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CROMODINÁMICA CUÁNTICA RELATIVISTA. HADRONIZACIÓN EN SUPERGRAVEDAD Y GRAVEDAD CUÁNTICAS. VOLUMEN I

RELATIVISTIC QUANTUM CHROMODYNAMICS. HADRONIZATION IN QUANTUM SUPERGRAVITY AND GRAVITY. VOLUME I

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Cromodinámica Cuántica Relativista. Hadronización En Supergravedad Y Gravedad Cuánticas. Volumen I

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RESUMEN

Como ha quedado demostrado en trabajos anteriores, las partículas elementales que conforman la fuerza nuclear fuerte, entre ellas los quarks y los gluones, pueden acreditar las características que son inherentes a una partícula oscura o blanca e incluso, una suprapartícula e hiperpartícula, ésta última, cuando alcanza o supera la velocidad de la luz, sin embargo, en este artículo, se propone un planteamiento alternativo para la hadronización. En sentido estricto, la hadronización es la combinación de quarks y gluones para la formación de otra partícula con distinta masa y energía, denominada hadrón, todo esto, debido al confinamiento que los une. Suponemos, que un plano cuántico – relativista, los quarks y los gluones, se combinan, por la deformación del espacio – tiempo cuántico, causado por cualquiera de éstos, sea en condición de partícula oscura o blanca, según sea el caso. Por tanto, en un espacio de gravedad cuántica o de supergravedad, la hadronización es posible, en la medida en que la gravedad en sí misma, interfiere transversalmente en el proceso de confinamiento, recalcando que, la hadronización puede formar partículas oscuras o blancas de naturaleza hadrónica.

Palabras clave: quarks, gluones, hadrones, hadronización, supergravedad cuántica relativista

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Relativistic Quantum Chromodynamics. Hadronization In Quantum Supergravity and Gravity. Volume I

ABSTRACT

As has been demonstrated in previous works, the elementary particles that make up the strong nuclear force, including quarks and gluons, can accredit the characteristics that are inherent to a dark or white particle and even a supraparticle and hyperparticle, the latter, when it reaches or exceeds the speed of light, however, in this article, An alternative approach to hadronization is proposed. Strictly speaking, hadronization is the combination of quarks and gluons to form another particle with different mass and energy, called a hadron, all due to the confinement that unites them. We suppose that a quantum-relativistic plane, quarks and gluons, are combined by the deformation of quantum space-time, caused by any of these, whether in the condition of a dark or white particle, as the case may be. Therefore, in a quantum gravity or supergravity space, hadronization is possible, insofar as gravity itself interferes transversally in the confinement process, emphasizing that hadronization can form dark or white particles of hadronic nature.

Keywords: quarks, gluones, hadrones, hadronización, supergravedad cuántica relativista

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INTRODUCCIÓN

El confinamiento, en cromodinámica cuántica, consiste en la combinación ineludible de los quarks y los gluones, en la medida en que, los primeros, no son susceptibles de aislamiento, por lo que, para su detección, éstos se asocian a los gluones, con la finalidad de formar hadrones (color neutro), los cuales pueden ser de carácter mesónico o bariónico. Ahora bien, la cromodinámica cuántica despierta interés para la formalización de la Teoría Cuántica de Campos Relativistas, en la medida en que: 1) posee la partícula subatómica con mayor masa en relación a las demás partículas que componen el modelo estándar, siendo ésta, el quark top; y, 2) para efectos de detectar, en especial los quarks, se requiere del proceso de hadronización, ya que aislar un quark, en especial el quark top, es de difícil realización (en el caso del primero) e imposible en el caso del segundo, pues, para aislar un quark, se requiere de un paquete de energía tan extremo, directamente proporcional a la distancia de su punto de origen, que finalmente se forma un par quark – antiquark, en tanto que, respecto del segundo, se ha dicho que es imposible la hadronización pues, al ser tan pesado, se desintegra rápidamente, sin que sea posible su detención, a diferencia de los otros tipos de quarks, los cuales, al ser ligeros, son susceptibles de hadronización. Ahora bien, esto ayuda a reforzar el hecho de que, en primer lugar, sí existen partículas supermasivas en el modelo estándar, siendo ésta, por ejemplo, el quark top, aunque el bosón de Higgs, también presenta esa particularidad, y por otro lado, que a nivel cronodinámico, la relatividad general y especial son posibles, en la medida en que, a mi criterio, es la gravedad la que impide la disociación del quark respecto del gluon, es decir, es la responsable del confinamiento de color que da lugar a los hadrones, los cuales, al ser de color neutro, podrían calificar como partículas blancas o estrella. De ahí la necesidad de la Cromodinámica Cuántica Relativista, como un modelo teórico apto para explicar los fenómenos antes referidos.

RESULTADOS Y DISCUSIÓN.

Procedo a desplegar el desarrollo matemático, el mismo que complementa el sistema de ecuaciones que se desglosan en el volumen II de este manuscrito.

Modelo de Hadronización en supergravedad cuántica relativista.

$$S = \int_{\Sigma} P_{\mu}\bar{\partial}X^{\mu} + \frac{1}{2}eP^{2} + \cdots$$

$$\sum_{j\neq i} \frac{k_i \cdot k_j}{z_i - z_j} = 0$$

$$S[\Psi] = \langle\Psi|c_0 Q|\Psi\rangle + \sum_{n>2} \frac{1}{n!}\{\Psi^{n}\}$$

$$\langle\Psi|c_0 Q|\Psi\rangle = \int \mathrm{d}^{10}x \left(\frac{1}{4}h_{\mu\nu} \square h^{\mu\nu} + \frac{1}{2}\left(\partial^{\nu}h_{\mu\nu}\right)^{2} + \frac{1}{2}h\partial^{\mu}\partial^{\nu}h_{\mu\nu} - \frac{1}{4}h \square h \right.$$
$$\left. -4\phi \square \phi + 2h \square \phi - 2\phi\partial^{\mu}\partial^{\nu}h_{\mu\nu} - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\right)$$

$$T_N^{*}M = \{(x,p) \in T^{*}M \mid p^{2} = 0\}$$

$$\mathcal{V} = p^{\mu}\frac{\partial}{\partial x^{\mu}},$$

$$\Upsilon = p_{\mu}\frac{\partial}{\partial p_{\mu}}$$

$$\mathcal{L} = P_{\mu}\bar{\partial}X^{\mu} + \frac{1}{2}eP^{2}$$

$$S = \int_{\Sigma} P_{\mu}\bar{\partial}X^{\mu} + \frac{1}{2}eP^{2} + b\bar{\partial}c$$

$$P_{\mu}(z)X^{\nu}(\omega) = \frac{\delta_{\mu}^{\nu}}{z-\omega} + \cdots, b(z)c(\omega) = \frac{1}{z-\omega} + \cdots$$

$$\delta(v)X^{\mu} = v\partial X^{\mu}, \delta(v)P_{\mu} = \partial\left(vP_{\mu}\right), \delta(v)e = v\partial e - e\partial v$$

$$\mathcal{T}(v) := \oint \mathrm{d}z v(z)T(z)$$

$$\tilde{\delta}(v)X^{\mu} = vP^{\mu}, \tilde{\delta}(v)P_{\mu} = 0, \tilde{\delta}(v)e = \bar{\partial}v$$

$$\mathcal{H}(v) := \oint \mathrm{d}z v(z)H(z), H(z) = \frac{1}{2}P^{2}(z)$$

$$[\mathcal{T}(v_1), \mathcal{T}(v_2)] = -\mathcal{T}([v_1, v_2]), [\mathcal{T}(v_1), \mathcal{H}(v_2)] = -\mathcal{H}([v_1, v_2])$$
$$[\mathcal{H}(v_1), \mathcal{H}(v_2)] = 0$$

$$Q = \oint \mathrm{d}z j(z)$$

$$j(z) = c(z)\left(T(z) + \tilde{T}_{\mathrm{gh}}(z) + \frac{1}{2}T_{\mathrm{gh}}(z)\right) + \tilde{c}(z)H(z)$$

$$e(z) = \sum_{a} s^{a}\mu^{a}(z)$$

$$\hat{S} = \int_{\Sigma} P_{\mu}\bar{\partial}X^{\mu} + b\bar{\partial}c + Q\tilde{b}F(e)$$

$$F(e) = e - \sum_{a=1}^{n-3} s_a\mu^{a}$$

$$Q\int_{\Sigma}\tilde{b}F(e)=\int_{\Sigma}\pi F(e)+\int_{\Sigma}\tilde{b}\bar{\partial}\tilde{c}-\sum_{a=1}^{n-3}q_a\int_{\Sigma}\tilde{b}\mu^a$$

$$\hat{S}=S-\frac{1}{2}\sum_{a=1}^{n-3}s_a\int_{\Sigma}\mu^a P^2-\sum_{a=1}^{n-3}\int_{\Sigma}q_a\tilde{b}\mu^a$$

$$S=\int_{\Sigma}\left(P_\mu\bar{\partial}X^\mu+b\bar{\partial}c+\tilde{b}\bar{\partial}\tilde{c}\right)$$

$$\prod_{a=1}^{n-3}\bar{\delta}\left(\int_{\Sigma}\mu^a(z)H(z)\right)\int_{\Sigma}\mu^a(z)\tilde{b}(z)\int_{\Sigma}\mu^a(z)b(z)$$

$$X^\mu(z)=x^\mu-\sum_{n\neq 0}\frac{\tilde{\alpha}_n^\mu}{n}z^{-n}$$

$$\partial X^\mu(z)=\sum_{n\neq 0}\tilde{\alpha}_n^\mu z^{-n-1}$$

$$P_\mu(z)=\sum_{n}\alpha_{n\mu}z^{-n-1}$$

$$[P_\mu(\sigma),X^\nu(\sigma')]=-i\delta_\mu^\nu\delta(\sigma-\sigma'),[P_\mu(\sigma),P_\nu(\sigma')]=0,[X^\mu(\sigma),X^\nu(\sigma')]=0$$

$$[\alpha_{n\mu},\tilde{\alpha}_m^\nu]=-in\delta_\mu^\nu\delta_{n+m,0},[\alpha_{n\mu},\alpha_{m\nu}]=0,[\tilde{\alpha}_n^\mu,\tilde{\alpha}_m^\nu]=0$$

$$[\alpha_{0\mu},\tilde{\alpha}_0^\nu]=[p_\mu,x^\nu]=-i\delta_\mu^\nu$$

$$\alpha_n|0\rangle=0,n\geq 0,\text{ and }\tilde{\alpha}_n|0\rangle=0,n>0.$$

$$\langle X^\mu(z)P_\nu(w)\rangle=\frac{\delta_\nu^\mu}{z-w}$$

$$\Box X_{\mathrm{cl}}^\mu=J^\mu.$$

$$X_{\mathrm{cl}}^\mu(z)=\sum_{i=1}^{n}k_i^\mu\ln|z-z_i|^2$$

$$P_{\mathrm{cl}}(z)=\sum_{i=1}^{n}\frac{k_i}{z-z_i}$$

$$\left[\alpha_{n\mu}^{(i)},\tilde{\alpha}_m^{(j)\nu}\right]=-in\delta^{ij}\delta_\mu^\nu\delta_{n+m,0},\left[\alpha_{n\mu}^{(i)},\alpha_{m\nu}^{(j)}\right]=0,\left[\tilde{\alpha}_n^{(i)\mu},\tilde{\alpha}_m^{(j)\nu}\right]=0$$

$$\left[\alpha_{0\mu}^{(i)},\tilde{\alpha}_0^{(j)\nu}\right]=\left[p_\mu^{(i)},x^{(j)\nu}\right]=-i\delta^{ij}\delta_\mu^\nu$$

$$\int_{\Sigma}\mu^a(z)b(z)=\sum_{i=1}^{n}\oint_{\partial\mathcal{D}_i}\mathrm{d}z_i v_i^a(z_i)b^{(i)}(z)$$

$$\prod_{a=1}^{n-3}\bar{\delta}\left(\mathcal{H}(\vec{v}^a)\right)\tilde{\mathbf{b}}(\vec{v}^a)\mathbf{b}(\vec{v}^a)$$

$$\mathbf{b}(\vec{v}^a)=\sum_{i=1}^{n}\oint_{\partial\mathcal{D}_i}\mathrm{d}z v_i^a(z)b^{(i)}(z),\tilde{\mathbf{b}}(\vec{v}^a)=\sum_{i=1}^{n}\oint_{\partial\mathcal{D}_i}\mathrm{d}z v_i^a(z)\tilde{b}^{(i)}(z)$$

$$\mathcal{H}(\vec{v}^a)=\sum_{i=1}^{n}\oint_{\partial\mathcal{D}_i}\mathrm{d}z v_i^a(z)H^{(i)}(z)$$

$$T(z)=\sum_n L_n z^{-n-2}, L_n=\oint \mathrm{d}z z^{n+1} T(z)$$

$$L_0=\frac{1}{2}\sum_{m>0}(\alpha_{-m}\cdot\tilde{\alpha}_m+\tilde{\alpha}_{-m}\cdot\alpha_m), L_n=\sum_{m\neq n}\tilde{\alpha}_{n-m}\cdot\alpha_m.$$

$$H(z)=\sum_n \tilde{L}_n z^{-n-2}$$

$$\tilde{L}_n=\frac{1}{2}\eta^{\mu\nu}\sum_m \alpha_{m\mu}\alpha_{n-m\nu}$$

$$c=\mp 3(2\lambda-1)^2\pm 1$$

$$c(z)=\sum_n c_n z^{-n+1}, b(z)=\sum_n b_n z^{-n-2}$$

$$Q=\sum_n c_{-n}\left(L_n^{(m)}+L_n^{(g)}+\tilde{L}_n^{(g)}\right)+\sum_n \tilde{c}_{-n}\tilde{L}_n^{(m)}$$

$$L_n^{(g)}=\sum_m (n-m){:}\,b_{n+m}c_{-m}{:}-\delta_{n,0}$$

$$\sum_n c_{-n}L_n^{(m)}=c_0(\alpha_{-1}\cdot\tilde{\alpha}_1+\tilde{\alpha}_{-1}\cdot\alpha_1)+\alpha_0\cdot(c_1\tilde{\alpha}_{-1}+c_{-1}\tilde{\alpha}_1)+\cdots$$

$$\sum_n \tilde{c}_{-n}\tilde{L}_n^{(m)}=\frac{1}{2}\tilde{c}_0\alpha_0^2+\frac{1}{2}\tilde{c}_0\alpha_{-1}\cdot\alpha_1+\alpha_0\cdot(\tilde{c}_{-1}\alpha_1+\tilde{c}_1\alpha_{-1})+\cdots$$

$$Q=c_0\mathcal{L}_0+\frac{1}{2}\tilde{c}_0\alpha_0^2+\frac{1}{2}\tilde{c}_0\alpha_{-1}\cdot\alpha_1+\alpha_0\cdot(c_1\tilde{\alpha}_{-1}+c_{-1}\tilde{\alpha}_1+\tilde{c}_{-1}\alpha_1+\tilde{c}_1\alpha_{-1})$$
$$-2b_0c_{-1}c_1+2\tilde{b}_0(c_1\tilde{c}_{-1}+\tilde{c}_1c_{-1})+\tilde{c}_0\left(c_{-1}\tilde{b}_1+c_1\tilde{b}_{-1}\right)+\cdots$$

$$\mathcal{L}_0=(\alpha_{-1}\cdot\tilde{\alpha}_1+\tilde{\alpha}_{-1}\cdot\alpha_1)+(b_{-1}c_1+c_{-1}b_1-1)+\left(\tilde{b}_{-1}\tilde{c}_1+\tilde{c}_{-1}\tilde{b}_1-1\right)+\cdots$$

$$\langle\Sigma|B(\vec{v})|V_i\rangle\ldots|V_n\rangle,$$

$$|V\rangle=\lim_{t\to 0}V(t)|0\rangle$$

$$V(z)=c(z)\tilde{c}(z)\varepsilon^{\mu\nu}P_\mu(z)P_\nu(z)e^{ik\cdot X(z)}$$

$$|V\rangle=c_1\tilde{c}_1\varepsilon_{\mu\nu}\alpha_{-1}^{\mu}\alpha_{-1}^{\nu}|k\rangle$$

$$M_n=\int_{\mathcal{M}_n}\langle\Sigma|B(\vec{v})|V_1\rangle\ldots|V_n\rangle$$

$$\langle\Sigma|=\int \mathrm{d}^n p\langle\vec{p}_n|\delta\left(\sum p_{(i)}\right)e^W\mathcal{Z}$$

$$V_{X,P}(z_1,\ldots z_n)=\sum_{i,j}\oint_0 \mathrm{d}t_i\oint_0 \mathrm{d}t_j\frac{X(t_i)\cdot P(t_j)}{h_i(t_i)-h_j(t_j)}$$

$$V_{\mathrm{gh}}(z_1,\ldots z_n)=\sum_{i,j}\oint_0 \mathrm{d}t_i\oint_0 \mathrm{d}t_j\frac{b(t_i)c(t_j)}{h_i(t_i)-h_j(t_j)}+\sum_{i,j}\oint_0 \mathrm{d}t_i\oint_0 \mathrm{d}t_j\frac{\tilde{b}(t_i)\tilde{c}(t_j)}{h_i(t_i)-h_j(t_j)}$$

$$\mathcal{Z}=\prod_{r=-1}^{+1}\mathcal{Z}_r\prod_{r=-1}^{+1}\tilde{\mathcal{Z}}_r$$

$$Z_r = \sum_{i=1}^{n} \sum_{m=-1}^{\infty} \mathcal{M}_{rm}(z_i) b_m^i$$

$$\mathcal{M}_{nm}(z_i) = \oint_{t_i=0} \frac{\mathrm{d}t_i}{2\pi i} t_i^{-m-2} (h_i'(t))^{-1} (h_i(t))^{n+1}$$

$$V_{X,P} = \sum_{m,n\geq 0} \sum_{i,j} \mathcal{S}^{mn}(z_i, z_j) \tilde{\alpha}_m^{(i)} \cdot \alpha_n^{(j)}$$

$$V_{\mathrm{gh}} = \sum_{i,j} \sum_{\substack{n\geq 2 \\ m\geq -1}} \mathcal{K}_{nm}(z_i, z_j) c_n^{(i)} b_m^{(j)} + \sum_{i,j} \sum_{\substack{n\geq 2 \\ m\geq -1}} \mathcal{K}_{nm}(z_i, z_j) \tilde{c}_n^{(i)} \tilde{b}_m^{(j)}.$$

$$\mathcal{S}_{mn}(z_i, z_j) = \oint \frac{\mathrm{d}t_i}{2\pi i} \oint \frac{\mathrm{d}t_j}{2\pi i} h_i'(t_i) t_i^{-m} t_j^{-n-1} \frac{1}{h_i(t_i) - h_j(t_j)}$$

$$\mathcal{K}_{nm}(z_i, z_j) = -\oint \frac{\mathrm{d}t_i}{2\pi i} \oint \frac{\mathrm{d}t_j}{2\pi i} t_i^{-n+1} t_j^{-m-2} (h_i'(t_i))^2 \left(h_j'(t_j)\right)^{-1} \frac{1}{h_i(t_i) - h_j(t_j)}$$

$$\Omega_{|\vec{V}\rangle}(\vec{v}) = \langle \Sigma | B_{n-3}(\vec{v}) | \vec{V} \rangle$$

$$B_{n-3}(\vec{v}) = \prod_{a=1}^{n-3} \tilde{\mathbf{b}}(\vec{v}_a) \prod_{a=1}^{n-3} \mathbf{b}(\vec{v}_a) \prod_{a=1}^{n-3} \bar{\delta}(\mathcal{H}(\vec{v}_a))$$

$$\langle V(z_1), \ldots, V(z_n) \rangle = \int_{\mathcal{M}_n} \Omega_{|\vec{V}\rangle}(\vec{v})$$

$$\langle \Sigma | \alpha_{-1}^{(i)} = \int \mathrm{d}^n p \langle \vec{p}_n | \delta\left(\sum p_{(j)}\right) e^W \sum_{j\neq i} \sum_{n\geq 0} \mathcal{S}_{1n}(z_i, z_j) \alpha_n^{(j)} \mathcal{Z}$$

$$\sum_{j\neq i} \sum_{n\geq 0} \mathcal{S}_{1n}(z_i, z_j) \alpha_n^{(j)} |k_j\rangle = \sum_{j\neq i} \frac{k_j}{z_i - z_j} |k_j\rangle$$

$$\alpha_{-1}^{(i)} \to \sum_{j\neq i} \frac{k_j}{z_i - z_j}$$

$$P_{\mathrm{cl}}(z) = \sum_j \frac{k_j}{z - z_j},$$

$$\prod_{a=1}^{n-3} \bar{\delta}(\mathcal{H}(\vec{v}_a)) \tilde{\mathbf{b}}(\vec{v}_a) \mathbf{b}(\vec{v}_a)$$

$$\langle \Sigma | \alpha_{-m}^{(i)} \alpha_{-n}^{(i)} = \int \mathrm{d}^n p \langle \vec{p}_n | \delta\left(\sum p_{(j)}\right) e^W A_{-m}^{(i)} A_{-n}^{(i)} \mathcal{Z}$$

$$A_{-m}^{(i)} \equiv \alpha_{-m}^{(i)} + \sum_{j\neq i} \sum_{n\geq 0} \mathcal{S}_{mn}(z_i, z_j) \alpha_n^{(j)}$$

$$z_i \to z_i + v_i^a \delta\tau_a$$

$$\mathcal{H}(\vec{v}^a) = \sum_{i=1}^{n} \oint \mathrm{d}z H^{(i)}(z) \delta_i^a = \tilde{L}_{-1}^{(a)}$$

$$\langle\Sigma|\tilde{L}_{-1}^{(a)} = \sum_{n\geq 0} \langle\Sigma|\alpha_n^{(a)} \cdot \alpha_{-n-1}^{(a)}$$

$$= \int \mathrm{d}^n p \langle\vec{p}_n|\delta\left(\sum p_{(j)}\right)\sum_{n\geq 0} \alpha_n^{(a)} \cdot \left(\sum_{j\neq a}\sum_{m\geq 0} \mathcal{S}_{1+n,m}(z_a, z_j)\alpha_m^{(j)}\right) e^W \mathcal{Z},$$

$$\sum_{n\geq 0} \alpha_n^{(a)} \cdot \left(\sum_{j\neq a}\sum_{m\geq 0} \mathcal{S}_{1+n,m}(z_a, z_j)\alpha_m^{(j)}\right)|k_1\rangle \ldots |k_n\rangle = \sum_{j\neq a} \frac{k_a \cdot k_j}{z_a - z_j}|k_1\rangle \ldots |k_n\rangle$$
$$= k_a \cdot P_{\mathrm{cl}}(z_a)|k_1\rangle \ldots |k_n\rangle$$

$$\int \mathrm{d}^n p \delta\left(\sum p_j\right)\langle p_1| \ldots \langle p_n|e^{V_{X,P}} \prod_{a=1}^{n-3} \bar{\delta}\left(\tilde{L}_{-1}^{(a)}\right)|k_1\rangle \ldots |k_n\rangle = \delta\left(\sum k_i\right)\prod_{a=1}^{n-3}\left(k_a \cdot P_{\mathrm{cl}}(z_a)\right)$$

$$\int_{\mathcal{M}_n} \Omega_{|\overline{\Psi}\rangle}(\vec{v}) = \int_{\mathcal{M}_n} \langle\Sigma| \prod_{i=1}^{3} c_1^{(i)}\tilde{c}_1^{(i)}\epsilon_i^{\mu\nu}\alpha_{-1\mu}^{(i)}\alpha_{-1\nu}^{(i)} \prod_{i=4}^{n} \bar{\delta}(k \cdot P_{\mathrm{cl}})\epsilon_i^{\mu\nu}\alpha_{-1\mu}^{(i)}\alpha_{-1\nu}^{(i)}|k_1\rangle \ldots |k_n\rangle,$$

$$M_N = \delta^D\left(\sum k_i\right)\int_{\mathcal{M}_N} \mathrm{d}^{N-3}z_i \frac{1}{\mathrm{d}\omega}\prod_{i=1}^{N} \epsilon_i^{\mu\nu} P_{\mathrm{cl}\mu}P_{\mathrm{cl}\nu} \prod_i{}' \bar{\delta}\left(k_i \cdot P_{\mathrm{cl}}(z_i)\right)$$

$$\prod_i{}' \bar{\delta}\left(k_i \cdot P_{\mathrm{cl}}(z_i)\right) = \frac{1}{\mathrm{d}\omega}\prod_{i=4}^{N} \bar{\delta}\left(k_i \cdot P_{\mathrm{cl}}(z_i)\right)$$

$$S_0 + \{Q, \mathcal{W}\}$$
$$S_0 = \int P_\mu \bar{\partial} X^\mu + b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c}$$
$$M_n = \int_{\Gamma\subset T^*\mathcal{M}} \widetilde{\Omega}_{|\vec{V}\rangle}(u, \tilde{u})$$

$$\widetilde{\Omega}_{|\vec{V}\rangle}(u, \tilde{u}) = \left\langle e^{-\{Q, \mathbf{b}(u) + \tilde{\mathbf{b}}(u, \tilde{u})\}} V_1 \ldots V_n \right\rangle_{S_0}$$

$$M_n = \sum_{\tau^*} \left\langle \prod_{a=1}^{n-3} \mathbf{b}(v_a) \prod_{a=1}^{n-3} \tilde{\mathbf{b}}(v_a)(\det\Phi)^{-1} V_1 \ldots V_n \right\rangle_{S_0}$$

$$M_n = \int_{\mathcal{M}} \langle\Sigma|B_{n-3}|V_1\rangle \ldots |V_n\rangle$$

$$M_n = \sum_{\tau^*} \langle\Sigma^*| \prod_{a=1}^{n-3} \mathbf{b}(v_a^*) \prod_{a=1}^{n-3} \tilde{\mathbf{b}}(v_a^*)(\det\Phi)^{-1}|V_1\rangle \ldots |V_n\rangle$$

$$M_n = \int_{\Gamma\subset T^*\mathcal{M}} \langle\tilde{\Sigma}||V_1\rangle \ldots |V_n\rangle$$

$$S[\Psi] = \langle\Psi|Q|\Psi\rangle + \frac{g}{3!}\langle\Sigma_3||\Psi\rangle|\Psi\rangle|\Psi\rangle$$

$$\frac{b_0}{L_0} = b_0 \int_0^\infty \mathrm{d}\tau e^{-\tau L_0}$$

$$S[\Psi] = \langle\Psi|c_0 Q|\Psi\rangle + \sum_{n>2} \frac{1}{n!}\{\Psi^n\}$$

$$|[\Psi_1, \ldots, \Psi_n]\rangle = \sum_r b_0^{(r)}|\phi_r\rangle\{\phi_r, \Psi_1, \ldots, \Psi_n\}$$

$$Q|\Psi\rangle + \sum_{n>2} \frac{1}{n!} |[\Psi_1, \dots, \Psi_n]\rangle = 0$$

$$Q[\Psi_1, \dots, \Psi_n] + \sum_{i=1}^{n} (-1)^{|\Psi_1|+\cdots+|\Psi_{i-1}|} [\Psi_1, \dots, Q\Psi_i, \dots, \Psi_n] + \sum_{\{i_\ell, j_k\}, l, k} \sigma(i_\ell, j_k) \left[\Psi_{i_1}, \dots, \Psi_{i_\ell}, [\Psi_{j_1}, \dots, \Psi_{j_k}]\right]$$

$$\delta|\Psi\rangle = Q|\Lambda\rangle + \sum_{n} \frac{1}{n!} |[\Lambda, \Psi_1, \dots, \Psi_n]\rangle,$$

$$\tilde{b}_0|\Psi\rangle = 0.$$

$$\langle\Psi|c_0 Q|\Psi\rangle = \langle\Psi|c_0 \tilde{c}_0 \tilde{L}_0|\Psi\rangle,$$

$$\frac{\delta(\mathcal{L}_0)}{\tilde{L}_0} \tilde{b}_0 b_0 |\mathcal{R}_{LR}\rangle$$

$$\{\Psi^n\} = \int_{\mathcal{V}_n} \Omega_{|\overline{\Psi}\rangle}(\vec{v})$$

$$|\Psi\rangle = \varepsilon_{\mu\nu} \alpha_{-1}^{\mu} \alpha_{-1}^{\nu} c_1 \tilde{c}_1 |k\rangle,$$

$$|\Psi\rangle = \int \mathrm{d}k \left(-\frac{1}{2} h_{\mu\nu}(k) \alpha_{-1}^{\mu} \alpha_{-1}^{\nu} c_1 \tilde{c}_1 + \cdots \right) |k\rangle,$$

$$|\Lambda\rangle = i \int \mathrm{d}k \lambda_\mu(k) \alpha_{-1}^{\mu} c_1 |k\rangle.$$

$$Q|\Lambda\rangle = \int \mathrm{d}k \left(\frac{i}{2} \tilde{c}_0 c_1 \alpha_0^2 \lambda_\mu(k) \alpha_{-1}^{\mu} + i \tilde{c}_1 c_1 \alpha_{0\mu} \lambda_\nu(k) \alpha_{-1}^{\mu} \alpha_{-1}^{\nu} + i c_{-1} c_1 \alpha_0^{\mu} \lambda_\mu(k) \right) |k\rangle.$$

$$|\Psi\rangle = \int \mathrm{d}k \left(-\frac{1}{2} h_{\mu\nu}(k) \alpha_{-1}^{\mu} \alpha_{-1}^{\nu} c_1 \tilde{c}_1 + \frac{1}{2} e(k) c_{-1} c_1 + i f_\mu(k) \alpha_{-1}^{\mu} \tilde{c}_0 c_1 \right) |k\rangle.$$

$$\delta h_{\mu\nu}(x) = \partial_\mu \lambda_\nu(x) + \partial_\nu \lambda_\mu(x), \delta f_\mu(x) = -\frac{1}{2} \Box \lambda_\mu(x), \delta e(x) = 2\partial^\mu \lambda_\mu(x)$$

$$e(x) = \eta^{\mu\nu} h_{\mu\nu}(x)$$

$$\Psi(z) = \int \mathrm{d}k \left(-\frac{1}{2} h_{\mu\nu}(k) P^\mu P^\nu c\tilde{c} + \frac{1}{2} e(k) \partial^2 c c + i f_\mu(k) P^\mu \partial \tilde{c} c \right) e^{ik\cdot X}$$

$$|\Psi\rangle = \lim_{z\to 0} \Psi(z)|0\rangle$$

$$\delta\Psi(z) = \oint_{C} \mathrm{d}\omega j(\omega) \Psi(z)$$

$$S_2[\Psi] = \langle\Psi|c_0 Q|\Psi\rangle = \langle R_{LR}|c_0 Q|\Psi_L\rangle|\Psi_R\rangle.$$

$$\mathcal{L}_0|\Psi\rangle = 0, b_0|\Psi\rangle = 0$$

$$\int \mathrm{d}k \langle -k| \left(-\frac{1}{2} h_{\mu\nu}(k) \alpha_1^{\mu} \alpha_1^{\nu} c_{-1} \tilde{c}_{-1} + \frac{1}{2} e(k) c_1 c_{-1} - i f_\mu(k) \alpha_1^{\mu} \tilde{c}_0 c_{-1} \right)$$

$$\langle\Psi| = \int \mathrm{d}k \langle -k| \left(-\frac{1}{2} h_{\mu\nu}(k) \tilde{\alpha}_1^{\mu} \tilde{\alpha}_1^{\nu} \tilde{c}_{-1} c_{-1} + \frac{1}{2} e(k) \tilde{c}_1 \tilde{c}_{-1} - i f_\mu(k) \tilde{\alpha}_1^{\mu} \tilde{c}_0 \tilde{c}_{-1} \right)$$

$$\langle k'|\tilde{c}_{-1} \tilde{c}_0 \tilde{c}_1 c_{-1} c_0 c_1 |k\rangle = \delta(k - k')$$

$$S_2[\Psi] = \int \mathrm{d}k \left(-\frac{1}{4} h_{\mu\nu}(-k) k^2 h^{\mu\nu}(k) + 2i h_{\mu\nu}(-k) k^{\mu} f^{\nu}(k) + \frac{1}{8} e(-k) k^2 e(k) \right.$$
$$\left. - ie(-k) k^{\mu} f_{\mu}(k) - 2 f_{\mu}(-k) f^{\mu}(k) \right)$$

$$S_2[h, e] = \int \mathrm{d}x \left(\frac{1}{4} h_{\mu\nu} \Box h^{\mu\nu} + 2 h_{\mu\nu} \partial^{\mu} f^{\nu} - \frac{1}{8} e \Box e - e \partial^{\mu} f_{\mu} - 2 f_{\mu} f^{\mu} \right)$$

$$f_{\mu} = -\frac{1}{2} \left(\partial^{\nu} h_{\mu\nu} - \frac{1}{2} \partial_{\mu} e \right).$$

$$S_2[h] = \int \mathrm{d}x \left(\frac{1}{4} h_{\mu\nu} \Box h^{\mu\nu} + \frac{1}{2} (\partial^{\nu} h_{\mu\nu})(\partial_{\lambda} h^{\mu\lambda}) + \frac{1}{2} h \partial_{\mu} \partial_{\nu} h^{\mu\nu} - \frac{1}{4} h \Box h \right),$$

$$\Omega_{|\overline{\Psi}\rangle}(\vec{v}) = \langle \Sigma | \mathbf{b}(\vec{v}_1) \ldots \mathbf{b}(\vec{v}_{2n-6}) | \overline{\Psi} \rangle,$$

$$\mathbf{b}(\vec{v}^{a}) = \sum_{i=1}^{n} \left(\oint \mathrm{d}z\, b^{(i)}(z) v_i^{a}(z) + \oint \mathrm{d}\bar{z}\, \bar{b}^{(i)}(\bar{z}) \bar{v}_i^{a}(\bar{z}) \right)$$

$$\begin{array}{ccc} \mathcal{T} & \hookrightarrow & \widehat{\mathcal{P}}_n \\ & & \downarrow \\ & & \mathcal{M}_n \end{array}$$

$$\Omega_{|\overline{\Psi}\rangle}(\vec{v}) = \langle \Sigma | B_{n-3}(\vec{v}) | \overline{\Psi} \rangle$$

$$\mathcal{N}_n \xrightarrow{\pi} \mathcal{M}_n,$$

$$\int_{\mathcal{M}_n} \Omega_{|\overline{\Psi}\rangle}(\vec{v})$$

$$\delta_{\vec{v}} \langle \Sigma | = \langle \Sigma | \mathcal{T}(\vec{v}) + \langle \Sigma | \mathcal{H}(\vec{v}),$$

$$\delta_{\vec{v}} \langle \Sigma | \delta(\mathcal{H}(\vec{v})) = \langle \Sigma | \mathcal{T}(\vec{v}) \delta(\mathcal{H}(\vec{v})),$$

$$\begin{array}{ccc} \mathcal{T} & \hookrightarrow & \widehat{\mathcal{A}}_n \\ & & \downarrow \\ & & T^{*}\mathcal{M}_n \end{array}$$

$$\{\Psi^n\} = \int_{\mathcal{V}_n} \Omega_{|\overline{\Psi}\rangle}(\vec{v})$$

$$\{\Psi^n\} = \int_{\Gamma_{\mathcal{V}_n}} \widetilde{\Omega}_{|\overline{\Psi}\rangle}(\vec{v}),$$

$$S_2[\Psi] = \langle \mathcal{R}_{LR}| c_0 \tilde{c}_0 \tilde{L}_0 |\Psi_L\rangle |\Psi_R\rangle$$

$$\tilde{b}_0 b_0 \frac{\delta(L_0 - 2)}{\tilde{L}_0} |\mathcal{R}_{L,R}\rangle,$$

$$\int \mathrm{d}s\, \mathrm{d}\tilde{s} b_0 \tilde{b}_0 e^{-\{Q, s b_0 + \tilde{s}\tilde{b}_0\}} |\mathcal{R}_{L,R}\rangle$$

$$\frac{\tilde{b}_0 b_0}{\tilde{L}_0 L_0} |\mathcal{R}_{L,R}\rangle.$$

$$\frac{\tilde{b}_0 b_0}{\tilde{L}_0} |\mathcal{R}_{L,R}\rangle$$

$$\begin{aligned} M_n^{\Gamma_{\mathcal{D}_n}} &= \int_{\mathcal{D}_n} \prod_{a=1}^{n-3} \mathrm{d}\tau_a \langle \Sigma_n | \prod_{a=1}^{n-3} \tilde{\mathbf{b}}(\vec{v}^a) \mathbf{b}(\vec{v}^a) \tilde{\delta}(\mathcal{H}(\vec{v}^a)) |\vec{\Psi}_n\rangle \\ &= \int_{\Gamma_{\mathcal{D}_n} \subset T^*\mathcal{D}_n} \langle e^{-\{Q,\mathbf{b}\}} e^{-\{Q,\tilde{\mathbf{b}}\}} \vec{\Psi}_n \rangle_{S_0} \end{aligned}$$

$$M_n^{\mathcal{R}_1} = \sum_{\sigma,\{n_L,n_R\}} \int_{\Gamma_L \in T^*\mathcal{M}_L} \int_{\Gamma_R \in T^*\mathcal{M}_R} \int \mathrm{d}s\, \mathrm{d}\tilde{s} \langle \tilde{\Sigma}_L | \langle \tilde{\Sigma}_R | e^{-\{Q, s b_0 + \tilde{s}\tilde{b}_0\}} |\mathcal{R}_{L,R}\rangle |\vec{\Psi}_L\rangle |\vec{\Psi}_R\rangle$$

$$\begin{aligned} M_n^{\mathcal{R}_1} = \sum_{\sigma,\{n_L,n_R\}} \int_{\Gamma_{\mathcal{R}_1}} \sum_{\Phi_L,\Phi_R} &\langle e^{-\{Q,\mathbf{b}(u_L)\}} e^{-\{Q,\tilde{\mathbf{b}}(u_L,\tilde{u}_L)\}} \vec{\Psi}_L \Phi_L \rangle_{S_0} \\ &\times \langle \Phi_L | e^{-\{Q, s b_0 + \tilde{s}\tilde{b}_0\}} |\Phi_R\rangle \langle e^{-\{Q,\mathbf{b}(u_R)\}} e^{-\{Q,\tilde{\mathbf{b}}(u_R,\tilde{u}_R)\}} \Phi_R \vec{\Psi}_R \rangle_{S_0}, \end{aligned}$$

$$M_n^{\mathcal{R}_1} = \sum_{\sigma,\{n_L,n_R\}} \int_{\Gamma_{\mathcal{R}_1}} \langle e^{-\{Q,\mathbf{b}(u)\}} e^{-\{Q,\tilde{\mathbf{b}}(u,\tilde{u})\}} \vec{\Psi}_{,n} \rangle_{S_0}$$

$$M_n^{\Gamma_{\mathcal{D}_n}} + M_n^{\mathcal{R}_1} + M_n^{\mathcal{R}_2} + \cdots = \int_{\Gamma_n} \langle e^{-\{Q,\mathbf{b}(u)\}} e^{-\{Q,\tilde{\mathbf{b}}(u,\tilde{u})\}} \vec{\Psi}_n \rangle_{S_0}$$

$$S = \int_\Sigma P_\mu \bar{\partial} X^\mu + b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c} + \eta_{\mu\nu}\psi^\mu \bar{\partial}\psi^\nu + \eta_{\mu\nu}\tilde{\psi}^\mu \bar{\partial}\tilde{\psi}^\nu + \chi P_\mu \psi^\mu + \tilde{\chi} P_\mu \tilde{\psi}^\mu$$

$$S = \int_\Sigma P_\mu \bar{\partial} X^\mu + \eta_{\mu\nu}\psi^\mu \bar{\partial}\psi^\nu + \eta_{\mu\nu}\tilde{\psi}^\mu \bar{\partial}\tilde{\psi}^\nu + b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c} + \beta\bar{\partial}\gamma + \tilde{\beta}\bar{\partial}\tilde{\gamma}$$

$$\psi^\mu(z)\psi^\nu(\omega) = \frac{\eta^{\mu\nu}}{z-\omega} + \cdots, \beta(z)\gamma(\omega) = \frac{1}{z-\omega} + \cdots$$

$$\delta_\epsilon X^\mu = \epsilon\psi^\mu, \delta_\epsilon \psi^\mu = \epsilon P^\mu, \delta_\epsilon \tilde{\psi}^\mu = 0, \delta_\epsilon P_\mu = 0$$

$$\tilde{\delta}_\epsilon X^\mu = \epsilon\tilde{\psi}^\mu, \tilde{\delta}_\epsilon \psi^\mu = 0, \tilde{\delta}_\epsilon \tilde{\psi}^\mu = \epsilon P^\mu, \tilde{\delta}_\epsilon P_\mu = 0$$

$$\mathcal{G}(\varepsilon) = \oint \mathrm{d}z \varepsilon(z) G(z)$$

$$\begin{gathered} [\mathcal{T}(v_1), \mathcal{T}(v_2)] = -\mathcal{T}([v_1, v_2]), [\mathcal{T}(v_1), \mathcal{H}(v_2)] = -\mathcal{H}([v_1, v_2]), \\ [\mathcal{T}(v), \mathcal{G}(\varepsilon)] = -\mathcal{G}([v, \varepsilon]), [\mathcal{T}(v), \tilde{\mathcal{G}}(\varepsilon)] = -\tilde{\mathcal{G}}([v, \varepsilon]), \\ [\mathcal{G}(\varepsilon_1), \mathcal{G}(\varepsilon_2)] = -\mathcal{H}([\varepsilon_1, \varepsilon_2]), [\tilde{\mathcal{G}}(\varepsilon_1), \tilde{\mathcal{G}}(\varepsilon_2)] = -\mathcal{H}([\varepsilon_1, \varepsilon_2]), \end{gathered}$$

$$\psi^\mu(z) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \psi_r^\mu z^{-r-\frac{1}{2}}, G(z) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} G_r z^{-r-\frac{3}{2}}$$

$$G_r = \sum_{n \in \mathbb{Z}} \alpha_{n\mu} \psi_{r-n}^\mu, \tilde{G}_r = \sum_{n \in \mathbb{Z}} \alpha_{n\mu} \tilde{\psi}_{r-n}^\mu$$

$$[L_m, L_n] = (m-n)L_{m+n} + \delta_{m+n,0}\frac{D}{6}m(m^2-1), [L_m, \tilde{L}_n] = (m-n)\tilde{L}_{m+n}, [\tilde{L}_m, \tilde{L}_n] = 0$$

$$[L_m, G_r] = \frac{(m-2r)}{2}G_{m+r}, [L_m, \tilde{G}_r] = \frac{(m-2r)}{2}\tilde{G}_{m+r}$$

$$\{G_r, G_s\} = 2\tilde{L}_{r+s}, \{G_r, \tilde{G}_s\} = 0, \{\tilde{G}_r, \tilde{G}_s\} = 2\tilde{L}_{r+s}.$$

$$S = -\frac{1}{2\pi}\int \mathrm{d}^2 z \partial_\alpha X^\mu \partial^\alpha \bar{X}_\mu - i\bar{\psi}\rho^\alpha \partial_\alpha \psi$$

$$[\alpha_m^\mu, \bar{\alpha}_n^\nu] = m\delta_{m+n}\eta^{\mu\nu}, [\alpha_m^\mu, \alpha_n^\nu] = 0, [\bar{\alpha}_m^\mu, \bar{\alpha}_n^\nu] = 0$$

$$Q = \oint \mathrm{d}z j(z)$$

$$j(z) = c(T_m + T_{\beta\gamma} + \tilde{T}_{\beta\gamma}) + \gamma G + \tilde{\gamma}\tilde{G} + bc\partial c + \tilde{b}\tilde{c}\partial\tilde{c} + \frac{1}{2}\gamma^2\tilde{b} + \frac{1}{2}\tilde{\gamma}^2\tilde{b} + \tilde{c}H,$$

$$\begin{aligned} Q =& c_0\mathcal{L}_0 + \frac{1}{2}\tilde{c}_0\alpha_0^2 + \frac{1}{2}\tilde{c}_0\alpha_{-1}\cdot\alpha_1 + \alpha_0\cdot(c_1\tilde{\alpha}_{-1} + c_{-1}\tilde{\alpha}_1 + \tilde{c}_{-1}\alpha_1 + \tilde{c}_1\alpha_{-1}) \\ & -2b_0c_{-1}c_1 + 2\tilde{b}_0(c_1\tilde{c}_{-1} + \tilde{c}_{-1}c_{-1}) + \tilde{c}_0(c_{-1}\tilde{b}_1 + c_1\tilde{b}_{-1}) \\ & +\gamma_{-\frac{1}{2}}\alpha_0\cdot\psi_{\frac{1}{2}} + \gamma_{\frac{1}{2}}\alpha_0\cdot\psi_{-\frac{1}{2}} + \tilde{\gamma}_{-\frac{1}{2}}\alpha_0\cdot\tilde{\psi}_{\frac{1}{2}} + \tilde{\gamma}_{\frac{1}{2}}\alpha_0\cdot\tilde{\psi}_{-\frac{1}{2}} \\ & -2\tilde{b}_0\left(\gamma_{-\frac{1}{2}}\gamma_{\frac{1}{2}} + \tilde{\gamma}_{-\frac{1}{2}}\tilde{\gamma}_{\frac{1}{2}}\right) + \cdots \end{aligned}$$

$$\mathcal{L}_0|\Psi\rangle = 0, b_0|\Psi\rangle = 0$$

$$\begin{aligned} \mathcal{L}_0 =& (\alpha_{-1}\cdot\tilde{\alpha}_1 + \tilde{\alpha}_{-1}\cdot\alpha_1) + \frac{1}{2}\left(\psi_{-\frac{1}{2}}\cdot\psi_{\frac{1}{2}} + \tilde{\psi}_{-\frac{1}{2}}\cdot\tilde{\psi}_{\frac{1}{2}}\right) + (b_{-1}c_1 + c_{-1}b_1) + (\tilde{b}_{-1}\tilde{c}_1 + \tilde{c}_{-1}\tilde{b}_1) \\ & -\frac{1}{2}\left(\gamma_{-\frac{1}{2}}\beta_{\frac{1}{2}} - \beta_{-\frac{1}{2}}\gamma_{\frac{1}{2}}\right) - \frac{1}{2}\left(\tilde{\gamma}_{-\frac{1}{2}}\tilde{\beta}_{\frac{1}{2}} - \tilde{\beta}_{-\frac{1}{2}}\tilde{\gamma}_{\frac{1}{2}}\right) - 1 \end{aligned}$$

$$\beta = \partial\xi e^{-\phi}, \gamma = \eta e^{\phi}, \tilde{\beta} = \partial\tilde{\xi}e^{-\tilde{\phi}}, \tilde{\gamma} = \tilde{\eta}e^{\tilde{\phi}}$$

$$\mathcal{L}_0|\Psi\rangle = 0, b_0|\Psi\rangle = 0, \eta_0|\Psi\rangle = 0, \tilde{\eta}_0|\Psi\rangle = 0$$

$$\Psi(z) = \int \mathrm{d}k\left(E_{\mu\nu}(k)\psi^\mu\tilde{\psi}^\nu e^{-\phi-\tilde{\phi}}c\tilde{c} + \cdots\right)e^{ik\cdot X}$$

$$\Lambda(z) = -\int \mathrm{d}k\left(i\lambda_\mu(k)\psi^\mu\partial\tilde{\xi}e^{-2\tilde{\phi}-\phi} - i\tilde{\lambda}_\mu(k)\tilde{\psi}^\mu\partial\xi e^{-2\phi-\tilde{\phi}} + \Omega(k)\partial\tilde{c}\partial\xi\partial\tilde{\xi}e^{-2\phi-2\tilde{\phi}}\right)c\tilde{c}e^{ik\cdot X}$$

$$\delta\Psi(z) = \oint_z \mathrm{d}\omega j(\omega)\Lambda(z)$$

$$\xi(z)\eta(\omega) = \frac{1}{z-\omega} + \cdots, e^{\ell_1\phi(z)}e^{\ell_2\phi(\omega)} = (z-\omega)^{-\ell_1\ell_2}e^{(\ell_1+\ell_2)\phi(\omega)} + \cdots$$

$$\begin{aligned} \Psi(z) = \int & \mathrm{d}k\left(E_{\mu\nu}(k)\psi^\mu\tilde{\psi}^\nu e^{-\phi-\tilde{\phi}} + 2e(k)\eta\partial\tilde{\xi}e^{-2\tilde{\phi}} + 2\tilde{e}(k)\tilde{\eta}\partial\xi e^{-2\phi}\right. \\ & \left. + if_\mu(k)\psi^\mu\partial\tilde{\xi}e^{-2\tilde{\phi}-\phi}\partial\tilde{c} + i\tilde{f}_\mu(k)\tilde{\psi}^\mu\partial\xi e^{-2\phi-\tilde{\phi}}\partial\tilde{c}\right)c\tilde{c}e^{ik\cdot X} \end{aligned}$$

$$\delta E_{\mu\nu}(k) = ik_\mu\tilde{\lambda}_\nu(k) + ik_\nu\lambda_\mu(k)\ \delta e(k) = -\frac{i}{2}k^\mu\lambda_\mu(k) + \Omega(k), \delta\tilde{e}(k) = \frac{i}{2}k^\mu\tilde{\lambda}_\mu(k) + \Omega(k)$$

$$\delta f_\mu(k) = \frac{1}{2}k^2\lambda_\mu(k) + ik_\mu\Omega(k), \delta\tilde{f}_\mu(k) = -\frac{1}{2}k^2\tilde{\lambda}_\mu(k) + ik_\mu\Omega(k)$$

$$\delta E_{\mu\nu}(x) = \partial_\mu\tilde{\lambda}_\nu(x) + \partial_\nu\lambda_\mu(x)\ \delta e(x) = -\frac{1}{2}\partial^\mu\lambda_\mu(x) + \Omega(x), \delta\tilde{e}(x) = \frac{1}{2}\partial^\mu\tilde{\lambda}_\mu(x) + \Omega(x)$$

$$\delta f_\mu(x) = -\frac{1}{2}\Box\lambda_\mu(x) + \partial_\mu\Omega(x), \delta\tilde{f}_\mu(x) = \frac{1}{2}\Box\tilde{\lambda}_\mu(x) + \partial_\mu\Omega(x),$$

$$\delta\tilde{f}_\mu(x) = \frac{1}{2}\partial^\nu\left(\delta E_{\nu\mu}(x)\right) + \partial_\mu(\delta e(x)), \delta f_\mu(x) = -\frac{1}{2}\partial^\nu\left(\delta E_{\mu\nu}(x)\right) + \partial_\mu(\delta\tilde{e}(x))$$

$$|\Psi\rangle = \int \mathrm{d}k \left(E_{\mu\nu}(k)\psi^{\mu}_{-\frac{1}{2}}\tilde{\psi}^{\nu}_{-\frac{1}{2}} + 2e(k)\gamma_{-\frac{1}{2}}\tilde{\beta}_{-\frac{1}{2}} + 2\tilde{e}(k)\tilde{\gamma}_{-\frac{1}{2}}\beta_{-\frac{1}{2}} \right.$$
$$\left. + i f_{\mu}(k)\psi^{\mu}_{-\frac{1}{2}}\tilde{\beta}_{-\frac{1}{2}}\tilde{c}_0 + i\tilde{f}_{\mu}(k)\tilde{\psi}^{\mu}_{-\frac{1}{2}}\beta_{-\frac{1}{2}}\tilde{c}_0 \right) c_1\tilde{c}_1|-1,-1,k\rangle$$

$$\langle\Psi| = \int \mathrm{d}k\langle -1,-1,-k|c_{-1}\tilde{c}_{-1} \left(E_{\mu\nu}(k)\psi^{\mu}_{\frac{1}{2}}\tilde{\psi}^{\gamma}_{\frac{1}{2}} + 2e(k)\gamma_{\frac{1}{2}}\tilde{\beta}_{\frac{1}{2}} + 2\tilde{e}(k)\tilde{\gamma}_{\frac{1}{2}}\beta_{-\frac{1}{2}} \right.$$
$$\left. + i f_{\mu}(k)\psi^{\mu}_{\frac{1}{2}}\tilde{\beta}_{\frac{1}{2}}\tilde{c}_0 + i\tilde{f}_{\mu}(k)\tilde{\psi}^{\mu}_{\frac{1}{2}}\beta_{\frac{1}{2}}\tilde{c}_0 \right)$$

$$S_2[\Psi] = \frac{1}{2}\langle\Psi|c_0 Q|\Psi\rangle$$

$$S_2[\Psi] = \int \mathrm{d}k\, \mathrm{d}k'\langle -1,-1,-k'|c_{-1}\tilde{c}_{-1}c_0 c_1\tilde{c}_1\mathcal{F}|-1,-1,k\rangle$$

$$\langle -1,-1,-k'|c_{-1}\tilde{c}_{-1}c_0\tilde{c}_0 c_1\tilde{c}_1|-1,-1,k\rangle = \delta(k+k')$$

$$S_2[\Psi] = \int \mathrm{d}k \left(-\frac{1}{4}E_{\mu\nu}(-k)k^2 E^{\mu\nu}(k) - 2\tilde{e}(-k)p^2 e(k) - i f^{\mu}(-k)k^{\nu}E_{\mu\nu}(k) + i\tilde{f}^{\nu}(-k)k^{\mu}E_{\mu\nu}(k) \right.$$
$$\left. + 2i f^{\mu}(-k)k_{\mu}\tilde{e}(k) + 2i\tilde{f}^{\mu}(-k)k_{\mu}e(k) - f_{\mu}(-k)f^{\mu}(k) - \tilde{f}^{\mu}(-k)\tilde{f}_{\mu}(k) \right)$$

$$S_2[\Psi] = \int \mathrm{d}x \left(\frac{1}{4}E_{\mu\nu}(x)\,\Box\, E^{\mu\nu}(x) + 2\tilde{e}(x)\,\Box\, e(x) - f_{\mu}(x)f^{\mu}(x) - \tilde{f}^{\mu}(x)\tilde{f}_{\mu}(x) \right.$$
$$\left. - f^{\mu}(x)\left[\partial^{\nu}E_{\mu\nu}(x) - 2\partial_{\mu}\tilde{e}(x)\right] + \tilde{f}^{\nu}(x)\left[\partial^{\mu}E_{\mu\nu}(x) + \partial_{\nu}e(x)\right] \right)$$

$$f_{\mu}(x) = -\frac{1}{2}\left(\partial^{\nu}E_{\mu\nu}(x) - 2\partial_{\mu}\tilde{e}(x)\right), \tilde{f}_{\mu}(x) = \frac{1}{2}\left(\partial^{\nu}E_{\nu\mu}(x) + 2\partial_{\mu}e(x)\right)$$

$$S_2[E,e,\tilde{e}] = \int \mathrm{d}x \left(\frac{1}{4}E_{\mu\nu}(x)\,\Box\, E^{\mu\nu}(x) + 2\tilde{e}(x)\,\Box\, e(x) + f_{\mu}(x)f^{\mu}(x) + \tilde{f}^{\mu}(x)\tilde{f}_{\mu}(x) \right)$$

$$\vartheta^{\pm} = \frac{1}{2}(e \pm \tilde{e})$$

$$\delta\vartheta^{+} = \frac{1}{2}\partial^{\mu}\epsilon_{\mu} + \Omega, \delta\vartheta^{-} = -\frac{1}{2}\partial^{\mu}\zeta_{\mu}$$

$$\zeta_{\mu} = \frac{1}{2}(\lambda_{\mu} + \tilde{\lambda}_{\mu}), \epsilon_{\mu} = -\frac{1}{2}(\lambda_{\mu} - \tilde{\lambda}_{\mu})$$

$$S_2 = \int \mathrm{d}x \left(\frac{1}{4}E_{\mu\nu}\,\Box\, E^{\mu\nu} - 4\vartheta^{-}\,\Box\,\vartheta^{-} + \frac{1}{4}\left(\partial^{\nu}E_{\mu\nu}\right)^2 - 2\vartheta^{-}\left(\partial^{\mu}\partial^{\nu}E_{\mu\nu}\right) + \frac{1}{4}\left(\partial^{\nu}E_{\nu\mu}\right)^2 \right)$$

$$S_2 = \int \mathrm{d}x \left(\frac{1}{4}h_{\mu\nu}\,\Box\, h^{\mu\nu} + \frac{1}{2}\left(\partial^{\nu}h_{\mu\nu}\right)^2 - 2\vartheta^{-}\left(\partial^{\mu}\partial^{\nu}h_{\mu\nu}\right) - 4\vartheta^{-}\,\Box\,\vartheta^{-} + \frac{1}{4}b_{\mu\nu}\,\Box\, b^{\mu\nu} + \frac{1}{2}\left(\partial^{\nu}b_{\mu\nu}\right)^2 \right)$$

$$\phi = \vartheta^{-} + \frac{1}{4}h$$

$$\frac{1}{4}b_{\mu\nu}\,\Box\, b^{\mu\nu} + \frac{1}{2}\left(\partial^{\nu}b_{\mu\nu}\right)^2 \approx -\frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}$$

$$S_2[h,b,\phi] = \int \mathrm{d}x \left(\frac{1}{4}h_{\mu\nu}\,\Box\, h^{\mu\nu} + \frac{1}{2}\left(\partial^{\nu}h_{\mu\nu}\right)^2 + \frac{1}{2}h\partial^{\mu}\partial^{\nu}h_{\mu\nu} - \frac{1}{4}h\,\Box\, h \right.$$
$$\left. - 4\phi\,\Box\,\phi + 2h\,\Box\,\phi - 2\phi\partial^{\mu}\partial^{\nu}h_{\mu\nu} - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} \right)$$

$$S = \int e^{-\phi}\left(R * 1 - \frac{1}{2}H_{(3)} \wedge * H_{(3)} + * \,\mathrm{d}\phi \wedge \mathrm{d}\phi \right)$$

$$\delta h_{\mu\nu} = \partial_{\mu}\zeta_{\nu} + \partial_{\nu}\zeta_{\mu}, \delta b_{\mu\nu} = \partial_{\mu}\epsilon_{\nu} - \partial_{\nu}\epsilon_{\mu}, \delta\phi = 0$$

$$S_2[\Psi] = \frac{1}{2}\langle\Psi|c_0\tilde{c}_0\tilde{L}_0|\Psi\rangle$$

$$j(z) = c(z)T(z) + \gamma(z)S(z) + \tilde{\gamma}(z)\tilde{S}(z) + \tilde{c}(z)H(z)$$

$$\mathcal{X}(z) = \oint_z \mathrm{d}\omega j(\omega)\xi(z), \tilde{\mathcal{X}}(z) = \oint_z \mathrm{d}\omega j(\omega)\tilde{\xi}(z)$$

$$\mathcal{X}(z) = c\partial\xi + e^{\phi}P_\mu\psi^\mu + \frac{1}{2}\partial\eta e^{2\phi}\tilde{b} + \frac{1}{2}\partial\left(\eta e^{2\phi}\tilde{b}\right)$$

$$\tilde{\mathcal{X}}(z) = c\partial\tilde{\xi} + e^{\tilde{\phi}}P_\mu\tilde{\psi}^\mu + \frac{1}{2}\partial\tilde{\eta}e^{2\tilde{\phi}}\tilde{b} + \frac{1}{2}\partial\left(\tilde{\eta}e^{2\tilde{\phi}}\tilde{b}\right)$$

$$\mathcal{X}_0 = \int_{\mathcal{C}} \frac{\mathrm{d}z}{z}\mathcal{X}(z)$$

$$\Psi^{(-1,0)}(z) = \tilde{\mathcal{X}}_0\Psi^{(-1,-1)}(z) = \oint_z \frac{\mathrm{d}\omega}{\omega - z}\tilde{\mathcal{X}}(\omega)\Psi^{(-1,-1)}(z)$$

$$\begin{aligned}\Psi^{(-1,0)}(z) = \int \mathrm{d}k\Big(&-e(k)\eta - \left(\tilde{e}(k)\partial\tilde{\xi}\partial^2 c\tilde{c} + i\tilde{f}_\mu(k)\tilde{\Pi}^\mu\partial\xi\partial\tilde{c}\tilde{c}\right)e^{-2\phi} \\ &+2\tilde{e}(k)(P\cdot\tilde{\psi} + ik\cdot\partial\tilde{\psi})\tilde{\eta}\partial\xi\tilde{c}e^{\tilde{\phi}-2\phi} + \left(E_{\mu\nu}(k)\tilde{\Pi}^\nu\psi^\mu\tilde{c} + \frac{i}{2}f_\mu(k)\psi^\mu\partial\tilde{c}\right)e^{-\phi} \\ &+\frac{1}{2}E_{\mu\nu}(k)\tilde{\eta}\psi^\mu\tilde{\psi}^\nu e^{-\phi-\tilde{\phi}} - \tilde{e}(k)\left(2\partial\tilde{\eta}\tilde{b}\tilde{c} + \frac{3}{2}\partial^2\tilde{\eta}\right)\tilde{\eta}\partial\xi e^{-2\phi+2\tilde{\phi}} \\ &+\frac{i}{2}\tilde{f}_\mu(k)\tilde{\psi}^\mu(\tilde{\eta}\partial\tilde{c} - 2\partial\tilde{\eta})\partial\xi e^{-2\phi+\tilde{\phi}}\Big)ce^{ik\cdot X}\end{aligned}$$

$$\Psi^{(0,0)}(z) = \mathcal{X}_0\tilde{\mathcal{X}}_0\Psi^{(-1,-1)}(z) := \oint_z \frac{\mathrm{d}\omega}{\omega - z}\oint_z \frac{\mathrm{d}\omega'}{\omega' - z}\mathcal{X}(\omega)\mathcal{X}(\omega')\Psi^{(-1,-1)}(z)$$

$$\begin{aligned}\Psi^{(0,0)}(z) = \int \mathrm{d}k\Big(&E_{\mu\nu}(k)\Pi^\mu\tilde{\Pi}^\nu\tilde{c} + \frac{1}{2}e(k)\partial^2 c + \frac{1}{2}\tilde{e}(k)\partial^2 c + \frac{i}{2}f_\mu(k)\Pi^\mu\partial\tilde{c} + \frac{i}{2}\tilde{f}_\mu(k)\tilde{\Pi}^\mu\partial\tilde{c} \\ &-\left(e(k)(P\cdot\psi + ik\cdot\partial\psi)\eta - \frac{1}{2}E_{\mu\nu}(k)\eta\tilde{\Pi}^\nu\psi^\mu + \frac{i}{2}f_\mu(k)\psi^\mu\partial\eta\right)e^{\phi} \\ &-\left(\tilde{e}(k)(P\cdot\tilde{\psi} + ik\cdot\partial\tilde{\psi})\eta - \frac{1}{2}E_{\mu\nu}(k)\Pi^\mu\tilde{\eta}\tilde{\psi}^\nu + \frac{i}{2}\tilde{f}_\mu(k)\tilde{\psi}^\nu\partial\tilde{\eta}\right)e^{\tilde{\phi}} \\ &-e(k)\partial\eta\tilde{b}\eta e^{2\phi} - \tilde{e}(k)\partial\tilde{\eta}\tilde{b}\tilde{\eta}e^{2\tilde{\phi}}\Big)ce^{ik\cdot X}\end{aligned}$$

$$\Pi^\mu = P^\mu + (k\cdot\psi)\psi^\mu, \tilde{\Pi}^\mu = P^\mu + (k\cdot\tilde{\psi})\tilde{\psi}^\mu$$

$$\langle\Sigma| = \int\prod_{i=1}^{n}\mathrm{d}p_{(i)}\delta\left(\sum p_{(i)}\right)\langle q_1; p_{(1)}|\ldots\langle q_n; p_{(n)}|\exp\left(V_m + V_{\mathrm{gh}} + \tilde{V}_{\mathrm{gh}}\right)\mathcal{Z}$$

$$V_m = \sum_{m,n}\sum_{i,j}\left(\mathcal{S}^{mn}(z_i,z_j)\tilde{\alpha}_m^{(i)}\cdot\alpha_n^{(j)} + \mathcal{K}^{rs}(z_i,z_j)\psi_r^{(i)}\cdot\psi_s^{(j)} + \mathcal{K}^{rs}(z_i,z_j)\tilde{\psi}_r^{(i)}\cdot\tilde{\psi}_s^{(j)}\right)$$

$$\mathcal{K}^{rs}(z_i,z_j) = \oint_{t_i=0}\mathrm{d}t_i\oint_{t_j=0}\mathrm{d}t_j t_i^{-m-\frac{1}{2}}t_j^{-n-\frac{1}{2}}\sqrt{h_i'h_j'}\frac{1}{h_i(t_i) - h_j(t_j)}$$

$$\{\Psi^3\} = \langle\Sigma|\left|\Psi^{(-1,-1)}\right\rangle\left|\Psi^{(-1,-1)}\right\rangle\left|\Psi^{(0,0)}\right\rangle,$$

$$\{\Psi^3\} = \left\langle\Psi^{(-1,-1)}(z_1)\Psi^{(-1,-1)}(z_2)\Psi^{(0,0)}(z_3)\right\rangle$$

$$S_3 = \int \mathrm{d}x \Big(-\frac{1}{8} E_{\mu\nu} \big(-(\partial_\lambda E^{\lambda\nu})(\partial_\rho E^{\mu\rho}) - (\partial_\lambda E^{\lambda\rho})(\partial_\rho E^{\mu\nu}) - 2(\partial^\mu E_{\lambda\rho})(\partial^\nu E^{\lambda\rho})$$

$$+2(\partial^\mu E_{\lambda\rho})(\partial^\rho E^{\lambda\nu}) + 2(\partial^\lambda E^{\mu\lambda})(\partial^\nu E_{\lambda\rho}) \big) + \frac{1}{2} E_{\mu\nu} f^\mu \tilde{f}^\nu - \frac{1}{2} f^\mu f_\mu \tilde{e} + \frac{1}{2} \tilde{f}^\mu \tilde{f}_\mu e$$

$$-\frac{1}{8} E_{\mu\nu} \big((\partial^\mu \partial^\nu e)\tilde{e} - (\partial^\mu e)(\partial^\nu \tilde{e}) - (\partial^\nu e)(\partial^\mu \tilde{e}) + e \partial^\mu \partial^\nu \tilde{e} \big)$$

$$-\frac{1}{4} f^\mu \left(E_{\mu\nu} \partial^\nu \tilde{e} + \partial^\nu (E_{\mu\nu} \tilde{e}) \right) + \frac{1}{4} f^\mu \left((\partial_\mu e)\tilde{e} - e \partial_\mu \tilde{e} \right)$$

$$-\frac{1}{4} \tilde{f}^\nu \left(E_{\mu\nu} \partial^\mu e + \partial^\mu (E_{\mu\nu} e) \right) + \frac{1}{4} \tilde{f}^\nu \big((\partial_\nu e)\tilde{e} - e \partial_\nu \tilde{e} \big) \Big)$$

$$\{\Psi^n\} = \int_{\mathcal{V}_n} \langle \Sigma | \mathcal{B}_{n-3}(\vec{v}) | \vec{\Psi} \rangle$$

$$\mathcal{B}_{n-3} = \sum_{r=0}^{n-3} B_{n-3}^{(r)} \wedge K_n^{n-3-r} \wedge \widetilde{K}_n^{n-3-r}$$

$$K_n^{(r)} \Big|_{\partial \mathcal{V}_n} = \sum_{s=0}^{r} K_{n_L}^{(r-s)} \wedge K_{n_R}^{(s)}$$

$$K_n^{(0)} = \sum_\alpha A^{(\alpha)}(\tau_1, \dots, \tau_{2n-6}) \mathcal{X}\left(w_1^{(\alpha)}(\tau) \right) \dots \mathcal{X}\left(w_{n-2}^{(\alpha)}(\tau) \right)$$

$$K^{(r)} = \left[\prod_{i=1}^{n-3} \left(\mathcal{X}(z_i) - \partial \xi(z_i) \mathrm{d}z_i \right) \right]^r$$

$$\mathcal{B}_{n-3} = \sum_{r=0}^{n-3} B_{n-3}^{(r)} \wedge K^{(r)} \wedge \widetilde{K}^{(r)}$$

$$S = -\frac{1}{2} \langle \Phi | c_0 Q \mathcal{G} | \Phi \rangle + \langle \Phi | c_0 Q | \Psi \rangle + \sum_{n=3}^{\infty} \frac{1}{n!} \{\Psi^n\}$$

$$Q|\Phi\rangle + \sum_{n=2}^{\infty} \frac{1}{n!} [\Psi^n] = 0$$

$$S = \int_\Sigma \Pi_\mu \bar{\partial} X^\mu + i \bar{\psi}_\mu \bar{\partial} \psi^\mu + \frac{1}{2} e P^2$$

$$\Pi_\mu = P_\mu + i \Gamma_{\mu\nu}^\lambda \psi^\nu \bar{\psi}_\lambda, P^2 = g^{\mu\nu}(X) P_\mu P_\nu, H(z) = \tfrac{1}{2} P^2(z)$$

$$\langle \Psi | c_0 Q | \Psi \rangle = \int \mathrm{d}x \left(\frac{1}{4} \nabla_\mu h_{\nu\lambda} \nabla^\mu h^{\nu\lambda} - \frac{1}{2} \nabla_\mu h_{\nu\lambda} \nabla^\nu h^{\mu\lambda} + \frac{1}{2} \nabla_\nu h \nabla_\mu h^{\mu\nu} - \frac{1}{4} \nabla_\mu h \nabla^\mu h + \cdots \right),$$

$$\phi(t) = \sum_n \phi_n t^{-n-d}$$

$$\phi(t) \to h[\phi(t)] = (h'(t))^d \phi(h(t)).$$

$$h[\phi(t)] = \sum_n h[\phi_n] t^{-n-d}$$

$$h[\phi_n] = \oint_{t=0} \frac{\mathrm{d}t}{2\pi i} t^{n+d-1} h[\phi(t)]$$

$$h[\phi_n] = \oint_{t=0} \frac{\mathrm{d}t}{2\pi i} t^{n+d-1} (h'(t))^d \phi(h(t))$$

$$h[\alpha_{-n}^{\mu}] = \oint_{t=0} \frac{\mathrm{d}t}{2\pi i} t^{n+d-1} h'(t) \partial X^{\mu}(h(t))$$

$$V_X = \sum_{i,j=1}^{n} \sum_{m,n>0} \mathcal{N}_{mn}(z_i, z_j) \alpha_n^{(i)} \cdot \alpha_m^{(j)}$$

$$\mathcal{N}_{mn}(z_i, z_j) = \frac{1}{2n} \langle 0 | \exp \left(\sum_{k,l} \sum_{p,q>0} \mathcal{N}_{pq}(z_i, z_j) \alpha_p^{(k)} \cdot \alpha_q^{(l)} \right) \alpha_{-m}^{(i)} \cdot \alpha_{-n}^{(j)} | 0 \rangle$$

$$\left\langle h_i \left[\alpha_{-n}^{\mu(i)} \right] h_j \left[\alpha_{-m}^{\nu(j)} \right] \right\rangle = \frac{1}{n} \oint_0 \frac{\mathrm{d}t_i}{2\pi i} t^{-n} h_i'(t_i) \oint_0 \frac{\mathrm{d}t_j}{2\pi i} t^{-m} h_j'(t_j) \frac{-\eta^{\mu\nu}}{\left(h_i(t_i) - h_j(t_j) \right)^2}$$

$$\mathcal{K}_{nm}(z_i, z_j) = -\oint \frac{\mathrm{d}t_i}{2\pi i} \oint \frac{\mathrm{d}t_j}{2\pi i} t_i^{-n+1} t_j^{-m-2} (h_i'(t_i))^2 \left(h_j'(t_j) \right)^{-1} \frac{1}{h_i(t_i) - h_j(t_j)}$$

$$\int_{\Sigma} \mathrm{d}^2 z \bar{\partial} b_{\mathrm{cl}} (c_{-1} z^2 + c_0 z + c_1) = \sum_{i=1}^{N} \oint_{z_i} b_{\mathrm{cl}}^{(i)} (c_{-1} z^2 + c_0 z + c_1)$$

$$\int_{\Sigma} \mathrm{d}^2 z \bar{\partial} b_{\mathrm{cl}} (c_{-1} z^2 + c_0 z + c_1) = \sum_{i} \sum_{n} \mathcal{M}_{nm}(z_i) b_m^{(i)} \mathcal{C}^n$$

$$\mathcal{M}_{nm}(z_i) = \oint_{t_i=0} \frac{\mathrm{d}t_i}{2\pi i} t_i^{-m-2} (h_i'(t))^{-1} (h_i(t))^{n+1}$$

$$\mathcal{S}_{nm}(z_i, z_j) = -\oint \frac{\mathrm{d}t_i}{2\pi i} \oint \frac{\mathrm{d}t_j}{2\pi i} t_i^{-n-\frac{1}{2}} t_j^{-m-\frac{1}{2}} \sqrt{h_i'(t_i) h_j'(t_j)} \frac{1}{h_i(t_i) - h_j(t_j)}$$

$$\langle \Sigma_{X,P} | = \langle p_1 | \ldots \langle p_n | e^{V_{X,P}}$$

$$\langle \Sigma_{X,P} | \alpha_{-p}^{(i)} = \langle p_1 | \ldots \langle p_n | \left(\alpha_{-p}^{(i)} + \left[V_{X,P}, \alpha_{-p}^{(i)} \right] + \frac{1}{2!} \left[V_{X,P}, \left[V_{X,P}, \alpha_{-p}^{(i)} \right] \right] + \cdots \right) e^{V_{X,P}}$$

$$\left[V_{X,P}, \alpha_{-p}^{(i)} \right] = \sum_{j \neq i} \sum_{n \geq 0} \mathcal{S}_{pn}(z_i, z_j) \alpha_n^{(j)}$$

$$\langle \Sigma_{X,P} | \alpha_{-p}^{(i)} = \begin{cases} \langle p_1 | \ldots \langle p_n | \sum_{j \neq i} \sum_{n \geq 0} \mathcal{S}_{pn}(z_i, z_j) \alpha_n^{(j)} e^{V_{X,P}}, & p > 0 \\ \langle p_1 | \ldots \langle p_n | \alpha_{-p}^{(i)} e^{V_{X,P}} & p \leq 0 \end{cases}$$

$$\langle \Sigma | \alpha_{-p}^{(i)} = \langle \Sigma | \sum_{j \neq i} \sum_{n \geq 0} \mathcal{S}_{pn}(z_i, z_j) \alpha_n^{(j)}, p > 0$$

$$\langle k_j | e^{V_{X,P}} \alpha_{-p}^{(i)} | k_j \rangle = \sum_{j \neq i} \mathcal{S}_{p0}(z_i, z_j) k_j$$

$$\langle k_j | e^{V_{X,P}} \alpha_{-1}^{(i)} | k_j \rangle = \sum_{j \neq i} \frac{k_j}{z_i - z_j},$$

$$\langle \Sigma_{X,P} | \alpha_{-p}^{(i)} \cdot \alpha_{-q}^{(i)} = \langle p_1 | \ldots \langle p_n | \left(\alpha_{-p}^{(i)} \cdot \alpha_{-q}^{(i)} + \alpha_{-q}^{(i)} \cdot S_p^{(i)} + \alpha_{-p}^{(i)} \cdot S_q^{(i)} + S_p^{(i)} \cdot S_q^{(i)} \right) e^{V_{X,(t)}}$$

$$S_p^{(i)} := \sum_{j \neq i} \sum_{n \geq 0} \mathcal{S}_{pn}(z_i, z_j) \alpha_n^{(j)}$$

$$V = c\tilde{c} E_{\mu\nu} \psi^\mu \tilde{\psi}^\nu e^{ip\cdot X}$$

$$|\Psi\rangle = \int \mathrm{d}p \left(E_{\mu\nu} \psi^\mu_{-\frac{1}{2}} \tilde{\psi}^\nu_{-\frac{1}{2}} + \cdots \right) c_1 \tilde{c}_1 | -1, -1, p\rangle$$

$$| -1, -1, p\rangle \equiv e^{-\phi(0) - \tilde{\phi}(0)} |p\rangle.$$

$$|\Lambda\rangle = -\int \mathrm{d}p \left(i\lambda_\mu \psi^\mu_{-\frac{1}{2}} \tilde{\beta}_{-\frac{1}{2}} - i\tilde{\lambda}_\mu \tilde{\psi}^\mu_{-\frac{1}{2}} \beta_{-\frac{1}{2}} + \Omega \tilde{c}_0 \beta_{-\frac{1}{2}} \tilde{\beta}_{-\frac{1}{2}} \right) c_1 \tilde{c}_1 | -1, -1, p\rangle$$

$$\delta|\Psi\rangle = Q|\Lambda\rangle$$

$$\begin{aligned} Q|\Lambda\rangle = -\int \mathrm{d}p \Big(& \frac{1}{2} \tilde{c}_0 \alpha_0^2 + \gamma_{-\frac{1}{2}} \alpha_0 \cdot \psi_{\frac{1}{2}} + \gamma_{\frac{1}{2}} \alpha_0 \cdot \psi_{-\frac{1}{2}} + \tilde{\gamma}_{-\frac{1}{2}} \alpha_0 \cdot \tilde{\psi}_{\frac{1}{2}} + \tilde{\gamma}_{\frac{1}{2}} \alpha_0 \cdot \tilde{\psi}_{-\frac{1}{2}} \\ & -2\tilde{b}_0 \left(\gamma_{-\frac{1}{2}} \gamma_{\frac{1}{2}} + \tilde{\gamma}_{-\frac{1}{2}} \tilde{\gamma}_{\frac{1}{2}} \right) + \cdots \Big) \\ & \times \left(i\lambda_\mu(p) \psi^\mu_{-\frac{1}{2}} \tilde{\beta}_{-\frac{1}{2}} - i\tilde{\lambda}_\mu(p) \tilde{\psi}^\mu_{-\frac{1}{2}} \beta_{-\frac{1}{2}} + \Omega(p) \tilde{c}_0 \beta_{-\frac{1}{2}} \tilde{\beta}_{-\frac{1}{2}} \right) c_1 \tilde{c}_1 | -1, -1, p\rangle \end{aligned}$$

$$\begin{aligned} Q|\Lambda\rangle = \int \mathrm{d}p \Big(& (ip_\mu \tilde{\lambda}_\nu + ip_\nu \lambda_\mu) \psi^\mu_{-\frac{1}{2}} \tilde{\psi}^\nu_{-\frac{1}{2}} \\ & + 2\left(-\frac{i}{2} p \cdot \lambda + \Omega \right) \gamma_{-\frac{1}{2}} \tilde{\beta}_{-\frac{1}{2}} + 2\left(\frac{i}{2} p \cdot \tilde{\lambda} + \Omega \right) \tilde{\gamma}_{-\frac{1}{2}} \beta_{-\frac{1}{2}} \\ & + \left(\frac{i}{2} p^2 \lambda_\mu - p_\mu \Omega \right) \psi^\mu_{-\frac{1}{2}} \tilde{\beta}_{-\frac{1}{2}} \tilde{c}_0 + \left(-\frac{i}{2} p^2 \tilde{\lambda}_\mu - p_\mu \Omega \right) \tilde{\psi}^\mu_{-\frac{1}{2}} \beta_{-\frac{1}{2}} \tilde{c}_0 \Big) c_1 \tilde{c}_1 | -1, -1, p\rangle \end{aligned}$$

$$\begin{aligned} |\Psi\rangle = \int \mathrm{d}p \Big(& E_{\mu\nu}(p) \psi^\mu_{-\frac{1}{2}} \tilde{\psi}^\nu_{-\frac{1}{2}} + 2e(p) \gamma_{-\frac{1}{2}} \tilde{\beta}_{-\frac{1}{2}} + 2\tilde{e}(p) \tilde{\gamma}_{-\frac{1}{2}} \beta_{-\frac{1}{2}} \\ & + if_\mu(p) \psi^\mu_{-\frac{1}{2}} \tilde{\beta}_{-\frac{1}{2}} \tilde{c}_0 + i\tilde{f}_\mu(p) \tilde{\psi}^\mu_{-\frac{1}{2}} \beta_{-\frac{1}{2}} \tilde{c}_0 \Big) c_1 \tilde{c}_1 | -1, -1, p\rangle \end{aligned}$$

$$T(z) = P_\mu \partial X^\mu, T_{\mathrm{gh}}(z) = (\partial b)c - 2\partial(bc), \tilde{T}_{\mathrm{gh}}(z) = (\partial \tilde{b})\tilde{c} - 2\partial(\tilde{b}\tilde{c})$$

$$\mathrm{d}^n p = \prod_{i=1}^n \mathrm{d}p_{(i)}$$

$$\mathbf{b}(\vec{v}^a) = \oint \mathrm{d}z b^{(a)}(z) = b_{-1}^{(a)}$$

$$14 \ \mathrm{d}\omega = \frac{\mathrm{d}z_i \ \mathrm{d}z_j \ \mathrm{d}z_k}{(z_i - z_j)(z_j - z_k)(z_k - z_i)}$$

$$G \mapsto \chi G, \chi = \mathbf{1} + \sum_{i=1}^{2n} \frac{\chi_i}{Z - Z_i}.$$

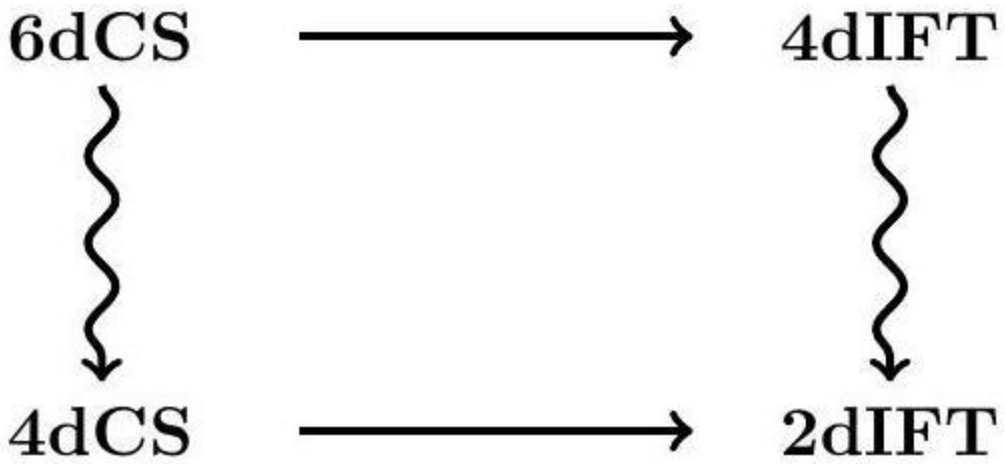
$D \geq 4$ vacuum gravity	$\mathrm{SL}(D-2, \mathbb{R})/\mathrm{SO}(D-2)$
4d EMd gravity	$\mathrm{SU}(2,1)/(\mathrm{SU}(2) \times \mathrm{U}(1))$
4d $N = 4$ from 10d SuGra	$\mathrm{SO}(8,8)/(\mathrm{SO}(8) \times \mathrm{SO}(8))$
4d $N = 8$ from 11d SuGra	$\mathrm{E}_{8(+8)}/\mathrm{SO}(16)$

$$\mathcal{L} \mapsto \chi \mathcal{L} \chi^{-1} - \mathrm{d}\chi \chi^{-1}.$$

$$S_{4\mathrm{dCS}} = \frac{1}{2\pi \mathrm{i}} \int_{\mathbb{CP}^1 \times \mathbb{R}^2} \omega \wedge \operatorname{tr}\left(A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A\right)$$

$$W = z + \frac{\rho}{2}(Z^{-1} - Z)$$

$$S_{6\mathrm{dCS}} = \frac{1}{2\pi \mathrm{i}} \int_{\mathbb{PT}} \Omega \wedge \operatorname{tr}\left(\mathcal{A} \wedge \bar{\partial}\mathcal{A} + \frac{2}{3}\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}\right),$$



$$\mathrm{d}s_4^2 = e^{2\nu}(\,\mathrm{d}\rho^2 + \mathrm{d}z^2) + \rho G_{mn}\,\mathrm{d}x^m\,\mathrm{d}x^n, m, n \in \{3,4\}$$

$$\det G = \epsilon, \epsilon = \pm 1.$$

$$U_\rho \equiv \rho \partial_\rho G G^{-1}, U_z \equiv \rho \partial_z G G^{-1}$$

$$\partial_\rho U_\rho + \partial_z U_z = 0$$

$$\partial_\rho \nu = \frac{1}{8\rho}\operatorname{tr}(U_\rho^2 - U_z^2) - \frac{1}{2\rho}, \partial_z \nu = \frac{1}{4\rho}\operatorname{tr}(U_\rho U_z)$$

$$\mathcal{L} = \frac{-\partial_\xi G G^{-1}}{1 - \mathrm{i}Z}\,\mathrm{d}\xi + \frac{-\partial_{\bar{\xi}} G G^{-1}}{1 + \mathrm{i}Z}\,\mathrm{d}\bar{\xi}, Z = \frac{2\mathrm{i}}{\xi - \bar{\xi}}\left(\frac{\xi + \bar{\xi}}{2} - W \pm \sqrt{(W - \xi)(W - \bar{\xi})}\right)$$

$$\partial_\xi \mathcal{L}_{\bar{\xi}} - \partial_{\bar{\xi}} \mathcal{L}_\xi + [\mathcal{L}_\xi, \mathcal{L}_{\bar{\xi}}] = 0\ \forall W \iff \text{Eq. (2.4).}$$

$$\nabla_\xi \Psi = 0, \nabla_{\bar{\xi}} \Psi = 0, \nabla \equiv\ \mathrm{d} + \mathcal{L}$$

$$\partial_\xi \mapsto \partial_\xi + (\xi - \bar{\xi})^{-1}\frac{1 + \mathrm{i}Z}{1 - \mathrm{i}Z} Z \partial_Z, \partial_{\bar{\xi}} \mapsto \partial_{\bar{\xi}} - (\xi - \bar{\xi})^{-1}\frac{1 - \mathrm{i}Z}{1 + \mathrm{i}Z} Z \partial_Z$$

$$\left(\partial_z - \frac{2\lambda^2}{\lambda^2 + \rho^2}\partial_\lambda\right)\Psi = \frac{\rho U_z - \lambda U_\rho}{\lambda^2 + \rho^2}\Psi, \left(\partial_\rho + \frac{2\lambda\rho}{\lambda^2 + \rho^2}\partial_\lambda\right)\Psi = \frac{\rho U_\rho + \lambda U_z}{\lambda^2 + \rho^2}\Psi$$

$$\sqrt{\det g^{(4)}} R^{(4)} = -\frac{\rho}{4}\operatorname{tr}\left[(G^{-1}\partial_\rho G)^2 + (G^{-1}\partial_z G)^2\right] + \mathcal{B}$$

$$x^M = (x^\mu, x^m), \mu = 1,2, m = 3,4,$$

$$E_M^A = \begin{pmatrix} \sqrt{\lambda} e_\mu^\alpha & 0 \\ \sqrt{\rho} \hat{e}_n^a B_\mu^n & \sqrt{\rho} \hat{e}_m^a \end{pmatrix}, g_{MN}^{(4)} = \eta_{AB} E_M^A E_N^B$$

$$\rho = \det E_m^a \iff \det \hat{e}_m^a = 1.$$

$$E^\alpha = \sqrt{\lambda} e^\alpha, E^a = \sqrt{\rho}(\hat{e}^a + \hat{e}_n^a B^n), \mathrm{d}s_4^2 = \eta_{\alpha\beta} E^\alpha E^\beta + \eta_{ab} E^a E^b,$$

$$x^m \mapsto x^m + \Gamma^m, B^n \mapsto B^n + \mathrm{d}\Gamma^n.$$

$$F^n_{\mu\nu} = \partial_\mu B^n_\nu - \partial_\nu B^n_\mu.$$

$$g^{(2)}_{\mu\nu} = \eta_{\alpha\beta} e^\alpha_\mu e^\beta_\nu, G_{mn} = \eta_{ab}\hat{e}^a_m \hat{e}^b_n$$

$$\begin{aligned}\sqrt{\det g^{(4)}}R^{(4)} = \rho\sqrt{\det g^{(2)}}\Big[R^{(2)} - \frac{1}{4}\mathrm{tr}\big(G^{-1}\partial_\mu G G^{-1}\partial^\mu G\big) \\ + \frac{1}{4\lambda}\rho F^T_{\mu\nu} G F^{\mu\nu} + \lambda^{-1}\partial_\mu\lambda\rho^{-1}\partial^\mu\rho\Big],\end{aligned}$$

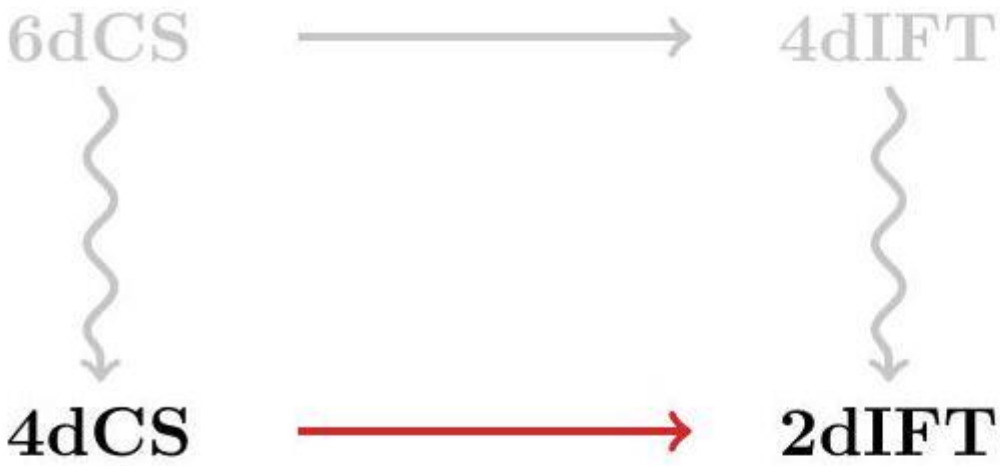
$$\nabla_\mu(\rho^2\lambda^{-1} G F^{\mu\nu}) = 0, \partial_\mu F_0 = 0$$

$$\begin{aligned}R^{(2)}_{\mu\nu} - \frac{1}{2} g^{(2)}_{\mu\nu} R^{(2)} =& \frac{1}{4}\mathrm{tr}\big(G^{-1}\partial_\mu G G^{-1}\partial_\nu G\big) - \lambda^{-1}\partial_{(\mu}\lambda\rho^{-1}\partial_{\nu)}\rho \\ & - \frac{1}{2} g^{(2)}_{\mu\nu}\Big[\frac{1}{4}\mathrm{tr}(G^{-1}\partial_\sigma G G^{-1}\partial^\sigma G) - \lambda^{-1}\partial_\sigma\lambda\rho^{-1}\partial^\sigma\rho\Big], \\ \nabla_\mu(\rho G^{-1}\partial^\mu G) =& 0, \\ \nabla_\mu\partial^\mu\rho =& 0.\end{aligned}$$

$$\partial_\mu(\rho G^{-1}\partial^\mu G) = 0,$$

$$\partial_z \log\lambda = \frac{\rho}{2}\mathrm{tr}\big(G^{-1}\partial_\rho G G^{-1}\partial_z G\big), \partial_\rho \log\lambda = \frac{\rho}{4}\mathrm{tr}\Big[\big(G^{-1}\partial_\rho G\big)^2 - (G^{-1}\partial_z G)^2\Big].$$

$$S = -\frac{1}{4}\int \mathrm{d}\rho\, \mathrm{d}z \rho \mathrm{tr}\Big[\big(G^{-1}\partial_\rho G\big)^2 + (G^{-1}\partial_z G)^2\Big]$$



$$S_{4\mathrm{dCS}} = \frac{1}{2\pi \mathrm{i}}\int_{M_4} \omega \wedge \mathrm{tr}\Big(A \wedge \mathrm{d}A + \frac{2}{3}A\wedge A\wedge A\Big).$$

$$\omega = -\frac{\rho}{2}\left(\frac{Z^2+1}{Z^2}\,\mathrm{d}Z\right) + \frac{Z^{-1}-Z}{2}\,\mathrm{d}\rho + \mathrm{d}z$$

$$Z = \frac{1}{\rho}\Big(z - W \pm \sqrt{(W-z)^2+\rho^2}\Big)$$

$$W = z + \frac{\rho}{2}(Z^{-1} - Z)$$

$$\omega = \frac{\mathrm{i}(\xi-\bar{\xi})}{4}\left(\frac{Z^2+1}{Z^2}\,\mathrm{d}Z\right) + \frac{\mathrm{i}(Z-\mathrm{i})^2}{4Z}\,\mathrm{d}\xi - \frac{\mathrm{i}(Z+\mathrm{i})^2}{4Z}\,\mathrm{d}\bar{\xi}$$

$$\delta S_{4\mathrm{dCS}} = \frac{2}{2\pi\mathrm{i}}\int_{M_4}\omega\wedge\mathrm{tr}(\delta A\wedge F) + \frac{1}{2\pi\mathrm{i}}\int_{M_4}\mathrm{d}\omega\wedge\mathrm{tr}(\delta A\wedge A)$$

$$\partial_{\bar{Z}}\left(\frac{1}{Z}\right) = -2\pi\mathrm{i}\delta(Z), \int_{\mathbb{CP}^1}\mathrm{d}Z\wedge\,\mathrm{d}\bar{Z}\delta(Z)f(Z) = f(0)$$

$$\frac{d\omega}{2\pi i} = \partial_Z \delta(Z) \frac{i(\xi - \bar{\xi})}{4} \, d\bar{Z} \wedge dZ + \delta(Z) \frac{i}{4} \, d\bar{Z} \wedge d\xi - \delta(Z) \frac{i}{4} \, d\bar{Z} \wedge d\bar{\xi}$$

$$A|_{Z=0} = 0, \ A|_{Z=\infty} = 0$$

$$d\epsilon|_{Z=0} = 0, \ \ d\epsilon|_{Z=\infty} = 0$$

$$A \mapsto A + C_\omega \omega.$$

$$A(Z) = \eta(A(-1/Z))$$

$$A = \mathcal{L}^{\hat{g}}, \mathcal{L}^{\hat{g}} \equiv \hat{g}^{-1} \mathcal{L} \hat{g} + \hat{g}^{-1} \, d\hat{g}$$

$$\mathcal{L} \mapsto \mathcal{L}^{\check{h}}, \hat{g} \mapsto \check{h}^{-1} \hat{g}$$

$$\partial_{\bar{Z}} \vee \mathcal{L} = 0 \ \Leftrightarrow \ \mathcal{L}_{\bar{Z}} = 0$$

$$S_{4dCS} = \frac{1}{2\pi i} \int_{M_4} \omega \wedge \mathrm{tr}(\mathcal{L} \wedge \, d\mathcal{L}) + \frac{1}{2\pi i} \int_{M_4} d\omega \wedge \mathrm{tr}(\mathcal{L} \wedge \, d\hat{g} \hat{g}^{-1}) - \frac{1}{6\pi i} \int_{M_4} \omega \wedge \mathrm{tr}(\hat{g}^{-1} \, d\hat{g})^3$$

$$\hat{g}|_{Z=0} = g, \ \hat{g}^{-1} \partial_Z \hat{g}|_{Z=0} = \phi, \ \hat{g}|_{Z=\infty} = \tilde{g}, \ \hat{g}^{-1} \partial_Z \hat{g}|_{Z=\infty} = \tilde{\phi}.$$

$$\int_{M_4} \omega \wedge \mathrm{tr}(\hat{g}^{-1} \, d\hat{g} \wedge \hat{g}^{-1} \, d\hat{g} \wedge \hat{g}^{-1} \, d\hat{g}) = \int_{M_5} d[\omega \wedge \mathrm{tr}(\hat{g}^{-1} \, d\hat{g} \wedge \hat{g}^{-1} \, d\hat{g} \wedge \hat{g}^{-1} \, d\hat{g})]$$

$$S_{4dCS} = \frac{1}{2\pi i} \int_{M_4} \omega \wedge \mathrm{tr}(\mathcal{L} \wedge \, d\mathcal{L}) + \frac{1}{2\pi i} \int_{M_4} d\omega \wedge [\mathrm{tr}(\mathcal{L} \wedge \, d\hat{g} \hat{g}^{-1}) - \mathrm{WZ}[\hat{g}]]$$

$$\mathrm{WZ}[\hat{g}] = \frac{1}{3} \int_{[0,1]} \mathrm{tr}(\hat{g}^{-1} \, d\hat{g} \wedge \hat{g}^{-1} \, d\hat{g} \wedge \hat{g}^{-1} \, d\hat{g})$$

$$\hat{g}|_{Z=0} = g, \ \hat{g}^{-1} \partial_Z \hat{g}|_{Z=0} = 0, \ \hat{g}|_{Z=\infty} = \eta(g), \ \hat{g}^{-1} \partial_Z \hat{g}|_{Z=\infty} = 0$$

$$\omega \wedge F = 0, F \equiv \ d\mathcal{L} + \frac{1}{2} [\mathcal{L}, \mathcal{L}]$$

$$dW \wedge F = 0 \ \Leftrightarrow \ \mathring{F}_{\xi\bar{\xi}} = 0, \mathring{F}_{\bar{W}\xi} = \mathring{F}_{\bar{W}\bar{\xi}} = 0$$

$$\mathring{\partial}_{\bar{W}} \mathring{\mathcal{L}}_\xi = 0, \mathring{\partial}_{\bar{W}} \mathring{\mathcal{L}}_{\bar{\xi}} = 0,$$

$$\mathring{\mathcal{L}}_\xi = \frac{1}{1 - iZ} U_\xi - \frac{iZ}{1 - iZ} V_\xi, \mathring{\mathcal{L}}_{\bar{\xi}} = \frac{1}{1 + iZ} U_{\bar{\xi}} + \frac{iZ}{1 + iZ} V_{\bar{\xi}},$$

$$U_\xi = -\partial_\xi g g^{-1}, U_{\bar{\xi}} = -\partial_{\bar{\xi}} g g^{-1}, V_\xi = -\partial_\xi \tilde{g} \tilde{g}^{-1}, V_{\bar{\xi}} = -\partial_{\bar{\xi}} \tilde{g} \tilde{g}^{-1}$$

$$\mathring{\mathcal{L}}_\xi = \frac{-\partial_\xi G G^{-1}}{1 - iZ}, \mathring{\mathcal{L}}_{\bar{\xi}} = \frac{-\partial_{\bar{\xi}} G G^{-1}}{1 + iZ}, G \equiv \tilde{g}^{-1} g$$

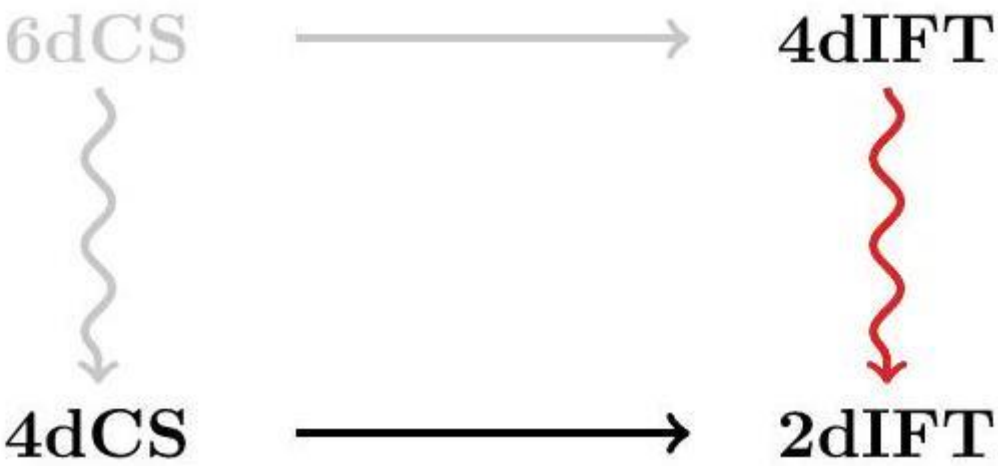
$$\frac{1}{2\pi i} \int_{M_4} d\omega \wedge \mathrm{WZ}[\hat{g}] = -\frac{i}{4} \int_{M_4} d\bar{Z} \wedge \ dZ \wedge \partial_Z (\mathrm{WZ}[\hat{g}]) \delta(Z) (\xi - \bar{\xi}),$$

$$\frac{1}{2\pi i} \int_{M_4} d\omega \ \wedge \mathrm{tr}(\mathcal{L} \wedge \ d\hat{g} \hat{g}^{-1}) = \frac{i}{4} \int_{M_4} dZ \wedge \ d\bar{Z} \wedge \ d\xi \wedge \ d\bar{\xi} \delta(Z) (\xi - \bar{\xi})$$
$$\times \mathrm{tr}\big[\big(\partial_Z \mathcal{L}_\xi + (\xi - \bar{\xi})^{-1} \mathcal{L}_Z\big) \partial_{\bar{\xi}} \hat{g} \hat{g}^{-1} - \big(\partial_Z \mathcal{L}_{\bar{\xi}} - (\xi - \bar{\xi})^{-1} \mathcal{L}_Z\big) \partial_\xi \hat{g} \hat{g}^{-1}\big]$$
$$\big(\partial_Z \mathcal{L}_\xi + (\xi - \bar{\xi})^{-1} \mathcal{L}_Z\big)\big|_{Z=0} = \partial_Z \dot{\mathcal{L}}_\xi\big|_{Z=0}$$
$$\big(\partial_Z \mathcal{L}_{\bar{\xi}} - (\xi - \bar{\xi})^{-1} \mathcal{L}_Z\big)\big|_{Z=0} = \partial_Z \dot{\mathcal{L}}_{\bar{\xi}}\big|_{Z=0}$$

$$\frac{1}{2} \int_{\mathbb{R}^2} d\xi \wedge \ d\bar{\xi} (\xi - \bar{\xi}) \mathrm{tr}\big(\partial_\xi g g^{-1} - \partial_\xi \tilde{g} \tilde{g}^{-1}\big)\big(\partial_{\bar{\xi}} g g^{-1} - \partial_{\bar{\xi}} \tilde{g} \tilde{g}^{-1}\big)$$

$$S_{2\mathrm{dIFT}} = -\frac{1}{2}\int_{\mathbb{R}^2} \mathrm{d}\rho \wedge \mathrm{d}z \rho \operatorname{tr}\left[\left(G^{-1}\partial_\rho G\right)^2 + (G^{-1}\partial_z G)^2\right]$$

$$\check{h}^{-1}\partial_{\bar{Z}}\check{h} = 0.$$



$$S_{4\mathrm{dWZW}} = \frac{1}{2}\int_{\mathbb{R}^4} \operatorname{tr}(G^{-1}\,\mathrm{d}G \wedge\star\, G^{-1}\,\mathrm{d}G) - \int_{\mathbb{R}^4} \mu \wedge \mathrm{WZ}[G],$$

$$R_0 = \frac{\mathrm{i}}{2}(u^1\partial_{u^1} - \bar{u}^1\partial_{\bar{u}^1} + u^2\partial_{u^2} - \bar{u}^2\partial_{\bar{u}^2}),$$
$$R_1 = \frac{\mathrm{i}}{2}(u^1\partial_{u^1} - \bar{u}^1\partial_{\bar{u}^1} - u^2\partial_{u^2} + \bar{u}^2\partial_{\bar{u}^2}),$$
$$R_2 = \frac{\mathrm{i}}{2}(u^2\partial_{u^1} - \bar{u}^2\partial_{\bar{u}^1} + u^1\partial_{u^2} - \bar{u}^1\partial_{\bar{u}^2}),$$
$$R_3 = \frac{1}{2}(u^2\partial_{u^1} + \bar{u}^2\partial_{\bar{u}^1} - u^1\partial_{u^2} - \bar{u}^1\partial_{\bar{u}^2}).$$

$$X_\phi = R_0 + R_1 = \mathrm{i}(u^1\partial_{u^1} - \bar{u}^1\partial_{\bar{u}^1}), X_\tau = \mathrm{i}(\partial_{u^2} - \partial_{\bar{u}^2}).$$

$$X_\phi = \partial_\phi, X_\tau = \partial_\tau$$

$$\partial_\phi G = 0, \partial_\tau G = 0.$$

$$\mu = 2\mathrm{i}(\rho\,\mathrm{d}\phi \wedge \mathrm{d}\rho + \mathrm{d}\tau \wedge \mathrm{d}z),$$

$$S_{2\mathrm{dIFT}} = -\frac{1}{2}\int_{\mathbb{R}^2} \mathrm{d}\rho \wedge \mathrm{d}z \rho \operatorname{tr}\left[\left(G^{-1}\partial_\rho G\right)^2 + (G^{-1}\partial_z G)^2\right]$$

$$\eta: \mathfrak{g} \to \mathfrak{g}, \eta: x \mapsto -x^T.$$

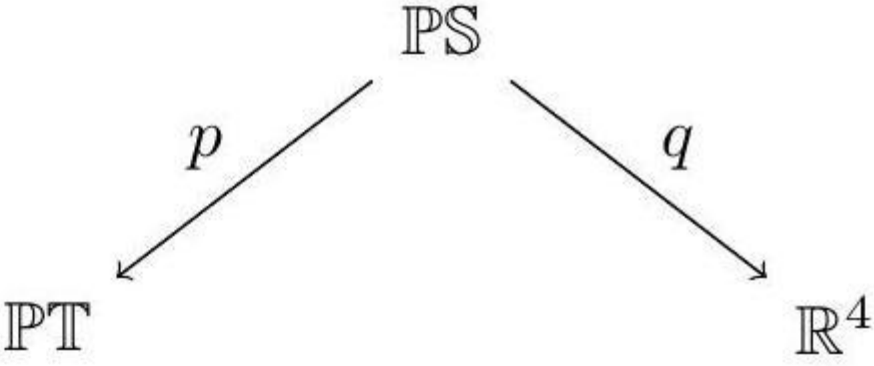
$$\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1, \mathfrak{g}_0 \cong \mathfrak{so}(2).$$

$$\sigma: (\rho, \phi, z, \tau) \mapsto (\rho, -\phi, z, -\tau)$$

$$\mathbb{PT} \cong \mathbb{CP}^1 \times \mathbb{R}^4$$

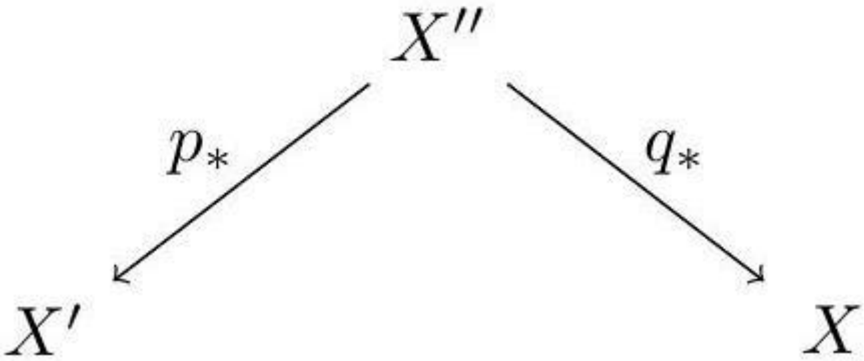
$$\zeta, v^1 = u^1 - \zeta\bar{u}^2, v^2 = u^2 + \zeta\bar{u}^1$$

$$\tilde{\zeta} = 1/\zeta, \tilde{v}^1 = v^1/\zeta, \tilde{v}^2 = v^2/\zeta$$



$$p:(\zeta,x^i)\mapsto(\zeta,v^1,v^2), q:(\zeta,x^i)\mapsto x^i.$$

$$X=X^{u^1}\partial_{u^1}+X^{\bar{u}^1}\partial_{\bar{u}^1}+X^{u^2}\partial_{u^2}+X^{\bar{u}^2}\partial_{\bar{u}^2}.$$



$$V_1=\partial_{\bar{u}^1}-\zeta\partial_{u^2}, V_2=\partial_{\bar{u}^2}+\zeta\partial_{u^1}.$$

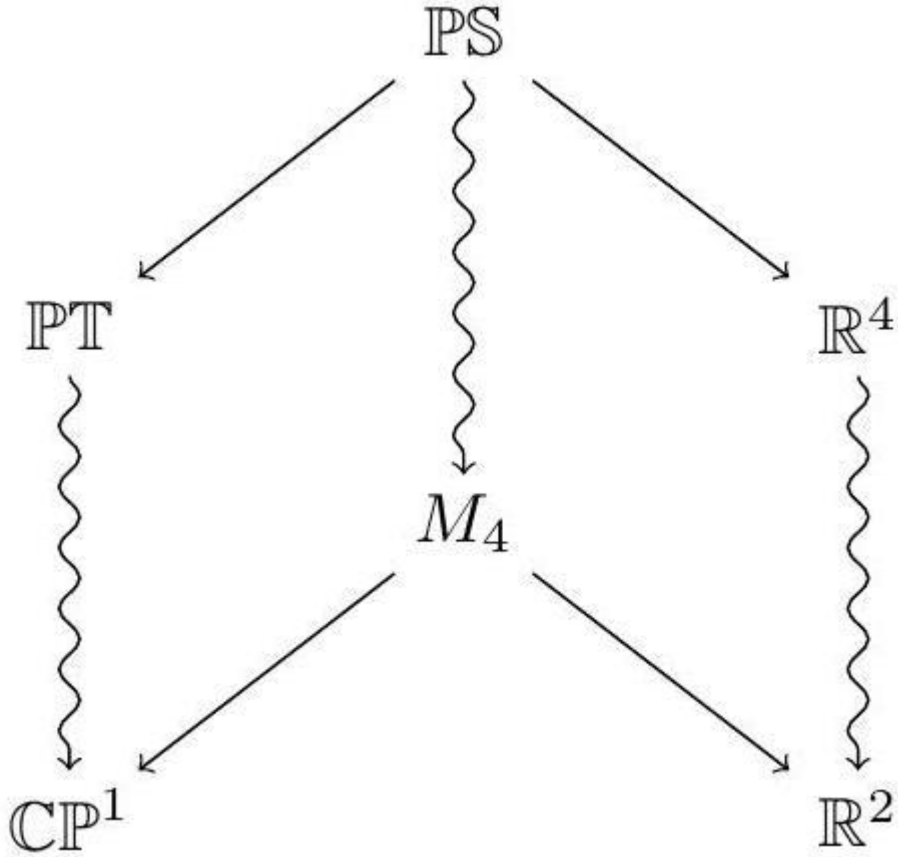
$$[X,V_1]=Q\partial_{u^2},[X,V_2]=-Q\partial_{u^1},\mathrm{mod}\{V_1,V_2\}$$

$$Q=\partial_{\bar{u}^2}X^{u^1}+\zeta\left(\partial_{u^1}X^{u^1}-\partial_{\bar{u}^2}X^{\bar{u}^2}\right)-\zeta^2\partial_{u^1}X^{\bar{u}^2}.$$

$$X''=X+Q\partial_\zeta+\bar{Q}\partial_{\bar{\zeta}},$$

$$X'=\left(X^{u^1}-\zeta X^{\bar{u}^2}-\bar{u}^2Q\right)\partial_{v^1}+\left(X^{u^2}+\zeta X^{\bar{u}^1}+\bar{u}^1Q\right)\partial_{v^2}+Q\partial_\zeta,$$

X	Q	X'
∂_{u^1}	-	∂_{v^1}
$\partial_{\bar{u}^1}$	-	$\zeta\partial_{v^2}$
∂_{u^2}	-	∂_{v^2}
$\partial_{\bar{u}^2}$	-	$-\zeta\partial_{v^1}$
R_0	$\mathrm{i}\zeta$	$(\mathrm{i}/2)(v^1\partial_{v^1}+v^2\partial_{v^2})+\mathrm{i}\zeta\partial_\zeta$
R_1	-	$(\mathrm{i}/2)(v^1\partial_{v^1}-v^2\partial_{v^2})$
R_2	-	$(\mathrm{i}/2)(v^2\partial_{v^1}+v^1\partial_{v^2})$
R_3	-	$(1/2)(v^2\partial_{v^1}-v^1\partial_{v^2})$



$$X''_{\phi} = \partial_{\phi} + \mathrm{i}\zeta\partial_{\zeta} - \mathrm{i}\bar{\zeta}\partial_{\bar{\zeta}}, X''_{\tau} = \partial_{\tau}.$$

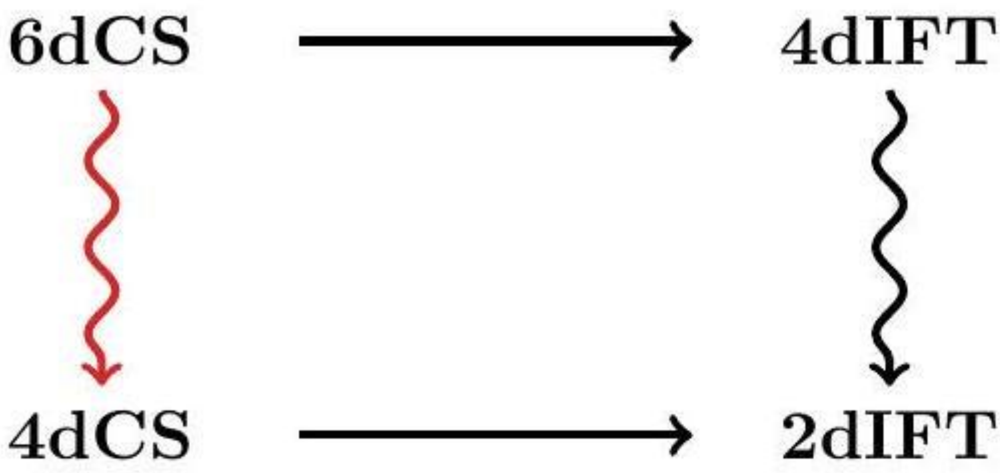
$$X'_{\phi} = \mathrm{i}\left(v^1\partial_{v^1} + \zeta\partial_{\zeta}\right), X'_{\tau} = \mathrm{i}(\partial_{v^2} - \zeta\partial_{v^1}).$$

$$W = \frac{1}{2}\left(v^2 + \frac{v^1}{\zeta}\right) = z + \frac{\rho}{2}(Z^{-1} - Z),$$

$$\sigma: (\rho, \phi, z, \tau) \mapsto (\rho, -\phi, z, -\tau).$$

$$\sigma'': (\zeta, \rho, \phi, z, \tau) \mapsto (-\zeta^{-1}, \rho, -\phi, z, -\tau).$$

$$\sigma': (\zeta, v^1, v^2) \mapsto \left(-\frac{1}{\zeta}, -\frac{v^2}{\zeta}, \frac{v^1}{\zeta}\right).$$



$$S_{6\mathrm{dCS}}=\frac{1}{2\pi \mathrm{i}}\int_{\mathbb{PT}}\Omega\wedge\operatorname{tr}\left(\mathcal{A}\wedge\bar{\partial}\mathcal{A}+\frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right)$$

$$\Omega=\frac{\mathrm{d}\zeta\wedge\ \mathrm{d}v^1\wedge\ \mathrm{d}v^2}{\zeta^2},\ \mathcal{A}|_{\zeta=0}=0,\ \mathcal{A}|_{\bar{\zeta}=0}=0$$

$$\bar{\partial}:\Omega^{p,q}\to\Omega^{p,q+1},\bar{\partial}=\pi_{p,q+1}\circ\ \mathrm{d}$$

$$X_{\phi}''=\partial_{\phi}+\mathrm{i}\zeta\partial_{\zeta}-\mathrm{i}\bar{\zeta}\partial_{\bar{\zeta}},X_{\tau}''=\partial_{\tau}$$

$$\mathcal{L}_{X_{\phi}''}\mathcal{A}=0,\mathcal{L}_{X_{\tau}''}\mathcal{A}=0$$

$$\eta^0=e^{-\mathrm{i}\phi}\ \mathrm{d}\zeta,\eta^1=e^{-\mathrm{i}\phi}(\ \mathrm{d}u^1-\zeta\mathrm{d}\bar{u}^2),\eta^2=\mathrm{d}u^2+\zeta\mathrm{d}\bar{u}^1$$

$$\bar{\eta}^0=e^{\mathrm{i}\phi}\ \mathrm{d}\bar{\zeta},\bar{\eta}^1=\frac{e^{\mathrm{i}\phi}(\ \mathrm{d}\bar{u}^1-\bar{\zeta}\mathrm{d}u^2)}{1+\zeta\bar{\zeta}},\bar{\eta}^2=\frac{\mathrm{d}\bar{u}^2+\bar{\zeta}\mathrm{d}u^1}{1+\zeta\bar{\zeta}}$$

$$\mathcal{A}=\mathcal{A}_0\bar{\eta}^0+\mathcal{A}_1\bar{\eta}^1+\mathcal{A}_2\bar{\eta}^2$$

$$X_{\phi}''\vee\mathcal{A}=0,X_{\tau}''\vee\mathcal{A}=0$$

$$\mathcal{A}\mapsto\mathcal{A}+(C_0\bar{\eta}^0+C_1\bar{\eta}^1+C_2\bar{\eta}^2)$$

$$\mathcal{A}+C=A_{\bar{Z}}\ \mathrm{d}\bar{Z}+A_{\xi}\mathrm{d}\xi+A_{\bar{\xi}}\mathrm{d}\bar{\xi}$$

$$\omega=\frac{1}{2}(X_{\tau}''\wedge X_{\phi}'')\vee\Omega=\mathrm{d}\left(z+\frac{\rho}{2}(Z^{-1}-Z)\right)$$

$$S_{4\mathrm{dCS}}=\frac{1}{2\pi \mathrm{i}}\int_{M_4}\omega\wedge\operatorname{tr}\left(A\wedge\ \mathrm{d}A+\frac{2}{3}A\wedge A\wedge A\right)$$

$$\sigma'':(\zeta,\rho,\phi,z,\tau)\mapsto(-\zeta^{-1},\rho,-\phi,z,-\tau)$$

$$A(Z)=\eta\big(A(-Z^{-1})\big)$$

$$F=\star F,F=\mathrm{d}A+\frac{1}{2}[A,A]$$

$$\mathrm{d}u^1\wedge\ \mathrm{d}u^2\wedge F=0,\ \mathrm{d}\bar{u}^1\wedge\ \mathrm{d}\bar{u}^2\wedge F=0,\mu\wedge F=0.$$

$$\mathrm{d}\zeta\wedge\ \mathrm{d}v^1\wedge\ \mathrm{d}v^2\wedge F=0\ \forall\zeta\in\mathbb{CP}^1$$

$$
\begin{aligned}
A^{1,0} &= \frac{A_{u^1} - \bar{\zeta} A_{\bar{u}^2}}{1+\zeta\bar{\zeta}} \mathrm{d}v^1 + \frac{A_{u^2} + \bar{\zeta} A_{\bar{u}^1}}{1+\zeta\bar{\zeta}} \mathrm{d}v^2 + \mathcal{A}_\zeta \mathrm{d}\zeta \\
A^{0,1} &= \frac{A_{\bar{u}^1} - \zeta A_{u^2}}{1+\zeta\bar{\zeta}} \mathrm{d}\bar{v}^1 + \frac{A_{\bar{u}^2} + \zeta A_{u^1}}{1+\zeta\bar{\zeta}} \mathrm{d}\bar{v}^2 + \mathcal{A}_{\bar{\zeta}} \mathrm{d}\bar{\zeta}
\end{aligned}
$$

$$
\mathrm{d}\zeta \wedge \mathrm{d}v^1 \wedge \mathrm{d}v^2 \wedge \mathcal{F} = 0 \Leftrightarrow \mathcal{F}^{0,2} = 0
$$

$$
\mathcal{A} = \frac{A_{\bar{u}^1} - \zeta A_{u^2}}{1+\zeta\bar{\zeta}} \mathrm{d}\bar{v}^1 + \frac{A_{\bar{u}^2} + \zeta A_{u^1}}{1+\zeta\bar{\zeta}} \mathrm{d}\bar{v}^2 + \mathcal{A}_{\bar{\zeta}} \mathrm{d}\bar{\zeta}
$$

$$
\mathrm{d}\zeta \wedge \mathrm{d}v^1 \wedge \mathrm{d}v^2 = -\frac{\mathrm{d}\tilde{\zeta} \wedge \mathrm{d}\tilde{v}^1 \wedge \mathrm{d}\tilde{v}^2}{\tilde{\zeta}^4}
$$

$$
\Omega = \frac{\mathrm{d}\zeta \wedge \mathrm{d}v^1 \wedge \mathrm{d}v^2}{\zeta^2} = -\frac{\mathrm{d}\tilde{\zeta} \wedge \mathrm{d}\tilde{v}^1 \wedge \mathrm{d}\tilde{v}^2}{\tilde{\zeta}^2}
$$

$$
S_{6\mathrm{dCS}}[\mathcal{A}] = \frac{1}{2\pi\mathrm{i}} \int_{\mathbb{PT}} \Omega \wedge \mathrm{tr}\left(\mathcal{A} \wedge \bar{\partial}\mathcal{A} + \frac{2}{3}\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}\right).
$$

$$
\delta S_{6\mathrm{dCS}} = \frac{2}{2\pi\mathrm{i}} \int_{\mathbb{PT}} \Omega \wedge \mathrm{tr}(\delta\mathcal{A} \wedge \mathcal{F}) + \frac{1}{2\pi\mathrm{i}} \int_{\mathbb{PT}} \bar{\partial}\Omega \wedge \mathrm{tr}(\delta\mathcal{A} \wedge \mathcal{A}).
$$

$$
\partial_{\bar{\zeta}}\left(\frac{1}{\zeta}\right) = -2\pi\mathrm{i}\delta(\zeta), \int_{\mathbb{CP}^1} \mathrm{d}\zeta \wedge \mathrm{d}\bar{\zeta}\delta(\zeta) f(\zeta) = f(0)
$$

$$
\begin{aligned}
\frac{1}{2\pi\mathrm{i}} \int_{\mathbb{PT}} \bar{\partial}\Omega \wedge \mathrm{tr}(\delta\mathcal{A} \wedge \mathcal{A}) &= \int_{\mathbb{PT}} \mathrm{d}\zeta \wedge \mathrm{d}\bar{\zeta}\delta(\zeta)\partial_\zeta[\mathrm{d}v^1 \wedge \mathrm{d}v^2 \wedge \mathrm{tr}(\delta\mathcal{A} \wedge \mathcal{A})] \\
&= \int_{\mathbb{R}^4} \left[\mu \wedge \mathrm{tr}(\delta\mathcal{A} \wedge \mathcal{A})|_{\zeta=0} + \mathrm{d}u^1 \wedge \mathrm{d}u^2 \wedge \partial_\zeta \mathrm{tr}(\delta\mathcal{A} \wedge \mathcal{A})\big|_{\zeta=0}\right]
\end{aligned}
$$

$$
\mathcal{A}|_{\zeta=0} = 0, \ \mathcal{A}|_{\tilde{\zeta}=0} = 0
$$

$$
\delta\mathcal{A} = \bar{\partial}\epsilon + [\mathcal{A}, \epsilon].
$$

$$
\delta S_{6\mathrm{dCS}} = \frac{1}{2\pi\mathrm{i}} \int_{\mathbb{PT}} \bar{\partial}\Omega \wedge \mathrm{tr}(\mathcal{A} \wedge \bar{\partial}\epsilon).
$$

$$
\frac{1}{2\pi\mathrm{i}} \int_{\mathbb{PT}} \bar{\partial}\Omega \wedge \mathrm{tr}(\mathcal{A} \wedge \bar{\partial}\epsilon) = \int_{\mathbb{R}^4} \mathrm{d}u^1 \wedge \mathrm{d}u^2 \wedge \mathrm{tr}\left(\partial_\zeta\mathcal{A} \wedge \bar{\partial}\epsilon\right)\Big|_{\zeta=0}.
$$

$$
\bar{\partial}\epsilon\big|_{\zeta=0} = 0, \ \bar{\partial}\epsilon\big|_{\tilde{\zeta}=0} = 0.
$$

$$
\mathrm{d}u^1 \wedge \mathrm{d}u^2 \wedge \bar{\partial}\left(\partial_\zeta\mathcal{A}\right)\big|_{\zeta=0} = 0, \ \ \mathrm{d}\tilde{u}^1 \wedge \mathrm{d}\tilde{u}^2 \wedge \bar{\partial}\left(\partial_{\tilde{\zeta}}\mathcal{A}\right)\big|_{\tilde{\zeta}=0} = 0.
$$

$$
\mathcal{A} \mapsto \mathcal{A}^g = g^{-1}\mathcal{A}g + g^{-1}\bar{\partial}g.
$$

$$
S_{6\mathrm{dCS}}[\mathcal{A}] \mapsto S_{6\mathrm{dCS}}[\mathcal{A}] + \frac{1}{2\pi\mathrm{i}} \int_{\mathbb{PT}} \bar{\partial}\Omega \wedge \mathrm{tr}\left(\mathcal{A} \wedge \bar{\partial}g g^{-1}\right) - \frac{1}{6\pi\mathrm{i}} \int_{\mathbb{PT}} \Omega \wedge \mathrm{tr}\left(g^{-1}\bar{\partial}g\right)^3.
$$

$$
g^{-1}\bar{\partial}g\big|_{\zeta=0} = 0, \ g^{-1}\bar{\partial}g\big|_{\tilde{\zeta}=0} = 0
$$

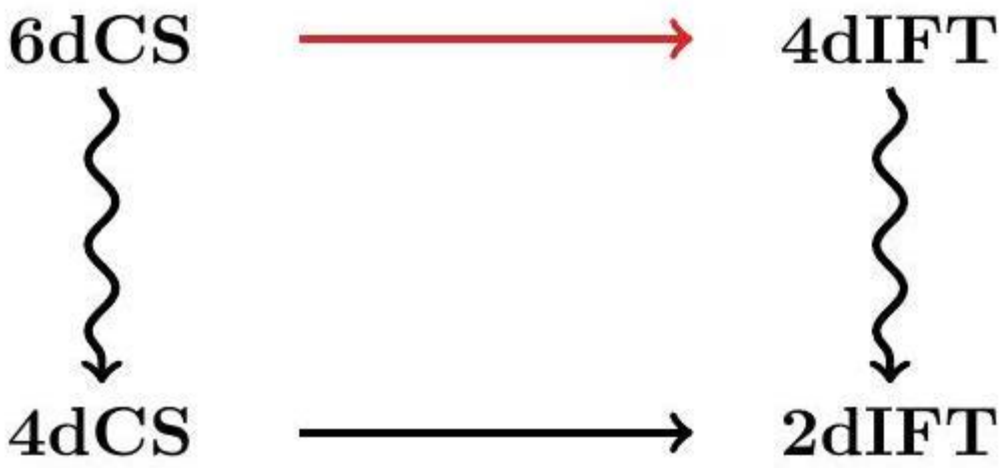
$$
\mathrm{WZ}[g] = \frac{1}{3} \int_{[0,1]} \mathrm{tr}(\tilde{g}^{-1}\,\mathrm{d}\tilde{g} \wedge \tilde{g}^{-1}\,\mathrm{d}\tilde{g} \wedge \tilde{g}^{-1}\,\mathrm{d}\tilde{g})
$$

$$
\frac{1}{3} \int_{\mathbb{PT}} \Omega \wedge \mathrm{tr}\left(g^{-1}\bar{\partial}g \wedge g^{-1}\bar{\partial}g \wedge g^{-1}\bar{\partial}g\right) = \int_{\mathbb{PT}} \bar{\partial}\Omega \wedge \mathrm{WZ}[g].
$$

$$
S_{6\mathrm{dCS}}[\mathcal{A}^g] = S_{6\mathrm{dCS}}[\mathcal{A}] + \frac{1}{2\pi\mathrm{i}} \int_{\mathbb{PT}} \bar{\partial}\Omega \wedge \left[\mathrm{tr}\left(\mathcal{A} \wedge \bar{\partial}g g^{-1}\right) - \mathrm{WZ}[g]\right].
$$

$$R_1'' = \frac{\mathrm{i}}{2}(u^1\partial_{u^1} - \bar{u}^1\partial_{\bar{u}^1} - u^2\partial_{u^2} + \bar{u}^2\partial_{\bar{u}^2}),$$
$$R_2'' = \frac{\mathrm{i}}{2}(u^2\partial_{u^1} - \bar{u}^2\partial_{\bar{u}^1} + u^1\partial_{u^2} - \bar{u}^1\partial_{\bar{u}^2}),$$
$$R_3'' = \frac{1}{2}(u^2\partial_{u^1} + \bar{u}^2\partial_{\bar{u}^1} - u^1\partial_{u^2} - \bar{u}^1\partial_{\bar{u}^2}).$$

$$R_0'' = \frac{\mathrm{i}}{2}(u^1\partial_{u^1} - \bar{u}^1\partial_{\bar{u}^1} + u^2\partial_{u^2} - \bar{u}^2\partial_{\bar{u}^2}) + \mathrm{i}\zeta\partial_\zeta.$$



$$\mathcal{A} = \mathcal{A}'^{\hat{g}} = \hat{g}^{-1}\mathcal{A}'\hat{g} + \hat{g}^{-1}\bar{\partial}\hat{g}.$$

$$\mathcal{A}' \mapsto \mathcal{A}'^{\check{h}}, \hat{g} \mapsto \check{h}^{-1}\hat{g}.$$

$$\partial_{\bar{\zeta}} \vee \mathcal{A}' = 0$$

$$S_{6\mathrm{dCS}}[\mathcal{A}', \hat{g}] = S_{6\mathrm{dCS}}[\mathcal{A}'] + \frac{1}{2\pi\mathrm{i}}\int_{\mathbb{PT}} \bar{\partial}\Omega \wedge \left[\mathrm{tr}\left(\mathcal{A}' \wedge \bar{\partial}\hat{g}\hat{g}^{-1}\right) - \mathrm{WZ}[\hat{g}]\right].$$

$$\hat{g}|_{\zeta=0} = g,\ \hat{g}^{-1}\partial_\zeta\hat{g}\big|_{\zeta=0} = \phi,\ \hat{g}|_{\tilde{\zeta}=0} = \tilde{g},\ \hat{g}^{-1}\partial_{\tilde{\zeta}}\hat{g}\big|_{\tilde{\zeta}=0} = \tilde{\phi}$$

$$\mathcal{A}' \mapsto \mathcal{A}', \hat{g} \mapsto \hat{g}\hat{h}$$

$$\hat{g}|_{\zeta=0} = g,\ \hat{g}^{-1}\partial_\zeta\hat{g}\big|_{\zeta=0} = 0,\ \hat{g}|_{\tilde{\zeta}=0} = \mathrm{id},\ \hat{g}^{-1}\partial_{\tilde{\zeta}}\hat{g}\big|_{\tilde{\zeta}=0} = 0$$

$$\mathcal{A}' \mapsto \mathcal{A}'^{h_\ell}, g \mapsto h_\ell^{-1} g h_r, \partial h_\ell = 0, \bar{\partial} h_r = 0$$

$$\bar{\partial}\epsilon\big|_{\zeta=0} = \bar{\partial}\left(\epsilon|_{\zeta=0}\right) = 0,\ \bar{\partial}\epsilon\big|_{\tilde{\zeta}=0} = \partial\left(\epsilon|_{\tilde{\zeta}=0}\right) = 0.$$

$$\Omega \wedge \mathcal{L}_{\bar{\zeta}}\mathcal{A}' = 0$$

$$\partial_{\bar{\zeta}} \vee \bar{\theta}^1 = 0, \Omega \wedge \mathcal{L}_{\bar{\zeta}}\bar{\theta}^1 = 0, \partial_{\bar{\zeta}} \vee \bar{\theta}^2 = 0, \Omega \wedge \mathcal{L}_{\bar{\zeta}}\bar{\theta}^2 = 0$$

$$\bar{\theta}^1 = \frac{\mathrm{d}\bar{u}^1 - \bar{\zeta}\mathrm{d}u^2}{1 + \zeta\bar{\zeta}}, \bar{\theta}^2 = \frac{\mathrm{d}\bar{u}^2 + \bar{\zeta}\mathrm{d}u^1}{1 + \zeta\bar{\zeta}}$$

$$\mathcal{A}' = (A_{\bar{u}^1} - \zeta A_{u^2})\bar{\theta}^1 + (A_{\bar{u}^2} + \zeta A_{u^1})\bar{\theta}^2$$

$$\mathcal{A}'|_{\zeta=0} = \bar{\partial}\hat{g}\hat{g}^{-1}\big|_{\zeta=0}, \mathcal{A}'|_{\tilde{\zeta}=0} = \bar{\partial}\hat{g}\hat{g}^{-1}\big|_{\tilde{\zeta}=0}$$

$$A_{u^1} = 0, A_{\bar{u}^1} = \partial_{\bar{u}^1} g g^{-1}, A_{u^2} = 0, A_{\bar{u}^2} = \partial_{\bar{u}^2} g g^{-1}$$

$$F^{2,0} = 0, F^{0,2} = 0$$

$$\mu \wedge F = 0 \iff \mu \wedge \partial(\bar{\partial} g g^{-1}) = 0$$

$$S_{6\mathrm{dCS}}[\mathcal{A}', \hat{g}] = S_{6\mathrm{dCS}}[\mathcal{A}'] + \frac{1}{2\pi \mathrm{i}} \int_{\mathbb{PT}} \bar{\partial}\Omega \wedge \left[\mathrm{tr}(\mathcal{A}' \wedge \bar{\partial}\hat{g}\hat{g}^{-1}) - \mathrm{WZ}[\hat{g}]\right].$$

$$\int_{\mathbb{R}^4} \left[\mu \wedge \mathrm{tr}(\mathcal{A}' \wedge \bar{\partial}\hat{g}\hat{g}^{-1})\big|_{\zeta=0} + \mathrm{d}u^1 \wedge \mathrm{d}u^2 \wedge \partial_\zeta \mathrm{tr}(\mathcal{A}' \wedge \bar{\partial}\hat{g}\hat{g}^{-1})\big|_{\zeta=0}\right]$$

$$\bar{\partial} f = \partial_{\bar{\zeta}} f \, \mathrm{d}\bar{\zeta} + (\partial_{\bar{u}^1} f - \zeta \partial_{u^2} f)\bar{\theta}^1 + (\partial_{\bar{u}^2} f + \zeta \partial_{u^1} f)\bar{\theta}^2$$

$$\int_{\mathbb{R}^4} \mathrm{d}u^1 \wedge \mathrm{d}u^2 \wedge \partial_\zeta \mathrm{tr}(\mathcal{A}' \wedge \bar{\partial}\hat{g}\hat{g}^{-1})\Big|_{\zeta=0} = \frac{1}{2}\int_{\mathbb{R}^4} \mathrm{tr}(g^{-1}\,\mathrm{d}g \wedge \star\, g^{-1}\,\mathrm{d}g)$$

$$\frac{1}{2}\int_{\mathbb{R}^4} \mathrm{tr}(g^{-1}\,\mathrm{d}g \wedge \star\, g^{-1}\,\mathrm{d}g) = \int_{\mathbb{R}^4} \mu \wedge \mathrm{tr}(g^{-1}\partial g \wedge g^{-1}\bar{\partial} g)$$

$$S_{4\mathrm{dWZW}}[g] = \frac{1}{2}\int_{\mathbb{R}^4} \mathrm{tr}(g^{-1}\,\mathrm{d}g \wedge \star\, g^{-1}\,\mathrm{d}g) - \int_{\mathbb{R}^4} \mu \wedge \mathrm{WZ}[g]$$

$$\delta S_{4\mathrm{dWZW}} = 0 \iff \mu \wedge \partial(\bar{\partial} g g^{-1}) = 0.$$

$$g \mapsto h_\ell^{-1} g h_r, \partial h_\ell = 0, \bar{\partial} h_r = 0$$

$$\mu \wedge \partial(\bar{\partial} g g^{-1}) = 0, \mu \wedge \bar{\partial}(g^{-1}\partial g) = 0.$$

$$S_{4\mathrm{dWZW}} = \frac{1}{2}\int_{\mathbb{R}^4} \mathrm{tr}(g^{-1}\,\mathrm{d}g \wedge \star\, g^{-1}\,\mathrm{d}g) - \int_{\mathbb{R}^4} \mu \wedge \mathrm{WZ}[g]$$

$$S_{2\mathrm{dPCM}} = \frac{1}{2}\int_{\mathbb{R}^2} \mathrm{tr}(g^{-1}\,\mathrm{d}g \wedge \star\, g^{-1}\,\mathrm{d}g)$$

$$X = \partial_{u^1} - \partial_{\bar{u}^2}, Y = \partial_{u^2} - \partial_{\bar{u}^1}$$

$$\sigma: (u^1, u^2) \mapsto (\bar{u}^2, \bar{u}^1)$$

$$\eta: \mathfrak{g} \to \mathfrak{g}, \eta^2 = \mathrm{id}$$

$$X' = (1+\zeta)\partial_{v^1}, Y' = (1-\zeta)\partial_{v^2}$$

$$\sigma'': (u^1, u^2, \zeta) \mapsto (\bar{u}^2, \bar{u}^1, \zeta^{-1})$$

$$\sigma': (\zeta, v^1, v^2) \mapsto \left(\frac{1}{\zeta}, -\frac{v^1}{\zeta}, \frac{v^2}{\zeta}\right)$$

$$W = \frac{1}{2}(\zeta + \zeta^{-1})$$

$$\zeta = W + \sqrt{W^2 - 1}$$

$$A(\zeta) = \eta(A(\zeta^{-1}))$$

$$\omega = \frac{1}{2}(X \wedge Y) \vee \Omega = \frac{1}{2}\frac{1-\zeta^2}{\zeta^2}\,\mathrm{d}\zeta = \mathrm{d}W$$

Modelo de Hadronización en gravedad cuántica relativista.

$$ds^2 = -dt^2 + d\vec{x}^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu;\ \Lambda^\mu{}_\nu \in SO(1,3)$$

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$$

$$ds^2 = d\theta^2 + \sin^2\ \theta d\phi^2$$

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^3$$

$$x_1^2 + x_2^2 + x_3^2 = R^2 \Rightarrow 2(x_1dx_1 + x_2dx_2 + x_3dx_3) = 0$$
$$\Rightarrow dx_3 = -\frac{x_1dx_1 + x_2dx_2}{x_3} = -\frac{x_1}{\sqrt{R^2 - x_1^2 - x_2^2}}dx_1 - \frac{x_2}{\sqrt{R^2 - x_1^2 - x_2^2}}dx_2$$

$$ds^2 = dx_1^2\left(1 + \frac{x_1^2}{R^2 - x_1^2 - x_2^2}\right) + dx_2^2\left(1 + \frac{x_2^2}{R^2 - x_1^2 - x_2^2}\right) + 2dx_1dx_2\frac{x_1x_2}{R^2 - x_1^2 - x_2^2} = g_{ij}dx^idx^j$$

$$ds^2 = dx^2 + dy^2 - dz^2;\ x^2 + y^2 - z^2 = -R^2$$

$$x'^\mu = x'^\mu(x^\nu) \Rightarrow ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = ds'^2 = g'_{\mu\nu}(x')dx'^\mu dx'^\nu$$

$$\Gamma^\mu_{\nu\rho} = \frac{1}{2}g^{\mu\sigma}\left(\partial_\rho g_{\nu\sigma} + \partial_\nu g_{\sigma\rho} - \partial_\sigma g_{\nu\rho}\right)$$

$$F^{ab}_{\mu\nu} = \partial_\mu A^{ab}_\nu - \partial_\nu A^{ab}_\mu + A^{ac}_\mu A^{cb}_\nu - A^{ac}_\nu A^{cb}_\mu$$

$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu + f^A{}_{BC}\left(A^B_\mu A^C_\nu - A^B_\nu A^C_\mu\right)$$

$$(R^\mu{}_\nu)_{\rho\sigma}(\Gamma) = \partial_\rho(\Gamma^\mu{}_\nu)_\sigma - \partial_\sigma(\Gamma^\mu{}_\nu)_\rho + (\Gamma^\mu{}_\lambda)_\rho\left(\Gamma^\lambda{}_\nu\right)_\sigma - (\Gamma^\mu{}_\lambda)_\sigma\left(\Gamma^\lambda{}_\nu\right)_\rho$$

$$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$$

$$A'^\mu = \frac{\partial x'^\mu}{\partial x^\nu}A^\nu$$

$$B'_\mu = \frac{\partial x^\nu}{\partial x'^\mu}B_\nu$$

$$D_\mu T^\rho_\nu \equiv \partial_\mu T^\rho_\nu + \Gamma^\rho{}_{\mu\sigma}T^\sigma_\nu - \Gamma^\sigma{}_{\mu\nu}T^\rho_\sigma$$

$$S_{\text{gravity}} = \frac{1}{16\pi G}\int\ d^dx\sqrt{-g}R$$

$$\frac{\delta\sqrt{-g}}{\sqrt{-g}} = -\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}$$

$$\frac{\delta S_{grav}}{\delta g^{\mu\nu}} = 0\text{: } R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0$$

$$S_{M,\phi} = -\frac{1}{2}\int\ d^4x(\partial_\mu\phi)(\partial_\nu\phi)\eta^{\mu\nu}$$

$$-\frac{1}{2}\int\ d^4x\sqrt{-g}(D_\mu\phi)(D_\nu\phi)g^{\mu\nu} = -\frac{1}{2}\int\ d^4x\sqrt{-g}(\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu}$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\text{matter}}}{\delta g^{\mu\nu}}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

$$T^\phi_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left(\partial_\rho\phi\right)^2$$

$$ds^2 = R^2(d\theta^2 + \sin^2\ \theta d\phi^2)$$

$$(D_\mu D_\nu - D_\nu D_\mu)A_\rho = R^\sigma_{\rho\mu\nu}A_\sigma$$

$$\delta R = \delta^\rho_\mu g^{\nu\sigma}\left(D_\rho \delta\Gamma^\mu_{\nu\sigma} - D_\sigma \delta\Gamma^\mu_{\nu\rho}\right) + R_{\nu\sigma}\delta g^{\nu\sigma}$$

$$g_{\mu\nu}(x) = e^a_\mu(x) e^b_\nu(x)\eta_{ab}$$

$$\delta_\xi g_{\mu\nu}(x) = \left(\xi^\rho \partial_\rho\right)g_{\mu\nu} + \left(\partial_\mu \xi^\rho\right)g_{\rho\nu} + (\partial_\nu \xi^\rho)g_{\rho\nu}$$

$$\delta_\xi e^a_\mu(x) = \left(\xi^\rho \partial_\rho\right)e^a_\mu + \left(\partial_\mu \xi^\rho\right)e^a_\rho$$

$$\delta_{l.L.} e^a_\mu(x) = \lambda^a{}_b(x) e^b_\mu(x)$$

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{4}\omega^{ab}_\mu \Gamma_{ab}\psi$$

$$T^a_{[\mu\nu]} \equiv 2D_{[\mu}e^a_{\nu]} = 2\partial_{[\mu}e^a_{\nu]} + 2\omega^{ab}_{[\mu}e^b_{\nu]} = 0$$

$$D_\mu e^a_\nu \equiv \partial_\mu e^a_\nu + \omega^{ab}_\mu e^b_\nu - \Gamma^\rho{}_{\mu\nu}e^a_\rho = 0$$

$$R^{ab}_{\mu\nu}(\omega) = \partial_\mu \omega^{ab}_\nu - \partial_\nu \omega^{ab}_\mu + \omega^{ab}_\mu \omega^{bc}_\nu - \omega^{ac}_\nu \omega^{cb}_\mu$$

$$R^{ab}_{\rho\sigma}(\omega(e)) = e^a_\mu e^{-1,\nu b} R^\mu_{\nu\rho\sigma}(\Gamma(e))$$

$$R = R^{ab}_{\mu\nu} e_a^{-1\mu} e_b^{-1\nu}$$

$$S_{EH} = \frac{1}{16\pi G}\int d^4x (\mathrm{det}e) R^{ab}_{\mu\nu}(\omega(e)) e_a^{-1,\mu} e_b^{-1,\nu}$$

$$-\int \left(\partial_{[\mu}A_{\nu]}\right)^2 \text{ to } \int \left[-2F^{\mu\nu}\left(\partial_{[\mu}A_{\nu]}\right) + F^2_{\mu\nu}\right]$$

$$(\mathrm{det}e) e_a^{-1\mu} e_b^{-1\nu} = \epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd} e^c_\rho e^d_\sigma$$

$$S_{EH} = \frac{1}{16\pi G}\int d^4x \epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd} R^{ab}_{\mu\nu}(\omega) e^c_\rho e^d_\sigma$$

$$\epsilon_{abcd}\epsilon^{\mu\nu\rho\sigma}\left(D_\nu e^c_\rho\right)e^d_\sigma = 0$$

$$T^a_{[\mu\nu]} \equiv 2D_{[\mu}e^a_{\nu]} = 0$$

$$ds^2 = -dX_0^2 + \sum_{i=1}^{d-1} dX_i^2 + dX_{d+1}^2$$

$$-X_0^2 + \sum_{i=1}^{d-1} X_i^2 + X_{d+1}^2 = R^2$$

$$ds^2 = -dX_0^2 + \sum_{i=1}^{d-1} dX_i^2 - dX_{d+1}^2$$

$$-X_0^2 + \sum_{i=1}^{d-1} X_i^2 - X_{d+1}^2 = -R^2$$

$$ds^2 = \frac{R^2}{x_0^2}\left(-dt^2 + \sum_{i=1}^{d-2} dx_i^2 + dx_0^2\right)$$

$$ds^2 = e^{-2y}\left(-dt^2 + \sum_{i=1}^{d-2} dx_i^2\right) + dy^2$$

$$t = \int dt = \int^{\infty} e^{-y} e^{-y} dy < \infty$$

$$ds_d^2 = R^2\left(-\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\vec{\Omega}_{d-2}^2\right)$$

$$ds_d^2 = R^2\left(\cos^2 \rho dw^2 + d\rho^2 + \sin^2 \rho d\vec{\Omega}_{d-2}^2\right)$$

$$ds_d^2 = \frac{R^2}{\cos^2 \theta}\left(-d\tau^2 + d\theta^2 + \sin^2 \theta d\vec{\Omega}_{d-2}^2\right)$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \Lambda g_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

$$ds^2 = -\left(1 - \frac{2MG}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2MG}{r}} + R^2 d\Omega_2^2$$

$$ds^2 \simeq -(1 + 2U)dt^2 + (1 - 2U)d\vec{x}^2 = -(1 + 2U)dt^2 + (1 - 2U)(dr^2 + r^2 d\Omega_2^2)$$

$$U = U_N(r) = -\frac{MG}{r}$$

$$dt = \frac{dr}{1 - \frac{2MG}{r}}$$

$$dt \simeq 2MG \frac{dr}{r - 2MG} \Rightarrow t \simeq 2MG \ln (r - 2MG) \to \infty$$

$$R \sim \frac{1}{r_H^2} = \frac{1}{(2MG)^2} = \text{ finite!}$$

$$ds^2 = -\frac{dt^2}{1 - \frac{2MG}{r}} = -d\tau^2 \Rightarrow d\tau = \sqrt{-g_{00}} dt = \frac{dt}{\sqrt{1 - \frac{2MG}{r}}}$$

$$g'_{\mu\nu}(x') = g_{\rho\sigma}(x) \frac{\partial x^\rho}{\partial x'^\mu} \frac{\partial x^\sigma}{\partial x'^\nu}$$

$$\partial_\xi g_{\mu\nu}(x) = \left(\xi^\rho \partial_\rho\right) g_{\mu\nu} + \left(\partial_\mu \xi^\rho\right) g_{\rho\nu} + \left(\partial_\nu \xi^\rho\right) g_{\rho\mu}$$

$$X_0 = R\cosh \rho \cos \tau;\ X_i = R\sinh \rho \Omega_i;\ X_{d+1} = R\cosh \rho \sin \tau$$

$$\omega_\mu^{ab}(e) = \frac{1}{2} e^{a\nu}\left(\partial_\mu e_\nu^b - \partial_\nu e_\mu^b\right) - \frac{1}{2} e^{b\nu}\left(\partial_\mu e_\nu^a - \partial_\nu e_\mu^a\right) - \frac{1}{2} e^{a\rho} e^{b\sigma}\left(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho}\right) e_\mu^c$$

$$ds^2 = R^2(-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega^2)$$

$$ds^2 = -\left(1 - \frac{\Lambda}{3} r^2\right) d\bar{t}^2 + \frac{dr^2}{1 - \frac{\Lambda}{3} r^2} + r^2 d\Omega^2$$

$$[T_r, T_s] = f_{rs}{}^t T_t$$

$$\left\{Q_\alpha^i, Q_\beta^j\right\} = \text{ other generators}$$

$$[\text{ even, even }] = \text{ even; } \{\text{ odd, odd }\} = \text{ even; } [\text{ even, odd }] = \text{ odd}$$

$$[Q_\alpha^i, J_{ab}] = (\dots) Q_\beta^i$$

$$\delta\ \text{quark} = \text{gluon}\ ;\ \delta\ \text{gluon} = \text{quark}$$

$$\chi^C \equiv \chi^T C = \bar{\chi} \equiv \chi^\dagger i\gamma^0$$

$$C^T = -C;\ C\gamma^m C^{-1} = -(\gamma^m)^T$$

$$S = -\frac{1}{2}\int d^2x\left[(\partial_\mu\phi)^2 + \bar{\psi}\,\partial\!\!\!/\,\psi\right]$$

$$\delta\phi = \bar{\epsilon}\psi = \bar{\epsilon}_\alpha\psi^\alpha = \epsilon^\beta C_{\beta\alpha}\psi^\alpha$$

$$\delta\psi = \partial\!\!\!/\,\phi\epsilon$$

$$\bar{\epsilon}\chi = +\bar{\chi}\epsilon; \quad 2)\ \bar{\epsilon}\gamma_\mu\chi = -\bar{\chi}\gamma_\mu\epsilon$$

$$\bar{\epsilon}\gamma_3\chi = -\bar{\chi}\gamma_3\epsilon; \qquad \bar{\epsilon}\gamma_5\chi = +\bar{\chi}\gamma_5\epsilon$$
$$\bar{\epsilon}\gamma_\mu\gamma_3\chi = -\bar{\chi}\gamma_\mu\gamma_3\epsilon \qquad \bar{\epsilon}\gamma_\mu\gamma_5\chi = +\bar{\chi}\gamma_\mu\gamma_5\epsilon$$

$$\delta S = -\int d^2x\left[-\phi\,\Box\,\delta\phi + \frac{1}{2}\delta\bar{\psi}\,\partial\!\!\!/\,\psi + \frac{1}{2}\bar{\psi}\,\partial\!\!\!/\,\delta\psi\right] = -\int d^2x[-\phi\,\Box\,\delta\phi + \bar{\psi}\,\partial\!\!\!/\,\delta\psi]$$

$$\delta S = -\int d^2x[-\phi\,\Box\,\bar{\epsilon}\psi + \bar{\psi}\,\partial\!\!\!/\,\partial\!\!\!/\,\phi\epsilon]$$

$$\partial\!\!\!/\,\partial\!\!\!/ = \partial_\mu\partial_\nu\gamma^\mu\gamma^\nu = \partial_\mu\partial_\nu\frac{1}{2}\{\gamma_\mu,\gamma_\nu\} = \partial_\mu\partial_\nu g^{\mu\nu} = \Box$$

$$\left\{Q^i_\alpha, Q^j_\beta\right\} = 2(C\gamma^\mu)_{\alpha\beta}P_\mu\delta^{ij} + \cdots$$

$$\epsilon_1^\alpha Q_\alpha Q_\beta\epsilon_2^\beta + \epsilon_1^\alpha Q_\beta Q_\alpha\epsilon_2^\beta = \epsilon_1^\alpha Q_\alpha Q_\beta\epsilon_2^\beta - \epsilon_2^\beta Q_\beta Q_\alpha\epsilon_1^\alpha = -[\delta_{\epsilon_1},\delta_{\epsilon_2}]$$

$$2\bar{\epsilon}_1\gamma^\mu\epsilon_1\partial_\mu = -(2\bar{\epsilon}_2\gamma^\mu\epsilon_1)\partial_\mu$$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}] = 2\bar{\epsilon}_2\gamma^\mu\epsilon_1\partial_\mu$$

$$\left[\delta_{\epsilon_{1,\alpha}},\delta_{\epsilon_{2\beta}}\right]\binom{\phi}{\psi} = 2\bar{\epsilon}_2\gamma^\mu\epsilon_1\partial_\mu\binom{\phi}{\psi}$$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]\phi = \delta_{\epsilon_1}(\bar{\epsilon}_2\psi) - 1 \leftrightarrow 2 = \bar{\epsilon}_2(\partial\!\!\!/\,\phi)\epsilon_1 - 1 \leftrightarrow 2 = 2\bar{\epsilon}_2\gamma^\rho\epsilon_1\partial_\rho\phi$$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]\psi = \delta_{\epsilon_1}(\partial\!\!\!/\,\phi)\epsilon_2 - 1 \leftrightarrow 2 = (\bar{\epsilon}_1\partial_\mu\psi)\gamma^\mu\epsilon_2 - 1 \leftrightarrow 2$$

$$M\chi(\bar{\psi}N\phi) = -\sum_j \frac{1}{2}MO_jN\phi(\bar{\psi}O_j\chi)$$

$$\delta_\alpha^\beta\delta_\gamma^\delta = \frac{1}{2}(O_i)_\alpha^\delta(O_i)_\gamma^\beta$$

$$M_\alpha^\delta = \frac{1}{2}\mathrm{Tr}(MO_i)(O_i)_\alpha^\delta$$

$$M\chi(\bar{\psi}N\phi) = -\sum_j \frac{1}{4}MO_jN\phi(\bar{\psi}O_j\chi)$$

$\gamma^\mu\epsilon_2(\bar{\epsilon}_1\partial_\mu\psi) - 1 \leftrightarrow 2$
$= -\frac{1}{2}[\gamma^\mu 1\partial_\mu\psi(\bar{\epsilon}_1 1\epsilon_2) + \gamma^\mu\gamma_\nu\partial_\mu\psi(\bar{\epsilon}_1\gamma^\nu\epsilon_2) + \gamma^\mu\gamma_3\partial_\mu\psi(\bar{\epsilon}_1\gamma_3\epsilon_2)] - 1 \leftrightarrow 2$
$= +\gamma^\mu\gamma_\nu\partial_\mu\psi(\bar{\epsilon}_2\gamma_\nu\epsilon_1) + \gamma^\mu\gamma_3\partial_\mu\psi(\bar{\epsilon}_2\gamma_3\epsilon_2)$
$= 2(\bar{\epsilon}_2\gamma^\mu\epsilon_1)\partial_\mu\psi - \gamma^\nu(\partial\!\!\!/\psi)(\bar{\epsilon}_2\gamma_\nu\epsilon_1) - \gamma_3(\partial\!\!\!/\psi)(\bar{\epsilon}_2\gamma_3\epsilon_1)$

$$S = -\frac{1}{2}\int d^2x\left[\left(\partial_\mu\phi\right)^2 + \bar{\psi}\,\partial\!\!\!/\psi - F^2\right]$$

$$\delta\phi = \bar{\epsilon}\psi;\ \delta\psi = \partial\!\!\!/\phi\epsilon + F\epsilon;\ \delta F = \bar{\epsilon}\,\partial\!\!\!/\psi$$

$$\delta_{\epsilon_1}\delta_{\epsilon_2}\phi = \delta_{\epsilon_1}(\bar{\epsilon}_2\psi) = \bar{\epsilon}_2\,\partial\!\!\!/\phi\epsilon_1 + \bar{\epsilon}_2\epsilon_1 F$$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]\phi = 2(\bar{\epsilon}_2\gamma^\mu\epsilon_1)\partial_\mu\phi$$

$$\delta_{\epsilon_1}\delta_{\epsilon_2}\psi = \delta_{\epsilon_1}(\partial\!\!\!/\phi\epsilon_2 + F\epsilon_2) = \gamma^\mu\epsilon_2(\bar{\epsilon}_1\partial_\mu\psi) + (\bar{\epsilon}_1\,\partial\!\!\!/\psi)\epsilon_2$$

$(\bar{\epsilon}_1\,\partial\!\!\!/\psi)\epsilon_2 = -\frac{1}{2}[1\cdot\partial\!\!\!/\psi(\bar{\epsilon}_1 1\epsilon_2) + \gamma^\mu\,\partial\!\!\!/\psi(\bar{\epsilon}_1\gamma_\mu\epsilon_2) + \gamma_3\,\partial\!\!\!/\psi(\bar{\epsilon}_1\gamma_3\epsilon_2)] - 1 \leftrightarrow 2$
$=-(\bar{\epsilon}_1\gamma_\mu\epsilon_2)\gamma^\mu\,\partial\!\!\!/\psi - (\bar{\epsilon}_1\gamma_3\epsilon_2)\gamma_3\,\partial\!\!\!/\psi$
$=(\bar{\epsilon}_2\gamma_\mu\epsilon_1)\gamma^\mu\,\partial\!\!\!/\psi + (\bar{\epsilon}_2\gamma_3\epsilon_1)\gamma_3\,\partial\!\!\!/\psi$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]\psi = 2(\bar{\epsilon}_2\gamma^\mu\epsilon_1)\partial_\mu\psi$$

$$S_0 = -\frac{1}{2}\int d^4x\left[\left(\partial_\mu A\right)^2 + \left(\partial_\mu B\right)^2 + \bar{\psi}\,\partial\!\!\!/\psi\right]$$

$$\delta A = \bar{\epsilon}\psi;\ \delta B = \bar{\epsilon}i\gamma_5\psi;\ \delta\psi = \partial\!\!\!/(A + i\gamma_5 B)\epsilon$$

$$S = S_0 + \int d^4x\left[\frac{F^2}{2} + \frac{G^2}{2}\right]$$

$$\delta A = \bar{\epsilon}\psi;\ \delta B = \bar{\epsilon}i\gamma_5\psi;\ \delta\psi = \partial\!\!\!/(A + i\gamma_5 B)\epsilon + (F + i\gamma_5 G)\epsilon$$
$$\delta F = \bar{\epsilon}\,\partial\!\!\!/\psi;\ \delta G = \bar{\epsilon}i\gamma_5\,\partial\!\!\!/\psi$$

$$C_{AB} = \begin{pmatrix}\epsilon^{\alpha\beta} & 0\\ 0 & \epsilon_{\dot{\alpha}\dot{\beta}}\end{pmatrix};\ \epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

$$\gamma^\mu = \begin{pmatrix}0 & \sigma^\mu\\ \bar{\sigma}^\mu & 0\end{pmatrix};\ (\sigma^\mu)_{\alpha\dot{\alpha}} = (1,\vec{\sigma})_{\alpha\dot{\alpha}};\ (\bar{\sigma}^\mu)^{\alpha\dot{\alpha}} = (1,-\vec{\sigma})^{\alpha\dot{\alpha}}$$

$$(\bar{\lambda}^a\gamma^\mu\lambda^c)(\bar{\epsilon}\gamma_\mu\lambda^b)f_{abc} = 0$$

$$S = -\frac{1}{2}\int d^2x\left[\left(\partial_\mu\phi\right)^2 + \bar{\psi}\,\partial\!\!\!/\psi - F^2\right]$$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]F = 2(\bar{\epsilon}_2\gamma^\mu\epsilon_1)\partial_\mu F$$

$$\psi = \begin{pmatrix}\psi_\alpha\\ \bar{\chi}^{\dot{\alpha}}\end{pmatrix}$$

$$C_{AB} = \begin{pmatrix}\epsilon^{\alpha\beta} & 0\\ 0 & \epsilon_{\dot{\alpha}\dot{\beta}}\end{pmatrix}$$

$$\gamma^\mu = \begin{pmatrix}0 & \sigma^\mu\\ \bar{\sigma}^\mu & 0\end{pmatrix}$$

$$\begin{pmatrix}\psi_\alpha\\ \bar{\psi}^{\dot{\alpha}}\end{pmatrix}$$

$$\{Q_A, Q_B\} = 2(C\gamma^\mu)_{AB}P_\mu$$

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2(\sigma^\mu)_{\alpha\dot{\alpha}} P_\mu$$
$$\{Q_\alpha, Q_\beta\} = 0;\ \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

$$Q_\alpha = \partial_\alpha - i(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$$
$$\bar{Q}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} + i(\sigma^\mu)_{\alpha\dot{\alpha}}\theta^\alpha\partial_\mu$$
$$P_\mu = i\partial_\mu$$

$$x^\mu \rightarrow x'^\mu = x^\mu + i\theta\sigma^\mu\bar{\xi} - i\xi\sigma^\mu\bar{\theta}$$
$$\theta \rightarrow \theta' = \theta + \xi$$
$$\bar{\theta} \rightarrow \bar{\theta}' = \bar{\theta} + \bar{\xi}$$

$$D_\alpha = \partial_\alpha + i(\sigma^\mu)_{\alpha\dot{\theta}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$$
$$\bar{D}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} - i(\sigma^\mu)_{\alpha\dot{\alpha}}\theta^\alpha\partial_\mu$$

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$$

$$\bar{D}_{\dot{\alpha}}\Phi = 0$$

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

$$\bar{D}_{\dot{\alpha}} y^\mu = 0;\ \bar{D}_{\dot{\alpha}}\theta^\beta = 0$$

$$\Phi = \Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

$$\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = 4$$

$$\phi(x) = \Phi|_{\theta=\bar{\theta}=0}$$
$$\psi(x) = \frac{1}{\sqrt{2}}D_\alpha\Phi\Big|_{\theta=\bar{\theta}=0}$$
$$F(x) = -\frac{D^2\Phi|_{\theta=\bar{\theta}=0}}{4}$$

$$\Phi = \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x)$$
$$+ i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) - \frac{i}{2}\theta^2(\partial_\mu\psi\sigma^\mu\bar{\theta}) - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi(x)$$

$$\int d\theta 1 = 0;\ \int d\theta\theta = 1$$

$$d^2\theta = -\frac{1}{4}d\theta^\alpha d\theta^\beta\epsilon_{\alpha\beta}$$

$$\int d^4x \int d^2\theta = -\frac{1}{4}\int d^4x D^2\Big|_{\theta\bar{\theta}=0} = -\frac{1}{4}\int d^4x D^\alpha D_\alpha\Big|_{\theta=\bar{\theta}=0}$$
$$\int d^4x \int d^2\bar{\theta} = -\frac{1}{4}\int d^4x \bar{D}^2\Big|_{\theta\bar{\theta}=0} = -\frac{1}{4}\int d^4x \bar{D}^\alpha\bar{D}_\alpha\Big|_{\theta=\bar{\theta}=0}$$

$$\mathcal{L} = \int d^4\theta K(\Phi, \Phi^\dagger) + \int d^2\theta W(\Phi) + \int d^2\bar{\theta}\bar{W}(\Phi^\dagger)$$

$$K = \Phi^\dagger\Phi$$
$$W = \lambda\Phi + \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3$$

$$\int d^4x \int d^2\theta\left(\lambda\Phi + \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3\right) = -\frac{1}{4}\int d^4x D^2\left(\lambda\Phi + \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3\right)\Big|_{\theta=\bar{\theta}=0}$$

$$D^2(\Phi^2)|_{\theta=\bar{\theta}=0} = 2(D^2\Phi)|_{\theta=\bar{\theta}=0}\Phi|_{\theta=\bar{\theta}=0} + 2(D^\alpha\Phi)|_{\theta=\bar{\theta}=0}(D_\alpha\Phi)|_{\theta=\bar{\theta}=0}$$
$$D^2(\Phi^3)|_{\theta=\bar{\theta}=0} = 3(D^2\Phi)|_{\theta=\bar{\theta}=0}\Phi|_{\theta=\bar{\theta}=0}\Phi|_{\theta=\bar{\theta}=0} + 6(D^\alpha\Phi)|_{\theta=\bar{\theta}=0}(D_\alpha\Phi)|_{\theta=\bar{\theta}=0}\Phi|_{\theta=\bar{\theta}=0}$$

$$\int d^4x \int d^2\theta W(\Phi) = -\frac{1}{4}\int d^4x[2m\psi\psi + 4g\phi\psi\psi - 4F(\lambda + m\phi + g\phi^2)]$$

$$\bar{D}^2 D^2 \Phi = 16 \square \Phi \Rightarrow D^2 \bar{D}^2 \Phi^\dagger = 16 \square \Phi^\dagger$$

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$$

$$D^2\bar{D}^2 = \bar{D}^2 D^2 + 8i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu \bar{D}^{\dot{\alpha}} D^\alpha + 16 \square$$

$$\frac{1}{16}\int d^4x D^2\bar{D}^2(\Phi^\dagger\Phi)\Big|_{\theta=\bar{\theta}=0} = \frac{1}{16}\int d^4x \Big[\left(D^2\bar{D}^2\Phi^\dagger\right)\big|_{\theta=\bar{\theta}=0}\Phi\Big|_{\theta=\bar{\theta}=0} + \left(\bar{D}^2\Phi^\dagger\right)\big|_{\theta=\bar{\theta}=0}(D^2\Phi)\Big|_{\theta=\bar{\theta}=0} + 8i(\sigma^\mu)^{\alpha\dot{\alpha}}\left(\partial_\mu\overline{D_{\dot{\alpha}}}\Phi^\dagger\right)\big|_{\theta=\bar{\theta}=0}(D_\alpha\Phi)\Big|_{\theta=\bar{\theta}=0}\Big]$$

$$\int d^4x\left[\phi^* \square \phi + F^*F + i\left(\partial_\mu\bar{\psi}^{\dot{\alpha}}\right)(\sigma^\mu)_{\alpha\dot{\alpha}}\psi^\alpha\right]$$

$$F = -(\lambda + m\phi + g\phi^2)$$

$$-\int d^4x(\lambda + m\phi + g\phi^2)^2$$

$$S = (-2)\int d^4x \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}\bar{\lambda}\not{D}\lambda + \frac{D^2}{2}\right]$$

$$\begin{aligned}
\delta A_\mu^a &= \bar{\epsilon}\gamma_\mu\lambda^a \\
\delta\lambda^a &= \left[-\frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu}^a + i\gamma_5 D^a\right]\epsilon \\
\delta D^a &= i\bar{\epsilon}\gamma_5\not{D}\lambda^a
\end{aligned}$$

$$\begin{aligned}
&\{Q_\alpha^i, \bar{Q}_{\dot{\alpha}j} = 2(\sigma^\mu)_{\alpha\dot{\alpha}}P_\mu\delta_j^i \\
&\left\{Q_\alpha^i, Q_\beta^j\right\} = \epsilon_{\alpha\beta}Z^{ij} \\
&\left\{\bar{Q}_{\dot{\alpha}i}\bar{Q}_{\dot{\beta}j}\right\} = \epsilon_{\dot{\alpha}\dot{\beta}}Z_{ij}^*
\end{aligned}$$

$$\Psi = \Phi(\tilde{y}, \theta) + \sqrt{2}\tilde{\theta}^\alpha W_\alpha(\tilde{y}, \theta) + \tilde{\theta}^2 G(\tilde{y}, \theta)$$

$$\tilde{y}^\mu \equiv x^\mu + i\theta\sigma^\mu\bar{\theta} + i\tilde{\theta}\sigma^\mu\bar{\tilde{\theta}}$$

$$\frac{1}{16\pi}\mathrm{Im}\int d^4x d^2\theta d^2\tilde{\theta}\mathcal{F}(\Psi)$$

$$\int d^4x d^4\theta d^4\bar{\theta}\mathcal{H}(\Psi)$$

$$\sim \phi\frac{1}{\square}\phi$$

$$\begin{aligned}
a_\alpha &= \frac{1}{\sqrt{2}}\left[Q_\alpha^1 + \epsilon_{\alpha\beta}\left(Q_\beta^2\right)^\dagger\right] \\
b_\alpha &= \frac{1}{\sqrt{2}}\left[Q_\alpha^1 - \epsilon_{\alpha\beta}\left(Q_\beta^2\right)^\dagger\right]
\end{aligned}$$

$$\begin{aligned}
\left\{a_\alpha, a_\beta^\dagger\right\} &= 2(M + |Z|)\delta_{\alpha\beta} \\
\left\{b_\alpha, b_\beta^\dagger\right\} &= 2(M - |Z|)\delta_{\alpha\beta}
\end{aligned}$$

$$\Phi = \phi(x) + \sqrt{2}\psi(x) + \theta^2 F(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) - \frac{i}{2}\theta^2\left(\partial_\mu\psi\sigma^\mu\bar{\theta}\right) - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi(x)$$

$$D^2\bar{D}^2\Phi = 16 \square \Phi$$

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\Phi_i^\dagger\Phi_i + \left(\int d^2\theta W(\Phi_i) + \text{ h.c.}\right)$$

$$\mathcal{L} = \left(\partial_\mu A_i\right)^\dagger \partial^\mu A_i - i\bar{\psi}_i\bar{\sigma}^\mu\partial_\mu\psi_i + F_i^\dagger F_i + \\ + \frac{\partial W}{\partial A_i}F_i + \frac{\partial \bar{W}}{\partial A_i^\dagger}F_i^\dagger - \frac{1}{2}\frac{\partial^2 W}{\partial A_i \partial A_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2 \bar{W}}{\partial A_i^\dagger \partial A_j^\dagger}\bar{\psi}_i\bar{\psi}_j$$

$$\binom{d}{r} - \binom{d-1}{r-1} = \frac{(d-1)\ldots(d-r)}{1\cdot 2\cdot \ldots r} = \binom{d-1}{r}$$

$$\mathcal{L} = \frac{1}{2}h_{\mu\nu,\rho}^2 + h_\mu^2 - h^\mu h_{,\mu} + \frac{1}{2}h_{,\mu}^2;\ h_\mu \equiv \partial^\nu h_{\nu\mu};\ h \equiv h^\mu{}_\mu$$

$$\partial^\nu \bar{h}_{\mu\nu} = 0;\ \bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \eta_{\mu\nu}\frac{h}{2}$$

$$\frac{d(d-1)}{2} - d = \frac{(d-1)(d-2)}{2} - 1$$

$$(p - m)u(p) = 0$$

$$\partial^{\mu_1}A_{\mu_1\ldots\mu_r} = 0$$

$$k^{\mu_1}\epsilon_{\mu_1\ldots\mu_r}(k) = 0$$

$$\binom{d-2}{r} = \frac{(d-2)\ldots(d-1-r)}{1\cdot 2\cdot \ldots r}$$

$$\delta e_\mu^a = \frac{k}{2}\bar{\epsilon}\gamma^a\psi_\mu$$

$$\delta\psi_\mu = \frac{1}{k}D_\mu\epsilon;\ D_\mu\epsilon = \partial_\mu\epsilon + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon$$

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}R(\Gamma)$$

$$S_{EH} = \frac{1}{2k^2}\int d^4x(\text{det}e)R_{\mu\nu}^{ab}(\omega)(e^{-1})_a^\mu(e^{-1})_b^\nu \\ = \frac{1}{2k^2}\int d^4x\epsilon_{abcd}\epsilon^{\mu\nu\rho\sigma}e_\mu^a e_\nu^b R_{\rho\sigma}^{cd} \\ \equiv \frac{1}{2k^2}\int d^4x\epsilon_{abcd}e^a \wedge e^b \wedge R^{cd}(\omega)$$

$$R^{ab} = d\omega^{ab} + \omega^{ac}\wedge\omega^{cb}$$

$$\left[D_\mu, D_\nu\right] = F_{\mu\nu} \equiv F_{\mu\nu}^a T_a$$

$$\left[D_\mu(\omega), D_\nu(\omega)\right] = R_{\mu\nu}^{rs}\frac{1}{4}\gamma_{rs}$$

$$[D_a, D_b] = \left(2e_a^\mu e_b^\nu D_{[\mu}e_{\nu]}^c\right)D_c + \left(e_a^\mu e_b^\nu R_{\mu\nu}^{rs}(\omega)\right)\frac{1}{4}\gamma_{rs} \\ \equiv T_{ab}^c D_c + R_{ab}^{rs}M_{rs}$$

$$T_{ab}^c \equiv e_a^\mu e_b^\nu T_{\mu\nu}^c = 2e_a^\mu e_b^\nu D_{[\mu}e_{\nu]}{}^c \\ R_{ab}^{cd} = e_a^\mu e_b^\nu R_{\mu\nu}^{cd}(\omega)$$

$$S_{RS} = -\frac{1}{2}\int d^dx\bar{\psi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho \\ = -\frac{i}{2}\int d^4x\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu\partial_\rho\psi_\sigma$$

$$S_{RS} = -\frac{1}{2}\int d^d x(\mathrm{dete})\bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho$$
$$= -\frac{i}{2}\int d^4 x \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma$$

$$S_{\mathcal{N}=1} = S_{EH}(\omega, e) + S_{RS}(\psi_\mu)$$

$$\delta e_\mu^a = \frac{k}{2}\bar{\epsilon}\gamma^a \psi_\mu;\ \delta\psi_\mu = \frac{1}{k}D_\mu \epsilon$$

$$\omega_\mu^{ab} = \omega_\mu^{ab}(e,\psi) = \omega_\mu^{ab}(e) + \psi\psi$$

$$\frac{\delta S_{\mathcal{N}=1}}{\delta \omega_\mu^{ab}} = 0 \Rightarrow \omega_\mu^{ab}(e,\psi)$$

$$\delta\omega_\mu^{ab} = -\frac{1}{4}\bar{\epsilon}\gamma_5\gamma_\mu\tilde{\psi}^{ab} + \frac{1}{8}\bar{\epsilon}\gamma_5\left(\gamma^\lambda \tilde{\psi}_\lambda^b e_\mu^a - \gamma^\lambda \tilde{\psi}_\lambda^a e_\mu^b\right)$$
$$\tilde{\psi}^{ab} \equiv \epsilon^{abcd}\psi_{cd};\ \psi_{ab} \equiv e_a{}^\mu e_b{}^\nu\left(D_\mu\psi_\nu - D_\nu\psi_\mu\right)$$

$$\sim \gamma^\lambda \tilde{\psi}_{\lambda\mu} = 0$$

$$\delta S = \frac{\delta S}{\delta e}\delta e + \frac{\delta S}{\delta\psi}\delta\psi + \frac{\delta S}{\delta\omega}\left(\frac{\delta\omega}{\delta e}\delta e + \frac{\delta\omega}{\delta\psi}\delta\psi\right)$$

$$\delta_E e_\mu^a = \xi^\nu \partial_\nu e_\mu^a + \left(\partial_\mu \xi^\nu\right)e_\nu^a$$
$$\delta_E \omega_\mu^{ab} = \xi^\nu \partial_\nu \omega_\mu^{ab} + \left(\partial_\mu \xi^\nu\right)\omega_\nu^{ab}$$
$$\delta_E \psi_\mu = \xi^\nu \partial_\nu \psi_\mu + \left(\partial_\mu \xi^\nu\right)\psi_\nu$$

$$\delta_{LL} e_\mu^a = \lambda^{ab} e_\mu^b$$
$$\delta_{LL}\omega_\mu^{ab} = D_\mu \lambda^{ab} = \partial_\mu \lambda^{ab} + \omega_\mu^{ac}\lambda^{cb} - \omega_\mu^{bc}\lambda^{ca}$$
$$\delta_{LL}\psi_\mu = -\lambda^{ab}\frac{1}{4}\gamma_{ab}\psi_\mu$$

$$D_\mu \eta^I = \pm\frac{i}{2}\gamma_\mu \eta^I$$

$$S_{EH} = -\frac{1}{8k^2}\int d^3 x e R$$

$$R = R_{\mu\nu}^{mn}(\omega)e_m^\mu e_n^\nu$$
$$R_{\mu\nu}^{mn}(\omega) = \partial_\mu \omega_\nu^{mn} - \partial_\nu \omega_\nu^{mn} + \omega_\mu^{mp}\omega_\nu^{pn} - \omega_\nu^{mp}\omega_\mu^{pn}$$

$$S_{RS} = -\frac{1}{2}\int d^3 x e \bar{\psi}\gamma^{\mu\nu\rho}D_\nu(\omega)\psi_\rho$$

$$D_\nu \psi_\rho = \partial_\nu \psi_\rho + \frac{1}{4}\omega_\nu^{mn}\gamma_{mn}\psi_\rho$$

$$S_{RS} = +\frac{1}{2}\int d^3 x \epsilon^{\mu\nu\rho}\bar{\psi}_\mu D_\nu \psi_\rho$$

$$\delta_E e_\mu^a = \xi^\nu \partial_\nu e_\mu^a + \left(\partial_\mu \xi^\nu\right)e_\nu^a$$
$$\delta_E \omega_\mu^{ab} = \xi^\nu \partial_\nu \omega_\mu^{ab} + \left(\partial_\mu \xi^\nu\right)\omega_\nu^{ab}$$
$$\delta_E \psi_\mu = \xi^\nu \partial_\nu \psi_\mu + \left(\partial_\mu \xi^\nu\right)\psi_\nu$$

$$\delta_{LL} e_\mu^a = \lambda^{ab} e_\mu^b$$
$$\delta_{LL}\omega_\mu^{ab} = D_\mu \lambda^{ab} = \partial_\mu \lambda^{ab} + \omega_\mu^{ac}\lambda^{cb} - \omega_\mu^{bc}\lambda^{ca}$$
$$\delta_{LL}\psi_\mu = -\lambda^{ab}\frac{1}{4}\gamma_{ab}\psi_\mu$$

$$\mathcal{L}_{3/2} = \bar{\psi}_\mu \mathcal{O}^{\mu\nu\rho}\partial_\nu \psi_\rho$$

$$S_S = -\frac{1}{2}\int d^3xeS^2$$

$$\delta_E S = \xi^\nu \partial_\nu S$$
$$\delta_{LL} S = 0$$

$$\delta\psi_\mu = \frac{1}{k}D_\mu\epsilon = \frac{1}{k}\left(\partial_\mu\epsilon + \frac{1}{4}\omega_\mu^{mn}\gamma_{mn}\right)\epsilon$$

$$\{Q_\alpha, Q_\beta\} = 2(C\gamma^\mu)_{\alpha\beta}P_\mu$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \xi^\mu\partial_\mu$$
$$\xi^\mu \equiv 2\bar{\epsilon}_2\gamma^\mu\epsilon_1$$

