Ciencia Latina Revista Científica Multidisciplinar, Ciudad de México, México.
ISSN 2707-2207 / ISSN 2707-2215 (en línea), septiembre-octubre 2025,
Volumen 9, Número 5.

https://doi.org/10.37811/cl_rcm.v9i5

CROMODINÁMICA CUÁNTICA RELATIVISTA. HADRONIZACIÓN EN SUPERGRAVEDAD Y GRAVEDAD CUÁNTICAS. VOLUMEN II

RELATIVISTIC QUANTUM CHROMODYNAMICS. HADRONIZATION IN SUPERGRAVITY AND QUANTUM GRAVITY. VOLUME II

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DOI: https://doi.org/10.37811/cl_rcm.v9i5.19770

Cromodinámica Cuántica Relativista. Hadronización en Supergravedad y Gravedad Cuánticas. Volumen II

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RESUMEN

Como ha quedado demostrado en trabajos anteriores, las partículas elementales que conforman la fuerza nuclear fuerte, entre ellas los quarks y los gluones, pueden acreditar las características que son inherentes a una partícula oscura o blanca e incluso, una suprapartícula e hiperpartícula, ésta última, cuando alcanza o supera la velocidad de la luz, sin embargo, en este artículo, se propone un planteamiento alternativo para la hadronización. En sentido estricto, la hadronización es la combinación de quarks y gluones para la formación de otra partícula con distinta masa y energía, denominada hadrón, todo esto, debido al confinamiento que los une. Suponemos, que un plano cuántico – relativista, los quarks y los gluones, se combinan, por la deformación del espacio – tiempo cuántico, causado por cualquiera de éstos, sea en condición de partícula oscura o blanca, según sea el caso. Por tanto, en un espacio de gravedad cuántica o de supergravedad, la hadronización es posible, en la medida en que la gravedad en sí misma, interfiere transversalmente en el proceso de confinamiento, recalcando que, la hadronización puede formar partículas oscuras o blancas de naturaleza hadrónica.

Palabras clave: quarks, gluones, hadrones, hadronización, supergravedad cuántica relativista

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Relativistic Quantum Chromodynamics. Hadronization in Supergravity and Quantum Gravity. Volume II

ABSTRACT

As has been demonstrated in previous works, the elementary particles that make up the strong nuclear force, including quarks and gluons, can accredit the characteristics that are inherent to a dark or white particle and even a supraparticle and hyperparticle, the latter, when it reaches or exceeds the speed of light, however, in this article, An alternative approach to hadronization is proposed. Strictly speaking, hadronization is the combination of quarks and gluons to form another particle with different mass and energy, called a hadron, all due to the confinement that unites them. We suppose that a quantum-relativistic plane, quarks and gluons, are combined by the deformation of quantum space-time, caused by any of these, whether in the condition of a dark or white particle, as the case may be. Therefore, in a quantum gravity or supergravity space, hadronization is possible, insofar as gravity itself interferes transversally in the confinement process, emphasizing that hadronization can form dark or white particles of hadronic nature.

Keywords: quarks, gluones, hadrones, hadronización, supergravedad cuántica relativista

Artículo recibido 11 agosto 2025
Aceptado para publicación: 13 setiembre 2025

INTRODUCCIÓN

El confinamiento, en cromodinámica cuántica, consiste en la combinación ineludible de los quarks y los gluones, en la medida en que, los primeros, no son susceptibles de aislamiento, por lo que, para su detección, éstos se asocian a los gluones, con la finalidad de formar hadrones (color neutro), los cuales pueden ser de carácter mesónico o bariónico. Ahora bien, la cromodinámica cuántica despierta interés para la formalización de la Teoría Cuántica de Campos Relativistas, en la medida en que: 1) posee la partícula subatómica con mayor masa en relación a las demás partículas que componen el modelo estándar, siendo ésta, el quark top; y, 2) para efectos de detectar, en especial los quarks, se requiere del proceso de hadronización, ya que aislar un quark, en especial el quark top, es de difícil realización (en el caso del primero) e imposible en el caso del segundo, pues, para aislar un quark, se requiere de un paquete de energía tan extremo, directamente proporcional a la distancia de su punto de origen, que finalmente se forma un par quark – antiquark, en tanto que, respecto del segundo, se ha dicho que es imposible la hadronización pues, al ser tan pesado, se desintegra rápidamente, sin que sea posible su detención, a diferencia de los otros tipos de quarks, los cuales, al ser ligeros, son susceptibles de hadronización. Ahora bien, esto ayuda a reforzar el hecho de que, en primer lugar, sí existen partículas supermasivas en el modelo estándar, siendo ésta, por ejemplo, el quark top, aunque el bosón de Higgs, también presenta esa particularidad, y por otro lado, que a nivel cronodinámico, la relatividad general y especial son posibles, en la medida en que, a mi criterio, es la gravedad la que impide la disociación del quark respecto del gluon, es decir, es la responsable del confinamiento de color que da lugar a los hadrones, los cuales, al ser de color neutro, podrían calificar como partículas blancas o estrella. De ahí la necesidad de la Cromodinámica Cuántica Relativista, como un modelo teórico apto para explicar los fenómenos antes referidos.

RESULTADOS Y DISCUSIÓN

Retomo el desarrollo matemático desplegado en el volumen I de este manuscrito.

Modelo de Hadronización de una partícula blanca u oscura en un plano cuántico relativista, bajo condiciones cromodrinámicas. Cálculos complementarios.

$$\mathcal{H}_{(2)}\left(\frac{2\pi}{3}\right) = \mathcal{H}([U(1)_6 \times SU(2)_{-1}]/\mathbb{Z}_2)$$

$$S_g[\phi] = S_g^{\mathrm{b}}[\phi] + \hbar S_g^{\mathrm{f}}[\phi]$$

$$\langle X[\phi]\rangle_g = \int \mathcal{D}\phi \exp\left\{\frac{\mathrm{i}}{\hbar} S_g[\phi]\right\} X[\phi]$$

$$T_g : \phi(x) \mapsto \phi'(x; g, \phi)$$

$$\langle X[\phi]\rangle_g = \langle X[T_g^{-1}\phi]\rangle_0 \ \forall X$$

$$\partial_g \langle X[\phi]\rangle_g = \langle(\partial_g + R_g[\phi])X[\phi]\rangle_g,$$

$$R_g[\phi] = \int \mathrm{d}x\left(\partial_g T_g^{-1} \circ T_g\right)\phi(x)\frac{\delta}{\delta\phi(x)} =: \int \mathrm{d}x K[\phi; x]\frac{\delta}{\delta\phi(x)}$$

$$(\partial_g + R_g[\phi])T_g\phi = 0$$

$$T_g\phi = \vec{\mathcal{P}}\exp\left\{-\int_0^g \mathrm{d}h R_h[\phi]\right\}\phi$$

$$S_0^{\mathrm{b}}[T_g\phi] = S_g^{\mathrm{b}}[\phi] \text{ and } S_0^{\mathrm{f}}[T_g\phi] - \mathrm{itrln}\frac{\delta T_g\phi}{\delta\phi} = S_g^{\mathrm{f}}[\phi]$$

$$(\partial_g + R_g[\phi])S_g^{\mathrm{b}}[\phi] = 0 \text{ and } (\partial_g + R_g[\phi])S_g^{\mathrm{f}}[\phi] = \int \mathrm{d}x\frac{\delta K[\phi; x]}{\delta\phi(x)}$$

$$(\partial_g + R_g[\phi])\mathcal{G}(\phi) = 0$$

$$\eta^{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1), \eta^{\Sigma\Theta} = \mathrm{diag}(-1, +1, \ldots, +1)$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{pmatrix}$$

$$\sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathrm{P}^+ = \frac{1}{2}(1 + \mathrm{i}\gamma^5) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathrm{P}^- = \frac{1}{2}(1 - \mathrm{i}\gamma^5) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\phi = \gamma^\mu a_\mu$$

$$\gamma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

$$a^{[\mu}b^{\nu]} = \frac{1}{2}(a^\mu b^\nu - a^\nu b^\mu)$$

$$f^{abc}f^{abd} = n_c\delta^{cd},$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + gA_\mu \times A_\nu \iff F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$$

$$\mathrm{D}_\mu = \partial_\mu + gA_\mu \times \iff (\mathrm{D}_\mu \ldots)^a = \partial_\mu(\ldots)^a + gf^{abc}A_\mu^b(\ldots)^c.$$

$$F_{\mu\nu}F^{\mu\nu} = F_{\mu\nu}^a F^{a\mu\nu},$$

$$\varphi\psi \times \lambda = f^{abc}\varphi^a\psi^b\lambda^c.$$

$$\partial^\rho C\varphi_i\partial_\mu CA_\rho \times \varphi_i \iff \int d^4y\, d^4z \partial^\rho C(x-y)(f^{abc}\varphi_i^b)(y)\partial_\mu C(y-z)(f^{cde}A_\rho^d)(z)\varphi_i^e(z)$$

$$C = \Box^{-1}$$

$$\mathcal{A}_\Gamma = (A_\mu, \varphi_i).$$

$$S_0 = \partial\!\!\!/^{-1} = -\partial\!\!\!/ C, G_0 = \left(\frac{\partial \mathcal{G}(\mathcal{A})}{\partial A_\mu}\partial_\mu\right)^{-1},$$

$$\Pi_\mu{}^\nu = \delta_\mu{}^\nu - \partial_\mu G_0 \frac{\partial \mathcal{G}(\mathcal{A})}{\partial \mathcal{A}_\nu},$$

$$\Pi_\Gamma{}^\Sigma = \delta_\Gamma{}^\Sigma - \partial_\Gamma G_0 \frac{\partial \mathcal{G}(\mathcal{A})}{\partial \mathcal{A}_\Sigma}$$

$$Ш_\mu{}^\nu = \delta_\mu{}^\nu - \partial_\mu C\partial^\nu.$$

$$A_\mu = A_\mu^{\mathrm{T}} + A_\mu^{\mathrm{L}}, A_\mu^{\mathrm{T}} = Ш_\mu{}^\nu A_\nu, A_\mu^{\mathrm{L}} = (\delta_\mu{}^\nu - Ш_\mu{}^\nu)A_\nu = \partial_\mu C\partial \cdot A,$$

$$A_\mu^* := A_\mu^{\mathrm{T}} - A_\mu^{\mathrm{L}} = A_\mu - 2\partial_\mu C\partial \cdot A,$$

$$S_{\mathrm{inv}} = \int d^4x\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\mathrm{D}_\mu\varphi_i\mathrm{D}^\mu\varphi_i - \frac{\mathrm{i}}{2}\bar{\chi}_A \not{D}\mathrm{P}^+\chi^A - \frac{\mathrm{i}}{2}\bar{\tilde{\chi}}^A \not{D}\mathrm{P}^-\tilde{\chi}_A$$
$$-\mathrm{i}gt^i{}_{AB}\bar{\tilde{\chi}}^A\mathrm{P}^+\varphi_i \times \chi^B + \mathrm{i}gt^{iAB}\bar{\chi}_A\mathrm{P}^-\varphi_i \times \tilde{\chi}_B - \frac{g^2}{4}(\varphi_i \times \varphi_j)^2\}$$

$$\varphi_{AB} = t^i{}_{AB}\varphi_i, \varphi^{AB} = t^{iAB}\varphi_i = (\varphi_{AB})^*$$

$$\psi^A = \mathrm{P}^+\chi^A + \mathrm{P}^-\tilde{\chi}_A, \bar{\psi}_A = \bar{\chi}_A\mathrm{P}^- + \bar{\tilde{\chi}}^A\mathrm{P}^+$$

$$S_{\mathrm{inv}} = \int d^4x\left\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\mathrm{D}_\mu\varphi_i\mathrm{D}^\mu\varphi_i - \frac{\mathrm{i}}{2}\bar{\psi}_A(\not{D}_B^A{}_B + g\Phi_B^A \times)\psi^B - \frac{g^2}{4}(\varphi_i \times \varphi_j)^2\right\}$$

$$\Phi_B^A := 2[t_{AB}^i\mathrm{P}^+ - t^{iAB}\mathrm{P}^-]\varphi_i \equiv (c^i)^A{}_B\varphi_i$$

$$\not{\mathcal{D}}^A{}_B := \not{D}\delta^A{}_B + g\Phi^A{}_B \times$$

$$V = (A_\mu, \lambda, \mathcal{D}), \Phi_I = (\phi_I, \psi_I, F_I) \text{ with } I = 1,2,3$$

$$\mathcal{L} = \frac{1}{g^2 n_c}\mathrm{tr}\left[\frac{1}{16}(W^\alpha W_\alpha|_{\theta\theta} + \text{h.c.}) + \mathrm{e}^{-2V}\Phi_I^\dagger\mathrm{e}^{2V}\Phi_I|_{\theta\theta\bar{\theta}\bar{\theta}} + \mathrm{i}\frac{\sqrt{2}}{3!}\left(\epsilon_{IJK}\Phi_I[\Phi_J, \Phi_K]|_{\theta\theta} + \text{h.c.}\right)\right]$$

$$W_\alpha = -\frac{1}{4}\overline{\mathrm{DD}}\mathrm{e}^{-2V}\mathrm{D}_\alpha\mathrm{e}^{2V}, \bar{W}^{\dot{\alpha}} = -\frac{1}{4}\mathrm{DD}\mathrm{e}^{-2V}\bar{\mathrm{D}}^{\dot{\alpha}}\mathrm{e}^{2V}$$

$$g^2\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\mathrm{i}}{2}\bar{\lambda}\gamma^\mu\mathrm{D}_\mu\lambda + \frac{1}{2}\mathcal{D}^2 - \frac{1}{\sqrt{2}}\epsilon_{IJK}(F_I\phi_J \times \phi_K + F_I^\dagger\phi_J^\dagger \times \phi_K^\dagger)$$
$$-\mathrm{D}_\mu\phi_I^\dagger\mathrm{D}^\mu\phi_I - \frac{\mathrm{i}}{2}\bar{\psi}_I\gamma^\mu\mathrm{D}_\mu\psi_I + F_I^\dagger F_I + \frac{1}{\sqrt{2}}\epsilon_{IJK}(\phi_I\bar{\psi}_J\mathrm{P}^+ \times \psi_K + \phi_I^\dagger\bar{\psi}_J\mathrm{P}^- \times \psi_K)$$
$$-\sqrt{2}(\bar{\psi}_I\mathrm{P}^-\lambda \times \phi_I + \bar{\psi}_I\mathrm{P}^+\lambda \times \phi_I^\dagger) - \mathrm{i}\phi_I^\dagger\mathcal{D} \times \phi_I$$

$$\delta_\alpha\phi_I = \sqrt{2}(\bar\psi_I \mathrm{P}^+)_\alpha$$
$$\delta_\alpha\phi_I^\dagger = \sqrt{2}(\bar\psi_I \mathrm{P}^-)_\alpha$$
$$\delta_\alpha(\mathrm{P}^+\psi_I)_\beta = -\mathrm{i}\sqrt{2}(\mathrm{P}^+\gamma^\mu)_{\beta\alpha}(\mathrm{D}_\mu\phi_I) - \sqrt{2}(\mathrm{P}^+)_{\beta\alpha}F_I$$
$$\delta_\alpha(\mathrm{P}^-\psi_I)_\beta = -\mathrm{i}\sqrt{2}(\mathrm{P}^-\gamma^\mu)_{\beta\alpha}(\mathrm{D}_\mu\phi_I^\dagger) - \sqrt{2}(\mathrm{P}^-)_{\beta\alpha}F_I^\dagger$$
$$\delta_\alpha F_I = -\mathrm{i}\sqrt{2}(\mathrm{D}_\mu\bar\psi_{I\beta})(\gamma^\mu \mathrm{P}^-)_{\beta\alpha} - 2\phi_I \times (\bar\lambda \mathrm{P}^-)_\alpha$$
$$\delta_\alpha F_I^\dagger = -\mathrm{i}\sqrt{2}(\mathrm{D}_\mu\bar\psi_{I\beta})(\gamma^\mu \mathrm{P}^+)_{\beta\alpha} - 2\phi_I^\dagger \times (\bar\lambda \mathrm{P}^+)_\alpha$$
$$\delta_\alpha A_\nu = -\mathrm{i}(\bar\lambda\gamma_\nu)_\alpha$$
$$\delta_\alpha \mathcal{D} = -\mathrm{i}(\mathrm{D}_\mu\bar\lambda_\beta)(\gamma_5\gamma^\mu)_{\beta\alpha}$$
$$\delta_\alpha\lambda_\beta = -\frac{1}{2}(\gamma^{\mu\nu})_{\beta\alpha}F_{\mu\nu} + \mathcal{D}(\gamma_5)_{\beta\alpha}$$

$$\mathring{\Delta}_\alpha = \frac{1}{4}\int \mathrm{d}^4x \Big\{-\mathcal{D}\gamma_5\lambda - \frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\lambda + 2\epsilon_{IJK}[\mathrm{P}^+\psi_I\phi_J \times \phi_K + \mathrm{P}^-\psi_I\phi_J^\dagger \times \phi_K^\dagger] + 2\mathrm{i}\gamma_5\phi_I^\dagger\lambda \times \phi_I + \mathrm{i}\sqrt{2}[\gamma^\mu \mathrm{P}^-\psi_I \mathrm{D}_\mu\phi_I + \gamma^\mu \mathrm{P}^+\psi_I \mathrm{D}_\mu\phi_I^\dagger] - \sqrt{2}[\mathrm{P}^+\psi_I F_I^\dagger + \mathrm{P}^-\psi_I F_I]\Big\}_\alpha$$

$$S_{\mathrm{inv}} = \int \mathrm{d}^4x\mathcal{L} = \frac{1}{2g^2}\delta_\alpha\mathring{\Delta}_\alpha$$

$$\mathcal{D} = -\mathrm{i}\phi_I^\dagger \times \phi_I, F_I = \frac{1}{\sqrt{2}}\epsilon_{IJK}\phi_J^\dagger \times \phi_K^\dagger,$$

$$S_{\mathrm{inv}} = \frac{1}{g^2}\int \mathrm{d}^4x\Big\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \mathrm{D}_\mu\phi_I^\dagger \mathrm{D}^\mu\phi_I - \frac{\mathrm{i}}{2}\bar\psi_A \not{D}\psi^A + \frac{1}{\sqrt{2}}\epsilon_{IJK}(\phi_I\bar\psi_J \mathrm{P}^+ \times \psi_K + \phi_I^\dagger\bar\psi_J \mathrm{P}^- \times \psi_K) - \sqrt{2}(\bar\psi_I \mathrm{P}^-\lambda \times \phi_I + \bar\psi_I \mathrm{P}^+\lambda \times \phi_I^\dagger) + \frac{1}{2}(\phi_I^\dagger \times \phi_I)^2 - \frac{1}{2}\epsilon_{IJK}\epsilon_{ILM}(\phi_J \times \phi_K)(\phi_L^\dagger \times \phi_M^\dagger)\Big\}$$

