{\ell_{+}}(z_{+})\chi_{\ell_{-}}(z_{-})\\&+q^{h+\frac{1}{2}}\left[\chi_{\ell_{+}+\frac{1}{2}}(z_{+})\chi_{\ell_{-}-\frac{1}{2}}(z_{-})+\chi_{\ell_{+}-\frac{1}{2}}(z_{+})\chi_{\ell_{-}+\frac{1}{2}}(z_{-})+\chi_{\ell_{+}-\frac{1}{2}}(z_{+})\chi_{\ell_{-}-\frac{1}{2}}(z_{-})\right]\\&+q^{h+1}\left[\chi_{\ell_{+}}(z_{+})\chi_{\ell_{-}-1}(z_{-})+\chi_{\ell_{+}-1}(z_{+})\chi_{\ell_{-}}(z_{-})+\chi_{\ell_{+}}(z_{+})\chi_{\ell_{-}}(z_{-})\right]\\&+q^{h+\frac{3}{2}}\chi_{l_{+}-\frac{1}{2}}(z_{+})\chi_{l_{-}-\frac{1}{2}}(z_{-})\end{aligned}$$

$$\chi_{\mathrm{S}}^{\mathrm{g}}\left(\frac{1}{2},\frac{1}{2}\right)=-q^{\frac{1}{2}}\chi_{\frac{1}{2}}(z_{+})\chi_{\frac{1}{2}}(z_{-})+q[1+\chi_{1}(z_{+})+\chi_{2}(z_{-})]-q^{\frac{3}{2}}\chi_{\frac{1}{2}}(z_{+})\chi_{\frac{1}{2}}(z_{-})+q^{2}$$

$$\sum_{j=0}^{\infty}\left|\chi_{\mathrm{S}}^{\mathrm{g}}(j,j)+\chi_{\mathrm{S}}^{\mathrm{g}}\left(j+\frac{1}{2},j+\frac{1}{2}\right)\right|^{2}+\text{ Long}$$

$$\left[-q^{\frac{1}{2}}\chi_{\frac{1}{2}}(z_{+})\chi_{\frac{1}{2}}(z_{-})+q[1+\chi_{1}(z_{+})+\chi_{2}(z_{-})]-q^{\frac{3}{2}}\chi_{\frac{1}{2}}(z_{+})\chi_{\frac{1}{2}}(z_{-})+q^{2}\right]\times[\bar{q}^{0}]$$

$$\left(\frac{1}{2},0,\frac{1}{2},0,\frac{1}{2},0\right)+(1,0,0,0,0,0)$$

$$(1,0,1,0,0,0)+(1,0,0,0,1,0)+\left(\frac{3}{2},0,\frac{1}{2},0,\frac{1}{2},0\right)+(2,0,0,0,0,0)$$

$$ds_{9\mathrm{D}}^{2}=d\rho^{2}+\cosh^{2}\,\rho dt_{\mathrm{E}}^{2}+R_{+}^{2}d\Omega_{+}^{2}+R_{-}^{2}d\Omega_{-}^{2}$$

$$\begin{gathered}\left(t_{\mathrm{E}},\varphi,\phi_{+,1},\phi_{+,2},\phi_{-,1},\phi_{-,2}\right)\sim\\\left(t_{\mathrm{E}}+\beta,\varphi+i\beta\mu,\phi_{+,1}+i\beta\nu_{+,1},\phi_{+,2}+i\beta\nu_{+,2},\phi_{-,1}+i\beta\nu_{-,1},\phi_{-,2}+i\beta\nu_{-,2}\right)\end{gathered}$$

$$\nu_{+}=\beta\nu_{+,2}-\beta\nu_{+,1}\rightarrow z_{+}=e^{2\pi i\nu_{+}},\nu_{-}=\beta\nu_{-,2}-\beta\nu_{-,1}\rightarrow z_{-}=e^{2\pi i\nu_{-}}$$

$$\frac{\tau_{2}}{2\pi}\sum_{m\in\mathbb{Z}}\int_{o}^{\infty}d\lambda\left[\hat{\chi}_{(j_{1},j_{2})}\left(e^{\frac{im\tau}{2}}\right)\chi_{\ell_{+}}(z_{+}^{m})\chi_{\bar{\ell}_{+}}(\bar{z}_{+}^{m})\chi_{\ell_{-}}(z_{-}^{m})\chi_{\bar{\ell}_{-}}(\bar{z}_{-}^{m})\right]e^{-(\lambda^{2}+s+1+M^{2})t}$$

$$\ln Z^{(3/2)} = -\sum_{m=1}^{\infty} \frac{(-1)^m}{m} \frac{q^{\frac{3m}{2}}}{1-q^m} \left(z_+^{1/2} + z_+^{-1/2}\right)\left(z_-^{1/2} + z_-^{-1/2}\right)$$

$$Z^{(3/2)} = \prod_{n=2}^{\infty} \left|1 - z_+^{1/2} z_-^{1/2} q^{n-1/2}\right|^2 \left|1 - z_+^{-1/2} z_-^{1/2} q^{n-1/2}\right|^2$$
$$\times \left|1 - z_+^{1/2} z_-^{-1/2} q^{n-1/2}\right|^2 \left|1 - z_+^{-1/2} z_-^{-1/2} q^{n-1/2}\right|^2$$

$$Z^{(3/2)} = \prod_{n=2}^{\infty} \prod_{\lambda_\pm \in \{\pm\}} \left|1 - z_+^{\lambda_+/2} z_-^{\lambda_-/2} q^{n-1/2}\right|^2$$

$$d\theta_+^2 + \sin^2\theta_+ \left(d\Phi_{+,1} + A_{+,1}\right)^2 + \cos^2\theta_+ \left(d\Phi_{+,2} + A_{+,2}\right)^2$$
$$+ d\theta_-^2 + \sin^2\theta_- \left(d\Phi_{-,1} + A_{-,1}\right)^2 + \cos^2\theta_- \left(d\Phi_{-,2} + A_{-,2}\right)^2$$

$$A_+ = A_{+,1} + A_{+,2}, \quad \bar{A}_+ = A_{+,2} - A_{+,1}$$
$$A_- = A_{-,1} + A_{-,2}, \quad \bar{A}_- = A_{-,2} - A_{-,1}$$

$$A_{+,\bar{w}} \propto \frac{\nu_+}{\tau}, \bar{A}_{+,w} \propto \frac{\bar{\nu}_+}{\tau}, A_{-,\bar{w}} \propto \frac{\nu_-}{\tau}, \bar{A}_{-,w} \propto \frac{\bar{\nu}_-}{\tau}$$

$$A_{+,w} \propto \frac{\nu_+}{\tau} + 2m_+, \bar{A}_{+,\bar{w}} \propto \frac{\bar{\nu}_+}{\tau} + 2\bar{m}_+, A_{-,w} \propto \frac{\nu_-}{\tau} + 2m_-, \bar{A}_{-,\bar{w}} \propto \frac{\bar{\nu}_-}{\tau} + 2\bar{m}_-,$$

$$L_0 \to L_0 + 2m_+ J_0^{+,3} + k_+ m_+^2 + 2m_- J_0^{-,3} + k_- m_-^2$$
$$J_n^{+,3} \to J_n^{+,3} + k_+ m_+ \delta_{n,0}$$
$$J_n^{-,3} \to J_n^{-,3} + k_- m_- \delta_{n,0}$$
$$J_n^{\pm} \to J_{n\pm m_\pm}^{\pm}$$

$$\mathrm{Z}_{(0,0)}^{\text{1-loop}} = \underbrace{\prod_{n=1}^{\infty} \prod_{\tilde{\lambda}_\pm \in \{\pm\}} \left|1 - z_+^{\tilde{\lambda}_+/2} z_-^{\tilde{\lambda}_-/2} q^{n-1/2}\right|^2}_{\text{4 spin-}\frac{1}{2}\text{ fermions}} \times \underbrace{\frac{1}{\prod_{n=1}^{\infty} |1-q^n|^2}}_{\text{u(1) vector}}$$

$$\mathrm{Z}_{(0,0)}^{\text{1- loop}} = \underbrace{\frac{1}{\prod_{n=1}^{\infty} \prod_{\mu_+ \in \{0,\pm\}} \left|1 - z_+^{\mu_+} q^n\right|^2}}_{3\mathfrak{su}(2)_+\text{vectors}} \times \underbrace{\frac{1}{\prod_{n=1}^{\infty} \prod_{\mu_- \in \{0,\pm\}} \left|1 - z_-^{\mu_-} q^n\right|^2}}_{3\mathfrak{su}(2)_-\text{vectors}}$$
$$\times \underbrace{\prod_{n=2}^{\infty} \prod_{\lambda_\pm \in \{\pm\}} \left|1 - z_+^{\lambda_+/2} z_-^{\lambda_-/2} q^{n-\frac{1}{2}}\right|^2}_{\text{4 gravitini}} \times \underbrace{\frac{1}{\prod_{n=2}^{\infty} |1-q^n|^2}}_{\text{graviton}}$$

$$\mathrm{Z}_{\text{SUGRA}}^{(1\text{ loop})} = \sum_{m_\pm, \bar{m}_\pm = 0}^{\infty} q^{k_+ m_+^2 + k_+ \bar{m}_+^2 + k_- m_-^2 + k_- \bar{m}_-^2} z^{k_+ m + k_+ \bar{m}_+ + k_- m_- + k_- \bar{m}_-}$$
$$\times \frac{\prod_{n=2}^{\infty} \prod_{\lambda_\pm \in \{\pm\}} \left|1 - z_+^{\lambda_+/2} z_-^{\lambda_-/2} q^{m_+ + m_- - 1/2}\right|^2 \prod_{n=1}^{\infty} \prod_{\tilde{\lambda}_\pm \in \{\pm\}} \left|1 - z_+^{\tilde{\lambda}_+/2} z_-^{\tilde{\lambda}_-/2} q^{m_+ + m_- - 1/2}\right|^2}{\prod_{n=1}^{\infty} \prod_{\mu_\pm \in \{0,\pm\}} \left|1 - z_\pm^{\mu_\pm} q^{2m_\pm + n}\right|^2 \prod_{n=2}^{\infty} |1-q^n|^2 \prod_{n=1}^{\infty} |1-q^n|^2}$$

$$\prod_{n=1}^{\infty}\frac{\left(1-z_+^{1/2}z_-^{1/2}q^{n-1/2}\right)\left(1-z_+^{-1/2}z_-^{1/2}q^{n-1/2}\right)\left(1-z_+^{1/2}z_-^{-1/2}q^{n-1/2}\right)\left(1-z_+^{-1/2}z_-^{-1/2}q^{n-1/2}\right)}{(1-z_+q^n)(1-z_+^{-1}q^n)(1-z_-q^n)(1-z_-^{-1}q^n)(1-q^n)^3\left(z_+^{1/2}-z_+^{-1/2}\right)\left(z_-^{1/2}-z_-^{-1/2}\right)}$$
$$\times\left[\frac{z_+^{1/2}z_-^{1/2}}{1+z_+^{1/2}z_-^{1/2}q^{1/2}}-\frac{z_+^{-1/2}z_-^{1/2}}{1+z_+^{-1/2}z_-^{1/2}q^{1/2}}-\frac{z_+^{1/2}z_-^{-1/2}}{1+z_+^{1/2}z_-^{-1/2}q^{1/2}}+\frac{z_+^{-1/2}z_-^{-1/2}}{1+z_+^{-1/2}z_-^{-1/2}q^{1/2}}\right]$$

$$\frac{(z_+-z_+^{-1})(z_--z_-^{-1})(1-q)}{(1+z_+z_-q^{1/2})(1+z_+^{-1}z_-q^{1/2})(1+z_+z_-^{-1}q^{1/2})(1+z_+^{-1}z_-^{-1}q^{1/2})},$$

$$\chi(0)|_{m_\pm=0}=q^{\frac{5}{8}}\frac{\vartheta_4^2\left(z_+^{1/2}z_-^{1/2},q\right)\vartheta_4^2\left(z_+^{1/2}z_-^{-1/2},q\right)}{\eta(q)^3\vartheta_1(z_+,q)\vartheta_1(z_-,q)}\sum_{\epsilon_\pm=\pm1}\epsilon_+\epsilon_-\mu\left(q,z_+^{\epsilon_+},z_-^{\epsilon_-}\right)\Big|_{m_\pm=0}$$

$$\mathfrak{f}(q,z)\to\sum_{m_\pm\in\mathbb{Z}}q^{k_+m_+^2+k_-m_-^2}z^{k_+m_++k_-m_-}\mathfrak{f}(q,z_+q^{2m_+},z_-q^{2m_-})$$

$$\mu(q,z_+,z_-)|_{m_\pm=0}\to\mu(q,z_+,z_-)\big|_{m_\pm}$$

$$\frac{\vartheta_4^2\left(z_+^{1/2}z_-^{1/2},\tau\right)\vartheta_4^2\left(z_+^{1/2}z_-^{-1/2},\tau\right)}{\eta(\tau)^3\vartheta_1(z_+,\tau)\vartheta_1(z_-,\tau)}\to\frac{\vartheta_4^2\left(z_+^{1/2}z_-^{1/2},\tau\right)\vartheta_4^2\left(z_+^{1/2}z_-^{-1/2},\tau\right)}{\eta(\tau)^3\vartheta_1(z_+,\tau)\vartheta_1(z_-,\tau)}$$

$$\mathrm{Z_{BTZ\ in\ R-R}}(\tau,\nu,\bar\tau,\bar\nu)=\mathrm{Z_{AdS}\ in\ NS\text{-}NS}\left(-\frac{1}{\tau},\nu\tau,-\frac{1}{\bar\tau},\bar\nu\bar\tau\right)$$

$$[L_m,L_n]=(m-n)L_{m+n}+\frac{1}{2}km(m^2-1)\delta_{m+n,0}$$
$$\left\{G_r^a,G_s^{\dot a}\right\}=2\delta^{a\dot a}L_{r+s}-2(r-s)\left(\sigma^i\right)^{a\dot a}J_{r+s}^i+\frac{1}{2}k(4r^2-1)\delta_{r+s,0}\delta^{a\dot a}$$
$$\left[J_m^i,J_n^j\right]=i\epsilon^{ijk}J_{m+n}^k+\frac{1}{2}km\delta_{m+n,0}\delta^{ij}$$
$$[L_m,G_r^a]=\left(\frac{1}{2}m-r\right)G_{m+r}^a$$
$$\left[L_m,J_n^i\right]=-nJ_{m+n}^i$$
$$\left[J_m^i,G_r^a\right]=-\frac{1}{2}\left(\sigma^i\right)^a{}_bG_{m+r}^b$$

$$L_0|h,\ell\rangle=h|h,\ell\rangle,J_0^3|h,\ell\rangle=j|h,\ell\rangle,J_0^+|h,\ell\rangle=0$$
$$J_n^i|h,\ell\rangle=G_r^a|h,\ell\rangle=G_r^{\dot a}|h,\ell\rangle=L_n|h,\ell\rangle=0,n,r>0$$

$$h\geq\ell$$

$$\boldsymbol{\chi}(h,\ell;\delta)=\mathrm{Tr_{NS}}\left(e^{i\pi\delta F}q^{L_0}z^{J_0^3}\right)$$

$$\chi(\ell;\delta)=q^\ell\frac{\prod_{n=1}\left(1+e^{i\pi\delta}z^{1/2}q^{n-1/2}\right)^2\left(1+e^{i\pi\delta}z^{-1/2}q^{n-1/2}\right)^2}{\prod_{n=1}(1-q^n)^2(1-z^{+1}q^n)(1-z^{-1}q^{n-1})}z^{-1/2}\left(\mu(q,z;\delta)-\mu(q,z^{-1};\delta)\right),$$

$$\mu(q,z;\delta)=\sum_{m\in\mathbb{Z}}\frac{z^{(k+1)m+\ell+1/2}q^{(k+1)m^2+(2\ell+1)m}}{(1+e^{i\pi\delta}z^{1/2}q^{m+1/2})^2}$$

$$\chi(\ell;\delta) = -iq^{\ell+1/4}\frac{\vartheta_{3+\delta}^2\left(z^{1/2},q\right)}{\eta(q)^3\vartheta_1(z,q)}\left(\mu(q,z;\delta)-\mu(q,z^{-1};\delta)\right),$$

$$\eta(q) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$

$$\vartheta_1(q,z) = -iq^{1/8}\left(z^{1/2}-z^{-1/2}\right)\prod_{n=1}^{\infty}(1-q^n)(1-zq^m)(1-z^{-1}q^m)$$

$$\vartheta_2(q,z) = q^{1/8}\left(z^{1/2}+z^{-1/2}\right)\prod_{n=1}^{\infty}(1-q^n)(1+zq^m)(1+z^{-1}q^m)$$

$$\vartheta_3(q,z) = \prod_{n=1}^{\infty}(1-q^n)\left(1+zq^{m-1/2}\right)\left(1+z^{-1}q^{m-1/2}\right)$$

$$\vartheta_4(q,z) = \prod_{n=1}^{\infty}(1-q^n)\left(1-zq^{m-1/2}\right)\left(1-z^{-1}q^{m-1/2}\right)$$

$$[L_m,L_n] = (m-n)L_{m+n}+\frac{c}{12}m(m^2-1)\delta_{m+n,0},$$

$$\{G_r^a,G_s^b\} = 2\delta_{ab}L_{r+s}+4(r-s)i\left(\gamma\left(\alpha^{+,i}\right)^{ab}J_{r+s}^{+,i}+(1-\gamma)\left(\alpha^{-,i}\right)^{ab}J_{r+s}^{-,i}\right)+\frac{c}{12}(4r^2-1)\delta_{r+s,0}\delta^{ab},$$

$$\left[J_m^{\pm,i},J_n^{\pm,j}\right] = i\epsilon^{ijk}J_{m+n}^{\pm,k}+\frac{1}{2}k^{\pm}m\delta_{m+n,0}\delta^{ij},$$

$$\left[J_m^{\pm,i},G_r^a\right] = i\left(\alpha^{\pm,i}\right)^{ab}G_{m+r}^b\mp\frac{2k^{\pm}}{k_++k_-}m\left(\alpha^{\pm,i}\right)^{ab}Q_{m+r}^b,$$

$$\{Q_r^a,Q_s^b\} = \frac{k_++k_-}{2}\delta^{ab}\delta_{r+s,0}$$

$$[U_m,U_n] = \frac{k_++k_-}{2}m\delta_{m+n,0}$$

$$\{Q_r^a,G_s^b\} = 2\alpha_{ab}^{+i}J_{r+s}^{+,i}-2\alpha_{ab}^{-i}J_{r+s}^{-,i}+\delta^{ab}U_{r+s}$$

$$\left[J_m^{\pm,i},Q_r^a\right] = i\alpha_{ab}^{\pm i}Q_{m+r}^b$$

$$[U_m,G_r^a] = mQ_{m+r}^a$$

$$\left(\alpha^{\pm,i}\right)^{ab} = \frac{1}{2}(\pm\delta_{ia}\delta_{b0}\mp\delta_{ib}\delta_{a0}+\epsilon_{iab}).$$

$$c = \frac{6k_+k_-}{k_++k_-},\gamma = \frac{k_-}{k_++k_-}\langle L_0,L_{\pm1},J_0^{\pm,i},G_{\pm\frac{1}{2}}^a\rangle$$

$$\mathcal{A}_\gamma = \tilde{\mathcal{A}}_\gamma\oplus\mathcal{A}_{\mathrm{QU}}$$

$$L_0|h,\ell_+,\ell_-\rangle = h|h,\ell_+,\ell_-\rangle, J_0^{\pm,3}|h,\ell_+,\ell_-\rangle = \ell_\pm|h,\ell_+,\ell_-\rangle, J_0^{\pm,+}|h,\ell_+,\ell_-\rangle = 0,$$
$$J_n^{\pm,i}|h,\ell_+,\ell_-\rangle = G_r^a|h,\ell_+,\ell_-\rangle = L_n|h,\ell_+,\ell_-\rangle = 0, n,r>0.$$

$$h \geq \frac{1}{k_++k_-}[k_+\ell_-+k_-\ell_++(\ell_+-\ell_-)^2]$$

$$\chi(h,\ell_+,\ell_-;\delta)=\mathrm{Tr}_{\mathrm{NS}}\left(e^{i\pi\delta F}q^{L_0}z_+^{J_0^{+,3}}z_-^{J_0^{-,3}}\right)$$

$$\tilde{\chi}(\ell_+,\ell_-;\delta)\ =q^h\frac{F^{NS}(q,z_+,z_-)B^{+-}(q,z_+,z_-)}{\prod_{n=1}^{\infty}\ (1-q^n)\left(z_+^{1/2}-z_+^{-1/2}\right)\left(z_-^{1/2}-z_-^{-1/2}\right)}$$
$$\times\sum_{\epsilon_\pm=\pm1}\epsilon_+\epsilon_-\mu\left(q,z_+^{\epsilon_+},z_-^{\epsilon_-}\right)$$

$$F^{NS}(q,z_+,z_-)=\prod_{n=1}^{\infty}\prod_{\epsilon_\pm\in\pm}\left(1+e^{i\pi\delta}z_+^{\epsilon_+/2}z_-^{\epsilon_-/2}q^{n-1/2}\right)$$

$$B^{+-}(q,z_+,z_-)=\prod_{n=1}^{\infty}\frac{1}{(1-q^n)^2(1-z_+q^n)(1-z_+^{-1}q^n)(1-z_-q^n)(1-z_-^{-1}q^n)},$$

$$\mu(q,z_+,z_-)=\sum_{m_\pm\in\mathbb{Z}}q^{m_+^2k_++m_-^2k_-+m_+(2l_++1)+m_-(2l_-+1)}\frac{z_+^{m_+k_++l_++1/2}z_-^{m_-k_-+l_-+1/2}}{1+e^{i\pi\delta}z_+^{1/2}z_-^{1/2}q^{m_++m_-+1/2}},$$

$$\tilde{\chi}(\ell_+,\ell_-;\delta)=-q^{h+1/2}\frac{\vartheta_{3+\delta}\left(q,z_+^{1/2}z_-^{1/2}\right)\vartheta_{3+\delta}\left(q,z_+^{1/2}z_-^{-1/2}\right)}{\eta(\tau)^3\vartheta_1(q,z_+)\vartheta_1(q,z_-)}\sum_{\epsilon_\pm=\pm}\epsilon_+\epsilon_-\mu\left(q,z_+^{\epsilon_+},z_-^{\epsilon_-}\right).$$

$$\chi_{QU}(\ell_+,\ell_-;\delta)=q^{-1/8}\frac{F^{NS}(q,z_+,z_-)}{\prod_{n=1}^{\infty}\ (1-q^n)},$$

$$\chi(\ell_+,\ell_-;\delta)=-q^{h+1/2}\frac{\vartheta_{3+\delta}\left(q,z_+^{1/2}z_-^{1/2}\right)\vartheta_{3+\delta}\left(q,z_+^{1/2}z_-^{-1/2}\right)}{\eta(\tau)^6\vartheta_1(q,z_+)\vartheta_1(q,z_-)}\sum_{\epsilon_\pm=\pm1}\epsilon_+\epsilon_-\mu\left(q,z_+^{\epsilon_+},z_-^{\epsilon_-}\right).$$

$$-\frac{1}{2}\ln\ \det\left(-\Delta_{(s)}+M^2\right)=$$
$$\sum_{m=1}^{\infty}(-1)^{m(\delta+1)}\frac{q^{sm}\chi_\ell(z^m)\chi_{\bar{\ell}}(\bar{z}^m)+\bar{q}^{sm}\chi_{\bar{\ell}}(z^m)\chi_\ell(\bar{z}^m)}{m|1-q^m|^2}|q|^{m\left(\sqrt{s+1+M^2}-(s-1)\right)}$$

$$Z_{1-\text{ loop}}\ =\det{}^{-1/2}\left(-\Delta_{(s)}+M^2\right)$$

$$M^2=m_p^2+m_s^2$$

$$\text{scalar fields}: h=\frac{1}{2}\left(1+\sqrt{1+m_p^2}\right)$$
$$\text{spinor fields}: h=\frac{1}{2}\left(m_p+\frac{3}{2}\right)$$
$$\text{vector fields}: h=1+\frac{m_p}{2}$$
$$\text{supergraviton}: h=\frac{1}{2}\left(m_p+\frac{5}{2}\right)$$
$$\text{white particle}: h=\frac{1}{2}\left(3+\sqrt{1+m_p^2}\right)$$

$$m_s^2 = \begin{cases} s(s-3) & s = 1,2 \\ -(s+1) & s = \frac{1}{2}, \frac{3}{2} \end{cases}$$

$$-\frac{1}{2}\ln\,\det\left(-\Delta_{(s)} + M^2\right) =$$
$$\sum_{m=1}^{\infty} (-1)^{m(\delta+1)} \frac{q^{sm}\chi_\ell(z^m)\chi_{\bar{\ell}}(\bar{z}^m) + \bar{q}^{sm}\chi_{\bar{\ell}}(z^m)\chi_\ell(\bar{z}^m)}{m|1-q^m|^2} |q|^{2(h-s)}$$

$$Z_{1-\text{ loop}} = \left| \prod_{n,\bar{n},p,\bar{p}=0}^{\infty} \frac{\left(1 + e^{i\pi\delta} z^{\ell+1+p} \bar{z}^{\bar{p}-\bar{\ell}} q^{h+n} \bar{q}^{h-s+\bar{n}}\right)}{\left(1 + e^{i\pi\delta} z^{\ell+1+p} \bar{z}^{\bar{\ell}+1+\bar{p}} q^{h+n} \bar{q}^{h-s+\bar{n}}\right)} \right.$$
$$\left. \frac{\left(1 + e^{i\pi\delta} z^{p-\ell} \bar{z}^{\bar{\ell}+1+\bar{p}} q^{h+n} \bar{q}^{h-s+\bar{n}}\right)}{\left(1 + e^{i\pi\delta} z^{p-\ell} \bar{z}^{\bar{p}-\bar{\ell}} q^{h+n} \bar{q}^{h-s+\bar{n}}\right)} \right|^{2(-1)^F}$$

$$Z_{1-\text{ loop}} = \left[\frac{\det^{\frac{1}{2}}\left(-\Delta_{(s-1)} + s(s-1)\right)^{TT}}{\det^{\frac{1}{2}}\left(-\Delta_{(s)} + s(s-3)\right)^{TT}} \right]^{(-1)^F} , s \geq 1$$

$$Z_{1-\text{loop}} = \left| \prod_{n,p,\bar{p}=0}^{\infty} \frac{\left(1 + e^{i\pi\delta} z^{\ell+1+p} \bar{z}^{\bar{p}-\bar{\ell}} q^{n+s}\right)\left(1 + e^{i\pi\delta} z^{p-\ell} \bar{z}^{\bar{\ell}+1+\bar{p}} q^{n+s}\right)}{\left(1 + e^{i\pi\delta} z^{\ell+1+p} \bar{z}^{\bar{\ell}+1+\bar{p}} q^{n+s}\right)\left(1 + e^{i\pi\delta} z^{p-\ell} \bar{z}^{\bar{p}-\bar{\ell}} q^{n+s}\right)} \right|^{2(-1)^F}$$

$$Z_{1-\text{loop}} = \prod_{n=2}^{\infty} \frac{1}{|1-q^n|^2}$$

$$Z_{1-\text{ loop}} = \prod_{n=2}^{\infty} \left| \left(1 + e^{i\pi\delta} z^{-\frac{1}{2}} q^{n-\frac{1}{2}}\right)\left(1 + e^{i\pi\delta} z^{\frac{1}{2}} q^{n-\frac{1}{2}}\right) \right|^2$$

$$Z_{1-\text{ loop}} = \prod_{n=1}^{\infty} \left| \frac{1}{(1-z^{-1}q^n)(1-q^n)(1-zq^n)} \right|^2$$

Existen diferentes tipos de supergravedad cuántica relativista, según la geometría de la gravedad y las supersimetría de gauge empleada, a más de sus propagadores y osciladores en relación al espacio – tiempo cuántico en dimensiones altas, siendo éstas:

Supergravedad cuántica relativista tipo I: Modelamiento Matemático.

$$S = \int_X \left(\langle y, \bar{D} y \rangle - \frac{1}{3} \langle y, [y,y] \rangle \right) \wedge \Omega$$

$$Q \simeq \Lambda^{1,0} \oplus \text{End}(V) \oplus T^{1,0}$$

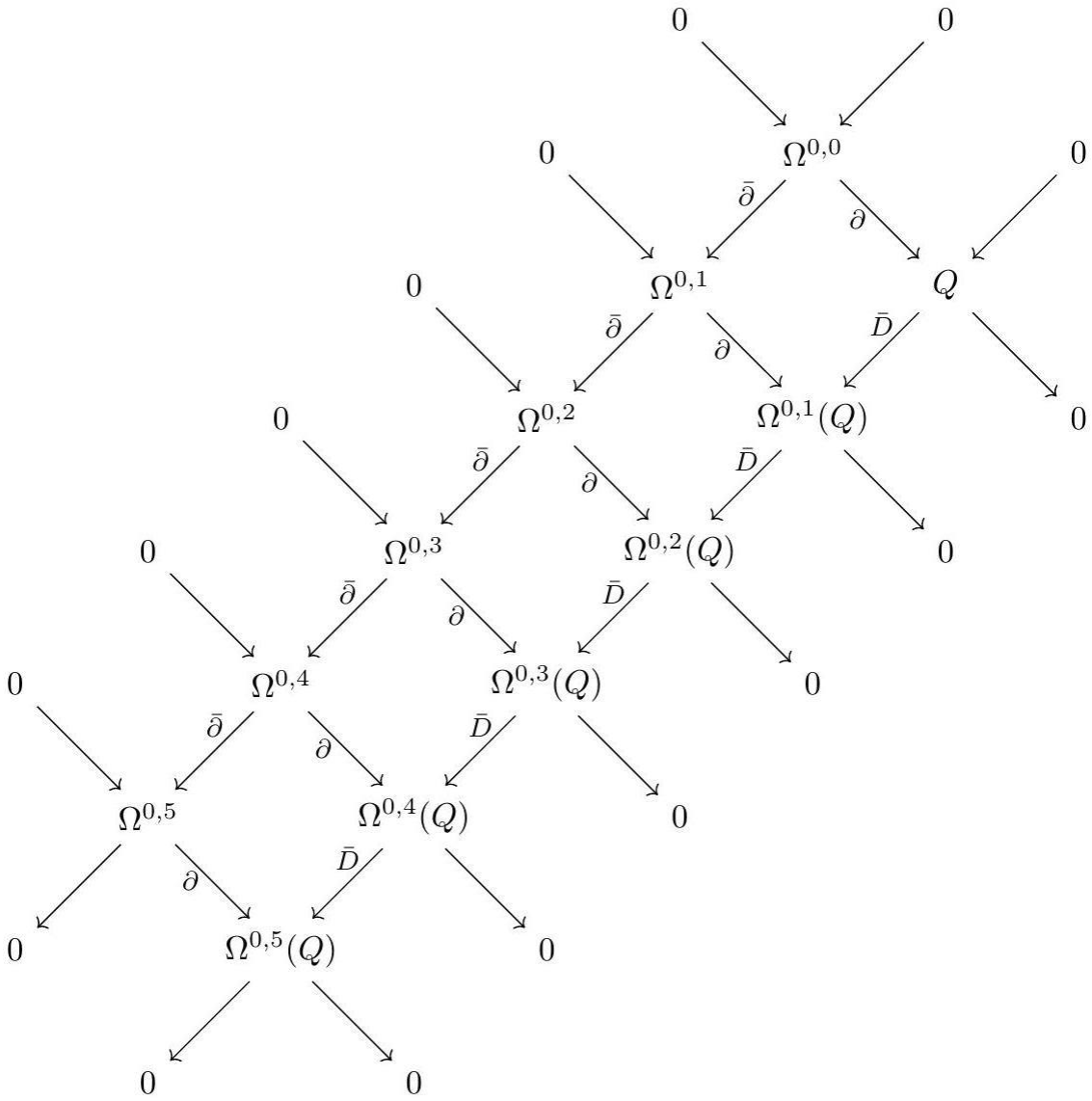
$$\bar{D} = \begin{pmatrix} \bar{\partial} & \alpha' \mathcal{F}^* & \mathcal{T} \\ 0 & \bar{\partial}_{\mathcal{A}} & \mathcal{F} \\ 0 & 0 & \bar{\partial} \end{pmatrix}$$

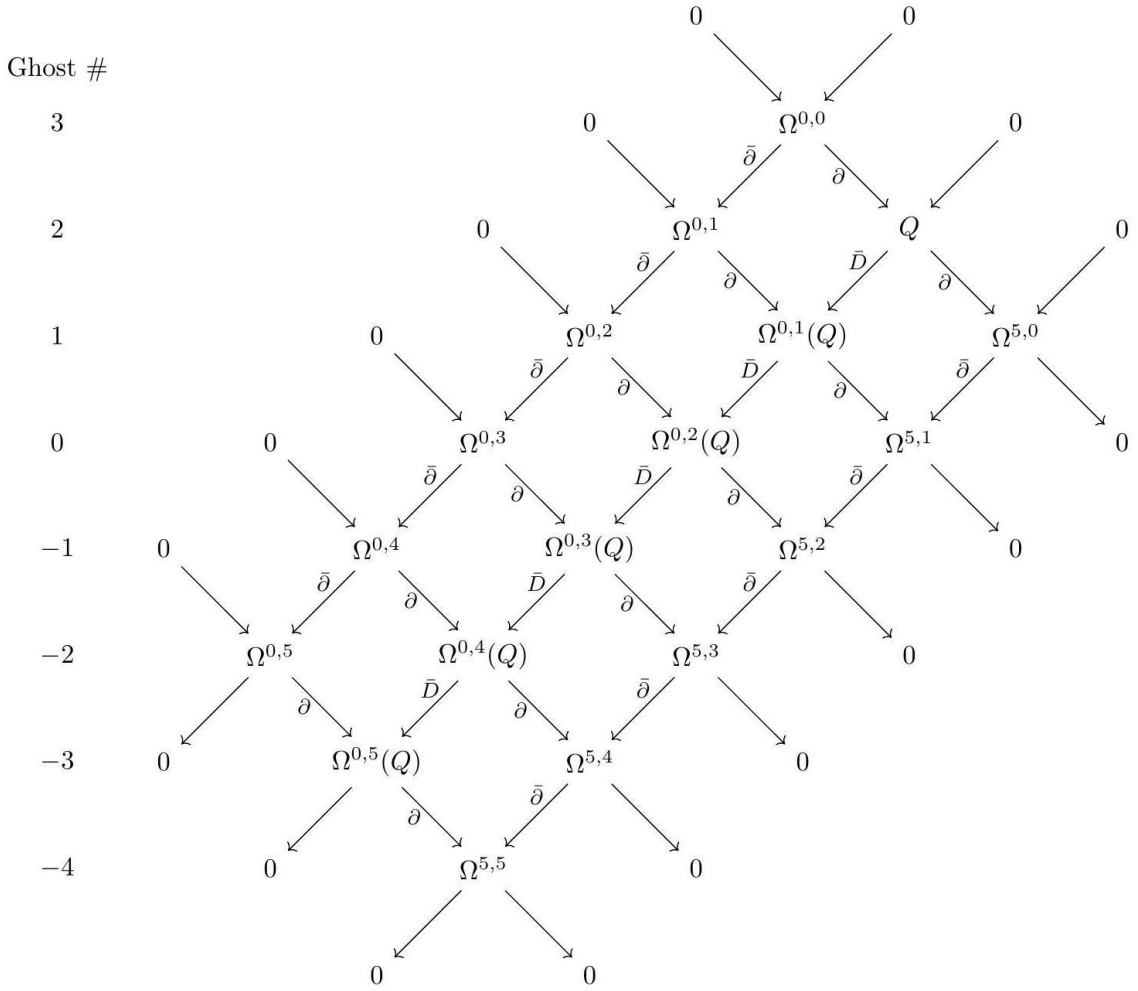
$$\begin{aligned} \mathcal{F}(\mu) &= F_a \wedge \mu^a \\ (\mathcal{F}^*(\alpha))_a &= \mathrm{tr}(F_a \wedge \alpha) \\ (\mathcal{T}(\mu))_a &= -2T_{ab} \wedge \mu^b \end{aligned}$$

$$\mathrm{i}\partial\bar{\partial}\omega = \frac{\alpha'}{4}\mathrm{tr}(F \wedge F),$$

$$\mathrm{i}\partial\bar{\partial}\omega = \frac{\alpha'}{4}(\mathrm{tr}(F \wedge F) - \mathrm{tr}(R \wedge R))$$

$$\omega^4 \wedge F = 0$$





$$Q \simeq \Lambda^{1,0} \oplus \mathrm{End}(V) \oplus T^{1,0} \simeq \Lambda^{1,0} \oplus \mathrm{End}(V) \oplus \Lambda^{4,0}$$

$$\left\langle b^{0,2}, c^{5,3} \right\rangle = \int_X b^{0,2} \wedge c^{5,3}$$

$$y^{0,\cdot} \in \Omega^{0,\cdot}(Q), b^{0,\cdot} \in \Omega^{0,\cdot}, c^{5,\cdot} \in \Omega^{5,\cdot}$$

$$S = \langle \Phi, \mathrm{d}_{\mathrm{total}} \Phi \rangle = \int_X \langle y, \bar{D} y \rangle \wedge \Omega + \int_X b \wedge \bar{\partial} c + \int_X \left(y \lrcorner \partial b \right) \wedge \Omega$$

$$y = (x, \alpha, \mu) \in \Omega^{0,\cdot}(Q) = \Omega^{0,\cdot}(\Lambda^{1,0}) \oplus \Omega^{0,\cdot}(\mathrm{End}(V)) \oplus \Omega^{0,\cdot}(T^{1,0}).$$

$$\langle y_1, y_2 \rangle = (\mu_1)^a \wedge (x_2)_a + (x_1)_a \wedge (\mu_2)^a + \mathrm{tr}(\alpha_1 \wedge \alpha_2)$$

$$\delta y = \bar{D}\gamma + \partial\beta, \delta b = \bar{\partial}\beta, \delta c = \partial(v \lrcorner \Omega) + \bar{\partial}\kappa,$$

$$\delta \Phi = \mathrm{d}_{\mathrm{total}} \Lambda,$$

$$S = \int_X \langle y, \bar{D}y\rangle \wedge \Omega + \int_X b \wedge \bar{\partial} c$$
$$= \int_X (\langle y^{0,2}, \bar{D}y^{0,2}\rangle + \langle y^{0,4}, \bar{D}y^{0,0}\rangle) \wedge \Omega + b^{0,3} \wedge \bar{\partial} c^{5,1} + b^{0,1} \wedge \bar{\partial} c^{5,3} \quad (\mathfrak{F})$$
$$+ \int_X \langle y^{0,3}, \bar{D}y^{0,1}\rangle \wedge \Omega + b^{0,4} \wedge \bar{\partial} c^{5,0} + b^{0,2} \wedge \bar{\partial} c^{5,2} + b^{0,0} \wedge \bar{\partial} c^{5,4} \quad (\mathfrak{B})$$

$$|Z| = |\bar{D}^{0,2}|^{+1/2} |\bar{D}^{0,1}|^{-1} |\bar{D}^{0,0}|^{+1} \left|\bar{\partial}^{0,0}\right|^{-1} \left|\bar{\partial}^{0,1}\right|^{+1} \left|\bar{\partial}^{0,2}\right|^{-1} \left|\bar{\partial}^{0,3}\right|^{+1} \left|\bar{\partial}^{0,4}\right|^{-1}$$
$$= \left[\prod_{p=0}^{5} \left|\Delta_{\bar{D}}^{0,p}\right|^{(-1)^{p+1}p}\right]^{1/4} \left[\prod_{q=0}^{5} \left|\Delta_{\bar{\partial}}^{0,q}\right|^{(-1)^{q+1}q}\right]^{-1/2}$$
$$= I_{\bar{D}}(Q)^{1/2} I(\Lambda^{0,0})^{-1}$$

$$S = \int_X \langle y^{0,1}, \bar{D}y^{0,3}\rangle \wedge \Omega + \int_X \left(b^{0,2} \wedge \bar{\partial} c^{5,2} + b^{0,4} \wedge \bar{\partial} c^{5,0}\right)$$
$$+ \int_X \left(y^{0,1} \lrcorner \partial b^{0,4} + y^{0,3} \lrcorner \partial b^{0,2}\right) \wedge \Omega$$

$$\delta\mu = \bar{\partial} v, \delta c = \partial(v \lrcorner \Omega).$$

$$v^a = \nabla^a w + \tilde{v}^a$$

$$\nabla_a \mu^a = 0.$$

$$S = \int_X \langle y^{0,1}, \bar{D}y^{0,3}\rangle \wedge \Omega$$

$$\int_X \langle y, y'\rangle \wedge \Omega$$

$$\mathrm{P}_{\sim}^{0,k} = \frac{\mathrm{P}^{0,k}}{\Omega^{1,k}_{\partial\text{-exact}}} \text{ where } \mathrm{P}^{0,k} = \{y \in \Omega^{0,k}(Q) : \nabla_a \mu^a = 0\} \subset \Omega^{0,\cdot}(Q).$$

$$\delta y = \bar{D}\epsilon \text{ for } \epsilon \in \mathrm{P}_{\sim}^{0,\cdot}.$$

$$S = \int_X \langle y, \bar{D}y\rangle \wedge \Omega$$

$$\left|\bar{D}^{0,p}_{\mathrm{P}}\right| = \frac{|\bar{D}^{0,p}|}{\left|\bar{\partial}^{1,p}_{\partial\text{-exact}}\right| \left|\bar{\partial}^{5,p}_{\partial\text{-exact}}\right|}$$

$$\left|\bar{\partial}^{1,p}_{\partial\text{-exact}}\right| = \left|\bar{\partial}^{0,p}\right|, \left|\bar{\partial}^{5,p}_{\partial\text{-exact}}\right| = \left|\bar{\partial}^{5,p}\right| = \left|\bar{\partial}^{0,p}\right|.$$

$$\left|\bar{D}^{0,p}_{\mathrm{P}}\right| = \frac{|\bar{D}^{0,p}|}{\left|\bar{\partial}^{0,p}\right|^2}$$

$$S = \int_X \left(\langle y, \bar{D}y\rangle - \frac{1}{3}\langle y, [y, y]\rangle\right) \wedge \Omega$$

$$[y_1, y_2]^a = \mu_1^b \wedge \partial_b \mu_2^a - (-1)^{|y_1||y_2|} \mu_2^b \wedge \partial_b \mu_1^a$$
$$[y_1, y_2]_\alpha = -\alpha_1 \wedge \alpha_2 + (-1)^{|y_1||y_2|} \alpha_2 \wedge \alpha_1$$
$$+\mu_1^b \wedge \nabla_b \alpha_2 - (-1)^{|y_1||y_2|} \mu_2^b \wedge \nabla_b \alpha_1$$
$$[y_1, y_2]_a = 2\mu_1^b \wedge \partial_{[b} x_{2,a]} - (-1)^{|y_1||y_2|} 2\mu_2^b \wedge \partial_{[b} x_{1,a]}$$
$$+\frac{1}{2}\mathrm{tr}(\nabla_a \alpha_1 \wedge \alpha_2) - (-1)^{|y_1||y_2|} \frac{1}{2}\mathrm{tr}(\nabla_a \alpha_2 \wedge \alpha_1),$$

$$\delta y = \bar{D}\lambda - [y, \lambda]$$

$$\bar{D}y - \frac{1}{2}[y, y] = 0$$

$$\bar{D}y^{0,1} - \frac{1}{2}[y^{0,1}, y^{0,1}] - [y^{0,0}, y^{0,2}] = 0.$$

$$\omega(\delta y, \delta y') = \int_X \langle \delta y', \delta y \rangle \wedge \Omega$$

$$Qy = \bar{D}y - \frac{1}{2}[y, y]$$

$$S(x, \mu) = 2\int_X \left(x_a \wedge \bar{\partial}\mu^a - \frac{1}{2}\mu^a \wedge \mu^b \wedge (\partial x)_{ab} \right) \wedge \Omega$$

$$\eta = \partial x$$

$$x = \partial^{-1}\eta = \Delta_\partial^{-1}\partial^\dagger \eta$$

$$S(x, \mu) = 2\int_X \left((\partial^{-1}\eta)_a \wedge \bar{\partial}\mu^a - \frac{1}{2}\eta_{ab} \wedge \mu^a \wedge \mu^b \right) \wedge \Omega$$

$$|Z|^2 = \frac{I_{\bar{D}}(Q)}{I(\Lambda^{0,0})^2}$$

$$\mathcal{F}_{\mathrm{Det}} = \frac{1}{2}\left[\int_X \mathrm{td}(X) \wedge \mathrm{ch}(Q) \right]_{(1,1)} - \left[\int_X \mathrm{td}(X) \right]_{(1,1)} := \left[-\int_X \mathcal{P} \right]_{(1,1)},$$

$$\mathcal{P}_{(6,6)} = \frac{\dim(G) - 496}{252}\mathrm{ch}_6(X) + \frac{\dim(G) - 64}{10368}\mathrm{ch}_2(X)^3$$
$$+\frac{\dim(G) + 224}{1440}\mathrm{ch}_2(X)\mathrm{ch}_4(X) - \frac{1}{288}\mathrm{ch}_2(X)^2\mathrm{ch}_2(\mathrm{End}(V))$$
$$-\frac{1}{120}\mathrm{ch}_4(X)\mathrm{ch}_2(\mathrm{End}(V)) + \frac{1}{12}\mathrm{ch}_2(X)\mathrm{ch}_4(\mathrm{End}(V)) - \mathrm{ch}_6(\mathrm{End}(V))$$

$$\mathcal{P}_{(6,6)} = \frac{1}{24}\mathrm{ch}_2(X)^3 + \frac{1}{2}\mathrm{ch}_2(X)\mathrm{ch}_4(X) - \frac{1}{288}\mathrm{ch}_2(X)^2\mathrm{ch}_2(\mathrm{End}(V))$$
$$-\frac{1}{120}\mathrm{ch}_4(X)\mathrm{ch}_2(\mathrm{End}(V)) + \frac{1}{12}\mathrm{ch}_2(X)\mathrm{ch}_4(\mathrm{End}(V)) - \mathrm{ch}_6(\mathrm{End}(V))$$

$$\mathcal{P}_{(6,6)} = \frac{1}{24}\mathrm{ch}_2(X)^3 + \frac{1}{2}\mathrm{ch}_2(X)\mathrm{ch}_4(X) - \frac{5}{48}\mathrm{ch}_2(X)^2\mathrm{ch}_2(V) - \frac{1}{4}\mathrm{ch}_4(X)\mathrm{ch}_2(V) + \frac{1}{24}\mathrm{ch}_2(X)\mathrm{ch}_2(V)^2 + 2\mathrm{ch}_2(X)\mathrm{ch}_4(V) - \mathrm{ch}_2(V)\mathrm{ch}_4(V)$$

$$\mathcal{P}_{(6,6)} = \frac{1}{24}\left(\mathrm{ch}_2(X) - \frac{1}{2}\mathrm{ch}_2(V)\right)(\mathrm{ch}_2(X)^2 + 12\mathrm{ch}_4(X) - 2\mathrm{ch}_2(X)\mathrm{ch}_2(V) + 48\mathrm{ch}_4(V)) = \frac{1}{24}\left(\mathrm{ch}_2(X) - \frac{1}{2}\mathrm{ch}_2(V)\right) \wedge \mathcal{P}_{(4,4)}^{SO(32)}$$

$$\mathcal{P}_{(6,6)} = \frac{1}{24}\mathrm{ch}_2(X)^3 + \frac{1}{2}\mathrm{ch}_2(X)\mathrm{ch}_4(X) - \frac{5}{48}\mathrm{ch}_2(X)^2\mathrm{ch}_2(V_1) - \frac{1}{4}\mathrm{ch}_4(X)\mathrm{ch}_2(V_1) + \frac{1}{8}\mathrm{ch}_2(X)\mathrm{ch}_2(V_1)^2 - \frac{1}{24}\mathrm{ch}_2(V_1)^3 - \frac{5}{48}\mathrm{ch}_2(X)^2\mathrm{ch}_2(V_2) - \frac{1}{4}\mathrm{ch}_4(X)\mathrm{ch}_2(V_2) + \frac{1}{8}\mathrm{ch}_2(X)\mathrm{ch}_2(V_2)^2 - \frac{1}{24}\mathrm{ch}_2(V_2)^3$$

$$\mathcal{P}_{(6,6)} = \frac{1}{48}\left(\mathrm{ch}_2(X) - \frac{1}{2}\mathrm{ch}_2(V)\right)(2\mathrm{ch}_2(X)^2 - 24\mathrm{ch}_4(X) - 4\mathrm{ch}_2(X)\mathrm{ch}_2(V_1) + 4\mathrm{ch}_2(V_1)^2 + 4\mathrm{ch}_2(X)\mathrm{ch}_2(V_2) - 4\mathrm{ch}_2(V_1)\mathrm{ch}_2(V_2) + 4\mathrm{ch}_2(V_2)^2) = \frac{1}{48}\left(\mathrm{ch}_2(X) - \frac{1}{2}\mathrm{ch}_2(V)\right) \wedge \mathcal{P}_{(4,4)}^{E_8 \times E_8}$$

$$\mathcal{P}_{12} = \mathrm{d}\mathcal{Q}_{11},$$

$$\delta\mathcal{Q}_{11} = \mathrm{d}\mathcal{P}_{10}(\gamma, \epsilon),$$

$$\delta\log\, Z = -\mathrm{i}\int_X \mathcal{P}_{10}(\gamma, \epsilon).$$

$$\delta\log\, Z = \mathrm{i}\beta'\int_X (\mathrm{tr}(R\gamma) - \mathrm{tr}(F\epsilon)) \wedge \mathcal{P}_{(4,4)},$$

$$\delta\mu^a = \bar{\partial}v^a$$

$$S_{\text{counter}}(\mu) = -\beta'\int_X R_b^a \nabla_a\left(\Delta_{\bar{\partial}}^{-1}\bar{\partial}^\dagger\mu^b\right) \wedge \mathcal{P}_{(4,4)}$$

$$\delta\alpha = \bar{\partial}_A\epsilon$$

$$S_{\text{counter}}(\alpha) = \beta'\int_X \mathrm{tr}\left(F\Delta_{\bar{\partial}_A}^{-1}\bar{\partial}_A^\dagger\alpha\right) \wedge \mathcal{P}_{(4,4)}$$

$$\delta x = \beta'(\mathrm{tr}(F\epsilon) - \mathrm{tr}(R\gamma)),$$

$$S_{\text{counter}}(x) = \int_X x \wedge \mathcal{P}_{(4,4)}$$

$$\bar{D} = \begin{pmatrix} \bar{\partial} & \beta'\mathcal{F}^* & \beta' R\cdot\nabla \\ 0 & \bar{\partial}_{\mathcal{A}} & \mathcal{F} \\ 0 & 0 & \bar{\partial} \end{pmatrix}$$

$$\begin{aligned}\mathcal{F}(\mu) &= F_a \wedge \mu^a,\\ (\mathcal{F}^*(\alpha))_a &= \mathrm{tr}(F_a \wedge \alpha),\\ (R\cdot\nabla\mu)_a &= -R_a{}^b{}_c \wedge \nabla_b \mu^c,\end{aligned}$$

$$S = \int_X \langle y, \bar{D}y\rangle \wedge \Omega + \int_X b \wedge \bar{\partial} c + \int_X \left(y \lrcorner \partial b\right) \wedge \Omega$$

$$\bar{D}^2 = 0 \;\Rightarrow\; \mathrm{tr}(F\wedge F) = \mathrm{tr}(R\wedge R).$$

$$\mathrm{tr}(F\wedge F) - \mathrm{tr}(R\wedge R) = i\partial\bar{\partial}\tau,$$

$$\omega = \omega_0 + \frac{\beta'}{4}\tau,$$

$$\mathrm{i}\partial\bar{\partial}\omega = \frac{\beta'}{4}(\mathrm{tr}(F\wedge F) - \mathrm{tr}(R\wedge R))$$

$$\bar{D} = \begin{pmatrix} \bar{\partial} & \beta'\mathcal{F}^* & \mathcal{T} + \beta' R\cdot\nabla \\ 0 & \bar{\partial}_{\mathcal{A}} & \mathcal{F} \\ 0 & 0 & \bar{\partial} \end{pmatrix}$$

$$(\mathcal{T}(\mu))_a = -4\mathrm{i}\partial_{[a}\omega_{b]}\wedge\mu^b = -\mathrm{i}\beta'\partial_{[a}\tau_{b]}\wedge\mu^b,$$

$$\delta S_{\text{counter}}(x) = 2\mathrm{i}\int_X \left(v \lrcorner \partial\omega\right)\wedge\mathcal{P}_{(4,4)} = 2\mathrm{i}\int_X \mathcal{L}_v\omega\wedge\mathcal{P}_{(4,4)}$$

$$\mathcal{L}_v\beta = v\lrcorner\partial\beta + \partial(v\lrcorner\beta)$$

$$\mathcal{P}_{(4,4)} = \frac{1}{3!}\omega^3\wedge\mathcal{P}^{\mathrm{p}}_{(1,1)} + \frac{1}{4!}\omega^4\mathcal{P}_0$$

$$\star\,\mathcal{P}_{(4,4)} = -\mathcal{P}^{\mathrm{p}}_{(1,1)} + \omega\mathcal{P}_0$$

$$\begin{aligned}\delta S_{\text{counter}}(x) &= 2\mathrm{i}\int_X \mathcal{L}_v\left(\frac{1}{4!}\omega^4\right)\wedge\left(\mathcal{P}^p_{(1,1)} + \frac{1}{4}\omega\mathcal{P}_0\right)\\ &= 2\mathrm{i}\int_X \mathcal{L}_v\left(\frac{1}{4!}\omega^4\right)\wedge\left(-\star\,\mathcal{P}_{(4,4)} + \frac{5}{4}\omega\mathcal{P}_0\right)\end{aligned}$$

$$\omega^4\wedge F = 0$$

$$\mathrm{d}\left(\mathrm{e}^{-2\phi}\omega^4\right) = 0$$

$$\mathrm{d}\star\omega = \mathrm{d}\left(\frac{1}{4!}\omega^4\right) = 0$$

$$\mathcal{L}_v\left(\frac{1}{4!}\omega^4\right) = \partial\left(v\lrcorner\frac{1}{4!}\omega^4\right) = \partial\left((v\lrcorner\omega)\wedge\frac{1}{3!}\omega^3\right)$$

$$\delta S_{\text{counter}}(x) = \frac{5}{2}\mathrm{i}\int_X \partial((v \lrcorner \omega) \wedge \frac{1}{3!}\omega^3)\wedge \omega\mathcal{P}_0 = 10\mathrm{i}\int_X \partial(v \lrcorner \omega)\wedge \frac{1}{4!}\omega^4\mathcal{P}_0$$
$$= 10\mathrm{i}\int_X \mathcal{L}_v\left(\frac{1}{5!}\omega^5\right)\mathcal{P}_0 = 10\mathrm{i}\int_X \frac{1}{|\Omega|^2}\mathcal{L}_v\Omega\wedge\bar{\Omega}\mathcal{P}_0$$

$$\bar{\partial}^\dagger \mathcal{P}_{(4,4)} = -\star\,\partial\star\,\mathcal{P}_{(4,4)} = 0$$

$$\frac{i}{|\Omega|^2}\Omega\wedge\bar{\Omega} = \frac{1}{5!}\omega^5.$$

$$|\Omega| = \mathrm{e}^{-2\phi},$$

$$\delta c^{5,0} = \partial(v \lrcorner \Omega) \equiv \mathcal{L}_v\Omega$$

$$S_{\text{counter}}(c^{5,0}) = -10\mathrm{i}\int_X \frac{1}{|\Omega|^2}c^{5,0}\wedge\bar{\Omega}\mathcal{P}_0$$

$$S_{\text{one-loop}} = S + S_{\text{counter}}(x) + S_{\text{counter}}(c^{5,0})$$

$$\delta\log Z = \mathrm{i}\beta'\int_X (\mathrm{tr}(R\gamma) - \mathrm{tr}(F\epsilon))\wedge\mathcal{P}_{(4,4)}$$

$$\frac{1}{4!}\omega^4\wedge\tilde{F} = \mathcal{P}_{(4,4)}(\tilde{F},\tilde{R})\wedge\tilde{F},$$
$$\frac{1}{4!}\omega^4\wedge\tilde{R} = \mathcal{P}_{(4,4)}(\tilde{F},\tilde{R})\wedge\tilde{R},$$

$$\delta\log Z = \mathrm{i}\beta'\int_X (\mathrm{tr}(\tilde{R}\gamma) - \mathrm{tr}(\tilde{F}\epsilon))\wedge\mathcal{P}_{(4,4)}(\tilde{F},\tilde{R})$$
$$= \mathrm{i}\beta'\int_X (\mathrm{tr}(\tilde{R}\gamma) - \mathrm{tr}(\tilde{F}\epsilon))\wedge\frac{1}{4!}\omega^4,$$

$$S^2_{\text{counter}}(x) = \frac{1}{4!}\int_X x\wedge\omega^4$$

$$\mathrm{e}^{-K(\omega)} = \frac{1}{5!}\int_X \omega^5$$

$$\mathcal{F}(\mu) = \tilde{F}_a\wedge\mu^a,$$
$$(\mathcal{F}^*(\alpha))_a = \mathrm{tr}(\tilde{F}_a\wedge\alpha),$$
$$(\tilde{R}\cdot\tilde{\nabla}\mu)_a = -\tilde{R}_a{}^b{}_c\wedge\tilde{\nabla}^+_b\mu^c.$$

$$\delta S^2_{\text{counter}}(x) = \beta'\int_X (\mathrm{tr}(\tilde{F}\epsilon) - \mathrm{tr}(\tilde{R}\gamma))\wedge\frac{1}{4!}\omega^4 + 2\mathrm{i}\int_X \mathcal{L}_v\omega\wedge\frac{1}{4!}\omega^4$$

$$\int_X \mathcal{L}_v(\omega)\wedge\frac{1}{4!}\omega^4 = \int_X \mathcal{L}_v\left(\frac{1}{5!}\omega^5\right) = 0$$

$$S_{\text{one-loop}} = S + S^2_{\text{counter}}(x)$$

$$\omega^5\wedge F(h) = 0$$

$$\Delta_h \delta h = \text{ Fifth and higher order in curvatures.}$$

$$\delta \log(Z) = \mathrm{i}\beta' \int_X \left(\mathrm{tr}(R^2) - \mathrm{tr}(F^2)\right) \wedge \mathcal{P}(\gamma, \epsilon)_{(3,3)}$$

$$\mathrm{tr}(R^2) = \partial \omega_{\mathrm{CS}}(\nabla)_{(1,2)}, \mathrm{tr}(F^2) = \partial \omega_{\mathrm{CS}}(A)_{(1,2)}$$

$$S_{\text{counter}}(\chi) = \int_X \left(\omega_{\mathrm{CS}}(A)_{(1,2)} - \omega_{\mathrm{CS}}(\nabla)_{(1,2)}\right) \wedge \chi$$

$$\delta\chi = \beta' \partial \mathcal{P}(\gamma, \epsilon)_{(3,3)}$$

$$\mathrm{tr}(R^2) - \mathrm{tr}(F^2) = \partial T,$$

$$\delta \log(Z) = \mathrm{i}\beta' \int_X \partial T \wedge \mathcal{P}(\gamma, \epsilon)_{(3,3)}.$$

$$S_{\text{counter}}(\chi) = -\int_X T \wedge \chi,$$

$$I(V)^2 = \prod_{p=0}^{n} \left|\Delta_{\bar{\partial}}^{0,p}\right|^{(-1)^{p+1} p}$$

$$\mathbf{S}_{\text{per.}} \sim \sum_{g,b,c=0}^{\infty} \int d^{10}x e^{(2g+b+c-2)\Phi} \sqrt{-G} \mathcal{L}_{(g,b,c)}(\alpha')$$

$$\mathbf{S}_{\text{per.}} \sim \sum_{g=0}^{\infty} \int d^{10}x e^{2(g-1)\Phi'} \sqrt{-G'} \mathcal{L}_g(\alpha')$$

$$\Phi' = -\Phi, G'_{\alpha\beta} = e^{-\Phi} G_{\alpha\beta}, B'_{\alpha\beta} = C_{\alpha\beta} \equiv B_{\alpha\beta}, A'_{\alpha i}{}^{j} = A_{\alpha i}{}^{j}$$

$$\mathbf{S}^{(0)}_{\text{het.}} = -\frac{2}{\kappa^2} \int d^{10}x \sqrt{-G'} e^{-2\Phi'} \left[R' + 4\nabla_\alpha \Phi' \nabla^\alpha \Phi' - \frac{1}{12} H'_{\alpha\beta\gamma} H'^{\alpha\beta\gamma} + \frac{1}{4} F'_{\alpha\beta i}{}^{j} F'^{\alpha\beta}{}_{j}{}^{i}\right],$$

$$\mathbf{S}^{(0)}_{\text{typeI}} = -\frac{2}{\kappa^2} \int d^{10}x \sqrt{-G} \left[e^{-2\Phi}(R + 4\nabla_\alpha \Phi \nabla^\alpha \Phi) - \frac{1}{12} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} + \frac{e^{-\Phi}}{4} F_{\alpha\beta i}{}^{j} F^{\alpha\beta}{}_{j}{}^{i}\right]$$

$$F^{j}_{\mu\nu i} = \partial_\mu A^{j}_{\nu i} - \partial_\nu A^{j}_{\mu i} + \frac{1}{\sqrt{\alpha'}} \left[A^{k}_{\mu i}, A^{j}_{\nu k}\right]$$

$$H_{\mu\nu\rho} = 3\partial_{[\mu} B_{\nu\rho]} + \frac{3}{2} A^{j}_{[\mu i} F^{i}_{\nu\rho]j}$$

$$\Omega_{\mu\nu\alpha} = \omega_{[\mu\mu_1}{}^{\nu_1} \partial_\nu \omega_{\alpha]\nu_1}{}^{\mu_1} + \frac{2}{3} \omega_{[\mu\mu_1}{}^{\nu_1} \omega_{\nu\nu_1}{}^{\alpha_1} \omega_{\alpha]\alpha_1}{}^{\mu_1}; \omega_{\mu\mu_1}{}^{\nu_1} = e^{\nu}{}_{\mu_1} \nabla_\mu e_\nu{}^{\nu_1},$$

$$\eta'_{\mu_1\nu_1} = e^{-\Phi} \eta_{\mu_1\nu_1}$$

$$\Omega'_{\mu\nu\rho} = \Omega_{\mu\nu\rho}$$

$$H'_{\alpha\beta\gamma} \to H'_{\alpha\beta\gamma} + \frac{3}{2}\alpha'\Omega'_{\alpha\beta\gamma}$$

$$\Phi' = -\Phi, G'_{\mu\nu} = e^{-2\Phi}G_{\mu\nu}, H' = e^{-2\Phi} * H$$

$$\begin{aligned}
\Phi' &= -\Phi + \alpha'\Delta\Phi(\Phi, G, B) \\
G'_{\alpha\beta} &= e^{-\Phi}\left[G_{\alpha\beta} + \alpha' G_{\alpha\beta}\Delta\Phi(\Phi, G, B) + \alpha'\Delta G_{\alpha\beta}(\Phi, G, B)\right] \\
B'_{\alpha\beta} &= B_{\alpha\beta} + \alpha'\Delta B_{\alpha\beta}(\Phi, G, B)
\end{aligned}$$

$$\begin{aligned}
\Delta\Phi(\Phi, G, B) - \Delta\Phi\left(-\Phi, e^{-\Phi}G, B\right) &= 0 \\
\Delta G_{\alpha\beta}(\Phi, G, B) + G_{\alpha\beta}\Delta\Phi\left(-\Phi, e^{-\Phi}G, B\right) + e^{\Phi}\Delta G_{\alpha\beta}\left(-\Phi, e^{-\Phi}G, B\right) &= 0 \\
\Delta B_{\alpha\beta}(\Phi, G, B) + \Delta B_{\alpha\beta}\left(-\Phi, e^{-\Phi}G, B\right) &= 0
\end{aligned}$$

$$\Delta\Phi = 0, \Delta G_{\alpha\beta} = 0, \Delta B_{\alpha\beta} = 0$$

$$\begin{aligned}
\mathbf{S}^{(1)}_{\text{het.}} =& -\frac{2\alpha'}{8\kappa^2}\int d^{10}x\sqrt{-G'}e^{-2\Phi'}\left[\frac{1}{4}F'^{\gamma kl}_{\alpha}F'^{\alpha\beta ij}F'^{\delta}_{\beta\ kl}F'_{\gamma\delta ij} - \frac{1}{2}F'^{\gamma}_{\alpha\ ij}F'^{\alpha\beta ij}F'^{\delta kl}_{\beta}F'_{\gamma\delta kl}\right. \\
&-\frac{1}{8}F'^{\ kl}_{\alpha\beta}F'^{\alpha\beta ij}F'_{\gamma\delta kl}F'^{\gamma\delta}_{\ \ ij} + \frac{1}{4}F'^{\alpha\beta ij}F'^{\gamma\delta}_{\ \ ij}H'_{\alpha\gamma\ \epsilon}H'^{\ \epsilon}_{\beta\delta} - \frac{1}{8}F'^{\alpha\beta ij}F'^{\gamma\delta}_{\ \ ij}H'_{\alpha\beta\ \epsilon}H'^{\ \epsilon}_{\gamma\delta} \\
&-\frac{1}{2}F'^{\gamma}_{\alpha\ ij}F'^{\alpha\beta ij}H'^{\delta\epsilon}_{\beta}H'_{\gamma\delta\epsilon} - \frac{1}{8}H'_{\alpha\beta\ \delta}H'^{\alpha\beta\gamma}H'^{\epsilon}_{\gamma}H'^{\delta}_{\delta\epsilon\varepsilon} + \frac{1}{24}H'^{\delta\epsilon}_{\alpha}H'^{\alpha\beta\gamma}H'_{\beta\delta\ \varepsilon}H'^{\ \varepsilon}_{\gamma\epsilon} \\
&\left.+R'_{\alpha\beta\gamma\delta}R'^{\alpha\beta\gamma\delta} - \frac{1}{2}H'^{\delta\epsilon}_{\alpha}H'^{\alpha\beta\gamma}R'_{\beta\gamma\delta\epsilon} + 2H'^{\alpha\beta\gamma}\Omega'_{\alpha\beta\gamma}\right]
\end{aligned}$$

$$\begin{aligned}
&2R\nabla_\alpha\nabla^\alpha\Phi + \frac{1}{4}H_{\beta\gamma\delta}H^{\beta\gamma\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi - 3R\nabla_\alpha\Phi\nabla^\alpha\Phi + 9\nabla_\alpha\nabla^\alpha\Phi\nabla_\beta\nabla^\beta\Phi - \frac{1}{2}H^{\gamma\delta}_{\alpha}H_{\beta\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi \\
&-2R_{\alpha\beta}\nabla^\alpha\Phi\nabla^\beta\Phi - 7\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi + 48\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\Phi - H^{\gamma\delta}_{\alpha}H_{\beta\gamma\delta}\nabla^\beta\nabla^\alpha\Phi
\end{aligned}$$

$$\begin{aligned}
\mathbf{S}^{(1)}_{\text{typeI}} =& -\frac{2\alpha'}{8\kappa^2}\int d^{10}x\sqrt{-G}\left(e^{\Phi}\left[\frac{1}{8}F^{\gamma kl}_{\alpha}F^{\alpha\beta ij}F^{\delta}_{\beta\ kl}F_{\gamma\delta ij}\right.\right. \\
&\left.-\frac{1}{16}F_{\alpha\beta\ }^{\ \ kl}F^{\alpha\beta ij}F_{\gamma\delta kl}F^{\gamma\delta}_{\ \ ij} + \frac{1}{1024}F_{\alpha\beta ij}F^{\alpha\beta ij}F_{\gamma\delta kl}F^{\gamma\delta kl}\right] \\
&+\left[\frac{1}{4}F^{\alpha\beta ij}F^{\gamma\delta}_{\ \ ij}\tilde{H}_{\alpha\gamma\ \epsilon}\tilde{H}_{\beta\delta}^{\ \ \epsilon} - \frac{1}{8}F_{\alpha\ \ ij}^{\ \gamma}F^{\alpha\beta ij}\tilde{H}_{\beta\ }^{\ \delta\epsilon}\tilde{H}_{\gamma\delta\epsilon} + \frac{1}{48}F_{\alpha\beta ij}F^{\alpha\beta ij}\tilde{H}_{\gamma\delta\epsilon}\tilde{H}^{\gamma\delta\epsilon}\right. \\
&\left.-\frac{1}{4}F^{\alpha\beta ij}\tilde{H}_{\beta\gamma\delta}\nabla_\alpha F^{\gamma\delta}_{\ \ ij} + \frac{3}{64}F_{\alpha\beta ij}F^{\alpha\beta ij}R - \frac{1}{2}F_{\alpha\ \ }^{\ \gamma ij}F_{\beta\gamma ij}R^{\alpha\beta}\right] \\
&+e^{-\Phi}\left[\frac{1}{16}R^2 - \frac{1}{16}\tilde{H}_{\alpha\beta\ }^{\ \ \delta}\tilde{H}^{\alpha\beta\gamma}\tilde{H}_{\gamma\ }^{\ \epsilon\varepsilon}\tilde{H}_{\delta\epsilon\varepsilon} + \frac{1}{96}\tilde{H}_{\alpha\beta\gamma}\tilde{H}^{\alpha\beta\gamma}\tilde{H}_{\delta\epsilon\varepsilon}\tilde{H}^{\delta\epsilon\varepsilon}\right. \\
&-R_{\alpha\beta}R^{\alpha\beta} + \frac{1}{24}\tilde{H}_{\alpha\ }^{\ \delta\epsilon}\tilde{H}^{\alpha\beta\gamma}\tilde{H}_{\beta\delta\ }^{\ \ \varepsilon}\tilde{H}_{\gamma\epsilon\varepsilon} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - \frac{1}{2}\tilde{H}_{\alpha\ }^{\ \delta\epsilon}\tilde{H}^{\alpha\beta\gamma}R_{\beta\gamma\delta\epsilon} \\
&\left.\left.+2\tilde{H}^{\alpha\beta\gamma}\Omega_{\alpha\beta\gamma}\right]\right)
\end{aligned}$$

$$ds^2 = G_{\mu\nu}(x)dx^\mu dx^\nu + g_{ab}(y)dy^a dy^b$$

$$\mathbf{S}_{\text{typeI}}^{(0)} = -\frac{2V}{\kappa^2}\int d^6x\sqrt{-G}e^{-2\Phi}\left[R + 4\nabla_\alpha\Phi\nabla^\alpha\Phi - \frac{1}{12}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}\right]$$

$$\int_{K3} d^4y\sqrt{g} = V$$

$$\mathbf{S}_{\text{typeI}}^{(3)} = \frac{2\alpha'^3\zeta(3)}{2^6\kappa^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R_{\mu\nu\rho\tau}R^{\mu\nu\rho\tau}\left(-\frac{5}{32}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\epsilon\varepsilon}H_{\delta\epsilon\varepsilon} + \frac{5}{4}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_{\beta\delta\gamma\epsilon}\right.\right.$$
$$\left.\left. - \frac{5}{24}\nabla_\delta H_{\alpha\beta\gamma}\nabla^\delta H^{\alpha\beta\gamma}\right) + \cdots\right]$$

$$\frac{1}{32\pi^2}\int_{K3} d^4y\sqrt{g}R_{abcd}R^{abcd}$$

$$\mathbf{S}_{\text{typeI}}^{(1)} = \frac{48\pi^2\alpha'^3\zeta(3)}{\kappa^2}\int d^6x\sqrt{-G}e^{-2\Phi}\left[\left(-\frac{5}{32}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\epsilon\varepsilon}H_{\delta\epsilon\varepsilon} + \frac{5}{4}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_{\beta\delta\gamma\epsilon}\right.\right.$$
$$\left.\left. - \frac{5}{24}\nabla_\delta H_{\alpha\beta\gamma}\nabla^\delta H^{\alpha\beta\gamma}\right)\right]$$

$$\delta\Phi D\Phi + \delta G_{\alpha\beta}DG^{\alpha\beta} + \delta B_{\alpha\beta}DB^{\alpha\beta}$$

$$D\Phi \equiv -2V\sqrt{-G}e^{-2\Phi}\left(R + 4\nabla_\alpha\nabla^\alpha\Phi - 4\nabla_\alpha\Phi\nabla^\alpha\Phi - \frac{1}{12}H^2\right) = 0,$$
$$DB^{\alpha\beta} \equiv \frac{V}{2}\sqrt{-G}\nabla_\gamma\left(e^{-2\Phi}H^{\alpha\beta\gamma}\right) = 0,$$
$$DG^{\alpha\beta} \equiv V\sqrt{-G}\left[\frac{1}{4}e^{-2\Phi}H^{\alpha\gamma\delta}H^{\beta}_{\gamma\delta} - e^{-2\Phi}\left(R^{\alpha\beta} + 2\nabla^\alpha\nabla^\beta\Phi\right)\right] - \frac{1}{4}D\Phi^{\alpha\beta} = 0,$$

$$\delta B_{\alpha\beta} = \frac{5c}{4}\nabla_\gamma H^{\gamma}_{\alpha\beta},$$
$$\delta\Phi = \frac{5c}{32}H^2 + \frac{5c}{2}\nabla_\alpha\Phi\nabla^\alpha\Phi,$$
$$\delta G_{\alpha\beta} = \frac{5c}{8}H_{\alpha\gamma\delta}H_{\beta}^{\gamma\delta} + 5c\nabla_\beta\nabla_\alpha\Phi,$$

$$\mathbf{S}_{\text{exact}} \sim \int d^{10}xe^{-2\Phi}\sqrt{-G}\mathcal{L}_{\text{exact}}(\alpha')$$

$$ds^2 = e^{2A}ds^2(\text{AdS}_2) + ds^2(\text{M}_8)$$
$$F = f_{\pm} + e^{2A}\text{vol}(\text{AdS}_2)\wedge\star_8\lambda(f_{\pm})$$
$$H = e^{2A}\text{vol}(\text{AdS}_2)\wedge H_1 + H_3$$

$$\chi_1^\dagger\gamma_a\chi_1 = \pm\chi_2^\dagger\gamma_a\chi_2, \chi_1^\dagger\hat{\gamma}\chi_1 = \pm\chi_2^\dagger\hat{\gamma}\chi_2, |\chi_1|^2 = |\chi_2|^2,$$

$$e^A = |\chi_1|^2, e^A\cos\beta = \chi_1^\dagger\hat{\gamma}\chi_1, e^A\sin\beta V = \chi_1^\dagger\gamma_a\chi_1 e^a,$$

$$\psi = \frac{1}{16}\sum_{n=0}^{8}\chi_2^\dagger\gamma_{a_n\ldots a_1}\chi_1 e^{a_1..a_n}, \hat{\psi} = \frac{1}{16}\sum_{n=0}^{8}\chi_2^\dagger\gamma_{a_n\ldots a_1}\hat{\gamma}\chi_1 e^{a_1..a_n},$$

$$e^{2A}H_1 = me^A \sin\,\beta V - d(e^{2A}\cos\,\beta), d(e^A \sin\,\beta V) = 0,$$

$$d_{H_3}\left(e^{-\Phi}\psi_\pm\right) = \pm\frac{1}{16}e^A \sin\,\beta V \wedge f_\pm,$$

$$d_{H_3}\left(e^{A-\Phi}\hat{\psi}_\mp\right) - me^{-\Phi}\psi_\pm = \mp\frac{1}{16}e^{2A}(\star_8\,\lambda f_\pm + \cos\,\beta f_\pm),$$

$$(\psi_\pm, f_\pm)_8 = \pm\frac{1}{4}e^{-\Phi}\left(m - \frac{1}{2}e^A \sin\,\beta\iota_V H_1\right)\mathrm{vol}(\mathrm{M}_8),$$

$$dH_3 = 0, \iota_V(df_\pm - H_3 \wedge f_\pm) = 0, d\left(e^{-2\Phi}\star_8 H_1\right) + \frac{1}{2}(f_\pm, f_\pm)_8 = 0$$

$$ds^2(\mathrm{M}_8) = e^{2C}ds^2(\mathbb{CP}^3) + ds^2(\mathrm{M}_2), H_3 = e^{2C}\tilde{H}_1 \wedge J_2$$

$$J_2, J_2 \wedge J_2, \mathrm{vol}(\mathbb{CP}^3) = \frac{1}{3!}J_2 \wedge J_2 \wedge J_2$$

$$ds^2(\,\mathrm{S}^7) = ds^2(\mathbb{CP}^3) + (e^\tau)^2, e^\tau = (d\tau + \eta), d\eta = 2J_2,$$

$$\nabla_a\xi_\pm = \pm\frac{i}{2}\gamma_a^{(7)}\xi_\pm$$

$$\xi_\pm = \sum_{I=1}^{8} c^I\xi_\pm^I$$

$$\xi_\pm^{I\dagger}\xi_\pm^J = \delta^{IJ}$$

$$\xi_+^I\colon \mathbf{8} \to \mathbf{6}_0 \oplus \mathbf{1}_{-2} \oplus \mathbf{1}_2, \xi_-^I\colon \mathbf{8} \to \mathbf{4}_{-1} \oplus \overline{\mathbf{4}}_1,$$

$$\Phi_3^I = -\frac{i}{3!}\xi_+^{(I)\dagger}\gamma_{a_1a_2a_3}^{(7)}\xi_+^{(I)}e^{a_1a_2a_3}, \star_7\,\Phi_3^I = -\frac{1}{4!}\xi_+^{(I)\dagger}\gamma_{a_1\dots a_4}^{(7)}\xi_+^{(I)}e^{a_1\dots a_4},$$
$$d\Phi_3^I = 4\,\star_7\,\Phi_3^I, \Phi_3^I \wedge \star_7\,\Phi_3^I = 7\mathrm{vol}(\,\mathrm{S}^7),$$

$$\mathbf{6}\text{ of SO(6)}: \xi_6^{\mathfrak{I}}, \mathbf{2}\text{ of SO(2)}: \xi_2^{\mathfrak{a}},$$

$$\Phi_3^{\mathfrak{a}} = J_2 \wedge e^\tau + \mathrm{Im}\Omega^{\mathfrak{a}}, \star_7\,\Phi_3^{\mathfrak{a}} = \frac{1}{2}J_2 \wedge J_2 - \mathrm{Re}\Omega^{\mathfrak{a}} \wedge e^\tau$$

$$de^\tau = 2J_2, d\Omega_3^{\mathfrak{a}} = 4ie^\tau \wedge \Omega_3^{\mathfrak{a}}$$
$$J_2 \wedge J_2 \wedge J_2 = \frac{3}{4}i\Omega_3^{(\mathfrak{a})} \wedge \bar{\Omega}_3^{(\mathfrak{a})}, J_2 \wedge \Omega_3^{\mathfrak{a}} = 0$$

$$\Phi_3^{\mathfrak{I}} = -\tilde{J}_2^{\mathfrak{I}} \wedge e^\tau + \mathrm{Im}\Omega^{\mathfrak{I}}, \star_7\,\Phi_3^{\mathfrak{I}} = \frac{1}{2}J_2^{(\mathfrak{I})} \wedge J_2^{(\mathfrak{I})} - \mathrm{Re}\Omega^{\mathfrak{I}} \wedge e^\tau$$

$$d\tilde{J}_2^{\mathfrak{I}} = 4\mathrm{Re}\Omega_3^{\mathfrak{I}}, d\mathrm{Im}\Omega_3^{\mathfrak{I}} = 6\tilde{J}_2^{\mathfrak{I}} \wedge J_2 - 2J_2 \wedge J_2, J_2 \wedge \Omega_3^{\mathfrak{I}} = 0$$
$$J_2 \wedge J_2 + \tilde{J}_2^{(\mathfrak{I})} \wedge \tilde{J}_2^{(\mathfrak{I})} = 2\tilde{J}_2 \wedge J_2, J_2 \wedge J_2 \wedge J_2 = -\tilde{J}_2^{(\mathfrak{I})} \wedge \tilde{J}_2^{(\mathfrak{I})} \wedge \tilde{J}_2^{(\mathfrak{I})} = \mathrm{vol}(\mathbb{CP}^3)$$

$$\tilde{J}_2^{(\mathcal{J})} \wedge \tilde{J}_2^{(\mathcal{J})} \wedge \tilde{J}_2^{(\mathcal{J})} = -\frac{3}{4} i\Omega_3^{(\mathcal{J})} \wedge \bar{\Omega}_3^{(\mathcal{J})}, \tilde{J}_2^{(\mathcal{J})} \wedge \Omega_3^{(\mathcal{J})} = 0$$

$$\gamma^{(6)} = \gamma_a^{(7)}, a = 1, \ldots, 6, \hat{\gamma}^{(6)} = \gamma_7^{(7)}$$

$$J_2 \xi_6^{\mathcal{J}} = -i\hat{\gamma}^{(6)} \xi_6^{\mathcal{J}}, J_2 \hat{\gamma}^{(6)} \xi_6^{\mathcal{J}} = -i\xi_6^{\mathcal{J}}.$$

$$\tilde{J}_2^{\mathcal{J}} = \frac{i}{2} \xi_6^{(\mathcal{J})\dagger} \gamma_{a_1 a_2} \hat{\gamma}^{(6)} \xi_6^{(\mathcal{J})} e^{a_1 a_2}, \mathrm{Re}\Omega_3^{\mathcal{J}} = \frac{1}{3!} \xi_6^{(\mathcal{J})\dagger} \gamma_{a_1 a_2 a_3} \hat{\gamma}^{(6)} \xi_6^{(\mathcal{J})} e^{a_1 a_2 a_3}, \mathrm{Im}\Omega_3^{\mathcal{J}}$$

$$= -\frac{i}{3!} \xi_6^{(\mathcal{J})\dagger} \gamma_{a_1 a_2 a_3} \xi_6^{(\mathcal{J})} e^{a_1 a_2 a_3}$$

$$\gamma_a = \sigma_3 \otimes \gamma_a^{(6)}, a = 1, \ldots, 6, \gamma_{7,8} = \sigma_{1,2} \otimes \mathbb{I}$$
$$\hat{\gamma} = \sigma_3 \otimes \hat{\gamma}^{(6)}, B = \sigma_1 \otimes B^{(6)}$$

$$\chi_{1,2}^{\mathcal{J}} = \frac{e^{\frac{A}{2}}}{2} \left(\eta_{1,2} \otimes \xi_6^{\mathcal{J}} + i\hat{\eta}_{1,2} \otimes \hat{\gamma}^{(6)} \xi_6^{\mathcal{J}} \right)$$

$$\left(\tilde{J}_2^{\mathcal{J}}, \Omega_3^{\mathcal{J}} \right) \to \left(\tilde{J}_2, \Omega_3 \right).$$

$$\eta_1 = i \begin{pmatrix} \cos\left(\frac{\alpha-\beta}{2}\right) + i\sin\left(\frac{\alpha+\beta}{2}\right) \\ -\left(\cos\left(\frac{\alpha-\beta}{2}\right) - i\sin\left(\frac{\alpha+\beta}{2}\right)\right) \end{pmatrix}, \hat{\eta}_1 = -\begin{pmatrix} \cos\left(\frac{\alpha+\beta}{2}\right) + \mathrm{i}\sin\left(\frac{\alpha-\beta}{2}\right) \\ \cos\left(\frac{\alpha+\beta}{2}\right) - \mathrm{i}\sin\left(\frac{\alpha-\beta}{2}\right) \end{pmatrix},$$

$$\eta_2 = -i \begin{pmatrix} \cos\left(\frac{\alpha+\beta}{2}\right) + i\cos\left(\frac{\alpha-\beta}{2}\right) \\ -\left(\cos\left(\frac{\alpha+\beta}{2}\right) - i\sin\left(\frac{\alpha-\beta}{2}\right)\right) \end{pmatrix}, \hat{\eta}_2 = -\begin{pmatrix} \cos\left(\frac{\alpha-\beta}{2}\right) + i\sin\left(\frac{\alpha+\beta}{2}\right) \\ \cos\left(\frac{\alpha-\beta}{2}\right) - i\sin\left(\frac{\alpha+\beta}{2}\right) \end{pmatrix},$$

$$\hat{\psi}_+ = -\frac{e^A \sin\beta}{16} \mathrm{Re}\psi_+^{(7)}, \psi_- = \frac{e^A}{16} \mathrm{Re}\left(\psi_-^{(7)} + \cos\beta \psi_+^{(7)} \wedge V\right)$$
$$\psi_+^{(7)} = -e^{-i\alpha} e^{ie^{2C}\tilde{J}_2} - e^{3C} i\Omega_3 \wedge U, \psi_-^{(7)} = -e^{-i\alpha} e^{ie^{2C}\tilde{J}_2} \wedge U - e^{3C} i\Omega_3,$$

$$\tilde{J}_2, J_2 \wedge \tilde{J}_2, \mathrm{Re}\Omega_3, \mathrm{Im}\Omega_3$$

$$e^C = -\frac{4e^A}{\sin\alpha \sin\beta}, me^{2C}\tilde{H} = 4d(e^{A+C}\cos\alpha \sin\beta) + 8e^A \sin\beta U$$
$$d(e^A \sin\beta U) = 0, d(e^A \sin\beta V) = 0$$
$$d\left(e^{A+3C-\Phi}\right) + 4e^{A+2C-\Phi}(\cos\alpha U - \sin\alpha\cos\beta V) = 0$$

$$U + iV = \frac{p}{e^A \sin\beta m}(dx_1 + idx_2)$$

$$e^{-\Phi} = e^{-(A+3C)}h$$

$$\frac{mpcos\ \beta}{e^{2A}\sin^2\ \beta} + \partial_{x_1}\log\ h = 0, \frac{mpcos\ \alpha}{e^{2A}\sin\ \alpha\sin^2\ \beta} + \partial_{x_2}\log\ h = 0$$

$$\cos\ \beta = -\frac{\partial_{x_1}h}{2h\left(\partial_{x_1}^2 + \partial_{x_2}^2\right)h - \partial_{x_2}h^2}$$

$$p = \frac{L^2}{m}$$

$$\left(e^{-\Phi}, F_-\right) \to c\left(e^{-\Phi}, F_-\right)$$

$$\frac{ds^2}{L^2} = \sqrt{\Delta_1}\left[\frac{h}{\Delta_2}ds^2(\mathrm{AdS}_2) + \frac{1}{m^2}\left(\frac{8}{\square\ h}ds^2(\mathbb{CP}^3) + \frac{1}{h}(dx_i)^2\right)\right], e^{-\Phi} = \frac{m^3c\sqrt{h\Delta_2}(\square\ h)^{\frac{3}{2}}}{16\sqrt{2}L^4\Delta_1}$$

$$H = dB, \frac{B}{L^2} = \left(\frac{h\partial_1 h}{\Delta_2} + x_1\right)\mathrm{vol}(\mathrm{AdS}_2) + \frac{8}{m^2}\left(\frac{\partial_2 h}{\square\ h} - (x_2 - l)\right)J_2$$

$$\square\ h = \left(\partial_{x_1}^2 + \partial_{x_2}^2\right)h, \partial_i = \partial_{x_i}$$

$$\Delta_1 = 2h\ \square\ h - (\partial_2 h)^2, \Delta_2 = 2h\ \square\ h - (\partial_1 h)^2 - (\partial_2 h)^2.$$

$$F_- = f_- + e^{2A}\mathrm{vol}(\mathrm{AdS}_2) \wedge \star_8\ \lambda(f_-)$$

$$\hat{F}_- = \hat{f}_- + \mathrm{vol}(\mathrm{AdS}_2) \wedge \hat{g}_-$$

$$\hat{f}_1 = F_1 = \frac{m^3c}{32L^4}\left[\partial_2(\square\ h)dx_1 - \partial_1(\square\ h)dx_2 - d\left(\frac{\partial_1 h\partial_2 h\ \square\ h}{\Delta_1}\right)\right]$$

$$\hat{f}_3 = \frac{mc}{4L^2}\left[\left((x_2 - l)\partial_2(\square\ h) - \square\ h\right)dx_1 - (x_2 - l)\partial_1(\square\ h)dx_2 + d\left(\frac{\partial_1 h\ \square\ h(2h - (x_2 - l)\partial_2 h)}{\Delta_1}\right)\right] \wedge J_2$$

$$\hat{f}_5 = \frac{c}{m}\left[\left(2\partial_2 h + (x_2 - l)(-2\ \square\ h + (x_2 - l)\partial_2(\square\ h))\right)dx_1 - (2\partial_1 h + (x_2 - l)^2\partial_1(\square\ h))dx_2\right.$$
$$\left. - d\left(\frac{\partial_1 h\left((x_2 - l)^2\partial_2 h\ \square\ h + 2h(\partial_2 h - 2(x_2 - l)\ \square\ h)\right)}{\Delta_1}\right)\right] \wedge J_2 \wedge J_2$$

$$\hat{f}_7 = \frac{8L^2c}{m^3}\left[\left(-2h + (x_2 - l)\left(2\partial_2 h + (x_2 - l)\left(\frac{1}{3}(x_2 - l)\partial_2(\square\ h) - \square\ h\right)\right)\right)dx_1\right.$$
$$-2\left((x_2 - l)\partial_1 h + \frac{1}{6}\partial_1(\square\ h)\right)dx_2$$
$$\left. + d\left(\frac{\partial_1 h\left(4h^2 - (x_2 - l)^3\partial_2 h\ \square\ h - 6(x_2 - l)h(\partial_2 h - (x_2 - l)\ \square\ h)\right)}{3\Delta_1}\right)\right] \wedge J_2 \wedge J_2 \wedge J_2.$$

$$d\hat{f}_{2n+1} \sim (x_2 - l)^n\ \square^2\ h dx_1 \wedge dx_2 \wedge J_2^n$$

$$\square^2\ h = (\partial_1^4 + 2\partial_1^2\partial_2^2 + \partial_2^4)h = 0$$

$$d\hat{g}_{2n+1} \sim x_1(x_2 - l)^n\ \square^2\ h dx_1 \wedge dx_2 \wedge J_2^n$$

$$\square^2\ h \sim \delta(x_2 - l)\delta(x_1)$$

$$z = x_1 + ix_2, \partial_z = \frac{1}{2}(\partial_1 - i\partial_2), \partial_{\bar{z}} = \frac{1}{2}(\partial_1 + i\partial_2), \Box = 4\partial_z\partial_{\bar{z}}$$

$$\partial_z\partial_{\bar{z}}\partial_z\partial_{\bar{z}}h = 0$$

$$\partial_z\partial_{\bar{z}}h = \frac{1}{2}\left(\partial_z\mathcal{A}_1(z) + \partial_{\bar{z}}\overline{\mathcal{A}}_1(\bar{z})\right)$$

$$h = \frac{1}{2}\left(\bar{z}\mathcal{A}_1(z) + z\overline{\mathcal{A}}_1(\bar{z}) + \mathcal{A}_2(z) + \overline{\mathcal{A}}_2(\bar{z})\right).$$

$$\mathcal{A}_{1,2} = h_{1,2} + ih_{1,2}^D$$

$$h = x_1h_1 + x_2h_1^D + h_2$$

$$ds^2(\mathrm{AdS}_3) = \frac{1}{4}[(2dx_1 + \eta)^2 + ds^2(\mathrm{AdS}_2)] \text{ with } d\eta = \mathrm{vol}(\mathrm{AdS}_2)$$

$$\frac{ds^2}{L^2} = \frac{h}{\sqrt{\Delta}}ds^2(\mathrm{AdS}_2) + \sqrt{\Delta}\left(\frac{8}{h''}ds^2(\mathbb{CP}^3) + \frac{1}{h}(dx_i)^2\right), e^{-\Phi} = \frac{c\sqrt{h}(h'')^{\frac{3}{2}}}{16\sqrt{2}L^4\sqrt{\Delta}}$$

$$H = dB, \frac{B}{L^2} = x_1\mathrm{vol}(\mathrm{AdS}_2) + 8\left(\frac{h'}{h''} - (x_2 - l)\right)J_2, \Delta = 2hh'' - h'^2$$

$$\hat{f}_1 = F_1 = \frac{c}{32L^4}h'''dx_1, \hat{f}_3 = \frac{c}{4L^2}\left((x_2 - l)h''' - h''\right)dx_1 \wedge J_2$$

$$\hat{f}_5 = c\left(2h' + (x_2 - l)(-2h'' + (x_2 - l)h''')\right)dx_1 \wedge J_2 \wedge J_2$$

$$x_1 \to r, x_2 \to \frac{1}{\pi}y, L^2 \to \frac{\pi}{2}, c \to 4\pi^2.$$

$$ds^2(\mathrm{S}^7/\mathbb{Z}_k) = \left(\frac{d\tau}{k} + \mathcal{A}\right)^2 + ds^2(\mathbb{CP}^3)$$

$$\frac{ds^2}{L^2} = \frac{\sqrt{2}h^{3/2}\sqrt{h''}}{\Delta}ds^2(\mathrm{AdS}_2) + \sqrt{2}\left(\frac{8\sqrt{h}}{\sqrt{h''}}ds^2(\mathbb{CP}^3) + \frac{\sqrt{h''}}{\sqrt{h}}(dx_i)^2\right), e^{-\Phi} = \frac{c\sqrt{h''}\sqrt{\Delta}}{32\sqrt{2}L^4\sqrt{h}}$$

$$H = dB, \frac{B}{L^2} = \left(\frac{hh'}{\Delta} + x_1\right)\mathrm{vol}(\mathrm{AdS}_2) - 8(x_2 - l)J_2, \Delta = 2hh'' - h'^2$$

$$\hat{f}_1 = F_1 = \frac{c}{32L^4}h'''dx_1, \hat{f}_3 = -\frac{c}{4L^2}(x_2 - l)h'''dx_2 \wedge J_2$$

$$\hat{f}_5 = c(d(2(x_2 - l)h') - 2(x_2 - l)h''dx_1 - (2h + h'''(x_2 - l)^2)dx_2)J_2 \wedge J_2$$

$$\hat{f}_7 = 8L^2c\left(dx_1(-(x_2 - l)^2h'' - 2h) - \frac{1}{3}dx_2(x_2 - l)(h'''(x_2 - l)^2 + 6h')\right.$$

$$\left. + d\left(\frac{h'(3(l - x_2)^2h'' + 2h)}{3h''}\right)\right).$$

$$L \to \frac{L}{22^{3/4}}, c \to \frac{L^2}{2\sqrt{2}c_0k}, x_1 \to -r, x_2 \to \frac{2\sqrt{2}k}{L^2}\tau,$$

$$\hat{h}(x_2),$$

$$\frac{ds^2}{L^2} = \frac{x_1^2\hat{h}\sqrt{\Delta}}{x_1^2\Delta - \hat{h}^2}ds^2(\mathrm{AdS}_2) + \frac{8\sqrt{\Delta}}{\hat{h}''}ds^2(\mathbb{CP}^3) + \frac{\sqrt{\Delta}}{\hat{h}}\left(dx_i^2\right)$$

$$e^{-\Phi} = \frac{c\sqrt{\hat{h}}\hat{h}''^{3/2}\sqrt{x_1^2\Delta - \hat{h}^2}}{16\sqrt{2}L^4\Delta}, \Delta = -\hat{h}'^2 + 2\hat{h}\hat{h}''$$

$$H = dB, \frac{B}{L^2} = x_1\left(1 + \frac{\hat{h}^2}{x_1^2\Delta - \hat{h}^2}\right)\mathrm{vol}(\mathrm{AdS}_2) + 8\left(\frac{\hat{h}'}{\hat{h}''} - (x_2 - l)\right)\wedge J_2$$

$$\hat{f}_1 = \frac{c}{L^4}\left(\hat{h}'''x_1dx_1 - \hat{h}''dx_2 - d\left(\frac{\hat{h}\hat{h}'\hat{h}''}{\Delta}\right)\right)$$

$$\hat{f}_3 = \frac{c}{4L^2}\left[d\left(\frac{\hat{h}\hat{h}''}{\Delta}\left(2\hat{h} - (x_2 - l)\hat{h}'\right)\right) - \left(\hat{h}'' - (x_2 - l)\hat{h}'''\right)x_1dx_1 - (x_2 - l)\hat{h}''dx_2\right]\wedge J_2$$

$$\hat{f}_5 = -\frac{c}{m}\left[x_1\left((x_2 - l)\left(2\hat{h}'' - (x_2 - l)\hat{h}'''\right) + 2\hat{h}'\right)dx_1 + \left(2\hat{h} + (x_2 - l)^2\hat{h}''\right)dx_2\right.$$
$$\left. + d\left(\frac{\hat{h}}{\Delta}\left((x_2 - l)\hat{h}''\left(4\hat{h} - (x_2 - l)\hat{h}'\right) - 2\hat{h}\hat{h}'\right)\right)\right]\wedge J_2\wedge J_2$$

$$x_1 \to \frac{\rho}{\pi}, x_2 \to r, L^2 \to \frac{\pi}{2}, c \to 4\pi^3.$$

$$x_1 \to r, x_2 \to \frac{1}{\pi}y, L^2 \to \frac{\pi}{2}, c \to 4\pi^2, \hat{h} \to h.$$

$$\frac{2}{\pi}ds^2 = \frac{h}{\sqrt{2hh'' - (h')^2}}ds^2(\mathrm{AdS}_2) + \frac{8}{h''}\sqrt{2hh'' - (h')^2}ds^2(\mathbb{CP}^3) + \frac{\sqrt{2hh'' - (h')^2}}{h}\left(dr^2 + \frac{1}{\pi^2}dy^2\right)$$

$$e^{\Phi} = \frac{\sqrt{2(2hh'' - (h')^2)}}{\sqrt{h(h'')^3}}, B = 4\pi\left(-(r - l) + \frac{h'}{h''}\right)J_2 + \frac{y}{2}\mathrm{vol}(\mathrm{AdS}_2)$$

$$\hat{f}_1 = -\frac{h'''}{2\pi}dy, \hat{F}_3 = 2(h'' - (r - l)h''')\Big)dy\wedge J_2$$

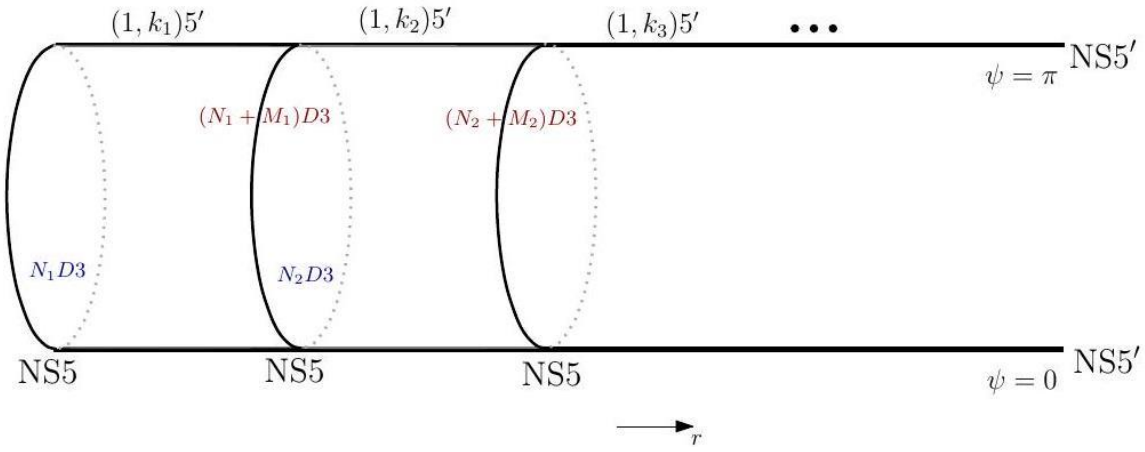
$$\hat{f}_5 = -4\pi\left(2h' + (r - l)(-2h'' + (r - l)h''')\right)dy\wedge J_2\wedge J_2$$

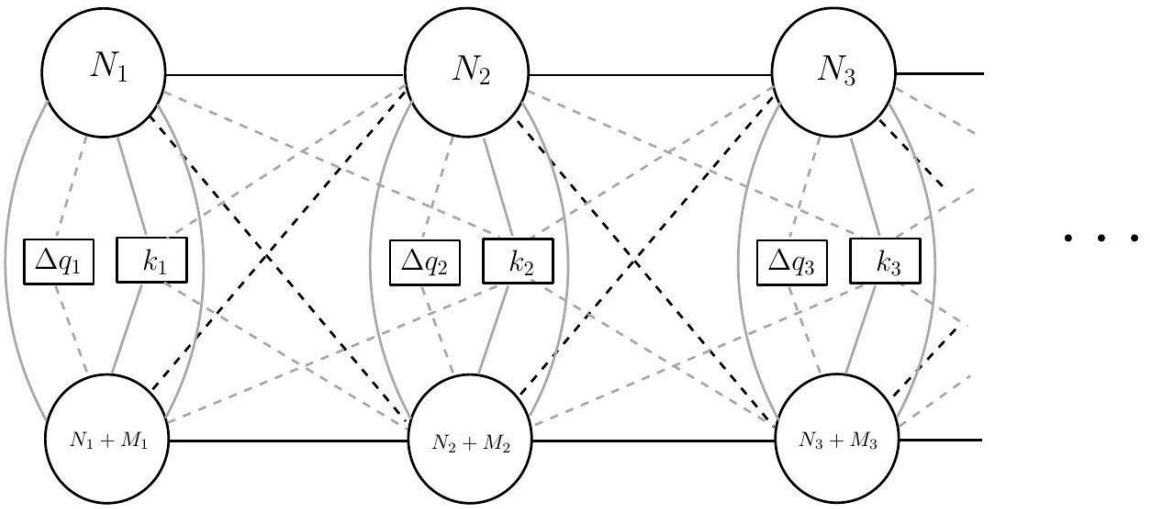
$$\hat{f}_7 = \frac{16\pi^2}{3}\left(6h - (r - l)\left(6h' + (r - l)((r - l)h''' - 3h'')\right)\right)dy\wedge J_2\wedge J_2\wedge J_2$$

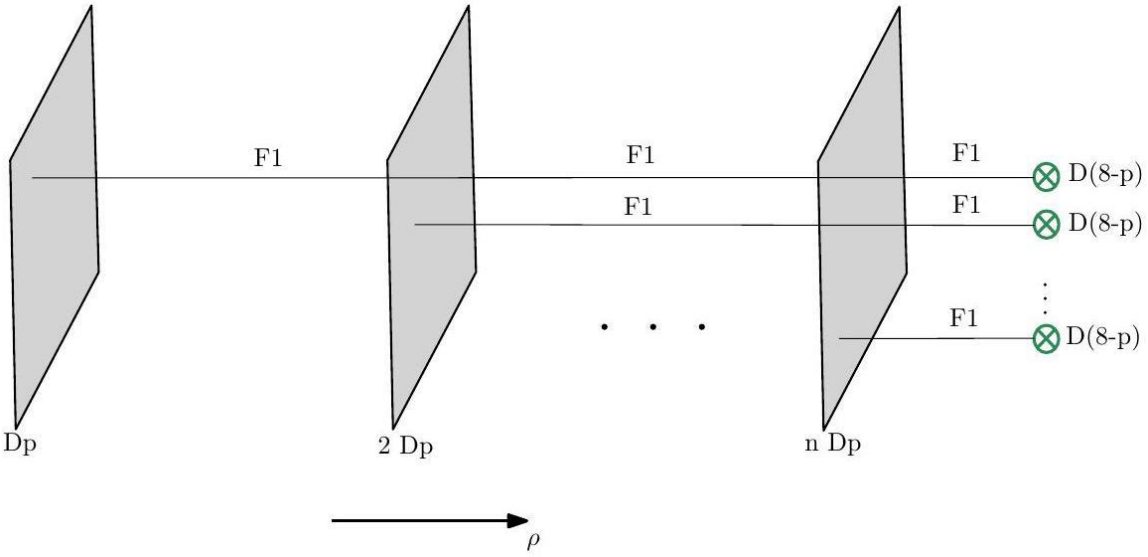
$$h'''' = 0.$$

$$\frac{1}{(2\pi)^2}\left|\int_{2\text{-cycle}} B\right| \in [0,1]$$

$$h = c_1 + c_2(r - r_0)^3$$
$$h = \big(c_1 + c_2(r - r_0)\big)(r - r_0)^2$$
$$h = c_1 + c_2(r - r_0) + \frac{c_2^2}{4c_1}(r - r_0)^2 + c_3(r - r_0)^3$$







Figuras 3, 4 y 5. Configuración de Supersimetría.

$$x_1 \to \frac{\rho}{\pi}, x_2 \to r, L^2 \to \frac{\pi}{2}, c \to 4\pi^3, \hat{h} \to h$$

$$\begin{aligned}
\frac{2}{\pi}ds^2 =& \frac{h}{\sqrt{2hh'' - (h')^2}}\frac{\rho^2}{\Xi}ds^2(\mathrm{AdS}_2) + \frac{8}{h''}\sqrt{2hh'' - (h')^2}ds^2(\mathbb{CP}^3) \\
&+ \frac{\sqrt{2hh'' - (h')^2}}{h}\left(dr^2 + \frac{1}{\pi^2}d\rho^2\right), e^{\Phi} = \frac{1}{\sqrt{\Xi}}\frac{\sqrt{2(2hh'' - (h')^2)}}{\sqrt{h(h'')^3}} \\
B =& 4\pi\left(-(r-l) + \frac{h'}{h''}\right)J_2 + \left(\frac{\rho^3}{2\Xi} - n\pi\right)\mathrm{vol}(\mathrm{AdS}_2), \Xi = \rho^2 - \frac{\pi^2 h^2}{2hh'' - (h')^2} \\
\hat{F}_1 =& \frac{h'''}{2\pi}\rho d\rho + \hat{F}_1^r \\
\hat{F}_3 =& -2(h'' - (r-l)h''')\rho d\rho \wedge J_2 + n\pi\mathrm{vol}(\mathrm{AdS}_2) \wedge \hat{F}_1^r + \hat{F}_3^{r,\mathbb{CP}^1} \\
\hat{F}_5 =& 4\pi\big(2h' + (r-l)(-2h'' + (r-l)h''')\big)\rho d\rho \wedge J_2 \wedge J_2 + n\pi\mathrm{vol}(\mathrm{AdS}_2) \wedge \hat{F}_3^{r,\mathbb{CP}^1} + \hat{F}_5^{r,\mathbb{CP}^2} \\
\hat{F}_7 =& \frac{16\pi^2}{3}\Big(6h - (r-l)\big(6h' + (r-l)((r-l)h''' - 3h'')\big)\Big)\rho d\rho \wedge J_2 \wedge J_2 \wedge J_2 \\
&+ n\pi\mathrm{vol}(\mathrm{AdS}_2) \wedge \hat{F}_5^{r,\mathbb{CP}^2} + \hat{F}_7^{r,\mathbb{CP}^3} \\
\hat{F}_9 =& n\pi\mathrm{vol}(\mathrm{AdS}_2) \wedge \hat{F}_7^{r,\mathbb{CP}^3}
\end{aligned}$$

$$\frac{\pi^2 h^2}{2hh'' - (h')^2} \to \frac{c_1\pi^2}{4c_2}(r - r_0)$$

$$S_{Dp} = -T_p \int_{(t, I_r, \mathbb{CP}^{(p-1)/2})} \hat{F}_p \wedge A_t = -T_{F1} Q'_{8-p} \int A_t$$

$$Q_1^{(n)} = \frac{1}{(2\pi)^6}\int \hat{F}_7^{\rho,\mathbb{CP}^3} = (2n+1)Q_1^{(0)}$$
$$Q_3^{(n)} = \frac{1}{(2\pi)^4}\int \hat{F}_5^{\rho,\mathbb{CP}^2} = (2n+1)Q_3^{(0)}$$
$$Q_5^{(n)} = \frac{1}{(2\pi)^2}\int \hat{F}_3^{\rho,\mathbb{CP}^1} = (2n+1)Q_5^{(0)}$$
$$Q_7^{(n)} = \int \hat{F}_1^{\rho} = (2n+1)Q_7^{(0)},$$

$$\pi h_l(r) = Q_{1,l}^{(0)} - Q_{3,l}^{(0)}(r-l) + \frac{1}{2}Q_{5,l}^{(0)}(r-l)^2 - \frac{1}{6}Q_{7,l}^{(0)}(r-l)^3 \text{ for } r \in [l, l+1]$$

$$Q_{1,l}^{(0)} = Q_{1,l-1}^{(0)} - Q_{3,l-1}^{(0)} + \frac{1}{2}Q_{5,l-1}^{(0)} - \frac{1}{6}Q_{7,l-1}^{(0)}$$
$$Q_{3,l}^{(0)} = Q_{3,l-1}^{(0)} - Q_{5,l-1}^{(0)} + \frac{1}{2}Q_{7,l-1}^{(0)}$$
$$Q_{5,l}^{(0)} = Q_{5,l-1}^{(0)} - Q_{7,l-1}^{(0)}$$

$$\hat{F}_{p+2}^{(\mathrm{AdS}_2, r, \mathbb{CP}^{(p-1)/2})} = n\pi \hat{F}_p^{(r,\mathbb{CP}^{(p-1)/2})} \wedge \mathrm{vol}_{\mathrm{AdS}_2},$$

$$Q_p^{(e)} = nQ'_{8-p} \text{ for } \rho \in [2n\pi, 2(n+1)\pi].$$

$$Q_{1,l}^{(n)} = N_l^{(n)} + \frac{1}{12}k_l^{(n)}$$
$$Q_{3,l}^{(n)} = M_l^{(n)} - \frac{1}{2}k_l^{(n)} + \frac{1}{12}q_l^{(n)}$$
$$Q_{5,l}^{(n)} = k_l^{(n)}$$
$$Q_{7,l}^{(n)} = -q_l^{(n)}$$

$$c_{hol} = \frac{3}{2^5\pi^8}V_{int} = \frac{3}{2^5\pi^8}\int d^8\xi e^{-2\Phi}\sqrt{\det(g_{ij})}$$

$$c_{hol} = \frac{3}{32\pi^4 m^2}\int dx_1 dx_2 (2h\,\square\, h - (\partial_1 h)^2 - (\partial_2 h)^2)$$

$$c_R = \frac{k(3k+13)}{k+3}$$

$$c_{1d} = \frac{3}{2^5\pi^4}\int dr d\rho(\rho^2(2hh'' - h'^2) - \pi^2 h^2)$$

$$c_{2d} = \frac{1}{2}\int dr(2hh'' - h'^2)$$

$$ds^2(\mathrm{S}^7) = ds^2(\mathbb{CP}^3) + (d\tau + \eta)^2, 4\eta = \cos^2\zeta(d\psi + 2\eta_1) - \sin^2\zeta(d\psi + 2\eta_2)$$
$$ds^2(\mathbb{CP}^3) = d\zeta^2 + \frac{1}{4}\cos^2\zeta ds^2(\mathrm{S}_1^2) + \frac{1}{4}\sin^2\zeta ds^2(\mathrm{S}_2^2) + \frac{1}{4}\cos^2\zeta\sin^2\zeta(d\psi + \eta_1 + \eta_2)^2$$
$$ds^2(\mathrm{S}_i^2) = d\theta_i^2 + \sin^2\theta_i d\phi_i^2, \eta_i = \cos\theta_i d\phi_i$$

$$J_2 = \frac{1}{2}\cos\ \zeta \sin\ \zeta d\zeta \wedge (d\psi + \eta_1 + \eta_2) - \frac{1}{4}\cos^2\ \zeta \mathrm{vol}(\ \mathrm{S}_1^2) + \frac{1}{4}\sin^2\ \zeta \mathrm{vol}(\ \mathrm{S}_2^2)$$

$$e^1 = d\zeta, e^2 = \frac{1}{2}\cos\ \zeta d\theta_1, e^3 = \frac{1}{2}\cos\ \zeta \sin\ \theta_1 d\phi_1, e^4 = \frac{1}{2}\sin\ \zeta d\theta_2, e^5 = \frac{1}{2}\sin\ \zeta \sin\ \theta_2 \phi_2$$
$$e^6 = \frac{1}{2}\cos\ \zeta \sin\ \zeta (d\psi + \eta_1 + \eta_2), e^7 = d\tau + \eta$$

$$\gamma_1 = \sigma_1 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2, \gamma_{2,3,4} = \sigma_2 \otimes \sigma_{1,2,3} \otimes \mathbb{I}_2, \gamma_{5,6,7} = \sigma_3 \otimes \mathbb{I}_2 \otimes \sigma_{1,2,3}$$

$$B = \sigma_3 \otimes \sigma_2 \otimes \sigma_2$$

$$\nabla_a \xi_\pm = \pm\frac{i}{2}\gamma_a \xi_\pm$$

$$\xi_\pm = \mathcal{M}_\pm \xi_\pm^0$$
$$\mathcal{M}_\pm = e^{\frac{\zeta}{2}(\pm i\gamma_7 + \gamma_{67})} e^{\frac{1}{8}(4\tau+\psi)(\pm\gamma_7 - \gamma_{23})} e^{\frac{\theta_1}{4}(\pm i\gamma_2 + \gamma_{37})} e^{\frac{\theta_1}{4}(\pm i\gamma_2 + \gamma_{23})} e^{\frac{1}{8}(-4\tau+\psi)(\gamma_{16} - \gamma_{45})} e^{\frac{\theta_2}{4}(\gamma_{14} + \gamma_{56})} e^{\frac{\phi_2}{4}(\gamma_{16} + \gamma_{45})}$$

$$\xi_{8\pm}^I = \mathcal{M}_\pm \eta^I, \eta^I = \sum_{J=1}^{8} \delta^{IJ}$$

$$\xi_{4-}^{\mathrm{i}} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i(\xi_{8-}^3 + \xi_{8-}^8) \\ -i(\xi_{8-}^2 - \xi_{8-}^5) \\ -(\xi_{8-}^1 + \xi_{8-}^6) \\ (\xi_{8-}^1 - \xi_{8-}^6) \end{pmatrix}^{\mathrm{i}}, \xi_{4+}^{\mathrm{i}} = \frac{1}{\sqrt{2}} \begin{pmatrix} i(\xi_{8-}^2 + \xi_{8-}^5) \\ i(\xi_{8-}^3 - \xi_{8-}^8) \\ (\xi_{8-}^4 + \xi_{8-}^7) \\ i(\xi_{8-}^4 - \xi_{8-}^7) \end{pmatrix}^{\mathrm{i}}$$

$$\partial_\tau \xi_{4\pm}^{\mathrm{i}} = \pm i\xi_{4\pm}^{\mathrm{i}}, \xi_{4-}^{\mathrm{i}c} = \xi_{4+}^{\mathrm{i}}$$

$$\xi_-^I = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_{4-}^{\mathrm{i}} + \xi_{4+}^{\mathrm{i}} \\ i(\xi_{4-}^{\mathrm{i}} - \xi_{4+}^{\mathrm{i}}) \end{pmatrix}$$

$$\partial_\tau(\xi_{8+}^2 - \xi_{8+}^5) = -2i(\xi_{8+}^2 - \xi_{8+}^5)\ \partial_\tau(\xi_{8+}^3 - \xi_{8+}^8) = 2i(\xi_{8+}^3 - \xi_{8+}^8),$$

$$\xi_2^{\mathrm{a}} = \frac{1}{2} \begin{pmatrix} \xi_{8+}^3 - \xi_{8+}^8 + \xi_{8+}^2 - \xi_{8+}^5 \\ i(\xi_{8+}^3 - \xi_{8+}^3 - \xi_{8+}^2 + \xi_{8+}^5) \end{pmatrix}, \xi_6^{\mathrm{J}} = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_{8+}^1 - \xi_{8+}^4 \\ i(\xi_{8+}^1 + \xi_{8+}^4) \\ \xi_{8+}^6 - \xi_{7+}^1 \\ i(\xi_{8+}^6 + \xi_{7+}^1) \\ \frac{1}{\sqrt{2}}(\xi_{8+}^2 + \xi_{8+}^5 + \xi_{8+}^3 + \xi_{8+}^8) \\ i\frac{1}{\sqrt{2}}(\xi_{8+}^2 + \xi_{8+}^5 - \xi_{8+}^3 - \xi_{8+}^8) \end{pmatrix}.$$

$$\psi_+ = \frac{e^A}{16}\mathrm{Re}\left(\psi_+^{(7)} + \cos\ \beta \psi_-^{(7)} \wedge V\right), \hat{\psi}_- = \frac{e^A \sin\ \beta}{16}\mathrm{Re}\psi_-^{(7)}$$
$$\psi_+^{(7)} = e^{i\alpha} e^{ie^{2C}\tilde{J}_2} + ie^{3C} e^{i\delta}\Omega_3 \wedge U, \psi_-^{(7)} = e^{i\alpha} e^{ie^{2C}\tilde{J}_2} \wedge U + ie^{3C} e^{i\delta}\Omega_3$$

$$\sin\ \alpha = 0, me^C = 6e^A \sin\ \beta \sin\ \delta$$

$$\cos\,\delta U \wedge V = 0$$
$$d\left(e^{2A+3C}e^{-\Phi}\sin\,\beta\cos\,\delta\right) + e^{A+3C}e^{-\Phi}m\cos\,\delta\cos\,\beta V + e^{A+2C}e^{-\Phi}(4e^{A}\sin\,\beta - e^{C}m\sin\,\delta)U = 0$$

$$\hat{g}_1 = \frac{cm^3}{32L^2}\left(d\left(\frac{\partial_2 h(x_1\partial_1 h - h)\,\square\, h}{\Delta_1}\right) - dx_1x_1\partial_2\,\square\, h + dx_2(x_1\partial_1\,\square\, h - \square\, h)\right),$$

$$\hat{g}_3 = cm\left(d\left(\frac{(2h-(x_2-l)\partial_2 h)(h - x_1\partial_1 h)\,\square\, h}{4\Delta_1}\right)\right.$$
$$\left. + \frac{1}{4}\left(dx_2(x_2-l)(x_1\partial_1\,\square\, h - \square\, h) - dx_1x_1\left((x_2-l)\partial_2\,\square\, h - \square\, h\right)\right)\right)\wedge J_2,$$

$$\hat{g}_5 = \frac{cL^2}{m}\left[-d\left(\frac{(h-x_1\partial_1 h)\left((l-x_2)^2\partial_2 h\,\square\, h + 2h(\partial_2 h + 2(l-x_2)\,\square\, h)\right)}{\Delta_1}\right)\right.$$
$$-dx_1x_1(-2(x_2-l)\,\square\, h + (x_2-l)^2\partial_2\,\square\, h + 2\partial_2 h)$$
$$\left. + dx_2\left((x_2-l)^2(x_1\partial_1\,\square\, h - \square\, h) - 2(h - x_1\partial_1 h)\right)\right]\wedge J_2\wedge J_2,$$

$$\hat{g}_7 = \left[-\frac{8cL^4}{3m^3}(dx_2((x_2-l)^3(\square\, h - x_1\partial_1\,\square\, h) - 6(x_2-l)x_1\partial_1 h + 4(x_2-l)h + 2(x_2-l)h)\right.$$
$$+dx_1(-3(x_2-l)^2x_1\,\square\, h + (x_2-l)^3x_1\partial_2\,\square\, h + 6(x_2-l)x_1\partial_2 h - 6x_1 h)\big)$$
$$\left. + \frac{8}{3}\frac{cL^4}{m^3}d\left(\frac{(h-x_1\partial_1 h)((x_2-l)^3(-\partial_2 h)\,\square\, h - 6(x_2-l)h(\partial_2 h - (x_2-l)\,\square\, h) + 4h^2)}{\Delta_1}\right)\right]$$
$$\wedge J_2\wedge J_2\wedge J_2.$$

$$\hat{F}_- = \hat{f}_- + \mathrm{vol}(\mathrm{AdS}_2)\wedge\hat{g}_-$$

$$\hat{g}_1 = -\frac{1}{3}\frac{c}{2^5L^2}\hat{h}'''d(x_1^3)$$
$$\hat{g}_3 = \frac{1}{3}\frac{c}{2^2}\left(\hat{h}'' - \hat{h}'''(x_2-l)\right)d(x_1^3)\wedge J_2$$

$$\frac{ds^2}{L^2} = \sqrt{\Delta_1}\left[\frac{x_2\hat{h}}{\Delta_2}ds^2(\mathrm{AdS}_2) + \frac{1}{m^2}\left(\frac{8}{x_2\hat{h}''}ds^2(\mathbb{CP}^3) + \frac{1}{x_2\hat{h}}(dx_i)^2\right)\right], e^{-\Phi} = \frac{m^3x_2^2\sqrt{\hat{h}}\hat{h}''^{3/2}}{16\sqrt{2}L^4\Delta_1}$$
$$H = dB, \frac{B}{L^2} = \left(\frac{x_2^2\hat{h}\hat{h}'}{\Delta_2} + x_1\right)\mathrm{vol}(\mathrm{AdS}_2) + \frac{8}{m^2}\left(\frac{\hat{h}}{x_2\hat{h}''} - (x_2-l)\right)J_2$$

$$\Delta_1 = 2x_2^2\hat{h}\hat{h}'' - \hat{h}^2, \Delta_2 = x_2^2\left(2\hat{h}\hat{h}'' - \hat{h}'^2\right) - \hat{h}^2$$

$$\hat{f}_1 = F_1 = \frac{m^3}{32L^4}\left[\hat{h}''dx_1 - x_2\hat{h}'''dx_2 - d\left(\frac{x_2^2\hat{h}\hat{h}'\hat{h}''}{\Delta_1}\right)\right]$$

$$\hat{f}_3 = \frac{m}{4L^2}\left[-l\hat{h}''dx_1 - x_2(x_2-l)\hat{h}'''dx_2 + d\left(\frac{x_2^2(x_2+l)\hat{h}\hat{h}'\hat{h}''}{\Delta_1}\right)\right] \wedge J_2$$

$$\hat{f}_5 = \frac{1}{m}\left[\left(2\hat{h} - (x_2-l)(x_2+l)\hat{h}''\right)dx_1 - x_2\left(2\hat{h}' + (x_2-l)^2\hat{h}'''\right)dx_2 + d\left(\frac{x_2^2\hat{h}\hat{h}'\left(-2\hat{h} + (x_2-l)(3x_2+l)\hat{h}''\right)}{\Delta_1}\right)\right] \wedge J_2 \wedge J_2$$

$$\hat{f}_7 = \frac{8L^2}{m^3}\left[\left(-2l\hat{h} - \frac{1}{3}(x_2-l)^2(2x_2+l)\hat{h}''\right)dx_1 - 2\left(x_2(x_2-l)\hat{h}' + \frac{1}{6}x_2(x_2-l)^3\hat{h}'''\right)dx_2 + d\left(\frac{x_2^2\hat{h}\hat{h}'\left(-(2x_2-6l)\hat{h} + (x_2-l)^2(5x_2+l)\hat{h}''\right)}{3\Delta_1}\right)\right] \wedge J_2 \wedge J_2 \wedge J_2$$

Supergravedad cuántica relativista tipo II: Modelamiento Matemático.

$$S_0 = \int_M \Re\sigma^2 + \bar{\psi}_\alpha \not{D}\psi^\alpha + \bar{\rho}\not{D}\rho + 2\bar{\rho}D_\alpha\psi^\alpha - \frac{1}{768}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\rho}\gamma^{abc}\rho) - \frac{1}{384}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\psi}\beta\gamma^{abc}\psi^\beta)$$

$$F(\delta_1\mathcal{L}, \delta_2\mathcal{L}) \sim [\delta_1\mathcal{L}, \delta_2\mathcal{L}]$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\rho = \delta_V\rho + \delta_\zeta\rho - \frac{1}{2}VEL[\rho]$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\psi_\alpha = \delta_V\psi_\alpha + \delta_\zeta\psi_\alpha + \left(\frac{1}{4}\epsilon_{[2}\bar{\epsilon}_{1]} - \frac{1}{2}V\right)EL[\psi]_\alpha$$

$$\begin{aligned}S = \int_M\ & \Re\sigma^2 + \bar{\psi}_\alpha\not{D}\psi^\alpha + \bar{\rho}\not{D}\rho + 2\bar{\rho}D_\alpha\psi^\alpha \\ & - \frac{1}{768}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\rho}\gamma^{abc}\rho) - \frac{1}{384}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)(\bar{\psi}_\beta\gamma^{abc}\psi^\beta) \\ & + \sigma^*\left[\mathcal{L}_\xi\sigma - \frac{1}{8}\sigma^{-1}(\bar{\rho}e)\right] + \mathcal{C}^*_{a\beta}\left[(\mathcal{L}_\xi\mathcal{G})^{a\beta} + \frac{1}{2}\sigma^{-2}(\bar{e}\gamma^a\psi^\beta)\right] \\ & + \bar{\rho}^*\left[\mathcal{L}_\xi\rho + \not{D}e + \frac{1}{192}\sigma^{-2}(\bar{\psi}_\beta\gamma_{abc}\psi^\beta)\gamma^{abc}e\right] \\ & + \bar{\psi}^*_\beta\left[(\mathcal{L}_\xi\psi)^\beta + D^\beta e + \frac{1}{8}\sigma^{-2}(\bar{\psi}^\beta\rho)e - \frac{1}{8}\sigma^{-2}(\bar{\psi}^\beta\gamma_a e)\gamma^a\rho\right] \\ & + \bar{e}^*\left[\mathcal{L}_\xi e + \frac{1}{16}\sigma^{-2}(\bar{e}\gamma_a e)\gamma^a\rho\right] + \left\langle\xi^*, \mathcal{D}f + \frac{1}{2}\mathcal{L}_\xi\xi\right\rangle - \frac{1}{8}\xi^*_a\sigma^{-2}(\bar{e}\gamma^a e) \\ & + \frac{1}{2}f^*\left(\mathcal{L}_\xi f + \frac{1}{8}\sigma^{-2}(\bar{e}\gamma_a e)\xi^a - \frac{1}{6}\langle\xi, \mathcal{L}_\xi\xi\rangle\right) \\ & - \frac{1}{64}\sigma^{-2}(\bar{e}\gamma_a e)(\bar{\psi}^*_\beta\gamma^a\psi^{*\beta}) - \frac{1}{32}\sigma^{-2}(\bar{e}\psi^*_\beta)(\bar{e}\psi^{*\beta}) - \frac{1}{64}\sigma^{-2}(\bar{e}\gamma_a e)(\bar{\rho}^*\gamma^a\rho^*)\end{aligned}$$

$$\{S, S\} = 0$$

$$\langle V, \mathcal{D}f\rangle := a(V)f.$$

$$0 \longrightarrow T^*M \xrightarrow{a^*} E \xrightarrow{a} TM \longrightarrow 0.$$

$$a([V,W]) = [a(V), a(W)].$$

$$\mathcal{L}_V W := [V,W]$$

$$\mathcal{L}_V f := L_{a(V)} f.$$

$$\mathcal{L}_{\mathcal{D}f} = 0, \forall f \in C^\infty(M).$$

$$\mathcal{L}_V \mu := L_{a(V)} \mu.$$

$$H \to M$$

$$\mathcal{H}^* := \{\sigma \in \Gamma(H) \mid \sigma \text{ everywhere nonzero }\}.$$

$$TU \oplus T^*U \oplus (\mathfrak{g} \times U),$$

$$\begin{aligned} a(v+\lambda+s) &:= v \\ \langle v_1+\lambda_1+s_1, v_2+\lambda_2+s_2\rangle &:= \lambda_1(v_2) + \lambda_2(v_1) + \mathrm{Tr}(s_1 s_2) \\ [v_1+\lambda_1+s_1, v_2+\lambda_2+s_2] &:= L_{v_1} v_2 + \left(L_{v_1}\lambda_2 - i_{v_2} d\lambda_1 + \mathrm{Tr}(s_2 d s_1)\right) + \left(L_{v_1} s_2 - L_{v_2} s_1 + [s_1,s_2]_{\mathfrak{g}}\right) \end{aligned}$$

$$E = C_+ \oplus C_-$$

$$C_+ = \left\{ v + \left(i_v g + i_v B - \frac{1}{2}\mathrm{Tr}(A i_v A) \right) + i_v A \,\middle|\, v \in TM \right\}$$

$$\sigma^2 = \sqrt{|g|} e^{-2\varphi}$$

$$e_A = \{e_a, e_\alpha\}, \langle e_A, e_B \rangle = \eta_{AB}$$

$$c_{ABC} := \langle [e_A, e_B], e_C \rangle$$

$$\rho \in \Gamma(\Pi S(C_+)_+ \otimes H), \psi \in \Gamma(\Pi S(C_+)_- \otimes C_- \otimes H)$$

$$\rho = \sqrt[4]{2}\sigma\rho, \psi = \sqrt[4]{2}\sigma\psi + \frac{1}{\sqrt[4]{2}}\sigma\chi$$

$$\begin{aligned} D: \Gamma(E) \times \Gamma(E) &\to \Gamma(E) \\ (V,W) &\mapsto D_V W, \end{aligned}$$

$$D_{fV} W = f D_V W, D_V(fW) = f D_V W + (a(V)f)W, D\langle\cdot,\cdot\rangle = 0$$

$$D_V f := a(V) f$$

$$D_V \mu := L_{a(V)} \mu - \nu (D_A V^A) \mu$$

$$D_{e_A} e_B := \Gamma_A^C{}_B e_C$$

$$DG = 0, D\sigma = 0$$

$$\Gamma_{[ABC]} = -\frac{1}{3}c_{ABC}$$

$$\mathrm{div}V := D_A V^A = \sigma^{-2}\mathcal{L}_V\sigma^2,$$

$$D_{V_+}:\Gamma(C_-)\to\Gamma(C_-), D_{V_-}:\Gamma(C_+)\to\Gamma(C_+)$$

$$\Gamma(S(C_+)\otimes H) \text{ or } \Gamma(S(C_+)\otimes C_-\otimes H),$$

$$\not{D} := \gamma^a D_a$$

$$\not{D}\rho, \not{D}\psi_\alpha, D_\alpha\rho, D_\alpha\psi^\alpha$$

$$[\not{D}, D_\alpha]\rho = \frac{1}{4}\mathcal{R}_{a\alpha}\gamma^a\rho$$

$$\left(\not{D}^2 + D_\alpha D^\alpha\right)\rho = -\frac{1}{8}\mathcal{R}\rho$$

$$\int \sigma^2\mathcal{R} = \int \sqrt{|g|}e^{-2\varphi}\left(R + 4|\nabla\varphi|^2 - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}\right),$$

$$\mathcal{S}_0 \to \mathcal{M}\times\mathcal{H}^*,$$

$$\Gamma(\Pi S(C_+)_+\otimes H)\oplus\Gamma(\Pi S(C_+)_-\otimes C_-\otimes H).$$

$$F(\delta_1\mathcal{L}_2, \delta_2\mathcal{L})\rho = \frac{1}{16}[\delta_1\mathcal{G}, \delta_2\mathcal{L}_{ab}\gamma^{ab}\rho$$

$$F\left(\delta_1\mathcal{L}, \delta_2\mathcal{L}\right)\psi^\alpha = \frac{1}{16}\Big[\delta_1\mathcal{L}, \delta_2\mathcal{L}_{ab}\gamma^{ab}\psi^\alpha + \frac{1}{4}\big[\delta_1\mathcal{L}, \delta_2\mathcal{L}^\alpha{}_\beta\psi^\beta.$$

$$\mathcal{L}_V\chi = D_V\chi + \frac{1}{2}(D_aV_b)\gamma^{ab}\chi$$

$$\mathcal{L}_V\lambda = D_V\lambda + \frac{1}{2}(D_aV_b)\gamma^{ab}\lambda + \frac{1}{2}(D_AV^A)\lambda$$

$$S_0 = \int_M \Re\sigma^2 + \bar\psi_\alpha\not{D}\psi^\alpha + \bar\rho\not{D}\rho + 2\bar\rho D_\alpha\psi^\alpha - \frac{1}{768}\sigma^{-2}(\bar\psi_\alpha\gamma_{abc}\psi^\alpha)\left(\bar\rho\gamma^{abc}\rho\right)$$
$$-\frac{1}{384}\sigma^{-2}(\bar\psi_\alpha\gamma_{abc}\psi^\alpha)\left(\bar\psi\beta\gamma^{abc}\psi^\beta\right)$$

$$\delta_V(\text{ field }) = \mathcal{L}_V(\text{ field }).$$

$$\delta_\epsilon\mathcal{G}_{ab} = \delta_\epsilon\mathcal{G}_{\alpha\beta} = 0, \delta_\epsilon\mathcal{G}_{a\beta} = \delta_\epsilon\mathcal{G}_{\beta a} = \frac{1}{2}\sigma^{-2}\bar\epsilon\gamma_a\psi_\beta$$
$$\delta_\epsilon\sigma = \frac{1}{8}\sigma^{-1}(\bar\rho\epsilon)$$
$$\delta_\epsilon\rho = \not{D}\epsilon + \frac{1}{192}\sigma^{-2}(\bar\psi_\alpha\gamma_{cde}\psi^\alpha)\gamma^{cde}\epsilon$$
$$\delta_\epsilon\psi_\alpha = D_\alpha\epsilon + \frac{1}{8}\sigma^{-2}(\bar\psi_\alpha\rho)\epsilon + \frac{1}{8}\sigma^{-2}(\bar\psi_\alpha\gamma_c\epsilon)\gamma^c\rho$$

$$\delta_\epsilon \rho = \not{\partial}\rho + \frac{1}{4}\delta \mathcal{C}_{a\alpha}\gamma^a \psi^\alpha$$

$$\delta_\epsilon \psi_\alpha = D_\alpha \epsilon + \frac{1}{8}\sigma^{-2}(\bar{\psi}_\alpha \rho)\epsilon - \frac{1}{4}\delta \mathcal{C}_{a\alpha}\gamma^a \rho$$

$$T_{\mathcal{G}}\mathcal{M} \cong \Gamma(C_+ \otimes C_-).$$

$$EL[\mathcal{L}]_{a\alpha} := \mathcal{R}_{a\alpha} + \sigma^{-2}\left(\frac{1}{2}\bar{\psi}_\beta \gamma_a D_\alpha \psi^\beta + \bar{\psi}_\alpha \gamma_a D_\beta \psi^\beta - \bar{\psi}_\beta \gamma_a D^\beta \psi_\alpha + \frac{1}{2}\bar{\rho}\gamma_a D_\alpha \rho - \frac{1}{2}\bar{\psi}_\alpha D_a \rho\right.$$
$$\left. + \frac{1}{4}\bar{\rho}\gamma_{ab} D^b \psi_\alpha - \frac{1}{4}\bar{\psi}_\alpha \gamma_{ab} D^b \rho\right) = 0$$

$$EL[\sigma] := \mathbb{R} + \sigma^{-2}(2\bar{\psi}^\alpha D_\alpha \rho + 2\bar{\rho} D_\alpha \psi^\alpha) + \sigma^{-4}\left[\frac{1}{768}(\bar{\psi}_\alpha \gamma_{cde}\psi^\alpha)(\bar{\rho}\gamma^{cde}\rho)\right.$$
$$\left. + \frac{1}{384}(\bar{\psi}_\alpha \gamma_{cde}\psi^\alpha)(\bar{\psi}_\beta \gamma^{cde}\psi^\beta)\right] = 0$$

$$EL[\rho] := \not{\partial}\rho + D_\alpha \psi^\alpha - \frac{1}{768}\sigma^{-2}(\bar{\psi}_\alpha \gamma_{cde}\psi^\alpha)\gamma^{cde}\rho = 0$$

$$EL[\psi]^\alpha := \not{\partial}^\alpha \psi^\alpha - D^\alpha \rho - \sigma^{-2}\left[\frac{1}{768}(\bar{\rho}\gamma_{cde}\rho)\gamma^{cde}\psi^\alpha + \frac{1}{192}(\bar{\psi}_\beta \gamma_{cde}\psi^\beta)\gamma^{cde}\psi^\alpha\right] = 0$$

$$\left[\delta_{V_1}, \delta_{V_2}\right] = \delta_{\mathcal{L}_{V_1}V_2}, [\delta_V, \delta_\epsilon] = \delta_{\mathcal{L}_V \epsilon}$$

$$\left[\delta_{\epsilon_1}, \delta_{\epsilon_2}\right] = \delta_V + \delta_\zeta$$

$$V^a := \frac{1}{4}\sigma^{-2}\bar{\epsilon}_2 \gamma^a \epsilon_1, \zeta := -\frac{1}{2}V\rho + \delta_{\epsilon_1}\epsilon_2 - \delta_{\epsilon_2}\epsilon_1$$

$$\left[\delta_{\epsilon_1}, \delta_{\epsilon_2}\right]\rho = \delta_V \rho + \delta_\zeta \rho - \frac{1}{2}VEL[\rho]$$
$$\left[\delta_{\epsilon_1}, \delta_{\epsilon_2}\right]\psi_\alpha = \delta_V \psi_\alpha + \delta_\zeta \psi_\alpha + \left(\frac{1}{4}\sigma^{-2}\epsilon_{[2}\bar{\epsilon}_{1]} - \frac{1}{2}V\right)EL[\psi]_\alpha$$

$$\mathcal{S} \to \mathcal{M} \times \mathcal{H}^*,$$

$$\Gamma(\Pi S(C_+)_+ \otimes H) \times \Gamma(\Pi S(C_+)_- \otimes C_- \otimes H) \times \Gamma(\Pi S(C_+)_- \otimes H)[1] \times \Gamma(E)[1] \times C^\infty(M)[2]$$

$$\mathcal{C}_{\mathcal{L}} \in \mathcal{M},$$
$$\sigma \in \mathcal{H}^*$$
$$\rho \in \Gamma(\Pi S(C_+)_+ \otimes H)$$
$$\psi \in \Gamma(\Pi S(C_+)_- \otimes C_- \otimes H)$$
$$\xi \in \Gamma(E)[1]$$
$$e \in \Gamma(\Pi S(C_+)_- \otimes H)[1]$$
$$f \in C^\infty(M)[2]$$

$$\mathcal{F}_{BV} := T^*[-1]\mathcal{S}.$$

$$T\mathcal{S} \cong \pi^* T(\mathcal{M} \times \mathcal{H}^*) \oplus \pi^* \mathcal{S}$$

$$T^*[-1]\mathcal{S} \cong \pi^* T^*[-1](\mathcal{M} \times \mathcal{H}^*) \oplus \pi^* \mathcal{S}^*[-1]$$

$$(\mathcal{G}, \sigma, \psi, \rho, \xi, e, f) \in \mathcal{S}$$

$$\mathcal{G}^* \in T^*_{\mathcal{G}}[-1]\mathcal{M} \cong \Gamma(C_+ \otimes C_- \otimes H^2)[-1],$$
$$\sigma^* \in T^*_{\sigma}[-1]\mathcal{H}^* \cong \Gamma(H)[-1],$$
$$\rho^* \in \Gamma(\Pi S(C_+)_- \otimes H)[-1],$$
$$\psi^* \in \Gamma(\Pi S(C_+)_+ \otimes C_- \otimes H)[-1],$$
$$\xi^* \in \Gamma(E \otimes H^2)[-2],$$
$$e^* \in \Gamma(\Pi S(C_+)_+ \otimes H)[-2],$$
$$f^* \in \Gamma(H^2)[-3].$$

$$E^* \cong E, C^*_{\pm} \cong C_{\pm}, S(C_+)^*_{\pm} \cong S(C_+)_{\mp}.$$

$$\frac{1}{2}\{S,S\} = \int_M \frac{\delta S}{\delta \mathcal{G}^*_{a\alpha}}\frac{\delta S}{\delta \mathcal{G}^{a\alpha}_{\alpha}} + \frac{\delta S}{\delta \sigma^*}\frac{\delta S}{\delta \sigma} + \frac{\delta S}{\delta \bar{\rho}^*}\frac{\delta S}{\delta \rho} + \frac{\delta S}{\delta \bar{\psi}^*_{\alpha}}\frac{\delta S}{\delta \psi^{\alpha}} + \frac{\delta S}{\delta \xi^*}\frac{\delta S}{\delta \xi} + \frac{\delta S}{\delta \bar{e}^*}\frac{\delta S}{\delta e} + \frac{\delta S}{\delta f^*}\frac{\delta S}{\delta f}$$
$$+ \frac{1}{16}\frac{\delta S}{\delta \mathcal{G}^*_{a\alpha}}\frac{\delta S}{\delta \mathcal{G}^{*\alpha}_{b}}(\bar{\rho}^*\gamma_{ab}\rho + \bar{\psi}^*_{\alpha}\gamma_{ab}\psi^{\alpha} + \bar{e}^*\gamma_{ab}e) + \frac{1}{4}\frac{\delta S}{\delta \mathcal{G}^*_{a\alpha}}\frac{\delta S}{\delta \mathcal{G}^{*a}_{\beta}}(\bar{\psi}^*_{\alpha}\psi_{\beta})$$

$$\frac{1}{2}\{S,S\} = \int_M \frac{\delta S}{\delta \phi^*_i}\frac{\delta S}{\delta \phi^i} + \frac{1}{2}\sum_{\phi \in \{\rho,\psi,e\}} \bar{\phi}^* F\left(\frac{\delta S}{\delta \mathcal{G}^*}, \frac{\delta S}{\delta \mathcal{G}^*}\right)\phi$$

$$\begin{aligned} S = \int_M \quad & \mathcal{R}\sigma^2 + \bar{\psi}_{\alpha}\not{\partial}\psi^{\alpha} + \bar{\rho}\not{\partial}_{\rho} + 2\bar{\rho}D_{\alpha}\psi^{\alpha} \\ & -\frac{1}{768}\sigma^{-2}(\bar{\psi}_{\alpha}\gamma_{abc}\psi^{\alpha})(\bar{\rho}\gamma^{abc}\rho) - \frac{1}{384}\sigma^{-2}(\bar{\psi}_{\alpha}\gamma_{abc}\psi^{\alpha})(\bar{\psi}_{\beta}\gamma^{abc}\psi^{\beta}) \\ & +\sigma^*\left[\mathcal{L}_{\xi}\sigma - \frac{1}{8}\sigma^{-1}(\bar{\rho}e)\right] + \mathcal{G}^*_{a\beta}\left[(\mathcal{L}_{\xi}\mathcal{G})^{a\beta} + \frac{1}{2}\sigma^{-2}(\bar{e}\gamma^{a}\psi^{\beta})\right] \\ & +\bar{\rho}^*\left[\mathcal{L}_{\xi}\rho + \not{\partial}e + \frac{1}{192}\sigma^{-2}(\bar{\psi}_{\beta}\gamma_{abc}\psi^{\beta})\gamma^{abc}e\right] \\ & +\bar{\psi}^*_{\beta}\left[(\mathcal{L}_{\xi}\psi)^{\beta} + D^{\beta}e + \frac{1}{8}\sigma^{-2}(\bar{\psi}^{\beta}\rho)e - \frac{1}{8}\sigma^{-2}(\bar{\psi}^{\beta}\gamma_{a}e)\gamma^{a}\rho\right] \\ & +\bar{e}^*\left[\mathcal{L}_{\xi}e + \frac{1}{16}\sigma^{-2}(\bar{e}\gamma_{a}e)\gamma^{a}\rho\right] + \left\langle \xi^*, \mathcal{D}f + \frac{1}{2}\mathcal{L}_{\xi}\xi\right\rangle - \frac{1}{8}\xi^*_{a}\sigma^{-2}(\bar{e}\gamma^{a}e) \\ & +\frac{1}{2}f^*\left(\mathcal{L}_{\xi}f + \frac{1}{8}\sigma^{-2}(\bar{e}\gamma_{a}e)\xi^{a} - \frac{1}{6}\langle \xi, \mathcal{L}_{\xi}\xi\rangle\right) \\ & -\frac{1}{64}\sigma^{-2}(\bar{e}\gamma_{a}e)(\bar{\psi}^*_{\beta}\gamma^{a}\psi^{*\beta}) - \frac{1}{32}\sigma^{-2}(\bar{e}\psi^*_{\beta})(\bar{e}\psi^{*\beta}) - \frac{1}{64}\sigma^{-2}(\bar{e}\gamma_{a}e)(\bar{\rho}^*\gamma^{a}\rho^*) \end{aligned}$$

$$\{S,S\} = 0$$

$$\sigma^*, \mathcal{L}^*, \rho^*, \psi^*, e^*, \xi^*, f^*$$

$$S = S_0 + S_{\text{lin}} + S_{\text{quad}}$$

$$\begin{aligned} \frac{1}{2}\{S,S\} = & \int_M \frac{\delta S_{\text{lin}}}{\delta \phi^*_i}\frac{\delta S_0}{\delta \phi^i} \\ & + \int_M \frac{\delta S_{\text{lin}}}{\delta \phi^*_i}\frac{\delta S_{\text{lin}}}{\delta \phi^i} + \frac{\delta S_{\text{quad}}}{\delta \phi^*_i}\frac{\delta S_0}{\delta \phi^i} + \frac{1}{2}\sum_{\phi \in \{\rho,\psi,e\}} \bar{\phi}^* F\left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}\right)\phi \\ & + \int_M \frac{\delta S_{\text{lin}}}{\delta \phi^*_i}\frac{\delta S_{\text{quad}}}{\delta \phi^i} + \frac{\delta S_{\text{quad}}}{\delta \phi^*_i}\frac{\delta S_{\text{lin}}}{\delta \phi^i} \\ & + \int_M \frac{\delta S_{\text{quad}}}{\delta \phi^*_i}\frac{\delta S_{\text{quad}}}{\delta \phi^i} \end{aligned}$$

$$X^a = X^a(\sigma, e) := \frac{1}{4}\sigma^{-2}(\bar{e}\gamma^a e)$$

$$\begin{aligned}
\{S, X^a\} &= \int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*}\frac{\delta X^a}{\delta \phi^i} = \int_M \frac{\delta S_{\text{lin}}}{\delta \sigma^*}\frac{\delta X^a}{\delta \sigma} + \frac{\delta S_{\text{lin}}}{\delta \bar{e}^*}\frac{\delta X^a}{\delta e} \\
&= -\frac{1}{2}\sigma^{-3}\left(\mathcal{L}_\xi \sigma - \frac{1}{8}\sigma^{-1}(\bar{\rho}e)\right)(\bar{e}\gamma^a e) + \frac{1}{2}\sigma^{-2}\left(\mathcal{L}_\xi \bar{e} - \frac{1}{16}(\bar{e}\gamma_c e)\bar{\rho}\gamma^c\right)\gamma^a e \\
&= \frac{1}{4}\left(\mathcal{L}_\xi \sigma^{-2}\right)(\bar{e}\gamma^a e) + \frac{1}{2}\sigma^{-2}\left(\mathcal{L}_\xi \bar{e}\right)\gamma^a e + \frac{1}{16}\sigma^{-4}\left[(\bar{\rho}e)(\bar{e}\gamma^a e) - \frac{1}{2}(\bar{\rho}\gamma^c\gamma^a e)(\bar{e}\gamma_c e)\right] \\
&= \left(\mathcal{L}_\xi X\right)^a + \frac{1}{32}\sigma^{-4}(\bar{\rho}\gamma^a\gamma^c e)(\bar{e}\gamma_c e) \\
&\overset{(\bar{e}\gamma_a e)\bar{e}\gamma^a = 0}{=} \left(\mathcal{L}_\xi X\right)^a.
\end{aligned}$$

$$XDe = 2\mathcal{L}_X e.$$

$$\mathcal{L}_{[V_1,V_2]}\lambda = \left(\mathcal{L}_{V_1}\mathcal{L}_{V_2} - \mathcal{L}_{V_2}\mathcal{L}_{V_1}\right)\lambda - F\left(\mathcal{L}_{V_1}\mathcal{L}, \mathcal{L}_{V_2}\mathcal{L}\right)\lambda$$

$$\mathcal{L}_{[\xi,\xi]}e = 2\mathcal{L}_\xi\mathcal{L}_\xi e - F\left(\mathcal{L}_\xi\mathcal{L}_\mathcal{L}, \mathcal{L}_\xi\mathcal{L}_\mathcal{L}\right)e.$$

$$\int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*}\frac{\delta}{\delta \phi^i}\left(\int_M \bar{e}^*\left(\mathcal{L}_\xi e + \frac{1}{4}X\rho\right)\right) + \frac{1}{2}\int_M \bar{e}^* F\left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}\right)e =: -\int_M \bar{e}^*\kappa_e$$

$$\begin{aligned}
\kappa_e &:= \int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*}\frac{\delta \mathcal{L}_\xi e}{\delta \phi^i} + \frac{1}{4}\left(\int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*}\frac{\delta X^a}{\delta \phi^i}\right)\gamma_a\rho + \frac{1}{4}X\left(\int_M \frac{\delta S_{\text{lin}}}{\delta \phi_i^*}\frac{\delta \rho}{\delta \phi^i}\right) - \frac{1}{2}F\left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}\right)e \\
&\overset{(\hat{\psi}_{\mu\nu}^\dagger)}{=} \mathcal{L}_{\mathcal{D}f + \frac{1}{2}\mathcal{L}_\xi\xi - \frac{1}{2}X}e - \mathcal{L}_\xi\left(\mathcal{L}_\xi e + \frac{1}{4}X\rho\right) + F\left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \mathcal{L}_\xi\mathcal{G}\right)e + \frac{1}{4}\left(\mathcal{L}_\xi X\right)^a\gamma_a\rho \\
&+ \frac{1}{4}X\left(\mathcal{L}_\xi\rho + De + \frac{1}{192}\sigma^{-2}(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)\gamma^{abc}e\right) - \frac{1}{2}F\left(\frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}, \frac{\delta S_{\text{lin}}}{\delta \mathcal{G}^*}\right)e
\end{aligned}$$

$$\begin{aligned}
\kappa_e &= -\frac{1}{2}F\left(\mathcal{L}_\xi\mathcal{G}, \mathcal{L}_\xi\mathcal{G}\right)e + F\left(\mathcal{L}_\xi\mathcal{L} + \delta_e\mathcal{G}, \mathcal{L}_\xi\mathcal{G}\right)e + \frac{1}{768}\sigma^{-2}X(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)\gamma^{abc}e \\
&\quad -\frac{1}{2}F\left(\mathcal{L}_\xi\mathcal{G} + \delta_e\mathcal{L}, \mathcal{L}_\xi\mathcal{G} + \delta_e\mathcal{L}\right)e \\
&= \frac{1}{768}\sigma^{-2}X(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)\gamma^{abc}e - \frac{1}{2}F(\delta_e\mathcal{L}, \delta_e\mathcal{G})e.
\end{aligned}$$

$$\frac{1}{768}\sigma^{-2}X(\bar{\psi}_\alpha\gamma_{abc}\psi^\alpha)\gamma^{abc}e^{\langle(\bar{\lambda}_1\gamma_a\lambda_2)(\bar{\lambda}_3\gamma^a\lambda_4)=\frac{1}{2}(\bar{\lambda}_1\gamma_a\lambda_3)(\bar{\lambda}_2\gamma^a\lambda_4)+\frac{1}{24}(\bar{\lambda}_1\gamma_{(3)}\lambda_3)(\bar{\lambda}_2\gamma^{(3)}\lambda_4)\rangle} - \frac{1}{32}\sigma^{-2}X_a\psi_\alpha(\bar{e}\gamma^a\psi^\alpha) = -\frac{1}{12}$$

$$\|(\bar{e}\gamma_a e)\bar{e}\gamma^a = 0\|\frac{1}{128}\sigma^{-4}\gamma_a\gamma_b\psi_\alpha(\bar{e}\gamma^b e)(\bar{e}\gamma^a\psi^\alpha)\gamma_a\psi_\alpha(\bar{e}\gamma^b e)($$

$$\overbrace{\phi''^\star_{\mu\nu}\sum_{\substack{0\le i\le m\\0<j<n}}\int P(i,j)}^{(\bar{e}\gamma_a e_2)(\bar{e}\gamma^a e_4)=-\frac{1}{2}(\bar{e}\gamma_a e)(e_2\gamma^a e_4)} - \frac{1}{64}\sigma^{-4}\gamma_a\gamma_b e(\bar{e}\gamma^b\psi_\alpha)($$

$$= -\frac{1}{16}\delta_e\mathcal{L}^b_\alpha\delta_e\mathcal{C}^{a\alpha}\gamma_a\gamma_b e = \frac{1}{32}[\delta_e\mathcal{L},\delta_e$$

$$\frac{1}{2}F(\delta_e$$

$$\int_M \frac{\delta S_{\text{lin}}}{\delta\phi_i^*}\frac{\delta}{\delta\phi^i}\left(\int_M\left\langle\xi^*,\mathcal{D}f+\frac{1}{2}\mathcal{L}_\xi\xi-\frac{1}{2}X\right\rangle\right) =: \int_M\langle\xi^*,\kappa_\xi\rangle$$

$$\kappa_\xi = \frac{1}{2}\mathcal{D}\left[\mathcal{L}_\xi f+\frac{1}{2}\langle X,\xi\rangle-\frac{1}{6}\langle\xi,\mathcal{L}_\xi\xi\rangle\right]+\frac{1}{2}\mathcal{L}_{\mathcal{D}f+\frac{1}{2}\mathcal{L}_\xi\xi-\frac{1}{2}X}\xi-\frac{1}{2}\mathcal{L}_\xi\left(\mathcal{D}f+\frac{1}{2}\mathcal{L}_\xi\xi-\frac{1}{2}X\right)-\frac{1}{2}\mathcal{L}_\xi X$$

$$= \frac{1}{2}(\mathcal{D}\mathcal{L}_\xi-\mathcal{L}_\xi\mathcal{D})f+\frac{1}{4}\left(\mathcal{D}\langle X,\xi\rangle-\frac{1}{4}\mathcal{L}_X\xi-\frac{1}{4}\mathcal{L}_\xi X\right)-\frac{1}{12}\mathcal{D}\langle\xi,\mathcal{L}_\xi\xi\rangle+\frac{1}{4}\mathcal{L}_{\mathcal{L}_\xi\xi}\xi-\frac{1}{4}\mathcal{L}_\xi\mathcal{L}_\xi\xi$$

$$= -\frac{1}{12}\left(\mathcal{L}_\xi\mathcal{L}_\xi\xi+\mathcal{L}_{\mathcal{L}_\xi\xi}\xi\right)+\frac{1}{4}\mathcal{L}_{\mathcal{L}_\xi\xi}\xi-\frac{1}{4}\mathcal{L}_\xi\mathcal{L}_\xi\xi = \frac{1}{6}\mathcal{L}_{\mathcal{L}_\xi\xi}\xi-\frac{1}{3}\mathcal{L}_\xi\mathcal{L}_\xi\xi = 0$$

$$\int_M \frac{\delta S_{\text{lin}}}{\delta\phi_i^*}\frac{\delta}{\delta\phi^i}\left(\int_M\frac{1}{2}f^*\left(\mathcal{L}_\xi f+\frac{1}{2}\langle X,\xi\rangle-\frac{1}{6}\langle\xi,\mathcal{L}_\xi\xi\rangle\right)\right) =: -\frac{1}{2}\int_M f^*\kappa_f,$$

$$\begin{aligned}\kappa_f =& \mathcal{L}_{\mathcal{D}_f+\frac{1}{2}\mathcal{L}_\xi\xi-\frac{1}{2}X}f-\mathcal{L}_\xi\left(\frac{1}{2}\mathcal{L}_\xi f+\frac{1}{4}\langle X,\xi\rangle-\frac{1}{12}\langle\xi,\mathcal{L}_\xi\xi\rangle\right)+\frac{1}{2}\langle\mathcal{L}_\xi X,\xi\rangle+\frac{1}{2}\left\langle X,\mathcal{D}f+\frac{1}{2}\mathcal{L}_\xi\xi-\right.\\ &-\frac{1}{6}\left\langle\mathcal{D}f+\frac{1}{2}\mathcal{L}_\xi\xi-\frac{1}{2}X,\mathcal{L}_\xi\xi\right\rangle+\frac{1}{6}\left\langle\xi,\mathcal{L}_{\mathcal{D}_f+\frac{1}{2}\mathcal{L}_\xi\xi-\frac{1}{2}X}\xi\right\rangle-\frac{1}{6}\left\langle\xi,\mathcal{L}_\xi\left(\mathcal{D}f+\frac{1}{2}\mathcal{L}_\xi\xi-\frac{1}{2}X\right)\right\rangle\\ =&\left(\frac{1}{2}-\frac{1}{6}\right)\mathcal{L}_{\mathcal{L}_\xi\xi}f-\left(\frac{1}{2}+\frac{1}{6}\right)\mathcal{L}_\xi\mathcal{L}_\xi f\\ &-\frac{1}{4}\mathcal{L}_\xi\langle X,\xi\rangle+\left(\frac{1}{2}-\frac{1}{12}\right)\langle\mathcal{L}_\xi X,\xi\rangle+\left(\frac{1}{4}+\frac{1}{12}\right)\langle X,\mathcal{L}_\xi\xi\rangle+\frac{1}{12}\langle\mathcal{L}_X\xi,\xi\rangle-\frac{1}{4}\langle X,X\rangle\\ &+\frac{1}{12}\left(\mathcal{L}_\xi\langle\xi,\mathcal{L}_\xi\xi\rangle-\langle\mathcal{L}_\xi\xi,\mathcal{L}_\xi\xi\rangle+\langle\xi,\mathcal{L}_{\mathcal{L}_\xi\xi}\xi\rangle-\langle\xi,\mathcal{L}_\xi\mathcal{L}_\xi\xi\rangle\right)\\ =&\frac{1}{3}\left(\mathcal{L}_{\mathcal{L}_\xi\xi}f-2\mathcal{L}_\xi\mathcal{L}_\xi f\right)-\frac{1}{4}\mathcal{L}_\xi\langle X,\xi\rangle+\frac{5}{12}\langle\mathcal{L}_\xi X,\xi\rangle+\frac{1}{3}\langle X,\mathcal{L}_\xi\xi\rangle+\frac{1}{12}\langle\mathcal{D}\langle X,\xi\rangle-\mathcal{L}_\xi X,\xi\rangle\\ &-\frac{1}{64}\sigma^{-4}(\bar{e}\gamma_a e)(\bar{e}\gamma^a e)+\frac{1}{12}\left(\mathcal{L}_\xi\langle\xi,\mathcal{L}_\xi\xi\rangle-\langle\mathcal{L}_\xi\xi,\mathcal{L}_\xi\xi\rangle+\langle\xi,\mathcal{L}_\xi\mathcal{L}_\xi\xi\rangle\right)\end{aligned}$$

$$\overbrace{\phi''^\star_{\mu\nu}\sum_{\substack{0\le\iota\le m\\0<\jmath<n}}\int P(\iota,\jmath)}^{\longleftarrow\atop(\bar{e}\gamma_a e_2)(\bar{e}\gamma^a e_4)=-\frac{1}{2}(\bar{e}\gamma_a e)(e_2\gamma^a e_4)} - \frac{1}{3}\left(\mathcal{L}_\xi\langle X,\xi\rangle-\langle\mathcal{L}_\xi X,\xi\rangle-\langle X,\mathcal{L}_\xi\xi\rangle\right) = 0.$$

$$\frac{1}{2}\{S,S\}=\int_M \frac{\delta S_{\text{lin}}}{\delta\phi_i^*}\frac{\delta S_{\text{quad}}}{\delta\phi^i}+\frac{\delta S_{\text{quad}}}{\delta\phi_i^*}\frac{\delta S_{\text{lin}}}{\delta\phi^i}$$
$$=\int_M \frac{\delta S_{\text{lin}}}{\delta\sigma^*}\frac{\delta S_{\text{quad}}}{\delta\sigma}+\frac{\delta S_{\text{lin}}}{\delta\bar{e}^*}\frac{\delta S_{\text{quad}}}{\delta e}+\frac{\delta S_{\text{quad}}}{\delta\bar{\rho}^*}\frac{\delta S_{\text{lin}}}{\delta\rho}+\frac{\delta S_{\text{quad}}}{\delta\bar{\psi}_\alpha^*}\frac{\delta S_{\text{lin}}}{\delta\psi^\alpha}$$

$$\frac{1}{2}\{S,S\}=\int_M\left[\mathcal{L}_\xi\sigma-\frac{1}{8}\sigma^{-1}(\bar{\rho}e)\right]\sigma^{-3}\left[\frac{1}{32}\bar{\psi}_\beta^*\psi\psi^{*\beta}+\frac{1}{16}\left(\bar{e}\psi_\beta^*\right)\left(\bar{e}\psi^{*\beta}\right)+\frac{1}{32}\bar{\rho}^*\psi\rho^*\right]$$
$$+\left[\mathcal{L}_\xi\bar{e}+\frac{1}{16}\sigma^{-2}\overline{\psi\rho}\right]\sigma^{-2}\left[-\frac{1}{32}\gamma_a e\left(\bar{\psi}_\beta^*\gamma^a\psi^{*\beta}\right)-\frac{1}{16}\psi_\beta^*\left(\bar{e}\psi^{*\beta}\right)-\frac{1}{32}\gamma_a e(\bar{\rho}^*\gamma^a\rho^*)\right]$$
$$-\left[\frac{1}{32}\sigma^{-2}\overline{\psi\rho^*}\right]\left[\frac{1}{8}\sigma^*\sigma^{-1}e-\mathcal{L}_\xi\rho^*+\frac{1}{8}\sigma^{-2}\psi^\beta\left(\bar{\psi}_\beta^*e\right)+\frac{1}{8}\sigma^{-2}\gamma^c\psi_\beta^*\left(\bar{\psi}^\beta\gamma_c e\right)-\frac{1}{16}\sigma^{-2}\psi e^*\right]$$
$$+\left[-\frac{1}{32}\sigma^{-2}\overline{\psi\psi^{*\alpha}}+\frac{1}{16}\sigma^{-2}(\bar{e}\psi^{*\alpha})\bar{e}\right]\left[-\frac{1}{2}\sigma^{-2}\mathcal{G}_{a\alpha}^*\gamma^a e+\frac{1}{96}\sigma^{-2}\gamma_{abc}\psi_\alpha\left(\bar{\rho}^*\gamma^{abc}e\right)\right.$$
$$\left.-\left(\mathcal{L}_\xi\psi^*\right)_\alpha+\frac{1}{8}\sigma^{-2}\rho(\bar{\psi}_\alpha^*e)-\frac{1}{8}\sigma^{-2}\gamma_a e(\bar{\psi}_\alpha^*\gamma^a\rho)\right]$$

$$\frac{1}{32}\sigma^{-2}(\bar{e}\gamma_e e)(\bar{\psi}^{*\alpha}\gamma^e\gamma_a e)+\frac{1}{16}\sigma^{-2}(\bar{e}\psi^{*\alpha})(\bar{e}\gamma_a e)=-\frac{1}{32}\sigma^{-2}(\bar{e}\gamma_e e)(\bar{\psi}^{*\alpha}\gamma_a\gamma^e e)\qquad \overbrace{\phi_{\mu\nu}''^\star\sum_{\substack{0\le\iota\le m\\0<\jmath<n}}\int P(\iota,\jmath)}^{\widetilde{(\bar{e}\gamma_a e_2)(\bar{e}\gamma^a e_4)=-\frac{1}{2}(\bar{e}\gamma_a e)(e_2\gamma^a e_4)}}$$