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = \delta_{g.c.}(\xi^\mu) + \delta_{LL}(\xi^\mu\omega_\mu^{mn}) + \delta_Q(-\xi^\mu\psi_\mu)$$
$$\xi^\mu \equiv 2\bar{\epsilon}_2\gamma^\mu\epsilon_1$$

$$\delta S_{EH} = -\frac{1}{8k^2}\int d^3xR_{\mu\nu}^{mn}(\omega)\delta[ee_m^\mu e_n^\nu]$$
$$= -\frac{1}{4k^2}\int d^3xe\left[R_\mu^m - \frac{1}{2}e_\mu^m R\right]\delta e_m^\mu$$

$$[D_\mu(\omega), D_\nu(\omega)] = \frac{1}{4}R_{\mu\nu}^{mn}\gamma_{mn}$$

$$\delta S_{RS} = \frac{1}{k}\int d^3x\epsilon^{\mu\nu\rho}\bar{\psi}_\mu D_\nu D_\rho\epsilon$$
$$= \frac{1}{8k}\int d^3x\epsilon^{\mu\nu\rho}R_{\mu\nu}^{mn}\bar{\psi}\gamma_{mn}\epsilon$$

$$\gamma_{mn} = -\epsilon_{mnr}\gamma^r$$

$$\epsilon^{\mu\nu\rho}\epsilon_{mnr} = -6ee_m^{[\mu}e_n^\nu e_r^{\rho]}$$

$$\epsilon^{\mu\nu\rho}R_{\nu\rho}^{mn}\gamma_{mn} = -\epsilon^{\mu\nu\rho}\epsilon_{mnr}R_{\nu\rho}^{mn}\gamma^r$$
$$= +6eR_{\nu\rho}^{mn}e_m^{[\mu}e_n^\nu e_r^{\rho]}\gamma^r$$
$$= 4ee_m^\mu\left(R_\rho^m - \frac{1}{2}Re_\rho^m\right)\gamma^r e_r^\rho$$

$$\delta S_{RS} = \frac{1}{2k}\int d^3xe\left(R_\rho^m - \frac{1}{2}Re_\rho^m\right)\bar{\psi}_m\gamma^\rho\epsilon$$

$$\delta e_m^\mu = 2k\bar{\psi}_m\gamma^\mu\epsilon$$

$$\bar{\psi}\chi = \bar{\chi}\psi$$
$$\bar{\psi}\gamma_m\chi = -\bar{\chi}\gamma_m\psi$$

$$\delta e_\mu^m = 2k\bar{\epsilon}\gamma^m\psi_\mu$$

$$[\delta_1, \delta_2]e_\mu^m = 2k\bar{\epsilon}_2\gamma^m\left(\frac{1}{k}D_\mu\epsilon_1\right) - (1 \leftrightarrow 2)$$
$$= 2\partial_\mu(\bar{\epsilon}_2\gamma^m\epsilon_1) + 2\left[\frac{1}{4}\omega_\mu^{rs}\bar{\epsilon}_2\gamma^m\gamma_r\gamma_s\epsilon_1 - (1 \leftrightarrow 2)\right]$$

$$[\delta_1, \delta_2]e_\mu^m = (\partial_\mu\xi^\nu)e_\nu^m + \xi^\nu(\partial_\mu e_\nu^m) + 2\left[\frac{1}{4}\omega_\mu^{rs}\bar{\epsilon}_2\gamma^m\gamma_r\gamma_s\epsilon_1 - (1 \leftrightarrow 2)\right]$$
$$= (\partial_\mu\xi^\nu)e_\nu^m + \xi^\nu\partial_\nu e_\mu^m + \xi^\nu(\partial_\mu e_\nu^m - \partial_\nu e_\mu^m) + 2\left[\frac{1}{4}\omega_\mu^{rs}\bar{\epsilon}_2\gamma^m\gamma_r\gamma_s\epsilon_1 - (1 \leftrightarrow 2)\right]$$

$$\partial_\mu e_\nu^m - \partial_\nu e_\mu^m + \omega_\mu^{mn}(e)e_\nu^n - \omega_\nu^{mn}(e)e_\mu^n = 0$$

$$[\delta_1,\delta_2]e_\mu^m = \delta_E(\xi^\nu)e_\mu^m + \xi^\nu\big(-\omega_\mu^{mn}(e)e_\nu^n + \omega_\nu^{mn}(e)e_\mu^n\big) + 2\left[\frac{1}{4}\omega_\mu^{rs}\bar{\epsilon}_2\gamma^m\gamma_r\gamma_s\epsilon_1 - (1\leftrightarrow 2)\right]$$

$$\gamma^m\gamma^r\gamma^s = \gamma^{mrs} + \eta^{mr}\gamma^s + \eta^{rs}\gamma^m - \eta^{ms}\gamma^r$$

$$2\omega_\mu^{rs}(e,\psi)\bar{\epsilon}_2\gamma_s\epsilon_1 = \omega_\mu^{rs}(e,\psi)\xi_s$$

$$[\delta_1,\delta_2]e_\mu^m = \delta_E(\xi^\nu)e_\mu^m + [\xi^\nu\omega_\nu^{mn}(e)]e_{\mu n} + [\omega_\mu^{ms}(e,\psi) - \omega_\mu^{ms}(e)]\xi_s$$

$$\omega_\mu^{mn}(e,\psi) = \omega_\mu^{mn}(e) + \omega(\psi)$$
$$\omega_\mu^{mn}(\psi) = k^2\big(\bar{\psi}_\mu\gamma_m\psi_n - \bar{\psi}_\mu\gamma^n\psi_m + \bar{\psi}_m\gamma_\mu\psi_n\big)$$

$$[\delta_1,\delta_2]e_\mu^m = \delta_E(\xi^\nu)e_\mu^m + [\xi^\nu\omega_\nu^{mn}(e,\psi)]e_{\mu n} + [\omega_{\mu s}^{m}(\psi) - \omega_s^{m}{}_{\mu}(\psi)]\xi_s$$

$$2k^2\bar{\psi}_\mu\gamma^m\psi_s\xi^s = \delta_Q(-k\xi^s\psi_s)$$

$$[\delta_1,\delta_2]e_\mu^m = \delta_E(\xi^\nu) + \delta_{LL}(\xi^\nu\omega_\nu^{mn}(e,\psi)) + \delta_Q(-k\xi^\nu\psi_\nu)$$

$$\delta\int d^3x\left(-\frac{1}{2}eS^2\right) = \int d^3x\big(-k\bar{\epsilon}\gamma^\mu\psi_\mu S^2 - eS\delta S\big)$$

$$\delta_S\psi_\mu = cS\gamma_\mu\epsilon$$

$$\delta_S I_{RS} = c\int d^3xS\epsilon^{\mu\nu\rho}[\bar{\epsilon}\gamma_\mu D_\nu(\omega)\psi_\rho]$$

$$\delta S = k\bar{\epsilon}\gamma^\mu\psi_\mu S + \frac{c}{e}\epsilon^{\mu\nu\rho}\bar{\epsilon}\gamma_\mu D_\nu(\omega)\psi_\rho$$

$$[\delta_1,\delta_2]e_\mu^m\big|_{extra} = 2ckS\bar{\epsilon}_2\gamma^m\gamma_\mu\epsilon_1 - (1\leftrightarrow 2) = 4ckS\bar{\epsilon}_2\gamma^{mn}\epsilon_1 e_{\mu n}$$

$$\delta_{LL}(4ckS\bar{\epsilon}_s\gamma^m{}_n\epsilon_1) = \delta_{LL}(-2ckS\epsilon^m{}_{ns}\xi^s)$$

$$[\delta_1,\delta_2]\psi_\mu = \frac{1}{4k}[\delta_1\omega_\mu^{mn}(e,\psi)]\gamma_{mn}\epsilon_2 + c(\delta_1 S)\gamma_\mu\epsilon_2 - (1\leftrightarrow 2)$$

$$[\delta_{g.c.}(\eta^\mu),\delta_{g.c.}(\xi^\mu)] = \delta_{g.c.}\big(\xi^\mu\partial_\mu\eta^\nu - \eta^\mu\partial_\mu\xi^\nu\big)$$

$$D_\mu e_n^a u = \partial_\mu e_\nu^a + \omega_\mu^{ab}e_\nu^b - \Gamma_{\mu\nu}^\rho(g)e_\rho^a$$

$$S = \int d^3x\mathrm{Tr}\left(dA\wedge A + \frac{2}{3}A\wedge A\wedge A\right)$$

$$S^n = \frac{SO(n+1)}{SO(n)}$$

$$[T_a,T_b] = f_{ab}^c T_c$$

$$[T_a,T_b\} = f_{ab}^c T_c$$

$$[T_a,T_b\}\colon [B,B];\ \{F,F\};\ [B,F]$$

$$[H_i,H_j] = f_{ij}{}^k H_k$$
$$[H_i,K_\alpha] = f_{i\alpha}{}^\beta K_\beta$$
$$[K_\alpha,K_\beta] = f_{\alpha\beta}{}^i H_i + f_{\alpha\beta}{}^\gamma K_\gamma$$

$$e^{z^\alpha K_\alpha}h, \forall h$$

$$h = e^{y^i H_i}$$

$$ge^{z^\alpha K_\alpha} = e^{z'^\alpha K_\alpha}h(z,g)$$

$$e^{x\cdot K}e^{dx\cdot K} = e^{x\cdot K + dx^m e_m^\mu(x)K_\mu} \times e^{dx^m e_m^\mu(x)\omega_\mu^i(x)H_i} + \mathcal{O}(dx^2)$$
$$e^{dg^a T_a}e^{x\cdot K} = e^{x\cdot K + dg^a f_a^\mu(x)K_\mu} \times e^{-dg^a \Omega_a^i(x)H_i} + \mathcal{O}(dg^2)$$

$$L^{-1}(x)\partial_\mu L(x) = e_\mu^m(x)K_m + \omega_\mu^i(x)H_i$$

$$V^\mu(x+dx) = V^\mu(x) - dx^\nu \Gamma_{\nu\rho}^\mu V^\rho(x)$$

$$V^m(x) = V^\mu(x)e_\mu^m(x)$$

$$V^m(x+dx) = V^m(x) - dx^\nu \omega_\nu^m{}_n(x)V^n(x)$$

$$D_\mu e_\nu^m = \partial_\mu e_\nu^m - \Gamma_{\mu\nu}^\rho e_\rho^m + \omega_\mu^m{}_n^m(x)e_\nu^n = 0$$

$$\omega_\mu{}^m{}_n(x) = e_\mu^r(x)\omega_r{}^m{}_n(0) + \omega_\mu^i(x)f_{in}{}^m$$

$$l = dg^A f_A^\mu(x)\partial_\mu$$

$$D_\mu v^m = \partial_\mu v^m + \omega_\mu^m{}_n^m(x)v^n$$

$$\omega_\mu{}^{\tilde{a}}_{\tilde{b}} = \omega_\mu^i(x)(H_i)^{\tilde{a}}{}_{\tilde{b}}$$

$$D_m\phi^{\tilde{a}}(x) = e_m^\mu(x)\left[\partial_\mu\phi^{\tilde{a}}(x) + \omega_\mu{}^{\tilde{a}}{}_{\tilde{b}}(x)\phi^{\tilde{b}}(x)\right]$$

$$\delta_{(g)}\phi^{\tilde{a}}(x) \equiv \mathcal{L}_H\phi^{\tilde{a}}(x) = l\phi^{\tilde{a}}(x) + dg^A\Omega_A^i(x)(H_i)^{\tilde{a}}{}_{\tilde{b}}\phi^{\tilde{b}}(x)$$

$$[D_m, \mathcal{L}_H] = 0$$

$$\int_M \mu(x)f(x)d^n x$$

$$\mu(x) = \left[\det e_\mu^m(x)\right]\mu(0)$$

$$e^{z^\alpha K_\alpha}e^{y^i H_i} = e^{\xi^\mu P_\mu + \epsilon^A Q_A + \bar{\epsilon}^{\dot{A}}Q_{\dot{A}}}e^{\lambda^{mn}M_{mn}}$$

$$Q_\alpha = \begin{pmatrix} Q^A \\ \bar{Q}_{\dot{A}} \end{pmatrix}$$

$$(\gamma^m)^\alpha{}_\beta = \begin{pmatrix} 0 & -i(\sigma^m)^{A\dot{B}} \\ i(\bar{\sigma}^m)_{\dot{A}B} & 0 \end{pmatrix}$$

$$e^{\epsilon^A Q_A + \bar{\epsilon}^{\dot{A}}Q_{\dot{A}} + \xi^\mu P_\mu + \frac{1}{2}\lambda_{mn}M^{mn}}e^{\bar{\theta}Q + x^\mu P_\mu} = e^{\bar{\theta}'Q + x'^\mu P_\mu}h$$

$$x'^\mu = x^\mu + \xi^\mu + \frac{1}{2}\bar{\theta}^{\dot{B}}\epsilon^A(-2i(\sigma^\mu)_{A\dot{B}}) + \frac{1}{2}\theta^B\bar{\epsilon}^{\dot{A}}(-2i(\sigma^\mu)_{B\dot{A}}) + \lambda_\nu^\mu x^\nu$$
$$\theta'^A = \theta^A + \epsilon^A + \frac{1}{4}\lambda^{mn}(\sigma_{mn})_B^A\theta^B$$
$$\bar{\theta}'^{\dot{A}} = \bar{\theta}^{\dot{A}} + \bar{\epsilon}^{\dot{A}} - \frac{1}{4}\lambda^{mn}(\bar{\sigma}_{mn})_{\dot{B}}^{\dot{A}}\bar{\theta}^{\dot{B}}$$

$$\omega_\Lambda^i = \Omega_\Lambda^i = 0$$

$$\delta z^\Lambda = lz^\Lambda = \Xi^\Sigma f_\Sigma^\Pi\partial_\Pi z^\Lambda = \Xi^\Sigma f_\Sigma^\Lambda = \epsilon^A l_A^\Lambda + \bar{\epsilon}^{\dot{A}}l_{\dot{A}}^\Lambda + \xi^\mu l_\mu^\Lambda$$

$$l_\Sigma = l_\Sigma^\Lambda\partial_\Lambda$$

$$l_A = \frac{\partial}{\partial\theta^A} + i(\sigma^\mu)_{A\dot{B}}\bar{\theta}^{\dot{B}}\partial_\mu$$
$$l_{\dot{A}} = \frac{\partial}{\partial\bar{\theta}^{\dot{A}}} + i(\sigma^\mu)_{B\dot{A}}\theta^B\partial_\mu$$
$$l_\mu = P_\mu = i\partial_\mu$$

$$[\mathcal{L}_H, D_M] = [l, D_M] = 0$$

$$\delta\Phi(x,\theta) = \left(\epsilon^A l_A + \bar{\epsilon}^{\dot{A}} l_{\dot{A}}\right)\Phi(x,\theta)$$

$$e^{z^\Lambda K_\Lambda} e^{dz^M K_M} = e^{z^\Lambda K_\Lambda + dz^M E_M^\Lambda K_\Lambda} + \mathcal{O}(dz^2)$$

$$E_M^\Lambda = \begin{pmatrix} \delta_m^\mu & 0 & 0 \\ -i\sigma_{A\dot{B}}^\mu \theta^{\dot{B}} & \delta_A^B & 0 \\ -i\sigma_{B\dot{A}}^\mu \theta^B & 0 & \delta_{\dot{A}}^{\dot{B}} \end{pmatrix}$$

$$\begin{aligned} D_M &\equiv (D_m, D_A, D_{\dot{A}}) \\ &= E_M^\Lambda\left(\partial_\Lambda + \omega_\Lambda^i T_i\right) = E_M^\Lambda \partial_\Lambda \end{aligned}$$

$$\begin{aligned} D_m &= \partial_m \\ D_A &= \partial_A - i\sigma_{A\dot{B}}^\mu \bar{\theta}^{\dot{B}} \partial_\mu \\ D_{\dot{A}} &= \partial_A - i\sigma_{B\dot{A}}^\mu \theta^B \partial_\mu \end{aligned}$$

$$[D_M, D_N\} = T_{MN}^P D_P + R_{MN}^i T_i$$

$$\begin{aligned} R_{MN}^i &= e_M^\Lambda e_N^\Sigma R_{\Lambda\Sigma}^i \\ R_{\Lambda\Sigma}^i &= \partial_\Lambda \omega_\Sigma^i - \partial_\Sigma \omega_\Lambda^i + f_{jk}{}^i \omega_\Lambda^j \Omega_\Sigma^k \\ T_{MN}^P &= e_M^\Lambda\left(D_\Lambda e_N^\Sigma\right) e_\Sigma^P - M \leftrightarrow N \\ D_\Lambda e_N^\Sigma &= \partial_\Lambda e_N^\Sigma + \omega_\Lambda^i f_{Ni} e_P^\Sigma \end{aligned}$$

$$T_{A\dot{B}}^m = f_{A\dot{B}}^m$$

$$\{D_A, D_{\dot{B}}\} = T_{A\dot{B}}^m D_m$$

$$\begin{pmatrix} x' \\ \theta' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix} \equiv M \begin{pmatrix} x \\ \theta \end{pmatrix}$$

$$\text{sdet}\, M \equiv \frac{\det(A - BD^{-1}C)}{\det D}$$

$$\int d^4x d^4\theta \mu(x,\theta) f(x,\theta) = \int d^4x d^4\theta J(x,\theta)\mu(0) f(x,\theta)$$

$$\mu = \text{sdet} E_\Lambda^M$$

$$\text{sdet}\, E_\Lambda^M = 1/\, \text{sdet}\, E_M^\Lambda = 1$$

$$V^m(x + dx) = V^m(x) - dx^\nu \omega_\nu^m{}_n V^n(x)$$

$$[D_m, \mathcal{L}_H] = 0$$

$$e^{x\cdot K} e^{dx\cdot K} = e^{x\cdot K + dx^m e_m^\mu(x) K_\mu} \times e^{dx^m e_m^\mu(x) \omega_\mu^i(x) H_i} + \mathcal{O}(dx^2)$$

$$\begin{aligned} L^{-1}\partial_\mu L &= e_\mu^m(x) K_m + \omega_\mu^i(x) H_i \\ L &\equiv e^{-x^\alpha K_\alpha} \end{aligned}$$

$$e^{\epsilon^A Q_A + \bar{\epsilon}^{\dot{A}} Q_{\dot{A}} + \xi^\mu P_\mu + \frac{1}{2}\lambda_{mn} M^{mn}} e^{\bar{\theta} Q + x^\mu P_\mu} = e^{\bar{\theta}' Q + x'^\mu P_\mu} h$$

$$x'^{\mu}= x^{\mu} + \xi^{\mu} + \frac{1}{2}\bar{\theta}^{\dot{B}}\epsilon^{A}(-2i(\sigma^{\mu})_{A\dot{B}}) + \frac{1}{2}\theta^{B}\bar{\epsilon}^{\dot{A}}(-2i(\sigma^{\mu})_{B\dot{A}}) + \lambda^{\mu}_{\nu}x^{\nu}$$
$$\theta'^{A}= \theta^{A} + \epsilon^{A} + \frac{1}{4}\lambda^{mn}(\sigma_{mn})^{A}_{B}\theta^{B}$$
$$\bar{\theta}'^{\dot{A}} = \bar{\theta}^{\dot{A}} + \bar{\epsilon}^{\dot{A}} - \frac{1}{4}\lambda^{mn}(\bar{\sigma}_{mn})^{\dot{A}}_{\dot{B}}\bar{\theta}^{\dot{B}}$$

$$\mathcal{D}_{M} \equiv \partial_{M} + A^{\tilde{a}}_{M}T_{\tilde{a}}$$

$$\mathcal{D}_{A} \equiv E^{M}_{A}\mathcal{D}_{M} = E^{M}_{A}\partial_{M} + E^{M}_{A}A_{M} = D_{A} + A_{A}$$

$$[\mathcal{D}_{A}, \mathcal{D}_{B}\} \equiv T^{C}_{AB}\mathcal{D}_{C} + \frac{1}{2}R^{rs}_{AB}M_{rs} + F^{\tilde{a}}_{AB}T_{\tilde{a}}$$

$$F_{AB} = D_{A}A_{B} - (-)^{AB}(A \leftrightarrow B) + [A_{A}, A_{B}\} - T^{C}_{AB}A_{C}$$

$$F_{\alpha\beta} = F_{\dot{\alpha}\dot{\beta}} = 0$$

$$F_{\alpha\dot{\beta}} = 0$$

$$[\mathcal{D}_{A}, [\mathcal{D}_{B}, \mathcal{D}_{C}\}\} + (-)^{A(B+C)}[\mathcal{D}_{B}, [\mathcal{D}_{C}, \mathcal{D}_{A}\}\} + (-)^{C(A+B)}[\mathcal{D}_{C}, [\mathcal{D}_{A}, \mathcal{D}_{B}\}\} = 0$$

$$\partial_{[\mu}F_{\nu\rho]} = 0$$

$$T_{I} = \{P_{\mu}, Q_{\alpha}, M_{rs}\}$$

$$\nabla_{A}\equiv D_{A} + H^{I}_{A}T_{I}$$

$$L_{\mu} = i\partial_{\mu}$$
$$L_{\alpha} = \partial_{\alpha} - i\theta^{\beta}(\gamma^{\mu})_{\beta\alpha}\partial_{\mu}$$

$$\frac{1}{2}L^{rs}\mathcal{L}_{rs} = L^{\mu}{}_{\nu}x^{\nu}\partial_{\mu}\frac{1}{4}L^{rs}(\gamma_{rs})^{\alpha}{}_{\beta}\theta^{\beta}\partial_{\alpha} + \frac{1}{2}L^{rs}M_{rs}$$

$$H^{I}_{A}T_{I} \equiv h^{M}_{A}D_{M} + \frac{1}{2}\phi^{rs}_{A}M_{rs}$$

$$\nabla_{A}= \delta^{M}_{A}D_{M} + h^{M}_{A}D_{M} + \frac{1}{2}\phi^{rs}_{A}M_{rs}$$
$$\equiv E^{M}_{A}D_{M} + \frac{1}{2}\phi^{rs}_{A}M_{rs}$$

$$E^{M}_{A} = \delta^{M}_{A} + h^{M}_{A}$$

$$[\nabla_{A}, \nabla_{B}\} = T^{C}_{AB}\nabla_{C} + \frac{1}{2}R^{rs}_{AB}M_{rs}$$

$$v_{ab} = (\gamma_{\mu})_{ab}v^{\mu}$$
$$v_{\mu} = -\frac{1}{2}(\gamma_{\mu})^{ab}v_{ab}$$

$$\{\nabla_{a}, \nabla_{b}\} = 2i\nabla_{ab}$$
$$T^{de}_{a,bc} = 0$$

$$T^{cd}_{a,b} = 2i\delta^{(c}_{a}\delta^{d)}_{b};\ T^{c}_{a,b} = 0;\ R^{rs}_{a,b} = 0$$

$$k^{\alpha\beta}(x,\theta) = \xi^{\alpha\beta}(x) + i\theta^{(\alpha}\epsilon^{\beta)}(x) + i\theta_{\gamma}\eta^{(\gamma\alpha\beta)}(x) + i\theta^{2}\zeta^{\alpha\beta}(x)$$

$$\psi(x,\theta)= h(x) + i\theta^{\alpha}\lambda_{\alpha}(x) + i\theta^{2}S$$
$$= e_{m\mu}(x)\delta^{\mu m} + i\theta^{\alpha}(\gamma^{\mu}\psi_{\mu})_{\alpha}(x) + i\theta^{2}S$$
$$E^{(\alpha\alpha\beta)}(x,\theta) = \chi^{(\alpha\alpha\beta)} + \theta^{(\alpha}X^{\alpha\beta)} + \delta^{\alpha\beta}_{bc}(\theta_{d}h^{(abcd)} + i\theta^{2}\psi^{(abc)})$$

$$\int d^3xd^2\theta \mathrm{sdet} E_M^A(x,\theta)$$

$$[\nabla_a, \nabla_{bc}] = \frac{1}{2}\epsilon_{ab}W_c + \frac{1}{2}\epsilon_{ac}W_b$$
$$W_a = W_a{}^b\nabla_b + \hat{W}_a^{bc}\nabla_{bc} + \frac{1}{2}W_a^{rs}M_{rs}$$
$$W_{ab} = \epsilon_{ab}R$$
$$\hat{W}_a^{bc} = 0$$
$$W_{a,bc} \equiv \frac{1}{4}W_a^{rs}(\gamma_{rs})_{bc} = G_{abc} - \frac{1}{3}\epsilon_{ab}\nabla_c R - \frac{1}{3}\nabla_b R$$

$$\nabla^a G_{abc} = \frac{2i}{3}\nabla_{bc}R$$

$$R = \Lambda;\ G_{abc} = 0$$

$$S = \frac{1}{k^2}\int d^3xd^2\theta \mathrm{sdet} E_A^M(R + \Lambda)$$

$$[D_A, D_B\} = T_{AB}^C D_C + \frac{1}{2}R_{AB}^{rs}M_{rs}$$

$$T_{\dot{A}\dot{B}}^m = f_{\dot{A}\dot{B}}^m$$

$$[\nabla_a, \{\nabla_b, \nabla_c\}] + \text{ super } - \text{ cyclic } = 0$$

$$T_{am}{}^b = T_{mn}{}^r = T_m{}^a = R_{am}{}^{rs} = 0$$

$$\delta e_\mu^m = \frac{k}{2}\bar{\epsilon}\gamma^m\psi_\mu$$
$$\delta\psi_\mu = \frac{1}{k}\left(D_\mu + \frac{ik}{2}A_\mu\gamma_5\right)\epsilon - \frac{1}{2}\gamma_\mu\eta\epsilon$$
$$\delta S = \frac{1}{4}\bar{\epsilon}\gamma^\mu R_\mu^{cov}$$
$$\delta P = -\frac{i}{4}\bar{\epsilon}\gamma_5\gamma^\mu R_\mu^{cov}$$
$$\delta A_m = \frac{3i}{4}\bar{\epsilon}\gamma_5\left(R_m^{cov} - \frac{1}{3}\gamma_m\gamma^\mu R_\mu^{cov}\right)$$

$$\eta \equiv -\frac{1}{3}(S - i\gamma_5 P - iA_\rho\gamma^\rho\gamma_5)$$

$$R^\mu = \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho\psi_\sigma$$

$$R^{\mu,cov} = \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\left[D_\rho\psi_\sigma - \frac{i}{2}A_\rho\gamma_5\psi_\sigma + \frac{1}{2}\gamma_\rho\eta\psi_\sigma\right]$$
$$\equiv \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\psi_{\rho\sigma}^{cov}$$

$$\mathcal{L} = -\frac{e}{2}R(e,\omega) - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma$$

$$-\frac{e}{3}(S^2 + P^2 - A_\mu^2)$$

$$0 = -2\frac{\delta I}{\delta e_{a\nu}} = e\left(R^{a\nu} - \frac{1}{2}e^{a\nu}R\right) - \frac{1}{4}\bar{\psi}_\lambda\gamma_5\gamma^a\tilde{\psi}^{\lambda\nu} - \frac{e}{3}e^{a\nu}(S^2 + P^2 - A_m^2)$$
$$\frac{\delta I}{\delta\bar{\psi}_\mu} = R^\mu = \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D\rho\psi_\sigma$$
$$\tilde{\psi}^{\lambda\nu} = 2\epsilon^{\lambda\nu\rho\sigma}D_\rho\psi_\sigma$$
$$S = P = A_m = 0$$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]e_\mu^m = \frac{1}{2}\bar{\epsilon}_2\gamma^m D_\mu\epsilon_1 - 1 \leftrightarrow 2$$
$$= \delta_E(\xi^\mu)e_\mu^m + \delta_{LL}\big(\xi^\mu\omega_\mu^{mn}(e,\psi)\big)e_\mu^m + \delta_Q\big(-k\xi^\mu\psi_\mu\big)e_\mu^m$$

$$\xi^\mu = \frac{1}{2}\bar{\epsilon}_2\gamma^\mu\epsilon_1$$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]\psi_\mu = \frac{1}{2}\sigma_{mn}\epsilon_2\delta_{\epsilon_1}\omega_\mu^{mn}$$

$$\delta\omega_{\mu ab} = \frac{1}{4}\bar{\epsilon}\big(\gamma_b\psi_{\mu a}^{cov} - \gamma_a\psi_{\mu b}^{cov} - \gamma_\mu\psi_{ab}^{cov}\big) + \frac{1}{2}\bar{\epsilon}(\sigma_{ab}\eta + \eta\sigma_{ab})\psi_\mu$$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]e_\mu^m = \text{ previous } + \frac{1}{4}\bar{\epsilon}_2\gamma^m\left(iA_\mu\gamma_5 + \frac{1}{3}\gamma_\mu\big(S - i\gamma_5 P - iA_\rho\gamma^\rho\gamma_5\big)\right)\epsilon_1 - 1 \leftrightarrow 2$$

$$\frac{k}{12}\bar{\epsilon}_2(\delta^{mn} + \gamma^{mn})(S - i\gamma_5 P)\epsilon_1 e_\mu^n - 1 \leftrightarrow 2$$

$$\frac{ik}{4}\left[A_\mu\bar{\epsilon}_2\gamma^m\gamma_5\epsilon_1 - \frac{1}{3}A_\rho\bar{\epsilon}_2\gamma^m\gamma_\mu\gamma_\rho\gamma_5\epsilon_1\right] - 1 \leftrightarrow 2$$

$$\bar{\epsilon}\chi = +\bar{\chi}\epsilon$$
$$\bar{\epsilon}\gamma_\mu\chi = -\bar{\chi}\gamma_\mu\epsilon$$

$$\bar{\epsilon}\gamma^{mn}\chi = -\bar{\chi}\gamma^{mn}\epsilon$$
$$\bar{\epsilon}\gamma_5\chi = +\bar{\chi}\gamma_5\epsilon$$
$$\bar{\epsilon}\gamma^m\gamma_5\chi = +\bar{\chi}\gamma^m\gamma_5\epsilon$$
$$\bar{\epsilon}\gamma^{mn}\gamma_5\chi = -\bar{\chi}\gamma^{mn}\gamma_5\epsilon$$

$$\gamma^m\gamma^\mu\gamma^\rho = \gamma^{m\mu\rho} + \eta^{m\mu}\gamma^\rho - \gamma^\mu\eta^{m\rho} + \gamma^m\eta^{\mu\rho}$$

$$\gamma_d\gamma_5 = \frac{i}{6}\epsilon_{abcd}\gamma^{abc}$$

$$\epsilon^{abcd}\gamma_d = i\gamma^{abc}\gamma_5$$

$$\frac{k}{6}\bar{\epsilon}_2\gamma^{mn}(S - i\gamma_5 P)\epsilon_1 e_\mu^n - \frac{ik}{6}A_p\bar{\epsilon}_2\gamma^{mnp}\epsilon_1 e_\mu^n$$

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}] = \delta_E(\xi^\mu) + \delta_Q\big(-\xi^\mu\psi_\mu\big) + \delta_{LL}\left[\xi^\mu\hat{\omega}_\mu^{mn} + \frac{1}{3}\bar{\epsilon}_2\sigma^{mn}(S - i\gamma_5 P)\epsilon_1\right]$$
$$\hat{\omega}_\mu^{mn} = \omega_\mu^{mn} - \frac{i}{3}\epsilon_\mu^{mnc}A_c$$
$$\xi^\mu = \frac{1}{2}\bar{\epsilon}_2\gamma^\mu\epsilon_1$$

$$\big[\delta_Q(\epsilon),\delta_{g.c.}(\xi^\mu)\big] = \delta_Q\big(\xi^\mu\partial_\mu\epsilon\big)$$

$$\psi_{\rho\sigma}^{cov} \equiv D_\rho\psi_\sigma^{cov} \equiv D_\rho\psi_\sigma - \frac{i}{2}A_\sigma\gamma_5\psi_\rho + \frac{1}{2}\gamma_\sigma\eta\psi_\rho$$

$$\delta S_\psi = \int R^\mu\delta\psi_\mu$$

$$\Lambda^{MN} = \begin{pmatrix} \Lambda^{mn} & 0 & 0 \\ 0 & -\frac{1}{4}(\sigma_{mn})_{AB}\Lambda^{mn} & 0 \\ 0 & 0 & \frac{1}{4}(\sigma_{mn})_{\dot{A}\dot{B}}\Lambda^{mn} \end{pmatrix}$$

$$\Omega_\Lambda^{MN} = \begin{pmatrix} \Omega_\Lambda^{mn} & 0 & 0 \\ 0 & -\frac{1}{4}(\sigma_{mn})_{AB}\Omega_\Lambda^{mn} & 0 \\ 0 & 0 & \frac{1}{4}(\sigma_{mn})_{\dot{A}\dot{B}}\Omega_\Lambda^{mn} \end{pmatrix}$$

$$D_\Lambda = \partial_\Lambda + \frac{1}{2}\Omega_\Lambda^{mn} M_{mn}$$

$$D_M = E_M^\Lambda D_\Lambda$$

$$[D_M, D_N\} = T_{MN}^P D_P + \frac{1}{2} R_{MN}^{mn} M_{mn}$$

$$[D_M, [D_N, D_P\}\} + \text{ supercyclic } = 0$$

$$E_\mu^m(x, \theta = 0) = e_\mu^m$$
$$E_\mu^a(x, \theta = 0) = \psi_\mu^a$$
$$\Omega_\mu^{mn}(x, \theta = 0) = \omega_\mu^{mn}$$

$$T_{mn}^p = 0$$
$$T_{AB}^C = 0;\ T_{\dot{A}\dot{B}}^{\dot{C}} = 0$$
$$T_{A\dot{B}}^m + 2i(\sigma^m)_{A\dot{B}} = 0$$
$$T_{Am}^n (\bar{\sigma}_n^m)_{B\dot{C}} = 0$$

$$T_{AB}^{\dot{C}} = T_{AB}^m = 0$$

$$T_A^m{}_m = 0$$

$$T_{AB}^{\dot{C}} = T_{AB}^m = 0$$

$$C_{MN}{}^P = E_M^\Lambda (\partial_\Lambda E_N^\Pi) E_\Pi^P - (-)^{MN}(M \leftrightarrow N)$$

$$\Omega_{mnr} = -\frac{1}{2}(C_{mnr} + C_{rnm} - C_{nrm})$$

$$\Omega_{ABC} = -\frac{1}{2}(C_{ABC} + C_{CBA} - C_{BCA})$$

$$T_{A(\dot{B}}\dot{C}) = 0$$

$$S = \frac{1}{2k^2}\int d^4x d^4\theta \ \text{sdet}\ E_\Lambda^M$$

$$E_A^\Lambda \to e^L E_A^\Lambda;\ E_{\dot{A}}^\Lambda \to e^{L^*} E_{\dot{A}}^\Lambda$$

$$R_{\dot{A}\dot{B}\dot{C}\dot{D}} = \frac{1}{6}(\epsilon_{\dot{D}\dot{B}}\epsilon_{\dot{C}\dot{A}} + \epsilon_{\dot{C}\dot{B}}\epsilon_{\dot{D}\dot{A}})R^*$$
$$T_{C\dot{C}\dot{A}D} = \frac{i}{12}\epsilon_{CD}\epsilon_{\dot{C}\dot{A}}T^*$$
$$T_{C\dot{C}DE} = \frac{1}{4}(\epsilon_{CE}G_{D\dot{C}} + 3\epsilon_{CD}G_{E\dot{C}} - 3\epsilon_{DE}G_{C\dot{C}})$$
$$T_{A\dot{A}B\dot{B}\dot{C}} = \epsilon_{AB}\left(W_{\dot{A}\dot{B}\dot{C}} - \frac{1}{2}\epsilon_{\dot{A}\dot{C}}D^E G_{E\dot{B}} - \frac{1}{2}\epsilon_{\dot{B}\dot{C}}D^E G_{E\dot{A}} - \frac{1}{2}\epsilon_{\dot{B}\dot{C}}G_{E\dot{A}}\right) + \epsilon_{\dot{A}\dot{B}}D_{(B}G_{C)\dot{A}}$$

$$T_{AB\dot{C}D\dot{D}} = T_{AM}^n(\sigma^m)_{B\dot{C}}(\sigma_n)_{D\dot{D}}$$
$$T_{A\dot{B}\dot{C}D} = T_{m\dot{C}D}(\sigma^m)_{A\dot{B}}$$

$$D^A G_{A\dot{A}} = \bar{D}_{\dot{A}} R^*$$
$$D^A W_{(ABC)} = D_B{}^{\dot{D}} G_{C\dot{D}} + D_C{}^{\dot{D}} G_{B\dot{D}}$$

$$\delta S=\int d^4xd^4\theta\left(\mathrm{sdet}E_\Lambda^M\right)[v^mG_m-RU-R^*U^*]$$

$$R=G_m=0$$

$$R(x,\theta=0)=M=S+iP$$
$$G_m(x,\theta=0)=A_m$$

$$[D_M,[D_N,D_P\}\}+\text{ supercyclic }=0$$

$$(\sigma^m)_{A\dot{B}}T_{mn}^{\dot{B}}(\theta=0)=0$$

$$\bar{D}_{\dot{A}}\Phi=0$$

$$\Phi=\Phi(x,\theta)=\phi(y)+\sqrt{2}\psi(y)+\theta^2F(y)$$

$$y^\mu=x^\mu+i\theta\sigma^\mu\bar{\theta}$$

$$\phi(x)=\Phi|_{\theta=\bar{\theta}=0};\ \psi(x)=\frac{D_A\Phi|_{\theta=\bar{\theta}=0}\,|}{\sqrt{2}};\ F(x)=-4D^2\Phi|_{\theta=\bar{\theta}=0}$$

$$V\rightarrow V+i\Lambda-i\Lambda^\dagger$$

$$V=-\theta\sigma^\mu\bar{\theta}A_\mu+i\theta^2(\bar{\theta}\bar{\lambda})-i\bar{\theta}^2(\theta\lambda)+\frac{\theta^2\bar{\theta}^2}{2}D$$

$$W_A=-\frac{1}{4}\bar{D}^2D_AV$$

$$D^AW_A=D^{\dot{A}}W_{\dot{A}}\ (\mathrm{Im}(D^AW_A)=0)$$

$$e^{-V}\rightarrow e^{i\Lambda^\dagger}e^{-V}e^{-i\Lambda}$$

$$W_A=\frac{1}{4}\bar{D}^2e^VD_Ae^{-V}$$

$$S=-\frac{1}{4}\int d^4xd^2\theta\mathrm{Tr}(W_AW^A+\text{ h.c.})$$
$$=-\frac{1}{4}\int d^4xF_{\mu\nu}^aF^{\mu\nu a}+\cdots$$

$$S_{\text{matter}}=\frac{1}{4g^2}\int d^4xd^2\theta d^2\bar{\theta}\mathrm{tr}\left(\Phi^\dagger e^V\Phi\right)$$

$$F_i=\frac{\partial W}{\partial\phi^i}$$
$$D^a=\Phi^\dagger T^a\Phi\equiv\phi^{\dagger i}(T^a)_{ij}\Phi^j$$

$$e^{-V}\rightarrow e^{i\Lambda^\dagger}e^{-V}e^{-i\Lambda}$$

$$\int d^4xd^2\bar{\theta}=\int d^4x\frac{1}{4}\left(\bar{D}^2-\frac{1}{3}R\right)\Big|_{\theta=\bar{\theta}=0}$$

$$W_A=\frac{1}{4}\left(\bar{D}^2-\frac{1}{3}R\right)e^VD_Ae^{-V}$$

$$\int d^4xd^4\theta E$$

$$\int d^4xd^4\theta\ E\ K\left(\Phi,\Phi^\dagger\right)$$

$$\mathcal{E} = e[1 + i\theta\sigma^m\bar{\psi}_m - \theta^2(M^* + \bar{\psi}_m\bar{\sigma}^m\bar{\psi}_n)]$$

$$\mathcal{E} = \frac{1}{4}\frac{\bar{D}^2 E}{R}$$

$$W = \int d^2\bar{\theta}U = \left(\bar{D}^2 - \frac{1}{3}R\right)U$$

$$-\frac{1}{3}\int d^4xd^4\theta EU = -\int d^4xd^4\theta\frac{E}{R}\bar{D}^2U + \int d^4xd^4\theta\frac{E}{R}W$$

$$\int d^4xd^4\theta\bar{D}_{\dot{A}}\left(\frac{\bar{D}_{\dot{A}}U}{R}\right)$$

$$\int d^4xd^4\theta ED_NV^N(-)^N = 0$$

$$\int d^4xd^4\theta V^N[-E_N^\Lambda\partial_\Lambda E - (-)^{\Lambda(\Lambda+N)}E\partial_\Lambda E_N^\Lambda]$$

$$\int d^4xd^4\theta V^N T_{NM}^M(-)^M$$

$$-\frac{1}{3}\int d^4xd^4\theta EU = \int d^4xd^4\theta\frac{E}{R}W$$

$$\int d^4xd^2\theta\left(d^2\bar{\theta}\frac{E}{R}\right)W$$

$$-\frac{1}{3}\int d^4xd^2\theta\mathcal{E}R = -\frac{1}{3}\int d^4xd^2\theta d^2\mathcal{E}\bar{\theta}$$

$$\int d^4xd^4\theta E$$

$$\begin{aligned} R =& M + \theta(\sigma^m\bar{\sigma}^n\psi_{mn} - i\sigma^m\bar{\psi}_m M + i\psi_m A^m) \\ &+\theta^2\left[-\frac{1}{2}R + \bar{\psi}^m\sigma^n\psi_{pq} + \frac{2}{3}MM^* + \frac{A_m^2}{3} - ie_m^\mu D_\mu A^m \right. \\ &\left. + \frac{\bar{\psi}\psi}{2}M - \frac{1}{2}\psi_m\sigma^m\bar{\psi}_n\sigma^n + \frac{1}{8}(\bar{\psi}_m\bar{\sigma}_n\psi_{pq} + \psi_m\sigma_n\bar{\psi}_{pq})\right] \end{aligned}$$

$$\begin{aligned} S =& \int d^4xd^4\theta E[K(\Phi,\Phi^\dagger) + \Phi^\dagger e^V\Phi] \\ &+\int d^4xd^4\theta\mathcal{E}[W(\Phi) + \mathrm{Tr}W^A W_A] + \text{ h.c.} \end{aligned}$$

$$\begin{aligned} S =& \int d^4xd^2\theta\mathcal{E}\left[\bar{D}^2 - \frac{1}{3}R\right][K(\Phi,\Phi^\dagger) + \Phi^\dagger e^V\Phi] \\ &+\int d^4xd^4\theta\mathcal{E}[W(\Phi) + \mathrm{Tr}W^A W_A] + \text{ h.c.} \end{aligned}$$

$$\begin{aligned} &\int d^4xd^4\theta E\Phi^\dagger e^V\Phi + \int d^4xd^2\theta\mathrm{Tr}[W_A W^A] + h.c. \\ \to &\int d^4xd^4\theta E(\Phi^\dagger e^V)^a F_a(\Phi) + \int d^4xd^2\theta[F_{ab}(\phi)W_A^a W^{Ab}] + h.c. \end{aligned}$$

$$-\frac{1}{3}\int d^4xd^2\theta\mathcal{E}R[a + \phi^\dagger(x)\phi(x)] + \cdots$$

$$g_{\mu\nu} = A(\phi)\tilde{g}_{\mu\nu}$$

$$S = \int d^4x\sqrt{-\tilde{g}}(R[\tilde{g}] + (\ldots)(\partial\phi)^2)$$

$$-\frac{1}{3}\int d^4xd^2\theta\mathcal{E}R\left[a+K\left(\phi(x),\phi^\dagger(x)\right)\right]$$

$$\int d^4xd^2\theta\mathcal{E}R\left[1-\frac{K}{3}\right]$$

$$1-\frac{K}{3}=e^{-\frac{k}{3}}$$

$$V=\sum_i |F_i|^2+\frac{g^2}{2}D^aD^a$$

$$V=\sum_i |F_i|^2+\frac{g^2}{2}D^aD^a-\frac{1}{3}(|M|^2+A_m^2)e^{-k/3}$$

$$M\sim\phi\frac{dW}{d\phi}-3W+F\frac{\partial K}{\partial\phi}$$

$$V=e^k\left[\sum_{i,\bar{j}}(g^{-1})^{i\bar{j}}\left(\frac{\partial W}{\partial\phi^i}+W\frac{\partial k}{\partial\phi^i}\right)\left(\frac{\partial W}{\partial\phi^j}+W\frac{\partial k}{\partial\phi^j}\right)^*-3|W|^2\right]$$
$$+\frac{1}{2}(F^{-1})^{ab}\left(\frac{\partial k}{\partial\phi^i}(T_a)_{ij}\phi^j\right)\left(\frac{\partial k}{\partial\phi^j}(T_b)_{kl}\phi_l\right)$$

$$g_{i\bar{j}}=\frac{\partial^2k}{\partial\phi^i\partial\bar{\phi}^{\bar{j}}}$$

$$\int d^4x\sqrt{-g}\left[g_{i\bar{j}}D_\mu\phi^i(D^\mu\phi)^{*\bar{j}}+g_{i\bar{j}}\psi^i\not{D}\bar{\psi}^{\bar{j}}\right]$$

$$-\frac{1}{4}\mathrm{Re}\left[F_{ab}(\phi)F^a_{\mu\nu}F^{b\mu\nu}\right]$$

$$D_i=\frac{\partial}{\partial\phi^i}+\frac{\partial k}{\partial\phi^i}$$

$$V=e^k\left[\sum_{i\bar{j}}(g^{-1})^{i\bar{j}}D_iW\left(D_jW\right)^*-3|W|^2\right]$$

$$-\frac{1}{2}\mathrm{Re}\left[F_{ab}(\phi)\bar{\lambda}^a\not{D}\lambda^b\right]+\frac{1}{2}e^{k/2}\mathrm{Re}\sum_{i\bar{j}}(g^{-1})^{i\bar{j}}D_iW\left(\frac{\partial F_{ab}}{\partial\phi^j}\right)^*(\bar{\lambda}^a\lambda^b)$$

$$k=-3\log\left(i(\rho-\bar{\rho})\right)$$
$$W(\rho)=W_0+Ae^{-ia\rho}+Be^{ib\rho}$$

$$\mathcal{E}=\frac{1}{4}\bar{D}^2\left(\frac{E}{R^*}\right)$$

$$\int d^4x\sqrt{-g}R[g]f(\phi)$$

$$\int d^4x\sqrt{-\tilde{g}}(R[\tilde{g}]+(\partial\phi)^2h(\phi))$$

$$g_{\mu\nu}=1/f(\phi)\tilde{g}_{\mu\nu}$$

$$g_{\Lambda\Sigma}(\vec{x},\vec{y})=\begin{pmatrix}g^{(0)}_{\mu\nu}(\vec{x}) & 0\\ 0 & g^{(0)}_{mn}(\vec{y})\end{pmatrix}$$

$$\phi(\vec{x},y)=\sum_{n}\ \phi_n(\vec{x})e^{\frac{2\pi iny}{R}}$$

$$\phi(\vec{x},\theta,\phi)=\sum_{lm}\ \phi_{lm}(\vec{x})Y_{lm}(\theta,\phi)$$

$$\partial_y^2 e^{\frac{2\pi iny}{R}}=-\left(\frac{2\pi n}{R}\right)^2 e^{\frac{2\pi iny}{R}}$$

$$\Delta_2 Y_{lm}(\theta,\phi)=-\frac{l(l+1)}{R^2}Y_{lm}(\theta,\phi)$$

$$\phi(\vec{x},\vec{y})=\sum_{q,I_q}\ \phi_q^{I_q}(\vec{x})Y_q^{I_q}(\vec{y})$$

$$\Delta_n Y_q^{I_q}(\vec{y})=-m_q^2 Y_q^{I_q}(\vec{y})$$

$$\Box_D\,\phi(\vec{x},\vec{y})=(\Box_4+\Delta_n)\phi(\vec{x},\vec{y})=(\Box_4-m_q^2)\phi(\vec{x},\vec{y})$$

$$\phi(\vec{x},\vec{y})=\phi_0(\vec{x})Y_0(\vec{y})$$

$$g_{\Lambda\Sigma}=\begin{pmatrix} g_{\mu\nu}^{(0)} & 0 \\ 0 & \delta_{mn}\end{pmatrix}$$

$$g_{\Lambda\Sigma}=\begin{pmatrix} g_{\mu\nu}(\vec{x},\vec{y})=g_{\mu\nu}^{(0)}(\vec{x})+\sum_{\{n_i\}}\ h_{\mu\nu}^{\{n_i\}}(\vec{x})e^{\frac{2\pi i n_i y_i}{R_i}}; & g_{\mu m}(\vec{x},\vec{y})=\sum_{\{n_i\}} B_\mu^{m,\{n_i\}}(\vec{x})e^{\frac{2\pi i n_i y_i}{R_i}} \\ g_{\mu m}(\vec{x},\vec{y}); & g_{mn}=\delta_{mn}+\sum_{\{n_i\}} h_{mn}^{\{n_i\}}(\vec{x})e^{\frac{2\pi i n_i y_i}{R_i}}\end{pmatrix}$$

$$Y_{\{n_i\}}(\vec{y})=\prod_i\ e^{\frac{2\pi i n_i y_i}{R_i}}$$

$$g_{\Lambda\Sigma}=\begin{pmatrix} g_{\mu\nu}^{(0)}(\vec{x})+h_{\mu\nu}^{\{0\}}(\vec{x}); & g_{\mu m}(\vec{x})=B_\mu^{m\{0\}}(\vec{x}) \\ g_{\mu m}(\vec{x}); & g_{mn}(\vec{x})=\delta_{mn}+h_{mn}^{\{0\}}(\vec{x})\end{pmatrix}$$

$$F_{\Lambda_1\ldots\Lambda_{p+2}}=(p+2)!\,\partial_{[\Lambda_1}A_{\Lambda_2\ldots\Lambda_{p+2}]}$$

$$\delta A_{\Lambda_1\ldots\Lambda_{p+1}}=\partial_{[\Lambda_1}\Lambda_{\Lambda_2\ldots\Lambda_{p+1}]}$$

$$\eta_A(\vec{x},\vec{y})=\eta_M(\vec{x})\epsilon_i(\vec{y})$$

$$\lambda=\int\ d^d\vec{x}\sqrt{\det g_{\mu\nu}^{(0)}}\phi_q^{I_q}(\vec{x})\phi_0^{I_0}(\vec{x})\phi_0^{J_0}(\vec{x})\times\int\ d^n\vec{y}\sqrt{\det g_{mn}^{(0)}}Y_q^{I_q}(\vec{y})Y_0^{I_0}(\vec{y})Y_0^{J_0}(\vec{y})$$

$$(\Box-m_q^2)\phi_q^{I_q}(\vec{x})=(\ldots)\phi_0^{I_0}(\vec{x})\phi_0^{J_0}(\vec{x})$$

$$\int\ d^n\vec{y}\sqrt{\det g_{mn}^{(0)}}Y_q^{I_q}(\vec{y})Y_0^{I_0}(\vec{y})Y_0^{J_0}(\vec{y})$$

$$\int\ dyY^{I_n}=\int\ dye^{\frac{2\pi iny}{R}}=0$$

$$\int\ Y_q^{I_q}(Y_0^{I_0}Y_0^{J_0})=0$$

$$\phi_q'=\phi_q+a\phi_0^2+\cdots$$
$$\phi_0'=\phi_0+\sum_{pq(\text{ including } 0)}\ c_{pq}\phi_p\phi_q$$

$$g_{\mu\nu}(\vec{x},\vec{y}) = g_{\mu\nu}(\vec{x})\left[\frac{\det g_{mn}(\vec{x},\vec{y})}{\det g^{(0)}_{mn}(\vec{y})}\right]^{-\frac{1}{d-2}}$$

$$\sqrt{g^{(D)}}\tilde{R} = \sqrt{g^{(d)}}\sqrt{\det g_{mn}}\lambda^{\frac{d}{2}-1}$$

$$g_{\Lambda\Sigma} = \begin{pmatrix} g_{\mu\nu}(\vec{x}) & g_{\mu 5} = B_\mu(\vec{x}) \\ g_{5\mu} = B_\mu(\vec{x}) & g_{55} = \phi(\vec{x}) \end{pmatrix}$$

$$g_{\Lambda\Sigma} = \begin{pmatrix} g_{\mu\nu}(\vec{x})\phi^{-1/2}(\vec{x}) & B_\mu(\vec{x})\phi(\vec{x}) \\ B_\mu(\vec{x})\phi(\vec{x}) & \phi(\vec{x}) \end{pmatrix}$$

$$g_{\Lambda\Sigma} = \Phi^{-1/3}(\vec{x})\begin{pmatrix} g_{\mu\nu}(\vec{x}) & B_\mu(\vec{x})\Phi(\vec{x}) \\ B_\mu(\vec{x})\Phi(\vec{x}) & \Phi(\vec{x}) \end{pmatrix}$$

$$g_{\mu m}(\vec{x},\vec{y}) = B^{AB}_\mu(\vec{x})V^{AB}_m(\vec{y})$$

$$E^\alpha_\mu(\vec{x},\vec{y}) = e^\alpha_\mu(\vec{x})\left[\frac{\det E^a_m(\vec{x},\vec{y})}{\det e^{(0)a}_m(\vec{y})}\right]^{-\frac{1}{d-2}}$$

$$E^a_\mu(\vec{x},\vec{y}) = B^m_\mu(\vec{x},\vec{y})E^a_m(\vec{x},\vec{y})$$
$$B^m_\mu(\vec{x},\vec{y}) = B^{AB}_\mu(\vec{x})V^{AB}_m(\vec{y})$$

$$\lambda_A(\vec{x},\vec{y}) = \lambda^i_M(\vec{x})$$

$$\lambda_A(\vec{x},\vec{y}) = \lambda^I_M(\vec{x})\eta^I_i(\vec{y})$$

$$D_{(\mu}V^{AB}_{\nu)} = 0$$

$$D_\mu\eta^I_i = c(\gamma_\mu\eta^I)_i \equiv ce^\alpha_\mu(\gamma_\alpha\eta^I)_i$$

$$V^{AB}_\mu = \bar{\eta}^I\gamma_\mu\eta^J(\gamma^{AB})_{IJ}$$

$$\delta_{\text{susy}}\,\lambda_A(\vec{x},\vec{y}) = 0$$

$$D_\mu\eta^I = (\text{ fields } \times\gamma \text{ matrices })_\mu\big|_{\text{bgr}}\eta^I$$

$$\lambda_A(\vec{x},\vec{y}) = \lambda^I_M(\vec{x})\eta^I_i(\vec{y})$$

$$\epsilon_{A_1\ldots A_5}dY^{A_1}\wedge dY^{A_2} = 3\sqrt{g^{(0)}}\epsilon_{\mu\nu\rho\sigma}dx^\mu\wedge dx^\nu\partial^\rho Y^{[A_3}\partial^\sigma Y^{A_4}Y^{A_5]}$$

$$g_{\Lambda\Sigma} = \phi^{-1/3}\begin{pmatrix} g_{\mu\nu} & B_\mu\phi \\ B_\mu\phi & \phi \end{pmatrix}$$

$$\xi_m(x,y) = \lambda^{AB}(x)V^{AB}_m(y)$$

$$S = \frac{1}{16\pi}\text{Im}\int d^4x d^2\theta d^2\tilde{\theta}F(\Psi^A)$$

$$\text{Im}\int d^4x\left[\int d^2\theta F_{AB}(\Phi)W^{A\alpha}W^B_\alpha + \int d^2\theta d^2\bar{\theta}\left(\Phi^\dagger e^{-2gV}\right)^A F_A(\Phi)\right]$$

$$F_A(\Phi) = \frac{\partial F}{\partial\Phi^A};\; F_{AB} = \frac{\partial^2 F}{\partial\Phi^A\partial\Phi^B}$$

$$\int d^2\theta d^2\bar{\theta}\left(\left(Q^\dagger e^{-2gV}\right)_i Q_i + \left(\tilde{Q}e^{2gV}\right)_i\tilde{Q}_i\right) + \int d^2\theta\left(\sqrt{2}(\tilde{Q}\Phi)_i Q_i + m_i\tilde{Q}_i Q_i\right) + h.c.$$

$$\mathcal{L} = g_{A\bar{B}}\partial_\mu X^A \partial^\mu \bar{X}^{\bar{B}} + g_{A\bar{B}}\bar{\lambda}^{iA}\,\partial\lambda_i^{\bar{B}} + \mathrm{Im}\left(F_{AB}\mathcal{F}_{\mu\nu}^{-A}\mathcal{F}_{\mu\nu}^{-B}\right)$$

$$g_{A\bar{B}} = \partial_A \partial_{\bar{B}} K$$

$$K(X,\bar{X}) = i(\bar{F}_A(\bar{X})X^A - F_A(X)\bar{X}^A)$$
$$F_A(X) = \partial_A F(X);\; F_{AB} = \partial_A\partial_B F(X)$$

$$\frac{\mathcal{L}_{\text{sugra}}}{\sqrt{-g}} = R[g] + g_{i\bar{j}}(z,\bar{z})\nabla^\mu z^i \nabla_\mu z^{\bar{j}} - 2\lambda h_{uv}(q)\nabla^\mu q^u \nabla_\mu q^v$$
$$+ i\left(\overline{\mathcal{N}}_{IJ}\mathcal{F}_{\mu\nu}^{-I}\mathcal{F}^{-J\mu\nu} - \mathcal{N}_{IJ}\mathcal{F}_{\mu\nu}^{+I}\mathcal{F}^{+J\mu\nu}\right) - g^2 V$$

$$V = \bar{X}^I\left(4k_I^u k_J^v h_{uv} + k_I^i k_J^{\bar{j}} g_{i\bar{j}}\right)X^J + (U^{IJ} - 3\bar{X}^I X^J)\mathcal{P}_I^x \mathcal{P}_J^x$$
$$U^{IJ} = -\frac{1}{2}(\mathrm{Im}\mathcal{N})^{-1IJ} - \bar{X}^I X^J$$
$$\nabla_\mu z^i = \partial_\mu z^i + gA_\mu^I k_I^i(z)$$
$$\nabla_\mu q^u = \partial_\mu q^u + gA_\mu^I k_I^u(z)$$

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K(z,\bar{z})$$

$$K = g_{i\bar{j}} dz^i \wedge dz^{\bar{j}}$$

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K(z,\bar{z})$$

$$J^x J^y = -1 + \epsilon^{xyz} J^z$$

$$K_{uv}^x = h_{uw}(J^x)^w$$
$$K^x = K_{uv}^x dq^u \wedge dq^v$$

$$z^i \to z^i + \epsilon^I k_I^i$$

$$\partial_{\bar{j}} k_I^i = 0$$

$$\nabla_i k_j + \nabla_j k_i = 0;\; \nabla_{\bar{\imath}} k_j + \nabla_j k_{\bar{\jmath}} = 0$$

$$k_j = g_{ij} k^{\bar{j}};\; k_{\bar{j}} = g_{\bar{j}i} k^i$$

$$[k_I, k_J] = f_{IJ}{}^K k_K$$

$$k_I^i = i g^{i\bar{j}} \partial_{\bar{j}} \mathcal{P}_I$$

$$\frac{i}{2} g_{i\bar{j}}\left(k_I^i k_J^{\bar{j}} - k_J^i k_I^{\bar{j}}\right) = \frac{1}{2} f_{IJ}{}^K \mathcal{P}_K$$

$$k_I^i \partial_i K + k_I^{\bar{\imath}} \partial_{\bar{\imath}} K = 0$$

$$i\mathcal{P}_I = k_I^i \partial_i K = -k_I^{\bar{\imath}} \partial_{\bar{\imath}} K$$

$$\left(k_K^i \partial_i + k_K^{\bar{\imath}} \partial_{\bar{\imath}}\right)\begin{pmatrix} X^I \\ F_I \end{pmatrix} = T_K \begin{pmatrix} X^I \\ F_I \end{pmatrix}$$

$$T_K = \begin{pmatrix} f_{KJ}^I & 0 \\ 0 & -f_{KJ}^I \end{pmatrix} \in Sp(2n+2, R)$$

$$F_I = \frac{\partial F}{\partial X^I}$$

$$i(\bar{X}^I F_I - \bar{F}_I X^I) = 1$$

$$\nabla_{\bar{\imath}} X^I \equiv \left(\partial_{\bar{\imath}} - \frac{1}{2}\partial_{\bar{\imath}} K\right) X^I = 0$$

$$X^I = e^{K/2} Z^I(z) \Rightarrow F_I = e^{K/2} F_I(Z(z))$$

$$e^{-K(z,\bar{z})} = i[\bar{Z}^I(z) F_I(Z(z)) - Z^I(z)\bar{F}_I(\bar{Z}(\bar{z}))]$$

$$R_{jk}^{i}{}_{l}^{l} = \delta_k^i \delta_k^l + \delta_k^i \delta_j^l - e^{2K} \mathcal{W}_{jkm} \overline{\mathcal{W}}^{mil}$$

$$\mathcal{W}_{ijk} = iF_{IJK}(Z(z)) \frac{\partial Z^I}{\partial z^i} \frac{\partial Z^J}{\partial z^j} \frac{\partial Z^K}{\partial z^k}$$

$$F_{IJK} = \frac{\partial}{\partial Z^I} \frac{\partial}{\partial Z^J} \frac{\partial}{\partial Z^K} F(Z)$$

$$\mathcal{N}_{IJ} = \bar{F}_{IJ} + 2i \frac{\mathrm{Im}(F_{IK}) \mathrm{Im}(F_{JL}) X^K X^L}{\mathrm{Im}(F_{KL}) X^K X^L}$$

$$\mathcal{N}_{IJ} X^J = F_{IJ} X^J$$

$$F_I = \mathcal{N}_{IJ} X^J$$

$$\partial_{\bar{\imath}} \bar{F}_I = \mathcal{N}_{IJ} \partial_{\bar{\imath}} \bar{X}^J$$

$$z^A = \frac{X^A}{X^0};\ A = 1, \dots, n$$

$$F(X) = \frac{d_{ABC} X^A X^B X^C}{X^0}$$

$$\mathcal{L}_1 = \frac{1}{4} (\mathrm{Im}\mathcal{N}_{IJ}) \mathcal{F}_{\mu\nu}^I \mathcal{F}^{\mu\nu J} - \frac{i}{8} (\mathrm{Re}\mathcal{N}_{IJ}) \epsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\mu\nu}^I \mathcal{F}_{\rho\sigma}^J = \frac{1}{2} \mathrm{Im}\left(\mathcal{N}_{IJ} \mathcal{F}_{\mu\nu}^{+I} \mathcal{F}^{+\mu\nu J}\right)$$

$$\mathcal{F}_{\mu\nu}^{\pm} = \frac{1}{2}\left(\mathcal{F}_{\mu\nu} \pm \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \mathcal{F}^{\rho\sigma}\right)$$

$$G_{+I}^{\mu\nu} \equiv 2i \frac{\partial \mathcal{L}}{\partial \mathcal{F}_{\mu\nu}^{+I}} = \mathcal{N}_{IJ} \mathcal{F}^{+J\mu\nu}$$

$$G_{-I}^{\mu\nu} \equiv -2i \frac{\partial \mathcal{L}}{\partial \mathcal{F}_{\mu\nu}^{-I}} = \bar{N}_{IJ} \mathcal{F}^{-J\mu\nu}$$

$$\begin{pmatrix} \mathcal{F}^+ \\ G^+ \end{pmatrix};\ \begin{pmatrix} X^I \\ F_I \end{pmatrix}$$

$$dF = 0;\ d * F = 0$$

$$M^T \Omega M = \Omega$$

$$\Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{F}^+ \\ G^+ \end{pmatrix} \to \begin{pmatrix} \tilde{\mathcal{F}}^+ \\ \tilde{G}^+ \end{pmatrix} = M \begin{pmatrix} \mathcal{F}^+ \\ G^+ \end{pmatrix}$$

$$\begin{pmatrix} X^I \\ F_I \end{pmatrix} \to \begin{pmatrix} \tilde{X}^I \\ \tilde{F}_I \end{pmatrix} = M \begin{pmatrix} X^I \\ F_I \end{pmatrix}$$

$$\tilde{F}_I = \frac{\partial \tilde{F}}{\partial \tilde{X}^I}$$

$$\delta\psi_\mu^i = D_\mu(\omega(e,\psi))\epsilon^i + g\gamma_\mu \epsilon^i + gA_\mu \epsilon^i$$

$$\mathcal{L} = -\frac{e}{2} R(e,\omega) - \frac{e}{2} \bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu(\omega) \psi_\rho - \frac{e}{48} F_{\mu\nu\rho\sigma}^2$$

$$F_{\mu\nu\rho\sigma} = 24 \partial_{[\mu} A_{\nu\rho\sigma]} \equiv \partial_\mu A_{\nu\rho\sigma} + \psi$$

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T$$

$$\bar{\lambda}\Gamma^{A_1} \dots \Gamma^{A_n}\chi = (-)^n \bar{\chi}\Gamma^{A_n} \dots \Gamma^{A_1}\lambda$$

$$\delta e_\mu^m = \frac{1}{2}\bar{\epsilon}\gamma^m\psi_\mu$$
$$\delta\psi_\mu = D_\mu(\omega)\epsilon + \mathfrak{K}\mathscr{k}$$

$$\hat{\omega}_{\mu mn} = \omega_{\mu mn}(e) + \frac{1}{4}\left(\bar{\psi}_\mu\gamma_m\psi_n - \bar{\psi}_\mu\gamma_n\psi_m + \bar{\psi}_m\gamma_\mu\psi_n\right)$$

$$\hat{F}_{\alpha\beta\gamma\delta} = 24\left[\partial_{[\alpha}A_{\beta\gamma\delta]} + \frac{1}{16\sqrt{2}}\bar{\psi}_{[\alpha}\Gamma_{\beta\gamma}\psi_{\delta]}\right]$$

$$\begin{aligned}\mathcal{L} =& -\frac{e}{2k^2}R(e,\omega) - \frac{e}{2}\bar{\psi}_\mu\Gamma^{\mu\nu\rho}D_\rho\left(\frac{\omega+\hat{\omega}}{2}\right) - \frac{e}{48}F^2_{\mu\nu\rho\sigma} \\ & -\frac{3D}{4}k\left[\bar{\psi}_\mu\Gamma^{\mu}{}_{\alpha\beta\gamma\delta\nu}\psi^\nu + 12\bar{\psi}^\alpha\gamma^{\beta\gamma}\psi^\delta\right]\left(F_{\alpha\beta\gamma\delta} + \hat{F}_{\alpha\beta\gamma\delta}\right) \\ & +Ck\epsilon^{\mu_1 \dots \mu_{11}}F_{\mu_1\dots\mu_4}F_{\mu_5\dots\mu_8}A_{\mu_9\mu_{10}\mu_{11}}\end{aligned}$$

$$\begin{aligned}\delta e_\mu^m &= \frac{k}{2}\bar{\epsilon}\gamma^m\psi_\mu \\ \delta\psi &= \frac{1}{k}D_\mu(\hat{\omega})\epsilon + D\left(\Gamma^{\alpha\beta\gamma\delta}{}_\mu - 8\delta_\mu^\alpha\Gamma^{\beta\gamma\delta}\right)\epsilon\hat{F}_{\alpha\beta\gamma\delta} \\ \delta A_{\mu\nu\rho} &= E\bar{\epsilon}\Gamma_{[\mu\nu}\psi_{\rho]}\end{aligned}$$

$$C = -\frac{\sqrt{2}}{6\cdot(24)^2};\ D = \frac{\sqrt{2}}{6\cdot 48};\ E = -\frac{\sqrt{2}}{8}$$

$$\omega_{\mu mn} = \hat{\omega}_{\mu mn} - \frac{1}{8}\left(\bar{\psi}^\alpha\Gamma_{\alpha\mu mn\beta}\psi^\beta\right)$$

$$\begin{aligned}\left[\delta_Q(\epsilon_1),\delta_Q(\epsilon_2)\right] &= \delta_E(\xi^\nu) + \delta_Q(-\xi^\nu\psi_\nu) + \delta_{LL}(\lambda_{mn}) + \delta_{\text{Maxwell}}\left(\Lambda_{\mu\nu}\right) \\ \lambda_{mn} &= \xi^\nu\hat{\omega}_\nu^{mn} + \bar{\epsilon}_2\left(\gamma^{mn\alpha\beta\gamma\delta} - 24e^{m\alpha}e^{n\beta}\gamma^{\gamma\delta}\right)\epsilon_1\hat{F}_{\alpha\beta\gamma\delta} \\ \Lambda_{\mu\nu} &= -\frac{1}{2}\bar{\epsilon}_2\Gamma_{\mu\nu}\epsilon_1 - \xi^\sigma A_{\sigma\mu\nu}\end{aligned}$$

$$A = A_{\Lambda\Pi\Sigma}dz^\Lambda \wedge dz^\Pi \wedge dz^\Sigma$$

$$H = E^M E^N E^P E^Q H_{MNPQ}$$

$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \\ 0 & -3 & 0 & 0 \end{pmatrix}$$

$$e^{-k(z,\bar{z})} = i\left[\bar{Z}^I(z)F_I(Z(z)) - Z^I(z)\bar{F}_I(\bar{Z}(z))\right]$$

$$\begin{aligned}R_{jk}^{i}{}^{l} &= \delta_j^i\delta_k^l + \delta_k^i\delta_j^l - e^{2k}W_{jkm}W^{mil} \\ W_{ijk} &= iF_{IJK}(Z(z))\frac{\partial Z^I}{\partial z^i}\frac{\partial Z^J}{\partial z^j}\frac{\partial Z^K}{\partial z^k}\end{aligned}$$

$$\bar{\psi}\Gamma^{A_1} \dots \Gamma^{A_n}\chi = (-)^n\bar{\chi}\Gamma^{A_n} \dots \Gamma^{A_1}\psi$$

$$\begin{aligned}\mathcal{L} =& -\frac{E}{2k^2}R(E,\Omega) - \frac{E}{2}\bar{\Psi}_\Lambda\Gamma^{\Lambda\Pi\Sigma}D_\Lambda\left(\frac{\Omega+\hat{\Omega}}{2}\right)\Psi_\Sigma + \frac{E}{48}\left(\mathcal{F}_{\Lambda\Pi\Sigma\Omega}\mathcal{F}^{\Lambda\Pi\Sigma\Omega} - 48\mathcal{F}^{\Lambda\Pi\Sigma\Omega}\partial_\Lambda A_{\Pi\Sigma\Omega}\right) \\ & -\frac{k\sqrt{2}}{6}\epsilon^{\Lambda_0\dots\Lambda_{10}}\partial_{\Lambda_0}A_{\Lambda_1\Lambda_2\Lambda_3}\partial_{\Lambda_4}A_{\Lambda_5\Lambda_6\Lambda_7}A_{\Lambda_8\Lambda_9\Lambda_{10}} \\ & -\frac{\sqrt{2}k}{8}E\left[\bar{\Psi}_\Pi\Gamma^{\Pi\Lambda_1\dots\Lambda_4}{}^2\Psi_\Sigma + 12\bar{\Psi}^{\Lambda_1}\Gamma^{\Lambda_2\Lambda_3}\Psi^{\Lambda_4}\right]\frac{1}{24}\left(\frac{F+\hat{F}}{2}\right)_{\Lambda_1\dots\Lambda_4}\end{aligned}$$

$$F_{\Lambda\Pi\Sigma\Omega} \equiv \partial_\Lambda A_{\Pi\Sigma\Omega} + 23 \text{ terms } = 24\partial_{[\Lambda}A_{\Pi\Sigma\Omega]}$$

$$\mathcal{F}_{\Lambda\Pi\Sigma\Omega} = F_{\Lambda\Pi\Sigma\Omega}$$

$$\mathcal{F}_{\Lambda\Pi\Sigma\Omega} = \partial_\Lambda A_{\Pi\Sigma\Omega} + 23 \text{ terms } + \frac{\mathcal{B}_{MNPQ}E^M_\Lambda \dots E^Q_\Omega}{\sqrt{E}}$$

$$\delta E^M_\Lambda = \frac{k}{2}\bar{\epsilon}\Gamma^M\Psi_\Lambda$$

$$\delta\Psi_\Lambda = \frac{D_\Lambda(\hat{\Omega})\epsilon}{k} + \frac{\sqrt{2}}{12}\left(\Gamma^{\Lambda_1\dots\Lambda_4}{}_\Lambda - 8\delta^{\Lambda_1}_\Lambda\Gamma^{\Lambda_2\Lambda_3\Lambda_4}\right)\epsilon\frac{\hat{F}_{\Lambda_1\dots\Lambda_4}}{24} + \frac{1}{24}\left(b\Gamma^{\Lambda_1\dots\Lambda_4}_\Lambda\frac{\mathcal{B}_{\Lambda_1\dots\Lambda_4}}{\sqrt{E}} - a\Gamma^{\Lambda_1\Lambda_2\Lambda_3}\frac{\mathcal{B}_{\Lambda\Lambda_1\Lambda_2\Lambda_3}}{\sqrt{E}}\right)\epsilon$$

$$\delta A_{\Lambda_1\Lambda_2\Lambda_3} = -\frac{\sqrt{2}}{8}\bar{\epsilon}\Gamma_{[\Lambda_1\Lambda_2}\Psi_{\Lambda_3]}$$

$$\delta\mathcal{B}_{MNPQ} = \sqrt{E}\bar{\epsilon}\left[a\Gamma_{MNP}E^\Lambda_Q R_\Lambda(\Psi) + b\Gamma_{MNPQ\Lambda}R^\Lambda(\Psi)\right]$$

$$R^\Lambda(\Psi) = \frac{1}{E}\frac{\delta\mathcal{L}}{\delta\bar{\Psi}} = -\Gamma^{\Lambda\Pi\Sigma}D_\Pi\Psi_\Sigma - \frac{\sqrt{2}}{4}k\left(\frac{\hat{F}_{\Lambda_1\dots\Lambda_4}}{24}\right)\Gamma^{\Lambda\Lambda_1\dots\Lambda_5}\Psi_{\Lambda_5} - 3\sqrt{2}k\frac{\hat{F}^{\Lambda\Pi\Sigma\Omega}}{24}\Gamma_{\Pi\Sigma}\Psi_\Omega$$

$$F_{\mu\nu\rho\sigma} = \frac{3}{\sqrt{2}}m\left(\det e^{(0)m}_\mu(x)\right)\epsilon_{\mu\nu\rho\sigma}$$

$$R_{\mu\nu} - \frac{1}{2}g^{(0)}_{\mu\nu}R = \frac{1}{6}\left(F_{\mu\Lambda\Pi\Sigma}F_\nu^{\Lambda\Pi\Sigma} - \frac{1}{8}g^{(0)}_{\mu\nu}F^2\right) = -\frac{9}{4}g^{(0)}_{\mu\nu}m^2$$

$$R_{\alpha\beta} - \frac{1}{2}g^{(0)}_{\alpha\beta}R = \frac{1}{48}g^{(0)}_{\alpha\beta}F^2 = \frac{9}{4}g^{(0)}_{\alpha\beta}m^2$$

$$R^{mn}_{\mu\nu}\left(e^{(0)4}\right) = m^2\left(e^{(0)m}_\mu(x)e^{(0)n}_\nu(x) - e^{(0)m}_\nu(x)e^{(0)n}_\mu(x)\right)$$

$$R^{ab}_{\alpha\beta}\left(e^{(0)4}\right) = -\frac{1}{4}m^2\left(e^{(0)a}_\alpha(x)e^{(0)b}_\beta(x) - e^{(0)a}_\beta(x)e^{(0)a}_\alpha(x)\right)$$

$$g_{\Lambda\Pi} = E^M_\Lambda E^M_\Pi = g^{(0)}_{\Lambda\Pi} + kh_{\Lambda\Pi}$$

$$h_{\alpha\beta}(y,x) = h_{\alpha\beta}(y) - \frac{g^{(0)}_{\alpha\beta}(y)}{5}\left(h_{\mu\nu}(y,x)g^{(0)\mu\nu}(x)\right)$$

$$h_{\mu\alpha}(y,x) = B_{\alpha,IJ}(y)V^{IJ}_\mu(x)$$

$$h_{\mu\nu}(y,x) = S_{IJKL}(y)\eta^{IJKL}_{\mu\nu}(x)$$

$$\Psi_\mu(y,x) = \lambda_{J,KL}(y)\gamma_5^{1/2}\eta^{JKL}_{\mu\nu}(x)$$

$$\sqrt{\gamma_5} \equiv \frac{i-1}{2}(1+i\gamma_5)$$

$$\Psi_\alpha(y,x) = \psi_{\alpha I}(y)\gamma_5^{\pm 1/2}\eta^I(x) - \frac{1}{5}\tau_\alpha\gamma_5\gamma^\mu\Psi_\mu(y,x)$$

$$A_{\mu\nu\rho}(y,x) = \frac{\sqrt{2}}{40}\sqrt{g^{(0)}}\epsilon_{\mu\nu\rho\sigma}D^\sigma h^\lambda_\lambda$$

$$A_{\alpha\mu\nu}(y,x) = \frac{i}{12\sqrt{2}}B_{\alpha,IJ}(y)\bar{\eta}^I(x)\gamma_{\mu\nu}\gamma_5\eta^J(x)$$

$$A_{\alpha\beta\mu} = 0$$

$$A_{\alpha\beta\gamma}(y,x) = \frac{1}{6}A_{\alpha\beta\gamma,IJ}(y)\phi^{IJ}_5(x)$$

$$B_{\alpha\beta\gamma,IJ}=\frac{1}{5}\left(S_{\alpha\beta\gamma,IJ}+\frac{1}{6}\epsilon_{\alpha\beta\gamma}{}^{\delta\epsilon\eta\zeta}D_{\delta}S_{\epsilon\eta\zeta,IJ}\right)$$

$$D_{\mu}^{(0)}\eta^{I}=\frac{i}{2}\gamma_{\mu}\eta^{I}$$

$$\bar{\eta}^{I}\eta^{J}=\Omega^{IJ}$$

$$\eta_{J}^{\alpha}\bar{\eta}_{\beta}^{I}=-\delta_{\beta}^{\alpha}$$

$$\eta_{J}^{\alpha}=\eta^{\alpha I}\Omega_{IJ}$$

$$\phi_{5}^{IJ}=\bar{\eta}^{I}\gamma_{5}\eta^{J}$$

$$Y^{A}=\frac{1}{4}(\gamma^{A})_{IJ}\phi_{5}^{IJ}$$

$$V_{\mu}^{IJ}=\bar{\eta}^{I}\gamma_{\mu}\eta^{J}$$

$$D_{(\mu}^{(0)}V_{\nu)}=0$$

$$V_{\mu}^{AB}=-\frac{i}{8}(\gamma^{AB})_{IJ}V_{\mu}^{IJ}$$

$$Y^{[A}D_{\mu}^{(0)}Y^{B]}$$

$$D_{(\mu}^{(0)}C_{\nu)}=\frac{1}{4}g_{\mu\nu}^{(0)}(D^{(0)\rho}C_{\rho})$$

$$C_{\mu}^{IJ}=\bar{\eta}^{I}\gamma_{\mu}\gamma_{5}\eta^{J}$$

$$C_{\mu}^{A}=\frac{i}{4}C_{\mu}^{IJ}(\gamma^{A})_{IJ}$$

$$C_{\mu}^{A}=D_{\mu}^{(0)}Y^{A}$$

$$\eta_{\mu\nu}^{IJKL}=\eta_{\mu\nu}^{IJKL}(-2)-\frac{1}{3}\eta_{\mu\nu}^{IJKL}(-10)$$

$$\square\,\eta_{\mu\nu}^{IJKL}(-2)=-2\eta_{\mu\nu}^{IJKL}(-2)$$
$$\square\,\eta_{\mu\nu}^{IJKL}(-10)=-10\eta_{\mu\nu}^{IJKL}(-10)$$

$$\eta_{\mu\nu}^{IJKL}(-2)=C_{(\mu}^{IJ}C_{\nu)}^{KL}-\frac{1}{4}g_{\mu\nu}^{(0)}C_{\lambda}^{IJ}C^{\lambda KL}$$
$$\eta_{\mu\nu}^{IJKL}(-10)=g_{\mu\nu}^{(0)}\left(\phi_{5}^{IJ}\phi_{5}^{KL}+\frac{1}{4}C_{\lambda}^{IJ}C^{\lambda KL}\right)$$

$$\eta_{\mu}^{JKL}=\eta_{\mu}^{JKL}(-2)+\eta_{\mu}^{JKL}(-6)$$

$$\gamma^{\nu}D_{\nu}^{(0)}\eta_{\mu}^{JKL}(-2)=-2\eta_{\mu}^{JKL}(-2)$$
$$\gamma^{\nu}D_{\nu}^{(0)}\eta_{\mu}^{JKL}(-6)=-6\eta_{\mu}^{JKL}(-6)$$

$$\eta_{\mu}^{JKL}(-2)=3\left(\eta^{J}C_{\mu}^{KL}-\frac{1}{4}\gamma_{\mu}\gamma^{\nu}\eta^{J}C_{\nu}^{KL}\right)$$
$$\eta_{\mu}^{JKL}(-6)=\gamma_{\mu}\left(\eta^{J}\phi_{5}^{KL}-\frac{1}{4}\gamma^{\nu}\eta^{J}C_{\nu}^{KL}\right)$$

$$E_{\alpha}^{a}(y,x)=e_{\alpha}^{a}(y)\Delta^{-1/5}(y,x)$$
$$\Delta(y,x)=\frac{\det E_{\mu}^{m}}{\det e_{\mu}^{(0)m}}$$

$$E_\alpha^m(y,x)= B_\alpha^\mu(y,x)E_\mu^m$$
$$B_\alpha^\mu(y,x) = -2B_\alpha^{AB}V^{\mu,AB}$$

$$\Psi_a= \Delta^{1/10}(\gamma_5)^{-p}\psi_a - \frac{A}{5}\tau_a\gamma_5\gamma^m\Delta^{1/10}(\gamma_5)^q\psi_m$$
$$\Psi_m= \Delta^{1/10}(\gamma_5)^q\psi_m$$
$$\epsilon(y,x) = \Delta^{-1/10}(\gamma_5)^{-p}\varepsilon(y,x)$$

$$\psi_\alpha(y,x) = \psi_{\alpha I'}(y)U^{I'}{}_I(y,x)\eta^I(x)$$

$$\psi_m(y,x)= \lambda_{J'K'L'}(y)U^{J'}{}_J(y,x)U^{K'}{}_K(y,x)U^{L'}{}_L(y,x)\eta_m^{JKL}(x)$$
$$\varepsilon(y,x)= \varepsilon_{I'}(y)U^{I'}{}_I(y,x)\eta'x)$$

$$\left(\tilde{\Omega}\cdot U^T\cdot\Omega\right)^I{}_{I'} = -(U^{-1})^I{}_{I'}$$

$$E_\mu^m = \frac{1}{4}\Delta^{2/5}\Pi_A^i C_\mu^A C^{mB}\mathrm{Tr}\left(U^{-1}\gamma^i U\gamma_B\right)$$

$$ds_{11}^2 = \Delta^{-2/5}g_{\alpha\beta}dy^\alpha dy^\beta + \Delta^{4/5}T_{AB}^{-1}(dY^A + 2B^{AC}Y^C)(dY^B + 2B^{BD}Y^D)$$

$$\frac{\sqrt{2}}{3}F_{(4)} =\epsilon_{ABCDE}\Bigg(-\frac{1}{3}DY^A DY^B DY^C DY^D\frac{(T\cdot Y)^E}{Y\cdot T\cdot Y}$$
$$+\frac{4}{3}DY^A DY^B DY^C D\left[\frac{(T\cdot Y)^D}{Y\cdot T\cdot Y}\right]Y^E$$
$$+2F_{(2)}^{AB}DY^C DY^D\frac{(T\cdot Y)^E}{Y\cdot T\cdot Y} + F_{(2)}^{AB}F_{(2)}^{CD}Y^E\Bigg) + d(\mathcal{A})$$

$$F_{(2)}^{AB} = 2(dB^{AB} + 2(B\cdot B)^{AB})$$
$$DY^A = dY^A + 2(B\cdot Y)^A$$

$$T^{AB} = (\Pi^{-1})_i^A(\Pi^{-1})_j^B\delta^{ij}$$

$$\mathcal{A}_{\alpha\beta\gamma} = \frac{8i}{\sqrt{3}}S_{\alpha\beta\gamma,B}Y^B$$

$$\frac{\mathcal{B}_{\alpha\beta\gamma\delta}}{\sqrt{E}} = \frac{i}{2\sqrt{3}}\epsilon_{\alpha\beta\gamma\delta\epsilon\eta\zeta}\frac{\delta S^{(7)}}{\delta S_{\epsilon\eta\zeta,A}}Y^A$$
$$= -24\sqrt{3}i\nabla_\alpha S_{\beta\gamma\delta,A}Y^A + \sqrt{3}i\epsilon_{\alpha\beta\gamma\delta}{}^{\epsilon\eta\zeta}T^{AB}S_{\epsilon\eta\zeta,B}Y^A + g\epsilon_{ABCDE}F_{[\alpha\beta}^{BC}F_{\gamma\delta]}^{DE}Y^A + 2 -$$ white particle/dark particle

$$\nu = \begin{pmatrix} u_{ij}{}^{IJ} & v_{ijKL} \\ v^{klIJ} & u^{kl}{}_{IJ}\end{pmatrix}$$

$$\mathcal{B}_\mu{}^i{}_j = \frac{2}{3}\left(u^{ik}{}_{IK}\partial_\mu u_{jk}{}^{IJ} - v^{ikIJ}\partial_\mu v_{jkIJ}\right)$$

$$D_\mu\nu\cdot\nu^{-1} = -\frac{1}{4}\sqrt{2}\begin{pmatrix} 0 & \mathcal{A}_\mu^{ijkl} \\ \mathcal{A}_{\mu mnpq} & 0\end{pmatrix}$$

$$\mathcal{A}_\mu^{ijkl} = -2\sqrt{2}\left(u^{ij}{}_{IJ}\partial_\mu v^{klIJ} - v^{ijIJ}\partial_\mu u^{kl}{}_{IJ}\right)$$

$$\mathcal{A}_\mu^{ijkl} = \frac{1}{24}\eta\epsilon^{ijklmnpq}\mathcal{A}_{\mu mnpq}$$

$$D_\mu\epsilon^i = \partial_\mu\epsilon^i - \frac{1}{2}\omega_{\mu ab}\sigma^{ab}\epsilon^i + \frac{1}{2}\mathcal{B}_\mu{}^i{}_j\epsilon^j$$

$$D_\mu u_{ij}{}^{IJ} = \partial_\mu u_{ij}{}^{IJ} + \mathcal{B}_\mu{}^k{}_{[i}u_{j]k}{}^{IJ} - 2gA_\mu^{K[I}u_{ij}{}^{J]K}$$

$$A_\mu^{\alpha\beta} = \mathbf{P}(15)^{\alpha\beta}{}_{\gamma\delta}A_\mu^{\gamma\delta} + \mathbf{P}^1(6)^{\alpha\beta}{}_{\gamma\delta}A_\mu^{\gamma\delta} + \mathbf{P}^2(6)^{\alpha\beta}{}_{\gamma\delta}A_\mu^{\gamma\delta}$$

$$(\Pi^{-1})^{\alpha\beta}_{ab}(\delta^{\gamma}_{\alpha}\delta^{\delta}_{\beta} + 2gB^{\gamma}_{\mu\alpha}\delta^{\delta}_{\beta})\Pi^{cd}_{\gamma\delta} = 2Q^{[c}_{\mu[a}\delta^{d]}_{b]} + 2P^{cd}_{\mu ab}$$

$$(\Pi^{-1})_i{}^A(\delta^B_A\partial_\alpha + gB_{\alpha A}{}^B)\Pi_B{}^k\delta_{kj} = Q_{\alpha ij} + P_{ij}$$

$$\gamma_5\eta^I\phi_5^{JK} - \gamma_\mu\gamma_5\eta^I C_\mu^{JK} = 4\eta^{[K}\Omega^{J]I} - \eta^I\Omega^{JK}$$

$$\mathcal{L} = -\frac{1}{2}m^2A_\mu A^\mu + \frac{1}{2}m\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$$

$$A_{\mu\nu\rho} = -\frac{1}{2\sqrt{2}}\frac{D^{(0)}_\sigma}{\Box^{(0)}}\left(\epsilon_{\mu\nu\rho\sigma}\sqrt{g^{(0)}}\right)$$

$$A_{\alpha\beta\gamma} = -\frac{i\sqrt{6}}{6}S_{\alpha\beta\gamma,A}Y^A$$

$$\frac{B_{\alpha\beta\gamma\delta}}{\sqrt{E}} = -24\sqrt{3}i\nabla_{[\alpha}S_{\beta\gamma\delta],A}Y^A$$

$$e^{-1}\mathcal{L} = \frac{1}{2}S_{\alpha\beta\gamma,A}S^{\alpha\beta\gamma}{}_{,B}\delta^{AB} + \frac{1}{48}me^{-1}\epsilon^{\alpha\beta\gamma\delta\epsilon\eta\zeta}\delta^{AB}S_{\alpha\beta\gamma,A}F_{\delta\epsilon\eta\zeta,B}$$

$$\delta\psi_M = \frac{D_M\eta}{k} + \frac{k}{32g^2\phi}(\Gamma_M^{NPQ} - 8\delta_M^N\Gamma^{PQ})\eta H_{NPQ} + (\mathfrak{RGF})^2$$
$$\delta\chi^a = -\frac{1}{4g\sqrt{\phi}}\Gamma^{MN}F^a_{MN}\eta + (\text{Fermi})^2$$
$$\delta\lambda == \frac{1}{\sqrt{2}\phi}(\Gamma\cdot\partial\phi)\eta + \frac{k}{8\sqrt{2}g^2\phi}\Gamma^{NPQ}\eta H_{NPQ} + (\mathfrak{RGF})^2$$

$$D_i\eta = 0$$
$$\Gamma^{ij}F_{ij}\eta = 0$$

$$\eta^\alpha \to U^\alpha{}_\beta\eta^\beta$$

$$U = P\exp\int_\gamma \omega\cdot dx$$

$$P\big(\omega(x_2)\omega(x_5)\omega(x_3)\big) = \omega(x_2)\omega(x_3)\omega(x_5)$$

$$g_{i\bar{j}} = \partial_i\partial_{\bar{j}}K$$

$$R_{i\bar{j}} = 0$$

$$\Box_{10}A = (\Box_4 + \Delta_K)A = \Box_4 A = 0$$

$$k_{ij} = \bar{\eta}\Gamma_{ij}\eta$$

$$J^i{}_j = g^{ik}k_{kj}$$

$$\Omega_{ijk} = \bar{\eta}\Gamma_{ijk}\eta$$

$$b_n = \sum_{p+q=n} h^{p,q}$$

$$\int_{A^I}\beta^J = \delta^J_I;\ \int_{B^J}\alpha_I = \delta^J_I$$

$$F_I = \int_{A_I} \Omega;\ Z^J = \int_{B^J} \Omega$$

$$F_I = N_{IJ} Z^J$$

$$N_{IJ} = \frac{\partial F_I}{\partial Z^J}$$

$$J = g_{i\bar{j}} dz^i \wedge dz^{\bar{j}}$$

$$K = J + iB$$

$$X_{I'} = \int_{A_{I'}} K;\ X^{J'} = \int_{B^{J'}} K$$

$$\tau = a + ie^{-\phi}$$

$$e^{-k} = i(F_I \bar{Z}^I - Z^I \bar{F}_I)$$

$$e^{-k} =< \Omega \mid \bar{\Omega} >$$

$$< A \mid \bar{B} >\equiv \int_K d^6 x A \wedge B$$

$$\Omega = Z^I \alpha_I + F_J \beta^J$$

$$G = F^{RR} - \tau H^{NS}$$

$$W = \int_{K_6} \Omega \wedge G$$

$$K(\rho) = -3\ln\left[-i(\rho - \bar{\rho})\right]$$
$$K(\tau, Z^\alpha) = -\ln\left[-i(\tau - \bar{\tau})\right] - \ln\left(-i \int_{K_6} \Omega \wedge \bar{\omega}\right)$$

$$dH = \mathrm{tr}(F \wedge F) - \mathrm{tr}(R \wedge R)$$

$$A_i^{AB} = \begin{pmatrix} 0 & 0 \\ 0 & \omega_i^{ab} \end{pmatrix}$$

$$F_{a\bar{b}} = -2iR_{a\bar{b}}$$

$$F_{ab} = F_{\bar{a}\bar{b}} = 0, g^{a\bar{b}} F_{a\bar{b}} = 0$$

$$\int_K F \wedge k \wedge \ldots \wedge k = (N-1)!^2 \int_K g^{a\bar{b}} F_{a\bar{b}} = 0$$

$$U_\gamma = P\exp \oint {}_\gamma A \cdot dx$$

$$\text{quarks: } \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$$
$$\text{gluons: } \begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

$$L_e(e) = L_e(\nu_e) = +1;\ L_e(e^+) = L_e(\bar{\nu}_e) = -1$$
$$L_\mu(\mu^-) = L_\mu(\nu_\mu) = +1;\ L_\mu(\mu^+) = L_\mu(\bar{\nu}_\mu) = -1$$
$$L_\tau(\tau^-) = L_\tau(\nu_\tau) = +1;\ L_\tau(\tau^+) = L_\tau(\bar{\nu}_\tau) = -1$$

$$L = L_e + L_\mu + L_\tau$$

$$B(q) = +1/3, B(\bar{q}) = -1/3$$

$$e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$$

$$\mathcal{E} = \begin{pmatrix} e_L \\ \epsilon e_L^* \end{pmatrix};\; E = \begin{pmatrix} -\epsilon e_R^* \\ e_R \end{pmatrix}$$

$$U_p \equiv (u, c, t)$$
$$D_p \equiv (d, s, b)$$
$$E_p \equiv (e, \mu, \tau)$$
$$\nu_p \equiv (\nu_e, \nu_\mu, \nu_\tau)$$

$$L_p \equiv \begin{pmatrix} \nu_p \\ \mathcal{E}_p \end{pmatrix}$$
$$Q_p \equiv \begin{pmatrix} \mathcal{U}_p \\ \mathcal{D}_p \end{pmatrix}$$

$$U_{Rp}: (3,1,+2/3)$$
$$D_{Rp}: (3,1,-1/3)$$
$$E_R: (1,1,-1)$$

$$L_{Lp} \equiv \begin{pmatrix} \nu_{Lp} \\ \mathcal{E}_{Lp} \end{pmatrix}: (1,2,-1/2)$$
$$Q_{Lp} \equiv \begin{pmatrix} \mathcal{U}_{Lp} \\ \mathcal{D}_{Lp} \end{pmatrix}: (3,2,+1/6)$$

$$U_{Lp}: (\overline{3}, 1, -2/3)$$
$$D_{Lp}: (\overline{3}, 1, +1/3)$$
$$E_{Lp}: (1,1,+1)$$
$$L_{Rp} \equiv \begin{pmatrix} \nu_{Rp} \\ \mathcal{E}_{Rp} \end{pmatrix}: (1,2,+1/2)$$
$$Q_{Rp} \equiv \begin{pmatrix} \mathcal{U}_{Rp} \\ \mathcal{D}_{Rp} \end{pmatrix}: (\overline{3}, 2, -1/6)$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}: (1,2,+1/2)$$

$$\tilde{\phi} = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix}: (1,2,-1/2)$$

$$D_\mu Q_p = \partial_\mu Q_p + \left[-ig_3 G_\mu^\alpha \frac{\lambda_\alpha}{2} - ig_2 W_\mu^a \frac{\tau^a}{2} - i\frac{g_1}{6} B_\mu\right] Q_{Lp} + \left[+ig_3 G_\mu^\alpha \frac{\lambda_\alpha^*}{2} + ig_2 W_\mu^a \frac{\tau_a^*}{2} + i\frac{g_1}{6} B_\mu\right] Q_{Rp}$$

$$D_\mu U_p = \partial_\mu U_p + \left[-ig_3 G_\mu^\alpha \frac{\lambda_\alpha}{2} - i\frac{2g_1}{3} B_\mu\right] U_{Rp} + \left[+ig_3 G_\mu^\alpha \frac{\lambda_\alpha^*}{2} + i\frac{2g_1}{3} B_\mu\right] U_{Lp}$$

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{kin} = -\frac{1}{4} G_{\mu\nu}^\alpha G^{\alpha\mu\nu} - \frac{1}{4} W^{a\mu\nu} W_{\mu\nu}^a - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{g_3^2}{64\pi^2} \theta_3 \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^\alpha G_{\rho\sigma}^\alpha - \frac{g_2^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} \theta_2 \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a - \frac{1}{2} \bar{L}_p \not{D} L_p - \frac{1}{2} \bar{E}_p \not{D} E_p - \frac{1}{2} \bar{Q}_p \not{D} Q_p - \frac{1}{2} \bar{U}_p \not{D} U_p - \frac{1}{2} \bar{D}_p \not{D} D_p$$

$$\mathcal{L}_{\text{Higgs}} = -(D_\mu\phi)^\dagger D^\mu\phi - \lambda\left[\phi^\dagger\phi - \frac{\mu^2}{2\lambda^2}\right]^2$$

$$D_\mu\phi = \partial_\mu\phi - ig_2W_\mu^a\frac{\tau}{2}\phi - i\frac{g_1}{2}B_\mu\phi$$

$$\mathcal{L}_{\text{Yukawa}} = -f_{pq}\bar{L}_p\frac{1-\gamma_5}{2}E_q\phi - h_{pq}\bar{Q}_p\frac{1-\gamma_5}{2}D_q\phi - g_{pq}\bar{Q}_p\frac{1-\gamma_5}{2}U_q\tilde{\phi}$$

$$\frac{1-\gamma_5}{2} = P_R = P_R^2 = \bar{P}_LP_R$$

$$\left(\bar{Q}_p\frac{1-\gamma_5}{2}D_q\right)\phi = (\bar{Q}_{Lp}D_{Rq})\phi$$

$$\phi = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$\phi = \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix}$$

$$m_H^2 = 2\lambda v^2 = 2\mu^2$$

$$\cos\,\theta_W \equiv \frac{g_2}{\sqrt{g_1^2+g_2^2}};\ \sin\,\theta_W \equiv \frac{g_1}{\sqrt{g_1^2+g_2^2}}$$

$$A_\mu = W_\mu^3\sin\,\theta_W + B_\mu\cos\,\theta_W$$
$$Z_\mu = W_\mu^3\cos\,\theta_W - B_\mu\sin\,\theta_W$$
$$W_\mu^\pm = \frac{W_\mu^1 \pm W_\mu^2}{2}$$

$$M_W = \frac{g_2v}{2}, M_Z = \frac{v}{2}\sqrt{g_1^2+g_2^2}$$

$$\mathcal{L}_{mF} = -\frac{v}{\sqrt{2}}\left[f_{pq}\overline{\mathcal{E}}_pE_{Rq} + g_{pq}\overline{\mathcal{U}}_pU_{Rq} + h_{pq}\overline{\mathcal{D}}_pD_{Rq}\right]$$

$$\mathcal{E}_L \text{ by } U^{(e)} \quad E_R \text{ by } V^{(e)}$$
$$\mathcal{U}_L \text{ by } U^{(u)} \quad U_R \text{ by } V^{(u)}$$

$$\mathcal{D}_L \text{ by } U^{(d)}\ D_R \text{ by } V^{(d)}$$

$$e_p = \mathcal{E}_{Lp} + E_{Rp}$$
$$d_p = \mathcal{D}_{Lp} + D_{Rp}$$
$$u_p = \mathcal{U}_{Lp} + U_{Rp}$$

$$\mathcal{L} = -\frac{v}{\sqrt{2}}(f_p\bar{e}_pe_p + g_p\bar{u}_pu_p + h_p\bar{d}_pd_p)$$

$$D_R + L_R = (3,1,-1/3) + (1,2,+1/2) = 5$$

$$U_R + E_R + Q_R = (3,1,+2/3) + (1,1,-1) + (\bar{3},2,-1/6) = \overline{10} = (\bar{5}\times\bar{5})_{a-\text{sym}}.$$

$$D_L + L_L = \bar{5}$$
$$U_L + E_L + Q_L = 10$$

$$24 = 8 + 3 + 1(+12 \text{ more })$$

$$16 \to 10_{-1} + \bar{5}_3 + 1_{-5}$$

$$10 \rightarrow 5 + \overline{5}$$

$$45 \rightarrow 24_0 + 10_4 + \overline{1}0_{-4} + 1_0$$

$$248 = (45,1) + (1,15) + (10,6) + (16,4) + (\overline{1}6,4)$$

$$Q{:}\ (3,2,+1/6)$$
$$L{:}\ (1,2,-1/2)$$
$$E^C{:}\ (1,1,+1)$$
$$U^C{:}\ (\overline{3},1,-2/3)$$
$$D^C{:}\ (\overline{3},1,+1/3)$$

$$H_u{:}\ (1,2,+1/2)$$
$$H_d{:}\ (1,2,-1/2)$$

$$W = \mu H_u H_d + y_u H_u Q U^C + y_d H_d Q D^C + y_l H_d L E^C$$

$$m_{1/2}\tilde{\lambda}\tilde{\lambda} + \text{ h.c.}$$

$$m_0 \phi^\dagger \phi$$

$$B_\mu h_u h_d + A h_u q u^C + A h_d q d^C + A h_d l e^C + h.c.$$

$$(dG)_{11IJKL} = -4\sqrt{2}\pi \left(\frac{k}{4\pi}\right)^{2/3} \left(J^{(1)}\delta(x_{11}) + J^{(2)}\delta(x_{11} - \pi R_{11}) + J + W\right)_{IJKL}$$

$$(dG)_{11IJKL} = -4\sqrt{2}\pi \left(\frac{k}{4\pi}\right)^{2/3} \left(\tilde{J}^{(1)}\delta(x_{11}) + \tilde{J}^{(2)}\delta(x_{11} - \pi R_{11}) + W\right)_{IJKL}$$

$$\tilde{J}^{(i)} = \frac{1}{16\pi^2}\left(\text{tr}F^{(i)} \wedge F^{(i)} - \frac{1}{2}\text{tr}R \wedge R\right)$$

$$\frac{\text{super} - \text{Poincaré}}{\text{Lorentz}} = \frac{\{T_I\}}{\{M_{rs}\}}$$

Modelo de una partícula oscura y/0 blanca hadrónica. Modelo Unificado.

$$ds^2 = -e^\lambda dr^2 - r^2\, d\theta^2 - r^2 \sin^2\, \theta\, d\phi^2 + c^2 e^\nu dt^2,$$

$$\frac{8\pi G}{c^4} P = e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2},$$
$$\frac{8\pi G}{c^4} \mathcal{E} = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2},$$
$$\frac{dP}{dr} = -\frac{1}{2}(\mathcal{E} + P)\nu',$$

$$\frac{dP}{dr} = -\frac{G(\mathcal{E} + P)(mc^2 + 4\pi r^3 P)}{rc^2(rc^2 - 2Gm)},$$
$$\frac{dm}{dr} = \frac{4\pi r^2 \mathcal{E}}{c^2}.$$

$$m(r = 0) = 0, (\, dP/dr)_{r=0} = 0.$$

$$m(r) = \frac{4\pi\mathcal{E}_0}{3c^2} r^3$$

$$\mathcal{E}_0 = \mathcal{E}(\rho = \bar{\rho})\Theta(\bar{\rho} - \rho) = \mathcal{A}(\bar{\rho})\Theta(R - r)$$

$$M = m(R) = \frac{4\pi\mathcal{E}_0}{3c^2} R^3$$

$$\mathrm{d}s^2 = -\frac{\mathrm{d}r^2}{1 - r^2/R_\mathrm{S}^2} - r^2\,\mathrm{d}\theta^2 - r^2 \sin^2\,\theta\,\mathrm{d}\phi^2$$
$$+c^2\left[A_\mathrm{S} - B_\mathrm{S}\sqrt{1 - r^2/R_\mathrm{S}^2}\right]^2\,\mathrm{d}t^2,$$

$$R_\mathrm{S} = \left(\frac{8\pi G\mathcal{E}_0}{3c^4}\right)^{-1/2}$$

$$A_\mathrm{S} = \frac{3}{2}\sqrt{1 - \frac{R^2}{R_\mathrm{S}^2}}, B_\mathrm{S} = \frac{1}{2}$$

$$P = \frac{\mathcal{E}_0}{3}\frac{3\zeta\sqrt{1 - (r/R_S)^2} - 1}{1 - \zeta\sqrt{1 - (r/R_S)^2}},$$

$$\zeta = \left|\frac{P_0 + \mathcal{E}_0/3}{P_0 + \mathcal{E}_0}\right|,$$

$$\mathcal{E}(\rho) = \mathcal{A}(\rho) + \mathcal{C}(\nabla\rho)^2$$

$$E = \int\,\mathrm{d}\mathcal{V}\mathcal{E}[\rho(\mathbf{r})]$$

$$\mathcal{A} = -b_V\rho + \varepsilon(\rho) + \frac{G}{4}\rho\Phi(\rho),$$

$$\varepsilon(\rho) = \frac{K}{18\bar{\rho}^2}\rho(\rho - \bar{\rho})^2$$

$$\left(\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\rho}\right)_{\rho=\bar{\rho}} = 0, \mathcal{W} = \frac{\mathcal{E}}{\rho}.$$

$$\mathcal{A} = -- b_V^{(G)}\rho + \varepsilon_G(\rho),$$

$$\varepsilon_G(\rho) = \varepsilon(\rho) + \frac{m}{2}\Phi_2(\rho - \bar{\rho})^2 = \frac{K_G}{18\bar{\rho}^2}\rho(\rho - \bar{\rho})^2.$$

$$K_G = K + 9m\bar{\rho}^2\Phi_2$$

$$N = \int\,\mathrm{d}\mathcal{V}\rho(\mathbf{r})$$

$$\frac{\delta\mathcal{E}}{\delta\rho} \equiv \frac{\partial\mathcal{A}}{\partial\rho} - 2\mathcal{C}\Delta\rho = \mu.$$

$$P = \rho^2\frac{\delta\mathcal{W}}{\delta\rho}.$$

$$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_1,$$

$$\mathcal{E}_1 = \mathcal{C}(\nabla\rho)^2.$$

$$P = P_0 + P_1, \lambda = \lambda_0 + \lambda_1, \nu = \nu_0 + \nu_1,$$

$$e^{-\lambda_0} = 1 - r^2/R_\mathrm{S}^2, r < R_\mathrm{S}.$$

$$\lambda_1(r) = -\frac{8\pi R_\mathrm{S}}{1 - R^2/R_\mathrm{S}^2}\mathcal{D}_1(r),$$

$$\mathcal{D}_1 \equiv \int \mathrm{d}r\mathcal{E}_1(r) \approx \frac{a}{R}\frac{RK_G}{9\bar{\rho}}\int \mathrm{d}\xi\left(\frac{\partial\rho}{\partial\xi}\right)^2.$$

$$\mathcal{D}_1 \propto \exp\left(-2\xi/a\right).$$

$$\mathcal{D}_1 \approx -\frac{a}{R}\frac{\bar{\rho}RK_G}{9}\mathcal{Y}(y).$$

$$\mathcal{Y}(y) = \int \mathrm{d}y\sqrt{y}(1-y)$$
$$= \frac{2}{3}y^{3/2} - \frac{2}{5}y^{5/2} + C_y \quad (32)$$

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{\mathcal{E}+P}{2r}\left[(8\pi Pr^2+1)e^{\lambda}-1\right],$$

$$\frac{\mathrm{d}P_1}{\mathrm{d}r} + \beta P_1 = \beta_1.$$

$$\beta = \frac{8\pi r^2(\mathcal{E}_0+P_0)+8\pi P_0 r^2+\tilde{r}^2}{2r(1-\tilde{r}^2)},$$

$$\beta_1 = -\frac{8\pi P_0 r^2+\tilde{r}^2}{2r(1-\tilde{r}^2)}\mathcal{E}_1 + \frac{(\mathcal{E}_0+P_0)(8\pi P_0 r^2+1)}{1-\tilde{r}^2}\lambda_1,$$

$$P_1 = -C_1^{(P)}(r)\exp\left(-\int \mathrm{d}r\beta(r)\right)$$

$$C_1^{(P)}(r) = \int \mathrm{d}r\beta_1(r)\exp\left[\int \mathrm{d}r\beta(r)\right]$$

$$\exp\left[\int \mathrm{d}r\beta(r)\right] = \frac{\left(1-\zeta\sqrt{1-\tilde{r}^2}\right)^2}{\sqrt{\tilde{r}(1-\tilde{r}^2)}}$$

$$C_1^{(P)}(r) = -\int \mathrm{d}r\left[\frac{8\pi P_0 r^2+\tilde{r}^2}{2r(1-\tilde{r}^2)}\mathcal{E}_1\right.$$
$$\left.+\frac{(\mathcal{E}_0+P_0)(8\pi P_0 r^2+1)}{1-\tilde{r}^2}\lambda_1\right]\exp\left[\int \mathrm{d}r\beta(r)\right].$$

$$\int \mathrm{d}r\lambda_1(r) \approx -\frac{a}{R}\frac{K_G\bar{\rho}}{9}\frac{4\pi R^2}{\sqrt{1-R^2/R_S^2}}$$
$$\times\int \mathrm{d}r\mathcal{Y}(y)$$

$$\int \mathrm{d}r\mathcal{Y}(y) \approx -\frac{a}{R}R\int \frac{\mathrm{d}y}{\sqrt{y}(1-y)}\mathcal{Y}(y).$$

$$\mathcal{Y}/(1-y) \to \mathcal{Y}'(y)/(-1) \to (1-y) \to 0$$

$$y\int \mathrm{d}y/\sqrt{y} = 2\sqrt{y}$$

$$C_1^{(P)}(r) \approx -\frac{\tilde{R}^2}{2R\left(1-\tilde{R}^2\right)}\mathcal{D}_1$$

$$P_1 = -\frac{3R^{3/2}}{4R_S^{5/2}\sqrt{1-R^2/R_S^2}}\mathcal{D}_1$$

$$P_1 = \frac{a}{R}\frac{\bar{\rho}K_G}{12}\frac{R^{5/2}}{R_S^{5/2}\sqrt{1-R^2/R_S^2}}\mathcal{Y}(y)$$

$$P \equiv P(\rho) = P_0(y) + P_1(y)$$

$$p_0 \equiv \frac{8\pi P_0 G R^2}{c^4} = \frac{R^2}{R_S^2}\frac{3\sqrt{1-r^2/R_S^2}/2 - A_S}{A_S - \sqrt{1-r^2/R_S^2}/2}$$

$$R/R_S = \sqrt{1-4A_S^2/9}$$

$$p_1 \equiv \frac{8\pi P_1 G R^2}{c^4} = \frac{3\kappa R^{9/2}}{R_S^{9/2}\sqrt{1-R^2/R_S^2}}\mathcal{Y}(y)\frac{a}{R},$$

$$\kappa \equiv \frac{\bar{\rho}K_G}{12\mathcal{E}_0} = -\frac{K + 9m\bar{\rho}^2\Phi_2}{12(b_V - m\Phi_0)}$$

$$p \approx p_0 + p_1,$$

$$8\pi\mathcal{E}_0 = e^{-\lambda_0}\left(\frac{1}{r}\frac{\mathrm{d}\lambda_0}{\mathrm{d}r} - \frac{1}{r^2}\right) + \frac{1}{r^2}.$$

$$8\pi\mathcal{E}_1 = \frac{1}{r}e^{-\lambda_0}\frac{\mathrm{d}\lambda_1}{\mathrm{d}r} - \lambda_1\left(8\pi\mathcal{E}_0 - \frac{1}{r^2}\right)$$

$$\frac{\mathrm{d}\lambda_1}{\mathrm{d}r} + \alpha\lambda_1 = \alpha_1$$

$$\begin{aligned}\alpha(r) &= re^{\lambda_0}\left(-8\pi\mathcal{E}_0 + \frac{1}{r^2}\right)\\ &= \frac{1}{r(1-\tilde{r}^2)} - \frac{8\pi\mathcal{E}_0 r}{1-\tilde{r}^2},\end{aligned}$$

$$\begin{aligned}\alpha_1 &= \frac{8\pi r\mathcal{E}_1}{1-\tilde{r}^2} \approx \frac{8\pi r\mathcal{C}}{1-\tilde{r}^2}\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)^2\\ &\approx \frac{8\pi r\varepsilon_G}{1-\tilde{r}^2}\end{aligned}$$

$$\mathcal{E}_1 \approx \mathcal{C}\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)^2 \approx \varepsilon_G;$$

$$\lambda_1 = C_1^{(\lambda)}(r)\exp\left(-\int \mathrm{d}r\alpha(r)\right),$$

$$C_1^{(\lambda)}(r) = \int \mathrm{d}r\alpha_1(r)\exp\left(\int \mathrm{d}r\alpha(r)\right).$$

$$\int \mathrm{d}r\alpha(r) = \ln\left(\frac{\tilde{r}}{\sqrt{1-\tilde{r}^2}}\right) + 8\pi\mathcal{E}_0 R_S^2 \ln\sqrt{1-\tilde{r}^2}$$

$$\exp\left(\int \mathrm{d}r\alpha(r)\right) = \tilde{r}(1-\tilde{r}^2)^{-1/2+4\pi\mathcal{E}_0 R_S^2}.$$

$$C_1^{(\lambda)}(r) = 8\pi\int r\,\mathrm{d}r\mathcal{E}_1(r)(1-\tilde{r}^2)^{4\pi\mathcal{E}_0 R_S^2 - 3/2}$$

$$C_1^{(\lambda)}(r) = 8\pi R\left(1-\tilde{R}^2\right)^{4\pi\mathcal{E}_0 R_S^2 - 3/2}\mathcal{D}_1,$$

$$R^a_{bcd} = \Gamma^a_{bd,c} - \Gamma^a_{bc,d} + \Gamma^e_{bd}\Gamma^a_{ce} - \Gamma^e_{bc}\Gamma^a_{de}$$

$$\Gamma^a_{bd} = \frac{1}{2} g^{ae}\left(g_{be,d} + g_{ed,b} - g_{bd,e}\right)$$

$$R_{ab} = g^{cd} R_{acbd}$$

$$T_{(ab)} = \frac{1}{2}(T_{ab} + T_{ba}), T_{[ab]} = \frac{1}{2}(T_{ab} - T_{ba})$$

$$\mathbb{A} = \frac{1}{2}\int d^4x\sqrt{-g}[R + 2\mathcal{L}_m]$$

$$\mathbb{A} = \frac{1}{2}\int d^4x\sqrt{-g}[f(R) + 2\mathcal{L}_m]$$

$$f' R_{ab} - \frac{1}{2} f g_{ab} - \nabla_b\nabla_a f' + g_{ab}\nabla_c\nabla^c f' = T^m_{ab}$$

$$G_{ab} = T^{\text{eff}}_{ab} = \tilde{T}^m_{ab} + T^R_{ab},$$

$$T^R_{ab} \equiv \frac{1}{f'}\left[\frac{1}{2}(f - Rf')g_{ab} + \nabla_b\nabla_a f' - g_{ab}\nabla_c\nabla^c f'\right],$$

$$\tilde{T}^m_{ab} \equiv T^m_{ab}/f'.$$

$$R = \frac{1}{f'}(3p^m - \mu^m) + \frac{2f}{f'} - 3\frac{f'''}{f'}\nabla^a R\nabla_a R - 3\frac{f''}{f'}\nabla^2 R + 3\Theta\dot{R}\frac{f''}{f'} +$$
$$+3\frac{f''}{f'}\ddot{R} + 3\frac{f'''}{f'}\dot{R}^2 - 3\dot{u}^c\frac{(\nabla_c f')}{f'},$$

$$\nabla^b\left(\frac{T^m_{ab}}{f'}\right) = -\nabla^b T^R_{ab} = -\frac{f''}{f'^2}T^m_{ab}\nabla^b R.$$

$$\mathbb{A} = \frac{1}{2}\int d^4x\sqrt{-g}[R + \alpha R^2 + 2\mathcal{L}_m].$$

$$h_{ab} = g_{ab} + u_a u_b$$

$$\dot{\psi}_{a\ldots b} \equiv u^d\nabla_d\psi_{a\ldots b}$$

$$\mathrm{D}_c\psi_{a\ldots b} \equiv h_c{}^d h_a{}^e \ldots h_b{}^f\nabla_d\psi_{e\ldots f}$$

$$e_a u^a = 0, e_a e^a = 1$$

$$N_{ab} \equiv h_{ab} - e_a e_b = g_{ab} + u_a u_b - e_a e_b, N^a_a = 2$$

$$v^a = Ve^a, V \equiv v^a e_a$$
$$\psi_{ab} = \psi_{\langle ab\rangle} = \Psi\left(e_a e_b - \frac{1}{2}N_{ab}\right)$$

$$\hat{\psi}_{a..b}{}^{c..d} \equiv e^f D_f\psi_{a..b}{}^{c..d}$$
$$\delta_f\psi_{a..b}{}^{c..d} \equiv N_a{}^f \ldots N_b{}^g N_h{}^c \ldots N_i{}^d N_f{}^j D_j\psi_{f..g}{}^{i..j}$$

$$\mathcal{A} \equiv e^a\dot{u}_a$$
$$\phi \equiv \delta_a e^a$$
$$\mathcal{E} \equiv C_{acbd}u^c u^d e^a e^b$$

$$T^{\text{tot}}_{ab} = \mu^{\text{tot}}u_a u_b + (p^{\text{tot}} + \Pi^{\text{tot}})e_a e_b + \left(p^{\text{tot}} - \frac{1}{2}\Pi^{\text{tot}}\right)N_{ab} + 2Q^{\text{tot}}e_{(a}u_{b)},$$

$$\hat{\phi} = -\frac{1}{2}\phi^2 - \frac{2}{3}\mu^{\text{tot}} - \frac{1}{2}\Pi^{\text{tot}} - \mathcal{E},$$
$$\hat{\mathcal{E}} - \frac{1}{3}\hat{\mu}^{\text{tot}} + \frac{1}{2}\hat{\Pi}^{\text{tot}} = -\frac{3}{2}\phi\left(\mathcal{E} + \frac{1}{2}\Pi^{\text{tot}}\right),$$
$$0 = -\mathcal{A}\phi + \frac{1}{3}(\mu^{\text{tot}} + 3p^{\text{tot}}) - \mathcal{E} + \frac{1}{2}\Pi^{\text{tot}},$$
$$\hat{p}^{\text{tot}} + \hat{\Pi}^{\text{tot}} = -\left(\frac{3}{2}\phi + \mathcal{A}\right)\Pi^{\text{tot}} - (\mu^{\text{tot}} + p^{\text{tot}})\mathcal{A},$$
$$\hat{\mathcal{A}} = -(A + \phi)\mathcal{A} + \frac{1}{2}(\mu^{\text{tot}} + 3p^{\text{tot}}),$$

$$K = \frac{1}{3}\mu^{\text{tot}} - \mathcal{E} - \frac{1}{2}\Pi^{\text{tot}} + \frac{1}{4}\phi^2.$$

$$T_{ab}^{\text{tot}} = T_{ab}^{\text{eff}}$$

$$\mu^{\text{tot}} = T_{ab}^{\text{eff}} u^a u^b = \frac{\mu^m}{f'} + \mu^R,$$
$$p^{\text{tot}} = \frac{1}{3}T_{ab}^{\text{eff}}(e^a e^b + 2N^{ab}) = \frac{p^m}{f'} + p^R,$$
$$\Pi^{\text{tot}} = \frac{2}{3}T_{ab}^{\text{eff}}(e^a e^b - N^{ab}) = \frac{\Pi_{ab}^m}{f'} + \Pi_{ab}^R,$$
$$Q^{\text{tot}} = -\frac{1}{2}T_{bc}^{\text{eff}} u^c e^b = -\frac{Q^m}{f'} + Q^R,$$

$$\mu^R = \frac{1}{f'}\left(\frac{1}{2}(Rf' - f) + f''\hat{X} + f''X\phi + f'''X^2\right)$$
$$p^R = \frac{1}{f'}\left(\frac{1}{2}(f - Rf') - \frac{2}{3}f''\hat{X} - \frac{2}{3}f''X\phi - \frac{2}{3}f'''X^2 - \mathcal{A}f''X\right)$$
$$\Pi^R = \frac{1}{f'}\left(\frac{2}{3}f''\hat{X} + \frac{2}{3}f'''X^2 - \frac{1}{3}f''X\phi\right)$$
$$Q^R = -\frac{1}{f'}(f'''\dot{R}X + f''(\dot{X} - \mathcal{A}\dot{R})) = 0$$

$$Rf' - 2f = 3p^m - \mu^m - 3f''\hat{X} - 3f''X\phi + -3f'''X^2 - 3\mathcal{A}f''X$$

$$\hat{X} = \frac{p^m}{f''} - \frac{1}{3}\frac{\mu^m}{f''} - \frac{1}{3}\frac{f'}{f''}R + \frac{2}{3}\frac{f}{f''} - \frac{f'''}{f''}X^2 - X(\phi + \mathcal{A}).$$

$$\hat{X} = \phi X_{,\rho}.$$

$$\rho = 2\ln\left(\frac{r}{r_0}\right)$$

$$\Xi = \frac{\phi,\rho}{\phi}, Y = \frac{\mathcal{A}}{\phi}$$
$$\frac{X}{\phi} \equiv \mathbb{X}, \mathcal{K} = \frac{K}{\phi^2}, E = \frac{\varepsilon}{\phi^2}$$
$$\tilde{\mathbb{M}}^m = \frac{\tilde{\mu}^m}{\phi^2}, \tilde{P}^m = \frac{\tilde{p}^m}{\phi^2}, \tilde{\mathbb{P}}^m = \frac{\tilde{\Pi}^m}{\phi^2}$$
$$\mathbb{M}^R = \frac{\mu^R}{\phi^2}, P^R = \frac{p^R}{\phi^2}, \mathbb{P}^R = \frac{\Pi^R}{\phi^2}$$

$$\mathbb{X}_{,\rho} + \mathbb{X}\Xi = \frac{P^m}{f''} - \frac{\mathbb{M}^m}{3f''} - \frac{f'}{3f''}\frac{R}{\phi^2} + \frac{2}{3}\frac{f}{f''\phi^2} - \frac{f'''}{f''}\mathbb{X} - \mathbb{X}(1 + Y),$$
$$P_{,\rho}^{\text{tot}} + \mathbb{P}_{,\rho}^{\text{tot}} = -Y(\mathbb{M}^{\text{tot}} + P^{\text{tot}}) - \mathbb{P}^{\text{tot}}\left(2\Xi + Y + \frac{3}{2}\right) - 2\Xi P^{\text{tot}},$$
$$Y_{,\rho} = -Y(\Xi + Y + 1) + \frac{1}{2}(\tilde{\mathbb{M}}^m + \mathbb{M}^R) + \frac{3}{2}(\tilde{P}^m + P^R),$$
$$\mathcal{K}_{,\rho} = -\mathcal{K}(1 + 2\Xi),$$

$$1+4Y-4\mathcal{K}-4(\tilde{P}^m+P^R)-4(\tilde{\mathbb{P}}^m+\mathbb{P}^R)=0$$
$$1+2\Xi-2Y+2(\tilde{\mathbb{M}}^m+\mathbb{M}^R)+2(\tilde{P}^m+P^R)+2(\tilde{\mathbb{P}}^m+\mathbb{P}^R)=0$$
$$2(\tilde{\mathbb{M}}^m+\mathbb{M}^R)-6Y-6E+6(\tilde{P}^m+P^R)+3(\tilde{\mathbb{P}}^m+\mathbb{P}^R)=0$$

$$ds^2=-k_1(\rho)dt^2+k_2(\rho)d\rho^2+k_3(\rho)d\Omega^2$$

$$k_3(\rho)=K_0e^\rho$$
$$d\Omega^2=d\theta^2+\sin^2\theta d\phi^2$$

$$\phi=\frac{1}{\sqrt{k_2}},Y=\frac{k_{1,\rho}}{2k_1}$$
$$\Xi=-\frac{k_{2,\rho}}{k_2},\mathcal{A}=\frac{k_{1,\rho}}{2k_1\sqrt{k_2}}$$
$$\mathcal{K}=\frac{k_2}{K_0e^\rho}$$

$$k_1(\rho)=k_1(r),k_2(\rho)=\frac{r^2}{4}k_2(r),r^2(\rho)=K_0e^\rho.$$

$$p_r=p+\Pi,p_\perp=p-\frac{1}{2}\Pi$$

$$\mu^m\geq 0,p_r^m\geq 0,p_\perp^m\geq 0.$$

$$\mu^m+p_r^m\geq 0$$

$$0\leq c_{m,r}^2=\frac{\partial p_{,r}^m}{\partial\mu^m}\leq 1,0\leq c_{m,\perp}^2=\frac{\partial p_{,\perp}^m}{\partial\mu^m}\leq 1$$

$$[\gamma_{ab}]_-^+=0$$
$$[K_{ab}]_-^+-\gamma_{ab}[K]_-^+=-S_{ab}$$

$$\{\chi\}=\frac{1}{2}(\chi^++\chi^-)$$

$$S_{ab}\{K^{ab}\}+[T_{ab}e^ae^b]_-^+=0$$

$$[p_r]_-^+=0$$

$$[\gamma_{ab}]_-^+=0$$
$$[K]_-^+=0$$
$$[R]_-^+=0$$
$$f'(R)[K_{ab}^*]_-^+=-S_{ab}^*$$
$$3f''(R)[e^a\nabla_aR]_-^+=S$$

$$K_{ab}^*=K_{ab}-\frac{1}{3}\gamma_{ab}K,$$
$$S_{ab}^*=S_{ab}-\frac{1}{3}\gamma_{ab}S.$$

$$S_{ab}\{K^{ab}\}+[T_{ab}^{\text{tot}}e^ae^b]_-^+=0$$

$$[T_{ab}^{\text{eff}}e^ae^b]_-^+=[p_r^{\text{eff}}]_-^+=0$$

$$\left[\frac{p_r^m}{f'}+p_r^R\right]_-^+=\left[\frac{p_r^m}{f'}\right]_-^++[p_r^R]_-^+=0$$

$$[p_r^R]_-^+=[p^R+\Pi^R]_-^+$$
$$=\left[\frac{f}{2f'}\right]_-^+-\left[\frac{f''}{f'}\right]_-^+\{X\}\{\phi\}-\left[\frac{f''}{f'}\right]_-^+\{X\}\{\mathcal{A}\}$$

$$[a+b]^{\pm}_{-} = [a]^{\pm}_{-} + [b]^{\pm}_{-}$$
$$[a\cdot b]^{\pm}_{-} = \{a\}[b]^{\pm}_{-} + \{b\}[a]^{\pm}_{-}$$
$$= \frac{1}{2}(a^{+}+a^{-})(b^{+}-b^{-}) + \frac{1}{2}(b^{+}+b^{-})(a^{+}-a^{-})$$

$$[p_r^m]^{\pm}_{-} = [p^m + \Pi^m]^{\pm}_{-} = 0.$$

$$R = 3p^{\text{tot}} - \mu^{\text{tot}}$$

$$0 = [R]^{\pm}_{-} = \left[\frac{3p^m - \mu^m}{f'} - \frac{f''}{f'}\hat{X}\right]^{+}_{-}$$
$$= [3p^m - \mu^m]^{\pm}_{-} - \frac{f''}{f'}[\hat{X}]^{\pm}_{-}$$

$$K_{ab} = \gamma_a^c \gamma_b^d \nabla_c e_d$$
$$= (N_a^c + u_a u^c)(N_b^d + u_b u^d)\nabla_c e_d$$

$$[K_{ab}]^{\pm}_{-} = \left[\frac{1}{2}\phi N_{ab} - u_a u_b \mathcal{A}\right]^{+}_{-}$$

$$S_{ab} = (N_{ab} + u_a u_b) f''[X]^{\pm}_{-} - f'[K_{ab}]^{\pm}_{-}$$
$$= (f'[\mathcal{A}]^{\pm}_{-} + f''[X]^{\pm}_{-}) u_a u_b + \left(f''[X]^{\pm}_{-} - \frac{f'}{2}[\phi]^{\pm}_{-}\right) N_{ab}$$

$$\mu^S = S_{ab} u^a u^b$$
$$p_\perp^S = \frac{1}{2} S_{ab} N^{ab}$$

$$\bar{S}_{ab} + \bar{\varsigma}_{ab} = S_{ab} + \varsigma_{ab} + 2\varsigma_{(a} e_{b)} + \varsigma e_a e_b + \bar{\varsigma}_{ab}$$

$$\varsigma_{ab} = f''\{K_{ab}\}[R]^{\pm}_{-}$$
$$\varsigma_a = f''(N_a^b + u^b u_a)\nabla_b [R]^{\pm}_{-}$$
$$\varsigma = f''\{K\}[R]^{\pm}_{-}$$

$$\bar{\zeta}_{ab} = f''\nabla_\rho [[R]^{\pm}_{-} \gamma_{ab} e^\rho \delta] = f'' \Delta_{ab}$$

$$\bar{S}_{ab} = \bar{\mu}^S u_a u_b + \bar{p}_r^S e_a e_b + \bar{p}_\perp^S N_{ab} + 2\bar{Q}^S u_{(a} e_{b)} + \bar{Q}^S_{(a} e_{b)},$$

$$\bar{\mu}^S = f'[\mathcal{A}]^{\pm}_{-} + f''[X]^{\pm}_{-} - f''\{\mathcal{A}\}[R]^{\pm}_{-}$$
$$\bar{p}_r^S = f''\{K\}[R]^{\pm}_{-}$$
$$\bar{p}_\perp^S = -\frac{1}{2} f'[\phi]^{\pm}_{-} + f''[X]^{\pm}_{-} + f''\{\phi\}[R]^{\pm}_{-}$$
$$\bar{Q}^S = f''(u^b \nabla_b [R]^{\pm}_{-}) u_a$$
$$\bar{Q}_a^S = f'' \delta_a [R]^{\pm}_{-}$$

$$\bar{\varsigma}_{ab} = f''\left(\Delta_u u_a u_b + \frac{1}{2}\Delta_N N_{ab}\right)$$

$$R = \frac{\phi^2\big(\mathcal{K}(4\mathcal{K} - 4Y_{,\rho} - 2Y(2Y+1) - 1) + 2(Y+1)\mathcal{K}_{,\rho}\big)}{2\mathcal{K}}$$
$$\widetilde{\mathbb{M}}^m = -\frac{-2\mathcal{K}_{,\rho} - 4\mathcal{K}^2 + \mathcal{K} + 4\mathcal{K}\mathbb{M}_R}{4\mathcal{K}}$$
$$\tilde{p}^m = -\frac{2\mathcal{K}_{,\rho} + 4\mathcal{K}^2 - \mathcal{K} + 12\mathcal{K}P_R - 8\mathcal{K}Y_{,\rho} + 4Y\mathcal{K}_{,\rho} - 8\mathcal{K}Y^2 - 4\mathcal{K}Y}{12\mathcal{K}}$$
$$\widetilde{\mathbb{P}}^m = -\frac{6\mathbb{P}_R\mathcal{K} - \mathcal{K}_{,\rho} + 4\mathcal{K}^2 - \mathcal{K} + 4\mathcal{K}Y_{,\rho} - 2Y\mathcal{K}_{,\rho} + 4\mathcal{K}Y^2 - 4\mathcal{K}Y}{6\mathcal{K}}$$

$$\mathbb{M}^R = \frac{R}{2\phi^2} + \frac{f''}{f'}\left(R_{,\rho\rho} + R_{,\rho} + \Xi R_{,\rho}\right) - \frac{1}{2\phi^2}\frac{f}{f'}$$
$$P^R = -\frac{R}{2\phi^2} - \frac{2}{3}\frac{f''}{f'}\left(R_{,\rho\rho} + R_{,\rho} + \Xi R_{,\rho} + \frac{3}{2}YR_{,\rho}\right) + \frac{1}{2\phi^2}\frac{f}{f'}$$
$$\mathbb{P}^R = \frac{2}{3}\frac{f''}{f'}\left(R_{,\rho\rho} - \frac{1}{2}R_{,\rho} + \Xi R_{,\rho}\right)$$

$$k_1(r) = a_0(c_1 + z)^2, k_2(r) = \frac{\mathcal{R}^2(A^2 + 2r^2)}{(\mathcal{R}^2 - r^2)(A^2 + r^2)}$$

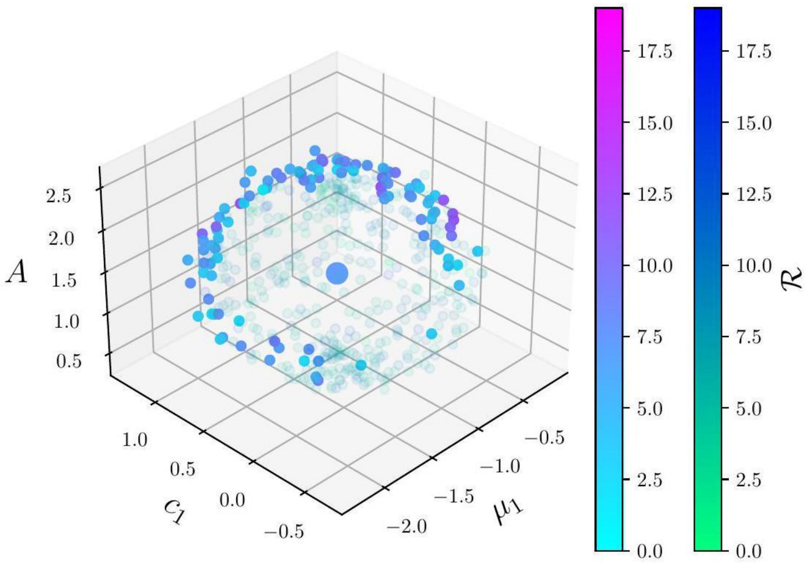
$$z = \sqrt{3 - \mu_1 r^2}$$

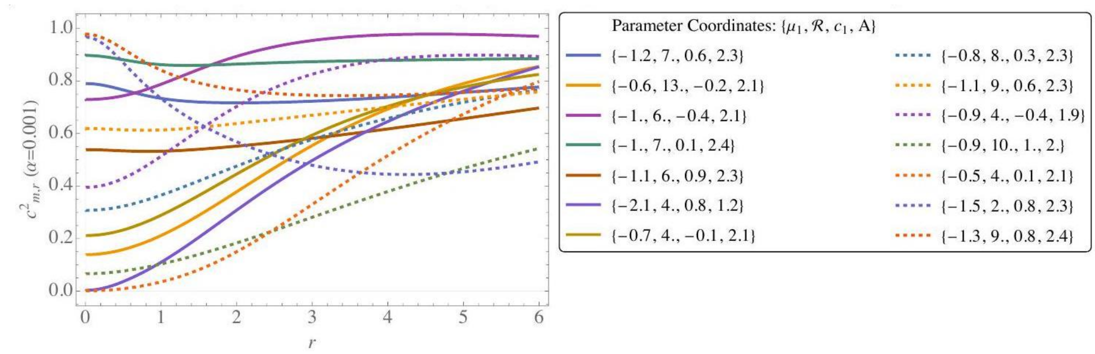
$$Y_{\mathrm{IS}} = \frac{\mu_1 e^\rho}{2\mu_1 e^\rho - 2c_1\sqrt{3 - \mu_1 e^\rho} - 6}$$
$$\mathcal{K}_{\mathrm{TIV}} = \frac{-\mathcal{R}^2(A^2 + 2e^\rho)}{4(A^2 + e^\rho)(e^\rho - \mathcal{R}^2)}$$
$$\phi = \frac{e^{-\frac{\rho}{2}}}{\sqrt{\mathcal{K}_{\mathrm{TIV}}}}$$

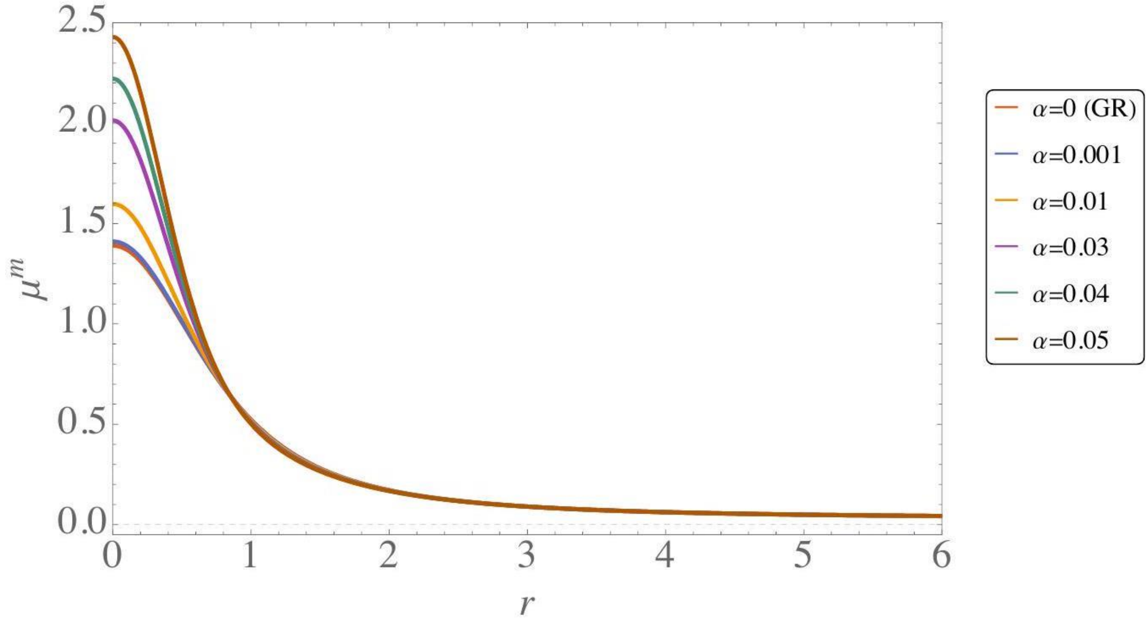
$$\bar{\mu}^{\mathcal{S}} = 2\alpha[X]^{\pm}_{-} + (1 + 2\alpha R)[\mathcal{A}]^{\pm}_{-} - 2\alpha\{\mathcal{A}\}[R]^{\pm}_{-}$$
$$\bar{p}^{\mathcal{S}}_r = 2\alpha(\{\phi\} + \{\mathcal{A}\})[R]^{\pm}_{-}$$
$$\bar{p}^{\mathcal{S}}_\perp = 2\alpha[X]^{\pm}_{-} - \frac{1}{2}(1 + 2\alpha R)[\phi]^{\pm}_{-} + f''\{\phi\}[R]^{\pm}_{-}$$

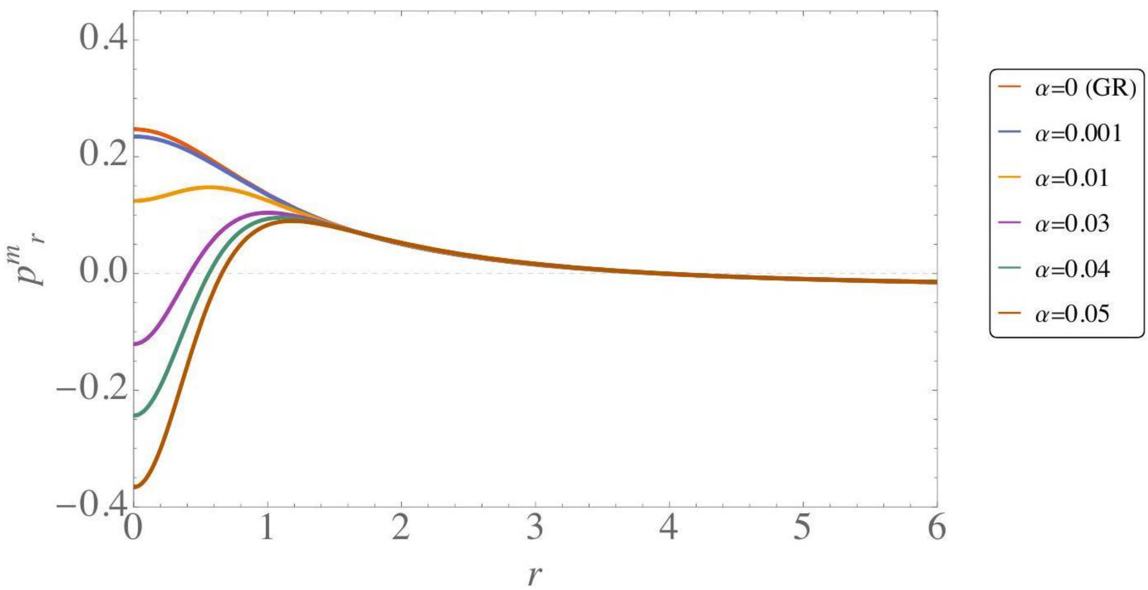
$$k_1(r) = 1 + \mathfrak{D}_1 r^2 + \mathfrak{D}_2 r^4, k_2(r) = \frac{1 + \mathfrak{D}_3 r^2}{1 + \mathfrak{D}_4 r^2 + \mathfrak{D}_5 r^4}$$

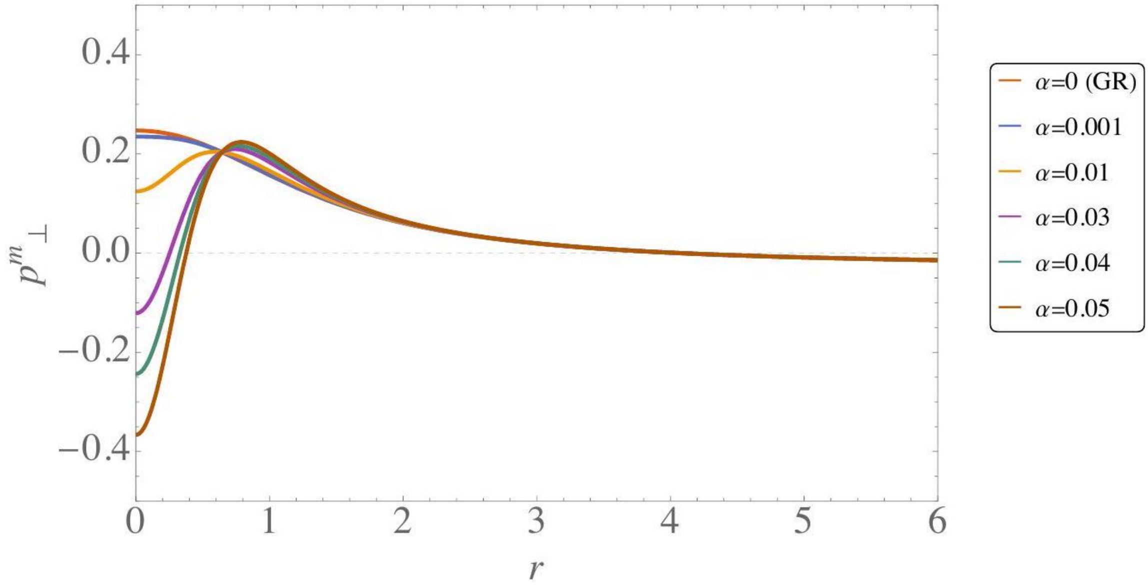
Figuras. Interacción de una partícula hadrónica blanca u oscura en un campo cuántico – relativista así como su morfología elemental (energía, masa, radio, presión, termodinámica, coordenadas vectoriales e isométricas, etc.)



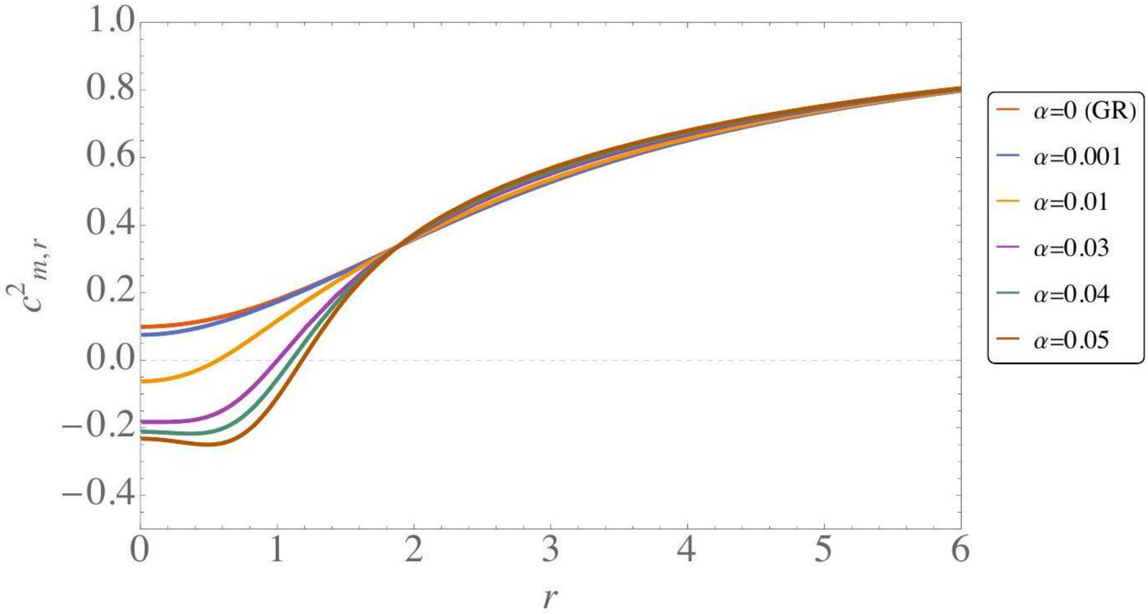

Parameter Coordinates: {μ_1, $\mathcal{R}$, c_1, A}
{−1.2, 7., 0.6, 2.3}
{−0.6, 13., −0.2, 2.1}
{−1., 6., −0.4, 2.1}
{−1., 7., 0.1, 2.4}
{−1.1, 6., 0.9, 2.3}
{−2.1, 4., 0.8, 1.2}
{−0.7, 4., −0.1, 2.1}
{−0.8, 8., 0.3, 2.3}
{−1.1, 9., 0.6, 2.3}
{−0.9, 4., −0.4, 1.9}
{−0.9, 10., 1., 2.}
{−0.5, 4., 0.1, 2.1}
{−1.5, 2., 0.8, 2.3}
{−1.3, 9., 0.8, 2.4}
$c^2_{m,r}$ (α=0.001)
r


μ^m
r
α=0 (GR)
α=0.001
α=0.01
α=0.03
α=0.04
α=0.05

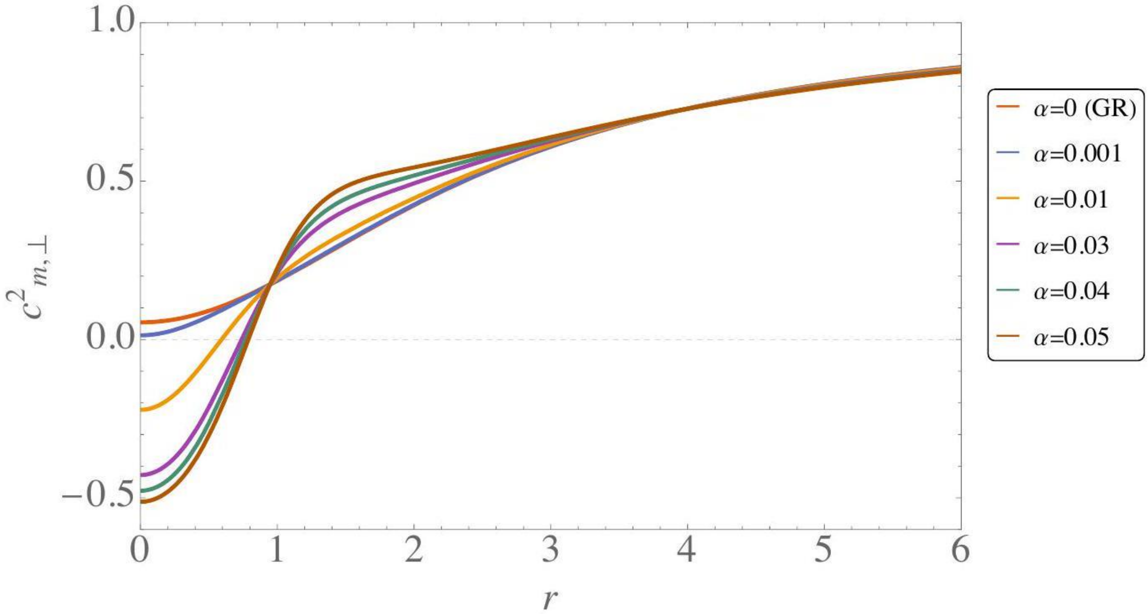

$p^m{}_r$
r
α=0 (GR)
α=0.001
α=0.01
α=0.03
α=0.04
α=0.05



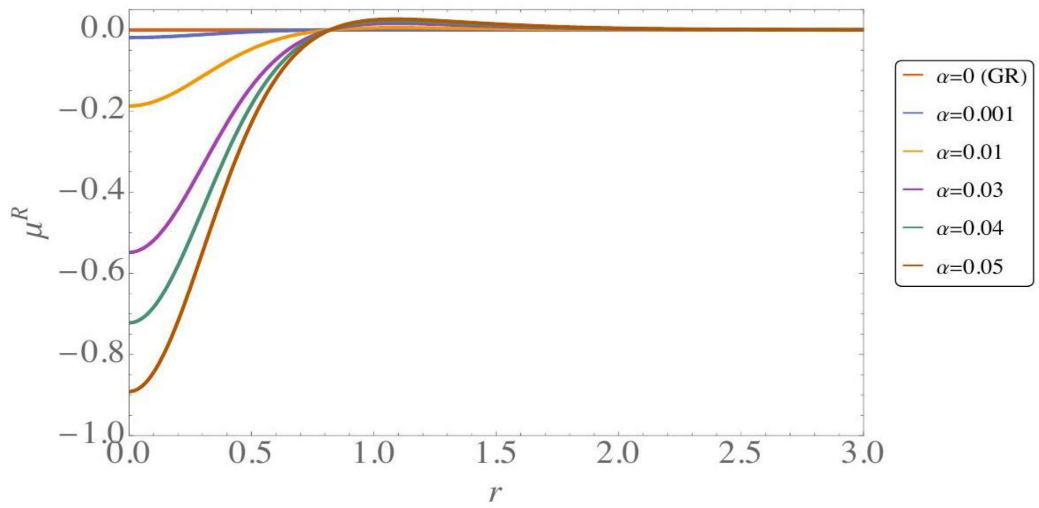
$p^m{}_\perp$
0.4
0.2
0.0
−0.2
−0.4
0
1
2
3
4
5
6
r
α=0 (GR)
α=0.001
α=0.01
α=0.03
α=0.04
α=0.05

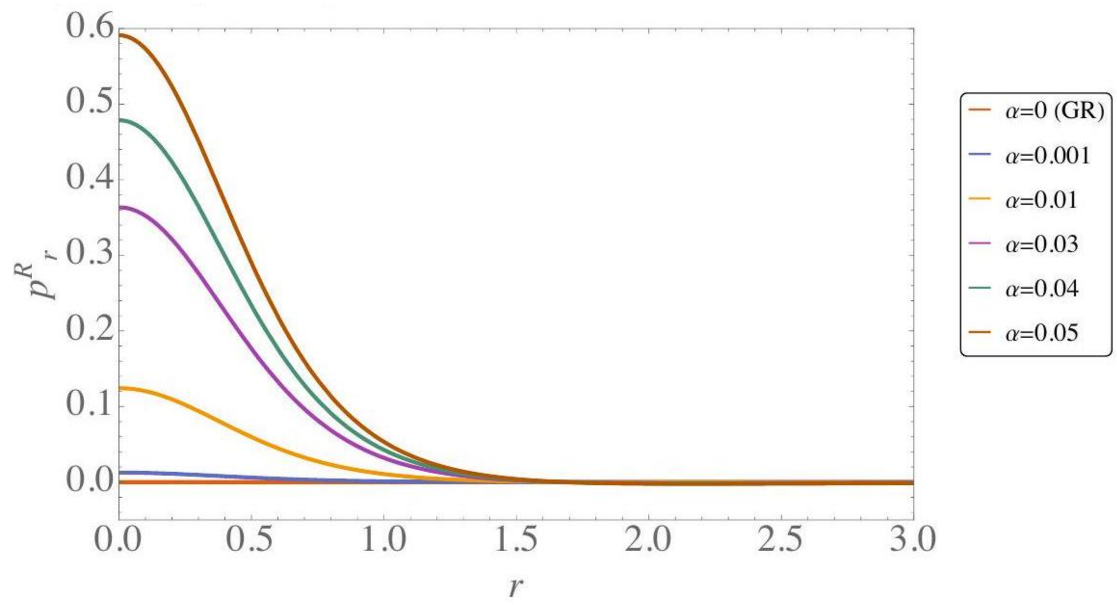


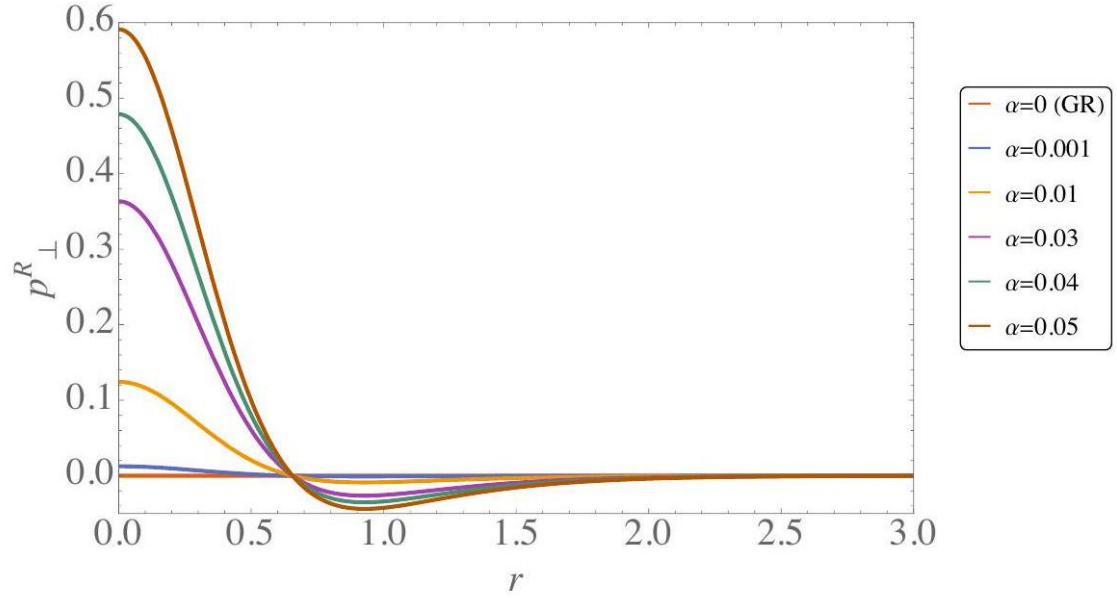
$c^2{}_{m,r}$
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0
1
2
3
4
5
6
r
α=0 (GR)
α=0.001
α=0.01
α=0.03
α=0.04
α=0.05

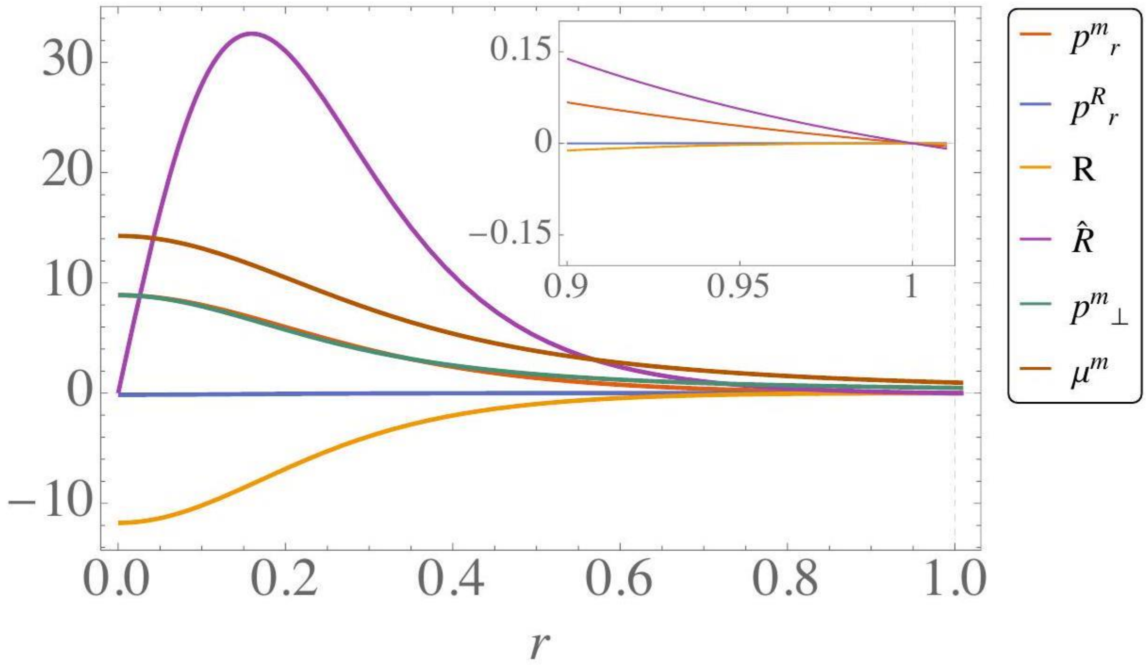


$c^2{}_{m,\perp}$
1.0
0.5
0.0
−0.5
0
1
2
3
4
5
6
r
α=0 (GR)
α=0.001
α=0.01
α=0.03
α=0.04
α=0.05

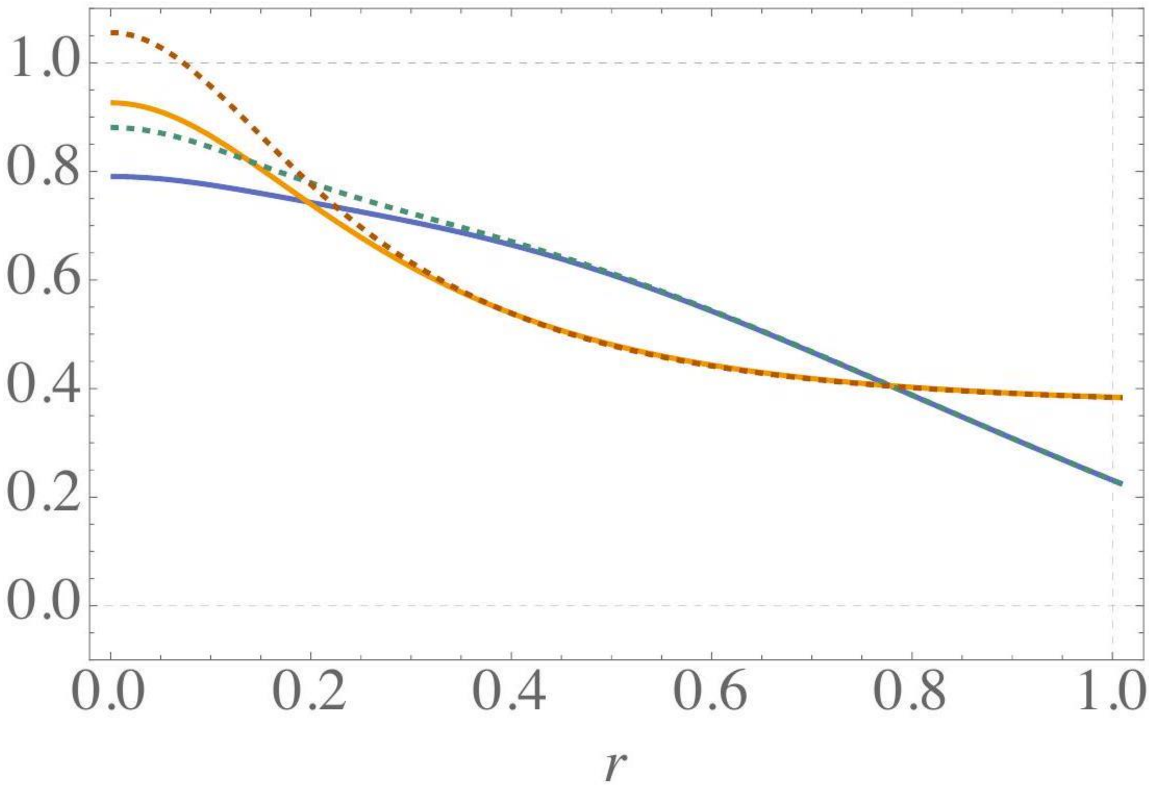

α=0 (GR)
α=0.001
α=0.01
α=0.03
α=0.04
α=0.05
μ^R
r


α=0 (GR)
α=0.001
α=0.01
α=0.03
α=0.04
α=0.05
p^R_r
r

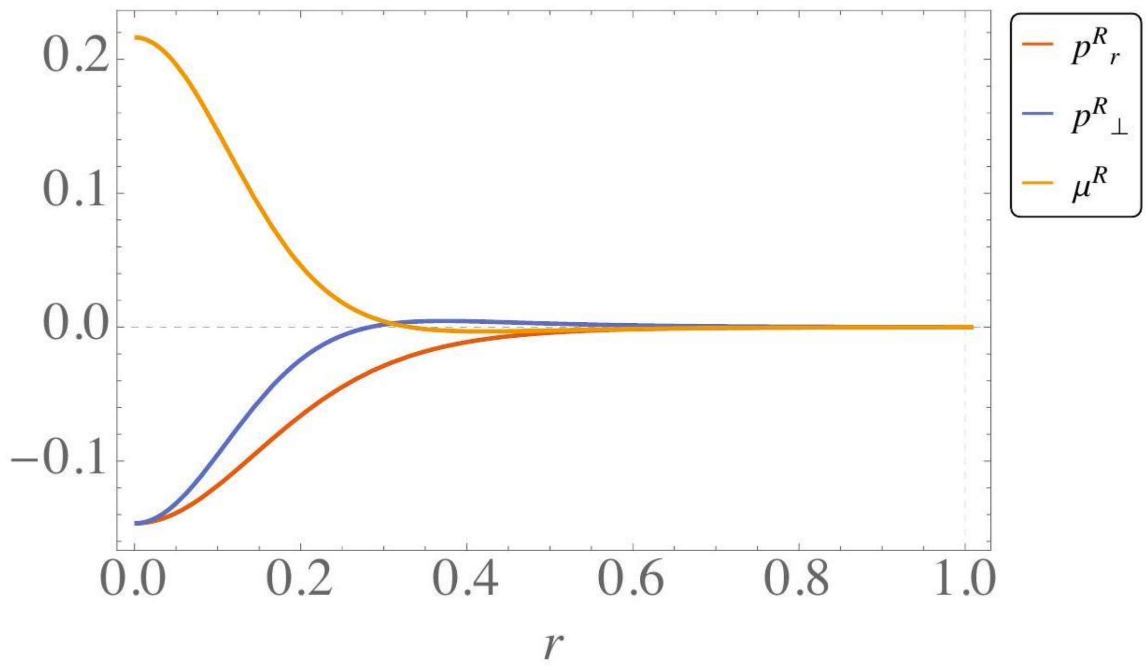

α=0 (GR)
α=0.001
α=0.01
α=0.03
α=0.04
α=0.05
p^R_⊥
r

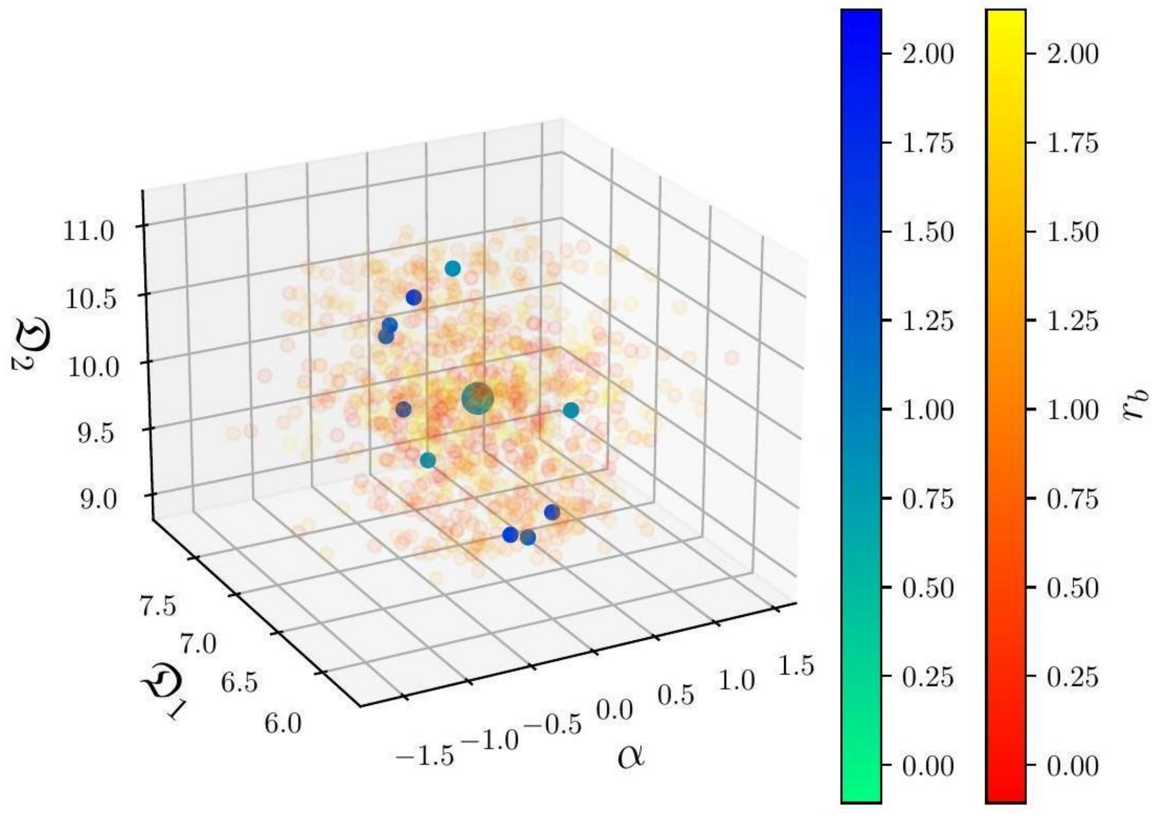


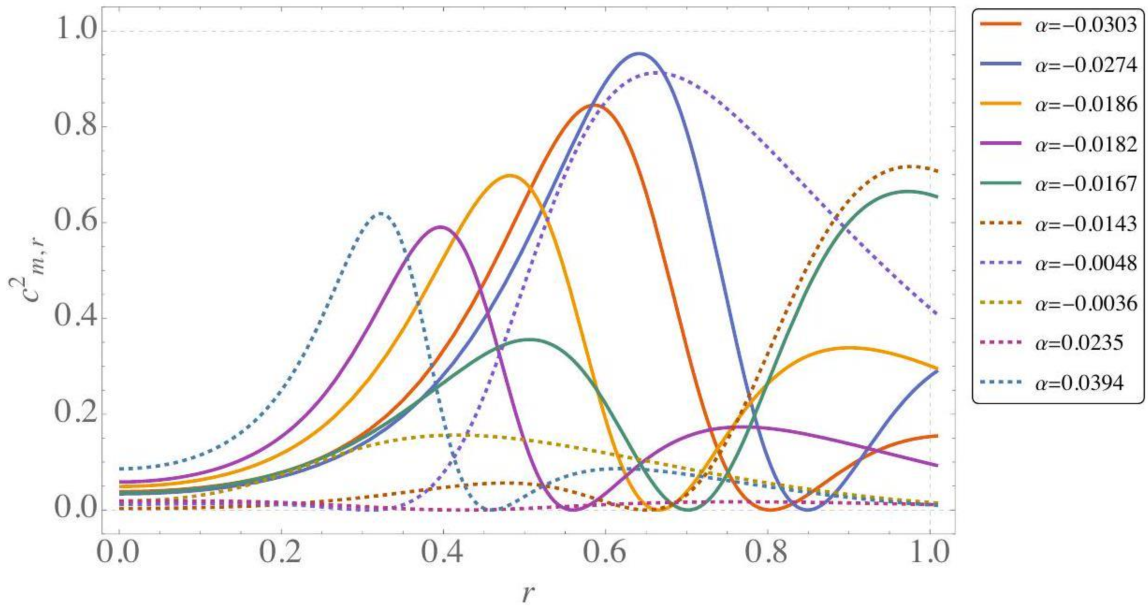
30
20
10
0
−10
0.0
0.2
0.4
0.6
0.8
1.0
r
0.15
0
−0.15
0.9
0.95
1
$p^m{}_r$
$p^R{}_r$
R
$\hat{R}$
$p^m{}_\perp$
μ^m



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
r


0.2
0.1
0.0
−0.1
0.0
0.2
0.4
0.6
0.8
1.0
r
$p^R{}_r$
$p^R{}_\perp$
μ^R


11.0
10.5
10.0
9.5
9.0
$\mathfrak{Q}_2$
7.5
7.0
6.5
6.0
$\mathfrak{Q}_1$
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
α
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
r_b



$$\mu^{\rm tot} = \frac{r k_2'(r) + k_2(r)2 - k_2(r)}{r^2 k_2(r)^2},$$

$$p_r^{\rm tot} = \frac{r k_1'(r) - k_1(r)k_2(r) + k_1(r)}{r^2 k_1(r)k_2(r)},$$

$$p_\perp^{\rm tot} = \frac{k_1''(r)}{2k_1(r)k_2(r)} - \frac{k_1'(r)k_2'(r)}{4k_1(r)k_2(r)^2} - \frac{k_1'(r)^2}{4k_1(r)^2k_2(r)} - \frac{k_2'(r)}{2rk_2(r)^2} + \frac{k_1'(r)}{2rk_1(r)k_2(r)}.$$

$$[\phi]_-^+ \equiv \phi^{\mathcal{S}} = \frac{\sqrt{A^2 + r_b^2}\sqrt{\mathcal{R}^2 - r_b^2}}{\mathcal{R}\sqrt{A^2 + 2r_b^2}}$$

$$[\mathcal{A}]_-^+ = \mathcal{A}^{\mathcal{S}} = \frac{\mu_1 r_b^2\sqrt{A^2 + r_b^2}\sqrt{\mathcal{R}^2 - r_b^2}}{\mathcal{R}\sqrt{A^2 + 2r_b^2}\left(-2c_1\sqrt{3 - \mu_1 r_b^2} + 2\mu_1 r_b^2 - 6\right)}$$

$$[X]_-^+ = X^{\mathcal{S}} = -\frac{2r_b\sqrt{\mathcal{R}^2 - r_b^2}\sqrt{A^2 + r_b^2}\left(-2x_{11} + 2x_{12} - c_1^2\sqrt{3 - r_b^2\mu_1}x_{13} + c_1 x_{10}\right)}{\mathcal{R}^3(A^2 + 2r_b^2)^{7/2}(3 - r_b^2\mu_1)^{3/2}\left(\mu_1 r_b^2 - c_1\sqrt{3 - r_b^2\mu_1} - 3\right)^3}$$

$$x_1 = 2\mathcal{R}^2A^6 + 12\mathcal{R}^2r_b^2A^4 + (4\mathcal{R}^2 - 6A^2)r_b^6 + (6A^2\mathcal{R}^2 - 9A^4)r_b^4,$$
$$x_2 = 8r_b^8 - 2(A^2 - 2\mathcal{R}^2)r_b^6 + (6A^2\mathcal{R}^2 - 15A^4)r_b^4 + 2(A^6 + 13\mathcal{R}^2A^4)r_b^2 + 4A^6\mathcal{R}^2,$$
$$x_3 = 16r_b^6 + 6(3A^2 - 2\mathcal{R}^2)r_b^4 - (A^4 + 26\mathcal{R}^2A^2)r_b^2 + 4A^4(A^2 + \mathcal{R}^2),$$
$$x_4 = 32r_b^8 + 2(A^2 - 26\mathcal{R}^2)r_b^6 - (71A^4 + 130\mathcal{R}^2A^2)r_b^4 + (8A^6 + 38\mathcal{R}^2A^4)r_b^2 + 5A^6\mathcal{R}^2,$$
$$x_5 = 16r_b^6 + 6(A^2 - 6\mathcal{R}^2)r_b^4 - (31A^4 + 86\mathcal{R}^2A^2)r_b^2 + 4A^4(A^2 + \mathcal{R}^2),$$
$$x_6 = 12r_b^8 - 6A^2r_b^6 - 2(16A^4 + 5\mathcal{R}^2A^2)r_b^4 + 3(A^6 + 13\mathcal{R}^2A^4)r_b^2 + 6A^6\mathcal{R}^2,$$
$$x_7 = 60r_b^8 + 4(A^2 - 5\mathcal{R}^2)r_b^6 - (103A^4 + 68\mathcal{R}^2A^2)r_b^4 + 3(5A^6 + 43\mathcal{R}^2A^4)r_b^2 + 19A^6\mathcal{R}^2,$$
$$x_8 = 112r_b^8 + (70A^2 - 92\mathcal{R}^2)r_b^6 - 3(35A^4 + 74\mathcal{R}^2A^2)r_b^4$$
$$+2(14A^6 + 53\mathcal{R}^2A^4)r_b^2 + 13A^6\mathcal{R}^2,$$
$$x_9 = 4r_b^6 + 3(A^2 - 2\mathcal{R}^2)r_b^4 - 2(2A^4 + 7\mathcal{R}^2A^2)r_b^2 + A^4(A^2 + \mathcal{R}^2),$$
$$x_{10} = r_b^4x_6\mu_1^4 - 3r_b^2x_7\mu_1^3 + 9x_8\mu_1^2 - 432x_9\mu_1 - 486(A^2 + 2\mathcal{R}^2)(5A^2 + 2r_b^2),$$
$$x_{11} = \left(r_b^2x_1\mu_1^3 - 3x_2\mu_1^2 + 9x_3\mu_1 + 27(A^2 + 2\mathcal{R}^2)(5A^2 + 2r_b^2)\right)(3 - r_b^2\mu_1)^{3/2},$$
$$x_{12} = (A^2 + 2\mathcal{R}^2)c_1^3(5A^2 + 2r_b^2)(r_b^2\mu_1 - 3)^3,$$
$$x_{13} = r_b^2(12r_b^8 + 2(A^2 - 8\mathcal{R}^2)r_b^6 - 6(4A^4 + 7\mathcal{R}^2A^2)r_b^4 + 3(A^6 + 5\mathcal{R}^2A^4)r_b^2 + 2A^6\mathcal{R}^2)\mu_1^3$$
$$-3x_4\mu_1^2 + 18x_5\mu_1 + 162(A^2 + 2\mathcal{R}^2)(5A^2 + 2r_b^2).$$

$$R(r)=\frac{1}{B}\Big(2\big(A^4(3c_1^2z^3+c_1a_1+za_2)+A^2(c_1^2z^3(7r^2+3\mathcal{R}^2)+c_1a_3+za_4)$$
$$+2r^2(c_1^2z^3(3r^2+\mathcal{R}^2)+3c_1a_5+za_6)\big)\Big)$$

$$a_1=9\mu_1^2r^4-2\mu_1r^2(\mu_1\mathcal{R}^2+24)+9(\mu_1\mathcal{R}^2+6)$$
$$a_2=6\mu_1^2r^4-2\mu_1r^2(\mu_1\mathcal{R}^2+15)+9(\mu_1\mathcal{R}^2+3)$$
$$a_3=22\mu_1^2r^6+\mu_1r^4(\mu_1\mathcal{R}^2-117)-6r^2(2\mu_1\mathcal{R}^2-21)+54\mathcal{R}^2$$
$$a_4=15\mu_1^2r^6-\mu_1r^4(2\mu_1\mathcal{R}^2+75)+r^2(6\mu_1\mathcal{R}^2+63)+27\mathcal{R}^2$$
$$a_5=3\mu_1^2r^6-16\mu_1r^4+r^2(18-\mu_1\mathcal{R}^2)+6\mathcal{R}^2$$
$$a_6=6\mu_1^2r^6-\mu_1r^4(\mu_1\mathcal{R}^2+30)+3r^2(\mu_1\mathcal{R}^2+9)+9\mathcal{R}^2$$
$$B=\mathcal{R}^2(A^2+2r^2)^2z(c_1z-\mu_1r^2+3)^2$$

$$\mu^{\mathrm{tot}}=\frac{3A^4+A^2(7r^2+3\mathcal{R}^2)+2r^2(3r^2+\mathcal{R}^2)}{\mathcal{R}^2(A^2+2r^2)^2}$$

$$p_r^{\mathrm{tot}}=\frac{4(A^2+r^2)(\mathcal{R}^2-r^2)}{r^2\mathcal{R}^2(A^2+2r^2)}\left(-\frac{\mathcal{R}^2(A^2+2r^2)}{4(A^2+r^2)(\mathcal{R}^2-r^2)}+\frac{\mu_1r^2}{-2c_1z+2\mu_1r^2-6}+\frac{1}{4}\right)$$

$$p^{\mathrm{tot}}=\frac{1}{3B}\Big(-\big(A^4(3c_1^2z^3+2c_1b_1+zb_2)\big)-A^2(c_1^2z^3(7r^2+3\mathcal{R}^2)+2c_1b_3+zb_4)$$
$$-2r^2(c_1^2z^3(3r^2+\mathcal{R}^2)+2c_1b_5+3zb_6)\Big),$$

$$b_1=\mu_1(6\mu_1r^4-2r^2(\mu_1\mathcal{R}^2+15)+9\mathcal{R}^2)+27,$$
$$b_2=\mu_1r^2(9\mu_1r^2-4\mu_1\mathcal{R}^2-42)+18\mu_1\mathcal{R}^2+27,$$
$$b_3=r^2(\mu_1r^2(15\mu_1r^2-2\mu_1\mathcal{R}^2-75)+6\mu_1\mathcal{R}^2+63)+27\mathcal{R}^2,$$
$$b_4=r^2(\mu_1r^2(23\mu_1r^2-7\mu_1\mathcal{R}^2-108)+30\mu_1\mathcal{R}^2+63)+27\mathcal{R}^2,$$
$$b_5=r^2(\mu_1r^2(6\mu_1r^2-\mu_1\mathcal{R}^2-30)+3\mu_1\mathcal{R}^2+27)+9\mathcal{R}^2,$$
$$b_6=r^2(\mu_1r^2(3\mu_1r^2-\mu_1\mathcal{R}^2-14)+4\mu_1\mathcal{R}^2+9)+3\mathcal{R}^2.$$

$$p_\perp^{\mathrm{tot}}=\frac{1}{B}\Big(A^4(-c_1^2z^3+c_1d_1+zd_2)-A^2(c_1^2z^3(2r^2+\mathcal{R}^2)+3c_1d_3+zd_4)$$
$$-2r^4(c_1^2z^3-c_1d_1+zd_5)\Big)$$

$$d_1=\mu_1r^2(-4\mu_1r^2+\mu_1\mathcal{R}^2+21)-6\mu_1\mathcal{R}^2-18$$
$$d_2=\mu_1r^2(-3\mu_1r^2+\mu_1\mathcal{R}^2+15)-6\mu_1\mathcal{R}^2-9$$
$$d_3=r^2(\mu_1(3\mu_1r^4-16r^2+\mathcal{R}^2)+12)+6\mathcal{R}^2$$
$$d_4=r^2(\mu_1r^2(7\mu_1r^2-\mu_1\mathcal{R}^2-36)+9\mu_1\mathcal{R}^2+18)+9\mathcal{R}^2$$
$$d_5=\mu_1r^2(3\mu_1r^2-\mu_1\mathcal{R}^2-15)+6\mu_1\mathcal{R}^2+9$$

$$\mathfrak{D}_3=\frac{3\left(12\mathfrak{D}_1^3r_b^4+\mathfrak{D}_1^2r_b^2(49\mathfrak{D}_2r_b^4+13)+10\mathfrak{D}_2r_b^2\left(3\mathfrak{D}_2^2r_b^8+1\right)+\mathfrak{D}_1(\mathfrak{D}_2r_b^4(61\mathfrak{D}_2r_b^4+30)+5)\right)}{2\mathfrak{D}_2r_b^4(\mathfrak{D}_2r_b^4(11\mathfrak{D}_2r_b^4+36)-11)+\mathfrak{D}_1r_b^2(\mathfrak{D}_2r_b^4(37\mathfrak{D}_2r_b^4+14)-3)+3\mathfrak{D}_1^2(\mathfrak{D}_2r_b^8+r_b^4)},$$
$$\mathfrak{D}_4=\frac{1}{\mathfrak{d}_1}\Big(36\mathfrak{D}_1^4r_b^6+3\mathfrak{D}_1^3r_b^4(69\mathfrak{D}_2r_b^4+25)+4\mathfrak{D}_1^2r_b^2(\mathfrak{D}_2r_b^4(71\mathfrak{D}_2r_b^4+80)+18)$$
$$+10\mathfrak{D}_2r_b^2\left(\mathfrak{D}_2r_b^4\left(23-\mathfrak{D}_2r_b^4(3\mathfrak{D}_2r_b^4+31)\right)+3\right)$$
$$+\mathfrak{D}_1(\mathfrak{D}_2r_b^4(\mathfrak{D}_2r_b^4(41\mathfrak{D}_2r_b^4+129)+287)+15)\Big)$$
$$\mathfrak{D}_5=\frac{1}{\mathfrak{d}_1}\Big(16\mathfrak{D}_2^2r_b^4(\mathfrak{D}_2r_b^4(2\mathfrak{D}_2r_b^4+7)-7)+8\mathfrak{D}_1\mathfrak{D}_2r_b^2(\mathfrak{D}_2r_b^4+1)(5\mathfrak{D}_2r_b^4-12)$$
$$-6\mathfrak{D}_1^3\left(5\mathfrak{D}_2r_b^6+r_b^2\right)-4\mathfrak{D}_1^2(\mathfrak{D}_2r_b^4(10\mathfrak{D}_2r_b^4+21)+3)\Big)$$

$$\mathfrak{d}_1=r_b^2\left(5\mathfrak{D}_2r_b^4+3\mathfrak{D}_1r_b^2+1\right)\Big(3\mathfrak{D}_1^2\left(\mathfrak{D}_2r_b^6+r_b^2\right)+\mathfrak{D}_1(\mathfrak{D}_2r_b^4(37\mathfrak{D}_2r_b^4+14)-3)$$
$$+2\mathfrak{D}_2r_b^2(\mathfrak{D}_2r_b^4(11\mathfrak{D}_2r_b^4+36)-11)\Big)$$

$$R(r)=\frac{2\left(-11\mathfrak{D}_2^2\mathfrak{D}_3\mathfrak{D}_5r^{12}+\mathfrak{D}_2\mathfrak{b}_1r^{10}-\mathfrak{b}_2r^8-\mathfrak{b}_3r^6+\mathfrak{b}_4r^4-\mathfrak{b}_5r^2-3(\mathfrak{D}_1-\mathfrak{D}_3+\mathfrak{D}_4)\right)}{(\mathfrak{D}_2r^4+\mathfrak{D}_1r^2+1)^2(\mathfrak{D}_3r^2+1)^2},$$

$$
\begin{aligned}
\mathfrak{b}_1 &= \mathfrak{D}_2\mathfrak{D}_3(\mathfrak{D}_3 - 7\mathfrak{D}_4) - 3(5\mathfrak{D}_2 + 6\mathfrak{D}_1\mathfrak{D}_3)\mathfrak{D}_5 \\
\mathfrak{b}_2 &= \mathfrak{D}_2\mathfrak{D}_3(\mathfrak{D}_2 - 2\mathfrak{D}_1\mathfrak{D}_3) + 11\mathfrak{D}_2(\mathfrak{D}_2 + \mathfrak{D}_1\mathfrak{D}_3)\mathfrak{D}_4 + 25\mathfrak{D}_1\mathfrak{D}_2\mathfrak{D}_5 \\
&\quad +6(\mathfrak{D}_1^2 + 3\mathfrak{D}_2)\mathfrak{D}_3\mathfrak{D}_5 \\
\mathfrak{b}_3 &= 6\mathfrak{D}_2^2 + 2(-\mathfrak{D}_3^2 + 6\mathfrak{D}_4\mathfrak{D}_3 + 9\mathfrak{D}_1\mathfrak{D}_4 + 12\mathfrak{D}_5)\mathfrak{D}_2 \\
&\quad +\mathfrak{D}_1\left(10\mathfrak{D}_3\mathfrak{D}_5 + \mathfrak{D}_1(-\mathfrak{D}_3^2 + 3\mathfrak{D}_4\mathfrak{D}_3 + 9\mathfrak{D}_5)\right) \\
\mathfrak{b}_4 &= 2(\mathfrak{D}_3 - 3\mathfrak{D}_4)\mathfrak{D}_1^2 - (-2\mathfrak{D}_3^2 + 5\mathfrak{D}_4\mathfrak{D}_3 + 9\mathfrak{D}_2 + 15\mathfrak{D}_5)\mathfrak{D}_1 - 2\mathfrak{D}_2(\mathfrak{D}_3 + 9\mathfrak{D}_4) \\
&\quad -3\mathfrak{D}_3\mathfrak{D}_5 \\
\mathfrak{b}_5 &= (2\mathfrak{D}_1^2 + (10\mathfrak{D}_4 - 4\mathfrak{D}_3)\mathfrak{D}_2 + \mathfrak{D}_3(\mathfrak{D}_4 - \mathfrak{D}_3) + 5\mathfrak{D}_5
\end{aligned}
$$

$$
\mu^{\text{tot}} = \frac{-3r^4\mathfrak{D}_3\mathfrak{D}_5 + r^2(\mathfrak{D}_3{}^2 - \mathfrak{D}_3\mathfrak{D}_4 - 5\mathfrak{D}_5) + 3\mathfrak{D}_3 - 3\mathfrak{D}_4}{(r^2\mathfrak{D}_3 + 1)^2}
$$

$$
\begin{aligned}
p_r^{\text{tot}} &= \frac{1}{(r^2\mathfrak{D}_3 + 1)(r^4\mathfrak{D}_2 + r^2\mathfrak{D}_1 + 1)}(5r^6\mathfrak{D}_2\mathfrak{D}_5 + r^4(3\mathfrak{D}_1\mathfrak{D}_5 - \mathfrak{D}_2\mathfrak{D}_3 + 5\mathfrak{D}_2\mathfrak{D}_4) \\
&\quad +r^2(-\mathfrak{D}_1\mathfrak{D}_3 + 3\mathfrak{D}_1\mathfrak{D}_4 + 4\mathfrak{D}_2 + \mathfrak{D}_5) + 2\mathfrak{D}_1 - \mathfrak{D}_3 + \mathfrak{D}_4)
\end{aligned}
$$

$$
\begin{aligned}
p_\perp^{\text{tot}} &= \frac{1}{(r^2\mathfrak{D}_3 + 1)^2(r^4\mathfrak{D}_2 + r^2\mathfrak{D}_1 + 1)^2}(7r^{12}\mathfrak{D}_2{}^2\mathfrak{D}_3\mathfrak{D}_5 + r^{10}\mathfrak{g}_1 + r^8\mathfrak{g}_2 + r^6\mathfrak{g}_3 + r^4\mathfrak{g}_4 \\
&\quad +r^2\mathfrak{g}_5 + 2\mathfrak{D}_1 - \mathfrak{D}_3 + \mathfrak{D}_4)
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{g}_1 &= \mathfrak{D}_2(11\mathfrak{D}_1\mathfrak{D}_3\mathfrak{D}_5 + 4\mathfrak{D}_2\mathfrak{D}_3\mathfrak{D}_4 + 10\mathfrak{D}_2\mathfrak{D}_5) \\
\mathfrak{g}_2 &= 3\mathfrak{D}_3\mathfrak{D}_5(\mathfrak{D}_1^2 + 4\mathfrak{D}_2) + 6\mathfrak{D}_1\mathfrak{D}_2\mathfrak{D}_3\mathfrak{D}_4 + 16\mathfrak{D}_1\mathfrak{D}_2\mathfrak{D}_5 + \mathfrak{D}_2^2(\mathfrak{D}_3 + 7\mathfrak{D}_4) \\
\mathfrak{g}_3 &= \mathfrak{D}_2(\mathfrak{D}_1\mathfrak{D}_3 + 11\mathfrak{D}_1\mathfrak{D}_4 + 8\mathfrak{D}_3\mathfrak{D}_4 + 16\mathfrak{D}_5) + \mathfrak{D}_1\left(\mathfrak{D}_1\mathfrak{D}_3\mathfrak{D}_4 + 5\mathfrak{D}_5(\mathfrak{D}_1 + \mathfrak{D}_3)\right) + \\
\mathfrak{g}_4 &= -\mathfrak{D}_1{}^2(\mathfrak{D}_3 - 3\mathfrak{D}_4) + 2\mathfrak{D}_1(3\mathfrak{D}_2 + \mathfrak{D}_3\mathfrak{D}_4 + 4\mathfrak{D}_5) + 4\mathfrak{D}_2(\mathfrak{D}_3 + 3\mathfrak{D}_4) + \mathfrak{D}_3\mathfrak{D}_5 \\
\mathfrak{g}_5 &= \mathfrak{D}_1{}^2 - \mathfrak{D}_1\mathfrak{D}_3 + 5\mathfrak{D}_1\mathfrak{D}_4 + 8\mathfrak{D}_2 + 2\mathfrak{D}_5
\end{aligned}
$$

$$
p^{\text{tot}} = \frac{19r^{12}\mathfrak{D}_2^2\mathfrak{D}_3\mathfrak{D}_5 + r^{10}\mathfrak{h}_1 + r^8\mathfrak{h}_2 + r^6\mathfrak{h}_3 + r^4\mathfrak{h}_4 + r^2\mathfrak{h}_5 + 6\mathfrak{D}_1 - 3\mathfrak{D}_3 + 3\mathfrak{D}_4}{3(r^2\mathfrak{D}_3 + 1)^2(r^4\mathfrak{D}_2 + r^2\mathfrak{D}_1 + 1)^2}
$$

$$
\begin{aligned}
\mathfrak{h}_1 &= \mathfrak{D}_2\left(5\mathfrak{D}_5(6\mathfrak{D}_1\mathfrak{D}_3 + 5\mathfrak{D}_2) - \mathfrak{D}_2\mathfrak{D}_3(\mathfrak{D}_3 - 13\mathfrak{D}_4)\right), \\
\mathfrak{h}_2 &= \mathfrak{D}_5(9\mathfrak{D}_1{}^2\mathfrak{D}_3 + 40\mathfrak{D}_1\mathfrak{D}_2 + 30\mathfrak{D}_2\mathfrak{D}_3) + \mathfrak{D}_2\left(\mathfrak{D}_2(5\mathfrak{D}_3 + 19\mathfrak{D}_4) - 2\mathfrak{D}_1\mathfrak{D}_3(\mathfrak{D}_3 - 10\mathfrak{D}_4)\right), \\
\mathfrak{h}_3 &= \mathfrak{D}_2(6\mathfrak{D}_1(\mathfrak{D}_3 + 5\mathfrak{D}_4) - 2\mathfrak{D}_3{}^2 + 22\mathfrak{D}_3\mathfrak{D}_4 + 38\mathfrak{D}_5) \\
&\quad +\mathfrak{D}_1(\mathfrak{D}_1(-\mathfrak{D}_3{}^2 + 5\mathfrak{D}_3\mathfrak{D}_4 + 13\mathfrak{D}_5) + 14\mathfrak{D}_3\mathfrak{D}_5) + 12\mathfrak{D}_2{}^2, \\
\mathfrak{h}_4 &= -\mathfrak{D}_1{}^2(\mathfrak{D}_3 - 9\mathfrak{D}_4) + 2\mathfrak{D}_1(9\mathfrak{D}_2 - \mathfrak{D}_3{}^2 + 4\mathfrak{D}_3\mathfrak{D}_4 + 10\mathfrak{D}_5) + 10\mathfrak{D}_2(\mathfrak{D}_3 + 3\mathfrak{D}_4) + 3\mathfrak{D}_3\mathfrak{D}_5, \\
\mathfrak{h}_5 &= 4\mathfrak{D}_1{}^2 - 2\mathfrak{D}_1(\mathfrak{D}_3 - 7\mathfrak{D}_4) + 20\mathfrak{D}_2 + \mathfrak{D}_3(\mathfrak{D}_4 - \mathfrak{D}_3) + 5\mathfrak{D}_5.
\end{aligned}
$$

Modelo de Hadronización en condiciones de supergravedad o gravedad cuánticas relativistas. Modelo Unificado.