$$\phi_I = \frac{1}{\sqrt{2}}(\varphi_{I+3} + \mathrm{i}\varphi_I), \phi_I^\dagger = \frac{1}{\sqrt{2}}(\varphi_{I+3} - \mathrm{i}\varphi_I),$$

$$S_{\mathrm{inv}} = \frac{1}{g^2}\int \mathrm{d}^4x\Big\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\mathrm{D}_\mu\varphi_i \mathrm{D}^\mu\varphi_i - \frac{\mathrm{i}}{2}\bar\psi_A \not{D}\psi^A + \frac{1}{2}\epsilon_{IJK}(\bar\psi_I\varphi_{J+3} \times \psi_K - \bar\psi_I\varphi_J\gamma_5 \times \psi_K) + \bar\psi_I\varphi_{I+3} \times \lambda + \bar\psi_I\varphi_I\gamma_5 \times \lambda - \frac{1}{4}(\varphi_i \times \varphi_j)^2\Big\}$$

$$(c^I)_4^J = \mathrm{i}\delta_{IJ}\gamma_5, (c^{I+3})_4^J = \mathrm{i}\delta_{IJ}\mathbb{1}_4, (c^I)_K^J = \mathrm{i}\epsilon_{IJK}\gamma_5, (c^{I+3})_K^J = -\mathrm{i}\epsilon_{IJK}\mathbb{1}_4$$

$$\delta_\alpha\varphi_i = -\mathrm{i}\bar\psi_J (c^i)^J{}_4$$

$$\Delta_\alpha = \frac{1}{4}\int \mathrm{d}^4x\Big\{-\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\lambda - (\Phi^4{}_A)^\dagger \not{\mathcal{D}}^A{}_B\psi^B + \frac{1}{2}(\Phi^4{}_A)^\dagger\Phi^A{}_B \times \psi^B\Big\} .$$

$$S^{(10)} = \frac{1}{g^2}\int \mathrm{d}^{10}x\Big\{-\frac{1}{4}F^{\Sigma\Theta}F_{\Sigma\Theta} - \frac{\mathrm{i}}{2}\bar\lambda\Gamma^\Sigma \mathrm{D}_\Sigma\lambda\Big\}$$

$$A_\Sigma = (A_\mu, \varphi_i)$$

$$-\frac{1}{4}F^{\Sigma\Theta}F_{\Sigma\Theta} \longrightarrow -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\mathrm{D}_\mu\varphi_i \mathrm{D}^\mu\varphi_i - \frac{1}{4}(\varphi_i \times \varphi_j)^2$$

$$\Gamma^\mu = \mathbb{1}_8 \otimes \gamma^\mu, \Gamma^{AB} = \begin{pmatrix} 0 & \rho^{AB} \\ \rho_{AB} & 0 \end{pmatrix} \otimes \mathrm{i}\gamma_5, A, B = 1,2,3,4$$

$$(\rho^{AB})_{CD} = \delta_{AC}\delta_{BD} - \delta_{AD}\delta_{BC}, (\rho_{AB})_{CD} = \frac{1}{2}\epsilon_{ABFG}(\rho^{FG})_{CD} = \epsilon_{ABCD}$$

$$\varphi_{I4} = \frac{1}{2}(\varphi_I + \mathrm{i}\varphi_{I+3}), \varphi^{AB} = \frac{1}{2}\epsilon^{ABCD}\varphi_{CD} = (\varphi_{AB})^*$$

$$(t^I)_{J4} = \frac{1}{2}\delta_{IJ} = (t^I)^{J4}, \quad (t^{I+3})_{J4} = \frac{\mathrm{i}}{2}\delta_{IJ} = -(t^{I+3})^{J4}$$
$$(t^I)_{JK} = \frac{1}{2}\epsilon_{IJK} = (t^I)^{JK}, \quad (t^{I+3})_{JK} = -\frac{\mathrm{i}}{2}\epsilon_{IJK} = -(t^{I+3})^{JK}$$

$$\lambda = (\mathrm{P}^+\chi^1, \dots, \mathrm{P}^+\chi^4, \mathrm{P}^-\tilde{\chi}_1, \dots, \mathrm{P}^-\tilde{\chi}_4)^{\mathrm{T}}, \text{ with } \tilde{\chi}_A = \mathrm{C}\bar{\chi}^{A\,\mathrm{T}},$$

$$-\frac{\mathrm{i}}{2}\bar{\lambda}\,\Gamma^{\Sigma}\,\mathrm{D}_{\Sigma}\,\lambda \quad \longrightarrow \quad -\frac{\mathrm{i}}{2}\bar{\psi}_A\,\mathscr{D}^A{}_B\,\psi^B\,,$$

$$\Phi^A_B = (c^i)^A_B\varphi_i = [(\rho^{CD})_{AB}\mathrm{P}^+ - (\rho_{CD})_{AB}\mathrm{P}^-]\varphi_{CD} = 2[t^i_{AB}\mathrm{P}^+ - t^{iAB}\mathrm{P}^-]\varphi_i$$

$$S_{\mathrm{SUSY}}[\tilde{A},\tilde{\varphi},\tilde{\psi},\tilde{\bar{\psi}},\tilde{C},\tilde{\bar{C}}] = S_{\mathrm{inv}}[\tilde{A},\tilde{\varphi},\tilde{\psi},\tilde{\bar{\psi}}] + S_{\mathrm{gf}}[\tilde{A},\tilde{\varphi},\tilde{C},\tilde{\bar{C}}]$$
$$S_{\mathrm{inv}} = \frac{1}{g^2}\int \mathrm{d}^4x\left\{-\frac{1}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} - \frac{1}{2}\tilde{\mathrm{D}}_\mu\tilde{\varphi}_i\tilde{\mathrm{D}}^\mu\tilde{\varphi}_i - \frac{\mathrm{i}}{2}\tilde{\bar{\psi}}_A\tilde{\Phi}^A{}_B\tilde{\psi}^B - \frac{1}{4}(\tilde{\varphi}_i\times\tilde{\varphi}_j)^2\right\}$$
$$S_{\mathrm{gf}} = \frac{1}{g^2}\int \mathrm{d}^4x\left\{-\frac{1}{2\xi}\mathcal{G}(\tilde{A},\tilde{\varphi})^2 + g\tilde{\bar{C}}\frac{\partial\mathcal{G}(\tilde{A},\tilde{\varphi})}{\partial\tilde{A}_\mu}\tilde{\mathrm{D}}_\mu\tilde{C} + g\tilde{\bar{C}}\frac{\partial\mathcal{G}(\tilde{A},\tilde{\varphi})}{\partial\tilde{\varphi}_i}\tilde{\varphi}_i\times\tilde{C}\right\}$$

$$S_{\mathrm{SUSY}}[\tilde{A},\tilde{\varphi},\tilde{\psi},\tilde{\bar{\psi}},\tilde{C},\tilde{\bar{C}}] = S_{\mathrm{inv}}[\tilde{A},\tilde{\varphi},\tilde{\psi},\tilde{\bar{\psi}}] + S_{\mathrm{gf}}[\tilde{A},\tilde{\varphi},\tilde{C},\tilde{\bar{C}}]\,,$$
$$S_{\mathrm{inv}} = \frac{1}{g^2}\int \mathrm{d}^4x\left\{-\tfrac{1}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} - \tfrac{1}{2}\tilde{\mathrm{D}}_\mu\tilde{\varphi}_i\tilde{\mathrm{D}}^\mu\tilde{\varphi}_i - \tfrac{\mathrm{i}}{2}\tilde{\bar{\psi}}_A\tilde{\mathscr{D}}^A{}_B\tilde{\psi}^B - \tfrac{1}{4}(\tilde{\varphi}_i\times\tilde{\varphi}_j)^2\right\}$$
$$S_{\mathrm{gf}} = \frac{1}{g^2}\int \mathrm{d}^4x\left\{-\tfrac{1}{2\xi}\mathcal{G}(\tilde{A},\tilde{\varphi})^2 + g\tilde{\bar{C}}\tfrac{\partial\mathcal{G}(\tilde{A},\tilde{\varphi})}{\partial\tilde{A}_\mu}\tilde{\mathrm{D}}_\mu\tilde{C} + g\tilde{\bar{C}}\tfrac{\partial\mathcal{G}(\tilde{A},\tilde{\varphi})}{\partial\tilde{\varphi}_i}\tilde{\varphi}_i\times\tilde{C}\right\}\,,$$

$$\partial_g S_{\mathrm{SUSY}} = -\frac{1}{g^3}\{\delta_\alpha\Delta_\alpha - \sqrt{g}\,s\Delta_{\mathrm{gh}}\}$$

$$\Delta_\alpha = \frac{1}{4}\int \mathrm{d}^4x\left\{-\frac{1}{2}\tilde{F}_{\mu\nu}\gamma^{\mu\nu}\tilde{\lambda} - (\tilde{\Phi}^4_A)^\dagger\mathcal{D}^A_B\tilde{\psi}^B + \frac{1}{2}(\tilde{\Phi}^4_A)^\dagger\tilde{\Phi}^A_B\times\tilde{\psi}^B\right\}$$

$$\Delta_\alpha = \frac{1}{4}\int \mathrm{d}^4x\left\{-\tfrac{1}{2}\tilde{F}_{\mu\nu}\gamma^{\mu\nu}\tilde{\lambda} - (\tilde{\Phi}^4{}_A)^\dagger\tilde{\mathscr{D}}^A{}_B\tilde{\psi}^B + \tfrac{1}{2}(\tilde{\Phi}^4{}_A)^\dagger\tilde{\Phi}^A{}_B\times\tilde{\psi}^B\right\}\,,$$

$$\Delta_{\mathrm{gh}} = \int \mathrm{d}^4x\{\tilde{\bar{C}}\mathcal{G}(\tilde{A},\tilde{\varphi})\}$$

$$s\tilde{A}_\mu = \sqrt{g}\tilde{\mathrm{D}}_\mu\tilde{C}, \quad s\tilde{\lambda} = \sqrt{g}\tilde{\lambda}\times\tilde{C}, \quad s\tilde{\bar{\lambda}} = \sqrt{g}\tilde{\bar{\lambda}}\times\tilde{C},$$
$$s\tilde{\mathcal{D}} = \sqrt{g}\tilde{\mathcal{D}}\times\tilde{C}, \quad s\tilde{C} = -\frac{\sqrt{g}}{2}\tilde{C}\times\tilde{C}, \quad s\tilde{\bar{C}} = \frac{1}{\sqrt{g}}\frac{1}{\xi}\mathcal{G}(\tilde{A},\tilde{\varphi}),$$
$$s\tilde{\varphi}_i = \sqrt{g}\tilde{\varphi}_i\times\tilde{C}, \quad s\tilde{\psi}_I = \sqrt{g}\tilde{\psi}_I\times\tilde{C}, \quad s\tilde{F} = \sqrt{g}\tilde{F}_I\times\tilde{C}$$

$$\tilde{R}[\tilde{\mathcal{A}}] = -\mathrm{i}\Delta_\alpha[\tilde{\mathcal{A}}]\delta_\alpha + \frac{\mathrm{i}}{\sqrt{g}}\Delta_{\mathrm{gh}}[\tilde{\mathcal{A}}]s - \frac{1}{\sqrt{g}}\Delta_\alpha[\tilde{\mathcal{A}}](\delta_\alpha\Delta_{\mathrm{gh}}[\tilde{\mathcal{A}}])s,$$

$$\tilde{R}[\mathscr{A}] = -\mathrm{i}\underbrace{\Delta_\alpha[\tilde{\mathscr{A}}]\,\delta_\alpha} + \tfrac{\mathrm{i}}{\sqrt{g}}\underbrace{\Delta_{\mathrm{gh}}[\tilde{\mathscr{A}}]\,s} - \tfrac{1}{\sqrt{g}}\underbrace{\Delta_\alpha[\tilde{\mathscr{A}}]}\,\underbrace{(\delta_\alpha\Delta_{\mathrm{gh}}[\tilde{\mathscr{A}}])\,s}\,,$$

$$\overleftarrow{R}_g[\mathscr{A}] = \frac{1}{8}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\,\mathrm{tr}\{(\mathscr{C}_\Sigma)^4{}_A S^A{}_B\mathscr{A}^B{}_C\times\mathscr{A}^{*C}{}_4\} + \frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}\Pi_\Gamma{}^\Sigma\mathscr{A}_\Sigma G\frac{\partial\mathcal{G}(\mathscr{A})}{\partial A_\nu}A^{\mathrm{L}}_\nu\,,$$

$$(\mathcal{C}_\Sigma)^A{}_B = \begin{cases}\delta^A{}_B\gamma_\mu & \text{for} \quad \Sigma = \mu = 0,1,2,3\\ (c^i)^A{}_B & \text{for} \quad \Sigma = 3 + i = 4,5,\dots,9\end{cases}$$

$$\mathscr{D}_\Gamma = (\mathrm{D}_\mu,\, g\varphi_i\times\,)\ , \qquad \not{\mathscr{D}}^A{}_B = \mathscr{D}^\Sigma(\mathscr{C}_\Sigma)^A{}_B = \not{\mathrm{D}}\delta^A{}_B + g\Phi^A{}_B\times\ ,$$

$$\underbrace{\psi^A(x)\bar\psi_B(y)} = -S^A{}_B(x,y;\mathscr{A})\ , \qquad \not{\mathscr{D}}^A{}_C S^C{}_B(x,y;\mathscr{A}) = \delta^A{}_B\delta(x-y)\ ,$$

$$\mathrm{i}\underbrace{C(x)\bar C(y)} = G(x,y;\mathscr{A})\ , \qquad \frac{\partial\mathcal{G}(\mathscr{A})}{\partial\mathscr{A}_\Gamma}\mathscr{D}_\Gamma G(x,y;\mathscr{A}) = \delta(x-y)\ ,$$

$$\not{\mathscr{A}}^A{}_B = \mathscr{A}^\Sigma(\mathscr{C}_\Sigma)^A{}_B = \not{A}\delta^A{}_B + \Phi^A{}_B\ , \qquad \not{\mathscr{A}}^{*A}{}_B := \not{A}^*\delta^A{}_B + (\Phi^A{}_B)^\dagger\ ,$$

$$P_\Gamma{}^\Sigma = \delta_\Gamma{}^\Sigma - \mathcal{D}_\Gamma G\frac{\partial\mathcal{G}(\mathcal{A})}{\partial\mathcal{A}_\Sigma}$$

$$\Pi_\Gamma{}^\Sigma = P_\Gamma{}^\Sigma\big|_{g=0}$$

$$R_g[\mathscr{A}] = \frac{1}{32}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P^{(10)}{}_\Gamma{}^\Sigma\ \mathrm{tr}^{(32)}\Big\{\Gamma_\Sigma S^{(10)}\not{\mathscr{A}}\times\not{\mathscr{A}}\Big\}\ ,$$

$$\mathcal{A} = \Gamma^\Sigma\mathcal{A}_\Sigma.$$

$$S^{(10)} = -\lambda\bar\lambda$$

$$S^{(10)} = \left(\begin{array}{c|c} (\mathrm{P}^+S^A{}_B\mathrm{P}^-)_{\alpha\beta} & (\mathrm{P}^+S^{AB}\mathrm{P}^+)_{\alpha\beta} \\ \hline (\mathrm{P}^-S_{AB}\mathrm{P}^-)_{\alpha\beta} & (\mathrm{P}^-S_A{}^B\mathrm{P}^+)_{\alpha\beta}\end{array}\right)\ ,$$

$$\not{\mathscr{A}} = \Gamma^\Sigma\mathscr{A}_\Sigma = \left(\begin{array}{c|c} \not{A}_{\alpha\beta}\delta^A{}_B & (\mathrm{i}\gamma_5)_{\alpha\beta}\varphi^{AB} \\ \hline (\mathrm{i}\gamma_5)_{\alpha\beta}\varphi_{AB} & \not{A}_{\alpha\beta}\delta_A{}^B\end{array}\right)\ .$$

$$\not{\mathscr{A}}\times\not{\mathscr{A}} = \left(\begin{array}{c|c} (\not{A}\times\not{A})_{\alpha\beta}\delta^A{}_B + (\mathbb{1}_4)_{\alpha\beta}\varphi^{AC}\times\varphi_{CB} & 2(\not{A}\mathrm{i}\gamma_5)_{\alpha\beta}\times\varphi^{AB} \\ \hline 2(\mathrm{i}\gamma_5\not{A})_{\alpha\beta}\times\varphi_{AB} & (\not{A}\times\not{A})_{\alpha\beta}\delta_A{}^B + (\mathbb{1}_4)_{\alpha\beta}\varphi_{AC}\times\varphi^{CB}\end{array}\right)$$

$$\begin{aligned}
&\mathrm{tr}^{(32)}\Big\{\Gamma_\mu\ S^{(10)}\ \mathbb{1}_8\otimes(\not{A}\times\not{A})\Big\} \\
&= \mathrm{tr}^{(32)}\left(\begin{array}{c|c} (\gamma_\mu)_{\alpha\gamma}(\mathrm{P}^+S^A{}_B\mathrm{P}^-)_{\gamma\delta}(\not{A}\times\not{A})_{\delta\beta} & (\gamma_\mu)_{\alpha\gamma}(\mathrm{P}^+S^{AB}\mathrm{P}^+)_{\gamma\delta}(\not{A}\times\not{A})_{\delta\beta} \\ \hline (\gamma_\mu)_{\alpha\gamma}(\mathrm{P}^-S_{AB}\mathrm{P}^-)_{\gamma\delta}(\not{A}\times\not{A})_{\delta\beta} & (\gamma_\mu)_{\alpha\gamma}(\mathrm{P}^-S_A{}^B\mathrm{P}^+)_{\gamma\delta}(\not{A}\times\not{A})_{\delta\beta}\end{array}\right) \\
&= \mathrm{tr}^{(4)}\{\gamma_\mu\mathrm{P}^+S^A{}_A\mathrm{P}^-\not{A}\times\not{A}\}\ +\ \mathrm{tr}^{(4)}\{\gamma_\mu\mathrm{P}^-S_A{}^A\mathrm{P}^+\not{A}\times\not{A}\} = \mathrm{tr}^{(4)}\{\gamma_\mu S^A{}_A\not{A}\times\not{A}\}\ ,
\end{aligned}$$

$$\mathrm{tr}^{(32)}\Big\{\Gamma_\Sigma S^{(10)}\not{\mathscr{A}}\times\not{\mathscr{A}}\Big\} = \mathrm{tr}^{(4)}\Big\{(\mathscr{C}_\Sigma)^A{}_B S^B{}_C\not{\mathscr{A}}^C{}_D\times\not{\mathscr{A}}^{*D}{}_A\Big\}\ ,$$