$$\frac{1}{2}\{S,S\}=:\frac{1}{128}\int_M\sigma^{-4}\kappa_{\text{quad}}$$

$$\begin{aligned}\kappa_{\text{quad}}&=-\frac{1}{2}(\bar{\rho}e)(\bar{\rho}^*\psi\rho^*)-\frac{1}{2}(\bar{\rho}e)(\bar{\psi}^*\psi\psi^*)-(\bar{\rho}e)(\bar{e}\psi^*)(\bar{e}\psi^*)+\frac{1}{4}(\bar{\rho}\psi\gamma_a e)(\bar{\rho}^*\gamma^a\rho^*)\\&\quad+\frac{1}{4}(\bar{\rho}\psi\gamma_a e)(\bar{\psi}^*\gamma^a\psi^*)+\frac{1}{2}(\bar{\rho}\psi\psi^*)(\bar{e}\psi^*)+\frac{1}{2}(\bar{\psi}^*e)(\bar{\psi}\psi\rho^*)-\frac{1}{2}\,(\bar{\psi}\gamma_a e)(\bar{\psi}^*\gamma^a\psi\rho^*)\\&\quad+\frac{1}{24}(\bar{\rho}^*\gamma_{abc}e)\left(\bar{\psi}\gamma^{abc}\psi\psi^*\right)-\frac{1}{12}\left(\bar{\rho}^*\gamma^{abc}e\right)(\bar{\psi}\gamma_{abc}e)(\bar{e}\psi^*)+\frac{1}{2}(\bar{\rho}\psi\psi^*)(\bar{\psi}^*e)\\&\quad-(\bar{\rho}e)(\bar{\psi}^*e)(\bar{e}\psi^*)+\frac{1}{2}(\bar{\psi}^*\gamma^a\rho)(\bar{e}\gamma_a\psi\psi^*)-(\bar{\psi}^*\psi\rho)(\bar{e}\psi^*)\\&=-\frac{1}{2}(\bar{\rho}e)(\bar{\rho}^*\psi\rho^*)-\frac{1}{2}(\bar{\rho}e)(\bar{\psi}^*\psi\psi^*)+\frac{1}{4}(\bar{\rho}\psi\gamma_a e)(\bar{\rho}^*\gamma^a\rho^*)+\frac{1}{4}(\bar{\rho}\psi\gamma_a e)(\bar{\psi}^*\gamma^a\psi^*)\\&\quad+\frac{1}{2}(\bar{\psi}^*e)(\bar{\psi}\psi\rho^*)-\frac{1}{2}(\bar{\psi}\gamma_a e)(\bar{\psi}^*\gamma^a\psi\rho^*)+\frac{1}{24}(\bar{\rho}^*\gamma_{abc}e)\left(\bar{\psi}\gamma^{abc}\psi\psi^*\right)\\&\quad-\frac{1}{12}\left(\bar{\rho}^*\gamma^{abc}e\right)(\bar{\psi}\gamma_{abc}e)(\bar{e}\psi^*)+\frac{1}{2}(\bar{\psi}^*\gamma^a\rho)(\bar{e}\gamma_a\psi\psi^*)-(\bar{\psi}^*\psi\rho)(\bar{e}\psi^*).\end{aligned}$$

$$\psi e=\bar{e}\psi=0$$

$$\kappa_{\text{quad}} = \frac{1}{2}(\bar{\psi}^* e)(\bar{\psi}\psi\rho^*) - \frac{1}{2}(\bar{\psi}\gamma_a e)(\bar{\psi}^*\gamma^a\psi\rho^*) + \frac{1}{24}(\bar{\rho}^*\gamma_{abc}e)\big(\bar{\psi}\gamma^{abc}\psi\psi^*\big) - \frac{1}{12}\big(\bar{\rho}^*\gamma^{abc}e\big)(\bar{\psi}\gamma_{abc}e)(\bar{e}\psi^*)$$

$$\phi''^{\dagger}_{mn}\overbrace{\sum_{\substack{0\le \iota \le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}^{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)} - (\bar{\psi}^* e)(\bar{\psi}\psi\rho^*) - \frac{1}{2}(\bar{\psi}\gamma_a e)(\bar{\psi}^*\gamma^a\psi\rho^*) + \frac{1}{24}(\bar{\rho}^*\gamma_{abc}e)\big(\bar{\psi}\gamma^{abc}\psi\psi^*\big)$$

$$\phi''^{\dagger}_{mn}\overbrace{\sum_{\substack{0\le \iota \le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}^{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)} - (\bar{\psi}^* e)(\bar{\psi}\psi\rho^*) - \frac{1}{2}(\bar{\psi}\gamma_a e)(\bar{\psi}^*\gamma^a\psi\rho^*) - \frac{1}{2}(\bar{\rho}^*\gamma_a\psi)(\bar{e}\gamma^a\psi\psi^*) + \frac{1}{2}(\bar{e}\gamma_a\psi)($$

$$= -\frac{1}{2}(\bar{\psi}\gamma_a e)(\bar{\psi}^*\gamma^a\psi\rho^*) + \frac{1}{2}(\bar{e}\gamma^a\psi)(\bar{\rho}^*\gamma_a\psi\psi^*)$$

$$= -\frac{1}{2}(\bar{\psi}\gamma_a e)(\bar{\psi}^*(\gamma^a\psi + \psi\gamma^a)\rho^*) = -(\bar{\psi}\psi e)(\bar{\psi}^*\rho^*) = 0.$$

$$\big[\delta_{\epsilon_1}, \delta_{\epsilon_2}\big]\Phi_{(0)} = \delta_{[\epsilon_1,\epsilon_2]}\Phi_{(0)} + M\big(\Phi_{(0)}, \epsilon_1, \epsilon_2\big)EL\big[\Phi_{(0)}\big]$$

$$Q_B\Phi_{(0)} := \delta_{\Phi_{(1)}}\Phi_{(0)}$$

$$Q_B\Phi_{(1)} := \delta_{\Phi_{(1)}}\Phi_{(1)} + \delta_{\Phi_{(2)}}\Phi_{(1)} = \frac{1}{2}\big[\Phi_{(1)}, \Phi_{(1)}\big] + \delta_{\Phi_{(2)}}\Phi_{(1)}$$

$$Q_B\Phi^i = \delta_\Phi\Phi^i$$

$$Q_B^2\Phi^i = \widetilde{M}(\Phi)^{ij}\frac{\partial S_0}{\partial\Phi^j}$$

$$Q_{BV}|_{\Phi^*=0}\Phi^i = Q_B\Phi^i$$

$$Q_{BV} = \{S,\cdot\} = \frac{\partial S}{\partial\Phi_i^*}\frac{\partial}{\partial\Phi^i} + \frac{\partial S}{\partial\Phi^i}\frac{\partial}{\partial\Phi_i^*}, \deg S = 0$$

$$S(\Phi,\Phi^*) = S_0(\Phi) + \sum_{k=1}^{N}\frac{1}{k!}\Phi^*_{i_1}\cdots\Phi^*_{i_k}S_k^{i_1\ldots i_k}(\Phi)$$

$$\{S,S\} = 0$$

$$S(\Phi,\Phi^*) = S_0(\Phi) + \Phi_i^* Q_B\Phi^i$$

$$S(\Phi,\Phi^*) = S_0(\Phi) + \Phi_i^* S_1^i(\Phi) + \frac{1}{2}\Phi_i^*\Phi_j^* S_2^{ij}(\Phi)$$

$$\delta_\Phi\Phi^i = Q_B\Phi^i = \big(Q_{BV}\Phi^i\big)\big|_{\Phi^*=0} = \big\{S,\Phi^i\big\}\big|_{\Phi^*=0} = S_1^i(\Phi)$$

$$\begin{aligned} Q_B S_0 &= 0,\\ Q_B^2\Phi^i + \big(\partial_j S_0\big)S_2^{ij} &= 0,\\ Q_B S_2^{ij} + S_2^{k[i}\partial_k\big(Q_B\Phi^{j]}\big) &= 0,\\ S_2^{i[j}\partial_i S_2^{kl]} &= 0, \end{aligned}$$

$$S_2^{ij} \sim \widetilde{M}(\Phi)^{ij}$$

$$\epsilon_{0\dots9} = -\epsilon^{0\dots9} = 1.$$

$$\{\gamma_a, \gamma_b\} = 2g_{ab}$$

$$C\gamma_a C^{-1} = -\gamma_a^T, C^T = -C$$

$$\bar{\psi} := \psi^T C$$

$$\bar{\psi}\chi = \bar{\chi}\psi.$$

$$C\gamma_{a_1\dots a_k}C^{-1} = (-1)^{\left[\frac{k+1}{2}\right]}\gamma^T_{a_1\dots b_k}$$

$$\bar{\psi}\gamma_{a_1\dots a_k}\chi = (-1)^{\left[\frac{k+1}{2}\right]}\bar{\chi}\gamma_{a_1\dots a_k}\psi$$

$$\gamma_{a_1\dots a_k}\gamma_* = (-1)^{\left[\frac{k}{2}\right]}\frac{1}{(10-k)!}\sqrt{-g}\epsilon_{a_1\dots a_k b_1\dots b_{10-k}}\gamma^{b_1\dots b_{10-k}}$$

$$\bar{\psi}\chi = 0 \ \text{ if } \mathrm{ch}\psi = \mathrm{ch}\chi$$

$$\mathrm{ch}\big(\gamma_{(p)}\psi\big) = (-1)^p\mathrm{ch}\psi$$

$$\gamma_{(p)} := \gamma_{a_1\dots a_p}$$

$$\bar{\lambda}\gamma_{(3)}\lambda,$$

$$\bar{\lambda}\gamma_{(1)}\lambda = \bar{\lambda}\gamma_{(5)}\lambda = 0$$

$$\frac{1}{p!}\left(\bar{\lambda}_1\gamma_{a_1\dots a_n b_1\dots b_p}\lambda_2\right)\left(\bar{\lambda}_3\gamma^{b_1\dots b_p}\lambda_4\right) = (\mathrm{ch}\lambda_2)(\mathrm{ch}\lambda_4)(-1)^{1+\left[\frac{n}{2}\right]}\frac{1}{q!}\left(\bar{\lambda}_1\gamma^{c_1\dots c_q}\lambda_2\right)\left(\bar{\lambda}_3\gamma_{a_1\dots a_n c_1\dots c_q}\lambda_4\right)$$

$$\frac{1}{(10-p)!}\big(\bar{\lambda}_1\gamma_{(10-p)}\lambda_2\big)\big(\bar{\lambda}_3\gamma^{(10-p)}\lambda_4\big) = -\frac{1}{p!}(\mathrm{ch}\lambda_2)(\mathrm{ch}\lambda_4)\big(\bar{\lambda}_1\gamma_{(p)}\lambda_2\big)\big(\bar{\lambda}_3\gamma^{(p)}\lambda_4\big)$$

$$\gamma_a\gamma^{b_1\dots b_p} = \gamma_a^{b_1\dots b_p} + p\delta_a^{[b_1}\gamma^{b_2\dots b_p]}$$

$$\big\{\gamma_a, \gamma^{b_1\dots b_{2p+1}}\big\} = 2(2p+1)\delta_a^{[b_1}\gamma^{b_2\dots b_{2p+1}]}$$

$$\lambda\bar{\psi} = \frac{1}{32}\sum_{p=0}^{10}\frac{(-1)^{p+1}}{p!}\gamma_{(p)}\psi\bar{\lambda}\gamma^{(p)}$$

$$\big(\bar{\lambda}_1\psi_1\big)\big(\bar{\lambda}_2\psi_2\big) = \frac{1}{16}\big(\bar{\lambda}_1\gamma_{(1)}\lambda_2\big)\big(\bar{\psi}_1\gamma^{(1)}\psi_2\big) + \frac{1}{96}\big(\bar{\lambda}_1\gamma_{(3)}\lambda_2\big)\big(\bar{\psi}_1\gamma^{(3)}\psi_2\big) + \frac{1}{3840}\big(\bar{\lambda}_1\gamma_{(5)}\lambda_2\big)\big(\bar{\psi}_1\gamma^{(5)}\psi_2\big)$$

$$\big(\bar{\lambda}_1\gamma_a\lambda_2\big)\big(\bar{\lambda}_3\gamma^a\lambda_4\big) = \frac{1}{2}\big(\bar{\lambda}_1\gamma_a\lambda_3\big)\big(\bar{\lambda}_2\gamma^a\lambda_4\big) + \frac{1}{24}\big(\bar{\lambda}_1\gamma_{(3)}\lambda_3\big)\big(\bar{\lambda}_2\gamma^{(3)}\lambda_4\big)$$

$$\big(\bar{\lambda}_1\gamma_{(3)}\lambda_2\big)\big(\bar{\lambda}_3\gamma^{(3)}\lambda_4\big) = 18\ell\big(\bar{\lambda}_1\gamma_a\lambda_3\big)\big(\bar{\lambda}_2\gamma^a\lambda_4\big) - \frac{1}{2}\big(\bar{\lambda}_1\gamma_{(3)}\lambda_3\big)\big(\bar{\lambda}_2\gamma^{(3)}\lambda_4\big)$$

$$\big(\bar{\lambda}_{[1}\gamma_{|a}\lambda_{2|}\big)\big(\bar{\lambda}_{3]}\gamma^a\lambda_4\big) = \frac{1}{2}\big(\bar{\lambda}_1\gamma_a\lambda_3\big)\big(\bar{\lambda}_2\gamma^a\lambda_4\big)$$

$$\left(\bar{\lambda}\gamma^{abc}\lambda\right)\bar{\lambda}\gamma_{abc}=0$$

$$(\bar{e}_1\gamma_a e_2)(\bar{e}_3\gamma^a e_4)=-\frac{1}{2}(\bar{e}_1\gamma_a e_3)(\bar{e}_2\gamma^a e_4)-\frac{1}{24}\left(\bar{e}_1\gamma_{(3)}e_3\right)\left(\bar{e}_2\gamma^{(3)}e_4\right)$$

$$(\bar{e}\gamma_a e_2)(\bar{e}\gamma^a e_4)=-\frac{1}{2}(\bar{e}\gamma_a e)(e_2\gamma^a e_4)$$

$$(\bar{e}\gamma_a e)\bar{e}\gamma^a=0$$

$$\frac{1}{24}\left(\bar{e}_1\gamma_{(3)}e_3\right)\left(\bar{e}_2\gamma^{(3)}e_4\right)=-\frac{1}{2}(\bar{e}_1\gamma_a e_2)(\bar{e}_3\gamma^a e_4)+\frac{1}{2}(\bar{e}_3\gamma_a e_2)(\bar{e}_1\gamma^a e_4)$$

$$\left(\bar{e}_1\gamma_{(3)}e\right)\left(\bar{e}_3\gamma^{(3)}e\right)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)$$

$$|A|\in\mathbb{Z}_2=\{0,1\}$$

$$|d|\equiv 1,|i_X|\equiv -1+|X|$$

$$L_X=[d,i_X],$$

$$L_X f=Xf$$

$$i_{X_H}\omega=-dH$$

$$\omega=dp_i\wedge dq^i$$

$$\{H,H'\}=X_H H'$$

$$df=dx^i\frac{\partial f}{\partial x^i}$$

$$i_X df=(-1)^{|X|}L_X f=(-1)^{|X|}Xf$$

$$|q|\equiv|p|+1\ \mathrm{mod}2$$

$$i_{X_H}\omega=dp\left(i_{A\partial_q}dq\right)+\left(i_{B\partial_p}dp\right)dq\quad\blacksquare\frac{\overset{\iota_X df=(-1)^{\widehat{|X|}L_{Xf}}=(-1)^{|X|}Xf}{\odot d\hbar''}}{\iiint\boxplus''_{(4)}\otimes\frac{\partial\mathfrak{G}}{\partial\mathfrak{M}}d^4\chi}-dpA-Bdq$$

$$dH=dq\left(\partial_q H\right)+dp\left(\partial_p H\right)=\left(\partial_q H\right)dq+dp\left(\partial_p H\right)$$

$$X_H=\frac{\partial H}{\partial p_i}\frac{\partial}{\partial q^i}+\frac{\partial H}{\partial q^i}\frac{\partial}{\partial p_i}$$

$$\{H,H'\}=\frac{\partial H}{\partial p_i}\frac{\partial H'}{\partial q^i}+\frac{\partial H}{\partial q^i}\frac{\partial H'}{\partial p_i}$$

$$\{F,F'\}=(-1)^{|F|\cdot|p_i|}\frac{\partial F}{\partial p_i}\frac{\partial F'}{\partial q^i}+(-1)^{|F|\cdot|q^i|}\frac{\partial F}{\partial q^i}\frac{\partial F'}{\partial p_i}$$

$$\{H,H'\}=\int_M\frac{\delta H}{\delta\bar{\lambda}^*}\frac{\delta H'}{\delta\lambda}+\frac{\delta H'}{\delta\bar{\lambda}^*}\frac{\delta H}{\delta\lambda}$$

$$\frac{\delta H}{\delta\left(\bar{\lambda}^*\right)_y}=\int_M \frac{\delta H}{\delta(\lambda^*)^x}\frac{\delta(\lambda^*)^x}{\delta\left(\bar{\lambda}^*\right)_y}=\frac{\delta H}{\delta(\lambda^*)^x}(C^{-1})^{yx}$$

$$\int_M \frac{\delta H}{\delta\left(\bar{\lambda}^*\right)_y}\frac{\delta H'}{\delta\lambda^y}=(C^{-1})^{yx}\int_M \frac{\delta H}{\delta(\lambda^*)^x}\frac{\delta H'}{\delta\lambda^y}=(C^{-1})^{yx}\int_M (C_{xz}\chi^z)\left(C_{yw}\varphi^w\right)=\int_M \chi^y C_{yw}\varphi^w$$

$$\left\{\int_M \bar{\lambda}^*\chi,\int_M \bar{\lambda}\varphi\right\}=\int_M \bar{\chi}\varphi$$

$$F^+(\delta_1\mathcal{L},\delta_2\mathcal{L})=\frac{1}{4}[\delta_1\mathcal{L},\delta_2\mathcal{L}]\colon \Gamma(C_+)\to\Gamma(C_+)$$

$$F^-\left(\delta_1\mathcal{L},\delta_2\mathcal{L}\right)=\frac{1}{4}[\delta_1\mathcal{G},\delta_2\mathcal{L}]\colon \Gamma(C_-)\to\Gamma(C_-)$$

$$\left(F^+\left(\delta_1\mathcal{L},\delta_2\mathcal{L}\right)u_+\right)^a=\frac{1}{4}(\delta_1\mathcal{L}^{a\alpha}\delta_2\mathcal{L}_{b\alpha}-\delta_2\mathcal{L}^{a\alpha}\delta_1\mathcal{L}_{b\alpha})u_+^b$$
$$\left(F^-\left(\delta_1\mathcal{L},\delta_2\mathcal{L}\right)u_-\right)^\alpha=\frac{1}{4}\left(\delta_1\mathcal{G}^{a\alpha}\delta_2\mathcal{G}_{a\beta}-\delta_2\mathcal{L}^{a\alpha}\delta_1\mathcal{G}_{a\beta}\right)u_-^\beta$$

$$F(\delta_1\mathcal{L}_2,\delta_2\mathcal{L})\rho=\frac{1}{16}[\delta_1\mathcal{L}_2,\delta_2\mathcal{L}_{ab}\gamma^{ab}\rho$$
$$F(\delta_1\mathcal{L}_2,\delta_2\mathcal{L})\psi^\alpha=\frac{1}{16}\Big[\delta_1\mathcal{L}_2,\delta_2\mathcal{L}_{ab}\gamma^{ab}\psi^\alpha+\frac{1}{4}\big[\delta_1\mathcal{L}_2,\delta_2\mathcal{L}^\alpha{}_\beta\psi^\beta.$$

$$C_+=\left\{\hat{u}_+ +\varphi(\hat{u}_+)\mid \hat{u}_+\in\hat{C}_+\right\}$$

$$\hat{p}\circ\left(p|_{C_+'}\right)^{-1}\circ\left(\hat{p}|_{C_+}\right)^{-1}\colon \hat{C}_+\to\hat{C}_+,$$

$$\hat{u}_+\mapsto\hat{u}_+-\varphi^*(1+\varphi\varphi^*)^{-1}\delta\varphi(\hat{u}_+),$$

$$A^+=\varphi^*(1+\varphi\varphi^*)^{-1}d\varphi,$$

$$F^+(\delta_1\varphi,\delta_2\varphi)=\delta_1\varphi^*\delta_2\varphi-\delta_2\varphi^*\delta_1\varphi\colon \hat{C}_+\to\hat{C}_+$$

$$F^+(\delta_1\mathcal{G},\delta_2\mathcal{L})=\frac{1}{4}[\delta_1\mathcal{G},\delta_2\mathcal{G}]\colon \hat{C}_+\to\hat{C}_+.$$

$$T_{\mathcal{G}}\mathcal{M}\cong\Gamma(C_+\otimes C_-).$$

$$T_{\mathcal{G}}\mathcal{M}\cong\Gamma(C_+\otimes C_-)\ni u,v\mapsto\int_M \sigma^2\langle u,v\rangle$$

$$TE=T_{\mathrm{hor}}E\oplus T_{\mathrm{ver}}E\cong\pi^*TM\oplus\pi^*E$$

$$B\in\Gamma(\pi^*TM),C\in\Gamma(\pi^*E).$$

$$B=[B_1,B_2]+C_1B_2-C_2B_1,C=[C_1,C_2]+\nabla_{B_1}C_2-\nabla_{B_2}C_1-F(B_1,B_2),$$

$$\nabla u=du+Au,\ \text{i.e.}\ \left(\nabla_{\partial_{x^i}}u\right)^a=\partial_{x^i}u^a+A_i{}^a{}_b u^b,$$

$$u(x_0+\Delta x)=u(x_0)-\varepsilon_a A_i{}^a{}_b(x_0)u^b(x_0)\Delta x^i.$$

$$\partial_{x^i}-A_i{}^a{}_b(x_0)u^b\partial_{y^a}.$$

$$\text{ver}=\text{span}\{e_a\},\ \text{hor}\ =\text{span}\{e_i\},\ e_a:=\partial_{y^a},e_i:=\partial_{x^i}-A_i{}^a{}_b(x)y^b\partial_{y^a}.$$

$$V(x,y)=B^i(x,y)e_i+C^a(x,y)e_a.$$

$$[e_a,e_b]=0,[e_a,e_i]=-A_i{}^b{}_a(x)e_b,\left[e_i,e_j\right]=-F_{ij}{}^a{}_b(x)e_a,$$

$$[V_1,V_2]=\left(B_1^j e_j B_2^i-B_2^j e_j B_1^i-C_2^a e_a B_1^i+C_1^a e_a B_2^i\right)e_i$$
$$+\left(B_1^i e_i C_2^a-B_2^i e_i C_1^a+C_1^b e_b C_2^a-C_2^b e_b C_1^a+A_i{}^a{}_b B_1^i C_2^b-A_i{}^a{}_b B_2^i C_1^b-B_1^i B_2^j F_{ij}{}^a{}_b y^b\right)e_a,$$

$$[\delta_1,\delta_2]:=\left[\delta_{\epsilon_1},\delta_{\epsilon_2}\right]$$

$$V^a:=\frac{1}{4}\sigma^{-2}(\bar\epsilon_2\gamma^a\epsilon_1)$$

$$[\delta_1,\delta_2]=\delta_V+\delta_\zeta$$

$$\zeta:=-\frac{1}{2}V\rho.$$

$$\begin{aligned}[\delta_1,\delta_2]\mathcal{L}_{a\alpha}=&-\frac{1}{8}\sigma^{-4}(\bar\rho\epsilon_1)(\bar\epsilon_2\gamma_a\psi_\alpha)+\frac{1}{2}\sigma^{-2}(\bar\epsilon_2\gamma_a D_\alpha\epsilon_1)+\frac{1}{16}\sigma^{-4}(\bar\psi_\alpha\rho)(\bar\epsilon_2\gamma_a\epsilon_1)\\&+\frac{1}{16}\sigma^{-4}\left(\bar\epsilon_2\gamma_a\gamma^b\rho\right)(\bar\psi_\alpha\gamma_b\epsilon_1)-1\leftrightarrow 2\\&\underbrace{\underbrace{\psi''^{\dagger}_{mn}\sum_{\substack{0\le \iota\le m\\0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar e_1\gamma_{(3)}e)(\bar e_3\gamma^{(3)}e)=-18\ell(\bar e_1\gamma_a e_3)(\bar e\gamma^a e)}}\ \frac{1}{2}\sigma^{-2}(\bar\epsilon_2\gamma_a D_\alpha\epsilon_1)+\frac{1}{4}\sigma^{-2}(\bar\psi_\alpha\rho)V_a+\frac{1}{16}\sigma^{-4}(\bar\epsilon_1\gamma_b\psi_\alpha)\left(\bar\epsilon_2\gamma^b\gamma_a\rho\right)-\\&=\frac{1}{2}\sigma^{-2}[(\bar\epsilon_2\gamma_a D_\alpha\epsilon_1)+(D_\alpha\bar\epsilon_2\gamma_a\epsilon_1)]+\frac{1}{2}\sigma^{-2}\left(\bar\psi_\alpha\rho\right)V_a+\frac{1}{8}\sigma^{-4}\left(\bar\epsilon_{[1}\gamma_{|b}\psi_{\alpha|}\right)\left(\bar\epsilon_{2]}\gamma^b\gamma_a\rho\right)\\&\underbrace{\underbrace{\psi''^{\star}_{mn}\sum_{\substack{0\le \iota\le m\\0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar e_1\gamma_{(3)}e)(\bar e_3\gamma^{(3)}e)=-18\ell(\bar e_1\gamma_a e_3)(\bar e\gamma^a e)}}\ 2D_\alpha V_a+\frac{1}{2}\sigma^{-2}(\bar\psi_\alpha\rho)V_a-\frac{1}{4}\sigma^{-2}V_b\left(\bar\psi_\alpha\gamma^b\gamma_a\rho\right)\\&\underbrace{\underbrace{\Gamma''^{\dagger}_{mn}\sum_{\substack{0\le \iota\le m\\0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar e_1\gamma_{(3)}e)(\bar e_3\gamma^{(3)}e)=-18\ell(\bar e_1\gamma_a e_3)(\bar e\gamma^a e)}}\ 2D_\alpha V_a+\frac{1}{4}\sigma^{-2}(\bar\psi_\alpha\gamma_a V\rho)\overset{(2.41)}{=}\delta_V\mathcal{C}_{a\alpha}+\delta_\zeta\mathcal{C}_{a\alpha},\end{aligned}$$

$$\left(\mathcal{L}_X\mathcal{L}\right)_{a\alpha}=2(D_\alpha X_a-D_a X_\alpha),$$

$$\begin{aligned}[\delta_1,\delta_2]\sigma=&\frac{1}{8}\delta_1[\sigma^{-1}(\bar\rho\epsilon_2)]-1\leftrightarrow 2\\&=-\frac{1}{64}\sigma^{-3}(\bar\rho\epsilon_1)(\bar\rho\epsilon_2)+\frac{1}{8}\sigma^{-1}\left(\overline{\not\partial\epsilon_1}\epsilon_2\right)+\frac{1}{8\times 192}\sigma^{-3}\left(\psi_\alpha\gamma_{(3)}\psi^\alpha\right)\left(\bar\epsilon_1\gamma^{(3)}\epsilon_2\right)-1\leftrightarrow 2\\&=\frac{1}{8}\sigma^{-1}(\bar\epsilon_2\not\partial\epsilon_1-\bar\epsilon_1\not\partial\epsilon_2)=\frac{1}{2}\sigma D_a\left(\frac{1}{4}\sigma^{-2}\bar\epsilon_2\gamma^a\epsilon_1\right)\overset{(2.26)}{=}\mathcal{L}_V\sigma=\delta_V\sigma=\delta_V\sigma+\delta_\zeta\sigma\end{aligned}$$

$$\delta_\zeta \sigma = -\frac{1}{16}\sigma^{-1}\bar{\rho}V\rho = 0,$$

$$(\delta \not{\partial})\rho = \frac{1}{2}\delta\mathcal{C}^{\alpha}{}_{a}\gamma^{a}D_{\alpha}\rho + \frac{1}{4}(D_{\alpha}\delta\mathcal{C}^{\alpha}{}_{a})\gamma^{a}\rho,$$
$$(\delta \not{\partial})\psi^{\alpha} = \frac{1}{2}\delta\mathcal{C}^{\gamma}{}_{a}\gamma^{a}D_{\gamma}\psi^{\alpha} + \frac{1}{4}\left(D_{\gamma}\delta\mathcal{C}^{\gamma}{}_{a}\right)\gamma^{a}\psi^{\alpha} + \left(D^{[\alpha}\delta\mathcal{C}^{\gamma]}{}_{a}\right)\gamma^{a}\psi_{\gamma},$$
$$(\delta D_{\alpha})\rho = -\frac{1}{2}\delta\mathcal{C}_{\alpha}{}^{a}D_{a}\rho - \frac{1}{4}(D_{b}\delta\mathcal{C}_{\alpha c})\gamma^{bc}\rho - \left(D_{\alpha}\frac{\delta\sigma}{\sigma}\right)\rho,$$

$$\mathcal{L}_V\lambda \overbrace{\underbrace{\Gamma''^{\dagger}_{mn}\sum_{\substack{0\le \iota \le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}}^{} V^a D_a\lambda + \frac{1}{2}D_aV_b\gamma^{ab}\lambda + \frac{1}{2}D_aV^a\lambda = \frac{1}{2}(\not{\partial}V + V\not{\partial})\lambda = \frac{1}{2}\{\not{\partial}, V\}\lambda$$

$$\delta_\epsilon\rho = \not{\partial}_\rho + \frac{1}{4}\delta\mathcal{C}_{a\alpha}\gamma^{a}\psi^{\alpha}$$
$$\delta_\epsilon\psi_\alpha = D_\alpha\epsilon + \frac{1}{8}\sigma^{-2}(\bar{\psi}_\alpha\rho)\epsilon - \frac{1}{4}\delta\mathcal{L}_{a\alpha}\gamma^{a}\rho$$

$$[\delta_1, \delta_2]\rho = \delta_V\rho + \delta_\zeta\rho - \frac{1}{2}VEL[\rho]$$

$$[\delta_1,\delta_2]\rho = \delta_1\left[\not{\partial}\epsilon_2 + \frac{1}{4}\delta_2\mathcal{G}_{a\alpha}\gamma^{a}\psi^{\alpha}\right] - 1 \leftrightarrow 2 - F(\delta_1\mathcal{G},\delta_2\mathcal{G})\rho$$
$$\overbrace{\underbrace{\Gamma''^{\dagger}_{mn}\sum_{\substack{0\le \iota \le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}}^{} (\delta_1\not{\partial})\epsilon_2 + \frac{1}{4}\delta_1\delta_2\mathcal{G}_{a\alpha}\gamma^{a}\psi^{\alpha} + \frac{1}{4}\delta_2\mathcal{G}_{a\alpha}\gamma^{a}\left[D^{\alpha}\epsilon_1 + \frac{1}{8}\sigma^{-2}(\bar{\psi}^{\alpha}\rho)\epsilon_1 - \frac{1}{4}\delta_1$$
$$-\frac{1}{16}[\delta_1\mathcal{G},\delta_2\mathcal{G}_{ab}\gamma^{ab}\rho$$
$$= (\delta_1\not{\partial})\epsilon_2 - \frac{1}{4}\delta_1\mathcal{G}_{a\alpha}\gamma^{a}D^{\alpha}\epsilon_2 + \frac{1}{32}\sigma^{-2}(\bar{\psi}^{\alpha}\rho)\delta_2\mathcal{G}_{a\alpha}\gamma^{a}\epsilon_1 - 1 \leftrightarrow 2 + \frac{1}{4}[\delta_1,\delta_2]\mathcal{G}_{a\alpha}\gamma^{a}\psi^{\alpha}.$$

$$(\delta_1\not{\partial})\epsilon_2 - \frac{1}{4}\delta_1\mathcal{G}_{a\alpha}\gamma^{a}D^{\alpha}\epsilon_2 - 1 \leftrightarrow 2 \overset{\blacksquare}{\iiint\boxplus''_{(4)}}\overset{\odot d\hbar''}{\otimes}\frac{\partial\mathfrak{G}}{\partial\mathfrak{W}}d^4\chi\,\frac{1}{4}\delta_1\mathcal{G}_{a\alpha}\gamma^{a}D^{\alpha}\epsilon_2 + \frac{1}{4}D_\alpha\delta_1\mathcal{G}_a^{\alpha}\gamma^{a}\epsilon_2 - 1 \leftrightarrow 2$$
$$= \frac{1}{4}D_\alpha[\delta_1\mathcal{G}_a^{\alpha}\gamma^{a}\epsilon_2 - 1 \leftrightarrow 2] = \frac{1}{8}D_\alpha[\sigma^{-2}(\bar{\epsilon}_1\gamma_a\psi^{\alpha})\gamma^{a}\epsilon_2 - 1 \leftrightarrow 2]$$
$$\overbrace{\underbrace{\Gamma''^{\dagger}_{mn}\sum_{\substack{0\le \iota \le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}}^{} - \frac{1}{8}D_\alpha[\sigma^{-2}(\bar{\epsilon}_2\gamma_a\epsilon_1)\gamma^{a}\psi^{\alpha}] = -\frac{1}{2}D_\alpha(V\psi^{\alpha}) = -\frac{1}{2}D_\alpha V_a\gamma^{a}\psi^{\alpha} - \frac{1}{2}VD_\alpha\psi^{\alpha}$$
$$= -\frac{1}{4}\mathcal{L}_V\mathcal{G}_{a\alpha}\gamma^{a}\psi^{\alpha} - \frac{1}{2}VD_\alpha\psi^{\alpha}.$$

$$[\delta_1,\delta_2]\rho = \frac{1}{4}\delta_\zeta\mathcal{G}_{a\alpha}\gamma^{a}\psi^{\alpha} - \frac{1}{2}VD_\alpha\psi^{\alpha} + \frac{1}{32}\sigma^{-2}[(\bar{\psi}_\alpha\rho)\delta_2\mathcal{G}_a{}^{\alpha}\gamma^{a}\epsilon_1 - 1 \leftrightarrow 2].$$

$$\frac{1}{4}\delta_\zeta \mathcal{C}_{a\alpha}\gamma^a\psi^\alpha = \delta_\zeta\rho - \not{\partial}\zeta = \delta_\zeta\rho + \frac{1}{2}\not{\partial}V\rho \overbrace{\underbrace{\Gamma''^{\dagger}_{mn}\sum_{\substack{0\le \iota \le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}}^{} \delta_\zeta\rho + \delta_V\rho - \frac{1}{2}V\not{\partial}\rho.$$

$$\frac{1}{32}\sigma^{-2}(\bar{\psi}_\alpha\rho)\delta_2\mathcal{G}^\alpha_a(\bar{\chi}\gamma^a\epsilon_1) - 1 \leftrightarrow 2 = -\frac{1}{32}\sigma^{-4}(\bar{\psi}_\alpha\rho)\big(\bar{\epsilon}_{[2}\gamma_a\psi^\alpha\big)\big(\bar{\epsilon}_{1]}\gamma^a\chi\big)$$

$$\overbrace{\underbrace{\Gamma''^{\dagger}_{mn}\sum_{\substack{0\le \iota \le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}}^{} \frac{1}{64}\sigma^{-4}(\bar{\psi}_\alpha\rho)(\bar{\epsilon}_2\gamma_a\epsilon_1)(\bar{\chi}\gamma^a\psi^\alpha) = \frac{1}{16}\sigma^{-2}(\bar{\psi}_\alpha\rho)\,(\bar{\chi}V\psi^\alpha)$$

$$\overbrace{\underbrace{\Gamma''^{\dagger}_{mn}\sum_{\substack{0\le \iota \le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}}^{} \frac{1}{1536}\sigma^{-2}\big(\bar{\chi}V\gamma_{(3)}\rho\big)\big(\bar{\psi}_\alpha\gamma^{(3)}\psi^\alpha\big)$$

$$[\delta_1,\delta_2]\rho = \delta_V\rho + \delta_\zeta\rho - \frac{1}{2}V\left[\not{\partial}\rho + D_\alpha\psi^\alpha - \frac{1}{768}\sigma^{-2}\big(\bar{\psi}_\alpha\gamma_{(3)}\psi^\alpha\big)\gamma^{(3)}\rho\right]$$
$$= \delta_V\rho + \delta_\zeta\rho - \frac{1}{2}VEL[\rho]$$

$$[\delta_1,\delta_2]\psi_\alpha = \delta_V\psi_\alpha + \delta_\zeta\psi_\alpha + \left(\frac{1}{4}\sigma^{-2}\epsilon_{[2}\bar{\epsilon}_{1]} - \frac{1}{2}V\right)EL[\psi]_\alpha$$

$$[\delta_1,\delta_2]\psi_\alpha = \delta_1\left(D_\alpha\epsilon_2+\frac{1}{8}\sigma^{-2}(\bar\psi_\alpha\rho)\epsilon_2-\frac{1}{4}\delta_2\mathcal{C}_{a\alpha}\gamma^a\rho\right)-1\leftrightarrow 2-F(\delta_1\mathcal{C},\delta_2\mathcal{C})\psi_\alpha$$

$$\overbrace{\zeta''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar e_1\gamma_{(3)}e)\overbrace{(\bar e_3\gamma^{(3)}e)=-18\ell(\bar e_1\gamma_a e_3)}(\bar e\gamma^a e)}(\delta_1 D_\alpha)\epsilon_2-\frac{1}{32}\sigma^{-4}(\bar\rho\epsilon_1)(\bar\psi_\alpha\rho)\epsilon_2+\frac{1}{8}\sigma^{-2}\bar\rho\left[D_\alpha\epsilon_1+\frac{1}{8}\sigma^{-2}(\bar\psi_\alpha\rho)\epsilon_1-\frac{1}{4}\delta_1\mathcal{C}_{a\alpha}\gamma^{a}\right.$$

$$+\frac{1}{8}\sigma^{-2}\bar\psi_\alpha\left(\not{D}\epsilon_1+\frac{1}{4}\delta_1\mathcal{G}_{a\beta}\gamma^a\psi^\beta\right)\epsilon_2-\frac{1}{4}\delta_1\delta_2\mathcal{G}_{a\alpha}\gamma^a\rho-\frac{1}{4}\delta_2\mathcal{G}_{a\alpha}\gamma^a\left(\not{D}\epsilon_1+\frac{1}{4}\delta_1\mathcal{G}_{b\beta}\gamma^b\psi^\beta\right)$$

$$-1\leftrightarrow 2-\frac{1}{16}[\delta_1\mathcal{L}_2,\delta_2\mathcal{L}_2]_{ab}\gamma^{ab}\psi_\alpha-\frac{1}{4}\big[\delta_1\mathcal{L}_2,\delta_2\mathcal{L}_{\alpha\beta}\psi^\beta$$

$$\overbrace{\zeta''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar e_1\gamma_{(3)}e)\overbrace{(\bar e_3\gamma^{(3)}e)=-18\ell(\bar e_1\gamma_a e_3)}(\bar e\gamma^a e)}\left[(\delta_1 D_\alpha)\epsilon_2+\frac{1}{8}\sigma^{-2}(\bar\rho D_\alpha\epsilon_1)\epsilon_2+\frac{1}{8}\sigma^{-2}(\bar\psi_\alpha\not{D}\epsilon_1)\epsilon_2-\frac{1}{4}\delta_2\mathcal{C}_{a\alpha}\gamma^a\not{D}\epsilon_1-1\leftrightarrow 2\right]$$

$$+\left[\frac{1-2}{64}\sigma^{-4}(\bar\psi_\alpha\rho)(\bar\rho\epsilon_1)\epsilon_2+\frac{1}{32}\sigma^{-2}\delta_1\mathcal{L}_{a\beta}\big(\bar\psi_\alpha\gamma^a\psi^\beta\big)\epsilon_2+\frac{1}{16}\delta_1\mathcal{L}_{a\alpha}\delta_2\mathcal{G}_{b\beta}\gamma^a\gamma^b\psi^\beta-1\leftrightarrow 2\right]$$

$$-\frac{1}{4}[\delta_1,\delta_2]\mathcal{G}_{a\alpha}\gamma^a\rho-\frac{1}{16}[\delta_1\mathcal{G},\delta_2\mathcal{G}]_{ab}\gamma^{ab}\psi_\alpha-\frac{1}{4}[\delta_1\mathcal{G},\delta_2\mathcal{G}]_{\alpha\beta}\psi^\beta$$

$$\overbrace{\zeta''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar e_1\gamma_{(3)}e)\overbrace{(\bar e_3\gamma^{(3)}e)=-18\ell(\bar e_1\gamma_a e_3)}(\bar e\gamma^a e)}\left[(\delta_1 D_\alpha)\epsilon_2+\frac{1}{8}\sigma^{-2}(\bar\rho D_\alpha\epsilon_1)\epsilon_2+\frac{1}{8}\sigma^{-2}(\bar\psi_\alpha\not{D}\epsilon_1)\epsilon_2-\frac{1}{4}\delta_2\mathcal{C}_{a\alpha}\gamma^a\not{D}\epsilon_1-1\leftrightarrow 2\right.$$

$$\left.-\frac{1}{4}\mathcal{L}_V\mathcal{G}_{a\alpha}\gamma^{a\ a}\right]\overbrace{\zeta''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar e_1\gamma_{(3)}e)\overbrace{(\bar e_3\gamma^{(3)}e)=-18\ell(\bar e_1\gamma_a e_3)}(\bar e\gamma^a e)}$$

$$+\left[\frac{1}{32}\sigma^{-2}\delta_1\mathcal{G}_{a\beta}\big(\bar\psi_\alpha\gamma^a\psi^\beta\big)\epsilon_2+\frac{1}{16}\delta_1\mathcal{G}_{a\alpha}\delta_2\mathcal{G}_{b\beta}\gamma^a\gamma^b\psi^\beta-1\leftrightarrow 2\right]-\frac{1}{4}\delta_\zeta\mathcal{G}_{a\alpha}\gamma^a\rho$$

$$-\frac{1}{3072}\sigma^{-4}\epsilon_{[2}\bar\epsilon_{1]}\big(\bar\rho\gamma_{(3)}\rho\big)\gamma^{(3)}\psi_\alpha-\frac{1}{16}[\delta_1\mathcal{L},\delta_2\mathcal{L}]_{ab}\gamma^{ab}\psi_\alpha-\frac{1}{4}[\delta_1\mathcal{G},\delta_2\mathcal{G}]_{\alpha\beta}\psi^\beta$$

$$[\delta_1,\delta_2]\psi_\alpha=\mathrm{Der}+\mathrm{Alg}$$

$$\mathrm{Der}\ {}^{\blacksquare\ \odot d\hbar''}_{\iint\boxplus''_{(4)}\otimes\frac{\partial\mathfrak{G}}{\partial\mathfrak{R}}d^4\chi}=\frac{1}{2}\delta_1\mathcal{L}_\alpha{}^a D_a\epsilon_2-\frac{1}{4}D_a\delta_1\mathcal{G}_{ab}\gamma^{ab}\epsilon_2-\frac{1}{8}\sigma^{-2}D_\alpha(\bar\rho\epsilon_1)\epsilon_2+\frac{1}{8}\sigma^{-2}(\bar\rho D_\alpha\epsilon_1)\epsilon_2$$
$$+\frac{1}{8}\sigma^{-2}(\bar\psi_\alpha\not{D}\epsilon_1)\epsilon_2-\frac{1}{4}\delta_2\mathcal{G}_{a\alpha}\gamma^a\not{D}\epsilon_1-1\leftrightarrow 2-\frac{1}{4}\mathcal{L}_V\mathcal{G}_{a\alpha}\gamma^a\rho$$

$$\zeta=-\frac{1}{2}\delta_1\mathcal{L}^a_{\alpha\ \alpha}D_a\epsilon_2-\frac{1}{4}\delta_2\mathcal{L}_{a\alpha}\gamma^a\gamma^b D_b\epsilon_1-\frac{1}{4}D_a\delta_1\mathcal{L}_{b\alpha}\gamma^{ab}\epsilon_2+\frac{1}{8}\sigma^{-2}(\bar\psi_\alpha\not{D}\epsilon_1)\epsilon_2$$
$$-\frac{1}{8}\sigma^{-2}\epsilon_2(\bar\epsilon_1 D_\alpha\rho)-1\leftrightarrow 2-\frac{1}{2}D_\alpha V_a\gamma^a\rho$$

$$\delta=-\frac{1}{2}\delta_1\mathcal{L}^a_\alpha D_a\epsilon_2+\frac{1}{4}\delta_1\mathcal{L}_{a\alpha}\gamma^a\gamma^b D_b\epsilon_2-\frac{1}{4}D_a\delta_1\mathcal{G}_{b\alpha}\gamma^{ab}\epsilon_2-\frac{1}{4}D_a\delta_1\mathcal{L}^a_\alpha\epsilon_2+\frac{1}{8}\sigma^{-2}(\bar\epsilon_1\not{D}\psi_\alpha)\epsilon_2$$
$$-\frac{1}{8}\sigma^{-2}\epsilon_2(\bar\epsilon_1 D_\alpha\rho)-1\leftrightarrow 2-\frac{1}{2}D_\alpha(V\rho)+\frac{1}{2}VD_\alpha\rho$$

$$\xi=-\frac{1}{4}\delta_1\mathcal{G}_{a\alpha}\gamma^b\gamma^a D_b\epsilon_2-\frac{1}{4}D_a\delta_1\mathcal{G}_{b\alpha}\gamma^a\gamma^b\epsilon_2+\frac{1}{8}\sigma^{-2}\epsilon_2\bar\epsilon_1(\not{D}\psi_\alpha-D_\alpha\rho)-1\leftrightarrow 2+D_\alpha\zeta+\frac{1}{2}VD_\alpha\rho$$

$$\varphi=-\frac{1}{4}D_a\big(\delta_1\mathcal{G}_{b\alpha}\gamma^a\gamma^b\epsilon_2\big)-1\leftrightarrow 2+\frac{1}{4}\sigma^{-2}\epsilon_{[2}\bar\epsilon_{1]}(\not{D}\psi_\alpha-D_\alpha\rho)+D_\alpha\zeta+\frac{1}{2}VD_\alpha\rho.$$

$$\delta_1\mathcal{G}_{b\alpha}\gamma^a\gamma^b\epsilon_2 - 1 \leftrightarrow 2 = -\sigma^{-2}\gamma^a\gamma^b\epsilon_{[2}\left(\bar{\psi}_\alpha\gamma_b\epsilon_{1]}\right)\overbrace{\underbrace{\zeta''^{\dagger}_{mn}\sum_{\substack{0\le \iota\le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}} - \frac{1}{2}\sigma^{-2}\gamma^a\gamma_b\psi_\alpha\left(\bar{\epsilon}_2\gamma^b\epsilon_1\right) = -2$$

$$\text{Der } = \frac{1}{2}\not{\partial}V\psi_\alpha + \frac{1}{4}\sigma^{-2}\epsilon_{[2}\bar{\epsilon}_{1]}(\not{\partial}\psi_\alpha - D_\alpha\rho) + D_\alpha\zeta + \frac{1}{2}VD_\alpha\rho$$

$$\overbrace{\underbrace{\zeta''^{\dagger}_{mn}\sum_{\substack{0\le \iota\le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}}\ \mathcal{L}_V\psi_\alpha + D_\alpha\zeta + \left(\frac{1}{4}\sigma^{-2}\epsilon_{[2}\bar{\epsilon}_{1]} - \frac{1}{2}V\right)(\not{\partial}\psi_\alpha - D_\alpha\rho)$$

$$\overbrace{\underbrace{\Lambda''^{\dagger}_{mn}\sum_{\substack{0\le \iota\le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}}\ \frac{1}{8}\sigma^{-2}(\bar{\psi}_\alpha\rho)\zeta = -\frac{1}{16}\sigma^{-2}(\bar{\psi}_\alpha\rho)V\rho \overset{(\text{B.19})}{=} -\frac{1}{1536}\sigma^{-2}V\left(\bar{\rho}\gamma_{(3)}\rho\right)\gamma^{(3)}\psi_\alpha$$

$$\frac{1}{8}\sigma^{-2}(\bar{\psi}_\alpha\rho)\zeta + \frac{1}{1536}\sigma^{-2}V\left(\bar{\rho}\gamma_{(3)}\rho\right)\gamma^{(3)}\psi_\alpha = 0$$

$$\begin{aligned}\text{Alg } =& \frac{1}{8}\sigma^{-2}(\bar{\psi}_\alpha\rho)\zeta - \frac{1}{4}\delta_\zeta\mathcal{G}_{a\alpha}\gamma^a\rho + \left(\frac{1}{4}\sigma^{-2}\epsilon_{[2}\bar{\epsilon}_{1]} - \frac{1}{2}V\right)\left[-\frac{1}{768}\sigma^{-2}\left(\bar{\rho}\gamma_{(3)}\rho\right)\gamma^{(3)}\psi_\alpha\right]\\ &+ \left[\frac{1}{16}\delta_1\mathcal{G}_{a\alpha}\delta_2\mathcal{G}_{b\beta}\gamma^a\gamma^b\psi^\beta + \frac{1}{32}\sigma^{-2}\delta_1\mathcal{G}_{a\beta}\left(\bar{\psi}_\alpha\gamma^a\psi^\beta\right)\epsilon_2 - 1 \leftrightarrow 2\right]\\ &- \frac{1}{16}\Big[\delta_1\mathcal{G}_2, \delta_2\mathcal{G}_{ab}\gamma^{ab}\psi_\alpha - \frac{1}{4}\big[\delta_1\mathcal{L}, \delta_2\mathcal{G}_{\alpha\beta}\psi^\beta\end{aligned}$$

$$\begin{aligned}&\frac{1}{16}\delta_1\mathcal{G}_{a\alpha}\delta_2\mathcal{G}_{b\beta}\gamma^a\gamma^b\psi^\beta + \frac{1}{32}\sigma^{-2}\delta_1\mathcal{G}_{a\beta}\left(\bar{\psi}_\alpha\gamma^a\psi^\beta\right)\epsilon_2 - 1 \leftrightarrow 2 - \frac{1}{16}\big[\delta_1\mathcal{G}_{,}\delta_2\mathcal{G}_{ab}\gamma^{ab}\psi_\alpha\\ &- \frac{1}{4}\Big[\delta_1\mathcal{G}_{,}\delta_2\mathcal{G}_{\alpha\beta}\psi^\beta = \left(\frac{1}{4}\sigma^{-2}\epsilon_{[2}\bar{\epsilon}_{1]} - \frac{1}{2}V\right)\Big[-\frac{1}{192}\sigma^{-2}\left(\bar{\psi}_\beta\gamma_{(3)}\psi^\beta\right)\gamma^{(3)}\psi_\alpha\Big]\end{aligned}$$

$$\begin{aligned}-(\bar{\epsilon}_1\gamma_a\psi_\alpha)(\bar{\lambda}\gamma^b\gamma^a\psi^\beta) &= (\bar{\epsilon}_1\gamma_a\psi_\alpha)\left(\overline{\gamma^b\lambda}\gamma^a\psi^\beta\right)\blacksquare^{\odot d\hbar''}_{\iint\boxplus''_{(4)}\otimes\frac{\partial\mathfrak{G}}{\partial\mathfrak{M}}d^4\chi}\left(\bar{\psi}_\alpha\gamma_a\gamma^b\lambda\right)\left(\bar{\epsilon}_1\gamma^a\psi^\beta\right) - \left(\bar{\epsilon}_1\gamma_a\gamma^b\lambda\right)\left(\bar{\psi}_\alpha\gamma^a\gamma^b\psi^\beta\right)\\ &= \bar{\lambda}\left[2\sigma^2\gamma^b\gamma^a\psi_\alpha\delta_1\mathcal{G}_a{}^\beta - \gamma^b\gamma_a\epsilon_1\left(\bar{\psi}_\alpha\gamma^a\psi^\beta\right)\right],\end{aligned}$$

$$\frac{1}{4}[\delta_1\mathcal{G}, \delta_2\mathcal{G}]_{\alpha\beta} = \frac{1}{16}\sigma^{-4}(\bar{\epsilon}_1\gamma^a\psi_\alpha)\left(\bar{\epsilon}_2\gamma_a\psi_\beta\right) - 1 \leftrightarrow 2\ \overbrace{\underbrace{\Lambda''^{\dagger}_{mn}\sum_{\substack{0\le \iota\le m\\ 0<\jmath<n}}\int P(\iota,\jmath)}_{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}} - \frac{1}{4}\sigma^{-2}V^a\left(\bar{\psi}_\alpha\gamma_a\psi_\beta\right)$$

$$\begin{aligned}\text{LHS}\ \blacksquare^{\odot d\hbar''}_{\iint\boxplus''_{(4)}\otimes\frac{\partial\mathfrak{G}}{\partial\mathfrak{M}}d^4\chi} &= \frac{1}{32}\delta_2\mathcal{L}_{b\beta}\sigma^{-2}(\bar{\epsilon}_1\gamma_a\psi_\alpha)\gamma^b\gamma^a\psi^\beta + \frac{1}{8}\delta_1\mathcal{G}_{a\alpha}\delta_2\mathcal{G}^a_\beta\psi^\beta - \frac{1}{32}\sigma^{-2}\delta_2\mathcal{G}^\beta_b\left(\bar{\psi}_\alpha\gamma^b\psi_\beta\right)\epsilon_1 - 1 \leftrightarrow 2\\ &- \frac{1}{16}\Big[\delta_1\mathcal{G}_{,}\delta_2\mathcal{G}_{ab}\gamma^{ab}\psi_\alpha - \frac{1}{4}\big[\delta_1\mathcal{G}_{,}\delta_2\mathcal{G}_{\alpha\beta}\psi^\beta\end{aligned}$$

$$\overbrace{\Lambda''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}\ \frac{1}{32}\delta_2\mathcal{C}_{b\beta}\sigma^{-2}\left[2\sigma^2\gamma^{ba}\psi_\alpha\delta_1\mathcal{C}_a{}^\beta-\gamma^b\gamma_a\epsilon_1\left(\bar{\psi}_\alpha\gamma^a\psi^\beta\right)\right]+\frac{1}{8}\delta_1\mathcal{C}_{a\alpha}\delta_2\mathcal{C}^a{}_\beta\psi^\beta$$

$$-\frac{1}{32}\sigma^{-2}\delta_2\mathcal{C}^\beta{}_b\left(\bar{\psi}_\alpha\gamma^b\psi_\beta\right)\epsilon_1-1\leftrightarrow 2-\frac{1}{16}[\delta_1\mathcal{L},\delta_2\mathcal{G}]_{ab}\,\gamma^{ab}\psi_\alpha-\frac{1}{4}[\delta_1\mathcal{C},\delta_2\mathcal{C}]_{\alpha\beta}\psi^\beta$$

$$=\frac{1}{32}\sigma^{-2}\delta_2\mathcal{G}_{b\beta}\gamma^b\gamma_a\epsilon_1\left(\bar{\psi}_\alpha\gamma^a\psi^\beta\right)-\frac{1}{32}\sigma^{-2}\delta_2\mathcal{G}^\beta{}_b\left(\bar{\psi}_\alpha\gamma^b\psi_\beta\right)\epsilon_1-1\leftrightarrow 2-\frac{1}{8}[\delta_1\mathcal{G},\delta_2\mathcal{G}]_{\alpha\beta}\psi^\beta$$

$$=-\frac{1}{32}\sigma^{-2}\delta_2\mathcal{G}_{b\beta}\gamma_a\gamma^b\epsilon_1\left(\bar{\psi}_\alpha\gamma^a\psi^\beta\right)+\frac{1}{32}\sigma^{-2}\delta_2\mathcal{G}_{a\beta}\left(\bar{\psi}_\alpha\gamma^a\psi^\beta\right)\epsilon_1-1\leftrightarrow 2-\frac{1}{8}[\delta_1\mathcal{G},\delta_2\mathcal{G}]_{\alpha\beta}\psi^\beta$$

$$=\frac{1}{64}\sigma^{-4}\gamma^a\gamma^b\epsilon_1\left(\bar{\psi}_\beta\gamma_b\epsilon_2\right)\left(\bar{\psi}_\alpha\gamma_a\psi^\beta\right)+\frac{1}{64}\sigma^{-4}\left(\bar{\epsilon}_2\gamma_a\psi^\beta\right)\left(\bar{\psi}_\alpha\gamma^a\psi_\beta\right)\epsilon_1-1\leftrightarrow 2-\frac{1}{8}[\delta_1\mathcal{L},\delta_2\mathcal{L}]_{\alpha\beta}\psi^\beta$$

$$\overbrace{\Lambda''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}\ \frac{1}{128}\sigma^{-4}\gamma^a\gamma^b\psi_\beta(\bar{\epsilon}_1\gamma_b\epsilon_2)\left(\bar{\psi}_\alpha\gamma_a\psi^\beta\right)+\frac{1}{64}\sigma^{-4}\left(\bar{\epsilon}_2\gamma_a\psi^\beta\right)\left(\bar{\psi}_\alpha\gamma^a\psi_\beta\right)\epsilon_1-1\leftrightarrow 2-$$

$$\overbrace{\Lambda''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}\ \frac{1}{16}\sigma^{-2}\gamma^a V\psi_\beta\left(\bar{\psi}_\alpha\gamma_a\psi^\beta\right)-\frac{1}{768}\sigma^{-4}\epsilon_{[2}\bar{\epsilon}_{1]}\left(\bar{\psi}_\beta\gamma_{(3)}\psi^\beta\right)\gamma^{(3)}\psi_\alpha-\frac{1}{8}\left[\delta_1\mathcal{L}_1,\delta_2\mathcal{C}_\right]_{\alpha\beta}$$

$$\overbrace{\Lambda''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}\ \frac{1}{16}\sigma^{-2}V\gamma^a\psi_\beta\left(\bar{\psi}_\alpha\gamma_a\psi^\beta\right)-\frac{1}{8}\sigma^{-2}\psi_\beta\left(\bar{\psi}_\alpha V\psi^\beta\right)-\frac{1}{768}\sigma^{-4}\epsilon_{[2}\bar{\epsilon}_{1]}\left(\bar{\psi}_\beta\gamma_{(3)}\psi^\beta\right)\gamma^{(3)}$$

$$-\frac{1}{8}[\delta_1\mathcal{C},\delta_2\mathcal{C}]_{\alpha\beta}\psi^\beta$$

$$\overbrace{\Lambda''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}\ \frac{1}{384}\sigma^{-2}V\left(\bar{\psi}_\beta\gamma_{(3)}\psi^\beta\right)\gamma^{(3)}\psi_\alpha-\frac{1}{8}\sigma^{-2}\psi_\beta\left(\bar{\psi}_\alpha V\psi^\beta\right)-\frac{1}{768}\sigma^{-4}\epsilon_{[2}\bar{\epsilon}_{1]}\left(\bar{\psi}_\beta\gamma_{(3)}\psi^\beta\right)$$

$$-\frac{1}{8}[\delta_1\mathcal{L},\delta_2\mathcal{G}]_{\alpha\beta}\psi^\beta$$

$$\overbrace{\Lambda''^{\dagger}_{mn}\sum_{\substack{0\le \imath\le m\\ 0<\jmath<n}}\int P(\imath,\jmath)}^{(\bar{e}_1\gamma_{(3)}e)(\bar{e}_3\gamma^{(3)}e)=-18\ell(\bar{e}_1\gamma_a e_3)(\bar{e}\gamma^a e)}\ \left(\frac{1}{4}\sigma^{-2}\epsilon_{[2}\bar{\epsilon}_{1]}-\frac{1}{2}V\right)\left[-\frac{1}{192}\sigma^{-2}\left(\bar{\psi}_\beta\gamma_{(3)}\psi^\beta\right)\gamma^{(3)}\psi_\alpha\right],$$

$$T^*E\cong\pi^*(T^*M\oplus E^*)$$

$$m\in M, e\in E_m, p\in T^*_mM, e^*\in E^*_m.$$

$$\begin{aligned}T_{(m,e,p,e^*)}T^*E&\cong T_{(m,e,p,e^*)}\pi^*(T^*M\oplus E^*)\\&\cong T_{(m,e)}E\oplus\pi^*(T^*M\oplus E^*)_{(m,e)}\\&\cong(T_mM\oplus E_m)\oplus(T^*M\oplus E^*)_m\\&\cong T_mM\oplus E_m\oplus T^*_mM\oplus E^*_m\end{aligned}$$

$$\pi=\varepsilon^i\wedge\varepsilon_i+\varepsilon^a\wedge\varepsilon_a-\frac{1}{2}\bar{T}^k{}_{ij}(m)p_k\varepsilon^i\wedge\varepsilon^j-\frac{1}{2}F_{ij}{}^a{}_b(m)e^b e^*_a\varepsilon^i\wedge\varepsilon^j,$$

$$\pi^*(T^*M \oplus E^*) \ni (m,e,p,e^*) \mapsto (m,e,\pi^*p + ie^*) \in T^*E,$$

$$\alpha(\dot\gamma) = \langle \gamma, \pi_*^X \dot\gamma \rangle$$

$$\alpha(\dot\gamma) = \langle (m,e)^{\cdot}; \pi^*p + ie^* \rangle = p(\dot m) + e^*(\mathrm{ver}(m,e)^{\cdot})$$

$$\dot\gamma = \dot\gamma^i \partial_{x^i} + \dot\gamma^a \partial_{y^a} + \dot\gamma_i \partial_{x_i^*} + \dot\gamma_a \partial_{y_a^*}.$$

$$\mathrm{ver}\partial_{x^i} = A_i^a{}_b(x) y^b \partial_{y^a}, \mathrm{ver}\partial_{y^a} = \partial_{y^a}$$

$$\alpha(\dot\gamma) = x_i^* \dot\gamma^i + y_a^* \left(A_i{}^a{}_b(x) y^b \dot\gamma^i + \dot\gamma^a \right),$$

$$\begin{aligned} \alpha &= \left(x_i^* + y_a^* A_i{}^a{}_b(x) y^b \right) dx^i + y_a^* dy^a \\ &= x_i^* dx^i + y_a^* \left(dy^a + A^a{}_b(x) y^b \right) \end{aligned}$$

$$\begin{aligned} \omega = d\alpha &= dx_i^* \wedge dx^i + dy_a^* \wedge dy^a + d\left(y_a^* A_i{}^a{}_b(x) y^b \right) \wedge dx^i \\ &= dx_i^* \wedge dx^i + \left(dy_a^* - A^b{}_a(x) y_b^* \right) \wedge \left(dy^a + A^a{}_c(x) y^c \right) + y_a^* F^a{}_b y^b. \end{aligned}$$

$$\pi = \partial_{x_i^*} \wedge \partial_{x^i} + \partial_{y_a^*} \wedge \partial_{y^a} + y_b^* A_i{}^b{}_a \partial_{x_i^*} \wedge \partial_{y_a^*} - A_i{}^a{}_b y^b \partial_{x_i^*} \wedge \partial_{y^a} - \frac{1}{2} y_a^* y^b F_{ij}{}^a{}_b \partial_{x_i^*} \wedge \partial_{x_j^*}$$

$$T_mM \oplus E_m \oplus T_m^*M \oplus E_m^* \to T_{(m,e,p,e^*)} \pi^*(T^*M \oplus E^*)$$

$$\begin{aligned} T_mM \ni v &\mapsto \lambda_{(m,e)}^{\pi^*(T^*M \oplus E^*) \to E} \lambda_e^{E \to M} v \\ E_m \ni \tilde e &\mapsto \lambda_{(m,e)}^{\pi^*(T^*M \oplus E^*) \to E} \left. \frac{d}{dt} \right|_{t=0} (e + t\tilde e) \\ T_m^*M \ni \beta &\mapsto \left. \frac{d}{dt} \right|_{t=0} (m,e,p + t\beta, e^*) \\ E_m^* \ni \tilde e^* &\mapsto \left. \frac{d}{dt} \right|_{t=0} (m,e,p,e^* + t\tilde e^*) \end{aligned}$$

$$\begin{aligned} \lambda_e^{E \to M} &: \partial_{x^i} \mapsto \partial_{x^i} - A_i{}^a{}_b(x) y^b \partial_{y^a} \\ \lambda_{(p,e^*)}^{T^*M \oplus E^* \to M} &: \partial_{x^i} \mapsto \partial_{x^i} + A_i{}^a{}_b y_a^* \partial_{y_b^*} + \bar A_i{}^j{}_k x_j^* \partial_{x_k^*} \end{aligned}$$

$$\lambda_{(p,e^*)}^{\pi^*(T^*M \oplus E^*) \to E} : \partial_{x^i} \mapsto \partial_{x^i} + A_i{}^a{}_b y_a^* \partial_{y_b^*} + \bar A_i{}^j{}_k x_j^* \partial_{x_k^*}, \partial_{y^a} \mapsto \partial_{y^a}$$

$$\begin{aligned} T_mM \ni \partial_{x^i} &\mapsto \partial_{x^i} + A_i{}^a{}_b y_a^* \partial_{y_b^*} + \bar A_i{}^j{}_k x_j^* \partial_{x_k^*} - A_i{}^a{}_b(x) y^b \partial_{y^a} \\ E_m \ni \varepsilon_a &\mapsto \partial_{y^a} \\ T_m^*M \ni dx^i &\mapsto \partial_{x_i^*} \\ E_m^* \ni \varepsilon^a &\mapsto \partial_{y_a^*} \end{aligned}$$

$$\pi = \varepsilon^i \wedge \varepsilon_i + \varepsilon^a \wedge \varepsilon_a - \bar A_i{}^j{}_k x_j^* \varepsilon^i \wedge \varepsilon^k - \frac{1}{2} y_a^* y^b F_{ij}{}^a{}_b \varepsilon^i \wedge \varepsilon^j$$

$$\bar A_{[i}{}^j{}_{k]} = \frac{1}{2} \bar T^j{}_{ik},$$

$$\delta(\mathcal{L}_V) = \frac{1}{16} [\delta\mathcal{L}, \mathcal{L}_V \mathcal{G}]_{ab} \gamma^{ab} = F(\delta\mathcal{L}, \mathcal{L}_V \mathcal{G}) \text{ on } \Gamma(S(C_+) \otimes H)$$

$$\Gamma\big(S(C_+)\big) \cong C^\infty(M) \otimes S_0$$

$$\delta e_a = \frac{1}{2}\delta\mathcal{G}_a^\alpha e_\alpha, \delta e_\alpha = -\frac{1}{2}\delta\mathcal{G}_\alpha^a e_a$$

$$\delta(\Gamma_{\alpha bc}) = -\frac{1}{2}\delta\mathcal{G}_\alpha{}^a\Gamma_{abc} - \partial_{[b}\delta\mathcal{G}_{c]\alpha} - \Gamma_{[bc]}{}^a\mathcal{G}_{a\alpha} - \Gamma_{[b|\alpha}{}^\beta\mathcal{G}_{|c]\beta},$$
$$\delta\big(\Gamma_{[abc]}\big) = \frac{1}{2}\delta\mathcal{G}_{[a|}{}^\alpha\Gamma_{\alpha|bc]}.$$

$$\begin{aligned}
\delta(\mathcal{L}_V) =& \delta\left(D_V + \frac{1}{2}(D_aV_b)\gamma^{ab}\right)\\
=& \delta\left(V^A a(e_A) + \frac{1}{4}V^A\Gamma_{Abc}\gamma^{bc} + \frac{1}{2}(a(e_a)V_b + \Gamma_{ab}{}^cV_c)\gamma^{ab}\right)\\
=& \delta\left(V^A a(e_A) + \frac{1}{2}(a(e_a)V_b)\gamma^{ab} + \frac{1}{4}V^\alpha\Gamma_{\alpha bc}\gamma^{bc} + \frac{3}{4}V^a\Gamma_{[abc]}\gamma^{bc}\right)\\
=& \frac{1}{2}(a(\delta e_a)V_b + a(e_a)\delta V_b)\gamma^{ab} + \frac{1}{4}(\delta V^\alpha\Gamma_{\alpha bc} + V^\alpha\delta\Gamma_{\alpha bc})\gamma^{bc} + \frac{3}{4}\big(\delta V^a\Gamma_{[abc]} + V^a\delta\Gamma_{[abc]}\big)\gamma^{bc}\\
=& \frac{1}{4}\big(\delta\mathcal{G}_a{}^\alpha\partial_\alpha V_b + \partial_a(\delta\mathcal{G}_b{}^\alpha V_\alpha)\big)\gamma^{ab} - \frac{1}{8}\delta\mathcal{G}_{\mathcal{L}}{}^\alpha{}_a V^a\Gamma_{\alpha bc}\gamma^{bc}\\
&+ \frac{1}{4}V^\alpha\left(-\frac{1}{2}\delta\mathcal{G}_\alpha{}^a\Gamma_{abc} - \partial_b\delta\mathcal{G}_{c\alpha} - \Gamma_{bc}{}^a\delta\mathcal{G}_{a\alpha} - \Gamma_{b\alpha}{}^\beta\delta\mathcal{G}_{c\beta}\right)\gamma^{bc}\\
&+ \frac{3}{8}\big(\delta\mathcal{G}^a{}_\alpha{}^\alpha V^a\Gamma_{[abc]} + V^a\delta\mathcal{G}_{[a|}{}^\alpha\Gamma_{\alpha|bc]}\big)\gamma^{bc}.
\end{aligned}$$

$$\begin{aligned}
\delta(\mathcal{L}_V) =& \frac{1}{4}(\delta\mathcal{G}_a{}^\alpha\partial_\alpha V_b + \delta\mathcal{G}_b{}^\alpha\partial_a V_\alpha)\gamma^{ab}\\
&+ \frac{1}{4}\Big(-\frac{1}{2}\delta\mathcal{G}^\alpha{}_a V^a\Gamma_{\alpha bc} - \frac{1}{2}V^\alpha\delta\mathcal{G}_\alpha{}^a\Gamma_{abc} - V^\alpha\Gamma_{bc}{}^a\delta\mathcal{G}_{a\alpha} - V^\alpha\Gamma_{b\alpha}{}^\beta\delta\mathcal{G}_{c\beta}\\
&+ \frac{1}{2}\delta\mathcal{G}^a{}_\alpha V^\alpha\Gamma_{abc} + \delta\mathcal{G}^a{}_\alpha V^\alpha\Gamma_{bca} + \frac{1}{2}V^a\delta\mathcal{G}_a{}^\alpha\Gamma_{\alpha bc} + V^a\delta\mathcal{G}_b{}^\alpha\Gamma_{\alpha ca}\Big)\gamma^{bc}\\
=& \frac{1}{4}\big(\delta\mathcal{G}_b{}^\alpha\partial_\alpha V_c - \delta\mathcal{G}_b{}^\alpha\partial_c V_\alpha - V^\alpha\Gamma_{b\alpha}{}^\beta\delta\mathcal{G}_{c\beta} + V^a\delta\mathcal{G}_b{}^\alpha\Gamma_{\alpha ca}\big)\gamma^{bc}\\
=& \frac{1}{4}\delta\mathcal{G}_b{}^\alpha(D_\alpha V_c - D_cV_\alpha)\gamma^{bc} = \frac{1}{8}\delta\mathcal{G}_b{}^\alpha(\mathcal{L}_V\mathcal{G}_l)_{c\alpha}\gamma^{bc} = \frac{1}{16}\big[\delta\mathcal{G},\mathcal{L}_V\mathcal{G}_{ab}\gamma^{ab}
\end{aligned}$$

$$\mathrm{d}s^2 = H_1^{\alpha_1}H_2^{\alpha_2}\,\mathrm{d}x^2 + H_1^{\alpha_1}H_2^{\beta_2}\,\mathrm{d}y_1^2 + H_1^{\beta_1}H_2^{\alpha 2}\,\mathrm{d}y_2^2 + H_1^{\beta_1}H_2^{\beta_2}\,\mathrm{d}z^2$$

$$\mathrm{d}\hat{s}_{10}^2 = H(y)^{\frac{p-7}{8}}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + H(y)^{\frac{p+7}{8}}\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J, e^{\hat{\Phi}} = H(y)^{\frac{3-p}{4}}$$

$$\hat{F}_{(p+2)}^{\mathrm{ele}} = \partial_I H^{-1}\,\mathrm{d}x^0\wedge\cdots\wedge\,\mathrm{d}x^p\wedge\,\mathrm{d}y^I, \hat{F}_{(8-p)}^{\mathrm{mag}} = \frac{1}{(8-p)!}\varepsilon_{I_1\cdots I_{9-p}}\partial_{I_1}H\,\mathrm{d}y^{I_2}\wedge\cdots\wedge\,\mathrm{d}y^{I_{9-p}}$$

$$\partial_I^2 H(y) = 0$$

$$H(y) = 1 + \frac{\alpha N}{r^{7-p}}$$

$$\mathrm{d}\hat{s}_{10}^2 = -H_1^{-\frac{3}{4}}H_2^{-\frac{1}{4}}\,\mathrm{d}t^2 + H_1^{-\frac{3}{4}}H_2^{\frac{3}{4}}\,\mathrm{d}x_1^2 + H_1^{\frac{1}{4}}H_2^{-\frac{1}{4}}(\,\mathrm{d}x_2^2 + \cdots + \mathrm{d}x_6^2) + H_1^{\frac{1}{4}}H_2^{\frac{3}{4}}(\,\mathrm{d}y_1^2 + \mathrm{d}y_2^2 + \mathrm{d}y_3^2)$$
$$\hat{H}_{(3)} = H_1^{-2}\partial_I H_1\,\mathrm{d}t\wedge\,\mathrm{d}x_1\wedge\,\mathrm{d}y^I, \tilde{F}_{(3)} = \frac{1}{2}\varepsilon_{IJK}\partial_I H_2\,\mathrm{d}x_1\wedge\,\mathrm{d}y^J\wedge\,\mathrm{d}y^K, e^{\hat{\Phi}} = H_1^{-\frac{1}{2}}H_2^{-\frac{1}{2}}$$

$$\partial_I^2 H_1(y) = \partial_I^2 H_2(y) = 0$$

$$\Gamma_{\underline{012345}}\varepsilon = c_1\varepsilon, \Gamma_{\underline{016789}}\varepsilon = c_2\varepsilon$$

$$\Gamma_{\underline{01,10}}\varepsilon = c_1 c_2 \varepsilon$$

$$\begin{aligned}
&\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{1}{2}}[-h^{-1}\,\mathrm{d}t^2 + h(\,\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2)] + H_0^{\frac{1}{2}}(\,\mathrm{d}y_1^2 + \cdots + \mathrm{d}y_6^2),\\
&\tilde{F}_{(5)} = hH_0^{-2}\partial_I H_0\,\mathrm{d}t \wedge \mathrm{d}x_1 \wedge \mathrm{d}x_2 \wedge \mathrm{d}x_3 \wedge \mathrm{d}y^I - \frac{1}{5!}\varepsilon_{I_1\cdots I_6}\partial_{I_1}H_0\,\mathrm{d}y^{I_2}\cdots\wedge\,\mathrm{d}y^{I_6},\\
&\hat{H}_{(3)} = \partial_a h^{-1}\,\mathrm{d}t \wedge \mathrm{d}x^a \wedge \mathrm{d}y^1, \tilde{F}_{(3)} = -\frac{1}{2!}\varepsilon_{abc}\partial_a h\,\mathrm{d}x^b \wedge \mathrm{d}x^c \wedge \mathrm{d}y^1,\\
&e^{\hat{\Phi}} = h^{-1}, h = h(x_1, x_2, x_3), H_0 = H_0(y_1, \cdots, y_6).
\end{aligned}$$

$$\partial_I^2 H_0(y) = 0, \partial_a^2 h(x) = 0$$

$$\tilde{F}_{(5)} = \mathrm{d}\hat{C}_{(4)} - \frac{1}{2}\hat{C}_{(2)} \wedge \hat{H}_{(3)} + \frac{1}{2}\hat{B}_{(2)} \wedge \mathrm{d}\hat{C}_{(2)}$$

$$\begin{aligned}
\mathrm{d}\hat{s}_{10}^2 = &-H_0^{-\frac{1}{2}}H_1^{-\frac{3}{4}}H_2^{-\frac{1}{4}}\,\mathrm{d}t^2 + H_0^{-\frac{1}{2}}H_1^{\frac{1}{4}}H_2^{\frac{3}{4}}(\,\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2)\\
&+H_0^{\frac{1}{2}}H_1^{-\frac{3}{4}}H_2^{\frac{3}{4}}\,\mathrm{d}y_1^2 + H_0^{\frac{1}{2}}H_1^{\frac{1}{4}}H_2^{-\frac{1}{4}}(\,\mathrm{d}y_2^2 + \cdots + \mathrm{d}y_6^2),
\end{aligned}$$

$$\mathrm{d}\tilde{F}_{(4)} = \hat{F}_{(2)} \wedge \hat{H}_{(3)}.$$

$$\begin{aligned}
&\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{1}{4}}\left[-h^{-\frac{5}{4}}\,\mathrm{d}t^2 + h^{\frac{3}{4}}(\,\mathrm{d}x_1^2 + \cdots + \mathrm{d}x_5^2)\right] + H_0^{\frac{3}{4}}h^{-\frac{1}{4}}(\,\mathrm{d}y_1^2 + \cdots + \mathrm{d}y_4^2)\\
&\tilde{F}_{(5)} = H_0 h^{-2}\partial_a h\,\mathrm{d}t \wedge \mathrm{d}y_2 \wedge \mathrm{d}y_3 \wedge \mathrm{d}y_4 \wedge \mathrm{d}x^a - \frac{1}{4!}\varepsilon_{abcde}\partial_a h\,\mathrm{d}y^1 \wedge \mathrm{d}x^b \wedge \mathrm{d}x^c \wedge \mathrm{d}x^d \wedge \mathrm{d}x^e\\
&\hat{H}_{(3)} = h^{-2}\partial_a h\,\mathrm{d}t \wedge \mathrm{d}y^1 \wedge \mathrm{d}x^a, \tilde{F}_{(3)} = \frac{1}{3!}\varepsilon_{IJKL}\partial_I H_0\,\mathrm{d}y^J \wedge \mathrm{d}y^K \wedge \mathrm{d}y^L\\
&e^{\hat{\Phi}} = h^{-\frac{1}{2}}H_0^{-\frac{1}{2}}, h = h(x_1, \cdots, x_5), H_0 = H_0(y_1, \cdots, y_4).
\end{aligned}$$

$$\begin{aligned}
&\mathrm{d}\hat{s}_{11}^2 = H_0^{-\frac{1}{3}}[h^{-1}(-\mathrm{d}t^2 + \mathrm{d}x^2) + h(\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_3^2)] + H_0^{\frac{2}{3}}(\,\mathrm{d}y_1^2 + \cdots + \mathrm{d}y_5^2)\\
&\hat{F}_{(4)} = -\frac{1}{4!}\varepsilon_{I_1\cdots I_5}\partial_{I_1}H_0\,\mathrm{d}y^{I_2}\wedge\cdots\wedge\,\mathrm{d}y^{I_5} - h^{-2}\partial_r h\,\mathrm{d}t\wedge\mathrm{d}t\wedge\,\mathrm{d}r\wedge\,\mathrm{d}y^1 - r^3\partial_r h\,\mathrm{d}\Omega_3\wedge\,\mathrm{d}y^1\\
&h = h(r), H_0 = H_0(y_1, \cdots, y_5)
\end{aligned}$$

$$\begin{aligned}
&\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{3}{8}}\left[-h^{-\frac{9}{8}}\,\mathrm{d}t^2 + h^{\frac{7}{8}}(\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_3^2)\right] + H_0^{\frac{5}{8}}h^{-\frac{1}{8}}(\,\mathrm{d}y_1^2 + \cdots + \mathrm{d}y_5^2),\\
&\tilde{F}_{(4)} = r^3\partial_r h\,\mathrm{d}y_1 \wedge \mathrm{d}\Omega_3 - \frac{1}{4!}\varepsilon_{I_1\cdots I_5}\partial_{I_1}H_0\,\mathrm{d}y^{I_2}\wedge\cdots\,\mathrm{d}y^{I_5},\\
&\hat{H}_{(3)} = h^{-2}\partial_r h\,\mathrm{d}t \wedge \mathrm{d}y_1 \wedge \mathrm{d}r,\\
&e^{\hat{\Phi}} = h^{-\frac{3}{4}}H_0^{-\frac{1}{4}}, h = h(r), H_0 = H_0(y_1, \cdots, y_5),
\end{aligned}$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{1}{4}}\left[-h^{-\frac{5}{4}}\,\mathrm{d}t^2 + h^{\frac{3}{4}}(\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_4^2)\right] + H_0^{\frac{3}{4}}h^{-\frac{1}{4}}(\,\mathrm{d}y_1^2 + \cdots + \mathrm{d}y_4^2)$$
$$\tilde{F}_{(5)} = H_0 h^{-2}\partial_r h\,\mathrm{d}t \wedge \mathrm{d}y_2 \wedge \mathrm{d}y_3 \wedge \mathrm{d}y_4 \wedge \mathrm{d}r - r^4\partial_r h\,\mathrm{d}y_1 \wedge \mathrm{d}\Omega_4$$
$$\hat{H}_{(3)} = -\frac{1}{3!}\varepsilon_{IJKL}\partial_I H_0\,\mathrm{d}y^J \wedge \mathrm{d}y^K \wedge \mathrm{d}y^L, \tilde{F}_{(3)} = h^{-2}\partial_r h\,\mathrm{d}t \wedge \mathrm{d}y_1 \wedge \mathrm{d}r$$
$$e^{\hat{\Phi}} = h^{\frac{1}{2}}H_0^{\frac{1}{2}}, h = h(r), H_0 = H_0(y_1, \cdots, y_4)$$

$$\mathrm{d}\hat{s}^2 = H(y)^{\frac{p-7}{8}}g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + H(y)^{\frac{p+7}{8}}\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J, \mathcal{R}_{\mu\nu}(g) = 0$$

$$ds_{p+1}^2 = h(r)^\alpha(-\mathrm{d}t^2 + \mathrm{d}x_1^2 + \cdots + \mathrm{d}x_s^2) + h(r)^\beta\left(\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_{p-s-1}^2\right),$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0(y)^{-\frac{1}{2}}g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + H_0(y)^{\frac{1}{2}}\delta_{IJ}\,\mathrm{d}y^I\mathrm{d}y^J, \hat{\Phi} = \phi, \hat{C}_0 = -\chi$$
$$\tilde{F}_{(5)} = -\mathrm{vol}_g \wedge \mathrm{d}H_0^{-1} - *_\delta\,\mathrm{d}H_0, \tilde{F}_{(3)} = -\frac{e^{-\phi}}{\sqrt{2}} *_g \mathcal{F}_{(2)}^I \wedge \mathrm{d}y^I, \hat{H}_{(3)} = \frac{1}{\sqrt{2}}\mathcal{F}_{(2)}^I \wedge \mathrm{d}y^I.$$

$$\mathrm{d}s_4^2 = -h^{-1}\,\mathrm{d}t^2 + h\delta_{ab}\,\mathrm{d}X^a\,\mathrm{d}X^b, \mathcal{A}_{(1)}^1 = -\sqrt{2}h^{-1}\,\mathrm{d}t, \phi = -\ln\,h,$$

$$\partial_X^2 h(X) = 0$$

$$\mathcal{F}_{(2)}^1 = \sqrt{2}\,\mathrm{d}t \wedge \mathrm{d}h^{-1}, * \mathcal{F}_{(2)}^1 = -\frac{\sqrt{2}}{h} *_X \mathrm{d}h,$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{1}{2}}(y)[-h^{-1}(X)\mathrm{d}t^2 + h(X)\delta_{ab}\,\mathrm{d}X^a\,\mathrm{d}X^b] + H_0^{\frac{1}{2}}(y)\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J, \hat{\Phi} = -\ln h,$$
$$\tilde{F}_{(5)} = -\mathrm{vol}_g \wedge \mathrm{d}H_0^{-1} - *_\delta\,\mathrm{d}H_0, \tilde{F}_{(3)} = -*_X(\,\mathrm{d}h) \wedge \mathrm{d}y^1, \hat{H}_{(3)} = \mathrm{d}t \wedge \mathrm{d}h^{-1} \wedge \mathrm{d}y^1.$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{1}{2}}(y)g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + H_0^{\frac{1}{2}}(y)\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J, \hat{\Phi} = \phi(x), \hat{C}_{(0)} = \alpha\chi(x)$$
$$\tilde{F}_{(5)} = H_0^{-2}\mathrm{vol}_g \wedge \mathrm{d}H_0 - *_\delta\,\mathrm{d}H_0, \tilde{F}_{(3)} = -\frac{e^{-\phi}}{\sqrt{2}} *_g \mathcal{F}_{(2)}^I \wedge \mathrm{d}y^I, \hat{H}_{(3)} = \frac{1}{\sqrt{2}}\mathcal{F}_{(2)}^I \wedge \mathrm{d}y^I.$$

$$\mathrm{d}\hat{s}_{10} = H_0^{-\frac{1}{2}}[-\mathrm{d}t^2 + \mathrm{d}x_0^2 + h(\,\mathrm{d}x_1^2 + \mathrm{d}x_2^2)] + H_0^{\frac{1}{2}}(\,\mathrm{d}y_1^2 + \cdots + \mathrm{d}y_6^2)$$
$$\tilde{F}_{(5)} = hH_0^{-2}\partial_I H_0\,\mathrm{d}t \wedge \mathrm{d}x_0 \wedge \mathrm{d}x_1 \wedge \mathrm{d}x_2 \wedge \mathrm{d}y^I - \frac{1}{5!}\varepsilon_{I_1\cdots I_6}\partial_{I_1}H_0\,\mathrm{d}y^{I_2} \wedge \cdots \wedge \mathrm{d}y^{I_6}$$
$$e^{\hat{\Phi}} = h^{-1}, \hat{F}_{(1)} = -\varepsilon_{ab}\partial^a h\,\mathrm{d}x^b, h = h(x_1, x_2)\; H_0 = H_0(y), a, b = 1,2$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x_0^2 + h(\,\mathrm{d}x_1^2 + \mathrm{d}x_2^2), e^\phi = h^{-1},$$
$$F_{(1)} = -\varepsilon_{ab}\partial^a h\,\mathrm{d}x^b, a, b = 1,2.$$

$$\mathrm{d}\hat{s}_{11}^2 = H_0^{-\frac{1}{3}}(y)g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + H_0^{\frac{2}{3}}(y)\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J, \hat{F}_{(4)} = \delta_{IJ}G_{(3)}^I \wedge \mathrm{d}y^J - *_\delta\,\mathrm{d}H_0, G_{(3)}^I = -*_g G_{(3)}^I.$$

$$\mathrm{d}s_6^2 = h^{-1}(-\mathrm{d}t^2 + \mathrm{d}x_1^2) + h\delta_{ab}\,\mathrm{d}X^a\,\mathrm{d}X^b, G_{(3)}^1 = -\frac{1}{h^2}\,\mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}h - *_X\,\mathrm{d}h,$$

$$\mathrm{d}\hat{s}_{11}^2 = H_0^{-\frac{1}{3}}(y)[h^{-1}(X)(-\mathrm{d}t^2 + \mathrm{d}x_1^2) + h(X)\delta_{ab}\,\mathrm{d}X^a\,\mathrm{d}X^b] + H_0^{\frac{2}{3}}(y)\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J$$

$$\tilde{F}_{(4)} = -\left[\frac{1}{h^2}\,\mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}h + *_X\,\mathrm{d}h\right] \wedge \mathrm{d}y^1 - *_\delta \mathrm{d}H_0$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{3}{8}}(y)e^{\frac{5}{8\sqrt{6}}\sigma}g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + H_0^{\frac{5}{8}}(y)e^{-\frac{3}{8\sqrt{6}}\sigma}\delta_{IJ}\,\mathrm{d}y^I\mathrm{d}y^J, e^{\hat{\Phi}} = H_0^{-\frac{1}{4}}e^{-\frac{9}{4\sqrt{6}}\sigma}$$

$$\tilde{F}_{(4)} = -*_\delta \mathrm{d}H_0 - \frac{1}{\sqrt{2}}e^{\frac{2}{\sqrt{6}}\sigma} *_g F_{(2)}^1 \wedge \mathrm{d}y^1, \hat{H}_{(3)} = \frac{1}{\sqrt{2}}F_{(2)}^1 \wedge \mathrm{d}y^1$$

$$\hat{F}_{(2)} = \alpha G_{(2)}.$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{3}{8}}(y)e^{\frac{5}{8\sqrt{6}}\sigma}g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + H_0^{\frac{5}{8}}(y)e^{-\frac{3}{8\sqrt{6}}\sigma}\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J, e^{\hat{\phi}} = H_0^{-\frac{1}{4}}e^{-\frac{9}{4\sqrt{6}}\sigma}$$

$$\tilde{F}_{(4)} = -*_\delta \mathrm{d}H_0 - \frac{1}{\sqrt{2}}e^{\frac{2}{\sqrt{6}}\sigma} *_g F_{(2)}^I \wedge dy^I, \hat{H}_{(3)} = \frac{1}{\sqrt{2}}F_{(2)}^I \wedge \mathrm{d}y^I, \hat{F}_{(2)} = -G_{(2)}.$$

$$\mathrm{d}s^2 = -H_0^{-\frac{3}{8}}\left[h^{-\frac{7}{8}}\,\mathrm{d}t^2 + h^{\frac{1}{8}}(\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_3^2)\right] + H_0^{\frac{5}{8}}h^{\frac{1}{8}}(\,\mathrm{d}y_1^2 + \cdots + \mathrm{d}y_5^2),$$

$$\tilde{F}_{(4)} = -*_\delta \mathrm{d}H_0, \hat{F}_{(2)} = -h^{-2}\partial_r h\,\mathrm{d}t \wedge \mathrm{d}r, e^{\hat{\Phi}} = H_0^{-\frac{1}{4}}h^{\frac{3}{4}},$$
$$H_0 = H_0(y), h = h(r),$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{1}{4}}(y)e^{-\frac{\sqrt{2}}{4}\varphi}g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + H_0^{\frac{3}{4}}(y)e^{\frac{\sqrt{2}}{4}\varphi}\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J$$

$$\hat{H}_{(3)} = -*_\delta \mathrm{d}H_0, \tilde{F}_{(3)} = -\frac{1}{\sqrt{2}}F_{(2)}^1(x) \wedge \mathrm{d}y^1, e^{\hat{\Phi}} = H_0^{\frac{1}{2}}e^{-\frac{\varphi}{\sqrt{2}}}$$

$$\tilde{F}_{(5)} = \frac{1}{\sqrt{2}}\left[-\frac{1}{3!}H_0\varepsilon_{1I_2\cdots I_4}F_{(2)}^1 \wedge \mathrm{d}y^{I_2} \wedge \mathrm{d}y^{I_3} \wedge \mathrm{d}y^{I_4} + e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^1 \wedge \mathrm{d}y^1\right].$$

$$\hat{H}_{(3)} = \alpha H_{(3)}(y).$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{1}{4}}(y)e^{-\frac{\sqrt{2}}{4}\varphi}g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + H_0^{\frac{3}{4}}(y)e^{\frac{\sqrt{2}}{4}\varphi}\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J$$

$$\hat{H}_{(3)} = -*_\delta \mathrm{d}H_0 - H_{(3)}(x), \tilde{F}_{(3)} = -\frac{1}{\sqrt{2}}F_{(2)}^I(x) \wedge \mathrm{d}y^I, e^{\hat{\phi}} = H_0^{\frac{1}{2}}e^{-\frac{\varphi}{\sqrt{2}}}$$

$$\tilde{F}_{(5)} = \frac{1}{\sqrt{2}}\left[-\frac{1}{3!}H_0\varepsilon_{I_1\cdots I_4}F_{(2)}^{I_1} \wedge \mathrm{d}y^{I_2} \wedge \mathrm{d}y^{I_3} \wedge \mathrm{d}y^{I_4} + e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^I \wedge \mathrm{d}y^I\right]$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{1}{4}}h^{-\frac{1}{4}}\left[h^{-\frac{1}{2}}(-\mathrm{d}t^2 + \mathrm{d}x^2) + h^{\frac{1}{2}}(\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_3^2)\right] + H_0^{\frac{3}{4}}h^{\frac{1}{4}}\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J,$$

$$\hat{H}_{(3)} = -*_\delta \mathrm{d}H_0 - h^{-2}\partial_r h\,\mathrm{d}t \wedge \mathrm{d}x \wedge \mathrm{d}r, e^{\hat{\Phi}} = H_0^{\frac{1}{2}}h^{-\frac{1}{2}}(r),$$
$$H_0 = H_0(y), h = h(r), I, J = 1,2,3,4,$$

$$\mathrm{d}\hat{s}_{10}^2 = H_0^{-\frac{1}{4}}(y)e^{-\frac{\sqrt{2}}{4}\varphi}g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + H_0^{\frac{3}{4}}(y)e^{\frac{\sqrt{2}}{4}\varphi}\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J$$

$$\hat{H}_{(3)} = \frac{1}{\sqrt{2}}F_{(2)}^I(x)\wedge \mathrm{d}y^I, \tilde{F}_{(3)} = -*_\delta \mathrm{d}H_0 - H_{(3)}(x), e^{\hat{\Phi}} = H_0^{-\frac{1}{2}}e^{\frac{\varphi}{\sqrt{2}}}$$

$$\tilde{F}_{(5)} = \frac{1}{\sqrt{2}}\left[-\frac{1}{3!}H_0\varepsilon_{I_1\cdots M_I}F_{(2)}^{I_1}\wedge \mathrm{d}y^{I_2}\wedge \mathrm{d}y^{I_3}\wedge \mathrm{d}y^{I_4} + e^{-\frac{\sqrt{2}}{2}\varphi}*_g F_{(2)}^I\wedge \mathrm{d}y^I\right].$$

$$S_{IIA} = \frac{1}{2\kappa^2}\int (*\hat{R} - \frac{1}{2}\,\mathrm{d}\hat{\Phi}\wedge * \,\mathrm{d}\hat{\Phi} - \frac{1}{2}e^{-\hat{\Phi}}\hat{H}_{(3)}\wedge * \hat{H}_{(3)} - \frac{1}{2}e^{\frac{3}{2}\hat{\Phi}}\hat{F}_{(2)}\wedge * \hat{F}_{(2)}$$
$$-\frac{1}{2}e^{\frac{\hat{\Phi}}{2}}\tilde{F}_{(4)}\wedge * \tilde{F}_{(4)}) - \frac{1}{4\kappa^2}\int \hat{F}_{(4)}\wedge\hat{F}_{(4)}\wedge\hat{B}_{(2)}$$

$$\mathrm{d}*\,\mathrm{d}\hat{\Phi} = \frac{3}{4}e^{\frac{3}{2}\hat{\Phi}}\hat{F}_{(2)}\wedge * \hat{F}_{(2)} - \frac{1}{2}e^{-\hat{\Phi}}\hat{H}_{(3)}\wedge * \hat{H}_{(3)} + \frac{1}{4}e^{\frac{\Phi}{2}}\tilde{F}_{(4)}\wedge * \tilde{F}_{(4)}$$

$$\mathrm{d}\left(e^{\frac{\hat{\Phi}}{2}} * \tilde{F}_{(4)}\right) = -\hat{H}_{(3)}\wedge\tilde{F}_{(4)}$$

$$\mathrm{d}\left(e^{-\hat{\Phi}} * \hat{H}_{(3)}\right) = e^{\frac{\hat{\Phi}}{2}}\hat{F}_{(2)}\wedge * \tilde{F}_{(4)} + \frac{1}{2}\tilde{F}_{(4)}\wedge\tilde{F}_{(4)}$$

$$\mathrm{d}\left(e^{\frac{3\hat{\Phi}}{2}} * \hat{F}_{(2)}\right) = -e^{\frac{\hat{\Phi}}{2}}\hat{H}_{(3)}\wedge * \tilde{F}_{(4)}$$

$$\hat{R}_{MN} = \frac{1}{2}\partial_M\hat{\Phi}\partial_N\hat{\Phi} + \frac{1}{2}e^{\frac{3\hat{\Phi}}{2}}\left(\hat{F}_{MA}\hat{F}_N^A - \frac{1}{8\cdot 2!}g_{MN}\hat{F}_{(2)}^2\right)$$
$$+\frac{1}{4}e^{-\hat{\Phi}}\left(\hat{H}_M^{P_1P_2}\hat{H}_{NP_1P_2} - \frac{1}{2\cdot 3!}\hat{g}_{MN}\hat{H}_{(3)}^2\right)$$
$$+\frac{1}{12}e^{\frac{\hat{\Phi}}{2}}\left(\tilde{F}_M^{P_1P_2P_3}\tilde{F}_{NP_1P_2P_3} - \frac{9}{4\cdot 4!}\hat{g}_{MN}\tilde{F}_{(4)}^2\right)$$

$$\delta\hat{\Psi}_M = \Big(\hat{D}_M - \frac{1}{4\cdot 2!}\hat{H}_{MPQ}\Gamma^{PQ}\Gamma^{(10)}$$
$$-\frac{1}{16}e^{\hat{\Phi}}\hat{F}_{PQ}\Gamma^{PQ}\Gamma_M\Gamma^{(10)} + \frac{1}{8\cdot 4!}e^{\hat{\Phi}}\tilde{F}_{NPQL}\Gamma^{NPQL}\Gamma_M\Big)\hat{\epsilon}$$
$$\delta\hat{\lambda} = \Big(-\frac{1}{3}\Gamma^M\partial_M\hat{\Phi}\Gamma^{(10)} + \frac{1}{6\cdot 3!}\hat{H}_{MNP}\Gamma^{MNP}$$
$$-\frac{1}{2\cdot 4}e^{\hat{\Phi}}\hat{F}_{MN}\Gamma^{MN} + \frac{1}{12\cdot 4!}e^{\hat{\Phi}}\tilde{F}_{MNPQ}\Gamma^{MNPQ}\Gamma^{(10)}\Big)\hat{\epsilon}$$

$$\hat{\epsilon} = \hat{\epsilon}_L + \hat{\epsilon}_R, \Gamma^{(10)}\hat{\epsilon}_L = \hat{\epsilon}_L, \Gamma^{(10)}\hat{\epsilon}_R = -\hat{\epsilon}_R$$

$$\mathrm{d}s_{10}^2 = H^{-\frac{3}{8}}g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + H^{\frac{5}{8}}\delta_{IJ}\,\mathrm{d}y^I\,\mathrm{d}y^J, e^\Phi = H^{-\frac{1}{4}}(y),$$
$$\tilde{F}_{(4)} = -\frac{1}{4!}\partial_{I_1}H\varepsilon_{I_1\cdots I_5}\,\mathrm{d}y^{I_2}\wedge\cdots\wedge\,\mathrm{d}y^{I_5}, I, J = 1,\cdots,5.$$

$$\Gamma_{\underline{01234}}\epsilon = -\Gamma^{(10)}\epsilon, \epsilon = H^{-3/32}\epsilon_0$$

$$\Gamma_{\underline{\mu}} = \sigma_1\otimes\gamma_{\underline{\mu}}\otimes\mathbf{1}, \Gamma_{\underline{I}} = \sigma_2\otimes\mathbf{1}\otimes\Sigma_{\underline{I}}$$

$$\epsilon = \begin{pmatrix}1\\-i\end{pmatrix}\otimes\varepsilon^i\otimes\eta_i, i = 1,2,3,4$$

$$(\sigma_2\otimes\mathbf{1}\otimes\mathbf{1})\epsilon = -\epsilon$$

$$S_{IIB}=\frac{1}{2\kappa_{10}^2}\int\ (*R\ -\frac{1}{2}\ \mathrm{d}\hat{\Phi}\wedge*\ \mathrm{d}\hat{\Phi}-\frac{1}{2}e^{2\hat{\Phi}}\hat{F}_{(1)}\wedge*\hat{F}_{(1)}-\frac{1}{2}e^{-\hat{\Phi}}\hat{H}_{(3)}\wedge*\hat{H}_{(3)}$$
$$-\frac{1}{2}e^{\hat{\Phi}}\tilde{F}_{(3)}\wedge*\tilde{F}_{(3)}-\frac{1}{4}\tilde{F}_{(5)}\wedge*\tilde{F}_{(5)}\Big)-\frac{1}{4\kappa_{10}^2}\int\ \hat{C}_{(4)}\wedge\hat{H}_{(3)}\wedge\tilde{F}_{(3)}$$

$$\mathrm{d}*\ \mathrm{d}\hat{\Phi}=e^{2\hat{\Phi}}\hat{F}_{(1)}\wedge*\hat{F}_{(1)}-\frac{1}{2}e^{-\hat{\Phi}}\hat{H}_{(3)}\wedge*\hat{H}_{(3)}+\frac{1}{2}e^{\hat{\Phi}}\tilde{F}_{(3)}\wedge*\tilde{F}_{(3)}$$
$$\mathrm{d}\left(e^{2\hat{\Phi}}*\hat{F}_{(1)}\right)=-e^{\hat{\Phi}}\hat{H}_{(3)}\wedge*\tilde{F}_{(3)}$$
$$\mathrm{d}\left(e^{\hat{\Phi}}*\tilde{F}_{(3)}\right)=\tilde{F}_{(5)}\wedge\hat{H}_{(3)}$$
$$\mathrm{d}\left(e^{-\hat{\Phi}}*\hat{H}_{(3)}\right)=e^{\hat{\Phi}}\hat{F}_{(1)}\wedge*\tilde{F}_{(3)}-\tilde{F}_{(5)}\wedge\tilde{F}_{(3)}$$
$$\mathrm{d}*\tilde{F}_{(5)}=-\tilde{F}_{(3)}\wedge\hat{H}_{(3)}$$
$$\hat{R}_{MN}=\frac{1}{2}\partial_M\hat{\Phi}\partial_N\hat{\Phi}+\frac{1}{2}e^{2\hat{\Phi}}\hat{F}_M\hat{F}_N+\frac{1}{4\cdot 4!}\tilde{F}_{MPQRS}\tilde{F}_N{}^{PQRS}$$
$$+\frac{1}{4}e^{\hat{\Phi}}\left(\hat{F}_{MPQ}\hat{F}_N{}^{PQ}-\frac{1}{12}G_{MN}\hat{F}_{PQR}\hat{F}^{PQR}\right)$$
$$+\frac{1}{4}e^{-\hat{\Phi}}\left(\hat{H}_{MPQ}\hat{H}_N{}^{PQ}-\frac{1}{12}G_{MN}\hat{H}_{PQR}\hat{H}^{PQR}\right)$$

$$\delta\hat{\lambda}=\frac{1}{2}\left(\partial_M\hat{\Phi}+ie^{\hat{\Phi}}\hat{F}_M\right)\Gamma^M\hat{\epsilon}-\frac{1}{4\cdot 3!}\left(e^{-\hat{\Phi}/2}\hat{H}_{MNP}+ie^{\hat{\Phi}/2}\hat{F}_{MNP}\right)\Gamma^{MNP}\hat{\epsilon}^c$$
$$\delta\hat{\Psi}_M=\hat{D}_M\hat{\epsilon}-\frac{i}{4}e^{\hat{\Phi}}\hat{F}_M\hat{\epsilon}+\frac{1}{96}\left(e^{-\hat{\Phi}/2}\hat{H}_{NPQ}-ie^{\hat{\Phi}/2}\hat{F}_{NPQ}\right)\left(\Gamma_M^{NPQ}-9\delta_M^N\Gamma^{PQ}\right)\hat{\epsilon}^c$$
$$-\frac{i}{16\cdot 5!}\hat{F}_{NPQRT}\Gamma^{NPQRT}\Gamma_M\hat{\epsilon}$$

$$\Gamma^{(10)}\hat{\Psi}_M=\hat{\Psi}_M\ \Gamma^{(10)}\hat{\epsilon}=\hat{\epsilon}$$

$$\Gamma^{(10)}\hat{\lambda}=-\hat{\lambda}$$

$$\hat{\epsilon}=\hat{\epsilon}_L+i\hat{\epsilon}_R,\hat{\epsilon}^c=\hat{\epsilon}_L-i\hat{\epsilon}_R,\Gamma^{(10)}\hat{\epsilon}_{L,R}=\hat{\epsilon}_{L,R}$$

$$\mathrm{d}s_{10}^2=H^{-\frac{1}{4}}\eta_\mu\mathrm{d}x^\mu\mathrm{d}x^\nu+H^{\frac{3}{4}}\delta_{IJ}\ \mathrm{d}y^I\ \mathrm{d}y^J,$$
$$\hat{H}_{(3)}=-*_\delta\mathrm{d}H,e^{\Phi}=H^{\frac{1}{2}},I,J=1,2,3,4.$$

$$\epsilon=-\Gamma_{\underline{012345}}\epsilon^c,\epsilon=H^{-\frac{1}{16}}\epsilon_0.$$

$$\Gamma_{\underline{\mu}}=\gamma_{\underline{\mu}}\otimes 1,\Gamma_{\underline{m}}=\gamma^{(6)}\otimes\Sigma_{\underline{m}},$$

$$\epsilon=\varepsilon^A\otimes\eta_A(y)+i\varepsilon^{\dot{A}}\otimes\eta_{\dot{A}}(y)$$

$$\varepsilon^A=-\gamma^{(6)}\varepsilon^A,\varepsilon^{\dot{A}}=\gamma^{(6)}\varepsilon^{\dot{A}}$$

$$\eta_A=-\Sigma^{(4)}\eta_A,\eta_{\dot{A}}=\Sigma^{(4)}\eta_{\dot{A}}$$

$$S=\frac{1}{2\kappa_6^2}\int\ *R\ -\frac{1}{2}e^{-\frac{\varphi}{\sqrt{2}}}\delta_{IJ}F_{(2)}^I\wedge*F_{(2)}^J-\frac{1}{2}e^{\sqrt{2}\varphi}H_{(3)}\wedge*H_{(3)}$$
$$-\frac{1}{2}\ \mathrm{d}\varphi\wedge*\ \mathrm{d}\varphi+\frac{1}{2}\delta_{IJ}B_{(2)}\wedge F_{(2)}^I\wedge F_{(2)}^J.$$

$$\mathrm{d}*\ \mathrm{d}\varphi = -\frac{1}{2\sqrt{2}} e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} F^I_{(2)} \wedge * F^J_{(2)} + \frac{\sqrt{2}}{2} e^{\sqrt{2}\varphi} H_{(3)} \wedge * H_{(3)}$$

$$\mathrm{d}\left(e^{-\frac{\varphi}{\sqrt{2}}} * F^I_{(2)} \right) = H_{(3)} \wedge F^I_{(2)}$$

$$\mathrm{d}\left(e^{\sqrt{2}\varphi} * H_{(3)} \right) = -\frac{1}{2} \delta_{IJ} F^I_{(2)} \wedge F^J_{(2)}$$

$$R_{\mu\nu} = \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{2} e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} \left(F^I_{\mu\rho} F^{J\rho}_{\nu} - \frac{1}{8} g_{\mu\nu} F^I_{\rho\sigma} F^{J\rho\sigma} \right) + \frac{1}{4} e^{\sqrt{2}\varphi} \left(H_{\mu\rho\sigma} H^{\rho\sigma}_{\nu} - \frac{1}{6} g_{\mu\nu} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right)$$

$$\delta\psi_{\mu i} = D_\mu \varepsilon_i + \frac{1}{16\sqrt{2}} e^{-\frac{\varphi}{2\sqrt{2}}} \left[F^0_{\rho\sigma} \epsilon_{ij} \gamma^{(6)} - i F^a_{\rho\sigma} \epsilon_{ik} (\sigma^a)^k_j \right] \left(\gamma^{\nu\lambda}_\mu - 6\delta^\nu_\mu \gamma^\lambda \right) \varepsilon^j + \frac{1}{48} e^{\frac{\varphi}{\sqrt{2}}} H_{\nu\lambda\rho} \gamma^{(6)} \left(\gamma^{\nu\lambda\rho}_\mu - 3\delta^\nu_\mu \gamma^{\lambda\rho} \right) \varepsilon_i$$

$$\delta\chi_i = -\frac{1}{4} \gamma^\mu \partial_\mu \varphi \varepsilon_i - \frac{1}{16} e^{-\frac{\varphi}{2\sqrt{2}}} \left[F^0_{\rho\sigma} \epsilon_{ij} \gamma^{(6)} + i F^a_{\rho\sigma} \epsilon_{ik} (\sigma^a)^k_j \right] \gamma^{\rho\sigma} \varepsilon^j + \frac{\sqrt{2}}{48} e^{\frac{\varphi}{\sqrt{2}}} H_{\mu\nu\lambda} \gamma^{(6)} \gamma^{\mu\nu\lambda} \varepsilon_i$$

$$\psi^i_\mu = \delta^i_A \psi^A_\mu + i\delta^i_{\dot{A}} \psi^{\dot{A}}_\mu, \chi^i = \delta^i_A \chi^A + i\delta^i_{\dot{A}} \chi^{\dot{A}}, \varepsilon^i = \delta^i_A \varepsilon^A + i\delta^i_{\dot{A}} \varepsilon^{\dot{A}}$$

$$\varepsilon^A = -\gamma^{(6)} \epsilon^A, \epsilon^{\dot{A}} = \gamma^{(6)} \epsilon^{\dot{A}}, \psi^A_\mu = -\gamma^{(6)} \psi^A_\mu, \psi^{\dot{A}}_\mu = \gamma^{(6)} \psi^{\dot{A}}_\mu$$

$$\chi^A = \gamma^{(6)} \chi^A, \chi^{\dot{A}} = -\gamma^{(6)} \chi^{\dot{A}}$$

$$\mathrm{d}s^2 = -H^{-\frac{3}{2}} \mathrm{d}t^2 + H^{\frac{1}{2}} (\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_4)$$

$$F_{(2)} = \sqrt{2} \partial_r H^{-1} \mathrm{d}t \wedge \mathrm{d}r, e^\varphi = H^{-\frac{1}{\sqrt{2}}}$$

$$\mathrm{d}s^2 = H^{-\frac{1}{2}} (-\mathrm{d}t^2 + \mathrm{d}x^2_1 + \mathrm{d}x^2_2) + H^{\frac{3}{2}} (\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_2),$$

$$F_{(2)} = \sqrt{2} r^2 \partial_r h \, \mathrm{d}\Omega_2, e^\varphi = H^{\frac{1}{\sqrt{2}}}.$$

$$\mathrm{d}s^2 = H^{-\frac{1}{2}} (-\mathrm{d}t^2 + \mathrm{d}x^2) + H^{\frac{1}{2}} (\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_3),$$

$$H_{(3)} = H^{-2} \partial_r h \, \mathrm{d}t \wedge \mathrm{d}x \wedge \mathrm{d}r, e^\varphi = H^{\frac{1}{\sqrt{2}}},$$

$$A^{ij}_\mu = -A^{ji}_\mu, A^{ij}_\mu \Omega_{ij} = 0$$

$$T^i = \Omega^{ij} T_j, T_i = T^j \Omega_{ji}$$

$$Q^{ij} = L^{Iik} \mathrm{d}L^j_{Ik}$$

$$S=\frac{1}{2\kappa_5^2}\int *R-\frac{1}{2}e^{\frac{2}{\sqrt{6}}\sigma}a_{IJ}F_{(2)}^{I}\wedge *F_{(2)}^{J}-\frac{1}{2}e^{-\frac{4}{\sqrt{6}}\sigma}G_{(2)}\wedge *G_{(2)}$$
$$-\frac{1}{2}\,\mathrm{d}\sigma\wedge *\,\mathrm{d}\sigma+\frac{1}{2}C_{IJ}F_{(2)}^{I}\wedge F_{(2)}^{J}\wedge a_{(1)}$$

$$\delta\psi_{\mu i}=D_\mu\varepsilon_i+\frac{i}{6\sqrt{2}}e^{\frac{1}{\sqrt{6}}\sigma}L_{Iij}F_{\rho\sigma}^{I}\left(\Gamma_\mu^{\rho\sigma}-4\delta_\mu^{\rho}\Gamma^{\sigma}\right)\varepsilon^j$$
$$+\frac{i}{24}e^{-\frac{2}{\sqrt{6}}\sigma}G_{\rho\sigma}\left(\Gamma_\mu^{\rho\sigma}-4\delta_\mu^{\rho}\Gamma^{\sigma}\right)\varepsilon_i$$
$$\delta\chi_i=-\frac{i}{2\sqrt{2}}\Gamma^\mu\partial_\mu\sigma\varepsilon_i+\frac{1}{2\sqrt{6}}e^{\frac{1}{\sqrt{6}}\sigma}L_{Iij}F_{\rho\sigma}^{I}\Gamma^{\rho\sigma}\varepsilon^j-\frac{1}{4\sqrt{3}}e^{-\frac{2}{\sqrt{6}}\sigma}G_{\rho\sigma}\Gamma^{\rho\sigma}\varepsilon_i$$

$$a_{IJ}=C_{IJ}=L_I^{ij}L_{Jij},$$

$$L_I{}^{ij}=a_{IJ}L^{Jij}$$

$$(L_I)_j^i=\frac{1}{2}\left(\Sigma_{\underline{I}}\right)_j^i,$$

$$\left\{\Sigma_{\underline{I}},\Sigma_{\underline{J}}\right\}=2\delta_{\underline{IJ}}.$$

$$a_{IJ}=C_{IJ}=\delta_{IJ}$$

$$\mathrm{d}*\,\mathrm{d}\sigma=\frac{1}{\sqrt{6}}e^{\frac{2}{\sqrt{6}}\sigma}\delta_{IJ}F_{(2)}^{I}\wedge *F_{(2)}^{J}-\frac{2}{\sqrt{6}}e^{-\frac{4}{\sqrt{6}}\sigma}G_{(2)}\wedge *G_{(2)}$$
$$\mathrm{d}\left(e^{-\frac{4}{\sqrt{6}}\sigma}*G_{(2)}\right)=\frac{1}{2}\delta_{IJ}F_{(2)}^{I}\wedge F_{(2)}^{J}$$
$$\mathrm{d}\left(e^{\frac{2}{\sqrt{6}}\sigma}*F_{(2)}^{I}\right)=F_{(2)}^{I}\wedge G_{(2)}$$

$$R_{\mu\nu}=\frac{1}{2}\partial_\mu\sigma\partial_\nu\sigma+\frac{1}{2}\delta_{IJ}e^{\frac{2}{\sqrt{6}}\sigma}\left(F_{\mu\rho}^{I}F_{\nu}^{J\rho}-\frac{1}{6}g_{\mu\nu}F_{\rho\sigma}^{I}F^{J\rho\sigma}\right)$$
$$+\frac{1}{2}e^{-\frac{4}{\sqrt{6}}\sigma}\left(G_{\mu\rho}G_{\nu}^{\rho}-\frac{1}{6}g_{\mu\nu}G_{\rho\sigma}G^{\rho\sigma}\right)$$

$$\mathrm{d}s_5^2=-h^{-4/3}\,\mathrm{d}t^2+h^{2/3}(\,\mathrm{d}x_1^2+\cdots+\mathrm{d}x_4^2),e^\sigma=h^{2/\sqrt{6}}$$
$$F_{(2)}^{1}=\sqrt{2}\partial_a h^{-1}\,\mathrm{d}t\wedge\,\mathrm{d}x^a,\partial^a\partial_a h(x_1,\cdots,x_4)=0,a,b=1,2,\cdots 4,$$

$$\mathrm{d}s_5^2=-h^{-2/3}\,\mathrm{d}t^2+h^{1/3}(\,\mathrm{d}x_1^2+\cdots+\mathrm{d}x_4^2),e^\sigma=h^{-2/\sqrt{6}}$$
$$G_{(2)}=h^{-2}\partial_a h\,\mathrm{d}t\wedge\,\mathrm{d}x^a,\partial^a\partial_a h(x_1,\cdots,x_4)=0,a,b=1,2,\cdots 4,$$

$$\epsilon=\Gamma_{\underline{012}}\epsilon,$$
$$\epsilon=\Gamma_{\underline{012345}}\epsilon,$$
$$\epsilon_L=\Gamma_{\underline{01}}\epsilon_L,\qquad \epsilon_R=-\Gamma_{\underline{01}}\epsilon_R,$$
$$\epsilon_L=\Gamma_{\underline{012345}}\epsilon_L,\quad \epsilon_R=\Gamma_{\underline{012345}}\epsilon_R,$$
$$\epsilon_L=\Gamma_{\underline{012345}}\epsilon_L,\quad \epsilon_R=-\Gamma_{\underline{012345}}\epsilon_R,$$
$$\epsilon_L=\Gamma_{\underline{01\cdots p}}\epsilon_R.$$

$$\epsilon = \varepsilon^A \otimes \eta_A$$

$$\hat{\Psi}_\mu = e^{\frac{5}{32\sqrt{6}}\sigma}\left\{\begin{pmatrix}1\\-i\end{pmatrix} \otimes \left(\psi_\mu^i + \frac{5i}{16\sqrt{3}}\gamma_\mu \chi^i\right) \otimes \eta_i\right\}$$

$$\hat{\Psi}_I = -\frac{\sqrt{3}}{16} H_0^{\frac{3}{16}} e^{-\frac{5}{16\sqrt{6}}\sigma} e_I^J \left(\sigma_3 \otimes \mathbf{1} \otimes \Sigma_{\underline{J}}\right)\left\{\begin{pmatrix}1\\-i\end{pmatrix} \otimes \chi^i \otimes \eta_i\right\}$$

$$\hat{\lambda} = -\frac{\sqrt{3}}{2} H_0^{\frac{3}{16}} e^{-\frac{5}{32\sqrt{6}}\sigma} \begin{pmatrix}1\\-i\end{pmatrix} \otimes \chi^i \otimes \eta_i, \hat{\epsilon} = e^{\frac{5}{32\sqrt{6}}\sigma} \begin{pmatrix}1\\-i\end{pmatrix} \otimes \varepsilon^i(x) \otimes \eta_i$$

$$\Sigma_{\underline{I}} \eta_i = 2\eta_j \left(L_{\underline{I}}\right)_i^j$$

$$\hat{\Psi}_\mu = e^{-\frac{\sqrt{2}}{16}\varphi}\left\{\left(\psi_\mu^A \otimes \eta_A + i\psi_\mu^{\dot{A}} \otimes \eta_{\dot{A}}\right) + \frac{\sqrt{2}}{4}\gamma_\mu\left(\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}}\right)\right\}$$

$$\hat{\Psi}_I = -\frac{1}{2\sqrt{2}} H_0^{\frac{1}{8}} e^{\frac{1}{8\sqrt{2}}\varphi} e_I^J \left(\gamma^{(6)} \otimes \Sigma_{\underline{J}}\right)\left(\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}}\right)$$

$$\hat{\lambda} = \sqrt{2} H_0^{\frac{1}{8}} e^{\frac{\sqrt{2}}{16}\varphi}\left(\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}}\right)$$

$$\hat{\epsilon} = e^{-\frac{\sqrt{2}}{16}\varphi}\varepsilon, \varepsilon = \varepsilon^A \otimes \eta_A + i\varepsilon^{\dot{A}} \otimes \eta_{\dot{A}}$$

$$* \tilde{F}_{(5)} = \frac{1}{\sqrt{2}}\left[e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^I \wedge \mathrm{d}y^I - \frac{1}{3!} H_0 \varepsilon_{I_1 \cdots I_4} F_{(2)}^{I_1} \wedge \mathrm{d}y^{I_2} \wedge \mathrm{d}y^{I_3}\, \mathrm{d}y^{I_4}\right]$$

$$* \hat{H}_{(3)} = H_0^{-\frac{3}{2}} e^{-\sqrt{2}\varphi} \partial_I H_0\, \mathrm{d}y^I \wedge \mathrm{vol}_g - H_0^{\frac{3}{2}} e^{\frac{\sqrt{2}}{2}\varphi} *_g H_{(3)} \wedge \mathrm{vol}_\delta$$

$$* \tilde{F}_{(3)} = -\frac{1}{\sqrt{2}}\left(\frac{1}{3!} H_0^{\frac{1}{2}} \varepsilon_{I_1 I_2 I_3 I_4} *_g F_{(2)}^{I_1}(x) \wedge \mathrm{d}y^{I_2} \wedge \cdots \wedge \mathrm{d}y^{I_4}\right)$$

$$\mathrm{d}H_{(3)} = 0$$

$$\mathrm{d}F_{(2)}^I = 0$$

$$\mathrm{d}\tilde{F}_{(5)} = -\frac{1}{\sqrt{2}} \partial_I H_0 F_{(2)}^I \wedge \mathrm{vol}_\delta + \frac{1}{\sqrt{2}}\, \mathrm{d}\left(e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^I\right) \wedge \mathrm{d}y^I$$

$$-\tilde{F}_{(3)} \wedge \hat{H}_{(3)} = -\frac{1}{\sqrt{2}} \partial_I H_0 F_{(2)}^I(x) \wedge \mathrm{vol}_\delta + \frac{1}{\sqrt{2}} H_{(3)}(x) \wedge F_{(2)}^I(x) \wedge \mathrm{d}y^I$$

$$\mathrm{d}\left(e^{-\frac{\sqrt{2}}{2}\varphi} *_g F_{(2)}^I\right) - H_{(3)}(x) \wedge F_{(2)}^I(x) = 0$$

$$\mathrm{d} * \mathrm{d}\hat{\Phi} = -\frac{1}{2} e^{-\frac{\sqrt{2}}{2}\varphi} H_0^{-2} \partial_I H_0 \partial_I H_0 \mathrm{vol}_\delta \wedge \mathrm{vol}_g - \frac{1}{\sqrt{2}} H_0\, \mathrm{d} * \mathrm{d}\varphi \wedge \mathrm{vol}_\delta$$

$$\frac{1}{2} e^{\hat{\Phi}} \tilde{F}_{(3)} \wedge * \tilde{F}_{(3)} = \frac{1}{4} H_0 e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} F_{(2)}^I(x) \wedge *_g F^J(x) \wedge \mathrm{vol}_\delta$$

$$-\frac{1}{2} e^{-\hat{\Phi}} \hat{H}_{(3)} \wedge * \hat{H}_{(3)} = -\frac{1}{2} H_0^{-2} e^{-\frac{\varphi}{\sqrt{2}}}\left(\partial_I H_0 \partial_I H_0 \mathrm{vol}_\delta \wedge \mathrm{vol}_g\right)$$
$$-\frac{1}{2} H_0 e^{\sqrt{2}\varphi} H_{(3)}(x) \wedge *_g H_{(3)} \wedge \mathrm{vol}_\delta$$

$$0 = \mathrm{d} * \mathrm{d}\varphi + \frac{1}{2\sqrt{2}} e^{-\frac{\varphi}{\sqrt{2}}} \delta_{IJ} F^I_{(2)}(x) \wedge *_g F^J_{(2)}(x) - \frac{\sqrt{2}}{2} e^{\sqrt{2}\varphi} H_{(3)}(x) \wedge *_g H_{(3)}$$

$$\begin{aligned} \mathrm{d}\left(e^{\hat{\Phi}} * \tilde{F}_{(3)}\right) =& -\frac{1}{\sqrt{2}} e^{-\frac{\varphi}{\sqrt{2}}} \partial_I H_0 *_g F^I_{(2)}(x) \wedge \mathrm{vol}_y \\ & -\frac{1}{\sqrt{2}} \frac{1}{3!} H_0 \varepsilon_{I_1 I_2 I_3 I_4} \mathrm{d}\left(e^{-\frac{\varphi}{\sqrt{2}}} *_g F^{I_1}_{(2)}(x)\right) \wedge \mathrm{d}y^{I_2} \wedge \cdots \wedge \mathrm{d}y^{I_4} \end{aligned}$$

$$\begin{aligned} \tilde{F}_{(5)} \wedge \hat{H}_{(3)} =& -\frac{1}{\sqrt{2}} e^{-\frac{\sqrt{2}}{2}\varphi} \partial_I H_0 *_g F^I_{(2)} \wedge \mathrm{vol}_y \\ & + \frac{1}{\sqrt{2}} \left[\frac{1}{3!} H_0 \varepsilon_{I_1 \cdots I_4} F^{I_1}_{(2)} \wedge \mathrm{d}y^{I_2} \wedge \mathrm{d}y^{I_3} \wedge \mathrm{d}y^{I_4}\right] \wedge H_{(3)}(x) \end{aligned}$$

$$\mathrm{d}\left(e^{-\hat{\Phi}} * \hat{H}_{(3)}\right) = -H_0 \, \mathrm{d}\left(e^{\sqrt{2}\varphi} * {}_g H_{(3)}\right) \wedge \mathrm{vol}_y$$

$$-\tilde{F}_{(5)} \wedge \tilde{F}_{(3)} = \frac{1}{2} H_0 \delta_{IJ} F^I_{(2)} \wedge F^J_{(2)}(x) \wedge \mathrm{vol}_y$$

$$\mathrm{d}\left(e^{\sqrt{2}\varphi} *_g H_{(3)}\right) + \frac{1}{2} \delta_{IJ} F^I_{(2)} \wedge F^J_{(2)}(x) = 0$$

$$\hat{e}^{\underline{\mu}} = H_0^{-\frac{1}{8}} e^{-\frac{\sqrt{2}}{8}\varphi} e^{\underline{\mu}}, \hat{e}^{\underline{I}} = H_0^{\frac{3}{8}} e^{\frac{\sqrt{2}}{8}\varphi} \, \mathrm{d}y^I$$

$$\begin{aligned} \hat{R}_{\mu\nu} &= R_{\mu\nu} - \frac{1}{4}\partial_\mu\varphi\partial_\nu\varphi + \frac{\sqrt{2}}{8}\nabla_\rho\left(g^{\rho\eta}\partial_\eta\varphi\right)g_{\mu\nu} + \frac{1}{8}g_{\mu\nu}H_0^{-1}e^{-\frac{\sqrt{2}}{2}\varphi}\partial_K H_0^{-1}\partial_K H_0, \\ \hat{R}_{IJ} &= -\frac{\sqrt{2}}{8}H_0 e^{\frac{\sqrt{2}}{2}\varphi}\nabla_\rho(\partial_\sigma\varphi g^{\rho\sigma})\delta_{IJ} + \frac{3}{8}\partial_J H_0^{-1}\partial_I H_0 - \frac{3}{8}\delta_{IJ}\partial_K H_0^{-1}\partial_K H_0, \\ \hat{R}_{\mu I} &= -\frac{\sqrt{2}}{8}H_0^{-1}\partial_I H_0\partial_\mu\varphi. \end{aligned}$$

$$\begin{aligned} \mathrm{RHS}_{\mu\nu} =& \frac{1}{4}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{4}e^{\sqrt{2}\varphi}\left(H_{\mu\rho\sigma}H_\nu^{\rho\sigma} - \frac{1}{12}g_{\mu\nu}H_{\rho\sigma\lambda}H^{\rho\sigma\lambda}\right) \\ & + \frac{1}{2}e^{-\frac{\varphi}{\sqrt{2}}}\delta_{IJ}\left(F^I_{\mu\rho}F_\nu^{J\rho} - \frac{3}{16}g_{\mu\nu}F^I_{\rho\sigma}F^{J\rho\sigma}\right) + \frac{1}{8}H^{-1}e^{-\frac{\varphi}{\sqrt{2}}}g_{\mu\nu}\partial_K H_0^{-1}\partial_K H_0, \\ \mathrm{RHS}_{IJ} =& \left(\frac{1}{32}H_0 F^K_{\rho\sigma}F^{K\rho\sigma} - \frac{1}{48}H_0 e^{\frac{3\sqrt{2}}{2}\varphi}H_{\rho\sigma\lambda}H^{\rho\sigma\lambda}\right)\delta_{IJ} + \frac{3}{8}\partial_I H_0^{-1}\partial_J H_0 - \frac{3}{8}\delta_{IJ}\partial_M H_0^{-1}\partial_M H_0, \\ \mathrm{RHS}_{\mu I} =& -\frac{1}{4\sqrt{2}}H_0^{-1}\partial_I H_0\partial_\mu\varphi. \end{aligned}$$

$$\begin{aligned} R_{\mu\nu} =& \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}e^{-\frac{\varphi}{\sqrt{2}}}\delta_{IJ}\left(F^I_{\mu\rho}F_\nu^{J\rho} - \frac{1}{8}g_{\mu\nu}F^I_{\rho\sigma}F^{J\rho\sigma}\right) \\ & + e^{\sqrt{2}\varphi}\frac{1}{4}\left(H_{\mu\rho\sigma}H_\nu^{\rho\sigma} - \frac{1}{6}g_{\mu\nu}H_{\rho\sigma\lambda}H^{\rho\sigma\lambda}\right). \end{aligned}$$

$$\delta\hat{\lambda} = \sqrt{2}H_0^{\frac{1}{8}}e^{\frac{\sqrt{2}}{16}\varphi}\left(\delta\chi^A \otimes \eta_A + i\delta\chi^{\dot{A}} \otimes \eta_{\dot{A}}\right)$$
$$= \sqrt{2}H_0^{\frac{1}{8}}e^{\frac{\sqrt{2}}{16}\varphi}\left\{-\frac{1}{4}\partial_\mu\varphi(\gamma^\mu \otimes \mathbf{1})\left(\varepsilon^A \otimes \eta_A + i\varepsilon^{\dot{A}} \otimes \eta_{\dot{A}}\right)\right.$$
$$\left. + \left(\frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\mu\nu\rho}(\gamma^{\mu\nu\rho} \otimes \mathbf{1}) + \frac{i}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F^I_{\mu\nu}\delta^J_{\underline{I}}\left(\gamma^{\mu\nu}\gamma^{(6)} \otimes \Sigma_{\underline{J}}\right)\right)\left(\varepsilon^A \otimes \eta_A - i\varepsilon^{\dot{A}} \otimes \eta_{\dot{A}}\right)\right\}.$$

$$\delta\hat{\Psi}_\mu = e^{-\frac{\sqrt{2}}{16}\varphi}\left\{\left(\psi^A_\mu \otimes \eta_A + i\psi^{\dot{A}}_\mu \otimes \eta_{\dot{A}}\right) + \frac{\sqrt{2}}{4}\gamma_\mu\left(\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}}\right)\right\}$$
$$= e^{-\frac{\sqrt{2}}{16}\varphi}\left\{D_\mu\hat{\epsilon} + \frac{i}{16\sqrt{2}}e^{-\frac{\varphi}{2\sqrt{2}}}F^I_{\rho\sigma}\delta^J_I\left(\left(\gamma_\mu^{\rho\sigma} - 6\delta_\mu^\rho\gamma^\sigma\right)\gamma^{(6)} \otimes \Sigma^{\underline{I}}\right)\hat{\epsilon}^c\right.$$
$$\left. - \frac{1}{48}e^{\frac{\varphi}{\sqrt{2}}}H_{\nu\rho\sigma}\left[\left(\gamma_\mu^{\nu\rho\sigma} - 3\delta_\mu^\nu\gamma^{\rho\sigma}\right) \otimes \mathbf{1}\right]\hat{\epsilon}^c\right\}$$
$$+ e^{-\frac{\sqrt{2}}{16}\varphi}\frac{\sqrt{2}}{4}\left(\gamma_\mu \otimes \mathbf{1}\right)\left\{-\frac{1}{4}\partial_\lambda\varphi\left(\gamma^\lambda \otimes \mathbf{1}\right)\hat{\epsilon}\right.$$
$$\left. + \left(\frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\rho\sigma\lambda}\left(\gamma^{\rho\sigma\lambda} \otimes \mathbf{1}\right) + \frac{i}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F^I_{\rho\sigma}\delta^J_I\left(\gamma^{\rho\sigma}\gamma^{(6)} \otimes \Sigma_{\underline{J}}\right)\right)\hat{\epsilon}^c\right\}$$

$$\delta\hat{\Psi}_I = -\frac{1}{2\sqrt{2}}H_0^{\frac{1}{8}}e^{\frac{1}{8\sqrt{2}}\varphi}e^{\underline{J}}_I\left(\gamma^{(6)} \otimes \Sigma_{\underline{J}}\right)\delta\left(\chi^A \otimes \eta_A + i\chi^{\dot{A}} \otimes \eta_{\dot{A}}\right)$$
$$= -\frac{1}{2\sqrt{2}}H_0^{\frac{1}{8}}e^{\frac{1}{8\sqrt{2}}\varphi}e^{\underline{J}}_I\left(\gamma^{(6)} \otimes \Sigma_{\underline{J}}\right)\left\{-\frac{1}{4}\partial_\nu\varphi(\gamma^\nu \otimes \mathbf{1})\hat{\epsilon}\right.$$
$$\left. + \left(\frac{1}{24\sqrt{2}}\left(e^{\frac{1}{\sqrt{2}}\varphi}H_{\mu\nu\rho}(\gamma^{\mu\nu\rho} \otimes \mathbf{1})\right) + \frac{i}{16}e^{-\frac{1}{2\sqrt{2}}\varphi}F^K_{\mu\nu}\delta^L_K\left(\gamma^{\mu\nu}\gamma^{(6)} \otimes \Sigma_{\underline{L}}\right)\right)\hat{\epsilon}^c\right\}$$

$$\Sigma^{\underline{0}} = \begin{pmatrix} & \delta^A_{\dot{B}} \\ \delta^{\dot{A}}_B & \end{pmatrix}, \Sigma^{\underline{a}} = \begin{pmatrix} & (\sigma^a)^A_{\dot{B}} \\ (\sigma^a)^{\dot{A}}_B & \end{pmatrix}.$$

$$\varepsilon^A \otimes \Sigma^{\underline{0}}\eta_A = \varepsilon^{\dot{A}} \otimes \eta_{\dot{A}}, \qquad \varepsilon^{\dot{A}} \otimes \Sigma^{\underline{a}}\eta_{\dot{A}} = \varepsilon^A \otimes \eta_A,$$
$$\varepsilon^A \otimes \Sigma^{\underline{a}}\eta_A = (\sigma^a)^{\dot{A}}_C\varepsilon^C \otimes \eta_{\dot{A}}, \quad \varepsilon^{\dot{A}} \otimes \Sigma^{\underline{a}}\eta_{\dot{A}} = (\sigma^a)^A_{\dot{C}}\varepsilon^{\dot{C}} \otimes \eta_A.$$

$$\delta\chi^A = -\frac{1}{4}\partial_\mu\varphi\gamma^\mu\varepsilon^A + \frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\mu\nu\rho}\gamma^{(6)}\gamma^{\mu\nu\rho}\varepsilon^A$$
$$- \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F^0_{\mu\nu}\gamma^{(6)}\gamma^{\mu\nu}\varepsilon^A + \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F^a_{\mu\nu}\gamma^{\mu\nu}(\sigma^a)^A_{\dot{C}}\varepsilon^{\dot{C}}$$
$$\delta\chi^{\dot{A}} = -\frac{1}{4}\partial_\mu\varphi\gamma^\mu\varepsilon^{\dot{A}} + \frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\mu\nu\rho}\gamma^{(6)}\gamma^{\mu\nu\rho}\varepsilon^{\dot{A}}$$
$$- \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F^I_{\mu\nu}\gamma^{(6)}\gamma^{\mu\nu}\varepsilon^{\dot{A}} - \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}F^a_{\mu\nu}\gamma^{\mu\nu}(\sigma^a)^{\dot{A}}_C\varepsilon^C$$

$$\delta\chi^i = -\frac{1}{4}\partial_\mu\varphi\gamma^\mu\varepsilon^i + \frac{1}{24\sqrt{2}}e^{\frac{\varphi}{\sqrt{2}}}H_{\mu\nu\rho}\gamma^{(6)}\gamma^{\mu\nu\rho}\varepsilon^i$$
$$- \frac{1}{16}e^{-\frac{\varphi}{2\sqrt{2}}}\left[F^0_{\mu\nu}\gamma^{(6)}\delta^i_j + iF^a_{\mu\nu}(\sigma^a)^i_j\right]\gamma^{\mu\nu}\varepsilon^j,$$

$$\delta\psi_\mu^i = D_\mu\varepsilon^i + \frac{1}{48}e^{\frac{\varphi}{\sqrt{2}}}H_{\nu\rho\sigma}\gamma^{(6)}\left(\gamma_\mu^{\nu\rho\sigma} - 3\delta_\mu^\nu\gamma^{\rho\sigma}\right)\varepsilon^i + \frac{1}{16\sqrt{2}}e^{-\frac{\varphi}{2\sqrt{2}}}\left[F^0_{\rho\sigma}\gamma^{(6)}\delta^i_j - iF^a_{\rho\sigma}(\sigma^a)^i_j\right]\left(\gamma_\mu^{\rho\sigma} - 6\delta_\mu^\rho\gamma^\sigma\right)\varepsilon^j,$$

$$\lambda_{ijk} = \beta_{ijk}\frac{\pi^{d/2}\sqrt{\mathcal{C}_{\Delta_1}\mathcal{C}_{\Delta_2}\mathcal{C}_{\Delta_3}}\Gamma\left(\frac{\Delta_i+\Delta_j-\Delta_k}{2}\right)\Gamma\left(\frac{\Delta_k+\Delta_i-\Delta_j}{2}\right)\Gamma\left(\frac{\Delta_j+\Delta_k-\Delta_i}{2}\right)\Gamma\left(\frac{\Delta_i+\Delta_j+\Delta_k-d}{2}\right)}{2\Gamma(\Delta_i)\Gamma(\Delta_j)\Gamma(\Delta_k)},$$

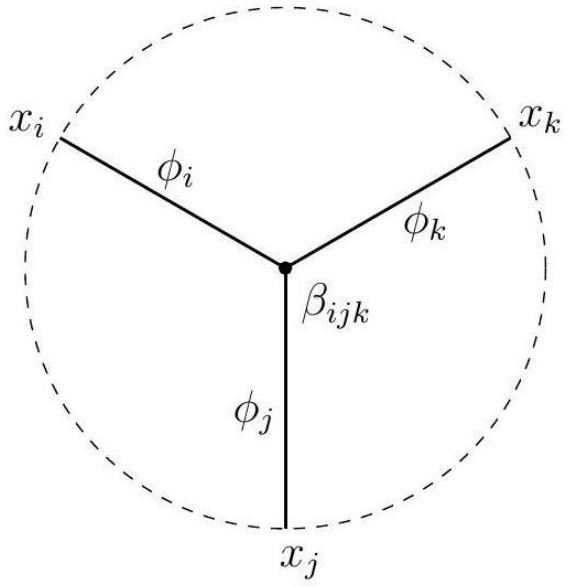


Figura 6. Diagramas de Witten.

$$M = \frac{1}{N}M_{F^2} + \frac{1}{N^2}\left[M_R + M_{F^2|F^2}(s,t) + \sum_i\left(b^i_{F^4,s}M^i_{F^4,s} + b^i_{F^4,tu}M^i_{F^4,tu}\right)\right] + \frac{\log N}{N^2}\left(b_{\log,s}M_{\log,s} + b_{\log,tu}M_{\log,tu}\right) + O\left(N^{-5/2}\right),$$

$$M_R = -\frac{p}{(p-2)!\,\boldsymbol{\Delta}}\left[\left[(p+1)H_{1-\frac{s}{2}} + \frac{4}{s-2}\right]\delta^{AB}\delta^{CD} + \left[(p+1)H_{\frac{2+p-u}{2}} + \frac{2p}{u-p}\right]\delta^{AC}\delta^{BD} + \left[(p+1)H_{\frac{2+p-t}{2}} + \frac{2p}{t-p}\right]\delta^{AD}\delta^{BC}\right]$$

$$\mathcal{A}_R^{ABCD} = -32\boldsymbol{\Delta}\pi^7\delta^{AB}\delta^{CD}\int\frac{d^2p_\perp}{(2\pi^2)}\frac{1}{-s+p_\perp^2} + \text{ crossed} = -8\boldsymbol{\Delta}\pi^6(\log(-s) + \cdots)\delta^{AB}\delta^{CD} + \text{ crossed}$$

$$\mathrm{d}s^2 = L^2\left(\mathrm{d}s^2_{\mathrm{Ads}_5} + \mathrm{d}\theta^2 + \cos^2\theta\,\mathrm{d}\phi^2 + \sin^2\theta\,\mathrm{d}s^2_{S^3}\right),$$

$$F_5 = \mathrm{d}C_4 = 4L^4(1+\star)\mathrm{vol}_{M_5}, \text{ with } \frac{1}{(2\pi\ell_s)^4 g_s}\int_{M_5}F_5 = N$$

$$(L/\ell_s)^4 = 4\pi\Delta g_s N$$

$$\mathrm{SO}(6) \to \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_R$$

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa^2}\int \mathrm{d}^{10}X\sqrt{g}\left(R + \frac{1}{5!}F_5^2\right), \frac{1}{2\kappa^2} = \frac{2\pi}{(2\pi\ell_s)^8 g_s^2} \equiv \frac{2\pi}{\left(2\pi\ell_p\right)^8}$$

$$S_7 = T_7\left(\int \mathrm{d}^8\hat{X}\sqrt{\det(\hat{g} + 2\pi\ell_s^2 F)} + \int \hat{C}_4 \wedge \mathrm{e}^{2\pi\ell_s^2 F}\right), T_7 = \frac{2\pi}{(2\pi\ell_s)^8 g_s}$$