$$
F(p, x; E) = 0
$$

$$
\Pi_A^{(\mathrm{c})} = 2\pi \mathrm{i}^{(\mathrm{c})} = \oiiint_{\mathcal{A}} p\, \mathrm{d}x
$$

$$
\Pi_B^{(\mathrm{c})} = \mathfrak{a}_D^{(\mathrm{c})} = \frac{\partial \mathcal{F}}{\partial \mathfrak{a}^{(\mathrm{c})}} = \oiiint_{\mathcal{B}} p\, \mathrm{d}x
$$

$$
\tilde{F}(\hat{p}, x; E) = 0, \hat{p} = -\mathrm{i}\partial_x
$$

$$
\tilde{F}(\hat{p}, x; E)\psi(x) = 0, \psi(x) = \exp\left(\mathrm{i}\int^x P\, P\, \mathrm{d}x'\right)
$$

$$
\Pi_A = 2\pi \mathrm{i}\mathfrak{a} = \oiint_{\mathcal{A}} P\, \mathrm{d}x, \Pi_B = \mathfrak{a}_D = \oiint_{\mathcal{B}} P\, \mathrm{d}x,
$$

$$
\Pi_I = 2\pi\left(n + \frac{1}{2}\right), n \in \mathbb{Z}
$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2}+\frac{1}{z^2(z-1)^2}\sum_{i=0}^{4}\hat{A}_i z^i\right]\Psi(z)=0$$

$$\begin{aligned}
\hat{A}_0 &= -\frac{1}{4}(m_1-m_2)^2+\frac{1}{4}\\
\hat{A}_1 &= -E-m_1m_2-\frac{1}{8}m_3\Lambda_3-\frac{1}{4}\\
\hat{A}_2 &= E+\frac{3}{8}m_3\Lambda_3-\frac{1}{64}\Lambda_3^2+\frac{1}{4}\\
\hat{A}_3 &= -\frac{1}{4}m_3\Lambda_3+\frac{1}{32}\Lambda_3^2\\
\hat{A}_4 &= -\frac{1}{64}\Lambda_3^2
\end{aligned}$$

$$E=\mathfrak{a}^2-\frac{\Lambda_{N_f}}{4-N_f}\frac{\partial\mathcal{F}_{\text{inst}}^{(N_f)}\left(\mathfrak{a},\boldsymbol{m},\Lambda_{N_f}\right)}{\partial\Lambda_{N_f}}$$

$$-p(z)^2+\frac{E}{z(z-1)}\simeq 0\ \Rightarrow\ p(z)\simeq\sqrt{\frac{E}{z(z-1)}},$$

$$\int_0^1 p(x+\mathrm{i}0^+)\mathrm{d}x+\int_1^0 p(x-\mathrm{i}0^+)\mathrm{d}x\simeq 2\pi\mathrm{i}\sqrt{E}$$

$$\begin{aligned}
&\int_1^R\sqrt{\frac{E}{x(x-1)}}\mathrm{d}x+\int_R^1\left[-\sqrt{\frac{E}{x(x-1)}}\right]\mathrm{d}x\\
&=2\sqrt{E}\log(4R)\simeq\sqrt{E}\log\left(\frac{E}{\Lambda_3^2}\right)
\end{aligned}$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2}+\frac{1}{z^4}\sum_{i=0}^{4}\tilde{A}_i z^i\right]\Psi(z)=0$$

$$2\int_{\tilde{R}^{-1}}^{\tilde{R}}\frac{\sqrt{E}}{x}\,\mathrm{d}x\simeq 2\sqrt{E}\log\left(\frac{E}{\Lambda_2^2}\right)$$

$$\begin{aligned}
&\Delta^{-s}\frac{\mathrm{d}}{\mathrm{d}r}\left[\Delta^{s+1}\frac{\mathrm{d}_sR_{\ell m}}{\mathrm{d}r}\right]+\frac{1}{\Delta}\left[K^2-\mathrm{i}s\frac{\mathrm{d}\Delta}{\mathrm{d}r}K\right.\\
&\left.+\Delta\left(2\mathrm{i}s\frac{\mathrm{d}K}{\mathrm{d}r}-\lambda_{\ell m}\right)\right]{}_sR_{\ell m}=0
\end{aligned}$$

$$K=(r^2+a^2)\omega-am,\Delta=(r-r_+)(r-r_-)$$

$$r_\pm=(1\pm b)/2,\ \text{where}\ b=\sqrt{1-4(a^2+Q^2)}$$

$$0\le a<\sqrt{1-4Q^2}/2$$

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}u}\left[(1-u^2)\frac{\mathrm{d}_sS_{\ell m}}{\mathrm{d}u}\right]+\left[(a\omega u)^2-2a\omega su+s+{}_sA_{\ell m}\right.\\
&\left.-\frac{(m+su)^2}{1-u^2}\right]{}_sS_{\ell m}=0
\end{aligned}$$

$${}_sA_{\ell m}=\ell(\ell+1)-s(s+1),\ \text{for}\ c=0$$

$${}_sR_{\ell m}=\Delta^{-(s+1)/2}\,{}_s\Psi_{\ell m}$$

$$r=r_++(r_+-r_-)(z-1)$$

$$A_0 = \frac{1}{4} + \frac{[2am - \mathrm{i}(r_+ - r_-)s + 2(Q^2 - r_-)\omega]^2}{4(r_+ - r_-)^2}$$

$$\begin{aligned}
A_1 =& {}_sA_{\ell m} + s(s+1) - 2(1-Q^2)\omega^2 \\
&+ \frac{1}{r_+ - r_-}\{[2 - 6Q^2 - (2r_- + 3)a^2]\omega^2 \\
&-2[am + \mathrm{i}s(2a^2 + Q^2)]\omega + 2\mathrm{i}ams\}, \\
A_2 =& - {}_sA_{\ell m} - s(s+1) - 3\mathrm{i}s(r_+ - r_-)\omega \\
&+(6r_- - 5a^2 - 6Q^2)\omega^2, \\
A_3 =& 2(\mathrm{i}s + 2r_-\omega)(r_+ - r_-)\omega, \\
A_4 =& (r_+ - r_-)^2\omega^2.
\end{aligned}$$

$$\begin{aligned}
\Lambda_3 &= -8\mathrm{i}(r_+ - r_-)\omega \\
E &= -\frac{1}{4} - {}_sA_{\ell m} - s(s+1) + (2 - a^2 - 2Q^2)\omega^2 \\
m_1 &= -s - \mathrm{i}\omega, m_3 = s - \mathrm{i}\omega \\
m_2 &= \mathrm{i}[2am - (1-2Q^2)\omega]/(r_+ - r_-)
\end{aligned}$$

$$\begin{aligned}
{}_sA_{\ell m} =& \ell(\ell+1) - s(s+1) - c^2 \\
&+ \Lambda_3\partial_{\Lambda_3}\mathcal{F}^{(3)}_{\mathrm{inst}}(\mathrm{i}\ell + \mathrm{i}/2, \boldsymbol{m}, \Lambda_3)\Big|_{\Lambda_3 = 16c} \\
\boldsymbol{m} =& \{-m, -s, -s\}
\end{aligned}$$

$$\begin{aligned}
\lim_{r\to r_+} {}_sR_{\ell m} &\simeq (r - r_+)^{-s-\mathrm{i}\sigma_+} \\
\lim_{r\to\infty} {}_sR_{\ell m} &\simeq r^{-1-2s+\mathrm{i}\omega}\mathrm{e}^{\mathrm{i}\omega r}
\end{aligned}$$

$$\begin{aligned}
{}_sR_{\ell m} =& \mathrm{e}^{\mathrm{i}\omega r}(r - r_-)^{-1-s+\mathrm{i}\omega+\mathrm{i}\sigma_+}(r - r_+)^{-s-\mathrm{i}\sigma_+} \\
&\times \sum_{n=0}^{\infty} a_n\left(\frac{r - r_+}{r - r_-}\right)^n
\end{aligned}$$

$$\begin{aligned}
&\alpha_0^r a_1 + \beta_0^r a_0 = 0 \\
&\alpha_n^r a_{n+1} + \beta_n^r a_n + \gamma_n^r a_{n-1} = 0, n = 1,2,\ldots
\end{aligned}$$

$$\begin{aligned}
\alpha_n^r &= n^2 + (c_0 + 1)n + c_0 \\
\beta_n^r &= -2n^2 + (c_1 + 2)n + c_3 \\
\gamma_n^r &= n^2 + (c_2 - 3)n + c_4 - c_2 + 2
\end{aligned}$$

$$\begin{aligned}
c_0 &= 1 - s - \mathrm{i}\omega - \frac{2\mathrm{i}}{b}\left[\frac{\omega}{2}(1-2Q^2) - am\right], \\
c_1 &= -4 + 2\mathrm{i}\omega(2+b) + \frac{4\mathrm{i}}{b}\left[\frac{\omega}{2}(1-2Q^2) - am\right], \\
c_2 &= s + 3 - 3\mathrm{i}\omega - \frac{2\mathrm{i}}{b}\left[\frac{\omega}{2}(1-2Q^2) - am\right], \\
c_3 &= \omega^2(4 + 2b - a^2 - 4Q^2) - 2am\omega - s - 1 + \mathrm{i}\omega(2+b) \\
&\quad - {}_sA_{\ell m} + \frac{4\omega + 2\mathrm{i}}{b}\left[\frac{\omega}{2}(1-2Q^2) - am\right], \\
c_4 &= s + 1 - 2\omega^2 - (2s+3)\mathrm{i}\omega - \frac{4\omega + 2\mathrm{i}}{b}\left[\frac{\omega}{2}(1-2Q^2)\right. \\
&\quad \left. - am\right].
\end{aligned}$$

$$0 = \beta_0^r - \frac{\alpha_0^r\gamma_1^r}{\beta_1^r -}\frac{\alpha_1^r\gamma_2^r}{\beta_2^r -}\frac{\alpha_2^r\gamma_3^r}{\beta_3^r -}\cdots$$

$$\mathcal{F}^{(N_f)}_{\mathrm{inst}} = -\lim_{\epsilon_2\to 0}\epsilon_2\log Z^{(N_f)}$$

$$A_0' = \frac{1}{4} - s^2$$
$$A_1' = {}_sA_{\ell m} + s(s+2) + a\omega(a\omega - 2m)$$
$$A_2' = -{}_sA_{\ell m} - s(s+1) - a\omega(5a\omega - 6m)$$
$$A_3' = 4\omega(-am + 2a^2\omega + Q^2\omega)$$
$$A_4' = (r_+ - r_-)^2\omega^2$$

$$\Psi(z) = \sqrt{f(z)}\phi(z), f(z) = 1 - z^{-1}$$

$$\left[f\frac{\mathrm{d}}{\mathrm{d}z}\left(f\frac{\mathrm{d}}{\mathrm{d}z}\right) + \omega^2 - V(z)\right]\phi(z) = 0$$

$$V = f\left[4(c^2 + d^2) + \frac{4c(m-c)}{z} + \frac{sA_{\ell m} + s(s+1) - c(2m-c)}{z^2} - \frac{s^2-1}{z^3}\right]$$

$$V = f\left[(2Q\omega)^2 + \frac{\ell(\ell+1)}{z^2} - \frac{s^2-1}{z^3}\right]$$

$$\Psi(z) \sim (z-1)^{\frac{1}{2}[1 \pm (m_1+m_2)]}.$$

$$R(r) \sim (r - r_+)^{-s-\mathrm{i}\sigma_+}, \Psi(z) \sim (z-1)^{\frac{1-s}{2}-\mathrm{i}\sigma_+}$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2} - \left(\frac{\Lambda_3^2}{64} + \frac{m_3\Lambda_3}{4z}\right)\right]\Psi(z) = 0, z \to \infty$$

$$\Psi(z) = \exp\left[-\frac{1}{8}\Lambda_3(z-1)\right] z^{\frac{1}{2}(-2m_3-1-m_1-m_2)} (z-1)^{\frac{1}{2}(1+m_1+m_2)} g(z)$$

$$\left\{z(z-1)\frac{\mathrm{d}^2}{\mathrm{d}z^2} + [B_1 z(z-1) + B_2(z-1) + B_3]\frac{\mathrm{d}}{\mathrm{d}z} + B_4\left(1 - \frac{1}{z}\right) + B_5\right\} g(z) = 0$$

$$S_{EH} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\mathcal{R}$$

$$8\pi G_N = \frac{\hbar c}{M_{pl}^2}$$

$$M_{pl} \approx 2 \times 10^{18}\mathrm{GeV}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_{pl}}h_{\mu\nu}$$

$$S_{EH} = \int d^4x(\partial h)^2 + \frac{1}{M_{pl}}h(\partial h)^2 + \frac{1}{M_{pl}^2}h^2(\partial h)^2 + \cdots$$

$$\Gamma \sim \frac{1}{\epsilon}\frac{1}{M_{pl}^4}\int d^4x\sqrt{-g}\mathcal{R}^{\mu\nu}_{\rho\sigma}\mathcal{R}^{\rho\sigma}_{\lambda\kappa}\mathcal{R}^{\lambda\kappa}_{\mu\nu}$$

$$S_{int} = \int d^4x\frac{1}{M_{pl}}h_{\mu\nu}T^{\mu\nu} + \mathcal{O}(h^2)$$

$$\frac{1}{M_{pl}^4}\int^{\infty} d^4\,k$$

$$\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, \dots, +1)$$

$$S = -m \int dt\sqrt{1 - \dot{\vec{x}} \cdot \dot{\vec{x}}}$$

$$\vec{p} = \frac{m\dot{\vec{x}}}{\sqrt{1 - \dot{\vec{x}} \cdot \dot{\vec{x}}}}, E = \sqrt{m^2 + \vec{p}^2},$$

$$S = -m \int d\tau \sqrt{-\dot{X}^\mu \dot{X}^\nu \eta_{\mu\nu}}$$

$$S = -m \int d\tilde{\tau} \sqrt{-\frac{dX^\mu}{d\tilde{\tau}} \frac{dX^\nu}{d\tilde{\tau}} \eta_{\mu\nu}}$$

$$\tau = X^0(\tau) \equiv t$$

$$p_\mu = \frac{\partial L}{\partial \dot{X}^\mu} = \frac{m\dot{X}^\nu \eta_{\mu\nu}}{\sqrt{-\dot{X}^\lambda \dot{X}^\rho \eta_{\lambda\rho}}}$$

$$p_\mu p^\mu + m^2 = 0$$

$$X^\mu \to \Lambda^\mu_\nu X^\nu + c^\mu$$

$$i\frac{\partial \Psi}{\partial \tau} = H\Psi$$

$$\left(-\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial X^\nu}\eta^{\mu\nu} + m^2\right)\Psi(X) = 0$$

$$S = \frac{1}{2}\int d\tau(e^{-1}\dot{X}^2 - em^2)$$

$$S = -\frac{1}{2}\int d\tau\sqrt{-g_{\tau\tau}}(g^{\tau\tau}\dot{X}^2 + m^2)$$

$$\tau \to \tilde{\tau} = \tau - \eta(\tau), \delta e = \frac{d}{d\tau}(\eta(\tau)e), \delta X^\mu = \frac{dX^\mu}{d\tau}\eta(\tau)$$

$$\sigma \in [0,2\pi)$$

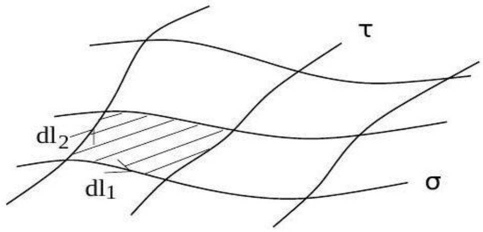
$$X^\mu(\sigma,\tau) = X^\mu(\sigma + 2\pi, \tau)$$

$$\gamma_{\alpha\beta} = \frac{\partial X^\mu}{\partial \sigma^\alpha}\frac{\partial X^\nu}{\partial \sigma^\beta}\eta_{\mu\nu}$$

$$S = -T\int d^2\sigma\sqrt{-\mathrm{det}\gamma}$$

$$\gamma_{\alpha\beta} = \begin{pmatrix} \dot{X}^2 & \dot{X} \cdot X' \\ \dot{X} \cdot X' & X'^2 \end{pmatrix}$$

$$S = -T\int d^2\sigma\sqrt{-(\dot{X})^2(X')^2 + (\dot{X} \cdot X')^2}$$



$$\vec{dl}_1 = \frac{\partial \vec{X}}{\partial \sigma}, \vec{dl}_2 = \frac{\partial \vec{X}}{\partial \tau}.$$

$$ds^2 = |\vec{dl}_1||\vec{dl}_2|\sin\,\theta = \sqrt{dl_1^2 dl_2^2(1-\cos^2\,\theta)} = \sqrt{dl_1^2 dl_2^2 - (\vec{dl}_1 \cdot \vec{dl}_2)^2}$$

$$S = -T\int\, d\tau d\sigma R\sqrt{(d\vec{x}/d\sigma)^2} = -T\int\, dt$$

$$T = \frac{1}{2\pi\alpha'}$$

$$\alpha' = l_s^2$$

$$T \sim (1\text{Gev})^2$$

$$T \lesssim M_{pl}^2 = (10^{18}\text{GeV})^2$$

$$\Pi_\mu^\tau = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = -T\frac{(\dot{X}\cdot X')X'_\mu - (X'^2)\dot{X}_\mu}{\sqrt{(\dot{X}\cdot X')^2 - \dot{X}^2 X'^2}}$$

$$\Pi_\mu^\sigma = \frac{\partial \mathcal{L}}{\partial X'^\mu} = -T\frac{(\dot{X}\cdot X')\dot{X}_\mu - (\dot{X}^2)X'_\mu}{\sqrt{(\dot{X}\cdot X')^2 - \dot{X}^2 X'^2}}$$

$$\frac{\partial \Pi_\mu^\tau}{\partial \tau} + \frac{\partial \Pi_\mu^\sigma}{\partial \sigma} = 0$$

$$\delta\sqrt{-\gamma} = \frac{1}{2}\sqrt{-\gamma}\gamma^{\alpha\beta}\delta\gamma_{\alpha\beta}$$

$$\partial_\alpha\left(\sqrt{-\det\gamma}\gamma^{\alpha\beta}\partial_\beta X^\mu\right) = 0$$

$$S = -\frac{1}{4\pi\alpha'}\int\, d^2\sigma\sqrt{-g}g^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

$$\partial_\alpha\left(\sqrt{-g}g^{\alpha\beta}\partial_\beta X^\mu\right) = 0$$

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\alpha\beta}\delta g^{\alpha\beta} = +\frac{1}{2}\sqrt{-g}g^{\alpha\beta}\delta g_{\alpha\beta})$$

$$\delta S = -\frac{T}{2}\int\, d^2\sigma\delta g^{\alpha\beta}\left(\sqrt{-g}\partial_\alpha X^\mu \partial_\beta X^\nu - \frac{1}{2}\sqrt{-g}g_{\alpha\beta}g^{\rho\sigma}\partial_\rho X^\mu \partial_\sigma X^\nu\right)\eta_{\mu\nu} = 0$$

$$g_{\alpha\beta} = 2f(\sigma)\partial_\alpha X \cdot \partial_\beta X$$

$$f^{-1} = g^{\rho\sigma}\partial_\rho X \cdot \partial_\sigma X$$

$$X^\mu \to \Lambda^\mu{}_\nu X^\nu + c^\mu$$

$$X^\mu(\sigma) \to \tilde{X}^\mu(\tilde{\sigma}) = X^\mu(\sigma)$$

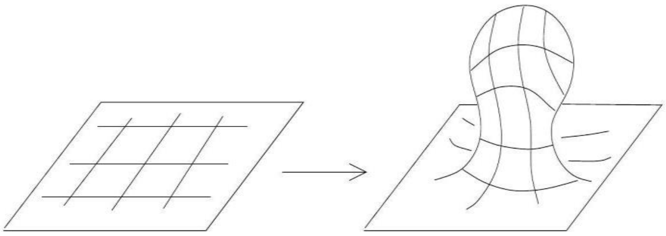
$$g_{\alpha\beta}(\sigma) \to \tilde{g}_{\alpha\beta}(\tilde{\sigma}) = \frac{\partial\sigma^\gamma}{\partial\tilde{\sigma}^\alpha}\frac{\partial\sigma^\delta}{\partial\tilde{\sigma}^\beta}g_{\gamma\delta}(\sigma)$$

$$\delta X^{\mu}(\sigma) = \eta^{\alpha}\partial_{\alpha}X^{\mu}$$
$$\delta g_{\alpha\beta}(\sigma) = \nabla_{\alpha}\eta_{\beta} + \nabla_{\beta}\eta_{\alpha}$$

$$\Gamma^{\sigma}_{\alpha\beta} = \frac{1}{2}g^{\sigma\rho}\left(\partial_{\alpha}g_{\beta\rho} + \partial_{\beta}g_{\rho\alpha} - \partial_{\rho}g_{\alpha\beta}\right)$$

$$g_{\alpha\beta}(\sigma) \to \Omega^{2}(\sigma)g_{\alpha\beta}(\sigma)$$

$$\delta g_{\alpha\beta}(\sigma) = 2\phi(\sigma)g_{\alpha\beta}(\sigma)$$



$$\int d^{2}\sigma\sqrt{-g}V(X)$$

$$\mu\int d^{2}\sigma\sqrt{-g}$$

$$g_{\alpha\beta} = e^{2\phi}\eta_{\alpha\beta}$$

$$g_{\alpha\beta} = \eta_{\alpha\beta}$$

$$\sqrt{g'}R' = \sqrt{g}(R - 2\nabla^{2}\phi).$$

$$R_{\alpha\beta\gamma\delta} = \frac{R}{2}\left(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}\right)$$

$$S = -\frac{1}{4\pi\alpha'}\int d^{2}\sigma\partial_{\alpha}X\cdot\partial^{\alpha}X$$

$$\partial_{\alpha}\partial^{\alpha}X^{\mu} = 0$$

$$T_{\alpha\beta} = -\frac{2}{T}\frac{1}{\sqrt{-g}}\frac{\partial S}{\partial g^{\alpha\beta}}$$

$$T_{\alpha\beta} = \partial_{\alpha}X\cdot\partial_{\beta}X - \frac{1}{2}\eta_{\alpha\beta}\eta^{\rho\sigma}\partial_{\rho}X\cdot\partial_{\sigma}X$$

$$T_{01} = \dot{X}\cdot X' = 0$$
$$T_{00} = T_{11} = \frac{1}{2}(\dot{X}^{2} + X'^{2}) = 0$$

$$X^{0} \equiv t = R\tau,$$

$$\ddot{\vec{x}} - \vec{x}'' = 0$$

$$\dot{\vec{x}}\cdot\vec{x}' = 0$$
$$\dot{\vec{x}}^{2} + \vec{x}'^{2} = R^{2}$$

$$\int d\sigma\sqrt{(d\vec{x}/d\sigma)^{2}} = 2\pi R$$

$$\sigma^{\pm} = \tau \pm \sigma$$

$$\partial_{+}\partial_{-}X^{\mu} = 0$$

$$X^\mu(\sigma,\tau) = X_L^\mu(\sigma^+) + X_R^\mu(\sigma^-)$$

$$X^\mu(\sigma,\tau) = X^\mu(\sigma + 2\pi,\tau)$$

$$X_L^\mu(\sigma^+) = \frac{1}{2}x^\mu + \frac{1}{2}\alpha' p^\mu \sigma^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0} \frac{1}{n}\tilde{\alpha}_n^\mu e^{-in\sigma^+}$$

$$X_R^\mu(\sigma^-) = \frac{1}{2}x^\mu + \frac{1}{2}\alpha' p^\mu \sigma^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0} \frac{1}{n}\alpha_n^\mu e^{-in\sigma^-}$$

$$\alpha_n^\mu = \left(\alpha_{-n}^\mu\right)^\star, \tilde{\alpha}_n^\mu = \left(\tilde{\alpha}_{-n}^\mu\right)^\star$$

$$(\partial_+ X)^2 = (\partial_- X)^2 = 0$$

$$\partial_- X^\mu = \partial_- X_R^\mu = \frac{\alpha'}{2}p^\mu + \sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0} \alpha_n^\mu e^{-in\sigma^-}$$
$$= \sqrt{\frac{\alpha'}{2}}\sum_{n} \alpha_n^\mu e^{-in\sigma^-}$$

$$\alpha_0^\mu \equiv \sqrt{\frac{\alpha'}{2}}p^\mu$$

$$(\partial_- X)^2 = \frac{\alpha'}{2}\sum_{m,p} \alpha_m \cdot \alpha_p e^{-i(m+p)\sigma^-}$$
$$= \frac{\alpha'}{2}\sum_{m,n} \alpha_m \cdot \alpha_{n-m} e^{-in\sigma^-}$$
$$\equiv \alpha' \sum_{n} L_n e^{-in\sigma^-} = 0$$

$$L_n = \frac{1}{2}\sum_{m} \alpha_{n-m} \cdot \alpha_m$$

$$\tilde{L}_n = \frac{1}{2}\sum_{m} \tilde{\alpha}_{n-m} \cdot \tilde{\alpha}_m$$

$$\tilde{\alpha}_0^\mu \equiv \sqrt{\frac{\alpha'}{2}}p^\mu$$

$$L_n = \tilde{L}_n = 0 \; n \in \mathbf{Z}$$

$$p_\mu p^\mu = -M^2.$$

$$M^2 = \frac{4}{\alpha'}\sum_{n>0} \alpha_n \cdot \alpha_{-n} = \frac{4}{\alpha'}\sum_{n>0} \tilde{\alpha}_n \cdot \tilde{\alpha}_{-n}$$

$$\dot{X} \cdot X' = \dot{X}^2 + X'^2 = 0.$$

$$[X^\mu(\sigma,\tau), \Pi_\nu(\sigma',\tau)] = i\delta(\sigma - \sigma')\delta_\nu^\mu$$
$$[X^\mu(\sigma,\tau), X^\nu(\sigma',\tau)] = \left[\Pi_\mu(\sigma,\tau), \Pi_\nu(\sigma',\tau)\right] = 0$$

$$[x^\mu, p_\nu] = i\delta_\nu^\mu \text{ and } \left[\alpha_n^\mu, \alpha_m^\nu\right] = \left[\tilde{\alpha}_n^\mu, \tilde{\alpha}_m^\nu\right] = n\eta^{\mu\nu}\delta_{n+m,0}$$

$$a_n = \frac{\alpha_n}{\sqrt{n}}, a_n^\dagger = \frac{\alpha_{-n}}{\sqrt{n}} \; n > 0$$

$$\alpha_n^\mu|0\rangle = \tilde{\alpha}_n^\mu|0\rangle = 0 \ \text{ for } n > 0$$

$$\hat{p}^\mu|0;p\rangle = p^\mu|0;p\rangle$$

$$\left(\alpha_{-1}^{\mu_1}\right)^{n_{\mu_1}}\left(\alpha_{-2}^{\mu_2}\right)^{n_{\mu_2}}\dots\left(\tilde{\alpha}_{-1}^{\nu_1}\right)^{n_{\nu_1}}\left(\tilde{\alpha}_{-2}^{\nu_2}\right)^{n_{\nu_2}}\dots|0;p\rangle$$

$$\left[\alpha_n^\mu, \alpha_m^{\nu\dagger}\right] = n\eta^{\mu\nu}\delta_{n,m}$$

$$\langle p';0|\alpha_1^0\alpha_{-1}^0|0;p\rangle \sim -\delta^D(p-p')$$

$$L_n = \frac{1}{2}\sum_m \alpha_{n-m}\cdot\alpha_m$$

$$\langle \text{phys}'|L_n|phys\rangle = \langle \text{phys}'|\tilde{L}_n|phys\rangle = 0$$

$$L_n|phys\rangle = \tilde{L}_n|phys\rangle = 0 \ \text{ for } n > 0$$

$$L_0 = \sum_{m=1}^{\infty}\alpha_{-m}\cdot\alpha_m + \frac{1}{2}\alpha_0^2\,, \tilde{L}_0 = \sum_{m=1}^{\infty}\tilde{\alpha}_{-m}\cdot\tilde{\alpha}_m + \frac{1}{2}\tilde{\alpha}_0^2$$

$$(L_0 - a)|phys\rangle = \left(\tilde{L}_0 - a\right)|phys\rangle = 0$$

$$M^2 = \frac{4}{\alpha'}\left(-a + \sum_{m=1}^{\infty}\alpha_{-m}\cdot\alpha_m\right) = \frac{4}{\alpha'}\left(-a + \sum_{m=1}^{\infty}\tilde{\alpha}_{-m}\cdot\tilde{\alpha}_m\right)$$

$$g_{\alpha\beta} = \eta_{\alpha\beta}$$

$$\eta_{\alpha\beta} \to \Omega^2(\sigma)\eta_{\alpha\beta}$$

$$\sigma^\pm = \tau \pm \sigma,$$

$$ds^2 = -d\sigma^+ d\sigma^-$$

$$\sigma^+ \to \tilde{\sigma}^+(\sigma^+)\,, \sigma^- \to \tilde{\sigma}^-(\sigma^-)$$

$$(\partial_+ X)^2 = (\partial_- X)^2 = 0$$

$$X^\pm = \sqrt{\frac{1}{2}}\left(X^0 \pm X^{D-1}\right)$$

$$ds^2 = -2dX^+dX^- + \sum_{i=1}^{D-2} dX^i dX^i$$

$$X^+ = X_L^+(\sigma^+) + X_R^+(\sigma^-).$$

$$X_L^+ = \frac{1}{2}x^+ + \frac{1}{2}\alpha' p^+\sigma^+\,, X_R^+ = \frac{1}{2}x^+ + \frac{1}{2}\alpha' p^+\sigma^-.$$

$$X^+ = x^+ + \alpha' p^+\tau.$$

$$\partial_+\partial_- X^- = 0$$

$$X^- = X_L^-(\sigma^+) + X_R^-(\sigma^-)$$

$$2\partial_+ X^-\partial_+ X^+ = \sum_{i=1}^{D-2}\partial_+ X^i\partial_+ X^i$$

$$\partial_+ X_L^- = \frac{1}{\alpha' p^+} \sum_{i=1}^{D-2} \partial_+ X^i \partial_+ X^i$$

$$\partial_- X_R^- = \frac{1}{\alpha' p^+} \sum_{i=1}^{D-2} \partial_- X^i \partial_- X^i$$

$$X_L^-(\sigma^+) = \frac{1}{2} x^- + \frac{1}{2} \alpha' p^- \sigma^+ + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \tilde{\alpha}_n^- e^{-in\sigma^+}$$

$$X_R^-(\sigma^-) = \frac{1}{2} x^- + \frac{1}{2} \alpha' p^- \sigma^- + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \alpha_n^- e^{-in\sigma^-}$$

$$\alpha_n^- = \sqrt{\frac{1}{2\alpha'} \frac{1}{p^+}} \sum_{m=-\infty}^{+\infty} \sum_{i=1}^{D-2} \alpha_{n-m}^i \alpha_m^i,$$

$$\frac{\alpha' p^-}{2} = \frac{1}{2p^+} \sum_{i=1}^{D-2} \left(\frac{1}{2} \alpha' p^i p^i + \sum_{n \neq 0} \alpha_n^i \alpha_{-n}^i \right)$$

$$\frac{\alpha' p^-}{2} = \frac{1}{2p^+} \sum_{i=1}^{D-2} \left(\frac{1}{2} \alpha' p^i p^i + \sum_{n \neq 0} \tilde{\alpha}_n^i \tilde{\alpha}_{-n}^i \right)$$

$$M^2 = 2p^+ p^- - \sum_{i=1}^{D-2} p^i p^i = \frac{4}{\alpha'} \sum_{i=1}^{D-2} \sum_{n>0} \alpha_{-n}^i \alpha_n^i = \frac{4}{\alpha'} \sum_{i=1}^{D-2} \sum_{n>0} \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i$$

$$[x^i, p^j] = i\delta^{ij}, [x^-, p^+] = -i$$

$$[\alpha_n^i, \alpha_m^j] = [\tilde{\alpha}_n^i, \tilde{\alpha}_m^j] = n\delta^{ij} \delta_{n+m,0}$$

$$[x^+, p^-] = -i$$

$$\hat{p}^\mu |0; p\rangle = p^\mu |0; p^\mu\rangle, \alpha_n^i |0; p\rangle = \tilde{\alpha}_n^i |0; p\rangle = 0 \text{ for } n > 0$$

$$\omega \sim \int d\sigma - d\dot{X}^+ \wedge dX^- - d\dot{X}^- \wedge dX^+ + 2d\dot{X}^i \wedge dX^i$$

$$M^2 = \frac{4}{\alpha'} \left(\sum_{i=1}^{D-2} \sum_{n>0} \alpha_{-n}^i \alpha_n^i - a \right) = \frac{4}{\alpha'} \left(\sum_{i=1}^{D-2} \sum_{n>0} \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i - a \right)$$

$$N = \sum_{i=1}^{D-2} \sum_{n>0} \alpha_{-n}^i \alpha_n^i, \tilde{N} = \sum_{i=1}^{D-2} \sum_{n>0} \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i$$

$$M^2 = \frac{4}{\alpha'} (N - a) = \frac{4}{\alpha'} (\tilde{N} - a)$$

$$\frac{1}{2} \sum_{n \neq 0} \alpha_{-n}^i \alpha_n^i = \frac{1}{2} \sum_{n<0} \alpha_{-n}^i \alpha_n^i + \frac{1}{2} \sum_{n>0} \alpha_{-n}^i \alpha_n^i$$

$$\frac{1}{2} \sum_{n<0} [\alpha_n^i \alpha_{-n}^i - n(D-2)] + \frac{1}{2} \sum_{n>0} \alpha_{-n}^i \alpha_n^i = \sum_{n>0} \alpha_{-n}^i \alpha_n^i + \frac{D-2}{2} \sum_{n>0} n.$$

$$\sum_{n=1}^{\infty} n \longrightarrow \sum_{n=1}^{\infty} n e^{-\epsilon n} = -\frac{\partial}{\partial \epsilon}\sum_{n=1}^{\infty} e^{-\epsilon n}$$
$$= -\frac{\partial}{\partial \epsilon}(1-e^{-\epsilon})^{-1}$$
$$= \frac{1}{\epsilon^2} - \frac{1}{12} + \mathcal{O}(\epsilon)$$

$$\sum_{n=1}^{\infty} n = -\frac{1}{12}$$

$$M^2 = \frac{4}{\alpha'}\left(N - \frac{D-2}{24}\right) = \frac{4}{\alpha'}\left(\tilde{N} - \frac{D-2}{24}\right)$$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$$

$$\zeta(-1) = -\frac{1}{12}$$

$$M^2 = -\frac{1}{\alpha'}\frac{D-2}{6}.$$

$$M^2 = \frac{\partial^2 V(T)}{\partial T^2}\bigg|_{T=0}$$

$$V(T) = \frac{1}{2}M^2T^2 + c_3T^3 + c_4T^4 + \cdots$$

$$\tilde{\alpha}_{-1}^{i}\alpha_{-1}^{j}|0;p\rangle,$$

$$M^2 = \frac{4}{\alpha'}\left(1 - \frac{D-2}{24}\right)$$

$$G_{\mu\nu}(X)\,, B_{\mu\nu}(X)\,, \Phi(X)$$

$$G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(X).$$

$$[a_{\mu\nu}, a_{\rho\sigma}^{\dagger}] \sim \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}$$

$$a_{0i}^{\dagger}|0\rangle$$

$$S_{EH} = \frac{M_{pl}^2}{2}\int d^4x\left[\partial_\mu h_\rho^\rho \partial_\nu h^{\mu\nu} - \partial^\rho h^{\mu\nu}\partial_\mu h_{\rho\nu} + \frac{1}{2}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} - \frac{1}{2}\partial_\mu h_\nu^\nu \partial^\mu h_\rho^\rho\right] + \cdots$$

$$h_{\mu\nu} \longrightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$(\alpha_{-1}^{i}\alpha_{-1}^{j} \oplus \alpha_{-2}^{i}) \otimes (\tilde{\alpha}_{-1}^{i}\tilde{\alpha}_{-1}^{j} \oplus \tilde{\alpha}_{-2}^{i})|0;p\rangle.$$

$$\frac{1}{2}(D-2)(D-1) + (D-2) = \frac{1}{2}D(D-1) - 1$$

$$X^\mu \to \Lambda_\nu^\mu X^\nu + c^\mu$$

$$P_\mu^\alpha = T\partial^\alpha X_\mu$$

$$J_{\mu\nu}^\alpha = P_\mu^\alpha X_\nu - P_\nu^\alpha X_\mu$$

$$M_{\mu\nu} = \int d\sigma J_{\mu\nu}^\tau$$

$$\mathcal{M}^{\mu\nu} = (p^\mu x^\nu - p^\nu x^\mu) - i\sum_{n=1}^{\infty}\frac{1}{n}(\alpha_{-n}^{\nu}\alpha_n^{\mu} - \alpha_{-n}^{\mu}\alpha_n^{\nu}) - i\sum_{n=1}^{\infty}\frac{1}{n}(\tilde{\alpha}_{-n}^{\nu}\tilde{\alpha}_n^{\mu} - \tilde{\alpha}_{-n}^{\mu}\tilde{\alpha}_n^{\nu})$$
$$\equiv l^{\mu\nu} + S^{\mu\nu} + \tilde{S}^{\mu\nu}$$

$$[\mathcal{M}^{\rho\sigma}, \mathcal{M}^{\tau\nu}] = \eta^{\sigma\tau}\mathcal{M}^{\rho\nu} - \eta^{\rho\tau}\mathcal{M}^{\sigma\nu} + \eta^{\rho\nu}\mathcal{M}^{\sigma\tau} - \eta^{\sigma\nu}\mathcal{M}^{\rho\tau}$$

$$[\mathcal{M}^{i-}, \mathcal{M}^{j-}] = 0$$

$$[\mathcal{M}^{i-}, \mathcal{M}^{j-}] = \frac{2}{(p^+)^2}\sum_{n>0}\left(\left[\frac{D-2}{24} - 1\right]n + \frac{1}{n}\left[a - \frac{D-2}{24}\right]\right)(\alpha_{-n}^{i}\alpha_n^{j} - \alpha_{-n}^{j}\alpha_n^{i}) + (\alpha \leftrightarrow \tilde{\alpha})$$

$$\sigma \in [0, \pi]$$

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \partial_\alpha X \cdot \partial^\alpha X$$

$$\delta S = -\frac{1}{2\pi\alpha'}\int_{\tau_i}^{\tau_f} d\tau \int_0^{\pi} d\sigma \partial_\alpha X \cdot \partial^\alpha \delta X$$
$$= \frac{1}{2\pi\alpha'}\int d^2\sigma(\partial^\alpha\partial_\alpha X)\cdot\delta X + \text{ total derivative}$$

$$\frac{1}{2\pi\alpha'}\left[\int_0^{\pi} d\sigma \dot{X}\cdot\delta X\right]_{\tau=\tau_i}^{\tau=\tau_f} - \frac{1}{2\pi\alpha'}\left[\int_{\tau_i}^{\tau_f} d\tau X'\cdot\delta X\right]_{\sigma=0}^{\sigma=\pi}$$

$$\partial_\sigma X^\mu \delta X_\mu = 0 \text{ at } \sigma = 0, \pi$$

$$\partial_\sigma X^\mu = 0 \text{ at } \sigma = 0, \pi$$

$$\dot{\vec{x}}\cdot\vec{x}' = 0 \text{ and } \dot{\vec{x}}^2 + \vec{x}'^2 = R^2$$

$$\delta X^\mu = 0 \text{ at } \sigma = 0, \pi$$

$$\partial_\sigma X^a = 0 \qquad \text{for } a = 0, \dots, p$$
$$X^I = c^I \qquad \text{for } I = p+1, \dots, D-1$$

$$SO(1, D-1) \to SO(1, p) \times SO(D-p-1)$$

$$X_L^\mu(\sigma^+) = \frac{1}{2}x^\mu + \alpha' p^\mu \sigma^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^{\mu} e^{-in\sigma^+}$$
$$X_R^\mu(\sigma^-) = \frac{1}{2}x^\mu + \alpha' p^\mu \sigma^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{\mu} e^{-in\sigma^-}$$

$$\alpha_n^a = \tilde{\alpha}_n^a$$

$$x^I = c^I, p^I = 0, \alpha_n^I = -\tilde{\alpha}_n^I$$

$$P^\mu = \int_0^\pi d\sigma (P^\tau)^\mu = \frac{1}{2\pi\alpha'}\int_0^\pi d\sigma \dot{X}^\mu = p^\mu$$

$$X^\pm = \sqrt{\frac{1}{2}}(X^0 \pm X^p)$$

$$M^2 = \frac{1}{\alpha'}\left(\sum_{i=1}^{p-1}\sum_{n>0}\alpha_{-n}^{i}\alpha_n^{i} + \sum_{i=p+1}^{D-1}\sum_{n>0}\alpha_{-n}^{i}\alpha_n^{i} - a\right)$$

$$\alpha_n^i|0;p\rangle = 0 \; n > 0$$

$$M^2 = -\frac{1}{\alpha'}$$

$$\alpha_{-1}^a|0;p\rangle \; a = 1, \dots, p-1$$

$$\alpha_{-1}^I|0;p\rangle \; I = p+1, \dots, D-1$$

$$M^2 = \frac{1}{\alpha'}(N-1)$$

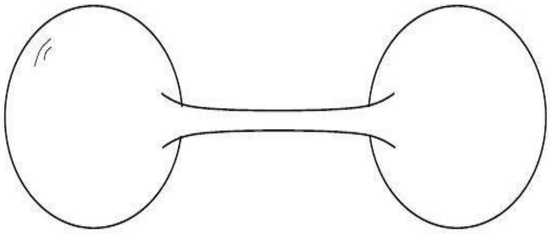
$$J_{\max} = N = \alpha' M^2 + 1$$

$$S_{Dp} = -T_p \int d^{p+1}\xi \sqrt{-\det\gamma}$$

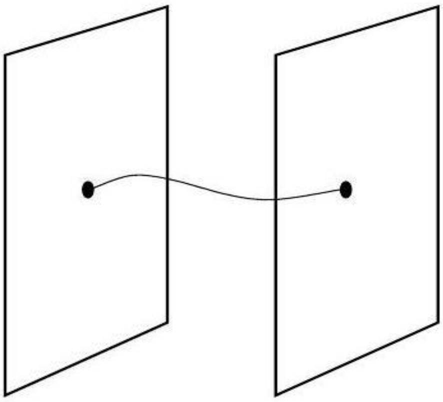
$$\gamma_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b}\eta_{\mu\nu}$$

$$X^a = \xi^a \; a = 0, \dots, p$$

$$X^I(\xi) = 2\pi\alpha'\phi^I(\xi) \; I = p+1, \dots, D-1$$



$$X^I(0,\tau) = c^I \text{ and } X^I(\pi,\tau) = d^I$$



$$X^I = c^I + \frac{(d^I - c^I)\sigma}{\pi} + \mathfrak{O}_{\text{oscillator modes}}$$

$$\partial_+ X \cdot \partial_+ X = \alpha'^2 p^2 + \frac{|\vec{d} - \vec{c}|^2}{4\pi^2} + \mathfrak{O}_{\text{oscillator modes}} = 0$$

$$M^2 = \frac{|\vec{d} - \vec{c}|^2}{(2\pi\alpha')^2} + \mathfrak{O}_{\text{oscillator modes}}$$

$$(\phi^I)_n^m, (A_a)_n^m$$

$$g_{\alpha\beta}(\sigma) \to \Omega^2(\sigma) g_{\alpha\beta}(\sigma)$$

$$z = \sigma^1 + i\sigma^2 \text{ and } \bar{z} = \sigma^1 - i\sigma^2$$

$$\partial_z \equiv \partial = \frac{1}{2}(\partial_1 - i\partial_2) \text{ and } \partial_{\bar{z}} \equiv \bar{\partial} = \frac{1}{2}(\partial_1 + i\partial_2)$$

$$ds^2 = (d\sigma^1)^2 + (d\sigma^2)^2 = dzd\bar{z}$$

$$g_{zz} = g_{\bar{z}\bar{z}} = 0 \text{ and } g_{z\bar{z}} = \frac{1}{2}$$

$$\int d^2 z\delta(z, \bar{z}) = 1$$

$$\int d^2\sigma\delta(\sigma) = 1$$

$$v^z = (v^1 + iv^2)$$

$$v^{\bar{z}} = (v^1 - iv^2)$$

$$v_z = \frac{1}{2}(v^1 - iv^2)$$

$$v_{\bar{z}} = \frac{1}{2}(v^1 + iv^2)$$

$$z \to z' = f(z) \text{ and } \bar{z} \to \bar{z}' = \bar{f}(\bar{z})$$

$$\delta\sigma^\alpha = \epsilon^\alpha$$

$$\delta S = \int d^2\sigma J^\alpha \partial_\alpha \epsilon$$

$$\partial_\alpha J^\alpha = 0$$

$$\delta\sigma^\alpha = \epsilon^\alpha(\sigma)$$

$$\delta g_{\alpha\beta} = \partial_\alpha \epsilon_\beta + \partial_\beta \epsilon_\alpha$$

$$\delta S = -\int d^2\sigma \frac{\partial S}{\partial g_{\alpha\beta}} \delta g_{\alpha\beta} = -2\int d^2\sigma \frac{\partial S}{\partial g_{\alpha\beta}} \partial_\alpha \epsilon_\beta$$

$$T_{\alpha\beta} = -\frac{4\pi}{\sqrt{g}} \frac{\partial S}{\partial g^{\alpha\beta}}$$

$$\delta g_{\alpha\beta} = \epsilon g_{\alpha\beta}$$

$$\delta S = \int d^2\sigma \frac{\partial S}{\partial g_{\alpha\beta}} \delta g_{\alpha\beta} = -\frac{1}{4\pi}\int d^2\sigma \sqrt{g}\epsilon T_\alpha^\alpha$$

$$T_\alpha^\alpha = 0$$

$$T_{z\bar{z}} = 0$$

$$\bar{\partial} T_{zz} = 0 \text{ and } \partial T_{\bar{z}\bar{z}} = 0$$

$$T_{zz}(z) \equiv T(z) \text{ and } T_{\bar{z}\bar{z}}(\bar{z}) \equiv \bar{T}(\bar{z})$$

$$z' = z + \epsilon(z), \bar{z}' = \bar{z} + \bar{\epsilon}(\bar{z})$$

$$
\begin{aligned}
\delta S &= -\int d^2\sigma \frac{\partial S}{\partial g^{\alpha\beta}} \delta g^{\alpha\beta} \\
&= \frac{1}{2\pi}\int d^2\sigma T_{\alpha\beta}\left(\partial^\alpha \delta\sigma^\beta\right) \\
&= \frac{1}{2\pi}\int d^2 z \frac{1}{2}[T_{zz}(\partial^z \delta z) + T_{\bar z\bar z}(\partial^{\bar z}\delta \bar z)] \\
&= \frac{1}{2\pi}\int d^2 z [T_{zz}\partial_{\bar z}\epsilon + T_{\bar z\bar z}\partial_z \bar\epsilon]
\end{aligned}
$$

$$\delta z = \epsilon(z)\,,\delta \bar z = 0$$

$$J^z = 0 \text{ and } J^{\bar z} = T_{zz}(z)\epsilon(z) \equiv T(z)\epsilon(z)$$

$$\bar J^z = \bar T(\bar z)\bar\epsilon(\bar z) \text{ and } \bar J^{\bar z} = 0$$

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma \partial_\alpha X \partial^\alpha X$$

$$X(\sigma) \to X(\lambda^{-1}\sigma) \text{ and } \frac{\partial X(\sigma)}{\partial \sigma^\alpha} \to \frac{\partial X(\lambda^{-1}\sigma)}{\partial\sigma^\alpha} = \frac{1}{\lambda}\frac{\partial X(\tilde\sigma)}{\partial\tilde\sigma}$$

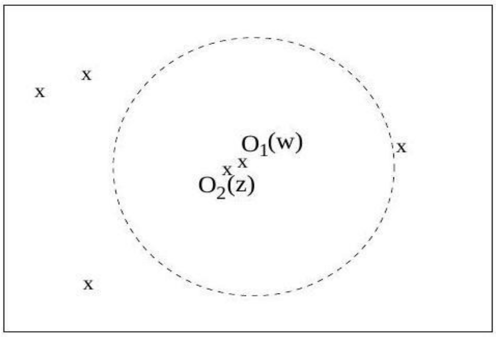
$$T_{\alpha\beta} = -\frac{1}{\alpha'}\left(\partial_\alpha X \partial_\beta X - \frac{1}{2}\delta_{\alpha\beta}(\partial X)^2\right)$$

$$T = -\frac{1}{\alpha'}\partial X \partial X \text{ and } \bar T = -\frac{1}{\alpha'}\bar\partial X \bar\partial X$$

$$X(z,\bar z) = X(z) + \bar X(\bar z)$$

$$\mathcal{O}_i(z,\bar z)\mathcal{O}_j(w,\bar w) = \sum_k C_{ij}^k(z-w,\bar z-\bar w)\mathcal{O}_k(w,\bar w)$$

$$\langle \mathcal{O}_i(z,\bar z)\mathcal{O}_j(w,\bar w)\ldots\rangle = \sum_k C_{ij}^k(z-w,\bar z-\bar w)\langle\mathcal{O}_k(w,\bar w)\ldots\rangle$$



$$Z = \int \mathcal{D}\phi e^{-S[\phi]}$$

$$\phi' = \phi + \epsilon\delta\phi$$

$$S[\phi'] = S[\phi] \text{ and } \mathcal{D}\phi' = \mathcal{D}\phi$$

$$
\begin{aligned}
Z &\longrightarrow \int \mathcal{D}\phi' \exp\left(-S[\phi']\right) \\
&= \int \mathcal{D}\phi \exp\left(-S[\phi] - \frac{1}{2\pi}\int J^\alpha \partial_\alpha \epsilon\right) \\
&= \int \mathcal{D}\phi e^{-S[\phi]}\left(1 - \frac{1}{2\pi}\int J^\alpha\partial_\alpha\epsilon\right)
\end{aligned}
$$

$$\int \mathcal{D}\phi e^{-S[\phi]}\left(\int J^{\alpha}\partial_{\alpha}\epsilon\right) = 0$$

$$\langle \partial_{\alpha} J^{\alpha} \rangle = 0$$

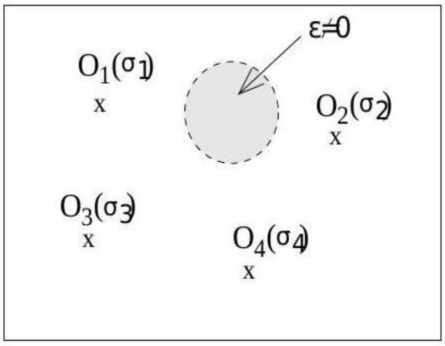
$$\langle \mathcal{O}_1(\sigma_1) \ldots \mathcal{O}_n(\sigma_n) \rangle = \frac{1}{Z}\int \mathcal{D}\phi e^{-S[\phi]}\mathcal{O}_1(\sigma_1) \ldots \mathcal{O}_n(\sigma_n)$$

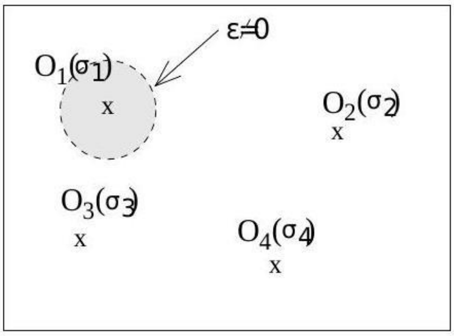
$$\mathcal{O}_i \to \mathcal{O}_i + \epsilon\delta\mathcal{O}_i$$

$$\delta\mathcal{O}_i(\sigma_i) = 0$$

$$\langle \partial_{\alpha} J^{\alpha}(\sigma)\mathcal{O}_1(\sigma_1) \ldots \mathcal{O}_n(\sigma_n) \rangle = 0 \text{ for } \sigma \neq \sigma_i$$

$$\partial_{\alpha} J^{\alpha} = 0$$





$$\frac{1}{Z}\int \mathcal{D}\phi e^{-S[\phi]}\left(1 - \frac{1}{2\pi}\int J^{\alpha}\partial_{\alpha}\epsilon\right)(\mathcal{O}_1 + \epsilon\delta\mathcal{O}_1)\mathcal{O}_2 \ldots \mathcal{O}_n$$

$$-\frac{1}{2\pi}\int_{\epsilon} \partial_{\alpha}\langle J^{\alpha}(\sigma)\mathcal{O}_1(\sigma_1) \ldots \rangle = \langle \delta\mathcal{O}_1(\sigma_1) \ldots \rangle$$

$$\int_{\epsilon} \partial_{\alpha} J^{\alpha} = \oint_{\partial\epsilon} J_{\alpha}\hat{n}^{\alpha} = \oint_{\partial\epsilon} (J_1 d\sigma^2 - J_2 d\sigma^1) = -i \oint_{\partial\epsilon} (J_z dz - J_{\bar{z}} d\bar{z})$$

$$\frac{i}{2\pi}\oint_{\partial\epsilon} dz\langle J_z(z,\bar{z})\mathcal{O}_1(\sigma_1) \ldots \rangle - \frac{i}{2\pi}\oint_{\partial\epsilon} d\bar{z}\langle J_{\bar{z}}(z,\bar{z})\mathcal{O}_1(\sigma_1) \ldots \rangle = \langle \delta\mathcal{O}_1(\sigma_1) \ldots \rangle$$

$$\frac{i}{2\pi}\oiint_{\partial\epsilon} dz J_z(z)\mathcal{O}_1(\sigma_1) = -\mathrm{Res}[J_z\mathcal{O}_1]$$

$$J_z(z)\mathcal{O}_1(w,\bar{w}) = \cdots + \frac{\mathrm{Res}[J_z\mathcal{O}_1(w,\bar{w})]}{z - w} + \cdots$$

$$\delta\mathcal{O}_1(\sigma_1) = -\mathrm{Res}[J_z(z)\mathcal{O}_1(\sigma_1)] = -\mathrm{Res}[\epsilon(z)T(z)\mathcal{O}_1(\sigma_1)]$$

$$\delta\mathcal{O}_1(\sigma_1) = -\mathrm{Res}[\bar{J}_{\bar{z}}(\bar{z})\mathcal{O}_1(\sigma_1)] = -\mathrm{Res}[\bar{\epsilon}(\bar{z})\bar{T}(\bar{z})\mathcal{O}_1(\sigma_1)]$$

$$\mathcal{O}(z-\epsilon) = \mathcal{O}(z) - \epsilon\partial\mathcal{O}(z) + \cdots$$

$$T(z)\mathcal{O}(w,\bar{w}) = \cdots + \frac{\partial\mathcal{O}(w,\bar{w})}{z-w} + \cdots$$

$$\bar{T}(\bar{z})\mathcal{O}(w,\bar{w}) = \cdots + \frac{\bar{\partial}\mathcal{O}(w,\bar{w})}{\bar{z}-\bar{w}} + \cdots$$

$$z \to z + \epsilon z \text{ and } \bar{z} \to \bar{z} + \bar{\epsilon}\bar{z}$$

$$\delta\mathcal{O} = -\epsilon(h\mathcal{O} + z\partial\mathcal{O}) - \bar{\epsilon}(\tilde{h}\mathcal{O} + \bar{z}\bar{\partial}\mathcal{O})$$

$$s = h - \tilde{h}$$

$$\Delta = h + \tilde{h}$$

$$L = -i(\sigma^1\partial_2 - \sigma^2\partial_1) = z\partial - \bar{z}\bar{\partial}$$

$$D = \sigma^\alpha\partial_\alpha = z\partial + \bar{z}\bar{\partial}$$

$$T(z)\mathcal{O}(w,\bar{w}) = \cdots + h\frac{\mathcal{O}(w,\bar{w})}{(z-w)^2} + \frac{\partial\mathcal{O}(w,\bar{w})}{z-w} + \cdots$$
$$\bar{T}(\bar{z})\mathcal{O}(w,\bar{w}) = \cdots + \tilde{h}\frac{\mathcal{O}(w,\bar{w})}{(\bar{z}-\bar{w})^2} + \frac{\bar{\partial}\mathcal{O}(w,\bar{w})}{\bar{z}-\bar{w}} + \cdots$$

$$T(z)\mathcal{O}(w,\bar{w}) = h\frac{\mathcal{O}(w,\bar{w})}{(z-w)^2} + \frac{\partial\mathcal{O}(w,\bar{w})}{z-w} + \text{ non-singular}$$
$$\bar{T}(\bar{z})\mathcal{O}(w,\bar{w}) = \tilde{h}\frac{\mathcal{O}(w,\bar{w})}{(\bar{z}-\bar{w})^2} + \frac{\bar{\partial}\mathcal{O}(w,\bar{w})}{\bar{z}-\bar{w}} + \text{ non-singular}$$

$$\begin{aligned}\delta\mathcal{O}(w,\bar{w}) &= -\mathrm{Res}[\epsilon(z)T(z)\mathcal{O}(w,\bar{w})] \\ &= -\mathrm{Res}\left[\epsilon(z)\left(h\frac{\mathcal{O}(w,\bar{w})}{(z-w)^2} + \frac{\partial\mathcal{O}(w,\bar{w})}{z-w} + \cdots\right)\right]\end{aligned}$$

$$\epsilon(z) = \epsilon(w) + \epsilon'(w)(z-w) + \cdots$$

$$\delta\mathcal{O}(w,\bar{w}) = -h\epsilon'(w)\mathcal{O}(w,\bar{w}) - \epsilon(w)\partial\mathcal{O}(w,\bar{w})$$

$$z \to \tilde{z}(z) \text{ and } \bar{z} \to \bar{\tilde{z}}(\bar{z})$$

$$\mathcal{O}(z,\bar{z}) \to \tilde{\mathcal{O}}(\tilde{z},\bar{\tilde{z}}) = \left(\frac{\partial\tilde{z}}{\partial z}\right)^{-h}\left(\frac{\partial\bar{\tilde{z}}}{\partial\bar{z}}\right)^{-\tilde{h}}\mathcal{O}(z,\bar{z})$$

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma\partial_\alpha X\partial^\alpha X$$

$$0 = \int \mathcal{D}X\frac{\delta}{\delta X(\sigma)}e^{-S} = \int \mathcal{D}Xe^{-S}\left[\frac{1}{2\pi\alpha'}\partial^2X(\sigma)\right]$$

$$\langle\partial^2X(\sigma)\rangle = 0$$

$$0 = \int \mathcal{D}X\frac{\delta}{\delta X(\sigma)}[e^{-S}X(\sigma')] = \int \mathcal{D}Xe^{-S}\left[\frac{1}{2\pi\alpha'}\partial^2X(\sigma)X(\sigma') + \delta(\sigma-\sigma')\right]$$

$$\langle\partial^2X(\sigma)X(\sigma')\rangle = -2\pi\alpha'\delta(\sigma-\sigma')$$

$$\partial^2\ln(\sigma-\sigma')^2 = 4\pi\delta(\sigma-\sigma')$$

$$\int d^2\sigma\partial^2 \ln\left(\sigma_1^2+\sigma_2^2\right) = \int d^2\sigma\partial^\alpha\left(\frac{2\sigma_\alpha}{\sigma_1^2+\sigma_2^2}\right) = 2\oint \frac{(\sigma_1 d\sigma^2 - \sigma_2 d\sigma^1)}{\sigma_1^2+\sigma_2^2}$$

$$2\int \frac{r^2 d\theta}{r^2} = 4\pi$$

$$\langle X(\sigma)X(\sigma')\rangle = -\frac{\alpha'}{2}\ln\left(\sigma-\sigma'\right)^2$$

$$X(\sigma)X(\sigma') = -\frac{\alpha'}{2}\ln\left(\sigma-\sigma'\right)^2 + \cdots$$

$$X(z,\bar{z}) = X(z) + \bar{X}(\bar{z})$$

$$X(z)X(w) = -\frac{\alpha'}{2}\ln\left(z-w\right) + \cdots$$

$$\partial X(z)\partial X(w) = -\frac{\alpha'}{2}\frac{1}{(z-w)^2} + \mathcal{ns}$$

$$\langle X(r)X(0)\rangle \sim \begin{cases} 1/r^{d-2} & d\neq 2 \\ \ln\ r & d=2 \end{cases}$$

$$T = -\frac{1}{\alpha'}\partial X\partial X$$

$$T = -\frac{1}{\alpha'}:\partial X\partial X:\equiv -\frac{1}{\alpha'}\mathrm{limit}_{z\to w}\left(\partial X(z)\partial X(w) - \langle\partial X(z)\partial X(w)\rangle\right)$$

$$T(z)\partial X(w) = -\frac{1}{\alpha'}:\partial X(z)\partial X(z):\partial X(w)$$

$$\overbrace{\partial X(z)\partial X(w)} = -\frac{\alpha'}{2}\frac{1}{(z-w)^2}$$

$$T(z)\partial X(w) = -\frac{2}{\alpha'}\partial X(z)\left(-\frac{\alpha'}{2}\frac{1}{(z-w)^2} + \mathcal{ns}\right)$$

$$T(z)\partial X(w) = \frac{\partial X(z)}{(z-w)^2} + \cdots = \frac{\partial X(w)}{(z-w)^2} + \frac{\partial^2 X(w)}{z-w} + \cdots$$

$$T(z)\partial^2 X(w) = \partial_w\left[\frac{\partial X(w)}{(z-w)^2} + \cdots\right] = \frac{2\partial X(w)}{(z-w)^3} + \frac{2\partial^2 X(w)}{(z-w)^2} + \cdots$$

$$\begin{aligned}\partial X(z):e^{ikX(w)}:&= \sum_{n=0}^{\infty}\frac{(ik)^n}{n!}\partial X(z):X(w)^n: \\ &= \sum_{n=1}^{\infty}\frac{(ik)^n}{(n-1)!}:X(w)^{n-1}:\left(-\frac{\alpha'}{2}\frac{1}{z-w}\right) + \cdots \\ &= -\frac{i\alpha' k}{2}\frac{e^{ikX(w)}}{z-w} + \cdots\end{aligned}$$

$$T(z):e^{ikX(w)}: = -\frac{1}{\alpha'}:\partial X(z)\partial X(z)::e^{ikX(w)}:$$

$$= \frac{\alpha' k^2}{4}\frac{:e^{ikX(w)}:}{(z-w)^2} + ik\frac{:\partial X(z)e^{ikX(w)}:}{z-w} + \cdots$$

$$T(z):e^{ikX(w)}: = \frac{\alpha' k^2}{4}\frac{:e^{ikX(w)}:}{(z-w)^2} + \frac{\partial_w :e^{ikX(w)}:}{z-w} + \cdots$$

$$T(z)T(w)=\frac{1}{\alpha'^2}:\partial X(z)\partial X(z)::\partial X(w)\partial X(w):$$
$$=\frac{2}{\alpha'^2}\left(-\frac{\alpha'}{2}\frac{1}{(z-w)^2}\right)^2-\frac{4}{\alpha'^2}\frac{\alpha'}{2}\frac{:\partial X(z)\partial X(w):}{(z-w)^2}+\cdots$$

$$T(z)T(w)=\frac{1/2}{(z-w)^4}+\frac{2T(w)}{(z-w)^2}-\frac{2}{\alpha'}\frac{\partial^2 X(w)\partial X(w)}{z-w}+\cdots$$
$$=\frac{1/2}{(z-w)^4}+\frac{2T(w)}{(z-w)^2}+\frac{\partial T(w)}{z-w}+\cdots$$

$$T(z)T(w)=\cdots+\frac{2T(w)}{(z-w)^2}+\frac{\partial T(w)}{z-w}+\cdots$$

$$\frac{\mathcal{O}_n}{(z-w)^n}$$

$$T(z)T(w)=\frac{c/2}{(z-w)^4}+\frac{2T(w)}{(z-w)^2}+\frac{\partial T(w)}{z-w}+\cdots$$

$$\bar{T}(\bar{z})\bar{T}(\bar{w})=\frac{\tilde{c}/2}{(\bar{z}-\bar{w})^4}+\frac{2\bar{T}(\bar{w})}{(\bar{z}-\bar{w})^2}+\frac{\bar{\partial}\bar{T}(\bar{w})}{\bar{z}-\bar{w}}+\cdots$$

$$T(w)T(z)=\frac{c/2}{(z-w)^4}+\frac{2T(z)}{(z-w)^2}+\frac{\partial T(z)}{w-z}+\cdots$$

$$T(w)T(z)=\frac{c/2}{(z-w)^4}+\frac{2T(w)+2(z-w)\partial T(w)}{(z-w)^2}-\frac{\partial T(w)}{z-w}+\cdots=T(z)T(w)$$

$$\delta T(w)=-\mathrm{Res}[\epsilon(z)T(z)T(w)]$$
$$=-\mathrm{Res}\left[\epsilon(z)\left(\frac{c/2}{(z-w)^4}+\frac{2T(w)}{(z-w)^2}+\frac{\partial T(w)}{z-w}+\cdots\right)\right]$$

$$\epsilon(z)=\epsilon(w)+\epsilon'(w)(z-w)+\frac{1}{2}\epsilon''(z-w)^2+\frac{1}{6}\epsilon'''(w)(z-w)^3+\cdots$$

$$\delta T(w)=-\epsilon(w)\partial T(w)-2\epsilon'(w)T(w)-\frac{c}{12}\epsilon'''(w)$$

$$\tilde{T}(\tilde{z})=\left(\frac{\partial\tilde{z}}{\partial z}\right)^{-2}\left[T(z)-\frac{c}{12}S(\tilde{z},z)\right]$$

$$S(\tilde{z},z)=\left(\frac{\partial^3\tilde{z}}{\partial z^3}\right)\left(\frac{\partial\tilde{z}}{\partial z}\right)^{-1}-\frac{3}{2}\left(\frac{\partial^2\tilde{z}}{\partial z^2}\right)^2\left(\frac{\partial\tilde{z}}{\partial z}\right)^{-2}$$

$$w=\sigma+i\tau\,,\sigma\in|0,2\pi\rangle$$

$$z=e^{-iw}$$

$$T_{\text{curvature}}(w)=-z^2T_{\text{supercurvature}}(z)+\frac{c}{24}$$

$$H\equiv\int d\sigma T_{\tau\tau}=-\int d\sigma(T_{ww}+\bar{T}_{\bar{w}\bar{w}})$$

$$E=-\frac{2\pi(c+\tilde{c})}{24}$$

$$E(L)=TL+a-\frac{\pi c}{24L}+\cdots$$

$$T^{\alpha}_{\alpha}=0$$

$$\langle T^{\alpha}_{\alpha}\rangle = -\frac{c}{12}R$$

$$R = -2e^{-2\omega}\partial^2\omega$$

$$\left\langle T^{\mu}{}_{\mu}\right\rangle_{4d} = \frac{c}{16\pi^2}C_{\rho\sigma\kappa\lambda}C^{\rho\sigma\kappa\lambda} - \frac{a}{16\pi^2}\tilde{R}_{\rho\sigma\kappa\lambda}\tilde{R}^{\rho\sigma\kappa\lambda}$$

$$\langle T^{\alpha}_{\alpha}\rangle = -\frac{\tilde{c}}{12}R$$

$$\partial T_{z\bar{z}} = -\bar{\partial}T_{zz}$$

$$\partial_z T_{z\bar{z}}(z,\bar{z})\partial_w T_{w\bar{w}}(w,\bar{w}) = \bar{\partial}_{\bar{z}}T_{zz}(z,\bar{z})\bar{\partial}_{\bar{w}}T_{ww}(w,\bar{w}) = \bar{\partial}_{\bar{z}}\bar{\partial}_{\bar{w}}\left[\frac{c/2}{(z-w)^4} + \cdots\right]$$

$$\bar{\partial}_{\bar{z}}\partial_z \ln\,|z-w|^2 = \bar{\partial}_{\bar{z}}\frac{1}{z-w} = 2\pi\delta(z-w,\bar{z}-\bar{w})$$

$$\bar{\partial}_{\bar{z}}\bar{\partial}_{\bar{w}}\frac{1}{(z-w)^4} = \frac{1}{6}\bar{\partial}_{\bar{z}}\bar{\partial}_{\bar{w}}\left(\partial_z^2\partial_w\frac{1}{z-w}\right) = \frac{\pi}{3}\partial_z^2\partial_w\bar{\partial}_{\bar{w}}\delta(z-w,\bar{z}-\bar{w})$$

$$T_{z\bar{z}}(z,\bar{z})T_{w\bar{w}}(w,\bar{w}) = \frac{c\pi}{6}\partial_z\bar{\partial}_{\bar{w}}\delta(z-w,\bar{z}-\bar{w})$$

$$\delta\langle T^{\alpha}_{\alpha}(\sigma)\rangle = \delta\int \mathcal{D}\phi e^{-S}T^{\alpha}_{\alpha}(\sigma)$$
$$= \frac{1}{4\pi}\int \mathcal{D}\phi e^{-S}\left(T^{\alpha}_{\alpha}(\sigma)\int d^2\sigma'\sqrt{g}\delta g^{\beta\gamma}T_{\beta\gamma}(\sigma')\right)$$

$$\delta\langle T^{\alpha}_{\alpha}(\sigma)\rangle = -\frac{1}{2\pi}\int \mathcal{D}\phi e^{-S}\left(T^{\alpha}_{\alpha}(\sigma)\int d^2\sigma'\omega(\sigma')T^{\beta}_{\beta}(\sigma')\right)$$

$$T^{\alpha}_{\alpha}(\sigma)T^{\beta}_{\beta}(\sigma') = 16T_{z\bar{z}}(z,\bar{z})T_{w\bar{w}}(w,\bar{w})$$

$$\partial_z\bar{\partial}_{\bar{w}}\delta(z-w,\bar{z}-\bar{w}) = -\partial^2\delta(\sigma-\sigma')$$

$$T^{\alpha}_{\alpha}(\sigma)T^{\beta}_{\beta}(\sigma') = -\frac{c\pi}{3}\partial^2\delta(\sigma-\sigma')$$

$$\delta\langle T^{\alpha}_{\alpha}\rangle = \frac{c}{6}\partial^2\omega \;\Rightarrow\; \langle T^{\alpha}_{\alpha}\rangle = -\frac{c}{12}R$$

$$\tau \in [0,\beta)$$

$$Z[\beta] = \mathrm{Tr}e^{-\beta H} = e^{-\beta F}$$

$$Z \to e^{c\beta/12} \text{ as } \beta \to \infty$$

$$\tau \to \frac{2\pi}{\beta}\tau\,, \sigma \to \frac{2\pi}{\beta}\sigma$$

$$Z[4\pi^2/\beta] = Z[\beta]$$

$$Z[\beta'] \to e^{c\pi^2/3\beta'} \text{ as } \beta' \to 0$$

$$e^{-\beta F} = \int dE\rho(E)e^{-\beta E} = \int dE\,e^{S(E)-\beta E}$$

$$S(E) \to N\sqrt{E}$$

$$F \sim N^2T^2$$

$$S(E) \sim \sqrt{cE}$$

$$S \to S + \alpha \int d^2\sigma \mathcal{O}(\sigma)$$

$$\omega = \sigma + i\tau \,, z = e^{-i\omega}$$

$$H = \partial_\tau$$

$$D = z\partial + \bar{z}\bar{\partial}$$

$$T_{\text{curvature}}(w) = -\sum_{m=-\infty}^{\infty} L_m e^{imw} + \frac{c}{24}$$

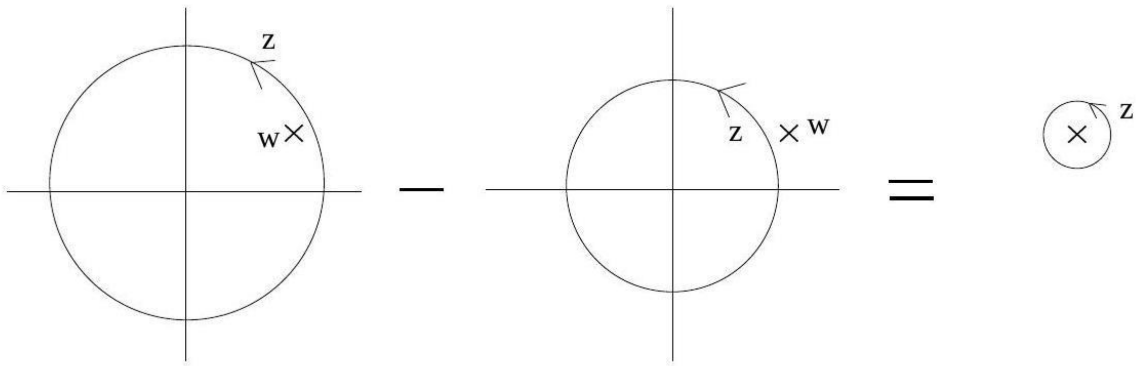
$$T(z) = \sum_{m=-\infty}^{\infty} \frac{L_m}{z^{m+2}}$$

$$\bar{T}(\bar{z}) = \sum_{m=-\infty}^{\infty} \frac{\tilde{L}_m}{\bar{z}^{m+2}}$$

$$L_n = \frac{1}{2\pi i}\oint dz z^{n+1} T(z),\ \tilde{L}_n = \frac{1}{2\pi i}\oint d\bar{z}\bar{z}^{n+1}\bar{T}(\bar{z})$$

$$D = L_0 + \tilde{L}_0$$

$$[L_m, L_n] = \left(\oint \frac{dz}{2\pi i} \oiint \frac{dw}{2\pi i} - \oiiint \frac{dw}{2\pi i} \iiint \frac{dz}{2\pi i}\right) z^{m+1} w^{n+1} T(z) T(w)$$



$$[L_m, L_n] = \oint \frac{dw}{2\pi i} \oint_w \frac{dz}{2\pi i} z^{m+1} w^{n+1} T(z) T(w)$$
$$= \oiint \frac{dw}{2\pi i} \operatorname{Res}\left[z^{m+1} w^{n+1}\left(\frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \cdots\right)\right]$$

$$z^{m+1} = w^{m+1} + (m+1)w^m(z-w) + \frac{1}{2}m(m+1)w^{m-1}(z-w)^2$$
$$+ \frac{1}{6}m(m^2-1)w^{m-2}(z-w)^3 + \cdots$$

$$[L_m, L_n] = \oint \frac{dw}{2\pi i} w^{n+1}\left[w^{m+1}\partial T(w) + 2(m+1)w^m T(w) + \frac{c}{12}m(m^2-1)w^{m-2}\right]$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

$$l_n = z^{n+1}\partial_z$$

$$[l_n, l_m] = (m-n)l_{m+n}$$

$$L_0|\psi\rangle = h|\psi\rangle\,, \tilde{L}_0|\psi\rangle = \tilde{h}|\psi\rangle$$

$$\frac{E}{2\pi} = h + \tilde{h} - \frac{c+\tilde{c}}{24}$$

$$L_0 L_n|\psi\rangle = (L_n L_0 - nL_n)|\psi\rangle = (h-n)L_n|\psi\rangle$$

$$L_n|\psi\rangle = \tilde{L}_n|\psi\rangle = 0 \ \text{ for all } n > 0$$

$$\begin{gathered} |\psi\rangle \\ L_{-1}|\psi\rangle \\ L_{-1}^2|\psi\rangle, L_{-2}|\psi\rangle \\ L_{-1}^3|\psi\rangle, L_{-1}L_{-2}|\psi\rangle, L_{-3}|\psi\rangle \end{gathered}$$

$$L_n|0\rangle = 0 \ \text{ for all } n \geq 0$$

$$L_{-2}|\psi\rangle - \frac{3}{2(2h+1)}L_{-1}^2|\psi\rangle$$

$$\mathcal{H} = T_{ww} + T_{\bar{w}\bar{w}} = \sum_n L_n e^{-in\sigma^+} + \tilde{L}_n e^{-in\sigma^-}$$

$$L_n = L_{-n}^{\dagger}$$

$$|L_{-1}|\psi\rangle|^2 = \langle\psi|L_{+1}L_{-1}|\psi\rangle = \langle\psi|[L_{+1}, L_{-1}]|\psi\rangle = 2h\langle\psi \mid \psi\rangle \geq 0$$

$$|L_{-n}|0\rangle|^2 = \langle 0|[L_n, L_{-n}]|0\rangle = \frac{c}{12}n(n^2-1) \geq 0$$

$$G(x_f, x_i) = \int_{x(\tau_i)=x_i}^{x(\tau_f)=x_f} \mathcal{D}x e^{iS}$$

$$\psi_f(x_f, \tau_f) = \int dx_i G(x_f, x_i)\psi_i(x_i, \tau_i)$$

$$\Psi_f[\phi_f(\sigma), \tau_f] = \int \mathcal{D}\phi_i \int_{\phi(\tau_i)=\phi_i}^{\phi(\tau_f)=\phi_f} \mathcal{D}\phi e^{-S[\phi]}\Psi_i[\phi_i(\sigma), \tau_i]$$

$$\Psi_f[\phi_f(\sigma), r_f] = \int \mathcal{D}\phi_i \int_{\phi(r_i)=\phi_i}^{\phi(r_f)=\phi_f} \mathcal{D}\phi e^{-S[\phi]}\Psi_i[\phi_i(\sigma), r_i]$$

$$\Psi[\phi_f; r] = \int^{\phi(r)=\phi_f} \mathcal{D}\,\phi e^{-S[\phi]}\mathcal{O}(z=0)$$

$$\begin{aligned} L_n|\mathcal{O}\rangle &= \oiint \frac{dz}{2\pi i} z^{n+1} T(z)\mathcal{O}(z=0) \\ &= \oiiint \frac{dz}{2\pi i} z^{n+1}\left(\frac{h\mathcal{O}}{z^2} + \frac{\partial\mathcal{O}}{z} + \cdots\right) \end{aligned}$$

$$L_{-1}|\mathcal{O}\rangle = |\partial\mathcal{O}\rangle \int \mathcal{D}\phi e^{-S[\phi]}$$

$$X(w, \bar{w}) = x + \alpha' p\tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n}\left(\alpha_n e^{inw} + \tilde{\alpha}_n e^{in\bar{w}}\right)$$

$$\partial_w X(w,\bar{w}) = -\sqrt{\frac{\alpha'}{2}}\sum_n \alpha_n e^{inw} \ \text{ with } \alpha_0 \equiv i\sqrt{\frac{\alpha'}{2}}p$$

$$\partial_z X(z) = \left(\frac{\partial z}{\partial w}\right)^{-1} \partial_w X(w) = -i\sqrt{\frac{\alpha'}{2}}\sum_n \frac{\alpha_n}{z^{n+1}}$$

$$\alpha_n = i\sqrt{\frac{2}{\alpha'}}\oint \frac{dz}{2\pi i} z^n \partial X(z)$$

$$\begin{aligned}[\alpha_m,\alpha_n] &= -\frac{2}{\alpha'}\left(\oint \frac{dz}{2\pi i}\bigwedge \frac{dw}{2\pi i} - \bigvee \frac{dw}{2\pi i}\iint \frac{dz}{2\pi i}\right) z^m w^n \partial X(z)\partial X(w) \\ &= -\frac{2}{\alpha'}\bigcup \frac{dw}{2\pi i}\mathrm{Res}_{z=w}\left[z^m w^n\left(\frac{-\alpha'/2}{(z-w)^2} + \cdots\right)\right] \\ &= m\coprod \frac{dw}{2\pi i} w^{m+n-1} = m\delta_{m+n,0}\end{aligned}$$

$$\prod_{m=1}^{\infty} \alpha_{-m}^{k_m}|0;p\rangle$$

$$\Psi_0[X_f] = \int^{X_f(r)} \mathcal{D}\,\mathcal{D}Xe^{-S[X]}$$

$$\alpha_m \Psi_0[X_f] = \int^{X_f} \mathcal{D}\,Xe^{-S[X]}\sum\int \frac{dw}{2\pi i} w^m \partial X(w)$$

$$\iiint \frac{dw}{2\pi i} w^m \partial X(w) = 0 \ \text{ for all } m \geq 0$$

$$\alpha_{-m}|0\rangle = \int \ \mathcal{D}Xe^{-S[X]}\partial^m X(z=0)$$

$$\alpha_n|\partial^m X\rangle \sim \int^{X_f(r)} \mathcal{D}\,Xe^{-S[X]} \oiiint \frac{dw}{2\pi i} w^n \partial X(w)\partial^m X(z=0)$$

$$\bigcap \frac{dw}{2\pi i} w^n \partial_z^{m-1} \frac{1}{(w-z)^2}\bigg|_{z=0} = m!\bigwedge \frac{dw}{2\pi i} w^{n-m-1} = 0 \ \text{ unless } m = n$$

$$|0;p\rangle \sim \int \ \mathcal{D}Xe^{-S[X]}e^{ipX(z=0)}$$

$$z = e^{-iw}$$

$$T_{\alpha\beta}n^\alpha t^\beta = 0 \ \text{ at } \mathrm{Im} z = 0$$

$$T_{zz} = T_{\bar{z}\bar{z}} \ \text{ at } \mathrm{Im} z = 0$$

$$T_{zz}(z) = T_{\bar{z}\bar{z}}(\bar{z})$$

$$L_n = \oint \frac{dz}{2\pi i} z^{n+1} T_{zz}(z)$$

$$\langle X(z,\bar{z})X(w,\bar{w})\rangle = G(z,\bar{z};w,\bar{w})$$

$$\partial^2 G = -2\pi\alpha'\delta(z-w,\bar{z}-\bar{w})$$

$$\partial_\sigma G(z,\bar{z};w,\bar{w})|_{\sigma=0} = 0$$

$$G(z,\bar{z};w,\bar{w}) = -\frac{\alpha'}{2}\ln\ |z-w|^2 - \frac{\alpha'}{2}\ln\ |z-\bar{w}|^2$$

$$\langle T^{\alpha}_{\alpha} \rangle = -\frac{c}{12} R$$

$$S_{\text{Poly}} = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \delta_{\mu\nu}$$

$$Z = \frac{1}{\text{Vol}} \int \mathcal{D}g \mathcal{D}X e^{-S_{\text{Poly}}[X,g]}$$

$$g_{\alpha\beta}(\sigma) \longrightarrow g^{\zeta}_{\alpha\beta}(\sigma') = e^{2\omega(\sigma)} \frac{\partial \sigma^\gamma}{\partial \sigma'^\alpha} \frac{\partial \sigma^\delta}{\partial \sigma'^\beta} g_{\gamma\delta}(\sigma)$$

$$\int \mathcal{D}\zeta \delta\left(g - \hat{g}^{\zeta}\right) = \Delta^{-1}_{FP}[g]$$

$$\mathcal{D}\zeta = \mathcal{D}(\zeta'\zeta) = \mathcal{D}(\zeta\zeta')$$

$$\Delta_{FP}[g] = \Delta_{FP}\left[g^{\zeta}\right]$$

$$\begin{aligned} \Delta^{-1}_{FP}\left[g^{\zeta}\right] &= \int \mathcal{D}\zeta' \delta\left(g^{\zeta} - \hat{g}^{\zeta'}\right) \\ &= \int \mathcal{D}\zeta' \delta\left(g - \hat{g}^{\zeta^{-1}\zeta'}\right) \\ &= \int \mathcal{D}\zeta'' \delta\left(g - \hat{g}^{\zeta''}\right) = \Delta^{-1}_{FP}[g] \end{aligned}$$

$$1 = \Delta_{FP}[g] \int \mathcal{D}\zeta \delta\left(g - \hat{g}^{\zeta}\right)$$

$$\begin{aligned} Z[\hat{g}] &= \frac{1}{\text{Vol}} \int \mathcal{D}\zeta \mathcal{D}X \mathcal{D}g \Delta_{FP}[g] \delta\left(g - \hat{g}^{\zeta}\right) e^{-S_{\text{Poly}}[X,g]} \\ &= \frac{1}{\text{Vol}} \int \mathcal{D}\zeta \mathcal{D}X \Delta_{FP}\left[\hat{g}^{\zeta}\right] e^{-S_{\text{Poly}}[X,\hat{g}^{\zeta}]} \\ &= \frac{1}{\text{Vol}} \int \mathcal{D}\zeta \mathcal{D}X \Delta_{FP}[\hat{g}] e^{-S_{\text{Poly}}[X,\hat{g}]} \end{aligned}$$

$$Z[\hat{g}] = \int \mathcal{D}X \Delta_{FP}[\hat{g}] e^{-S_{\text{Poly}}[X,\hat{g}]}$$

$$\delta \hat{g}_{\alpha\beta} = 2\omega \hat{g}_{\alpha\beta} + \nabla_\alpha v_\beta + \nabla_\beta v_\alpha$$

$$\Delta^{-1}_{FP}[\hat{g}] = \int \mathcal{D}\omega \mathcal{D}v \delta\left(2\omega \hat{g}_{\alpha\beta} + \nabla_\alpha v_\beta + \nabla_\beta v_\alpha\right)$$

$$\Delta^{-1}_{FP}[\hat{g}] = \int \mathcal{D}\omega \mathcal{D}v \mathcal{D}\beta \exp\left(2\pi i \int d^2\sigma \sqrt{\hat{g}} \beta^{\alpha\beta}\left[2\omega \hat{g}_{\alpha\beta} + \nabla_\alpha v_\beta + \nabla_\beta v_\alpha\right]\right)$$

$$\beta^{\alpha\beta} \hat{g}_{\alpha\beta} = 0$$

$$\Delta^{-1}_{FP}[\hat{g}] = \int \mathcal{D}v \mathcal{D}\beta \exp\left(4\pi i \int d^2\sigma \sqrt{\hat{g}} \beta^{\alpha\beta} \nabla_\alpha v_\beta\right)$$

$$\begin{aligned} \beta_{\alpha\beta} &\longrightarrow b_{\alpha\beta} \\ v^\alpha &\longrightarrow c^\alpha \end{aligned}$$

$$\Delta_{FP}[g] = \int \mathcal{D}b \mathcal{D}c \exp\left[iS_{\text{ghost}}\right]$$

$$S_{\text{ghost}} = \frac{1}{2\pi} \int d^2\sigma \sqrt{g} b_{\alpha\beta} \nabla^\alpha c^\beta$$

$$Z[\hat{g}] = \int \mathcal{D}X \mathcal{D}b \mathcal{D}c \exp\left(-S_{\text{Poly}}[X,\hat{g}] - S_{\text{ghost}}[b,c,\hat{g}]\right)$$

$$\hat{g}_{\alpha\beta} = e^{2\omega} \delta_{\alpha\beta}$$

$$S_{\text{ghost}} = \frac{1}{2\pi}\int d^2z(b_{zz}\nabla_{\bar{z}}c^z + b_{\bar{z}\bar{z}}\nabla_z c^{\bar{z}})$$

$$\nabla_{\bar{z}}c^z = \partial_{\bar{z}}c^z + \Gamma^z_{\bar{z}\alpha}c^\alpha$$

$$\Gamma^z_{\bar{z}\alpha} = \frac{1}{2}g^{z\bar{z}}(\partial_{\bar{z}}g_{\alpha\bar{z}} + \partial_\alpha g_{\bar{z}\bar{z}} - \partial_{\bar{z}}g_{\bar{z}\alpha}) = 0 \ \text{ for } \alpha = z, \bar{z}$$

$$S_{\text{ghost}} = \frac{1}{2\pi}\int d^2z b_{zz}\partial_{\bar{z}}c^z + b_{\bar{z}\bar{z}}\partial_z c^{\bar{z}}$$

$$b = b_{zz} \quad , \quad \bar{b} = b_{\bar{z}\bar{z}}$$
$$c = c^z \quad , \quad \bar{c} = c^{\bar{z}}$$

$$S_{\text{ghost}} = \frac{1}{2\pi}\int d^2z(b\bar{\partial}c + \bar{b}\partial\bar{c})$$

$$\bar{\partial}b = \partial\bar{b} = \bar{\partial}c = \partial\bar{c} = 0$$

$$T = 2(\partial c)b + c\partial b\,, \bar{T} = 2(\bar{\partial}\bar{c})\bar{b} + \bar{c}\bar{\partial}\bar{b}$$

$$0 = \int \mathcal{D}b\mathcal{D}c\frac{\delta}{\delta b(\sigma)}[e^{-S_{\text{ghost}}}b(\sigma')] = \int \mathcal{D}b\mathcal{D}cs^{-S_{\text{ghost}}}\left[-\frac{1}{2\pi}\bar{\partial}c(\sigma)b(\sigma') + \delta(\sigma-\sigma')\right]$$

$$\bar{\partial}c(\sigma)b(\sigma') = 2\pi\delta(\sigma-\sigma')$$

$$\bar{\partial}b(\sigma)c(\sigma') = 2\pi\delta(\sigma-\sigma')$$

$$b(z)c(w) = \frac{1}{z-w} + \cdots$$
$$c(w)b(z) = \frac{1}{w-z} + \cdots$$

$$T(z) = 2{:}\partial c(z)b(z){:} + {:}c(z)\partial b(z){:}$$

$$\begin{aligned}T(z)c(w) &= 2{:}\partial c(z)b(z){:}c(w) + {:}c(z)\partial b(z){:}c(w)\\ &= \frac{2\partial c(z)}{z-w} - \frac{c(z)}{(z-w)^2} + \cdots = -\frac{c(w)}{(z-w)^2} + \frac{\partial c(w)}{z-w} + \cdots\end{aligned}$$

$$\begin{aligned}T(z)b(w) &= 2{:}\partial c(z)b(z){:}b(w) + {:}c(z)\partial b(z){:}b(w)\\ &= -2b(z)\left(\frac{-1}{(z-w)^2}\right) - \frac{\partial b(z)}{z-w} = \frac{2b(w)}{(z-w)^2} + \frac{\partial b(w)}{z-w} + \cdots\end{aligned}$$

$$\begin{aligned}T(z)T(w) =& 4{:}\partial c(z)b(z){:}{:}\partial c(w)b(w){:} + 2{:}\partial c(z)b(z){:}{:}c(w)\partial b(w){:}\\ &+ 2{:}c(z)\partial b(z){:}{:}\partial c(w)b(w){:} + {:}c(z)\partial b(z){:}{:}c(w)\partial b(w){:}\end{aligned}$$

$$\begin{aligned}T(z)T(w) =& \frac{-4}{(z-w)^4} + \frac{4{:}\partial c(z)b(w){:}}{(z-w)^2} - \frac{4{:}b(z)\partial c(w){:}}{(z-w)^2}\\ &- \frac{4}{(z-w)^4} + \frac{2{:}\partial c(z)\partial b(w){:}}{z-w} - \frac{4{:}b(z)c(w){:}}{(z-w)^3}\\ &- \frac{4}{(z-w)^4} - \frac{4{:}c(z)b(w){:}}{(z-w)^3} + \frac{2{:}\partial b(z)\partial c(w){:}}{z-w}\\ &- \frac{1}{(z-w)^4} - \frac{{:}c(z)\partial b(w){:}}{(z-w)^2} + \frac{\partial b(z)c(w){:}}{(z-w)^2} + \cdots\end{aligned}$$

$$T(z)T(w) = \frac{-13}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \cdots$$

$$S_{\text{non-critical}} = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}\left(g^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu + \mu\right)$$

$$\hat{g}_{\alpha\beta} = e^{2\omega} g_{\alpha\beta}$$

$$\begin{aligned}\frac{1}{Z}\frac{\partial Z}{\partial \omega} &= \frac{1}{Z}\int \mathcal{D}\phi e^{-S}\left(-\frac{\partial S}{\partial \hat{g}_{\alpha\beta}}\frac{\partial \hat{g}_{\alpha\beta}}{\partial \omega}\right)\\ &= \frac{1}{Z}\int \mathcal{D}\phi e^{-S}\left(-\frac{1}{2\pi}\sqrt{\hat{g}}T_{\alpha}^{\alpha}\right)\\ &= \frac{c}{24\pi}\sqrt{\hat{g}}\hat{R} - \frac{1}{2\pi\alpha'}\mu e^{2\omega}\\ &= \frac{c}{24\pi}\sqrt{g}(R - 2\nabla^2\omega) - \frac{1}{2\pi\alpha'}\mu e^{2\omega}\end{aligned}$$

$$Z[\hat{g}] = Z[g]\exp\left[-\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}\left(2\mu e^{2\omega} - \frac{c\alpha'}{6}\left(g^{\alpha\beta}\partial_\alpha\omega\partial^\beta\omega + R\omega\right)\right)\right]$$

$$\begin{aligned}L_n|\Psi\rangle &= 0\\ L_0|\Psi\rangle &= a|\Psi\rangle\end{aligned}$$

$$\begin{aligned}\tilde{L}_n|\Psi\rangle &= 0 \qquad \text{for } n > 0\\ \tilde{L}_0|\Psi\rangle &= \tilde{a}|\Psi\rangle\end{aligned}$$

$$V \sim \int d^2z\mathcal{O}$$

$$V_{\text{dark particle/white particle}} \sim \int d^2z : e^{ip\cdot X}:$$

$$M^2 \equiv -p^2 = -\frac{4}{\alpha'}$$

$$V_{\text{excited}} \sim \int d^2z : e^{ip\cdot X}\partial X^\mu\bar{\partial}X^\nu : \zeta_{\mu\nu}$$

$$p^2 = 0$$

$$p^\mu\zeta_{\mu\nu} = p^\nu\zeta_{\mu\nu} = 0$$

$$V_{\text{dark particle/white particle}} \sim \int_{\partial\mathcal{M}} ds : e^{ip\cdot X}:$$

$$\begin{aligned}\partial X(z) : e^{ipX(w,\bar{w})} : &= \sum_{n=1}^{\infty}\frac{(ip)^n}{(n-1)!} : X(w,\bar{w})^{n-1} : \left(-\frac{\alpha'}{2}\frac{1}{z-w} - \frac{\alpha'}{2}\frac{1}{z-\bar{w}}\right) + \cdots\\ &= -\frac{i\alpha' p}{2} : e^{ipX(w,\bar{w})} : \left(\frac{1}{z-w} + \frac{1}{z-\bar{w}}\right) + \cdots\end{aligned}$$

$$T(z) : e^{ipX(w,\bar{w})} : = \frac{\alpha' p^2}{4} : e^{ipX} : \left(\frac{1}{z-w} + \frac{1}{z-\bar{w}}\right)^2 + \cdots$$

$$T(z) : e^{ipX(w,\bar{w})} : = \frac{\alpha' p^2 : e^{ipX(w,\bar{w})} :}{(z-w)^2} + \cdots$$

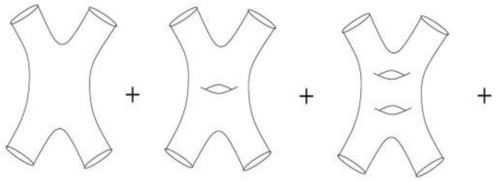
$$M^2 \equiv -p^2 = -\frac{1}{\alpha'}$$

$$V_{\text{dark particle/white particle}} \sim \int_{\partial\mathcal{M}} ds\zeta_a : \partial X^a e^{ip\cdot X}:$$

$$\frac{\alpha' p^2}{4} = 1 - h$$

$$M^2 = \frac{4}{\alpha'}(h-1)$$

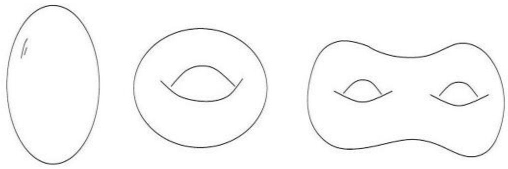
$$\langle \phi(x_1) \dots \phi(x_n) \rangle$$



$$S_{\text{survature}} = S_{\text{Poly}} + \lambda\chi$$

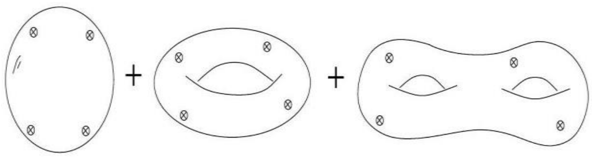
$$\chi = \frac{1}{4\pi}\int d^2\sigma\sqrt{g}R$$

$$\chi = 2 - 2h = 2(1-g)$$



$$\sum_{\substack{\text{topologies}\\ \text{metrics}}} e^{-S_{\text{curvature}}} \sim \sum_{\text{topologies}} e^{-2\lambda(1-g)} \int \mathcal{D}X\mathcal{D}g e^{-S_{\text{supercurvature}}}$$

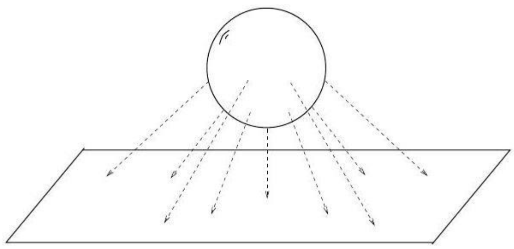
$$g_s = e^{\lambda}$$



$$(g_s^2)^{g-1}$$

$$\mathcal{A}^{(m)}(\Lambda_i, p_i) = \sum_{\text{topologies}} g_s^{-\chi} \frac{1}{\text{Vol}} \int \mathcal{D}X\mathcal{D}g e^{-S_{\text{supercurvature}}} \prod_{i=1}^{m} V_{\Lambda_i}(p_i)$$

$$\mathcal{A}^{(m)} = \frac{1}{g_s^2}\frac{1}{\text{Vol}} \int \mathcal{D}X\mathcal{D}g e^{-S_{\text{supercurvature}}} \prod_{i=1}^{m} V_{\Lambda_i}(p_i)$$



$$ds^2 = \frac{4R^2}{(1+|z|^2)^2} dz d\bar{z}$$

$$l_n = z^{n+1}\partial_z$$

$$u = \frac{1}{z}$$

$$l_n = z^{n+1}\partial_z = \frac{1}{u^{n+1}}\frac{\partial u}{\partial z}\partial_u = -u^{1-n}\partial_u$$

$$l_{-1}: z \to z + \epsilon$$
$$l_0: z \to (1+\epsilon)z$$
$$l_1: z \to (1+\epsilon z)z$$

$$l_{-1}: z \to z + \alpha$$
$$l_0: z \to \lambda z$$
$$l_1: z \to \frac{z}{1-\beta z}$$

$$z \to \frac{az+b}{cz+d}$$

$$ad - bc = 1$$

$$\mathcal{A}^{(m)}(p_1, \dots, p_m) = \frac{1}{g_s^2}\frac{1}{\mathrm{Vol}(SL(2;\mathbf{C}))}\int \mathcal{D}X e^{-S_{\mathrm{Poly}}}\prod_{i=1}^{m} V(p_i)$$

$$V(p_i) = g_s \int d^2 z e^{ip_i \cdot X} \equiv g_s \int d^2 z \hat{V}(z, p_i)$$

$$\mathcal{A}^{(m)}(p_1, \dots, p_m) = \frac{g_s^{m-2}}{\mathrm{Vol}(SL(2;\mathbf{C}))}\int \prod_{i=1}^{m} d^2 z_i \langle \hat{V}(z_1, p_1) \dots \hat{V}(z_m, p_m) \rangle$$

$$\langle \hat{V}(z_1, p_1) \dots \hat{V}(z_m, p_m) \rangle = \int \mathcal{D}X \exp\left(-\frac{1}{2\pi\alpha'}\int d^2 z \partial X \cdot \bar{\partial} X\right) \exp\left(i\sum_{i=1}^{m} p_i \cdot X(z_i, \bar{z}_i)\right)$$

$$\int \mathcal{D}X \exp\left(\int d^2 z \frac{1}{2\pi\alpha'} X \cdot \partial\bar{\partial} X + iJ \cdot X\right) \sim \exp\left(\frac{\pi\alpha'}{2}\int d^2 z d^2 z' J(z, \bar{z}) \frac{1}{\partial\bar{\partial}} J(z', \bar{z}')\right)$$

$$\partial\bar{\partial} G(z, \bar{z}; z', \bar{z}') = \delta(z - z', \bar{z} - \bar{z}')$$

$$G(z, \bar{z}; z', \bar{z}') = \frac{1}{2\pi}\ln |z - z'|^2$$

$$J(z, \bar{z}) = \sum_{i=1}^{m} p_i \delta(z - z_i, \bar{z} - \bar{z}_i)$$

$$\mathcal{A}^{(m)} \sim \frac{g_s^{m-2}}{\mathrm{Vol}(SL(2;\mathbf{C}))}\int \prod_{i=1}^{m} d^2 z_i \exp\left(\frac{\alpha'}{2}\sum_{j,l} p_j \cdot p_l \ln |z_j - z_l|\right)$$

$$\mathcal{A}^{(m)} \sim \frac{g_s^{m-2}}{\mathrm{Vol}(SL(2;\mathbf{C}))}\int \prod_{i=1}^{m} d^2 z_i \prod_{j<l} |z_j - z_l|^{\alpha' p_j \cdot p_l}$$

$$\int dx \exp\left(i\sum_{i=1}^{m} p_i \cdot x\right) \sim \delta^{26}\left(\sum_{i=1}^{m} p_i\right)$$

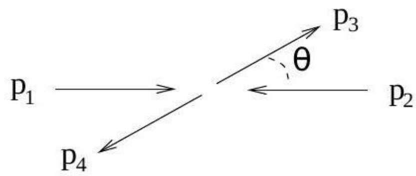
$$\mathcal{A}^{(m)} \sim \frac{g_s^{m-2}}{\mathrm{Vol}(SL(2;\mathbf{C}))}\delta^{26}\left(\sum_i p_i\right)\int \prod_{i=1}^{m} d^2 z_i \prod_{j<l} |z_j - z_l|^{\alpha' p_j \cdot p_l}$$

$$z_1 = \infty, z_2 = 0, z_3 = z, z_4 = 1$$

$$\mathcal{A}^{(4)} \sim g_s^2 \delta^{26}\left(\sum_i p_i\right)\int d^2 z |z|^{\alpha' p_2 \cdot p_3} |1 - z|^{\alpha' p_3 \cdot p_4}$$

$$\int d^2z|z|^{2a-2}|1-z|^{2b-2} = \frac{2\pi\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(1-a)\Gamma(1-b)\Gamma(1-c)}$$

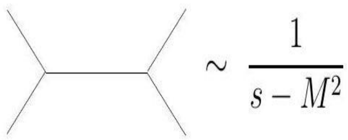
$$s = -(p_1+p_2)^2\,, t = -(p_1+p_3)^2\,, u = -(p_1+p_4)^2$$

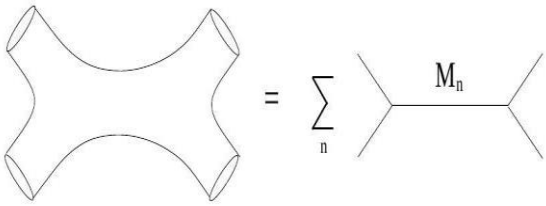


$$s+t+u = -\sum_i p_i^2 = \sum_i M_i^2 = -\frac{16}{\alpha'}$$

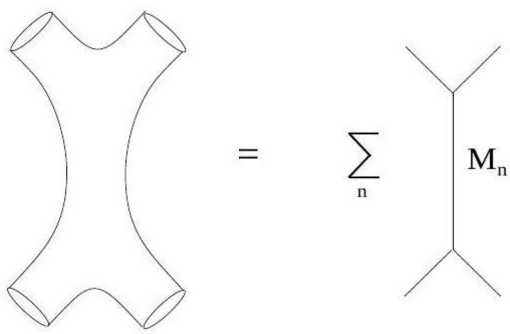
$$\mathcal{A}^{(4)} \sim g_s^2\delta^{26}\left(\sum_i p_i\right)\frac{\Gamma(-1-\alpha' s/4)\Gamma(-1-\alpha' t/4)\Gamma(-1-\alpha' u/4)}{\Gamma(2+\alpha' s/4)\Gamma(2+\alpha' t/4)\Gamma(2+\alpha' u/4)}$$

$$-1-\frac{\alpha' s}{4} = 0 \;\Rightarrow\; s = -\frac{4}{\alpha'}$$





$$\mathcal{A}^{(4)} \sim \sum_{n=0}^{\infty}\frac{t^{2n}}{s-M_n^2}$$

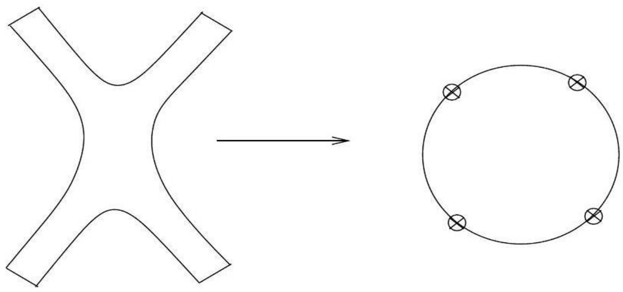


$$p_1 = \frac{\sqrt{s}}{2}(1,1,0,\dots), p_2 = \frac{\sqrt{s}}{2}(1,-1,0,\dots)$$
$$p_3 = \frac{\sqrt{s}}{2}(1,\cos\,\theta,\sin\,\theta,\dots)\,, p_4 = \frac{\sqrt{s}}{2}(1,-\cos\,\theta,-\sin\,\theta,\dots)$$

$$\mathcal{A}^{(4)} \sim g_s^2\delta^{26}\left(\sum_i p_i\right)\exp\left(-\frac{\alpha'}{2}(s\ln\,s + t\ln\,t + u\ln\,u)\right) \quad \text{as } s\to\infty$$

$$S=\frac{1}{2\kappa^2}\int d^{26}X\sqrt{-G}\mathcal{R}$$

$$\kappa^2\approx g_s^2(\alpha')^{12}$$



$$\chi=\frac{1}{4\pi}\int_{\mathcal{M}}d^2\sigma\sqrt{g}R+\frac{1}{2\pi}\int_{\partial\mathcal{M}}dsk$$

$$k=-t^{\alpha}n_{\beta}\nabla_{\alpha}t^{\beta}$$

$$\chi=2-2h-b$$

$$g_{\text{open}}^2=g_s$$

$$z\rightarrow\frac{az+b}{cz+d}$$

$$V(p_i)=\sqrt{g_s}\int dxe^{ip_i\cdot X}$$

$$\mathcal{A}^{(4)}\sim\frac{g_s}{\text{Vol}(SL(2;\mathbf{R}))}\delta^{26}\left(\sum_i p_i\right)\int\prod_{i=1}^{4}dx_i\prod_{j<l}\left|x_i-x_j\right|^{2\alpha' p_i\cdot p_j}$$

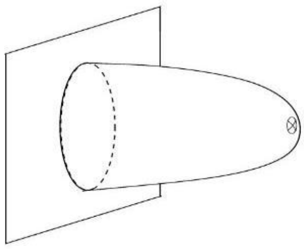
$$\mathcal{A}^{(4)}\sim g_s\int_0^1 dx|x|^{2\alpha' p_1\cdot p_2}|1-x|^{2\alpha' p_2\cdot p_3}$$

$$B(a,b)=\int_0^1 dxx^{a-1}(1-x)^{b-1}=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

$$\mathcal{A}^{(4)}\sim g_s[B(-\alpha' s-1,-\alpha' t-1)+B(-\alpha' s-1,-\alpha' u-1)+B(-\alpha' t-1,-\alpha' u-1)]$$

$$s=\frac{n-1}{\alpha'}\ n=0,1,2,\ldots$$

$$T_p\sim\frac{1}{l_s^{p+1}}\frac{1}{g_s}$$



$$z\equiv z+2\pi\ \text{ and }\ z\equiv z+2\pi\tau$$

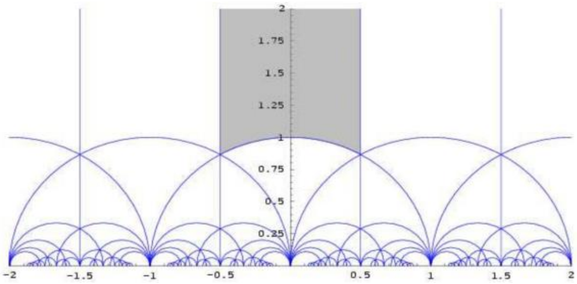
$$ds^2=dzd\bar{z}$$

$$z \equiv z + 2\pi \text{ and } z \equiv z + 2\pi(\tau + 1) \equiv z + 2\pi\tau$$

$$\tau \to \frac{a\tau + b}{c\tau + d} \text{ with } ad - bc = 1$$

$$\mathcal{M} \cong \mathbf{C}/SL(2;\mathbf{Z})$$

$$\mathrm{Re}\tau \in \left[-\frac{1}{2}, +\frac{1}{2}\right]$$



$$|\tau| \geq 1 \text{ and } \mathrm{Re}\tau \in \left[-\frac{1}{2}, +\frac{1}{2}\right]$$

$$\int \frac{d^2\tau}{(\mathrm{Im}\tau)^2}$$

$$d^2\tau \to \frac{d^2\tau}{|c\tau + d|^4} \text{ and } \mathrm{Im}\tau \to \frac{\mathrm{Im}\tau}{|c\tau + d|^2}$$

$$\dim\mathcal{M}_g = 3g - 2$$

$$Z[\tau] = \mathrm{Tr} e^{-2\pi(\mathrm{Im}\tau)H}$$

$$H = L_0 + \tilde{L}_0 - \frac{c + \tilde{c}}{24}$$

$$P = L_0 - \tilde{L}_0$$

$$Z[\tau] = \mathrm{Tr} e^{-2\pi(\mathrm{Im}\tau)(L_0 + \tilde{L}_0)} e^{-2\pi i(\mathrm{Re}\tau)(L_0 - \tilde{L}_0)} e^{2\pi(\mathrm{Im}\tau)(c + \tilde{c})/24}$$

$$q = e^{2\pi i\tau}, \bar{q} = e^{-2\pi i\bar{\tau}}$$

$$Z[\tau] = \mathrm{Tr} q^{L_0 - c/24} \bar{q}^{\tilde{L}_0 - \tilde{c}/24}$$

$$\sum_{d=0}^{\infty} q^{nd} = \frac{1}{1 - q^n}$$

$$\mathrm{Tr} q^{L_0 - c/24} = \frac{1}{q^{1/24}} \prod_{n=1}^{\infty} \frac{1}{1 - q^n}$$

$$\frac{1}{4\pi\alpha'} \int d\sigma(\alpha' p)^2 = \frac{1}{2}\alpha' p^2$$

$$\int \frac{dp}{2\pi} e^{-\pi\alpha'(\mathrm{Im}\tau)p^2} \sim \frac{1}{\sqrt{\alpha' \mathrm{Im}\tau}}$$

$$Z_{\mathrm{scalar}}[\tau] \sim \frac{1}{\sqrt{\alpha' \mathrm{Im}\tau}} \frac{1}{(q\bar{q})^{1/24}} \prod_{n=1}^{\infty} \frac{1}{1 - q^n} \prod_{n=1}^{\infty} \frac{1}{1 - \bar{q}^n}$$

$$Z_{\text{supercurvature}} = \int d^2\tau \frac{1}{(\text{Im}\tau)} \frac{1}{(\alpha' \text{Im}\tau)^{13}} \frac{1}{q\bar{q}} \left(\prod_{n=1}^{\infty} \frac{1}{1-q^n} \right)^{24} \left(\prod_{n=1}^{\infty} \frac{1}{1-\bar{q}^n} \right)^{24}$$

$$\eta(q) = q^{1/24} \prod_{n=1}^{\infty} (1-q^n)$$

$$\eta(\tau+1) = e^{2\pi i/24}\eta(\tau) \text{ and } \eta(-1/\tau) = \sqrt{-i\tau}\eta(\tau)$$

$$Z_{\text{supercurvature}} = \int \frac{d^2\tau}{(\text{Im}\tau)^2} \left(\frac{1}{\sqrt{\text{Im}\tau}} \frac{1}{\eta(q)} \frac{1}{\bar{\eta}(\bar{q})} \right)^{24}$$

$$Z = \int \mathcal{D}\phi \exp\left(-\frac{1}{2} \int d^D x \phi(-\partial^2 + m^2)\phi \right)$$
$$\sim \det^{-1/2}(-\partial^2 + m^2)$$
$$= \exp\left(\frac{1}{2} \int \frac{d^D p}{(2\pi)^D} \ln(p^2 + m^2) \right)$$

$$Z = \exp(Z_1) = \sum_{n=0}^{\infty} \frac{Z_1^n}{n!}$$

$$Z_1 = \frac{1}{2} \int \frac{d^D p}{(2\pi)^D} \ln(p^2 + m^2)$$

$$\int_0^{\infty} dl e^{-xl} = \frac{1}{x} \Rightarrow \int_0^{\infty} dl \frac{e^{-xl}}{l} = -\ln x$$

$$Z_1 = \int \frac{d^D p}{(2\pi)^D} \int_0^{\infty} \frac{dl}{2l} e^{-(p^2+m^2)l}$$

$$Z_1 = \int_0^{\infty} dl \frac{1}{l^{1+D/2}} e^{-m^2 l}$$

$$Z = \int_0^{\infty} dl \frac{1}{l^{14}} \sum_{n=0}^{\infty} e^{-m_n^2 l}$$

$$m^2 = \frac{4}{\alpha'}(L_0 - 1) = \frac{4}{\alpha'}(\tilde{L}_0 - 1) = \frac{2}{\alpha'}(L_0 + \tilde{L}_0 - 2)$$

$$\frac{1}{2\pi} \int_{-1/2}^{+1/2} ds e^{2\pi i s(L_0 - \tilde{L}_0)} = \delta_{L_0, \tilde{L}_0}$$

$$Z = \int_0^{\infty} dl \frac{1}{l^{14}} \int_{-1/2}^{+1/2} ds \text{Tr} e^{2\pi i s(L_0 - \tilde{L}_0)} e^{-2(L_0 + \tilde{L}_0 - 2)l/\alpha'}$$

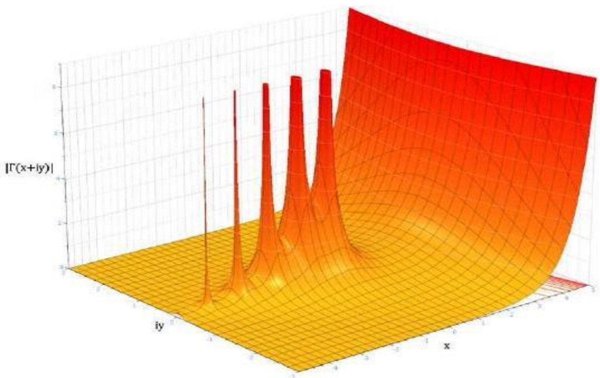
$$\tau = s + \frac{2li}{\alpha'}$$

$$Z_{\text{supercurvature}} = \int \frac{d^2\tau}{(\text{Im}\tau)^2} \left(\frac{1}{\sqrt{\text{Im}\tau}} \frac{1}{\eta(q)} \frac{1}{\bar{\eta}(\bar{q})} \right)^{24}$$

$$\int^{\infty} \frac{dl}{l^{14}} e^{+4l/\alpha'}$$

$$Z = \int \mathcal{D}\Phi e^{iS[\Phi(X(\sigma))]}$$

$$\Gamma(z) = \int_0^{\infty} dt t^{z-1} e^{-t}$$



$$\Gamma(n) = (n-1)!\ n \in \mathbf{Z}^+$$

$$\Gamma(z) \approx \frac{1}{z+n}\frac{(-1)^n}{n!}$$

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$B(x,y) = \int_0^1 dt t^{x-1}(1-t)^{y-1}$$

$$\Gamma(x)\Gamma(y) = \int_0^\infty du \int_0^\infty dv e^{-u}u^{x-1}e^{-v}v^{y-1}$$

$$\begin{aligned}\Gamma(x)\Gamma(y) &= 4\int_0^\infty da \int_0^\infty db e^{-(a^2+b^2)}a^{2x-1}b^{2y-1} \\ &= \int_{-\infty}^\infty da \int_{-\infty}^\infty db e^{-(a^2+b^2)}|a|^{2x-1}|b|^{2y-1}\end{aligned}$$

$$\begin{aligned}\Gamma(x)\Gamma(y) &= \int_0^\infty r dr e^{-r^2}r^{2x+2y-2}\int_0^{2\pi} d\theta|\cos\ \theta|^{2x-1}|\sin\ \theta|^{2y-1} \\ &= \frac{1}{2}\Gamma(x+y)\times 4\int_0^{\pi/2} d\theta(\cos\ \theta)^{2x-1}(\sin\ \theta)^{2y-1} \\ &= \Gamma(x+y)\int_0^1 dt(1-t)^{y-1}t^{x-1}\end{aligned}$$

$$C(a,b) = \int\ d^2z|z|^{2a-2}|1-z|^{2b-2}$$

$$|z|^{2a-2} = \frac{1}{\Gamma(1-a)}\int_0^\infty dt t^{-a}e^{-|z|^2 t}$$

$$|1-z|^{2b-2} = \frac{1}{\Gamma(1-b)}\int_0^\infty du u^{-b}e^{-|1-z|^2 u}$$

$$\begin{aligned}C(a,b) &= \int\ \frac{d^2z du dt}{\Gamma(1-a)\Gamma(1-b)}t^{-a}u^{-b}e^{-|z|^2 t}e^{-|1-z|^2 u} \\ &= 2\int\ \frac{dx dy du dt}{\Gamma(1-a)\Gamma(1-b)}t^{-a}u^{-b}e^{-(t+u)(x^2+y^2)+2xu-u} \\ &= 2\int\ \frac{dx dy du dt}{\Gamma(1-a)\Gamma(1-b)}t^{-a}u^{-b}\exp\left(-(t+u)\left[\left(x-\frac{u}{t+u}\right)^2+y^2\right]-u+\frac{u^2}{t+u}\right)\end{aligned}$$

$$C(a,b) = \frac{2\pi}{\Gamma(1-a)\Gamma(1-b)}\int_0^\infty du dt \frac{t^{-a}u^{-b}}{t+u}e^{-tu/(t+u)}$$

$$C(a,b) = \frac{2\pi}{\Gamma(1-a)\Gamma(1-b)} \int d\alpha d\beta \frac{\alpha^{1-a-b}}{\alpha} \beta^{-a}(1-\beta)^{-b} e^{-\alpha\beta(1-\beta)}$$

$$\int_0^\infty d\alpha \alpha^{-a-b} e^{-\beta\alpha(1-\beta)} = [\beta(1-\beta)]^{a+b-1}\Gamma(1-a-b)$$

$$C(a,b) = \frac{2\pi\Gamma(c)}{\Gamma(1-a)\Gamma(1-b)} \int_0^1 d\beta(1-\beta)^{a-1}\beta^{b-1}$$

$$C(a,b) = \frac{2\pi\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(1-a)\Gamma(1-b)\Gamma(1-c)}$$

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X)$$

$$G_{\mu\nu}(X) = \delta_{\mu\nu} + h_{\mu\nu}(X)$$

$$Z = \int \mathcal{D}X\mathcal{D}g e^{-S_{\text{Poly}}-V} = \int \mathcal{D}X\mathcal{D}g e^{-S_{\text{Poly}}} \left(1 - V + \frac{1}{2}V^2 + \cdots\right)$$

$$V = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu h_{\mu\nu}(X)$$

$$h_{\mu\nu}(X) = \zeta_{\mu\nu} e^{ip\cdot X}$$

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma G_{\mu\nu}(X) \partial_\alpha X^\mu \partial^\alpha X^\nu$$

$$X^\mu(\sigma) = \bar{x}^\mu + \sqrt{\alpha'} Y^\mu(\sigma)$$

$$G_{\mu\nu}(X)\partial X^\mu \partial X^\nu = \alpha' \left[G_{\mu\nu}(\bar{x}) + \sqrt{\alpha'} G_{\mu\nu,\omega}(\bar{x}) Y^\omega + \frac{\alpha'}{2} G_{\mu\nu,\omega\rho}(\bar{x}) Y^\omega Y^\rho + \cdots \right] \partial Y^\mu \partial Y^\nu$$

$$\frac{\partial G}{\partial X} \sim \frac{1}{r_c}$$

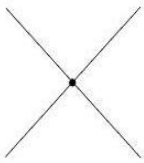
$$\frac{\sqrt{\alpha'}}{r_c}$$

$$\beta_{\mu\nu}(G) \sim \mu \frac{\partial G_{\mu\nu}(X;\mu)}{\partial \mu}$$

$$\beta_{\mu\nu}(G) = 0$$

$$G_{\mu\nu}(X) = \delta_{\mu\nu} - \frac{\alpha'}{3} \mathcal{R}_{\mu\lambda\nu\kappa}(\bar{x}) Y^\lambda Y^\kappa + \mathcal{O}(Y^3)$$

$$S = \frac{1}{4\pi} \int d^2\sigma \partial Y^\mu \partial Y^\nu \delta_{\mu\nu} - \frac{\alpha'}{3} \mathcal{R}_{\mu\lambda\nu\kappa} Y^\lambda Y^\kappa \partial Y^\mu \partial Y^\nu$$



$$\sim \mathcal{R}_{\mu\lambda\nu\kappa}\left(k^\mu \cdot k^\nu\right)$$

$$\langle Y^\lambda(\sigma)Y^\kappa(\sigma')\rangle = -\frac{1}{2}\delta^{\lambda\kappa}\ln|\sigma-\sigma'|^2$$

$$\langle Y^\lambda(\sigma)Y^\kappa(\sigma')\rangle = 2\pi\delta^{\lambda\kappa}\int \frac{d^{2+\epsilon}k}{(2\pi)^{2+\epsilon}}\frac{e^{ik\cdot(\sigma-\sigma')}}{k^2}$$
$$\longrightarrow \frac{\delta^{\lambda\kappa}}{\epsilon} \text{ as } \sigma \to \sigma'$$

$$\mathcal{R}_{\mu\lambda\nu\kappa}Y^\lambda Y^\kappa \partial Y^\mu \partial Y^\nu \to \mathcal{R}_{\mu\lambda\nu\kappa}Y^\lambda Y^\kappa \partial Y^\mu \partial Y^\nu - \frac{1}{\epsilon}\mathcal{R}_{\mu\nu}\partial Y^\mu \partial Y^\nu$$

$$G_{\mu\nu} \to G_{\mu\nu} + \frac{\alpha'}{\epsilon}\mathcal{R}_{\mu\nu}$$

$$\beta_{\mu\nu}(G) = \alpha'\mathcal{R}_{\mu\nu} = 0$$

$$g_{\alpha\beta} = e^{2\phi}\delta_{\alpha\beta}$$

$$S = \frac{1}{4\pi\alpha'}\int d^{2+\epsilon}\sigma e^{\phi\epsilon}\partial_\alpha X^\mu \partial^\alpha X^\nu G_{\mu\nu}(X)$$
$$\approx \frac{1}{4\pi\alpha'}\int d^{2+\epsilon}\sigma(1+\phi\epsilon)\partial_\alpha X^\mu \partial^\alpha X^\nu G_{\mu\nu}(X)$$

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma\partial_\alpha X^\mu \partial^\alpha X^\nu [G_{\mu\nu}(X) + \alpha'\phi\mathcal{R}_{\mu\nu}(X)]$$

$$T_{\alpha\beta} = +\frac{4\pi}{\sqrt{g}}\frac{\partial S}{\partial g^{\alpha\beta}} = -2\pi\frac{\partial S}{\partial\phi}\delta_{\alpha\beta} \Rightarrow T^\alpha_\alpha = -\frac{1}{2}\mathcal{R}_{\mu\nu}\partial X^\mu \partial X^\nu$$

$$T^\alpha_\alpha = -\frac{1}{2\alpha'}\beta_{\mu\nu}\partial X^\mu \partial X^\nu$$

$$\beta_{\mu\nu} = \alpha'\mathcal{R}_{\mu\nu}$$

$$\mu\frac{\partial G_{\mu\nu}}{\partial\mu} = \alpha'\mathcal{R}_{\mu\nu}$$

$$\mu\frac{\partial r^2}{\partial\mu} = \frac{\alpha'}{2\pi}$$

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}\left(G_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu g^{\alpha\beta} + iB_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu \epsilon^{\alpha\beta}\right)$$

$$\int d\tau A_\mu(X)\dot{X}^\mu$$

$$\int dt A_0(X) + A_i(X)\dot{X}^i$$

$$\int d^2\sigma B_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu \epsilon^{\alpha\beta}$$

$$B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu C_\nu - \partial_\nu C_\mu$$

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}\left(G_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu g^{\alpha\beta} + iB_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu \epsilon^{\alpha\beta} + \alpha'\Phi(X)R^{(2)}\right)$$

$$\Phi(X) = \lambda,$$

$$S_{\text{superparticle}} = \lambda\chi$$

$$\Phi_0 = \text{limit}_{X\to\infty}\Phi(X)$$

$$g_s = e^{\Phi_0}$$

$$\langle T^\alpha_\alpha\rangle = -\frac{1}{2\alpha'}\beta_{\mu\nu}(G)g^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu - \frac{i}{2\alpha'}\beta_{\mu\nu}(B)\epsilon^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu - \frac{1}{2}\beta(\Phi)R^{(2)}$$

$$\beta_{\mu\nu}(G) = \alpha'\mathcal{R}_{\mu\nu} + 2\alpha'\nabla_\mu\nabla_\nu\Phi - \frac{\alpha'}{4}H_{\mu\lambda\kappa}H^{\lambda\kappa}_\nu$$
$$\beta_{\mu\nu}(B) = -\frac{\alpha'}{2}\nabla^\lambda H_{\lambda\mu\nu} + \alpha'\nabla^\lambda\Phi H_{\lambda\mu\nu}$$
$$\beta(\Phi) = -\frac{\alpha'}{2}\nabla^2\Phi + \alpha'\nabla_\mu\Phi\nabla^\mu\Phi - \frac{\alpha'}{24}H_{\mu\nu\lambda}H^{\mu\nu\lambda}$$

$$\beta_{\mu\nu}(G) = \beta_{\mu\nu}(B) = \beta(\Phi) = 0$$

$$S = \frac{1}{2\kappa_0^2}\int d^{26}X\sqrt{-G}e^{-2\Phi}\left(\mathcal{R} - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\Phi\partial^\mu\Phi\right)$$

$$\delta S = \frac{1}{2\kappa_0^2\alpha'}\int d^{26}X\sqrt{-G}e^{-2\Phi}(\delta G_{\mu\nu}\beta^{\mu\nu}(G) - \delta B_{\mu\nu}\beta^{\mu\nu}(B)$$
$$-\left(2\delta\Phi + \frac{1}{2}G^{\mu\nu}\delta G_{\mu\nu}\right)\left(\beta^\lambda_\lambda(G) - 4\beta(\Phi)\right)\Big)$$

$$\tilde{\Phi} = \Phi - \Phi_0$$

$$\tilde{G}_{\mu\nu}(X) = e^{-4\tilde{\Phi}/(D-2)}G_{\mu\nu}(X)$$

$$\tilde{\mathcal{R}} = e^{-2\omega}(\mathcal{R} - 2(D-1)\nabla^2\omega - (D-2)(D-1)\partial_\mu\omega\partial^\mu\omega)$$

$$S = \frac{1}{2\kappa^2}\int d^{26}X\sqrt{-\tilde{G}}\left(\tilde{\mathcal{R}} - \frac{1}{12}e^{-\tilde{\Phi}/3}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{1}{6}\partial_\mu\tilde{\Phi}\partial^\mu\tilde{\Phi}\right)$$

$$\kappa^2 = \kappa_0^2 e^{2\Phi_0} \sim l_s^{24}g_s^2$$

$$8\pi G_N = \kappa^2$$

$$g_s \ll 1 \Rightarrow l_p \ll l_s$$

$$\beta_{\mu\nu} = \alpha'\mathcal{R}_{\mu\nu} + \frac{1}{2}\alpha'^2\mathcal{R}_{\mu\lambda\rho\sigma}\mathcal{R}_\nu{}^{\lambda\rho\sigma} + \cdots = 0$$

$$S_{\text{supercurvature}} = S_1 + S_2 + S_{\text{superparticle}}$$

$$S_1 = \frac{1}{2\kappa_0^2}\int d^{10}X\sqrt{-G}e^{-2\Phi}\left(\mathcal{R} - \frac{1}{2}\left|\tilde{H}_3\right|^2 + 4\partial_\mu\Phi\partial^\mu\Phi\right)$$

$$S_2 = -\frac{1}{4\kappa_0^2}\int d^{10}X\left[\sqrt{-G}\left(|F_2|^2 + \left|\tilde{F}_4\right|^2\right) + B_2\wedge F_4\wedge F_4\right]$$

$$S_2 = -\frac{1}{4\kappa_0^2}\int d^{10}X\left[\sqrt{-G}\left(|F_1|^2 + \left|\tilde{F}_3\right|^2 + \frac{1}{2}\left|\tilde{F}_5\right|^2\right) + C_4\wedge H_3\wedge F_3\right]$$

$$\tilde{F}_5 = {}^\star\tilde{F}_5$$

$$S_2 = \frac{\alpha'}{8\kappa_0^2}\int d^{10}X\sqrt{-G}\,\text{Tr}|F_2|^2$$

$$\omega_3 = \text{Tr}\left(A_1\wedge dA_1 + \frac{2}{3}A_1\wedge A_1\wedge A_1\right)$$

$$\beta_{\mu\nu}(G) = \beta_{\mu\nu}(B) = \beta(\Phi) = 0$$

$$\mathbf{R}^{1,3} \times \mathbf{X}$$

$$S_{EH} = \frac{1}{2\kappa^2}\int d^{26}X\sqrt{-\tilde{G}}\tilde{\mathcal{R}} = \frac{\mathrm{Vol}(\mathbf{X})}{2\kappa^2}\int d^4X\sqrt{-G_{4d}}\mathcal{R}_{4d}$$

$$8\pi G_N^{4d} = \frac{\kappa^2}{\mathrm{Vol}(\mathbf{X})}$$

$$l_p^{(4d)} \sim g_s l_s^{12}/\sqrt{\mathrm{Vol}(\mathbf{X})}$$

$$ds^2 = f(r)^{-1}(-dt^2 + dX_1^2) + \sum_{i=1}^{25} dX_i^2$$
$$B = (f(r)^{-1} - 1)dt \wedge dX_1\, , e^{2\Phi} = f(r)^{-1}$$

$$f(r) = 1 + \frac{g_s^2 N l_s^{22}}{r^{22}}$$

$$\frac{1}{g_s^2}\int_{\mathbf{R}^{26}} H_3 \wedge^\star H_3 + \int_{\mathbf{R}^2} B_2 = \frac{1}{g_s^2}\int_{\mathbf{R}^{26}} H_3 \wedge^\star H_3 + g_s^2 B_2 \wedge \delta(\omega)$$

$$d^\star H_3 \sim g_s^2 \delta(\omega)$$

$$\frac{1}{g_s^2}\int_{\mathbf{S}^{23}}^\star H_3 = 1$$

$$\frac{1}{g_s^2}\int_{\mathbf{S}^{23}}^\star H_3 = N$$

$$q = \int_{\mathbf{S}^2} \vec{E}\cdot d\vec{S} = \int_{\mathbf{S}^2} {}^\star F_2$$

$$g = \int_{\mathbf{S}^2} \vec{B}\cdot d\vec{S} = \int_{\mathbf{S}^2} F_2$$

$$\mu\int_W C_{p+1}$$

$$G_{p+2} = dC_{p+1}$$

$$q = \int_{\mathbf{S}^{D-p-2}}^\star G_{p+2}$$

$${}^\star G_{p+2} = \tilde{G}_{D-p-2} = d\tilde{C}_{D-p-3}$$

$$\tilde{\mu}\int_{\tilde{W}} \tilde{C}_{D-p-3}$$

$$ds^2 = \left(-dt^2 + \sum_{i=1}^{21} dX_i^2\right) + h(r)(dX_{22}^2 + \cdots dX_{25}^2)$$
$$\tilde{B}_{22} = (1 - h(r)^{-2})dt \wedge dX_1 \wedge \ldots \wedge dX_{21}$$
$$e^{2\Phi} = h(r)$$

$$h(r) = 1 + \frac{N l_s^2}{r^2}$$

$$T \sim \frac{N}{l_s^{22}}\frac{1}{g_s^2}$$

$$\beta(\Phi) = \frac{D-26}{6} - \frac{\alpha'}{2}\nabla^2\Phi + \alpha'\nabla_\mu\Phi\nabla^\mu\Phi - \frac{\alpha'}{24}H_{\mu\nu\lambda}H^{\mu\nu\lambda} = 0$$

$$S = \frac{1}{2\kappa_0^2}\int d^{26}X\sqrt{-G}e^{-2\Phi}\left(\mathcal{R} - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{2(D-26)}{3\alpha'}\right)$$

$$\partial_\mu\Phi\partial^\mu\Phi = \frac{26-D}{6\alpha'}$$

$$\Phi = \sqrt{\frac{26-D}{6\alpha'}}X^1 \quad D < 26$$

$$\Phi = \sqrt{\frac{D-26}{6\alpha'}}X^0 \quad D > 26$$

$$S_{\text{dark particle}} = \frac{1}{4\pi}\int d^2\sigma\sqrt{g}\Phi(X)R^{(2)}$$

$$T_{\alpha\beta} = -4\pi\frac{\partial S}{\partial g^{\alpha\beta}}\bigg|_{g_{\alpha\beta}=\delta_{\alpha\beta}}$$

$$\delta\left(\sqrt{g}g^{\alpha\beta}R_{\alpha\beta}\right) = \sqrt{g}g^{\alpha\beta}\delta R_{\alpha\beta} = \sqrt{g}\nabla^\alpha v_\alpha$$

$$v_\alpha = \nabla^\beta\left(\delta g_{\alpha\beta} - g^{\gamma\delta}\nabla_\alpha\delta g_{\gamma\delta}\right)$$

$$\delta S_{\text{white particle}} = \frac{1}{4\pi}\int d^2\sigma\sqrt{g}\left(\nabla^\alpha\nabla^\beta\Phi - \nabla^2\Phi g^{\alpha\beta}\right)\delta g_{\alpha\beta}$$

$$T_{\alpha\beta}^{\text{dark particle}} = -\partial_\alpha\partial_\beta\Phi + \partial^2\Phi\delta_{\alpha\beta}$$

$$T^{\text{white particle}} = -\partial^2\Phi\,,\bar{T}^{\text{white particle}} = -\bar{\partial}^2\Phi$$

$$\Phi = QX$$

$$T = -\frac{1}{\alpha'}:\partial X\partial X: -Q\partial^2 X$$

$$T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \cdots$$

$$c = D + 6\alpha' Q^2$$

$$V_{\text{superparticle}} \sim \int d^2\sigma\sqrt{g}e^{ip\cdot X}$$

$$S_{\text{potential}} = \int d^2\sigma\sqrt{g}\alpha' V(X)$$

$$V_{\text{white particle}} \sim \int_{\partial\mathcal{M}} d\tau\zeta_a\partial^\tau X^a e^{ip\cdot X}$$

$$S_{\text{end-point}} = \int_{\partial\mathcal{M}} d\tau A_a(X)\frac{dX^a}{d\tau}$$

$$S = S_{\text{Neumann}} + S_{\text{Dirichlet}}$$

$$S_{\text{Neumann}} = \frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\sigma\partial^\alpha X^a\partial_\alpha X^b\delta_{ab} + i\int_{\partial\mathcal{M}} d\tau A_a(X)\dot{X}^a$$

$$S_{\text{Dirichlet}} = \frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\sigma\partial^\alpha X^I\partial_\alpha X^J\delta_{IJ}$$

$$X^a(\sigma) = \bar{x}^a(\sigma) + \sqrt{\alpha'}Y^a(\sigma)$$

$$\partial^2 \bar{x}^a = 0$$

$$\partial_\sigma \bar{x}^a + 2\pi\alpha' i F^{ab}\partial_\tau \bar{x}_b = 0 \text{ at } \sigma = 0$$

$$F_{ab}(X) = \frac{\partial A_b}{\partial X^a} - \frac{\partial A_a}{\partial X^b} \equiv \partial_a A_b - \partial_b A_a$$

$$S[\bar{x} + \sqrt{\alpha'}Y] = S[\bar{x}] + \frac{1}{4\pi}\int_{\mathcal{M}} d^2\sigma \partial Y^a \partial Y^b \delta_{ab}$$
$$+ i\alpha' \int_{\partial\mathcal{M}} d\tau \left(\partial_a A_b Y^a \dot{Y}^b + \frac{1}{2}\partial_a \partial_b A_c Y^a Y^b \dot{\bar{x}}^c\right) + \cdots$$

$$\int d\tau (\partial_a A_b) Y^a \dot{Y}^b = \frac{1}{2}\int d\tau \partial_a A_b Y^a \dot{Y}^b - \partial_a A_b \dot{Y}^a Y^b - \partial_c \partial_a A_b Y^a Y^b \dot{\bar{x}}^c$$

$$S[\bar{x} + \sqrt{\alpha'}Y] = S[\bar{x}] + \frac{1}{4\pi}\int_{\mathcal{M}} d^2\sigma \partial Y^a \partial Y^b \delta_{ab}$$
$$+ \frac{i\alpha'}{2}\int_{\partial\mathcal{M}} d\tau (F_{ab} Y^a \dot{Y}^b + \partial_b F_{ac} Y^a Y^b \dot{\bar{x}}^c) + \cdots$$

$$\langle Y^a(z,\bar{z}) Y^b(w,\bar{w})\rangle = G^{ab}(z,\bar{z}; w,\bar{w})$$

$$\partial\bar{\partial} G^{ab}(z,\bar{z}) = -2\pi\delta^{ab}\delta(z,\bar{z})$$

$$\partial_\sigma G^{ab}(z,\bar{z}; w,\bar{w}) + 2\pi\alpha' i F^a_c \partial_\tau G^{cb}(z,\bar{z}; w,\bar{w}) = 0 \text{ at } \sigma = 0$$

$$G^{ab} = -\delta^{ab} \ln |z - w| - \frac{1}{2}\left(\frac{1 - 2\pi\alpha' F}{1 + 2\pi\alpha' F}\right)^{ab} \ln |z - \bar{w}| - \frac{1}{2}\left(\frac{1 + 2\pi\alpha' F}{1 - 2\pi\alpha' F}\right)^{ab} \ln |\bar{z} - w|$$

$$-\frac{1}{\epsilon}\left[\delta^{ab} + \frac{1}{2}\left(\frac{1 - 2\pi\alpha' F}{1 + 2\pi\alpha' F}\right)^{ab} + \frac{1}{2}\left(\frac{1 + 2\pi\alpha' F}{1 - 2\pi\alpha' F}\right)^{ab}\right] = -\frac{2}{\epsilon}\left(\frac{1}{1 - 4\pi^2\alpha'^2 F^2}\right)^{ab}$$

$$-\frac{i2\pi\alpha'^2}{\epsilon}\int_{\partial\mathcal{M}} d\tau \partial_b F_{ac}\left[\frac{1}{1 - 4\pi^2\alpha'^2 F^2}\right]^{ab} \dot{\bar{x}}^c$$

$$\partial_b F_{ac}\left[\frac{1}{1 - 4\pi^2\alpha'^2 F^2}\right]^{ab} = 0$$

$$S = -T_p \int d^{p+1}\xi \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab})}$$

$$S = -T_p \int d^{p+1}\xi \left(1 + \frac{(2\pi\alpha')^2}{4} F_{ab}F^{ab} + \cdots\right)$$

$$S_{DBI} = -T_p \int d^{p+1}\xi \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab})}$$

$$\gamma_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b}\eta_{\mu\nu}$$

$$X^a = \xi^a \; a = 0, \ldots, p$$

$$\gamma_{ab} = \eta_{ab} + \frac{\partial X^I}{\partial \xi^a}\frac{\partial X^J}{\partial \xi^b}\delta_{IJ}$$

$$S = -(2\pi\alpha')^2 T_p \int d^{p+1}\xi \left(\frac{1}{4}F_{ab}F^{ab} + \frac{1}{2}\partial_a \phi^I \partial^a \phi^I + \cdots\right)$$

$$S_{DBI} = -T_p \int d^{p+1}\xi e^{-\tilde{\Phi}} \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab} + B_{ab})}$$

$$\gamma_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} G_{\mu\nu}$$

$$T_p \sim 1/g_s$$

$$g_s^{\text{eff}} = e^{\Phi(X)} = g_s e^{\tilde{\Phi}(X)}$$

$$B_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} B_{\mu\nu}$$

$$\frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\sigma \epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu} + \int_{\partial\mathcal{M}} d\tau A_a \dot{X}^a$$

$$B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu C_\nu - \partial_\nu C_\mu$$

$$\begin{aligned} S_B &= \frac{1}{4\pi\alpha'}\int_{\mathcal{M}} d^2\sigma \epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu} \\ &\longrightarrow S_B + \frac{1}{2\pi\alpha'}\int_{\mathcal{M}} d\sigma d\tau \epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \partial_\mu C_\nu \\ &= S_B + \frac{1}{2\pi\alpha'}\int_{\mathcal{M}} d\sigma d\tau \epsilon^{\alpha\beta}\partial_\alpha(\partial_\beta X^\nu C_\nu) \\ &= S_B + \frac{1}{2\pi\alpha'}\int_{\partial\mathcal{M}} d\tau \dot{X}^\nu C_\nu = S_B + \frac{1}{2\pi\alpha'}\int_{\partial\mathcal{M}} d\tau \dot{X}^a C_a \end{aligned}$$

$$A_a \to A_a - \frac{1}{2\pi\alpha'} C_a$$

$$B_{ab} + 2\pi\alpha' F_{ab}$$

$$(A_a)^m_n$$

$$F_{ab} = \partial_a A_b - \partial_b A_a + i[A_a, A_b]$$

$$S = -(2\pi\alpha')^2 T_p \int d^{p+1}\xi \text{Tr}\left(\frac{1}{4}F_{ab}F^{ab} + \frac{1}{2}\mathcal{D}_a\phi^I \mathcal{D}^a\phi^I - \frac{1}{4}\sum_{I\neq J}[\phi^I,\phi^J]^2\right)$$

$$g_{YM}^2 \sim l_s^{p-3} g_s$$

$$\mathcal{D}_a\phi^I = \partial_a\phi^I + i[A_a, \phi^I]$$

$$S(1, D-1) \to SO(1,p) \times SO(D-p-1)$$

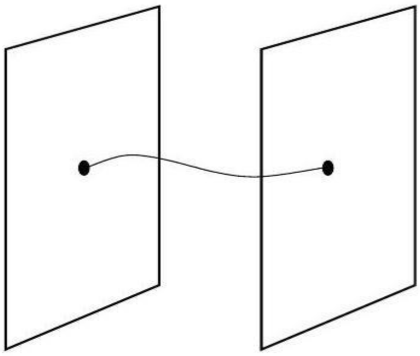
$$V = -\frac{1}{4}\sum_{I\neq J}\text{Tr}[\phi^I,\phi^J]^2$$

$$\phi^I = \begin{pmatrix} \phi_1^I & & \\ & \ddots & \\ & & \phi_N^I \end{pmatrix}$$

$$\vec{X}_n = 2\pi\alpha'\vec{\phi}_n$$

$$\phi = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix}$$

$$A_a = \begin{pmatrix} A_a^{11} & W_a \\ W_a^\dagger & A_a^{22} \end{pmatrix}$$



$$\frac{1}{2}\mathrm{Tr}[A_a,\phi]^2 = -(\phi_2-\phi_1)^2|W_a|^2$$

$$M_W^2 = (\phi_2-\phi_1)^2 = T^2|X_2-X_1|^2$$

$$M_W = |\vec{\phi}_n - \vec{\phi}_m| = T|\vec{X}_n - \vec{X}_m|$$

$$\mathbf{R}^{1,24}\times\mathbf{S}^1$$

$$X^{25} \equiv X^{25} + 2\pi R$$

$$ds^2 = \tilde{G}_{\mu\nu}dX^\mu dX^\nu + e^{2\sigma}\left(dX^{25} + A_\mu dX^\mu\right)^2$$

$$\delta G_{\mu\nu} = \nabla_\mu\Lambda_\nu + \nabla_\nu\Lambda_\mu$$

$$\delta A_\mu = \partial_\mu\Lambda$$

$$\mathcal{R}^{(26)} = \mathcal{R} - 2e^{-\sigma}\nabla^2 e^\sigma - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu}$$

$$S = \frac{1}{2\kappa^2}\int d^{26}X\sqrt{-\tilde{G}^{(26)}}\mathcal{R}^{(26)} = \frac{2\pi R}{2\kappa^2}\int d^{25}X\sqrt{-\tilde{G}}e^\sigma\left(\mathcal{R} - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu} + \partial_\mu\sigma\partial^\mu\sigma\right)$$

$$\Phi(X^\mu;X^{25}) = \sum_{n=-\infty}^{\infty}\Phi_n(X^\mu)e^{inX^{25}/R}$$

$$\Phi_n^\star = \Phi_{-n}$$

$$\int d^{26}X\partial_\mu\Phi\partial^\mu\Phi + (\partial_{25}\Phi)^2 = 2\pi R\int d^{25}X\sum_{n=-\infty}^{\infty}\left(\partial_\mu\Phi_n\partial^\mu\Phi_{-n} + \frac{n^2}{R^2}|\Phi_n|^2\right)$$

$$M_n^2 = \frac{n^2}{R^2}$$

$$\Phi_n \to \exp\left(\frac{in\Lambda}{R}\right)\Phi_n$$

$$p^{25} = \frac{n}{R}\; n\in\mathbf{Z}$$

$$X^{25}(\sigma+2\pi) = X^{25}(\sigma) + 2\pi mR\; m\in\mathbf{Z}$$

$$X^{25}(\sigma,\tau) = x^{25} + \frac{\alpha' n}{R}\tau + mR\sigma + \text{ oscillator modes}$$

$$p_L = \frac{n}{R} + \frac{mR}{\alpha'}, p_R = \frac{n}{R} - \frac{mR}{\alpha'}$$

$$X_L^{25}(\sigma^+) = \frac{1}{2}x^{25} + \frac{1}{2}\alpha' p_L \sigma^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^{25}e^{-in\sigma^+}$$

$$X_R^{25}(\sigma^-) = \frac{1}{2}x^{25} + \frac{1}{2}\alpha' p_R \sigma^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{25}e^{-in\sigma^-}$$

$$M^2 = -\sum_{\mu=0}^{24} p_\mu p^\mu$$

$$M^2 = p_L^2 + \frac{4}{\alpha'}(\tilde{N}-1) = p_R^2 + \frac{4}{\alpha'}(N-1)$$

$$N - \tilde{N} = nm$$

$$M^2 = \frac{n^2}{R^2} + \frac{m^2R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2)$$

$$V_\pm(p) \sim \int d^2z\zeta_\mu\left(\partial X^\mu \bar{\partial}\bar{X}^{25} \pm \partial X^{25}\bar{\partial}\bar{X}^\mu\right)e^{ip\cdot X}$$

$$V_{m,n}(p) \sim \int d^2z e^{ip\cdot X}e^{ip_L X^{25} + ip_R \bar{X}^{25}}$$

$$\langle V_\pm(p_1)V_{m,n}(p_2)V_{-m,-n}(p_3)\rangle \sim \delta^{25}\left(\sum_i p_i\right)\zeta_\mu\left(p_2^\mu - p_3^\mu\right)(p_L \pm p_R)$$

$$M^2 = \left(\frac{mR}{\alpha'}\right)^2 - \frac{4}{\alpha'}$$

$$M^2 = \frac{n^2}{R^2} - \frac{4}{\alpha'}$$

$$R = \sqrt{\alpha'}$$

$$U(1)\times U(1) \to SU(2)\times SU(2)$$

$$R \leftrightarrow \frac{\alpha'}{R}$$

$$m \leftrightarrow n$$

$$p_L \to p_L\,, p_R \to -p_R$$

$$Y^{25} = X_L^{25}(\sigma^+) - X_R^{25}(\sigma^-)$$

$$\partial_\alpha X = \epsilon_{\alpha\beta}\partial^\beta Y$$

$$\frac{2\pi R}{2l_s^{24}g_s^2}\int d^{25}X\sqrt{-\tilde{G}}e^{\sigma}\mathcal{R} + \cdots$$

$$g_s \to \tilde{g}_s = \frac{\sqrt{\alpha'}g_s}{R}$$

$$S[\varphi] = \frac{R^2}{4\pi\alpha'}\int d^2\sigma\partial_\alpha\varphi\partial^\alpha\varphi$$

$$\partial_\alpha\varphi \to \mathcal{D}_\alpha\varphi = \partial_\alpha\varphi + A_\alpha$$

$$S[\varphi,\theta,A] = \frac{R^2}{4\pi\alpha'}\int d^2\sigma\mathcal{D}_\alpha\varphi\mathcal{D}^\alpha\varphi + \frac{i}{2\pi}\int d^2\sigma\theta\epsilon^{\alpha\beta}\partial_\alpha A_\beta$$

$$Z = \frac{1}{\text{Vol}} \int \mathcal{D}\varphi \mathcal{D}\theta \mathcal{D}A e^{-S[\varphi,\theta,A]}$$

$$Z = \int \mathcal{D}\theta \mathcal{D}A \exp\left(-\frac{R^2}{4\pi\alpha'} \int d^2\sigma A_\alpha A^\alpha - \frac{i}{2\pi} \int d^2\sigma \epsilon^{\alpha\beta} (\partial_\alpha \theta) A_\beta\right)$$

$$Z = \int \mathcal{D}\theta \exp\left(-\frac{\tilde{R}^2}{4\pi\alpha'} \int d^2\sigma \partial_\alpha \theta \partial^\alpha \theta\right)$$

$$X = \text{const} \Rightarrow \partial_\tau X^{25} = 0 \ \text{ at } \sigma = 0, \pi$$

$$\partial_\sigma Y = 0 \ \text{ at } \sigma = 0, \pi$$

$$T = \frac{1}{2\pi\alpha'}$$

$$t_p = \text{ Planck Time } = \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} \approx 5.391 \times 10^{-44} \text{ s}$$

$$l_p = \text{ Planck Length } = \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} \approx 1.616 \times 10^{-33} \text{ cm}$$

$$m_p = \text{ Planck Mass } = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} \approx 2.177 \times 10^{-5} \text{ g}$$

$$A = -m \int_{\tau_1}^{\tau_2} d\tau \sqrt{-\eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}$$

$$\delta A = m \int_{\tau_1}^{\tau_2} d\tau \frac{\partial}{\partial \tau} \left[\frac{-\dot{X}_\mu}{\sqrt{-\eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}}\right] \delta X^\mu + m \left[\frac{\dot{X}_\mu \delta X^\mu}{\sqrt{-\eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}}\right]_{\tau_1}^{\tau_2}$$

$$\frac{\partial}{\partial \tau} \left[\frac{-m\dot{X}_\mu}{\sqrt{-\eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}}\right] = 0$$

$$X^\mu \to \Lambda^\mu_\nu X^\nu + a^\mu$$

$$A = -m \int_{\tau_1}^{\tau_2} \sqrt{-\eta_{\mu\nu} dX^\mu dX^\nu}$$

$$\hat{A} = \frac{1}{2} \int_{\tau_1}^{\tau_2} d\tau \left(e^{-1} \eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu - e m^2\right)$$

$$\frac{\partial}{\partial \tau} (-e^{-1} \dot{X}^\mu) = 0$$

$$\frac{\eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}{e^2} + m^2 = 0$$

$$e(\tau) = \sqrt{g_{\tau\tau}} \Rightarrow e(\tau) d\tau = \sqrt{g_{\tau\tau} d\tau^2}$$

$$g_{\tau\tau} d\tau d\tau \equiv g_{\tilde{\tau}\tilde{\tau}} d\tilde{\tau} d\tilde{\tau}$$

$$e(\tau) d\tau = \sqrt{g_{\tilde{\tau}\tilde{\tau}}} d\tilde{\tau} = e(\tilde{\tau}) d\tilde{\tau}$$

$$\hat{A} \to \hat{\tilde{A}} = \frac{1}{2} \int_{\tilde{\tau}_1}^{\tilde{\tau}_2} d\tilde{\tau} (\tilde{e}^{-1} (\partial_{\tilde{\tau}} X)^2 - \tilde{e} m^2)$$

$$A_{NG} = -T \int d^2\xi \sqrt{-\det\left(\eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right)}$$

$$A_{BDH} = -\frac{T}{2}\int d^2\xi \sqrt{-\det\gamma}\gamma^{\alpha\beta}\left(\eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right)$$

$$\gamma_{\alpha\beta} = f(\sigma,\tau)\left(\eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right) \equiv f(\sigma,\tau)\hat{\gamma}_{\alpha\beta}$$

$$\delta X^\mu = \zeta^\alpha \partial_\alpha X^\mu,$$

$$\delta\gamma_{\alpha\beta} = 2\nabla_{(\alpha}\zeta_{\beta)} = \zeta^\rho \partial_\rho \gamma_{\alpha\beta} + 2\gamma_{\rho(\beta}\partial_{\alpha)}\zeta^\rho$$
$$\delta(\sqrt{-\det\gamma}) = \partial_\alpha\left(\zeta^\alpha\sqrt{-\det\gamma}\right)$$

$$\delta\gamma_{\alpha\beta} = \Lambda\gamma_{\alpha\beta},$$

$$T_{\alpha\beta} = -\frac{2}{T}\frac{1}{\sqrt{-\det\gamma}}\frac{\delta A_{BDH}}{\delta\gamma^{\alpha\beta}},$$

$$T_{\alpha\beta} = \hat{\gamma}_{\alpha\beta} - \frac{1}{2}\gamma_{\alpha\beta}\gamma^{\epsilon\rho}\hat{\gamma}_{\epsilon\rho}$$
$$\frac{\delta A_{BDH}}{\delta\gamma^{\alpha\beta}} = 0 \Rightarrow T_{\alpha\beta} = 0$$

$$\gamma_{\alpha\beta} = e^{\phi(\sigma,\tau)}\eta_{\alpha\beta} = e^{\phi(\sigma,\tau)}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\gamma_{\alpha\beta} = \eta_{\alpha\beta}$$

$$A_{BDH} = -\frac{T}{2}\int d^2\xi \eta^{\alpha\beta}\eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$$
$$= \frac{T}{2}\int d^2\xi (\dot{X}^2 - \dot{X}^2)$$

$$\delta A_{BDH} = T\int d^2\xi[-\partial_\tau^2 X^\mu + \partial_\sigma^2 X^\nu]\delta X_\mu$$
$$+T\int d\sigma \dot{X}^\mu \delta X_\mu\Big|_{\tau=-\infty}^{\tau=\infty} - T\int d\tau \dot{X}^\mu \delta X_\mu\Big|_\sigma$$

$$\square X^\mu = 0$$

$$\dot{X}^\mu\big|_{-\infty} = \dot{X}^\mu\big|_\infty$$
$$X^\mu(\sigma,\tau) = X^\mu(\sigma + 2\pi,\tau)$$
$$X^\mu(\sigma,\tau) = X^\mu(\sigma + 2\pi,\tau)$$

$$\dot{X}^\mu(0,\tau)\delta X_\mu(0,\tau) = \dot{X}^\mu(\pi,\tau)\delta X_\mu(\pi,\tau)$$

$$\mathfrak{N}_{\text{Neumann}} \Rightarrow \dot{X}^\mu = 0$$
$$\mathfrak{D}_{\text{Dirichlet}} \Rightarrow \delta X^\mu = 0 \text{ (i.e. } \dot{X}^\mu) = 0$$

$$X^\mu(\sigma,\tau) = X_R^\mu(\tau - \sigma) + X_L^\mu(\tau + \sigma)$$
$$= X_R^\mu(\xi^-) + X_L^\mu(\xi^+)$$

$$\xi^\pm = \tau \pm \sigma$$
$$\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$$

$$\gamma_{\pm\pm} = \gamma^{\pm\pm} = 0;$$
$$\gamma_{+-} = \gamma_{-+} = -\frac{1}{2};$$
$$\gamma^{+-} = \gamma^{-+} = -2,$$

$$U^+ = \eta^{+-}U_- = -2U_-;$$
$$U_+ = \eta_{+-}U^- = -\frac{1}{2}U^-$$

$$X_R^\mu = \frac{1}{2}x^\mu + \frac{1}{2}\alpha' p_R^\mu \xi^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-in\xi^-}$$
$$X_L^\mu = \frac{1}{2}x^\mu + \frac{1}{2}\alpha' p_L^\mu \xi^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n^\mu}{n}e^{-in\xi^+}$$

$$X^\mu = x^\mu + \alpha' p^\mu \tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^\mu e^{-in\xi^-} + \tilde{\alpha}_n^\mu e^{-in\xi^+}\right)$$

$$X_R^\mu = \frac{1}{2}x^\mu + \alpha' p_R^\mu \xi^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-2in\xi^-}$$
$$X_L^\mu = \frac{1}{2}x^\mu + \alpha' p_L^\mu \xi^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n^\mu}{n}e^{-2in\xi^+},$$

$$X^\mu = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^\mu e^{-2in\xi^-} + \tilde{\alpha}_n^\mu e^{-2in\xi^+}\right)$$

$$X^\mu = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-in\tau}\cos n\sigma.$$

$$X^\mu = a^\mu + \frac{1}{\pi}(b^\mu - a^\mu)\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-in\tau}\sin n\sigma.$$

$$X^\mu = b^\mu + i\sqrt{2\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\frac{\alpha_r^\mu}{r}e^{-ir\tau}\cos r\sigma.$$

$$X^\mu = a^\mu + \sqrt{2\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\frac{\alpha_r^\mu}{r}e^{-ir\tau}\sin r\sigma.$$

$$X^\mu = (X^\mu)^\dagger \Rightarrow \left(\alpha_n^\mu\right)^\dagger = \alpha_{-n}^\mu$$
$$\left(\tilde{\alpha}_n^\mu\right)^\dagger = \tilde{\alpha}_{-n}^\mu,$$

$$P^\mu \equiv \frac{\delta A_{BDH}}{\delta \dot{X}_\mu} = T\dot{X}^\mu.$$

$$P^\mu = T\left\{2\alpha' p^\mu + 2\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\left(\alpha_n^\mu e^{-2in\xi^-} + \tilde{\alpha}_n^\mu e^{-2in\xi^+}\right)\right\}$$
$$= T\sqrt{2\alpha'}\sum_{n=-\infty}^{\infty}\left(\alpha_n^\mu e^{-2in\xi^-} + \tilde{\alpha}_n^\mu e^{-2in\xi^+}\right)$$

$$\alpha_0^\mu = \tilde{\alpha}_0^\mu = \sqrt{\alpha'/2}p^\mu$$

$$P^\mu = T\sqrt{2\alpha'}\sum_{n=-\infty}^{\infty}\alpha_n^\mu e^{-in\tau}\cos n\sigma,$$

$$[X^\mu(\sigma), X^\nu(\sigma')]_{P.B.} = [P^\mu(\sigma), P^\nu(\sigma')]_{P.B.} = 0$$
$$[X^\mu(\sigma), P^\nu(\sigma')]_{P.B.} = \delta(\sigma - \sigma')\eta^{\mu\nu},$$

$$[\alpha_m^\mu, \alpha_n^\nu]_{P.B.} = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu]_{P.B.} = -im\delta_{m+n}\eta^{\mu\nu}$$
$$[\alpha_m^\mu, \tilde{\alpha}_n^\nu]_{P.B.} = 0$$
$$[x^\mu, p^\nu]_{P.B.} = \eta^{\mu\nu},$$

$$\delta_{m+n} = \begin{cases} 1 & m+n=0 \\ 0 & m+n \neq 0 \end{cases}.$$

$$H = \int d\sigma(\dot{X}^\mu P_\mu - \mathcal{L})$$
$$= \frac{T}{2}\int d\sigma(\dot{X}^2 + X'^2)$$

$$H = \alpha' T \left\{ \frac{1}{2}\pi \sum_{n\neq 0} \alpha_n \cdot \alpha_{-n} + \frac{1}{2}\pi \sum_{n \neq 0} \alpha_n \cdot \alpha_{-n} + \pi\alpha_0 \cdot \alpha_0 \right\}$$
$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{-n} \cdot \alpha_n$$

$$H = \frac{1}{2}\sum_{n=-\infty}^{\infty} (\alpha_{-n} \cdot \alpha_n + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n)$$

$$T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2}\eta_{\alpha\beta}\eta^{\epsilon\rho}\partial_\epsilon X^\mu \partial_\rho X_\mu,$$

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu$$
$$T_{--} = \partial_- X^\mu \partial_- X_\mu$$
$$T_{+-} = T_{-+} \equiv 0$$

$$\partial_+ X^\mu \partial_+ X_\mu = \partial_- X^\mu \partial_- X_\mu = 0,$$

$$\partial^\alpha T_{\alpha\beta} = 0$$
$$\Rightarrow \partial_- T_{++} = \partial_+ T_{--} = 0.$$

$$T_{++} = 2\alpha' \sum_{n,m\in\mathbb{Z}} \tilde{\alpha}_n \cdot \tilde{\alpha}_{-m} e^{-2in\xi^+} e^{2im\xi^+}$$
$$T_{--} = 2\alpha' \sum_{n,m\in\mathbb{Z}} \alpha_n \cdot \alpha_{-m} e^{-2in\xi^-} e^{2im\xi^-}.$$

$$T_{\pm\pm} = \frac{\alpha'}{2} \sum_{n,m\in\mathbb{Z}} \alpha_n \cdot \alpha_{-m} e^{-in\xi^\pm} e^{im\xi^\pm}.$$

$$T_{++} = 4\alpha' \sum_{n=-\infty}^{\infty} \tilde{L}_n e^{-2in\xi^+}$$
$$T_{--} = 4\alpha' \sum_{n=-\infty}^{\infty} L_n e^{-2in\xi^-}$$

$$T_{\pm\pm} = \alpha' \sum_{n=-\infty}^{\infty} L_n e^{-in\xi^\pm}$$

$$\tilde{L}_m = \frac{1}{4\pi\alpha'}\int_0^\pi d\sigma e^{2im\sigma} T_{++}\Big|_{\tau=0} = \frac{1}{2}\sum_{n=-\infty}^{\infty} \tilde{\alpha}_{m-n} \cdot \tilde{\alpha}_n$$
$$L_m = \frac{1}{4\pi\alpha'}\int_0^\pi d\sigma e^{-2im\sigma} T_{--}\Big|_{\tau=0} = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n} \cdot \alpha_n$$

$$L_m = \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma e^{im\sigma} T_{++}\Big|_{\tau=0}$$
$$\equiv \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma e^{-im\sigma} T_{--}\Big|_{\tau=0}$$
$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n}\cdot\alpha_n$$

$$\tilde{L}_m = L_m = 0\ \forall\ m.$$

$$[f,g]_{P.B.} \to -i[\hat{f},\hat{g}],$$

$$[X^\mu(\sigma), X^\nu(\sigma')] = [P^\mu(\sigma), P^\nu(\sigma')] = 0$$
$$[X^\mu(\sigma), P^\nu(\sigma')] = i\delta(\sigma-\sigma')\eta^{\mu\nu},$$

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\delta_{m+n}\eta^{\mu\nu}$$
$$[\alpha_m^\mu, \tilde{\alpha}_n^\nu] = 0$$
$$[x^\mu, p^\nu] = i\eta^{\mu\nu}.$$

$$(\alpha_m^\mu)^\dagger \equiv \alpha_{-m}^\mu \quad m>0$$
$$(\tilde{\alpha}_m^\mu)^\dagger \equiv \tilde{\alpha}_{-m}^\mu \quad m>0.$$

$$\alpha_m^\mu \quad m>0$$
$$\tilde{\alpha}_m^\mu \quad m>0.$$

$$|\phi; p^\mu\rangle$$

$$\alpha_m^\mu|\phi; p^\mu\rangle = \tilde{\alpha}_m^\mu|\phi; p^\mu\rangle = 0\ m>0$$

$$\alpha_{-m}^\mu|\phi; p^\mu\rangle\ ;\ \tilde{\alpha}_{-m}^\mu|\phi; p^\mu\rangle$$

$$L_m|\phi\rangle = \tilde{L}_m|\phi\rangle = 0\ \forall\ m$$

$$\langle\phi|L_m|\phi\rangle = \langle\phi|\tilde{L}_m|\phi\rangle = 0\ \forall\ m,$$

$$L_m|\phi\rangle = \tilde{L}_m|\phi\rangle = 0\ \forall\ m>0$$

$$\langle\phi|L_{-m} = \langle\phi|\tilde{L}_{-m} = 0\ \forall\ m>0$$

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n}\cdot\alpha_n$$

$$L_0 = \frac{1}{2}\alpha_0^2 + \frac{1}{2}\sum_{n\neq 0} :\alpha_{-n}\cdot\alpha_n:$$
$$= \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n$$

$$(L_0 - a)|\phi\rangle = 0$$
$$(\tilde{L}_0 - \tilde{a})|\phi\rangle = 0$$

$$L_0 = \alpha' p^2 + \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n = 0$$

$$M^2 = \frac{1}{\alpha'}\sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n = \frac{1}{\alpha'}N$$

$$N = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n$$

$$L_0 - \tilde{L}_0 = 0 \Rightarrow N = \tilde{N}$$

$$L_0 + \tilde{L}_0 = 0 \Rightarrow \frac{1}{2}(\alpha_0^2 + \tilde{\alpha}_0^2) = -(N + \tilde{N})$$

$$M^2 = \frac{2}{\alpha'} \sum_{n=1}^{\infty} (\alpha_{-n} \cdot \alpha_n + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n) = \frac{2}{\alpha'}(N + \tilde{N})$$

$$(L_0 - a)|\phi\rangle = 0 \Rightarrow \frac{1}{2}\alpha_0^2|\phi\rangle = \left(a - \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n\right)|\phi\rangle$$

$$\Rightarrow M^2|\phi\rangle = -\frac{1}{\alpha'}\left(a - \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n\right)|\phi\rangle$$

$$M^2 = -\frac{1}{\alpha'}\left(a - \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n\right),$$

$$N - a = \tilde{N} - \tilde{a},$$

$$M^2 = \frac{2}{\alpha'}(N + \tilde{N} - a - \tilde{a}).$$

$$[L_m, \alpha_n^\mu]_{P.B.} = in\alpha_{m+n}^\mu,$$

$$L_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} \alpha_{m-n} \cdot \alpha_n,$$

$$[\alpha_m^\mu, \alpha_n^\nu]_{P.B.} = -im\delta_{m+n}\eta^{\mu\nu}.$$

$$\begin{aligned}
[L_m, L_n]_{P.B.} &= \frac{1}{2}\eta_{\mu\nu}\left[\sum_{p=-\infty}^{\infty} \alpha_{m-p}^\mu \alpha_p^\nu, L_n\right]_{P.B.} \\
&= \frac{1}{2}\eta_{\mu\nu} \sum_{p=-\infty}^{\infty} \left(\alpha_{m-p}^\mu [\alpha_p^\nu, L_n]_{P.B.} + [\alpha_{m-p}^\mu, L_n]_{P.B.} \alpha_p^\nu\right) \\
&= \frac{-i}{2}\eta_{\mu\nu} \sum_{p=-\infty}^{\infty} \left(p\alpha_{m-p}^\mu \alpha_{p+n}^\nu + (m-p)\alpha_{m+n-p}^\mu \alpha_p^\nu\right) \\
&= -i\left(m\frac{1}{2}\sum_{p=-\infty}^{\infty} \alpha_{m+n-p} \cdot \alpha_p - \frac{1}{2}\sum_{p=-\infty}^{\infty} p\alpha_{m+n-p} \cdot \alpha_p + \frac{1}{2}\sum_{p=-\infty}^{\infty} p\alpha_{m-p} \cdot \alpha_{p+n}\right) \\
&= -i\left(m\frac{1}{2}\sum_{p=-\infty}^{\infty} \alpha_{m+n-p} \cdot \alpha_p - \frac{1}{2}\sum_{p=-\infty}^{\infty} p\alpha_{m+n-p} \cdot \alpha_p + \frac{1}{2}\sum_{q=-\infty}^{\infty} (q-n)\alpha_{m+n-q} \cdot \alpha_q\right) \\
&= -i\left(m\frac{1}{2}\sum_{p=-\infty}^{\infty} \alpha_{m+n-p} \cdot \alpha_p - \frac{1}{2}\sum_{p=-\infty}^{\infty} p\alpha_{m+n-p} \cdot \alpha_p + \frac{1}{2}\sum_{p=-\infty}^{\infty} (p-n)\alpha_{m+n-p} \cdot \alpha_p\right) \\
&= -i(m-n)\underbrace{\frac{1}{2}\sum_{p=-\infty}^{\infty} \alpha_{m+n-p} \cdot \alpha_p}_{L_{m+n}}.
\end{aligned}$$

$$[L_m, L_n]_{P.B.} = -i(m-n)L_{m+n}.$$

$$[L_m, L_n] = (m-n)L_{m+n},$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{d}{12}(m^3 - m)\delta_{m+n}$$

$$L_m|\phi;p\rangle = 0 \ \forall \ m > 0$$
$$(L_0 - a)|\phi;p\rangle = 0.$$

$$\alpha' p^2 = a \Rightarrow M^2 = -\frac{a}{\alpha'}.$$

$$\zeta_\mu \alpha^\mu_{-1}|0;p\rangle,$$

$$(L_0 - a)\zeta_\mu \alpha^\mu_{-1}|0;p\rangle = 0 \Rightarrow M^2 = -\frac{a}{\alpha'}(a-1).$$

$$L_1 \zeta_\mu \alpha^\mu_{-1}|0;p\rangle = \zeta_\mu \alpha^\mu_0|0;p\rangle = \sqrt{2\alpha'}\zeta_\mu p^\mu|0;p\rangle.$$

$$\zeta_\mu p^\mu = 0.$$

$$\begin{aligned}\langle 0;p|\zeta_\mu \alpha^\mu_1 \zeta_\nu \alpha^\nu_{-1}|0;p\rangle &= \zeta_\mu \zeta_\nu \{\langle 0;p|\alpha^\mu_{-1}\alpha^\nu_1|0;p\rangle + \eta^{\mu\nu}\langle 0;p \mid 0;p\rangle\} \\ &= \zeta_\mu \zeta_\nu \eta^{\mu\nu}\langle 0;p \mid 0;p\rangle \\ &= \zeta \cdot \zeta,\end{aligned}$$

$$\zeta_\mu p^\mu = \omega(\zeta_0 + \zeta_{d-1}) = 0 \Rightarrow \zeta_0 = -\zeta_{d-1}$$

$$\zeta \cdot \zeta = -\zeta_0^2 + \zeta_i^2 + \zeta_{d-1}^2 = \zeta_i^2 \geq 18$$

$$\langle \phi \mid \psi \rangle = 0$$

$$|\psi\rangle = \sum_{n>0} L_{-n}|\chi_n\rangle$$

$$|\psi\rangle = L_{-1}|\chi_1\rangle + L_{-2}|\chi_2\rangle.$$

$$\langle \phi \mid \psi \rangle = 0\,;\ L_m|\psi\rangle = 0, \forall\ m > 0\,;\ (L_0 - a)|\psi\rangle = 0.$$

$$\langle \psi \mid \psi \rangle = \sum_{m>0} \langle \chi_m|L_m|\psi\rangle = 0$$

$$|\psi\rangle = L_{-1}|\tilde{\chi}\rangle$$

$$L_1|\psi\rangle = L_1 L_{-1}|\tilde{\chi}\rangle = 2L_0|\tilde{\chi}\rangle$$

$$|\psi\rangle = (L_{-2} + \gamma L_{-1}^2)|\tilde{\chi}\rangle$$

$$|\phi\rangle = |\phi_S\rangle + |\phi_T\rangle,$$

$$|0;p\rangle\ M^2 = -\frac{1}{\alpha'}$$

$$\zeta_\mu \alpha^\mu_{-1}|0;p\rangle\ M^2 = 0$$

$$\left(\zeta^{(2)}_{\mu\nu}\alpha^\mu_{-1}\alpha^\nu_{-1} + \zeta^{(1)}_\mu \alpha^\mu_{-2}\right)|0;p\rangle\ M^2 = 1/\alpha'$$

$$\sqrt{2\alpha'}p^\mu \zeta^{(2)}_{\mu\nu} = -\zeta^{(1)}_\nu$$
$$\eta^{\mu\nu}\zeta^{(2)}_{\mu\nu} = -2\sqrt{2\alpha'}\zeta^{(1)}_\mu p^\mu = 4\alpha' p^\mu p^\nu \zeta^{(2)}_{\mu\nu}$$

$$L_m = \frac{1}{2}\sum_n \alpha_{m-n}\cdot\alpha_n\ m \neq 0\,;\quad L_0 = N + \frac{\alpha_0^2}{2}$$
$$\tilde{L}_m = \frac{1}{2}\sum_n \tilde{\alpha}_{m-n}\cdot\tilde{\alpha}_n\ m \neq 0\,;\quad L_0 = \tilde{N} + \frac{\tilde{\alpha}_0^2}{2}$$

$$L_m|\phi\rangle = \tilde{L}_m|\phi\rangle = 0 \; m \neq 0$$
$$(L_0 - a)|\phi\rangle = (\tilde{L}_0 - \tilde{a})|\phi\rangle = 0.$$

$$|\phi, \tilde{\phi}\rangle_{\text{closed}} \equiv |\phi\rangle_{\text{open}} \otimes |\tilde{\phi}\rangle_{\text{open}},$$

$$\left(L_0 + \tilde{L}_0 - (a + \tilde{a})\right)|\phi\rangle = 0$$
$$\left(L_0 - \tilde{L}_0 - (a - \tilde{a})\right)|\phi\rangle = 0$$

$$(L_0 - a)|0;p\rangle = 0 \Rightarrow \left(N + \frac{\alpha' p^2}{4} - a\right)|0;p\rangle = 0$$
$$\Rightarrow M^2 = -p^2 = -\frac{4a}{\alpha'}.$$

$$M^2 = -\frac{4\tilde{a}}{\alpha'}.$$

$$\zeta_{\mu\nu}\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0;p\rangle,$$

$$p^{\mu}\zeta_{\mu\nu} = p^{\nu}\zeta_{\mu\nu} = 0.$$

$$p^{\mu}\zeta_{\mu\nu} = 0 \Rightarrow \zeta_{0\nu} + \zeta_{(d-1)\nu} = 0$$
$$p^{\nu}\zeta_{\mu\nu} \Rightarrow \zeta_{\mu 0} + \zeta_{\mu(d-1)} = 0$$

$$\zeta_{ij} = \zeta_{(ij)} + \zeta_{[ij]} + \zeta^{i}_{i}.$$

$$|0;p\rangle \; M^2 = -\frac{4}{\alpha'}.$$

$$\zeta_{\mu\nu}\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0;p\rangle \; M^2 = 0$$

$$X^+ = \frac{X^0 + X^{d-1}}{\sqrt{2}} \; X^- = \frac{X^0 - X^{d-1}}{\sqrt{2}}$$

$$X^{\mu} \rightarrow X^+, X^-, X^i \; i = 1 \ldots d-2$$
$$\eta_{ij} = 1, \eta_{+-} = \eta_{-+} = \eta^{+-} = \eta^{-+} = -1.$$

$$V^{\pm} = \frac{1}{\sqrt{2}}(V^0 \pm V^{d-1}), V^i \; i = 1 \ldots d-2$$

$$V^{\mu}W_{\mu} = V^iW^i - V^+W^- - V^-W^+$$

$$\xi^{\pm} \rightarrow \tilde{\xi}^{\pm}(\xi^{\pm})$$

$$\tilde{\tau} = \frac{1}{2}\left[\tilde{\xi}^+(\xi^+) + \tilde{\xi}^-(\xi^-)\right]$$
$$\tilde{\sigma} = \frac{1}{2}\left[\tilde{\xi}^+(\xi^+) - \tilde{\xi}^-(\xi^-)\right].$$

$$(\partial^2_{\sigma} - \partial^2_{\tau})\tilde{\tau} = 0$$

$$X^+(\sigma, \tau) = x^+ + 2\alpha' p^+ \tau$$

$$T_{\pm\pm} = \frac{1}{4}(\dot{X} \pm X')^2 = 0$$
$$\Rightarrow \left(\dot{X}^i \pm X^{i'}\right) - 2\left(\dot{X}^+ \pm X^{+'}\right)\left(\dot{X}^- \pm X^{-'}\right) = 0$$
$$\Rightarrow \left(\dot{X}^- \pm X^{-'}\right) = \frac{1}{4p^+\alpha'}\left(\dot{X}^i \pm X^{i'}\right)^2$$

$$\alpha^-_n = \frac{1}{\sqrt{2\alpha' p^+}}\left\{\frac{1}{2}\sum_{i=1}^{d-2}\sum_{m=-\infty}^{\infty} :\alpha^i_{n-m}\alpha^i_m: - a\delta_n\right\}.$$

$$\alpha_0^+ = \sqrt{2\alpha'}p^+ \; ; \; \alpha_0^- = \sqrt{2\alpha'}p^-$$
$$\alpha_n^+ = 0 \; \forall n \neq 0,$$

$$\alpha_0^+ = \sqrt{\frac{\alpha'}{2}}p^+ \; ; \; \alpha_0^- = \sqrt{\frac{\alpha'}{2}}p^-$$
$$\alpha_n^+ = 0 \; \forall n \neq 0,$$

$$\begin{aligned} L_m &= \frac{1}{2}\sum_{-\infty}^{\infty} \alpha_{m-n}\cdot\alpha_n \\ &= \frac{1}{2}\sum_{-\infty}^{\infty} \{\alpha_{m-n}^i\alpha_n^i - \alpha_{m-n}^+\alpha_n^- - \alpha_{m-n}^-\alpha_n^+\} \\ &= -\alpha_0^+\alpha_m^- + \frac{1}{2}\sum_{-\infty}^{\infty} \alpha_{m-n}^i\alpha_n^i, \end{aligned}$$

$$L_0 = -2\alpha' p^+p^- + N + \alpha' p^i p^i$$

$$[(-2p^+p^- + p^ip^i)\alpha' + N - a]|\phi\rangle = 0$$
$$\Rightarrow M^2 = \frac{1}{\alpha'}(N-a)$$

$$|0;p\rangle \; M^2 = -\frac{a}{\alpha'}$$

$$\zeta_i\alpha_{-1}^i|0;p\rangle \; M^2 = \frac{1-a}{\alpha'}$$

$$\left(\zeta_{ij}^{(2)}\alpha_{-1}^i\alpha_{-1}^j + \zeta_i^{(1)}\alpha_{-2}^i\right)|0;p\rangle \; M^2 = \frac{2-a}{\alpha'}.$$

$$\begin{aligned} L_0 = \frac{1}{2}\sum_{-\infty}^{\infty} \alpha_{-n}^i\alpha_n^i &= \frac{1}{2}\left\{\sum_{n=1}^{\infty} \alpha_{-n}^i\alpha_n^i + \sum_{n=-1}^{-\infty} \alpha_{-n}^i\alpha_n^i + \alpha_0^2\right\} \\ &= \frac{1}{2}\left\{\sum_{n=1}^{\infty} \alpha_{-n}^i\alpha_n^i + \sum_{n=1}^{\infty} \alpha_n^i\alpha_{-n}^i + \alpha_0^2\right\} \\ &= \frac{1}{2}\left\{\sum_{n=1}^{\infty} \alpha_{-n}^i\alpha_n^i + \sum_{n=1}^{\infty} \alpha_{-n}^i\alpha_n^i + \eta_{ij}\eta^{ij}\sum_{n=1}^{\infty} n + \alpha_0^2\right\} \\ &= \frac{1}{2}\left\{\sum_{n=1}^{\infty} \alpha_{-n}^i\alpha_n^i + \sum_{n=-1}^{-\infty} \alpha_n^i\alpha_{-n}^i + \alpha_0^2 + (d-2)\sum_{n=1}^{\infty} n\right\} \\ &= \frac{1}{2}\sum_{-\infty}^{\infty} :\alpha_{-n}^i\alpha_n^i: + \frac{d-2}{2}\sum_{n=1}^{\infty} n \end{aligned}$$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$$

$$\sum_{n=1}^{\infty} n = -\frac{1}{12} \; (+\infty)$$

$$\tau_2(x_1,x_2) = \langle\phi(x_1)\phi(x_2)\rangle = \frac{1}{N}\int \mathcal{D}\phi\phi(x_1)\phi(x_2)e^{iS[\phi]},$$

$$N = \int \mathcal{D}\phi e^{iS[\phi]}$$

$$Z = \int \mathcal{D}X^\mu(\tau)\mathcal{D}e e^{iS[X,e]},$$

$$Z = \int \mathcal{D}X^{\mu}(\sigma,\tau)\mathcal{D}\gamma_{\alpha\beta}(\sigma,\tau)e^{iS[X,\gamma]},$$

$$Z_E = \int \mathcal{D}X\mathcal{D}\gamma e^{-S_E[X,\gamma]}.$$

$$G = \mathbb{D}_{\text{Diffeomorphisms}} \otimes \mathfrak{W}_{\text{Weyl Transformations}}$$

$$\gamma_{\alpha\beta} \to \tilde{\gamma}_{\alpha\beta} = e^{\phi}\begin{pmatrix} \mp 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{aligned} \delta X^{\mu} &= \zeta^{\alpha}\partial_{\alpha}X^{\mu} \\ \delta\gamma_{\alpha\beta} &= 2\nabla_{(\alpha}\zeta_{\beta)}, \end{aligned}$$

$$\delta\gamma_{\alpha\beta} = 2\Lambda\gamma_{\alpha\beta}$$

$$\begin{aligned} \delta\gamma_{\alpha\beta} &= 2\nabla_{(\alpha}\zeta_{\beta)} + 2\Lambda\gamma_{\alpha\beta} \\ &= (\mathcal{P}\zeta)_{\alpha\beta} + 2\tilde{\Lambda}\gamma_{\alpha\beta}, \end{aligned}$$

$$\begin{aligned} (\mathcal{P}\zeta)_{\alpha\beta} &= 2\nabla_{(\alpha}\zeta_{\beta)} - (\nabla_{\gamma}\zeta^{\gamma})\gamma_{\alpha\beta} \\ 2\tilde{\Lambda} &= 2\Lambda + \nabla_{\gamma}\zeta^{\gamma} \end{aligned}$$

$$\mathcal{D}\gamma_{\alpha\beta} \equiv \mathcal{D}(\mathcal{P}\zeta)\mathcal{D}\tilde{\Lambda}$$

$$\mathcal{D}\gamma_{\alpha\beta} \to \mathcal{D}\zeta\mathcal{D}\Lambda$$
$$\text{i.e. } \mathcal{D}(\mathcal{P}\zeta)\mathcal{D}\tilde{\Lambda} \to \mathcal{D}\zeta\mathcal{D}\Lambda.$$

$$\mathcal{D}(\mathcal{P}\zeta)\mathcal{D}\tilde{\Lambda} = \mathcal{D}\zeta\mathcal{D}\Lambda\left|\frac{\partial(\mathcal{P}\zeta,\tilde{\Lambda})}{\partial(\zeta,\Lambda)}\right|.$$

$$\begin{vmatrix} \mathcal{P} & 0 \\ * & 1 \end{vmatrix},$$

$$Z_E = \underbrace{\int \mathcal{D}G}_{\text{Vol}_G}\int \mathcal{D}Xe^{-S_E[X,\tilde{\gamma}]}\text{det}\mathcal{P},$$

$$\text{det}M = \int d\theta_1 \ldots d\theta_n \int d\bar{\theta}_1 \ldots d\bar{\theta}_n e^{-\bar{\theta}_i M_{ij}\theta_j},$$

$$\text{det}\mathcal{P} = \int \mathcal{D}b \int \mathcal{D}ce^{-\int b\mathcal{P}c},$$

$$\mathcal{P}^{\rho}_{\alpha\beta} = \delta^{\rho}_{\alpha}\nabla_{\beta} + \delta^{\rho}_{\beta}\nabla_{\alpha} - \tilde{\gamma}_{\alpha\beta}\nabla^{\rho}$$

$$Z_E = \text{Vol}_G\int \mathcal{D}X\mathcal{D}b\mathcal{D}c\exp\left\{-\frac{T}{2}\int d^2\xi\sqrt{\text{det}\tilde{\gamma}}b_{\alpha\beta}\left(2\tilde{\gamma}^{\beta(\alpha}\nabla^{\rho)} - \tilde{\gamma}^{\alpha\rho}\nabla^{\beta}\right)c_{\rho}\right\},$$

$$Z = \text{Vol}_G\int \mathcal{D}X\mathcal{D}b\mathcal{D}ce^{-A_q}$$

$$\begin{aligned} A_{\mathrm{q}} &= A + A^{(bc)} \\ A &= -\frac{1}{2\pi}\int d^2\xi\partial^{\alpha}X^{\mu}\partial_{\alpha}X_{\mu} \\ A^{(bc)} &= -\frac{i}{2\pi}\int d^2\xi\sqrt{-\text{det}\tilde{\gamma}}\tilde{\gamma}^{\alpha\beta}c^{\gamma}\nabla_{\alpha}b_{\beta\gamma} \end{aligned}$$

$$T^{(bc)}_{\alpha\beta} = -i\left[\frac{1}{2}c^{\gamma}\nabla_{(\alpha}b_{\beta)\gamma} + (\nabla_{(\alpha}c^{\gamma})b_{\beta)\gamma} - \frac{1}{2}\left(\frac{1}{2}c^{\gamma}\nabla^{\rho}b_{\rho\gamma} + (\nabla^{\rho}c^{\gamma})b_{\rho\gamma}\right)\tilde{\gamma}_{\alpha\beta}\right].$$

$$A^{(bc)} = \frac{i}{\pi}\int d^2\xi(c^{+}\partial_{-}b_{++} + c^{-}\partial_{+}b_{--})$$

$$T_{\pm\pm}^{(bc)} = -i\left[\frac{1}{2}c^{\pm}\partial_{\pm}b_{\pm\pm} + (\partial_{\pm}c^{\pm})b_{\pm\pm}\right]$$

$$\begin{aligned} \partial_{\pm}c^{\mp} &= 0, \\ \partial_{\pm}b_{\mp\mp} &= 0. \end{aligned}$$

$$\Pi^{(b_{\pm\pm})} = \frac{\delta A^{(bc)}}{\delta(\partial_\tau b_{\pm\pm})} = \frac{ic^{\pm}}{2\pi},$$

$$\begin{aligned} \left\{b_{\pm\pm}(\sigma,\tau), \Pi^{(b_{\pm\pm})}(\sigma',\tau)\right\} &= i\delta(\sigma-\sigma') \\ \Rightarrow \{b_{\pm\pm}(\sigma,\tau), c^{\pm}(\sigma',\tau)\} &= 2\pi\delta(\sigma-\sigma') \end{aligned}$$

$$\begin{aligned} c^{\pm} &= c^{\pm}(\xi^{\pm}) \\ b_{\pm\pm} &= b_{\pm\pm}(\xi^{\pm}) \end{aligned}$$

$$c^{\pm} = \sum_{-\infty}^{\infty} c_n e^{-in\xi^{\pm}}$$

$$b_{\pm\pm} = \sum_{-\infty}^{\infty} b_n e^{-in\xi^{\pm}}$$

$$\{c_m, b_n\} = \delta_{m+n}$$
$$\{c_m, c_n\} = \{b_m, b_n\} = 0$$

$$c^{+} = \sqrt{2}\sum_{-\infty}^{\infty} \tilde{c}_n e^{-2in\xi^{+}}$$
$$c^{-} = \sqrt{2}\sum_{-\infty}^{\infty} c_n e^{-2in\xi^{-}}$$
$$b_{++} = \sqrt{2}\sum_{-\infty}^{\infty} \tilde{b}_n e^{-2in\xi^{+}}$$
$$b_{--} = \sqrt{2}\sum_{-\infty}^{\infty} b_n e^{-2in\xi^{-}}$$

$$\tilde{L}_m^{(bc)} = \frac{1}{2\pi}\int_0^{\pi} d\sigma e^{2im\sigma} T_{++}\bigg|_{\tau=0}$$
$$L_m^{(bc)} = \frac{1}{2\pi}\int_0^{\pi} d\sigma e^{-2im\sigma} T_{--}\bigg|_{\tau=0}$$

$$L_m^{(bc)} = \frac{1}{\pi}\int_0^{2\pi} d\sigma e^{im\sigma} T_{++}\bigg|_{\tau=0}$$

$$L_m^{(bc)} = \sum_{n=-\infty}^{\infty} (m-n): b_{m+n}c_{-n}:$$

$$\begin{aligned} \left[L_m^{(bc)}, b_n\right] &= (m-n)b_{m+n} \\ \left[L_m^{(bc)}, c_n\right] &= -(2m+n)c_{m+n} \\ \left[L_m^{(bc)}, L_n^{(bc)}\right] &= (m-n)L_{m+n}^{(bc)} + \frac{1}{6}(m-13m^3)\delta_{m+n} \end{aligned}$$

$$L_m^{(\text{tot})} = L_m^{(\alpha)} + L_m^{(bc)} - a\delta_m,$$

$$\left[L_m^{(\text{tot})}, L_n^{(\text{tot})}\right] = (m-n)L_{m+n}^{(\text{tot})} + \left[\frac{d}{12}(m^3-m) + \frac{1}{6}(m-13m^3) + 2am\right]\delta_{m+n}$$

$$\left[L_m^{(\text{tot})}, L_n^{(\text{tot})}\right] = (m-n)L_{m+n}^{(\text{tot})}$$

$$\delta_\epsilon X^\mu = \epsilon(c^+\partial_+ + c^-\partial_-)X^\mu,$$
$$\delta_\epsilon b_{\pm\pm} = 2i\epsilon T^{(\text{tot})}_{\pm\pm},$$
$$\delta_\epsilon c^\pm = \epsilon(c^+\partial_+ + c^-\partial_-)c^\pm,$$
$$T^{(\text{tot})}_{\pm\pm} \equiv T^{(\alpha)}_{\pm\pm} + T^{(bc)}_{\pm\pm}\delta_\epsilon T^{(\text{tot})}_{\pm\pm} = 0$$

$$J_{B\pm} = 2c^\pm\left(T^{(\alpha)}_{\pm\pm} + \frac{1}{2}T^{(bc)}_{\pm\pm}\right).$$

$$Q_B = \frac{1}{2\pi}\int_0^\pi d\sigma(J_{B+} + J_{B-})\Big|_{\tau=0}$$

$$\tilde{Q}_B = \frac{1}{2\pi}\int_0^{2\pi} d\sigma J_B + \Big|_{\tau=0},$$
$$Q_B = \frac{1}{2\pi}\int_0^{2\pi} d\sigma J_B - \Big|_{\tau=0},$$

$$Q_B \equiv Q = \sum_{-\infty}^{\infty} L^{(\alpha)}_{-m}c_m - \frac{1}{2}\sum_{-\infty}^{\infty}(m-n){:}c_{-m}c_{-n}b_{m+n}{:} - ac_0,$$

$$Q = \sum_{-\infty}^{\infty} {:}\left(L^{(\alpha)}_{-m} + \frac{1}{2}L^{(bc)}_{-m} - a\delta_m\right)c_m{:}.$$

$$j_\pm = c^\pm b_{\pm\pm},$$

$$U = \frac{1}{2\pi}\int_0^\pi d\sigma(j_+ + j_-)\Big|_{\tau=0}$$

$$U_\pm = \frac{1}{2\pi}\int_0^{2\pi} d\sigma j_\pm\Big|_{\tau=0}$$

$$U \equiv U_- = \sum_n {:}c_n b_{-n}{:} = \frac{1}{2}(c_0b_0 - b_0c_0) + \sum_{n=1}^{\infty}(c_{-n}b_n - b_{-n}c_n)$$
$$U \equiv U_+ = \sum_n {:}\tilde{c}_n\tilde{b}_{-n}{:} = \frac{1}{2}(\tilde{c}_0\tilde{b}_0 - \tilde{b}_0\tilde{c}_0) + \sum_{n=1}^{\infty}(\tilde{c}_{-n}\tilde{b}_n - \tilde{b}_{-n}\tilde{c}_n)$$

$$|\uparrow\rangle \;\; |\downarrow\rangle,$$

$$c_0|\uparrow\rangle = 0$$
$$b_0|\downarrow\rangle = 0$$
$$c_0|\downarrow\rangle = |\uparrow\rangle$$
$$b_0|\uparrow\rangle = |\downarrow\rangle$$

$$b_n|\uparrow\rangle = c_n|\uparrow\rangle = b_n|\downarrow\rangle = c_n|\downarrow\rangle = 0 \;\forall\; n > 0.$$

$$U|\uparrow\rangle = \frac{1}{2}|\uparrow\rangle$$
$$U|\downarrow\rangle = -\frac{1}{2}|\downarrow\rangle,$$

$$U|\uparrow\rangle = \left(\frac{1}{2}(c_0b_0 - b_0c_0) + \sum_{n=1}^{\infty}(c_{-n}b_n - b_{-n}c_n)\right)|\uparrow\rangle$$
$$= \frac{1}{2}c_0b_0|\uparrow\rangle$$
$$= \frac{1}{2}c_0|\downarrow\rangle$$
$$= \frac{1}{2}|\uparrow\rangle.$$

$$n^c - n^b \pm \frac{1}{2},$$

$$U(b_{-1}|\downarrow\rangle) = -\frac{3}{2}b_{-1}|\downarrow\rangle,$$

$$[K_i, K_j] = f_{ij}{}^k K_k,$$

$$\{c^i, b_j\} = \delta^i{}_j$$

$$U = \sum_i c^i b_i$$

$$Q = c^i K_i - \frac{1}{2} f_{ij}{}^k c^i c^j b_k$$

$$f_{ij}{}^m f_{mk}{}^l + f_{jk}{}^m f_{mi}{}^l + f_{ki}{}^m f_{mj}{}^l = 0$$

$$Q^2 = \frac{1}{2}\{Q, Q\} = \frac{1}{2}\sum_{-\infty}^{\infty} ([L_m^{\text{tot}}, L_n^{\text{tot}}] - (m-n)L_{m+n}^{\text{tot}})c_{-m}c_{-n}$$

$$c_n|\chi\rangle = b_n|\chi\rangle = 0 \;\forall\; n > 0$$

$$Q|\chi\rangle = \left[\sum_n \left(c_n L_{-n}^{(\alpha)} + \frac{1}{2} : c_n L_{-n}^{(bc)} :\right) - c_0\right]|\chi\rangle.$$

$$Q|\chi\rangle = \left[c_0\left(L_0^{(\alpha)} - 1\right) + \sum_{n>0} c_{-n} L_n^{(\alpha)}\right]|\chi\rangle.$$

$$|\chi\rangle = |\phi\rangle_\alpha \otimes |\downarrow\rangle_{bc}.$$

$$|\chi\rangle \to |\chi'\rangle = |\chi\rangle + Q|\chi\rangle,$$

$$|\chi\rangle = \sum_{n>0} L_{-n}^{(\alpha)}|\lambda_n\rangle$$

$$\langle\chi \mid \chi\rangle = \left\langle \sum_{n>0} L_n \lambda_n \mid \chi \right\rangle = \sum_{n>0} \langle\lambda_n|L_n|\chi\rangle = 0.$$

$$S = S_1 + S_2 + S_3,$$

$$S_1 = -\frac{T}{2}\int d^2\xi\sqrt{-\det\gamma}\,\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$$
$$S_2 = -\frac{T}{2}\int d^2\xi\,\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}$$
$$S_3 = \frac{T}{2}\int d^2\xi\sqrt{-\det\gamma}\,\alpha'\Phi R^{(2)}$$

$$\chi = \frac{1}{4\pi}\int d^2\xi\sqrt{-\det\gamma}\,R^{(2)}$$

$$(\gamma_{\alpha\beta} \to e^\rho \gamma_{\alpha\beta}), R^{(2)} \to R^{(2)} - \Box\,\rho$$

$$S_1 \to \tilde{S}_1 = -\frac{T}{2}\int d^{2+\varepsilon}\xi\, e^{\varepsilon\phi}\partial_\alpha X^\mu \partial^\alpha X^\nu g_{\mu\nu}$$

$$g_{\mu\nu}(X) = \eta_{\mu\nu} - \frac{\alpha'}{3}R_{\mu\lambda\nu\rho}(X_0)X^\lambda X^\rho + O(X^3).$$

$$\tilde{S}_1 = -\frac{T}{2}\int d^{2+\varepsilon}\xi\left(1+\varepsilon\phi+\mathcal{O}(\phi^2)\right)(\partial_\alpha X^\mu \partial^\alpha X^\nu)$$
$$\left(\eta_{\mu\nu} - \frac{\alpha'}{3}R_{\mu\lambda\nu\rho}(X_0)X^\lambda X^\rho + \mathcal{O}(X^3)\right)$$

$$\langle X^\mu X^\nu\rangle \sim \frac{\eta^{\mu\nu}}{\varepsilon}$$

$$\frac{\alpha'}{3}R_{\mu\lambda\nu\rho}X^\lambda X^\rho \partial_\alpha X^\mu \partial^\alpha X^\nu (1+\varepsilon\phi)$$

$$\frac{\alpha'}{3}R_{\mu\lambda\nu\rho}\frac{\eta^{\lambda\rho}}{\varepsilon}\partial_\alpha X^\mu \partial^\alpha X^\nu(1+\varepsilon\phi) = \frac{\alpha'}{3\varepsilon}R_{\mu\nu}\partial_\alpha X^\mu \partial^\alpha X^\nu (1+\varepsilon\phi)$$

$$R_{\mu\nu}(X) = 0$$
$$\Rightarrow R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \equiv G_{\mu\nu} = 0$$

$$0 = R_{\mu\nu} + \frac{1}{4}H_\mu{}^{\lambda\rho}H_{\nu\lambda\rho} - 2D_\mu D_\nu \Phi + \mathcal{O}(\alpha')$$
$$0 = D_\lambda H^\lambda{}_{\mu\nu} - 2(D_\lambda\Phi)H^\lambda{}_{\mu\nu} + \mathcal{O}(\alpha')$$
$$0 = 4D_\mu\Phi D^\mu\Phi - 4D_\mu D^\mu\Phi + R + \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{d-26}{3\alpha'} + \mathcal{O}(\alpha')$$