$$\overleftarrow{R}_g[\mathscr{A}] = \frac{1}{32}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^A{}_B S^B{}_C\not{\mathscr{A}}^C{}_D\times\not{\mathscr{A}}^{*D}{}_A\Big\}\ .$$

$$\overleftarrow{R}_g[\mathscr{A}] = \frac{1}{8}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^4{}_B S^B{}_C\not{\mathscr{A}}^C{}_D\times\not{\mathscr{A}}^{*D}{}_4\Big\}$$

$$\overleftarrow{R}_g[\mathscr{A}] = \frac{1}{32}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^A{}_B S^B{}_C\not{\mathscr{A}}^C{}_D\times\not{\mathscr{A}}^{*D}{}_A\Big\}$$

$$R_g[\phi] = \int\ \mathrm{d}xK[\phi;x]\frac{\delta}{\delta\phi(x)}$$

$$\overleftarrow{R}_g[\mathscr{A}] = \frac{\overleftarrow{\delta}}{\delta \mathscr{A}_\Gamma} K_\Gamma \ ,$$

$$(\partial_g + R_g[\phi])S_g^{\mathrm{b}}[\phi] = 0, (\partial_g + R_g[\phi])S_g^{\mathrm{f}}[\phi] = \int \mathrm{d}x \frac{\delta K[\phi; x]}{\delta \phi(x)}, (\partial_g + R_g[\phi])\mathcal{G}(\phi) = 0$$

$$R_g' := pR_g^{(1)} + qR_g^{(2)} \text{ with } p, q \in \mathbb{R} \text{ and } p + q = 1$$

$$\overleftarrow{R}_g[\mathscr{A}] = \frac{1}{32} \frac{\overleftarrow{\delta}}{\delta \mathscr{A}_\Gamma} P_\Gamma{}^\Sigma \ \mathrm{tr}\Big\{ (\mathscr{C}_\Sigma)^A{}_B S^B{}_C \mathscr{A}^C{}_D \times \mathscr{A}^{*D}{}_E (\delta^E{}_A + \mathrm{L}^E{}_A) \Big\}$$

$$\mathrm{L} = \mathrm{diag}(-1, -1, -1, +3) \text{ and } \mathrm{L} = 0,$$

$$\Big\{ \mathrm{L} = \mathrm{diag}(q_1, q_2, q_3, q_4) \text{ with } \sum_i q_i = 0 \Big\} =: \mathfrak{h},$$

$$\mathrm{L} = L\mathrm{P}^- + L^*\mathrm{P}^+$$

$$L \longrightarrow ULU^\dagger, \text{ with } U \in \mathrm{SU}(4)$$

$$\mathrm{tr}L = \sum q_i = 0, \mathrm{tr}L^2 = \sum q_i^2, \mathrm{tr}L^3 = \sum q_i^3, \mathrm{tr}L^4 = \sum q_i^4$$

$$\mathrm{SU}(4) / \mathrm{U}(1)^3$$

$$T_g \mathcal{A}_\Sigma = \mathcal{A}_\Sigma - gC^\Theta \mathcal{A}_\Sigma \mathcal{A}_\Theta + \frac{3}{2} g^2 C^\Theta \mathcal{A}^\Gamma C_{[\Sigma} \mathcal{A}_\Theta \mathcal{A}_{\Gamma]} + \mathcal{O}(g^3),$$

$$T_g A_\mu = A_\mu - gC^\rho A_\mu A_\rho + \frac{3}{2} g^2 C^\rho A^\lambda C_{[\mu} A_\rho A_{\lambda]} + g^2 C^\rho \varphi_i C_{[\mu} A_{\rho]} \varphi_i + \mathcal{O}(g^3)$$
$$T_g \varphi_i = \varphi_i - gC^\rho \varphi_i A_\rho + g^2 C^{[\rho} A^{\lambda]} C_\lambda \varphi_i A_\rho + \frac{1}{2} g^2 C^\rho \varphi_j C_\rho \varphi_j \varphi_i + \mathcal{O}(g^3)$$

$$L = \mathrm{diag}(+3, -1, -1, -1), \ldots, \mathrm{diag}(-1, -1, -1, +3)$$

$$\overleftarrow{R}_g{}^{(A)}[\mathscr{A}] = \frac{1}{8} \frac{\overleftarrow{\delta}}{\delta \mathscr{A}_\Gamma} P_\Gamma{}^\Sigma \ \mathrm{tr}\Big\{ (\mathscr{C}_\Sigma)^A{}_B S^B{}_C \mathscr{A}^C{}_D \times \mathscr{A}^{*D}{}_A \Big\}$$

$$R_g[\mathcal{A}] = \sum_{k=1}^{\infty} g^{k-1} \mathrm{R}_k[\mathcal{A}] = \mathrm{R}_1[\mathcal{A}] + g\mathrm{R}_2[\mathcal{A}] + g^2 \mathrm{R}_3[\mathcal{A}] + \cdots$$

$$T_g \mathcal{A} = \mathcal{A} - g\mathrm{R}_1 \mathcal{A} - \frac{1}{2} g^2 (\mathrm{R}_2 - \mathrm{R}_1^2) \mathcal{A} + \mathcal{O}(g^3).$$

$$\begin{aligned} T_g^{(4)} A_\mu =& A_\mu - gC^\rho A_\mu A_\rho + \frac{3}{2} g^2 C^\rho A^\lambda C_{[\mu} A_\rho A_{\lambda]} + g^2 C^\rho \varphi_i C_{[\mu} A_{\rho]} \varphi_i \\ & - \frac{1}{2} g^2 \Pi_\mu{}^\nu \epsilon_{\nu\lambda\rho\sigma} \sum_{J=1}^{3} \left[C^\lambda \varphi_J C^\rho \varphi_{J+3} A^\sigma - C^\lambda \varphi_{J+3} C^\rho \varphi_J A^\sigma + C^\lambda A^\rho C^\sigma \varphi_{J+3} \varphi_J \right] + \mathcal{O}(g^3) \\ T_g^{(K)} A_\mu =& A_\mu - gC^\rho A_\mu A_\rho + \frac{3}{2} g^2 C^\rho A^\lambda C_{[\mu} A_\rho A_{\lambda]} + g^2 C^\rho \varphi_i C_{[\mu} A_{\rho]} \varphi_i \\ & + \frac{1}{2} g^2 \Pi_\mu{}^\nu \epsilon_{\nu\lambda\rho\sigma} \sum_{J=1}^{3} (-)^{\delta_{KJ}} \left[C^\lambda \varphi_J C^\rho \varphi_{J+3} A^\sigma - C^\lambda \varphi_{J+3} C^\rho \varphi_J A^\sigma + C^\lambda A^\rho C^\sigma \varphi_{J+3} \varphi_J \right] + \mathcal{O}(g^3) \end{aligned}$$

$$
\begin{aligned}
T_g^{(4)}\varphi_I =& \varphi_I - gC^\rho\varphi_I A_\rho + g^2 C^{[\rho}A^{\lambda]}C_\lambda\varphi_I A_\rho + \frac{1}{2}g^2 C^\rho\varphi_j C_\rho\varphi_j\varphi_I \\
&-\frac{1}{4}g^2\epsilon_{\mu\nu\rho\lambda}\left[C^\mu\varphi_{I+3}C^\nu A^\rho A^\lambda + 2C^\mu A^\nu C^\rho\varphi_{I+3}A^\lambda\right] \\
&-\frac{1}{2}g^2 C^\rho\sum_{J=1}^{3}\left[\varphi_{I+3}C_\rho\varphi_{J+3}\varphi_J + \varphi_J C_\rho\varphi_{I+3}\varphi_{J+3} - \varphi_{J+3}C_\rho\varphi_{I+3}\varphi_J\right] + \mathcal{O}(g^3)
\end{aligned}
$$

$$
\begin{aligned}
T_g^{(K)}\varphi_I =& \varphi_I - gC^\rho\varphi_I A_\rho + g^2 C^{[\rho}A^{\lambda]}C_\lambda\varphi_I A_\rho + \frac{1}{2}g^2 C^\rho\varphi_j C_\rho\varphi_j\varphi_I \\
&+\frac{1}{4}g^2\epsilon_{\mu\nu\rho\lambda}(-)^{\delta_{IK}}\left[C^\mu\varphi_{I+3}C^\nu A^\rho A^\lambda + 2C^\mu A^\nu C^\rho\varphi_{I+3}A^\lambda\right] \\
&-\frac{1}{2}g^2 C^\rho(-)^{\delta_{IK}}\sum_{J=1}^{3}\left[\varphi_{I+3}C_\rho\varphi_{J+3}\varphi_J + \varphi_J C_\rho\varphi_{I+3}\varphi_{J+3} - \varphi_{J+3}C_\rho\varphi_{I+3}\varphi_J\right] \\
&+g^2 C^\rho\left[\varphi_{I+3}C_\rho\varphi_{K+3}\varphi_K + \varphi_K C_\rho\varphi_{I+3}\varphi_{K+3} - \varphi_{K+3}C_\rho\varphi_{I+3}\varphi_K\right] + \mathcal{O}(g^3)
\end{aligned}
$$

$$R_g := \frac{1}{4}\left(R_g^{(1)} + R_g^{(2)} + R_g^{(3)} + R_g^{(4)}\right),$$

$$\overleftarrow{R}_g[\mathscr{A}] = \frac{1}{32}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\left\{(\mathscr{C}_\Sigma)^A{}_B S^B{}_C \not{\mathscr{A}}^C{}_D \times \not{\mathscr{A}}^{*D}{}_E(\delta^E{}_A + \mathrm{L}^E{}_A)\right\},$$

$$\mathrm{L} = L\mathrm{P}^- + L^*\mathrm{P}^+$$

$$
\begin{aligned}
V =& \theta\sigma^\mu\bar\theta A_\mu(x) - \mathrm{i}\theta^2\bar\theta\bar\lambda(x) + \mathrm{i}\bar\theta^2\theta\lambda(x) - \frac{1}{2}\theta^2\bar\theta^2\mathcal{D}(x) \\
=& \theta\sigma^\mu\bar\theta A_\mu(y) - \mathrm{i}\theta^2\bar\theta\bar\lambda(y) + \mathrm{i}\bar\theta^2\theta\lambda(y) - \frac{1}{2}\theta^2\bar\theta^2\left[\mathcal{D}(y) - \mathrm{i}\mathrm{D}^\mu A_\mu(y)\right] \\
=& \theta\sigma^\mu\bar\theta A_\mu(y^\dagger) - \mathrm{i}\theta^2\bar\theta\bar\lambda(y^\dagger) + \mathrm{i}\bar\theta^2\theta\lambda(y^\dagger) - \frac{1}{2}\theta^2\bar\theta^2\left[\mathcal{D}(y^\dagger) + \mathrm{i}\mathrm{D}^\mu A_\mu(y^\dagger)\right]
\end{aligned}
$$

$$V^2 = -\frac{1}{2}\theta^2\bar\theta^2 A_\mu A^\mu$$

$$\mathrm{e}^{2V} = 1 + 2V + 2V^2$$

$$
\begin{aligned}
W_\alpha &= -\tfrac{1}{4}\bar{\mathrm{D}}\bar{\mathrm{D}}\mathrm{e}^{-2V}\mathrm{D}_\alpha\mathrm{e}^{2V} = +2\mathrm{i}\lambda_\alpha(y) - 2\left[\delta_\alpha{}^\beta\mathcal{D}(y) - \mathrm{i}\sigma^{\mu\nu}{}_\alpha{}^\beta F_{\mu\nu}(y)\right]\theta_\beta - 2\theta^2\not{\mathrm{D}}_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}(y), \\
\bar W^{\dot\alpha} &= -\tfrac{1}{4}\mathrm{D}\mathrm{D}\mathrm{e}^{-2V}\bar{\mathrm{D}}^{\dot\alpha}\mathrm{e}^{2V} = -2\mathrm{i}\bar\lambda^{\dot\alpha}(y^\dagger) - 2\left[\delta^{\dot\alpha}{}_{\dot\beta}\mathcal{D}(y^\dagger) + \mathrm{i}\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}F_{\mu\nu}(y^\dagger)\right]\bar\theta^{\dot\beta} + 2\bar\theta^2\bar{\not{\mathrm{D}}}^{\dot\alpha\alpha}\lambda_\alpha(y^\dagger),
\end{aligned}
$$

$$\Phi_I = \phi_I(y) + \sqrt{2}\theta\psi_I(y) + \theta^2 F_I(y), \Phi_I^\dagger = \phi_I^\dagger(y^\dagger) + \sqrt{2}\bar\theta\bar\psi_I(y^\dagger) + \bar\theta^2 F_I^\dagger(y^\dagger)$$

$$
\begin{aligned}
\Phi_I &= \phi_I(x) + \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi_I(x) + \frac{1}{4}\theta^2\bar\theta^2\,\Box\,\phi_I(x) + \sqrt{2}\theta\psi_I(x) - \frac{\mathrm{i}}{\sqrt{2}}\theta^2\partial_\mu\psi_I(x)\sigma^\mu\bar\theta + \theta^2 F_I(x) \\
\Phi_I^\dagger &= \phi_I^\dagger(x) - \mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu\phi_I^\dagger(x) + \frac{1}{4}\theta^2\bar\theta^2\,\Box\,\phi_I^\dagger(x) + \sqrt{2}\bar\theta\bar\psi_I(x) + \frac{\mathrm{i}}{\sqrt{2}}\bar\theta^2\theta\sigma^\mu\partial_\mu\bar\psi_I(x) + \bar\theta^2 F_I^\dagger(x)
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{4}W^\alpha W_\alpha &= -\lambda^2 + \left[-2\mathrm{i}\mathcal{D}\lambda - 2F_{\mu\nu}\lambda\sigma^{\mu\nu}\right]\theta + \left[-2\mathrm{i}\lambda\sigma^\mu\mathrm{D}_\mu\bar\lambda - \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \mathcal{D}^2 + \frac{\mathrm{i}}{4}F^{\mu\nu}F^{\rho\lambda}\epsilon_{\mu\nu\rho\lambda}\right]\theta^2 \\
\frac{1}{4}\bar W_{\dot\alpha}\bar W^{\dot\alpha} &= -\bar\lambda^2 + \left[+2\mathrm{i}\mathcal{D}\bar\lambda - 2F_{\mu\nu}\bar\lambda\bar\sigma^{\mu\nu}\right]\bar\theta + \left[+2\mathrm{i}\mathrm{D}_\mu\lambda\sigma^\mu\bar\lambda - \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \mathcal{D}^2 - \frac{\mathrm{i}}{4}F^{\mu\nu}F^{\rho\lambda}\epsilon_{\mu\nu\rho\lambda}\right]\bar\theta^2
\end{aligned}
$$

$$\epsilon_{IJK}\mathrm{tr}\Phi_I[\Phi_J,\Phi_K] = \mathrm{i}\epsilon_{IJK}f^{abc}\left[\phi_I^a\phi_J^b\phi_K^c + 3\sqrt{2}\theta\psi_I^a\phi_J^b\phi_K^c + 3\theta^2\left(F_I^a\phi_J^b\phi_K^c - \phi_I^a\psi_J^b\psi_K^c\right)\right],$$

$$\frac{1}{n_c}\mathrm{tr}e^{-2V}\Phi_I^\dagger e^{2V}\Phi_I = \Phi_I^{a\dagger}\Phi_I^a + \frac{2}{n_c}\mathrm{tr}[T^a,T^b]T^c\Phi_I^{a\dagger}V^b\Phi_I^c + \frac{2}{n_c}\mathrm{tr}[T^a,T^b][T^c,T^d]\Phi_I^{a\dagger}V^bV^c\Phi_I^d$$

$$= \cdots + \theta^2\bar{\theta}[-\mathrm{i}\sqrt{2}\bar{\sigma}^\mu\psi_I^a\mathrm{D}_\mu\phi_I^{a\dagger} + \sqrt{2}F_I^a\bar{\psi}_I^a + 2f^{abc}\phi_I^{a\dagger}\bar{\lambda}^b\phi_I^c]$$
$$+\bar{\theta}^2\theta[-\mathrm{i}\sqrt{2}\sigma^\mu\bar{\psi}_I^a\mathrm{D}_\mu\phi_I^a + \sqrt{2}F_I^{a\dagger}\psi_I^a - 2f^{abc}\phi_I^{a\dagger}\lambda^b\phi_I^c]$$
$$+\theta^2\bar{\theta}^2[-\mathrm{D}_\mu\phi_I^{a\dagger}\mathrm{D}^\mu\phi_I^a + F_I^{a\dagger}F_I^a + \mathrm{i}\mathrm{D}_\mu\bar{\psi}_I^a\bar{\sigma}^\mu\psi_I$$
$$-f^{abc}(\mathrm{i}_I^{a\dagger}\mathcal{D}^b\phi_I^c - \sqrt{2}\phi_I^{a\dagger}\lambda^b\psi_I^c + \sqrt{2}\bar{\psi}_I^a\bar{\lambda}^b\phi_I^c)]$$
$$+ \text{total derivatives}$$

$$[T^a,T^b] = \mathrm{i}f^{abc}T^c, \mathrm{tr}T^aT^b = n_c\delta^{ab}$$

$$g^2\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^aF^{a\mu\nu} - \mathrm{i}\lambda^a\sigma^\mu\mathrm{D}_\mu\bar{\lambda}^a + \frac{1}{2}\mathcal{D}^2 - \frac{1}{\sqrt{2}}\epsilon_{IJK}f^{abc}(F_I^a\phi_J^b\phi_K^c + F_I^{a\dagger}\phi_J^{b\dagger}\phi_K^{c\dagger})$$
$$-\mathrm{D}_\mu\phi_I^{a\dagger}\mathrm{D}^\mu\phi_I^a - \mathrm{i}\psi_I^a\sigma^\mu\mathrm{D}_\mu\bar{\psi}_I + F_I^{a\dagger}F_I^a + \frac{1}{\sqrt{2}}\epsilon_{IJK}f^{abc}(\phi_I^a\psi_J^b\psi_K^c + \phi_I^{a\dagger}\bar{\psi}_J^b\bar{\psi}_K^c)$$
$$-\sqrt{2}f^{abc}(\psi_I^a\lambda^b\phi_I^{c\dagger} + \bar{\psi}_I^a\bar{\lambda}^b\phi_I^c) - \mathrm{i}f^{abc}\phi_I^{a\dagger}\mathcal{D}^b\phi_I^c$$