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa_5^2}\int \mathrm{d}^5x\sqrt{g}R + \cdots, \text{ where } \frac{1}{2\kappa_5^2} = \frac{\mathrm{vol}_{M_5}}{2\kappa^2} = \frac{\Delta N^2}{8\pi^2 L^3}$$

$$\left\langle T_{\hat{\mu}\hat{\nu}}(z)T_{\hat{\rho}\hat{\sigma}}(0)\right\rangle = \frac{c}{640\pi^2 z^8}\left(I_{\hat{\mu}(\hat{\rho}}I_{\hat{\sigma})\hat{\nu}} - \text{trace}\right), c \approx \frac{\pi^2 L^3}{\kappa_5^2} = \frac{\Delta}{4}N^2$$

$$S_7 \approx -\int \mathrm{d}^5x\frac{1}{4\,\mathrm{g}_{5d}^2}\left(F_{\mu\nu}^A\right)^2 + \cdots, \text{ where } \frac{1}{4\,\mathrm{g}_{5d}^2} = \frac{N\Delta}{8\pi^2 L}$$

$$\mathrm{tr}T^A T^B = 2\delta^{AB}$$

$$\left\langle J_{\hat{\mu}}^A(x)J_{\hat{\nu}}^B(0)\right\rangle = \frac{3\mathbf{k}}{2\pi^2}\frac{1}{z^6}\delta^{AB}I_{\hat{\mu}\hat{\nu}}, \mathbf{k} = \frac{8\pi^2 L}{\mathrm{g}_{5d}^2} = 2\Delta N$$

$$\delta g_{ab} = h_{ab} = h'_{ab} + \frac{h}{5}g_{ab}, \text{ with } g^{ab}h'_{ab} = 0, h'_{ab} = h'_{ab},$$
$$\delta g_{\mu\nu} = h_{\mu\nu} = h'_{\mu\nu} - \frac{3h}{25}g_{\mu\nu}, \text{ with } g^{\mu\nu}h'_{\mu\nu} = 0, h'_{\mu\nu} = h'_{\nu\mu},$$

$$\delta C_4 = \left((\star\,\mathrm{d})_{X_5} - (\star\,\mathrm{d})_{\mathrm{AdS}_5}\right)a.$$

$$h(x,y) = \sum_k h_k(x)\mathcal{Y}^{I_k}(y),$$
$$h'_{(\mu\nu)}(x,y) = \sum_k h'_{(\mu\nu),k}(x)\mathcal{Y}^{I_k}(y),$$
$$a(x,y) = \sum_k a_k(x)\mathcal{Y}^{I_k}(y),$$

$$\Delta_{S^5}\mathcal{Y}^{I_{k_p}} = -k_p\left(k_p + 4\right), \text{ and } \Delta_{S^3}\mathcal{Y}^{I_{k_p}} = -p(p-2)$$

$$k_p = \begin{cases} p, p+2, p+4, \ldots & p = 2,3 \\ p-2, p, p+2, \ldots & p > 3 \end{cases}$$

$$\mathcal{Y}^{I_{k_p}} = T_{i_1 i_2 \ldots i_{p-2}}y_{i_1}y_{i_2}\cdots y_{i_{p-2}}\,{}_2F_1\left(\frac{p-2-k_p}{2}, \frac{p+2+k_p}{2}, 1, \rho^2\right),$$

$$h_{k_p}(x) = 10k_p s_{k_p}(x) + 20\left(k_p + 4\right)t_{k_p}(x), a_{k_p}(x) = t_{k_p}(x) - s_{k_p}(x),$$

$$\left(\Box - k_p\left(k_p - 4\right)\right)s_{k_p} = \left(\Box - \left(k_p + 8\right)\left(k_p + 4\right)\right)t_{k_p} = 0$$

$$S^{(2)}\left(s_{k_p}\right)=\frac{1}{2\kappa^2}\frac{64k_p\left(k_p-1\right)}{\left(k_p+1\right)(p-1)}\frac{\pi^3}{\Delta}\int\ d^5x\sqrt{g}\left(\left(\partial_\mu s_{k_p}\right)^2+m_{k_p}^2s_{k_p}^2\right)$$

$$h'_{(\mu\nu),k_p}=H'_{(\mu\nu),k_p}+\frac{4\nabla_{(\mu}\nabla_{\nu)}s_{k_p}}{k_p+1},$$

$$S^{(3)}_{\text{brane}}=-2\pi^2\ell_s^4T_7\int\ \left[\mathrm{d}^8\hat{X}\sqrt{\hat{g}}\left(F^{A,\hat{M}\hat{N}}F^A_{\hat{M}\hat{N}}-2F^{A,\hat{M}\hat{N}}F^A_{\hat{N}\hat{P}}\hat{h}^{\hat{P}}{}_{\hat{M}}\right)-2\hat{C}_4\wedge F^A\wedge F^A\right]$$

$$A^A_\mu=\sum_I\ a^{A,I}_\mu(x)\hat{\mathcal{Y}}^I(\hat{y}),A^A_{\hat{a}}=\varphi^A(x)\cdot\mathcal{X}_{\hat{a}}(\hat{y})$$

$$\mathcal{X}^p_{\hat{a}}(\hat{y}):\left(\frac{p}{2}-1,\frac{p}{2}\right)_0\ \text{and}\ \tilde{\mathcal{X}}^p_{\hat{a}}(\hat{y}):\left(\frac{p}{2},\frac{p}{2}-1\right)_0$$

$$\nabla^{\hat{b}}\nabla_{\hat{b}}\mathcal{X}^p_{\hat{a}}=(2-p^2)\mathcal{X}^p_{\hat{a}},\epsilon^{\hat{b}\hat{c}}_{\hat{a}}\nabla_{\hat{b}}\mathcal{X}^p_{\hat{c}}=-p\mathcal{X}^p_{\hat{a}},\nabla^{\hat{a}}\mathcal{X}^p_{\hat{a}}=0$$

$$S^{(2)}(\varphi_p)=-\frac{4\pi^4\ell_s^4T_7}{p-1}\int\ \ \mathrm{d}^5x\sqrt{g}\left(\left(\partial_\mu\varphi^A_{(p)}\right)^2+m_p^2\left(\varphi^A_{(p)}\right)^2\right),m_p^2=p(p-4)$$

$$S^{(3)}=\int\ \mathrm{d}^5x\sqrt{g}\left(\sum_{p=2}^{\infty}\left(-\frac{1}{2}\left(\partial_\mu\varphi^A_{(p)}\right)^2-\frac{1}{2}m_p^2\left(\varphi^A_{(p)}\right)^2\right)+\sum_{p=2}^{\infty}\sum_{k_p}\left(-\frac{1}{2}\left(\partial_\mu s_{(k_p)}\right)^2-\frac{1}{2}m_{k_p}^2s_{(k_p)}^2\right)\right.$$
$$\left.+\sum_{p,q=2}^{\infty}\sum_r\sum_{k_r}\frac{1}{2}\beta_{pqk_r}s_{(k_r)}\varphi^A_{(p)}\varphi^A_{(q)}\right)$$

$$r=|p-q|+2,|p-q|+4,\ldots,p+q-2$$

$$\beta_{pqk_r}=\frac{\pi}{2N}\sqrt{\frac{(p-1)(q-1)(r-1)}{2\Delta k_r(k_r-1)(k_r+1)}}(k_r^2-(p-q)^2)(k_r+p+q-4)(k_r+p+q-2)$$
$$\times\frac{\Gamma(p-1)\Gamma(q-1)\Gamma(r-1)}{\Gamma\left(\frac{p+q-r}{2}\right)\Gamma\left(\frac{p-q+r}{2}\right)\Gamma\left(\frac{q-p+r}{2}\right)\Gamma\left(\frac{p+q+r-2}{2}\right)}$$

$$\lambda_{pqk_r}=\frac{1}{N}\sqrt{\frac{r-1}{\Delta k_r(k_r+1)}}\frac{\Gamma\left(\frac{2+k_r+p-q}{2}\right)\Gamma\left(\frac{2+k_r-p+q}{2}\right)\Gamma\left(\frac{p+q-k_r}{2}\right)\Gamma\left(\frac{k_r+p+q}{2}\right)\Gamma(r-1)}{\Gamma(k_r)\Gamma\left(\frac{p+q-r}{2}\right)\Gamma\left(\frac{p-q+r}{2}\right)\Gamma\left(\frac{q-p+r}{2}\right)\Gamma\left(\frac{p+q+r-2}{2}\right)}+O(1/N^2$$

$$\lambda_{pp(k_2=2)}=\frac{p}{N\sqrt{6\Delta}}+O(1/N^2)$$

$$\left\langle\phi_2^A(y_1,x_1)\phi_2^B(y_2,x_2)\phi_p^C(y_3,\bar{y}_3,x_3)\phi_p^D(y_4,\bar{y}_4,x_4)\right\rangle$$
$$=\frac{\langle y_1,y_2\rangle^2\langle y_3,y_4\rangle^p\langle\bar{y}_3,\bar{y}_4\rangle^{p-2}}{x_{12}^4x_{34}^{2p}}\sum_{\mathbf{r}\in\mathfrak{g}\otimes\mathfrak{g}}G_{\mathbf{r}}(U,V;w)P_{\mathbf{r}}^{ABCD}$$
$$\left\langle\phi_2^A(y_1,x_1)\phi_p^B(y_2,x_2)\phi_2^C(y_3,\bar{y}_3,x_3)\phi_p^D(y_4,\bar{y}_4,x_4)\right\rangle$$
$$=\frac{\langle y_1,y_2\rangle^2\langle y_3,y_4\rangle^2\langle y_2,y_4\rangle^{p-2}}{x_{12}^4x_{34}^4x_{24}^{2(p-2)}}\sum_{\mathbf{r}\in\mathfrak{g}\otimes\mathfrak{g}}\tilde{G}_{\mathbf{r}}(U,V;w)P_{\mathbf{r}}^{ABCD}$$

$$G_{\mathbf{r}}(U,V;w)=\frac{z(w-\bar{z})f_r(\bar{z})-\bar{z}(w-z)f_r(z)}{w(z-\bar{z})}+\left(1-\frac{z}{w}\right)\left(1-\frac{\bar{z}}{w}\right)\mathcal{G}_{\mathbf{r}}(U,V),$$

$$\mathcal{G}_{\mathbf{r}}(U,V)=U^{-1}\left[\sum_{\ell}\sum_{\Delta\geq\ell+2}\lambda^{(2,p)}_{\Delta,\ell,\mathbf{r}}g^{0,0}_{\Delta+2,\ell}(U,V)+\sum_{\ell}\lambda^{(2,p)}_{\ell+3,\ell+1,\mathbf{r}}g^{0,0}_{\ell+5,\ell+1}(U,V)+\lambda^{(2,p)}_{2,0,\mathbf{r}}g^{0,0}_{4,0}(U,V)\right]$$

$$f_{\mathbf{r}}(z)=\sum_{\ell}\lambda^{(2,p)}_{\ell+3,\ell+1,\mathbf{r}}k^{0,0}_{2\ell+6}(z)+\lambda^{(2,p)}_{2,0,\mathbf{r}}k^{0,0}_{4}(z)+\delta_{\mathbf{r},\mathbf{1}}\dim(G)-\delta_{\mathbf{r},\mathrm{Adj}}\frac{4h^{\vee}}{\mathbf{k}}k^{0,0}_{2}(z)-\delta_{\mathbf{r},\mathbf{1}}\frac{p\dim(G)}{12c}k^{0,0}_{4}(z),$$

$$g^{\Delta_{12},\Delta_{34}}_{\Delta,\ell}(U,V)=\frac{z\bar{z}}{z-\bar{z}}\left(k^{\Delta_{12},\Delta_{34}}_{\Delta+\ell}(z)k^{\Delta_{12},\Delta_{34}}_{\Delta-\ell-2}(\bar{z})-k^{\Delta_{12},\Delta_{34}}_{\Delta+\ell}(\bar{z})k^{\Delta_{12},\Delta_{34}}_{\Delta-\ell-2}(z)\right),$$

$$k^{\Delta_{12},\Delta_{34}}_{h}(z)\equiv z^{\frac{h}{2}}\,{}_2F_1(h/2+\Delta_{12}/2,h/2-\Delta_{34}/2,h,z).$$

$$c=\frac{\Delta}{4}N^2+\frac{3(\Delta-1)}{4}N-\frac{1}{12},\mathbf{k}=2\Delta N.$$

$$\lambda^{(2,p)}_{2,0,1}=2\delta_{2,p}+\frac{p\dim(G_F)}{12c}-\frac{4h^{\vee}}{\mathbf{k}}.$$

$$\tilde{\mathcal{G}}_{\mathbf{r}}(U,V)=U^{-\frac{p}{2}}\sum_{\ell}\sum_{\Delta\geq\ell+p}\lambda^{2}_{2p(\Delta,\ell,\mathbf{r})}g^{2-p,2-p}_{\Delta+2,\ell}(U,V)+\cdots,$$

$$\mathcal{G}^{ABCD}_{\mathrm{conn}}(U,V)=\int\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t-p-2}{2}}M^{ABCD}(s,t)\Gamma\left[2-\frac{s}{2}\right]\Gamma\left[p-\frac{s}{2}\right]\Gamma\left[\frac{2+p-t}{2}\right]^2\Gamma\left[\frac{2+p-u}{2}\right]^2$$

$$\tilde{\mathcal{G}}^{ABCD}_{\mathrm{conn}}(U,V)=\int\frac{dsdt}{(4\pi i)^2}U^{\frac{s+2-p}{2}}V^{\frac{t-p-2}{2}}M^{ABCD}(s,t)\Gamma\left[\frac{2+p-s}{2}\right]^2\Gamma\left[\frac{2+p-t}{2}\right]^2\Gamma\left[2-\frac{u}{2}\right]\Gamma\left[p-\frac{u}{2}\right]$$

$$\mathcal{G}^{ABCD}_{\mathrm{conn}}(U,V)=\mathcal{G}^{ABCD}(U,V)-\left(\delta^{AB}\delta^{CD}+\delta_{2,p}U\delta^{AC}\delta^{BD}+\delta_{2,p}\frac{U}{V^2}\delta^{AD}\delta^{BC}\right),$$

$$\tilde{\mathcal{G}}^{ABCD}_{\mathrm{conn}}(U,V)=\tilde{\mathcal{G}}^{ABCD}(U,V)-\left(\delta_{2,p}\delta^{AB}\delta^{CD}+U^{\frac{p}{2}}\delta^{AC}\delta^{BD}+\delta_{2,p}\frac{U}{V^2}\delta^{AD}\delta^{BC}\right).$$

$$M_{F^2}(s,t)=-\frac{4}{(p-2)!\,\Delta}\left[\frac{\mathrm{c}_s}{(s-2)(u-p)}-\frac{\mathrm{c}_t}{(t-p)(u-p)}\right],$$

$$\mathrm{c}_s^{ABCD}=f^{ABJ}f^{JCD},\mathrm{c}_t^{ABCD}=f^{ADJ}f^{JBC},\mathrm{c}_u^{ABCD}=f^{ACJ}f^{JDB}$$

$$M_{F^2|F^2}(s,t)=-\frac{p}{4(p-2)!\,\Delta^2}\left[\,\mathrm{d}_{st}\mathcal{B}^{p}_{st}+\mathrm{d}_{su}\mathcal{B}^{p}_{su}+\mathrm{d}_{tu}\mathcal{B}^{p}_{tu}\right]$$

$$\mathrm{d}^{ABCD}_{st}=f^{IAJ}f^{JBK}f^{KCL}f^{LDI}$$
$$\mathrm{d}^{ABCD}_{su}=f^{IAJ}f^{JBK}f^{KDL}f^{LCI}$$
$$\mathrm{d}^{ABCD}_{tu}=f^{IAJ}f^{JCK}f^{KBL}f^{LDI}$$

$$\mathcal{B}_{st}^{p}=\sum_{m,n=0}^{\infty}\frac{(m-n)(m+2n+3)-m(n+1)(p+1)(m+n+2)}{54(m+n)(m+n+1)(m+n+2)(s-4-2m)(t-2-p-2n)}$$

$$\mathcal{B}_{su}^{p}=\sum_{m,n=0}^{\infty}\frac{(m-n)(m+2n+3)-m(n+1)(p+1)(m+n+2)}{54(m+n)(m+n+1)(m+n+2)(s-4-2m)(u-2-p-2n)}$$

$$\mathcal{B}_{tu}^{p}=\sum_{m,n=0}^{\infty}\frac{(m-n)^2-(m+1)(n+1)(p+1)(m+n)}{54(m+n)(m+n+1)(m+n+2)(t-2-p-2m)(u-2-p-2n)}$$

$$\mathcal{B}_{st}^{2}=R_0(s,t)\left[\psi^{(1)}\left(2-\frac{s}{2}\right)+\psi^{(1)}\left(2-\frac{t}{2}\right)-\left(\psi\left(2-\frac{s}{2}\right)-\psi\left(2-\frac{t}{2}\right)\right)^2\right]$$
$$+R_1(s,t)\psi^{(0)}\left(2-\frac{s}{2}\right)+R_1(t,s)\psi^{(0)}\left(2-\frac{t}{2}\right)+\pi^2R_2(s,t)+\frac{4}{27(s+t-8)}-\frac{\gamma}{9}$$

$$R_0(s,t)=\frac{3s^2t-8s^2+3st^2-32st+60s-8t^2+60t-96}{54(s+t-8)(s+t-6)(s+t-4)}$$
$$R_1(s,t)=-\frac{3s^2+3st-26s-10t+48}{27(s+t-8)(s+t-4)},R_2(s,t)=-R_0(s,t)$$

$$M^1_{F^4,s}(s,t)=M_{\log,s}(s,t)=\mathrm{t}_1,M^1_{F^4,tu}(s,t)=M_{\log,tu}(s,t)=\mathrm{t}_2+\mathrm{t}_3$$

$$\mathrm{t}_1^{ABCD}=\delta^{AB}\delta^{CD},\mathrm{t}_2^{ABCD}=\delta^{AC}\delta^{DB},\mathrm{t}_3^{ABCD}=\delta^{AD}\delta^{BC}$$

$$p=2\colon M^1_{F^4}(s,t)=M_{\log}(s,t)=\mathrm{t}_1+\mathrm{t}_2+\mathrm{t}_3$$

$$b_{\log}=\frac{6}{\Delta}.$$

$$-\lambda_{22k_2}\lambda_{ppk_2}\sum_{m=0}\frac{1}{s-2m-k_2}\frac{4^{k_2+1}\Gamma\left(\frac{k_2+1}{2}\right)\Gamma\left(\frac{k_2+3}{2}\right)}{\pi(k_2/2)!^2\,m!\,(1-m-k_2/2)!\,(p-1-m-k_2/2)!\,(m+k_2)!},$$

$$-\lambda_{2pk_p}^2\sum_{m=0}\frac{1}{s-2m-k_p}\frac{2k_p!\,(k_p+1)!}{m!\,(k_p+m)!\,\Gamma\left(\frac{k_p}{2}-\frac{p}{2}+2\right)^2\Gamma\left(\frac{k_p}{2}+\frac{p}{2}\right)^2\Gamma\left(1-\frac{k_p}{2}-m+\frac{p}{2}\right)^2},$$

$$\lambda_{22k_2}\lambda_{ppk_2}=\beta_{22k_2}\beta_{ppk_2}\frac{\Gamma(2-k_2/2)\Gamma(k_2/2)^5\Gamma(p-k_2/2)\Gamma(k_2/2+p-2)}{32\pi^2\Gamma(k_2)\Gamma(k_2-1)\Gamma(p-1)\Gamma(p)}$$

$$\lambda_{2pk_p}^2=\beta_{2pk_p}^2\frac{\Gamma\left(\frac{1}{2}(k_p-p+2)\right)^2\Gamma\left(\frac{1}{2}(-k_p+p+2)\right)^2\Gamma\left(\frac{1}{2}(k_p+p-2)\right)^4}{32\pi^2\Gamma(k_p-1)\Gamma(k_p)\Gamma(p-1)\Gamma(p)}.$$

$$M_R \approx \frac{2}{\Delta}\sum_{m=0}\left[\sum_{k_2}\frac{k_2^2(k_2^2-4)\Gamma\left(\frac{k_2}{2}+m-1\right)\Gamma\left(\frac{k_2}{2}+p\right)\Gamma\left(\frac{k_2}{2}+m-p+1\right)}{8(k_2+2m-s)\Gamma(m+1)\Gamma(p-1)\Gamma(p)\Gamma(k_2+m+1)\Gamma\left(\frac{k_2}{2}-p+1\right)}\mathrm{t}_1\right.$$
$$+\sum_{k_p}\frac{k_p(p-1)\Gamma\left(\frac{1}{2}(k_p+p+2)\right)^2\Gamma\left(\frac{k_p}{2}+m-\frac{p}{2}\right)^2}{(k_p+2m-u)\Gamma(m+1)\Gamma(p)^2\Gamma(k_p+m+1)\Gamma\left(\frac{k_p-p}{2}\right)^2}\mathrm{t}_2$$
$$\left.+\sum_{k_p}\frac{k_p(p-1)\Gamma\left(\frac{1}{2}(k_p+p+2)\right)^2\Gamma\left(\frac{k_p}{2}+m-\frac{p}{2}\right)^2}{(k_p+2m-t)\Gamma(m+1)\Gamma(p)^2\Gamma(k_p+m+1)\Gamma\left(\frac{k_p-p}{2}\right)^2}\mathrm{t}_3\right]$$

$$\mathcal{A}(s,t)=(u+s/w)^2\left[\ell_P^4\mathcal{A}_{F^2}+\ell_P^8\left[\mathcal{A}_{F^4}(\tau)+\mathcal{A}_{F^2|F^2}+\mathcal{A}_R+\mathcal{A}_{\log}\log\,\ell_P^2+O(\ell_P^{10})\right]\right]$$

$$\mathcal{A}^{ABCD}(s,t)=\lim_{L\to\infty}8\pi^4(p-1)!\,(p-2)!\,L^4\int\frac{d\beta}{2\pi i}\frac{e^\beta}{\beta^{2+p}}\frac{L^4(u+s/w)^2}{16}M^{ABCD}\left(\frac{L^2}{2\beta}s,\frac{L^2}{2\beta}t\right)$$

$$\mathcal{A}_{F^2}(s,t)=-\frac{(2\pi)^5}{stu}(t\mathrm{c}_s-s\mathrm{c}_t)$$

$$\mathcal{A}_{\log}(s,t)=-16\pi^6\boldsymbol{\Delta}(\mathrm{t}_1+\mathrm{t}_2+\mathrm{t}_3),$$

$$\mathcal{A}_{F^2|F^2}(s,t)=-\frac{2\pi^6}{3}[\,\mathrm{d}_{st}f_{\mathrm{box}}(s,t)+\mathrm{d}_{su}f_{\mathrm{box}}(s,u)+\mathrm{d}_{tu}f_{\mathrm{box}}(t,u)],$$

$$f_{\mathrm{box}}(s,t)\sim\mathcal{B}_{st}^2\sim\frac{st\log^2\left(\frac{-s}{-t}\right)}{(s+t)^2}+2\frac{s\log(-s/2)+t\log(-t/2)}{s+t}+\pi^2\frac{st}{(s+t)^2}+2\gamma.$$

$$\mathcal{A}_R=-8\Delta\pi^6[\mathrm{t}_1(\log(-s/2)+\gamma)+\mathrm{t}_2(\log(-t/2)+\gamma)+\mathrm{t}_3(\log(-u/2)+\gamma)]$$

$$:\phi_2^A\,\Box^{n-2}\,\phi_2^A:,:\phi_3^A\,\Box^{n-3}\,\phi_3^A:,\ldots,:\phi_n^A\phi_n^A:,$$

$$\Delta_a=4+\frac{1}{N}\gamma_a^{(1)}+\frac{1}{N^2}\gamma_a^{(2)}+\mathcal{O}\left(\frac{1}{N^3}\right),$$

$$\lambda_{2,2,\mathcal{O}_a}=\lambda_{2,2,\mathcal{O}_a}^{(0)}+\frac{1}{N}\lambda_{2,2,\mathcal{O}_a}^{(1)}+\frac{1}{N^2}\lambda_{2,2,\mathcal{O}_a}^{(2)}+\mathcal{O}\left(\frac{1}{N^3}\right)$$

$$\mathcal{G}_1^{2,2}(U,V) = U^{-1} \quad \left[\left\langle\left(\lambda^{(0)}\right)^2\right\rangle + \frac{1}{N}\left(2\left\langle\lambda^{(0)}\lambda^{(1)}\right\rangle + \frac{1}{2}\left\langle\left(\lambda^{(0)}\right)^2\gamma^{(1)}\right\rangle\left(\log U + 2\partial_\Delta^{\text{no log}}\right)\right)\right.$$
$$+\frac{1}{N^2}\left(\left\langle 2\lambda^{(0)}\lambda^{(2)} + \left(\lambda^{(1)}\right)^2\right\rangle + \frac{1}{2}\left\langle 2\lambda^{(0)}\lambda^{(1)}\gamma^{(1)} + \left(\lambda^{(0)}\right)^2\gamma^{(2)}\right\rangle\left(\log U + 2\partial_\Delta^{\text{no log}}\right)\right.$$
$$\left.\left.+\frac{1}{8}\left\langle\left(\lambda^{(0)}\right)^2\left(\gamma^{(1)}\right)^2\right\rangle\left(\log^2 U + 4\log U\partial_\Delta + 4\left(\partial_\Delta^{\text{no log}}\right)^2\right)\right)\right.$$
$$\left.+\mathcal{O}\left(\frac{1}{N^3}\right)\right] g_{\Delta+2,0}(U,V)\big|_{\Delta=4}$$
$$+\cdots$$

$$\left\langle\left(\lambda^{(0)}\right)^2\right\rangle = \sum_{a=1}^{2}\left(\lambda_a^{(0)}\right)^2.$$

$$\left\langle\left(\lambda^{(0)}\right)^2\right\rangle = \frac{2}{3}$$
$$\left\langle\left(\lambda^{(0)}\right)^2\gamma^{(1)}\right\rangle = -\frac{4}{3}\frac{h^\vee}{\Delta} = \frac{8(1-\Delta)}{\Delta}$$
$$\left\langle\left(\lambda^{(0)}\right)^2\left(\gamma^{(1)}\right)^2\right\rangle = \left\langle\left(\lambda^{(0)}\right)^2\left(\gamma^{(1)}\right)^2\right\rangle_{\text{superparticle}} + \left\langle\left(\lambda^{(0)}\right)^2\left(\gamma^{(1)}\right)^2\right\rangle_{\text{grav}}$$

$$\left\langle\left(\lambda^{(0)}\right)^2\left(\gamma^{(1)}\right)^2\right\rangle_{\text{superparticle}} = \frac{3}{2}\left(-\frac{4}{3}\frac{h^\vee}{\Delta}\right)^2 = \frac{96(1-\Delta)^2}{\Delta^2}$$
$$\left\langle\left(\lambda^{(0)}\right)^2\left(\gamma^{(1)}\right)^2\right\rangle_{\text{grav}} = \frac{16}{5}\frac{\dim(G_F)}{\Delta} = \frac{32}{5}\frac{(6-5\Delta)(5-6\Delta)}{\Delta^2}$$

$$h^\vee = 6(\boldsymbol{\Delta}-1), \dim(G_F) = \frac{2(6-5\boldsymbol{\Delta})(5-6\boldsymbol{\Delta})}{\boldsymbol{\Delta}}$$

$$\begin{pmatrix} \langle\rho_4(x)\rho_4(y)\rangle & \frac{1}{\mathcal{N}}\langle\rho_4(x){:}\,\phi_2\phi_2{:}\,(y)\rangle \\ \frac{1}{\mathcal{N}}\langle{:}\,\phi_2\phi_2{:}\,(x)\rho_4(y)\rangle & \frac{1}{\mathcal{N}^2}\langle{:}\,\phi_2\phi_2{:}\,(x){:}\,\phi_2\phi_2{:}\,(y)\rangle \end{pmatrix}$$
$$= \frac{1}{|x-y|^8}\left[\delta_{ab} - \frac{1}{N}M_{ab}\log\left(|x-y|^2\right) + \mathcal{O}\left(\frac{1}{N^2}\right)\right]$$

$$M = \frac{1}{\langle(\lambda^{(0)})^2\rangle}\begin{pmatrix} 0 & \sqrt{\langle(\lambda^{(0)})^2(\gamma^{(1)})^2\rangle_{\text{grav}}\,\langle(\lambda^{(0)})^2\rangle} \\ \sqrt{\langle(\lambda^{(0)})^2(\gamma^{(1)})^2\rangle_{\text{grav}}\,\langle(\lambda^{(0)})^2\rangle} & \left\langle\left(\lambda^{(0)}\right)^2\gamma^{(1)}\right\rangle \end{pmatrix}$$
$$= \begin{pmatrix} 0 & \frac{4}{\Delta}\sqrt{\frac{3}{5}}\sqrt{(6-5\Delta)(5-6\Delta)} \\ \frac{4}{\Delta}\sqrt{\frac{3}{5}}\sqrt{(6-5\Delta)(5-6\Delta)} & \frac{12(1-\Delta)}{\Delta} \end{pmatrix}$$

$$\gamma_1^{(1)} = \frac{6}{\Delta}\left(1-\Delta-\frac{1}{\sqrt{15}}\sqrt{135-274\Delta+135\Delta^2}\right)$$
$$\gamma_2^{(1)} = \frac{6}{\Delta}\left(1-\Delta+\frac{1}{\sqrt{15}}\sqrt{135-274\Delta+135\Delta^2}\right)$$

$$\left(\lambda^{(0)}_{2,2,\mathcal{O}_1}\right)^2 = \frac{2}{3}\left(\frac{-\sqrt{15}(1-\Delta)+\sqrt{135-274\Delta+135\Delta^2}}{2\sqrt{135-274\Delta+135\Delta^2}}\right)$$
$$\left(\lambda^{(0)}_{2,2,\mathcal{O}_2}\right)^2 = \frac{2}{3}\left(\frac{\sqrt{15}(1-\Delta)+\sqrt{135-274\Delta+135\Delta^2}}{2\sqrt{135-274\Delta+135\Delta^2}}\right)$$

$$\gamma_1^{(1)} = -106.132\ldots, \gamma_2^{(1)} = 72.274\ldots, \gamma_3^{(1)} = -4.542\ldots$$

$$\left(\lambda^{(0)}_{2,2,\mathcal{O}_1}\right)^2 = 0.0086\ldots, \quad \left(\lambda^{(0)}_{2,2,\mathcal{O}_2}\right)^2 = 0.0016\ldots, \left(\lambda^{(0)}_{2,2,\mathcal{O}_3}\right)^2 = 0.0898\ldots,$$
$$\left(\lambda^{(0)}_{3,3,\mathcal{O}_1}\right)^2 = 0.1283\ldots, \quad \left(\lambda^{(0)}_{3,3,\mathcal{O}_2}\right)^2 = 0.0985\ldots, \left(\lambda^{(0)}_{3,3,\mathcal{O}_3}\right)^2 = 0.0232\ldots$$

$$28 \otimes 28 = 1 \oplus 28 \oplus 35_{\mathrm{v}} \oplus 35_{\mathrm{c}} \oplus 35_{\mathrm{s}} \oplus 300 \oplus 350$$

$$P_{\mathbf{1}} = \left(\frac{1}{28} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right)\cdot \mathrm{t}$$
$$P_{\mathbf{28}} = \left(0 \quad 0 \quad 0 \quad -\frac{1}{12} \quad \frac{1}{12} \quad 0 \quad 0\right)\cdot \mathrm{t}$$
$$P_{\mathbf{35c}} = \left(0 \quad \frac{1}{12} \quad \frac{1}{12} \quad 0 \quad \frac{1}{12} \quad -\frac{1}{12} \quad \frac{1}{2}\right)\cdot \mathrm{t}$$
$$P_{\mathbf{35s}} = \left(0 \quad \frac{1}{12} \quad \frac{1}{12} \quad 0 \quad \frac{1}{12} \quad -\frac{1}{12} \quad -\frac{1}{2}\right)\cdot \mathrm{t}$$
$$P_{\mathbf{35v}} = \left(-\frac{1}{12} \quad 0 \quad 0 \quad \frac{1}{12} \quad -\frac{1}{12} \quad \frac{1}{6} \quad 0\right)\cdot \mathrm{t}$$
$$P_{\mathbf{350}} = \left(0 \quad -\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{12} \quad -\frac{1}{12} \quad 0 \quad 0\right)\cdot \mathrm{t}$$
$$P_{\mathbf{300}} = \left(\frac{1}{21} \quad \frac{1}{3} \quad \frac{1}{3} \quad -\frac{1}{12} \quad -\frac{1}{12} \quad 0 \quad 0\right)\cdot \mathrm{t}$$

$$\mathrm{t}_i = \left(\begin{array}{l}(\delta^{AB}\delta^{CD}, \delta^{AC}\delta^{BD}, \delta^{AD}\delta^{BC}, f^{ACE}f^{BDE}, f^{ADE}f^{BCE} \\ \mathrm{tr}(T^A T^B T^C T^D), \frac{1}{4\cdot 4!}\epsilon_{a_1a_2b_1b_2c_1c_2d_1d_2}T^A_{a_1a_2}T^B_{b_1b_2}T^C_{c_1c_2}T^D_{d_1d_2}\end{array}\right).$$

$$M^2_{F^4} = \mathrm{t}_6 + \frac{1}{3}\mathrm{t}_4 - \frac{2}{3}\mathrm{t}_5, M^3_{F^4} = \mathrm{t}_7$$

$$-\partial^4_{\mu_1} F\big|_{\mu=0} = k^2 I[M_1 + M_2 + M_3 + 2M_6],$$
$$-\partial^2_{\mu_1}\partial^2_{\mu_2} F\big|_{\mu=0} = \frac{k^2}{3} I[M_1 + M_2 + M_3],$$
$$-\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4} F\big|_{\mu=0} = \frac{k^2}{6} I[M_7],$$

$$I[M_{\mathbf{r}}] \equiv -\int \frac{dsdt}{(4\pi i)^2}[M_{\mathbf{r}}(s,t)\Gamma[2-s/2]\Gamma[s/2]\Gamma[2-t/2]\Gamma[t/2]\Gamma[2-u/2]\Gamma[u/2] \times\left(\frac{H_{\frac{s}{2}-1}+H_{1-\frac{s}{2}}}{(t-2)(u-2)}+\frac{H_{\frac{t}{2}-1}+H_{1-\frac{t}{2}}}{(s-2)(u-2)}+\frac{H_{\frac{u}{2}-1}+H_{1-\frac{u}{2}}}{(s-2)(t-2)}\right)]$$

$$\mathcal{F}_{\mathbf{v}} \equiv -4\partial_{\mu_1}^2\partial_{\mu_2}^2 F\big|_{\mu=0},$$
$$\mathcal{F}_{\mathbf{c}} \equiv -\partial_{\mu_1}^4 F\big|_{\mu=0} - \partial_{\mu_1}^2\partial_{\mu_2}^2 F\big|_{\mu=0} + 2\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4} F\big|_{\mu=0},$$
$$\mathcal{F}_{\mathbf{s}} \equiv -\partial_{\mu_1}^4 F\big|_{\mu=0} - \partial_{\mu_1}^2\partial_{\mu_2}^2 F\big|_{\mu=0} - 2\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4} F\big|_{\mu=0},$$

$$S:\quad \tau \to -1/\tau \quad\Leftrightarrow\quad \mathbf{35}_{\mathrm{v}} \leftrightarrow \mathbf{35}_{\mathrm{c}} \quad\Leftrightarrow\quad \mathcal{F}_{\mathrm{v}} \leftrightarrow \mathcal{F}_{\mathrm{c}},$$
$$T:\quad \tau \to \tau+1 \quad\Leftrightarrow\quad \mathbf{35}_{\mathrm{c}} \leftrightarrow \mathbf{35}_{\mathrm{s}} \quad\Leftrightarrow\quad \mathcal{F}_{\mathrm{c}} \leftrightarrow \mathcal{F}_{\mathrm{s}}.$$

$$\mathcal{F}_{\mathbf{v}} = -8\log[\tau_2|\theta_3(\tau)\theta_4(\tau)|^2] + 8\log[8\pi N] + 4f(N) + O(e^{-N}),$$
$$\mathcal{F}_{\mathbf{c}} = -8\log[\tau_2|\theta_2(\tau)\theta_3(\tau)|^2] + 8\log[8\pi N] + 4f(N) + O(e^{-N}),$$
$$\mathcal{F}_{\mathbf{s}} = -8\log[\tau_2|\theta_2(\tau)\theta_4(\tau)|^2] + 8\log[8\pi N] + 4f(N) + O(e^{-N}),$$

$$f(N) = \frac{16}{3} - 4\zeta(3) + 4\gamma - 4\log(4\pi) - 2\log(2N) + 2\psi(2N+5/2) + (N+1/4)\psi^{(1)}(N+5/4) + (N+3/4)\psi^{(1)}(N+7/4)$$
$$= \frac{22}{3} - 4\zeta(3) + 4\gamma - 4\log(4\pi) + \frac{1}{N} - \frac{7}{48N^2} - \frac{1}{48N^3} + \frac{67}{2560N^4} + O(N^{-5})$$

$$I[1] = \frac{1}{24}, I[\mathcal{B}(s,t)] = \frac{11}{1296} - \frac{\zeta(3)}{108}\; I\left[3H_{1-\frac{s}{2}} + \frac{4}{s-2}\right] = -\frac{11}{96},$$

$$b_{F^4}^1 = 6(\gamma - \log(4\pi)) - 3\log[\tau_2|\theta_3(\tau)\theta_4(\tau)|^2],$$
$$b_{F^4}^2 = -3\log[\tau_2|\theta_2(\tau)\theta_3(\tau)|^2] - 3\log[\tau_2|\theta_2(\tau)\theta_4(\tau)|^2] + 6\log[\tau_2|\theta_3(\tau)\theta_4(\tau)|^2],$$
$$b_{F^4}^3 = 18\log[\tau_2|\theta_2(\tau)\theta_3(\tau)|^2] - 18\log[\tau_2|\theta_2(\tau)\theta_4(\tau)|^2],$$

$$\int \frac{d\beta}{2\pi i}\frac{e^{\beta}}{\beta^4}\log(1/(2\beta)) = -\frac{11}{36} + \frac{\gamma}{6} - \frac{\log(2)}{6}$$

$$\mathcal{A}_{F^4} = a_{F^4}^1(\mathrm{t}_1+\mathrm{t}_2+\mathrm{t}_3) + a_{F^4}^2\left(\mathrm{t}_6 + \frac{1}{3}\mathrm{t}_4 - \frac{2}{3}\mathrm{t}_5\right) + a_{F^4}^1\mathrm{t}_7$$

$$a_{F^4}^1 = -\frac{16}{3}\pi^6(6\log(2\pi) - 11) - 16\pi^6\log[\tau_2|\theta_3(\tau)\theta_4(\tau)|^2]$$
$$a_{F^4}^2 = -16\pi^6\log[\tau_2|\theta_2(\tau)\theta_3(\tau)|^2] - 16\pi^6\log[\tau_2|\theta_2(\tau)\theta_4(\tau)|^2] + 32\pi^6\log[\tau_2|\theta_3(\tau)\theta_4(\tau)|^2]$$
$$a_{F^4}^3 = 96\pi^6\log[\tau_2|\theta_2(\tau)\theta_3(\tau)|^2] - 96\pi^6\log[\tau_2|\theta_2(\tau)\theta_4(\tau)|^2]$$

$$\gamma_{\mathbf{1},\ell}^{(2)} = -\frac{8(\ell^6+15\ell^5+56\ell^4+7\ell^3+357\ell^2+2360\ell+1536)}{\ell(\ell+1)^3(\ell+4)^3(\ell+5)}$$
$$\gamma_{\mathbf{35}_*,\ell}^{(2)} = -\frac{8(\ell^6+15\ell^5+82\ell^4+203\ell^3+295\ell^2+400\ell+192)}{\ell(\ell+1)^3(\ell+4)^3(\ell+5)}$$
$$\gamma_{\mathbf{300},\ell}^{(2)} = -\frac{8(\ell^6+15\ell^5+91\ell^4+287\ell^3+511\ell^2+505\ell+192)}{\ell(\ell+1)^3(\ell+4)^3(\ell+5)}$$

$$\gamma^{(2)}_{\mathbf{28},\ell} = -\frac{8(\ell^6+15\ell^5+77\ell^4+163\ell^3+273\ell^2+665\ell+384)}{\ell(\ell+1)^3(\ell+4)^3(\ell+5)}$$

$$\gamma^{(2)}_{\mathbf{350},\ell} = -\frac{8(\ell+2)(\ell+3)}{\ell(\ell+1)(\ell+4)(\ell+5)}$$

$$\gamma^{(2)}_{\mathbf{300},0} = \frac{12}{5}\left(2\log\left[\sqrt{\tau_2}|\eta(\tau)|^2\right] - 2\gamma + 2\log\,\pi + 5\log\,2\right) + \frac{723}{200},$$

$$\gamma^{(2)}_{\mathbf{35},0} = -\frac{6}{5}\left(8\log\left[\sqrt{\tau_2}|\eta(\tau)|^2\right] - 12\log\left[\sqrt{\tau_2}|\theta_2(\tau)|^2\right] + 4\gamma - 4\log\,\pi + 2\log\,2\right) + \frac{126}{25},$$

$$\gamma^{(2)}_{\mathbf{35},0} = -\frac{6}{5}\left(8\log\left[\sqrt{\tau_2}|\eta(\tau)|^2\right] - 12\log\left[\sqrt{\tau_2}|\theta_4(\tau)|^2\right] + 4\gamma - 4\log\,\pi + 2\log\,2\right) + \frac{126}{25},$$

$$\gamma^{(2)}_{\mathbf{35},0} = -\frac{6}{5}\left(8\log\left[\sqrt{\tau_2}|\eta(\tau)|^2\right] - 12\log\left[\sqrt{\tau_2}|\theta_3(\tau)|^2\right] + 4\gamma - 4\log\,\pi + 2\log\,2\right) + \frac{126}{25},$$

$$y_1^2 + y_2^2 + y_3^2 + y_4^2 = 1-\rho^2, y_5^2 + y_6^2 = \rho^2$$

$$SO(6) \cong SU(4) \longrightarrow SO(4)\times SO(2) \cong SU(2)_L \times SU(2)_R \times U(1)$$

$$\int_{S^D} d\Omega_D y_{I_1} y_{I_2} \dots y_{I_{2n}}$$
$$= \frac{\mathrm{Vol}(S^D)}{(D+1)(D+3)\dots(D+2n-1)}\left(\delta_{I_1I_2}\dots\delta_{I_{2n-1}I_{2n}} + \wp_{\text{all other contractions}}\right)$$

$$y^{\bar{\alpha}\alpha} = (\sigma_i)^{\bar{\alpha}\alpha} y_i$$

$$(\sigma_i)^{\bar{\alpha}\alpha}(\sigma_i)^{\bar{\beta}\beta} = 2\epsilon^{\bar{\alpha}\bar{\beta}}\epsilon^{\alpha\beta}$$

$$\int_{\bar{S}^5} d\Omega_5 y^{\bar{\alpha}_1\alpha_1}\dots y^{\bar{\alpha}_{2n}\alpha_{2n}} f(\rho^2)$$
$$= \frac{2\pi^3}{(n+1)!\,\boldsymbol{\Delta}}\left(\int_0^1 dt(1-t)^{n+1} f(t)\right)\left(\epsilon^{\bar{\alpha}_1\bar{\alpha}_2}\epsilon^{\alpha_1\alpha_2}\dots\epsilon^{\bar{\alpha}_{2n-1}\bar{\alpha}_{2n}}\epsilon^{\alpha_{2n-1}\alpha_{2n}} + \wp_{\text{all other contractions}}\right)$$

$$\epsilon^{\bar{\alpha}_1\bar{\alpha}_2}\epsilon^{\alpha_1\alpha_2}\epsilon^{\bar{\alpha}_3\bar{\alpha}_4}\epsilon^{\alpha_3\alpha_4} + \epsilon^{\bar{\alpha}_1\bar{\alpha}_3}\epsilon^{\alpha_1\alpha_3}\epsilon^{\bar{\alpha}_2\bar{\alpha}_4}\epsilon^{\alpha_2\alpha_4} + \epsilon^{\bar{\alpha}_1\bar{\alpha}_4}\epsilon^{\alpha_1\alpha_4}\epsilon^{\bar{\alpha}_2\bar{\alpha}_3}\epsilon^{\alpha_2\alpha_3}$$

$$\int_{S^3} d\Omega_3 y^{\bar{\alpha}_1\alpha_1}\dots y^{\bar{\alpha}_{2n}\alpha_{2n}} = \frac{2\pi^2}{(n+1)!}\left(\epsilon^{\bar{\alpha}_1\bar{\alpha}_2}\epsilon^{\alpha_1\alpha_2}\dots\epsilon^{\bar{\alpha}_{2n-1}\bar{\alpha}_{2n}}\epsilon^{\alpha_{2n-1}\alpha_{2n}} + \wp_{\text{all other contractions}}\right)$$

$$k_p = \begin{cases} p, p+2, p+4, \dots & p = 2,3 \\ p-2, p, p+2, \dots & p > 3 \end{cases}$$

$$s_{I_1I_2\dots I_k} y_{I_1} y_{I_2}\dots y_{I_k}$$

$$k_p\colon s_{i_1i_2\dots i_{p-2}} y_{i_1} y_{i_2}\dots y_{i_{p-2}}(1-\rho^2)^{(k_p-p+2)/2} - (SO(6)\mathfrak{T}),$$

$$k_p - (p-2) = 0,2,4,\dots$$

$$k_p\colon s_{i_1i_2\dots i_{p-2}} y_{i_1} y_{i_2}\dots y_{i_{p-2}}(1-\rho^2)^{(k_p-p+2)/2} f_{k_p}\left(\frac{\rho^2}{1-\rho^2}\right)$$

$$\left(k_p - p + 2\right)\left(k_p + p\right)f_{k_p}(z) + 4\left(1 - k_p z\right)f'_{k_p}(z) + 4z(1+z)f''_{k_p}(z) = 0$$

$$f_{k_p}(z) = {}_2F_1\left(-\frac{k_p + p}{2}, \frac{p - k_p - 2}{2}, 1; -z\right)$$

$$s_{(k_p)}(x,y) = \frac{1}{\mathcal{N}\left(k_p\right)}\left(s_{(k_p)}\right)_{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_{p-2}}(x) y^{\bar{\alpha}_1 \alpha_1} \ldots y^{\bar{\alpha}_{p-2}\alpha_{p-2}}\, {}_2F_1\left(\frac{p + k_p + 2}{2}, \frac{p - k_p - 2}{2}, 1; \rho^2\right),$$

$$\delta g_{\mu\nu} = \frac{2}{k_p + 1}\left(\nabla_\mu \nabla_\nu + \nabla_\nu \nabla_\mu - \frac{2}{5} g_{\mu\nu}\, \Box\right) s_{(k_p)} - \frac{6}{5} k_p g_{\mu\nu} s_{(k_p)}$$

$$\delta g_{ab} = 2k_p g_{ab} s_{(k_p)}$$

$$\delta g_{\mu a} = 0$$

$$\delta C = \frac{1}{6}\left(\varepsilon_{\mu\nu\rho\sigma\tau}\left(\nabla^\tau s_{(k_p)}\right) dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma - \varepsilon_{abcde}\left(\nabla^e s_{(k_p)}\right) dy^a \wedge dy^b \wedge dy^c \wedge dy^d\right)$$

$$\mathcal{N}\left(k_p\right)^2 \int_{\tilde{S}^5} d\Omega_5 \left(s_{(k_p)}\right)^2 = \frac{2\pi^3}{(p-1)\Delta}\frac{1}{k_p + 2}\left(s_{(k_p)}\right)_{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_{p-2}} \left(s_{(k_p)}\right)^{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_{p-2}}$$

$$\int_0^1 dt (1-t)^{p-1}\left[{}_2F_1\left(\frac{p + k_p + 2}{2}, \frac{p - k_p - 2}{2}; 1; t\right)\right]^2 = \frac{1}{k_p + 2}$$

$$\mathcal{N}\left(k_p\right)^2 = \frac{32\pi^3}{\Delta\kappa^2}\frac{k_p\left(k_p - 1\right)}{\left(k_p + 1\right)(p-1)},$$

$$S^{(k_p)}_{\text{kinetic}} = \int_{\text{AdS}_5} d^5x \sqrt{-g_{\text{AdS}}}\left(-\frac{1}{2}\nabla_\mu \left(s_{(k_p)}\right)_{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_{p-2}} \nabla^\mu \left(s_{(k_p)}\right)^{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_{p-2}}\right.$$
$$\left. -\frac{1}{2}k_p\left(k_p - 4\right)\left(s_{(k_p)}\right)_{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_{p-2}} \left(s_{(k_p)}\right)^{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_{p-2}}\right)$$

$$\delta g_{\mu\nu} = \frac{2}{k_p + 1}\left(\nabla_\mu \nabla_\nu + \nabla_\nu \nabla_\mu - \frac{2}{5} g_{\mu\nu}\, \Box\right) s_{(k_p)} - \frac{6}{5} k_p g_{\mu\nu} s_{(k_p)}$$

$$\delta g_{\hat{a}\hat{b}} = 2k_p g_{\hat{a}\hat{b}} s_{(k_p)}$$

$$\delta g_{\mu\hat{a}} = 0$$

$$\delta C = \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\left(\nabla^\tau s_{(k_p)}\right) dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma$$

$$s_{(k_p)}(x,y) = \frac{1}{\mathcal{N}\left(k_p\right)}\left(s_{(k_p)}\right)_{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \alpha_1 \ldots \alpha_{p-2}}(x) y^{\bar{\alpha}_1 \alpha_1} \ldots y^{\bar{\alpha}_{p-2}\alpha_{p-2}}$$

$$g^{\mu\nu}\delta g_{\mu\nu} + g^{\hat{a}\hat{b}}\delta g_{\hat{a}\hat{b}} = 0$$

$$\nabla_{\hat{a}}\nabla^{\hat{a}} s_{k_p} = -p(p-2) s_{k_p}$$

$$V_p = \varphi^{(p)}_{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \beta\gamma\alpha_1 \ldots \alpha_{p-2}}(x) y^{\bar{\alpha}_1 \alpha_1} \ldots y^{\bar{\alpha}_{p-2}\alpha_{p-2}}\left(\left(\sigma_j\right)^{\bar{\delta}(\beta}\left(\sigma_k\right)^{\gamma)}_{\bar{\delta}}\right) y_j dy_k$$
$$= \varphi^{(p)}_{\bar{\alpha}_1 \ldots \bar{\alpha}_{p-2}, \beta\gamma\alpha_1 \ldots \alpha_{p-2}}(x) y^{\bar{\alpha}_1 \alpha_1} \ldots y^{\bar{\alpha}_{p-2}\alpha_{p-2}}\left(\sigma_j\right)^{\bar{\delta}\beta}\left(\sigma_k\right)^{\gamma}_{\bar{\delta}} y_j dy_k$$

$$\left((\sigma_i)^{\bar{\delta}(\beta}(\sigma_j)_{\bar{\delta}}^{\gamma)}\right) = -\left((\sigma_j)^{\bar{\delta}(\beta}(\sigma_i)_{\bar{\delta}}^{\gamma\gamma)}\right) = -\frac{1}{2}\epsilon_{ijkl}\left((\sigma_k)^{\bar{\delta}(\beta}(\sigma_l)_{\bar{\delta}}^{\gamma)}\right).$$

$$A = \frac{1}{\mathcal{B}(p)} V_p^*$$

$$\nabla_{\hat{b}}\nabla^{\hat{b}} A_{\hat{a}} = (2-p^2)A_{\hat{a}}$$
$$\epsilon_{\hat{a}\hat{b}\hat{c}}\nabla^{\hat{b}} A^{\hat{c}} = -pA_{\hat{a}}$$
$$\nabla_{\hat{a}} A^{\hat{a}} = 0$$

$$g^{\hat{a}\hat{b}}\left(V_p^*\right)_{\hat{a}}\left(V_q^*\right)_{\hat{b}} = \left(V_p\right)_i\left(V_q\right)_i$$
$$= 2\varphi^{(p)}_{\bar{\alpha}_1\ldots\bar{\alpha}_{p-2},\gamma_1\gamma_2\alpha_1\ldots\alpha_{p-2}}\varphi^{(q)}_{\bar{\beta}_1\ldots\bar{\beta}_{q-2},}{}^{\gamma_1\gamma_2}_{\beta_1\ldots\beta_{q-2}} y^{\bar{\alpha}_1\alpha_1}\ldots y^{\bar{\alpha}_{p-2}\alpha_{p-2}} y^{\bar{\beta}_1\beta_1}\ldots y^{\bar{\beta}_{q-2}}$$

$$S^{(p)}_{\text{kinetic}} = \mu\text{Tr}\int_{\text{AdS}_5\times S^3}\left(-\frac{1}{2}F\wedge\star_8 F + \frac{1}{2}C_4\wedge F\wedge F\right)\Big|_{\text{quadratic}}$$
$$= \text{Tr}\int_{\text{AdS}_5} d^5x\sqrt{-g_{\text{AdS}}}\left(-\frac{1}{2}\nabla_\mu\left(\varphi_{(p)}\right)_{\bar{\alpha}_1\ldots\bar{\alpha}_{p-2},\alpha_1\ldots\alpha_p}\nabla^\mu\left(\varphi_{(p)}\right)^{\bar{\alpha}_1\ldots\bar{\alpha}_{p-2},\alpha_1\ldots\alpha_p}\right.$$
$$\left.-\frac{1}{2}p(p-4)\left(\varphi_{(p)}\right)_{\bar{\alpha}_1\ldots\bar{\alpha}_{p-2},\alpha_1\ldots\alpha_p}\left(\varphi_{(p)}\right)^{\bar{\alpha}_1\ldots\bar{\alpha}_{p-2},\alpha_1\ldots\alpha_p}\right)$$

$$\mathcal{B}(p)^2 = \frac{4\pi^2}{p-1}\mu$$

$$pqk_r$$

$$r = |p-q|+2, |p-q|+4, \ldots, p+q-2.$$

$$\nabla_\mu\nabla_\nu f_1\nabla^\mu f_2\nabla^\nu f_3 = \frac{1}{4}\left((\Box^2 f_1)f_2f_3 - f_1(\Box^2 f_2)f_3 - f_1f_2(\Box^2 f_3) + 2f_1(\Box f_2)(\Box f_3)\right) + \nabla_\mu(\ldots),$$
$$f_1\nabla^\mu f_2\nabla_\mu f_3 = \frac{1}{2}\left((\Box f_1)f_2f_3 - f_1(\Box f_2)f_3 - f_1f_2(\Box f_3)\right) + \nabla_\mu(\ldots),$$

$$S^{(p,q,k_r)}_{\text{cubic}} = \text{Tr}\int_{\text{AdS}_5} d^5x\sqrt{-}g_{\text{AdS}}$$
$$\times\frac{1}{1+\delta_{p,q}}\beta_{pqk_r}\left(\varphi_{(p)}\right)_{\bar{\alpha}_1\ldots\bar{\alpha}_{(p+q-r-2)/2}\bar{\beta}_1\ldots\bar{\beta}_{(p-q+r-2)/2},\gamma\delta\alpha_1\ldots\alpha_{(p+q-r-2)/2}\beta_1\ldots\beta_{(p-q+r-2)/2}}$$
$$\times\left(\varphi_{(q)}\right)^{\bar{\alpha}_1\ldots\bar{\alpha}_{(p+q-r-2)/2}\bar{\beta}_{(p-q+r)/2}\ldots\bar{\beta}_{r-2},\,\gamma\delta\alpha_1\ldots\alpha_{(p+q-r-2)/2}\beta_{(p-q+r)/2}\ldots\beta_{r-2}}$$
$$\times\left(s_{(k_r)}\right)^{\bar{\beta}_1\ldots\bar{\beta}_{r-2},\beta_1\ldots\beta_{r-2}}$$

$$\beta_{pqk_r} = \frac{4\pi^2\mu}{\mathcal{N}(k_r)\mathcal{B}(p)\mathcal{B}(q)}\frac{(k_r+p-q)(k_r+q-p)(k_r+p+q-2)(k_r+p+q-4)}{k_r+1}$$
$$\times\frac{\Gamma(p-1)\Gamma(q-1)\Gamma(r-1)}{\Gamma\left(\frac{p+q-r}{2}\right)\Gamma\left(\frac{r+p-q}{2}\right)\Gamma\left(\frac{r+q-p}{2}\right)\Gamma\left(\frac{p+q+r-2}{2}\right)}$$

$$:\phi_p\,\Box^{n-p}\,\phi_p:, p = 2,3,\ldots,n,$$

$$\Delta_{n,a} = 2n + \frac{1}{N}\gamma_{n,a}^{(1)} + \frac{1}{N^2}\gamma_{n,a}^{(2)} + \mathcal{O}\left(\frac{1}{N^3}\right)$$

$$\lambda_{p,p,\mathcal{O}_{n,a}} = \lambda_{p,p,\mathcal{O}_{n,a}}^{(0)} + \frac{1}{N}\lambda_{p,p,\mathcal{O}_{n,a}}^{(1)} + \frac{1}{N^2}\lambda_{p,p,\mathcal{O}_{n,a}}^{(2)} + \mathcal{O}\left(\frac{1}{N^3}\right)$$

$$\begin{aligned}&\left\langle \phi_p^A(y_1,x_1)\phi_p^B(y_2,x_2)\phi_q^C(y_3,\bar{y}_3,x_3)\phi_q^D(y_4,\bar{y}_4,x_4)\right\rangle \\ &= \frac{\langle y_1,y_2\rangle^p\langle y_3,y_4\rangle^q\langle \bar{y}_1,\bar{y}_2\rangle^{p-2}\langle \bar{y}_3,\bar{y}_4\rangle^{q-2}}{x_{12}^{2p}x_{34}^{qq}}\sum_{\mathbf{r}\in \mathfrak{g}\otimes\mathfrak{g}} G_{\mathbf{r}}^{p,q}(U,V;w)P_{\mathbf{r}}^{ABCD}\end{aligned}$$

$$G_{\mathbf{r}}^{p,q}(U,V;w) = \frac{z(w-\bar{z})f_r(\bar{z}) - \bar{z}(w-z)f_r(z)}{w(z-\bar{z})} + \left(1-\frac{z}{w}\right)\left(1-\frac{\bar{z}}{w}\right)\mathcal{G}_{\mathbf{r}}^{p,q}(U,V).$$

$$\begin{aligned}\mathcal{G}_1^{p,q}(U,V) = \sum_{n=2,3,\dots} U^{-1}\Bigg[&\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\lambda_{q,q,\mathcal{O}_n}^{(0)}\right\rangle \\ &+\frac{1}{N}\left(\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\lambda_{q,q,\mathcal{O}_n}^{(1)} + \lambda_{p,p,\mathcal{O}_n}^{(1)}\lambda_{q,q,\mathcal{O}_n}^{(0)}\right\rangle + \frac{1}{2}\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\gamma_n^{(1)}\lambda_{q,q,\mathcal{O}_n}^{(0)}\right\rangle(\log U + 2\partial_\Delta)\right) \\ &+\frac{1}{N^2}\Bigg(\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\lambda_{q,q,\mathcal{O}_n}^{(2)} + \lambda_{p,p,\mathcal{O}_n}^{(2)}\lambda_{q,q,\mathcal{O}_n}^{(0)} + \lambda_{p,p,\mathcal{O}_n}^{(1)}\lambda_{q,q,\mathcal{O}_n}^{(1)}\right\rangle \\ &+\frac{1}{2}\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\gamma_n^{(1)}\lambda_{q,q,\mathcal{O}_n}^{(1)} + \lambda_{p,p,\mathcal{O}_n}^{(1)}\gamma_n^{(1)}\lambda_{q,q,\mathcal{O}_n}^{(0)} + \lambda_{p,p,\mathcal{O}_n}^{(0)}\gamma_n^{(2)}\lambda_{q,q,\mathcal{O}_n}^{(0)}\right\rangle(\log U + 2\partial_\Delta) \\ &+\frac{1}{8}\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\left(\gamma_n^{(1)}\right)^2\lambda_{q,q,\mathcal{O}_n}^{(0)}\right\rangle\left(\log^2 U + 4\log U\partial_\Delta + 4\partial_\Delta^2\right)\Bigg) \\ &+\mathcal{O}\left(\frac{1}{N^3}\right)\Bigg] g_{\Delta+2,0}(U,V)\Big|_{\Delta=2n} \\ &+\cdots\end{aligned}$$

$$\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\gamma_n^{(1)}\lambda_{q,q,\mathcal{O}_n}^{(0)}\right\rangle = \sum_{a=1}^{n}\lambda_{p,p,\mathcal{O}_{n,a}}^{(0)}\gamma_{n,a}^{(1)}\lambda_{q,q,\mathcal{O}_{n,a}}^{(0)}$$

$$\begin{aligned}\left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\lambda_{q,q,\mathcal{O}_n}^{(0)}\right\rangle &= a_n^p a_n^q \delta_{p,q} \\ \left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\gamma^{(1)}\lambda_{q,q,\mathcal{O}_n}^{(0)}\right\rangle &= a_n^p a_n^q b_n^{p,q} \\ \left\langle \lambda_{p,p,\mathcal{O}_n}^{(0)}\left(\gamma^{(1)}\right)^2\lambda_{q,q,\mathcal{O}_n}^{(0)}\right\rangle &= a_n^p a_n^q\left(\sum_{s=2}^{n} b_n^{p,s}b_n^{q,s} + c_n^p c_n^q\right).\end{aligned}$$

$$\begin{aligned}&\begin{pmatrix}\langle \rho_{2n}(x)\rho_{2n}(y)\rangle & \langle \rho_{2n}(x)\colon \phi_q\,\square^{n-q}\,\phi_q\colon(y)\rangle \\ \langle \colon \phi_p\,\square^{n-p}\,\phi_p\colon(x)\rho_{2n}(y)\rangle & \langle \colon \phi_p\,\square^{n-p}\,\phi_p\colon(x)\colon \phi_q\,\square^{n-q}\,\phi_q\colon(y)\rangle\end{pmatrix} \\ &= \frac{1}{|x-y|^{4n}}\left[\delta_{ab} - \frac{1}{N}(M_n)_{ab}\log\left(|x-y|^2\right) + \mathcal{O}\left(\frac{1}{N^2}\right)\right]\end{aligned}$$

$$M_n = \begin{pmatrix} d_n & c_n^2 & c_n^3 & \dots & c_n^n \\ c_n^2 & b_n^{2,2} & b_n^{2,3} & \dots & b_n^{2,n} \\ c_n^3 & b_n^{3,2} & b_n^{3,3} & \dots & b_n^{3,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_n^n & b_n^{n,2} & b_n^{n,3} & \dots & b_n^{n,n}\end{pmatrix}$$

$$d_n = 0$$

$$\left\langle \gamma_n^{(1)} \right\rangle = \sum_{p=2}^{n} \frac{\left\langle \lambda^{(0)}_{p,p,\mathcal{O}_n} \gamma_n^{(1)} \lambda^{(0)}_{p,p,\mathcal{O}_n} \right\rangle}{\left\langle \left(\lambda^{(0)}_{p,p,\mathcal{O}_n} \right)^2 \right\rangle}$$

$$\gamma^{(1)}_{n,a} = \mu_{n,a}$$
$$\lambda^{(0)}_{p,p,\mathcal{O}_{n,a}} = a_n^p \left(v_{n,a}\right)^p$$

$$\left\langle \lambda^{(0)}_{p,p,\mathcal{O}_n} \lambda^{(0)}_{p,p,\mathcal{O}_n} \right\rangle = \left(a_n^p\right)^2 = \frac{2(2n-1)\Gamma(n)^4}{(n^2-1)\Gamma(2n)^2} \frac{\Gamma(n+p-1)\Gamma(n+p)}{\Gamma(p)^4 \Gamma(n-p+2)\Gamma(n-p+1)}.$$

$$\left\langle \lambda^{(0)}_{2,2,\mathcal{O}_n} \gamma^{(1)} \lambda^{(0)}_{q,q,\mathcal{O}_n} \right\rangle = a_n^2 a_n^q b_n^{2,q} = -8 \frac{h^\vee}{(\mathbf{k}/N)} \frac{(-1)^q (q-1) n (2n-1) \Gamma(n)^4 \Gamma(n+q-1)}{\Gamma(2n)^2 \Gamma(q)^2 \Gamma(n-q+1)}$$
$$= \frac{24(1-\Delta)}{\Delta} \frac{(-1)^q (q-1) n (2n-1) \Gamma(n)^4 \Gamma(n+q-1)}{\Gamma(2n)^2 \Gamma(q)^2 \Gamma(n-q+1)}$$

$$a_2^2 = \sqrt{\frac{2}{3}}$$

$$\left\langle \lambda^{(0)}_{2,2,\mathcal{O}_2} \gamma^{(1)} \lambda^{(0)}_{2,2,\mathcal{O}_2} \right\rangle = (a_2^2)^2 b_2^{2,2} = 8 \frac{(1-\boldsymbol{\Delta})}{\boldsymbol{\Delta}}.$$

$$\left\langle \lambda^{(0)}_{2,2,\mathcal{O}_2} \left(\gamma^{(1)}\right)^2 \lambda^{(0)}_{2,2,\mathcal{O}_2} \right\rangle_{\text{grav}} = (a_2^2)^2 (c_2^2)^2 = \frac{16}{5} \frac{\dim(G_F)}{\Delta} = \frac{32}{5} \frac{(6-5\Delta)(5-6\Delta)}{\Delta^2},$$

$$a_3^2 = \sqrt{\frac{1}{10}}, a_3^3 = \sqrt{\frac{1}{4}}$$

$$\left\langle \lambda^{(0)}_{2,2,\mathcal{O}_3} \gamma^{(1)} \lambda^{(0)}_{2,2,\mathcal{O}_3} \right\rangle = (a_3^2)^2 b_3^{2,2} = \frac{12}{5} \frac{(1-\boldsymbol{\Delta})}{\boldsymbol{\Delta}}$$
$$\left\langle \lambda^{(0)}_{2,2,\mathcal{O}_3} \gamma^{(1)} \lambda^{(0)}_{3,3,\mathcal{O}_3} \right\rangle = a_3^2 a_3^3 b_3^{2,3} = -\frac{24}{5} \frac{(1-\boldsymbol{\Delta})}{\boldsymbol{\Delta}}.$$

$$\left\langle \lambda^{(0)}_{3,3,\mathcal{O}_3} \gamma^{(1)} \lambda^{(0)}_{3,3,\mathcal{O}_3} \right\rangle = (a_3^3)^2 b_3^{3,3} = \frac{66}{5} \frac{(1-\Delta)}{\Delta}.$$

$$\left\langle \lambda^{(0)}_{2,2,\mathcal{O}_3} \left(\gamma^{(1)}\right)^2 \lambda^{(0)}_{2,2,\mathcal{O}_3} \right\rangle_{\text{grav}} = (a_3^2)^2 (c_3^2)^2 = \frac{864}{175} \frac{\dim(G_F)}{\boldsymbol{\Delta}} = \frac{1728}{175} \frac{(6-5\boldsymbol{\Delta})(5-6\boldsymbol{\Delta})}{\boldsymbol{\Delta}^2},$$
$$\left\langle \lambda^{(0)}_{2,2,\mathcal{O}_3} \left(\gamma^{(1)}\right)^2 \lambda^{(0)}_{3,3,\mathcal{O}_3} \right\rangle_{\text{grav}} = a_3^2 a_3^3 c_3^2 c_3^3 = -\frac{864}{35} \frac{\dim(G_F)}{\boldsymbol{\Delta}} = -\frac{1728}{35} \frac{(6-5\boldsymbol{\Delta})(5-6\boldsymbol{\Delta})}{\boldsymbol{\Delta}^2}.$$

$$M_3=\begin{pmatrix} 0 & \frac{24}{\Delta}\sqrt{\frac{6}{35}}\sqrt{(6-5\Delta)(5-6\Delta)} & -\frac{48}{\Delta}\sqrt{\frac{3}{7}}\sqrt{(6-5\Delta)(5-6\Delta)} \\ \frac{24}{\Delta}\sqrt{\frac{6}{35}}\sqrt{(6-5\Delta)(5-6\Delta)} & 24\frac{(1-\Delta)}{\Delta} & -48\sqrt{\frac{2}{5}}\frac{(1-\Delta)}{\Delta} \\ -\frac{48}{\Delta}\sqrt{\frac{3}{7}}\sqrt{(6-5\Delta)(5-6\Delta)} & -48\sqrt{\frac{2}{5}}\frac{(1-\Delta)}{\Delta} & \frac{264}{5}\frac{(1-\Delta)}{\Delta} \end{pmatrix}$$

Supergravedad cuántica relativista Tipo IIA: Modelamiento Matemático.

$$\mathbf{S}_0=-\frac{2}{\kappa_{11}^2}\left[\int d^{11}x\sqrt{-g}\left(R-\frac{1}{2.4!}F_{abcd}F^{abcd}\right)-\frac{1}{6}\int A^{(3)}\wedge F^{(4)}\wedge F^{(4)}\right],$$

$$g_{ab}=e^{-2\Phi/3}\begin{pmatrix} G_{\mu\nu}+e^{2\Phi}C_\mu C_\nu & e^{2\Phi}C_\mu \\ e^{2\Phi}C_\nu & e^{2\Phi}\end{pmatrix};\ g^{ab}=e^{2\Phi/3}\begin{pmatrix} G^{\mu\nu} & -C^\mu \\ -C^\nu & e^{-2\Phi}+C_\alpha C^\alpha\end{pmatrix}$$

$$A_{\mu\nu\alpha}=C_{\mu\nu\alpha};A_{\mu\nu y}=B_{\mu\nu}$$

$$\sqrt{-g}=e^{-8\Phi/3}\sqrt{-G}$$

$$R=e^{2\Phi/3}\left(R-\frac{16}{3}\nabla_\mu\Phi\nabla^\mu\Phi+\frac{14}{3}\nabla_\mu\nabla^\mu\Phi-\frac{1}{2.2!}e^{2\Phi}F_{\mu\nu}F^{\mu\nu}\right)$$

$$\sqrt{-g}R=e^{-2\Phi}\sqrt{-G}\left(R+4\nabla_\mu\Phi\nabla^\mu\Phi-\frac{1}{2.2!}e^{2\Phi}F_{\mu\nu}F^{\mu\nu}\right)$$

$$-\frac{1}{2.4!}\sqrt{-g}F_{abcd}F^{abcd}=e^{-2\Phi}\sqrt{-G}\left(-\frac{1}{2.3!}H_{\mu\nu\alpha}H^{\mu\nu\alpha}-\frac{1}{2.4!}e^{2\Phi}\bar{F}_{\mu\nu\alpha\beta}\bar{F}^{\mu\nu\alpha\beta}\right)$$

$$-\frac{1}{6}A^{(3)}\wedge F^{(4)}\wedge F^{(4)}=-\frac{1}{2}e^{-2\Phi}\left[B^{(2)}\wedge e^{\Phi}F^{(4)}\wedge e^{\Phi}F^{(4)}\right]$$

$$S_0=-\frac{2}{\kappa_{10}^2}\left[\int d^{10}x\sqrt{-G}\left(e^{-2\Phi}\left(R+4\nabla_\mu\Phi\nabla^\mu\Phi-\frac{1}{2}|H|^2\right)-\frac{1}{2}\left|F^{(2)}\right|^2-\frac{1}{2}\left|\bar{F}^{(4)}\right|^2\right)-\frac{1}{2}\int B^{(2)}\wedge F^{(4)}\wedge F^{(4)}\right]$$

$$\mathbf{S}_6=-\frac{2}{\kappa_{11}^2}\frac{\pi^2\ell_p^6}{2^{11}\cdot 3^2}\int d^{11}x\sqrt{-g}\left[\left(t_8t_8-\frac{1}{4!}\epsilon_{11}\epsilon_{11}\right)R^4+\cdots\right]$$

$$\begin{aligned} t_8^{abcdefgh}=&-2g^{af}g^{be}g^{gd}g^{ch}+8g^{ad}g^{be}g^{gf}g^{ch}+8g^{ah}g^{be}g^{gd}g^{cf}\\ &+8g^{ah}g^{bc}g^{gf}g^{de}-2g^{ah}g^{bg}g^{cf}g^{de}-2g^{ad}g^{bc}g^{gf}g^{eh}\end{aligned}$$

$$\begin{aligned}\mathbf{S}_6=-\frac{2}{\kappa_{11}^2}\frac{\pi^2\ell_p^6}{2^3.3}\int d^{11}x\sqrt{-g}\Big[&\frac{1}{4}R_{ab}{}^{de}R^{abcf}R_{cd}{}^{gh}R_{fegh}-R_{ab}{}^{de}R^{abcf}R_c{}^g{}_d{}^hR_{fgeh}\\ &+\frac{1}{16}R_{ab}{}^{de}R^{abcf}R_{cf}{}^{gh}R_{degh}+\frac{1}{2}R_a{}^d{}_c{}^eR^{abcf}R_b{}^g{}_f{}^hR_{dgeh}\\ &-R_{abc}{}^dR^{abcf}R_f{}^{egh}R_{dgeh}+\frac{1}{32}R_{abcf}R^{abcf}R_{degh}R^{degh}\Big]\end{aligned}$$

$$S_6 = -\frac{2}{\kappa^2}\frac{\pi^2\ell_s^6}{2^3.3}\int d^{10}x\sqrt{-G}\left[\frac{1}{4}R_{\alpha\beta}{}^{\epsilon\varepsilon}R^{\alpha\beta\gamma\delta}R_{\gamma\epsilon}{}^{\mu\nu}R_{\delta\varepsilon\mu\nu} - R_{\alpha\beta}{}^{\epsilon\varepsilon}R^{\alpha\beta\gamma\delta}R_{\gamma}{}^{\mu}{}_{\epsilon}{}^{\nu}R_{\delta\mu\varepsilon\nu}\right.$$
$$+\frac{1}{16}R_{\alpha\beta}{}^{\epsilon\varepsilon}R^{\alpha\beta\gamma\delta}R_{\gamma\delta}{}^{\mu\nu}R_{\epsilon\varepsilon\mu\nu} + \frac{1}{2}R_{\alpha}{}^{\epsilon}{}_{\gamma}{}^{\varepsilon}R^{\alpha\beta\gamma\delta}R_{\beta}{}^{\mu}{}_{\delta}{}^{\nu}R_{\epsilon\mu\varepsilon\nu}$$
$$\left.-R_{\alpha\beta\gamma}{}^{\epsilon}R^{\alpha\beta\gamma\delta}R_{\delta}{}^{\varepsilon\mu\nu}R_{\epsilon\mu\varepsilon\nu} + \frac{1}{32}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}R_{\epsilon\varepsilon\mu\nu}R^{\epsilon\varepsilon\mu\nu} + \cdots\right].$$

$$S_6 \sim -\frac{2}{\kappa^2}\frac{\pi^2\ell_s^6}{2^3\cdot 3}\int d^{10}x\sqrt{-G}\mathcal{L}$$

$c_1 = 2341/4608, c_2 = -209747/921600, c_3 = -1027/7200, c_4 = -880957/1843200$
$c_5 = 4625129/29491200, c_6 = -66793/28800, c_7 = -817/960, c_8 = 385121/230400$
$c_9 = -3433/46080, c_{10} = 194693/368640, c_{11} = 24653/19200, c_{12} = -979/1600$
$c_{13} = 4307/3840, c_{14} = 2979/1600, c_{15} = -43241/9600, c_{16} = 3351/3200, c_{17} = -1151/800$
$c_{18} = -30283/4800, c_{19} = 4, c_{20} = -7167/3200, c_{21} = -7/2, c_{22} = -3/2, c_{23} = 1/2$
$c_{24} = 89527/115200, c_{25} = 9721/115200, c_{26} = 1, c_{27} = 1027/640, c_{28} = -3849/3200$
$c_{29} = -5873/3840, c_{30} = -75041/38400, c_{31} = 17/128, c_{32} = 16717/20480, c_{33} = 3$
$c_{34} = 1/2, c_{35} = 7167/6400, c_{36} = -13/2, c_{37} = -3/4, c_{38} = -3/8, c_{39} = 0, c_{40} = -1$
$c_{41} = -1, c_{42} = 1/32, c_{43} = 1119/6400, c_{44} = 1693/3840, c_{45} = -4451/12800$
$c_{46} = -647/3200, c_{47} = -4193/7680, c_{48} = -3/2, c_{49} = 11/4, c_{50} = 11/2, c_{51} = -7/8$
$c_{52} = 3/2, c_{53} = -5/4, c_{54} = 1/16, c_{55} = 1/4, c_{56} = 0, c_{57} = 3391/11520$
$c_{58} = -173339/115200, c_{59} = 3449/800, c_{60} = -5/2, c_{61} = 73027/115200$
$c_{62} = 43031/57600, c_{63} = -267211/115200, c_{64} = -29399/230400, c_{65} = 246391/921600$
$c_{66} = -6197/3200, c_{67} = 1559/9600, c_{68} = 2159/9600, c_{69} = -8857/12800, c_{70} = 3/4$
$c_{71} = -1/4, c_{72} = 5/4, c_{73} = 14741/800, c_{74} = 19219/1600, c_{75} = 41/4, c_{76} = -6767/600$
$c_{77} = 6767/1200, c_{78} = -5497/400, c_{79} = -1217/400, c_{80} = 14781/3200, c_{81} = -57/16$
$c_{82} = -45/8, c_{83} = -186973/28800, c_{84} = 144301/115200, c_{85} = 58361/7680$
$c_{86} = -1711343/921600, c_{87} = -16457/1600, c_{88} = 23339/1600, c_{89} = -2259/1600$
$c_{90} = 12251/800, c_{91} = 181/128, c_{92} = -5167/400, c_{93} = 5, c_{94} = 3567/400, c_{95} = 3, c_{96} = 2$

$c_{97} = 6367/200, c_{98} = 14377/1600, c_{99} = -681/128, c_{100} = -1, c_{101} = 1, c_{102} = -145/64,$
$c_{103} = -5471/640, c_{104} = -973/1600, c_{105} = -5667/400, c_{106} = 3/2, c_{107} = 6367/400,$
$c_{108} = -6167/3200, c_{109} = 4367/400, c_{110} = -3/4, c_{111} = 1, c_{112} = -6367/200,$
$c_{113} = 141/100, c_{114} = -2761/800, c_{115} = -131/10, c_{116} = 9/2, c_{117} = 8/5,$
$c_{118} = -125353/230400, c_{119} = 293867/1843200, c_{120} = -22253/19200, c_{121} = 5/16,$
$c_{122} = 37/1152, c_{123} = 21661/57600, c_{124} = -5213/9600, c_{125} = -3827/1920, c_{126} = 1,$
$c_{127} = 1129/800, c_{128} = -10889/14400, c_{129} = -849/3200, c_{130} = 1049/400,$
$c_{131} = -2879/800, c_{132} = 2929/800, c_{133} = -7/2, c_{134} = -11/16, c_{135} = -3587/1920,$
$c_{136} = 89/40, c_{137} = 2, c_{138} = 2081/800, c_{139} = -13/8, c_{140} = -1/5, c_{141} = -357/200,$
$c_{142} = 0, c_{143} = -1/4, c_{144} = -5/16, c_{145} = -3/4, c_{146} = -1/2, c_{147} = -5, c_{148} = 0,$
$c_{149} = 3, c_{150} = 1, c_{151} = 0, c_{152} = 3/2, c_{153} = 3/4, c_{154} = 0, c_{155} = 1/4, c_{156} = 0, c_{157} = 0,$
$c_{158} = -14331/6400, c_{159} = -26579/800, c_{160} = 5647/160, c_{161} = -1501/60,$
$c_{162} = 7041/100, c_{163} = -563/60, c_{164} = -2927/200, c_{165} = 125/4, c_{166} = 2127/200,$
$c_{167} = -61/2, c_{168} = 85/2, c_{169} = 6927/100, c_{170} = 2271/80, c_{171} = -1543/80, c_{172} = 0,$
$c_{173} = -77/4, c_{174} = -1531/80, c_{175} = -1255/64, c_{176} = 319/15, c_{177} = -3727/200,$
$c_{178} = -49/8, c_{179} = 6927/200, c_{180} = -4527/1600, c_{181} = 2427/200, c_{182} = 61/8, c_{183} = 2,$
$c_{184} = -6927/100, c_{185} = -552/5, c_{186} = 0, c_{187} = 65, c_{188} = 4/5, c_{189} = -727/16,$
$c_{190} = -52291/3200, c_{191} = 467/20, c_{192} = -3/10, c_{193} = -1/5, c_{194} = 342/5,$
$c_{195} = -1/16, c_{196} = 3/8, c_{197} = 4, c_{198} = -1/4, c_{199} = -39/5, c_{200} = 553/200,$
$c_{201} = -13/5, c_{202} = 0, c_{203} = -27/16, c_{204} = -31/4, c_{205} = -5/2, c_{206} = -1779/25,$
$c_{207} = -26781/800, c_{208} = 161/4, c_{209} = 173/8, c_{210} = -4559/100, c_{211} = -147/2,$
$c_{212} = 129/2, c_{213} = -29, c_{214} = -2599/50, c_{215} = 353/40, c_{216} = -23/2,$
$c_{217} = -811/50, c_{218} = 294/5, c_{219} = -979/50, c_{220} = -17383/200, c_{221} = -107,$
$c_{222} = 157/2, c_{223} = 8/5, c_{224} = 53/10, c_{225} = -392/5, c_{226} = -499/100, c_{227} = 499/100,$
$c_{228} = 557/100, c_{229} = -657/400, c_{230} = 1573/800, c_{231} = 5/2, c_{232} = -5/2,$
$c_{233} = -121/5, c_{234} = -8/5, c_{235} = 8/5, c_{236} = -1/2, c_{237} = 13/4, c_{238} = 4/5, c_{239} = 133/5,$
$c_{240} = -197/80, c_{241} = 2953/400, c_{242} = -512/5, c_{243} = 106, c_{244} = 168/5, c_{245} = -38,$
$c_{246} = 19, c_{247} = 84/5, c_{248} = -342/5, c_{249} = -128/5, c_{250} = 5/4, c_{251} = 63/20, c_{252} = 15/4,$
$c_{253} = 5/4, c_{254} = 0, c_{255} = -5/2, c_{256} = 71/40, c_{257} = -261/200, c_{258} = 252/5,$
$c_{259} = -41/100, c_{260} = 39/160, c_{261} = -8/5, c_{262} = -32/5, c_{263} = 4/5, c_{264} = 16,$
$c_{265} = 1071/200, c_{266} = -10, c_{267} = 807/20, c_{268} = 617/10, c_{269} = -103/5, c_{270} = -7,$
$c_{271} = 13, c_{272} = -316/5, c_{273} = -264/5, c_{274} = 15029/14400, c_{275} = -6749/14400,$
$c_{276} = 3389/9600, c_{277} = -1177/2304, c_{278} = 967/3840, c_{279} = -1667/960, c_{280} = 2387/960,$
$c_{281} = -389/320, c_{282} = 1/4, c_{283} = -3/2, c_{284} = -3827/960, c_{285} = 5/2, c_{286} = 17/32,$
$c_{287} = 3587/1920, c_{288} = -5/4, c_{289} = 1/4, c_{290} = -119/40, c_{291} = 3, c_{292} = 0,$

$c_{293} = -11/10, c_{294} = -57/20, c_{295} = 5, c_{296} = -31, c_{297} = -9/10, c_{298} = 1/4, c_{299} = -4,$
$c_{300} = 1403/1600, c_{301} = 6449/28800, c_{302} = -5769/3200, c_{303} = 343/128, c_{304} = -1327/1920,$
$c_{305} = 1427/1920, c_{306} = 1427/1920, c_{307} = -11/16, c_{308} = 2987/960, c_{309} = -3827/960,$
$c_{310} = -3, c_{311} = 3347/1920, c_{312} = -5/2, c_{313} = -1/3, c_{314} = 2827/960, c_{315} = 3667/960,$
$c_{316} = 6, c_{317} = -8, c_{318} = -67/40, c_{319} = 2, c_{320} = -3, c_{321} = -5/4, c_{322} = -283/5,$
$c_{323} = -1/128, c_{324} = -41/40, c_{325} = -13/40, c_{326} = 235/4, c_{327} = 85, c_{328} = 112/5,$
$c_{329} = -2168/5, c_{330} = -3844/5, c_{331} = -502, c_{332} = -4, c_{333} = 4/15, c_{334} = 0, c_{335} = -32,$
$c_{336} = -333/400, c_{337} = 0, c_{338} = 84, c_{339} = -368/5, c_{340} = 102, c_{341} = -112/5, c_{342} = 0,$
$c_{343} = 6, c_{344} = 2, c_{345} = 11/20, c_{346} = -2, c_{347} = 2, c_{348} = 2, c_{349} = -10, c_{350} = 5, c_{351} = -1,$
$c_{352} = 4, c_{353} = -2, c_{354} = 0, c_{355} = -3/2, c_{356} = 7/4, c_{357} = 6, c_{358} = -7/2, c_{359} = -5/4,$
$c_{360} = -5/2, c_{361} = -5/2, c_{362} = 5/4, c_{363} = 7/2, c_{364} = -3/4, c_{365} = 7, c_{366} = -9/2,$
$c_{367} = 4, c_{368} = 2, c_{369} = -9/2, c_{370} = -1/2, c_{371} = 639/200, c_{372} = -4/5, c_{373} = -3,$
$c_{374} = 4, c_{375} = 1/16, c_{376} = 3/32, c_{377} = 0$

$$[\Phi''^4]_2 = -\frac{8}{5}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\nabla^\beta\Phi\nabla_\beta\nabla^\delta\Phi\nabla_\delta\nabla_\gamma\Phi + \frac{4}{5}\nabla^\alpha\nabla^\beta\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\gamma\nabla^\delta\Phi\nabla_\delta\nabla_\gamma\Phi.$$

$$\begin{aligned}S_{hhCC} =& -\frac{2}{\kappa^2}\frac{\pi^2\ell_s^6}{2^6.3}\int d^{10}x\sqrt{-G}\Big[\frac{1}{4}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\partial_\gamma F_{\alpha\beta}\partial^\gamma F^{\alpha\beta} + \frac{1}{2}R^{\alpha\beta}_{\tau\kappa}R^{\tau\kappa\mu\nu}\partial_\gamma F_{\alpha\beta}\partial^\gamma F_{\mu\nu}\\ &-R^{\mu\nu}{}_\tau{}^\gamma R^{\tau\kappa\alpha\beta}\partial_\gamma F_{\mu\nu}\partial_\kappa F_{\alpha\beta} + 2R^{\beta\mu}{}_{\kappa\rho}R^{\tau\kappa}{}_{\alpha\beta}\partial^\alpha F^{\rho\gamma}\partial_\mu F_{\tau\gamma} + 2R_{\beta\mu\kappa\rho}R^{\tau\kappa\alpha\beta}\partial_\alpha F_{\tau\gamma}\partial^\mu F^{\rho\gamma}\\ &-4R_\alpha{}^{\nu\rho\gamma}R^{\tau\kappa\alpha\beta}\partial_\beta F_{\kappa\rho}\partial_\nu F_{\tau\gamma} + 2R^{\alpha\beta\kappa\rho}R^\tau{}_{\kappa\alpha\beta}\partial_\nu F_{\tau\gamma}\partial^\nu F_\rho{}^\gamma + R^{\alpha\beta\rho\gamma}R^{\tau\kappa}{}_{\alpha\beta}\partial_\nu F_{\kappa\rho}\partial^\nu F_{\tau\gamma}\\ &+2R_\beta{}^\mu{}_{\kappa\rho}R^{\tau\kappa\alpha\beta}\partial^\rho F_\mu{}^\nu\partial_\tau F_{\alpha\nu} + R^{\mu\nu}{}_\kappa{}^\rho R^{\tau\kappa\alpha\beta}\partial_\rho F_{\alpha\beta}\partial_\tau F_{\mu\nu} + R^{\alpha\beta\kappa}{}_\rho R^\tau{}_{\kappa\alpha\beta}\partial^\rho F^{\mu\nu}\partial_\tau F_{\mu\nu}\\ &+2R^{\beta\mu\kappa\rho}R_{\tau\kappa}{}^\alpha{}_\beta\partial_\rho F_{\alpha\nu}\partial^\tau F_\mu{}^\nu\Big]\end{aligned}$$

$$S_{R^2F'^2} = -\frac{2}{\kappa^2}\frac{\pi^2\ell_s^6}{2^3.3}\int d^{10}x\sqrt{-G}[R^2F^2]_{13}$$

$$\begin{aligned}[R^2F^2]_{13} =& -\frac{5}{2}e^{2\Phi}R_\gamma{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma} + \frac{3}{4}e^{2\Phi}R_{\varepsilon\mu\delta\epsilon}R^{\varepsilon\mu}{}_{\beta\gamma}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\\ &-\frac{1}{4}e^{2\Phi}R_{\varepsilon\mu\gamma\epsilon}R^{\varepsilon\mu}{}_{\beta\delta}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + \frac{5}{16}e^{2\Phi}R^{\delta\epsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma}\\ &+e^{2\Phi}R_\gamma{}^{\epsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta - \frac{7}{2}e^{2\Phi}R_\delta{}^\varepsilon{}_\alpha{}^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}\\ &-\frac{3}{2}e^{2\Phi}R_\alpha{}^\varepsilon{}_\delta{}^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + \frac{5}{2}e^{2\Phi}R_\gamma{}^\varepsilon{}_\alpha{}^\mu R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon\\ &-\frac{5}{4}e^{2\Phi}R_{\varepsilon\mu\gamma\delta}R^{\varepsilon\mu}{}_{\alpha\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon - 3e^{2\Phi}R_\beta{}^\mu{}_{\alpha\gamma}R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}\\ &-\frac{5}{2}e^{2\Phi}R_\alpha{}^\mu{}_{\delta\epsilon}R_{\varepsilon\mu\beta\gamma}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon} + 6e^{2\Phi}R_{\beta\epsilon\alpha}{}^\mu R_{\varepsilon\mu\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}\\ &-8e^{2\Phi}R_\beta{}^\mu{}_{\alpha\epsilon}R_{\varepsilon\mu\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}\end{aligned}$$

$$\begin{aligned}S_{h\Phi CC} = -\frac{2}{\kappa^2}\frac{\pi^2\ell_s^6}{2^6.3}\int d^{10}x\sqrt{-G}\big[&-8R_{\beta\mu\kappa\nu}\partial^\beta\partial^\alpha\Phi\partial_\gamma F^{\mu\nu}\partial^\kappa F_\alpha^\gamma + 24R_{\beta\mu\gamma\nu}\partial^\beta\partial^\alpha\Phi\partial_\kappa F^{\mu\nu}\partial^\kappa F_\alpha^\gamma\\ &-8R_{\alpha\kappa\beta\nu}\partial^\beta\partial^\alpha\Phi\partial_\gamma F_\mu^\nu\partial^\mu F^{\gamma\kappa} + 8R_{\beta\mu\gamma\nu}\partial^\beta\partial^\alpha\Phi\partial^\kappa F_\alpha^\gamma\partial^\nu F_\kappa^\mu\big]\end{aligned}$$

$$S_{RF'^2\Phi''} = -\frac{2}{\kappa^2}\frac{\pi^2\ell_s^6}{2^3.3}\int d^{10}x\sqrt{-G}[RF^2\Phi'']_8$$

$$\begin{aligned}[RF'^2\Phi'']_8 =& -10e^{2\Phi}R_{\epsilon\varepsilon\alpha\gamma}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}\nabla_\delta\nabla^\varepsilon\Phi + 3e^{2\Phi}R_{\delta\varepsilon\gamma\epsilon}\nabla_\alpha\nabla^\varepsilon\Phi\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon\\ &+5e^{2\Phi}R_{\epsilon\varepsilon\alpha\delta}\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla^\varepsilon\Phi\nabla^\delta F_\beta{}^\epsilon + 2e^{2\Phi}R_{\epsilon\varepsilon\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla_\beta\nabla_\alpha\Phi\nabla^\delta F^{\epsilon\varepsilon}\\ &+2e^{2\Phi}R_{\delta\varepsilon\beta\gamma}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}\nabla_\epsilon\nabla_\alpha\Phi - 2e^{2\Phi}R_{\gamma\varepsilon\alpha\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}\nabla_\epsilon\nabla_\beta\Phi\\ &-3e^{2\Phi}R_{\gamma\epsilon\delta\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla^\epsilon\nabla^\varepsilon\Phi + 4e^{2\Phi}R_{\delta\epsilon\alpha\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\gamma{}^\delta\nabla^\epsilon\nabla^\varepsilon\Phi.\end{aligned}$$

$$S_{\Phi\Phi CC} = -\frac{2}{\kappa^2}\frac{\pi^2\ell_s^6}{2^6.3}\int d^{10}x\sqrt{-G}\big[8\partial^\theta\partial^\eta\Phi\partial^\kappa\partial_\eta\Phi\partial_\nu F_{\kappa\mu}\partial^\nu F_\theta^\mu - 2\partial_\theta\partial_\eta\Phi\partial^\theta\partial^\eta\Phi\partial_\nu F_{\kappa\mu}\partial^\nu F^{\kappa\mu}\big]$$

$$S_{RF'^2\Phi''} = -\frac{2}{\kappa^2}\frac{\pi^2\ell_s^6}{2^3\cdot 3}\int d^{10}x\sqrt{-G}[F^2\Phi''^2]_6$$

$$\begin{aligned}[F'^2\Phi''^2]_6 =& 2e^{2\Phi}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\beta\Phi\nabla_\epsilon\nabla_\gamma\Phi + 2e^{2\Phi}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\alpha\Phi\nabla^\delta F_\beta{}^\epsilon\nabla_\epsilon\nabla_\gamma\Phi\\ &+5e^{2\Phi}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla^\epsilon\Phi\nabla_\epsilon\nabla_\delta\Phi - e^{2\Phi}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla_\gamma\nabla^\epsilon\Phi\nabla_\epsilon\nabla_\delta\Phi\\ &-2e^{2\Phi}\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla_\alpha\Phi\nabla^\delta F_\beta{}^\epsilon\nabla_\epsilon\nabla_\delta\Phi - \frac{3}{2}e^{2\Phi}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma}\nabla^\delta\nabla^\epsilon\Phi\nabla_\epsilon\nabla_\delta\Phi.\end{aligned}$$

$$ds^2 = G_{\mu\nu}(x)dx^\mu dx^\nu + g_{ab}(y)dy^a dy^b$$

$$\mathbf{S}^{(0)} = -\frac{2V}{\kappa_{10}^2}\int d^6x\sqrt{-G}e^{-2\Phi}\left[R + 4\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{4}e^{2\Phi}F_{\mu\nu}F^{\mu\nu}\right]$$

$$ds'^2 = G'_{MN}(x)dx^M dx^N + g'_{ij}(z)dz^i dz^j$$

$$G'_{MN} = \begin{pmatrix} G'_{\mu\nu} + R_1^{-2}C'_\mu C'_\nu & C'_\mu \\ C'_\nu & R_1^2 \end{pmatrix}$$

$$\mathbf{S}'^{(0)} = -\frac{2V'}{\kappa_{10}'^2}\int d^6x\sqrt{-G'}e^{-2\Phi'}\left[R' + 4\nabla_\mu\Phi'\nabla^\mu\Phi' - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu}\right],$$

$$\kappa_{10}^2/V = \kappa_{10}'^2/V'.$$

$$G'_{\mu\nu} = e^{-2\Phi}G_{\mu\nu}, \Phi' = -\Phi, C'_\mu = C_\mu,$$

$$\frac{1}{32\pi^2}\int_{K3} d^4y\sqrt{g}R_{abcd}R^{abcd}$$

$$S_{R^2} = -\frac{2}{\kappa^2}\frac{4\pi^4\ell_s^6}{3}\int d^6x\sqrt{-G}\left[\frac{3}{2}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right].$$

$$S'_{R^2} = -\frac{2}{\kappa_{10}'^2}\frac{\ell_s'^2}{8}\int d^{10}x\sqrt{-G'}e^{-2\Phi'}R'_{\mu\nu\alpha\beta}R'^{\mu\nu\alpha\beta}.$$

$$S'_{R^2} = -\frac{2}{\kappa_{10}'^2}\frac{\ell_s'^2V'}{8}\int d^6x\sqrt{-G'}e^{-2\Phi'}R'_{\mu\nu\alpha\beta}R'^{\mu\nu\alpha\beta}$$

$$\frac{2\pi V'}{(2\pi\ell'_s)^6 g_s'^2} = \frac{1}{2\pi\ell_s^2}$$

$$\begin{aligned} S_{6D} =& -\frac{2}{\kappa^2}\frac{4\pi^4\ell_s^6}{3}\int d^6x\sqrt{-G}\left[\frac{3}{2}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} + \frac{51}{16}e^{4\Phi}F_\alpha^\gamma F^{\alpha\beta}F_\beta^\delta F_{\gamma\delta} + \frac{50151}{2560}e^{4\Phi}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}\right. \\ & -3e^{2\Phi}F^{\alpha\beta}F^{\gamma\delta}R_{\alpha\beta\gamma\delta} - 3e^{2\Phi}F^{\alpha\beta}F^{\gamma\delta}R_{\alpha\gamma\beta\delta} + \frac{44343}{400}e^{2\Phi}F_{\beta\gamma}F^{\beta\gamma}\nabla_\alpha\Phi\nabla^\alpha\Phi \\ & -\frac{13581}{200}e^{2\Phi}F_\alpha^\gamma F_{\beta\gamma}\nabla^\alpha\Phi\nabla^\beta\Phi - \frac{36}{5}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi - \frac{24}{5}\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\Phi \\ & -\frac{24}{5}\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\nabla^\alpha\Phi + \frac{15}{2}e^{2\Phi}F^{\beta\gamma}\nabla^\alpha\Phi\nabla_\gamma F_{\alpha\beta} - \frac{18501}{400}e^{2\Phi}F_\alpha^\gamma F^{\alpha\beta}\nabla_\gamma\nabla_\beta\Phi \\ & \left.+\frac{15}{2}e^{2\Phi}\nabla_\beta F_{\alpha\gamma}\nabla^\gamma F^{\alpha\beta}\right]. \end{aligned}$$

$$S_{6D} = -\frac{2}{\kappa^2}\frac{4\pi^4\ell_s^6}{3}\int d^6x\sqrt{-G}\left[\frac{3}{2}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} + a_1 e^{4\Phi}F_\alpha^\gamma F^{\alpha\beta}F_\beta^\delta F_{\gamma\delta} + a_2 e^{4\Phi}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}\right.$$
$$+\left(-\frac{9}{8} - 2a_1 + 2a_{19} - \frac{1}{2}a_3\right)e^{2\Phi}F_\alpha^\gamma F^{\alpha\beta}R_{\beta\gamma} + \frac{1}{4800}(-65403 + 800a_1 + 1800a_{10}$$
$$+900a_{11} + 150a_{12} - 825a_{13} - 450a_{14} + 450a_{15} - 525a_{16} + 300a_{17} + 1200a_{19}$$
$$+3200a_2 - 1200a_3)e^{2\Phi}F_{\alpha\beta}F^{\alpha\beta}R + \frac{1}{16}(-12 + 4a_{10} + 4a_{11} - 2a_{13} - a_{14} + a_{15}$$
$$-2a_{16} - 8a_3)R^2 + \left(-\frac{3}{4} - a_{19}\right)e^{2\Phi}F^{\alpha\beta}F^{\gamma\delta}R_{\alpha\beta\gamma\delta} + a_3 R_{\alpha\beta}R^{\alpha\beta} + \left(\frac{20001}{400} + \frac{2}{3}a_1\right.$$
$$\left.-\frac{3}{8}a_{12} + \frac{5}{16}a_{13} + \frac{3}{8}a_{14} - \frac{3}{16}a_{15} + \frac{1}{16}a_{16} - \frac{3}{4}a_{17} - 3a_{19} + \frac{8}{3}a_2\right)e^{2\Phi}F_{\beta\gamma}F^{\beta\gamma}\nabla_\alpha\Phi\nabla^\alpha\Phi$$
$$+a_{10}R\nabla_\alpha\Phi\nabla^\alpha\Phi + a_{11}R^{\alpha\beta}\nabla_\beta\nabla_\alpha\Phi + a_{12}e^{2\Phi}F_\alpha^\gamma F_{\beta\gamma}\nabla^\alpha\Phi\nabla^\beta\Phi + a_{13}R_{\alpha\beta}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+a_{14}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi + a_{15}\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\Phi + a_{16}\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\nabla^\alpha\Phi$$
$$+a_{17}e^{2\Phi}F^{\beta\gamma}\nabla^\alpha\Phi\nabla_\gamma F_{\alpha\beta} + \frac{1}{4}(-3 - 16a_1 + 2a_{12} + a_{13} - a_{16})e^{2\Phi}F_\alpha^\gamma F^{\alpha\beta}\nabla_\gamma\nabla_\beta\Phi$$
$$\left.+a_{19}e^{2\Phi}\nabla_\gamma F_{\alpha\beta}\nabla^\gamma F^{\alpha\beta}\right]$$

$$S'_{6D} = -\frac{2}{\kappa'^2_{10}}\frac{\ell'^2_s V'}{8}\int d^6x\sqrt{-G'}e^{-2\Phi'}\left[R'_{\alpha\beta\gamma\delta}R'^{\alpha\beta\gamma\delta} + \frac{5}{8}F_\alpha'^\gamma F'^{\alpha\beta}F_\beta'^\delta F'_{\gamma\delta} + \frac{3}{8}F'_{\alpha\beta}F'^{\alpha\beta}F'_{\gamma\delta}F'^{\gamma\delta}\right.$$
$$\left.-F'^{\alpha\beta}F'^{\gamma\delta}R'_{\alpha\beta\gamma\delta} - F'^{\alpha\beta}F'^{\gamma\delta}R'_{\alpha\gamma\beta\delta} + \nabla_\gamma F'_{\alpha\beta}\nabla^\gamma F'^{\alpha\beta}\right]$$

$$S'_{6D} = -\frac{2}{\kappa^2}\frac{4\pi^4\ell_s^6}{3}\int d^6x\sqrt{-G'}e^{-2\Phi'}\left[\frac{3}{2}R'_{\alpha\beta\gamma\delta}R'^{\alpha\beta\gamma\delta} + b_1 F_\alpha'^\gamma F'^{\alpha\beta}F_\beta'^\delta F'_{\gamma\delta} + b_2 F'_{\alpha\beta}F'^{\alpha\beta}F'_{\gamma\delta}F'^{\gamma\delta}\right.$$
$$+\frac{1}{4}(2b_{11} + 2b_{12} + b_{13} - b_{16})R'_{\alpha\beta}R'^{\alpha\beta} + \frac{1}{8}(-9 - 16b_1 - 2b_{11} - 2b_{12} - b_{13} + b_{16}$$
$$+16b_{19})F_\alpha'^\gamma F'^{\alpha\beta}R'_{\beta\gamma} + \frac{1}{192}(162 + 32b_1 - 36b_{10} - 12b_{12} - 6b_{13} + 9b_{14} + 2b_{16} + 8b_{17}$$

$$+8b_{18} - 32b_{19} - 256b_2)F'_{\alpha\beta}F'^{\alpha\beta}R + \frac{1}{16}(4b_{10} - b_{14})R'^2 + \left(-\frac{3}{4} - b_{19}\right)F'^{\alpha\beta}F'^{\gamma\delta}R'_{\alpha\beta\gamma\delta} +$$
$$+\frac{1}{48}(162 + 32b_1 - 12b_{12} - 9b_{14} - 3b_{15} + 2b_{16} + 8b_{17} + 8b_{18} - 32b_{19}$$
$$-256b_2)F'_{\beta\gamma}F'^{\beta\gamma}\nabla_\alpha\Phi'\nabla^\alpha\Phi' + b_{10}R'\nabla_\alpha\Phi'\nabla^\alpha\Phi' + b_{11}R'^{\alpha\beta}\nabla_\beta\nabla_\alpha\Phi' + b_{12}F_\alpha'^\gamma F'_{\beta\gamma}\nabla^\alpha\Phi'\nabla^\beta\Phi'$$
$$+b_{13}R'_{\alpha\beta}\nabla^\alpha\Phi'\nabla^\beta\Phi' + b_{14}\nabla_\alpha\Phi'\nabla^\alpha\Phi'\nabla_\beta\Phi'\nabla^\beta\Phi' + b_{15}\nabla^\alpha\Phi'\nabla_\beta\nabla_\alpha\Phi'\nabla^\beta\Phi'$$
$$\left.+b_{16}\nabla_\beta\nabla_\alpha\Phi'\nabla^\beta\nabla^\alpha\Phi' + b_{17}F'^{\beta\gamma}\nabla^\alpha\Phi'\nabla_\gamma F'_{\alpha\beta} + b_{18}F_\alpha'^\gamma F'^{\alpha\beta}\nabla_\gamma\nabla_\beta\Phi' + b_{19}\nabla_\gamma F'_{\alpha\beta}\nabla^\gamma F'^{\alpha\beta}\right]$$

$a_2 = 65403/6400 - a_1/4 + (3a_{13})/64 + (3a_{14})/64 - (3a_{15})/128, a_3 = -(20901/200) + a_{10}$
$+a_{11}/2 - (3a_{13})/4 - a_{14}/2 + (3a_{15})/8 - a_{16}/4, b_1 = (4a_1)/3, b_{10} = 13734/25 - 4a_{10}$
$-(4a_{11})/3 + 4a_{13} + (8a_{14})/3 - 2a_{15} + (4a_{16})/3, b_{11} = 27268/25 + (8a_{10})/3 + (20a_{11})/3$
$+(4a_{13})/3 + 2a_{14} - (2a_{15})/3 - 4a_{16}, b_{12} = 6767/25 - (8a_{10})/3 - (4a_{11})/3 + (4a_{13})/3$
$+(4a_{14})/3 - a_{15} + (4a_{16})/3, b_{13} = -(13834/25) + 12a_{10} + (16a_{11})/3 - 6a_{13} - (13a_{14})/3$
$+(11a_{15})/3 - (14a_{16})/3, b_{14} = 27468/25 - (32a_{10})/3 - (16a_{11})/3 + (32a_{13})/3 + (20a_{14})/3$
$-(16a_{15})/3 + (16a_{16})/3, b_{15} = 26468/25 + (88a_{10})/3 + (64a_{11})/3 - (68a_{13})/3 - (34a_{14})/3$
$+(34a_{15})/3 - 20a_{16}, b_{16} = 13634/5 + (20a_{10})/3 + (40a_{11})/3 + (10a_{13})/3 + 5a_{14} - (5a_{15})/3$
$-(26a_{16})/3, b_{17} = 130273/100 - (32a_1)/3 + 4a_{10} + 2a_{11} + (5a_{12})/3 + (9a_{13})/2 + (14a_{14})/3$
$-(11a_{15})/6 - (11a_{16})/6 + 2a_{17} + (8a_{19})/3, b_{18} = 6742/25 - (16a_1)/3 - (8a_{10})/3 - (4a_{11})/3$
$-(2a_{12})/3 + (5a_{13})/3 + (4a_{14})/3 - a_{15} + a_{16} + (32a_{19})/3, b_{19} = (4a_{19})/3, b_2 = 21801/1600$
$-a_1/3 + a_{13}/16 + a_{14}/16 - a_{15}/32$

$$\mathcal{L}' = c_1' R_\beta{}^{\delta\epsilon\nu} R_{\gamma\epsilon}{}^{\mu\zeta} R_{\delta\nu\mu\zeta} F_\alpha{}^\beta F^{\alpha\gamma} e^{2\Phi} + \cdots$$

$$\mathcal{J} \equiv \nabla_\alpha(\mathcal{J}^\alpha)$$

$$\mathcal{K} \equiv e^\Phi \nabla_\beta F^{\beta\alpha} \delta C_\alpha - \frac{1}{8} e^{2\Phi} F_{\alpha\beta} F^{\alpha\beta} \delta G^\mu{}_\mu - \left(R^{\alpha\beta} - \frac{1}{2} e^{2\Phi} F^{\alpha\gamma} F^\beta{}_\gamma + 2\nabla^\beta \nabla^\alpha \Phi \right) \delta G_{\alpha\beta}$$
$$-2(R + 4\nabla_\alpha \nabla^\alpha \Phi - 4\nabla_\alpha \Phi \nabla^\alpha \Phi) \left(\delta\Phi - \frac{1}{4} \delta G^\mu{}_\mu \right)$$

$$\begin{aligned} G_{\mu\nu} &\to G_{\mu\nu} + \ell_s^6 e^{2\Phi} \delta G_{\mu\nu} \\ C_\mu &\to C_\mu + \ell_s^6 e^\Phi \delta C_\mu \\ \Phi &\to \Phi + \ell_s^6 e^{2\Phi} \delta\Phi \end{aligned}$$

$$\Delta - \mathcal{J} - \mathcal{K} = 0$$

$$\begin{aligned} R_{\alpha[\beta\gamma\delta]} &= 0, \\ \nabla_{[\mu} R_{\alpha\beta]\gamma\delta} &= 0, \\ \nabla_{[\mu} F_{\alpha\beta]} &= 0, \\ [\nabla, \nabla]\mathcal{O} - R\mathcal{O} &= 0. \end{aligned}$$

$$[R^2 R'']_1 = c_{56} R^{\alpha\beta\gamma\delta} R^{\mu\nu}{}_{\gamma\delta} \nabla_\alpha \nabla_\mu R_{\beta\nu}$$

$$\begin{aligned} \mathcal{L} = &[R^4]_7 + [F'^4]_5 + [R\Phi'^2 F'^2]_{11} + [RF'^2\Phi'']_9 + [R^2 F'^2]_{13} + [F^2 R^3]_{16} + [R^3\Phi'']_3 + [R^3\Phi'^2]_5 \\ &+[F^2 R\Phi''\Phi'^2]_8 + [R^2\Phi''^2]_5 + [R^2\Phi''\Phi'^2]_4 + [R^2\Phi'^4]_3 + [F^3 F'\Phi'^3]_9 + [F^4 R^2]_{20} + [F^2 R^2\Phi'']_{14} \\ &+[F^2 R^2\Phi'^2]_{19} + [F^4 R\Phi'^2]_{12} + [F^2 RF'^2]_{33} + [F^6 R]_7 + [F^6\Phi'']_4 + [F^6\Phi'^2]_1 + [FR^2 F'\Phi']_2 \\ &+[FR^2 F'\Phi']_{15} + [F^4\Phi'^4]_5 + [F^2 R\Phi'^4]_4 + [FR\Phi'^3 F']_{10} + [F^4 R\Phi'']_9 + [F^2\Phi'^6]_2 + [F^4\Phi''^2]_8 \\ &+[F^2 R\Phi''^2]_9 + [F^2 F'^2\Phi'']_{21} + [F^3 R\Phi' F']_1 + [FR\Phi''\Phi' F']_2 + [F^2\Phi''^2\Phi'^2]_4 + [F^2\Phi'^2 F'^2]_{17} \\ &+[FF'\Phi'\Phi''^2]_1 + [FF'\Phi'^3\Phi'']_5 + [R\Phi'^2\Phi''^2]_2 + [\Phi'^4 F'^2]_4 + [\Phi''^4]_2 + [FF'\Phi'^5]_2 + [F^2\Phi'^4\Phi'']_2 \\ &+[R\Phi''^3]_1 + [R\Phi'^4\Phi'']_1 + [F'^2\Phi''^2]_6 + [F'^2\Phi'^2\Phi'']_4 + [FF'^3\Phi']_1 + [\Phi'^4\Phi''^2]_2 + [\Phi'^6\Phi'']_1 \\ &+[F^8]_5 + [F^4 F'^2]_{20} + [R^2 R'']_1 \end{aligned}$$

$$\begin{aligned} [R^4]_7 = &c_{23} R^{\alpha\beta\gamma\delta} R_\gamma{}^\epsilon{}_\varepsilon{}^\varepsilon R_\delta{}^\mu{}_\beta{}^\nu R_{\varepsilon\nu\epsilon\mu} + c_{39} R_\alpha{}^{\epsilon\varepsilon\mu} R^{\alpha\beta\gamma\delta} R_\gamma{}^\nu{}_{\beta\varepsilon} R_{\mu\nu\delta\epsilon} + c_{40} R_\alpha{}^\epsilon{}_\gamma{}^\varepsilon R^{\alpha\beta\gamma\delta} R_{\beta\epsilon}{}^{\mu\nu} R_{\mu\nu\delta\varepsilon} \\ &+c_{41} R_{\alpha\gamma\beta}{}^\epsilon R^{\alpha\beta\gamma\delta} R_\delta{}^{\varepsilon\mu\nu} R_{\mu\nu\epsilon\varepsilon} + c_{42} R^{\alpha\beta\gamma\delta} R_{\gamma\delta\alpha\beta} R^{\epsilon\varepsilon\mu\nu} R_{\mu\nu\epsilon\varepsilon} + c_{54} R_{\alpha\beta}{}^{\epsilon\varepsilon} R^{\alpha\beta\gamma\delta} R_{\mu\nu\epsilon\varepsilon} R^{\mu\nu}{}_{\gamma\delta} \\ &+c_{55} R_{\alpha\beta}{}^{\epsilon\varepsilon} R^{\alpha\beta\gamma\delta} R_{\mu\nu\delta\varepsilon} R^{\mu\nu}{}_{\gamma\epsilon} \end{aligned}$$

$$[F'^4]_5 = e^{4\Phi}c_{254}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla_\gamma F^{\epsilon\varepsilon}\nabla_\delta F_{\epsilon\varepsilon} + e^{4\Phi}c_{323}\nabla_\alpha F_{\beta\gamma}\nabla^\alpha F^{\beta\gamma}\nabla_\delta F_{\epsilon\varepsilon}\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{342}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\gamma{}^\delta\nabla_\delta F^{\epsilon\varepsilon}\nabla_\epsilon F_{\alpha\varepsilon} + e^{4\Phi}c_{375}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla^\epsilon F_\gamma{}^\varepsilon\nabla_\varepsilon F_{\delta\epsilon}$$
$$+e^{4\Phi}c_{376}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\varepsilon F_{\gamma\epsilon}\nabla^\varepsilon F_{\beta\delta},$$
$$[R\Phi'^2F'^2]_{11} = e^{2\Phi}c_{198}R_{\epsilon\varepsilon\beta\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta}\nabla_\gamma F^{\epsilon\varepsilon} + e^{2\Phi}c_{253}R_{\alpha\epsilon\beta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F^{\delta\epsilon}\nabla_\delta F_\gamma{}^\varepsilon$$
$$+e^{2\Phi}c_{255}R_{\alpha\varepsilon\beta\gamma}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F^{\delta\epsilon}\nabla_\delta F_\epsilon{}^\varepsilon + e^{2\Phi}c_{321}R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha F_\beta{}^\gamma\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{2\Phi}c_{359}R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\epsilon F_\beta{}^\varepsilon + e^{2\Phi}c_{362}R_{\delta\varepsilon\gamma\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla^\epsilon F_\beta{}^\varepsilon$$
$$+e^{2\Phi}c_{364}R_{\delta\varepsilon\beta\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta}\nabla^\epsilon F_\gamma{}^\varepsilon + e^{2\Phi}c_{365}R_{\delta\varepsilon\beta\epsilon}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\epsilon F_\gamma{}^\varepsilon$$
$$+e^{2\Phi}c_{366}R_{\epsilon\varepsilon\beta\delta}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\epsilon F_\gamma{}^\varepsilon + e^{2\Phi}c_{369}R_{\delta\varepsilon\beta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla^\epsilon F_\gamma{}^\varepsilon$$
$$+e^{2\Phi}c_{370}R_{\epsilon\varepsilon\beta\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla^\epsilon F_\gamma{}^\varepsilon,$$
$$[RF'^2\Phi'']_9 = e^{2\Phi}c_{266}R_{\epsilon\varepsilon\alpha\gamma}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}\nabla_\delta\nabla^\varepsilon\Phi + e^{2\Phi}c_{291}R_{\delta\varepsilon\gamma\epsilon}\nabla_\alpha\nabla^\varepsilon\Phi\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon$$
$$+e^{2\Phi}c_{292}R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha\nabla^\varepsilon\Phi\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{2\Phi}c_{295}R_{\epsilon\varepsilon\alpha\delta}\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla^\varepsilon\Phi\nabla^\delta F_\beta{}^\epsilon$$
$$+e^{2\Phi}c_{319}R_{\epsilon\varepsilon\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla_\beta\nabla_\alpha\Phi\nabla^\delta F^{\epsilon\varepsilon} + e^{2\Phi}c_{344}R_{\delta\varepsilon\beta\gamma}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}\nabla_\epsilon\nabla_\alpha\Phi$$
$$+e^{2\Phi}c_{346}R_{\gamma\varepsilon\alpha\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}\nabla_\epsilon\nabla_\beta\Phi + e^{2\Phi}c_{373}R_{\gamma\epsilon\delta\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla^\epsilon\nabla^\varepsilon\Phi$$
$$+e^{2\Phi}c_{374}R_{\delta\epsilon\alpha\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\gamma{}^\delta\nabla^\epsilon\nabla^\varepsilon\Phi,$$
$$[R^2F'^2]_{13} = e^{2\Phi}c_{60}R_\gamma{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma} + e^{2\Phi}c_{70}R_{\varepsilon\mu\delta\epsilon}R^{\varepsilon\mu}{}_{\beta\gamma}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}$$
$$+e^{2\Phi}c_{71}R_{\varepsilon\mu\gamma\epsilon}R^{\varepsilon\mu}{}_{\beta\delta}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + e^{2\Phi}c_{121}R^{\delta\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma}$$
$$+e^{2\Phi}c_{126}R_\gamma{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta + e^{2\Phi}c_{133}R_\delta{}^\varepsilon{}_\alpha{}^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}$$
$$+e^{2\Phi}c_{283}R_\alpha{}^\varepsilon\delta^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{2\Phi}c_{285}R_\gamma{}^\varepsilon{}_\alpha{}^\mu R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon$$
$$+e^{2\Phi}c_{288}R_{\varepsilon\mu\gamma\delta}R^{\varepsilon\mu}{}_{\alpha\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{2\Phi}c_{310}R_\beta{}^\mu{}_{\alpha\gamma}R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{2\Phi}c_{312}R_\alpha{}^\mu{}_{\delta\epsilon}R_{\varepsilon\mu\beta\gamma}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon} + e^{2\Phi}c_{316}R_{\beta\epsilon\alpha}{}^\mu R_{\varepsilon\mu\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{2\Phi}c_{317}R_\beta{}^\mu{}_{\alpha\epsilon}R_{\varepsilon\mu\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon},$$
$$[F^2R^3]_{16} = e^{2\Phi}c_{19}F^{\alpha\beta}F^{\gamma\delta}R_\alpha{}^\epsilon{}_\gamma{}^\varepsilon R_\epsilon{}^\mu{}_\beta{}^\nu R_{\varepsilon\nu\delta\mu} + e^{2\Phi}c_{20}F_{\alpha\beta}F^{\alpha\beta}R^{\gamma\delta\epsilon\varepsilon}R_\epsilon{}^\mu{}_\gamma{}^\nu R_{\varepsilon\nu\delta\mu}$$
$$+e^{2\Phi}c_{21}F^{\alpha\beta}F^{\gamma\delta}R_\alpha{}^\epsilon{}_\gamma{}^\varepsilon R_\delta{}^\mu{}_\beta{}^\nu R_{\varepsilon\nu\epsilon\mu} + e^{2\Phi}c_{22}F_\alpha{}^\gamma F^{\alpha\beta}R_\delta{}^\mu{}_\gamma{}^\nu R^{\delta\epsilon}{}_\beta{}^\varepsilon R_{\varepsilon\nu\epsilon\mu}$$
$$+e^{2\Phi}c_{33}F^{\alpha\beta}F^{\gamma\delta}R_\alpha{}^{\epsilon\varepsilon\mu}R_\varepsilon{}^\nu{}_{\beta\epsilon}R_{\mu\nu\gamma\delta} + e^{2\Phi}c_{34}F^{\alpha\beta}F^{\gamma\delta}R_\alpha{}^{\epsilon\varepsilon\mu}R_\varepsilon{}^\nu{}_{\beta\gamma}R_{\mu\nu\delta\epsilon}$$
$$+e^{2\Phi}c_{35}F_{\alpha\beta}F^{\alpha\beta}R_{\gamma\epsilon}{}^{\mu\nu}R^{\gamma\delta\epsilon\varepsilon}R_{\mu\nu\delta\varepsilon} + e^{2\Phi}c_{36}F^{\alpha\beta}F^{\gamma\delta}R_\alpha{}^\epsilon{}_{\beta\gamma}R_\delta{}^{\varepsilon\mu\nu}R_{\mu\nu\epsilon\varepsilon}$$
$$+e^{2\Phi}c_{37}F_\alpha{}^\gamma F^{\alpha\beta}R_\beta{}^\delta{}_\gamma{}^\epsilon R_\delta{}^{\varepsilon\mu\nu}R_{\mu\nu\epsilon\varepsilon} + e^{2\Phi}c_{38}F^{\alpha\beta}F^{\gamma\delta}R_{\alpha\gamma\beta\delta}R^{\epsilon\varepsilon\mu\nu}R_{\mu\nu\epsilon\varepsilon}$$
$$+e^{2\Phi}c_{48}F^{\alpha\beta}F^{\gamma\delta}R_\alpha{}^\epsilon{}_\gamma{}^\varepsilon R_{\mu\nu\epsilon\varepsilon}R^{\mu\nu}{}_{\beta\delta} + e^{2\Phi}c_{49}F^{\alpha\beta}F^{\gamma\delta}R^{\epsilon\varepsilon}{}_{\alpha\gamma}R_{\mu\nu\delta\varepsilon}R^{\mu\nu}{}_{\beta\epsilon}$$
$$+e^{2\Phi}c_{50}F^{\alpha\beta}F^{\gamma\delta}R_\alpha{}^\epsilon{}_\gamma{}^\varepsilon R_{\mu\nu\delta\epsilon}R^{\mu\nu}{}_{\beta\varepsilon} + e^{2\Phi}c_{51}F^{\alpha\beta}F^{\gamma\delta}R_\alpha{}^\epsilon{}_\beta{}^\varepsilon R_{\mu\nu\epsilon\varepsilon}R^{\mu\nu}{}_{\gamma\delta}$$
$$+e^{2\Phi}c_{52}F_\alpha{}^\gamma F^{\alpha\beta}R_\beta{}^{\delta\epsilon\varepsilon}R_{\mu\nu\delta\varepsilon}R^{\mu\nu}{}_{\gamma\epsilon} + e^{2\Phi}c_{53}F^{\alpha\beta}F^{\gamma\delta}R^{\epsilon\varepsilon}{}_{\alpha\beta}R_{\mu\nu\delta\varepsilon}R^{\mu\nu}{}_{\gamma\epsilon},$$
$$[R^3\Phi'']_3 = c_{97}R_\gamma{}^\varepsilon{}_\beta{}^\mu R^{\gamma\delta}{}_\alpha{}^\epsilon R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha\nabla^\beta\Phi + c_{107}R_\alpha{}^\gamma{}_\beta{}^\delta R_\gamma{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi$$
$$+c_{112}R_\alpha{}^{\gamma\delta\epsilon}R_{\varepsilon\mu\gamma\epsilon}R^{\varepsilon\mu}{}_{\beta\delta}\nabla^\alpha\nabla^\beta\Phi,$$

$$[R^3\Phi'^2]_5 = c_{76}R^{\beta\gamma\delta\epsilon}R_{\delta\ \beta\ \mu}^{\ \ \ \ \ \varepsilon}R_{\epsilon\mu\gamma\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi + c_{77}R_{\beta\delta}^{\ \ \varepsilon\mu}R^{\beta\gamma\delta\epsilon}R_{\varepsilon\mu\gamma\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi$$
$$+c_{169}R_{\gamma\ \beta}^{\ \ \varepsilon\ \ \mu}R^{\gamma\delta}_{\ \ \ \alpha\ \ \epsilon}R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi + c_{179}R_{\alpha\ \beta}^{\ \ \gamma\ \ \delta}R_{\gamma}^{\ \ \epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+c_{184}R_{\alpha}^{\ \gamma\delta\epsilon}R_{\varepsilon\mu\gamma\epsilon}R^{\varepsilon\mu}_{\ \ \beta\delta}\nabla^\alpha\Phi\nabla^\beta\Phi,$$
$$[F^2R\Phi''\Phi'^2]_8 = e^{2\Phi}c_{185}F_\delta^\varepsilon F^{\delta\epsilon}R_{\gamma\epsilon\beta\varepsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\Phi\nabla^\beta\Phi + e^{2\Phi}c_{186}F_\gamma^\delta F^{\epsilon\varepsilon}R_{\epsilon\varepsilon\beta\delta}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{187}F_\beta^{\ \delta}F^{\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\Phi\nabla^\beta\Phi + e^{2\Phi}c_{241}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}R_{\alpha\gamma\beta\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi$$
$$+e^{2\Phi}c_{242}F_\alpha^{\ \epsilon}F_\epsilon^{\ \varepsilon}R_{\gamma\varepsilon\beta\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + e^{2\Phi}c_{243}F_\alpha^{\ \epsilon}F_\gamma^{\ \varepsilon}R_{\delta\varepsilon\beta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi$$
$$+e^{2\Phi}c_{245}F_\alpha^{\ \epsilon}F_\gamma^{\ \varepsilon}R_{\epsilon\varepsilon\beta\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + e^{2\Phi}c_{246}F_{\alpha\gamma}F^{\epsilon\varepsilon}R_{\epsilon\varepsilon\beta\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi,$$
$$[R^2\Phi''^2]_5 = c_{117}R_\beta^{\ \delta\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\nabla^\beta\Phi + c_{140}R^{\gamma\delta\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla^\alpha\nabla^\beta\Phi\nabla_\beta\nabla_\alpha\Phi$$
$$+c_{234}R_{\alpha\ \beta}^{\ \epsilon\ \ \varepsilon}R_{\gamma\varepsilon\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + c_{235}R_{\alpha\ \gamma}^{\ \epsilon\ \ \varepsilon}R_{\delta\varepsilon\beta\epsilon}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi$$
$$+c_{238}R_{\epsilon\varepsilon\beta\delta}R^{\epsilon\varepsilon}_{\ \ \alpha\gamma}\nabla^\alpha\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi,$$
$$[R^2\Phi''\Phi'^2]_4 = c_{188}R_\beta^{\ \delta\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\Phi\nabla^\beta\Phi + c_{193}R^{\gamma\delta\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\Phi$$
$$+c_{244}R_{\gamma\ \alpha}^{\ \epsilon\ \ \varepsilon}R_{\delta\varepsilon\beta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + c_{247}R_{\epsilon\varepsilon\beta\delta}R^{\epsilon\varepsilon}_{\ \ \alpha\gamma}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi,$$
$$[R^2\Phi'^4]_3 = c_{192}R^{\gamma\delta\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi + c_{223}R_\beta^{\ \delta\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi$$
$$+c_{328}R_{\alpha\ \beta}^{\ \epsilon\ \ \varepsilon}R_{\gamma\varepsilon\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta\Phi,$$
$$[F^3F'\Phi'^3]_9 = e^{4\Phi}c_{206}F_\gamma^{\ \epsilon}F_\delta^{\ \varepsilon}F_{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\beta^{\ \delta} + e^{4\Phi}c_{207}F_{\gamma\delta}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\beta^{\ \delta}$$
$$+e^{4\Phi}c_{209}F_\beta^{\ \varepsilon}F_{\gamma\delta}F_{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F^{\delta\epsilon} + e^{4\Phi}c_{210}F_{\beta\delta}F_\gamma^{\ \varepsilon}F_{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F^{\delta\epsilon}$$
$$+e^{4\Phi}c_{214}F_\gamma^{\ \epsilon}F_\delta^{\ \varepsilon}F_{\epsilon\varepsilon}\nabla_\alpha F_\beta^{\ \delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi + e^{4\Phi}c_{215}F_{\gamma\delta}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla_\alpha F_\beta^{\ \delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi$$
$$+e^{4\Phi}c_{217}F_{\beta\delta}F_\gamma^{\ \varepsilon}F_{\epsilon\varepsilon}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi + e^{4\Phi}c_{267}F_\beta^{\ \varepsilon}F_{\gamma\varepsilon}F_{\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta F_\alpha^{\ \epsilon}$$
$$+e^{4\Phi}c_{268}F_{\beta\delta}F_\gamma^{\ \varepsilon}F_{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta F_\alpha^{\ \epsilon},$$
$$[F^4R^2]_{20} = e^{4\Phi}c_{11}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\varepsilon}F^{\mu\nu}R_{\alpha\gamma\beta\delta}R_{\epsilon\mu\varepsilon\nu} + e^{4\Phi}c_{12}F_\alpha^{\ \gamma}F^{\alpha\beta}F_\delta^{\ \varepsilon}F^{\delta\epsilon}R_{\beta\ \ \gamma\ \ \nu}^{\ \ \mu}R_{\epsilon\nu\varepsilon\mu}$$
$$+e^{4\Phi}c_{13}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\varepsilon}F^{\mu\nu}R_{\alpha\gamma\epsilon\mu}R_{\varepsilon\nu\beta\delta} + e^{4\Phi}c_{14}F_\alpha^{\ \gamma}F^{\alpha\beta}F_\delta^{\ \varepsilon}F^{\delta\epsilon}R_{\beta\ \ \epsilon\ \ \nu}^{\ \ \mu}R_{\varepsilon\nu\gamma\mu}$$
$$+e^{4\Phi}c_{15}F_\alpha^{\ \gamma}F^{\alpha\beta}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\delta\ \ \beta\epsilon}^{\ \ \nu}R_{\varepsilon\nu\gamma\mu} + e^{4\Phi}c_{16}F_{\alpha\beta}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\varepsilon}R_{\gamma\ \ \epsilon\ \ \nu}^{\ \ \mu}R_{\varepsilon\nu\delta\mu}$$
$$+e^{4\Phi}c_{17}F_\alpha^{\ \gamma}F^{\alpha\beta}F_\beta^{\ \delta}F^{\epsilon\varepsilon}R_{\epsilon\ \ \gamma\ \ \nu}^{\ \ \mu}R_{\varepsilon\nu\delta\mu} + e^{4\Phi}c_{18}F_\alpha^{\ \gamma}F^{\alpha\beta}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\beta\ \ \gamma\delta}^{\ \ \nu}R_{\varepsilon\nu\epsilon\mu}$$
$$+e^{4\Phi}c_{26}F_\alpha^{\ \gamma}F^{\alpha\beta}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\beta\delta\varepsilon}^{\ \ \ \nu}R_{\mu\nu\gamma\epsilon} + e^{4\Phi}c_{27}F_\alpha^{\ \gamma}F^{\alpha\beta}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\delta\ \ \beta\varepsilon}^{\ \ \nu}R_{\mu\nu\gamma\epsilon}$$
$$+e^{4\Phi}c_{28}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\varepsilon}F^{\mu\nu}R_{\alpha\beta\gamma\epsilon}R_{\mu\nu\delta\varepsilon} + e^{4\Phi}c_{29}F_\alpha^{\ \gamma}F^{\alpha\beta}F_\beta^{\ \delta}F_\gamma^{\ \epsilon}R_{\delta}^{\ \varepsilon\mu\nu}R_{\mu\nu\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{30}F_{\alpha\beta}F^{\alpha\beta}F_\gamma^{\ \epsilon}F^{\gamma\delta}R_\delta^{\ \varepsilon\mu\nu}R_{\mu\nu\epsilon\varepsilon} + e^{4\Phi}c_{31}F_\alpha^{\ \gamma}F^{\alpha\beta}F_\beta^{\ \delta}F_{\gamma\delta}R^{\epsilon\varepsilon\mu\nu}R_{\mu\nu\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{32}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}R^{\epsilon\varepsilon\mu\nu}R_{\mu\nu\epsilon\varepsilon} + e^{4\Phi}c_{43}F_\alpha^{\ \gamma}F^{\alpha\beta}F_\delta^{\ \varepsilon}F^{\delta\epsilon}R_{\mu\nu\gamma\varepsilon}R^{\mu\nu}_{\ \ \beta\epsilon}$$
$$+e^{4\Phi}c_{44}F_\alpha^{\ \gamma}F^{\alpha\beta}F_\beta^{\ \delta}F^{\epsilon\varepsilon}R_{\mu\nu\epsilon\varepsilon}R^{\mu\nu}_{\ \ \gamma\delta} + e^{4\Phi}c_{45}F_{\alpha\beta}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\varepsilon}R_{\mu\nu\epsilon\varepsilon}R^{\mu\nu}_{\ \ \gamma\delta}$$
$$+e^{4\Phi}c_{46}F_\alpha^{\ \gamma}F^{\alpha\beta}F_\beta^{\ \delta}F^{\epsilon\varepsilon}R_{\mu\nu\delta\varepsilon}R^{\mu\nu}_{\ \ \gamma\epsilon} + e^{4\Phi}c_{47}F_{\alpha\beta}F^{\alpha\beta}F^{\gamma\delta}F^{\epsilon\varepsilon}R_{\mu\nu\delta\varepsilon}R^{\mu\nu}_{\ \ \gamma\epsilon},$$

$$[F^2R^2\Phi'']_{14} = e^{2\Phi}c_{92}F_\gamma{}^\epsilon F^{\gamma\delta}R_\alpha{}^\varepsilon{}_\beta{}^\mu R_{\delta\mu\epsilon\varepsilon}\nabla^\alpha\nabla^\beta\Phi + e^{2\Phi}c_{93}F^{\gamma\delta}F^{\epsilon\varepsilon}R_\gamma{}^\mu{}_{\alpha\delta}R_{\epsilon\mu\beta\varepsilon}\nabla^\alpha\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{94}F_\gamma{}^\epsilon F^{\gamma\delta}R_\delta{}^\varepsilon{}_\alpha{}^\mu R_{\epsilon\mu\beta\varepsilon}\nabla^\alpha\nabla^\beta\Phi + e^{2\Phi}c_{95}F_\alpha{}^\gamma F^{\delta\epsilon}R_\delta{}^\varepsilon{}_\beta{}^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla^\alpha\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{96}F^{\gamma\delta}F^{\epsilon\varepsilon}R_\alpha{}^\mu{}_{\beta\gamma}R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha\nabla^\beta\Phi + e^{2\Phi}c_{100}F^{\gamma\delta}F^{\epsilon\varepsilon}R_{\alpha\gamma\epsilon}{}^\mu R_{\varepsilon\mu\beta\delta}\nabla^\alpha\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{101}F^{\gamma\delta}F^{\epsilon\varepsilon}R_\gamma{}^\mu{}_{\alpha\epsilon}R_{\varepsilon\mu\beta\delta}\nabla^\alpha\nabla^\beta\Phi + e^{2\Phi}c_{103}F_{\gamma\delta}F^{\gamma\delta}R_\alpha{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\beta\epsilon}\nabla^\alpha\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{105}F_\alpha{}^\gamma F_\gamma{}^\delta R_\beta{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi + e^{2\Phi}c_{106}F_\alpha{}^\gamma F_\beta{}^\delta R_\gamma{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{108}F_\alpha{}^\gamma F_{\beta\gamma}R^{\delta\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\nabla^\beta\Phi + e^{2\Phi}c_{109}F_\gamma{}^\epsilon F^{\gamma\delta}R_{\varepsilon\mu\beta\epsilon}R^{\varepsilon\mu}{}_{\alpha\delta}\nabla^\alpha\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{110}F_\alpha{}^\gamma F^{\delta\epsilon}R_{\varepsilon\mu\delta\epsilon}R^{\varepsilon\mu}{}_{\beta\gamma}\nabla^\alpha\nabla^\beta\Phi + e^{2\Phi}c_{111}F_\alpha{}^\gamma F^{\delta\epsilon}R_{\varepsilon\mu\gamma\epsilon}R^{\varepsilon\mu}{}_{\beta\delta}\nabla^\alpha\nabla^\beta\Phi,$$
$$[F^2R^2\Phi'^2]_{19} = e^{2\Phi}c_{75}F^{\beta\gamma}F^{\delta\epsilon}R_\beta{}^\varepsilon{}_\delta{}^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi + e^{2\Phi}c_{79}F_\beta{}^\delta F^{\beta\gamma}R_\gamma{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi$$
$$+e^{2\Phi}c_{80}F_{\beta\gamma}F^{\beta\gamma}R^{\delta\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi + e^{2\Phi}c_{81}F^{\beta\gamma}F^{\delta\epsilon}R_{\varepsilon\mu\delta\epsilon}R^{\varepsilon\mu}{}_{\beta\gamma}\nabla_\alpha\Phi\nabla^\alpha\Phi$$
$$+e^{2\Phi}c_{82}F^{\beta\gamma}F^{\delta\epsilon}R_{\varepsilon\mu\gamma\epsilon}R^{\varepsilon\mu}{}_{\beta\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi + e^{2\Phi}c_{164}F_\gamma{}^\epsilon F^{\gamma\delta}R_\alpha{}^\varepsilon{}_\beta{}^\mu R_{\delta\mu\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{165}F^{\gamma\delta}F^{\epsilon\varepsilon}R_\gamma{}^\mu{}_{\alpha\delta}R_{\epsilon\mu\beta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{2\Phi}c_{166}F_\gamma{}^\epsilon F^{\gamma\delta}R_\delta{}^\varepsilon{}_\alpha{}^\mu R_{\epsilon\mu\beta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{167}F_\alpha{}^\gamma F^{\delta\epsilon}R_\delta{}^\varepsilon{}_\beta{}^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{2\Phi}c_{168}F^{\gamma\delta}F^{\epsilon\varepsilon}R_\alpha{}^\mu{}_{\beta\gamma}R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{172}F^{\gamma\delta}F^{\epsilon\varepsilon}R_{\alpha\gamma\epsilon}{}^\mu R_{\varepsilon\mu\beta\delta}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{2\Phi}c_{173}F^{\gamma\delta}F^{\epsilon\varepsilon}R_\gamma{}^\mu{}_{\alpha\epsilon}R_{\varepsilon\mu\beta\delta}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{175}F_{\gamma\delta}F^{\gamma\delta}R_\alpha{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\beta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{2\Phi}c_{177}F_\alpha{}^\gamma F_\gamma{}^\delta R_\beta{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{178}F_\alpha{}^\gamma F_\beta{}^\delta R_\gamma{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{2\Phi}c_{180}F_\alpha{}^\gamma F_{\beta\gamma}R^{\delta\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{181}F_\gamma{}^\epsilon F^{\gamma\delta}R_{\varepsilon\mu\beta\epsilon}R^{\varepsilon\mu}{}_{\alpha\delta}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{2\Phi}c_{182}F_\alpha{}^\gamma F^{\delta\epsilon}R_{\varepsilon\mu\delta\epsilon}R^{\varepsilon\mu}{}_{\beta\gamma}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{2\Phi}c_{183}F_\alpha{}^\gamma F^{\delta\epsilon}R_{\varepsilon\mu\gamma\epsilon}R^{\varepsilon\mu}{}_{\beta\delta}\nabla^\alpha\Phi\nabla^\beta\Phi,$$
$$[F^4R\Phi'^2]_{12} = e^{4\Phi}c_{73}F_\beta{}^\delta F^{\beta\gamma}F_\epsilon{}^\mu F^{\epsilon\varepsilon}R_{\gamma\varepsilon\delta\mu}\nabla_\alpha\Phi\nabla^\alpha\Phi + e^{4\Phi}c_{74}F_{\beta\gamma}F^{\beta\gamma}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\delta\varepsilon\epsilon\mu}\nabla_\alpha\Phi\nabla^\alpha\Phi$$
$$+e^{4\Phi}c_{78}F_\beta{}^\delta F^{\beta\gamma}F_\gamma{}^\epsilon F^{\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi + e^{4\Phi}c_{159}F_\gamma{}^\epsilon F^{\gamma\delta}F_\delta{}^\varepsilon F_\epsilon{}^\mu R_{\alpha\varepsilon\beta\mu}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{4\Phi}c_{160}F_{\gamma\delta}F^{\gamma\delta}F_\epsilon{}^\mu F^{\epsilon\varepsilon}R_{\alpha\varepsilon\beta\mu}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{4\Phi}c_{161}F_\alpha{}^\gamma F_\beta{}^\delta F_\epsilon{}^\mu F^{\epsilon\varepsilon}R_{\gamma\varepsilon\delta\mu}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{4\Phi}c_{162}F_\alpha{}^\gamma F_\gamma{}^\delta F_\epsilon{}^\mu F^{\epsilon\varepsilon}R_{\delta\varepsilon\beta\mu}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{4\Phi}c_{163}F_\alpha{}^\gamma F_{\beta\gamma}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\delta\varepsilon\epsilon\mu}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{4\Phi}c_{170}F_\alpha{}^\gamma F_\delta{}^\varepsilon F^{\delta\epsilon}F_\epsilon{}^\mu R_{\varepsilon\mu\beta\gamma}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{4\Phi}c_{171}F_\alpha{}^\gamma F_{\delta\epsilon}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\varepsilon\mu\beta\gamma}\nabla^\alpha\Phi\nabla^\beta\Phi$$
$$+e^{4\Phi}c_{174}F_\alpha{}^\gamma F_\gamma{}^\delta F_\delta{}^\epsilon F^{\varepsilon\mu}R_{\varepsilon\mu\beta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi + e^{4\Phi}c_{176}F_\alpha{}^\gamma F_\beta{}^\delta F_\gamma{}^\epsilon F^{\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi,$$
$$[F^2RF'^2]_{33} = e^{4\Phi}c_{59}F_\gamma{}^\epsilon F^{\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma} + e^{4\Phi}c_{66}F_\beta{}^\varepsilon F_\delta{}^\mu R_{\varepsilon\mu\gamma\epsilon}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}$$
$$+e^{4\Phi}c_{67}F_{\beta\delta}F^{\varepsilon\mu}R_{\varepsilon\mu\gamma\epsilon}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + e^{4\Phi}c_{68}F_\beta{}^\varepsilon F_\gamma{}^\mu R_{\varepsilon\mu\delta\epsilon}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}$$
$$+e^{4\Phi}c_{69}F_{\beta\gamma}F^{\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + e^{4\Phi}c_{120}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\delta\varepsilon\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma}$$
$$+e^{4\Phi}c_{124}F_\epsilon{}^\mu F^{\epsilon\varepsilon}R_{\gamma\varepsilon\delta\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta + e^{4\Phi}c_{125}F_\gamma{}^\epsilon F^{\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta$$
$$+e^{4\Phi}c_{127}F_\epsilon{}^\mu F^{\epsilon\varepsilon}R_{\delta\varepsilon\alpha\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\gamma{}^\delta + e^{4\Phi}c_{129}F_{\varepsilon\mu}F^{\varepsilon\mu}R_{\delta\epsilon\alpha\gamma}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}$$
$$+e^{4\Phi}c_{130}F_\delta{}^\varepsilon F_\varepsilon{}^\mu R_{\epsilon\mu\alpha\gamma}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon} + e^{4\Phi}c_{131}F_\gamma{}^\varepsilon F_\delta{}^\mu R_{\epsilon\mu\alpha\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}$$
$$+e^{4\Phi}c_{132}F_\alpha{}^\varepsilon F_\delta{}^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon} + e^{4\Phi}c_{134}F_\alpha{}^\varepsilon F_\delta{}^\mu R_{\varepsilon\mu\gamma\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}$$
$$+e^{4\Phi}c_{135}F_{\alpha\delta}F^{\varepsilon\mu}R_{\varepsilon\mu\gamma\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon} + e^{4\Phi}c_{278}F_{\varepsilon\mu}F^{\varepsilon\mu}R_{\gamma\epsilon\alpha\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon$$
$$+e^{4\Phi}c_{279}F_\alpha{}^\varepsilon F_\varepsilon{}^\mu R_{\delta\mu\gamma\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{4\Phi}c_{280}F_\alpha{}^\varepsilon F_\epsilon{}^\mu R_{\delta\mu\gamma\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon$$

$$+e^{4\Phi}c_{281}F_\gamma{}^\varepsilon F_\varepsilon{}^\mu R_{\epsilon\mu\alpha\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{4\Phi}c_{282}F_\alpha{}^\varepsilon F_\varepsilon{}^\mu R_{\epsilon\mu\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon$$
$$+e^{4\Phi}c_{284}F_\alpha{}^\varepsilon F_\gamma{}^\mu R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{4\Phi}c_{286}F_\alpha{}^\varepsilon F_\delta{}^\mu R_{\varepsilon\mu\gamma\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon$$
$$+e^{4\Phi}c_{287}F_{\alpha\delta}F^{\varepsilon\mu}R_{\varepsilon\mu\gamma\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{4\Phi}c_{304}F_\beta{}^\mu F_{\epsilon\mu}R_{\gamma\varepsilon\alpha\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{305}F_\alpha{}^\mu F_{\epsilon\mu}R_{\delta\varepsilon\beta\gamma}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon} + e^{4\Phi}c_{306}F_\alpha{}^\mu F_{\beta\mu}R_{\epsilon\varepsilon\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{307}F_{\alpha\delta}F_\beta{}^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon} + e^{4\Phi}c_{308}F_{\alpha\beta}F_\delta{}^\mu R_{\epsilon\mu\gamma\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{309}F_{\alpha\beta}F_\gamma{}^\mu R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon} + e^{4\Phi}c_{311}F_{\alpha\epsilon}F_\delta{}^\mu R_{\varepsilon\mu\beta\gamma}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{313}F_\alpha{}^\mu F_{\beta\epsilon}R_{\varepsilon\mu\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon} + e^{4\Phi}c_{314}F_{\alpha\epsilon}F_\beta{}^\mu R_{\varepsilon\mu\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{315}F_{\alpha\beta}F_\epsilon{}^\mu R_{\varepsilon\mu\gamma\delta}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon},$$
$$[F^6R]_7 = e^{6\Phi}c_6F_\alpha{}^\gamma F^{\alpha\beta}F_\beta{}^\delta F_\epsilon{}^\mu F^{\epsilon\varepsilon}F_\varepsilon{}^\nu R_{\gamma\mu\delta\nu} + e^{6\Phi}c_7F_\alpha{}^\gamma F^{\alpha\beta}F_\beta{}^\delta F_\gamma{}^\epsilon F_\varepsilon{}^\nu F^{\varepsilon\mu}R_{\delta\mu\epsilon\nu}$$
$$+e^{6\Phi}c_8F_{\alpha\beta}F^{\alpha\beta}F_\gamma{}^\epsilon F^{\gamma\delta}F_\varepsilon{}^\nu F^{\varepsilon\mu}R_{\delta\mu\epsilon\nu} + e^{6\Phi}c_9F_\alpha{}^\gamma F^{\alpha\beta}F_\beta{}^\delta F_{\gamma\delta}F^{\epsilon\varepsilon}F^{\mu\nu}R_{\epsilon\mu\varepsilon\nu}$$
$$+e^{6\Phi}c_{10}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}F^{\epsilon\varepsilon}F^{\mu\nu}R_{\epsilon\mu\varepsilon\nu} + e^{6\Phi}c_{24}F_\alpha{}^\gamma F^{\alpha\beta}F_\beta{}^\delta F_\gamma{}^\epsilon F_\delta{}^\varepsilon F^{\mu\nu}R_{\mu\nu\epsilon\varepsilon}$$
$$+e^{6\Phi}c_{25}F_{\alpha\beta}F^{\alpha\beta}F_\gamma{}^\epsilon F^{\gamma\delta}F_\delta{}^\varepsilon F^{\mu\nu}R_{\mu\nu\epsilon\varepsilon},$$
$$[F^6\Phi'']_4 = e^{6\Phi}c_{83}F_\alpha{}^\gamma F_\beta{}^\delta F_\gamma{}^\epsilon F_\delta{}^\varepsilon F_\epsilon{}^\mu F_{\varepsilon\mu}\nabla^\alpha\nabla^\beta\Phi + e^{6\Phi}c_{84}F_\alpha{}^\gamma F_{\beta\gamma}F_\delta{}^\varepsilon F^{\delta\epsilon}F_\epsilon{}^\mu F_{\varepsilon\mu}\nabla^\alpha\nabla^\beta\Phi$$
$$+e^{6\Phi}c_{85}F_\alpha{}^\gamma F_\beta{}^\delta F_\gamma{}^\epsilon F_{\delta\epsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla^\alpha\nabla^\beta\Phi + e^{6\Phi}c_{86}F_\alpha{}^\gamma F_{\beta\gamma}F_{\delta\epsilon}F^{\delta\epsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla^\alpha\nabla^\beta\Phi,$$
$$[F^6\Phi'^2]_1 = e^{6\Phi}c_{158}F_\alpha^\gamma F_{\beta\gamma}F_{\delta\epsilon}F^{\delta\epsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla^\alpha\Phi\nabla^\beta\Phi,$$
$$[F^4\Phi'^2\Phi'']_2 = e^{4\Phi}c_{239}F_{\alpha\gamma}F_\beta{}^\epsilon F_\delta{}^\varepsilon F_{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + e^{4\Phi}c_{240}F_{\alpha\gamma}F_{\beta\delta}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi,$$
$$[FR^2F'\Phi']_{15} = e^{2\Phi}c_{72}F_\beta{}^\delta R_\gamma{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla_\alpha F^{\beta\gamma}\nabla^\alpha\Phi + e^{2\Phi}c_{143}F_\beta{}^\delta R_\gamma{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta F_\alpha{}^\gamma$$
$$+e^{2\Phi}c_{144}F_{\beta\gamma}R^{\delta\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta F_\alpha{}^\gamma + e^{2\Phi}c_{145}F^{\delta\epsilon}R_{\varepsilon\mu\delta\epsilon}R^{\varepsilon\mu}{}_{\beta\gamma}\nabla^\alpha\Phi\nabla^\beta F_\alpha{}^\gamma$$
$$+e^{2\Phi}c_{146}F^{\delta\epsilon}R_{\varepsilon\mu\gamma\epsilon}R^{\varepsilon\mu}{}_{\beta\delta}\nabla^\alpha\Phi\nabla^\beta F_\alpha{}^\gamma + e^{2\Phi}c_{147}F^{\epsilon\varepsilon}R_\beta{}^\mu{}_{\alpha\gamma}R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta}$$
$$+e^{2\Phi}c_{148}F_\alpha{}^\epsilon R_\gamma{}^\varepsilon{}_\beta{}^\mu R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta} + e^{2\Phi}c_{149}F^{\epsilon\varepsilon}R_\gamma{}^\mu{}_{\alpha\beta}R_{\epsilon\mu\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta}$$
$$+e^{2\Phi}c_{150}F^{\epsilon\varepsilon}R_\gamma{}^\mu{}_{\beta\epsilon}R_{\varepsilon\mu\alpha\delta}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta} + e^{2\Phi}c_{151}F^{\epsilon\varepsilon}R_\beta{}^\mu{}_{\alpha\epsilon}R_{\varepsilon\mu\gamma\delta}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta}$$
$$+e^{2\Phi}c_{152}F^{\epsilon\varepsilon}R_\epsilon{}^\mu{}_{\alpha\beta}R_{\varepsilon\mu\gamma\delta}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta} + e^{2\Phi}c_{153}F_{\alpha\gamma}R_\beta{}^{\epsilon\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta}$$
$$+e^{2\Phi}c_{154}F_\gamma{}^\epsilon R_{\varepsilon\mu\beta\epsilon}R^{\varepsilon\mu}{}_{\alpha\delta}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta} + e^{2\Phi}c_{155}F_\beta{}^\epsilon R_{\varepsilon\mu\gamma\delta}R^{\varepsilon\mu}{}_{\alpha\epsilon}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta}$$
$$+e^{2\Phi}c_{156}F_\alpha{}^\epsilon R_{\varepsilon\mu\gamma\delta}R^{\varepsilon\mu}{}_{\beta\epsilon}\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta},$$
$$[F^4\Phi'^4]_5 = e^{4\Phi}c_{189}F_\gamma{}^\epsilon F^{\gamma\delta}F_\delta{}^\varepsilon F_{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi + e^{4\Phi}c_{190}F_{\gamma\delta}F^{\gamma\delta}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi$$
$$+e^{4\Phi}c_{219}F_\beta{}^\delta F_\gamma{}^\epsilon F_\delta{}^\varepsilon F_{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi + e^{4\Phi}c_{220}F_\beta{}^\delta F_{\gamma\delta}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi$$
$$+e^{4\Phi}c_{326}F_\alpha{}^\epsilon F_{\beta\epsilon}F_\gamma{}^\varepsilon F_{\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta\Phi,$$
$$[F^2R\Phi'^4]_4 = e^{2\Phi}c_{191}F^{\gamma\delta}F^{\epsilon\varepsilon}R_{\gamma\epsilon\delta\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi + e^{2\Phi}c_{221}F_\delta{}^\varepsilon F^{\delta\epsilon}R_{\beta\epsilon\gamma\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi$$
$$+e^{2\Phi}c_{222}F_\beta{}^\delta F^{\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi + e^{2\Phi}c_{327}F_\alpha{}^\epsilon F_\beta{}^\varepsilon R_{\gamma\epsilon\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta\Phi,$$

$$[FR\Phi'^{3}F']_{10} = e^{2\Phi}c_{208}F^{\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}F_{\beta}{}^{\delta} + e^{2\Phi}c_{211}F_{\gamma}{}^{\varepsilon}R_{\delta\varepsilon\beta\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}F^{\delta\epsilon}$$
$$+e^{2\Phi}c_{212}F_{\beta}{}^{\varepsilon}R_{\delta\varepsilon\gamma\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}F^{\delta\epsilon} + e^{2\Phi}c_{213}F_{\delta}{}^{\varepsilon}R_{\epsilon\varepsilon\beta\gamma}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}F^{\delta\epsilon}$$
$$+e^{2\Phi}c_{216}F^{\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla_{\alpha}F_{\beta}{}^{\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi + e^{2\Phi}c_{218}F_{\delta}{}^{\varepsilon}R_{\beta\varepsilon\gamma\epsilon}\nabla_{\alpha}F^{\delta\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi$$
$$+e^{2\Phi}c_{269}F_{\delta}{}^{\varepsilon}R_{\beta\varepsilon\gamma\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi\nabla^{\delta}F_{\alpha}{}^{\epsilon} + e^{2\Phi}c_{270}F_{\beta}{}^{\varepsilon}R_{\delta\varepsilon\gamma\epsilon}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi\nabla^{\delta}F_{\alpha}{}^{\epsilon}$$
$$+e^{2\Phi}c_{271}F_{\beta}{}^{\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi\nabla^{\delta}F_{\alpha}{}^{\epsilon} + e^{2\Phi}c_{322}F_{\alpha\epsilon}R_{\beta\varepsilon\gamma\delta}\nabla^{\alpha}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi\nabla^{\delta}F^{\epsilon\varepsilon},$$
$$[F^{4}R\Phi'']_{9} = e^{4\Phi}c_{87}F_{\gamma}{}^{\epsilon}F^{\gamma\delta}F_{\delta}{}^{\varepsilon}F_{\epsilon}{}^{\mu}R_{\alpha\varepsilon\beta\mu}\nabla^{\alpha}\nabla^{\beta}\Phi + e^{4\Phi}c_{88}F_{\gamma\delta}F^{\gamma\delta}F_{\epsilon}{}^{\mu}F^{\epsilon\varepsilon}R_{\alpha\varepsilon\beta\mu}\nabla^{\alpha}\nabla^{\beta}\Phi$$
$$+e^{4\Phi}c_{89}F_{\alpha}{}^{\gamma}F_{\beta}{}^{\delta}F_{\epsilon}{}^{\mu}F^{\epsilon\varepsilon}R_{\gamma\varepsilon\delta\mu}\nabla^{\alpha}\nabla^{\beta}\Phi + e^{4\Phi}c_{90}F_{\alpha}{}^{\gamma}F_{\gamma}{}^{\delta}F_{\epsilon}{}^{\mu}F^{\epsilon\varepsilon}R_{\delta\varepsilon\beta\mu}\nabla^{\alpha}\nabla^{\beta}\Phi$$
$$+e^{4\Phi}c_{91}F_{\alpha}{}^{\gamma}F_{\beta\gamma}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\delta\varepsilon\epsilon\mu}\nabla^{\alpha}\nabla^{\beta}\Phi + e^{4\Phi}c_{98}F_{\alpha}{}^{\gamma}F_{\delta}{}^{\varepsilon}F^{\delta\epsilon}F_{\epsilon}{}^{\mu}R_{\varepsilon\mu\beta\gamma}\nabla^{\alpha}\nabla^{\beta}\Phi$$
$$+e^{4\Phi}c_{99}F_{\alpha}{}^{\gamma}F_{\delta\epsilon}F^{\delta\epsilon}F^{\varepsilon\mu}R_{\varepsilon\mu\beta\gamma}\nabla^{\alpha}\nabla^{\beta}\Phi + e^{4\Phi}c_{102}F_{\alpha}{}^{\gamma}F_{\gamma}{}^{\delta}F_{\delta}{}^{\epsilon}$$
$$+e^{4\Phi}c_{104}F_{\alpha}{}^{\gamma}F_{\beta}{}^{\delta}F_{\gamma}{}^{\epsilon}F^{\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^{\alpha}\nabla^{\beta}\Phi, F^{\varepsilon\mu}R_{\varepsilon\mu\beta\epsilon}\nabla^{\alpha}\nabla^{\beta}\Phi$$
$$[F^{2}\Phi'^{6}]_{2} = e^{2\Phi}c_{224}F_{\delta\epsilon}F^{\delta\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla_{\beta}\Phi\nabla^{\beta}\Phi\nabla_{\gamma}\Phi\nabla^{\gamma}\Phi + e^{2\Phi}c_{330}F_{\gamma}{}^{\epsilon}F_{\delta\epsilon}\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi\nabla_{\beta}\Phi\nabla^{\beta}\Phi\nabla^{\gamma}\Phi\nabla^{\delta}\Phi,$$
$$[F^{4}\Phi'^{2}]_{8} = e^{4\Phi}c_{113}F_{\beta}{}^{\delta}F_{\gamma}{}^{\epsilon}F_{\delta}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla_{\alpha}\nabla^{\gamma}\Phi\nabla^{\alpha}\nabla^{\beta}\Phi + e^{4\Phi}c_{114}F_{\beta}{}^{\delta}F_{\gamma\delta}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla_{\alpha}\nabla^{\gamma}\Phi\nabla^{\alpha}\nabla^{\beta}\Phi$$
$$+e^{4\Phi}c_{137}F_{\gamma}{}^{\epsilon}F^{\gamma\delta}F_{\delta}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla_{\beta}\nabla_{\alpha}\Phi + e^{4\Phi}c_{138}F_{\gamma\delta}F^{\gamma\delta}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla_{\beta}\nabla_{\alpha}\Phi$$
$$+e^{4\Phi}c_{226}F_{\alpha}{}^{\epsilon}F_{\beta}{}^{\varepsilon}F_{\gamma\epsilon}F_{\delta\varepsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi + e^{4\Phi}c_{227}F_{\alpha}{}^{\epsilon}F_{\beta\epsilon}F_{\gamma}{}^{\varepsilon}F_{\delta\varepsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi$$
$$+e^{4\Phi}c_{228}F_{\alpha\gamma}F_{\beta}{}^{\epsilon}F_{\delta}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi + e^{4\Phi}c_{229}F_{\alpha\gamma}F_{\beta\delta}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi,$$
$$[F^{2}R\Phi'^{2}]_{9} = e^{2\Phi}c_{115}F_{\delta}{}^{\varepsilon}F^{\delta\epsilon}R_{\beta\epsilon\gamma\varepsilon}\nabla_{\alpha}\nabla^{\gamma}\Phi\nabla^{\alpha}\nabla^{\beta}\Phi + e^{2\Phi}c_{116}F_{\beta}{}^{\delta}F^{\epsilon\varepsilon}R_{\epsilon\varepsilon\gamma\delta}\nabla_{\alpha}\nabla^{\gamma}\Phi\nabla^{\alpha}\nabla^{\beta}\Phi$$
$$+e^{2\Phi}c_{139}F^{\gamma\delta}F^{\epsilon\varepsilon}R_{\gamma\epsilon\delta\varepsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla_{\beta}\nabla_{\alpha}\Phi + e^{2\Phi}c_{230}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}R_{\alpha\gamma\beta\delta}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi$$
$$+e^{2\Phi}c_{231}F_{\alpha}{}^{\epsilon}F_{\gamma}{}^{\varepsilon}R_{\beta\varepsilon\delta\epsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi + e^{2\Phi}c_{232}F_{\alpha}{}^{\epsilon}F_{\beta}{}^{\varepsilon}R_{\gamma\epsilon\delta\varepsilon}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi$$
$$+e^{2\Phi}c_{233}F_{\alpha}{}^{\epsilon}F_{\epsilon}{}^{\varepsilon}R_{\gamma\varepsilon\beta\delta}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi + e^{2\Phi}c_{236}F_{\alpha}{}^{\epsilon}F_{\gamma}{}^{\varepsilon}R_{\epsilon\varepsilon\beta\delta}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi$$
$$+e^{2\Phi}c_{237}F_{\alpha\gamma}F^{\epsilon\varepsilon}R_{\epsilon\varepsilon\beta\delta}\nabla^{\alpha}\nabla^{\beta}\Phi\nabla^{\gamma}\nabla^{\delta}\Phi,$$
$$[F^{2}F'^{2}\Phi'']_{21} = e^{4\Phi}c_{136}F_{\gamma\delta}F_{\epsilon\varepsilon}\nabla_{\alpha}\nabla^{\varepsilon}\Phi\nabla^{\alpha}F^{\beta\gamma}\nabla_{\beta}F^{\delta\epsilon} + e^{4\Phi}c_{141}F_{\gamma\delta}F_{\epsilon\varepsilon}\nabla_{\alpha}F^{\delta\epsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\beta}\nabla^{\varepsilon}\Phi$$
$$+e^{4\Phi}c_{200}F_{\delta}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla_{\alpha}F_{\beta}{}^{\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\gamma}\nabla^{\epsilon}\Phi + e^{4\Phi}c_{201}F_{\delta}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\beta}F_{\alpha}{}^{\delta}\nabla_{\gamma}\nabla^{\epsilon}\Phi$$
$$+e^{4\Phi}c_{256}F_{\gamma}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\beta}F^{\delta\epsilon}\nabla_{\delta}\nabla_{\alpha}\Phi + e^{4\Phi}c_{257}F_{\gamma}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla_{\alpha}F^{\delta\epsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\delta}\nabla_{\beta}\Phi$$
$$+e^{4\Phi}c_{259}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla_{\alpha}F_{\beta}{}^{\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\delta}\nabla_{\gamma}\Phi + e^{4\Phi}c_{260}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\beta}F_{\alpha}{}^{\delta}\nabla_{\delta}\nabla_{\gamma}\Phi$$
$$+e^{4\Phi}c_{265}F_{\alpha\gamma}F_{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\beta}F^{\delta\epsilon}\nabla_{\delta}\nabla^{\varepsilon}\Phi + e^{4\Phi}c_{289}F_{\gamma\varepsilon}F_{\delta\epsilon}\nabla_{\alpha}\nabla^{\varepsilon}\Phi\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F_{\beta}{}^{\epsilon}$$
$$+e^{4\Phi}c_{290}F_{\gamma\delta}F_{\epsilon\varepsilon}\nabla_{\alpha}\nabla^{\varepsilon}\Phi\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F_{\beta}{}^{\epsilon} + e^{4\Phi}c_{293}F_{\delta}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\gamma}\nabla_{\alpha}\Phi\nabla^{\delta}F_{\beta}{}^{\epsilon}$$
$$+e^{4\Phi}c_{294}F_{\alpha\delta}F_{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\gamma}\nabla^{\varepsilon}\Phi\nabla^{\delta}F_{\beta}{}^{\epsilon} + e^{4\Phi}c_{297}F_{\gamma}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\delta}\nabla_{\alpha}\Phi\nabla^{\delta}F_{\beta}{}^{\epsilon}$$
$$+e^{4\Phi}c_{318}F_{\gamma\delta}F_{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\beta}\nabla_{\alpha}\Phi\nabla^{\delta}F^{\epsilon\varepsilon} + e^{4\Phi}c_{324}F_{\beta\epsilon}F_{\gamma\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\delta}\nabla_{\alpha}\Phi\nabla^{\delta}F^{\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{325}F_{\beta\gamma}F_{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\delta}\nabla_{\alpha}\Phi\nabla^{\delta}F^{\epsilon\varepsilon} + e^{4\Phi}c_{336}F_{\delta}{}^{\varepsilon}F_{\epsilon\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\beta}F_{\alpha\gamma}\nabla^{\delta}\nabla^{\epsilon}\Phi$$
$$+e^{4\Phi}c_{345}F_{\alpha\delta}F_{\gamma\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\delta}F^{\epsilon\varepsilon}\nabla_{\epsilon}\nabla_{\beta}\Phi + e^{4\Phi}c_{371}F_{\gamma\epsilon}F_{\delta\varepsilon}\nabla_{\alpha}F_{\beta}{}^{\delta}\nabla^{\alpha}F^{\beta\gamma}\nabla^{\epsilon}\nabla^{\varepsilon}\Phi$$
$$+e^{4\Phi}c_{372}F_{\gamma\epsilon}F_{\delta\varepsilon}\nabla^{\alpha}F^{\beta\gamma}\nabla_{\beta}F_{\alpha}{}^{\delta}\nabla^{\epsilon}\nabla^{\varepsilon}\Phi,$$

$$[F^3R\Phi'F']_1 = e^{4\Phi}c_{142}F_\beta{}^\delta F_\gamma{}^\epsilon F^{\varepsilon\mu}R_{\varepsilon\mu\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta F_\alpha{}^\gamma,$$

$$[FR\Phi''\Phi'F']_2 = e^{2\Phi}c_{157}F_\beta{}^\varepsilon R_{\epsilon\varepsilon\gamma\delta}\nabla_\alpha\nabla^\epsilon\Phi\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta} + e^{2\Phi}c_{337}F_\delta{}^\varepsilon R_{\epsilon\varepsilon\beta\gamma}\nabla^\alpha\Phi\nabla^\beta F_\alpha{}^\gamma\nabla^\delta\nabla^\epsilon\Phi,$$

$$[F^2\Phi''^2\Phi'^2]_4 = e^{2\Phi}c_{194}F_\gamma{}^\epsilon F_{\delta\epsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\Phi\nabla_\beta\nabla^\delta\Phi\nabla^\beta\Phi + e^{2\Phi}c_{199}F_{\delta\epsilon}F^{\delta\epsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma\nabla_\beta\Phi$$
$$+e^{2\Phi}c_{248}F_\gamma{}^\epsilon F_{\delta\epsilon}\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\nabla^\delta\Phi + e^{2\Phi}c_{338}F_{\beta\delta}F_{\gamma\epsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\delta\nabla^\epsilon\Phi,$$

$$[F^2\Phi'^2F'^2]_{17} = e^{4\Phi}c_{195}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta}\nabla_\gamma F_{\beta\delta} + e^{4\Phi}c_{196}F_\delta{}^\varepsilon F_{\epsilon\varepsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta F^{\gamma\delta}\nabla_\gamma F_\beta{}^\epsilon$$
$$+e^{4\Phi}c_{197}F_\delta{}^\varepsilon F_{\epsilon\varepsilon}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma F_\beta{}^\epsilon + e^{4\Phi}c_{203}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma F_{\beta\delta}\nabla^\gamma F_\alpha{}^\delta$$
$$+e^{4\Phi}c_{204}F_\delta{}^\varepsilon F_{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma F_\beta{}^\epsilon\nabla^\gamma F_\alpha{}^\delta + e^{4\Phi}c_{205}F_{\beta\epsilon}F_{\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma F^{\epsilon\varepsilon}\nabla^\gamma F_\alpha{}^\delta$$
$$+e^{4\Phi}c_{250}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla_\delta F_{\beta\gamma} + e^{4\Phi}c_{252}F_\beta{}^\varepsilon F_{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla_\delta F_\gamma{}^\epsilon$$
$$+e^{4\Phi}c_{298}F_\delta{}^\varepsilon F_{\epsilon\varepsilon}\nabla_\alpha F_\beta{}^\gamma\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\delta F_\gamma{}^\epsilon + e^{4\Phi}c_{320}F_{\gamma\delta}F_{\epsilon\varepsilon}\nabla_\alpha F_\beta{}^\gamma\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{4\Phi}c_{356}F_\delta{}^\varepsilon F_{\epsilon\varepsilon}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\epsilon F_{\beta\gamma} + e^{4\Phi}c_{358}F_{\gamma\epsilon}F_{\delta\varepsilon}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\epsilon F_\beta{}^\varepsilon$$
$$+e^{4\Phi}c_{360}F_{\gamma\epsilon}F_{\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla^\epsilon F_\beta{}^\varepsilon + e^{4\Phi}c_{361}F_{\gamma\delta}F_{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla^\epsilon F_\beta{}^\varepsilon$$
$$+e^{4\Phi}c_{363}F_\beta{}^\varepsilon F_{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla^\epsilon F_{\gamma\delta} + e^{4\Phi}c_{367}F_{\beta\epsilon}F_{\delta\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla^\epsilon F_\gamma{}^\varepsilon$$
$$+e^{4\Phi}c_{368}F_{\beta\delta}F_{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma F_\alpha{}^\delta\nabla^\epsilon F_\gamma{}^\varepsilon,$$

$$[FF'\Phi'\Phi''^2]_1 = e^{2\Phi}c_{202}F_{\delta\epsilon}\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta F^{\gamma\delta}\nabla_\gamma\nabla^\epsilon\Phi,$$

$$[FF'\Phi'^3\Phi'']_5 = e^{2\Phi}c_{225}F_{\delta\epsilon}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma\nabla^\epsilon\Phi\nabla^\gamma\Phi + e^{2\Phi}c_{258}F_{\gamma\epsilon}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla_\delta\nabla_\beta\Phi$$
$$+e^{2\Phi}c_{264}F_{\gamma\epsilon}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla_\delta\nabla^\epsilon\Phi + e^{2\Phi}c_{272}F_{\delta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma\nabla_\beta\Phi\nabla^\gamma\Phi\nabla^\delta F_\alpha{}^\epsilon$$
$$+e^{2\Phi}c_{273}F_{\gamma\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla_\delta\nabla_\beta\Phi\nabla^\delta F_\alpha{}^\epsilon,$$

$$[R\Phi'^2\Phi''^2]_2 = c_{249}R_{\alpha\delta\beta\epsilon}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma\nabla^\epsilon\Phi\nabla^\gamma\nabla^\delta\Phi + c_{339}R_{\gamma\delta\beta\epsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\delta\nabla^\epsilon\Phi,$$

$$[\Phi'^4F'^2]_4 = e^{2\Phi}c_{251}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi\nabla^\gamma F^{\delta\epsilon}\nabla_\delta F_{\gamma\epsilon} + e^{2\Phi}c_{296}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla_\delta F_{\gamma\epsilon}\nabla^\delta F_\beta{}^\epsilon$$
$$+e^{2\Phi}c_{332}\nabla_\alpha F_\beta{}^\epsilon\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma F_{\delta\epsilon}\nabla^\gamma\Phi\nabla^\delta\Phi + e^{2\Phi}c_{343}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta F_\beta{}^\epsilon\nabla_\epsilon F_{\gamma\delta},$$

$$[\Phi''^4]_2 = c_{261}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\nabla^\beta\Phi\nabla_\beta\nabla^\delta\Phi\nabla_\delta\nabla_\gamma\Phi + c_{263}\nabla^\alpha\nabla^\beta\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\gamma\nabla^\delta\Phi\nabla_\delta\nabla_\gamma\Phi,$$

$$[FF'\Phi'^5]_2 = e^{2\Phi}c_{299}F_{\delta\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta F_\gamma{}^\epsilon + e^{2\Phi}c_{329}F_{\delta\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta F_\gamma{}^\epsilon\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta\Phi,$$

$$[F^2\Phi'^4\Phi'']_2 = e^{2\Phi}c_{331}F_\gamma{}^\epsilon F_{\delta\epsilon}\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta\Phi + e^{2\Phi}c_{340}F_{\beta\delta}F_{\gamma\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta\nabla^\epsilon\Phi,$$

$$[R\Phi''^3]_1 = c_{335}R_{\beta\delta\gamma\epsilon}\nabla_\alpha\nabla^\gamma\Phi\nabla^\alpha\nabla^\beta\Phi\nabla^\delta\nabla^\epsilon\Phi,$$
$$[R\Phi'^4\Phi'']_1 = c_{341}R_{\beta\delta\gamma\epsilon}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla^\delta\nabla^\epsilon\Phi,$$
$$[F'^2\Phi''^2]_6 = e^{2\Phi}c_{347}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\beta\Phi\nabla_\epsilon\nabla_\gamma\Phi + e^{2\Phi}c_{348}\nabla^\alpha F^{\beta\gamma}\nabla_\delta\nabla_\alpha\Phi\nabla^\delta F_\beta{}^\epsilon\nabla_\epsilon\nabla_\gamma\Phi$$
$$+e^{2\Phi}c_{350}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla^\epsilon\Phi\nabla_\epsilon\nabla_\delta\Phi + e^{2\Phi}c_{351}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta\nabla_\gamma\nabla^\epsilon\Phi\nabla_\epsilon\nabla_\delta\Phi$$
$$+e^{2\Phi}c_{353}\nabla^\alpha F^{\beta\gamma}\nabla_\gamma\nabla_\alpha\Phi\nabla^\delta F_\beta{}^\epsilon\nabla_\epsilon\nabla_\delta\Phi + e^{2\Phi}c_{355}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma}\nabla^\delta\nabla^\epsilon\Phi\nabla_\epsilon\nabla_\delta\Phi,$$
$$[F'^2\Phi'^2\Phi'']_4 = e^{2\Phi}c_{349}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma F_\beta{}^\epsilon\nabla_\epsilon\nabla_\delta\Phi + e^{2\Phi}c_{352}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\gamma F_\beta{}^\epsilon\nabla^\gamma F_\alpha{}^\delta\nabla_\epsilon\nabla_\delta\Phi$$
$$+e^{2\Phi}c_{354}\nabla_\alpha F_\beta{}^\gamma\nabla^\alpha\Phi\nabla^\beta\Phi\nabla^\delta F_\gamma{}^\epsilon\nabla_\epsilon\nabla_\delta\Phi + e^{2\Phi}c_{357}\nabla_\alpha F^{\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\nabla_\epsilon\nabla_\delta\Phi\nabla^\epsilon F_{\beta\gamma},$$
$$[FF'^3\Phi']_1 = e^{4\Phi}c_{377}F_{\epsilon\varepsilon}\nabla^\alpha\Phi\nabla^\beta F_\alpha{}^\gamma\nabla^\delta F_\beta{}^\epsilon\nabla^\varepsilon F_{\gamma\delta},$$
$$[\Phi'^4\Phi''^2]_2 = c_{262}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\nabla^\delta\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla_\delta\nabla_\gamma\Phi + c_{334}\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\Phi\nabla^\gamma\Phi\nabla_\delta\nabla_\gamma\Phi\nabla^\delta\Phi,$$
$$[\Phi'^6\Phi'']_1 = c_{333}\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi\nabla_\gamma\Phi\nabla^\gamma\Phi\nabla_\delta\Phi\nabla^\delta\Phi,$$
$$[F^8]_5 = e^{8\Phi}c_1F_\alpha{}^\gamma F^{\alpha\beta}F_\beta{}^\delta F_\gamma{}^\epsilon F_\delta{}^\varepsilon F_\epsilon{}^\mu F_\varepsilon{}^\nu F_{\mu\nu} + e^{8\Phi}c_2F_{\alpha\beta}F^{\alpha\beta}F_\gamma{}^\epsilon F^{\gamma\delta}F_\delta{}^\varepsilon F_\epsilon{}^\mu F_\varepsilon{}^\nu F_{\mu\nu}$$
$$+e^{8\Phi}c_3F_\alpha{}^\gamma F^{\alpha\beta}F_\beta{}^\delta F_{\gamma\delta}F_\epsilon{}^\mu F^{\epsilon\varepsilon}F_\varepsilon{}^\nu F_{\mu\nu} + e^{8\Phi}c_4F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}F_\epsilon{}^\mu F^{\epsilon\varepsilon}F_\varepsilon{}^\nu F_{\mu\nu}$$
$$+e^{8\Phi}c_5F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}F_{\epsilon\varepsilon}F^{\epsilon\varepsilon}F_{\mu\nu}F^{\mu\nu},$$
$$[F^4F^2]_{20} = e^{6\Phi}c_{57}F_\gamma{}^\epsilon F_\delta{}^\varepsilon F_\epsilon{}^\mu F_{\varepsilon\mu}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma} + e^{6\Phi}c_{58}F_\gamma{}^\epsilon F_{\delta\epsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla_\alpha F_\beta{}^\delta\nabla^\alpha F^{\beta\gamma}$$
$$+e^{6\Phi}c_{61}F_\beta{}^\varepsilon F_\gamma{}^\mu F_{\delta\varepsilon}F_{\epsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + e^{6\Phi}c_{62}F_{\beta\delta}F_\gamma{}^\varepsilon F_\epsilon{}^\mu F_{\varepsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}$$
$$+e^{6\Phi}c_{63}F_{\beta\gamma}F_\delta{}^\varepsilon F_\epsilon{}^\mu F_{\varepsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + e^{6\Phi}c_{64}F_{\beta\delta}F_{\gamma\epsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma}$$
$$+e^{6\Phi}c_{65}F_{\beta\gamma}F_{\delta\epsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla_\alpha F^{\delta\epsilon}\nabla^\alpha F^{\beta\gamma} + e^{6\Phi}c_{118}F_\delta^\varepsilon F^{\delta\epsilon}F_\epsilon^\mu F_{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma}$$
$$+e^{6\Phi}c_{119}F_{\delta\epsilon}F^{\delta\epsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_{\alpha\gamma} + e^{6\Phi}c_{122}F_\gamma{}^\epsilon F_\delta{}^\varepsilon F_\epsilon{}^\mu F_{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta$$
$$+e^{6\Phi}c_{123}F_\gamma{}^\epsilon F_{\delta\epsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F_\alpha{}^\delta + e^{6\Phi}c_{128}F_{\alpha\delta}F_\gamma{}^\varepsilon F_\epsilon{}^\mu F_{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla_\beta F^{\delta\epsilon}$$
$$+e^{6\Phi}c_{274}F_\alpha{}^\varepsilon F_\gamma{}^\mu F_{\delta\varepsilon}F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{6\Phi}c_{275}F_\alpha{}^\varepsilon F_{\gamma\varepsilon}F_\delta{}^\mu F_{\epsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon$$
$$+e^{6\Phi}c_{276}F_{\alpha\delta}F_\gamma{}^\varepsilon F_\epsilon{}^\mu F_{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon + e^{6\Phi}c_{277}F_{\alpha\delta}F_{\gamma\epsilon}F_{\varepsilon\mu}F^{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F_\beta{}^\epsilon$$
$$+e^{6\Phi}c_{300}F_{\alpha\epsilon}F_{\beta\delta}F_\gamma{}^\mu F_{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon} + e^{6\Phi}c_{301}F_{\alpha\delta}F_{\beta\epsilon}F_\gamma{}^\mu F_{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon}$$
$$+e^{6\Phi}c_{302}F_{\alpha\beta}F_\gamma{}^\mu F_{\delta\epsilon}F_{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon} + e^{6\Phi}c_{303}F_{\alpha\beta}F_{\gamma\epsilon}F_\delta{}^\mu F_{\varepsilon\mu}\nabla^\alpha F^{\beta\gamma}\nabla^\delta F^{\epsilon\varepsilon},$$

$$ds^2 = \frac{u}{\sqrt{h_4h_8}}\left(ds^2(\mathrm{AdS}_3) + \frac{h_8h_4}{g_1}ds^2(\mathrm{S}^2)\right) + \frac{\sqrt{h_4h_8}}{u}d\rho^2 + \sqrt{\frac{h_4}{h_8}}ds^2(\mathrm{CY}_2)$$
$$e^\Phi = \frac{2h_4^{1/4}}{h_8^{3/4}}\sqrt{\frac{u}{g_1}}, B_2 = (g_2 + 2\pi k)\mathrm{vol}(\mathrm{S}^2), g_1 = 4h_4h_8 + u'^2, g_2 = -\frac{1}{2}\left(\rho - \frac{uu'}{g_1}\right)$$

$$F_0 = h_8', F_2 = -\frac{1}{2}\left(h_8 - \frac{h_8'uu'}{g_1}\right)\mathrm{vol}(\mathrm{S}^2), F_8 = \star_{10}F_2, F_{10} = -\star_{10}F_0$$
$$F_4 = -\left(\left(\frac{uu'}{2h_4}\right)' + 2h_8\right)d\rho\wedge\mathrm{vol}(\mathrm{AdS}_3) - h_4'\mathrm{vol}(\mathrm{CY}_2), F_6 = -\star_{10}F_4$$

$$h_{4,8}(\rho) = h_{4,8}^{(k)}\Theta[k, k+1],$$
$$h_{4,8}(\rho)\big|_{\rho=\rho_i,\rho_f} = 0,$$
$$h_{4,8}^{(k-1)}(\rho)\Big|_{\rho=k} = h_{4,8}^{(k)}(\rho)\Big|_{\rho=k},$$
$$u^{(k-1)}(\rho)\big|_{\rho=k} = u^{(k)}(\rho)\big|_{\rho=k},$$
$$u^{(k-1)}(\rho)\big|_{\rho=k} = u^{(k)}(\rho)\big|_{\rho=k}$$

$$ds^2 = e^{2A}ds^2(\mathrm{AdS}_3) + ds^2(M_7), F = f + e^{3A}\mathrm{vol}(\mathrm{AdS}_3) \wedge \star_7 \lambda(f),$$

$$\epsilon_1 = \sum_{I=1}^{4} \xi^I \otimes \begin{pmatrix}1\\0\end{pmatrix} \otimes \chi_1^I, \epsilon_2 = \sum_{I=1}^{4} \xi^I \otimes \begin{pmatrix}0\\1\end{pmatrix} \otimes \chi_2^I$$

$$ds^2 = e^{2A}ds(\mathrm{AdS}_3) + e^{2C}ds^2(\mathrm{S}^2) + ds^2(M_5)$$

$$d_H\left(e^{A-\Phi}\Psi_-\right) = 0$$
$$d_H\left(e^{2A-\Phi}\Psi_+\right) - 2me^{A-\Phi}\Psi_- = \frac{e^{3A}}{8} \star_7 \lambda(f)$$
$$(\Psi_-, f)_7 = \mp\frac{m}{2}e^{-\Phi}\mathrm{vol}(M_7)$$

$$\Psi_+ + i\Psi_- = \frac{1}{8}\sum_{j=0}^{d=7} \frac{1}{j!}\left(\chi_2^\dagger \gamma_{a_1,\ldots,a_j}\chi_1\right) e^{a_1} \wedge \ldots \wedge e^{a_j}$$

$$\chi_1^I = \frac{1}{2}e^{\frac{A}{2}}\mathcal{M}_{\delta\sigma}^I\left(\left(\cos\frac{\beta}{2} + \sin\frac{\beta}{2}\right)\xi^\sigma + \left(\cos\frac{\beta}{2} - \sin\frac{\beta}{2}\right)\hat{\xi}^\delta\right) \otimes \eta_1^\sigma$$
$$\chi_2^I = \frac{1}{2}e^{\frac{A}{2}}\mathcal{M}_{\delta\sigma}^I\left(\left(\cos\frac{\beta}{2} - \sin\frac{\beta}{2}\right)\xi^\delta + \left(\cos\frac{\beta}{2} + \sin\frac{\beta}{2}\right)\hat{\xi}^\delta\right) \otimes \eta_2^\sigma$$

$$\eta_2 = a_1\eta + a_2\eta^c + \frac{a_3}{2}\bar{w}\eta, |\eta| = 1,$$

$$\psi_+^1 = \left(\eta \otimes \eta^\dagger\right)_+ = \frac{1}{4}e^{-ij}, \qquad \psi_+^2 = \left(\eta \otimes \eta^{c\dagger}\right)_+ = \frac{1}{4}\omega$$
$$\psi_-^1 = \left(\eta \otimes \eta^\dagger\right)_- = \frac{1}{4}v \wedge e^{-ij}, \quad \psi_-^2 = \left(\eta \otimes \eta^{c\dagger}\right)_- = \frac{1}{4}v \wedge \omega$$

$$j = e^7 \wedge e^8 + e^9 \wedge e^{10}, \omega = (e^7 + ie^8) \wedge (e^9 + ie^{10})$$

$$j \wedge \omega = \omega \wedge \omega = 0, \omega \wedge \bar{\omega} = 2j \wedge j = \mathrm{vol}(M_4),$$
$$\iota_v v = 1, \iota_v j = \iota_v \omega = 0,$$

$$e^\rho = \frac{(h_8h_4)^{1/4}}{\sqrt{u}}d\rho, e^i = \left(\frac{h_4}{h_8}\right)^{1/4} dz_i, i = 7, \ldots, 10$$

$$\Psi_+^{(2)} = \frac{1}{2}\mathrm{Re}\left[\psi_+^1 \wedge \left(\left(1 + ie^{2C}y_3\mathrm{vol}(\mathrm{S}^2)\right)\cos\beta + i\left(y_3 + ie^{2C}\mathrm{vol}(\mathrm{S}^2)\right)\sin\beta\right) + i\psi_-^1 \wedge K_3e^C\right.$$
$$\left. + \psi_+^2 \wedge \left(e^{2C}\mathrm{vol}(\mathrm{S}^2)\right)\cos\beta + \sin\beta)(iy_1 + y_2)e^{2C} + \psi_-^2 \wedge (K_2 + iK_1)e^C\right]$$
$$\Psi_-^{(2)} = -\frac{1}{2}\mathrm{Im}\left[\psi_+^1 \wedge dy_3e^C - \psi_-^1 \wedge \left(\left(y_3 + ie^{2C}\mathrm{vol}(\mathrm{S}^2)\right)\cos\beta + \left(i - y_3e^{2C}\mathrm{vol}(\mathrm{S}^2)\right)\sin\beta\right)\right.$$
$$\left. + \psi_+^2 \wedge (dy_1 - idy_2)e^C - \psi_-^2 \wedge \left(\cos\beta - e^{2C}\mathrm{vol}(\mathrm{S}^2)\right)\sin\beta)(y_1 - iy_2)\right]$$