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu}$$

$$S_{26} = -\frac{1}{2\kappa^2}\int d^{26}X\sqrt{-\det g}\,e^{-2\Phi}\left\{R - 4D_\mu\Phi D^\mu\Phi + \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\right\}$$

$$\Delta = \frac{1}{L_0-1} = \int_0^1 dz z^{L_0-2}$$
$$V_0(\xi,k) = e^{ik\cdot X(\xi)}$$

$$V_0(y,k) = e^{ik\cdot X(y)}$$

$$V_{\zeta_\mu}(y,k) = \zeta_\mu \frac{dX^\mu}{dy}e^{ik\cdot X(y)}$$

$$V_{\zeta_{\mu\nu}}(\xi,k) = \zeta_{\mu\nu}\partial_\alpha X^\mu \partial^\alpha X^\nu e^{ik\cdot X(\xi)}$$

$$A_n \sim g_s^{n-2}\int \prod_{i=1}^n dy_i \langle V_{\Lambda_1}(y_1,k_1)V_{\Lambda_2}(y_2,k_2)\ldots V_{\Lambda_n}(y_n,k_n)\rangle$$

$$A_n \sim g_s^{n-2}\int \prod_{i=1}^n d^2\xi_i \langle V_{\Lambda_1}(\xi_1,k_1)V_{\Lambda_2}(\xi_2,k_2)\ldots V_{\Lambda_n}(\xi_n,k_n)\rangle$$

$$A_n(k_1,\ldots,k_n) = \frac{1}{\mathrm{Vol}_{SL(2,\mathbb{C})}}g_s^{n-2}\int \prod_{i=1}^n d^2\xi_i \langle V_0(\xi_1,k_1)V_0(\xi_2,k_2)\ldots V_0(\xi_n,k_n)\rangle$$

$$A_4 = \frac{1}{\mathrm{Vol}_{SL(2,\mathbb{C})}}g_s^2\int \prod_{i=1}^4 d^2\xi_i \langle V_0(\xi_1,k_1)V_0(\xi_2,k_2)V_0(\xi_3,k_3)V_0(\xi_4,k_4)\rangle,$$

$$V_0(\xi,k) = e^{ik_\mu X^\mu(\xi)}$$

$$S = \frac{1}{4\pi\alpha'}\int d^2\xi \partial_\alpha X^\mu \partial^\alpha X_\mu$$

$$\langle V_0(1)V_0(2)V_0(3)V_0(4)\rangle = \int \mathcal{D}X e^{-\frac{1}{4\pi\alpha'}\int d^2\xi \partial_\alpha X^\mu \partial^\alpha X_\mu}e^{i\sum_{j=1}^4 k_j^\mu X_\mu(\xi_j)},$$

$$\sum_{j=1}^{4} k_j^{\mu} X_{\mu}(\xi_j)= \int d^2\xi \sum_{j=1}^{4} k_j^{\mu} X_{\mu}(\xi)\delta^2(\xi - \xi_j)$$
$$= \int d^2\xi J^{\mu}(\xi) X_{\mu}(\xi)$$

$$J^{\mu}(\xi) = \sum_{j=1}^{4} k_j^{\mu} \delta^2(\xi - \xi_j).$$

$$\therefore \langle V_0(1)V_0(2)V_0(3)V_0(4)\rangle = \int \mathcal{D}X e^{\frac{1}{4\pi\alpha'}\int d^2\xi X^{\mu}\partial^2 X_{\mu} + i\int d^2\xi J^{\mu}X_{\mu}}.$$

$$\int du_1 \dots du_n e^{-\frac{1}{2}u^{\mathsf{T}}Au + b^{\mathsf{T}}u} = \frac{(2\pi)^{n/2}}{\sqrt{\det A}} e^{\frac{1}{2}b^{\mathsf{T}}A^{-1}b}.$$

$$\langle V_0(1)V_0(2)V_0(3)V_0(4)\rangle \sim \frac{1}{\sqrt{\det(-\partial^2/2\pi\alpha')}} e^{\pi\alpha'\int d^2\xi d^2\xi' J^{\mu}\frac{1}{\partial^2}J_{\mu}}$$

$$G = \frac{1}{2\pi}\log|\xi_i - \xi_j|$$

$$\frac{1}{4\pi\alpha'}\left(X^{\mu} + i2\pi\alpha' J^{\mu}\frac{1}{\partial^2}\right)\partial^2\left(X_{\mu} + i2\pi\alpha'\frac{1}{\partial^2}J_{\mu}\right) + \pi\alpha' J^{\mu}\frac{1}{\partial^2}J_{\mu}$$

$$A_4 = g_s^2\mathcal{N}\int \prod_{i=1}^{4} d^2\xi_i \exp\left\{\frac{\alpha'}{2}\int d^2\xi d^2\xi' \sum_{j,l} k_j \cdot k_l \delta^2(\xi_j - \xi)\log|\xi - \xi'|\delta^2(\xi' - \xi_l)\right\},$$

$$A_4 = g_s^2\mathcal{N}\int \prod_{i=1}^{4} d^2\xi_i \exp\left\{\frac{\alpha'}{2}\sum_{j\neq l} k_j \cdot k_l \log|\xi_j - \xi_l|\right\}$$
$$= g_s^2\mathcal{N}\int \prod_{i=1}^{4} d^2\xi_i \exp\left\{\log\left(\prod_{j\neq l}|\xi_j - \xi_l|^{\frac{\alpha' k_j\cdot k_l}{2}}\right)\right\}$$
$$= g_s^2\mathcal{N}\int \prod_{i=1}^{4} d^2\xi_i \prod_{j\neq l}|\xi_j - \xi_l|^{\frac{\alpha' k_j\cdot k_l}{2}}$$
$$= g_s^2\mathcal{N}\int \prod_{i=1}^{4} d^2\xi_i \prod_{j<l}|\xi_j - \xi_l|^{\alpha' k_j\cdot k_l}$$

$$z_i \to \frac{az_i + b}{cz_i + d}$$

$$g_s^2\mathcal{N}\int \prod_{i=1}^{4} d^2 z_i |\tilde{z}_1\tilde{z}_2\tilde{z}_3\tilde{z}_4|^4 \prod_{j<l}|\tilde{z}_j - \tilde{z}_l|^{\alpha' k_j\cdot k_l}$$
$$\prod_{i=1}^{4}\left(\tilde{z}_i\bar{\tilde{z}}_i\right)^{-2} = \prod_{i=1}^{4}|\tilde{z}_i|^{-4}$$

$$g_s^2\mathcal{N}\int d^2 z |z_1|^{-4}\left|1 - \frac{1}{z_1}\right|^{\alpha' k_1\cdot k_2}|z|^{\alpha' k_3\cdot k_4}\left|1 - \frac{z}{z_1}\right|^{\alpha' k_1\cdot k_3}|1 - z|^{\alpha' k_2\cdot k_3}$$
$$\sim g_s^2\tilde{\mathcal{N}}\int d^2 z |z|^{\alpha' k_3\cdot k_4}|1 - z|^{\alpha' k_2\cdot k_3}$$

$$\int d^2 z |z|^{-A}|1 - z|^{-B} = B\left(1 - \frac{A}{2}, 1 - \frac{B}{2}, \frac{A + B}{2} - 1\right)$$

$$B(a, b, c) = \pi\frac{\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(a + b)\Gamma(b + c)\Gamma(c + a)}$$

$$k_i^2 = \frac{4}{\alpha'}$$

$$s = -(k_1 + k_2)^2$$
$$t = -(k_1 + k_4)^2$$
$$u = -(k_1 + k_3)^2$$

$$s + t + u = -\sum_{i=1}^{4} k_i^2$$
$$= -\frac{16}{\alpha'}$$

$$A_4 \sim g_s^2 \frac{\Gamma\left(-1 - \frac{\alpha' s}{4}\right)\Gamma\left(-1 - \frac{\alpha' t}{4}\right)\Gamma\left(-1 - \frac{\alpha' u}{4}\right)}{\Gamma\left(2 + \frac{\alpha' s}{4}\right)\Gamma\left(2 + \frac{\alpha' t}{4}\right)\Gamma\left(2 + \frac{\alpha' u}{4}\right)}$$

$$A_4^{open} \sim g_s \frac{\Gamma(-1 - \alpha' s)\Gamma(-1 - \alpha' t)}{\Gamma(-2 - \alpha' s - \alpha' t)}.$$

$$A_4^{\text{open}} \sim g_s B(-1 - \alpha' s, -1 - \alpha' t)$$

$$S = A^{(X)} + A^{(\psi)}$$
$$= -\frac{T}{2}\int d^2\xi\{\partial_\alpha X^\mu \partial^\alpha X_\mu + \bar{\psi}^\mu \gamma^\alpha \partial_\alpha \psi_\mu\}$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} ;\ \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ;\ \gamma_3 \equiv \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\{\gamma^\alpha, \gamma^\beta\} = 2\eta^{\alpha\beta}.$$

$$\gamma^\alpha \gamma^\beta \gamma_\alpha = 0.$$

$$\bar{\psi} \equiv \psi^\dagger \gamma^0 = \psi^T \gamma^0,$$

$$\bar{\chi}\psi = \bar{\psi}\chi,$$

$$\bar{\chi}\gamma^\alpha \psi = -\bar{\psi}\gamma^\alpha \chi.$$

$$\delta X^\mu = \bar{\epsilon}\psi^\mu,$$
$$\delta\psi^\mu = \gamma^\alpha \epsilon \partial_\alpha X^\mu.$$

$$\delta S = -\frac{1}{2\pi\alpha'}\int d^2\xi\left(-\bar{\psi}^\mu \epsilon \partial^2 X_\mu + \bar{\psi}^\mu \epsilon \partial^2 X_\mu\right) = 0$$

$$\partial\psi = 0,$$

$$[\delta_1, \delta_2]Y^\mu = a^\alpha \partial_\alpha Y^\mu$$

$$\delta S = \frac{1}{\pi\alpha'}\int d^2\xi(\partial_\alpha \bar{\epsilon})_a J_a^\alpha$$

$$J^\alpha = \frac{1}{2}\gamma^\beta \gamma^\alpha \psi^\mu \partial_\beta X_\mu$$

$$\partial_\alpha J^\alpha = 0$$

$$\gamma^\alpha J_\alpha = 0$$

$$T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2}\bar{\psi}^\mu \gamma_{(\alpha}\partial_{\beta)}\psi_\mu - \frac{1}{2}\left(\partial_\rho X^\mu \partial^\rho X_\mu - \frac{1}{2}\bar{\psi}^\mu \gamma_\rho \partial^\rho \psi_\mu\right)\gamma_{\alpha\beta}$$

$$\psi \equiv \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\not\partial = \gamma \cdot \partial$$
$$= \gamma^0 \partial_0 + \gamma^1 \partial_1$$
$$= \begin{pmatrix} 0 & \partial_\tau \\ -\partial_\tau & 0 \end{pmatrix} + \begin{pmatrix} 0 & \partial_\sigma \\ \partial_\sigma & 0 \end{pmatrix}$$
$$= -2 \begin{pmatrix} 0 & -\partial_+ \\ \partial_- & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -\partial_+ \\ \partial_- & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} -\partial_+ \psi_R \\ \partial_- \psi_L \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\psi_L = \psi_L(\xi^+)$$
$$\psi_R = \psi_R(\xi^-)$$

$$P^+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$P^- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$P^+ \psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$$
$$P^- \psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

$$\gamma^\pm = \frac{1}{2}(\gamma^0 \pm \gamma^1)$$

$$\{\gamma^+, \gamma^-\}_{ab} = \mathbb{I}_{ab}$$

$$\gamma^+ \partial_+ \psi_R = 0;$$
$$\gamma^- \partial_- \psi_L = 0,$$

$$A^{(\psi)} \sim \int d\sigma d\tau \bar{\psi} \not\partial \psi$$
$$\sim \int d\sigma d\tau (\psi_L \partial_- \psi_L + \psi_R \partial_+ \psi_R)$$
$$\sim \int d\sigma d\tau (\psi_L (\partial_\tau - \partial_\sigma) \psi_L + \psi_R (\partial_\tau + \partial_\sigma) \psi_R)$$

$$\int d\tau \{\psi_R \delta\psi_R - \psi_L \delta\psi_L\}\Big|_\sigma + \int d\sigma \{\psi_R \delta\psi_R + \psi_L \delta\psi_L\}\Big|_\tau,$$

$$\psi_R(\pi, \tau) = \psi_L(\pi, \tau),$$

$$\psi_R^\mu(\sigma, \tau) = \sum_{n \in \mathbb{Z}} \psi_n^\mu e^{-in(\tau - \sigma)};$$
$$\psi_L^\mu(\sigma, \tau) = \sum_{n \in \mathbb{Z}} \psi_n^\mu e^{-in(\tau + \sigma)},$$

$$\psi_R(\pi, \tau) = -\psi_L(\pi, \tau),$$

$$\psi_R^\mu(\sigma, \tau) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \psi_r^\mu e^{-ir(\tau - \sigma)}$$
$$\psi_L^\mu(\sigma, \tau) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} \psi_r^\mu e^{-ir(\tau + \sigma)}$$

$$\psi_R^\mu = \sum \psi_n^\mu e^{-in(\tau - \sigma)} \text{ or } \psi_R^\mu = \sum \psi_r^\mu e^{-ir(\tau - \sigma)}$$

$$\psi_L^\mu = \sum \tilde{\psi}_n^\mu e^{-in(\tau + \sigma)} \text{ or } \psi_L^\mu = \sum \tilde{\psi}_r^\mu e^{-ir(\tau + \sigma)}.$$

$$L_m = L_m^{(\alpha)} + L_m^{(\psi)},$$

$$L_m = \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma e^{im\sigma} T_{++}\Big|_{\tau=0}$$

$$F_m = \frac{2}{\pi\alpha'}\int_0^{2\pi} d\sigma e^{im\sigma} J_+\Big|_{\tau=0};$$
$$G_r = \frac{2}{\pi\alpha'}\int_0^{2\pi} d\sigma e^{ir\sigma} J_+\Big|_{\tau=0}$$

$$L_m^{(\alpha)} = \frac{1}{2}\sum_{-\infty}^{\infty} :\alpha_{-n}\cdot\alpha_{m+n}$$
$$L_m^{(\psi_n)} = \frac{1}{2}\sum_{-\infty}^{\infty}\left(n+\frac{1}{2}m\right):\psi_{-n}\cdot\psi_{m+n}$$
$$L_m^{(\psi_r)} = \frac{1}{2}\sum_{-\infty}^{\infty}\left(r+\frac{1}{2}m\right):\psi_{-r}\cdot\psi_{m+r}$$
$$F_m = \frac{1}{2}\sum_{-\infty}^{\infty} :\alpha_{-n}\cdot\psi_{m+n}$$
$$G_r = \frac{1}{2}\sum_{-\infty}^{\infty} :\alpha_{-n}\cdot\psi_{r+n}$$
$$[L_m, L_n] = (m-n)L_{m+n} + \frac{d}{8}m^3\delta_{m+n};$$
$$[L_m, F_n] = \left(\frac{m}{2}-n\right)F_{m+n};$$
$$\{F_m, F_n\} = 2L_{m+n} + \frac{d}{2}m^2\delta_{m+n}.$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{d}{8}(m^3-m)\delta_{m+n}$$
$$[L_m, G_r] = \left(\frac{m}{2}-r\right)G_{m+r}$$
$$\{G_r, G_s\} = 2L_{r+s} + \frac{d}{2}\left(r^2-\frac{1}{4}\right)$$

$$\frac{\delta S}{\delta(\partial_\tau\psi)} \sim \frac{T}{2}\psi,$$

$$\{\psi_a^\mu(\sigma,\tau), \psi_b^\nu(\sigma',\tau)\}_{P.B.} = 2\pi\delta_{ab}\delta(\sigma-\sigma')\eta^{\mu\nu}$$

$$\{\psi_a^\mu(\sigma,\tau), \psi_b^\nu(\sigma',\tau)\} = -2\pi i\delta_{ab}\delta(\sigma-\sigma')\eta^{\mu\nu}$$

$$\{\psi_m^\mu, \psi_n^\nu\} = -i\delta_{m+n}\eta^{\mu\nu}$$
$$\{\psi_r^\mu, \psi_s^\nu\} = -i\delta_{r+s}\eta^{\mu\nu}$$

$$\tilde{\psi}_{n,r}\psi_{-n,-r} = \psi_{n,r}^\dagger\tilde{\psi}_{-n,-r} = \tilde{\psi}_{n,r}^\dagger\{\psi_0^\mu, \psi_0^\nu\} = -i\eta^{\mu\nu}\psi_0^\mu = \gamma^\mu/(\sqrt{2}e^{i3\pi/4})$$

$$L_m|\phi\rangle_R = F_m|\phi\rangle_R = 0 \; m > 0$$
$$L_m|\phi\rangle_{NS} = G_r|\phi\rangle_{NS} = 0 \; m, r > 0$$

$$(L_0 - a_R)|\phi\rangle_R = F_0|\phi\rangle_R = 0$$
$$(L_0 - a_{NS})|\phi\rangle_{NS} = 0$$

$$\alpha_n^\mu \to \alpha_n^i \; (i = 1, \ldots, d-2) \; n \neq 0$$
$$\psi_{n,r}^\mu \to \psi_{n,r}^i \; (i = 1, \ldots, d-2) \; n \neq 0$$

$$\begin{aligned} (L_0 - a_{NS})|0;p\rangle &= 0 \\ \Rightarrow \left(N^{(\alpha)} + N^{(\psi)} + \alpha' p^2 - a_{NS}\right)|0;p\rangle &= 0 \\ \Rightarrow M^2 = -\frac{a_{NS}}{\alpha'} \end{aligned}$$

$$\underbrace{\zeta_i^{(1)}\psi_{-1/2}^{i}}_{8 \text{ states in } d=10}|0;p\rangle$$

$$M^2 = -\frac{1}{\alpha'}\left(a_{NS} - \frac{1}{2}\right)$$

$$\underbrace{\zeta_i^{(2)}\alpha_{-1}^{i}}_{8 \text{ states}}|0;p\rangle + \underbrace{\zeta_{ij}^{(2)}\psi_{-1/2}^{i}\psi_{-1/2}^{j}}_{\binom{8}{2}=28 \text{ states}}|0;p\rangle$$

$$M^2 = -\frac{1}{\alpha'}(a_{NS} - 1)$$

$$N^{(\alpha)} + N^{(\psi)} = 3/2,$$

$$\underbrace{\zeta_i^{(2)}\zeta_j^{(1)}\alpha_{-1}^{i}\psi_{-1/2}^{j}}_{8\times 8=64 \text{ states}}|0;p\rangle + \underbrace{\zeta_{ijk}^{(3)}\psi_{-1/2}^{i}\psi_{-1/2}^{j}\psi_{-1/2}^{k}}_{\binom{8}{3}=56 \text{ states}}|0;p\rangle + \underbrace{\zeta_i^{(1)}\psi_{-3/2}^{i}}_{8 \text{ states}}|0;p\rangle,$$

$$a_{NS} = \frac{d-2}{16},$$

$$a_{NS} = \frac{1}{2} \Rightarrow d = 10.$$

$$\begin{aligned} \sum_{1/2}^{\infty} r &= \frac{1}{2}(1 + 3 + 5 + \cdots) \\ &= \frac{1}{2}\left(\sum_{n=1}^{\infty} n - \sum_{n=1}^{\infty} 2n\right) \\ &= -\frac{1}{2}\sum_{1}^{\infty} n \\ &= \frac{1}{24}, \end{aligned}$$

$$L_0|\phi\rangle = 0 \Rightarrow \alpha' p^2 = 0 \Rightarrow M^2 = 0$$

$$\underbrace{\zeta_i^{(1)}\alpha_{-1}^{i}u_a}_{8\times 16=64 \text{ states}}|a,0;p\rangle + \underbrace{\zeta_i^{(2)}\psi_{-1}^{i}u_a}_{8\times 16=64 \text{ states}}|a,0;p\rangle,$$

$$\begin{aligned} P^{NS} &= \frac{1}{2}(1 + (-1)^F) \\ F &= \sum_{r=1/2}^{\infty} \psi_r^{\dagger}\cdot\psi_r \end{aligned}$$

$$\begin{aligned} P^{R} &= \frac{1}{2}(1 \pm (-1)^G\gamma_{11}) \\ G &= \sum_{n>0} \psi_n^{\dagger}\cdot\psi_n \end{aligned}$$

$$S = \int d^{10}X\left(-\frac{1}{4}F^2 + \frac{1}{2}\bar{\psi}\not{D}\psi\right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g_{YM}\left[A_\mu, A_\nu\right]$$

$$\left(D_\mu\psi\right)^A = \partial_\mu\psi^A + g_{YM}f^{A}{}_{BC}A_\mu^B\psi^C$$

$$\delta A_\mu = \frac{1}{2}\bar{\epsilon}\gamma_\mu\psi,$$
$$\delta\psi = -\frac{1}{4}F_{\mu\nu}\gamma^{\mu\nu}\epsilon,$$
$$\gamma^{\mu\nu} = (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2$$
$$\zeta_{ij}|i\rangle \otimes |\tilde{j}\rangle,$$

$$\zeta_{a\tilde{a}}|a\rangle \otimes |\tilde{a}\rangle,$$

$$\frac{1}{2}(1+\hat{\gamma}_9)|a\rangle = \frac{1}{2}(1-\hat{\gamma}_9)|\tilde{a}\rangle = 0$$

$$\hat{\gamma}_9 = \prod_{i=1}^{8} \hat{\gamma}_i$$

$$\mathbf{8_s} \otimes \mathbf{8_c} = \mathbf{8_v} + \mathbf{566_v}$$

$$\zeta_{a\tilde{a}} = \underbrace{\hat{\gamma}^i_{b\tilde{b}}\hat{\zeta}_{\tilde{b}b}}_{\text{8 states}}\,\hat{\gamma}^i_{a\tilde{a}} + \underbrace{\hat{\gamma}^{ijk}_{b\tilde{b}}\hat{\zeta}_{\tilde{b}b}}_{\binom{8}{3}=56\text{ states}}\,\hat{\gamma}^{ijk}_{a\tilde{a}}$$
$$\frac{1}{2}(1+\hat{\gamma}_9)|a\rangle = \frac{1}{2}(1+\hat{\gamma}_9)|\tilde{a}\rangle = 0$$
$$\zeta_{a\tilde{a}} = \underbrace{\delta_{b\tilde{b}}\hat{\zeta}_{\tilde{b}b}}_{\text{1 state}}\,\delta_{a\tilde{a}} + \underbrace{\hat{\gamma}^{ij}_{\tilde{b}\tilde{b}}\hat{\zeta}_{\tilde{b}b}}_{\binom{8}{2}=28\text{ states}}\,\hat{\gamma}^{ij}_{a\tilde{a}} + \underbrace{\hat{\gamma}^{ijkl}_{b\tilde{b}}\hat{\zeta}_{\tilde{b}b}}_{\frac{1}{2}\binom{8}{4}=35\text{ states}}\,\hat{\gamma}^{ijkl}_{a\tilde{a}}$$

$$\zeta_{i\tilde{a}}|i\rangle \otimes |\tilde{a}\rangle;$$
$$\zeta_{ai}|a\rangle \otimes |\tilde{\imath}\rangle.$$

$$F(\omega) = \sum_{n=0}^{\infty} d_n\omega^n = \mathrm{Tr}\omega^N$$

$$\mathrm{Tr}\omega^N = \mathrm{Tr}\omega^{\sum_{m=1}^{\infty}\alpha_{-m}\cdot\alpha_m} = \prod_{m=1}^{\infty}\mathrm{Tr}\omega^{\alpha_{-m}\cdot\alpha_m}$$
$$= \prod_{m=1}^{\infty}\prod_{i=1}^{24}\mathrm{Tr}\omega^{\alpha^i_{-m}\alpha^i_m} = \left(\prod_{m=1}^{\infty}\mathrm{Tr}\omega^{\alpha^1_{-m}\alpha^1_m}\right)^{24}$$

$$\mathrm{Tr}\omega^{\alpha^1_{-m}\alpha^1_m} = \sum_{n=0}^{\infty}\omega^{mn} = \frac{1}{1-\omega^m}$$

$$\therefore \mathrm{Tr}\omega^N = \left[\prod_{m=1}^{\infty}(1-\omega^m)\right]^{-24} = F(\omega)$$

$$\phi(t) = \prod_{k=1}^{\infty}(1-t^k)$$

$$\phi(\omega) = \prod_{m=1}^{\infty}(1-\omega^m) = \exp\left(\sum_{m=1}^{\infty}\log(1-\omega^m)\right)$$
$$= \exp\left(-\sum_{m,n}\frac{\omega^{mn}}{n}\right) = \exp\left(-\sum_{n=1}^{\infty}\frac{\omega^n}{n(1-\omega^n)}\right)$$

$$\Rightarrow -\sum_{n=1}^{\infty}\frac{\omega^n}{n(1-\omega^n)} \sim -\sum_n \frac{1-n\epsilon}{n^2\epsilon} \sim -\sum_n \frac{1}{n^2\epsilon} = -\frac{1}{1-\omega}\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\Rightarrow \phi(\omega) \sim \exp\left(-\frac{\pi^2}{6(1-\omega)}\right)$$

$$\Rightarrow F(\omega) \sim \exp\left(\frac{4\pi^2}{1-\omega}\right)$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \zeta(2) = \frac{\pi^2}{6}$$

$$\eta(\tau) := e^{i\pi\tau/12}\prod_{n=1}^{\infty}\left(1-e^{2\pi i n\tau}\right)$$

$$\omega = e^{2\pi i\tau} \text{ (i.e. } \tau = \tfrac{\log\,\omega}{2\pi i}\text{)}$$

$$\eta(\tau) = \omega^{1/24}\prod_{n=1}^{\infty}(1-\omega^n).$$

$$\eta(-1/\tau) = (-i\tau)^{1/2}\eta(\tau)$$

$$\eta(-1/\tau) = e^{-i\pi/12\tau}\prod_{n=1}^{\infty}\left(1-e^{-2\pi i n/\tau}\right)$$

$$\eta(-1/\tau) = (q^2)^{1/24}\prod_{n=1}^{\infty}(1-(q^2)^n)$$

$$\begin{aligned}\phi(\omega) &= \omega^{-1/24}\eta(\tau)\\ &= \omega^{-1/24}(-i\tau)^{-1/2}\eta(-1/\tau)\\ &= \left(\frac{-\log\,\omega}{2\pi}\right)^{-1/2}\omega^{-1/24}(q^2)^{1/24}\phi(q^2)\end{aligned}$$

$$q^2 = \exp\left(\frac{4\pi^2}{\log\,\omega}\right)$$

$$\omega \to 1, \log\,\omega \to \omega - 1$$

$$(q^2)^{1/24} \to \exp\left(-\frac{\pi^2}{6(1-\omega)}\right)$$

$$\phi(\omega) \sim (2\pi)^{1/2}(1-\omega)^{-1/2}\exp\left(\frac{-\pi^2}{6(1-\omega)}\right)$$

$$F(\omega) \sim (2\pi)^{-12}(1-\omega)^{12}\exp\left(\frac{4\pi^2}{1-\omega}\right)$$

$$d_n = \frac{1}{2\pi i}\oint d\omega\,\frac{F(\omega)}{\omega^{n+1}}\ .$$

$$d_n \sim \frac{1}{2\pi i}\oint\ d\omega\left(\frac{-\log\,\omega}{2\pi}\right)^{12}\exp\left[-(n+1)\left(\log\,\omega + \frac{4\pi^2}{(n+1)\log\,\omega}\right)\right].$$

$$\begin{aligned}s &\to n+1\\ f(z) &\to f(\omega) = -\left(\log\,\omega + \frac{4\pi^2}{(n+1)\log\,\omega}\right)\\ g(z) &\to g(\omega) = \left(\frac{-\log\,\omega}{2\pi}\right)^{12},\end{aligned}$$

$$f''(\omega_0) = f''(\omega)|_{\omega=e^{-2\pi/\sqrt{n+1}}} = \left(\frac{\sqrt{n+1}}{\pi}\right)e^{4\pi/\sqrt{n+1}}$$

$$g(\omega_0) = g(\omega)|_{\omega=e^{-2\pi/\sqrt{n+1}}} = \left(\frac{1}{\sqrt{n+1}}\right)^{12}.$$

$$d_n \sim \frac{1}{\sqrt{2}}(n+1)^{-27/4}e^{4\pi\sqrt{n}}e^{-2\pi/\sqrt{n+1}},$$

$$d_n \sim n^{-27/4}e^{4\pi\sqrt{n}}.$$

$$Z = \sum_n d_n e^{-E_n/(k_B T)} \to \int dn d_n e^{-E_n/(k_B T)}$$
$$\sim 2\alpha' \int dE_n d_n E_n e^{-\beta E_n}$$

$$Z \sim 2\alpha' \int dE n^{-27/4}e^{4\pi\sqrt{n}}E e^{-\beta E}$$
$$\sim 2\alpha'(\alpha')^{-27/4} \int dE E^{-27/2} E e^{4\pi\sqrt{\alpha'}E-\beta E}$$
$$\sim \int dE E^{-25/2} e^{(4\pi\sqrt{\alpha'}-\beta)E}$$

$$T_H = \frac{1}{4\pi k_B\sqrt{\alpha'}}.$$

$$X^\mu(\sigma,\tau) = X_L^\mu(\sigma+\tau) + X_R^\mu(\sigma-\tau)$$
$$X_L^\mu = \frac{1}{2}x^\mu + \frac{1}{2}\alpha' p_L^\mu \xi^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n^\mu}{n}e^{-in\xi^+}$$
$$X_R^\mu = \frac{1}{2}x^\mu + \frac{1}{2}\alpha' p_R^\mu \xi^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-in\xi^-}$$

$$X^{25}(\sigma,\tau) = X^{25}(\sigma+2\pi,\tau) + 2\pi R n^{25};$$
$$X^\nu(\sigma,\tau) = X^\nu(\sigma+2\pi,\tau)\ \nu = 0,1,\ldots,24,$$

$$X^{25}(\sigma+2\pi,\tau) = x^{25} + \frac{\alpha'}{2}p_L^{25}\xi^+ + \frac{\alpha'}{2}p_R^{25}\xi^- + \frac{\alpha'}{2}2\pi\left(p_L^{25}-p_R^{25}\right)$$
$$+i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left[\alpha_n^{25}e^{-in\xi^+} + \tilde{\alpha}_n^{25}e^{-in\xi^-}\right]$$
$$= X^{25}(\sigma,\tau) + \pi\alpha'\left(p_L^{25}-p_R^{25}\right)$$
$$\Rightarrow p_L^{25} - p_R^{25} = -\frac{2Rn^{25}}{\alpha'}$$

$$p_L^{25} = \frac{m^{25}}{R} - \frac{Rn^{25}}{\alpha'};$$
$$p_R^{25} = \frac{m^{25}}{R} + \frac{Rn^{25}}{\alpha'}.$$

$$\left(N + \tilde{N} + \frac{\alpha'}{4}(p_L^\nu p_{L\nu} - p_R^\nu p_{R\nu}) + \frac{\alpha'}{4}\left(\left(p_L^{25}\right)^2 - \left(p_R^{25}\right)^2\right)\right)|\phi\rangle = 0$$

$$\left(N - \tilde{N} - m^{25}n^{25}\right)|\phi\rangle = 0$$

$$\left(\frac{N+\tilde{N}}{2} + \frac{(m^{25})^2\alpha'}{4R^2} + \frac{(n^{25})^2R^2}{4\alpha'} + \frac{\alpha'}{4}p_A^\nu p_{A\nu} - 1\right)|\phi\rangle$$

$$M^2 = -q^2 = \frac{2(N+\tilde{N})}{\alpha'} - \frac{4}{\alpha'} + \frac{(m^{25})^2}{R^2} + \frac{(n^{25})^2R^2}{\alpha'^2}$$

$$\alpha_0^{25} = \left(\frac{m^{25}}{R} - \frac{Rn^{25}}{\alpha'}\right)\sqrt{\frac{\alpha'}{2}}$$

$$\tilde{\alpha}_0^{25} = \left(\frac{m^{25}}{R} + \frac{Rn^{25}}{\alpha'}\right)\sqrt{\frac{\alpha'}{2}}$$

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n} \cdot \alpha_n$$

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n} \cdot \alpha_n:$$

$$:\alpha_p^{\mu}\alpha_q^{\nu}: = \begin{cases} \alpha_p^{\mu}\alpha_q^{\nu} & q \geq 0 \\ \alpha_q^{\mu}\alpha_p^{\nu} & q < 0 \end{cases}$$

$$[L_m, \alpha_n^{\mu}] = -n\alpha_{m+n}^{\mu},$$

$$\begin{aligned}
[L_m, L_n] &= \left[\frac{1}{2}\sum_{p=-\infty}^{\infty} :\alpha_{m-p} \cdot \alpha_p:, L_n\right] \\
&= \frac{1}{2}\sum_{p=-\infty}^{-1} [\alpha_p \cdot \alpha_{m-p}, L_n] + \frac{1}{2}\sum_{p=0}^{\infty} [\alpha_{m-p} \cdot \alpha_p, L_n] \\
&= \frac{1}{2}\sum_{p=-\infty}^{-1} \big((m-p)\alpha_p \cdot \alpha_{m+n-p} + p\alpha_{n+p} \cdot \alpha_{m-p}\big) \\
&\quad + \frac{1}{2}\sum_{p=0}^{\infty} \big(p\alpha_{m-p} \cdot \alpha_{n+p} + (m-p)\alpha_{m+n-p} \cdot \alpha_p\big)
\end{aligned}$$

$$\begin{aligned}
[L_m, L_n] &= \frac{1}{2}\left(\sum_{p=-\infty}^{-1} (m-p)\alpha_p \cdot \alpha_{m+n-p} + \sum_{p=-\infty}^{n-1} (p-n)\alpha_p \cdot \alpha_{n+m-p}\right) \\
&\quad + \frac{1}{2}\left(\sum_{p=n}^{\infty} (p-n)\alpha_{n+m-p} \cdot \alpha_p + \sum_{p=0}^{\infty} (m-p)\alpha_{m+n-p} \cdot \alpha_p\right).
\end{aligned}$$

$$\begin{aligned}
\sum_{p=n}^{\infty} (p-n)\alpha_{n+m-p} \cdot \alpha_p &= \sum_{p=n}^{-1} (p-n)\alpha_{n+m-p} \cdot \alpha_p + \sum_{p=0}^{\infty} (p-n)\alpha_{n+m-p} \cdot \alpha_p \\
&= \sum_{p=n}^{-1} (p-n)\big(\alpha_p \cdot \alpha_{n+m-p} + \eta_{\mu\nu}[\alpha_{n+m-p}^{\mu}, \alpha_p^{\nu}]\big) \\
&\quad + \sum_{p=0}^{\infty} (p-n)\alpha_{n+m-p} \cdot \alpha_p \\
&= \sum_{p=n}^{-1} (p-n)\alpha_p \cdot \alpha_{n+m-p} + \sum_{p=n}^{-1} (p-n)(n+m-p)\delta_{n+m}\eta_{\mu\nu}\eta^{\mu\nu} \\
&\quad + \sum_{p=0}^{\infty} (p-n)\alpha_{n+m-p} \cdot \alpha_p
\end{aligned}$$

$$\sum_{p=0}^{\infty} (m-n)\alpha_{n+m-p} \cdot \alpha_p$$

$$\sum_{p=-\infty}^{-1} (p-n)\alpha_p \cdot \alpha_{n+m-p}$$

$$\sum_{p=-\infty}^{-1} (m-n)\alpha_p \cdot \alpha_{n+m-p}$$

$$\frac{1}{2}\sum_{p=-\infty}^{\infty} (m-n){:}\alpha_{n+m-p} \cdot \alpha_p{:} = L_{m+n}$$

$$\begin{aligned}
\sum_{p=n}^{-1} (p-n)(n+m-p)\delta_{n+m}\eta_{\mu\nu}\eta^{\mu\nu} &= \sum_{p=-m}^{-1} d(p+m)(-p)\delta_{n+m} \\
&= \sum_{-q=-m}^{-1} d(-q+m)(q)\delta_{n+m} \\
&= \sum_{q=m}^{1} d(mq-q^2)\delta_{n+m} \\
&= \sum_{q=1}^{m} d(mq-q^2)\delta_{n+m} \\
&= dm\delta_{n+m}\sum_{q=1}^{m} q - d\delta_{n+m}\sum_{q=1}^{m} q^2 \\
&= d\delta_{n+m}\left(m\frac{1}{2}m(m+1) - \frac{1}{6}m(m+1)(2m+1)\right) \\
&= \frac{d}{6}m\delta_{n+m}(3m^2+3m-2m^2-3m-1) \\
&= \frac{d}{6}(m^3-m)\delta_{n+m}
\end{aligned}$$

$$[L_m, L_n] = L_{m+n} + \frac{d}{12}(m^3-m)\delta_{m+n},$$

$$\frac{1}{2}\sum_{p=0}^{n-1} p(p-n)d\delta_{m+n},$$

$$\{c_m, b_n\} = \delta_{m+n}$$

$$L_m^{(bc)} = \sum_{p=-\infty}^{\infty} {:}b_{m+p}c_{-p}{:},$$

$$:b_ic_j: = \begin{cases} b_ic_j & j \geq 0 \\ -c_jb_i & j < 0 \end{cases}$$

$$\left[L_m^{(bc)}, b_n\right] = (m-n)b_{m+n},$$
$$\left[L_m^{(bc)}, c_n\right] = -(2m+n)c_{m+n}.$$

$$\begin{aligned}
\left[L_m^{(bc)}, L_n^{(bc)}\right] = &\sum_{p=-\infty}^{0} (m-p)(2n-p)b_{m+p}c_{n-p} - \sum_{p=-\infty}^{0} (m-p)(n-m-p)b_{m+n+p}c_{-p} \\
&+ \sum_{p=1}^{\infty} (m-p)(n-m-p)c_{-p}b_{m+n+p} - \sum_{p=1}^{\infty} (m-p)(2n-p)c_{n-p}b_{m+p}
\end{aligned}$$

$$\begin{aligned}
\left[L_m^{(bc)}, L_n^{(bc)}\right] = &\sum_{p=-\infty}^{-n} (m-n-p)(n-p)b_{m+n+p}c_{-p} - \sum_{p=-\infty}^{0} (m-p)(n-m-p)b_{m+n+p}c_{-p} \\
&+ \sum_{p=1}^{\infty} (m-p)(n-m-p)c_{-p}b_{m+n+p} - \sum_{p=-n+1}^{\infty} (m-n-p)(n-p)c_{-p}b_{m+n+p}.
\end{aligned}$$

$$\sum_{p=-\infty}^{0} ((m-n-p)(n-p)+(m+p-n)(m-p))b_{m+n+p}c_{-p}$$
$$-\sum_{p=1}^{\infty} ((m+p-n)(m-p)+(m-n-p)(n-p))c_{-p}b_{m+n+p}$$
$$=(m-n)\sum_{p=-\infty}^{\infty} (m+n-p){:}\, b_{m+n+p}c_{-p}$$
$$:=(m-n)L_{m+n}^{(bc)}$$

$$\sum_{p=1}^{-n} (m-n-p)(n-p)\delta_{m+n} = \frac{1}{6}(m-13m^3)\delta_{m+n}$$

$$\left[L_m^{(bc)}, L_n^{(bc)}\right] = (m-n)L_{m+n}^{(bc)} + \frac{1}{6}(m-13m^3)\delta_{m+n}$$

$$X^\mu = x^\mu + \alpha' p^\mu \tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^\mu e^{-in(\tau-\sigma)} + \tilde{\alpha}_n^\mu e^{-in(\tau+\sigma)}\right).$$

$$X^\mu = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-in\tau}\cos n\sigma.$$

$$X^\mu = a^\mu + \frac{1}{\pi}(b^\mu - a^\mu)\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-in\tau}\sin n\sigma.$$

$$X^\mu = b^\mu + i\sqrt{2\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\frac{\alpha_r^\mu}{r}e^{-ir\tau}\cos r\sigma.$$

$$X^\mu = a^\mu + \sqrt{2\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\frac{\alpha_r^\mu}{r}e^{-ir\tau}\sin r\sigma$$

$$\psi_R(\pi,\tau) = \psi_L(\pi,\tau),$$

$$\psi_R^\mu(\sigma,\tau) = \sum_{n\in\mathbb{Z}} \psi_n^\mu e^{-in(\tau-\sigma)};$$
$$\psi_L^\mu(\sigma,\tau) = \sum_{n\in\mathbb{Z}} \psi_n^\mu e^{-in(\tau+\sigma)},$$

$$\psi_R(\pi,\tau) = -\psi_L(\pi,\tau),$$

$$\psi_R^\mu(\sigma,\tau) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} \psi_r^\mu e^{-ir(\tau-\sigma)};$$
$$\psi_L^\mu(\sigma,\tau) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} \psi_r^\mu e^{-ir(\tau+\sigma)},$$

$$\psi_R^\mu = \sum \psi_n^\mu e^{-in(\tau-\sigma)} \text{ or } \psi_R^\mu = \sum \psi_r^\mu e^{-ir(\tau-\sigma)}$$

$$\psi_L^\mu = \sum \tilde{\psi}_n^\mu e^{-in(\tau+\sigma)} \text{ or } \psi_L^\mu = \sum \tilde{\psi}_r^\mu e^{-ir(\tau+\sigma)}.$$

$$\Omega{:}\, \sigma \to l-\sigma; \tau \to \tau,$$

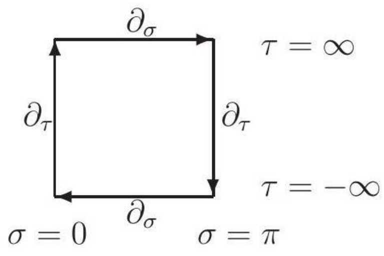
$$\Omega{:}\, \alpha_n^\mu \leftrightarrow \tilde{\alpha}_n^\mu,$$

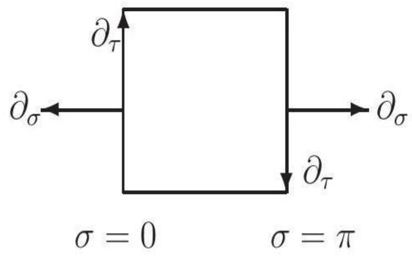
$$\Omega: \alpha_n^\mu \to (-1)^n \alpha_n^\mu.$$

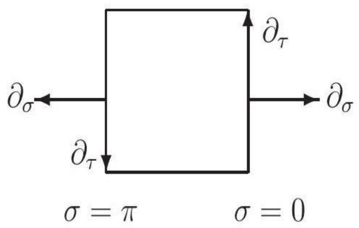
$$\Omega: \alpha_n^\mu \to (-1)^n \alpha_n^\mu; a^\mu \leftrightarrow b^\mu.$$

$$\text{ND} \quad \Omega: \alpha_r^\mu \to e^{i\pi r} \alpha_r^\mu$$
$$\text{DN} \quad \Omega: \alpha_r^\mu \to e^{-i\pi r} \alpha_r^\mu$$

$$\Omega: \psi_n^\mu \to (-1)^n \psi_n^\mu$$
$$\Omega: \psi_r^\mu \to e^{\pm i\pi r} \psi_r^\mu,$$







$$\Omega: |n;k\rangle \to (-1)^{-n} |n;k\rangle.$$

$$|0\rangle_\alpha \otimes |0\rangle_\psi (n;k;ij)$$

$$|n;k;a\rangle = \sum_{i,j=1}^{N_c} |n;k;ij\rangle \lambda_{ji}^a,$$

$$\Omega: |n;k;ij\rangle \to \omega_n |n;k;ij\rangle^T$$
$$= \omega_n |n;k;ji\rangle,$$

$$\Omega: |n;k;a\rangle \to \omega_n \sum_{i,j} |n;k;ij\rangle^T \left(\lambda_{ji}^a\right)^T$$
$$= \omega_n \sum_{i,j} |n;k;ji\rangle (\lambda^a)^T{}_{ij}$$
$$= \omega_n \sum_{i,j} |n;k;ij\rangle (\lambda^a)^T{}_{ji}$$
$$= \omega_n s^a \sum_{i,j} |n;k;ij\rangle \lambda_{ji}^a$$
$$= \omega_n s^a |n;k;a\rangle,$$

$$\Omega_\gamma : |n;k;ij\rangle \to \omega_n \gamma_{jj'} |n;k;j'i'\rangle \gamma^{-1}_{i'i}.$$

$$\begin{aligned}\Omega_\gamma^2 : |n;k;ij\rangle &\to \omega_n^2 \gamma_{ii''} \left[\gamma_{j''j'} |n;k;j'i'\rangle \gamma^{-1}_{i'i''}\right]^T \gamma^{-1}_{j''j} \\ &= \omega_n^2 \gamma_{ii''} \left(\gamma^{-1}_{i'i''}\right)^T |n;k;i'j'\rangle \left(\gamma_{j''j'}\right)^T \gamma^{-1}_{j''j} \\ &= \omega_n^2 \gamma_{ii''} (\gamma^T)^{-1}_{i''i'} |n;k;i'j'\rangle \gamma^T_{j'j''} \gamma^{-1}_{j''j} \\ &= \omega_n^2 (\gamma(\gamma^T)^{-1})_{ii'} |n;k;i'j'\rangle (\gamma^T \gamma^{-1})_{j'j}.\end{aligned}$$

$$\gamma = M = i\begin{pmatrix} 0 & I_k \\ -I_k & 0 \end{pmatrix},$$

$$\lambda \to M\lambda^T M$$

$$\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} \to \begin{pmatrix} D_k^T & -B_k^T \\ -C_k^T & A_k^T \end{pmatrix} = -\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix}$$

$$\lambda = \begin{pmatrix} A_k & B_k \\ C_k & -A_k^T \end{pmatrix},$$

$$X^I(\sigma, \tau) \to -X^I(l - \sigma, \tau),$$

$$Q_{Op} = \pm 2 \cdot 2^{p-5} Q_{Dp},$$

$$\frac{1}{2}(\psi_6 + i\psi_7)_{-1/2}(\psi_8 + i\psi_9)_{-1/2}|0\rangle,$$

$$\begin{aligned}\Omega^2_{\gamma_5\gamma_9} : |\phi; iJ\rangle &\to \Omega_{\gamma_5\gamma_9} : \omega_{59} \gamma_{9JJ'} |\phi; J'i'\rangle \gamma_5^{-1}{}_{i'i} \\ &\to \omega_{59}^2 \gamma_{5ii''} \left[\gamma_{9J''J'} |\phi; J'i'\rangle \gamma_5^{-1}{}_{i'i''}\right]^T \gamma_9^{-1}{}_{J''J} \\ &= \omega_{59}^2 \gamma_{5ii''} (\gamma_5^{-1}{}_{i'i''})^T |\phi; i'J'\rangle \left(\gamma_{9J''J'}\right)^T \gamma_9^{-1}{}_{J''J} \\ &= \omega_{59}^2 \gamma_{5ii''} (\gamma_5^T)^{-1}_{i''i'} |\phi; i'J'\rangle \gamma_9^T{}_{J'J''} \gamma_9^{-1}{}_{J''J} \\ &= \omega_{59}^2 (\gamma_5 \gamma_5^T)_{ii'} |\phi; i'J'\rangle (\gamma_9^T \gamma_9^{-1})_{J'J},\end{aligned}$$

$$\begin{aligned}N_{NS} &= \tilde{N}^{(\alpha)} + \tilde{N}^{(\psi)} \\ &= \frac{1}{2}\sum_{n\in\mathbb{Z}} \alpha^i_{-n}\alpha^i_n + \frac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}} r\psi^i_{-r}\psi^i_r \\ &= \frac{1}{2}\sum_1^\infty \alpha^i_{-n}\alpha^i_n + \frac{1}{2}\sum_{-\infty}^{-1} \alpha^i_{-n}\alpha^i_n + \frac{1}{2}\sum_{1/2}^\infty r\psi^i_{-r}\psi^i_r + \frac{1}{2}\sum_{-\infty}^{-1/2} r\psi^i_{-r}\psi^i_r \\ &= \frac{1}{2}\sum_1^\infty \alpha^i_{-n}\alpha^i_n + \frac{1}{2}\sum_1^\infty \alpha^i_n\alpha^i_{-n} + \frac{1}{2}\sum_{1/2}^\infty r\psi^i_{-r}\psi^i_r + \frac{1}{2}\sum_{1/2}^\infty -r\psi^i_r\psi^i_{-r} \\ &= \frac{1}{2}\sum_1^\infty \alpha^i_{-n}\alpha^i_n + \frac{1}{2}\sum_1^\infty \left(\alpha^i_{-n}\alpha^i_n + [\alpha^i_n, \alpha^i_{-n}]\right) \\ &\quad + \frac{1}{2}\sum_{1/2}^\infty r\psi^i_{-r}\psi^i_r + \frac{1}{2}\sum_{1/2}^\infty \left(r\psi^i_{-r}\psi^i_r - r\{\psi^i_r, \psi^i_{-r}\}\right) \\ &= \sum_1^\infty \alpha^i_{-n}\alpha^i_n + \sum_{1/2}^\infty r\psi^i_{-r}\psi^i_r + \frac{1}{2}\sum_1^\infty n\eta^{ii} - \frac{1}{2}\sum_{1/2}^\infty r\eta^{ii} \\ &= N^{(\alpha)} + N^{(\psi)} + \frac{d-2}{2}\left(\sum_1^\infty n - \sum_{1/2}^\infty r\right) \\ &= N^{(\alpha)} + N^{(\psi)} - a_{NS}\end{aligned}$$

$$\sum_{1/2}^\infty r = \frac{1}{24}$$

$$\psi_a^{\pm} = \frac{1}{\sqrt{2}}(\psi_{2a} \pm i\psi_{2a+1}),$$

$$\begin{aligned} &|0\rangle \\ &\psi_a^{+}|0\rangle \\ &\psi_{a_1}^{+}\psi_{a_2}^{+}|0\rangle \\ &\psi_{a_1}^{+}\psi_{a_2}^{+}\psi_{a_3}^{+}|0\rangle \\ &\psi_{a_1}^{+}\psi_{a_2}^{+}\psi_{a_3}^{+}\psi_{a_4}^{+}|0\rangle \end{aligned}$$

$$SO(9,1) \to SO(3,1) \times SO(6)$$

$$16 = 8_s + 8_c \to (2,4) + (2',4').$$

Directions	0-3	4-9
3-3 b.cs	NN	DD
NS $a_{NS} = 1/2$ \#	α_{-n}/ψ_{-r} $\psi^{\mu}_{-1/2}\|0\rangle$ 2	α_{-n}/ψ_{-r} $\psi^{I}_{-1/2}\|0\rangle$ 6
R $a_R = 0$ \#	α_{-n}/ψ_{-r} $\psi^{\mu}_{0}\|0\rangle$ 8 total	α_{-n}/ψ_{-r} $\psi^{I}_{0}\|0\rangle$

Directions	$0-3$	$4-7$	$8-9$
b.cs	NN	DD	DD
NS $a_{NS} = 1/2$ \# $USp(N_c)$ rep	α_{-n}/ψ_{-r} $\psi^{\mu}_{-1/2}\|0\rangle$ 2 $N_c(N_c+1)/2$	α_{-n}/ψ_{-r} $\psi^{M}_{-1/2}\|0\rangle$ 4 $N_c(N_c-1)/2$	α_{-n}/ψ_{-r} $\psi^{m}_{-1/2}\|0\rangle$ 2 $N_c(N_c+1)/2$
R $a_R = 0$ \# $USp(N_c)$ rep	α_{-n}/ψ_{-r} $\psi^{\mu}_{0}\|0\rangle$ 2 $N_c(N_c+1)/2$	α_{-n}/ψ_{-r} $\psi^{M}_{0}\|0\rangle$ 4 $N_c(N_c-1)/2$	α_{-n}/ψ_{-r} $\psi^{m}_{0}\|0\rangle$ 2 $N_c(N_c+1)/2$

$$-a_{NS} = \frac{4}{2}\left(\sum_{1}^{\infty} n - \sum_{1/2}^{\infty} r\right) + \frac{4}{2}\left(\sum_{1/2}^{\infty} r - \sum_{1}^{\infty} n\right) = 0$$

$$-a_{R} = \frac{4}{2}\left(\sum_{1}^{\infty} n - \sum_{1}^{\infty} n\right) + \frac{4}{2}\left(\sum_{1/2}^{\infty} r - \sum_{1/2}^{\infty} r\right) = 0$$

Directions	$0-3$	$4-7$	$8-9$
b.cs	NN	DN	DD
NS $a_{NS} = 0$ \# $SO(8)$ rep	α_{-n}/ψ_{-r} – – –	α_{-r}/ψ_{-n} $\psi^{M}_{0}\|0\rangle$ 2 **8**	α_{-n}/ψ_{-r} – – –
R $a_R = 0$ \# $SO(8)$ rep	α_{-n}/ψ_{-n} $\psi^{\mu}_{0}\|0\rangle$	α_{-r}/ψ_{-r} – 2 total **8**	α_{-n}/ψ_{-n} $\psi^{m}_{0}\|0\rangle$

Directions	$0-5$	$6-9$
b.cs	NN	DD
NS $a_{NS}=1/2$ # $USp(N_c)$ rep	α_{-n}/ψ_{-r} $\psi^{\mu}_{-1/2}\|0\rangle$ 4 $N_c(N_c+1)/2$	α_{-n}/ψ_{-r} $\psi^{M}_{-1/2}\|0\rangle$ 4 $N_c(N_c-1)/2$
R $a_R=0$ # $USp(N_c)$ rep	α_{-n}/ψ_{-r} $\psi^{\mu}_{0}\|0\rangle$ 4 $N_c(N_c+1)/2$	α_{-n}/ψ_{-r} $\psi^{M}_{0}\|0\rangle$ 4 $N_c(N_c-1)/2$

Directions	$0-5$	$6-9$
b.cs	NN	DN
NS $a_{NS}=0$ # $SO(32)$ rep	α_{-n}/ψ_{-r} – – –	α_{-r}/ψ_{-n} $\psi^{M}_{0}\|0\rangle$ 2 **32**
R $a_R=0$ # $SO(32)$ rep	α_{-n}/ψ_{-n} $\psi^{\mu}_{0}\|0\rangle$ 2 **32**	α_{-r}/ψ_{-r} – – –

$$\gamma_{d+1}=\prod_{i=0}^{d-1}\gamma^i, d\in 2\mathbb{Z}$$

$$P^{\pm}=\frac{1}{2}(1\pm\gamma_{d+1})$$

$$\sum_{p=1}^{n}1=n$$
$$\sum_{p=1}^{n}p=\frac{1}{2}n(n+1)$$
$$\sum_{p=1}^{n}p^2=\frac{1}{6}n(n+1)(2n+1)$$

$$\delta(x-x')=\frac{1}{2\pi}\int_{-\infty}^{\infty}dpe^{\pm ip(x-x')}$$

$$\delta(\sigma-\sigma')=\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}e^{\pm in(\sigma-\sigma')}$$

$$\int_{x_0}^{x_0+L} dx\sin\left(\frac{2\pi rx}{L}\right)\cos\left(\frac{2\pi px}{L}\right) = 0\ \forall r,p$$

$$\int_{x_0}^{x_0+L} dx\cos\left(\frac{2\pi rx}{L}\right)\cos\left(\frac{2\pi px}{L}\right) = \begin{cases} L & r=p=0 \\ \frac{1}{2}L & r=p>0 \\ 0 & r\neq p \end{cases}$$

$$\int_{x_0}^{x_0+L} dx\sin\left(\frac{2\pi rx}{L}\right)\sin\left(\frac{2\pi px}{L}\right) = \begin{cases} 0 & r=p=0 \\ \frac{1}{2}L & r=p>0 \\ 0 & r\neq p \end{cases}$$

$$\int_{x_0}^{x_0+L} dx\exp\left(\frac{i2\pi rx}{L}\right)\exp\left(\frac{i2\pi px}{L}\right) = \begin{cases} L & r=p=0 \\ 0 & r=p>0 \\ 0 & r\neq p \end{cases}$$

$$\int_{x_0}^{x_0+L} dx\exp\left(\frac{i2\pi rx}{L}\right)\exp\left(\frac{-i2\pi px}{L}\right) = \begin{cases} L & r=p=0 \\ L & r=p>0 \\ 0 & r\neq p \end{cases} = L\delta_{r-p}$$

$$[AB,C] = A[B,C] + [A,C]B$$
$$[PQ,R] = P\{Q,R\} - \{P,R\}Q$$
$$[AP,B] = A[P,B] + [A,B]P$$
$$\{AP,Q\} = A\{P,Q\} - [A,Q]P$$

$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$$
$$[P,\{Q,R\}] + [Q,\{R,P\}] + [R,\{P,Q\}] = 0$$
$$[A,[B,P]] + [B,[P,A]] + [P,[A,B]] = 0$$
$$[\{P,Q\},A] + \{[A,P],Q\} + \{[A,Q],P\} = 0$$

$$\Gamma(z+1) = z! = \int_0^\infty e^{-t}t^z dt\ (\Re(z) > -1)$$

$$\Gamma(z-1) = \frac{\Gamma(z)}{(z-1)}$$

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt\ (\Re(x),\Re(y) > 0)$$

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$\int_C dzg(z)e^{sf(z)} \approx \sqrt{\frac{2\pi}{s|f''(z_0)|}}g(z_0)e^{sf(z_0)}e^{i\alpha}\ \text{ as } s\to\infty,$$

$$\alpha = \left(\pi - \arg f''(z_0)\right)/2$$

$$e^{-1} = \sqrt{g_{\tau\tau}g^{\tau\tau}} = \sqrt{g^{\tau\tau}}\,g$$

$$\delta_\eta X^\mu = \eta\partial_\tau X^\mu; \delta_\eta e = \partial_\tau(\eta e)$$

$$\xi^0 = \tau, \xi^1 = \sigma$$

$$M\!: \frac{\delta|\mathrm{det}M|}{|\mathrm{det}M|} = \mathrm{tr}(M^{-1}\delta M)$$

$$A_1 = \lambda\int d^2\xi\sqrt{-\mathrm{det}\gamma}$$

$$A_2 = \frac{1}{2\pi}\int d^2\xi\sqrt{-\mathrm{det}\gamma}R^{(2)}(\gamma)$$

$$\Box := -\partial_\tau^2 + \partial_\sigma^2$$

$$f(\sigma,\tau) \equiv f(\xi^+,\xi^-)$$

$$[f,g]_{P.B.} \equiv \frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p^i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q^i}$$

$$[q,p]_{P.B.} = 1, [q,q]_{P.B.} = 0, [p,p]_{P.B.} = 0$$

$$A_{(\alpha\beta)} \equiv \frac{1}{2}\left(A_{\alpha\beta} + A_{\beta\alpha}\right)$$

$$N^{(\psi)} = \sum_{r=1/2}^{\infty} r\psi_{-r}\cdot\psi_r$$

$$\hat{\gamma}_{i_1 i_2 \dots i_k} \sim \hat{\gamma}_{[i_1}\hat{\gamma}_{i_2}\dots\hat{\gamma}_{i_k]}$$

IIA: $(\mathbf{8_v} + \mathbf{8_s}) \otimes (\mathbf{8_v} + \mathbf{8_c})$; IIB: $(\mathbf{8_v} + \mathbf{8_s}) \otimes (\mathbf{8_v} + \mathbf{8_s})$

$$[\tilde{J}_A, \tilde{J}_B] = \epsilon_{ABC}\tilde{J}^C, [\tilde{J}_A, \tilde{P}_B] = \epsilon_{ABC}\tilde{P}^C, [\tilde{P}_A, \tilde{P}_B] = \frac{1}{\ell^2}\epsilon_{ABC}\tilde{J}^C$$

$$[J, G_a] = \epsilon_{ab}G_b, \qquad [J, P_a] = \epsilon_{ab}P_b,$$
$$[H, G_a] = \epsilon_{ab}P_b, \qquad [G_a, G_b] = -\epsilon_{ab}J$$
$$[P_a, P_b] = -\frac{1}{\ell^2}\epsilon_{ab}J, \qquad [H, P_a] = \frac{1}{\ell^2}\epsilon_{ab}G_b$$

$$J = \tilde{J}_0, G_a = \tilde{J}_a, H = \tilde{P}_0, P_a = \tilde{P}_a$$

$$[V_0, V_0] \subset V_0, [V_0, V_1] \subset V_1, [V_1, V_1] \subset V_0$$

$$\lambda_i\lambda_j = \begin{cases}\lambda_{i+j} & \text{if } i+j \leq 2,\\ \lambda_2 & \text{if } i+j > 2,\end{cases}$$

$$S_0 = \{\lambda_0, \lambda_2\}, S_1 = \{\lambda_1, \lambda_2\}$$

$$S_0 \cdot S_0 \subset S_0, S_0 \cdot S_1 \subset S_1, S_1 \cdot S_1 \subset S_0$$

$$\lambda_i\lambda_j = \begin{cases}\lambda_{i+j} & \text{if } i+j \leq 3\\ \lambda_3 & \text{if } i+j > 3\end{cases}$$

$$S_0 = \{\lambda_0, \lambda_2, \lambda_3\}, S_1 = \{\lambda_1, \lambda_3\}$$

$$\mathfrak{G} = (S_0 \times V_0) \oplus (S_1 \times V_1),$$

$$[\tilde{J}_A, \tilde{J}_B] = \epsilon_{ABC}\tilde{J}^C$$
$$[\tilde{J}_A, \tilde{P}_B] = \epsilon_{ABC}\tilde{P}^C$$
$$[\tilde{P}_A, \tilde{P}_B] = \frac{1}{\ell^2}\epsilon_{ABC}\tilde{J}^C$$
$$[\tilde{J}_A, \tilde{Q}^i_\alpha] = -\frac{1}{2}(\gamma_A)_\alpha^\beta\tilde{Q}^i_\beta$$
$$[\tilde{P}_A, \tilde{Q}^i_\alpha] = -\frac{1}{2\ell}(\gamma_A)_\alpha^\beta\tilde{Q}^i_\beta$$
$$[\tilde{\mathcal{T}}, \tilde{Q}^i_\alpha] = \frac{1}{2}\epsilon^{ij}\tilde{Q}^j_\beta$$
$$\left\{\tilde{Q}^i_\alpha, \tilde{Q}^j_\beta\right\} = -\frac{1}{\ell}\delta_{ij}(\gamma^A C)_{\alpha\beta}\tilde{J}_A - \delta_{ij}(\gamma^A C)_{\alpha\beta}\tilde{P}_A - C_{\alpha\beta}\epsilon^{ij}\left(\tilde{\mathcal{U}} + \frac{1}{\ell}\tilde{\mathcal{T}}\right)$$

$$
\begin{aligned}
\langle \tilde{J}_A \tilde{J}_B \rangle &= \tilde{\alpha}_0 \eta_{AB}, & \langle \tilde{J}_A \tilde{P}_B \rangle &= \tilde{\alpha}_1 \eta_{AB} \\
\langle \tilde{P}_A \tilde{P}_B \rangle &= \frac{\tilde{\alpha}_0}{\ell^2} \eta_{AB}, & \langle \tilde{\mathcal{T}} \tilde{\mathcal{T}} \rangle &= \tilde{\alpha}_0 \\
\langle \tilde{\mathcal{T}} \tilde{\mathcal{U}} \rangle &= \tilde{\alpha}_1, & \langle \tilde{\mathcal{U}} \tilde{\mathcal{U}} \rangle &= -\frac{\tilde{\alpha}_1}{\ell} \\
\left\langle \tilde{Q}^i_\alpha \tilde{Q}^j_\beta \right\rangle &= 2\left(\alpha_1 + \frac{\alpha_0}{\ell}\right) C_{\alpha\beta} \delta^{ij}.
\end{aligned}
$$

$$
T = \tilde{\mathcal{T}}, U = \tilde{\mathcal{U}}, Q^\pm_\alpha = \frac{1}{\sqrt{2}} (\tilde{Q}^1_\alpha \pm (\gamma^0)_{\alpha\beta} \tilde{Q}^2_\beta)
$$

$$
[J, G_a] = \epsilon_{ab} G_b, [J, P_a] = \epsilon_{ab} P_b, [G_a, G_b] = -\epsilon_{ab} J
$$

$$
\begin{aligned}
[H, G_a] &= \epsilon_{ab} P_b\,, & [G_a, P_b] &= -\epsilon_{ab} H\,, & [H, P_a] &= \frac{1}{\ell^2} \epsilon_{ab} G_b\,, \\
[P_a, P_b] &= -\frac{1}{\ell^2} \epsilon_{ab} J\,, & [J, Q^\pm_\alpha] &= -\frac{1}{2} (\gamma_0)_\alpha^{\ \beta} \tilde{Q}^\pm_\beta\,, & [H, Q^\pm_\alpha] &= -\frac{1}{2\ell} (\gamma_0)_\alpha^{\ \beta} \tilde{Q}^\pm_\beta\,, \\
[G_a, Q^\pm_\alpha] &= -\frac{1}{2} (\gamma_a)_\alpha^{\ \beta} \tilde{Q}^\mp_\beta\,, & [P_a, Q^\pm_\alpha] &= -\frac{1}{2\ell} (\gamma_a)_\alpha^{\ \beta} \tilde{Q}^\mp_\beta\,, & [T, Q^\pm_\alpha] &= \pm\frac{1}{2} (\gamma_0)_{\alpha\beta} \tilde{Q}^\pm_\beta\,,
\end{aligned}
$$

$$
\begin{aligned}
\{Q^+_\alpha, Q^+_\beta\} &= -(\gamma^0 C)_{\alpha\beta} \left(\frac{1}{\ell} J + H\right) - (\gamma^0 C)_{\alpha\beta} \left(U + \frac{1}{\ell} T\right)\,, \\
\{Q^+_\alpha, Q^-_\beta\} &= -(\gamma^a C)_{\alpha\beta} \left(\frac{1}{\ell} G_a + P_a\right)\,, \\
\{Q^-_\alpha, Q^-_\beta\} &= -(\gamma^0 C)_{\alpha\beta} \left(\frac{1}{\ell} J + H\right) + (\gamma^0 C)_{\alpha\beta} \left(U + \frac{1}{\ell} T\right)\,.
\end{aligned}
$$

$$
V_0 = \{J, P_a, T\}, V_1 = \{Q^+_\alpha, Q^-_\alpha\}, V_2 = \{H, G_a, U\}
$$

$$
\begin{aligned}
&[V_0, V_0] \subset V_0, \quad [V_1, V_1] \subset V_0 \oplus V_2, \\
&[V_0, V_1] \subset V_1, \quad [V_1, V_2] \subset V_1, \\
&[V_0, V_2] \subset V_2, \quad [V_2, V_2] \subset V_0.
\end{aligned}
$$

$$
S_0 = \{\lambda_0, \lambda_2, \lambda_3\}, S_1 = \{\lambda_1, \lambda_3\}, S_2 = \{\lambda_2, \lambda_3\}
$$

$$
\begin{aligned}
&S_0 \cdot S_0 \subset S_0, \quad S_1 \cdot S_1 \subset S_0 \cap S_2 \\
&S_0 \cdot S_1 \subset S_1, \quad S_1 \cdot S_2 \subset S_1 \\
&S_0 \cdot S_2 \subset S_2, \quad S_2 \cdot S_2 \subset S_0
\end{aligned}
$$

$$
\mathfrak{G}_R = (S_0 \times V_0) \oplus (S_1 \times V_1) \oplus (S_2 \times V_2)
$$

$$
\begin{aligned}
&\check{S}_0 = \{\lambda_0\}, \quad \hat{S}_0 = \{\lambda_2, \lambda_3\} \\
&\check{S}_1 = \{\lambda_1\}, \quad \hat{S}_1 = \{\lambda_3\} \\
&\check{S}_2 = \{\lambda_2\}, \quad \hat{S}_2 = \{\lambda_3\}
\end{aligned}
$$

$$
\begin{aligned}
&\check{S}_0 \cdot \hat{S}_0 \subset \hat{S}_0, \quad \check{S}_1 \cdot \hat{S}_1 \subset \hat{S}_0 \cap \hat{S}_2, \\
&\check{S}_0 \cdot \hat{S}_1 \subset \hat{S}_1, \quad \check{S}_1 \cdot \hat{S}_2 \subset \hat{S}_1, \\
&\check{S}_0 \cdot \hat{S}_2 \subset \hat{S}_2, \quad \check{S}_2 \cdot \hat{S}_2 \subset \hat{S}_0.
\end{aligned}
$$

$$
\begin{aligned}
\check{\mathfrak{G}}_R &= (\check{S}_0 \times V_0) \oplus (\check{S}_1 \times V_1) \oplus (\check{S}_2 \times V_2), \\
\hat{\mathfrak{G}}_R &= (\hat{S}_0 \times V_0) \oplus (\hat{S}_1 \times V_1) \oplus (\hat{S}_2 \times V_2),
\end{aligned}
$$

$$
[\check{\mathfrak{G}}_R, \hat{\mathfrak{G}}_R] \subset \hat{\mathfrak{G}}_R.
$$

$$
\begin{aligned}
[\mathrm{J}, \mathrm{G}_a] &= \epsilon_{ab}\mathrm{G}_b\,, & [\mathrm{J}, \mathrm{P}_a] &= \epsilon_{ab}\mathrm{P}_b\,, & [\mathrm{P}_a, \mathrm{P}_b] &= -\frac{1}{\ell^2}\epsilon_{ab}\mathrm{J}\,,\\
[\mathrm{H}, \mathrm{P}_a] &= \frac{1}{\ell^2}\epsilon_{ab}\mathrm{G}_b\,, & [\mathrm{G}_a, \mathrm{P}_a] &= -\epsilon_{ab}\mathrm{H}\,, & [\mathrm{J}, \mathrm{Q}^{\pm}_\alpha] &= -\frac{1}{2}\left(\gamma_0\right)_\alpha^{\ \beta}\mathrm{Q}^{\pm}_\beta\,,\\
\left[\mathrm{P}_a, \mathrm{Q}^{\pm}_\alpha\right] &= -\frac{1}{2\ell}\left(\gamma_a\right)_\alpha^{\ \beta}\mathrm{Q}^{\mp}_\beta\,, & \left[\mathrm{T}, \mathrm{Q}^{\pm}_\alpha\right] &= \pm\frac{1}{2}\left(\gamma_0\right)_\alpha^{\ \beta}\mathrm{Q}^{\pm}_\beta\,, & &\\
\{\mathrm{Q}^{+}_\alpha, \mathrm{Q}^{+}_\beta\} &= -\left(\gamma^0 C\right)_{\alpha\beta}(\mathrm{H}+\mathrm{U})\,, & & & &
\end{aligned}
$$

$$
\begin{aligned}
\{\mathrm{Q}^{+}_\alpha, \mathrm{Q}^{-}_\beta\} &= -\frac{1}{\ell}(\gamma^a C)_{\alpha\beta}\mathrm{G}_a,\\
\{\mathrm{Q}^{-}_\alpha, \mathrm{Q}^{-}_\beta\} &= -(\gamma^0 C)_{\alpha\beta}(\mathrm{H}-\mathrm{U}).
\end{aligned}
$$

$$
\lambda_\alpha\lambda_\beta = \begin{cases}\lambda_{\alpha+\beta} & \text{if } \alpha+\beta \le 2N+1,\\ \lambda_{2N+1} & \text{if } \alpha+\beta > 2N+1,\end{cases}
$$

$$
S_0 = \{\lambda_{2m}, \text{ with } m = 0, \dots, N\} \cup \{\lambda_{2N+1}\},
$$

$$
\begin{aligned}
S_1 &= \{\lambda_{2m+1}, \text{ with } m = 0, \dots, N-1\} \cup \{\lambda_{2N+1}\}\\
S_2 &= \{\lambda_{2m+2}, \text{ with } m = 0, \dots, N-1\} \cup \{\lambda_{2N+1}\}
\end{aligned}
$$

$$
\mathfrak{G}_R = (S_0 \times V_0) \oplus (S_1 \times V_1) \oplus (S_2 \times V_2)
$$

$$
\begin{aligned}
\check{S}_0 &= \{\lambda_{2m}, \text{ with } m = 0, \dots, N-1\}, & \hat{S}_0 &= \{\lambda_{2N}, \lambda_{2N+1}\}\\
\check{S}_1 &= \{\lambda_{2m+1}, \text{ with } m = 0, \dots, N-1\}, & \hat{S}_1 &= \{\lambda_{2N+1}\}\\
\check{S}_2 &= \{\lambda_{2m+2}, \text{ with } m = 0, \dots, N-1\}, & \hat{S}_2 &= \{\lambda_{2N+1}\}
\end{aligned}
$$

$$
[\check{\mathfrak{G}}_R, \hat{\mathfrak{G}}_R] \subset \hat{\mathfrak{G}}_R,
$$

$$
\begin{aligned}
\check{\mathfrak{G}}_R &= (\check{S}_0 \times V_0) \oplus (\check{S}_1 \times V_1) \oplus (\check{S}_2 \times V_2),\\
\hat{\mathfrak{G}}_R &= (\hat{S}_0 \times V_0) \oplus (\hat{S}_1 \times V_1) \oplus (\hat{S}_2 \times V_2).
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{J}^{(m)} &= \lambda_{2m}J\,, & \mathrm{P}_a^{(m)} &= \lambda_{2m}P_a\,, & \mathrm{T}^{(m)} &= \lambda_{2m}T\,,\\
\mathrm{H}^{(m)} &= \lambda_{2m+2}H\,, & \mathrm{G}_a^{(m)} &= \lambda_{2m+2}G_a\,, & \mathrm{U}^{(m)} &= \lambda_{2m+2}U\,,\\
\mathrm{Q}_\alpha^{+\,(m)} &= \lambda_{2m+1}Q^{+}_\alpha\,, & \mathrm{Q}_\alpha^{-\,(m)} &= \lambda_{2m+1}Q^{-}_\alpha\,, & &
\end{aligned}
$$

$$
\begin{aligned}
\left[\mathrm{J}^{(m)}, \mathrm{G}_a^{(n)}\right] &= \epsilon_{ab}\mathrm{G}_b^{(m+n)}, & \left[\mathrm{G}_a^{(m)}, \mathrm{G}_b^{(n)}\right] &= -\epsilon_{ab}\,\mathrm{J}^{(m+n+2)}\\
\left[\mathrm{J}^{(m)}, \mathrm{P}_a^{(n)}\right] &= \epsilon_{ab}\mathrm{P}_b^{(m+n)}, & \left[\mathrm{G}_a^{(m)}, \mathrm{P}_a^{(n)}\right] &= -\epsilon_{ab}\mathrm{H}^{(m+n)}\\
\left[\mathrm{H}^{(m)}, \mathrm{P}_a^{(n)}\right] &= \frac{1}{\ell^2}\epsilon_{ab}\mathrm{G}_b^{(m+n)}, & \left[\mathrm{P}_a^{(m)}, \mathrm{P}_b^{(n)}\right] &= -\frac{1}{\ell^2}\epsilon_{ab}\,\mathrm{J}^{(m+n)}\\
\left[\mathrm{H}^{(m)}, \mathrm{G}_a^{(n)}\right] &= \epsilon_{ab}\mathrm{P}_b^{(m+n+2)}, & \left[\mathrm{J}^{(m)}, \mathrm{Q}_\alpha^{\pm(n)}\right] &= -\frac{1}{2}(\gamma_0)_\alpha^\beta\mathrm{Q}_\beta^{\pm(m+n)}\\
\left[\mathrm{H}^{(m)}, \mathrm{Q}_\alpha^{\pm(n)}\right] &= -\frac{1}{2\ell}(\gamma_0)_\alpha^\beta\mathrm{Q}_\beta^{\pm(m+n+1)}, & \left[\mathrm{P}_a^{(m)}, \mathrm{Q}_\alpha^{\pm(n)}\right] &= -\frac{1}{2\ell}(\gamma_a)_\alpha^\beta\mathrm{Q}_\beta^{\mp(m+n)}
\end{aligned}
$$

$$
\left[\mathrm{G}_a^{(m)}, \mathrm{Q}_\alpha^{\pm(n)}\right] = -\frac{1}{2}(\gamma_a)_\alpha^\beta\mathrm{Q}_\beta^{\mp(m+n+1)}, \left[\mathrm{T}^{(m)}, \mathrm{Q}_\alpha^{\pm(n)}\right] = \pm\frac{1}{2}(\gamma_0)_\alpha^\beta\mathrm{Q}_\beta^{\pm(m+n)}
$$

$$
\begin{aligned}
\left\{\mathrm{Q}_\alpha^{+(m)}, \mathrm{Q}_\beta^{+(n)}\right\} &= -(\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{J}^{(m+n+1)} + \mathrm{H}^{(m+n)}\right) - (\gamma^0 C)_{\alpha\beta}\left(\mathrm{U}^{(m+n)} + \frac{1}{\ell}\,\mathrm{T}^{(m+n+1)}\right)\\
\left\{\mathrm{Q}_\alpha^{+(m)}, \mathrm{Q}_\beta^{-(n)}\right\} &= -(\gamma^a C)_{\alpha\beta}\left(\frac{1}{\ell}\mathrm{G}_a^{(m+n)} + \mathrm{P}_a^{(m+n+1)}\right)\\
\left\{\mathrm{Q}_\alpha^{-(m)}, \mathrm{Q}_\beta^{-(n)}\right\} &= -(\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{J}^{(m+n+1)} + \mathrm{H}^{(m+n)}\right) + (\gamma^0 C)_{\alpha\beta}\left(\mathrm{U}^{(m+n)} + \frac{1}{\ell}\,\mathrm{T}^{(m+n+1)}\right)
\end{aligned}
$$

$$
[\mathrm{J}, \mathrm{G}_a] = \epsilon_{ab}\mathrm{G}_b, [\mathrm{G}_a, \mathrm{P}_a] = -\epsilon_{ab}\mathrm{H}, [\,\mathrm{J}, \mathrm{P}_a] = \epsilon_{ab}\mathrm{P}_b
$$

$$[\mathrm{H},\mathrm{P}_a]=\frac{1}{\ell^2}\epsilon_{ab}\mathrm{G}_b,\qquad [\mathrm{P}_a,\mathrm{P}_b]=-\frac{1}{\ell^2}\epsilon_{ab}\,\mathrm{J},$$
$$[\mathrm{J},\mathrm{Z}_a]=\epsilon_{ab}\mathrm{Z}_b,\qquad [\mathrm{J},\mathrm{M}_a]=\epsilon_{ab}\mathrm{M}_b,\mathrm{M}_b]=-\frac{1}{\ell^2}\epsilon_{ab}\,\mathrm{K},$$
$$[\mathrm{Z},\mathrm{P}_a]=\frac{1}{\ell^2}\epsilon_{ab}\mathrm{Z}_b,\qquad [\mathrm{K},\mathrm{G}_a]=\epsilon_{ab}\mathrm{Z}_b,$$
$$[\mathrm{H},\mathrm{M}_a]=\frac{1}{\ell^2}\epsilon_{ab}\mathrm{Z}_b,\quad [\mathrm{P}_a,\mathrm{Z}_b]=-\epsilon_{ab}\mathrm{Z},\qquad [\mathrm{K},\mathrm{P}_a]=\epsilon_{ab}\mathrm{M}_b,$$
$$[\mathrm{P}_a,\mathrm{Q}_\alpha^\pm]=-\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta\mathrm{Q}_\beta^\mp,\quad [\mathrm{H},\mathrm{Q}_\alpha^\pm]=-\frac{1}{2\ell}(\gamma_0)_\alpha{}^\beta\mathrm{F}_\beta^\pm,\qquad [\mathrm{G}_a,\mathrm{Q}_\alpha^\pm]=-\frac{1}{2}(\gamma_a)_\alpha{}^\beta\mathrm{F}_\beta^\mp$$
$$[\mathrm{J},\ \mathrm{F}_\alpha^\pm]=-\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{F}_\beta^\pm,\quad [\mathrm{P}_a,\ \mathrm{F}_\alpha^\pm]=-\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta\mathrm{F}_\beta^\mp,\qquad [\mathrm{K},\mathrm{Q}_\alpha^\pm]=-\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{F}_\beta^\pm$$
$$[\mathrm{M}_a,\mathrm{Q}_\alpha^\pm]=-\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta\mathrm{F}_\beta^\mp,\quad [\mathrm{T}_1,\mathrm{Q}_\alpha^\pm]=\pm\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{Q}_\beta^\pm,\qquad [\mathrm{T}_1,\ \mathrm{F}_\alpha^\pm]=\pm\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{F}_\beta^\pm$$
$$[\mathrm{T}_2,\mathrm{Q}_\alpha^\pm]=\pm\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{F}_\beta^\pm,$$

$$\{\mathrm{Q}_\alpha^\pm,\mathrm{Q}_\beta^\pm\}=-(\gamma^0C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{K}+\mathrm{H}\right)\mp(\gamma^0C)_{\alpha\beta}\left(\mathrm{U}_1+\frac{1}{\ell}\,\mathrm{T}_2\right)$$
$$\{\mathrm{Q}_\alpha^+,\mathrm{Q}_\beta^-\}=-(\gamma^aC)_{\alpha\beta}\left(\frac{1}{\ell}\mathrm{G}_a+\mathrm{M}_a\right)$$
$$\{\mathrm{Q}_\alpha^\pm,\mathrm{F}_\beta^\pm\}=-(\gamma^0C)_{\alpha\beta}(\mathrm{Z}\pm\mathrm{U}_2)$$
$$\{\mathrm{Q}_\alpha^\pm,\mathrm{F}_\beta^\mp\}=-\frac{1}{\ell}(\gamma^aC)_{\alpha\beta}\mathrm{Z}_a$$

$$\begin{array}{cccc}\mathrm{J}^{(0)}=\mathrm{J}, & \mathrm{J}^{(1)}=\mathrm{K}, & \mathrm{H}^{(0)}=\mathrm{H}, & \mathrm{H}^{(1)}=\mathrm{Z}\\ \mathrm{G}_a^{(0)}=\mathrm{G}_a, & \mathrm{G}_a^{(1)}=\mathrm{Z}_a, & \mathrm{P}_a^{(0)}=\mathrm{P}_a, & \mathrm{P}_a^{(1)}=\mathrm{M}_a\\ \mathrm{T}^{(0)}=\mathrm{T}_1, & \mathrm{T}^{(1)}=\mathrm{T}_2, & \mathrm{U}^{(0)}=\mathrm{U}_1, & \mathrm{U}^{(1)}=\mathrm{U}_2\\ \mathrm{Q}_\alpha^{\pm(0)}=\mathrm{Q}_\alpha^\pm, & \mathrm{Q}_\alpha^{\pm(1)}=\mathrm{F}_\alpha^\pm. & & \end{array}$$

$$V_0=\{J,H,T,U,Q_\alpha^+\},V_1=\{G_a,P_a,Q_\alpha^-\}$$

$$S_0=\{\lambda_0,\lambda_2,\lambda_3\},S_1=\{\lambda_1,\lambda_3\}$$

$$\mathfrak{G}=(S_0\times V_0)\oplus(S_1\times V_1),$$

$$[\mathrm{G}_a,\mathrm{G}_b]=-\epsilon_{ab}\,\mathrm{S},[\mathrm{J},\mathrm{G}_a]=\epsilon_{ab}\mathrm{G}_b,[\mathrm{J},\mathrm{P}_a]=\epsilon_{ab}\mathrm{P}_b,$$
$$[\mathrm{G}_a,\mathrm{P}_b]=-\epsilon_{ab}\mathrm{M},[\mathrm{H},\mathrm{G}_a]=\epsilon_{ab}\mathrm{P}_b,[\mathrm{H},\mathrm{P}_a]=\frac{1}{\ell^2}\epsilon_{ab}\mathrm{G}_b,$$
$$[\mathrm{P}_a,\mathrm{P}_a]=-\frac{1}{\ell^2}\epsilon_{ab}\,\mathrm{S},[\mathrm{J},\mathrm{Q}_\alpha^\pm]=-\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{Q}_\beta^\pm,[\mathrm{J},\mathrm{R}_\alpha]=-\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{R}_\beta,$$
$$[\mathrm{H},\mathrm{Q}_\alpha^\pm]=-\frac{1}{2\ell}(\gamma_0)_\alpha{}^\beta\mathrm{Q}_\beta^\pm,[\mathrm{H},\mathrm{R}_\alpha]=-\frac{1}{2\ell}(\gamma_0)_\alpha{}^\beta\mathrm{R}_\beta,[\mathrm{S},\mathrm{Q}_\alpha^+]=-\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{R}_\beta,$$
$$[\mathrm{M},\mathrm{Q}_\alpha^+]=-\frac{1}{2\ell}(\gamma_0)_\alpha{}^\beta\mathrm{R}_\beta,[\mathrm{G}_a,\mathrm{Q}_\alpha^+]=-\frac{1}{2}(\gamma_a)_\alpha{}^\beta\mathrm{Q}_\beta^-,[\mathrm{G}_a,\mathrm{Q}_\alpha^-]=-\frac{1}{2}(\gamma_a)_\alpha{}^\beta\mathrm{R}_\beta,$$
$$[\mathrm{P}_a,\mathrm{Q}_\alpha^+]=-\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta\mathrm{Q}_\beta^-,[\mathrm{P}_a,\mathrm{Q}_\alpha^-]=-\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta\mathrm{R}_\beta,[\mathrm{T}_1,\mathrm{Q}_\alpha^\pm]=\pm\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{Q}_\beta^\pm,$$
$$[\mathrm{T}_1,\mathrm{R}_\alpha]=\pm\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{R}_\beta,[\mathrm{T}_2,\mathrm{Q}_\alpha^+]=\pm\frac{1}{2}(\gamma_0)_\alpha{}^\beta\mathrm{R}_\beta,$$

$$\{\mathrm{Q}_\alpha^+,\mathrm{Q}_\beta^+\}=-(\gamma^0C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{J}+\mathrm{H}\right)-(\gamma^0C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{T}_1+\mathrm{U}_1\right),$$
$$\{\mathrm{Q}_\alpha^+,\mathrm{Q}_\beta^-\}=-(\gamma^aC)_{\alpha\beta}\left(\frac{1}{\ell}\mathrm{G}_a+\mathrm{P}_a\right),$$
$$\{\mathrm{Q}_\alpha^-,\mathrm{Q}_\beta^-\}=-(\gamma^0C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{S}+\mathrm{M}\right)+(\gamma^0C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{T}_2+\mathrm{U}_2\right),$$
$$\{\mathrm{Q}_\alpha^+,\mathrm{R}_\beta\}=-(\gamma^0C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{S}+\mathrm{M}\right)-(\gamma^0C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{T}_2+\mathrm{U}_2\right).$$

$$V_0 = \{\mathrm{J}, \mathrm{H}, \mathrm{T}, \mathrm{U}, \mathrm{Q}_\alpha^+\}, V_1 = \{\mathrm{G}_a, \mathrm{P}_a, \mathrm{Q}_\alpha^-\}$$

$$[\mathrm{J}, \mathrm{G}_a] = \epsilon_{ab}\mathrm{G}_b, [\mathrm{J}, \mathrm{P}_a] = \epsilon_{ab}\mathrm{P}_b, [\mathrm{P}_a, \mathrm{P}_b] = -\frac{1}{\ell^2}\epsilon_{ab}\,\mathrm{S},$$
$$[\mathrm{H}, \mathrm{P}_a] = \frac{1}{\ell^2}\epsilon_{ab}\mathrm{G}_b, [\mathrm{G}_a, \mathrm{P}_a] = -\epsilon_{ab}\mathrm{M}, [\mathrm{J}, \mathrm{Q}_\alpha^\pm] = -\frac{1}{2}(\gamma_0)_\alpha^\beta \mathrm{Q}_\beta^\pm,$$
$$[\mathrm{J}, \mathrm{R}_\alpha] = -\frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{R}_\beta\ [\mathrm{S}, \mathrm{Q}_\alpha^+] = -\frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{R}_\beta\ [\mathrm{P}_a, \mathrm{Q}_\alpha^+] = -\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta \mathrm{Q}_\beta^-,$$
$$[\mathrm{P}_a, \mathrm{Q}_\alpha^-] = -\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta \mathrm{R}_\beta, [\mathrm{T}_1, \mathrm{Q}_\alpha^\pm] = \pm\frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{Q}_\beta^\pm, [\mathrm{T}_1, \mathrm{R}_\alpha] = \frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{R}_\beta,$$
$$[\mathrm{T}_2, \mathrm{Q}_\alpha^+] = \frac{1}{2}(\gamma_0)_\alpha^\beta \mathrm{R}_\beta,$$

$$\{\mathrm{Q}_\alpha^+, \mathrm{Q}_\beta^+\} = -(\gamma^0 C)_{\alpha\beta}(\mathrm{H} + \mathrm{U}_1), \quad \{\mathrm{Q}_\alpha^+, \mathrm{Q}_\beta^-\} = -\frac{1}{\ell}(\gamma^a C)_{\alpha\beta}\mathrm{G}_a$$
$$\{\mathrm{Q}_\alpha^-, \mathrm{Q}_\beta^-\} = -(\gamma^0 C)_{\alpha\beta}(\mathrm{M} - \mathrm{U}_2), \quad \{\mathrm{Q}_\alpha^+, \mathrm{R}_\beta\} = -(\gamma^0 C)_{\alpha\beta}(\mathrm{M} + \mathrm{U}_2)$$

$$V_0 = \{\mathrm{J}, \mathrm{S}, \mathrm{P}_a, \mathrm{T}_1, \mathrm{T}_2\}, V_1 = \{\mathrm{Q}_\alpha^+, \mathrm{Q}_\alpha^-, \mathrm{R}_\alpha\}, V_2 = \{\mathrm{H}, \mathrm{M}, \mathrm{G}_a, \mathrm{U}_1, \mathrm{U}_2\}$$

$$S_0 = \{\lambda_{2m}, \text{with } m = 0, \dots, N\} \cup \{\lambda_{2N+1}\},$$
$$S_1 = \{\lambda_{2m+1}, \text{with } m = 0, \dots, N-1\} \cup \{\lambda_{2N+1}\},$$

$$\mathfrak{G}_R = (S_0 \times V_0) \oplus (S_1 \times V_1),$$

$$\mathrm{J}^{(m)} = \lambda_{2m}J, \quad \mathrm{P}_a^{(m)} = \lambda_{2m+1}P_a, \quad \mathrm{T}^{(m)} = \lambda_{2m}T,$$
$$\mathrm{H}^{(m)} = \lambda_{2m}H, \quad \mathrm{G}_a^{(m)} = \lambda_{2m+1}G_a, \quad \mathrm{U}^{(m)} = \lambda_{2m}U,$$
$$\mathrm{Q}_\alpha^{+\,(m)} = \lambda_{2m}Q_\alpha^+, \quad \mathrm{Q}_\alpha^{-\,(m)} = \lambda_{2m+1}Q_\alpha^-.$$

$$\left[\mathrm{J}^{(m)}, \mathrm{G}_a^{(n)}\right] = \epsilon_{ab}\mathrm{G}_b^{(m+n)}, \quad \left[\mathrm{G}_a^{(m)}, \mathrm{G}_b^{(n)}\right] = -\epsilon_{ab}\,\mathrm{J}^{(m+n+1)}$$
$$\left[\mathrm{J}^{(m)}, \mathrm{P}_a^{(n)}\right] = \epsilon_{ab}\mathrm{P}_b^{(m+n)}, \quad \left[\mathrm{G}_a^{(m)}, \mathrm{P}_a^{(n)}\right] = -\epsilon_{ab}\mathrm{H}^{(m+n+1)}$$
$$\left[\mathrm{H}^{(m)}, \mathrm{P}_a^{(n)}\right] = \frac{1}{\ell^2}\epsilon_{ab}\mathrm{G}_b^{(m+n)}, \quad \left[\mathrm{P}_a^{(m)}, \mathrm{P}_b^{(n)}\right] = -\frac{1}{\ell^2}\epsilon_{ab}\,\mathrm{J}^{(m+n+1)}$$
$$\left[\mathrm{H}^{(m)}, \mathrm{G}_a^{(n)}\right] = \epsilon_{ab}\mathrm{P}_b^{(m+n)}, \quad \left[\mathrm{J}^{(m)}, \mathrm{Q}_\alpha^{\pm(n)}\right] = -\frac{1}{2}(\gamma_0)_\alpha^\beta \mathrm{Q}_\beta^{\pm(m+n)}$$
$$\left[\mathrm{H}^{(m)}, \mathrm{Q}_\alpha^{\pm(n)}\right] = -\frac{1}{2\ell}(\gamma_0)_\alpha^\beta \mathrm{Q}_\beta^{\pm(m+n)}, \quad \left[\mathrm{P}_a^{(m)}, \mathrm{Q}_\alpha^{+(n)}\right] = -\frac{1}{2\ell}(\gamma_a)_\alpha^\beta \mathrm{Q}_\beta^{-(m+n)}$$
$$\left[\mathrm{P}_a^{(m)}, \mathrm{Q}_\alpha^{-(n)}\right] = -\frac{1}{2\ell}(\gamma_a)_\alpha^\beta \mathrm{Q}_\beta^{+(m+n+1)}, \quad \left[\mathrm{G}_a^{(m)}, \mathrm{Q}_\alpha^{+(n)}\right] = -\frac{1}{2}(\gamma_a)_\alpha^\beta \mathrm{Q}_\beta^{-(m+n)}$$
$$\left[\mathrm{G}_a^{(m)}, \mathrm{Q}_\alpha^{-(n)}\right] = -\frac{1}{2}(\gamma_a)_\alpha^\beta \mathrm{Q}_\beta^{+(m+n+1)}, \quad \left[\mathrm{T}^{(m)}, \mathrm{Q}_\alpha^{\pm(n)}\right] = \pm\frac{1}{2}(\gamma_0)_\alpha^\beta \mathrm{Q}_\beta^{\pm(m+n)}$$

$$\left\{\mathrm{Q}_\alpha^{+(m)}, \mathrm{Q}_\beta^{+(n)}\right\} = -(\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{J}^{(m+n)} + \mathrm{H}^{(m+n)}\right) - (\gamma^0 C)_{\alpha\beta}\left(\mathrm{U}^{(m+n)} + \frac{1}{\ell}\,\mathrm{T}^{(m+n)}\right)$$
$$\left\{\mathrm{Q}_\alpha^{+(m)}, \mathrm{Q}_\beta^{-(n)}\right\} = -(\gamma^a C)_{\alpha\beta}\left(\frac{1}{\ell}\mathrm{G}_a^{(m+n)} + \mathrm{P}_a^{(m+n)}\right)$$
$$\left\{\mathrm{Q}_\alpha^{-(m)}, \mathrm{Q}_\beta^{-(n)}\right\} = -(\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\,\mathrm{J}^{(m+n+1)} + \mathrm{H}^{(m+n+1)}\right) + (\gamma^0 C)_{\alpha\beta}\left(\mathrm{U}^{(m+n+1)} + \frac{1}{\ell}\,\mathrm{T}^{(m+n+1)}\right)$$

$$
\begin{array}{lll}
[\mathrm{G}_a,\ \mathrm{B}_b] = -\epsilon_{ab}\mathrm{Z}, & [\,\mathrm{J},\ \mathrm{B}_a] = \epsilon_{ab}\ \mathrm{B}_b, & [\,\mathrm{J},\ \mathrm{T}_a] = \epsilon_{ab}\ \mathrm{T}_b, \\
[\mathrm{G}_a,\ \mathrm{T}_b] = -\epsilon_{ab}\mathrm{Y}, & [\mathrm{H},\ \mathrm{B}_a] = \epsilon_{ab}\ \mathrm{T}_b, & [\mathrm{H},\ \mathrm{T}_a] = \frac{1}{\ell^2}\epsilon_{ab}\ \mathrm{B}_b, \\
[\mathrm{P}_a,\ \mathrm{B}_a] = -\epsilon_{ab}\mathrm{Y}, & [\,\mathrm{S},\mathrm{G}_a] = \epsilon_{ab}\ \mathrm{B}_b, & [\,\mathrm{S},\mathrm{P}_a] = \epsilon_{ab}\ \mathrm{T}_b, \\
[\mathrm{P}_a,\ \mathrm{T}_a] = -\frac{1}{\ell^2}\epsilon_{ab}\mathrm{Z}, & [\mathrm{M},\mathrm{G}_a] = \epsilon_{ab}\ \mathrm{T}_b, & [\mathrm{M},\mathrm{P}_a] = \frac{1}{\ell^2}\epsilon_{ab}\ \mathrm{B}_b, \\
[\,\mathrm{J},\ \mathrm{W}_\alpha^\pm] = -\frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^\pm, & [\mathrm{S},\mathrm{Q}_\alpha^-] = -\frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^-, & [\mathrm{S},\mathrm{R}_\alpha] = -\frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^+, \\
[\mathrm{H},\ \mathrm{W}_\alpha^\pm] = -\frac{1}{2\ell}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^\pm, & [\mathrm{M},\mathrm{Q}_\alpha^-] = -\frac{1}{2\ell}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^-, & [\mathrm{M},\mathrm{R}_\alpha] = -\frac{1}{2\ell}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^+, \\
[\mathrm{Z},\mathrm{Q}_\alpha^+] = -\frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^+, & [\mathrm{Y},\mathrm{Q}_\alpha^+] = -\frac{1}{2\ell}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^+, & [\mathrm{G}_a,\mathrm{R}_\alpha] = -\frac{1}{2}(\gamma_a)_\alpha{}^\beta \mathrm{W}_\beta^- \\
[\mathrm{G}_a,\ \mathrm{W}_\alpha^-] = -\frac{1}{2}(\gamma_a)_\alpha{}^\beta \mathrm{W}_\beta^+, & [\mathrm{P}_a,\mathrm{R}_\alpha] = -\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta \mathrm{W}_\beta^-, & [\mathrm{P}_a,\ \mathrm{W}_\alpha^-] = -\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta \mathrm{W}_\beta^+ \\
[\mathrm{B}_a,\mathrm{Q}_\alpha^\pm] = -\frac{1}{2}(\gamma_a)_\alpha{}^\beta \mathrm{W}_\beta^\mp, & [\mathrm{T}_a,\mathrm{Q}_\alpha^\pm] = -\frac{1}{2\ell}(\gamma_a)_\alpha{}^\beta \mathrm{W}_\beta^\mp, & [\mathrm{T}_1,\ \mathrm{W}_\alpha^\pm] = \pm\frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^\pm \\
[\mathrm{T}_2,\mathrm{Q}_\alpha^-] = -\frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^-, & [\mathrm{T}_2,\mathrm{R}_\alpha] = \frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^+, & [\mathrm{T}_3,\mathrm{Q}_\alpha^+] = \frac{1}{2}(\gamma_0)_\alpha{}^\beta \mathrm{W}_\beta^+
\end{array}
$$

$$
\begin{aligned}
\{\mathrm{Q}_\alpha^+,\mathrm{W}_\beta^+\} &= -(\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\mathrm{Z}+\mathrm{Y}\right) - (\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\ \mathrm{T}_3+\mathrm{U}_3\right) \\
\{\mathrm{Q}_\alpha^+,\mathrm{W}_\beta^-\} &= -(\gamma^a C)_{\alpha\beta}\left(\frac{1}{\ell}\ \mathrm{B}_a+\mathrm{T}_a\right) \\
\{\mathrm{Q}_\alpha^-,\mathrm{R}_\beta\} &= -(\gamma^a C)_{\alpha\beta}\left(\frac{1}{\ell}\ \mathrm{B}_a+\mathrm{T}_a\right) \\
\{\mathrm{Q}_\alpha^-,\mathrm{W}_\beta^-\} &= -(\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\mathrm{Z}+\mathrm{Y}\right) + (\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\ \mathrm{T}_3+\mathrm{U}_3\right) \\
\{\mathrm{R}_\alpha,\mathrm{R}_\beta\} &= -(\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\mathrm{Z}+\mathrm{Y}\right) - (\gamma^0 C)_{\alpha\beta}\left(\frac{1}{\ell}\ \mathrm{T}_3+\mathrm{U}_3\right)
\end{aligned}
$$

$$
\begin{array}{llll}
\mathrm{J} = \mathrm{J}^{(0)}, & \mathrm{G}_a = \mathrm{G}_a^{(0)}, & \mathrm{T}_1 = \mathrm{T}^{(0)}, & \mathrm{U}_1 = \mathrm{U}^{(0)}, \\
\mathrm{S} = \mathrm{J}^{(1)}, & \mathrm{B}_a = \mathrm{G}_a^{(1)}, & \mathrm{T}_2 = \mathrm{T}^{(1)}, & \mathrm{U}_2 = \mathrm{U}^{(1)}, \\
\mathrm{Z} = \mathrm{J}^{(2)}, & \mathrm{P}_a = \mathrm{P}_a^{(0)}, & \mathrm{T}_3 = \mathrm{T}^{(2)}, & \mathrm{U}_3 = \mathrm{U}^{(2)}, \\
\mathrm{H} = \mathrm{H}^{(0)}, & \mathrm{T}_a = \mathrm{P}_a^{(1)}, & \mathrm{Q}_a^+ = \mathrm{Q}_a^{+(0)}, & \mathrm{R}_a = \mathrm{Q}_a^{+(1)}, \\
\mathrm{M} = \mathrm{H}^{(1)}, & \mathrm{W}_a^+ = \mathrm{Q}_a^{+(2)}, & \mathrm{Q}_a^- = \mathrm{Q}_a^{-(0)}, & \mathrm{W}_a^- = \mathrm{Q}_a^{-(1)}, \\
\mathrm{Y} = \mathrm{H}^{(2)}, & & &
\end{array}
$$

$$
\begin{array}{llll}
\mathrm{J}^{(m)} = \lambda_{2m}\,\mathrm{J}, & \mathrm{P}_a^{(m)} = \lambda_{2m+1}\mathrm{P}_a, & \mathrm{T}^{(m)} = \lambda_{2m}\,\mathrm{T}, & \mathrm{Q}_\alpha^{+(m)} = \lambda_{2m}\mathrm{Q}_\alpha^+ \\
\mathrm{H}^{(m)} = \lambda_{2m}\mathrm{H}, & \mathrm{G}_a^{(m)} = \lambda_{2m+1}\mathrm{G}_a, & \mathrm{U}^{(m)} = \lambda_{2m}\mathrm{U}, & \mathrm{Q}_\alpha^{-(m)} = \lambda_{2m+1}\mathrm{Q}_\alpha^-
\end{array}
$$

$$
\begin{array}{ll}
\left[\mathrm{J}^{(m)},\mathrm{G}_a^{(n)}\right] = \epsilon_{ab}\mathrm{G}_b^{(m+n)}, & \left[\mathrm{J}^{(m)},\mathrm{P}_a^{(n)}\right] = \epsilon_{ab}\mathrm{P}_b^{(m+n)} \\
\left[\mathrm{P}_a^{(m)},\mathrm{P}_b^{(n)}\right] = -\frac{1}{\ell^2}\epsilon_{ab}\ \mathrm{J}^{(m+n+1)}, & \left[\mathrm{H}^{(m)},\mathrm{P}_a^{(n)}\right] = \frac{1}{\ell^2}\epsilon_{ab}\mathrm{G}_b^{(m+n)} \\
\left[\mathrm{G}_a^{(m)},\mathrm{P}_a^{(n)}\right] = -\epsilon_{ab}\mathrm{H}^{(m+n+1)}, & \left[\mathrm{J}^{(m)},\mathrm{Q}_\alpha^{\pm(n)}\right] = -\frac{1}{2}(\gamma_0)_\alpha^\beta\mathrm{Q}_\beta^{\pm(m+n)} \\
\left[\mathrm{P}_a^{(m)},\mathrm{Q}_\alpha^{+(n)}\right] = -\frac{1}{2\ell}(\gamma_a)_\alpha^\beta\mathrm{Q}_\beta^{(m+n)}, & \left[\mathrm{P}_a^{(m)},\mathrm{Q}_\alpha^{-(n)}\right] = -\frac{1}{2\ell}(\gamma_a)_\alpha^\beta\mathrm{Q}_\beta^{+(m+n+1)}
\end{array}
$$

$$
\begin{aligned}
\left[\mathrm{T}^{(m)},\mathrm{Q}_\alpha^{\pm(n)}\right] &= \pm\frac{1}{2}(\gamma_0)_\alpha^\beta\mathrm{Q}_\beta^{\pm(m+n)} \\
\left\{\mathrm{Q}_\alpha^{+(m)},\mathrm{Q}_\beta^{+(n)}\right\} &= -(\gamma^0 C)_{\alpha\beta}\left(\mathrm{H}^{(m+n)}+\mathrm{U}^{(m+n)}\right) \\
\left\{\mathrm{Q}_\alpha^{+(m)},\mathrm{Q}_\beta^{-(n)}\right\} &= -\frac{1}{\ell}(\gamma^a C)_{\alpha\beta}\mathrm{G}_a^{(m+n)} \\
\left\{\mathrm{Q}_\alpha^{-(m)},\mathrm{Q}_\beta^{-(n)}\right\} &= -(\gamma^0 C)_{\alpha\beta}\left(\mathrm{H}^{(m+n+1)}-\mathrm{U}^{(m+n+1)}\right)
\end{aligned}
$$

$$
I_{CS} = \frac{k}{4\pi}\int_{\mathcal{M}}\left\langle AdS + \frac{2}{3}A^3\right\rangle
$$

$$
A = \omega\mathrm{J} + \omega^a\mathrm{G}_a + \tau\mathrm{H} + e^a\mathrm{P}_a + t\,\mathrm{T} + u\mathrm{U} + \bar{\psi}^+\mathrm{Q}^+ + \bar{\psi}^-\mathrm{Q}^-,
$$

$$F = F(\omega)\mathrm{J} + F^a(\omega^b)\mathrm{G}_a + F(\tau)\mathrm{H} + F^a(e^b)\mathrm{P}_a + F(t)\mathrm{T} + F(u)\mathrm{U} + \nabla\bar{\psi}^+\mathrm{Q}^+ + \nabla\bar{\psi}^-\mathrm{Q}^-,$$

$$F(\omega) = d\omega + \frac{1}{2\ell^2}\epsilon^{ac}e_a e_c = R(\omega) + \frac{1}{2\ell^2}\epsilon^{ac}e_a e_c$$
$$F^a(\omega^b) = d\omega^a + \epsilon^{ac}\omega\omega_c + \frac{1}{\ell^2}\epsilon^{ac}\tau e_c + \frac{1}{\ell}\bar{\psi}^+\gamma^a\psi^- = R^a(\omega^b) + \frac{1}{\ell^2}\epsilon^{ac}\tau e_c + \frac{1}{\ell}\bar{\psi}^+\gamma^a\psi^-,$$

$$F(\tau) = d\tau + \epsilon^{ac}\omega_a e_c + \frac{1}{2}\bar{\psi}^+\gamma^0\psi^+ + \frac{1}{2}\bar{\psi}^-\gamma^0\psi^- = R(\tau) + \frac{1}{2}\bar{\psi}^+\gamma^0\psi^+ + \frac{1}{2}\bar{\psi}^-\gamma^0\psi^-$$
$$F^a(e^b) = de^a + \epsilon^{ac}\omega e_c = R^a(e^b)$$
$$F(t) = dt$$
$$F(u) = du + \frac{1}{2}\bar{\psi}^+\gamma^0\psi^+ - \frac{1}{2}\bar{\psi}^-\gamma^0\psi^-$$
$$\nabla\psi^+ = d\psi^+ + \frac{1}{2}\omega\gamma_0\psi^+ + \frac{1}{2\ell}e^a\gamma_a\psi^- - \frac{1}{2}t\gamma_0\psi^+$$
$$\nabla\psi^- = d\psi^- + \frac{1}{2}\omega\gamma_0\psi^- + \frac{1}{2\ell}e^a\gamma_a\psi^+ + \frac{1}{2}t\gamma_0\psi^-$$

$$\begin{aligned} \langle \mathrm{JJ}\rangle &= -\alpha_0, & \langle \mathrm{P}_a\mathrm{P}_b\rangle &= \frac{\alpha_0}{\ell^2}\delta_{ab} \\ \langle \mathrm{JH}\rangle &= -\alpha_1, & \langle \mathrm{G}_a\mathrm{P}_b\rangle &= \alpha_1\delta_{ab} \\ \langle \mathrm{TT}\rangle &= \alpha_0, & \langle \mathrm{TU}\rangle &= \alpha_1 \\ \langle \mathrm{Q}^+_\alpha\mathrm{Q}^+_\beta\rangle &= 2\alpha_1 C_{\alpha\beta}, & \langle \mathrm{Q}^-_\alpha\mathrm{Q}^-_\beta\rangle &= 2\alpha_1 C_{\alpha\beta} \end{aligned}$$

$$\alpha_0 = \lambda_0\tilde{\alpha_0}, \alpha_1 = \lambda_2\tilde{\alpha_1}$$

$$\begin{aligned} I^{\mathcal{N}=2}_{\text{ads}-\text{car}} = \frac{k}{4\pi}\int \; & \alpha_0\left[\frac{1}{\ell^2}e_a R^a(e^b) - \omega R(\omega) + t dt\right] \\ & + \alpha_1\left[2e_a R^a(\omega^b) - 2\tau R(\omega) + \frac{1}{\ell^2}\epsilon^{ac}\tau e_a e_c + 2t du - 2\bar{\psi}^+\nabla\psi^+ - 2\bar{\psi}^-\nabla\psi^-\right]. \end{aligned}$$

$$\delta\omega = 0$$

$$\begin{aligned} \delta\omega^a &= \frac{1}{\ell}\bar{\varepsilon}^+\gamma^a\psi^- + \frac{1}{\ell}\bar{\varepsilon}^-\gamma^a\psi^+, \\ \delta\tau &= \bar{\varepsilon}^+\gamma^0\psi^+ + \bar{\varepsilon}^-\gamma^0\psi^-, \\ \delta e^a &= 0, \\ \delta t &= 0, \\ \delta u &= \bar{\varepsilon}^+\gamma^0\psi^+ - \bar{\varepsilon}^-\gamma^0\psi^-, \\ \delta\psi^+ &= d\varepsilon^+ + \frac{1}{2}\omega\gamma_0\varepsilon^+ + \frac{1}{2\ell}e^a\gamma_a\varepsilon^- - \frac{1}{2}t\gamma_0\varepsilon^+, \\ \delta\psi^- &= d\varepsilon^- + \frac{1}{2}\omega\gamma_0\varepsilon^- + \frac{1}{2\ell}e^a\gamma_a\varepsilon^+ + \frac{1}{2}t\gamma_0\varepsilon^-. \end{aligned}$$

$$\begin{aligned} A = \sum_{m=0}^{N-1} & \left(\omega^{(m)}\mathrm{J}^{(m)} + \omega^{a(m)}\mathrm{G}_a^{(m)} + \tau^{(m)}\mathrm{H}^{(m)} + e^{a(m)}\mathrm{P}_a^{(m)} + t^{(m)}\mathrm{T}^{(m)} + u^{(m)}\mathrm{U}^{(m)} \right. \\ & \left. + \bar{\psi}^{+(m)}\mathrm{Q}^{+(m)} + \bar{\psi}^{-(m)}\mathrm{Q}^{-(m)}\right) \end{aligned}$$

$$\begin{aligned} F = \sum_{m=0}^{N-1} & \left[F\left(\omega^{(m)}\right)\mathrm{J}^{(m)} + F^a\left(\omega^{b(m)}\right)\mathrm{G}_a^{(m)} + F\left(\tau^{(m)}\right)\mathrm{H}^{(m)} + F^a\left(e^{b(m)}\right)\mathrm{P}_a^{(m)} \right. \\ & \left. + F\left(t^{(m)}\right)\mathrm{T}^{(m)} + F\left(u^{(m)}\right)\mathrm{U}^{(m)} + \nabla\bar{\psi}^{+(m)}\mathrm{Q}^{+(m)} + \nabla\bar{\psi}^{-(m)}\mathrm{Q}^{-(m)}\right] \end{aligned}$$

$$F(\omega^{(m)}) = d\omega^{(m)} + \frac{1}{2\ell^2}\sum_{n,p=0}^{N-1} \delta^m_{n+p}\epsilon^{ac}e_a^{(n)}e_c^{(p)} + \frac{1}{2\ell}\sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\bar{\psi}^{\pm(n)}\gamma^0\psi^{\pm(p)}$$
$$+\frac{1}{2}\sum_{n,p=0}^{N-1} \delta^m_{n+p+2}\epsilon^{ac}\omega_a^{(n)}\omega_c^{(p)}$$
$$F^a(\omega^{b(m)}) = d\omega^{a(m)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p}\left(\epsilon^{ac}\omega^{(n)}\omega_c^{(p)} + \frac{1}{\ell^2}\epsilon^{ac}\tau^{(n)}e_c^{(p)} + \frac{1}{\ell}\bar{\psi}^{+(n)}\gamma^a\psi^{-(p)}\right)$$
$$F(\tau^{(m)}) = d\tau^{(m)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p}\left(\epsilon^{ac}\omega_a^{(n)}e_c^{(p)} + \frac{1}{2}\bar{\psi}^{+(n)}\gamma^0\psi^{+(p)} + \frac{1}{2}\bar{\psi}^{-(n)}\gamma^0\psi^{-(p)}\right)$$

$$F^a(e^{b(m)}) = de^{a(m)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p}\epsilon^{ac}\omega^{(n)}e_c^{(p)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p+2}\epsilon^{ac}\tau^{(n)}\omega_c^{(p)}$$
$$F(t^{(m)}) = dt^{(m)} + \frac{1}{2\ell}\sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\bar{\psi}^{\pm(n)}\gamma^0\psi^{\pm(p)}$$
$$F(u^{(m)}) = du^{(m)} + \frac{1}{2}\sum_{n,p=0}^{N-1} \delta^m_{n+p}\bar{\psi}^{\pm(n)}\gamma^0\psi^{\pm(p)}$$
$$\nabla\psi^{\pm(m)} = d\psi^{\pm(m)} + \frac{1}{2}\sum_{n,p=0}^{N-1} \delta^m_{n+p}\left(\omega^{(n)}\gamma_0\psi^{\pm(p)} - \frac{1}{\ell}e^{a(n)}\gamma_a\psi^{\mp(p)} \mp t^{(n)}\gamma_0\psi^{\pm(p)}\right)$$
$$+\frac{1}{2}\sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\left(\omega^{a(n)}\gamma_a\psi^{\mp(p)} + \frac{1}{\ell}\tau^{(n)}\gamma_0\psi^{\pm(p)}\right)$$

$$\langle \mathrm{J}^{(m)}\mathrm{H}^{(n)}\rangle = -\alpha_{m+n+1}, \qquad \langle \mathrm{G}_a^{(m)}\mathrm{P}_b^{(n)}\rangle = \alpha_{m+n+1}\delta_{ab}$$
$$\langle \mathrm{T}^{(m)}\mathrm{U}^{(n)}\rangle = \alpha_{m+n+1}, \qquad \langle \mathrm{U}^{(m)}\mathrm{U}^{(n)}\rangle = -\frac{1}{\ell}\alpha_{m+n+2}$$
$$\langle \mathrm{Q}_\alpha^{+\,(m)}\mathrm{Q}_\beta^{+(n)}\rangle = 2\alpha_{m+n+1}C_{\alpha\beta}, \qquad \langle \mathrm{Q}_\alpha^{-(m)}\mathrm{Q}_\beta^{-(n)}\rangle = 2\alpha_{n+m+1}C_{\alpha\beta}$$

$$I^{\mathcal{N}=2}_{\text{ads-car}(N)} = \frac{k}{4\pi}\int \sum_{p=1}^{N} \alpha_p\delta^p_{m+n+1}\Big[2e_a^{(m)}R^a(\omega^{b(n)}) - 2\tau^{(m)}R(\omega^{(n)}) + \frac{\delta^n_{q+r}}{\ell^2}\epsilon^{ac}\tau^{(m)}e_a^{(q)}e_c^{(r)}$$
$$+2t^{(m)}du^{(n)} - \frac{\delta^n_{q+1}}{\ell}u^{(m)}du^{(q)} - 2\bar{\psi}^{+(m)}\nabla\psi^{+(n)} - 2\bar{\psi}^{-(m)}\nabla\psi^{-(n)}\Big]$$

$$R^a(\omega^{b(m)}) = d\omega^{a(m)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p}\epsilon^{ac}\omega^{(n)}\omega_c^{(p)}$$
$$R(\omega^{(m)}) = d\omega^{(m)} + \frac{1}{2}\sum_{n,p=0}^{N-1} \delta^m_{n+p+2}\epsilon^{ac}\omega_a^{(n)}\omega_c^{(p)}$$

$$\delta\omega^{(m)} = \frac{1}{\ell}\sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\bar{\varepsilon}^{\pm(n)}\gamma^0\psi^{\pm(p)}$$

$$\delta\omega^{a(m)} = \frac{1}{\ell}\sum_{n,p=0}^{N-1} \delta^m_{n+p}\bar{\varepsilon}^{\pm(n)}\gamma^a\psi^{\mp(p)}$$

$$\delta\tau^{(m)} = \sum_{n,p=0}^{N-1} \delta^m_{n+p}\bar{\varepsilon}^{\pm(n)}\gamma^0\psi^{\pm(p)}$$

$$\delta e^{a(m)} = \sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\bar{\varepsilon}^{\pm(n)}\gamma^a\psi^{\mp(p)}$$

$$\delta t^{(m)} = \pm\frac{1}{\ell}\sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\bar{\varepsilon}^{\pm(n)}\gamma^0\psi^{\pm(p)}$$

$$\delta u^{(m)} = \pm\frac{1}{\ell}\sum_{n,p=0}^{N-1} \delta^m_{n+p}\bar{\varepsilon}^{\pm(n)}\gamma^0\psi^{\pm(p)}$$

$$\delta\psi^{\pm(m)} = d\varepsilon^{\pm(m)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p}\left(\frac{1}{2}\omega^{(n)}\gamma_0\varepsilon^{\pm(p)} + \frac{1}{2\ell}e^{a(n)}\gamma_a\varepsilon^{\mp(p)} \mp \frac{1}{2}t^{(n)}\gamma_0\varepsilon^{\pm(p)}\right) + \sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\left(\frac{1}{2}\omega^{a(n)}\gamma_a\varepsilon^{\mp(p)} + \frac{1}{2\ell}\tau^{(n)}\gamma_0\varepsilon^{\pm(p)}\right)$$

$$I^{\mathcal{N}=2}_{\text{ads-car}} = \frac{k}{4\pi}\int \sum_{p=1}^{N} \alpha_p \mathcal{L}^{\mathcal{N}=2}_{\text{ads-car}(p)}$$

$$\mathcal{L}^{\mathcal{N}=2}_{\text{ads-car}} = \Big[2e_aR^a(k^b) + 2m_aR^a(\omega^b) - 2\tau R(k) - 2hR(\omega) + \frac{2}{\ell^2}\epsilon^{ac}\tau e_a m_c + \frac{1}{\ell^2}\epsilon^{ac}he_ae_c + 2t_1du_2 + 2t_2du_1 - \frac{1}{\ell}u_1du_1 - 2\bar{\psi}^{\pm}\nabla\chi^{\pm} - 2\bar{\chi}^{\pm}\nabla\psi^{\pm}\Big]$$

$$R^a(k^b) = dk^a + \epsilon^{ac}\omega k_c + \epsilon^{ac}k\omega_c$$

$$R(k) = dk$$

$$\nabla\chi^{\pm} = d\chi^{\pm} + \frac{1}{2}\omega\gamma_0\chi^{\pm} + \frac{1}{2}\omega^a\gamma_a\psi^{\mp} + \frac{1}{2}k\gamma_0\psi^{\pm} + \frac{1}{2\ell}e^a\gamma_a\chi^{mp} + \frac{1}{2\ell}\tau\gamma_0\psi^{\pm} + \frac{1}{2\ell}m^a\gamma_a\psi^{\pm} \mp \frac{1}{2}t_1\gamma_0\chi^{\pm} \mp \frac{1}{2}t_2\gamma_0\psi^{\pm}$$

$$\omega^{(0)} = \omega\,,\quad \omega^{(1)} = k\,,\quad \tau^{(0)} = \tau\,,\quad \tau^{(1)} = h\,,$$
$$\omega_a^{(0)} = \omega_a\,,\quad \omega_a^{(1)} = k_a\,,\quad e_a^{(0)} = e_a\,,\quad e_a^{(1)} = m_a\,,$$
$$t^{(0)} = t_1\,,\quad t^{(1)} = t_2\,,\quad u^{(0)} = u_1\,,\quad u^{(1)} = u_2\,,$$
$$\psi_\alpha^{\pm(0)} = \psi_\alpha^{\pm}\,,\quad \psi_\alpha^{\pm(1)} = \chi_\alpha^{\pm}\,.$$

$$A = \omega \mathrm{J} + \omega^a \mathrm{G}_a + s\,\mathrm{S} + \tau \mathrm{H} + e^a \mathrm{P}_a + m\mathrm{M} + t_1\,\mathrm{T}_1 + t_2\,\mathrm{T}_2 + u_1\mathrm{U}_1 + u_2\mathrm{U}_2 + \bar{\psi}^+\mathrm{Q}^+ + \bar{\psi}^-\mathrm{Q}^- + \bar{\rho}\mathrm{R},$$

$$\langle \mathrm{JS}\rangle = -\sigma_0,\qquad \langle \mathrm{G}_a\mathrm{G}_b\rangle = \sigma_0\delta_{ab},$$
$$\langle \mathrm{JM}\rangle = \langle \mathrm{HS}\rangle = -\beta_1,\qquad \langle \mathrm{G}_a\mathrm{P}_b\rangle = \beta_1\delta_{ab},$$
$$\langle \mathrm{HM}\rangle = -\frac{\sigma_0}{\ell^2},\qquad \langle \mathrm{P}_a\mathrm{P}_b\rangle = \frac{\sigma_0}{\ell^2}\delta_{ab},$$
$$\langle \mathrm{T}_1\,\mathrm{T}_2\rangle = \sigma_0,\qquad \langle \mathrm{T}_1\mathrm{U}_2\rangle = \langle \mathrm{T}_2\mathrm{U}_1\rangle = \beta_1,$$
$$\langle \mathrm{U}_1\mathrm{U}_2\rangle = -\frac{\beta_1}{\ell},\qquad \langle \mathrm{Q}_\alpha^-\mathrm{Q}_\beta^-\rangle = \langle \mathrm{Q}_\alpha^+\mathrm{R}_\beta\rangle = 2\left(\beta_1 + \frac{\sigma_0}{\ell}\right)C_{\alpha\beta}.$$

$$\sigma_0 = \lambda_2\tilde{\alpha}_0, \beta_1 = \lambda_2\tilde{\alpha}_1$$

$$
\begin{aligned}
I^{\mathcal{N}=2}_{\text{extended } \mathfrak{n}\mathfrak{h}} =& \frac{k}{4\pi}\int \beta_1 \Big[2e_a R^a(\omega^b) - 2mR(\omega) - 2\tau R(s) + \frac{1}{\ell^2}\epsilon^{ac}\tau e_a e_c + 2t_1 du_2 + 2t_2 du_1 \\
& - \frac{2}{\ell}u_1 du_2 - 2\bar{\psi}^-\nabla\psi^- - 2\bar{\psi}^+\nabla\rho - 2\bar{\rho}\nabla\psi^+\Big]
\end{aligned}
$$

$$
\begin{aligned}
R(\omega) =& d\omega \\
R^a(\omega^a) =& d\omega^a + \epsilon^{ac}\omega\omega_c \\
R(s) =& ds + \frac{1}{2}\epsilon^{ac}\omega_a\omega_c \\
\nabla\psi^+ =& d\psi^+ + \frac{1}{2}\omega\gamma_0\psi^+ + \frac{1}{2\ell}\tau\gamma_0\psi^+ - \frac{1}{2}t_1\gamma_0\psi^+ \\
\nabla\psi^- =& d\psi^- + \frac{1}{2}\omega\gamma_0\psi^- + \frac{1}{2\ell}\tau\gamma_0\psi^- + \frac{1}{2}\omega^a\gamma_a\psi^+ + \frac{1}{2\ell}e^a\gamma_a\psi^+ + \frac{1}{2}t_1\gamma_0\psi^- \\
\nabla\rho =& d\rho + \frac{1}{2}\omega\gamma_0\rho + \frac{1}{2}s\gamma_0\psi^+ + \frac{1}{2}\omega^a\gamma_a\psi^- + \frac{1}{2\ell}e^a\gamma_a\psi^- + \frac{1}{2\ell}\tau\gamma_0\rho \\
& + \frac{1}{2\ell}m\gamma_0\psi^+ - \frac{1}{2}t_1\gamma_0\rho - \frac{1}{2}t_2\gamma_0\psi^+
\end{aligned}
$$

$$
\begin{aligned}
F(\omega) &= R(\omega) + \frac{1}{2\ell}\bar{\psi}^+\gamma^0\psi^+ \\
F(\omega^a) &= R^a(\omega^b) + \frac{1}{\ell^2}\epsilon^{ac}\tau e_c + \frac{1}{\ell}\bar{\psi}^+\gamma^a\psi^- \\
F(\tau) &= d\tau + \frac{1}{2}\bar{\psi}^+\gamma^0\psi^+ \\
F(e^a) &= de^a + \epsilon^{ac}\omega e_c + \epsilon^{ac}\tau\omega_c + \bar{\psi}^+\gamma^a\psi^- \\
F(s) &= R(s) + \frac{1}{2\ell^2}\epsilon^{ac}e_a e_c + \frac{1}{2\ell}\bar{\psi}^-\gamma^0\psi^- + \frac{1}{\ell}\bar{\psi}^+\gamma^0\rho \\
F(m) &= dm + \epsilon^{ac}\omega_a e_c + \frac{1}{2}\bar{\psi}^-\gamma^0\psi^- + \bar{\psi}^+\gamma^0\rho \\
F(t_1) &= dt_1 + \frac{1}{2\ell}\bar{\psi}^+\gamma^0\psi^+ \\
F(t_2) &= dt_2 - \frac{1}{2\ell}\bar{\psi}^-\gamma^0\psi^- + \frac{1}{\ell}\bar{\psi}^+\gamma^0\rho
\end{aligned}
$$

$$
\begin{aligned}
F(u_1) &= du_1 + \frac{1}{2}\bar{\psi}^+\gamma^0\psi^+ \\
F(u_2) &= du_2 - \frac{1}{2}\bar{\psi}^-\gamma^0\psi^- + \bar{\psi}^+\gamma^0\rho
\end{aligned}
$$

$$
\begin{aligned}
A =& \sum_{m=0}^{N}\left(\omega^{(m)}\mathrm{J}^{(m)} + \tau^{(m)}\mathrm{H}^{(m)} + t^{(m)}\mathrm{T}^{(m)} + u^{(m)}\mathrm{U}^{(m)} + \bar{\psi}^{+(m)}\mathrm{Q}^{+(m)}\right) \\
& + \sum_{m=0}^{N-1}\left(\omega^{a(m)}\mathrm{G}_a^{(m)} + e^{a(m)}\mathrm{P}_a^{(m)} + \bar{\psi}^{-(m)}\mathrm{Q}^{-(m)}\right)
\end{aligned}
$$

$$
\begin{aligned}
\left\langle \mathrm{J}^{(m)}\mathrm{H}^{(n)}\right\rangle &= -\beta_{m+n}, & \left\langle \mathrm{G}_a^{(m)}\mathrm{P}_b^{(n)}\right\rangle &= \beta_{m+n+1}\delta_{ab} \\
\left\langle \mathrm{T}^{(m)}\mathrm{U}^{(n)}\right\rangle &= \beta_{m+n}, & \left\langle \mathrm{U}^{(m)}\mathrm{U}^{(n)}\right\rangle &= -\frac{1}{\ell}\beta_{m+n} \\
\left\langle \mathrm{Q}_\alpha^{-(m)}\mathrm{Q}_\beta^{-(n)}\right\rangle &= 2\beta_{m+n+1}C_{\alpha\beta}, & \left\langle \mathrm{Q}_\alpha^{+(m)}\mathrm{Q}_\beta^{+(n)}\right\rangle &= 2\beta_{n+m}C_{\alpha\beta}
\end{aligned}
$$

$$
\beta_p = \lambda_{2p}\tilde{\alpha}_1
$$

$$
\begin{aligned}
I^{\mathcal{N}=2}_{\mathfrak{n}\mathfrak{h}^{(N)}} =& \frac{k}{4\pi}\int \sum_{p=1}^{N}\beta_p\delta^p_{m+n}\Big[-2\tau^{(m)}R\big(\omega^{(n)}\big) + \frac{\delta^n_{q+r+1}}{\ell^2}\epsilon^{ac}\tau^{(m)}e_a^{(q)}e_c^{(r)} + 2t^{(m)}du^{(n)} - \frac{1}{\ell}u^{(m)}du^{(n)} \\
& - 2\bar{\psi}^{+(m)}\nabla\psi^{+(n)}\Big] + \beta_p\delta^p_{m+n+1}\Big[2e_a^{(m)}R^a\big(\omega^{b(n)}\big) - 2\bar{\psi}^{-(m)}\nabla\psi^{-(n)}\Big]
\end{aligned}
$$

$$R\left(\omega^{(m)}\right)=d\omega^{(m)}+\frac{1}{2}\sum_{p,q=0}^{N-1}\delta^{m}_{p+q+1}\epsilon^{ac}\omega_a^{(p)}\omega_c^{(q)}$$

$$R^a\left(\omega^{a(m)}\right)=d\omega^{a(m)}+\sum_{n,p=0}^{N}\delta^{m}_{n+p}\epsilon^{ac}\omega^{(n)}\omega_c^{(p)}$$

$$\nabla\psi^{+(m)}=d\psi^{+(m)}+\frac{1}{2}\sum_{n,p=0}^{N}\delta^{m}_{n+p}\left(\omega^{(n)}\gamma_0\psi^{+(p)}+\frac{1}{\ell}\tau^{(n)}\gamma_0\psi^{+(p)}-t^{(n)}\gamma_0\psi^{+(p)}\right)$$
$$+\sum_{n,p=0}^{N-1}\delta^{m}_{n+p+1}\left(\omega^{a(n)}\gamma_a\psi^{-(p)}+\frac{1}{\ell}e^{a(n)}\gamma_a\psi^{-(p)}\right)$$

$$\nabla\psi^{-(m)}=d\psi^{-(m)}+\frac{1}{2}\sum_{n,p=0}^{N-1}\delta^{m}_{n+p}\left(\omega^{(n)}\gamma_0\psi^{-(p)}+\frac{1}{\ell}\tau^{(n)}\gamma_0\psi^{-(p)}+t^{(n)}\gamma_0\psi^{-(p)}\right.$$
$$\left.+\omega^{a(n)}\gamma_a\psi^{+(p)}+\frac{1}{\ell}e^{a(n)}\gamma_a\psi^{+(p)}\right)$$

$$I^{\mathcal{N}=2}_{\mathfrak{n}\mathfrak{h}^{(N)}}=\frac{k}{4\pi}\int\sum_{p=1}^{N}\beta_p\mathcal{L}^{\mathcal{N}=2}_{\mathfrak{n}\mathfrak{h}^{(p)}}$$

$$F\left(\omega^{(m)}\right)=R\left(\omega^{(m)}\right)+\frac{1}{2}\sum_{n,p=0}^{N-1}\delta^{m}_{n+p+1}\left(\frac{1}{\ell^2}\epsilon^{ac}e_a^{(n)}e_c^{(p)}+\frac{1}{\ell}\bar{\psi}^{-(n)}\gamma^0\psi^{-(p)}\right)$$
$$+\frac{1}{2\ell}\sum_{n,p=0}^{N}\delta^{m}_{n+p}\bar{\psi}^{+(n)}\gamma^0\psi^{+(p)}$$

$$F^a\left(\omega^{b(m)}\right)=R\left(\omega^{a(m)}\right)+\sum_{n,p=0}^{N-1}\delta^{m}_{n+p}\left(\frac{1}{\ell^2}\epsilon^{ac}\tau^{(n)}e_c^{(p)}+\frac{1}{\ell}\bar{\psi}^{+(n)}\gamma^a\psi^{-(p)}\right)$$

$$F\left(\tau^{(m)}\right)=d\tau^{(m)}+\sum_{n,p=0}^{N-1}\delta^{m}_{n+p+1}\left(\epsilon^{ac}\omega_a^{(n)}e_c^{(p)}+\frac{1}{2}\bar{\psi}^{-(n)}\gamma^0\psi^{-(p)}\right)$$
$$+\frac{1}{2}\sum_{n,p=0}^{N}\delta^{m}_{n+p}\bar{\psi}^{+(n)}\gamma^0\psi^{+(p)}$$

$$F^a\left(e^{b(m)}\right)=de^{a(m)}+\sum_{n,p=0}^{N-1}\delta^{m}_{n+p}\left(\epsilon^{ac}\omega^{(n)}e_c^{(p)}+\epsilon^{ac}\tau^{(n)}\omega_c^{(p)}+\bar{\psi}^{+(n)}\gamma^a\psi^{-(p)}\right)$$

$$F\left(t^{(m)}\right)=dt^{(m)}+\frac{1}{2\ell}\sum_{n,p=0}^{N}\delta^{m}_{n+p}\bar{\psi}^{+(n)}\gamma^0\psi^{+(p)}+\frac{1}{2\ell}\sum_{n,p=0}^{N-1}\delta^{m}_{n+p+1}\bar{\psi}^{-(n)}\gamma^0\psi^{-(p)}$$

$$F\left(u^{(m)}\right)=du^{(m)}+\frac{1}{2}\sum_{n,p=0}^{N}\delta^{m}_{n+p}\bar{\psi}^{+(n)}\gamma^0\psi^{+(p)}+\frac{1}{2}\sum_{n,p=0}^{N-1}\delta^{m}_{n+p+1}\bar{\psi}^{-(n)}\gamma^0\psi^{-(p)}$$

$$\begin{array}{ccc}
\omega^{(0)}=\omega, & \omega^{(1)}=s, & \omega^{(2)}=z\\
\tau^{(0)}=\tau, & \tau^{(1)}=m, & \tau^{(2)}=t\\
t^{(0)}=t_1, & t^{(1)}=t_2, & t^{(2)}=t_3\\
u^{(0)}=u_1, & u^{(1)}=u_2, & u^{(2)}=u_3\\
\psi_\alpha^{+(0)}=\psi_\alpha^{+}, & \psi_\alpha^{+(1)}=\rho_\alpha, & \psi_\alpha^{+(2)}=\phi_\alpha^{+}\\
\omega_a^{(0)}=\omega_a, & e_a^{(0)}=e_a, & \psi_\alpha^{-(0)}=\psi_\alpha^{-}\\
\omega_a^{(1)}=b_a, & e_a^{(1)}=t_a, & \psi_\alpha^{-(1)}=\phi_\alpha^{-}
\end{array}$$

$$\mathcal{L}^{\mathcal{N}=2}_{\mathfrak{n}\mathfrak{h}^{(2)}}=2e^aR^a(b^b)+2t^aR^a(\omega^b)-2\tau R(z)-2mR(s)-2yR(\omega)+\frac{2}{\ell^2}\epsilon^{ac}\tau e_a t_c+\frac{1}{\ell^2}\epsilon^{ac}me_ae_c$$
$$+2t_1du_3+2t_2du_2+2t_3du_1-\frac{2}{\ell}u_1du_3-\frac{1}{\ell}u_2du_2-2\bar{\psi}^{\pm}\nabla\phi^{\pm}-2\bar{\phi}^{\pm}\nabla\psi^{\pm}-2\bar{\rho}\nabla\rho$$

$$R^a(b^b) = db^a + \epsilon^{ac}\omega b_c + \epsilon^{ac} s\omega_c$$
$$R(z) = dz + \epsilon^{ac}\omega_a b_c$$
$$\nabla\phi^+ = d\phi^+ + \frac{1}{2}\omega\gamma_0\phi^+ + \frac{1}{2}s\gamma_0\rho + \frac{1}{2}\omega^a\gamma_a\phi^- + \frac{1}{2}b^a\gamma_a\psi^- + \frac{1}{2\ell}e^a\gamma_a\phi^- + \frac{1}{2\ell}\tau\gamma_0\phi^+ + \frac{1}{2\ell}m\gamma_0\rho + \frac{1}{2\ell}t^a\gamma_a\psi^- + \frac{1}{2}z\gamma_0\psi^+ + \frac{1}{2\ell}y\gamma_0\psi^+ - \frac{1}{2}t_1\gamma_0\phi^+ - \frac{1}{2}t_2\gamma_0\rho - \frac{1}{2}t_3\gamma_0\psi^+$$
$$\nabla\phi^- = d\phi^- \frac{1}{2}\omega\gamma_0\phi^- + \frac{1}{2}s\gamma_0\psi^- + \frac{1}{2}\omega^a\gamma_a\rho + \frac{1}{2}b^a\gamma_a\psi^+ + \frac{1}{2\ell}e^a\gamma_a\rho + \frac{1}{2\ell}\tau\gamma_0\phi^- + \frac{1}{2\ell}m\gamma_0\psi^- + \frac{1}{2\ell}t^a\gamma_a\psi^+ - \frac{1}{2}t_1\gamma_0\phi^- - \frac{1}{2}t_2\gamma_0\psi^-$$

$$I_{\text{AdS}}^{\mathcal{N}=2} = \int \tilde{\alpha}_1\left[2e_A R^A + \frac{1}{3\ell^2}\epsilon_{ABC}e^A e^B e^C + \mathrm{t}d\mathbf{u} - \frac{1}{\ell^2}\mathbf{u}d\mathbf{u} - 2\bar{\Psi}^i\nabla\Psi^i\right]$$

$$R^A = d\omega^A + \frac{1}{2}\epsilon^{ABC}\omega_B\omega_C$$
$$T^A = de^A + \frac{1}{2}\epsilon^{ABC}\omega_B e_C$$
$$\nabla\Psi^i = d\Psi^i + \frac{1}{2}\omega^A\gamma_A\Psi^i + \frac{1}{2\ell}e^A\gamma_A\Psi^i + \mathrm{t}\epsilon^{ij}\Psi^j$$

$$\omega^{(m)} = \lambda_{2m}\omega_0, \quad \tau^{(m)} = \lambda_{2m}e_0 \quad \psi^{+(m)} = \lambda_{2m}\Psi^+,$$
$$\omega_a^{(m)} = \lambda_{2m+1}\omega_a \quad e_a^{(m)} = \lambda_{2m+1}e_a \quad \psi^{-(m)} = \lambda_{2m+1}\Psi^-$$

$$\Psi_\alpha^\pm = \frac{1}{\sqrt{2}}\left(\Psi_\alpha^1 \pm (\gamma^0)_{\alpha\beta}\Psi_\beta^2\right)$$

$$A = \omega\mathrm{J} + \omega^a\mathrm{G}_a + s\,\mathrm{S} + \tau\mathrm{H} + e^a\mathrm{P}_a + m\mathrm{M} + t_1\,\mathrm{T}_1 + t_2\,\mathrm{T}_2 + u_1\mathrm{U}_1 + u_2\mathrm{U}_2 + \bar{\psi}^+\mathrm{Q}^+ + \bar{\psi}^-\mathrm{Q}^- + \bar{\rho}\mathrm{R}.$$

$$F = \mathcal{F}(\omega)\mathrm{J} + \mathcal{F}^a(\omega^b)\mathrm{G}_a + \mathcal{F}(\tau)\mathrm{H} + \mathcal{F}^a(e^b)\mathrm{P}_a + \mathcal{F}(t_1)\mathrm{T}_1 + \mathcal{F}(t_2)\mathrm{T}_2 + \mathcal{F}(u_1)\mathrm{U}_1 + \mathcal{F}(u_2)\mathrm{U}_2 + \nabla\bar{\psi}^+\mathrm{Q}^+ + \nabla\bar{\psi}^-\mathrm{Q}^- + \nabla\bar{\rho}\mathrm{R}$$

$$\mathcal{F}(\omega) = d\omega = \mathcal{R}(\omega)$$
$$\mathcal{F}^a(\omega^b) = d\omega^a + \epsilon^{ac}\omega\omega_c + \frac{1}{\ell^2}\epsilon^{ac}\tau e_c + \frac{1}{\ell}\bar{\psi}^+\gamma^a\psi^- = \mathcal{R}^a(\omega^b) + \frac{1}{\ell^2}\epsilon^{ac}\tau e_c + \frac{1}{\ell}\bar{\psi}^+\gamma^a\psi^-$$
$$\mathcal{F}(\tau) = d\tau + \frac{1}{2}\bar{\psi}^+\gamma^0\psi^+$$
$$\mathcal{F}(e^a) = de^a + \epsilon^{ac}\omega e_c$$
$$\mathcal{F}(s) = ds + \frac{1}{2\ell^2}\epsilon^{ac}e_a e_c = \mathcal{R}(s) + \frac{1}{2\ell^2}\epsilon^{ac}e_a e_c$$
$$\mathcal{F}(m) = dm + \epsilon^{ac}\omega_a e_c + \frac{1}{2}\bar{\psi}^-\gamma^0\psi^-$$

$$\mathcal{F}(t_1) = dt_1$$
$$\mathcal{F}(t_2) = dt_2$$
$$\mathcal{F}(u_1) = du_1 + \frac{1}{2}\bar{\psi}^+\gamma^0\psi^+$$
$$\mathcal{F}(u_2) = du_2 + \frac{1}{2}\bar{\psi}^-\gamma^0\psi^- + \bar{\psi}^+\gamma^0\rho$$
$$\nabla\psi^+ = d\psi^+ + \frac{1}{2}\omega\gamma_0\psi^+ - \frac{1}{2}t_1\gamma_0\psi^+$$
$$\nabla\psi^- = d\psi^- + \frac{1}{2}\omega\gamma_0\psi^- + \frac{1}{2\ell}e^a\gamma_a\psi^+ + \frac{1}{2}t_1\gamma_0\psi^-$$
$$\nabla\rho = d\rho + \frac{1}{2}\omega\gamma_0\rho + \frac{1}{2}s\gamma_0\psi^+ + \frac{1}{2\ell}e^a\gamma_a\psi^- - \frac{1}{2}t_1\gamma_0\rho - \frac{1}{2}t_2\gamma_0\psi^+$$

$$\langle \mathrm{JS}\rangle = -\nu_0, \qquad \langle \mathrm{P}_a \mathrm{P}_b\rangle = \frac{\nu_0}{\ell^2}\delta_{ab}$$
$$\langle \mathrm{JM}\rangle = \langle \mathrm{HS}\rangle = -\mu_1, \qquad \langle \mathrm{G}_a \mathrm{P}_b\rangle = \mu_1 \delta_{ab}$$
$$\langle \mathrm{T}_1\, \mathrm{T}_2\rangle = \nu_0, \qquad \langle \mathrm{T}_1 \mathrm{U}_2\rangle = \langle \mathrm{T}_2 \mathrm{U}_1\rangle = \mu_1$$
$$\left\langle \mathrm{Q}_\alpha^- \mathrm{Q}_\beta^- \right\rangle = 2\mu_1 C_{\alpha\beta}, \qquad \left\langle \mathrm{Q}_\alpha^+ \mathrm{R}_\beta \right\rangle = 2\mu_1 C_{\alpha\beta}$$

$$\nu_0 = \lambda_2 \alpha_0, \mu_1 = \lambda_2 \alpha_1$$

$$\nu_0 = \lambda_0 \sigma_0, \mu_1 = \lambda_2 \beta_1$$

$$I_{\text{aas-stat}}^{\mathcal{N}=2} = \frac{k}{4\pi}\int \mu_1 \Big[2e_a \mathcal{R}^a(\omega^b) - 2m\mathcal{R}(\omega) - 2\tau\mathcal{R}(s) + \frac{1}{\ell^2}\epsilon^{ac}\tau e_a e_c + 2t_1 du_2 + 2t_2 du_1 - 2\bar{\psi}^+ \nabla\rho - 2\bar{\rho}\nabla\psi^+ - 2\bar{\psi}^- \nabla\psi^- \Big]$$

$$\begin{aligned}
\delta\omega &= 0\\
\delta\omega^a &= \frac{1}{\ell}\bar{\varepsilon}^+ \gamma^a \psi^- + \frac{1}{\ell}\bar{\varepsilon}^- \gamma^a \psi^+\\
\delta\tau &= \bar{\varepsilon}^+ \gamma^0 \psi^+\\
\delta e^a &= 0\\
\delta s &= 0\\
\delta m &= \bar{\varepsilon}^- \gamma^0 \psi^- + \bar{\varepsilon}^+ \gamma^0 \rho + \bar{\varrho}\gamma^0 \psi^+\\
\delta t_1 &= 0\\
\delta t_2 &= 0\\
\delta u_1 &= \bar{\varepsilon}^+ \gamma^0 \psi^+\\
\delta u_2 &= \bar{\varepsilon}^+ \gamma^0 \rho + \bar{\varrho}^+ \gamma^0 \psi^+ - \bar{\varepsilon}^- \gamma^0 \psi^-\\
\delta\psi^+ &= d\varepsilon^+ + \frac{1}{2}\omega\gamma_0 \varepsilon^+ + \frac{1}{2\ell}e^a \gamma_a \varepsilon^- - \frac{1}{2}t_1 \gamma_0 \varepsilon^+\\
\delta\psi^- &= d\varepsilon^- + \frac{1}{2}\omega\gamma_0 \varepsilon^- + \frac{1}{2\ell}e^a \gamma_a \varepsilon^+ + \frac{1}{2}t_1 \gamma_0 \varepsilon^-\\
\delta\rho &= d\varrho + \frac{1}{2}\omega\gamma_0 \varrho + \frac{1}{2}s\gamma_0 \varepsilon^+ + \frac{1}{2\ell}e^a \gamma_a \varepsilon^- - \frac{1}{2}t_1 \gamma_0 \varrho - \frac{1}{2}t_2 \gamma_0 \varepsilon^+
\end{aligned}$$

$$\begin{aligned}
\left\langle \mathrm{J}^{(m)} \mathrm{H}^{(n)} \right\rangle &= -\mu_{m+n}, & \left\langle \mathrm{G}_a^{(m)} \mathrm{P}_b^{(n)} \right\rangle &= \mu_{m+n+1}\delta_{ab}\\
\left\langle \mathrm{T}^{(m)} \mathrm{U}^{(n)} \right\rangle &= \mu_{m+n}, & \left\langle \mathrm{Q}_\alpha^{-(m)} \mathrm{Q}_\beta^{-(n)} \right\rangle &= 2\mu_{m+n+1} C_{\alpha\beta}\\
\left\langle \mathrm{Q}_\alpha^{+(m)} \mathrm{Q}_\beta^{+(n)} \right\rangle &= 2\mu_{n+m} C_{\alpha\beta}, &&
\end{aligned}$$

$$\mu_p = \lambda_{2p}\alpha_1$$

$$\mu_p = \lambda_2 \beta_p$$

$$A = \sum_{m=0}^{N} \left(\omega^{(m)}\mathrm{J}^{(m)} + \tau^{(m)}\mathrm{H}^{(m)} + t^{(m)}\mathrm{T}^{(m)} + u^{(m)}\mathrm{U}^{(m)} + \bar{\psi}^{+(m)}\mathrm{Q}^{+(m)}\right) + \sum_{m=0}^{N-1} \left(\omega^{a(m)}\mathrm{G}_a^{(m)} + e^{a(m)}\mathrm{P}_a^{(m)} + \bar{\psi}^{-(m)}\mathrm{Q}^{-(m)}\right)$$

$$\mathcal{F}\left(\omega^{(m)}\right) = d\omega^{(m)} + \frac{1}{2\ell^2}\sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\epsilon^{ac}e^{(n)}_a e^{(p)}_c$$

$$\mathcal{F}^a\left(\omega^{b(m)}\right) = d\omega^{a(m)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p}\left(\epsilon^{ac}\omega^{(n)}\omega^{(p)}_c + \frac{1}{\ell^2}\epsilon^{ac}\tau^{(n)}e^{(p)}_c + \frac{1}{\ell}\bar{\psi}^{+(n)}\gamma^a\psi^{-(p)}\right)$$

$$\mathcal{F}\left(\tau^{(m)}\right) = d\tau^{(m)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\left(\epsilon^{ac}\omega^{(n)}_a e^{(p)}_c + \frac{1}{2}\bar{\psi}^{-(n)}\gamma^0\psi^{-(p)}\right)$$

$$+\frac{1}{2}\sum_{n,p=0}^{N} \delta^m_{n+p}\bar{\psi}^{+(n)}\gamma^0\psi^{+(p)}$$

$$\mathcal{F}^a\left(e^{b(m)}\right) = de^{a(m)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p}\epsilon^{ac}\omega^{(n)}e^{(p)}_c$$

$$\mathcal{F}\left(t^{(m)}\right) = dt^{(m)},$$

$$\mathcal{F}\left(u^{(m)}\right) = du^{(m)} + \frac{1}{2}\sum_{n,p=0}^{N} \delta^m_{n+p}\bar{\psi}^{+(n)}\gamma^0\psi^{+(p)} + \frac{1}{2}\sum_{n,p=0}^{N-1} \delta^m_{n+p+1}\bar{\psi}^{-(n)}\gamma^0\psi^{-(p)}$$

$$\nabla\psi^{+(m)} = d\psi^{+(m)} + \frac{1}{2}\sum_{n,p=0}^{N} \delta^m_{n+p}\left(\omega^{(n)}\gamma_0\psi^{+(p)} - t^{(n)}\gamma_0\psi^{+(p)}\right)$$

$$+\frac{1}{\ell}\sum_{n,p=0}^{N-1} \delta^m_{n+p+1}e^{a(n)}\gamma_a\psi^{-(p)}$$

$$\nabla\psi^{-(m)} = d\psi^{-(m)} + \frac{1}{2}\sum_{n,p=0}^{N-1} \delta^m_{n+p}\left(\omega^{(n)}\gamma_0\psi^{-(p)} + t^{(n)}\gamma_0\psi^{-(p)} + \frac{1}{\ell}e^{a(n)}\gamma_a\psi^{+(p)}\right)$$

$$I^{\mathcal{N}=2}_{\mathfrak{ads-stat}} = \frac{k}{4\pi}\int\sum_{p=1}^{N} \mu_p\delta^p_{m+n+1}\left[2e^{(m)}_a\mathcal{R}^a\left(\omega^{b(n)}\right) - 2\bar{\psi}^{-(m)}\nabla\psi^{-(n)}\right]$$

$$+\mu_p\delta^p_{m+n}\left[-2\tau^{(m)}\mathcal{R}\left(\omega^{(n)}\right) + \frac{\delta^n_{q+r+1}}{\ell^2}\epsilon^{ac}\tau^{(m)}e^{(q)}_a e^{(r)}_c + 2t^{(m)}du^{(n)} - 2\bar{\psi}^{+(m)}\nabla\psi^{+(n)}\right]$$

$$\mathcal{R}\left(\omega^{(n)}\right) = d\omega^{(n)}$$

$$\mathcal{R}^a\left(\omega^{b(n)}\right) = d\omega^{a(m)} + \sum_{n,p=0}^{N-1} \delta^m_{n+p}\epsilon^{ac}\omega^{(n)}\omega^{(p)}_c$$

$$I^{\mathcal{N}=2}_{\mathfrak{ads-stat}} = \frac{k}{4\pi}\int\sum_{p=1}^{N} \mu_p\mathcal{L}^{\mathcal{N}=2}_{\mathfrak{ads-stat}}$$

$$\omega^{(m)} = \lambda_0\omega^{(m)},\quad \tau^{(m)} = \lambda_2\tau^{(m)}\quad \psi^{+(m)} = \lambda_1\psi^{+(m)},$$
$$\omega^{(m)}_a = \lambda_2\omega^{(m)}_a\quad e^{(m)}_a = \lambda_0 e^{(m)}_a\quad \psi^{-(m)} = \lambda_0 t^{(m)}, u^{(m)} = \lambda_2 u^{(m)}$$

$$\omega^{(m)} = \lambda_{2m}\omega,\quad \tau^{(m)} = \lambda_{2m}\tau\quad \psi^{+(m)} = \lambda_{2m}\psi^{+},$$
$$\omega^{(m)}_a = \lambda_{2m+1}\omega_a\quad e^{(m)}_a = \lambda_{2m+1}e_a\quad \psi^{-(m)} = \lambda_{2m}t, u^{(m)} = \lambda_{2m}u$$

$$\Lambda = \mathcal{C}^{(1)} \oplus \mathcal{C}^{(d-3)} \subset D$$

$$S_{\text{SymTFT}} = \frac{1}{2\pi}\sum_{p,i,j}\int_{M_{d+1}} \kappa^{(p)}_{ij}B^i_p dC^j_{d-p} + \mathcal{A}\left(\{B^i_p\}\right)$$

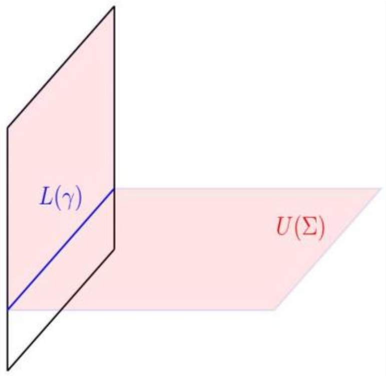
$$S_{\text{SymTFT}} = \frac{N}{2\pi}\int_{M_{d+1}} B_p dC_{d-p}$$

$$B_p \to B_p + d\lambda_{p-1}$$
$$C_{d-p} \to C_{d-p} + d\lambda_{d-p-1}$$

$$U(M_p) = e^{i\int_{M_p} B_p}, V(M_{d-p}) = e^{i\int_{M_{d-p}} C_{d-p}}$$

$$C_{d-p} \to C_{d-p} + \frac{1}{N}\xi_{d-p}, B_p \to B_p + \frac{1}{N}\xi_p$$

$$U(M_p)V(M_{d-p}) = \exp\left(\frac{2\pi i L(M_p, M_{d-p})}{N}\right)V(M_{d-p})U(M_p)$$



$$D = \bigoplus_{n=1}^{d-1} D^{(n)}$$

$$\langle\cdot,\cdot\rangle: D \times D \to U(1).$$

$$\Lambda = \bigoplus_{n=1}^{d-1} \Lambda_n, \Lambda_n \subseteq D^{(n)}$$

$$\mathrm{SNF}(\kappa^{(p)}) = \mathrm{diag}\left\{l_1^{(p)}, \dots, l_i^{(p)}, \dots\right\}$$

$$D = \bigoplus_{p,i} \left(\mathbb{Z}_{l_i^{(p)}}^{(p-1)} \oplus \mathbb{Z}_{l_i^{(p)}}^{(d-p-1)}\right)$$

$$D = \bigoplus_{p,i} \left(\mathbb{Z}_{\kappa_{ii}}^{(p-1)} \oplus \mathbb{Z}_{\kappa_{ii}}^{(d-p-1)}\right)$$

$$\mathbb{R}^{1,d-1} \times X_{D-d},$$

$$H_{m+1}(\partial X) = H^{\tilde{d}-(m+2)}(\partial X) = 0 = H_m(X, \partial X) = H^{\tilde{d}-m}(X)$$

$$0 \to H_{m+1}(X) \overset{J_{m+1}}{\to} H_{m+1}(X, \partial X) \overset{D_{m+1}}{\to} H_m(\partial X) \overset{I_m}{\to} H_m(X) \to 0.$$

$$0 \to H^{\tilde{d}-(m+1)}(X, \partial X) \overset{J_{m+1}^*}{\to} H^{\tilde{d}-(m+1)}(X) \overset{D_{m+1}^*}{\to} H^{\tilde{d}-(m+1)}(\partial X) \overset{I_m^*}{\to} H^{\tilde{d}-m}(X, \partial X) \to 0,$$

$$H^{\tilde{d}-(m+1)}(\partial X) = H^{\tilde{d}-(m+1)}(X)/\mathrm{im}(J_{m+1}^*)$$

$$H^{\tilde{d}-(m+1)}(X, \partial X) \cong H_{m+1}(X)$$

$$H^{\tilde{d}-(m+1)}(X) = \mathrm{Hom}(H_{m+1}(X), \mathbb{Z}) \oplus \mathrm{Ext}(H_m(X), \mathbb{Z})$$

$$\mathrm{Ext}(H_m(X), \mathbb{Z}) =: H_m(X)^\vee \cong \mathrm{Tor}(H_m(X))$$

$$H_{m+1}(X) \cong H^{\tilde{d}-(m+1)}(X, \partial X)$$

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \frac{\mathrm{Hom}(H_{m+1}(X),\mathbb{Z})}{J_{m+1}(H_{m+1}(X))} \oplus H_m(X)^\vee & \xrightarrow{\widetilde{D}^*_{m+1}} & H^{\tilde{d}-(m+1)}(\partial X) & \xrightarrow{I^*_m} & H^{\tilde{d}-m}(X,\partial X) & \longrightarrow & 0 \\ & & \downarrow\cong & & \downarrow\cong & & \downarrow\cong & & \\ 0 & \longrightarrow & \frac{H_{m+1}(X,\partial X)}{J_{m+1}(H_{m+1}(X))} \oplus H_m(X)^\vee & \xrightarrow{\widetilde{D}_{m+1}} & H_m(\partial X) & \xrightarrow{I_m} & H_m(X) & \longrightarrow & 0 \end{array}$$

$$\widetilde{D}^*_{m+1}\left(\frac{\mathrm{Hom}(H_{m+1}(X),\mathbb{Z})}{J_{m+1}(H_{m+1}(X))}\right) = \frac{\mathrm{Hom}(H_{m+1}(X),\mathbb{Z})}{J_{m+1}(H_{m+1}(X))} \subset H^{\tilde{d}-(m+1)}(\partial X)$$

$$\widetilde{D}^*_{m+1}(H_m(X)^\vee) \cong H_m(X)^\vee \subset H^{\tilde{d}-(m+1)}(X)$$

$$\check{\gamma}_{\tilde{d}-(m+1)} \in H^{\tilde{d}-(m+1)}(\partial X)$$

$$\check{\gamma}_{\tilde{d}-(m+1)} \in H^{\tilde{d}-(m+1)}(X)$$

$$D = \bigoplus_{n=1}^{d-1} D^{(n)}, D^{(n)} = \bigoplus_p \frac{H_{p-n+1}(X,\partial X)}{J_{p-n+1}\big(H_{p-n+1}(X)\big)} \cong \bigoplus_p \frac{H_{p-n+1}(X,\partial X)}{H_{p-n+1}(X)},$$

$$\ell: H_k(\partial X) \times H_{\tilde{d}-1-k}(\partial X) \to \mathbb{Q}/\mathbb{Z}$$

$$\langle \Sigma_{p-n+1}, \Sigma_{q-m+1} \rangle = \ell\left(D_{p-n+1}(\Sigma_{p-n+1}), D_{q-m+1}(\Sigma_{q-m+1})\right).$$

$$\sigma_{p-n} = D_{p-n+1}(\Sigma_{p-n+1})$$

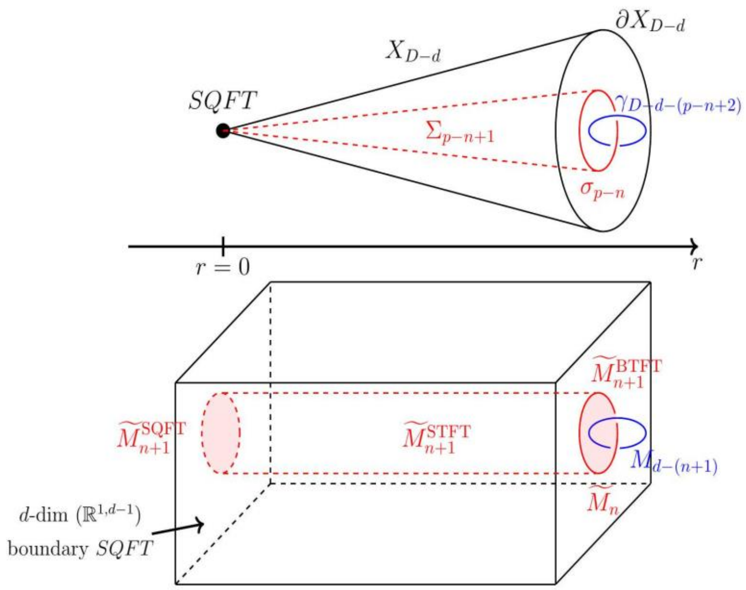
$$\ell(\sigma_{p-n}, \tilde{\sigma}_{q-m})$$

$$\sigma_{q-m} = D_{q-m+1}(\Sigma_{q-m+1})$$

$$0 \to H_2(X^{\mathrm{loc}}) \xrightarrow{J_2} H_2(X^{\mathrm{loc}}, \partial X^{\mathrm{loc}}) \xrightarrow{D_2} H_1(\partial X^{\mathrm{loc}}) \xrightarrow{I_1} H_1(X^{\mathrm{loc}}) \to 0.$$

$$0 \to \mathbb{Z}^{16} \xrightarrow{J_2} \mathbb{Z}_2^{16} \xrightarrow{D_2} \mathbb{Z}_2^{16} \xrightarrow{I_1} 0 \to 0$$

$$D = \left(\mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(4)}\right)^{16}$$



$$\partial \widetilde{M}^{\mathrm{BTFT}}_{n+1} = \widetilde{M}_n$$

$$\widetilde{M}_{n+1} = \widetilde{M}_{n+1}^{\text{SQFT}} \cup \widetilde{M}_{n+1}^{\text{STFT}} \cup \widetilde{M}_{n+1}^{\text{BTFT}}$$

$$\sigma_{p-n} \times \widetilde{M}_{n+1}$$

$$\sigma_{p-n} \in \ker(I_{p-n})$$

$$\gamma_{D-d-(p-n+2)} \times M_{d-(n+1)}$$

$$H_1(\partial X^{\text{loc}}) = H_2(X^{\text{loc}}, \partial X^{\text{loc}})/H_2(X^{\text{loc}}))$$

$$H_2(X^{\text{loc}}, \partial X^{\text{loc}}) = (\mathbf{A}_1^*)^{\oplus 16}$$

$$e^* \cdot e^* = \frac{1}{2}$$

$$H_2(X^{\text{loc}}, \partial X^{\text{loc}})$$

$$\{0, \sigma_1^i\} = \mathbb{Z}_2^i \subset \mathbb{Z}_2^{16} = H_1(\partial X^{\text{loc}})$$

$$\langle \sigma_1^i, \sigma_1^j \rangle = \frac{\delta ij}{2}$$

$$S_{\text{SymTFT}} = \int_{M_8} 2 \sum_{i=1}^{16} B_2^{(i)} dC_5^{(i)}$$

$$H_1(\partial X)^{(1)} \oplus H_1(\partial X)^{(4)}$$

$$\mathbb{R}^{1,d-1} \times X_{D-d}^{\circ}$$

$$X_{D-d}^{\circ} \equiv X^{\circ}$$

$$\frac{H_{p-n+1}(X^{\circ}, \partial X^{\circ})}{H_{p-n+1}(X^{\circ})} \neq H_{p-n}(\partial X^{\circ})$$

$$\widetilde{M}_{n+1} = \widetilde{M}_{n+1}^{\text{SQFT}} \cup \widetilde{M}_{n+1}^{\text{STFT}} \cup \widetilde{M}_{n+1}^{\text{BTFT}}$$