$$\delta\phi_I = \sqrt{2}\theta\psi_I, \delta\psi_I = \mathrm{i}\sqrt{2}\sigma^\mu\bar{\theta}\mathrm{D}_\mu\phi_I + \sqrt{2}\theta F_I, \delta F_I = \mathrm{i}\sqrt{2}\bar{\theta}\bar{\sigma}^\mu\mathrm{D}_\mu\psi_I - 2\phi_I\times\bar{\lambda}\bar{\theta}$$
$$\delta A^\mu = -\mathrm{i}\bar{\lambda}\bar{\sigma}^\mu\theta + \mathrm{i}\bar{\theta}\bar{\sigma}^\mu\lambda, \delta\lambda = \sigma^{\mu\nu}\theta F_{\mu\nu} + \mathrm{i}\theta\mathcal{D}, \delta\mathcal{D} = -\theta\sigma^\mu\mathrm{D}_\mu\bar{\lambda} - \mathrm{D}_\mu\lambda\sigma^\mu\bar{\theta}$$

$$\lambda^{(\mathrm{M})} = \begin{pmatrix}\lambda_\alpha\\ \bar{\lambda}^{\dot{\alpha}}\end{pmatrix}, \bar{\lambda}^{(\mathrm{M})} = (\lambda^\alpha, \bar{\lambda}_{\dot{\alpha}}), \alpha = \begin{pmatrix}\theta_\alpha\\ \bar{\theta}^{\dot{\alpha}}\end{pmatrix}, \bar{\alpha} = (\theta^\alpha, \bar{\theta}_{\dot{\alpha}})$$
$$\gamma_\mu = \begin{pmatrix}0 & \sigma_\mu\\ \bar{\sigma}_\mu & 0\end{pmatrix}, \gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}, \text{ etc.}$$

$$\bar{\lambda}^{(\mathrm{M})}\lambda^{(\mathrm{M})} = \lambda\lambda + \bar{\lambda}\bar{\lambda}, \bar{\lambda}^{(\mathrm{M})}\mathrm{i}\gamma_5\lambda^{(\mathrm{M})} = \lambda\lambda - \bar{\lambda}\bar{\lambda}$$
$$\bar{\lambda}^{(\mathrm{M})}\gamma^\mu\lambda^{(\mathrm{M})} = \lambda\sigma^\mu\bar{\lambda} + \bar{\lambda}\bar{\sigma}^\mu\lambda = 2\lambda\sigma^\mu\bar{\lambda}, \frac{1}{2}\bar{\lambda}^{(\mathrm{M})}\gamma^{\mu\nu}\alpha = \lambda\sigma^{\mu\nu}\theta + \bar{\lambda}\bar{\sigma}^{\mu\nu}\bar{\theta},$$

$$\mathrm{P}^\pm = \frac{1}{2}(1\pm\mathrm{i}\gamma_5), \bar{\lambda}^{(\mathrm{M})}\mathrm{P}^+\lambda^{(\mathrm{M})} = \lambda\lambda, \bar{\lambda}^{(\mathrm{M})}\mathrm{P}^-\lambda^{(\mathrm{M})} = \bar{\lambda}\bar{\lambda}$$

$$\mathring{\Delta} = \bar{\alpha}\Big\{-\mathcal{D}\gamma_5\lambda - \frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\lambda + 2\epsilon_{IJK}f^{abc}[\mathrm{P}^+\psi_I^a\phi_J^b\phi_K^c + \mathrm{P}^-\psi_I^a\phi_J^{b\dagger}\phi_K^{c\dagger}] + 2\mathrm{i}f^{abc}\gamma_5\phi_I^{a\dagger}\lambda^b\phi_I^c$$
$$+\mathrm{i}\sqrt{2}[\gamma^\mu\mathrm{P}^-\psi_I^a\mathrm{D}_\mu\phi_I^a + \gamma^\mu\mathrm{P}^+\psi_I^a\mathrm{D}_\mu\phi_I^{a\dagger}] - \sqrt{2}[\mathrm{P}^+\psi_I^aF_I^{a\dagger} + \mathrm{P}^-\psi_I^aF_I^a]\Big\}$$

$$\bar{\chi}\xi = \bar{\xi}\chi, \bar{\chi}\gamma^\mu\xi = -\bar{\xi}\gamma^\mu\chi, \bar{\chi}\gamma_5\xi = \bar{\xi}\gamma_5\chi, \bar{\chi}\gamma^\mu\gamma_5\xi = \bar{\xi}\gamma^\mu\gamma_5\chi$$
$$\bar{\chi}\gamma^{\mu\nu}\xi = -\bar{\xi}\gamma^{\mu\nu}\chi, \bar{\chi}\gamma^{\mu\nu}\gamma_5\xi = -\bar{\xi}\gamma^{\mu\nu}\gamma_5\chi, \bar{\chi}\gamma^{\rho\lambda}\gamma_\mu\xi = \bar{\xi}\gamma_\mu\gamma^{\rho\lambda}\chi$$

$$\frac{1}{4}\delta\mathring{\Delta}\Big|_{\bar{\alpha}\alpha} = g^2\mathcal{L}$$

$$4\xi\bar{\chi} = -(\bar{\chi}\xi) + \gamma_\mu(\bar{\chi}\gamma^\mu\xi) + \frac{1}{2}\gamma_{\mu\nu}(\bar{\chi}\gamma^{\mu\nu}\xi) + \gamma_5\gamma_\mu(\bar{\chi}\gamma_5\gamma^\mu\xi) + \gamma_5(\bar{\chi}\gamma_5\xi)$$

$$\{Q_\alpha, \bar{Q}_\beta\} = 2(\gamma^\mu)_{\alpha\beta}P_\mu = -2\mathrm{i}(\gamma^\mu)_{\alpha\beta}\partial_\mu,$$

$$[\delta^{(1)}, \delta^{(2)}] = [\bar{\alpha}_{1\alpha}Q_\alpha, \bar{Q}_\beta\alpha_{2\beta}] = \bar{\alpha}_{1\alpha}\{Q_\alpha, \bar{Q}_\beta\}\alpha_{2\beta},$$

$$\{Q_\alpha, \bar{Q}_\beta\} = -2\mathrm{i}(\gamma^\mu)_{\alpha\beta}\partial_\mu - [\omega,\cdot]_{\alpha\beta} + G_{\alpha\beta}(A),$$

$$\bar{\chi}\delta\lambda = \bar{\chi}_\beta M_{\beta\alpha}\alpha_\alpha = \bar{\chi}_\beta\delta_\alpha\alpha_\alpha\lambda_\beta = -\bar{\chi}_\beta\delta_\alpha\lambda_\beta\alpha_\alpha \Rightarrow \delta_\alpha\lambda_\beta = -M_{\beta\alpha},$$

$$\delta_\alpha X[\tilde{\mathcal{A}}] = -\mathrm{i}\int \mathrm{d}^4x\left(\tilde{\bar{\psi}}_4\gamma_\mu\frac{\delta}{\delta\tilde{A}_\mu} + \tilde{\bar{\psi}}_J(c^i)^J{}_4\frac{\delta}{\delta\tilde{\varphi}_i}\right)_\alpha X[\tilde{\mathcal{A}}]$$
$$= -\mathrm{i}\int \mathrm{d}^4x\left(\tilde{\bar{\psi}}_A(\hat{\mathcal{C}}_\Sigma)^A{}_4\frac{\delta}{\delta\tilde{\mathcal{A}}_\Sigma}\right)_\alpha X[\tilde{\mathcal{A}}]$$

$$(\hat{\mathcal{C}}_\Sigma)^A{}_4 = \begin{cases}\delta^A{}_4\gamma_\mu & \text{for } \Sigma = \mu = 0,1,2,3\\ (c^i)^A{}_4 & \text{for } \Sigma = 3+i = 4,5,\dots,9'\end{cases}$$

$$\delta_\alpha^{(4)}\tilde{\mathcal{A}}_\Sigma = -\mathrm{i}\left(\tilde{\bar{\psi}}_A(\hat{\mathcal{C}}_\Sigma)^A{}_4\right)_\alpha,$$

$$sX[\tilde{\mathcal{A}}] = \sqrt{g}\int \mathrm{d}^4x\tilde{\mathcal{D}}_\Gamma\tilde{C}\frac{\delta}{\delta\tilde{\mathcal{A}}_\Gamma}X[\tilde{\mathcal{A}}],$$

$$\tilde{\psi}^A(x)\tilde{\bar{\psi}}_B(y) = -\tilde{S}^A{}_B(x,y;\tilde{\mathscr{A}}) , \qquad \tilde{\mathscr{D}}^A{}_C\tilde{S}^C{}_B(x,y;\tilde{\mathscr{A}}) = \delta^A{}_B\delta(x-y) ,$$

$$\mathrm{i}\tilde{C}(x)\tilde{\bar{C}}(y) = \tilde{G}(x,y;\tilde{\mathscr{A}}) , \qquad \frac{\partial\mathcal{G}(\tilde{\mathscr{A}})}{\partial\tilde{\mathscr{A}}_\Gamma}\tilde{\mathscr{D}}_\Gamma\tilde{G}(x,y;\tilde{\mathscr{A}}) = \delta(x-y) ,$$

$$\overleftarrow{\tilde{R}}[\tilde{\mathscr{A}}] = \frac{\overleftarrow{\delta}}{\delta\tilde{\mathscr{A}}_\Gamma}\tilde{P}_\Gamma{}^\Sigma\tilde{R}_\Sigma + \frac{\overleftarrow{\delta}}{\delta\tilde{\mathscr{A}}_\Gamma}\tilde{\mathscr{D}}_\Gamma\tilde{G}\,\mathcal{G}(\tilde{\mathscr{A}}) ,$$

$$\tilde{P}_\Gamma{}^\Sigma = \delta_\Gamma{}^\Sigma - \tilde{\mathcal{D}}_\Gamma\tilde{G}\frac{\partial\mathcal{G}(\tilde{\mathcal{A}})}{\partial\mathcal{A}_\Sigma},$$

$$\tilde{R}_\Sigma = -\frac{1}{4}\mathrm{tr}\left\{\left[\frac{1}{2}\tilde{F}_{\mu\nu}\gamma^{\mu\nu}\tilde{S}_C^4 + (\tilde{\Phi}_A^4)^\dagger\Phi_B^A{}_B\tilde{S}_C^B - \frac{1}{2}(\tilde{\Phi}_A^4)^\dagger\tilde{\Phi}_B^A\times\tilde{S}_C^B\right](\hat{\mathcal{C}}_\Sigma)^C{}_4\right\},$$

$$R_g[\mathcal{A}] = \frac{1}{g}(\tilde{R}[\tilde{\mathcal{A}}] - E) \text{ with } E = \tilde{\mathcal{A}}_\Gamma\frac{\delta}{\delta\tilde{\mathcal{A}}_\Gamma}$$

$$\gamma^{\rho\lambda}\tilde{F}_{\rho\lambda} = 2\tilde{\not{D}}\tilde{\not{A}} + 2\partial\cdot\tilde{A} - \tilde{\not{A}}\times\tilde{\not{A}} ,$$
$$\tilde{\not{D}}\tilde{S}^4{}_C = \delta^4{}_C - \tilde{\Phi}^4{}_B\times\tilde{S}^B{}_C ,$$

$$(\tilde{\Phi}^4{}_A)^\dagger(\hat{\mathcal{C}}_\Sigma)^A{}_4 = \begin{cases} 0 & \text{for} \quad \Sigma = \mu \\ -\mathbb{1}_4\tilde{\varphi}_I - \gamma_5\tilde{\varphi}_{I+3} & \text{for} \quad \Sigma = 3+I. \\ +\gamma_5\tilde{\varphi}_I - \mathbb{1}_4\tilde{\varphi}_{I+3} & \text{for} \quad \Sigma = 6+I \end{cases}$$

$$\tilde{R}_\Sigma = \tilde{\mathcal{A}}_\Sigma - \frac{1}{4}\mathrm{tr}\left\{(\mathcal{C}_\Sigma)^4{}_A\left[\frac{1}{2}\tilde{S}^A{}_4(2\partial\cdot\tilde{A} - \tilde{A}\times\tilde{A}) - \tilde{S}^A{}_B\tilde{\Phi}^B{}_4\times\tilde{A} - \frac{1}{2}\tilde{S}^A{}_B\tilde{\Phi}^B{}_C\times(\tilde{\Phi}^C{}_4)^\dagger\right]\right\}$$

$$\bar{\psi} = \psi^\dagger\gamma_0, (\gamma_0)^2 = \mathbb{1}_4, (\tilde{S}_B^A)^\dagger = \gamma_0\tilde{S}_A^B\gamma_0, (\mathcal{C}_\Sigma)^4{}_A := \gamma_0\left((\hat{\mathcal{C}}_\Sigma)^A{}_4\right)^\dagger\gamma_0,$$
$$\gamma_\mu^\dagger = \gamma_0\gamma_\mu\gamma_0, \gamma_5^\dagger = \gamma_0\gamma_5\gamma_0 = -\gamma_5, \gamma_0\tilde{\Phi}^A{}_B\gamma_0 = (\tilde{\Phi}^B{}_A)^\dagger.$$

$$(\mathcal{C}_\Sigma)^4{}_A = \begin{cases} \delta^4{}_A\gamma_\mu & \text{for } \Sigma = \mu = 0,1,2,3 \\ (c^i)^4{}_A & \text{for } \Sigma = 3+i = 4,5,\ldots,9 \end{cases}.$$

$$\overleftarrow{R}_g[\mathscr{A}] = -\frac{1}{4}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\left\{(\mathscr{C}_\Sigma)^4{}_A\left[\tfrac{1}{2}S^A{}_4(\tfrac{2}{g}\partial\cdot A - \not{A}\times\not{A}) - S^A{}_B\Phi^B{}_4\times\not{A} - \tfrac{1}{2}S^A{}_B\Phi^B{}_C\times(\Phi^C{}_4)^\dagger\right]\right\}$$

$$\mathscr{D}^A{}_B = \not{D}\delta^A{}_B + g\Phi^A{}_B\times , \qquad \text{with} \qquad \mathrm{D}_\mu = \partial_\mu + gA_\mu\times ,$$

$$S^A{}_B = S_0\delta^A{}_B - gS_0\mathscr{A}^A{}_CS^C{}_B = \sum_{l=0}^{\infty}(-gS_0\mathscr{A})^{l\,A}{}_BS_0 ,$$

$$\mathscr{A}^A{}_B = \not{A}\delta^A{}_B + \Phi^A{}_B$$

$$2S_0\partial\cdot A = -2\not{A}^{\mathrm{L}} = \not{A}^* - \not{A}$$

$$
\begin{aligned}
& -\tfrac{1}{4}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\{(\mathscr{C}_\Sigma)^4{}_A\big[\tfrac{1}{2}S^A{}_4(\tfrac{2}{g}\partial\cdot A-\not{A}\times\not{A})\big]\} \\
= & -\tfrac{1}{8}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^4{}_A\Big[\sum_{l=0}^{\infty}(-gS_0\not{\mathscr{A}})^l{}^A{}_4S_0(\tfrac{2}{g}\partial\cdot A-\not{A}\times\not{A})\Big]\Big\} \\
= & -\tfrac{1}{8}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^4{}_A\Big[\tfrac{1}{g}\sum_{l=0}^{\infty}(-gS_0\not{\mathscr{A}})^l{}^A{}_4S_0(\not{A}^*-\not{A})-\sum_{l=0}^{\infty}(-gS_0\not{\mathscr{A}})^l{}_{A4}S_0\not{A}\times\not{A}\Big]\Big\} \\
= & -\tfrac{1}{8}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^4{}_A\Big[\tfrac{1}{g}\sum_{l=0}^{\infty}(-gS_0\not{\mathscr{A}})^l{}^A{}_4S_0(\not{A}^*-\not{A})-\sum_{l=0}^{\infty}(-gS_0\not{\mathscr{A}})^l{}^A{}_BS_0(\not{\mathscr{A}}^B{}_4-\Phi^B{}_4)\times\not{A}\Big]\Big\} \\
= & -\tfrac{1}{8}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^4{}_A\Big[\tfrac{1}{g}\sum_{l=0}^{\infty}(-gS_0\not{\mathscr{A}})^l{}^A{}_4S_0(\not{A}^*-\not{A})+\tfrac{1}{g}\sum_{l=1}^{\infty}(-gS_0\not{\mathscr{A}})^l{}^A{}_4\times\not{A}+S^A{}_B\Phi^B{}_4\times\not{A}\Big]\Big\} \\
= & -\tfrac{1}{8}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^4{}_A\Big[\tfrac{1}{g}\delta^A{}_4S_0(\not{A}^*-\not{A})+\tfrac{1}{g}\sum_{l=1}^{\infty}(-gS_0\not{\mathscr{A}})^l{}^A{}_4\times\not{A}^*+S^A{}_B\Phi^B{}_4\times\not{A}\Big]\Big\} \\
= & -\tfrac{1}{8}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^4{}_AS^A{}_B\Big[-\not{\mathscr{A}}^B{}_4\times\not{A}^*+\Phi^B{}_4\times\not{A}\Big]\Big\}-\tfrac{1}{g}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\nu A^{\mathrm{L}}_\nu \\
= & -\tfrac{1}{4}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^4{}_AS^A{}_B\Big[-\tfrac{1}{2}\not{\mathscr{A}}^B{}_4\times\not{A}^*+\tfrac{1}{2}\Phi^B{}_4\times\not{A}\Big]\Big\}+\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}\Pi_\Gamma{}^\Sigma\mathscr{A}_\Sigma G\frac{\partial\mathcal{G}(\mathscr{A})}{\partial A_\nu}A^{\mathrm{L}}_\nu\ ,
\end{aligned}
$$

$$
\overleftarrow{R}_g[\mathscr{A}]=\tfrac{1}{8}\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}P_\Gamma{}^\Sigma\ \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^4{}_AS^A{}_B\big[\not{\mathscr{A}}^B{}_4\times\not{A}^*+\Phi^B{}_4\times\not{A}+\Phi^B{}_C\times\Phi^{\dagger C}{}_4\big]\Big\}+\frac{\overleftarrow{\delta}}{\delta\mathscr{A}_\Gamma}\Pi_\Gamma{}^\Sigma\mathscr{A}_\Sigma G\frac{\partial\mathcal{G}(\mathscr{A})}{\partial A_\nu}A^{\mathrm{L}}_\nu\ ,
$$

$$
\not{\mathscr{A}}^{*A}{}_B=\not{A}^*\delta^A{}_B+(\Phi^A{}_B)^\dagger\ ,
$$

$$
(\partial_g+R_g)S_g^{\mathrm{b}}[\mathcal{A}]=0
$$

$$
S_g^{\mathrm{b}}[\mathcal{A}]=\int\ \mathrm{d}^4x\Big\{-\frac{1}{4}\mathcal{F}^{\Sigma\Theta}\mathcal{F}_{\Sigma\Theta}\Big\}
$$