$$S_{\mathrm{DBI}} = -T_p\int e^{-\Phi}\sqrt{|\mathcal{P}[g + \mathcal{F}]|}dX^{p+1}, S_{\mathrm{WZ}} = \pm T_p\int \mathcal{P}[C \wedge e^{\mathcal{F}} + F_0\sigma]$$

$$ds^2(\mathrm{AdS}_3) = r^2(-dt^2 + dx^2) + \frac{dr^2}{r^2}$$

$$C_1 = \frac{1}{2}(h_8 - (\rho - 2\pi k)h_8')\eta, d\eta = -\mathrm{vol}(\mathrm{S}^2)$$

$$C_3 = \frac{1}{2h_4^2} dt \wedge d\rho \wedge \left((h_4 g_1 - uu' h_4')rxdr + \sqrt{h_4^3 h_8 g_1}\, d(r^2 x) \right) - z_7 h_4' \mathrm{vol}(\mathrm{T}^3)$$

$$C_5 = -\frac{u^2}{4h_4}\mathrm{vol}(\mathrm{AdS}_3) \wedge \mathrm{vol}(\mathrm{S}^2) + \frac{1}{2}(h_4 - \rho h_4')\mathrm{vol}(\mathrm{T}^4) \wedge \eta$$
$$- \left(\frac{2h_4 u' u g_1 - h_4' u^2 (g_1 + 4h_4 h_8)}{8h_4^2 g_1} r^2 dt \wedge dx \wedge d\rho - g_2 z_7 h_4' \mathrm{vol}(\mathrm{T}^3) \right) \wedge \mathrm{vol}(\mathrm{S}^2)$$

$$C_7 = \frac{h_4}{h_8}\left(C_3 + \frac{2g_1 h_4 h_8 - uu' \partial_\rho (h_4 h_8)}{2h_4^2} rxdt \wedge dr \wedge d\rho \right) \wedge \mathrm{vol}(\mathrm{T}^4)$$

$$C_9 = \frac{h_4}{h_8}\left(C_5 - \frac{2f_1 h_4' h_8 - u'^2 \partial_\rho (h_4 h_8)}{8h_4^2 h_8 f_1} u^2 r^2 dt \wedge dx \wedge d\rho \right) \wedge \mathrm{vol}(\mathrm{S}^2) \wedge \mathrm{vol}(\mathrm{T}^4)$$

$$e^{-\Phi}\sqrt{|\mathcal{P}[g + B_2]|}\Big|_{\Sigma^{(3)}} = \frac{1}{2} r^2 \sqrt{\frac{h_8}{h_4} g_1}$$

$$C_3 - C_1 \wedge B_2|_{\Sigma^{(3)}} = \frac{1}{2} r^2 \sqrt{\frac{h_8}{h_4} g_1}$$

$$e^{-\Phi}\sqrt{|\mathcal{P}[g + B]|}\Big|_{\Sigma^{(7)}} = \frac{1}{2} r^2 \sqrt{\frac{h_4}{h_8} g_1}$$

$$C_7 - C_5 \wedge B_2|_{\Sigma^{(7)}} = -\frac{1}{2} r^2 \sqrt{\frac{h_4}{h_8} g_1}$$

$$e^{-\Phi}\sqrt{|\mathcal{P}[g + B]|}\Big|_{\Sigma^{(5)}} = \frac{1}{4} \frac{u\sqrt{4h_4 h_8 u^2 + (uu' - g_1(\rho - 2\pi k))^2}\, r \sin\theta}{h_4 \sqrt{g_1}}$$

$$C_5 - C_3 \wedge B_2|_{\Sigma^{(5)}} = -\frac{1}{4}\frac{u^2}{h_4} r \sin\theta$$

$$e^{-\Phi}\sqrt{|\mathcal{P}[g + B]|}\Big|_{\Sigma^{(9)}} = \frac{1}{4} \frac{u\sqrt{4h_4 h_8 u^2 + (uu' - g_1(\rho - 2\pi k))^2}\, r \sin\theta}{h_8 \sqrt{g_1}}$$

$$C_9 - C_7 \wedge B_2|_{\Sigma^{(9)}} = -\frac{1}{4}\frac{u^2}{h_8} r \sin\theta$$

$$e^{-\Phi}\sqrt{|\mathcal{P}[g + \mathcal{F}]|}\Big|_{\Sigma} = \pm \Psi_{Dp}^{(\mathrm{cal})}\Big|_{\Sigma} = \pm 8 e^{3A - \Phi} \mathrm{vol}(\mathrm{AdS}_3) \wedge \Psi_- \wedge e^{-\mathcal{F}}\Big|_{\Sigma}$$

$$\Psi_{D4}^{(\mathrm{cal})} = \frac{u\big(4h_4 h_8 u + u'(uu' - g_1(\rho - 2\pi k))\big)}{4h_4 g_1} \mathrm{vol}(\mathrm{AdS}_3) \wedge \mathrm{vol}(\mathrm{S}^2)$$

$$\Psi_{D8}^{(\mathrm{cal})} = \frac{u\big(4h_4 h_8 u + u'(uu' - g_1(\rho - 2\pi k))\big)}{4h_8 g_1} \mathrm{vol}(\mathrm{AdS}_3) \wedge \mathrm{vol}(\mathrm{S}^2) \wedge \mathrm{vol}(\mathrm{T}^4)$$

$$\frac{ds^2}{2\pi}=\frac{|h|}{\sqrt{\Delta}}ds^2(\mathrm{AdS}_3)+\sqrt{\Delta}\left[\frac{1}{4|h|}d\rho^2+\frac{2}{|h''|}ds^2(\mathbb{CP}^3)\right]$$
$$e^{-\Phi}=\frac{(|h''|)^{\frac{3}{2}}}{2\sqrt{\pi}\Delta^{\frac{1}{4}}},B_2=c(\rho)J,c(\rho)=4\pi\left(-(\rho-k)+\frac{h'}{h''}\right)$$

$$ds^2(\mathbb{CP}^3)=d\xi^2+\frac{1}{4}\cos^2\,\xi ds^2(\,\mathrm{S}_1^2)+\frac{1}{4}\sin^2\,\xi ds^2(\,\mathrm{S}_2^2)+\frac{1}{4}\sin^2\,\xi\cos^2\,\xi(d\psi+\eta_1-\eta_2)^2$$
$$d\eta_i=-\mathrm{vol}\left(\mathrm{S}_i^2\right)$$

$$J=-\frac{1}{4}\sin^2\,\xi\mathrm{vol}(\,\mathrm{S}_2^2)-\frac{1}{4}\cos^2\,\xi\mathrm{vol}(\,\mathrm{S}_1^2)-\frac{1}{2}\sin\,\xi\cos\,\xi d\xi\wedge(d\psi+\eta_1-\eta_2)$$

$$e^1=2\sqrt{\pi}\frac{\Delta^{1/4}}{\sqrt{h''}}d\xi,e^2=\sqrt{\pi}\frac{\Delta^{1/4}}{\sqrt{h''}}d\theta_1,e^3=\sqrt{\pi}\frac{\Delta^{1/4}}{\sqrt{h''}}\sin\,\theta_1 d\phi_1,e^4=\sqrt{\pi}\frac{\Delta^{1/4}}{\sqrt{h''}}d\theta_2$$
$$e^5=\sqrt{\pi}\frac{\Delta^{1/4}}{\sqrt{h''}}\sin\,\theta_2 d\phi_2.e^6=\sqrt{\pi}\frac{\Delta^{1/4}}{\sqrt{h''}}(d\psi+\eta_1-\eta_2),e^7=\sqrt{\frac{\pi}{2}}\frac{\Delta^{1/4}}{\sqrt{h}}d\rho$$

$$F_0=-\frac{1}{2\pi}h''',F_2=B_2F_0+2(h''-(\rho-k)h''')J$$
$$F_4=\pi d\left(h'+\frac{hh'h''}{\Delta}\right)\wedge\mathrm{vol}(\mathrm{AdS}_3)+B_2\wedge F_2-\frac{1}{2}B_2\wedge B_2F_0$$
$$-4\pi\big(2h'+(\rho-k)(-2h''+(\rho-k)h''')\big)J\wedge J$$

$$h=Q_{\mathrm{D}_2}-Q_{\mathrm{D}_4}\rho+\frac{1}{2}Q_{\mathrm{D}_6}\rho^2$$

$$\zeta_6^{\mathcal{J}}=\frac{1}{\sqrt{2}}\begin{pmatrix}i(\zeta_+^1+\zeta_+^4)\\ \zeta_+^1+\zeta_+^4\\ i(\zeta_+^6+\zeta_+^7)\\ \zeta_+^6-\zeta_+^7\\ \frac{1}{\sqrt{2}}\left(\zeta_+^2-\zeta_+^5+(\zeta_+^3-\zeta_+^8)\right)\\ \frac{i}{\sqrt{2}}\left(\zeta_+^2-\zeta_+^5-(\zeta_+^3-\zeta_+^8)\right)\end{pmatrix}^{\mathcal{J}},\hat{\zeta}_6^{\mathcal{J}}=i\gamma_7\zeta_6^{\mathcal{J}}$$

$$\chi_1^{\mathcal{J}}=\cos\left(\frac{\beta_1+\beta_2}{2}\right)\zeta_6^{\mathcal{J}}+\sin\left(\frac{\beta_1+\beta_2}{2}\right)\hat{\zeta}_6^{\mathcal{J}}$$
$$\chi_2^{\mathcal{J}}=\cos\left(\frac{\beta_1-\beta_2}{2}\right)\zeta_6^{\mathcal{J}}+\sin\left(\frac{\beta_1-\beta_2}{2}\right)\hat{\zeta}_6^{\mathcal{J}}$$

$$\Psi_+^{(5)}=\frac{1}{8}\mathrm{Re}\left[e^{i\beta_2}e^{-i\mathcal{J}}-e^{i\beta_1}\Omega\wedge V\right],\Psi_-^{(5)}=\frac{1}{8}\mathrm{Im}\left[e^{i\beta_2}e^{-i\mathcal{J}}\wedge V+e^{i\beta_1}\Omega\right]$$

$$
\begin{aligned}
\mathcal{J} =& -\left(e^1\wedge e^6 + c_{2\xi}(e^2\wedge e^3 - e^4\wedge e^5) + c_\psi s_{2\xi}(e^2\wedge e^4 + e^3\wedge e^5)\right)\\
&+s_{2\xi}s_\psi(e^3\wedge e^4 - e^2\wedge e^5)\\
V =& e^7\\
\Omega =& s_{2\xi}(e^1\wedge e^2\wedge e^3 - e^1\wedge e^4\wedge e^5) - c_\psi c_{2\xi}(e^1\wedge e^2\wedge e^4 + e^1\wedge e^3\wedge e^5)\\
&+c_{2\xi}s_\psi(e^1\wedge e^2\wedge e^5 - e^1\wedge e^3\wedge e^4) - s_\psi(e^2\wedge e^4\wedge e^6 + e^3\wedge e^5\wedge e^6)\\
&-c_\psi(e^2\wedge e^5\wedge e^6 - e^3\wedge e^4\wedge e^6) + i\big[s_{2\xi}(e^4\wedge e^5\wedge e^6 - e^2\wedge e^3\wedge e^6)\\
&+c_\psi c_{2\xi}(e^2\wedge e^4\wedge e^6 + e^3\wedge e^5\wedge e^6) + c_{2\xi}s_\psi(e^3\wedge e^4\wedge e^6 - e^2\wedge e^5\wedge e^6)\\
&-s_\psi(e^1\wedge e^2\wedge e^4 + e^1\wedge e^3\wedge e^5) + c_\psi(e^1\wedge e^3\wedge e^4 - e^1\wedge e^2\wedge e^5)\big]
\end{aligned}
$$

$$
\iota_V\mathcal{J} = \iota_V\Omega = 0, \mathcal{J}\wedge\Omega = 0, \mathcal{J}\wedge\mathcal{J}\wedge\mathcal{J} = \frac{3i}{4}\Omega\wedge\bar{\Omega}
$$

$$
\begin{aligned}
\zeta_6^5 =& \frac{e^{-i\left(\xi+\frac{\psi}{2}\right)}}{4}\left(1-e^{2i\xi}, 1+e^{2i\xi}, e^{i\psi}\left(1+e^{2i\xi}\right), e^{i\psi}\left(1-e^{2i\xi}\right), -\left(1+e^{2i\xi}\right), -\left(1-e^{2i\xi}\right)\right.\\
&\left.-e^{i\psi}\left(1-e^{2i\xi}\right), -e^{i\psi}\left(1+e^{2i\xi}\right)\right)^T\\
\zeta_6^6 =& \frac{ie^{-i\left(\xi+\frac{\psi}{2}\right)}}{4}\left(1-e^{2i\xi}, 1+e^{2i\xi}, -e^{i\psi}\left(1+e^{2i\xi}\right), -e^{i\psi}\left(1-e^{2i\xi}\right), -\left(1+e^{2i\xi}\right), -\left(1-e^{2i\xi}\right)\right.\\
&\left.e^{i\psi}\left(1-e^{2i\xi}\right), e^{i\psi}\left(1+e^{2i\xi}\right)\right)^T
\end{aligned}
$$

$$
ds^2 = \frac{u}{\sqrt{h_4h_8}}\left(ds^2(\mathrm{AdS}_3) + \frac{h_8h_4}{g_1}ds^2(\mathrm{S}^2)\right) + \frac{\sqrt{h_4h_8}}{u}d\rho^2 + \sqrt{\frac{h_4}{h_8}}\left(dz_{7,10}^2 + Gdz_{9,8}^2\right)
$$

$$
e^{\tilde{\Phi}} = \sqrt{G}e^{\Phi}, \tilde{B}_2 = B_2 - \gamma Gg dz_9\wedge dz_8, G = (1+\gamma^2 g)^{-1}, g = \frac{h_4}{h_8}
$$

$$
\begin{aligned}
\tilde{F}_0 =& h_8', \tilde{F}_2 = F_2 - \gamma h_4' dz_7\wedge dz_{10} - \gamma Ggh_8' dz_9\wedge dz_8\\
\tilde{F}_4 =& F_4 + \gamma^2 h_4' Gg\mathrm{vol}(\mathrm{T}^4) +\\
&\frac{\gamma}{2}\left(Gg\left(h_8 - \frac{uu'h_8'}{g_1}\right)dz_9\wedge dz_8 + \left(h_4 - \frac{uu'h_4'}{f_1}\right)dz_7\wedge dz_{10}\right)\wedge\mathrm{vol}(\mathrm{S}^2)\\
\tilde{F}_6 =& -\star_{10}\tilde{F}_4, \tilde{F}_8 = \star_{10}\tilde{F}_2, \tilde{F}_{10} = -\star_{10}\tilde{F}_0
\end{aligned}
$$

$$
\begin{aligned}
\tilde{C}_1 =& C_1 - \gamma h_4' z_7 dz_{10}\\
\tilde{C}_3 =& C_3 + \tilde{C}_1\wedge\tilde{B}_2 + \frac{\gamma}{2}(h_4 - \rho h_4')dz_7\wedge dz_{10}\wedge\eta + \gamma k\pi z_7 dz_{10}\wedge\mathrm{vol}(\mathrm{S}^2)\\
\tilde{C}_5 =& C_5 - \frac{\gamma}{2}\sqrt{\frac{h_4}{h_8}g_1}r^2 dt\wedge dx\wedge d\rho\wedge(dz_7\wedge dz_{10} - Gdz_9\wedge dz_8)\\
&-\frac{\gamma}{2h_8}rxdt\wedge dr\wedge d\rho\wedge\left(\left(-g_1 + \frac{uu'h_8'}{h_8} + 2\sqrt{h_4h_8g_1}\right)dz_7\wedge dz_{10}\right.\\
&\left.-G\left(g_1 - \frac{uu'h_4'}{h_4} + 2\sqrt{h_4h_8g_1}\right)dz_9\wedge dz_8\right)\\
&-\frac{\gamma^2}{2}(h_4 - \rho h_4')Gg\mathrm{vol}(T^4)\wedge\eta + \gamma^2 Ggh_4' g_2 z_7 dz_{10}\wedge\mathrm{vol}(T^2)\wedge\mathrm{vol}(S^2)
\end{aligned}
$$

$$Q_{\tilde{\mathrm{D}}6}^{(k)} = \gamma Q_{\mathrm{D}4}^{(k)}, \in \mathbb{Z}\; Q_{\tilde{\mathrm{D}}4}^{(k)} = \gamma Q_{\mathrm{D}2}^{(k)}, \in \mathbb{Z}$$

$$ds^2 = \frac{c_0}{\sqrt{c_4 c_8}}\left(ds^2(\mathrm{AdS}_3) + \frac{1}{4}\left(ds^2(\mathrm{S}^2) + \gamma^2 \sqrt{\frac{c_4}{c_8}} G(d\psi + \cos\,\theta d\phi)^2 \right)\right)$$
$$+ \sqrt{\frac{c_4}{c_8}} ds^2(\mathrm{T}^3) + \frac{\sqrt{c_4 c_8}}{c_0} G d\rho^2$$
$$\tilde{B}_2 = \frac{G}{2}(d\psi + \cos\,\theta d\phi) \wedge d\rho, e^{\tilde{\Phi}} = \frac{\sqrt{G c_0}}{\left(c_8^5 c_4\right)^{1/4}}, G = (1+\gamma^2 g)^{-1}, g = \frac{c_4}{c_0}$$

$$\tilde{F}_2 = -\frac{1}{2} c_8 \mathrm{vol}(\mathrm{S}^2), \tilde{F}_4 = 2c_8 \mathrm{vol}(\mathrm{AdS}_3) \wedge d\rho + \tilde{F}_2 \wedge \tilde{B}_2$$
$$\tilde{F}_6 = -\star_{10} \tilde{F}_4, \tilde{F}_8 = \star_{10} \tilde{F}_2$$

$$\frac{ds^2}{2\pi} = \frac{|h|}{\sqrt{\Delta}} ds^2(\mathrm{AdS}_3) + \sqrt{\Delta}\left[\frac{1}{4|h|} d\rho^2 + \frac{2}{|h''|} ds^2\left(\widetilde{\mathbb{CP}}^3\right)\right]$$
$$e^{\tilde{\Phi}} = 2\Delta^{1/4} (h'')^{-3/2} \sqrt{\pi G}, \tilde{B}_2 = B_2 + \gamma\left(\frac{c(\rho)^2}{16} Z_1 - \frac{\pi^2 \Delta}{(h'')^2} W\right)$$

$$ds^2\left(\widetilde{\mathbb{CP}}^3\right) = d\xi^2 + \frac{1}{4}\cos^2\,\xi d\theta_1^2 + \frac{1}{4}\sin^2\,\xi d\theta_2^2 + \gamma^2 G \frac{\Delta \cos^4\,\xi \sin^4\,\xi \sin^2\,\theta_1 \sin^2\,\theta_2}{4h''^2} d\psi^2$$
$$+ \frac{1}{4}\sin^2\,\xi \cos^2\,\xi G\left(d\psi + \eta_1 - \eta_2 - \frac{\gamma}{4} c(\rho) \cos\,\theta_1 \sin\,\theta_2 \sin^2\,\xi d\theta_2 - \frac{\gamma}{4} c(\rho) \cos\,\theta_2 \sin\,\theta_1 \cos^2\,\xi d\theta_1\right)^2$$
$$+ \frac{1}{4}\cos^2\,\xi \sin^2\,\theta_1 G\left(d\phi_1 + \frac{\gamma}{4} c(\rho) \cos\,\theta_2 \sin\,2\xi d\xi - \frac{\gamma}{4} c(\rho) \sin\,\theta_2 \sin^2\,\xi d\theta_2\right)^2$$
$$+ \frac{1}{4}\sin^2\,\xi \sin^2\theta_2 G\left(d\phi_2 + \frac{\gamma}{4} c(\rho) \cos\,\theta_1 \sin\,2\xi d\xi + \frac{\gamma}{4} c(\rho) \sin\,\theta_1 \cos^2\,\xi d\theta_1\right)^2$$
$$G = \left(1 + \gamma^2 \frac{\pi^2 \Delta}{h''^2} M\right)^{-1}, M = s_{\theta_1}^2 c_{\theta_2}^2 c_\xi^2 + s_{\theta_1}^2 s_{\theta_2}^2 + c_{\theta_1}^2 s_{\theta_2}^2 s_\xi^2$$

$$\tilde{F}_0 = -\frac{h'''}{2\pi},$$
$$\tilde{F}_2 = \frac{2((h'')^2 - h' h''')}{h''} J + \frac{\gamma}{2}\left(\frac{\pi \Delta h'''}{(h'')^2} W + \left(\frac{\pi h''' (h')^2}{(h'')^2} + \frac{c(\rho)}{2h''}((h'')^2 - h' h''')\right) Z_1\right)$$
$$\tilde{F}_4 = \pi d\left(h' + \frac{h h' h''}{\Delta}\right) \wedge \mathrm{vol}(\mathrm{AdS}_3) - \frac{4\pi h''' (h')^2}{(h'')^2} J \wedge J,$$
$$- \frac{\gamma \pi \Delta}{2(h'')^3 M}\left[\left(3(h'')^2 + \frac{h'^2 h'''}{\Delta}(h' - 2(\rho - k) h'')\right) Z_1 \wedge Z_2\right.$$
$$\left. + \left(((h'')^2 - h' h''') Z_2 + \gamma \pi^2 M \frac{(h')^2 h'''}{h''} Z_1\right) \wedge W\right].$$

$$Z_1 = c_\xi s_\xi \left(c_\xi s_\xi s_{\theta_1} s_{\theta_2} d\theta_1 \wedge d\theta_2 - 2\left(s_{\theta_1} c_{\theta_2} c_\xi^2 d\theta_1 + c_{\theta_1} s_{\theta_2} s_\xi^2 d\theta_2\right) \wedge d\xi\right)$$

$$Z_2 = \pi c_\xi s_\xi s_{\theta_1} s_{\theta_2} \left(c_{\theta_1} s_{\theta_2} s_\xi c_\xi d\theta_1 - s_{\theta_1}\left(c_{\theta_2} c_\xi s_\xi d\theta_2 + 2 s_{\theta_2} d\xi\right)\right) \wedge d\psi$$

$$W = G c_\xi^2 s_\xi^2 \left(M d\phi_1 \wedge d\phi_2 - \left(c_{\theta_2} s_{\theta_1}^2 c_\xi^2 d\phi_1 + c_{\theta_1} s_{\theta_2}^2 s_\xi^2 d\phi_2\right) \wedge d\psi + \gamma c(\rho)\left(MJ - \frac{Z_2}{4\pi}\right) + \gamma^2 \frac{c(\rho)^2}{16} M Z_1\right)$$

$$\tilde{\psi}_+^1 = \sqrt{G}(\psi_+^1 + ib\psi_+^2), \quad \tilde{\psi}_+^2 = \sqrt{G}(\psi_+^2 + ib\psi_+^1),$$
$$\tilde{\psi}_-^1 = \sqrt{G}(\psi_-^1 + ib\psi_-^2), \quad \tilde{\psi}_-^2 = \sqrt{G}(\psi_-^2 + ib\psi_-^1).$$

$$\tilde{\psi}_+^1 = \sqrt{G}\psi_+^1 \wedge e^{4ib\psi_+^2}, \quad \tilde{\psi}_+^2 = ib\sqrt{G}\psi_+^1 \wedge e^{\left(-4ib^{-1}\psi_+^2\right)}$$
$$\tilde{\psi}_-^1 = \sqrt{G}\psi_-^1 \wedge e^{4ib\psi_+^2}, \quad \tilde{\psi}_-^2 = ib\sqrt{G}\psi_-^1 \wedge e^{\left(-4ib^{-1}\psi_+^2\right)}$$

$$\tilde{\psi}_+^1 = \frac{\sqrt{G}}{4} e^{-ij - bw \wedge v}, \qquad \tilde{\psi}_+^2 = -\frac{\sqrt{G}}{4} u \wedge e^{-ij} \wedge (w - bv)$$
$$\tilde{\psi}_-^1 = \frac{\sqrt{G}}{4}(v + bw) \wedge e^{-ij - bw \wedge v}, \quad \tilde{\psi}_-^2 = -\frac{\sqrt{G}}{4} u \wedge \left(w \wedge v + b e^{-ij}\right)$$

$$v = e^\rho, w = e^{10} - ie^9, u = e^8 - ie^7$$

$$j = \frac{i}{2}(w \wedge \bar{w} + u \wedge \bar{u}), \omega = w \wedge u$$

$$O = \begin{pmatrix} \cos\kappa_1 & 0 & \sin\kappa_1 \\ 0 & 1 & 0 \\ -\sin\kappa_1 & 0 & \cos\kappa_1 \end{pmatrix}, O' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\kappa_2 & -\sin\kappa_2 \\ 0 & \sin\kappa_2 & \cos\kappa_2 \end{pmatrix}$$
$$\kappa_1 = \arctan(\sin\xi \cot\theta_1) \kappa_2 = \arctan\left(\frac{\sin\theta_1 \cos\xi \cot\theta_2}{\sqrt{\sin\theta_1^2 + \cos\theta_1^2 \sin^2\xi}}\right)$$

$$\hat{e}^3 = \frac{c_\xi}{\sqrt{s_{\theta_1}^2 + c_{\theta_1}^2 s_\xi^2}} \left(\frac{1}{2} d\phi_1 \left(s_{\theta_1}^2 + c_{\theta_1}^2 s_\xi^2\right) - c_{\theta_1} s_\xi^2 (d\psi - \eta_2)\right)$$
$$\hat{e}^5 = \frac{s_\xi}{4c_\xi \sqrt{M\left(s_{\theta_1}^2 + c_{\theta_1}^2 s_\xi^2\right)}} \left(c_\xi s_{\theta_1} M d\phi_2 - s_{\theta_1}^3 c_{\theta_2}^2 c_\xi^3 d\psi\right)$$
$$\hat{e}^6 = \frac{s_{2\xi} s_{\theta_1} s_{\theta_2}}{8\sqrt{M}} d\psi$$

$$\Omega_O = \cos\frac{\kappa_1}{2} + \sin\frac{\kappa_1}{2}\Gamma^{36}, \Omega_{O'} = \cos\frac{\kappa_2}{2} - \sin\frac{\kappa_2}{2}\Gamma^{56},$$

$$\Psi_\pm^{(I)} \to \Omega_{O'} \Omega_O \Psi_\pm^{(I)} (\Omega_{O'} \Omega_O)^{-1}, I = 5{,}6$$

$$\tilde{\Psi}_+^{(5)} = \Psi_+^{(5)} + \frac{bc_\alpha}{8}\left[s_{\beta_2} - c_{\beta_2}\mathcal{J}_b - \frac{c_\alpha^2 c_{\beta_2}^2}{\varkappa}\left(\frac{s_{\beta_2}}{2} - \frac{c_{\beta_2}}{6}\mathcal{J}_b\right)\wedge\mathcal{J}_b\wedge\mathcal{J}_b - \frac{c_\alpha^2 c_{\beta_2}^3}{\varkappa^{3/2}}\mathrm{Re}[\Omega_b]\wedge V_b\right]$$

$$\tilde{\Psi}_-^{(5)} = \Psi_-^{(5)} - \frac{bc_\alpha}{8}\left[\left(c_{\beta_2} + s_{\beta_2}\mathcal{J}_b - \frac{c_\alpha^2 c_{\beta_2}^2}{\varkappa}\left(\frac{c_{\beta_2}}{2} + \frac{s_{\beta_2}c_\alpha^2 c_{\beta_2}^2}{6\varkappa}\mathcal{J}_b\right)\wedge\mathcal{J}_b\wedge\mathcal{J}_b\right)\wedge V_b - \frac{c_{\beta_2}}{\sqrt{\varkappa}}\mathrm{Im}[\Omega_b]\right]$$

$$c_\alpha = \frac{s_{2\xi}c_\psi s_{\theta_1}s_{\theta_2}}{\sqrt{M}}, \varkappa = 1 - c_\alpha^2 s_{\beta_2}^2$$

$$\tilde{\Psi}_+ = \frac{1}{8}\mathrm{Re}\left[e^{i\beta_2}e^{-i(j+U_1\wedge U_2)}\wedge(1 - ibc_\alpha e^{-\tan\alpha\omega}) - \left(e^{i\beta_1}U\wedge\omega - ibs_\alpha e^{-i\beta_1}\bar{U}\wedge e^{ij+\cot\alpha\bar{\omega}}\right)\wedge V\right],$$

$$\tilde{\Psi}_- = \frac{1}{8}\mathrm{Im}\left[e^{i\beta_2}e^{-i(j+U_1\wedge U_2)}\wedge(1 - ibc_\alpha e^{-\tan\alpha\omega})\wedge V + e^{i\beta_1}U\wedge\omega + ibs_\alpha e^{-i\beta_1}\bar{U}\wedge e^{ij+\cot\alpha\bar{\omega}}\right]$$

$$\mathcal{J} = j + U_1\wedge U_2, \qquad \Omega = U\wedge\omega, \qquad \omega = \omega_R + i\omega_I,$$
$$U = U_1 + iU_2, \qquad \iota_{U_1}U_2 = 0, \qquad \iota_{U_2}U_2 = \iota_{U_1}U_1 = 1,$$

$$V_b = V - \frac{\tan\alpha}{c_{\beta_2}}\mathrm{Re}\left[e^{i\beta_1}U\right]$$

$$\Omega_b = \frac{\sqrt{\varkappa}}{c_{\beta_2}}\left[\frac{\tan\alpha\left(s_\alpha^2 + c_\alpha^2 c_{\beta_2}^2\right)}{\varkappa c_{\beta_2}}\mathrm{Im}\left[e^{i\beta_1}U\right]\wedge\left(s_{\beta_2}j + s_{\beta_2}\tan\alpha\omega_I - \frac{c_{\beta_2}}{c_\alpha s_\alpha}\omega_R\right)\right.$$
$$+\left(\sec\beta_2\tan\alpha V + \mathrm{Re}\left[e^{i\beta_1}U\right]\right)\wedge(\tan\alpha j - \omega_I) + \frac{i}{c_\alpha^2 c_{\beta_2}^2}\left(s_\alpha V\wedge\left(c_\alpha c_{\beta_2}\omega_R + s_\alpha s_{\beta_2}U_1\wedge U_2\right)\right.$$
$$\left.\left.+c_\alpha c_{\beta_2}^2\mathrm{Im}\left[e^{i\beta_1}U\right]\wedge(s_\alpha j - c_\alpha\omega_I) - \mathrm{Re}\left[e^{i\beta_1}U\right]\wedge\left(\left(s_\alpha^2 - c_{\beta_2}^2\right)\omega_R + s_\alpha c_{\beta_2}s_{\beta_2}(c_\alpha j + s_\alpha\omega_I)\right)\right)\right]$$

$$\mathcal{J}_b = \mathcal{J} + \tan\alpha\left(\omega_I + \tan\beta_2\omega_R - \sec\beta_2\mathrm{Im}\left[e^{i\beta_1}U\right]\wedge V\right)$$

$$(\mathrm{N_{D8}},\ \mathrm{N_{D6}},\ \mathrm{N_{D4}},\ \mathrm{N_{D2}}) \overset{\mathrm{TsT}}{\rightarrow} (\mathrm{N_{D8}},\ \mathrm{N_{D6}} + \gamma\mathrm{N_{D4}},\ \mathrm{N_{D4}} + \gamma\mathrm{N_{D2}},\ \mathrm{N_{D2}})$$

$$e^{-\tilde{\Phi}}\sqrt{\left|\tilde{g} + \tilde{B}_2\right|}\Big|_{\Sigma^{(3)}} = \frac{1}{2}r^2\sqrt{\frac{h_8 g_1}{h_4 G}}, \tilde{C}_3 - \tilde{C}_1\wedge\tilde{B}_2\big|_{\Sigma^{(3)}} = \frac{1}{2}r^2\sqrt{\frac{h_8}{h_4}g_1},$$

$$e^{-\tilde{\Phi}}\sqrt{\left|\tilde{g} + \tilde{B}_2\right|}\Big|_{\Sigma^{(7)}} = \frac{1}{2}r^2\sqrt{\frac{h_4}{h_8}g_1}, \tilde{C}_7 - \tilde{C}_5\wedge\tilde{B}_2\big|_{\Sigma^{(7)}} = -\frac{1}{2}r^2\sqrt{\frac{h_4}{h_8}g_1},$$

$$e^{-\tilde{\Phi}}\sqrt{\left|\tilde{g} + \tilde{B}_2\right|}\Big|_{\Sigma^{(3)}\times\mathrm{T}^2} = \frac{1}{2}r^2\sqrt{\frac{h_8 g_1}{h_4 G}}, \tilde{C}_5 - \tilde{C}_3\wedge\tilde{B}_2 + \frac{1}{2}\tilde{C}_1\wedge\tilde{B}_2\wedge\tilde{B}_2\Big|_{\Sigma^{(3)}\times\mathrm{T}^2}$$

$$e^{-\tilde{\Phi}}\sqrt{\left|\tilde{g} + \tilde{B}_2\right|}\Big|_{\Sigma^{(3)}\times\mathrm{T}^2} = \frac{1}{\gamma}\frac{r^2}{2}\sqrt{\frac{h_8}{h_4}g_1}, -\tilde{C}_3\wedge f_2\big|_{\Sigma^{(3)}\times\mathrm{T}^2} = \frac{1}{\gamma}\frac{r^2}{2}\sqrt{\frac{h_8}{h_4}g_1},$$

$$e^{-\Phi}\sqrt{\left|\tilde{g}+\tilde{B}_2\right|}\Big|_{\Sigma^{(5)}} = \frac{1}{\sqrt{G}}\frac{u^2}{4h_4} r\sin\,\theta$$

$$\tilde{\Psi}_{D4}^{(\mathrm{cal})} = \frac{u^2}{4h_4}\mathrm{vol}(\mathrm{AdS}_3)\wedge\mathrm{vol}(\mathrm{S}^2)$$

$$e^{-\tilde{\Phi}}\sqrt{\left|\tilde{g}+\tilde{B}_2\right|}\Big|_{\Sigma^{(9)}} = \frac{u^2}{4h_8} r\sin\,\theta$$

$$\tilde{\Psi}_{D8}^{(\mathrm{cal})} = \frac{u^2}{4h_8}\mathrm{vol}(\mathrm{AdS}_3)\wedge\mathrm{vol}(\mathrm{S}^2)$$

$$e^{-\tilde{\Phi}}\sqrt{\left|\tilde{g}+\tilde{B}_2\right|}\Big|_{\Sigma^{(5)}\times \mathrm{T}^2} = \frac{1}{\gamma}\frac{u^2}{4h_4} r\sin\,\theta = \tilde{\Psi}_{\tilde{\mathrm{D}}6}^{(\mathrm{cal})} = \frac{1}{\gamma}\Psi_{D4}^{(\mathrm{cal})},$$

$$2Q_{\tilde{\mathrm{D}}4}^{(k)} = \Delta Q_{\tilde{\mathrm{D}}6}^{(k)}, 2Q_{\mathrm{D}6}^{(k)} = \Delta Q_{\mathrm{D}8}^{(k)}$$

$$e^{A} = \left(e^{a}{}_{\mu}dx^{\mu}, e^{C_1}(du+A_1), e^{C_2}(dv+A_2)\right), a = 1,\ldots,8$$

$$\hat{e}_v^m = \frac{1}{g_{vv}}e_v^m, \hat{e}_\sigma^m = e^m{}_\sigma - \frac{1}{g_{vv}}(g_{\sigma v}+B_{\sigma v})e^m{}_v, \sigma \neq v$$

$$e^{\hat{\Phi}} = e^{\Phi - C_2}$$

$$\frac{1}{k!}F_{a_1,\ldots,a_k}e^{a_1}\wedge\ldots\wedge e^{a_k} \leftrightarrow \frac{1}{k!}F_{a_1,\ldots,a_k}\Gamma^{a_1\ldots a_k} \equiv F_k$$

$$e^{\hat{\Phi}}\hat{\not{F}} = e^{\Phi}\not{F}\Gamma^{v}, \not{F}\Gamma^{a} = (-1)^k(e^a\wedge -\iota^a)\not{F}.$$

$$e^a = O_b^a e^b, O = \frac{1}{|O|}\begin{pmatrix} 1 & -\gamma\sqrt{g} \\ \gamma\sqrt{g} & 1 \end{pmatrix},$$

$$\hat{F}\to\Omega^{-1}\hat{F}\Omega.$$

$$e^{\tilde{\Phi}}\tilde{\not{F}} = e^{\Phi}\not{F}\Omega, \Omega = \sqrt{G}\left(1+\gamma\sqrt{g}\Gamma^{vu}\right),$$

$$e^{\tilde{\Phi}}\tilde{\phi} = e^{\Phi} \not\subset \Omega$$

$$\Psi\to\tilde{\Psi} = \Omega\Psi, \Gamma^a\Psi = (e^a\wedge +\iota^a)\Psi$$

$$Q_{Dp} = \frac{1}{(2\pi)^{7-p}}\int_{\Sigma_{8-p}}\hat{f}_{8-p}d\vec{x}$$

	$0\le\rho\le 2\pi$	$2\pi j\le\rho\le 2\pi(j+1)$	$2\pi P\le\rho\le 2\pi(P+1)$
h_8	$\frac{\nu_0}{2\pi}\rho$	$\mu_j+\frac{\nu_j}{2\pi}(\rho-2\pi j)$	$\mu_P-\frac{\mu_P}{2\pi}(\rho-2P\pi)$

h_4	$\frac{\beta_0}{2\pi}\rho$	$\alpha_j+\frac{\beta_j}{2\pi}(\rho-2\pi j)$	$\alpha_P-\frac{\alpha_P}{2\pi}(\rho-2P\pi)$
u	$\frac{b_0}{2\pi}\rho$		

$$\alpha_k=\sum_{j=0}^{k-1}\beta_j,\mu_k=\sum_{j=0}^{k-1}\nu_j.$$

$$\hat{f}_0=h_8',\hat{f}_2=-\frac{1}{2}(h_8-h_8'(\rho-2\pi k))\mathrm{vol}(\mathrm{S}^2)$$
$$\hat{f}_4=h_4'\mathrm{vol}(\mathrm{T}^4),\hat{f}_6=\frac{1}{2}(h_4-h_4'(\rho-2\pi k))\mathrm{vol}(\mathrm{S}^2)\wedge\mathrm{vol}(\mathrm{T}^4)$$

$$Q_{\mathrm{NS5}}^{(k)}=\frac{1}{(2\pi)^2}\int_{(\rho,S^2)}H_3=1,Q_{D8}^{(k)}=2\pi h_8'=\nu_k,Q_{D6}^{(k)}=\frac{1}{2\pi}\int_{S^2}\hat{f}_2=\mu_k$$
$$Q_{D4}^{(k)}=\frac{1}{(2\pi)^3}\int_{T^4}\hat{f}_4=\beta_k,Q_{D2}^{(k)}=\frac{1}{(2\pi)^5}\int_{(S^2,T^4)}\hat{f}_6=\alpha_k$$

$$d\hat{f}_0=2\pi h_8''d\rho,\qquad d\hat{f}_2=0$$
$$d\hat{f}_4=h_4''d\rho\wedge\mathrm{vol}(\mathrm{T}^4),\quad d\hat{f}_6=0$$

$$\hat{\tilde{f}}_2=-\frac{1}{2}(h_8-h_8'(\rho-2\pi k))\mathrm{vol}(S^2)+\gamma h_4'dz_7\wedge dz_{10}$$
$$\hat{\tilde{f}}_4=h_4'\mathrm{vol}(\mathrm{T}^4)-\frac{\gamma}{2}(h_4-h_4'(\rho-2\pi k))\mathrm{vol}(S^2)\wedge dz_7\wedge dz_{10}$$
$$\hat{\tilde{f}}_6=-\frac{1}{2}(h_4-h_4'(\rho-2\pi k))\mathrm{vol}(S^2)\wedge\mathrm{vol}(\mathrm{T}^4)$$

$$Q_{\tilde{\mathrm{D}}6}^{(k)}=\frac{1}{2\pi}\int_{z_7z_{10}}\hat{\tilde{f}}_2=\gamma Q_{D4}^{(k)},Q_{\tilde{\mathrm{D}}4}^{(k)}=\frac{1}{(2\pi)^3}\int_{(S^2,z_7z_{10})}\hat{\tilde{f}}_4=\gamma Q_{D2}^{(k)}$$

$$d\hat{\tilde{f}}_2=\gamma h_4''d\rho\wedge dz_7\wedge dz_{10},d\hat{\tilde{f}}_4=0$$

$$d\hat{\tilde{f}}\Big|_{(z_7,z_{10})}=(2\pi)^3N_{D6}\delta(\rho-\rho_0)e^{\tilde{f}_2}d\rho\wedge dz_7\wedge dz_{10}$$

$$\gamma\Delta h_4'=(2\pi)^3N_{D6}=(2\pi)^3\gamma N_{D4},\tilde{f}_2=\frac{1}{\gamma}dz_9\wedge dz_8$$

$$ds^2(S^7)=d\xi^2+\cos^2\xi ds^2(S_1^3)+\sin^2\xi ds^2(S_2^3)$$

$$\nabla_a\xi_\pm=\pm\frac{i}{2}\gamma_a\xi_\pm$$

$$e^1 = d\xi, e^2 = \frac{\cos\,\xi}{2} d\theta_1, e^3 = \frac{\cos\,\xi}{2} \sin\,\theta_1 d\phi_1$$
$$e^4 = \frac{\sin\,\xi}{2} d\theta_2, e^5 = \frac{\sin\,\xi}{2} \sin\,\theta_2 d\phi_2$$
$$e^6 = -\frac{\cos\,\xi \sin\,\xi}{2} (d[\psi_1 - \psi_2] + \eta_1 - \eta_2)$$
$$e^7 = \frac{1}{2}\left(\cos^2\,\xi(d\psi_1 + \eta_1) + \sin^2\,\xi(d\psi_2 + \eta_2)\right)$$

$$\xi_{\pm} = \mathcal{M}_{\pm}\xi_{\pm}^0$$
$$\mathcal{M}_{\pm} = e^{\frac{\xi}{2}(\pm i\gamma_1 - \gamma_{67})} e^{\frac{\psi_1}{4}(\pm i\gamma_7 - \gamma_{23})} e^{\frac{\theta_1}{4}(\pm i\gamma_2 + \gamma_{37})} e^{\frac{\phi_1}{4}(\pm i\gamma_7 + \gamma_{23})}$$
$$\times e^{\frac{\psi_2}{4}(\gamma_{16} - \gamma_{45})} e^{\frac{\theta_2}{4}(\gamma_{14} + \gamma_{56})} e^{\frac{\phi_2}{4}(\gamma_{16} + \gamma_{45})}$$

$$\xi_+^A = \mathcal{M}_+\eta^A, A = 1, \ldots, 8$$

$$ds^2(S^7) = ds^2(\mathbb{CP}^3) + \frac{1}{16}\left(d\tilde{\psi} + \cos^2\,\xi(d\psi + 2\eta_1) + \sin^2\,\xi(d\psi - 2\eta_2)\right)^2$$

$$\tilde{\psi} = \frac{1}{2}(\psi_1 + \psi_2), \psi = \frac{1}{2}(\psi_1 - \psi_2).$$

$$j=\frac{1}{p\sqrt{M}}\left(c_{\theta_2}c_\xi s_{\theta_1}^2\left(e^1\wedge\hat{e}^5-s_{2\xi}c_\psi\hat{e}^3\wedge\hat{e}^6\right)+\left(p_1e^4\wedge\hat{e}^5-p_2c_\xi s_\xi e^2\wedge\hat{e}^5\right)\right)$$
$$-\frac{1}{p}\left(s_{\theta_1}\left(c_{2\xi}e^2\wedge\hat{e}^3+s_{2\xi}s_\psi\hat{e}^3\wedge e^4\right)+c_{\theta_1}s_\xi\left(e^1\wedge\hat{e}^3+s_{2\xi}c_\psi\hat{e}^5\wedge\hat{e}^6\right)\right)$$
$$-c_\alpha\hat{e}^3\wedge\hat{e}^5+\frac{c_\alpha}{s_\alpha^2\sqrt{M}}\left(c_\xi p_3(e^1\wedge e^2-c_\alpha e^4\wedge\hat{e}^6)-s_\xi p_4\sqrt{M}(e^1\wedge e^4+c_\alpha e^2\wedge\hat{e}^6)\right.$$
$$\left.-s_{\theta_1}s_{\theta_2}\sqrt{M}\left(1-s_{2\xi}^2c_\psi^2\right)e^2\wedge e^4-s_{2\xi}c_\psi\left(c_{\theta_2}^2s_{\theta_1}^2c_\xi^2+c_{\theta_1}^2s_{\theta_2}^2s_\xi^2\right)e^1\wedge\hat{e}^6\right),$$
$$U=\frac{1}{Ms_\alpha^2}\left(s_{\theta_1}s_{\theta_2}s_\psi e^1-s_\xi\left(2c_{\theta_2}s_{\theta_1}c_\xi^2+c_{\theta_1}s_{\theta_2}s_\psi c_{2\xi}\right)e^2+c_\xi\left(2c_{\theta_1}s_{\theta_2}s_\xi^2-c_{\theta_2}s_{\theta_1}s_\psi c_{2\xi}\right)e^4-c_\psi c_{2\xi}\sqrt{M}\hat{e}^6\right)$$
$$+i\left(c_\psi s_{\theta_1}s_{\theta_2}c_{2\xi}e^1-c_{\theta_1}c_\psi s_{\theta_2}s_\xi e^2-c_{\theta_2}c_\xi c_\psi s_{\theta_1}e^4+s_\psi\sqrt{M}\hat{e}^6\right)\Big),$$
$$\omega=\frac{1}{2ps_\alpha\sqrt{M}}\left[2c_{\theta_2}c_\xi s_{\theta_1}^2\left(e^1\wedge\hat{e}^3+s_{2\xi}c_\psi\hat{e}^5\wedge\hat{e}^6\right)+s_{2\xi}\left(2s_{\theta_2}s_\psi\left(1-c_{\theta_1}^2c_\xi^2\right)-c_{\theta_1}c_{\theta_2}s_{\theta_1}c_{2\xi}\right)e^2\wedge\hat{e}^3\right.$$
$$-\left(2s_{\theta_2}c_{2\xi}\left(1-c_{\theta_1}^2c_\xi^2\right)+c_{\theta_1}c_{\theta_2}s_{\theta_1}s_{2\xi}^2s_\psi\right)\hat{e}^3\wedge e^4+2\sqrt{M}\left(c_{\theta_1}s_\xi\left(e^1\wedge\hat{e}^5-s_{2\xi}c_\psi\hat{e}^3\wedge\hat{e}^6\right)\right.$$
$$\left.\left.+s_{\theta_1}\left(c_{2\xi}e^2\wedge\hat{e}^5-s_{2\xi}s_\psi e^4\wedge\hat{e}^5\right)\right)\right]+i\left[-s_\alpha\hat{e}^3\wedge\hat{e}^5+\frac{\cot\alpha}{p}\left(c_{\theta_1}s_\xi e^1\wedge\hat{e}^3+s_{\theta_1}c_{2\xi}e^2\wedge\hat{e}^3\right.\right.$$
$$+s_{\theta_1}s_{2\xi}s_\psi\hat{e}^3\wedge e^4)+\frac{\cot\alpha}{2p\sqrt{M}}\left(s_{2\xi}\left(c_{\theta_1}c_{\theta_2}s_{\theta_1}c_{2\xi}-2s_{\theta_2}s_\psi\left(1-c_{\theta_1}^2c_\xi^2\right)\right)e^2\wedge\hat{e}^5\right.$$
$$+2s_{2\xi}c_\psi\left(c_{\theta_2}c_\xi s_{\theta_1}^2\hat{e}^3\wedge\hat{e}^6+c_{\theta_1}s_\xi\sqrt{M}\hat{e}^5\wedge\hat{e}^6\right)-2s_{\theta_1}^2c_\xi c_{\theta_2}e^1\wedge\hat{e}^5-\left(2s_{\theta_2}c_{2\xi}\left(1-c_{\theta_1}^2c_\xi^2\right)\right.$$
$$\left.+s_{2\xi}^2c_{\theta_1}c_{\theta_2}s_{\theta_1}s_\psi\right)e^4\wedge\hat{e}^5)-\frac{1}{Ms_\alpha}\left(s_{2\xi}c_\psi\left(c_{\theta_2}^2c_\xi^2s_{\theta_1}^2+s_{\theta_1}^2s_{\theta_2}^2s_\psi^2\right)e^1\wedge\hat{e}^6+s_{\theta_1}s_{\theta_2}\sqrt{M}\left(1-c_\psi^2s_{2\xi}^2\right)e^2\wedge e^4\right)$$
$$\left.+\frac{1}{s_\alpha\sqrt{H}}\left(p_3c_\xi(e^1\wedge e^2-c_\alpha e^4\wedge\hat{e}^6)-p_4s_\xi(e^1\wedge e^4+c_\alpha e^2\wedge\hat{e}^6)\right)\right],$$
$$p=\sqrt{s_\xi^2c_{\theta_1}^2+s_{\theta_1}^2},M=c_{\theta_2}^2c_\xi^2s_{\theta_1}^2+s_{\theta_2}^2p^2,$$
$$p_1=\frac{s_{2\xi}^2c_{\theta_1}s_{\theta_1}c_{\theta_2}s_\psi}{2}+c_{2\xi}s_{\theta_2}p^2,p_2=c_{\theta_1}s_{\theta_1}c_{\theta_2}c_{2\xi}-2s_{\theta_2}s_\psi p^2,$$
$$p_3=c_{\theta_2}s_{\theta_1}c_{2\xi}-2c_{\theta_1}s_{\theta_2}s_\psi s_\xi^2,p_4=c_{\theta_1}s_{\theta_2}c_{2\xi}+2c_{\theta_2}s_{\theta_1}s_\psi c_\xi^2.$$

Supergravedad cuántica relativista Tipo IIB: Modelamiento Matemático.

$$\mathfrak{so}(3;\mathbb{R})\oplus\mathfrak{so}(1,6;\mathbb{R})$$
$$\mathfrak{so}(3;\mathbb{R})\oplus\mathfrak{so}(2,5;\mathbb{R})$$
$$\mathfrak{so}(1,2;\mathbb{R})\oplus\mathfrak{so}(7;\mathbb{R})$$
$$\mathfrak{so}(1,2;\mathbb{R})\oplus\mathfrak{so}(3,4;\mathbb{R})$$

$$\Lambda''^{\langle\dagger\rangle}=\nabla_M\varepsilon-\frac{1}{8}e^\phi(\Gamma\cdot\partial\chi)\Gamma_Ms^2\varepsilon-\frac{1}{480}e^\phi\left(\Gamma\cdot F_{(5)}\right)\Gamma_Ms^2\varepsilon$$
$$-\frac{1}{8}H_{(3)MNP}\Gamma^{NP}s^3\varepsilon-\frac{1}{48}e^\phi\left(\Gamma\cdot\tilde{F}_{(3)}\right)\Gamma_Ms^1\varepsilon$$
$$\Lambda'^{\langle\star\rangle}=(\Gamma\cdot\partial\phi)\varepsilon+e^\phi(\Gamma\cdot\partial\chi)s^2\varepsilon-\frac{1}{12}\left(\Gamma\cdot H_{(3)}\right)s^3\varepsilon+\frac{1}{12}e^\phi\left(\Gamma\cdot\tilde{F}_{(3)}\right)s^1\varepsilon$$

$$H_{(3)}=dB_{(2)}\qquad\tilde{F}_{(3)}=F_{(3)}-\chi H_{(3)}$$
$$F_{(3)}=dC_{(2)}\qquad F_{(5)}=dC_{(4)}+\frac{1}{8}\left(B_{(2)}\wedge dC_{(2)}-C_{(2)}\wedge dB_{(2)}\right)$$

$$\varepsilon=\begin{pmatrix}\varepsilon_+\\ \varepsilon_-\end{pmatrix}\ s^1=\begin{pmatrix}0&1\\1&0\end{pmatrix}\ s^2=\begin{pmatrix}0&1\\-1&0\end{pmatrix}\ s^3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$g^*_{MN} = g_{MN}\ \phi^* = \phi$$

$$(\Gamma_A)^* = \mathcal{B}\Gamma_A\mathcal{B}^{-1}\ \mathcal{B} = \mathcal{C}\Gamma^0$$

$$(e^A)^* = \eta_{AA}e^A$$

$$(e^A\Gamma_A)^* = \mathcal{B}_\eta(e^A\Gamma_A)\mathcal{B}_\eta^{-1}$$

$$\mathcal{B}_\eta = u\mathcal{C}\prod_{i\in\mathcal{T}}\Gamma^i\ \mathcal{B}_\eta^*\mathcal{B}_\eta = (-)^{\frac{1}{2}t(t-1)}I_{32}$$

$$\mathcal{B}_\eta^{-1}\varepsilon^* = S\varepsilon$$
$$\mathcal{B}_\eta^{-1}\lambda^* = S\lambda$$
$$\mathcal{B}_\eta^{-1}\psi^* = S\psi$$

$$S = \begin{cases} I_2 & \text{for} \quad \text{Type IIB}_{\mathbb{R}} \\ s^1 & \text{for} \quad \text{Type IIB}' \\ s^2 & \text{for} \quad \text{Type IIB}_3 \\ s^3 & \text{for} \quad \text{Type IIB}^\star \end{cases}$$

$$t \equiv 1(\text{mod}4) \quad S = I_2, s^1, s^3$$
$$t \equiv 3(\text{mod}4) \quad S = s^2$$

$$ds^2 = f_6^2 ds^2_{\mathcal{M}_{6\mathbb{C}}} + f_2^2 ds^2_{\mathcal{M}_{2\mathbb{C}}} + ds^2_{\Sigma_{\mathbb{C}}}$$

$$e^m = f_6\hat{e}^m\ ds^2_{\mathcal{M}_{6\mathbb{C}}} = \eta_{mn}\hat{e}_m\hat{e}_n\ m,n = 0,..,5$$
$$e^i = f_2\hat{e}^i\ ds^2_{\mathcal{M}_{2\mathbb{C}}} = \delta_{ij}\hat{e}^i\hat{e}^j\ i,j = 6,7$$
$$e^a\ ds^2_{\Sigma_{\mathbb{C}}} = \delta_{ab}e^ae^b\ a,b = 8,9$$

$$F_{(5)} = 0\ \tilde{F}_{(3)} = g_ae^a \wedge e^{67}\ H_{(3)} = h_ae^a \wedge e^{67}$$

$$d\phi = \varphi_ae^a\ d\chi = \chi_ae^a$$

$$\mathcal{M}_{n\mathbb{C}} = \{(z_1,\cdots,z_{n+1}) \in \mathbb{C}^{n+1} \text{ such that } z_1^2 + \cdots + z_{n+1}^2 = 1\}$$

$$\mathfrak{so}(n+1;\mathbb{C}) = \mathfrak{so}(n;\mathbb{C}) \oplus \mathfrak{M}_{n\mathbb{C}}$$

$$[t_{AB}, t_{CD}] = \eta_{AD}t_{BC} + \eta_{BC}t_{AD} - \eta_{AC}t_{BD} - \eta_{BD}t_{AC}$$
$$[t_{AB}, u_C] = \eta_{BC}u_A - \eta_{AC}u_B$$
$$[u_A, u_B] = -t_{AB}$$

$$\Omega = \frac{1}{2}\hat{\omega}^{AB}t_{AB} + \hat{e}^Au_A$$

$$d\hat{e}^A + \eta_{BC}\hat{\omega}^{AB} \wedge \hat{e}^C = 0$$
$$d\hat{\omega}^{AB} + \eta_{CD}\hat{\omega}^{AC} \wedge \hat{\omega}^{DB} = \hat{e}^A \wedge \hat{e}^B$$

$$\left(d+\frac{1}{4}\hat{\omega}^{mn}\gamma_{mn}+\frac{k_1}{2}\hat{e}^{m}\gamma_m\gamma_{(1)}\right)\chi_6=0$$
$$\left(d+\frac{1}{4}\hat{\omega}^{ij}\gamma_{ij}+\frac{k_2}{2}\hat{e}^{i}\gamma_i\gamma_{(2)}\right)\chi_2=0$$

$$\left(\hat{\nabla}_m-\frac{k_1}{2}\eta_1\gamma_m\otimes I_2\right)\chi^{\eta_1,\eta_2}=0$$
$$\left(\hat{\nabla}_i-\frac{k_2}{2}\eta_2 I_8\otimes\gamma_i\right)\chi^{\eta_1,\eta_2}=0$$

$$\varepsilon=\sum_{\eta_1,\eta_2=\pm}\chi^{\eta_1,\eta_2}\otimes\zeta_{\eta_1,\eta_2}\quad \varepsilon=\begin{pmatrix}\varepsilon_+\\ \varepsilon_-\end{pmatrix}\quad \zeta_{\eta_1,\eta_2}=\begin{pmatrix}\zeta_{+,\eta_1,\eta_2}\\ \zeta_{-,\eta_1,\eta_2}\end{pmatrix}$$

$$\left(\tau^{(ij)}\zeta\right)_{\eta_1,\eta_2}=\sum_{\eta_1',\eta_2'=\pm}\left(\tau^{i}\right)_{\eta_1\eta_1'}\left(\tau^{j}\right)_{\eta_2\eta_2'}\zeta_{\eta_1'\eta_2'}$$

$$\left(I_2\otimes\tau^{(11)}\otimes\gamma_{(3)}\right)\zeta=\zeta\quad\left(I_2\otimes\gamma_{(3)}\right)\zeta_{-\eta_1,-\eta_2}=\zeta_{\eta_1,\eta_2}$$

$$\nabla_m\varepsilon=\hat{\nabla}_m\varepsilon+\frac{D_af_6}{2f_6}\Gamma_m\Gamma^a\varepsilon\quad\nabla_i\varepsilon=\hat{\nabla}_i\varepsilon+\frac{D_af_2}{2f_2}\Gamma_i\Gamma^a\varepsilon$$

$$(m)\quad 0=-\frac{ik_1}{2f_6}\tau^{(20)}\zeta+\left(\frac{D_af_6}{2f_6}-\frac{1}{8}e^{\phi}\chi_a s^2\right)\tau^{(01)}\gamma^a\zeta-\frac{i}{8}e^{\phi}g_a s^1\gamma^a\zeta$$
$$(i)\quad 0=\frac{k_2}{2f_2}\tau^{(03)}\zeta+\left(\frac{D_af_2}{2f_2}-\frac{1}{8}e^{\phi}\chi_a s^2\right)\tau^{(01)}\gamma^a\zeta+\left(-\frac{i}{4}h_a s^3+\frac{i}{8}e^{\phi}g_a s^1\right)\gamma^a\zeta$$
$$(a)\quad 0=\left(D_a+\frac{i}{2}\hat{\omega}_a\gamma_{(3)}\right)\zeta+\frac{1}{8}e^{\phi}\chi_b s^2\gamma^b\gamma_a\zeta+\left(-\frac{i}{4}h_a s^3+\frac{i}{8}e^{\phi}g_b s^1\gamma^b\gamma_a\right)\tau^{(01)}\zeta$$
$$(\lambda)\quad 0=\left(\varphi_a-e^{\phi}\chi_a s^2\right)\tau^{(01)}\gamma^a\zeta-\frac{i}{2}\left(h_a s^3+e^{\phi}g_a s^1\right)\gamma^a\zeta$$

$$I_2\otimes\tau^{(11)}\otimes\gamma_{(3)}\quad\gamma_{(3)}=\sigma^3$$
$$is^2\otimes\tau^{(32)}\otimes I_2$$
$$iI_2\otimes\tau^{(20)}\otimes I_2$$

$$\mathcal{I}=I_2\otimes\tau^{(11)}\otimes\gamma_{(3)}$$
$$\mathcal{J}=is^2\otimes\tau^{(32)}\otimes I_2$$
$$\mathcal{I}\mathcal{J}=is^2\otimes\tau^{(23)}\otimes\gamma_{(3)}$$

$$\left(I_2\otimes\tau^{(11)}\otimes\gamma_{(3)}\right)\zeta=\zeta$$
$$\left(is^2\otimes\tau^{(32)}\otimes I_2\right)\zeta=\nu\zeta$$

$$\left(I_2\otimes\tau^{(11)}\otimes\gamma_{(3)}\right)\zeta_{s,\eta_1,\eta_2,\sigma}=\sigma\zeta_{s,-\eta_1,-\eta_2,\sigma}=\zeta_{s,\eta_1,\eta_2,\sigma}$$
$$\left(is^2\otimes\tau^{(32)}\otimes I_2\right)\zeta_{s,\eta_1,\eta_2,\sigma}=s\eta_1\eta_2\zeta_{-s,\eta_1,-\eta_2,\sigma}=\nu\zeta_{s,\eta_1,\eta_2,\sigma}$$

$$\alpha=\zeta_{++++}=+\zeta_{+--+}=+\nu\zeta_{-+-+}=+\nu\zeta_{--++}$$
$$\beta=\zeta_{+---}=-\zeta_{+++-}=-\nu\zeta_{-+--}=+\nu\zeta_{--+-}$$
$$\gamma=\zeta_{-+++}=+\zeta_{---+}=-\nu\zeta_{++-+}=-\nu\zeta_{+-++}$$
$$\delta=\zeta_{----}=-\zeta_{-++-}=+\nu\zeta_{++--}=-\nu\zeta_{+-+-}$$

$$e^{z}=\frac{1}{2}(e^{8}+ie^{9})\;e^{\tilde{z}}=\frac{1}{2}(e^{8}-ie^{9})\;\delta_{z\tilde{z}}=2\;\delta^{z\tilde{z}}=\frac{1}{2}$$

$$\gamma^{z}=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}=\frac{1}{2}\gamma_{\tilde{z}}\;\gamma^{\tilde{z}}=\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}=\frac{1}{2}\gamma_{z}$$

$$0=\varphi_{z}\beta-e^{\phi}\chi_{z}\delta+\frac{i\nu}{2}\left(h_{z}\delta-e^{\phi}g_{z}\beta\right)$$
$$0=\varphi_{z}\delta+e^{\phi}\chi_{z}\beta+\frac{i\nu}{2}\left(h_{z}\beta+e^{\phi}g_{z}\delta\right)$$
$$0=\varphi_{\tilde{z}}\alpha-e^{\phi}\chi_{\tilde{z}}\gamma+\frac{i\nu}{2}\left(h_{\tilde{z}}\gamma-e^{\phi}g_{\tilde{z}}\alpha\right)$$
$$0=\varphi_{\tilde{z}}\gamma+e^{\phi}\chi_{\tilde{z}}\alpha+\frac{i\nu}{2}\left(h_{\tilde{z}}\alpha+e^{\phi}g_{\tilde{z}}\gamma\right)$$

$$\Lambda''^{\langle\dagger\rangle}=+\frac{k_{1}}{2f_{6}}\alpha+\frac{D_{z}f_{6}}{2f_{6}}\beta-\frac{1}{8}e^{\phi}\chi_{z}\delta-\frac{i\nu}{8}e^{\phi}g_{z}\beta$$
$$\Lambda'^{\langle\star\rangle}=+\frac{k_{1}}{2f_{6}}\gamma+\frac{D_{z}f_{6}}{2f_{6}}\delta+\frac{1}{8}e^{\phi}\chi_{z}\beta+\frac{i\nu}{8}e^{\phi}g_{z}\delta$$
$$\Lambda''^{\langle\dagger\rangle}=-\frac{k_{1}}{2f_{6}}\beta+\frac{D_{\tilde{z}}f_{6}}{2f_{6}}\alpha-\frac{1}{8}e^{\phi}\chi_{\tilde{z}}\gamma-\frac{i\nu}{8}e^{\phi}g_{\tilde{z}}\alpha$$
$$\Lambda'^{\langle\star\rangle}=-\frac{k_{1}}{2f_{6}}\delta+\frac{D_{\tilde{z}}f_{6}}{2f_{6}}\gamma+\frac{1}{8}e^{\phi}\chi_{\tilde{z}}\alpha+\frac{i\nu}{8}e^{\phi}g_{\tilde{z}}\gamma$$

$$\Lambda''^{\langle\dagger\rangle}=-\frac{k_{2}\nu}{2f_{2}}\gamma+\frac{D_{z}f_{2}}{2f_{2}}\beta-\frac{1}{8}e^{\phi}\chi_{z}\delta+\frac{i\nu}{8}e^{\phi}g_{z}\beta+\frac{i\nu}{4}h_{z}\delta$$
$$\Lambda'^{\langle\star\rangle}=+\frac{k_{2}\nu}{2f_{2}}\alpha+\frac{D_{z}f_{2}}{2f_{2}}\delta+\frac{1}{8}e^{\phi}\chi_{z}\beta-\frac{i\nu}{8}e^{\phi}g_{z}\delta+\frac{i\nu}{4}h_{z}\beta$$
$$\Lambda''^{\langle\dagger\rangle}=-\frac{k_{2}\nu}{2f_{2}}\delta+\frac{D_{\tilde{z}}f_{2}}{2f_{2}}\alpha-\frac{1}{8}e^{\phi}\chi_{\tilde{z}}\gamma+\frac{i\nu}{8}e^{\phi}g_{\tilde{z}}\alpha+\frac{i\nu}{4}h_{\tilde{z}}\gamma$$
$$\Lambda'^{\langle\star\rangle}=+\frac{k_{2}\nu}{2f_{2}}\beta+\frac{D_{\tilde{z}}f_{2}}{2f_{2}}\gamma+\frac{1}{8}e^{\phi}\chi_{\tilde{z}}\alpha-\frac{i\nu}{8}e^{\phi}g_{\tilde{z}}\gamma+\frac{i\nu}{4}h_{\tilde{z}}\alpha$$

$$\Lambda''^{\langle\dagger\rangle}=D_{\tilde{z}}\alpha+\frac{i}{2}\hat{\omega}_{\tilde{z}}\alpha+\frac{i\nu}{4}h_{\tilde{z}}\gamma$$
$$\Lambda'^{\langle\star\rangle}=D_{\tilde{z}}\gamma+\frac{i}{2}\hat{\omega}_{\tilde{z}}\gamma+\frac{i\nu}{4}h_{\tilde{z}}\alpha$$
$$\Lambda''^{\langle\dagger\rangle}=D_{z}\beta-\frac{i}{2}\hat{\omega}_{z}\beta+\frac{i\nu}{4}h_{z}\delta$$
$$\Lambda'^{\langle\star\rangle}=D_{z}\delta-\frac{i}{2}\hat{\omega}_{z}\delta+\frac{i\nu}{4}h_{z}\beta$$

$$\Lambda''^{\langle\dagger\rangle}=D_{z}\alpha+\frac{i}{2}\hat{\omega}_{z}\alpha+\frac{1}{4}e^{\phi}\chi_{z}\gamma+\frac{i\nu}{4}e^{\phi}g_{z}\alpha+\frac{i\nu}{4}h_{z}\gamma$$
$$\Lambda'^{\langle\star\rangle}=D_{z}\gamma+\frac{i}{2}\hat{\omega}_{z}\gamma-\frac{1}{4}e^{\phi}\chi_{z}\alpha-\frac{i\nu}{4}e^{\phi}g_{z}\gamma+\frac{i\nu}{4}h_{z}\alpha$$
$$\Lambda''^{\langle\dagger\rangle}=D_{\tilde{z}}\beta-\frac{i}{2}\hat{\omega}_{\tilde{z}}\beta+\frac{1}{4}e^{\phi}\chi_{\tilde{z}}\delta+\frac{i\nu}{4}e^{\phi}g_{\tilde{z}}\beta+\frac{i\nu}{4}h_{\tilde{z}}\delta$$
$$\Lambda'^{\langle\star\rangle}=D_{\tilde{z}}\delta-\frac{i}{2}\hat{\omega}_{\tilde{z}}\delta-\frac{1}{4}e^{\phi}\chi_{\tilde{z}}\beta-\frac{i\nu}{4}e^{\phi}g_{\tilde{z}}\delta+\frac{i\nu}{4}h_{\tilde{z}}\beta$$

$$f_6 = c_6(\alpha\delta - \beta\gamma)$$
$$f_2 = c_2(\alpha\beta + \gamma\delta)$$

$$\Lambda''^{\langle\dagger\rangle} = \frac{k_1}{2c_6} - \frac{1}{8}e^{\phi}\chi_z(\beta^2 + \delta^2) - \frac{i\nu}{4}e^{\phi}g_z\beta\delta$$
$$\Lambda'^{\langle\star\rangle} = \frac{k_1}{2c_6} - \frac{1}{8}e^{\phi}\chi_{\tilde{z}}(\alpha^2 + \gamma^2) - \frac{i\nu}{4}e^{\phi}g_{\tilde{z}}\alpha\gamma$$

$$\Lambda''^{\langle\dagger\rangle} = \frac{k_2\nu}{2c_2} + \frac{1}{8}e^{\phi}\chi_z(\beta^2 + \delta^2) - \frac{i\nu}{4}e^{\phi}g_z\beta\delta + \frac{i\nu}{4}h_z(\beta^2 - \delta^2)$$
$$\Lambda'^{\langle\star\rangle} = \frac{k_2\nu}{2c_2} + \frac{1}{8}e^{\phi}\chi_{\tilde{z}}(\alpha^2 + \gamma^2) - \frac{i\nu}{4}e^{\phi}g_{\tilde{z}}\alpha\gamma + \frac{i\nu}{4}h_{\tilde{z}}(\alpha^2 - \gamma^2)$$

$$\Lambda''^{\langle\dagger\rangle} = \frac{D_z f_6}{2f_6}(\alpha\delta - \beta\gamma) + \frac{1}{8}e^{\phi}\chi_z(\alpha\beta + \gamma\delta) + \frac{i\nu}{8}e^{\phi}g_z(\alpha\delta + \beta\gamma)$$
$$\Lambda'^{\langle\star\rangle} = \frac{D_{\tilde{z}} f_6}{2f_6}(\alpha\delta - \beta\gamma) - \frac{1}{8}e^{\phi}\chi_t z(\alpha\beta + \gamma\delta) - \frac{i\nu}{8}e^{\phi}g_{\tilde{z}}(\alpha\delta + \beta\gamma)$$

$$D_z(\alpha\delta - \beta\gamma) = -\frac{1}{4}e^{\phi}\chi_z(\alpha\beta + \gamma\delta) - \frac{i\nu}{4}e^{\phi}g_z(\alpha\delta + \beta\gamma)$$
$$D_{\tilde{z}}(\alpha\delta - \beta\gamma) = \frac{1}{4}e^{\phi}\chi_{\tilde{z}}(\alpha\beta + \gamma\delta) + \frac{i\nu}{4}e^{\phi}g_{\tilde{z}}(\alpha\delta + \beta\gamma)$$

$$\frac{D_z f_6}{f_6}(\alpha\delta - \beta\gamma) - D_z(\alpha\delta - \beta\gamma) = 0$$
$$\frac{D_{\tilde{z}} f_6}{f_6}(\alpha\delta - \beta\gamma) - D_{\tilde{z}}(\alpha\delta - \beta\gamma) = 0$$

$$\Lambda''^{\langle\dagger\rangle} = e^{\phi}\chi_z(\beta^2 + \delta^2) + \frac{i\nu}{2}h_z(\beta^2 - \delta^2) + i\nu e^{\phi}g_z\beta\delta$$
$$\Lambda'^{\langle\star\rangle} = e^{\phi}\chi_{\tilde{z}}(\alpha^2 + \gamma^2) + \frac{i\nu}{2}h_{\tilde{z}}(\alpha^2 - \gamma^2) + i\nu e^{\phi}g_{\tilde{z}}\alpha\gamma$$

$$\Lambda''^{\langle\dagger\rangle} = \frac{k_2\nu}{2c_2} - \frac{3}{8}e^{\phi}\chi_z(\beta^2 + \delta^2) - \frac{3i\nu}{4}e^{\phi}g_z\beta\delta$$
$$\Lambda'^{\langle\star\rangle} = \frac{k_2\nu}{2c_2} - \frac{3}{8}e^{\phi}\chi_{\tilde{z}}(\alpha^2 + \gamma^2) - \frac{3i\nu}{4}e^{\phi}g_{\tilde{z}}\alpha\gamma$$

$$\frac{k_2\nu}{c_2} = \frac{3k_1}{c_6}$$

$$\Lambda''^{\langle\dagger\rangle} = \frac{k_1}{c_6} + \frac{1}{4}e^{\phi}\chi_z(\beta^2 + \delta^2) + \frac{i\nu}{4}h_z(\beta^2 - \delta^2)$$
$$\Lambda'^{\langle\star\rangle} = \frac{k_1}{c_6} + \frac{1}{4}e^{\phi}\chi_{\tilde{z}}(\alpha^2 + \gamma^2) + \frac{i\nu}{4}h_{\tilde{z}}(\alpha^2 - \gamma^2)$$

$$e^{z} = \rho dw \quad D_z = \rho^{-1}\partial_w \quad \hat{\omega}_z = +i\rho^{-2}\partial_w\rho$$
$$e^{\tilde{z}} = \rho d\tilde{w} \quad D_{\tilde{z}} = \rho^{-1}\partial_{\tilde{w}} \quad \hat{\omega}_{\tilde{z}} = -i\rho^{-2}\partial_{\tilde{w}}\rho$$

$$\Lambda''^{\langle\dagger\rangle} = D_{\tilde{z}}(\alpha^2 - \gamma^2) + i\hat{\omega}_{\tilde{z}}(\alpha^2 - \gamma^2)$$
$$\Lambda'^{\langle\star\rangle} = D_z(\beta^2 - \delta^2) - i\hat{\omega}_z(\beta^2 - \delta^2)$$

$$\partial_{\tilde{w}}\big(\rho(\alpha^2-\gamma^2)\big)=0$$
$$\partial_{w}\big(\rho(\beta^2-\delta^2)\big)=0$$

$$\rho(\alpha^2-\gamma^2)=2\kappa_1$$
$$\rho(\beta^2-\delta^2)=2\tilde{\kappa}_1$$

$$\Lambda''^{\langle\dagger\rangle}=D_z(\beta\delta)-i\hat{\omega}_z\beta\delta-\varphi_z\beta\delta-\frac{1}{2}e^{\phi}\chi_z(\beta^2-\delta^2)$$
$$\Lambda'^{\langle\star\rangle}=D_{\tilde{z}}(\alpha\gamma)+i\hat{\omega}_{\tilde{z}}\alpha\gamma-\varphi_{\tilde{z}}\alpha\gamma-\frac{1}{2}e^{\phi}\chi_{\tilde{z}}(\alpha^2-\gamma^2)$$

$$\varphi_z=\rho^{-1}\partial_w\phi\quad\chi_z=\rho^{-1}\partial_w\chi$$
$$\varphi_{\tilde{z}}=\rho^{-1}\partial_{\tilde{w}}\phi\quad\chi_{\tilde{z}}=\rho^{-1}\partial_{\tilde{w}}\chi$$

$$\partial_w\big(e^{-\phi}\rho\beta\delta+\chi\tilde{\kappa}_1\big)=\Lambda''^{\langle\dagger\rangle}$$
$$\partial_{\tilde{w}}\big(e^{-\phi}\rho\alpha\gamma+\chi\kappa_1\big)=\Lambda'^{\langle\star\rangle}$$

$$\rho\beta\delta=e^{\phi}(\tilde{\kappa}_1\chi-\tilde{\kappa}_2)$$
$$\rho\alpha\gamma=e^{\phi}(\kappa_1\chi-\kappa_2)$$

$$\tau_{\pm}=\chi\pm ie^{-\phi}\;\xi=\frac{\kappa_2}{\kappa_1}\;\tilde{\xi}=\frac{\tilde{\kappa}_2}{\tilde{\kappa}_1}$$

$$\rho(\alpha\pm i\gamma)^2=\mp4\kappa_1\frac{\tau_{\mp}-\xi}{\tau_+-\tau_-}$$
$$\rho(\beta\pm i\delta)^2=\mp4\tilde{\kappa}_1\frac{\tau_{\mp}-\tilde{\xi}}{\tau_+-\tau_-}$$

$$\rho f_6=-2c_6\frac{(\kappa_1\tilde{\kappa}_1)^{\frac{1}{2}}}{\tau_+-\tau_-}\Big\{(\tau_--\xi)^{\frac{1}{2}}(\tau_+-\tilde{\xi})^{\frac{1}{2}}-(\tau_+-\xi)^{\frac{1}{2}}(\tau_--\tilde{\xi})^{\frac{1}{2}}\Big\}$$
$$\rho f_2=2ic_2\frac{(\kappa_1\tilde{\kappa}_1)^{\frac{1}{2}}}{\tau_+-\tau_-}\Big\{(\tau_--\xi)^{\frac{1}{2}}(\tau_+-\tilde{\xi})^{\frac{1}{2}}+(\tau_+-\xi)^{\frac{1}{2}}(\tau_--\tilde{\xi})^{\frac{1}{2}}\Big\}$$

$$\rho\big(h_z\pm ie^{\phi}g_z\big)=2ive^{\phi}\frac{\beta\pm i\delta}{\beta\mp i\delta}\partial_w\tau_{\pm}=\pm4iv\left(\frac{\tau_{\mp}-\tilde{\xi}}{\tau_{\pm}-\tilde{\xi}}\right)^{\frac{1}{2}}\frac{\partial_w\tau_{\pm}}{\tau_+-\tau_-}$$
$$\rho\big(h_{\tilde{z}}\pm ie^{\phi}g_{\tilde{z}}\big)=2ive^{\phi}\frac{\alpha\pm i\gamma}{\alpha\mp i\gamma}\partial_{\tilde{w}}\tau_{\pm}=\pm4iv\left(\frac{\tau_{\mp}-\xi}{\tau_{\pm}-\xi}\right)^{\frac{1}{2}}\frac{\partial_{\tilde{w}}\tau_{\pm}}{\tau_+-\tau_-}$$

$$\Lambda''^{\langle\dagger\rangle}=\frac{(D_z+i\hat{\omega}_z)(\alpha^2-\gamma^2)}{\alpha^2-\gamma^2}+\frac{\chi-\xi}{2}e^{2\phi}\chi_z+\frac{1}{X}\big(\varphi_z-e^{2\phi}(\chi-\tilde{\xi})\chi_z\big)$$
$$\Lambda'^{\langle\star\rangle}=\frac{(D_{\tilde{z}}-i\hat{\omega}_{\tilde{z}})(\beta^2-\delta^2)}{\beta^2-\delta^2}+\frac{\chi-\tilde{\xi}}{2}e^{2\phi}\chi_{\tilde{z}}+X\big(\varphi_{\tilde{z}}-e^{2\phi}(\chi-\xi)\chi_{\tilde{z}}\big)$$
$$\Lambda''^{\langle\dagger\rangle}=\frac{(D_z+i\hat{\omega}_z)(\alpha\gamma)}{\alpha^2-\gamma^2}-\frac{1}{4}e^{\phi}\chi_z-\frac{1}{2X}e^{\phi}\big((\chi-\tilde{\xi})\varphi_z+\chi_z\big)$$
$$\Lambda'^{\langle\star\rangle}=\frac{(D_{\tilde{z}}-i\hat{\omega}_{\tilde{z}})(\beta\delta)}{\beta^2-\delta^2}-\frac{1}{4}e^{\phi}\chi_{\tilde{z}}-\frac{1}{2}Xe^{\phi}((\chi-\xi)\varphi_{\tilde{z}}+\chi_{\tilde{z}})$$

$$X = \frac{\beta^2 + \delta^2}{\alpha^2 + \gamma^2}\frac{\kappa_1}{\tilde{\kappa}_1}$$

$$\Lambda''^{\langle\dagger\rangle} = -\frac{k_1}{c_6}\frac{\beta^2 + \delta^2}{\tilde{\kappa}_1^2}e^{-\phi}\rho^2 + \left(1 - e^{2\phi}(\chi - \tilde{\xi})^2\right)\chi_z + 2(\chi - \tilde{\xi})\varphi_z$$
$$\Lambda'^{\langle\star\rangle} = -\frac{k_1}{c_6}\frac{\alpha^2 + \gamma^2}{\kappa_1^2}e^{-\phi}\rho^2 + \left(1 - e^{2\phi}(\chi - \xi)^2\right)\chi_{\tilde{z}} + 2(\chi - \xi)\varphi_{\tilde{z}}$$

$$\alpha^2 + \gamma^2 = \frac{4\epsilon_\kappa\kappa_1}{\rho(\tau_+ - \tau_-)}\sqrt{-(\tau_+ - \xi)(\tau_- - \xi)} \qquad \epsilon_\kappa = \frac{\sqrt{\kappa_1^2}}{\kappa_1}$$
$$\beta^2 + \delta^2 = \frac{4\tilde{\epsilon}_\kappa\tilde{\kappa}_1}{\rho(\tau_+ - \tau_-)}\sqrt{-\left(\tau_+ - \tilde{\xi}\right)\left(\tau_- - \tilde{\xi}\right)} \qquad \tilde{\epsilon}_\kappa = \frac{\sqrt{\tilde{\kappa}_1^2}}{\tilde{\kappa}_1}$$

$$(\tau_\mp - \xi)\partial_w \ln\,\hat{\rho}^2 + \partial_w\xi \mp \frac{\partial_w\tau_\mp}{\tau_+ - \tau_-}\left\{\tau_\pm - \xi - \frac{2}{X}\left(\tau_\pm - \tilde{\xi}\right)\right\} = \Lambda''^{\langle\dagger\rangle}$$
$$\left(\tau_\mp - \tilde{\xi}\right)\partial_{\tilde{w}} \ln\,\hat{\rho}^2 + \partial_{\tilde{w}}\tilde{\xi} \mp \frac{\partial_{\tilde{w}}\tau_\mp}{\tau_+ - \tau_-}\{\tau_\pm - \tilde{\xi} - 2X(\tau_\pm - \xi)\} = \Lambda'^{\langle\star\rangle}$$

$$X = \frac{\tilde{\epsilon}_\kappa}{\epsilon_\kappa}\sqrt{\frac{\tau_+ - \tilde{\xi}}{\tau_+ - \xi}\cdot\frac{\tau_- - \tilde{\xi}}{\tau_- - \xi}}\;\rho^2 = \frac{\kappa_1\tilde{\kappa}_1\hat{\rho}^2}{(\tau_+ - \tau_-)^{\frac{1}{2}}}$$

$$\frac{k_1}{c_6}\tilde{\epsilon}_\kappa\kappa_1\hat{\rho}^2\left(\frac{1}{\tau_- - \tilde{\xi}} - \frac{1}{\tau_+ - \tilde{\xi}}\right)^{\frac{3}{2}} + \partial_w\left(\frac{1}{\tau_- - \tilde{\xi}} + \frac{1}{\tau_+ - \tilde{\xi}}\right) = \Lambda''^{\langle\dagger\rangle}$$
$$\frac{k_1}{c_6}\epsilon_\kappa\tilde{\kappa}_1\hat{\rho}^2\left(\frac{1}{\tau_- - \xi} - \frac{1}{\tau_+ - \xi}\right)^{\frac{3}{2}} + \partial_{\tilde{w}}\left(\frac{1}{\tau_- - \xi} + \frac{1}{\tau_+ - \xi}\right) = \Lambda'^{\langle\star\rangle}$$

$$Z_\pm^2 = \frac{\tau_\pm - \tilde{\xi}}{\tau_\pm - \xi}\frac{1}{R^2} = -\frac{k_1}{c_6}\frac{(Z_- + \varepsilon_\kappa Z_+)^{\frac{3}{2}}\hat{\rho}^2}{X^{\frac{1}{2}}(Z_- - \varepsilon_\kappa Z_+)^{\frac{1}{2}}(\xi - \tilde{\xi})^{\frac{1}{2}}} \quad \varepsilon_\kappa = \frac{\tilde{\epsilon}_\kappa}{\epsilon_\kappa}$$

$$X = \varepsilon_\kappa Z_+ Z_- \quad Y = -\frac{(Z_- + \varepsilon_\kappa Z_+)^2}{X}$$

$$\partial_w\left(2R^2X^{\frac{1}{2}}(\tilde{\xi} - \xi)\right) = -\epsilon_\kappa\left(\kappa_2 - \kappa_1\tilde{\xi}\right)$$
$$\partial_{\tilde{w}}\left(2R^2X^{-\frac{1}{2}}(\xi - \tilde{\xi})\right) = +\epsilon_\kappa(\tilde{\kappa}_2 - \tilde{\kappa}_1\xi)$$

$$\partial_w \ln\left(R^2X^{-\frac{1}{2}}\right) - X\partial_w \ln\,(\tilde{\xi} - \xi) = \Lambda''^{\langle\dagger\rangle}$$
$$\partial_{\tilde{w}} \ln\left(R^2X^{\frac{1}{2}}\right) - X^{-1}\partial_{\tilde{w}} \ln\,(\tilde{\xi} - \xi) = \Lambda'^{\langle\star\rangle}$$

$$\partial_w Y + 2(Y + 3)X\partial_w \ln\,(\tilde{\xi} - \xi) + (Y + 6)\partial_w \ln\,(X(\tilde{\xi} - \xi)) = \Lambda''^{\langle\dagger\rangle}$$
$$\partial_{\tilde{w}} Y + 2(Y + 3)/X\partial_{\tilde{w}} \ln\,(\tilde{\xi} - \xi) + (Y + 6)\partial_{\tilde{w}} \ln\left(X^{-1}(\tilde{\xi} - \xi)\right) = \Lambda'^{\langle\star\rangle}$$

$$-\epsilon_\kappa \kappa_1 = \partial_w \mathcal{A}_1 \qquad \epsilon_\kappa \tilde{\kappa}_1 = \partial_{\tilde{w}} \tilde{\mathcal{A}}_1$$
$$-\epsilon_\kappa \kappa_2 = \partial_w \mathcal{A}_2 \qquad \epsilon_\kappa \tilde{\kappa}_2 = \partial_{\tilde{w}} \tilde{\mathcal{A}}_2$$
$$\xi = \frac{\kappa_2}{\kappa_1} \qquad \tilde{\xi} = \frac{\tilde{\kappa}_2}{\tilde{\kappa}_1}$$

$$2R^2 X^{+\frac{1}{2}}(\tilde{\xi} - \xi) = \tilde{\mathcal{A}}_0 + \mathcal{A}_2 - \tilde{\xi}\mathcal{A}_1$$
$$2R^2 X^{-\frac{1}{2}}(\xi - \tilde{\xi}) = \mathcal{A}_0 + \tilde{\mathcal{A}}_2 - \xi\tilde{\mathcal{A}}_1$$

$$\mathcal{A}_0 = -\xi\mathcal{A}_1 + \mathcal{A}_2$$
$$\tilde{\mathcal{A}}_0 = -\tilde{\xi}\tilde{\mathcal{A}}_1 + \tilde{\mathcal{A}}_2$$

$$2R^2 X^{+\frac{1}{2}}(\tilde{\xi} - \xi) = \tilde{\mathcal{L}} \quad \tilde{\mathcal{L}} = -\tilde{\xi}\left(\tilde{\mathcal{A}}_1 + \mathcal{A}_1\right) + \tilde{\mathcal{A}}_2 + \mathcal{A}_2$$
$$2R^2 X^{-\frac{1}{2}}(\xi - \tilde{\xi}) = \mathcal{L} \quad \mathcal{L} = -\xi\left(\mathcal{A}_1 + \tilde{\mathcal{A}}_1\right) + \mathcal{A}_2 + \tilde{\mathcal{A}}_2$$

$$R^4 = -\frac{\mathcal{L}\tilde{\mathcal{L}}}{4(\xi - \tilde{\xi})^2} \; X = -\frac{\tilde{\mathcal{L}}}{\mathcal{L}}$$

$$\partial_w\left(Y(\tilde{\xi} - \xi)R^4\right) = \frac{3}{2}\left(-\mathcal{A}_2\partial_w\mathcal{A}_1 + \mathcal{A}_1\partial_w\mathcal{A}_2 + \tilde{\mathcal{A}}_1\partial_w\mathcal{A}_2 - \tilde{\mathcal{A}}_2\partial_w\mathcal{A}_1\right)$$
$$\partial_{\tilde{w}}\left(Y(\xi - \tilde{\xi})R^4\right) = \frac{3}{2}\left(-\tilde{\mathcal{A}}_2\partial_{\tilde{w}}\tilde{\mathcal{A}}_1 + \tilde{\mathcal{A}}_1\partial_{\tilde{w}}\tilde{\mathcal{A}}_2 + \mathcal{A}_1\partial_{\tilde{w}}\tilde{\mathcal{A}}_2 - \mathcal{A}_2\partial_{\tilde{w}}\tilde{\mathcal{A}}_1\right)$$

$$\partial_w \mathcal{B} = \mathcal{A}_2\partial_w\mathcal{A}_1 - \mathcal{A}_1\partial_w\mathcal{A}_2$$
$$\partial_{\tilde{w}}\tilde{\mathcal{B}} = \tilde{\mathcal{A}}_2\partial_{\tilde{w}}\tilde{\mathcal{A}}_1 - \tilde{\mathcal{A}}_1\partial_{\tilde{w}}\tilde{\mathcal{A}}_2$$

$$Y(\tilde{\xi} - \xi)R^4 = \frac{3}{2}\left(-\mathcal{B} + \tilde{\mathcal{B}} + \tilde{\mathcal{A}}_1\mathcal{A}_2 - \mathcal{A}_1\tilde{\mathcal{A}}_2\right)$$

$$Y = -6\frac{\tilde{\xi} - \xi}{\mathcal{L}\tilde{\mathcal{L}}}\left(-\mathcal{B} + \tilde{\mathcal{B}} + \tilde{\mathcal{A}}_1\mathcal{A}_2 - \mathcal{A}_1\tilde{\mathcal{A}}_2\right)$$

$$\tau_\pm - \xi = \frac{\xi - \tilde{\xi}}{Z_\pm^2 - 1} \; \tau_\pm - \tilde{\xi} = \frac{Z_\pm^2(\xi - \tilde{\xi})}{Z_\pm^2 - 1}$$

$$Z_\pm^2 = -\frac{1}{2}X(Y + 2) \pm \frac{1}{2}XYT$$

$$T^2 = \frac{Y + 4}{Y}$$

$$\rho^2 = \frac{c_6}{k_1}\frac{\kappa_1\tilde{\kappa}_1}{R^2 Y}\left(X + \frac{1}{X} + Y + 2\right)^{\frac{1}{2}}$$

$$\left(\frac{f_6}{c_6} \pm i\frac{f_2}{c_2}\right)^2 = -(\alpha \pm i\gamma)^2(\beta \mp i\delta)^2 = \frac{16\kappa_1\tilde{\kappa}_1 Z_\pm^2}{\rho^2 XY(Y + 4)}\left(X + \frac{1}{X} + Y + 2\right)$$

$$\rho^2 f_2 f_6 = 2c_2 c_6 e^{\phi}(\tilde{\kappa}_1\kappa_2 - \kappa_1\tilde{\kappa}_2)$$

$$\frac{f_6^2}{c_6^2}-\frac{f_2^2}{c_2^2}=-\frac{8\kappa_1\tilde{\kappa}_1}{\rho^2 Y}\frac{Y+2}{Y+4}\left(X+\frac{1}{X}+Y+2\right)$$

$$\frac{f_6^2}{c_6^2}+\frac{f_2^2}{c_2^2}=\frac{16\eta\kappa_1\tilde{\kappa}_1}{\rho^2 Y(Y+4)}\left(X+\frac{1}{X}+Y+2\right)$$

$$f_6^2=-4c_6^2\frac{\kappa_1\tilde{\kappa}_1}{\rho^2 Y}\frac{Y+2-2\eta}{Y+4}\left(X+\frac{1}{X}+Y+2\right)$$
$$f_2^2=+4c_2^2\frac{\kappa_1\tilde{\kappa}_1}{\rho^2 Y}\frac{Y+2+2\eta}{Y+4}\left(X+\frac{1}{X}+Y+2\right)$$

$$\left(\rho^{\mathrm{E}}\right)^2=+\frac{c_6}{k_1}\kappa_1\tilde{\kappa}_1\left(-\frac{i(\tilde{\xi}-\xi)}{2R^4 Y}T\right)^{\frac{1}{2}}$$
$$(f_6^E)^2=+2c_6^2\frac{i\kappa_1\tilde{\kappa}_1(\tilde{\xi}-\xi)}{(\rho^{\mathrm{E}})^2}T^{-\eta}$$
$$(f_2^E)^2=-2c_2^2\frac{i\kappa_1\tilde{\kappa}_1(\tilde{\xi}-\xi)}{(\rho^{\mathrm{E}})^2}T^{\eta}$$

$$\mathcal{G}=i\left(-\mathcal{B}+\tilde{\mathcal{B}}+\tilde{\mathcal{A}}_1\mathcal{A}_2-\mathcal{A}_1\tilde{\mathcal{A}}_2\right)$$
$$\kappa^2=i\left(\partial_w\mathcal{A}_1\partial_{\tilde{w}}\tilde{\mathcal{A}}_2-\partial_{\tilde{w}}\tilde{\mathcal{A}}_1\partial_w\mathcal{A}_2\right)=-\partial_w\partial_{\tilde{w}}\mathcal{G}$$

$$Y=\frac{6\kappa^2\mathcal{G}}{\partial_w\mathcal{G}\partial_{\tilde{w}}\mathcal{G}}\quad R^4Y=-\frac{3i\mathcal{G}}{2(\tilde{\xi}-\xi)}\quad X=\frac{\partial_w\mathcal{A}_1\partial_{\tilde{w}}\mathcal{G}}{\partial_{\tilde{w}}\tilde{\mathcal{A}}_1\partial_w\mathcal{G}}$$

$$T^2=1+\frac{2}{3}\frac{\partial_w\mathcal{G}\partial_{\tilde{w}}\mathcal{G}}{\kappa^2\mathcal{G}}$$

$$\left(\rho^{\mathrm{E}}\right)^2=-\frac{c_6}{k_1}\kappa^2\sqrt{-\frac{T}{3\mathcal{G}}}\quad(f_6^E)^2=-\frac{2c_6^2\kappa^2}{(\rho^{\mathrm{E}})^2}\frac{1}{T^{\eta}}\quad(f_2^E)^2=+\frac{2c_2^2\kappa^2}{(\rho^{\mathrm{E}})^2}T^{\eta}$$

$$\tau_{\pm}=\frac{\partial_{\tilde{w}}\tilde{\mathcal{A}}_2(T\pm1)\partial_w\mathcal{G}+\partial_w\mathcal{A}_2(T\mp1)\partial_{\tilde{w}}\mathcal{G}}{\partial_{\tilde{w}}\tilde{\mathcal{A}}_1(T\pm1)\partial_w\mathcal{G}+\partial_w\mathcal{A}_1(T\mp1)\partial_{\tilde{w}}\mathcal{G}}$$

$$\begin{pmatrix}\mathcal{A}_2'\\ \mathcal{A}_1'\end{pmatrix}=M\begin{pmatrix}\mathcal{A}_2\\ \mathcal{A}_1\end{pmatrix}\quad\begin{pmatrix}\tilde{\mathcal{A}}_2'\\ \tilde{\mathcal{A}}_1'\end{pmatrix}=M\begin{pmatrix}\tilde{\mathcal{A}}_2\\ \tilde{\mathcal{A}}_1\end{pmatrix}\quad M=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$$

$$\tau_{\pm}'=\frac{a\tau_{\pm}+b}{c\tau_{\pm}+d}$$

$$\kappa_1'=(c\xi+d)\kappa_1\quad\xi'=\frac{a\xi+b}{c\xi+d}\quad\mathcal{L}'=\frac{\mathcal{L}}{c\xi+d}$$
$$\tilde{\kappa}_1'=(c\tilde{\xi}+d)\tilde{\kappa}_1\quad\tilde{\xi}'=\frac{a\tilde{\xi}+b}{c\tilde{\xi}+d}\quad\tilde{\mathcal{L}}'=\frac{\tilde{\mathcal{L}}}{c\tilde{\xi}+d}$$

$$\rho'\left(h_z'\pm ie^{\phi'}g_z'\right)=(c\tau_{\mp}+d)^{\frac{1}{2}}(c\tau_{\pm}+d)^{-\frac{1}{2}}\rho\left(h_z\pm ie^{\phi}g_z\right)$$
$$\rho'\left(h_{\tilde{z}}'\pm ie^{\phi'}g_{\tilde{z}}'\right)=(c\tau_{\pm}+d)^{\frac{1}{2}}(c\tau_{\mp}+d)^{-\frac{1}{2}}\rho\left(h_{\tilde{z}}\pm ie^{\phi}g_{\tilde{z}}\right)$$

$$\mathcal{B}_\eta^{-1}\varepsilon^* = S\varepsilon\, S = I_2, s^1, s^2, s^3$$

$$\varepsilon = \sum_{\eta_1,\eta_2=\pm} \chi^{\eta_1,\eta_2} \otimes \zeta_{\eta_1,\eta_2}$$

$$\mathcal{B}_\eta^{-1}\varepsilon^* = \sum_{\eta_1,\eta_2=\pm} \chi^{\eta_1,\eta_2} \otimes \left(\mathfrak{b}_\eta^{-1}\zeta^*\right)_{\eta_1,\eta_2}$$

$$\mathcal{K}\zeta^* = \zeta\,\mathcal{K} = S^{-1} \otimes \mathfrak{b}_\eta^{-1}$$

$$\zeta \to \mathcal{K}\zeta^\star\,\mathcal{K} = S^{-1} \otimes \mathfrak{b}_\eta^{-1}$$

$$\tilde{F}_{(3)} = g_a e^a \wedge e^{67}\, H_{(3)} = h_a e^a \wedge e^{67}\, d\phi = \varphi_a e^a\, d\chi = \chi_a e^a$$

$$(e^A)^* = \eta_{AA} e^A$$

$$\mathfrak{b}_\eta^{-1} = K_1 K_2 K_3 \tau^{(02)} \otimes \sigma^2$$

$$K_1 = \begin{cases} \tau^{(00)} \otimes I_2 & \text{for} \quad (\mathbb{k}_1, \mathbb{k}_2) = (1, i) \\ \tau^{(01)} \otimes I_2 & \text{for} \quad (\mathbb{k}_1, \mathbb{k}_2) = (1,1) \\ \tau^{(10)} \otimes I_2 & \text{for} \quad (\mathbb{k}_1, \mathbb{k}_2) = (i, i) \\ \tau^{(11)} \otimes I_2 & \text{for} \quad (\mathbb{k}_1, \mathbb{k}_2) = (i, 1) \end{cases}$$

$$\mathbb{k}_1 = k_1 \frac{|f_6|}{f_6}\ \mathbb{k}_2 = k_2 \frac{|f_2|}{f_2}$$

$$K_2 = \begin{cases} \tau^{(00)} \otimes I_2 & \text{for} \quad \mathcal{M}_{0,2}, \mathcal{M}_{2,0} \\ -i\tau^{(32)} \otimes I_2 & \text{for} \quad \mathcal{M}_{1,1} \end{cases} \quad K_3 = \begin{cases} \tau^{(00)} \otimes I_2 & \text{for} \quad \Sigma_{0,2} \\ i\tau^{(20)} \otimes \sigma^1 & \text{for} \quad \Sigma_{1,1} \\ \tau^{(00)} \otimes \sigma^3 & \text{for} \quad \Sigma_{2,0} \end{cases}$$

$$\begin{gathered} (\hat{e}^m \gamma_m)^* = B_{\eta(1)} (\hat{e}^m \gamma_m) B_{\eta(1)}^{-1} \\ \left(\hat{e}^i \gamma_i\right)^* = -B_{\eta(2)} \left(\hat{e}^i \gamma_i\right) B_{\eta(2)}^{-1} \\ (e^a \gamma_a)^* = -B_{\eta(3)} (e^a \gamma_a) B_{\eta(3)}^{-1} \end{gathered}$$

$$\begin{array}{lll} \mathcal{M}_{0,6} & B_{\eta(1)}^{-1} = \gamma_{(1)} \gamma^0 B_{(1)}^{-1} & \alpha_1 = 1 \\ \mathcal{M}_{1,5} & B_{\eta(1)}^{-1} = B_{(1)}^{-1} & \alpha_1 = 0 \\ \mathcal{M}_{2,4} & B_{\eta(1)}^{-1} = \gamma_{(1)} \gamma^1 B_{(1)}^{-1} & \alpha_1 = 1 \\ \mathcal{M}_{3,3} & B_{\eta(1)}^{-1} = \gamma^1 \gamma^2 B_{(1)}^{-1} & \alpha_1 = 0 \\ \mathcal{M}_{4,2} & B_{\eta(1)}^{-1} = \gamma_{(1)} \gamma^1 \gamma^2 \gamma^3 B_{(1)}^{-1} & \alpha_1 = 1 \\ \mathcal{M}_{5,1} & B_{\eta(1)}^{-1} = \gamma^1 \gamma^2 \gamma^3 \gamma^4 B_{(1)}^{-1} & \alpha_1 = 0 \end{array}$$

$$\begin{array}{lll} \mathcal{M}_{0,2} & B_{\eta(2)}^{-1} = B_{(2)}^{-1} = \sigma^2 & \alpha_2 = 1 \\ \mathcal{M}_{1,1} & B_{\eta(2)}^{-1} = \gamma_{(2)} \gamma^7 B_{(2)}^{-1} = \sigma^3 & \alpha_2 = 0 \\ \mathcal{M}_{2,0} & B_{\eta(2)}^{-1} = \gamma^6 \gamma^7 B_{(2)}^{-1} = \sigma^1 & \alpha_2 = 1 \end{array}$$

$$\begin{array}{lll}\Sigma_{0,2} & B_{\eta(3)}^{-1} = B_{(3)}^{-1} = \sigma^2 & \alpha_3 = 1 \\ \Sigma_{1,1} & B_{\eta(3)}^{-1} = \gamma_{(3)}\gamma^9 B_{(3)}^{-1} = \sigma^3 & \alpha_3 = 0 \\ \Sigma_{2,0} & B_{\eta(3)}^{-1} = i\gamma^8\gamma^9 B_{(3)}^{-1} = i\sigma^1 & \alpha_3 = 1\end{array}$$

$$\begin{array}{l}\gamma_{(1)}B_{\eta(1)} = (-)^{\alpha_1}B_{\eta(1)}\gamma_{(1)} \\ \gamma_{(2)}B_{\eta(2)} = (-)^{\alpha_2}B_{\eta(2)}\gamma_{(2)} \\ \gamma_{(3)}B_{\eta(3)} = (-)^{\alpha_3}B_{\eta(3)}\gamma_{(3)}\end{array}$$

$$\mathcal{B}_\eta = \tilde{B}_{\eta(1)} \otimes \tilde{B}_{\eta(2)} \otimes \tilde{B}_{\eta(3)}$$

$$\mathcal{B}_\eta\left(\hat{e}^i\Gamma_i\right)\mathcal{B}_\eta^{-1} = B_{\eta(1)}\gamma_{(1)}B_{\eta(1)}^{-1} \otimes \tilde{B}_{\eta(2)}\left(\hat{e}^i\gamma_i\right)\tilde{B}_{\eta(2)}^{-1} \otimes I_2 = \gamma_{(1)} \otimes \left(\hat{e}^i\gamma_i\right)^* \otimes I_2$$

$$\tilde{B}_{\eta(2)}\left(\hat{e}^i\gamma_i\right)\tilde{B}_{\eta(2)}^{-1} = (-)^{\alpha_1}\left(\hat{e}^i\gamma_i\right)^* = (-)^{1-\alpha_1}B_{\eta(2)}\left(\hat{e}^i\gamma_i\right)B_{\eta(2)}^{-1}$$

$$\mathcal{B}_\eta = B_{\eta(1)} \otimes \left(B_{\eta(2)}\gamma_{(2)}^{\alpha_1+1}\right) \otimes \left(B_{\eta(3)}\gamma_{(3)}^{\alpha_1+\alpha_2+1}\right)$$

$$\alpha_1 + \alpha_2 + \alpha_3 \equiv 0(\mathrm{mod}2)$$

$$\mathcal{B}_\eta = \left(B_{\eta(1)}\gamma_{(1)}^{1+\alpha_3}\right) \otimes \left(B_{\eta(2)}\gamma_{(2)}^{\alpha_2}\right) \otimes B_{\eta(3)}$$

$$\left(\mathcal{B}_{\eta(1)}^{-1} \otimes \mathcal{B}_{\eta(2)}^{-1}\right)(\chi^{+,+})^* = \chi^{k_1^2,-k_2^2}$$

$$\begin{array}{l}\left(\gamma_{(1)} \otimes I_2\right)\chi^{\eta_1,\eta_2} = \chi^{-\eta_1,\eta_2} \\ \left(I_8 \otimes \gamma_{(2)}\right)\chi^{\eta_1,\eta_2} = \chi^{\eta_1,-\eta_2}\end{array}$$

$$\left(\mathcal{B}_{\eta(1)}^{-1} \otimes \mathcal{B}_{\eta(2)}^{-1}\right)(\chi^{\eta_1,\eta_2})^* = \eta_1^{\alpha_1}\eta_2^{\alpha_2}\chi^{k_1^2\eta_1,-k_2^2\eta_2}$$

$$\mathcal{B}_\eta^{-1}\varepsilon^* = \sum_{\eta_1,\eta_2} \eta_1^{\alpha_2+\alpha_3}\eta_2^{\alpha_2}\gamma_{(1)}^{1+\alpha_3} \otimes \gamma_{(2)}^{\alpha_2}\chi^{k_1^2\eta_1,-k_2^2\eta_2} \otimes B_{\eta(3)}^{-1}\zeta_{\eta_1,\eta_2}^*$$

$$\begin{array}{ll}(\alpha_2,\alpha_3) = (0,0) & \mathcal{B}_\eta^{-1}\varepsilon^* = \sum \chi^{\eta_1,\eta_2} \otimes B_{\eta(3)}^{-1}\zeta_{-k_1^2\eta_1,-k_2^2\eta_2}^* \\ (\alpha_2,\alpha_3) = (0,1) & \mathcal{B}_\eta^{-1}\varepsilon^* = \sum_{\eta_1,\eta_2} \chi^{\eta_1,\eta_2} \otimes (k_1^2\eta_1)B_{\eta(3)}^{-1}\zeta_{k_1^2\eta_1,-k_2^2\eta_2}^* \\ (\alpha_2,\alpha_3) = (1,0) & \mathcal{B}_\eta^{-1}\varepsilon^* = \sum_{\eta_1,\eta_2} \chi^{\eta_1,\eta_2} \otimes (-k_1^2\eta_1k_2^2\eta_2)B_{\eta(3)}^{-1}\zeta_{-k_1^2\eta_1,k_2^2\eta_2}^* \\ (\alpha_2,\alpha_3) = (1,1) & \mathcal{B}_\eta^{-1}\varepsilon^* = \sum_{\eta_1,\eta_2} \chi^{\eta_1,\eta_2} \otimes (k_2^2\eta_2)B_{\eta(3)}^{-1}\zeta_{k_1^2\eta_1,k_2^2\eta_2}^*\end{array}$$

$$\hat{K}_1 = \begin{cases}\tau^{(00)} \otimes I_2 & \text{for} \quad (k_1,k_2) = (1,i) \\ \tau^{(01)} \otimes I_2 & \text{for} \quad (k_1,k_2) = (1,1) \\ \tau^{(10)} \otimes I_2 & \text{for} \quad (k_1,k_2) = (i,i) \\ \tau^{(11)} \otimes I_2 & \text{for} \quad (k_1,k_2) = (i,1)\end{cases}$$

$$(\alpha_2,\alpha_3)=(0,0)\quad \mathfrak{b}_\eta^{-1}=\hat{K}_1\tau^{(10)}\otimes\sigma^3$$
$$(\alpha_2,\alpha_3)=(0,1)\quad \mathfrak{b}_\eta^{-1}=-i\hat{K}_1\tau^{(30)}\otimes B_{\eta(3)}^{-1}$$
$$(\alpha_2,\alpha_3)=(1,0)\quad \mathfrak{b}_\eta^{-1}=-\hat{K}_1\tau^{(22)}\otimes\sigma^3$$
$$(\alpha_2,\alpha_3)=(1,1)\quad \mathfrak{b}_\eta^{-1}=\hat{K}_1\tau^{(02)}\otimes B_{\eta(3)}^{-1}$$

$$\mathcal{K}\zeta^*=\zeta\,\mathcal{K}=S^{-1}\otimes\mathfrak{b}_\eta^{-1}\,\mathcal{K}^*=\mathcal{K}$$

$$\mathcal{J}\mathcal{K}\mathcal{J}\zeta=\mathcal{K}\zeta\,\mathcal{K}^*\mathcal{K}\zeta=\mathcal{K}^2\zeta=\zeta$$

$$\mathcal{J}S\mathcal{J}=S\cdot\begin{cases}+1 & \mathrm{IIB}_{\mathbb{R}}\quad \text{or } \mathrm{IIB}_3\\ -1 & \mathrm{IIB}^\star\quad \text{or } \mathrm{IIB}'\end{cases}$$
$$\mathcal{J}K_1\mathcal{J}=K_1\cdot\begin{cases}+1 & (\Bbbk_1,\Bbbk_2)\in\{(1,i),(i,1)\}\\ -1 & (\Bbbk_1,\Bbbk_2)\in\{(1,1),(i,i)\}\end{cases}$$
$$\mathcal{J}K_2\mathcal{J}=K_2$$
$$\mathcal{J}K_3\mathcal{J}=K_3\cdot\begin{cases}+1 & \Sigma_{2,0}\text{ or }\Sigma_{0,2}\\ -1 & \Sigma_{1,1}\end{cases}$$

$$\begin{array}{lll}
\mathrm{IIB}_{\mathbb{R}},\mathrm{IIB}_3: & (\Bbbk_1,\Bbbk_2)\in\{(1,i),(i,1)\} & \{\Sigma_{2,0}\text{ or }\Sigma_{0,2}\}\times\{\mathcal{M}_{0,2}\text{ or }\mathcal{M}_{1,1}\}\\
 & (\Bbbk_1,\Bbbk_2)\in\{(1,1),(i,i)\} & \Sigma_{1,1}\times\mathcal{M}_{2,0}\\
 & & \Sigma_{1,1}\times\{\mathcal{M}_{0,2}\text{ or }\mathcal{M}_{1,1}\}\\
 & & \{\Sigma_{2,0}\text{ or }\Sigma_{0,2}\}\times\mathcal{M}_{2,0}\\
\mathrm{IIB}^\star,\mathrm{IIB}': & (\Bbbk_1,\Bbbk_2)\in\{(1,1),(i,i)\} & \{\Sigma_{2,0}\text{ or }\Sigma_{0,2}\}\times\{\mathcal{M}_{0,2}\text{ or }\mathcal{M}_{1,1}\}\\
 & & \Sigma_{1,1}\times\mathcal{M}_{2,0}\\
 & (\Bbbk_1,\Bbbk_2)\in\{(1,i),(i,1)\} & \Sigma_{1,1}\times\{\mathcal{M}_{0,2}\text{ or }\mathcal{M}_{1,1}\}\\
 & & \{\Sigma_{2,0}\text{ or }\Sigma_{0,2}\}\times\mathcal{M}_{2,0}
\end{array}$$

$$S^{-2}=\begin{cases}+I_2 & \text{for} \quad \mathrm{IIB}_{\mathbb{R}},\mathrm{IIB}^\star,\mathrm{IIB}'\\ -I_2 & \text{for} \quad \mathrm{IIB}_3\end{cases}\Bigg\}$$

$$\mathcal{K}^2\zeta=\zeta\begin{cases}\Bbbk_2^2, & \mathcal{M}_{0,2},\mathcal{M}_{2,0}\\ \Bbbk_1^2, & \mathcal{M}_{1,1}\end{cases}\Bigg\}\begin{cases}-1,\Sigma_{0,2}\\ 1,\Sigma_{2,0}\end{cases}\Bigg\}\begin{cases}+1,\mathrm{IIB}_{\mathbb{R}},\mathrm{IIB}^\star,\mathrm{IIB}'\\ -1,\mathrm{IIB}_3\end{cases}\Bigg\}$$

$$\mathcal{K}^2\zeta=\zeta\begin{cases}-\Bbbk_1^2\Bbbk_2^2, & \{\mathcal{M}_{0,2}\text{ or }\mathcal{M}_{2,0}\}\times\Sigma_{1,1}\\ 1, & \mathcal{M}_{1,1}\times\Sigma_{1,1}\end{cases}\Bigg\}\begin{cases}+1, & \mathrm{IIB}_{\mathbb{R}},\mathrm{IIB}^\star,\mathrm{IIB}'\\ -1, & \mathrm{IIB}_3\end{cases}\Bigg\}$$

$$S^n=\{\mu_{ij}x^ix^j=+1;ds^2=\mu_{ij}dx^idx^j;\mu=(+_{n+1})\}$$
$$H_n=\{\mu_{ij}x^ix^j=-1;ds^2=\mu_{ij}dx^idx^j;\mu=(-_1,+_n)\}$$
$$AdS_{r,s}=\{\mu_{ij}x^ix^j=-1;ds^2=\mu_{ij}dx^idx^j;\mu=(-_{r+1},+_s)\}$$
$$dS_{r,s}=\{\mu_{ij}x^ix^j=+1;ds^2=\mu_{ij}dx^idx^j;\mu=(-_r,+_{s+1})\}$$

$$-dS_{p,q}=AdS_{q,p}\ -AdS_{p,q}=dS_{q,p}$$

k_1	k_2	$f_2^2\mathcal{M}_2$	Σ	real form	geometry	bosonic subalgebra
1	i	(0,2)	(0,2)	$\mathrm{IIB}_{\mathbb{R}}$	$AdS_{1,5}\times S^2\times\Sigma_{0,2}$	$\mathfrak{so}(2,5)\oplus\mathfrak{so}(3)$

i	1	(1,1)	(0,2)	$\mathrm{IIB}_{\mathbb{R}}$	$S^6 \times AdS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	i	(0,2)	(0,2)	IIB*, IIB $'$	$dS_{1,5} \times S^2 \times \Sigma_{0,2}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
1	i	(0,2)	(1,1)	IIB *, IIB $'$	$H^6 \times S^2 \times \Sigma_{1,1}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	1	(0,2)	(1,1)	IIB *, IIB $'$	$S^6 \times H^2 \times \Sigma_{1,1}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	i	(1,1)	(0,2)	IIB *, IIB $'$	$S^6 \times dS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	1	(0,2)	(0,2)	IIB_3	$dS_{3,3} \times H^2 \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(0,2)	(1,1)	IIB_3	$AdS_{2,4} \times H^2 \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	i	(0,2)	(1,1)	IIB_3	$dS_{2,4} \times S^2 \times \Sigma_{1,1}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	i	(0,2)	(2,0)	IIB_3	$AdS_{1,5} \times S^2 \times \Sigma_{2,0}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	i	(1,1)	(0,2)	IIB_3	$AdS_{2,4} \times dS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(1,1)	(2,0)	IIB_3	$S^6 \times AdS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(0,2)	IIB_3	$dS_{1,5} \times (-S^2) \times \Sigma_{0,2}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
1	1	(2,0)	(1,1)	IIB_3	$H^6 \times (-S^2) \times \Sigma_{1,1}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	i	(2,0)	(1,1)	IIB_3	$S^6 \times (-H^2) \times \Sigma_{1,1}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$

$$\mathcal{M}_2 = S^2 \to \mathcal{M}_2 = AdS_{2,0} \;\; \mathcal{M}_2 = H^2 \to \mathcal{M}_2 = dS_{2,0}$$

$$\left(AdS_{p,q}, dS_{p,q}, \Sigma_{r,2-r}\right) \longrightarrow \left(-AdS_{p,q}, -dS_{p,q}, -\Sigma_{r,2-r}\right) = \left(dS_{q,p}, AdS_{q,p}, \Sigma_{2-r,r}\right)$$

$\mathbb{k}_1$	$\mathbb{k}_2$	$f_2^2\mathcal{M}_2$	Σ	real form	geometry	bosonic subalgebra
1	i	(0,2)	(0,2)	IIB_5	$AdS_{5,1} \times S^2 \times \Sigma_{0,2}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	1	(0,2)	(2,0)	IIB_5	$dS_{3,3} \times H_2 \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$

i	1	(1,1)	(0,2)	IIB_5	$dS_{4,2} \times AdS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(1,1)	(2,0)	IIB_5	$AdS_{2,4} \times dS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(1,1)	(1,1)	IIB_5	$AdS_{3,3} \times AdS_{1,1} \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	i	(1,1)	(1,1)	IIB_5	$dS_{3,3} \times dS_{1,1} \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(2,0)	(0,2)	IIB_5	$AdS_{3,3} \times (-H_2) \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(2,0)	IIB_5	$dS_{1,5} \times (-S^2) \times \Sigma_{2,0}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	i	(0,2)	(0,2)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$dS_{5,1} \times S^2 \times \Sigma_{0,2}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	i	(0,2)	(1,1)	$\mathrm{IIB}\ {}^{\star}_{5}, \mathrm{IIB}_5'$	$AdS_{4,2} \times S^2 \times \Sigma_{1,1}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
i	1	(0,2)	(1,1)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$dS_{4,2} \times H^2 \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(0,2)	(2,0)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$AdS_{3,3} \times H^2 \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	i	(1,1)	(0,2)	$\mathrm{IIB}\ {}^{\star}_{5}, \mathrm{IIB}\ {}^{\prime}_{5}$	$dS_{4,2} \times dS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(1,1)	(2,0)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$AdS_{2,4} \times AdS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(1,1)	(1,1)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$AdS_{3,3} \times dS_{1,1} \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(1,1)	(1,1)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$dS_{3,3} \times AdS_{1,1} \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(2,0)	(1,1)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$AdS_{2,4} \times (-H^2) \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(2,0)	(2,0)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$AdS_{1,5} \times (-S^2) \times \Sigma_{2,0}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
i	i	(2,0)	(0,2)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$dS_{3,3} \times (-H^2) \times \Sigma_{0,2}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(1,1)	$\mathrm{IIB}_5^{\star}, \mathrm{IIB}_5'$	$dS_{2,4} \times (-S^2) \times \Sigma_{1,1}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$

$$\left(AdS_{5,1} \times S^2 \times \Sigma_{2,0}, dS_{5,1} \times (-S^2) \times \Sigma_{0,2}\right)$$

k_1	k_2	$f^2\mathcal{M}_2$	Σ	real form	geometry	bosonic subalgebra
1	i	(0,2)	(2,0)	IIB_7	$AdS_{5,1} \times S^2 \times \Sigma_{2,0}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
1	1	(0,2)	(1,1)	IIB_7	$(-S^6) \times H^2 \times \Sigma_{1,1}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	i	(0,2)	(1,1)	IIB_7	$(-H^6) \times S^2 \times \Sigma_{1,1}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	1	(1,1)	(2,0)	IIB_7	$dS_{4,2} \times AdS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	i	(1,1)	(0,2)	IIB_7	$(-S^6) \times dS_{1,1} \times \Sigma_{0,2}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
1	i	(2,0)	(2,0)	IIB_7	$AdS_{3,3} \times (-H^2) \times \Sigma_{2,0}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
1	1	(2,0)	(1,1)	IIB_7	$AdS_{4,2} \times (-S^2) \times \Sigma_{1,1}$	$\mathfrak{so}(5,2) \oplus \mathfrak{so}(3)$
i	i	(2,0)	(1,1)	IIB_7	$dS_{4,2} \times (-H^2) \times \Sigma_{1,1}$	$\mathfrak{so}(3,4) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(0,2)	IIB_7	$dS_{5,1} \times (-S^2) \times \Sigma_{0,2}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	i	(1,1)	(2,0)	IIB_9	$(-S^6) \times dS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
i	1	(2,0)	(2,0)	IIB_9	$dS_{5,1} \times (-S^2) \times \Sigma_{2,0}$	$\mathfrak{so}(2,5) \oplus \mathfrak{so}(3)$
1	1	(1,1)	(2,0)	$IIB_9^\star, IIB_9'$	$(-S^6) \times AdS_{1,1} \times \Sigma_{2,0}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
1	1	(2,0)	(1,1)	$IIB\ {}^\star_9, IIB\ {}'_9$	$(-S^6) \times (-H^2) \times \Sigma_{1,1}$	$\mathfrak{so}(7) \oplus \mathfrak{so}(1,2)$
1	i	(2,0)	(2,0)	$IIB_9^\star, IIB_9'$	$AdS_{5,1} \times (-S^2) \times \Sigma_{2,0}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$
i	i	(2,0)	(1,1)	$IIB_9^\star, IIB_9'$	$(-H^6) \times (-S^2) \times \Sigma_{1,1}$	$\mathfrak{so}(1,6) \oplus \mathfrak{so}(3)$

$$\rho^\star = \Lambda_\rho \rho$$

geometry	real form	$\{\alpha, \beta, \gamma, \delta\}^\star$	$\Lambda_\rho\{\tilde{\kappa}_1, \kappa_1, \tilde{\kappa}_2, \kappa_2\}^\star$

$AdS_{1,5} \times S^2 \times \Sigma_{0,2}$	$\text{IIB}_{\mathbb{R}}$	$\{-\delta\nu, \gamma\nu, \beta\nu, -\alpha\nu\}$	$\{-\kappa_1, -\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
$S^6 \times AdS_{1,1} \times \Sigma_{0,2}$	$\text{IIB}_{\mathbb{R}}$	$\{\beta, \alpha, \delta, \gamma\}$	$\{+\kappa_1, +\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$dS_{1,5} \times S^2 \times \Sigma_{0,2}$	IIB*	$\{-\beta, -\alpha, \delta, \gamma\}$	$\{+\kappa_1, +\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
	IIB $'$	$\{-\delta, -\gamma, -\beta, -\alpha\}$	$\{-\kappa_1, -\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$H^6 \times S^2 \times \Sigma_{1,1}$	IIB*	$\{\alpha, \beta, -\gamma, -\delta\}$	$\{+\tilde{\kappa}_1, +\kappa_1, -\tilde{\kappa}_2, -\kappa_2\}$
	IIB $'$	$\{\gamma, \delta, \alpha, \beta\}$	$\{-\tilde{\kappa}_1, -\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$S^6 \times H^2 \times \Sigma_{1,1}$	IIB*	$\{\alpha, -\beta, -\gamma, \delta\}$	$\{+\tilde{\kappa}_1, +\kappa_1, -\tilde{\kappa}_2, -\kappa_2\}$
	IIB $'$	$\{\gamma, -\delta, \alpha, -\beta\}$	$\{-\tilde{\kappa}_1, -\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$S^6 \times dS_{1,1} \times \Sigma_{0,2}$	IIB*	$\{-\delta\nu, -\gamma\nu, -\beta\nu, -\alpha\nu\}$	$\{-\kappa_1, -\kappa_1, +\kappa_2, +\tilde{\kappa}_2\}$
	IIB $'$	$\{\beta\nu, \alpha\nu, -\delta\nu, -\gamma\nu\}$	$\{+\kappa_1, +\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
$dS_{3,3} \times H^2 \times \Sigma_{0,2}$	IIB_3	$\{\beta\nu, \alpha\nu, \delta\nu, \gamma\nu\}$	$\{+\kappa_1, +\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$AdS_{2,4} \times H^2 \times \Sigma_{1,1}$	IIB_3	$\{-\alpha\nu, -\beta\nu, -\gamma\nu, -\delta\nu\}$	$\{+\tilde{\kappa}_1, +\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$dS_{2,4} \times S^2 \times \Sigma_{1,1}$	IIB_3	$\{-\alpha\nu, \beta\nu, -\gamma\nu, \delta\nu\}$	$\{+\tilde{\kappa}_1, +\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$AdS_{1,5} \times S^2 \times \Sigma_{2,0}$	IIB_3	$\{\beta\nu, \alpha\nu, \delta\nu, \gamma\nu\}$	$\{+\kappa_1, +\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$AdS_{2,4} \times dS_{1,1} \times \Sigma_{0,2}$	IIB_3	$\{\delta, -\gamma, -\beta, \alpha\}$	$\{-\kappa_1, -\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
$S^6 \times AdS_{1,1} \times \Sigma_{2,0}$	IIB_3	$\{\delta, -\gamma, -\beta, \alpha\}$	$\{-\kappa_1, -\tilde{\kappa}_1, -\kappa_2, -\tilde{\kappa}_2\}$
$dS_{1,5} \times (-S^2) \times \Sigma_{0,2}$	IIB_3	$\{\beta\nu, \alpha\nu, \delta\nu, \gamma\nu\}$	$\{+\kappa_1, +\tilde{\kappa}_1, +\kappa_2, +\tilde{\kappa}_2\}$
$H^6 \times (-S^2) \times \Sigma_{1,1}$	IIB_3	$\{-\alpha\nu, -\beta\nu, -\gamma\nu, -\delta\nu\}$	$\{+\tilde{\kappa}_1, +\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$
$S^6 \times (-H^2) \times \Sigma_{1,1}$	IIB_3	$\{-\alpha\nu, \beta\nu, -\gamma\nu, \delta\nu\}$	$\{+\tilde{\kappa}_1, +\kappa_1, +\tilde{\kappa}_2, +\kappa_2\}$

$$AdS_6 \times S^2 \times \Sigma: \quad (\alpha^\star, \beta^\star) = \nu(-\delta, \gamma) \quad \kappa_{1/2}^\star = -\tilde{\kappa}_{1/2} \quad c_6 \in \mathbb{R}$$
$$S^6 \times AdS_2 \times \Sigma: \quad (\alpha^\star, \gamma^\star) = (\beta, \delta) \qquad \kappa_{1/2}^\star = +\tilde{\kappa}_{1/2} \quad c_6 \in i\mathbb{R}$$

$$\epsilon_\kappa = \mp\tilde{\epsilon}_\kappa \; \varepsilon_\kappa = \mp 1$$

$$(\mathcal{A}_1)^\star = -\varepsilon_\kappa \tilde{\mathcal{A}}_1 \; (\mathcal{A}_2)^\star = -\varepsilon_\kappa \tilde{\mathcal{A}}_2$$

$$h_z = \frac{2i\left(2\beta\delta\phi_z + e^{\phi}\chi_z(\beta^2 - \delta^2)\right)}{\nu(\beta^2 + \delta^2)} \qquad h_{\bar{z}} = \frac{2i\left(2\alpha\gamma\phi_{\bar{z}} + e^{\phi}\chi_{\bar{z}}(\alpha^2 - \gamma^2)\right)}{\nu(\alpha^2 + \gamma^2)}$$
$$g_z = \frac{2i\left((\delta^2 - \beta^2)e^{-\phi}\phi_z + 2\beta\delta\chi_z\right)}{\nu(\beta^2 + \delta^2)} \qquad g_{\bar{z}} = \frac{2i\left((\gamma^2 - \alpha^2)e^{-\phi}\phi_{\bar{z}} + 2\alpha\gamma\chi_{\bar{z}}\right)}{\nu(\alpha^2 + \gamma^2)}$$

$$(h_z)^\star = \pm h_{\bar{z}} \; (g_z)^\star = \pm g_{\bar{z}}$$

$$\hat{f}_2^2 = \frac{c^2\hat{\kappa}^2}{9\hat{\rho}^2}\frac{1}{\hat{T}} \; \hat{f}_6^2 = \frac{c^2\hat{\kappa}^2}{\hat{\rho}^2}\hat{T} \; \hat{\rho}^2 = c\hat{\kappa}^2\sqrt{\frac{\hat{T}}{6\Lambda\hat{\mathcal{G}}}}$$

$$\hat{\mathcal{G}} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \hat{\mathcal{B}} + \hat{\mathcal{B}}^\star\partial_w\hat{\mathcal{B}} = \mathcal{A}_+\partial_w\mathcal{A}_- - \mathcal{A}_-\partial_w\mathcal{A}_+$$
$$\hat{\kappa}^2 = -|\partial_w\mathcal{A}_+|^2 + |\partial_w\mathcal{A}_-|^2\hat{T} = \frac{1 + \Lambda\hat{R}}{1 - \Lambda\hat{R}} = \sqrt{1 + \frac{2\left|\partial_w\hat{\mathcal{G}}\right|^2}{3\hat{\kappa}^2\hat{\mathcal{G}}}}$$

$$B = \frac{1 + i\hat{\tau}}{1 - i\hat{\tau}} = \frac{\partial_w\mathcal{A}_+\partial_{\bar{w}}\hat{\mathcal{G}} - \Lambda\hat{R}\partial_{\bar{w}}\overline{\mathcal{A}}_-\partial_w\hat{\mathcal{G}}}{\Lambda\hat{R}\partial_{\bar{w}}\overline{\mathcal{A}}_+\partial_w\hat{\mathcal{G}} - \partial_w\mathcal{A}_-\partial_{\bar{w}}\hat{\mathcal{G}}}$$
$$\mathcal{C} = \frac{4ic}{9}\Lambda\left[\frac{\partial_{\bar{w}}\overline{\mathcal{A}} - \partial_w\hat{\mathcal{G}}}{\hat{\kappa}^2} - 2\Lambda\hat{R}\frac{\partial_{\bar{w}}\hat{\mathcal{G}}\partial_w\mathcal{A}_+ + \partial_w\hat{\mathcal{G}}\partial_{\bar{w}}\overline{\mathcal{A}}_-}{(1 + \Lambda\hat{R})^2\hat{\kappa}^2} - \overline{\mathcal{A}}_- - 2\mathcal{A}_+\right]$$

$$\mathcal{A}_1 = \sqrt{-\varepsilon_\kappa}(a_1\mathcal{A}_+ - a_1^\star\mathcal{A}_-) \quad \mathcal{A}_2 = \sqrt{-\varepsilon_\kappa}(a_2\mathcal{A}_+ - a_2^\star\mathcal{A}_-)$$
$$\tilde{\mathcal{A}}_1 = \sqrt{-\varepsilon_\kappa}(a_1^\star\mathcal{A}_+^\star - a_1\mathcal{A}_-^\star) \quad \tilde{\mathcal{A}}_2 = \sqrt{-\varepsilon_\kappa}(a_2^\star\mathcal{A}_+^\star - a_2\mathcal{A}_-^\star)$$

$$\kappa^2 = -\varepsilon_\kappa i(a_1^\star a_2 - a_1 a_2^\star)\hat{\kappa}^2 \; \mathcal{G} = -\varepsilon_\kappa i(a_1^\star a_2 - a_1 a_2^\star)\hat{\mathcal{G}}$$

$$\tau_+ = \frac{\partial_{\tilde{w}}\mathcal{G}(T-1)(a_2\partial_w\mathcal{A}_+ - a_2^\star\partial_w\mathcal{A}_-) + \partial_w\mathcal{G}(T+1)\left(a_2^\star\partial_{\tilde{w}}\tilde{\mathcal{A}}_+ - a_2\partial_{\tilde{w}}\tilde{\mathcal{A}}_-\right)}{\partial_{\tilde{w}}\mathcal{G}(T-1)(a_1\partial_w\mathcal{A}_+ - a_1^\star\partial_w\mathcal{A}_-) + \partial_w\mathcal{G}(T+1)\left(a_1^\star\partial_{\tilde{w}}\tilde{\mathcal{A}}_+ - a_1\partial_{\tilde{w}}\tilde{\mathcal{A}}_-\right)}$$

$$T = -\hat{T} \; a_1 \in \mathbb{R} \; a_2 = -ia_1$$

$$\left(\rho^{\mathrm{E}}\right)^2 = \frac{2a_1c_6}{k_1}\varepsilon_\kappa\hat{\kappa}^2\sqrt{\frac{\hat{T}}{6(-\varepsilon_\kappa)\hat{G}}} \; (f_6^E)^2 = -\frac{4\varepsilon_\kappa c_6^2 a_1^2}{(\rho^{\mathrm{E}})^2}\frac{\hat{\kappa}^2}{\hat{T}^\eta} \; (f_2^E)^2 = \frac{4c_2^2a_1^2\varepsilon_\kappa}{9(\rho^{\mathrm{E}})^2}\hat{\kappa}^2\hat{T}^\eta$$

$$\tilde{c} = \frac{2a_1\varepsilon_\kappa c_6}{k_1}$$

$$\left(\rho^{\mathrm{E}}\right)^2 = \tilde{c}\hat{\kappa}^2\sqrt{\frac{\hat{T}}{6(-\varepsilon_\kappa)\hat{G}}} \; (f_6^E)^2 = \frac{\tilde{c}^2}{(\rho^{\mathrm{E}})^2}\frac{\hat{\kappa}^2}{\hat{T}^\eta} \; (f_2^E)^2 = \frac{\tilde{c}^2\hat{\kappa}^2}{9(\rho^{\mathrm{E}})^2}\hat{T}^\eta$$

$$\begin{array}{lllll} \text{IIB}^\star: & (\alpha^\star,\gamma^\star)=(-\beta,+\delta) & (\kappa_1,\kappa_2)^\star=(+\tilde{\kappa}_1,-\tilde{\kappa}_2) & c_6\in\mathbb{R} & c_2\in\mathbb{R} \\ \text{IIB}': & (\alpha^\star,\beta^\star)=(-\delta,-\gamma) & (\kappa_1,\kappa_2)^\star=(-\tilde{\kappa}_1,+\tilde{\kappa}_2) & c_6\in i\mathbb{R} & c_2\in i\mathbb{R} \end{array}$$

$$\epsilon_\kappa=\pm\tilde{\epsilon}_\kappa\ \varepsilon_\kappa=\pm 1\ \xi^\star=-\tilde{\xi}$$

$$\mathcal{A}_1^\star=\mp\tilde{\mathcal{A}}_1\ \mathcal{A}_2^\star=\pm\tilde{\mathcal{A}}_2$$

$$(h_z)^\star=\pm h_{\tilde{z}}\ (g_z)^\star=\mp g_{\tilde{z}}$$

$$dS_{1,5}\to S^6$$

$$\begin{array}{llll} \{\xi,\tilde{\xi}\}^*=\{\tilde{\xi},\xi\} & \text{IIB}_{\mathbb{R}},\text{IIB}_3 & \Sigma_{(0,2)},\Sigma_{(2,0)} & M^*=\pm M \\ \{\xi,\tilde{\xi}\}^*=\{-\tilde{\xi},-\xi\} & \text{IIB}^\star,\text{IIB}' & \Sigma_{(0,2)},\Sigma_{(2,0)} & M^*=\pm\sigma^3M\sigma^3 \\ \{\xi,\tilde{\xi}\}^*=\{\xi,\tilde{\xi}\} & \text{IIB}_3 & \Sigma_{(1,1)} & M^*=\pm M \\ \{\xi,\tilde{\xi}\}^*=\{-\xi,-\tilde{\xi}\} & \text{IIB}^\star,\text{IIB}' & \Sigma_{(1,1)} & M^*=\pm\sigma^3M\sigma^3 \end{array}$$

$$S_{\text{IKKT}}=\text{tr}\left(-\frac{1}{4}[X_\mu,X_\nu][X^\mu,X^\nu]+\frac{1}{2}\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[X_\mu,\Psi_\beta]\right)$$

$$\begin{aligned}\delta X^\mu&=-\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}\varepsilon_\beta\\ \delta\Psi_\alpha&=\frac{1}{2}\Gamma^{\mu\nu}_{\alpha\beta}[X_\mu,X_\nu]\varepsilon_\beta+v_\alpha\mathbf{1}\end{aligned}$$

$$S_\Omega=S_{\text{IKKT}}+\text{tr}\left(\frac{\Omega^2}{4^3}X^AX_A+\frac{3\Omega^2}{4^3}X_aX^a+i\Omega X_8[X_9,X_{10}]+i\frac{\Omega}{8}\Psi_\alpha(\mathcal{C}\mathfrak{N})_{\alpha\beta}\Psi_\beta\right)$$

$$\begin{aligned}\delta X^\mu&=-\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}\varepsilon_\beta\\ \delta\Psi_\alpha&=\frac{1}{2}\Gamma^{\mu\nu}_{\alpha\beta}[X_\mu,X_\nu]\varepsilon_\beta+i\frac{\Omega}{8}(\Gamma^\mu\mathfrak{N}+2\mathfrak{N}\Gamma^\mu)X_\mu\varepsilon_\beta\end{aligned}$$

$$\begin{array}{llll} \Gamma^m & =\gamma^m\otimes I_2\otimes I_2 & m & =0,1,2,3,4,5 \\ \Gamma^i & =\gamma_{(1)}\otimes\gamma^i\otimes I_2 & i & =6,7 \\ \Gamma^a & =\gamma_{(1)}\otimes\gamma_{(2)}\otimes\gamma^a & a & =8,9 \end{array}$$

$$\begin{array}{ll} \gamma^0=-i\sigma^2\otimes I_2\otimes I_2 & \\ \gamma^1=\sigma^1\otimes I_2\otimes I_2 & \\ \gamma^2=\sigma^3\otimes\sigma^2\otimes I_2 & \gamma^6=\sigma^1 \\ \gamma^3=\sigma^3\otimes\sigma^1\otimes I_2 & \gamma^7=\sigma^2 \\ \gamma^4=\sigma^3\otimes\sigma^3\otimes\sigma^1 & \gamma^8=\sigma^1 \\ \gamma^5=\sigma^3\otimes\sigma^3\otimes\sigma^2 & \gamma^9=\sigma^2 \end{array}$$

$$\gamma_{(1)}=\sigma^3\otimes\sigma^3\otimes\sigma^3\ \gamma_{(2)}=\sigma^3\ \gamma_{(3)}=\sigma^3$$

$$\Gamma^{012345}=-\gamma_{(1)}\otimes I_2\otimes I_2 \qquad \begin{array}{l}\Gamma^{67}=iI_6\otimes\gamma_{(2)}\otimes I_2\\ \Gamma^{89}=iI_6\otimes I_2\otimes\gamma_{(3)}\end{array}$$

$$\Gamma^{11}=\Gamma^{0123456789}=\gamma_{(1)}\otimes\gamma_{(2)}\otimes\gamma_{(3)}=\sigma^3\otimes\sigma^3\otimes\sigma^3\otimes\sigma^3\otimes\sigma^3$$

$$(\Gamma^A)^t=-\mathcal{C}\Gamma^A\mathcal{C}^{-1}$$

$$\mathcal{C} = i\sigma^2 \otimes \sigma^1 \otimes \sigma^2 \otimes \sigma^1 \otimes \sigma^2$$

$$\mathcal{C}^* = \mathcal{C}\,\mathcal{C}^t = -\mathcal{C}\,\mathcal{C}^2 = -I_{32}\,\{\mathcal{C}, \Gamma^{11}\} = 0$$

$$(\gamma^m)^* = +B_{(1)}\gamma^m B_{(1)}^{-1} \qquad \left(B_{(1)}\right)^* B_{(1)} = -I_6 \qquad B_{(1)} = -i\gamma^2\gamma^5 = I_2 \otimes \sigma_1 \otimes \sigma_2$$
$$\left(\gamma^i\right)^* = -B_{(2)}\gamma^i B_{(2)}^{-1} \qquad \left(B_{(2)}\right)^* B_{(2)} = -I_2 \qquad B_{(2)} = \gamma^7 = \sigma^2$$
$$(\gamma^a)^* = -B_{(3)}\gamma^a B_{(3)}^{-1} \qquad \left(B_{(3)}\right)^* B_{(3)} = -I_2 \qquad B_{(3)} = \gamma^9 = \sigma^2$$

$$(\Gamma^M)^* = \mathcal{B}\Gamma^M\mathcal{B}^{-1}\,\mathcal{B}^*\mathcal{B} = I_{32}\,[\mathcal{B}, \Gamma^{11}] = 0$$

$$\mathcal{B} = -iB_{(1)} \otimes \left(B_{(2)}\gamma_{(2)}\right) \otimes B_{(3)} = I_2 \otimes \sigma^1 \otimes \sigma^2 \otimes \sigma^1 \otimes \sigma^2$$

$$dP - 2iQ \wedge P = 0 \qquad dG - iQ \wedge G + P \wedge \bar{G} = 0$$
$$dQ + iP \wedge \bar{P} = 0 \qquad dF_{(5)} - \frac{i}{8}G \wedge \bar{G} = 0$$

$$\delta\psi_M = \nabla_M\varepsilon - \frac{i}{2}Q_M\varepsilon + \frac{i}{480}(\Gamma \cdot F_{(5)})\Gamma_M\varepsilon - \frac{1}{96}(\Gamma_M(\Gamma \cdot G) + 2(\Gamma \cdot G)\Gamma_M)\mathcal{B}^{-1}\varepsilon^*$$
$$\delta\lambda = i(\Gamma \cdot P)\mathcal{B}^{-1}\varepsilon^* - \frac{i}{24}(\Gamma \cdot G)\varepsilon$$

$$P \to e^{2i\theta}P \qquad \lambda \to e^{3i\theta/2}\lambda$$
$$Q \to Q + d\theta \qquad \psi_M \to e^{i\theta/2}\psi_M$$
$$G \to e^{i\theta}G \qquad \varepsilon \to e^{i\theta/2}\varepsilon$$

$$P = \frac{1}{2}d\phi + \frac{i}{2}e^{\phi}d\chi\; Q = -\frac{1}{2}e^{\phi}d\chi\; G = e^{-\phi/2}H_{(3)} + ie^{\phi/2}\tilde{F}_{(3)}$$

$$\varepsilon = \varepsilon_+ + i\varepsilon_- \qquad \mathcal{B}^{-1}\varepsilon^* = \varepsilon_+ - i\varepsilon_-$$
$$\lambda = \lambda_+ + i\lambda_- \qquad \mathcal{B}^{-1}\lambda^* = \lambda_+ - i\lambda_-$$
$$\psi = \psi_+ + i\psi_- \qquad \mathcal{B}^{-1}\psi^* = \psi_+ - i\psi_-$$

$$\delta\psi_{\pm M} = \nabla_M\varepsilon_\pm \mp \frac{e^{\phi}}{4}\partial_M\chi\varepsilon_\mp \mp \frac{e^{-\phi/2}}{96}\{\Gamma_M(\Gamma \cdot H_{(3)}) + 2(\Gamma \cdot H_{(3)})\Gamma_M\}\varepsilon_\pm$$
$$\mp \frac{1}{480}(\Gamma \cdot F_{(5)})\Gamma_M\varepsilon_\mp - \frac{e^{\phi/2}}{96}\{\Gamma_M(\Gamma \cdot \tilde{F}_{(3)}) + 2(\Gamma \cdot \tilde{F}_{(3)})\Gamma_M\}\varepsilon_\mp$$
$$\delta\lambda_\pm = \frac{1}{2}(\Gamma \cdot \partial\phi)\varepsilon_\mp \mp \frac{e^{\phi}}{2}(\Gamma \cdot \partial\chi)\varepsilon_\pm \pm \frac{e^{-\phi/2}}{24}(\Gamma \cdot H_{(3)})\varepsilon_\mp + \frac{e^{\phi/2}}{24}(\Gamma \cdot \tilde{F}_{(3)})\varepsilon_\pm$$

$$\lambda = \begin{pmatrix}\lambda_+\\ \lambda_-\end{pmatrix}\; \psi_M = \begin{pmatrix}\psi_{+M}\\ \psi_{-M}\end{pmatrix}\; \varepsilon = \begin{pmatrix}\varepsilon_+\\ \varepsilon_-\end{pmatrix}$$

$$s^1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\; s^2 = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\; s^3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\delta\psi_M = \nabla_M\varepsilon - \frac{e^{\phi}}{4}\partial_M\chi s^2\varepsilon - \frac{e^{-\phi/2}}{96}\{\Gamma_M(\Gamma \cdot H_{(3)}) + 2(\Gamma \cdot H_{(3)})\Gamma_M\}s^3\varepsilon$$
$$-\frac{1}{480}(\Gamma \cdot F_{(5)})\Gamma_M s^2\varepsilon - \frac{e^{\phi/2}}{96}\{\Gamma_M(\Gamma \cdot \tilde{F}_{(3)}) + 2(\Gamma \cdot \tilde{F}_{(3)})\Gamma_M\}s^1\varepsilon$$
$$\delta\lambda = \frac{1}{2}(\Gamma \cdot \partial\phi)s^1\varepsilon - \frac{e^{\phi}}{2}(\Gamma \cdot \partial\chi)s^3\varepsilon + \frac{e^{-\phi/2}}{24}(\Gamma \cdot H_{(3)})s^2\varepsilon + \frac{e^{\phi/2}}{24}(\Gamma \cdot \tilde{F}_{(3)})\varepsilon$$

$$g^E_{MN} = e^{-\phi/2} g^S_{MN}$$

$$\Gamma^M \to e^{\phi/4}\Gamma^M \; \Gamma_M \to e^{-\phi/4}\Gamma_M$$

$$\omega^E_{Mab}\Gamma^{ab} = \omega^S_{Mab}\Gamma^{ab} - \frac{1}{4}\Gamma_M(\Gamma\cdot\partial\phi) + \frac{1}{4}(\Gamma\cdot\partial\phi)\Gamma_M$$

$$\delta\psi_M = \nabla_M\varepsilon - \frac{1}{8}(\Gamma_M(\Gamma\cdot\partial\phi) - (\Gamma\cdot\partial\phi)\Gamma_M)\varepsilon - \frac{e^{\phi}}{4}\partial_M\chi s^2\varepsilon$$
$$-\frac{1}{96}\{\Gamma_M(\Gamma\cdot H_{(3)}) + 2(\Gamma\cdot H_{(3)})\Gamma_M\}s^3\varepsilon$$
$$-\frac{e^{\phi}}{480}(\Gamma\cdot F_{(5)})\Gamma_M s^2\varepsilon - \frac{e^{\phi}}{96}\{\Gamma_M(\Gamma\cdot\tilde{F}_{(3)}) + 2(\Gamma\cdot\tilde{F}_{(3)})\Gamma_M\}s^1\varepsilon$$
$$e^{-\phi/4}\delta\lambda = \frac{1}{2}(\Gamma\cdot\partial\phi)s^1\varepsilon + \frac{1}{24}(\Gamma\cdot H_{(3)})s^2\varepsilon - \frac{e^{\phi}}{2}(\Gamma\cdot\partial\tau_1)s^3\varepsilon + \frac{e^{\phi}}{24}(\Gamma\cdot\tilde{F}_{(3)})\varepsilon$$

$$\delta\tilde{\psi}_M = \delta\psi_M + \frac{1}{4}e^{-\phi/4}s^1\Gamma_M\delta\lambda$$

$$\begin{array}{llll} \Gamma\cdot\partial\phi & = \varphi_a\Gamma^a & \Gamma\cdot H_{(3)} = 3!\,h_a\Gamma^{67a} & \Gamma^a = \gamma_{(1)}\otimes\gamma_{(2)}\otimes\gamma^a \\ \Gamma\cdot\partial\tau_1 & = \chi_a\Gamma^a & \Gamma\cdot\tilde{F}_{(3)} = 3!\,g_a\Gamma^{67a} & \Gamma^{67} = iI_8\otimes\gamma_{(2)}\otimes I_2 \end{array}$$

$$\Gamma^{67}\chi^{\eta_1,\eta_2} = i\chi^{\eta_1,-\eta_2}$$
$$\Gamma^{67a}\chi^{\eta_1,\eta_2} = i\gamma^a\chi^{\eta_1,-\eta_2}$$

$$0 = (\varphi_a - e^{\phi}\chi_a s^2)\Gamma^a\varepsilon - \frac{1}{2}(h_a s^3 - e^{\phi}g_a s^1)\Gamma^{a67}\varepsilon$$

$$0 = \sum_{\eta_1,\eta_2}\chi^{\eta_1,\eta_2}\otimes\left[(\varphi_a - e^{\phi}\chi_a s^2)\gamma^a\tau^{(11)}\zeta - \frac{1}{2}(h_a s^3 + e^{\phi}g_a s^1)i\tau^{(10)}\gamma^a\zeta\right]$$

$$H_{(3)mNP}\Gamma^{NP} = 0$$
$$H_{(3)iNP}\Gamma^{NP} = 2h_a\Gamma_i\Gamma^{67}\Gamma^a$$
$$H_{(3)aNP}\Gamma^{NP} = 2h_a\Gamma^{67}$$

$$(m)\quad 0 = \hat{\nabla}_m\varepsilon + \left(\frac{D_a f_6}{2f_6} - \frac{1}{8}e^{\phi}\chi_a s^2 - \frac{1}{8}e^{\phi}g_a\Gamma^{67}s^1\right)\Gamma_m\Gamma^a\varepsilon$$
$$(i)\quad 0 = \hat{\nabla}_i\varepsilon + \left(\frac{D_a f_2}{2f_2} - \frac{1}{4}h_a\Gamma^{67}s^3 - \frac{1}{8}e^{\phi}\chi_a s^2 - \frac{1}{8}e^{\phi}g_a\Gamma^{67}s^1\right)\Gamma_i\Gamma^a\varepsilon$$
$$(a)\quad 0 = \nabla_a\varepsilon - \frac{1}{4}h_a\Gamma^{67}s^3\varepsilon + \frac{1}{8}e^{\phi}\chi_b\Gamma^b\Gamma_a s^2\varepsilon + \frac{1}{8}e^{\phi}g_b\Gamma^{67}\Gamma^b\Gamma_a s^1\varepsilon$$

$$0 = \Gamma_m\sum_{\eta_1,\eta_2}\chi^{\eta_1,\eta_2}\otimes\left[\frac{k_1}{2f_6}\eta_1\zeta_{\eta_1,\eta_2} + \left(\frac{D_a f_6}{2f_6} - \frac{1}{8}e^{\phi}\chi_a s^2\right)\gamma^a\zeta_{-\eta_1,-\eta_2} - \frac{i}{8}e^{\phi}g_a\gamma^a s^1\zeta_{-\eta_1,\eta_2}\right]$$
$$0 = \Gamma_i\sum_{\eta_1,\eta_2}\chi^{\eta_1,\eta_2}\otimes\left[\frac{k_2}{2f_2}\eta_2\zeta_{-\eta_1,\eta_2} + \left(\frac{D_a f_2}{2f_2} - \frac{1}{8}e^{\phi}\chi_a s^2\right)\gamma^a\zeta_{-\eta_1,-\eta_2}\right.$$
$$\left. - \frac{i}{4}h_a\gamma^a s^3\zeta_{-\eta_1,\eta_2} + \frac{i}{8}e^{\phi}g_a\gamma^a s^1\zeta_{-\eta_1,\eta_2}\right]$$

$$0=\sum_{\eta_1,\eta_2}\chi^{\eta_1,\eta_2}\otimes[\left(D_a+\frac{i}{2}\hat{\omega}_a\sigma^3\right)\zeta_{\eta_1,\eta_2}-\frac{i}{4}h_a s^3\zeta_{\eta_1,-\eta_2}$$
$$+\frac{1}{8}e^{\phi}\chi_b\gamma^b\gamma_a s^2\zeta_{\eta_1,\eta_2}+\frac{i}{8}e^{\phi}g_b\gamma^b\gamma_a s^1\zeta_{\eta_1,-\eta_2}]$$

$$g_{mn}=e_m^a e_n^b\delta_{ab}\ a,b=1,2$$

$$g_{mn}=2e_m^z e_n^{\tilde{z}}+2e_m^{\tilde{z}}e_n^z$$

$$g_{mn}(\xi^1,\xi^2)d\xi^m d\xi^n=4\rho(w,\tilde{w})^2 dwd\tilde{w}$$

$$e_m{}^z d\xi^m=\rho dw$$
$$e_m{}^{\tilde{z}}d\xi^m=\tilde{\rho}d\tilde{w}$$

$$D_z f=e_z{}^m\partial_m f=\rho^{-1}\partial_w f\quad D_z f=0\text{ is solved by }\quad f(\tilde{w})$$
$$D_{\tilde{z}}f=e_{\tilde{z}}{}^m\partial_m f=\rho^{-1}\partial_{\tilde{w}}f\quad D_{\tilde{z}}f=0\text{ is solved by }\quad f(w)$$

$$Z_\pm^2=\frac{\tau_\pm-\tilde{\xi}}{\tau_\pm-\xi}\ X=\varepsilon_\kappa Z_+Z_-\ \varepsilon_\kappa=\frac{\tilde{\epsilon}_\kappa}{\epsilon_\kappa}$$

$$\tau_\pm-\xi=\frac{\xi-\tilde{\xi}}{Z_\pm^2-1}\ \tau_\pm-\tilde{\xi}=\frac{Z_\pm^2(\xi-\tilde{\xi})}{Z_\pm^2-1}$$

$$\frac{\partial_w\tau_\pm}{\tau_+-\tau_-}=\frac{Z_\pm^2(Z_\mp^2-1)}{Z_-^2-Z_+^2}\partial_w\ln(\xi-\tilde{\xi})-\frac{Z_\mp^2-1}{Z_\pm^2-1}\cdot\frac{\partial_w Z_\pm^2}{Z_-^2-Z_+^2}$$

$$\tau_\pm-\xi+\frac{2}{Z_+Z_-}(\tau_\pm-\tilde{\xi})=\frac{\xi-\tilde{\xi}}{Z_\mp^2(Z_\pm^2-1)}(Z_\mp^2+2Z_+Z_-)$$
$$\tau_\pm-\tilde{\xi}+2Z_+Z_-(\tau_\pm-\xi)=\frac{\xi-\tilde{\xi}}{Z_\pm^2-1}(Z_\pm^2+2Z_+Z_-)$$

$$\partial_w\ln\hat{\rho}^2\mp\frac{Z_\pm^2-2X}{Z_-^2-Z_+^2}(Z_\mp^2-1)\partial_w\ln(\xi-\tilde{\xi})\pm\frac{Z_\mp^2-2X}{Z_-^2-Z_+^2}\cdot\frac{\partial_w Z_\mp^2}{Z_\mp^2}=0$$
$$\partial_{\tilde{w}}\ln\hat{\rho}^2\mp\frac{Z_\mp^2-2X}{Z_\mp^2(Z_-^2-Z_+^2)}(Z_\mp^2-1)\partial_{\tilde{w}}\ln(\xi-\tilde{\xi})\pm\frac{Z_\pm^2-2X}{Z_-^2-Z_+^2}\cdot\frac{\partial_{\tilde{w}}Z_\mp^2}{Z_\mp^2}=0$$

$$0=2\partial_w\ln\hat{\rho}^2+\partial_w\ln\frac{(Z_-+\varepsilon_\kappa Z_+)^3}{(Z_--\varepsilon_\kappa Z_+)}-(1-2X)\partial_w\ln(\xi-\tilde{\xi})$$
$$0=2\partial_{\tilde{w}}\ln\hat{\rho}^2+\partial_{\tilde{w}}\ln\frac{(Z_-+\varepsilon_\kappa Z_+)^3}{(Z_--\varepsilon_\kappa Z_+)}-2\partial_{\tilde{w}}\ln X-\left(1-\frac{2}{X}\right)\partial_{\tilde{w}}\ln(\xi-\tilde{\xi})$$

$$\frac{1}{R^2}=-\frac{k_1}{c_6}\frac{(Z_-+\varepsilon_\kappa Z_+)^{\frac{3}{2}}\hat{\rho}^2}{X^{\frac{1}{2}}(Z_--\varepsilon_\kappa Z_+)^{\frac{1}{2}}(\xi-\tilde{\xi})^{\frac{1}{2}}}$$

$$\partial_w\ln R^2-\frac{1}{2}\partial_w\ln X-X\partial_w\ln(\xi-\tilde{\xi})=0$$
$$\partial_{\tilde{w}}\ln R^2+\frac{1}{2}\partial_w\ln X-\frac{1}{X}\partial_{\tilde{w}}\ln(\xi-\tilde{\xi})=0$$

$$0 =(2X^2+4X-(Z_+^2+Z_-^2)(1+2X))\partial_w \ln(\xi-\tilde{\xi})-\partial_w(Z_+^2+Z_-^2-4X)$$
$$0 =\left(2+4X-(Z_+^2+Z_-^2)(1+2X^{-1})\right)\partial_{\tilde{w}}\ln(\xi-\tilde{\xi})-\partial_{\tilde{w}}(Z_+^2+Z_-^2)$$
$$+2(Z_+^2+Z_-^2-2X)\partial_{\tilde{w}}\ln X$$

$$0 =\frac{\tilde{\epsilon}_\kappa \kappa_1}{\epsilon_\kappa R^2}(Z_+^2+Z_-^2-2X)X^{-\frac{1}{2}}-\partial_w(Z_+^2+Z_-^2)+2(Z_+^2+Z_-^2)\partial_w \ln X$$
$$-(2X^2-Z_+^2-Z_-^2)\partial_w\ln(\xi-\tilde{\xi})$$
$$0 =\frac{\epsilon_\kappa \tilde{\kappa}_1}{R^2}(Z_+^2+Z_-^2-2X)X^{\frac{1}{2}}-\partial_{\tilde{w}}(Z_+^2+Z_-^2)+(Z_+^2+Z_-^2-2)\partial_{\tilde{w}}\ln(\xi-\tilde{\xi})$$

$$0=-\frac{\epsilon_\kappa \kappa_1}{R^2X^{\frac{1}{2}}}+2\partial_w \ln X+2(1+X)\partial_w\ln(\xi-\tilde{\xi})$$
$$0=\frac{\epsilon_\kappa \tilde{\kappa}_1}{R^2}X^{\frac{1}{2}}-2\partial_{\tilde{w}}\ln X+2\left(1+\frac{1}{X}\right)\partial_{\tilde{w}}\ln(\xi-\tilde{\xi})$$

$$0=-\frac{\epsilon_\kappa \kappa_1}{R^2X^{\frac{1}{2}}}+2\partial_w \ln R^2+\partial_w\ln X+2\partial_w\ln(\xi-\tilde{\xi})$$
$$0=\frac{\epsilon_\kappa \tilde{\kappa}_1}{R^2}X^{\frac{1}{2}}+2\partial_{\tilde{w}}\ln R^2-\partial_{\tilde{w}}\ln X+2\partial_{\tilde{w}}\ln(\xi-\tilde{\xi})$$

$$0=-\epsilon_\kappa(\kappa_2-\kappa_1\tilde{\xi})+\partial_w\left(2R^2X^{\frac{1}{2}}(\xi-\tilde{\xi})\right)$$
$$0=\epsilon_\kappa(\tilde{\kappa}_2-\tilde{\kappa}_1\xi)+\partial_{\tilde{w}}\left(2R^2X^{-\frac{1}{2}}(\tilde{\xi}-\xi)\right)$$

$$\partial_w\left(R^2X^{-\frac{1}{2}}(\tilde{\xi}-\xi)\right)-\left(R^2X^{\frac{1}{2}}+R^2X^{-\frac{1}{2}}\right)\partial_w(\tilde{\xi}-\xi)=0$$
$$\partial_{\tilde{w}}\left(R^2X^{\frac{1}{2}}(\tilde{\xi}-\xi)\right)-\left(R^2X^{-\frac{1}{2}}+R^2X^{\frac{1}{2}}\right)\partial_{\tilde{w}}(\tilde{\xi}-\xi)=0$$

$$(\tilde{\xi}-\xi)\frac{\partial_w\mathcal{A}_0}{\partial_w\xi}+\mathcal{A}_0-\tilde{\mathcal{A}}_0-\mathcal{A}_2+\tilde{\mathcal{A}}_2+\tilde{\xi}(\mathcal{A}_1-\tilde{\mathcal{A}}_1)=0$$
$$(\tilde{\xi}-\xi)\frac{\partial_{\tilde{w}}\tilde{\mathcal{A}}_0}{\partial_{\tilde{w}}\tilde{\xi}}+\mathcal{A}_0-\tilde{\mathcal{A}}_0-\mathcal{A}_2+\tilde{\mathcal{A}}_2+\xi(\mathcal{A}_1-\tilde{\mathcal{A}}_1)=0$$

$$\frac{\partial_w\mathcal{A}_0}{\partial_w\xi}+\mathcal{A}_1=\frac{\partial_{\tilde{w}}\tilde{\mathcal{A}}_0}{\partial_{\tilde{w}}\tilde{\xi}}+\tilde{\mathcal{A}}_1$$

$$\partial_w\mathcal{A}_0=-\mathcal{A}_1\partial_w\xi$$
$$\partial_{\tilde{w}}\tilde{\mathcal{A}}_0=-\tilde{\mathcal{A}}_1\partial_{\tilde{w}}\tilde{\xi}$$

$$\tilde{\mathcal{A}}_0-\tilde{\mathcal{A}}_2+\tilde{\xi}\tilde{\mathcal{A}}_1=\mathcal{A}_0-\mathcal{A}_2+\xi\mathcal{A}_1$$

$$\eta_{ab}=(-)^{s_a}\delta_{ab}\;\tilde{\eta}_{mn}=(-)^{\tilde{s}_m}\delta_{mn}$$

$$\{\gamma_a,\gamma_b\}=2\eta_{ab}\qquad \gamma_{ab}=\frac{1}{2}[\gamma_a,\gamma_b]\qquad \sigma_{ab}=\frac{1}{2}\gamma_{ab}$$
$$\{\tilde{\gamma}_m,\tilde{\gamma}_n\}=2\tilde{\eta}_{mn}\qquad \tilde{\gamma}_{mn}=\frac{1}{2}[\tilde{\gamma}_m,\tilde{\gamma}_n]\qquad \tilde{\sigma}_{mn}=\frac{1}{2}\tilde{\gamma}_{mn}$$

$$\gamma_a^t = -C\gamma_a C^{-1} \quad C^t = \varepsilon_C C \quad (C\gamma_{ab})^t = -\varepsilon_C C\gamma_{ab}$$
$$\tilde{\gamma}_m^t = -\tilde{C}\gamma_m C^{-1} \quad \tilde{C}^t = \varepsilon_{\tilde{C}}\tilde{C} \quad \left(\tilde{C}\tilde{\gamma}_{mn}\right)^t = -\varepsilon_{\tilde{C}}\tilde{C}\tilde{\gamma}_{mn}$$

$$C = \gamma_2 \qquad \varepsilon_C = -1$$
$$\tilde{C} = \tilde{\gamma}_2\tilde{\gamma}_4\tilde{\gamma}_6 \quad \varepsilon_{\tilde{C}} = +1$$

$$[\mathcal{A}_{ab}, \mathcal{A}_{cd}] = \eta_{ad}\mathcal{A}_{bc} + \eta_{bc}\mathcal{A}_{ad} - \eta_{ac}\mathcal{A}_{bd} - \eta_{bd}\mathcal{A}_{ac}$$
$$\left[\tilde{\mathcal{A}}_{mn}, \tilde{\mathcal{A}}_{pq}\right] = \tilde{\eta}_{mq}\tilde{\mathcal{A}}_{np} + \tilde{\eta}_{np}\tilde{\mathcal{A}}_{mq} - \tilde{\eta}_{mp}\tilde{\mathcal{A}}_{nq} - \tilde{\eta}_{nq}\tilde{\mathcal{A}}_{mp}$$

$$[\mathcal{A}_{ab}, \mathcal{S}_{\alpha\mu}] = \mathcal{S}_{\beta\mu}(\sigma_{ab})_{\beta\alpha}$$
$$\left[\tilde{\mathcal{A}}_{mn}, \mathcal{S}_{\alpha\mu}\right] = \mathcal{S}_{\alpha\nu}(\tilde{\sigma}_{mn})_{\nu\mu}$$

$$\{\mathcal{S}_{\alpha\mu}, \mathcal{S}_{\beta\nu}\} = s\tilde{C}_{\mu\nu}\left(C\gamma^{ab}\right)_{\alpha\beta}\mathcal{A}_{ab} + tC_{\alpha\beta}\left(\tilde{C}\tilde{\gamma}^{mn}\right)_{\mu\nu}\tilde{\mathcal{A}}_{mn}$$

$$\left[\{\mathcal{S}_{\alpha\mu}, \mathcal{S}_{\beta\nu}\}, \mathcal{S}_{\gamma\rho}\right] + \text{cycl}(\alpha\mu, \beta\nu, \gamma\rho) = 0$$

$$s\tilde{C}_{\mu\nu}\left(C\gamma^{ab}\right)_{\alpha\beta}[\mathcal{A}_{ab}, \mathcal{S}_{\gamma\rho}] + tC_{\alpha\beta}\left(\tilde{C}\tilde{\gamma}^{mn}\right)_{\mu\nu}[\tilde{\mathcal{A}}_{mn}, \mathcal{S}_{\gamma\rho}] + \text{cycl}(\alpha\mu, \beta\nu, \gamma\rho) = 0$$

$$s\left(C\gamma^{ab}\right)_{\alpha\beta}(\gamma_{ab})_{\delta\gamma}\tilde{C}_{\mu\nu}\delta_{\sigma\rho} + tC_{\alpha\beta}\delta_{\gamma\delta}\left(\tilde{C}\tilde{\gamma}^{mn}\right)_{\mu\nu}(\tilde{\gamma}_{mn})_{\sigma\rho} + \text{cycl}(\alpha\mu, \beta\nu, \gamma\rho) = 0$$

$$\left(C\gamma^{ab}\right)_{\alpha\beta}(\gamma_{ab})_{\delta\gamma} = 2C_{\alpha\beta}\delta_{\gamma\delta} - 4C_{\alpha\gamma}\delta_{\beta\delta}$$

$$\left(\tilde{C}\tilde{\gamma}^{mn}\right)_{\mu\nu}(\tilde{\gamma}_{mn})_{\rho\sigma} + \left(\tilde{C}\tilde{\gamma}^{mn}\right)_{\mu\sigma}(\tilde{\gamma}_{mn})_{\rho\nu} = 6\delta_{\mu\rho}\tilde{C}_{\nu\sigma} - 3\delta_{\nu\rho}\tilde{C}_{\mu\sigma} - 3\delta_{\rho\sigma}\tilde{C}_{\mu\nu}$$

$$\{\mathcal{S}_{\alpha\mu}, \mathcal{S}_{\beta\nu}\} = -\frac{1}{2}\tilde{C}_{\mu\nu}\left(C\gamma^{ab}\right)_{\alpha\beta}\mathcal{A}_{ab} + \frac{1}{3}C_{\alpha\beta}\left(\tilde{C}\tilde{\gamma}^{mn}\right)_{\mu\nu}\tilde{\mathcal{A}}_{mn}$$

$$(\gamma_a^E)^* = -C\gamma_a^E C^{-1} \; (\tilde{\gamma}_m^E)^* = -\tilde{C}\tilde{\gamma}_m^E\tilde{C}^{-1}$$

$$\eta_{ab} = (-)^{s_a}\,\delta_{ab} \qquad \gamma_a = i^{s_a}\,\gamma_a^E \qquad (\gamma_a)^* = -(-)^{s_a}C\gamma_a C^{-1}$$
$$\tilde{\eta}_{mn} = (-)^{\tilde{s}_m}\,\delta_{mn} \qquad \tilde{\gamma}_m = i^{\tilde{s}_m}\,\tilde{\gamma}_m^E \qquad (\tilde{\gamma}_m)^* = -(-)^{\tilde{s}_m}\tilde{C}\tilde{\gamma}_m\tilde{C}^{-1}$$

$$\gamma_1^E = \sigma_1 \quad \tilde{\gamma}_1^E = \sigma_1 \otimes I_2 \otimes I_2 \quad \tilde{\gamma}_2^E = \sigma_2 \otimes I_2 \otimes I_2$$
$$\gamma_2^E = \sigma_2 \quad \tilde{\gamma}_3^E = \sigma_3 \otimes \sigma_1 \otimes I_2 \quad \tilde{\gamma}_4^E = \sigma_3 \otimes \sigma_2 \otimes I_2$$
$$\gamma_3^E = \sigma_3 \quad \tilde{\gamma}_5^E = \sigma_3 \otimes \sigma_3 \otimes \sigma_1 \quad \tilde{\gamma}_6^E = \sigma_3 \otimes \sigma_3 \otimes \sigma_2$$
$$C = \sigma_2 \quad \tilde{\gamma}_7^E = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \quad \tilde{C} = \sigma_2 \otimes \sigma_1 \otimes \sigma_2$$

$$\mathcal{A}_{ab}^* = \mathcal{A}_{ab}\;\tilde{\mathcal{A}}_{mn}^* = \tilde{\mathcal{A}}_{mn}\;\mathcal{S}_{\alpha\mu}^* = S_{\beta\nu}X_{\beta\alpha}Y_{\nu\mu}$$

$$XX^* \otimes YY^* = I_2 \otimes I_8$$

$$\gamma_{ab}X = X\gamma_{ab}^*\;\tilde{\gamma}_{mn}Y = Y\tilde{\gamma}_{mn}^*$$

$$XX^* = x_0 I_2\; YY^* = y_0 I_8$$

$$\tilde{C}^*_{\mu\nu}\left(C\gamma^{ab}\right)^*_{\alpha\beta}=\tilde{C}_{\rho\sigma}\left(C\gamma^{ab}\right)_{\gamma\delta}X_{\gamma\alpha}X_{\delta\beta}Y_{\rho\mu}Y_{\sigma\nu}$$
$$C^*_{\alpha\beta}\left(\tilde{C}\tilde{\gamma}^{mn}\right)^*_{\mu\nu}=C_{\gamma\delta}\left(\tilde{C}\tilde{\gamma}^{mn}\right)_{\rho\sigma}X_{\gamma\alpha}X_{\delta\beta}Y_{\rho\mu}Y_{\sigma\nu}$$

$$\tilde{C}^*=\lambda Y^t\tilde{C}Y \qquad \left(C\gamma^{ab}\right)^*=\lambda^{-1}X^tC\gamma^{ab}X$$
$$C^*=\mu X^tCX \qquad \left(\tilde{C}\tilde{\gamma}^{mn}\right)^*=\mu^{-1}Y^t\tilde{C}\tilde{\gamma}^{mn}Y$$

$$\gamma^E_{ab}X=(-)^{s_a+s_b+a+b}X\gamma^E_{ab}$$
$$\tilde{\gamma}^E_{mn}X=(-)^{\tilde{s}_m+\tilde{s}_n+m+n}X\tilde{\gamma}^E_{mn}$$

$$(+++) \quad \mathfrak{so}(3;\mathbb{R}) \quad X=x\sigma^2 \quad XX^*=-x^2I_2$$
$$(++-) \quad \mathfrak{so}(2,1;\mathbb{R}) \quad X=x\sigma^3 \quad XX^*=+x^2I_2$$

$$(+++++++) \quad \mathfrak{so}(7;\mathbb{R}) \quad Y=y\sigma^2\otimes\sigma^1\otimes\sigma^2 \quad YY^*=+y^2I_8$$
$$(++++++-) \quad \mathfrak{so}(6,1;\mathbb{R}) \quad Y=y\sigma^3\otimes\sigma^1\otimes\sigma^2 \quad YY^*=-y^2I_8$$
$$(+++++--) \quad \mathfrak{so}(5,2;\mathbb{R}) \quad Y=y\sigma^1\otimes\sigma^1\otimes\sigma^2 \quad YY^*=-y^2I_8$$
$$(++++---) \quad \mathfrak{so}(4,3;\mathbb{R}) \quad Y=y\sigma^2\otimes\sigma^1\otimes\sigma^2 \quad YY^*=+y^2I_8$$

$$\mathfrak{so}(3;\mathbb{R})\oplus\mathfrak{so}(1,6;\mathbb{R})$$
$$\mathfrak{so}(3;\mathbb{R})\oplus\mathfrak{so}(2,5;\mathbb{R})$$
$$\mathfrak{so}(1,2;\mathbb{R})\oplus\mathfrak{so}(7;\mathbb{R})$$
$$\mathfrak{so}(1,2;\mathbb{R})\oplus\mathfrak{so}(3,4;\mathbb{R})$$

$$\mathrm{d}_\xi=\mathrm{d}-\xi\lrcorner$$

$$\int_\Gamma\Phi=\sum_\Sigma\frac{1}{d_{2k}}\int_\Sigma\frac{f^*\Phi}{e_\xi(\mathcal{N})}$$

$$\int_\Gamma\Phi=\sum_\Sigma\frac{1}{d_{2k}}\frac{(2\pi)^k}{\prod_{i=1}^k\epsilon_i}\int_\Sigma\frac{f^*\Phi}{\left[1+\frac{2\pi}{\epsilon_i}c_1(L_i)\right]}$$

$$\int_{\Gamma_2}\Phi_2=\sum_{\Sigma_0}\frac{2\pi}{d_0}\frac{\Phi_0}{\epsilon_1}\Bigg|_{\Sigma_0}$$

$$\int_{\Gamma_4}\Phi_4=\sum_{\Sigma_0}\frac{(2\pi)^2}{d_0}\frac{\Phi_0}{\epsilon_1\epsilon_2}\Bigg|_{\Sigma_0}+\sum_{\Sigma_2}\frac{2\pi}{d_2}\int_{\Sigma_2}\left[\frac{\Phi_2}{\epsilon_1}-\frac{2\pi\Phi_0}{\epsilon_1^2}c_1(L)\right]$$

$$\int_{M_6}\Phi_6=\sum_{\Sigma_0}\frac{(2\pi)^3}{d_0}\frac{\Phi_0}{\epsilon_1\epsilon_2\epsilon_3}\Bigg|_{\Sigma_0}+\sum_{\Sigma_2}\frac{(2\pi)^2}{d_2}\frac{1}{\epsilon_1\epsilon_2}\int_{\Sigma_2}\left[\Phi_2-2\pi\Phi_0\left(\frac{c_1(L_1)}{\epsilon_1}+\frac{c_1(L_2)}{\epsilon_2}\right)\right]$$
$$+\sum_{\Sigma_4}\frac{2\pi}{d_4}\int_{\Sigma_4}\left[\frac{\Phi_4}{\epsilon_1}-\frac{2\pi\Phi_2}{\epsilon_1^2}\wedge c_1(L)+\frac{(2\pi)^2\Phi_0}{\epsilon_1^3}c_1(L)\wedge c_1(L)\right]$$

$$\int_\Gamma\Phi=\mathrm{BV}-\mathrm{AB}-\int_{\partial\Gamma}f^*\frac{\xi\wedge\Phi}{\mathrm{d}_\xi\xi}$$

$$\frac{1}{\mathrm{d}_\xi \xi} = -\frac{1}{|\xi|^2(1-|\xi|^{-2}\,\mathrm{d}\xi)} = -\frac{1}{|\xi|^2}\left[1+\sum_{a=1}^{\infty}\left(\frac{\mathrm{d}\xi}{|\xi|^2}\right)^a\right].$$

$$\mathrm{d}s^2 = \mathrm{d}\alpha^2 + \sin^2\,\alpha(\,\mathrm{d}\theta^2 + \sin^2\,\theta\,\mathrm{d}\phi_1^2 + \cos^2\,\theta\,\mathrm{d}\phi_2^2)$$

$$\xi = b_1\partial_{\phi_1} + b_2\partial_{\phi_2}$$

$$|\xi|^2 = \sin^2\,\alpha(b_1^2\sin^2\,\theta + b_2^2\cos^2\,\theta),$$

$$\mathrm{Vol}(X_{\alpha_0}) = \frac{4\pi^2}{3} + \frac{2\pi^2\cos\,\alpha_0(\cos\,\alpha_0^2 - 3)}{3}$$

$$\Phi_4 = \frac{1}{b_1 b_2}\mathrm{vol} = \frac{1}{b_1 b_2}\sin^3\,\alpha\sin\,\theta\cos\,\theta\,\mathrm{d}\alpha\wedge\,\mathrm{d}\theta\wedge\,\mathrm{d}\phi_1\wedge\,\mathrm{d}\phi_2$$
$$\Phi_2 = -\frac{1}{2}\sin^3\,\alpha\left(\frac{1}{b_1}\sin^2\,\theta\,\mathrm{d}\phi_1 + \frac{1}{b_2}\cos^2\,\theta\,\mathrm{d}\phi_2\right)\wedge\mathrm{d}\alpha$$
$$\Phi_0 = \frac{1}{6}(3\cos\,\alpha - \cos^3\,\alpha)$$

$$\int_{X_{\alpha_0}}\frac{1}{b_1 b_2}\mathrm{vol}(X_{\alpha_0}) = \frac{(2\pi)^2}{b_1 b_2}\Phi_0\Big|_{\alpha=0} + \int_{\partial X_{\alpha_0}} f^*\frac{1}{|\xi|^2}\left[\xi\wedge\Phi_2 + \frac{\Phi_0}{|\xi|^2}\xi\wedge\,\mathrm{d}\xi\right]$$
$$= \frac{4\pi^2}{3b_1 b_2} + \int_{\partial X_{\alpha_0}}\frac{b_1 b_2\cos\,\alpha_0(3-\cos^2\,\alpha_0)\cos\,\theta\sin\,\theta}{3(b_1^2\sin^2\,\theta + b_2^2\cos^2\,\theta)^2}\,\mathrm{d}\theta\wedge\,\mathrm{d}\phi_1\wedge\,\mathrm{d}\phi_2$$

$$\int_{X_{\alpha_0}}\frac{1}{b_1 b_2}\mathrm{vol}(X_{\alpha_0}) = \frac{2\pi^2(2-3\cos\,\alpha_0+\cos^3\,\alpha_0)}{3b_1 b_2}$$