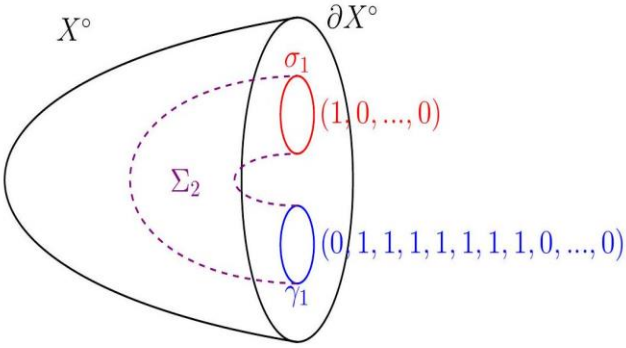
$$0 \to H_2(X^{\circ}) \xrightarrow{J_2} H_2(X^{\circ}, \partial X^{\circ}) \xrightarrow{D_2} H_1(\partial X^{\circ}) \xrightarrow{I_1} H_1(X^{\circ}) \to 0,$$

$$0 \to \mathbb{Z}^6 \xrightarrow{J_2} \mathbb{Z}^6 \oplus \mathbb{Z}_2^5 \xrightarrow{D_2} \mathbb{Z}_2^{16} \xrightarrow{I_1} \mathbb{Z}_2^5 \to 0$$

$$D = \left(\mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(4)}\right)^{11}$$

$$D_{\text{SymTFT}} = \left(\mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(4)}\right)^{16}$$

$$S_{\text{SymTFT}} = \int_{M_8} 2 \sum_{i=1}^{16} B_2^{(i)} dC_5^{(i)}$$



$$H_2(X) \times H_2(X) \to \mathbb{Z}, (\Sigma, \tilde{\Sigma}) \mapsto \Sigma \cdot \tilde{\Sigma},$$

$$\mathrm{Hom}(H_2(X), \mathbb{Z}) = H_2(X)^* := \left\{ \sum_i \lambda_i \Sigma_i, \lambda_i \in \mathbb{Q} \mid \forall \tilde{\Sigma} \in H_2(X): \sum_i \lambda_i \Sigma_i \cdot \tilde{\Sigma}_i \in \mathbb{Z} \right\},$$

$$\ell\left(D_2 \Sigma_2^{\mathrm{rel}}, D_2 \tilde{\Sigma}_2^{\mathrm{rel}}\right) = \Sigma_2^{\mathrm{rel}} \cdot \tilde{\Sigma}_2^{\mathrm{rel}} \mathrm{mod}\mathbb{Z} \ \text{ for } \ \Sigma^{\mathrm{rel}}, \tilde{\Sigma}^{\mathrm{rel}} \in \mathrm{Hom}(H_2(X), \mathbb{Z})$$

$$0 \to \underbrace{\frac{H_2(X^\circ)^*}{H_2(X^\circ)}}_{\cong H_2(X^\circ, \partial X^\circ)/H_2(X^\circ)} \oplus H_1(X^\circ)^\vee \overset{\tilde{D}_2}{\to} H_1(\partial X^\circ) \overset{\iota_1}{\to} H_1(X^\circ) \to 0$$

$$\tilde{D}_2(H_1(X^\circ)^\vee) \cong H_1(X^\circ)^\vee \subset H_1(\partial X^\circ)$$

$$H_1(\partial X^\circ)/\mathrm{im}(\tilde{D}_2) \cong H_1(X^\circ)$$

$$H_1(X^\circ)^\vee \times H_1(X^\circ) \to \mathbb{Q}/\mathbb{Z}$$

$$\ell(\tilde{\sigma}_1, \tilde{\sigma}_1') = 0 \ \text{ for } \ \tilde{\sigma}_1, \tilde{\sigma}_1' \in H_1(X^\circ)^\vee \subset H_1(\partial X^\circ)$$

$$\tilde{D}_2[H_2(X^\circ)^*/H_2(X^\circ)]$$

$$v_1 \in \tilde{D}_2[H_2(X^\circ)^*/H_2(X^\circ)]$$

$$\tilde{\sigma}_1 \in \tilde{D}_2(H_1(X^\circ)^\vee)$$

$$\tilde{D}_2[H_2(X^\circ)^*/H_2(X^\circ)]$$

$$H_1(X^\circ)^{(1)} \oplus H_1(X^\circ)^{(4)}$$

$$[H_2(X^\circ)^*/H_2(X^\circ)]^{(1)} \oplus [H_2(X^\circ)^*/H_2(X^\circ)]^{(4)}$$

$$H_1(\partial X^\circ)^{(1)} \times H_1(\partial X^\circ)^{(4)}$$

$$X^{\mathrm{loc}} = \bigcup_i \hat{X}_i, L_{\mathrm{g}} = H_2(X^{\mathrm{loc}})$$

$$D = D^{(1)} \oplus D^{(4)}.$$

$$\Lambda_{\text{maximally mixed}} = \Lambda_1 \oplus \Lambda_4, \Lambda_1 \cong \Lambda_4$$

$$\Lambda_{\text{maximally mixed}} = \Lambda_1 \oplus \Lambda_{d-3}, \Lambda_1 \cong \Lambda_{d-3}.$$

$$0 \to H_2(X^\circ) \oplus H_2(X^{\mathrm{loc}}) \overset{J_2}{\to} \underbrace{H_2(X)}_{= \mathrm{II}_{3,19}} \overset{\partial_2}{\to} H_1(\underbrace{\partial X^{\mathrm{loc}}}_{= \partial X^\circ}) \overset{\iota_1}{\to} H_1(X^\circ) \to 0,$$

$$H_1(X^{\mathrm{loc}} \cap X^\circ) = H_1(\partial X^{\mathrm{loc}}) = H_1(\partial X^\circ)$$

$$H_2(X^\circ) \text{ and } H_2(X^{\text{loc}}) = L_g$$

$$\text{im}(J_2) \cong H_2(X^\circ) \oplus H_2(X^{\text{loc}})$$

$$J_2\left(H_2(X^{\text{loc}})\right) \cong H_2(X^{\text{loc}})$$

$$H_2(X^{\text{loc}})^* = H_2(X^{\text{loc}}, \partial X^{\text{loc}})$$

$$\frac{M}{J_2(H_2(X^{\text{loc}}))} = \frac{M}{H_2(X^{\text{loc}})} \subset \frac{H_2(X^{\text{loc}})^*}{H_2(X^{\text{loc}})} = \mathcal{Z}(\tilde{G}),$$

$$\frac{M}{H_2(X^{\text{loc}})} \times H_1(X^\circ) \to \mathbb{Q}/\mathbb{R}$$

$$M/H_2(X^{\text{loc}}) \cong H_1(X^\circ)^\vee$$

$$0 \to \text{coker}(J_2) \overset{\tilde{\partial}_2}{\to} H_1(\partial X^\circ) \overset{\iota_1}{\to} H_1(X^\circ) \to 0$$

$$\text{Hom}(H_2(X^\circ), \mathbb{Z}) = H_2(X^\circ)^*$$

$$0 \to H_2(X^\circ) \overset{J_2^\circ}{\to} H_2(X^\circ, \partial X^\circ) \cong H_2(X^\circ)^* \oplus H_1(X^\circ)^\vee \overset{D_2^\circ}{\to} H_1(\partial X^\circ) \overset{I_1}{\to} H_1(X^\circ) \to 0$$
$$\Rightarrow\ 0 \to \frac{H_2(X^\circ)^*}{H_2(X^\circ)} \oplus H_1(X^\circ)^\vee \overset{\tilde{D}_2^\circ}{\to} H_1(\partial X^\circ) \overset{I_1}{\to} H_1(X^\circ) \to 0$$

$$\text{coker}(J_2) = \frac{H_2(X)}{H_2(X^\circ) \oplus H_2(X^{\text{loc}})} \cong \frac{H_2(X^\circ)^*}{H_2(X^\circ)} \oplus H_1(X^\circ)^\vee \cong \frac{H_2(X^\circ)^*}{H_2(X^\circ)} \oplus \frac{M}{H_2(X^{\text{loc}})}$$

$$\Sigma_2^\circ \in H_2(X^\circ, \partial X^\circ)$$

$$\Sigma_2^{\text{loc}} \in H_2(X^{\text{loc}}, \partial X^{\text{loc}})$$

$$\sigma_1 \in \ker(\iota_1) = \text{im}(\tilde{\partial}_2) \cong \text{coker}(J_2)$$

$$\Sigma_2 \in H_2(X)$$

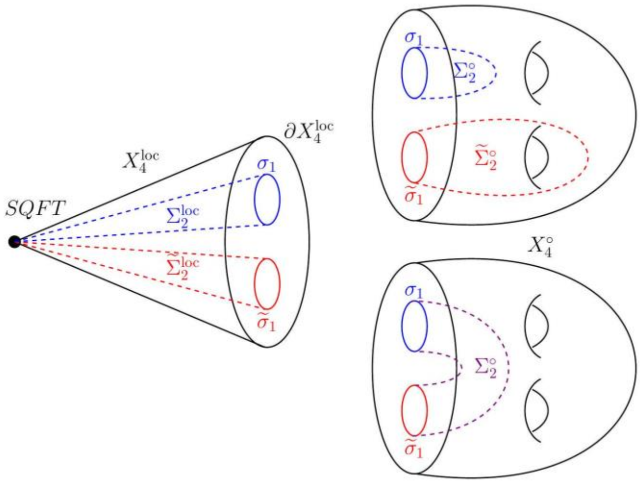
$$\text{im}(J_2) = \ker(\partial_2) = H_2(X^\circ) \oplus H_2(X^{\text{loc}})$$

$$\sigma_1 = D_2^{\text{loc}}(\Sigma_2^{\text{loc}}) \in H_1(X^\circ)^\vee \subset \text{coker}(J_2)$$

$$\Sigma_2^\circ \in \text{Ext}\big(\text{Tor}(H_1(X^\circ), \mathbb{Z})\big) \cong H_1(X^\circ)^\vee \subset H_2(X^\circ, \partial X^\circ) \text{ with } D_2^\circ(\Sigma_2^\circ) = \sigma_1$$

$$\Sigma_2 \in H_2(X) \text{ with } \partial_2 \Sigma_2 = \sigma_1$$

$$\Sigma_2 \simeq \Sigma_2^{\text{loc}} \cup_{\sigma_1} \Sigma_2^\circ$$



$$\Sigma_2 \simeq \Sigma_2^{\mathrm{loc}} \cup_{\sigma_1} \Sigma_2^{\circ}$$

$$\tilde{\Sigma}_2 \simeq \tilde{\Sigma}_2^{\mathrm{loc}} \cup_{\tilde{\sigma}_1} \tilde{\Sigma}_2^{\circ}$$

$$\Sigma_2 \in \frac{M}{H_2(X^{\mathrm{loc}})} \subset H_2(X)$$

$$\tilde{\Sigma}_2 \notin \frac{M}{H_2(X^{\mathrm{loc}})}$$

$$\sigma_1, \tilde{\sigma}_1 \notin \ker(\iota_1)$$

$$[\sigma_1] + [\tilde{\sigma}_1] = 0 \in H_1(X^{\circ})$$

$$\tilde{\sigma}_1 = D_2^{\mathrm{loc}}\left(\tilde{\Sigma}_2^{\mathrm{loc}}\right) \in \frac{H_2(X^{\circ})^*}{H_2(X^{\circ})} \subset \mathrm{coker}(\jmath_2)$$

$$\tilde{\Sigma}_2^{\circ} \in H_2(X^{\circ})^* = \mathrm{Hom}(H_2(X^{\circ}), \mathbb{Z}) \subset H_2(X^{\circ}, \partial X^{\circ}) \text{ with } D_2^{\circ}\big(\tilde{\Sigma}_2^{\circ}\big) = \tilde{\sigma}_1$$

$$\tilde{\Sigma}_2 \simeq \tilde{\Sigma}_2^{\mathrm{loc}} \cup_{\tilde{\sigma}_1} \tilde{\Sigma}_2^{\circ}$$

$$0 \neq \sigma_1 = D_2^{\mathrm{loc}}\big(\Sigma_2^{\mathrm{loc}}\big) \notin \mathrm{coker}(\jmath_2) = \ker(\iota_1) = \ker(I_1)$$

$$H_1(X^{\circ}) = \mathrm{Tor}\big(H_1(X^{\circ})\big)$$

$$0 \neq [\tilde{\sigma}_1 \mathrm{mod} \ker(I_1)] \in H_1(X^{\circ})$$

$$[\sigma_1] + [\tilde{\sigma}_1] = 0 \in H_1(X^{\circ})$$

$$D_2^{\circ}(\Sigma_2^{\circ}) = \sigma_1 + \tilde{\sigma}_1 \in \ker(I_1) = \mathrm{Im}(D_2^{\circ})$$

$$\left[\left(\tilde{G}/\mathcal{C}_{\mathrm{loc}}\right) \times U(1)^b\right]/\mathcal{C}_{\mathrm{extra}}$$

$$\mathcal{C}_{\mathrm{loc}} \cong H_1(X^{\circ})^{\vee} = \frac{M}{H_2(X^{\mathrm{loc}})}, \mathcal{C}_{\mathrm{extra}} \cong \frac{H_2(X^{\circ})^*}{H_2(X^{\circ})} = \frac{\mathrm{coker}(\jmath_2)}{H_1(X^{\circ})^{\vee}}$$

$$\mathcal{C}_{\mathrm{loc}} \oplus \mathcal{C}_{\mathrm{extra}}$$

$$\mathrm{rank}(H_2(X)) = \mathrm{rank}\big(H_2(X^{\circ})\big) + \mathrm{rank}\left(H_2\big(X^{\mathrm{loc}}\big)\right)$$

$$H_2(X) \otimes \mathbb{Q} = (H_2(X^{\circ}) \otimes \mathbb{Q}) \oplus \left(H_2\left(X^{\mathrm{loc}}\right) \otimes \mathbb{Q}\right)$$

$$\Sigma_2 \in H_2(X)$$

$$\Sigma_2 = S^{\mathrm{loc}} + S^{\circ}, S^{\mathrm{loc}} \in H_2\big(X^{\mathrm{loc}}\big) \otimes \mathbb{Q}, S^{\circ} \in H_2(X^{\circ}) \otimes \mathbb{Q},$$

$$\Sigma_2 \cdot \tilde{\Sigma}_2 = S^{\mathrm{loc}} \cdot \tilde{S}^{\mathrm{loc}} + S^{\circ} \cdot \tilde{S}^{\circ}$$

$$\Sigma_2 \simeq \Sigma_2^{\mathrm{loc}} \cup_{\sigma_1} \Sigma_2^{\circ}$$

$$\Sigma_2^{\phi} \in H_2(X^*)^* = \mathrm{Hom}(H_2(X^*), \mathbb{Z}) \subset H_2(X^*, \partial X^*)$$

$$X^* = X^{\mathrm{loc}}$$

$$\Sigma_2^{\mathrm{loc}}\, H_2\left(X^{\mathrm{loc}}, \partial X^{\mathrm{loc}}\right) = H_2\left(X^{\mathrm{loc}}\right)^*$$

$$\Sigma_2^{\mathrm{loc}} = S^{\mathrm{loc}} \in H_2\left(X^{\mathrm{loc}}, \partial X^{\mathrm{loc}}\right) \subset H_2\left(X^{\mathrm{loc}}\right) \otimes \mathbb{Q}$$

$$\Sigma_2^{\circ} \in \mathrm{Hom}(H_2(X^{\circ}), \mathbb{Z}) \subsetneq H_2(X^{\circ}, \partial X^{\circ})$$

$$D_2^{\circ}\Sigma_2^{\circ} \in \mathcal{C}_{\mathrm{extra}} \subset H_1(\partial X^{\circ}) = H_1\left(\partial X^{\mathrm{loc}}\right)$$

$$\Sigma_2 \simeq \Sigma_2^{\mathrm{loc}} \cup_{\sigma_1} \Sigma_2^{\circ}$$

$$\partial_2\Sigma_2 = \sigma_1 \in \mathcal{C}_{\mathrm{extra}} \subset \mathrm{coker}(\jmath_2)$$

$$\Sigma_2 = S^{\mathrm{loc}} + S^{\circ}$$

$$S^{\mathrm{loc}} = \Sigma_2^{\mathrm{loc}} \in H_2\left(X^{\mathrm{loc}}, \partial X^{\mathrm{loc}}\right) = H_2\left(X^{\mathrm{loc}}\right)^*, S^{\circ} = \Sigma_2^{\circ} \in \mathrm{Hom}(H_2(X^{\circ}), \mathbb{Z}) = H_2(X^{\circ})^* \subset H_2(X^{\circ}, \partial X^{\circ})$$

$$\Sigma_2 \simeq \Sigma_2^{\mathrm{loc}} \cup_{\sigma_1} \Sigma_2^{\circ} \text{ with } \partial_2\Sigma_2 \in \mathcal{C}_{\mathrm{loc}}$$

$$S^{\mathrm{loc}} = \Sigma_2^{\mathrm{loc}} \in H_2\left(X^{\mathrm{loc}}\right)^*$$

$$S^{\circ} = 0 \in H_2(X^{\circ})^*$$

$$\widetilde{D}_2^{\circ}[H_2(X^{\circ})^*/H_2(X^{\circ})] \cong \mathcal{C}_{\mathrm{extra}}$$

$$\widetilde{D}_2^{\circ}(H_1(X^{\circ})^{\vee}) \cong \mathcal{C}_{\mathrm{loc}}$$

$$\widetilde{D}_2^{\circ}(H_1(X^{\circ})^{\vee}) \subset H_1(\partial X^{\circ})$$

$$H_1(\partial X^{\circ})/\ker(I_1) \cong H_1(X^{\circ})$$

$$\partial X^{\circ} = \partial X^{\mathrm{loc}}$$

$$H_1\left(\partial X^{\mathrm{loc}}\right) = H_2\left(X^{\mathrm{loc}}\right)^*/H_2\left(X^{\mathrm{loc}}\right)$$

$$\tilde{\partial}_2\left[M/\jmath_2\left(H_2\left(X^{\mathrm{loc}}\right)\right)\right] \subset H_1\left(\partial X^{\mathrm{loc}}\right) \text{ with } \widetilde{D}_2^{\circ}[H_1(X^{\circ})^{\vee}] \subset H_1(\partial X^{\circ})$$

$$\Sigma_2 \in M \subset H_2(X) \text{ with } \partial_2\Sigma_2 = \sigma_1 \in \widetilde{D}_2^{\circ}[H_1(\partial X^{\circ})^{\vee}] = \mathcal{C}_{\mathrm{loc}}$$

$$\jmath_2\left(H_2\left(X^{\mathrm{loc}}\right)\right) \cong H_2\left(X^{\mathrm{loc}}\right)$$

$$H_2\left(X^{\mathrm{loc}}\right)^* = H_2\left(X^{\mathrm{loc}}, \partial X^{\mathrm{loc}}\right)$$

$$\sigma_1 \in \widetilde{D}_2^{\circ}[H_1(X^{\circ})^{\vee}] = \mathcal{C}_{\mathrm{loc}}$$

$$\tilde{\sigma}_1 \text{ as } \ell(\sigma_1, \tilde{\sigma}_1) = \Sigma_2^{\mathrm{loc}} \cdot \tilde{\Sigma}_2^{\mathrm{loc}} \bmod \mathbb{Z}$$

$$\Sigma_2 = \Sigma_2^{\mathrm{loc}} \in M \subset H_2\left(X^{\mathrm{loc}}\right)^* \text{ and } \tilde{\Sigma}_2^{\mathrm{loc}} \in H_2\left(X^{\mathrm{loc}}\right)^* \text{ with } D_2^{\mathrm{loc}}\left(\Sigma_2^{\mathrm{loc}}\right) = \sigma_1 \text{ and } D_2^{\mathrm{loc}}\left(\tilde{\Sigma}_2^{\mathrm{loc}}\right) = \tilde{\sigma}_1$$

$$\ell(\sigma_1, \tilde{\sigma}_1) = \Sigma_2^{\mathrm{loc}} \cdot \tilde{\Sigma}_2^{\mathrm{loc}} = 0 \bmod \mathbb{Z} \text{ if } \tilde{\sigma}_1 \in \mathcal{C}_{\mathrm{loc}} \Leftrightarrow \tilde{\Sigma}_2^{\mathrm{loc}} \in M$$

$$\tilde{\sigma}_1 = \partial_2\tilde{\Sigma}_2 \in \mathcal{C}_{\mathrm{extra}}$$

$$\tilde{\Sigma}_2 = \tilde{\Sigma}_2^{\mathrm{loc}} + \tilde{\Sigma}_2^{\circ}$$

$$\ell(\sigma_1, \tilde{\sigma}_1) = \Sigma_2^{\mathrm{loc}} \cdot \tilde{\Sigma}_2^{\mathrm{loc}} \bmod \mathbb{Z} = \Sigma_2^{\mathrm{loc}} \cdot \tilde{\Sigma}_2 \bmod \mathbb{Z} = \Sigma_2 \cdot \tilde{\Sigma}_2 \bmod \mathbb{Z} = 0 \bmod \mathbb{Z},$$

$$X^{\mathrm{loc}} \coprod X^{\circ}$$

$$\Sigma_2 = \Sigma_2^{\mathrm{loc}} + \Sigma_2^{\circ}(\Sigma_2^{\mathrm{loc}}, \Sigma_2^{\circ})$$

$$\mathrm{coker}(\jmath_2) \cong \mathrm{im}(\partial_2)$$

$$(\sigma_1^{\mathrm{loc}}, \sigma_1^{\circ}) = (D_2^{\mathrm{loc}}\Sigma_2^{\mathrm{loc}}, D_2^{\circ}\Sigma_2^{\circ}) = (\partial_2\Sigma_2, \partial_2\Sigma_2) = (\sigma_1, \sigma_1) \in H_1(\partial X^{\mathrm{loc}}) \times H_1(\partial X^{\circ}).$$

$$\left\langle (\Sigma_2^{\mathrm{loc}}, \Sigma_2^{\circ})_{M2}, (\tilde{\Sigma}_2^{\mathrm{loc}}, \tilde{\Sigma}_2^{\circ})_{M5} \right\rangle = \ell(\sigma_1^{\mathrm{loc}}, \tilde{\sigma}_1^{\mathrm{loc}}) + \ell(\sigma_1^{\circ}, \tilde{\sigma}_1^{\circ})$$
$$= \Sigma_2^{\mathrm{loc}} \cdot \tilde{\Sigma}_2^{\mathrm{loc}} + \Sigma_2^{\circ} \cdot \tilde{\Sigma}_2^{\circ} \mathrm{mod}\mathbb{Z} = \Sigma_2 \cdot \tilde{\Sigma}_2 \ \mathrm{mod}\mathbb{Z} = 0$$

$$(0, \sigma_1) \in H_1(\partial X^{\mathrm{loc}}) \oplus H_1(\partial X^{\circ})$$

$$\sigma_1 \in H_1(X^{\circ})^{\vee}$$

$$\mathrm{coker}(\jmath_2) \oplus \{(0, \sigma_1) \mid \sigma_1 \in H_1(X^{\circ})^{\vee}\} \subset H_1(\partial X^{\mathrm{loc}}) \oplus H_1(\partial X^{\circ})$$

$$|\mathrm{coker}(\jmath_2)| \times |H_1(X^{\circ})^{\vee}| = |H_1(\partial X^{\circ})| = |H_1(\partial X^{\mathrm{loc}})| = \sqrt{|H_1(\partial X^{\mathrm{loc}}) \oplus H_1(\partial X^{\circ})|}$$

$$\Lambda_{\text{maximally mixed}} = [\mathrm{coker}(\jmath_2) \oplus H_1(X^{\circ})^{\vee}]^{(1)} \oplus [\mathrm{coker}(\jmath_2) \oplus H_1(X^{\circ})^{\vee}]^{(4)}$$
$$\subset D = [H_1(\partial X^{\mathrm{loc}}) \oplus H_1(\partial X^{\circ})]^{(1)} \oplus [H_1(\partial X^{\mathrm{loc}}) \oplus H_1(\partial X^{\circ})]^{(4)}$$

$$[H_1(\partial X^{\mathrm{loc}}) \oplus H_1(\partial X^{\circ})]/\Lambda$$

$$[(\tilde{G}/\mathcal{C}_{\mathrm{loc}}) \times U(1)^b]/\mathcal{C}_{\mathrm{extra}}$$

$$H_2(X^{\circ})^* \subset H_2(X^{\circ}) \otimes \mathbb{Q}$$

$$\sigma_1 \in \mathcal{C}_{\mathrm{loc}} = \{\partial_2\Sigma_2 \mid \Sigma_2 \in M\}$$

$$X = T^4/\mathbb{Z}_2$$

$$X^{\mathrm{loc}} \cong \textstyle\coprod_{i=1}^{16} \mathbb{C}^2/\mathbb{Z}_2^{(i)} \text{ and } X^{\circ} = X \setminus X^{\mathrm{loc}}$$

$$X^{\mathrm{loc}} \coprod X^{\circ} \text{ is } \mathfrak{su}(2)^{16} \oplus \mathfrak{u}(1)^6$$

$$[H_1(\partial X^{\mathrm{loc}}) \oplus H_1(\partial X^{\circ})]^{(1)} \oplus [H_1(\partial X^{\mathrm{loc}}) \oplus H_1(\partial X^{\circ})]^{(4)} = \bigoplus_{i=1}^{32} [\mathbb{Z}_{2,i}^{(1)} \oplus \mathbb{Z}_{2,i}^{(4)}]$$

$$(\mathbb{Z}_2^{(1)} \oplus \mathbb{Z}_2^{(4)})^{\oplus 5} \cong (H_1(X^{\circ})^{\vee})^{(1)} \oplus (H_1(X^{\circ})^{\vee})^{(4)}$$

$$(\mathbb{Z}_2^{(1)} \oplus \mathbb{Z}_2^{(4)})^{\oplus 5} \cong H_1(X^{\circ})^{(1)} \oplus H_1(X^{\circ})^{(4)}$$

$$M \subset H_2(X^{\mathrm{loc}})^*$$

$$M/H_2(X^{\mathrm{loc}}) \cong \mathbb{Z}_2^{\oplus 5}$$

$$M/H_2(X^{\mathrm{loc}}) \cong \mathbb{Z}_2^{\oplus 5}$$

$$\mathbb{Z}_2^{\oplus 5} \subset H_1(\partial X^{\mathrm{loc}})$$

$$M/H_2(X^{\mathrm{loc}})$$

$$\mathcal{C}_{\mathrm{loc}}^{(1)} \oplus \mathcal{C}_{\mathrm{loc}}^{(4)} \cong (\mathbb{Z}_2^{(1)} \oplus \mathbb{Z}_2^{(4)})^{\oplus 5}$$

$$[\mathbb{Z}_2^{(1)} \oplus \mathbb{Z}_2^{(4)}] H_1(\partial X^{\mathrm{loc}})^{(1)} \oplus H_1(\partial X^{\mathrm{loc}})^{(4)}$$

$$H_1(\partial X^\circ)^{(1)} \oplus H_1(\partial X^\circ)^{(4)} H_2(X) \ni F_\alpha \simeq \left(\frac{1}{2}T_\alpha\right) \cup_{\sigma_\alpha} \left(\sum_{I_\alpha} \frac{1}{2} e_{I_\alpha}\right)$$

$$H_2(X^\circ) \tfrac{1}{2}T_\alpha \in H_2(X^\circ)^* \subset H_2(X^\circ, \partial X^\circ) \ \textstyle\sum_{I_\alpha} \tfrac{1}{2} e_{I_\alpha} \in H_2\left(X^{\mathrm{loc}}\right)^* \sigma_\alpha = \partial_2 F_\alpha = D_2^\circ\left(\tfrac{1}{2}T_\alpha\right) = D_2^{\mathrm{loc}}\left(\textstyle\sum_{I_\alpha} \tfrac{1}{2} e_{I_\alpha}\right) \in H_1(\partial X^\circ) = H_1\left(X^{\mathrm{loc}}\right)$$

$$(\sigma_\alpha, \sigma_\alpha) \in H_1\left(\partial X^{\mathrm{loc}}\right) \oplus H_1(\partial X^\circ)$$

$$\mathcal{C}_{\mathrm{extra}}^{(1)} \oplus \mathcal{C}_{\mathrm{extra}}^{(4)} \cong \left(\mathbb{Z}_2{}^{(1)} \oplus \mathbb{Z}_2{}^{(4)}\right)^{\oplus 6}$$

$$\left[SU(2)^{16}/\mathbb{Z}_2^5 \times U(1)^6\right]/\mathbb{Z}_2^6 \cong \left[SU(2)^{16} \times U(1)^6\right]/\left[\left(\mathbb{Z}_2^{\mathrm{loc}}\right)^5 \times (\mathbb{Z}_2^{\mathrm{extra}})^6\right]$$

$$\left(\mathbb{Z}_2{}^{(1)} \oplus \mathbb{Z}_2{}^{(4)}\right)^{\oplus 6} \mathbb{Z}_2{}^{\oplus 11} \mathbb{Z}_2{}^{\oplus 22}/\mathbb{Z}_2{}^{\oplus 11} = \mathbb{Z}_2{}^{\oplus 11}$$

$$\mathrm{Ext}(H_1(X^\circ), \mathbb{Z}) = H_1(X^\circ)^\vee \subset H_2(X^\circ, \partial X^\circ) \text{ and } H_1(X^\circ)$$

$$\widetilde{D}_2^\circ[H_1(X^\circ)^\vee] \subset H_1(\partial X^\circ)$$

$$H_1(\partial X^\circ)/\mathrm{im}\left(\widetilde{D}_2^\circ\right) = H_1(X^\circ)$$

$$\left(\frac{H_2(X^\circ)^*}{H_2(X^\circ)}\right)^{(1)} \oplus \left(\frac{H_2(X^\circ)^*}{H_2(X^\circ)}\right)^{(4)} \subset H_1(\partial X^\circ)^{(1)} \oplus H_1(\partial X^\circ)^{(4)}$$

$$\mathrm{Hom}(H_2(X^\circ), \mathbb{Z}) = H_2(X^\circ)^*$$

$$H_2(X^\circ)^*/H_2(X^\circ) \equiv Z$$

$$\left[\frac{H_2\left(X^{\mathrm{loc}}\right)^*}{H_2\left(X^{\mathrm{loc}}\right)} \oplus \frac{H_2(X^\circ)^*}{H_2(X^\circ)}\right]^{(1)} \oplus \left[\frac{H_2\left(X^{\mathrm{loc}}\right)^*}{H_2\left(X^{\mathrm{loc}}\right)} \oplus \frac{H_2(X^\circ)^*}{H_2(X^\circ)}\right]^{(4)} = [\mathcal{Z}(\tilde{G}) \oplus Z]^{(1)} \oplus [\mathcal{Z}(\tilde{G}) \oplus Z]^{(4)}$$

$$\mathcal{Z}(\tilde{G}) \cong H_1\left(\partial X^{\mathrm{loc}}\right)$$

$$H_2\left(X^{\mathrm{loc}}\right) \subset H_2\left(X^{\mathrm{loc}}\right)^* \text{ and } H_2(X^\circ) \subset H_2(X^\circ)^*$$

$$(a_e, b_e) \in [\mathcal{Z}(\tilde{G}) \oplus Z]^{(1)}$$

$$(a_m, b_m) \in [\mathcal{Z}(\tilde{G}) \oplus Z]^{(4)}$$

$$\langle (a_e, b_e), (a_m, b_m) \rangle = \vec{a}_e \cdot \vec{a}_m + \vec{b}_e \cdot \vec{b}_m \bmod \mathbb{Z}$$

$$\mathcal{C}_{\mathrm{loc}} \oplus \mathcal{C}_{\mathrm{extra}} \subset \mathcal{Z}(\tilde{G}) \oplus Z$$

$$\Lambda = [\mathcal{C}_{\mathrm{loc}} \oplus \mathcal{C}_{\mathrm{extra}}]^{(1)} \oplus [\mathcal{C}_{\mathrm{loc}} \oplus \mathcal{C}_{\mathrm{extra}}]^{(4)}$$

$$\Sigma_2 \cdot \tilde{\Sigma}_2 \bmod \mathbb{Z} = \Sigma_2 = \Sigma_2^{\mathrm{loc}} \cup_{\sigma_1} \Sigma_2^\circ$$

$$H_2\left(X^{\mathrm{loc}}\right)^* \ni \Sigma_2^{\mathrm{loc}} \equiv \vec{a}_e H_2(X^\circ)^* = H_2(X^\circ, \partial X^\circ)/H_1(X^\circ)^\vee \ni \Sigma_2^\circ \equiv \vec{b}_e(\vec{a}_m, \vec{b}_m)$$

$$|\mathcal{C}_{\mathrm{loc}} \oplus \mathcal{C}_{\mathrm{extra}}|^2 = \left|\frac{M}{H_2\left(X^{\mathrm{loc}}\right)}\right|^2 \times \left|\frac{H_2(X^\circ)^*}{H_2(X^\circ)}\right|^2 = |H_1(X^\circ)^\vee|^2 \times \left|\frac{H_2(X^\circ)^*}{H_2(X^\circ)}\right|^2$$

$$= |H_1(X^\circ)^\vee| \times |H_1(X^\circ)| \times \left|\frac{H_2(X^\circ)^*}{H_2(X^\circ)}\right|^2 = \left|H_1\left(\partial X^{\mathrm{loc}}\right)\right| \times \left|\frac{H_2(X^\circ)^*}{H_2(X^\circ)}\right| = |\mathcal{Z}(\tilde{G}) \oplus Z|,$$

$$(X^\circ) \oplus H_2\left(X^{\mathrm{loc}}\right) \to H_2(X)$$

$$M \subset H_2\left(X^{\mathrm{loc}}, \partial X^{\mathrm{loc}}\right)$$

$$\mathfrak{g} \oplus \mathfrak{u}(1)^b$$

$$H_1(X^\circ)^\vee \subset H_2(X^\circ, \partial X^\circ)$$

$$\mathfrak{g} \oplus \mathfrak{u}(1)^b$$

$$H_1(\partial X^\circ)/\mathrm{im}(\widetilde{D}_2^\circ)$$

$$H_1(\partial Y)^{(1)} \oplus H_1(\partial Y)^{(4)}$$

$$\mathrm{CS}(\partial Y, \sigma_1) \int_{M_8} dC_3 \cup B_2 \cup B_2$$

$$\mathrm{CS}[\partial Y, \sigma_1] = \frac{1}{2} \int_{\partial Y} \breve{\sigma} \star \breve{\sigma}$$

$$\mathrm{CS}(\partial Y, \sigma_1) = \frac{1}{2} \Sigma_2 \cdot \Sigma_2 \bmod \mathbb{Z},$$

$$(\mathcal{L}^*/\mathcal{L})^{(1)} \times (\mathcal{L}^*/\mathcal{L})^{(4)}$$

$$\mathcal{L} = H_2\left(X^{\mathrm{loc}}\right) \oplus H_2(X^\circ)$$

$$\sigma_1 = [v \mathrm{mod} \mathcal{L}] \in \mathcal{L}^*/\mathcal{L}$$

$$q([v]) := \frac{1}{2} v \cdot v \bmod \mathbb{Z},$$

$$\ell(\sigma_1, \tilde{\sigma}_1) = q([v] + [\tilde{v}]) - q([v]) - q([\tilde{v}]) \bmod \mathbb{Z}.$$

$$\mathcal{L} = H_2\left(X^{\mathrm{loc}}\right) \oplus H_2(X^\circ) \mathcal{L}^* = H_2\left(X^{\mathrm{loc}}\right)^* \oplus H_2(X^\circ)^*$$

$$q([v]) = q\left(\left[v^{\mathrm{loc}} + v^\circ\right]\right) = q^{\mathrm{loc}}\left(\left[v^{\mathrm{loc}}\right]\right) + q^\circ([v^\circ])$$

$$\mathcal{C}_{\mathrm{loc}} \oplus \mathcal{C}_{\mathrm{extra}} = \mathrm{coker}(\jmath_2)$$

$$\Sigma_2 = \Sigma_2^{\mathrm{loc}} \cup_{\sigma_1} \Sigma_2^\circ$$

$$\begin{aligned} q(\sigma_1) &\equiv q^{\mathrm{loc}}\left(\left[\Sigma_2^{\mathrm{loc}}\right]\right) + q^\circ([\Sigma_2^\circ]) \\ &= \frac{1}{2} \Sigma_2^{\mathrm{loc}} \cdot \Sigma_2^{\mathrm{loc}} + \frac{1}{2} \Sigma_2^\circ \cdot \Sigma_2^\circ \bmod \mathbb{Z} = \frac{1}{2} \Sigma_2 \cdot \Sigma_2 \bmod \mathbb{Z} = 0 \bmod \mathbb{Z} \end{aligned}$$

$$[\mathcal{C}_{\mathrm{loc}} \oplus \mathcal{C}_{\mathrm{extra}}]^{(1)} \mathcal{C} \mathcal{Z}(\tilde{G}) \oplus Z \mathcal{C}^{(1)} \oplus \mathcal{C}^{(4)}$$

$$|\mathcal{Z}(\tilde{G}) \oplus Z| = |\mathrm{coker}(\jmath_2)|^2 \left[\tilde{G} \times U(1)^b\right] / \mathcal{C} \mathcal{Z}(\tilde{G}) \times Z \supset \mathcal{C}$$

$$\mathfrak{g} \oplus \mathfrak{u}(1)^b$$

$$L_{\mathfrak{g}}^*/L_{\mathfrak{g}} \equiv \mathcal{Z}(\tilde{G}) \mathcal{Z}(\tilde{G}) \oplus Z$$

$$q = q_{\mathfrak{g}} \oplus q_Z |\mathcal{Z}(\tilde{G}) \oplus Z|$$

$$q_L([v_I^M]) + q_T([v_I^T]) = 0 \bmod \mathbb{Z}.$$

$$q_M([v \quad \mathrm{mod} M]) = \frac{1}{2} v \cdot v \bmod \mathbb{Z} = q_L([v \quad \mathrm{mod} L]) \text{ for } v \in M^* \subset L^*.$$

$$\mathrm{II}_{\tilde{d}, 16+\tilde{d}} \equiv \mathrm{II} T \oplus L w_a \equiv 0 + w_a \in T^* \oplus L^* v_I = v_I^T + v_I^M \in T^* \oplus L^*$$

$$\mathrm{II} = T \oplus L + \{w_a, v_I\}_{\mathbb{Z}} \subset T^* \oplus L^*$$

$$[(\tilde{G}/\mathcal{C}_{\text{loc}}) \times U(1)^{\tilde{d}}]/\mathcal{C}_{\text{extra}}$$

$$\mathcal{C}_{\text{loc}} \cong \frac{M}{L}, \mathcal{C}_{\text{extra}} \cong \frac{\pi_L(\text{II})}{L} \cong \frac{T^*}{T}$$

$$\pi_L: \text{II} \to L^* T \oplus L \subset \text{II}$$

$$v_I^M \in M^* \subset L^* \bigsqcup^{15}$$

$$\mathfrak{g} \oplus \mathfrak{u}(1)^{\tilde{d}} [Z \oplus \mathcal{Z}(\tilde{G})]^{(1)} \oplus [Z \oplus \mathcal{Z}(\tilde{G})]^{(d-3)} \mathcal{Z}(\tilde{G}) = L^*/LZ = T^*/T$$

$$\langle ([s_T],[s_L])^{(1)}, ([t_T],[t_L])^{(4)} \rangle = s_T \cdot t_T + s_L \cdot t_L \bmod \mathbb{Z}$$

$$\text{II} \ni s \mapsto (\pi_T(s), \pi_L(s)) \in T^* \oplus L^*.$$

$$([\pi_T(s)][\pi_L(s)]) \in Z \oplus \mathcal{Z}(\tilde{G})$$

$$[Z \oplus \mathcal{Z}(\tilde{G})]^{(1)} \oplus [Z \oplus \mathcal{Z}(\tilde{G})]^{(d-3)}$$

$$\{([\pi_T(s)],[\pi_L(s)]) \mid s \in \text{II}\}$$

$$T \oplus L \overset{J_2}{\hookrightarrow} \text{II}$$

$$\{([\pi_T(s)],[\pi_L(s)]) \in Z \oplus \mathcal{Z}(\tilde{G}) \mid s \in \text{II}\} = \text{coker}(J_2) = \frac{\text{II}}{T \oplus L} \subset \frac{T^*}{T} \oplus \frac{L^*}{L}$$

$$[w_a] = (0,[w_a]) \in T^*/T \text{ and } [v_I] = ([v_I^T],[v_I^M]) \in L^*/L$$

$$q_{T \oplus L} = q_T \oplus q_L T^*/T \oplus L^*/L$$

$$\text{coker}(J_2)^{(1)} \oplus \text{coker}(J_2)^{(d-3)}$$

$$|\mathcal{M}/\mathcal{L}|^2 = d(\mathcal{L})/d(\mathcal{M})\ d(\mathcal{L}) = |\mathcal{L}^*/\mathcal{L}|$$

$$|\text{coker}(J_2)|^2 = \left|\frac{\text{II}}{T \oplus L}\right|^2 = \frac{d(T \oplus L)}{d(\text{II})} = d(T \oplus L) = \left|\frac{T^*}{T}\right| \times \left|\frac{L^*}{L}\right| = |Z \oplus \mathcal{Z}(\tilde{G})|$$

$$d(\text{II}) = |\text{II}^*/\text{II}| = 1$$

$$\Lambda_{\text{maximally mixed}} = [\text{coker}(J_2)]^{(1)} \oplus [\text{coker}(J_2)]^{(d-3)}$$

$$\mathfrak{g} = \mathfrak{e}_8 \oplus \mathfrak{e}_8$$

$$[\mathbb{Z}_2 \oplus \mathbb{Z}_2]^{(1)} \oplus [\mathbb{Z}_2 \oplus \mathbb{Z}_2]^{(4)}$$

$$q: \mathbb{Z}_2 \oplus \mathbb{Z}_2 \to \mathbb{Q}/\mathbb{Z}$$

$$q((1,0)) = q((0,1)) = 0, q((1,1)) = \frac{1}{2}$$

$$\text{II}_{\mathfrak{e}} \oplus U = \text{II}_{1,17} = \text{II}_{\mathfrak{so}} \oplus U,$$

$$T^*/T = \mathbb{Z}_{2m} \overset{!}{\cong} M^*/M$$

$$d(M) = |M^*/M| = \frac{d(L)}{|M/L|^2} = \frac{18}{|\mathbb{Z}_3|^2} = 2$$

$$\mathcal{Z}(SU(18)) \oplus Z \cong \mathbb{Z}_9 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

$$v = (v^T, v^M) \in T^* \oplus L^*$$

$$[(SU(18)/\mathbb{Z}_3)\times U(1)]/\mathbb{Z}_2$$

$$\mathfrak{g}=\mathfrak{su}(7)\oplus\mathfrak{so}(22)$$

$$\mathbb{Z}_7\times\mathbb{Z}_4\cong\mathbb{Z}_{28}$$

$$q_L(k)=k^2\left(\frac{3}{7}+\frac{11}{4}\right)\bmod\mathbb{Z}=\frac{5}{28}k^2\bmod\mathbb{Z}.$$

$$\mathfrak{g}=\mathfrak{su}(7)\oplus\mathfrak{so}(22)$$

$$\mathbb{Z}_{10}\oplus\mathbb{Z}_{10}=L^*/L$$

$$L=\left(L_{\mathfrak{su}(10)}\right)^{\oplus 2}$$

$$q_L\big((k_1,k_2)\big)=\frac{1}{2}(k_1w_1,k_2w_1)\cdot(k_1w_1,k_2w_1)\bmod\mathbb{Z}=\frac{9}{20}(k_1^2+k_2^2)\bmod\mathbb{Z},$$

$$(w,\widetilde{w})\in\left(L^*_{\mathfrak{su}(10)}\right)^{\oplus 2}=L^*$$

$$\mathbb{Z}_5^{\text{iso}}=\{(0,0),(2,4),(4,8),(8,6),(6,2)\}$$

$$M=L\text{ and }M=L+\{(w_2,w_4)\}_{\mathbb{Z}}$$

$$(x_1,x_2)\cdot(y_1,y_2):=(x_1,x_2)\begin{pmatrix}-10 & 0\\ 0 & -10\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$$

$$v_1=\left(v_1^T,(w_1,0)\right)$$

$$v_2=\left(v_2^T,(0,w_1)\right)\in T^*\oplus\left[\left(L_{\mathfrak{su}(10)}\right)^*\oplus\left(L_{\mathfrak{su}(10)}\right)^*\right]$$

$$M=L+\{(w_2,w_4)\}_{\mathbb{Z}}$$

$$M^*/M=\mathbb{Z}_2\oplus\mathbb{Z}_2$$

$$v_1^M=(w_5,0)\in M^*\subset\left(L_{\mathfrak{su}(10)}\right)^*\oplus\left(L_{\mathfrak{su}(10)}\right)^*$$

$$v_2^M=(0,w_5)\in M^*$$

$$q_L(v_I^M)=\tfrac{1}{2}w_5^2\bmod\mathbb{Z}=\tfrac{1}{4}\text{ for }I=1,2$$

$$(x_1,x_2)\cdot(y_1,y_2):=(x_1,x_2)\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}.$$

$$v_1=\left(\left(\frac{1}{2},0\right),(w_5,0)\right)$$

$$v_2=\left(\left(0,\frac{1}{2}\right),(0,w_5)\right)\left[SU(10)^2/\mathbb{Z}_5^{\text{iso}}\times U(1)^2\right]/\mathbb{Z}_2^2$$

$$\frac{1}{2}C_{d-4}\wedge\left(\sum_g\operatorname{tr}(\mathcal{F}_g^2)+\sum_{i,j}T_{ij}F_i\wedge F_j\right)=C_{d-4}\wedge\left(\sum_g c_2(\mathcal{F}_g)-\sum_{i,j}\frac{T_{ij}}{2}c_1(F_i)\wedge c_1(F_j)\right).$$

$$c_2(\mathcal{F}_g)=\frac{1}{2}\operatorname{tr}(\mathcal{F}_g^2)$$

$$\mathfrak{g}=\bigoplus_g\mathfrak{g}_g$$

$$[w_a] \in \mathcal{C}_{\text{loc}} \subset L^*/L = \mathcal{Z}(\tilde{G})$$

$$\sum_g c_2(\mathcal{F}_g) \equiv -q_L([w_a])B \cup B \bmod \mathbb{Z}$$

$$[v_I] = ([v_I^T], [v_I^M]) \in T^*/T \oplus L^*/L = Z \oplus \mathcal{Z}(\tilde{G})$$

$$\sum_g c_2(\mathcal{F}_g) - \sum_{i,j} \frac{T_{ij}}{2} c_1(F_i) \wedge c_1(F_j) \equiv \left(-q_L([v_I^M]) - q_T([v_I^T])\right)B \cup B \bmod \mathbb{Z}$$

$$T^*/T = \mathcal{C}_{\text{extra}} \subset \mathcal{Z}(U(1)^{10-d})$$

$$\mathcal{C}_{\text{extra}} \subset \mathcal{Z}(\tilde{G})$$

$$\mathcal{C}_{\text{extra}} \subset \mathcal{Z}(U(1)^{10-d})$$

$$\mathcal{C} \subset \left[\mathcal{Z}(\tilde{G}) \times \mathcal{Z}(U(1)^{10-d})\right]^{(1)}$$

$$X^{\text{loc}} = \bigcup_i X_i$$

$$X = X^{\text{loc}} \cup X^{\circ}$$

$$\partial X^{\text{loc}} = \partial X^{\circ}$$

$$\partial X^{\text{loc}} \coprod \partial X^{\circ}$$

$$X = X^{\text{loc}} \cup X^{\circ}$$

$$0 \to H_{p-n+1}(X^{\circ}) \oplus H_{p-n+1}(X^{\text{loc}}) \overset{J_{p-n+1}}{\to} H_{p-n+1}(X) \overset{\partial_{p-n+1}}{\to} H_{p-n}(\underbrace{\partial X^{\text{loc}}}_{=\partial X^{\circ}}) \overset{\iota_{p-n}}{\to} H_{p-n}(X^{\circ}) \to 0$$

$$\Sigma_{p-n+1}^{\text{loc}} \in H_{p-n+1}(X^{\text{loc}}, \partial X^{\text{loc}})$$

$$\gamma_{D-d-(p-n+2)} \in H_{D-d-(p-n+2)}(\partial X^{\text{loc}})$$

$$D^{\text{loc}} \Sigma_{p-n+1}^{\text{loc}} = \sigma_{p-n} \in H_{p-n}(\partial X^{\text{loc}}) \partial X^{\text{loc}}$$

$$H_{p-n+1}(X) = \mathbb{Z}_N$$

$$H_{D-d-(p-n+2)}(\partial X^*)$$

$$\partial X^{\text{loc}} \coprod \partial X^{\circ}$$

$$\partial X^{\text{loc}} \coprod \partial X^{\circ}$$

$$D - d = 2(p - n + 1)$$

$$H_{p-n}(\partial X^{\text{loc}}) = H_{p-n}(\partial X^{\circ})$$

$$D - d = 2(p - n + 1)$$

$$D = 2p + 4$$

$$\mathbb{R}^{1,5} \times X_4^{\text{loc}}$$

$$X_4^{\text{loc}} = \bigcup_i \mathbb{C}^2/\Gamma_i$$

$$S_{\text{SymTFT}}^{(0-4)} = \frac{1}{4\pi}\sum_{i,j} K_{ij}\int_{M_7}\left(B_5^i dA_1^j + \tilde{B}_5^i d\tilde{A}_1^j\right)$$

$$\mathbb{C}^2/\Gamma_i\sqrt{25}(\tilde{B}_5^i)\; A_1^i(\tilde{A}_1^i)$$

$$S_{\text{SymTFT}}^{(2)} = \frac{1}{4\pi}\sum_{i,j} K_{ij}\int_{M_7} C_3^i dC_3^j$$

$$U_\lambda(M_3) = e^{i\int_{M_3}\lambda_i C_3^i}$$

$$\langle U_\lambda(M_3)U_{\lambda'}(M_3')\rangle = e^{-2\pi i\lambda'^T(K^{-1})\lambda\cdot \text{Link}_{M_7}(M_3,M_3')},$$

$$\langle\cdot,\cdot\rangle: D^{(2)}\times D^{(2)} \to U(1).$$

$$q: \lambda \to \frac{1}{2}\lambda^T K^{-1}\lambda,$$

$$\langle\lambda,\lambda'\rangle = \exp\left[-2\pi i\big(q(\lambda+\lambda') - q(\lambda) - q(\lambda')\big)\right].$$

ADE Algebra	q
A_{n-1}	$\frac{n-1}{2n}$
D_{2n}	$q(v) = \frac{1}{2},\ q(s) = q(c) = \frac{2n}{8}$
D_{2n+1}	$\frac{2n+1}{8}$
E_6	$\frac{2}{3}$
E_7	$\frac{3}{4}$

$$q(a) = \frac{N-1}{2N}a^2 \in \frac{1}{2}\mathbb{Z}.$$

$$S_{\text{SymTFT}} = \frac{N}{4\pi}\int C_3 dC_3$$

$$S_{\text{SymTFT}} + S_\delta = \frac{N}{4\pi}\int C_3 dC_3 + \frac{n}{4\pi}\int_{\partial M_7}(C_3 - B_3)Y_3$$

$$\frac{n^2}{4\pi}\int B_3 dB_3$$

$$S_{\text{top}} \supset 2\pi\int_{M_{10}}\left(-\frac{1}{2}F_5\wedge B_2\wedge F_3\right)$$

$$\frac{S_{\text{top}}}{2\pi} \supset \int_{M_{10}}\breve{I}_{11}\text{mod}1 = \int_{M_{10}}\left(-\frac{1}{2}\breve{F}_5 * \breve{H}_3 * \breve{F}_3\right)\text{mod}1$$

$$H^0(S^3/\Gamma,\mathbb{Z}) = H^3(S^3/\Gamma,\mathbb{Z}) = \mathbb{Z}, H^1(S^3/\Gamma,\mathbb{Z}) = 0, H^2(S^3/\Gamma,\mathbb{Z}) = \Gamma_{ab}$$

$$\breve{F}_5 = \breve{f}_5 * \breve{1} + \sum_i \breve{c}_3^{(i)} * \breve{t}_{2(i)} + \breve{f}_2 * \breve{v}_3,$$

$$\breve{F}_3 = \breve{h}_3 * \breve{1} + \sum_i \breve{a}_1^{(i)} * \breve{t}_{2(i)} + \breve{f}_0 * \breve{v}_3,$$

$$\breve{H}_3 = \breve{h}_3 * \breve{1} + \sum_i \breve{a}_1^{(i)} * \breve{t}_{2(i)} + \breve{h}_0 * \breve{v}_3,$$

$$\check{1} \leftrightarrow H^0(S^3/\Gamma,\mathbb{Z}) = \mathbb{Z}, \check{t}_{2(i)} \leftrightarrow H^2(S^3/\Gamma,\mathbb{Z}) = \Gamma_{ab}, \check{v} \leftrightarrow H^3(S^3/\Gamma,\mathbb{Z}) = \mathbb{Z}.$$

$$\frac{S_{\text{SymTFT}}}{2\pi} \supset -\sum_{i,j} \text{CS}[S^3/\Gamma]_{ij} \int_{M_7} \check{c}_3^{(i)} * \left(\check{h}_3 * \check{a}_1^{(j)} + \check{h}_3 * \check{\tilde{a}}_1^{(j)}\right)$$

$$\text{CS}[S^3/\Gamma]_{ij} = \frac{1}{2}\int_{S^3/\Gamma} \check{t}_{2(i)}\check{t}_{2(j)} \text{ mod} 1$$

$$\check{c}_3^{(i)}, \check{a}_1^{(j)} \left(\check{\tilde{a}}_1^{(j)}\right)$$

ADE Algebra	Γ_{ab}	$-\text{CS}[S^3/\Gamma]_{ij}$
A_{n-1}	$\mathbb{Z}_n$	$\frac{n-1}{2n}$
D_{2n}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\frac{1}{4}\begin{pmatrix} n & n-1 \\ n-1 & n \end{pmatrix}$
D_{2n+1}	$\mathbb{Z}_4$	$\frac{2n+1}{8}$
E_6	$\mathbb{Z}_3$	$\frac{2}{3}$
E_7	$\mathbb{Z}_2$	$\frac{3}{4}$
E_8	0	0

$$\forall x, y \in \mathcal{L} \subset \mathbb{R}^n : x \cdot y = \langle x, y\rangle$$

$$G_{ab} = e_a \cdot e_b = G_{ba}$$

$$\mathcal{L} \hookrightarrow \mathbb{R}^n$$

$$\mathbb{R}^n \supset \mathcal{L}^* := \{v \in \mathbb{R}^n \mid \forall x \in \mathcal{L} \subset \mathbb{R}^n : \langle v, x\rangle \in \mathbb{Z}\}$$

$$\ell: \frac{\mathcal{L}^*}{\mathcal{L}} \times \frac{\mathcal{L}^*}{\mathcal{L}} \to \mathbb{Q}/\mathbb{Z}, \ell([v\text{mod}\mathcal{L}], [w\text{mod}\mathcal{L}]) := v \cdot w \text{ mod}\mathbb{Z}.$$

$$\ell(z, z') = q(z + z') - q(z) - q(z').$$

$$q_{\mathcal{L}}: \frac{\mathcal{L}^*}{\mathcal{L}} \to \mathbb{Q}/\mathbb{Z}, q_{\mathcal{L}}([v\text{mod}\mathcal{L}]) := \frac{1}{2}v^2 \text{ mod}Z.$$

$$\mathfrak{g} = \bigoplus_g \mathfrak{g}_g$$

$$L_{\mathfrak{g}} = \bigoplus_g L_{\mathfrak{g}_g}$$

$$c_2(\mathcal{F}) \equiv -\alpha B \cup B$$

$$L_{\mathfrak{g}}^*/L_{\mathfrak{g}} = \mathcal{Z}(\tilde{G})$$

$$\mathcal{C} \subset \mathcal{Z}(\tilde{G})$$

$$[w] \in L^*/L$$

$$[w] = ([w_g]) \in L^*/L = \bigoplus_g \left(L_{\mathfrak{g}_g}^*/L_{\mathfrak{g}_g}\right)$$

$$\sum_g c_2(\mathcal{F}_g) \equiv -\sum_g q_g([w_g])B \cup B \text{ mod}\mathbb{Z} = -q_L([w])B \cup B \text{ mod}\mathbb{Z}$$

$$\frac{T_{ij}}{2}c_1(F_i)\wedge c_1(F_j)$$

$$V=\sum_i c_1(F_i)t_i$$

$$\mathbb{Z}_N=T^*/T$$

$$c_1(F)\equiv\frac{1}{N}B\text{mod}\mathbb{Z}$$

$$c_1(F_i)\equiv\lambda_i B\text{mod}\mathbb{Z}$$

$$v=\sum_i \lambda_i t_i\in T^*$$

$$\begin{aligned}\frac{T_{ij}}{2}c_1(F_i)\wedge c_1(F_j)&=\frac{T_{ij}}{2}(p_iA+\lambda_iB)\cup(p_jA+\lambda_jB)\\&=\frac{T_{ij}}{2}p_ip_jA\cup A+T_{ij}p_i\lambda_jA\cup B+\frac{T_{ij}}{2}\lambda_i\lambda_jB\cup B\\&\equiv\left(\frac{1}{2}v\cdot v\right)B\cup B+\text{ integer 4-cocycles }=q_T([v])B\cup B\text{ mod}\mathbb{Z},\end{aligned}$$

$$\mathfrak{g}\oplus\mathfrak{u}(1)^b$$

$$\partial X\cong\coprod_i S^3/\Gamma_i$$

$$\Gamma_i\subset SU(2)$$

$$H_1(\partial X^\circ)=\prod_i \text{Ab}[\Gamma_i]$$

$$0\to\mathcal{C}_{\text{loc}}\to\mathcal{C}_{\text{full}}:=\text{coker}(\jmath_2)\to\mathcal{C}_{\text{extra}}\to 0$$

$$c_2(\mathcal{F}_g)\to\tfrac{1}{2}\textstyle\sum_{a,b\leq\text{rk}(\mathfrak{g}_g)}K_{ab}^{(g)}\mathcal{F}_{g,a}\wedge\mathcal{F}_{g,b},\text{ with }K_{ab}^{(g)}$$

$$0\subset D_1\subset\cdots\subset D_k=TM$$

$$[\Gamma(D_i),\Gamma(D_j)]\subset\Gamma(D_{i+j}).$$

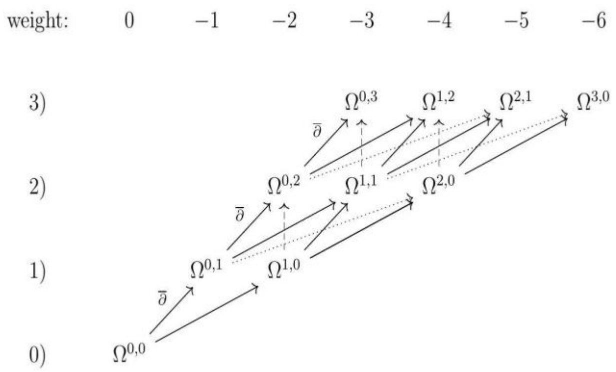
$$[X,fY]=X(f)Y+(-)^{|f|\cdot|X|}f[X,Y].$$

$$F_nT_{\mathbb{C}}^*=(T_{\mathbb{C}}/D_{-1-n})^\vee.$$

$$\text{Gr}_pT_{\mathbb{C}}^*=\left(\text{Gr}_{-p}T_{\mathbb{C}}\right)^\vee$$

$$D_0=0\subset D_1=T^{(0,1)}\subset D_2=T_{\mathbb{C}}.$$

$$F_{-3}T^*=0\subset F_{-2}T^*=\left(T_{\mathbb{C}}/T^{(0,1)}\right)^\vee\subset F_{-1}T^*M=T_{\mathbb{C}}^*$$



$$\phi\colon \mathrm{Gr}TM \to TM$$

$$\phi\left(\bigoplus_{1\leq j\leq k} \mathrm{Gr}_j TM\right) = D_k.$$

$$\mathrm{d} = \mathrm{d}_1 + \mathrm{d}_0 + \mathrm{d}_{-1} + \cdots,$$

$$\mathrm{d} = \mathrm{d}_1 + \mathrm{d}_0 + \mathrm{d}_{-1} + \cdots$$

$$W^\bullet := H^\bullet\left(\mathrm{Gr}\, F_\bullet(\Omega^\bullet)\right) = H^\bullet\left(\Omega^\bullet, \mathrm{d}_1\right).$$

$$h \circlearrowright \left(\Omega^\bullet\,,\ \mathrm{d}_1\right) \underset{i}{\overset{p}{\rightleftarrows}} \left(W^\bullet, 0\right).$$

$$F_k^+(W^j) := F_{k-j}(W^j),$$

$$\left(\Omega^\bullet(X)\,,\ \mathrm{d} = \partial + \bar{\partial}\right).$$

$$0 \subset D_1 = T^{(0,1)}X \subset D_2 = T_{\mathbb{C}}X$$

$$T_1X = T^{(0,1)}X, T_2X = T^{(1,0)}X.$$

$$\mathrm{d}_1 = 0,\ \mathrm{d}_0 = \bar{\partial}, \mathrm{d}_{-1} = \partial,$$

$$\mathrm{d} = \bar{\mu} + \bar{\partial} + \partial + \mu$$

$$\mathrm{d}_1 = \bar{\mu}, \mathrm{d}_0 = \bar{\partial}, \mathrm{d}_{-1} = \partial, \mathrm{d}_{-2} = \mu.$$

$$W^\bullet = H^\bullet(\Omega^\bullet(X), \bar{\mu}).$$

$$(\Omega^\bullet(N), \mathrm{d_{dR}}) = \left(C^\infty(N_+)[\theta, \mathrm{d}\theta, \mathrm{d}x]\,,\ \mathrm{d}x\frac{\partial}{\partial x} + \mathrm{d}\theta\frac{\partial}{\partial \theta}\right)$$

$$\lambda = \mathrm{d}\theta, v = \mathrm{d}x + \lambda\theta$$

$$\mathrm{d_{dR}} = \lambda^2\frac{\partial}{\partial v} + \lambda\left(\frac{\partial}{\partial \theta} - \theta\frac{\partial}{\partial x}\right) + v\frac{\partial}{\partial x}.$$

$$\mathrm{d}_1 = \lambda^2 \frac{\partial}{\partial v}$$
$$\mathrm{d}_0 = \lambda \frac{\partial}{\partial \theta} - \lambda\theta \frac{\partial}{\partial x}$$
$$\mathrm{d}_{-1} = v \frac{\partial}{\partial x}$$

$$W^\bullet = \Omega^\bullet(X), \quad \mathrm{d}_0 = \overline{\partial}, \quad \mathrm{d}_{-1} = \partial.$$

$$\pi : (\Omega^{2,\bullet}(X)\,,\,\bar{\partial}) \longrightarrow (\Omega^{0,\bullet}(X)\,,\,\bar{\partial})\,, \qquad \alpha \mapsto \pi \vee \alpha.$$

$$\{\alpha, \beta\} = (-1)^{|\alpha|}(\Delta(\alpha\beta) - \Delta(\alpha)\beta) - \alpha\Delta(\beta)$$

$$[-,-]_\partial := \{\partial(-), -\}.$$

$$(\Omega^{0,\bullet}(X)\,,\,\bar{\partial}\,,\,[-,-]_\partial)$$

$$[\alpha, \beta]_\partial = \{\partial\alpha, \beta\} = \pi(\partial\alpha \wedge \partial\beta)$$

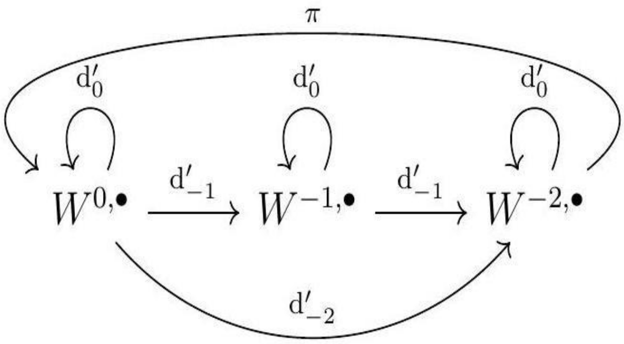
$$S_{BV}(\alpha) = \int_{X\times M} \Omega \wedge \alpha \left(\frac{1}{2}(\bar{\partial}_X + \mathrm{d}_M)\alpha + \frac{1}{6}[\alpha, \alpha]_\partial\right).$$

$$\mathrm{d}' = \mathrm{d}'_0 + \mathrm{d}'_{-1} + \mathrm{d}'_{-2}$$

$$\mathrm{d}'_0 = i \circ \mathrm{d}_0 \circ p$$
$$\mathrm{d}'_{-1} = i \circ (\mathrm{d}_0 h\, \mathrm{d}_0 + \mathrm{d}_{-1}) \circ p$$
$$\mathrm{d}'_{-2} = i \circ ((\mathrm{d}_0 h)^2\, \mathrm{d}_0 + \mathrm{d}_0 h\, \mathrm{d}_{-1} + \mathrm{d}_{-1} h\, \mathrm{d}_0) \circ p$$

$$(\mathrm{d}'_0)^2 = 0$$
$$[\mathrm{d}'_{-1}, \mathrm{d}'_0] = 0$$
$$(\mathrm{d}'_{-1})^2 + [\mathrm{d}'_0, \mathrm{d}'_{-2}] = 0$$
$$[\mathrm{d}'_{-1}, \mathrm{d}'_{-2}] = 0$$
$$(\mathrm{d}'_{-2})^2 = 0$$

$$\pi : (W^{-2,\bullet}\,,\,\mathrm{d}'_0) \longrightarrow (W^{0,\bullet}\,,\,\mathrm{d}'_0)$$



$$\Delta = \frac{1}{t}\sum_{k=1}^{\infty} t^k \Delta_k$$

$$\Delta^2 = 0 \text{ and } \Delta(1) = 0.$$

$$\{a_1, \ldots, a_n\}_t = [\ldots[\Delta, a_1], \ldots, a_n](1).$$

$$\{a_1, a_2\} = \Delta(a_1 a_2) - \Delta(a_1)a_2 - (-1)^{|a_1|} a_1 \Delta(a_2).$$

$$\{a_1, \dots, a_n\} = \lim_{t\to 0} \frac{1}{t^{n-1}} \{a_1, \dots, a_n\}_t$$

$$\{a_1, \dots, a_n\} = [\dots[\Delta_n, a_1], \dots, a_n](1).$$

$$\Delta = \Delta_1 + t\Delta_2 + t^2\Delta_3 = \mathrm{d}'_0 + t[\pi, \mathrm{d}'_{-1}] + t^2[\pi, [\pi, \mathrm{d}'_{-2}]]$$

$$[\pi, \mathrm{d}'_{-1}]^2 + [\mathrm{d}'_0, \pi \mathrm{d}'_{-2}\pi].$$

$$\pi(\mathrm{d}'_{-1})^2\pi = -\pi[\mathrm{d}'_0, \mathrm{d}'_{-2}]\pi = -[\mathrm{d}'_0, \pi \mathrm{d}'_{-2}\pi]$$

$$\mathrm{d}^t = \mathrm{d}'_0 + t\, \mathrm{d}'_{-1} + t^2\, \mathrm{d}'_{-2}$$

$$\mu_n^t(a_1, \dots, a_n) = \{\dots\{\mathrm{d}^t, a_1\}, \dots a_n\}$$

$$\mu_n = \lim_{t\to 0} \frac{1}{t^{n-1}} \mu_n^t$$

$$\mu_1(\alpha) = \mathrm{d}'_0\alpha$$
$$\mu_2(\alpha, \beta) = \{\mathrm{d}'_{-1}\alpha, \beta\}$$
$$\mu_3(\alpha, \beta, \gamma) = \{\{\mathrm{d}'_{-2}\alpha, \beta\}, \gamma\}.$$

$$\mu_2(\alpha, \beta) = \pi(\mathrm{d}'_{-1}\alpha \cdot \mathrm{d}'_{-1}\beta)$$
$$\mu_3(\alpha, \beta, \gamma) = \pi\big(\mathrm{d}'_{-2}\alpha \cdot \pi(\mathrm{d}'_{-1}\beta \cdot \mathrm{d}'_{-1}\gamma)\big)$$

$$\{\mathrm{d}'_{-1}\alpha, \beta\} = (-1)^{|\alpha|}(\pi \mathrm{d}'_{-1}(\mathrm{d}'_{-1}\alpha \cdot \beta) - (\pi(\mathrm{d}'_{-1})^2\alpha) \cdot \beta)$$

$$\begin{aligned}\{\mathrm{d}'_{-2}\alpha, \beta\} &= (-1)^{|\alpha|}\left(\mathrm{d}'_{-1}\pi(\mathrm{d}'_{-2}\alpha \cdot \beta) - (\mathrm{d}'_{-1}\pi \mathrm{d}'_{-2}\alpha) \cdot \beta\right) \\ &= (\pi \mathrm{d}'_{-2}\alpha) \cdot \mathrm{d}'_{-1}\beta \in W^{-1,\bullet},\end{aligned}$$

$$\begin{aligned}\{\{\mathrm{d}'_{-2}\alpha, \beta\}, \gamma\} &= \{(\pi \mathrm{d}'_{-2}\alpha)\mathrm{d}'_{-1}\beta, \gamma\} \\ &= (-1)^{|\alpha|+|\beta|}\big[\pi \mathrm{d}'_{-1}\big((\pi \mathrm{d}'_{-2}\alpha)(\mathrm{d}'_{-1}\beta)\gamma\big) - \pi \mathrm{d}'_{-1}\big((\pi \mathrm{d}'_{-2}\alpha)(\mathrm{d}'_{-1}\beta)\big) \cdot \gamma\big] \\ &= \pi\big((\pi \mathrm{d}'_{-2}\alpha)\mathrm{d}'_{-1}\beta \cdot \mathrm{d}'_{-1}\gamma\big)\end{aligned}$$

$$\mathfrak{n} = \Pi\mathfrak{n}_1 \oplus \mathfrak{n}_2,$$

$$0 \to \mathfrak{n} \to \mathfrak{p} \to \mathfrak{p}_0 \to 0,$$

$$Y = \mathrm{Spec}(R/I),$$

$$I = \left(\lambda^\alpha f^\mu_{\alpha\beta}\lambda^\beta\right), \mu = 1 \dots d, \alpha, \beta = 1 \dots n.$$

$$\mathfrak{n} = \Pi S(-1) \oplus V(-2),$$

$$[-,-]: \mathrm{Sym}^2(S) \longrightarrow V,$$

$$A^\bullet_{R/I} : \mathsf{Mod}^{\mathfrak{p}_0}_{R/I} \longrightarrow \mathsf{Mult}_{\mathfrak{p}}$$

$$A^\bullet := (C^\infty(N) \otimes_{\mathbb{C}} R/I\,,\ \lambda^a D_a)\,,$$

$$D = \frac{\partial}{\partial\theta} - \theta\frac{\partial}{\partial x},$$

$$A^\bullet_{R/I}(\Gamma) := A^\bullet \otimes_{R/I} \Gamma,$$

$$\begin{array}{ccc} \mathsf{Mod}^{\mathfrak{p}_0}_{R/I} & \xrightarrow{A^\bullet_{R/I}} & \mathsf{Mult}_{\mathfrak{p}} \\ \downarrow & \overset{\hat{A}^\bullet}{\nearrow} \; \underset{C^\bullet}{\swarrow} & \\ \mathsf{Mod}^{\mathfrak{p}_0}_{C^\bullet(\mathfrak{n})} & & \end{array}$$

$$C^\bullet(\mathfrak{n}) \cong \Omega^\bullet(N)^N \subset \Omega^\bullet(N).$$

$$\hat{A}^\bullet(C^\bullet(\mathfrak{n})) \cong (\Omega^\bullet(N), \mathrm{d}_{\mathrm{dR}}).$$

$$\mathrm{Gr}\, F^+_\bullet W^\bullet = A^\bullet_{R/I}(H^\bullet(\mathfrak{n}))$$

$$(W^{0,\bullet}, \mathrm{d}'_0) = A^\bullet_{R/I}(R/I),$$

$$\mathfrak{n}_Q = H^{>0}(\mathfrak{p}, [Q, -])$$

$$\tilde{\mathfrak{n}}_Q = (\mathfrak{p}^1/\mathrm{Im}([Q,-])(-1) \oplus \mathfrak{p}^2(-2), [Q,-]).$$

$$A^\bullet(\mathcal{O}_Y)^Q \cong A^\bullet(\mathcal{O}_{Y_Q}),$$

$$A^\bullet(\mathcal{O}_Y)^Q = \Omega^{0,\bullet}(\mathbb{C}^n) \otimes \Omega^\bullet(\mathbb{R}^{d-2n}).$$

$$\mathrm{def}(\mathfrak{n}) = \dim(Y) - \big(\dim(\mathfrak{n}_1) - \dim(\mathfrak{n}_2)\big) = \dim(\mathfrak{n}_2) - \mathrm{codim}(Y)$$

$$\mathrm{depth}(I, R) = \dim(V) - n.$$

$$\mathrm{def}(Q) = \dim(V) - \mathrm{codim}(P_0 \cdot Q)$$

$$H^2(\mathfrak{n}, [Q,-]) \cong V/\mathrm{Im}([Q,-]).$$

$$\mathfrak{n}_1/\ker([Q,-]) \longrightarrow \mathrm{Im}([Q,-]) \subseteq V.$$

$$i^* T_Q \mathfrak{n}_1 \cong T_Q(P_0 \cdot Q) \oplus N_Q(P_0 \cdot Q).$$

$$\mathrm{codim}(P_0 \cdot Q) = \dim\left(N_Q(P_0 \cdot Q)\right) = \dim(\mathfrak{n}_1/\ker([Q,-])).$$

$$\begin{aligned} \mathrm{def}(Q) &= \dim(V) - \dim(\mathfrak{n}_1/\ker([Q,-])) \\ &= \dim(V/\mathrm{Im}([Q,-])) = \dim(H^2(\mathfrak{n}, [Q,-])), \end{aligned}$$

$$H^0(\mathfrak{n}) = R/I$$

$$H^{-2}(\mathfrak{n}) \cong \mathrm{Ext}_R^{\mathrm{codim}(Y)}(R/I, R) \cong R/I,$$

$$\pi : \left(A^\bullet(H^{-2}(\mathfrak{n}))\ ,\ \mathrm{d}_0'\right) \longrightarrow \left(A^\bullet(H^0(\mathfrak{n}))\ ,\ \mathrm{d}_0'\right).$$

$$\mathfrak{p} = \mathfrak{so}(V) \oplus S(-1) \oplus V(-2).$$

$$\mathrm{Spin}(11) \supset \mathrm{Spin}(7) \times \mathrm{Spin}(4) \supset G_2 \times \mathrm{SU}(2)_+ \times \mathrm{SU}(2)_- \supset G_2 \times \mathrm{SU}(2)_+ \times \mathrm{U}(1),$$

$$V = V_7 \oplus L \oplus L^\vee,$$

$$S = \left(\mathbf{1}_{G_2} \oplus V_7\right) \otimes (\mathbf{2}^0 \oplus \mathbf{1}^1 \oplus \mathbf{1}^{-1}).$$

$$Q \in \mathbf{1}_{G_2} \otimes \mathbf{1}^{-1}.$$

$$\wedge^2 V_7 \cong V_7 \oplus \mathfrak{g}_2,$$

$$V_7 \longrightarrow V_7 \otimes \mathbf{1}^{-1}$$

$$\mathfrak{g}_2 \qquad V_7 \otimes \mathbf{1}^1 \longrightarrow V_7$$

$$V_7 \otimes \mathbf{2}^1 \longrightarrow V_7 \otimes \mathbf{2}^0 \qquad \mathbf{2}^1$$

$$V_7 \otimes \mathbf{2}^{-1} \qquad \mathbf{2}^0 \longrightarrow \mathbf{2}^{-1}$$

$$\mathbf{1}^0 \longrightarrow \mathbf{1}^{-1}$$

$$\mathbf{1}^2 \longrightarrow \mathbf{1}^1$$

$$\mathbf{1}^{-2}$$

$$\mathbf{3}^0$$

$$\mathfrak{p}_Q = H^\bullet(\mathfrak{p}, [Q, -]) = \left(\mathfrak{g}_2 \oplus \mathfrak{sl}(L) \oplus V_7 \otimes \mathbf{2}^{-1} \oplus \mathbf{1}^{-2}\right) \oplus L(-2).$$

$$V_7 \otimes \mathbf{1}^1 \longrightarrow V_7$$
$$\mathbf{2}^0 \longrightarrow \mathbf{2}^{-1}$$
$$\mathbf{2}^1$$

$$\Omega^\bullet = \mathbb{C}[z_1, z_2][\mathrm{d}z_1, \mathrm{d}z_2],$$

$$A^\bullet(\mathcal{O}_{Y_Q}) \simeq \left(\Omega^{0,\bullet}(\mathbb{C}^2) \otimes \Omega^\bullet(\mathbb{R}^7)\ ,\ \bar{\partial}_{\mathbb{C}^2} + \mathrm{d}_{\mathbb{R}^7}\right)$$

$$V = L \oplus L^\vee \oplus \mathbb{C}.$$

$$\mathfrak{n}_Q \cong \Pi \wedge^2 L(-1) \oplus \wedge^4 L(-2),$$

$$\mathrm{def}(\mathfrak{n}_Q) = 7 - (10 - 5) = 2.$$

$$C^\bullet(\mathfrak{n}_Q) \cong \wedge^\bullet L^\vee \otimes R,$$

$$\mathrm{Sym}^\bullet(\mathfrak{n}_1^\vee) = R = \mathbb{C}[\lambda^{ab}].$$

$$(\mathfrak{n}_Q)_1^\vee = (\wedge^2 L)^\vee$$

$$\mathrm{d}_{CE} = \lambda^{ab}\lambda^{cd}\varepsilon_{abcde}\frac{\partial}{\partial(\mathrm{d}z_e)}$$

$$H^k(\mathfrak{n}_Q) \cong \begin{cases} R/I & k \in \{0,-2\} \\ M & k = -1 \\ 0 & \text{else} \end{cases}$$

$$\phi: R \otimes \wedge^2 L \longrightarrow R \otimes L\; e_a \wedge e_b \mapsto \varepsilon^{abcde}\lambda_{cd}e_e$$

$$\left(\Omega^\bullet_{\mathrm{dR}}(\mathbb{C}^5) \otimes \mathbb{C}[\lambda^{ab}, \theta^{ab}]\,,\; \partial_{\mathbb{C}^5} + \bar{\partial}_{\mathbb{C}^5} + \mathscr{R} + \mathrm{d}_{CE}\right) \otimes \left(\Omega^\bullet(\mathbb{R})\,,\; \mathrm{d}_{\mathbb{R}}\right),$$

$$\mathcal{R} = \lambda\left(\frac{\partial}{\partial\theta} - \theta\frac{\partial}{\partial x}\right)$$

$$\mathrm{d}_1 = \mathrm{d}_{CE},\; \mathrm{d}_0 = \mathcal{R} + \bar{\partial} + \mathrm{d}_{\mathbb{R}}, \mathrm{d}_{-1} = \partial$$

$$\mathrm{d}'_{-2} : A^\bullet(H^0(\mathfrak{n}_Q)) \longrightarrow A^\bullet(H^{-2}(\mathfrak{n}_Q)),$$

$$\pi : \left(W^{-2,\bullet}, \mathrm{d}'_0\right) \longrightarrow \left(W^{0,\bullet}, \mathrm{d}'_0\right).$$

$$\pi(\lambda_{ab}\,\mathrm{d}z^a\,\mathrm{d}z^b) = 1$$

$$C^\bullet(\mathfrak{n}) = (\wedge^\bullet V^\vee \otimes R\,,\; \mathrm{d}_{CE}),$$

$$R = \mathrm{Sym}^\bullet(S^\vee) = \mathbb{C}[\lambda^\alpha]$$

$$\mathrm{d}_{CE} = \lambda^\alpha\Gamma^\mu_{\alpha\beta}\lambda^\beta\frac{\partial}{\partial v^\mu}.$$

$$\left(\lambda^\alpha\Gamma^{\mu\nu}_{\alpha\beta}\lambda^\beta\right)v_\mu v_\nu$$

$$\mathrm{d}'_{-1} : W^{0,\bullet} \longrightarrow W^{-1,\bullet},$$

$$\mathrm{d}'_{-2} : W^{0,\bullet} \longrightarrow W^{-2,\bullet}$$

$$\pi\left(\lambda^\alpha\Gamma^{\mu\nu}_{\alpha\beta}\lambda^\beta v_\mu v_\nu\right) = 1$$

$$A^{(p)} = \frac{1}{p!}A_{\mu_1\ldots\mu_p}dx^{\mu_1} \wedge dx^{\mu_p}$$

$$\mu^A: \mathcal{M}_4 \rightarrow G$$

$$[J_{ab}, J_{cd}] = -2\eta_{a[c}J_{d]b} + 2\eta_{b[c}J_{d]a}, [J_{ab}, P_c] = -2P_{[a}\eta_{b]c}, [P_a, P_b] = 0$$

$$\omega^{ab}(J_{cd}) = 2\delta^{ab}_{cd}, V^a(P_b) = \delta^a_b$$

$$\begin{aligned} d\omega^{ab} - \omega^a_c \wedge \omega^{cb} &= 0 \\ dV^a - \omega^a{}_b \wedge V^b &= 0 \end{aligned}$$

$$\begin{aligned} R^{ab} &\equiv d\omega^{ab} - \omega^a_c \wedge \omega^{cb} \\ \mathring{T}^a &\equiv dV^a - \omega^a{}_b \wedge V^b = \mathcal{D}V^a \end{aligned}$$

$$\mathcal{A}[\omega^{ab},V^a] = \frac{1}{4\kappa^2}\int_{\mathcal{M}_4} R^{ab}\wedge V^c \wedge V^d \epsilon_{abcd}$$

$$R^{ab} = R^{ab}{}_{cd}V^c \wedge V^d = R^{ab}{}_{\mu\nu}dx^\mu \wedge dx^\nu,$$

$$\begin{aligned} R^{ab}\wedge V^c \wedge V^d \epsilon_{abcd} &= R^{ab}{}_{ij}V^i V^j V^c V^d \epsilon_{abcd} \\ &= R^{ab}{}_{ij}V^i{}_\mu V^j{}_\nu V^c V^d{}_\sigma d^4x \epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd} \\ &= -4R^{ij}{}_{ij}\det V d^4x. \end{aligned}$$

$$R^{ij}{}_{ij} \equiv R^{\mu\nu}{}_{\mu\nu} = \mathcal{R},$$

$$\det(V) = \sqrt{-g}\left(g = \det(g_{\mu\nu})\right)$$

$$\int_{\mathcal{M}_4} R^{ab}\wedge V^c \wedge V^d \epsilon_{abcd} = -4\int_{\mathcal{M}_4}\mathcal{R}\sqrt{-g}dx^4$$

$$\frac{\delta\mathcal{A}}{\delta\omega^{ab}} = 0 \quad : \mathring{T}^c \wedge V^d \epsilon_{abcd} = 0$$
$$\frac{\delta\mathcal{A}}{\delta V^d} = 0 \quad : R^{ab}\wedge V^c \epsilon_{abcd} = 0$$

$$\mathring{T}^c = \mathring{T}^c{}_{\ell m}V^\ell \wedge V^m$$

$$\mathring{T}^c{}_{\ell m}V^\ell \wedge V^m \wedge V^d \epsilon_{abcd} = 0$$

$$V^\ell \wedge V^m \wedge V^d = \epsilon^{\ell m d p}\Omega_p^{(3)}$$
$$\mathring{T}^c{}_{\ell m}\epsilon^{\ell m d p}\epsilon_{abcd}\Omega_p^{(3)} = 0,$$

$$\left(\mathring{T}^p{}_{ab} + 2\mathring{T}^c{}_{c[a}\delta^p_{b]}\right)\Omega_p^{(3)} = 0$$

$$R^{ab}\wedge V^c \epsilon_{abcd} = 0 \;\Rightarrow\; R^{ab}{}_{\ell m}V^\ell \wedge V^m \wedge V^c \epsilon_{abcd} = 0$$

$$-6R^{ab}{}_{\ell m}\delta^{\ell m p}_{abd} = 0,$$

$$\mathcal{R}^a{}_b - \frac{1}{2}\delta^a_b\mathcal{R} = 0$$

$$\delta^{(\text{gauge})}\mu^A = (\nabla\epsilon)^A,$$

$$\delta^{(\text{gauge})}\omega^{ab} = \mathcal{D}\epsilon^{ab},$$
$$\delta^{(\text{gauge})}V^a = \mathcal{D}\epsilon^a + \epsilon^{ab}V_b,$$

$$\delta^{(\text{gauge})}\int R^{ab}\wedge V^c \wedge V^d \epsilon_{abcd} = 2\int R^{ab}\wedge\mathcal{D}\epsilon^c \wedge V^d \epsilon_{abcd} = -2\int \epsilon^c R^{ab}\wedge\mathring{T}^d \epsilon_{abcd} \neq 0$$

$$\delta^{(\text{gauge})}R^{ab} = \mathcal{D}^2\epsilon^{ab} = 2R^a_c\epsilon^{cb}$$

$$R^A = R^A_{BC}\mu^B \wedge \mu^C = R^A_{bc}V^b \wedge V^c + R^A_{bcC}\omega^{bc}\wedge\mu^C$$

$$R^A_{abC} = 0$$

$$R^A = R^A_{ab}V^a \wedge V^b$$

$$G = \overline{\mathrm{OSp}(1 \mid 4)}$$

$$T_A = (J_{ab}, P_a, Q_\alpha)$$

$$[J_{ab}, J_{cd}] = -2\eta_{a[c}J_{d]b} + 2\eta_{b[c}J_{d]a}, [J_{ab}, P_c] = -2P_{[a}\eta_{b]c}, [P_a, P_b] = 0$$
$$[J_{ab}, Q_\alpha] = \frac{1}{2}(\gamma_{ab})_\alpha{}^\beta Q_\beta, [P_a, Q_\alpha] = 0$$
$$\{Q_\alpha, Q_\beta\} = -i(C\gamma^a)_{\alpha\beta}P_a.$$

$$R^A(x,\theta) = R^A{}_{\hat{a}\hat{b}}(x,\theta)E^{\hat{a}} \wedge E^{\hat{b}}$$
$$= R^A{}_{(2|0)ab}V^a \wedge V^b + R^A{}_{(1|1)a\alpha}V^a \wedge \psi^\alpha + R^A{}_{(0|2)\alpha\beta}\psi^\alpha \wedge \psi^\beta.$$

$$R^{ab} \equiv d\omega^{ab} - \omega^a_c \wedge \omega^{cb}$$
$$T^a \equiv dV^a - \omega^a_b \wedge V^b - \frac{i}{2}\psi^\alpha(C \cdot \gamma^a)_{\alpha\beta} \wedge \psi^\beta = \mathcal{D}V^a - \frac{i}{2}\bar{\psi}\gamma^a \wedge \psi$$
$$\rho^\alpha \equiv d\psi^\alpha - \frac{1}{4}(\gamma_{ab})^\alpha{}_\beta\omega^{ab} \wedge \psi^\beta = \mathcal{D}\psi^\alpha$$

$$\mathcal{D}R^{ab} = 0$$
$$\mathcal{D}T^a + R^a{}_b \wedge V^b - i\bar{\psi}\gamma^a \wedge \rho = 0$$
$$\mathcal{D}\rho^\alpha + \frac{1}{4}(\gamma_{ab})^\alpha{}_\beta R^{ab} \wedge \psi^\beta = 0$$

$$D_a(V^b) = \delta^b_a;\ D_\alpha(\psi^\beta) = \delta^\beta_\alpha;\ D_a(\psi^\alpha) = D_\alpha(V^a) = 0$$

$$\delta_\epsilon\mu^A = \ell_\epsilon\mu^A \equiv d(\iota_\epsilon\mu^A) + \iota_\epsilon(d\mu^A)$$

$$\iota_\epsilon\psi^\alpha = \epsilon^\alpha, \iota_\epsilon V^a = 0$$

$$\bar{\psi}\gamma^5\gamma_a\mathcal{D}\psi V^a = -i\bar{\psi}_\mu\gamma^{\mu\nu\rho}\mathcal{D}_\nu\psi_\rho\sqrt{g}d^4x$$

$$\mathcal{A}^{\mathcal{N}=1}_{D=4} = \frac{1}{4\kappa^2}\int_{\mathcal{M}_4\subset\mathcal{M}^{[4|4]}}[R^{ab}V^cV^d\epsilon_{abcd} + \alpha\bar{\psi}\gamma_5\gamma_a\mathcal{D}\psi V^a]$$

$$\frac{\delta\mathcal{A}}{\delta\omega^{ab}} = 0\colon \epsilon_{abcd}\mathcal{D}V^c \wedge V^d + \frac{\alpha}{4}\bar{\psi}\gamma_5\gamma_c\gamma_{ab}\psi V^c = 0,$$
$$\epsilon_{abcd}\left(\mathcal{D}V^a + \frac{\mathrm{i}\alpha}{8}\bar{\psi}\gamma^a\psi\right) \wedge V^d = 0$$
$$\frac{\delta\mathcal{A}}{\delta V^a} = 0\colon 2R^{ab} \wedge V^c\epsilon_{abcd} - \alpha\bar{\psi} \wedge \gamma^5\gamma_d\rho = 0$$
$$\frac{\delta\mathcal{A}}{\delta\bar{\psi}} = 0\colon 2\gamma^5\gamma_a\rho \wedge V^a - \gamma_5\gamma_a\psi \wedge T^a = 0$$

$$T^c \wedge V^d\epsilon_{abcd} = 0$$

$$T^a = T^a_{(2|0)bc}V^bV^c + T^a_{(1|1)c\alpha}\psi_\alpha V^c + \psi^\alpha T^a_{(0|2)\alpha\beta}\psi^\beta$$
$$\rho^\alpha = \rho^\alpha_{(2|0)ab}V^aV^b + \rho^\alpha_{(1|1)a}\psi^\alpha V^a + \rho^\alpha_{(0|2)\beta\gamma}\psi^\beta\psi^\gamma$$
$$R^{ab} = R^{ab}_{(2|0)cd}V^cV^d + \bar{\Theta}^{ab}_c\psi V^c + \bar{\psi}K^{ab}\psi$$

$$R^A_{(2|0)ab} \equiv \tilde{R}^A_{ab}$$

$$\rho^\alpha_{\mu\nu} = \tilde{\rho}^\alpha{}_{ab}V^a_\mu V^b_\nu + \rho^\alpha_{(1|1)a}\psi^\alpha_{[\mu}V^a_{\nu]} + \rho^\alpha_{(0|2)\alpha\beta}\psi^\alpha_\mu\psi^\beta_\nu,$$

$$T^a = \tilde{T}^a{}_{bc}V^b \wedge V^c = T^a{}_{bc}V^b \wedge V^c$$

$$T^a = 0$$

$$2\gamma^5\gamma_a\rho \wedge V^a = 0$$

$$\rho^\alpha = \rho^\alpha_{(2|0)ab}V^aV^b + \rho^\alpha_{(1|1)a}\psi^\alpha V^a + \rho^\alpha_{(0|2)\beta\gamma}\psi^\beta\psi^\gamma,$$

$$\rho = \rho_{ab}V^aV^b = \rho_{\mu\nu}dx^\mu \wedge dx^\nu$$

$$\gamma^5\gamma_a\rho \wedge V^a = 0 \ \Rightarrow\ \gamma^5\gamma_a\rho_{bc}V^a_\sigma V^b_\mu V^c_\nu\sqrt{g}d^3x\epsilon^{\mu\nu\sigma\lambda} = 0$$

$$\epsilon^{\mu\nu\sigma\lambda}\gamma_5\gamma_\sigma\rho_{\mu\nu} = 0, \text{ or, equivalently: } \epsilon^{\mu\nu\sigma\lambda}\gamma_5\gamma_\sigma\mathcal{D}_\mu\psi_\nu = 0$$

$$\bar{\Theta}^{ab}_c = -\epsilon^{abrs}\bar{\rho}_{rs}\gamma_5\gamma_c - \delta^{[a}_c\epsilon^{b]mst}\bar{\rho}_{st}\gamma_5\gamma_m$$

$$\begin{aligned} R^{ab} &= \tilde{R}^{ab}_{cd}V^cV^d + \bar{\Theta}^{ab}_c\psi V^c \\ T^a &= 0 \\ \rho &= \rho_{ab}V^aV^b \end{aligned}$$

$$\begin{aligned} \tilde{R}^{ac}{}_{bc} - \frac{1}{2}\delta^a_b\tilde{R}^{cd}{}_{cd} &= 0, \\ \tilde{T}^a_{bc} &= 0, \\ \epsilon^{abcd}\gamma^5\gamma_c\tilde{\rho}_{ab} &= 0. \end{aligned}$$

$$\begin{aligned} R^{ab}_{\mu\nu} &= \tilde{R}^{ab}{}_{cd}V^c_\mu V^d_\nu + \bar{\Theta}^{ab}_c\psi_{[\mu}V^c_{\nu]} = \tilde{R}^{ab}{}_{\mu\nu} + \bar{\Theta}^{ab}_{[\nu}\psi_{\mu]}, \\ T^a_{\mu\nu} &= \tilde{T}^a{}_{bc}V^b_\mu V^c_\nu = \tilde{T}^a{}_{\mu\nu}, \\ \rho_{\mu\nu} &= \tilde{\rho}_{ab}V^a_\mu V^b_\nu = \tilde{\rho}_{\mu\nu}, \end{aligned}$$

$$\mathcal{A}^{\mathcal{N}=1}_{D=4} = \frac{1}{4\kappa^2}\int_{\mathcal{M}_4\subset\mathcal{M}^{[4|4]}}[R^{ab}V^cV^d\epsilon_{abcd} + 4\bar{\psi}\gamma_5\gamma_a\mathcal{D}\psi V^a].$$

$$\delta_\epsilon\mathcal{L} \equiv \ell_\epsilon\mathcal{L} = \iota_\epsilon d\mathcal{L} + d(\iota_\epsilon\mathcal{L}) = 0,$$

$$\begin{aligned} d\mathcal{L} = \mathcal{D}\mathcal{L} = \frac{1}{4\kappa^2}\Big[&2R^{ab}\left(T^c + \frac{\mathrm{i}}{2}\bar{\psi}\gamma^c\psi\right)V^d\epsilon_{abcd} + 4\bar{\rho}\gamma^5\gamma_a\rho V^a + \\ &+\bar{\psi}\gamma^5\gamma_c\gamma_{ab}\psi R^{ab}V^c - 4\bar{\psi}\gamma^5\gamma_a\rho\left(T^a + \frac{\mathrm{i}}{2}\bar{\psi}\gamma^a\psi\right)\Big]. \end{aligned}$$

$$d\mathcal{L} = R^{ab}T^cV^d\epsilon_{abcd} + \bar{\rho}\gamma^5\gamma_a\rho V^a - 4\bar{\psi}\gamma^5\gamma_a\rho T^a$$

$$\begin{aligned} &\iota_\epsilon(d\mathcal{L}) = 2(\iota_\epsilon R^{ab})T^cV^d\epsilon_{abcd} + 2R^{ab}(\iota_\epsilon T^c)V^d\epsilon_{abcd} + 8(\iota_\epsilon\bar{\rho})\gamma^5\gamma_a\rho V^a \\ &-4\bar{\epsilon}\gamma^5\gamma_a\rho T^a - 4\bar{\psi}\gamma_5\gamma_a(\iota_\epsilon\rho)T^a - 4\bar{\psi}\gamma^5\gamma_a\rho(\iota_\epsilon T^a) \end{aligned}$$

$$\iota_\epsilon(d\mathcal{L}) = d(\text{3-form })$$

$$\iota_\epsilon T^a = 0;\ \iota_\epsilon\rho = 0$$

$$2(\iota_\epsilon R^{ab})V^d\epsilon_{abcd} - 4\bar{\epsilon}\gamma_5\gamma_c\rho = 0$$

$$\iota_\epsilon R^{ab} = \bar{\Theta}^{ab}_c\epsilon V^c,$$

$$\begin{aligned} \delta_\epsilon\omega^{ab} &= (\nabla\epsilon)^{ab} + \epsilon^cV^dR^{ab}_{cd} + \bar{\Theta}^{ab}_c\psi\epsilon^c + \bar{\Theta}^{ab}_c\epsilon V^c, \\ \delta_\epsilon V^a &= (\nabla\epsilon)^a, \\ \delta_\epsilon\psi^\alpha &= (\nabla\epsilon)^\alpha + \epsilon^a\rho^\alpha_{ab}V^b. \end{aligned}$$

$$\begin{aligned} \delta_\epsilon\omega^{ab} &= (\nabla\epsilon)^{ab} + \bar{\Theta}^{ab}_c\epsilon V^c, \\ \delta_\epsilon V^a &= (\nabla\epsilon)^a, \\ \delta_\epsilon\psi^\alpha &= (\nabla\epsilon)^\alpha. \end{aligned}$$

$$\begin{aligned} \delta_\epsilon\omega^{ab}_\mu &= \bar{\Theta}^{ab}_c\epsilon V^c_\mu \\ \delta_\epsilon V^a_\mu &= -\mathrm{i}\bar{\psi}_\mu\gamma^a\epsilon \\ \delta_\epsilon\psi_\mu &= \mathcal{D}_\mu\epsilon \end{aligned}$$

$$[\ell_{\tilde{T}_A}, \ell_{\tilde{T}_B}] = \ell_{[\tilde{T}_A,\tilde{T}_B]}$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]V^a_\mu = -i\mathcal{D}_\mu(\overline{\epsilon_1}\gamma^a\epsilon_2)$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\psi_\mu = -i(\overline{\epsilon_1}\gamma^\nu\epsilon_2)\mathcal{D}_{[\mu}\psi_{\nu]} + \alpha(x)_\mu{}^\lambda\gamma_{\lambda\nu\rho}\rho^{\nu\rho},$$

$$\alpha(x)_\mu{}^\lambda = \frac{1}{8}(2\overline{\epsilon_1}\gamma^\nu\epsilon_2A^\lambda{}_{\mu\nu} + \overline{\epsilon_1}\gamma^{\nu\sigma}\epsilon_2B^\lambda{}_{\mu\nu\sigma}),$$

$$\delta^{(\text{gauge})}\omega^{ab} = (\nabla\kappa)^{ab} = \mathcal{D}\kappa^{ab},$$
$$\delta^{(\text{gauge})}V^{a} = (\nabla\kappa)^{a} = \mathcal{D}\kappa^{ab} + \kappa^{ab}V_b - \mathrm{i}\bar{\psi}\gamma^a\kappa,$$
$$\delta^{(\text{gauge})}\psi = (\nabla\kappa) = \mathcal{D}\kappa - \frac{1}{4}\kappa^{ab}\gamma_{ab}\psi,$$

$$\delta_\kappa^{(\text{gauge})}\omega_\mu^{ab} = 0$$
$$\delta_\kappa^{(\text{gauge})}V_\mu^{a} = -\mathrm{i}\bar{\psi}_\mu\gamma^a\kappa$$
$$\delta_\kappa^{(\text{gauge})}\psi_\mu = \mathcal{D}_\mu\kappa$$

$$\begin{aligned} R^{ab} &\equiv d\omega_b^a + \omega_c^a\omega_b^c \\ &= \tilde{R}_{cd}^{ab}V^cV^d + \bar{\Theta}_{i|c}^{ab}\psi^iV^c - \bar{\psi}_i\left(\tilde{F}^{ab} + \frac{\mathrm{i}}{2}\tilde{F}_{cd}\epsilon^{abcd}\gamma_5\right)\psi_j\epsilon^{ij} \\ T^a &\equiv \mathcal{D}V^a - \frac{\mathrm{i}}{2}\bar{\psi}_i\gamma\psi^i \\ &= 0 \\ F &\equiv d\mathcal{A} + \epsilon^{ij}\bar{\psi}_i\psi_j \\ &= \tilde{F}_{ab}V^aV^b \\ \rho_i &\equiv \mathcal{D}\psi_i \\ &= \tilde{\rho}_{i|ab}V^aV^b + \left(\gamma^a\tilde{F}_{ab} + \mathrm{i}\gamma_5\gamma^a\frac{\mathrm{i}}{2}\tilde{F}^{cd}\epsilon_{abcd}\right)\epsilon_{ij}\psi^jV^b. \end{aligned}$$

$$\delta_\epsilon\omega^{ab} = \bar{\Theta}_{i|c}^{ab}\epsilon^iV^c$$
$$\delta_\epsilon V^a = -\mathrm{i}\bar{\psi}_i\gamma^a\epsilon^i$$
$$\delta_\epsilon\psi_i = \mathcal{D}\epsilon_i + \mathrm{i}\epsilon_{ij}F^{ab}V^b\gamma^a\epsilon_j + \mathrm{i}\frac{1}{2}\epsilon_{ij}\epsilon_{abcd}F^{cd}V^b\gamma_5\gamma^a\epsilon^j$$
$$\delta_\epsilon\mathcal{A} = 2\epsilon^{ij}\bar{\psi}_i\epsilon_j$$

$$F\wedge^* F \propto F_{\mu_1\ldots\mu_{p+1}}F^{\mu_1\ldots\mu_{p+1}}\sqrt{g}d^Dx$$

$$\mathcal{A} = -\int F^{\mu\nu}F_{\mu\nu}\sqrt{-g}d^4x = \frac{1}{2}\int F\wedge^* F$$

$$\frac{-1}{4!}\int \hat{F}_{ab}\hat{F}^{ab}\epsilon_{pqrs}V^{pqrs} + \alpha\int \hat{F}^{ab}FV^{cd}\epsilon_{abcd}$$

$$\hat{F}_{ab} = F_{ab}$$

$$\mathcal{D}_\mu F^{\mu\nu} = \mathcal{D}_a F^{ab} = 0$$

$$R_{\alpha C}^{A} = C_{\alpha C|B}^{A|mn}R_{mn}^{B}$$

$$\delta\mu^A = (\nabla\epsilon)^A + 2\bar{\epsilon}C_{\alpha C|B}^{A|mn}R_{mn}^{B}$$

$$R^A \equiv d\mu^A + \frac{1}{2}C_{BC}^{A}\mu^B\wedge\mu^C$$

$$dR^A + C^A{}_{BC}\mu^B\wedge R^C = 0$$

$$D(T_A,T_B) \equiv [T_A,T_B] = C_{AB}^{C}T_C;$$

$$d\sigma^A + \frac{1}{2}C_{BC}^{A}\sigma^B\wedge\sigma^C = 0$$

$$d\sigma^A(T_B,T_C) = -\frac{1}{2}\sigma^A([T_B,T_C])$$

$$d\Theta^{A(p)} + \sum_{n=1}^{N}\frac{1}{n!}C^{A(p)}{}_{B_1(p_1)B_2(p_2)\ldots B_n(p_n)}\Theta^{B_1(p_1)}\wedge\Theta^{B_2(p_2)}\wedge\cdots\wedge\Theta^{B_n(p_n)} = 0.$$

$$B_i(p_i)B_{i+1}(p_{i+1}) = (-1)^{|B_i||B_{i+1}|+p_ip_{i+1}}B_{i+1}(p_{i+1})B_i(p_i),$$

$$d^2\Theta^{A(p)} = -\sum_{n=1}^{N} \frac{1}{(n-1)!} \sum_{m=1}^{N} \frac{1}{m!} C^{A(p)}{}_{B_{1(p_1)}B_{2(p_2)}\dots B_{n(p_n)}} C^{B_1(p_1)}_{D_{1(q_1)}D_{2(q_2)}\dots D_{m(q_m)}} \times$$
$$\Theta^{D_1(q_1)} \wedge \Theta^{D_2(q_2)} \wedge \cdots \wedge \Theta^{D_m(q_m)} \wedge \Theta^{B_2(p_2)} \wedge \dots \Theta^{B_n(p_n)} = 0.$$

$$C^{A(p)}{}_{B_{1(p_1)}} \left[B_{2(p_2)\dots B_{n(p_n)}} C^{B_1(p_1)} D_{1(q_1)} D_{2(q_2)\dots D_{m(q_m)}}\right] = 0$$

$$d\sigma^A + \frac{1}{2} C^A_{BC} \sigma^B \wedge \sigma^C = 0$$

$$\Omega^i_{(n,p)} = C^i_{A_1,\dots,A_n} \sigma^{A_1} \wedge, \dots, \wedge \sigma^{A_n}$$

$$\left(\nabla^{(n)}\right)^i_j = d\delta^i_j + \sigma^A \wedge D^{(n)}(T_A)^i_j$$

$$\nabla^{(n)}\nabla^{(n)} = 0$$

$$\Omega^i_{(n,p)} = \nabla^{(n)} \tilde{\Omega}^i_{(n,p-1)}$$

$$\nabla^{(n)} A^i_{(n,p-1)} + \Omega^i_{(n,p)} = 0,$$

$$\hat{\Omega}^i_{(n,p)}[\sigma, A] = C^i_{A_1,\dots,A_r i_1,\dots,i_s} \times \sigma^{A_1}, \dots, \sigma^{A_r} \wedge A^{i_1}_{(n_1,p_1)}, \dots, A^{i_s}_{(n_s,p_s)}.$$

$$d\omega^{ab} - \omega^a_c \wedge \omega^{cb} = 0$$
$$\mathcal{D}V^a - \frac{i}{2} \bar{\Psi} \Gamma^a \wedge \Psi = 0$$
$$\mathcal{D}\Psi \equiv d\Psi - \frac{1}{4} \Gamma_{ab} \omega^{ab} \wedge \Psi = 0$$

$$\Omega_{(V,\Psi)} = \frac{1}{2} \bar{\Psi} \wedge \Gamma^{ab} \Psi \wedge V^a \wedge V^b$$

$$d\Omega = \frac{i}{2} \bar{\Psi} \wedge \Gamma^{ab} \Psi \bar{\Psi} \wedge \Gamma_a \Psi \wedge V^b = 0$$

$$\bar{\Psi} \wedge \Gamma_{ab} \Psi \wedge \bar{\Psi} \Gamma^a \Psi = 0$$

$$dA^{(3)} - \frac{1}{2} \bar{\Psi} \wedge \Gamma^{ab} \Psi \wedge V^a \wedge V^b = 0$$

$$\Omega'(V, \Psi, A) = \frac{i}{2} \bar{\Psi} \Gamma^{a_1,\dots,a_5} \wedge \Psi \wedge V^{a_1} \wedge \cdots \wedge V^{a_5} + \frac{15}{2} \bar{\Psi} \wedge \Gamma^{ab} \Psi \wedge V^a \wedge A^{(3)}$$

$$dB^{(6)} = \frac{i}{2} \bar{\Psi} \Gamma^{a_1 \dots a_5} \wedge \Psi \wedge V^{a_1} \cdots \wedge V^{a_5} + \frac{15}{2} \bar{\Psi} \wedge \Gamma^{ab} \Psi \wedge V^a \wedge V^b \wedge A^{(3)}$$

$$\Pi^{A(p)} = \left(\omega^{ab}, V^a, \Psi, A^{(3)}, B^{(6)}\right)$$

$$R^{A(p+1)} \equiv d\Pi^{A(p)} + \sum_{i=1}^{N} \frac{1}{n!} C^{A(p)}{}_{B_{1(p_1)}B_{2(p_2)}\dots B_{n(p_n)}} \Pi^{B_1(p_1)} \wedge \Pi^{B_2(p_2)} \wedge \cdots \wedge \Pi^{B_n(p_n)}$$

$$\nabla R^{A(p+1)} = dR^{A(p+1)} - \sum_{i=1}^{N} \frac{1}{(n-1)!} C^{A(p)}{}_{B_{1(p_1)}B_{2(p_2)}\dots B_{n(p_n)}}$$
$$\times R^{B_1(p_1+1)} \wedge \Pi^{B_2(p_2)} \wedge \cdots \wedge \Pi^{B_n(p_n)} = 0$$

$$R_b^a \equiv d\omega_b^a - \omega_c^a \wedge \omega_b^c$$
$$T^a \equiv \mathcal{D}V^a - \frac{\mathrm{i}}{2}\bar{\Psi}\Gamma^a \wedge \Psi$$
$$\rho \equiv \mathcal{D}\Psi = d\Psi - \frac{1}{4}\omega^{ab} \wedge \Gamma_{ab}\Psi$$
$$F^{(4)} \equiv dA^{(3)} - \frac{1}{2}\bar{\Psi}\Gamma^{ab} \wedge \Psi \wedge V^a \wedge V^b,$$
$$F^{(7)} \equiv dB^{(6)} - \frac{i}{2}\bar{\Psi}\Gamma^{a_1\ldots a_5} \wedge \Psi \wedge V^{a_1,\ldots,\wedge V^{a_5}} - \frac{15}{2}\bar{\Psi} \wedge \Gamma^{ab}\Psi \wedge V^a \wedge V^b \wedge A + -15F^{(4)} \wedge A^{(3)}$$

$$A^{(3)} \to A^{(3)} + d\phi^{(2)},$$
$$B^{(6)} \to B^{(6)} + d\lambda^{(5)},$$

$$\mathcal{D}B_{a_1a_2} = \frac{1}{2}\bar{\Psi} \wedge \Gamma_{a_1a_2}\Psi,$$
$$\mathcal{D}B_{a_1\ldots a_5} = \frac{i}{2}\bar{\Psi} \wedge \Gamma_{a_1\ldots a5}\Psi,$$
$$\mathcal{D}\eta = iE_1\Gamma_a\Psi \wedge V^a + E_2\Gamma^{ab}\Psi \wedge B_{ab} + iE_3\Gamma^{a_1\ldots a_5}\Psi \wedge B_{a_1\ldots a_5},$$

$$\left(\omega^{ab}, V^a, \Psi, B_{ab}, B_{a_1\cdots a_5}, \eta\right)$$

$$E_1 + 10E_2 - 720E_3 = 0$$

$$A_{par}^{(3)} = T_0B_{ab} \wedge V^a \wedge V^b + T_1B_{ab} \wedge B_c^b \wedge B^{ca} + T_2B_{b_1a_1\ldots a_4} \wedge B_{b_2}^{b_1} \wedge B^{b_2a_1\ldots a_4}$$
$$+T_3\epsilon_{a_1\ldots a_5b_1\ldots b_5m}B^{a_1\ldots a_5} \wedge B^{b_1\ldots b_5} \wedge V^m$$
$$+T_4\epsilon_{m_1\ldots m_6n_1\ldots n_5}B^{m_1m_2m_3p_1p_2} \wedge B^{m_4m_5m_6p_1p_2} \wedge B^{n_1\ldots n_5}$$
$$+iS_1\bar{\Psi}\Gamma_a\eta \wedge V^a + S_2\bar{\Psi}\Gamma^{ab}\eta \wedge B_{ab} + iS_3\bar{\Psi}\Gamma^{a_1\cdots a_5}\eta \wedge B_{a_1\ldots a_5}$$

$$dA_{par}^{(3)} - \frac{1}{2}\bar{\Psi} \wedge \Gamma_{ab}\Psi \wedge V^a \wedge V^b = 0$$

$$d\omega^{ab} = \omega^{ac} \wedge \omega_c^b,$$
$$\mathcal{D}V^a = \frac{\mathrm{i}}{2}\bar{\Psi} \wedge \Gamma^a\Psi,$$
$$\mathcal{D}\Psi = 0,$$
$$\mathcal{D}B_{a_1a_2} = \frac{1}{2}\bar{\Psi} \wedge \Gamma_{a_1a_2}\Psi,$$
$$\mathcal{D}B_{a_1\ldots a_5} = \frac{\mathrm{i}}{2}\bar{\Psi} \wedge \Gamma_{a_1\ldots a_5}\Psi,$$
$$\mathcal{D}\eta = \mathrm{i}E_1\Gamma_a\Psi \wedge V^a + E_2\Gamma^{ab}\Psi \wedge B_{ab} + \mathrm{i}E_3\Gamma^{a_1\cdots a_5}\Psi \wedge B_{a_1\ldots a_5}.$$

$$R_b^a \equiv d\omega_b^a - \omega_c^a \wedge \omega_b^c,$$
$$T^a \equiv \mathcal{D}V^a - \frac{\mathrm{i}}{2}\bar{\Psi}\Gamma^a \wedge \Psi,$$
$$\rho \equiv \mathcal{D}\Psi = d\Psi - \frac{1}{4}\omega^{ab} \wedge \Gamma_{ab}\Psi$$
$$F^{(4)} = dA^{(3)} - \frac{1}{2}\bar{\Psi}\Gamma^{ab} \wedge \Psi \wedge V^a \wedge V^b.$$

$$T_A \equiv \{P_a, Q, J_{ab}, Z^{ab}, Z^{a_1\ldots a_5}, Q'\}$$

$$\omega^{ab}(J_{cd}) = 2\delta_{cd}^{ab}, V^a(P_b) = \delta_b^a, \Psi^\alpha\left(Q_\beta\right) = \delta_\beta^\alpha$$
$$B^{ab}(Z_{cd}) = 2\delta_{cd}^{ab}, B^{a_1\cdots a_5}\left(Z_{b_1\cdots b_5}\right) = 5!\,\delta_{b_1\cdots b_5}^{a_1\cdots a_5}, \eta^\alpha\left(Q'_\beta\right) = \delta_\beta^\alpha$$

$$[J_{ab}, J_{cd}] = -2\eta_{a[c}J_{d]b} + 2\eta_{b[c}J_{d]a}$$
$$[J_{ab}, P_c] = -2P_{[a}\eta_{b]c}$$
$$\{Q, \bar{Q}\} = -\left(\mathrm{i}\Gamma^a P_a + \frac{1}{2}\Gamma^{ab}Z_{ab} + \frac{\mathrm{i}}{5!}\Gamma^{a_1\dots a_5}Z_{a_1\dots a_5}\right)$$
$$[Q', \bar{Q}'] = 0$$
$$[Q, P_a] = -2\mathrm{i}E_1\Gamma_a Q'$$
$$[Q, Z^{ab}] = -4E_2\Gamma^{ab}Q'$$
$$[Q, Z^{a_1\dots a_5}] = -2(5!)\mathrm{i}E_3\Gamma^{a_1\dots a_5}Q'$$
$$[J_{ab}, Z^{cd}] = -8\delta^{[c}_{[a}Z^{d]}_{b]}$$
$$[J_{ab}, Z^{c_1\dots c_5}] = -20\delta^{[c_1}_{[a}Z^{c_2\dots c_5]}_{b]}$$
$$[J_{ab}, Q] = -\Gamma_{ab}Q$$
$$[J_{ab}, Q'] = -\Gamma_{ab}Q'$$

$$A^{(3)}_{par} = A^{(3)}_{(0)} + \alpha A^{(3)}_{(e)}$$

$$dA^{(3)}_{(0)} = \frac{1}{2}\bar{\psi} \wedge \Gamma_{ab}\psi \wedge V^a \wedge V^b$$
$$dA^{(3)}_{(e)} = 0.$$

$$\{Q, \bar{Q}\} = -\left(\mathrm{i}\Gamma^a P_a + \frac{1}{2}\Gamma^{ab}Z_{ab} + \frac{\mathrm{i}}{5!}\Gamma^{a_1\dots a_5}Z_{a_1\dots a_5}\right),$$

$$dB^{(6)}_{par} = \frac{i}{2}\bar{\Psi}\Gamma^{a_1\dots a_5} \wedge \Psi \wedge V^{a_1} \cdots \wedge V^{a_5} + \frac{15}{2}\bar{\Psi} \wedge \Gamma^{ab}\Psi \wedge V^a \wedge V^b \wedge A^{(3)}_{par},$$

$$\eta_{ab} = \mathrm{diag}(+, -, \cdots, -).$$

$$\gamma^a \equiv \begin{pmatrix} \sigma^a & 0 \\ 0 & \bar{\sigma}^a \end{pmatrix}, \gamma_5 \equiv -\frac{i}{4!}\epsilon_{abcd}\gamma^a\gamma^b\gamma^c\gamma^d, \gamma^{a_1\dots a_k} \equiv \gamma^{[a_1}\dots\gamma^{a_k]}$$

$$\gamma_0^\dagger = \gamma_0, \gamma_0\gamma^a\gamma_0 = (\gamma^a)^\dagger,$$
$$\gamma_5^\dagger = \gamma_5, \gamma_5^* = \gamma_5, (\gamma_5)^2 = \mathbb{I},$$
$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, [\gamma^a, \gamma^b] = 2\gamma^{ab}, \gamma^a\gamma^b = \eta^{ab} + \gamma^{ab}.$$

$$C^2 = -1, C^T = -C, (C\gamma_a)^T = C\gamma_a, (C\gamma_5)^T = -C\gamma_5$$
$$(C\gamma_5\gamma_a)^T = -C\gamma_5\gamma_a, (C\gamma_{ab})^T = C\gamma_{ab}$$

$$\bar{\psi} \equiv \psi^\dagger\gamma^0 = \psi^t C$$

$$\gamma_a\psi\bar{\psi}\gamma^a\psi = 0.$$

$$5984 \to 32 + 320 + 1408 + 4224$$

$$\Xi^{(32)} \in \mathbf{32}, \Xi^{(320)}_a \in \mathbf{320}, \Xi^{(1408)}_{a_1a_2} \in \mathbf{1408}, \Xi^{(4224)}_{a_1\dots a_5} \in \mathbf{4224},$$

$$\Psi \wedge \bar{\Psi} \wedge \Gamma_a\Psi = \Xi^{(320)}_a + \frac{1}{11}\Gamma_a\Xi^{(32)}$$
$$\Psi \wedge \bar{\Psi}\Gamma_{a_1a_2}\Psi = \Xi^{(1408)}_{a_1a_2} - \frac{2}{9}\Gamma_{[a_2}\Xi^{(320)}_{a_2]} + \frac{1}{11}\Gamma_{a_1a_2}\Xi^{(32)}$$
$$\Psi \wedge \bar{\Psi} \wedge \Gamma_{a_1\dots a_5}\Psi = \Xi^{(4224)}_{a_1\dots a_5} + 2\Gamma_{[a_1a_2a_3}\Xi^{(1408)}_{a_4a_5]} + \frac{5}{9}\Gamma_{[a_1\dots a_4}\Xi^{(320)}_{a_5]} - \frac{1}{77}\Gamma_{a_1\dots a_5}\Xi^{(32)}$$

$$[T_A, T_B] = C^C{}_{AB}T_C,$$

$$[[T_A, T_B], T_C] + [[T_B, T_C], T_A] + [[T_C, T_A], T_B] = 0$$

$$C^L{}_{[AB}C^D{}_{C]L} = 0.$$

$$\sigma^A(T_B) = \delta^A_B$$

$$d\sigma^{C}+\frac{1}{2}C^{C}{}_{AB}\sigma^{A}\wedge\sigma^{B}=0.$$

$$0=d^{2}\sigma^{C}=-\frac{1}{2}C_{AB}^{C}d(\sigma^{A}\wedge\sigma^{B})$$
$$=-C_{AB}^{C}d\sigma^{A}\wedge\sigma^{B}=-\frac{1}{2}C_{AB}^{C}C_{LM}^{A}\sigma^{L}\wedge\sigma^{M}\wedge\sigma^{B}$$

$$d\sigma^{C}(T_{L},T_{M})=-\frac{1}{2}\sigma^{C}([T_{L},T_{M}])=-\frac{1}{2}C_{LM}^{C}$$

$$\sigma\equiv g^{-1}dg=\sigma^{A}T_{A}\in\mathfrak{G},g=\exp\ \alpha^{A}T_{A}\in G$$

$$d\sigma\qquad=dg^{-1}\wedge dg=-g^{-1}dg\wedge g^{-1}dg$$
$$=-\sigma\wedge\sigma\text{(B.9)}$$

$$d\sigma^{C}T_{C}=-\sigma^{A}\wedge\sigma^{B}T_{A}\cdot T_{B}=-\frac{1}{2}\sigma^{A}\wedge\sigma^{B}[T_{A},T_{B}]=-\frac{1}{2}C_{AB}^{C}\sigma^{A}\wedge\sigma^{B}T_{C}$$

$$\mu(g,x)=g^{-1}\mathring{\mu}(x)g+g^{-1}dg$$

$$R(x,g)\ \equiv d\mu+\mu\wedge\mu$$
$$=g^{-1}[d\mathring{\mu}(x)+\mathring{\mu}(x)\wedge\mathring{\mu}(x)]g=g^{-1}\mathring{R}(x)g=R^{A}(x,g)T_{A}$$

$$\mathring{R}(x)\equiv d\grave{\mu}(x)+\check{\mu}(x)\wedge\check{\mu}(x),$$

$$\mathring{R}(x)=\mathring{R}^{C}(x)T_{C}=\left(d\overleftrightarrow{\mu}^{C}+\frac{1}{2}C_{AB}{}^{C}\overleftrightarrow{\mu}^{A}\wedge\overleftrightarrow{\mu}^{B}\right)T_{C}$$

$$R^{A}(x,g)=R_{BC}^{A}(x,g)\mu^{B}\wedge\mu^{C}$$

$$d\mu^{C}+\frac{1}{2}[C_{AB}^{C}-2R_{AB}^{C}(x,g)]\mu^{A}\wedge\mu^{B}=0$$

$$\mathcal{C}^{C}{}_{AB}(x)\equiv C^{C}{}_{AB}-2R^{C}{}_{AB}(x,g)$$

$$\epsilon=\epsilon^{A}\tilde{T}_{A},$$

$$\ell_{\epsilon}\mu^{A}=(\iota_{\epsilon}d+d\iota_{\epsilon})\mu^{A}$$
$$=\iota_{\epsilon}d\mu^{A}+d\left(\iota_{(\epsilon)}\mu^{A}\right)$$
$$=\iota_{\epsilon}d\mu^{A}+d\epsilon^{A}.$$

$$\nabla\epsilon^{A}=d\epsilon^{A}+C_{BC}^{A}\mu^{B}\epsilon^{C},$$

$$\ell_{\epsilon}\mu^{A}=\iota_{\epsilon}\left(d\mu^{A}+\frac{1}{2}C_{BC}^{A}\mu^{B}\wedge\mu^{C}\right)-\epsilon^{B}C_{BC}^{A}\mu^{C}+d\epsilon^{A}.$$

$$\ell_{\epsilon}\mu^{A}=(\nabla\epsilon)^{A}+\iota_{\epsilon}R^{A}.$$

$$\iota_{\epsilon}R^{A}\equiv\epsilon^{B}R^{A}{}_{BC}\mu^{C}=0,$$

$$l_{n}(\dots)=[-,-,\dots,-]:g\bigotimes\ \cdots\bigotimes\ g\to g$$

$$l_{n}\left(v_{\sigma_{1}},v_{\sigma_{2}}\dots v_{\sigma_{n}}\right)=\chi(\sigma,v_{1},\dots v_{n})l_{n}(v_{1},v_{2}\dots v_{n})$$

$$\sum_{i,j(i+j=n+1)}\left[\sum_{\sigma\varepsilon Unsh(i,j-1)}\chi(\sigma,v_{1},\dots v_{n})(-1)^{i(j-1)}l_{j}\left(l_{i}\left(v_{\sigma_{1}},v_{\sigma_{2}}\dots v_{\sigma_{i}}\right),v_{\sigma_{i+1}},\dots v_{\sigma_{n}}\right)\right]=0$$

$$dt^{a}=-\sum_{k=1}^{\infty}\frac{1}{k!}\left[t_{a_{1}}\dots t_{a_{k}}\right]^{a}t^{a_{1}}\wedge\cdots\wedge t^{a_{k}}$$

$$ddt^a = -d\sum_{k=1}^{\infty}\frac{1}{k!}\left[t_{a_1}\dots t_{a_k}\right]^a t^{a_1}\wedge\dots\wedge t^{a_k}$$

$$= \sum_{k,l}^{\infty}\frac{1}{k-1)!\,l!}\left[\left[t_{b_1}\dots t_{b_l}\right]t_{a_2}\dots t_{a_k}\right]^a t^{b_1}\wedge\dots\wedge t^{b_l}\wedge t^{a_2}\wedge\dots\wedge t^{a_k} = 0$$

$$\sum_{k,l}^{\infty}\frac{1}{(k+l-1)!}\sum_{\sigma\varepsilon Unsh(l,k-1)}(-1)^{\sigma}\frac{1}{(k-1)!\,l!}\left[\left[t_{b_1}\dots t_{b_l}\right]t_{a_2}\dots t_{a_k}\right]^a\wedge\dots\wedge t_{b_l}\wedge t^{a_2}\wedge\dots t^{a_k} = 0$$

$$\sum_{k,l=1}^{\infty}\frac{1}{(k+l-1)!}\sum_{\sigma\varepsilon Unsh(l,k-1)}(-1)^{\sigma}\left[\left[t_{a_1}\dots t_{a_l}\right]t_{a_{l+1}}\dots t_{a_{k+l-1}}\right]t^{a_1}\wedge\dots\wedge t^{a_{k+l-1}} = 0$$

$$\sum_{k+l=n-1}\ \sum_{\sigma\varepsilon Unsh(l,k-1)}(-1)^{\sigma}\left[\left[t_{a_1}\dots t_{a_l}\right]t_{a_{l+1}}\dots t_{a_{k+l-1}}\right] = 0$$

$$\begin{cases} T_0 - 2S_1E_1 - 1 = 0 \\ T_0 - 2S_1E_2 - 2S_2E_1 = 0 \\ 3T_1 - 8S_2E_2 = 0 \\ T_2 + 10S_2E_3 + 10S_3E_2 = 0 \\ 120T_3 - S_3E_1 - S_1E_3 = 0 \\ T_2 + 1200S_3E_3 = 0 \\ T_3 - 2S_3E_3 = 0 \\ 9T_4 + 10S_3E_3 = 0 \\ S_1 + 10S_2 - 720S_3 = 0 \\ E_1 + 10E_2 - 720E_3 = 0 \end{cases}$$

$$\begin{cases} T_0 = \frac{1}{6} + \alpha \\ T_1 = -\frac{1}{90} + \frac{1}{3}\alpha \\ T_2 = -\frac{1}{4!}\alpha \\ T_3 = \frac{1}{(5!)^2}\alpha \\ T_4 = -\frac{1}{3[2!\cdot(3!)^2\cdot 5!]}\alpha \end{cases}$$

$$\begin{cases} S_1 = \frac{1}{2C}\left(\frac{10}{5!} + \sqrt{\frac{\alpha}{5!}}\right) \\ S_2 = \frac{1}{2C}\left(-\frac{1}{5!} + \frac{1}{2}\sqrt{\frac{\alpha}{5!}}\right) \\ S_3 = \frac{1}{2C}\frac{1}{5!}\sqrt{\frac{\alpha}{5!}} \end{cases}$$

$$\begin{cases} E_1 = 5!\,C\left(-\frac{10}{5!} + \sqrt{\frac{\alpha}{5!}}\right) \\ E_2 = 5!\,C\left(\frac{1}{5!} + \frac{1}{2}\sqrt{\frac{\alpha}{5!}}\right) \\ E_3 = 5!\,C\left(\frac{1}{5!}\sqrt{\frac{\alpha}{5!}}\right) \end{cases}$$

$$\partial_\mu g_{v\rho} - \Gamma^{\sigma}_{\mu v}g_{\sigma\rho} - \Gamma^{\sigma}_{\mu\rho}g_{v\sigma} = 0,$$

$$\Gamma^{\rho}_{(\mu v)} = \frac{1}{2}g^{\rho\sigma}\left(2\partial_{(\mu}g_{v)\sigma} - \partial_\sigma g_{\mu v}\right) + g^{\rho\tau}T^{\sigma}_{\tau(\mu}g_{v)\sigma}.$$

$$g_{\mu v} = E_\mu{}^{\hat{A}}E_v{}^{\hat{B}}\eta_{\hat{A}\hat{B}}$$

$$\delta E_\mu{}^{\hat{A}} = \Lambda^{\hat{A}}{}_{\hat{B}}E_\mu{}^{\hat{B}},$$

$$\delta\Omega_\mu{}^{\hat{A}\hat{B}} = \partial_\mu\Lambda^{\hat{A}\hat{B}} + 2\Lambda^{[\hat{A}|\hat{C}|}\Omega_{\mu\hat{C}}{}^{\hat{B}]}.$$

$$\partial_\mu E_\nu{}^{\hat{A}} - \Omega_\mu{}^{\hat{A}\hat{B}}E_{\nu\hat{B}} - \Gamma^\rho_{\mu\nu}E_\rho{}^{\hat{A}} = 0.$$

$$R_{\mu\nu}{}^{\hat{A}}(P) \equiv 2\partial_{[\mu}E_{\nu]}{}^{\hat{A}} - 2\Omega_{[\mu}{}^{\hat{A}\hat{B}}E_{\nu]\hat{B}} = T^\rho_{\mu\nu}E_\rho{}^{\hat{A}}.$$

$$\Omega_\mu^{\hat{A}\hat{B}} = -2E^{[\hat{A}|\nu}\partial_{[\mu}E_{\nu]}^{\hat{B}]} + E^{\hat{A}\nu}E^{\hat{B}\rho}E_{\mu\hat{C}}\partial_{[\nu}E_{\rho]}^{\hat{C}} - \frac{1}{2}E^{\hat{A}\nu}E^{\hat{B}\rho}T^\sigma_{\nu\rho}g_{\sigma\mu}$$
$$+E^{[\hat{A}|\nu|}E_\rho{}^{\hat{B}]}T^\rho_{\mu\nu}$$

$$D_\mu\psi = \partial_\mu\psi - \frac{1}{4}\Omega_\mu^{\hat{A}\hat{B}}\gamma_{\hat{A}\hat{B}}\psi$$

$$[J_{ab}, P_c] = -2\delta_{c[a}P_{b]}, \qquad [J_{ab}, G_c] = -2\delta_{c[a}G_{b]}, \qquad [G_a, H] = -P_a,$$
$$[J_{ab}, J_{cd}] = 4\delta_{[a[c}J_{d]b]}, \qquad [G_a, P_b] = -\delta_{ab}M.$$

$$\delta\tau_\mu = 0, \delta e_\mu^a = \lambda^{ab}e_{\mu b} + \lambda^a\tau_\mu, \delta m_\mu = \partial_\mu\sigma + \lambda^a e_{\mu a}$$
$$\delta\omega_\mu^{ab} = \partial_\mu\lambda^{ab} + 2\lambda^{[a|c|}\omega_{\mu c}^{b]}, \delta\omega_\mu^a = \partial_\mu\lambda^a - \omega_\mu^{ab}\lambda_b + \lambda^{ab}\omega_{\mu b}$$

$$R_{\mu\nu}(H) \equiv 2\partial_{[\mu}\tau_{\nu]}$$
$$R_{\mu\nu}^a(P) \equiv 2\partial_{[\mu}e_{\nu]}^a - 2\omega_{[\mu}^{ab}e_{\nu]b} - 2\omega_{[\mu}^a\tau_{\nu]}$$
$$R_{\mu\nu}(M) \equiv 2\partial_{[\mu}m_{\nu]} - 2\omega_{[\mu}^a e_{\nu]a}$$
$$R_{\mu\nu}{}^{ab}(J) \equiv 2\partial_{[\mu}\omega_{\nu]}^{ab} - 2\omega_{[\mu}^{[a|c]}\omega_{\nu]c}^{b]}$$
$$R_{\mu\nu}{}^a(G) \equiv 2\partial_{[\mu}\omega_{\nu]}{}^a - 2\omega_{[\mu}^{ab}\omega_{\nu]b}$$

$$\tau^\mu\tau_\mu = 1, \qquad \tau^\mu e_\mu^a = 0,$$
$$e_a^\mu e_\mu^b = \delta_a^b, \quad \tau_\mu\tau^\nu + e_\mu^a e_a^\nu = \delta_\mu^\nu$$

$$\delta\tau^\mu = -\lambda^a e_a^\mu, \delta e_a^\mu = \lambda_a^b e_b^\mu$$

$$\tau_{\mu\nu} = \tau_\mu\tau_\nu$$

$$h^{\mu\nu} = e_a{}^\mu e_b{}^\nu\delta^{ab}$$

$$\Delta t \equiv \int_0^1 \mathrm{d}t\sqrt{\dot{x}^\mu\dot{x}^\nu\tau_{\mu\nu}} = \int_0^1 \mathrm{d}t\dot{x}^\mu\tau_\mu = \int_\gamma \mathrm{d}x^\mu\tau_\mu.$$

$$\ell \equiv \int_0^1 \mathrm{d}s\sqrt{x'^\mu x'^\nu h_{\mu\nu}}, \text{ with } h_{\mu\nu} = e_\mu^a e_\nu^b\delta_{ab}$$

$$h^{\mu\nu}h_{\nu\rho} = \delta_\rho^\mu - \tau^\mu\tau_\rho,$$

$$\delta h_{\mu\nu} = 2\lambda^a\tau_{(\mu}e_{\nu)a}.$$

$$R_{\mu\nu}(P^a) \equiv 2\partial_{[\mu}e_{\nu]}^a - 2\omega_{[\mu}^{ab}e_{\nu]b} - 2\omega_{[\mu}^a\tau_{\nu]} = T_{\mu\nu}^a,$$
$$R_{\mu\nu}(M) \equiv 2\partial_{[\mu}m_{\nu]} - 2\omega_{[\mu}^a e_{\nu]a} = T_{\mu\nu}^{(m)}.$$

$$\omega_\mu^a = \tau_\mu\tau^\nu e^{a\rho}\partial_{[\nu}m_{\rho]} + e^{a\nu}\partial_{[\mu}m_{\nu]} + e_{\mu b}e^{a\nu}\tau^\rho\partial_{[\nu}e_{\rho]}^b + \tau^\nu\partial_{[\mu}e_{\nu]}^a$$
$$-\tau_\mu\tau^\nu e^{a\rho}T_{\nu\rho}^{(m)} + e_{\mu b}\tau^\nu e^{(a|\rho|}T_{\nu\rho}^{b)} - \frac{1}{2}e_{\mu b}e^{b\nu}e^{a\rho}T_{\nu\rho}^{(m)}$$

$$\omega_\mu^{ab} = -2e^{[a|\nu|}\partial_{[\mu}e_{\nu]}^{b]} + e_{\mu c}e^{a\nu}e^{b\rho}\partial_{[\nu}e_{\rho]}^c - \tau_\mu e^{a\nu}e^{b\rho}\partial_{[\nu}m_{\rho]}$$
$$+\frac{1}{2}\tau_\mu e^{a\nu}e^{b\rho}T_{\nu\rho}^{(m)} + e^{[a|\nu|}T_{\mu\nu}^{b]} - \frac{1}{2}e_{\mu c}e^{a\nu}e^{b\rho}T_{\nu\rho}^c$$

$$\delta T_{\mu\nu}^a = \lambda_b^a T_{\mu\nu}^b + 2\lambda^a\partial_{[\mu}\tau_{\nu]}, \delta T_{\mu\nu}^{(m)} = \lambda_a T_{\mu\nu}^a$$

$$\partial_\mu \tau_\nu - \Gamma^\rho_{\mu\nu}\tau_\rho = 0, \partial_\mu e_\nu{}^a - \omega_\mu{}^{ab} e_{\nu b} - \omega_\mu{}^a \tau_\nu - \Gamma^\rho_{\mu\nu} e_\rho{}^a = 0$$

$$\nabla_\mu \tau_{\nu\rho} \equiv \partial_\mu \tau_{\nu\rho} - \Gamma^\sigma_{\mu\nu}\tau_{\sigma\rho} - \Gamma^\sigma_{\mu\rho}\tau_{\nu\sigma} = 0$$
$$\nabla_\mu h^{\nu\rho} \equiv \partial_\mu h^{\nu\rho} + \Gamma^\nu_{\mu\sigma} h^{\sigma\rho} + \Gamma^\rho_{\mu\sigma} h^{\nu\sigma} = 0$$

$$\Gamma^\rho_{\mu\nu} = \tau^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\sigma}\left(\partial_\mu h_{\sigma\nu} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu}\right) + h^{\rho\sigma}\tau_\mu \partial_{[\sigma} m_{\nu]} + h^{\rho\sigma}\tau_\nu \partial_{[\sigma} m_{\mu]}$$
$$+ h^{\rho\sigma}\tau_{(\mu} T^m_{\nu)\sigma} - h^{\rho\sigma} e_{(\mu|a|} T_{\nu)\sigma}{}^a + \frac{1}{2} e_a{}^\rho T_{\mu\nu}{}^a$$

$$2\Gamma^\rho_{[\mu\nu]} = 2\tau^\rho \partial_{[\mu}\tau_{\nu]} + e^\rho_a T^a_{\mu\nu}.$$

$$\partial_{[\mu}\tau_{\nu]} = 0$$

$$\delta\tau_\mu = z\Lambda_D \tau_\mu, \delta e^a_\mu = \Lambda_D e^a_\mu$$

$$\tau_{[\mu}\partial_\nu \tau_{\rho]} = 0 \ \Leftrightarrow \ \tau_{ab} \equiv e_a{}^\mu e_b{}^\nu \partial_{[\mu}\tau_{\nu]} = 0$$

$$\delta\tau_\mu{}^A = \lambda_M \varepsilon^A{}_B \tau_\mu{}^B, \qquad \delta e_\mu{}^a = \lambda^a{}_b e_\mu{}^b - \lambda_A{}^a \tau_\mu{}^A,$$
$$\delta b_{\mu\nu} = -2\varepsilon_{AB}\lambda^A{}_a \tau_{[\mu}{}^B e_{\nu]}{}^a.$$

$$\delta b_{\mu\nu} = 2\partial_{[\mu}\theta_{\nu]}.$$

$$\tau_A{}^\mu \tau_\mu{}^B = \delta^B_A, \quad \tau_A{}^\mu e_\mu{}^a = 0, e_a{}^\mu \tau_\mu{}^A = 0,$$
$$e_\mu{}^a e_b{}^\mu = \delta^a_b, \quad \tau_\mu{}^A \tau_A{}^\nu + e_\mu{}^a e_a{}^\nu = \delta^\nu_\mu.$$

$$\tau_{\mu\nu} = \tau_\mu{}^A \tau_\nu{}^B \eta_{AB}, h^{\mu\nu} = e_a{}^\mu e_b{}^\nu \delta^{ab}$$

$$\delta\omega_\mu = \partial_\mu \lambda_M, \delta\omega^{ab}_\mu = \partial_\mu \lambda^{ab} + 2\lambda^{[a|c|}\omega^{b]}_{\mu c}$$
$$\delta\omega^{Aa}_\mu = \partial_\mu \lambda^{Aa} + \lambda_M \varepsilon^A{}_B \omega^{Ba}_\mu + \lambda^a{}_b \omega_\mu{}^{Ab} - \varepsilon^A{}_B \lambda^{Ba}\omega_\mu + \lambda^{Ab}\omega_{\mu b}{}^a$$

$$2\partial_{[\mu}\tau^A_{\nu]} - 2\varepsilon^A_B \omega_{[\mu}\tau^B_{\nu]} = T^A_{\mu\nu}$$
$$2\partial_{[\mu}e^a_{\nu]} - 2\omega^{ab}_{[\mu} e_{\nu]b} + 2\omega^{Aa}_{[\mu}\tau_{\nu]A} = T^a_{\mu\nu}$$
$$3\partial_{[\mu} b_{\nu\rho]} + 6\varepsilon_{AB}\omega^{Ab}_{[\mu}\tau^B_\nu e_{\rho]b} = T^{(b)}_{\mu\nu\rho}$$

$$2\tau_{(A|}{}^\mu e_a{}^\nu \partial_{[\mu}\tau_{\nu]|B)} = \tau_{(A|}{}^\mu e_a{}^\nu T_{\mu\nu|B)}, 2e_a{}^\mu e_b{}^\nu \partial_{[\mu}\tau_{\nu]}{}^A = e_a{}^\mu e_b{}^\nu T_{\mu\nu}{}^A,$$
$$3e_a{}^\mu e_b{}^\nu e_c{}^\rho \partial_{[\mu} b_{\nu\rho]} = e_a{}^\mu e_b{}^\nu e_c{}^\rho T^{(b)}_{\mu\nu\rho},$$

$$\tau^\mu_{\{A|}\omega^a_{\mu|B\}} \equiv \tau^\mu_{(A|}\omega^a_{\mu|B)} - \frac{1}{2}\eta_{AB}\tau^\mu_C \omega^{Ca}_\mu,$$

$$\partial_\mu \tau_\nu{}^A - \varepsilon^A{}_B \omega_\mu \tau_\nu{}^B - \Gamma^\rho_{\mu\nu}\tau_\rho{}^A = 0$$
$$\partial_\mu e_\nu{}^a - \omega_\mu{}^{ab} e_{\nu b} + \omega_\mu{}^{Aa}\tau_{\nu A} - \Gamma^\rho_{\mu\nu} e_\rho{}^a = 0$$

$$\nabla_\mu \tau_{\nu\rho} \equiv \partial_\mu \tau_{\nu\rho} - \Gamma^\sigma_{\mu\nu}\tau_{\sigma\rho} - \Gamma^\sigma_{\mu\rho}\tau_{\nu\sigma} = 0$$
$$\nabla_\mu h^{\nu\rho} \equiv \partial_\mu h^{\nu\rho} + \Gamma^\nu_{\mu\sigma} h^{\sigma\rho} + \Gamma^\rho_{\mu\sigma} h^{\nu\sigma} = 0$$

$$\delta T_{\mu\nu}{}^A = \lambda_M \varepsilon^A{}_B T_{\mu\nu}{}^B, \delta T_{\mu\nu}{}^a = \lambda^a{}_b T^b_{\mu\nu} - \lambda_A{}^a T_{\mu\nu}{}^A,$$
$$\delta T^{(b)}_{\mu\nu\rho} = -3\varepsilon_{AB}\lambda^A{}_a T_{[\mu\nu}{}^B e_{\rho]}{}^a + 3\varepsilon_{AB}\lambda^A{}_a T_{[\mu\nu}{}^a \tau_{\rho]}{}^B.$$

$$2\Gamma^\rho_{[\mu\nu]} = \tau_A{}^\rho T_{\mu\nu}{}^A + e_a{}^\rho T_{\mu\nu}{}^a.$$

$$\delta E_\mu{}^{\hat{A}} = \xi^\nu \partial_\nu E_\mu{}^{\hat{A}} + \partial_\mu \xi^\nu E_\nu{}^{\hat{A}} + \Lambda^{\hat{A}}{}_{\hat{B}} E_\mu{}^{\hat{B}}$$
$$\delta\Psi_{\mu i} = \xi^\nu \partial_\nu \Psi_{\mu i} + \partial_\mu \xi^\nu \Psi_{\nu i} + \frac{1}{4}\Lambda^{\hat{A}\hat{B}}\gamma_{\hat{A}\hat{B}}\Psi_{\mu i}$$

$$\delta E_\mu{}^{\hat{A}} = \frac{1}{2}\delta^{ij}\bar{\eta}_i\gamma^{\hat{A}}\Psi_{\mu j}$$

$$\delta\Psi_{\mu i} = D_\mu\eta_i = \partial_\mu\eta_i - \frac{1}{4}\Omega_\mu^{\hat{A}\hat{B}}(E,\Psi_i)\gamma_{\hat{A}\hat{B}}\eta_i$$

$$\Omega_\mu^{\hat{A}\hat{B}}(E,\Psi_i) = -2E^{\nu[\hat{A}}\left(\partial_{[\mu}E_{\nu]}^{\hat{B}]} - \frac{1}{4}\delta^{ij}\bar{\Psi}_{[\mu i}\gamma^{\hat{B}]}\Psi_{\nu]j}\right)$$
$$+E_{\mu\hat{C}}E^{\nu\hat{A}}E^{\rho\hat{B}}\left(\partial_{[\nu}E_{\rho]}{}^{\hat{C}} - \frac{1}{4}\delta^{ij}\bar{\Psi}_{[\nu i}\gamma^{\hat{C}}\Psi_{\rho]j}\right).$$

$$\delta\Omega_\mu{}^{\hat{A}\hat{B}}(E,\Psi_i) = -\frac{1}{2}\delta^{ij}E^{\nu[\hat{A}}\bar{\eta}_i\gamma^{\hat{B}]}\hat{\Psi}_{\mu\nu j} + \frac{1}{4}\delta^{ij}E_{\mu\hat{C}}E^{\nu\hat{A}}E^{\rho\hat{B}}\bar{\eta}_i\gamma^{\hat{C}}\hat{\Psi}_{\nu\rho j}$$

$$\hat{\Psi}_{\mu\nu i} = 0.$$

$$\delta M_\mu = \xi^\nu\partial_\nu M_\mu + \partial_\mu\xi^\nu M_\nu + \partial_\mu\Lambda$$

$$\delta M_\mu = \frac{1}{2}\varepsilon^{ij}\bar{\eta}_i\Psi_{\mu j}$$

$$\hat{F}_{\mu\nu}(M) \equiv 2\partial_{[\mu}M_{\nu]} - \frac{1}{2}\varepsilon^{ij}\bar{\Psi}_{[\mu i}\Psi_{\nu]j} = 0$$

$$\hat{F}_{\mu\nu}(M) = 0 \rightarrow \hat{\Psi}_{\mu\nu i} = 0 \rightarrow \hat{R}_{\mu\nu}{}^{\hat{A}\hat{B}}(\Omega) = 0.$$

$$[M_{\hat{A}\hat{B}}, P_{\hat{C}}] = -2\eta_{\hat{C}[\hat{A}}P_{\hat{B}]}, \qquad [M_{\hat{A}\hat{B}}, M_{\hat{C}\hat{D}}] = 4\eta_{[\hat{A}[\hat{C}}M_{\hat{D}]\hat{B}]},$$
$$[M_{\hat{A}\hat{B}}, Q^i] = -\frac{1}{2}\gamma_{\hat{A}\hat{B}}Q^i, \qquad \{Q^i, Q^j\} = -\gamma^{\hat{A}}C^{-1}P_{\hat{A}}\delta^{ij} + C^{-1}\mathcal{Z}\varepsilon^{ij}$$

$$Q_\pm = \frac{1}{\sqrt{2}}(Q^1 \pm \gamma_0 Q^2)$$

$$Q_- \rightarrow \sqrt{\omega}Q_-, Q_+ \rightarrow \frac{1}{\sqrt{\omega}}Q_+,$$
$$\mathcal{Z} \rightarrow -\omega M + \frac{1}{2\omega}H, P_0 \rightarrow \omega M + \frac{1}{2\omega}H,$$

$$[J_{ab}, P_c] = -2\delta_{c[a}P_{b]}, \qquad [J_{ab}, G_c] = -2\delta_{c[a}G_{b]}$$
$$[G_a, H] = -P_a, \qquad [G_a, P_b] = -\delta_{ab}M$$
$$[J_{ab}, Q_\pm] = -\frac{1}{2}\gamma_{ab}Q_\pm, \qquad [G_a, Q_+] = -\frac{1}{2}\gamma_{a0}Q_-$$
$$\{Q_+, Q_+\} = -\gamma^0 C^{-1}H, \qquad \{Q_+, Q_-\} = -\gamma^a C^{-1}P_a$$
$$\{Q_-, Q_-\} = -2\gamma^0 C^{-1}M$$

$$E_\mu^0 = \omega\tau_\mu + \frac{1}{2\omega}m_\mu, E_\mu^a = e_\mu^a$$
$$M_\mu = \omega\tau_\mu - \frac{1}{2\omega}m_\mu$$

$$\tau_\mu = \frac{1}{2\omega}(E_\mu{}^0 + M_\mu), \qquad m_\mu = \omega(E_\mu{}^0 - M_\mu)$$

$$\lambda^{ab} = \Lambda^{ab}, \lambda^a = \omega\Lambda_0^a, \sigma = -\omega\Lambda,$$

$$\Psi_\pm = \frac{1}{\sqrt{2}}(\Psi_1 \pm \gamma_0\Psi_2)$$

$$\Psi_{\mu+} = \sqrt{\omega}\psi_{\mu+}, \quad \eta_+ = \sqrt{\omega}\varepsilon_+$$
$$\Psi_{\mu-} = \frac{1}{\sqrt{\omega}}\psi_{\mu-}, \quad \eta_- = \frac{1}{\sqrt{\omega}}\varepsilon_-$$

$$\delta\tau_\mu = \frac{1}{2}\bar{\varepsilon}_+\gamma^0\psi_{\mu+},$$
$$\delta e_\mu^a = \frac{1}{2}\bar{\varepsilon}_+\gamma^a\psi_{\mu-} + \frac{1}{2}\bar{\varepsilon}_-\gamma^a\psi_{\mu+},$$
$$\delta m_\mu = \bar{\varepsilon}_-\gamma^0\psi_{\mu-},$$

$$\delta\psi_{\mu+} = \partial_\mu\varepsilon_+ - \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\varepsilon_+$$
$$\delta\psi_{\mu-} = \partial_\mu\varepsilon_- - \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\varepsilon_- + \frac{1}{2}\omega_\mu^a\gamma_{a0}\varepsilon_+.$$

$$\delta\psi_{\mu+} = \frac{1}{4}\lambda^{ab}\gamma_{ab}\psi_{\mu+}$$
$$\delta\psi_{\mu-} = \frac{1}{4}\lambda^{ab}\gamma_{ab}\psi_{\mu-} - \frac{1}{2}\lambda^a\gamma_{a0}\psi_{\mu+}$$

$$\omega_\mu^{ab}(e,\tau,m,\psi_\pm) = -2e^{[a|\nu|}\left(\partial_{[\mu}e_{\nu]}^{b]} - \frac{1}{2}\bar{\psi}_{[\mu+}\gamma^{b]}\psi_{\nu]-}\right)$$
$$+e_\mu^c e^{a\nu}e^{b\rho}\left(\partial_{[\nu}e_{\rho]}^c - \frac{1}{2}\bar{\psi}_{[\nu+}\gamma^c\psi_{\rho]-}\right)$$
$$-\tau_\mu e^{a\nu}e^{b\rho}\left(\partial_{[\nu}m_{\rho]} - \frac{1}{2}\bar{\psi}_{[\nu-}\gamma^0\psi_{\rho]-}\right)$$
$$\omega_\mu^a(e,\tau,m,\psi_\pm) = \tau^\nu\left(\partial_{[\mu}e_{\nu]}^a - \frac{1}{2}\bar{\psi}_{[\mu+}\gamma^a\psi_{\nu]-}\right)$$
$$+e_{\mu b}e^{a\nu}\tau^\rho\left(\partial_{[\nu}e_{\rho]}^b - \frac{1}{2}\bar{\psi}_{[\nu+}\gamma^b\psi_{\rho]-}\right)$$
$$+e^{a\nu}\left(\partial_{[\mu}m_{\nu]} - \frac{1}{2}\bar{\psi}_{[\mu-}\gamma^0\psi_{\nu]-}\right)$$
$$-\tau_\mu e^{a\nu}\tau^\rho\left(\partial_{[\nu}m_{\rho]} - \frac{1}{2}\bar{\psi}_{[\nu-}\gamma^0\psi_{\rho]-}\right)$$

$$T_{\mu\nu}^a = \bar{\psi}_{[\mu+}\gamma^a\psi_{\nu]-} \text{ and } T_{\mu\nu}^{(m)} = \bar{\psi}_{[\mu-}\gamma^0\psi_{\nu]-}$$

$$\hat{R}_{\mu\nu}^a(P) \equiv R_{\mu\nu}^a(P) - \bar{\psi}_{[\mu+}\gamma^a\psi_{\nu]-} = 0$$
$$\hat{R}_{\mu\nu}(M) \equiv R_{\mu\nu}(M) - \bar{\psi}_{[\mu-}\gamma^0\psi_{\nu]-} = 0$$

$$\hat{R}_{\mu\nu}^a(G) \equiv 2\partial_{[\mu}\omega_{\nu]}^a - 2\omega_{[\mu}^{ab}\omega_{\nu]b}$$
$$\hat{R}_{\mu\nu}^{ab}(J) \equiv 2\partial_{[\mu}\omega_{\nu]}^{ab}$$
$$\hat{R}_{\mu\nu}(H) \equiv 2\partial_{[\mu}\tau_{\nu]} - \frac{1}{2}\bar{\psi}_{[\mu+}\gamma^0\psi_{\nu]+}$$
$$\hat{\psi}_{\mu\nu+} \equiv 2\partial_{[\mu}\psi_{\nu]+} - \frac{1}{2}\omega_{[\mu}^{ab}\gamma_{ab}\psi_{\nu]+}$$
$$\hat{\psi}_{\mu\nu-} \equiv 2\partial_{[\mu}\psi_{\nu]-} - \frac{1}{2}\omega_{[\mu}^{ab}\gamma_{ab}\psi_{\nu]-} + \omega_{[\mu}^a\gamma_{a0}\psi_{\nu]+}$$

$$\delta_Q\omega_\mu^{ab}(e,\tau,m,\psi_\pm) = \frac{1}{2}\bar{\varepsilon}_+\gamma^{[b}\hat{\psi}_{\mu-}^{a]} + \frac{1}{4}e_{\mu c}\bar{\varepsilon}_+\gamma^c\hat{\psi}_{--}^{ab} - \frac{1}{2}\tau_\mu\bar{\varepsilon}_-\gamma^0\hat{\psi}_-^{ab}$$
$$+\frac{1}{2}\bar{\varepsilon}_-\gamma^{[b}\hat{\psi}_{\mu+}^{a]} + \frac{1}{4}e_{\mu c}\bar{\varepsilon}_-\gamma^c\hat{\psi}_+^{ab}$$
$$\delta_Q\omega_\mu^a(e,\tau,m,\psi_\pm) = \frac{1}{2}\bar{\varepsilon}_-\gamma^0\hat{\psi}_{\mu\ -}^a + \frac{1}{2}\tau_\mu\bar{\varepsilon}_-\gamma^0\hat{\psi}_0^a + \frac{1}{4}e_{\mu b}\bar{\varepsilon}_+\gamma^b\hat{\psi}_{0-}^a$$
$$+\frac{1}{4}\bar{\varepsilon}_+\gamma^a\hat{\psi}_{\mu0-} + \frac{1}{4}e_{\mu b}\bar{\varepsilon}_-\gamma^b\hat{\psi}_{0+}^a + \frac{1}{4}\bar{\varepsilon}_-\gamma^a\hat{\psi}_{\mu0+}$$

$$\hat{R}_{\mu\nu}(H) = 0.$$

$$\hat{\psi}_{ab-} = 0$$
$$\hat{R}_{\mu\nu}(H) = 0 \rightarrow \hat{\psi}_{\mu\nu+} = 0 \rightarrow \hat{R}_{\mu\nu}{}^{ab}(J) = 0$$
$$\gamma^a\hat{\psi}_{a0-} = 0 \rightarrow \hat{R}_{0a}{}^a(G) = 0.$$

$$\hat{R}_{ab}^c(G) = 0, \hat{R}_{0[a}^{b]}(G) = 0$$

$$[\delta_Q(\varepsilon_1), \delta_Q(\varepsilon_2)] = \delta_{\text{g.c.t.}}(\xi^\mu) + \delta_{J_{ab}}(\lambda^a{}_b) + \delta_{G_a}(\lambda^a) + \delta_{Q_+}(\varepsilon_+) + \delta_{Q_-}(\varepsilon_-) + \delta_M(\sigma)$$

$$\xi^\mu = \frac{1}{2}(\bar{\varepsilon}_{2+}\gamma^0\varepsilon_{1+})\tau^\mu + \frac{1}{2}(\bar{\varepsilon}_{2+}\gamma^a\varepsilon_{1-} + \bar{\varepsilon}_{2-}\gamma^a\varepsilon_{1+})e_a^\mu$$
$$\lambda_b^a = -\xi^\mu\omega_\mu^a{}_b$$
$$\lambda^a = -\xi^\mu\omega_\mu^a$$
$$\varepsilon_\pm = -\xi^\mu\psi_{\mu\pm}$$
$$\sigma = -\xi^\mu m_\mu + (\bar{\varepsilon}_{2-}\gamma^0\varepsilon_{1-})$$

$$\tau_\mu(x^\nu) = \delta_\mu{}^\emptyset, \omega_\mu{}^{ab}(x^\nu) = 0.$$

$$\xi^\emptyset(x^\nu) = \xi^\emptyset, \lambda^{ab}(x^\nu) = \lambda^{ab}$$

$$e_i{}^a(x^\nu) = \delta_i{}^a.$$

$$\xi^a(x^\nu) = \xi^a(t) - \lambda_{ai}x^i.$$

$$\tau_\mu(x^\nu) = \delta_\mu{}^\emptyset, \qquad e_\mu{}^a(x^\nu) = (-\tau^a(x^\nu), \delta_i^a),$$
$$\tau^\mu(x^\nu) = (1, \tau^a(x^\nu)), \quad e_a{}^\mu(x^\nu) = (0, \delta^i{}_a),$$

$$\delta\tau^a(x^\nu) = \lambda^a{}_b\tau^b(x^\nu) - \lambda^c{}_d x^d\partial_c\tau^a(x^\nu) + \xi^\emptyset\partial_\emptyset\tau^a(x^\nu) + \xi^j(t)\partial_j\tau^a(x^\nu) - \dot{\xi}^a(t) - \lambda^a(x^\nu)$$
$$\delta m_i(x^\nu) = \xi^\emptyset\partial_\emptyset m_i(x^\nu) + \xi^j(t)\partial_j m_i(x^\nu) + \lambda_i{}^j m_j(x^\nu) - \lambda^j{}_k x^k\partial_j m_i(x^\nu) + \lambda_i(x^\nu) + \partial_i\sigma(x^\nu)$$
$$\delta m_\emptyset(x^\nu) = \xi^\emptyset\partial_\emptyset m_\emptyset(x^\nu) + \dot{\xi}^i(t)m_i(x^\nu) + \xi^i(t)\partial_i m_\emptyset(x^\nu) - \lambda^i{}_j x^j\partial_i m_\emptyset(x^\nu) - \lambda^a(x^\nu)\tau_a(x^\nu) + \dot{\sigma}(x^\nu)$$

$$\partial_{[i}\tau_{j]}(x^\nu) + \partial_{[i}m_{j]}(x^\nu) = 0$$

$$\tau_i(x^\nu) + m_i(x^\nu) = \partial_i m(x^\nu)$$

$$\delta m(x^\nu) = \xi^\emptyset\partial_\emptyset m(x^\nu) - \dot{\xi}^k(t)x^k + \xi^j(t)\partial_j m(x^\nu) - \lambda^j{}_k x^k\partial_j m(x^\nu) + \sigma(x^\nu) + Y(t)$$

$$m(x^\nu) = 0$$

$$\sigma(x^\mu) = \sigma(t) + \dot{\xi}^a(t)x_a$$

$$\delta\tau^i(x^\nu) = \lambda^i{}_j\tau^j(x^\nu) - \lambda^j{}_k x^k\partial_j\tau^i(x^\nu) + \xi^\emptyset\partial_\emptyset\tau^i(x^\nu) + \xi^j(t)\partial_j\tau^i(x^\nu) - \dot{\xi}^i(t) - \lambda^i(x^\nu)$$
$$\delta m_\emptyset(x^\nu) = \xi^\emptyset\partial_\emptyset m_\emptyset(x^\nu) - \dot{\xi}^i(t)\tau_i(x^\nu) + \xi^i(t)\partial_i m_\emptyset(x^\nu) + \ddot{\xi}^k(t)x^k - \lambda^i{}_j x^j\partial_i m_\emptyset(x^\nu) - \lambda^i(x^\nu)\tau_i(x^\nu) + \dot{\sigma}(t)$$

$$\tau^a(x^\nu) = 0.$$

$$\lambda^i(x^\nu) = -\dot{\xi}^i(t)$$

$$m_\emptyset(x^\nu) \equiv \Phi(x^\nu)$$

$$\delta\Phi(x^\nu) = \xi^\emptyset\partial_\emptyset\Phi(x^\nu) + \xi^i(t)\partial_i\Phi(x^\nu) + \ddot{\xi}^k(t)x^k - \lambda^i{}_j x^j\partial_i\Phi(x^\nu) + \dot{\sigma}(t)$$

$$\omega_\emptyset^a(x^\nu) = -\partial^a\Phi(x^\nu)$$

$$\triangle\Phi = \partial_a\partial_a\Phi = 0$$

$$[\delta_{\xi^{\emptyset}}, \delta_{\xi^i(t)}]\Phi(x^\nu) = \delta_{\xi^i(t)}(-\xi^{\emptyset}\dot{\xi}^i(t))\Phi(x^\nu)$$
$$[\delta_{\xi^{\emptyset}}, \delta_{\sigma(t)}]\Phi(x^\nu) = \delta_{\sigma(t)}(-\xi^{\emptyset}\dot{\sigma}(t))\Phi(x^\nu)$$
$$\left[\delta_{\xi_1^i(t)}, \delta_{\xi_2^i(t)}\right]\Phi(x^\nu) = \delta_{\sigma(t)}(\dot{\xi}_1^j(t)\xi_2^j(t) - \dot{\xi}_2^j(t)\xi_1^j(t))\Phi(x^\nu)$$
$$[\delta_{\xi^i(t)}, \delta_{\lambda jk}]\Phi(x^\nu) = \delta_{\xi^i(t)}(\lambda^i{}_j\xi^j(t))\Phi(x^\nu)$$

gauge condition/restriction	compensating transformation
$\tau_\mu(x^\nu) = \delta_\mu{}^\emptyset$ $\omega_\mu{}^{ab}(x^\nu) = 0$ $e_i{}^a(x^\nu) = \delta_i^a$ $\tau_i(x^\nu) + m_i(x^\nu) = \partial_i m(x^\nu)$ $m(x^\nu) = 0$ $\tau^a(x^\nu) = 0$	$\xi^\emptyset(x^\nu) = \xi^\emptyset$ $\lambda^{ab}(x^\nu) = \lambda^{ab}$ $\xi^a(x^\nu) = \xi^a(t) - \lambda_{ai}x^i$ – $\sigma(x^\nu) = \sigma(t) + \dot{\xi}^a(t)x_a$ $\lambda^i(x^\nu) = -\dot{\xi}^i(t)$
$m_\emptyset(x^\nu) = \Phi(x^\nu)$	$\omega_\emptyset{}^a(x^\nu) = -\partial^a\Phi(x^\nu)$

$$\tau_\mu(x^\nu) = \delta_\mu^\emptyset, \omega_\mu^{ab}(x^\nu) = 0, \psi_{\mu+}(x^\nu) = 0$$

$$\xi^\emptyset(x^\nu) = \xi^\emptyset, \lambda^{ab}(x^\nu) = \lambda^{ab}, \varepsilon_+(x^\nu) = \varepsilon_+.$$

$$e_i^a(x^\nu) = \delta_i^a$$

$$\partial_{[i}\omega_{j]}^a = 0, \partial_{[i}\Psi_{j]-} = 0$$

$$\omega_i^a = \partial_i\omega^a$$

$$\psi_{i-}(x^\nu) = 0$$

$$\varepsilon_-(x^\nu) = \varepsilon_-(t) - \frac{1}{2}\omega^a\gamma_{a0}\varepsilon_+$$

$$\xi^i(x^\nu) = \xi^i(t) - \lambda^i{}_j x^j$$

$$\partial_{[i}(\tau_{j]} + m_{j]})(x^\nu) = 0$$

$$\tau_i(x^\nu) + m_i(x^\nu) = \partial_i m(x^\nu)$$

$$\delta\partial_i m = \xi^\emptyset\partial_\emptyset\partial_i m + \xi^j(t)\partial_j\partial_i m + \lambda_i^j\partial_j m - \lambda^m{}_n x^n\partial_m\partial_i m + \partial_i\sigma(x^\nu) - \dot{\xi}^i(t)$$
$$-\frac{1}{2}\bar{\varepsilon}_+\gamma_i\psi_{\emptyset-}$$

$$m(x^\nu) = 0$$

$$\partial_i\sigma(x^\nu) = \dot{\xi}^i(t) + \frac{1}{2}\bar{\varepsilon}_+\gamma_i\psi_{\emptyset-}(x^\nu)$$

$$\delta\tau_i = \xi^\emptyset\partial_\emptyset\tau_i + \xi^j(t)\partial_j\tau_i - \dot{\xi}^i(t) + \lambda_{ij}\tau^j - \lambda^k{}_l x^l\partial_k\tau_i - \lambda_i(x^\nu) - \frac{1}{2}\bar{\varepsilon}_+\gamma_i\psi_{\emptyset-}$$
$$\delta\partial_i m_\emptyset = \xi^\emptyset\partial_\emptyset\partial_i m_\emptyset + \xi^j(t)\partial_j\partial_i m_\emptyset + \ddot{\xi}^i(t) - \dot{\xi}^j(t)\partial_i\tau_j + \lambda_i{}^j\partial_j m_\emptyset - \lambda^m{}_n x^n\partial_m\partial_i m_\emptyset -$$
$$-\partial_i(\lambda^j(x^\nu)\tau_j) + \bar{\varepsilon}_-(t)\gamma^0\partial_i\psi_{\emptyset-} + \frac{1}{2}\partial_i(\omega^a\bar{\varepsilon}_+\gamma_a\psi_{\emptyset-}) + \frac{1}{2}\bar{\varepsilon}_+\gamma_i\dot{\psi}_{\emptyset-},$$
$$\delta\psi_{\emptyset-} = \xi^\emptyset\partial_\emptyset\psi_{\emptyset-} + \xi^i(t)\partial_i\psi_{\emptyset-} - \lambda^i{}_j x^j\partial_i\psi_{\emptyset-} + \frac{1}{4}\lambda^{ab}\gamma_{ab}\psi_{\emptyset-}$$
$$+\dot{\varepsilon}_-(t) + \frac{1}{2}(\omega_\emptyset{}^a - \dot{\omega}^a)\gamma_{a0}\varepsilon_+$$

$$\omega_i^a \equiv \partial_i \omega^a = -\partial_i \tau^a \ \rightarrow \ \omega^a = -\tau^a,$$
$$\omega_\emptyset^a = -\dot{\tau}^a - \partial_a \left(m_\emptyset - \frac{1}{2}\tau^i \tau^i \right).$$

$$\tau^i(x^v) = 0$$

$$\lambda^i(x^v) = -\dot{\xi}^i(t) - \frac{1}{2}\bar{\varepsilon}_+ \gamma_i \psi_{\emptyset -}(x^v)$$

$$\omega^a = 0, \omega_\emptyset{}^a = -\partial^a m_\emptyset \equiv -\partial^a \Phi$$

$$\Phi_i = \partial_i \Phi, \Psi = \psi_{\emptyset -}$$

$$\delta\Phi_i = \xi^\emptyset \partial_\emptyset \Phi_i + \xi^j(t)\partial_j \Phi_i + \ddot{\xi}^i(t) + \lambda_i^j \Phi_j - \lambda^m{}_n x^n \partial_m \Phi_i + \bar{\varepsilon}_-(t)\gamma^0 \partial_i \Psi$$
$$+ \frac{1}{2}\bar{\varepsilon}_+ \gamma_i \dot{\Psi},$$
$$\delta\Psi = \xi^\emptyset \partial_\emptyset \Psi + \xi^i(t)\partial_i \Psi - \lambda^i{}_j x^j \partial_i \Psi + \frac{1}{4}\lambda^{ab}\gamma_{ab}\Psi + \dot{\varepsilon}_-(t) - \frac{1}{2}\Phi^i \gamma_{i0}\varepsilon_+.$$

$$\partial_{[i}\Phi_{j]}(x^v) = 0$$

$$\gamma^i \partial_i \Psi(x^v) = 0 \ \Leftrightarrow \ \partial_{[i}\gamma_{j]}\Psi(x^v) = 0.$$

$$\partial^i \Phi_i(x^v) = 0$$

$$\gamma_i \Psi = \partial_i \chi$$

$$\gamma^1 \partial_1 \chi = \gamma^2 \partial_2 \chi$$

$$\Psi = \gamma^1 \partial_1 \chi = \gamma^2 \partial_2 \chi = \frac{1}{2}\gamma^i \partial_i \chi$$

$$\delta\Phi_i = \partial_i(\delta\Phi)$$

$$\delta\Phi = \xi^\emptyset \partial_\emptyset \Phi + \xi^i(t)\partial_i \Phi + \ddot{\xi}^i(t)x^i - \lambda^m{}_n x^n \partial_m \Phi + \frac{1}{2}\bar{\varepsilon}_-(t)\gamma^{0i}\partial_i \chi + \frac{1}{2}\bar{\varepsilon}_+ \dot{\chi} + \sigma(t)$$

$$\gamma_i \delta\Psi = \partial_i(\delta\chi)$$

$$-\frac{1}{2}\gamma_i \Phi^j \gamma_{j0}\varepsilon_+ = -\frac{1}{2}\gamma_i \partial^j \Phi \gamma_{j0}\varepsilon_+ = -\frac{1}{2}\partial^j \Phi \gamma_{ij0}\varepsilon_+ - \frac{1}{2}\partial_i \Phi \gamma_0 \varepsilon_+$$

$$\partial_i \Phi = \varepsilon_{ij}\partial^j \Xi, \partial_i \Xi = -\varepsilon_{ij}\partial^j \Phi$$

$$-\frac{1}{2}\gamma_i \Phi^j \gamma_{j0}\varepsilon_+ = \frac{1}{2}\partial_i \Xi \varepsilon_+ - \frac{1}{2}\partial_i \Phi \gamma_0 \varepsilon_+.$$

$$\delta\chi = \xi^\emptyset \partial_\emptyset \chi + \xi^i(t)\partial_i \chi - \lambda^m{}_n x^n \partial_m \chi + \frac{1}{4}\lambda^{mn}\gamma_{mn}\chi$$
$$+ x^i \gamma_i \dot{\varepsilon}_-(t) + \frac{1}{2}\Xi\varepsilon_+ - \frac{1}{2}\Phi\gamma_0 \varepsilon_+ + \eta(t)$$

$$\partial_i(\delta\Xi) = -\varepsilon_{ij}\partial^j(\delta\Phi)$$

$$\delta\Xi = \xi^\emptyset \partial_\emptyset \Xi + \xi^i(t)\partial_i \Xi + \ddot{\xi}^i(t)\varepsilon_{ij}x^j - \lambda^m{}_n x^n \partial_m \Xi$$
$$+ \frac{1}{2}\bar{\varepsilon}_-(t)\gamma^i \partial_i \chi - \frac{1}{2}\bar{\varepsilon}_+ \gamma_0 \dot{\chi} + \tau(t)$$

$$[\delta_{\varepsilon_{1-}(t)},\delta_{\varepsilon_{2-}(t)}]=\delta_{\sigma(t)}\left(\frac{\mathrm{d}}{\mathrm{d}t}(\bar{\varepsilon}_{2-}(t)\gamma^0\varepsilon_{1-}(t))\right)$$

$$[\delta_{\varepsilon_{1+}},\delta_{\varepsilon_{2+}}]=\delta_{\xi^{\emptyset}}\left(\frac{1}{2}(\bar{\varepsilon}_{2+}\gamma^0\varepsilon_{1+})\right)$$

$$[\delta_{\varepsilon_{+}},\delta_{\varepsilon_{-}(t)}]=\delta_{\xi^{i}(t)}\left(\frac{1}{2}(\bar{\varepsilon}_{-}(t)\gamma^i\varepsilon_{+})\right)$$

$$[\delta_{\eta(t)},\delta_{\varepsilon_{+}}]=\delta_{\sigma(t)}\left(\frac{1}{2}(\bar{\varepsilon}_{+}\dot{\eta}(t))\right)$$

$$[\delta_{\xi^i(t)},\delta_{\varepsilon_+}]=\delta_{\varepsilon_-(t)}\left(\frac{1}{2}\dot{\xi}^i(t)\gamma_{0i}\varepsilon_+\right),\qquad [\delta_{\lambda^{ij}},\delta_{\varepsilon_+}]=\delta_{\varepsilon_+}\left(-\frac{1}{4}\lambda^{ij}\gamma_{ij}\varepsilon_+\right)$$

$$[\delta_{\xi^{\emptyset}},\delta_{\varepsilon_-(t)}]=\delta_{\varepsilon_-(t)}\left(-\xi^{\emptyset}\dot{\varepsilon}_-(t)\right),\qquad [\delta_{\xi^i(t)},\delta_{\varepsilon_-(t)}]=\delta_{\eta(t)}\left(-\xi^i(t)\gamma_i\dot{\varepsilon}_-(t)\right)$$

$$[\delta_{\lambda^{ij}},\delta_{\varepsilon_-(t)}]=\delta_{\varepsilon_-(t)}\left(-\frac{1}{4}\lambda^{ij}\gamma_{ij}\varepsilon_-(t)\right),\qquad [\delta_{\sigma(t)},\delta_{\varepsilon_+}]=\delta_{\eta(t)}\left(\frac{1}{2}(\sigma(t)\gamma^0\varepsilon_+)\right)$$

$$[\delta_{\xi^{\emptyset}},\delta_{\eta(t)}]=\delta_{\eta(t)}\left(-\xi^{\emptyset}\dot{\eta}(t)\right),\qquad [\delta_{\lambda^{ij}},\delta_{\eta(t)}]=\delta_{\eta(t)}\left(-\frac{1}{4}\lambda^{ij}\gamma_{ij}\eta(t)\right)$$

gauge condition/restriction	compensating transformation
$\tau_\mu(x^\nu)=\delta_\mu{}^\emptyset$	$\xi^\emptyset(x^\nu)=\xi^\emptyset$
$\omega_\mu{}^{ab}(x^\nu)=0$	$\lambda^{ab}(x^\nu)=\lambda^{ab}$
$\psi_{\mu+}(x^\nu)=0$	$\varepsilon_+(x^\nu)=\varepsilon_+$
$e_i{}^a(x^\nu)=\delta_i^a$	$\xi^i(x^\nu)=\xi^i(t)-\lambda^i{}_jx^j$
$\psi_{i-}(x^\nu)=0$	$\varepsilon_-(x^\nu)=\varepsilon_-(t)-\frac{1}{2}\omega^a(x^\nu)\gamma_{a0}\varepsilon_+$
$\tau_i(x^\nu)+m_i(x^\nu)=\partial_i m(x^\nu)$	$-$
$m(x^\nu)=0$	$\partial_i\sigma(x^\nu)=\dot{\xi}^i(t)+\frac{1}{2}\bar{\varepsilon}_+\gamma_i\psi_{\emptyset-}(x^\nu)$
$\tau^a(x^\nu)=0$	$\lambda^i(x^\nu)=-\dot{\xi}^i(t)-\frac{1}{2}\bar{\varepsilon}_+\gamma_i\psi_{\emptyset-}(x^\nu)$
$m_\emptyset(x^\nu)=\Phi(x^\nu),\ \ \omega_\emptyset{}^a(x^\nu)=-\partial^a\Phi(x^\nu)$	$\psi_{\emptyset-}(x^\nu)=\Psi(x^\nu)$

$$[H,G_a]=-\varepsilon_{ab}P_b\,,\quad [J,G_a]=-\varepsilon_{ab}G_b\,,\quad [J,P_a]=-\varepsilon_{ab}P_b\,,$$
$$[G_a,G_b]=\varepsilon_{ab}S\,,\quad [G_a,P_b]=\varepsilon_{ab}M\,.$$

$$\mathrm{tr}(G_aP_b)=\delta_{ab},\mathrm{tr}(HS)=\mathrm{tr}(MJ)=-1.$$

$$S=\frac{k}{4\pi}\int\ \mathrm{tr}\left(A\wedge dA+\frac{2}{3}A\wedge A\wedge A\right),$$

$$A_\mu=\tau_\mu H+e_\mu{}^aP_a+\omega_\mu J+\omega_\mu{}^aG_a+m_\mu M+s_\mu S.$$

$$S=\frac{k}{4\pi}\int\ \mathrm{d}^3x\left(\varepsilon^{\mu\nu\rho}e^a_\mu R_{\nu\rho a}(G)-\varepsilon^{\mu\nu\rho}m_\mu R_{\nu\rho}(J)-\varepsilon^{\mu\nu\rho}\tau_\mu R_{\nu\rho}(S)\right),$$

$$R^a_{\mu\nu}(G)\equiv 2\partial_{[\mu}\omega^a_{\nu]}+2\varepsilon^{ab}\omega_{[\mu}\omega_{\nu]b},R_{\mu\nu}(J)\equiv 2\partial_{[\mu}\omega_{\nu]}$$
$$R_{\mu\nu}(S)\equiv 2\partial_{[\mu}s_{\nu]}+\varepsilon^{ab}\omega_{[\mu a}\omega_{\nu]b}$$

$$R^a_{\mu\nu}(P)\equiv 2\partial_{[\mu}e^a_{\nu]}+2\varepsilon^{ab}\omega_{[\mu}e_{\nu]b}-2\varepsilon^{ab}\omega_{[\mu b}\tau_{\nu]}=0$$
$$R_{\mu\nu}(M)\equiv 2\partial_{[\mu}m_{\nu]}+2\varepsilon^{ab}\omega_{[\mu a}e_{\nu]b}=0$$

$$[J,Q^\pm]=-\frac{1}{2}\gamma_0Q^\pm,\qquad [J,R]=-\frac{1}{2}\gamma_0R,\qquad [G_a,Q^+]=-\frac{1}{2}\gamma_aQ^-$$
$$[G_a,Q^-]=-\frac{1}{2}\gamma_aR,\qquad [S,Q^+]=-\frac{1}{2}\gamma_0R$$
$$\{Q^+_\alpha,Q^+_\beta\}=(\gamma_0C^{-1})_{\alpha\beta}H,\quad \{Q^+_\alpha,Q^-_\beta\}=-(\gamma^aC^{-1})_{\alpha\beta}P_a$$
$$\{Q^-_\alpha,Q^-_\beta\}=(\gamma_0C^{-1})_{\alpha\beta}M,\quad \{Q^+_\alpha,R_\beta\}=(\gamma_0C^{-1})_{\alpha\beta}M$$

$$\mathrm{tr}\left(Q_\alpha^+ R_\beta\right) = 2(C^{-1})_{\alpha\beta}, \mathrm{tr}\left(Q_\alpha^- Q_\beta^-\right) = 2(C^{-1})_{\alpha\beta}$$

$$A_\mu = \tau_\mu H + e_\mu{}^a P_a + \omega_\mu J + \omega_\mu{}^a G_a + m_\mu M + s_\mu S$$
$$+ \bar{\psi}_\mu^+ Q^+ + \bar{\psi}_\mu^- Q^- + \bar{\rho}_\mu R$$

$$S = \frac{k}{4\pi} \int \mathrm{d}^3 x \big(\varepsilon^{\mu\nu\rho} e_\mu^a R_{\nu\rho a}(G) - \varepsilon^{\mu\nu\rho} m_\mu R_{\nu\rho}(J) - \varepsilon^{\mu\nu\rho} \tau_\mu R_{\nu\rho}(S)$$
$$+ \varepsilon^{\mu\nu\rho} \bar{\psi}_\mu^+ \hat{\rho}_{\nu\rho} + \varepsilon^{\mu\nu\rho} \bar{\rho}_\mu \hat{\psi}_{\nu\rho}^+ + \varepsilon^{\mu\nu\rho} \bar{\psi}_\mu^- \hat{\psi}_{\nu\rho}^- \big)$$

$$\hat{\psi}_{\mu\nu}^+ = 2\partial_{[\mu} \psi_{\nu]}^+ + \omega_{[\mu} \gamma_0 \psi_{\nu]}^+$$
$$\hat{\psi}_{\mu\nu}^- = 2\partial_{[\mu} \psi_{\nu]}^- + \omega_{[\mu} \gamma_0 \psi_{\nu]}^- + \omega_{[\mu}{}^a \gamma_a \psi_{\nu]}^+$$
$$\hat{\rho}_{\mu\nu} = 2\partial_{[\mu} \rho_{\nu]} + \omega_{[\mu} \gamma_0 \rho_{\nu]} + \omega_{[\mu}{}^a \gamma_a \psi_{\nu]}^- + s_{[\mu} \gamma_0 \psi_{\nu]}^+$$

$$\delta\tau_\mu = -\bar{\varepsilon}^+ \gamma_0 \psi_\mu^+$$
$$\delta e_\mu^a = \bar{\varepsilon}^+ \gamma^a \psi_\mu^- + \bar{\varepsilon}^- \gamma^a \psi_\mu^+$$
$$\delta m_\mu = -\bar{\varepsilon}^- \gamma_0 \psi_\mu^- - \bar{\varepsilon}^+ \gamma_0 \rho_\mu - \bar{\eta} \gamma_0 \psi_\mu^+$$
$$\delta\psi_\mu^+ = \partial_\mu \varepsilon^+ + \frac{1}{2} \omega_\mu \gamma_0 \varepsilon^+$$
$$\delta\psi_\mu^- = \partial_\mu \varepsilon^- + \frac{1}{2} \omega_\mu \gamma_0 \varepsilon^- + \frac{1}{2} \omega_\mu^a \gamma_a \varepsilon^+$$
$$\delta\rho_\mu = \partial_\mu \eta + \frac{1}{2} \omega_\mu \gamma_0 \eta + \frac{1}{2} \omega_\mu^a \gamma_a \varepsilon^- + \frac{1}{2} s_\mu \gamma_0 \varepsilon^+$$

$$F^\alpha \equiv \mathrm{d}A^\alpha + \frac{1}{2} f_{\beta\gamma}^\alpha A^\beta \wedge A^\gamma = 0$$

$$[V_0, V_0] \subset V_0, [V_0, V_1] \subset V_1, [V_1, V_1] \subset V_0$$

$$F^{\alpha_0} = \mathrm{d}A^{\alpha_0} + \frac{1}{2} f_{\beta_0\gamma_0}^{\alpha_0} A^{\beta_0} \wedge A^{\gamma_0} + \frac{1}{2} f_{\beta_1\gamma_1}^{\alpha_0} A^{\beta_1} \wedge A^{\gamma_1} = 0$$
$$F^{\alpha_1} = \mathrm{d}A^{\alpha_1} + f_{\beta_0\gamma_1}^{\alpha_1} A^{\beta_0} \wedge A^{\gamma_1} = 0$$

$$A^{\alpha_0} = \sum_{n=0}^{\infty} \lambda^{2n} A_{(2n)}^{\alpha_0} = A_{(0)}^{\alpha_0} + \lambda^2 A_{(2)}^{\alpha_0} + \lambda^4 A_{(4)}^{\alpha_0} + \cdots,$$
$$A^{\alpha_1} = \sum_{n=0}^{\infty} \lambda^{2n+1} A_{(2n+1)}^{\alpha_1} = \lambda A_{(1)}^{\alpha_1} + \lambda^3 A_{(3)}^{\alpha_1} + \lambda^5 A_{(5)}^{\alpha_1} + \cdots.$$

$$A^{\alpha_0} = \sum_{n=0}^{N_0/2} \lambda^{2n} A_{(2n)}^{\alpha_0} = A_{(0)}^{\alpha_0} + \lambda^2 A_{(2)}^{\alpha_0} + \cdots + \lambda^{N_0} A_{(N_0)}^{\alpha_0}$$
$$A^{\alpha_1} = \sum_{n=0}^{(N_1-1)/2} \lambda^{2n+1} A_{(2n+1)}^{\alpha_1} = \lambda A_{(1)}^{\alpha_1} + \lambda^3 A_{(3)}^{\alpha_1} + \cdots + \lambda^{N_1} A_{(N_1)}^{\alpha_1}$$

$$\mathrm{d}A_{(2n_0)}^{\alpha_0} + \frac{1}{2} f_{\beta_0\gamma_0}^{\alpha_0} \sum_{r=0}^{n_0} A_{(2r)}^{\beta_0} \wedge A_{(2(n_0-r))}^{\gamma_0} + \frac{1}{2} f_{\beta_1\gamma_1}^{\alpha_0} \sum_{r=1}^{n_0} A_{(2r-1)}^{\beta_1} \wedge A_{(2(n_0-r)+1)}^{\gamma_1} = 0$$
$$\mathrm{d}A_{(2n_1+1)}^{\alpha_1} + f_{\beta_0\gamma_1}^{\alpha_1} \sum_{r=0}^{n_1} A_{(2r)}^{\beta_0} \wedge A_{(2(n_1-r)+1)}^{\gamma_1} = 0$$

$$N_1 = N_0 \pm 1.$$

$$A_\mu = E_\mu{}^{\hat{A}} P_{\hat{A}} + \Omega_\mu{}^{\hat{A}} J_{\hat{A}} + \bar{\varepsilon}^i Q_i$$

$$R_{\mu\nu}{}^{\hat{A}}(P) \equiv 2\partial_{[\mu} E_{\nu]}{}^{\hat{A}} + 2\varepsilon^{\hat{A}}{}_{\hat{B}\hat{C}} \Omega_{[\mu}{}^{\hat{B}} E_{\nu]}{}^{\hat{C}} - \bar{\psi}_\mu{}^i \gamma^{\hat{A}} \psi_\nu^j \delta_{ij}$$
$$R_{\mu\nu}{}^{\hat{A}}(J) \equiv 2\partial_{[\mu} \Omega_{\nu]}{}^{\hat{A}} + \varepsilon^{\hat{A}}{}_{\hat{B}\hat{C}} \Omega_\mu{}^{\hat{B}} \Omega_\nu{}^{\hat{C}}$$
$$R_{\mu\nu}(Q^i) \equiv 2\partial_{[\mu} \psi_{\nu]}^i + \Omega_{[\mu}{}^{\hat{A}} \gamma_{\hat{A}} \psi_{\nu]}^i$$

$$Q_{\pm}=\frac{1}{\sqrt{2}}(Q^1\pm\gamma_0 Q^2),\psi_{\mu\pm}=\frac{1}{\sqrt{2}}(\psi_\mu^1\pm\gamma_0\psi_\mu^2)$$

$$\begin{aligned}
R_{\mu\nu}{}^{0}(P)&\equiv 2\partial_{[\mu}E_{\nu]}{}^{0}+2\varepsilon_{ab}\Omega_{[\mu}{}^{a}E_{\nu]}{}^{b}-\bar\psi_{\mu+}\gamma^0\psi_{\nu+}-\bar\psi_{\mu-}\gamma^0\psi_{\nu-}=0\\
R_{\mu\nu}{}^{0}(J)&\equiv 2\partial_{[\mu}\Omega_{\nu]}{}^{0}+\varepsilon_{ab}\Omega_{[\mu}{}^{a}\Omega_{\nu]}{}^{b}=0\\
R_{\mu\nu}(Q_+)&\equiv 2\partial_{[\mu}\psi_{\nu]+}+\Omega_{[\mu}{}^{0}\gamma_0\psi_{\nu]+}+\Omega_{[\mu}{}^{a}\gamma_a\psi_{\nu]-}=0\\
R_{\mu\nu}{}^{a}(P)&\equiv 2\partial_{[\mu}E_{\nu]}{}^{a}+2\varepsilon^{a}{}_{b}\Omega_{[\mu}{}^{0}E_{\nu]}{}^{b}-2\varepsilon^{a}{}_{b}\Omega_{[\mu}{}^{b}E_{\nu]}{}^{0}-2\bar\psi_{[\mu+}\gamma^a\psi_{\nu]-}=0\\
R_{\mu\nu}{}^{a}(J)&\equiv 2\partial_{[\mu}\Omega_{\nu]}{}^{a}+2\varepsilon^{a}{}_{b}\Omega_{[\mu}{}^{0}\Omega_{\nu]}{}^{b}=0\\
R_{\mu\nu}(Q_-)&\equiv 2\partial_{[\mu}\psi_{\nu]-}+\Omega_{[\mu}{}^{0}\gamma_0\psi_{\nu]-}+\Omega_{[\mu}{}^{a}\gamma_a\psi_{\nu]+}=0
\end{aligned}$$

$$V_0=\{P^0,J^0,Q_+\}\ \text{and}\ V_1=\{P^a,J^a,Q_-\}$$

$$\begin{aligned}
&E_\mu{}^{0}=\tau_{\mu(0)}+\lambda^2\tau_{\mu(2)}, && \Omega_\mu{}^{0}=\omega_{\mu(0)}+\lambda^2\omega_{\mu(2)}, && \psi_{\mu+}=\psi_{\mu+(0)}+\lambda^2\psi_{\mu+(2)}\\
&E_\mu{}^{a}=\lambda e_{\mu(1)}{}^{a}, && \Omega_\mu{}^{a}=\lambda\omega_{\mu(1)}{}^{a}, && \psi_{\mu-}=\lambda\psi_{\mu-(1)}
\end{aligned}$$

$$\begin{aligned}
&2\partial_{[\mu}\tau_{\nu](0)}-\bar\psi_{\mu+(0)}\gamma^0\psi_{\nu+(0)}=0,\\
&2\partial_{[\mu}\tau_{\nu](2)}+2\varepsilon^{a}{}_{b}\omega_{[\mu(1)}{}^{a}e_{\nu](1)}{}^{b}-\bar\psi_{\mu-(1)}\gamma^0\psi_{\nu-(1)}-2\bar\psi_{[\mu+(0)}\gamma^0\psi_{\nu]+(2)}=0,\\
&2\partial_{[\mu}\omega_{\nu](0)}=0,\\
&2\partial_{[\mu}\omega_{\nu](2)}+\varepsilon_{ab}\omega_{[\mu(1)}{}^{a}\omega_{\nu](1)}{}^{b}=0,\\
&2\partial_{[\mu}\psi_{\nu]+(0)}+\omega_{[\mu(0)}\gamma_0\psi_{\nu]+(0)}=0,\\
&2\partial_{[\mu}\psi_{\nu]+(2)}+\omega_{[\mu(0)}\gamma_0\psi_{\nu]+(2)}+\omega_{[\mu(2)}\gamma_0\psi_{\nu]+(0)}+\omega_{[\mu(1)}{}^{a}\gamma_a\psi_{\nu]-(1)}=0,\\
&2\partial_{[\mu}e_{\nu](1)}{}^{a}+2\varepsilon^{a}{}_{b}\omega_{[\mu(0)}e_{\nu](1)}{}^{b}-2\varepsilon^{a}{}_{b}\omega_{[\mu(1)}{}^{b}\tau_{\nu](0)}-2\bar\psi_{\mu+(0)}\gamma^a\psi_{\nu]-(1)}=0,\\
&2\partial_{[\mu}\omega_{\nu](1)}{}^{a}+2\varepsilon^{a}{}_{b}\omega_{[\mu(0)}\omega_{\nu](1)}{}^{b}=0,\\
&2\partial_{[\mu}\psi_{\nu]-(1)}+\omega_{[\mu(0)}\gamma_0\psi_{\nu]-(1)}+\omega_{[\mu(1)}{}^{a}\gamma_a\psi_{\nu]+(0)}=0.
\end{aligned}$$

$$\begin{aligned}
&\tau_{\mu(0)}\to\tau_\mu,\tau_{\mu(2)}\to m_\mu,e^a_{\mu(1)}\to e^a_\mu\\
&\omega_{\mu(0)}\to\omega_\mu,\omega_{\mu(2)}\to s_\mu,\omega^a_{\mu(1)}\to\omega^a_\mu\\
&\psi_{\mu+(0)}\to\psi_{\mu+},\psi_{\mu+(2)}\to\rho_\mu,\psi_{\mu-(1)}\to\psi_{\mu-}
\end{aligned}$$

$$\{E_\mu^{\hat A},B_{\mu\nu},\Phi;\Psi_\mu,\lambda\}$$

$$S=\frac{1}{2\kappa^2}\int\ \mathrm{d}^{10}xE\mathrm{e}^{-2\Phi}\left\{\mathcal{R}+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{12}\mathcal{H}_{\mu\nu\rho}\mathcal{H}^{\mu\nu\rho}\right\}$$

$$\mathcal{H}_{\mu\nu\rho}=3\partial_{[\mu}B_{\nu\rho]}$$

$$\begin{aligned}
\delta E_\mu{}^{\hat A}&=\Lambda^{\hat A}{}_{\hat B}E_\mu{}^{\hat B}+\bar\varepsilon\Gamma^{\hat A}\Psi_\mu\\
\delta B_{\mu\nu}&=2\partial_{[\mu}\Theta_{\nu]}+2\bar\varepsilon\Gamma_{[\mu}\Psi_{\nu]},\delta\Phi=\frac{1}{2}\bar\varepsilon\lambda\\
\delta\Psi_\mu&=\frac{1}{4}\Lambda^{\hat A\hat B}\Gamma_{\hat A\hat B}\Psi_\mu+D_\mu(\Omega^{(+)})\varepsilon\\
\delta\lambda&=\frac{1}{4}\Lambda^{\hat A\hat B}\Gamma_{\hat A\hat B}\lambda+\Gamma^\mu\varepsilon\partial_\mu\Phi-\frac{1}{12}\Gamma^{\hat A\hat B\hat C}\varepsilon\mathcal{H}_{\hat A\hat B\hat C}
\end{aligned}$$

$$D_\mu(\Omega^{(+)})\varepsilon=\partial_\mu\varepsilon-\frac{1}{4}\Omega_\mu^{(+)\hat A\hat B}\Gamma_{\hat A\hat B}\varepsilon\ \text{with}\ \Omega_\mu^{(+)\hat A\hat B}=\Omega_\mu^{\hat A\hat B}+\frac{1}{2}\mathcal{H}_\mu^{\hat A\hat B}$$

$$\begin{aligned}
&\tau_\mu{}^{A}=\omega^{-1}E_\mu{}^{A}, && e_\mu{}^{a}=E_\mu{}^{a}, && b_{\mu\nu}=B_{\mu\nu}+\varepsilon_{AB}E_\mu{}^{A}E_\nu{}^{B},\\
&\phi=\Phi-\log\omega, && \psi_{\mu\pm}=\omega^{\mp1/2}\Pi_\pm\Psi_\mu, && \lambda_\pm=\omega^{\mp1/2}\Pi_\pm\lambda,
\end{aligned}$$

$$\chi_\pm=\Pi_\pm\chi\ \text{with}\ \Pi_\pm=\frac{1}{2}(\mathbb{1}\pm\Gamma_{01})\ \text{for any spinor}\ \chi$$

$$\delta_D\tau_\mu{}^{A}=\lambda_D\tau_\mu{}^{A},\delta e_\mu{}^{a}=0$$

$$\delta\phi=\lambda_D.$$

$$S=\omega^2S^{(2)}+S^{(0)}+\omega^{-2}S^{(-2)}+\omega^{-4}S^{(-4)}$$

$$S^{(2)} = 0$$

$$\delta_Q F = \omega^2 \delta_Q^{(2)} F + \delta_Q^{(0)} F + \omega^{-2} \delta_Q^{(-2)} F$$

$$\delta_Q S = \omega^2 \delta_Q^{(2)} S^{(0)} + \omega^0 \left(\delta_Q^{(0)} S^{(0)} + \delta_Q^{(2)} S^{(-2)} \right) + \mathcal{O}(\omega^{-2})$$

$$(1)\ \ \delta_Q^{(2)} S^{(0)} = 0$$
$$(2)\ \ \delta_Q^{(0)} S^{(0)} = -\delta_Q^{(2)} S^{(-2)}$$

$$\delta_S \psi_{\mu+} = \frac{1}{2} \tau_\mu^+ \Gamma_+ \eta_-, \quad \delta_S \lambda_- = \eta_-$$
$$\delta_T \psi_{\mu-} = \tau_\mu^+ \rho_-$$

$$T_{ab}{}^A, T_a{}^{\{AB\}}, h_{abc},$$

$$\omega_{\{AB\}a} \text{ and } b_A$$

$$T_{ab}^- = T_{a+}^- = 0$$

$$\tau_{[\mu}^- \partial_\nu \tau_{\rho]}^- = 0$$

$$[\bar{\varepsilon} Q, \bar{\eta} S \text{ or } \bar{\eta} T] \sim \lambda_D D$$

$$\delta_D \psi_{\mu\pm} = \pm \frac{1}{2} \lambda_D \psi_{\mu\pm}, \delta_D \lambda_\pm = \pm \frac{1}{2} \lambda_D \lambda_\pm$$

$$\delta_G M \sim B \text{ but not } \delta_G B \sim M$$

$$\delta_Q M \sim B \text{ and } \delta_Q B \sim M.$$

$$S_{NR} = S_B + S_{\lambda\lambda} + S_{\lambda\psi} + S_{\psi\psi} + \mathfrak{Q}$$

$$S_B = \frac{1}{2\kappa^2} \int \mathrm{d}^{10} x e e^{-2\phi} \Big(\mathrm{R}(J) + 4 \partial_a \phi \partial^a \phi - \frac{1}{12} h_{abc} h^{abc}$$
$$-4 e_a{}^\mu \left(\partial_\mu b^a - \omega_\mu{}^{ab} b_b - \omega_\mu{}^{Ab} \tau^a{}_{bA} \right)$$
$$-4 b_a b^a - 4 \tau_{a\{AB\}} \tau^{a\{AB\}} \Big)$$

$$\mathrm{R}_{\mu\nu}(J)^{ab} = 2 \partial_{[\mu} \omega_{\nu]}^{ab} + 2 \omega_{[\mu}^{ac} \omega_{\nu]}^{b}{}_c$$
$$+2 e_{[\mu}^c \left(2 \omega_{\nu]}^{C[a} \tau^{b]}{}_{cC} - \omega_{\nu]Cc} \tau^{abC} \right)$$
$$+8 \tau_{[\mu}^A \left(\omega_{\nu]}^{B[a} \tau^{b]}{}_{\{AB\}} - \frac{1}{8} \varepsilon_A^B \omega_{\nu]Bc} h^{abc} \right)$$

$$b_\mu = e_\mu{}^a \tau_{aA}{}^A + \tau_\mu{}^A \partial_A \phi,$$
$$\omega_\mu = \left(\tau_\mu{}^{AB} - \frac{1}{2} \tau_\mu{}^C \tau^{AB}{C} \right) \varepsilon_{AB} - \tau_\mu{}^A \varepsilon_{AB} \partial^B \phi,$$
$$\omega_\mu{}^{Aa} = -e_\mu{}^{Aa} + e_{\mu b} e^{Aab} + \frac{1}{2} \varepsilon^A{}_B h_\mu{}^{Ba} + \tau_{\mu B} W^{BAa},$$
$$\omega_\mu{}^{ab} = -2 e_\mu{}^{[ab]} + e_{\mu c} e^{abc} - \frac{1}{2} \tau_\mu{}^A \varepsilon_{AB} h^{Bab},$$

$$\tau_{\mu\nu}{}^A = \partial_{[\mu} \tau_{\nu]}{}^A \text{ and } e_{\mu\nu}{}^a = \partial_{[\mu} e_{\nu]}{}^a.$$

$$S_{\lambda\lambda} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10} x e e^{-2\phi} \Big(2 \bar{\lambda}_\pm \Gamma^a D_a \lambda_\mp + 2 \bar{\lambda}_+ \Gamma^A D_A \lambda_+$$
$$- \frac{1}{6} h_{abc} (\bar{\lambda}_+ \Gamma^{abc} \lambda_-) + \tau_{bcA} (\bar{\lambda}_- \Gamma^{bcA} \lambda_-) \Big)$$

$$
\begin{aligned}
S_{\lambda\psi} = \frac{1}{2\kappa^2}\int \quad & \mathrm{d}^{10}x e \mathrm{e}^{-2\phi}(-4\bar{\lambda}_{\pm}\Gamma^{ab}e_a^{\mu}e_b^{\nu}D_{[\mu}\psi_{\nu]\mp} - 8\bar{\lambda}_{+}\Gamma^{Ab}\tau_A^{\mu}e_b^{\nu}D_{[\mu}\psi_{\nu]+} \\
& -4\bar{\lambda}_{\pm}\Gamma^{ab}\psi_{a\mp}D_b\phi - 4\bar{\lambda}_{+}\Gamma^{Ab}\psi_{A+}D_b\phi \\
& +\frac{1}{6}h_{abc}\left(\bar{\lambda}_{\pm}\Gamma^{abcd}\psi_{d\mp}\right) + \frac{1}{2}h_{abc}\left(\bar{\lambda}_{+}\Gamma^{abcD}\psi_{D+}\right) \\
& -(\eta^{DA} + \varepsilon^{DA})\tau_{bcD}\left(\bar{\lambda}_{-}\Gamma^{bc}\psi_{A+} - \bar{\lambda}_{+}\Gamma^{bc}\psi_{A-}\right) \\
& +2\tau_{bc}{}^{A}\bar{\lambda}_{\pm}\Gamma^{bc}\psi_{A\mp} + 2\tau^{c\{AB\}}\bar{\lambda}_{+}\Gamma_{cA}\psi_{B+} \\
& -2\tau_{bcA}\bar{\lambda}_{-}\Gamma^{Abcd}\psi_{d-})
\end{aligned}
$$

$$
\begin{aligned}
S_{\psi\psi} = \frac{1}{2\kappa^2}\int \quad & \mathrm{d}^{10}x e \mathrm{e}^{-2\phi}(-2\bar{\psi}_{A+}\Gamma^{Abc}e_b{}^{\mu}e_c{}^{\nu}D_{[\mu}\psi_{\nu]+} - 4\bar{\psi}_{a+}\Gamma^{abC}e_b{}^{\mu}\tau_C{}^{\nu}D_{[\mu}\psi_{\nu]+} \\
& -2\bar{\psi}_{a\pm}\Gamma^{abc}e_b{}^{\mu}e_c{}^{\nu}D_{[\mu}\psi_{\nu]\mp} + \frac{1}{2}h^{abc}(\bar{\psi}_{a\pm}\Gamma_b\psi_{c\mp}) \\
& -\frac{1}{6}h_{abc}\left(\bar{\psi}_{d+}\Gamma^{abcdE}\psi_{E+} + \frac{1}{2}\bar{\psi}_{d\pm}\Gamma^{abcde}\psi_{e\mp}\right) + \\
& -4(\bar{\psi}_{a\pm}\Gamma^{a}\psi_{b\pm} + \bar{\psi}_{A+}\Gamma^{A}\psi_{b+})D^{b}\phi \\
& -2(\eta^{AD} + \varepsilon^{AD})\tau^{bc}{}_{D}\bar{\psi}_{c\pm}\Gamma_b\psi_{A\mp} + 2\tau^{bcA}(\bar{\psi}_{b-}\Gamma_A\psi_{C'-}) \\
& -2(\eta_{BC} - \varepsilon_{BC})\tau^{c\{AB\}}\bar{\psi}^{C}{}_{A}\psi_{c+} \\
& +(\eta_{AB} + \varepsilon_{AB})\tau_{bc}{}^{A}\bar{\psi}_{d\pm}\Gamma^{BbcdE}\psi_{E\mp} \\
& +\tau_{bc}{}^{A}\bar{\psi}_{d-}\Gamma_A\Gamma^{bcde}\psi_{e-})
\end{aligned}
$$

$$
\begin{aligned}
\delta\tau_{\mu}{}^{A} &= \bar{\varepsilon}_{+}\Gamma^{A}\psi_{\mu+} \\
\delta e_{\mu}{}^{a} &= \bar{\varepsilon}_{+}\Gamma^{a}\psi_{\mu-} + \bar{\varepsilon}_{-}\Gamma^{a}\psi_{\mu+} \\
\delta\phi &= \frac{1}{2}(\bar{\varepsilon}_{+}\lambda_{-} + \bar{\varepsilon}_{-}\lambda_{+}) \\
\delta b_{\mu\nu} &= 4\tau_{[\mu}{}^{A}\bar{\varepsilon}_{-}\Gamma_A\psi_{\nu]-} + 2\left(e_{[\mu}{}^{a}\bar{\varepsilon}_{+}\Gamma_a\psi_{\nu]-} + e_{[\mu}{}^{a}\bar{\varepsilon}_{-}\Gamma_a\psi_{\nu]+}\right)
\end{aligned}
$$

$$
\begin{aligned}
\delta\psi_{\mu+} &= \delta_{+}\psi_{\mu+} + \delta_{-}\psi_{\mu-} \\
\delta\psi_{\mu-} &= \delta_{+}\psi_{\mu-} + \delta_{-}\psi_{\mu-} \\
\delta\lambda_{+} &= \delta_{+}\lambda_{+} + \delta_{-}\lambda_{+} \\
\delta\lambda_{-} &= \delta_{+}\lambda_{-} + \delta_{-}\lambda_{-}
\end{aligned}
$$

$$
\begin{aligned}
\delta_{+}\psi_{\mu+} &= \mathcal{D}_{\mu}\varepsilon_{+} - \frac{1}{8}e_{\mu c}h^{cab}\Gamma_{ab}\varepsilon_{+} \\
\delta_{-}\psi_{\mu+} &= \left(e_{\mu b}\tau^{ba+} + \tau_{\mu}{}^{-}\tau^{a++}\right)\Gamma_{a+}\varepsilon_{-} \\
\delta_{+}\psi_{\mu-} &= -\frac{1}{2}\omega_{\mu}{}^{-a}\Gamma_{-a}\varepsilon_{+} \\
\delta_{-}\psi_{\mu-} &= \mathcal{D}_{\mu}\varepsilon_{-} - \frac{1}{8}e_{\mu c}h^{cab}\Gamma_{ab}\varepsilon_{-} \\
\delta_{+}\lambda_{+} &= \left(\partial_a\phi\Gamma^{a} - b_a\Gamma^{a} - \frac{1}{12}h^{abc}\Gamma_{abc}\right)\varepsilon_{+} \\
\delta_{-}\lambda_{+} &= \frac{1}{2}\tau^{ab+}\Gamma_{ab+}\varepsilon_{-} \\
\delta_{+}\lambda_{-} &= 0 \\
\delta_{-}\lambda_{-} &= \left(\partial_a\phi\Gamma^{a} - b_a\Gamma^{a} - \frac{1}{12}h^{abc}\Gamma_{abc}\right)\varepsilon_{-}
\end{aligned}
$$