$$
\mathcal{F}_{\Sigma\Theta}=\partial_\Sigma\mathcal{A}_\Theta-\partial_\Theta\mathcal{A}_\Sigma+g\mathcal{A}_\Sigma\times\mathcal{A}_\Theta,\partial_{3+i}=0,\mathcal{A}_\mu=A_\mu,\mathcal{A}_{3+i}=\varphi_i
$$

$$
\partial_gS_g^{\mathrm{b}}=-\frac{1}{2}\mathcal{F}^{\Sigma\Theta}\mathcal{A}_\Sigma\times\mathcal{A}_\Theta\ \text{ and }\ \frac{\delta S_g^{\mathrm{b}}}{\mathcal{A}_\Sigma}=\mathcal{D}_\Theta\mathcal{F}^{\Theta\Sigma},
$$

$$
(\partial_g+R_g)S_g^{\mathrm{b}}[\mathscr{A}]=-\tfrac{1}{2}\mathcal{F}^{\Sigma\Theta}\mathscr{A}_\Sigma\times\mathscr{A}_\Theta+\tfrac{1}{8}\mathscr{D}_\Theta\mathcal{F}^{\Theta\Sigma}\ \mathrm{tr}\{(\mathscr{C}_\Sigma)^4{}_BS^B{}_C\not{\mathscr{A}}^C{}_D\times\not{\mathscr{A}}^{*D}{}_4\}
$$

$$
\begin{aligned}
&\tfrac{1}{4}\mathrm{tr}\{(\mathscr{C}_\Sigma)^4{}_B(\bar{\mathscr{C}}_\Theta)^B{}_C(\mathscr{C}_\Gamma)^C{}_D(\bar{\mathscr{C}}_\Psi)^D{}_4\}=\eta_{\Sigma\Psi}\eta_{\Theta\Gamma}-\eta_{\Sigma\Gamma}\eta_{\Theta\Psi}+\eta_{\Sigma\Theta}\eta_{\Gamma\Psi}\ ,\\
&(\mathscr{C}^\Gamma)^A{}_B\mathscr{D}_\Gamma\ S^B{}_C=\not{\mathscr{D}}^A{}_B\ S^B{}_C=\delta^A{}_C\ ,\\
&(\mathscr{C}_{[\Sigma})^A{}_B(\bar{\mathscr{C}}_{\Theta]})^B{}_C(\mathscr{C}_\Gamma)^C{}_D=-2(\mathscr{C}_{[\Sigma})^A{}_D\ \eta_{\Theta]\Gamma}+(\mathscr{C}_{[\Sigma})^A{}_B(\bar{\mathscr{C}}_\Theta)^B{}_C(\mathscr{C}_{\Gamma]})^C{}_D\ ,
\end{aligned}
$$

$$
\begin{aligned}
&\mathscr{C}_\mu=\mathbb{1}_4\gamma_\mu\ ,\qquad \mathscr{C}_{3+i}=2[(t^i)^*\mathrm{P}^+-t^i\mathrm{P}^-]\ ,\quad \not{\mathscr{A}}^A{}_B=\mathscr{A}^\Gamma(\mathscr{C}_\Gamma)^A{}_B=\not{A}+\Phi^A{}_B\\
&\bar{\mathscr{C}}_\mu=\mathbb{1}_4\gamma_\mu\ ,\qquad \bar{\mathscr{C}}_{3+i}=2[t^i\mathrm{P}^+-(t^i)^*\mathrm{P}^-]\ ,\quad \not{\mathscr{A}}^{*A}{}_B=\mathscr{A}^\Gamma(\bar{\mathscr{C}}_\Gamma)^A{}_B=\not{A}+(\Phi^A{}_B)^\dagger
\end{aligned}
$$

$$
\Gamma_\mu=\mathbb{1}_8\otimes\gamma_\mu\ \text{ and }\ \Gamma_{3+i}=2\begin{pmatrix}0 & t^i\\ (t^i)^* & 0\end{pmatrix}\otimes(\mathrm{P}^+-\mathrm{P}^-)
$$

$$
\{t^i,(t^j)^*\}=-\frac{1}{2}\delta^{ij}\mathbb{1}_4
$$

$$
\begin{aligned}
& \tfrac{1}{16}\mathcal{F}^{\Sigma\Theta}\mathrm{tr}\{(\mathscr{C}_\Sigma)^4{}_B(\bar{\mathscr{C}}_\Theta)^B{}_C(\mathscr{C}_\Gamma)^C{}_D(\bar{\mathscr{C}}_\Psi)^D{}_4\}\mathscr{A}^\Gamma\times\mathscr{A}^\Psi \\
& \tfrac{1}{16}\mathcal{F}^{\Sigma\Theta}\mathrm{tr}\{(\mathscr{C}_\Sigma)^4{}_B(\bar{\mathscr{C}}_\Theta)^B{}_C(\mathscr{C}_\Gamma)^C{}_D\mathscr{D}^\Gamma\ S^D{}_E\not{\mathscr{A}}^E{}_F\times\not{\mathscr{A}}^{*F}{}_4\} \\
& -\tfrac{1}{16}\mathscr{D}^\Gamma\mathcal{F}^{\Sigma\Theta}\mathrm{tr}\{(\mathscr{C}_\Sigma)^4{}_B(\bar{\mathscr{C}}_\Theta)^B{}_C(\mathscr{C}_\Gamma)^C{}_D S^D{}_E\not{\mathscr{A}}^E{}_F\times\not{\mathscr{A}}^{*F}{}_4\} \\
-\tfrac{1}{2}\mathcal{F}^{\Sigma\Theta}\mathscr{A}_\Sigma\times\mathscr{A}_\Theta \quad & -\tfrac{1}{16}\mathscr{D}^\Gamma\mathcal{F}^{\Sigma\Theta}\mathrm{tr}\{[-2(\mathscr{C}_\Sigma)^4{}_D\ \eta_{\Theta\Gamma}+(\mathscr{C}_{[\Sigma})^4{}_B(\bar{\mathscr{C}}_\Theta)^B{}_C(\mathscr{C}_{\Gamma]})^C{}_D]S^D{}_E\not{\mathscr{A}}^E{}_F\times\not{\mathscr{A}}^{*F}{}_4\} \\
= \quad & -\tfrac{1}{8}\mathscr{D}_\Theta\mathcal{F}^{\Theta\Sigma}\mathrm{tr}\{(\mathscr{C}_\Sigma)^4{}_D S^D{}_E\not{\mathscr{A}}^E{}_F\times\not{\mathscr{A}}^{*F}{}_4\}\ ,
\end{aligned}
$$

$$0=\left(\partial_g+R_g[\mathcal{A}']\right)S_g^{\mathrm{b}}[\mathcal{A}']=\left(\partial_g+R_g[\mathcal{A}']\right)S_g^{\mathrm{b}}[\mathcal{A}],$$

$$\frac{1}{4}\mathrm{tr}\left\{(\mathcal{C}_\mu)^4{}_B\left(\overline{\mathcal{C}}_\nu\right)^B{}_C\left(\mathcal{C}_\rho\right)^C{}_D\left(\overline{\mathcal{C}}_\sigma\right)^D{}_4\right\}=\frac{1}{4}\mathrm{tr}\{\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\}=\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\nu}\eta_{\rho\sigma}$$

$$
\begin{aligned}
\frac{1}{4}\mathrm{tr}\left\{(\mathcal{C}_{3+i})^4{}_B\left(\overline{\mathcal{C}}_{3+j}\right)^B{}_C\left(\mathcal{C}_\mu\right)^C{}_D\left(\overline{\mathcal{C}}_\nu\right)^D{}_4\right\} &= \frac{1}{4}\mathrm{tr}\left\{(\mathcal{C}_{3+i})^4{}_B\left(\overline{\mathcal{C}}_{3+j}\right)^B{}_4\gamma_\mu\gamma_\nu\right\} \\
&= \left(t^i\right)_{4J}\left(t^j\right)^{J4}\mathrm{tr}\{\mathrm{P}^+\gamma_\mu\gamma_\nu\}+\left(t^i\right)^{4J}\left(t^j\right)_{J4}\mathrm{tr}\{\mathrm{P}^-\gamma_\mu\gamma_\nu\} \\
&= -2\left[\left(t^i\right)_{4J}\left(t^j\right)^{J4}\eta_{\mu\nu}+\text{ c.c.}\right]=\delta_{ij}\eta_{\mu\nu},
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{4}\mathrm{tr}\left\{(\mathcal{C}_{3+i})^4_B\left(\overline{\mathcal{C}}_\mu\right)^B_C(\mathcal{C}_{3+j})^C_D\left(\overline{\mathcal{C}}_\nu\right)^D{}_4\right\} &= \mathrm{tr}\left\{\left[\left(t^i\right)_{4J}\mathrm{P}^+-\left(t^i\right)^{4J}\mathrm{P}^-\right]\gamma_\mu\left[\left(t^j\right)_{J4}\mathrm{P}^+-\left(t^j\right)^{J4}\mathrm{P}^-\right]\gamma_\nu\right\} \\
&= -\left(t^i\right)_{4J}\left(t^j\right)^{J4}\mathrm{tr}\{\mathrm{P}^+\gamma_\mu\gamma_\nu\}-\left(t^i\right)^{4J}\left(t^j\right)_{J4}\mathrm{tr}\{\mathrm{P}^-\gamma_\mu\gamma_\nu\} \\
&= 2\left(t^i\right)_{4J}\left(t^j\right)^{J4}\eta_{\mu\nu}+\text{ c.c. }=-\delta_{ij}\eta_{\mu\nu},
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{4}\mathrm{tr}\left\{(\mathcal{C}_{3+i})^4_B\left(\overline{\mathcal{C}}_\mu\right)^B_C(\mathcal{C}_\nu)^C_D\left(\overline{\mathcal{C}}_{3+j}\right)^D{}_4\right\} &= \mathrm{tr}\left\{\left[\left(t^i\right)_{4J}\mathrm{P}^+-\left(t^i\right)^{4J}\mathrm{P}^-\right]\gamma_\mu\gamma_\nu\left[\left(t^j\right)^{J4}\mathrm{P}^+-\left(t^j\right)_{J4}\mathrm{P}^-\right]\right\} \\
&= \left(t^i\right)_{4J}\left(t^j\right)^{J4}\mathrm{tr}\{\mathrm{P}^+\gamma_\mu\gamma_\nu\}+\left(t^i\right)^{4J}\left(t^j\right)_{J4}\mathrm{tr}\{\mathrm{P}^-\gamma_\mu\gamma_\nu\} \\
&= -2\left[\left(t^i\right)_{4J}\left(t^j\right)^{J4}\eta_{\mu\nu}+\text{ c.c.}\right]=\delta_{ij}\eta_{\mu\nu},
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{4}\mathrm{tr}\left\{(\mathcal{C}_{3+i})^4{}_B\left(\overline{\mathcal{C}}_{3+j}\right)^B{}_C(\mathcal{C}_{3+k})^C{}_D\left(\overline{\mathcal{C}}_{3+l}\right)^D{}_4\right\} =& 4\left(t^i\right)_{4I}\left(t^j\right)^{IC}(t^k)_{CK}(t^l)^{K4}\mathrm{tr}\mathrm{P}^+ \\
&+4\left(t^i\right)^{4I}\left(t^j\right)_{IC}(t^k)^{CK}(t^l)_{K4}\mathrm{tr}\mathrm{P}^- \\
=& 8\left(t^i\right)_{4I}\left(t^j\right)^{IC}(t^k)_{CK}(t^l)^{K4}+\text{ c.c.} \\
=& 8\left[\left(t^i\right)_{4I}\left(t^j\right)^{I4}(t^k)_{4K}(t^l)^{K4}\right. \\
&\left.+\left(t^i\right)_{4I}\left(t^j\right)^{IJ}(t^k)_{JK}(t^l)^{K4}\right]+\text{ c.c.}
\end{aligned}
$$

$$\epsilon_{IJM}\epsilon^{MKL}=\delta_I{}^K\delta_J{}^L-\delta_I{}^L\delta_J{}^K$$

$$\frac{1}{4}\mathrm{tr}\left\{(\mathcal{C}_{3+I})^4{}_B\left(\overline{\mathcal{C}}_{6+J}\right)^B{}_C(\mathcal{C}_{3+K})^C{}_D\left(\overline{\mathcal{C}}_{6+L}\right)^D{}_4\right\}=\delta_{IL}\delta_{JK}-\delta_{IK}\delta_{JL}-\delta_{IJ}\delta_{KL},$$

$$(\varphi_I\times\varphi_J)(\varphi_{I+3}\times\varphi_{J+3})+(\varphi_I\times\varphi_{J+3})(\varphi_J\times\varphi_{I+3})+(\varphi_I\times\varphi_{I+3})(\varphi_{J+3}\times\varphi_J)=0$$

$$\sum_B\Phi^A{}_B\times(\Phi^B{}_A)^\dagger=\begin{cases}-2\gamma_5\sum_J\varphi_J\times\varphi_{J+3} & \text{for } A=4\\ 2\gamma_5\sum_J(-)^{\delta_{KJ}}\varphi_J\times\varphi_{J+3} & \text{for } A=K'\end{cases}$$

$$\sum_B\Phi^A{}_BS_0\Phi^B{}_A=\begin{cases}\sum_j\varphi_jS_0\varphi_j-\gamma_5\sum_J(\varphi_{J+3}S_0\varphi_J-\varphi_JS_0\varphi_{J+3}) & \text{for } A=4\\ \sum_j\varphi_jS_0\varphi_j+\gamma_5\sum_J(-)^{\delta_{KJ}}(\varphi_{J+3}S_0\varphi_J-\varphi_JS_0\varphi_{J+3}) & \text{for } A=K'\end{cases}$$

$$\sum_B(c^I)^A{}_BS_0\Phi^B{}_A=\begin{cases}S_0\varphi_I+\gamma_5S_0\varphi_{I+3} & \text{for } A=4\\ S_0\varphi_I-\gamma_5S_0\varphi_{I+3}(-)^{\delta_{IK}} & \text{for } A=K'\end{cases}$$

$$\sum_B(c^{I+3})^A{}_BS_0\Phi^B{}_A=\begin{cases}-\gamma_5S_0\varphi_I+S_0\varphi_{I+3} & \text{for } A=4\\ \gamma_5S_0\varphi_I(-)^{\delta_{IK}}+S_0\varphi_{I+3} & \text{for } A=K'\end{cases}$$

$$\sum_{B,C,D} \mathrm{tr}\{(c^I)^4{}_B S_0 \Phi^B{}_C S_0 \Phi^C{}_D \times (\Phi^D{}_4)^\dagger\} = \mathrm{tr}\Big\{-2S_0 \sum_j \varphi_j S_0 \varphi_j \varphi_I$$
$$-2S_0 \sum_J [\varphi_{I+3} S_0 \varphi_J \varphi_{J+3} - \varphi_J S_0 \varphi_{I+3} \varphi_{J+3} + \varphi_{J+3} S_0 \varphi_{I+3} \varphi_J]\Big\}$$

$$\sum_{B,C,D} \mathrm{tr}\{(c^I)^K{}_B S_0 \Phi^B{}_C S_0 \Phi^C{}_D \times (\Phi^D{}_K)^\dagger\} = \mathrm{tr}\Big\{-2S_0 \sum_j \varphi_j S_0 \varphi_j \varphi_I$$
$$-2S_0 \sum_J [\varphi_{I+3} S_0 \varphi_J \varphi_{J+3} - \varphi_J S_0 \varphi_{I+3} \varphi_{J+3} + \varphi_{J+3} S_0 \varphi_{I+3} \varphi_J](-)^{\delta_{IK}}$$
$$+4S_0[\varphi_{I+3} S_0 \varphi_K \varphi_{K+3} - \varphi_K S_0 \varphi_{I+3} \varphi_{K+3} + \varphi_{K+3} S_0 \varphi_{I+3} \varphi_K]\}$$

$$\sum_{B,C,D} \mathrm{tr}\{(c^{I+3})^4{}_B S_0 \Phi^B{}_C S_0 \Phi^C{}_D \times (\Phi^D{}_4)^\dagger\} = \mathrm{tr}\Big\{-2S_0 \sum_j \varphi_j S_0 \varphi_j \varphi_{I+3}$$
$$-2S_0 \sum_J [\varphi_I S_0 \varphi_{J+3} \varphi_J - \varphi_{J+3} S_0 \varphi_I \varphi_J + \varphi_J S_0 \varphi_I \varphi_{J+3}]\Big\}$$

$$\mathrm{tr}\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma = -4\epsilon^{\mu\nu\rho\sigma}$$
$$\mathrm{tr}\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma = 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})$$
$$\mathrm{tr}\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\lambda \gamma^\eta = -\eta^{\mu\nu}\mathrm{tr}\gamma^\rho \gamma^\sigma \gamma^\lambda \gamma^\eta + \eta^{\mu\rho}\mathrm{tr}\gamma^\nu \gamma^\sigma \gamma^\lambda \gamma^\eta \mp \cdots$$

$$\overleftarrow{R}_1{}^{(A)} = \frac{1}{8} \frac{\overleftarrow{\delta}}{\delta \mathscr{A}_\Gamma} \Pi_\Gamma{}^\Sigma \; \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^A{}_B S_0 \not{\mathscr{A}}^B{}_C \times \not{\mathscr{A}}^{*C}{}_A\Big\}$$

$$\overleftarrow{R}_1{}^{(A)} = \frac{1}{8} \frac{\overleftarrow{\delta}}{\delta \mathscr{A}_\Gamma} \Pi_\Gamma{}^\Sigma \; \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^A{}_B S_0 \Big[\not{A} \times \not{A}\delta^B{}_A + 2\Phi^B{}_A \times \not{A} + \Phi^B{}_C \times (\Phi^C{}_A)^\dagger\Big]\Big\}$$

$$R_1{}^{(A)} A_\mu = \tfrac{1}{8} \Pi_\mu{}^\nu \; \mathrm{tr}\Big\{\gamma_\nu S_0 \not{A} \times \not{A}\Big\} = C^\rho A_\mu A_\rho \,, \quad \forall \; A = 1,2,3,4$$

$$R_1{}^{(A)} \varphi_i = \tfrac{1}{4} \; \mathrm{tr}\Big\{(c^i)^A{}_B S_0 \Phi^B{}_A \times \not{A}\Big\} = C^\rho \varphi_i A_\rho \,, \quad \forall \; A = 1,2,3,4$$

$$\left(R_1^{(A)}\right)^2 A_\mu = 2C^\rho A_{[\mu} C^\lambda A_{\rho]} A_\lambda, \forall A = 1,2,3,4,$$
$$\left(R_1^{(A)}\right)^2 \varphi_i = C_\rho \varphi_i C_\lambda A^\rho A^\lambda - C_\rho A^\rho C_\lambda \varphi_i A^\lambda \; \forall A = 1,2,3,4.$$