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2A}\left[\,\mathrm{d}s_{\mathrm{AdS}_4}^2 + \mathrm{d}s_{M_6}^2\right]$$

$$\mathrm{d}s_{M_6}^2 = \frac{1}{\mathrm{e}^{4A}-y^2}\,\mathrm{d}y^2 + \frac{1}{4}(1-\mathrm{e}^{-4A}y^2)D\psi^2 + \frac{y}{F_0^2\mathrm{e}^{4A}y^2 + l^2}g_{ij}^{(4)}(y,x)\mathrm{d}x^i\,\mathrm{d}x^j$$

$$\partial_\psi j = 0, \quad \partial_y j = \frac{1}{2}(F_0^2 y^2 + l^2 y^{-2})\mathrm{d}_4\rho, \quad \mathrm{d}_4 j = 0$$
$$\partial_\psi \omega = 0, \quad \partial_y \omega = -\frac{1}{2}(F_0^2 y^2 + l^2 y^{-2})T\omega, \quad \mathrm{d}_4\omega = \mathrm{i}P\wedge\omega$$

$$P \equiv -\rho + \mathrm{i}\frac{2\mathrm{e}^{4A}(F_0^2 y^4 + l^2)}{(\mathrm{e}^{4A}-y^2)(F_0^2\mathrm{e}^{4A}y^2 + l^2)}\mathrm{d}_4 A$$
$$T \equiv \frac{\partial_y(\mathrm{e}^{4A}y^2)}{(\mathrm{e}^{4A}-y^2)(F_0^2\mathrm{e}^{4A}y^2 + l^2)}$$

$$n_0 = 2\pi\ell_s F_0 \in \mathbb{Z}$$

$$F_2 = l\left(\frac{1}{2y}\,\mathrm{d}\rho - \frac{1}{2}\,\mathrm{d}(\mathrm{e}^{-4A}yD\psi) + \frac{1}{F_0^2\mathrm{e}^{4A}y^2 + l^2}j\right)$$

$$F_4 = -\frac{F_0}{2}\left[\frac{\mathrm{e}^{4A}y^2}{(F_0^2\mathrm{e}^{4A}y^2 + l^2)^2}j\wedge j\right.$$

$$\left.-(\mathrm{d}(\mathrm{e}^{-4A}yD\psi) - y^{-1}\,\mathrm{d}\rho)\wedge\left(\frac{\mathrm{e}^{4A}y^2}{F_0^2\mathrm{e}^{4A}y^2 + l^2}j + \frac{y^2}{2}\,\mathrm{d}y\wedge D\psi\right)\right]$$

$$F_6 = \frac{3l}{4}\frac{y^2}{(F_0^2\mathrm{e}^{4A}y^2 + l^2)^2}\,\mathrm{d}y\wedge D\psi\wedge j\wedge j$$

$$B_1 = -\frac{F_0}{l}\left(\frac{\mathrm{e}^{4A}y^2}{F_0^2\mathrm{e}^{4A}y^2 + l^2}j + \frac{y^2}{2}\,\mathrm{d}y\wedge D\psi\right)$$

$$B_2 = \frac{l}{F_0}\left(\frac{1}{F_0^2\mathrm{e}^{4A}y^2 + l^2}j + \frac{1}{2y^2}\,\mathrm{d}y\wedge D\psi\right)$$

$$\mathrm{e}^{2\phi} = \frac{\mathrm{e}^{6A}}{F_0^2\mathrm{e}^{4A}y^2 + l^2}$$

$$\frac{1}{(2\pi\ell_s)^{p-1}}\int_{\Sigma_p} f_p \in \mathbb{Z}$$

$$f_2 = \frac{l}{2}\,\mathrm{d}[y^{-1}(1-\mathrm{e}^{-4A}y^2)D\psi] - F_0(B_1 - B_2)$$

$$f_4 = \frac{F_0}{2l^2}\left[\frac{\mathrm{e}^{4A}y^2}{F_0^2\mathrm{e}^{4A}y^2 + l^2}j\wedge j + y^2\,\mathrm{d}y\wedge D\psi\wedge j\right]$$

$$f_6 = \frac{y^2}{4l^3}\frac{F_0^2\mathrm{e}^{4A}y^2 + 3l^2}{F_0^2\mathrm{e}^{4A}y^2 + l^2}\,\mathrm{d}y\wedge D\psi\wedge j\wedge j$$

$$\mathrm{d}s^2 \sim \frac{1}{\sqrt{r}}\,\mathrm{d}s_9^2 + \sqrt{r}\,\mathrm{d}r^2$$

$$\mathrm{e}^{2\phi} \sim r^{-5/2}$$

$$\mathrm{e}^{2A} \sim y^{-1/2}$$

$$\mathrm{d}s^2 \sim \frac{1}{\sqrt{y}}\left(\mathrm{d}s^2_{\mathrm{AdS}_4} + \frac{1}{4}D\psi^2 + \frac{1}{F_0^2}g_{ij}^{(4)}(0,x)\mathrm{d}x^i\,\mathrm{d}x^j\right) + \sqrt{y}\,\mathrm{d}y^2,$$

$$\mathrm{e}^{2\phi} \sim y^{-5/2}.$$

$$\mathrm{e}^{2A} \sim \frac{a_1}{\sqrt{y}} + a_2\sqrt{y}$$

$$\mathcal{F} = \frac{16\pi^3}{(2\pi\ell_s)^8}\int_{M_6}\mathrm{e}^{8A-2\phi}\mathrm{vol}_{M_6}$$

$$\Phi^{\mathcal{F}} = \mathrm{e}^{8A-2\phi}\star_1 - \frac{1}{12}\left(\frac{y^3}{F_0^2\mathrm{e}^{4A}y^2 + l^2}j\wedge j + y\,\mathrm{d}y\wedge D\psi\wedge j\right)$$

$$+\frac{1}{24}\left(y^2 j + \frac{1}{2}(F_0^2y^4 + l^2)\mathrm{d}y\wedge D\psi\right) - \frac{y}{48}\left(\frac{F_0^2y^4}{5} + l^2\right)$$

$$\Phi^{f_2}= f_2 - \frac{y}{6l}(F_0^2 y^2 - 3l^2 \mathrm{e}^{-4A})$$

$$\Phi^{f_4}= f_4 - \frac{F_0}{6l^2}\left(y^3 j + \frac{F_0^2 y^5 + l^2 y}{2}\,\mathrm{d}y \wedge D\psi\right) + \frac{F_0 y^2}{72l^2}(F_0^2 y^4 + 3l^2)$$

$$\Phi^{f_6} = f_6 - \frac{1}{12l^3}\left(y^3 \frac{F_0^2 \mathrm{e}^{4A} y^2 + 3l^2}{F_0^2 \mathrm{e}^{4A} y^2 + l^2} j \wedge j + \left(F_0^2 y^5 + 3l^2 y\right)\mathrm{d}y \wedge D\psi \wedge j\right)$$
$$+ \frac{1}{144l^3}(2y^2(F_0^2 y^4 + 9l^2)j + (F_0^2 y^4 + 9l^2)(F_0^2 y^4 + l^2)\mathrm{d}y \wedge D\psi)$$
$$- \frac{y}{1296l^3}(F_0^2 y^4 + 9l^2)^2$$

$$Y = j + \frac{F_0^2 y^2 + l^2 y^{-2}}{2}\,\mathrm{d}y \wedge D\psi$$

$$\Phi^Y = Y - \frac{1}{6y}(F_0^2 y^4 - 3l^2)$$

$$Z = j \wedge j + (F_0^2 y^2 + l^2 y^{-2})\mathrm{d}y \wedge D\psi \wedge j,$$

$$\Phi^Z = Z - \frac{1}{6}\left(F_0^2 y^3 - \frac{3l^2}{y}\right)(2j + (F_0^2 y^2 + l^2 y^{-2})\mathrm{d}y \wedge D\psi) + \frac{(F_0^2 y^4 - 3l^2)^2}{36y^2}$$

$$F_4 = -\frac{F_0}{2}\left[\frac{1}{F_0^4 \mathrm{e}^{4A} y^2} j \wedge j - (\mathrm{d}(\mathrm{e}^{-4A} y D\psi) - y^{-1}\,\mathrm{d}\rho) \wedge \left(\frac{1}{F_0^2} j + \frac{y^2}{2}\,\mathrm{d}y \wedge D\psi\right)\right]$$

$$\Phi^{F_4} = F_4 - \frac{F_0}{2}\left[\frac{y}{F_0^2 \mathrm{e}^{4A}} j + \frac{y}{2}\,\mathrm{d}y \wedge D\psi\right] + \frac{F_0 y^2}{8}$$

$$\mathcal{F}= \frac{1}{24\pi^2 \ell_s^8}\left\{-\sum_{\Sigma_0} \frac{1}{d_0}\frac{F_0^2}{20}\frac{y^5}{\epsilon_1\epsilon_2\epsilon_3}\bigg|_{\Sigma_0} + \sum_{\Sigma_2} \frac{1}{d_2}\frac{1}{\epsilon_1\epsilon_2}\int_{\Sigma_2}\left[\frac{y^2}{4\pi} j + \frac{F_0^2 y^5}{20}\left(\frac{c_1(L_1)}{\epsilon_1} + \frac{c_1(L_2)}{\epsilon_2}\right)\right]\right.$$
$$\left. - \sum_{\Sigma_4} \frac{1}{d_4}\int_{\Sigma_4}\left[\frac{1}{F_0^2}\frac{1}{(2\pi)^2\epsilon_1}\frac{1}{y} j \wedge j + \frac{y^2}{4\pi\epsilon_1^2} j \wedge c_1(L) + \frac{F_0^2 y^5}{20\epsilon_1^3} c_1(L) \wedge c_1(L)\right]\right\}$$

$$N^I = \frac{1}{(2\pi\ell_s)^3}\int_{\Gamma_4^I} F_4 = \frac{F_0}{4\pi\ell_s^3}\left[\sum_{\Sigma_0} \frac{1}{d_0}\frac{y^2}{4\epsilon_1\epsilon_2}\bigg|_{\Sigma_0} - \sum_{\Sigma_2} \frac{1}{d_2}\int_{\Sigma_2}\left[\frac{1}{2\pi\epsilon_1}\frac{1}{F_0^2 y} j + \frac{y^2}{4\epsilon_1^2} c_1(L)\right]\right]$$

$$N = \frac{1}{(2\pi\ell_s)^3}\int_{\Sigma_4} F_4 = -\frac{1}{(2\pi\ell_s)^3}\int_{\Sigma_4} \frac{1}{2F_0^3 y^4} j \wedge j$$

$$\Delta(\Sigma_2) = \frac{1}{(2\pi)^2 \ell_s^3}\int_{\Sigma_2} F_0 \mathrm{e}^{2A} y \mathrm{vol}_{\Sigma_2} = \frac{1}{(2\pi)^2 \ell_s^3}\int_{\Sigma_2} Y'$$

$$Y' = F_0\left[\frac{y}{F_0^2 \mathrm{e}^{4A}} j + \frac{y}{2}\,\mathrm{d}y \wedge D\psi\right] = -2\Phi_2^{F_4}$$

$$\Delta(\Sigma_2) = -\frac{F_0}{2\pi\ell_s^3}\sum_{\Sigma_0}\frac{1}{d_0}\frac{y^2}{4\epsilon_1}, \Delta(\Sigma_2) = \frac{1}{(2\pi)^2\ell_s^3}\int_{\Sigma_2}\frac{1}{F_0 y}j$$

$$\mathcal{O}(-p_1)\oplus\mathcal{O}(-p_2)\to\Sigma_g$$

$$\xi = \sum_{i=1}^{2} b_i\partial_{\varphi_i}$$

$$\int_{\Sigma_g} c_1(\mathcal{L}_i) = -p_i$$

$$N = \frac{1}{(2\pi\ell_s)^3}\int_{HS^4} F_4 = \frac{1}{2\pi\ell_s^3}\frac{1}{b_1 b_2}\frac{F_0 y_p^2}{8}$$

$$y_p = 4\sqrt{\frac{\pi l_s^3}{F_0}b_1 b_2 N}$$

$$N_i = \frac{1}{(2\pi\ell_s)^3}\int_{C_4^i} F_4 = \frac{1}{(2\pi\ell_s)^3}\left(\int_{\Sigma_g^p} -\frac{2\pi}{b_i}\frac{1}{2F_0 y_p}j + \left(\frac{2\pi}{b_i}\right)^2\frac{F_0 y_p^2}{8}p_i\right)$$

$$\int_{\Sigma_g^p} j = 32\sqrt{\pi^5\ell_s^9 F_0 b_1 b_2 N^3}(b_1 p_2 + b_2 p_1)$$

$$\mathcal{F} = -\frac{16\pi^3}{(2\pi\ell_s)^8}\frac{(2\pi)^2}{b_1 b_2}\left(\frac{y_p^2}{24}\int_{\Sigma_g^p} j - \frac{2\pi}{b_1 b_2}\frac{F_0^2 y_p^5}{240}\left(\frac{p_1}{b_1}+\frac{p_2}{b_2}\right)\right)$$

$$= -\frac{16\sqrt{2}\pi}{5n_0^{1/2}}\sqrt{b_1 b_2}(b_1 p_2 + b_2 p_1)N^{5/2}$$

$$b_1 = \frac{1}{2}(1+\varepsilon), b_2 = \frac{1}{2}(1-\varepsilon)$$

$$p_1 = (1-g)(1+\kappa^{-1}z), p_2 = (1-g)(1-\kappa^{-1}z)$$

$$\mathcal{F} = -\frac{8\sqrt{2}\pi}{5\kappa n_0^{1/2}}\sqrt{(1-\varepsilon^2)}(g-1)(\varepsilon z - \kappa)N^{5/2}$$

$$\varepsilon_{\pm} = \frac{\kappa \pm \sqrt{\kappa^2 + 8z^2}}{4z}$$

$$\mathcal{F} = \frac{2\pi(1-g)N^{5/2}}{5\kappa n_0^{1/2}}\left(\sqrt{\kappa^2+8z^2} - 3\kappa\right)\sqrt{1 - \frac{\kappa\left(\sqrt{\kappa^2+8z^2}+\kappa\right)}{4z^2}}.$$

$$\Delta(HS^{2,1}) = -\frac{F_0}{2\pi\ell_s^3}\frac{y_p^2}{4b_1} = 2b_2N = (1-\varepsilon)N$$
$$\Delta(HS^{2,2}) = -\frac{F_0}{2\pi\ell_s^3}\frac{y_p^2}{4b_2} = 2b_1N = (1+\varepsilon)N$$
$$\Delta(\Sigma_g^p) = -\frac{1}{(2\pi)^2\ell_s^3}\int_{\Sigma_g^p}\frac{1}{F_0 y}j = -2(b_1p_2 + b_2p_1)N = \frac{2(1-g)}{\kappa}(z\varepsilon - \kappa)N$$

$$\Delta(HS^{2,i}) = \left(1 \mp \frac{\sqrt{\kappa^2+8z^2}+\kappa}{4z}\right)N$$
$$\Delta(\Sigma_g^p) = \frac{(1-g)}{2\kappa}\left(\sqrt{\kappa^2+8z^2}-3\kappa\right)N$$

$$\Delta(HS^{2,i}) = \left(1 \mp \frac{\kappa-\sqrt{\kappa^2+8z^2}}{4z}\right)N$$
$$\Delta(\Sigma_g^p) = \left(\frac{(g-1)\left(3\kappa+\sqrt{\kappa^2+8z^2}\right)}{2\kappa}\right)N$$

$$\xi = \sum_{i=1}^{2} b_i\partial_{\varphi_i} + \varepsilon\partial_{\varphi_0}$$

$$N_\pm = \frac{1}{(2\pi\ell_s)^3}\int_{HS^4/\mathbb{Z}_{n_\pm}} F_4 = \frac{1}{2\pi\ell_s^3}\frac{1}{n_\pm}\frac{1}{b_1^\pm b_2^\pm}\frac{F_0 y_\pm^2}{8}$$

$$n_+N_+ = n_-N_- \equiv N$$

$$y_+ = 4\sqrt{\frac{\pi l_s^3}{F_0}b_1^+b_2^+N}, y_- = 4\sigma\sqrt{\frac{\pi l_s^3}{F_0}b_1^-b_2^-N}$$

$$\mathcal{F} = -\frac{1}{24\pi^2\ell_s^8}\frac{F_0^2}{20}\left(\frac{1}{n_+}\frac{y_+^5}{b_1^+b_2^+(-\varepsilon/n_+)} + \frac{1}{n_-}\frac{y_-^5}{b_1^-b_2^-(\varepsilon/n_-)}\right)$$

$$= \frac{32\sqrt{2}\pi}{15n_0^{1/2}}\frac{(b_1^+b_2^+)^{3/2} - \sigma(b_1^-b_2^-)^{3/2}}{\varepsilon}N^{5/2}.$$

$$b_i^\pm = b_i + \frac{\varepsilon}{2}\left(\pm\frac{p_i}{n_+n_-} + \frac{n_+ - \sigma n_-}{2n_+n_-}\right), b_1 + b_2 = 1$$

$$b_1^\pm + b_2^\pm = 1 \pm \frac{\varepsilon}{n_\pm}, b_i^+ - b_i^- = \frac{p_i}{n_+n_-}\varepsilon$$

$$N_i = \frac{1}{(2\pi\ell_s)^3}\int_{C_4^i} F_4 = \frac{1}{2\pi\ell_s^3}\frac{F_0}{8}\left(\frac{1}{n_+}\frac{y_+^2}{b_i^+(-\varepsilon/n_+)} + \frac{1}{n_-}\frac{y_-^2}{b_i^-(\varepsilon/n_-)}\right)$$

$$N_1 = \frac{b_2^- - b_2^+}{\varepsilon}N = \frac{p_2}{n_+n_-}N, N_2 = \frac{b_1^- - b_1^+}{\varepsilon}N = \frac{p_1}{n_+n_-}N$$

$$\Delta(\Sigma^p) = -\frac{F_0}{2\pi\ell_s^3}\left(\frac{1}{n_+}\frac{y_+^2}{4(-\varepsilon/n_+)} + \frac{1}{n_-}\frac{y_-^2}{4(\varepsilon/n_-)}\right) = -2\frac{b_1^- b_2^- - b_1^+ b_2^+}{\varepsilon}N.$$

$$\Delta\left(HS_{\pm}^{2,i}\right) = \frac{F_0}{2\pi\ell_s^3}\frac{y_{\pm}^2}{4b_i^{\pm}} = 2\frac{b_1^{\pm}b_2^{\pm}}{b_i^{\pm}}N.$$

$$HS^2 \hookrightarrow M_6 \to B_4$$

$$c_\alpha = \int_{\Gamma_\alpha} j, n_\alpha = \int_{\Gamma_\alpha} c_1(\mathcal{L})$$

$$N_\alpha = \frac{1}{(2\pi\ell_s)^3}\int_{C_4^\alpha} F_4 = \frac{F_0}{4\pi\ell_s^3}\left[\frac{1}{2\pi}\frac{1}{F_0^2 y_p}c_\alpha - \frac{y_p^2}{4}n_\alpha\right]$$

$$c_\alpha = \frac{\pi F_0 y_p}{2}\left(F_0 y_p^2 n_\alpha - 16\pi\ell_s^3 N_\alpha\right)$$

$$N_p = \frac{1}{(2\pi\ell_s)^3}\int_{B_p^4} F_4 = -\frac{1}{(2\pi\ell_s)^3}\frac{1}{2F_0^3 y_p^4}\langle c, c\rangle$$
$$= \frac{1}{2}\langle N, n\rangle - \frac{F_0 y_p^2}{64\pi\ell_s^3}\langle n, n\rangle - \frac{4\pi\ell_s^3}{F_0 y_p^2}\langle N, N\rangle$$

$$N_p = -\langle N, n\rangle + M$$

$$y_p = 4\sqrt{\frac{\pi\ell_s^3}{F_0}\frac{3\langle N, n\rangle - 2M \pm \sqrt{(3\langle N, n\rangle - 2M)^2 - \langle N, N\rangle\langle n, n\rangle}}{\langle n, n\rangle}}$$

$$\mathcal{F} = -\frac{1}{24\pi^2\ell_s^8}\left[\frac{1}{(2\pi F_0)^2 y_p}\langle c, c\rangle - \frac{y_p^2}{4\pi}\langle c, n\rangle + \frac{F_0^2 y_p^5}{20}\langle n, n\rangle\right]$$
$$= \frac{2y_p}{3\ell_s^2}\langle N, N\rangle - \frac{F_0^2 y_p^5}{1920\pi^2\ell_s^8}\langle n, n\rangle$$
$$= \frac{16\sqrt{2}\pi}{15 n_0^{1/2}}\frac{1}{\langle n, n\rangle^{3/2}}\sqrt{3\langle N, n\rangle - 2M \pm \sqrt{(3\langle N, n\rangle - 2M)^2 - \langle N, N\rangle\langle n, n\rangle}}$$
$$\times\left(3\langle N, N\rangle\langle n, n\rangle + (3\langle N, n\rangle - 2M)\left(3\langle N, n\rangle \pm \sqrt{(3\langle N, n\rangle - 2M)^2 - \langle N, N\rangle\langle n, n\rangle}\right)\right)$$

$$\Delta(HS^2) = \frac{F_0}{2\pi\ell_s^3}\frac{y_p^2}{4} = \frac{2}{\langle n, n\rangle}\left(3\langle N, n\rangle - 2M \pm \sqrt{(3\langle N, n\rangle - 2M)^2 - \langle N, N\rangle\langle n, n\rangle}\right)$$

$$\Delta(\Gamma_\alpha) = \frac{1}{(2\pi)^2\ell_s^3}\frac{c_\alpha}{F_0 y_p}$$
$$= \frac{2}{\langle n, n\rangle}\left((3\langle N, n\rangle - 2M)n_\alpha + \left(\langle n, n\rangle \pm \sqrt{(3\langle N, n\rangle - 2M)^2 - \langle N, N\rangle\langle n, n\rangle}\right)N_\alpha\right)$$

$$\langle n, n\rangle = \bar{M}, \langle N, n\rangle = k\bar{M}, \langle N, N\rangle = k^2\bar{M}$$

$$k = \frac{N}{h}.$$

$$\mathcal{F} = \frac{32\sqrt{2}\pi}{5n_0^{1/2}}(3 \pm 2\sqrt{2})\bar{M}\left(\frac{N}{h}\right)^{5/2}$$

$$\Delta(HS^2) = \frac{2(3 \pm 2\sqrt{2})}{h}N, \Delta(\Gamma_\alpha) = \frac{4(1 \pm \sqrt{2})}{h}Nn_\alpha,$$

$$\langle n, n\rangle = 2\chi_1\chi_2, \langle N, n\rangle = \chi_1 N_2 + \chi_2 N_1, \langle N, N\rangle = 2N_1N_2.$$

$$\mathcal{F} = \frac{32\pi}{5n_0^{1/2}}\sqrt{3 \pm \sqrt{8+z^2}}\left(\sqrt{8+z^2} \pm (2+z^2)\right)N^{5/2}$$

$$\Delta(HS^2) = \left(3 \pm \sqrt{8+z^2}\right)N$$
$$\Delta(S_1^2) = 2\left(2 - z \pm \sqrt{8+z^2}\right)N, \Delta(S_2^2) = 2\left(2 + z \pm \sqrt{8+z^2}\right)N,$$

$$L^N = \mathcal{L}^{-1}, L^S = \mathcal{L}$$

$$[\Gamma_\alpha^S] - [\Gamma_\alpha^N] = n_\alpha\left[S^2_{\text{fibre}}\right] \in H^2(M_6, \mathbb{Z})$$

$$n_\alpha \equiv \int_{\Gamma_\alpha} c_1(\mathcal{L}) \in \mathbb{Z}$$

$$[B_4^S] - [B_4^N] = \sum_{\alpha=1}^{b_2} n_\alpha [C_\alpha]$$

$$c_\alpha^{N,S} \equiv \int_{\Gamma_\alpha^{N,S}} j$$

$$c_\alpha^S - c_\alpha^N = \frac{l^2\pi}{\epsilon}\left(\frac{1}{y^N} - \frac{1}{y^S}\right)n_\alpha$$

$$k_2 = \frac{1}{2\pi\ell_s}\int_{S^2_{\text{fibre}}} f_2 = \frac{l}{2\epsilon\ell_s}\left[\frac{1}{y_N} - \frac{1}{y_S}\right]$$
$$k_\alpha^{N,S} = \frac{1}{2\pi\ell_s}\int_{\Gamma_\alpha^{N,S}} f_2 = \frac{c_\alpha^{N,S}}{2l\ell_s\pi}$$

$$c_\alpha^S - c_\alpha^N = 2l\ell_s\pi k_2 n_\alpha \;\Leftrightarrow\; k_\alpha^S - k_\alpha^N = k_2 n_\alpha$$

$$y_N = \frac{l y_S}{l + 2\ell_s k_2 y_S \epsilon}$$

$$N = -\frac{1}{(2\pi\ell_s)^5}\int_{M_6} f_6$$
$$= \frac{l^4 k_2^2\pi^2(y_S - y_N) + l^2 k_2\epsilon\pi(c_N y_N^2 - c_S y_S^2) + \epsilon^2\left(c_S^2 y_S^3 - c_N^2 y_N^3\right)}{64 l^3\epsilon^3\pi^4\ell_s^5}$$

$$l = -L y_S k_2 \ell_s$$

$$N = \frac{y_S^2(\langle k^N, k^N\rangle L^2 + \langle k^S, k^S\rangle (L-2)^2 + \langle k^N, k^S\rangle L(L-2))}{8L(L-2)^3 \ell_s^4 k_2 \pi^2}$$

$$\mathcal{F} = \frac{4\pi}{3}\sqrt{\frac{2k_2^3 L^3 (L-2)^3}{\langle k^N, k^N\rangle L^2 + \langle k^S, k^S\rangle (L-2)^2 + \langle k^N, k^S\rangle L(L-2)}} N^{3/2}$$

$$\langle k^N, k^N\rangle L^2(2L-1) + \langle k^S, k^S\rangle (L-L)^2(2L-3) + 2\langle k^N, k^S\rangle L(2-3L+L^2) = 0$$

$$\mathcal{F} = \sqrt{\frac{2\pi^6}{27\mathrm{Vol}\left(Y^{p,k}(\mathrm{KE}_4)\right)}} N^{3/2}$$

$$\mathrm{Vol}\left(Y^{p,k}(\mathrm{KE}_4)\right) = \mathrm{Vol}(\mathrm{KE}_4)\frac{2\pi^2}{3\cdot 4^4}\frac{x_2 - x_1}{p(x_2-1)(1-x_1)}(x_2^3 - x_1^3),$$

$$0 = 3p^3 x_1^3 + 2p^2(6b-5p)x_1^2 + p(18b^2 - 28bp + 11p^2)x_1 + 4(3b^3 + 4bp^2 - 6b^2 p - p^2)$$
$$0 = 3p^3 x_2^3 + 2p^2(p-6b)x_2^2 + p(18b^2 - 8bp + p^2)x_2 + 4b(3bp - 3b^2 - p^2)$$

$$k_2 \equiv p, k_\alpha^S = \frac{k}{h} n_\alpha \equiv b n_\alpha, k_\alpha^N = (b-p)n_\alpha, \mathrm{Vol}(\mathrm{KE}_4) = \frac{\pi^2}{2}\langle n, n\rangle,$$

$$\xi = \sum_{i=1}^{3} \epsilon_i \partial_{\varphi_i}$$

$$0 = \int_{S_i^4} \Phi^Z = \frac{(2\pi)^2 \epsilon_i}{\epsilon_1 \epsilon_2 \epsilon_3}\left[\frac{(F_0^2 y_N^4 - 3l^2)^2}{36 y_N^2} + \frac{(F_0^2 y_S^4 - 3l^2)^2}{36 y_S^2}\right]$$

$$|y_N| = |y_S| = \left(\frac{3l^2}{F_0^2}\right)^{1/4}$$

$$0 = \int_{S_i^2} \Phi^Y = -\frac{\pi}{3\epsilon_i}\left[\frac{1}{y_N}(F_0^2 y_N^4 - 3l^2) - \frac{1}{y_S}\left(F_0^2 y_N^S - 3l^2\right)\right]$$

$$N = \frac{1}{(2\pi\ell_s)^5}\int_{M_6} f_6 = \frac{(2\pi)^3}{1296 l^3 (2\pi\ell_s)^5 \epsilon_1 \epsilon_2 \epsilon_3}[y_N(F_0^2 y_N^4 + 9l^2)^2 - y_S(F_0^2 y_S^4 + 9l^2)^2]$$

$$\ell_s^{-1} = \frac{2^{1/9} 3^{7/18} n_0^{1/9} N^{2/9} \pi^{1/3} (\epsilon_1 \epsilon_2 \epsilon_3)^{2/9}}{l^{1/3}}, F_0 = \frac{n_0}{2\pi\ell_s}$$

$$\mathcal{F} = \frac{(2\pi)^3}{240\epsilon_1\epsilon_2\epsilon_3}\frac{16\pi^3}{(2\pi\ell_s)^8}[y_N(F_0^2 y_N^4 + 5l^2) - y_S(F_0^2 y_S + 5l^2)]$$
$$= \frac{9}{5} 2^{1/3} 3^{1/6} n_0^{1/3} N^{5/3} \pi (\epsilon_1 \epsilon_2 \epsilon_3)^{2/3}$$

$$\epsilon_i = \frac{1}{3}$$

$$\mathcal{F} = \frac{2^{1/3}3^{1/6}n_0^{1/3}\pi^3}{5\mathrm{Vol}(S^5)^{2/3}}N^{5/3}$$

$$b_i = 3\epsilon_i$$

$$\mathrm{Vol}(S^5)[b_i] = \frac{\pi^3}{b_1 b_2 b_3},$$

$$\mathcal{F} = \frac{2^{1/3}3^{1/6}n_0^{1/3}\pi^3}{5(\mathrm{Vol}(S^5)[b_i])^{2/3}}N^{5/3}$$

$$|y_N| = |y_S| = \left(\frac{3l^2}{F_0^2}\right)^{1/4}$$

$$\begin{aligned} N &= \frac{1}{(2\pi\ell_s)^2}\int_{S(\mathrm{SE}_5)} f_6 \\ &= \frac{1}{1296(2\pi\ell_s)^5}\left[\frac{y_N(F_0^2 y_N^4 + 9l^2)^2}{e(\mathcal{N}_N)} + \frac{y_S(F_0^2 y_S^4 + 9l^2)^2}{e(\mathcal{N}_S)}\right] \\ &= \frac{1}{9(2\pi\ell_s)^5}\left[\frac{y_N}{e(\mathcal{N}_N)} + \frac{y_S}{e(\mathcal{N}_S)}\right] \end{aligned}$$

$$\ell_s^{-1} = \frac{2\cdot 3^{7/18}\pi n_0^{1/9}N^{2/9}}{\mathcal{E}}, F_0 = \frac{n_0}{2\pi\ell_s}$$

$$\mathcal{E} = \frac{1}{e(\mathcal{N}_N)} - \frac{1}{e(\mathcal{N}_S)}$$

$$\begin{aligned} \mathcal{F} &= \frac{\pi^3}{15(2\pi\ell_s)^8}\left[\frac{y_N(F_0^2 y_N^4 + 5l^2)}{e(\mathcal{N}_N)} + \frac{y_S(F_0^2 y_S^4 + 5l^2)}{e(\mathcal{N}_S)}\right] \\ &= \frac{8\pi^3 y_N}{15(2\pi\ell_s)^8}\left[\frac{1}{e(\mathcal{N}_N)} - \frac{1}{e(\mathcal{N}_S)}\right] \end{aligned}$$

$$\mathcal{F} = \frac{72\times 3^{1/6}n_0^{1/3}\pi^3}{5\mathcal{E}^{2/3}}N^{5/3}$$

$$\mathcal{E} = 2^4 3^3 \mathrm{Vol}(\mathrm{SE}_5)$$

$$\mathcal{F} = \frac{2^{1/3}3^{1/6}\pi^3}{5\mathrm{Vol}(\mathrm{SE}_5)^{2/3}}n_0^{1/3}N^{5/3}$$

$$\Phi^{\mathrm{Thom}} = \frac{1}{2\pi}\,\mathrm{d}(\rho(r)\eta)$$

$$s^*\Phi^{\mathrm{Thom}} = -e$$

$$\Psi_p \equiv \mathrm{PD}[B_4] = \Phi^{\mathrm{Thom}}$$

$$\tau = \sum_{\alpha=1}^{b_2} \hat{C}_\alpha \Psi_\alpha$$

$$C_\alpha \equiv \int_{\Gamma_\alpha} \tau$$

$$C_\alpha = \int_{B_4} \tau \wedge \Psi_\alpha = \sum_{\beta=1}^{b_2} Q_{\alpha\beta} \hat{C}$$

$$Q_{\alpha\beta} \equiv \int_{B_4} \Psi_\alpha \wedge \Psi_\beta = [\Gamma_\alpha] \cap [\Gamma_\beta]$$

$$\int_{B_4} \tau \wedge \tau = \sum_{\alpha,\beta=1}^{b_2} Q_{\alpha\beta} \hat{C}_\alpha \hat{C}_\beta = \sum_{\alpha,\beta=1}^{b_2} I_{\alpha\beta} C_\alpha C_\beta$$

$$c_1(\mathcal{L}) = \sum_{\alpha=1}^{b_2} \hat{n}_\alpha \Psi_\alpha$$

$$\tau = \sum_{\alpha=1}^{b_2} \hat{C}_\alpha \Psi_\alpha, \omega = \sum_{\alpha=1}^{b_2} \hat{N}_\alpha \Psi_\alpha$$

$$\int_{B_4} \tau \wedge \omega = \sum_{\alpha,\beta=1}^{b_2} Q_{\alpha\beta} \hat{C}_\alpha \hat{N}_\beta = \sum_{\alpha,\beta=1}^{b_2} I_{\alpha\beta} C_\alpha N_\beta \equiv \langle C, N \rangle$$

$$F_4 = \Phi^{\text{Thom}} \wedge \sum_{\alpha=1}^{b_2} \hat{N}_\alpha \Psi_\alpha + M\widetilde{\text{vol}}(B_4)$$

$$\begin{aligned}(2\pi\ell_s)^3 N &= \int_{B_4} F_4 \\ &= M + \hat{N}_\alpha \int_{B_4} s^*\left(\Phi^{\text{Thom}}\right) \wedge \Psi_\alpha \\ &= M - \hat{N}_\alpha \int_{B_4} e \wedge \Psi_\alpha \\ &= M - \langle N, n \rangle\end{aligned}$$

$$\text{PD}[\Gamma_\alpha] = \pi^* \Psi_\alpha \wedge \Phi^{\text{Thom}} \in H^4(M_6)$$

$$\int_{B_4} \pi^* \Psi_\alpha \wedge \Phi^{\text{Thom}} = n_\alpha$$

$$[\Gamma_\alpha] = n_\alpha \left[HS^2_{\text{fibre}}\right]$$

$$t_8t_8\hat{R}^4.$$

$$\hat{e}^a = e^{-\Phi/3}e^a, \hat{e}^{10} = e^{2\Phi/3}(dx^{10} + A), \hat{F} \equiv d\hat{C} = F_4 + H \wedge (dx^{10} + A)$$

$$\hat{e}^a = e^{2\Phi/3}\hat{e}'^a$$

$$\hat{e}'^a = e'^a, \hat{e}'^{10} = dx^{10} + A$$

$$e'^a = e^{-\Phi}e^a$$

$$\hat{F}_{abc10} = H_{abc}, \hat{R}'_{abc10} = -\frac{1}{2}\nabla_c F_{ab}$$

$$e^a = e^{k\Phi}e'^a$$

$$R_{abcd} = e^{-2k\Phi}(R'_{abcd} + D_{abcd} + \cdots) \text{ with } D^{ab}{}_{cd} = 4k\delta^{[a}_{[c}\nabla_{d]}\nabla^{b]}\Phi$$

$$t_8t_8R^4 = t_8t_8R'^4 + 96k(2l - k(D-2))\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi X_{abcd} + \cdots$$

$$\begin{gathered}X^{cd}_{ab} = 8R'_{aghb}R'^{cghd} + 8R'_{agh}{}^{c}R'^{hgd}_{b} - 8\delta^d_a R'_{begh}R'^{cegh} + \delta^c_a\delta^d_b R'_{efgh}R'^{efgh} \\ +2k(2l - k(D-2))[3\nabla_a\nabla_b\Phi\nabla^c\nabla^d\Phi - 2\nabla_a\nabla^c\Phi\nabla_b\nabla^d\Phi]\end{gathered}$$

$$R_{ab} = l\nabla_a\nabla_b\Phi.$$

$$t_8t_8\hat{R}^4 = t_8t_8\hat{R}'^4 - 2^7 3\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi\hat{X}_{abcd} + \cdots,$$

$$\begin{gathered}\hat{X}_{abcd} = 8\hat{R}'_{aghb}\hat{R}'^{gh}_{c\ \ d} + 8\hat{R}'_{aghc}\hat{R}'^{hg}_{b\ \ d} - 8\eta_{ad}\hat{R}'_{begh}\hat{R}'^{egh}_{c} + \eta_{ac}\eta_{bd}\hat{R}'_{efgh}\hat{R}'^{efgh} \\ -24\nabla_a\nabla_b\Phi\nabla_c\nabla_d\Phi + 16\nabla_a\nabla_c\Phi\nabla_b\nabla_d\Phi\end{gathered}$$

$$\begin{gathered}t_8t_8R^4 = 12\big(16R_{efab}R^{fgbc}R_{ghcd}R^{heda} + 32R_{efab}R^{hebc}R^{fg}{}_{cd}R_{gh}{}^{da} - 16R_{efab}R^{fgab}R_{ghcd}R^{hecd} \\ -8R_{efab}R_{gh}{}^{ab}R^{hecd}R^{fg}{}_{cd} + R_{abcd}R^{abcd}R_{efgh}R^{efgh} + 2R_{efab}R^{ghab}R^{efcd}R_{ghcd}\big)\end{gathered}$$

$$t_8t_8R'^4 + L_{R'^2F_2^2} + L_{F_2^4} - 2^7 3\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi\big(X_{abcd} + \tilde{X}_{abcd}\big)$$

$$\begin{gathered}L_{R'^2F_2^2} = 12\big(-32R'_{efab}R'^{heda}\nabla^f F^{bc}\nabla_h F_{cd} - 32R'_{fgcd}R'^{ghda}\nabla^f F_{ab}\nabla_h F^{bc} \\ -32R'_{hebc}R'^{gh}{}_{da}\nabla^e F^{ab}\nabla_g F^{cd} - 16R'_{ghcd}R'^{hecd}\nabla_a F_{ef}\nabla^a F^{fg} \\ +8R'_{ghcd}R'^{hecd}\nabla_e F_{ab}\nabla^g F^{ab} + 8R'_{efab}R'^{hecd}\nabla^f F^{ab}\nabla_h F_{cd} \\ -8R'_{efab}R'_{gh}{}^{ab}\nabla_c F^{he}\nabla^c F^{fg} + 8R'_{ghab}R'^{fgcd}\nabla_f F^{ab}\nabla^h F_{cd} \\ +4R'_{efab}R'^{efcd}\nabla^g F^{ab}\nabla_g F_{cd} + 2R'_{efab}R'^{efab}\nabla_g F_{cd}\nabla^g F^{cd}\big),\end{gathered}$$

$$\begin{gathered}\tilde{X}_{abcd} = 4\nabla_a F_{be}\nabla_c F_d{}^e + 4\nabla_a F_{ce}\nabla_d F_b{}^e - 4\eta_{ad}\nabla_e F_{fb}\nabla^e F^f{}_c - 2\eta_{ad}\nabla_b F_{ef}\nabla_c F^{ef} \\ + \eta_{ac}\eta_{bd}\nabla_g F_{ef}\nabla^g F^{ef}\end{gathered}$$

$$k = -1, l = -\frac{2}{3}(11 - 2) = -6$$

$$t_8t_8R^4 + L_{R'^2F_2^2} + L_{F_2^4} - 2^7 3\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi\tilde{X}_{abcd} + \cdots$$

$$R'_{abcd} = R_{abcd} - 4\delta^{[a}_{[c}\nabla_{d]}\nabla^{b]}\Phi$$

$$t_8t_8\hat{R}^4 \to t_8t_8R^4 + L_{R^2F_2^2} + L_{F_2^4} + L_{\Phi RF_2^2} + L_{\Phi^2F_2^2} + \cdots$$

$$\begin{aligned} L_{\Phi RF_2^2} &= 2^8 3\nabla_a\nabla^b\Phi\big(2R^{aecd}\nabla^f F_{bc}\nabla_e F_{df} + 2R^{ae}_{cd\ \ b}F^{cf}\nabla^d F_{ef} + R^{ac}_{db}\nabla_e F_{cf}\nabla^e F^{fd}\big) \\ &= -2^9 3\Phi R^{ab}_{cd}\nabla_a\nabla^e F^{fc}\nabla^d\nabla_f F_{eb} + \cdots, \end{aligned}$$

$$\begin{aligned} L_{\Phi^2F_2^2} =& 2^7 3\nabla_a\nabla^b\Phi\nabla_c\nabla_d\Phi\big(2\nabla^a F^{ce}\nabla^d F_{be} - 2\nabla^a F_{be}\nabla^c F^{de} + 2\nabla_e F^{ac}\nabla^e F^d_b \\ &+4\eta^{ad}\nabla^e F_{bf}\nabla^f F^{ce} + 3\eta^{ad}\nabla_b F^{ef}\nabla^c F_{ef} - \eta^{ad}\delta^c_b\nabla_g F_{ef}\nabla^g F^{ef}\big) \\ =& -2^8 3\Phi\nabla_a\nabla^b\Phi\nabla^c\nabla^d F^{ea}\nabla_c\nabla_d F_{eb} + \cdots \end{aligned}$$

$$\hat{L}_{\hat{R}'^2\hat{F}^2} + \hat{L}_{\Phi\hat{R}'\hat{F}^2} + \hat{L}_{\Phi^2\hat{F}^2} + \cdots$$

$$L_{\Phi R'F^2} = 48k\Phi R'_{abcd}\big(-3\nabla^4\big(F^{abef}F^{cd}{}_{ef}\big) + 8\nabla^2\big(\nabla^c F^{aefg}\nabla^d F^{b}{}_{efg}\big) - 4\nabla^a\nabla^c F^{efgh}\nabla^b\nabla^d F_{efgh}\big)$$

$$\begin{aligned} L_{\Phi^2F^2} = 16k(2l - k(D-2))\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi\big(&-12\nabla_g F_{acef}\nabla^g F^{ef}_{bd} + 8\nabla_a F_{befg}\nabla_c F^{efg}_d \\ &-8\eta_{ac}\nabla_h F_{befg}\nabla^h F_d{}^{efg} - 2\eta_{ac}\nabla_b F_{efgh}\nabla_d F^{efgh} + \eta_{ac}\eta_{bd}\nabla_k F_{efgh}\nabla^k F^{efgh}\big) \\ &-288k^2\Phi\nabla_a\nabla^b\Phi\nabla^4\big(F_{befg}F^{aefg}\big). \end{aligned}$$

$$\hat{L}_{\Phi\hat{R}'\hat{F}^2} = 32\Phi\hat{R}'_{abcd}\big(-3\nabla^4\big(\hat{F}^{abef}\hat{F}^{cd}{}_{ef}\big) + 8\nabla^2\big(\nabla^c\hat{F}^{aefg}\nabla^d\hat{F}^{b}{}_{efg}\big) - 4\nabla^a\nabla^c\hat{F}^{efgh}\nabla^b\nabla^d\hat{F}_{efgh}\big)$$

$$\begin{aligned} \hat{L}_{\Phi^2\hat{F}^2} = 64\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi\big(&12\nabla_g\hat{F}_{acef}\nabla^g\hat{F}^{ef}_{bd} - 8\nabla_a\hat{F}_{befg}\nabla_c\hat{F}^{efg}_d \\ &+8\eta_{ac}\nabla_h\hat{F}_{befg}\nabla^h\hat{F}^{efg}_d + 2\eta_{ac}\nabla_b\hat{F}_{efgh}\nabla_d\hat{F}^{efgh} - \eta_{ac}\eta_{bd}\nabla_k\hat{F}_{efgh}\nabla^k\hat{F}^{efgh}\big) \\ &-128\Phi\nabla_a\nabla^b\Phi\nabla^4\big(\hat{F}_{befg}\hat{F}^{aefg}\big) \end{aligned}$$

$$L_{R'^2H^2} + L_{\Phi R'H^2} + L_{\Phi^2H^2} + L_{R'^2F_4^2} + L_{\Phi R'F_4^2} + L_{\Phi^2F_4^2} + L_{R'HF_2F_4} + L_{\Phi HF_2F_4} + L_{H^2F_2^2} + L_{F_2^2F_4^2},$$

$$\begin{aligned} L_{R'^2H^2} = 16\big(&48R'_{abcd}R'_{efg}{}^d\nabla^a H^{cg}{}_h\nabla^b H^{efh} - 12R'_{abcd}R'_{efg}{}^d\nabla^c H^{ab}{}_h\nabla^g H^{efh} \\ &-12R'_{abcd}R'_{efg}{}^d\nabla_i H^{efc}\nabla^i H^{abg} + 12R'_{abcd}R'_{efg}{}^d\nabla^c H^{ef}{}_h\nabla^g H^{abh} \\ &+24R'_{abcd}R'^{eafc}\nabla_i H^b{}_{fg}\nabla^i H_e{}^{dg} + 24R'_{abcd}R'^{eafc}\nabla_i H^{bdg}\nabla^i H_{efg} \\ &+24R'_{abcd}R'^{eafc}\nabla^b H^{dgh}\nabla_e H_{fgh} - 12R'_{abcd}R'^{efcd}\nabla^a H_{egh}\nabla^b H_f{}^{gh} \\ &+12R'_{abcd}R'^{eacd}\nabla_i H^{bgh}\nabla^i H_{egh} + 4R'_{abcd}R'_{eacd}\nabla^b H_{fgh}\nabla^e H^{fgh} \\ &+3R'_{abcd}R'^{abcd}\nabla_e H_{fgh}\nabla^f H^{egh}\big) \end{aligned}$$

$$L_{\Phi R'H^2} = 64\Phi R'_{abcd}\big(-3\nabla^4\big(H^{abe}H^{cd}{}_e\big) + 12\nabla^2\big(\nabla^c H^{aef}\nabla^d H^{b}{}_{ef}\big) - 8\nabla^a\nabla^c H^{efg}\nabla^b\nabla^d H_{efg}\big)$$

$$\begin{aligned} L_{\Phi^2H^2} = 2^8\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi\big(&6\nabla_g H_{ace}\nabla^g H^e_{bd} - 6\nabla_a H_{bef}\nabla_c H^{ef}_d \\ &+6\eta_{ac}\nabla_h H_{bef}\nabla^h H^{ef}_d + 2\eta_{ac}\nabla_b H_{efg}\nabla_d H^{efg} - \eta_{ac}\eta_{bd}\nabla_k H_{efg}\nabla^k H^{efg}\big) \\ &-2^7 3\Phi\nabla_a\nabla^b\Phi\nabla^4\big(H_{bef}H^{aef}\big) \end{aligned}$$

$$L_{R'HF_2F_4} = 16\big(24R'_{abcd}\nabla^h H^{cef}\nabla_e F^{gd}\nabla_h F^{ab}{}_{fg} - 12R'^{ab}{}_{cd}\nabla^f H_{abe}\nabla^c F_{gh}\nabla_f F^{degh}$$
$$-12R'_{abcd}\nabla^e H_{fgh}\nabla^f F^{ec}\nabla^d F^{abgh} - 12R'_{abcd}\nabla^e H^{cgh}\nabla^d F_{ef}\nabla^f F^{ab}{}_{gh}$$
$$+12R'_{abcd}\nabla^c H_{egh}\nabla^e F^{df}\nabla_f F^{abgh} + 12R'_{abcd}\nabla^a H^{cef}\nabla^d F^{gh}\nabla^b F_{efgh}$$
$$-24R'_{abcd}\nabla^a H^{egh}\nabla_e F_f{}^c\nabla^b F^{dfgh} + 24R'_{ab}{}^{cd}\nabla^e H^{agh}\nabla_c F^{bf}\nabla_e F_{fdgh}$$
$$-6R'_{abcd}\nabla^e H_{fgh}\nabla^f F^{ab}\nabla_e F^{cdgh} - 8R'_{abcd}\nabla_e H_{fgh}\nabla^c F^{ea}\nabla^b F^{dfgh}$$
$$+4R'_{abcd}\nabla^a H_{fgh}\nabla^e F^{cd}\nabla^b F_{efgh} - 4R'_{abcd}\nabla_e H_{fgh}\nabla^a F^{cd}\nabla^e F^{bfgh}\big)$$

$$L_{\Phi HF_2F_4} = 64\Phi\big[3\nabla^e F_{ab}\nabla^4\big(F^{abcd}H_{ecd}\big) + 4\nabla_f F_{ab}\nabla^2\big(\nabla^a F^{fcde}\nabla^b H_{cde}\big)\big].$$

$$L_{R^2H^2} + L_{R^2F_4^2} + L_{RHF_2F_4} + L_{\Phi RF_4^2} + L_{\Phi^2F_4^2} + L_{\Phi HF_2F_4} + L_{H^2F_2^2} + L_{F_2^2F_4^2},$$

$$L_{\Phi RF_4^2} = 16\Phi R_{abcd}\big(3\nabla^4\big(F^{abef}F^{cd}{}_{ef}\big) - 8\nabla^2\big(\nabla^c F^{aefg}\nabla^d F^b{}_{efg}\big) + 4\nabla^a\nabla^c F^{efgh}\nabla^b\nabla^d F_{efgh}\big),$$

$$L_{\Phi^2F_4^2} = -32\Phi\nabla_a\nabla^b\Phi\nabla^4\big(F^{acde}F_{bcde}\big)$$

$$L_{\Phi HF_2F_4} = 64\Phi\big[3\nabla_a H_{bef}\nabla^2\big(\nabla^b F_{gh}\nabla^a F^{efgh}\big) - 2\nabla_a H_{cde}\nabla^2\big(\nabla^f F^{ag}\nabla_g F_{fcde}\big)\big].$$

$$L_{\mathrm{NSNSRR}} = c\left(A_1 + \frac{1}{2}A_2 + \frac{1}{4}A_3\right)$$

$$A_1 = \mathcal{R}_{aefb}\mathcal{R}^{cef}{}_d\mathrm{tr}\big(\gamma^a\nabla_c\boldsymbol{F}^{(b}\nabla^{d)}\boldsymbol{F}^T\big)$$
$$A_2 = \mathcal{R}_{abeg}\mathcal{R}_{cdf}{}^g\mathrm{tr}\big(\gamma^{abc}\nabla^d\not{\boldsymbol{F}}^{(e}\nabla^{f)}\boldsymbol{F}^T\big) + \mathcal{R}_{egab}\mathcal{R}_f{}^g{}_{cd}\mathrm{tr}\big(\gamma^{abc}\nabla^d\boldsymbol{F}^T\gamma^{(e}\nabla^{f)}\not{\boldsymbol{F}}'\big)$$
$$A_3 = \mathcal{R}_{abcd}\mathcal{R}_{efgh}\mathrm{tr}\big(\gamma^{abe}\nabla^f\not{\boldsymbol{F}}^{[cdg}\nabla^{h]}\boldsymbol{F}^T\big)$$

$$\mathcal{R}_{abcd} = R_{abcd} + \nabla_{[a}H_{b]cd}$$

$$\not{F} = \frac{1}{2}F_{ab}\gamma^{ab} + \frac{1}{4!}F_{abcd}\gamma^{abcd}, \not{F}^T = -\frac{1}{2}F_{ab}\gamma^{ab} + \frac{1}{4!}F_{abcd}\gamma^{abcd}.$$

$$[A_1]_\Phi = -\frac{1}{2}R^{ef}\mathcal{R}_{cefd}\mathrm{tr}\big(\nabla^c\not{F}^dF^T\big)$$
$$[A_2]_\Phi = R_{ag}\mathcal{R}_{cdf}{}^g\mathrm{tr}\big(\gamma^{ac}\nabla^d\not{F}^fF^T\big) + R_{ag}\mathcal{R}_f{}^g{}_{cd}\mathrm{tr}\left(\gamma^{ac}\nabla^d\not{F}^{T\,f}\nabla^f\right)$$
$$+\frac{1}{2}R_{dg}\mathcal{R}_{abe}{}^g\mathrm{tr}\big(\gamma^{ab}\nabla^d\nabla^eF^T\big) + \frac{1}{2}R_{dg}\mathcal{R}_e{}^g{}_{ab}\mathrm{tr}\big(\gamma^{ab}\nabla^dF^T\nabla^e\not{F}'\big)$$
$$[A_3]_\Phi = 2R_{ad}\mathcal{R}_{efgh}\mathrm{tr}\big(\gamma^{ae}\nabla^f\not{F}^{dg}\nabla^hF^T\big) + \frac{1}{2}R_{fh}\mathcal{R}_{abcd}\mathrm{tr}\big(\gamma^{ab}\nabla^f\not{F}^{cd}\nabla^hF^T\big)$$
$$+R_{ad}\mathcal{R}_{efgh}\mathrm{tr}\big(\gamma^{ae}\nabla^f\not{F}^{gh}\nabla^dF^T\big) + R_{fh}\mathcal{R}_{abcd}\mathrm{tr}\big(\gamma^{ab}\nabla^f\not{F}^{hc}\nabla^dF^T\big)$$
$$+2R_{ad}R_{fh}\mathrm{tr}\big(\gamma^a\nabla^f\not{F}^d\gamma^hF^T\big) - 2R_{ad}R_{fh}\mathrm{tr}\big(\gamma^a\nabla^f\not{F}^h\nabla^dF^T\big)$$

$$[A_1]_{\Phi F_2^2} = 8\nabla^a\nabla^b\Phi R_{acbd}\nabla^cF_{ef}\nabla^dF^{ef}$$
$$[A_2]_{\Phi F_2^2} = 32\nabla_a\nabla^b\Phi\big(R_{becd}\nabla^eF^{fa}\nabla_fF^{cd} + 2R_{becd}\nabla_fF^{ac}\nabla^eF^{fd}\big)$$
$$[A_3]_{\Phi F_2^2} = 32\nabla^a\nabla^b\Phi\big(R_{acbd}\nabla^cF_{ef}\nabla^dF^{ef} - 4R^{cdef}\nabla_aF_{be}\nabla_fF_{cd} + 4R^{cdef}\nabla_eF_{ca}\nabla_dF_{fb}$$
$$-R_{cdef}\nabla_aF^{cd}\nabla_bF^{ef} - 2R_{becd}\nabla^eF_{af}\nabla^fF^{cd} + 4R_{be}{}^{cd}\nabla^fF_{ac}\nabla^eF_{df}$$
$$+8R_{be}{}^{cd}\nabla_aF_{fc}\nabla_dF^{fe}\big) + 64\nabla^a\nabla^b\Phi\nabla^c\nabla_d\Phi\big(4\nabla_aF_{ce}\nabla_bF^{de}$$
$$-2\nabla_aF_{be}\nabla_cF^{de} - 2\nabla_cF_{ae}\nabla_bF^{de} + \delta_a^d\nabla_bF_{ef}\nabla_cF^{ef}\big)$$

$$[A_1]_{\Phi F_4^2} = \frac{2}{3}\nabla^a\nabla^b\Phi R_{acbd}\nabla^c F_{efgh}\nabla^d F^{efgh}$$

$$[A_2]_{\Phi F_4^2} = \frac{32}{3}\nabla^a\nabla^b\Phi R_{becd}\left(\nabla^e F_{afgh}\nabla^c F^{dfgh} - \nabla^c F_{afgh}\nabla^e F^{dfgh} + \nabla_a F^{cfgh}\nabla^e F^d_{fgh}\right)$$

$$\begin{aligned}[A_3]_{\Phi F_4^2} = \frac{8}{3}\nabla^a\nabla^b\Phi\big(&R_{acbd}\nabla^c F_{efgh}\nabla^d F^{efgh} - 12R_{cdef}\nabla_a F^{cdgh}\nabla_b F^{ef}{}_{gh} - 24R^{cdef}\nabla_e F_{cagh}\nabla_d F_{bf}{}^{gh}\\ &-24R^{cdef}\nabla_c F_{dagh}\nabla_e F_{fb}{}^{gh} - 24R^{cdef}\nabla_e F_{fagh}\nabla_b F_{cd}{}^{gh} - 24R^{cdef}\nabla_a F_{begh}\nabla_f F_{cd}{}^{gh}\\ &+8R_{becd}\nabla^e F_{afgh}\nabla^c F^{dfgh} + 8R_{becd}\nabla^c F_{afgh}\nabla^e F^{dfgh} + 8R_{becd}\nabla_a F^{cfgh}\nabla^e F^d{}_{fgh}\\ &-16R_{becd}\nabla_a F^{efgh}\nabla^c F^d{}_{fgh}\big) + \frac{16}{3}\nabla^a\nabla_b\Phi\nabla^c\nabla^d\Phi\big(8\nabla_c F_{aefg}\nabla_d F^{befg}\\ &-4\nabla_a F_{cefg}\nabla_d F^{befg} - 4\nabla_a F^{befg}\nabla_c F_{defg} + \eta_{ad}\nabla_c F_{efgh}\nabla^b F^{efgh}\big)\end{aligned}$$

$$[A_1]_{\Phi F_2 F_4} = 0,$$

$$\begin{aligned}[A_2]_{\Phi F_2 F_4} = 16\nabla_a\nabla^b\Phi(\nabla H)_{becd}\big(&2\nabla^c F_{gh}\nabla^e F^{adgh} - 2\nabla^e F_{gh}\nabla^c F^{adgh} - \nabla^a F_{gh}\nabla^e F^{cdgh}\\ &+\nabla^e F_{gh}\nabla^a F^{cdgh}\big)\end{aligned}$$

$$\begin{aligned}[A_3]_{\Phi F_2 F_4} = 64\nabla^a\nabla^b\Phi\Big(&\frac{1}{3}\nabla_f H_{cde}\nabla_a F_{bg}\nabla^f F^{gcde} - \frac{2}{3}\nabla^f H_{cde}\nabla_a F_{fg}\nabla_b F^{gcde}\\ &+\frac{1}{3}\nabla_f H_{cde}\nabla^f F_{ag}\nabla_b F^{gcde} + 4(\nabla H)^{cdef}\nabla_e F_{ag}\nabla^c F_{bfd}{}^g\\ &-2(\nabla H)^{cdef}\nabla_a F_{cg}\nabla^e F_{bfd}{}^g - 2(\nabla H)^{cdef}\nabla_e F_{cg}\nabla^a F_{bfd}{}^g\\ &+(\nabla H)^{cdef}\nabla_a F_{eg}\nabla_f F_{bcd}{}^g - (\nabla H)^{cdef}\nabla_e F_{fg}\nabla_a F_{bcd}{}^g\\ &+(\nabla H)_{becd}\nabla^e F^{gh}\nabla^c F_a^{dgh} + (\nabla H)_{becd}\nabla^c F_{gh}\nabla^e F_a^{dgh}\\ &+(\nabla H)_{becd}\nabla_a F_{gh}\nabla^c F^{degh} + (\nabla H)_{becd}\nabla^c F_{gh}\nabla_a F^{degh}\Big)\end{aligned}$$

$$-24\left(A_1 + \frac{1}{2}A_2 + \frac{1}{4}A_3\right)_{\Phi} = L_{\Phi R F_2^2} + L_{\Phi^2 F_2^2} + L_{\Phi R F_4^2} + L_{\Phi^2 F_4^2} + L_{\Phi H F_2 F_4} + \cdots,$$

$$-24\left(A_1 + \frac{1}{2}A_2 + \frac{1}{4}A_3\right)_{R^2 F_2^2} = L_{R^2 F_2^2} + 12\cdot 6!\, R_{[ab}{}^{ab}R_{cd}{}^{cd}\nabla_e F^{eg}\nabla_{f]}F^f{}_g + 24\cdot 6!\, R_{[ab}{}^{ag}R_{cd}{}^{bc}\nabla_e F^{de}\nabla_{f]}F^f{}_g,$$

$$\mathcal{R}_{abcd} = R_{abcd} + \nabla_{[a}H_{b]cd} + \frac{1}{2}H_{a[c}{}^e H_{d]be}$$

$$t_8 t_8\mathcal{R}^4 \equiv t_{a_1\cdots a_8}t^{b_1\cdots b_8}\mathcal{R}^{a_1a_2}{}_{b_1b_2}\cdots\mathcal{R}^{a_7a_8}{}_{b_7b_8}$$

$$t_8 M_1M_2M_3M_4 = 8\mathrm{tr}(M_1M_2M_3M_4) - 2\mathrm{tr}(M_1M_2)\mathrm{tr}(M_3M_4) + \mathrm{cycl}(2,3,4).$$

$$\mathcal{R} = \frac{1}{4}R_{abcd}\Gamma^{ab}\otimes\Gamma^{cd} + \frac{1}{4}\nabla_a H_{bcd}\Gamma^{ab}\otimes\Gamma^{cd}\Gamma_{11}.$$

$$\begin{aligned}t_{8s}M_1M_2M_3M_4 = -\frac{3}{4}\mathrm{str}(M_1\cdots M_4) + \frac{1}{64}[\mathrm{tr}(M_1M_2)\mathrm{tr}(M_3M_4) + \mathrm{cycl}(2,3,4)]\\ +\frac{1}{256}\left[\mathrm{tr}\left(M_1\{M_2,\hat{\Gamma}^a\}\right)\mathrm{tr}\left(\{\hat{\Gamma}_a,M_3\}M_4\right) + \mathrm{cycl}(2,3,4)\right]\end{aligned}$$

$$\mathrm{str}(ABCD) = \frac{1}{6}\mathrm{tr}(ABCD + \mathrm{perm}(B,C,D))$$

$$t_{8s}M_1M_2M_3M_4 = t_8M_1M_2M_3M_4$$

$$t_{8s}t_{8s}\mathbb{R}^4.$$

$$\hat{\mathcal{R}} = \frac{1}{4}\hat{R}_{abcd}\hat{\Gamma}^{ab} \otimes \hat{\Gamma}^{cd} + \nabla_a \hat{F}_{bcde}\left(\frac{1}{12}\hat{\Gamma}^{ab} \otimes \hat{\Gamma}^{cde} + \frac{a}{48}\hat{\Gamma}^{a} \otimes \hat{\Gamma}^{bcde}\right),$$

$$\hat{\mathcal{R}} \to \mathbb{R} + \mathcal{R}' \text{ with } \mathbb{R}' = \frac{1}{12}\nabla_a H_{bcd}\left(-\Gamma^a\Gamma_{11} \otimes \Gamma^{bcd} + a\Gamma^a \otimes \Gamma^{bcd}\Gamma_{11}\right).$$

$$t_{8s}t_{8s}\hat{\mathbb{R}}^4 \to t_{8s}t_{8s}\mathbb{R}^4 + 6t_{8s}t_{8s}\mathbb{R}^2\mathbb{R}'^2 + t_{8s}t_{8s}\mathbb{R}'^4$$

$$\begin{aligned}6t_{8s}t_{8s}\mathbb{R}^2\mathbb{R}'^2 =& -\frac{3}{32}(1+a^2)R_{abcd}R_{efgh}t_{8s}\left(\Gamma^{ab}\Gamma^{ef}\Gamma^i\Gamma^j\right)t_{8s}\left(\Gamma^{cd}\Gamma^{gh}\nabla_i \not{H}\nabla_j \not{H}\right)\\ &-\frac{3}{32}(1+a^2)(\nabla H)_{abcd}(\nabla H)_{efgh}t_{8s}\left(\Gamma^{ab}\Gamma^{ef}\Gamma^i\Gamma^j\right)t_{8s}\left(\Gamma^{cd}\Gamma_{11}\Gamma^{gh}\Gamma_{11}\nabla_i \not{H}\nabla_j \not{H}\right)\\ &+192a^2R^a{}_{bcd}R^{bgef}\nabla_{(a}H^{hcd}\nabla_{g)}H_{hef} + 48a^2R_{abcd}R^{abef}\nabla^g H^{hcd}\nabla_g H_{hef},\end{aligned}$$

$$a = i$$

$$\begin{aligned}\hat{L}_1 =& 96\hat{R}^a_{bcd}\hat{R}^{bgef}\nabla_{(a}\hat{F}^{cdij}\nabla_{g)}\hat{F}_{efij} + 24\hat{R}_{abcd}\hat{R}^{abef}\nabla^g\hat{F}^{cdij}\nabla_g\hat{F}_{efij}\\ =& -\frac{9}{8}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{e}\hat{\Gamma}^{f}\right)\hat{R}_{abgh}\hat{R}_{cdij}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}^{ij}{}_{kl},\end{aligned}$$

$$\begin{aligned}t_{8s}t_{8s}\mathbb{R}'^4 =& -3\mathrm{tr}\left(\nabla_a \not{H}^a \not{H}\nabla_b \not{H}\nabla^b \not{H}\right) + \frac{3}{2}\mathrm{tr}\left(\nabla_a \not{H}\nabla_b \not{H}\nabla^a \not{H}\nabla^b \not{H}\right)\\ &+\frac{3}{32}\mathrm{tr}\left(\nabla_a \not{H}\nabla^a \not{H}\right)\mathrm{tr}\left(\nabla_b \not{H}\nabla^b \not{H}\right).\end{aligned}$$

$$\hat{L}_2 = 3\mathrm{tr}\left(\nabla_a \not{\hat{F}}\nabla^a \not{\hat{F}}\nabla_b \not{\hat{F}}\nabla^b \not{\hat{F}}\right) - \frac{3}{2}\mathrm{tr}\left(\nabla_a \hat{F}\nabla_b \hat{F}\nabla^a \not{\hat{F}}\nabla^b \not{\hat{F}}\right) - \frac{3}{32}\mathrm{tr}\left(\nabla_a \hat{F}\nabla^a \not{\hat{F}}\right)\mathrm{tr}\left(\nabla_b \hat{F}\nabla^b \not{\hat{F}}\right),$$

$$\begin{aligned}\hat{L} =& t_{8s}t_{8s}\hat{\mathbb{R}}^4 - \frac{9}{8}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{e}\hat{\Gamma}^{f}\right)\hat{R}_{abgh}\hat{R}_{cdij}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}^{ij}{}_{kl} + 3\mathrm{tr}\left(\nabla_a \not{\hat{F}}\nabla^a \not{\hat{F}}\nabla_b \hat{F}\nabla^b \hat{F}\right)\\ &-\frac{3}{2}\mathrm{tr}\left(\nabla_a \hat{F}\nabla_b \hat{F}\nabla^a \hat{F}\nabla^b \hat{F}\right) - \frac{3}{32}\mathrm{tr}\left(\nabla_a \hat{F}\nabla^a \hat{F}\right)\mathrm{tr}\left(\nabla_b \hat{F}\nabla^b \hat{F}\right) + \hat{L}',\end{aligned}$$

$$\hat{L}' = b\hat{R}_{abcd}\hat{R}_{efgh}\nabla^k\hat{F}^{abef}\nabla_k\hat{F}^{cdgh} + \sum_i c_i Y_i$$

$$\begin{aligned}Y_1 &= C_2 + 12C_7 + 4C_9 - 24C_{10} + 6C_{14} + 48C_{17} - 6C_{18} + 24C_{19} - 6C_{21} + 6C_{23}\\ Y_2 &= 3C_4 - 3C_5 + C_6 + 9C_7 - 6C_8 + 12C_{17} - 3C_{18}\\ Y_3 &= 3C_{10} + C_{11} - 3C_{13} - 3C_{15} - 6C_{17} + 6C_{20} + 3C_{21}\\ Y_4 &= C_4 + 3C_7 + C_9 - 6C_{15} + 12C_{19}\\ Y_5 &= C_1 + 4C_2 - 16C_3 - 16C_6 - 144C_7 + 144C_8\\ Y_6 &= C_2 - 6C_4 - 2C_9 + 6C_{11} + 36C_{12} - 18C_{13} + 18C_{15}\\ Y_7 &= C_2 - 12C_4 + 12C_5 + 4C_9\\ Y_8 &= C_9 - 9C_{15} - C_{22} + 9C_{23}\\ Y_9 &= C_{21} + C_{24}\\ Y_{10} &= C_{16} - 36C_{20} - 9C_{21} - C_{22}\end{aligned}$$

$$b = -48, c_2 = 12, c_3 = 24, c_5 = \frac{1}{3}, c_7 = -\frac{10}{3}, c_8 = \frac{68}{3}, c_9 = 12$$

$$\frac{9}{8}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{e}\hat{\Gamma}^{f}\right)\nabla_a\hat{F}_b^{ghm}\nabla_c\hat{F}_{dijm}\nabla_e\hat{F}_{ghkl}\nabla_f\hat{F}^{ijkl}$$
$$= 96\nabla_{[a}\hat{F}_{b]ecd}\nabla^{[a}\hat{F}^{f]egh}\nabla_{(b}\hat{F}^{cdij}\nabla_{f)}\hat{F}_{ghij} - 24\nabla_a\hat{F}_{becd}\nabla^{[a}\hat{F}^{b]egh}\nabla^{f}\hat{F}^{cdij}\nabla_f\hat{F}_{ghij}$$
$$= -16C_4 + 8C_5 - 24C_7 + 24C_8 - 48C_{12} - 24C_{13} + 12C_{14} - 12C_{18} - 48C_{19}$$
$$-36C_{21} - 60C_{23}$$
$$\frac{9}{8}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{e}\hat{\Gamma}^{f}\right)\nabla_m\hat{F}_{abgh}\nabla_m\hat{F}_{cdij}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}^{ij}{}_{kl}$$
$$= 96\nabla_a\hat{F}^{be}{}_{gh}\nabla^a\hat{F}_b{}^{fij}\nabla_{(e}\hat{F}^{ghkl}\nabla_{f)}\hat{F}_{ijkl} - 24\nabla_a\hat{F}_{efcd}\nabla^a\hat{F}^{efgh}\nabla^b\hat{F}^{cdij}\nabla_b\hat{F}_{ghij}$$
$$= -32C_4 + 32C_5 + 96C_7 - 96C_8 - 192C_{12} - 96C_{13} + 48C_{14} - 48C_{18} - 192C_{19}$$
$$+48C_{21} - 48C_{23}$$

$$t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{ef}\hat{\Gamma}^{gh}\right)\nabla_a\hat{F}_{bijk}\nabla_c\hat{F}_d^{ijk}\nabla_e\hat{F}_{flmn}\nabla_g\hat{F}_h^{lmn} = -\frac{9}{32}C_1 + C_2 + C_3 - 3C_4 - 9C_7$$
$$-4C_9 - 9C_{15} - 9C_{24}$$

$$\sum_i c_iY_i = \sum_i d_i'\tilde{Z}_i - \frac{3}{2}\hat{L}_2 - \frac{9}{8}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{e}\hat{\Gamma}^{f}\right)\nabla_a\hat{F}_{bghm}\nabla_c\hat{F}_d^{ijm}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}_{ijkl}$$
$$-\frac{9}{8}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{e}\hat{\Gamma}^{f}\right)\nabla_m\hat{F}_{abgh}\nabla_m\hat{F}_{cdij}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}^{ij}{}_{kl}$$
$$-\frac{4}{3}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{ef}\hat{\Gamma}^{gh}\right)\nabla_a\hat{F}_{bijk}\nabla_c\hat{F}_d^{ijk}\nabla_e\hat{F}_{flmn}\nabla_g\hat{F}_h^{lmn}$$
$$-15\nabla_a\hat{F}_{cdef}\nabla^a\hat{F}^{efgh}\nabla_b\hat{F}^{cdij}\nabla^b\hat{F}_{ghij} + \frac{9}{2}\nabla_a\hat{F}_{cdef}\nabla^a\hat{F}_{ghij}\nabla_b\hat{F}^{cdij}\nabla^b\hat{F}^{efgh}$$
$$+\frac{64}{3}\nabla_a\hat{F}_{bcde}\nabla^g\hat{F}^{hcde}\nabla^a\hat{F}^{bijk}\nabla_{(g}\hat{F}_{h)ijk} - 2\nabla_a\hat{F}_{bcde}\nabla^a\hat{F}^{fcde}\mathrm{tr}\left(\nabla^b\hat{F}\nabla_f\hat{F}\right)$$
$$+\frac{3}{32}\mathrm{tr}\left(\nabla_a\hat{F}\nabla^a\hat{F}\right)\mathrm{tr}\left(\nabla_b\hat{F}\nabla^b\hat{F}\right) - \frac{15}{32}\mathrm{tr}\left(\nabla_a\hat{F}\nabla_b\hat{F}\right)\mathrm{tr}\left(\nabla^a\hat{F}\nabla^b\hat{F}\right)$$

$$\hat{L} = t_{8s}t_{8s}\hat{\mathbb{R}}^4 - \frac{9}{8}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{e}\hat{\Gamma}^{f}\right)\hat{R}_{abgh}\hat{R}_{cdij}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}^{ij}{}_{kl} - 48\hat{R}_{abcd}\hat{R}_{efgh}\nabla^i\hat{F}^{abef}\nabla_i\hat{F}^{cdgh}$$
$$-\frac{9}{8}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{e}\hat{\Gamma}^{f}\right)\nabla_a\hat{F}_{bghm}\nabla_c\hat{F}_d^{ijm}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}_{ijkl}$$
$$-\frac{9}{8}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{e}\hat{\Gamma}^{f}\right)\nabla_m\hat{F}_{abgh}\nabla_m\hat{F}_{cdij}\nabla_e\hat{F}^{ghkl}\nabla_f\hat{F}^{ij}kl$$
$$-\frac{4}{3}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{ef}\hat{\Gamma}^{gh}\right)\nabla_a\hat{F}_{bijk}\nabla_c\hat{F}_d^{ijk}\nabla_e\hat{F}_{flmn}\nabla_g\hat{F}_h^{lmn}$$
$$-\frac{3}{2}\mathrm{tr}\left(\nabla_a\hat{F}\nabla_a\hat{F}\nabla_b\hat{F}\nabla_b\hat{F}\right) + \frac{3}{4}\mathrm{tr}\left(\nabla_a\hat{F}\nabla_b\hat{F}\nabla_a\hat{F}\nabla_b\hat{F}\right)$$
$$-\frac{15}{32}\mathrm{tr}\left(\nabla_a\hat{F}\nabla_b\hat{F}\right)\mathrm{tr}\left(\nabla^a\hat{F}\nabla^b\hat{F}\right) + \frac{9}{64}\mathrm{tr}\left(\nabla_a\hat{F}\nabla^a\hat{F}\right)\mathrm{tr}\left(\nabla_b\hat{F}\nabla^b\hat{F}\right)$$
$$-15\nabla_a\hat{F}_{cdef}\nabla^a\hat{F}^{efgh}\nabla_b\hat{F}^{cdij}\nabla^b\hat{F}_{ghij} + \frac{9}{2}\nabla_a\hat{F}_{cdef}\nabla^a\hat{F}_{ghij}\nabla_b\hat{F}^{cdij}\nabla^b\hat{F}^{efgh}$$
$$+\frac{64}{3}\nabla_a\hat{F}_{bcde}\nabla^g\hat{F}^{hcde}\nabla^a\hat{F}^{bijk}\nabla_{(g}\hat{F}_{h)ijk} - 2\nabla_a\hat{F}_{bcde}\nabla^a\hat{F}^{fcde}\mathrm{tr}\left(\nabla^b\hat{F}\nabla_f\hat{F}\right).$$

$$\mathbb{M}^{(n)} = \frac{1}{n!}M_{a_1\cdots a_n}\Gamma^{a_1\cdots a_n}, M^{(n')} = \frac{1}{n!}M_{a_1\cdots a_n}\Gamma^{a_1\cdots a_n}\Gamma_{11}$$

$$
\begin{aligned}
t_{8s}\left(M_1^{(1')}M_2^{(1')}M_3^{(1')}M_4^{(1')}\right) &= t_{8s}\left(M_1^{(1)}M_2^{(1)}M_3^{(1)}M_4^{(1)}\right)\\
t_{8s}\left(M_1^{(1)}M_2^{(1)}M_3^{(1')}M_4^{(1')}\right) &= -t_{8s}\left(M_1^{(1)}M_2^{(1)}M_3^{(1)}M_4^{(1)}\right) + 16M_1^aM_{2a}M_3^bM_{4b}\\
t_{8s}\left(M_1^{(2)}M_2^{(2)}M_3^{(1')}M_4^{(1')}\right) &= -t_{8s}\left(M_1^{(2)}M_2^{(2)}M_3^{(1)}M_4^{(1)}\right)\\
t_{8s}\left(M_1^{(2)}M_2^{(2)}M_3^{(3')}M_4^{(3')}\right) &= -t_{8s}\left(M_1^{(2)}M_2^{(2)}M_3^{(3)}M_4^{(3)}\right) + 8M_1^{ab}M_{2cd}M_{3abe}M_4^{cde}\\
t_{8s}\left(M_1^{(2')}M_2^{(2')}M_3^{(3')}M_4^{(3')}\right) &= -t_{8s}\left(M_1^{(2')}M_2^{(2')}M_3^{(3)}M_4^{(3)}\right)\\
t_{8s}\left(M_1^{(3')}M_2^{(3')}M_3^{(3')}M_4^{(3')}\right) &= t_{8s}\left(M_1^{(3)}M_2^{(3)}M_3^{(3)}M_4^{(3)}\right)\\
t_{8s}\left(M_1^{(3)}M_2^{(3)}M_3^{(3')}M_4^{(3')}\right) &= -t_{8s}\left(M_1^{(3)}M_2^{(3)}M_3^{(3)}M_4^{(3)}\right) - \frac{1}{2}\mathrm{tr}\left(M_1^{(3)}M_4^{(3)}M_2^{(3)}M_3^{(3)}\right)\\
&\quad + \frac{1}{64}\mathrm{tr}\left(M_1^{(3)}M_3^{(3)}\right)\mathrm{tr}\left(M_4^{(3)}M_2^{(3)}\right)\\
&\quad + \frac{1}{64}\mathrm{tr}\left(M_1^{(3)}M_4^{(3)}\right)\mathrm{tr}\left(M_2^{(3)}M_3^{(3)}\right)
\end{aligned}
$$

$$
t_{8s}\left(\Gamma^a\Gamma^b\Gamma^c\Gamma^d\right) = 24\eta^{a(b}\eta^{cd)}
$$

$$
\begin{aligned}
B_1 &= \hat{R}_{abcd}\hat{R}_{efgh}\nabla^e\hat{F}^{agh}{}_i\nabla^c\hat{F}^{bdfi}B_{13}\\
B_2 &= \hat{R}_{abcd}\hat{R}_{efgh}\nabla^e\hat{F}^{acg}{}_i\nabla^h\hat{R}^{bdfi\,a}{}_f{}^c\nabla_i\hat{F}^{bf}{}_{gh}\nabla^i\hat{F}^{degh}\\
B_3 &= \hat{R}_{abcd}\hat{R}_{efgh}\nabla^e\hat{F}^{acg}{}_i\nabla^f\hat{F}^{bdhi}\\
B_4 &= \hat{R}_{abcd}\hat{R}_{efgh}\nabla_i\hat{F}^{cdgh}\nabla^f\hat{F}^{iabe}\\
B_5 &= \hat{R}_{abcd}\hat{R}_{efg}{}^d\nabla^a\hat{F}^{bc}{}_{hi}\nabla^e\hat{F}^{fghi}\\
B_6 &= \hat{R}_{abcd}\hat{R}_{efg}{}^d\nabla^a\hat{F}^{be}{}_{hi}\nabla^c\hat{F}^{fghi}\\
B_7 &= \hat{R}_{abcd}\hat{R}_{efg}{}^d\nabla^a\hat{F}^{be}{}_{hi}\nabla^g\hat{F}^{cfhi}\\
B_8 &= \hat{R}_{abcd}\hat{R}_{efg}{}^d\nabla^aF^{ce}{}_{hi}\nabla^b\hat{F}^{fghi}\\
B_9 &= \hat{R}_{abcd}\hat{R}_{efg}{}^d\nabla^a\hat{F}^{ce}{}_{hi}\nabla^f\hat{F}^{bghi}\\
B_{10} &= \hat{R}_{abcd}\hat{R}_{efg}{}^d\nabla_i\hat{F}^{cegh}\nabla^i\hat{F}^{abfh}\\
B_{11} &= \hat{R}_{abcd}\hat{R}_{efg}{}^d\nabla_h\hat{F}^{abf}{}_i\nabla^i\hat{F}^{cegh}\\
B_{12} &= \hat{R}_{abcd}\hat{R}_{efg}{}^d\nabla^c\hat{F}^{ef}{}_{hi}\nabla^g\hat{F}^{bahi}
\end{aligned}
$$

$$
\begin{aligned}
B_{14} &= \hat{R}_{abcd}\hat{R}_e{}^a{}_f{}^c\nabla_i\hat{F}^{bd}{}_{gh}\nabla^i\hat{F}^{efgh}\\
B_{15} &= \hat{R}_{abcd}\hat{R}_e{}^a{}_f{}^c\nabla^b\hat{F}^f{}_{ghi}\nabla^e\hat{F}^{dghi}\\
B_{16} &= \hat{R}_{abcd}\hat{R}_e{}^a{}_f{}^c\nabla^b\hat{F}^d{}_{ghi}\nabla^e\hat{F}^{fghi}\\
B_{17} &= \hat{R}_{abcd}\hat{R}_e{}^a{}_f{}^c\nabla^bF^e{}_{ghi}\nabla^d\hat{F}^{fghi}\\
B_{18} &= \hat{R}_{abcd}\hat{R}_e{}^a{}_f{}^c\nabla_i\hat{F}^{ef}{}_{gh}\nabla^d\hat{F}^{bghi}\\
B_{19} &= \hat{R}_{abcd}\hat{R}_{ef}{}^{cd}\nabla_i\hat{F}^{ae}{}_{gh}\nabla^i\hat{F}^{bfgh}\\
B_{20} &= \hat{R}_{abcd}\hat{R}_{ef}{}^{cd}\nabla^a\hat{F}^e{}_{ghi}\nabla^b\hat{F}^{fghi}\\
B_{21} &= \hat{R}_{abcd}\hat{R}_{ef}{}^{cd}\nabla^a\hat{F}^e{}_{ghi}\nabla^f\hat{F}^{bghi}\\
B_{22} &= \hat{R}_{abcd}\hat{R}_e{}^{acd}\nabla^b\hat{F}_{fghi}\nabla^e\hat{F}^{fghi}\\
B_{23} &= \hat{R}_{abcd}\hat{R}_e{}^{acd}\nabla_i\hat{F}^b{}_{fgh}\nabla^i\hat{F}^{efgh}\\
B_{24} &= \hat{R}_{abcd}\hat{R}^{abcd}\nabla_e\hat{F}_{fghi}\nabla^f\hat{F}^{eghi}
\end{aligned}
$$

$$
\begin{aligned}
Z_1 &= 48B_1 + 48B_2 - 48B_3 + 36B_4 + 96B_6 + 48B_7 - 48B_8 + 96B_{10}\\
&\quad + 12B_{12} + 24B_{13} - 12B_{14} + 8B_{15} + 8B_{16} - 16B_{17} + 6B_{19} + 2B_{22} + B_{24},\\
Z_2 &= -48B_1 - 48B_2 - 24B_4 - 24B_5 + 48B_6 - 48B_8 - 24B_9 - 72B_{10}\\
&\quad - 24B_{13} + 24B_{14} - B_{22} + 4B_{23},\\
Z_3 &= 12B_1 + 12B_2 - 24B_3 + 9B_4 + 48B_6 + 24B_7 - 24B_8 + 24B_{10} + 6B_{12} + 6B_{13}\\
&\quad + 4B_{15} - 4B_{17} + 3B_{19} + 2B_{21},\\
Z_4 &= 12B_1 + 12B_2 - 12B_3 + 9B_4 + 24B_6 + 12B_7 - 12B_8 + 24B_{10} + 3B_{12}\\
&\quad + 6B_{13} + 4B_{15} - 4B_{17} + 2B_{20},\\
Z_5 &= 4B_3 - 8B_6 - 4B_7 + 4B_8 - B_{12} - 2B_{14} + 4B_{18},\\
Z_6 &= B_4 + 2B_{11}.
\end{aligned}
$$

$$C_1 = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bcde} \nabla_f \hat{F}_{ghij} \nabla^f \hat{F}^{ghij}$$
$$C_2 = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bcd}{}_f \nabla^e \hat{F}_{ghij} \nabla^f \hat{F}^{ghij}$$
$$C_3 = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bcd}{}_f \nabla_g \hat{F}^e{}_{hij} \nabla^g \hat{F}^{fhij}$$
$$C_4 = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bc}{}_{fg} \nabla^d \hat{F}^e{}_{hij} \nabla^f \hat{F}^{ghij}$$
$$C_5 = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla_i \hat{F}^{cde}{}_j \nabla^f \hat{F}^{ghij}$$
$$C_6 = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla_i \hat{F}^{cde}{}_j \nabla^j \hat{F}^{fghi}$$
$$C_7 = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{ac}{}_{fg} \nabla^d \hat{F}^e{}_{hij} \nabla^h \hat{F}^{fgij}$$
$$C_8 = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{ac}{}_{fg} \nabla_h \hat{F}^{de}{}_{ij} \nabla^i \hat{F}^{fghj}$$
$$C_9 = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}_{fghi} \nabla^f \hat{F}^{ghi}{}_j \nabla^j \hat{F}^{acde}$$
$$C_{10} = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bc}{}_{fg} \nabla_h \hat{F}^{df}{}_{ij} \nabla^i \hat{F}^{eghj}$$
$$C_{11} = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bc}{}_{fg} \nabla^d \hat{F}^f{}_{hij} \nabla^g \hat{F}^{ehij}$$
$$C_{12} = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla^c \hat{F}^{df}{}_{ij} \nabla^i \hat{F}^{eghj}$$

$$C_{13} = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla^c \hat{F}^{fg}{}_{ij} \nabla^d \hat{F}^{ehij}$$
$$C_{14} = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^b{}_{fgh} \nabla_i \hat{F}^{cdf}{}_j \nabla^i \hat{F}^{eghj}$$
$$C_{15} = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}_{fghi} \nabla^b \hat{F}^{cdf}{}_j \nabla^g \hat{F}^{ehij}$$
$$C_{16} = \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}_{fghi} \nabla^b \hat{F}^{fgh}{}_j \nabla^i \hat{F}^{cdej}$$
$$C_{17} = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{ac}{}_{fg} \nabla_h \hat{F}^{df}{}_{ij} \nabla^i \hat{F}^{eghj}$$
$$C_{18} = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^a{}_{fgh} \nabla_i \hat{F}^{cdf}{}_j \nabla^j \hat{F}^{eghi}$$
$$C_{19} = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^c{}_{fgh} \nabla^f \hat{F}^{dg}{}_{ij} \nabla^i \hat{F}^{aehj}$$
$$C_{20} = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^c{}_{fgh} \nabla^f \hat{F}^{ad}{}_{ij} \nabla^i \hat{F}^{eghj}$$
$$C_{21} = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^c{}_{fgh} \nabla^d \hat{F}^{fg}{}_{ij} \nabla^h \hat{F}^{aeij}$$
$$C_{22} = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}_{fghi} \nabla^f \hat{F}^{cde}{}_j \nabla^j \hat{F}^{aghi}$$
$$C_{23} = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}_{fghi} \nabla^f \hat{F}^{cdg}{}_j \nabla^j \hat{F}^{aehi}$$
$$C_{24} = \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}_{fghi} \nabla^f \hat{F}^{acd}{}_j \nabla^g \hat{F}^{ehij}$$

$$\tilde{Z}_1 = -C_3 + 12C_4 - 6C_5 + 72C_7 - 9C_8 - C_9 + 54C_{10} - 6C_{11} - 144C_{12} + 18C_{14} - 27C_{18} + 18C_{21},$$
$$\tilde{Z}_2 = C_3 - 6C_5 - 18C_7 + 9C_8 + C_9 + 6C_{11} + 9C_{18} + 18C_{23},$$
$$\tilde{Z}_3 = C_1 + 96C_4 - 96C_5 + 32C_6 + 288C_7 + 64C_9 + 32C_{22},$$
$$\tilde{Z}_4 = -C_{10} + 2C_{12} + 2C_{20},$$
$$\tilde{Z}_5 = C_7 + C_{10} + 4C_{19},$$
$$\tilde{Z}_6 = -C_7 - C_{10} + 2C_{17},$$
$$\tilde{Z}_7 = C_1 - 8C_2 + 32C_6 + 32C_9 + 32C_{16},$$
$$\tilde{Z}_8 = -C_2 - 12C_4 + 12C_5 - 4C_9 - 12C_{11} + 36C_{15},$$
$$\tilde{Z}_9 = C_{10} - 2C_{12} + C_{13}.$$

$$t_{8s} t_{8s} \hat{\mathcal{R}}^4 = t_{8s} t_{8s} \hat{R}^4 + 6 t_{8s} t_{8s} \hat{\mathcal{R}}^2 \hat{\mathcal{R}}'^2 + t_{8s} t_{8s} \hat{\mathcal{R}}'^4,$$

$$\hat{\mathbb{R}}' = \nabla_a \hat{F}_{bcde} \left(\frac{1}{12} \hat{\Gamma}^{ab} \otimes \hat{\Gamma}^{cde} + \frac{i}{48} \hat{\Gamma}^a \otimes \hat{\Gamma}^{bcde} \right)$$

$$t_{8s} t_{8s} \hat{R}^4 = t_8 t_8 \hat{R}^4$$

$$6t_{8s}t_{8s}\hat{R}^2\hat{R}'^2=\frac{1}{24}t_{a_1\cdots a_8}\hat{R}^{a_1a_2}{}_{ab}\hat{R}^{a_3a_4}{}_{cd}\nabla^{a_5}\hat{F}^{a_6}{}_{efg}\nabla^{a_7}\hat{F}^{a_8}{}_{hij}t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}\hat{\Gamma}^{efg}\hat{\Gamma}^{hij}\right)$$
$$-\frac{3}{32}\hat{R}^{ab}{}_{ef}\hat{R}^{cd}{}_{gh}t_{8s}\left(\hat{\Gamma}_{ab}\hat{\Gamma}_{cd}\hat{\Gamma}^{i}\hat{\Gamma}^{j}\right)t_{8s}\left(\hat{\Gamma}^{ef}\hat{\Gamma}^{gh}\nabla_i\hat{F}\nabla_j\hat{F}\right).$$

$$t_{8s}\left(\hat{\Gamma}^{ab}\hat{\Gamma}^{cd}M_1M_2\right)=-\frac{1}{4}\mathrm{tr}\left(\hat{\Gamma}^{abcd}M_1M_2\right)-\frac{1}{8}\mathrm{tr}\left(\hat{\Gamma}^{ab}M_1\hat{\Gamma}^{cd}M_2\right)+\frac{1}{64}\mathrm{tr}\left(\hat{\Gamma}^{ab}M_1\right)\mathrm{tr}\left(\hat{\Gamma}^{cd}M_2\right)$$
$$+\frac{1}{256}\mathrm{tr}\left(\hat{\Gamma}^{ab}\left\{M_1,\hat{\Gamma}^{e}\right\}\right)\mathrm{tr}\left(\left\{\hat{\Gamma}_e,M_2\right\}\hat{\Gamma}^{cd}\right)+(1\leftrightarrow 2)$$

$$6t_{8s}t_{8s}\hat{\mathcal{R}}^2\hat{\mathcal{R}}'^2=8(24B_1+24B_2-6B_4-48B_6-48B_8+24B_9-24B_{10}+24B_{11}$$
$$-18B_{12}-12B_{13}+8B_{16}+8B_{17}-8B_{20}+4B_{21}+B_{22}+4B_{23}+B_{24}),$$

$$16(-12B_5+3B_{12}+6B_{14})-4bB_4,$$

$$16(-24B_5-48B_8-24B_{10}-6B_{12}-12B_{13}+12B_{14}+8B_{16}-4B_{20}+B_{22}+4B_{23}+B_{24}).$$

$$t_{8s}t_{8s}\hat{\mathcal{R}}'^4=\frac{1}{6^4}t_8^{a_1a_2b_1b_2c_1c_2d_1d_2}\nabla_{a_1}\hat{F}_{a_2a_3a_4a_5}\nabla_{b_1}\hat{F}_{b_2b_3b_4b_5}\nabla_{c_1}\hat{F}_{c_2c_3c_4c_5}\nabla_{d_1}\hat{F}_{d_2d_3d_4d_5}$$
$$\times t_{8s}\left(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}^{b_3b_4b_5}\hat{\Gamma}^{c_3c_4c_5}\hat{\Gamma}^{d_3d_4d_5}\right)$$
$$+\frac{2}{3}\nabla_{[a}\hat{F}_{c]a_3a_4a_5}\nabla^{[a}\hat{F}^{d]b_3b_4b_5}t_{8s}\left(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}_{b_3b_4b_5}\nabla^{c}\hat{\mathcal{F}}\nabla_{d}\hat{\mathcal{F}}\right)$$
$$-\frac{1}{6}\nabla_{[a}\hat{F}_{b]a_3a_4a_5}\nabla^{a}\hat{F}^{bb_3b_4b_5}t_{8s}\left(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}_{b_3b_4b_5}\nabla_{c}\hat{\mathcal{F}}\nabla^{c}\hat{\mathcal{F}}\right)$$
$$+\frac{3}{2}t_{8s}\left(\nabla_a\hat{\mathcal{F}}\nabla^{a}\hat{\mathcal{F}}\nabla_b\hat{\mathcal{F}}\nabla^{b}\hat{\mathcal{F}}\right).$$

$$-36t_8^{a_1a_2b_1b_2c_1c_2d_1d_2}\nabla_{a_1}\hat{F}_{a_2cd}{}^{a}\nabla_{b_1}\hat{F}_{b_2efa}\nabla_{c_1}\hat{F}_{c_2}{}^{b[cd}\nabla_{d_1}\hat{F}_{d_2b}{}^{ef]}$$
$$+\frac{2}{3}t_8^{a_1a_2b_1b_2c_1c_2d_1d_2}\nabla_{a_1}\hat{F}_{a_2efg}\nabla_{b_1}\hat{F}_{b_2}{}^{efg}\nabla_{c_1}\hat{F}_{c_2hij}\nabla_{d_1}\hat{F}_{d_2}{}^{hij}$$
$$=-\frac{3}{16}C_1+\frac{4}{3}C_2+\frac{10}{3}C_3-2C_4-2C_5-6C_6+6C_7-18C_8-\frac{10}{3}C_9-72C_{10}$$
$$+12C_{11}-144C_{12}-48C_{13}+66C_{14}+60C_{15}+4C_{16}+84C_{17}-96C_{19}+48C_{20}$$
$$+18C_{21}+2C_{22}+18C_{23}+36C_{24}$$

$$\frac{2}{3}\nabla_{[a}\hat{F}_{c]a_3a_4a_5}\nabla^{[a}\hat{F}^{d]b_3b_4b_5}t_{8s}\left(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}_{b_3b_4b_5}\nabla^{c}\hat{\mathcal{F}}\nabla_{d}\hat{\mathcal{F}}\right)$$
$$=C_2-\frac{4}{3}C_3-4C_4+8C_5-4C_6-72C_7+72C_8-\frac{4}{3}C_9-24C_{10}+4C_{11}-96C_{12}-24C_{13}$$
$$-6C_{14}+36C_{15}+\frac{4}{3}C_{16}-30C_{18}-120C_{19}+48C_{20}-30C_{21}+\frac{8}{3}C_{22}-90C_{23}-24C_{24}$$

$$-\frac{1}{6}\nabla_{[a}\hat{F}_{b]a_3a_4a_5}\nabla^{a}\hat{F}^{bb_3b_4b_5}t_{8s}\left(\hat{\Gamma}^{a_3a_4a_5}\hat{\Gamma}_{b_3b_4b_5}\nabla_{c}\hat{\mathcal{F}}\nabla^{c}\hat{\mathcal{F}}\right)$$
$$=-\frac{1}{8}C_1+\frac{1}{3}C_2+\frac{8}{3}C_3-24C_4+4C_5+36C_7-36C_8+36C_{10}+6C_{14}-24C_{15}$$
$$-48C_{17}-6C_{18}-24C_{19}+6C_{21}-6C_{23}-24C_{24}$$

$$\frac{3}{2}t_{8s}\left(\nabla_a\hat{F}\nabla^a\hat{F}\nabla_b\hat{F}\nabla^b\hat{F}\right)$$
$$=-\frac{1}{48}C_1+\frac{2}{3}C_3-6C_4+2C_5+\frac{2}{3}C_6+18C_7-18C_8+\frac{2}{3}C_9+12C_{10}+6C_{14}$$
$$-12C_{15}-12C_{17}+18C_{21}-\frac{2}{3}C_{22}+18C_{23}$$

$$-\frac{1}{12}C_1+\frac{8}{3}C_3+8C_4+8C_5+\frac{8}{3}C_6-24C_7+24C_8-\frac{8}{3}C_9-144C_{10}+24C_{14}$$
$$+144C_{15}+144C_{17}-24C_{21}+8C_{22}-24C_{23}$$

$$\hat{L}_{\hat{F}^4}+\sum_i d_i\tilde{Z}_i=\frac{8}{3}(3C_5+C_6-9C_8+C_9-72C_{12}+9C_{14}+18C_{17}-9C_{18}-72C_{19}-C_{22})$$

$$c_2=12,c_3=24,c_5=\frac{1}{3},c_7=-\frac{10}{3},c_8=\frac{68}{3},c_9=12$$

$$d_1=-4,d_2=-\frac{20}{3},d_3=\frac{1}{4},d_4=-120,d_5=12,d_6=-60,d_7=-\frac{1}{6}$$
$$d_8=2,d_9=144$$

$$t_8t_8R^4$$

$$e^a=e^{k\Phi}e'^a$$

$$R_{abcd}=e^{-2k\Phi}(R'_{abcd}+D_{abcd}+\cdots)\;D^{ab}{}_{cd}=4k\delta^{[a}_{[c}\nabla_{d]}\nabla^{b]}\Phi$$

$$t_8t_8R^4=t_8t_8R^4+4t_8t_8DR^3+6t_8t_8D^2R^2+4t_8t_8D^3R'+t_8t_8D^4+\cdots$$

$$t_8t_8DR^{(1)}R^{(2)}R^{(3)},$$

$$4k\Phi t_8t_8\nabla^2\left(R^{(1)}R^{(2)}R^{(3)}\right)$$

$$\begin{aligned}t_8t_8\nabla^2\left(R^{(1)}R^{(2)}R^{(3)}\right)=&32\delta^e_{[a}\nabla_{b]}\nabla^f\left(R^{(1)bc}{}_{fg}R^{(2)gh}_{cd}R^{(3)da}{}_{he}\right)+64\delta^{[e}_{[a}\nabla_{b]}\nabla^{f]}\left(R^{(1)}_{cdfg}R^{(2)dagh}R^{(3)bc}{}_{he}\right)\\&-32\delta^e_{[a}\nabla_{b]}\nabla^f\left(R^{(1)bc}{}_{ef}R^{(2)}_{cdgh}R^{(3)dagh}\right)-16\delta^e_{[a}\nabla_{b]}\nabla^f\left(R^{(1)bcgh}R^{(2)}_{cdef}R^{(3)da}{}_{gh}\right)\\&+4\delta^e_a\nabla_b\nabla^f\left(R^{(1)abgh}R^{(2)}_{cdef}R^{(3)cd}{}_{gh}\right)+2\delta^e_a\nabla_b\nabla^f\left(R^{(1)ab}{}_{ef}R^{(2)}_{cdgh}R^{(3)cdgh}\right)\\&+\text{perm}(1,2,3).\end{aligned}$$

$$\begin{aligned}&32\delta^e_{[a}\nabla_{b]}\nabla^f\left(R^{(1)bc}_{fg}R^{(2)gh}_{cd}R^{(3)da}_{he}\right)+\text{perm}(1,2,3)\\&=16\nabla_b\nabla_f\left(R^{(1)bcfg}R^{(2)}_{cdgh}R^{(3)dh}\right)-16\nabla_a\nabla^f\left(R^{(1)bc}_{fg}R^{(2)gh}_{cd}R^{(3)da}_{hb}\right)+\text{perm}(1,2,3),\end{aligned}$$

$$\begin{aligned}&-16\nabla_a\nabla^f\left(R^{(1)bc}{}_{fg}R^{(2)gh}_{cd}R^{(3)da}{}_{hb}\right)+\text{perm}(1,2,3)\\&=8\nabla_a\left(R^{(1)bc}{}_{fg}\nabla^hR^{(2)fg}_{cd}R^{(3)da}{}_{hb}\right)+8\nabla_a\left(R^{(1)bc}{}_{fg}R^{(2)gh}_{cd}\nabla^aR^{(3)fd}{}_{hb}\right)+\text{perm}(1,2,3)+\cdots,\end{aligned}$$

$$8\nabla_a\nabla^h\left(R^{(1)bc}{}_{fg}R^{(2)fg}_{cd}R^{(3)da}{}_{hb}\right)+4\nabla^a\left(\nabla_f R^{(1)cd}{}_{gh}R^{(2)fbgh}R^{(3)}_{bacd}\right)$$
$$-8\nabla_a\left(R^{(1)cb}_{f}{}_{g}R^{(2)gh}_{cd}\nabla^a R^{(3)df}{}_{hb}\right)+2\nabla_a\left(R^{(1)}_{cdef}R^{(2)cdgh}\nabla^a R^{(3)ef}{}_{gh}\right)+\text{perm}(1,2,3)+\cdots$$
$$=8\nabla_a\nabla^h\left(R^{(1)bc}{}_{fg}R^{(2)fg}_{cd}R^{(3)da}{}_{hb}\right)+4\nabla^a\nabla_f\left(R^{(1)cd}{}_{gh}R^{(2)fbgh}R^{(3)}_{bacd}\right)$$
$$-\frac{8}{3}\nabla^2\left(R^{(1)cb}_{f}{}_{g}R^{(2)gh}_{cd}R^{(3)df}{}_{hb}\right)+\frac{4}{3}\nabla^2\left(R^{(1)}_{cdef}R^{(2)cdgh}R^{(3)ef}{}_{gh}\right)+\text{perm}(1,2,3)+\cdots$$

$$32\delta^e_{[a}\nabla_{b]}\nabla^f\left(R^{(1)bc}{}_{fg}R^{(2)gh}_{cd}R^{(3)da}{}_{he}\right)+\text{perm}(1,2,3)$$
$$=16\nabla_b\nabla_f\left(R^{(1)bcfg}R^{(2)}_{cdgh}R^{(3)dh}\right)+8\nabla_a\nabla^h\left(R^{(1)bc}{}_{fg}R^{(2)fg}_{cd}R^{(3)da}{}_{hb}\right)$$
$$+4\nabla^a\nabla_f\left(R^{(1)cd}{}_{gh}R^{(2)fbgh}R^{(3)}_{bacd}\right)-\frac{8}{3}\nabla^2\left(R^{(1)cb}_{f}{}_{g}R^{(2)gh}_{cd}R^{(3)df}{}_{hb}\right)$$
$$+\frac{4}{3}\nabla^2\left(R^{(1)}_{cdef}R^{(2)cdgh}R^{(3)ef}{}_{gh}\right)+\text{perm}(1,2,3)+\cdots$$

$$64\delta^{[e}_{[a}\nabla_{b]}\nabla^{f]}\left(R^{(1)}_{cdfg}R^{(2)dagh}R^{(3)bc}_{he}\right)$$
$$=16\nabla_b\nabla^f\left(R^{(1)}_{cdfg}R^{(2)dagh}R^{(3)bc}_{ha}\right)-16\nabla_b\nabla^e\left(R^{(1)}_{cdag}R^{(2)dagh}R^{(3)bc}{}_{he}\right)$$
$$+16\nabla_a\nabla^e\left(R^{(1)}_{cdbg}R^{(2)dagh}R^{(3)bc}{}_{he}\right)+16\nabla_a\nabla^f\left(R^{(1)}_{cdfg}R^{(2)dagh}R^{(3)c}{}_{h}\right)$$
$$=8\nabla_b\nabla^e\left(R^{(1)}_{cdag}R^{(2)dhag}R^{(3)bc}{}_{he}\right)+4\nabla_b\nabla^f\left(R^{(1)}_{cfdg}R^{(2)dgah}R^{(3)bc}{}_{ha}\right)$$
$$-4\nabla_a\nabla^e\left(R^{(1)}_{cdgh}R^{(2)ghab}R^{(3)cd}{}_{be}\right)+16\nabla_a\nabla^f\left(R^{(1)}_{cdfg}R^{(2)dagh}R^{(3)c}{}_{h)}\right.$$

$$t_8t_8\nabla^2\left(R^{(1)}R^{(2)}R^{(3)}\right)=16\nabla_b\nabla_f\left(R^{(1)dh}R^{(2)bcfg}R^{(3)}_{cdgh}\right)+16\nabla_b\nabla^f\left(R^{(1)d}{}_{h}R^{(2)}_{cdfg}R^{(3)bcgh}\right)$$
$$+16\nabla_b\nabla_f\left(R^{(1)cf}R^{(2)}_{cdgh}R^{(3)dbgh}\right)+2\nabla^b\nabla^f\left(R^{(1)}_{bf}R^{(2)}_{cdgh}R^{(3)cdgh}\right)$$
$$-\frac{8}{3}\nabla^2\left(R^{(1)cb}_{f}{}_{g}R^{(2)gh}_{cd}R^{(3)df}{}_{hb}\right)+\frac{4}{3}\nabla^2\left(R^{(1)}_{cdef}R^{(2)cdgh}R^{(3)ef}{}_{gh}\right)$$
$$+\text{perm}(1,2,3)+\cdots$$

$$t_8t_8DR^{(1)}R^{(2)}R^{(3)}=4k\Phi t_8t_8\nabla^2\left(R^{(1)}R^{(2)}R^{(3)}\right)+\cdots$$
$$=8k\nabla^a\nabla^b\Phi\left[8R^{(1)cd}R^{(2)}_{aghb}R^{(3)cghd}+8R^{(1)}_{cd}R^{(2)hc}_{ag}R^{(3)gd}_{bh}\right.$$
$$\left.-8R^{(1)}_{ac}R^{(2)cdgh}R^{(3)}_{bdgh}+R^{(1)}_{ab}R^{(2)}_{cdgh}R^{(3)cdgh}+\text{perm}(1,2,3)\right]+\cdots,$$

$$8k\nabla^a\nabla^b\Phi\left(8R^{(1)cd}D_{aghb}R^{(3)cghd}+8R^{(1)}_{cd}D^{hc}_{ag}R^{(3)gd}_{bh}-8R^{(1)}_{ac}D^{cdgh}R^{(3)}_{bdgh}+R^{(1)}_{ab}D_{cdgh}R^{(3)cdgh}\right)$$
$$=32k^2\nabla^a\nabla^b\Phi\left(6R^{(1)cd}\nabla_a\nabla^e\Phi R^{(3)}_{bcde}+3R^{(1)}_{ab}\nabla^c\nabla^d\Phi R^{(3)}_{cd}-2R^{(1)cd}\nabla_a\nabla_c\Phi R^{(3)}_{bd}\right)+\cdots$$
$$=32k^2\nabla^a\nabla^b\Phi\left(3R^{(1)}_{ab}\nabla^c\nabla^d\Phi R^{(3)}_{cd}-2R^{(1)cd}\nabla_a\nabla_c\Phi R^{(3)}_{bd}\right)+\cdots$$

$$R_{ef}=R'_{ef}+D_{ef}=l\nabla_e\nabla_f\Phi+\cdots,$$

$$D_{ef}=k(D-2)\nabla_e\nabla_f\Phi+\cdots,$$

$$t_8t_8R^4=t_8t_8R^4+96k(2l-k(D-2))\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi X_{abcd}+\cdots$$

$$X_{ab}^{cd} = 8R'_{aghb}R'^{cghd} + 8R'_{ag}{}^{hc}R'_{bh}{}^{gd} - 8\delta_a^c R'^{dfgh}R'_{bfgh} + \delta_a^c\delta_b^d R'_{efgh}R'^{efgh}$$
$$+2k(2l - k(D-2))[3\nabla_a\nabla_b\Phi\nabla^c\nabla^d\Phi - 2\nabla_a\nabla^c\Phi\nabla_b\nabla^d\Phi]$$

$$d'_1 = d'_2 = \frac{16}{3}, d'_3 = -\frac{1}{3}, d'_4 = 72, d'_5 = -60, d'_6 = 108, d'_7 = 0, d'_8 = -2, d'_9 = -192$$

$$\begin{aligned}\nabla^a\nabla^b\Phi\nabla^c\nabla^d\Phi\nabla_a\nabla^e\Phi R^{(3)}_{bcde} &= -\nabla^d\nabla^a\nabla^b\Phi\nabla^c\Phi\nabla_a\nabla^e\Phi R^{(3)}_{bcde} + \cdots \\ &= -\frac{1}{2}\nabla^d\nabla^a\nabla^b\Phi\nabla_a(\nabla^c\Phi\nabla^e\Phi)R^{(3)}_{bcde} + \cdots \\ &= -\frac{1}{4}\nabla^2\big(\nabla^d\nabla^b\Phi\nabla^c\Phi\nabla^e\Phi\big)R^{(3)}_{bcde} + \frac{1}{4}\nabla^d\nabla^b\Phi\nabla^2(\nabla^c\Phi\nabla^e\Phi)R^{(3)}_{bcde} + \cdots \\ &= -\frac{1}{2}\nabla^d\nabla^c\Phi\nabla^a\nabla^b\Phi\nabla_a\nabla^e\Phi R^{(3)}_{bcde} + \cdots,\end{aligned}$$

Supergravedad cuántica relativista en D=11.

$$\langle T_{\mu\nu}\rangle_{\mathcal{M}}.$$

$$G_{\mu\nu} = 8\pi G\big[T^{\mathrm{Cl}}_{\mu\nu} + \langle T^{\mu\nu}\rangle_{\mathcal{M}}\big]$$

$$T^{\mathrm{Cl}}_{\mu\nu} \sim \langle T^{\mu\nu}\rangle_{\mathcal{M}} \sim \epsilon M_P^D, \epsilon \ll 1.$$

$$g^{(0)} \to g^{(0)} + \epsilon g^{(1)} + \cdots, T_{\mu\nu} = \epsilon T^{(0)}_{\mu\nu} + \epsilon^2 T^{(1)}_{\mu\nu} + \cdots$$

$$G^{(n)}_{\mu\nu} = 8\pi G T^{(n-1)}_{\mu\nu},$$

$$\mathcal{M} = \mathbb{R}^{D-k} \times \mathcal{N}_k$$

$$ds^2_{\mathcal{M}} = ds^2_{\Lambda} + ds^2_{\mathcal{N}_k}$$

$$\langle T_{\mu\nu}\rangle_{\mathbb{R}^{D-k}\times\mathcal{N}_k} = \hat{\rho} g^{\mathbb{R}^{D-k},\Lambda}_{\mu\nu} + T^{\mathcal{N}_k}_{\mu\nu}$$

$$V_{\mathrm{Cas}} = \int d^k z\sqrt{-g_0}\hat{\rho}$$

$$V_{\mathrm{Cas}} \sim -\frac{\mathcal{C}}{R^d},$$

$$V^{(d)} = V_{\mathrm{Fluxes}} + V_{\mathrm{Cas}},$$

$$\frac{n_p^2}{R^{2p-k}}$$

$$\frac{V^{(d)}}{M_P^d} \sim \frac{1}{R^{\frac{d}{d-2}k}}\left(\sum_p \frac{n_p^2}{R^{2p-k}} - \frac{\mathcal{C}}{R^d}\right)$$

$(R,-R,0,0),(0,R,-R,0),(0,0,R,-R)$ and $(0,0,0,2R)$

$$\frac{V^{(7d)}}{M_7^7}=\left(\frac{\ell_{11}^9}{2R^4}\right)^{\frac{7}{5}}\left[\left(\frac{\pi}{2}\right)^{\frac{2}{3}}\frac{n_4^2}{2R^4\ell_{11}^3}-\frac{\mathcal{C}}{R^7}\right],\mathcal{C}\approx 8.827$$

$$M_7^5\equiv\frac{\mathrm{Vol}(T^4)}{\ell_{11}^9}=\frac{2R^4}{\ell_{11}^9}$$

$$R\approx\frac{2.58\ell_{11}}{n_4^{2/3}},\frac{V^{(7d)}}{M_7^7}\approx 6.85\cdot 10^{-6}n_4^{42/5}$$

$$\frac{1}{H_0}=\sqrt{\frac{15}{V^{(7d)}}}\approx 1480\text{ (7 d Planck lengths),}$$

$$\int|G_4|^{2n}$$

$$\int R^k_{\mu\nu\rho\sigma}|G_4|^{2n}$$

$$\mathcal{F}_k=\mathbb{R}^k/\mathcal{B}$$

$$b\in ISO(k):\vec{z}\to\mathbf{D}\vec{z}+\vec{b},\mathbf{D}\in SO(k)$$

$$T^k=\mathbb{R}^k/\Lambda$$

$$\Gamma\equiv\mathcal{B}/\sim,b\sim b'\text{ iff }b'\circ b^{-1}\in\Lambda.$$

$$0\longrightarrow\Lambda\longrightarrow\mathcal{B}\longrightarrow\Gamma\longrightarrow 1$$

$$\mathbf{D}_\gamma\vec{z}+\vec{b}_\gamma=\vec{z}+\vec{l},\vec{l}\in\Lambda,$$

$$\left(\mathbf{D}_\gamma-\mathbf{I}\right)\vec{z}+\vec{b}_\gamma=0\mathrm{mod}\Lambda$$

$$z\to-\bar{z}+\frac{i}{2},z\to z+\beta$$

$$b_1:\vec{z}\to\begin{pmatrix}-1&0\\0&1\end{pmatrix}\vec{z}+\begin{pmatrix}0\\1/2\end{pmatrix},b_2:\vec{z}\to\vec{z}+\begin{pmatrix}\beta\\0\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&-1&0\\0&0&1\end{pmatrix}\text{ or }\begin{pmatrix}1&0&1\\0&-1&0\\0&0&1\end{pmatrix}$$

$$\mathbf{D}_\gamma\sim\begin{pmatrix}\mathbf{B}_1&0&0&\vdots&\vdots&\vdots\\0&\ddots&0&\vec{v}_1&\cdots&\vec{v}_l\\0&0&\mathbf{B}_i&\vdots&\vdots&\vdots\\0&\cdots&0&\ddots&\vdots&\\\vdots&0&\vdots&\cdots&\mathbf{I}_{l\times l}&\cdots\\0&\cdots&0&&\vdots&\ddots\end{pmatrix}$$

$$\mathbf{D}_\gamma \vec{z} + \vec{b}_\gamma \to \mathbf{U} \cdot \mathbf{D}_\gamma \cdot \mathbf{U}^{-1}\vec{z} - \mathbf{U} \cdot \mathbf{D}_\gamma \cdot \mathbf{U}^{-1}\vec{u} + \mathbf{U}\vec{b}_\gamma + \vec{u}$$

$$\vec{b}_\gamma \to \mathbf{D}_\gamma^q \vec{b}_\gamma + \left(\mathbf{I} - \mathbf{D}_\gamma\right)\vec{u} = \mathbf{D}_\gamma^q \vec{b}_\gamma + \vec{u}_\perp$$

$$\vec{b}_\gamma = \mathbf{D}_\gamma \vec{b}_\gamma \bmod \Lambda_{\vec{u}_\perp}$$

$$\frac{k(k+1)}{2}$$

$$\mathbf{D}_\gamma^T \cdot \mathbf{G} \cdot \mathbf{D}_\gamma = \mathbf{G}, \forall \mathbf{D}_\gamma$$

$$\delta\mathbf{G} \cdot \mathbf{D}_\gamma = \left(\mathbf{D}_\gamma^T\right)^{-1} \cdot \delta\mathbf{G}$$

$$\mathcal{F}_k = T^{l-1} \times \left(\frac{T^{k-l+1} \times S^1}{\mathbb{Z}_n}\right)$$

$$\psi(\vec{z} + \vec{n}) = e^{2\pi i \vec{h} \cdot \vec{n}} \psi(\vec{z})$$

$$\psi(\vec{z}) \to \mathcal{D}_\mathbf{g} \psi\left(\mathbf{D}_\mathbf{g} \vec{z}\right)$$

$$\mathcal{D}_\mathbf{g} \psi\left(\mathbf{D}_\mathbf{g}(\vec{z} + \vec{n})\right) = e^{2\pi i \vec{h} \cdot \mathbf{D}_\mathbf{g} \vec{n}} \mathcal{D}_\mathbf{g} \psi\left(\mathbf{D}_\mathbf{g} \vec{z}\right)$$

$$\left(\mathbf{D}_g^T - \mathbf{I}\right) \cdot \vec{h} \in \mathbb{Z}^{k-l+1}$$

$$s_\mathbf{g} \equiv 2p\vec{h} \cdot \vec{b}_g \bmod 2\mathbb{Z}$$

$$S = \int \sqrt{-g}\mathcal{L} = \frac{1}{2}\int \sqrt{-g}\phi \cdot \mathcal{D} \cdot \phi = 0$$

$$\mathcal{D}\phi = 0$$

$$T_{\mu\nu} \equiv \frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \phi \cdot \left[\frac{\delta \mathcal{D}}{\delta g^{\mu\nu}} - \frac{1}{2} g_{\mu\nu}\mathcal{D}\right]\phi$$

$$\left\langle T_{\mu\nu}(x)\right\rangle = \lim_{y \to x}\left[\frac{\delta \mathcal{D}}{\delta g^{\mu\nu}} - \frac{1}{2} g_{\mu\nu}\mathcal{D}\right]\langle \phi(x)\phi(y)\rangle$$

$$T_{\mu\nu}^{sc} = \frac{1}{2}\left(2\nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu}\nabla^\rho \phi \nabla_\rho \phi\right),$$

$$\left\langle T_{\mu\nu}^{sc}\right\rangle \to \frac{1}{2}\lim_{x \to y}\left(\nabla_\mu \nabla'_\nu + \nabla'_\mu \nabla_\nu - g_{\mu\nu}\nabla^\rho \nabla'_\rho\right)G(x, y),$$

$$\hat{\rho} = -\langle T_{00}\rangle = -\sum_{\text{Polarizations}} (-1)^F \langle T_{00}^{sc}\rangle,$$

$$\mathcal{N}_k = \frac{\tilde{\mathcal{N}}_k}{\mathcal{B}}$$

$$G_{\mathcal{N}_k}(x,y) = \sum_{b\in\mathcal{B}} G_{\tilde{\mathcal{N}}_k}(x, b(y))$$

$$G_{\mathcal{N}_k}(b(x),y) = \sum_{b\in\mathcal{B}} G_{\tilde{\mathcal{N}}_k}(b(x), b(y)) = \sum_{b\in\mathcal{B}} G_{\tilde{\mathcal{N}}_k}(x, b^{-1}\circ b(y))$$

$$G^{\text{ren.}}_{\mathcal{N}_k}(x,y) = \sum_{b\neq \mathbf{I}\in\mathcal{B}} G_{\tilde{\mathcal{N}}_k}(x, b(y)),$$

$$G^{\text{ren.}}_{\mathcal{N}_k,\varphi}(x,y) = \sum_{b\neq \text{Id.}\in\mathcal{B}} e^{i\varphi(b)} G_{\tilde{\mathcal{N}}_k}(x, b(y)),$$

$$G^{\text{ren.}}_{\mathcal{N}_k,\varphi}(b_0(x), b_0(x)) = \sum_{b\neq \text{ Id. }\in\mathcal{B}} e^{i\varphi(b)} G_{\tilde{\mathcal{N}}_k}(b_0(x), b\circ b_0(y)) = \sum_{b\neq \text{ Id. }\in\mathcal{B}} e^{i\varphi(b)} G_{\tilde{\mathcal{N}}_k}(x, b_0^{-1}\circ b\circ b_0(y))$$

$$e^{i\varphi\left(b_0^{-1}\circ b\circ b_0(y)\right)} = e^{i\varphi(b)}, \forall b_0, b$$

$$G^{\text{ren.}ab}_{\mathcal{N}_k}(x,y) = \sum_{b\neq \mathbf{I}\in\mathcal{B}} R^a_c(b) G^{cb}_{\tilde{\mathcal{N}}_k}(x, b(y)),$$

$$1 \longrightarrow \mathcal{Z} \longrightarrow \hat{\mathcal{B}} \longrightarrow \mathcal{B} \longrightarrow 1$$

$$\phi^a(\vec{z}) \to R^a_c(b)\phi^c(b(\vec{z})) = R^a_c(b)\phi^c\left(\mathbf{D}_\gamma \vec{z} + \vec{b}_\gamma + \vec{n}\right)$$

$$\hat{\rho}(\vec{z}) = -\frac{\Gamma\left(\frac{D}{2}\right)}{2\pi^{D/2}} \sum_{\substack{\gamma\in\Gamma \\ \vec{n}\in\mathbb{Z}^k}}^{\sim} \frac{\text{Tr}(\mathbf{R})}{\left|\vec{z} - \left(\mathbf{D}_\gamma \vec{z} + \vec{b}_\gamma + \vec{n}\right)\right|^D}$$

$$SO(D-1,1) \to SO(D-k-1,1) \times SO(k)$$

$$\mathbf{r}_{SO(D-1,1)} \to \sum_i \mathbf{r}^i_{SO(D-k-1,1)} \otimes \tilde{\mathbf{r}}^i_{SO(k)}$$

$$\text{Tr}_{\mathbf{r}_{SO(D-1,1)}} = \sum_i g_{\mathbf{r}^i_{SO(D-k-1,1)}} \text{Tr}_{\tilde{\mathbf{r}}^i_{SO(k)}}$$

$$T_{\vec{n}} \to e^{2\pi i \vec{h}\cdot\vec{n}} \cdot \mathbf{I}$$

$$\hat{\rho}(\vec{z}) = -\frac{\Gamma\left(\frac{D}{2}\right)}{2\pi^{D/2}} \sum_{\substack{\gamma\in\Gamma \\ \vec{n}\in\mathbb{Z}^k}}^{\sim} \frac{\mathrm{Tr}_{\mathbf{r}}(\mathbf{D}_\gamma) e^{2\pi i\vec{h}\cdot\vec{n}}}{\left|\vec{z} - \left(\mathbf{D}_\gamma\vec{z} + \vec{b}_\gamma + \vec{n}\right)\right|^D}$$

$$V_{\mathrm{Cas}} = \int_{\mathcal{F}_k} d^k\vec{z}\sqrt{G}\hat{\rho}(\vec{z}) = -\frac{\Gamma(s)}{2\pi^s}\frac{1}{|\Gamma|}\sum_{\substack{\gamma\in\Gamma \\ \vec{n}\in\mathbb{Z}^k}}^{\sim} \int_{[0,1]^k} d^k\vec{z}\sqrt{G}\frac{\mathrm{Tr}_{\mathbf{r}}(\mathbf{D}_\gamma) e^{2\pi i\vec{h}\cdot\vec{n}}}{\left|\vec{z} - \left(\mathbf{D}_\gamma\vec{z} + \vec{b}_\gamma + \vec{n}\right)\right|^D}$$

$$\mathcal{E}(\gamma) \equiv -\frac{\Gamma(s)}{2\pi^s}\frac{1}{|\Gamma|}\sum_{\vec{n}\in\mathbb{Z}^k}^{\sim} \int_{[0,1]^k} dV \frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{\left|\left(\mathbf{I} - \mathbf{D}_\gamma\right)\vec{z} + \vec{b}_\gamma + \vec{n}\right|^{2s}}$$

$$dV = \sqrt{G}dz^1 \wedge \ldots \wedge dz^k$$

$$V_{\mathrm{Cas}} = \sum_{\mathbf{r}}\sum_{\gamma\in\Gamma} \mathrm{Tr}_{\mathbf{r}}(\mathbf{D}_\gamma)\mathcal{E}(\gamma)$$

$$M_s[f] \equiv \frac{1}{\Gamma(s)}\int_0^\infty \mathrm{d}t t^{s-1} f$$

$$\frac{1}{\left|\left(\mathbf{I} - \mathbf{D}_\gamma\right)\vec{z} - \vec{b}_\gamma + \vec{n}\right|^{2s}} = \frac{1}{\Gamma(s)}\int_0^\infty \mathrm{d}t t^{s-1} e^{-\left|\left(\mathbf{I}-\mathbf{D}_\gamma\right)\vec{z}-\vec{b}_\gamma+\vec{n}\right|^2 t}$$

$$\mathcal{E}(\gamma) = -\frac{\Gamma(s)}{2\pi^s}\cdot\frac{\sqrt{G}}{|\Gamma|}\frac{1}{\Gamma(s)}\int_0^\infty \mathrm{d}t t^{s-1}\int_{[0,1]^k}\mathrm{d}^k\vec{z}\sum_{\vec{n}\in\mathbb{Z}^k} e^{-\left|\left(\mathbf{I}-\mathbf{D}_\gamma\right)\vec{z}+\vec{b}_\gamma+\vec{n}\right|^2 t + 2\pi i\vec{h}\cdot\vec{n}}$$

$$\begin{aligned}-|\vec{n}+\vec{\alpha}|^2 t + 2\pi i\vec{h}\cdot\vec{n} =& -(\vec{n}+\vec{\alpha})^T(t\mathbf{G})(\vec{n}+\vec{\alpha}) + 2\pi i\vec{h}\cdot\vec{n} \\ =& -\left(\vec{n}+\vec{\alpha} - \frac{\pi i}{t}\mathbf{G}^{-1}\vec{h}\right)^T(t\mathbf{G})\left(\vec{n}+\vec{\alpha} - \frac{\pi i}{t}\mathbf{G}^{-1}\vec{h}\right) \\ & -2\pi i\vec{\alpha}\cdot\vec{h} - \frac{\pi^2}{t}\vec{h}^T\mathbf{G}^{-1}\vec{h}\end{aligned}$$

$$\sum_{\vec{n}\in\mathbb{Z}^k}{}' e^{-|\vec{\alpha}+\vec{n}|^2 t + 2\pi i\vec{h}\cdot\vec{n}} = e^{-2\pi i\vec{\alpha}\cdot\vec{h} - \frac{\pi^2}{t}\vec{h}^T\mathbf{G}^{-1}\vec{h}}\sum_{\vec{n}\in\mathbb{Z}^k} e^{-\left|\vec{n}+\vec{\alpha}-\frac{\pi i}{t}\mathbf{G}^{-1}\vec{h}\right|^2 t}$$

$$\sum_{\vec{n}\in\Lambda} f(\vec{n}) = \sum_{\vec{q}\in\Lambda^*}\mathcal{F}[f](\vec{q}),$$

$$\Lambda^* \equiv \{\vec{q}\in\mathbb{R}^k \mid \vec{n}\cdot\vec{q}\in\mathbb{Z}, \forall\vec{n}\in\Lambda\}$$

$$\mathcal{F}[f(\mathbf{A}\vec{z}+\vec{b})](\vec{\xi}) = \frac{e^{2\pi i\vec{b}\cdot\left(\mathbf{A}^{-1}\right)^T\vec{\xi}}}{|\det\mathbf{A}|}\mathcal{F}[f(\vec{z})]\left((\mathbf{A}^T)^{-1}\vec{\xi}\right)$$

$$\vec{v}^T(t\mathbf{G})\vec{v} = (\sqrt{t}\mathbf{M}\vec{v})^T(\sqrt{t}\mathbf{M}\vec{v})$$

$$\mathcal{F}[f((\sqrt{t}\mathbf{M})(\vec{n}+\vec{\sigma}))](\vec{\xi}) = \frac{e^{2\pi i\vec{\sigma}\cdot\vec{q}}}{|\det\sqrt{t}\mathbf{M}|}\cdot\mathcal{F}[f(\vec{n})]\left(\left(\sqrt{t}\mathbf{M}^T\right)^{-1}\vec{q}\right)$$

$$\mathcal{F}\left[e^{-\left|\vec{n}+\vec{\alpha}-\frac{\pi i}{t}\mathbf{G}^{-1}\vec{h}\right|^2 t}\right] = \frac{\pi^{\frac{k}{2}}}{t^{\frac{k}{2}}|\det\mathbf{M}|}\cdot e^{2\pi i\left(\vec{\alpha}-\frac{\pi i}{t}\mathbf{G}^{-1}\vec{h}\right)\cdot\vec{q}}\cdot e^{-\frac{\pi^2\vec{q}^T\mathbf{G}^{-1}\vec{q}}{t}}$$

$$= \frac{\pi^{\frac{k}{2}}}{t^{\frac{k}{2}}\sqrt{G}}\cdot e^{-\frac{\pi^2}{t}\left(\vec{q}^T\mathbf{G}^{-1}\vec{q}-2\mathbf{G}^{-1}\vec{h}\cdot\vec{q}\right)}\cdot e^{2\pi i\vec{\alpha}\cdot\vec{q}}$$

$$\mathcal{E}(\gamma) = -\frac{\Gamma(s)}{2\pi^s}\cdot\frac{\sqrt{G}}{|\Gamma|}\frac{\pi^{\frac{k}{2}}}{\Gamma(s)}\int_0^\infty \mathrm{d}t\,\frac{t^{s-\frac{k}{2}-1}}{\sqrt{G}}\sum_{\vec{q}\in\mathbb{Z}^k} e^{-\frac{\pi^2}{t}(\vec{q}-\vec{h})^T\mathbf{G}^{-1}(\vec{q}-\vec{h})+2\pi i\vec{b}_\gamma\cdot(\vec{q}-\vec{h})}\cdot\mathcal{I}_k(\mathbf{D}_\gamma)$$

$$\mathcal{I}_k(\mathbf{D}_\gamma) = \int_{[0,1]^k}\mathrm{d}^k\vec{z}\,e^{2\pi i[(\mathbf{I}-\mathbf{D}_\gamma)\vec{z}]\cdot(\vec{q}-\vec{h})} = \int_{[0,1]^k}\mathrm{d}^k\vec{z}\,e^{2\pi i\vec{z}\cdot\left[(\mathbf{I}-\mathbf{D}_\gamma)^T\cdot(\vec{q}-\vec{h})\right]}$$

$$\mathcal{I}_k(\mathbf{D}_\gamma) = \prod_{j=1}^{k}\frac{e^{2\pi i\left[(\mathbf{I}-\mathbf{D}_\gamma)^T\cdot(\vec{q}-\vec{h})\right]_j}-1}{\left[(\mathbf{I}-\mathbf{D}_\gamma)^T\cdot(\vec{q}-\vec{h})\right]_j}$$

$$(\mathbf{I}-\mathbf{D}_\gamma)^T\cdot\vec{h}\in\mathbb{Z}^k$$

$$\mathcal{I}_k(\mathbf{D}_\gamma) = \prod_{j=1}^{k}\delta_{0,\left[(\mathbf{I}-\mathbf{D}_\gamma)^T\cdot(\vec{q}-\vec{h})\right]_j}$$

$$(\mathbf{I}-\mathbf{D}_\gamma)^T(\vec{q}-\vec{h}) = 0$$

$$(\mathbf{I}-\mathbf{D}_\gamma)^T\vec{\eta} = (\mathbf{I}-\mathbf{D}_\gamma)^T\vec{h}$$

$$\mathcal{E}(\gamma) = -\hat{\delta}_{\vec{h}}\frac{\Gamma(s)}{2\pi^s}\cdot\frac{\sqrt{G}}{|\Gamma|}\frac{\pi^{\frac{k}{2}}}{\Gamma(s)}\int_0^\infty \mathrm{d}t\,\frac{t^{s-\frac{k}{2}-1}}{\sqrt{G}}\sum_{\vec{\kappa}\in\Lambda_\parallel} e^{-\frac{\pi^2}{t}(\vec{\kappa}-\vec{\beta})^T\mathbf{G}_\parallel^{-1}(\vec{\kappa}-\vec{\beta})+2\pi i\vec{b}_\gamma^\parallel\cdot(\vec{\kappa}-\vec{\beta})}$$

$$\mathcal{E}(\gamma) = -\hat{\delta}_{\vec{h}}\frac{\Gamma(s)}{2\pi^s}\cdot\frac{\sqrt{G}}{|\Gamma|}\frac{\pi^{\frac{k-k'}{2}}}{\Gamma(s)}\int_0^\infty \mathrm{d}t\,t^{s-\frac{k-k'}{2}-1}\frac{\sqrt{G_\parallel}}{\sqrt{G}}\sum_{\vec{\xi}\in\Xi} e^{-\left|\vec{\xi}+\vec{b}_\gamma^\parallel\right|_\parallel^2 t+2\pi i\vec{\beta}\cdot\vec{\xi}}$$

$$\left(\mathbf{D}_\gamma^T\vec{\kappa}\right)\cdot\vec{\xi} = \vec{\kappa}\cdot\left(\mathbf{D}_\gamma\vec{\xi}\right)$$

$$\Xi = \left(\Lambda^\parallel\right)^* \equiv \left\{\vec{\xi}\in\mathbb{R}^k \mid \vec{\xi}\cdot\vec{\kappa}\in\mathbb{Z}, \mathbf{D}_\gamma\vec{\xi} = \vec{\xi}, \forall\vec{\kappa}\in\Lambda^\parallel\right\}$$

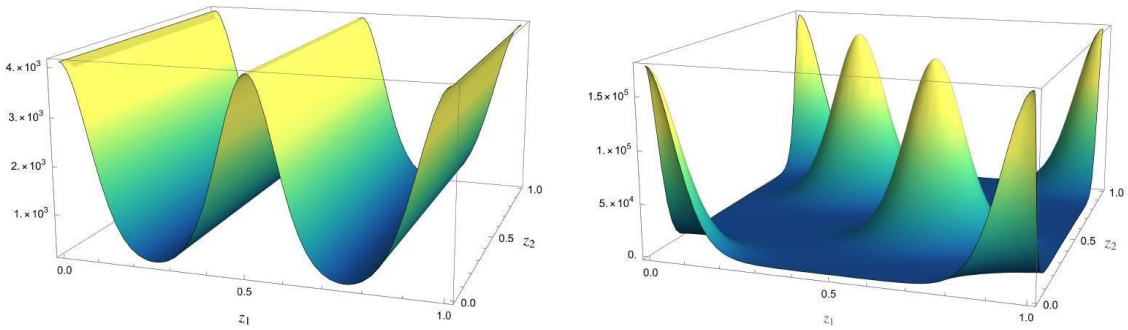
$$\vec{\xi} = \vec{\xi}^\perp + \vec{\xi}^{\mathrm{Inv}}$$

$$\vec{\kappa}\cdot\vec{\xi} = \vec{\kappa}\cdot\vec{\xi}^{\mathrm{Inv}}$$

$$V_{\text{Cas}} = \sum_{\mathbf{r}} \sum_{\gamma \in \Gamma} \text{Tr}_{\mathbf{r}}(\mathbf{D}_\gamma)\mathcal{E}(\gamma), \mathcal{E}(\gamma) = -\hat{\delta}_{\vec{h}} \frac{\Gamma(s_\gamma)}{2\pi^{s_\gamma}} \cdot \frac{\sqrt{G_\parallel}}{|\Gamma|} \sum_{\vec{\xi} \in \Xi_\gamma} \frac{e^{2\pi i \vec{\beta}_\gamma \cdot \vec{\xi}}}{\left|\vec{\xi} + \vec{b}_\gamma^{\parallel}\right|_\parallel^{2s_\gamma}},$$

$$\mathbf{D}_{\text{g}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \vec{b}_{\text{g}} = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$$

$$\mathbf{G} = R^2 \begin{pmatrix} x & 0 \\ 0 & x^{-1} \end{pmatrix}$$



(a) $T^2/\mathbb{Z}_2$ (Klein bottle) (b) $T^3/\mathbb{Z}_3$

Figura 1. Superespacios de Klein.

$$(\mathbf{I} - \mathbf{D}_{\mathbf{g}})\vec{z} = \begin{pmatrix} 2z_1 \\ 0 \end{pmatrix} \stackrel{!}{=} 0 \text{mod} \mathbb{Z}^2 \implies z_1 \in \left\{0, \frac{1}{2}\right\}, z_2 \in [0,1).$$

$$2h_1 \in \mathbb{Z}, s_g = 2h_2 \text{mod} 2\mathbb{Z}.$$

$$\mathcal{E}(\mathbf{g}) = -\frac{\Gamma(s_{\mathbf{g}})}{2\pi^{s_{\mathbf{g}}}} \cdot \frac{x^{s_{\mathbf{g}}-\frac{1}{2}}}{2R^{2s_{\mathbf{g}}-1}} \sum_{\xi \in \mathbb{Z}} \frac{e^{2\pi i h_2 \xi}}{\left|\xi + \frac{1}{2}\right|^{2s_{\mathbf{g}}}} = -\frac{\Gamma(s_{\mathbf{g}})}{2\pi^{s_{\mathbf{g}}}} \cdot \frac{x^{s_{\mathbf{g}}-\frac{1}{2}}}{R^{2s_{\mathbf{g}}-1}} (2^{2s_{\mathbf{g}}} - 1)\zeta(2s_{\mathbf{g}})\delta_{h_2,0}$$

$$\mathcal{E}(\mathbf{g}) = -\frac{124\pi^5}{945} \frac{x^{9/2}}{R^9} \approx -\frac{40.2x^{9/2}}{R^9}$$

$$\text{Tr}_{\mathbf{44}}(\mathbf{D}_{\mathbf{g}}) = 16, \text{Tr}_{\mathbf{84}}(\mathbf{D}_{\mathbf{g}}) = 0, \text{Tr}_{\mathbf{128}}(\mathbf{D}_{\mathbf{g}}) = 0$$

$$V_{\text{Cas}}^{(\mathbf{I})} = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{128x^{2s}}{2R^{2s-2}} \sum_{\vec{n} \in \mathbb{Z}^2} \frac{1 - e^{2\pi i \vec{h} \cdot \vec{n}}}{|x^2 n_1^2 + n_2^2|^{2s}}$$

$$V_{\text{Cas}}^{(\mathbf{I})} = 0, \text{ for } \vec{h} = (0,0)$$

$$V_{\text{Cas}}^{(\mathbf{I})} = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{128x^{2s}}{2R^{2s-2}} \cdot 4\beta(s)[\zeta(s) - \eta(s)] \approx -\frac{0.56}{R^9}, \text{ for } \vec{h} = \left(\frac{1}{2}, 0\right)$$

$$\mathbf{D_g} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \vec{b}_{\mathbf{g}} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{3} \end{pmatrix}.$$

$$\mathbf{G} = R^2 x^{4/3} \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & x^{-2} \end{pmatrix},$$

$$(\mathbf{I} - \mathbf{D_g})\vec{z} = \begin{pmatrix} z_1 + z_2 \\ -z_1 + 2z_2 \\ 0 \end{pmatrix} \stackrel{!}{=} 0 \mathrm{mod} \mathbb{Z}^3$$

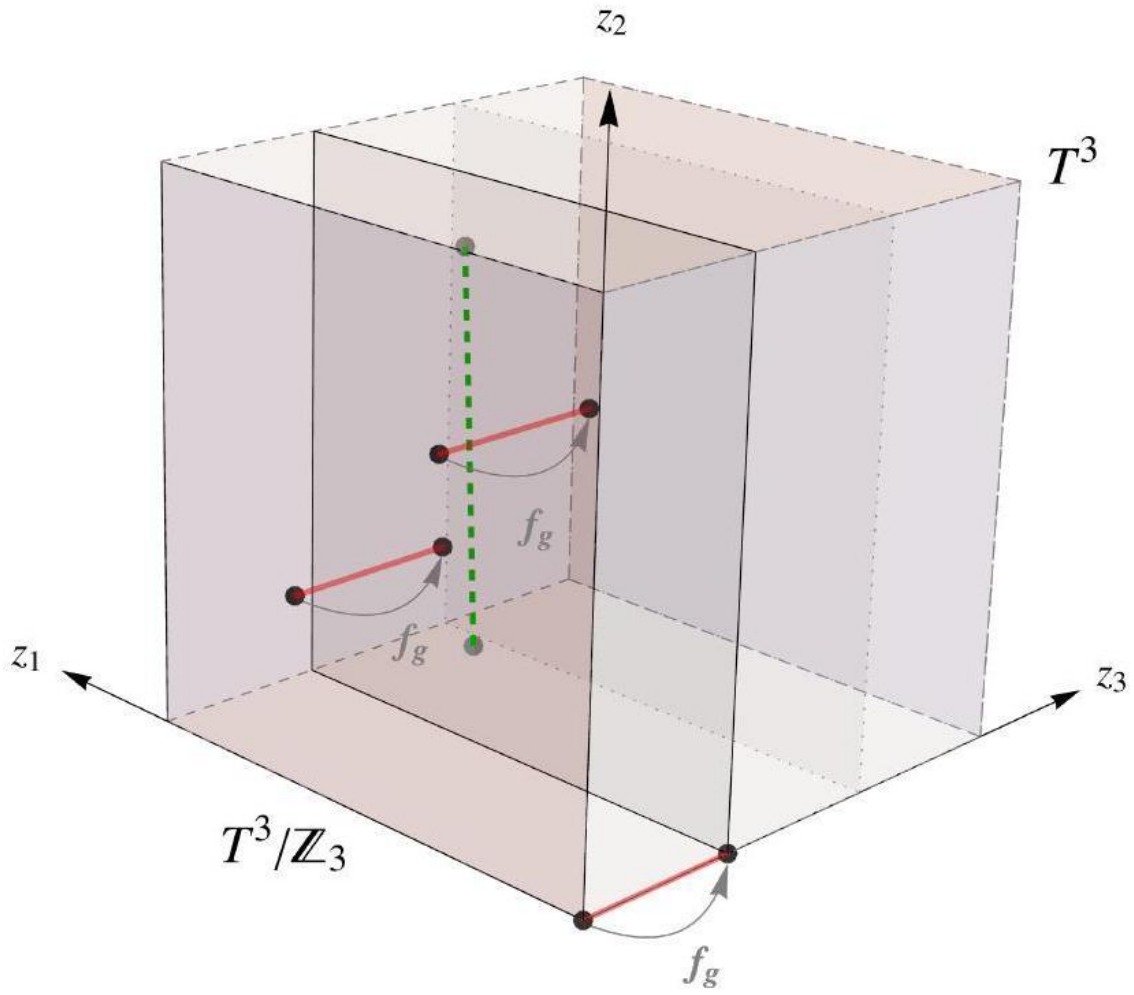


Figura 2. Multidimensiones.

$$\Longrightarrow (z_1, z_2) \in \left\{(0,0), \left(\frac{1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{1}{3}\right)\right\}, z_3 \in [0,1)$$

$$s_g = 2h_3 \mathrm{mod} 2\mathbb{Z}$$

$$\mathcal{E}(\mathbf{g}) = -\frac{\Gamma(s_{\mathbf{g}})}{2\pi^{s_{\mathbf{g}}}} \cdot \frac{x^{\frac{2s_{\mathbf{g}}-1}{3}}}{3R^{2s_{\mathbf{g}}-1}} \sum_{\xi \in \mathbb{Z}} \frac{e^{2\pi i h_3 \xi}}{\left|\xi + \frac{1}{3}\right|^{2s_{\mathbf{g}}}}$$

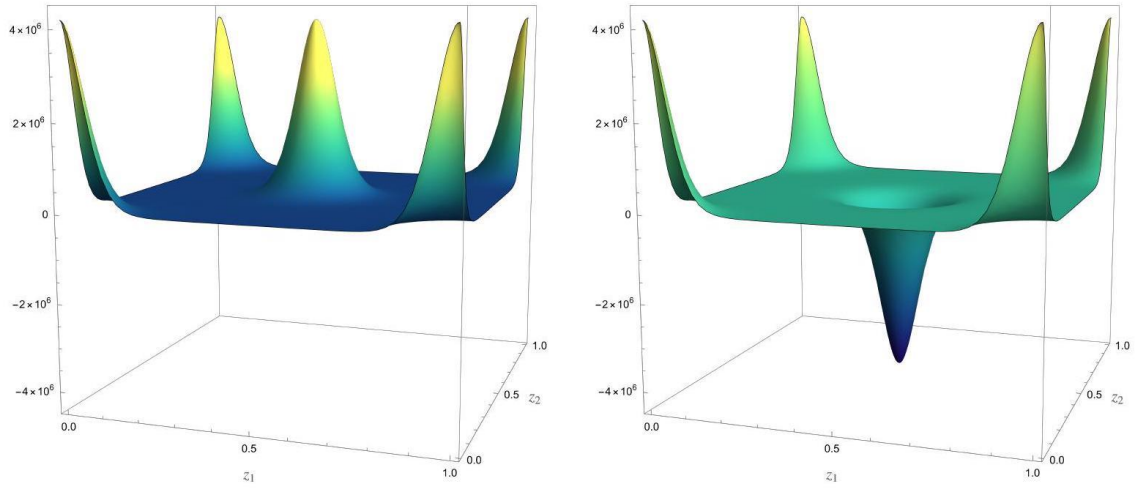
$$\mathcal{E}(\mathbf{g}) = -\frac{\Gamma(s_{\mathbf{g}})}{2\pi^{s_{\mathbf{g}}}} \cdot \frac{x^{\frac{2s_{\mathbf{g}}-1}{3}}}{3R^{2s_{\mathbf{g}}-1}} \left[\zeta\left(2s_{\mathbf{g}}, \frac{1}{3}\right) + \zeta\left(2s_{\mathbf{g}}, \frac{2}{3}\right)\right] \approx -\frac{221.4x^{\frac{8}{3}}}{R^8}$$

$$\mathcal{E}(\mathbf{g}) = -\frac{\Gamma(s_{\mathbf{g}})}{2\pi^{s_{\mathbf{g}}}} \cdot \frac{x^{s_{\mathbf{g}}-\frac{1}{3}}}{3R^{2s_{\mathbf{g}}-1}} \frac{1}{2^{2s}} \left[\zeta\left(2s_{\mathbf{g}}, \frac{1}{6}\right) - \zeta\left(2s_{\mathbf{g}}, \frac{1}{3}\right) - \zeta\left(2s_{\mathbf{g}}, \frac{2}{3}\right) + \zeta\left(2s_{\mathbf{g}}, \frac{5}{6}\right)\right]$$

$$\mathrm{Tr}_{\mathbf{44}}(\mathbf{D}_{\mathbf{g}}) = 20, \mathrm{Tr}_{\mathbf{84}}(\mathbf{D}_{\mathbf{g}}) = 21, \mathrm{Tr}_{\mathbf{128}}(\mathbf{D}_{\mathbf{g}}) = 40$$

$$\mathcal{E}(\mathbf{g}^j) = -\mathcal{E}(\mathbf{g}^{p-j})$$

$$\mathbf{D}_{\mathrm{g}} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \vec{b}_{\mathrm{g}} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \frac{1}{4} \end{pmatrix}$$



(a) Periodic $\vec{h} = (0, 0, 0)$. (b) Anti-periodic $\vec{h} = (\frac{1}{2}, \frac{1}{2}, 0)$.