$$
\mathcal{D}_{\mu}\varepsilon_{\pm} = \left(\partial_{\mu} - \frac{1}{4}\omega_{\mu}^{ab}\Gamma_{ab} \pm \frac{1}{2}\omega_{\mu} \mp \frac{1}{2}b_{\mu}\right)\varepsilon_{\pm}
$$

$$
\mathrm{d}s^2 = e^{2A}\,\mathrm{d}s^2(\mathcal{M}_2) + \mathrm{d}s^2(\mathcal{M}_8)
$$

$$
F^{\mathrm{tot}} = \mathrm{vol}_2 \wedge F^{\mathrm{el}} + F
$$

$$
F^{\mathrm{el}} = e^{2A}\star_8\sigma(F),
$$

$$
\chi(\mathcal{M}_8) = \frac{1}{2}\int_{\mathcal{M}_8}\left(p_2 - \frac{1}{4}p_1^2\right)
$$

$$
\epsilon_1 = \frac{\alpha}{\sqrt{2}}\zeta_{+}\otimes(\eta + \eta^{c}), \epsilon_2 = \frac{\alpha}{\sqrt{2}}\zeta_{-}\otimes\left(e^{i\theta}\eta + e^{-i\theta}\eta^{c}\right)
$$

$$\nabla_\mu \psi = W \gamma_\mu \psi$$

$$R = -8W^2,$$

$$\delta\lambda^1 = \left(\underline{\partial}\phi + \frac{1}{2}\underline{H}\right)\epsilon_1 - \frac{1}{16}e^{\phi}\Gamma^M \underline{F}^{\text{tot}}\Gamma_M\epsilon_2 = 0$$
$$\delta\lambda^2 = \left(\underline{\partial}\phi - \frac{1}{2}\underline{H}\right)\epsilon_2 - \frac{1}{16}e^{\phi}\Gamma^M \sigma(\underline{F}^{\text{tot}})\Gamma_M\epsilon_1 = 0$$
$$\delta\psi_M^1 = \left(\nabla_M + \frac{1}{4}\underline{H}_M\right)\epsilon_1 - \frac{1}{16}e^{\phi}\underline{F}^{\text{tot}}\Gamma_M\epsilon_2 = 0$$
$$\delta\psi_M^2 = \left(\nabla_M - \frac{1}{4}\underline{H}_M\right)\epsilon_2 - \frac{1}{16}e^{\phi}\sigma(\underline{F}^{\text{tot}})\Gamma_M\epsilon_1 = 0$$

$$\underline{S}_{M_1\ldots M_q} \equiv \frac{1}{p!}\Gamma^{N_1\ldots N_p}S_{N_1\ldots N_p M_1\ldots M_q}$$

$$W = 0$$
$$2\,\mathrm{d}A + h_1 = 0$$
$$\left(2\,\underline{\mathrm{d}\phi} + \underline{h_1} + \underline{h_3}\right)(\eta + \eta^c) = 0$$
$$\left(2\,\underline{\mathrm{d}\phi} + \underline{h_1} - \underline{h_3}\right)\left(e^{i\theta}\eta + e^{-i\theta}\eta^c\right) = 0$$
$$\left(\nabla_m + \partial_m \log\,\alpha + \frac{1}{4}h_{1|m} + \frac{1}{4}\underline{h_{3|m}}\right)(\eta + \eta^c) = 0$$
$$\left(\nabla_m + \partial_m \log\,\alpha + \frac{1}{4}h_{1|m} - \frac{1}{4}\underline{h_{3|m}}\right)\left(e^{i\theta}\eta + e^{-i\theta}\eta^c\right) = 0$$

$$\underline{\sigma(F)}(\eta + \eta^c) = 0$$
$$\underline{\sigma(F)}\gamma_m(\eta + \eta^c) = 0$$
$$\underline{F}\left(e^{i\theta}\eta + e^{-i\theta}\eta^c\right) = 0$$
$$\underline{F}\gamma_m\left(e^{i\theta}\eta + e^{-i\theta}\eta^c\right) = 0$$

$$W = 0$$
$$\alpha = e^{\frac{1}{2}A}$$
$$h_1 = -2\,\mathrm{d}A$$

$$F = \star_8\,\sigma(F)$$

$$f_4 = \frac{1}{6}f_0 + \frac{4}{3}e^{-i\theta}\cos\,\theta\tilde{f}_4$$
$$f_2 = 2e^{-i\theta}\sin\,\theta\tilde{f}_4$$
$$\sin\,\theta f_{2|mn}^{(2,0)} = -\cos\,\theta f_{4|mn}^{(2,0)} - \frac{1}{8}e^{i\theta}\Omega_{mn}{}^{pq}f_{4|pq}^{(0,2)}$$
$$\sin\,\theta f_{4|mn}^{(2,0)} = \cos\,\theta f_{2|mn}^{(2,0)} - \frac{1}{8}e^{i\theta}\Omega_{mn}{}^{pq}f_{2|pq}^{(0,2)}$$

$$e^{\phi} = g_s e^A$$
$$h_3^{(1,0)} = \tilde{h}_3^{(1,0)} = 0$$
$$h_3^{(2,1)} = 0$$
$$W_1 = -\frac{3i}{4}W_4$$
$$W_3 = \frac{1}{2}W_2$$
$$W_5 = \frac{3}{2}W_4$$

$$
\begin{aligned}
h_3^{(1,0)} &= 0\\
\tilde{h}_3^{(1,0)} &= \frac{1}{4}\partial^+(A-\phi)\\
W_1 &= 0\\
W_2 &= -2ih_3^{(2,1)}\\
W_3 &= 0\\
W_4 &= \partial^+(\phi - A)\\
W_5 &= \frac{3}{2}\partial^+(\phi - A),
\end{aligned}
$$

$$
\begin{aligned}
e^{\phi} &= g_s e^{A}\cos\,\theta\\
h_3^{(1,0)} &= \frac{2}{3}\partial^+\theta\\
\tilde{h}_3^{(1,0)} &= \frac{1}{4}(i+\tan\,\theta)\partial^+\theta\\
W_1^{(1,0)} &= \frac{1}{4}(1+i\cot\,\theta)\partial^+\theta\\
W_2^{(2,1)} &= 2(-i+\cot\,\theta)h_3^{(2,1)}\\
W_3^{(2,1)} &= \cot\,\theta h_3^{(2,1)}\\
W_4^{(1,0)} &= -\left(\tan\,\theta + \frac{1}{3}\cot\,\theta\right)\partial^+\theta\\
W_5^{(1,0)} &= \left(i - \frac{1}{2}\cot\,\theta - \frac{3}{2}\tan\,\theta\right)\partial^+\theta,
\end{aligned}
$$

$$
\begin{aligned}
E_{MN} &\equiv R_{MN} + 2\nabla_M\nabla_N\phi - \frac{1}{2}H_M\cdot H_N - \frac{1}{4}e^{2\phi}F_M^{\mathrm{tot}}\cdot F_N^{\mathrm{tot}}\\
\delta H &\equiv e^{2\phi} *_{10}\left[\mathrm{d}\left(e^{-2\phi} *_{10} H\right) - \frac{1}{2}(*_{10} F^{\mathrm{tot}}\wedge F^{\mathrm{tot}})_8\right]\\
D &\equiv 2R - H^2 + 8(\nabla^2\phi - (\partial\phi)^2)
\end{aligned}
$$

$$E_{MN} = 0;\ \delta H_{MN} = 0;\ D = 0$$

$$\mathrm{d}H = 0;\ \mathrm{d}_H F^{\mathrm{tot}} = 0$$

$$\mathcal{E}^{+}_{NP}\Gamma^P\epsilon_1 = 0;\ \mathcal{E}^{-}_{NP}\Gamma^P\epsilon_2 = 0;\ D = 0$$

$$\mathcal{E}^{\pm}_{NP} \equiv -2E_{NP} \pm \delta H_{NP}$$

$$\sum_{P=0}^{9} \mathcal{E}^{\pm}_{NP}\mathcal{E}^{\pm P}_{N} = 0,\ \text{no sum over } N$$

$$\sum_{P=1}^{9} \mathcal{E}^{\pm}_{NP}\mathcal{E}^{\pm P}_{N} = 0, N \neq 0$$

$$ds^2 = e^{2A}ds^2(\mathbb{R}^{1,1}) + ds^2(\mathcal{M}_8)$$

$$e^{\phi} = g_s e^{A};\ H = -\mathrm{vol}_2\wedge\ \mathrm{d}e^{2A}$$

$$
\begin{aligned}
&F_0 = f_0;\ F_2 = f_2 J + f_2^{(1,1)} + \left(f_2^{(2,0)} + \text{ c.c. }\right)\\
&F_4 = \star_8 F_4 = f_4^{(2,2)} + f_4 J\wedge J + \left(\tilde{f}_4\Omega + f_4^{(2,0)}\wedge J + \text{ c.c}\right)\\
&F_6 = -\star_8 F_2;\ F_8 = \star_8 F_0
\end{aligned}
$$

$$\mathrm{d}F = \mathrm{d}\star_8 F = 0$$

$$-\mathrm{d}\star_8\ \mathrm{d}e^{-2A} + \frac{g_s^2}{2}F\wedge\sigma(F)\Big|_8 = 0$$

$$\frac{1}{2}\int_{\mathcal{M}_8} F\wedge\sigma(F)=\int_{\mathcal{M}_8}\left(F_0\wedge\star F_0+F_2\wedge\star F_2+\frac{1}{2}F_4\wedge\star F_4\right)\geq 0,$$

$$f_0=0,$$

$$f_{2|mn}^{(1,1)}=f_{2|mn}^{(2,0)}=0;\ f_2=0,$$

$$ds^2=e^{\frac{4A}{3}}ds^2(\mathbb{R}^{1,2})+e^{-\frac{2A}{3}}ds^2(\mathcal{M}_8),$$

$$G=\mathrm{vol}_3\wedge\ \mathrm{d}(e^{2A})+f_4\left(J\wedge J+\frac{3}{2}\mathrm{Re}\Omega\right)+\left(f_4^{(2,0)}-\frac{1}{4}f_4^{(2,0)}\lrcorner\Omega^*\right)\wedge J+f_4^{(2,2)},$$

$$f_4^{(2,0)*}=-\frac{1}{4}f_4^{(2,0)}\lrcorner\Omega^*$$

$$W_3=\frac{1}{2}W_2;\ W_4=\frac{4i}{3}W_1;\ W_5=2iW_1$$

$$\begin{aligned} ds^2 &= e^{\frac{4A}{3}}\left(ds^2(\mathbb{R}^{1,2})+\widetilde{ds}^2(\mathcal{M}_8)\right)\\ G &= F+e^{2A}\mathrm{vol}_3\wedge f\end{aligned}$$

$$\tilde{g}_{mn}=e^{-2A}g_{mn}$$

$$\epsilon=e^{-\frac{A}{3}}\zeta\otimes(\eta+\eta^c)$$

$$\begin{aligned}\nabla_m\eta+\text{ c.c.}&=0\\ 2\partial_mA-f_m&=0\\ F_{mpqr}\gamma^{pqr}\eta+\text{ c.c. }&=0,\end{aligned}$$

$$\begin{aligned}0&=\mathrm{d}F=\mathrm{d}\star_8 F\\ 0&=-\mathrm{d}\star_8\ \mathrm{d}e^{-2A}+\frac{1}{2}F\wedge F\end{aligned}$$

$$\epsilon=e^{-\frac{A}{3}}(\zeta\otimes\eta+\text{ c.c. }),$$

$$\begin{aligned}\nabla_m\eta&=0\\ 2\partial_mA-f_m&=0\\ F_{mpqr}\gamma^{pqr}\eta&=0\end{aligned}$$

$$G=\mathrm{vol}_3\wedge\ \mathrm{d}(e^{2A})+f_4^{(2,2)}$$

$$-\mathrm{d}\star_8\ \mathrm{d}e^{-2A}+\frac{1}{2}F\wedge F+(2\pi)^2\sum_{i=1}^{N_{\mathrm{M2}}}\delta^{(8)}(y-y_i)\mathrm{vol}_8=(2\pi)^2X_8$$

$$X_8=\frac{1}{48}\left(p_2-\frac{1}{4}p_1^2\right),$$

$$\frac{1}{8\pi^2}\int_{\mathcal{M}_8}F\wedge F+N_{\mathrm{M2}}=\frac{\chi(\mathcal{M}_8)}{24}$$

$$ds^2=e^{\frac{4}{3}A}ds^2(\mathbb{R}^{1,2})+e^{-\frac{2}{3}A}ds^2(\mathcal{M}_8)$$

$$G=\mathrm{vol}_3\wedge\ \mathrm{d}(e^{2A})+F;\ F=f_4\left(J\wedge J+\frac{3}{2}\mathrm{Re}\Omega\right)+f_4^{(2,2)}$$

$$\mathrm{d}F=\mathrm{d}\star_8 F=0$$

$$-\mathrm{d}\star_8\ \mathrm{d}e^{-2A}+\frac{1}{2}F\wedge F+(2\pi)^2\sum_{i=1}^{N_{\mathrm{M2}}}\delta^{(8)}(y-y_i)\mathrm{vol}_8=(2\pi)^2X_8$$

$$\frac{1}{8\pi^2}\int_{\mathcal{M}_8}F\wedge F+N_{\mathrm{M2}}=\frac{\chi(\mathcal{M}_8)}{24}.$$

$$\frac{[F]}{2\pi}-\frac{p_1}{4}\in H^4(\mathcal{M}_8,\mathbb{Z}),$$

$$-\int_S\frac{p_1}{2}=\int_S c_2=\chi(S)=24$$

$$\frac{1}{2\pi}\int_{\mathcal{C}_4}F\in\mathbb{Z},$$

$$g_{mn}=t^2g_{mn}^{(0)}+g_{mn}^{(2)}+\cdots,$$

$$F=f_4^{(2,2)}+f_4J\wedge J+\left(\tilde{f}_4\Omega+\text{ c.c. }\right)$$

$$W=\int_{\mathcal{M}_8}F\wedge\Omega$$

$$\hat{W}=\frac{1}{2}\int_{\mathcal{M}_8}F\wedge J\wedge J$$

$$W=\hat{W}=0;\ D_\alpha W=D_A\hat{W}=0$$

$$D_\alpha W=\partial_\alpha W-(\partial_\alpha K)W;\ D_A\hat{W}=\partial_A\hat{W}-\frac{1}{2}(\partial_A K)\hat{W}$$

$$K_\Omega=\log\int_{\mathcal{M}_8}\frac{1}{16}\Omega\wedge\Omega^*;\ K_J=\log\int_{\mathcal{M}_8}\frac{1}{4!}J^4$$

$$G=f_4^{(2,2)}+f_4^{(1,1)}\wedge J+f_4J\wedge J+\left(f_4^{(3,1)}+f_4^{(2,0)}\wedge J+\tilde{f}_4\Omega+\text{ c.c. }\right)$$

$$W=16\tilde{f}_4^*\mathrm{vol}(\mathcal{M}_8);\hat{W}=12f_4\mathrm{vol}(\mathcal{M}_8)$$

$$D_\alpha W=\int_{\mathcal{M}_8}\left(f_4^{(1,3)}+f_4^{(0,2)}\wedge J\right)\wedge e_\alpha;D_A\hat{W}=\int_{\mathcal{M}_8}f_4^{(1,1)}\wedge J^2\wedge E_A$$

$$e^{\frac{1}{2}K_\Omega}\partial_{R^\alpha}\left(e^{-\frac{1}{2}K_\Omega}|W|\right)=\mathrm{Re}\left(e^{i\theta}\int_{\mathcal{M}_8}\left(f_4^{(1,3)}+f_4^{(0,2)}\wedge J\right)\wedge e_\alpha\right)$$

$$e^{\frac{1}{2}K_J}\partial_A\left(e^{-\frac{1}{2}K_J}\hat{W}\right)=\int_{\mathcal{M}_8}f_4^{(1,1)}\wedge J^2\wedge E_A$$

$$|W|-\hat{W}=0;\ \partial_{R^\alpha}\left(e^{-\frac{1}{2}K_\Omega}|W|\right)=0;\ \partial_A\left(e^{-\frac{1}{2}K_J}\hat{W}\right)=0$$

$$\mathcal{W}=e^{-\frac{1}{2}K_\Omega}|W|-e^{-\frac{1}{2}K_J}\hat{W}$$

$$\partial_{R^\alpha}\mathcal{W}=\partial_A\mathcal{W}=\mathcal{W}=0,$$

$$\eta=\eta_S\otimes\eta_{\tilde{S}}$$

$$H^2(K3,\mathbb{R})=\mathcal{H}^+(K3,\mathbb{R})\oplus\mathcal{H}^-(K3,\mathbb{R})$$

$$j=\rho^a j^a;\ \omega=c^a j^a$$

$$\mathrm{vol}_4=\frac{1}{2}j\wedge j=\frac{1}{4}\omega\wedge\omega^*;\ j\wedge\omega=0$$

$$\star_4 j^a = j^a;\ \star_4 l^\alpha = -l^\alpha$$
$$\frac{1}{2} j^a \wedge j^b = \delta^{ab}\mathrm{vol}_4;\ \frac{1}{2} l^\alpha \wedge l^\beta = -\delta^{\alpha\beta}\mathrm{vol}_4$$

$$\rho^a c^a = 0;\ c^a c^a = 0;\ \rho^a \rho^a = 1;\ c^a c^{*a} = 2$$

$$J = j + \tilde{j};\ \Omega = \omega \wedge \tilde{\omega}$$

$$\frac{1}{4!} J^4 = \frac{1}{16}\Omega \wedge \Omega^* = \mathrm{vol}_4 \wedge \widetilde{\mathrm{vol}}_4;\ J \wedge \Omega = 0$$

$$G = f^{ab} j^a \wedge \tilde{j}^b + f^{\alpha\beta} l^\alpha \wedge \tilde{l}^\beta + f\left(\mathrm{vol}_4 + \widetilde{\mathrm{vol}}_4\right),$$

$$f + 2(\rho^a \tilde{\rho}^b - c^a \tilde{c}^b) f^{ab} = 0$$

$$\rho^a \tilde{c}^b f^{ab} = c^a \tilde{\rho}^b f^{ab} = 0,$$

$$f^{ab} = (\rho^d \tilde{\rho}^e f^{de}) \rho^a \tilde{\rho}^b + \hat{f}^{ab},$$

$$G = Cj \wedge \tilde{j} + A\mathrm{Re}(\omega \wedge \tilde{\omega}) + \mathrm{Re}(B\omega^* \wedge \tilde{\omega}) + (4A - 2C)\left(\mathrm{vol}_4 + \widetilde{\mathrm{vol}}_4\right) + f^{\alpha\beta} l^\alpha \wedge \tilde{l}^\beta,$$

$$\left(f_4^{(2,0)} + f_4^{(0,2)}\right) \wedge J = \alpha\omega \wedge \tilde{j} + \alpha^* \tilde{\omega} \wedge j + \text{ c.c. },$$

$$G = Cj \wedge \tilde{j} + \mathrm{Re}(B\omega^* \wedge \tilde{\omega}) - 2C\left(\mathrm{vol}_4 + \widetilde{\mathrm{vol}}_4\right) + f^{\alpha\beta} l^\alpha \wedge \tilde{l}^\beta.$$

$$\vec{J}_{mn} \equiv -\frac{i}{2}\vec{\sigma}_{ij}\left(\tilde{\eta}_i^c \gamma_{mn} \eta_j\right)$$

$$\delta\eta_i = \frac{i}{2}\delta\vec{\theta} \cdot \vec{\sigma}_{ij}\eta_j$$

$$\delta\vec{J} = -\delta\vec{\theta} \times \vec{J}$$

$$(\omega, \tilde{\omega}) \longrightarrow \left(e^{i\theta}\omega, e^{i\tilde{\theta}}\tilde{\omega}\right)$$

$$(\omega, \tilde{\omega}) \longrightarrow e^{i\theta}(\omega, \tilde{\omega})$$

$$\epsilon = e^{-\frac{A}{3}}\zeta \otimes \eta + \text{ c.c.}$$

$$G = Aj \wedge \tilde{j} + A\mathrm{Re}(\omega \wedge \tilde{\omega}) + 2A\left(\mathrm{vol}_4 + \widetilde{\mathrm{vol}}_4\right) + f^{\alpha\beta} l^\alpha \wedge \tilde{l}^\beta$$

$$\begin{pmatrix} j \\ \mathrm{Re}\omega \\ \mathrm{Im}\omega \end{pmatrix} \longrightarrow R_{\hat{n}}(\theta) \cdot \begin{pmatrix} j \\ \mathrm{Re}\omega \\ \mathrm{Im}\omega \end{pmatrix};\begin{pmatrix} \tilde{j} \\ \mathrm{Re}\tilde{\omega} \\ \mathrm{Im}\tilde{\omega} \end{pmatrix} \longrightarrow R_{\hat{n}'}(\theta) \cdot \begin{pmatrix} \tilde{j} \\ \mathrm{Re}\tilde{\omega} \\ \mathrm{Im}\tilde{\omega} \end{pmatrix}$$

$$\begin{pmatrix} \eta_S \\ \eta_S^c \end{pmatrix} \longrightarrow e^{-\frac{i}{2}\theta\hat{n}\cdot\vec{\sigma}} \cdot \begin{pmatrix} \eta_S \\ \eta_S^c \end{pmatrix};\ \begin{pmatrix} \eta_{\tilde{S}} \\ \eta_{\tilde{S}}^c \end{pmatrix} \longrightarrow e^{\frac{i}{2}\theta\hat{n}'\cdot\vec{\sigma}} \cdot \begin{pmatrix} \eta_{\tilde{S}} \\ \eta_{\tilde{S}}^c \end{pmatrix}$$

$$\eta + \eta^c \longrightarrow a\eta_S \otimes \eta_{\tilde{S}} + b\eta_S \otimes \eta_{\tilde{S}}^c + \text{ c.c. },$$

$$\epsilon = e^{-\frac{A}{3}}\left(\zeta_1 \otimes \eta_S \otimes \eta_{\tilde{S}} + \zeta_2 \otimes \eta_S \otimes \eta_{\tilde{S}}^c + \text{ c.c. }\right)$$

$$I = U \oplus U \oplus U \oplus (-\mathrm{e}_8) \oplus (-\mathrm{e}_8)$$

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$e_{1,I} \wedge e_{2,J} = \delta_{IJ} v_0;\ e_{1,I} \wedge e_{1,J} = e_{2,I} \wedge e_{2,J} = 0;\ e_i \wedge e_j = I_{ij} v_0$$

$$\int_S v_0 = 1$$

$$e_{\pm I} \equiv e_{1,I} \pm e_{2,I}$$

$$\int_{\mathcal{C}_{a,I}} e_{b,J} = \delta_{ab}\delta_{IJ}; \int_{\mathcal{C}_i} e_j = \delta_{ij}$$

$$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \oplus U \oplus (-\mathrm{e}_8) \oplus (-\mathrm{e}_8)$$

$$j = \sqrt{2\pi v}e_{+1}; \quad \omega = \sqrt{2\pi v}(e_{+2} + ie_{+3})$$
$$\tilde{j} = \sqrt{2\pi\tilde{v}}\tilde{e}_{+1}; \quad \tilde{\omega} = \sqrt{2\pi\tilde{v}}(\tilde{e}_{+2} + i\tilde{e}_{+3})$$

$$\int_S \mathrm{vol}_4 = 2\pi v; \int_{\tilde{S}} \widetilde{\mathrm{vol}}_4 = 2\pi\tilde{v}$$

$$C = \pm\frac{2}{\sqrt{v\tilde{v}}}$$

$$\frac{1}{2\pi}\int_{\mathcal{C}_{a,1}\times\mathcal{C}_{b,1}} G = \pm 2; \frac{1}{2\pi}\int_{\mathcal{C}_{a,2}\times\mathcal{C}_{b,2}} G = \pm 1; \frac{1}{2\pi}\int_{\mathcal{C}_{a,3}\times\mathcal{C}_{b,3}} G = \mp 1$$

$$C = \pm\frac{2}{\sqrt{v\tilde{v}}}$$

$$\frac{1}{2\pi}\int_{\mathcal{C}_{a,1}\times\mathcal{C}_{b,1}} G = \pm 2; \frac{1}{2\pi}\int_S G = \mp 4\sqrt{\frac{v}{\tilde{v}}}; \frac{1}{2\pi}\int_{\tilde{S}} G = \mp 4\sqrt{\frac{\tilde{v}}{v}},$$

$$v = 4^n\tilde{v}$$

$$C = \pm\frac{1}{\sqrt{v\tilde{v}}}$$

$$\frac{1}{2\pi}\int_{\mathcal{C}_{a,1}\times\mathcal{C}_{b,1}} G = \pm 1; \frac{1}{2\pi}\int_{\mathcal{C}_{a,2}\times\mathcal{C}_{b,2}} G = \pm 1; \frac{1}{2\pi}\int_{\mathcal{C}_{a,3}\times\mathcal{C}_{b,3}} G = \mp 1$$
$$\frac{1}{2\pi}\int_S G = \frac{1}{2\pi}\int_{\tilde{S}} G = \pm 2$$

$$\tilde{\psi} \equiv \psi^{Tr}C^{-1}$$

$$\bar{\psi} \equiv \psi^\dagger\Gamma^0$$

$$(\Gamma^M)^\dagger = \Gamma^0\Gamma^M\Gamma^0,$$

$$\Gamma^{(n)}_{M_1\dots M_n} \equiv \Gamma_{[M_1}\dots\Gamma_{M_n]}.$$

$$C^{Tr} = -C;\ (C\gamma^\mu)^{Tr} = C\gamma^\mu;\ C^* = -C^{-1}.$$

$$\zeta^c \equiv \gamma_0 C\zeta^*$$

$$\gamma_3 \equiv -\gamma_0\gamma_1.$$

$$\star\,\gamma_{(n)}\gamma_3 = -(-1)^{\frac{1}{2}n(n+1)}\gamma_{(2-n)}.$$

$$C^{Tr} = C;\ (C\gamma^\mu)^{Tr} = C\gamma^\mu;\ C^* = C^{-1}$$

$$\eta^c \equiv C\eta^*.$$

$$\gamma_9 \equiv \gamma_1\dots\gamma_8.$$

$$\star\,\gamma_{(n)}\gamma_9 = (-)^{\frac{1}{2}n(n+1)}\gamma_{(8-n)}.$$

$$C^{Tr} = -C;\ (C\Gamma^M)^{Tr} = C\Gamma^M;\ C^* = -C^{-1}$$

$$\bar{\epsilon} = \tilde{\epsilon}$$

$$\Gamma_{11} \equiv -\Gamma_0 \dots \Gamma_9$$

$$\begin{cases}\Gamma^\mu = \gamma^\mu \otimes \mathbb{1} & ,\mu = 0,1 \\ \Gamma^m = \gamma_3 \otimes \gamma^{m-1}, & m = 2\dots 9\end{cases}$$

$$C_{10} = C_2 \otimes C_8;\ \Gamma_{11} = \gamma_3 \otimes \gamma_9$$

$$\star\,\Gamma_{(n)}\Gamma_{11} = -(-1)^{\frac{1}{2}n(n+1)}\Gamma_{(10-n)}$$

$$\tilde{\eta}\eta = 0.$$

$$\eta = \frac{1}{\sqrt{2}}(\eta_R + i\eta_I);\ \tilde{\eta}_R\eta_R = \tilde{\eta}_I\eta_I = 1$$

$$iJ_{mn} = \widetilde{\eta^c}\gamma_{mn}\eta$$
$$\Omega_{mnpq} = \tilde{\eta}\gamma_{mnpq}\eta.$$

$$J \wedge \Omega = 0$$
$$\frac{1}{16}\Omega \wedge \Omega^* = \frac{1}{4!}J^4 = \mathrm{vol}_8$$

$$J_m^p J_p^n = -\delta_m^n$$

$$(\Pi^\pm)_m^n \equiv \frac{1}{2}(\delta_m^n \mp iJ_m^n)$$

$$(\Pi^+)_m^i\Omega_{inpq} = \Omega_{mnpq};\ (\Pi^-)_m^i\Omega_{inpq} = 0$$

$$\xi_+ = \varphi\eta + \chi\eta^c + \varphi_{mn}\gamma^{mn}\eta^c$$
$$\xi_- = \lambda_m\gamma^m\eta + \chi_m\gamma^m\eta^c$$

$$\tau \in (\mathbf{4} \oplus \bar{\mathbf{4}}) \otimes (\mathbf{1} \oplus \mathbf{6} \oplus \mathbf{6})$$
$$\sim (\mathbf{4} \oplus \bar{\mathbf{4}}) \oplus (\mathbf{20} \oplus \overline{\mathbf{20}}) \oplus (\mathbf{20} \oplus \overline{\mathbf{20}}) \oplus (\mathbf{4} \oplus \bar{\mathbf{4}}) \oplus (\mathbf{4} \oplus \bar{\mathbf{4}})$$

$$\mathrm{d}J = W_1 \lrcorner \Omega^* + W_3 + W_4 \wedge J + \text{ c.c.}$$
$$\mathrm{d}\Omega = \frac{8i}{3}W_1 \wedge J \wedge J + W_2 \wedge J + W_5^* \wedge \Omega$$

$$\nabla_m\eta = \left(\frac{3}{4}W_{4m} - \frac{1}{2}W_{5m} - \text{ c.c. }\right)\eta + \frac{i}{24}\Omega^*_{mnkl}W_1^n\gamma^{kl}\eta$$
$$+\left(-\frac{i}{16}W_{2mkl} - \frac{1}{32}\Omega_{mnkl}W_4^{n*} + \frac{i}{64}W^*_{3mnp}\Omega^{np}kl\right)\gamma^{kl}\eta^c$$

$$\nabla_m\eta = \varphi_m\eta + \vartheta_m\eta^c + \Psi_{m,pq}\Omega^{pqrs}\gamma_{rs}\eta^c$$

$$\Psi_{m,pq} = \Omega^*_{mpqr}A^r + \Omega^*_{pq}{}^{rs}\tilde{\varphi}_{rsm} + (\Pi^+)_{m[p}B^*_{q]} + (\Pi^+)_m{}^n\psi^*_{npq}$$

$$\mathbf{28} \to (\mathbf{6} \oplus \mathbf{6}) \oplus (\mathbf{1} \oplus \mathbf{15})$$
$$\mathbf{56} \to (\mathbf{4} \oplus \bar{\mathbf{4}}) \oplus (\mathbf{4} \oplus \mathbf{20}) \oplus (\bar{\mathbf{4}} \oplus \overline{\mathbf{20}})$$
$$\mathbf{35}^+ \to (\mathbf{1} \oplus \mathbf{1}) \oplus (\mathbf{6} \oplus \mathbf{6}) \oplus \mathbf{20}' \oplus \mathbf{1}$$
$$\mathbf{35}^- \to (\mathbf{10} \oplus \overline{\mathbf{10}}) \oplus \mathbf{15}.$$

$$F_{mn} = f^{(1,1)}_{2|mn} + f_2 J_{mn} + \left(f^{(2,0)}_{2|mn} + \text{ c.c. }\right)$$

$$\varphi_{mn}^{(2,0)} = \frac{1}{8} e^{i\theta} \Omega_{mn}{}^{pq} \varphi_{mn}^{(0,2)}$$

$$F_{mnp} = f_{3|mnp}^{(2,1)} + 3 f_{3|[m}^{(1,0)} J_{np]} + \tilde{f}_{3|s}^{(1,0)} \Omega^{s*}{}_{mnp} + \text{ c.c.}$$

$$F_{mnpq}^{+} = f_{4|mnpq}^{(2,2)} + 6 f_4 J_{[mn} J_{pq]} + \left(6 f_{4|[mn}^{(2,0)} J_{pq]} + \tilde{f}_4 \Omega_{mnps} + \text{ c.c.} \right)$$

$$F_{mnpq}^{-} = 6 f_{4|[mn}^{(1,1)} J_{pq]} + \left(f_{4|mnpq}^{(3,1)} + \text{ c.c.} \right)$$

$$\Omega_{[s}^{*mnp} f_{4|q]mnp}^{(3,1)} = 0$$

$$(\star_8 F_6)_{mn} = f_{6|mn}^{(1,1)} + f_6 J_{mn} + \left(f_{6|mn}^{(2,0)} + \text{ c.c.} \right)$$

$$\star_8 F_8 = f_8$$

$$H = e^{2A} \mathrm{vol}_2 \wedge h_1 + h_3,$$

$$h_{1|m} = h_{1|m}^{(1,0)} + \text{ c.c.} \,,$$

$$h_{3|mnp} = h_{3|mnp}^{(2,1)} + 3 h_{3|[m}^{(1,0)} J_{np]} + \tilde{h}_{3|s}^{(1,0)} \Omega^{s*}{}_{mnp} + \text{ c.c.} \,,$$

$$\frac{1}{4! \times 2^4} \Omega_{rstu} \Omega^{*rstu} = 1$$
$$\frac{1}{6 \times 2^4} \Omega_{irst} \Omega^{*mrst} = (\Pi^+)_i^m$$
$$\frac{1}{4 \times 2^4} \Omega_{ijrs} \Omega^{*mnrs} = (\Pi^+)_{[i}^m (\Pi^+)_{j]}^n$$
$$\frac{1}{6 \times 2^4} \Omega_{ijkr} \Omega^{*mnpr} = (\Pi^+)_{[i}^m (\Pi^+)_j^n (\Pi^+)_{k]}^p$$
$$\frac{1}{4! \times 2^4} \Omega_{ijkl} \Omega^{*mnpq} = (\Pi^+)_{[i}^m (\Pi^+)_j^n (\Pi^+)_k^p (\Pi^+)_{l]}^q,$$

$$\widetilde{\eta^c} \eta = 1; \tilde{\eta} \eta = 0$$
$$\widetilde{\eta^c} \gamma_{mn} \eta = i J_{mn}; \tilde{\eta} \gamma_{mn} \eta = 0$$
$$\widetilde{\eta^c} \gamma_{mnpq} \eta = -3 J_{[mn} J_{pq]}; \tilde{\eta} \gamma_{mnpq} \eta = \Omega_{mnpq}$$
$$\widetilde{\eta^c} \gamma_{mnpqrs} \eta = -15 i J_{[mn} J_{pq} J_{rs]}; \tilde{\eta} \gamma_{mnpqrs} \eta = 0$$
$$\widetilde{\eta^c} \gamma_{mnmpqstu} \eta = 105 J_{[mn} J_{pq} J_{rs} J_{tu]}; \tilde{\eta} \gamma_{mnpqrstu} \eta = 0,$$

$$\sqrt{g} \varepsilon_{\mathrm{mnpqrstu}} J^{rs} J^{tu} = 24 J_{[mn} J_{pq]}$$
$$\sqrt{g} \varepsilon_{\mathrm{mnpqrstu}} J^{tu} = 30 J_{[mn} J_{pq} J_{rs]}$$
$$\sqrt{g} \varepsilon_{\mathrm{mnpqrstu}} = 105 J_{[mn} J_{pq} J_{rs} J_{tu]}$$

$$\Omega_{[ijkl} \Omega_{mnpq]}^{*} = \frac{8}{35} \sqrt{g} \varepsilon_{ijklmnpq}$$

$$\gamma_m \eta = (\Pi^+)_m^n \gamma_n \eta$$
$$\gamma_{mn} \eta = i J_{mn} \eta - \frac{1}{8} \Omega_{mnpq} \gamma^{pq} \eta^c$$
$$\gamma_{mnp} \eta = 3 i J_{[mn} \gamma_{p]} \eta - \frac{1}{2} \Omega_{mnpq} \gamma^q \eta^c$$
$$\gamma_{mnpq} \eta = -3 J_{[mn} J_{pq]} \eta - \frac{3i}{4} J_{[mn} \Omega_{pq]ij} \gamma^{ij} \eta^c + \Omega_{mnpq} \eta^c$$

$$F_0 \eta = f_0 \eta$$
$$\underline{F_2} \eta = 4 i f_2 \eta - \frac{1}{16} f_{2|mn}^{(0,2)} \Omega^{mnpq} \gamma_{pq} \eta^c$$
$$\underline{F_4} \eta = -12 f_4 \eta + 16 \tilde{f}_4^* \eta^c - \frac{i}{8} f_{4|mn}^{(0,2)} \Omega^{mnpq} \gamma_{pq} \eta^c$$

$$\underline{h_1}\eta = h^{(0,1)}_{1|m}\gamma^m\eta$$
$$\underline{h_3}\eta = 3ih^{(0,1)}_{3|m}\gamma^m\eta + 8\tilde{h}^{(1,0)}_{3|m}\gamma_m\eta^c$$

$$\underline{h_{3|m}\eta} = 3i\left(h^{(1,0)}_{3|m} + h^{(0,1)}_{3|m}\right)\eta - \left(\frac{i}{8}h^{(0,1)}_{3|n}\Omega_m{}^{nrs} + \frac{1}{16}h^{(1,2)}_{3|mpq}\Omega^{pqrs}\right)\gamma_{rs}\eta^c - \frac{1}{2}\tilde{h}^{(1,0)}_{3|n}\Omega^*_m{}^{npq}\gamma_{pq}\eta$$

$$F_0\gamma_m\eta = f_0\gamma_m\eta$$
$$\underline{F_2}\gamma_m\eta = \left(2if_2\gamma_m - 2f^{(1,1)}_{2|mn}\gamma^n\right)\eta - \frac{1}{4}f^{(0,2)}_{2|np}\gamma_q\Omega^{npq}_m\eta^c$$
$$\underline{F_4}\gamma_m\eta = -4if^{(1,1)}_{4|mn}\gamma^n\eta + \frac{1}{6}f^{(1,3)}_{4|mnpq}\gamma_r\Omega^{npqr}\eta^c$$

$$\epsilon_1 = \frac{\alpha}{\sqrt{2}}\zeta_{1+}\otimes(\eta+\eta^c) + \frac{\beta}{\sqrt{2}}\zeta_{2+}\otimes\left(e^{i\theta_2}\eta + e^{-i\theta_2}\eta^c\right)$$
$$\epsilon_2 = \frac{\gamma}{\sqrt{2}}\zeta_{1-}\otimes\left(e^{i\theta}\eta + e^{-i\theta}\eta^c\right) + \frac{\delta}{\sqrt{2}}\zeta_{2-}\otimes\left(e^{i\theta_3}\eta + e^{-i\theta_3}\eta^c\right)$$

$$W = 0$$
$$2\,\mathrm{d}A + h_1 = 0$$
$$\left(2\,\underline{\mathrm{d}\phi} + \underline{h_1} + \underline{h_3}\right)(\eta+\eta^c) = 0$$
$$\left(2\,\underline{\mathrm{d}\phi} + \underline{h_1} + \underline{h_3}\right)\left(e^{i\theta_2}\eta + e^{-i\theta_2}\eta^c\right) = 0$$
$$\left(2\,\underline{\mathrm{d}\phi} + \underline{h_1} - \underline{h_3}\right)\left(e^{i\theta}\eta + e^{-i\theta}\eta^c\right) = 0$$
$$\left(\nabla_m + \partial_m\log\,\alpha + \frac{h_1}{4} - \underline{h_3}\right)\left(e^{i\theta_3}\eta + e^{-i\theta_3}\eta^c\right) = 0$$
$$\left(\nabla_m + \partial_m\log\,\beta + \frac{1}{4}h_{1|m} + \frac{1}{4}\underline{h_{3|m}}\right)\left(e^{i\theta_2}\eta + e^{-i\theta_2}\eta^c\right) = 0$$
$$\left(\nabla_m + \partial_m\log\,\gamma + \frac{1}{4}h_{1|m} - \frac{1}{4}\underline{h_{3|m}}\right)\left(e^{i\theta}\eta + e^{-i\theta}\eta^c\right) = 0$$
$$\left(\nabla_m + \partial_m\log\,\delta + \frac{1}{4}h_{1|m} - \frac{1}{4}\underline{h_{3|m}}\right)\left(e^{i\theta_3}\eta + e^{-i\theta_3}\eta^c\right) = 0$$

$$\underline{\sigma(F)}(\eta+\eta^c) = 0$$
$$\underline{\sigma(F)}\left(e^{i\theta_2}\eta + e^{-i\theta_2}\eta^c\right) = 0$$
$$\frac{\sigma(F)}{(F)}\gamma_m(\eta+\eta^c) = 0$$
$$\underline{\sigma(F)}\gamma_m\left(e^{i\theta_2}\eta + e^{-i\theta_2}\eta^c\right) = 0$$
$$\underline{F}\left(e^{i\theta}\eta + e^{-i\theta}\eta^c\right) = 0$$
$$\underline{F}\left(e^{i\theta_3}\eta + e^{-i\theta_3}\eta^c\right) = 0$$
$$\underline{F}\gamma_m\left(e^{i\theta}\eta + e^{-i\theta}\eta^c\right) = 0$$
$$\underline{F}\gamma_m\left(e^{i\theta_3}\eta + e^{-i\theta_3}\eta^c\right) = 0$$

$$f_8 = f_0 = 6f_4$$
$$f_6 = f_2 = 0$$
$$\tilde{f}_4 = 0$$
$$f^{(2,0)}_{2|mn} = f^{(2,0)}_{6|mn} = f^{(2,0)}_{4|mn} = 0$$
$$f^{(1,1)}_{4|mn} = 0$$
$$f^{(1,1)}_{6|mn} = -f^{(1,1)}_{2|mn}$$
$$f^{(3,1)}_{4|mn} = 0$$

$$W = 0$$
$$W_1 = W_2 = W_3 = W_4 = W_5 = 0$$
$$\alpha = e^{\frac{1}{2}A}$$
$$\beta = C_1 e^{\frac{1}{2}A}$$
$$\gamma = C_2 e^{\frac{1}{2}A}$$
$$\delta = C_3 e^{\frac{1}{2}A}$$
$$h_3 = 0$$
$$d\phi = dA = -\frac{1}{2}h_1$$
$$d\theta = d\theta_2 = d\theta_3 = 0$$

$$\epsilon_1 = \frac{\alpha}{\sqrt{2}}\zeta_+ \otimes (\eta + \eta^c), \epsilon_2 = 0$$

$$W = 0$$
$$2\,\mathrm{d}A + h_1 = 0$$
$$\left(2\,\underline{\mathrm{d}\phi} + \underline{h_1} + \underline{h_3}\right)(\eta + \eta^c) = 0$$
$$\left(\nabla_m + \partial_m \log\, \alpha + \frac{1}{4}h_{1|m} + \frac{1}{4}\underline{h_{3|m}}\right)(\eta + \eta^c) = 0$$

$$\underline{\sigma(F)}(\eta + \eta^c) = 0$$
$$\underline{\sigma(F)}\gamma_m(\eta + \eta^c) = 0$$

$$W = 0$$
$$\alpha = e^{\frac{1}{2}A}$$
$$h_1 = -2\,\mathrm{d}A$$
$$\tilde{h}^{(1,0)}_{3|m} = \frac{1}{4}\partial^+_m(A - \phi) - \frac{3}{8}ih^{(1,0)}_{3|m}$$
$$W_1 = -\frac{3}{8}h^{(1,0)}_{3|m} - \frac{1}{2}iW_5 - \frac{3}{4}i\partial^+_m(A - \phi)$$
$$W_2 = 2W_3 - 2ih^{(2,1)}_3$$
$$W_4 = \frac{2}{3}W_5 - ih^{(1,0)}_3$$

$$f_8 = f_0$$
$$f_6 = -f_2$$
$$\tilde{f}_4 = \frac{3}{4}f_0 - \frac{1}{8}f_4 + \frac{i}{2}f_2$$
$$f^{(2,0)}_{2|mn} = -f^{(2,0)}_{6|mn}$$
$$\left(f^{(2,0)}_{2|mn} - \frac{1}{8}\Omega_{mn}{}^{pq} f^{(0,2)}_{2|pq}\right) = -i\left(f^{(2,0)}_{4|mn} + \frac{1}{8}\Omega_{mn}{}^{pq} f^{(0,2)}_{4|pq}\right)$$
$$f^{(1,1)}_{4|mn} = 0$$
$$f^{(1,1)}_{6|mn} = -f^{(1,1)}_{2|mn}$$
$$f^{(3,1)}_{4|mn} = 0$$

$$S = \int \mathrm{d}^{11}x\sqrt{-g_{11}}\left(R_{11} - \frac{1}{48}G^2\right) - \frac{1}{6}\int A_3 \wedge G \wedge G$$

$$G = \mathrm{d}A_3.$$

$$\mathrm{d}s^2_{11} = e^{-\frac{1}{6}\phi}\,\mathrm{d}s^2_{10} + e^{\frac{4}{3}\phi}(C_1 + \mathrm{dz})^2,$$

$$A_3 = C_3 + B \wedge \mathrm{dz}$$

$$\mathcal{L} = R_{10} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{\frac{3}{2}\phi}F_2^2 - \frac{1}{12}e^{-\phi}H^2 - \frac{1}{48}e^{\frac{1}{2}\phi}F_4^2 + \text{ C.S. },$$

$$F_4 = \mathrm{d}C_3 - H \wedge C_1;\ H = \mathrm{d}B;\ F_2 = \mathrm{d}C_1$$

$$G = F_4 + H \wedge (\mathrm{d}z + C_1).$$

$$\mathrm{d}s_E^2 = e^{-\frac{1}{2}\phi}\, \mathrm{d}s_{\mathrm{str}}^2.$$

$$\star_{10}\, \omega_p = \frac{1}{p!\,(10-p)!}\sqrt{-g}\epsilon_{M_1 \ldots M_{10}} \omega^{M_{11-p} \ldots M_{10}}\, \mathrm{d}x^{M_1} \wedge \ldots \wedge\, \mathrm{d}x^{M_{10-p}}$$

$$\xi^+ = \eta + \eta^c, m = \xi^- = 0$$

$$\tilde{\nabla}\xi^+ = \nabla_m \xi^+ - \frac{1}{2}\gamma_m{}^n \partial_n A$$

$$\mathfrak{M} = \mathfrak{M}_c \times \mathfrak{M}_k$$

$$\varphi \lrcorner \chi = \frac{1}{p!(q-p)!}\varphi^{m_1 \ldots m_p} \chi_{m_1 \ldots m_p n_1 \ldots n_{q-p}} \mathrm{d}x^{n_1} \wedge \cdots \wedge \mathrm{d}x^{n_{q-p}} \ .$$

$$\mathcal{L}_{\mathrm{SYM}} = \mathrm{tr}\left[-\frac{1}{2}F_{\alpha\beta}F^{\alpha\beta} - D^\alpha\phi_a D_\alpha\phi_a + \frac{g^2}{2}[\phi_a, \phi_b]^2 + i\bar{\psi}_p\gamma^\alpha D_\alpha\psi_p - g\big(\bar{\psi}_p(\Gamma_a^{pq}P_+ + \bar{\Gamma}_a^{pq}P_-)[\phi_a, \psi_q]\big)\right],$$

$$\delta_\epsilon A_\alpha = i\bar{\psi}_p\gamma_\alpha\epsilon_p, \delta_\epsilon\phi_a = -i\bar{\psi}_p\big(\Gamma_a^{pq}P_+ + \bar{\Gamma}_a^{pq}P_-\big)\epsilon_q$$
$$\delta_\epsilon\psi_p = iF_{\alpha\beta}\Sigma^{\alpha\beta}\epsilon_p + \gamma^\alpha D_\alpha\phi_a\big(\Gamma_a^{pq}P_+ + \bar{\Gamma}_a^{pq}P_-\big)\epsilon_q - g[\phi_a, \phi_b]\big(\Gamma_{ab}^{pq}P_+ + \bar{\Gamma}_{ab}^{pq}P_-\big)\epsilon_q$$

$$\Gamma_{ab}^{pq} = v_p^-\Sigma^{ab}v_q^+, \bar{\Gamma}_{pq}^{ab} = v_p^+\Sigma^{ab}v_q^-$$

$$\Gamma_{ab}^{pq} = \frac{i}{4}\big(\bar{\Gamma}_a^{pr}\Gamma_b^{rq} - \bar{\Gamma}_b^{pr}\Gamma_a^{rq}\big), \bar{\Gamma}_{ab}^{pq} = \frac{i}{4}\big(\Gamma_a^{pr}\bar{\Gamma}_b^{rq} - \Gamma_b^{pr}\bar{\Gamma}_a^{rq}\big)$$

$$\Gamma_1^{pq} = i\big(\delta_{p1}\delta_{q4} - \delta_{p4}\delta_{q1} + \delta_{p2}\delta_{q3} - \delta_{p3}\delta_{q2}\big), \quad \Gamma_2^{pq} = i\big(\delta_{p1}\delta_{q2} - \delta_{p2}\delta_{q1} + \delta_{p3}\delta_{q4} - \delta_{p4}\delta_{q3}\big),$$
$$\Gamma_3^{pq} = i\big(\delta_{p1}\delta_{q3} - \delta_{p3}\delta_{q1} - \delta_{p2}\delta_{q4} + \delta_{p4}\delta_{q2}\big), \quad \Gamma_4^{pq} = -\big(\delta_{p1}\delta_{q4} - \delta_{p4}\delta_{q1} - \delta_{p2}\delta_{q3} + \delta_{p3}\delta_{q2}\big),$$
$$\Gamma_5^{pq} = \big(\delta_{p1}\delta_{q2} - \delta_{p2}\delta_{q1} - \delta_{p3}\delta_{q4} + \delta_{p4}\delta_{q3}\big), \quad \Gamma_6^{pq} = -\big(\delta_{p1}\delta_{q3} - \delta_{p3}\delta_{q1} + \delta_{p2}\delta_{q4} - \delta_{p4}\delta_{q2}\big),$$

$$\epsilon_p = \delta_{p4}\epsilon$$

$$\delta_\epsilon A_\alpha = i\bar{\psi}_4\gamma_\alpha\epsilon, \delta_\epsilon\phi_a = -i\bar{\psi}_p\big(\Gamma_a^{p4}P_+ + \bar{\Gamma}_a^{p4}P_-\big)\epsilon$$
$$\delta_\epsilon\psi_p = iF_{\alpha\beta}\Sigma^{\alpha\beta}\delta_{p4}\epsilon + \gamma^\alpha D_\alpha\phi_a\big(\Gamma_a^{p4}P_+ + \bar{\Gamma}_a^{p4}P_-\big)\epsilon - g[\phi_a, \phi_b]\big(\Gamma_{ab}^{p4}P_+ + \bar{\Gamma}_{ab}^{p4}P_-\big)\epsilon$$

$$\delta'_\epsilon\psi_p = \mu_{pq}\phi_a\big(\Gamma_a^{q4}P_+ + \bar{\Gamma}_a^{q4}P_-\big)\epsilon$$

$$(\delta_\epsilon + \delta'_\epsilon)\mathcal{L}_{\mathrm{SYM}} = \mathrm{tr}\big(2i\mu_{pq}\bar{\psi}_p(\delta_\epsilon + \delta'_\epsilon)\psi_q + 2M_{ab}\phi_a\delta_\epsilon\phi_b - 3igT_{abc}[\phi_b, \phi_c]\delta_\epsilon\phi_a\big)$$

$$T_{234} = \frac{1}{3}(\mu_1 - \mu_2 - \mu_3), \quad T_{126} = \frac{1}{3}(\mu_1 - \mu_2 + \mu_3),$$
$$T_{135} = \frac{1}{3}(\mu_1 + \mu_2 - \mu_3), \quad T_{456} = \frac{1}{3}(\mu_1 + \mu_2 + \mu_3).$$

$$\mathcal{L}_\mu = \mathrm{tr}\big(-i\mu_{pq}\bar{\psi}_p\psi_q - M_{ab}\phi_a\phi_b + igT_{abc}\phi_a[\phi_b, \phi_c]\big)$$

$$(\delta_\epsilon + \delta'_\epsilon)\big(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu\big) = 0$$

$$(\delta_\epsilon + \delta'_\epsilon)\big(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu\big) = \delta'_\epsilon\mathcal{L}_{\mathrm{SYM}} + \delta_\epsilon\mathcal{L}_\mu + \delta'_\epsilon\mathcal{L}_\mu$$

$$\delta'_\epsilon\mathcal{L}_{\mathrm{SYM}} = \mathrm{tr}[2i(\partial_\alpha\mu_m)\bar{\psi}_m\gamma^\alpha\phi_a(\Gamma_a^{m4}P_+ + \bar{\Gamma}_a^{m4}P_-)\epsilon + 2i\mu_m\bar{\psi}_m\gamma^\alpha D_\alpha\phi_a(\Gamma_a^{m4}P_+ + \bar{\Gamma}_a^{m4}P_-)\epsilon$$
$$-2g\mu_m\bar{\psi}_p[\phi_a, \phi_b]\big(\Gamma_a^{pm}\Gamma_b^{m4}P_+ + \bar{\Gamma}_a^{pm}\bar{\Gamma}_b^{m4}P_-\big)\epsilon]$$

$$\begin{aligned}\delta_\epsilon \mathcal{L}_\mu =& \mathrm{tr}[-2i\mu_m D_\alpha\phi_a\bar{\psi}_m\gamma^\alpha(\Gamma_a^{m4}P_+ + \bar{\Gamma}_a^{m4}P_-)\epsilon - g\mu_m[\phi_a,\phi_b]\bar{\psi}_m(\bar{\Gamma}_a^{mp}\Gamma_b^{p4}P_+ + \Gamma_a^{mp}\bar{\Gamma}_b^{p4}P_-)\epsilon \\ &+2iM_{ab}\phi_a\bar{\psi}_p(\Gamma_b^{p4}P_+ + \bar{\Gamma}_b^{p4}P_-)\epsilon + 3gT_{abc}[\phi_b,\phi_c])\bar{\psi}_p(\Gamma_a^{p4}P_+ + \bar{\Gamma}_a^{p4}P_-)\epsilon] \\ \delta'_\epsilon \mathcal{L}_\mu =& \mathrm{tr}[-2i\mu_{pr}\mu_{rq}\phi_a\bar{\psi}_p(\Gamma_a^{q4}P_+ + \bar{\Gamma}_a^{q4}P_-)\epsilon]\end{aligned}$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) = 2i(\partial_\alpha\mu_m)\mathrm{tr}\left[\left(-\sum_{a=1}^{3} + \sum_{a=4}^{6}\right)\phi_a\bar{\psi}_m(\Gamma_a^{m4}P_+ + \bar{\Gamma}_a^{m4}P_-)\right]\gamma^\alpha\epsilon.$$

$$\begin{aligned}(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) &= 2i(\partial_\alpha\mu_p)\mathrm{tr}\left[\left(-\sum_{a=1}^{3} + \sum_{a=4}^{6}\right)\phi_a\bar{\psi}_p(\Gamma_a^{p4}P_+ + \bar{\Gamma}_a^{p4}P_-)\right]\gamma^\alpha\epsilon \\ &= \mathrm{tr}[-2i(\partial_\alpha J_{ab})\phi_a\bar{\psi}_p(\Gamma_b^{p4}P_+ + \bar{\Gamma}_b^{p4}P_-)\gamma^\alpha\epsilon]\end{aligned}$$

$$J_{ab} = \mathrm{diag}(\mu_1,\mu_3,\mu_2,-\mu_1,-\mu_3,-\mu_2)$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) = \mathrm{tr}[-2iJ'_{ab}\phi_a\bar{\psi}_p(\Gamma_b^{p4}P_+ + \bar{\Gamma}_b^{p4}P_-)\gamma^1\epsilon]$$

$$\gamma^1\epsilon = \epsilon$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) = \mathrm{tr}\left[2J'_{ab}\phi_a\left(-i\bar{\psi}_p\left(\Gamma_b^{p^4}P_+ + \bar{\Gamma}_b^{p^4}P_-\right)\epsilon\right)\right] = \mathrm{tr}[2J'_{ab}\phi_a\delta\phi_b]$$

$$\mathcal{L}_J = -\mathrm{tr}(J'_{ab}\phi_a\phi_b)$$

$$S = \int d^4x(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu + \mathcal{L}_J)$$

$$T_{\mu\nu} = \mathrm{tr}\left[2\partial_\mu\phi_a\partial_\nu\phi_a + g_{\mu\nu}\left(-\partial^\alpha\phi_a\partial_\alpha\phi_a + \frac{g^2}{2}[\phi_a,\phi_b]^2 - (M_{ab} + J'_{ab})\phi_a\phi_b + igT_{abc}\phi_a[\phi_b,\phi_c]\right)\right].$$

$$E_0 = \int d^3xT_{00} = \int d^3x\mathrm{tr}\left[\phi'_a\phi'_a - \frac{g^2}{2}[\phi_a,\phi_b]^2 + (M_{ab} + J'_{ab})\phi_a\phi_b - igT_{abc}\phi_a[\phi_b,\phi_c]\right]$$

$$\begin{aligned}(\delta_\epsilon + \delta'_\epsilon)\psi_p =& iF_{\alpha\beta}\Sigma^{\alpha\beta}\delta_{p4}\epsilon + D_\alpha\phi_a(\Gamma_a^{p4}P_- + \bar{\Gamma}_a^{p4}P_+)\gamma^\alpha\epsilon - g[\phi_a,\phi_b](\Gamma_{ab}^{p4}P_+ + \bar{\Gamma}_{ab}^{p4}P_-)\epsilon \\ &+\mu_{pq}\phi_a(\Gamma_a^{q4}P_+ + \bar{\Gamma}_a^{q4}P_-)\epsilon\end{aligned}$$

$$(\delta_\epsilon + \delta'_\epsilon)\psi_p = \left[\phi'_a\left(\Gamma_a^{p^4}P_- + \bar{\Gamma}_a^{p^4}P_+\right) - g[\phi_a,\phi_b]\left(\Gamma_{ab}^{p^4}P_+ + \bar{\Gamma}_{ab}^{p^4}P_-\right) + \mu_{pq}\phi_a\left(\Gamma_a^{q^4}P_+ + \bar{\Gamma}_a^{q^4}P_-\right)\right]\epsilon$$

$$\begin{aligned}&\mathrm{tr}\left[\left|\phi'_a\left(\Gamma_a^{p^4}P_- + \bar{\Gamma}_a^{p^4}P_+\right) - g[\phi_a,\phi_b]\left(\Gamma_{ab}^{p^4}P_+ + \bar{\Gamma}_{ab}^{p^4}P_-\right) + \mu_{pq}\phi_a\left(\Gamma_a^{q^4}P_+ + \bar{\Gamma}_a^{q^4}P_-\right)\right|^2\right] \\ &= \mathrm{tr}\left[\phi'_a\phi'_a - \frac{ig}{3}(\tilde{T}_{abc}\phi_a[\phi_b,\phi_c])' - J_{ab}(\phi_a\phi_b)' - \frac{g^2}{2}[\phi_a,\phi_b]^2 - igT_{abc}\phi_a[\phi_b,\phi_c] + M_{ab}\phi_a\phi_b\right]\end{aligned}$$

$$|A|^2 \equiv AA^\dagger$$

$$\Gamma_a^{pq*} = \bar{\Gamma}_a^{pq}$$

$$\tilde{T}_{126} = \tilde{T}_{135} = -\tilde{T}_{234} = -\tilde{T}_{456} = -1$$

$$\begin{aligned}E_0 =& \int d^3x\mathrm{tr}\left[\left|\phi'_a\left(\Gamma_a^{p^4}P_- + \bar{\Gamma}_a^{p^4}P_+\right) - g[\phi_a,\phi_b]\left(\Gamma_{ab}^{p^4}P_+ + \bar{\Gamma}_{ab}^{p^4}P_-\right) + \mu_{pq}\phi_a\left(\Gamma_a^{q^4}P_+ + \bar{\Gamma}_a^{q^4}P_-\right)\right|^2\right] \\ &+ \int d^3x\mathcal{K}'\end{aligned}$$

$$\mathcal{K} = \mathrm{tr}\left(J_{ab}\phi_a\phi_b + \frac{ig}{3}\tilde{T}_{abc}\phi_a[\phi_b,\phi_c]\right)$$

$$\phi'_a(\Gamma_a^{p4}P_- + \bar{\Gamma}_a^{p4}P_+) - g[\phi_a,\phi_b](\Gamma_{ab}^{p4}P_+ + \bar{\Gamma}_{ab}^{p4}P_-) + \mu_{pq}\phi_a(\Gamma_a^{q4}P_+ + \bar{\Gamma}_a^{q4}P_-) = 0.$$

$$\phi'_a\bar{\Gamma}_a^{p4} - g[\phi_a,\phi_b]\Gamma_{ab}^{p4} + \mu_{pq}\phi_a\Gamma_a^{q4} = 0, \phi'_a\Gamma_a^{p4} - g[\phi_a,\phi_b]\bar{\Gamma}_{ab}^{p4} + \mu_{pq}\phi_a\bar{\Gamma}_a^{q4} = 0.$$

$$\phi_a'\bar{\Gamma}_a^{p4} - \frac{ig}{2}[\phi_a, \phi_b]\bar{\Gamma}_a^{pr}\Gamma_b^{r4} + \mu_{pq}\phi_a\Gamma_a^{q4} = 0$$

$$\int_{x_L}^{x_R} dx\mathcal{K}' = 0 \Leftrightarrow \mathcal{K}|_{x\to x_L} = \mathcal{K}|_{x\to x_R}$$

$$i\phi_1' + \phi_4' - g\big(i([\phi_2, \phi_3] + [\phi_5, \phi_6]) + ([\phi_2, \phi_6] + [\phi_3, \phi_5])\big) - \mu_1(i\phi_1 - \phi_4) = 0,$$
$$i\phi_3' - \phi_6' + g\big(-i([\phi_1, \phi_2] - [\phi_4, \phi_5]) + ([\phi_1, \phi_5] - [\phi_2, \phi_4])\big) - \mu_2(i\phi_3 + \phi_6) = 0,$$
$$i\phi_2' + \phi_5' + g\big(i([\phi_1, \phi_3] + [\phi_4, \phi_6]) + ([\phi_1, \phi_6] + [\phi_3, \phi_4])\big) - \mu_3(i\phi_2 - \phi_5) = 0,$$
$$[\phi_1, \phi_4] + [\phi_2, \phi_5] - [\phi_3, \phi_6] = 0.$$

$$\Phi_1 = g(\phi_1 + i\phi_4), \Phi_3 = g(\phi_2 + i\phi_5), \Phi_2 = g(\phi_3 - i\phi_6)$$

$$\Phi_i^{\dagger'} + \frac{1}{2}\sum_{j,k=1}^{3} \epsilon_{ijk}[\Phi_j, \Phi_k] - \mu_i\Phi_i = 0, \sum_{i=1}^{3} [\Phi_i, \Phi_i^\dagger] = 0.$$

$$\lim_{x_L\to-\infty} \mu_i(x_L) = \mu_{Li}, \lim_{x_R\to\infty} \mu_i(x_R) = \mu_{Ri}, (i = 1,2,3)$$

$$[\Phi_i, \Phi_j] - \epsilon_{ijk}(\mu_{0k}\Phi_k) = 0, \sum_{i=1}^{3} [\Phi_i^\dagger, \Phi_i] = 0$$

$$\Phi_1 = -i\sqrt{\mu_{02}\mu_{03}}T_1, \Phi_2 = -i\sqrt{\mu_{01}\mu_{03}}T_2, \Phi_3 = -i\sqrt{\mu_{01}\mu_{02}}T_3$$

$$T_i = \begin{pmatrix} T_i^{(n_1)} & & \\ & \ddots & \\ & & T_i^{(n_l)} \end{pmatrix}$$

$$\sum_{k=1}^{l} n_k = N$$

$$\sum_{n=1}^{\infty} nN_n = N$$

$$\phi_1 = \phi_2 = \phi_3 = 0, \phi_4 = -\frac{\sqrt{\mu_{02}\mu_{03}}}{g}T_1, \phi_5 = -\frac{\sqrt{\mu_{01}\mu_{02}}}{g}T_3, \phi_6 = \frac{\sqrt{\mu_{01}\mu_{03}}}{g}T_2$$

$$\mathcal{K}|_{\text{vac}} = -\frac{1}{3g^2}\mu_{01}\mu_{02}\mu_{03}\text{tr}(T_1^2 + T_2^2 + T_3^2)$$
$$= -\frac{1}{12g^2}\mu_{01}\mu_{02}\mu_{03}\sum_{n=1}^{\infty} n(n^2 - 1)N_n$$

$$\left(T_1^{(n)}\right)^2 + \left(T_1^{(n)}\right)^2 + \left(T_1^{(n)}\right)^2 = c_2(n)\mathbb{1}_{n\times n}$$

$$c_2(n) = \frac{1}{4}(n^2 - 1)$$

$$\mu_{L1}\mu_{L2}\mu_{L3}\sum_{n=1}^{\infty} n(n^2 - 1)N_n^{(L)} = \mu_{R1}\mu_{R2}\mu_{R3}\sum_{n=1}^{\infty} n(n^2 - 1)N_n^{(R)}$$

$$R_k^2 \sim \frac{\mu_{01}\mu_{02}\mu_{03}}{4N}n_k\left(n_k^2 - 1\right)$$

$$\mu_i(x) = m_{i0} + m_i(x) \text{ with } \int_0^{\tau} m_i(x)dx = 0$$

$$\Phi_i(x) = e^{K_i(x)}\tilde{\Phi}_i(x)$$
$$K_i(x) = m_{i0}(\xi_i - x) - \Lambda_i(x)$$
$$\Lambda_i = \int^x m_i\, m_i(x')dx'$$

$$\frac{d\xi_i}{dx} = e^{K_i - \Sigma_{i'\neq i} K_{i'}}$$

$$e^{\Sigma_{i'\neq i} K_{i'}}\left(\frac{d\tilde{\Phi}_i^\dagger}{d\xi_i} + \frac{1}{2}\sum_{j,k=1}^{3} \epsilon_{ijk}[\tilde{\Phi}_j, \tilde{\Phi}_k] + m_{i0}\tilde{\Phi}_i^\dagger\right) = e^{\Sigma_{i'\neq i} K_{i'}}\mu_i(\tilde{\Phi}_i^\dagger + \tilde{\Phi}_i)$$
$$\sum_{i=1}^{3} e^{2K_i}[\tilde{\Phi}_i, \tilde{\Phi}_i^\dagger] = 0$$

$$\frac{d\tilde{\Phi}_i}{d\xi_i} - \frac{1}{2}\sum_{j,k=1}^{3} \epsilon_{ijk}[\tilde{\Phi}_j, \tilde{\Phi}_k] + m_{i0}\tilde{\Phi}_i = 0$$

$$\Phi_i(x) = e^{K(x)}\tilde{\Phi}_i(x)$$
$$K(x) = m_0(\xi - x) - \Lambda(x) \ \text{ with } \Lambda = \int^x m\, m(x')dx'$$

$$\left(\frac{d\xi}{dx}\right)(x) = e^{-K(x)} = e^{-m_0(\xi - x)+\Lambda(x)}$$

$$\Phi_i(x) = \left(\frac{dx}{d\xi}\right)\tilde{\Phi}_i(x)$$

$$\left(\frac{dx}{d\xi}\right)(x) = m_0\int_{-\infty}^{x} e^{m_0(x'-x)+(\Lambda(x')-\Lambda(x))}dx'$$

$$\left(\frac{dx}{d\xi}\right)(x+\tau) = m_0\int_{-\infty}^{x+\tau} e^{m_0(x'-x-\tau)+(\Lambda(x')-\Lambda(x))}dx'$$
$$= m_0\int_{-\infty}^{x} e^{m_0(x''-x)+(\Lambda(x'')-\Lambda(x))}dx'' = \left(\frac{dx}{d\xi}\right)(x)$$

$$\frac{d\tilde{\Phi}_i}{d\xi_i} - \frac{1}{2}\sum_{j,k=1}^{3} \epsilon_{ijk}[\tilde{\Phi}_j, \tilde{\Phi}_k] = 0$$

$$\tilde{\Phi}_D^i = \mathrm{diag}(a_1^i, a_2^i, \cdots, a_N^i)$$

$$\mu(x) = m_1 \sin\, qx$$

$$\Phi_i(x) = e^{\frac{m_1}{q}\cos\, qx}\tilde{\Phi}_D^i$$

$$\Phi_i(x) = \left(\frac{dx}{d\xi}\right)\tilde{\Phi}_i(x)$$

$$\left(\frac{dx}{d\xi}\right)_x = m_0\int_{-\infty}^{x} e^{m_0(x'-x)-\frac{m_1}{q}(\cos\, qx' - \cos\, qx)}dx'$$
$$\tilde{\Phi}_i(x) = -im_0 T_i$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) = 2i(\partial_\alpha\mu)\mathrm{tr}\left[\left(-\sum_{a=1}^{3} + \sum_{a=4}^{6}\right)\phi_a\bar{\psi}_m(\Gamma_a^{mi}P_+ + \bar{\Gamma}_a^{mi}P_-)\gamma^\alpha\epsilon_i\right]$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) = 2i(\partial_\alpha\mu)\mathrm{tr}\left[\left(-\sum_{a=1,3} + \sum_{a=4,6}\right)\phi_a\bar{\psi}_p(\Gamma_a^{pi}P_+ + \bar{\Gamma}_a^{pi}P_-)\gamma^\alpha\epsilon_i\right]$$
$$= \mathrm{tr}[-2i\partial_\alpha J_{ab}\phi_a\bar{\psi}_p(\Gamma_b^{pi}P_+ + \bar{\Gamma}_b^{pi}P_-)\gamma^\alpha\epsilon_i]$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\text{SYM}} + \mathcal{L}_\mu) = \text{tr}[2J'_{ab}\phi_a(-i\bar{\psi}_p(\Gamma_b^{pi}P_+ + \bar{\Gamma}_b^{pi}P_-)\epsilon_i)] = \text{tr}(2J'_{ab}\phi_a\delta\phi_b).$$

$$\mathcal{L}_J = -\text{tr}(J'_{ab}\phi_a\phi_b)$$

$$\phi'_a\bar{\Gamma}_a^{pi} - \frac{ig}{2}[\phi_a,\phi_b]\bar{\Gamma}_a^{pr}\Gamma_b^{ri} + \mu_{pr}\phi_a\Gamma_a^{ri} = 0,\ \left(J_{ab}\phi_a\phi_b + \frac{ig}{3}\tilde{T}_{abc}\phi_a[\phi_b,\phi_c]\right)\Big|_{\text{boundary}} = 0,$$

$$\begin{array}{ll}
\phi'_3 - ig[\phi_1,\phi_5] - \mu\phi_3 = 0, & [\phi_1,\phi_2] = 0,\\
\phi'_6 - ig[\phi_4,\phi_5] + \mu\phi_6 = 0, & [\phi_2,\phi_4] = 0,\\
\phi'_4 - ig[\phi_5,\phi_6] + \mu\phi_4 = 0, & [\phi_2,\phi_6] = 0,\\
\phi'_1 + ig[\phi_3,\phi_5] - \mu\phi_1 = 0, & [\phi_2,\phi_3] = 0,\\
[\phi_1,\phi_4] - [\phi_3,\phi_6] = 0, & [\phi_2,\phi_5] = 0,\\
\phi'_5 + ig([\phi_1,\phi_3] + [\phi_4,\phi_6]) = 0, & \phi'_2 = 0,\\
[\phi_1,\phi_6] + [\phi_3,\phi_4] = 0. &
\end{array}$$

$$\begin{aligned}\mathcal{L}_{\text{SYM}} =& \text{tr}\Big[-\frac{1}{2}F_{\alpha\beta}F^{\alpha\beta} - D^\alpha\phi_a D_\alpha\phi_a + \frac{g^2}{2}[\phi_a,\phi_b]^2 + i\bar{\psi}_p\gamma^\alpha D_\alpha\psi_p\\ &-g(\bar{\psi}_p(\Gamma_a^{pq}P_+ + \bar{\Gamma}_a^{pq}P_-)[\phi_a,\psi_q])\Big]\end{aligned}$$

$$\delta_\epsilon A_\alpha = i\bar{\psi}_p\gamma_\alpha\epsilon_p, \delta_\epsilon\phi_a = -i\bar{\psi}_p(\Gamma_a^{pq}P_+ + \bar{\Gamma}_a^{pq}P_-)\epsilon_q$$
$$\delta_\epsilon\psi_p = iF_{\alpha\beta}\Sigma^{\alpha\beta}\epsilon_p + \gamma^\alpha D_\alpha\phi_a(\Gamma_a^{pq}P_+ + \bar{\Gamma}_a^{pq}P_-)\epsilon_q - g[\phi_a,\phi_b](\Gamma_{ab}^{pq}P_+ + \bar{\Gamma}_{ab}^{pq}P_-)\epsilon_q \quad (2.2)$$

$$\Sigma^{\alpha\beta} = \frac{i}{4}[\gamma^\alpha,\gamma^\beta]$$

$$\Gamma_{ab}^{pq} = v_p^-\Sigma^{ab}v_q^+, \bar{\Gamma}_{pq}^{ab} = v_p^+\Sigma^{ab}v_q^- \text{with } \Sigma^{ab} = -\frac{i}{4}[\Gamma^a,\Gamma^b]$$

$$\Gamma_{ab}^{pq} = \frac{i}{4}(\bar{\Gamma}_a^{pr}\Gamma_b^{rq} - \bar{\Gamma}_b^{pr}\Gamma_a^{rq}), \bar{\Gamma}_{ab}^{pq} = \frac{i}{4}(\Gamma_a^{pr}\bar{\Gamma}_b^{rq} - \Gamma_b^{pr}\bar{\Gamma}_a^{rq})$$

$$\begin{array}{ll}
\Gamma_1^{pq} = i(\delta_{p1}\delta_{q4} - \delta_{p4}\delta_{q1} + \delta_{p2}\delta_{q3} - \delta_{p3}\delta_{q2}), & \Gamma_2^{pq} = i(\delta_{p1}\delta_{q2} - \delta_{p2}\delta_{q1} + \delta_{p3}\delta_{q4} - \delta_{p4}\delta_{q3}),\\
\Gamma_3^{pq} = i(\delta_{p1}\delta_{q3} - \delta_{p3}\delta_{q1} - \delta_{p2}\delta_{q4} + \delta_{p4}\delta_{q2}), & \Gamma_4^{pq} = -(\delta_{p1}\delta_{q4} - \delta_{p4}\delta_{q1} - \delta_{p2}\delta_{q3} + \delta_{p3}\delta_{q2}),\\
\Gamma_5^{pq} = (\delta_{p1}\delta_{q2} - \delta_{p2}\delta_{q1} - \delta_{p3}\delta_{q4} + \delta_{p4}\delta_{q3}), & \Gamma_6^{pq} = -(\delta_{p1}\delta_{q3} - \delta_{p3}\delta_{q1} + \delta_{p2}\delta_{q4} - \delta_{p4}\delta_{q2}),
\end{array}$$

$$\epsilon_p = \delta_{p4}\epsilon$$

$$\begin{aligned}\delta_\epsilon A_\alpha &= i\bar{\psi}_4\gamma_\alpha\epsilon, \delta_\epsilon\phi_a = -i\bar{\psi}_p(\Gamma_a^{p4}P_+ + \bar{\Gamma}_a^{p4}P_-)\epsilon\\ \delta_\epsilon\psi_p &= iF_{\alpha\beta}\Sigma^{\alpha\beta}\delta_{p4}\epsilon + \gamma^\alpha D_\alpha\phi_a(\Gamma_a^{p4}P_+ + \bar{\Gamma}_a^{p4}P_-)\epsilon - g[\phi_a,\phi_b](\Gamma_{ab}^{p4}P_+ + \bar{\Gamma}_{ab}^{p4}P_-)\epsilon\end{aligned}$$

$$\delta'_\epsilon\psi_p = \mu_{pq}\phi_a(\Gamma_a^{q4}P_+ + \bar{\Gamma}_a^{q4}P_-)\epsilon$$

$$(\delta_\epsilon + \delta'_\epsilon)\mathcal{L}_{\text{SYM}} = \text{tr}(2i\mu_{pq}\bar{\psi}_p(\delta_\epsilon + \delta'_\epsilon)\psi_q + 2M_{ab}\phi_a\delta_\epsilon\phi_b - 3igT_{abc}[\phi_b,\phi_c]\delta_\epsilon\phi_a)$$

$$M_{ab} = \text{diag}(\mu_1^2,\mu_3^2,\mu_2^2,\mu_1^2,\mu_3^2,\mu_2^2)$$

$$\begin{aligned}T_{234} &= \frac{1}{3}(\mu_1 - \mu_2 - \mu_3), \quad T_{126} = \frac{1}{3}(\mu_1 - \mu_2 + \mu_3)\\ T_{135} &= \frac{1}{3}(\mu_1 + \mu_2 - \mu_3), \quad T_{456} = \frac{1}{3}(\mu_1 + \mu_2 + \mu_3)\end{aligned}$$

$$\mathcal{L}_\mu = \text{tr}(-i\mu_{pq}\bar{\psi}_p\psi_q - M_{ab}\phi_a\phi_b + igT_{abc}\phi_a[\phi_b,\phi_c])$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\text{SYM}} + \mathcal{L}_\mu) = 0$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\text{SYM}} + \mathcal{L}_\mu) = \delta'_\epsilon\mathcal{L}_{\text{SYM}} + \delta_\epsilon\mathcal{L}_\mu + \delta'_\epsilon\mathcal{L}_\mu,$$

$$\begin{aligned}\delta'_\epsilon\mathcal{L}_{\text{SYM}} =& \text{tr}[2i(\partial_\alpha\mu_m)\bar{\psi}_m\gamma^\alpha\phi_a(\Gamma_a^{m4}P_+ + \bar{\Gamma}_a^{m4}P_-)\epsilon + 2i\mu_m\bar{\psi}_m\gamma^\alpha D_\alpha\phi_a(\Gamma_a^{m4}P_+ + \bar{\Gamma}_a^{m4}P_-)\epsilon\\ &-2g\mu_m\bar{\psi}_p[\phi_a,\phi_b](\Gamma_a^{pm}\Gamma_b^{m4}P_+ + \bar{\Gamma}_a^{pm}\bar{\Gamma}_b^{m4}P_-)\epsilon],\end{aligned}$$

$$\delta_\epsilon \mathcal{L}_\mu = \mathrm{tr}[-2i\mu_m D_\alpha \phi_a \bar{\psi}_m \gamma^\alpha (\Gamma_a^{m4} P_+ + \bar{\Gamma}_a^{m4} P_-)\epsilon - g\mu_m [\phi_a, \phi_b] \bar{\psi}_m (\bar{\Gamma}_a^{mp} \Gamma_b^{p4} P_+ + \Gamma_a^{mp} \bar{\Gamma}_b^{p4} P_-)\epsilon$$
$$+2iM_{ab}\phi_a \bar{\psi}_p (\Gamma_b^{p4} P_+ + \bar{\Gamma}_b^{p4} P_-)\epsilon + 3gT_{abc}[\phi_b, \phi_c])\bar{\psi}_p (\Gamma_a^{p4} P_+ + \bar{\Gamma}_a^{p4} P_-)\epsilon]$$
$$\delta'_\epsilon \mathcal{L}_\mu = \mathrm{tr}[-2i\mu_{pr}\mu_{rq}\phi_a \bar{\psi}_p (\Gamma_a^{q4} P_+ + \bar{\Gamma}_a^{q4} P_-)\epsilon].$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) = 2i(\partial_\alpha \mu_m)\mathrm{tr}\left[\left(-\sum_{a=1}^{3} + \sum_{a=4}^{6}\right)\phi_a \bar{\psi}_m (\Gamma_a^{m4} P_+ + \bar{\Gamma}_a^{m4} P_-)\right]\gamma^\alpha \epsilon.$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) = 2i(\partial_\alpha \mu_p)\mathrm{tr}\left[\left(-\sum_{a=1}^{3} + \sum_{a=4}^{6}\right)\phi_a \bar{\psi}_p (\Gamma_a^{p4} P_+ + \bar{\Gamma}_a^{p4} P_-)\right]\gamma^\alpha \epsilon$$
$$= \mathrm{tr}[-2i(\partial_\alpha J_{ab})\phi_a \bar{\psi}_p (\Gamma_b^{p4} P_+ + \bar{\Gamma}_b^{p4} P_-)\gamma^\alpha \epsilon]$$

$$J_{ab} = \mathrm{diag}(\mu_1, \mu_3, \mu_2, -\mu_1, -\mu_3, -\mu_2).$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) = \mathrm{tr}[-2iJ'_{ab}\phi_a \bar{\psi}_p (\Gamma_b^{p4} P_+ + \bar{\Gamma}_b^{p4} P_-)\gamma^1 \epsilon],$$

$$\gamma^1 \epsilon = \epsilon$$

$$(\delta_\epsilon + \delta'_\epsilon)(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu) = \mathrm{tr}[2J'_{ab}\phi_a(-i\bar{\psi}_p (\Gamma_b^{p4} P_+ + \bar{\Gamma}_b^{p4} P_-)\epsilon)] = \mathrm{tr}[2J'_{ab}\phi_a \delta\phi_b]$$

$$\mathcal{L}_J = -\mathrm{tr}(J'_{ab}\phi_a \phi_b).$$

$$S = \int d^4x (\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu + \mathcal{L}_J)$$

$$T_{\mu\nu} = \mathrm{tr}[2\partial_\mu \phi_a \partial_\nu \phi_a$$
$$+g_{\mu\nu}\left(-\partial^\alpha \phi_a \partial_\alpha \phi_a + \frac{g^2}{2}[\phi_a, \phi_b]^2 - (M_{ab} + J'_{ab})\phi_a \phi_b + igT_{abc}\phi_a[\phi_b, \phi_c]\right)\Big].$$

$$E_0 = \int d^3x T_{00} = \int d^3x \mathrm{tr}\left[\phi'_a \phi'_a - \frac{g^2}{2}[\phi_a, \phi_b]^2 + (M_{ab} + J'_{ab})\phi_a \phi_b - igT_{abc}\phi_a[\phi_b, \phi_c]\right]$$

$$(\delta_\epsilon + \delta'_\epsilon)\psi_p = iF_{\alpha\beta}\Sigma^{\alpha\beta}\delta_{p4}\epsilon + D_\alpha \phi_a (\Gamma_a^{p4} P_- + \bar{\Gamma}_a^{p4} P_+)\gamma^\alpha \epsilon - g[\phi_a, \phi_b](\Gamma_{ab}^{p4} P_+ + \bar{\Gamma}_{ab}^{p4} P_-)\epsilon$$
$$+\mu_{pq}\phi_a (\Gamma_a^{q4} P_+ + \bar{\Gamma}_a^{q4} P_-)\epsilon$$

$$(\delta_\epsilon + \delta'_\epsilon)\psi_p = [\phi'_a (\Gamma_a^{p4} P_- + \bar{\Gamma}_a^{p4} P_+) - g[\phi_a, \phi_b](\Gamma_{ab}^{p4} P_+ + \bar{\Gamma}_{ab}^{p4} P_-) + \mu_{pq}\phi_a (\Gamma_a^{q4} P_+ + \bar{\Gamma}_a^{q4} P_-)]\epsilon$$

$$\mathrm{tr}\left[\left|\phi'_a (\Gamma_a^{p4} P_- + \bar{\Gamma}_a^{p4} P_+) - g[\phi_a, \phi_b](\Gamma_{ab}^{p4} P_+ + \bar{\Gamma}_{ab}^{p4} P_-) + \mu_{pq}\phi_a (\Gamma_a^{q4} P_+ + \bar{\Gamma}_a^{q4} P_-)\right|^2\right]$$
$$= \mathrm{tr}\left[\phi'_a \phi'_a - \frac{ig}{3}(\tilde{T}_{abc}\phi_a[\phi_b, \phi_c])' - J_{ab}(\phi_a \phi_b)' - \frac{g^2}{2}[\phi_a, \phi_b]^2 - igT_{abc}\phi_a[\phi_b, \phi_c] + M_{ab}\phi_a \phi_b\right]$$

$$|A|^2 \equiv AA^\dagger$$

$$\Gamma_a^{pq*} = \bar{\Gamma}_a^{pq}$$

$$\tilde{T}_{126} = \tilde{T}_{135} = -\tilde{T}_{234} = -\tilde{T}_{456} = -1.$$

$$E_0 = \int d^3x \mathrm{tr}[|\ \phi'_a (\Gamma_a^{p4} P_- + \bar{\Gamma}_a^{p4} P_+) - g[\phi_a, \phi_b](\Gamma_{ab}^{p4} P_+ + \bar{\Gamma}_{ab}^{p4} P_-)$$
$$+\mu_{pq}\phi_a (\Gamma_a^{q4} P_+ + \bar{\Gamma}_a^{q4} P_-)|^2\Big] + \int d^3x \mathcal{K}'$$

$$\mathcal{K} = \mathrm{tr}\left(J_{ab}\phi_a \phi_b + \frac{ig}{3}\tilde{T}_{abc}\phi_a[\phi_b, \phi_c]\right)$$

$$\phi'_a (\Gamma_a^{p4} P_- + \bar{\Gamma}_a^{p4} P_+) - g[\phi_a, \phi_b](\Gamma_{ab}^{p4} P_+ + \bar{\Gamma}_{ab}^{p4} P_-) + \mu_{pq}\phi_a (\Gamma_a^{q4} P_+ + \bar{\Gamma}_a^{q4} P_-) = 0$$

$$\phi'_a \bar{\Gamma}_a^{p4} - g[\phi_a, \phi_b]\Gamma_{ab}^{p4} + \mu_{pq}\phi_a \Gamma_a^{q4} = 0, \phi'_a \Gamma_a^{p4} - g[\phi_a, \phi_b]\bar{\Gamma}_{ab}^{p4} + \mu_{pq}\phi_a \bar{\Gamma}_a^{q4} = 0.$$

$$\phi_a' \bar{\Gamma}_a^{p4} - \frac{ig}{2}[\phi_a, \phi_b]\bar{\Gamma}_a^{pr}\Gamma_b^{r4} + \mu_{pq}\phi_a\Gamma_a^{q4} = 0$$

$$\int_{x_L}^{x_R} dx\mathcal{K}' = 0 \Leftrightarrow \mathcal{K}|_{x\to x_L} = \mathcal{K}|_{x\to x_R}$$

$$i\phi_1' + \phi_4' - g\big(i([\phi_2,\phi_3] + [\phi_5,\phi_6]) + ([\phi_2,\phi_6] + [\phi_3,\phi_5])\big) - \mu_1(i\phi_1 - \phi_4) = 0$$

$$i\phi_3' - \phi_6' + g\big(-i([\phi_1,\phi_2] - [\phi_4,\phi_5]) + ([\phi_1,\phi_5] - [\phi_2,\phi_4])\big) - \mu_2(i\phi_3 + \phi_6) = 0$$

$$i\phi_2' + \phi_5' + g\big(i([\phi_1,\phi_3] + [\phi_4,\phi_6]) + ([\phi_1,\phi_6] + [\phi_3,\phi_4])\big) - \mu_3(i\phi_2 - \phi_5) = 0$$

$$[\phi_1,\phi_4] + [\phi_2,\phi_5] - [\phi_3,\phi_6] = 0$$

$$\Phi_1 = g(\phi_1 + i\phi_4), \Phi_3 = g(\phi_2 + i\phi_5), \Phi_2 = g(\phi_3 - i\phi_6)$$

$$\Phi_i^{\dagger'} + \frac{1}{2}\sum_{j,k=1}^{3} \epsilon_{ijk}[\Phi_j, \Phi_k] - \mu_i\Phi_i = 0, \sum_{i=1}^{3} [\Phi_i, \Phi_i^\dagger] = 0.$$

$$\lim_{x_L\to-\infty} \mu_i(x_L) = \mu_{Li}, \lim_{x_R\to\infty} \mu_i(x_R) = \mu_{Ri}, (i = 1,2,3)$$

$$[\Phi_i, \Phi_j] - \epsilon_{ijk}(\mu_{0k}\Phi_k) = 0, \sum_{i=1}^{3} [\Phi_i^\dagger, \Phi_i] = 0$$

$$\Phi_1 = -i\sqrt{\mu_{02}\mu_{03}}T_1, \Phi_2 = -i\sqrt{\mu_{01}\mu_{03}}T_2, \Phi_3 = -i\sqrt{\mu_{01}\mu_{02}}T_3$$

$$T_i = \begin{pmatrix} T_i^{(n_1)} & & \\ & \ddots & \\ & & T_i^{(n_l)} \end{pmatrix}$$

$$\sum_{k=1}^{l} n_k = N$$

$$\sum_{n=1}^{\infty} nN_n = N$$

$$\phi_1 = \phi_2 = \phi_3 = 0, \phi_4 = -\frac{\sqrt{\mu_{02}\mu_{03}}}{g}T_1, \phi_5 = -\frac{\sqrt{\mu_{01}\mu_{02}}}{g}T_3, \phi_6 = \frac{\sqrt{\mu_{01}\mu_{03}}}{g}T_2$$

$$\begin{aligned}\mathcal{K}|_{\text{vac}} &= -\frac{1}{3g^2}\mu_{01}\mu_{02}\mu_{03}\text{tr}(T_1^2 + T_2^2 + T_3^2) \\ &= -\frac{1}{12g^2}\mu_{01}\mu_{02}\mu_{03}\sum_{n=1}^{\infty} n(n^2-1)N_n\end{aligned}$$

$$\left(T_1^{(n)}\right)^2 + \left(T_1^{(n)}\right)^2 + \left(T_1^{(n)}\right)^2 = c_2(n)\mathbb{1}_{n\times n}$$

$$c_2(n) = \frac{1}{4}(n^2 - 1)$$

$$\mu_{L1}\mu_{L2}\mu_{L3}\sum_{n=1}^{\infty} n(n^2-1)N_n^{(L)} = \mu_{R1}\mu_{R2}\mu_{R3}\sum_{n=1}^{\infty} n(n^2-1)N_n^{(R)}$$

$$\Phi_1(x) = -if_1(x)\sqrt{\mu_2(x)\mu_3(x)}T_1, \Phi_2(x) = -if_2(x)\sqrt{\mu_1(x)\mu_3(x)}T_2,$$
$$\Phi_3(x) = -if_3(x)\sqrt{\mu_1(x)\mu_2(x)}T_3,$$

$$-\frac{d}{dx}\left(\sqrt{\mu_2\mu_3}f_1\right) - \mu_1\sqrt{\mu_2\mu_3}f_2f_3 + \mu_1\sqrt{\mu_2\mu_3}f_1 = 0$$
$$-\frac{d}{dx}\left(\sqrt{\mu_1\mu_3}f_2\right) - \mu_2\sqrt{\mu_1\mu_3}f_1f_3 + \mu_2\sqrt{\mu_1\mu_3}f_2 = 0$$
$$-\frac{d}{dx}\left(\sqrt{\mu_1\mu_2}f_3\right) - \mu_3\sqrt{\mu_1\mu_2}f_1f_2 + \mu_3\sqrt{\mu_1\mu_2}f_3 = 0$$

$$f^{-2}\frac{df}{dx} - P(x)f^{-1} = -\mu(x),$$

$$\frac{dg}{dx} + P(x)g = \mu(x),$$

$$g(x) = \frac{1}{u(x)}\left(\int u(x)\mu(x)dx + C\right),$$

$$u(x) = e^{\int P(x)dx}$$

$$\lim_{x\to\pm\infty} f(x) = 1$$

$$R_k^2 \sim \frac{\mu_{01}\mu_{02}\mu_{03}}{4N} n_k\left(n_k^2 - 1\right)$$

$$\mu_i(x) = m_{i0} + m_i(x) \text{ with } \int_0^\tau m_i(x)dx = 0$$

$$\Phi_i(x) = e^{K_i(x)}\tilde{\Phi}_i(x)$$
$$K_i(x) = m_{i0}(\xi_i - x) - \Lambda_i(x)$$
$$\Lambda_i = \int^x m_i\, m_i(x')dx'$$

$$\frac{d\xi_i}{dx} = e^{K_i - \sum_{i'\neq i} K_{i'}}$$

$$e^{\sum_{i'\neq i} K_{i'}}\left(\frac{d\tilde{\Phi}_i^\dagger}{d\xi_i} + \frac{1}{2}\sum_{j,k=1}^{3} \epsilon_{ijk}\left[\tilde{\Phi}_j, \tilde{\Phi}_k\right] + m_{i0}\tilde{\Phi}_i^\dagger\right) = e^{\sum_{i'\neq i} K_{i'}}\mu_i\left(\tilde{\Phi}_i^\dagger + \tilde{\Phi}_i\right)$$
$$\sum_{i=1}^{3} e^{2K_i}\left[\tilde{\Phi}_i, \tilde{\Phi}_i^\dagger\right] = 0$$

$$\frac{d\tilde{\Phi}_i}{d\xi_i} - \frac{1}{2}\sum_{j,k=1}^{3} \epsilon_{ijk}\left[\tilde{\Phi}_j, \tilde{\Phi}_k\right] + m_{i0}\tilde{\Phi}_i = 0$$

$$\Phi_i(x) = e^{K(x)}\tilde{\Phi}_i(x)$$
$$K(x) = m_0(\xi - x) - \Lambda(x) \text{ with } \Lambda = \int^x m\, m(x')dx'$$

$$\left(\frac{d\xi}{dx}\right)(x) = e^{-K(x)} = e^{-m_0(\xi - x) + \Lambda(x)}$$

$$\Phi_i(x) = \left(\frac{dx}{d\xi}\right)\tilde{\Phi}_i(x)$$

$$\left(\frac{dx}{d\xi}\right)(x) = m_0\int_{-\infty}^{x} e^{m_0(x'-x) + (\Lambda(x') - \Lambda(x))}dx'$$

$$\left(\frac{dx}{d\xi}\right)(x+\tau) = m_0\int_{-\infty}^{x+\tau} e^{m_0(x'-x-\tau) + (\Lambda(x') - \Lambda(x))}dx'$$
$$= m_0\int_{-\infty}^{x} e^{m_0(x''-x) + (\Lambda(x'') - \Lambda(x))}dx'' = \left(\frac{dx}{d\xi}\right)(x)$$

$$\frac{d\tilde{\Phi}_i}{d\xi_i} - \frac{1}{2}\sum_{j,k=1}^{3} \epsilon_{ijk}\left[\tilde{\Phi}_j, \tilde{\Phi}_k\right] = 0$$

$$\tilde{\Phi}_D^i = \mathrm{diag}\left(a_1^i, a_2^i, \cdots, a_N^i\right)$$

$$\mu(x) = m_1 \sin\ qx,$$

$$\Phi_i(x) = e^{\frac{m_1}{q}\cos\ qx}\tilde{\Phi}_D^i$$

$$\Phi_i(x) = \left(\frac{dx}{d\xi}\right)\tilde{\Phi}_i(x)$$

$$\left(\frac{dx}{d\xi}\right)_x = m_0 \int_{-\infty}^{x} e^{m_0(x'-x)-\frac{m_1}{q}(\cos\ qx' - \cos\ qx)} dx'$$
$$\tilde{\Phi}_i(x) = -i m_0 T_i$$

$$\mathcal{O}^{(\Delta)} = \mathrm{tr}\left((P^a \phi_a)^\Delta\right)$$

$$P^a = \left(0,0,0,1, e^{\frac{2}{3}\pi i}, e^{\frac{4}{3}\pi i}\right)$$

$$\langle \mathcal{O}^{(2)} \rangle = \mathrm{tr}\left(\phi_4^2 + e^{-\frac{2i\pi}{3}}\phi_5^2 + e^{\frac{2i\pi}{3}}\phi_6^2\right), \langle \mathcal{O}^{(3)} \rangle = 6\mathrm{tr}(\phi_4\phi_5\phi_6)$$

$$\phi_4 = -\frac{m_0}{g}\left(\frac{dx}{d\xi}\right)T_1, \phi_5 = -\frac{m_0}{g}\left(\frac{dx}{d\xi}\right)T_3, \phi_6 = \frac{m_0}{g}\left(\frac{dx}{d\xi}\right)T_2$$

$$\langle \mathcal{O}^{(2)} \rangle = 0, \langle \mathcal{O}^{(3)} \rangle = 3N\left(\frac{m_0}{g}\right)^3\left(\frac{dx}{d\xi}\right)^3$$

$$(\delta_\epsilon + \delta'_\epsilon)\left(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu\right) = 2i(\partial_\alpha\mu)\mathrm{tr}\left[\left(-\sum_{a=1}^{3} + \sum_{a=4}^{6}\right)\phi_a \bar{\psi}_m\left(\Gamma_a^{mi}P_+ + \bar{\Gamma}_a^{mi}P_-\right)\gamma^\alpha\epsilon_i\right]$$

$$(\delta_\epsilon + \delta'_\epsilon)\left(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu\right) = 2i(\partial_\alpha\mu)\mathrm{tr}\left[\left(-\sum_{a=1,3} + \sum_{a=4,6}\right)\phi_a \bar{\psi}_p\left(\Gamma_a^{pi}P_+ + \bar{\Gamma}_a^{pi}P_-\right)\gamma^\alpha\epsilon_i\right]$$
$$= \mathrm{tr}\left[-2i\partial_\alpha J_{ab}\phi_a\bar{\psi}_p\left(\Gamma_b^{pi}P_+ + \bar{\Gamma}_b^{pi}P_-\right)\gamma^\alpha\epsilon_i\right]$$

$$J_{ab} = \mathrm{diag}(\mu, 0, \mu, -\mu, 0, -\mu)$$

$$(\delta_\epsilon + \delta'_\epsilon)\left(\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_\mu\right) = \mathrm{tr}\left[2J'_{ab}\phi_a\left(-i\bar{\psi}_p\left(\Gamma_b^{pi}P_+ + \bar{\Gamma}_b^{pi}P_-\right)\epsilon_i\right)\right] = \mathrm{tr}(2J'_{ab}\phi_a\delta\phi_b)$$

$$\mathcal{L}_J = -\mathrm{tr}(J'_{ab}\phi_a\phi_b)$$

$$\phi'_a\bar{\Gamma}_a^{pi} - \frac{ig}{2}[\phi_a, \phi_b]\bar{\Gamma}_a^{pr}\Gamma_b^{ri} + \mu_{pr}\phi_a\Gamma_a^{ri} = 0,\ \left(J_{ab}\phi_a\phi_b + \frac{ig}{3}\tilde{T}_{abc}\phi_a[\phi_b, \phi_c]\right)\Big|_{\mathrm{boundary}} = 0$$

$$\begin{aligned}
&\phi'_3 - ig[\phi_1, \phi_5] - \mu\phi_3 = 0, [\phi_1, \phi_2] = 0,\\
&\phi'_6 - ig[\phi_4, \phi_5] + \mu\phi_6 = 0, [\phi_2, \phi_4] = 0,\\
&\phi'_4 - ig[\phi_5, \phi_6] + \mu\phi_4 = 0, [\phi_2, \phi_6] = 0,\\
&\phi'_1 + ig[\phi_3, \phi_5] - \mu\phi_1 = 0, [\phi_2, \phi_3] = 0,\\
&[\phi_1, \phi_4] - [\phi_3, \phi_6] = 0, [\phi_2, \phi_5] = 0,\\
&\phi'_5 + ig([\phi_1, \phi_3] + [\phi_4, \phi_6]) = 0, \phi'_2 = 0,\\
&[\phi_1, \phi_6] + [\phi_3, \phi_4] = 0.
\end{aligned}$$

$$R_6 = \frac{g_{YM}^2}{8\pi^2}$$

$$F = -\frac{(9/4 + m^2)^2}{96\pi}\lambda N^2$$

$$\lambda = g_{YM}^2 N/r$$

$$I_{AdS} = -\frac{5\pi R_6}{12r} N^3$$

$$\langle W \rangle \sim \exp\left(\frac{\lambda}{8\pi}\right)$$

$$\langle W \rangle \sim \exp\left((9/4 + m^2)\frac{\lambda}{8\pi}\right).$$

$$\langle W \rangle_{AdS} \sim \exp\left(\frac{2\pi N R_6}{r}\right).$$

$$R_6 = \frac{5 g_{YM}^2}{32\pi^2}$$

$$L_{\text{vector}} = \frac{1}{g_{YM}^2}\text{Tr}\left[\frac{1}{2}F_{mn}F^{mn} - D_m\sigma D^m\sigma - \frac{1}{2}D_{IJ}D^{IJ} + \frac{2}{r}\sigma t^{IJ}D_{IJ} - \frac{10}{r^2}t^{IJ}t_{IJ}\sigma^2 \right.$$
$$\left. + i\lambda_I\Gamma^m D_m\lambda^I - \lambda_I[\sigma, \lambda^I] - \frac{i}{r}t^{IJ}\lambda_I\lambda_J\right]$$

$$L_{vector} = \frac{1}{g_{YM}^2}\left[\frac{1}{2}F_{mn}F^{mn} - D_m\sigma D^m\sigma - \frac{4}{r^2}\sigma^2 + \cdots\right]$$

$$L_{\text{scalar}} = D_m\phi D^m\phi + \frac{d-2}{4(d-1)}\mathcal{R}\phi^2$$

$$g_{mn} \to e^{2\Omega}g_{mn}$$

$$\phi \to e^{\frac{2-d}{2}\Omega}\phi$$

$$\mathcal{R} = \frac{d(d-1)}{r^2}$$

$$L_{\text{scalar}} = D_m\phi D^m\phi + \frac{15}{4r^2}\phi^2$$

$$L_{\text{matter}} = \epsilon^{IJ}D_m\bar{q}_I D^m q_J - \epsilon^{IJ}\bar{q}_I\sigma^2 q_J + \frac{15}{4r^2}\epsilon^{IJ}\bar{q}_I q_J - 2i\bar{\psi}\not{D}\psi - 2\bar{\psi}\sigma\psi$$
$$-4\epsilon^{IJ}\bar{\psi}\lambda_I q_J - iq_I D^{IJ}q_J$$

$$L_{mass} = -M^2\epsilon^{IJ}\bar{q}_I q_J + \frac{2i}{r}Mt^{IJ}\bar{q}_I q_J - 2M\bar{\psi}\psi$$

$$-\frac{4}{r^2}\sigma^2 + \left(\frac{15}{4r^2} - M^2\right)\epsilon^{IJ}\bar{q}_I q_J + \frac{2i}{r}Mt^{IJ}\bar{q}_I q_J$$

$$-\frac{4}{r^2}\sigma^2 + \frac{3}{r^2}\bar{q}_1 q^1 + \frac{4}{r^2}\bar{q}_2 q^2$$

$$Z = \int_{\text{Cartan}} [d\phi] e^{-\frac{8\pi^3 r}{g_{YM}^2}\text{Tr}(\phi^2) - \frac{\pi k}{3}\text{Tr}(\phi^3)} Z_{1-\text{loop}}^{\text{vect}}(\phi) Z_{1-\text{loop}}^{\text{hyper}}(\phi) + \mathcal{O}\left(e^{-\frac{16\pi^3 r}{g_{YM}^2}}\right)$$

$$Z_{1-\text{loop}}^{\text{vect}}(\phi) = \prod_{\beta}\prod_{t\neq 0}(t - \langle\beta, i\phi\rangle)^{\left(1+\frac{3}{2}t+\frac{1}{2}t^2\right)}$$

$$Z_{1-\text{loop}}^{\text{hyper}}(\phi) = \prod_{\mu}\prod_{t}\left(t - \langle i\phi, \mu\rangle + \frac{3}{2}\right)^{-\left(1+\frac{3}{2}t+\frac{1}{2}t^2\right)}$$

$$S=\frac{1}{g_{YM}^2}\int_{S^5}\mathrm{Tr}(F\wedge *F)+\cdots+\frac{ik}{24\pi^2}\int_{S^5}\mathrm{Tr}(A\wedge dA\wedge dA)+\cdots$$

$$\mathcal{P}=x\prod_{t=1}^{\infty}(t+x)^{\left(1+\frac{3}{2}t+\frac{1}{2}t^2\right)}(t-x)^{\left(1-\frac{3}{2}t+\frac{1}{2}t^2\right)}$$

$$\log\,\mathcal{P}=\sum_{t=1}^{\infty}\left(3x-\frac{x^2}{2}\right)+\text{ convergent part}$$

$$S_3(x)=2\pi e^{-\zeta'(-2)}xe^{\frac{x^2}{4}-\frac{3}{2}x}\prod_{t=1}^{\infty}\left(\left(1+\frac{x}{t}\right)^{\left(1+\frac{3}{2}t+\frac{1}{2}t^2\right)}\left(1-\frac{x}{t}\right)^{\left(1-\frac{3}{2}t+\frac{1}{2}t^2\right)}e^{\frac{x^2}{2}-3x}\right)$$

$$\begin{aligned}\log\left(Z_{1-\text{ loop}}^{\text{vect}}(\phi)Z_{1-\text{ loop}}^{\text{hyper}}(\phi)\right)&=-\frac{\pi\Lambda r}{2}\sum_{\beta}(\langle\beta,i\phi\rangle)^2+\frac{\pi\Lambda r}{2}\sum_{\mu}(\langle i\phi,\mu\rangle)^2+\text{ convergent part}\\&=\pi\Lambda r(C_2(\text{adj})-C_2(R))\mathrm{Tr}(\phi^2)+\text{ convergent part},\end{aligned}$$

$$\mathrm{Tr}(T_AT_B)=C_2(R)\delta_{AB}$$

$$\sum_{\mu}(\langle\phi,\mu\rangle)^2=2C_2(R)\mathrm{Tr}(\phi^2)$$

$$\frac{1}{g_{eff}^2}=\frac{1}{g_{YM}^2}-\frac{\Lambda}{8\pi^2}(C_2(\text{adj})-C_2(R))$$

$$S_3(x)e^{-\frac{x^2}{4}+\frac{3}{2}x}$$

$$Z=\int d\phi e^{-\frac{8\pi^3r}{g_{YM}^2}\mathrm{Tr}(\phi^2)-\frac{\pi k}{3}\mathrm{Tr}(\phi^3)}\det_{Ad}(S_3(i\phi))\det_R^{-1}\left(S_3\left(i\phi+\frac{3}{2}\right)\right)$$

$$S_3(-x)=S_3(x+3),S_3\left(x+\frac{3}{2}\right)=S_3\left(-x+\frac{3}{2}\right)$$

$$\det_R\left(S_3\left(i\phi+\frac{3}{2}\right)\right)=\det_R\left(S_3\left(-i\phi+\frac{3}{2}\right)\right)=\det_{\bar{R}}\left(S_3\left(i\phi+\frac{3}{2}\right)\right).$$

$$Z=\int d\phi e^{-\frac{8\pi^3r}{g_{YM}^2}\mathrm{Tr}(\phi^2)-\frac{\pi k}{3}\mathrm{Tr}(\phi^3)}\det_{Ad}(S_3(i\phi))\det_R^{-1}\left(S_3\left(i\phi+im+\frac{3}{2}\right)\right),$$

$$\det_R\left(S_3\left(i\phi+im+\frac{3}{2}\right)\right)=\det_{\bar{R}}\left(S_3\left(i\phi-im+\frac{3}{2}\right)\right).$$

$$\begin{aligned}&\int d\phi e^{-\frac{8\pi^3r}{g_{YM}^2}\mathrm{Tr}(\phi^2)-\frac{\pi k}{3}\mathrm{Tr}(\phi^3)}\det_{Ad}(S_3(i\phi))\\&\qquad\times\det_R^{-1/2}\left(S_3\left(i\phi+im+\frac{3}{2}\right)\right)\det_{\bar{R}}^{-1/2}\left(S_3\left(i\phi-im+\frac{3}{2}\right)\right)\end{aligned}$$

$$\int d\phi e^{-\mathcal{F}}$$

$$\mathcal{F}=\frac{8\pi^3r}{g_{YM}^2}\mathrm{Tr}(\phi^2)+\frac{\pi k}{3}\mathrm{Tr}(\phi^3)-\sum_{\beta}\log\,S_3(\langle i\phi,\beta\rangle)+\sum_{\mu}\log\,S_3\left(\langle i\phi,\mu\rangle+im+\frac{3}{2}\right)$$

$$\log\,S_3(z)\sim-\mathrm{sgn}(\mathrm{Im}z)\pi i\left(\frac{1}{6}z^3-\frac{3}{4}z^2+z+\cdots\right)$$

$$\frac{1}{2\pi r^3}\mathcal{F} = \frac{4\pi^2}{g_{YM}^2}\mathrm{Tr}(\phi^2) + \frac{k}{6}\mathrm{Tr}(\phi^3) + \frac{1}{12}\left(\sum_\beta |\langle\phi,\beta\rangle|^3 - \sum_\mu |\langle\phi,\mu\rangle + m|^3\right) + O(r^{-2})$$

$$\mathcal{F} = \frac{8\pi^3 r}{g_{YM}^2}\mathrm{Tr}(\phi^2) + \frac{\pi k}{3}\mathrm{Tr}(\phi^3) + \frac{\pi}{6}\left(\sum_\beta |\langle\phi,\beta\rangle|^3 - \sum_\mu |\langle\phi,\mu\rangle|^3\right)$$
$$-\frac{\pi}{2}m\sum_\mu \mathrm{sgn}(\langle\phi,\mu\rangle)(\langle\phi,\mu\rangle)^2 - \pi\sum_\beta |\langle\phi,\beta\rangle| - \frac{\pi}{2}\left(m^2 + \frac{1}{4}\right)\sum_\mu |\langle\phi,\mu\rangle| + \cdots$$

$$\mathcal{F} = \frac{8\pi^3 r}{g_{YM}^2}\mathrm{Tr}(\phi^2) + \frac{\pi k}{3}\mathrm{Tr}(\phi^3) - \sum_\beta \log S_3(\langle i\phi,\beta\rangle) - \mathrm{sgn}(m)\frac{\pi}{2}\sum_\mu \left(\frac{1}{3}(\langle\phi,\mu\rangle)^3 + m(\langle\phi,\mu\rangle)^2\right)$$

$$\mathrm{Tr}(T_A T_B T_C + T_A T_C T_B) = C_3(R) d_{ABC}$$

$$\sum_\mu (\langle\phi,\mu\rangle)^3 = C_3(R)\mathrm{Tr}(\phi^3)$$

$$k_{eff} = k - \mathrm{sgn}(m)\frac{C_3(R)}{2},$$

$$\frac{r}{g_{eff}^2} = \frac{r}{g_{YM}^2} - \frac{|m|}{8\pi^2}C_2(R)$$

$$S_3(z) = 2e^{-\zeta'(-2)}\sin(\pi z)e^{\frac{1}{2}f(z)}e^{\frac{3}{2}l(z)}$$

$$l(z) = -z\log\left(1 - e^{2\pi i z}\right) + \frac{i}{2}\left(\pi z^2 + \frac{1}{\pi}\mathrm{Li}_2\left(e^{2\pi i z}\right)\right) - \frac{i\pi}{12}$$

$$f(z) = \frac{i\pi z^3}{3} + z^2\log\left(1 - e^{-2\pi i z}\right) + \frac{iz}{\pi}\mathrm{Li}_2\left(e^{-2\pi i z}\right) + \frac{1}{2\pi^2}\mathrm{Li}_3\left(e^{-2\pi i z}\right) - \frac{\zeta(3)}{2\pi^2}.$$

$$Z = \int_{\mathrm{Cartan}} [d\phi] e^{-\frac{8\pi^3 r}{g_{YM}^2}\mathrm{Tr}(\phi^2)} \prod_\beta \left(\sin(\pi\langle\beta, i\phi\rangle) e^{-\frac{1}{4}l\left(\frac{1}{2} - im - \langle\beta,i\phi\rangle\right) - \frac{1}{4}l\left(\frac{1}{2} - im + \langle\beta,i\phi\rangle\right)}\right.$$
$$\left.\times\ e^{\frac{1}{2}f(\langle\beta,i\phi\rangle) - \frac{1}{4}f\left(\frac{1}{2} - im - \langle\beta,i\phi\rangle\right) - \frac{1}{4}f\left(\frac{1}{2} - im + \langle\beta,i\phi\rangle\right)}\right) + \cdots$$

$$\lambda = \frac{g_{YM}^2 N}{r},$$

$$Z \sim \int \prod_{i=1}^N d\phi_i \exp\left(-\frac{8\pi^3 N}{\lambda}\sum_i \phi_i^2 + \sum_{j\neq i}\sum_i \left[\log\left[\sinh\left(\pi(\phi_i - \phi_j)\right)\right]\right.\right.$$
$$-\frac{1}{4}l\left(\frac{1}{2} - im + i(\phi_i - \phi_j)\right) - \frac{1}{4}l\left(\frac{1}{2} - im - i(\phi_i - \phi_j)\right) + \frac{1}{2}f\left(i(\phi_i - \phi_j)\right) -$$
$$\left.\left.-\frac{1}{4}f\left(\frac{1}{2} - im + i(\phi_i - \phi_j)\right) - \frac{1}{4}f\left(\frac{1}{2} - im - i(\phi_i - \phi_j)\right)\right]\right)$$

$$l(z) = -l(-z), f(z) = f(-z)$$

$$\frac{df(z)}{dz} = \pi z^2\cot(\pi z); \quad \frac{dl(z)}{dz} = -\pi z\cot(\pi z);$$

$$\lim_{|x|\to\infty}\mathrm{Re}f\left(\frac{1}{2} + ix\right) = -\frac{\pi}{3}|x|^3 + \frac{\pi}{4}|x|; \quad \lim_{x\to\infty}\mathrm{Im}f\left(\frac{1}{2} \pm ix\right) = \pm\frac{\pi}{2}x^2$$
$$\lim_{|x|\to\infty}\mathrm{Re}l\left(\frac{1}{2} + ix\right) = -\frac{\pi}{2}|x|; \quad \lim_{x\to\infty}\mathrm{Im}l\left(\frac{1}{2} \pm ix\right) = \mp\frac{\pi}{2}x^2$$
$$\lim_{|x|\to\infty}\mathrm{Re}f(ix) = -\frac{\pi}{3}|x|^3; \quad \mathrm{Im}f(ix) = 0$$

$$\frac{16\pi^3 N}{\lambda}\phi_i = \pi \sum_{j\neq i} [(2-(\phi_i-\phi_j)^2)\coth\left(\pi(\phi_i-\phi_j)\right)$$
$$+\frac{1}{2}\left(\frac{1}{4}+(\phi_i-\phi_j-m)^2\right)\tanh\left(\pi(\phi_i-\phi_j-m)\right)$$
$$+\frac{1}{2}\left(\frac{1}{4}+(\phi_i-\phi_j+m)^2\right)\tanh\left(\pi(\phi_i-\phi_j+m)\right)\Big].$$

$$\frac{16\pi^3 N}{\lambda}\phi_i \approx 2\sum_{j\neq i}\frac{1}{\phi_i-\phi_j}$$

$$\rho(\phi) \equiv \frac{1}{N}\frac{dn}{d\phi} = \frac{2}{\pi\phi_0^2}\sqrt{\phi_0^2-\phi^2}\ \phi_0 = \sqrt{\frac{\lambda}{4\pi^3}},$$

$$\int \rho(\phi)d\phi = 1$$

$$F = -\log Z \approx -N^2 \log\sqrt{\lambda}$$

$$\frac{16\pi^3 N}{\lambda}\phi_i = \pi\left(\frac{9}{4}+m^2\right)\sum_{j\neq i}\operatorname{sign}(\phi_i-\phi_j).$$

$$\phi_i = \frac{(9+4m^2)\lambda}{64\pi^2 N}(2i-N)$$

$$\rho(\phi) = \frac{32\pi^2}{(9+4m^2)\lambda} \qquad |\phi| \leq \phi_m, \phi_m = \frac{(9+4m^2)\lambda}{64\pi^2}$$
$$= 0\ |\phi| > \phi_m.$$

$$Z \sim \int \prod_i d\phi_i e^{-\frac{8\pi^3 N}{\lambda}\sum_i \phi_i^2 + \frac{\pi}{2}\left(\frac{9}{4}+m^2\right)\sum_{j\neq i}\sum_i |\phi_i-\phi_j|}$$

$$F \equiv -\log Z \approx -\frac{g_{YM}^2 N^3}{96\pi r}\left(\frac{9}{4}+m^2\right)^2$$

$$\sum_{i=1}^{N}(2i-N)^2 \approx \frac{1}{3}N^3, \sum_{j\neq i}\sum_{i=1}^{N}|i-j| \approx \frac{1}{3}N^3.$$

$$F = -\frac{25 g_{YM}^2 N^3}{384\pi r}$$

$$\frac{16\pi^3 N}{\lambda}\phi_i = \pi\sum_{j\neq i}\left[\left(2-(\phi_i-\phi_j)^2\right)\coth\left(\pi(\phi_i-\phi_j)\right) + 2m(\phi_i-\phi_j)\right]$$
$$= 2\pi m N\phi_i + \pi\sum_{j\neq i}\left(2-(\phi_i-\phi_j)^2\right)\coth\left(\pi(\phi_i-\phi_j)\right)$$

$$\frac{16\pi^3 N}{\lambda_{eff}}\phi_i = \pi\sum_{j\neq i}\left(2-(\phi_i-\phi_j)^2\right)\coth\left(\pi(\phi_i-\phi_j)\right),$$

$$\frac{1}{\lambda_{eff}} = \frac{1}{\lambda} - \frac{m}{8\pi^2}.$$

$$\langle W\rangle = \frac{1}{N}\langle \mathrm{Tr} e^{2\pi\phi_i}\rangle.$$

$$\langle W\rangle \sim \frac{1}{N}\int \prod_i d\phi_i \sum_i e^{2\pi\phi_i} e^{-\frac{8\pi^3 N}{\lambda}\sum_i \phi_i^2 + \frac{\pi}{2}\left(\frac{9}{4}+m^2\right)\sum_{j\neq i}\sum_i|\phi_i-\phi_j|}$$

$$\langle W\rangle = \int d\phi\rho(\phi)e^{2\pi\phi}$$

$$\langle W\rangle \approx \int d\phi\rho(\phi)(1+2\pi^2\phi^2) = 1+\frac{\lambda}{8\pi} \approx \exp\left(\frac{\lambda}{8\pi}\right).$$

$$\langle W\rangle \approx \frac{32\pi^2}{(9+4m^2)\lambda}\int_{-\phi_m}^{\phi_m} e^{2\pi\phi}d\phi \sim \exp\left(\frac{\lambda}{8\pi}\left(\frac{9}{4}+m^2\right)\right)$$

$$\frac{16\pi^3 N}{\lambda}\psi_i^{(r)} = \pi\left[\sum_{j\neq i}\left(2-\left(\psi_i^{(r)}-\psi_j^{(r)}\right)^2\right)\coth\left(\pi\left(\psi_i^{(r)}-\psi_j^{(r)}\right)\right)\right.$$

$$+\left(\sum_j\left[\frac{1}{4}\left(\frac{1}{4}+\left(\psi_i^{(r)}-\psi_j^{(r+1)}-m\right)^2\right)\tanh\left(\pi\left(\psi_i^{(r)}-\psi_j^{(r+1)}-m\right)\right)\right.\right.$$

$$\left.\left.+\frac{1}{4}\left(\frac{1}{4}+\left(\psi_i^{(r)}-\psi_j^{(r-1)}-m\right)^2\right)\tanh\left(\pi\left(\psi_i^{(r)}-\psi_j^{(r-1)}-m\right)\right)\right]\right]$$

$$+(m\to -m))$$

$$\psi_i^{(r)} = \frac{(9+4m^2)\lambda}{64\pi^2 N}(2i-N/k)$$

$$F \approx -k\frac{g_{YM}^2 N^3}{96\pi r k^3}\left(\frac{9}{4}+m^2\right)^2 = -\frac{g_{YM}^2 N^3}{96\pi r k^2}\left(\frac{9}{4}+m^2\right)^2$$

$$\langle W\rangle \approx \exp\left(\frac{\lambda}{8\pi k}\right)$$

$$\langle W\rangle \sim \exp\left(\frac{\lambda}{8\pi k}\left(\frac{9}{4}+m^2\right)\right),$$

$$\tau\frac{d\phi_i}{dt} = -\frac{\partial\mathcal{F}}{\partial\phi_i}.$$

$$a_3 = c_1 + c_2 m + c_3 m^2 + c_4 m^3 + \cdots$$

$$ds^2 = \ell^2(\cosh^2\,\rho d\tau^2 + d\rho^2 + \sinh^2\,\rho d\Omega_5^2),$$

$$I_{AdS} = I_{\text{bulk}} + I_{\text{surface}} + I_{\text{ct}},$$

$$I_{\text{bulk}} = -\frac{1}{16\pi G_N}\text{Vol}(S^4)\int d^7x\sqrt{g}(R-2\Lambda)$$

$$R-2\Lambda = -\frac{12}{\ell^2}$$

$$I_{\text{bulk}} = -\frac{1}{256\pi^8\ell_{pl}^9}\left(\frac{\pi^2\ell^4}{6}\right)\frac{2\pi R_6}{r}\pi^3(-12\ell^5)\int_0^{\rho_0}\cosh\,\rho\sinh^5\,\rho d\rho = \frac{4\pi R_6}{3r}N^3\sinh^6\,\rho_0$$

$$\sinh^6\,\rho_0 = \frac{1}{64}\epsilon^{-6} - \frac{3}{32}\epsilon^{-4} + \frac{15}{64}\epsilon^{-2} - \frac{5}{16} + O(\epsilon^2)$$

$$I_{AdS} = -\frac{5\pi R_6}{12r}N^3$$

$$\langle W\rangle \sim e^{-T^{(2)}\int dV}$$

$$T^{(2)} = \frac{1}{(2\pi)^2 l_p^3}$$

$$\int dV = l^3 \int_0^{\frac{2\pi R_6}{r}} d\tau \int_0^{2\pi} d\phi \int_0^{\rho_0} d\rho \sinh(\rho) \cosh(\rho)$$

$$T^{(2)} \int dV = \frac{\pi N R_6}{r}\left(\frac{1}{\epsilon} - 2 + \epsilon\right)$$

$$\langle W \rangle \sim \exp\left(\frac{2\pi N R_6}{r}\right)$$

$$R_6 = \frac{g_{YM}^2}{16\pi^2}\left(\frac{9}{4} + m^2\right)$$

$$R_6 = \frac{g_{YM}^2}{16\pi^2}\frac{5}{2}$$

$$I_{AdS} = -\frac{5\pi R_6}{12rk}N^3$$

$$R_6 = \frac{g_{YM}^2}{16\pi^2 k}\left(\frac{9}{4} + m^2\right)$$

$$I = \frac{1}{e^2}\int d^{10}x \mathrm{Tr}\left(\frac{1}{2}F_{IJ}F^{IJ} - i\bar{\Psi}\Gamma^I D_I \Psi\right)$$

$$J^I = \frac{1}{2}\mathrm{Tr}\Gamma^{JK}F_{JK}\Gamma^I\Psi$$

$$\begin{aligned} \delta A_I &= i\bar{\varepsilon}\Gamma_I\Psi \\ \delta\Psi &= \frac{1}{2}\Gamma^{IJ}F_{IJ}\varepsilon. \end{aligned}$$

$$M = \begin{pmatrix} S & T \\ U & V \end{pmatrix}$$

$$V = \begin{pmatrix} e^{i\beta} & 0 \\ 0 & 1 \end{pmatrix}, \beta \in \mathbb{R}$$

$$\begin{aligned} B_0 &= \Gamma_{456789} \\ B_1 &= \Gamma_{3456} \\ B_2 &= \Gamma_{3789} \end{aligned}$$

$$\begin{aligned} B_0 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ B_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ B_2 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned}$$

$$\mathrm{Tr}\bar{\varepsilon}\Gamma^{IJ}F_{IJ}\Gamma_3\Psi = 0$$

$$\bar{\Gamma}\Psi' = -\Psi'$$

$$\begin{aligned} 0 &= \bar{\varepsilon}\left(\Gamma^{\mu\nu}F_{\mu\nu} + 2\Gamma^{3\mu}F_{3\mu}\right)\Psi' \\ 0 &= \sum_{\mu=0,1,2} \bar{\varepsilon}\left(\Gamma^{\mu a}D_\mu X_a\right)\Psi' \\ 0 &= \sum_{\mu=0,1,2} \bar{\varepsilon}\left(\Gamma^{\mu m}D_\mu Y_m\right)\Psi' \\ 0 &= \bar{\varepsilon}\Gamma^{am}[X_a, Y_m]\Psi' \\ 0 &= \bar{\varepsilon}(2\Gamma^{3a}D_3X_a + \Gamma^{ab}[X_a, X_b])\Psi' \\ 0 &= \bar{\varepsilon}(2\Gamma^{3m}D_3Y_a + \Gamma^{mn}[Y_m, Y_n])\Psi'. \end{aligned}$$

$$0 = \bar{\varepsilon}_0\left(F_{\mu\nu} - \epsilon_{\mu\nu\lambda}F^{3\lambda}B_0\right)\cdot\vartheta$$
$$0 = D_\mu X_a \cdot \bar{\varepsilon}_0 B_2\vartheta$$
$$0 = D_\mu Y_m \cdot \bar{\varepsilon}_0 B_1\vartheta$$
$$0 = [X_a, Y_m]\cdot\bar{\varepsilon}_0 B_0\vartheta$$
$$0 = \bar{\varepsilon}_0([X_b, X_c] - \epsilon_{abc}D_3X_aB_1)\vartheta$$
$$0 = \bar{\varepsilon}_0\left([Y_m, Y_n] - \epsilon_{pmn}D_3Y_pB_2\right)\vartheta$$

$$\epsilon_{\lambda\mu\nu}F^{3\lambda} + \gamma F_{\mu\nu} = 0$$

$$\bar{\varepsilon}_0(1 + \gamma B_0)\vartheta = 0.$$

$$D_3X_a + \frac{u}{2}\epsilon_{abc}[X_b, X_c] = 0$$

$$0 = \bar{\varepsilon}_0B_2\vartheta = \bar{\varepsilon}_0(1 + uB_1)\vartheta$$

$$\vartheta = \binom{a}{1}$$

$$\gamma = -\frac{2a}{1 - a^2}, u = -\frac{2a}{1 + a^2}$$

$$\vec{Y}(0) = \vec{w}.$$

$$I = \frac{1}{e^2}\int \mathrm{d}^4x\mathrm{Tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu}\right) + \frac{\theta}{8\pi^2}\int \mathrm{Tr}F\wedge F$$

$$A_\mu(x^3) = A_\mu^\tau(-x^3), \mu = 0,1,2$$
$$A_3(x^3) = -A_3^\tau(-x^3)$$
$$\vec{X}(x^3) = -\vec{X}^\tau(-x^3)$$
$$\vec{Y}(x^3) = \vec{Y}^\tau(-x^3)$$

$$F_{3\mu}^+ | = F_{\mu\nu}^- | = 0$$
$$D_3X^- | = X^+ | = 0$$
$$Y^- | = D_3Y^+ | = 0$$

$$\bar{\varepsilon}_0\vartheta^+ = \bar{\varepsilon}_0B_1\vartheta^+ = 0$$
$$\bar{\varepsilon}_0B_0\vartheta^- = \bar{\varepsilon}_0B_2\vartheta^- = 0.$$

$$[\mathfrak{g}^+, \mathfrak{g}^+] = \mathfrak{g}^+, [\mathfrak{g}^+, \mathfrak{g}^-] = \mathfrak{g}^-, [\mathfrak{g}^-, \mathfrak{g}^-] = \mathfrak{g}^+,$$

$$\vec{X}^+(0) = \vec{v}$$
$$\vec{Y}^-(0) = \vec{w}.$$

$$\frac{1}{2e^2}\int_{\mathbb{R}^{1,2}} \mathrm{d}^3x \int_L \mathrm{d}y \sum_{\mu,\nu=0,1,2} \mathrm{Tr}F_{\mu\nu}F^{\mu\nu}$$

$$ds^2 = -\int_L \mathrm{d}y\mathrm{Tr}\left(\delta A_3^2 + \sum_i \delta X_i^2\right)$$

$$\omega_i = \int_L \mathrm{d}y\mathrm{Tr}(\delta A_3\wedge\delta X_i + \delta X_{i+1}\wedge\delta X_{i-1}), i = 1,2,3$$

$$\langle a, b\rangle = -\int \mathrm{d}ye(y)^{-2}\mathrm{Tr}ab$$

$$\iota_{V(\alpha)}\omega_i = \int \mathrm{d}y\mathrm{Tr}(-D_3\alpha\delta X_i - \delta A_3[\alpha, X_i] + \alpha[X_{i+1}, \delta X_{i-1}] - \alpha[\delta X_{i+1}, X_{i-1}])$$

$$\mu_i(\alpha) = \int \mathrm{d}y\mathrm{Tr}\left(\alpha\left(\frac{DX_i}{Dy} + [X_{i+1}, X_{i-1}]\right)\right) + \mathrm{Tr}\alpha X_i(0)$$

$$\vec{\mu}(y)=\frac{D\vec{X}}{Dy}+\vec{X}\times\vec{X}(y)+\delta(y)\vec{X}(0)$$

$$\int \mathrm{d}^3x(\vec{\mu},\vec{\mu})$$

$$-\int_{\mathbb{R}^{2,1}}\mathrm{d}^3x\int_L \mathrm{d}y\mathrm{Tr}\vec{\mu}^2$$

$$\vec{\mu}(y)=\frac{D\vec{X}}{Dy}+\vec{X}\times\vec{X}(y)+\delta(y)\big(\vec{X}(0)+\vec{\mu}^Z\big).$$

$$\vec{X}(0)+\vec{\mu}^Z=0.$$

$$\vec{X}^+(0)+\vec{\mu}^Z=0,$$

$$\vec{\mu}(y)=\frac{D\vec{X}}{Dy}+\vec{X}\times\vec{X}(y)+\delta(y)\big(\vec{X}(0)+\vec{\mu}^Z-\vec{v}\big),$$

$$\vec{X}^+(0)+\vec{\mu}^Z=\vec{v}$$

$$\delta\bar{\Psi}=\frac{1}{2}\bar{\varepsilon}\Gamma^{IJ}F_{IJ}$$

$$\bar{\varepsilon}\Gamma^{IJ}F_{IJ}=0$$

$$\begin{aligned}
0&=[X_a,Y_m]\cdot\bar{\varepsilon}_0B_0\\
0&=\bar{\varepsilon}_0([X_b,X_c]-\epsilon_{abc}D_3X_aB_1)\\
0&=\bar{\varepsilon}_0\big([Y_m,Y_n]-\epsilon_{pmn}D_3Y_pB_2\big).
\end{aligned}$$

$$\frac{DX^1}{Dy}=\pm[X^2,X^3]$$

$$\frac{D\vec{Y}}{Dy}=[\vec{Y},\vec{Y}]=[\vec{Y},\vec{X}]=0$$

$$\frac{\mathrm{d}X^1}{\mathrm{d}y}+[X^2,X^3]=0$$

$$X^i(y)=\frac{t^i}{y}$$

$$\boldsymbol{f}=\boldsymbol{f}^+\oplus\boldsymbol{f}^-=\mathfrak{h}$$

$$\begin{aligned}
F^+_{3\mu}\mid&=F^-_{\mu\nu}\mid=0\\
D_3X^-\mid&=X^+\mid=0\\
Y^-\mid&=D_3Y^+\mid=0
\end{aligned}$$

$$\begin{aligned}
\vec{X}^+\mid+\vec{\mu}^Z&=\vec{v}\\
\vec{Y}^-\mid&=\vec{w}.
\end{aligned}$$

$$\frac{\mathrm{d}X_i}{\mathrm{d}y}+[X_{i+1},X_{i-1}]=0, i=1{,}2{,}3$$

$$\omega_i=\int_L \mathrm{d}y\mathrm{Tr}(\delta A\wedge\delta X_i+\delta X_{i+1}\wedge\delta X_{i-1}), i=1{,}2{,}3.$$

$$\mu_i=\frac{DX_i}{Dy}+[X_{i+1},X_{i-1}], i=1{,}2{,}3,$$

$$\vec{\mu} = \int \mathrm{d}y \vec{\mu}(y)$$

$$\vec{\mu} = \vec{X}(0).$$

$$X_i(y) = \frac{t_i}{y} + \cdots.$$

$$\vec{\mu} = \vec{X}_{\mathfrak{h}}(0).$$

$$\frac{\mathcal{D}\mathcal{X}}{\mathcal{D}y} = 0.$$

$$\mathcal{D}\mathcal{X}/\mathcal{D}y = \mathrm{d}\mathcal{X}/\mathrm{d}y + [\mathcal{A}, \mathcal{X}]$$

$$x = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$w_u = \begin{pmatrix} 0 & 1 \\ u/2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$ad - bc = 0$$

$$X_i(y) = g \frac{t_i}{y + f^{-1}} g^{-1}$$

$$\frac{\mathcal{D}\mathcal{X}}{\mathcal{D}y} = 0$$

$$x = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

$$\mathfrak{g}_{\mathbb{C}} = \bigoplus_{j=1}^{s} \mathcal{T}_j.$$

$$\frac{\mathcal{D}\mathcal{X}}{\mathcal{D}y} = 0.$$

$$\begin{aligned} \mathcal{X} &= \frac{t_1 + it_2}{y} + \cdots = \frac{t_+}{y} + \cdots \\ \mathcal{A} &= \frac{it_3}{y} + \cdots \end{aligned}$$

$$[it_3, v_\alpha] = m_\alpha v_\alpha$$

$$\mathcal{X} = \sum_\alpha \epsilon_\alpha \frac{v_\alpha}{y^{m_\alpha}}$$

$$\mathcal{X} = \frac{t_+}{y} + \sum_{m_\alpha \leq 0} \epsilon_\alpha v_\alpha y^{-m_\alpha}$$

$$\phi = \sum_\alpha f_\alpha v_\alpha y^{-m_\alpha}$$

$$\phi = \sum_{m_\alpha < 0} f_\alpha v_\alpha y^{-m_\alpha}$$

$$\mathcal{X} = \frac{t_+}{y} + \sum_{\alpha \in P_-} \epsilon_\alpha v_\alpha y^{-m_\alpha},$$

$$t_+ + \sum_{\alpha \in P_-} \epsilon_\alpha v_\alpha$$

$$x = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \dots & 1 \\ a_n & a_{n-1} & a_{n-2} & \dots & 0 \end{pmatrix},$$

$$\vec{\mu}(y) = \frac{D\vec{X}}{Dy} + \vec{X} \times \vec{X} + \delta(y)\vec{\mu}^Z$$

$$\frac{D\vec{X}}{Dy} + \vec{X} \times \vec{X} + \delta(y)\vec{\mu}^Z = 0$$

$$\Delta\vec{X} + \vec{\mu}^Z = 0.$$

$$\frac{\mathcal{D}\mathcal{X}}{\mathcal{D}y} + \delta(y)\mu_{\mathbb{C}}^Z = 0$$

$$\frac{\mathrm{d}\mathcal{X}}{\mathrm{d}y} + \delta(y)\mu_{\mathbb{C}}^Z = 0$$

$$\frac{D\vec{X}}{Dy} + \vec{X} \times \vec{X} + \sum_{\alpha=1}^{k} \delta(y - y_\alpha)\vec{\mu}^{Z_\alpha} = 0.$$

$$\frac{\mathrm{d}\mathcal{X}}{\mathrm{d}y} + \sum_{\alpha=1}^{k} \delta(y - y_\alpha)\mu_{\mathbb{C}}^{Z_\alpha} = 0$$

$$\mathcal{X}'' = \mathcal{X}' - M.$$

$$\begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ C_1 & C_2 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} * & * \\ * & * \end{pmatrix}.$$

$$\begin{pmatrix} * & * & \times \\ * & * & \times \\ \times & \times & \times \end{pmatrix}.$$

$$0 = \frac{\mathcal{D}\mathcal{X}}{\mathcal{D}y} = \frac{\mathrm{d}\mathcal{X}}{\mathrm{d}y} + [\mathcal{A}, \mathcal{X}]$$

$$\mathcal{X}_{\infty,+} = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & W \end{pmatrix},$$

$$\mathcal{X}(y) = \begin{pmatrix} * & * & W^{1/2}B^1 \\ * & * & W^{1/2}B^2 \\ W^{1/2}C_1 & W^{1/2}C_2 & W \end{pmatrix}$$

$$\mathcal{X}_{\infty,-} - M$$

$$\mathcal{X}_{\infty,-} - M = \mathcal{X}''$$

$$M = \begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \times \\ \times & \times & W+\times \end{pmatrix}$$

$$\vec{X} = \frac{\vec{t}}{y + c^{-1}}$$

$$\vec{X} = \frac{\vec{t}}{y}$$

$$\begin{pmatrix} \vec{A} & \vec{B} \\ \vec{C} & \vec{D} \end{pmatrix}$$

$$\vec{D} = \frac{\vec{t}}{y} + \cdots;$$

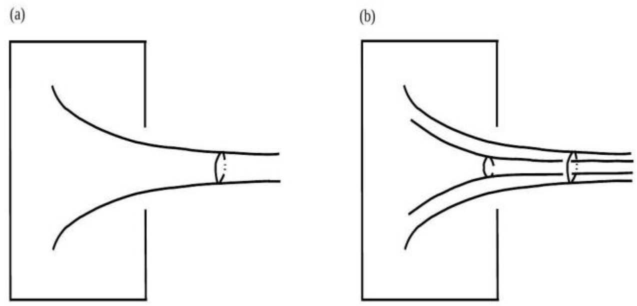
$$\vec{\mu}^{+} = \vec{A},$$

$$\dim \mathcal{M}_{+} = m^2 + m.$$

$$\dim \mathcal{M}_{-} = m^2 - m.$$

$$\vec{\mu}^{-} = -\vec{X}(0) = -\lim_{y \to 0^-} \vec{X}(y)$$

$$(s-n)/2 = \sum_{j=1}^{k} (j-1) n_j.$$



$$\mathcal{X} = \begin{pmatrix} a & y^{-1} & 0 \\ by & a & cy^{1/2} \\ dy^{1/2} & 0 & e \end{pmatrix},$$

$$\mathcal{X} = \begin{pmatrix} a & y^{-1} \\ by & a \end{pmatrix}.$$

$$\mathcal{X} = \begin{pmatrix} a & y^{-1} & c \\ by & a & d \\ e & f & g \end{pmatrix}.$$

$$\mathcal{X} = \begin{pmatrix} a & (y-y_1)^{-1} & 0 \\ b(y-y_1) & a & c(y-y_1)^{1/2} \\ d(y-y_1)^{1/2} & 0 & e \end{pmatrix}$$

$$\vec{\mu} = \vec{X}(0)_f$$

$$\mathcal{M}_{\rho,H} = \mathcal{M}_{\rho}///H.$$

$$\mathcal{M}_{\rho,H,B} = \left(\mathcal{M}_{\rho}(\vec{X}_{\infty}) \times \mathcal{H}\right)///H.$$

$$\frac{D\vec{Y}}{Dy} = [\vec{Y},\vec{Y}] = [\vec{Y},\vec{X}] = 0$$

$$\vec{Y}^{-}(0) = \vec{w}.$$

$$\vec{Y}(0) = \vec{w}\mathrm{mod}\mathfrak{h}.$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & * & * \\ 0 & 0 & * & * \end{pmatrix}$$

$$\vec{Y}_{\infty} = \vec{w} + h$$

$$\mathcal{M}_{H,\vec{Y}} = \bigcup_{\alpha} \mathcal{M}_{\vec{Y}}^{\alpha}///H_{\vec{Y}}$$

$$\varepsilon \to \frac{1 - \Gamma_{0123}}{\sqrt{2}}\varepsilon$$

$$\frac{D\vec{X}}{Dy} + \vec{X} \times \vec{X} = 0$$

$$\vec{\mu}_{L} = \vec{X}(0)$$
$$\vec{\mu}_{R} = -\vec{X}(\ell).$$

$$\mathcal{G}_{\ell} \times_{G} \mathcal{G}_{\ell'} = \mathcal{G}_{\ell+\ell'}$$

$$\frac{\mathcal{D}\mathcal{X}}{\mathcal{D}y} = 0$$

$$g = P\exp\left(-\int_{0}^{\ell} \mathcal{A}\right)$$

$$\vec{X} \sim \frac{\vec{t}}{y - \ell}$$

$$\vec{\mu} = \vec{X}(0).$$

$$\mathcal{G}_{\ell} \times_{G} \mathcal{T}_{\ell'}^{\rho} = \mathcal{T}_{\ell+\ell'}^{\rho}$$

$$g = \lim_{\delta\to 0}\left[(-\delta)^{it_3} P\exp\left(-\int_{0}^{\ell-\delta} \mathcal{A}\right)\right].$$

$$\mathcal{X}(y) = \frac{t_{+}}{y - \ell} + \sum_{\alpha} \epsilon_{\alpha} v_{\alpha} (y - \ell)^{-m_{\alpha}}$$

$$\eta = (-\delta)^{it_3}(\mathcal{X}(\ell - \delta) + t_{+}/\delta)(-\delta)^{-it_3}$$

$$\mathcal{S}_{\ell+\ell'}^{\rho,\rho'} = \left(\mathcal{T}_{\ell}^{\rho} \times \mathcal{T}_{\ell'}^{\rho'}\right)///G.$$

$$-\vec{X}_{L}(0) + \vec{\mu}_{L} = 0$$
$$\vec{X}_{R}(0) + \vec{\mu}_{R} = 0$$

$$\mathcal{X}_L(0) = AB$$

$$\mathcal{X}_R(0) = BA.(3.80)$$

$$\varepsilon_R = \Gamma_{0123}\varepsilon_L = -B_0\varepsilon_L.$$

$$\varepsilon_R = -\exp\left(\Gamma^{IJ}\boldsymbol{F}_{IJ}/4\right)B_0B_1\varepsilon_L$$

$$\varepsilon_L = -B_0 w B_0 B_1 \varepsilon_L$$

$$B_1 w B_1 = w^{-1}$$

$$B_1\boldsymbol{F} = -\boldsymbol{F}B_1.$$

$$h = \exp\left(\Gamma^{IJ}\boldsymbol{F}_{IJ}/8\right)$$

$$h^{-1}B_1 h\varepsilon_L = \varepsilon_L$$

$$X^a \sim \frac{t^a}{y} + \cdots$$

$$B_1\varepsilon = \varepsilon$$

$$\frac{DX^a}{Dy} + \frac{1}{2}\epsilon^{abc}[X_b, X_c] = 0$$

$$h^{-1}B_1 h\varepsilon = \varepsilon$$

$$B_1 h\varepsilon = h\varepsilon.$$

$$\Gamma'\varepsilon_\pm = \pm\varepsilon_\pm.$$

$$h\varepsilon_- = B_1 h\varepsilon_+.$$

$$h\varepsilon_- = -\Gamma^* h\varepsilon_+.$$

$$\begin{pmatrix}\varepsilon_R\\ \varepsilon_L\end{pmatrix} = -\exp\left(\frac{1}{4}\Gamma^{IJ}\boldsymbol{F}_{IJ}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right)B_0B_1\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\varepsilon_R\\ \varepsilon_L\end{pmatrix}.$$

$$\begin{pmatrix}\varepsilon'_R\\ \varepsilon'_L\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\begin{pmatrix}\varepsilon_R\\ \varepsilon_L\end{pmatrix}.$$

$$\begin{pmatrix}\varepsilon'_R\\ \varepsilon'_L\end{pmatrix} = -\exp\left(-\frac{1}{4}\Gamma^{IJ}\boldsymbol{F}_{IJ}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right)B_0B_1\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\varepsilon'_R\\ \varepsilon'_L\end{pmatrix}$$

$$\varepsilon'_R = -B_0\varepsilon'_L$$

$$\left(1 - \cosh\left(\Gamma^{IJ}\boldsymbol{F}_{IJ}/4\right)B_2 + \sinh\left(\Gamma^{IJ}\boldsymbol{F}_{IJ}/4\right)B_1\right)\varepsilon'_L = 0.$$

$$\Gamma'\varepsilon_\pm = \pm\varepsilon_\pm$$

$$\varepsilon_- = \frac{1}{1 - \cosh\left(\Gamma^{IJ}\boldsymbol{F}_{IJ}/4\right)B_2}\sinh\left(\Gamma^{IJ}\boldsymbol{F}_{IJ}/4\right)B_1\varepsilon_+$$

$$\varepsilon_- = \frac{1}{1 - \cosh\left(\Gamma^{IJ}\boldsymbol{F}_{IJ}/4\right)}\sinh\left(\Gamma^{IJ}\boldsymbol{F}_{IJ}/4\right)B_1\varepsilon_+$$

$$\varepsilon_- = \frac{1}{4}\Gamma^{IJ}\boldsymbol{F}_{IJ}B_1\varepsilon_+$$

$$\varepsilon_- = \sum_{I<J<K} \Gamma^{IJK} q_{IJK} \Gamma_3 \varepsilon_+$$

$$q = \frac{(\star_{012} + \star_{456})\boldsymbol{F}}{8}$$

$$\epsilon_{\lambda\mu\nu} F^{3\lambda} + \gamma F_{\mu\nu} = 0$$

$$-\frac{\gamma}{2e^2}\int_{\partial M} \mathrm{d}^3 x \epsilon^{\mu\nu\lambda} \mathrm{Tr}\left(A_\mu \partial_\nu A_\lambda + \frac{2}{3} A_\mu A_\nu A_\lambda\right)$$

$$-\frac{u}{3e^2}\int_{\partial M} \mathrm{d}^3 x \epsilon_{abc} \mathrm{Tr} X^a [X^b, X^c]$$

$$\frac{DX_a}{Dy} + \frac{u}{2}\epsilon_{abc}[X_b, X_c] = 0$$

$$\int_{\partial M} \mathrm{d}^3 x q_{IJK} \mathrm{Tr} Z^I [Z^J, Z^K]$$

$$\gamma = -\frac{2a}{1-a^2}, u = -\frac{2a}{1+a^2}$$

$$q = u(\mp \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 + \mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6)$$

$$\varepsilon_- = \sum_{I<J<K} \Gamma^{IJK} q_{IJK} \Gamma_3 \varepsilon_+$$

$$\varepsilon \to \left(\frac{|c\tau + d|}{c\tau + d}\right)^{-i\Gamma^*/2} \varepsilon,$$

$$\varepsilon \to \frac{1 - \Gamma^*}{\sqrt{2}} \varepsilon.$$

$$q = \frac{1}{4}(\mathrm{d}x^0 + \mathrm{d}x^4) \wedge (\mathrm{d}x^1 \wedge \mathrm{d}x^5 + \mathrm{d}x^2 \wedge \mathrm{d}x^6)$$

$$\Gamma' \varepsilon_\pm = \pm \varepsilon_\pm$$

$$\varepsilon_- = \sum_{I<J<K} q_{IJK} \Gamma^{3IJK} \varepsilon_+ = \frac{1}{4}\Gamma_3(-\Gamma_0 + \Gamma_4)(\Gamma_{15} + \Gamma_{26})\varepsilon_+.$$

$$\varepsilon_+ = \frac{1}{\sqrt{2}}(\tilde{\varepsilon}_+ + \Gamma^* \tilde{\varepsilon}_-)$$
$$\varepsilon_- = \frac{1}{\sqrt{2}}(\tilde{\varepsilon}_- + \Gamma^* \tilde{\varepsilon}_+)$$

$$(1 - M\Gamma^*)\tilde{\varepsilon}_- = -\Gamma^*(1 + \Gamma^* M)\tilde{\varepsilon}_+$$

$$P\tilde{\varepsilon}_- = -\Gamma^* P \tilde{\varepsilon}_+$$

$$P = 1 + \frac{\Gamma_{16} - \Gamma_{25}}{2}$$

$$T = \frac{1 - \Gamma_{1256}}{2} + \frac{1 + \Gamma_{1256}}{2\sqrt{2}}$$

$$TP\tilde{\varepsilon}_- = -\Gamma^* TP \tilde{\varepsilon}_+$$

$$TP = \left(\frac{1 - \Gamma_{1256}}{2} + \frac{1 + \Gamma_{1256}}{2\sqrt{2}}\right)\left(1 + \frac{\Gamma_{16} - \Gamma_{25}}{2}\right) = \exp\left(\frac{\pi}{8}(\Gamma_{16} - \Gamma_{25})\right)$$

$$\bar{\varepsilon}\Gamma_3\tilde{\varepsilon} = 0$$

$$(\varepsilon, \tilde{\varepsilon}) = \langle\mu, \tilde{\mu}\rangle\langle\nu, \tilde{\nu}\rangle'$$

$$4 \otimes 4 = \mathbf{6}_A \oplus 10_S$$
$$4' \otimes 4' = \mathbf{6}_A \oplus 10'_S$$
$$4 \otimes 4' = \mathbf{1} \oplus \mathbf{15}.$$

$$B_2\varepsilon = -\varepsilon,$$

$$B_2\varepsilon = \varepsilon.$$

$$\Gamma'\eta = \eta, \Gamma'\zeta = -\zeta$$

$$\langle\mu, \tilde{\mu}\rangle = \sum_{a=1}^{4} \left(\eta^a\tilde{\zeta}_a - \zeta_a\tilde{\eta}^a\right)$$

$$F(\mu) = \sum_{a=1}^{4} \eta^a\zeta_a$$

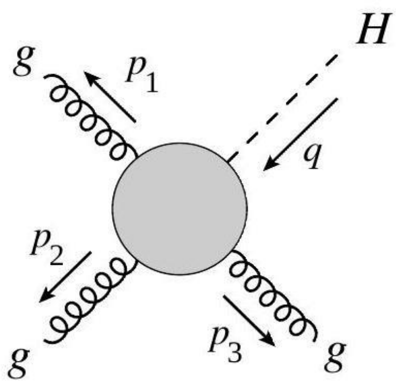
$$\zeta_a = \sum_b f_{ab}\eta^b$$

$$\zeta = \sum_{I<J<K} q_{IJK}\Gamma^{IJK}\eta.$$

$$\varepsilon_- = \sum_{I<J<K} q_{IJK}\Gamma^{IJK}\Gamma^3\varepsilon_+.$$

$$\eta^a = \sum_b g^{ab}\zeta_b$$

$$\eta = \sum_{I<J<K} \tilde{q}_{IJK}\Gamma^{IJK}\zeta$$



$$u = \frac{(p_1 + p_2)^2}{q^2}, v = \frac{(p_2 + p_3)^2}{q^2}, w = \frac{(p_3 + p_1)^2}{q^2}$$

$$\mathcal{F}(u, v) = 1 + \sum_{L=1}^{\infty} g^{2L}\mathcal{F}^{(L)}(u, v)$$

$$dF = \sum_i F_i \mathrm{dlog}\ l_i \Rightarrow \mathcal{S}(F) = \sum_i \mathcal{S}(F_i) \otimes l_i$$

$$a=\sqrt{\frac{u}{vw}}, b=\sqrt{\frac{v}{uw}}, c=\sqrt{\frac{w}{uv}}, d=\frac{1-u}{u}, e=\frac{1-v}{v}, f=\frac{1-w}{w}.$$

$$\mathcal{S}\left[\mathcal{F}^{(1)}\right]=(-2)[b\otimes d+c\otimes e+a\otimes f+b\otimes f+c\otimes d+a\otimes e]$$

$$\begin{aligned}\mathcal{S}\left[\mathcal{F}^{(2)}\right]=8[&b\otimes d\otimes d\otimes d+c\otimes e\otimes e\otimes e+a\otimes f\otimes f\otimes f\\ &+b\otimes f\otimes f\otimes f+c\otimes d\otimes d\otimes d+a\otimes e\otimes e\otimes e]\\ &+16[b\otimes b\otimes b\otimes d+c\otimes c\otimes c\otimes e+a\otimes a\otimes a\otimes f\\ &+b\otimes b\otimes b\otimes f+c\otimes c\otimes c\otimes d+a\otimes a\otimes a\otimes e].\end{aligned}$$

$$\mathcal{S}\left[\mathcal{F}^{(L)}\right]=\sum_{l_{i_1},\dots,l_{i_{2L}}\in\mathcal{L}} C^{l_{i_1},\dots,l_{i_{2L}}} l_{i_1}\otimes\cdots\otimes l_{i_{2L}}$$

cycle: {a, b, c, d, e, f} → {b, c, a, e, f, d}
flip: {a, b, c, d, e, f} → {b, a, c, e, d, f}

$$\sum_{i,j=1}^{6} C_{ij}\,c\left(\mathrm{Y}\mathrm{x}_i\mathrm{x}_j\mathrm{Z}\right)=0,$$

$$\sum_{i_1,i_2,i_3=1}^{6} C_{i_1i_2i_3}\,c\left(\mathrm{Y}\mathrm{x}_{i_1}\mathrm{x}_{i_2}\mathrm{x}_{i_3}\mathrm{Z}\right)=0,$$

$$\sum_{i_1,\dots,i_k=1}^{6} \hat{C}_{i_1\dots i_k}\,c\left(\mathrm{x}_{i_1}\dots\mathrm{x}_{i_k}\mathrm{Z}\right)=0$$

$$\sum_{i_1,\dots,i_k=1}^{6} \tilde{C}_{i_1\dots i_k}\,c\left(\mathrm{Y}\mathrm{x}_{i_1}\dots\mathrm{x}_{i_k}\right)=0$$

$$c(\xi)=0$$
$$c(\zeta)=0$$

$$c(\gamma)=0$$

$$c(\Lambda^{n-1})=c(\Xi^k\Upsilon^{n-k}),$$

$$c(\psi)=c(\phi)=-c(\varphi),$$

$$c_{L=1,2,3,4,5,6,\dots}(\tau)=-2,16,-384,15360,-860160,61931520,\dots$$

$$c_L(\sigma)=\frac{(-4)^L}{2}\frac{[2(L-1)]!}{(L-1)!}=(-1)^L2^{3L-2}(2L-3)!!,$$

$$n!!=\prod_{k=0}^{[n/2]-1}(n-2k)$$

$$c_L(\mathrm{af}\dots\mathrm{f})=(-1)^L2^{2L-1}(2L-3)!!$$

$$(-1)^L2^{2L-2\lceil m/2\rceil}(2L-1-2\lceil m/2\rceil)!!$$

$$c_L(\mathbf{X}_8\mathbf{f}\dots\mathbf{f})=p_L(\mathbf{X}_8\mathbf{f}\dots\mathbf{f})\times(-1)^L2^{2L-8}(2L-9)!!,$$

$$
\begin{aligned}
p_L(\daleth) &= 0 \\
p_L(\gimel) &= 32(L-2)(2L-5)(2L-7), \\
p_L(\beth) &= \frac{16}{3}(4L-9)(2L-5)(2L-7), \\
p_L(\aleph) &= -\frac{4}{5}(2L-7)(7L^2+22L-140), \\
p_L(\mathscr{o}) &= -\frac{8}{3}(L-4)(L^2-47L+135), \\
p_L(\mathfrak{I}) &= -\frac{2}{45}(163L^3-2220L^2+15977L-36660).
\end{aligned}
$$

$$c_L(\hbar) = (12-8L)c_{L-1}(\mathfrak{G})$$

$$c_L(\mathfrak{H}) = (24-16L)c_{L-1}(\mathfrak{H})$$

$$0 = c_L(\lambda) = C \times [c_{L-1}(\mathcal{G}) - c_{L-1}(\mathfrak{B})]$$

$$
\begin{aligned}
c_L(\varrho) &= (20-8L)c_{L-1}(\mathscr{g}) - c_{L-1}(\mathfrak{F}) \\
c_L(\varsigma) &= (20-8L)c_{L-1}(\mathscr{f}) + 2c_{L-1}(\mathfrak{R})
\end{aligned}
$$

$$c_L(\mathscr{k}) = (20-8L)c_{L-1}(\mathscr{h}) + (1-C)c_{L-1}(\mathscr{s}) + Cc_{L-1}(\mathscr{w})$$

$$
\begin{aligned}
c_L(\mathfrak{U}) &= (12-8L)c_{L-1}(\mathfrak{L}) \\
c_L(\mathfrak{T}) &= (28-8L)c_{L-1}(\mathfrak{S}) + 16c_{L-1}(\mathfrak{M}) \\
c_L(\mathfrak{N}) &= (24-8L)c_{L-1}(\mathfrak{P}) - 4c_{L-1}(\mathfrak{O})
\end{aligned}
$$

$$
\begin{aligned}
c_L(\mathfrak{V}) =& (7C-8L)c_{L-1}(\mathfrak{W}) + (-60+15C)c_{L-1}(\mathfrak{X}) \\
&+(-102+20C)c_{L-1}(\mathfrak{Y}) + (30-6C)c_{L-1}(\mathfrak{Z})
\end{aligned}
$$

$$
\begin{aligned}
c_L(\mathbb{G}) = (28-8L)c_{L-1}(\mathbb{D}) + 40c_{L-1}(\mathbb{B}) \\
+12c_{L-1}(\mathbb{A})
\end{aligned}
$$

$$c_L(\mathbb{M}) = \left(12 - \frac{12}{-4+L} - 8L\right)c_{L-1}(\mathbb{N})$$

$$c_L(\mathbb{S}) = (12-8L)c_{L-1}(\mathbb{U}) - 4c_{L-2}(\mathbb{T})$$

$$c(\mathrm{YabZ}) + c(\mathrm{YacZ}) - c(\mathrm{YbaZ}) - c(\mathrm{YcaZ}) = 0,$$

$$
\begin{aligned}
c(\mathrm{YabZ}) + c(\mathrm{YacZ}) &= c(\mathrm{YbaZ}) + c(\mathrm{YcaZ}), \\
c(\mathrm{YcaZ}) + c(\mathrm{YcbZ}) &= c(\mathrm{YacZ}) + c(\mathrm{YbcZ}),
\end{aligned}
$$

$$
\begin{aligned}
&c(\mathrm{YdbZ}) + c(\mathrm{YcdZ}) + c(\mathrm{YecZ}) + c(\mathrm{YaeZ}) + c(\mathrm{YfaZ}) + c(\mathrm{YbfZ}) + 2c(\mathrm{YcbZ}) \\
&= c(\mathrm{YbdZ}) + c(\mathrm{YdcZ}) + c(\mathrm{YceZ}) + c(\mathrm{YeaZ}) + c(\mathrm{YafZ}) + c(\mathrm{YfbZ}) + 2c(\mathrm{YbcZ}),
\end{aligned}
$$

$$c(\mathrm{YaabZ}) + c(\mathrm{YabbZ}) + c(\mathrm{YacbZ}) = 0,$$

$$c(\mathrm{Y(abc)Z}) \equiv c(\mathrm{YaZ}) + c(\mathrm{YbZ}) + c(\mathrm{YcZ}),$$

$$c(\mathrm{Y(abc)aZ}) = c(\mathrm{Ya(abc)Z}).$$

$$c(\mathrm{YA_1(abc)A_2Z}) = c(\mathrm{Y(abc)AZ}) = c(\mathrm{YA(abc)Z}).$$

$$c(\mathrm{YA_1(abc)A_2Z}) = c(\mathrm{YA(abc)Z}) = c(\mathrm{YAaZ}) + c(\mathrm{YAbZ}).$$

$$c(\mathrm{Ya(abc)bZ}) = 0.$$

$$c(\mathrm{YA_1(abc)A_2Z}) = c(\mathrm{Y(abc)AZ}) = c(\mathrm{YA(abc)Z}) = 0.$$

$$c(\mathrm{YfaAbfZ}) = c(\mathrm{YfbAafZ}) = -c(\mathrm{YfbAbfZ}).$$

$$c(\mathrm{Yfa}^{n+1}\mathrm{bfZ}) = c(\mathrm{Yfba}^{n+1}\mathrm{fZ}) = -c(\mathrm{Yfba}^{n}\mathrm{bfZ}).$$

$$c(\mathrm{Yfa}^{n+1}\mathrm{bfZ}) = c(\mathrm{Yfa}^{n+1}(\mathrm{abc})\mathrm{fZ}) - c(\mathrm{Yfa}^{n+2}\mathrm{fZ})$$
$$= c(\mathrm{Yf}^{2}(\mathrm{abc})\mathrm{a}^{n+1}\mathrm{fZ}) - c(\mathrm{Yfa}^{n+2}\mathrm{fZ}) = c(\mathrm{Yfba}^{n+1}\mathrm{fZ}).$$

$$c(\mathrm{YfAafZ}) + c(\mathrm{YfAbfZ}) = 0,$$

$$c(\mathrm{YfaAfZ}) + c(\mathrm{YfbAfZ}) = 0.$$

$$c(\mathrm{YfaAbfZ}) = c(\mathrm{YfbAafZ}) = -c(\mathrm{YfbAbfZ}) = -c(\mathrm{YfaAafZ}),$$

$$c(\mathrm{Xba} \ldots \mathrm{af}) = 0$$
$$c(\mathrm{Xca} \ldots \mathrm{af}) = 0$$

$$c(\mathrm{XbaaY}) + c(\mathrm{XbbaY}) + c(\mathrm{XbcaY}) = 0 = c(\mathrm{XcaaY}) + c(\mathrm{XcbaY}) + c(\mathrm{XccaY}).$$

$$U = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$N_{\mathrm{adj}}(j) = IU^{j-1}F_1^T$$

$$N_{\mathrm{adj}}(j) = N_j^d = \hat{I}\hat{U}^{j-1}\hat{F}_1^T, N_j^a = \hat{I}\hat{U}^{j-1}\hat{A}^T$$

$$\hat{U} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}, \hat{I} = (3{,}0), \hat{F}_1 = (0{,}1), \hat{A} = (1{,}0)$$

$$N_j^a = 3N_{j-1}^a + 2N_{j-1}^d$$
$$N_j^d = 2N_{j-1}^a + N_{j-1}^d$$

$$N_j^d - 4N_{j-1}^d = 2N_{j-1}^a + (1-4)N_{j-1}^d = 2\left(3N_{j-2}^a + 2N_{j-2}^d\right) - 3\left(2N_{j-2}^a + N_{j-2}^d\right) = N_{j-2}^d$$

$$N_j^a = 4N_{j-1}^a + N_{j-2}^a$$
$$N_j^d = 4N_{j-1}^d + N_{j-2}^d$$

$$N_{\mathrm{adj}}(j) = 4N_{\mathrm{adj}}(j-1) + N_{\mathrm{adj}}(j-2),$$

$$\frac{N_{\mathrm{adj}}(j)}{N_{\mathrm{adj}}(j-1)} \sim 2 + \sqrt{5} = 4.236 \ldots$$

$$c(\mathrm{Xbaf}) = c(\mathrm{Xcaf}) = 0,$$

$$N(k, L) = \hat{I}\hat{U}^{2L-k}\hat{F}_k^T, \text{ for } k \leq 2L.$$

$$F_k^a = 4F_{k-1}^a + F_{k-2}^a,$$
$$F_k^d = 4F_{k-1}^d + F_{k-2}^d - 8.$$

$$F_k^d = 2F_{k-1}^a + F_{k-1}^d$$

$$F_k^a = 3(F_{k-1}^a - 2) + 2 + 2F_{k-1}^d$$
$$= 3F_{k-1}^a + 2F_{k-1}^d - 4,$$

$$F_k^a - 4F_{k-1}^a = -F_{k-1}^a + 2F_{k-1}^d - 4$$
$$= -\left(3F_{k-2}^a + 2F_{k-2}^d - 4\right) + 2\left(2F_{k-2}^a + F_{k-2}^d\right) - 4$$
$$= F_{k-2}^a$$

$$a = \sqrt{\frac{u}{v}}, b = \sqrt{\frac{v}{u}}, c = \sqrt{\frac{1}{uv}}, d = \frac{1}{u}, e = \frac{1}{v},$$

$$b \to \frac{1}{a}, d \to \frac{c}{a}, e \to a \times c.$$

$$\mathrm{Disc}_w = \frac{1}{2}[-\mathrm{Disc}_a - \mathrm{Disc}_b + \mathrm{Disc}_c] - \mathrm{Disc}_f.$$

$$\mathcal{A}_5^{(L)} = i^L g^{3+2L} \sum_{i\in\Gamma_3} \int \frac{d^{LD}p}{(2\pi)^{LD}} \frac{1}{S_i} \frac{N_i C_i}{l_{i_1}^2 l_{i_2}^2 l_{i_3}^2 \cdots l_{i_m}^2}$$

$$d^{LD}p = \prod_{j=1}^{L} d^D p_j$$

$$\tilde{f}^{abc} = i\sqrt{2} f^{abc} = \mathrm{Tr}([T^a, T^b] T^c)$$

$$\begin{aligned} N_i + N_j + N_k = 0 \quad &\Leftrightarrow C_i + C_j + C_k = 0, \\ N_i \to -N_i \quad &\Leftrightarrow C_i \to -C_i \end{aligned}$$

$$\mathcal{M}_5^{(L)} = i^{L+1} \left(\frac{\kappa}{2}\right)^{3+2L} \sum_{i\in\Gamma_3} \int \frac{d^{LD}p}{(2\pi)^{LD}} \frac{1}{S_i} \frac{N_i \widetilde{N}_i}{l_{i_1}^2 l_{i_2}^2 l_{i_3}^2 \cdots l_{i_m}^2}$$

$$\mathcal{K}(1,2,3,4) \equiv s_{12} s_{23} A_4^{\mathrm{tree}}(1,2,3,4) = -i\delta^{(8)}(Q) \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

$$Q^{\alpha A} = \sum_{i=1}^{m} \lambda_i^\alpha \eta_i^A$$

$$N_i \sim \mathcal{K}(1,2,3,4) \times (\text{ local momentum factor }),$$

$$\beta_{12345} \equiv \delta^{(8)}(Q) \frac{[12][23][34][45][51]}{4\varepsilon(1,2,3,4)},$$

$$\varepsilon(1,2,3,4) \equiv \varepsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\nu k_3^\rho k_4^\sigma = \mathrm{Det}(k_i^\mu)$$

$$\gamma_{12} \equiv \gamma_{12345} \equiv \delta^{(8)}(Q) \frac{[12]^2[34][45][35]}{4\varepsilon(1,2,3,4)}$$

$$\sum_{i=1}^{5} \gamma_{ij} = 0, \gamma_{ij} = -\gamma_{ji}$$

$$\begin{aligned} \gamma_{12} &= \beta_{12345} - \beta_{21345} \\ \beta_{12345} &= \frac{1}{2}(\gamma_{12} + \gamma_{13} + \gamma_{14} + \gamma_{23} + \gamma_{24} + \gamma_{34}) \end{aligned}$$

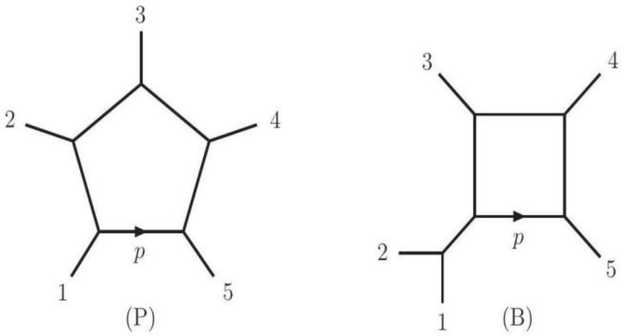
$$0 = (\gamma_{12} + \gamma_{13})(s_{23} - s_{45}) + \gamma_{23}(s_{12} - s_{23}) + \gamma_{45}(s_{14} - s_{15})$$

$$N_i = \sum_{j,k,n} a_{i;jk;n} \gamma_{jk} M_n^{(L)}$$

$$M^{(L)} = \left\{ \prod_{l}^{L-1} m_l \mid m_l \in \{s_{ij}, \tau_{ij}\} \right\}$$

$$s_{ij} = (k_i + k_j)^2 = 2k_i \cdot k_j, \tau_{ip} = 2k_i \cdot p, \tau_{iq} = 2k_i \cdot q, \tau_{pq} = 2p \cdot q$$

$$N^{(\mathrm{P})}(1,2,3,4,5;p) \text{ and } N^{(\mathrm{B})}(1,2,3,4,5;p)$$



$$N^{(\mathrm{B})}(1,2,3,4,5;p) = N^{(\mathrm{P})}(1,2,3,4,5;p) - N^{(\mathrm{P})}(2,1,3,4,5;p),$$
$$0 = N^{(\mathrm{tri}_2)}(1,2,3,4,5;p) = N^{(\mathrm{B})}(1,2,3,4,5;p) - N^{(\mathrm{B})}(1,2,4,3,5;p),$$
$$0 = N^{(\mathrm{tri}_1)}(1,2,3,4,5;p) = N^{(\mathrm{B})}(1,2,3,4,5;p) - N^{(\mathrm{B})}(1,2,4,5,3;p+k_3).$$

$$N^{(\mathrm{P})}(2,3,4,5,1;p+k_1) = N^{(\mathrm{P})}(1,2,3,4,5;p),$$
$$N^{(\mathrm{P})}(5,4,3,2,1;-p) = -N^{(\mathrm{P})}(1,2,3,4,5;p).$$

$$N^{(\mathrm{B})}(2,1,3,4,5;p) = -N^{(\mathrm{B})}(1,2,3,4,5;p),$$
$$N^{(\mathrm{B})}(1,2,5,4,3;-k_1-k_2-p) = N^{(\mathrm{B})}(1,2,3,4,5;p).$$

$$N^{(\mathrm{P})} \propto \delta^{(8)}(Q)$$

$$N^{(\mathrm{P})}(1,2,3,4,5;p) = \beta_{12345} + \alpha_{12345}l_1^2 + \alpha_{23451}l_2^2 + \alpha_{34512}l_3^2 + \alpha_{45123}l_4^2 + \alpha_{51234}l_5^2$$

$$\alpha_{abcde} = -\alpha_{baedc}, \beta_{abcde} = \beta_{bcdea}, \beta_{abcde} = -\beta_{baedc}$$

$$N^{(\mathrm{B})}(1,2,3,4,5;p) = \gamma_{12345}$$

$$\gamma_{abcde} = -\gamma_{bacde}, \gamma_{abcde} = \gamma_{abedc}$$

$$0 = \gamma_{12345} - \gamma_{12435}, 0 = \gamma_{12345} - \gamma_{12453},$$

$$\gamma_{ab} \equiv \gamma_{abcde}$$

$$\gamma_{12345} = \beta_{[12]345} + \alpha_{12345}l_1^2 - \alpha_{21345}(l_5^2 + l_2^2 - l_1^2 - s_{12}) + \alpha_{3[12]54}l_2^2 + \alpha_{345[12]}l_3^2$$
$$+\alpha_{45[12]3}l_4^2 + \alpha_{5[12]34}l_5^2$$

$$l_1^2|_{k_1\leftrightarrow k_2} = (p+k_2)^2 = l_5^2 + l_2^2 - l_1^2 - s_{12}$$

$$\alpha_{(12)345} = 0, \alpha_{345[12]} = 0, \alpha_{45[12]3} = 0,$$
$$\alpha_{5[12]34} - \alpha_{21345} = 0, \alpha_{3[12]54} - \alpha_{21345} = 0,$$
$$\gamma_{12345} = \beta_{[12]345} + s_{12}\alpha_{21345},$$

$$\alpha_{ab} = \alpha_{a1} + \alpha_{1b},$$

$$\beta_{[12]} + \beta_{[13]} + \beta_{[14]} + \beta_{[15]} = 0,$$

$$\beta_{[ab]} \equiv \beta_{[ab]cde} = \beta_{\mathrm{abcde}} - \beta_{\mathrm{bacde}}$$

$$\beta_{12345} = \frac{1}{2}\left(\beta_{[12]} + \beta_{[13]} + \beta_{[14]} + \beta_{[23]} + \beta_{[24]} + \beta_{[34]}\right),$$

$$\frac{\beta_{12345} + \alpha_{12}l_1^2}{l_1^2} + \frac{\gamma_{12}}{s_{12}} = \frac{\beta_{12345}}{(p+k_1)^2} + \frac{\beta_{[12]345}}{s_{12}}.$$

$$p = \frac{(k_1+k_2)|3\rangle\langle 5|}{\langle 35\rangle}$$
$$(p+k_1)^2 = \frac{\langle 5|k_1(k_1+k_2)|3\rangle}{\langle 35\rangle} = \frac{\langle 51\rangle[12]\langle 23\rangle}{\langle 35\rangle}.$$

$$i\delta^{(8)}(Q)\frac{s_{34}s_{45}}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle} = \beta_{12345}\frac{\langle 35\rangle}{\langle 51\rangle[12]\langle 23\rangle} + \frac{\beta_{[12]345}}{s_{12}}$$
$$0 = \beta_{12345}\frac{[35]}{[51]\langle 12\rangle[23]} + \frac{\beta_{[12]345}}{s_{12}}$$

$$\beta_{12345} = i\delta^{(8)}(Q)\frac{[12][23][34][45][51]}{\langle 12\rangle[23]\langle 35\rangle[51] - [12]\langle 23\rangle[35]\langle 51\rangle} = \delta^{(8)}(Q)\frac{[12][23][34][45][51]}{4\varepsilon(1,2,3,4)},$$

$$N^{(\mathrm{B})} = \gamma_{12} = \beta_{12345} - \beta_{21345} = \delta^{(8)}(Q)\frac{[12]^2[34][45][35]}{4\varepsilon(1,2,3,4)},$$

$$\beta_{12345} = \frac{1}{2}(\gamma_{12}+\gamma_{13}+\gamma_{14}+\gamma_{23}+\gamma_{24}+\gamma_{34})$$

$$\sum_{i=1}^{5}\gamma_{ij} = 0$$

$$\mathcal{A}_5^{(1)} = ig^5\sum_{S_5}\left(\frac{1}{10}\beta_{12345}C^{(\mathrm{P})}I^{(\mathrm{P})} + \frac{1}{4}\gamma_{12}C^{(\mathrm{B})}I^{(\mathrm{B})}\right)$$

$$I^{(\mathrm{P})} = \int\frac{d^Dp}{(2\pi)^D}\frac{1}{p^2(p+k_1)^2(p+k_1+k_2)^2(p-k_4-k_5)^2(p-k_5)^2},$$
$$I^{(\mathrm{B})} = \frac{1}{s_{12}}\int\frac{d^Dp}{(2\pi)^D}\frac{1}{p^2(p+k_1+k_2)^2(p-k_4-k_5)^2(p-k_5)^2},$$

$$C^{(\mathrm{P})} = \tilde{f}^{ga_1b}\tilde{f}^{ba_2c}\tilde{f}^{ca_3d}\tilde{f}^{da_4e}\tilde{f}^{ea_5g}$$
$$C^{(\mathrm{B})} = \tilde{f}^{a_1a_2b}\tilde{f}^{bcg}\tilde{f}^{ca_3d}\tilde{f}^{da_4e}\tilde{f}^{ea_5g}$$

$$\mathcal{M}_5^{(1)} = -\left(\frac{\kappa}{2}\right)^5\sum_{S_5}\left(\frac{1}{10}\beta_{12345}\tilde{\beta}_{12345}I^{(\mathrm{P})} + \frac{1}{4}\gamma_{12}\tilde{\gamma}_{12}I^{(\mathrm{B})}\right)$$

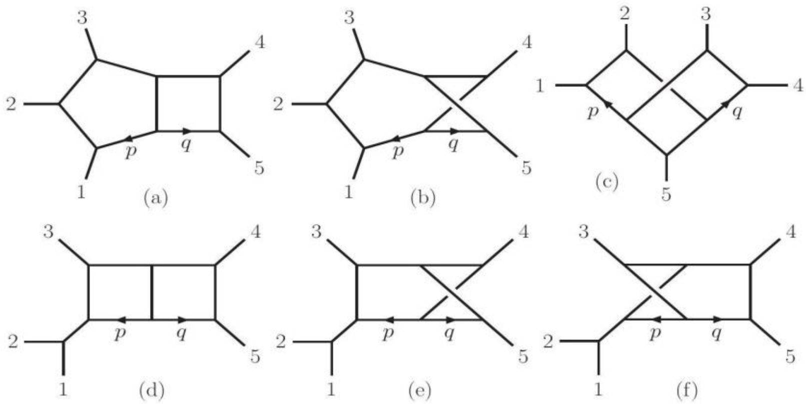
$$I^{(\mathrm{B})}\Big|_{\mathrm{UVpole}} = \frac{i}{6(4\pi)^4\epsilon}\frac{1}{s_{12}}$$

$$\mathcal{A}_5^{(1)}\Big|_{\mathrm{UV}} = -g^5\frac{1}{6(4\pi)^4\epsilon}\Big[N_c\mathrm{Tr}_{12345}\left(\frac{\gamma_{12}}{s_{12}}+\frac{\gamma_{23}}{s_{23}}+\frac{\gamma_{34}}{s_{34}}+\frac{\gamma_{45}}{s_{45}}+\frac{\gamma_{51}}{s_{15}}\right)$$
$$+6\mathrm{Tr}_{123}\mathrm{Tr}_{45}\left(\frac{\gamma_{12}}{s_{12}}+\frac{\gamma_{23}}{s_{23}}+\frac{\gamma_{31}}{s_{13}}\right) + \text{ perms }\Big]$$

$$\mathcal{M}_5^{(1)}\Big|_{\mathrm{UV}} = -i\left(\frac{\kappa}{2}\right)^5\frac{1}{6(4\pi)^4\epsilon}\left[\frac{\gamma_{12}^2}{s_{12}}+\frac{\gamma_{13}^2}{s_{13}}+\frac{\gamma_{14}^2}{s_{14}}+\frac{\gamma_{15}^2}{s_{15}}+\frac{\gamma_{23}^2}{s_{23}}+\frac{\gamma_{24}^2}{s_{24}}+\frac{\gamma_{25}^2}{s_{25}}+\frac{\gamma_{34}^2}{s_{34}}+\frac{\gamma_{35}^2}{s_{35}}+\frac{\gamma_{45}^2}{s_{45}}\right]$$

$$\gamma_{ij}^2 \equiv \gamma_{ij}\tilde{\gamma}_{ij} = \gamma_{ij}\left(\gamma_{ij}\big|_{\eta_i^A\to\eta_i^{A+4}}\right)$$

$$N^{(x)} = N^{(x)}(1,2,3,4,5;p,q)$$



$$N^{(a)} = N^{(b)}, N^{(d)} = N^{(e)} = N^{(f)}.$$

$$N^{(c)}(1,2,3,4,5;p,q) = N^{(a)}(1,2,5,4,3;p,k_{3,4}-q) - N^{(a)}(5,4,3,1,2;k_5+q,k_{1,2}-p),$$
$$N^{(d)}(1,2,3,4,5;p,q) = N^{(a)}(1,2,3,4,5;p,q) - N^{(a)}(2,1,3,4,5;p,q),$$

$$N^{(b)}(1,2,3,4,5;p,q) = -N^{(c)}(1,2,5,3,4;p,k_{3,5}-q) - N^{(c)}(1,2,4,3,5;p,k_{3,4}+p+q)$$

$$\begin{aligned}
N^{(a)}(1,2,3,4,5;p,q) &= -N^{(a)}(3,2,1,5,4;k_{1,2,3}-p,k_{4,5}-q),\\
N^{(b)}(1,2,3,4,5;p,q) &= -N^{(b)}(3,2,1,4,5;k_{1,2,3}-p,k_5-q),\\
N^{(b)}(1,2,3,4,5;p,q) &= N^{(b)}(1,2,3,5,4;p,p+q+k_4),\\
N^{(c)}(1,2,3,4,5;p,q) &= -N^{(c)}(4,3,2,1,5;q,p),\\
N^{(c)}(1,2,3,4,5;p,q) &= N^{(c)}(3,4,1,2,5;k_{3,4}-q,k_{1,2}-p),\\
N^{(d)}(1,2,3,4,5;p,q) &= -N^{(d)}(2,1,3,4,5;p,q),\\
N^{(e)}(1,2,3,4,5;p,q) &= -N^{(e)}(2,1,3,4,5;p,q),\\
N^{(e)}(1,2,3,4,5;p,q) &= N^{(e)}(1,2,3,5,4;p,p+q+k_4),\\
N^{(f)}(1,2,3,4,5;p,q) &= -N^{(f)}(2,1,3,4,5;p,q),\\
N^{(f)}(1,2,3,4,5;p,q) &= -N^{(f)}(1,2,3,5,4;k_{1,2}-p,k_{4,5}-q).
\end{aligned}$$

$$\gamma_{12}, \gamma_{13}, \gamma_{14}, \gamma_{23}, \gamma_{24}, \gamma_{34}$$

$$N^{(a)} = \gamma_{12}m_1 + \gamma_{13}m_2 + \gamma_{14}m_3 + \gamma_{23}m_4 + \gamma_{24}m_5 + \gamma_{34}m_6$$

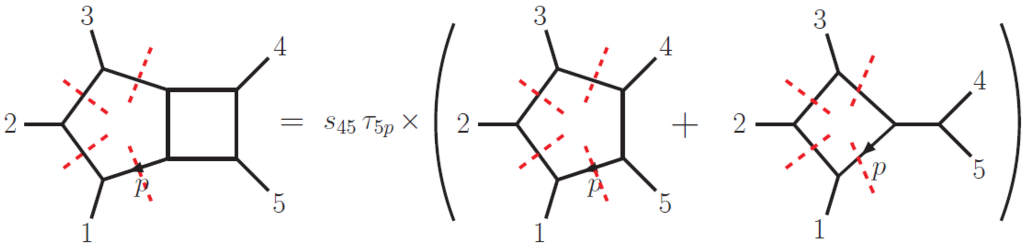
$$m_j = a_{1j}s_{12} + a_{2j}s_{13} + a_{3j}s_{14} + a_{4j}s_{23} + a_{5j}s_{24} + a_{6j}\tau_{1p} + a_{7j}\tau_{2p} + a_{8j}\tau_{3p} + a_{9j}\tau_{4p}$$

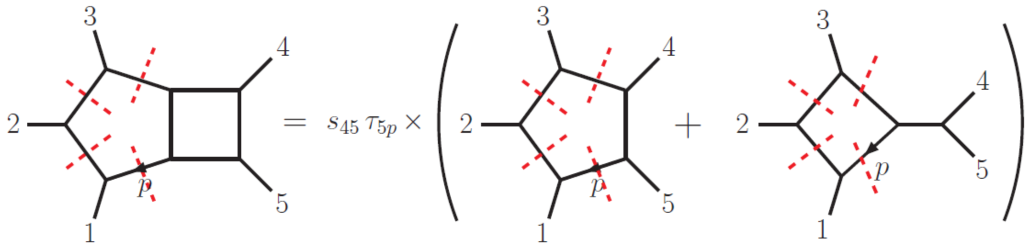
$$a_{45} = a_{26} = a_{36} = a_{46} = a_{56} = 0$$

$$p^2 = (p-k_1)^2 = (p-k_1+k_2)^2 = (p-k_1-k_2-k_3)^2 = 0$$

$$N^{(a)}\big|_{\text{cut}} = s_{45}\tau_{5p}\left(\frac{\beta_{12345}}{\tau_{5p}} + \frac{\gamma_{45}}{s_{45}}\right)$$

$$\{\tau_{1p} \to 0, \tau_{2p} \to s_{12}, \tau_{3p} \to s_{45} - s_{12}, \tau_{4p} \to -\tau_{5p} - s_{45}\}$$





$$a_{12} = a_{21} = a_{41} = \frac{1}{2}, a_{14} = \frac{3}{4}, a_{93} = a_{95} = a_{96} = -1, a_{61} = a_{62} = a_{74} = -\frac{1}{4}$$

$$a_{11} = a_{22} = a_{24} = a_{42} = a_{44} = a_{71} = a_{82} = a_{84} = \frac{1}{4}$$

$$a_{63} = a_{65} = a_{66} = a_{73} = a_{75} = a_{76} = a_{83} = a_{85} = a_{86} = -\frac{1}{2}$$

$$a_{13} = a_{15} = a_{16} = a_{23} = a_{25} = a_{31} = a_{32} = a_{33} = a_{34} = a_{35} = a_{43} = a_{51} = a_{52} = a_{53}$$
$$= a_{54} = a_{55} = a_{64} = a_{72} = a_{81} = a_{91} = a_{92} = a_{94} = 0$$

$$N^{(\mathrm{a})}(1,2,3,4,5;p,q) = \frac{1}{4}\big(\gamma_{12}(2s_{45} - s_{12} + \tau_{2p} - \tau_{1p}) + \gamma_{23}(s_{45} + 2s_{12} - \tau_{2p} + \tau_{3p})$$
$$+2\gamma_{45}(\tau_{5p} - \tau_{4p}) + \gamma_{13}(s_{12} + s_{45} - \tau_{1p} + \tau_{3p})\big)$$

$$\mathcal{A}_5^{(2)} = -g^7 \sum_{S_5} \left(\frac{1}{2}\mathcal{J}^{(\mathrm{a})} + \frac{1}{4}\mathcal{J}^{(\mathrm{b})} + \frac{1}{4}\mathcal{J}^{(\mathrm{c})} + \frac{1}{2}\mathcal{J}^{(\mathrm{d})} + \frac{1}{4}\mathcal{J}^{(\mathrm{e})} + \frac{1}{4}\mathcal{J}^{(\mathrm{f})}\right),$$

$\mathcal{I}^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator
(a),(b)	$\frac{1}{4}\Big(\gamma_{12}(2s_{45} - s_{12} + \tau_{2p} - \tau_{1p}) + \gamma_{23}(s_{45} + 2s_{12} - \tau_{2p} + \tau_{3p})$ $+ 2\gamma_{45}(\tau_{5p} - \tau_{4p}) + \gamma_{13}(s_{12} + s_{45} - \tau_{1p} + \tau_{3p})\Big)$
(c)	$\frac{1}{4}\Big(\gamma_{15}(\tau_{5p} - \tau_{1p}) + \gamma_{25}(s_{12} - \tau_{2p} + \tau_{5p}) + \gamma_{12}(s_{34} + \tau_{2p} - \tau_{1p} + 2s_{15} + 2\tau_{1q} - 2\tau_{2q})$ $+ \gamma_{45}(\tau_{4q} - \tau_{5q}) - \gamma_{35}(s_{34} - \tau_{3q} + \tau_{5q}) + \gamma_{34}(s_{12} + \tau_{3q} - \tau_{4q} + 2s_{45} + 2\tau_{4p} - 2\tau_{3p})\Big)$
(d)-(f)	$\gamma_{12}s_{45} - \frac{1}{4}\Big(2\gamma_{12} + \gamma_{13} - \gamma_{23}\Big)s_{12}$