$$P_\Gamma{}^\Sigma = \delta_\Gamma{}^\Sigma - \mathcal{D}_\Gamma G \partial^\Sigma = \Pi_\Gamma{}^\Theta \Big\{\delta_\Theta{}^\Sigma - g\mathcal{A}_\Theta \sum_{k=0}^\infty (-g\partial \cdot AC)^k C\partial^\Sigma\Big\},$$

$$\overleftarrow{R}_2{}^{(A)} = -\frac{1}{8} \frac{\overleftarrow{\delta}}{\delta \mathscr{A}_\Gamma} \Pi_\Gamma{}^\Sigma \, \mathrm{tr}\Big\{(\mathscr{C}_\Sigma)^A{}_B S_0 \not{\mathscr{A}}^B{}_C S_0 \not{\mathscr{A}}^C{}_D \times \not{\mathscr{A}}^{*D}{}_A\Big\} - \frac{1}{8} \frac{\overleftarrow{\delta}}{\delta \mathscr{A}_\Gamma} \Pi_\Gamma{}^\Theta \mathscr{A}_\Theta S_0 \partial^\sigma \mathrm{tr}\Big\{\gamma_\sigma S_0 \not{\mathscr{A}}^A{}_B \times \not{\mathscr{A}}^{*B}{}_A\Big\}$$

$$R_2{}^{(4)} A_\mu = 3C^\rho A^\lambda C_{[\mu} A_\lambda A_{\rho]} + 2C^\rho A_{[\mu} C^\lambda A_{\rho]} A_\lambda + 2C^\rho \varphi_i C_{[\rho} A_{\mu]} \varphi_i$$
$$+\Pi_\mu{}^\nu \epsilon_{\nu\lambda\rho\sigma} \sum_{J=1}^3 [C^\lambda \varphi_J C^\rho \varphi_{J+3} A^\sigma - C^\lambda \varphi_{J+3} C^\rho \varphi_J A^\sigma + C^\lambda A^\rho C^\sigma \varphi_{J+3} \varphi_J],$$
$$R_2{}^{(K)} A_\mu = 3C^\rho A^\lambda C_{[\mu} A_\lambda A_{\rho]} + 2C^\rho A_{[\mu} C^\lambda A_{\rho]} A_\lambda + 2C^\rho \varphi_i C_{[\rho} A_{\mu]} \varphi_i$$
$$-\Pi_\mu{}^\nu \epsilon_{\nu\lambda\rho\sigma} \sum_{J=1}^3 (-)^{\delta_{KJ}} [C^\lambda \varphi_J C^\rho \varphi_{J+3} A^\sigma - C^\lambda \varphi_{J+3} C^\rho \varphi_J A^\sigma + C^\lambda A^\rho C^\sigma \varphi_{J+3} \varphi_J],$$

$$R_2{}^{(4)}\varphi_I = C^\rho\varphi_I C^\lambda A_\rho A_\lambda - C_\rho A^\rho C_\lambda \varphi_I A^\lambda + 2C^{[\rho}A^{\lambda]}C_\rho\varphi_I A_\lambda + C^\rho\varphi_j C_\rho \varphi_I \varphi_j$$
$$+\frac{1}{2}\epsilon_{\mu\nu\rho\lambda}\left[C^\mu\varphi_{I+3}C^\nu A^\rho A^\lambda + 2C^\mu A^\nu C^\rho \varphi_{I+3}A^\lambda\right]$$
$$+C_\rho\sum_{J=1}^{3}\left[\varphi_{I+3}C_\rho\varphi_{J+3}\varphi_J + \varphi_J C_\rho \varphi_{I+3}\varphi_{J+3} - \varphi_{J+3}\varphi_{I+3}\varphi_J\right]$$
$$R_2{}^{(K)}\varphi_I = C^\rho\varphi_I C^\lambda A_\rho A_\lambda - C_\rho A^\rho C_\lambda \varphi_I A^\lambda + 2C^{[\rho}A^{\lambda]}C_\rho\varphi_I A_\lambda + C^\rho\varphi_j C_\rho \varphi_I \varphi_j$$
$$-\frac{1}{2}\epsilon_{\mu\nu\rho\lambda}(-)^{\delta_{IK}}\left[C^\mu\varphi_{I+3}C^\nu A^\rho A^\lambda + 2C^\mu A^\nu C^\rho \varphi_{I+3}A^\lambda\right]$$
$$+C^\rho(-)^{\delta_{IK}}\sum_{J=1}^{3}\left[\varphi_{I+3}C_\rho\varphi_{J+3}\varphi_J + \varphi_J C_\rho \varphi_{I+3}\varphi_{J+3} - \varphi_{J+3}C_\rho\varphi_{I+3}\varphi_J\right]$$
$$-2C^\rho\left[\varphi_{I+3}C_\rho\varphi_{K+3}\varphi_K + \varphi_K C_\rho\varphi_{I+3}\varphi_{K+3} - \varphi_{K+3}C_\rho\varphi_{I+3}\varphi_K\right]$$
$$R_2{}^{(4)}\varphi_{I+3} = C^\rho\varphi_{I+3}C^\lambda A_\rho A_\lambda - C_\rho A^\rho C_\lambda \varphi_{I+3} A^\lambda + 2C^{[\rho}A^{\lambda]}C_\rho\varphi_{I+3} A_\lambda + C^\rho\varphi_j C_\rho \varphi_{I+3} \varphi_j$$
$$-\frac{1}{2}\epsilon_{\mu\nu\rho\lambda}\left[C^\mu\varphi_I C^\nu A^\rho A^\lambda + 2C^\mu A^\nu C^\rho \varphi_I A^\lambda\right]$$
$$-C_\rho\sum_{J=1}^{3}\left[\varphi_I C_\rho\varphi_{J+3}\varphi_J + \varphi_J C_\rho \varphi_I\varphi_{J+3} - \varphi_{J+3}\varphi_I\varphi_J\right]$$

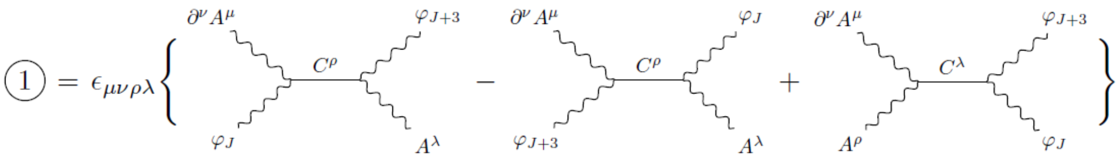
$$R_2^{(K)}\varphi_{I+3} = C^\rho\varphi_{I+3}C^\lambda A_\rho A_\lambda - C_\rho A^\rho C_\lambda \varphi_{I+3} A^\lambda + 2C^{[\rho}A^{\lambda]}C_\rho\varphi_{I+3} A_\lambda + C^\rho\varphi_j C_\rho \varphi_{I+3} \varphi_j$$
$$+\frac{1}{2}\epsilon_{\mu\nu\rho\lambda}(-)^{\delta_{IK}}\left[C^\mu\varphi_I C^\nu A^\rho A^\lambda + 2C^\mu A^\nu C^\rho \varphi_I A^\lambda\right]$$
$$-C_\rho(-)^{\delta_{IK}}\sum_{J=1}^{3}\left[\varphi_I C_\rho\varphi_{J+3}\varphi_J + \varphi_J C_\rho \varphi_I\varphi_{J+3} - C^\rho\varphi_{J+3}\varphi_I\varphi_J\right]$$
$$+2\left[\varphi_I C_\rho\varphi_{K+3}\varphi_K + \varphi_K C_\rho\varphi_I\varphi_{K+3} - C^\rho\varphi_{K+3}\varphi_I\varphi_K\right]$$

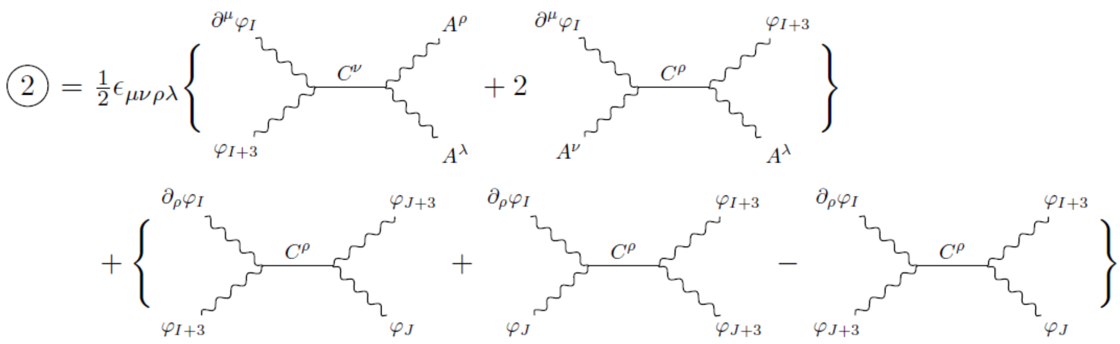
$$S_0[A'_\mu,\varphi'_I,\varphi'_{I+3}] = S_g^{\mathrm{b}}[A_\mu,\varphi_I,\varphi_{I+3}]$$

$$S_g^{\text{particle}} = \int \mathrm{d}^4x\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\mathrm{D}_\mu\varphi_i\mathrm{D}^\mu\varphi_i - \frac{g^2}{4}\left(\varphi_i\times\varphi_j\right)^2\right\}$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + gA_\mu\times A_\nu$$
$$\mathrm{D}_\mu = \partial_\mu + gA_\mu\times$$

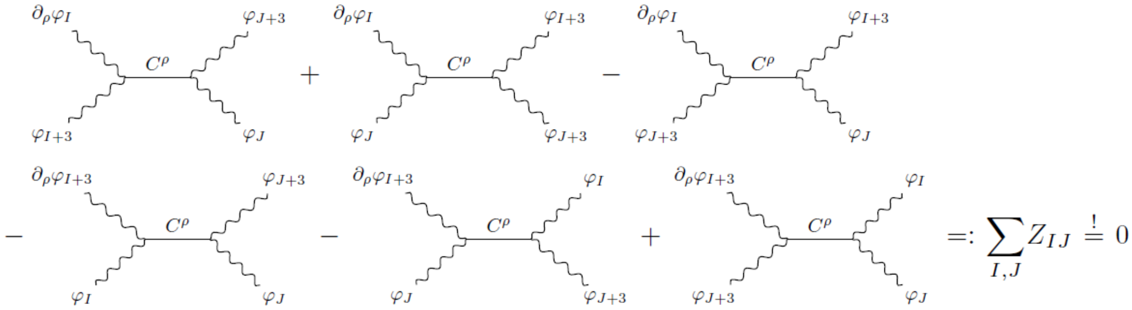
$$\int \mathrm{d}^4x\left\{\frac{1}{2}A'_\mu\Big|_{\mathcal{O}(g)}(\Box\,\eta^{\mu\nu} - \partial^\mu\partial^\nu)A'_\nu\Big|_{\mathcal{O}(g)} + A_\mu(\Box\,\eta^{\mu\nu} - \partial^\mu\partial^\nu)A'_\nu|_{\mathcal{O}(g^2)} + \frac{1}{2}\varphi'_i\Big|_{\mathcal{O}(g)}\Box\,\varphi'_i\Big|_{\mathcal{O}(g)} + \varphi_i\,\Box\,\varphi'_i|_{\mathcal{O}(g^2)}\right\}$$
$$= \int \mathrm{d}^4x\left\{-\frac{1}{4}\left(A_\mu\times A_\nu\right)^2 - \frac{1}{2}\left(A_\mu\times\varphi_i\right)^2 - \frac{1}{4}\left(\varphi_i\times\varphi_j\right)^2\right\}$$

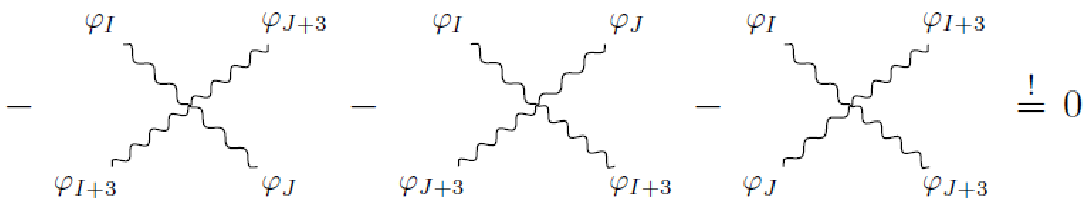
$$\int \mathrm{d}^4x\left\{A_\mu\,\Box\,A'^\mu\big|_{\mathcal{O}(g^2)} + \varphi_I\,\Box\,\varphi'_I|_{\mathcal{O}(g^2)} + \varphi_{I+3}\,\Box\,\varphi'_{I+3}|_{\mathcal{O}(g^2)}\right\}_{\text{blue terms}} = 0$$





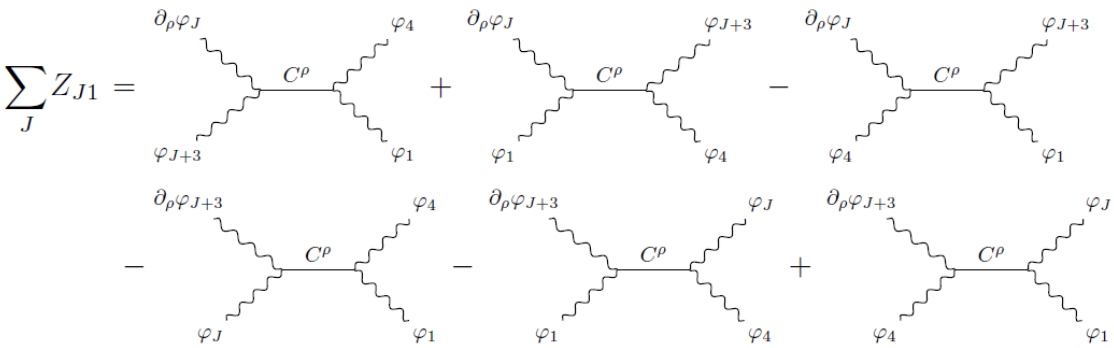
$$③ = -[② \text{ with } (I \leftrightarrow I+3)] \; .$$

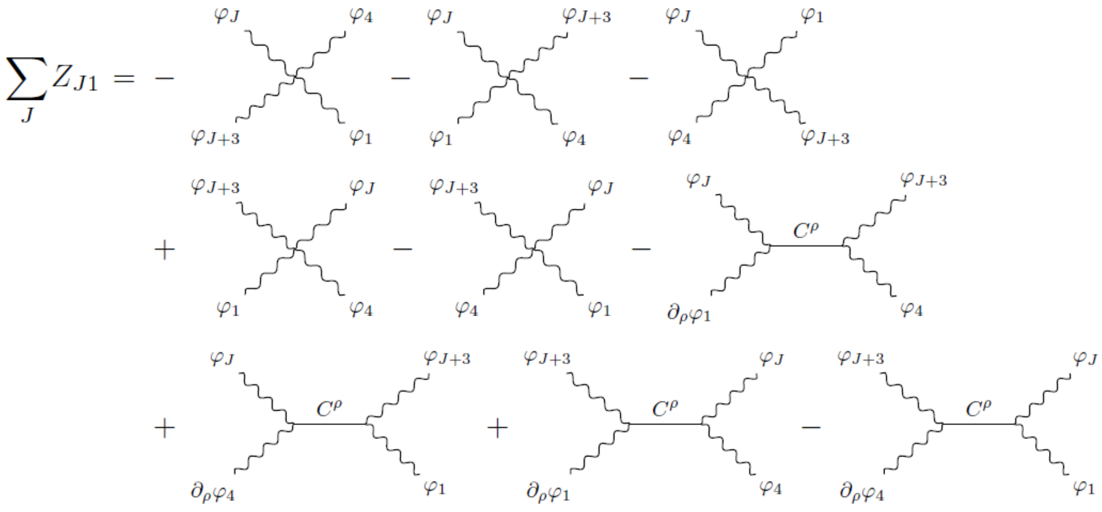


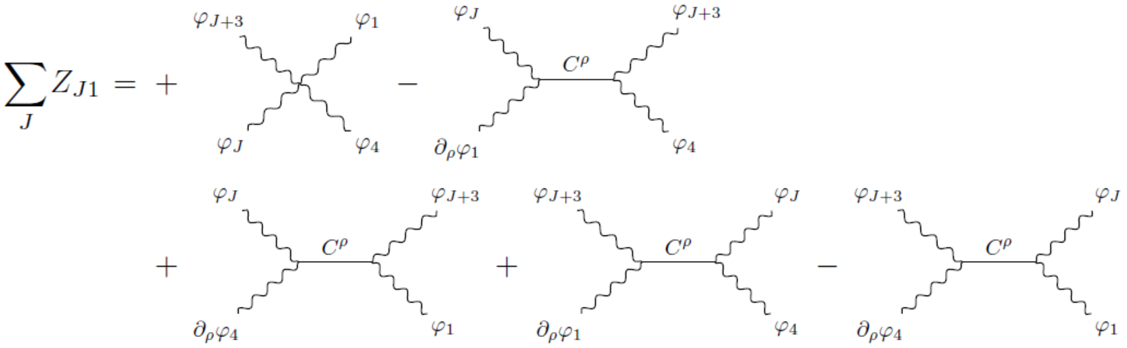


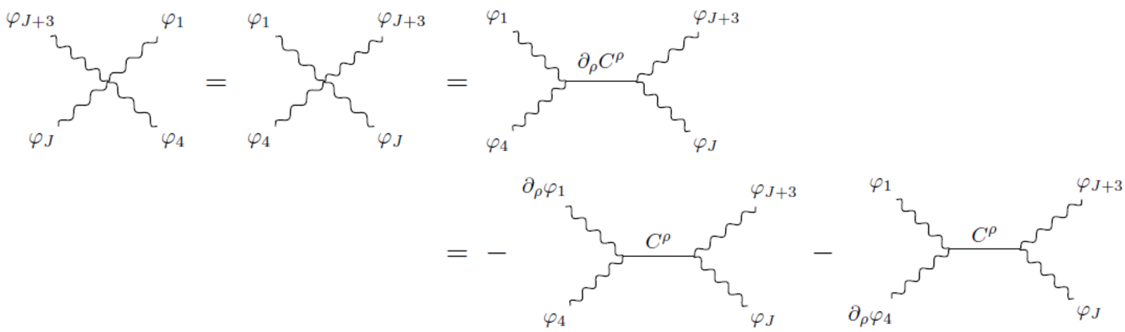
$$\sum_{I,J} (-)^{\delta_{KI}} Z_{IJ} - 2\sum_{I} Z_{IK} = \sum_{J} \left(-Z_{KJ} - 2Z_{JK} + \sum_{I \neq K} Z_{IJ} \right) \stackrel{!}{=} 0.$$