Figura 3. Deformación intensa del espacio – tiempo cuántico.

$$s_{\mathbf{g}} = 2h_3 \mathrm{mod} 2\mathbb{Z}.$$

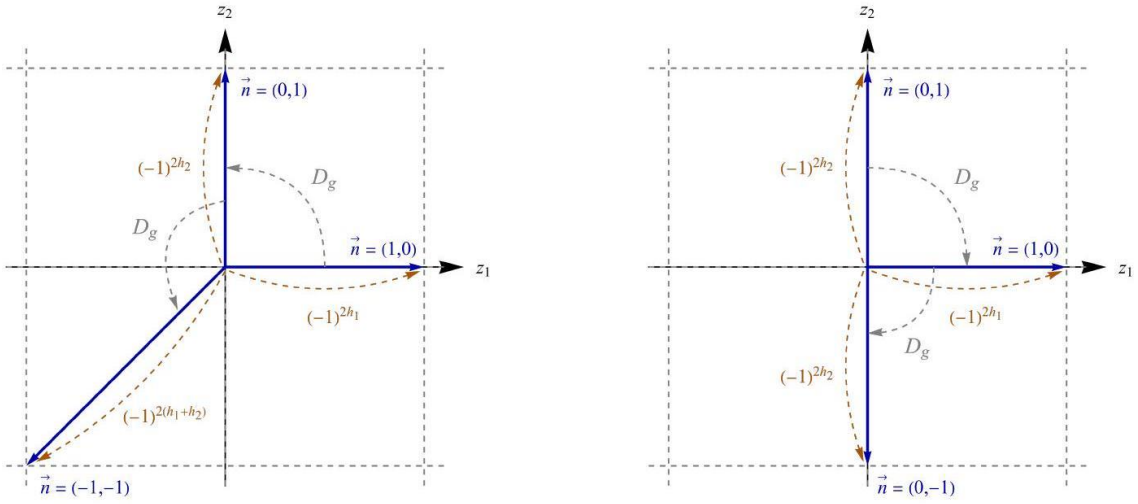


Figura 4. Difeomorfismos.

$$V_{\text{Cas}}^{\text{M-theory on } S^1/\mathbb{Z}_2} = -\frac{\mathcal{J}}{L^{10}}, \mathcal{J} = \frac{\Gamma\left(\frac{11}{2}\right)}{2\pi^{11/2}}\frac{2^{11}-1}{2^{13}}\zeta(11) \approx 1.206 \cdot 10^{-2}.$$

$$V_{\text{Cas}}^{\text{M-theory on } T^4} = -\frac{\Gamma\left(\frac{11}{2}\right)}{2\pi^{11/2}}\sqrt{G}\sum_{\vec{n}\in\mathbb{Z}^4}^{\sim}\frac{\text{Tr}_{\mathbf{44}}(\mathbf{I}) + \text{Tr}_{\mathbf{84}}(\mathbf{I}) - \text{Tr}_{\mathbf{128}}(\mathbf{I})e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}|^{11}},$$

$$\mathbf{G} = R^2\begin{pmatrix} 2 & -1 & 0 & 1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 1 & 0 & -1 & 2 \end{pmatrix}$$

$$V_{\text{Cas}}^{\text{M-theory on } T^4} = -128 \cdot \frac{945}{64\pi^5} \cdot 2R^4 \sum_{\vec{n}\in\mathbb{Z}^4}^{\sim}\frac{1-(-1)^{n_1+n_2+n_3+n_4}}{|\vec{n}|^{11}} \approx -\frac{8.827}{R^7},$$

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix}$$

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

$$\left(\mathbf{I} - \mathbf{D}_\gamma\right)\cdot\vec{e} \in \mathbb{Z}^k$$

$$e^{2\pi i\left[\left(\mathbf{I}-\mathbf{D_g}\right)^T\vec{h}\right]\cdot\vec{e}}$$

$$\vec{h}\cdot\left(\mathbf{I} - \mathbf{D}_\gamma\right)\vec{e} \in \mathbb{Z}, \text{ whenever } \left(\mathbf{I} - \mathbf{D}_\gamma\right)\vec{e} \in \mathbb{Z}^k$$

$$\left(\mathbf{I} - \mathbf{D}_\gamma\right)^T\vec{\eta} + \vec{\xi}_{\text{Inv}}, \vec{\eta} \in \mathbb{Z}^k, \mathbf{D}_\gamma^T\vec{\xi}_{\text{Inv}} = \vec{\xi}_{\text{Inv}}$$

$$V_{\text{Cas}} \sim \int_{\text{RFM}} \hat{\rho}(\vec{z}) dV$$

$$\hat{\rho}(\vec{z}+\vec{e}) = e^{i\varphi(\vec{e})}\hat{\rho}(\vec{z}),$$

$$V^{\text{1-loop}} \sim \int_{\mathcal{F}} \mathcal{Z}(\tau)\frac{d^2\tau}{\tau_2}$$

$$\mathcal{Z}\left(-\frac{1}{2\tau}\right) = e^{i\mathcal{A}}\mathcal{Z}(\tau)$$

$$\sum_{\vec{n}\neq-\vec{c}} \frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}.$$

$$\sum_{\vec{n}\neq-\vec{c}} \frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}} = \sum_{\vec{n}\neq-\vec{c}} \frac{F(\vec{n})e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}} + \sum_{\vec{n}} \frac{(1-F(\vec{n}))e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}},$$

$$\lim_{\vec{y}\to-\vec{c}} \frac{1-F(\vec{y})}{|\vec{y}+\vec{c}|^{2s}}\, F(\vec{y}) \to 0 \text{ as } |\vec{y}| \to \infty$$

$$\sum_{\vec{n}+\vec{c}\neq 0} \frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}} =$$

$$\sum_{\vec{n}+\vec{c}\neq 0} \frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}} \frac{\Gamma(s,\alpha|\vec{n}+\vec{c}|^2)}{\Gamma(s)} + \frac{\pi^{2s-\frac{k}{2}}}{\Gamma(s)\sqrt{G}} \sum_{\vec{k}-\vec{h}\neq 0} \Gamma\left(\frac{k}{2}-s, \frac{\pi^2|\vec{h}-\vec{k}|_D^2}{\alpha}\right) \frac{e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{c}}}{|\vec{h}-\vec{k}|_D^{k-2s}}$$

$$+\delta_{\vec{h},\vec{0}} \frac{\pi^{\frac{k}{2}}\alpha^{s-k/2}}{\sqrt{G}\Gamma(s)\left(s-\frac{k}{2}\right)} - \frac{\alpha^s e^{-2\pi i\vec{h}\cdot\vec{c}}}{\Gamma(s+1)} \chi_{\mathbb{Z}^k}(\vec{c})$$

$$\Gamma(s,z) \equiv \int_z^\infty t^{s-1}e^{-t}dt$$

$$\Gamma(s,z) \approx \Gamma(s) - \frac{z^s}{s} + \mathcal{O}(z^{s+1}), \quad |\vec{z}| \ll 1$$
$$\Gamma(s,z) \approx e^{-z}z^{s-1}[1+\mathcal{O}(z^{-1})] \quad |\vec{z}| \gg 1$$

$$\sum_{\text{KK spectrum}} m_{\text{KK}}^d,$$

$$V^{(5d)} = V_{\text{Cas}} + V_{G_4}$$

$$z_i \sim z_i + 1$$

$$\vec{z} \to \iota_{\mathbf{g}}(\vec{z}) = \mathbf{D}_{\mathbf{g}}\vec{z} + \vec{b}_{\mathbf{g}}, \text{ with } \mathbf{D}_{\mathbf{g}} \equiv \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \vec{b}_{\mathbf{g}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{8} \end{pmatrix}$$

$$\mathcal{F}_6 \equiv T^6/\mathbb{Z}_8$$

$$ds^2 = G_{ij}dz^i dz^j$$

$$\mathbf{G} = \begin{pmatrix} (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & (\gamma+1)R_2^2 & 0 & 0 \\ -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & 0 & 0 \\ 0 & -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & 0 \\ (\gamma+1)R_2^2 & 0 & -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta R_1^2 & \alpha R_1^2 \\ 0 & 0 & 0 & 0 & \alpha R_1^2 & \beta^{-1}R_1^2 \end{pmatrix}.$$

$$f\left(\iota_{\mathbf{g}}(\vec{z})\right) = \iota_{\mathbf{g}}^{n_1+1}(f(\vec{z})).$$

$$\mathbf{A} \cdot \mathbf{D}_{\mathbf{g}} \cdot \mathbf{A}^{-1} = \mathbf{D}_{\mathbf{g}}^{n_1}, n_1 \in \mathbb{Z}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\mathbf{G} \to \mathbf{A}^T \cdot \mathbf{G} \cdot \mathbf{A} = \begin{pmatrix} (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & (\gamma+1)R_2^2 & 0 & 0 \\ -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & 0 & 0 \\ 0 & -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & -(\gamma+1)R_2^2 & 0 & 0 \\ (\gamma+1)R_2^2 & 0 & -(\gamma+1)R_2^2 & (\gamma+2)R_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta^{-1}R_1^2 & -\alpha R_1^2 \\ 0 & 0 & 0 & 0 & -\alpha R_1^2 & \beta R_1^2 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{G} = R_1^2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} + R_2^2 \begin{pmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

$$e_1 \to e_1, e_2 \leftrightarrow e_4, e_3 \to -e_3$$

$$(0,0,0,0,0,0) \text{ or } \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0\right)$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{D_g}\vec{b}_\mathbf{g} = \vec{b}_\mathbf{g} \bmod \mathbb{Z}$$

$$\vec{b}_\mathbf{g} = \left(0,0,0,0,\frac{a}{8},\frac{b}{8}\right) \text{ or } \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{a}{8},\frac{b}{8}\right)$$

$$\text{Vol}(T^6) = 2R_2^4R_1^2$$

$$R^6 \equiv R_2^4R_1^2, x \equiv \frac{R_2}{R_1}$$

$$R_1 = \frac{R}{x^{2/3}}, R_2 = Rx^{1/3}$$

$$V_{\text{Cas}} = -\frac{\mathcal{C}(x)}{4R^5},$$

$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s} \cdot \int_{\mathcal{F}_6} d^6z\sqrt{G} \sum_{j=0}^{7} \sum_{\vec{n}\in\mathbb{Z}^6} \frac{\text{Tr}_\mathbf{B}\left(\mathbf{D}_\mathbf{g}^j\right) - \text{Tr}_\mathbf{F}\left(\mathbf{D}_\mathbf{g}^j\right) \cdot e^{2\pi i\vec{h}\cdot\vec{n}}}{\left|\left(\mathbf{I} - \mathbf{D}_\mathbf{g}^j\right)\vec{z} - j\vec{b}_\mathbf{g} + \vec{n}\right|^{2s}},$$

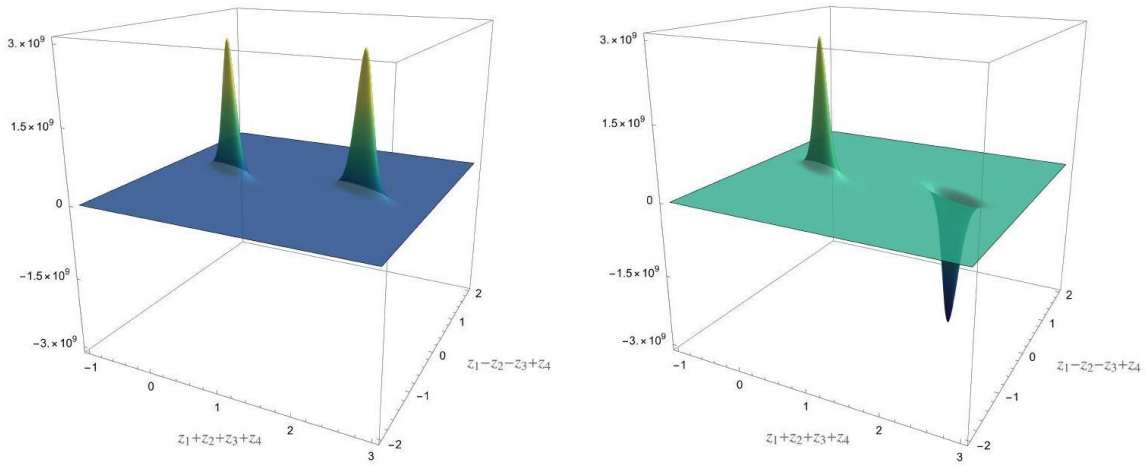


Figura 5. Torsión del espacio – tiempo cuántico.

$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s}\cdot\frac{2R_1^2R_2^4}{|\mathbb{Z}_8|}\cdot\sum_{j=0}^{7}\int_{T^6}d^4z\sum_{\substack{\vec{n}\in\mathbb{Z}^4\\ \vec{m}\in\mathbb{Z}^2}}\frac{\mathrm{Tr}_{\mathbf{B}}\left(\mathbf{D}_{\mathbf{g}}^j\right)-\mathrm{Tr}_{\mathbf{F}}\left(\mathbf{D}_{\mathbf{g}}^j\right)\cdot e^{2\pi i\vec{h}\cdot(\vec{n},\vec{m})}}{\left[R_2^2\left|\left(\mathbf{I}-\mathbf{D}_{\mathbf{g}}^j\right)\vec{z}+\vec{n}\right|^2+R_1^2\left|\vec{m}-j\vec{b}_{\mathbf{g}}\right|^2\right]^s}.$$

$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s}\cdot\frac{x^{\frac{4s}{3}}}{4\cdot R^{2s-6}}\cdot\sum_{j=0}^{7}\int_{T^6}d^4z\sum_{\substack{\vec{n}\in\mathbb{Z}^4\\ \vec{m}\in\mathbb{Z}^2}}\frac{\mathrm{Tr}_{\mathbf{B}}\left(\mathbf{D}_{\mathbf{g}}^j\right)-\mathrm{Tr}_{\mathbf{F}}\left(\mathbf{D}_{\mathbf{g}}^j\right)\cdot e^{2\pi i\vec{h}\cdot(\vec{n},\vec{m})}}{\left[x^2\left|\left(\mathbf{I}-\mathbf{D}_{\mathbf{g}}^j\right)\vec{z}+\vec{n}\right|^2+\left|\vec{m}-j\vec{b}_{\mathbf{g}}\right|^2\right]^s}$$

$$\mathcal{C}(x) = \frac{\Gamma(s)}{2\pi^s}\cdot x^{\frac{22}{3}}\sum_{j=0}^{7}\int_{T^6}d^4z\sum_{\substack{\vec{n}\in\mathbb{Z}^4\\ \vec{m}\in\mathbb{Z}^2}}\frac{\mathrm{Tr}_{\mathbf{B}}\left(\mathbf{D}_{\mathbf{g}}^j\right)-\mathrm{Tr}_{\mathbf{F}}\left(\mathbf{D}_{\mathbf{g}}^j\right)\cdot e^{\pi i(n_1+n_2+n_3+n_4)}}{\left[x^2\left|\left(\mathbf{I}-\mathbf{D}_{\mathbf{g}}^j\right)\vec{z}+\vec{n}\right|^2+\left|\vec{m}-j\vec{b}_{\mathbf{g}}\right|^2\right]^s}$$

$$\mathcal{C}_{\text{twisted}}(x) = \frac{\Gamma(s-2)}{2\pi^{s-2}}\cdot x^{\frac{10}{3}}\sum_{j=1}^{7}\sum_{\vec{m}\in\mathbb{Z}^2}\frac{\mathrm{Tr}_{\mathbf{B}}\left(\mathbf{D}_{\mathbf{g}}^j\right)}{\left[m_1^2+\left(m_2-\frac{j}{8}\right)^2\right]^{s-2}}$$

$$= 1.538\times 10^6\cdot x^{\frac{10}{3}}$$

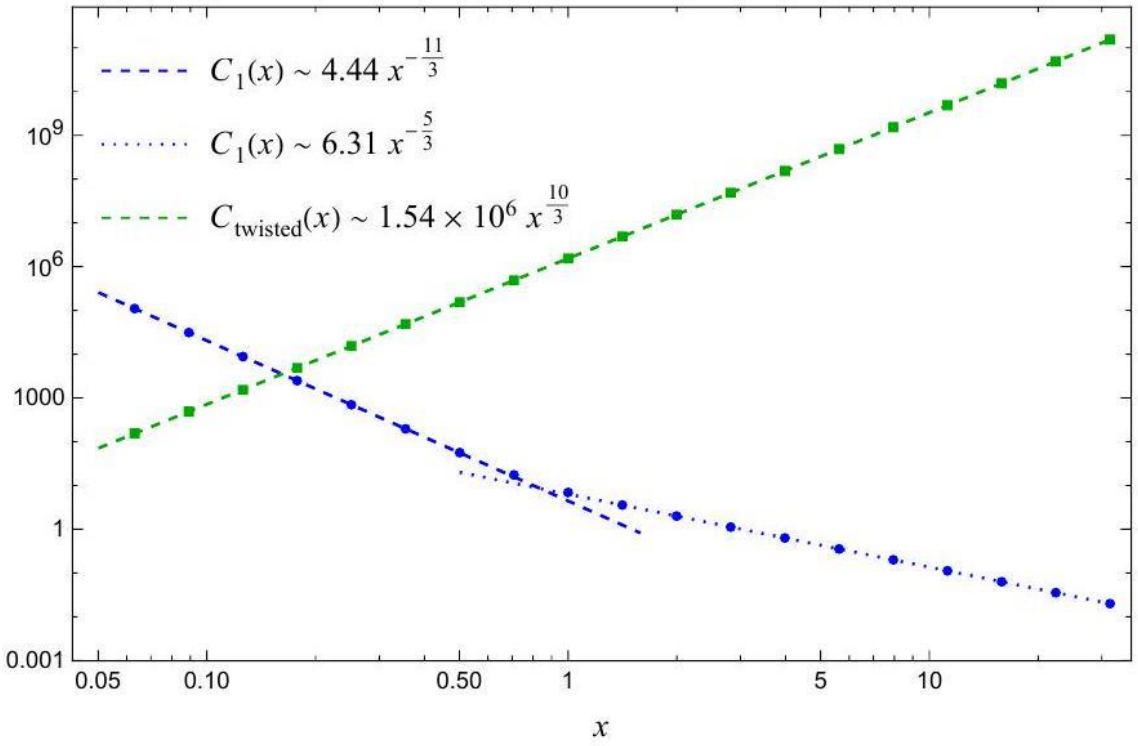
k	0	1	2	3	4	5	6	7
$\mathrm{Tr}_{\mathbf{44}}\left(\mathbf{D}_{\mathbf{g}}^j\right)$	44	14	12	14	4	14	12	14
$\mathrm{Tr}_{\mathbf{84}}\left(\mathbf{D}_{\mathbf{g}}^j\right)$	84	10	20	10	-4	10	20	10
$\mathrm{Tr}_{\mathbf{128}}\left(\mathbf{D}_{\mathbf{g}}^j\right)$	128	$16\sqrt{2}$	-32	$-16\sqrt{2}$	0	$-16\sqrt{2}$	-32	$16\sqrt{2}$

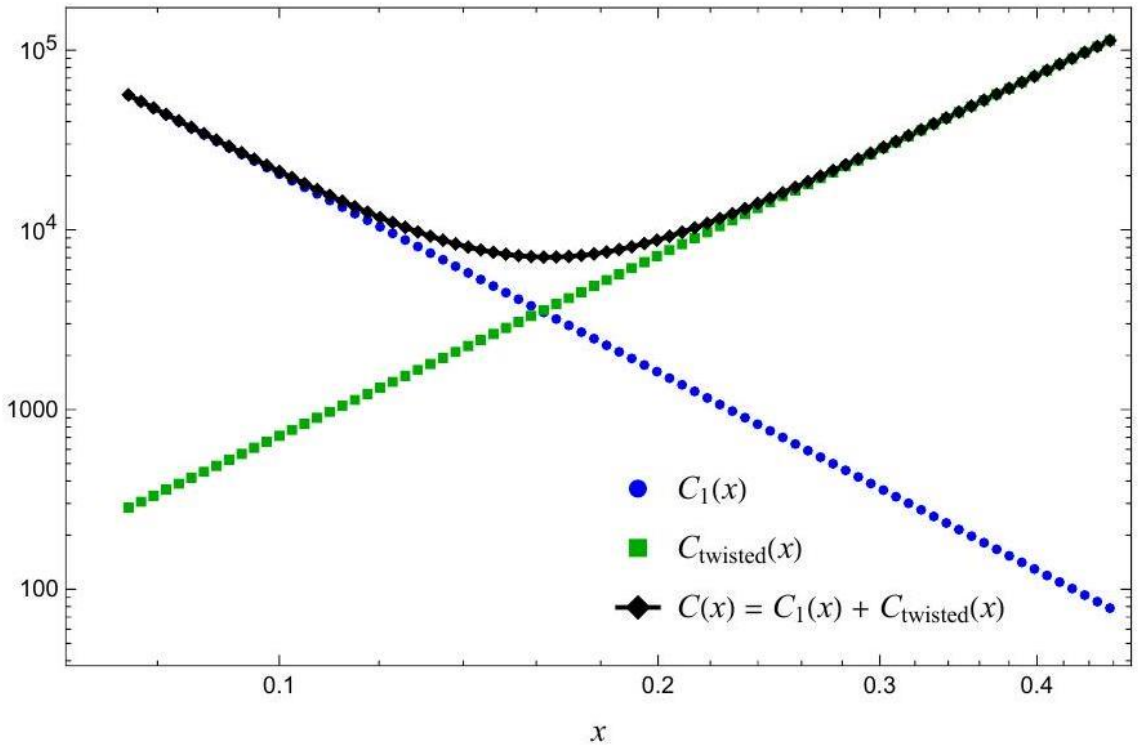
$$\mathcal{C}_{\mathbf{I}}(x) = \frac{\Gamma(s)}{2\pi^s}\cdot 128\cdot x^{\frac{22}{3}}\sum_{\substack{\vec{n}\in\mathbb{Z}^4\\ \vec{m}\in\mathbb{Z}^2}}\frac{1-e^{\pi i(n_1+n_2+n_3+n_4)}}{[x^2|\vec{n}|^2+|\vec{m}|^2]^s}$$

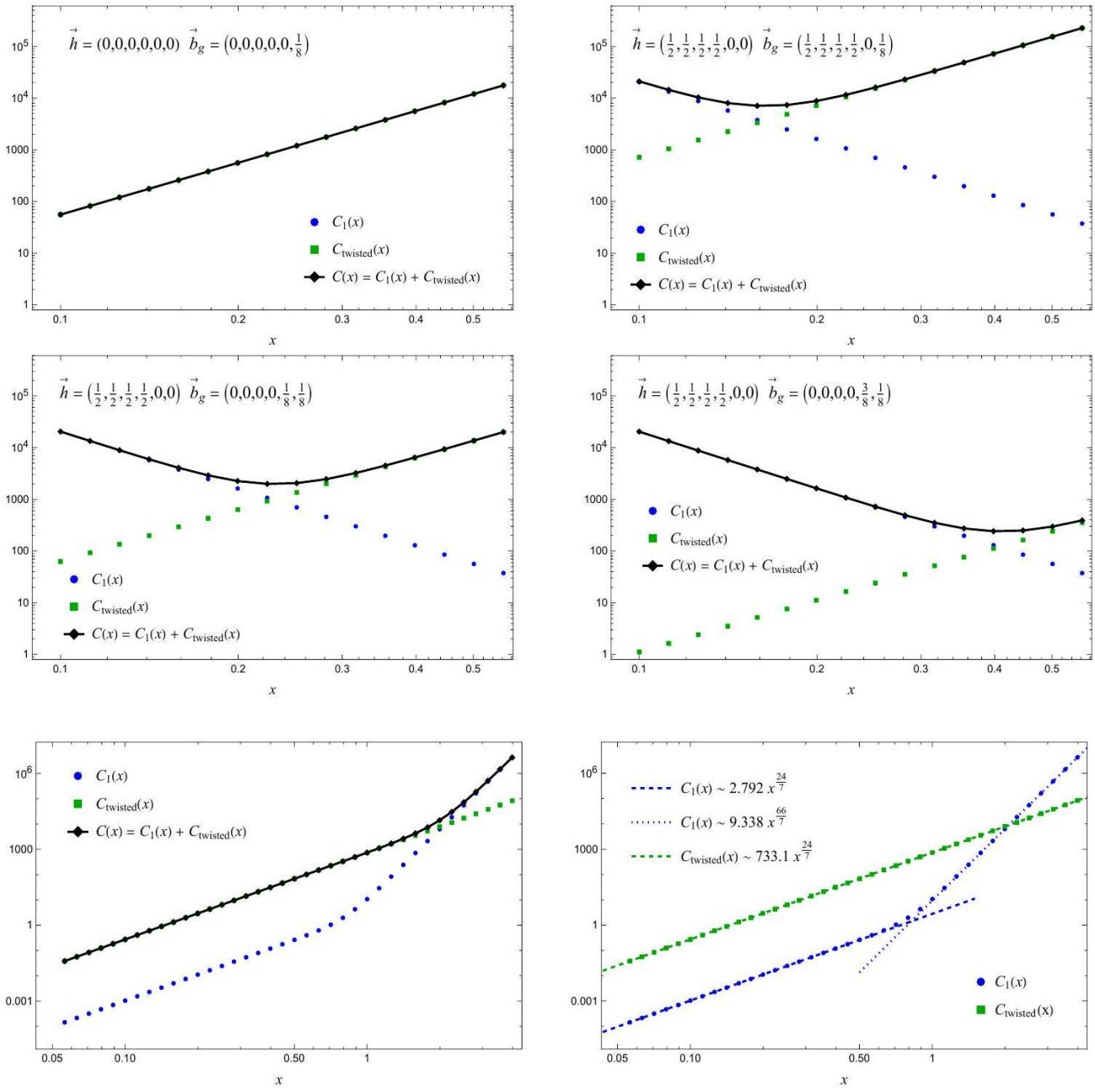
$$\mathcal{C}_{\mathbf{I}}(x)\sim 6.31\cdot x^{-5/3},\ \text{as}\ x\to\infty$$

$$\mathcal{C}_{\mathbf{I}}(x)\sim\frac{\Gamma(s)}{2\pi^s}\cdot 128\cdot x^{\frac{22}{3}-2s}\sum_{\vec{n}\in\mathbb{Z}^4}\frac{1-e^{\pi i(n_1+\cdots+n_4)}}{|\vec{n}|^{2s}}\approx 4.414\cdot x^{-\frac{11}{3}},\ \text{as}\ x\to 0$$

$$\mathcal{C}(x_{\min}=0.1637)\approx 7046.54.$$


$C_1(x) \sim 4.44\, x^{-\frac{11}{3}}$
$C_1(x) \sim 6.31\, x^{-\frac{5}{3}}$
$C_{\text{twisted}}(x) \sim 1.54 \times 10^6\, x^{\frac{10}{3}}$
10^9
10^6
1000
1
0.001
0.05
0.10
0.50
1
5
10
x


10^5
10^4
1000
100
$C_1(x)$
$C_{\text{twisted}}(x)$
$C(x) = C_1(x) + C_{\text{twisted}}(x)$
0.1
0.2
0.3
0.4
x



Figuras 6, 7, 8, 9 y 10. Paquetes de energía en dimensiones altas.

$$\int_{\mathcal{C}_4} G_4 \in \mathbb{Z}, \forall \mathcal{C}_4 \in H_4(\mathcal{F}_6, \mathbb{Z})$$

$$\int_{\mathcal{F}_6} \alpha \wedge G_4 \in \mathbb{Z}, \forall \alpha \in H^2(\mathcal{F}_6, \mathbb{Z})$$

$$G_4 = \sum_{i,j} n^i \mathcal{G}_{ij}^{-1} * \alpha_j, n^i \in \mathbb{Z}$$

$$\mathcal{G}_{ij} \equiv \int_{\mathcal{F}^6} \alpha_i \wedge * \, \alpha_j$$

$$\int_{\mathcal{F}^6} |G_4|^2 = \sum_{i,j} \mathcal{G}_{ij}^{-1} n^i n^j$$

$$\int_{\mathcal{F}^6} \alpha_i \wedge * \, \alpha_j = \frac{1}{8} \int_{T^6} p^*(\alpha_i) \wedge * \, p^*(\alpha_j)$$

$\omega_1 \equiv dz_1 \wedge dz_2 + dz_1 \wedge dz_4 + dz_2 \wedge dz_3 + dz_3 \wedge dz_4 - dz_1 \wedge dz_3 - dz_2 \wedge dz_4$
$\omega_2 \equiv dz_1 \wedge dz_2 + dz_1 \wedge dz_4 + dz_2 \wedge dz_3 + dz_3 \wedge dz_4 - 2dz_1 \wedge dz_3 - 2dz_2 \wedge dz_4$
$\omega_3 \equiv dz_5 \wedge dz_6$

$$\omega_1 \to \omega_1, \omega_2 \to -\omega_2, \omega_3 \to \omega_3$$

$$W(\ell) = \exp\left(2\pi i \int_\ell A\right)$$

$$\vec{n} \cdot \mathbf{F} \cdot \vec{m} \in \mathbb{Z}$$

$$\ell_1 = (1-t)\vec{z}_0 + t\left(\mathbf{D}_\mathbf{g}^j \vec{z}_0 + j\vec{b}_\mathbf{g}\right) \, t \in [0,1)$$

$$\left[\left(\mathbf{D}_\mathbf{g}^l \vec{z}_0 + l\vec{b}_\mathbf{g} + \vec{n}\right)\right] \cdot \mathbf{F} \cdot \left[\left(\mathbf{D}_\mathbf{g}^j - \mathbf{I}\right)\vec{z}_0 + j\vec{b}_\mathbf{g}\right]$$

$$\vec{n} \cdot \mathbf{F} \cdot \vec{b}_\mathbf{g} \in \mathbb{Z}$$

$$\kappa_1 = \frac{1}{R_2^4}, \kappa_2 = \frac{2}{R_2^4}, \kappa_3 = \frac{64}{R_1^4}$$

$$\int_{\mathcal{F}^6} |G_4|^2 = \frac{1}{R^2}\left(4n_1^2 x^{4/3} + \frac{n_3^2}{16x^{8/3}}\right)$$

$$V_{G_4} = \frac{(2\pi^2)^{\frac{1}{3}}}{2\ell_{11}^3} \int_{\mathcal{F}_6} |G_4|^2 = \frac{(2\pi^2)^{\frac{1}{3}}}{2R^2\ell_{11}^3}\left(4n_1^2 x^{4/3} + \frac{n_3^2}{16x^{8/3}}\right)$$

$$V^{(5d)} = \frac{2^{\frac{10}{3}}}{R^{10}}\left[\frac{(2\pi^2)^{\frac{1}{3}}}{2R^2}\left(4n_1^2 x^{4/3} + \frac{n_3^2}{16x^{8/3}}\right) - \frac{\mathcal{C}(x)}{4R^5}\right]$$

$$\begin{pmatrix} \frac{2(\gamma^2+2)}{(\gamma^2-2)^2 R_2^4} & -\frac{8\gamma}{(\gamma^2-2)^2 R_2^4} \\ -\frac{8\gamma}{(\gamma^2-2)^2 R_2^4} & \frac{4(\gamma^2+2)}{(\gamma^2-2)^2 R_2^4} \end{pmatrix}$$

$$R^3 = \frac{\mathcal{C}}{(2\pi^2)^{1/3}} \cdot \frac{10x^{8/3}}{64n_1^2 x^4 + n_3^2}$$
$$\frac{x\mathcal{C}'}{\mathcal{C}} = \frac{10}{3} \cdot \frac{32n_1^2 x^4 - n_3^2}{64n_1^2 x^4 + n_3^2}$$

$$\frac{x\mathcal{C}'(x)}{\mathcal{C}(x)} = -\frac{10}{3} \text{ and } \frac{x\mathcal{C}'(x)}{\mathcal{C}(x)} = \frac{5}{3}$$

$$R^3 \approx \frac{\mathcal{C}(x_{\max})}{(2\pi^2)^{\frac{1}{3}}} \frac{10}{96 n_1^2 x_{\max}^{4/3}} \approx \frac{3034}{n_1^2}.$$

n_1	n_3	R	x	$\frac{1}{8}R_1$	$\sqrt{2}R_2$	$\mathrm{vol}(\mathcal{F}_6)$	V_{saddle}
0	1	5.45	0.105	3.06	3.64	6.59×10^3	9.99×10^{-8}
1	0	16.2	0.191	6.11	13.18	4.50×10^6	3.68×10^{-15}
7	1	4.00	0.166	1.65	3.12	1.03×10^3	4.06×10^{-6}

Property	Value
5d vacuum energy	$V^{(5d)} \approx 3.68 \cdot 10^{-15} \ell_5^{-3}$
Hubble radius	$H_0^{-1} \sim 4.04 \cdot 10^7 \, \ell_5 = 2.45 \cdot 10^5 \, \ell_{11}$
Internal space volume	$\mathcal{V} = 4.50 \times 10^6 \, \ell_{11}^6 = 9.1 \cdot 10^{19} \, \ell_5^6$
Internal manifold radii	$\frac{1}{8} R_1 \sim 6.1 \, \ell_{11} = 1009 \, \ell_5$, $\sqrt{2} \, R_2 \sim 13.2 \, \ell_{11} = 2176 \, \ell_5$
Scale separation	Yes: $\frac{1}{8} R_1 = 2.50 \cdot 10^{-5} \, H_0^{-1}$
Geometric moduli	Five, with masses $m_i^2 =$ $\{90.45 \, , \, 23.82 \, , \, -309.55 \, , \, -47.43 \, , \, -45.16\} \cdot H_0^2$
Axions	Five, with negligible masses
Vectors	Seven: two KK photons, five from C_3
Light fermions	None
11d average energy density	$\rho_{11d} = 1.00 \cdot 10^{-10} \ell_{11}^{-11}$
Flux & Casimir contributions to ρ_{11d}	$\rho_{\text{Cas}} \approx -4.02 \cdot 10^{-10} \, \ell_{11}^{-11}$, $\rho_{G_4} \approx 5.02 \cdot 10^{-10} \, \ell_{11}^{-11}$
Internal gradients	$\langle \rho_{11d} \rangle^2 \sim 10^{-9}$

$$\frac{R_1}{8} \text{ and } \sqrt{2} R_2$$

$$\frac{1}{2\kappa_d^2} \int d^d x \sqrt{-g_d} \left[\left(\frac{D-2}{k(d-2)} \right) (d \log \sqrt{\det \mathbf{G}}) \right)^2 + \frac{1}{4} \mathrm{Tr}((\mathbf{G}^{-1} \partial \mathbf{G})^2) \right]$$

$$\{90.45, 23.82, -309.55, -47.43, -45.16\} \times H_0^2,$$

p	0	1	2	3	4	5	6
$H^p(\mathcal{F}_6, \mathbb{R})$	$\mathbb{R}$	$2\mathbb{R}$	$3\mathbb{R}$	$4\mathbb{R}$	$3\mathbb{R}$	$2\mathbb{R}$	$\mathbb{R}$

$$m^2 \gtrsim \frac{gq}{\sqrt{G}} H$$

$$\frac{g}{\sqrt{G}} \sim \frac{\mathrm{Vol}(\omega_2)}{\ell_{11}^3} \sim \frac{R^2}{\ell_{11}^3} \gg H$$

$$\frac{g}{\sqrt{G}} \sim \frac{\mathrm{Vol}(\omega_5)}{\ell_{11}^6} \sim \frac{R^5}{\ell_{11}^6} \gg H$$

$$H_0^{-1} = \sqrt{\frac{6}{V_{\mathrm{Saddle}}}} \approx 4.04 \cdot 10^7 \ell_5$$

$$\frac{\ell_{11}}{\ell_5} = \frac{R^2}{4^{1/3}} \approx 165.14$$

$$\rho_{11d} = 1.00 \cdot 10^{-10} \ell_{11}^{-11}$$

$$\rho_{\mathrm{Cas}} \approx -4.02 \cdot 10^{-10} \ell_{11}^{-11}\, \rho_{G_4} \approx 5.02 \cdot 10^{-10} \ell_{11}^{-11}.$$

$$\int P\left(R_{\mu\nu\rho\sigma}, |G_4|\right)$$

$$\hat{\rho}(0) \approx 1.28 \cdot 10^{-9} \ell_{11}^{-11}$$

$$V_{\mathrm{Cas}} \sim \frac{1}{4R^5}\left[\mathcal{C} + \sum_{l=1}^{\infty} \mathcal{C}_i \left(\frac{\ell_{11}}{R}\right)^{9l}\right]$$

$$\left(\frac{\ell_{11}}{R}\right)^9 \sim 1.31 \cdot 10^{-11}$$

$$\mathcal{C}_i \gtrsim 10^{6+11l}$$

$$e^{-S_{\mathrm{M2}}} \sim e^{-T_{\mathrm{M2}}\left(\frac{R_1 R_2^2}{4}\right)} \approx e^{-716.9}, e^{-S_{\mathrm{M5}}} \sim e^{-T_{\mathrm{M5}}\frac{R^6}{4}} \approx e^{-5.24 \cdot 10^6}$$

$$\delta V^{(5d)} = \frac{2^{\frac{3}{10}}}{R^{10}} \frac{R^6}{4}\left[\sum_{n=1}^{\infty} \frac{(2\mu^{11}) K_{\frac{11}{2}}(\mu n R_1)}{(2\pi)^{\frac{11}{2}} (\mu n R_1)^{\frac{11}{2}}}\right]$$

$$\sqrt{T} \sim (2\pi^2)^{1/3} \frac{R_1}{8} \approx 16.48$$

$$\mathcal{F}_6 = T^6/\mathbb{Z}_8$$

$$\mathcal{F}_7 = \frac{T^7}{\Gamma}$$

$$\left\{\mathbf{g}: \mathbf{D_g} = \begin{pmatrix} \mathbf{A} & 0 \\ 0 & 1 \end{pmatrix}, \vec{b}_{\mathbf{g}} = \begin{pmatrix} \vec{0} \\ |\mathbf{A}|^{-1} \end{pmatrix}\right\},$$

$$\mathbf{D_g} = \left(\begin{array}{cccccc|c} 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right), \quad \vec{b}_{\mathbf{g}} = \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{7} \end{array}\right)$$

$$\mathbf{D_g} = \left(\begin{array}{ccc|ccc|c} 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right), \quad \vec{b}_{\mathbf{g}} = \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{30} \end{array}\right)$$

$$\mathbf{G} = \left(\begin{array}{cccccc|c} 2\,R_2^2 & \gamma - R_2^2 & \alpha & -\alpha-\gamma & -\alpha-\gamma & \alpha & 0 \\ \gamma - R_2^2 & 2\,R_2^2 & \gamma - R_2^2 & \alpha & -\alpha-\gamma & -\alpha-\gamma & 0 \\ \alpha & \gamma - R_2^2 & 2\,R_2^2 & \gamma - R_2^2 & \alpha & -\alpha-\gamma & 0 \\ -\alpha-\gamma & \alpha & \gamma - R_2^2 & 2\,R_2^2 & \gamma - R_2^2 & \alpha & 0 \\ -\alpha-\gamma & -\alpha-\gamma & \alpha & \gamma - R_2^2 & 2\,R_2^2 & \gamma - R_2^2 & 0 \\ \alpha & -\alpha-\gamma & -\alpha-\gamma & \alpha & \gamma - R_2^2 & 2\,R_2^2 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & R_1^2 \end{array}\right)$$

$$\mathbf{Q} = \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\mathbf{G} = R_1^2 \left(\begin{array}{cccccc|c} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) + R_2^2 \left(\begin{array}{cccccc|c} 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s} \cdot \int_{\mathcal{F}_7} d^7 z \sqrt{G} \sum_{j=0}^{6} \sum_{\vec{n} \in \mathbb{Z}^7} \frac{\text{Tr}_{\mathbf{B}}\left(\mathbf{D}_{\mathbf{h}}^j\right) - \text{Tr}_{\mathbf{F}}\left(\mathbf{D}_{\mathbf{g}}^j\right) \cdot e^{2\pi i \vec{h} \cdot \vec{n}}}{\left|\left(\mathbf{I} - \mathbf{D}_{\mathbf{g}}^j\right)\vec{z} - j\vec{b}_{\mathbf{g}} + \vec{n}\right|^{2s}}$$

$$(0,0,0,0,0,0), \left(\frac{6}{7}, \frac{5}{7}, \frac{4}{7}, \frac{3}{7}, \frac{2}{7}, \frac{1}{7}\right), \left(\frac{5}{7}, \frac{3}{7}, \frac{1}{7}, \frac{6}{7}, \frac{4}{7}, \frac{2}{7}\right), \left(\frac{4}{7}, \frac{1}{7}, \frac{5}{7}, \frac{2}{7}, \frac{6}{7}, \frac{3}{7}\right)$$
$$\left(\frac{3}{7}, \frac{6}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}, \frac{4}{7}\right), \left(\frac{2}{7}, \frac{4}{7}, \frac{6}{7}, \frac{1}{7}, \frac{3}{7}, \frac{5}{7}\right), \left(\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}\right)$$

$$V_{\text{Cas}} = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{\sqrt{7} R_1 R_2^6}{|\mathbb{Z}_7|} \cdot \sum_{j=0}^{6} \int_{T^7} d^6 z \sum_{\substack{\vec{n} \in \mathbb{Z}^6 \\ m \in \mathbb{Z}}} \frac{\text{Tr}_{\mathbf{B}}\left(\mathbf{D}_{\mathbf{g}}^j\right) - \text{Tr}_{\mathbf{F}}\left(\mathbf{D}_{\mathbf{g}}^j\right) \cdot e^{2\pi i m h_7}}{\left[R_2^2 \left|\left(\mathbf{I} - \mathbf{D}_{\mathbf{g}}^j\right)\vec{z} + \vec{n}\right|^2 + R_1^2 \left|m - \frac{j}{7}\right|^2\right]^s},$$

$$R_1 = \frac{R}{x^{6/7}}, R_2 = R x^{1/7}$$

$$V_{\mathrm{Cas}} = -\frac{\Gamma(s)}{2\pi^s}\cdot\frac{x^{\frac{66}{7}}}{\sqrt{7}\cdot R^4}\cdot\sum_{j=0}^{6}\int_{T^6} d^6z \sum_{\substack{\vec{n}\in\mathbb{Z}^6\\ m\in\mathbb{Z}}} \frac{\mathrm{Tr}_{\mathbf{B}}\left(\mathbf{D}_{\mathbf{g}}^{j}\right) - \mathrm{Tr}_{\mathbf{F}}\left(\mathbf{D}_{\mathbf{g}}^{j}\right)\cdot e^{2\pi i m h_7}}{\left[x^2\left|\left(\mathbf{I}-\mathbf{D}_{\mathbf{g}}^{j}\right)\vec{z}+\vec{n}\right|^2+\left|m-\frac{j}{7}\right|^2\right]^s}$$

$$s_{\mathbf{g}} = 2\cdot 7\vec{h}\cdot\vec{b}_{\mathbf{g}}\mathrm{mod}2\mathbb{Z} = 2h_7\mathrm{mod}2\mathbb{Z}$$

j	0	1	2	3	4	5	6
$\mathrm{Tr}_{\mathbf{B}}\left(\mathbf{D}_{\mathbf{g}}^{j}\right)$	128	2	2	2	2	2	2
$\mathrm{Tr}_{\mathbf{F}}\left(\mathbf{D}_{\mathbf{g}}^{j}\right)$	128	± 2	2	± 2	2	± 2	2

$$V_{\mathrm{Cas}}^{\mathrm{twisted}} = -\frac{\Gamma(s-3)}{2\pi^{s-3}}\cdot\frac{x^{\frac{24}{7}}}{7R^4}\,2\sum_{j=1}^{6}\sum_{m\in\mathbb{Z}}\frac{1-(-1)^{j+m}}{\left(m-\frac{j}{7}\right)^{2s-6}}$$

$$= -\frac{\Gamma(s-3)}{2\pi^{s-3}}\cdot\frac{x^{\frac{24}{7}}}{7R^4}\cdot\frac{2^{2s-6}-1}{2^{2s-7}}(7^{2s-6}-1)\zeta(2s-6)$$

$$V_{\mathrm{Cas}} = -\frac{\mathcal{C}_{\mathbf{I}}(x)}{R^4} - \underbrace{\frac{\Gamma\left(\frac{5}{2}\right)}{2\pi^{\frac{5}{2}}}\cdot\frac{2^5-1}{2^4}\cdot\frac{7^5-1}{7}\zeta(5)}_{\approx 733.1}\cdot\frac{x^{\frac{24}{7}}}{R^4}$$

$$\mathcal{C}_{\mathbf{I}}(x) = \frac{\Gamma(s)}{2\pi^s}\cdot\frac{128x^{\frac{66}{7}}}{\sqrt{7}}\sum_{\substack{\vec{n}\in\mathbb{Z}^6\\ m\in\mathbb{Z}}}\frac{1-(-1)^m}{[x^2|\vec{n}|^2+m^2]^s}$$

$$\mathcal{C}_{\mathbf{I}}(x) \sim \frac{\Gamma(s)}{2\pi^s}\cdot\frac{x^{\frac{66}{7}}}{\sqrt{7}\cdot 2^{2s-8}}\sum_{m\in\mathbb{Z}}\frac{1}{\left|m+\frac{1}{2}\right|^{2s}} = \underbrace{\frac{\Gamma\left(\frac{11}{2}\right)}{2\pi^{11/2}}\cdot\frac{(2^{11}-1)\zeta(11)}{4\sqrt{7}}}_{\approx 9.337}x^{\frac{66}{7}} \text{ as } x\to\infty$$

$$\mathcal{C}_{\mathbf{I}}(x) \sim \frac{186\zeta(5)}{7\pi^2}x^{\frac{24}{7}} \approx 2.791x^{\frac{24}{7}} \text{ as } x\to 0$$

$$\frac{x\mathcal{C}'}{\mathcal{C}} = \frac{d(k+4)-8}{k(k+(d-2)p)}\frac{(k-p)n_p^2x^2-p\tilde{n}_p^2}{n_p^2x^2+\tilde{n}_p^2},$$

$$\frac{x\mathcal{C}'}{\mathcal{C}} = \frac{12}{35}\cdot\frac{3n_p^2x^2-4\tilde{n}_p^2}{n_p^2x^2+\tilde{n}_p^2}\in\left[-\frac{48}{35},\frac{36}{35}\right]$$

$$\frac{x\mathcal{C}'}{\mathcal{C}}\in\left[\frac{24}{7},\frac{66}{7}\right] = \left[\frac{120}{35},\frac{330}{35}\right]$$

$$\mathbf{D_g} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \vec{b}_{\mathbf{g}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{8} \end{pmatrix}$$

$$2\kappa_{11}^2 S_{\text{TCCR}} = \int d^{11}x\sqrt{-g}\left[R - \frac{1}{2}\left|\tilde{G}_4\right|^2\right] - \frac{1}{6}\int \tilde{G}_4 \wedge \tilde{G}_4 \wedge \tilde{C}_3$$

$$\frac{1}{2\kappa_{11}^2}\left(\int \tilde{G}_4\right)^3 = 2\pi n, n \in \mathbb{Z}$$

$$S_{\text{TCCR}} = \int d^{11}x\sqrt{-g}\left[\frac{1}{2\kappa_{11}^2}R - \frac{1}{2g_3^2}|G_4|^2\right] - \frac{1}{24\pi^2}\int G_4 \wedge G_4 \wedge C_3$$

$$g_3^2 \equiv 2 \cdot (2\pi^2)^{2/3}\kappa_{11}^{2/3} = 2 \cdot (2\pi^2)^{2/3}\ell_{11}^6 \approx (3.822\ell_{11}^3)^2$$

$$T_{\text{M2}} = \sqrt{\frac{1}{2}\frac{g_3^2}{\kappa_{11}^2}} = \frac{(2\pi^2)^{1/3}}{\ell_{11}^3}, T_{\text{M5}} = \sqrt{\frac{1}{2}\frac{4\pi^2}{g_3^2\kappa_{11}^2}} = \left(\frac{\pi}{2}\right)^{\frac{1}{3}}\frac{1}{\ell_{11}^6}.$$

$$\int \frac{1}{2g_3^2}|G_4|^2 = \left(\frac{\pi}{2}\right)^{\frac{2}{3}}\frac{n_4^2}{\ell_{11}^3}\int |\omega_4|^2$$

$$\mathcal{E}(\gamma) = -\frac{\Gamma(s_\gamma)}{2\pi^{s_\gamma}} \cdot \frac{G_{\|}^{-\left(s_\gamma - \frac{1}{2}\right)}}{|\Gamma|}\sum_{n\in\mathbb{Z}}\frac{e^{2\pi i\beta n}}{\left|n - b_\gamma^{\|}\right|^{2s_\gamma}}$$

$$\sum_{n\in\mathbb{Z}}\frac{e^{2\pi i\beta n}}{\left|n - b_\gamma^{\|}\right|^{2s_\gamma}} = \begin{cases} \zeta\left(2s_\gamma, b_\gamma^{\|}\right) + \zeta\left(2s_\gamma, 1 - b_\gamma^{\|}\right) & \text{if } \beta = 0\text{mod}\mathbb{Z} \\ 2^{2s_\gamma}\left[\zeta\left(2s_\gamma, \frac{b_\gamma^{\|}}{2}\right) - \zeta\left(2s_\gamma, \frac{1 - b_\gamma^{\|}}{2}\right)\right. & \\ \left. + \zeta\left(2s_\gamma, 1 - \frac{b_\gamma^{\|}}{2}\right) - \zeta\left(2s_\gamma, 1 - \frac{1 - b_\gamma^{\|}}{2}\right)\right] & \text{if } \beta = \frac{1}{2}\text{mod}\mathbb{Z} \end{cases}$$

$$\left(\mathbf{I} - \mathbf{D}_\gamma^{-1}\right)\vec{n} = 0\text{mod}\mathbb{Z} \Leftrightarrow \left(\mathbf{I} - \mathbf{D}_\gamma\right)\vec{n} = 0\text{mod}\mathbb{Z},$$

$$V_{\text{Cas}} = \sum_{\mathbf{r}}\sum_{\gamma\in\Gamma}\text{Tr}_{\mathbf{r}}(\mathbf{D}_\gamma)\mathcal{E}(\gamma) = \frac{1}{2}\sum_{\mathbf{r}}\sum_{\gamma\in\Gamma}\left(\text{Tr}_{\mathbf{r}}(\mathbf{D}_\gamma) + (-1)^{2\beta_{\mathbf{r}}}\text{Tr}_{\mathbf{r}}\left(\mathbf{D}_\gamma^{-1}\right)\right)\mathcal{E}(\gamma).$$

$$\text{Tr}_{\text{Spinor}}\left(\mathbf{D}_\gamma^{-1}\right) = (-1)^{\sum n_i}\text{Tr}_{\text{Spinor}}\left(\mathbf{D}_\gamma\right), \text{ with } p \cdot \theta_i = 2\pi n_i,$$

$$\sum_{\vec{n}+\vec{c}\neq 0}\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n} + \vec{c}|^{2s}}$$

$$|\vec{n}|^2 = G_{ij}n^i n^j$$

$$\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}$$

$$\mathcal{F}\left[\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}\right](\vec{k}) \equiv \int d^k\vec{n}\frac{e^{2\pi i(\vec{h}-\vec{k})\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}} = \frac{2^{k-2s}\pi^{k/2}\Gamma\left(\frac{k}{2}-s\right)}{\Gamma(s)}\frac{e^{-2\pi i\vec{h}\cdot\vec{c}}}{|\vec{h}-\vec{k}|^{k-2s}}$$

$$F(\vec{n}) \equiv \frac{\Gamma(s,\alpha|\vec{n}+\vec{c}|^2)}{\Gamma(s)}, \Gamma(s,x) \equiv \int_x^\infty t^{s-1}e^{-t}dt$$

$$f(\vec{n}) \equiv 1 - F(\vec{n}) = \frac{\gamma(s,\alpha|\vec{n}+\vec{c}|^2)}{\Gamma(s)}$$

$$\lim_{\vec{n}+\vec{c}\to\vec{0}} f(\vec{n})\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}} = \frac{\alpha^s e^{-2\pi i\vec{h}\cdot\vec{c}}}{\Gamma(s+1)}\chi_{\mathbb{Z}^k}(\vec{c})$$

$$\mathcal{F}\left[\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}\cdot\frac{\gamma(s,\alpha|\vec{n}+\vec{c}|^2)}{\Gamma(s)}\right](\vec{k}) \equiv \int d^k\vec{n}e^{-2\pi i\vec{k}\cdot\vec{n}}\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}\cdot\frac{\gamma(s,\alpha|\vec{n}+\vec{c}|^2)}{\Gamma(s)}$$

$$= \frac{1}{\Gamma(s)}\int d^k\vec{n}\frac{e^{2\pi i(\vec{h}-\vec{k})\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}\gamma(s,\alpha|\vec{n}+\vec{c}|^2) = \frac{e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{c}}}{\Gamma(s)}\int d^k\overrightarrow{n'}\frac{e^{2\pi i(\vec{h}-\vec{k})\cdot\overrightarrow{n'}}}{\left|\overrightarrow{n'}\right|^{2s}}\gamma\left(s,\alpha\left|\overrightarrow{n'}\right|^2\right)$$

$$\mathbf{G} = \mathbf{M}^T\mathbf{M}, \overrightarrow{m} = \mathbf{M}\overrightarrow{n'}$$

$$\vec{q} \equiv \mathbf{G}^{-1}(\vec{h}-\vec{k})$$

$$\mathcal{F}\left[\frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}}\cdot\frac{\gamma(s,\alpha|\vec{n}+\vec{c}|^2)}{\Gamma(s)}\right](\vec{k}) = \frac{e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{c}}}{\Gamma(s)\sqrt{G}}\int d^k\overrightarrow{m}\frac{e^{2\pi i\left(\left(\mathbf{M}^{-1}\right)^T(\vec{h}-\vec{k})\right)\cdot\overrightarrow{m}}}{|\overrightarrow{m}|^{2s}}\gamma(s,\alpha|\overrightarrow{m}|^2)$$

$$= \frac{e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{c}}}{\Gamma(s)\sqrt{G}}\mathcal{S}_{k-2}\int_0^\infty dr\frac{\gamma(s,\alpha r^2)}{r^{2s-k+1}}\int_0^\pi d\theta(\sin\theta)^{k-2}e^{-2\pi i|\vec{q}|r\cos\theta}$$

$$= \frac{(2\pi)^{\frac{k}{2}}e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{c}}}{\Gamma(s)\sqrt{G}}\frac{1}{(2\pi|\vec{q}|)^{k-1-2s}}\int_0^\infty dr\frac{\gamma(s,\alpha r^2)}{(2\pi|\vec{q}|r)^{2s-\frac{k}{2}}}J_{\frac{k-2}{2}}(2\pi|\vec{q}|r)$$

$$= \frac{(2\pi)^{\frac{k}{2}}e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{c}}}{\Gamma(s)\sqrt{G}}\frac{1}{(2\pi|\vec{q}|)^{k-2s}}\int_0^\infty d\tau\frac{\gamma\left(s,\frac{\alpha}{(2\pi|\vec{q}|)^2}\tau^2\right)}{\tau^{2s-\frac{k}{2}}}J_{\frac{k-2}{2}}(\tau)$$

$$= \frac{2^{k-2s}\pi^{\frac{k}{2}}}{\Gamma(s)}\Gamma\left(\frac{k}{2}-s,\frac{\pi^2|\vec{q}|^2}{\alpha}\right)\frac{e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{c}}}{\sqrt{G}(2\pi|\vec{q}|)^{k-2s}}$$

$$J_\nu(t) = \frac{t^\nu}{(2\pi)^{\nu+1}}\mathcal{S}_{2\nu}\int_0^\pi e^{-it\cos\theta}\sin^{2\nu}\theta d\theta$$

$$\frac{\mathcal{S}_{k-2}}{\Gamma(s)\sqrt{G}}\int_0^\infty dr\frac{\gamma(s,\alpha r^2)}{r^{2s-k+1}}\int_0^\pi d\theta(\sin\theta)^{k-2} = \frac{\pi^{\frac{k}{2}}\alpha^{s-k/2}}{\sqrt{G}\Gamma(s)\left(s-\frac{k}{2}\right)}$$

$$\sum_{\vec{n}+\vec{c}\neq 0} \frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}} =$$

$$\sum_{\vec{n}+\vec{c}\neq 0} \frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{|\vec{n}+\vec{c}|^{2s}} \frac{\Gamma(s,\alpha|\vec{n}+\vec{c}|^2)}{\Gamma(s)} + \frac{\pi^{2s-\frac{k}{2}}}{\Gamma(s)\sqrt{G}} \sum_{\vec{k}-\vec{h}\neq 0} \Gamma\left(\frac{k}{2}-s, \frac{\pi^2|\vec{h}-\vec{k}|_D^2}{\alpha}\right) \frac{e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{c}}}{|\vec{h}-\vec{k}|_D^{k-2s}}$$

$$+\delta_{\vec{h},\vec{0}} \frac{\pi^{\frac{k}{2}}\alpha^{s-k/2}}{\sqrt{G}\Gamma(s)\left(s-\frac{k}{2}\right)} - \frac{\alpha^s e^{-2\pi i\vec{h}\cdot\vec{c}}}{\Gamma(s+1)} \chi_{\mathbb{Z}^k}(\vec{c})$$

$$\sum_{\vec{n}\neq\vec{0}} \frac{(-1)^m}{m^2+mn+n^2} = -\frac{4\pi \ln 2}{3\sqrt{3}}, \sum_{\vec{n}\neq\vec{0}} \frac{(-1)^{m+n}}{m^2+5n^2} = -\frac{\pi}{\sqrt{5}} \ln(1+\sqrt{5}),$$

$$\sum_{\vec{n}\in\mathbb{R}^3} \frac{(-1)^{n_1+n_2+n_3}}{\left[\left(n_1+\frac{1}{6}\right)^2+\left(n_2+\frac{1}{6}\right)^2+\left(n_3+\frac{1}{6}\right)^2\right]^s} = 12^s\beta(2s-1),$$

$$\sum_{\vec{n}\in\mathbb{R}^8-\{\vec{0}\}} \frac{(-1)^{\sum_i n_i}}{|\vec{n}|^{2s}} = -16\zeta(s)\eta(s-3), \sum_{\vec{n}\in\mathbb{R}^8} \frac{1}{|\vec{n}+\vec{1}/2|^{2s}} = \frac{256}{2^s}\zeta(s-3)\lambda(s),$$

$$\frac{\Gamma\left(\frac{k}{2}-s, \frac{\pi^2\lambda}{\alpha}\right)}{\lambda^{k/2-s}} \leq \frac{\Gamma(k/2)}{2\pi^{k/2}} \frac{\epsilon}{\lambda^{\frac{k-1}{2}}}.$$

$$\frac{\Gamma\left(\frac{k}{2}-s, \frac{\pi^2\lambda}{\alpha}\right)}{\left(\frac{\pi^2}{\alpha}\lambda\right)^{1/2-s}} \leq \frac{\Gamma(k/2)}{2\pi^{k/2}} \left(\frac{\pi^2}{\alpha}\right)^{s-1/2} \epsilon$$

$$\frac{\Gamma(a,z)}{z^a} \leq \frac{e^{-z}}{z}$$

$$\frac{e^{-\frac{\pi^2\lambda}{\alpha}}}{\frac{\pi^2\lambda}{\alpha}} \leq \frac{\Gamma(k/2)}{2\pi^{k/2}} \left(\frac{\pi^2}{\alpha}\right)^{s-1/2} \epsilon$$

$$\lambda \geq \frac{\alpha}{\pi^2} W\left(\frac{2\pi^{k/2}}{\Gamma(k/2)\epsilon}\left(\frac{\alpha}{\pi^2}\right)^{s-1/2}\right)$$

$$\int_{[0,1]^k} dV \sum_{n\in\mathbb{Z}^k} \frac{e^{2\pi i\vec{h}\cdot\vec{n}}}{\left|\left(\mathbf{I}-\mathbf{D}_\gamma\right)\vec{z}+\vec{b}_\gamma+\vec{n}\right|^{2s}}$$

$$\Lambda^\perp \equiv \left(\mathbf{I}-\mathbf{D}_\gamma\right)\mathbb{Z}^k \subset \mathbb{Z}^k$$

$$\Lambda^\parallel \equiv \left\{\vec{n}\in\mathbb{Z}^k \mid \left(\mathbf{I}-\mathbf{D}_\gamma\right)\vec{n}=\vec{0}\right\}$$

$$\Lambda \equiv \Lambda^\parallel \oplus \Lambda^\perp \supset \mathbb{Z}^k$$

$$\Gamma_\Lambda \equiv \mathbb{Z}^k/\Lambda$$

$$\vec{n} = \vec{k} + \vec{l} + \vec{\gamma}$$

$$\sum_{\substack{\vec{k}\in\Lambda^{\perp} \\ \vec{l}\in\Lambda^{\parallel},\gamma\in\Gamma_{\Lambda}}} \int_{[0,1]^k} dV \frac{e^{2\pi i\vec{h}\cdot[\vec{k}+\vec{l}+\vec{\gamma}]}}{\left|(\mathbf{I}-\mathbf{D}_{\gamma})\vec{x}+\vec{k}+\vec{b}_{\gamma}+\vec{l}+\vec{\gamma}\right|^{2s}}$$

$$\vec{h}\cdot(\mathbf{I}-\mathbf{D}_{\gamma})\vec{l} = 0 \mathrm{mod}\mathbb{Z}$$

$$\vec{y} \equiv (\mathbf{I}-\mathbf{D}_{\gamma})\vec{x}_{\perp} + \vec{k}$$

$$\frac{V_{\parallel}}{\left|\det(\mathbf{I}-\mathbf{D}_{\gamma})\right|} \sum_{\substack{\vec{l}\in\Lambda^{\parallel} \\ \gamma\in\Gamma_{\Lambda}}} \int_{\mathbb{R}^{k''}} dV \frac{e^{2\pi i\vec{h}\cdot(\vec{\gamma}+\vec{l})}}{\left|\vec{y}+\vec{b}_{\gamma}+\vec{l}+\vec{\gamma}\right|^{2s}}$$

$$\vec{y}+\vec{b}_{\gamma}+\vec{l}+\vec{\gamma} \to \vec{y}+\vec{b}_{\gamma}^{\parallel}+\vec{l}+\vec{\gamma}^{\parallel}$$

$$\left|\vec{y}+\vec{b}\right\|_{\gamma}^{\parallel}+\vec{l}+\vec{\gamma}^{\parallel}\Big|^2 = |\vec{y}|^2+\alpha^2, \alpha^2 \equiv \left|\vec{b}\right\|_{\gamma}^{\parallel}+\vec{l}+\vec{\gamma}^{\parallel}\Big|^2$$

$$\int_{\mathbb{R}^{k''}} \frac{dV}{(|\vec{y}|^2+\alpha^2)^s} = \frac{2\pi^{\frac{k''}{2}}}{\Gamma(k''/2)} \int_0^{\infty} \frac{y^{k''-1}dy}{(y^2+\alpha^2)^s} = \frac{2\pi^{\frac{k''}{2}}}{\Gamma(k''/2)} = \pi^{\frac{k-k'}{2}} \frac{\Gamma(s')}{\Gamma(s)} \frac{1}{\alpha^{2s'}}$$

$$\pi^{\frac{k-k'}{2}} \frac{\Gamma(s')}{\Gamma(s)} \frac{V_{\parallel}}{\left|\det(\mathbf{I}-\mathbf{D}_{\gamma})\right|} \sum_{\substack{\vec{l}\in\Lambda_{\parallel}^{\parallel} \\ \gamma\in\Gamma_{\Lambda}}} \frac{e^{2\pi i\vec{h}\cdot(\vec{\gamma}+\vec{l})}}{\left|\vec{b}_{\gamma}^{\parallel}+\vec{l}+\vec{\gamma}\right\|^{2s'}}$$

$$\sum_{\gamma\in\Gamma_{\Sigma}} e^{2\pi i\vec{h}\cdot\vec{\gamma}} = \left|\det(\mathbf{I}-\mathbf{D}_{\gamma})\right|\hat{\delta}_{\vec{h}}$$

$\hat{\delta}_{\vec{h}} \equiv 1$ iff $\vec{h}\cdot\vec{l} \in \mathbb{Z}, \forall\vec{l} \in \Sigma^{\perp}$, and 0 otherwise.

$$(\mathbf{I}-\mathbf{D}_{\gamma})^T \cdot \vec{h} = (\mathbf{I}-\mathbf{D}_{\gamma})^T \cdot \vec{\eta}$$

$$\vec{h}\cdot\vec{w} = \vec{\beta}\cdot\vec{w} + \vec{\eta}\cdot\vec{w} = \vec{\beta}\cdot\vec{w} \mathrm{mod}\mathbb{Z}$$

$$\sqrt{G_{\parallel}}\pi^{\frac{k-k'}{2}} \frac{\Gamma(s')}{\Gamma(s)} \sum_{\vec{w}\in\Xi} \frac{e^{2\pi i\vec{\beta}\cdot\vec{w}}}{\left|\vec{w}+\vec{b}_{\gamma}^{\parallel}\right|^{2s'}},$$

$$\vec{c} = (\mathbf{I}-\mathbf{D}_{\gamma})\vec{z} + \vec{b}_{\gamma}$$

$$\sqrt{G}\int_{[0,1]^k} d^k z e^{-2\pi i(\vec{h}-\vec{k})\cdot[(\mathbf{I}-\mathbf{D}_{\gamma})\vec{z}+\vec{b}_{\gamma}]} = \sqrt{G}e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{b}_{\gamma}} \int_{[0,1]^k} d^k z e^{-2\pi i(\vec{h}-\vec{k})\cdot[(\mathbf{I}-\mathbf{D}_{\gamma})\vec{z}]}$$

$$= \sqrt{G}e^{-2\pi i(\vec{h}-\vec{k})\cdot\vec{b}_{\gamma}} \prod_{l=1}^{k} \frac{e^{2\pi i\vec{q}_l}-1}{2\pi i\vec{q}_l}, \vec{q} \equiv -(\mathbf{I}-\mathbf{D}_{\gamma})^T(\vec{h}-\vec{k})$$

$$\vec{h}\cdot\left(\mathbf{I}-\mathbf{D}_{\gamma}\right)\vec{l}\in\mathbb{Z},\forall\vec{l}\in\mathbb{Z}^{k}$$

$$\frac{e^{2\pi i\vec{q}_{l}}-1}{2\pi i\vec{q}_{l}}\rightarrow\delta_{q_{l},0}\ \text{ for }\ q_{l}\in\mathbb{Z}$$

$$\left(\mathbf{I}-\mathbf{D}_{\gamma}\right)^{T}(\vec{h}-\vec{k})=0,\Rightarrow\ \vec{k}=\vec{\eta}+\vec{\kappa}$$

$$\left(\mathbf{I}-\mathbf{D}_{\gamma}\right)^{T}\vec{\eta}=\left(\mathbf{I}-\mathbf{D}_{\gamma}\right)^{T}\vec{h}$$

$$(\vec{\beta}-\vec{k})\cdot\vec{b}_{\gamma}=(\vec{\beta}-\vec{k})\cdot\vec{b}_{\gamma}^{\parallel}$$

$$\mathcal{S}_{\text{position}}^{\alpha}+\frac{\pi^{2s-\frac{k}{2}}}{\Gamma(s)}\sum_{\substack{k\in\Lambda\\ \kappa-\vec{\beta}\neq\vec{0}}}\Gamma\left(\frac{k}{2}-s,\frac{\pi^{2}|\vec{\beta}-\vec{\kappa}|_{D}^{2}}{\alpha}\right)\frac{e^{-2\pi i(\vec{\beta}-\vec{\kappa})\cdot\vec{b}_{\gamma}^{\parallel}}}{|\vec{\beta}-\vec{\kappa}|_{D}^{k-2s}}$$
$$+\delta_{\vec{h},\vec{0}}\frac{\pi^{\frac{k}{2}}\alpha^{s-k/2}}{\Gamma(s)\left(s-\frac{k}{2}\right)}-\frac{\sqrt{G}\alpha^{s}}{\Gamma(s+1)}\delta_{(\mathbf{D}_{\gamma},\vec{b}_{\gamma})}$$

$$|\vec{v}|_{I}\equiv\vec{v}^{T}\cdot\mathbf{G}_{I}\cdot\vec{v},\mathbf{G}_{I}\equiv\mathbf{V}^{T}\mathbf{G}_{D}\mathbf{V},\vec{d}\equiv\mathbf{V}^{-1}\vec{\beta},\vec{v}_{\gamma}\equiv\mathbf{V}^{T}\vec{b}_{\gamma}^{\parallel},s'=\frac{k'-k}{2}+s$$

$$\mathcal{S}_{\text{position}}^{\alpha}+\frac{\pi^{2s'-k'+\frac{k}{2}}}{\Gamma(s)}\sum_{\substack{\vec{l}\in\mathbb{Z}^{k'}\\ \vec{l}-\vec{d}\neq\vec{0}}}\Gamma\left(\frac{k'}{2}-s',\frac{\pi^{2}|\vec{d}-\vec{l}|_{I}^{2}}{\alpha}\right)\frac{e^{-2\pi i(\vec{d}-\vec{l})\cdot\vec{v}_{\gamma}}}{|\vec{d}-\vec{l}|_{I}^{k'-2s'}}+\delta_{\vec{d},\vec{0}}\frac{\pi^{\frac{k}{2}}\alpha^{s'-k'/2}}{\Gamma(s)\left(s'-\frac{k'}{2}\right)}$$

$$\frac{\Gamma(s)}{\pi^{k/2}}\frac{\pi^{k'/2}}{\Gamma(s')}\frac{1}{\sqrt{G_{I}}}\mathcal{S}_{\text{position}}^{\alpha}+\frac{\pi^{2s'-\frac{k'}{2}}}{\Gamma(s')\sqrt{G_{I}}}\sum_{\substack{\vec{l}\in\mathbb{Z}^{k'}\\ \vec{l}-\vec{d}\neq0}}\Gamma\left(\frac{k'}{2}-s',\frac{\pi^{2}\mid\vec{d}-\vec{l}_{I}^{2}}{\alpha}\right)\frac{e^{-2\pi i(\vec{d}-\vec{l})\cdot\vec{v}_{\gamma}}}{|\vec{d}-\vec{l}|_{I}^{k'-2s'}}$$
$$+\delta_{\vec{d},\vec{0}}\frac{\pi^{\frac{k'}{2}}\alpha^{s'-k'/2}}{\sqrt{G_{I}}\Gamma(s')\left(s'-\frac{k'}{2}\right)}$$

$$\mathcal{E}(\gamma)=-\hat{\delta}_{\vec{h}}\frac{\Gamma(s')}{2\pi^{s'}}\cdot\frac{\sqrt{G_{\parallel}}}{|\Gamma|}\sum_{\vec{m}}\frac{e^{2\pi i\vec{d}\cdot\vec{m}}}{\left|\vec{m}+\vec{v}_{\gamma}\right|_{\parallel}^{2s'}},$$

$$\mathbf{G}_{\parallel}=\mathbf{V}^{-1}\mathbf{G}(\mathbf{V}^{-1})^{T},\vec{v}_{\gamma}=\mathbf{V}^{T}\cdot\vec{b}_{\gamma}^{\parallel},\vec{d}\equiv\mathbf{V}^{-1}\vec{\beta},s'=\frac{k'-k}{2}+s$$

$$\mathcal{E}(\gamma)=-\hat{\delta}_{\vec{h}}\frac{\Gamma(s')}{2\pi^{s'}}\cdot\frac{\sqrt{G_{\parallel}}}{|\Gamma|}\sum_{\vec{\xi}\in\Xi}\frac{e^{2\pi i\vec{\beta}\cdot\vec{\xi}}}{\left|\vec{\xi}+\vec{b}_{\gamma}^{\parallel}\right|^{2s'}}$$

$$SO(D-2)\rightarrow SO(k)\times SO(d-2)$$

$$v^a w^b \to M_c^a M_d^b v^c w^d$$

$$\mathrm{Tr}(\mathbf{M}^{\text{Tens.}}) = \delta_a^c \delta_b^d M_c^a M_d^b = (\mathrm{Tr}(\mathbf{M}))^2$$

$$\mathbb{P}^{\text{Sym.}}\, T^{ab} = \frac{1}{2}\left(T^{ab} + T^{ba}\right) - \frac{T}{n}\delta^{ab}$$
$$\mathbb{P}^{\text{ASym.}}\, T^{ab} = \frac{1}{2}\left(T^{ab} - T^{ba}\right)$$
$$\mathbb{P}^{\text{Sing.}}\, T^{ab} = \frac{T}{n}\delta^{ab}$$

$$\mathrm{Tr}\left(\mathbf{M}^{\text{Sym.}}\right) = \frac{1}{2}\left(\mathrm{Tr}(\mathbf{M})^2 + \mathrm{Tr}(\mathbf{M}^2)\right) - 1$$

$$\mathrm{Tr}\left(\mathbf{I}^{\text{Sym.}}\right) = \frac{n^2}{2}\left(\frac{1}{2} - \frac{1}{n}\right) \cdot n = \frac{n(n+1)}{2} - 1$$

$$\mathbb{P}^{3-\text{ Antisym.}}\, T^{abc} = \frac{1}{6}\left(T^{abc} + T^{bca} + T^{cab} - T^{bac} - T^{acb} - T^{cba}\right),$$

$$\mathrm{Tr}\left(\mathbf{M}^{3\text{-Antisym.}}\right) = \frac{1}{6}\left(\mathrm{Tr}(\mathbf{M})^3 - 3\mathrm{Tr}(\mathbf{M}^2) \cdot \mathrm{Tr}(\mathbf{M}) + 2\mathrm{Tr}(\mathbf{M}^3)\right)$$

$$\mathrm{Tr}\left(\mathbf{I}^{3-\text{ Antisym.}}\right) = \frac{1}{6}(n^3 - 3n^2 + 2n) = \frac{n(n-1)(n-2)}{6} = \binom{n}{3}$$

$$\sum_{k=0}^{\infty} x^k \mathrm{Tr}\left(\mathbf{M}^{k\text{-Antisym.}}\right) = \prod_{l=1}^{\infty} \exp\left(-(-x)^l \frac{\mathrm{Tr}\left(\mathbf{M}^l\right)}{l}\right)$$

$$\mathbb{P}^{\text{R.S}} \rho_a^\alpha = \rho_a^\alpha - \frac{1}{n^2}\delta^{ab}(\gamma_b)_\beta^\alpha \rho_a^\beta$$

$$\mathrm{Tr}\left(\mathbf{M}^{\text{R.S.}}\right) = (\mathrm{Tr}(\mathbf{M}) - 1)\mathrm{Tr}\left(\mathbf{M}^{\text{Spinor}}\right),$$

$$\mathbf{2^k} \otimes \mathbf{2^k} = \sum_{l=1}^{k} \binom{\mathbf{n}}{\mathbf{l}}$$

$$\begin{aligned}\mathrm{Tr}\left(\mathbf{M}^{\text{Spinor}}\right)^2 = {} & 1 + \mathrm{Tr}(\mathbf{M}) + \frac{1}{2}\left(\mathrm{Tr}(\mathbf{M})^2 - \mathrm{Tr}(\mathbf{M}^2)\right) \\ & + \frac{1}{6}\left(\mathrm{Tr}(\mathbf{M})^3 - 3\mathrm{Tr}(\mathbf{M})\mathrm{Tr}(\mathbf{M}^2) + 2\mathrm{Tr}(\mathbf{M}^3)\right) \\ & + \frac{1}{24}\left(\mathrm{Tr}(\mathbf{M})^4 - 6\mathrm{Tr}(\mathbf{M})^2\mathrm{Tr}(\mathbf{M}^2) + 3\mathrm{Tr}(\mathbf{M}^2)^2 + 8\mathrm{Tr}(\mathbf{M})\mathrm{Tr}(\mathbf{M}^3) - 6\mathrm{Tr}(\mathbf{M}^4)\right)\end{aligned}$$

$$\mathbf{M} = \begin{pmatrix} \cos\,\theta_1 & \sin\,\theta_1 & 0 & 0 & 0 & 0 \\ -\sin\,\theta_1 & \cos\,\theta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cos\,\theta_4 & \sin\,\theta_4 & 0 \\ 0 & 0 & \cdots & -\sin\,\theta_4 & \cos\,\theta_4 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix} = \exp\,(i\theta_\alpha E_\alpha^3)$$

$$\mathrm{Tr}_{\mathbf{R}}(\mathbf{M}) = \sum_{\vec{\mu}\in R} \exp\,(i\vec{\theta}\cdot\vec{\mu})$$

$$\mathrm{Tr}_{\mathbf{9}}(\mathbf{M}) = 2\sum_{i=1}^{4} \cos\,\theta_i + 1$$

$$\mathrm{Tr}_{\mathrm{Spinor}}\,(\mathbf{M}) = \prod_{i=1}^{4} 2\cos\,\frac{\theta_i}{2}$$

$$\mathrm{Tr}_{\mathrm{Spinor}}\,(-\mathcal{M}) = \prod_{i=1}^{4} 2\cos\left(\frac{\theta_i'}{2} + \pi\eta_i\right)$$

$$= (-1)^{\Sigma\,\eta_i} \prod_{i=1}^{4} 2\cos\,\frac{\theta_i'}{2} = (-1)^{\Sigma\,\eta_i}\mathrm{Tr}_{\mathrm{Spinor}}\,(\mathcal{M})$$

$$\mathrm{Tr}_{\mathbf{9}}\left(\mathbf{D}_{\mathbf{g}}^{j}\right) = 2\sum_{i=1}^{4} \cos\left(j\cdot\theta_i^{\mathbf{g}}\right) + 1$$

$$\mathrm{Tr}_{\mathrm{Spinor}}\left(\mathbf{D}_{\mathbf{g}}^{p}\right) = \prod_{i=1}^{4} 2\cos\,(\pi n_i) = (-1)^{\Sigma\,n_i}\cdot 16$$

$$\mathrm{Tr}_{\mathbf{44}}\left(\mathbf{D}_{\mathbf{g}}^{j}\right) = \sum_{a=1}^{4}\left[2\cos\left(2j\theta_a^{\mathbf{g}}\right) + 2\cos\left(j\theta_a^{\mathbf{g}}\right) + 1 + 2\sum_{\substack{b=1 \\ j\neq i}}^{4} \cos\left(j\theta_a^{\mathbf{g}}\right)\cos\left(j\theta_b^{\mathbf{g}}\right)\right]$$

$$\mathrm{Tr}_{\mathbf{84}}\left(\mathbf{D}_{\mathbf{g}}\right) = \sum_{a=1}^{4}\left[1 + 6\cos\left(j\theta_a^{\mathbf{g}}\right) + 2\sum_{\substack{b=1 \\ j\neq i}}^{4} \cos\left(j\theta_a^{\mathbf{g}}\right)\cos\left(j\theta_b^{\mathbf{g}}\right) + \frac{4}{3}\sum_{\substack{b,c=1 \\ b\neq c\neq a}}^{4} \cos\left(j\theta_a^{\mathbf{g}}\right)\cos\left(j\theta_b^{\mathbf{g}}\right)\cos\left(j\theta_c^{\mathbf{g}}\right)\right]$$

$$\mathrm{Tr}_{\mathbf{128}}\left(\mathbf{D}_{\mathbf{g}}^{j}\right) = \left(2\sum_{a=1}^{4} \cos\left(j\theta_a^{\mathbf{g}}\right)\right)\prod_{a=1}^{4} 2\cos\left(j\frac{\theta_a^{\mathbf{g}}}{2}\right)$$

$$\mathrm{Tr}_{\mathbf{128}}\left(\mathbf{D}_{\mathbf{g}}^{p-j}\right) = (-1)^{\sum n_i}\left(2\sum_{a=1}^{4}\cos\left(j\theta_a^{\mathbf{g}}\right)\right)\prod_{a=1}^{4}2\cos\left(j\frac{\theta_a^{\mathbf{g}}}{2}\right),\ \text{with}\ p\cdot\theta_i = 2\pi n_i$$

$\mathbf{D_g}$	$\vec{b}_{\mathbf{g}}$	$\lvert\Gamma\rvert$	Moduli	$\vec{h}$
$\begin{pmatrix}0&-1&0&0&0&0&0\\1&-1&0&0&0&0&0\\0&0&0&0&0&-1&0\\0&0&1&0&0&1&0\\0&0&0&1&0&-1&0\\0&0&0&0&1&1&0\\0&0&0&0&0&0&1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\\0\\0\\\frac{1}{30}\end{pmatrix}$	30	4	(0, 0, 0, 0, 0, 0, 0)
$\begin{pmatrix}0&-1&0&0&0&0&0\\1&1&0&0&0&0&0\\0&0&0&0&0&-1&0\\0&0&1&0&0&1&0\\0&0&0&1&0&-1&0\\0&0&0&0&1&1&0\\0&0&0&0&0&0&1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\\0\\0\\\frac{1}{30}\end{pmatrix}$	30	4	(0,0,0,0,0,0,0)
$\begin{pmatrix}0&0&0&-1&0&0&0\\1&0&0&-1&0&0&0\\0&1&0&-1&0&0&0\\0&0&1&-1&0&0&0\\0&0&0&0&0&-1&0\\0&0&0&0&1&1&0\\0&0&0&0&0&0&1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\\0\\0\\\frac{1}{30}\end{pmatrix}$	30	4	(0,0,0,0,0,0,0)
$\begin{pmatrix}0&-1&0&0&0&0&0\\1&-1&0&0&0&0&0\\0&0&0&0&0&-1&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&0&1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\\0\\0\\\frac{1}{24}\end{pmatrix}$	24	4	$(0,0,h_3,h_3,h_3,h_3,0)$

$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{24} \end{pmatrix}$	24	4	$(0,0,h_3,h_3,h_3,h_3,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{20} \end{pmatrix}$	20	4	$(h_1, h_1, 0,0,0,0,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{20} \end{pmatrix}$	20	4	$(h_1, h_1, 0,0,0,0,0)$

$\mathbf{D_g}$	$\vec{b}_g$	|Γ|	Moduli	$\vec{h}$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{18} \end{pmatrix}$	18	4	(0, 0, 0, 0, 0, 0, 0)

$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \overline{15} \end{pmatrix}$	15	4	(0,0,0,0,0,0, h_7)
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \overline{14} \end{pmatrix}$	14	4	(0,0,0,0,0,0,0)
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \overline{12} \end{pmatrix}$	12	4	(h_1, h_1, 0,0,0,0,0)
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \overline{12} \end{pmatrix}$	12	4	(0,0,0,0,0,0,0)
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \overline{12} \end{pmatrix}$	12	4	(0,0,0,0,0,0,0)

$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_2, 0,0,0,0,0)$

$\mathbf{D_g}$	$\vec{b}_{\mathbf{g}}$	$\lvert\Gamma\rvert$	Moduli	$\vec{h}$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	8	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_1, h_3, h_3, 0,0,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_1, 0,0,0,0,0)$

$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(0,0,h_3,h_3,h_5,h_5,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	4	$(0,0,h_3,h_3,0,0,0)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(0,0,0,0,h_5,h_5,0)$

$\mathbf{D_g}$	$\vec{b}_g$	$\lvert\Gamma\rvert$	Moduli	$\vec{h}$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1,h_2,h_3,h_3,0,0,0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1,h_2,0,0,h_5,h_5,0)$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_2, h_3, h_3, 0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	6	$(h_1, h_2, 0,0, h_5, h_5, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	8	$(h_1, h_2, h_3, h_3, 0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{12} \end{pmatrix}$	12	8	$(h_1, h_2, 0,0, h_5, h_5, 0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{10} \end{pmatrix}$	10	6	$(h_1, h_2, 0,0,0,0,0)$

$\mathbf{D_g}$	$\vec{b}_\mathbf{g}$	$\lvert\Gamma\rvert$	Moduli	$\vec{h}$

$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{10} \end{pmatrix}$	10	6	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{10} \end{pmatrix}$	10	6	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{10} \end{pmatrix}$	10	6	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{10} \end{pmatrix}$	10	8	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{9} \end{pmatrix}$	9	4	$(0,0,0,0,0,0, h_7)$

$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{8} \end{pmatrix}$	8	4	$(h_1, h_1, h_3, h_3, h_3, h_3, 0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{8} \end{pmatrix}$	8	6	$(h_1, h_2, h_3, h_3, h_3, h_3, 0)$

$\mathbf{D_g}$	$\vec{b}_{\mathbf{g}}$	$\lvert\Gamma\rvert$	Moduli	$\vec{h}$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{8} \end{pmatrix}$	8	6	$(h_1, h_2, h_3, h_3, h_3, h_3, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{8} \end{pmatrix}$	8	8	$(h_1, h_2, h_3, h_3, h_3, h_3, 0)$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{7} \end{pmatrix}$	7	4	$(0,0,0,0,0,0, h_7)$

$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	6	(0,0,0,0,0,0,0)
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	6	(0, 0, 0, 0, 0, 0, 0)
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	10	(0, 0, 0, 0, 0, 0, 0)
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	8	(h_1, h_2, 0,0,0,0,0)

D_g	$\vec{b}_g$	|Γ|	Moduli	$\vec{h}$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	6	(h_1, h_2, 0,0,0,0,0)

$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{6} \end{pmatrix}$	6	8	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{6} \end{pmatrix}$	6	8	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{6} \end{pmatrix}$	6	6	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{6} \end{pmatrix}$	6	8	$(h_1, h_2, 0,0,0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{6} \end{pmatrix}$	6	8	$(h_1, h_2, 0,0,0,0,0)$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	10	$(h_1, h_2, 0,0,0,0,0)$

$\mathbf{D_g}$	$\vec{b}_{\mathbf{g}}$	$\lvert\Gamma\rvert$	Moduli	$\vec{h}$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	12	$(h_1, h_2, h_3, h_4, 0,0,0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	12	$(h_1, h_2, h_3, h_4, 0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	10	$(h_1, h_2, h_3, h_4, 0,0,0)$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	10	$(h_1, h_2, h_3, h_4, 0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	10	$(h_1, h_2, h_3, h_4, 0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	10	$(h_1, h_2, h_3, h_4, 0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	12	$(h_1, h_2, h_3, h_4, 0,0,0)$

$\mathbf{D_g}$	$\vec{b}_g$	$\lvert\Gamma\rvert$	Moduli	$\vec{h}$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	12	$(h_1, h_2, h_3, h_4, 0,0,0)$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{6} \end{pmatrix}$	6	16	$(h_1, h_2, h_3, h_4, 0,0,0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{5} \end{pmatrix}$	5	8	$(h_1, h_2, 0,0,0,0, h_7)$
$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{4} \end{pmatrix}$	4	10	$(h_1, h_1, h_3, h_3, h_5, h_5, 0)$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{4} \end{pmatrix}$	4	8	$(h_1, h_2, h_3, h_3, h_5, h_5, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{4} \end{pmatrix}$	4	8	$(h_1, h_2, h_3, h_3, h_5, h_5, 0)$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{4} \end{pmatrix}$	4	10	$(h_1, h_2, h_3, h_3, h_5, h_5, 0)$

$\mathbf{D_g}$	$\vec{b}_g$	$\lvert\Gamma\rvert$	Moduli	$\vec{h}$
$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{4} \end{pmatrix}$	4	12	$(h_1, h_2, h_3, h_4, h_5, h_5, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{4} \end{pmatrix}$	4	10	$(h_1, h_2, h_3, h_4, h_5, h_5, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{4} \end{pmatrix}$	4	10	$(h_1, h_2, h_3, h_4, h_5, h_5, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{4} \end{pmatrix}$	4	12	$(h_1, h_2, h_3, h_4, h_5, h_5, 0)$

$\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&0&-1&0\\0&0&0&0&1&0&0\\0&0&0&0&0&0&1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\\0\\0\\\frac{1}{4}\end{pmatrix}$	4	16	$(\ h_1, h_2, h_3, h_4, h_5, h_5, 0\ )$
$\begin{pmatrix}0&-1&0&0&0&0&0\\1&-1&0&0&0&0&0\\0&0&0&-1&0&0&0\\0&0&1&-1&0&0&0\\0&0&0&0&0&-1&0\\0&0&0&0&1&-1&0\\0&0&0&0&0&0&1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\\0\\0\\\frac{1}{3}\end{pmatrix}$	3	10	$(\ 0,0,0,0,0,0, h_7\ )$
$\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&0&-1&0&0&0\\0&0&1&-1&0&0&0\\0&0&0&0&0&-1&0\\0&0&0&0&1&-1&0\\0&0&0&0&0&0&1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\\0\\0\\\frac{1}{3}\end{pmatrix}$	3	10	$(\ h_1, h_2, 0,0,0,0, h_7\ )$

$\mathbf{D_g}$	$\vec{b}_g$	$\lvert\Gamma\rvert$	Moduli	$\vec{h}$
$\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&0&-1&0\\0&0&0&0&1&-1&0\\0&0&0&0&0&0&1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\\0\\0\\\frac{1}{3}\end{pmatrix}$	3	16	$(\ h_1, h_2, h_3, h_4, 0,0, h_7\ )$
$\begin{pmatrix}-1&0&0&0&0&0&0\\0&-1&0&0&0&0&0\\0&0&-1&0&0&0&0\\0&0&0&-1&0&0&0\\0&0&0&0&-1&0&0\\0&0&0&0&0&-1&0\\0&0&0&0&0&0&1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\\0\\0\\\frac{1}{2}\end{pmatrix}$	2	22	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	18	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	16	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	16	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	18	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	2	22	$(h_1, h_2, h_3, h_4, h_5, h_6, 0)$

$$\mathbf{D_g}\left(\mathbf{D_g}^{-1}\vec{z} - \mathbf{D_g}^{-1}\vec{b}_\mathbf{g} + \vec{k}\right) + \vec{b}_\mathbf{g} = \vec{z} + \mathbf{D_g} \cdot \vec{k}$$

$$\Phi_n(x) = \prod_{1 \le k \le n} \left(x - e^{2\pi i \frac{k}{n}}\right)$$

Supergravedad cuántica relativista en D=8.

$$S=\int_{\Sigma}\mathcal{Z}^{a}\cdot\bar{\partial}\mathcal{Z}_{a}+A_{ab}\mathcal{Z}^{a}\cdot\mathcal{Z}^{b}+a\langle\lambda_{A}\lambda_{B}\rangle\Omega^{AB}+\tilde{a}\left(\langle\lambda_{A}\lambda_{B}\rangle\Omega^{AB}+\langle\eta_{\mathcal{J}}\eta_{\mathcal{J}}\rangle\Omega^{\mathcal{J}\mathcal{J}}\right)$$
$$+S_{\rho_{1}}+S_{(\rho_{2},\tau)}+S_{\text{aux}}$$
$$S_{\rho}=\int_{\Sigma}\frac{1}{2}\langle\rho_{A}\bar{\partial}\rho_{B}\rangle\Omega^{AB}+B_{ab}\lambda_{A}^{a}\rho_{B}^{b}\Omega^{AB}+b\langle\lambda_{A}\rho_{B}\rangle\Omega^{AB}$$
$$S_{(\rho,\tau)}=\int_{\Sigma}\frac{1}{2}\left(\langle\rho_{A}\bar{\partial}\rho_{B}\rangle\Omega^{AB}+\langle\tau_{\mathcal{J}}\bar{\partial}\tau_{\mathcal{J}}\rangle\Omega^{\mathcal{J}\mathcal{J}}\right)+D_{ab}\left(\lambda_{A}^{a}\rho_{B}^{b}\Omega^{AB}+\eta_{\mathcal{J}}^{a}\tau_{\mathcal{J}}^{b}\Omega^{\mathcal{J}\mathcal{J}}\right)$$
$$+d\left(\langle\lambda_{A}\rho_{B}\rangle\Omega^{AB}+\langle\eta_{\mathcal{J}}\tau_{\mathcal{J}}\rangle\Omega^{\mathcal{J}\mathcal{J}}\right)$$
$$S_{\text{aux}}=\sum_{i=3}^{4}\int_{\Sigma}\mathcal{Z}_{i}^{\mathrm{a}}\cdot\bar{\partial}\mathcal{Z}_{i\mathrm{a}}+\mathcal{A}_{i\mathrm{ab}}\mathcal{Z}_{i}^{\mathrm{a}}\cdot\mathcal{Z}_{i}^{\mathrm{b}}+\sum_{j=1}^{2}\left(\frac{1}{2}\rho_{ij\mathrm{a}}^{A}\bar{\partial}\rho_{ijA}^{\mathrm{a}}+\mathcal{B}_{ij}^{\mathrm{ab}}\lambda_{i\mathrm{a}}^{A}\rho_{ijA\,\mathrm{b}}+b_{ij}\lambda_{iA}^{\mathrm{a}}\rho_{ij\mathrm{a}}^{A}+\tilde{\tau}_{ij}^{A}\bar{\partial}\tau_{ijA}\right)$$
$$+a_{i}\lambda_{i\mathrm{a}}^{\alpha}\lambda_{i\alpha}^{\mathrm{a}}+\tilde{a}_{i}\tilde{\lambda}_{i\mathrm{a}}^{\dot{\alpha}}\tilde{\lambda}_{i\dot{\alpha}}^{\mathrm{a}}$$

$$\mathcal{Z}=(\lambda_{A},\mu^{A},\eta_{\iota}):\lambda_{A}=\left(\lambda_{\alpha},\tilde{\lambda}_{\dot{\alpha}}\right),\mu^{A}=\left(\tilde{\mu}^{\alpha},\mu^{\dot{\alpha}}\right),\eta_{\iota}=\left(\eta_{\mathcal{J}},\tilde{\eta}^{\mathcal{J}}\right),\eta_{\mathcal{J}}=(\eta_{I},\tilde{\eta}_{\dot{I}}),\tilde{\eta}^{\mathcal{J}}=\left(\tilde{\eta}'^{I},\eta'^{\dot{I}}\right),$$
$$I,\dot{I}=1\ldots\frac{\mathcal{N}}{2},\mathcal{J}=1,\ldots,\mathcal{N},\iota=1,\ldots,2\mathcal{N},\mathcal{N}=8,$$

$$\mathcal{Z}_{a}\cdot\mathcal{Z}_{b}=\frac{1}{2}\left(\tilde{Z}_{a}\cdot Z_{b}+\tilde{Z}_{b}\cdot Z_{a}+\tilde{\eta}_{a}^{\mathcal{J}}\eta_{b\mathcal{J}}+\tilde{\eta}_{b}^{\mathcal{J}}\eta_{a\mathcal{J}}\right),\tilde{Z}_{a}\cdot Z_{b}=\tilde{\mu}_{a}^{\alpha}\lambda_{b\alpha}+\tilde{\lambda}_{a\dot{\alpha}}\mu_{b}^{\dot{\alpha}}=\mu_{a}^{A}\lambda_{bA},$$

$$\Omega^{AB}=\Omega^{(\alpha,\dot{\alpha})(\beta,\dot{\beta})}\overset{\text{e.g.}}{=}\varepsilon^{\alpha\beta}\oplus\varepsilon^{\dot{\beta}\dot{\alpha}},\Omega_{AB}\overset{\text{e.g.}}{=}\varepsilon_{\beta\alpha}\oplus\varepsilon_{\dot{\alpha}\dot{\beta}},\Omega^{AB}\Omega_{BC}=\delta_{C}^{A}.$$

$$\Omega^{\mathcal{J}\mathcal{J}}=\begin{pmatrix}0 & \mathbb{1}^{I\dot{J}}\\ \mathbb{1}^{\dot{I}J} & 0\end{pmatrix}$$

$$\rho_{rA}^{a}(r=1,2)$$

$$\underbrace{\left|\mathcal{Z}_{\iota\mathrm{a}}=\left(\mu_{\mathrm{ia}}^{A},\lambda_{\iota\mathrm{a}A},\eta_{\iota a\mathcal{J}}\otimes\|\lambda_{\iota A}^{\mathrm{a}},\mu_{\mathrm{ia}}^{A},\|\odot\left\|\rho_{\iota jA}^{\mathrm{a}},\tau_{\iota jA},\tilde{\tau}_{ij}^{A}A\,\tilde{\tau}_{ij}^{A}\widehat{\mathrm{SL}(2,\mathbb{C}),}\right\|\circledast\left[\!\left[\rho_{\iota jA}^{\mathrm{a}},\tau_{\iota jA},\tilde{\tau}_{ij}^{A}\varepsilon_{ab}\varepsilon^{ab}\right]\!\right]\right|\boxplus(\alpha,\dot{\alpha})\,\eta_{\iota\mathcal{J}}^{\mathrm{a}}\,\Omega_{\iota}^{\mathcal{J}\mathcal{J}}.\tau_{\iota jA}\left|\right)\right\rangle}$$
$$\left\langle\left(B_{ab},b,D_{ab},d,\mathcal{B}_{j\iota}^{\mathrm{ab}},b_{j\iota}\right)\boxtimes\left\|\lambda_{A}^{(a}\rho_{1B}^{b)}\Omega^{AB}\,\langle\lambda_{A}\rho_{1B}\rangle\Omega^{AB}\,\lambda_{A}^{(a}\rho_{2B}^{b)}\Omega^{AB}\right\|\boxdot\left[\!\left[\eta_{\mathcal{J}}^{(a}\tau_{\mathcal{J}}^{b)}\Omega^{\mathcal{J}\mathcal{J}}\langle\lambda_{A}\rho_{2B}\rangle\Omega^{AB}\right]\!\right]\boxplus\langle\eta_{\mathcal{J}}\tau_{\mathcal{J}}\rangle\Omega^{\mathcal{J}\mathcal{J}}\lambda_{\iota A}^{(\mathrm{a}}\rho_{\iota jB}^{\mathrm{b})}\Omega^{AB}\lambda_{\iota\mathrm{a}A}\rho_{\iota jB}^{\mathrm{a}}\Omega^{AB}A_{ab}A_{(ab)}\mathcal{Z}_{\iota\mathrm{a}}\mathcal{A}_{\iota\mathrm{ab}}\mathcal{A}_{\iota(\mathrm{ab})}a\tilde{a}a_{\iota}\tilde{a}\lambda_{\iota A}^{\mathrm{a}}\right|$$

$$\mathcal{Z}_{a}^{A}(z)\cdot\mathcal{Z}_{B}^{b}(0)=\frac{\delta_{B}^{A}\delta_{a}^{b}}{z}+\cdots\tilde{\tau}_{a}^{\mathcal{J}}(z)\tau_{\mathcal{J}}^{b}(0)=\frac{\delta_{\mathcal{J}}^{\mathcal{J}}\delta_{a}^{b}}{z}+\cdots$$

$$\left\{\left(M^{ab},N_{ab}\right),\left(\mathcal{M}_{i}^{\mathrm{ab}},\mathcal{N}_{i\mathrm{ab}}\right),(m,n),(\tilde{m},\tilde{n}),(m_{i},n_{i}),(\tilde{m}_{i},\tilde{n}_{i})\right\}$$

$$\left\{\left(\beta^{ab},\gamma_{ab}\right),(\beta,\gamma),\left(\tilde{\beta}^{ab},\tilde{\gamma}ab\right),(\tilde{\beta},\tilde{\gamma}),\left(\beta_{ij}^{\mathrm{ab}},\gamma_{ij\mathrm{ab}}\right),\left(\beta_{ij},\gamma_{ij}\right)\right\}.$$

$$J_{BRST}=cT+N_{ab}\left(J^{ab}+M_{c}^{a}N^{bc}\right)+n\langle\lambda_{A}\lambda^{A}\rangle+\tilde{n}\left(\langle\lambda_{A}\lambda^{A}\rangle+\langle\eta_{\mathcal{J}}\eta^{\mathcal{J}}\rangle\right)+\gamma_{ab}\lambda^{aA}\rho_{1A}^{b}+\gamma\langle\lambda^{A}\rho_{1A}\rangle$$
$$+m\gamma\gamma+\tilde{\gamma}_{ab}\left(\lambda^{aA}\rho_{2A}^{b}+\eta^{a\mathcal{J}}\tau_{\mathcal{J}}^{b}\right)+\tilde{\gamma}\left(\langle\lambda^{A}\rho_{2A}\rangle+\langle\eta^{\mathcal{J}}\tau_{\mathcal{J}}\rangle\right)+\tilde{m}\tilde{\gamma}\tilde{\gamma}+\sum_{i=3}^{4}\left(\mathcal{N}_{iab}\left(J_{i}^{ab}+\mathcal{M}_{ic}^{\mathrm{a}}\mathcal{N}_{i}^{bc}\right)\right.$$
$$\left.+\sum_{j=1}^{2}\left(\gamma_{ij\mathrm{ab}}\lambda_{i}^{\mathrm{a}A}\rho_{ijA}^{\mathrm{b}}+\gamma_{ij}\lambda_{iA}^{\mathrm{a}}\rho_{ij\mathrm{a}}^{A}+(m_{i}-\tilde{m}_{i})\gamma_{ij}\gamma_{ij}\right)+n_{i}\lambda_{i\mathrm{a}}^{\alpha}\lambda_{i\alpha}^{\mathrm{a}}+\tilde{n}_{i}\tilde{\lambda}_{i\mathrm{a}}^{\dot{\alpha}}\tilde{\lambda}_{i\dot{\alpha}}^{\mathrm{a}}\right),$$
$$T=\mathcal{Z}^{a}\cdot\partial\mathcal{Z}_{a}+\frac{1}{2}\sum_{r=1,2}\langle\rho_{rA}\partial\rho_{r}^{A}\rangle+\beta^{ab}\partial\gamma_{ab}+\beta\partial\gamma+\frac{1}{2}\langle\tau_{\mathcal{J}}\partial\tau^{\mathcal{J}}\rangle+\tilde{\beta}^{ab}\partial\tilde{\gamma}_{ab}+\tilde{\beta}\partial\tilde{\gamma}$$

$$+M^{ab}\partial N_{ab}+m\partial n+\tilde{m}\partial\tilde{n}+\frac{1}{2}(b\partial c+\partial(bc))+\sum_{i=3}^{4}\left(\mathcal{M}_i^{\mathrm{ab}}\partial\mathcal{N}_{i\mathrm{ab}}+\mathcal{Z}_i^{\mathrm{a}}\cdot\partial\mathcal{Z}_{i\mathrm{a}}\right.$$

$$\left.+\sum_{j=1}^{2}\left(\frac{1}{2}\left(\rho_{ij\mathrm{a}}^{A}\partial\rho_{ijA}^{\mathrm{a}}+\beta_{ij}^{\mathrm{ab}}\partial\gamma_{ij\mathrm{ab}}+\beta_{ji}\partial\gamma_{ij}+\tilde{\tau}_{ij}^{A}\partial\tau_{ijA}\right)+m_i\partial n_i+\tilde{m}_i\partial\tilde{n}_i\right)\right),$$

$$J^{ab}=\mathcal{Z}^a\cdot\mathcal{Z}^b+\frac{1}{2}\sum_{r=1,2}\rho_{rA}^{(a}\rho_r^{b)A}+\beta^{c(a}\gamma_c^{b)}+\frac{1}{2}\tau_{\mathcal{J}}^{(a}\tau^{b)\mathcal{J}}+\tilde{\beta}^{c(a}\tilde{\gamma}_c^{b)},$$

$$J_i^{\mathrm{ab}}=\mathcal{Z}_i^{\mathrm{a}}\cdot\mathcal{Z}_i^{\mathrm{b}}+\sum_{j=1}^{2}\left(\frac{1}{2}\rho_{ijA}^{(\mathrm{a}}\rho_{ij}^{\mathrm{b})A}+\beta_{ij}^{\mathrm{c(a}}\gamma_{ijc}^{\mathrm{b)}}\right),$$

$$c_{\mathrm{aux}}=\sum_{i=3}^{4}\left(-6_{\mathcal{M}_i\mathcal{N}_i}-(8-\mathcal{N})_{\mathcal{Z}_i}-2_{m_in_i}-2_{\tilde{m}_i\tilde{n}_i}+2\left(4_{\rho_{ij}}+8_{\beta_{ij}\gamma_{ij}}-4_{\tau_{ij}}\right)\right)=2(\mathcal{N}-2)$$

$$\left(\mathrm{tr}_{\mathrm{adj}}\left(t^kt^k\right)=6,\mathrm{trf}_{\mathrm{F}}\left(t^kt^k\right)=\frac{3}{2}\right)a_i^{sl2}$$

$$=\frac{1}{2}\mathrm{trf}_{\mathrm{F}}\left(t^kt^k\right)\left((8-\mathcal{N})_{\mathcal{Z}_i}-4_{\rho_{i1}}-4_{\rho_{i2}}\right)$$

$$+\mathrm{tr}_{\mathrm{adj}}\left(t^kt^k\right)\left(-1_{\mathcal{M}_i\mathcal{N}_i}+1_{\beta_{i1}\gamma_{i1}}+1_{\beta_{i2}\gamma_{i2}}\right)=\frac{3}{4}(8-\mathcal{N})$$

$$\mathcal{V}=\int\mathrm{d}^2u\,\mathrm{d}^2v\mathcal{W}(u)\mathcal{W}_{\mathrm{aux}}\bar{\delta}(\langle v\epsilon\rangle-1)\bar{\delta}^4(\langle u\lambda_A\rangle-\langle v\kappa_A\rangle)\bar{\delta}^{\mathcal{N}}(\langle u\eta_{\mathcal{J}}\rangle-\langle v\zeta_{\mathcal{J}}\rangle)e^{\langle u\mu^A\rangle\epsilon_A+\langle u\tilde{\eta}^{\mathcal{J}}\rangle q_{\mathcal{J}}}$$

$$\mathcal{W}(u)=\mathrm{cn}\tilde{n}\delta(\gamma)\delta\left(\gamma_{ab}u^au^b\right)\delta(\tilde{\gamma})\delta\left(\tilde{\gamma}_{ab}u^au^b\right)$$

$$\mathcal{W}_{\mathrm{aux}}=\prod_{i=3}^{4}n_i\tilde{n}_i\int\mathrm{d}u_i'^2\bar{\delta}\left(u_i'^{\mathrm{a}}-\chi_i^A\lambda_{iA}^{\mathrm{a}}\right)\prod_{j=1}^{2}\delta(\gamma_{ij})\delta\left(\gamma_{ij\mathrm{ab}}u_i'^{\mathrm{a}}u_i'^{\mathrm{b}}\right)\frac{1}{2}\left(\chi_i^A\rho_{ijA}^{\mathrm{a}}\right)\left(\chi_i^A\rho_{ijA\mathrm{a}}\right)$$

$$\Upsilon_\beta(z)=\delta(\beta)[Q_B,\beta],\qquad\Upsilon_{\hat{u}\beta}(z)=\delta\left(\beta^{ab}\hat{u}_a\hat{u}_b\right)[Q_B,\beta^{ab}\hat{u}_a\hat{u}_b]$$

$$\Upsilon_{\tilde{\beta}}(z)=\delta(\tilde{\beta})\left[Q_B,\tilde{\beta}\right],\qquad\Upsilon_{\hat{u}\tilde{\beta}}(z)=\delta\left(\tilde{\beta}^{ab}\hat{u}_a\hat{u}_b\right)\left[Q_B,\tilde{\beta}^{ab}\hat{u}_a\hat{u}_b\right]$$

$$\Upsilon_m(z)=m\delta(\{Q_B,m\}),\qquad\Upsilon_{\tilde{m}}(z)=\tilde{m}\delta(\{Q_B,\tilde{m}\}),\Upsilon_{M^{ab}}(z)=M^{ab}\delta\left(\{Q_B,M^{ab}\}\right)$$

$$\Upsilon_{m_i}(z)=m_i\delta(\{Q_B,m_i\}),\quad\Upsilon_{\tilde{m}_i}(z)=\tilde{m}_i\delta(\{Q_B,\tilde{m}_i\}),\Upsilon_{\mathcal{M}_i^{\mathrm{ab}}}(z)=\mathcal{M}_i^{\mathrm{ab}}\delta\left(\left\{Q_B,\mathcal{M}_i^{\mathrm{ab}}\right\}\right),$$

$$\Upsilon_{\beta_{ij}}(z)=\delta\left(\beta_{ij}\right)\left[Q_B,\beta_{ij}\right],\quad\Upsilon_{\hat{u}'\beta_{ij}}(z)=\delta\left(\beta_{ij}^{\mathrm{ab}}\hat{u}_{i\mathrm{a}}'\hat{u}_{i\,\mathrm{b}}'\right)\left[Q_B,\beta_{ij}^{\mathrm{ab}}\hat{u}_{i\mathrm{a}}'\hat{u}_{i\,\mathrm{b}}'\right].$$

$$\lim_{x,y\to\sigma}\int \mathrm{d}u_i'^2\,\bar{\delta}\big(u_i'^{\mathrm{a}}-\chi_i^A\lambda_{iA}^{\mathrm{a}}\big)\Upsilon_{\beta_{ij}}(x)\Upsilon_{u_i'\beta_{ij}}(y)\delta\big(\gamma_{ij}(\sigma)\big)\delta\big(\gamma_{ij\mathrm{ab}}(\sigma)u_i'^{\mathrm{a}}u_i'^{\mathrm{b}}\big)\frac{1}{2}\big(\chi_i^A\rho_{ijA}^{\mathrm{a}}(\sigma)\big)\big(\chi_i^A\rho_{ijA\mathrm{a}}(\sigma)\big)$$

$$=\lim_{x,y\to\sigma}\int \mathrm{d}u_i'^2\,\bar{\delta}\big(u_i'^{\mathrm{a}}-\chi_i^A\lambda_{iA}^{\mathrm{a}}\big)\delta\big(\beta_{ij}(x)\big)\delta\big(\beta_{ij}^{\mathrm{ab}}(y)\hat{u}_{i\mathrm{a}}'\hat{u}_{i\,\mathrm{b}}'\big)\delta\big(\gamma_{ij}(\sigma)\big)\delta\big(\gamma_{ij\mathrm{ab}}(\sigma)u_i'^{\mathrm{a}}u_i'^{\mathrm{b}}\big)$$

$$\big(\lambda_{i\mathrm{a}}^A(x)\rho_{ijA}^{\mathrm{a}}(x)\big)\big(\lambda_{iB}^{\mathrm{a}}(y)\rho_{ij}^{\mathrm{b}B}(y)\hat{u}_{i\mathrm{a}}'\hat{u}_{i\,\mathrm{b}}'\big)\frac{1}{2}\big(\chi_i^A\rho_{ijA}^{\mathrm{a}}(\sigma)\big)\big(\chi_i^A\rho_{ijA\mathrm{a}}(\sigma)\big)$$

$$=\int \mathrm{d}u_i'^2\,\bar{\delta}\big(u_i'^{\mathrm{a}}-\chi_i^A\lambda_{iA}^{\mathrm{a}}\big)\frac{\big(\chi_i^A\lambda_{iA}^{\mathrm{a}}(\sigma)\hat{u}_{i\mathrm{a}}'\big)^2}{\big(u_{i\mathrm{a}}'\hat{u}_i'^{\mathrm{a}}\big)^2}=1$$

$$\lim_{x,y\to\sigma} n_i\Upsilon_{m_i}(x)\tilde{n}_i\Upsilon_{\tilde{m}_i}(y)=\delta\big(\mathrm{Res}_\sigma(\lambda_i^\alpha\lambda_{i\alpha})\big)\delta\left(\mathrm{Res}_\sigma\big(\tilde{\lambda}_i^{\dot{\alpha}}\tilde{\lambda}_{i\dot{\alpha}}\big)\right)$$

$$V=\int \mathrm{d}\sigma\,\mathrm{d}^2u\,\mathrm{d}^2v w(u)\bar{\delta}(\langle v\epsilon\rangle-1)\bar{\delta}^4(\langle u\lambda_A\rangle-\langle v\kappa_A\rangle)\bar{\delta}^{\mathcal{N}}(\langle u\eta_\jmath\rangle-\langle v\zeta_\jmath\rangle)e^{\langle u\mu^A\rangle\epsilon_A+\langle u\tilde{\eta}^\jmath\rangle q_\jmath}$$

$$w(u)=\delta\big(\mathrm{Res}_\sigma(\langle\lambda_A\lambda_B\rangle\Omega^{AB})\big)\delta\left(\mathrm{Res}_\sigma\big(\langle\eta_\jmath\eta_\jmath\rangle\Omega^{\jmath\jmath}\big)\right)\left(\frac{\langle\hat{u}\lambda_A\rangle\epsilon^A}{\langle u\hat{u}\rangle}+\frac{\langle\rho_{1A}\rho_{1B}\rangle}{2}\epsilon^A\epsilon^B\right)$$

$$\left(\frac{\langle\hat{u}\lambda_A\rangle\epsilon^A}{\langle u\hat{u}\rangle}+\frac{\langle\rho_{2A}\rho_{2B}\rangle}{2}\epsilon^A\epsilon^B+\frac{\langle\hat{u}\eta_\jmath\rangle q^\jmath}{\langle u\hat{u}\rangle}+\frac{\langle\tau_\jmath\mathcal{T}_\jmath\rangle}{2}q^\jmath q^\jmath\right)$$

$$\int \mathrm{d}u_1'^2\,\mathrm{d}u_2'^2\,\mathrm{d}u_3'^2\frac{\bar{\delta}\left(u_1'^{\mathrm{a}}-\chi_{i1}^A\lambda_{iA}^{\mathrm{a}}(\sigma_1)\right)\bar{\delta}\left(u_2'^{\mathrm{a}}-\chi_{i2}^A\lambda_{iA}^{\mathrm{a}}(\sigma_2)\right)\bar{\delta}\left(u_3'^{\mathrm{a}}-\chi_{i3}^A\lambda_{iA}^{\mathrm{a}}(\sigma_3)\right)}{[u_1'u_2']^2[u_2'u_3']^2[u_3'u_1']^2}\sigma_{12}^2\sigma_{31}^2$$

$$\prod_{j=1}^{2}\frac{1}{2}\big(\chi_{i1}^A[\rho_{ijA}(\sigma_1)\rho_{ijB}(\sigma_1)]\chi_{i1}^B\big)\big(\chi_{i2}^A[\rho_{ijA}(\sigma_2)\lambda_{iB}(\sigma_2)]\chi_{i2}^B\big)\big(\chi_{i3}^A[\rho_{ijA}(\sigma_3)\lambda_{iB}(\sigma_3)]\chi_{i3}^B\big)$$

$$=\int \mathrm{d}u_1'^2\,\mathrm{d}u_2'^2\,\mathrm{d}u_3'^2\frac{\prod_{k=1}^3\bar{\delta}\left(u_k'^{\mathrm{a}}-\chi_{ik}^A\lambda_{iA}^{\mathrm{a}}(\sigma_k)\right)}{[u_1'u_2']^2[u_2'u_3']^2[u_3'u_1']^2}(\chi_{i1}\chi_{i2})^2(\chi_{i1}\chi_{i3})^2\big(\chi_{i2}^A[\lambda_{iA}(\sigma_2)\lambda_{iB}(\sigma_3)]\chi_{i3}^B\big)^2$$

$$=\frac{(\chi_{i1}\chi_{i2})^2(\chi_{i1}\chi_{i3})^2}{\big(\chi_{i1}^A[\lambda_{iA}(\sigma_1)\lambda_{iB}(\sigma_2)]\chi_{i2}^B\big)^2\big(\chi_{i3}^A[\lambda_{iA}(\sigma_3)\lambda_{iB}(\sigma_1)]\chi_{i1}^B\big)^2}$$

$$=\frac{(\chi_{i1}\chi_{i2})^2(\chi_{i1}\chi_{i3})^2}{\big(\chi_{i1}^A[\kappa_{i1A}'\kappa_{i2B}']\chi_{i2}^B\big)^2\big(\chi_{i1}^A[\kappa_{i1A}'\kappa_{i3B}']\chi_{i3}^B\big)^2}$$

$$\lim_{\kappa_{ij}',\chi_{ij}\to\kappa_i',\chi_i}\frac{(\chi_{i1}\chi_{i2})^2(\chi_{i1}\chi_{i3})^2}{\big(\chi_{i1}^A[\kappa_{i1A}'\kappa_{i2B}']\chi_{i2}^B\big)^2\big(\chi_{i1}^A[\kappa_{i1A}'\kappa_{i3B}']\chi_{i3}^B\big)^2}=\frac{1}{16\big(k_i'^0\big)^4}$$

$$\oiiint_{a_i}\omega_j=\delta_{ij},\oiiint_{b_i}\omega_i=\tau_{ij}.$$

$$M=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\in \mathrm{Sp}(2g,\mathbb{Z})$$

$$\theta[\alpha](\mathbf{z})=\sum_{m\in\mathbb{Z}^g}\exp\left[i\pi(m+\alpha')^T\tau(m+\alpha')+2\pi i(m+\alpha')(\mathbf{z}+\alpha'')\right],$$

$$\theta[\alpha](-\mathbf{z})=(-1)^{4\alpha'\cdot\alpha''}\theta[\alpha](\mathbf{z}),\alpha=\begin{pmatrix}\alpha'\\ \alpha''\end{pmatrix},\alpha',\alpha''\in(\mathbb{Z}/2\mathbb{Z})^g,$$

$$\sum_{i=j}^{n} c_j z_j \to \mathbf{z} = \sum_{i=j}^{n} c_j \int_{z_0}^{z_j} \omega_i \in \mathbb{C}^g$$

$$\Delta \in \mathbb{C}^g, \Delta_i = \frac{1+\tau_{ii}}{2} - \sum_{j \neq i} \oint_{a_j} \omega_j(z) \int_{z_0}^{z} \omega_i$$

$$\theta(z) = 0 \iff \exists z_1, \dots, z_{g-1} \text{ with } z = \Delta - \sum_{i=1}^{g-1} z_i.$$

$$E(z,w) = \frac{\theta[\alpha](z-w)}{h_\alpha(z)h_\alpha(w)}, h_\alpha(z) = \left(\sum_{i=1}^{g} \partial_i \theta[\alpha](0)\omega_i(z)\right)^{\frac{1}{2}},$$

$$S_\alpha(z,w \mid \tau) = \frac{\theta[\alpha](z-w)}{E(z,w)\theta[\alpha](0)}, \qquad \alpha \text{ even}$$

$$S_\alpha(z,w \mid \tau) = \frac{1}{E(z,w)} \frac{\sum_{i=1}^{g} \partial_i \theta[\alpha](z-w)\omega_i(y))}{\sum_{i=1}^{g} \partial_i \theta[\alpha](0)\omega_i(y)}, \qquad \alpha \text{ odd}$$

$$Z_\lambda(z_1, \dots, z_G) = \int \mathrm{D}b\mathrm{D}c b(z_1) \dots b(z_G) \exp\left(-\frac{1}{2\pi}\int \mathrm{d}^2 z \sqrt{g} b(z) \bar{\nabla}_{1-\lambda} c(z)\right), \qquad \lambda > 1,$$

$$Z_1(z_1, \dots, z_g, w) = \int \mathrm{D}\beta\mathrm{D}\gamma \beta(z_1) \dots \beta(z_g)\gamma(w) \exp\left(-\frac{1}{2\pi}\int \mathrm{d}^2 z \sqrt{g} \beta(z) \bar{\nabla}_{1-\lambda} \gamma(z)\right),$$

$$Z^{\text{even}}_{\frac{1}{2},\alpha} = \int \mathrm{D}\chi\mathrm{D}\psi \exp\left(-\frac{1}{2\pi}\int \mathrm{d}^2 z \sqrt{g} \chi(z) \bar{\nabla}_{\frac{1}{2}} \psi(z)\right),$$

$$Z^{\text{odd}}_{\frac{1}{2},\alpha}(w) = \int \mathrm{D}\chi\mathrm{D}\psi \chi(w)\psi(w) \exp\left(-\frac{1}{2\pi}\int \mathrm{d}^2 z \sqrt{g} \chi(z) \bar{\nabla}_{\frac{1}{2}} \psi(z)\right),$$

$$Z_{\lambda>1}(z_1, \dots, z_G) = \tilde{Z}_1^{-\frac{1}{2}} \theta[\alpha]\left(\sum_{i=1}^{G} z_i - Q\Delta\right) \prod_{j>i=1}^{G} E(z_i, z_j) \prod_{i=1}^{G} \sigma(z_i)^Q$$

$$Z_1(z_1, \dots, z_g, w) = \tilde{Z}_1^{-\frac{1}{2}} \theta\left(\sum_{i=1}^{g} z_i - w - \Delta\right) \frac{\prod_{j>i=1}^{g} E(z_i, z_j) \prod_{i=1}^{g} \sigma(z_i)}{\prod_{i=1}^{g} E(z_i, w)\sigma(w)}$$

$$Z^{\text{even}}_{\frac{1}{2},\alpha} = \tilde{Z}_1^{-\frac{1}{2}} \theta[\alpha](0), Z^{\text{odd}}_{\frac{1}{2},\alpha}(z) = \left(\tilde{Z}_1^{\frac{1}{2}} \det{}_\omega\right)^{-1} h_\alpha(z)^2$$

$$\tilde{Z}_1 = \frac{Z_1(z_1, \dots, z_g, w)}{\det_\omega}, \det{}_\omega \equiv \det\left\|\omega_i(z_j)\right\|, \int_\Sigma \mathrm{d}^2 z \bar{\omega}_i \omega_j(z) = <\omega_i \mid \omega_j> = 2i\mathrm{Im}\tau_{ij}$$

$$\frac{\sigma(z)}{\sigma(w)} = \frac{\theta\left(z - \sum_{i=1}^{g} r_i - \Delta\right)}{\theta\left(w - \sum_{i=1}^{g} r_i - \Delta\right)} \prod_{i=1}^{g} \frac{E(w, r_i)}{E(z, r_i)}$$

$$\det\bar{\nabla}_{1-\lambda}=\frac{Z_\lambda(z_1,\ldots,z_G)}{\det\|\phi_i(z_j)\|}\cdot\phi_1\wedge\ldots\wedge\phi_G \qquad \lambda>1$$

$$\det\bar{\nabla}_0=\frac{Z_1(z_1,\ldots,z_g,w)}{\det_\omega e}\cdot\omega_1\wedge\ldots\wedge\omega_g\otimes e=\tilde{Z}_1\cdot\omega_1\wedge\ldots\wedge\omega_g\otimes e$$

$$\det\bar{\nabla}^{\mathrm{ev}}_{\frac{1}{2},\alpha}=Z^{\mathrm{even}}_{\frac{1}{2},\alpha},\det\bar{\nabla}^{\mathrm{odd}}_{\frac{1}{2},\alpha}=\frac{Z^{\mathrm{odd}}_{\frac{1}{2},\alpha}(z)}{h_\alpha(z)^2}\cdot h_\alpha\otimes h_\alpha$$

$$\det_{[\mathrm{bos}]}\bar{\nabla}_{1-\lambda}=\left(\det_{[\mathrm{ferm}]}\bar{\nabla}_{1-\lambda}\right)^{-1}$$

$$Z_\lambda(\tilde{z}_1,\ldots,\tilde{z}_G)=\epsilon^{\frac{2}{3}}Z_\lambda(z_1,\ldots,z_G),(\lambda\in\mathbb{Z}),$$

$$Z^{\mathrm{odd}}_{\frac{1}{2},\tilde{\alpha}}(\tilde{z})=\epsilon^{\frac{2}{3}}e^{i\pi\phi(\alpha)}\det\|C\tau+D\|Z^{\mathrm{odd}}_{\frac{1}{2},\alpha}(z),$$

$$Z^{\mathrm{even}}_{\frac{1}{2},\tilde{\alpha}}=\epsilon^{\frac{2}{3}}e^{i\pi\phi(\alpha)}Z^{\mathrm{even}}_{\frac{1}{2},\alpha},$$

$$\text{with }\tilde{z}_i=(\tau C^T+D^T)^{-1}_{ij}z_j,\tilde{\omega}_i=\omega_j(C\tau+D)^{-1}_{ji},\tilde{\Delta}-\tilde{\tau}\delta_1-\delta_2=(\tau C^T+D^T)^{-1}\Delta,$$

$$\tilde{\alpha}=\alpha'+\delta,\begin{bmatrix}\alpha'_1\\ \alpha'_2\end{bmatrix}=\begin{bmatrix}D & -C\\ -B & A\end{bmatrix}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix},\begin{bmatrix}\delta_1\\ \delta_2\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\mathrm{diag}CD^T\\ \mathrm{diag}AB^T\end{bmatrix},$$

$$\epsilon^8=1,\phi(\alpha)=\alpha'_1\cdot\alpha'_2-\alpha_1\cdot\alpha_2+2\alpha'_1\cdot\delta_2\in\frac{\mathbb{Z}_2}{4}.$$

$$\tilde{Z}_2\det_\omega=\frac{Z_2}{\det_\omega}=Z_1=\tilde{Z}_1\det_\omega=\tilde{Z}_1^{-\frac{1}{2}}\frac{\theta(z-w)}{E(z,w)}=\tilde{Z}_1^{-\frac{1}{2}}\theta'(0)\det_\omega=Z^{\mathrm{odd}}_{\frac{1}{2},\alpha}(z)\det_\omega,$$

$$\tilde{Z}_2=\frac{Z_2}{\det^2_\omega}=\frac{Z_1}{\det_\omega}=\tilde{Z}_1=Z^{\mathrm{odd}}_{\frac{1}{2}}=\eta(\tau)^2,Z^{\mathrm{even}}_{\frac{1}{2},\alpha}=\frac{\theta[\alpha](0)}{\eta(\tau)}$$

$$\mathrm{d}(\mathrm{WP})\left(\det\hat{P}_1^\dagger\hat{P}_1\right)^{\frac{1}{2}}=W\wedge\bar{W},W=\mathrm{d}m^{3(g-1)}\int\mathrm{d}b\,\mathrm{d}c\prod_{i=1}^{3(g-1)}(\mu_i,b)\exp\left[\int\mathrm{d}^2zb(z)\bar{\nabla}_{-1}c(z)\right]$$

$$W=\mathrm{d}m^{3(g-1)}\frac{\det\|(\mu_i,\phi_j)\|}{\det\|\phi_i(z_j)\|}Z_2\left(z_1,\ldots,z_{3(g-1)}\right)=\det\bar{\nabla}_{-1}$$

$$\mathrm{d}\mu_g=\frac{\phi_1\wedge\ldots\wedge\phi_{3(g-1)}}{\det\|\phi_i(z_j)\|}=\mathrm{d}m^{3(g-1)}\frac{\det\|(\mu_i,\phi_j)\|}{\det\|\phi_i(z_j)\|}$$

$$Z_{g\geq 2}=\left\langle \prod_{i=1}^{3(g-1)} b(y_i)\prod_{j=1}^{g}\Upsilon_m(x_j)\Upsilon_{\tilde{m}}(\tilde{x}_j)\Upsilon_{M_{00}}(w_{1j})\Upsilon_{M_{11}}(w_{2j})\Upsilon_{M_{(01)}}(w_{3j})\right.$$

$$\left(\prod_{k=3}^{4}\Upsilon_{m_k}(x_{kj})\Upsilon_{\tilde{m}_k}(\tilde{x}_{kj})\Upsilon_{\mathcal{M}_{k00}}(w_{1kj})\Upsilon_{\mathcal{M}_{i11}}(w_{2kj})\Upsilon_{\mathcal{M}_{i(01)}}(w_{3kj})\right.$$

$$\left.\left.\prod_{l=1}^{2}\Upsilon_{\beta_{kl}}(r_{klj})\Upsilon_{(0,1)\beta_{kl}}(r_{1klj})\Upsilon_{(-1,1)\beta_{kl}}(r_{2klj})\Upsilon_{(-1,0)\beta_{kl}}(r_{3klj})\right)\right.$$

$$\left.\Upsilon_{\beta}(z_j)\Upsilon_{(0,1)\beta}(z_{1j})\Upsilon_{(-1,1)\beta}(z_{2j})\Upsilon_{(-1,0)\beta}(z_{3j})\Upsilon_{\tilde{\beta}}(\tilde{z}_j)\Upsilon_{(0,1)\tilde{\beta}}(\tilde{z}_{1j})\Upsilon_{(-1,1)\tilde{\beta}}(\tilde{z}_{2j})\Upsilon_{(-1,0)\tilde{\beta}}(\tilde{z}_{3j})\right\rangle$$

$$=\sum_{\alpha}\int \mathrm{d}\mu_g\frac{Z_2((y))Z_1((x))^6Z_1((w))^9}{Z_1((z))^8Z_1((r))^{16}}\left\langle\prod^{g}\left(\bar{\delta}_{\mathrm{tot}}j_{\mathrm{tot}}\right)\right\rangle_{\alpha}$$

$$\bar{\delta}_{\mathrm{tot}}=\bar{\delta}\left(\langle\lambda^A\lambda_A\rangle+\langle\eta^{\mathcal{I}}\eta_{\mathcal{I}}\rangle\right)\ldots, j_{\mathrm{tot}}=\langle\lambda^A\rho_{2A}+\eta^{\mathcal{I}}\tau_{\mathcal{I}}\rangle\ldots$$

$$Z^{\mathrm{ev}}_{g=1}=\sum_{\alpha}\int\frac{\mathrm{d}\mu_1Z_2Z_1^{-9}\left(Z^{\mathrm{even}}_{\frac{1}{2},\alpha}\right)^8}{\mathrm{Vol(CKV)_{chir}}}\langle\bar{\delta}_{\mathrm{tot}}j_{\mathrm{tot}}\rangle_{\alpha}=\sum_{\alpha}\int\frac{\mathrm{d}\mu_1\det_{\omega}^{-7}}{\mathrm{Vol(CKV)_{chir}}}\left(\frac{\theta_{\alpha}(0)}{\eta(\tau)}\right)^8\frac{\langle\bar{\delta}_{\mathrm{tot}}j_{\mathrm{tot}}\rangle_{\alpha}}{\eta(\tau)^{16}},$$

$$Z^{\mathrm{odd}}_{g=1}=\int\frac{\mathrm{d}\mu_1Z_2Z_1^{-9}\left(Z^{\mathrm{odd}}_{\frac{1}{2}}\right)^8}{\mathrm{Vol(CKV)_{chir}}}\mathrm{d}^{16}\psi\langle\bar{\delta}_{\mathrm{tot}}j_{\mathrm{tot}}\rangle=\int\frac{\mathrm{d}\mu_1\det_{\omega}^{-7}\,\mathrm{d}^{16}\psi}{\mathrm{Vol(CKV)_{chir}}}\langle\bar{\delta}_{\mathrm{tot}}j_{\mathrm{tot}}\rangle$$

$$\mathrm{d}\mu_1=\mathrm{d}\tau\det_{\omega}^{-2}$$

$$Z^{\mathrm{ev}}_{g=1}=\sum_{\tilde{\alpha}}\int\mathrm{d}\tilde{\tau}\frac{\langle\bar{\delta}_{\mathrm{tot}}j_{\mathrm{tot}}\rangle_{\alpha}}{\det_{\omega}^{10}}\left(\frac{\theta_{\tilde{\alpha}}(0)}{\eta(\tilde{\tau})^3}\right)^8=\sum_{\alpha}\int\frac{\mathrm{d}\tau}{M_{\tau}^2}\frac{\langle\bar{\delta}_{\mathrm{tot}}j_{\mathrm{tot}}\rangle_{\alpha}}{(\det_{\omega}M_{\tau}^{-1})^{10}}\left(\frac{\theta_{\alpha}(0)}{\eta(\tau)^3M_{\tau}}\right)^8$$

$$Z^{\mathrm{odd}}_{g=1}=\int\mathrm{d}\tilde{\tau}\frac{\mathrm{d}^{16}\tilde{\psi}}{\det_{\tilde{\omega}}^{10}}\langle\bar{\delta}_{\mathrm{tot}}j_{\mathrm{tot}}\rangle=\int\frac{\mathrm{d}\tau}{M_{\tau}^2}\frac{\mathrm{d}^{16}\psi M_{\tau}^{-8}}{(\det_{\omega}M_{\tau}^{-1})^{10}}\langle\bar{\delta}_{\mathrm{tot}}j_{\mathrm{tot}}\rangle$$

$$\tilde{\tau}=\frac{A\tau+B}{C\tau+D},M_{\tau}=\det\|C\tau+D\|$$

$$A_{g,n}=\langle V_1\ldots V_n\rangle=\sum_{\alpha}\int\mathrm{d}\mu_gZ_g^{\mathrm{chir}}((z),(y),(x),(w),(r))A_g^{\alpha}((z),(x),(w),(r))$$

$$Z_g^{\mathrm{chir}}=\frac{Z_2((y))Z_1((x))^6Z_1((w))^9}{Z_1((z))^8Z_1((r))^{16}\tilde{Z}_1^4}=\frac{Y_3(y)Y(x)Y(\tilde{x})\prod_{k=3}^{4}Y(x_k)Y(x_k)\prod_{j=1}^{3}Y(w_j)\prod_{k=3}^{4}Y(w_{jk})}{Y(z)Y(\tilde{z})\prod_{j=1}^{3}Y(z_j)Y(\tilde{z}_j)\prod_{k=3}^{4}\prod_{l=1}^{2}Y(r_{kl})\prod_{j=1}^{3}Y(r_{jkl})}$$

$$A_g^{\alpha}((z),(x),(w),(r))=\int\mathrm{d}m_0\tilde{Z}_1^4\left\langle\prod_{j=1}^{g}\left(\bar{\delta}_{x_j}\bar{\delta}'_{w_j}j_{z_j}\tilde{j}_{\tilde{z}_j}j'_{r_j}\right)V_1\ldots V_n\right\rangle_{\alpha}$$

$$Y_Q(y)\equiv\theta\left(\Delta_y^Q\right)\prod_{j>i=1}^{Q(g-1)}E(y_i,y_j)\prod_{i=1}^{Q(g-1)}\sigma(y_i)^Q, Y(a)\equiv\theta(\Delta_a)\frac{\prod_{j>i=1}^{g}E(a_i,a_j)\prod_{i=1}^{g}\sigma(a_i)}{\prod_{i=1}^{g}E(a_i,a_0)\sigma(a_0)}$$

$$\Delta_a^Q \equiv \sum_{i=1}^{Q(g-1)} a_i - Q\Delta, \Delta_a \equiv \sum_{i=1}^{g} a_i - a_0 - \Delta$$

$$\bar{\delta}_{x_j} = \bar{\delta}(\langle \lambda^A(x_j)\lambda_A(x_j)\rangle)\bar{\delta}(\langle \lambda^A(\tilde{x}_j)\lambda_A(\tilde{x}_j)\rangle + \langle \eta^{\jmath}(\tilde{x}_j)\eta_{\jmath}(\tilde{x}_j)\rangle)$$

$$\prod_{k=3}^{4} \bar{\delta}\left(\lambda_k^{\alpha}(x_{kj})\lambda_{k\alpha}(x_{kj})\right)\bar{\delta}\left(\tilde{\lambda}_k^{\dot{\alpha}}(\tilde{x}_{kj})\tilde{\lambda}_{k\dot{\alpha}}(\tilde{x}_{kj})\right)$$

$$\bar{\delta}'_{w_j} = \bar{\delta}(\{Q_B, M^{00}(w_{1j})\})\bar{\delta}(\{Q_B, M^{01}(w_{2j})\})\bar{\delta}(\{Q_B, M^{11}(w_{3j})\})$$

$$\prod_{k=3}^{4} \bar{\delta}(\{Q_B, \mathcal{M}^{00}(w_{1kj})\})\bar{\delta}(\{Q_B, \mathcal{M}^{01}(w_{2kj})\})\bar{\delta}(\{Q_B, \mathcal{M}^{11}(w_{3kj})\})$$

$$j_{z_j} = \langle \lambda^A(z_j)\rho_{1A}(z_j)\rangle\left(\lambda^{A0}(z_{1j})\rho_{1A}^0(z_{1j})\right)\left(\lambda^{A(0}(z_{2j})\rho_{1A}^{1)}(z_{2j})\right)\left(\lambda^{A1}(z_{3j})\rho_{1A}^1(z_{3j})\right),$$

$$\tilde{j}_{\tilde{z}_j} = \left(\langle \lambda^A(\tilde{z}_j)\rho_{2A}(\tilde{z}_j)\rangle + \langle \eta^{\jmath}(\tilde{z}_j)\tau_{\jmath}(\tilde{z}_j)\rangle\right)\left(\lambda^{A0}(\tilde{z}_{1j})\rho_{2A}^0(\tilde{z}_{1j}) + \eta^{\jmath 0}(\tilde{z}_{1j})\tau_{\jmath}^0(\tilde{z}_{1j})\right)$$
$$\left(\lambda^{A(0}(\tilde{z}_{2j})\rho_{2A}^{1)}(\tilde{z}_{2j}) + \eta^{\jmath(0}(\tilde{z}_{2j})\tau_{\jmath}^{1)}(\tilde{z}_{2j})\right)\left(\lambda^{A1}(\tilde{z}_{3j})\rho_{2A}^1(\tilde{z}_{3j}) + \eta^{\jmath 1}(\tilde{z}_{3j})\tau_{\jmath}^1(\tilde{z}_{3j})\right)$$

$$j'_{r_j} = \prod_{k=3}^{4}\prod_{l=1}^{2}\left(\left(\lambda_{k\mathrm{a}}^A(r_{klj})\rho_{klA}^{\mathrm{a}}(r_{klj})\right)\left(\lambda^{A0}(r_{1klj})\rho_{klA}^0(r_{1klj})\right)\left(\lambda_k^{A(0}(r_{2klj})\rho_{klA}^{1)}(r_{2klj})\right)\right.$$
$$\left.\left(\lambda_k^{A1}(r_{3klj})\rho_{klA}^1(r_{3klj})\right)\right)$$

$$\bar{\delta}'_{w_j} j_{z_j}\tilde{j}_{z_j} j'_{r_j} \to j_{z_j}^u \tilde{j}_{\tilde{z}_j}^u j_{r_j}^{\prime u}$$

$$j_{z_j}^u = \langle \lambda^A(z_j)\rho_{1A}(z_j)\rangle \int \frac{\mathrm{d}^2u_{1j}\,\mathrm{d}^2u_{2j}\,\mathrm{d}^2u_{3j}e^{-\sum_{k=1}^3\|u_{kj}\|^2}}{\langle u_{1j}u_{2j}\rangle\langle u_{2j}u_{3j}\rangle\langle u_{3j}u_{1j}\rangle}\prod_{k=1}^{3}\left(\langle u_{kj}\lambda^A(z_{kj})\rangle\langle u_{kj}\rho_{1A}(z_{kj})\rangle\right)$$

$$\tilde{j}_{\tilde{z}_j}^u = \left(\langle \lambda^A(\tilde{z}_j)\rho_{2A}(\tilde{z}_j)\rangle + \langle \eta^{\jmath}(\tilde{z}_j)\tau_{\jmath}(\tilde{z}_j)\rangle\right)\int \frac{\mathrm{d}^2\tilde{u}_{1j}\,\mathrm{d}^2\tilde{u}_{2j}\,\mathrm{d}^2\tilde{u}_{3j}}{\langle \tilde{u}_{1j}\tilde{u}_{2j}\rangle\langle \tilde{u}_{2j}\tilde{u}_{3j}\rangle\langle \tilde{u}_{3j}\tilde{u}_{1j}\rangle}e^{-\sum_{k=1}^3\|\tilde{u}_{kj}\|^2}$$
$$\prod_{k=1}^{3}\left(\langle \tilde{u}_{kj}\lambda^A(\tilde{z}_{kj})\rangle\langle \tilde{u}_{kj}\rho_{2A}(\tilde{z}_{kj})\rangle + \langle \tilde{u}_{kj}\eta^{\jmath}(\tilde{z}_{kj})\rangle\langle \tilde{u}_{kj}\tau_{\jmath}(\tilde{z}_{kj})\rangle\right)$$

$$j_{r_j}^{\prime u} = \prod_{k=3}^{4}\prod_{l=1}^{2}\left(\left[\lambda_k^A(r_{klj})\rho_{klA}(r_{klj})\right]\int \frac{\mathrm{d}^2u'_{1kj}\mathrm{d}^2u'_{2kj}\mathrm{d}^2u'_{3kj}}{[u'_{1kj}u'_{2jk}][u'_{2kj}u'_{3kj}][u'_{3kj}u'_{1kj}]}e^{-\sum_{s=1}^3\left\|u'_{skj}\right\|^2}\right.$$
$$\left.\prod_{s=1}^{3}[u'_{skj}\lambda_{kj}^A(r_{sklj})][u'_{skj}\rho_{kljA}(r_{sklj})]\right)$$

$$\langle v_r\kappa_r^A\rangle = \langle u_r\lambda^A(\sigma_r)\rangle, r = 1, \ldots, n, \lambda_A^a(\sigma) = \lambda_{0A}^a\frac{h_\alpha(\sigma)}{\sqrt{\mathrm{d}\sigma}} + \sum_{l=1}^{n} u_l^a\epsilon_{lA}\frac{S_\alpha(\sigma,\sigma_l \mid \tau)}{\sqrt{\mathrm{d}\sigma}\sqrt{\mathrm{d}\sigma_l}}$$

$$\left\langle v_r\zeta_r^{\jmath}\right\rangle = \left\langle u_r\eta^{\jmath}(\sigma_r)\right\rangle, r = 1, \ldots, n, \eta_{\jmath}^a(\sigma) = \eta_{0A}^a\frac{h_\alpha(\sigma)}{\sqrt{\mathrm{d}\sigma}} + \sum_{l=1}^{n} u_l^a q_{l\jmath}\frac{S_\alpha(\sigma,\sigma_l \mid \tau)}{\sqrt{\mathrm{d}\sigma}\sqrt{\mathrm{d}\sigma_l}}$$

$$\mathcal{B} = \begin{pmatrix} A & C \\ -C^T & H \end{pmatrix}$$

$$A_{ij} = \langle \Lambda_{iA}(z_i')\Lambda_j^A(z_j') \rangle \frac{S_\alpha(z_i', z_j' \mid \tau)}{\sqrt{\mathrm{d}z_i'}\sqrt{\mathrm{d}z_j'}}, A = 4g \times 4g \text{ antisymmetric matrix,}$$

$$C_{ij}^a = \Lambda_{iA}^a(z_i')\epsilon_j^A \frac{S_\alpha(z_i', \sigma_j \mid \tau)}{\sqrt{\mathrm{d}z_i'}\sqrt{\mathrm{d}\sigma_j}},$$

$$H_{ij} = \epsilon_{iA}\epsilon_j^A \frac{S_\alpha(\sigma_i, \sigma_j \mid \tau)}{\sqrt{\mathrm{d}\sigma_i}\sqrt{\mathrm{d}\sigma_j}}, (i \neq j = 1, \dots, n), u_i^a H_{ii} = -\lambda_A^a(\sigma_i)\epsilon_i^A$$

$$\Lambda_{iA}^a(z_i') = \begin{cases} \lambda_A^a(z_k), i = 4k, \\ u_{lk}^a \langle u_{lk}\lambda_A(z_{lk}) \rangle, i = 4k + l, l = 1,2,3, \end{cases} k = 1, \dots, g$$

$$\tilde{\mathcal{B}} = \begin{pmatrix} \tilde{A} & \tilde{C} \\ -\tilde{C}^T & G \end{pmatrix}$$

$$\tilde{A}_{ij} = \left(\langle \tilde{\Lambda}_{iA}(\tilde{z}_i')\tilde{\Lambda}_j^A(\tilde{z}_j') \rangle + \langle \tilde{H}_{i\jmath}(\tilde{z}_i')\tilde{H}_j^\jmath(\tilde{z}_j') \rangle \right) \frac{S_\alpha(\tilde{z}_i', \tilde{z}_j' \mid \tau)}{\sqrt{\mathrm{d}\tilde{z}_i'}\sqrt{\mathrm{d}\tilde{z}_j'}}, \tilde{A} = 4g \times 4g \text{ antisymmetric matrix,}$$

$$\tilde{C}_{ij}^a = \left(\tilde{\Lambda}_{iA}^a(\tilde{z}_i')\epsilon_j^A + \tilde{H}_{i\jmath}^a(\tilde{z}_i')q_j^\jmath \right) \frac{S_\alpha(\tilde{z}_i', \sigma_j \mid \tau)}{\sqrt{\mathrm{d}\tilde{z}_i'}\sqrt{\mathrm{d}\sigma_j}},$$

$$\langle \tilde{C}_{ij}\tilde{C}_{kl} \rangle \equiv \left(\epsilon_j^A \langle \tilde{\Lambda}_{iA}(\tilde{z}_i')\tilde{\Lambda}_{kB}(\tilde{z}_k') \rangle \epsilon_l^B + q_j^\jmath \langle \tilde{H}_{i\jmath}(\tilde{z}_i')\tilde{H}_{k\mathcal{J}}(\tilde{z}_k') \rangle q_l^\mathcal{J} \right) \frac{S_\alpha(\tilde{z}_i', \sigma_j \mid \tau)}{\sqrt{\mathrm{d}\tilde{z}_i'}\sqrt{\mathrm{d}\sigma_j}} \frac{S_\alpha(\tilde{z}_k', \sigma_l \mid \tau)}{\sqrt{\mathrm{d}\tilde{z}_k'}\sqrt{\mathrm{d}\sigma_l}}$$

$$G_{ij} = \left(\epsilon_{iA}\epsilon_j^A + q_{i\jmath}q_j^\jmath \right) \frac{S_\alpha(\sigma_i, \sigma_j \mid \tau)}{\sqrt{\mathrm{d}\sigma_i}\sqrt{\mathrm{d}\sigma_j}}, u_i^a G_{ii} = -\lambda_A^a(\sigma_i)\epsilon_i^A - \eta_\jmath^a(\sigma_i)q_i^\jmath,$$

$$\tilde{\Lambda}_{iA}^a(\tilde{z}_i') = \left\{ \lambda_A^a(\tilde{z}_k), i = 4k, \quad \tilde{H}_{i\jmath}^a(\tilde{z}_i') = \begin{cases} \eta_\jmath^a(\tilde{z}_k), i = 4k, \\ \tilde{u}_{lk}^a \langle \tilde{u}_{lk}\lambda_A(\tilde{z}_{lk}) \rangle, \end{cases} \tilde{u}_{lk}\eta_\jmath(\tilde{z}_{lk}) \right\}, i = 4k + l, l = 1,2,3$$

$$A^{\alpha}_{g,V,n} = \left\langle \prod_{j=1}^{g} \left(j^{u}_{z_j} \tilde{j}^{u}_{\tilde{z}_j} j'^{u}_{r_j} \right) V_1 \dots V_n \right\rangle_{\alpha}$$

$$= \prod_{l=1}^{n} \frac{\mathrm{d}\sigma_l \, \mathrm{d}^2 u_l \, \mathrm{d}^2 v_l}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C}))_u} \prod_{r=1}^{n} \bar{\delta}(\langle v_r \epsilon_r \rangle - 1) \bar{\delta}^4(\langle u_r \lambda^A \rangle - \langle v_r \kappa^A_r \rangle) \bar{\delta}^{\mathcal{N}}\left(\langle u_r \eta^{\mathcal{I}} \rangle - \langle v_r \zeta^{\mathcal{I}}_r \rangle \right) \mathcal{H}\mathcal{G} \bar{\delta}_r j'_r,$$

$$\bar{\delta}_r = \prod_{r=2}^{n} \bar{\delta}\left(\mathrm{Res}_{\sigma_r}(\langle \lambda_A \lambda_B \rangle \Omega^{AB}) \right) \bar{\delta}\left(\mathrm{Res}_{\sigma_r}(\langle \eta_{\mathcal{I}} \eta_{\mathcal{J}} \rangle \Omega^{\mathcal{I}\mathcal{J}}) \right)$$

$$j'_r = \left\langle \prod_{j=1}^{g} j'^{u}_{r_j} \prod_{i=3}^{4} \prod_{k=s,t} \int \mathrm{d}u'^{2}_{ki} \bar{\delta}\left(u'^{\mathrm{a}}_{ki} - \chi^{A}_{ki} \lambda^{\mathrm{a}}_{iA}(\sigma_k) \right) \frac{\prod_{r=1}^{2} \prod_{p=s,t} \frac{1}{2} \left(\chi^{A}_{pi} \rho^{\mathrm{a}}_{irA}(\sigma_p) \right) \left(\chi^{A}_{pi} \rho_{irA\mathrm{a}}(\sigma_p) \right)}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C}))_{u'_{si}} \left[u'_{si} u'_{ti} \right]^4} \right\rangle_{\alpha}$$

$$\mathcal{H} = \prod_{j=1}^{g} \int \frac{\mathrm{d}^2 u_{1j} \, \mathrm{d}^2 u_{2j} \, \mathrm{d}^2 u_{3j}}{\langle u_{1j} u_{2j} \rangle \langle u_{2j} u_{3j} \rangle \langle u_{3j} u_{1j} \rangle} e^{-\sum_{k=1}^{3} \|u_{kj}\|^2} \sum_{I,J} \mathrm{Pf}(A^I_J) \sum_{D} \det{}' H^I_{DJ},$$

$$\mathcal{G} = \prod_{j=1}^{g} \int \frac{\mathrm{d}^2 \tilde{u}_{1j} \, \mathrm{d}^2 \tilde{u}_{2j} \, \mathrm{d}^2 \tilde{u}_{3j}}{\langle \tilde{u}_{1j} \tilde{u}_{2j} \rangle \langle \tilde{u}_{2j} \tilde{u}_{3j} \rangle \langle \tilde{u}_{3j} \tilde{u}_{1j} \rangle} e^{-\sum_{k=1}^{3} \|\tilde{u}_{kj}\|^2} \sum_{I,J} \mathrm{Pf}(\tilde{A}^I_J) \sum_{D} \det{}' G^I_{DJ},$$

$$\det{}' H^I_{DJ} = \frac{\det\left(H^{[ij]}_{[kl]} \right)^I_{DJ}}{\langle u_i u_j \rangle \langle u_k u_l \rangle}, \qquad \det{}' G^I_{DJ} = \frac{\det\left(G^{[pr]}_{[st]} \right)^I_{DJ}}{\langle u_p u_r \rangle \langle u_s u_t \rangle},$$

$$H_{ij} = \epsilon_{iA} \epsilon^A_j \frac{S_\alpha(\sigma_i, \sigma_j \mid \tau)}{\sqrt{\mathrm{d}\sigma_i} \sqrt{\mathrm{d}\sigma_j}}, \qquad u^a_i H_{ii} = -\lambda^a_A(\sigma_i) \epsilon^A_i,$$

$$G_{ij} = \left(\epsilon_{iA} \epsilon^A_j + q_{i\mathcal{J}} q^{\mathcal{J}}_j \right) \frac{S_\alpha(\sigma_i, \sigma_j \mid \tau)}{\sqrt{\mathrm{d}\sigma_i} \sqrt{\mathrm{d}\sigma_j}}, \qquad u^a_i G_{ii} = -\lambda^a_A(\sigma_i) \epsilon^A_i - \eta^a_{\mathcal{J}}(\sigma_i) q^{\mathcal{J}}_i.$$

$$0 = \sum_i \epsilon^{[A}_i \left(\left\langle v_i \kappa^{B]}_i \right\rangle - \langle u_i \lambda(\sigma_i)^{B]} \rangle \right) = \sum_i \langle \kappa^A_i \kappa^B_i \rangle - \sum_{ij} \langle u_i u_j \rangle \epsilon^{[A}_i \epsilon^{B]}_j S_\alpha(\sigma_i, \sigma_j \mid \tau) = \sum_i k^{AB}_i$$

$$\sum_i \langle \kappa^A_i \zeta^{\mathcal{J}}_i \rangle = \sum_i \left(\langle k^A_i v_i \rangle \langle \epsilon_i \zeta^{\mathcal{J}}_i \rangle - \langle k^A_i \epsilon_i \rangle \langle v_i \zeta^{\mathcal{J}}_i \rangle \right) = \sum_i \left(\langle u_i \lambda(\sigma_i)^A \rangle q^{\mathcal{J}}_i - \epsilon^A_i \langle u_i \eta(\sigma_i)^{\mathcal{J}} \rangle \right)$$

$$= \sum_{ij} \left(\epsilon^A_j q^{\mathcal{J}}_i - \epsilon^A_i q^{\mathcal{J}}_j \right) \langle u_i u_j \rangle S_\alpha(\sigma_i, \sigma_j \mid \tau) = 0$$

$$A^{\mathrm{even}}_{g,n} = \sum_{\alpha} \int \mathrm{d}\mu_g Z^{\alpha}_g((z),(y),(x),(w),(r)) \left(\prod_{j=1}^{g} \bar{\delta}_{x_j} \right) A^{\alpha}_{g,V,n}$$

$$Z^{\alpha}_g = \frac{Z_2((y)) Z_1((x))^6 Z_1((w))^9 \left(Z^{\mathrm{even}}_{\frac{1}{2},\alpha} \right)^8}{Z_1((z))^8 Z_1((r))^{16}}$$

$$= \frac{Y_3(y) Y(x) Y(\tilde{x}) \prod_{k=3}^{4} Y(x_k) Y(x_k) \prod_{j=1}^{3} Y(w_j) \prod_{k=3}^{4} Y(w_{jk}) \theta[\alpha](0)^8}{Y(z) Y(\tilde{z}) \prod_{j=1}^{3} Y(z_j) Y(\tilde{z}_j) \prod_{k=3}^{4} \prod_{l=1}^{2} Y(r_{kl}) \prod_{j=1}^{3} Y(r_{jkl})}$$

$$A^{\mathrm{even}}_{1,n} = \sum_{\alpha} \int \frac{\mathrm{d}\tau}{\det{}^{10}_{\omega}} \left(\frac{\theta[\alpha](0)}{\eta(\tau)^3} \right)^8 \bar{\delta}_{x_1} \frac{A^{\alpha}_{1,V,n}}{\mathrm{vol}(\mathrm{GL}(1,\mathbb{C}))_{\sigma}}$$

$$\mu_0^{a\dot\beta}\frac{h_\alpha}{\sqrt{\mathrm{d}\sigma}} = x_0^{\beta\dot\beta}\lambda_\beta^a - \Theta_0^{\jmath\dot\beta}\eta_\jmath^a, \tilde\mu_0^{a\beta}\frac{h_\alpha}{\sqrt{\mathrm{d}\sigma}} = -x_0^{\beta\dot\beta}\tilde\lambda_{\dot\beta}^a - \Theta_0^{\jmath\beta}\eta_\jmath^a, \tilde\eta_0^{a\jmath}\frac{h_\alpha}{\sqrt{\mathrm{d}\sigma}} = \Theta_0^{\jmath A}\lambda_A^a$$

$$\int \frac{\mathrm{d}^8\mu_0^{aA}\,\mathrm{d}^{2\mathcal{N}}\tilde\eta_0^{a\jmath}}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times\mathbb{C})_{\mu_0}}\exp\left[-\sum_{i=1}^n U_i\right]$$

$$U_i = \left(\langle u_i\mu_0^A\rangle\epsilon_{iA} + \langle u_i\tilde\eta_0^\jmath\rangle q_{i\jmath}\right)\frac{h_\alpha}{\sqrt{\mathrm{d}\sigma}}$$

$$= x_0^{\beta\dot\beta}\left(\epsilon_{i\dot\beta}\langle u_i\lambda_{i\beta}\rangle - \epsilon_{i\beta}\langle u_i\tilde\lambda_{i\dot\beta}\rangle\right) - \Theta_0^{\jmath A}\langle u_i\eta_\jmath\rangle\epsilon_{iA} + \Theta_0^{\jmath A}\langle u_i\lambda_{iA}\rangle q_{i\jmath}$$

$$= x_0^{\beta\dot\beta}\left(\langle v_i\kappa_{i\beta}\rangle\langle\epsilon_i\tilde\kappa_{i\dot\beta}\rangle - \langle\epsilon_i\kappa_{i\beta}\rangle\langle v_i\tilde\kappa_{i\dot\beta}\rangle\right) + \Theta_0^{\jmath A}\left(-\langle\epsilon_i\kappa_{iA}\rangle\langle v_i\zeta\jmath\rangle + \langle v_i\kappa_{iA}\rangle\langle e_i\zeta_{i\jmath}\rangle\right)$$

$$= x_0^{\beta\dot\beta}\langle\kappa_{i\beta}\tilde\kappa_{i\dot\beta}\rangle + \Theta_0^{\jmath A}\langle\kappa_{iA}\zeta_{i\jmath}\rangle = x_0\cdot k_i + \Theta_0^{\jmath A}\langle\kappa_{iA}\zeta_{i\jmath}\rangle$$

$$A_{g,n}^{\mathrm{odd}} = \delta\left(\sum_{i=1}^n k_i\right)\delta\left(\sum_{i=1}^n\langle\kappa_{iA}\zeta_{i\jmath}\rangle\right)\sum_\alpha\int \mathrm{d}\mu_g Z_g^\alpha((z),(y),(x),(w),(r))\int \mathrm{d}m_0\left(\prod_{j=1}^g\bar\delta_{x_j}\right)A_{g,V,n}^\alpha$$

$$Z_g^\alpha = \frac{Z_2((y))Z_1((x))^6Z_1((w))^9\left(Z_{\frac{1}{2},\alpha}^{\mathrm{odd}}\right)^8}{Z_1((z))^8Z_1((r))^{16}}$$

$$= \frac{Y_3(y)Y(x)Y(\tilde x)\prod_{k=3}^4 Y(x_k)Y(x_k)\prod_{j=1}^3 Y(w_j)\prod_{k=3}^4 Y(w_{jk})}{Y(z)Y(\tilde z)\prod_{j=1}^3 Y(z_j)Y(\tilde z_j)\prod_{k=3}^4\prod_{l=1}^2 Y(r_{kl})\prod_{j=1}^3 Y(r_{jkl})}\left(\frac{h_\alpha(z)^2}{\det_\omega}\right)^8$$

$$\mathrm{d}m_0 = \frac{\mathrm{d}^8\lambda_{0A}^a\,\mathrm{d}^{2\mathcal{N}_\eta}\eta_{0\jmath}^a}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times\mathbb{C})_{\lambda_0}}$$

$$A_{1,n}^{\mathrm{odd}} = \delta\left(\sum_{i=1}^n k_i\right)\delta\left(\sum_{i=1}^n\langle\kappa_{iA}\zeta_{i\jmath}\rangle\right)\int\frac{\mathrm{d}\tau}{\det_\omega^{10}}\frac{\mathrm{d}^8\lambda_{0A}^a\,\mathrm{d}^{2\mathcal{N}}\eta_{0\jmath}^a}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times\mathbb{C})_{\lambda_0}}\bar\delta_{x_1}\frac{A_{1,V,n}}{\mathrm{vol}(\mathrm{GL}(1,\mathbb{C}))_\sigma}$$

$$A_{1,n}^{\mathrm{odd}} = \delta\left(\sum_{i=1}^n k_i\right)\delta\left(\sum_{i=1}^n\langle\kappa_{iA}\zeta_{iI}\rangle\right)\int\frac{\mathrm{d}\tau}{\mathrm{d}^2z}\frac{\mathrm{d}^8\left(\lambda_{0A}^a\sqrt{\mathrm{d}z}\right)\mathrm{d}^{2\mathcal{N}}\left(\eta_{0\jmath}^a\sqrt{\mathrm{d}z}\right)}{\left(\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times\mathbb{C})_{\lambda_0}\,\mathrm{d}^4z\right)}\bar\delta_{x_1}\frac{A_{1,V,n}^\alpha}{\mathrm{vol}(\mathrm{GL}(1,\mathbb{C}))_\sigma}.$$

$$\tau = \begin{pmatrix}\tau_{ij}^{(g-1)} & \int_{\sigma_-}^{\sigma_+}\omega_i^{(g-1)}\\ \int_{\sigma_-}^{\sigma_+}\omega_j^{(g-1)} & \frac{1}{\pi i}\ln q_{gg} + \mathrm{const}\end{pmatrix} + O\left(q_{gg}^2\right)$$

$$\omega_i = \omega_i^{(g-1)} + O\left(q_{gg}^2\right)\text{ for } i < g$$
$$\omega_g = \omega_{+-}^{(g-1)} + O\left(q_{gg}^2\right)$$
$$S_\alpha(\sigma,\sigma_*\mid\tau) = S_{\alpha^{(g-1)}}\left(\sigma,\sigma_*\mid\tau^{(g-1)}\right) + O\left(q_{gg}\right) \equiv S_\alpha^{(g-1)}(\sigma,\sigma_*\mid\tau) + O\left(q_{gg}\right)$$

$$P^\mu = \sum_{I=1}^g c_i^\mu\omega_i + \sum_{j=1}^n k_j^\mu\omega_{j*}, \omega_{j*} = S_\alpha\left(\sigma,\sigma_j\mid\tau\right) - S_\alpha(\sigma,\sigma_*\mid\tau),$$

$$P^{\mu} \to \sum_{I=1}^{g-1} c_i^{\mu}\omega_i^{(g-1)} + k_+^{\mu}S_{\alpha}^{(g-1)}(\sigma,\sigma_+ \mid \tau) + k_-^{\mu}S_{\alpha}^{(g-1)}(\sigma,\sigma_+ \mid \tau) + \sum_{j=1}^{n} k_j^{\mu}S_{\alpha}^{(g-1)}(\sigma,\sigma_j \mid \tau)$$
$$= \sum_{I=1}^{g-1} c_i^{\mu}\omega_i^{(g-1)} + \sum_{j=1}^{n+2} k_j^{\mu}S_{\alpha}^{(g-1)}(\sigma,\sigma_j \mid \tau)$$

$$\lambda_A^a(\sigma) \to \sum_{i=1}^{n+2} u_i^a \epsilon_{iA} \frac{S_{\alpha}^{(g-1)}(\sigma,\sigma_i \mid \tau)}{\sqrt{\mathrm{d}\sigma}\sqrt{\mathrm{d}\sigma_i}}, \kappa_{n+1A}^a = \kappa_{+A}^a = (-1)^{\delta_{a2}}\kappa_{-A}^a = (-1)^{\delta_{a2}}\kappa_{n+2A}^1,$$
$$\eta_{\mathcal{J}}^a(\sigma) \to \sum_{i=1}^{n+2} u_i^a q_{i\mathcal{J}} \frac{S_{\alpha}^{(g-1)}(\sigma,\sigma_i \mid \tau)}{\sqrt{\mathrm{d}\sigma}\sqrt{\mathrm{d}\sigma_i}}, \zeta_{n+1\mathcal{J}}^a = \zeta_{+\mathcal{J}}^a = (-1)^{\delta_{a2}}\zeta_{-\mathcal{J}}^a = (-1)^{\delta_{a2}}\zeta_{n+2\mathcal{J}}^1,$$

$$P^{AB}(\sigma) = \langle \lambda^A(\sigma)\lambda^B(\sigma)\rangle,$$

$P^{AB}(\sigma) = \epsilon_+^{[A}\langle u_+\lambda^{B]}(\sigma_+)\rangle \frac{S_{\alpha}(\sigma,\sigma_+|\tau)}{\sqrt{\mathrm{d}\sigma}\sqrt{\mathrm{d}\sigma_+}} + \epsilon_-^{[A}\langle u_-\lambda^{B]}(\sigma_-)\rangle \frac{S_{\alpha}(\sigma,\sigma_-|\tau)}{\sqrt{\mathrm{d}\sigma}\sqrt{\mathrm{d}\sigma_-}} + \sum_{i=1}^{n} k_i^{AB} \frac{S_{\alpha}(\sigma,\sigma_i|\tau)}{\sqrt{\mathrm{d}\sigma}\sqrt{\mathrm{d}\sigma_i}}, P_A^A(\sigma) =$