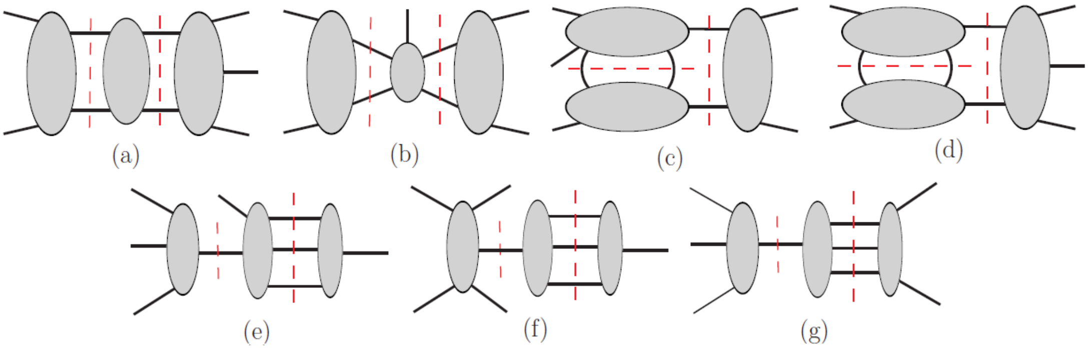
$$\mathcal{J}^{(x)} = \int \frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{C^{(x)}N^{(x)}(1,2,3,4,5;p,q)}{l_1^2 l_2^2 l_3^2 l_4^2 l_5^2 l_6^2 l_7^2 l_8^2},$$

$$C^{(\mathrm{a})} = c_{(4,10,8)}c_{(5,7,10)}c_{(6,1,12)}c_{(7,6,9)}c_{(8,9,11)}c_{(11,13,3)}c_{(12,2,13)},$$
$$C^{(\mathrm{b})} = c_{(4,9,10)}c_{(5,7,8)}c_{(6,1,12)}c_{(8,9,11)}c_{(10,7,6)}c_{(11,13,3)}c_{(12,2,13)},$$
$$C^{(\mathrm{c})} = c_{(1,6,8)}c_{(2,12,8)}c_{(6,9,11)}c_{(7,4,13)}c_{(10,9,5)}c_{(11,13,3)}c_{(12,7,10)},$$
$$C^{(\mathrm{d})} = c_{(4,10,8)}c_{(5,7,10)}c_{(6,13,12)}c_{(7,6,9)}c_{(8,9,11)}c_{(11,13,3)}c_{(12,2,1)},$$
$$C^{(\mathrm{e})} = c_{(4,10,8)}c_{(5,9,7)}c_{(6,13,12)}c_{(7,10,6)}c_{(8,9,11)}c_{(11,13,3)}c_{(12,2,1)},$$
$$C^{(\mathrm{f})} = c_{(2,1,8)}c_{(6,9,7)}c_{(7,13,5)}c_{(8,6,11)}c_{(10,3,9)}c_{(11,12,10)}c_{(12,4,13)},$$

$$\mathcal{M}_5^{(2)} = -i\left(\frac{\kappa}{2}\right)^7 \sum_{S_5} \left(\frac{1}{2}I^{(\mathrm{a})} + \frac{1}{4}I^{(\mathrm{b})} + \frac{1}{4}I^{(\mathrm{c})} + \frac{1}{2}I^{(\mathrm{d})} + \frac{1}{4}I^{(\mathrm{e})} + \frac{1}{4}I^{(\mathrm{f})}\right),$$

$$I^{(x)} = \int \frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{N^{(x)}(1,2,3,4,5;p,q)\tilde{N}^{(x)}(1,2,3,4,5;p,q)}{l_1^2 l_2^2 l_3^2 l_4^2 l_5^2 l_6^2 l_7^2 l_8^2},$$

$$\tilde{N}^{(x)}(1,2,3,4,5;p,q) = N^{(x)}(1,2,3,4,5;p,q)\big|_{\eta_i^A \to \eta_i^{A+4}}$$

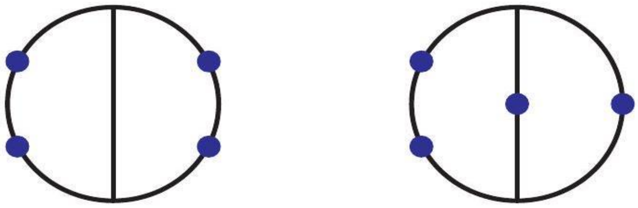


$$\mathcal{I}^{(d)}\Big|_{\text{UV pole}} = -\frac{1}{s_{12}} N^{(d)} C^{(d)} V^{(P)},\ \mathcal{I}^{(e)}\Big|_{\text{UV pole}} = -\frac{1}{s_{12}} N^{(e)} C^{(e)} V^{(NP)},$$
$$\mathcal{I}^{(f)}\Big|_{\text{UV pole}} = -\frac{1}{s_{12}} N^{(f)} C^{(f)} V^{(NP)},$$

$$V^{(P)} = -\frac{\pi}{20(4\pi)^7\epsilon}, V^{(NP)} = -\frac{\pi}{30(4\pi)^7\epsilon}.$$

$$\mathcal{A}_5^{(2)}\Big|_{\text{UV}} = -g^7\Big[\Big(N_c^2 V^{(P)} + 12\big(V^{(P)} + V^{(NP)}\big)\Big)\text{Tr}_{12345}\Big(5\beta_{12345} + \frac{\gamma_{12}}{s_{12}}(s_{35} - 2s_{12}) + \frac{\gamma_{23}}{s_{23}}(s_{14} - 2s_{23}) + \frac{\gamma_{34}}{s_{34}}(s_{25} - 2s_{34}) + \frac{\gamma_{45}}{s_{45}}(s_{13} - 2s_{45}) + \frac{\gamma_{51}}{s_{15}}(s_{24} - 2s_{15})\Big) - 12N_c\big(V^{(P)} + V^{(NP)}\big)\text{Tr}_{123}\text{Tr}_{45}s_{45}\Big(\frac{\gamma_{12}}{s_{12}} + \frac{\gamma_{23}}{s_{23}} + \frac{\gamma_{31}}{s_{13}}\Big) + \text{ perms }\Big],$$

$$\mathcal{M}_5^{(2)}\Big|_{\text{UV}} = i\left(\frac{\kappa}{2}\right)^7 \frac{1}{6}\big(V^{(P)} + V^{(NP)}\big)\sum_{S_5} \frac{\gamma_{12}^2}{s_{12}}(s_{34}^2 + s_{35}^2 + s_{45}^2 - 3s_{12}^2),$$



$$\gamma_{ij}^2 \equiv \gamma_{ij}\tilde{\gamma}_{ij} = \gamma_{ij}\left(\gamma_{ij}\big|_{\eta_i^A \to \eta_i^{A+4}}\right)$$

$$0 = \sum_{S_5} s_{12}(\gamma_{13}\gamma_{23} + \gamma_{14}\gamma_{24} + \gamma_{15}\gamma_{25} - 2\gamma_{12}^2),$$
$$0 = \sum_{S_5} s_{12}(\gamma_{43}\gamma_{35} + \gamma_{34}\gamma_{45} + \gamma_{35}\gamma_{54}).$$

$$ds_{10}^2 = (dx^{a+4})^2 + \left(dx^\mu + \Omega_a^\mu dx^{a+4}\right)^2$$

$$\Omega_\mu{}^\rho{}_a\Omega_{\rho\nu b} - \Omega_\mu{}^\rho{}_b\Omega_{\rho\nu a} = 0.$$

$$\Omega_{\mu\nu a} = \begin{pmatrix} 0 & \epsilon_a^1 & 0 & 0 \\ -\epsilon_a^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\epsilon_a^2 \\ 0 & 0 & \epsilon_a^2 & 0 \end{pmatrix},$$

$$
\begin{aligned}
S = \frac{1}{\kappa g^2}\int \ d^4x\mathrm{Tr}[&\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \Lambda^A\sigma^\mu D_\mu\bar{\Lambda}_A + \frac{1}{2}\left(D_\mu\varphi_a - F_{\mu\nu}\Omega_a^\nu\right)^2 \\
&-\frac{1}{2}(\Sigma_a)^{AB}\bar{\Lambda}_A[\varphi_a,\bar{\Lambda}_B] - \frac{1}{2}(\bar{\Sigma}_a)_{AB}\Lambda^A[\varphi_a,\Lambda^B] \\
&-\frac{1}{4}\Big([\varphi_a,\varphi_b] + i\Omega_a^\mu D_\mu\varphi_b - i\Omega_b^\mu D_\mu\varphi_a - iF_{\mu\nu}\Omega_a^\mu\Omega_b^\nu \\
&-\frac{1}{2}((\Sigma_b\bar{\Sigma}_c)^A{}_B\varphi_c(\mathcal{A}_a)^B{}_A - (\Sigma_a\bar{\Sigma}_c)^A{}_B\varphi_c(\mathcal{A}_b)^B{}_A)\Big)^2 \\
&-\frac{i}{2}\Omega_a^\mu\left((\Sigma_a)^{AB}\bar{\Lambda}_A D_\mu\bar{\Lambda}_B + (\bar{\Sigma}_a)_{AB}\Lambda^A D_\mu\Lambda^B\right) \\
&+\frac{i}{4}\Omega_{\mu\nu a}((\Sigma_a)^{AB}\bar{\Lambda}_A\bar{\sigma}^{\mu\nu}\bar{\Lambda}_B + (\bar{\Sigma}_a)_{AB}\Lambda^A\sigma^{\mu\nu}\Lambda^B) \\
&+\frac{1}{2}(\Sigma_a)^{AB}\bar{\Lambda}_A\bar{\Lambda}_D(\mathcal{A}_a)^D{}_B - \frac{1}{2}(\bar{\Sigma}_a)_{AB}\Lambda^A(\mathcal{A}_a)^B{}_D\Lambda^D\Big]
\end{aligned}
$$

$$SO(4) \simeq SU(2)_L \times SU(2)_R$$

$$\epsilon_{\alpha\beta}, \epsilon_{\dot{\alpha}\dot{\beta}}\ \epsilon^{12} = -\epsilon_{12} = 1$$

$$\sigma^\mu, \bar{\sigma}^\mu, (\Sigma_a)^{AB}, (\bar{\Sigma}_a)_{AB}$$

$$D_\mu * = \partial_\mu * + i[A_\mu, *]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$$

$$
\begin{aligned}
[\varphi_a,\varphi_b] \longrightarrow & [\varphi_a,\varphi_b] + i\Omega_a^\mu D_\mu\varphi_b - i\Omega_b^\mu D_\mu\varphi_a - iF_{\mu\nu}\Omega_a^\mu\Omega_b^\nu \\
&-\frac{1}{2}((\Sigma_b\bar{\Sigma}_c)^A{}_B\varphi_c(\mathcal{A}_a)^B{}_A - (\Sigma_a\bar{\Sigma}_c)^A{}_B\varphi_c(\mathcal{A}_b)^B{}_A), \\
D_\mu\varphi_a \longrightarrow & D_\mu\varphi_a - F_{\mu\nu}\Omega_a^\nu, \\
[\varphi_a,\Lambda^A] \longrightarrow & [\varphi_a,\Lambda^A] + i\Omega_a^\mu D_\mu\Lambda^A - \frac{i}{2}\Omega_{\mu\nu a}\sigma^{\mu\nu}\Lambda^A + (\mathcal{A}_a)^A{}_B\Lambda^B, \\
[\varphi_a,\bar{\Lambda}_A] \longrightarrow & [\varphi_a,\bar{\Lambda}_A] + i\Omega_a^\mu D_\mu\bar{\Lambda}_A - \frac{i}{2}\Omega_{\mu\nu a}\bar{\sigma}^{\mu\nu}\bar{\Lambda}_A - \bar{\Lambda}_B(\mathcal{A}_a)^B{}_A.
\end{aligned}
$$

$$\bar{\Lambda}_{\dot{\alpha}}^{A'} = \frac{1}{2}\delta_{\dot{\alpha}}^{A'}\bar{\Lambda} + \frac{1}{2}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}}^{A'}\bar{\Lambda}_{\mu\nu}, \Lambda_\alpha^{A'} = \frac{1}{2}\epsilon^{A'B'}(\sigma^\mu)_{\alpha B'}\Lambda_\mu$$

$$\varphi^{AB} = \frac{i}{\sqrt{2}}(\Sigma_a)^{AB}\varphi_a, \bar{\varphi}_{AB} = -\frac{i}{\sqrt{2}}(\bar{\Sigma}_a)_{AB}\varphi_a$$

$$\varphi^{AB} = \begin{pmatrix} \epsilon^{A'B'}\varphi & \varphi^{A'\hat{B}} \\ \varphi^{\hat{A}B'} & -\epsilon^{\hat{A}\hat{B}}\bar{\varphi} \end{pmatrix}, \bar{\varphi}_{AB} = \begin{pmatrix} \epsilon_{A'B'}\bar{\varphi} & \bar{\varphi}_{A'\hat{B}} \\ \bar{\varphi}_{\hat{A}B'} & -\epsilon_{\hat{A}\hat{B}}\varphi \end{pmatrix}$$

$$\Omega_{\mu\nu a'} = \begin{pmatrix} 0 & \epsilon_{a'}^1 & 0 & 0 \\ -\epsilon_{a'}^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\epsilon_{a'}^2 \\ 0 & 0 & \epsilon_{a'}^2 & 0 \end{pmatrix}, (\mathcal{A}_{a'})^A{}_B = \begin{pmatrix} \frac{1}{4}(\epsilon_{a'}^1 + \epsilon_{a'}^2)\tau^3 & 0 \\ 0 & m_{a'}\tau^3 \end{pmatrix},$$

$$\Omega_{\mu\nu\hat{a}} = (\mathcal{A}_{\hat{a}})^A{}_B = 0, (a' = 1,2, \hat{a} = 3,4,5,6),$$

$$
\begin{aligned}
\mathcal{A} = \frac{1}{\sqrt{2}}(\mathcal{A}_1 - i\mathcal{A}_2), &\quad \overline{\mathcal{A}} = \frac{1}{\sqrt{2}}(\mathcal{A}_1 + i\mathcal{A}_2) \\
m = \frac{1}{\sqrt{2}}(m_1 - im_2), &\quad \bar{m} = \frac{1}{\sqrt{2}}(m_1 + im_2)
\end{aligned}
$$

$$\Omega_{\mu\nu}=\begin{pmatrix}0 & \epsilon^1 & 0 & 0\\ -\epsilon^1 & 0 & 0 & 0\\ 0 & 0 & 0 & -\epsilon^2\\ 0 & 0 & \epsilon^2 & 0\end{pmatrix}, \bar{\Omega}_{\mu\nu}=\begin{pmatrix}0 & \bar{\epsilon}^1 & 0 & 0\\ -\bar{\epsilon}^1 & 0 & 0 & 0\\ 0 & 0 & 0 & -\bar{\epsilon}^2\\ 0 & 0 & \bar{\epsilon}^2 & 0\end{pmatrix},$$

$$\epsilon^i=\frac{1}{\sqrt{2}}\left(\epsilon_1^i-i\epsilon_2^i\right), \bar{\epsilon}^i=\frac{1}{\sqrt{2}}\left(\epsilon_1^i+i\epsilon_2^i\right), (i=1,2)$$

$$\bar{Q}A_\mu=\Lambda_\mu$$
$$\bar{Q}\Lambda_\mu=-2\sqrt{2}\left(D_\mu\varphi-F_{\mu\nu}\Omega^\nu\right)$$
$$\bar{Q}\varphi=\Omega^\mu\Lambda_\mu$$
$$\bar{Q}\bar{\varphi}=-\sqrt{2}\bar{\Lambda}+\bar{\Omega}^\mu\Lambda_\mu$$
$$\bar{Q}\bar{\Lambda}=-2i\left([\varphi,\bar{\varphi}]+i\Omega^\mu D_\mu\bar{\varphi}-i\bar{\Omega}^\mu D_\mu\varphi+i\bar{\Omega}^\mu\Omega^\nu F_{\mu\nu}\right)$$
$$\bar{Q}\bar{\Lambda}_{\mu\nu}=-2F_{\mu\nu}^{-}-i\left(\bar{\sigma}_{\mu\nu}\right)^{\dot{\beta}}{}_{\dot{\alpha}}\left[\varphi^{\dot{\alpha}\hat{A}},\bar{\varphi}_{\hat{A}\dot{\beta}}\right]$$
$$\bar{Q}\varphi^{\dot{\alpha}\hat{A}}=-\sqrt{2}\bar{\Lambda}^{\dot{\alpha}\hat{A}}$$
$$\bar{Q}\bar{\Lambda}^{\dot{\alpha}\hat{A}}=-2i\left([\varphi,\varphi^{\dot{\alpha}\hat{A}}]+i\Omega^\mu D_\mu\varphi^{\dot{\alpha}\hat{A}}+M^{\hat{A}}{}_{\hat{B}}\varphi^{\dot{\alpha}\hat{B}}\right)-\Omega^{\mu\nu}\left(\bar{\sigma}_{\mu\nu}\right)^{\dot{\alpha}}{}_{\beta}\varphi^{\dot{\beta}\hat{A}}$$
$$\bar{Q}\Lambda_\alpha^{\hat{A}}=\sqrt{2}(\sigma^\mu)_{\alpha\dot{\alpha}}D_\mu\varphi^{\dot{\alpha}\hat{A}}$$

$$\Omega^\mu=\Omega^{\mu\nu}x_\nu, \bar{\Omega}^\mu=\bar{\Omega}^{\mu\nu}x_\nu$$

$$X_{\mu\nu}^{\pm}=\frac{1}{2}\left(X_{\mu\nu}\pm\tilde{X}_{\mu\nu}\right)$$

$$\Lambda_{\alpha A'}=\frac{1}{2}(\sigma^\mu)_{\alpha A'}\Lambda_\mu, \Lambda_{\alpha\hat{A}}=\frac{1}{2}(\sigma^\mu)_{\alpha\hat{A}}\hat{\Lambda}_\mu$$
$$\bar{\Lambda}_{\dot{\alpha}}^{A'}=\frac{1}{2}\delta_{\dot{\alpha}}^{A'}\bar{\Lambda}+\frac{1}{2}(\bar{\sigma}^{\mu\nu})^{A'}{}_{\dot{\alpha}}\bar{\Lambda}_{\mu\nu}, \bar{\Lambda}_{\dot{\alpha}}^{\hat{A}}=\frac{1}{2}\delta_{\dot{\alpha}}^{\hat{A}}\hat{\bar{\Lambda}}+\frac{1}{2}(\bar{\sigma}^{\mu\nu})^{\hat{A}}{}_{\dot{\alpha}}\hat{\bar{\Lambda}}_{\mu\nu}$$

$$\varphi^{A'B'}=\epsilon^{A'B'}\varphi, \varphi^{\hat{A}\hat{B}}=-\epsilon^{\hat{A}\hat{B}}\bar{\varphi}, \varphi^{A'\hat{A}}=\frac{1}{2}\epsilon^{A'\hat{A}}\hat{\varphi}+\frac{1}{2}\epsilon^{\hat{A}\hat{B}}(\bar{\sigma}^{\mu\nu})^{A'}{}_{\hat{B}}\hat{\varphi}_{\mu\nu}$$

$$\Omega_{\mu\nu a'}=\begin{pmatrix}0 & \epsilon_{a'}^1 & 0 & 0\\ -\epsilon_{a'}^1 & 0 & 0 & 0\\ 0 & 0 & 0 & -\epsilon_{a'}^2\\ 0 & 0 & \epsilon_{a'}^2 & 0\end{pmatrix}, \Omega_{\mu\nu\hat{a}}=\begin{pmatrix}0 & \epsilon_{\hat{a}}^1 & 0 & 0\\ -\epsilon_{\hat{a}}^1 & 0 & 0 & 0\\ 0 & 0 & 0 & -\epsilon_{\hat{a}}^2\\ 0 & 0 & \epsilon_{\hat{a}}^2 & 0\end{pmatrix},$$

$$(\mathcal{A}_{a'})^A{}_B=\begin{pmatrix}\frac{1}{4}\left(\epsilon_{a'}^1+\epsilon_{a'}^2\right)\tau^3 & 0\\ 0 & \frac{1}{4}\left(\epsilon_{a'}^1+\epsilon_{a'}^2\right)\tau^3\end{pmatrix}, (\mathcal{A}_{\hat{a}})^A{}_B=\begin{pmatrix}\frac{1}{4}\left(\epsilon_{\hat{a}}^1+\epsilon_{\hat{a}}^2\right)\tau^3 & 0\\ 0 & \frac{1}{4}\left(\epsilon_{\hat{a}}^1+\epsilon_{\hat{a}}^2\right)\tau^3\end{pmatrix},$$

$(a'=1,2, \hat{a}=5,6)$,

$\Omega_{\mu\nu 3}=\Omega_{\mu\nu 4}=(\mathcal{A}_3)^A{}_B=(\mathcal{A}_4)^A{}_B=0.$

$$\Omega_{\mu\nu}^{AB}=\frac{i}{\sqrt{2}}(\Sigma_a)^{AB}\Omega_{\mu\nu}^a, \bar{\Omega}_{AB}^{\mu\nu}=-\frac{i}{\sqrt{2}}(\bar{\Sigma}_a)_{AB}\Omega^{a\mu\nu},$$
$$\Omega_\mu^{AB}=\Omega_{\mu\nu}^{AB}x^\nu, \bar{\Omega}_{AB}^\mu=\bar{\Omega}_{AB}^{\mu\nu}x_\nu.$$

$$\Omega_{\mu\nu}^{A'B'}=\epsilon^{A'B'}\Omega_{\mu\nu}, \Omega_{\mu\nu}^{\hat{A}\hat{B}}=-\epsilon^{\hat{A}\hat{B}}\bar{\Omega}_{\mu\nu}, \Omega_{\mu\nu}^{A'\hat{A}}=\frac{1}{2}\epsilon^{A'\hat{A}}\hat{\Omega}_{\mu\nu}+\frac{1}{2}\epsilon^{\hat{A}\hat{B}}(\bar{\sigma}^{\rho\sigma})^{A'}{}_{\hat{B}}\hat{\Omega}_{\mu\nu,\rho\sigma},$$

$$\hat{\Omega}_{\mu\nu}=\sqrt{2}\begin{pmatrix}0 & \epsilon_6^1 & 0 & 0\\ -\epsilon_6^1 & 0 & 0 & 0\\ 0 & 0 & 0 & -\epsilon_6^2\\ 0 & 0 & \epsilon_6^2 & 0\end{pmatrix}, \hat{\Omega}_{\mu\nu,12}=-\hat{\Omega}_{\mu\nu,34}=-\frac{1}{\sqrt{2}}\begin{pmatrix}0 & \epsilon_5^1 & 0 & 0\\ -\epsilon_5^1 & 0 & 0 & 0\\ 0 & 0 & 0 & -\epsilon_5^2\\ 0 & 0 & \epsilon_5^2 & 0\end{pmatrix}$$

$\hat{\Omega}_{\mu\nu,\rho\sigma}=0, ((\rho,\sigma)\neq(1,2),(3,4)).$

$$\bar{Q}A_\mu = \Lambda_\mu, \bar{Q}\Lambda_\mu = -2\sqrt{2}\left(D_\mu\varphi - F_{\mu\nu}\Omega^\nu\right),$$
$$\bar{Q}\varphi = \Omega^\mu\Lambda_\mu,$$
$$\bar{Q}\bar{\varphi} = -\sqrt{2}\bar{\Lambda} + \bar{\Omega}^\mu\Lambda_\mu, \bar{Q}\bar{\Lambda} = -2i\left([\varphi,\bar{\varphi}] + i\Omega^\mu D_\mu\bar{\varphi} - i\bar{\Omega}^\mu D_\mu\varphi + i\bar{\Omega}^\mu\Omega^\nu F_{\mu\nu}\right),$$
$$\bar{Q}\bar{\Lambda}_{\mu\nu} = -2F^-_{\mu\nu} + i\left([\hat{\varphi},\hat{\varphi}_{\mu\nu}] + i\hat{\Omega}^\rho D_\rho\hat{\varphi}_{\mu\nu} - i\hat{\Omega}^\rho{}_{\mu\nu}D_\rho\hat{\varphi} + i\hat{\Omega}^\rho{}_{\mu\nu}\hat{\Omega}^\sigma F_{\rho\sigma} - i\hat{\Omega}_\mu{}^\rho\hat{\varphi}_{\rho\nu} + i\hat{\Omega}_\nu{}^\rho\hat{\varphi}_{\rho\mu}\right),$$
$$-\frac{i}{2}\left([\hat{\varphi}_{\mu\rho},\hat{\varphi}_\nu{}^\rho] + i\hat{\Omega}^\rho{}_{\mu\sigma}D_\rho\hat{\varphi}_\nu{}^\sigma - i\hat{\Omega}^\rho{}_{\nu\sigma}D_\rho\hat{\varphi}_\mu{}^\sigma + \hat{\Omega}^-_{\mu\nu,\rho\sigma}\hat{\varphi}^{\rho\sigma} - \hat{\Omega}_{\rho\sigma}{}^{\rho\sigma}\hat{\varphi}_{\mu\nu}\right),$$
$$\bar{Q}\hat{\Lambda}_\mu = -\sqrt{2}\left(D_\mu\hat{\varphi} - F_{\mu\nu}\hat{\Omega}^\nu\right) - 2\sqrt{2}\left(D^\nu\hat{\varphi}_{\mu\nu} - F^{\nu\rho}\hat{\Omega}_{\rho,\mu\nu}\right),$$
$$\bar{Q}\hat{\varphi} = -\sqrt{2}\hat{\bar{\Lambda}} + \hat{\Omega}^\mu\Lambda_\mu, \bar{Q}\hat{\bar{\Lambda}} = -2i\left([\varphi,\hat{\varphi}] + i\Omega^\mu D_\mu\hat{\varphi} - i\hat{\Omega}^\mu D_\mu\varphi + i\hat{\Omega}^\mu\Omega^\nu F_{\mu\nu}\right),$$
$$\bar{Q}\hat{\varphi}_{\mu\nu} = -\sqrt{2}\hat{\bar{\Lambda}}_{\mu\nu} + \hat{\Omega}^{\rho,}{}_{\mu\nu}\Lambda_\rho,$$
$$\bar{Q}\hat{\bar{\Lambda}}_{\mu\nu} = -2i\left([\varphi,\hat{\varphi}_{\mu\nu}] + i\Omega^\rho D_\rho\hat{\varphi}_{\mu\nu} - i\hat{\Omega}^{\rho,}{}_{\mu\nu}D_\rho\varphi + i\hat{\Omega}^{\rho,}{}_{\mu\nu}\Omega^\sigma F_{\rho\sigma} - i\Omega_\mu{}^\rho\hat{\varphi}_{\rho\nu} + i\Omega_\nu{}^\rho\hat{\varphi}_{\rho\mu}\right).$$

$$\hat{\Omega}^\mu = \hat{\Omega}^{\mu\nu}x_\nu, \hat{\Omega}^{\rho,}{}_{\mu\nu} = \hat{\Omega}^{\rho\sigma,}{}_{\mu\nu}x_\sigma,$$

$$\hat{\bar{Q}}A_\mu = \hat{\Lambda}_\mu, \hat{\bar{Q}}\hat{\Lambda}_\mu = 2\sqrt{2}\left(D_\mu\bar{\varphi} - F_{\mu\nu}\bar{\Omega}^\nu\right),$$
$$\hat{\bar{Q}}\bar{\varphi} = \bar{\Omega}^\mu\hat{\Lambda}_\mu,$$
$$\hat{\bar{Q}}\varphi = \sqrt{2}\hat{\bar{\Lambda}} + \Omega^\mu\hat{\Lambda}_\mu, \hat{\bar{Q}}\hat{\bar{\Lambda}} = 2i\left([\varphi,\bar{\varphi}] + i\Omega^\mu D_\mu\bar{\varphi} - i\bar{\Omega}^\mu D_\mu\varphi + i\bar{\Omega}^\mu\Omega^\nu F_{\mu\nu}\right),$$
$$\hat{\bar{Q}}\hat{\Lambda}_{\mu\nu} = -2F^-_{\mu\nu} - i\left([\hat{\varphi},\hat{\varphi}_{\mu\nu}] + i\hat{\Omega}^\rho D_\rho\hat{\varphi}_{\mu\nu} - i\hat{\Omega}^{\rho,}{}_{\mu\nu}D_\rho\hat{\varphi} + i\hat{\Omega}^{\rho,}{}_{\mu\nu}\hat{\Omega}^\sigma F_{\rho\sigma} - i\hat{\Omega}_\mu{}^\rho\hat{\varphi}_{\rho\nu} + i\hat{\Omega}_\nu{}^\rho\hat{\varphi}_{\rho\mu}\right),$$
$$-\frac{i}{2}\left([\hat{\varphi}_{\mu\rho},\hat{\varphi}_\nu{}^\rho] + i\hat{\Omega}^{\rho,}{}_{\mu\sigma}D_\rho\hat{\varphi}_\nu{}^\sigma - i\hat{\Omega}^{\rho,}{}_{\nu\sigma}D_\rho\hat{\varphi}_\mu{}^\sigma + \hat{\Omega}^-_{\mu\nu,\rho\sigma}\hat{\varphi}^{\rho\sigma} - \hat{\Omega}_{\rho\sigma}{}^{\rho\sigma}\hat{\varphi}_{\mu\nu}\right),$$
$$\hat{\bar{Q}}\Lambda_\mu = -\sqrt{2}\left(D_\mu\hat{\varphi} - F_{\mu\nu}\hat{\Omega}^\nu\right) + 2\sqrt{2}\left(D^\nu\hat{\varphi}_{\mu\nu} - F^{\nu\rho}\hat{\Omega}_{\rho,\mu\nu}\right),$$
$$\hat{\bar{Q}}\hat{\varphi} = \sqrt{2}\bar{\Lambda} + \hat{\Omega}^\mu\hat{\Lambda}_\mu, \hat{\bar{Q}}\bar{\Lambda} = -2i\left([\bar{\varphi},\hat{\varphi}] + i\bar{\Omega}^\mu D_\mu\hat{\varphi} - i\hat{\Omega}^\mu D_\mu\bar{\varphi} + i\hat{\Omega}^\mu\bar{\Omega}^\nu F_{\mu\nu}\right),$$
$$\hat{\bar{Q}}\hat{\varphi}_{\mu\nu} = \sqrt{2}\bar{\Lambda}_{\mu\nu} + \hat{\Omega}^{\rho,}{}_{\mu\nu}\hat{\Lambda}_\rho,$$
$$\hat{\bar{Q}}\hat{\bar{\Lambda}}_{\mu\nu} = -2i\left([\bar{\varphi},\hat{\varphi}_{\mu\nu}] + i\bar{\Omega}^\rho D_\rho\hat{\varphi}_{\mu\nu} - i\hat{\Omega}^{\rho,}{}_{\mu\nu}D_\rho\bar{\varphi} + i\hat{\Omega}^{\rho,}{}_{\mu\nu}\bar{\Omega}^\sigma F_{\rho\sigma} - i\bar{\Omega}_\mu{}^\rho\hat{\varphi}_{\rho\nu} + i\bar{\Omega}_\nu{}^\rho\hat{\varphi}_{\rho\mu}\right).$$

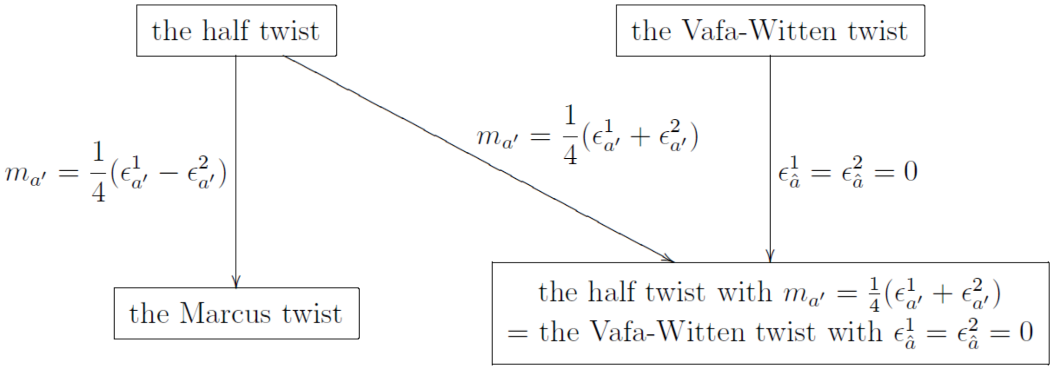
$$\Lambda^{A'}_\alpha = \frac{1}{2}\epsilon^{A'\hat{B}}(\sigma^\mu)_{\alpha\hat{B}}\Lambda_\mu, \bar{\Lambda}^{\dot{\alpha}}_{A'} = -\frac{1}{2}\delta^{\dot{\alpha}}_{A'}\bar{\Lambda} + \frac{1}{2}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{A'}\bar{\Lambda}_{\mu\nu}$$
$$\bar{\Lambda}^{\dot{\alpha}\hat{A}} = \frac{1}{2}(\bar{\sigma}^\mu)^{\dot{\alpha}\hat{A}}\bar{\Lambda}_\mu, \Lambda^{\hat{A}}_\alpha = \frac{1}{2}\delta^{\hat{A}}_\alpha\Lambda + \frac{1}{2}(\sigma^{\mu\nu})_\alpha{}^{\hat{A}}\Lambda_{\mu\nu}$$

$$\varphi_\mu = (\sigma_\mu)_{\hat{B}A'}\varphi^{A'\hat{B}}$$

$$\Omega_{\mu\nu a'} = \begin{pmatrix} 0 & \epsilon^1_{a'} & 0 & 0 \\ -\epsilon^1_{a'} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\epsilon^2_{a'} \\ 0 & 0 & \epsilon^2_{a'} & 0 \end{pmatrix}, (\mathcal{A}_{a'})^A{}_B = \begin{pmatrix} \frac{1}{4}(\epsilon^1_{a'} + \epsilon^2_{a'})\tau^3 & 0 \\ 0 & \frac{1}{4}(\epsilon^1_{a'} - \epsilon^2_{a'})\tau^3 \end{pmatrix},$$
$\Omega_{\mu\nu\hat{a}} = (\mathcal{A}_{\hat{a}})^A{}_B = 0, (a' = 1,2, \hat{a} = 3,4,5,6).$

$$\bar{Q}A_\mu = \Lambda_\mu, \bar{Q}\Lambda_\mu = -2\sqrt{2}\left(D_\mu\varphi - F_{\mu\nu}\Omega^\nu\right)$$
$$\bar{Q}\varphi = \Omega^\mu\Lambda_\mu,$$
$$\bar{Q}\bar{\varphi} = -\sqrt{2}\bar{\Lambda} + \bar{\Omega}^\mu\Lambda_\mu, \bar{Q}\bar{\Lambda} = -2i\left([\varphi,\bar{\varphi}] + i\Omega^\mu D_\mu\bar{\varphi} - i\bar{\Omega}^\mu D_\mu\varphi + i\bar{\Omega}^\mu\Omega^\nu F_{\mu\nu}\right)$$
$$\bar{Q}\varphi_\mu = -\sqrt{2}\bar{\Lambda}_\mu, \bar{Q}\bar{\Lambda}_\mu = -2i\left([\varphi,\varphi_\mu] + i\Omega^\nu D_\nu\varphi_\mu - i\Omega^\nu_\mu\varphi_\nu\right)$$
$$\bar{Q}\Lambda = \sqrt{2}D_\mu\varphi^\mu, \bar{Q}\Lambda_{\mu\nu} = \sqrt{2}\left(D_\mu\varphi_\nu - D_\nu\varphi_\mu\right)^+$$
$$\bar{Q}\bar{\Lambda}_{\mu\nu} = -2F^-_{\mu\nu} + i[\varphi_\mu,\varphi_\nu]^-$$

$$QA_\mu = \bar{\Lambda}_\mu, Q\bar{\Lambda}_\mu = -2\sqrt{2}\left(D_\mu\varphi - F_{\mu\nu}\Omega^\nu\right)$$
$$Q\varphi = \Omega^\mu\bar{\Lambda}_\mu$$
$$Q\bar{\varphi} = \sqrt{2}\Lambda + \bar{\Omega}^\mu\bar{\Lambda}_\mu, Q\Lambda = 2i\left([\varphi,\bar{\varphi}] + i\Omega^\mu D_\mu\bar{\varphi} - i\bar{\Omega}^\mu D_\mu\varphi + i\bar{\Omega}^\mu\Omega^\nu F_{\mu\nu}\right)$$
$$Q\varphi_\mu = \sqrt{2}\Lambda_\mu, Q\Lambda_\mu = 2i\left([\varphi,\varphi_\mu] + i\Omega^\nu D_\nu\varphi_\mu - i\Omega^\nu_\mu\varphi_\nu\right)$$
$$Q\bar{\Lambda} = \sqrt{2}D_\mu\varphi^\mu, Q\Lambda_{\mu\nu} = -2F^+_{\mu\nu} + i[\varphi_\mu,\varphi_\nu]^+$$
$$Q\bar{\Lambda}_{\mu\nu} = -\sqrt{2}\left(D_\mu\varphi_\nu - D_\nu\varphi_\mu\right)^-$$



$$S_1 = S + \frac{1}{\kappa g^2}\int d^4x \mathrm{Tr}\left[-\frac{1}{2}(D_{\mu\nu})^2 + \frac{1}{2}K^{\hat{A}}_{\alpha}K^{\alpha}_{\hat{A}}\right].$$

$$\begin{aligned}\bar{Q}\bar{\Lambda}_{\mu\nu} &= 2D_{\mu\nu} - 2F^{-}_{\mu\nu} - i(\bar{\sigma}_{\mu\nu})^{\dot{\beta}}_{\dot{\alpha}}[\varphi^{\dot{\alpha}\hat{A}}, \bar{\varphi}_{\hat{A}\dot{\beta}}],\\ \bar{Q}\Lambda^{\hat{A}}_{\alpha} &= 2K^{\hat{A}}_{\alpha} + \sqrt{2}(\sigma^{\mu})_{\alpha\dot{\alpha}}D_{\mu}\varphi^{\dot{\alpha}\hat{A}}.\end{aligned}$$

$$\begin{aligned}\bar{Q}D_{\mu\nu} =& (D_{\mu}\Lambda_{\nu} - D_{\nu}\Lambda_{\mu})^{-} - \sqrt{2}i(\bar{\sigma}_{\mu\nu})^{\dot{\beta}}{}_{\dot{\alpha}}[\bar{\Lambda}^{\dot{\alpha}\hat{A}}, \bar{\varphi}_{\hat{A}\dot{\beta}}]\\ &+\sqrt{2}i[\varphi, \bar{\Lambda}_{\mu\nu}] - \sqrt{2}\Omega^{\rho}D_{\rho}\bar{\Lambda}_{\mu\nu} + \sqrt{2}(\Omega_{\mu}{}^{\rho}\bar{\Lambda}_{\rho\nu} - \Omega_{\nu}{}^{\rho}\bar{\Lambda}_{\rho\mu}),\\ \bar{Q}K^{\hat{A}}_{\alpha} =& (\sigma^{\mu})_{\alpha\dot{\alpha}}D_{\mu}\bar{\Lambda}^{\dot{\alpha}\hat{A}} - \frac{i}{\sqrt{2}}(\sigma^{\mu})_{\alpha\dot{\alpha}}[\Lambda_{\mu}, \varphi^{\dot{\alpha}\hat{A}}]\\ &+\sqrt{2}i\left([\varphi, \Lambda^{\hat{A}}_{\alpha}] + i\Omega^{\mu}D_{\mu}\Lambda^{\hat{A}}_{\alpha} + M^{\hat{A}}{}_{\hat{B}}\Lambda^{\hat{B}}{}_{\alpha} - \frac{i}{2}\Omega^{\mu\nu}(\sigma_{\mu\nu})_{\alpha}{}^{\beta}\Lambda^{\hat{A}}_{\beta}\right).\end{aligned}$$

$$\begin{aligned}H_{\mu\nu} &= D_{\mu\nu} - F^{-}_{\mu\nu} - \frac{i}{2}(\bar{\sigma}_{\mu\nu})^{\dot{\beta}}{}_{\dot{\alpha}}\left[\varphi^{\dot{\alpha}\hat{A}}, \bar{\varphi}_{\hat{A}\dot{\beta}}\right]\\ G^{\hat{A}}_{\alpha} &= K^{\hat{A}}_{\alpha} + \frac{1}{\sqrt{2}}(\sigma^{\mu})_{\alpha\dot{\alpha}}D_{\mu}\varphi^{\dot{\alpha}\hat{A}}\end{aligned}$$

$$\bar{Q}\bar{\Lambda}_{\mu\nu} = 2H_{\mu\nu}, \bar{Q}\Lambda^{\hat{A}}_{\alpha} = 2G^{\hat{A}}_{\alpha}$$

$$\begin{aligned}\bar{Q}H_{\mu\nu} &= \sqrt{2}i[\varphi, \bar{\Lambda}_{\mu\nu}] - \sqrt{2}\Omega^{\rho}D_{\rho}\bar{\Lambda}_{\mu\nu} + \sqrt{2}(\Omega_{\mu}{}^{\rho}\bar{\Lambda}_{\rho\nu} - \Omega_{\nu}{}^{\rho}\bar{\Lambda}_{\rho\mu})\\ \bar{Q}G^{\hat{A}}_{\alpha} &= \sqrt{2}i\left([\varphi, \Lambda^{\hat{A}}_{\alpha}] + i\Omega^{\mu}D_{\mu}\Lambda^{\hat{A}}_{\alpha} + M^{\hat{A}}{}_{\hat{B}}\Lambda^{\hat{B}}{}_{\alpha} - \frac{i}{2}\Omega^{\mu\nu}(\sigma_{\mu\nu})_{\alpha}{}^{\beta}\Lambda^{\hat{A}}_{\beta}\right)\end{aligned}$$

$$\bar{Q}^2\Psi = 2\sqrt{2}(\delta_{\text{gauge}}(\varphi) + \delta_{\text{Lorentz}}(\Omega) + \delta_{\text{flavor}}(M))\Psi$$

$$S_1 = \bar{Q}\Xi_1 + \int d^4x \frac{1}{\kappa g^2}\mathrm{Tr}\left[\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

$$\begin{aligned}\Xi_1 =& \int d^4x \frac{1}{\kappa g^2}\mathrm{Tr}\Big[-\frac{1}{2}F^{-}_{\mu\nu}\bar{\Lambda}^{\mu\nu} - \frac{1}{4}H_{\mu\nu}\bar{\Lambda}^{\mu\nu} - \frac{1}{2\sqrt{2}}\Lambda^{\mu}(D_{\mu}\bar{\varphi} - F_{\mu\nu}\bar{\Omega}^{\nu})\\ &+\frac{i}{4}\bar{\Lambda}([\varphi, \bar{\varphi}] + i\Omega^{\mu}D_{\mu}\bar{\varphi} - i\bar{\Omega}^{\mu}D_{\mu}\varphi + i\bar{\Omega}^{\mu}\Omega^{\nu}F_{\mu\nu})\\ &+\frac{1}{2}\Lambda^{\alpha}_{\hat{A}}G^{\hat{A}}_{\alpha} - \frac{1}{\sqrt{2}}\Lambda^{\alpha}_{\hat{A}}(\sigma^{\mu})_{\alpha\dot{\alpha}}D_{\mu}\varphi^{\dot{\alpha}\hat{A}}\\ &-\frac{i}{2}\bar{\Lambda}^{\dot{\alpha}\hat{A}}\left([\bar{\varphi}, \bar{\varphi}_{\hat{A}\dot{\alpha}}] + i\bar{\Omega}^{\mu}D_{\mu}\bar{\varphi}_{\hat{A}\dot{\alpha}} - \frac{i}{2}\bar{\Omega}_{\mu\nu}(\bar{\sigma}^{\mu\nu})^{\dot{\beta}}{}_{\dot{\alpha}}\bar{\varphi}_{\hat{A}\dot{\beta}} + \bar{M}^{\hat{B}}{}_{\hat{A}}\bar{\varphi}_{\hat{B}\dot{\alpha}}\right)\\ &-\frac{i}{4}\bar{\Lambda}^{\mu\nu}(\bar{\sigma}_{\mu\nu})^{\dot{\beta}}{}_{\dot{\alpha}}[\varphi^{\dot{\alpha}\hat{A}}, \bar{\varphi}_{\hat{A}\dot{\beta}}]\Big].\end{aligned}$$

$$S_2 = S + \int d^4x \frac{1}{\kappa g^2}\mathrm{Tr}\left[-\frac{1}{2}(D_{\mu\nu})^2 - \frac{1}{2}(K_{\mu})^2\right]$$

$$\begin{aligned}\bar{Q}\bar{\Lambda}_{\mu\nu} =& 2D_{\mu\nu} - 2F_{\mu\nu}^{-}\\ &+i\left([\hat{\varphi},\hat{\varphi}_{\mu\nu}] + i\hat{\Omega}^{\rho}D_{\rho}\hat{\varphi}_{\mu\nu} - i\hat{\Omega}^{\rho,}{}_{\mu\nu}D_{\rho}\hat{\varphi} + i\hat{\Omega}^{\rho,}{}_{\mu\nu}\hat{\Omega}^{\sigma}F_{\rho\sigma} - i\hat{\Omega}_{\mu}{}^{\rho}\hat{\varphi}_{\rho\nu} + i\hat{\Omega}_{\nu}{}^{\rho}\hat{\varphi}_{\rho\mu}\right)\\ &-\frac{i}{2}\left([\hat{\varphi}_{\mu\rho},\hat{\varphi}_{\nu}{}^{\rho}] + i\hat{\Omega}^{\rho,}{}_{\mu\sigma}D_{\rho}\hat{\varphi}_{\nu}{}^{\sigma} - i\hat{\Omega}^{\rho,}{}_{\nu\sigma}D_{\rho}\hat{\varphi}_{\mu}{}^{\sigma} + \hat{\Omega}_{\mu\nu,\rho\sigma}^{-}\hat{\varphi}^{\rho\sigma} - \hat{\Omega}_{\rho\sigma,}{}^{\rho\sigma}\hat{\varphi}_{\mu\nu}\right)\\ \bar{Q}\hat{\Lambda}_{\mu} =& 2K_{\mu} - \sqrt{2}\left(D_{\mu}\hat{\varphi} - F_{\mu\nu}\hat{\Omega}^{\nu}\right) - 2\sqrt{2}\left(D^{\nu}\hat{\varphi}_{\mu\nu} - F^{\nu\rho}\hat{\Omega}_{\rho,\mu\nu}\right)\end{aligned}$$

$$\begin{aligned}\bar{Q}D_{\mu\nu} =& (D_{\mu}\Lambda_{\nu} - D_{\nu}\Lambda_{\mu})^{-} - \frac{i}{2\sqrt{2}}\left[\hat{\varphi}_{\mu\rho},\hat{\bar{\Lambda}}_{\nu}^{\rho}\right] + \frac{i}{2\sqrt{2}}\left[\hat{\varphi}_{\nu\rho},\hat{\bar{\Lambda}}_{\mu}^{\rho}\right]\\ &+\frac{1}{2\sqrt{2}}\hat{\Omega}^{\rho,}{}_{\mu\sigma}D_{\rho}\hat{\bar{\Lambda}}_{\nu}^{\sigma} - \frac{1}{2\sqrt{2}}\hat{\Omega}^{\rho,}{}_{\nu\sigma}D_{\rho}\bar{\hat{\bar{\Lambda}}}_{\mu}^{\sigma} - \frac{i}{2\sqrt{2}}\hat{\Omega}_{\mu\nu,\rho\sigma}^{-}\hat{\bar{\Lambda}}^{\rho\sigma} + \frac{i}{2\sqrt{2}}\hat{\Omega}_{\rho\sigma,}{}^{\rho\sigma}\hat{\bar{\Lambda}}_{\mu\nu}\\ &+\frac{i}{\sqrt{2}}\left[\hat{\varphi},\hat{\bar{\Lambda}}_{\mu\nu}\right] - \frac{i}{\sqrt{2}}\hat{\Omega}^{\rho}D_{\rho}\hat{\bar{\Lambda}}_{\mu\nu} + \frac{i}{\sqrt{2}}\left(\hat{\Omega}_{\mu}{}^{\rho}\hat{\bar{\Lambda}}_{\rho\nu} - \hat{\Omega}_{\nu}{}^{\rho}\hat{\bar{\Lambda}}_{\rho\mu}\right)\\ &+\sqrt{2}i[\varphi,\bar{\Lambda}_{\mu\nu}] - \sqrt{2}\Omega^{\rho}D_{\rho}\bar{\Lambda}_{\mu\nu} + \sqrt{2}\left(\Omega_{\mu}{}^{\rho}\bar{\Lambda}_{\rho\nu} - \Omega_{\nu}{}^{\rho}\bar{\Lambda}_{\rho\mu}\right)\\ &-\frac{i}{\sqrt{2}}\left[\hat{\varphi}_{\mu\nu},\hat{\bar{\Lambda}}\right] + \frac{1}{\sqrt{2}}\hat{\Omega}^{\rho,}{}_{\mu\nu}D_{\rho}\hat{\bar{\Lambda}}\\ \bar{Q}K_{\mu} =& -D_{\mu}\hat{\bar{\Lambda}} - 2D^{\nu}\hat{\bar{\Lambda}}_{\mu\nu} - \sqrt{2}i[\hat{\varphi}_{\mu\nu},\Lambda^{\nu}] + \sqrt{2}\hat{\Omega}^{\rho,}{}_{\mu\nu}D_{\rho}\Lambda^{\nu} + \sqrt{2}\hat{\Omega}^{\rho\nu,}{}_{\mu\nu}\Lambda_{\rho}\\ &-\frac{i}{\sqrt{2}}[\hat{\varphi},\Lambda_{\mu}] - \frac{1}{\sqrt{2}}\hat{\Omega}^{\rho}D_{\rho}\Lambda_{\mu} + \frac{1}{\sqrt{2}}\hat{\Omega}_{\mu}{}^{\nu}\Lambda_{\nu}\\ &+\sqrt{2}i[\varphi,\hat{\Lambda}_{\mu}] - \sqrt{2}\Omega^{\rho}D_{\rho}\hat{\Lambda}_{\mu} + \sqrt{2}\Omega_{\mu}{}^{\nu}\hat{\Lambda}_{\nu}\end{aligned}$$

$$\begin{aligned}H_{\mu\nu} =& D_{\mu\nu} - F_{\mu\nu}^{-}\\ &+\frac{i}{2}\left([\hat{\varphi},\hat{\varphi}_{\mu\nu}] + i\hat{\Omega}^{\rho}D_{\rho}\hat{\varphi}_{\mu\nu} - i\hat{\Omega}^{\rho,}{}_{\mu\nu}D_{\rho}\hat{\varphi} + i\hat{\Omega}^{\rho,}{}_{\mu\nu}\hat{\Omega}^{\sigma}F_{\rho\sigma} - i\hat{\Omega}_{\mu}{}^{\rho}\hat{\varphi}_{\rho\nu} + i\hat{\Omega}_{\nu}{}^{\rho}\hat{\varphi}_{\rho\mu}\right)\\ &-\frac{i}{4}\left([\hat{\varphi}_{\mu\rho},\hat{\varphi}_{\nu}{}^{\rho}] + i\hat{\Omega}^{\rho,}{}_{\mu\sigma}D_{\rho}\hat{\varphi}_{\nu}{}^{\sigma} - i\hat{\Omega}^{\rho,}{}_{\nu\sigma}D_{\rho}\hat{\varphi}_{\mu}{}^{\sigma} + \hat{\Omega}_{\mu\nu,\rho\sigma}^{-}\hat{\varphi}^{\rho\sigma} - \hat{\Omega}_{\rho\sigma}{}^{\rho\sigma}\hat{\varphi}_{\mu\nu}\right),\\ G_{\mu} =& K_{\mu} - \frac{1}{\sqrt{2}}\left(D_{\mu}\hat{\varphi} - F_{\mu\nu}\hat{\Omega}^{\nu}\right) - \sqrt{2}\left(D^{\nu}\hat{\varphi}_{\mu\nu} - F^{\nu\rho}\hat{\Omega}_{\rho,\mu\nu}\right),\end{aligned}$$

$$\bar{Q}\bar{\Lambda}_{\mu\nu} = 2H_{\mu\nu}, \bar{Q}\hat{\Lambda}_{\mu} = 2G_{\mu}$$

$$\begin{aligned}\bar{Q}H_{\mu\nu} &= \sqrt{2}i[\varphi,\bar{\Lambda}_{\mu\nu}] - \sqrt{2}\Omega^{\rho}D_{\rho}\bar{\Lambda}_{\mu\nu} + \sqrt{2}\left(\Omega_{\mu}{}^{\rho}\bar{\Lambda}_{\rho\nu} - \Omega_{\nu}{}^{\rho}\bar{\Lambda}_{\rho\mu}\right)\\ \bar{Q}G_{\mu} &= \sqrt{2}i[\varphi,\hat{\Lambda}_{\mu}] - \sqrt{2}\Omega^{\rho}D_{\rho}\hat{\Lambda}_{\mu} + \sqrt{2}\Omega_{\mu}{}^{\nu}\hat{\Lambda}_{\nu}\end{aligned}$$

$$S_2 = \bar{Q}\Xi_2 + \int d^4x\frac{1}{\kappa g^2}\mathrm{Tr}\left[\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}\right],$$

$$\begin{aligned}\Xi_2 = \int d^4x\frac{1}{\kappa g^2}\mathrm{Tr}[&-\frac{1}{2}F_{\mu\nu}^{-}\bar{\Lambda}^{\mu\nu} - \frac{1}{4}H_{\mu\nu}\bar{\Lambda}^{\mu\nu} - \frac{1}{2\sqrt{2}}\Lambda^{\mu}\left(D_{\mu}\bar{\varphi} - F_{\mu\nu}\bar{\Omega}^{\nu}\right)\\ &+\frac{i}{4}\bar{\Lambda}\left([\varphi,\bar{\varphi}] + i\Omega^{\mu}D_{\mu}\bar{\varphi} - i\bar{\Omega}^{\mu}D_{\mu}\varphi + i\bar{\Omega}^{\mu}\Omega^{\nu}F_{\mu\nu}\right)\\ &-\frac{1}{4}G_{\mu}\hat{\Lambda}^{\mu} - \frac{1}{2\sqrt{2}}\hat{\Lambda}^{\mu}\left(D_{\mu}\hat{\varphi} - F_{\mu\nu}\hat{\Omega}^{\nu}\right) + \frac{1}{\sqrt{2}}\hat{\Lambda}_{\nu}\left(D_{\mu}\hat{\varphi}^{\mu\nu} - F_{\mu\rho}\hat{\Omega}^{\rho,\mu\nu}\right)\\ &+\frac{i}{4}\hat{\bar{\Lambda}}\left([\bar{\varphi},\hat{\varphi}] + i\bar{\Omega}^{\mu}D_{\mu}\hat{\varphi} - i\hat{\Omega}^{\mu}D_{\mu}\bar{\varphi} + i\hat{\Omega}^{\mu}\bar{\Omega}^{\nu}F_{\mu\nu}\right)\\ &-\frac{i}{4}\hat{\bar{\Lambda}}^{\mu\nu}\left([\bar{\varphi},\hat{\varphi}_{\mu\nu}] + i\bar{\Omega}^{\rho}D_{\rho}\hat{\varphi}_{\mu\nu} - i\hat{\Omega}^{\rho,}{}_{\mu\nu}D_{\rho}\bar{\varphi} + i\hat{\Omega}^{\rho,}{}_{\mu\nu}\bar{\Omega}^{\sigma}F_{\rho\sigma} - i\bar{\Omega}_{\mu}{}^{\rho}\hat{\varphi}_{\rho\nu} + i\bar{\Omega}_{\nu}{}^{\rho}\hat{\varphi}_{\rho\mu}\right)\\ &+\frac{i}{4}\bar{\Lambda}^{\mu\nu}\left([\hat{\varphi},\hat{\varphi}_{\mu\nu}] + i\hat{\Omega}^{\rho}D_{\rho}\hat{\varphi}_{\mu\nu} - i\hat{\Omega}^{\rho,}{}_{\mu\nu}D_{\rho}\hat{\varphi} + i\hat{\Omega}^{\rho,}{}_{\mu\nu}\hat{\Omega}^{\sigma}F_{\rho\sigma} - i\hat{\Omega}_{\mu}{}^{\rho}\hat{\varphi}_{\rho\nu} + i\hat{\Omega}_{\nu}{}^{\rho}\hat{\varphi}_{\rho\mu}\right)\\ &-\frac{i}{8}\bar{\Lambda}^{\mu\nu}\left([\hat{\varphi}_{\mu\rho},\hat{\varphi}_{\nu}{}^{\rho}] + i\hat{\Omega}^{\rho,}{}_{\mu\sigma}D_{\rho}\hat{\varphi}_{\nu}{}^{\sigma} - i\hat{\Omega}^{\rho,}{}_{\nu\sigma}D_{\rho}\hat{\varphi}_{\mu}{}^{\sigma} + \hat{\Omega}_{\mu\nu,\rho\sigma}\hat{\varphi}^{\rho\sigma} - \hat{\Omega}_{\rho\sigma}{}^{\rho\sigma}\hat{\varphi}_{\mu\nu}\right)\Big].\end{aligned}$$

$$\begin{aligned}\hat{\bar{Q}}\hat{\bar{\Lambda}}_{\mu\nu} =& 2D_{\mu\nu} - 2F_{\mu\nu}^{-}\\ &-i\left([\hat{\varphi},\hat{\varphi}_{\mu\nu}] + i\hat{\Omega}^{\rho}D_{\rho}\hat{\varphi}_{\mu\nu} - i\hat{\Omega}^{\rho,}{}_{\mu\nu}D_{\rho}\hat{\varphi} + i\hat{\Omega}^{\rho,}{}_{\mu\nu}\hat{\Omega}^{\sigma}F_{\rho\sigma} - i\hat{\Omega}_{\mu}{}^{\rho}\hat{\varphi}_{\rho\nu} + i\hat{\Omega}_{\nu}{}^{\rho}\hat{\varphi}_{\rho\mu}\right)\\ &-\frac{i}{2}\left([\hat{\varphi}_{\mu\rho},\hat{\varphi}_{\nu}{}^{\rho}] + i\hat{\Omega}^{\rho,}{}_{\mu\sigma}D_{\rho}\hat{\varphi}_{\nu}{}^{\sigma} - i\hat{\Omega}^{\rho,}{}_{\nu\sigma}D_{\rho}\hat{\varphi}_{\mu}{}^{\sigma} + \hat{\Omega}_{\mu\nu,\rho\sigma}^{-}\hat{\varphi}^{\rho\sigma} - \hat{\Omega}_{\rho\sigma,}{}^{\rho\sigma}\hat{\varphi}_{\mu\nu}\right)\\ \hat{\bar{Q}}\Lambda_{\mu} =& -2K_{\mu} - \sqrt{2}\left(D_{\mu}\hat{\varphi} - F_{\mu\nu}\hat{\Omega}^{\nu}\right) + 2\sqrt{2}\left(D^{\nu}\hat{\varphi}_{\mu\nu} - F^{\nu\rho}\hat{\Omega}_{\rho,\mu\nu}\right)\end{aligned}$$

$$\hat{\bar{Q}} D_{\mu\nu} = (D_\mu \hat{\bar{\Lambda}}_\nu - D_\nu \hat{\bar{\Lambda}}_\mu)^- + \frac{i}{2\sqrt{2}}[\hat{\varphi}_{\mu\rho}, \bar{\Lambda}_\nu{}^\rho] - \frac{i}{2\sqrt{2}}[\hat{\varphi}_{\nu\rho}, \bar{\Lambda}_\mu{}^\rho]$$
$$- \frac{1}{2\sqrt{2}} \hat{\Omega}^{\rho,}{}_{\mu\sigma} D_\rho \bar{\Lambda}_\nu{}^\sigma + \frac{1}{2\sqrt{2}} \hat{\Omega}^{\rho,}{}_{\nu\sigma} D_\rho \bar{\Lambda}_\mu{}^\sigma + \frac{i}{2\sqrt{2}} \hat{\Omega}^-_{\mu\nu,\rho\sigma} \bar{\Lambda}^{\rho\sigma} - \frac{i}{2\sqrt{2}} \hat{\Omega}_{\rho\sigma,}{}^{\rho\sigma} \bar{\Lambda}_{\mu\nu}$$
$$+ \frac{i}{\sqrt{2}}[\hat{\varphi}, \bar{\Lambda}_{\mu\nu}] - \frac{i}{\sqrt{2}} \hat{\Omega}^\rho D_\rho \bar{\Lambda}_{\mu\nu} + \frac{i}{\sqrt{2}}(\hat{\Omega}_\mu{}^\rho \bar{\Lambda}_{\rho\nu} - \hat{\Omega}_\nu{}^\rho \bar{\Lambda}_{\rho\mu})$$
$$- \sqrt{2} i \left[\bar{\varphi}, \hat{\bar{\Lambda}}_{\mu\nu}\right] + \sqrt{2} \bar{\Omega}^\rho D_\rho \hat{\bar{\Lambda}}_{\mu\nu} - \sqrt{2}\left(\bar{\Omega}_\mu{}^\rho \hat{\bar{\Lambda}}_{\rho\nu} - \bar{\Omega}_\nu{}^\rho \hat{\bar{\Lambda}}_{\rho\mu}\right)$$
$$+ \frac{i}{\sqrt{2}}[\hat{\varphi}_{\mu\nu}, \bar{\Lambda}] - \frac{1}{\sqrt{2}} \hat{\Omega}^{\rho,}{}_{\mu\nu} D_\rho \bar{\Lambda}$$
$$\hat{\bar{Q}} K_\mu = D_\mu \bar{\Lambda} + 2 D^\nu \bar{\Lambda}_{\mu\nu} - \sqrt{2} i [\bar{\varphi}_{\mu\nu}, \hat{\Lambda}^\nu] + \sqrt{2} \hat{\Omega}^{\rho,}{}_{\mu\nu} D_\rho \hat{\Lambda}^\nu + \sqrt{2} \hat{\Omega}^{\rho\nu}{}_{\mu\nu} \hat{\Lambda}_\rho$$
$$+ \frac{i}{\sqrt{2}}[\hat{\varphi}, \hat{\Lambda}_\mu] - \frac{1}{\sqrt{2}} \hat{\Omega}^\nu D_\nu \hat{\Lambda}_\mu + \frac{1}{\sqrt{2}} \hat{\Omega}_\mu{}^\nu \hat{\Lambda}_\nu$$
$$+ \sqrt{2} i [\bar{\varphi}, \Lambda_\mu] - \sqrt{2} \bar{\Omega}^\nu D_\nu \Lambda_\mu + \sqrt{2} \bar{\Omega}_\mu{}^\nu \Lambda_\nu$$

$$\hat{H}_{\mu\nu} = D_{\mu\nu} - F^-_{\mu\nu}$$
$$- \frac{i}{2}\left([\hat{\varphi}, \hat{\varphi}_{\mu\nu}] + i \hat{\Omega}^\rho D_\rho \hat{\varphi}_{\mu\nu} - i \hat{\Omega}^{\rho,}{}_{\mu\nu} D_\rho \hat{\varphi} + i \hat{\Omega}^{\rho,}{}_{\mu\nu} \hat{\Omega}^\sigma F_{\rho\sigma} - i \hat{\Omega}_\mu{}^\rho \hat{\varphi}_{\rho\nu} + i \hat{\Omega}_\nu{}^\rho \hat{\varphi}_{\rho\mu}\right)$$
$$- \frac{i}{4}\left([\hat{\varphi}_{\mu\rho}, \hat{\varphi}_\nu{}^\rho] + i \hat{\Omega}^{\rho,}{}_{\mu\sigma} D_\rho \hat{\varphi}_\nu{}^\sigma - i \hat{\Omega}^{\rho,}{}_{\nu\sigma} D_\rho \hat{\varphi}_\mu{}^\sigma + \hat{\Omega}^-_{\mu\nu,\rho\sigma} \hat{\varphi}^{\rho\sigma} - \hat{\Omega}_{\rho\sigma}{}^{\rho\sigma} \hat{\varphi}_{\mu\nu}\right)$$
$$\hat{G}_\mu = K_\mu + \frac{1}{\sqrt{2}}(D_\mu \hat{\varphi} - F_{\mu\nu} \hat{\Omega}^\nu) - \sqrt{2}(D^\nu \hat{\varphi}_{\mu\nu} - F^{\nu\rho} \hat{\Omega}_{\rho,\mu\nu}),$$

$$\hat{\bar{Q}} \hat{\bar{\Lambda}}_{\mu\nu} = 2 \hat{H}_{\mu\nu}, \hat{\bar{Q}} \Lambda_\mu = -2 \hat{G}_\mu$$

$$\hat{\bar{Q}} \hat{H}_{\mu\nu} = -\sqrt{2} i \left[\bar{\varphi}, \hat{\bar{\Lambda}}_{\mu\nu}\right] + \sqrt{2} \bar{\Omega}^\rho D_\rho \hat{\bar{\Lambda}}_{\mu\nu} - \sqrt{2}\left(\bar{\Omega}_\mu{}^\rho \hat{\bar{\Lambda}}_{\rho\nu} - \bar{\Omega}_\nu{}^\rho \hat{\bar{\Lambda}}_{\rho\mu}\right)$$
$$\hat{\bar{Q}} \hat{G}_\mu = \sqrt{2} i [\bar{\varphi}, \Lambda_\mu] - \sqrt{2} \bar{\Omega}^\nu D_\nu \Lambda_\mu + \sqrt{2} \bar{\Omega}_\mu{}^\nu \Lambda_\nu$$

$$S_2 = \hat{\bar{Q}} \Xi'_2 + \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}\left[\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}\right],$$
$$\Xi'_2 = \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}[-\frac{1}{2} F^-_{\mu\nu} \hat{\bar{\Lambda}}^{\mu\nu} - \frac{1}{4} \hat{H}_{\mu\nu} \hat{\bar{\Lambda}}^{\mu\nu} + \frac{1}{2\sqrt{2}} \hat{\Lambda}^\mu (D_\mu \varphi - F_{\mu\nu} \Omega^\nu)$$
$$- \frac{i}{4} \hat{\bar{\Lambda}}([\varphi, \bar{\varphi}] + i \Omega^\mu D_\mu \bar{\varphi} - i \bar{\Omega}^\mu D_\mu \varphi + i \bar{\Omega}^\mu \Omega^\nu F_{\mu\nu})$$
$$- \frac{1}{4} \hat{G}_\mu \Lambda^\mu + \frac{1}{2\sqrt{2}} \Lambda^\mu (D_\mu \hat{\varphi} - F_{\mu\nu} \hat{\Omega}^\nu) - \frac{1}{\sqrt{2}} \Lambda_\nu (D_\mu \hat{\varphi}^{\mu\nu} - F_{\mu\rho} \hat{\Omega}^{\rho,\mu\nu})$$
$$+ \frac{i}{4} \bar{\Lambda}([\varphi, \hat{\varphi}] + i \Omega^\mu D_\mu \hat{\varphi} - i \hat{\Omega}^\mu D_\mu \varphi + i \hat{\Omega}^\mu \Omega^\nu F_{\mu\nu})$$
$$- \frac{i}{4} \bar{\Lambda}^{\mu\nu}([\varphi, \hat{\varphi}_{\mu\nu}] + i \Omega^\rho D_\rho \hat{\varphi}_{\mu\nu} - i \hat{\Omega}^\rho{}_{\mu\nu} D_\rho \varphi + i \hat{\Omega}^\rho{}_{\mu\nu} \Omega^\sigma F_{\rho\sigma} - i \Omega_\mu{}^\rho \hat{\varphi}_{\rho\nu} + i \Omega_\nu{}^\rho \hat{\varphi}_{\rho\mu})$$
$$- \frac{i}{4} \hat{\bar{\Lambda}}^{\mu\nu}([\hat{\varphi}, \hat{\varphi}_{\mu\nu}] + i \hat{\Omega}^\rho D_\rho \hat{\varphi}_{\mu\nu} - i \hat{\Omega}^{\rho,}{}_{\mu\nu} D_\rho \hat{\varphi} + i \hat{\Omega}^{\rho,}{}_{\mu\nu} \hat{\Omega}^\sigma F_{\rho\sigma} - i \hat{\Omega}_\mu{}^\rho \hat{\varphi}_{\rho\nu} + i \hat{\Omega}_\nu{}^\rho \hat{\varphi}_{\rho\mu})$$
$$+ \frac{i}{8} \hat{\bar{\Lambda}}^{\mu\nu}([\hat{\varphi}_{\mu\rho}, \hat{\varphi}_\nu{}^\rho] + i \hat{\Omega}^{\rho,}{}_{\mu\sigma} D_\rho \hat{\varphi}_\nu{}^\sigma - i \hat{\Omega}^{\rho,}{}_{\nu\sigma} D_\rho \hat{\varphi}_\mu{}^\sigma + \hat{\Omega}_{\mu\nu,\rho\sigma} \hat{\varphi}^{\rho\sigma} - \hat{\Omega}_{\rho\sigma}{}^{\rho\sigma} \hat{\varphi}_{\mu\nu})\Big].$$

$$\bar{Q}^2 \Psi = 2\sqrt{2}(\delta_{\text{gauge}}(\varphi) + \delta_{\text{Lorentz}}(\Omega))\Psi$$
$$\hat{\bar{Q}}^2 \Psi = -2\sqrt{2}(\delta_{\text{gauge}}(\bar{\varphi}) + \delta_{\text{Lorentz}}(\bar{\Omega}))\Psi$$
$$\{\bar{Q}, \hat{\bar{Q}}\}\Psi = 2\sqrt{2}(\delta_{\text{gauge}}(\hat{\varphi}) + \delta_{\text{Lorentz}}(\hat{\Omega}))\Psi.$$

$$S_2 = \bar{Q} \hat{\bar{Q}} \mathcal{F} + \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}\left[\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}\right]$$

$$\mathcal{F} = \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}\Big[-\frac{1}{2\sqrt{2}} \hat{\varphi}^{\mu\nu} F^-_{\mu\nu} + \frac{1}{8} \bar{\Lambda}^{\mu\nu} \hat{\bar{\Lambda}}_{\mu\nu} + \frac{1}{8} \Lambda^\mu \Lambda_\mu - \frac{1}{8} \bar{\Lambda} \hat{\bar{\Lambda}} + \frac{i}{24\sqrt{2}} \hat{\varphi}^{\mu\nu} [\hat{\varphi}_\mu{}^\lambda, \hat{\varphi}_{\lambda\nu}]$$
$$+ \frac{1}{16\sqrt{2}} \hat{\varphi}^{\mu\nu} (\hat{\Omega}^{\rho,}{}_{\mu\sigma} D_\rho \hat{\varphi}_\nu{}^\sigma - \hat{\Omega}^{\rho,}{}_{\nu\sigma} D_\rho \hat{\varphi}_\mu{}^\sigma - i \hat{\Omega}_{\mu\nu,\rho\sigma} \hat{\varphi}^{\rho\sigma} + i \hat{\Omega}_{\rho\sigma,}{}^{\rho\sigma} \hat{\varphi}_{\mu\nu})$$
$$+ \frac{3}{2\sqrt{2}} \hat{\Omega}^{[\rho,\mu\nu]} \left(A_{[\mu} F_{\nu\rho]} - \frac{i}{3} A_{[\mu} A_\nu A_{\rho]}\right)\Big].$$

$$\delta_{\text{gauge}}(\alpha)\mathcal{F} = \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}\left[-\frac{1}{2\sqrt{2}}(\hat{\Omega}_\rho^{\mu,\nu\rho} - \hat{\Omega}_\rho^{\nu,\mu\rho})F_{\mu\nu}\alpha\right]$$

$$S_3 = S + \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}\left[-\frac{1}{2}(D_{\mu\nu})^2 - \frac{1}{2}(K_{\mu\nu})^2 - \frac{1}{2}K^2\right]$$

$$\bar{Q}\Lambda = 2K + \sqrt{2}D_\mu\varphi^\mu$$
$$\bar{Q}\Lambda_{\mu\nu} = 2K_{\mu\nu} + \sqrt{2}(D_\mu\varphi_\nu - D_\nu\varphi_\mu)^+$$
$$\bar{Q}\bar{\Lambda}_{\mu\nu} = 2D_{\mu\nu} - 2F_{\mu\nu}^- + i[\varphi_\mu, \varphi_\nu]^-$$

$$\bar{Q}K = D_\mu\bar{\Lambda}^\mu - \frac{i}{\sqrt{2}}[\Lambda_\mu, \varphi^\mu] + \sqrt{2}i([\varphi, \Lambda] + i\Omega^\mu D_\mu\Lambda),$$

$$\bar{Q}K_{\mu\nu} = (D_\mu\bar{\Lambda}_\nu - D_\nu\bar{\Lambda}_\mu)^+ + \frac{i}{\sqrt{2}}([\varphi_\mu, \Lambda_\nu] - [\varphi_\nu, \Lambda_\mu])^+$$
$$+\sqrt{2}i[\varphi, \Lambda_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda\Lambda_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda\Lambda_{\lambda\nu} - \Omega_\nu{}^\lambda\Lambda_{\lambda\mu}),$$

$$\bar{Q}D_{\mu\nu} = (D_\mu\Lambda_\nu - D_\nu\Lambda_\mu)^- - \frac{i}{\sqrt{2}}([\varphi_\mu, \bar{\Lambda}_\nu] - [\varphi_\nu, \bar{\Lambda}_\mu])^-$$
$$+\sqrt{2}i[\varphi, \bar{\Lambda}_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda\bar{\Lambda}_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda\bar{\Lambda}_{\lambda\nu} - \Omega_\nu{}^\lambda\bar{\Lambda}_{\lambda\mu})$$

$$G = K + \frac{1}{\sqrt{2}}D_\mu\varphi^\mu,$$
$$G_{\mu\nu} = K_{\mu\nu} + \frac{1}{\sqrt{2}}(D_\mu\varphi_\nu - D_\nu\varphi_\mu)^+,$$
$$H_{\mu\nu} = D_{\mu\nu} - F_{\mu\nu}^- + \frac{i}{2}[\varphi_\mu, \varphi_\nu]^-,$$

$$\bar{Q}\Lambda = 2G, \bar{Q}\Lambda_{\mu\nu} = 2G_{\mu\nu}, \bar{Q}\bar{\Lambda}_{\mu\nu} = 2H_{\mu\nu}$$

$$\bar{Q}G = \sqrt{2}i([\varphi, \Lambda] + i\Omega^\mu D_\mu\Lambda)$$
$$\bar{Q}G_{\mu\nu} = \sqrt{2}i[\varphi, \Lambda_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda\Lambda_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda\Lambda_{\lambda\nu} - \Omega_\nu{}^\lambda\Lambda_{\lambda\mu})$$
$$\bar{Q}H_{\mu\nu} = \sqrt{2}i[\varphi, \bar{\Lambda}_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda\bar{\Lambda}_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda\bar{\Lambda}_{\lambda\nu} - \Omega_\nu{}^\lambda\bar{\Lambda}_{\lambda\mu})$$

$$S_3 = \bar{Q}\Xi_3 + \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}\left[\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

$$\Xi_3 = \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}\Big[-\frac{1}{2}F_{\mu\nu}^-\bar{\Lambda}^{\mu\nu} + \frac{i}{4}\bar{\Lambda}^{\mu\nu}[\varphi_\mu, \varphi_\nu]^- - \frac{1}{4}H_{\mu\nu}\bar{\Lambda}^{\mu\nu}$$
$$-\frac{1}{4}\Lambda_{\mu\nu}G^{\mu\nu} + \frac{1}{2\sqrt{2}}\Lambda^{\mu\nu}(D_\mu\varphi_\nu - D_\nu\varphi_\mu)^+$$
$$-\frac{1}{4}\Lambda G + \frac{i}{4}\bar{\Lambda}([\varphi, \bar{\varphi}] + i\Omega^\mu D_\mu\bar{\varphi} - i\bar{\Omega}^\mu D_\mu\varphi + i\bar{\Omega}^\mu\Omega^\nu F_{\mu\nu})$$
$$+\frac{1}{2\sqrt{2}}\Lambda D_\mu\varphi^\mu - \frac{1}{2\sqrt{2}}\Lambda^\mu(D_\mu\bar{\varphi} - F_{\mu\nu}\bar{\Omega}^\nu)$$
$$-\frac{i}{4}\bar{\Lambda}^\mu([\bar{\varphi}, \varphi_\mu] + i\bar{\Omega}^\nu D_\nu\varphi_\mu - i\bar{\Omega}_\mu{}^\nu\varphi_\nu)\Big]$$

$$Q\bar{\Lambda} = 2K + \sqrt{2}D_\mu\varphi^\mu$$
$$Q\Lambda_{\mu\nu} = 2K_{\mu\nu} - 2F_{\mu\nu}^+ + i[\varphi_\mu, \varphi_\nu]^+$$
$$Q\bar{\Lambda}_{\mu\nu} = 2D_{\mu\nu} - \sqrt{2}(D_\mu\varphi_\nu - D_\nu\varphi_\mu)^-$$

$$QK = -D_\mu \Lambda^\mu - \frac{i}{\sqrt{2}}[\bar{\Lambda}_\mu, \varphi^\mu] + \sqrt{2}i([\varphi, \bar{\Lambda}] + i\Omega^\mu D_\mu \bar{\Lambda}),$$

$$QK_{\mu\nu} = (D_\mu \bar{\Lambda}_\nu - D_\nu \bar{\Lambda}_\mu)^+ + \frac{i}{\sqrt{2}}([\varphi_\mu, \Lambda_\nu] - [\varphi_\nu, \Lambda_\mu])^+$$
$$+\sqrt{2}i[\varphi, \Lambda_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda \Lambda_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda \Lambda_{\lambda\nu} - \Omega_\nu{}^\lambda \Lambda_{\lambda\mu})$$

$$QD_{\mu\nu} = (D_\mu \Lambda_\nu - D_\nu \Lambda_\mu)^- - \frac{i}{\sqrt{2}}([\varphi_\mu, \bar{\Lambda}_\nu] - [\varphi_\nu, \bar{\Lambda}_\mu])^-$$
$$+\sqrt{2}i[\varphi, \bar{\Lambda}_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda \bar{\Lambda}_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda \bar{\Lambda}_{\lambda\nu} - \Omega_\nu{}^\lambda \bar{\Lambda}_{\lambda\mu}).$$

$$G'_{\mu\nu} = K_{\mu\nu} - F^+_{\mu\nu} + \frac{i}{2}[\varphi_\mu, \varphi_\nu]^+$$
$$H'_{\mu\nu} = D_{\mu\nu} - \frac{1}{\sqrt{2}}(D_\mu \varphi_\nu - D_\nu \varphi_\mu)^-$$

$$Q\bar{\Lambda} = 2G, Q\Lambda_{\mu\nu} = 2G'_{\mu\nu}, Q\bar{\Lambda}_{\mu\nu} = 2H'_{\mu\nu}$$

$$QG = \sqrt{2}i([\varphi, \bar{\Lambda}] + i\Omega^\mu D_\mu \bar{\Lambda})$$
$$QG'_{\mu\nu} = \sqrt{2}i[\varphi, \Lambda_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda \Lambda_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda \Lambda_{\lambda\nu} - \Omega_\nu{}^\lambda \Lambda_{\lambda\mu})$$
$$QH'_{\mu\nu} = \sqrt{2}i[\varphi, \bar{\Lambda}_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda \bar{\Lambda}_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda \bar{\Lambda}_{\lambda\nu} - \Omega_\nu{}^\lambda \bar{\Lambda}_{\lambda\mu})$$

$$S_3 = Q\Xi_3 - \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}\left[\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}\right]$$

$$\Xi_3 = \int d^4x \frac{1}{\kappa g^2} \mathrm{Tr}\Big[-\frac{1}{2} F^+_{\mu\nu} \Lambda^{\mu\nu} + \frac{i}{4} \Lambda^{\mu\nu} [\varphi_\mu, \varphi_\nu]^+ - \frac{1}{4} G'_{\mu\nu} \Lambda^{\mu\nu}$$
$$-\frac{1}{4} \bar{\Lambda}^{\mu\nu} H'_{\mu\nu} - \frac{1}{2\sqrt{2}} \bar{\Lambda}^{\mu\nu} (D_\mu \varphi_\nu - D_\nu \varphi_\mu)^-$$
$$-\frac{1}{4} \bar{\Lambda} G - \frac{i}{4} \Lambda([\varphi, \bar{\varphi}] + i\Omega^\mu D_\mu \bar{\varphi} - i\bar{\Omega}^\mu D_\mu \varphi + i\bar{\Omega}^\mu \Omega^\nu F_{\mu\nu})$$
$$+\frac{1}{2\sqrt{2}} \bar{\Lambda} D_\mu \varphi^\mu - \frac{1}{2\sqrt{2}} \bar{\Lambda}^\mu (D_\mu \bar{\varphi} - F_{\mu\nu} \bar{\Omega}^\nu)$$
$$+\frac{i}{4} \bar{\Lambda}^\mu ([\bar{\varphi}, \varphi_\mu] + i\bar{\Omega}^\nu D_\nu \varphi_\mu - i\bar{\Omega}_\mu{}^\nu \varphi_\nu)\Big]$$

$$\bar{Q}^2 \Psi = Q^2 \Psi = 2\sqrt{2}(\delta_{\text{gauge}}(\varphi) + \delta_{\text{Lorentz}}(\Omega))\Psi$$
$$\{Q, \bar{Q}\}\Psi = 0$$

$$\mathcal{Q} = uQ + v\bar{Q}, u, v \in \mathbb{C}$$

$$\mathcal{Q}^2 \Psi = 2\sqrt{2}(u^2 + v^2)(\delta_{\text{gauge}}(\varphi) + \delta_{\text{Lorentz}}(\Omega))\Psi.$$

$$\mathcal{Q}\Lambda_{\mu\nu} = 2U_{\mu\nu}, \mathcal{Q}\bar{\Lambda}_{\mu\nu} = 2V_{\mu\nu},$$

$$U_{\mu\nu} \equiv -uF^+_{\mu\nu} + \frac{i}{2} u[\varphi_\mu, \varphi_\nu]^+ + \frac{1}{\sqrt{2}} v(D_\mu \varphi_\nu - D_\nu \varphi_\mu)^+$$
$$V_{\mu\nu} \equiv -vF^-_{\mu\nu} + \frac{i}{2} v[\varphi_\mu, \varphi_\nu]^- - \frac{1}{\sqrt{2}} u(D_\mu \varphi_\nu - D_\nu \varphi_\mu)^-$$

$$\mathcal{Q}\Lambda_{\mu\nu} = 2\sqrt{u^2 + v^2} K_{\mu\nu} + 2U_{\mu\nu},$$
$$\mathcal{Q}\bar{\Lambda}_{\mu\nu} = 2\sqrt{u^2 + v^2} D_{\mu\nu} + 2V_{\mu\nu}.$$

$$\mathcal{G}_{\mu\nu} = \sqrt{u^2 + v^2} K_{\mu\nu} + U_{\mu\nu}$$
$$\mathcal{H}_{\mu\nu} = \sqrt{u^2 + v^2} D_{\mu\nu} + V_{\mu\nu}$$

$$\mathcal{Q}\Lambda_{\mu\nu} = 2\mathcal{G}_{\mu\nu}, \mathcal{Q}\bar{\Lambda}_{\mu\nu} = 2\mathcal{H}_{\mu\nu}$$

$$\mathcal{Q}\mathcal{G}_{\mu\nu} = (u^2+v^2)\left(\sqrt{2}i[\varphi,\Lambda_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda \Lambda_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda \Lambda_{\lambda\nu} - \Omega_\nu{}^\lambda \Lambda_{\lambda\mu})\right),$$
$$\mathcal{Q}\mathcal{H}_{\mu\nu} = (u^2+v^2)\left(\sqrt{2}i[\varphi,\bar{\Lambda}_{\mu\nu}] - \sqrt{2}\Omega^\lambda D_\lambda \bar{\Lambda}_{\mu\nu} + \sqrt{2}(\Omega_\mu{}^\lambda \bar{\Lambda}_{\lambda\nu} - \Omega_\nu{}^\lambda \bar{\Lambda}_{\lambda\mu})\right).$$

$$\hat{\Xi}^{(1)} = \frac{1}{u^2+v^2}\int d^4x \frac{1}{\kappa g^2}\mathrm{Tr}\left[\left(\frac{1}{2}U_{\mu\nu}\Lambda^{\mu\nu} - \frac{1}{2}\mathcal{G}_{\mu\nu}\Lambda^{\mu\nu}\right) + \left(\frac{1}{2}V_{\mu\nu}\bar{\Lambda}^{\mu\nu} - \frac{1}{2}\mathcal{H}_{\mu\nu}\bar{\Lambda}^{\mu\nu}\right)\right].$$

$$\hat{\Xi}^{(2)} = a\bar{\Xi}_3' + b\Xi_3'.$$

$$\Xi_3' = QV, \bar{\Xi}_3' = -\bar{Q}V$$

$$V = \int d^4x \frac{1}{\kappa g^2}\mathrm{Tr}\left[\frac{1}{8}\Lambda\bar{\Lambda} - \frac{1}{4}\varphi^\mu\left(D_\mu\bar{\varphi} - F_{\mu\nu}\bar{\Omega}^\nu\right)\right]$$

$$\hat{\Xi}^{(2)} = \frac{1}{u^2+v^2}(-u\bar{Q} + vQ)V,$$

$$S_3 = \mathcal{Q}\left(\hat{\Xi}^{(1)} + \hat{\Xi}^{(2)}\right) + \int d^4x \frac{1}{\kappa g^2}\mathrm{Tr}\left[\frac{u^2-v^2}{4(u^2+v^2)}F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

$$\mathcal{V}_\mu = A_\mu + \frac{i}{\sqrt{2}}\varphi_\mu, \qquad \overline{\mathcal{V}}_\mu = A_\mu - \frac{i}{\sqrt{2}}\varphi_\mu$$
$$\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{V}_\nu - \partial_\nu\mathcal{V}_\mu + i[\mathcal{V}_\mu,\mathcal{V}_\nu], \qquad \overline{\mathcal{F}}_{\mu\nu} = \partial_\mu\overline{\mathcal{V}}_\nu - \partial_\nu\overline{\mathcal{V}}_\mu + i[\overline{\mathcal{V}}_\mu,\overline{\mathcal{V}}_\nu]$$
$$\psi_\mu = \Lambda_\mu - i\bar{\Lambda}_\mu, \qquad \bar{\psi}_\mu = \Lambda_\mu + i\bar{\Lambda}_\mu$$
$$\eta = \Lambda - i\bar{\Lambda}, \qquad \bar{\eta} = \Lambda + i\bar{\Lambda}$$
$$\chi_{\mu\nu} = \Lambda_{\mu\nu} - i\bar{\Lambda}_{\mu\nu}, \qquad G^+ = G + [\varphi,\bar{\varphi}]$$
$$\mathcal{I}_{\mu\nu} = \mathcal{G}_{\mu\nu} - i\mathcal{H}_{\mu\nu}, \qquad \phi = \varphi - \frac{i}{\sqrt{2}}\Omega^\mu\varphi_\mu$$

$$\mathcal{Q}\mathcal{V}_\mu = 2i\psi_\mu, \qquad \mathcal{Q}\psi_\mu = 0$$
$$\mathcal{Q}\overline{\mathcal{V}}_\mu = 0,$$
$$\mathcal{Q}\bar{\psi}_\mu = -4\sqrt{2}i(\overline{\mathcal{D}}_\mu\phi - \overline{\mathcal{F}}_{\mu\nu}\Omega^\nu), \qquad \mathcal{Q}\phi = 0$$
$$\mathcal{Q}\bar{\varphi} = \sqrt{2}\eta, \qquad \mathcal{Q}\eta = 0$$
$$\mathcal{Q}\bar{\eta} = 4iG^+, \qquad \mathcal{Q}G^+ = 0$$
$$\mathcal{Q}\chi_{\mu\nu} = 2\mathcal{I}_{\mu\nu}, \qquad \mathcal{Q}\mathcal{I}_{\mu\nu} = 0$$

$$\mathcal{D}_\mu * = \partial_\mu * + i[\mathcal{V}_\mu, *], \overline{\mathcal{D}}_\mu * = \partial_\mu * + i[\overline{\mathcal{V}}_\mu, *]$$

$$\mathcal{Q}\chi_{\mu\nu} = -2\overline{\mathcal{F}}_{\mu\nu}.$$

$$\hat{\Xi}^{(1)} = \int d^4x \frac{1}{\kappa g^2}\mathrm{Tr}\left[-\frac{1}{8}\chi^{\mu\nu}\mathcal{F}_{\mu\nu}\right]$$

$$S_3 = \mathcal{Q}\left(\hat{\Xi}^{(1)} + \hat{\Xi}^{(2)}\right) + S_3'$$
$$S_3' = \int d^4x \frac{1}{\kappa g^2}\mathrm{Tr}\left[-\frac{i}{4}\tilde{\chi}^{\mu\nu}(\overline{\mathcal{D}}_\mu\bar{\psi}_\nu - \overline{\mathcal{D}}_\nu\bar{\psi}_\mu) + \frac{i}{2\sqrt{2}}\tilde{\chi}^{\mu\nu}[\phi,\chi_{\mu\nu}] - \frac{1}{2\sqrt{2}}\Omega^\lambda\tilde{\chi}^{\mu\nu}\overline{\mathcal{D}}_\lambda\chi_{\mu\nu} + \frac{1}{2\sqrt{2}}\tilde{\chi}^{\mu\nu}(\Omega_\mu{}^\lambda\chi_{\lambda\nu} - \Omega_\nu{}^\lambda\chi_{\lambda\mu})\right].$$

$$\sigma^\mu = (i\tau_1, i\tau_2, i\tau_3, \mathbf{1}_2), \bar{\sigma}^\mu = (-i\tau_1, -i\tau_2, -i\tau_3, \mathbf{1}_2)$$

$$\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu), \bar{\sigma}^{\mu\nu} = \frac{1}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)$$

$$\Sigma_1 = \begin{pmatrix} i\tau^2 & 0 \\ 0 & i\tau^2 \end{pmatrix}, \Sigma_2 = \begin{pmatrix} \tau^2 & 0 \\ 0 & -\tau^2 \end{pmatrix}, \Sigma_3 = \begin{pmatrix} 0 & -\tau^3 \\ \tau^3 & 0 \end{pmatrix}$$

$$\Sigma_4=\begin{pmatrix}0 & i\mathbf{1}_2\\ -i\mathbf{1}_2 & 0\end{pmatrix},\quad \Sigma_5=\begin{pmatrix}0 & -\tau^1\\ \tau^1 & 0\end{pmatrix},\quad \Sigma_6=\begin{pmatrix}0 & \tau^2\\ \tau^2 & 0\end{pmatrix}$$
$$\bar{\Sigma}_1=\begin{pmatrix}-i\tau^2 & 0\\ 0 & -i\tau^2\end{pmatrix},\quad \bar{\Sigma}_2=\begin{pmatrix}\tau^2 & 0\\ 0 & -\tau^2\end{pmatrix},\quad \bar{\Sigma}_3=\begin{pmatrix}0 & \tau^3\\ -\tau^3 & 0\end{pmatrix}$$
$$\bar{\Sigma}_4=\begin{pmatrix}0 & i\mathbf{1}_2\\ -i\mathbf{1}_2 & 0\end{pmatrix},\quad \bar{\Sigma}_5=\begin{pmatrix}0 & \tau^1\\ -\tau^1 & 0\end{pmatrix},\quad \bar{\Sigma}_6=\begin{pmatrix}0 & \tau^2\\ \tau^2 & 0\end{pmatrix}$$

$$\mathrm{S_{inv}}[A_\mu^a,\lambda^a,D^a]=\int \mathrm{d}x\left[-\frac{1}{4}F_{\mu\nu}^aF^{a\mu\nu}-\frac{i}{2}\bar{\lambda}^a\gamma^\mu(\mathrm{D}_\mu\lambda)^a+\frac{1}{2}D^aD^a\right]$$

$$F_{\mu\nu}^a:=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+gf^{abc}A_\mu^bA_\nu^c$$
$$(\mathrm{D}_\mu\lambda)^a:=\partial_\mu\lambda^a+gf^{abc}A_\mu^b\lambda^c$$

$$\delta_\alpha A_\mu^a=(i\bar{\lambda}^a\gamma_\mu)_\alpha,\delta_\alpha\lambda_\beta^a=-\frac{1}{2}(\gamma^{\mu\nu})_{\beta\alpha}F_{\mu\nu}^a+i(\gamma^5)_{\beta\alpha}D^a,\delta_\alpha D^a=(\mathrm{D}^\mu\bar{\lambda}^a\gamma^5\gamma_\mu)_\alpha$$

$$\mathrm{S_{inv}}=\delta_\alpha\Delta_\alpha$$

$$\Delta_\alpha:=\int \mathrm{d}x\left[-\frac{i}{16}(\gamma^{\mu\nu}\lambda^a)_\alpha F_{\mu\nu}^a+\frac{1}{8}(\gamma^5\lambda^a)_\alpha D^a\right]$$

$$\mathrm{S_{gf}}[A_\mu^a,C^a,\bar{C}^a]=\int \mathrm{d}x\left[\frac{1}{2\xi}\mathcal{G}^a[A]\mathcal{G}^a[A]+\bar{C}^a\frac{\delta\mathcal{G}^a[A]}{\delta A_\mu^b}(\mathrm{D}_\mu C)^b\right]$$

$$s(A_\mu^a)=(\mathrm{D}_\mu C)^a,\qquad s(F_{\mu\nu}^a)=f^{abc}F_{\mu\nu}^bC^c,\quad s(\lambda^a)=f^{abc}\lambda^bC^c$$
$$s(C^a)=-\frac{g}{2}f^{abc}C^bC^c,\quad s(\bar{C}^a)=-\frac{1}{\xi}\mathcal{G}^a[A],\quad s(D^a)=f^{abc}D^bC^c$$

$$\mathcal{G}^a[A]=g\mathcal{G}^a[g^{-1}A]$$

$$\mathcal{G}^a[A](x)\equiv\mathcal{G}^\mu A_\mu^a(x)$$

$$\tilde{A}_\mu^a=gA_\mu^a,\tilde{\lambda}^a=g\lambda^a,\tilde{D}^a=gD^a,\tilde{C}^a=gC^a,\bar{\tilde{C}}^a=g\bar{C}^a$$

$$\tilde{\mathrm{S}}_{\mathrm{inv}}[\tilde{A}_\mu^a,\tilde{\lambda}^a,\tilde{D}^a]=\frac{1}{g^2}\int \mathrm{d}x\left[-\frac{1}{4}\tilde{F}_{\mu\nu}^a\tilde{F}^{a\mu\nu}-\frac{i}{2}\bar{\tilde{\lambda}}^a\gamma^\mu(\mathrm{D}_\mu\tilde{\lambda})^a+\frac{1}{2}\tilde{D}^a\tilde{D}^a\right]$$

$$\tilde{F}_{\mu\nu}^a\equiv\partial_\mu\tilde{A}_\nu^a-\partial_\nu\tilde{A}_\mu^a+f^{abc}\tilde{A}_\mu^b\tilde{A}_\nu^c$$
$$\mathrm{D}_\mu\tilde{\lambda}^a\equiv\partial_\mu\tilde{\lambda}^a+f^{abc}\tilde{A}_\mu^b\tilde{\lambda}^c$$

$$\langle\langle X[A]\rangle\rangle_g=\int \mathcal{D}A\mathcal{D}\lambda\mathcal{D}C\mathcal{D}\bar{C}X[A]e^{-iS[g,A,\lambda,C,\bar{C}]}$$

$$\left\langle\left\langle\tilde{A}_{\mu_1}^{a_1}(x_1)\dots\tilde{A}_{\mu_n}^{a_n}(x_n)\right\rangle\right\rangle_g=g^n\left\langle\left\langle A_{\mu_1}^{a_1}(x_1)\dots A_{\mu_n}^{a_n}(x_n)\right\rangle\right\rangle_g.$$

$$\langle X[A]\rangle_g=\int \mathcal{D}_g[A]X[A]\,(=\langle\langle X[A]\rangle\rangle_g)$$

$$\langle X[A]\rangle_g=\left\langle X[\mathcal{T}_g^{-1}[A]]\right\rangle_0=\int \mathcal{D}_0[A]X[\mathcal{T}_g^{-1}[A]]$$

$$\mathrm{S_{YM}}[A,g]=\mathrm{S_{YM}}[\mathcal{T}_g[A],0]$$

$$\det\left(\frac{\delta\mathcal{T}_g[A]}{\delta A}\right)=\Delta_{\mathrm{MSS}}[A]\Delta_{\mathrm{FP}}[A]$$

$$(\mathcal{T}_g^{-1}A)_\mu^a(x)=\sum_{n=0}^{\infty}\frac{g^n}{n!}\left((\mathcal{R}_g^nA)_\mu^a(x)\Big|_{g=0}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}g}\langle X\rangle_g = \langle \mathcal{R}_g(X)\rangle_g$$

$$\mathcal{R}_g(XY) = \mathcal{R}_g(X)Y + X\mathcal{R}_g(Y)$$

$$\mathcal{R}_g\left(\int \mathrm{d}x F^a_{\mu\nu}F^{a\mu\nu}\right) = 0$$

$$\mathcal{R}_g(\mathcal{G}^a[A]) = 0.$$

$$\mathcal{R}_g = \mathcal{R}_{\text{inv}} + \mathcal{R}_{\text{gf}}$$

$$\mathcal{R}_{\text{inv}} := \frac{\mathrm{d}}{\mathrm{d}g} - \frac{1}{2r}\int \mathrm{d}x\,\mathrm{d}y \mathrm{Tr}\big(\gamma_\mu S^{ab}(x,y;A)\gamma^{\rho\lambda}\big) f^{bcd} A^c_\rho(y) A^d_\lambda(y) \frac{\delta}{\delta A^a_\mu(x)}$$

$$\mathcal{R}_{\text{gf}} := -\frac{1}{2r}\int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \big(D_\mu G\big)^{ae}(x,z;A)\mathrm{Tr}\big(\gamma^\nu \partial_\nu S^{eb}(z,y;A)\gamma^{\rho\lambda}\big) f^{bcd} A^c_\rho(y) A^d_\lambda(y)\frac{\delta}{\delta A^a_\mu(x)}$$

$$\gamma^\mu \big(\mathrm{D}_\mu S\big)^{ab}(x,y;A) = \delta^{ab}\delta(x-y)$$

$$\frac{\delta \mathcal{G}^a}{\delta A^c_\mu}\big(\mathrm{D}_\mu G\big)^{cb}(x,y;A) \equiv \partial^\mu \big(\mathrm{D}_\mu G\big)^{ab}(x,y;A) = \delta^{ab}\delta(x-y)$$

$$S^{ab}(x,y;A) = \delta^{ab}S_0(x-y) - gf^{acd}\int \mathrm{d}z S_0(x-z)\gamma^\mu A^c_\mu(z) S^{db}(z,y;A)$$

$$G^{ab}(x,y;A) = \delta^{ab}G_0(x-y) - gf^{acd}\int \mathrm{d}z G_0(x-z)\mathcal{G}^\mu A^c_\mu(z) G^{db}(z,y;A)$$

$$\tilde{\mathcal{R}}_{\text{inv}} = \frac{\mathrm{d}}{\mathrm{d}g} + \frac{1}{8g}\int \mathrm{d}x\mathrm{d}y\mathrm{Tr}\big(\gamma_\mu \tilde{S}^{ab}(x,y;\tilde{A})\gamma^{\rho\lambda}\big)\tilde{F}^b_{\rho\lambda}(y)\frac{\delta}{\delta \tilde{A}^a_\mu(x)} + \frac{1}{g}\int \mathrm{d}x \tilde{D}^a(x)\frac{\delta}{\delta \tilde{D}^a(x)}$$
$$+\frac{i}{4g}\int \mathrm{d}x\mathrm{Tr}\big(\gamma_5\gamma_\mu \tilde{S}^{ab}(x,y;\tilde{A})\big)\tilde{D}^b(x)\frac{\delta}{\delta \tilde{A}^a_\mu(x)}$$

$$\tilde{\mathcal{R}}_{\text{gf}} = -g\int \mathrm{d}x\,\mathrm{d}y \big(\mathrm{D}_\mu \tilde{G}\big)^{ab}(x,y;\tilde{A})\tilde{\mathcal{R}}_{\text{inv}}\left(\frac{1}{g}\mathcal{G}^b[\tilde{A}](y)\right)\frac{\delta}{\delta \tilde{A}^a_\mu(x)}$$
$$+gf^{abc}\int \mathrm{d}x\,\mathrm{d}y \tilde{G}^{bd}(x,y;\tilde{A})\tilde{\mathcal{R}}_{\text{inv}}\left(\frac{1}{g}\mathcal{G}^d[\tilde{A}](y)\right)\tilde{D}^c(x)\frac{\delta}{\delta \tilde{D}^a(x)}$$

$$\big(\mathcal{T}_g^{-1}(A)\big)^a_\mu \equiv \left(\tilde{\mathcal{T}}_g^{-1}\left(\frac{1}{g}\tilde{A}\right)\right)^a_\mu (x) := \sum_{n=0}^{\infty}\frac{g^n}{n!}\left[\left(\tilde{\mathcal{R}}^n_g\left(\frac{1}{g}\tilde{A}\right)\right)^a_\mu (x)\bigg|_{\tilde{A}=gA}\right|_{g=0}\Bigg]$$

$$\gamma^{\rho\lambda}\tilde{F}^b_{\rho\lambda} = 2\gamma^\rho\gamma^\lambda\big(\mathrm{D}_\rho \tilde{A}_\lambda\big)^b - 2\partial^\lambda \tilde{A}^b_\lambda - f^{bde}\gamma^{\rho\lambda}\tilde{A}^d_\rho \tilde{A}^e_\lambda$$

$$\tilde{\mathcal{R}}_g = \tilde{\mathcal{R}}_0 + \tilde{\mathcal{R}}_1 + \tilde{\mathcal{R}}_2$$

$$\tilde{\mathcal{R}}_0 := \frac{\mathrm{d}}{\mathrm{d}g} + \frac{1}{g}\int \mathrm{d}x \tilde{A}^a_\mu(x)\frac{\delta}{\delta \tilde{A}^a_\mu(x)}$$

$$\tilde{\mathcal{R}}_1 := -\frac{1}{8g}\int \mathrm{d}x\,\mathrm{d}y\mathrm{Tr}\big(\gamma_\mu \tilde{S}^{ab}(x,y;\tilde{A})\gamma^{\rho\lambda}\big) f^{bcd}\tilde{A}^c_\rho(y)\tilde{A}^d_\lambda(y)\frac{\delta}{\delta \tilde{A}^a_\mu(x)}$$
$$-\frac{1}{8g}\int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\big(\mathrm{D}_\mu \tilde{G}\big)^{ae}(x,z;\tilde{A})\mathrm{Tr}\left(\gamma_\nu \frac{\delta \mathcal{G}^e}{\delta \tilde{A}^f_\nu}\tilde{S}^{fb}(z,y;\tilde{A})\gamma^{\rho\lambda}\right) f^{bcd}\tilde{A}^c_\rho(y)\tilde{A}^d_\lambda(y)\frac{\delta}{\delta \tilde{A}^a_\mu(x)}$$

$$\tilde{\mathcal{R}}_2 := -\frac{1}{4g}\int \mathrm{d}x\,\mathrm{d}y\mathrm{Tr}\big(\gamma_\mu \tilde{S}^{ab}(x,y;\tilde{A})\big)\partial^\lambda \tilde{A}^b_\lambda(y)\frac{\delta}{\delta \tilde{A}^a_\mu(x)}$$
$$+\frac{1}{4g}\int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\big(\mathrm{D}_\mu \tilde{G}\big)^{ab}(x,y;\tilde{A})\mathrm{Tr}\left(\gamma_\nu \frac{\delta \mathcal{G}^b}{\delta \tilde{A}^e_\nu}\tilde{S}^{ec}(y,z;\tilde{A})\right)\partial^\lambda \tilde{A}^c_\lambda(z)\frac{\delta}{\delta \tilde{A}^a_\mu(x)}$$

$$\tilde{\mathcal{R}}_0(A_\mu^a) \equiv \tilde{\mathcal{R}}_0\left(\frac{1}{g}\tilde{A}_\mu^a\right) = 0$$

$$\tilde{\mathcal{R}}_0\big(\tilde{S}^{ab}(x,y;\tilde{A})\big) = -f^{cde}\int \mathrm{d}z\tilde{S}^{ac}(x,z;\tilde{A})\gamma^\mu \tilde{A}_\mu^d(z)\tilde{S}^{eb}(z,y;\tilde{A}) \text{ etc.}$$

$$\begin{aligned}
\tilde{S}^{ab}(x,y;\tilde{A}) &= -\delta^{ab}\gamma^\rho\partial_\rho C(x-y) + \mathcal{O}(\tilde{A}) \\
\tilde{G}^{ab}(x,y;\tilde{A}) &= \delta^{ab}\tilde{G}_0(x-y) + \mathcal{O}(\tilde{A}) \\
\frac{\delta\mathcal{G}^a[\tilde{A}](x)}{\delta\tilde{A}_\mu^b(y)} &= \delta^{ab}\mathcal{G}^\mu\delta(x-y) + \mathcal{O}(\tilde{A})
\end{aligned}$$

$$\Box\, C(x) = -\delta(x)$$

$$\mathrm{Tr}\big(\gamma_\mu n^\mu S_0(y-z)\big) = -4n^\mu\partial_\mu C(y-z)$$

$$\lim_{g\to 0}\tilde{\mathcal{R}}_2\left(\frac{1}{g}\tilde{A}_\mu^a\right)\bigg|_{n\cdot A^a=0} \neq 0$$

$$\begin{aligned}
(\mathcal{T}_g A)_\mu^a(x) = A_\mu^a(x) &+ gf^{abc}\int \mathrm{d}y\,\mathrm{d}z\big(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y)n_\nu\big) \\
&\times\big\{A^{b\nu}(y)C(y-z)\partial\cdot A^c(z) + \partial^\lambda C(y-z)A^{b\nu}(z)A_\lambda^c(z)\big\} \\
&+2gf^{abc}\int \mathrm{d}y\,\mathrm{d}z\,\mathrm{d}w\big(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y)n_\nu\big) \\
&\times\partial_\lambda C(y-z)A^{b[\nu}(z)\partial^{\lambda]}C(z-w)\partial\cdot A^c(w) \\
&+\frac{g^2}{2}f^{abc}f^{bde}\int \mathrm{d}y\,\mathrm{d}z\,\mathrm{d}w\big(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y)n_\nu\big)\{ \\
&-2A^{c\nu}(y)C(y-z)A_\lambda^d(z)\partial^\lambda C(z-w)\partial\cdot A^e(w) \\
&-A^{c\nu}(y)C(y-z)\partial\cdot A^d(z)C(z-w)\partial\cdot A^e(w) \\
&-\frac{1}{2}C(y-z)\partial\cdot A^c(z)\partial^\lambda C(z-w)A^{d\nu}(w)A_\lambda^e(w) \\
&+\frac{1}{2}C(y-z)\partial\cdot A^c(z)\partial^\lambda C(y-w)A^{d\nu}(w)A_\lambda^e(w) \\
&-\frac{1}{2}C(y-z)A^{d\nu}(z)A_\lambda^e(z)\partial^\lambda C(z-w)\partial\cdot A^c(w) \\
&+\frac{1}{2}\partial^\lambda C(y-z)A^{d\nu}(z)A_\lambda^e(z)C(z-w)\partial\cdot A^c(w) \\
&-2\partial_\lambda C(y-z)A^{c[\nu}(z)A^{d\lambda]}(z)C(z-w)\partial\cdot A^e(w) \\
&+3\partial_\rho C(y-z)A_\lambda^c(z)\partial^{[\nu}C(z-w)A^{d\lambda}(w)A^{e\rho]}(w)\big\} \\
&+\frac{g^2}{2}f^{abc}f^{bde}\int \mathrm{d}y\,\mathrm{d}z\,\mathrm{d}w\,\mathrm{d}v\big(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y)n_\nu\big)\{ \\
&-C(y-z)A^{d[\nu}(z)\partial^{\lambda]}C(z-w)\partial\cdot A^e(w)\partial_\lambda C(z-v)\partial\cdot A^c(v) \\
&-C(y-z)\partial\cdot A^c(z)\partial_\lambda C(z-w)A^{d[\nu}(w)\partial^{\lambda]}C(w-v)\partial\cdot A^e(v) \\
&+C(y-z)\partial\cdot A^c(z)\partial_\lambda C(y-w)A^{d[\nu}(w)\partial^{\lambda]}C(w-v)\partial\cdot A^e(v) \\
&-\partial_\lambda C(y-z)A^{d[\nu}(z)\partial^{\lambda]}C(z-w)\partial\cdot A^e(w)C(z-v)\partial\cdot A^c(v) \\
&-\partial^\lambda C(y-z)\partial^\nu C(z-w)\partial\cdot A^c(w)\partial^\rho C(z-v)A_\lambda^d(v)A_\rho^e(v) \\
&+2\partial_\lambda C(y-z)\partial^{[\nu}A^{d\lambda]}(z)C(z-w)\partial\cdot A^e(w)C(z-v)\partial\cdot A^c(v) \\
&-2\partial_\lambda C(y-z)A^{c[\nu}(z)\partial^{\lambda]}C(z-w)\partial\cdot A^d(w)C(w-v)\partial\cdot A^e(v) \\
&-4\partial_\lambda C(y-z)A^{c[\nu}(z)\partial^{\lambda]}C(z-w)A_\rho^d(w)\partial^\rho C(w-v)\partial\cdot A^e(v) \\
&+6\partial_\rho C(y-z)A_\lambda^c(z)\partial^{[\nu}C(z-w)A^{d\lambda}(w)\partial^{\rho]}C(w-v)\partial\cdot A^e(v)\big\} \\
&-g^2 f^{abc}f^{bde}\int \mathrm{d}y\,\mathrm{d}z\,\mathrm{d}w\,\mathrm{d}v\,\mathrm{d}u\big(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y)n_\nu\big) \\
&\times\partial^\lambda C(y-z)\partial^\nu C(z-w)\partial\cdot A^c(w)\partial^\rho C(z-v)A_{[\lambda}^d(v)\partial_{\rho]}C(v-u)\partial\cdot A^e(u) \\
&+\mathcal{O}(g^3)
\end{aligned}$$

$$
\begin{aligned}
(\mathcal{T}_g A)^a_\mu(x) = A^a_\mu(x) + g f^{abc} \int \mathrm{d}y \ \mathrm{d}z \ \left(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y)n_\nu\right) \\
\times \left\{ A^{b\,\nu}(y) C(y-z)\, \partial\cdot A^c(z) + \partial^\lambda C(y-z) A^{b\,\nu}(z) A^c_\lambda(z) \right\} \\
+ 2g f^{abc} \int \mathrm{d}y \ \mathrm{d}z \ \mathrm{d}w \ \left(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y)n_\nu\right) \\
\times \partial_\lambda C(y-z) A^{b\,[\nu}(z)\partial^{\lambda]} C(z-w)\, \partial\cdot A^c(w) \\
+ \frac{g^2}{2} f^{abc} f^{bde} \int \mathrm{d}y \ \mathrm{d}z \ \mathrm{d}w \ \left(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y)n_\nu\right) \Bigg\{
\end{aligned}
$$

$$
\begin{aligned}
& - 2A^{c\,\nu}(y) C(y-z) A^d_\lambda(z) \partial^\lambda C(z-w)\, \partial\cdot A^e(w) \\
& - A^{c\,\nu}(y) C(y-z)\, \partial\cdot A^d(z) C(z-w)\, \partial\cdot A^e(w) \\
& - \frac{1}{2} C(y-z)\, \partial\cdot A^c(z) \partial^\lambda C(z-w) A^{d\,\nu}(w) A^e_\lambda(w) \\
& + \frac{1}{2} C(y-z)\, \partial\cdot A^c(z) \partial^\lambda C(y-w) A^{d\,\nu}(w) A^e_\lambda(w) \\
& - \frac{1}{2} C(y-z) A^{d\,\nu}(z) A^e_\lambda(z) \partial^\lambda C(z-w)\, \partial\cdot A^c(w) \\
& + \frac{1}{2} \partial^\lambda C(y-z) A^{d\,\nu}(z) A^e_\lambda(z) C(z-w)\, \partial\cdot A^c(w) \\
& - 2\partial_\lambda C(y-z) A^{c\,[\nu}(z) A^{d\,\lambda]}(z) C(z-w)\, \partial\cdot A^e(w) \\
& + 3\partial_\rho C(y-z) A^c_\lambda(z) \partial^{[\nu} C(z-w) A^{d\,\lambda}(w) A^{e\,\rho]}(w) \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
+ \frac{g^2}{2} f^{abc} f^{bde} \int \mathrm{d}y \ \mathrm{d}z \ \mathrm{d}w \ \mathrm{d}v \ \left(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y)n_\nu\right) \Bigg\{ \\
& - C(y-z) A^{d\,[\nu}(z) \partial^{\lambda]} C(z-w)\, \partial\cdot A^e(w) \partial_\lambda C(z-v)\, \partial\cdot A^c(v) \\
& - C(y-z)\, \partial\cdot A^c(z) \partial_\lambda C(z-w) A^{d\,[\nu}(w) \partial^{\lambda]} C(w-v)\, \partial\cdot A^e(v) \\
& + C(y-z)\, \partial\cdot A^c(z) \partial_\lambda C(y-w) A^{d\,[\nu}(w) \partial^{\lambda]} C(w-v)\, \partial\cdot A^e(v) \\
& - \partial_\lambda C(y-z) A^{d\,[\nu}(z) \partial^{\lambda]} C(z-w)\, \partial\cdot A^e(w) C(z-v)\, \partial\cdot A^c(v) \\
& - \partial^\lambda C(y-z) \partial^\nu C(z-w)\, \partial\cdot A^c(w) \partial^\rho C(z-v) A^d_\lambda(v) A^e_\rho(v) \\
& + 2\partial_\lambda C(y-z) \partial^{[\nu} A^{d\,\lambda]}(z) C(z-w)\, \partial\cdot A^e(w) C(z-v)\, \partial\cdot A^c(v) \\
& - 2\partial_\lambda C(y-z) A^{c\,[\nu}(z) \partial^{\lambda]} C(z-w)\, \partial\cdot A^d(w) C(w-v)\, \partial\cdot A^e(v) \\
& - 4\partial_\lambda C(y-z) A^{c\,[\nu}(z) \partial^{\lambda]} C(z-w) A^d_\rho(w) \partial^\rho C(w-v)\, \partial\cdot A^e(v) \\
& + 6\partial_\rho C(y-z) A^c_\lambda(z) \partial^{[\nu} C(z-w) A^{d\,\lambda}(w) \partial^{\rho]} C(w-v)\, \partial\cdot A^e(v) \Bigg\}
\end{aligned}
$$

$$
-g^2 f^{abc} f^{bde} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \mathrm{d}u\ \left(\eta_{\mu\nu}\delta(x-y) - \partial_\mu G_0(x-y) n_\nu\right)
\times \partial^\lambda C(y-z)\partial^\nu C(z-w)\,\partial\cdot A^c(w)\partial^\rho C(z-v) A^d_{[\lambda}(v)\partial_{\rho]} C(v-u)\,\partial\cdot A^e(u)
+\mathcal{O}(g^3)\,.
$$

$$
G_0(x) = \varepsilon(n\cdot x)\delta^{(3)}(x^\perp) = -G_0(-x)
$$

$$
\Pi_{\mu\nu}(x) := \eta_{\mu\nu}\delta(x) - \partial_\mu G_0(x) n_\nu
$$

$$
\int \mathrm{d}y \Pi_{\mu\nu}(x-y)\partial^\nu F(y) = 0
$$

$$
n^\mu A'^a_\mu(x) = n^\mu A^a_\mu(x)
$$

$$
\frac{1}{2}\int \mathrm{d}x A'^a_\mu(x)(-\Box\,\eta^{\mu\nu} + \partial^\mu\partial^\nu) A'^a_\nu(x) = \frac{1}{4}\int \mathrm{d}x F^a_{\mu\nu}(x) F^{a\mu\nu}(x) + \mathcal{O}(g^3)
$$

$$
\frac{1}{2}\int \mathrm{d}x A'^a_\mu(x)(-\Box\,\eta^{\mu\nu} + \partial^\mu\partial^\nu) A'^a_\nu(x)\Big|_{\mathcal{O}(g^1)}
= f^{abc}\int \mathrm{d}x\ \mathrm{d}y\{A^b_\mu(x) C(x-y)\partial\cdot A^c(y) + \partial^\lambda C(x-y) A^b_\mu(y) A^c_\lambda(y)\}
\times(-\Box\,\eta^{\mu\nu} + \partial^\mu\partial^\nu) A'^a_\nu(x)
+2 f^{abc}\int \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z \partial^\lambda C(x-y) A^b_{[\mu}(y)\partial_{\lambda]} C(y-z)\partial\cdot A^c(z)
\times(-\Box\,\eta^{\mu\nu} + \partial^\mu\partial^\nu) A'^a_\nu(x)
$$

$$
\frac{1}{2}\int \mathrm{d}x A'^a_\mu(x)(-\Box\,\eta^{\mu\nu} + \partial^\mu\partial^\nu) A'^a_\nu(x)\Big|_{\mathcal{O}(g^1)}
= -f^{abc}\int \mathrm{d}x \partial_\lambda A^a_\mu(x) A^{b\mu}(x) A^{c\lambda}(x)
+ f^{abc}\int \mathrm{d}x\ \mathrm{d}y\{-\Box\, A^a_\mu(x) A^{b\mu}(x) C(x-y)\partial\cdot A^c(y)
+\partial_\mu\partial\cdot A^a(x) A^{b\mu}(x) C(x-y)\partial\cdot A^c(y) - 2\partial_\lambda A^a_\mu(x) A^{b[\mu}(x)\partial^{\lambda]} C(x-y)\partial\cdot A^c(y)\}
= f^{abc}\int \mathrm{d}x \partial_\mu A^a_\lambda(x) A^{b\mu}(x) A^{c\lambda}(x) = \frac{1}{4}\int \mathrm{d}x F^a_{\mu\nu}(x) F^{a\mu\nu}(x)\Big|_{\mathcal{O}(g^1)}
$$

$$
\frac{1}{2}\int \mathrm{d}x A'^a_\mu(x)(-\Box\,\eta^{\mu\nu} + \partial^\mu\partial^\nu) A'^a_\nu(x)\Big|_{\mathcal{O}(g^2)}
= \int \mathrm{d}x A'^a_\mu(x)\Big|_{\mathcal{O}(g^2)} (-\Box\,\eta^{\mu\nu} + \partial^\mu\partial^\nu) A'^a_\nu(x)\Big|_{\mathcal{O}(g^0)}
+\frac{1}{2}\int \mathrm{d}x A'^a_\mu(x)\Big|_{\mathcal{O}(g^1)} (-\Box\,\eta^{\mu\nu} + \partial^\mu\partial^\nu) A'^a_\nu(x)\Big|_{\mathcal{O}(g^1)}
= -\frac{g^2}{4} f^{abc} f^{bde}\int \mathrm{d}x A^a_\mu(x) A^c_\lambda(x) A^{d\mu}(x) A^{e\lambda}(x)
-\frac{g^2}{2}\int \mathrm{d}x\ \mathrm{d}y A^a_\mu(x) A^e_\lambda(x)\partial^\lambda A^{d\mu}(x) C(x-y)\partial\cdot A^c(y)
\times(f^{abc} f^{bde} + f^{eba} f^{bdc} + f^{cbe} f^{bda})
$$

$$
= -\frac{g^2}{4} f^{abc} f^{bde}\int \mathrm{d}x A^a_\mu(x) A^c_\lambda(x) A^{d\mu}(x) A^{e\lambda}(x) = \frac{1}{4}\int \mathrm{d}x F^a_{\mu\nu}(x) F^{a\mu\nu}(x)\Big|_{\mathcal{O}(g^2)}
$$

$$
0 = f^{abc} f^{bde} + f^{eba} f^{bdc} + f^{cbe} f^{bda}
$$

$$
\log\ \det\left(\frac{\delta A'^a_\mu(x)}{\delta A^b_\nu(y)}\right) \stackrel{!}{=} \log\left(\Delta_{\mathrm{MSS}}[A]\Delta_{\mathrm{FP}}[A]\right)
$$

$$
\mathbf{X}^{ab}(x,y;A) = g f^{abc} G_0(x-y)\, n\cdot A^c(y),
$$

$$\log\ \det(1-\mathbf{X}) = \mathrm{Tr}\log\left(1-\mathbf{X}\right)$$

$$\log\ \det(1-\mathbf{X}) = \frac{g^2}{2}N\int\ \mathrm{d}x\,\mathrm{d}y G_0(x-y)n\cdot A^a(y)G_0(y-x)n\cdot A^a(x) + \mathcal{O}(g^3)$$

$$\mathbf{Y}^{ab}_{\alpha\beta}(x,y;A) = gf^{abc}\partial_\rho C(x-y)\left(\gamma^\rho\gamma^\lambda\right)_{\alpha\beta}A^c_\lambda(y)$$

$$\frac{1}{2}\log\ \det(1-\mathbf{Y}) = \frac{g^2}{4}N\mathrm{Tr}\left(\gamma^\rho\gamma^\lambda\gamma^\sigma\gamma^\nu\right)\int\ \mathrm{d}x\,\mathrm{d}y\partial_\rho C(x-y)A^a_\lambda(y)\partial_\sigma C(y-x)A^a_\nu(x) + \mathcal{O}(g^3)$$

$$\log\left(\Delta_{\mathrm{MSS}}[A]\Delta_{\mathrm{FP}}[A]\right)|_{\mathcal{O}(g^2)} = \frac{g^2}{2}N\int\ \mathrm{d}x\,\mathrm{d}y\{ +r\partial^\mu C(x-y)A^a_\mu(y)\partial^\nu C(y-x)A^a_\nu(x) -\frac{r}{2}\partial_\mu C(x-y)A^a_\nu(y)\partial^\mu C(y-x)A^{a\nu}(x) +G_0(x-y)n\cdot A^a(y)G_0(y-x)n\cdot A^a(x)\}$$

$$\log\ \det\left(\frac{\delta A'^a_\mu(x)}{\delta A^b_\nu(y)}\right)\Bigg|_{\mathcal{O}(g^2)} = \mathrm{Tr}\left[\frac{\delta A'}{\delta A}\bigg|_{\mathcal{O}(g^2)}\right] - \frac{1}{2}\mathrm{Tr}\left[\frac{\delta A'}{\delta A}\bigg|_{\mathcal{O}(g^1)}\frac{\delta A'}{\delta A}\bigg|_{\mathcal{O}(g^1)}\right]$$

$$\begin{aligned}-\frac{1}{2}\mathrm{Tr}\left[\frac{\delta A'}{\delta A}\bigg|_{\mathcal{O}(g^1)}\frac{\delta A'}{\delta A}\bigg|_{\mathcal{O}(g^1)}\right] = Ng^2\int\ \mathrm{d}x\,\mathrm{d}y\{ &+\frac{D}{2}\partial^\mu C(x-y)A^a_\mu(y)\partial^\nu C(x-y)A^a_\nu(x)\\ &+\frac{1}{2}G_0(x-y)n\cdot A^a(y)G_0(y-x)n\cdot A^a(x)\\ &+C(x-y)\partial^\mu\left(A^a_\mu(y)G_0(y-x)\right)n\cdot A^a(x)\\ &+\frac{2-D}{8}C(x-y)\partial\cdot A^a(y)(C(y-x)-2C(0))\partial\cdot A^a(x)\Big\}\\ &+Ng^2\int\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\{\\ &-\frac{1}{4}G_0(x-z)n^\mu C(z-x)\partial\cdot A^a(y)\partial_\mu C(y-x)\partial\cdot A^a(x)\\ &-2G_0(x-z)\partial^\mu C(z-x)\partial\cdot A^a(y)n^\nu\partial_{\{\nu}C(y-x)A^a_{\mu\}}(x)\\ &+2G_0(x-z)\partial^\mu C(z-y)A^a_\nu(y)n^\lambda\partial^\nu\partial_{\{\lambda}C(y-x)A^a_{\mu\}}(x)\\ &+\frac{1-D}{2}\delta(0)C(z-y)\partial\cdot A^a(y)C(z-x)\partial\cdot A^a(x)\Big\}\end{aligned}$$

$$\begin{aligned}\mathrm{Tr}\left[\frac{\delta A'}{\delta A}\bigg|_{\mathcal{O}(g^2)}\right] = Ng^2\int\ \mathrm{d}x\,\mathrm{d}y\{ &-\frac{4-D}{2}\partial^\mu C(x-y)A^a_\mu(y)\partial^\nu C(y-x)A^a_\nu(x)\\ &+\frac{2-D}{2}\partial_\mu C(x-y)A^a_\nu(y)\partial^\mu C(y-x)A^{a\nu}(x)\\ &-C(x-y)\partial^\mu\left(A^a_\mu(y)G_0(y-x)\right)n\cdot A^a(x)\\ &-\frac{2-D}{8}C(x-y)\partial\cdot A^a(y)(C(y-x)-2C(0))\partial\cdot A^a(x)\Big\}\\ &+Ng^2\int\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\{\\ &+\frac{1}{4}G_0(x-z)n^\mu C(z-x)\partial\cdot A^a(y)\partial_\mu C(y-x)\partial\cdot A^a(x)\\ &+2G_0(x-z)\partial^\mu C(z-x)\partial\cdot A^a(y)n^\nu\partial_{\{\nu}C(y-x)A^a_{\mu\}}(x)\\ &-2G_0(x-z)\partial^\mu C(z-y)A^a_\nu(y)n^\lambda\partial^\nu\partial_{\{\lambda}C(y-x)A^a_{\mu\}}(x)\\ &-\frac{1-D}{2}\delta(0)C(z-y)\partial\cdot A^a(y)C(z-x)\partial\cdot A^a(x)\Big\}\end{aligned}$$

$$\frac{r}{2} = \frac{D}{2} - \frac{4-D}{2} = D - 2$$
$$-\frac{r}{4} = \frac{2-D}{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}g}\langle\tilde{X}\rangle_g = \frac{\mathrm{d}}{\mathrm{d}g}\langle\langle\tilde{X}\rangle\rangle_g = \left\langle\left\langle\frac{\mathrm{d}\tilde{X}}{\mathrm{d}g}\right\rangle\right\rangle_g - i\left\langle\left\langle\frac{\mathrm{d}(\tilde{S}_{\mathrm{inv}} + \tilde{S}_{\mathrm{gf}})}{\mathrm{d}g}\tilde{X}\right\rangle\right\rangle_g =: \langle\tilde{\mathcal{R}}\tilde{X}\rangle_g.$$

$$\frac{\mathrm{d}\tilde{S}_{\mathrm{inv}}}{\mathrm{d}g} = -\frac{2\tilde{S}_{\mathrm{inv}}}{g} = -\frac{2}{g^3}\delta_\alpha\tilde{\Delta}_\alpha,$$

$$\frac{\mathrm{d}}{\mathrm{d}g}\langle\tilde{X}\rangle_g = \left\langle\left\langle\frac{\mathrm{d}\tilde{X}}{\mathrm{d}g}\right\rangle\right\rangle_g + \frac{2i}{g^3}\left\langle\langle(\delta_\alpha\tilde{\Delta}_\alpha)\tilde{X}\rangle\right\rangle_g + \frac{2i}{g}\left\langle\langle\,\tilde{S}_{\mathrm{gf}}\tilde{X}\rangle\right\rangle_g$$

$$\left\langle\langle(\delta_\alpha\tilde{\Delta}_\alpha)\tilde{X}\rangle\right\rangle_g = \left\langle\langle\delta_\alpha(\tilde{\Delta}_\alpha\tilde{X})\rangle\right\rangle_g + \left\langle\langle\tilde{\Delta}_\alpha\delta_\alpha\tilde{X}\rangle\right\rangle_g$$

$$\left\langle\langle\delta_\alpha\tilde{Y}\rangle\right\rangle_g = -i\left\langle\langle(\delta_\alpha\tilde{S}_{\mathrm{gf}})\tilde{Y}\rangle\right\rangle_g$$

$$\tilde{S}_{\mathrm{gf}} = -s\left(\frac{1}{g^2}\int \mathrm{d}x\overline{\tilde{C}}^a\mathcal{G}^a[\tilde{A}]\right)$$

$$\delta_\alpha\tilde{S}_{\mathrm{gf}} = -s\left(\frac{1}{g^2}\int \mathrm{d}x\overline{\tilde{C}}^a\delta_\alpha\mathcal{G}^a[\tilde{A}]\right)$$

$$\langle\langle\delta_\alpha Y\rangle\rangle_g = -\left\langle\left\langle\frac{i}{g^2}\int \mathrm{d}x\overline{\tilde{C}}^a(x)\delta_\alpha\mathcal{G}^a(\tilde{A})s(\tilde{Y})\right\rangle\right\rangle_g$$

$$s(\tilde{\Delta}_\alpha X) = -\tilde{\Delta}_\alpha s(\tilde{X})$$

$$\frac{\mathrm{d}}{\mathrm{d}g}\langle\langle\tilde{X}\rangle\rangle_g = \left\langle\left\langle\frac{\mathrm{d}\tilde{X}}{\mathrm{d}g}\right\rangle\right\rangle_g - \frac{2i}{g^3}\left\langle\langle\tilde{\Delta}_\alpha\delta_\alpha\tilde{X}\rangle_g - \frac{2}{g^5}\left\langle\left\langle\int \mathrm{d}x\tilde{\tilde{C}}^a(x)\delta_\alpha\mathcal{G}^a(\tilde{A})\tilde{\Delta}_\alpha s(\tilde{X})\right\rangle\right\rangle_g\right.$$
$$+\frac{2}{g^3}\left\langle\left\langle\int \mathrm{d}x\tilde{\tilde{C}}^a(x)\mathcal{G}^a(\tilde{A})s(\tilde{X})\right\rangle\right\rangle_g$$

$$\tilde{\mathcal{R}}\tilde{X} = \frac{\mathrm{d}X}{\mathrm{d}g} + \frac{2i}{g}\delta_\alpha\tilde{X}\tilde{\Delta}_\alpha - \frac{2}{g}\int \mathrm{d}x\overset{\tilde{\bar{c}}}{L^a}(x)\delta_\alpha\mathcal{G}^a(\tilde{A})\tilde{\Delta}_\alpha s(\tilde{X}) + \frac{2}{g}\int \mathrm{d}x\overset{\bar{\tilde{c}}}{L^a}(x)\mathcal{G}^a(\tilde{A})s(\tilde{X})$$

$$\tilde{\mathcal{R}}\tilde{X} \equiv \tilde{\mathcal{R}}_{\mathrm{inv}}\tilde{X} := \frac{\mathrm{d}\tilde{X}}{\mathrm{d}g} + \frac{2i}{g}\delta_\alpha\tilde{X}\tilde{\Delta}_\alpha$$

$$-\frac{1}{4g^2}\int \mathrm{d}x\tilde{F}^a_{\mu\nu}\tilde{F}^{a\mu\nu} \text{ and } -\frac{1}{2g^2}\int \mathrm{d}x\tilde{D}^a\tilde{D}^a$$

$$i\tilde{\lambda}^a(x)\overline{\tilde{\lambda}}^b(y) = \tilde{S}^{ab}(x,y;\tilde{A}) \text{ and } \tilde{C}^a(x)\tilde{\bar{C}}^b(y) = \tilde{G}^{ab}(x,y;\tilde{A})$$

$$
\begin{aligned}
\left(\mathcal{T}_g A\right)^a_\mu(x) =& A^a_\mu(x) + g f^{abc} \int \mathrm{d}y \partial^\rho C(x-y) A^b_\mu(y) A^c_\rho(y) \\
&+\frac{3g^2}{2} f^{abc} f^{bde} \int \mathrm{d}y\, \mathrm{d}z \partial^\rho C(x-y) A^{\lambda c}(y) \partial_{[\rho} C(y-z) A^d_\mu(z) A^e_{\lambda]}(z) \\
&+\frac{g^3}{2} f^{abc} f^{bde} f^{cfg} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w \partial^\rho C(x-y) \\
&\quad \times \partial^\lambda C(y-z) A^d_\lambda(z) A^{\sigma e}(z) \partial_{[\rho} C(y-w) A^f_\mu(w) A^g_{\sigma]}(w) \\
&+g^3 f^{abc} f^{bde} f^{dfg} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w \partial^\rho C(x-y) A^{\lambda c}(y) \{ \\
&\quad -\partial^\sigma C(y-z) A^e_\sigma(z) \partial_{[\rho} C(z-w) A^f_\mu(w) A^g_{\lambda]}(w) \\
&\quad +\partial_{[\rho} C(y-z) A^e_\mu(z) \partial^\sigma C(z-w) A^f_{\lambda]}(w) A^g_\sigma(w) \Big\} \\
&+\frac{g^3}{3} f^{abc} f^{bde} f^{dfg} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w \{ \\
&\quad +6\partial_\rho C(x-y) A^{\lambda c}(y) \partial^{[\rho} C(y-z) A^{\sigma]}(z) \partial_{[\lambda} C(z-w) A^f_\mu(w) A^g_{\sigma]}(w) \\
&\quad -6\partial^\rho C(x-y) A^c_\lambda(y) \partial^{[\lambda} C(y-z) A^{\sigma]e}(z) \partial_{[\rho} C(z-w) A^f_\mu(w) A^g_{\sigma]}(w) \\
&\quad -6\partial_\rho C(x-y) A^c_\lambda(y) \partial_{[\sigma} C(y-z) A^e_{\mu]}(z) \partial^{[\rho} C(z-w) A^{\lambda f}(w) A^{\sigma]g}(w) \\
&\quad +2\partial^\rho C(x-y) A^c_{[\rho}(y) \partial_{\mu]} C(y-z) A^{\lambda e}(z) \partial^\sigma C(z-w) A^f_\lambda(w) A^g_\sigma(w) \\
&\quad -\partial_\mu C(x-y) \partial^\rho \big(A^c_\rho(y) C(y-z)\big) A^{\lambda e}(z) \partial^\sigma C(z-w) A^f_\lambda(w) A^g_\sigma(w) \} \\
&-\frac{g^3}{3} f^{abc} f^{bde} f^{dfg} \int \mathrm{d}y\, \mathrm{d}z A^c_\mu(x) C(x-y) A^{\rho e}(y) \partial^\lambda C(y-z) A^f_\rho(z) A^g_\lambda(z) \\
&+\frac{g^4}{12} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w \\
&\quad \times C(x-y) A^{\lambda e}(y) \partial^\rho C(y-z) A^f_\lambda(z) A^g_\rho(z) \partial^\sigma C(x-w) A^h_\sigma(w) A^i_\mu(w) \\
&+\frac{g^4}{8} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial^\lambda C(x-y) \partial_\rho C(y-z) \{ \\
&\quad -9A^e_\sigma(z) \partial^{[\rho} C(z-w) A^{\sigma f}(w) A^{\nu]g}(w) \partial_{[\mu} C(y-v) A^h_\lambda(v) A^i_{\nu]}(v) \\
&\quad +4A^{[\rho e}(z) \partial^{|\sigma|} C(z-w) A^f_\sigma(w) A^{\nu]g}(w) \partial_{[\mu} C(y-v) A^h_\lambda(v) A^i_{\nu]}(v) \\
&\quad -2A^{\rho e}(z) \partial_{[\mu} C(z-w) A^f_\lambda(w) A^g_{\nu]}(w) \partial^\sigma C(y-v) A^h_\sigma(v) A^{\nu i}(v) \Big\}
\end{aligned}
$$

$$-\frac{g^4}{12} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial_\mu C(x-y) \partial^\lambda C(y-z)$$

$$\times A^{\rho e}(z) \partial^\sigma C(z-w) A^f_\sigma(w) A^g_\rho(w) \partial^\tau C(y-v) A^h_\tau(v) A^i_\lambda(v)$$

$$+\frac{g^4}{2} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial_\lambda C(x-y) \{$$

$$+\partial_{[\mu} C(y-z) A^e_{\rho]}(z) \partial^{[\lambda} C(z-w) A^{\rho f}(w) A^{\nu] g}(w) \partial^\sigma C(y-v) A^h_\sigma(v) A^i_\nu(v)$$

$$-\partial^{[\lambda} C(y-z) A^{\rho] e}(z) \partial_{[\mu} C(z-w) A^f_\rho(w) A^g_{\nu]}(w) \partial^\sigma C(y-v) A^h_\sigma(v) A^i_\nu(v) \}$$

$$+\frac{g^4}{6} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial^\lambda C(x-y) \{$$

$$+3\partial^{[\rho} C(y-z) A^{\nu] e}(z) \partial_{[\mu} C(z-w) A^f_\lambda(w) A^g_{\rho]}(w) \partial^\sigma C(y-v) A^h_\sigma(v) A^i_\nu(v)$$

$$+\partial_{[\lambda} C(y-z) A^{\nu e}(z) \partial^\sigma C(z-w) A^f_{|\sigma}(w) A^g_\nu(w) \partial^\rho C(y-v) A^h_{\rho|}(v) A^i_{\mu]}(v) \}$$

$$-\frac{g^4}{3} f^{abc} f^{bde} f^{dfg} f^{ehi} \int \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w$$

$$\times A^c_\mu(x) C(x-y) \partial^\lambda C(y-z) A^f_\lambda(z) A^{\rho g}(z) \partial^\sigma C(y-w) A^h_\sigma(w) A^i_\rho(w)$$

$$-\frac{g^4}{3} f^{abc} f^{bde} f^{dfg} f^{ehi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial_\mu C(x-y)$$

$$\times \partial^\lambda (A^c_\lambda(y) C(y-z)) \partial^\rho C(z-w) A^f_\rho(w) A^{\nu g}(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^i_\nu(v)$$

$$+\frac{g^4}{12} f^{abc} f^{bde} f^{dfg} f^{ehi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial^\lambda C(x-y) A^{\rho c}(y) \{$$

$$-3\partial_\rho C(y-z) \partial_{[\mu} C(z-w) A^f_\lambda(w) A^g_{\nu]}(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^i_\nu(v)$$

$$-3\partial^\nu C(y-z) \partial_{[\mu} C(z-w) A^f_\rho(w) A^g_{\nu]}(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^i_\lambda(v)$$

$$+3\partial^\nu C(y-z) \partial_{[\lambda} C(z-w) A^f_\rho(w) A^g_{\nu]}(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^i_\mu(v)$$

$$-3\partial_\mu C(y-z) \partial_{[\lambda} C(z-w) A^f_\rho(w) A^g_{\nu]}(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^i_\nu(v)$$

$$+3\partial_\lambda C(y-z) \partial_{[\mu} C(z-w) A^f_\rho(w) A^g_{\nu]}(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^{\nu i}(v)$$

$$-2\partial_{[\lambda} C(y-z) \partial^\nu C(z-w) A^f_{|\nu|}(w) A^g_{\mu]}(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^i_\rho(v)$$

$$+\partial_\rho C(y-z) \partial^\nu C(z-w) A^f_\nu(w) A^g_\mu(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^i_\lambda(v) \}$$

$$+\frac{g^4}{6} f^{abc} f^{bde} f^{dfg} f^{ehi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial^\lambda C(x-y) \{$$

$$-7A^c_{[\mu}(y) \partial_{\lambda]} C(y-z) \partial^\rho C(z-w) A^f_\rho(w) A^{\nu g}(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^i_\nu(v)$$

$$+3A^{[\nu c}(y) \partial^{\rho]} C(y-z) \partial_{[\mu} C(z-w) A^f_\lambda(w) A^g_{\rho]}(w) \partial^\sigma C(z-v) A^h_\sigma(v) A^i_\nu(v) \}$$

$$-\frac{g^4}{2} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w$$

$$\times \partial^\lambda C(x-y) A^c_{[\mu}(y) A^e_{\lambda]}(y) C(y-z) A^{\rho g}(z) \partial^\sigma C(z-w) A^h_\sigma(w) A^i_\rho(w)$$

$$+\frac{g^4}{12} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w A^c_\mu(x) C(x-y) \{$$

$$+9A^{\lambda e}(y)\partial^\rho C(y-z) A^{\sigma g}(z)\partial_{[\lambda} C(z-w) A^h_\rho(w) A^i_{\sigma]}(w)$$

$$+4A^{[\lambda e}(y)\partial^{\rho]} C(y-z) A^g_\lambda(z)\partial^\sigma C(z-w) A^h_\sigma(w) A^i_\rho(w)$$

$$-3A^e_\lambda(y)\partial^\lambda C(y-z) A^{\rho g}(z)\partial^\sigma C(z-w) A^h_\sigma(w) A^i_\rho(w)$$

$$-3\partial^\lambda (A^e_\lambda(y) C(y-z)) A^{\rho g}(z)\partial^\sigma C(z-w) A^h_\sigma(w) A^i_\rho(w)\}$$

$$+\frac{g^4}{12} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial_\mu C(x-y) \partial^\lambda (A^c_\lambda(y) C(y-z)) \{$$

$$+9A^{\rho e}(z)\partial^\sigma C(z-w) A^{\nu g}(w)\partial_{[\rho} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v)$$

$$+4A^{[\rho e}(z)\partial^{\nu]} C(z-w) A^g_\rho(w)\partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v)$$

$$-3\partial^\rho \big(A^e_\rho(z) C(z-w)\big) A^{\nu g}(w)\partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v)$$

$$-3A^e_\rho(z)\partial^\rho C(z-w) A^{\nu g}(w)\partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v)\}$$

$$+\frac{g^4}{2} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial^\lambda C(x-y) A^c_{[\mu}(y)\partial_{\lambda]} C(y-z) \{$$

$$-\partial^\rho \big(A^e_\rho(z) C(z-w)\big) A^{\nu g}(w)\partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v)$$

$$-A^e_\rho(z)\partial^\rho C(z-w) A^{\nu g}(w)\partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v)\}$$

$$+\frac{2g^4}{3} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial^\lambda C(x-y)$$

$$\times A^c_{[\mu}(y)\partial_{\lambda]} C(y-z) A^{\rho e}(z)\partial^\nu C(z-w) A^g_{[\rho}(w)\partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v)$$

$$+\frac{3g^4}{2} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial_\lambda C(x-y) \{$$

$$+4A^{\rho c}(y)\partial^{[\lambda} C(y-z) A^{e\nu]}(z)\partial^\sigma C(z-w) A^g_{[\mu}(w)\partial_\rho C(w-v) A^h_\sigma(v) A^i_{\nu]}(v)$$

$$-4A^c_r(y)\partial_{[\mu} C(y-z) A^e_{\nu]}(z)\partial_\sigma C(z-w) A^{g[\lambda}(w)\partial^\rho C(w-v) A^{h\sigma}(v) A^{i\nu]}(v)$$

$$-A^{\rho c}(y)\partial_{[\rho} C(y-z) A^e_{\sigma]}(z)\partial_\mu C(z-w) A^g_\nu(w)\partial^{[\lambda} C(w-v) A^{h\sigma}(v) A^{i\nu]}(v)\}$$

$$+\frac{3g^4}{2} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial^\lambda C(x-y) \{$$

$$-A^c_{[\lambda}(y)\partial_{\mu]} C(y-z) A^{\rho e}(z)\partial^\sigma C(z-w) A^{\nu g}(w)\partial_{[\rho} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v)$$

$$-A^{[\rho c}(y)\partial_\lambda C(y-z) A^{\sigma]e}(z)\partial_\rho C(z-w) A^{\nu g}(w)\partial_{[\mu} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v)$$

$$+A^{[\rho c}(y)\partial_\mu C(y-z) A^{\sigma]e}(z)\partial_\rho C(z-w) A^{\nu g}(w)\partial_{[\lambda} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v)$$

$$+A^{[\rho c}(y)\partial^{\sigma]} C(y-z) A^e_\lambda(z)\partial_\rho C(z-w) A^{\nu g}(w)\partial_{[\mu} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v)$$

$$-A^{[\rho c}(y)\partial^{\sigma]} C(y-z) A^e_\mu(z)\partial_\rho C(z-w) A^{\nu g}(w)\partial_{[\lambda} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v)\}$$

$$
\begin{aligned}
&+\frac{g^4}{2} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial^\lambda C(x-y) A^{\rho c}(y) \{ \\
&\quad -8\partial^\nu C(y-z) A^e_{[\mu}(z) \partial_\lambda C(z-w) A^g_\rho(w) \partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v) \\
&\quad -2\partial_\lambda C(y-z) A^{\nu e}(z) \partial_{[\mu} C(z-w) A^g_\rho(w) \partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v) \\
&\quad -2\partial_\rho C(y-z) A^{\nu e}(z) \partial_{[\mu} C(z-w) A^g_\lambda(w) \partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v) \\
&\quad +2\partial_\mu C(y-z) A^{\nu e}(z) \partial_{[\lambda} C(z-w) A^g_\rho(w) \partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v) \\
&\quad -\frac{3}{2} \partial_{[\mu} C(y-z) A^e_{\rho]}(z) \partial^\sigma C(z-w) A^{\nu g}(w) \partial_{[\lambda} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +\frac{3}{2} \partial_{[\lambda} C(y-z) A^e_{\rho]}(z) \partial^\sigma C(z-w) A^{\nu g}(w) \partial_{[\mu} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +\frac{3}{2} \partial_{[\mu} C(y-z) A^e_{\lambda]}(z) \partial^\sigma C(z-w) A^{\nu g}(w) \partial_{[\rho} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +\partial_{[\mu} C(y-z) A^e_\lambda(z) \partial^\nu C(z-w) A^g_{\rho]}(w) \partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v) \\
&\quad -\partial_{[\mu} C(y-z) A^e_\lambda(z) \partial^\nu C(z-w) A^g_{|\nu}(w) \partial^\sigma C(w-v) A^h_{\sigma|}(v) A^i_{\rho]}(v) \} \\
&+\frac{3g^4}{4} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}w\, \mathrm{d}v \partial^\lambda C(x-y) A^{\rho c}(y) \{ \\
&\quad -20\partial^\nu C(y-z) A^{\sigma e}(z) \partial_{[\nu} C(z-w) A^g_\mu(w) \partial_\sigma C(w-v) A^h_\lambda(v) A^i_{\rho]}(v) \\
&\quad -4\partial_\rho C(y-z) A^{\nu e}(z) \partial^\sigma C(z-w) A^g_{[\mu}(w) \partial_\lambda C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +4\partial^\sigma C(y-z) A^e_\sigma(z) \partial^\nu C(z-w) A^g_{[\mu}(w) \partial_\lambda C(w-v) A^h_\rho(v) A^i_{\nu]}(v) \\
&\quad -2\partial_{[\mu} C(y-z) A^{\sigma e}(z) \partial_{\lambda]} C(z-w) A^{\nu g}(w) \partial_{[\rho} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +2\partial^\sigma C(y-z) A^e_{[\mu}(z) \partial_{\lambda]} C(z-w) A^{\nu g}(w) \partial_{[\rho} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad -2\partial^\sigma C(y-z) A^e_{[\sigma}(z) \partial_{\rho]} C(z-w) A^{\nu g}(w) \partial_{[\mu} C(w-v) A^h_\lambda(v) A^i_{\nu]}(v) \\
&\quad -2\partial_{[\mu} C(y-z) A^e_\lambda(z) \partial_{\rho]} C(z-w) A^{\nu g}(w) \partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v) \\
&\quad -2\partial^\sigma C(y-z) A^e_\rho(z) \partial^\nu C(z-w) A^g_{[\mu}(w) \partial_\lambda C(w-v) A^h_{\sigma]}(v) A^i_\nu(v) \\
&\quad -\partial_\rho C(y-z) A^e_\sigma(z) \partial^\sigma C(z-w) A^{\nu g}(w) \partial_{[\mu} C(w-v) A^h_\lambda(v) A^i_{\nu]}(v) \\
&\quad -\partial^\sigma C(y-z) A^e_\sigma(z) \partial_\mu C(z-w) A^{\nu g}(w) \partial_{[\lambda} C(w-v) A^h_\rho(v) A^i_{\nu]}(v) \\
&\quad -\partial^\sigma C(y-z) A^e_\rho(z) \partial^\nu C(z-w) A^g_{[\mu}(w) \partial_{|\nu|} C(w-v) A^h_\lambda(v) A^i_{\sigma]}(v) \\
&\quad +\partial^\sigma C(y-z) A^e_\rho(z) \partial^\nu C(z-w) A^g_\nu(w) \partial_{[\mu} C(w-v) A^h_\lambda(v) A^i_{\sigma]}(v) \\
&\quad +\partial^\sigma C(y-z) A^e_\sigma(z) \partial_\lambda C(z-w) A^{\nu g}(w) \partial_{[\mu} C(w-v) A^h_\rho(v) A^i_{\nu]}(v) \} \\
&+\mathcal{O}(g^5)
\end{aligned}
$$

$$
\begin{aligned}
(\mathcal{T}_g A)^a_\mu(x) = A^a_\mu(x) &+ g f^{abc} \int \mathrm{d}y\; \partial^\rho C(x-y) A^b_\mu(y) A^c_\rho(y) \\
&+ \frac{3g^2}{2} f^{abc} f^{bde} \int \mathrm{d}y\; \mathrm{d}z\; \partial^\rho C(x-y) A^{\lambda c}(y) \partial_{[\rho} C(y-z) A^d_\mu(z) A^e_{\lambda]}(z) \\
&+ \frac{g^3}{2} f^{abc} f^{bde} f^{cfg} \int \mathrm{d}y\; \mathrm{d}z\; \mathrm{d}w\; \partial^\rho C(x-y) \\
&\quad \times \partial^\lambda C(y-z) A^d_\lambda(z) A^{\sigma e}(z) \partial_{[\rho} C(y-w) A^f_\mu(w) A^g_{\sigma]}(w) \\
&+ g^3 f^{abc} f^{bde} f^{dfg} \int \mathrm{d}y\; \mathrm{d}z\; \mathrm{d}w\; \partial^\rho C(x-y) A^{\lambda c}(y) \Big\{ \\
&\quad - \partial^\sigma C(y-z) A^e_\sigma(z) \partial_{[\rho} C(z-w) A^f_\mu(w) A^g_{\lambda]}(w) \\
&\quad + \partial_{[\rho} C(y-z) A^e_\mu(z) \partial^\sigma C(z-w) A^f_{\lambda]}(w) A^g_\sigma(w) \Big\}
\end{aligned}
$$

$$+\frac{g^3}{3} f^{abc} f^{bde} f^{dfg} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w \Big\{$$

$$+6\,\partial_\rho C(x-y)A^{\lambda\,c}(y)\partial^{[\rho}C(y-z)A^{\sigma]}(z)\partial_{[\lambda}C(z-w)A^f_\mu(w)A^g_{\sigma]}(w)$$

$$-6\,\partial^\rho C(x-y)A^c_\lambda(y)\partial^{[\lambda}C(y-z)A^{\sigma]\,e}(z)\partial_{[\rho}C(z-w)A^f_\mu(w)A^g_{\sigma]}(w)$$

$$-6\,\partial_\rho C(x-y)A^c_\lambda(y)\partial_{[\sigma}C(y-z)A^e_{\mu]}(z)\partial^{[\rho}C(z-w)A^{\lambda\,f}(w)A^{\sigma]\,g}(w)$$

$$+2\,\partial^\rho C(x-y)A^c_{[\rho}(y)\partial_{\mu]}C(y-z)A^{\lambda\,e}(z)\partial^\sigma C(z-w)A^f_\lambda(w)A^g_\sigma(w)$$

$$-\,\partial_\mu C(x-y)\,\partial^\rho\left(A^c_\rho(y)C(y-z)\right)A^{\lambda\,e}(z)\partial^\sigma C(z-w)A^f_\lambda(w)A^g_\sigma(w)\Big\}$$

$$-\frac{g^3}{3} f^{abc} f^{bde} f^{dfg} \int \mathrm{d}y\ \mathrm{d}z\ A^c_\mu(x)C(x-y)A^{\rho\,e}(y)\partial^\lambda C(y-z)A^f_\rho(z)A^g_\lambda(z)$$

$$+\frac{g^4}{12} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w$$

$$\times\, C(x-y)A^{\lambda\,e}(y)\partial^\rho C(y-z)A^f_\lambda(z)A^g_\rho(z)\partial^\sigma C(x-w)A^h_\sigma(w)A^i_\mu(w)$$

$$+\frac{g^4}{8} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial^\lambda C(x-y)\partial_\rho C(y-z)\Big\{$$

$$-9A^e_\sigma(z)\partial^{[\rho}C(z-w)A^{\sigma\,f}(w)A^{\nu]\,g}(w)\partial_{[\mu}C(y-v)A^h_\lambda(v)A^i_{\nu]}(v)$$

$$+4A^{[\rho\,e}(z)\partial^{|\sigma|}C(z-w)A^f_\sigma(w)A^{\nu]\,g}(w)\partial_{[\mu}C(y-v)A^h_\lambda(v)A^i_{\nu]}(v)$$

$$-2A^{\rho\,e}(z)\partial_{[\mu}C(z-w)A^f_\lambda(w)A^g_{\nu]}(w)\partial^\sigma C(y-v)A^h_\sigma(v)A^{\nu\,i}(v)\Big\}$$

$$-\frac{g^4}{12} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial_\mu C(x-y)\partial^\lambda C(y-z)$$

$$\times\, A^{\rho\,e}(z)\partial^\sigma C(z-w)A^f_\sigma(w)A^g_\rho(w)\partial^\tau C(y-v)A^h_\tau(v)A^i_\lambda(v)$$

$$+\frac{g^4}{2} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial_\lambda C(x-y)\Big\{$$

$$+\partial_{[\mu}C(y-z)A^e_{\rho]}(z)\partial^{[\lambda}C(z-w)A^{\rho\,f}(w)A^{\nu]\,g}(w)\partial^\sigma C(y-v)A^h_\sigma(v)A^i_\nu(v)$$

$$-\,\partial^{[\lambda}C(y-z)A^{\rho]\,e}(z)\partial_{[\mu}C(z-w)A^f_\rho(w)A^g_{\nu]}(w)\partial^\sigma C(y-v)A^h_\sigma(v)A^i_\nu(v)\Big\}$$

$$+\frac{g^4}{6} f^{abc} f^{bde} f^{dfg} f^{chi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial^\lambda C(x-y)\Big\{$$

$$+3\partial^{[\rho}C(y-z)A^{\nu]\,e}(z)\partial_{[\mu}C(z-w)A^f_\lambda(w)A^g_{\rho]}(w)\partial^\sigma C(y-v)A^h_\sigma(v)A^i_\nu(v)$$

$$+\partial_{[\lambda}C(y-z)A^{\nu\,e}(z)\partial^\sigma C(z-w)A^f_{|\sigma}(w)A^g_\nu(w)\partial^\rho C(y-v)A^h_{\rho|}(v)A^i_{\mu]}(v)\Big\}$$

$$-\frac{g^4}{3} f^{abc} f^{bde} f^{dfg} f^{ehi} \int \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w$$

$$\times\, A^c_\mu(x)C(x-y)\partial^\lambda C(y-z)A^f_\lambda(z)A^{\rho\,g}(z)\partial^\sigma C(y-w)A^h_\sigma(w)A^i_\rho(w)$$

$$-\frac{g^4}{3} f^{abc} f^{bde} f^{dfg} f^{ehi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial_\mu C(x-y)$$

$$\times\,\partial^\lambda\left(A^c_\lambda(y)C(y-z)\right)\partial^\rho C(z-w)A^f_\rho(w)A^{\nu\, g}(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^i_\nu(v)$$

$$-\frac{g^4}{12}f^{abc}f^{bde}f^{dfg}f^{ehi}\int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial^\lambda C(x-y)A^{\rho\, c}(y)\Big\{$$

$$-3\partial_\rho C(y-z)\partial_{[\mu}C(z-w)A^f_\lambda(w)A^g_{\nu]}(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^i_\nu(v)$$

$$-3\partial^\nu C(y-z)\partial_{[\mu}C(z-w)A^f_\rho(w)A^g_{\nu]}(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^i_\lambda(v)$$

$$+3\partial^\nu C(y-z)\partial_{[\lambda}C(z-w)A^f_\rho(w)A^g_{\nu]}(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^i_\mu(v)$$

$$-3\partial_\mu C(y-z)\partial_{[\lambda}C(z-w)A^f_\rho(w)A^g_{\nu]}(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^i_\nu(v)$$

$$+3\partial_\lambda C(y-z)\partial_{[\mu}C(z-w)A^f_\rho(w)A^g_{\nu]}(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^{\nu\, i}(v)$$

$$-2\partial_{[\lambda}C(y-z)\partial^\nu C(z-w)A^f_{|\nu|}(w)A^g_{\mu]}(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^i_\rho(v)$$

$$+\partial_\rho C(y-z)\partial^\nu C(z-w)A^f_\nu(w)A^g_\mu(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^i_\lambda(v)\Big\}$$

$$+\frac{g^4}{6}f^{abc}f^{bde}f^{dfg}f^{ehi}\int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial^\lambda C(x-y)\Big\{$$

$$-7A^c_{[\mu}(y)\partial_{\lambda]}C(y-z)\partial^\rho C(z-w)A^f_\rho(w)A^{\nu\, g}(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^i_\nu(v)$$

$$+3A^{[\nu\, c}(y)\partial^{\rho]}C(y-z)\partial_{[\mu}C(z-w)A^f_\lambda(w)A^g_{\rho]}(w)\partial^\sigma C(z-v)A^h_\sigma(v)A^i_\nu(v)\Big\}$$

$$-\frac{g^4}{2}f^{abc}f^{bde}f^{dfg}f^{fhi}\int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w$$

$$\times\,\partial^\lambda C(x-y)A^c_{[\mu}(y)A^e_{\lambda]}(y)C(y-z)A^{\rho\, g}(z)\partial^\sigma C(z-w)A^h_\sigma(w)A^i_\rho(w)$$

$$+\frac{g^4}{12}f^{abc}f^{bde}f^{dfg}f^{fhi}\int \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ A^c_\mu(x)C(x-y)\Big\{$$

$$+9A^{\lambda\, e}(y)\partial^\rho C(y-z)A^{\sigma\, g}(z)\partial_{[\lambda}C(z-w)A^h_\rho(w)A^i_{\sigma]}(w)$$

$$+4A^{[\lambda\, e}(y)\partial^{\rho]}C(y-z)A^g_\lambda(z)\partial^\sigma C(z-w)A^h_\sigma(w)A^i_\rho(w)$$

$$-3A^e_\lambda(y)\partial^\lambda C(y-z)A^{\rho\, g}(z)\partial^\sigma C(z-w)A^h_\sigma(w)A^i_\rho(w)$$

$$-3\partial^\lambda\left(A^e_\lambda(y)C(y-z)\right)A^{\rho\, g}(z)\partial^\sigma C(z-w)A^h_\sigma(w)A^i_\rho(w)\Big\}$$

$$+\frac{g^4}{12}f^{abc}f^{bde}f^{dfg}f^{fhi}\int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial_\mu C(x-y)\partial^\lambda\left(A^c_\lambda(y)C(y-z)\right)\Big\{$$

$$+9A^{\rho\, e}(z)\partial^\sigma C(z-w)A^{\nu\, g}(w)\partial_{[\rho}C(w-v)A^h_\sigma(v)A^i_{\nu]}(v)$$

$$+4A^{[\rho\, e}(z)\partial^{\nu]}C(z-w)A^g_\rho(w)\partial^\sigma C(w-v)A^h_\sigma(v)A^i_\nu(v)$$

$$-3\partial^\rho\left(A^e_\rho(z)C(z-w)\right)A^{\nu\, g}(w)\partial^\sigma C(w-v)A^h_\sigma(v)A^i_\nu(v)$$

$$-3A^e_\rho(z)\partial^\rho C(z-w)A^{\nu\, g}(w)\partial^\sigma C(w-v)A^h_\sigma(v)A^i_\nu(v)\Big\}$$

$$
\begin{aligned}
&+\frac{g^4}{2} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial^\lambda C(x-y) A^c_{[\mu}(y)\partial_{\lambda]} C(y-z) \Big\{ \\
&\quad - \partial^\rho \left(A^e_\rho(z) C(z-w)\right) A^{\nu\, g}(w) \partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v) \\
&\quad - A^e_\rho(z) \partial^\rho C(z-w) A^{\nu\, g}(w) \partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v) \Big\} \\
&+\frac{2g^4}{3} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial^\lambda C(x-y) \\
&\quad \times A^c_{[\mu}(y)\partial_{\lambda]} C(y-z) A^{\rho\, e}(z) \partial^\nu C(z-w) A^g_{[\rho}(w) \partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v) \\
&+\frac{3g^4}{2} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial_\lambda C(x-y) \Big\{ \\
&\quad + 4 A^{\rho\, c}(y) \partial^{[\lambda} C(y-z) A^{e\, \nu]}(z) \partial^\sigma C(z-w) A^g_{[\mu}(w) \partial_\rho C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad - 4 A^c_r(y) \partial_{[\mu} C(y-z) A^e_{\nu]}(z) \partial_\sigma C(z-w) A^{g\, [\lambda}(w) \partial^\rho C(w-v) A^{h\, \sigma}(v) A^{i\, \nu]}(v) \\
&\quad - A^{\rho\, c}(y) \partial_{[\rho} C(y-z) A^e_{\sigma]}(z) \partial_\mu C(z-w) A^g_\nu(w) \partial^{[\lambda} C(w-v) A^{h\, \sigma}(v) A^{i\, \nu]}(v) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{3g^4}{2} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial^\lambda C(x-y) \Big\{ \\
&\quad - A^c_{[\lambda}(y) \partial_{\mu]} C(y-z) A^{\rho\, e}(z) \partial^\sigma C(z-w) A^{\nu\, g}(w) \partial_{[\rho} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad - A^{[\rho\, c}(y) \partial_\lambda C(y-z) A^{\sigma]\, e}(z) \partial_\rho C(z-w) A^{\nu\, g}(w) \partial_{[\mu} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad + A^{[\rho\, c}(y) \partial_\mu C(y-z) A^{\sigma]\, e}(z) \partial_\rho C(z-w) A^{\nu\, g}(w) \partial_{[\lambda} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad + A^{[\rho\, c}(y) \partial^{\sigma]} C(y-z) A^e_\lambda(z) \partial_\rho C(z-w) A^{\nu\, g}(w) \partial_{[\mu} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad - A^{[\rho\, c}(y) \partial^{\sigma]} C(y-z) A^e_\mu(z) \partial_\rho C(z-w) A^{\nu\, g}(w) \partial_{[\lambda} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{g^4}{2} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial^\lambda C(x-y) A^{\rho c}(y) \Bigg\{ \\
&\quad -8\partial^\nu C(y-z) A^e_{[\mu}(z) \partial_\lambda C(z-w) A^g_\rho(w) \partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v) \\
&\quad -2\partial_\lambda C(y-z) A^{\nu e}(z) \partial_{[\mu} C(z-w) A^g_\rho(w) \partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v) \\
&\quad -2\partial_\rho C(y-z) A^{\nu e}(z) \partial_{[\mu} C(z-w) A^g_\lambda(w) \partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v) \\
&\quad +2\partial_\mu C(y-z) A^{\nu e}(z) \partial_{[\lambda} C(z-w) A^g_\rho(w) \partial^\sigma C(w-v) A^h_{|\sigma|}(v) A^i_{\nu]}(v) \\
&\quad -\frac{3}{2}\partial_{[\mu} C(y-z) A^e_{\rho]}(z) \partial^\sigma C(z-w) A^{\nu g}(w) \partial_{[\lambda} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +\frac{3}{2}\partial_{[\lambda} C(y-z) A^e_{\rho]}(z) \partial^\sigma C(z-w) A^{\nu g}(w) \partial_{[\mu} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +\frac{3}{2}\partial_{[\mu} C(y-z) A^e_{\lambda]}(z) \partial^\sigma C(z-w) A^{\nu g}(w) \partial_{[\rho} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +\partial_{[\mu} C(y-z) A^e_\lambda(z) \partial^\nu C(z-w) A^g_{\rho]}(w) \partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v) \\
&\quad -\partial_{[\mu} C(y-z) A^e_\lambda(z) \partial^\nu C(z-w) A^g_{|\nu}(w) \partial^\sigma C(w-v) A^h_{\sigma|}(v) A^i_{\rho]}(v) \Bigg\} \\
&+\frac{3g^4}{4} f^{abc} f^{bde} f^{dfg} f^{fhi} \int \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}w\ \mathrm{d}v\ \partial^\lambda C(x-y) A^{\rho c}(y) \Bigg\{ \\
&\quad -20\partial^\nu C(y-z) A^{\sigma e}(z) \partial_{[\nu} C(z-w) A^g_\mu(w) \partial_\sigma C(w-v) A^h_\lambda(v) A^i_{\rho]}(v) \\
&\quad -4\partial_\rho C(y-z) A^{\nu e}(z) \partial^\sigma C(z-w) A^g_{[\mu}(w) \partial_\lambda C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +4\partial^\sigma C(y-z) A^e_\sigma(z) \partial^\nu C(z-w) A^g_{[\mu}(w) \partial_\lambda C(w-v) A^h_\rho(v) A^i_{\nu]}(v) \\
&\quad -2\partial_{[\mu} C(y-z) A^{\sigma e}(z) \partial_{\lambda]} C(z-w) A^{\nu g}(w) \partial_{[\rho} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad +2\partial^\sigma C(y-z) A^e_{[\mu}(z) \partial_{\lambda]} C(z-w) A^{\nu g}(w) \partial_{[\rho} C(w-v) A^h_\sigma(v) A^i_{\nu]}(v) \\
&\quad -2\partial^\sigma C(y-z) A^e_{[\sigma}(z) \partial_{\rho]} C(z-w) A^{\nu g}(w) \partial_{[\mu} C(w-v) A^h_\lambda(v) A^i_{\nu]}(v) \\
&\quad -2\partial_{[\mu} C(y-z) A^e_\lambda(z) \partial_{\rho]} C(z-w) A^{\nu g}(w) \partial^\sigma C(w-v) A^h_\sigma(v) A^i_\nu(v) \\
&\quad -2\partial^\sigma C(y-z) A^e_\rho(z) \partial^\nu C(z-w) A^g_{[\mu}(w) \partial_\lambda C(w-v) A^h_{\sigma]}(v) A^i_\nu(v) \\
&\quad -\partial_\rho C(y-z) A^e_\sigma(z) \partial^\sigma C(z-w) A^{\nu g}(w) \partial_{[\mu} C(w-v) A^h_\lambda(v) A^i_{\nu]}(v) \\
&\quad -\partial^\sigma C(y-z) A^e_\sigma(z) \partial_\mu C(z-w) A^{\nu g}(w) \partial_{[\lambda} C(w-v) A^h_\rho(v) A^i_{\nu]}(v) \\
&\quad -\partial^\sigma C(y-z) A^e_\rho(z) \partial^\nu C(z-w) A^g_{[\mu}(w) \partial_{|\nu|} C(w-v) A^h_\lambda(v) A^i_{\sigma]}(v) \\
&\quad +\partial^\sigma C(y-z) A^e_\rho(z) \partial^\nu C(z-w) A^g_\nu(w) \partial_{[\mu} C(w-v) A^h_\lambda(v) A^i_{\sigma]}(v) \\
&\quad +\partial^\sigma C(y-z) A^e_\sigma(z) \partial_\lambda C(z-w) A^{\nu g}(w) \partial_{[\mu} C(w-v) A^h_\rho(v) A^i_{\nu]}(v) \Bigg\} \\
&+\mathcal{O}(g^5)\,.
\end{aligned}
$$

$$\star F = -\kappa \wedge F,$$

$$\iota_R F = 0, F_H^+ = 0,$$

$$\star F = -\kappa \wedge F.$$

$$\mathcal{A} = \iota_R F - \left(d_A^\dagger B\right)^H = 0,$$
$$\mathcal{B} = F_H^+ - \frac{1}{4} B \times B - \frac{1}{2} \iota_R d_A B = 0.$$

$$F_H^+ = 1/2(F_H + \iota_R \star F_H)$$

$$(\alpha, \beta) = \mathrm{Tr} \int_{M_5} \alpha \wedge \star \beta, \|\alpha\|^2 = (\alpha, \alpha).$$

$$(X \times Y)_{mn} = X_{mp} Y_n^p - X_{np} Y_m^p, X, Y \in \Omega_H^{2+}$$

$$(X \times Y)_{mn}^a = \frac{1}{2} f_{bc}^a \left(X_{mp}^b (Y^c)_n^p - X_{np}^b (Y^c)_m^p\right)$$

$$\kappa \wedge \hat{F} = - \star \hat{F} + d_A B$$

$$\hat{F} = F - \frac{1}{4} B \times B$$

$$\hat{F} - \kappa \iota_R \hat{F} = -\iota_R \star \hat{F} + \iota_R d_A B$$

$$\left(d_A^\dagger B\right)^H = \iota_R \star d_A B$$

$$d_A^\dagger B = \star\, d_A \star B = \star\, d_A(\kappa \wedge B) = \star\,(d\kappa \wedge B) - \star\,(\kappa \wedge d_A B)$$

$$\left(d_A^\dagger B\right)^H = -\left(\star\,(\kappa \wedge d_A B)\right)^H = (\iota_R \star d_A B)^H = \iota_R \star d_A B$$

$$g \to \lambda^2 g, \kappa \to \lambda\kappa, R \to \lambda^{-1} R, B \to \lambda B, A \to A,$$

$$\|\mathcal{A}\|^2 = (\mathcal{A}, \mathcal{A}) = \|\iota_R F\|^2 - 2\left(\iota_R F, d_A^\dagger B\right) + \left\|\left(d_A^\dagger B\right)^H\right\|^2,$$

$$\left(\iota_R F, d_A^\dagger B\right) = (d_A \iota_R F, B) = (\mathcal{L}_R^A F, B)$$

$$\|\mathcal{B}\|^2 = \left\|F_H^+ - \frac{1}{4} B \times B\right\|^2 - \left(F_H^+ - \frac{1}{4} B \times B, \iota_R d_A B\right) + \frac{1}{4} \|\iota_R d_A B\|^2,$$

$$\left(F_H^+ - \frac{1}{4} B \times B, \iota_R d_A B\right) = \left(F_H^+ - \frac{1}{4} B \times B, \mathcal{L}_R^A B\right)$$

$$\left(F_H^+ - \frac{1}{4} B \times B, \mathcal{L}_R^A B\right) = (F_H^+, \mathcal{L}_R^A B) - \frac{1}{4}(B \times B, \mathcal{L}_R^A B) = (F, \mathcal{L}_R^A B) - \frac{1}{4}(B \times B, \mathcal{L}_R^A B)$$

$$E = \frac{1}{2}\|\mathcal{A}\|^2 + \|\mathcal{B}\|^2 = \frac{1}{2}\|\iota_R F\| + \frac{1}{2}\left\|\left(d_A^\dagger B\right)^H\right\|^2 + \frac{1}{4}\|\iota_R d_A B\|^2 + \left\|F_H^+ - \frac{1}{4} B \times B\right\|^2.$$

$$\iota_R F = 0$$
$$F_H^+ - \frac{1}{4} B \times B = 0$$
$$\left(d_A^\dagger B\right)^H = 0$$
$$\iota_R d_A B = 0$$

$$\star\, d_A B = \mathcal{L}_R^A B + \kappa d_A^\dagger B,$$

$$\iota_R \star d_A B = \iota_R\left(\kappa d_A^\dagger B\right) = \left(d_A^\dagger B\right)^H = 0$$

$$\iota_R \star d_A B = \iota_R d_A B = 0$$

$$\mathcal{L}_R^A B + \kappa d_A^\dagger B = \mathcal{L}_R^A B = \{\iota_R, d_A\} B\ \kappa d_A^\dagger B \left(d_A^\dagger B\right)^H$$

$$\iota_R F = 0$$
$$F_H^+ - \frac{1}{4} B \times B = 0$$
$$d_A B = 0$$

$$\left(d_A^\dagger C, d_A^\dagger B\right) =-\frac{1}{2}\left(\mathcal{L}_R^A C, \mathcal{L}_R^A B\right) + \frac{1}{2}(J \cdot C, J \cdot B) + (F, C \times B)$$
$$+\mathrm{Tr}\int_{M_5}\left(\frac{1}{4}\nabla C \cdot \nabla B - \frac{1}{4} C_{ij} X^{ijkl} B_{kl}\right)\mathrm{vol}_g$$

$$J \cdot B = J^{pq} B_{pq}, \nabla C \cdot \nabla B = \left(\nabla_i C_{jk}\right)\left(\nabla^i B^{jk}\right)$$

$$X_{pqrs} = R_{pqrs} - \frac{1}{2}(Ric \barwedge g)_{pqrs} = W_{pqrs} - \frac{1}{6}(Ric \barwedge g)_{pqrs} - \frac{s}{24}(g \barwedge g)_{pqrs},$$

$$d_A^\dagger B = \left(d_A^\dagger B\right)^H - \kappa(J \cdot B)$$

$$E =\frac{1}{2}\|\iota_R F\|^2 + \|F_H^+\|^2 + \frac{1}{16}\|B \times B\|^2 + \frac{1}{8}\mathrm{Tr}\int_{M_5}(\nabla B \cdot \nabla B)\mathrm{vol}_g$$
$$-\frac{1}{4}\|J \cdot B\|^2 - \frac{1}{8}\mathrm{Tr}\int_{M_5}\left(B_{ij} X^{ijkl} B_{kl}\right)\mathrm{vol}_g$$

$$\left\|\left(d^\dagger w\right)^H\right\|^2 = -\frac{1}{2}\|\mathcal{L}_R w\|^2 - \frac{1}{2}\|J \cdot w\|^2 - \frac{1}{4}\int_{M_5}\left(w_{ij} X^{ijkl} w_{kl}\right)\mathrm{vol}_g + \frac{1}{4}\int_{M_5}\nabla w \cdot \nabla w \mathrm{vol}_g$$

$$\left\|\left(d^\dagger w\right)^H\right\|^2 = -\frac{1}{2}\|J \cdot w\|^2 - \frac{1}{4}\int_{M_5}\left(w_{ij} X^{ijkl} w_{kl}\right)\mathrm{vol}_g + \frac{1}{4}\int_{M_5}\nabla w \cdot \nabla w \mathrm{vol}_g$$

$$R^j \nabla_i (d\kappa)_{jk} = \nabla_i\left(R^j (d\kappa)_{jk}\right) - \left(\nabla_i R^j\right)(d\kappa)_{jk} = 2\left(g_{ik} - R_i R_k\right) \neq 0.$$

$$\nabla B_3 \cdot \nabla C = 0$$

$$\nabla_i\left(J_{jk} f\right)\nabla^i C^{jk} = \left(f \nabla_i J_{jk} + J_{jk}\nabla_i f\right)\nabla^i C^{jk} = \left(-2 f R_j g_{ik} + J_{jk}\nabla_i f\right)\nabla^i C^{jk}$$
$$= 2f\left(\nabla^i{}_{R_j}\right) g_{ik} C^{jk} - \left(\nabla^i J_{jk}\right)\nabla_i f C^{jk}$$
$$= -2 f J^i{}_j g_{ik} C^{jk} - \nabla_i f\left(-2 R_j \delta_k^i\right) C^{jk} = 0,$$

$$B_3^{ij} X_{ijkl} C^{kl} = 0$$

$$B_3^{ij} X_{ijkl} C^{kl} = B_3^{ij}\left(R_{ijkl} - 2 R_{pi}{}^p{}_k g_{jl}\right) C^{kl}.$$

$$\left(\mathcal{L}_R^A B_3, \mathcal{L}_R^A C\right) = 0$$

$$\left\|\left(d_A^\dagger B\right)^H\right\|^2 - \left\|\left(d_A^\dagger B_3\right)^H\right\|^2 = \left\|d_A^\dagger \tilde{B}\right\|^2 + 2\left(d_A^\dagger \tilde{B}, \left(d_A^\dagger B_3\right)^H\right),$$

$$\left\|d_A^\dagger \tilde{B}\right\|^2 + 2\left(d_A^\dagger \tilde{B}, d_A^\dagger B_3\right) =\frac{1}{4}\mathrm{Tr}\int\left(\nabla\tilde{B} \cdot \nabla\tilde{B} - \tilde{B} X \tilde{B}\right)\mathrm{vol}_g$$
$$+\left(F, \tilde{B} \times \tilde{B}\right) + 2\left(F, \tilde{B} \times B_3\right) - \frac{1}{2}\left\|\mathcal{L}_R \tilde{B}\right\|^2,$$

$$\left(F, \tilde{B} \times \tilde{B}\right) + 2\left(F, \tilde{B} \times B_3\right) = (F, B \times B).$$

$$E =\frac{1}{2}\|\iota_R F\| + \frac{1}{2}\left\|\left(d_A^\dagger B_3\right)^H\right\|^2 + \frac{1}{4}\left\|\mathcal{L}_R^A B_3\right\|^2 + \|F_H^+\|^2 + \frac{1}{16}\|B \times B\|^2$$
$$+\frac{1}{8}\mathrm{Tr}\int\left(\nabla\tilde{B} \cdot \nabla\tilde{B} - \tilde{B} X \tilde{B}\right)\mathrm{vol}_g.$$

$$Z^{mn} R_{mnpq} \bar{Z}^{pq}\mathrm{vol}_g = 4 Z \wedge\star \bar{Z}, Z^{mn}(R \barwedge g)_{mnpq}\bar{Z}^{pq}\mathrm{vol}_g = 2(s-4) Z \wedge\star \bar{Z}.$$

$$Z^{mn}\bar{Z}^{pq}(R\,\bar{\wedge}\,g)_{mnpq} = -4Z^{mn}\bar{Z}^{pq}J_p{}^rJ_q{}^sR_{mr}g_{ns}$$
$$= -4Z^{mn}\bar{Z}^{pq}J_p{}^rJ_{qn}R_{mr} = -4Z^{mn}\bar{Z}^{pq}(c_1)_{mp}J_{nq}$$

$$Z^{mn}\bar{Z}^{pq}(c_1)_{mp}J_{nq} = \frac{1}{16}Z^{[mn}\bar{Z}^{pq]}(c_1)_{mp}J_{nq} = -\frac{1}{24}(Z\wedge\bar{Z})^{mnpq}(c_1\wedge J)_{mnpq}$$

$$c_1\wedge J = \frac{1}{4}(s-4)J\wedge J$$

$$Z^{mn}(R\,\bar{\wedge}\,g)_{mnpq}\bar{Z}^{pq}\mathrm{vol}_g = 2(s-4)Z\wedge\star\,\bar{Z}$$

$$\mathrm{Tr}\int_{M_5}(\tilde{B}_{ij}X^{ijkl}\tilde{B}_{kl})\mathrm{vol}_g = \mathrm{Tr}\int_{M_5}(8-s)\tilde{B}\wedge\star\,\tilde{B}$$

$$B_3^{mn}X_{mnpq}B_3^{pq}\mathrm{vol}_g = -12B_3\star B_3$$

$$X = 0 - \frac{1}{6}\cdot 4g\,\bar{\wedge}\,g - \frac{20}{24}g\,\bar{\wedge}\,g = -\frac{3}{2}g\,\bar{\wedge}\,g,$$

$$\kappa\to a\kappa, R\to a^{-1}R, g\to ag+(a^2-a)\kappa\otimes\kappa$$

$$s\to a^{-1}(s+4)-4.$$

$$\iota_R F = F_H^+ = d_A B = 0,$$

$$\nabla\tilde{B}\cdot\nabla\tilde{B} = (\nabla\tilde{B})_H\cdot(\nabla\tilde{B})_H + 12\tilde{B}*\tilde{B}$$

$$\bar{\partial}^B:\Omega_H^{p,q}\to\Omega_H^{p,q+1};\ \partial^B:\Omega_H^{p,q}\to\Omega_H^{p+1,q},$$

$$\Delta^B = \{d^B,(d^B)^\dagger\}, \Delta^B_{\bar{\partial}^B} = \left\{\bar{\partial}^B,\left(\bar{\partial}^B\right)^\dagger\right\};\ \Delta^B = 2\Delta^B_{\bar{\partial}^B} = 2\Delta^B_{\partial^B}$$

$$d_A B = \left(d_A^\dagger B\right)^H = 0$$

$$0 = d_A B_3 = d_A(Jf) = (dJ)f + Jd_A f = Jd_A f$$

$$\begin{aligned}
&\delta_\epsilon A = \bar{\epsilon}\Psi + \epsilon\bar{\Psi},\\
&\delta_\epsilon\sigma = -2\bar{\epsilon}\iota_R\bar{\Psi},\\
&\delta_\epsilon\bar{\sigma} = -2\epsilon\iota_R\Psi,\\
&\delta_\epsilon B = -\bar{\epsilon}\chi + \epsilon\bar{\chi},\\
&\delta_\epsilon H = -i\mathcal{L}_R^A(\bar{\epsilon}\chi+\epsilon\bar{\chi}) - \epsilon[\sigma,\chi] - \bar{\epsilon}[\bar{\sigma},\bar{\chi}] - \iota_R[B,\bar{\epsilon}\Psi-\epsilon\bar{\Psi}],\\
&\delta_\epsilon\tilde{\mathcal{F}} = \epsilon(i\iota_R d_A\bar{\Psi} - [\sigma,\Psi^H]) + \bar{\epsilon}(-i\iota_R d_A\Psi + [\bar{\sigma},\bar{\Psi}^H]),\\
&\delta_\epsilon\Psi = \epsilon\left(2\tilde{\mathcal{F}} - 2i\iota_R F + \kappa[\sigma,\bar{\sigma}]\right) + 2i\bar{\epsilon}d_A\bar{\sigma},\\
&\delta_\epsilon\bar{\Psi} = \bar{\epsilon}\left(-2\tilde{\mathcal{F}} - 2i\iota_R F - \kappa[\sigma,\bar{\sigma}]\right) + 2i\epsilon d_A\sigma,\\
&\delta_\epsilon\chi = 2\epsilon(H - i\mathcal{L}_R^A B) + \bar{\epsilon}[\bar{\sigma},B],\\
&\delta_\epsilon\bar{\chi} = 2\bar{\epsilon}(H + i\mathcal{L}_R^A B) - \epsilon[\sigma,B].
\end{aligned}$$

$$\langle\Psi,B\rangle_l = [\Psi_j,B_{kl}]g^{jk}$$

$$\{\delta_\epsilon,\delta_\eta\} = -4\eta\epsilon G_{\bar{\sigma}} - 4\bar{\eta}\bar{\epsilon}G_\sigma - 4i(\eta\bar{\epsilon}+\epsilon\bar{\eta})\left(\mathcal{L}_R - iG_{\iota_R A}\right),$$

$$\begin{aligned}
G_\phi A &= -id_A\phi\\
G_\phi - &= [\phi,-]
\end{aligned}$$

$$\begin{aligned}
&\delta A = \Psi, && \delta\Psi = -i\iota_R F + id_A\sigma\\
&\delta\chi = H, && \delta H = -i\mathcal{L}_R^A\chi - [\sigma,\chi]\\
&\delta\sigma = -\iota_R\Psi,
\end{aligned}$$

$$\delta^2 = -i\mathcal{L}_R - G_{\sigma+\iota_R A},$$

$$J_{pq} = -\frac{1}{2}\nabla_{[p}R_{q]}.$$

$$\delta q = iP_+\psi$$
$$\delta\psi = -\frac{1}{4r}J_{pq}(\Gamma^{pq}q) + (\not{D} + i\sigma)q + \mathcal{F}$$
$$\delta\mathcal{F} = -iP_-\not{D}\psi - \sigma P_-\psi + i\Psi^m(\Gamma_m + R_m)q$$

$$D_m\xi = -\frac{i}{2}\Gamma_m\xi,$$

$$P_-\xi = 0\ , \quad \left((JX)\cdot\Gamma - \frac{i}{2}(\not{X} + \not{R}\not{X})\right)\xi = 0\ ,$$
$$\not{J}\xi = -4i\xi\ .$$

$$\not{J} = -4i(1 - \deg)\ , \quad J_{\bar{i}}{}^{\bar{j}} = i\delta_{\bar{i}}^{\bar{j}}\ ,$$

$$q \Rightarrow \not{B}\xi + f\xi\ , \quad \psi_+ \Rightarrow \not{\Sigma}\xi + \lambda\xi\ ,$$
$$\psi_- \Rightarrow \psi_{-p}\Gamma^p\xi\ , \quad \mathcal{F} \Rightarrow \mathcal{F}_p\Gamma^p\xi\ ,$$

$$\delta\psi = \left(-\mathcal{L}_R^A B - \frac{3i}{2}B + i\sigma B\right)_{pq}\Gamma^{pq}\xi$$
$$+\left(-4d_A^\dagger B + (d_A f)^{(0,1)}\right)_p\Gamma^p\xi + \left(-\mathcal{L}_R^A f - \frac{3i}{2}f + i\sigma f\right)\xi + \mathcal{F},$$

$$\delta\Sigma = -\mathcal{L}_R^A B - \frac{3i}{2}B + i\sigma B$$
$$\delta\lambda = -\mathcal{L}_R^A f - \frac{3i}{2}f + i\sigma f$$

$$\delta\mathcal{F} = -i\mathcal{L}_R^A\psi_- + \frac{3}{2}\psi_- - ((d_A\lambda)^{0,1}\cdot\Gamma)\xi + 4i(d_A^\dagger\Sigma\cdot\Gamma)\xi - \sigma\psi_- + 4i\langle\Psi, B\rangle + i\Psi^{0,1}f,$$

$$\delta f = i\lambda, \delta B = i\Sigma,$$
$$\delta\mathcal{F} = -i\left(\mathcal{L}_R^A\psi_- + (d_A\lambda)^{0,1} - 4d_A^\dagger\Sigma\right) - \sigma\psi_- + 4i\langle\Psi, B\rangle + i\Psi^{0,1}f,$$
$$\delta\Sigma = -\mathcal{L}_R^A B + i\sigma B, \delta\lambda = -\mathcal{L}_R^A f + i\sigma f,$$
$$\delta\psi_- = -4d_A^\dagger B + (d_A f)^{(0,1)} + \mathcal{F}.$$

$$\delta_1 A = \Psi_1,$$
$$\delta_1 B = -\chi_2, \delta_1 H = -i\mathcal{L}_R^A\chi_1 - [\sigma_1, \chi_1],$$
$$\delta_1\mathcal{F} = \iota_R d_A\Psi_2 - 4\left(d_A^\dagger\chi_2\right)^H - i\sigma_1\Psi_2^H + 4i\langle\Psi_1, B\rangle + i[\Psi_1^H, \sigma_2],$$
$$\delta_1\sigma_1 = -\iota_R\Psi_1, \delta_1\sigma_2 = \iota_R\Psi_2,$$
$$\delta_1\Psi_1 = -i\iota_R F + id_A\sigma_1, \delta_1\Psi_2 = 4i\left(d_A^\dagger B\right)^H - id_A\sigma_2 - i\mathcal{F} - \kappa[\sigma_1, \sigma_2],$$
$$\delta_1\chi_1 = H, \delta_1\chi_2 = i\mathcal{L}_R^A B + [\sigma_1, B],$$

$$\rho\chi = i\chi;\ \rho\Psi = i\Psi;\ \rho\sigma = -2i\sigma$$
$$\delta A_m = i\epsilon\Gamma_m\lambda$$
$$\delta\lambda = \frac{1}{2}\not{F}\epsilon + \sum_{i=1}^{7}\nu_j G_j$$
$$\delta G_i = -i\nu_i\not{D}\lambda$$

$$\nu_j\Gamma_m\nu_k - \delta_{jk}\epsilon\Gamma_m\epsilon = 0 = \nu_j\Gamma_m\epsilon$$

field	δ_1	δ_2
A	Ψ_1	Ψ_2
σ_1	$-\iota_R\Psi_1$	$\iota_R\Psi_2$
σ_2	$\iota_R\Psi_2$	$\iota_R\Psi_1$
B	$-\chi_2$	χ_1
H	$-i\mathcal{L}_R^A\chi_1-[\sigma_1,\chi_1]-\delta_1[\sigma_2,B]$	$-i\mathcal{L}_R^A\chi_2+[\sigma_1,\chi_2]+\delta_2[\sigma_2,B]$
χ_1	$H+[\sigma_2,B]$	$-i\mathcal{L}_R^A B+[\sigma_1,B]$
χ_2	$i\mathcal{L}_R^A B+[\sigma_1,B]$	$H-[\sigma_2,B]$
Ψ_1	$-i\iota_R F+id_A\sigma_1$	$-4i(d_A^\dagger B)^H-id_A\sigma_2+i\mathcal{F}+\kappa[\sigma_1,\sigma_2]$
Ψ_2	$4i(d_A^\dagger B)^H-id_A\sigma_2-i\mathcal{F}-\kappa[\sigma_1,\sigma_2]$	$-i\iota_R F-id_A\sigma_1$
$\mathcal{F}$	$\iota_R d_A\Psi_2-4(d_A^\dagger\chi_2)^H-i[\sigma_1,\Psi_2^H]$ $+4i\langle\Psi_1,B\rangle-i[\sigma_2,\Psi_1^H]$	$-\iota_R d_A\Psi_1+4(d_A^\dagger\chi_1)^H-i[\sigma_1,\Psi_1^H]$ $+4i\langle\Psi_2,B\rangle+i[\sigma_2,\Psi_2^H]$