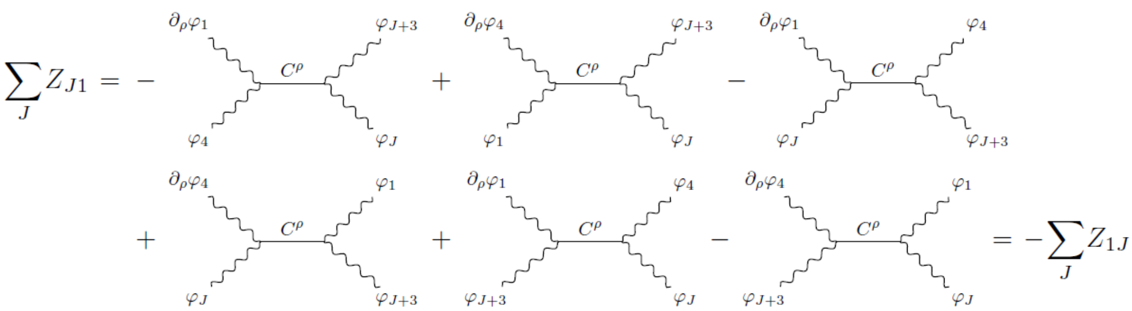
$$\sum_{J} Z_{JK} = -\sum_{J} Z_{KJ}, \text{ for any } K = 1,2,3$$











$$\log\ \det\frac{\delta\mathcal{A}'}{\delta\mathcal{A}} = \log\ \Delta_{\mathrm{MSS}}[A]\Delta_{\mathrm{FP}}[A]$$

$$\log\ \det\frac{\delta\mathcal{A}'}{\delta\mathcal{A}}\bigg|_{\mathcal{O}(g^2)} = \mathrm{tr}\frac{\delta\mathcal{A}'}{\delta\mathcal{A}}\bigg|_{\mathcal{O}(g^2)} - \frac{1}{2}\mathrm{tr}\frac{\delta\mathcal{A}'}{\delta\mathcal{A}}\bigg|_{\mathcal{O}(g)}\frac{\delta\mathcal{A}'}{\delta\mathcal{A}}\bigg|_{\mathcal{O}(g)}$$

$$\mathrm{tr}\frac{\delta\mathcal{A}'}{\delta\mathcal{A}} = \int \mathrm{d}^4x\, \mathrm{d}^4y\,\delta^{(4)}(x-y)\delta^{ab}\delta^{\Sigma}{}_{\Delta}\frac{\delta\mathcal{A}'^{a}_{\Sigma}(x)}{\delta\mathcal{A}^{b}_{\Delta}(y)}$$
$$= \int \mathrm{d}^4x\, \mathrm{d}^4y\,\delta^{(4)}(x-y)\delta^{ab}\left\{\frac{\delta A'^{a}_{\mu}(x)}{\delta A^{b}_{\nu}(y)}\delta^{\mu}{}_{\nu} + \frac{\delta\varphi'^{a}_{I}(x)}{\delta\varphi^{b}_{J}(y)}\delta_{IJ} + \frac{\delta\varphi'^{a}_{I+3}(x)}{\delta\varphi^{b}_{J+3}(y)}\delta_{IJ}\right\}$$

$$S_{\text{on-shell}} = \frac{1}{g_{YM}^2}\int d^{10}x\,\mathrm{Tr}\left[\frac{1}{4}F_{MN}F^{MN} - \frac{1}{2}\Psi\Gamma^M D_M\Psi\right]$$

$$\delta_{\epsilon}^{(\text{on-shell})}A_M = \epsilon\Gamma_M\Psi$$
$$\delta_{\epsilon}^{(\text{on-shell})}\Psi = \frac{1}{2}F_{MN}\Gamma^{MN}\epsilon$$

$$S_{10d} = \frac{1}{g_{YM}^2}\int d^{10}x\,\mathrm{Tr}\left[\frac{1}{4}F_{MN}F^{MN} - \frac{1}{2}\Psi\Gamma^M D_M\Psi - \frac{1}{2}K_m K_m\right]$$

$$\delta_{\epsilon}^{(10d)}A_M = \epsilon\Gamma_M\Psi$$
$$\delta_{\epsilon}^{(10d)}\Psi = \frac{1}{2}F_{MN}\Gamma^{MN}\epsilon + K_m\nu_m$$
$$\delta_{\epsilon}^{(10d)}K_m = -\nu_m\Gamma^M D_M\Psi$$

$$\epsilon\Gamma^M\nu_m = 0$$
$$\frac{1}{2}(\epsilon\Gamma_M\epsilon)\tilde{\Gamma}^M_{\alpha\beta} = \nu^m_\alpha\nu^m_\beta + \epsilon_\alpha\epsilon_\beta$$
$$\nu_m\Gamma^M\nu_n = \delta_{mn}\epsilon\Gamma^M\epsilon$$
$$S_{\text{curvature}} = \frac{1}{g_{YM}^2}\int d^dx\,\mathrm{Tr}\left[\frac{1}{4}F_{MN}F^{MN} - \frac{1}{2}\Psi\Gamma^M D_M\Psi - \frac{1}{2}K_m K_m\right].$$

$$A_{M=\mu} = A_\mu, A_{M=A} = \Phi_A,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu], F_{\mu A} = D_\mu\Phi_A, F_{AB} = [\Phi_A, \Phi_B]$$

$$D_\mu\Psi = \partial_\mu\Psi + [A_\mu, \Psi], D_A\Psi = [\Phi_A, \Psi].$$

$$\delta_{\epsilon}^{(\text{curvature})}A_M = \epsilon\Gamma_M\Psi$$
$$\delta_{\epsilon}^{(\text{curvature})}\Psi = \frac{1}{2}F_{MN}\Gamma^{MN}\epsilon + K_m\nu_m$$
$$\delta_{\epsilon}^{(\text{curvature})}K_m = -\nu_m\Gamma^M D_M\Psi$$

$$\Gamma_{09}\epsilon = \epsilon$$

$$\nu_m = \Gamma_{m8}\epsilon$$

$$\Gamma^0 = \begin{pmatrix}\mathbf{1}_{8\times8} & 0\\ 0 & \mathbf{1}_{8\times8}\end{pmatrix}, \Gamma^9 = \begin{pmatrix}\mathbf{1}_{8\times8} & 0\\ 0 & -\mathbf{1}_{8\times8}\end{pmatrix}$$
$$\Gamma^8 = \begin{pmatrix}0 & \mathbf{1}_{8\times8}\\ \mathbf{1}_{8\times8} & 0\end{pmatrix}, \Gamma^m = \begin{pmatrix}0 & \lambda_m\\ -\lambda_m & 0\end{pmatrix}$$

$$\lambda_m\lambda_n + \lambda_n\lambda_m = -2\delta_{mn}$$

$$\nu_m^{(+)} = \lambda_m\epsilon^{(+)}, \nu_m^{(-)} = \epsilon^{(-)} = 0$$

$$\nu_m^{(+)} = n\lambda_m n\epsilon^{(+)}, \nu_m^{(-)} = -\lambda_m\epsilon^{(-)} = 0, n\epsilon^{(-)} = \epsilon^{(-)}$$

$$\Gamma_{45}\epsilon = \Gamma_{67}\epsilon = \Gamma_{89}\epsilon,$$
$$\nu_1 = \Gamma_{68}\epsilon = -\Gamma_{79}\epsilon, \nu_2 = -\Gamma_{69}\epsilon = -\Gamma_{78}\epsilon,$$
$$\nu_3 = \Gamma_{84}\epsilon = -\Gamma_{95}\epsilon, \nu_4 = -\Gamma_{85}\epsilon = -\Gamma_{94}\epsilon,$$
$$\nu_5 = \Gamma_{46}\epsilon = -\Gamma_{57}\epsilon, \nu_6 = -\Gamma_{47}\epsilon = -\Gamma_{56}\epsilon,$$
$$\nu_7 = -\Gamma_{45}\epsilon = -\Gamma_{67}\epsilon = -\Gamma_{89}\epsilon,$$

$$\nabla_\mu \epsilon = \alpha \tilde{\Gamma}_\mu \Gamma \epsilon$$

$$R_{\mu\nu} = -4\alpha^2 (-1)^\sharp (\Gamma)^2 (d-1) g_{\mu\nu}.$$

$$\nabla_\mu \epsilon = \frac{1}{d} \Gamma_\mu D \epsilon$$

$$\nabla_\mu (D\epsilon) = d\lambda^2 \tilde{\Gamma}_\mu \epsilon \text{ with } \lambda^2 = -\alpha^2 (\Gamma)^2.$$

$$\nabla_\mu \epsilon_\pm = \pm \lambda \Gamma_\mu \epsilon_\pm$$

$$S_0 = \frac{1}{g_{YM}^2} \int d^d x \sqrt{g} \mathrm{Tr} \left[\frac{1}{4} F_{MN} F^{MN} - \frac{1}{2} \Psi \Gamma^M D_M \Psi - \frac{1}{2} K_m K_m \right]$$

$$\begin{aligned} \delta_\epsilon^{(0)} A_M &= \epsilon \Gamma_M \Psi \\ \delta_\epsilon^{(0)} \Psi &= \frac{1}{2} F_{MN} \Gamma^{MN} \epsilon + K_m \nu_m \\ \delta_\epsilon^{(0)} K_m &= -\nu_m \Gamma^M D_M \Psi \end{aligned}$$

$$\begin{aligned} g_{\mathrm{YM}}^2 \delta_\epsilon^{(0)} S_0 &= \int d^d x \sqrt{g} \mathrm{Tr} \left[-\frac{1}{2} F_{NP} (\nabla_\mu \epsilon) \Gamma^{NP\mu} \Psi + F^{\mu N} (\nabla_\mu \epsilon) \Gamma_N \Psi \right] \\ &= (-1)^{\Gamma+1} \int d^d x \sqrt{g} \mathrm{Tr} \left[\frac{\alpha(d-4)}{2} F_{\mu\nu} (\epsilon \Gamma \Gamma^{\mu\nu} \Psi) + \alpha(d-2) F_{\mu A} (\epsilon \Gamma \Gamma^{\mu A} \Psi) \right. \\ &\qquad \left. + \frac{\alpha d}{2} F_{AB} (\epsilon \Gamma \Gamma^{AB} \Psi) \right] \end{aligned}$$

$$\Gamma^T = (-1)^\Gamma \Gamma$$

$$S_1 = S_0 + \frac{1}{g_{YM}^2} \int d^d x \sqrt{g} \mathrm{Tr} \left[\frac{c_\Phi}{2} \Phi_A \Phi^A \right]$$

$$\begin{aligned} \delta_\epsilon^{(1)} A_M &= \epsilon \Gamma_M \Psi \\ \delta_\epsilon^{(1)} \Psi &= \frac{1}{2} F_{MN} \Gamma^{MN} \epsilon + c \Phi_A \tilde{\Gamma}^A \Gamma \epsilon + K_m \nu_m \\ \delta_\epsilon^{(1)} K_m &= -\nu_m \Gamma^M D_M \Psi \end{aligned}$$

$$\begin{aligned} & g_{\mathrm{YM}}^2 \delta_\epsilon^{(1)} S_1 \\ &= (-1)^{\Gamma+1} \int d^3 x \sqrt{g} \mathrm{Tr} \left[\frac{\alpha(d-4)}{2} F_{\mu\nu} (\epsilon \Gamma \Gamma^{\mu\nu} \Psi) + (\alpha(d-2) + c) F_{\mu A} (\epsilon \Gamma \Gamma^{\mu A} \Psi) \right. \\ &\quad \left. + \left(\frac{\alpha d}{2} + c \right) F_{AB} (\epsilon \Gamma \Gamma^{AB} \Psi) + (-1)^{\Gamma+1} (c_\Phi + c\alpha d (\Gamma)^2) \Phi_A (\epsilon \Gamma^A \Psi) \right] \end{aligned}$$

$$\alpha(d-4) = 0, \alpha(d-2) + c = 0, \frac{\alpha d}{2} + c = 0, c_\Phi + c\alpha d (\Gamma)^2 = 0$$

$$c = -2\alpha, c_\Phi = 8\alpha^2 (\Gamma)^2$$

$$\begin{aligned} S_{4d} = \frac{1}{g_{YM}^2} \int d^4 x \sqrt{g} \mathrm{Tr} [& \frac{1}{4} F_{MN} F^{MN} + 4\alpha^2 (\Gamma)^2 \Phi_A \Phi^A \\ & - \frac{1}{2} \Psi \Gamma^M D_M \Psi - \frac{1}{2} K_m K_m \Big] \end{aligned}$$

$$\begin{aligned} \delta_\epsilon^{(4d)} A_M &= \epsilon \Gamma_M \Psi \\ \delta_\epsilon^{(4d)} \Psi &= \frac{1}{2} F_{MN} \Gamma^{MN} \epsilon - 2\alpha \Phi_A \tilde{\Gamma}^A \Gamma \epsilon + K_m \nu_m \\ \delta_\epsilon^{(4d)} K_m &= -\nu_m \Gamma^M D_M \Psi \end{aligned}$$

$$\nabla_\mu \epsilon = \tilde{\Gamma}_\mu \tilde{\epsilon}$$

$$S_2 = S_0 + \frac{1}{g_{YM}^2}\int d^dx\sqrt{g}\mathrm{Tr}\left[\frac{c_\Phi}{2}\Phi_A\Phi^A + \frac{c'_\Phi}{2}\Phi_p\Phi^p + c_Y\epsilon_{pqr}\Phi^p[\Phi^q,\Phi^r] + \frac{c_\Psi}{2}\Psi\Gamma^{789}\Psi\right]$$

$$\delta_\epsilon^{(2)}A_M = \epsilon\Gamma_M\Psi$$

$$\delta_\epsilon^{(2)}\Psi = \frac{1}{2}F_{MN}\Gamma^{MN}\epsilon + c\Phi_A\tilde{\Gamma}^A\Gamma^{789}\epsilon + c'\Phi_p\tilde{\Gamma}^p\Gamma^{789}\epsilon + K_m\nu_m$$

$$\delta_\epsilon^{(2)}K_m = -\nu_m\Gamma^M D_M\Psi + c_K\nu_m\Gamma^{789}\Psi$$

$$\begin{aligned}
&g_{YM}^2\delta_\epsilon^{(2)}S_2\\
&=\int d^dx\sqrt{g}\mathrm{Tr}\Big[\frac{\alpha(d-4)-c_\Psi}{2}F_{\mu\nu}(\epsilon\Gamma^{789}\Gamma^{\mu\nu}\Psi)\\
&+(\alpha(d-2)+c+c_\Psi+c')F_{\mu p}(\epsilon\Gamma^{789}\Gamma^{\mu p}\Psi)+(\alpha(d-2)+c-c_\Psi)F_{\mu\hat{A}}(\epsilon\Gamma^{789}\Gamma^{\mu\hat{A}}\Psi)\\
&+\left(\frac{\alpha d-c_\Psi}{2}+c-3c_Y+c'\right)F_{pq}(\epsilon\Gamma^{789}\Gamma^{pq}\Psi)+(\alpha d+c_\Psi+2c+c')F_{p\hat{A}}(\epsilon\Gamma^{789}\Gamma^{p\hat{A}}\Psi)\\
&+\left(\frac{\alpha d-c_\Psi}{2}+c\right)F_{\hat{A}\hat{B}}(\epsilon\Gamma^{789}\Gamma^{\hat{A}\hat{B}}\Psi)\\
&+(c_\Phi-c\alpha d+c_\Psi c-c'\alpha d+c_\Psi c'+c'_\Phi)\Phi_p(\epsilon\Gamma^p\Psi)+(c_\Phi-c\alpha d-c_\Psi c)\Phi_{\hat{A}}(\epsilon\Gamma^{\hat{A}}\Psi)\\
&+(c_K-c_\Psi)K_m\Psi\Gamma^{789}\nu_m\Big]
\end{aligned}$$

$$\begin{aligned}
&c_\Psi = \alpha(d-4), \alpha(d-2)+c+c_\Psi+c' = 0, \alpha(d-2)+c-c_\Psi = 0\\
&\alpha d - c_\Psi + 2c - 6c_Y + 2c' = 0, \alpha d + c_\Psi + 2c + c' = 0, \alpha d - c_\Psi + 2c = 0\\
&c_\Phi - c\alpha d + c_\Psi c - c'\alpha d + c_\Psi c' + c'_\Phi = 0, c_\Phi - c\alpha d - c_\Psi c = 0, c_K = c_\Psi
\end{aligned}$$

$$\begin{aligned}
&c_\Phi = -4\alpha^2(d-2), c'_\Phi = -4\alpha^2(d-4), c_\Psi = \alpha(d-4), c_Y = -\frac{2\alpha(d-4)}{3}\\
&c = -2\alpha, c' = -2\alpha(d-4), c_K = c_\Psi = \alpha(d-4)
\end{aligned}$$

$$\begin{aligned}
S_2 = &\frac{1}{g_{YM}^2}\int d^dx\sqrt{g}\mathrm{Tr}\Big[\frac{1}{4}F_{MN}F^{MN} - 2\alpha^2(d-2)\Phi_A\Phi^A - 2\alpha^2(d-4)\Phi_p\Phi^p\\
&-\frac{2\alpha(d-4)}{3}\epsilon_{pqr}\Phi^p[\Phi^q,\Phi^r] - \frac{1}{2}\Psi\Gamma^M D_M\Psi + \frac{\alpha(d-4)}{2}\Psi\Gamma^{789}\Psi - \frac{1}{2}K_mK_m\Big]
\end{aligned}$$

$$\delta_\epsilon^{(2)}A_M = \epsilon\Gamma_M\Psi$$

$$\delta_\epsilon^{(2)}\Psi = \frac{1}{2}F_{MN}\Gamma^{MN}\epsilon - 2\alpha\Phi_A\tilde{\Gamma}^A\Gamma^{789}\epsilon - 2\alpha(d-4)\Phi_p\tilde{\Gamma}^p\Gamma^{789}\epsilon + K_m\nu_m$$

$$\delta_\epsilon^{(2)}K_m = -\nu_m\Gamma^M D_M\Psi + \alpha(d-4)\nu_m\Gamma^{789}\Psi$$

$$\left(\delta_\epsilon^{(4d)}\right)^2 A_\mu = -v^\nu F_{\nu\mu} - [v^A\Phi_A, D_\mu]$$

$$\left(\delta_\epsilon^{(4d)}\right)^2 \Phi_A = -v^\mu D_\mu\Phi_{\hat{A}} - [v^B\Phi_B,\Phi_A] - \bar{R}_{AB}\Phi^B - \Omega\Phi_A$$