$0, P^2(\sigma) = -P_A^A(\sigma)P_B^B(\sigma) = 0$

$$\langle v_{\pm}\kappa_{\pm}^A\rangle = \langle u_{\pm}\lambda^A(\sigma_{\pm})\rangle, k_{\pm}\cdot P(\sigma_{\pm}) = 0$$

$$\delta^4\left(\sum_{i=1}^{n} k_i^{\mu}\right)\delta^{4\mathcal{N}}\left(\sum_{i=1}^{n}\langle \kappa_{iA}\zeta_{i\mathcal{J}}\rangle\right)\frac{\mathrm{d}^8\lambda_{0A}^a\,\mathrm{d}^{2\mathcal{N}}\eta_{0\mathcal{J}}^a}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times\mathbb{C})_{\lambda_0}}\frac{1}{(2\pi i)^3}\frac{\mathrm{d}q_{gg}}{q_{gg}}\frac{\mathrm{d}q_1}{q_1}\frac{\mathrm{d}q_2}{q_2}\bar{\delta}(\langle\lambda^A(x_j)\lambda_A(x_j)\rangle)$$
$$\bar{\delta}(\langle\lambda^A(\tilde{x}_j)\lambda_A(\tilde{x}_j)\rangle)\bar{\delta}(\langle\eta^{\mathcal{J}}(\tilde{x}_j)\eta_{\mathcal{J}}(\tilde{x}_j)\rangle)$$
$$= \int \frac{\prod_{l=n+1}^{n+2}\mathrm{d}^4\epsilon_l^A\,\mathrm{d}^{\mathcal{N}}q_l^{\mathcal{J}}}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times\mathbb{C})_{\epsilon}}\frac{\prod_{l=n+1}^{n+2}\mathrm{d}\sigma_l\,\mathrm{d}^2u_l\,\mathrm{d}^2v_l}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})_{\sigma}\times\mathrm{SL}(2,\mathbb{C})_u)}\frac{1}{\langle u_{n+1}u_{n+2}\rangle^4}$$
$$\prod_{r=n+1}^{n+2}\bar{\delta}(\langle v_r\epsilon_r\rangle - 1)\bar{\delta}^4(\langle u_r\lambda^A\rangle - \langle v_r\kappa_r^A\rangle)\bar{\delta}^{\mathcal{N}}(\langle u_r\eta^{\mathcal{J}}\rangle - \langle v_r\zeta_r^{\mathcal{J}}\rangle)$$
$$\bar{\delta}(\langle\lambda^A(x_j)\lambda_A(x_j)\rangle)\bar{\delta}(\langle\lambda^A(\tilde{x}_j)\lambda_A(\tilde{x}_j)\rangle + \langle\eta^{\mathcal{J}}(\tilde{x}_j)\eta_{\mathcal{J}}(\tilde{x}_j)\rangle)$$

$$1 = \int \frac{\prod_{l=n+1}^{n+2}\mathrm{d}^4\epsilon_l^A\,\mathrm{d}^{\mathcal{N}}q_l^{\mathcal{J}}}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{C})\times\mathbb{C})_{\epsilon}}\frac{\mathcal{W}_{\mathrm{vac}}}{\mathrm{d}\sigma_*\,\mathrm{d}^3u}\prod_{l=n+1}^{n+2}\mathrm{d}^2u_l\,\mathrm{d}^2v_l\bar{\delta}(\langle v_l\epsilon_l\rangle - 1)\bar{\delta}^4(\langle u_l\lambda_A(\sigma_l)\rangle - \langle v_l\kappa_{lA}\rangle)$$
$$\bar{\delta}^{\mathcal{N}}(\langle u_l\eta_{\mathcal{J}}(\sigma_l)\rangle - \langle v_l\zeta_{l\mathcal{J}}\rangle)e^{\langle u_l\mu^A(\sigma_l)\rangle\epsilon_{lA} + \langle u_l\tilde{\eta}^{\mathcal{J}}(\sigma_l)\rangle q_{l\mathcal{J}}}$$
$$\prod_{i=3}^{4}\frac{\prod_{l=n+1}^{n+2}\mathrm{d}u_{il}'^2}{\mathrm{d}^3u_i'}\bar{\delta}\left(u_{il}'^{\mathrm{a}} - \chi_{il}^A\lambda_{lA}^{\mathrm{a}}(\sigma_l)\right)\prod_{j=1}^{2}\frac{1}{2}\left(\chi_{il}^A\rho_{ijA}^{\mathrm{a}}(\sigma_l)\right)\left(\chi_{il}^A\rho_{ijA\mathrm{a}}(\sigma_l)\right),$$
$$\kappa_{n+1A}^1 = \kappa_{n+2A}^1, \kappa_{n+1A}^2 = -\kappa_{n+2A}^2, \zeta_{n+1\mathcal{J}}^1 = \zeta_{n+2\mathcal{J}}^1, \zeta_{n+1\mathcal{J}}^2 = -\zeta_{n+2\mathcal{J}}^2$$
$$\mathcal{W}_{\mathrm{vac}}(\sigma_{n+1},\sigma_{n+2},\sigma_*) = c(\sigma_{n+1})c(\sigma_{n+2})c(\sigma_*)n\tilde{n}\prod_{i=3}^{4}(n_i\tilde{n}_i)\delta(\gamma)\delta(\tilde{\gamma})\prod_{i=3}^{4}\prod_{j=1}^{2}\delta(\gamma_{ij})$$

$$\prod_{a,b}\left(\delta(\gamma_{ab})\delta(\tilde{\gamma}_{ab})N_{ab}\prod_{i=3}^{4}\mathcal{N}_{\mathrm{iab}}\prod_{j=1}^{2}\delta(\gamma_{ij\mathrm{ab}})\right),$$
$$\langle 0|\mathcal{W}_{\mathrm{vac}}(0,0,0)|0\rangle = 1, \langle\epsilon_{n+1}\epsilon_{n+2}\rangle = 1,$$

$$\left(\delta\left(\gamma_{ab}u_1^a u_1^b\right)\delta\left(\beta_{ab}\hat{u}^a\hat{u}^b\right)\langle\lambda_A(\sigma_1)\hat{u}\rangle\langle\rho_1^A\hat{u}\rangle\right)\left(\delta(\gamma)\delta(\beta)\langle\lambda_A(\sigma_1)\rho_1^A\rangle\right)e^{\langle u_1\mu^A(\sigma_1)\rangle\epsilon_{1A}} \Rightarrow$$
$$\left(\frac{\langle\hat{u}\lambda_A(\sigma_1)\rangle\epsilon_1^A}{\langle u_1\hat{u}\rangle}+\frac{\langle\rho_1^A\rho_1^B\rangle}{2}\epsilon_{1A}\epsilon_{1B}\right)e^{\langle u_1\mu^A(\sigma_1)\rangle\epsilon_{1A}}$$

$$\left(\delta\left(\gamma_{ab}u_2^a u_2^b\right)\delta\left(\beta_{ab}\hat{u}^a\hat{u}^b\right)\langle\lambda_A(\sigma_2)\hat{u}\rangle\langle\rho_1^A\hat{u}\rangle\right)\left(\langle u_2\hat{u}\rangle\delta\left(\gamma_{ab}u_2^a\hat{u}^b\right)\delta\left(\beta_{ab}\hat{u}^a u_2^b\right)\langle\lambda_A(\sigma_2)\hat{u}\rangle\langle\rho_1^A u_2\rangle\right)$$
$$e^{\langle u_2\mu^A(\sigma_2)\rangle\epsilon_{2A}} \Rightarrow$$
$$\left(\frac{\langle\hat{u}\lambda_A(\sigma_2)\rangle\epsilon_2^A}{\langle u_2\hat{u}\rangle}+\frac{\langle\rho_1^A\rho_1^B\rangle}{2}\epsilon_{2A}\epsilon_{2B}\right)e^{\langle u_2\mu^A(\sigma_2)\rangle\epsilon_{2A}}$$

$$A_{g,n}^{\text{odd}}=A_{g-1,n+2}^{\text{even}}=\sum_\alpha\int \mathrm{d}\mu_{g-1}Z_{g-1}^\alpha((z),(y),(x),(w),(r))\int \mathrm{d}m_{\text{fw}}\prod_{j=1}^{g-1}\bar{\delta}_{x_j}A_{g-1,V,n+2}^\alpha$$
$$\mathrm{d}m_{\text{fw}}=\frac{\prod_{l=n+1}^{n+2}\mathrm{d}^4\epsilon_l^A\,\mathrm{d}^{\mathcal{N}}q_l^\jmath}{\text{vol}(\text{SL}(2,\mathbb{C})\times\mathbb{C})_\epsilon}$$

$$A_{g,n}^{\text{odd}}=A_{g-1,n+2}^{\text{odd}}=\delta\left(\sum_{i=1}^{n+2}k_i\right)\delta\left(\sum_{i=1}^{n+2}\langle\kappa_{iA}\zeta_{i\jmath}\rangle\right)\sum_\alpha\int \mathrm{d}\mu_{g-1}Z_{g-1}^\alpha((z),(y),(x),(w),(r))$$
$$\int \mathrm{d}m_0\,\mathrm{d}m_{\text{fw}}\prod_{j=1}^{g-1}\bar{\delta}_{x_j}A_{g-1,V,n+2}^\alpha$$
$$\mathrm{d}m_0=\frac{\mathrm{d}^8\lambda_{0A}^a\,\mathrm{d}^{2\mathcal{N}}\eta_{0\jmath}^a}{\text{vol}(\text{SL}(2,\mathbb{C})\times\mathbb{C})_{\lambda_0}},\ \mathrm{d}m_{\text{fw}}=\frac{\prod_{l=n+1}^{n+2}\mathrm{d}^4\epsilon_l^A\,\mathrm{d}^{\mathcal{N}}q_l^\jmath}{\text{vol}(\text{SL}(2,\mathbb{C})\times\mathbb{C})_\epsilon}$$

$$A_{g,n}^{\text{even}}=A_{g-1,n+2}^{\text{even}}=\sum_\alpha\int \mathrm{d}\mu_{g-1}Z_{g-1}^\alpha((z),(y),(x),(w),(r))\int \mathrm{d}m_{\text{fw}}\prod_{j=1}^{g-1}\bar{\delta}_{x_j}A_{g-1,V,n+2}^\alpha$$

$$\mathrm{d}m_{\text{fw}}\delta\left(\left\langle\kappa_{n+1}^A\kappa_{n+1A}\right\rangle\right)\to\frac{\mathrm{d}^4p^\mu}{p^2}\,\mathrm{d}^{\mathcal{N}}q_{n+1}^\jmath\mathrm{d}^{\mathcal{N}}q_{n+2}^\jmath$$

$$\det{}'H_{DJ}^I\det{}'G_{DJ}^I=\frac{\det\left(H_{[+-]}^{[+-]}\right)_{DJ}^I\det\left(G_{[+-]}^{[+-]}\right)_{DJ}^I}{\langle u_+u_-\rangle^4}$$

$$\frac{1}{\langle\lambda^A(x)\lambda_A(x)\rangle}\frac{1}{\langle\lambda^A(x)\lambda_A(x)\rangle}=\frac{1}{P_A^A(x)^2}=-\frac{1}{P^{AB}P_{BA}(x)}=-\frac{1}{P^2(x)}.$$

$$\epsilon_j^A(u_{+a}\epsilon_{+A}+u_{-a}\epsilon_{-A})\sum_i u_i^a\left(H_{[+-]}^{[+-]}\right)_{ij}=\epsilon_j^A(u_{+a}\epsilon_{+A}+u_{-a}\epsilon_{-A})\left(-\frac{u_+^a\epsilon_+^B\epsilon_{jB}}{\sigma_+-\sigma_j}-\frac{u_-^a\epsilon_-^B\epsilon_{jB}}{\sigma_--\sigma_j}\right)$$
$$=\epsilon_j^A\epsilon_{+A}\epsilon_j^B\epsilon_{-A}\langle u_+u_-\rangle\left(\frac{1}{\sigma_+-\sigma_j}-\frac{1}{\sigma_--\sigma_j}\right)=O(\Lambda^{-1})$$

$$\text{SO}(10)\to\text{SO}(8)\times\text{U}(1)\to\text{SU}(2)\times\text{SU}(2)\times\text{SU}(2)\times\text{SU}(2)\times\text{U}(1)$$

$$S=\sum_{i=1}^{4}\int_{\Sigma}\mathcal{Z}_i^{\mathrm{a}}\cdot\bar{\partial}\mathcal{Z}_{i\mathrm{a}}+\mathcal{A}_{i\mathrm{ab}}\mathcal{Z}_i^{\mathrm{a}}\cdot\mathcal{Z}_i^{\mathrm{b}}+\sum_{j=1}^{2}\left(\frac{1}{2}\rho_{ij\mathrm{a}}^{A}\bar{\partial}\rho_{ijA}^{\mathrm{a}}+\mathcal{B}_{ij}^{\mathrm{ab}}\lambda_{i\mathrm{a}}^{A}\rho_{ijA\,\mathrm{b}}+b_{ij}\lambda_{iA}^{\mathrm{a}}\rho_{ij\mathrm{a}}^{A}+\tilde{\tau}_{ij}^{A}\bar{\partial}\tau_{ijA}\right)$$
$$+\begin{cases}a_i\lambda_{i\mathrm{a}}^{A}\lambda_{iA}^{\mathrm{a}} & i=1\\ a_i\lambda_{i\mathrm{a}}^{\alpha}\lambda_{i\alpha}^{\mathrm{a}}+\tilde{a}_i\tilde{\lambda}_{i\mathrm{a}}^{\dot{\alpha}}\tilde{\lambda}_{i\dot{\alpha}}^{\mathrm{a}} & i>1\end{cases}$$
$$\mathcal{Z}_i^{a}=\left(\lambda_{iA}^{a},\mu_i^{aA},\eta_{i\mathcal{J}}^{a}\right),A=1,\dots,4,I=1,\dots,8,a=1,2$$

$$S=\int_{\Sigma}\mathcal{Z}^{a}\cdot\bar{\partial}\mathcal{Z}_a+A_{ab}\mathcal{Z}^{a}\cdot\mathcal{Z}^{b}+a\langle\lambda_A\lambda_B\rangle\Omega^{AB}+\tilde{a}\left(\langle\lambda_A\lambda_B\rangle\Omega^{AB}+\langle\eta_{\mathcal{I}}\eta_{\mathcal{J}}\rangle\Omega^{\mathcal{I}\mathcal{J}}\right)$$
$$+S_{\rho_1}+S_{(\rho_2,\tau)}+S_{\mathrm{aux}}$$
$$S_{\rho}=\int_{\Sigma}\frac{1}{2}\langle\rho_A\bar{\partial}\rho_B\rangle\Omega^{AB}+B_{ab}\lambda_A^{a}\rho_B^{b}\Omega^{AB}+b\langle\lambda_A\rho_B\rangle\Omega^{AB}$$
$$S_{(\rho,\tau)}=\int_{\Sigma}\frac{1}{2}\left(\langle\rho_A\bar{\partial}\rho_B\rangle\Omega^{AB}+\langle\tau_{\mathcal{I}}\bar{\partial}\tau_{\mathcal{J}}\rangle\Omega^{\mathcal{I}\mathcal{J}}\right)+D_{ab}\left(\lambda_A^{a}\rho_B^{b}\Omega^{AB}+\eta_{\mathcal{I}}^{a}\tau_{\mathcal{J}}^{b}\Omega^{\mathcal{I}\mathcal{J}}\right)$$
$$+d\left(\langle\lambda_A\rho_B\rangle\Omega^{AB}+\langle\eta_{\mathcal{I}}\tau_{\mathcal{J}}\rangle\Omega^{\mathcal{I}\mathcal{J}}\right)$$
$$S_{\mathrm{aux}}=\sum_{i=3}^{4}\int_{\Sigma}\mathcal{Z}_i^{\mathrm{a}}\cdot\bar{\partial}\mathcal{Z}_{i\mathrm{a}}+\mathcal{A}_{i\mathrm{ab}}\mathcal{Z}_i^{\mathrm{a}}\cdot\mathcal{Z}_i^{\mathrm{b}}+\sum_{j=1}^{2}\left(\frac{1}{2}\rho_{ij\mathrm{a}}^{A}\bar{\partial}\rho_{ijA}^{\mathrm{a}}+\mathcal{B}_{ij}^{\mathrm{ab}}\lambda_{i\mathrm{a}}^{A}\rho_{ijA\,\mathrm{b}}+b_{ij}\lambda_{iA}^{\mathrm{a}}\rho_{ij\mathrm{a}}^{A}+\tilde{\tau}_{ij}^{A}\bar{\partial}\tau_{ijA}\right)$$
$$+a_i\lambda_{i\mathrm{a}}^{\alpha}\lambda_{i\alpha}^{\mathrm{a}}+\tilde{a}_i\tilde{\lambda}_{i\mathrm{a}}^{\dot{\alpha}}\tilde{\lambda}_{i\dot{\alpha}}^{\mathrm{a}}$$

$$p^{\alpha\dot{\alpha}}=\sigma_{\mu}^{\alpha\dot{\alpha}}p^{\mu}=\begin{pmatrix}p^0+p^3 & p^1-ip^2\\ p^1+ip^2 & p^0-p^3\end{pmatrix}=\lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}$$

$$(z,\bar{z})\mapsto\left(\frac{az+b}{cz+d},\frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}\right),\begin{pmatrix}a & b\\ c & d\end{pmatrix}\in\mathrm{SL}(2,\mathbb{C})$$

$$p^{\mu}=\omega q^{\mu},q^{\mu}=\frac{1}{2}(1+|z|^2,z+\bar{z},-i(z-\bar{z}),1-|z|^2)$$

$$\omega\mapsto(cz+d)(\bar{c}\bar{z}+\bar{d})\omega,q^{\mu}\mapsto q'^{\mu}=(cz+d)^{-1}(\bar{c}\bar{z}+\bar{d})^{-1}\Lambda^{\mu}{}_{\nu}q^{\nu}.$$

$$q^{\alpha\dot{\alpha}}=\sigma_{\mu}^{\alpha\dot{\alpha}}q^{\mu}=\begin{pmatrix}1 & \bar{z}\\ z & z\bar{z}\end{pmatrix}=\begin{pmatrix}1\\ z\end{pmatrix}(1\quad\bar{z})$$

$$\lambda^{\alpha}\equiv\left\langle p|^{\alpha}=\sqrt{\omega}\begin{pmatrix}1\\ z\end{pmatrix}=\sqrt{\omega}\langle q|^{\alpha},\tilde{\lambda}^{\dot{\alpha}}\equiv|p\right]^{\dot{\alpha}}=\sqrt{\omega}\begin{pmatrix}1\\ \bar{z}\end{pmatrix}=\sqrt{\omega}\left|\,q\right]^{\dot{\alpha}},$$

$$\left\langle q|^{\alpha}=\begin{pmatrix}1\\ z\end{pmatrix},\left|\,q\right]^{\dot{\alpha}}=\begin{pmatrix}1\\ \bar{z}\end{pmatrix}.$$

$$\langle ij\rangle=-\sqrt{\omega_i\omega_j}z_{ij},[ij]=\sqrt{\omega_i\omega_j}\bar{z}_{ij}$$

$$\varphi_{\Delta}^{\pm}(X^{\mu},z,\bar{z})=\int_0^{\infty}d\omega\omega^{\Delta-1}e^{\pm i\omega q\cdot X-\epsilon\omega}=\frac{(\mp i)^{\Delta}\Gamma(\Delta)}{(-q\cdot X\mp i\epsilon)^{\Delta}}$$

$$\psi^{\pm}_{\Delta,\ell=-1/2;\alpha}(X,z,\bar{z}) = |q\rangle_{\alpha}\varphi^{\pm}_{\Delta+\frac{1}{2}}(X,z,\bar{z})$$

$$\psi^{\pm\dot{\alpha}}_{\Delta,\ell=1/2}(X,z,\bar{z}) = |\ q]^{\dot{\alpha}}\varphi^{\pm}_{\Delta+\frac{1}{2}}(X,z,\bar{z})$$

$$V^{\mu\pm}_{\Delta,\ell=\pm1}(X,z,\bar{z}) = \epsilon^{\mu}_{\ell=\pm1}(q,r)\varphi^{\pm}_{\Delta}(X,z,\bar{z})$$

$$H^{\mu\nu\pm}_{\Delta,\ell=\pm2}(X,z,\bar{z}) = \epsilon^{\mu}_{\ell=\pm1}(q,r)V^{\nu\pm}_{\Delta,\ell=\pm1}(X,z,\bar{z})$$

$$\psi^{\mu\pm}_{\Delta,\ell=-3/2}(X,z,\bar{z}) = \epsilon^{\mu}_{\ell=-1}(q,r)\psi^{\pm}_{\Delta,\ell=-1/2}(X,z,\bar{z})$$

$$\bar{\psi}^{\mu\pm}_{\Delta,\ell=+3/2}(X,z,\bar{z}) = \epsilon^{\mu}_{\ell=+1}(q,r)\bar{\psi}^{\pm}_{\Delta,\ell=+1/2}(X,z,\bar{z})$$

$$\epsilon^{\mu}_{\ell=+1}(q,r) = \frac{\langle r|\sigma^{\mu}\mid q]}{\sqrt{2}\langle rq\rangle}, \epsilon^{\mu}_{\ell=-1}(q,r) = \frac{[r|\bar{\sigma}^{\mu}|q\rangle}{\sqrt{2}[qr]}$$

$$\left\langle \prod_{n=1}^{N} \mathcal{O}_{\Delta_n,\ell_n}(z_n,\bar{z}_n) \right\rangle \equiv \left(\prod_{n=1}^{N} \int d\omega_n \omega_n^{\Delta_n-1}\right)\delta^{(4)}\left(\sum_{n=1}^{N} \omega_n q_n\right) A_{\ell_1\dots\ell_N}(\omega_n,z_n,\bar{z}_n),$$

$$\left\langle \prod_{n=1}^{N} \mathcal{O}_{\Delta_n,\ell_n}\left(\frac{az_n+b}{cz_n+d},\frac{\bar{a}\bar{z}_n+\bar{b}}{\bar{c}\bar{z}_n+\bar{d}}\right)\right\rangle = \prod_{i=1}^{N} (cz_i+d)^{\Delta_i+\ell_i}\left(\bar{c}\bar{z}_i+\bar{d}\right)^{\Delta_i-\ell_i}\left\langle \prod_{n=1}^{N} \mathcal{O}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle.$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{C}).$$

$$\begin{aligned}\Psi(p,\eta) = {} & H^{+}(p) + \eta_A\psi^{A}_{+}(p) + \eta_{AB}G^{AB}_{+}(p) + \eta_{ABC}\chi^{ABC}_{+}(p) \\ & + \eta_{ABCD}\Phi^{ABCD}(p) + \tilde{\eta}^{ABC}\chi^{-}_{ABC}(p) + \tilde{\eta}^{AB}G^{-}_{AB}(p) + \tilde{\eta}^{A}\psi^{-}_{A}(p) + \tilde{\eta}H^{-}(p)\end{aligned}$$

$$\begin{aligned}\eta_{A_1\dots A_n} &\equiv \frac{1}{n!}\eta_{A_1}\dots\eta_{A_2} \\ \tilde{\eta}^{A_1\dots A_n} &\equiv \epsilon^{A_1\dots A_nB_1\dots B_{8-n}}\eta_{B^1\dots B^{8-n}} \\ \tilde{\eta} &\equiv \prod_{A=1}^{8}\eta^{A}\end{aligned}$$

$$\mathcal{M}_n(\{p_1,\eta^1\},\dots\{p_n,\eta^n\}) = \langle\Psi_1(p_1,\eta^1)\dots\Psi_n(p_n,\eta^n)\rangle$$

$$\begin{aligned}\Psi_{\Delta}(z,\bar{z},\eta) = {} & H^{+}_{\Delta}(z,\bar{z}) + \eta_A\psi^{A}_{\Delta}(z,\bar{z}) + \eta_{AB}G^{AB}_{\Delta}(z,\bar{z}) + \eta_{ABC}\chi^{ABC}_{\Delta}(z,\bar{z}) \\ & + \eta_{ABCD}\Phi^{ABCD}_{\Delta}(z,\bar{z}) + \tilde{\eta}^{ABC}\bar{\chi}_{ABC}\Delta(z,\bar{z}) + \tilde{\eta}^{AB}\bar{G}_{AB}\Delta(z,\bar{z}) \\ & + \tilde{\eta}^{A}\bar{\psi}_{A}\Delta(z,\bar{z}) + \tilde{\eta}H^{-}_{\Delta}(z,\bar{z})\end{aligned}$$

$$\begin{aligned}\left\langle \mathcal{O}_{\Delta_1,+2}\mathcal{O}_{\Delta_2,+2}\dots\mathcal{O}_{\Delta_n,\ell_n}\right\rangle &= \left(\prod_{j=1}^{n}\int_0^{\infty} d\omega_j\omega_j^{\Delta_j-1}\right)\delta^4\left(\sum_i \omega_i q_i\right)M_n(1^{+2},2^{+2},\dots,n) \\ &= \left(\prod_{j=3}^{n}\int_0^{\infty} d\omega_j\omega_j^{\Delta_j-1}\int_0^{\infty} d\omega_1\int_0^{\infty} d\omega_2\omega_1^{\Delta_1-1}\omega_2^{\Delta_2-1}\right) \\ &\quad \times\delta^4\left(\sum_{i=3}^{n}\omega_i q_i + \omega_p q_p\right)\frac{\omega_p^2}{\omega_1\omega_2}\frac{\bar{z}_{12}}{z_{12}}M_{n-1}(p^{+2},\dots,n)\end{aligned}$$

$$M_n(1^{+2},2^{+2},\dots,n)=\frac{\omega_p^2}{\omega_1\omega_2}\frac{\bar{z}_{12}}{z_{12}}M_{n-1}(p^{+2},\dots,n)$$

$$\int_0^\infty d\omega_2\omega_2^{\Delta_2-1}\int_0^\infty d\omega_1\omega_1^{\Delta_1-1}\omega_1^\alpha\omega_2^\beta\omega_p^\gamma f(\omega_p)=B(\Delta_1+\alpha,\Delta_2+\beta)\int_0^\infty d\omega_p\omega_p^{\Delta_p-1}f(\omega_p)$$

$$B(x,y)=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$\begin{aligned}\langle\mathcal{O}_{\Delta_1,+2}\mathcal{O}_{\Delta_2,+2}\dots\mathcal{O}_{\Delta_n,\ell_n}\rangle&=\frac{\bar{z}_{12}}{z_{12}}B(\Delta_1-1,\Delta_2-1)\left(\prod_{j=3}^n\int_0^\infty d\omega_j\omega_j^{\Delta_j-1}\int_0^\infty d\omega_p\omega_p^{\Delta_1+\Delta_2-1}\right)\\&\times\delta^4\left(\sum_{i=3}^n\omega_iq_i+\omega_pq_p\right)M_{n-1}(p^{+2},3,\dots,n)\\&=\frac{\bar{z}_{12}}{z_{12}}B(\Delta_1-1,\Delta_2-1)\langle\mathcal{O}_{\Delta_1+\Delta_2,+2}\mathcal{O}_{\Delta_3,\ell_3}\dots\mathcal{O}_{\Delta_n,\ell_n}\rangle\end{aligned}$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2)\sim\frac{\bar{z}_{12}}{z_{12}}B(\Delta_1-1,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)$$

$$M_n(1^{-2},2^{-2},\dots)=\frac{\omega_p^2}{\omega_1\omega_2}\frac{z_{12}}{\bar{z}_{12}}M_{n-1}(p^{-2},\dots,n)$$

$$\mathcal{O}_{\Delta_1,-2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,-2}(z_2,\bar{z}_2)\sim\frac{z_{12}}{\bar{z}_{12}}B(\Delta_1-1,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)$$

$$\begin{aligned}M_n(1^{+2},2^{-2},3,\dots,n)&=\frac{\omega_1^3}{\omega_p^2\omega_2}\frac{\bar{z}_{12}}{z_{12}}M_{n-1}(p^{-2},3,\dots,n)\\&+\frac{\omega_2^3}{\omega_p^2\omega_1}\frac{z_{12}}{\bar{z}_{12}}M_{n-1}(p^{+2},3,\dots,n)\end{aligned}$$

$$\begin{aligned}\mathcal{O}_{\Delta_1,+2}(z_1,z_1)\mathcal{O}_{\Delta_2,-2}(z_2,\bar{z}_2)&=B(\Delta_1+3,\Delta_2-1)\frac{\bar{z}_{12}}{z_{12}}\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar{z}_2)\\&+B(\Delta_1-1,\Delta_2+3)\frac{z_{12}}{\bar{z}_{12}}\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)\end{aligned}$$

$$\left\langle\prod_{n=1}^N\mathcal{O}_{\Delta_n}(z_n,\bar{z}_n,\eta^n)\right\rangle\equiv\left(\prod_{n=1}^N\int d\omega_n\omega_n^{\Delta_n-1}\right)\delta^{(4)}\left(\sum_{n=1}^N\omega_nq_n\right)\times\mathcal{M}_N(\{\omega_1,z_1,\bar{z}_1,\eta^1\},\dots\{\omega_N,z_N,\bar{z}_N,\eta^N\})$$

$$\mathcal{M}_N(\cdots,j-1,j,j+1,\cdots)\overset{\omega_j\to0}{\to}\frac{1}{\omega_j}\sum_{\substack{i=1\\i\neq j}}^N\omega_iz_{ji}\bar{z}_{ji}\left[\frac{z_{j-1,i}^2}{z_{j-1,j}^2z_{j,i}^2}+\frac{\bar{z}_{j-1,i}^2}{\bar{z}_{j-1}^2\bar{z}_{j,i}^2}\delta^4(\eta^j)\right]\times\mathcal{M}_{N-1}(\cdots,j-1,j+1,\cdots)$$

$$\left\langle \prod_{n=1}^{N} \lim_{\Delta_j \to 1} (\Delta_j - 1)\mathcal{O}_{\Delta_n}(z_n, \bar{z}_n, \eta^n) \right\rangle = \left(\prod_{\substack{n=1 \\ n \neq j}}^{N} \int d\omega_n \omega_n^{\Delta_n - 1} \right) \lim_{\Delta_j \to 1} \int_0^{\infty} d\omega_j (\Delta_j - 1) \omega_j^{\Delta_j - 1}$$

$$\times \delta^{(4)} \left(\sum_{\substack{k=1 \\ k \neq j}}^{N} \omega_k q_k \right) \mathcal{M}_N(1, \dots, n, \dots, N)$$

$$= \left(\prod_{\substack{n=1 \\ n \neq j}}^{N} \int d\omega_n \omega_n^{\Delta_n - 1} \right) \int_0^{\infty} d\omega_j \frac{d}{d\omega_j} \left(\lim_{\Delta_j \to 1} \omega_j^{\Delta_j - 1} \right) \delta^{(4)} \left(\sum_{\substack{k=1 \\ k \neq j}}^{N} \omega_k q_k \right) \omega_j \mathcal{M}_N(1, \dots, n, \dots, N)$$

$$\frac{d}{d\omega_j} \left(\lim_{\Delta_j \to 1} \omega_j^{\Delta_j - 1} \right) = \frac{d}{d\omega_j} \theta(\omega_j) = \delta(\omega_j)$$

$$\left\langle \prod_{n=1}^{N} \lim_{\Delta_j \to 1} (\Delta_j - 1)\mathcal{O}_{\Delta_n}(z_n, \bar{z}_n, \eta^n) \right\rangle = \left(\prod_{\substack{n=1 \\ n \neq j}}^{N} \int d\omega_n \omega_n^{\Delta_n - 1} \right) \delta^{(4)} \left(\sum_{\substack{k=1 \\ k \neq j}}^{N} \omega_k q_k \right)$$

$$\times \lim_{\omega_j \to 0} \omega_j \mathcal{M}_N(1, \dots, n, \dots, N)$$

$$\left\langle \prod_{n=1}^{N} \lim_{\Delta_j \to 1} (\Delta_j - 1)\mathcal{O}_{\Delta_n}(z_n, \bar{z}_n, \eta^n) \right\rangle = \sum_{\substack{i=1 \\ i \neq j}}^{N} \omega_i \left\{ \frac{z_{j-1,i}^2 \bar{z}_{ji}}{z_{j-1,j}^2 z_{ji}} + \frac{\bar{z}_{j-1,i}^2 z_{ji}}{\bar{z}_{j-1,j}^2 \bar{z}_{ji}} \delta^4(\eta^j) \right\}$$

$$\times \left(\prod_{\substack{n=1 \\ n \neq j}}^{N} \int_0^{\infty} d\omega_k \omega_k^{\Delta_k - 1} \right) \delta^{(4)} \left(\sum_{\substack{k=1 \\ k \neq j}}^{N} \omega_k q_k \right) \mathcal{M}_{N-1}(1, \dots, i, \dots, N)$$

$$= \sum_{\substack{i=1 \\ i \neq j}}^{n} \left\{ \frac{z_{j-1,i}^2 \bar{z}_{ji}}{z_{j-1,j}^2 z_{ji}} + \frac{\bar{z}_{j-1,i}^2 z_{ji}}{\bar{z}_{j-1,j}^2 \bar{z}_{ji}} \delta^8(\eta^j) \right\} \left[\prod_{\substack{n=1 \\ n \neq i}}^{N} \int_0^{\infty} d\omega_k \omega_k^{\Delta_k - 1} \int_0^{\infty} d\omega_i \omega_i^{\Delta_i} \right.$$

$$= \sum_{\substack{i=1 \\ i \neq j}}^{n} \left\{ \frac{z_{j-1,i}^2 \bar{z}_{ji}}{z_{j-1,j}^2 z_{ji}} + \frac{\bar{z}_{j-1,i}^2 z_{ji}}{\bar{z}_{j-1,j}^2 \bar{z}_{ji}} \delta^8(\eta^j) \right\} \left\langle \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1, \eta_1), \cdots \mathcal{O}_{\Delta_i + 1}(z_i, \bar{z}_i, \eta^i), \dots \right\rangle$$

$$\left\langle J_1(z, \bar{z}, \eta) \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1, \eta^1) \cdots \mathcal{O}_{\Delta_N}(z_N, \bar{z}_N, \eta^N) \right\rangle$$

$$= \sum_{i=1}^{N} \left\{ \frac{(\bar{z} - \bar{z}_i)}{(z - z_i)} \frac{(z_N - z_i)^2}{(z_N - z)^2} + \frac{(z - z_i)}{(\bar{z} - z_i)} \frac{(\bar{z}_N - \bar{z}_i)^2}{(\bar{z}_N - \bar{z})^2} \delta^8(\eta) \right\}$$

$$\times \left\langle \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1, \eta^1), \cdots \mathcal{O}_{\Delta_i + 1}(z_i, \bar{z}_i, \eta^i), \cdots, \mathcal{O}_{\Delta_N}(z_N, \bar{z}_N, \eta^N) \right\rangle$$

$$J_1(z, \bar{z}, \eta) = \lim_{\Delta \to 1} (\Delta - 1)\mathcal{O}_{\Delta}(z, \bar{z}, \eta)$$

$$\langle J_1(z,\bar{z},\eta)\mathcal{O}_{\Delta_1}(z_1,\bar{z}_1,\eta^1)\cdots\mathcal{O}_{\Delta_N}(z_N,\bar{z}_N,\eta^N)\rangle$$
$$=\sum_{i=1}^{N}\left\{\frac{(\bar{z}-\bar{z}_i)}{(z-z_i)}\frac{(\xi-z_i)^2}{(\xi-z)^2}+\frac{(z-z_i)}{(\bar{z}-\bar{z}_i)}\frac{\left(\bar{\xi}-\bar{z}_i\right)^2}{(\bar{\xi}-\bar{z})^2}\delta^8(\eta)\right\}$$
$$\times\langle\mathcal{O}_{\Delta_1}(z_1,\bar{z}_1,\eta^1),\cdots\mathcal{O}_{\Delta_i+1}(z_i,\bar{z}_i,\eta^i),\cdots,\mathcal{O}_{\Delta_N}(z_N,\bar{z}_N,\eta^N)\rangle$$

$$\left\langle J_1(z,\bar{z})\prod_{n=1}^{N}\mathcal{O}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle=\sum_{i=1}^{N}\frac{(\bar{z}-\bar{z}_i)(\xi-z_i)^2}{(z-z_i)(\xi-z)^2}\langle\mathcal{O}_{\Delta_1,\ell_1}(z_1,\bar{z}_1)$$
$$\cdots\mathcal{O}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots,\mathcal{O}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\rangle$$

$$\left\langle\bar{J}_1(z,\bar{z})\prod_{n=1}^{N}\mathcal{O}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle=\sum_{i=1}^{N}\frac{(z-z_i)}{(\bar{z}-\bar{z}_i)}\frac{\left(\bar{\xi}-\bar{z}_i\right)^2}{(\bar{\xi}-\bar{z})^2}\langle\mathcal{O}_{\Delta_1,\ell_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots,\mathcal{O}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\rangle$$

$$J_1(z,\bar{z})=\lim_{\Delta\to1}(\Delta-1)\mathcal{O}_{\Delta,+2}(z,\bar{z}),\bar{J}_1(z,\bar{z})=\lim_{\Delta\to1}(\Delta-1)\mathcal{O}_{\Delta,-2}(z,\bar{z})$$

$$\langle J_0(z,\bar{z},\eta)\mathcal{O}_{\Delta_1}(z,\bar{z},\eta^1)\cdots\mathcal{O}_{\Delta_N}(z_N,\bar{z}_N,\eta^N)\rangle$$
$$=\sum_{i=1}^{N}\left\{\frac{(\bar{z}-\bar{z}_i)}{(z-z_i)}\frac{(\xi-z_i)}{(\xi-z)}\left((\bar{z}-\bar{z}_i)\partial_{\bar{z}_i}-2\bar{h}_i\right)+\frac{(z-z_i)}{(\bar{z}-\bar{z}_i)}\frac{\left(\bar{\xi}-\bar{z}_i\right)}{(\bar{\xi}-\bar{z})}\delta^8(\eta)\left((z-z_i)\partial_{z_i}-2h_i\right)\right\}$$
$$\times\langle\mathcal{O}_{\Delta_1}(z_1,\bar{z}_1,\eta^1),\cdots\mathcal{O}_{\Delta_i}\left(z_i,\bar{z}_i,\eta^i\right)\cdots,\mathcal{O}_{\Delta_N}(z_N,\bar{z}_N,\eta^N)\rangle$$

$$J_0(z,\bar{z},\eta)=\lim_{\Delta\to0}\Delta\left[\mathcal{O}_{\Delta,+2}(z,\bar{z})+\delta^8(\eta)\mathcal{O}_{\Delta,-2}(z,\bar{z})\right].$$

$$\left\langle J_0(z,\bar{z})\prod_{n=1}^{N}\mathcal{O}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle=\sum_{i=1}^{N}\frac{(\bar{z}-\bar{z}_i)}{(z-z_i)}\frac{(\xi-z_i)}{(\xi-z)}\left((\bar{z}-\bar{z}_i)\partial_{\bar{z}_i}-2\bar{h}_i\right)$$
$$\times\langle\cdots\mathcal{O}_{\Delta_i,\ell_i}(z_i,\bar{z}_i)\cdots\rangle$$

$$\left\langle\bar{J}_0(z,\bar{z})\prod_{n=1}^{N}\mathcal{O}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle=\sum_{i=1}^{N}\frac{(z-z_i)}{(\bar{z}-\bar{z}_i)}\frac{\left(\bar{\xi}-\bar{z}_i\right)}{(\bar{\xi}-\bar{z})}\left((z-z_i)\partial_{z_i}-2h_i\right)$$
$$\times\langle\cdots\mathcal{O}_{\Delta_i,\ell_i}(z_i,\bar{z}_i)\cdots\rangle$$

$$J_0(z,\bar{z})=\lim_{\Delta\to0}\Delta\mathcal{O}_{\Delta,+2}(z,\bar{z}),\bar{J}_0(z,\bar{z})=\lim_{\Delta\to0}\Delta\mathcal{O}_{\Delta,-2}(z,\bar{z})$$

$$\mathcal{M}_{N+1}(\psi_{s+}^{A},\{p_1,\eta^1\},\ldots\{p_N,\eta^N\})=\sum_{i=1}^{N}\frac{[si]\langle ri\rangle}{\langle si\rangle\langle rs\rangle}\frac{\partial}{\partial\eta_{iA}}\mathcal{M}_N(\{p_1,\eta^1\},\ldots\{p_N,\eta^N\})$$

$$\ell_i\in\{-3/2,-1,-1/2,0,+1/2,+1,+3/2,+2\}$$

$$\ell_i^c\in\{-2,-3/2,-1,-1/2,0,+1/2,+1,+3/2\}$$

$$\left\langle J^A_{1/2}(z,\bar{z})\prod_{n=1}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle=\sum_{i=1}^{N}f(A,\ell_i,*_i,*_i')(-1)^{\sigma_i}\frac{(\bar{z}-\bar{z}_i)}{(z-z_i)}\frac{(\xi-z_i)}{(\xi-z)}\left\langle\cdots\mathcal{O}^{*_i'}_{\Delta_i+\frac{1}{2},\ell_i^c}(z_i,\bar{z}_i),\cdots\right\rangle$$

$$\left\langle \bar{J}_{1/2A}(z,\bar{z})\prod_{n=1}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n^c}(z_n,\bar{z}_n)\right\rangle=\sum_{i=1}^{N}\bar{f}(A,\ell_i^c,*_i,*_i')(-1)^{\sigma_i}\frac{(z-z_i)}{(\bar{z}-\bar{z}_i)}\frac{(\bar{\xi}-\bar{z}_i)}{(\bar{\xi}-\bar{z})}\left\langle\cdots\mathcal{O}^{*_i'}_{\Delta_i+\frac{1}{2},\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$

$$J^A_{1/2}(z,\bar{z})=\lim_{\Delta\to\frac{1}{2}}\left(\Delta-\frac{1}{2}\right)\mathcal{O}^A_{\Delta,+\frac{3}{2}}(z,\bar{z}),\bar{J}_{1/2A}(z,\bar{z})=\lim_{\Delta\to\frac{1}{2}}\left(\Delta-\frac{1}{2}\right)\mathcal{O}_{\Delta,-\frac{3}{2},A}(z,\bar{z})$$

$$T_0(z,\bar{z})=\lim_{\Delta\to0}\frac{3!\,\Delta}{2\pi}\int d^2z'\frac{1}{(z-z')^4}\mathcal{O}_{\Delta,-2}(z',\bar{z}')$$
$$\bar{T}_0(z,\bar{z})=\lim_{\Delta\to0}\frac{3!\,\Delta}{2\pi}\int d^2z'\frac{1}{(\bar{z}-\bar{z}')^4}\mathcal{O}_{\Delta,+2}(z',\bar{z}')$$

$$\bar{\partial}T_0=-\frac{1}{2}\partial^3\bar{J}_0(z,\bar{z})$$

$$T_{\text{mod}}:=T_0+\frac{1}{2}\partial^3\epsilon_{\bar{J}_0}$$

$$\epsilon_{\bar{J}_0}:=\int_{\bar{z}_0}^{\bar{z}}d\bar{w}\bar{J}_0(z,\bar{w})$$

$$T(z)\mathcal{O}_{\Delta,\ell}(w,\bar{w})=\frac{h}{(z-w)^2}\mathcal{O}_{\Delta,\ell}(w,\bar{w})+\frac{1}{z-w}\partial_w\mathcal{O}_{\Delta,\ell}(w,\bar{w})$$
$$\bar{T}(\bar{z})\mathcal{O}_{\Delta,\ell}(w,\bar{w})=\frac{\bar{h}}{(\bar{z}-\bar{w})^2}\mathcal{O}_{\Delta,\ell}(w,\bar{w})+\frac{1}{\bar{z}-\bar{w}}\partial_{\bar{w}}\mathcal{O}_{\Delta,\ell}(w,\bar{w})$$

$$P(z)=\lim_{\Delta\to1}\frac{(\Delta-1)}{4}\partial_{\bar{z}}\mathcal{O}_{\Delta,+2}(z,\bar{z})$$
$$\bar{P}(\bar{z})=\lim_{\Delta\to1}\frac{(\Delta-1)}{4}\partial_z\mathcal{O}_{\Delta,-2}(z,\bar{z})$$

$$P(z)\mathcal{O}_{\Delta,\ell}(w,\bar{w})=\frac{1}{z-w}\mathcal{O}_{\Delta+1,\ell}(w,\bar{w})$$

$$P(z)=\frac{1}{2\pi i}\oint d\bar{z}\mathcal{P}(z,\bar{z}),\bar{P}(\bar{z})=\frac{1}{2\pi i}\oint dz\mathcal{P}(z,\bar{z})$$

$$\mathcal{P}(z,\bar{z})\mathcal{O}_{\Delta,\ell}(w,\bar{w})=\frac{1}{z-w}\frac{1}{\bar{z}-\bar{w}}\mathcal{O}_{\Delta+1,\ell}(w,\bar{w})$$

$$S_A(z)= \lim_{\Delta\to\frac{1}{2}} \frac{\Delta-\frac{1}{2}}{\pi}\int\ d^2z' \frac{1}{(z-z')^3}\mathcal{O}_{A;\Delta,-\frac{3}{2}}(z',\bar{z}')$$

$$\bar{S}^A(\bar{z}) = \lim_{\Delta\to\frac{1}{2}} \frac{\Delta-\frac{1}{2}}{\pi}\int\ d^2z' \frac{1}{(\bar{z}-\bar{z}')^3}\mathcal{O}^A_{\Delta,+\frac{3}{2}}(z',\bar{z}')$$

$$\bar{\partial}S_A(z,\bar{z}) = \lim_{\Delta\to 1/2}\left(\Delta-\frac{1}{2}\right)\partial_z^2\mathcal{O}_{A;\Delta,-\frac{3}{2}}(z,\bar{z}) = \partial^2\bar{J}_{1/2A}(z,\bar{z}) \neq 0$$

$$\epsilon_{\bar{J}_{1/2}A}(z,\bar{z}) := \int_{\bar{z}_0}^{z} d\bar{w}\bar{J}_{1/2A}(w,\bar{w})$$

$$S^A_{\text{mod}} := S^A - \partial^2\epsilon^A_{\bar{J}_{1/2}}$$

$$T(z)S_A(w) = \frac{3}{2}\frac{S_A(w)}{(z-w)^2} + \frac{\partial S_A(w)}{z-w}$$

$$\bar{T}(\bar{z})\bar{S}^A(\bar{w}) = \frac{3}{2}\frac{\bar{S}^A(\bar{z})}{(\bar{z}-\bar{w})^2} + \frac{\overline{\partial\bar{S}^A}(\bar{w})}{\bar{z}-\bar{w}}$$

$$:\{S_B(z),\bar{S}^A(\bar{z})\}: =: S_B(z)\bar{S}^A(\bar{z}) + \bar{S}^A(\bar{z})S_B(z): = \delta^A_B\mathcal{P}(z,\bar{z})$$

$$\left\langle S_B(z)\bar{S}^A(\bar{w})\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$

$$= \delta^A_B\sum_{i=3}^{N}\left[\frac{1}{(\bar{w}-\bar{z})^2}\frac{\bar{z}-\bar{z}_i}{z-z_i} + \frac{1}{\bar{z}-\bar{w}}\frac{1}{z-z_i} + \frac{1}{\bar{w}-\bar{z}_i}\frac{1}{z-z_i}\right]\left\langle\cdots\mathcal{O}^{*i}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$

$$-\sum_{i=3}^{N} f(A,\ell_i,*_i,*_i')\bar{f}(B,\ell_i-1/2,*_i',*_i'')\frac{1}{z-z_i}\frac{1}{\bar{w}-\bar{z}_i}\left\langle\cdots\mathcal{O}^{*_i'}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$

$$+\sum_{\substack{i,j=3\\ i\neq j}}^{N}(-1)^{\sigma_i+\sigma_j}f(A,\ell_i,*_i,*_i')\bar{f}\left(B,\ell_j,*_j,*_j'\right)\frac{1}{z-z_i}\frac{1}{\bar{w}-\bar{z}_j}\left\langle\cdots\mathcal{O}^{*_i'}_{\Delta_i+\frac{1}{2},\ell_i-\frac{1}{2}}(z_i,\bar{z}_i),\cdots\right.$$

$$\left.\mathcal{O}^{*_j'}_{\Delta_j+\frac{1}{2},\ell_j+\frac{1}{2}}(z_j,\bar{z}_j)\cdots\right\rangle$$

$$\left\langle \bar{S}^A(\bar{z})S_B(w)\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$=\delta_B^A\sum_{i=3}^{N}\left[\frac{1}{(w-z)^2}\frac{z-z_i}{\bar{z}-\bar{z}_i}+\frac{1}{z-w}\frac{1}{\bar{z}-\bar{z}_i}+\frac{1}{\bar{z}-\bar{z}_i}\frac{1}{w-z_i}\right]\left\langle\cdots\mathcal{O}^{*i}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$
$$-\sum_{i=3}^{N}\bar{f}(B,\ell_i,*_i,*_i')f(A,\ell_i+1/2,*_i',*_i'')\frac{1}{w-z_i}\frac{1}{\bar{z}-\bar{z}_i}\left\langle\cdots\mathcal{O}^{*_i'}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$
$$-\sum_{\substack{i,j=3\\ i\neq j}}^{N}(-1)^{\sigma_i+\sigma_j}\bar{f}(B,\ell_i,*_i,*_i')f\left(A,\ell_j,*_j,*_j'\right)\frac{1}{w-z_i}\frac{1}{\bar{z}-\bar{z}_j}\left\langle\cdots\mathcal{O}^{*_i'}_{\Delta_i+\frac{1}{2},\ell_i+\frac{1}{2}}(z_i,\bar{z}_i),\cdots\right.$$
$$\left.\mathcal{O}^{*_j'}_{\Delta_j+\frac{1}{2},\ell_j-\frac{1}{2}}\left(z_j,\bar{z}_j\right)\cdots\right\rangle$$

$$\left\langle(\bar{S}^A(\bar{z})S_B(w)+S_B(z)\bar{S}^A(\bar{w}))\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$=\delta_B^A\sum_{i=3}^{N}\left[\frac{1}{(w-z)^2}\frac{z-z_i}{\bar{z}-\bar{z}_i}+\frac{1}{z-w}\frac{1}{\bar{z}-\bar{z}_i}+\frac{1}{\bar{z}-\bar{z}_i}\frac{1}{w-z_i}\right]\left\langle\cdots\mathcal{O}^{*i}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$
$$+\delta_B^A\sum_{i=3}^{N}\left[\frac{1}{(\bar{w}-\bar{z})^2}\frac{\bar{z}-\bar{z}_i}{z-z_i}+\frac{1}{\bar{z}-\bar{w}}\frac{1}{z-z_i}+\frac{1}{\bar{w}-\bar{z}_i}\frac{1}{z-z_i}\right]\left\langle\cdots\mathcal{O}^{*i}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$
$$-\sum_{i=3}^{N}f(A,\ell_i,*_i,*_i')\bar{f}(B,\ell_i-1/2,*_i',*_i'')\frac{1}{z-z_i}\frac{1}{\bar{w}-\bar{z}_i}\left\langle\cdots\mathcal{O}^{*_i''}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$
$$-\sum_{i=3}^{N}\bar{f}(B,\ell_i,*_i,*_i')f(A,\ell_i+1/2,*_i',*_i'')\frac{1}{w-z_i}\frac{1}{\bar{z}-\bar{z}_i}\left\langle\cdots\mathcal{O}^{*_i''}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$

$$\left\langle:\{S_B(z),\bar{S}^A(\bar{z})\}:\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$=2\delta_B^A\sum_{i=3}^{N}\frac{1}{\bar{z}-\bar{z}_i}\frac{1}{z-z_i}\left\langle\cdots\mathcal{O}^{*i}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$
$$-\sum_{i=3}^{N}f(A,\ell_i,*_i,*_i')\bar{f}(B,\ell_i-1/2,*_i',*_i'')\frac{1}{z-z_i}\frac{1}{\bar{z}-\bar{z}_i}\left\langle\cdots\mathcal{O}^{*_i''}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$
$$-\sum_{i=3}^{N}\bar{f}(B,\ell_i,*_i,*_i')f(A,\ell_i+1/2,*_i',*_i'')\frac{1}{z-z_i}\frac{1}{\bar{z}-\bar{z}_i}\left\langle\cdots\mathcal{O}^{*_i''}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle.$$

$$f(A,+2,-,*_i')\bar{f}(B,+3/2,*_i',-)\mathcal{O}_{\Delta_i+1,+2}(z_i,\bar{z}_i)=\delta_B^A\mathcal{O}_{\Delta_i+1,+2}(z_i,\bar{z}_i).$$

$$f(A,+3/2,C,*_i')\bar{f}(B,+1,*_i',*_i'')\mathcal{O}^{*_i''}_{\Delta_i+1,+3/2}(z_i,\bar{z}_i)=2!\,\delta_B^{[A}\mathcal{O}^{C]}_{\Delta_i+1,+3/2}(z_i,\bar{z}_i)$$

$$\bar{f}(B,+3/2,C,-)f(A,+2,-,*_i'')\mathcal{O}^{*_i''}_{\Delta_i+1,+3/2}(z_i,\bar{z}_i)=\delta_B^C\mathcal{O}^A_{\Delta_i+1,+3/2}(z_i,\bar{z}_i)$$

$$f(A,+1,CD,*_i')\bar{f}(B,+1/2,*_i',*_i'')\mathcal{O}^{*_i''}_{\Delta_i+1,+2}(z_i,\bar{z}_i)$$
$$= \bar{f}(B,+1/2,*_i',ACD)\mathcal{O}^{ACD}_{\Delta_i+\frac{1}{2},+1/2}(z_i,\bar{z}_i)$$
$$= 3\delta_B^{[A}\mathcal{O}^{CD]}_{\Delta_i+1,+1}(z_i,\bar{z}_i)$$

$$\bar{f}(B,+1,CD,*_i')f(A,+3/2,*_i',*_i'')\mathcal{O}^{*_i''}_{\Delta_i+1,+3/2}(z_i,\bar{z}_i) = -2!\,\delta_B^{[C}\mathcal{O}^{D]A}_{\Delta_i+1,+1}(z_i,\bar{z}_i)$$

$$3\delta_B^{[A}\mathcal{O}^{CD]} - 2!\,\delta_B^{[C}\mathcal{O}^{D]A} = \frac{1}{2}\left[\left(\delta_B^A\mathcal{O}^{CD} - \delta_B^A\mathcal{O}^{DC}\right) + \left(\delta_B^C\mathcal{O}^{DA} - \delta_B^C\mathcal{O}^{AD}\right) + \left(\delta_B^D\mathcal{O}^{AC} - \delta_B^D\mathcal{O}^{CA}\right)\right]$$
$$-\delta_B^C\mathcal{O}^{DA} + \delta_B^D\mathcal{O}^{CA}$$
$$= \delta_B^A\mathcal{O}^{CD}$$

$$\left\langle :\{S_B(z),\bar{S}^A(\bar{z})\}:\prod_{n=3}^{N}\mathcal{O}^{*_n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle = \delta_B^A\sum_{i=3}^{N}\frac{1}{\bar{z}-\bar{z}_i}\frac{1}{z-z_i}\left\langle\cdots\mathcal{O}^{*_i}_{\Delta_i+1,\ell_i}(z_i,\bar{z}_i),\cdots\right\rangle$$

$$:\{S_B(z),\bar{S}^A(\bar{z})\}:\mathcal{O}_{\Delta,\ell}(w,\bar{w}) = \frac{1}{z-w}\frac{1}{\bar{z}-\bar{w}}\mathcal{O}_{\Delta+1,\ell}(w,\bar{w})$$

$$:\{S_B(z),\bar{S}^A(\bar{z})\}: = \delta_B^A\mathcal{P}(z,\bar{z}).$$

$$\left[T_B^A,T_D^C\right] = \delta_D^AT_B^C - \delta_B^CT_D^A,$$

$$\left(T_B^A\right)^C{}_D = \delta_D^A\delta_B^C - \frac{1}{8}\delta_B^A\delta_D^C$$

$$[T_B^A,Q_\alpha^C] = \left(T_B^A\right)^C_DQ_\alpha^D, [T_B^A,\bar{Q}_{\dot{\alpha}C}] = -\left(T_B^A\right)^D_C\bar{Q}_{\dot{\alpha}D}.$$

$$\tilde{\mathcal{G}}_B^A(z,\bar{z})S_C(w) \sim ((z-w)\text{ singularity })\left(T_B^A\right)^D_CS_D(w)$$
$$\tilde{\mathcal{G}}_B^A(z,\bar{z})\bar{S}^C(w) \sim -((\bar{z}-\bar{w})\text{ singularity })\left(T_B^A\right)^C_D\bar{S}^D(w).$$

$$G_{AB}(z,\bar{z}) = \lim_{\Delta\to0}\frac{\Delta}{\pi}\int d^2z'\frac{1}{(z-z')^a}\frac{1}{(\bar{z}-\bar{z}')^b}\mathcal{O}_{AB;\Delta,-1}(z',\bar{z}')$$
$$\bar{G}^{CD}(z,\bar{z}) = \lim_{\Delta\to0}\frac{\Delta}{\pi}\int d^2z'\frac{1}{(\bar{z}-\bar{z}')^{a'}}\frac{1}{(z-z')^{b'}}\mathcal{O}^{CD}_{\Delta,+1}(z',\bar{z}')$$

$$\left\langle G_{AB}(z,\bar{z})\prod_{n=2}^{N}\mathcal{O}^{*_n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$= \lim_{\Delta_1\to0}\frac{\Delta_1}{\pi}\int d^2z_1\frac{1}{(z-z_1)^a}\frac{1}{(\bar{z}-\bar{z}_1)^b}\left\langle\mathcal{O}_{AB\Delta_1,-1}(z_1,\bar{z}_1)\prod_{n=2}^{N}\mathcal{O}^{*_n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$= \frac{1}{\pi}\int d^2z_1\frac{1}{(z-z_1)^a}\frac{1}{(\bar{z}-\bar{z}_1)^b}\left[\sum_{n=2}^{N}f(A,B,\ell_i,*_n,*_n')\frac{z_1-z_n}{\bar{z}_1-\bar{z}_n}\left\langle\cdots\mathcal{O}^{*_n'}_{\Delta_n,\ell_n+1}(z_n,\bar{z}_n)\right\rangle\right]$$

$$\int d^2z_1 \frac{1}{(z-z_1)^A}\frac{1}{(\bar{z}-\bar{z}_1)^B}\frac{(\bar{z}_1-\bar{z}_j)^s}{z_1-z_j} = C_s(A,B)\frac{1}{(z_j-z)^A(\bar{z}_j-\bar{z})^{B-s-1}}$$

$$\int d^2z_1 \frac{1}{(\bar{z}-\bar{z}_1)^A}\frac{1}{(z_1-z_1)^B}\frac{(z_1-z_j)^s}{\bar{z}_1-\bar{z}_j} = C_s(A,B)\frac{1}{(\bar{z}_j-\bar{z})^A}\frac{1}{(z_j-z)^{B-s-1}}$$

$$C_s(A,B) = \frac{(-1)^{s+A+B}(-\pi)s!}{(-B+1)(-B+2)\cdots(-B+s+1)}$$

$$\int d^2z_1 \frac{1}{(z-z_1)^a}\frac{1}{(\bar{z}-\bar{z}_1)^b}\frac{z_1-z_n}{\bar{z}_1-\bar{z}_n} = C_1(b,a)\frac{1}{(\bar{z}_n-\bar{z})^b}\frac{1}{(z_n-z)^{a-2}}$$

$$\left\langle G_{AB}(z,\bar{z})\prod_{n=2}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$

$$= \sum_{i=2}^{N} f(A,B,\ell_i,*_i,*_i')C_1(b,a)\frac{1}{(\bar{z}_i-\bar{z})^b}\frac{1}{(z_i-z)^{a-2}}\left\langle\cdots\mathcal{O}^{*_i'}_{\Delta_i,\ell_i+1}(z_i,\bar{z}_i)\right\rangle$$

$$\ell_n \in \left\{-2,-\frac{3}{2},-1,-\frac{1}{2},0,+\frac{1}{2},+1\right\}$$

$$\ell_n' \in \left\{-1,-\frac{1}{2},0,+\frac{1}{2},+1,+\frac{3}{2},+2\right\}$$

$$\left\langle \bar{G}^{CD}(z,\bar{z})\prod_{n=2}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n'}(z_n,\bar{z}_n)\right\rangle$$

$$= \sum_{n=2}^{N} \bar{f}(C,D,\ell_i',*_n,*_n')C(b',a')\frac{1}{(\bar{z}_n-\bar{z})^{b'}}\frac{1}{(z_n-z)^{a'-2}}\left\langle\cdots\mathcal{O}^{*_n'}_{\Delta_n,\ell_n'-1}(z_n,\bar{z}_n)\right\rangle$$

$$G_{AB}(z)\mathcal{O}^{*}_{\Delta,\ell}(w,\bar{w}) \sim f(A,B,\ell,*,*')C_1(b,a)\frac{1}{(\bar{w}-\bar{z})^b}\frac{1}{(w-z)^{a-2}}\mathcal{O}^{*'}_{\Delta,\ell'}(z,\bar{w})$$

$$\bar{G}^{CD}(z)\mathcal{O}^{*}_{\Delta,\ell'}(w,\bar{w}) \sim \bar{f}(C,D,\ell',*,*')C_1(b',a')\frac{1}{(\bar{w}-\bar{z})^{b'}}\frac{1}{(z-z)^{a'-2}}\mathcal{O}^{*'}_{\Delta,\ell}(w,\bar{w})$$

$$\mathcal{G}^{CD}_{AB}(z,\bar{z};w,\bar{w}) := G_{AB}(z)\bar{G}^{CD}(\bar{w}) - \bar{G}^{CD}(\bar{w})G_{AB}(z) \equiv [G_{AB}(z),\bar{G}^{CD}(\bar{w})].$$

$$\mathcal{G}^{CD}_{AB}(z,\bar{z}) =: \mathcal{G}^{CD}_{AB}(z,\bar{z};z,\bar{z}) :=: G_{AB}(z)\bar{G}^{CD}(\bar{z}) - \bar{G}^{CD}(\bar{z})G^{AB}(z) :$$
$$\equiv : [G_{AB}(z),\bar{G}^{CD}(\bar{z})] :$$

$$\left\langle \mathcal{G}_{AB}^{CD}(z,\bar{z})\prod_{n=3}^{N}\mathcal{O}_{\Delta_n,\ell_n}^{*n}(z_n,\bar{z}_n)\right\rangle$$
$$=(-1)^{a+b+a'+b'}C_1(b,a)C_1(b',a')\Big[f\big(A,B,\ell_j,*_j,*_j'\big)\bar{f}\big(C,D,\ell_j+1,*_j',*_j''\big)$$
$$\times\frac{1}{(z-z_j)^{a+b'-2}}\frac{1}{(\bar{z}-\bar{z}_j)^{a'+b-2}}\left\langle\mathcal{O}_{\Delta_3,\ell_3}^{*_3}(z_3,\bar{z}_3)\cdots\mathcal{O}_{\Delta_j,\ell_j}^{*_j''}(z_j,\bar{z}_j)\cdots\mathcal{O}_{\Delta_N,\ell_N}^{*N}(z_N,\bar{z}_N)\right\rangle$$
$$-\bar{f}\big(C,D,\ell_j,*_j,*_j'\big)f\big(A,B,\ell_j-1,*_j',*_j''\big)$$
$$\times\frac{1}{(z-z_j)^{a'+b-2}}\frac{1}{(\bar{z}-\bar{z}_j)^{a+b'-2}}\left\langle\mathcal{O}_{\Delta_3,\ell_3}^{*_3}(z_3,\bar{z}_3)\cdots\mathcal{O}_{\Delta_j,\ell_j}^{*_j''}(z_j,\bar{z}_j)\cdots\mathcal{O}_{\Delta_N,\ell_N}^{*N}(z_N,\bar{z}_N)\right\rangle\Big]$$

$$\mathcal{G}_{AB}^{CD}(z,\bar{z})\mathcal{O}_{E\Delta,-\frac{3}{2}}(w,\bar{w})\sim-\delta_{AB}^{CD}\frac{(-1)^{a+b+a'+b'}C_1(b,a)C_1(b',a')}{(z-w)^{a+b'-2}(\bar{z}-\bar{w})^{a'+b-2}}\mathcal{O}_{E\Delta,-\frac{3}{2}}(w,\bar{w})$$
$$\mathcal{G}_{AB}^{CD}(z,\bar{z})\mathcal{O}_{\Delta,+\frac{3}{2}}^{E}(w,\bar{w})\sim\delta_{AB}^{CD}\frac{(-1)^{a+b+a'+b'}C_1(b,a)C_1(b',a')}{(z-w)^{a'+b-2}(\bar{z}-\bar{w})^{a+b'-2}}\mathcal{O}_{\Delta,+\frac{3}{2}}^{E}(w,\bar{w})$$

$$\lim_{\Delta_1\to0}\Delta_1\mathcal{O}_{AB;\Delta_1,-1}(z,\bar{z})\mathcal{O}_{E,\Delta,-\frac{3}{2}}(z_1,\bar{z}_1)=\frac{z-z_1}{\bar{z}-\bar{z}_1}\mathcal{O}_{ABE;\Delta,-\frac{1}{2}}(z_1,\bar{z}_1)$$

$$\lim_{\Delta_2\to0}\Delta_2\mathcal{O}_{\Delta_{2,+1}}^{cd}(w,\bar{w})\mathcal{O}_{abr;\Delta,-\frac{1}{2}}(z_1,\bar{z}_1)=-\delta_{ab}^{cd}\frac{\bar{w}-\bar{z}_1}{w-z_1}\mathcal{O}_{r,\Delta,-\frac{3}{2}}(z_1,\bar{z}_1)$$

$$\big(\tilde{\mathcal{G}}_B^A\big)_D^C(z,\bar{z}):=-\left(\frac{1}{7}\delta_D^A\sum_{E=1}^{8}\mathcal{G}_{BE}^{EC}(z,\bar{z})+\frac{1}{56}\delta_B^A\sum_{E=1}^{8}\mathcal{G}_{ED}^{EC}(z,\bar{z})\right).$$

$$\delta_{b_1\dots b_n}^{a_1\dots a_n}=\sum_{\sigma\in S_n}\operatorname{sign}(\sigma)\delta_{b_{\sigma(1)}}^{a_1}\dots\delta_{b_{\sigma(n)}}^{a_n}$$

$$\sum_{E=1}^{8}\delta_{BE}^{EC}=-7\delta_B^C,\sum_{E=1}^{8}\delta_{ED}^{EC}=7\delta_D^C.$$

$$\big(\tilde{\mathcal{G}}_A^C\big)_B^D(z,\bar{z})\mathcal{O}_{D\Delta,-\frac{3}{2}}(w,\bar{w})\sim\frac{(-1)^{a+b+a'+b'}C_1(b,a)C_1(b',a')}{(z-w)^{a+b'-2}(\bar{z}-\bar{w})^{a'+b-2}}\big(T_A^C\big)_B^D\mathcal{O}_{D\Delta,-\frac{3}{2}}(w,\bar{w})$$
$$\big(\tilde{\mathcal{G}}_A^C\big)_B^D(z,\bar{z})\mathcal{O}_{\Delta,+\frac{3}{2}}^{B}(w,\bar{w})\sim-\frac{(-1)^{a+b+a'+b'}C_1(b,a)C_1(b',a')}{(z-w)^{a'+b-2}(\bar{z}-\bar{w})^{a+b'-2}}\big(T_A^C\big)_B^D\mathcal{O}_{\Delta,+\frac{3}{2}}^{B}(w,\bar{w})$$

$$\big(\tilde{\mathcal{G}}_A^C\big)_B^D(z,\bar{z})S_D(w)\sim\big(T_A^C\big)_B^D\lim_{\Delta\to\frac{1}{2}}\frac{\Delta-\frac{1}{2}}{\pi}(-1)^{a+b+a'+b'}C_1(b,a)C_1(b',a')$$
$$\times\int d^2z_1\frac{1}{(w-z_1)^3}\frac{1}{(z-z_1)^{a+b'-2}}\frac{1}{(\bar{z}-\bar{z}_1)^{a'+b-2}}\mathcal{O}_{D,\Delta,-\frac{3}{2}}(z_1,\bar{z}_1)$$

$$\big(\tilde{\mathcal{G}}_A^C\big)_B^D(z,\bar{z})\bar{S}^B(\bar{w})\sim-\big(T_A^C\big)_B^D\lim_{\Delta\to\frac{1}{2}}\frac{\Delta-\frac{1}{2}}{\pi}(-1)^{a+b+a'+b'}C_1(b,a)C_1(b',a')$$
$$\times\int d^2z_1\frac{1}{(\bar{w}-\bar{z}_1)^3}\frac{1}{(z-z_1)^{a'+b-2}}\frac{1}{(\bar{z}-\bar{z}_1)^{a+b'-2}}\mathcal{O}_{\Delta,+\frac{3}{2}}^{B}(z_1,\bar{z}_1)$$

$$a + b' - 2 = 0 \text{ and } a' + b - 2 = 0$$

$$\left(\tilde{\mathcal{G}}_A^C\right)_B^D(z,\bar{z})S_D(w) \sim \varsigma,\ \left(\tilde{\mathcal{G}}_A^C\right)_B^D(z,\bar{z})\bar{S}^B(\bar{w}) \sim \varrho$$

$$T(z) = \sum_{n\in\mathbb{Z}} L_n z^{-n-2}, \bar{T}(\bar{z}) = \sum_{n\in\mathbb{Z}} \bar{L}_n \bar{z}^{-n-2};$$

$$\mathcal{P}(z,\bar{z}) \equiv \sum_{n,m\in\mathbb{Z}} P_{n-\frac{1}{2},m-\frac{1}{2}} z^{-n-1}\bar{z}^{-m-1}$$

$$P(z) = \sum_{n\in\mathbb{Z}} P_{n-\frac{1}{2}} z^{-n-1}, \bar{P}(\bar{z}) = \sum_{m\in\mathbb{Z}} \bar{P}_{m-\frac{1}{2}} z^{-m-1}$$

$$P_{n-\frac{1}{2},-\frac{1}{2}} = P_{n-\frac{1}{2}}, P_{-\frac{1}{2},m-\frac{1}{2}} = \bar{P}_{m-\frac{1}{2}}$$

$$P_{n-\frac{1}{2},m-\frac{1}{2}} = \frac{1}{i\pi(m+1)} \oint dw\bar{w}^{m+1}\left[\bar{T}(\bar{w}), P_{n-\frac{1}{2},-\frac{1}{2}}\right]$$
$$= \frac{1}{i\pi(m+1)} \oint dw w^{n+1}\left[T(w), \bar{P}_{-\frac{1}{2},m-\frac{1}{2}}\right].$$

$$[L_m, L_n] = (m-n)L_{m+n}, [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n}$$
$$[L_n, P_{kl}] = \left(\frac{1}{2}n - k\right)P_{n+k,l}, [\bar{L}_n, P_{kl}] = \left(\frac{1}{2}n - l\right)P_{k,n+l}$$

$$S_A(z) = \sum_{k\in\mathbb{Z}+\frac{1}{2}} \frac{(S_A)_k}{z^{k+\frac{3}{2}}}, \text{ with } (S_A)_k = \frac{1}{2\pi i}\oint dz z^{k+\frac{1}{2}} S_A(z)$$

$$\bar{S}^A(\bar{z}) = \sum_{l\in\mathbb{Z}+\frac{1}{2}} \frac{\bar{S}_l^A}{z^{l+\frac{3}{2}}}, \text{ with } \bar{S}_l^A = \frac{1}{2\pi i}\oint d\bar{z}\bar{z}^{l+\frac{1}{2}}\bar{S}^A(\bar{z})$$

$$[L_n, (S_A)_m] = \left(\frac{n}{2} - m\right)(S_A)_{m+n}, [\bar{L}_n, (S_A)_m] = 0$$
$$[L_n, \bar{S}_m^A] = 0, [\bar{L}_n, \bar{S}_m^A] = \left(\frac{n}{2} - m\right)\bar{S}_{m+n}^A$$

$$\{(S_B)_m, \bar{S}_n^A\} = \delta_B^A P_{mn}, m, n \in \mathbb{Z} + \frac{1}{2}$$

$$\left(\tilde{\mathcal{G}}_B^A\right)_D^C(z,\bar{z}) = \sum_{n,m\in\mathbb{Z}} \left\{\left(\tilde{\mathcal{G}}_B^A\right)_D^C\right\}_{mn} z^{-m-h}\bar{z}^{-n-\bar{h}}$$

$$\left\{\left(\tilde{\mathcal{G}}_B^A\right)_D^C\right\}_{mn} = \frac{1}{(2\pi i)^2}\oint dz \oint d\bar{z} z^{m+h-1}\bar{z}^{n+\bar{h}+1}\left(\tilde{\mathcal{G}}_B^A\right)_D^C(z,\bar{z})$$

$$\left(\tilde{\mathcal{G}}_B^A\right)_D^C(z,\bar{z})S_C(w) \sim \frac{1}{z-w}\frac{1}{\bar{z}-\bar{w}}\left(T_B^A\right)_D^C S_C(w)$$

$$\left[\left(\tilde{\mathcal{G}}_B^A\right)_D^C\right\}_{mn}, (S_C)_k\Big] = \bar{w}^n\left(T_B^A\right)_D^C\{S_C\}_{m+k}$$

$$[L_m, L_n] = (m-n)L_{m+n}, [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n}$$
$$[L_n, P_{kl}] = \left(\frac{1}{2}n - k\right)P_{n+k,l}, [\bar{L}_n, P_{kl}] = \left(\frac{1}{2}n - l\right)P_{k,n+l}$$
$$[L_n, (S_A)_m] = \left(\frac{n}{2} - m\right)(S_A)_{m+n}, [\bar{L}_n, (S_A)_m] = 0$$
$$[L_n, \bar{S}^A_m] = 0, [\bar{L}_n, \bar{S}^A_m] = \left(\frac{n}{2} - m\right)\bar{S}^A_{m+n}$$
$$\{(S_B)_m, \bar{S}^A_n\} = \delta^A_B P_{mn}$$

$$\mathcal{O}^A_{\Delta_1,+\frac{3}{2}}(z_1,\bar{z}_1)\mathcal{O}^B_{\Delta_2,+\frac{3}{2}}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - \frac{1}{2}, \Delta_2 - \frac{1}{2}\right)\mathcal{O}^{AB}_{\Delta_1+\Delta_2,+1}(z_2,\bar{z}_2)$$
$$\mathcal{O}^A_{\Delta_1,+\frac{3}{2}}(z_1,\bar{z}_1)\mathcal{O}_{B\Delta_2,-\frac{3}{2}}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}}\delta^A_B B\left(\Delta_1 - \frac{1}{2}, \Delta_2 + \frac{5}{2}\right)\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar{z}_2)$$
$$+ \frac{z_{12}}{\bar{z}_{12}}\delta^A_B B\left(\Delta_1 + \frac{5}{2}, \Delta_2 - \frac{1}{2}\right)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{AB}_{\Delta_1,+1}(z_1,\bar{z}_1)\mathcal{O}^{CD}_{\Delta_2,+1}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1,\Delta_2)\mathcal{O}^{ABCD}_{\Delta_1+\Delta_2,0}(z_2,\bar{z}_2)$$
$$\mathcal{O}^{AB}_{\Delta_1,+1}(z_1,\bar{z}_1)\mathcal{O}_{CD;\Delta_2,-1}(z_2,\bar{z}_2) \sim -\delta^{AB}_{CD}\left[\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1,\Delta_2+2)\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar{z}_2)\right.$$
$$\left. + \frac{z_{12}}{\bar{z}_{12}} B(\Delta_1+2,\Delta_2)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)\right]$$

$$\mathcal{O}^{ars}_{\Delta_1,+\frac{1}{2}}(z_1,\bar{z}_1)\mathcal{O}^{btu}_{\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2) \sim \epsilon^{rstu}\epsilon^{abcd}\frac{z_{12}}{\bar{z}_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + \frac{1}{2}\right)\mathcal{O}_{cd;\Delta_1+\Delta_2,-1}(z_2,\bar{z}_2)$$
$$\mathcal{O}^{ars}_{\Delta_1,+\frac{1}{2}}(z_1,\bar{z}_1)\mathcal{O}^{bct}_{\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2) \sim \epsilon^{rstu}\epsilon^{abcd}\frac{z_{12}}{\bar{z}_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + \frac{1}{2}\right)\mathcal{O}_{ud;\Delta_1+\Delta_2,-1}(z_2,\bar{z}_2)$$
$$\mathcal{O}^{rst}_{\Delta_1,+\frac{1}{2}}(z_1,\bar{z}_1)\mathcal{O}^{abc}_{\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2) \sim \epsilon^{rstu}\epsilon^{abcd}\frac{z_{12}}{\bar{z}_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + \frac{1}{2}\right)\mathcal{O}_{ud;\Delta_1+\Delta_2,-1}(z_2,\bar{z}_2)$$
$$\mathcal{O}^{ars}_{\Delta_1,+\frac{1}{2}}(z_1,\bar{z}_1)\mathcal{O}_{btu;\Delta_2,-\frac{1}{2}}(z_2,\bar{z}_2) \sim \epsilon_{tuvw}\epsilon^{rsw}\delta^a_b\left[\frac{z_{12}}{\bar{z}_{12}} B\left(\Delta_1 + \frac{3}{2}, \Delta_2 + \frac{1}{2}\right)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)\right.$$
$$\left. + \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 + \frac{1}{2}, \Delta_2 + \frac{3}{2}\right)\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar{z}_2)\right]$$

$$\mathcal{O}^{abrs}_{\Delta_1,0}(z_1,\bar{z}_1)\mathcal{O}^{cdtu}_{\Delta_2,0}(z_2,\bar{z}_2) \sim \epsilon^{abcd}\epsilon^{rstu} B(\Delta_1+1,\Delta_2+1)\left[\frac{z_{12}}{\bar{z}_{12}}\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)\right.$$
$$\left. + \frac{\bar{z}_{12}}{z_{12}}\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar{z}_2)\right]$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}^{A}_{\Delta_2,+\frac{3}{2}}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-1,\Delta_2-\frac{1}{2}\right)\mathcal{O}^{A}_{\Delta_1+\Delta_2,+\frac{3}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{A;\Delta_2,-\frac{3}{2}}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-1,\Delta_2+\frac{5}{2}\right)\mathcal{O}_{A;\Delta_1+\Delta_2,-\frac{3}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}^{AB}_{\Delta_2,+1}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1-1,\Delta_2)\mathcal{O}^{AB}_{\Delta_1+\Delta_2,+1}(z_2,\bar{z}_2)$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{AB;\Delta_2,-1}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1-1,\Delta_2+2)\mathcal{O}_{AB;\Delta_1+\Delta_2,-1}(z_2,\bar{z}_2)$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}^{abr}_{\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-1,\Delta_2+\frac{1}{2}\right)\mathcal{O}^{abr}_{\Delta_1+\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}^{abc}_{\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-1,\Delta_2+\frac{1}{2}\right)\mathcal{O}^{abc}_{\Delta_1+\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{abc;\Delta_2,-\frac{1}{2}}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-1,\Delta_2+\frac{3}{2}\right)\mathcal{O}_{abc;\Delta_1+\Delta_2,-\frac{1}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}^{ABCD}_{\Delta_2,0}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1-1,\Delta_2+1)\mathcal{O}^{ABCD}_{\Delta_1+\Delta_2,0}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{A}_{\Delta_1,+\frac{3}{2}}(z_1,\bar{z}_1)\mathcal{O}^{BC}_{\Delta_2,+1}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-\frac{1}{2},\Delta_2\right)\mathcal{O}^{ABC}_{\Delta_1+\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{A}_{\Delta_1,+\frac{3}{2}}(z_1,\bar{z}_1)\mathcal{O}_{BC;\Delta_2,-1}(z_2,\bar{z}_2) \sim 2!\,\delta^{A}_{[B}\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-\frac{1}{2},\Delta_2\right)\mathcal{O}_{C];\Delta_1+\Delta_2,-\frac{3}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{A}_{\Delta_1,+\frac{3}{2}}(z_1,\bar{z}_1)\mathcal{O}^{BCD}_{\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-\frac{1}{2},\Delta_2+\frac{1}{2}\right)\mathcal{O}^{ABCD}_{\Delta_1+\Delta_2,0}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{A}_{\Delta_1,+\frac{3}{2}}(z_1,\bar{z}_1)\mathcal{O}_{BCD\Delta_2,-\frac{1}{2}}(z_2,\bar{z}_2) \sim 3\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-\frac{1}{2},\Delta_2+\frac{3}{2}\right)\delta^{A}_{[B}\mathcal{O}_{CD];\Delta_1+\Delta_2,-1}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{A}_{\Delta_1,+\frac{3}{2}}(z_1,\bar{z}_1)\mathcal{O}^{BCDE}_{\Delta_2,0}(z_2,\bar{z}_2) \sim -\frac{1}{6}\epsilon^{ABCDEFGH}\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-\frac{1}{2},\Delta_2+1\right)\mathcal{O}_{FGH;\Delta_1+\Delta_2,-\frac{1}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{A}_{\Delta_1,+\frac{3}{2}}(z_1,\bar{z}_1)\mathcal{O}_{BCDE\Delta_2,0}(z_2,\bar{z}_2) \sim 3!\,\delta^{A}_{[B}\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1-\frac{1}{2},\Delta_2+1\right)\mathcal{O}_{CDE];\Delta_1+\Delta_2,-\frac{1}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{ab}_{\Delta_1,+1}(z_1,\bar{z}_1)\mathcal{O}^{cdr}_{\Delta_2,+\frac{1}{2}}(z_2,\bar{z}_2) \sim \frac{1}{3!}\epsilon^{rstu}\epsilon^{abcd}\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1,\Delta_2+\frac{1}{2}\right)\mathcal{O}_{stu;\Delta_1+\Delta_2,-\frac{1}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{AB}_{\Delta_1,+1}(z_1,\bar{z}_1)\mathcal{O}_{CDE;\Delta_2,-\frac{1}{2}}(z_2,\bar{z}_2) \sim -\delta^{AB}_{CD}\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1,\Delta_2+\frac{3}{2}\right)\mathcal{O}_{E;\Delta_1+\Delta_2,-\frac{3}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{ab}_{\Delta_1,+1}(z_1,\bar{z}_1)\mathcal{O}^{cdrs}_{\Delta_2,0}(z_2,\bar{z}_2) \sim \epsilon^{rstu}\epsilon^{abcd}\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1,\Delta_2+1)\mathcal{O}_{tu;\Delta_1+\Delta_2,-1}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{ab}_{\Delta_1,+1}(z_1,\bar{z}_1)\mathcal{O}^{cdef}_{\Delta_2,0}(z_2,\bar{z}_2) \sim -\epsilon^{cdef}\epsilon^{abgh}\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1,\Delta_2+1)\mathcal{O}_{gh;\Delta_1+\Delta_2,-1}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{abr}_{\Delta_1,+\frac{1}{2}}(z_1,\bar{z}_1)\mathcal{O}^{cdst}_{\Delta_2,0}(z_2,\bar{z}_2) \sim \epsilon^{rstu}\epsilon^{abcd}\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1+\frac{1}{2},\Delta_2+1\right)\mathcal{O}_{u;\Delta_1+\Delta_2,-\frac{3}{2}}(z_2,\bar{z}_2)$$

$$\mathcal{O}^{abr}_{\Delta_1,+\frac{1}{2}}(z_1,\bar{z}_1)\mathcal{O}^{cstu}_{\Delta_2,0}(z_2,\bar{z}_2) \sim -\epsilon^{rstu}\epsilon^{abcd}\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1+\frac{1}{2},\Delta_2+1\right)\mathcal{O}_{d;\Delta_1+\Delta_2,-\frac{3}{2}}(z_2,\bar{z}_2)$$

$$\left\langle G_{AB}(z,\bar{z})\bar{G}^{CD}(w,\bar{w})\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$=\lim_{\substack{\Delta_1\to 0\\ \Delta_2\to 0}}\frac{\Delta_1\Delta_2}{\pi^2}\int d^2z_1\frac{1}{(z-z_1)^a}\frac{1}{(\bar{z}-\bar{z}_1)^b}\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}$$
$$\times\left\langle \mathcal{O}_{AB}\Delta_1,-1(z_1,\bar{z}_1)\mathcal{O}^{CD}_{\Delta_2,+1}(z_2,\bar{z}_2)\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$

$$\left\langle G_{AB}(z)\bar{G}^{CD}(\bar{w})\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$=\lim_{\Delta_2\to 0}\frac{\Delta_2}{\pi^2}\int d^2z_1\int d^2z_2\frac{1}{(z-z_1)^a}\frac{1}{(\bar{z}-\bar{z}_1)^b}\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}$$
$$\times\left[-\delta^{CD}_{AB}\frac{z_1-z_2}{\bar{z}_1-\bar{z}_2}\left\langle\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2)\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle\right.$$
$$\left.+\sum_{j=3}^{N}f\left(A,B,\ell_j,*_j,*'_j\right)\frac{z_1-z_j}{\bar{z}_1-\bar{z}_j}\left\langle\mathcal{O}^{CD}_{\Delta_2,+1}(z_2,\bar{z}_2)\cdots\mathcal{O}^{*'}_{\Delta_j,\ell_j+1}(z_j,\bar{z}_j)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle\right]$$

$$\left\langle G_{AB}(z)\bar{G}^{CD}(\bar{w})\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$=\lim_{\Delta_2\to 0}\frac{\Delta_2}{\pi}\int d^2z_2\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}$$
$$\times\left[-\delta^{CD}_{AB}C_1(b,a)\frac{1}{(z_2-z)^{a-2}(\bar{z}_2-\bar{z})^b}\left\langle\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2)\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle\right.$$
$$+\sum_{j=3}^{N}f\left(A,B,\ell_j,*_j,*'_j\right)C_1(b,a)\frac{1}{\left(z_j-z\right)^{a-2}}\frac{1}{\left(\bar{z}_j-\bar{z}\right)^b}$$
$$\left.\times\left\langle\mathcal{O}^{CD}_{\Delta_2,+1}(z_2,\bar{z}_2)\cdots\mathcal{O}^{*'_j}_{\Delta_j,\ell_j+1}(z_j,\bar{z}_j)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle\right]$$

$$\lim_{\Delta_2\to 0}\frac{\Delta_2}{\pi}\int\ d^2z_2\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}\left[-\delta_{AB}^{CD}C_1(b,a)\frac{1}{(z_2-z)^{a-2}(\bar{z}_2-\bar{z})^{b}}\right.$$
$$\left.\times\left\langle\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2)\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle\right]=-\delta_{AB}^{CD}C_1(b,a)\lim_{\Delta_2\to 0}\frac{\Delta_2}{\pi}\int\ d^2z_2\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}$$
$$\frac{1}{(w-z_2)^{b'}}\frac{1}{(z_2-z)^{a-2}(\bar{z}_2-\bar{z})^{b}}$$
$$\times\sum_{i=3}^{N}B\big(\Delta_2-1,f(\Delta_i)\big)\frac{\bar{z}_2-\bar{z}_i}{z_2-z_i}\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*_i}_{\Delta_i,\ell_i}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$
$$=-\delta_{AB}^{CD}C_1(b,a)\frac{1}{\pi}\int\ d^2z_2\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}\frac{1}{(z_2-z)^{a-2}(\bar{z}_2-\bar{z})^{b}}$$
$$\times\sum_{i=3}^{N}\big(1-f(\Delta_i)\big)\frac{\bar{z}_2-\bar{z}_i}{z_2-z_i}\times\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*_i}_{\Delta_i,\ell_i}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$

$$\lim_{\Delta_2\to 0}\Delta_2 B\big(\Delta_2-1,f(\Delta_i)\big)=1-f(\Delta_i)$$

$$\lim_{\Delta_2\to 0}\frac{\Delta_2}{\pi}\sum_{j=3}^{N}f\big(A,B,\ell_j,*_j,*_j'\big)\frac{1}{\big(z_j-z\big)^{a-2}}\frac{1}{\big(\bar{z}_j-\bar{z}\big)^{b}}\int\ d^2z_2\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}$$
$$\times\left\langle\mathcal{O}^{CD}_{\Delta_2,+1}(z_2,\bar{z}_2)\cdots\mathcal{O}^{*_j'}_{\Delta_j,\ell_j+1}(z_j,\bar{z}_j)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle=\frac{1}{\pi}\sum_{j=3}^{N}f\big(A,B,\ell_j,*_j,*_j'\big)\bar{f}\big(C,D,\ell_j+1,*_j',*_j''\big)$$
$$C_1(b,a)\frac{1}{\big(z_j-z\big)^{a-2}}\frac{1}{\big(\bar{z}_j-\bar{z}\big)^{b}}\times\int\ d^2z_2\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}$$
$$\frac{\bar{z}_2-\bar{z}_j}{z_2-z_j}\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*_j''}_{\Delta_j,\ell_j}(z_j,\bar{z}_j)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle+\frac{1}{\pi}\sum_{i,j=3}^{N}f\big(A,B,\ell_j,*_j,*_j'\big)$$
$$\bar{f}(C,D,\ell_i,*_i,*_i')C_1(b,a)\frac{1}{\big(z_j-z\big)^{a-2}}\frac{1}{\big(\bar{z}_j-\bar{z}\big)^{b}}\times\int\ d^2z_2\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}$$
$$\frac{\bar{z}_2-\bar{z}_i}{z_2-z_i}\left\langle\cdots\mathcal{O}^{*_j'}_{\Delta_j,\ell_j+1}(z_j,\bar{z}_j)\cdots\mathcal{O}^{*_i'}_{\Delta_i,\ell_i-1}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle=\sum_{j=3}^{N}f\big(A,B,\ell_j,*_j,*_j'\big)\bar{f}$$
$$\big(C,D,\ell_j+1,*_j',*_j''\big)C_1(b',a')\frac{1}{\big(z_j-z\big)^{a-2}}\frac{1}{\big(\bar{z}_j-\bar{z}\big)^{b}}\times\frac{1}{\big(z_j-w\big)^{b'}}\frac{1}{\big(\bar{z}_j-\bar{w}\big)^{a'-2}}$$
$$\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*_j''}_{\Delta_j,\ell_j}(z_j,\bar{z}_j)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle+\sum_{i,j=3}^{N}f\big(A,B,\ell_j,*_j,*_j'\big)$$
$$\bar{f}(C,D,\ell_i,*_i,*_i')C_1(b,a)C_1(b',a')\frac{1}{\big(z_j-z\big)^{a-2}}\frac{1}{\big(\bar{z}_j-\bar{z}\big)^{b}}$$
$$\times\frac{1}{(z_i-w)^{b'}}\frac{1}{(\bar{z}_i-\bar{w})^{a'-2}}\left\langle\cdots\mathcal{O}^{*_j'}_{\Delta_j,\ell_j+1}(z_j,\bar{z}_j)\cdots\mathcal{O}^{*_i'}_{\Delta_i,\ell_i-1}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$

$$\left\langle G_{AB}(z,\bar{z})\bar{G}^{CD}(w,\bar{w})\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$=-\delta^{CD}_{AB}C_1(b,a)\frac{1}{\pi}\int d^2z_2\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}\frac{1}{(z_2-z)^{a-2}(\bar{z}_2-\bar{z})^{b}}$$
$$\times\sum_{i=3}^{N}\left(1-f(\Delta_i)\right)\frac{\bar{z}_2-\bar{z}_i}{z_2-z_i}\times\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*_i}_{\Delta_i,\ell_i}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$
$$+\sum_{j=3}^{N}f\left(A,B,\ell_j,*_j,*'_j\right)\bar{f}\left(C,D,\ell_j+1,*'_j,*''_j\right)C_1(b',a')\frac{1}{\left(z_j-z\right)^{a-2}}\frac{1}{\left(\bar{z}_j-\bar{z}\right)^{b}}$$
$$\times\frac{1}{\left(z_j-w\right)^{b'}}\frac{1}{\left(\bar{z}_j-\bar{w}\right)^{a'-2}}\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*''_j}_{\Delta_j,\ell_j}\left(z_j,\bar{z}_j\right)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$
$$+\sum_{i,j=3}^{N}f\left(A,B,\ell_j,*_j,*'_j\right)\bar{f}(C,D,\ell_i,*_i,*'_i)C_1(b,a)C_1(b',a')\frac{1}{\left(z_j-z\right)^{a-2}}\frac{1}{\left(\bar{z}_j-\bar{z}\right)^{b}}$$
$$\times\frac{1}{(z_i-w)^{b'}}\frac{1}{(\bar{z}_i-\bar{w})^{a'-2}}\left\langle\cdots\mathcal{O}^{*'_j}_{\Delta_j,\ell_j+1}\left(z_j,\bar{z}_j\right)\cdots\mathcal{O}^{*'_i}_{\Delta_i,\ell_i-1}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$

$$\int d^2z_2\frac{1}{(\bar{w}-\bar{z}_2)^{a'}}\frac{1}{(w-z_2)^{b'}}\frac{1}{(z_2-z)^{a-2}}\frac{1}{(\bar{z}_2-\bar{z})^{b}}\frac{\bar{z}_2-\bar{z}_i}{z_2-z_i}\overset{z=w}{\rightarrow}$$
$$=(-1)^{a+b-2}\int d^2z_2\frac{1}{(\bar{z}-\bar{z}_2)^{a'+b}}\frac{1}{(z-z_2)^{b'+a-2}}\frac{\bar{z}_2-\bar{z}_i}{z_2-z_i}$$
$$=(-1)^{a+b}C_1(a+b'-2,a'+b)\frac{1}{(z_i-z)^{a'+b-2}}\frac{1}{(\bar{z}_i-\bar{z})^{a+b'-2}}$$

$$\left\langle:G_{AB}(z,\bar{z})\bar{G}^{CD}(z,\bar{z}):\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$
$$=\frac{(-1)^{a+b+1}}{\pi}\delta^{CD}_{AB}C_1(b,a)C(a+b'-2,a'+b)\sum_{i=3}^{N}\left(1-f(\Delta_i)\right)$$
$$\times\frac{1}{(z_i-z)^{a'+b-2}}\frac{1}{(\bar{z}_i-\bar{z})^{a+b'-2}}\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*_i}_{\Delta_i,\ell_i}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell'_N}(z_N,\bar{z}_N)\right\rangle$$
$$+\sum_{j=3}^{N}f\left(A,B,\ell_j,*_j,*'_j\right)\bar{f}\left(C,D,\ell_j+1,*'_j,*''_j\right)C_1(b,a)C_1(b',a')(-1)^{a+b+a'+b'}$$
$$\times\frac{1}{\left(z-z_j\right)^{a+b'-2}}\frac{1}{\left(\bar{z}-\bar{z}_j\right)^{a'+b-2}}\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*''_j}_{\Delta_j,\ell_j}\left(z_j,\bar{z}_j\right)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$
$$+\sum_{\substack{i,j=3\\ i\neq j}}^{N}f\left(A,B,\ell_j,*_j,*'_j\right)\bar{f}(C,D,\ell_i,*_i,*'_i)C_1(b,a)C_1(b',a')(-1)^{a+b+a'+b'}\frac{1}{\left(z-z_j\right)^{a-2}}\frac{1}{\left(\bar{z}-\bar{z}_j\right)^{b}}$$
$$\times\frac{1}{(z-z_i)^{b'}}\frac{1}{(\bar{z}-\bar{z}_i)^{a'-2}}\left\langle\cdots\mathcal{O}^{*'_j}_{\Delta_j,\ell_j+1}\left(z_j,\bar{z}_j\right)\cdots\mathcal{O}^{*'_i}_{\Delta_i,\ell_i-1}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$

$$\left\langle :\bar{G}^{CD}(z,\bar{z})G_{AB}(z,\bar{z}):\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$

$$=\frac{(-1)^{a'+b'+1}}{\pi}\delta^{CD}_{AB}C_1(b',a')C(a'+b-2,a+b')\sum_{i=3}^{N}\left(1-f(\Delta_i)\right)$$

$$\times\frac{1}{(z_i-z)^{a+b'-2}}\frac{1}{(\bar{z}_i-\bar{z})^{a'+b-2}}\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*_i}_{\Delta_i,\ell_i}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$

$$+\sum_{j=3}^{N}\bar{f}\left(C,D,\ell_j,*_j,*'_j\right)f\left(A,B,\ell_j-1,*'_j,*''_j\right)C_1(b',a')C_1(b,a)(-1)^{a+b+a'+b'}$$

$$\times\frac{1}{\left(z-z_j\right)^{a'+b-2}}\frac{1}{\left(\bar{z}-\bar{z}_j\right)^{a+b'-2}}\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*''_j}_{\Delta_j,\ell_j}\left(z_j,\bar{z}_j\right)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$

$$+\sum_{\substack{i,j=3\\ i\neq j}}^{N}\bar{f}\left(C,D,\ell_j,*_j,*'_j\right)f(A,B,\ell_i,*_i,*'_i)C_1(b,a)C_1(b',a')(-1)^{a+b+a'+b'}\frac{1}{\left(\bar{z}-\bar{z}_j\right)^{a'-2}}\frac{1}{\left(z-z_j\right)^{b'}}$$

$$\times\frac{1}{(\bar{z}-\bar{z}_i)^{b}}\frac{1}{(z-z_i)^{a-2}}\left\langle\cdots\mathcal{O}^{*'_j}_{\Delta_j,\ell_j-1}\left(z_j,\bar{z}_j\right)\cdots\mathcal{O}^{*'_i}_{\Delta_i,\ell_i+1}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$

$$\left\langle\mathcal{G}^{CD}_{AB}(z,\bar{z})\prod_{n=3}^{N}\mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n)\right\rangle$$

$$=-\delta^{CD}_{AB}\sum_{i=3}^{N}\left(1-f(\Delta_i)\right)\left[\frac{(-1)^{a+b}}{\pi}C_1(b,a)C(a+b'-2,a'+b)\frac{1}{(z_i-z)^{a'+b-2}}\frac{1}{(\bar{z}_i-\bar{z})^{a+b'-2}}\right.$$

$$\times\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*_i}_{\Delta_i,\ell_i}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$

$$-\frac{(-1)^{a'+b'}}{\pi}C_1(b',a')C_1(a'+b-2,a+b')\frac{1}{(z_i-z)^{a+b'-2}}\frac{1}{(\bar{z}_i-\bar{z})^{a'+b-2}}$$

$$\left.\times\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*_i}_{\Delta_i,\ell_i}(z_i,\bar{z}_i)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle\right]$$

$$+(-1)^{a+b+a'+b'}C_1(b,a)C_1(b',a')\left[f\left(A,B,\ell_j,*_j,*'_j\right)\bar{f}\left(C,D,\ell_j+1,*'_j,*''_j\right)\right.$$

$$\times\frac{1}{\left(z-z_j\right)^{a+b'-2}}\frac{1}{\left(\bar{z}-\bar{z}_j\right)^{a'+b-2}}\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*''_j}_{\Delta_j,\ell_j}\left(z_j,\bar{z}_j\right)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle$$

$$\left.\times\frac{-\bar{f}\left(C,D,\ell_j,*_j,*'_j\right)f\left(A,B,\ell_j-1,*'_j,*''_j\right)}{\left(z-z_j\right)^{a'+b-2}}\frac{1}{\left(\bar{z}-\bar{z}_j\right)^{a+b'-2}}\left\langle\mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3)\cdots\mathcal{O}^{*''_j}_{\Delta_j,\ell_j}\left(z_j,\bar{z}_j\right)\cdots\mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N)\right\rangle\right]$$

$$C_1(b',a')C_1(a'+b-2,a+b')(-1)^{a+b}=C_1(b,a)C_1(a+b'-2,a'+b)(-1)^{a'+b'}$$

$$a'+b-2=a+b'-2$$

$$(-a'-b+1)(-a'-b+2)(-a+1)(-a+2)$$

$$=(-a'+1)(-a'+2)(-a-b'+1)(-a-b'+2)$$

$$(-a+1)(-a+2)=(-a'+1)(-a'+2)$$

$$\left\langle \mathcal{G}_{AB}^{CD}(z,\bar{z}) \prod_{n=3}^{N} \mathcal{O}^{*n}_{\Delta_n,\ell_n}(z_n,\bar{z}_n) \right\rangle = (-1)^{a+b+a'+b'} C_1(b,a) C_1(b',a') \Big[f\big(A,B,\ell_j,*_j,*_j'\big) \bar{f}\big(C,D,\ell_j+1,*_j',*_j''\big)$$

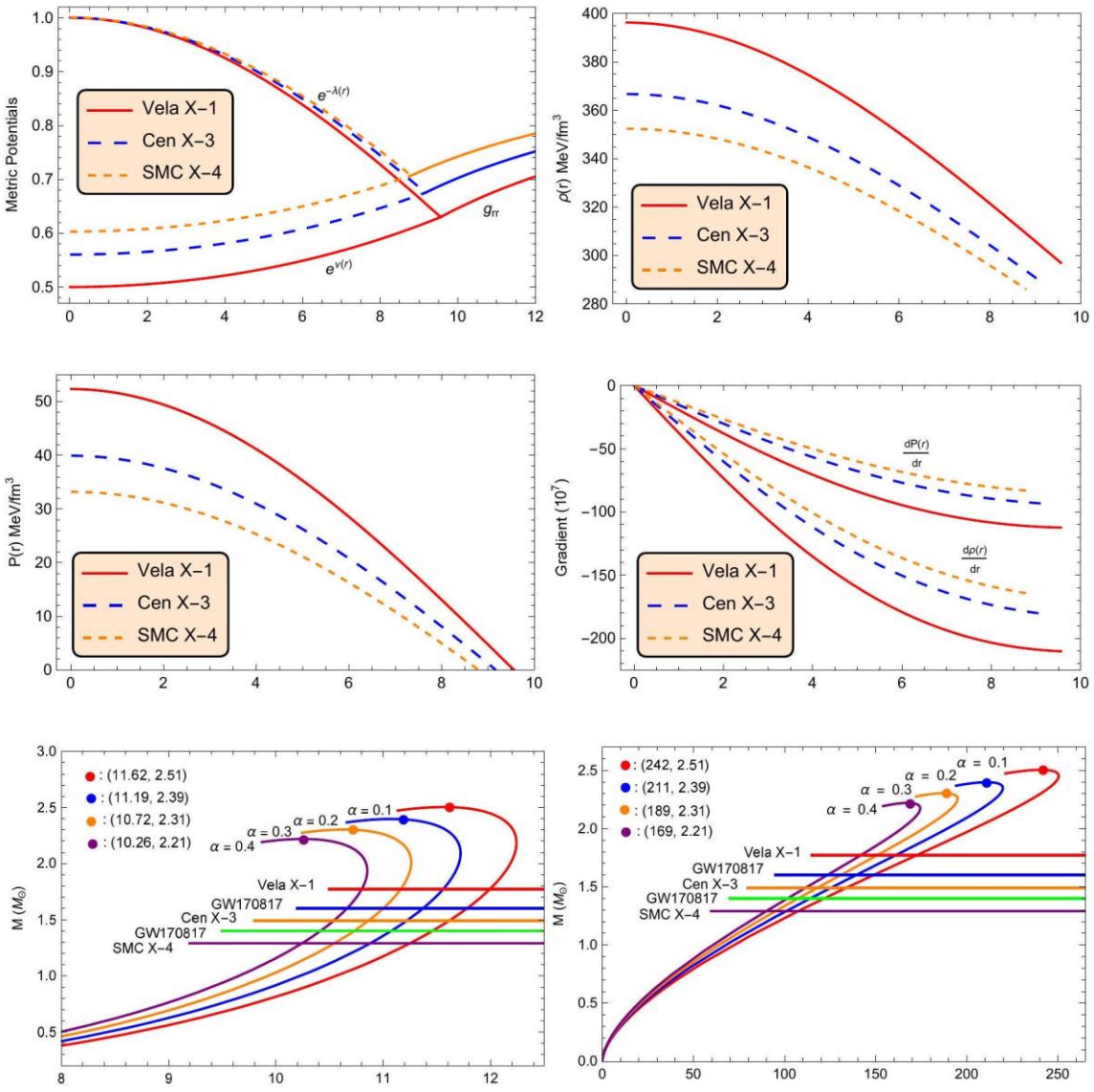
$$\times \frac{1}{(z-z_j)^{a+b'-2}} \frac{1}{(\bar{z}-\bar{z}_j)^{a'+b-2}} \left\langle \mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3) \cdots \mathcal{O}^{*_j''}_{\Delta_j,\ell_j}(z_j,\bar{z}_j) \cdots \mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N) \right\rangle$$

$$- \bar{f}\big(C,D,\ell_j,*_j,*_j'\big) f\big(A,B,\ell_j-1,*_j',*_j''\big)$$

$$\times \frac{1}{(z-z_j)^{a'+b-2}} \frac{1}{(\bar{z}-\bar{z}_j)^{a+b'-2}} \left\langle \mathcal{O}^{*_3}_{\Delta_3,\ell_3}(z_3,\bar{z}_3) \cdots \mathcal{O}^{*_j''}_{\Delta_j,\ell_j}(z_j,\bar{z}_j) \cdots \mathcal{O}^{*N}_{\Delta_N,\ell_N}(z_N,\bar{z}_N) \right\rangle \Bigg]$$

Partícula Blanca o Partícula Estrella. Modelo TOV.

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\,\theta d\phi^2).$$



Figuras 1, 2 y 3. Densidad del centro de masa – energía de una partícula blanca o estrella.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu},$$

$$T_0^0 = \rho^{\text{eff}}(r) = \rho(r) + \rho^{de}(r)$$
$$T_1^1 = T_2^2 = T_3^3 = -P^{\text{eff}}(r) = -[P(r) + P^{de}(r)]$$

$$\rho(r) + \rho^{de}(r) = \frac{e^{-\lambda}}{8\pi}\left\{\frac{\lambda'}{r} - \frac{1}{r^2}\right\} + \frac{1}{8\pi r^2},$$
$$P(r) + P^{de}(r) = \frac{e^{-\lambda}}{8\pi}\left\{\frac{\nu'}{r} + \frac{1}{r^2}\right\} - \frac{1}{8\pi r^2},$$

$$P(r) + P^{de}(r) = \frac{e^{-\lambda}}{32\pi}\left\{2\nu'' + \nu'^2 + \frac{2(\nu' - \lambda')}{r} - \nu'\lambda'\right\}.$$

$$e^{\lambda(r)} = \left[1 - \frac{3ar^2}{2}\left(\frac{1 + C(1 + 4ar^2)^{-\frac{1}{2}}}{1 + ar^2}\right)\right]^{-1}$$

$$r^2(2\nu''(r) + \nu'^2(r) - \nu'(r)\lambda'(r)) - 2r(\nu'(r) + \lambda'(r)) + 4\left(e^{\lambda(r)} - 1\right) = 0.$$

$$e^{\nu(r)} = A^2(1 + ar^2)^3,$$

$$\rho(r) + \rho^{de}(r) = \frac{3a\left[C(3 + 9ar^2) + (3 + ar^2)(1 + 4ar^2)^{3/2}\right]}{16\pi(1 + ar^2)^2(1 + 4ar^2)^{3/2}}$$
$$P(r) + P^{de}(r) = \frac{3a\left[3(1 - ar^2)\sqrt{1 + 4ar^2} - C - 7aCr^2\right]}{16\pi(1 + ar^2)^2\sqrt{1 + 4ar^2}}$$

$$P^{de}(r) = -\rho^{de}(r),$$

$$\rho^{de}(r) = \alpha\rho(r),$$

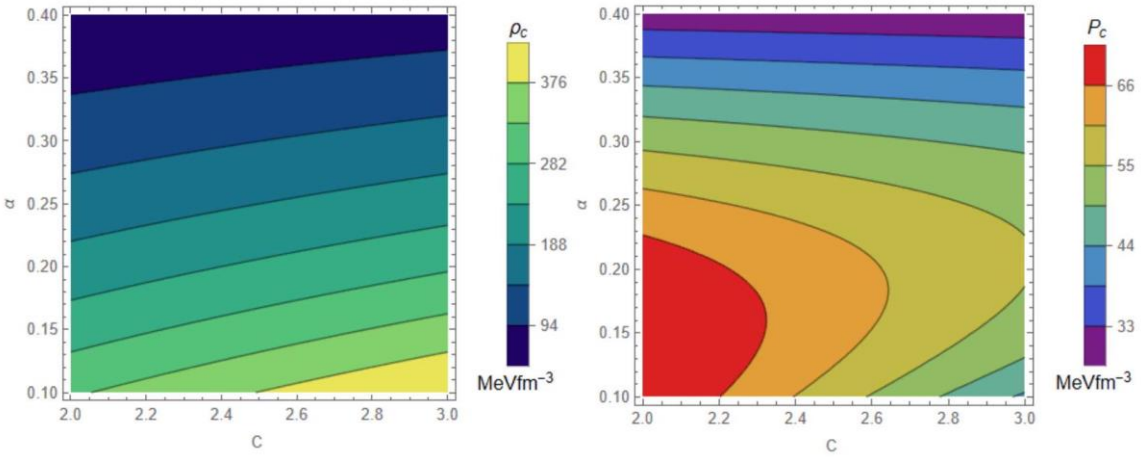


Figura 4. Fluctuaciones de radiación de una partícula estrella o blanca.

$$\rho(r) = \frac{3a\left[C(3+9ar^2)+(3+ar^2)(1+4ar^2)^{3/2}\right]}{16\pi(1+\alpha)(1+ar^2)^2(1+4ar^2)^{3/2}},$$

$$P(r) = \frac{3a\left[(1+4ar^2)^{3/2}(3+6\alpha-ar^2(3+2\alpha))-C(1-2\alpha+28a^2r^4(1+\alpha)+ar^2(11+2\alpha))\right]}{16\pi(1+\alpha)(1+ar^2)^2(1+4ar^2)^{3/2}}.$$

$$\rho^{de}(r) = \frac{3a\alpha\left[C(3+9ar^2)+(3+ar^2)(1+4ar^2)^{3/2}\right]}{16\pi(1+\alpha)(1+ar^2)^2(1+4ar^2)^{3/2}},$$

$$P^{de}(r) = -\frac{3a\alpha\left[C(3+9ar^2)+(3+ar^2)(1+4ar^2)^{3/2}\right]}{16\pi(1+\alpha)(1+ar^2)^2(1+4ar^2)^{3/2}}.$$

$$\rho^{\text{eff}}(r) = \rho(r)+\rho^{de}(r) = \frac{3a\left[C(3+9ar^2)+(3+ar^2)(1+4ar^2)^{3/2}\right]}{16\pi(1+ar^2)^2(1+4ar^2)^{3/2}},$$

$$P^{\text{eff}}(r) = P(r)+P^{de}(r) = \frac{3a\left[3(1-ar^2)\sqrt{1+4ar^2}-C-7aCr^2\right]}{16\pi(1+ar^2)^2\sqrt{1+4ar^2}}.$$

$$ds^2 = g_{tt}(R)dt^2 - g_{rr}(R)dr^2 - r^2(d\theta^2+\sin^2\theta d\phi^2),$$

$$A^2(1+aR^2)^3 = 1-\frac{2M}{R},$$

$$1-\frac{3aR^2\left[1+C(1+4aR^2)^{-1/2}\right]}{2(1+aR^2)} = 1-\frac{2M}{R}.$$

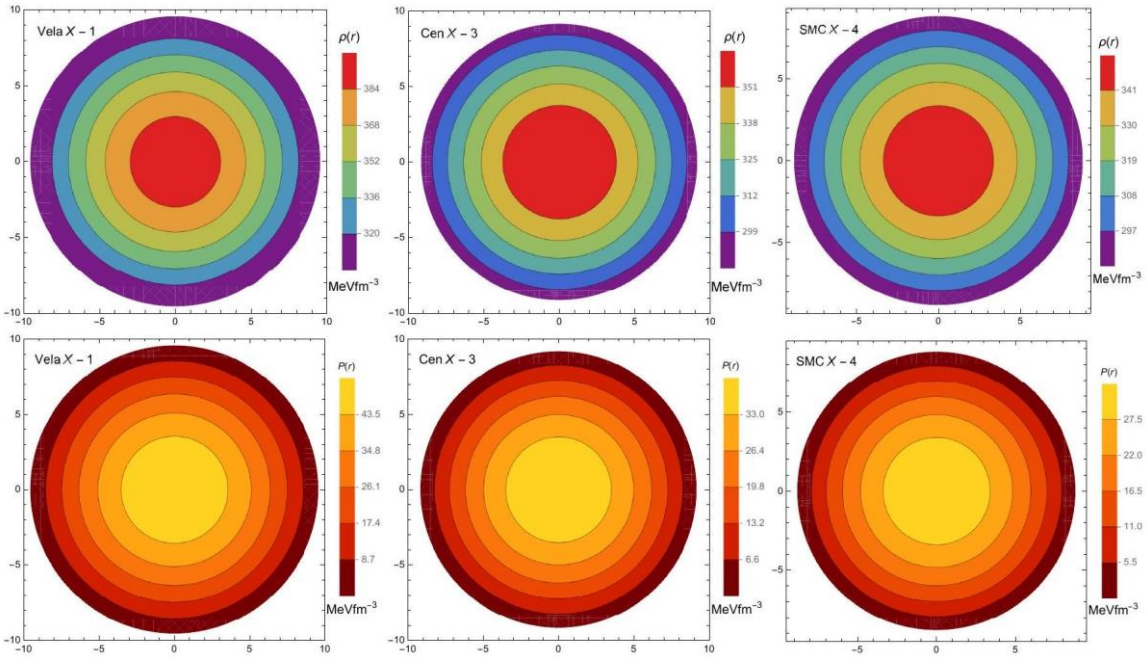


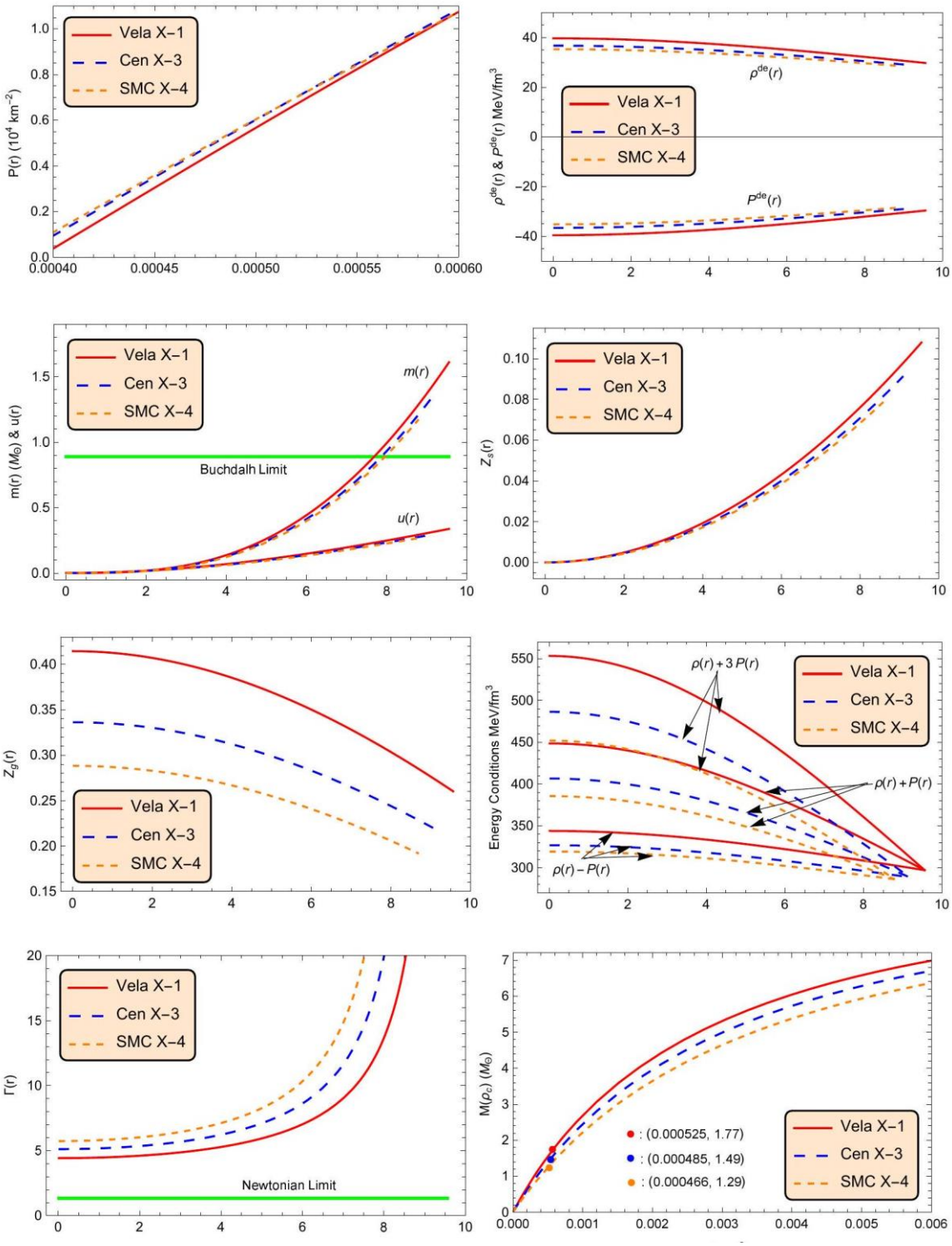
Figura 5. Interior de una partícula estrella o blanca.

$$C[1-2\alpha+28a^2R^4(1+\alpha)+aR^2(11+2\alpha)]+(1+4aR^2)^{3/2}[aR^2(3+2\alpha)-6\alpha-3]=0.$$

$$a = \frac{M(11+2\alpha) - 3R(1+\alpha) + \sqrt{3M^2(3+52\alpha+76\alpha^2) - 6MR(3+19\alpha+22\alpha^2) + 9R^2(1+\alpha)^2}}{2R^2[3R(4+5\alpha) - 28M(1+\alpha)]},$$

$$C = \frac{\sqrt{1+4aR^2}[4M(1+aR^2) - 3aR^3]}{3aR^3},$$

$$A = \frac{\sqrt{R-2M}}{\sqrt{R(1+aR^2)^3}}.$$



Figuras 6, 7, 8 y 9. Formación de materia oscura por interacción de la partícula estrella con el campo supergravitónico. Ondas cuánticas gravitacionales extremas.

$$\left(e^{\nu(r)}\right)' = \frac{12ar\left[C - 2a^2Cr^4 + 2aCr^2 + (4ar^2+1)^{3/2}\right]}{\sqrt{4ar^2+1}\left[ar^2\left(\sqrt{4ar^2+1}+3C\right) - 2\sqrt{4ar^2+1}\right]^2}$$

$$\left(e^{\lambda(r)}\right)' = 6aA^2r(1+ar^2)^2$$

$$\rho_c = \rho(0) = \frac{9a(1+C)}{16\pi(1+\alpha)} > 0$$

$$P_c = P(0) = \frac{3a[3+6\alpha - C(1-2\alpha)]}{16\pi(1+\alpha)} > 0$$

$$\frac{d\rho(r)}{dr} = -\frac{3a^2r\left[3C(30a^2r^4+23ar^2+5) + (ar^2+5)(4ar^2+1)^{5/2}\right]}{8\pi(\alpha+1)(ar^2+1)^3(4ar^2+1)^{5/2}}$$

$$\frac{dP(r)}{dr} = \frac{3a^2r\left[(4ar^2+1)^{5/2}(a(2\alpha+3)r^2 - 14\alpha - 9) + 3c\xi_1\right]}{8\pi(\alpha+1)(ar^2+1)^3(4ar^2+1)^{5/2}}$$

$$\xi_1 = 56a^3(\alpha+1)r^6 - 6a^2(2\alpha-3)r^4 - a(26\alpha+3)r^2 - 6\alpha - 1.$$

$$\rho^{de}_{\max} = \rho^{de}(0) = \frac{9a\alpha(1+C)}{16\pi(1+\alpha)} > 0.$$

$$P^{de}_{\min} = -\rho^{de}(0) = -\frac{9a\alpha(1+C)}{16\pi(1+\alpha)} < 0.$$

$$m(r) = 4\pi\int_0^r r^{*2}\rho(r^*)dr^* = \frac{3ar^3\left(1+4ar^2+C\sqrt{4ar^2+1}\right)}{4(1+\alpha)(1+5ar^2+4a^2r^4)},$$

$$u(r) = \frac{2m(r)}{r} = \frac{3ar^2\left(1+4ar^2+C\sqrt{4ar^2+1}\right)}{2(1+\alpha)(1+5ar^2+4a^2r^4)}$$

$$Z_s(r) = [1-u(r)]^{-1/2} - 1 = \left[1 - \frac{3ar^2\left(1+4ar^2+C\sqrt{1+4ar^2}\right)}{2(1+\alpha)(1+5ar^2+4a^2r^4)}\right]^{-1/2} - 1.$$

$$Z_g(r) = e^{-\nu(r)/2} - 1 = \frac{1}{A}(ar^2+1)^{-3/2} - 1.$$

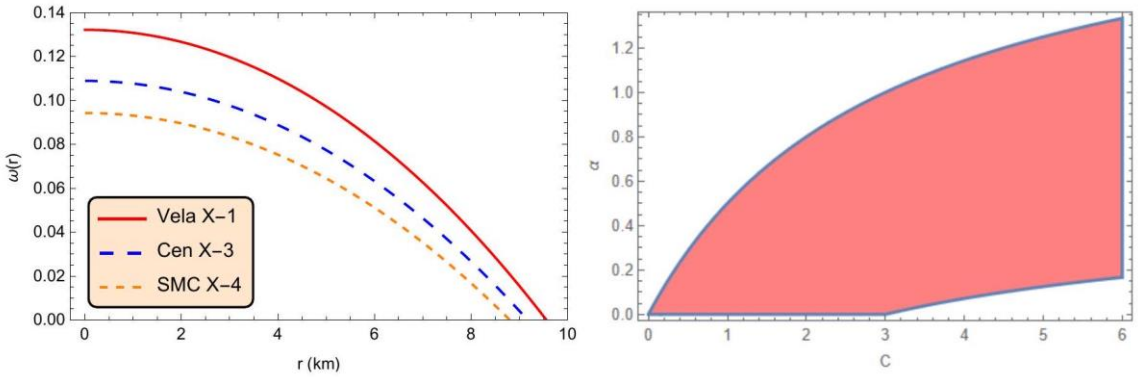


Figura 10. Fluctuaciones gravitacionales extremas provocadas por la partícula estrella o blanca.

$$\text{NEC: } \rho(r)+P(r)\geq 0,\ \text{WEC}: \rho(r)\geq 0, \rho(r)+P(r)\geq 0,$$
$$\text{DEC: } \rho(r)\geq 0, \rho(r)-P(r)\geq 0,\ \text{SEC}: \rho(r)+3P(r)\geq 0.$$

$$\rho(r)+P(r) = \frac{3a\left[C(1-14a^2r^4-ar^2)+(3-4a^2r^4+11ar^2)\sqrt{4ar^2+1}\right]}{8\pi(1+ar^2)^2(1+4ar^2)^{3/2}},$$

$$\rho(r)-P(r) = \frac{3a\left[C(14a^2(\alpha+1)r^4+a(\alpha+10)r^2-\alpha+2)+(4ar^2+1)^{3/2}(a(\alpha+2)r^2-3\alpha)\right]}{8\pi(\alpha+1)(ar^2+1)^2(4ar^2+1)^{3/2}},$$

$$\rho(r)+3P(r) = \frac{3a}{8\pi(1+\alpha)(1+ar^2)^2(1+4ar^2)^{3/2}}\left[3\left((2+3\alpha)\sqrt{1+4ar^2}+\alpha C\right)-2a^2r^4\left(2(4+3\alpha)\sqrt{1+4ar^2}\right.\right.$$
$$\left.\left.+21(1+\alpha)C\right)-ar^2\left(3(4+\alpha)C-(33\alpha+20)\sqrt{1+4ar^2}\right)\right].$$

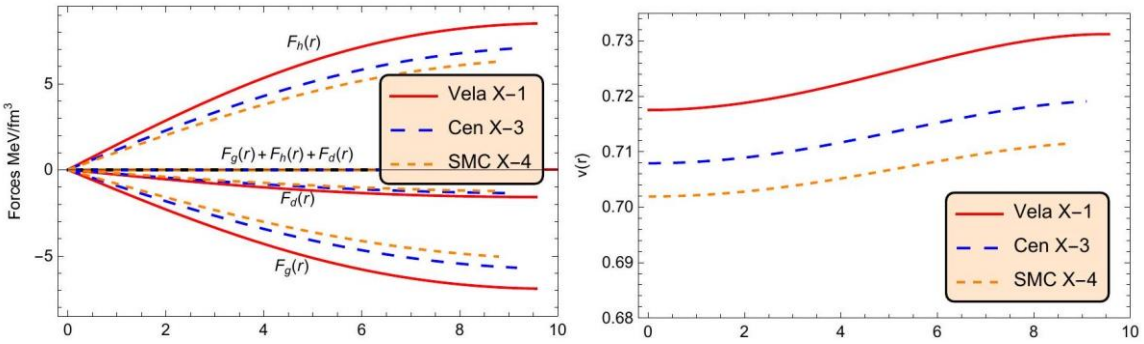


Figura 11. Cinética y coordenadas radiales de una partícula blanca o estrella.

$$\alpha < \frac{2C}{3+C}.$$

$$\alpha > \frac{C-3}{2(C+3)}.$$

$$\frac{C-3}{2(C+3)} < \alpha < \frac{2C}{3+C}.$$

$$-\frac{\nu'(r)}{2}[\rho(r)+P(r)]-\frac{dP(r)}{dr}-\frac{dP^{de}(r)}{dr}=0.$$

$$F_g(r)+F_h(r)+F_d(r)=0,$$

$$F_g(r) = -\frac{\nu'(r)}{2}[\rho(r) + P(r)],$$
$$F_h(r) = -\frac{dP(r)}{dr},$$
$$F_d(r) = -\frac{dP^{de}(r)}{dr},$$

$$F_g(r) = \frac{9a^2r\left[C(14a^2r^4 + ar^2 - 1) + (4a^2r^4 - 11ar^2 - 3)\sqrt{1 + 4ar^2}\right]}{8\pi(1 + ar^2)^3(1 + 4ar^2)^{3/2}},$$

$$F_h(r) = \frac{3a^2r}{8\pi(1+\alpha)(1 + ar^2)^3(1 + 4ar^2)^{5/2}}[3C(-56a^3r^6(1+\alpha) + 6a^2r^4(2\alpha - 3) + ar^2(3 + 26\alpha) + 6\alpha + 1)$$
$$+(1 + 4ar^2)^{5/2}(9 + 14\alpha - ar^2(3 + 2\alpha))],$$
$$F_d(r) = -\frac{3a^2\alpha r\left[3C(5 + 23ar^2 + 30a^2r^4) + (5 + ar^2)(1 + 4ar^2)^{5/2}\right]}{8\pi(1+\alpha)(1 + ar^2)^3(1 + 4ar^2)^{5/2}}.$$

$$V(r) = \left[\frac{dP(r)}{d\rho(r)}\right]^{1/2} = \left[\frac{3C(1 + 6\alpha + 6a^2r^4(2\alpha - 3) - 56a^3r^6(1+\alpha) + ar^2(3 + 26\alpha)) + \xi_2}{3C(30a^2r^4 + 23ar^2 + 5) + (5 + ar^2)(1 + 4ar^2)^{\frac{5}{2}}}\right]^{\frac{1}{2}},$$

$$\xi_2 = (1 + 4ar^2)^{\frac{5}{2}}(9 + 14\alpha - ar^2(2\alpha + 3))$$

$$\Gamma(r) = \left(1 + \frac{\rho(r)}{P(r)}\right)V^2(r) = \frac{[3C(ar^2(3 - 2ar^4(9 + 28ar^2(\alpha+1) - 6\alpha) + 26\alpha) + 6\alpha + 1) + \xi_2]\xi_3}{3C\left(5 + ar^2(30ar^2 + 23)\right) + (5 + ar^2)(1 + 4ar^2)^{5/2}},$$

$$\xi_3 = 1 - \frac{C(3 + 9ar^2) + (3 + ar^2)(1 + 4ar^2)^{3/2}}{C(ar^2(28ar^2(\alpha+1) + 2\alpha + 11) - 2\alpha + 1) + (1 + 4ar^2)^{3/2}(ar^2(2\alpha + 3) - 6\alpha - 3)}.$$

$$M(\rho_c) = \frac{\pi\rho_c R^3\left[C\sqrt{82944\pi\rho_c R^2(1+\alpha)(1+C) + 11664(1+C)^2} + 108(1+C) + 768\pi\rho_c R^2(1+\alpha)\right]}{16\pi\rho_c R^2(1+\alpha)(45C + 64\pi\rho_c R^2(1+\alpha) + 45) + 81(1+C)^2}.$$

$$I = \frac{8\pi}{3}\int_0^R r^4[\rho(r) + P(r)]e^{[\lambda(r)-\nu(r)]/2}\frac{\bar{\omega}}{\Omega}dr$$

$$\frac{d}{dr}\left(r^4\mathcal{H}\frac{d\bar{\omega}}{dr}\right) = -4r^3\bar{\omega}\frac{d\mathcal{H}}{dr}$$

$$I = \frac{2}{5}(1 + x)MR^2$$

$$p = \frac{1}{3}(\rho - 4B_g)$$

$$\mathcal{S} = \frac{1}{16\pi}\int \sqrt{-g}f(\tilde{R}, T)d^4x + \int \sqrt{-g}\mathcal{L}_{\mathcal{M}}d^4x$$

$$f_R R_{ij} - \frac{1}{2} g_{ij} f(\tilde{R}, T) + f_R\left[g_{ij} \square - \nabla_i \nabla_j\right] = 8\pi T_{ij} - f_T T_{ij} - f_T \Theta_{ij}.$$

$$f_R = \frac{df(\tilde{R},T)}{dR}, f_T = \frac{df(\tilde{R},T)}{dT}, \square \equiv \frac{\partial_i\left(\sqrt{-g} g^{ij} \partial_j\right)}{\sqrt{-g}}, \Theta_{ij} = g^{ij} \frac{\delta T_{ij}}{\delta g_{ij}}$$

$$G_{ij} + 2\alpha R R_{ij} - \frac{1}{2} g_{ij} \alpha R^2 + 2\alpha\left[g_{ij} \square - \nabla_i \nabla_j\right] R = 8\pi T_{ij} - f_T T_{ij} - f_T \Theta_{ij} + g_{ij} \beta T,$$

$$ds^2 = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2\, \theta d\phi^2)$$

$$T_{ij} = (p + \rho) u_i u_j + p g_{ij},$$

$$\nabla^i T_{ij} = \frac{f_T(\tilde{R}, T)}{8\pi - f_T(\tilde{R}, T)} \left[\left(T_{ij} + \Theta_{ij}\right) \nabla^i \ln\, f_T(\tilde{R}, T) + \nabla^i \Theta_{ij} - \frac{1}{2} g_{ij} \nabla^i T\right]$$

$$\nabla^i T_{ij} = \frac{2\beta}{8\pi + 2\beta} \left[\nabla^i \left(p g_{ij} - \frac{1}{2} g_{ij} \nabla^i T\right)\right].$$

$$\frac{2\lambda' e^{-2\lambda}}{r} + \frac{1 - e^{-2\lambda}}{r^2} + 2\alpha R \left[e^{-2\lambda} \left\{-\nu'' - \nu'^2 + \lambda' \nu' - \frac{2\nu'}{r}\right\}\right] + \frac{1}{2} \alpha R^2 - 2\alpha \left[e^{-2\lambda} \left\{R'' + \left(\nu' - \lambda' + \frac{2}{r}\right) R'\right\}\right] = 8\pi\rho -$$

$$\frac{2\nu' e^{-2\lambda}}{r} - \frac{1 - e^{-2\lambda}}{r^2} + 2\alpha R \left[e^{-2\lambda} \left\{\nu'' + \nu'^2 - \lambda' \nu' - \frac{2\lambda'}{r}\right\}\right] - \frac{1}{2} \alpha R^2 + 2\alpha \left[e^{-2\lambda} \left\{R'' + \left(\nu' - \lambda' + \frac{2}{r}\right) R'\right\}\right] - 2\alpha[R''$$
$$= 8\pi p + 2p$$

$$\frac{dm}{dr} = 4\pi r^2 \rho + \frac{\beta r^2}{2} [3\rho - p] + \frac{r^2}{2} \left[-2\alpha R \left[e^{-2\lambda} \left\{-\nu'' - \nu'^2 + \lambda' \nu' - \frac{2\nu'}{r}\right\}\right] - \frac{1}{2} \alpha R^2 + 2\alpha \left[e^{-2\lambda} \left\{R'' + \left(\nu' - \lambda' + \frac{2}{r}\right.\right.\right.$$

$$\frac{dp}{dr} = \left[\frac{\beta}{8\pi + 3\beta}\right] \rho' - \left[\frac{8\pi + 2\beta}{8\pi + 3\beta}\right] (\rho + p) \left[\frac{m(r) + 4\pi r^3 p_{eff}}{r[r - 2m(r)]}\right]$$

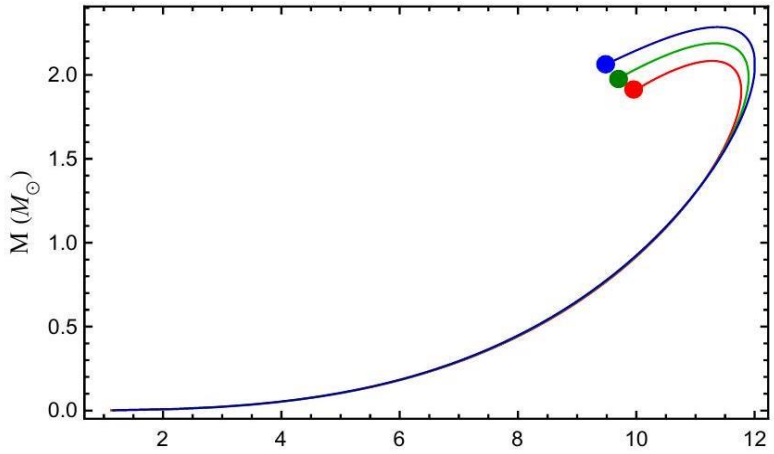


Figura 12. Trayectorias escalares y vectoriales de una partícula blanca o estrella en relación a su centro de masa – energía extremadamente denso.

β	α	Maximum mass $M_\odot$	Radius (nm)
	10	2.08	11.08
0	30	2.23	11.35
	50	2.41	11.64

β	α	Maximum mass $M_\odot$	Radius (nm)
	0	1.94	10.67
0.5	2.5	1.87	10.66
	5	1.81	10.64

β	α	Maximum mass $M_{\odot}$	Radius (nm)
	0	1.62	8.78
-0.5	2.5	1.76	8.84
	5	1.90	8.88

β	α	Maximum mass $M_{\odot}$	Radius (nm)
	0	1.56	8.53
0	2.5	1.58	8.57
	5	1.61	8.61

β	α	Maximum mass $M_{\odot}$	Radius (nm)
	10	1.65	8.69
0	30	1.87	9.05
	50	2.16	9.54

β	α	Maximum mass $M_{\odot}$	Radius (nm)
	0	1.51	8.30
0.5	2.5	1.43	8.28
	5	1.35	8.24

α	β	Maximum mass $M_{\odot}$	Radius (nm)
	-0.82	2.13	11.50
0	0	2.012	10.96
	0.82	1.90	10.49

α	β	Maximum mass $M_{\odot}$	Radius (nm)
	-0.61	2.17	11.39
1.5	0	2.02	10.98
	0.61	1.87	10.59

α	β	Maximum mass $M_{\odot}$	Radius (nm)
	-4.1	2.13	11.32
0	0	1.56	8.53
	4.1	1.21	7.00

α	β	Maximum mass $M_{\odot}$	Radius (nm)
	-2.2	2.17	9.73
1.5	0	1.58	8.55
	1.9	1.16	7.57

$$\mathcal{S}(\Psi, g) = \int d^4x\sqrt{-g}\left\{\frac{c^3}{16\pi G}f(R) + \mathcal{L}_m(\Psi, g)\right\}$$

$$T_{\mu\nu} = cg_{\mu\nu}\mathcal{L}_m - 2c\frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}}$$

$$f_R R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f + \left(g_{\mu\nu}\,\square - \nabla_\mu\nabla_\nu\right)f_R = \kappa T_{\mu\nu}$$

$$\kappa = \frac{8\pi G}{c^4},\ f_R = \frac{\partial f}{\partial R}$$

$$\nabla_\mu\left[f_R R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu f + \left(\delta^\mu_\nu\,\square - \nabla^\mu\nabla_\nu\right)f_R\right] = 0$$

$$f(R) = R + \alpha R^2$$

$$G_{\mu\nu} + 2\alpha R\left(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R\right) + 2\alpha\left(g_{\mu\nu}\,\square - \nabla_\mu\nabla_\nu\right)R = \kappa T_{\mu\nu}$$

$$6\alpha\,\square\,R = R + \kappa T,$$

$$ds^2 = -e^{\nu(r)}c^2dt^2 + e^{\lambda(r)}dr^2 + r^2d\theta^2 + r^2\sin^2\,\theta d\varphi^2$$

$$T_{\mu\nu} = \frac{(\varepsilon + P)}{c^2}u_\mu u_\nu + g_{\mu\nu}P$$

$$\lambda' = \frac{1 - e^\lambda}{r} + \frac{re^\lambda}{6}\left(\frac{2R + 3\alpha R^2}{1 + 2\alpha R}\right) - \left(\frac{\alpha r R'}{1 + 2\alpha R}\right)\nu' + \frac{\kappa r e^\lambda}{3}\left(\frac{T + 3\varepsilon}{1 + 2\alpha R}\right)$$

$$\gamma\nu' = \frac{e^\lambda - 1}{r} + \frac{\kappa r e^\lambda P}{1 + 2\alpha R} - \frac{r\alpha e^\lambda R^2}{2(1 + 2\alpha R)} - \frac{4}{r}\left(\frac{\alpha r R'}{1 + 2\alpha R}\right)$$

$$\gamma = 1 + \frac{\alpha r R'}{1 + 2\alpha R}$$

$$R'' = \left(\frac{\lambda' - \nu'}{2} - \frac{2}{r}\right)R' + \frac{\kappa}{6\alpha}e^\lambda T + \frac{e^\lambda R}{6\alpha}$$

$$P' = -\frac{\nu'}{2}(P + \varepsilon)$$

$$m(r) = \frac{c^2}{2G}r\left(1 - e^{-\lambda(r)}\right)$$

$$R'' = \left(\frac{\lambda' - \nu'}{2} - \frac{2}{r}\right)R' + \frac{e^\lambda R}{6\alpha}$$

$$R'' + \frac{2}{r}R' - \frac{R}{6\alpha} = 0$$

$$R(r) = \frac{c_1}{r}e^{-\frac{r}{\sqrt{6\alpha}}} + \frac{c_2}{2r}\sqrt{6\alpha}e^{\frac{r}{\sqrt{6\alpha}}}$$

$$R(r) = \frac{c_1}{r}e^{-\frac{r}{\sqrt{6\alpha}}}$$

$$\zeta = \frac{a_1 + a_2\xi + a_3\xi^3}{(a_4\xi + 1)(e^{a_5(\xi - a_6)} + 1)} + \frac{a_7 + a_8\xi}{e^{a_9(a_{10} - \xi)} + 1} + \frac{a_{11} + a_{12}\xi}{e^{a_{13}(a_{14} - \xi)} + 1} + \frac{a_{15} + a_{16}\xi}{e^{a_{17}(a_{18} - \xi)} + 1}$$

$$\zeta = \frac{a_1 + a_2\xi + a_3\xi^3}{(a_4\xi + 1)(e^{a_5(\xi - a_6)} + 1)} + \frac{a_7 + a_8\xi}{e^{a_9(a_6 - \xi)} + 1} + \frac{a_{10} + a_1\xi}{e^{a_{12}(a_{13} - \xi)} + 1} + \frac{a_{14} + a_{15}\xi}{e^{a_{16}(a_{17} - \xi)} + 1} + \frac{a_{18}}{[a_{19}(\xi - a_{20})]^2 + 1} + \frac{a_{21}}{[a_{22}(\xi - a_{23})]^2 + 1}$$

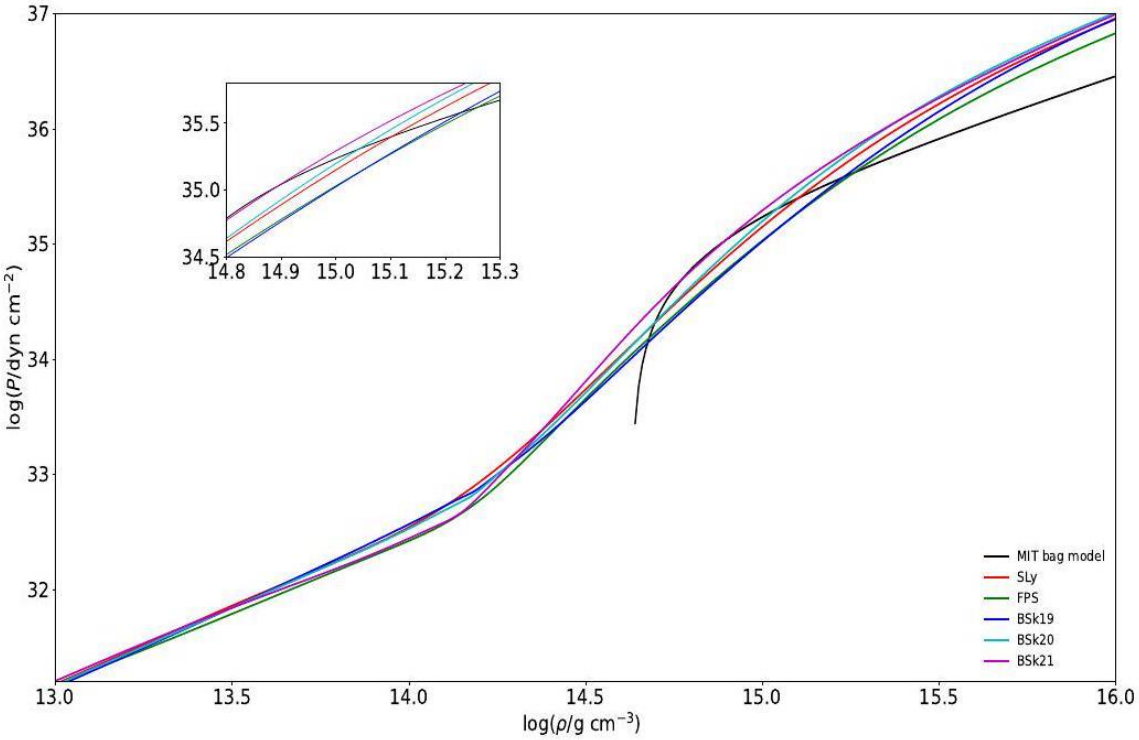


Figura 13. MIT bag model aplicable a un partícula blanca o estrella. Parametrizaciones.

$$P = k(\rho c^2 - 4B),$$

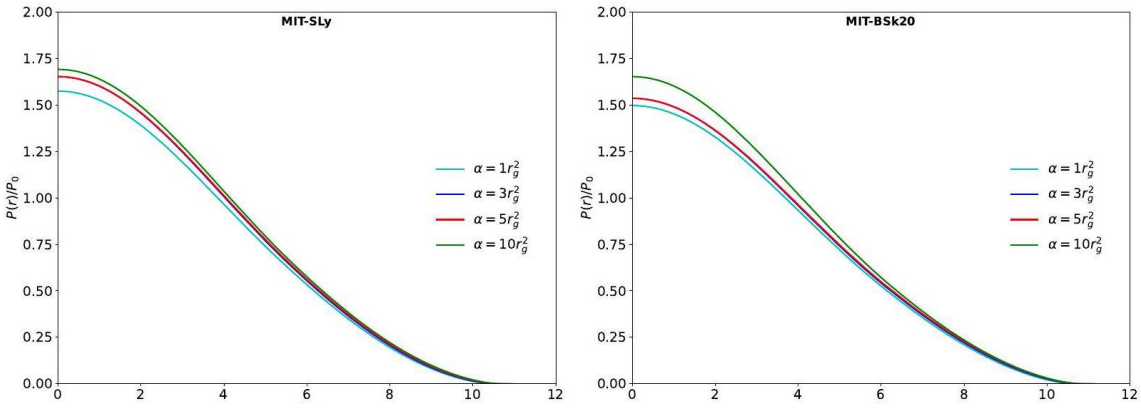
$$\eta = \frac{r}{r_g}, \tilde{P} = \frac{P}{P_0}, \tilde{\rho} = \frac{\rho}{\rho_0}, \chi = R r_g^2, \tilde{T} = \frac{T}{P_0}, \Omega = \frac{d\chi}{d\eta},$$

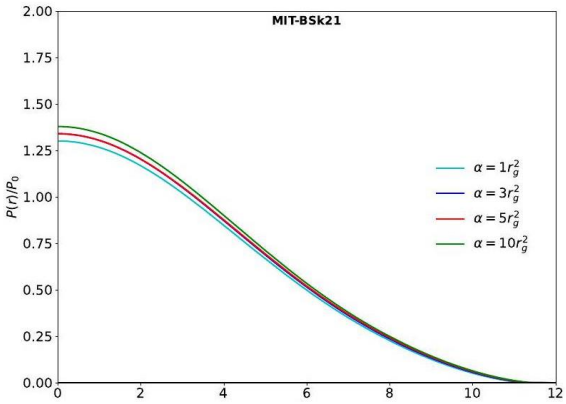
$$M = \frac{c^2}{2G} r_\infty \left(1 - e^{-\lambda(r_\infty)}\right)$$

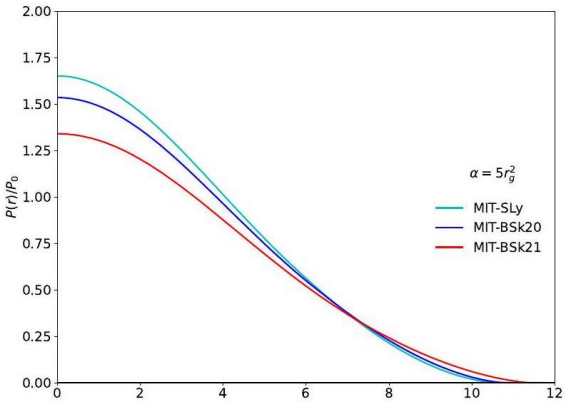
$$M_s = m(r_s) = \frac{c^2}{2G} r_s \left(1 - e^{-\lambda(r_s)}\right)$$

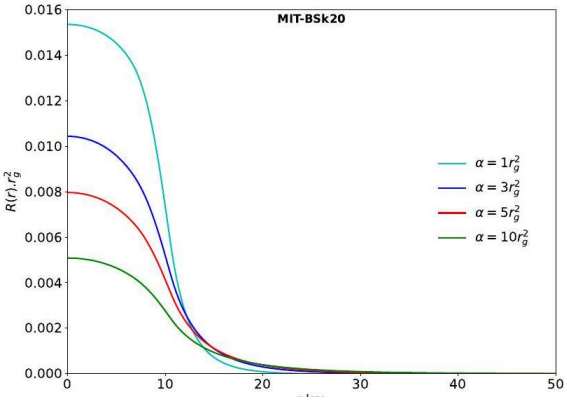
Equation of state	α	$\rho_c \times 10^{15}$	R_c	$m(r_s)$	r_s	M_∞
	r_g^2	g. nm $^{-3}$	r_g^2	$M_\odot$	nm	$M_\odot$
MIT-SLy	GR	2.39	-	1.85	10.75	1.85
	1.0	2.45	0.016	1.78	10.81	1.88
	3.0	2.50	0.010	1.73	10.89	1.91

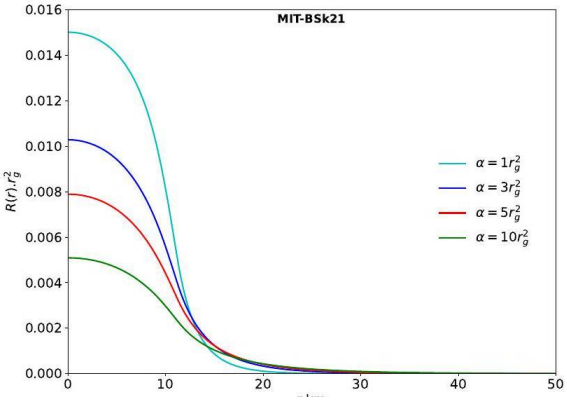
	5.0	2.55	0.008	1.71	10.92	1.92
	10.0	2.55	0.005	1.69	11.01	1.95
MIT-BSk20	GR	2.28	-	1.89	10.93	1.89
	1.0	2.35	0.015	1.81	10.98	1.92
	3.0	2.40	0.010	1.76	11.07	1.94
	5.0	2.40	0.008	1.74	11.13	1.96
	10.0	2.55	0.005	1.72	11.14	1.98
MIT-BSk21	GR	2.03	-	1.98	11.60	1.98
	1.0	2.10	0.015	1.90	11.65	2.00
	3.0	2.15	0.010	1.84	11.72	2.03
	5.0	2.20	0.008	1.82	11.75	2.04
	10.0	2.20	0.005	1.79	11.84	2.07

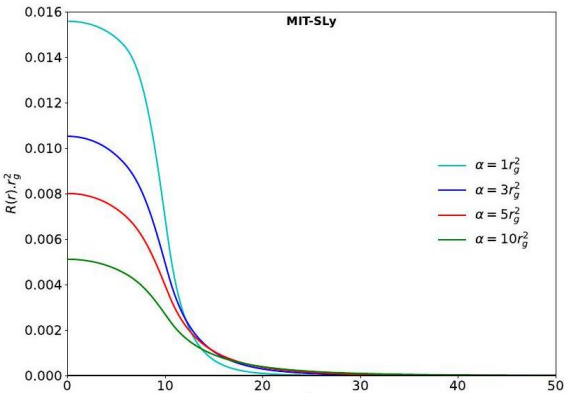


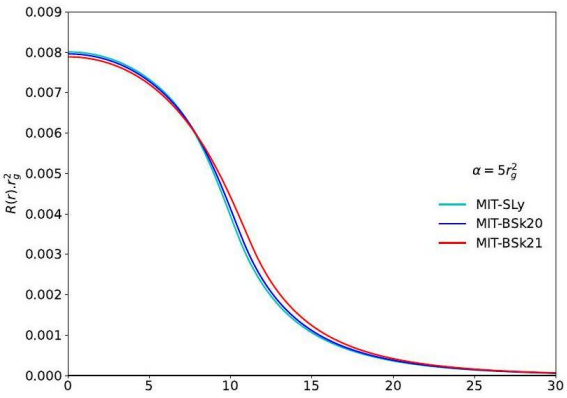

MIT-BSk21
P(r)/P0
α = 1r_g^2
α = 3r_g^2
α = 5r_g^2
α = 10r_g^2

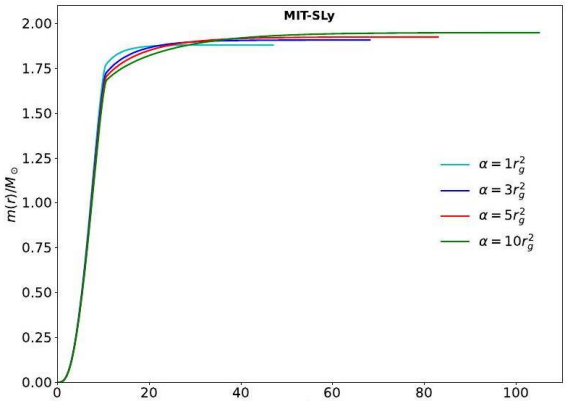

P(r)/P0
α = 5r_g^2
MIT-SLy
MIT-BSk20
MIT-BSk21

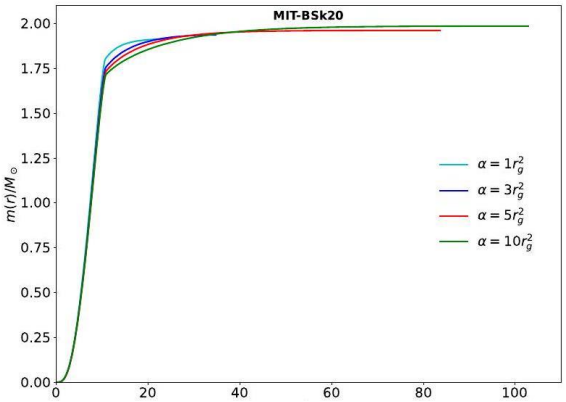

MIT-BSk20
R(r),r_g^2
α = 1r_g^2
α = 3r_g^2
α = 5r_g^2
α = 10r_g^2

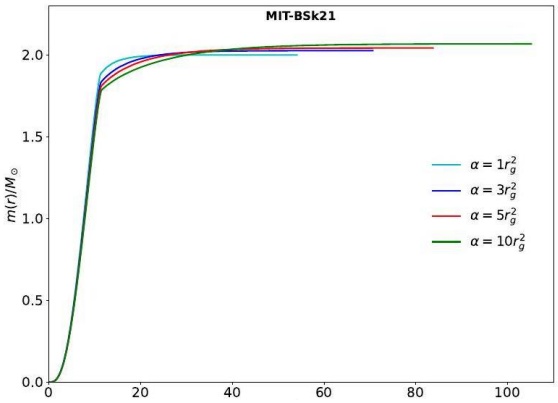

MIT-BSk21
R(r),r_g^2
α = 1r_g^2
α = 3r_g^2
α = 5r_g^2
α = 10r_g^2

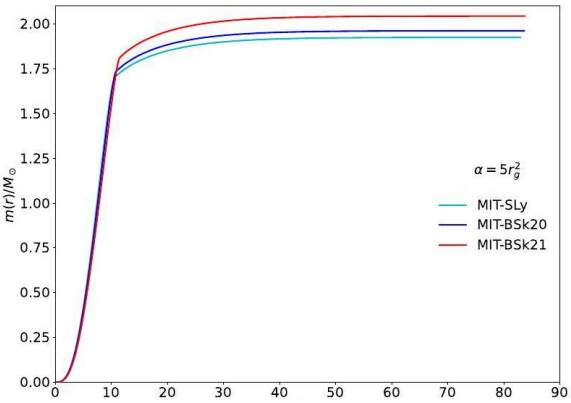

MIT-SLy
R(r),r_g^2
α = 1r_g^2
α = 3r_g^2
α = 5r_g^2
α = 10r_g^2

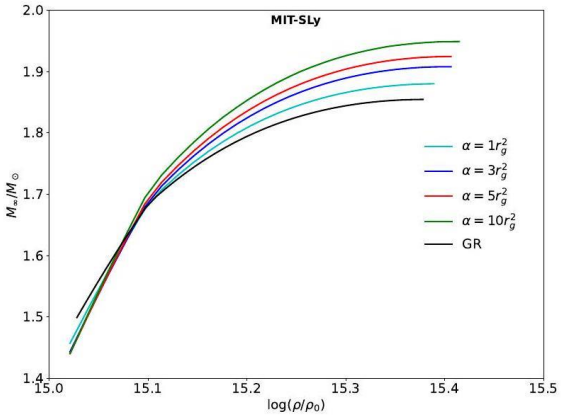

R(r),r_g^2
α = 5r_g^2
MIT-SLy
MIT-BSk20
MIT-BSk21

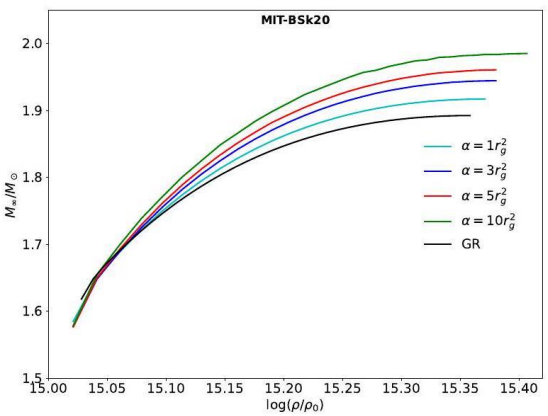

MIT-SLy
m(r)/M⊙
α = 1r_g^2
α = 3r_g^2
α = 5r_g^2
α = 10r_g^2

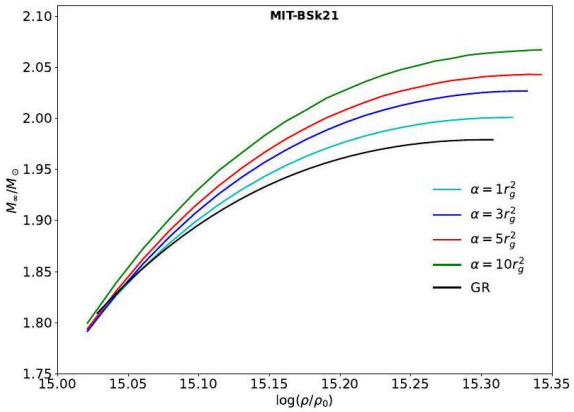

MIT-BSk20
m(r)/M⊙
α = 1r_g^2
α = 3r_g^2
α = 5r_g^2
α = 10r_g^2

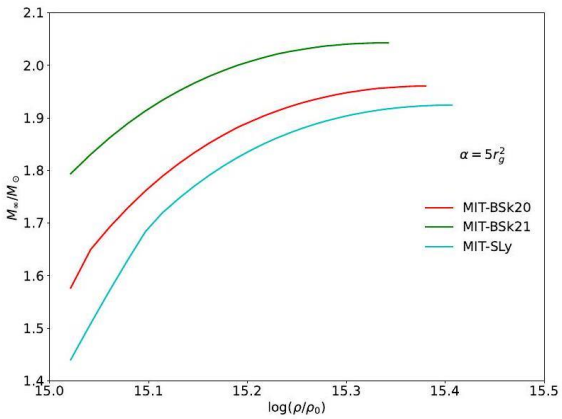

MIT-BSk21
$m(r)/M_\odot$
$\alpha = 1r_g^2$
$\alpha = 3r_g^2$
$\alpha = 5r_g^2$
$\alpha = 10r_g^2$

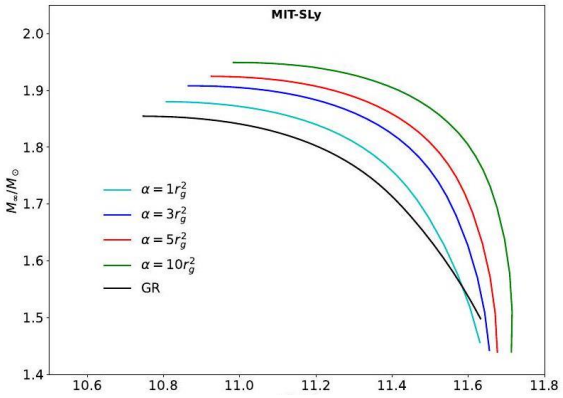

$m(r)/M_\odot$
$\alpha = 5r_g^2$
MIT-SLy
MIT-BSk20
MIT-BSk21

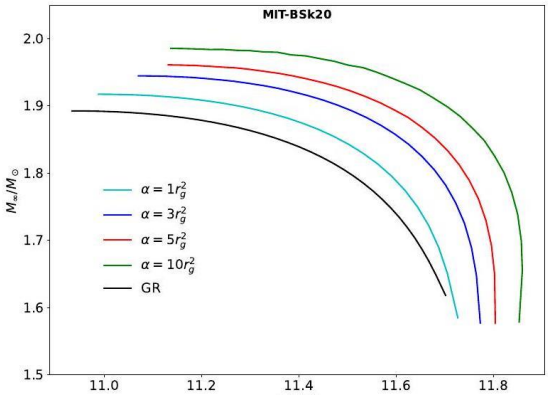

MIT-SLy
$M_*/M_\odot$
$\log(\rho/\rho_0)$
$\alpha = 1r_g^2$
$\alpha = 3r_g^2$
$\alpha = 5r_g^2$
$\alpha = 10r_g^2$
GR

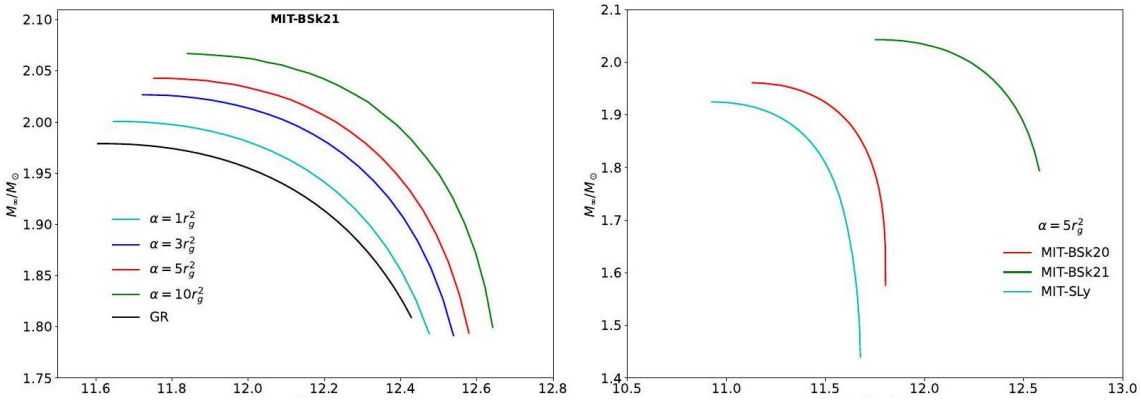

MIT-BSk20
$M_*/M_\odot$
$\log(\rho/\rho_0)$
$\alpha = 1r_g^2$
$\alpha = 3r_g^2$
$\alpha = 5r_g^2$
$\alpha = 10r_g^2$
GR


MIT-BSk21
$M_*/M_\odot$
$\log(\rho/\rho_0)$
$\alpha = 1r_g^2$
$\alpha = 3r_g^2$
$\alpha = 5r_g^2$
$\alpha = 10r_g^2$
GR


$M_*/M_\odot$
$\log(\rho/\rho_0)$
$\alpha = 5r_g^2$
MIT-BSk20
MIT-BSk21
MIT-SLy


MIT-SLy
$M_*/M_\odot$
$\alpha = 1r_g^2$
$\alpha = 3r_g^2$
$\alpha = 5r_g^2$
$\alpha = 10r_g^2$
GR


MIT-BSk20
$M_*/M_\odot$
$\alpha = 1r_g^2$
$\alpha = 3r_g^2$
$\alpha = 5r_g^2$
$\alpha = 10r_g^2$
GR



Figuras 14, 15, 16, 17 y 18. Perfiles de presión radial de una superpartícula o partícula blanca o estrella en gravedad extrema o supergravedad.

$$p(k_F) = \frac{1}{3\pi^2\hbar^3}\int_0^{k_F} \frac{k^4}{\sqrt{k^2 + m_\varepsilon^2}}dk + p_L(Z)$$

$$p_L(Z) = \frac{1}{3}Ce^2 n_e^{4/3} Z^{2/3}$$

$$\begin{aligned}\epsilon(k_F) =& \epsilon_i + \epsilon_e + \epsilon_L - \epsilon_\epsilon \\ =& n_i M(Z,A)c^2 + \frac{1}{\pi^2\hbar^3}\int_0^{k_F}\sqrt{k^2 + m_e^2}k^2 dk \\ &+ Ce^2 n_e^{4/3} Z^{2/3} - n_e m_e c^2\end{aligned}$$

$$A(N,Z) + e^- \rightarrow A(N+1, Z-1) + \nu_e$$

$$\epsilon_e + p_e = n_e \mu_e$$

$$g(A,Z) = m_n c^2 + \frac{M(Z,A)c^2}{A} + \gamma_e\left[\mu_e - m_e c^2 + \frac{4}{3}\frac{\epsilon_L}{n_e}\right],$$

$$g(A,Z) \geq g(A, Z-1)$$

$$\mu_e + Ce^2 n_e^{1/3} f(Z, Z-1) \geq \mu_e^\beta$$

$$\mu_e^\beta(A,Z) \equiv M(Z-1,A)c^2 - M(Z,A)c^2 + m_e c^2$$

$$f(Z, Z-1) = Z^{5/3} - (Z-1)^{5/3} + \frac{1}{3}Z^{2/3}$$

$$n_e = \frac{k_F^3}{3\pi^2\hbar^3}$$

$$\rho = \frac{1}{\gamma_e} m n_e$$

$$k_F = \hbar\left(\frac{3\pi^2\rho}{m_n}\frac{Z}{A}\right)^{1/3}$$

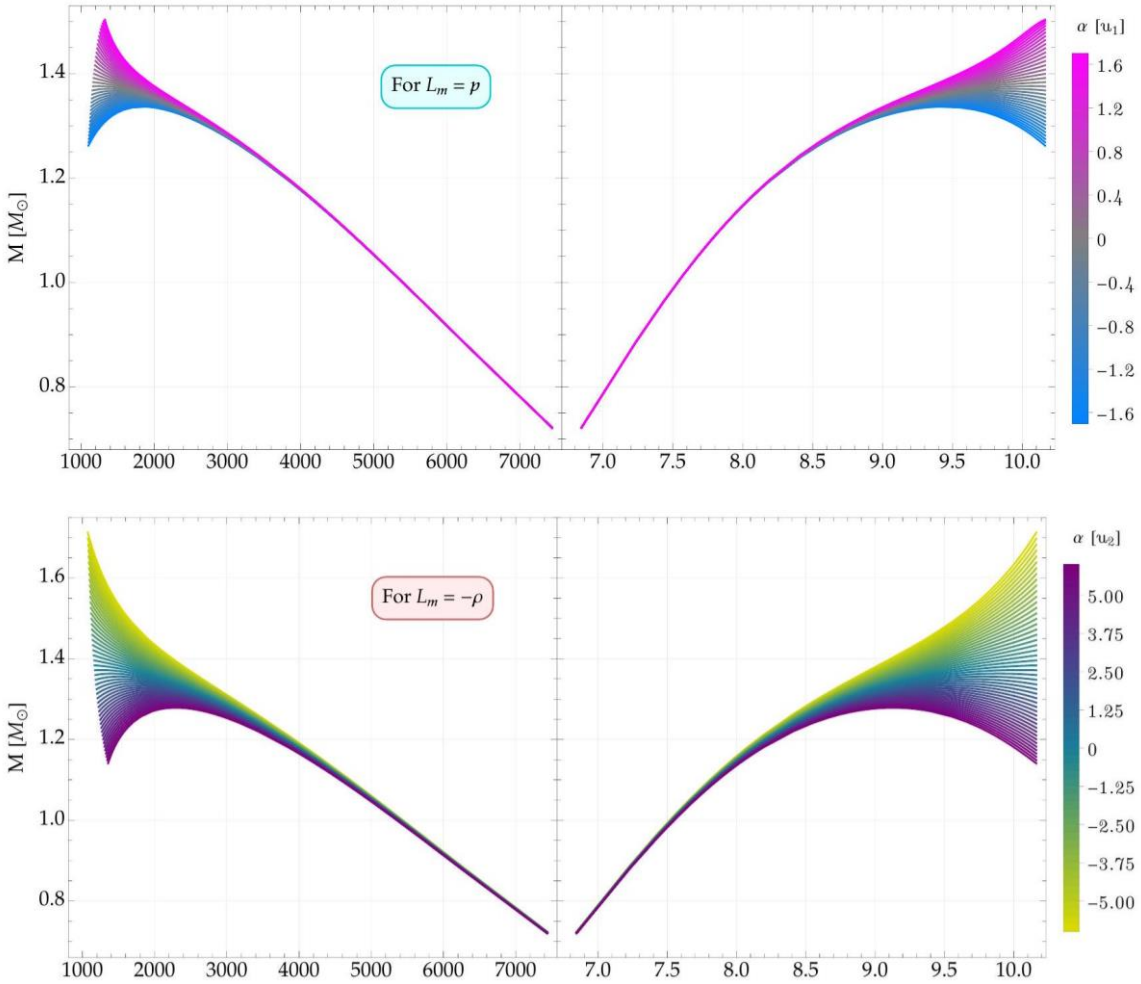
$$\frac{dm}{dr} = 4\pi r^2\rho + \frac{\alpha r^2}{2}\left[\frac{\rho}{2}(5p-\rho) + p^2\right]$$

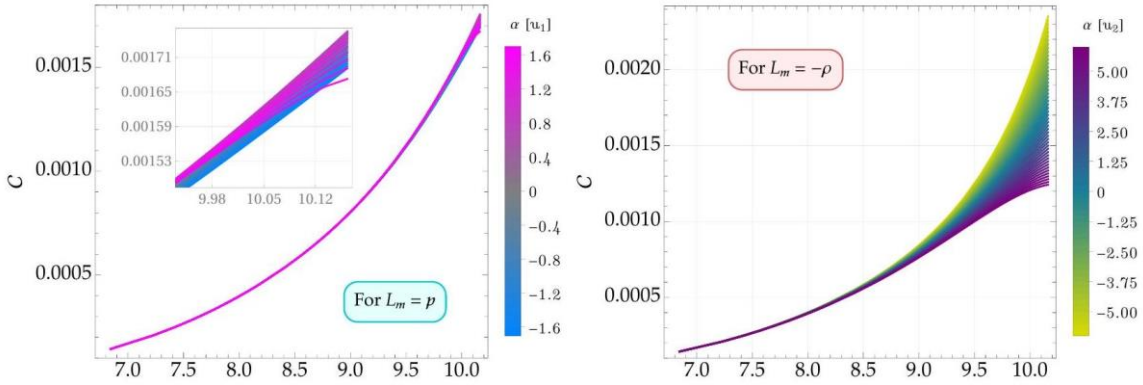
$$\frac{dp}{dr} = -\frac{(\rho+p)\left[4\pi r p + \frac{m}{r^2} + \frac{\alpha r}{4}(3p-\rho)p\right]}{\left(1-\frac{2m}{r}\right)\left[1+\frac{\alpha p}{16\pi+\alpha(5p-\rho)}\left(1-\frac{d\rho}{dp}\right)\right]}$$

$$\frac{dm}{dr} = 4\pi r^2\rho + \frac{\alpha r^2}{4}(3p-\rho)\rho$$

$$\frac{dp}{dr} = -\frac{(\rho+p)\left[4\pi r p + \frac{m}{r^2} + \frac{3\alpha r}{4}(p-\rho)p - \frac{\alpha r}{2}\rho^2\right]}{\left(1-\frac{2m}{r}\right)\left\{1+\frac{\alpha[3p(1-d\rho/dp)-4\rho(d\rho/dp)]}{16\pi+3\alpha(p-\rho)}\right\}}$$

$$m(0) = 0, \rho(0) = \rho_c$$





Figuras 19, 20 y 21. Densidad del centro de masa – energía de una partícula blanca o estrella.

$$S_{\text{grav}}\left[g_{\mu\nu},\varphi\right]=\int d^4x\sqrt{-g}\left(K+Q\nabla^{\mu}\nabla_{\mu}\varphi+FR+\sum_{i=1}^{5}A_iL_i\right)$$

$$L_1=\nabla^{\nu}\nabla^{\mu}\varphi\nabla_{\nu}\nabla_{\mu}\varphi \quad L_2=\left(\nabla^{\mu}\nabla_{\mu}\varphi\right)^2 \quad L_3=\nabla^{\alpha}\nabla_{\alpha}\varphi\nabla_{\nu}\nabla_{\mu}\varphi\nabla^{\nu}\varphi\nabla^{\mu}\varphi$$

$$L_4=\nabla^{\nu}\varphi\nabla_{\nu}\nabla_{\mu}\varphi\nabla^{\mu}\nabla^{\alpha}\varphi\nabla_{\alpha}\varphi, \quad L_5=\left(\nabla^{\nu}\varphi\nabla_{\nu}\nabla_{\mu}\varphi\nabla^{\mu}\varphi\right)^2$$

$$K=Q=A_1=A_2=A_5=0, A_3=-A_4=-4\frac{F_X}{X}$$

$$S_{\text{grav}}=\int d^4x\sqrt{-g}\left(FR-4\frac{F_X}{X}(L_3-L_4)\right)$$

$$T_{\alpha\beta}=(P+\rho)u_{\alpha}u_{\beta}+Pg_{\alpha\beta},$$

$$S_{\text{m}}=\int d^4x\sqrt{-g}P(\mu)$$

$$\mu=\sqrt{-g^{\alpha\beta}\mu_{\alpha}\mu_{\beta}}\,)$$

$$\mu_{\alpha}=\mu u_{\alpha}=\partial_{\alpha}L+A\partial_{\alpha}B$$

$$ds^2=-f(r)dt^2+h(r)dr^2+r^2(d\theta^2+\sin^2\,\theta d\phi^2)$$

$$\varphi(t,r)=qt+\psi(r)$$

$$S_{\text{bgd}}=4\pi\int drdt\left[r^2\sqrt{fh}\left(\frac{2Fh'}{h^2r}+\frac{2F(h-1)}{hr^2}-\frac{4F_XX'(fX+q^2)}{fhrX}+P\right)+\lambda_0(\mu\sqrt{f})'\right]$$

$$X=\psi'^2/h-q^2/f$$

$$\frac{\mu'}{\mu}=-\frac{f'}{2f},$$

$$h'\left(\frac{F_{\mathrm{X}}(-4f^2(\psi')^4+8fhq^2(\psi')^2+4h^2q^4)}{fh^2q^2-f^2h(\psi')^2}+2F\right)-\frac{8F_{\mathrm{X}}(f(\psi')^3-2hq^2\psi')}{f(\psi')^2-hq^2}\psi''$$
$$+\frac{4hq^2F_{\mathrm{X}}(f(h+1)(\psi')^2-(h-1)hq^2)}{fr(f(\psi')^2-hq^2)}+\frac{2F(h-1)h}{r}-h^2r\rho=0$$
$$f'\left(\frac{F_{\mathrm{X}}(-4f^2(\psi')^4+8fhq^2(\psi')^2+4h^2q^4)}{f^2h^2q^2-f^3h(\psi')^2}+\frac{2F}{f}\right)+\frac{8q^2F_{\mathrm{X}}\psi'\psi''}{hq^2-f(\psi')^2}$$
$$+\frac{F_{\mathrm{X}}(4(h-1)hq^2(\psi')^2-4f(h+1)(\psi')^4)}{hr(hq^2-f(\psi')^2)}+\frac{F(2-2h)}{r}-hrP=0$$

$$\lambda_0'=r^2\sqrt{h}\frac{dP}{d\mu},$$

$$\rho=\mu\frac{dP}{d\mu}-P.$$

$$\frac{d}{dr}\left(\sqrt{fh}r^2J^r\right)=0$$

$$J^r=\frac{2\sqrt{fX+q^2}}{f^3h^3r^2X}\{-2F_X[hrf'(fX-q^2)+f(fh(h+1)X-q^2rh'+2hq^2)]\},$$

$$J^r=0$$

$$P'=-\frac{f'(P+\rho)}{2f}$$

$$F=\frac{\kappa}{2}+\sigma X,\kappa\equiv(8\pi G)^{-1}$$

$$\begin{aligned}ds^2=&-f(r)dt^2+h(r)dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)\\&+2r^2\sin\theta\partial_\theta Y_{l0}(\theta)(H_0(r,t)dtd\phi+H_1(r,t)drd\phi)\end{aligned}$$

$$u^\mu=\bar{u}^\mu+\delta u^\mu,\bar{u}^\mu=\frac{\delta_0^\mu}{f^{1/2}},\delta u^\mu=\left\{0,0,0,v(r,t)\frac{\partial_\theta Y_{l0}(\theta)}{f^{\frac{1}{2}}\sin\theta}\right\}.$$

$$\delta T^{t\phi}=\frac{\partial_\theta Y_{l0}(\theta)}{\sin\theta f}[PH_0+(\rho+P)v]$$
$$\delta T^{r\phi}=-\frac{\partial_\theta Y_{l0}(\theta)}{\sin\theta h}PH_1$$

$$v=-H_0.$$

$$S_{\text{axial}}=l(l+1)\int drdt[Q_2H_0^2-Q_1H_1^2+Q_3H_1H_0+Q_0(H_0'-\dot{H}_1)^2]$$

$$Q_0=\frac{Fr^4}{2\sqrt{fh}},Q_1=\frac{\sqrt{f}Fr^2(l(l+1)-2)}{2\sqrt{h}},Q_2=\frac{h}{f}Q_1$$
$$Q_3=-\frac{2\sqrt{f}\sqrt{h}qr^2F_{\mathrm{X}}\psi'\left(f(\psi')^2(4F+Pr^2)-q^2(4F(h-1)+hr^2(2P+\rho))\right)}{fFh(f(\psi')^2-hq^2)+2F_{\mathrm{X}}(f^2(\psi')^4-2fhq^2(\psi')^2-h^2q^4)}$$

$$\chi = \dot{H}_1 - H_0'.$$

$$S'_{\text{axial}} = l(l+1)\int drdt[Q_2H_0^2 - Q_1H_1^2 + Q_0\chi^2 + \lambda(\chi + H_0' - \dot{H}_1)].$$

$$\lambda = -2Q_0\chi, \lambda' = 2Q_2H_0, \dot{\lambda} = 2Q_1H_1.$$

$$\dot{\chi} = 0, (Q_0\chi)' = 0,$$

$$\chi = \chi_0\frac{\sqrt{fh}}{Fr^4},$$

$$S'_{\text{axial}} = \int drdt\frac{l(l+1)}{4Q_1}\left[\dot{\lambda}^2 - \frac{f}{h}\lambda'^2 - \frac{Q_1}{Q_0}\lambda^2\right]$$

$$\ddot{\lambda} - Q_1\frac{d}{dr}\left[\frac{1}{Q_1}\frac{f}{h}\lambda'\right] + \frac{Q_1}{Q_0}\lambda = 0.$$

$$S''_{\text{axial}} = \frac{1}{2}\int dtdr_*\frac{1}{Fr^2}\left[\dot{\lambda}^2 - (\partial_*\lambda)^2 - \frac{Q_1}{Q_0}\lambda^2\right]$$

$$dr_* = \sqrt{\frac{h}{f}}dr$$

$$\ddot{\lambda} - Fr^2\frac{\partial}{\partial r_*}\left[\frac{1}{Fr^2}\frac{\partial\lambda}{\partial r_*}\right] + \frac{f}{r^2}(l(l+1) - 2)\lambda$$

$$\lambda(t,r) = \int d\omega e^{i\omega t}\lambda(\omega,r)$$

$$U \equiv \frac{\lambda}{r\sqrt{F}}$$

$$-\frac{d^2U}{dr_*^2} + V_lU = \omega^2U$$

$$V_l = \frac{f}{r^2}(l(l+1) - 2) - r\sqrt{F}\partial_*\left(\frac{\partial_*(r\sqrt{F})}{r^2F}\right)$$

$$\mathrm{d}\tilde{s}^2 = F[-fdt^2 + hdr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

$$F = \frac{\kappa}{2} + \sigma X$$

$$p \equiv q^2\sigma$$

$$\log_{10}\left(\frac{P}{\mathrm{g\,nm^{-3}}}\right) = \frac{b_1 + b_2\xi + b_3\xi^3}{1 + b_4\xi}u[b_5(\xi - b_6)] + (b_7 + b_8\xi)u[b_9(b_{10} - \xi)] + (b_{11} + b_{12}\xi)u[b_{13}(b_{14} - \xi)]$$
$$+(b_{15} + b_{16}\xi)u[b_{17}(b_{18} - \xi)] + \frac{b_{19}}{b_{20}^2(b_{21} - \xi)^2 + 1} + \frac{b_{22}}{b_{23}^2(b_{24} - \xi)^2 + 1}$$

$$u[x] \equiv \frac{1}{e^x + 1}, \xi = \log_{10}(\rho/g\ \mathrm{nm}^{-3})$$

$$b_i = a_i^{\mathrm{HP}} \text{ for } 1 \le i \le 18, b_j = 0 \text{ for } 19 \le j \le 24,$$

$$b_i = a_i^{\mathrm{PFCPG}} \text{ for } 1 \le i \le 9, b_{10} = a_6^{\mathrm{PFCPG}}, b_j = a_{j-1}^{\mathrm{PFCPG}} \text{ for } 11 \le j \le 24,$$

$$f = \frac{1}{h} = 1 - \frac{2M}{r},$$

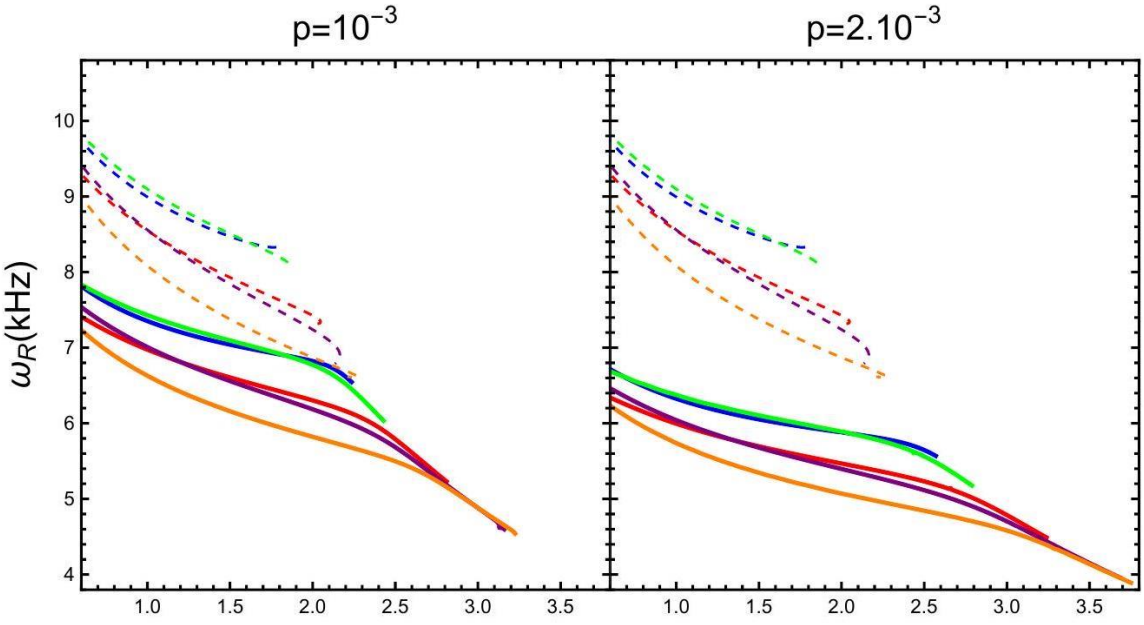
$$\psi' = \frac{q\sqrt{2Mr}}{r - 2M},$$

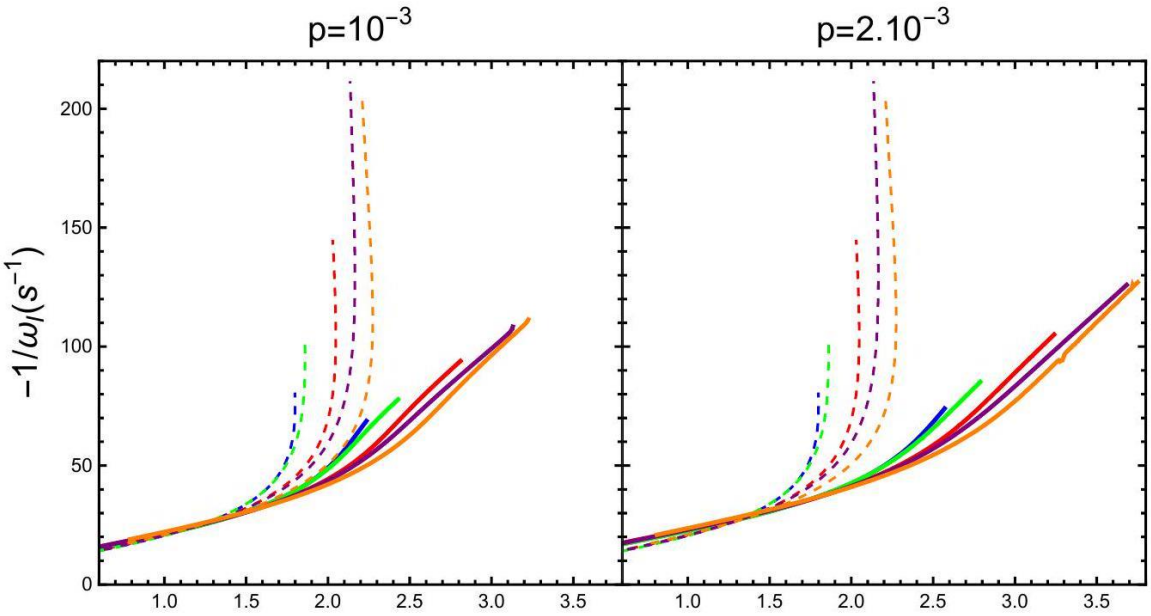
$$Q_1 \frac{d}{dr}\left[\frac{1}{Q_1}\frac{f}{h}\lambda'\right] + \left(\omega^2 - \frac{Q_1}{Q_0}\right)\lambda = 0.$$

$$\lambda(\omega, r) \sim \lambda_- r^{1-l} + \lambda_+ r^{l+2}\ (r \to 0),$$

$$\lambda(\omega, r) \sim \lambda_{\mathrm{in}}\, e^{i\omega r_*} + \lambda_{\mathrm{out}}\, e^{-i\omega r_*}\ (r_* \to \infty),$$

$$r = r_s + \eta e^{i\Theta}$$





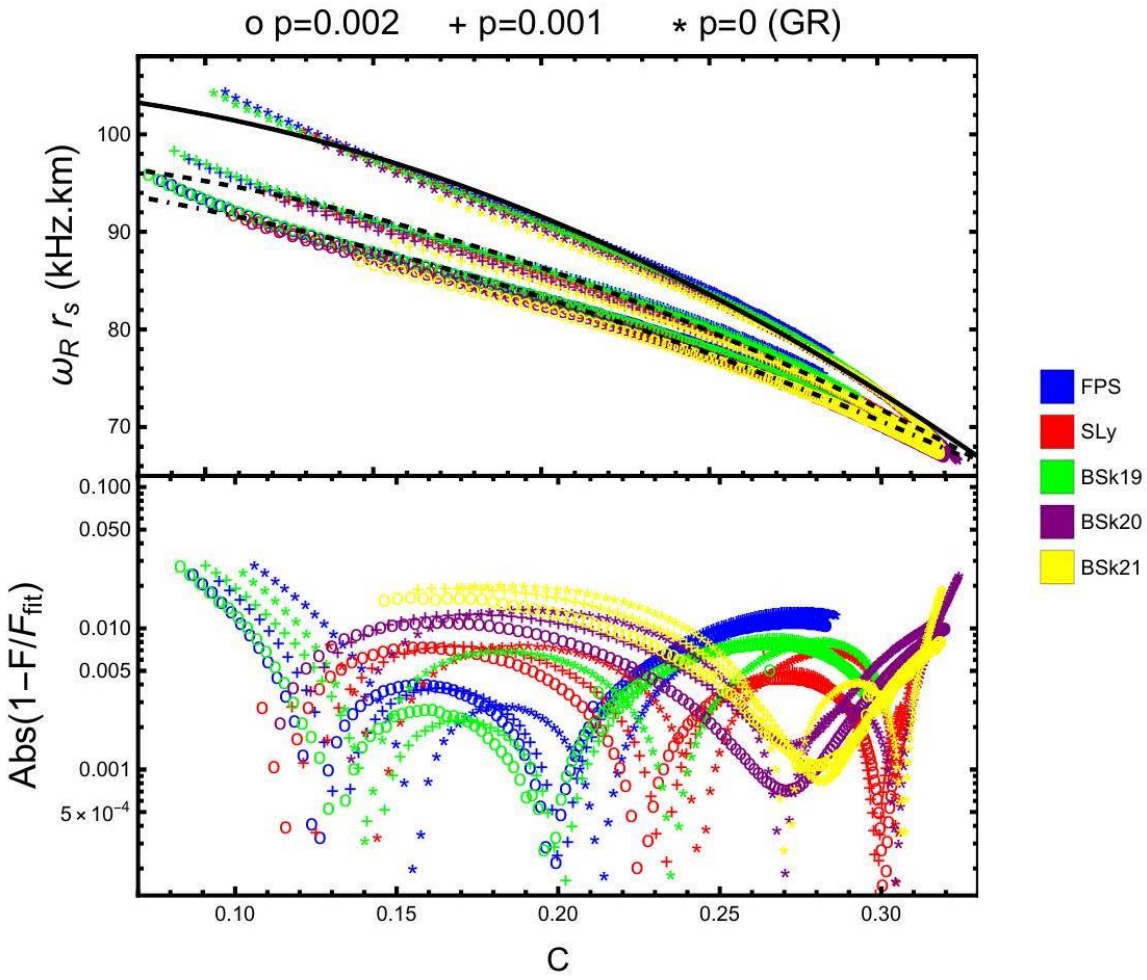
Figuras 22 y 23. Densidad de masa extrema de una partícula blanca o estrella.