,

$$\delta_2=[\rho,\delta_1],$$

$$\delta_2=e^{\frac{\pi}{2}\rho}\delta_1 e^{-\frac{\pi}{2}\rho}.$$

$$\delta_1^2=-i\mathcal{L}_R-G_{\sigma_1+\iota_R A},$$
$$\delta_2^2=-i\mathcal{L}_R-G_{-\sigma_1+\iota_R A}.$$

$$\epsilon=\frac{1}{2}(\epsilon_1+i\epsilon_2),\bar{\epsilon}=\frac{1}{2}(\epsilon_1-i\epsilon_2),$$

$$\begin{aligned}
\delta_\epsilon A &=\bar{\epsilon}\Psi+\epsilon\bar{\Psi},\\
\delta_\epsilon\sigma &=-2\bar{\epsilon}\iota_R\bar{\Psi},\\
\delta_\epsilon\bar{\sigma} &=-2\epsilon\iota_R\Psi,\\
\delta_\epsilon B &=-i(\epsilon\bar{\chi}-\bar{\epsilon}\chi),\\
\delta_\epsilon H &=-i\mathcal{L}_R^A(\bar{\epsilon}\chi+\epsilon\bar{\chi})-\epsilon[\sigma,\chi]-\bar{\epsilon}[\bar{\sigma},\bar{\chi}]-i\iota_R[B,\bar{\epsilon}\Psi-\epsilon\bar{\Psi}],\\
\delta_\epsilon\mathcal{F} &=\epsilon\left(i\iota_R d_A\bar{\Psi}-4i\left(d_A^\dagger\bar{\chi}\right)^H+4i\langle\bar{\Psi},B\rangle-[\sigma,\Psi^H]\right)\\
&\quad+\bar{\epsilon}\left(-i\iota_R d_A\Psi+4i\left(d_A^\dagger\chi\right)^H+4i\langle\Psi,B\rangle+[\bar{\sigma},\bar{\Psi}^H]\right),\\
\delta_\epsilon\Psi &=\epsilon\left(2\mathcal{F}-2i\iota_R F-8\left(d_A^\dagger B\right)^H+\kappa[\sigma,\bar{\sigma}]\right)+2i\bar{\epsilon}d_A\bar{\sigma},\\
\delta_\epsilon\bar{\Psi} &=\bar{\epsilon}\left(-2\mathcal{F}-2i\iota_R F+8\left(d_A^\dagger B\right)^H-\kappa[\sigma,\bar{\sigma}]\right)+2i\epsilon d_A\sigma,\\
\delta_\epsilon\chi &=2\epsilon(H-\mathcal{L}_R^A B)+2i\bar{\epsilon}[\bar{\sigma},B],\\
\delta_\epsilon\bar{\chi} &=2\bar{\epsilon}(H+\mathcal{L}_R^A B)-2i\epsilon[\sigma,B].
\end{aligned}$$

$$\frac{1}{2}\{Q,\bar{Q}\}=\frac{1}{2}\{\delta_1+i\delta_2,\delta_1-i\delta_2\}=\delta_1^2+\delta_2^2=\delta_1^2+e^{\frac{\pi}{2}\rho}\delta_1^2e^{-\frac{\pi}{2}\rho}=-2i\left(\mathcal{L}_R-iG_{\iota_R A}\right),$$

$$\begin{aligned}
Q^2&=(\delta_1+i\delta_2)^2=\delta_1^2-\delta_2^2+i\{\delta_1,\delta_2\}=\delta_1^2-e^{\frac{\pi}{2}\rho}\delta_1^2e^{-\frac{\pi}{2}\rho}+i\{\delta_1,[\rho,\delta_1]\}\\
&=-2G_{\sigma_1}+2iG_{\sigma_2}=-2G_{\bar{\sigma}}
\end{aligned}$$

$$\begin{aligned}
S_1&=-\frac{1}{4}\bar{Q}\left(\Psi,-\mathcal{F}-i\iota_R F-4\left(d_A^\dagger B\right)^H-\frac{1}{2}\kappa[\sigma,\bar{\sigma}]\right)\\
&\quad-\frac{1}{4}Q\left(\bar{\Psi},\mathcal{F}-i\iota_R F+4\left(d_A^\dagger B\right)^H+\frac{1}{2}\kappa[\sigma,\bar{\sigma}]\right),\\
S_2&=\frac{1}{2}\bar{Q}\left(\chi,H+\mathcal{L}_R^A B+\tilde{F}\right)+\frac{1}{2}Q\left(\bar{\chi},H-\mathcal{L}_R^A B+\tilde{F}\right),\\
S_3&=-\frac{1}{8}Q(\Psi,\bar{Q}\bar{\Psi})-\frac{1}{8}\bar{Q}(\bar{\Psi},Q\Psi),
\end{aligned}$$

$$\tilde{F}=2i\left(F_H^{2+}+\frac{1}{4}B\times B\right),$$

$$
\begin{aligned}
S_1|_{\text{bos}} &= \left\|\mathcal{F}+\frac{1}{2}\kappa[\sigma,\bar{\sigma}]\right\|^2 + \|\iota_R F\|^2 - 16\left\|\left(d_A^\dagger B\right)^H\right\|^2 \\
S_2|_{\text{bos}} &= 2\|H\|^2 - 2\left\|\mathcal{L}_R^A B\right\|^2 + 2(H,\tilde{F}) \\
S_3|_{\text{bos}} &= (d_A\bar{\sigma}, d_A\sigma)
\end{aligned}
$$

$$S = S_1 + S_2 + S_3$$

$$
\begin{aligned}
S =& \|\iota_R F\|^2 + 2\left\|F_H^+ + \frac{1}{4}B\times B\right\|^2 + (d_A\bar{\sigma}, d_A\sigma) - 16\left\|\left(d_A^\dagger B\right)^H\right\|^2 \\
&-2\left\|\mathcal{L}_R^A B\right\|^2 + \frac{1}{4}\|[\sigma,\bar{\sigma}]\|^2 + \|\mathcal{F}\|^2,
\end{aligned}
$$

$$
\begin{aligned}
S = \|\iota_R F\|^2 &+ 2\left\|F_H^+ - \frac{1}{4}B\times B\right\|^2 \\
&+(d_A\bar{\sigma}, d_A\sigma) + 16\left\|\left(d_A^\dagger B\right)^H\right\|^2 + 2\left\|\mathcal{L}_R^A B\right\|^2 + \frac{1}{4}\|[\sigma,\bar{\sigma}]\|^2
\end{aligned}
$$

$$\tilde{\mathcal{F}} = \mathcal{F} - 4\left(d_A^\dagger B\right)^H$$

$$\kappa \wedge (d\kappa)^n \neq 0$$

$$g = \frac{1}{2}d\kappa(J-,-) + \kappa \otimes \kappa$$

$$\iota_R\left(\star\,\omega_p\right) = (-1)^p \star \left(\kappa \wedge \omega_p\right)$$

$$\text{vol}_g = \frac{(-1)^n}{2^n n!}\kappa \wedge (d\kappa)^n$$

$$G = r^2 g + dr \otimes dr$$

$$\mathcal{J}_R = -r\partial_r, \mathcal{J}r\partial_r = R$$

$$\langle Z, (\nabla_X J)Y\rangle = -\kappa(Z)\langle X, Y\rangle + \langle Z, X\rangle\kappa(Y),$$

$$R_{XY} = [\nabla_X, \nabla_Y] - \nabla_{[X,Y]}$$

$$\langle U, R_{XY}V\rangle - \langle JU, R_{XY}JV\rangle = \langle X, U\rangle\langle Y, V\rangle - \langle X, JU\rangle\langle Y, JV\rangle - (X \leftrightarrow Y)$$

$$R_{XY} \star J = \text{vol}_g(\langle X, c_1 Y\rangle - (2n-1)\langle X, JY\rangle)$$

$$W_{ijkl} = R_{ijkl} - \frac{s}{4n(2n+1)}g \barwedge g - \frac{1}{2n-1}\left(Ric - \frac{s}{2n+1}g\right)\barwedge g$$

$$(A \barwedge B)_{ijkl} = A_{ik}B_{jl} - A_{jk}B_{il} - A_{il}B_{jk} + A_{jl}B_{ik}$$

$$
\begin{aligned}
\Omega^p(M_5) &= \Omega_V^p(M_5) \oplus \Omega_H^p(M_5) \\
\alpha &= \kappa\iota_R\alpha + \iota_R(\kappa \wedge \alpha)
\end{aligned}
$$

$$(\alpha, \beta) = \int_{M_5} \alpha \wedge\star\, \beta$$

$$\star_R\, \alpha = \iota_R \star \alpha = (-1)^{\deg\alpha} \star (\kappa \wedge \alpha).$$

$$\Omega_H^2(M_5) = \Omega_H^{2+}(M_5) \oplus \Omega_H^{2-}(M_5)$$

$$\iota_R \star \omega_H^\pm = \pm\omega_H^\pm$$

$$\Omega_B(M) = \{\alpha \in \Omega(M) \mid \iota_R\alpha = 0 = \mathcal{L}_R\alpha\}$$

$$(d^B)^\dagger\alpha = \left(d^\dagger\alpha\right)^H$$

$$0 \to \mathbb{R} \overset{d\kappa}{\to} H_B^2 \to H^2 \to 0,$$

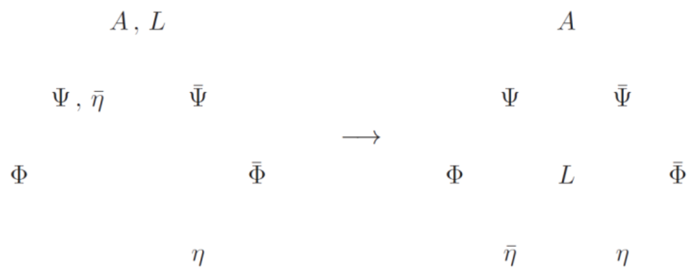
$$\Gamma_{i_1\cdots i_k} = \frac{1}{k!}\Gamma_{[i_1} \cdots \Gamma_{i_k]}.$$

$$A = A \cdot \Gamma = A_{i_1\cdots i_k}\Gamma^{i_1\cdots i_k}.$$

$$(d + s + \delta - i_\kappa)(A + c + |\kappa|\bar{c}) + (A + c + |\kappa|\bar{c})^2 = F + \Psi + g(\kappa)\eta + i_\kappa\chi + \Phi + |\kappa|^2\bar{\Phi}$$

$$s^2 = \delta^2 = 0 \, \{s, \delta\} = \mathcal{L}_\kappa$$

$$(d + s + \bar{s})(A + c + \bar{c}) + (A + c + \bar{c})^2 = F + \Psi + \bar{\Psi} + \Phi + L + \bar{\Phi}$$



$$\phi_p, \tilde{\phi}_p = \sum_{0\le G\le p} \sum_{0\le g\le G} \phi_{p-G}^{g,G-g}$$

$$\begin{array}{ll}
sA = \Psi - d_A c & \bar{s}A = \bar{\Psi} - d_A\bar{c} \\
s\Psi = -d_A\Phi - [c, \Psi] & \bar{s}\Psi = -T - d_A L - [\bar{c}, \Psi] \\
s\Phi = -[c, \Phi] & \bar{s}\Phi = -\bar{\eta} - [\bar{c}, \Phi] \\
s\bar{\Phi} = \eta - [c, \bar{\Phi}] & \bar{s}\bar{\Phi} = -[\bar{c}, \bar{\Phi}] \\
s\eta = [\Phi, \bar{\Phi}] - [c, \eta] & \bar{s}\eta = -[\bar{\Phi}, L] - [\bar{c}, \eta] \\
sL = \bar{\eta} - [c, L] & \bar{s}L = -\eta - [\bar{c}, L] \\
s\bar{\eta} = [\Phi, L] - [c, \bar{\eta}] & \bar{s}\bar{\eta} = [\Phi, \bar{\Phi}] - [\bar{c}, \bar{\eta}] \\
s\bar{\Psi} = T - [c, \bar{\Psi}] & \bar{s}\bar{\Psi} = -d_A\bar{\Phi} - [\bar{c}, \bar{\Psi}] \\
sT = [\Phi, \bar{\Psi}] - [c, T] & \bar{s}T = -d_A\eta + [L, \bar{\Psi}] - [\bar{\Phi}, \Psi] - [\bar{c}, T] \\
sc = \Phi - c^2 & \bar{s}c = L - b \\
s\bar{c} = b - [c, \bar{c}] & \bar{s}\bar{c} = \bar{\Phi} - \bar{c}^2 \\
sb = [\Phi, \bar{c}] - [c, \bar{c}] & \bar{s}b = \eta + [\bar{c}, L]
\end{array}$$

$$s_{(c)}^{\alpha} A = \Psi^{\alpha}$$

$$s_{(c)}^{\alpha} \Psi_\beta = \delta_\beta^\alpha T - \sigma^i{}_\beta{}^\alpha d_A \Phi_i$$

$$s_{(c)}^{\alpha} \Phi_i = \frac{1}{2}\sigma_i{}^{\alpha\beta}\eta_\beta$$

$$s_{(c)}^{\alpha} \eta_\beta = -2\sigma^{ij}{}_\beta{}^\alpha[\Phi_i, \Phi_j]$$

$$s_{(c)}^{\alpha} T = \frac{1}{2}d_A\eta^\alpha + \sigma^{i\,\alpha\beta}[\Phi_i, \Psi_\beta]$$

$$\gamma\Psi_\mu^\alpha = -\kappa_\mu\eta^\alpha - C_{\mu\nu}{}^\sigma\kappa^\nu\Psi_\sigma^\alpha$$

$$\gamma\eta^\alpha = \kappa^\mu\Psi_\mu^\alpha$$

$$i_\kappa \star C^\star w_1 = -C_{\mu\nu}{}^\sigma\kappa^\nu w_\sigma dx^\mu$$

$$\left\{s_{(c)}^\alpha, s_{(c)}^\beta\right\} = \sigma^{i\alpha\beta}\delta_{\text{gauge}}(\Phi_i)$$

$$\left\{e^{t\gamma}s_{(c)}^\alpha e^{-t\gamma}, e^{t\gamma}s_{(c)}^\beta e^{-t\gamma}\right\} = \left\{s_{(c)}^\alpha, s_{(c)}^\beta\right\}$$

$$\delta_{(c)}^\alpha \equiv [s_{(c)}^\alpha, \gamma]$$

$$\left[[s_{(c)}^{\alpha}, \mathrm{Y}], \mathrm{Y}\right] = -s_{(c)}^{\alpha}$$

$$e^{t\gamma} s_{(c)}^{\alpha} e^{-t\gamma} = \cos\, t s_{(c)}^{\alpha} - \sin\, t\delta_{(c)}^{\alpha}$$

$$\gamma T = -i_\kappa F - 2 i_\kappa \star C^\star T$$

$$\begin{aligned}
&\delta_{(c)}^{\alpha} A = g(\kappa)\eta^{\alpha} + i_\kappa \star C^\star \Psi^{\alpha} \\
&\delta_{(c)}^{\alpha} \Psi_\beta = \delta_\beta^\alpha i_\kappa (F + \star C^\star T) + \sigma^i{}_\beta{}^\alpha i_\kappa \star C^\star d_A \Phi_i - 2g(\kappa)\sigma^{ij}{}_\beta{}^\alpha [\Phi_i, \Phi_j] \\
&\delta_{(c)}^{\alpha} \Phi_i = -\frac{1}{2}\sigma_i{}^{\alpha\beta} i_\kappa \Psi_\beta \\
&\delta_{(c)}^{\alpha} \eta_\beta = -\delta_\beta^\alpha i_\kappa T + \sigma^i{}_\beta{}^\alpha \mathcal{L}_\kappa \Phi_i \\
&\delta_{(c)}^{\alpha} T = \frac{1}{2} d_A i_\kappa \Psi^\alpha - i_\kappa \star C^\star d_A \eta^\alpha + g(\kappa)\sigma^{i\alpha\beta}[\Phi_i, \eta_\beta] - \sigma^{i\alpha\beta} i_\kappa \star C^\star [\Phi_i, \Psi_\beta] - \mathcal{L}_\kappa \Psi^\alpha
\end{aligned}$$

$$\left\{s_{(c)}^{\alpha}, \delta_{(c)}^{\beta}\right\} = \varepsilon^{\alpha\beta}\left(\mathcal{L}_\kappa + \delta_{\text{gauge}}(i_\kappa A)\right) \left\{\delta_{(c)}^{\alpha}, \delta_{(c)}^{\beta}\right\} = 2\sigma^{i\alpha\beta}\delta_{\text{gauge}}(\Phi_i)$$

$$\begin{aligned}
&\delta A = g(\kappa)\eta + i_\kappa \star C^\star \bar\Psi - |\kappa| d_A \gamma && \delta\bar\Phi = -[|\kappa|\gamma, \bar\Phi] \\
&\delta\Psi = i_\kappa (F - \star C^\star T) + g(\kappa)[\Phi, \bar\Phi] - [|\kappa|\gamma, \Psi] && \delta\eta = \mathcal{L}_\kappa \bar\Phi - [|\kappa|\gamma, \eta] \\
&\delta\Phi = i_\kappa \Psi - [|\kappa|\gamma, \Phi] && \delta L = i_\kappa \bar\Psi - [|\kappa|\gamma, L] \\
& && \delta\bar\eta = i_\kappa (T + d_A L) - [|\kappa|\gamma, \bar\eta] \\
& && \delta\bar\Psi = i_\kappa \star C^\star d_A \bar\Phi - g(\kappa)[\bar\Phi, L] - [|\kappa|\gamma, \bar\Psi] \\
& && \delta T = \mathcal{L}_\kappa \bar\Psi + i_\kappa \star C^\star (d_A \eta + [\bar\Phi]) + g(\kappa)[L, \eta] - g(\kappa)[\bar\Phi, \bar\eta] - [|\kappa|\gamma, T]
\end{aligned}$$

$$\begin{aligned}
&\delta A = g(\kappa)\eta + i_\kappa \star C^\star \bar\Psi - |\kappa| d_A \gamma && \delta\bar\Phi = -[|\kappa|\gamma, \bar\Phi] \\
&\delta\Psi = i_\kappa (F - \star C^\star T) + g(\kappa)[\Phi, \bar\Phi] - [|\kappa|\gamma, \Psi] && \delta\eta = \mathscr{L}_\kappa \bar\Phi - [|\kappa|\gamma, \eta] \\
&\delta\Phi = i_\kappa \Psi - [|\kappa|\gamma, \Phi] && \delta L = i_\kappa \bar\Psi - [|\kappa|\gamma, L] \\
& && \delta\bar\eta = i_\kappa (T + d_A L) - [|\kappa|\gamma, \bar\eta] \\
& && \delta\bar\Psi = i_\kappa \star C^\star d_A \bar\Phi - g(\kappa)[\bar\Phi, L] - [|\kappa|\gamma, \bar\Psi] \\
& && \delta T = \mathscr{L}_\kappa \bar\Psi + i_\kappa \star C^\star \left(d_A \eta + [\bar\Phi]\right) + g(\kappa)[L, \eta] - g(\kappa)[\bar\Phi, \bar\eta] - [|\kappa|\gamma, T]
\end{aligned}$$

$$[\bar s, \mathrm{Y}] = \delta$$

$$\gamma T = i_\kappa F - 2 i_\kappa \star C^\star T - i_\kappa \star C^\star d_A L$$

$$\gamma \bar c = -|\kappa|\gamma \qquad \gamma\gamma = \frac{1}{|\kappa|}\bar c$$

$$\begin{aligned}
(d + s + \bar s + \delta + \bar\delta)&\left(A + c + \bar c + |\kappa|\gamma + |\kappa|\bar\gamma\right) + \left(A + c + \bar c + |\kappa|\gamma + |\kappa|\bar\gamma\right)^2 \\
&= F + \Psi + \bar\Psi + g(\kappa)\left(\eta + \bar\eta\right) + i_\kappa \star C^\star\left(\Psi + \bar\Psi\right) + \left(1 + |\kappa|^2\right)\left(\Phi + L + \bar\Phi\right)
\end{aligned}$$

$$\{s, \delta\} + \{\bar s, \bar\delta\} = 0$$

$$\{s, \delta\} = \mathcal{L}_\kappa \, \{\bar s, \bar\delta\} = -\mathcal{L}_\kappa$$

$$\begin{aligned}
&(d + s + \delta - i_\kappa)(A + c + |\kappa|\gamma) + (A + c + |\kappa|\gamma)^2 \\
&\qquad = F + \Psi + g(\kappa)\eta + i_\kappa \star C^\star(\bar\Psi) + (\Phi + |\kappa|^2\bar\Phi) \\
&(d + \bar s + \bar\delta + i_\kappa)(A + \bar c + |\kappa|\bar\gamma) + (A + \bar c + |\kappa|\bar\gamma)^2 \\
&\qquad = F + \bar\Psi + g(\kappa)\bar\eta + i_\kappa \star C^\star(\Psi) + (\bar\Phi + |\kappa|^2\Phi)
\end{aligned}$$

$$S = s\mathbf{\Psi} - \frac{1}{2}\int_M C_\wedge \mathrm{Tr} F_\wedge F$$

$$\mathbf{\Psi} = 2\int_M \mathrm{Tr}(C_\wedge^\star \bar{\Psi}_\wedge F + \bar{\Psi}_\star (d_A L + T) + \Psi \star d_A \bar{\Phi} + \star\, \eta[\Phi, \bar{\Phi}] + \star\, \bar{\eta}[\bar{\Phi}, L])$$

$$\sigma^{i\alpha\beta}\delta_{(c)\alpha}\mathbf{\Psi}_\beta = 0$$

$$\mathbf{\Psi}_\alpha = \delta_{(c)\alpha}\mathcal{G}.S = s_{(c)}^\alpha \mathbf{\Psi}_\alpha - \frac{1}{2}\int_M C_\wedge \mathrm{Tr} F_\wedge F$$

$$\mathbf{\Psi} \propto \mathbf{\Psi}_1$$

$$
\begin{aligned}
s_{(c)}^\alpha A &= \Psi^\alpha \\
s_{(c)}^\alpha \Psi_\beta &= \delta_\beta^\alpha T - \sigma^i{}_\beta{}^\alpha d_A \Phi_i \\
s_{(c)}^\alpha \Phi_i &= \frac{1}{2}\sigma_i{}^{\alpha\beta}\eta_\beta \\
s_{(c)}^\alpha \eta_\beta &= -2\sigma^{ij}{}_\beta{}^\alpha [\Phi_i, \Phi_j] \\
s_{(c)}^\alpha T &= \frac{1}{2} d_A \eta^\alpha + \sigma^{i\,\alpha\beta}[\Phi_i, \Psi_\beta]
\end{aligned}
\qquad
\begin{aligned}
s_{(c)}^\alpha h^I &= \chi^{\alpha\, I} \\
s_{(c)}^\alpha \chi_\beta^I &= \delta_\beta^\alpha H^I + \sigma^i{}_\beta{}^\alpha [\Phi_i, h^I] \\
s_{(c)}^\alpha H^I &= \frac{1}{2}[\eta^\alpha, h^I] + \sigma^{i\,\alpha\beta}[\Phi_i, \chi_\beta^I]
\end{aligned}
$$

$$s_{(c)}^{\{\alpha} s_{(c)}^{\beta\}} = \sigma^{i\alpha\beta}\delta_{\mathrm{gauge}}(\Phi_i)$$

$$
\begin{aligned}
\delta_{(c)}^\alpha A &= g(\kappa)\eta^\alpha + g(J_I\kappa)\chi^{\alpha I} \\
\delta_{(c)}^\alpha \Psi_\beta &= \delta_\beta^\alpha (i_\kappa F - g(J_I\kappa)H^I) + \sigma^i{}_\beta{}^\alpha g(J_I\kappa)[\Phi_i, h^I] - 2\sigma^{ij}{}_\beta{}^\alpha g(\kappa)[\Phi_i, \Phi_j] \\
\delta_{(c)}^\alpha \Phi_i &= -\frac{1}{2}\sigma_i{}^{\alpha\beta} i_\kappa \Psi_\beta \\
\delta_{(c)}^\alpha \eta_\beta &= -\delta_\beta^\alpha i_\kappa T + \sigma^i{}_\beta{}^\alpha \mathcal{L}_\kappa \Phi_i \\
\delta_{(c)}^\alpha T &= \frac{1}{2} d_A i_\kappa \Psi^\alpha - g(J_I\kappa)([\eta^\alpha, h^I] + \sigma^{i\alpha\beta}[\Phi_i, \chi_\beta^I]) + g(\kappa)\sigma^{i\alpha\beta}[\Phi_i, \eta_\beta] - \mathcal{L}_\kappa \Psi^\alpha \\
\delta_{(c)}^\alpha h^I &= -i_{J^I\kappa}\Psi^\alpha \\
\delta_{(c)}^\alpha \chi_\beta^I &= \delta_\beta^\alpha (\mathcal{L}_\kappa h^I + i_{J^I\kappa} T) + \sigma^i{}_\beta{}^\alpha \mathcal{L}_{J^I\kappa}\Phi_i \\
\delta_{(c)}^\alpha H^I &= \frac{1}{2}[i_\kappa \Psi^\alpha, h^I] + \mathcal{L}_{J^I\kappa}\eta^\alpha + \sigma^{i\alpha\beta}[\Phi_i, i_{J^I\kappa}\Psi_\beta] - \mathcal{L}_\kappa \chi^{\alpha I}
\end{aligned}
$$

$$
\begin{aligned}
\delta_{(c)}^\alpha A &= g(\kappa)\eta^\alpha + g(J_I\kappa)\chi^{\alpha\, I} \\
\delta_{(c)}^\alpha \Psi_\beta &= \delta_\beta^\alpha \left(i_\kappa F - g(J_I\kappa)H^I\right) + \sigma^i{}_\beta{}^\alpha g(J_I\kappa)[\Phi_i, h^I] - 2\sigma^{ij}{}_\beta{}^\alpha g(\kappa)[\Phi_i, \Phi_j] \\
\delta_{(c)}^\alpha \Phi_i &= -\frac{1}{2}\sigma_i{}^{\alpha\beta} i_\kappa \Psi_\beta \\
\delta_{(c)}^\alpha \eta_\beta &= -\delta_\beta^\alpha i_\kappa T + \sigma^i{}_\beta{}^\alpha \mathscr{L}_\kappa \Phi_i \\
\delta_{(c)}^\alpha T &= \frac{1}{2} d_A i_\kappa \Psi^\alpha - g(J_I\kappa)\left([\eta^\alpha, h^I] + \sigma^{i\,\alpha\beta}[\Phi_i, \chi_\beta^I]\right) + g(\kappa)\sigma^{i\,\alpha\beta}[\Phi_i, \eta_\beta] - \mathscr{L}_\kappa \Psi^\alpha \\
\delta_{(c)}^\alpha h^I &= -i_{J^I\kappa}\Psi^\alpha \\
\delta_{(c)}^\alpha \chi_\beta^I &= \delta_\beta^\alpha \left(\mathscr{L}_\kappa h^I + i_{J^I\kappa} T\right) + \sigma^i{}_\beta{}^\alpha \mathscr{L}_{J^I\kappa}\Phi_i \\
\delta_{(c)}^\alpha H^I &= \frac{1}{2}[i_\kappa \Psi^\alpha, h^I] + \mathscr{L}_{J^I\kappa}\eta^\alpha + \sigma^{i\,\alpha\beta}[\Phi_i, i_{J^I\kappa}\Psi_\beta] - \mathscr{L}_\kappa \chi^{\alpha\, I}
\end{aligned}
$$

$$\delta_{(c)}^{\{\alpha}\delta_{(c)}^{\beta\}} = |\kappa|^2 \sigma^{i\alpha\beta}\delta_{\mathrm{gauge}}(\Phi_i)\delta_{(c)}^\alpha s_{(c)}^\alpha$$

$$\left\{s_{(c)}^\alpha, \delta_{(c)}^\beta\right\} = \varepsilon^{\alpha\beta}\left(\mathcal{L}_\kappa + \delta_{\mathrm{gauge}}(i_\kappa A)\right)$$

$$\delta^{\alpha}_{(c)} = [s^{\alpha}_{(c)}, \gamma]$$

$$\gamma\Psi_\alpha = -g(\kappa)\eta_\alpha - g(J_I\kappa)\chi^I_\alpha \qquad \gamma T = -i_\kappa F + 2g(J_I\kappa)H^I$$

$$\gamma\eta_\alpha = i_\kappa\Psi_\alpha \qquad \gamma H^I = -\mathscr{L}_\kappa h^I - 2i_{J^I\kappa}T$$

$$\gamma\chi^I_\alpha = i_{J^I\kappa}\Psi_\alpha$$

$$e^{t\gamma}s^{\alpha}_{(c)}e^{-t\gamma} = \cos\, ts^{\alpha}_{(c)} - \sin\, t\delta^{\alpha}_{(c)} e^{t\gamma}\delta^{\alpha}_{(c)}e^{-t\gamma} = \cos\, t\delta^{\alpha}_{(c)} + \sin\, ts^{\alpha}_{(c)}$$

$$\sigma^{i\alpha\beta}\delta_{(c)\alpha}\boldsymbol{\Psi}_\beta = 0$$

$$S = -\frac{1}{2}\int_M \mathrm{Tr} F_\wedge F + s^{\alpha}_{(c)}\Psi_\alpha$$

$$\boldsymbol{\Psi}_\alpha = \int_M \mathrm{Tr}(\star\,\chi^I_\alpha H_I + \chi^I_\alpha J_I \star F - \Psi_\alpha \star T + J^I \star \Psi_{\alpha\wedge} d_A h_I - \sigma^i{}_\alpha{}^\beta \Psi_\beta \star d_A\Phi_i$$
$$-2\star\sigma^{ij}{}_\alpha{}^\beta\eta_\beta[\Phi_i,\Phi_j] - \star\,\sigma^i{}_\alpha{}^\beta\chi^I_\beta[\Phi_i,h_I] + \frac{1}{2}\star\varepsilon_{IJK}\chi^I_\alpha[h^J,h^K]\Big)$$

$$S = -\frac{1}{2}\int_M \mathrm{Tr}F_\wedge F + s^{\alpha}_{(c)}s_{(c)}\alpha\mathcal{F} = -\frac{1}{2}\int_M \mathrm{Tr}F_\wedge F + s^{\alpha}_{(c)}\delta_{(c)\alpha}\mathcal{G}$$

$$\mathcal{F} = \int_M \mathrm{Tr}\left(\star\, h_I H^I + h_I J^I \star F + \frac{1}{3}\varepsilon_{IJK}h^I h^J h^K - \frac{1}{2}\Psi^\alpha \star \Psi_\alpha + \frac{1}{2}\star\eta^\alpha\eta_\alpha\right)$$
$$\mathcal{G} = \int_M \mathrm{Tr}\Big(-\frac{1}{2}g(\kappa)_\wedge\Big((A-\mathring{A})_\wedge(F+\mathring{F}) - \frac{1}{3}(A-\mathring{A})^3\Big)$$
$$-\frac{1}{2}\star\varepsilon_{IJK}h^I\mathcal{L}_{J^J\kappa}h^K + \star\, s^{\alpha}_{(c)}\delta_{(c)\alpha}\left(\frac{1}{2}h_I h^I - \frac{2}{3}\Phi^i\Phi_i\right)\Big)$$

$$S = -\frac{1}{2}\int_M \mathrm{Tr}\ F_\wedge F + s^{\alpha}_{(c)}s_{(c)\,\alpha}\mathscr{F} = -\frac{1}{2}\int_M \mathrm{Tr}\ F_\wedge F + s^{\alpha}_{(c)}\delta_{(c)\,\alpha}\mathscr{G}$$

$$\mathscr{F} = \int_M \mathrm{Tr}\ \left(\star h_I H^I + h_I J^I \star F + \frac{1}{3}\varepsilon_{IJK}h^I h^J h^K - \frac{1}{2}\Psi^\alpha \star \Psi_\alpha + \frac{1}{2}\star\eta^\alpha\eta_\alpha\right)$$

$$\mathscr{G} = \int_M \mathrm{Tr}\ \Big(-\frac{1}{2}g(\kappa)_\wedge((A-\mathring{A})_\wedge(F+\mathring{F}) - \tfrac{1}{3}(A-\mathring{A})^3)$$
$$-\frac{1}{2}\star\varepsilon_{IJK}h^I\mathscr{L}_{J_{J}\kappa}h^K + \star\, s^{\alpha}_{(c)}\delta_{(c)\,\alpha}\left(\tfrac{1}{2}h_I h^I - \tfrac{2}{3}\Phi^i\Phi_i\right)\Big)$$

$$J^I \star F + \frac{1}{2}\star\varepsilon^I_{JK}[h^J,h^K] = 0$$
$$d_A \star h_I J^I = 0$$

$$S \approx \int_M \mathrm{Tr}\Big(-\frac{1}{2}F\star F + \frac{1}{4}d_A h_I \star d_A h^I + 2d_A\Phi^i \star d_A\Phi_i - 2\chi^\alpha_I J^I \star d_A\Psi_\alpha + 2\Psi^\alpha \star d_A\eta_\alpha$$
$$+2\star\eta^\alpha[h_I,\chi^I_\alpha] + J_I \star \Psi^\alpha[h^I,\Psi_\alpha] + \star\,\varepsilon_{IJK}\chi^{\alpha I}[h^J,\chi^K_\alpha]$$
$$-2\star\sigma^{i\alpha\beta}\chi_{\alpha I}[\Phi_i,\chi^I_\beta] - 2\star\sigma^{i\alpha\beta}\eta_\alpha[\Phi_i,\eta_\beta] - 2\sigma^{i\alpha\beta}\Psi_\alpha \star [\Phi_i,\Psi_\beta]$$
$$-\frac{1}{8}\star[h_I,h_J][h^I,h^J] - 2\star[\Phi^i,h_I][\Phi_i,h^I] - 4\star[\Phi^i,\Phi^j][\Phi_i,\Phi_j]\Big)$$

$$S \approx \int_M \mathrm{Tr}\Bigg(-\frac{1}{2}F\star F + \frac{1}{4}d_A h_I \star d_A h^I + 2 d_A\Phi^i \star d_A\Phi_i - 2\chi_I^\alpha J^I \star d_A\Psi_\alpha + 2\Psi^\alpha \star d_A\eta_\alpha$$
$$+ 2\star\eta^\alpha[h_I, \chi_\alpha^I] + J_I\star\Psi^\alpha[h^I, \Psi_\alpha] + \star\varepsilon_{IJK}\chi^{\alpha\, I}[h^J, \chi_\alpha^K]$$
$$- 2\star\sigma^{i\,\alpha\beta}\chi_{\alpha\, I}[\Phi_i, \chi_\beta^I] - 2\star\sigma^{i\,\alpha\beta}\eta_\alpha[\Phi_i, \eta_\beta] - 2\sigma^{i\,\alpha\beta}\Psi_\alpha\star[\Phi_i, \Psi_\beta]$$
$$- \frac{1}{8}\star[h_I, h_J][h^I, h^J] - 2\star[\Phi^i, h_I][\Phi_i, h^I] - 4\star[\Phi^i, \Phi^j][\Phi_i, \Phi_j]\Bigg)$$

$$(d + s + \bar{s} + \delta + \bar{\delta})(A + c + \bar{c} + |\kappa|\gamma + |\kappa|\bar{\gamma}) + (A + c + \bar{c} + |\kappa|\gamma + |\kappa|\bar{\gamma})^2$$
$$= F + \Psi + \bar{\Psi} + g(\kappa)(\eta + \bar{\eta}) + g(J_I\kappa)(\chi^I + \bar{\chi}^I) + (1 + |\kappa|^2)(\Phi + L + \bar{\Phi})$$
$$(d_A + s_{(c)} + \bar{s}_{\bar{c}} + \delta_\gamma + \bar{\delta}_{\bar{\gamma}})h^I = d_A h^I + \bar{\chi}^I - \chi^I + i_{J^I\kappa}(\bar{\Psi} - \Psi)$$
$$(d + s + \delta - i_\kappa)(A + c + |\kappa|\gamma) + (A + c + |\kappa|\gamma)^2 = F + \Psi + g(\kappa)\eta + g(J_I\kappa)\chi^I + \Phi + |\kappa|^2\bar{\Phi}$$
$$(d_A + s_{(c)} + \delta_\gamma - i_\kappa)h^I = d_A h^I + \bar{\chi}^I + i_{J^I\kappa}\bar{\Psi}$$

$$(d_A + s_{(c)} + \delta_{(c)} - i_\kappa)h^\alpha = d_A h^\alpha + \lambda_+^\alpha + \not{\kappa}\lambda_-^\alpha$$

$$s_{(c)}h^\alpha = \lambda_+^\alpha \qquad \delta_{(c)}h^\alpha = \not{\kappa}\lambda_-^\alpha$$
$$s_{(c)}\lambda_+^\alpha = [\Phi, h^\alpha] \qquad \delta_{(c)}\lambda_+^\alpha = \not{\kappa}D^\alpha + \mathscr{L}_\kappa h^\alpha$$
$$s_{(c)}\lambda_-^\alpha = D^\alpha \qquad \delta_{(c)}\lambda_-^\alpha = \not{\kappa}[\bar{\Phi}, h^\alpha]$$
$$s_{(c)}D^\alpha = [\Phi, \lambda_-^\alpha] \qquad \delta_{(c)}D^\alpha = \not{\kappa}[\eta, h^\alpha] + \not{\kappa}[\bar{\Phi}, \lambda_+^\alpha] + \mathscr{L}_\kappa\lambda_-^\alpha$$

$$(d_A + s_{(c)} + \delta_{(c)} - i_\kappa)V = d_A V + \bar{\Psi} + g(\kappa)\bar{\eta} + i_\kappa\bar{\chi}$$

$$s_{(c)}V = \bar{\Psi} \qquad \delta_{(c)}V = g(\kappa)\bar{\eta} + i_\kappa\bar{\chi}$$
$$s_{(c)}\bar{\Psi} = [\Phi, V] \qquad \delta_{(c)}\bar{\Psi} = g(\kappa)h + i_\kappa\bar{H} + \mathcal{L}_\kappa V$$
$$s_{(c)}\bar{\eta} = h \qquad \delta_{(c)}\bar{\eta} = -[\bar{\Phi}, i_\kappa V]$$
$$s_{(c)}h = [\Phi, \bar{\eta}] \qquad \delta_{(c)}h = [\eta, i_\kappa V] - [\bar{\Phi}, i_\kappa\bar{\Psi}] + \mathcal{L}_\kappa\bar{\eta}$$
$$s_{(c)}\bar{\chi} = \bar{H} \qquad \delta_{(c)}\bar{\chi} = -2[\bar{\Phi}, (g(\kappa)V)^+]$$
$$s_{(c)}\bar{H} = [\Phi, \bar{\chi}] \qquad \delta_{(c)}\bar{H} = 2[\eta, (g(\kappa)V)^+] - 2[\bar{\Phi}, (g(\kappa)\bar{\Psi})^+] + \mathcal{L}_\kappa\bar{\chi}$$

$$s_{(c)}V = \bar{\Psi} \qquad \delta_{(c)}V = g(\kappa)\bar{\eta} + i_\kappa\bar{\chi}$$
$$s_{(c)}\bar{\Psi} = [\Phi, V] \qquad \delta_{(c)}\bar{\Psi} = g(\kappa)h + i_\kappa\bar{H} + \mathscr{L}_\kappa V$$
$$s_{(c)}\bar{\eta} = h \qquad \delta_{(c)}\bar{\eta} = -[\bar{\Phi}, i_\kappa V]$$
$$s_{(c)}h = [\Phi, \bar{\eta}] \qquad \delta_{(c)}h = [\eta, i_\kappa V] - [\bar{\Phi}, i_\kappa\bar{\Psi}] + \mathscr{L}_\kappa\bar{\eta}$$
$$s_{(c)}\bar{\chi} = \bar{H} \qquad \delta_{(c)}\bar{\chi} = -2[\bar{\Phi}, \big(g(\kappa)V\big)^+]$$
$$s_{(c)}\bar{H} = [\Phi, \bar{\chi}] \qquad \delta_{(c)}\bar{H} = 2[\eta, \big(g(\kappa)V\big)^+] - 2[\bar{\Phi}, \big(g(\kappa)\bar{\Psi}\big)^+] + \mathscr{L}_\kappa\bar{\chi}$$

$$\tau \equiv \frac{4\pi i}{g_{\mathrm{YM}}^2} + \frac{\theta}{2\pi}$$

$$\langle \mathcal{O}_1\mathcal{O}_2\cdots\rangle \equiv \mathrm{tr}\big((-1)^F\hat{\mathcal{S}}\hat{\mathcal{R}}e^{-2\pi RH}\mathcal{O}_1\mathcal{O}_2\cdots\big),$$

$$\begin{pmatrix}\mathbf{a} & \mathbf{b}\\ \mathbf{c} & \mathbf{d}\end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

$$\tau \rightarrow \frac{\mathbf{a}\tau + \mathbf{b}}{\mathbf{c}\tau + \mathbf{d}}$$

A_μ	gauge field	$\mu = 0, \ldots, 3,$
Φ^I	adjoint-valued scalars	$I = 1, \ldots, 6,$
ψ^a_α	adjoint-valued spinors	$a = 1, \ldots, 4$ and $\alpha = 1,2,$
$\bar{\psi}_{a\dot{\alpha}}$	complex conjugate spinors	$a = 1, \ldots, 4$ and $\dot{\alpha} = \dot{1}, \dot{2},$
$Q_{a\alpha}$	SUSY generators	$a = 1, \ldots, 4$ and $\alpha = 1,2,$
$\bar{Q}^a_{\dot{\alpha}}$	complex conjugate generators	$a = 1, \ldots, 4$ and $\dot{\alpha} = \dot{1}, \dot{2}.$

$$Z^j \equiv \Phi^j + i\Phi^{3+j}, j = 1,2,3$$

$$\tau = \frac{\mathbf{a}\tau + \mathbf{b}}{\mathbf{c}\tau + \mathbf{d}}$$

$$\mathbf{c}\tau + \mathbf{d} = e^{iv}$$

$$|v| = \frac{2\pi}{\mathbf{r}}$$

$$\mathbf{g}: Q_{a\alpha} \rightarrow \left(\frac{\mathbf{c}\tau + \mathbf{d}}{|\mathbf{c}\tau + \mathbf{d}|}\right)^{-\frac{1}{2}} Q_{a\alpha} = e^{-\frac{iv}{2}} Q_{a\alpha}$$

$$\gamma \equiv \begin{pmatrix} e^{i\varphi_1} & & & \\ & e^{i\varphi_2} & & \\ & & e^{i\varphi_3} & \\ & & & e^{i\varphi_4} \end{pmatrix} \in SU(4)_R, \left(\sum_a \varphi_a = 0\right)$$

$$\gamma(\psi^a_\alpha) = e^{i\varphi_a}\psi^a_\alpha, \gamma(\bar{\psi}_{a\dot{\alpha}}) = e^{-i\varphi_a}\bar{\psi}_{a\dot{\alpha}}, a = 1, \ldots, 4,$$

$$\gamma(A_\mu) = A_\mu, \gamma(Z^j) = e^{i(\varphi_j + \varphi_4)} Z^j, j = 1,2,3.$$

$$\begin{aligned} \psi^a_\alpha(x_0, x_1, x_2, x_3 + 2\pi R) &= e^{i\varphi_a}\Lambda^{-1}\psi^a_\alpha(x_0, x_1, x_2, x_3)\Lambda, \\ Z^j(x_0, x_1, x_2, x_3 + 2\pi R) &= e^{i(\varphi_j + \varphi_4)}\Lambda^{-1} Z^j(x_0, x_1, x_2, x_3)\Lambda, \\ A_\mu(x_0, x_1, x_2, x_3 + 2\pi R) &= \Lambda^{-1} A_\mu(x_0, x_1, x_2, x_3)\Lambda + \Lambda^{-1}\partial_\mu\Lambda, \end{aligned}$$

$$\gamma\mathbf{g}(Q_{a\alpha}) = e^{i\left(\varphi_a - \frac{v}{2}\right)} Q_{a\alpha}$$

$$\gamma = \begin{pmatrix} e^{\frac{i}{2}v} & & & \\ & e^{\frac{i}{2}v} & & \\ & & e^{\frac{i}{2}v} & \\ & & & e^{-\frac{3i}{2}v} \end{pmatrix} \in SU(4)_R$$

$$\begin{pmatrix} E_i \\ B_i \end{pmatrix} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix} \begin{pmatrix} E_i \\ B_i \end{pmatrix},$$

$$Z^j = e^{i(\varphi_j + \varphi_4)} Z^j = e^{-iv} Z^j, j = 1,2,3$$

$$\begin{pmatrix} E_i^{(\sigma(l))} \\ B_i^{(\sigma(l))} \end{pmatrix} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix} \begin{pmatrix} E_i^{(l)} \\ B_i^{(l)} \end{pmatrix}, i = 1, \ldots, 3, l = 1, \ldots, n$$

$$Z^{j,\sigma(l)} = e^{-iv} Z^{j,l}, j = 1, \ldots, 3, l = 1, \ldots, n,$$

$$L_1, L_2, R \gg \alpha'^{1/2}$$

$$\mathcal{A} = (2\pi)^2 \alpha'^2 \tau_2^{-1} L_1^{-2} = (2\pi)^2 M_{\mathrm{p}}^{-3} L_1^{-1}$$

$$g_{\mathrm{hadronization}} = \left(M_{\mathrm{p}} L_2\right)^{3/2} = \tau_2^{1/2} L_1^{1/2} L_2^{3/2} \alpha'^{-1}$$

$$\alpha'_{\ \mathrm{hadronization}} = M_{\mathrm{p}}^{-3} L_2^{-1} = \alpha'^2 \tau_2^{-1} L_1^{-1} L_2^{-1}$$

$$\mathcal{A} \gg \alpha'_{\ \mathrm{hadronization}}, g_{\mathrm{hadronization}} \ll 1, R \gg \alpha'^{1/2}$$

$$z \sim z + 1 \sim z + \tau$$

$$-\infty < x_3 < \infty,$$

$$(\zeta_1, \zeta_2, \zeta_3), \zeta_1, \zeta_2, \zeta_3 \in \mathbb{C}$$

$$(z, x_3, \zeta_1, \zeta_2, \zeta_3) \sim \left(e^{iv} z, x_3 + 2\pi R, e^{iv}\zeta_1, e^{iv}\zeta_2, e^{iv}\zeta_3\right)$$

$$W: (z, x_3) \sim (z + \tau, x_3) \sim (z + 1, x_3) \sim \left(e^{iv} z, x_3 + 2\pi R\right).$$

$$\mathcal{H}(n, v) = \bigoplus_{\substack{n_1 \geq n_2 \geq \cdots \geq n_p > 0 \\ n_1 + n_2 + \cdots + n_p = n}} \mathcal{H}_{(n_1, \ldots, n_p)}(v)$$

$$\mathcal{H}(n, v) = \bigoplus_{[\sigma]} \mathcal{H}_{[\sigma]}(v), (\sigma \in S_n)$$

$$(z, \zeta_1, \zeta_2, \zeta_3) \sim \left(e^{iv} z, e^{iv}\zeta_1, e^{iv}\zeta_2, e^{iv}\zeta_3\right)$$

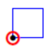
$$e^{i\tilde{n}v} \zeta_{M_a, M_b} = \zeta_{M_a, M_b} + M_a + M_b \tau,$$

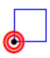
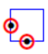
$$\zeta_{M_a, M_b} - \zeta_{M_a', M_b'} \in \mathbb{Z} + \mathbb{Z}\tau$$

$$\left|\left[z, e^{iv} z, \ldots, e^{(\tilde{n}-1)iv} z\right]\right\rangle$$

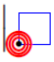
$$\left|\left\{\left[z_1, e^{iv} z_1, \ldots, e^{(\tilde{n}_1 - 1)iv} z_1\right], \left[z_2, e^{iv} z_2, \ldots, e^{(\tilde{n}_2 - 1)iv} z_2\right], \ldots, \left[z_p, e^{iv} z_p, \ldots, e^{(\tilde{n}_p - 1)iv} z_p\right]\right\}\right\rangle$$

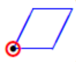
$$\left.\begin{array}{l} k = 1 \text{ when } \mathbf{r} = 6, v = \dfrac{\pi}{3}, \tau = e^{\pi i/3}, \mathbf{g} = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \\ k = 2 \text{ when } \mathbf{r} = 4, v = \dfrac{\pi}{2}, \tau = i, \mathbf{g} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\ k = 3 \text{ when } \mathbf{r} = 3, v = \dfrac{2\pi}{3}, \tau = e^{\pi i/3}, \mathbf{g} = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} \end{array}\right\}$$

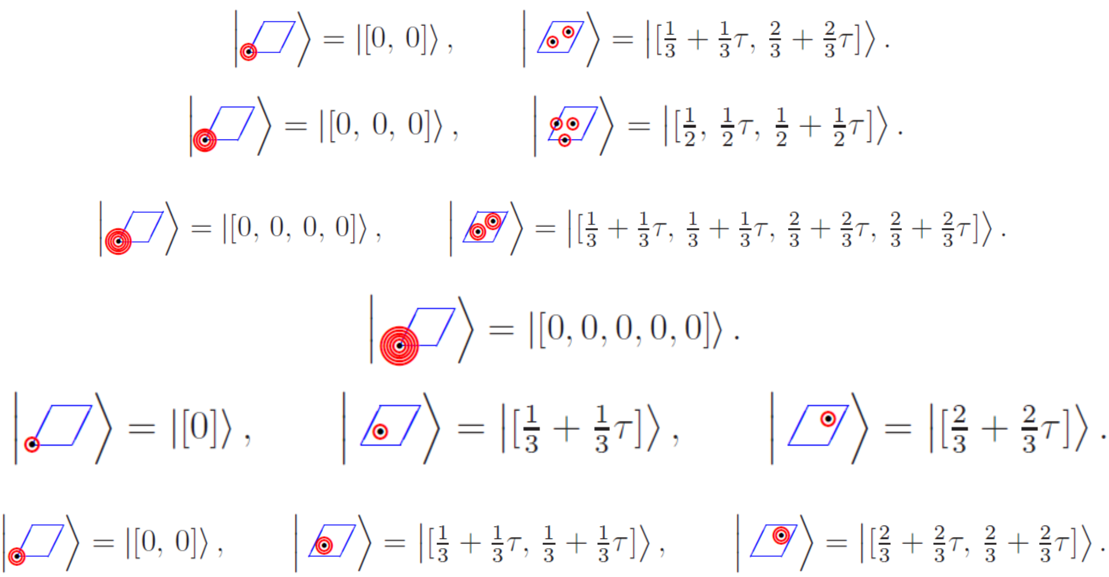
|  $\rangle = |[0]\rangle$, |  $\rangle = \left|\left[\frac{1}{2} + \frac{1}{2}i\right]\right\rangle$.

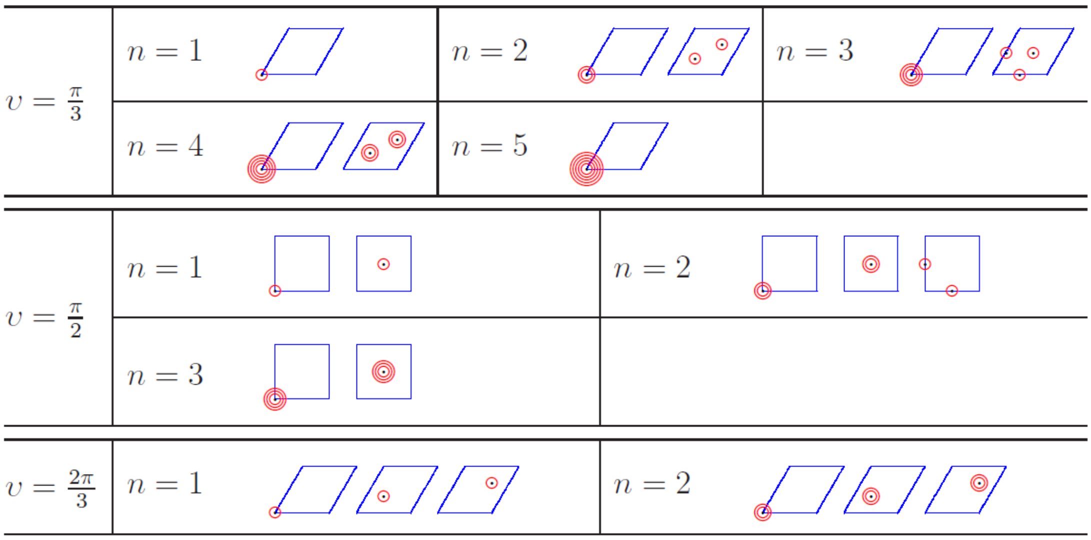
|  $\rangle = |[0, 0]\rangle$, |  $\rangle = \left|\left[\frac{1}{2}, \frac{1}{2}i\right]\right\rangle$, |  $\rangle = \left|\left[\frac{1}{2} + \frac{1}{2}i, \frac{1}{2} + \frac{1}{2}i\right]\right\rangle$.

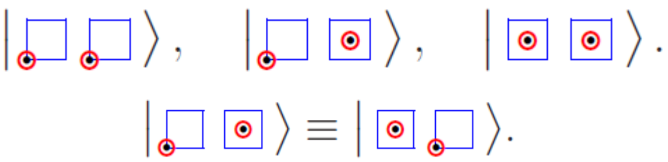
$$\zeta_{0,0} = 0, \zeta_{1,0} = \tfrac{1}{2}, \zeta_{0,1} = \tfrac{1}{2}i, \zeta_{1,1} = \tfrac{1}{2} + \tfrac{1}{2}i, (\mathrm{mod}\,\mathbb{Z} + \mathbb{Z}i)$$

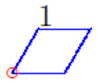
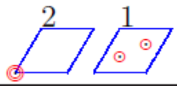
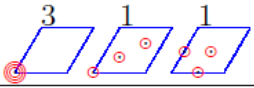
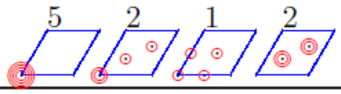
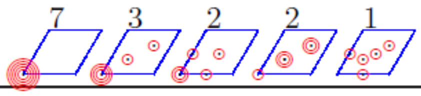
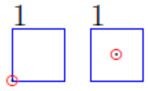
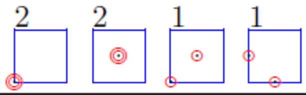
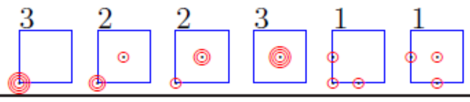
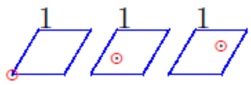
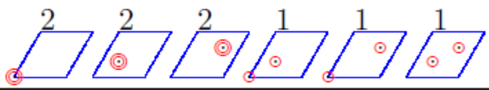
|  $\rangle = |[0, 0, 0]\rangle$, |  $\rangle = \left|\left[\frac{1}{2} + \frac{1}{2}i, \frac{1}{2} + \frac{1}{2}i, \frac{1}{2} + \frac{1}{2}i\right]\right\rangle$.

|  $\rangle = |[0]\rangle$.


$= |[0,\ 0]\rangle\,,$ $= \left|\left[\tfrac{1}{3}+\tfrac{1}{3}\tau,\ \tfrac{2}{3}+\tfrac{2}{3}\tau\right]\right\rangle.$
$= |[0,\ 0,\ 0]\rangle\,,$ $= \left|\left[\tfrac{1}{2},\ \tfrac{1}{2}\tau,\ \tfrac{1}{2}+\tfrac{1}{2}\tau\right]\right\rangle.$
$= |[0,\ 0,\ 0,\ 0]\rangle\,,$ $= \left|\left[\tfrac{1}{3}+\tfrac{1}{3}\tau,\ \tfrac{1}{3}+\tfrac{1}{3}\tau,\ \tfrac{2}{3}+\tfrac{2}{3}\tau,\ \tfrac{2}{3}+\tfrac{2}{3}\tau\right]\right\rangle.$
$= |[0, 0, 0, 0, 0]\rangle\,.$
$= |[0]\rangle\,,$ $= \left|\left[\tfrac{1}{3}+\tfrac{1}{3}\tau\right]\right\rangle,$ $= \left|\left[\tfrac{2}{3}+\tfrac{2}{3}\tau\right]\right\rangle.$
$= |[0,\ 0]\rangle\,,$ $= \left|\left[\tfrac{1}{3}+\tfrac{1}{3}\tau,\ \tfrac{1}{3}+\tfrac{1}{3}\tau\right]\right\rangle,$ $= \left|\left[\tfrac{2}{3}+\tfrac{2}{3}\tau,\ \tfrac{2}{3}+\tfrac{2}{3}\tau\right]\right\rangle.$

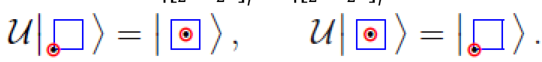

$\upsilon = \frac{\pi}{3}$
$n=1$
$n=2$
$n=3$
$n=4$
$n=5$
$\upsilon = \frac{\pi}{2}$
$n=1$
$n=2$
$n=3$
$\upsilon = \frac{2\pi}{3}$
$n=1$
$n=2$



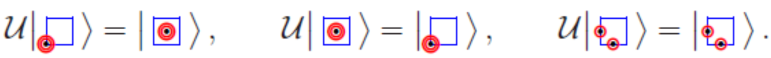
$\upsilon = \frac{\pi}{3}$	$n = 1$		$N_s = 1$
	$n = 2$		$N_s = 3$
	$n = 3$		$N_s = 5$
	$n = 4$		$N_s = 10$
	$n = 5$		$N_s = 15$
$\upsilon = \frac{\pi}{2}$	$n = 1$		$N_s = 2$
	$n = 2$		$N_s = 6$
	$n = 3$		$N_s = 12$
$\upsilon = \frac{2\pi}{3}$	$n = 1$		$N_s = 3$
	$n = 2$		$N_s = 9$

$$\mathcal{U}: (z, x_3) \mapsto \left(z + \frac{1}{k} + \frac{1}{k}\tau, x_3\right),$$

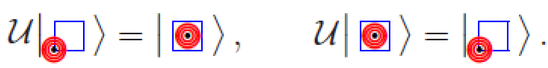
$$\mathcal{U}|[0]\rangle = \left|\left[\frac{1}{2} + \frac{1}{2}i\right]\right\rangle, \mathcal{U}\left|\left[\frac{1}{2} + \frac{1}{2}i\right]\right\rangle = |[0]\rangle,$$



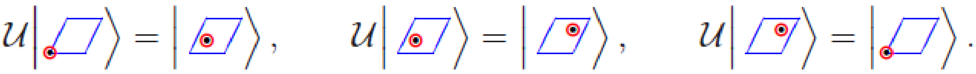
$$\mathcal{U}|[0,0]\rangle = \left|[\tfrac{1}{2} + \tfrac{1}{2}i, \tfrac{1}{2} + \tfrac{1}{2}i]\right\rangle, \quad \mathcal{U}\left|[\tfrac{1}{2}, \tfrac{1}{2}i]\right\rangle = \left|[\tfrac{1}{2}, \tfrac{1}{2}i]\right\rangle, \quad \mathcal{U}\left|[\tfrac{1}{2} + \tfrac{1}{2}i, \tfrac{1}{2} + \tfrac{1}{2}i]\right\rangle = |[0,0]\rangle .$$



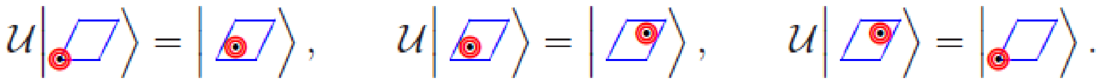
$$\mathcal{U}|[0,0,0]\rangle = \left|[\tfrac{1}{2} + \tfrac{1}{2}i, \tfrac{1}{2} + \tfrac{1}{2}i, \tfrac{1}{2} + \tfrac{1}{2}i]\right\rangle, \quad \mathcal{U}[\tfrac{1}{2} + \tfrac{1}{2}i, \tfrac{1}{2} + \tfrac{1}{2}i, \tfrac{1}{2} + \tfrac{1}{2}i] = |[0,0,0]\rangle .$$



$$\mathcal{U}|[0]\rangle = \left|[\tfrac{1}{3} + \tfrac{1}{3}\tau]\right\rangle, \qquad \mathcal{U}\left|[\tfrac{1}{3} + \tfrac{1}{3}\tau]\right\rangle = \left|[\tfrac{2}{3} + \tfrac{2}{3}\tau]\right\rangle, \qquad \mathcal{U}\left|[\tfrac{2}{3} + \tfrac{2}{3}\tau]\right\rangle = |[0]\rangle .$$



$$\left.\begin{aligned} \mathcal{U}|[0,0]\rangle &= \left|[\tfrac{1}{3}+\tfrac{1}{3}\tau, \tfrac{1}{3}+\tfrac{1}{3}\tau]\right\rangle \\ \mathcal{U}\left|[\tfrac{1}{3}+\tfrac{1}{3}\tau, \tfrac{1}{3}+\tfrac{1}{3}\tau]\right\rangle &= \left|[\tfrac{2}{3}+\tfrac{2}{3}\tau, \tfrac{2}{3}+\tfrac{2}{3}\tau]\right\rangle \\ \mathcal{U}\left|[\tfrac{2}{3}+\tfrac{2}{3}\tau, \tfrac{2}{3}+\tfrac{2}{3}\tau]\right\rangle &= |[0,0]\rangle \end{aligned}\right\}$$



$$\left.\begin{aligned} \eta &= [t \mapsto (z=0, x_3 = 2\pi R t)] \\ \alpha_a &= [t \mapsto (z=t, x_3=0)] \\ \alpha_b &= [t \mapsto (z=t\tau, x_3=0)] \end{aligned}\right\} 0 \le t < 1$$

$$\alpha_a\alpha_b = \alpha_b\alpha_a, \eta^{-1}\alpha_a\eta = \alpha_a^{\mathbf{a}}\alpha_b^{\mathbf{b}}, \eta^{-1}\alpha_b\eta = \alpha_a^{\mathbf{c}}\alpha_b^{\mathbf{d}},$$

$$\bar{\alpha}_a = \bar{\alpha}_a^{\mathbf{a}}\bar{\alpha}_b^{\mathbf{b}}, \bar{\alpha}_b = \bar{\alpha}_a^{\mathbf{c}}\bar{\alpha}_b^{\mathbf{d}}$$

$$\beta_a = \mathbf{a}\beta_a + \mathbf{b}\beta_b, \beta_b = \mathbf{c}\beta_a + \mathbf{d}\beta_b$$

$$\tau = i, v = \tfrac{\pi}{2}, \mathbf{g} = \mathbf{g}' \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, k = 2, \beta_a = -\beta_b = -\beta_a, H_1(W) = \mathbb{Z} \oplus \mathbb{Z}_2, \varrho \in \mathbb{Z}, \beta_a \in \mathbb{Z}_2$$

$$\tau = e^{\pi i/3}, v = \tfrac{\pi}{3}, \mathbf{g} = \mathbf{g}'' \equiv \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}, k = 1, \beta_b = \beta_a, \beta_a = \beta_a - \beta_b, \beta_a = \beta_b = 0, H_1(W) = \mathbb{Z}\varrho$$

$$\tau = e^{\pi i/3}, v = \tfrac{2\pi}{3}, \mathbf{g} = -\mathbf{g}''^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}, k = 3, \beta_b = -\beta_a, \beta_a = \beta_b - \beta_a, H_1(W) = \mathbb{Z} \oplus \mathbb{Z}_3, \varrho \in \mathbb{Z}, \beta_a \in \mathbb{Z}_3$$

$$[t \mapsto (z = 0, x_3 = 2\pi t R)]$$

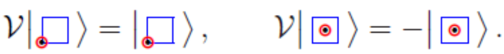
$$\left[t \mapsto \left(z = \frac{1}{2} + \frac{1}{2}i, x_3 = 2\pi t R\right)\right]$$

$$\mathcal{V} \to e^{i\phi}\mathcal{V}$$

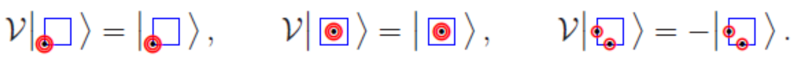
$$\mathcal{V}^k = \mathcal{U}^k = 1.$$

$$\mathcal{V}\mathcal{U}\mathcal{V}^{-1}\mathcal{U}^{-1} = e^{\frac{2\pi i n}{k}},$$

$$\mathcal{V}|[0]\rangle = |[0]\rangle, \mathcal{V}\left|\left[\frac{1}{2}+\frac{1}{2}i\right]\right\rangle = -\left|\left[\frac{1}{2}+\frac{1}{2}i\right]\right\rangle.$$

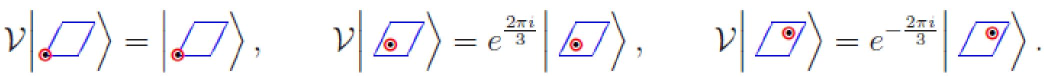


$$\mathcal{V}|[0,0]\rangle = |[0,0]\rangle, \quad \mathcal{V}\left|[\tfrac{1}{2}, \tfrac{1}{2}i]\right\rangle = -\left|[\tfrac{1}{2}, \tfrac{1}{2}i]\right\rangle, \quad \mathcal{V}\left|[\tfrac{1}{2}+\tfrac{1}{2}i, \tfrac{1}{2}+\tfrac{1}{2}i]\right\rangle = \left|[\tfrac{1}{2}+\tfrac{1}{2}i, \tfrac{1}{2}+\tfrac{1}{2}i]\right\rangle.$$



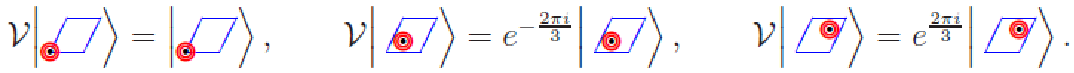
$$\mathcal{V}|[0,0,0]\rangle = |[0,0,0]\rangle, \quad \mathcal{V}\left|[\tfrac{1}{2}+\tfrac{1}{2}i, \tfrac{1}{2}+\tfrac{1}{2}i, \tfrac{1}{2}+\tfrac{1}{2}i]\right\rangle = -\left|[\tfrac{1}{2}+\tfrac{1}{2}i, \tfrac{1}{2}+\tfrac{1}{2}i, \tfrac{1}{2}+\tfrac{1}{2}i]\right\rangle.$$

$$\mathcal{V}|[0]\rangle = |[0]\rangle, \quad \mathcal{V}\left|[\tfrac{1}{3}+\tfrac{1}{3}\tau]\right\rangle = e^{\frac{2\pi i}{3}}\left|[\tfrac{1}{3}+\tfrac{1}{3}\tau]\right\rangle, \quad \mathcal{V}\left|[\tfrac{2}{3}+\tfrac{2}{3}\tau]\right\rangle = e^{-\frac{2\pi i}{3}}\left|[\tfrac{2}{3}+\tfrac{2}{3}\tau]\right\rangle.$$



$$\mathcal{V}|[0,0]\rangle = |[0,0]\rangle\,, \qquad \mathcal{V}\Big|[\tfrac{1}{3}+\tfrac{1}{3}\tau,\tfrac{1}{3}+\tfrac{1}{3}\tau]\Big\rangle = e^{-\frac{2\pi i}{3}}\Big|[\tfrac{1}{3}+\tfrac{1}{3}\tau,\tfrac{1}{3}+\tfrac{1}{3}\tau]\Big\rangle$$

$$\mathcal{V}\Big|[\tfrac{2}{3}+\tfrac{2}{3}\tau,\tfrac{2}{3}+\tfrac{2}{3}\tau]\Big\rangle = e^{\frac{2\pi i}{3}}\Big|[\tfrac{2}{3}+\tfrac{2}{3}\tau,\tfrac{2}{3}+\tfrac{2}{3}\tau]\Big\rangle\,.$$



$$(z,x_3,\zeta_1,\zeta_2,\zeta_3) \sim \left(e^{i\tilde{n}v}z, x_3+2\pi\tilde{n}R, e^{i\tilde{n}v}\zeta_1, e^{i\tilde{n}v}\zeta_2, e^{i\tilde{n}v}\zeta_3\right), v \equiv \frac{2\pi}{\mathbf{r}}$$

$$(z,y) \mapsto \left(e^{iv}z, y+2\pi R\right)$$

$$Z(\sigma+2\pi,\eta) = e^{i\tilde{n}v}Z(\sigma,\eta) + M_a + M_b\tau, Y(\sigma+2\pi,\eta) = Y(\sigma,\eta) + 2\pi\tilde{n}R$$

$$Y = y_0 + P_y\eta + \tilde{n}\sigma R + \sum_{n'\neq 0}\frac{i}{n'}\gamma_{-n'}e^{in'(\eta-\sigma)} + \sum_{n'\neq 0}\frac{i}{n'}\tilde{\gamma}_{-n'}e^{in'(\eta+\sigma)}$$

$$Z = \zeta_{M_a,M_b} + \sum_{n'\in\mathbb{Z}}\frac{i}{n'-\frac{\tilde{n}}{\mathbf{r}}}\alpha_{-n'+\frac{\tilde{n}}{\mathbf{r}}}e^{i\left(n'-\frac{\tilde{n}}{\mathbf{r}}\right)(\eta-\sigma)} + \sum_{n'\in\mathbb{Z}}\frac{i}{n'+\frac{\tilde{n}}{\mathbf{r}}}\tilde{\alpha}_{-n'-\frac{\tilde{n}}{\mathbf{r}}}e^{i\left(n'+\frac{\tilde{n}}{\mathbf{r}}\right)(\eta+\sigma)}$$

$$e^{iv}\zeta_{M_a,M_b} = \zeta_{\mathbf{d}M_a+\mathbf{b}M_b,\mathbf{c}M_a+\mathbf{a}M_b}$$

$$\mathcal{R}|\zeta_{M_a,M_b}\rangle = |e^{iv}\zeta_{M_a,M_b}\rangle$$

$$\sum_{j=0}^{\tilde{n}-1}|e^{ijv}\zeta_{M_a,M_b}\rangle.$$

$$|e^{i\tilde{n}v}\zeta_{M_a,M_b}\rangle = |\zeta_{M_a,M_b}\rangle$$

$$e^{i\tilde{n}v}\zeta - \zeta \in \mathbb{Z} + \mathbb{Z}\tau$$

$$\tilde{\mathcal{U}}(\zeta)|\zeta_{M_a,M_b}\rangle = |\zeta_{M_a,M_b}+\zeta\rangle, \tilde{\mathcal{V}}(\zeta)|\zeta_{M_a,M_b}\rangle = e^{4\pi i\mathrm{Re}\left(\zeta^*\zeta_{M_a,M_b}\right)}|\zeta_{M_a,M_b}\rangle$$

$$\tilde{\mathcal{U}}(\zeta)\tilde{\mathcal{U}}(\zeta') = \tilde{\mathcal{U}}(\zeta')\tilde{\mathcal{U}}(\zeta), \tilde{\mathcal{V}}(\zeta)\tilde{\mathcal{V}}(\zeta') = \tilde{\mathcal{V}}(\zeta')\tilde{\mathcal{V}}(\zeta)$$

$$\tilde{\mathcal{U}}(\zeta)\tilde{\mathcal{V}}(\zeta') = e^{-4\pi i\mathrm{Re}(\zeta^*\zeta')}\tilde{\mathcal{V}}(\zeta')\tilde{\mathcal{U}}(\zeta)$$

$$\mathcal{U} = \tilde{\mathcal{U}}\left(\frac{1}{k}+\frac{1}{k}\tau\right), \mathcal{V} = \tilde{\mathcal{V}}\left(\frac{1}{k}+\frac{1}{k}\tau\right).$$

$$\sum_{j=0}^{\tilde{n}-1}\tilde{\mathcal{U}}(e^{ijv}\zeta), \sum_{j=0}^{\tilde{n}-1}\tilde{\mathcal{V}}(e^{ijv}\zeta), \prod_{j=0}^{\tilde{n}-1}\tilde{\mathcal{U}}(e^{ijv}\zeta)\tilde{\mathcal{V}}(e^{ijv}\zeta), \cdots$$

$$\zeta_{0,0} = 0, \zeta_{1,0} = \frac{1}{2}, \zeta_{0,1} = \frac{1}{2}i, \zeta_{1,1} = \frac{1}{2}(1+i)\ (\mathrm{mod}\mathbb{Z}+\mathbb{Z}\tau).$$

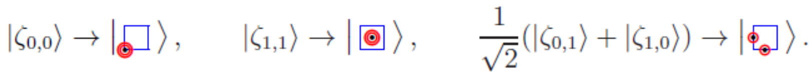
$$\tilde{\mathcal{V}}_a \equiv \tilde{\mathcal{V}}(\zeta_{1,0}), \tilde{\mathcal{U}}_a \equiv \tilde{\mathcal{U}}(\zeta_{1,0}), \tilde{\mathcal{V}}_b \equiv \tilde{\mathcal{V}}(\zeta_{0,1}), \tilde{\mathcal{U}}_b \equiv \tilde{\mathcal{U}}(\zeta_{0,1}),$$

$$\left.\begin{array}{l}\tilde{\mathcal{V}}_a|\zeta_{M_a,M_b}\rangle = (-1)^{M_a}|\zeta_{M_a,M_b}\rangle, \tilde{\mathcal{V}}_b|\zeta_{M_a,M_b}\rangle = (-1)^{M_b}|\zeta_{M_a,M_b}\rangle \\ \tilde{\mathcal{U}}_a|\zeta_{M_a,M_b}\rangle = |\zeta_{M_a+1,M_b}\rangle, \tilde{\mathcal{U}}_b|\zeta_{M_a,M_b}\rangle = |\zeta_{M_a,M_b+1}\rangle,\end{array}\right\}$$

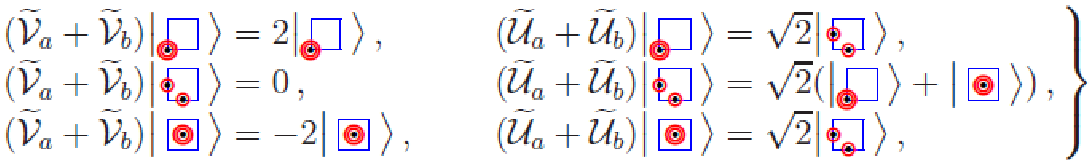
$$|K_a,K_b\rangle \equiv \frac{1}{2}\sum_{M_a,M_b\in\mathbb{Z}_2}(-1)^{K_aM_a+K_bM_b}|\zeta_{M_a,M_b}\rangle$$

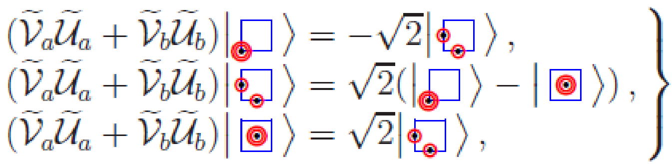
$$\mathcal{U} \equiv \tilde{\mathcal{U}}(\zeta_{1,1}) = \tilde{\mathcal{U}}_a\tilde{\mathcal{U}}_b, \mathcal{V} \equiv \tilde{\mathcal{V}}(\zeta_{1,1}) = \tilde{\mathcal{V}}_a\tilde{\mathcal{V}}_b$$

$$\bar{M}_a \equiv M_a \;(\text{mod}2), \bar{M}_b \equiv M_b \;(\text{mod}2).$$



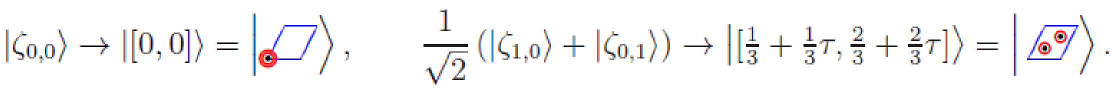
$$\tilde{\mathcal{U}}_a + \tilde{\mathcal{U}}_b, \tilde{\mathcal{V}}_a + \tilde{\mathcal{V}}_b, \tilde{\mathcal{V}}_a \tilde{\mathcal{U}}_a + \tilde{\mathcal{V}}_b \tilde{\mathcal{U}}_b, \ldots$$





$$\zeta_{0,0} = 0, \zeta_{1,0} = \frac{i}{\sqrt{3}}, \zeta_{0,1} = -\frac{i}{\sqrt{3}} \;(\text{mod}\mathbb{Z} + \mathbb{Z}\tau)$$

$$|\zeta_{0,0}\rangle, \frac{1}{\sqrt{2}}(|\zeta_{1,0}\rangle + |\zeta_{0,1}\rangle)$$

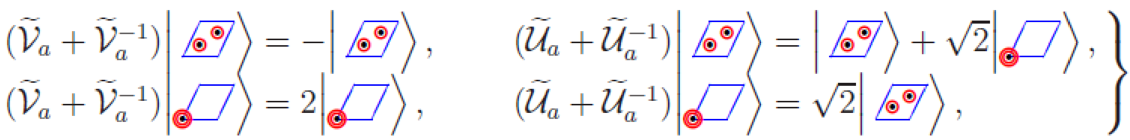


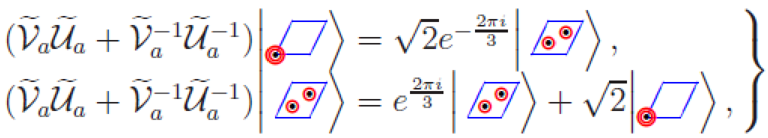
$$\tilde{\mathcal{V}}_a \equiv \tilde{\mathcal{V}}(\zeta_{1,0}), \tilde{\mathcal{U}}_a \equiv \tilde{\mathcal{U}}(\zeta_{1,0}).$$

$$\tilde{\mathcal{V}}_a^3 = \tilde{\mathcal{U}}_a^3 = 1,$$

$$\tilde{\mathcal{V}}_a|\zeta_{M_a,M_b}\rangle \equiv e^{\frac{2\pi i}{3}(M_b - M_a)}|\zeta_{M_a,M_b}\rangle, \tilde{\mathcal{U}}_a|\zeta_{M_a,M_b}\rangle = |\zeta_{M_a+1,M_b}\rangle.$$

$$\tilde{\mathcal{U}}_a + \tilde{\mathcal{U}}_a^{-1}, \tilde{\mathcal{V}}_a + \tilde{\mathcal{V}}_a^{-1}, \tilde{\mathcal{V}}_a \tilde{\mathcal{U}}_a + \tilde{\mathcal{V}}_a^{-1} \tilde{\mathcal{U}}_a^{-1}, \ldots$$





$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \mathcal{G}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \mathcal{G} \equiv \begin{pmatrix} \tilde{\mathbf{a}}\tilde{\mathbf{b}} \\ \tilde{\mathbf{c}}\tilde{\mathbf{d}} \end{pmatrix} \in \text{SL}(2,\mathbb{Z}),$$

$$\rho \to \frac{\tilde{\mathbf{a}}\rho + \tilde{\mathbf{b}}}{\tilde{\mathbf{c}}\rho + \tilde{\mathbf{d}}}$$

$$\text{SL}(2,\mathbb{Z}) \setminus \text{SL}(2,\mathbb{R})/SO(2)$$

$$\rho = \frac{i}{\alpha'_{\text{IIA}}}\text{Area}(T^2) + \frac{1}{2\pi}\int_{T^2} B$$

$$\mathcal{S} \to \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}), \mathcal{S}: \rho \to -\frac{1}{\rho}$$

$$\mathcal{T} \to \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}), \mathcal{T}: \rho \to \rho + 1$$

$$\mathcal{S}^{-1}\mathcal{V}\mathcal{S} = \mathcal{U}, \mathcal{S}^{-1}\mathcal{U}\mathcal{S} = \mathcal{V}^{-1}.$$

$$\mathcal{G}^{-1}\mathcal{V}\mathcal{G} = e^{i\phi_1}\mathcal{V}^{\tilde{\mathrm{d}}}\mathcal{U}^{-\tilde{\mathrm{c}}}, \mathcal{G}^{-1}\mathcal{U}\mathcal{G} = e^{i\phi_2}\mathcal{V}^{-\tilde{\mathrm{b}}}\mathcal{U}^{\tilde{\mathrm{a}}}.$$

$$\mathcal{G} \to \mathcal{U}^{\tilde{\mathrm{p}}}\mathcal{V}^{\tilde{\mathrm{q}}}\mathcal{G},$$

$$\phi_1 \to \phi_1 + \frac{2\pi n}{k}\tilde{\mathbf{p}}, \phi_2 \to \phi_2 - \frac{2\pi n}{k}\tilde{\mathbf{q}}.$$

$$\mathcal{T}^{-1}\mathcal{V}\mathcal{T} = \mathcal{V}, \mathcal{T}^{-1}\mathcal{U}\mathcal{T} = e^{\frac{i\pi n(-1)^{k+1}(k-1)}{k}}\mathcal{U}\mathcal{V}^{-1}.$$

$$\zeta_{1,1} \simeq \zeta_{0,0} = 0, \zeta_{1,0} \simeq \zeta_{0,1} \simeq \frac{1}{2}(1+i)\ (\mathrm{mod}\mathbb{Z} + \mathbb{Z}\tau)$$

$$\frac{1}{\sqrt{2}}\left(\left|\zeta_{0,0}\right\rangle \pm \left|\zeta_{1,1}\right\rangle\right)$$

$$Z \to Z + \tfrac{1}{2}(1+i).$$

$$\left|\zeta_{0,0}\right\rangle \tfrac{1}{\sqrt{2}}\left(\left|\zeta_{0,0}\right\rangle + \left|\zeta_{1,1}\right\rangle\right)$$

$$\left|\zeta_{1,1}\right\rangle \text{ to } \tfrac{1}{\sqrt{2}}\left(\left|\zeta_{0,0}\right\rangle - \left|\zeta_{1,1}\right\rangle\right).$$

$$\mathcal{S}^{-1}\tilde{\mathcal{V}}_a\mathcal{S} = \tilde{\mathcal{U}}_b, \mathcal{S}^{-1}\tilde{\mathcal{V}}_b\mathcal{S} = \tilde{\mathcal{U}}_a^{-1}, \mathcal{S}^{-1}\tilde{\mathcal{U}}_a\mathcal{S} = \tilde{\mathcal{V}}_b, \mathcal{S}^{-1}\tilde{\mathcal{U}}_b\mathcal{S} = \tilde{\mathcal{V}}_a^{-1}$$
$$\mathcal{T}^{-1}\tilde{\mathcal{V}}_a\mathcal{T} = \tilde{\mathcal{V}}_a, \mathcal{T}^{-1}\tilde{\mathcal{V}}_b\mathcal{T} = \tilde{\mathcal{V}}_b, \mathcal{T}^{-1}\tilde{\mathcal{U}}_a\mathcal{T} = \tilde{\mathcal{U}}_a\tilde{\mathcal{V}}_b^{-1}, \mathcal{T}^{-1}\tilde{\mathcal{U}}_b\mathcal{T} = \tilde{\mathcal{U}}_b\tilde{\mathcal{V}}_a$$

$$\mathcal{S}\left|\zeta_{K_a,K_b}\right\rangle = \frac{1}{2}\sum_{M_a,M_b\in\mathbb{Z}_2}(-1)^{K_aM_a+K_bM_b}\left|\zeta_{M_a,M_b}\right\rangle \equiv |K_a,K_b\rangle$$

$$\mathcal{T}\left|\zeta_{K_a,K_b}\right\rangle = (-1)^{K_aK_b}\left|\zeta_{K_a,K_b}\right\rangle.$$

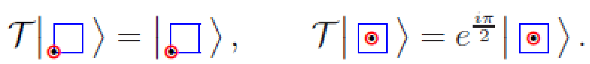
$$\mathcal{S}^{-1}\tilde{\mathcal{V}}_a\mathcal{S} = \tilde{\mathcal{U}}_a, \mathcal{S}^{-1}\tilde{\mathcal{U}}_a\mathcal{S} = \tilde{\mathcal{V}}_a^{-1}, \mathcal{T}^{-1}\tilde{\mathcal{V}}_a\mathcal{T} = \tilde{\mathcal{V}}_a, \mathcal{T}^{-1}\tilde{\mathcal{U}}_a\mathcal{T} = e^{\frac{2\pi i}{3}}\tilde{\mathcal{U}}_a\tilde{\mathcal{V}}_a,$$

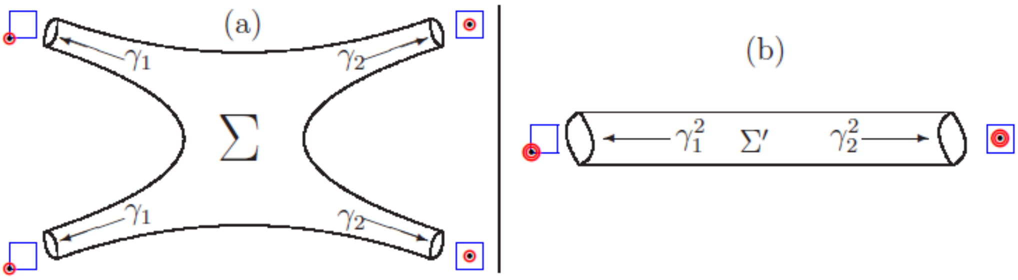
$$\omega_F \equiv \frac{1}{2i\mathrm{Im}\tau}dz \wedge d\bar{z}$$

$$B \to B + \omega_F$$

$$\partial\Sigma = \left(\bigcup_{i=1}^{p}\gamma_i^{-1}\right)\bigcup\left(\bigcup_{j=1}^{q}\gamma_j'\right),$$

$$e^{i\Phi(\gamma_1^{-1},\gamma_2^{-1},\cdots,\gamma_p^{-1},\gamma_1',\gamma_2',\cdots,\gamma_q')} \equiv \exp\left(i\int_\Sigma \omega_F\right)$$





$$0 \leq t < 1,$$

$$\gamma_1 = [t \mapsto (z = 0, x_3 = 2\pi R t)], \gamma_2 = \left[t \mapsto \left(z = \frac{1}{2}(1+\tau), x_3 = 2\pi R t\right)\right].$$

$$\gamma_1^2 = [t \mapsto (z = 0, x_3 = 4\pi R t)], \gamma_2^2 = \left[t \mapsto \left(z = \frac{1}{2}(1+\tau), x_3 = 4\pi R t\right)\right].$$

$$\alpha_a = [t \mapsto (z = t, x_3 = 0)], \alpha_b = [t \mapsto (z = t\tau, x_3 = 0)],$$
$$\alpha_{a+b} = [t \mapsto (z = t(\tau+1), x_3 = 0)].$$

$$\Phi(\gamma_1^{-1}, \gamma_1^{-1}, \gamma_2, \gamma_2) \equiv \Phi(\gamma_1^{-2}, \gamma_2^2)\ (\mathrm{mod} 2\pi).$$

$$\Phi(\gamma_1^{-2}, \gamma_2^2, \alpha_{a+b}) \equiv 0\ (\mathrm{mod} 2\pi).$$

$$\Sigma = \left[(\sigma, \eta) \mapsto \left(z = \frac{1}{2}\sigma(1+\tau), x_3 = 4\pi R \eta\right)\right], 0 \leq \sigma, \eta < 1,$$

$$\partial\Sigma = \gamma_1^{-2} \cup \gamma_2^2 \cup \alpha_{a+b} \int_\Sigma \omega_F = 0$$

$$\Phi(\alpha_a^{-1}, \alpha_b^{-1}) \equiv 0\ (\mathrm{mod} 2\pi)$$

$$\Sigma = [(\sigma, \eta) \mapsto (z = \sigma, x_3 = 2\pi R \eta)], 0 \leq \sigma, \eta < 1$$

$$\Phi(\alpha_a^{-1}, \alpha_b^{-1}, \alpha_{a+b}) \equiv \pi\ (\mathrm{mod} 2\pi)$$

$$\Phi(\gamma_1^{-2}, \gamma_2^2) \equiv \Phi(\alpha_{a+b}) \equiv \pi\ (\mathrm{mod} 2\pi)$$

$$\gamma_3 = \left[t \mapsto \left(z = \frac{1}{2}, x_3 = 4\pi R t\right)\right]$$

$$\Sigma_1 = \left[(\sigma, \eta) \mapsto \left(z = \frac{1}{2}\sigma, x_3 = 2\pi R \eta\right)\right], 0 \leq \sigma, \eta < 1$$

$$\Sigma_2 = \left[(\sigma, \eta) \mapsto \left(z = \frac{1}{2}\sigma\tau, x_3 = 2\pi R \eta\right)\right], 0 \leq \sigma, \eta < 1$$

$$\delta = \left[t \mapsto \begin{cases} (z = 2t, x_3 = 0) & 0 \leq t \leq \frac{1}{4} \\ \left(z = \frac{1}{2} + 2\left(t - \frac{1}{4}\right)\tau, x_3 = 0\right) & \frac{1}{4} \leq t \leq \frac{1}{2} \\ \left(z = 2\left(\frac{3}{4} - t\right) + \frac{1}{2}\tau, x_3 = 0\right) & \frac{1}{2} \leq t \leq \frac{3}{4} \\ (z = 2(1-t)\tau, x_3 = 0) & \frac{3}{4} \leq t \leq 1 \end{cases}\right]$$

$$\partial(\Sigma_1 \cup \Sigma_2) = \gamma_1^{-1} \cup \gamma_2^{-1} \cup \gamma_3 \cup \delta, \text{ and } \int_{\Sigma_1 \cup \Sigma_2} \omega_F = 0$$

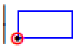
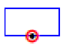
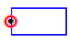
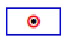
$$\Phi(\gamma_1^{-1}, \gamma_2^{-1}, \gamma_3) \equiv -\Phi(\delta) \equiv -\frac{\pi}{2}\ (\mathrm{mod} 2\pi)$$

$$\begin{aligned}\left[\bar{\psi}^{\dot\alpha}_{a}(x_0,x_1,x_2,x_3+2\pi R)\right]^* &= i\Lambda^{-1}\psi^{a}_{\alpha}(x_0,x_1,x_2,x_3)\Lambda, a=1,\dots,4\\ [\Phi^I(x_0,x_1,x_2,x_3+2\pi R)]^* &= -\Lambda^{-1}\Phi^I(x_0,x_1,x_2,x_3)\Lambda, I=1\dots 6\\ -A^*_\mu(x_0,x_1,x_2,x_3+2\pi R) &= \Lambda^{-1}A_\mu(x_0,x_1,x_2,x_3)\Lambda - i\Lambda^{-1}\partial_\mu\Lambda\end{aligned}$$

$$g = P\exp\left(i\oint_C A\right)$$

$$g \mapsto [\Omega^{-1}g\Omega]^*$$

$$g = [\Omega^{-1}g\Omega]^*$$

$\Big|$  $\Big\rangle = |\{0\}\rangle$, $\Big|$  $\Big\rangle = |\{\tfrac{1}{2}\}\rangle$, $\Big|$  $\Big\rangle = |\{\tfrac{1}{2}\tau\}\rangle$, $\Big|$  $\Big\rangle = |\{\tfrac{1}{2}(1+\tau)\}\rangle$.

$$\left.\begin{aligned}\mathcal{U}_1\left|\left[\frac{1}{2}M+\frac{1}{2}N\tau\right]\right\rangle &= \left|\left[\frac{1}{2}(1-M)+\frac{1}{2}N\tau\right]\right\rangle\\ \mathcal{U}_2\left|\left[\frac{1}{2}M+\frac{1}{2}N\tau\right]\right\rangle &= \left|\left[\frac{1}{2}M+\frac{1}{2}(1-N)\tau\right]\right\rangle\end{aligned}\right\}$$

$$\left.\begin{aligned}\mathcal{V}_1\left|\left[\frac{1}{2}M+\frac{1}{2}N\tau\right]\right\rangle &= (-1)^M\left|\left[\frac{1}{2}M+\frac{1}{2}N\tau\right]\right\rangle\\ \mathcal{V}_2\left|\left[\frac{1}{2}M+\frac{1}{2}N\tau\right]\right\rangle &= (-1)^N\left|\left[\frac{1}{2}M+\frac{1}{2}N\tau\right]\right\rangle\end{aligned}\right\}$$

$$\mathcal{U}_1 = (-1)^{\mathbf{m}_1}, \mathcal{U}_2 = (-1)^{\mathbf{e}_1}, \mathcal{V}_1 = (-1)^{\mathbf{e}_2}, \mathcal{V}_2 = (-1)^{\mathbf{m}_2}$$

$$A_1 \to A_1, A_2 \to A_2 + \frac{1}{2L_2}$$

$$A_1 \to A_1 + \frac{1}{2L_1}, A_2 \to A_2$$

$$\mathcal{V}_1|w_1,w_2\rangle = |w_1,-w_2\rangle, \mathcal{U}_2|w_1,w_2\rangle = |-w_1,w_2\rangle$$

$$\left|\left[\frac{1}{2}M+\frac{1}{2}N\tau\right]\right\rangle_{\boxtimes} = \frac{1}{\sqrt{2}}\sum_{M'=0,1}(-1)^{M'M}|(-1)^N,(-1)^{M'}\rangle_{\boxdot}$$

$$\mathcal{U}_1|w_1,w_2\rangle = w_2|w_1,w_2\rangle, \mathcal{V}_2|w_1,w_2\rangle = w_1|w_1,w_2\rangle.$$

$$\Lambda \to \Lambda e^{\frac{ix_1}{L_1}}$$

$$\mathcal{S}|w_1,w_2\rangle = |w_2,w_1\rangle, \mathcal{T}|w_1,w_2\rangle = |w_1,w_1w_2\rangle$$

$$|[z,-z]\rangle, \left|\left[z,\frac{1}{2}-z\right]\right\rangle, \left|\left[z,\frac{1}{2}\tau-z\right]\right\rangle, \left|\left[z,\frac{1}{2}+\frac{1}{2}\tau-z\right]\right\rangle,$$

$$\left.\begin{aligned}\overline{\mathcal{U}}_1\left|\left\{\left[\frac{1}{2}M+\frac{1}{2}N\tau\right],\left[\frac{1}{2}M'+\frac{1}{2}N'\tau\right]\right\}\right\rangle &= \left|\left[\frac{1}{2}(1-M)+\frac{1}{2}N\tau\right],\left[\frac{1}{2}(1-M')+\frac{1}{2}N'\tau\right]\right\rangle\\ \overline{\mathcal{U}}_2\left|\left\{\left[\frac{1}{2}M+\frac{1}{2}N\tau\right],\left[\frac{1}{2}M'+\frac{1}{2}N'\tau\right]\right\}\right\rangle &= \left|\left[\frac{1}{2}M+\frac{1}{2}(1-N)\tau\right],\left[\frac{1}{2}M'+\frac{1}{2}(1-N')\tau\right]\right\rangle\end{aligned}\right\},$$

$$\left.\begin{aligned}\overline{\mathcal{V}}_1\left|\left\{\left[\frac{1}{2}M+\frac{1}{2}N\tau\right],\left[\frac{1}{2}M'+\frac{1}{2}N'\tau\right]\right\}\right\rangle &= (-1)^{M+M'}\left|\left\{\left[\frac{1}{2}M+\frac{1}{2}N\tau\right],\left[\frac{1}{2}M'+\frac{1}{2}N'\tau\right]\right\}\right\rangle\\ \overline{\mathcal{V}}_2\left|\left\{\left[\frac{1}{2}M+\frac{1}{2}N\tau\right],\left[\frac{1}{2}M'+\frac{1}{2}N'\tau\right]\right\}\right\rangle &= (-1)^{N+N'}\left|\left\{\left[\frac{1}{2}M+\frac{1}{2}N\tau\right],\left[\frac{1}{2}M'+\frac{1}{2}N'\tau\right]\right\}\right\rangle\end{aligned}\right\}.$$

$$-\phi^* = (i\sigma_2)^{-1}\phi(i\sigma_2).$$

$$\mathbf{e}'_1\mathbf{m}'_2 - \mathbf{e}'_2\mathbf{m}'_1 = \mathbf{e}'_1\mathbf{m}'_3 - \mathbf{e}'_3\mathbf{m}'_1 = \mathbf{e}'_2\mathbf{m}'_3 - \mathbf{e}'_3\mathbf{m}'_2 = 0.$$

$$\mathbf{e}_1'\mathbf{m}_2' - \mathbf{e}_2'\mathbf{m}_1' = 0.$$

$$A^{U(2)} = \begin{pmatrix} \frac{\mathbf{m}_2}{2L_1}dx_1 + \frac{\mathbf{m}_1}{2L_2}dx_2 & 0 \\ 0 & 0 \end{pmatrix}, \Lambda = \begin{pmatrix} e^{-\frac{i\mathbf{m}_2 x_1}{L_1} - \frac{i\mathbf{m}_1 x_2}{L_2}} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{U}_1 \equiv (-1)^{\mathbf{m}_1}, \mathcal{V}_2 \equiv (-1)^{\mathbf{m}_2}$$

$$\Lambda = e^{-\frac{i\mathbf{m}_2 x_1}{2L_1} - \frac{i\mathbf{m}_1 x_2}{2L_2}} \begin{pmatrix} e^{-\frac{i\mathbf{m}_2 x_1}{2L_1} - \frac{i\mathbf{m}_1 x_2}{2L_2}} & 0 \\ 0 & e^{\frac{i\mathbf{m}_2 x_1}{2L_1} + \frac{i\mathbf{m}_1 x_2}{2L_2}} \end{pmatrix}$$

$$0 = \mathbf{m}_1 + \mathbf{m}_1' = \mathbf{m}_2 + \mathbf{m}_2' \ (\text{mod}2).$$

$$\tilde{\Lambda} = \begin{pmatrix} e^{\frac{ix_1}{L_1}} & 0 \\ 0 & 1 \end{pmatrix} = e^{\frac{ix_1}{2L_1}} \begin{pmatrix} e^{\frac{ix_1}{2L_1}} & 0 \\ 0 & e^{-\frac{ix_1}{2L_1}} \end{pmatrix}$$

$$\mathcal{U}_2^2(-1)^{\mathbf{e}_1'} = \mathcal{V}_1^2(-1)^{\mathbf{e}_2'} = 1$$

$$\begin{pmatrix} e^{\frac{ix_1}{2L_1}} & 0 \\ 0 & e^{\frac{ix_1}{2L_1}} \end{pmatrix}$$

$$\mathcal{U}_1^2 = \mathcal{V}_2^2 = 1, \mathcal{U}_1\mathcal{U}_2 = \mathcal{U}_2\mathcal{U}_1, \mathcal{V}_1\mathcal{V}_2 = \mathcal{V}_2\mathcal{V}_1,$$
$$\mathcal{U}_i\mathcal{V}_j = (-1)^{\delta_{ij}}\mathcal{V}_j\mathcal{U}_i, i, j = 1,2,$$

$$\mathcal{V}_1^4 = \mathcal{U}_2^4 = 1,$$

$$\mathcal{U}_2^2 = (-1)^{\mathbf{e}_1'}, \mathcal{V}_1^2 = (-1)^{\mathbf{e}_2'}$$

$$\begin{cases} \mathcal{U}_1|w_1, w_2, \mathbf{e}_1, \mathbf{e}_2\rangle = w_2|w_1, w_2, \mathbf{e}_1, \mathbf{e}_2\rangle \\ \mathcal{U}_2|w_1, w_2, \mathbf{e}_1, \mathbf{e}_2\rangle = i^{\mathbf{e}_1}|-w_1, w_2, \mathbf{e}_1, \mathbf{e}_2\rangle \\ \mathcal{V}_1|w_1, w_2, \mathbf{e}_1, \mathbf{e}_2\rangle = i^{\mathbf{e}_2}|w_1, -w_2, \mathbf{e}_1, \mathbf{e}_2\rangle \\ \mathcal{V}_2|w_1, w_2, \mathbf{e}_1, \mathbf{e}_2\rangle = w_1|w_1, w_2, \mathbf{e}_1, \mathbf{e}_2\rangle \end{cases}$$

$$|\mathbf{e}_1', \mathbf{e}_2', \mathbf{m}_1', \mathbf{m}_2'\rangle_{U(2)} \equiv$$
$$|\, w_1 = (-1)^{\mathbf{m}_2'}, w_2 = (-1)^{\mathbf{m}_1'}, \mathbf{e}_1', \mathbf{e}_2'\rangle_{U(1)} \otimes |\mathbf{e}_1', \mathbf{e}_2', 0, \mathbf{m}_1', \mathbf{m}_2', 0\rangle_{SU(2)},$$

$$\mathbf{e}_1', \mathbf{e}_2', \mathbf{m}_1', \mathbf{m}_2' \in \mathbb{Z}_2, \mathbf{e}_3' = \mathbf{m}_3' = 0$$

$$\left|\left\{\left[\frac{1}{2}M + \frac{1}{2}N\tau\right], \left[\frac{1}{2}M' + \frac{1}{2}N'\tau\right]\right\}\right\rangle$$

$$\overline{\mathcal{U}}_1 = \mathcal{U}_1, \overline{\mathcal{U}}_2 = \mathcal{U}_2^2, \overline{\mathcal{V}}_1 = \mathcal{V}_1^2, \overline{\mathcal{V}}_2 = \mathcal{V}_2.$$

$$\left|\left\{\left[\frac{1}{2}M + \frac{1}{2}N\tau\right], \left[\frac{1}{2}M' + \frac{1}{2}N'\tau\right]\right\}\right\rangle_{\text{hadronization}} =$$
$$\frac{1}{2}\sum_{K=0}^{1}\sum_{L=0}^{1} (-1)^{MK+NL}|\mathbf{e}_1' = L, \mathbf{e}_2' = M - M', \mathbf{m}_1' = K, \mathbf{m}_2' = N - N'\rangle_{U(2)}$$

$$|\mathbf{e}_1', \mathbf{e}_2', \mathbf{m}_1', \mathbf{m}_2'\rangle_{U(2)} =$$
$$\frac{1}{2}\sum_{M=0}^{1}\sum_{N=0}^{1} (-1)^{M\mathbf{m}_1' + N\mathbf{e}_1'} \left|\left\{\left[\frac{1}{2}M + \frac{1}{2}N\tau\right], \left[\frac{1}{2}(M + \mathbf{e}_2') + \frac{1}{2}(N + \mathbf{m}_2')\tau\right]\right\}\right\rangle_{\text{hadronization}}.$$

$$\psi_\alpha^a(x_0, x_1, x_2, x_3 + 2\pi R) = e^{i\varphi_a}\psi_\alpha^a(x_0, x_1, x_2, x_3), a = 1, \dots, 4.$$
$$Z^j(x_0, x_1, x_2, x_3 + 2\pi R) = e^{i(\varphi_j + \varphi_4)}Z^j(x_0, x_1, x_2, x_3), j = 1,2,3.$$

$$A(0) \equiv \sum_{\mu=0}^{2} A_\mu(x_0, x_1, x_2, x_3 = 0)dx^\mu, A(2\pi R) \equiv \sum_{\mu=0}^{2} A_\mu(x_0, x_1, x_2, x_3 = 2\pi R)dx^\mu$$

$$I_S(\mathbf{g}') = \frac{1}{2\pi}\int A(0) \wedge dA(2\pi R)$$

$$\Psi(A) \to \widetilde{\Psi}(\tilde{A}) = \int [\mathcal{D}A]\exp\left\{\frac{i}{2\pi}\int A \wedge d\tilde{A}\right\}\Psi(A)$$

$$E_i \to B_i, B_i \to -E_i$$

$$I_S(-\mathbf{g}') = -\frac{1}{2\pi}\int A(0) \wedge dA(2\pi R)$$

For $\mathbf{g} = \mathbf{g}'' \equiv \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\mathbf{g}'$ we get

$$I_S(\mathbf{g}'') = \frac{1}{4\pi}\int \{-A(2\pi R) \wedge dA(2\pi R) + 2A(0) \wedge dA(2\pi R)\}$$

$$\Psi(A) \to \exp\left\{-\frac{i}{4\pi}\int A \wedge dA\right\}\Psi(A)$$

$$\mathbf{g} = -\mathbf{g}'' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}(\mathbf{g}')^{-1}$$

$$I_S(-\mathbf{g}'') = \frac{1}{4\pi}\int \{-A(2\pi R) \wedge dA(2\pi R) - 2A(0) \wedge dA(2\pi R)\}$$

$$I_S \to \frac{2 - \mathbf{a} - \mathbf{d}}{4\pi\mathbf{c}}\int A \wedge dA$$

$$k \equiv (2 - \mathbf{a} - \mathbf{d})/\mathbf{c}$$

$$\tau = i, v = \frac{\pi}{2}, \mathbf{g} = \mathbf{g}' \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\tau = e^{\pi i/3}, v = \frac{\pi}{3}, \mathbf{g} = \mathbf{g}'' \equiv \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\tau = e^{\pi i/3}, v = \frac{2\pi}{3}, \mathbf{g} = -\mathbf{g}''^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$$

$$0 \le x_0 < 2\pi L_0, 0 \le x_1 < 2\pi L_1, 0 \le x_2 < 2\pi L_2$$

$$A = \frac{\overline{\mathbf{m}}_0 x_1}{2\pi L_1 L_2}dx_2 + \frac{\overline{\mathbf{m}}_1 x_2}{2\pi L_2 L_0}dx_0 + \frac{\overline{\mathbf{m}}_2 x_0}{2\pi L_0 L_1}dx_1 + A'$$

$$\tilde{A} = \frac{\tilde{\overline{\mathbf{m}}}_0 x_1}{2\pi L_1 L_2}dx_2 + \frac{\tilde{\overline{\mathbf{m}}}_1 x_2}{2\pi L_2 L_0}dx_0 + \frac{\tilde{\overline{\mathbf{m}}}_2 x_0}{2\pi L_0 L_1}dx_1 + \tilde{A}'$$

$$\begin{aligned}
&\Psi(A'; \overline{\mathbf{m}}_0, \overline{\mathbf{m}}_1, \overline{\mathbf{m}}_2) \to \\
&\widetilde{\Psi}(\tilde{A}; \tilde{\overline{\mathbf{m}}}_0, \tilde{\overline{\mathbf{m}}}_1, \tilde{\overline{\mathbf{m}}}_2) = \sum_{\overline{\mathbf{m}}_0 \in \mathbb{Z}} \sum_{\overline{\mathbf{m}}_1 \in \mathbb{Z}} \sum_{\overline{\mathbf{m}}_2 \in \mathbb{Z}} \int [\mathcal{D}A']\exp\{ \\
&i\int A' \wedge \left(\frac{\tilde{\overline{\mathbf{m}}}_0}{4\pi^2 L_1 L_2}dx_1 \wedge dx_2 + \frac{\tilde{\overline{\mathbf{m}}}_1}{4\pi^2 L_2 L_0}dx_2 \wedge dx_0 + \frac{\tilde{\overline{\mathbf{m}}}_2}{4\pi^2 L_0 L_1}dx_0 \wedge dx_1\right) \\
&-i\int \tilde{A}' \wedge \left(\frac{\overline{\mathbf{m}}_0}{4\pi^2 L_1 L_2}dx_1 \wedge dx_2 + \frac{\overline{\mathbf{m}}_1}{4\pi^2 L_2 L_0}dx_2 \wedge dx_0 + \frac{\overline{\mathbf{m}}_2}{4\pi^2 L_0 L_1}dx_0 \wedge dx_1\right) \\
&+\frac{i}{2\pi}\int A' \wedge d\tilde{A}'\Big\}\Psi(A'; \overline{\mathbf{m}}_0, \overline{\mathbf{m}}_1, \overline{\mathbf{m}}_2)
\end{aligned}$$

$$\begin{aligned}
\frac{i}{2\pi}\int A \wedge d\tilde{A} + i\pi(\overline{\mathbf{m}}_0\tilde{\overline{\mathbf{m}}}_1 + \overline{\mathbf{m}}_1\tilde{\overline{\mathbf{m}}}_2 + \overline{\mathbf{m}}_2\tilde{\overline{\mathbf{m}}}_0) + \frac{i\overline{\mathbf{m}}_0}{L_2}\int \tilde{A}_0(x_0, 0, x_2)dx_0 dx_2 \\
+\frac{i\overline{\mathbf{m}}_1}{L_0}\int \tilde{A}_1(x_0, x_1, 0)dx_0 dx_1 + \frac{i\overline{\mathbf{m}}_2}{L_1}\int \tilde{A}_2(0, x_1, x_2)dx_1 dx_2
\end{aligned}$$

$$\overline{\mathbf{m}}_0 = \int dA \wedge \delta(x_0) dx_0$$

$$\overline{\mathbf{m}}_1 = \int dA \wedge \delta(x_1) dx_1 = \frac{1}{2\pi L_1}\int dA \wedge dx_1$$
$$\overline{\mathbf{m}}_2 = \int dA \wedge \delta(x_2) dx_2 = \frac{1}{2\pi L_2}\int dA \wedge dx_2$$

$$\tilde{\overline{\mathbf{m}}}_0 = \int d\tilde{A} \wedge \delta(x_0) dx_0$$

$$\tilde{\overline{\mathbf{m}}}_1 = \int d\tilde{A} \wedge \delta(x_1) dx_1 = \frac{1}{2\pi L_1}\int d\tilde{A} \wedge dx_1$$
$$\tilde{\overline{\mathbf{m}}}_2 = \int d\tilde{A} \wedge \delta(x_2) dx_2 = \frac{1}{2\pi L_2}\int d\tilde{A} \wedge dx_2$$

$$(-1)^{\mathbf{e}_1+\mathbf{m}_1}$$

$$e^{\frac{2\pi i}{k}(\mathbf{e}_j+\mathbf{m}_j)}$$

$$A \to A + \frac{dx_1}{2L_1}$$

$$A' \to A' + \frac{dx_1}{2L_1}, \tilde{A}' \to \tilde{A}' + \frac{dx_1}{2L_1}$$

$$\frac{i}{2}\int [F_{02}(x_3 = 0) - F_{02}(x_3 = 2\pi R)] dx_0 dx_2$$

$$i\pi \int [A_2(x_3 = 2\pi R, x_0 = \infty) - A_2(x_3 = 0, x_0 = \infty)] dx^2$$
$$-i\pi \int [A_2(x_3 = 2\pi R, x_0 = -\infty) - A_2(x_3 = 0, x_0 = -\infty)] dx^2$$

$$\exp\left\{i\pi \int F_{23} dx_2 dx_3\right\} \overset{A_3=0}{\to} \exp\left\{i\pi\left(\int [A_2(x_3 = 2\pi R) - A_2(x_3 = 0)] dx^2\right)\right\}$$

$$I = \frac{k}{4\pi}\int A \wedge dA$$

$$\omega \equiv e^{\frac{2\pi i}{k}}$$

$$\mathcal{W}_1 = \exp\left\{\int_0^{2\pi L_1} A_1(t,0) dt\right\}, \mathcal{W}_2 = \exp\left\{\int_0^{2\pi L_2} A_2(0,t) dt\right\}$$

$$\mathcal{W}_1 = \begin{pmatrix} 1 & & & & \\ & \omega & & & \\ & \ddots & & & \\ & & \omega^{k-2} & & \\ & & & \omega^{k-1} & \end{pmatrix}, \mathcal{W}_2 = \begin{pmatrix} & & & 1 \\ 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}$$

$$\mathcal{W}_1\mathcal{W}_2 = \omega \mathcal{W}_2\mathcal{W}_1, \mathcal{W}_1^k = \mathcal{W}_2^k = 1$$

$$|p\rangle, p = 0, \dots, k-1 \in \mathbb{Z}/k\mathbb{Z},$$

$$\mathcal{W}_1|p\rangle = \omega^p|p\rangle, \mathcal{W}_2|p\rangle = |p+1\rangle$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \mathcal{G}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \mathcal{G} \equiv \begin{pmatrix} \tilde{\mathbf{a}}\tilde{\mathbf{b}} \\ \tilde{\mathbf{c}}\tilde{\mathbf{d}} \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}),$$

$$\mathcal{G}^{-1}\mathcal{W}_1\mathcal{G} = \mathcal{W}_2^{-\tilde{\mathrm{b}}}\mathcal{W}_1^{\tilde{\mathrm{a}}}, \mathcal{G}^{-1}\mathcal{W}_2\mathcal{G} = \mathcal{W}_2^{\tilde{\mathrm{d}}}\mathcal{W}_1^{-\tilde{\mathrm{c}}}$$

$$\mathcal{S}|p\rangle = \frac{1}{\sqrt{k}}\sum_{q} \omega^{pq}|q\rangle$$

$$\mathcal{S}^{-1}\mathcal{W}_1\mathcal{S} = \mathcal{W}_2, \mathcal{S}^{-1}\mathcal{W}_2\mathcal{S} = \mathcal{W}_1^{-1}$$

$$\mathcal{T} \equiv \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{T}|p\rangle = e^{\frac{\pi i}{k}p^2}|p\rangle, k \equiv 0 \ (\mathrm{mod}2)$$

$$\exp\left(\frac{\pi i}{k}p^2\right)$$

$$\mathcal{T}|p\rangle = e^{\frac{\pi i}{k}p(p+k)}|p\rangle.$$

$$\Lambda_1(x_1, x_2) = e^{\frac{ivx_1}{L_1}}, \Lambda_2(x_1, x_2) = e^{\frac{ivx_2}{L_2}}$$

$$\Omega_1 = e^{2\pi i v \mathrm{e}_1}, \Omega_2 = e^{2\pi i v \mathrm{e}_2}$$

$$\Omega_1^{-1}\mathcal{W}_1\Omega_1 = e^{2\pi i v}\mathcal{W}_1, \Omega_2^{-1}\mathcal{W}_1\Omega_2 = \mathcal{W}_1, \Omega_1^{-1}\mathcal{W}_2\Omega_1 = \mathcal{W}_2, \Omega_2^{-1}\mathcal{W}_2\Omega_2 = e^{2\pi i v}\mathcal{W}_2.$$

$$v = \frac{1}{k}, \Lambda_1(x_1, x_2) = e^{\frac{ix_1}{kL_1}}, \Lambda_2(x_1, x_2) = e^{\frac{ix_2}{kL_2}},$$

$$\Omega_1 \equiv \mathcal{W}_2, \Omega_2 \equiv \mathcal{W}_1^{-1},$$

$$\Omega_1|p\rangle = |p+1\rangle, \Omega_2|p\rangle = \omega^{-p}|p\rangle$$

$$\frac{1}{\sqrt{k}}\sum_{j=0}^{k-1} \omega^{-qj}|j\rangle$$

$$\mathfrak{a} \equiv -\frac{i\rho_2}{\pi}A_{\bar{z}} = \frac{1}{2\pi}(-\rho A_1 + A_2),$$

$$\hat{\mathfrak{a}}^\dagger = \frac{\rho_2}{\pi k}\frac{\partial}{\partial \mathfrak{a}}, \hat{\mathfrak{a}} = \mathfrak{a}$$

$$\langle \psi \mid \psi \rangle = \int \ e^{-\frac{\pi k}{\rho_2}|\mathfrak{a}|^2}|\psi|^2 d^2\mathfrak{a}$$

$$\psi_p(\mathfrak{a}) = \theta(k\mathfrak{a} + p\rho; k\rho)e^{\frac{\pi k}{2\rho_2}\mathfrak{a}^2 + \frac{1}{k}\pi i\rho p^2 + 2\pi i p\mathfrak{a}}$$

$$\theta(\mathfrak{a}; \rho) \equiv \sum_{n=-\infty}^{\infty} e^{\pi i\rho n^2 + 2\pi i n\mathfrak{a}}$$

$$\mathcal{W}_1\psi(\mathfrak{a}) = e^{-\frac{\pi}{\rho_2}\mathfrak{a} - \frac{\pi}{2k\rho_2}}\psi\left(\mathfrak{a} + \frac{1}{k}\right), \mathcal{W}_2\psi(\mathfrak{a}) = e^{-\frac{\pi\bar{\rho}}{\rho_2}\mathfrak{a} - \frac{\pi|\rho|^2}{2k\rho_2}}\psi\left(\mathfrak{a} + \frac{\rho}{k}\right)$$

$$e^{-\frac{\pi}{\rho_2}\mathfrak{a} - \frac{\pi}{2k\rho_2}}e^{-\frac{\pi\bar{\rho}}{\rho_2}\mathfrak{a} - \frac{\pi|\rho|^2}{2k\rho_2}}\mathfrak{a} \to \mathfrak{a} + \frac{1}{k}\mathfrak{a}^\dagger \to \mathfrak{a}^\dagger + \frac{1}{k}e^{-\frac{\pi}{\rho_2}\mathfrak{a}}$$

$$\mathcal{T}|p\rangle = e^{\frac{\pi i}{k}p^2}|p\rangle$$

$$\psi_p(\mathfrak{a}, \rho + 1) = e^{\frac{1}{k}\pi i p^2 + \pi i p}e^{-\frac{\pi k}{2\rho_2}\mathfrak{a}}e^{-\frac{\pi k}{8\rho_2}}\psi_p\left(\mathfrak{a} + \frac{1}{2}, \rho\right).$$

$$\mathcal{T}\psi(\mathfrak{a}, \rho) \equiv e^{-\frac{\pi k}{2\rho_2}\mathfrak{a} - \frac{\pi k}{8\rho_2}}\psi\left(\mathfrak{a} + \frac{1}{2}, \rho + 1\right)$$

$$\mathcal{T}\psi_p = e^{\frac{1}{k}\pi i p^2 + \pi i p}\psi_p$$

$$\mathcal{V} = \mathcal{W}_1, \mathcal{U} = \mathcal{W}_2.$$

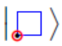
$$\mathcal{W}_1 = \omega^{-\mathbf{e}_2}, \mathcal{W}_2 = \omega^{\mathbf{e}_1}$$

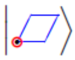
$$\mathcal{U} = e^{\frac{2\pi i}{k}(\mathbf{e}_1 + \mathbf{m}_1)}.$$

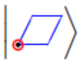
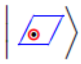
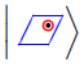
$$\mathcal{V} = e^{-\frac{2\pi i}{k}(\mathbf{e}_2 + \mathbf{m}_2)}$$

$$e^{2\pi i(p_1 + p_{10})/k} \exp\left[\frac{2\pi i}{k}(\mathbf{e}_j + \mathbf{m}_j)\right] \exp\left[\frac{2\pi i}{k}\mathbf{e}_j\right]$$

$$\mathcal{U} = \omega^{\mathbf{e}_1}, \mathcal{V} = \omega^{-\mathbf{e}_2},$$

 $= |0\rangle$,  $= |1\rangle$.

 $= |0\rangle$.

 $= |0\rangle$,  $= |1\rangle$,  $= |2\rangle$.

$$A \to \begin{pmatrix} A & & \\ & A & \\ & & \ddots \end{pmatrix}$$

$$|p\rangle_{U(1)}, p = 0, \dots, k' - 1$$

$$\mathcal{W}_1 = \exp\left\{\int_0^{2\pi L_1} A_1(x_1, 0) dx_1\right\}$$

$$\mathcal{W}_1 |p\rangle_{U(1)} = e^{\frac{2\pi i p}{kn}} |p\rangle_{U(1)}$$

$$|\psi\rangle_{U(n)} = \sum_{p=0}^{kn-1} |\psi; p\rangle_{SU(n)} \otimes |p\rangle_{U(1)}$$

$$\Omega_j(x_1, x_2) = \mathrm{diag}\left(e^{\frac{i x_j}{L_j}}, 1, \cdots, 1\right),$$

$$\Omega_j'(x_1, x_2) = \mathrm{diag}\left(e^{\frac{i x_j}{n L_j}}, e^{\frac{i x_j}{n L_j}} \cdots, e^{\frac{i x_j}{n L_j}}\right) \in U(1),$$

$$\Omega_j''(x_1, x_2) = \mathrm{diag}\left(e^{\frac{(n-1) i x_j}{n L_j}}, e^{-\frac{i x_j}{n L_j}}, \cdots, e^{-\frac{i x_j}{n L_j}}\right) \in SU(n),$$

$$\Omega_1' |p\rangle_{U(1)} = |p + k\rangle_{U(1)}, \Omega_2' |p\rangle_{U(1)} = e^{-\frac{2\pi i p}{n}} |p\rangle_{U(1)}$$

$$\Omega_1'' |\psi; p\rangle_{SU(n)} = |\psi; p + k\rangle_{SU(n)}, \Omega_2'' |\psi; p\rangle_{SU(n)} = e^{\frac{2\pi i p}{n}} |\psi; p\rangle_{SU(n)}.$$

$$\Upsilon_j^{(\alpha)}(x_1, x_2) = \mathrm{diag}\left(e^{\frac{i\alpha x_j}{k' L_j}}, \cdots, e^{\frac{i\alpha x_j}{k' L_j}}\right) \in U(1), j = 1,2,$$

$$\left(\Upsilon_1^{(\alpha)}\right)^{-1} \mathcal{W}_1 \Upsilon_1^{(\alpha)} = e^{\frac{2\pi i \alpha}{k'}} \mathcal{W}_1, \left(\Upsilon_1^{(\alpha)}\right)^{-1} \mathcal{W}_2 \Upsilon_1^{(\alpha)} = \mathcal{W}_2$$

$$\Upsilon_1^{(\alpha)} = \mathcal{W}_2^{\alpha}, \Upsilon_2^{(\alpha)} = \mathcal{W}_1^{-\alpha}$$

$$\Upsilon_2^{(\alpha)}|\psi\rangle_{U(n)} = \sum_{p=0}^{kn-1} e^{-\frac{2\pi i\alpha p}{kn}}|\psi;p\rangle_{SU(n)} \otimes |p\rangle_{U(1)}$$

$$e^{\frac{2\pi i\alpha p}{kn}}|\psi;p\rangle_{SU(n)}$$

$$\Upsilon_1^{(\alpha)}|\psi\rangle_{U(n)} = \sum_{p=0}^{kn-1} |\psi;p-\alpha\rangle_{SU(n)} \otimes |p\rangle_{U(1)}$$

$$\mathcal{K}_1 \equiv \Upsilon_1^{(n)} = \mathcal{W}_2^n, \mathcal{K}_2 \equiv \Upsilon_2^{(n)} = \mathcal{W}_1^{-n}$$

$$\mathcal{K}_1|\psi\rangle_{U(n)} = \sum_{p=0}^{kn-1} |\psi;p-n\rangle_{SU(n)} \otimes |p\rangle_{U(1)}$$
$$\mathcal{K}_2|\psi\rangle_{U(n)} = \sum_{p=0}^{kn-1} e^{-\frac{2\pi ip}{k}}|\psi;p\rangle_{SU(n)} \otimes |p\rangle_{U(1)}$$

$$\mathcal{K}_1\mathcal{K}_2 = e^{\frac{2\pi in}{k}}\mathcal{K}_2\mathcal{K}_1, (\mathcal{K}_1)^k = (\mathcal{K}_2)^k = 1$$

$$\mathcal{U} = \mathcal{K}_1, \mathcal{V} = \mathcal{K}_2^{-1},$$

$$\mathcal{V}|\psi\rangle_{U(n)} = \sum_{p=0}^{kn-1} e^{\frac{2\pi ip}{k}}|\psi;p\rangle_{SU(n)} \otimes |p\rangle_{U(1)}$$
$$\mathcal{U}|\psi\rangle_{U(n)} = \sum_{p=0}^{kn-1} |\psi;p-n\rangle_{SU(n)} \otimes |p\rangle_{U(1)}$$

$$\mathcal{T} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \mathcal{S} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\mathcal{S}|p\rangle_{U(1)} = \frac{1}{\sqrt{kn}} \sum_{q=0}^{kn-1} e^{\frac{2\pi i}{kn}pq}|q\rangle_{U(1)}$$
$$\mathcal{T}|p\rangle_{U(1)} = \begin{cases} e^{\frac{i\pi}{kn}p^2}|p\rangle_{U(1)} & \text{for even } kn \\ e^{\frac{i\pi}{kn}p(p+kn)}|p\rangle_{U(1)} & \text{for odd } kn \end{cases}$$

$$\mathcal{T}|\psi\rangle_{U(n)} = \begin{cases} \sum_{p=0}^{kn-1} e^{\frac{\pi ip^2}{kn}}\mathcal{T}|\psi;p\rangle_{SU(n)} \otimes |p\rangle_{U(1)} & \text{for even } kn \\ \sum_{p=0}^{kn-1} e^{\frac{\pi ip(p+kn)}{kn}}\mathcal{T}|\psi;p\rangle_{SU(n)} \otimes |p\rangle_{U(1)} & \text{for odd } kn \end{cases}$$

$$\mathcal{T}^{-1}\Omega_2''\mathcal{T}|\psi;p\rangle_{SU(n)} = e^{\frac{2\pi ip}{n}}|\psi;p\rangle_{SU(n)}$$

$$\mathcal{T}^{-1}\Omega_1''\mathcal{T}|\psi;p\rangle_{SU(n)} = \begin{cases} e^{\frac{2\pi ip}{n}+\frac{\pi ik}{n}}|\psi;p+k\rangle_{SU(n)} & \text{for even } kn \\ e^{\frac{2\pi ip}{n}+\frac{\pi ik(n+1)}{n}}|\psi;p+k\rangle_{SU(n)} & \text{for odd } kn \end{cases}$$

$$\mathcal{T}^{-1}\Omega_2''\mathcal{T} = \Omega_2'', \mathcal{T}^{-1}\Omega_1''\mathcal{T} = \begin{cases} e^{-\frac{\pi ik}{n}}\Omega_2''\Omega_1'' & \text{for even } kn \\ e^{\frac{\pi ik(n-1)}{n}}\Omega_2''\Omega_1'' & \text{for odd } kn \end{cases}$$

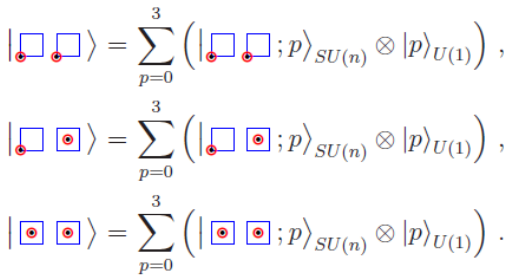
$$\mathcal{S}^{-1}\Omega_1''\mathcal{S} = \Omega_2'', \mathcal{S}^{-1}\Omega_2''\mathcal{S} = (\Omega_1'')^{-1}$$

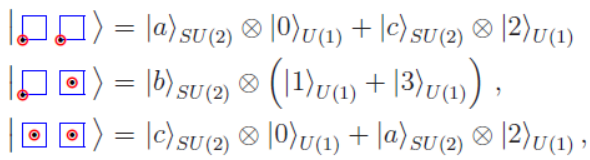
$$[U(1)_{k'} \times SU(n)_k]/\mathbb{Z}_n$$

$$\mathcal{H}([U(1)_{kn} \times SU(n)_k]/\mathbb{Z}_n) \simeq \mathcal{H}(U(1)_k)^{\otimes n}/S_n$$

$$\dim\mathcal{H}([U(1)_{kn} \times SU(n)_k]/\mathbb{Z}_n) = kn\binom{n+k-1}{k}\frac{1}{n^2} = \binom{n+k-1}{k-1} = \dim\left[\mathcal{H}(U(1)_k)^{\otimes n}\right]_S.$$

$$\Psi_{p_1,\dots,p_n}(\mathfrak{a}_1,\dots,\mathfrak{a}_n) \equiv \sum_{\sigma\in S_n}\prod_{i=1}^{n}\psi_{p_i}(\mathfrak{a}_{\sigma(i)}).$$





$$\mathcal{S} = \begin{pmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix}, \mathcal{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{\pi i}{4}} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\Omega_1'' = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \Omega_2'' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

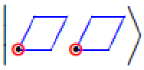
$$\psi_{p,k}(\mathfrak{a}) = \theta(k\mathfrak{a} + p\rho; k\rho)e^{\frac{\pi k}{2\rho_2}\mathfrak{a}^2 + \frac{1}{k}\pi i\rho p^2 + 2\pi i p\mathfrak{a}}, p = 0, \dots, k-1.$$

$$\theta(z_1;\tau)\theta(z_2;\tau) = \theta(z_1+z_2;2\tau)\theta(z_1-z_2;2\tau) + e^{\pi i(\tau+2z_2)}\theta(z_1+z_2+\tau;2\tau)\theta(z_1-z_2-\tau;2\tau)$$

$$\begin{aligned}\psi_{p_1,k}(\mathfrak{a}_1)\psi_{p_2,k}(\mathfrak{a}_2) + \psi_{p_1,k}(\mathfrak{a}_2)\psi_{p_2,k}(\mathfrak{a}_1) \\ =\psi_{p_1+p_2,2k}\left(\frac{\mathfrak{a}_1+\mathfrak{a}_2}{2}\right)\left[\psi_{p_1-p_2,2k}\left(\frac{\mathfrak{a}_1-\mathfrak{a}_2}{2}\right) + \psi_{p_2-p_1,2k}\left(\frac{\mathfrak{a}_1-\mathfrak{a}_2}{2}\right)\right] \\ +\psi_{p_1+p_2+k,2k}\left(\frac{\mathfrak{a}_1+\mathfrak{a}_2}{2}\right)\left[\psi_{p_1-p_2-k,2k}\left(\frac{\mathfrak{a}_1-\mathfrak{a}_2}{2}\right) + \psi_{p_2-p_1+k,2k}\left(\frac{\mathfrak{a}_1-\mathfrak{a}_2}{2}\right)\right]\end{aligned}$$

$$|p\rangle_{U(1)} \to \psi_{p,2k}\left(\frac{\mathfrak{a}_1+\mathfrak{a}_2}{2}\right)$$

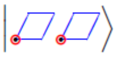
$$\begin{aligned}|a\rangle_{SU(2)} &\to \psi_{0,4}\left(\frac{\mathfrak{a}_1-\mathfrak{a}_2}{2}\right) \\ |b\rangle_{SU(2)} &\to \frac{1}{\sqrt{2}}\left[\psi_{1,4}\left(\frac{\mathfrak{a}_1-\mathfrak{a}_2}{2}\right) + \psi_{3,4}\left(\frac{\mathfrak{a}_1-\mathfrak{a}_2}{2}\right)\right] \\ |c\rangle_{SU(2)} &\to \psi_{2,4}\left(\frac{\mathfrak{a}_1-\mathfrak{a}_2}{2}\right)\end{aligned}$$



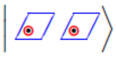
$$\left| \square \right\rangle = |a\rangle_{SU(2)} \otimes |0\rangle_{U(1)} + |b\rangle_{SU(2)} \otimes |1\rangle_{U(1)} .$$

$$\mathcal{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\pi i/2} \end{pmatrix}, \mathcal{S} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

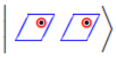
$$\Omega_1'' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \Omega_2'' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$



$$\left| \square \right\rangle = |a\rangle_{SU(2)} \otimes |0\rangle_{U(1)} + |b\rangle_{SU(2)} \otimes |3\rangle_{U(1)} ,$$

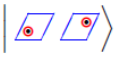


$$\left| \square \right\rangle = |a\rangle_{SU(2)} \otimes |2\rangle_{U(1)} + |b\rangle_{SU(2)} \otimes |5\rangle_{U(1)} ,$$



$$\left| \square \right\rangle = |a\rangle_{SU(2)} \otimes |4\rangle_{U(1)} + |b\rangle_{SU(2)} \otimes |1\rangle_{U(1)} ,$$

$$\left| \square \right\rangle = |c\rangle_{SU(2)} \otimes |1\rangle_{U(1)} + |d\rangle_{SU(2)} \otimes |4\rangle_{U(1)} ,$$



$$\left| \square \right\rangle = |c\rangle_{SU(2)} \otimes |3\rangle_{U(1)} + |d\rangle_{SU(2)} \otimes |0\rangle_{U(1)} ,$$

$$\left| \square \right\rangle = |c\rangle_{SU(2)} \otimes |5\rangle_{U(1)} + |d\rangle_{SU(2)} \otimes |2\rangle_{U(1)} ,$$

$$\mathcal{T} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{\pi i/2} & 0 & 0 \\ 0 & 0 & e^{-5\pi i/6} & 0 \\ 0 & 0 & 0 & e^{2\pi i/3} \end{pmatrix}, \quad \mathcal{S} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 1 & \sqrt{2} & \sqrt{2} \\ 1 & -1 & -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 1 & -1 \\ \sqrt{2} & \sqrt{2} & -1 & -1 \end{pmatrix},$$

$$\Omega_1'' = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad \Omega_2'' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$\mathcal{S}_V|m\rangle = \sqrt{\frac{2}{k+2}} \sum_{m'} \sin \frac{\pi(m+1)(m'+1)}{k+2} |m'\rangle, m = 0, \dots, k$$

$$\mathcal{T}_V|m\rangle = e^{\frac{\pi i m(m+2)}{2(k+2)}} |m\rangle \; m = 0, \dots, k$$

$$\mathcal{H}_{(1,1,\dots,1)}(v) \simeq \mathcal{H}(U(1)_k)^{\otimes n}/S_n.$$

$$\mathrm{ch}_k^{\lambda}(\mathfrak{a}) = \sum_{m=1-k}^{k} C_m^{\lambda}(\rho)\Theta_{m,k}, \Theta_{m,k} \equiv \sum_{n \in \mathbb{Z}+m/2k} e^{2\pi i(n^2\rho - n\mathfrak{a})}$$

$$C_m^{\lambda} = 0 \; \forall \lambda \neq m \; (\mathrm{mod} 2), C_m^{\lambda} = C_{-m}^{\lambda}, C_m^{\lambda} = C_{k+m}^{k-\lambda}$$

$$C_+ \equiv C_0^0 + C_2^0 = \frac{\eta(q)}{\eta(\sqrt{q})\eta(q^2)}, C_- \equiv C_0^0 - C_2^0 = \frac{\eta(\sqrt{q})}{[\eta(q)]^2}, C_1^1 = \frac{\eta(q^2)}{[\eta(q)]^2},$$

$$|a\rangle_{SU(2)} = \frac{e^{\pi k \mathfrak{a}_-^2/2\rho_2}}{C_+ C_-}\left(C_0^0 \mathrm{ch}_2^0(\mathfrak{a}_-) - C_2^0 \mathrm{ch}_2^2(\mathfrak{a}_-)\right),$$

$$|c\rangle_{SU(2)} = \frac{e^{\pi k \mathfrak{a}_-^2/2\rho_2}}{C_+ C_-}\left(-C_2^0 \mathrm{ch}_2^0(\mathfrak{a}_-) + C_0^0 \mathrm{ch}_2^2(\mathfrak{a}_-)\right),$$

$$|b\rangle_{SU(2)} = \frac{e^{\pi k \mathfrak{a}_-^2/2\rho_2}}{C_1^1} \mathrm{ch}_2^1(\mathfrak{a}_-),$$

$$C_1^1 \to C_1^1, C_- \to e^{-\frac{\pi i}{8}} C_+, C_+ \to e^{-\frac{\pi i}{8}} C_-$$

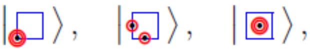
$$\mathcal{H}_{(2,1)}\left(\frac{\pi}{2}\right) \simeq \mathcal{H}_{(1)}\left(\frac{\pi}{2}\right) \otimes \mathcal{H}_{(2)}\left(\frac{\pi}{2}\right)$$

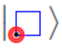
$$n_1 > \cdots > n_{p-q} > n_{p-q+1} = \cdots = n_p = 1$$

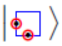
$$U(n')_{kn',k''} \equiv \mathcal{H}([U(1)_{kn'} \times SU(n')_{k''}]/\mathbb{Z}_{n'})$$

$$\mathcal{H}_{(n_1,\dots,n_p)}(v) \simeq U(n_1)_{kn_1,k_1} \otimes \cdots \otimes U(n_{p-q})_{kn_{p-q},k_{p-q}} \otimes U(q)_{kq,k},$$

$$\mathfrak{a}_1 \to \mathfrak{a}_1 + \zeta, \mathfrak{a}_2 \to \mathfrak{a}_2 + \zeta, \dots$$



 $= |a\rangle_{SU(2)} \otimes |0\rangle_{U(1)} + |c\rangle_{SU(2)} \otimes |2\rangle_{U(1)}$

 $= |b\rangle_{SU(2)} \otimes \left(|1\rangle_{U(1)} + |3\rangle_{U(1)}\right),$

 $= |c\rangle_{SU(2)} \otimes |0\rangle_{U(1)} + |a\rangle_{SU(2)} \otimes |2\rangle_{U(1)}.$

$$\mathcal{T}|a\rangle_{SU(2)} = |a\rangle_{SU(2)}, \mathcal{T}|b\rangle_{SU(2)} = e^{-\frac{\pi i}{4}}|b\rangle_{SU(2)}, \mathcal{T}|c\rangle_{SU(2)} = -|c\rangle_{SU(2)},$$

$$\left.\begin{aligned} \mathcal{S}|a\rangle_{SU(2)} &= \frac{1}{2}|a\rangle_{SU(2)} + \frac{1}{\sqrt{2}}|b\rangle_{SU(2)} + \frac{1}{2}|c\rangle_{SU(2)}, \\ \mathcal{S}|b\rangle_{SU(2)} &= \frac{1}{\sqrt{2}}|a\rangle_{SU(2)} - \frac{1}{\sqrt{2}}|c\rangle_{SU(2)}, \\ \mathcal{S}|c\rangle_{SU(2)} &= \frac{1}{2}|a\rangle_{SU(2)} - \frac{1}{\sqrt{2}}|b\rangle_{SU(2)} + \frac{1}{2}|c\rangle_{SU(2)}, \end{aligned}\right\}$$

$$\Omega_1''|a\rangle_{SU(2)} = |c\rangle_{SU(2)}, \Omega_1''|b\rangle_{SU(2)} = |b\rangle_{SU(2)}, \Omega_1''|c\rangle_{SU(2)} = |a\rangle_{SU(2)},$$

$$\Omega_2''|a\rangle_{SU(2)} = |a\rangle_{SU(2)}, \Omega_2''|b\rangle_{SU(2)} = -|b\rangle_{SU(2)}, \Omega_2''|c\rangle_{SU(2)} = |c\rangle_{SU(2)}$$

 $= |a\rangle_{SU(3)} \otimes |0\rangle_{U(1)} + |b\rangle_{SU(3)} \otimes |2\rangle_{U(1)} + |c\rangle_{SU(3)} \otimes |4\rangle_{U(1)},$

 $= |a\rangle_{SU(3)} \otimes |3\rangle_{U(1)} + |b\rangle_{SU(3)} \otimes |5\rangle_{U(1)} + |c\rangle_{SU(3)} \otimes |1\rangle_{U(1)},$

$$\mathcal{T}|a\rangle_{SU(3)} = |a\rangle_{SU(3)}, \mathcal{T}|b\rangle_{SU(3)} = e^{-\frac{2\pi i}{3}}|b\rangle_{SU(3)}, \mathcal{T}|c\rangle_{SU(3)} = e^{-\frac{2\pi i}{3}}|c\rangle_{SU(3)}$$

$$\left.\begin{aligned} \mathcal{S}|a\rangle_{SU(3)} &= \frac{1}{\sqrt{3}}\left(|a\rangle_{SU(3)} + |b\rangle_{SU(3)} + |c\rangle_{SU(3)}\right) \\ \mathcal{S}|b\rangle_{SU(3)} &= \frac{1}{\sqrt{3}}\left(|a\rangle_{SU(3)} + e^{\frac{2\pi i}{3}}|b\rangle_{SU(3)} + e^{-\frac{2\pi i}{3}}|c\rangle_{SU(3)}\right) \\ \mathcal{S}|c\rangle_{SU(3)} &= \frac{1}{\sqrt{3}}\left(|a\rangle_{SU(3)} + e^{-\frac{2\pi i}{3}}|b\rangle_{SU(3)} + e^{\frac{2\pi i}{3}}|c\rangle_{SU(3)}\right) \end{aligned}\right\}.$$

$$\Omega_2''|a\rangle_{SU(3)} = |a\rangle_{SU(3)}, \Omega_2''|b\rangle_{SU(3)} = e^{-\frac{2\pi i}{3}}|b\rangle_{SU(3)}, \Omega_2''|c\rangle_{SU(3)} = e^{\frac{2\pi i}{3}}|c\rangle_{SU(3)},$$

$$\Omega_1''|a\rangle_{SU(3)} = |b\rangle_{SU(3)}, \Omega_1''|b\rangle_{SU(3)} = |c\rangle_{SU(3)}, \Omega_1''|c\rangle_{SU(3)} = |a\rangle_{SU(3)}$$

$$|0\rangle_{U(1)_{-3}} \otimes |a'\rangle_{SU(3)_{-1}} + |1\rangle_{U(1)_{-3}} \otimes |b'\rangle_{SU(3)_{-1}} + |2\rangle_{U(1)_{-3}} \otimes |c'\rangle_{SU(3)_{-1}}$$

$$U(3)_{6,2} \oplus U(3)_{6,-1} \oplus [U(1)_2 \otimes U(2)_{4,-2}].$$

$$\mathcal{W}(C, x_3) = \mathcal{M}(C, x_3 + 2\pi R) = \mathcal{W}(C, x_3 + 4\pi R)^\dagger = \mathcal{M}(C, x_3 + 6\pi R)^\dagger = \mathcal{W}(C, x_3 + 8\pi R),$$

$$\mathcal{V}^{(p)}(C, x_3) \equiv \mathcal{W}(C, x_3) + i^p\mathcal{M}(C, x_3) + (-1)^p\mathcal{W}(C, x_3)^\dagger + (-i)^p\mathcal{M}(C, x_3)^\dagger, p = 0,1,2,3.$$

$$\mathcal{V}^{(p)}(C,x_3)=\sum_{m\in\mathbb{Z}}\hat{\mathcal{V}}^{(p)}_{m+\frac{p}{4}}(C)e^{\left(m+\frac{p}{4}\right)\frac{ix_3}{R}}$$

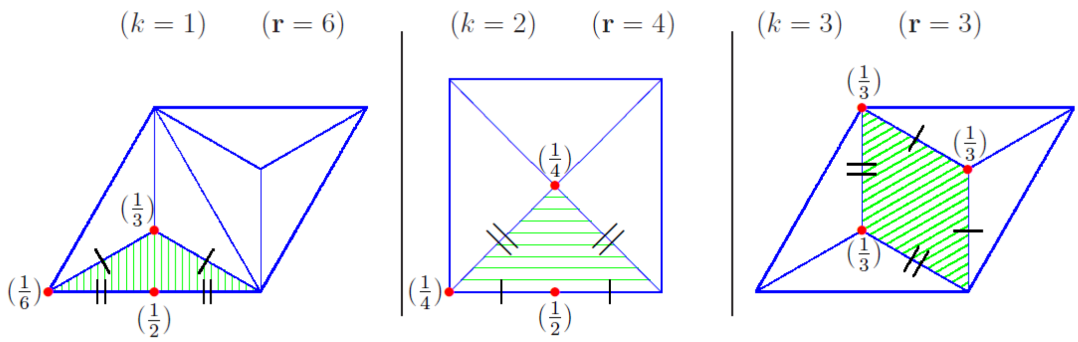
$$\mathcal{W}(C,x_3),\mathcal{M}(C,x_3),\mathcal{W}(C,x_3)^{\dagger},\mathcal{M}(C,x_3)^{\dagger}\overset{\text{IR}}{\to}\frac{1}{4}\hat{\mathcal{V}}^{(0)}_0.$$

$$\mathcal{V}^{(0)}(C,x_3)\equiv\mathcal{W}(C,x_3)+\mathcal{M}(C,x_3)+\mathcal{W}(C,x_3)^{\dagger}+\mathcal{M}(C,x_3)^{\dagger}$$

$$\mathcal{A}\ll R^2,$$

$$\gamma'=\begin{pmatrix}e^{\frac{i}{2}v} & & & \\ & e^{\frac{i}{2}v} & & \\ & & e^{i\epsilon-\frac{i}{2}v} & \\ & & & e^{-i\epsilon-\frac{i}{2}v}\end{pmatrix}\in SU(4)_R$$

$$\frac{\mathcal{A}}{R^2}\to\infty$$



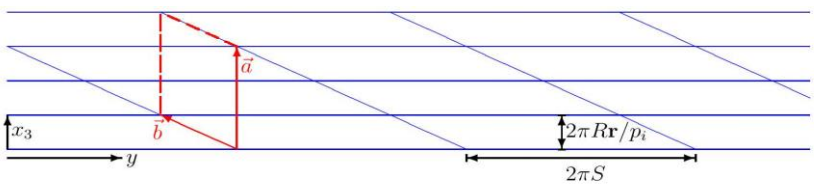
$$(p_1,p_2,p_3)=\begin{cases}(3,3,3)\text{ for }\mathbf{r}=3\left(\tau=e^{\pi i/3}\right),\\(4,4,2)\text{ for }\mathbf{r}=4(\tau=i),\\(6,3,2)\text{ for }\mathbf{r}=6\left(\tau=e^{\pi i/3}\right).\end{cases}$$

$$1=\sum_{i=1}^{3}\frac{1}{p_i}.$$

$$g_{\text{YM}}^{(5\text{D})}=2\pi(2R\mathbf{r})^{\frac{1}{2}}.$$

$$\mathbb{Z}_{p_i}:(x_3,\zeta_1,\zeta_2)\mapsto\left(x_3+\frac{2\pi R\mathbf{r}}{p_i},e^{\frac{2\pi i}{p_i}}\zeta_1,e^{-\frac{2\pi i}{p_i}}\zeta_2\right)$$

$$ds^2=\left(1+\frac{S}{2r}\right)^{-1}(dy+\cos\,\theta d\phi)^2+\left(1+\frac{S}{2r}\right)\left(dr^2+r^2(d\theta^2+\sin^2\,\theta d\phi^2)\right)$$



$$(x_3,y)\simeq(x_3,y+2\pi S)\simeq\left(x_3+\frac{2\pi R\mathbf{r}}{p_i},y-\frac{2\pi S}{p_i}\right).$$

$$w=\frac{1}{2\pi R\mathbf{r}}(x_3+iy)$$

$$w \simeq w+1 \simeq w-\frac{1}{p_i}+i\frac{S}{p_i R\mathbf{r}}.$$

$$\tau_{\mathrm{IIB}}=-\frac{1}{p_i}+i\frac{S}{p_i R\mathbf{r}}.$$

$$-\frac{1}{4\pi p_i}\int_{D3}\mathrm{tr}(F\wedge F)=\frac{1}{4\pi p_i}\int\left(\mathbf{C}\wedge d\mathbf{C}+\frac{2}{3}\mathbf{C}\wedge\mathbf{C}\wedge\mathbf{C}\right),$$

$$\frac{1}{4\pi}\mathrm{tr}\int\left(\mathbf{C}\wedge d\mathbf{C}+\frac{2}{3}\mathbf{C}\wedge\mathbf{C}\wedge\mathbf{C}\right)$$

$$\sum_{i=1}^{3}\frac{p_i}{4\pi}\mathrm{tr}\int\left(\mathbf{B}_i\wedge d\mathbf{B}_i+\frac{2}{3}\mathbf{B}_i\wedge\mathbf{B}_i\wedge\mathbf{B}_i\right)$$

$$\frac{1}{4\pi}\int(p\mathbf{B}\wedge d\mathbf{B}+2\mathbf{B}\wedge d\mathbf{C}_b)$$

$$I=I_{\mathrm{sp}}+\frac{1}{2\pi}\int(\mathbf{B}_1+\mathbf{B}_2+\mathbf{B}_3)\wedge d\mathbf{C}$$

$$\left.\begin{aligned}I_{\mathrm{sp}}&=\frac{1}{4\pi}\int(6\mathbf{B}_1\wedge d\mathbf{B}_1+3\mathbf{B}_2\wedge d\mathbf{B}_2+2\mathbf{B}_3\wedge d\mathbf{B}_3),\quad(k=1)\\ I_{\mathrm{sp}}&=\frac{1}{4\pi}\int(4\mathbf{B}_1\wedge d\mathbf{B}_1+4\mathbf{B}_2\wedge d\mathbf{B}_2+2\mathbf{B}_3\wedge d\mathbf{B}_3),\quad(k=2)\\ I_{\mathrm{sp}}&=\frac{1}{4\pi}\int(3\mathbf{B}_1\wedge d\mathbf{B}_1+3\mathbf{B}_2\wedge d\mathbf{B}_2+3\mathbf{B}_3\wedge d\mathbf{B}_3),\quad(k=3)\end{aligned}\right\}$$

$$I=\frac{1}{4\pi}\int\left(\sum_{i=1}^{3}p_i\mathbf{B}_i\wedge d\mathbf{B}_i+2d\mathbf{C}\wedge\sum_{i=1}^{3}\mathbf{B}_i\right)$$

$$\frac{1}{4\pi}\int\sum_{i,j}h_{ij}\mathbf{B}_i\wedge d\mathbf{B}_j$$

$$N_{\mathrm{states}}=\det\{h_{ij}\}$$

$$\det\begin{pmatrix}p_1&0&0&1\\0&p_2&0&1\\0&0&p_3&1\\1&1&1&1\end{pmatrix}=p_1p_2p_3\left(1-\frac{1}{p_1}-\frac{1}{p_2}-\frac{1}{p_3}\right)=0$$

$$\mathbf{B}_1\equiv\mathbf{B}_1',\mathbf{B}_2\equiv\mathbf{B}_2'+\frac{p_1}{p_2}\mathbf{B}_1',\mathbf{B}_3\equiv\mathbf{B}_3'+\frac{p_1}{p_3}\mathbf{B}_1',\mathbf{C}\equiv\mathbf{C}'-p_1\mathbf{B}_1'$$

$$I=\frac{1}{4\pi}\int\left(\sum_{i=2}^{3}p_i\mathbf{B}_i'\wedge d\mathbf{B}_i'+2d\mathbf{C}'\wedge\sum_{i=2}^{3}\mathbf{B}_i'\right)$$

$$\begin{pmatrix}3&0&1\\0&2&1\\1&1&1\end{pmatrix},\begin{pmatrix}4&0&1\\0&2&1\\1&1&1\end{pmatrix},\begin{pmatrix}3&0&1\\0&3&1\\1&1&1\end{pmatrix}$$

$$\mathcal{W}_{1i}=e^{i\oint_{\alpha_a'}\mathbf{B}_i},\qquad\mathcal{W}_{2i}=e^{i\oint_{\alpha_b'}\mathbf{B}_i}.$$

$$\mathcal{W}_{1i}\mathcal{W}_{2j}=e^{2\pi i h^{ij}}\mathcal{W}_{2j}\mathcal{W}_{1i}$$

$$X_i\equiv\prod_{j=1}^{d}\mathcal{W}_{1j}^{h_{ij}}\ \text{and}\ Y_i\equiv\prod_{j=1}^{d}\mathcal{W}_{2j}^{h_{ij}}$$

$$\mathcal{W}(\alpha_a{}' \times \gamma_i) \to e^{i \oint_{\alpha_a{}'} \mathbf{B}_i}\,, \qquad \mathcal{W}(\alpha_a{}' \times \gamma_0) \to e^{i \oint_{\alpha_a{}'} \mathbf{C}}\,,$$

$$\mathcal{W}(\alpha_b{}' \times \gamma_i) \to e^{i \oint_{\alpha_b{}'} \mathbf{B}_i}\,, \qquad \mathcal{W}(\alpha_b{}' \times \gamma_0) \to e^{i \oint_{\alpha_b{}'} \mathbf{C}}\,.$$

$$I = \frac{1}{4\pi}\int \left\{\sum_{i=1}^{3} p_i \mathrm{tr}\left(\mathbf{B}_i \wedge d\mathbf{B}_i + \frac{2}{3}\mathbf{B}_i \wedge \mathbf{B}_i \wedge \mathbf{B}_i\right) + \mathrm{tr}\left(\mathbf{C} \wedge d\mathbf{C} + \frac{2}{3}\mathbf{C} \wedge \mathbf{C} \wedge \mathbf{C}\right)\right\} + \sum_{i=1}^{3} I_i^{[\mathcal{T}(U(2))]}(\mathbf{B}_i, \mathbf{C})$$

$$P\exp \oint_{Q_i} \mathbf{C}' = \begin{pmatrix} 1 & \\ & (-1)^{\mathrm{r}/p_i} \end{pmatrix}.$$

$$U(n_j)_{k_j',k_j''} \simeq \left[U(1)_{k_j'} \times SU(n_j)_{k_j''}\right]/\mathbb{Z}_{n_j}.$$

$\upsilon = \frac{\pi}{3}$ $(k=1)$	$n=1$	$U(1)_1$
	$n=2$	$U(2)_{2,1} \oplus U(2)_{2,-3}$
	$n=3$	$U(3)_{3,1} \oplus [U(1)_1 \times U(2)_{2,-3}] \oplus U(3)_{3,-2}$
	$n=4$	$U(4)_{4,1} \oplus 2[U(2)_{2,1} \times U(2)_{2,-3}] \oplus [U(1)_1 \times U(3)_{3,-2}] \oplus \mathcal{H}_{(2,2)}$
	$n=5$	$U(5)_{5,1} \oplus U(5)_{5,1} \oplus 2[U(3)_{3,1} \times U(2)_{2,-3}] \oplus [U(1)_1 \times \mathcal{H}_{(2,2)}] \oplus$ $[U(2)_{2,1} \times U(3)_{3,-2}] \oplus [U(2)_{2,-3} \times U(3)_{3,-2}]$
$\upsilon = \frac{\pi}{2}$ $(k=2)$	$n=1$	$U(1)_2$
	$n=2$	$U(2)_{4,2} \oplus U(2)_{4,-2}$
	$n=3$	$U(3)_{6,2} \oplus [U(1)_2 \times U(2)_{4,-2}] \oplus U(3)_{6,-1}$
$\upsilon = \frac{2\pi}{3}$ $(k=3)$	$n=1$	$U(1)_3$
	$n=2$	$U(2)_{6,3} \oplus U(2)_{6,-1}$

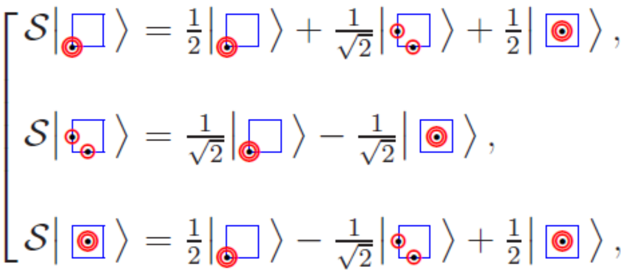
$$\mathcal{T} \to e^{i\phi}\mathcal{U}^{\tilde{p}}\mathcal{V}^{\tilde{q}}\mathcal{T}, \mathcal{S} \to e^{i\phi'}\mathcal{U}^{\tilde{p}}\mathcal{V}^{\tilde{q}}\mathcal{S},$$

$$\mathcal{S}|\square\rangle = \frac{1}{\sqrt{2}}\left(|\square\rangle + |\boxdot\rangle\right)\,, \qquad \mathcal{S}|\boxdot\rangle = \frac{1}{\sqrt{2}}\left(|\square\rangle - |\boxdot\rangle\right)\,,$$

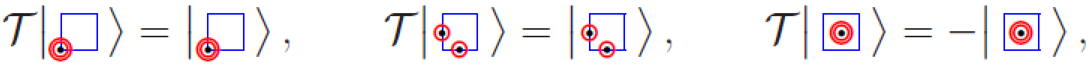
$$\mathcal{S}|[0]\rangle = \frac{1}{\sqrt{2}}\left(|[0]\rangle + \left|\left[\frac{1}{2}+\frac{1}{2}i\right]\right\rangle\right), \mathcal{S}\left|\left[\frac{1}{2}+\frac{1}{2}i\right]\right\rangle = \frac{1}{\sqrt{2}}\left(|[0]\rangle - \left|\left[\frac{1}{2}+\frac{1}{2}i\right]\right\rangle\right)$$

$$\mathcal{T}|\square\rangle = |\square\rangle\,, \qquad \mathcal{T}|\boxdot\rangle = e^{\frac{i\pi}{2}}|\boxdot\rangle\,,$$

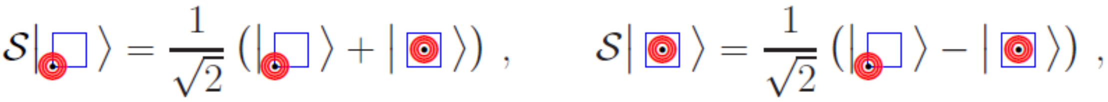
$$\mathcal{T}|[0]\rangle = |[0]\rangle, \mathcal{T}\left|\left[\frac{1}{2}+\frac{1}{2}i\right]\right\rangle = e^{\frac{i\pi}{2}}\left|\left[\frac{1}{2}+\frac{1}{2}i\right]\right\rangle.$$



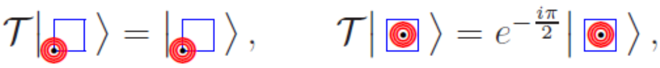
$$\begin{cases} \mathcal{S}\left|[0,0]\right\rangle = \frac{1}{2}\left|[0,0]\right\rangle + \frac{1}{\sqrt{2}}\left|\left[\frac{1}{2},\frac{1}{2}i\right]\right\rangle + \frac{1}{2}\left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle \\ \mathcal{S}\left|\left[\frac{1}{2},\frac{1}{2}i\right]\right\rangle = \frac{1}{\sqrt{2}}\left|[0,0]\right\rangle - \frac{1}{\sqrt{2}}\left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle \\ \mathcal{S}\left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle = \frac{1}{2}\left|[0,0]\right\rangle - \frac{1}{\sqrt{2}}\left|\left[\frac{1}{2},\frac{1}{2}i\right]\right\rangle + \frac{1}{2}\left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle \end{cases}$$



$$\mathcal{T}\left|[0,0]\right\rangle = \left|[0,0]\right\rangle, \mathcal{T}\left|\left[\frac{1}{2},\frac{1}{2}i\right]\right\rangle = \left|\left[\frac{1}{2},\frac{1}{2}i\right]\right\rangle, \mathcal{T}\left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle = -\left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle$$

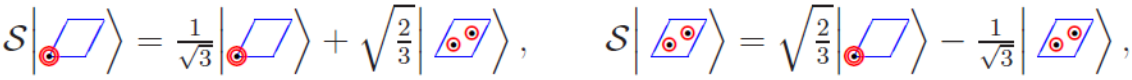


$$\begin{cases} \mathcal{S}\left|[0,0,0]\right\rangle = \frac{1}{\sqrt{2}}\left(\left|[0,0,0]\right\rangle + \left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle\right), \\ \mathcal{S}\left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle = \frac{1}{\sqrt{2}}\left(\left|[0,0,0]\right\rangle - \left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle\right). \end{cases}$$

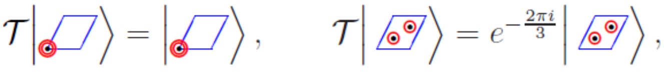


$$\begin{cases} \mathcal{T}\left|[0,0,0]\right\rangle = \left|[0,0,0]\right\rangle \\ \mathcal{T}\left|\left[\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle = e^{-\frac{i\pi}{2}}\left|\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i,\frac{1}{2}+\frac{1}{2}i\right]\right\rangle \end{cases}$$

$$\mathcal{S}\left|[0]\right\rangle = \left|[0]\right\rangle, \mathcal{T}\left|[0]\right\rangle = \left|[0]\right\rangle.$$

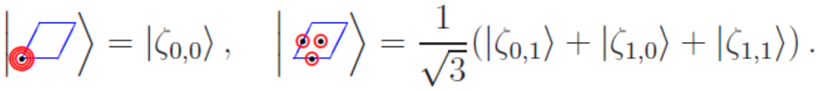


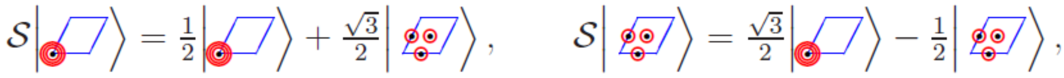
$$\begin{cases} \mathcal{S}\left|[0,0]\right\rangle = \frac{1}{\sqrt{3}}\left|[0,0]\right\rangle + \sqrt{\frac{2}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}},\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle, \\ \mathcal{S}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}},\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = \sqrt{\frac{2}{3}}\left|[0,0]\right\rangle - \frac{1}{\sqrt{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}},\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle. \end{cases}$$



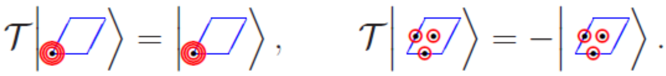
$$\mathcal{T}|[0,0]\rangle = |[0,0]\rangle, \mathcal{T}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = e^{-\frac{2\pi i}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle.$$

$$\mathcal{S}^{-1}\tilde{\mathcal{V}}_a\mathcal{S} = \tilde{\mathcal{U}}_b, \mathcal{S}^{-1}\tilde{\mathcal{V}}_b\mathcal{S} = \tilde{\mathcal{U}}_a^{-1}, \mathcal{S}^{-1}\tilde{\mathcal{U}}_a\mathcal{S} = \tilde{\mathcal{V}}_b, \mathcal{S}^{-1}\tilde{\mathcal{U}}_b\mathcal{S} = \tilde{\mathcal{V}}_a^{-1}$$
$$\mathcal{T}^{-1}\tilde{\mathcal{V}}_a\mathcal{T} = \tilde{\mathcal{V}}_a, \mathcal{T}^{-1}\tilde{\mathcal{V}}_b\mathcal{T} = -\tilde{\mathcal{V}}_b, \mathcal{T}^{-1}\tilde{\mathcal{U}}_a\mathcal{T} = -\tilde{\mathcal{U}}_a\mathcal{V}_b^{-1}, \mathcal{T}^{-1}\tilde{\mathcal{U}}_b\mathcal{T} = \tilde{\mathcal{U}}_b\mathcal{V}_a$$

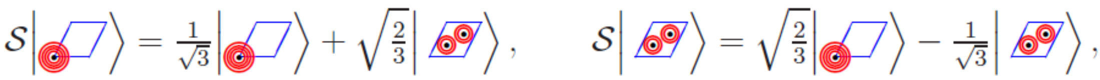




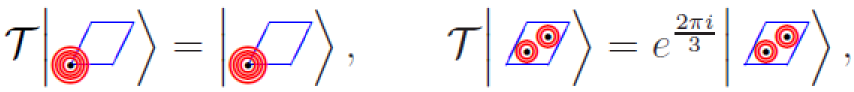
$$\begin{cases} \mathcal{S}|[0,0,0]\rangle = \frac{1}{2}|[0,0,0]\rangle + \frac{\sqrt{3}}{2}\left|\left[\frac{1}{2}, \frac{1}{2}\tau, \frac{1}{2} + \frac{1}{2}\tau\right]\right\rangle \\ \mathcal{S}\left|\left[\frac{1}{2}, \frac{1}{2}\tau, \frac{1}{2} + \frac{1}{2}\tau\right]\right\rangle = \frac{\sqrt{3}}{2}|[0,0,0]\rangle - \frac{1}{2}\left|\left[\frac{1}{2}, \frac{1}{2}\tau, \frac{1}{2} + \frac{1}{2}\tau\right]\right\rangle \end{cases}$$



$$\mathcal{T}|[0,0,0]\rangle = |[0,0,0]\rangle, \mathcal{T}\left|\left[\frac{1}{2}, \frac{1}{2}\tau, \frac{1}{2} + \frac{1}{2}\tau\right]\right\rangle = -\left|\left[\frac{1}{2}, \frac{1}{2}\tau, \frac{1}{2} + \frac{1}{2}\tau\right]\right\rangle.$$

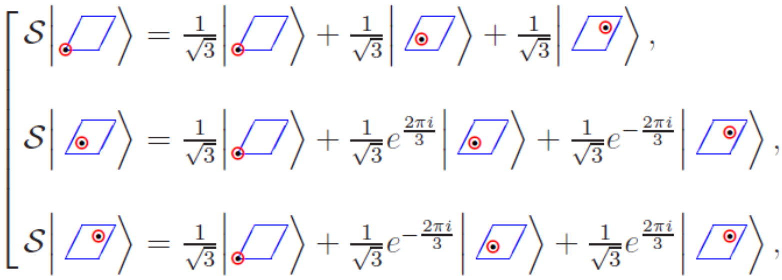


$$\begin{cases} \mathcal{S}|[0,0,0,0]\rangle = \frac{1}{\sqrt{3}}|[0,0,0,0]\rangle + \sqrt{\frac{2}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle, \\ \mathcal{S}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = \sqrt{\frac{2}{3}}|[0,0,0,0]\rangle - \frac{1}{\sqrt{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle, \end{cases}$$

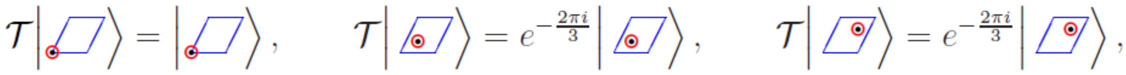


$$\begin{cases} \mathcal{T}|[0,0,0,0]\rangle = |[0,0,0,0]\rangle \\ \mathcal{T}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = e^{\frac{2\pi i}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle. \end{cases}$$

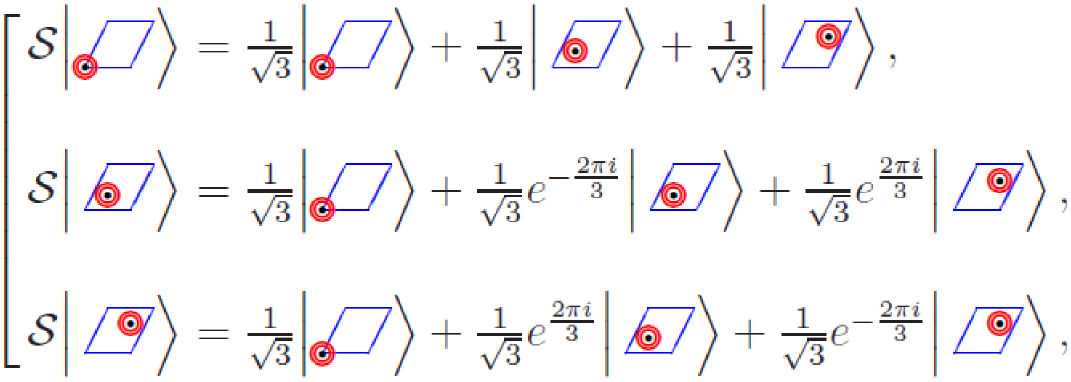
$$\mathcal{S}|[0,0,0,0,0]\rangle = |[0,0,0,0,0]\rangle, \mathcal{T}|[0,0,0,0,0]\rangle = |[0,0,0,0,0]\rangle.$$



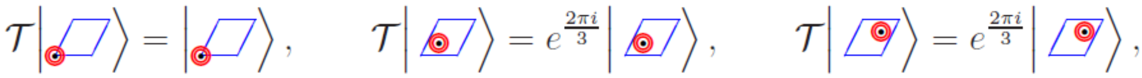
$$\left[\begin{array}{l} \mathcal{S}|[0]\rangle = \frac{1}{\sqrt{3}}|[0]\rangle + \frac{1}{\sqrt{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle + \frac{1}{\sqrt{3}}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle \\ \mathcal{S}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = \frac{1}{\sqrt{3}}|[0]\rangle + \frac{1}{\sqrt{3}}e^{\frac{2\pi i}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle + \frac{1}{\sqrt{3}}e^{-\frac{2\pi i}{3}}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle \\ \mathcal{S}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = \frac{1}{\sqrt{3}}|[0]\rangle + \frac{1}{\sqrt{3}}e^{-\frac{2\pi i}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle + \frac{1}{\sqrt{3}}e^{\frac{2\pi i}{3}}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle \end{array}\right.$$



$$\mathcal{T}|[0]\rangle = |[0]\rangle, \mathcal{T}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = e^{-\frac{2\pi i}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle, \mathcal{T}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = e^{-\frac{2\pi i}{3}}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle$$



$$\left[\begin{array}{l} \mathcal{S}|[0,0]\rangle = \frac{1}{\sqrt{3}}|[0,0]\rangle + \frac{1}{\sqrt{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle + \frac{1}{\sqrt{3}}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle \\ \mathcal{S}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = \frac{1}{\sqrt{3}}|[0,0]\rangle + \frac{1}{\sqrt{3}}e^{-\frac{2\pi i}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle + \frac{1}{\sqrt{3}}e^{\frac{2\pi i}{3}}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle \\ \mathcal{S}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = \frac{1}{\sqrt{3}}|[0,0]\rangle + \frac{1}{\sqrt{3}}e^{\frac{2\pi i}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle + \frac{1}{\sqrt{3}}e^{-\frac{2\pi i}{3}}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle \end{array}\right.$$



$$\left[\begin{array}{l} \mathcal{T}|[0,0]\rangle = |[0,0]\rangle \\ \mathcal{T}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = e^{\frac{2\pi i}{3}}\left|\left[\frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{1}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle \\ \mathcal{T}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle = e^{\frac{2\pi i}{3}}\left|\left[\frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}, \frac{2}{\sqrt{3}}e^{\frac{i\pi}{6}}\right]\right\rangle \end{array}\right.$$

$$\sum_{\sigma\in S_n}\left|p_{\sigma(1)},\dots,p_{\sigma(n)}\right\rangle=\sum_{p=0}^{kn-1}|p_1,\dots,p_n;p\rangle_{SU(n)}|p\rangle_{U(1)}$$

$$\langle p_1,\dots,p_n;p\mid p_1,\dots,p_n;p\rangle=\frac{1}{n}N_{p_1\dots p_n}$$

$$N_{p_1\dots p_n}=\frac{n!}{\prod_j m_j!}$$

$$\sum_{\sigma\in S_n}\left|p_{\sigma(1)}+1,\dots,p_{\sigma(n)}+1\right\rangle=\mathcal{K}_1\sum_{\sigma\in S_n}\left|p_{\sigma(1)},\dots,p_{\sigma(n)}\right\rangle$$
$$=\sum_{p=0}^{kn-1}|p_1,\dots,p_n;p\rangle_{SU(n)}|p+n\rangle_{U(1)}$$

$$e^{-\frac{2\pi i}{k}\sum_i p_i}\sum_{\sigma\in S_n}\left|p_{\sigma(1)},\dots,p_{\sigma(n)}\right\rangle=\mathcal{K}_2\sum_{\sigma\in S_n}\left|p_{\sigma(1)},\dots,p_{\sigma(n)}\right\rangle$$
$$=\sum_{p=0}^{kn-1}e^{-\frac{2\pi i}{k}}|p_1,\dots,p_n;p\rangle_{SU(n)}|p\rangle_{U(1)}$$

$$|p_1,\dots,p_n;p-n\rangle=|p_1+1,\dots,p_n+1;p\rangle$$

$$e^{\frac{i\pi}{k}\sum_i p_i^2}\sum_{\sigma\in S_n}\left|p_{\sigma(1)},\dots,p_{\sigma(n)}\right\rangle=\sum_{\sigma\in S_n}\mathcal{T}\left|p_{\sigma(1)},\dots,p_{\sigma(n)}\right\rangle$$
$$=\sum_{p=0}^{kn-1}e^{\frac{i\pi}{kn}p^2}\mathcal{T}|p_1,\dots,p_n;p\rangle_{SU(n)}|p\rangle_{U(1)}$$

$$\mathcal{T}|p_1,\dots,p_n;p\rangle_{SU(n)}=e^{\frac{i\pi}{k}\left(\sum_{i=1}^n p_i^2-\frac{1}{n}p^2\right)}|p_1,\dots,p_n;p\rangle_{SU(n)}$$

$$\mathcal{T}|p_1,\dots,p_n;p\rangle_{SU(n)}=(-1)^{p-\sum_{i=1}^n p_i}e^{\frac{i\pi}{k}\left(\sum_{i=1}^n p_i^2-\frac{1}{n}p^2\right)}|p_1,\dots,p_n;p\rangle_{SU(n)}$$

$$\frac{1}{k^{n/2}}\sum_{\sigma\in S_n}\sum_{q_1=0}^{k-1}\cdots\sum_{q_n=0}^{k-1}e^{\frac{2\pi i}{k}\sum_{i=1}^n q_i p_{\sigma(i)}}|q_1,\dots,q_n\rangle=\sum_{\sigma\in S_n}\mathcal{S}\left|p_{\sigma(1)},\dots,p_{\sigma(n)}\right\rangle$$
$$=\frac{1}{\sqrt{kn}}\sum_{q=0}^{kn-1}e^{\frac{2\pi i}{kn}pq}\mathcal{S}|p_1,\dots,p_n;p\rangle_{SU(n)}|q\rangle_{U(1)}$$

$$\frac{1}{\sqrt{kn}}\sum_{q=0}^{kn-1}e^{\frac{2\pi i}{kn}pq}|q\rangle_{U(1)}$$

$$\mathcal{S}|p_1,\dots,p_n;p\rangle_{SU(n)}=$$
$$\frac{1}{\sqrt{k^{n+1}n}}\sum_{q_1=0}^{k-1}\cdots\sum_{q_n=0}^{k-1}e^{\frac{2\pi i}{k}\sum_i q_i\left(p_i-\frac{1}{n}p\right)}\sum_{m=0}^{n-1}e^{-\frac{2\pi i}{n}pm}\left|q_1,\dots,q_n;mk+\sum_i q_i\right\rangle_{SU(n)}$$

$$\Omega_1''|p_1,\dots,p_n;p\rangle_{SU(n)}=|p_1,\dots,p_n;p+k\rangle_{SU(n)}$$
$$\Omega_2''|p_1,\dots,p_n;p\rangle_{SU(n)}=e^{-\frac{2\pi i}{n}p}|p_1,\dots,p_n;p\rangle_{SU(n)}$$

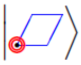
$$\left|p_1,p_2;3m+\sum p_i\right\rangle_{SU(2)},0\le p_1,p_2<3,0\le m<2.$$

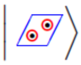
$$\left|p_1,p_2;3m+\sum p_i\right\rangle_{SU(2)}|q\rangle_{U(1)}$$

$$q \equiv 3m + \sum_{i=1}^{2} p_i \equiv m + \sum_{i=1}^{2} p_i \ (\mathrm{mod}2)$$

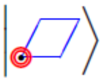
$$|p_1, p_2\rangle_s \equiv \sum_{m=0}^{1} \left|p_1, p_2; 3m + \sum p_i\right\rangle_{SU(2)} \left|3m + \sum p_i\right\rangle_{U(1)}$$

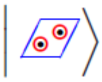
$$\mathcal{T}|p_1, p_2\rangle_s = (-1)^{\sum p_i} e^{\frac{\pi i}{3}[2(\sum p_i)^2 - \sum p_i^2]} |p_1, p_2\rangle_s$$

 $= \sqrt{2}|0,0\rangle_s = \sqrt{2}|1,1\rangle_s = \sqrt{2}|2,2\rangle_s\,,$

 $= |0,1\rangle_s = |1,2\rangle_s = |2,0\rangle_s = |1,0\rangle_s = |2,1\rangle_s = |0,2\rangle_s\,,$

$$\mathcal{H}\left(2, \frac{\pi}{3}\right) = \mathcal{H}_{(1,1)}\left(\frac{\pi}{3}\right) \oplus \mathcal{H}_{(2)}\left(\frac{\pi}{3}\right)$$

 $= |a\rangle_{SU(2)} \otimes |0\rangle_{U(1)} + |b\rangle_{SU(2)} \otimes |1\rangle_{U(1)}\,,$

 $= |d\rangle_{SU(2)} \otimes |0\rangle_{U(1)} + |c\rangle_{SU(2)} \otimes |1\rangle_{U(1)}\,,$

$$\mathcal{T}|a\rangle = |a\rangle, \mathcal{T}|b\rangle = e^{-\frac{\pi i}{2}}|b\rangle, \mathcal{T}|c\rangle = e^{\frac{5\pi i}{6}}|c\rangle, \mathcal{T}|d\rangle = e^{-\frac{2\pi i}{3}}|d\rangle.$$

$$(\mathcal{T}\mathcal{S})^3 = e^{-\frac{\pi i}{4}}$$

$$\mathcal{S}|a\rangle = \frac{1}{\sqrt{6}}|a\rangle + \frac{1}{\sqrt{6}}|b\rangle + \sqrt{\frac{1}{3}}|c\rangle + \sqrt{\frac{1}{3}}|d\rangle,$$

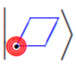
$$\mathcal{S}|b\rangle = \frac{1}{\sqrt{6}}|a\rangle - \frac{1}{\sqrt{6}}|b\rangle - \sqrt{\frac{1}{3}}|c\rangle + \sqrt{\frac{1}{3}}|d\rangle,$$

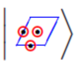
$$\mathcal{S}|c\rangle = \sqrt{\frac{1}{3}}|a\rangle - \sqrt{\frac{1}{3}}|b\rangle + \frac{1}{\sqrt{6}}|c\rangle - \frac{1}{\sqrt{6}}|d\rangle,$$

$$\mathcal{S}|d\rangle = \sqrt{\frac{1}{3}}|a\rangle + \sqrt{\frac{1}{3}}|b\rangle - \frac{1}{\sqrt{6}}|c\rangle - \frac{1}{\sqrt{6}}|d\rangle,$$

$$\Omega_1''|a\rangle = |b\rangle, \quad \Omega_1''|b\rangle = |a\rangle, \quad \Omega_1''|c\rangle = |d\rangle, \Omega_1''|d\rangle = |c\rangle,$$

$$\mathcal{H}_{(2)}\left(\frac{\pi}{3}\right) = \mathcal{H}([U(1)_2 \times SU(2)_{-3}]/\mathbb{Z}_2).$$

 $= |a\rangle_{SU(3)} \otimes |0\rangle_{U(1)} + |b\rangle_{SU(3)} \otimes |1\rangle_{U(1)} + |c\rangle_{SU(3)} \otimes |2\rangle_{U(1)}\,,$

 $= |d\rangle_{SU(3)} \otimes |0\rangle_{U(1)} + |e\rangle_{SU(3)} \otimes |1\rangle_{U(1)} + |f\rangle_{SU(3)} \otimes |2\rangle_{U(1)}\,.$

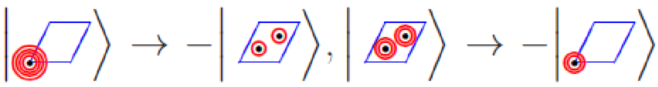
$$\mathcal{T}|a\rangle_{SU(3)} = -|a\rangle_{SU(3)}, \quad \mathcal{T}|b\rangle_{SU(3)} = -e^{\frac{2\pi i}{3}}|b\rangle_{SU(3)}, \quad \mathcal{T}|c\rangle_{SU(3)} = -e^{\frac{2\pi i}{3}}|c\rangle_{SU(3)}$$

$$\mathcal{T}|d\rangle_{SU(3)} = |d\rangle_{SU(3)}, \quad \mathcal{T}|e\rangle_{SU(3)} = e^{\frac{2\pi i}{3}}|e\rangle_{SU(3)}, \quad \mathcal{T}|f\rangle_{SU(3)} = e^{\frac{2\pi i}{3}}|f\rangle_{SU(3)}$$

$$\mathcal{S}=\frac{1}{2}\begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 1 & 1 & 1\\ \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} & 1 & \omega & \omega^2\\ \frac{1}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & 1 & \omega^2 & \omega\\ 1 & 1 & 1 & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}\\ 1 & \omega & \omega^2 & -\frac{1}{\sqrt{3}} & -\frac{\omega}{\sqrt{3}} & -\frac{\omega^2}{\sqrt{3}}\\ 1 & \omega^2 & \omega & -\frac{1}{\sqrt{3}} & -\frac{\omega^2}{\sqrt{3}} & -\frac{\omega}{\sqrt{3}}\end{pmatrix},$$

$$\begin{aligned}\Omega_1''|a\rangle_{SU(3)} &= |b\rangle_{SU(3)}, \quad \Omega_1''|b\rangle_{SU(3)} = |c\rangle_{SU(3)},\\ \Omega_1''|d\rangle_{SU(3)} &= |e\rangle_{SU(3)}, \quad \Omega_1''|c\rangle_{SU(3)} = |f\rangle_{SU(3)},\\ \Omega_1''|f\rangle_{SU(3)} &= |a\rangle_{SU(3)}\end{aligned}$$

$$\Omega_2''|a\rangle_{SU(3)} = |a\rangle_{SU(3)}, \quad \Omega_2''|b\rangle_{SU(3)} = e^{\frac{2\pi i}{3}}|b\rangle_{SU(3)}, \quad \Omega_2''|c\rangle_{SU(3)} = e^{-\frac{2\pi i}{3}}|c\rangle_{SU(3)}$$
$$\Omega_2''|d\rangle_{SU(3)} = |d\rangle_{SU(3)}, \quad \Omega_2''|e\rangle_{SU(3)} = e^{\frac{2\pi i}{3}}|e\rangle_{SU(3)}, \quad \Omega_2''|f\rangle_{SU(3)} = e^{-\frac{2\pi i}{3}}|f\rangle_{SU(3)}$$



$$\mathcal{H}_{(2,2)}\left(\frac{\pi}{3}\right) \simeq \mathcal{H}_{(2)}\left(\frac{\pi}{3}\right)^{\otimes 2}/S_2$$

$$\mathcal{H}_{(2)}\left(\frac{\pi}{3}\right) \simeq U(2)_{2,-3}$$

$$\mathcal{H}_{(2,2)}\left(\frac{\pi}{3}\right)$$

$$\mathcal{H}_{(2,2)}\left(\frac{\pi}{3}\right) \simeq U(2)_{2,-3}^{\otimes 2}/S_2.$$

$$\mathcal{H}_{(2,2)}\left(\frac{\pi}{3}\right) = \left[U(1)_4 \times \tilde{\mathcal{H}}_{(2,2)}\left(\frac{\pi}{3}\right)\right]/\mathbb{Z}_4$$

$$\mathcal{H}[SU(2)_{k'}]^{\otimes n'}/S_{n'} \simeq \mathcal{H}[Sp(n')_{k'}],$$

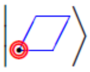
$$|a,a\rangle + |b,b\rangle, |a,d\rangle + |b,c\rangle, |c,c\rangle + |d,d\rangle$$

$$\begin{aligned}|a,a\rangle &\equiv |a\rangle_{SU(2)} \otimes |a\rangle_{SU(2)}\\ |a,d\rangle &\equiv \frac{1}{\sqrt{2}}\left(|a\rangle_{SU(2)} \otimes |d\rangle_{SU(2)} + |d\rangle_{SU(2)} \otimes |a\rangle_{SU(2)}\right)\end{aligned}$$

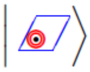
$$\mathcal{T}^2 = \begin{pmatrix}1 & 0 & 0\\ 0 & e^{\frac{4\pi i}{3}} & 0\\ 0 & 0 & e^{\frac{2\pi i}{3}}\end{pmatrix}, \mathcal{S} = \frac{1}{3}\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{pmatrix}$$

$$\mathcal{H}_{(5)}\left(\frac{\pi}{3}\right) = \mathcal{H}[U(5)_{5,1}].$$

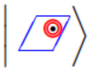
$$\mathcal{H}\left(2, \frac{2\pi}{3}\right) = \mathcal{H}_{(1,1)}\left(\frac{2\pi}{3}\right) \oplus \mathcal{H}_{(2)}\left(\frac{2\pi}{3}\right)$$



$$= |a\rangle_{SU(2)} \otimes |0\rangle_{U(1)} + |b\rangle_{SU(2)} \otimes |3\rangle_{U(1)} ,$$



$$= |a\rangle_{SU(2)} \otimes |2\rangle_{U(1)} + |b\rangle_{SU(2)} \otimes |5\rangle_{U(1)} ,$$



$$= |a\rangle_{SU(2)} \otimes |4\rangle_{U(1)} + |b\rangle_{SU(2)} \otimes |1\rangle_{U(1)} ,$$

$$\mathcal{S}|a\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle), \mathcal{S}|b\rangle = \frac{1}{\sqrt{2}}(|a\rangle - |b\rangle),$$

$$\mathcal{T}|a\rangle = |a\rangle, \mathcal{T}|b\rangle = e^{\frac{\pi i}{2}}|b\rangle.$$

$$\Omega_1''|a\rangle = |a\rangle, \Omega_1''|b\rangle = -|b\rangle, \Omega_2''|a\rangle = |b\rangle, \Omega_2''|b\rangle = |a\rangle$$

CONCLUSIONES

Según los resultados antes referidos, se concluye que, en un campo cuántico – relativista, los quarks y gluones, para lograr el confinamiento de color, intermedia la gravedad o supergravedad cuánticas, según el caso, es decir, según la naturaleza de la partícula subatómica de origen (partícula oscura o blanca, según corresponda), lo que permite la formación de hadrones (mesones y bariones respectivamente), los cuales, al igual que las partículas que los originan, pueden ser partículas blancas u oscuras, según sea el caso. Téngase en cuenta, que si bien es cierto, que el quark top, es un candidato genuino para actuar como partícula oscura, debido a su enorme masa, sin embargo, no es susceptible de hadronización, debido a que se desintegra inmediatamente sin que le sea posible confinarse, más, como se ha dicho, esta partícula supermasiva, es capaz de producir gravedad, incluso en condiciones extremas y entrópicas.

Aclaraciones Finales

Algunas aclaraciones finales a tener en consideración y aplicar, a propósito de la Teoría Cuántica de Campos Relativistas o Curvos (TCCR) propuesta por este autor:

1. En todos los casos, este símbolo † será reemplazado por este símbolo†o por este símbolo †, equivaliendo lo mismo.

Símbolo a ser reemplazado.	Símbolos de reemplazo.
†	†
	†

2. En todos los casos, este símbolo ‡, será reemplazado por este símbolo‡o por este símbolo ‡.

Símbolo a ser reemplazado.	**Símbolos de reemplazo.**
‡	‡
	‡

3. En todos los casos, se añadirá y por ende, se calculará la magnitud§que equivale a un campo de Yang – Mills y por ende, a la teoría de Yang – Mills en sentido amplio, en relación a la Teoría Cuántica de Campos Relativistas propuesta por este autor.

4. Este símbolo • podrá usarse como exponente u operador, según sea el caso.

Las aclaraciones antes referidas aplican tanto a este trabajo como a todos los trabajos previos y posteriores publicados por este autor, según corresponda.

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