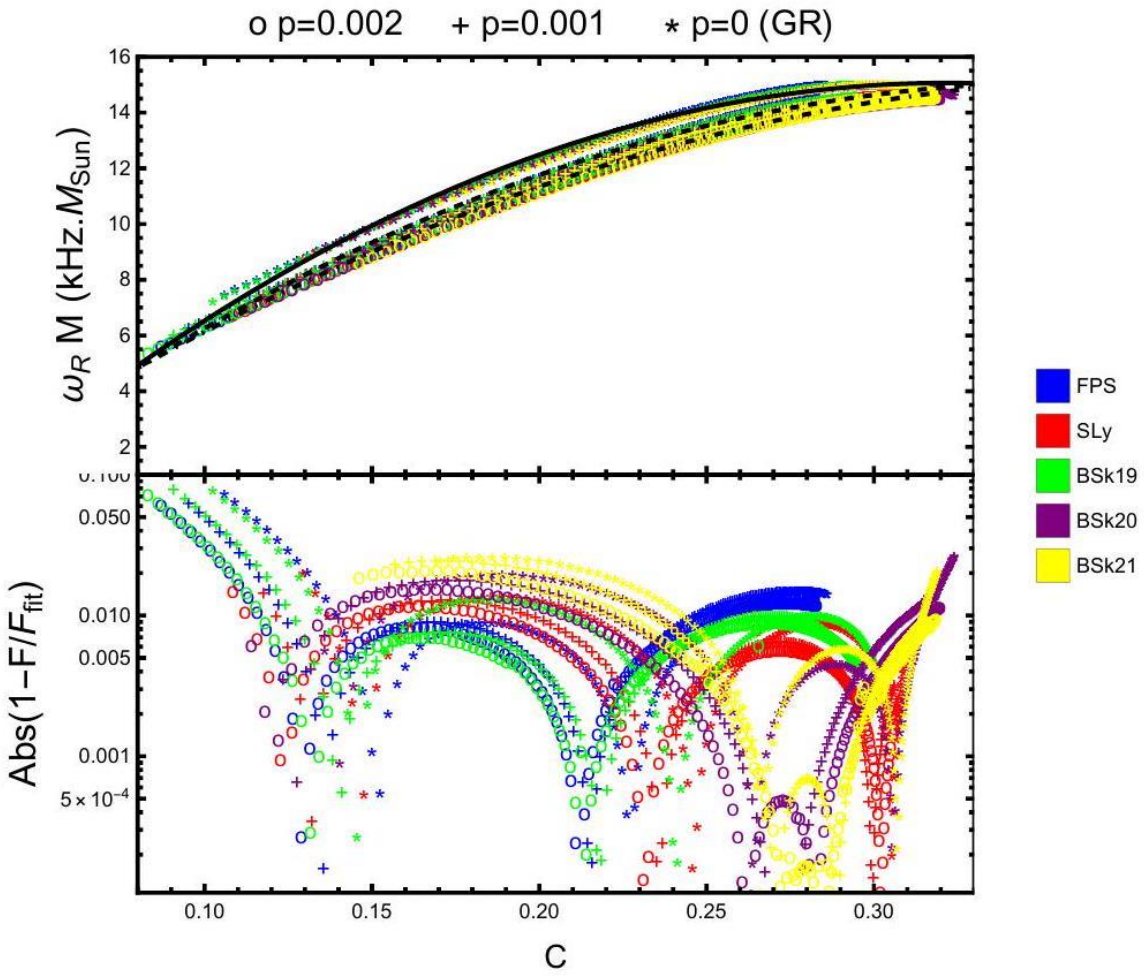
$$\lambda(\omega, r) \sim \lambda_{\text{in}} e^{i(\omega r_s + \eta(\omega_R \cos(\Theta) - \omega_I \sin(\Theta)))} + \lambda_{\text{out}} e^{-i(\omega r_s + \eta(\omega_R \cos(\Theta) - \omega_I \sin(\Theta)))}$$

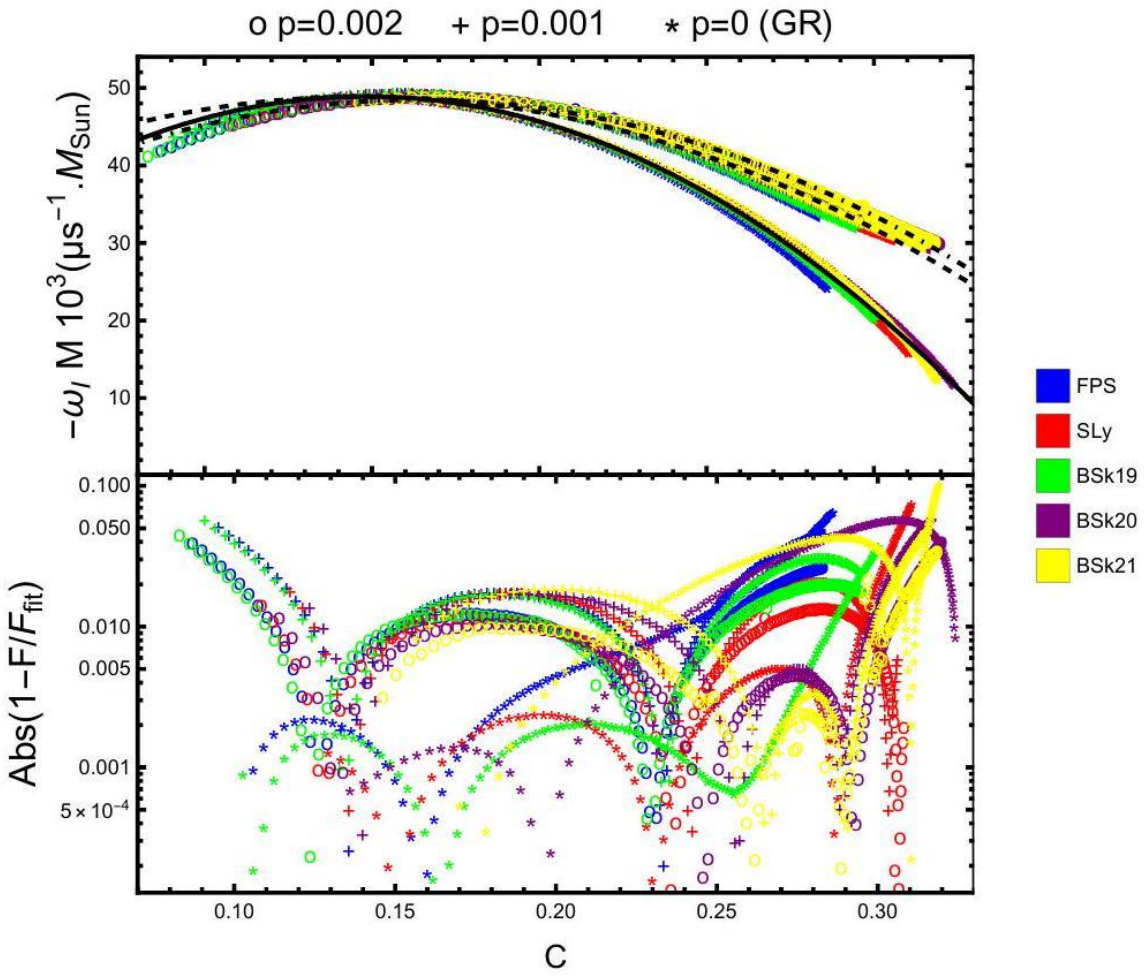
$$\frac{d\lambda}{dr}(r = r_s) = e^{-i\Theta} \frac{d\lambda}{d\eta}(\eta = 0)$$

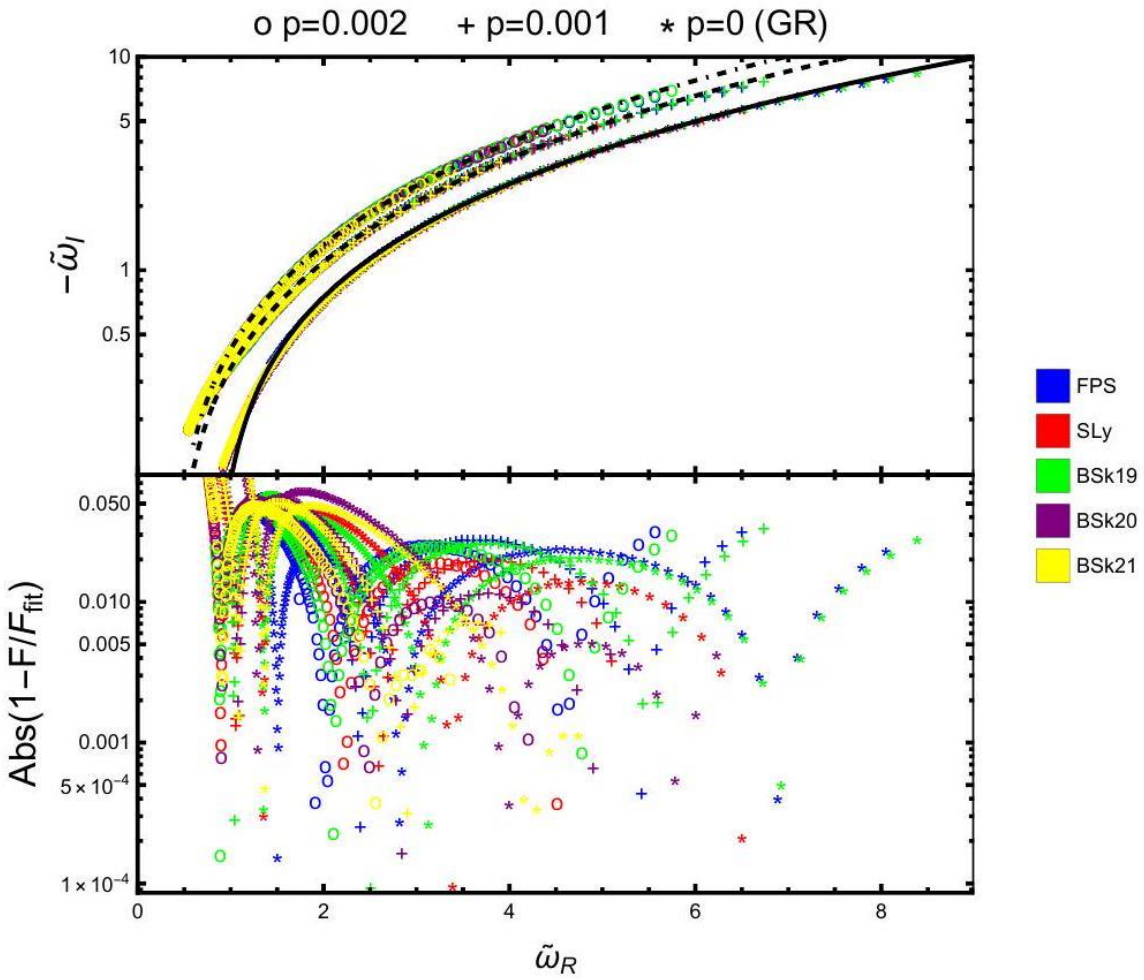
$$C \equiv \frac{M}{r_s},$$

$$\omega_R r_s = \begin{cases} 105.1(\pm 0.5) + 6C(\pm 10) - 369(\pm 22)C^2, & p = 0 \text{ (GR)} \\ 98.6(\pm 0.9) + 7(\pm 8)C - 273(\pm 18)C^2 & p = 10^{-3} \\ 97.5(\pm 0.7) - 34(\pm 6)C - 182.6(\pm 14.2)C^2, & p = 2 \times 10^{-3}. \end{cases}$$


o p=0.002 + p=0.001 * p=0 (GR)
$\omega_R\, r_s$ (kHz.km)
100
90
80
70
Abs(1−F/F_{fit})
0.100
0.050
0.010
0.005
0.001
5×10^{-4}
0.10
0.15
0.20
0.25
0.30
C
FPS
SLy
BSk19
BSk20
BSk21


o p=0.002 + p=0.001 * p=0 (GR)
ω_R M (kHz.M_{Sun})
Abs(1−F/F_{fit})
C
FPS
SLy
BSk19
BSk20
BSk21


o p=0.002 + p=0.001 ∗ p=0 (GR)
$-\omega_I$ M 10^3(μs^{-1}.M_{Sun})
50
40
30
20
10
Abs(1−F/F_{fit})
0.100
0.050
0.010
0.005
0.001
5×10^{-4}
0.10
0.15
0.20
0.25
0.30
C
FPS
SLy
BSk19
BSk20
BSk21



Figuras 24, 25, 26 y 27. Densidad de energía extrema de una partícula blanca o estrella.

$$\omega_R M = \begin{cases} -2.9(\pm 0.2) + 111.5(\pm 2.1)C - 173.2(\pm 4.7)C^2, & p = 0 \text{ (GR)} \\ -1.9(\pm 0.2) + 94.4(\pm 1.6)C - 131.3(\pm 3.5)C^2, & p = 10^{-2} \\ -1.2(\pm 0.1) + 84.4(\pm 1.2)C - 108.0(\pm 2.6)C^2, & p = 2 \times 10^{-2}. \end{cases}$$

$$\omega_I M = \begin{cases} 22.5(\pm 1.1) + 356.4(\pm 9.7)C - 1203(\pm 21)C^2, & p = 0 \text{ (GR)} \\ 32.9(\pm 1.1) + 216.5(\pm 10.7)C - 733(\pm 23)C^2 & p = 10^{-3} \\ 27.0(\pm 0.6) + 262.7(\pm 6.1)C - 803(\pm 14)C^2, & p = 2 \times 10^{-3} \end{cases}$$

$$\tilde{\omega}_I = \begin{cases} -0.379(\pm 0.016) + 0.400(\pm 0.011)\tilde{\omega}_R + 0.082(\pm 0.002)\tilde{\omega}_R^2, & p = 0 \text{ (GR)} \\ -0.151(\pm 0.017) + 0.374(\pm 0.015)\tilde{\omega}_R + 0.124(\pm 0.003)\tilde{\omega}_R^2, & p = 10^{-3} \\ -0.174(\pm 0.015) + 0.456(\pm 0.015)\tilde{\omega}_R + 0.144(\pm 0.003)\tilde{\omega}_R^2, & p = 2 \times 10^{-3} \end{cases}$$

$$S_m[L, A, B] = \int d^4x\sqrt{-g}P(\mu)$$

$$\mu = \sqrt{-g^{\alpha\beta}(\partial_\alpha L + A\partial_\alpha B)\left(\partial_\beta L + A\partial_\beta B\right)}$$

$$u_\alpha = \frac{1}{\mu}(\partial_\alpha L + A\partial_\alpha B).$$

$$\nabla_\alpha\left(\frac{dP}{d\mu}u^\alpha\right) = 0, u^\alpha\partial_\alpha A = 0, u^\alpha\partial_\alpha B = 0$$

$$\nabla_{[\alpha}(\mu u_{\beta]}) = \partial_{[\alpha}A\partial_{\beta]}B$$

$$u^\alpha\nabla_{[\alpha}(\mu u_{\beta]}) = 0$$

$$T_{\alpha\beta} = (\rho + P)u_\alpha u_\beta + Pg_{\alpha\beta}$$

$$\bar{u}^\alpha = \left\{\frac{1}{\sqrt{f}}, 0,0,0\right\}, \bar{u}_\alpha = \{-\sqrt{f}, 0,0,0\}$$

$$L(t,r) = -\sqrt{f}\mu t$$

$$(\mu\sqrt{f})' = 0 \text{ or } \frac{\mu'}{\mu} = -\frac{f'}{2f}$$

$$\delta^2\mu(t,r,\theta) = -\frac{r^2\mu}{2f}v^2(\partial_\theta Y_{l0})^2$$

$$\delta^2\sqrt{-g} = \frac{1}{2}r^4\left(\sqrt{\frac{h}{f}}H_0^2 - \sqrt{\frac{f}{h}}H_1^2\right)\sin\,\theta(\partial_\theta Y_{l0})^2.$$

$$S_{\text{m, axial}} = \int d^4x\left(P\delta^2\sqrt{-g} + \sqrt{-g}\frac{dP}{d\mu}\delta^2\mu\right)$$
$$= -\frac{l(l+1)}{2}\int drdt\sqrt{fh}r^4\left(\frac{P}{h}H_1^2 + \frac{\rho}{f}H_0^2\right)$$

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}$$

$$h_{at} = r^2\sum_{lm} H_0^{lm}(r,t)\sin\,\theta\epsilon_{ab}\nabla^b Y_{lm}$$
$$h_{ar} = r^2\sum_{lm} H_1^{lm}(r,t)\sin\,\theta\epsilon_{ab}\nabla^b Y_{lm}$$
$$h_{ab} = \frac{1}{2}r^2\sum_{lm} Q^{lm}(r,t)\sin\,\theta(\epsilon_a^c\nabla_c\nabla_b Y_{lm} + \epsilon_b^c\nabla_c\nabla_a Y_{lm})$$

$$\xi^a = r^2\sum_{lm}\Lambda^{lm}(r,t)\sin\,\theta\epsilon_b^a\nabla^b Y_{lm},$$

$$H_0 \to H_0 + \dot{\Lambda}, H_1 \to H_1 + \Lambda', Q \to Q + 2\Lambda,$$

$$\delta u^a = r^2 f^{-1/2}\sum_{lm} v^{lm}(r,t)\sin\,\theta\epsilon_b^a\nabla^b Y_{lm}$$

$$\lambda'' - \frac{2}{r}\lambda' - \frac{(l^2+l-2)}{r^2}\lambda$$

$$\mathcal{L}_{NL} = \mathcal{L}_{nm} + \mathcal{L}_{\sigma} + \mathcal{L}_{\omega} + \mathcal{L}_{\rho} + \mathcal{L}_{int},$$

$$\begin{aligned}
\mathcal{L}_{nm} =& \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - M\right)\psi + g_{\sigma}\sigma\bar{\psi}\psi - g_{\omega}\bar{\psi}\gamma^{\mu}\omega_{\mu}\psi \\
& - \frac{g_{\rho}}{2}\bar{\psi}\gamma^{\mu}\vec{\rho}_{\mu}\vec{\tau}\psi \\
\mathcal{L}_{\sigma} =& \frac{1}{2}\left(\partial^{\mu}\sigma\partial_{\mu}\sigma - m_{\sigma}^2\sigma^2\right) - \frac{\mathrm{A}}{3}\sigma^3 - \frac{\mathrm{B}}{4}\sigma^4 \\
\mathcal{L}_{\omega} =& -\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega^{\mu}\omega_{\mu} + \frac{\mathrm{C}}{4}\left(g_{\omega}^2\omega_{\mu}\omega^{\mu}\right)^2 \\
\mathcal{L}_{\rho} =& -\frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_{\rho}^2\vec{\rho}_{\mu}\vec{\rho}^{\mu} \\
\mathcal{L}_{int} =& \frac{1}{2}\Lambda_v g_{\omega}^2 g_{\rho}^2\omega_{\mu}\omega^{\mu}\vec{\rho}_{\mu}\vec{\rho}^{\mu}
\end{aligned}$$

$$\Omega_{\mu\nu} = \partial_v\omega_{\mu} - \partial_{\mu}\omega_v, \vec{B}_{\mu\nu} = \partial_v\vec{\rho}_{\mu} - \partial_{\mu}\vec{\rho}_v - g_{\rho}\left(\vec{\rho}_{\mu}\times\vec{\rho}_v\right)$$

$$\begin{aligned}
\mathcal{E}_{HM} =& \frac{1}{\pi^2}\int_0^{k_p,k_n} dk k^2\sqrt{k^2+(M^*)^2} + \frac{1}{2}m_{\sigma}^2\sigma^2 - \frac{1}{2}m_{\omega}^2\omega^2 - \frac{1}{2}m_{\rho}^2\rho^2 + \frac{\mathrm{A}}{3}\sigma^3 + \frac{\mathrm{B}}{4}\sigma^4 \\
& - \frac{\mathrm{C}}{4}g_{\omega}^4\omega^4 + g_{\omega}\omega\left(\rho_p+\rho_n\right) - \frac{\Lambda_v}{2}\left(g_{\rho}g_{\omega}\rho\omega\right)^2 + \frac{g_{\rho}}{2}\rho\left(\rho_p-\rho_n\right) \\
P_{HM} =& \frac{1}{3\pi^2}\int_0^{k_p,k_n} dk\frac{k^4}{\sqrt{k^2+(M^*)^2}} - \frac{1}{2}m_{\sigma}^2\sigma^2 + \frac{1}{2}m_{\omega}^2\omega^2 + \frac{1}{2}m_{\rho}^2\rho^2 - \frac{\mathrm{A}}{3}\sigma^3 - \frac{\mathrm{B}}{4}\sigma^4 \\
& + \frac{\mathrm{C}}{4}g_{\omega}^4\omega^4 + \frac{\Lambda_v}{2}\left(g_{\rho}g_{\omega}\rho\omega\right)^2
\end{aligned}$$

Model	Interaction	g_{σ}	g_{ω}	g_{ρ}	A	B	C	Λ_v
BITSH-E	with ω^4 ($\mathrm{C} \neq 0$)	10.0	12.52	10.32	10.62	-14.07	0.0027	0.08
BITSH-I	with ω^4 ($\mathrm{C} \neq 0$)	9.05	10.69	9.95	15.29	-13.56	0.0006	0.06
BITSH-E	without ω^4 ($\mathrm{C} = 0$)	9.58	11.58	10.44	15.13	-38.21	-	0.1702
BITSH-I	without ω^4 ($\mathrm{C} = 0$)	8.96	10.35	9.95	19.71	-46.44	-	0.0726

$$\mathcal{L}_{\chi} = \bar{\chi}_D\left[\gamma_{\mu}\left(i\partial^{\mu} - g_{vd}V_D^{\mu}\right) - M_D\right]\chi_D - \frac{1}{4}V_{\mu\nu,D}V_D^{\mu\nu} + \frac{1}{2}m_{vd}^2V_{\mu,D}V_D^{\mu}.$$

$$\mathcal{E}_{DM}=\frac{1}{\pi^2}\int_0^k dk k^2\sqrt{k^2+(M_D)^2}+\frac{g_{vd}^2}{2m_{vd}^2}\rho_D^2,$$
$$P_{DM}=\frac{1}{3\pi^2}\int_0^k dk\frac{k^4}{\sqrt{k^2+(M_D)^2}}+\frac{g_{vd}^2}{2m_{vd}^2}\rho_D^2,$$

$$\frac{dP_{HM}}{dr}=-(P_{HM}+\mathcal{E}_{HM})\frac{4\pi r^3(P_{HM}+P_{DM})+m(r)}{r\big(r-2m(r)\big)},$$
$$\frac{dP_{DM}}{dr}=-(P_{DM}+\mathcal{E}_{DM})\frac{4\pi r^3(P_{HM}+P_{DM})+m(r)}{r(r-2m(r))},$$
$$\frac{dm(r)}{dr}=4\pi(\mathcal{E}_{HM}(r)+\mathcal{E}_{DM}(r))r^2.$$

$$f_{DM}=\frac{M_{DM}}{M_{\text{Total}}}$$

$$M_{DM}(R_{DM})=4\pi\int_0^{R_{DM}} r^2\mathcal{E}_{DM}(r)dr$$

$$M_{\text{Total}}\,(R)=4\pi\int_0^{R} r^2(\mathcal{E}_{HM}(r)+\mathcal{E}_{DM}(r))dr$$

$$\lambda=\tfrac{2}{3}k_2R^5,\ \Lambda=\tfrac{2}{3}k_2\mathcal{C}^{-5},\ \mathcal{C}=M/R$$

$$k_2=\frac{8\mathcal{C}^5}{5}(1-2\mathcal{C})^2[2+2\mathcal{C}(y_R-1)-y_R]\times\big(2\mathcal{C}\big(6-3y_R+3\mathcal{C}(5y_R-8)\big)$$
$$+4\mathcal{C}^3[13-11y_R+\mathcal{C}(3y_R-2)+2\mathcal{C}^2(1+y_R)]$$
$$+3(1-2\mathcal{C})^2\big(2-y_R+2\mathcal{C}(y_R-1)\big)\log(1-2\mathcal{C})\big)^{-1}$$

$$r\frac{dy(r)}{dr}+y^2(r)+y(r)F(r)+r^2Q(r)=0.$$

$$F(r)=\frac{r-4\pi r^3(\mathcal{E}_{HM}(r)+\mathcal{E}_{DM}(r))-(P_{HM}(r)+P_{DM}(r))}{r-2m(r)},$$

$$Q(r)=\frac{4\pi r\left(5(\mathcal{E}_{HM}(r)+\mathcal{E}_{DM}(r))+9(P_{HM}(r)+P_{DM}(r))+\frac{\mathcal{E}_{HM}(r)+P_{HM}(r)}{\partial P_{HM}(r)/\partial\mathcal{E}_{HM}(r)}+\frac{\mathcal{E}_{DM}(r)+P_{DM}(r)}{\partial P_{DM}(r)/\partial\mathcal{E}_{DM}(r)}-\frac{6}{4\pi r^2}\right)}{r-2m(r)}$$
$$-4\left[\frac{m(r)+4\pi r^3(P_{HM}(r)+P_{DM}(r))}{r^2(1-2m(r)/r)}\right]^2$$

$$c_s^2=\frac{1}{3}-c_1\exp\left[-\frac{(n-c_2)^2}{n_{bl}^2}\right]+h_p\exp\left[\frac{(n-n_p)^2}{w_p^2}\right]\left(1+\operatorname{erf}\left[s_p\frac{(n-n_p)}{w_p}\right]\right).$$

$$P(\boldsymbol{\Theta}\mid D)=\frac{\mathcal{L}(D\mid\boldsymbol{\Theta})P(\boldsymbol{\Theta})}{\mathcal{Z}},$$

$$\mathcal{L}(D\mid\boldsymbol{\Theta})=\mathcal{L}_{Obs}(D\mid\boldsymbol{\Theta})\times\mathcal{L}_{GW}(D\mid\boldsymbol{\Theta}).$$

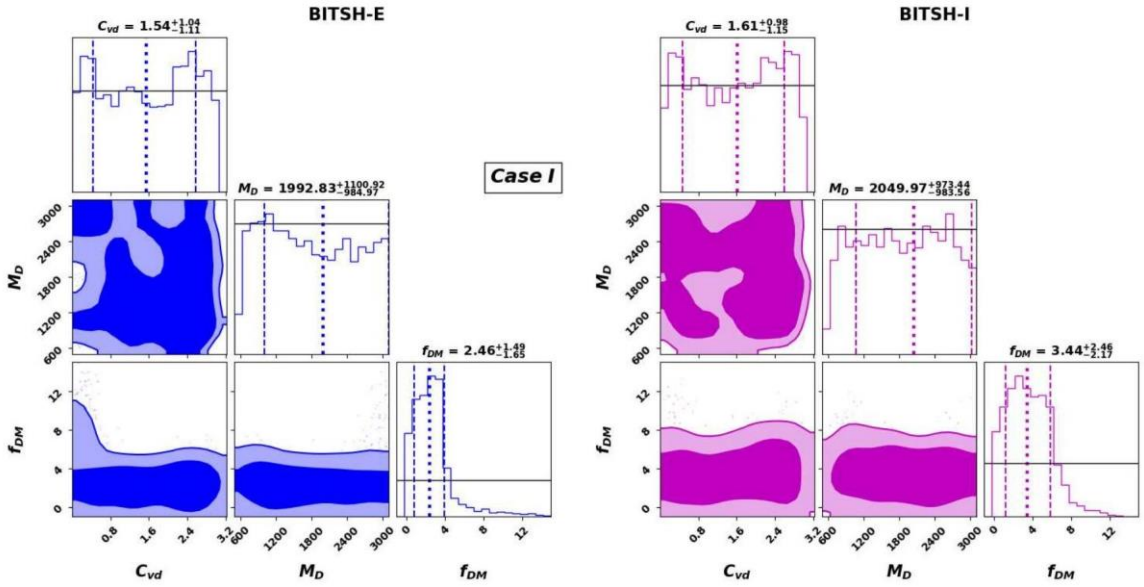
$$\mathcal{L}_{\sigma}(D \mid \Theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(D(\Theta) - D_{Obs})^2}{2\sigma^2}\right)$$

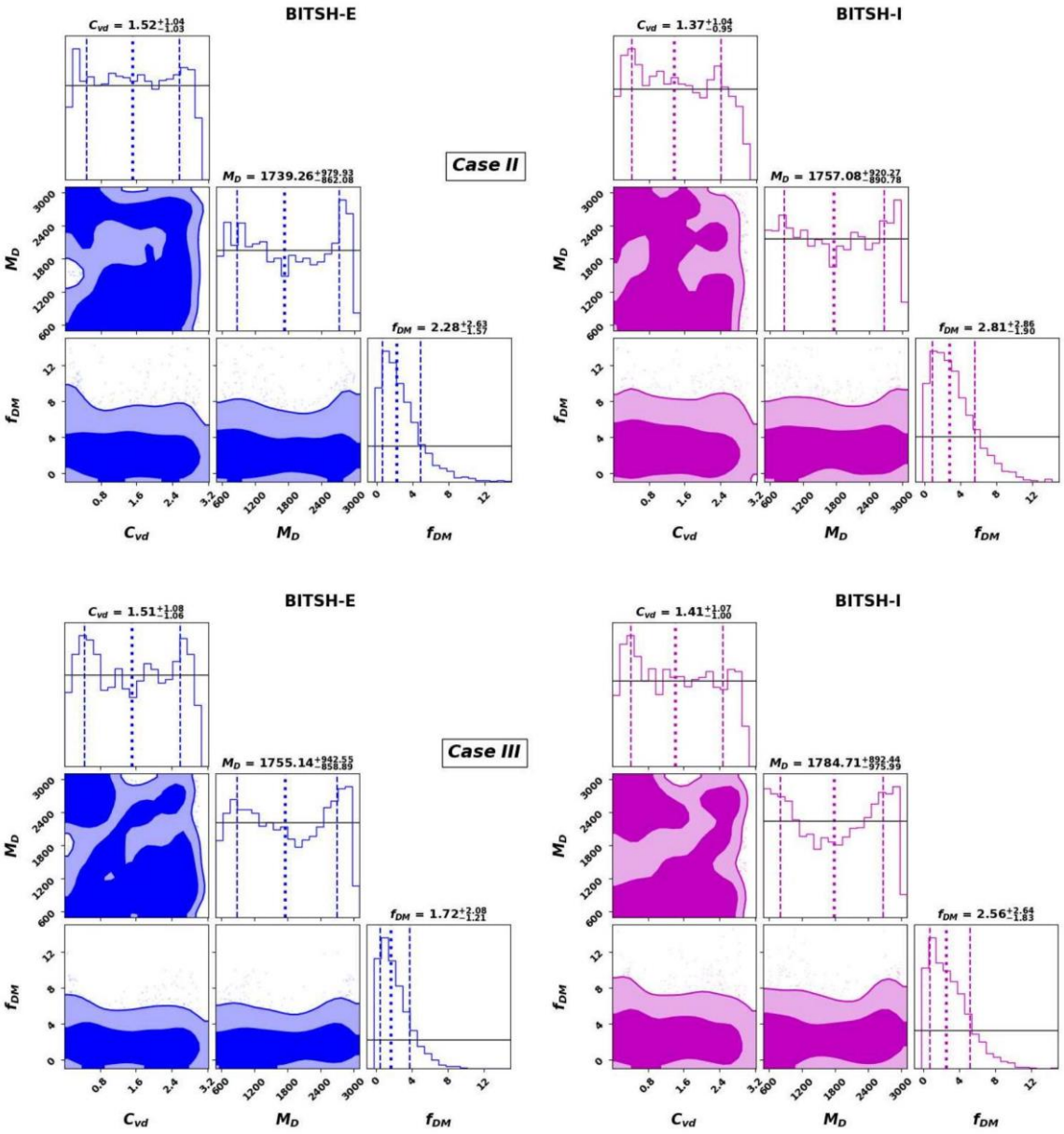
$$\mathcal{L}_{Obs}(D \mid \Theta) = \mathcal{L}_{NICER}^{\text{high}} \times \mathcal{L}_{\text{NICER}}^{\text{low}}$$

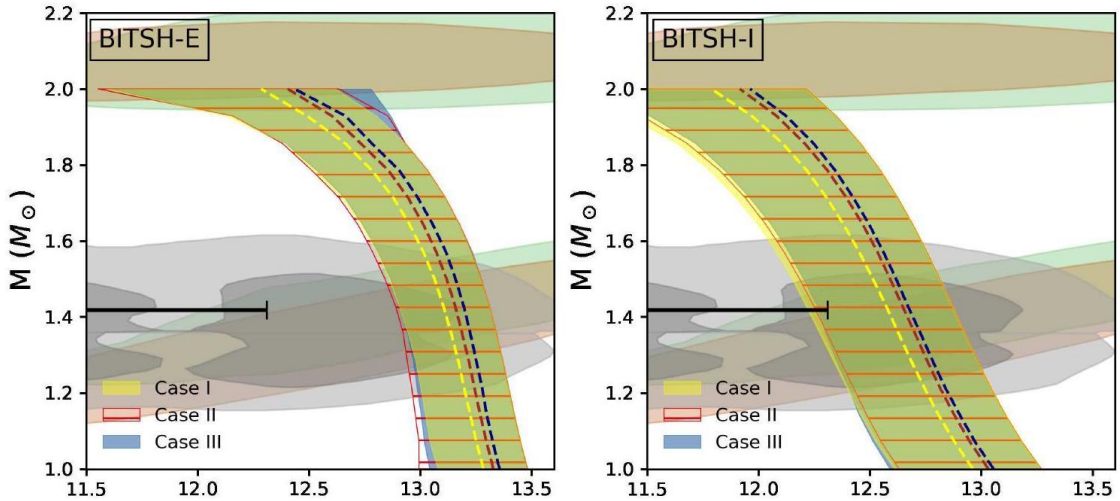
$$\mathcal{L}_{NICER}(D \mid \boldsymbol{\Theta}) = \int_{M_0}^{M_{\max}} dm P(m \mid \boldsymbol{\Theta}) \times P(D \mid m, R(m, \boldsymbol{\Theta}))$$

$$\mathcal{L}_{GW}(D \mid \boldsymbol{\Theta}) = \int_{m_l}^{M_u} dm_1 \int_{M_l}^{m_1} dm_2 P(m_1, m_2 \mid \boldsymbol{\Theta}) \times P\big(d_{GW} \mid m_1, m_2, \Lambda_1(m_1, \boldsymbol{\Theta}), \Lambda_2(m_2, \boldsymbol{\Theta})\big)$$

$$P(m \mid \Theta) = \begin{cases} \dfrac{1}{M_{\max} - M_0} & \text{if } M_0 \le m \le M_{\max} \\ 0 & \text{else} \end{cases}$$




BITSH-E
BITSH-I
Case II
Case III
C_{vd}
M_D
f_{DM}


BITSH-E
BITSH-I
M ($M_\odot$)
Case I
Case II
Case III

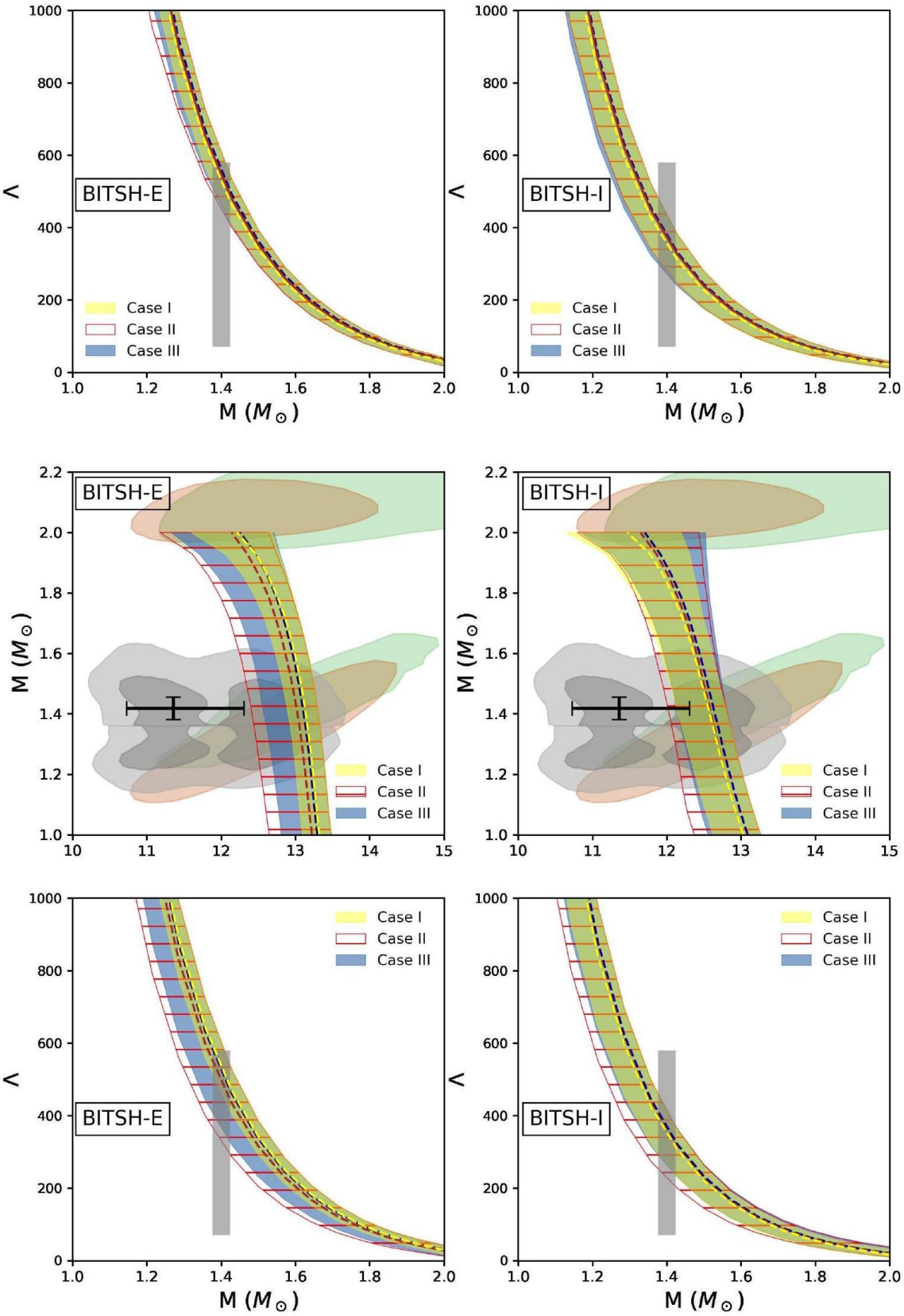


Figura 28, 29, 30, 31, 32, 33 y 34. Formación de materia oscura por colapso de una partícula estrella o blanca.

Models	BITSH-E			BITSH-I		
DM parameter	$C_{vd}(\mathrm{MeV}^{-1})$ ($\times 10^{-2}$)	$M_D(\mathrm{MeV})$	$f_{DM}(\%)$	$C_{vd}(\mathrm{MeV}^{-1})$ ($\times 10^{-2}$)	$M_D(\mathrm{MeV})$	$f_{DM}(\%)$
Fixed HM EoS						
Case I	$1.54^{+1.04}_{-1.11}$	$1992.83^{+1100.91}_{-984.97}$	$2.46^{+1.49}_{-1.65}$	$1.61^{+0.98}_{-1.15}$	$2049.96^{+973.43}_{-983.56}$	$3.44^{+2.46}_{-2.17}$
Case II	$1.52^{+1.04}_{-1.03}$	$1739.25^{+979.92}_{-862.07}$	$2.28^{+2.63}_{-1.57}$	$1.37^{+1.04}_{-0.95}$	$1757.07^{+920.26}_{-890.77}$	$2.81^{+2.86}_{-1.90}$
Case III	$1.51^{+1.08}_{-1.06}$	$1755.13^{+942.55}_{-858.89}$	$1.72^{+2.08}_{-1.21}$	$1.41^{+1.07}_{-1.00}$	$1784.70^{+892.43}_{-975.98}$	$2.56^{+2.64}_{-1.83}$
HM EoS with HDU						
Case I	$1.53^{+0.97}_{-1.04}$	$1962.62^{+1069}_{-938.55}$	$2.36^{+1.38}_{-1.52}$	$1.48^{+0.97}_{-0.96}$	$2099.68^{+935.17}_{-986.67}$	$3.65^{+2.22}_{-2.28}$
Case II	$1.50^{+0.95}_{-0.94}$	$1706.20^{+836.3}_{-740.55}$	$3.36^{+3.37}_{-2.19}$	$1.48^{+0.97}_{-0.99}$	$1832.88^{+767.58}_{-836.21}$	$2.85^{+3.11}_{-1.91}$
Case III	$1.49^{+0.94}_{-1.00}$	$1849.82^{+774.37}_{-888.29}$	$2.40^{+2.73}_{-1.57}$	$1.43^{+1.06}_{-0.95}$	$1818.82^{+779.63}_{-846.20}$	$2.28^{+2.75}_{-1.57}$
HM EoS ($C = 0$) with HDU						
Case I	$1.58^{+0.91}_{-0.96}$	$1936.31^{+939.70}_{-812.02}$	$8.12^{+5.74}_{-4.86}$	$1.57^{+0.94}_{-1.01}$	$2056.07^{+968.79}_{-972.54}$	$4.37^{+3.52}_{-2.83}$

Case II	$1.58^{+0.93}_{-0.95}$	$1783.23^{+826}_{-762.95}$	$4.82^{+4.73}_{-2.97}$	$1.54^{+0.93}_{-1.04}$	$1833.26^{+834.36}_{-903.34}$	$2.92^{+3.51}_{-1.99}$
Case III	$1.53^{+0.95}_{-1.01}$	$1689.08^{+858.60}_{-782.98}$	$3.58^{+4.00}_{-2.41}$	$1.58^{+0.92}_{-1.10}$	$1820.58^{+800.18}_{-883.46}$	$2.35^{+2.92}_{-1.63}$

Interaction terms	Models	$C_{vd}(\mathrm{MeV}^{-1})$ $(\times 10^{-2})$	$M_D(\mathrm{MeV})$	$f_{DM}(\%)$
$\mathrm{C}=0,\Lambda_v=0$	GM1	$1.63^{+0.85}_{-0.92}$	$1835.18^{+732.33}_{-787.64}$	$5.34^{+4.12}_{-3.28}$
	NL3	$1.41^{+0.89}_{-0.76}$	$1841.52^{+697}_{-700.78}$	$15.39^{+3.08}_{-4.38}$
$\mathrm{C}\neq 0,\Lambda_v=0$	MS1	$1.71^{+0.84}_{-1.02}$	$1704.71^{+759.38}_{-704.62}$	$5.63^{+4.89}_{-2.95}$
	TM1	$1.69^{+0.86}_{-0.94}$	$1580.95^{+865}_{-668.57}$	$5.19^{+3.87}_{-2.68}$
$\mathrm{C}\neq 0,\Lambda_v\neq 0$	BITSH-E	$1.50^{+0.95}_{-0.94}$	$1706.20^{+836.3}_{-740.55}$	$3.36^{+3.37}_{-2.19}$
	BITSH-I	$1.48^{+0.97}_{-0.99}$	$1832.88^{+767.58}_{-836.21}$	$2.85^{+3.11}_{-1.91}$
$\mathrm{C}=0,\Lambda_v\neq 0$	BITSH-E	$1.58^{+0.93}_{-0.95}$	$1783.23^{+826}_{-762.95}$	$4.82^{+4.73}_{-2.97}$
	BITSH-I	$1.54^{+0.93}_{-1.04}$	$1833.26^{+834}_{-903.34}$	$2.92^{+3.51}_{-1.99}$

$$\mathcal{Q}_{\psi\gamma\Upsilon}=\nabla_{\psi}g_{\gamma\Upsilon}=\partial_{\psi}g_{\gamma\Upsilon}-\Gamma^{\sigma}_{\psi\gamma}g_{\sigma\Upsilon}-\Gamma^{\sigma}_{\psi\Upsilon}g_{\gamma\sigma}.$$

$$\Gamma^{\sigma}_{\gamma\Upsilon}=\left\{{}^{\sigma}{}_{\gamma\Upsilon}\right\}+\zeta^{\sigma}_{\gamma\Upsilon}+L^{\sigma}_{\gamma\Upsilon},$$

$$\left\{{}^{\sigma}{}_{\gamma\Upsilon}\right\}\equiv\frac{1}{2}g^{\sigma\Psi}\left(\partial_{\gamma}g_{\Psi\Upsilon}+\partial_{\Upsilon}g_{\Psi\Upsilon}-\partial_{\Psi}g_{\gamma\Upsilon}\right),$$

$$\zeta^{\sigma}_{\gamma\Upsilon}\equiv\frac{1}{2}\mathcal{T}^{\sigma}\gamma\Upsilon+\mathcal{T}_{(\gamma}{}^{\sigma}{}_{\Upsilon)}$$

$$L^{\sigma}_{\gamma\Upsilon} \equiv \frac{1}{2}\mathcal{Q}^{\sigma}_{\gamma\Upsilon} - \mathcal{Q}_{(\gamma\ \ \ \Upsilon)}^{\ \ \ \sigma}$$

$$H^{\psi}_{\gamma\Upsilon} = \frac{-1}{4}\mathcal{Q}^{\psi}_{\gamma\Upsilon} + \frac{1}{2}\mathcal{Q}^{\psi}_{(\gamma\Upsilon)} + \frac{1}{4}\left(\mathcal{Q}^{\psi} - \tilde{\mathcal{Q}}^{\psi}\right)g_{\gamma\Upsilon} - \frac{1}{4}\delta^{\psi}_{(\gamma\Upsilon)}$$

$$\mathcal{Q}_{\psi} \equiv \mathcal{Q}^{\gamma}_{\psi\gamma}\ \tilde{\mathcal{Q}}_{\psi} \equiv \mathcal{Q}^{\gamma\ \ \gamma}_{\ \ \psi\gamma},$$

$$\mathcal{Q} = -\mathcal{Q}_{\psi\Upsilon\gamma}F^{\psi\gamma\Upsilon}.$$

$$S = \int \sqrt{-g}d^4x\left[\frac{1}{2}\mathcal{F}(\mathcal{Q}) + \sigma^{\Psi\gamma\Upsilon}_{\psi}\mathcal{R}^{\psi}_{\Psi\gamma\Upsilon} + \sigma^{\gamma\Upsilon}_{\psi}\mathcal{T}^{\psi}_{\gamma\Upsilon} + \mathcal{L}_n\right]$$

$$\mathcal{T}_{\gamma\Upsilon} = \frac{2}{\sqrt{-g}}\Delta_{\psi}\left(\sqrt{-g}\mathcal{F}_{\mathcal{Q}}H^{\psi}_{\gamma\Upsilon}\right) + \frac{1}{2}g_{\gamma\Upsilon}\mathcal{F} + \mathcal{F}_{\mathcal{Q}}\left(H_{\gamma\psi\Psi}\mathcal{Q}^{\psi\Psi}_{\Upsilon} - 2\mathcal{Q}_{\psi\Psi\gamma}H^{\psi\Psi}_{\Upsilon}\right).$$

$$\mathcal{F}_{\mathcal{Q}} = \frac{\partial\mathcal{F}}{\partial\mathcal{Q}}$$

$$\mathcal{T}_{\gamma\Upsilon} \equiv \frac{2}{\sqrt{g}}\frac{\delta(\sqrt{-g})\mathcal{L}_n}{\delta g^{\gamma\Upsilon}}.$$

$$\nabla_{\rho}\sigma^{\Upsilon\gamma\rho}_{\psi} + \sigma^{\gamma\Upsilon}_{\psi} = \sqrt{-g\mathcal{F}_{\mathcal{Q}}}H^{\psi}_{\gamma\Upsilon} + J^{\gamma\Upsilon}_{\psi}.$$

$$J^{\gamma\Upsilon}_{\psi} = \frac{-1}{2}\frac{\delta\mathcal{L}_n}{\delta T^{\psi}_{\gamma\Upsilon}}.$$

$$\nabla_{\gamma}\nabla_{\Upsilon}\left(\sqrt{-g}\mathcal{F}_{\mathcal{Q}}H^{\gamma\Upsilon}{}_{\psi} + J_{\psi}{}^{\gamma\Upsilon}\right) = 0.$$

$$\nabla_{\gamma}\nabla_{\Upsilon}\left(\sqrt{-g}\mathcal{F}_{\mathcal{Q}}H^{\gamma\Upsilon\ \Upsilon}_{\ \ \psi}\right) = 0.$$

$$\Gamma^{\psi}_{\gamma\Upsilon} = \left(\frac{\partial x^{a}}{\partial\eta^{\sigma}}\right)\partial_{\gamma}\partial\Upsilon\eta^{\sigma}.$$

$$\mathcal{Q}_{\psi\gamma\Upsilon} = \partial_{\psi}g_{\gamma\Upsilon}.$$

$$ds^2 = -e^{\Upsilon(r)}dt^2 + e^{\sigma(r)}dr^2 + r^2dw^2.$$

$$\mathcal{Q}(r) = -\frac{2e^{-\sigma}}{r}\left(\Upsilon' + \frac{1}{r}\right),$$

$$\mathcal{T}_{\gamma\Upsilon} = (\rho + p_t)u_{\gamma}u_{\Upsilon} + p_t g_{\gamma\Upsilon} + (p_r - p_t)v_{\gamma}v_{\Upsilon},$$

$$\rho = -\frac{\mathcal{F}}{2} + \mathcal{F}_{\mathcal{Q}}\left[\mathcal{Q} + \frac{1}{r^2} + \frac{e^{-\sigma}}{r}(\Upsilon' + \sigma')\right],$$

$$p_r = \frac{\mathcal{F}}{2} - \mathcal{F}_{\mathcal{Q}}\left[\mathcal{Q} + \frac{1}{r^2}\right],$$

$$p_t = \frac{\mathcal{F}}{2} - \mathcal{F}_{\mathcal{Q}}\frac{\mathcal{Q}}{2} - \mathcal{F}_{\mathcal{Q}}e^{-\sigma}\frac{\mathcal{Q}}{2}$$

$$-\mathcal{F}_{\mathcal{Q}}e^{-\sigma}\left[\frac{\Upsilon''}{2} + \left(\frac{\Upsilon'}{4} + \frac{1}{2r}\right)(\Upsilon' - \sigma')\right],$$

$$0 = \frac{\cot\,\theta}{2}\mathcal{Q}'\mathcal{F}_{\mathcal{Q}\mathcal{Q}}.$$

$$\frac{\cot\,\theta}{2}\mathcal{Q}'\mathcal{F}_{\mathcal{Q}\mathcal{Q}} = 0.$$

$$\mathcal{F}_{\mathcal{Q}\mathcal{Q}} = 0 \Rightarrow \mathcal{F}(\mathcal{Q}) = \delta_1\mathcal{Q} + \delta_2.$$

$$\rho = \frac{1}{16\pi r^2}[2\delta_1 + 2e^{-\sigma}\delta_1(r\sigma' - 1) - r^2\delta_2],$$

$$p_r = \frac{1}{16\pi r^2}[-2\delta_1 + 2e^{-\sigma}\delta_1(r\Upsilon' + 1) + r^2\delta_2],$$

$$p_t = \frac{e^{-\sigma}}{32\pi r}[2e^{\sigma}r\delta_2 + \delta_1(2 + r\Upsilon')(\Upsilon' - \sigma') + 2r\delta_1\Upsilon'']$$

$$\Delta(r) = \frac{\delta_1}{8\pi}\left[e^{-\sigma}\left(\frac{\Upsilon''}{2} - \frac{\sigma'\Upsilon'}{4} + \frac{(\Upsilon')^2}{4} - \frac{\Upsilon' + \sigma'}{2r} - \frac{1}{r^2}\right) + \frac{1}{r^2}\right].$$

$$R_{1414} = \frac{R_{1212}R_{3434} - R_{1224}R_{1334}}{R_{2323}},$$

$$R_{2323} = \frac{\sin^2\,\theta(e^{\sigma} - 1)r^2}{e^{-\sigma}}, R_{1212} = \frac{\sigma' r}{2}$$

$$R_{2424} = -\frac{\Upsilon' r e^{\Upsilon - \sigma}}{4}, R_{1334} = R_{1224}\sin^2\,\theta = 0$$

$$R_{1414} = -\frac{1}{4e^{\Upsilon}}[2\Upsilon'' + (\Upsilon')^2 - \sigma'\Upsilon'], R_{3434} = \sin^2\,\theta R_{2424}$$

$$\frac{2\Upsilon'' + (\Upsilon')^2}{\Upsilon'} = \frac{\sigma' e^{\sigma}}{(e^{\sigma} - 1)}.$$

$$e^{\Upsilon} = \left[A_1 + B_1\int\,\sqrt{(e^{\sigma(r)} - 1)}dr\right]^2$$

$$e^{\sigma} = b^2r^2\mathrm{csch}(fr^2 + h)\mathrm{csch}(fr^2 + h) + 1,$$

$$e^{\Upsilon} = \left[A_1 + \frac{B_1 b}{2f}\log\left(\frac{e^{(fr^2+h)} - 1}{e^{(fr^2+h)} + 1}\right)\right]^2.$$

$$8\pi\rho = -\frac{b^4r^2(\delta_2 r^2 - 2\delta_1)\text{csch}^4(fr^2+h) + \delta_2 + \phi_1}{2(b^2r^2\text{csch}^2(fr^2+h)+1)^2}$$

$$8\pi p_r = \frac{8B_1 b\delta_1 fr^2 \log(e) e^{fr^2+h} + \delta_1\phi_2}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)} + \frac{\delta_2}{2} - \frac{\delta_1}{r^2},$$

$$8\pi p_t = \frac{\delta_2\phi_5(b^4r^4\text{csch}^4(fr^2+h)+1) + 2b^2\phi_6 + \phi_4}{2\phi_7\left(e^{fr^2+h}-1\right)^2\left(e^{fr^2+h}+1\right)^2}$$

$$8\pi\Delta = \frac{\delta_2\phi_5(b^4r^4\text{csch}^4(fr^2+h)+1) + 2b^2\phi_6 + \phi_4}{2\phi_7\left(e^{fr^2+u}-1\right)^2\left(e^{fr^2+h}+1\right)^2} - \frac{8B_1 b\delta_1 fr^2 \log(e) e^{fr^2+h} + \delta_1\phi_2}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)} - \frac{\delta_2}{2} + \frac{\delta_1}{r^2}$$

Compact particle star	$M(M_0)$	R(nm)	δ_1	$b(nm^{-1})$	$f(\text{nm}^{-1})$	h	$\frac{M}{R}$	A_1	B_1
Cen X-3	1.48	11.6	0.2	0.107	-0.0011	1.39	0.128291379	0.0269800	0.0263286
Cen X-3	1.49	11.6	0.4	0.107	-0.0011	1.39	0.128481897	0.0235791	0.0264463
Cen X-3	1.49	11.6	0.6	0.107	-0.0011	1.39	0.128544828	0.0224455	0.0264855
Cen X-3	1.49	11.6	0.8	0.107	-0.0011	1.39	0.128576724	0.0218787	0.0265051
Cen X-3	1.49	11.6	1.0	0.107	-0.0011	1.39	0.128671552	0.0201782	0.026564

Expression	Remarks
ρ, p_r, p_t	> 0, acceptable

$\underline{p}_{rc}$	≤ 0
	> 0, acceptable
$\frac{d\rho}{dr}, \frac{dp_r}{dr}, \frac{dp_t}{dr}$	< 0, acceptable
F_a, F_h, F_g	balanced
$\rho \pm p_r$	> 0, acceptable
$\rho \pm p_t$	> 0, acceptable
$\rho + p_r$ $+ 2p_t$	> 0, acceptable
$m(r)$	> 0, acceptable
$u(r)$	$0 < u(r) < \frac{8}{9}$, acceptable
Z_S	$0 < Z_S < 5$, acceptable
ω_r, ω_t	$0 < \omega_r, \omega_t < 1$, acceptable
V_r^2, V_t^2	$0 < V_r^2, V_t^2 < 1$, acceptable
$V_t^2 - V_r^2$	$-1 < \lvert V_t^2 - V_r^2 \rvert < 1$, acceptable
Γ	$> \frac{4}{3}$, acceptable

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\left(1-\frac{2M}{r}-\frac{\Lambda}{3}r^2\right)=e^{\Upsilon(R)}$$
$$\left(1-\frac{2M}{r}-\frac{\Lambda}{3}r^2\right)=e^{-\sigma(R)}$$
$$p_r(R)=0$$

$$\left(\frac{\partial w_{ss}^{-}}{\partial r}\right)_{r=R}=\left(\frac{\partial w_{ss}^{+}}{\partial r}\right)_{r=R}$$

$$w_{ss}^{-}=e^{\Upsilon}, w_{ss}^{+}=\left(1-\frac{2M}{r}-\frac{\Lambda}{3}r^2\right).$$

$$\Upsilon'(R)=\frac{6M-2\Lambda R^3}{3R^2-6MR-\Lambda R^4}$$

$$\Upsilon'(R)=\frac{6M-2\Lambda R^3}{3R^2-6MR-\Lambda R^4}$$

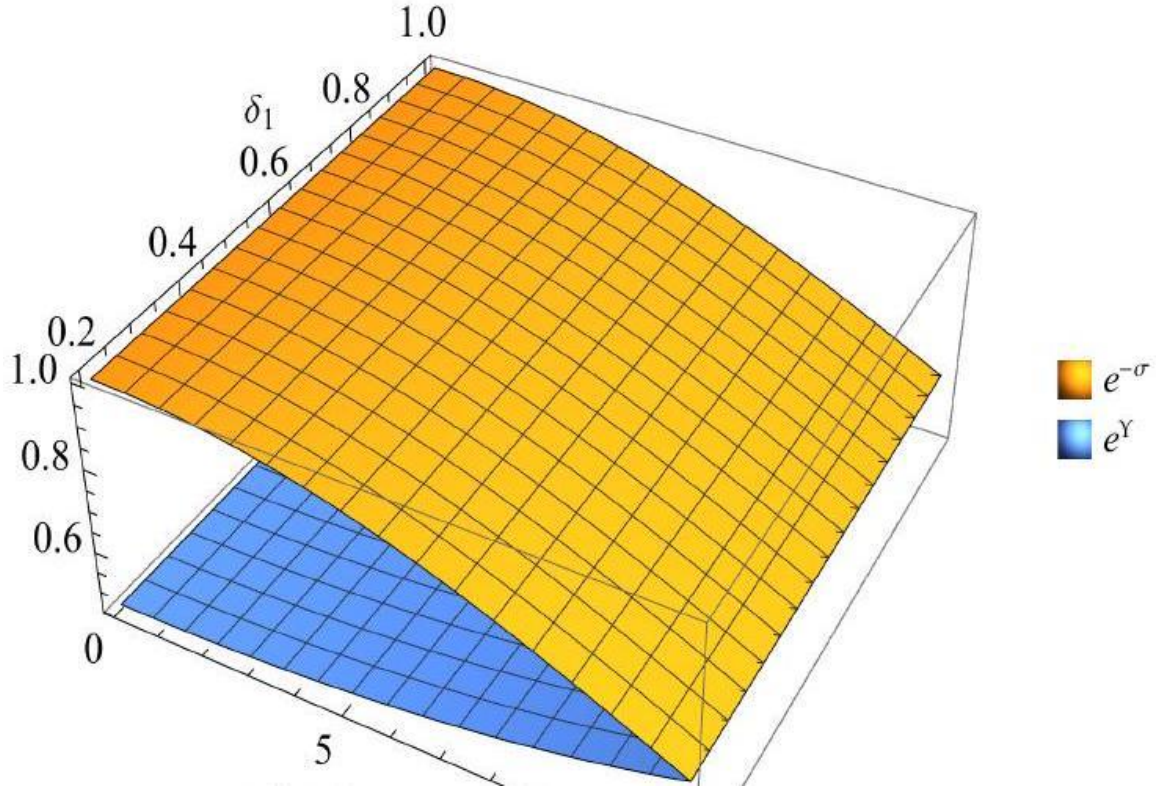


Figura 35. Deformación extrema del espacio – tiempo cuántico por interacción de una partícula blanca o estrella.

$$A_1=\frac{B_1 b\left(-\phi_{10}(\delta_2+\phi_{11})-16\delta_1 f\log(e)e^{fR^2+h}\right)}{2f\phi_{12}\left(e^{2(fR^2+h)}-1\right)}$$
$$B_1=-\frac{\phi_{13}e^{-fR^2-h}\left(e^{2(fR^2+h)}-1\right)}{8b\delta_1\log(e)}$$

$$\omega_r = \left(\frac{8B_1 b\delta_1 f r^2 \log(e) e^{fr^2+h} + \delta_1\phi_2}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)} + A\right)B$$

$$\omega_t = \left(\frac{\delta_2\phi_5(b^4 r^4 \operatorname{csch}^4(fr^2+h)+1)+C}{2\phi_7\left(e^{fr^2+h}-1\right)^2\left(e^{fr^2+h}+1\right)^2}\right)B$$

$$\begin{aligned} NEC&: \rho \geq 0, \\ WEC&: \rho + p_r \geq 0, \rho + p_t \geq 0, \\ DEC&: \rho - p_r \geq 0, \rho - p_t \geq 0, \\ SEC&: \rho + p_r + 2p_t \geq 0, \\ TEC&: \rho - p_r - 2p_t \geq 0. \end{aligned}$$

$$\rho + p_r, \rho + p_t, \rho - p_r, \rho - p_t, \rho + p_r + 2p_t, \rho - p_r - 2p_t$$

$$\Gamma < \frac{4}{3} - \left[\frac{4}{3}\frac{p_r - p_t}{|p_r'|r}\right]_{\max}$$

$$\frac{dp_r}{dr} + \frac{\Upsilon'(\rho + p_r)}{2} - \frac{2(p_t - p_r)}{r} = 0$$

$$F_g + F_h + F_a = 0; F_g = -\frac{\Upsilon'(\rho+p_r)}{2}, F_h = -\frac{dp_r}{dr}, F_a = \frac{2(p_t-p_r)}{r}$$

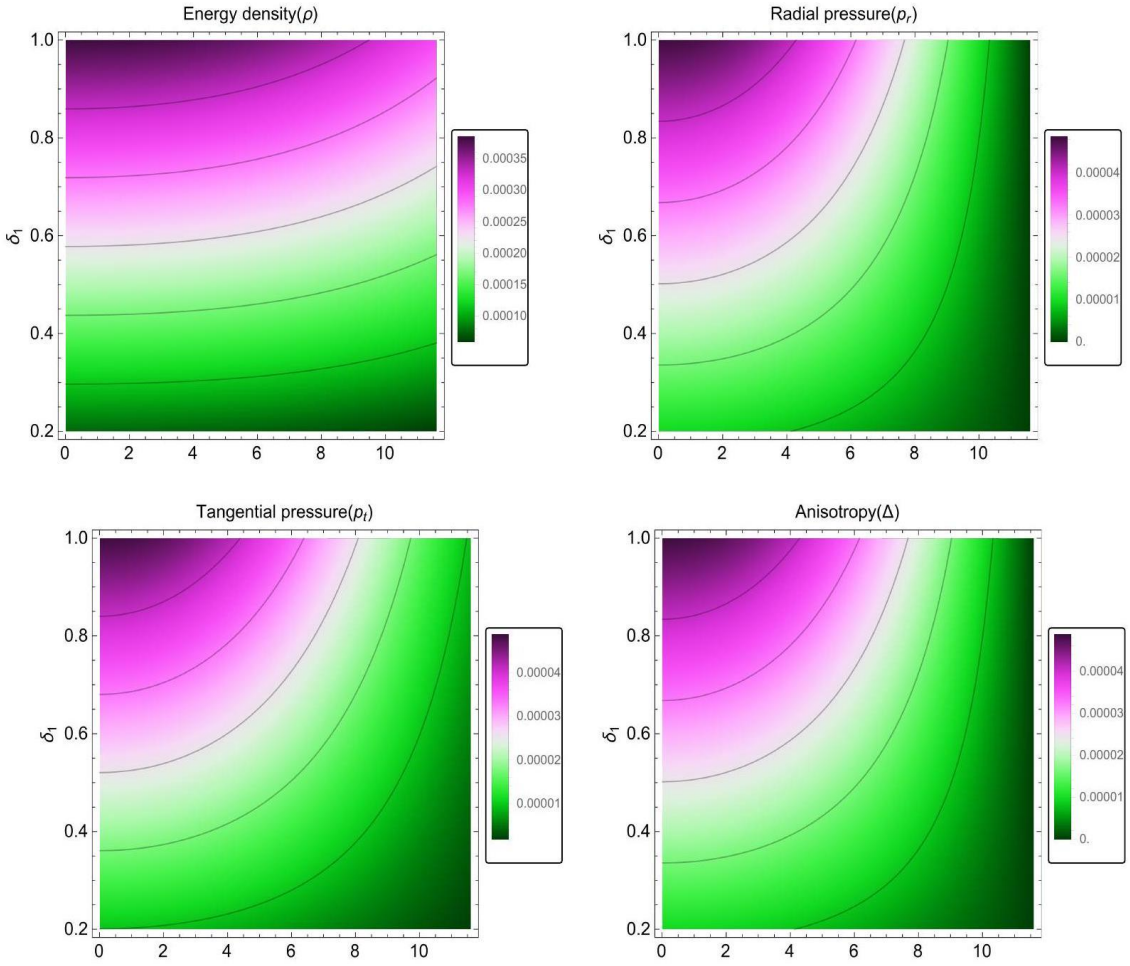
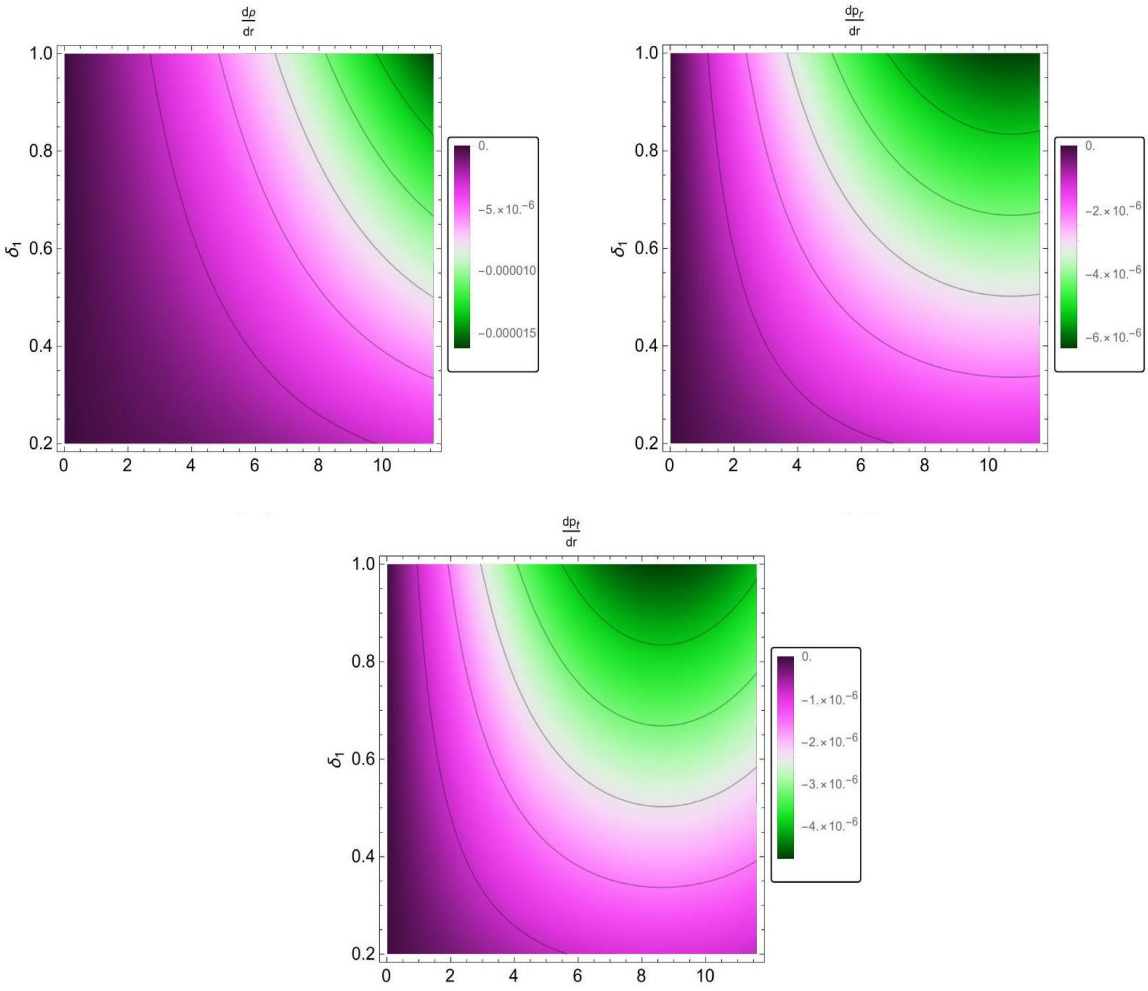


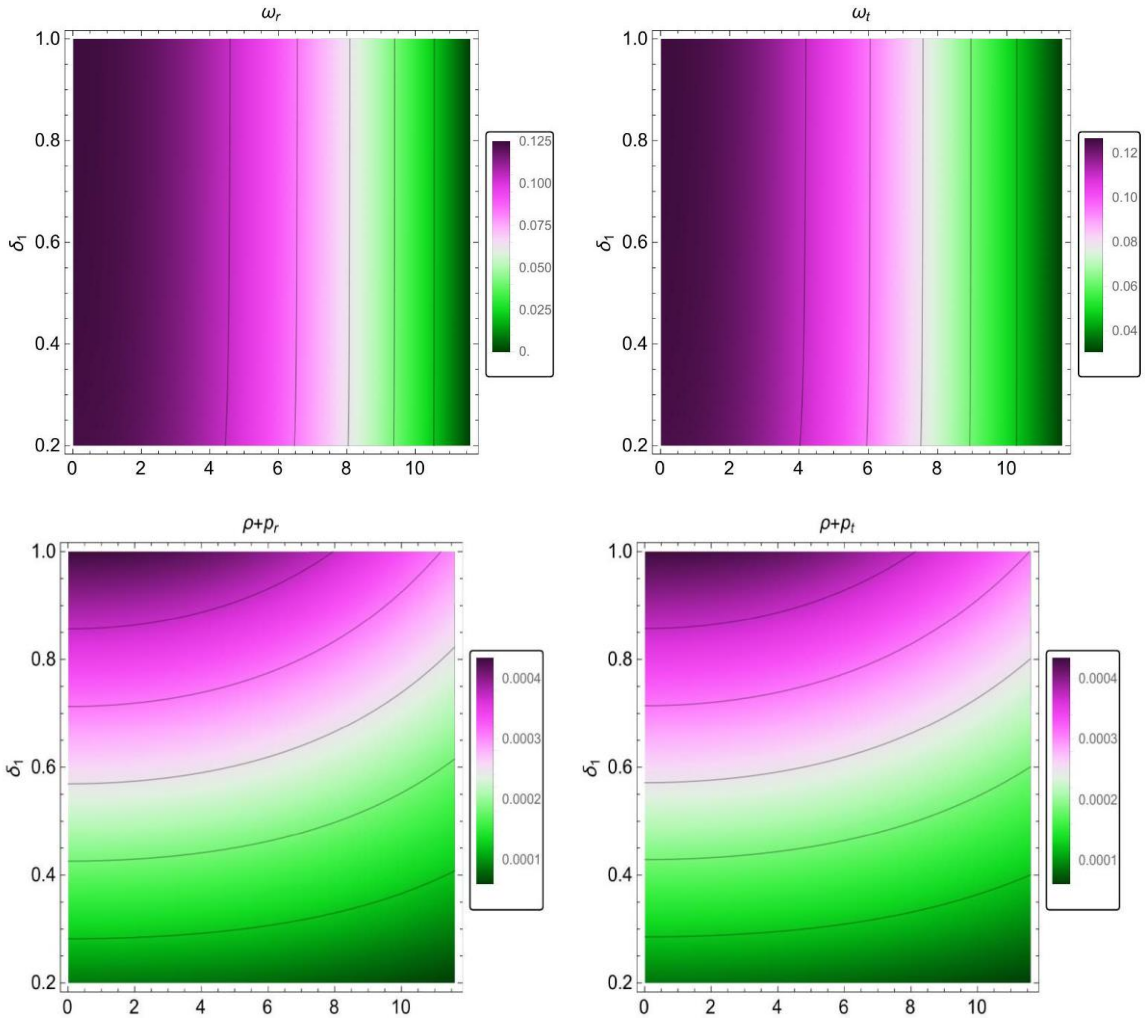
Figura 36. Densidad de energía, presión radial, presión tangencial y asintropía de una partícula blanca o estrella.

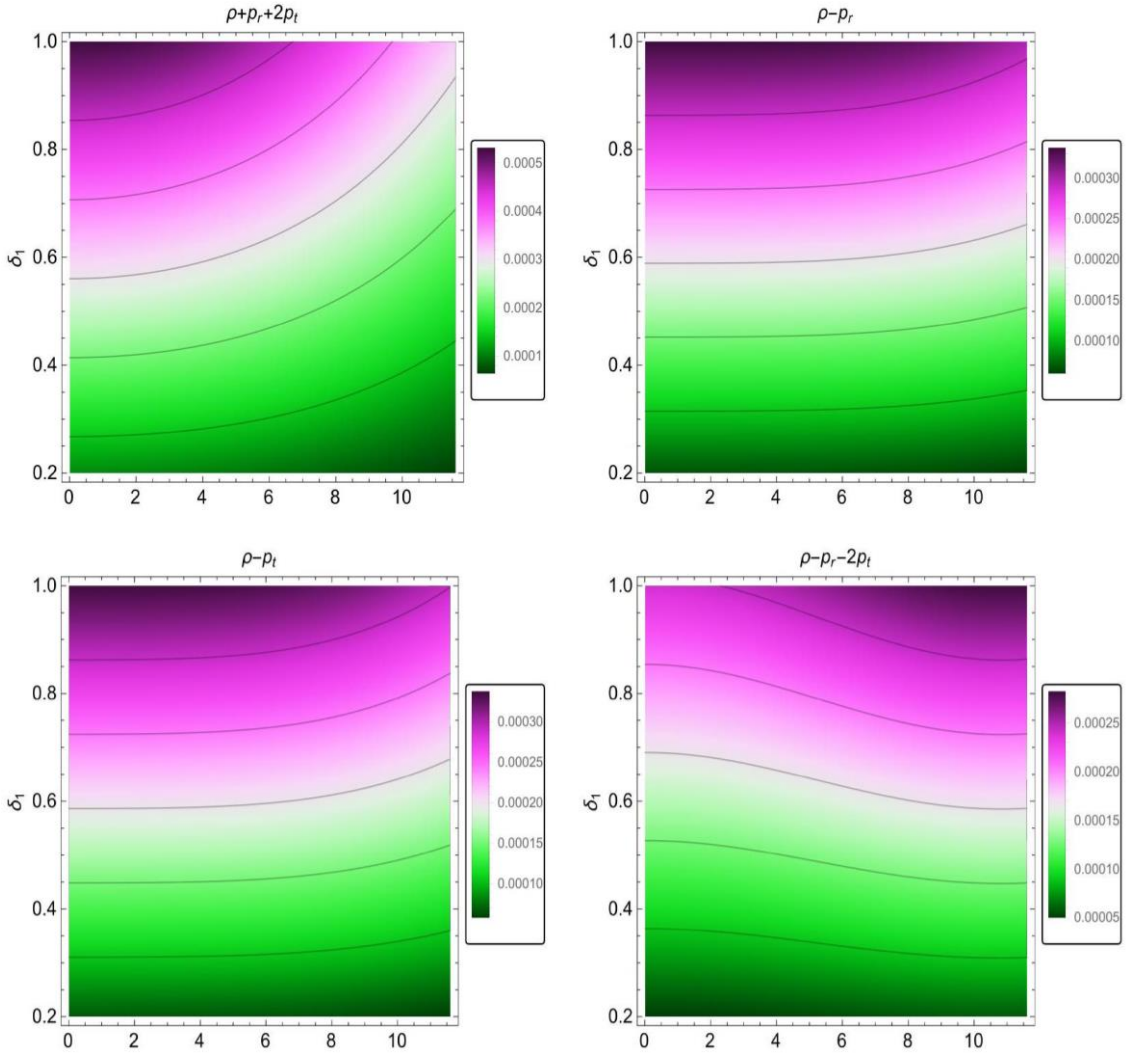
$$V_r^2 = \frac{dp_r}{d\rho}, V_t^2 = \frac{dp_t}{d\rho}$$

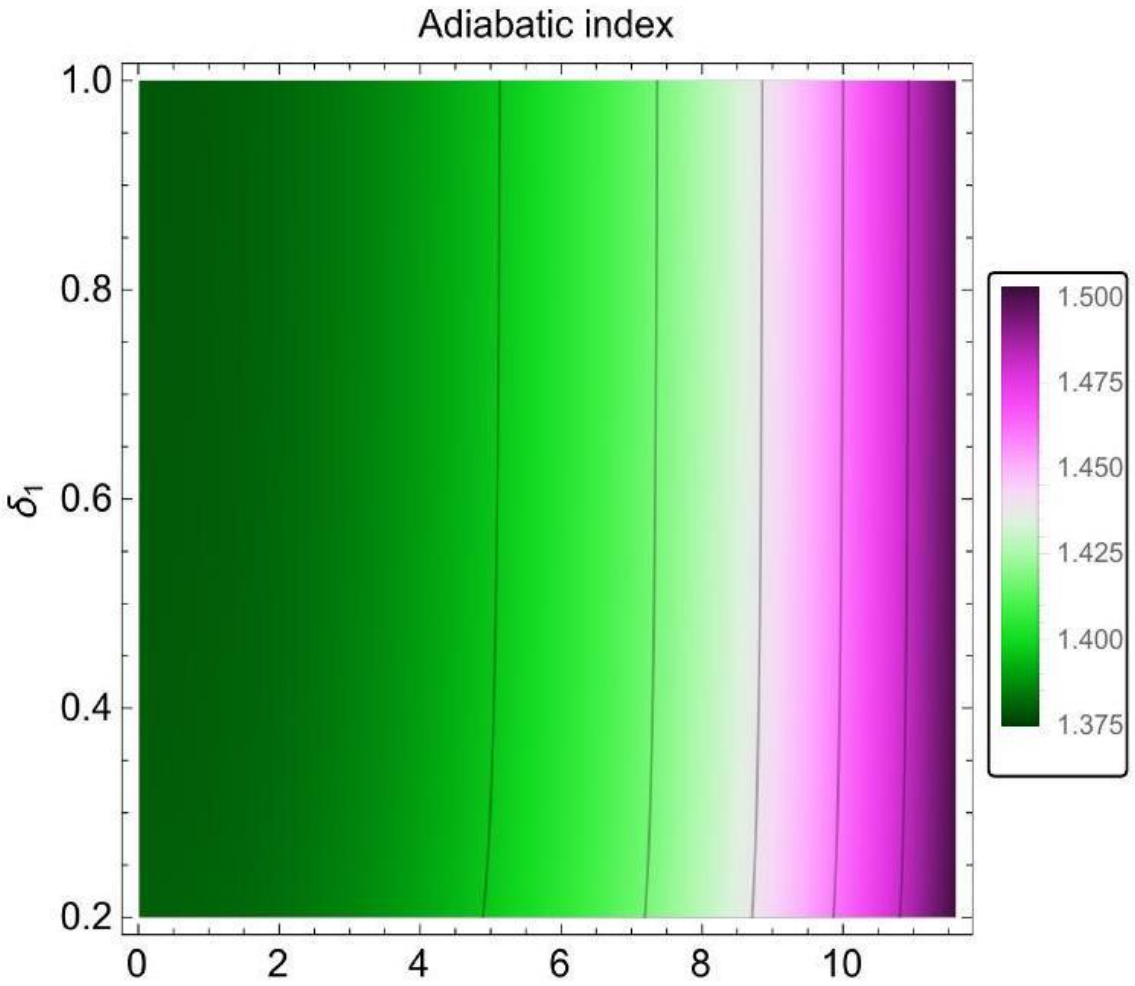
$$V_r^2 = \frac{dp_r}{d\rho} V_t^2 = \frac{dp_r}{d\rho}$$

$$\rho''(r) \leq 0$$




ω_r
ω_t
$\rho+p_r$
$\rho+p_t$
δ_1
0.125
0.100
0.075
0.050
0.025
0.
0.12
0.10
0.08
0.06
0.04
0.0004
0.0003
0.0002
0.0001


$\rho+p_r+2p_t$
$\rho-p_r$
$\rho-p_t$
$\rho-p_r-2p_t$
δ_1
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10
0.0005
0.0004
0.0003
0.0002
0.0001
0.00030
0.00025
0.00020
0.00015
0.00010
0.00005



Figuras. 37, 38, 39, 40, 41, 42 y 43. Gradiente de la densidad de energía $\left(\frac{d\rho}{dr}\right)$, gradiente de la presión radial $\left(\frac{dp_r}{dr}\right)$, gradiente de la presión tangencial $\left(\frac{dp_t}{dr}\right)$ y la indexación adiabática $\left(\frac{dp_{index\ adiabatic}}{dr}\right)$ de una partícula blanca o estrella.

$$M(\rho_c) = \frac{R}{2}\left(-\frac{D}{E(2\rho_c + \delta_2) + D} - \frac{\delta_2 R^2}{6\delta_1} + 1\right).$$

$$\frac{dM}{d\rho_c} = \frac{4EDR}{(4E\rho_c + 2E\delta_2 + 12\delta_1 \operatorname{csch}^2(h))^2}$$

$$Z_G = \frac{1}{\sqrt{|e^{\Upsilon(r)}|}} - 1 = \frac{1}{A_1 + \frac{B_1 b}{2f}\log\left(\frac{e^{(fr^2+h)} - 1}{e^{(fr^2+h)} + 1}\right)} - 1$$

$$Z_S = \frac{1}{\sqrt{1 - 2u(r)}} - 1$$

$$m(r) = \frac{\kappa}{2}\int_0^r \rho r^2 dr$$

$$u(r) = \frac{\kappa}{2r}\int_0^r \rho r^2 dr$$

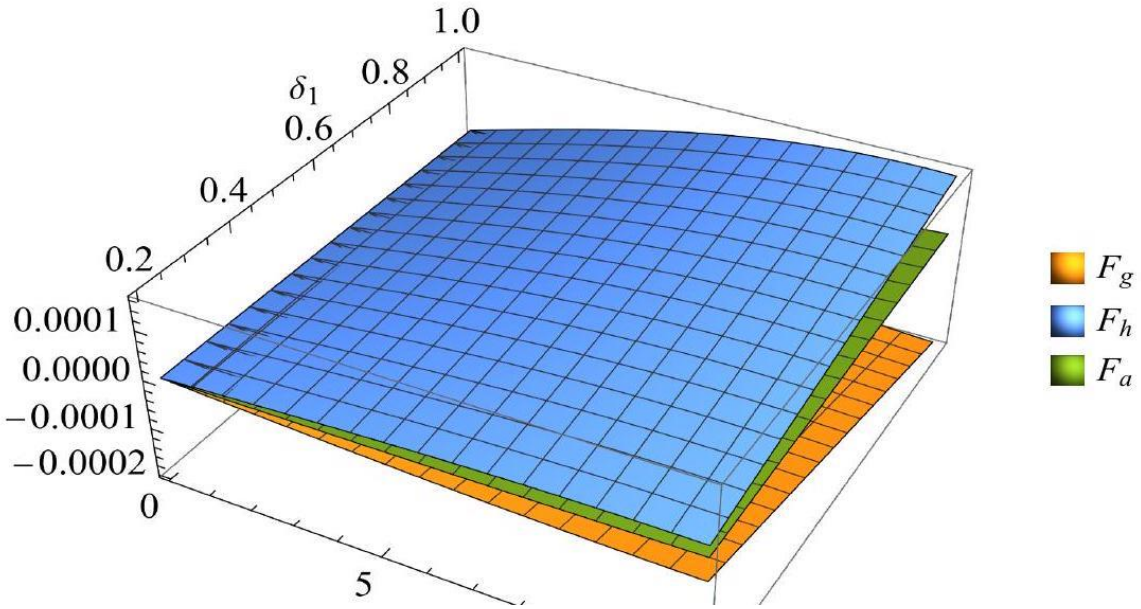
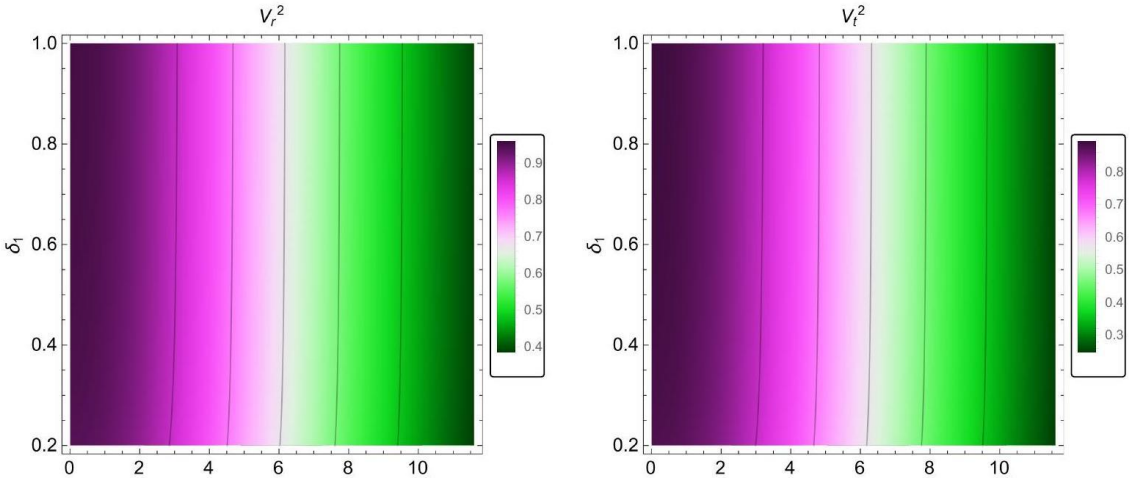


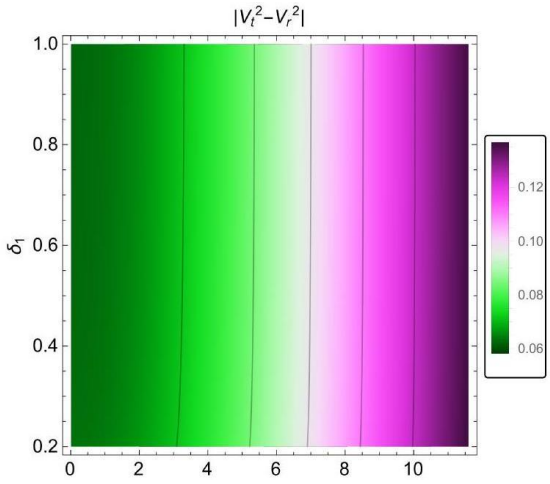
Figura. 44. Fuerza gravitacional (F_g), fuerza hidrostática (F_h) y fuerza anisotrópica (F_a) de una partícula blanca o estrella.

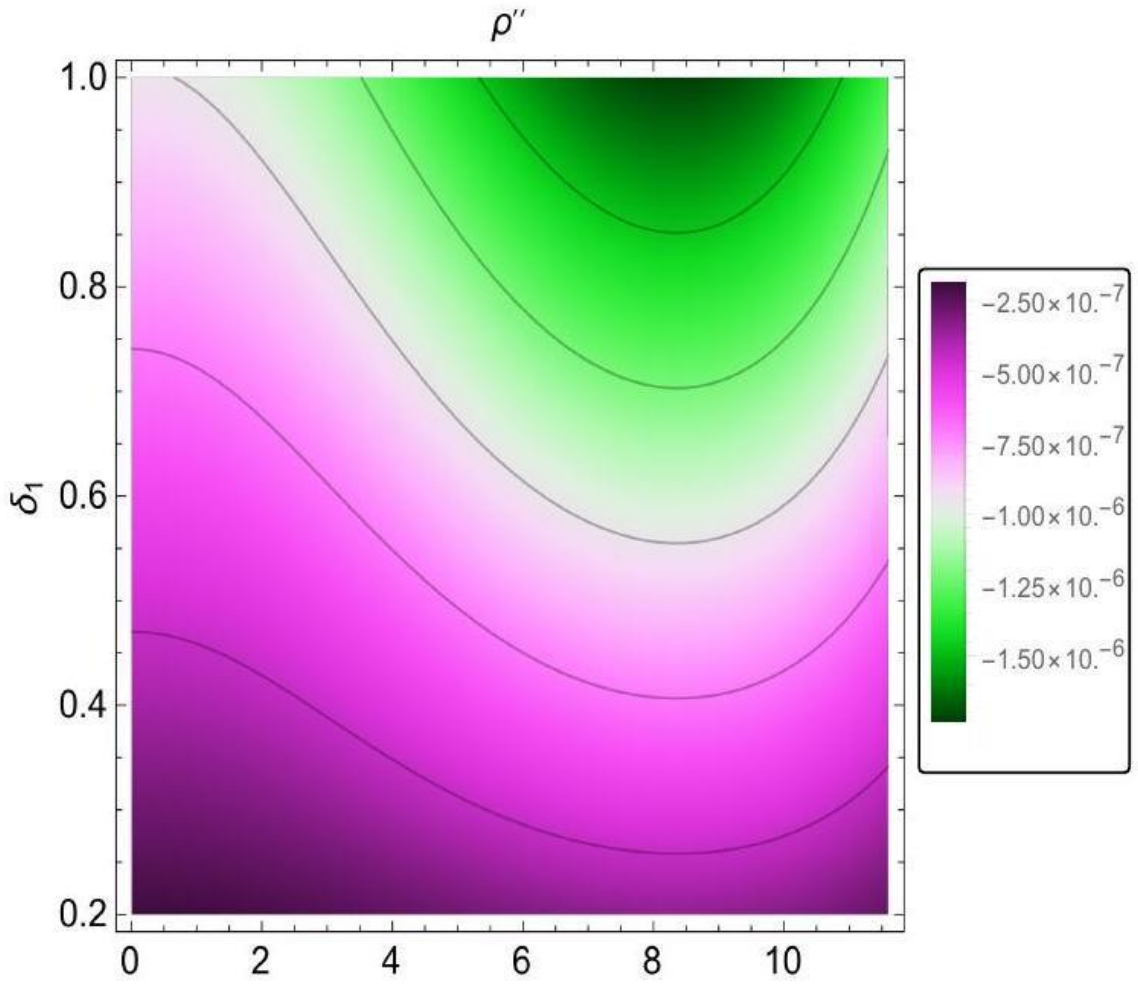
Compact particle star	δ_1	density value at r = 0(gm/ nm^3)	density value at r = R(gm/nm^3)	pressure value at r = 0(dyne /nm^2)
Cen X-3	0.2	0.041349×10^{14}	0.03222×10^{14}	0.043506×10^{34}
Cen X-3	0.4	0.082698×10^{14}	0.06444×10^{14}	0.091846×10^{34}
Cen X-3	0.6	0.124047×10^{14}	0.09666×10^{14}	0.140186×10^{34}
Cen X-3	0.8	0.165396×10^{14}	0.12888×10^{14}	0.188526×10^{34}
Cen X-3	1.0	0.206745×10^{14}	0.16110×10^{14}	0.236866×10^{34}

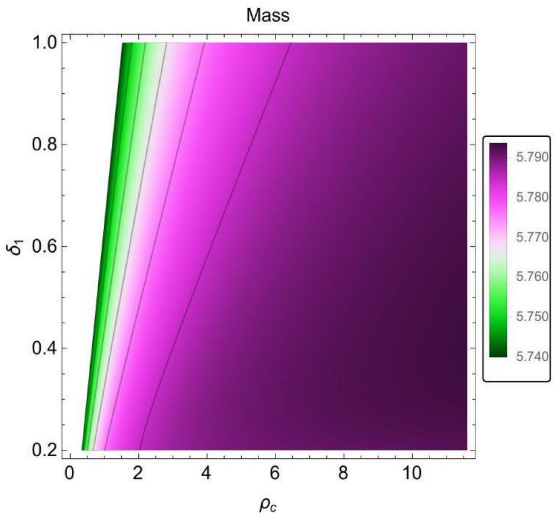
$$m(r) = -\frac{2\pi r^3\left(2b^2(\delta_2 r^2 - 6\delta_1) + \delta_2(\phi_{14} - 1)\right)}{3(2b^2r^2 + \phi_{14} - 1)}$$

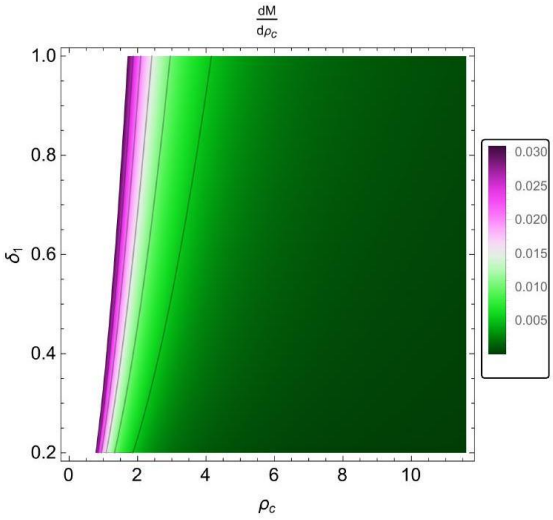
$$u(r) = -\frac{2\pi r^2\left(2b^2(\delta_2 r^2 - 6\delta_1) + \delta_2(\phi_{14} - 1)\right)}{3(2b^2r^2 + \phi_{14} - 1)}$$

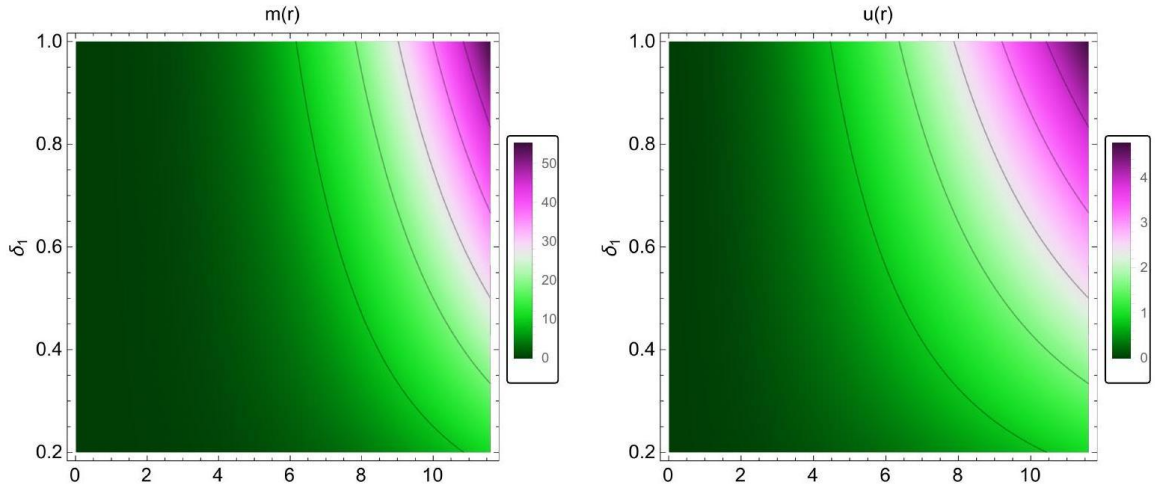





ρ''
1.0
0.8
0.6
0.4
0.2
δ1
0
2
4
6
8
10
−2.50×10.−7
−5.00×10.−7
−7.50×10.−7
−1.00×10.−6
−1.25×10.−6
−1.50×10.−6


Mass
1.0
0.8
0.6
0.4
0.2
δ1
0
2
4
6
8
10
ρc
5.790
5.780
5.770
5.760
5.750
5.740


dM/dρc
1.0
0.8
0.6
0.4
0.2
δ1
0
2
4
6
8
10
ρc
0.030
0.025
0.020
0.015
0.010
0.005



Figuras 45, 46, 47 y 48. Velocidad radial (V_r^2), velocidad tangencial (V_t^2) y velocidad resultante ($|V_t^2 - V_r^2|$) y comportamiento de la masa $\frac{dM}{d\rho_c}$ de una partícula estrella o blanca.

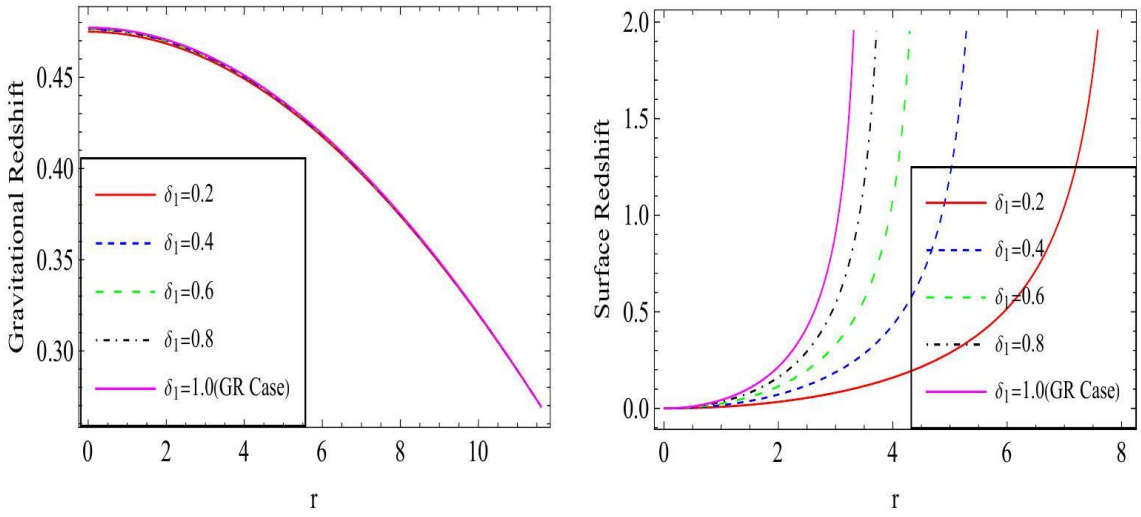


Figura 49. Gravitational Redshift (Z_G) y Surface Redshift (Z_S) de una partícula blanca o estrella.

$$\phi_1 = 2b^2\text{csch}^2(fr^2+h)(-3\delta_1 + 4\delta_1 fr^2\coth(fr^2+h) + \delta_2 r^2)$$

$$\phi_2 = \left(e^{2(fr^2+h)}-1\right)\left(2A_1 f + B_1 b\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right)\right)$$

$$\phi_3 = (b^2r^2\text{csch}^2(fr^2+h)+1)\left(2A_1 f + B_1 b\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right)\right)$$

$$\phi_4 = 16B_1 b\delta_1 f\log(e)e^{fr^2+h}\left(e^{2(fr^2+h)}-1\right) - 16B_1 b\delta_1 f^2 r^2\log^2(e)e^{fr^2+h}\left(e^{2(fr^2+h)}+1\right)$$

$$\phi_5 = \left(e^{2(fr^2+h)}-1\right)^2\left(2A_1 f + B_1 b\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right)\right)$$

$$\phi_6 = \text{csch}^2(fr^2+h)\left(4B_1 b\delta_1 fr^2\log(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h} - 8B_1\phi_8 + 2\delta_1\phi_9 - \phi_5(\delta_1 - \delta_2 r^2)\right)$$

$$\phi_7 = (b^2r^2\text{csch}^2(fr^2+h)+1)^2\left(2A_1 f + B_1 b\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right)\right)$$

$$\phi_8 = b\delta_1 f^2 r^4\log^2(e)e^{fr^2+h}\left(e^{2(fr^2+h)}+1\right)$$

$$\phi_9 = 2\delta_1 fr^2\left(e^{2(fr^2+h)}-1\right)\coth(fr^2+h)\left(4B_1 bfr^2\log(e)e^{fr^2+h} + \phi_2\right)$$

$$\phi_{10} = \left(e^{2(fR^2+h)}-1\right)\log\left(\frac{e^{fR^2+h}-1}{e^{fR^2+h}+1}\right)$$

$$\phi_{11} = b^2(2\delta_1 - \delta_2 R^2)\text{csch}^2(fR^2+h)$$

$$\phi_{12} = \delta_1 - b^2(2\delta_1 - \delta_2 R^2)\text{csch}^2(fR^2+h)$$

$$\phi_{13} = \sqrt{\frac{1}{b^2R^2\text{csch}^2(fR^2+h)+1}}\left(\delta_2 - b^2(2\delta_1 - \delta_2 R^2)\text{csch}^2(fR^2+h)\right)$$

$$\phi_{14} = \cosh\left(2(fr^2+h)\right)$$

$$A = \frac{\delta_2}{2} - \frac{\delta_1}{r^2}$$

$$B = -\frac{2(b^2r^2\text{csch}^2(fr^2+h)+1)^2}{b^4r^2(\delta_2 r^2 - 2\delta_1)\text{csch}^4(fr^2+h) + \delta_2 + \phi_1}$$

$$C = 2b^2\phi_6 + \phi_4$$

$$D = 6\delta_1\text{csch}^2(h)$$

$$E = R^2\text{csch}(fR^2+h)\text{csch}(fR^2+h)$$

$$\phi_{15} = 1 + b^2r^2\text{csch}(fr^2+h)^2$$

$$\phi_{16} = aB_1\delta_1\left(\frac{2fr\log(e)e^{fr^2+h}}{e^{fr^2+h}+1} - \frac{2fr\log(e)e^{fr^2+h}\left(e^{fr^2+h}-1\right)}{\left(e^{fr^2+h}+1\right)^2}\right)\left(8bB_1 fr^2\log(e)e^{fr^2+h} + \phi_2\right)$$

$$\phi_{18} = \frac{2\delta_1 f\log(e)e^{fr^2+h}\left(8bB_1 fr^2\log(e)e^{fr^2+h} + \phi_2\right)}{r\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)^2}$$

$$\phi_{19} = \delta_1\left(2b^2 fr\text{csch}^2(fr^2+h) - 4b^2 fr^3\coth(fr^2+h)\text{csch}^2(fr^2+h)\right)\left(8bB_1 fr^2\log(e)e^{fr^2+h} + \phi_2\right)$$

$$\phi_{20} = A - \frac{b^4r^2(\delta_2 r^2 - 2\delta_1)\text{csch}^4(fr^2+h) + \delta_2 + \phi_1}{2\phi_{15}^2} + \frac{\delta 1\left(8bB_1 fr^2\log(e)e^{fr^2+h} + \phi_2\right)}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}$$

$$\phi_{21} = A + \frac{\delta_1\left(8bB_1 fr^2\log(e)e^{fr^2+h} + \phi_2\right)}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}$$

$$\phi_{22} = \frac{2fr\log(e)e^{fr^2+h}}{e^{fr^2+h}+1} - \frac{2fr\log(e)e^{fr^2+h}\left(e^{fr^2+h}-1\right)}{\left(e^{fr^2+h}+1\right)^2}$$

$$\phi_{23} = \frac{bB_1\phi_{22}\left(e^{fr^2+h}+1\right)\left(e^{2(fr^2+h)}-1\right)}{e^{fr^2+h}-1}$$

$$\phi_{24} = \frac{2\delta_1\left(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2\right)}{r^3\phi_3\left(e^{br^2+h}+1\right)\left(e^{fr^2+h}-1\right)}$$

$$\phi_{25} = \frac{4fr\log(e)e^{2(fr^2+h)}\left(2A_1f+bB_1\log\left(\frac{e^{br^2+c}-1}{e^{b^2+c}+1}\right)\right)}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}$$

$$\phi_{26} = b^4r^2(\delta_2r^2-2\delta_1)\text{csch}^4(fr^2+h)+2b^2\text{csch}^2(fr^2+h)(-3\delta_1+4\delta_1fr^2\coth(fr^2+h)+\delta_2r^2)+\delta_2$$

$$\phi_{27} = 2b^4r(\delta_2r^2-2\delta_1)\text{csch}^4(fr^2+h)+2b^4\delta_2r^3\text{csch}^4(fr^2+h)$$
$$-8b^4fr^3(\delta_2r^2-2\delta_1)\coth(fr^2+h)\text{csch}^4(fr^2+h)$$
$$+2b^2\text{csch}^2(fr^2+h)(-8\delta_1f^2r^3\text{csch}^2(fr^2+h)+8\delta_1fr\coth(fr^2+h)+2\delta_2r)$$
$$-8b^2fr\coth(fr^2+h)\text{csch}^2(fr^2+h)(-3\delta_1+4\delta_1fr^2\coth(fr^2+h)+\delta_2r^2)$$

$$\phi_{28} = 2b^2r\text{csch}^2(fr^2+h)-4b^2fr^3\coth(fr^2+h)\text{csch}^2(fr^2+h)$$

$$\phi_{29} = -\frac{\phi_{19}}{r^2\phi_7\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}$$

$$\phi_{30} = \frac{2\delta_1}{r^3}-\frac{\phi_{16}}{r^2\phi_5\phi_{15}}-2\phi_{18}-\phi_{24}-\phi_{29}$$

$$\Gamma = \frac{\phi_{20}\phi_{28}\phi_{26}\left(\delta_1\left(16bB_1f^2r^3\log^2(e)e^{fr^2+h}+16bB_1fr\log(e)e^{fr^2+h}+\phi_{23}+\phi_{25}\right)+\phi_{30}\right)}{\phi_{21}\phi_{15}^3}-\frac{\phi_{27}}{2\phi_{15}^2}$$

$$\rho+p_r = A-\frac{b^4r^2(\delta_2r^2-2\delta_1)\text{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+\frac{\delta_1\left(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2\right)}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}$$

$$W_1 = b^4\delta_2r^4\phi_5\text{csch}^4(fr^2+h)-16bB_1\delta_1f^2r^2\log^2(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h}$$
$$+16bB_1\delta_1f\log(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h}+\delta_2\phi_5$$

$$W_2 = -8bB_1\delta_1f^2r^4\log^2(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h}$$
$$+2\delta_1fr^2\left(e^{2(fr^2+h)}-1\right)\coth(fr^2+h)\left(4bB_1fr^2\log(e)e^{fr^2+h}+\phi_2\right)$$
$$+4bB_1\delta_1fr^2\log(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h}-\phi_5(\delta_1-\delta_2r^2)$$

$$\rho+p_t = -\frac{b^4r^2(\delta_2r^2-2\delta_1)\text{csch}^4(br^2+c)+\delta_2+\phi_1}{2\phi_{15}^2}+\frac{2b^2W_2\text{csch}^2(fr^2+h)}{2\phi_7\left(e^{br^2+c}-1\right)^2\left(e^{br^2+c}+1\right)^2}+W_1$$

$$\rho+p_r+2p_t = -\frac{b^4r^2(\delta_2r^2-2\delta_1)\text{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+\frac{2b^2W_2\text{csch}^2(fr^2+h)}{\phi_7\left(e^{fr^2+h}-1\right)^2\left(e^{fr^2+h}+1\right)^2}$$
$$+\frac{\delta_1\left(8bf\ \text{B1}r^2\log(e)e^{fr^2+h}+\phi_2\right)}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}+A+W_1$$

$$\rho-p_r = -\frac{b^4r^2(\delta_2r^2-2\delta_1)\text{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+A-\frac{\delta_1\left(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2\right)}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}$$

$$\rho-p_t = -\frac{b^4r^2(\delta_2r^2-2\delta_1)\text{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+A+\frac{2b^2W_2\text{csch}^2(fr^2+h)}{\phi_7\left(e^{fr^2+h}-1\right)^2\left(e^{fr^2+h}+1\right)^2}$$
$$+\frac{\delta_1\left(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2\right)}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}+W_1$$

$$W_3 = b^4\delta_2r^4\phi_5\text{csch}^4(fr^2+h)-16bB_1\delta_1f^2r^2\log^2(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h}$$
$$-16bB_1\delta_1f\log(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h}+\delta_2\phi_5$$

$$\rho-p_r-2p_t = A-\frac{b^4r^2(\delta_2r^2-2\delta_1)\text{csch}^4(fr^2+h)+\delta_2+\phi_1}{2\phi_{15}^2}+b^4\delta_2r^4\phi_5\text{csch}^4(fr^2+h)$$
$$+\frac{2b^2W_2\text{csch}^2(fr^2+h)}{\phi_7\left(e^{fr^2+h}-1\right)^2\left(e^{fr^2+h}+1\right)^2}-\frac{\delta1\left(8bB_1fr^2\log(e)e^{fr^2+h}+\phi_2\right)}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}-W_3$$

$$Z_1 = A - \frac{b^4 r^2(\delta_2 r^2 - 2\delta_1)\text{csch}^4(fr^2+h) + \delta_2 + \phi_1}{2\phi_{15}^2} + \frac{\phi_{24}}{2}$$

$$Z_2 = \frac{2\delta_1 f \log(e) e^{fr^2+h}\left(8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2\right)}{r\phi_3\left(e^{fr^2+h}-1\right)^2\left(e^{fr^2+h}+1\right)}$$

$$Z_3 = 16bB_1 f^2 r^3 \log^2(e) e^{fr^2+h} + 16bB_1 fr \log(e) e^{fr^2+h}$$

$$Z_4 = \left(A_1 + \frac{bB_1 \log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right)}{2f}\right)$$

$$Z_5 = \frac{bB_1\phi_{22}\left(e^{fr^2+h}+1\right)\left(e^{2(fr^2+h)}-1\right)}{e^{fr^2+h}-1}$$

$$Z_6 = 2b^4\delta_2 r^3 \text{csch}^4(fr^2+h) - 8b^2 fr\coth(fr^2+h)\text{csch}^2(fr^2+h)(-3\delta_1 + 4\delta_1 fr^2\coth(fr^2+h) + \delta_2 r^2)$$

$$Z_7 = 2b^4 r(\delta_2 r^2 - 2\delta_1)\text{csch}^4(fr^2+h) - 8b^4 fr^3(\delta_2 r^2 - 2\delta_1)\coth(fr^2+h)\text{csch}^4(fr^2+h)$$
$$+2b^2\text{csch}^2(fr^2+h)(-8\delta_1 f^2 r^3\text{csch}^2(fr^2+h) + 8\delta_1 fr\coth(fr^2+h) + 2\delta_2 r)$$

$$Z_8 = -8b^2 f\coth(fr^2+h)\text{csch}^2(fr^2+h)(-3\delta_1 + 4\delta_1 fr^2\coth(fr^2+h) + \delta_2 r^2)$$

$$Z_9 = 32b^2 f^2 r^2\coth^2(fr^2+h)\text{csch}^2(fr^2+h)(-3\delta_1 + 4\delta_1 fr^2\coth(fr^2+h) + \delta_2 r^2)$$

$$Z_{10} = 10b^4\delta_1 r^2\text{csch}^4(fr^2+h) + 2b^4(\delta_2 r^2 - 2\delta_1)\text{csch}^4(fr^2+h) - 32b^4\delta_2 fr^4\coth(fr^2+h)\text{csch}^4(fr^2+h)$$

$$Z_{11} = 16b^4 f^2 r^4(\delta_2 r^2 - 2\delta_1)\text{csch}^6(fr^2+h) + 64b^4 f^2 r^4(\delta_2 r^2 - 2\delta_1)\coth^2(fr^2+h)\text{csch}^4(fr^2+h)$$
$$+40b^4 fr^2(\delta_2 r^2 - 2\delta_1)\coth(fr^2+h)\text{csch}^4(fr^2+h) + 16b^2 f^2 r^2\text{csch}^4(fr^2+h)(4b\delta_1 r^2\coth(fr^2+h) - 3\delta_1 + \delta_2 r^2)$$

$$Z_{12} = 16b^2 fr\coth(fr^2+h)\text{csch}^2(fr^2+h)(-8\delta_1 f^2 r^3\text{csch}^2(br^2+c) + 8\delta_1 fr\coth(fr^2+h) + 2\delta_1 r)$$

$$Z_{13} = 2b^2\text{csch}^2(fr^2+h)\left(2\delta_2 - 40\delta_2 f^2 r^2\text{csch}^2(fr^2+h) + 8\delta_1 f\coth(fr^2+f)\right)$$

$$Z_{14} = b^4 r^2(\delta_2 r^2 - 2\delta_1)\text{csch}^4(fr^2+h) + \delta_2 + \phi_1$$

$$Z_{15} = 8b^2 f^2 r^4\text{csch}^4(fr^2+h) + 16b^2 f^2 r^4\coth^2(fr^2+h)\text{csch}^2(fr^2+h) + 2b^2\text{csch}^2(fr^2+h) - 20b^2 fr^2\coth(fr^2+h)$$
$$\times\text{csch}^2(fr^2+h)$$

$$F_g = -\frac{bB_1 Z_1\phi_{22}\left(e^{fr^2+h}+1\right)}{2f\left(e^{fr^2+h}-1\right)Z_4}$$

$$F_h = \frac{aB_1\delta_1\phi_{22}\left(8bB_1 fr^2\log(e)e^{fr^2+h} + \phi_2\right)}{r^2\phi_2\phi_5}$$
$$+\frac{\delta_1\left(2b^2 r\text{csch}^2(fr^2+h) - 4b^2 br^3\coth(fr^2+h)\text{csch}^2(fr^2+h)\right)\left(8bB_1 fr^2\log(e)e^{fr^2+h} + \phi_2\right)}{r^2\phi_7\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)}$$
$$-\frac{\delta_1\left(4frZ_4\log(e)e^{2(fr^2+h)} + Z_3 + Z_5\right)}{r^2\phi_3\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)} - \frac{2\delta_1}{r^3} + Z_2 + \phi_{18} + \phi_{24}$$

$$F_a = \frac{2}{r}\left(\frac{2a^2 W_2\text{csch}^2(br^2+c)}{2\phi_7\left(e^{br^2+c}-1\right)^2\left(e^{br^2+c}+1\right)^2} + A - \frac{r\phi_{24}}{2} + W_1\right)$$

$$\rho'' = \frac{-Z_{15}(a^4 r^2(\delta 2 r^2 - 2\delta 1)\text{csch}^4(br^2+c) + \delta 2 + \phi_1)}{\phi_{15}^7}$$
$$-\frac{3Z_{14}\left(2a^2 r\text{csch}^2(br^2+c) - 4a^2 br^3\coth(br^2+c)\text{csch}^2(br^2+c)\right)^2}{\phi_{15}^4}$$
$$-\frac{2(Z_6+Z_7)\left(2a^2 r\text{csch}^2(br^2+c) - 4a^2 br^3\coth(br^2+c)\text{csch}^2(br^2+c)\right)}{\phi_{15}^7}$$
$$+\frac{Z_8 + Z_9 + Z_{10} - Z_{11} - Z_{12} + Z_{13} + Z_{14}}{2\phi_{15}^6}$$

$$T_1 = b^3\left(e^{2(fr^2+h)}-1\right)^2\text{csch}^4(fr^2+h)\left(2A_1 f + bB_1\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right)\right)^2$$

$$T_2 = 2A_1 f\left(e^{2(fr^2+h)}+1\right) + 2bB_1 e^{fr^2+h} + bB_1\left(e^{2(fr^2+h)}+1\right)\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right)$$

$$T_3 = 2bf\phi_2 \coth(fr^2+h)\operatorname{csch}^2(fr^2+h)\left(8bB_1 fr^2 \log(e) e^{fr^2+h} + \phi_2\right)$$

$$T_4 = fr^2 \log(e)\left(2A_1 f\left(e^{2(fr^2+h)}+1\right) + 2bB_1 e^{fr^2+h} + bB_1\left(e^{2(fr^2+h)}+1\right)\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right)\right) + \phi_2$$

$$T_5 = \operatorname{csch}^2(fr^2+h)(b^2r^2(b^2-4f^2r^2)\operatorname{csch}^2(fr^2+h) + 5b^2 - 4f^2r^2)$$
$$+8f^2r^2\coth^2(fr^2+h)(b^2r^2\operatorname{csch}^2(fr^2+h) - 1) + 2f\coth(fr^2+h)\left(5 - 3b^2r^2\operatorname{csch}^2(fr^2+h)\right)$$

$$D_1 = b^4\delta_2 r^4\phi_5 \operatorname{csch}^4(fr^2+h) - 16b^3B_1\delta_1 f^2 r^4 \log^2(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h}\operatorname{csch}^2(fr^2+h)$$
$$+4b^2\delta_1 fr^2\left(e^{2(fr^2+h)}-1\right)\coth(fr^2+h)\operatorname{csch}^2(fr^2+h)\left(4bB_1 fr^2\log(e)e^{fr^2+h}+\phi_2\right)$$
$$+8bb^2B_1\delta_1 fr^2\log(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h}\operatorname{csch}^2(fr^2+h) - 2b^2\phi_5(\delta_1-\delta_2 r^2)\operatorname{csch}^2(fr^2+h)$$
$$-16bB_1\delta_1 f^2r^2\log^2(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h} + 16bB_1\delta_1 f\log(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h} + \delta_2\phi_5$$

$$D_2 = 4b^5B_1\delta_2 fr^5\log(e)\left(-e^{fr^2+h}\right)\left(e^{fr^2+h}+1\right)e^{fr^2+h}\operatorname{csch}^4(fr^2+h) + 8b^4\delta_2 fr^5\phi_2\log(e)e^{2(fr^2+h)}\operatorname{csch}^4(fr^2+h)$$
$$-8b^4\delta_2 fr^5\phi_5\coth(fr^2+h)\operatorname{csch}^4(fr^2+h) + 4b^4\delta_2 r^3\phi_5\operatorname{csch}^4(fr^2+h)$$
$$-64b^3B_1\delta_1 f^3 rr^4\log^2(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h}\coth(fr^2+h)\operatorname{csch}^2(fr^2+h)$$
$$-32b^3B_1\delta_1 f^2 rr^2\log(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h}\coth(fr^2+h)\operatorname{csch}^2(fr^2+h)$$
$$+16b^2\delta_1 f^2r^3\left(e^{2(fr^2+h)}-1\right)\coth^2(fr^2+h)\operatorname{csch}^3(fr^2+h)\left(4bB_1 fr^2\log(e)e^{fr^2+h}+\phi_2\right)$$
$$-8b^2fr\phi_5(\delta_1-\delta_2 r^2)\coth(fr^2+h)\operatorname{csch}^2(fr^2+h) - 64bB_1\delta_1 f^3r^3\log^3(e)e^{3(br^2+c)}$$
$$-32bB_1\delta_1 f^3r^3\log^3(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h} - 32bB_1\delta_1 f^2 r\log^2(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h}$$
$$+32bB_1\delta_1 f^2 r\log^2(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h} + 64bB_1\delta_1 f^2 r\log^2(e)e^{3(fr^2+h)}$$
$$+4bB_1\delta_2 fr\log(e)\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)e^{fr^2+h} + 8b\delta_2 r\phi_2\log(e)e^{2(fr^2+h)}$$

$$D_3 = 8\delta_1 f^2r^3\log(e)e^{fr^2+h}\left(e^{2(fr^2+h)}-1\right)\coth(fr^2+h)$$

$$D_4 = D_3\left(2A_1 fe^{fr^2+h} + bB_1 e^{fr^2+h}\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right) + 2bB_1 fr^2\log(e) + 3B_1 b\right)$$
$$-16bB_1\delta_1 f^3r^5\log^3(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h} - 32bB_1\delta_1 f^3r^5\log^3(e)e^{3(fr^2+h)}$$
$$+8\delta_1 f^2r^3\log(e)e^{2(fr^2+h)}\coth(fr^2+h)\left(4bB_1 fr^2\log(e)e^{fr^2+h}+\phi_2\right)$$
$$-4\delta_1 f^2r^3\left(e^{2(fr^2+h)}-1\right)\operatorname{csch}^2(fr^2+h)\left(4bB_1 fr^2\log(e)e^{fr^2+h}+\phi_2\right)$$
$$-32bB_1\delta_1 f^2r^3\log^2(e)\left(e^{2(fr^2+h)}+1\right)e^{fr^2+h} + 8bB_1\delta_1 f^2r^3\log^2(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h}$$
$$+16bB_1\delta_1 f^2r^3\log^2(e)e^{3(fr^2+h)} + 4\delta_1 fr\left(e^{2(fr^2+h)}-1\right)\coth(fr^2+h)\left(4bB_1 fr^2\log(e)e^{fr^2+h}+\phi_2\right)$$
$$+8bB_1\delta_1 fr\log(e)\left(e^{2(fr^2+h)}-1\right)e^{fr^2+h} + 4bB_1 fr\log(e)(\delta_2 r^2-\delta_1)\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)e^{fr^2+h}$$
$$+8fr\phi_2\log(e)(\delta 2r^2-\delta 1)e^{2(br^2+c)} + 2\delta_2 r\phi_5$$

$$D_5 = -4b^4r^2 - 2(b^2(8f^2r^4+5) - 4f^2r^2)\cosh\left(2(fr^2+h)\right) + 12b^2fr^2\sinh\left(2(fr^2+h)\right) + 10b^2$$
$$-5f\sinh\left(4(br^2+c)\right) + 4f^2r^2\cosh\left(4(fr^2+h)\right) - 12f^2r^2 + 10f\sinh\left(2(fr^2+h)\right)$$

$$D_6 = 4b^2r\left(e^{fr^2+h}-1\right)\left(e^{fr^2+h}+1\right)\left(1-2fr^2\coth(fr^2+h)\right)\operatorname{csch}^2(fr^2+h)$$

$$V_r^2 = -\frac{\phi_{15}\sinh^2(fr^2+h)\left(-8b^2fB_1T_4\log(e)e^{fr^2+h}\operatorname{csch}^2(fr^2+h) - 8b^2B_1T_2\log^2(e)e^{fr^2+h} + T_1 + T_3\right)}{aT_5\left(e^{fr^2+h}-1\right)^2\left(e^{fr^2+h}+1\right)^2\left(fB_1\log\left(\frac{e^{fr^2+h}-1}{e^{fr^2+h}+1}\right) + 2A_1 b\right)^2}$$

$$V_t^2 = \frac{\phi_3\left(e^{\mathrm{bf}r^2+h}-1\right)\left(e^{\mathrm{bf}r^2+h}+1\right)\sinh^6(\mathrm{bf}r^2+h)(2b^2D_3\mathrm{csch}^2(\mathrm{bf}r^2+h)+D_2)}{b^2\delta_1 D_5 r\left(e^{2(\mathrm{bf}r^2+h)}-1\right)^3\left(2A_1 f+bB_1\log\left(\frac{e^{\mathrm{ef}rr^2+h}-1}{e^{\mathrm{bf}r^2+h}+1}\right)\right)^2}$$

$$-4D_1 fr\phi_{15}\log(e)\left(e^{\mathrm{bf}r^2+h}+1\right)e^{\mathrm{bf}r^2+h}\sinh^6(\mathrm{bf}r^2+h)\left(2A_1 f+bB_1\log\left(\frac{e^{\mathrm{bf}r^2+h}-1}{e^{\mathrm{bf}r^2+h}+1}\right)\right)$$

$$-D_6 D_1\sinh^6(\mathrm{bf}r^2+h)\left(2A_1 f+bB_1\log\left(\frac{e^{\mathrm{bf}rr^2+h}-1}{e^{\mathrm{bf}rr^2+h}+1}\right)\right)-4bB_1 D_1 fr\phi_{15}\log(e)e^{\mathrm{bf}r^2+h}\sinh^6(\mathrm{bf}r^2+h)$$

$$-4D_1 fr\phi_3\log(e)\left(e^{\mathrm{bf}r^2+h}-1\right)e^{\mathrm{bf}r^2+h}\sinh^6(\mathrm{bf}r^2+h)$$

En el volumen 2 de este trabajo, continúa el desarrollo matemático.

CONCLUSIONES.

De los resultados obtenidos en el apartado anterior, se concluye: 1) que la supergravedad cuántica relativista consiste en la deformación extrema del espacio – tiempo cuántico; 2) que la deformación extrema del espacio – tiempo cuántico, se desarrolla en dimensiones altas; 3) que las dimensiones o supermembranas formadas por la referida distorsión, comportan otros puntos lejanos del espacio – tiempo cuántico, en los que, la dimensión temporal es diferente para cada caso; 4) la formación de dimensiones altas, supone la existencia de múltiples regiones que no convergen en un mismo espacio – tiempo cuántico, sino en distintos sectores espaciales, por lo que, la transdimensionalidad y la multidimensionalidad ocurren a escalas infinitas y en ramificación, pues parten de una dimensión cero o de origen; 5) la supergravedad cuántica relativista, obedece a la interacción de una superpartícula, llamada también partícula blanca o estrella, es decir, aquella cuyo centro de masa y energía es extremadamente denso, capaz de deformar el espacio – tiempo cuántico en forma extrema, formando así, la supergravedad. Sin embargo, esta partícula también puede alcanzar la velocidad de la luz, por lo que, se la denomina suprapartícula, con las mismas propiedades que la aquí referida. Asimismo, la superpartícula o suprapartícula, según sea el caso, también produce supergravedad cuando interactúa con el supergravitón, esto es, con la supercompañera del gravitón, por lo que el espacio – tiempo cuántico distorsionado, se ve permeado y por ende, mutado por el campo supergravitónico vinculante; y, 6) la supersimetría de gauge y su modelamiento matemático, permite conciliar la relatividad general y especial y la mecánica cuántica, en la medida en que, estamos ante campos cuánticos en los que interfiere la dualidad holográfica y el principio cosmológico.

ACLARACIONES FINALES:

Algunas aclaraciones finales a tener en consideración y aplicar, a propósito de la Teoría Cuántica de Campos Relativistas o Curvos (TCCR) propuesta por este autor:

1. En todos los casos, este símbolo † será reemplazado por este símbolo†o por este símbolo †, equivaliendo lo mismo.

Símbolo a ser reemplazado.	**Símbolos de reemplazo.**
†	†
	†

2. En todos los casos, este símbolo ‡, será reemplazado por este símbolo‡o por este símbolo ‡.

Símbolo a ser reemplazado.	**Símbolos de reemplazo.**
‡	‡
	‡

3. En todos los casos, se añadirá y por ende, se calculará la magnitud§que equivale a un campo de Yang – Mills y por ende, a la teoría de Yang – Mills en sentido amplio, en relación a la Teoría Cuántica de Campos Relativistas propuesta por este autor.

4. Este símbolo • podrá usarse como exponente u operador, según sea el caso.

Las aclaraciones antes referidas aplican tanto a este trabajo como a todos los trabajos previos y posteriores publicados por este autor, según corresponda.

REFERENCIAS BIBLIOGRÁFICAS.

Masafumi Shimojo, Satoshi Ishihara, Hironobu Kataoka, Atsuko Matsukawa y Hikaru Sato, The superspace representation of Super Yang–Mills theory on NCG, arXiv:1801.01172v1 [hep-th] 25 Dec 2017.

Jiří Vohánka y Mir Faizal, Super-Yang-Mills Theory in SIM(1) Superspace, arXiv:1409.6334v2 [hep-th] 21 Jan 2015.

L. A. Ferreira y H. Malavazzi, The Hidden Symmetries of Yang-Mills Theory in (3 + 1)-dimensions, arXiv:2506.15832v1 [hep-th] 18 Jun 2025.

Shuichi Yokoyama, More on BPS States in N = 4 Supersymmetric Yang-Mills Theory on $R \times S^3$, arXiv:1406.6694v2 [hep-th] 9 Sep 2014.

Z. Bern, L. Dixon, D.C. Dunbar, M. Perelstein y J.S. Rozowsky, On the Relationship between Yang-Mills Theory and Gravity and its Implication for Ultraviolet Divergences, arXiv:hep-th/9802162v2 25 May 1998.

Lukas Kaufmann, Stefano Lanza y Timo Weigand, Asymptotics of 5d Supergravity Theories and the Emergent String Conjecture, arXiv:2412.12251v1 [hep-th] 16 Dec 2024.

Laura Andrianopoli, D = 4, N = 2 Supergravity: An Unconventional Application, arXiv:2502.17324v1 [hep-th] 24 Feb 2025.

Remigiusz Durka, DEFORMED BF THEORY AS A THEORY OF GRAVITY AND SUPERGRAVITY, arXiv:1208.5185v1 [gr-qc] 26 Aug 2012.

Sat Paul, Jitendra Kumar y S. K. Maurya, Study on physical properties and characteristics of an anisotropic compact star model using Karmarkar Condition in F(Q) gravity, arXiv:2505.15853v2 [gr-qc] 23 May 2025.

Payaswinee Arvikar, Sakshi Gautam, Anagh Venneti y Sarmistha Banik, Exploring Fermionic Dark Matter Admixed Neutron Stars in the Light of Astrophysical Observations, arXiv:2506.20736v1 [astro-ph.HE] 25 Jun 2025.

Hamza Boumaza y David Langlois, Neutron stars in degenerate higher-order scalar-tensor theories: Axial perturbations, arXiv:2507.02749v1 [gr-qc] 3 Jul 2025.

Edson Otoniel, Juan M. Z. Pretel, Clésio E. Mota, César O. V. Flores y Victor B. T. Alves, White dwarf structure in f(R, T, Lm) gravity: beyond the Chandrasekhar mass limit, arXiv:2507.18745v1 [gr-qc] 24 Jul 2025.

Muhammed Shafeeque, Arun Mathew y Malay K. Nandy, Supermassive Neutron Stars in Starobinsky Gravity with Causal Hybrid Stellar Matter, arXiv:2508.09861v1 [astro-ph.HE] 13 Aug 2025.

Debadri Bhattacharjee y Pradip Kumar Chattopadhyay, Revisiting the maximum mass limit of strange stars in higher-order curvature-matter coupled gravity, arXiv:2508.10524v1 [gr-qc] 14 Aug 2025.

Susmita Sarkar y Nayan Sarkar, Relativistic compact stars coupled with dark energy in Heintzmann spacetime, arXiv:2508.10930v1 [gr-qc] 11 Aug 2025.

Nabamita Banerjee, Tabasum Rahnuma y Ranveer Kumar Singh, Asymptotic Symmetry algebra of N = 8 Supergravity, arXiv:2212.12133v1 [hep-th] 23 Dec 2022.

Pietro Capuozzo, John Estes, Brandon Robinson y Benjamin Suzzoni, From Large to Small N = (4, 4) Superconformal Surface Defects in Holographic 6d SCFTs, arXiv:2402.11745v2 [hep-th] 8 Mar 2024.

Grigorios Giotopoulos, Hisham Sati y Urs Schreiber, Flux Quantization on 11-dimensional Superspace, arXiv:2403.16456v4 [hep-th] 1 May 2025.

Christian Kunz, Extended Massive Ambitwistor String II, arXiv:2502.03581v2 [hep-th] 10 Feb 2025.

Bruno Valeixo Bento y Miguel Montero, An M-theory dS maximum from Casimir energies on Riemann-flat manifolds, arXiv:2507.02037v2 [hep-th] 21 Jul 2025.

Sergei M. Kuzenko, Emmanouil S. N. Raptakis, Igor B. Samsonov Y Gabriele Tartaglino-Mazzucchelli, On three-dimensional N = 4 supersymmetry: maximally supersymmetric backgrounds and massive deformations, arXiv:2504.20712v2 [hep-th] 28 Jul 2025.

Aviral Aggarwal, Subhroneel Chakrabarti, Steven Weilong Hsia, Ahmed Rakin Kamal y Linus Wulff, One loop in D = 11 vs D = 10: 4-point check, arXiv:2506.16391v2 [hep-th] 31 Jul 2025.

Shai M. Chester, Rishi Mouland y Jesse van Muiden, Extremal couplings, graviton exchange, and gluon scattering in AdS, arXiv:2505.23948v2 [hep-th] 1 Jul 2025.

Andrea Conti, Yolanda Lozano y Niall T. Macpherson, N = 6 supersymmetric $AdS_2 \times \mathbb{CP}^3 \times \Sigma_2$, arXiv:2503.23585v1 [hep-th] 30 Mar 2025.

Christopher Couzens y Alice Lüscher, A geometric dual of F-maximization in massive type IIA, arXiv:2406.15547v2 [hep-th] 6 Sep 2024.

GABRIELE CASAGRANDE, Linear and nonlinear supersymmetry in field and string theory, NNT : 2025IPPAX011.

Anayeli Ramírez y Salomón Zacarías, On deformations of AdS3 solutions, supersymmetry and G-structures, arXiv:2504.11207v1 [hep-th] 15 Apr 2025.

Jieming Lin, Torben Skrzypek y K.S. Stelle, Interplay of Intersecting Branes and Consistent Embeddings, arXiv:2508.00987v1 [hep-th] 1 Aug 2025.

Mohammad R. Garousi, Type IIA supergravity at one loop: α'^3 terms in the metric-dilaton-RR one-form sector, arXiv:2508.10529v1 [hep-th] 14 Aug 2025.

Julian Kupka, Charles Strickland-Constable y Fridrich Valach, Geometry of supergravity and the Batalin–Vilkovisky formulation of the N = 1 theory in ten dimensions, arXiv:2508.06398v1 [hep-th] 8 Aug 2025.

Mohammad R. Garousi, S-duality in higher-derivative corrections of heterotic supergravity, arXiv:2502.00357v4 [hep-th] 7 Aug 2025.

Anthony Ashmore, Javier José Murgas Ibarra, Charles Strickland-Constable y Eirik Eik Svanes, Holomorphic supergravity in ten dimensions and anomaly cancellation, arXiv:2507.06003v2 [hep-th] 23 Jul 2025.

Eric D'Hoker, Michael Gutperle y Christoph F. Uhlemann, Exact solutions to complex Type IIB supergravity for complex superalgebra F(4) and its real forms, arXiv:2508.14959v1 [hep-th] 20 Aug 2025.

Arthur Lipstein, Romain Ruzziconi y Akshay Yelleshpur Srikant, Towards a Flat Space Carrollian Hologram from AdS4/CFT3, arXiv:2504.10291v2 [hep-th] 17 Jun 2025.

Ilija Rakic y Lorenzo Toni, One-loop determinants in AdS3 supergravity with extended supersymmetry, arXiv:2508.15701v1 [hep-th] 21 Aug 2025.

Nikolay Bobev y Hynek Paul, Holographic Correlators Beyond Maximal Supersymmetry, arXiv:2505.20400v1 [hep-th] 26 May 2025.

Augustus Brown, Daniele Dorigoni, Francesco Galvagno y Congkao Wen, Universality of giant graviton correlators, arXiv:2508.15657v1 [hep-th] 21 Aug 2025.

Sameer Murthy y Mukund Rangamani, Strings and near-extremal black holes in theories with large N = 4 superconformal symmetry, arXiv:2505.14380v1 [hep-th] 20 May 2025.

Emil J. Martinec, Stefano Massai y David Turton, Cogito, ergo - strings: Supersymmetric ergoregions and their stringy excitations, arXiv:2508.05420v1 [hep-th] 7 Aug 2025.

Jun Nian, Leopoldo A. Pando Zayas y Wenni Zheng, Explorations of Universality in the Entropy and Hawking Radiation of Non-Extremal Kerr AdS4 Black Holes, arXiv:2508.05322v1 [hep-th] 7 Aug 2025.

Roberto Bonezzi, Giuseppe Casale y Olaf Hohm, The Double Copy of Maximal Supersymmetry in D = 10, arXiv:2508.11753v1 [hep-th] 15 Aug 2025.

Carlos Nunez y Dibakar Roychowdhury, Holographic Timelike Entanglement Across Dimensions, arXiv:2508.13266v1 [hep-th] 18 Aug 2025.

C.N. Pope, D.O. Rohrer y B.F. Whiting, Perturbations of Black Holes in Einstein-Maxwell-Dilaton-Axion (EMDA) Theories, arXiv:2508.04589v1 [hep-th] 6 Aug 2025.

Petarpa Boonserm1, Napasorn Jongjittanon y Tritos Ngampitipan, The Modified Tolman-Oppenheimer-Volkov (TOV) Equation and the Effect of Charge on Pressure in Charge Anisotropy, American Journal of Physics and Applications, 2016.

Tetu Makino, ON THE TOLMAN-OPPENHEIMER-VOLKOFF-DE SITTER EQUATION, arXiv:1508.06506v11 [math.AP] 8 Mar 2016.

Carlo Alberto Mantica y Luca Guido Molinari, Tolman-Oppenheimer-Volkoff equation and static spheres in Conformal Killing gravity, arXiv:2409.18663v2 [gr-qc] 27 Jan 2025.

D. Momeni, Tolman-Oppenheimer-Volkoff Equations in Modified Gauss-Bonnet Gravity, arXiv:1408.3626v2 [gr-qc] 2 Oct 2014.

Kostas Glampedakis, George Pappas, Hector O. Silva y Emanuele Berti, Post-Tolman-Oppenheimer-Volko_ formalism for relativistic stars, arXiv: 1504.02455v2 [gr-qc] 1 Aug 2015.

Franciele M. da Silva, Fábio Köpp, Marcelo D. Alloy, Luis C. N. Santos, Adamu Issifu , Clésio E. Mota y Débora P. Menezes, Constraining the Generalized Tolman-Oppenheimer-Volkoff (GTOV) equation with Bayesian análisis, arXiv:2408.10425v3 [astro-ph.HE] 11 Apr 2025.

Dániel Barta, The representation and computational efficiency of the Tolman–Oppenheimer–Volkoff equations in isotropic coordinates, arXiv:2406.07319v1 [gr-qc] 11 Jun 2024.

Davood Momeni, H. Gholizade, Muhammad Raza y Ratbay Myrzakulov, Tolman-Oppenheimer-Volkoff equations in non-local f(R) gravity, arXiv:1502.05000v3 [gr-qc] 4 Apr 2015.

Shuichi Yokoyama, Relativistic Hydrostatic Structure Equations and Analytic Multilayer Stellar Model, arXiv:2306.16647v3 [gr-qc] 13 Aug 2023.

Carlo Maccaferri, Fabio Marino y Beniamino Valsesia, Introduction to String Theory, arXiv:2311.18111v2 [hep-th] 14 Jan 2024.

Albuja Bustamante, M. I. (2024). Demostración del Espectro Hamiltoniano para un Campo de Yang-Mills no Abeliano que Poseen una Brecha de Masa Finita con Respecto al Estado de Vacío. *Ciencia Latina Revista Científica Multidisciplinar*, *8*(1).

Manuel Ignacio Albuja Bustamante , Teoría de Campos: Reforzamiento Teórico – Matemático al Modelo Estándar de Partículas, bajo la estructura ecuacional de Yang – Mills , Ciencia Latina Revista Científica Multidisciplinar: Vol. 8 Núm. 2 (2024).

Manuel Ignacio Albuja Bustamante, La brecha de masa y la curvatura de los campos cuánticos , Ciencia Latina Revista Científica Multidisciplinar: Vol. 8 Núm. 4 (2024).

Manuel Ignacio Albuja Bustamante, Formalización Matemática y en Física de Partículas, en Relación a la Brecha de Masa y la Curvatura Geométrica de los Campos Cuánticos , Ciencia Latina Revista Científica Multidisciplinar: Vol. 8 Núm. 5 (2024).

Manuel Ignacio Albuja Bustamante, Campos Cuánticos Relativistas: Aproximaciones Teórico – Matemáticas Relativas a los Espacios Cuánticos Geométricamente Deformados o Perforados por Partículas y Antipartículas Supermasivas y Masivas E Hiperpartículas y Suprapartículas , Ciencia Latina Revista Científica Multidisciplinar: Vol. 9 Núm. 1 (2025).

Manuel Ignacio Albuja Bustamante, Teoría Cuántica de Campos Relativistas. Formalización Teórica , Ciencia Latina Revista Científica Multidisciplinar: Vol. 9 Núm. 3 (2025)

Manuel Ignacio Albuja Bustamante, Supergravedad cuántica relativista. Teorización inicial , Ciencia Latina Revista Científica Multidisciplinar: Vol. 9 Núm. 4 (2025)

Manuel Ignacio Albuja Bustamante, Supergravedad cuántica relativista. Dimensiones en R^η. , Ciencia Latina Revista Científica Multidisciplinar: Vol. 9 Núm. 4 (2025)

Manuel Ignacio Albuja Bustamante, Gravedad cuántica relativista , Ciencia Latina Revista Científica Multidisciplinar: Vol. 9 Núm. 4 (2025)

Manuel Ignacio Albuja Bustamante, Agujeros Negros Cuántico-Relativistas Masivos o Supermasivos , Ciencia Latina Revista Científica Multidisciplinar: Vol. 9 Núm. 4 (2025).

Manuel Ignacio Albuja Bustamante, Cromodinámica Cuántica Relativista , Ciencia Latina Revista Científica Multidisciplinar: Vol. 9 Núm. 4 (2025).

Albuja Bustamante, M. I. (2025). Las partículas supermasivas: naturaleza fenomenológica de la partícula oscura. *Ciencia Latina Revista Científica Multidisciplinar*, *9*(3).