$$\left(\delta_\epsilon^{(4d)}\right)^2 \Psi = -v^\mu D_\mu\Psi - [v^A\Phi_A,\Psi] - \frac{1}{4}(-\bar{R}_{\mu\nu}\Gamma^{\mu\nu} + \bar{R}_{AB}\Gamma^{AB})\Psi - \frac{3}{2}\Omega\Psi$$

$$\left(\delta_\epsilon^{(4d)}\right)^2 K_m = -v^\mu D_\mu K_m - [v^A\Phi_A, K_m] - (\nu_{[m}\not{v}_{n]})K_n - 2\Omega K_m$$

$$v^M = \epsilon\Gamma^M\epsilon, \bar{R}_{MN} = 2\alpha(\epsilon\tilde{\Gamma}_{MN}\Gamma\epsilon), \Omega = 2\alpha(\epsilon\Gamma\epsilon)$$

$$\left(\delta_\epsilon^{(4d)}\right)^2 = -L_v - G_\Phi - \bar{R} - \Omega$$

$$\left(\delta_\epsilon^{(2)}\right)^2 A_\mu = -v^\nu F_{\nu\mu} - [v^A\Phi_A, D_\mu]$$

$$\left(\delta_\epsilon^{(2)}\right)^2 \Phi_{\hat{A}} = -v^\nu D_\nu\Phi_{\hat{A}} - [v^B\Phi_B,\Phi_{\hat{A}}] - \bar{R}_{\hat{A}\hat{B}}\Phi^{\hat{B}}$$

$$\left(\delta_\epsilon^{(2)}\right)^2 \Phi_p = -v^\nu D_\nu\Phi_p - [v^B\Phi_B,\Phi_p] - (d-3)\bar{R}_{pq}\Phi^q$$

$$\left(\delta_\epsilon^{(2)}\right)^2 \Psi = -v^\nu D_\nu\Psi - [v^A\Phi_A,\Psi] - \frac{1}{4}(-\bar{R}_{\mu\nu}\Gamma^{\mu\nu} + \bar{R}_{\hat{A}\hat{B}}\Gamma^{\hat{A}\hat{B}} + (d-3)\bar{R}_{pq}\Gamma^{pq})\Psi$$

$$\left(\delta_\epsilon^{(2)}\right)^2 K_m = -v^\mu D_\mu K_m - [v^A\Phi_A, K_m] - (\nu_{[m}\not{D}\nu_{n]})K_n$$

$$\bar{R}_{MN} = 2\alpha\left(\epsilon\tilde{\Gamma}_{MN}\Gamma^{789}\epsilon\right)$$

$$\left(\delta_\epsilon^{(2)}\right)^2 = -L_v - G_\Phi - \bar{R}$$

$$\nabla_\mu\epsilon = \alpha\tilde{\Gamma}_\mu\Gamma^{789}\epsilon$$

$$\nabla_\mu\epsilon = \frac{i}{2l}\Gamma_\mu\epsilon \text{ or } \nabla_\mu\epsilon = -\frac{i}{2l}\Gamma_\mu\epsilon$$

$$R = \frac{d(d-1)}{l^2}$$

$$R = -\frac{d(d-1)}{l^2}$$

$$\begin{aligned} ds^2_{\mathbb{R}^4} &= dr^2 + r^2 d\Omega_3^2 \\ &= e^{\frac{2}{l}\tau}(d\tau^2 + l^2 d\Omega_3^2)\ \left(r = le^{\frac{\tau}{l}}\right) \\ &= e^{\frac{2}{l}\tau} ds^2_{\mathbb{R}\times S^3} \end{aligned}$$

$$g_{\mu\nu}^{\mathbb{R}\times S^3} = e^{2\omega} g_{\mu\nu}^{\mathbb{R}^4}$$

$$\begin{aligned} S_{\mathbb{R}\times S^3} = &\frac{1}{g_{YM}^2}\int d\tau d\Omega_3 \mathrm{Tr}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\Phi_A D^\mu\Phi^A + \frac{1}{2l^2}\Phi_A\Phi^A \right. \\ &\left. + \frac{1}{4}[\Phi_A,\Phi_B][\Phi^A,\Phi^B] - \frac{1}{2}\Psi\Gamma^\mu D_\mu\Psi - \frac{1}{2}\Psi\Gamma^A[\Phi_A,\Psi] - \frac{1}{2}K_m K_m\right] \end{aligned}$$

$$\begin{aligned} \delta_\epsilon^{(\mathbb{R}\times S^3)} A_M &= \epsilon\Gamma_M\Psi \\ \delta_\epsilon^{(\mathbb{R}\times S^3)}\Psi &= \frac{1}{2}F_{MN}\Gamma^{MN}\epsilon + \frac{1}{2}\Gamma^{\mu A}\Phi_A\nabla_\mu\epsilon + K_m\nu_m \end{aligned}$$

$$\delta_\epsilon^{(\mathbb{R}\times S^3)} K_m = -\nu_m\Gamma^M D_M\Psi$$

$$\nabla_\mu\epsilon = \tilde{\Gamma}_\mu\tilde{\epsilon}, \nabla_\mu\tilde{\epsilon} = -\frac{1}{8l^2}\Gamma_\mu\epsilon$$

$$\epsilon = e^{\frac{1}{2}\omega}(\epsilon_s + x^\mu\tilde{\Gamma}_\mu\epsilon_c), \tilde{\epsilon} = e^{\frac{1}{2}\omega}\left(\frac{1}{2}e^{-\omega}\epsilon_c - \frac{1}{2l^2}e^{\omega}\Gamma^a x_a\epsilon_s\right)$$

$$\begin{aligned} \left(\delta_\epsilon^{(\mathbb{R}\times S^3)}\right)^2 A_\mu &= -v^\nu F_{\nu\mu} - [v^A\Phi_A, D_\mu] \\ \left(\delta_\epsilon^{(\mathbb{R}\times S^3)}\right)^2 \Phi_A &= -v^\mu D_\mu\Phi_{\hat{A}} - [v^B\Phi_B,\Phi_A] - \bar{R}_{AB}\Phi^B - \Omega\Phi_A \\ \left(\delta_\epsilon^{(\mathbb{R}\times S^3)}\right)^2 \Psi &= -v^\mu D_\mu\Psi - [v^A\Phi_A,\Psi] - \frac{1}{4}\left(-\bar{R}_{\mu\nu}\Gamma^{\mu\nu} + \bar{R}_{AB}\Gamma^{AB}\right)\Psi - \frac{3}{2}\Omega\Psi \\ \left(\delta_\epsilon^{(\mathbb{R}\times S^3)}\right)^2 K_m &= -v^\mu D_\mu K_m - [v^A\Phi_A, K_m] - \left(\nu_{[m}\not{D}\nu_{n]}\right)K_n - 2\Omega K_m \end{aligned}$$

$$v^M = \epsilon\Gamma^M\epsilon, \bar{R}_{MN} = 2\left(\epsilon\tilde{\Gamma}_{MN}\tilde{\epsilon}\right), \Omega = 2(\epsilon\tilde{\epsilon})$$

$$\begin{aligned} F &= dA + A\wedge A \\ &= \left(-i\frac{2}{l}(J_i\phi) + (\partial_\tau A_i) + [\phi, A_i]\right) d\tau\wedge e^i \\ &\quad + \frac{1}{2}\epsilon_{ijk}\left\{\frac{2}{l}i\epsilon_{klm}J_l A_m + \frac{2}{l}A_k + \frac{1}{2}\epsilon_{klm}[A_l, A_m]\right\} e^i\wedge e^j \end{aligned}$$

$$\Gamma^\mu D_\mu\Psi = \Gamma^\tau D_\tau\Psi + \frac{2i}{l}\Gamma^i J_i\Psi + \frac{3i}{4l}\Gamma^{123}\Psi + \Gamma^i[A_i,\Psi]$$

$$S_3 = \frac{1}{g_{YM}^2}\int d\Omega_3 \mathrm{Tr}\left[\frac{1}{2}\left(\tilde{D}_i\phi\right)^2 + \frac{1}{2}f_{ij}f^{ij} + \frac{1}{2}\tilde{D}_i\Phi_A\tilde{D}^i\Phi^A + \frac{1}{2l^2}\Phi_A\Phi^A + \frac{1}{2}[\phi,\Phi_A][\phi,\Phi^A]\right]$$

$$+\frac{1}{4}[\Phi_A,\Phi_B][\Phi^A,\Phi^B]-\frac{1}{2}\Psi\Gamma^i\tilde{D}_i\Psi-\frac{3i}{8l}\Psi\Gamma^{123}\Psi-\frac{1}{2}\Psi\Gamma^\tau[\phi,\Psi]-\frac{1}{2}\Psi\Gamma^A[\Phi_A,\Psi]-\frac{1}{2}K_mK_m\Big]$$

$$\tilde{D}_i\phi=i\frac{2}{l}J_i\phi+[A_i,\phi],f_{ij}=\frac{1}{2}\epsilon_{ijk}\left(\frac{2}{l}i\epsilon_{klm}J_lA_m+\frac{2}{l}A_k+\frac{1}{2}\epsilon_{klm}[A_l,A_m]\right)$$

$$\delta_\epsilon^{(3)}A_i=\epsilon\Gamma_i\Psi,\delta_\epsilon^{(3)}\phi=\epsilon\Gamma_\tau\Psi,\delta_\epsilon^{(3)}\Phi_A=\epsilon\Gamma_A\Psi$$
$$\delta_\epsilon^{(3)}\Psi=\tilde{D}_i\phi\Gamma^{i\tau}\epsilon+\frac{1}{2}f_{ij}\Gamma^{ij}\epsilon+\tilde{D}_i\Phi_A\Gamma^{iA}\epsilon+[\phi,\Phi_A]\Gamma^{\tau A}\epsilon+\frac{1}{2}[\Phi_A,\Phi_B]\Gamma^{AB}\epsilon-2\Phi_A\tilde{\Gamma}^A\tilde{\epsilon}+K_m\nu_m$$
$$\delta_\epsilon^{(3)}K_m=-\nu_m\Gamma^i\tilde{D}_i\Psi-\frac{3i}{4l}\nu_m\Gamma^{123}\Psi-\nu_m\Gamma^\tau[\phi,\Psi]-\nu_m\Gamma^A[\Phi_A,\Psi]$$

$$S_{\mathrm{BMN}}=\frac{1}{g_{\mathrm{YM}}^2}\int d\tau\mathrm{Tr}\Bigg[\frac{1}{2}D_\tau X_iD^\tau X^i+\frac{1}{2}\left(\frac{2}{l}X_i+\frac{1}{2}\epsilon_{ijk}[X_j,X_k]\right)^2$$
$$+\frac{1}{2}D_\tau\Phi_AD^\tau\Phi^A+\frac{1}{2l^2}\Phi_A\Phi^A+\frac{1}{2}[X_i,\Phi_A][X^i,\Phi^A]+\frac{1}{4}[\Phi_A,\Phi_B][\Phi^A,\Phi^B]$$
$$-\frac{1}{2}\Psi\Gamma^\tau D_\tau\Psi-\frac{3i}{8l}\Psi\Gamma^{123}\Psi-\frac{1}{2}\Psi\Gamma^i[X_i,\Psi]-\frac{1}{2}\Psi\Gamma^A[\Phi_A,\Psi]-\frac{1}{2}K_mK_m\Bigg].$$

$$\delta_\epsilon^{(\mathrm{BMN})}A_\tau=\epsilon\Gamma_\tau\Psi,\delta_\epsilon^{(\mathrm{BMN})}X_i=\epsilon\Gamma_i\Psi,\delta_\epsilon^{(\mathrm{BMN})}\Phi_A=\epsilon\Gamma_A\Psi$$
$$\delta_\epsilon^{(\mathrm{BMN})}\Psi=D_\tau X_i\Gamma^{\tau i}\epsilon+\frac{1}{2}f_{ij}\Gamma^{ij}\epsilon+D_\tau\Phi_A\Gamma^{\tau A}\epsilon$$
$$+[X_i,\Phi_A]\Gamma^{iA}\epsilon+\frac{1}{2}[\Phi_A,\Phi_B]\Gamma^{AB}\epsilon-2\Phi_A\tilde{\Gamma}^A\tilde{\epsilon}+K_m\nu_m$$
$$\delta_\epsilon^{(\mathrm{BMN})}K_m=-\nu_m\Gamma^\tau D_\tau\Psi-\frac{3i}{4l}\nu_m\Gamma^{123}\Psi-\nu_m\Gamma^i\tau[X_i,\Psi]-\nu_m\Gamma^A[\Phi_A,\Psi]$$

$$(\nabla_\mu-i\tilde{A}_\mu)\epsilon=-iV_\mu\epsilon-iV^\nu\Gamma_{\mu\nu}\epsilon$$

$$\{\gamma^M,\gamma^N\}=2g^{MN}$$

$$\gamma^{11}=\gamma^1\gamma^2\cdots\gamma^9\gamma^0$$

$$\gamma^M=\begin{pmatrix}0&\tilde{\Gamma}^M\\\Gamma^M&0\end{pmatrix}$$

$$\tilde{\Gamma}^{\{M}\Gamma^{N\}}=g^{MN},\Gamma^{\{M}\tilde{\Gamma}^{N\}}=g^{MN}$$

$$(\Gamma_M)_{\alpha_1\{\alpha_2}(\Gamma^M)_{\alpha_3\alpha_4\}}=0$$

$$(\epsilon_1A\epsilon_2)(\epsilon_3B\epsilon_4)=\frac{1}{16}\Big[(\epsilon_1\epsilon_4)(\epsilon_3BA\epsilon_2)-\frac{1}{2}(\epsilon_1\tilde{\Gamma}_{MN}\epsilon_4)(\epsilon_3B\tilde{\Gamma}^{MN}A\epsilon_2)$$
$$+\frac{1}{4!}(\epsilon_1\tilde{\Gamma}_{MNKL}\epsilon_4)(\epsilon_3B\tilde{\Gamma}^{MNKL}A\epsilon_2)\Big]$$

$$\tilde{\Gamma}^{NP}\Gamma^M=\Gamma^{NPM}-2g^{M[N}\Gamma^{P]},\tilde{\Gamma}_\rho\Gamma^{NP\rho}=\tilde{\Gamma}_\rho(\tilde{\Gamma}^{NP}\Gamma^\rho+2g^{\rho[N}\Gamma^{P]})$$
$$\tilde{\Gamma}_\rho\tilde{\Gamma}^{\mu\nu}\Gamma^\rho=(d-4)\Gamma^{\mu\nu},\tilde{\Gamma}_\rho\tilde{\Gamma}^{\mu A}\Gamma^\rho=(d-2)\Gamma^{\mu A},\tilde{\Gamma}_\rho\tilde{\Gamma}^{AB}\Gamma^\rho=d\Gamma^{AB}$$

$$\frac{L^4}{\ell_s^4}=g_{\mathrm{YM}}^2N,\tau=\tau_s$$

$$\langle S_{I_1J_1}(\vec{x}_1)S_{I_2J_2}(\vec{x}_2)S_{I_3J_3}(\vec{x}_3)S_{I_4J_4}(\vec{x}_4)\rangle$$

$$U\equiv\frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2},V\equiv\frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2},$$

$$\mathcal{T}(V,U)=\frac{V^2}{U^2}\mathcal{T}(U,V),\mathcal{T}\left(\frac{U}{V},\frac{1}{V}\right)=V^2\mathcal{T}(U,V)$$

$$\mathcal{T}(U,V)=\mathcal{T}_{\mathrm{long}}(U,V)+\mathcal{T}_{\mathrm{short}}(U,V)$$

$$\mathcal{T}_{\text{long}}(U,V)=\frac{1}{U^2}\sum_{\mathcal{O}_{\Delta,\ell}}\lambda_{\Delta,\ell}^2 G_{\Delta+4,\ell}(U,V)$$

$$G_{\Delta,\ell}(U,V)=\frac{z\bar{z}}{z-\bar{z}}\left(k_{\Delta+\ell}(z)k_{\Delta-\ell-2}(\bar{z})-k_{\Delta+\ell}(\bar{z})k_{\Delta-\ell-2}(z)\right)$$
$$k_h(z)\equiv z^{\frac{h}{2}}\,{}_2F_1(h/2,h/2,h,z)$$

$$\mathcal{F}_2(\tau,\bar{\tau})\equiv\frac{1}{8c}\frac{\partial_m^2\partial_\tau\partial_{\bar{\tau}}F}{\partial_\tau\partial_{\bar{\tau}}F}\bigg|_{m=0}=I_2[\mathcal{T}]$$
$$\mathcal{F}_4(\tau,\bar{\tau})\equiv 48\zeta(3)c^{-1}+c^{-2}\partial_m^4F|_{m=0}=I_4[\mathcal{T}]$$

$$I_2[f]\equiv-\frac{2}{\pi}\int dRd\theta\frac{R^3\sin^2\theta f(U,V)}{U^2}\bigg|_{\substack{U=1+R^2-2R\cos\theta\\ V=R^2}}$$
$$I_4[f]\equiv-\frac{32}{\pi}\int dRd\theta R^3\sin^2\theta(U^{-1}+U^{-2}V+U^{-2})\bar{D}_{1,1,1,1}(U,V)f(U,V)\bigg|_{\substack{U=1+R^2-2R\cos\theta\\ V=R^2}}$$

$$\bar{D}_{1,1,1,1}(U,V)=\frac{1}{z-\bar{z}}\left(\log(z\bar{z})\log\frac{1-z}{1-\bar{z}}+2\mathrm{Li}(z)-2\mathrm{Li}(\bar{z})\right).$$

$$Z(m,\tau,\bar{\tau})=\int\frac{d^{N-1}a}{N!}\frac{\prod_{i<j}a_{ij}^2H^2(a_{ij})}{H(m)^{N-1}\prod_{i\neq j}H(a_{ij}+m)}e^{-\frac{8\pi^2}{g_{\mathrm{YM}}^2}\sum_i a_i^2}\left|Z_{\text{inst}}(m,\tau,a_{ij})\right|^2,$$

$$|\tau|\geq 1,|\Re(\tau)|\leq\frac{1}{2}$$

$$\begin{aligned}\mathcal{F}_2(\tau,\bar{\tau})\approx&\\ &-\frac{\tau_2^2}{4c^2}\partial_{\tau_2}^2\int_0^\infty dw\frac{e^{-\frac{w^2}{\pi\tau_2}}}{2\sinh^2 w}\left[\left[L_{N-1}^{(1)}\left(\frac{w^2}{\pi\tau_2}\right)\right]^2-\sum_{i,j=1}^{N}(-1)^{i-j}L_{i-1}^{(j-i)}\left(\frac{w^2}{\pi\tau_2}\right)L_{j-1}^{(i-j)}\left(\frac{w^2}{\pi\tau_2}\right)\right]\\ &+\frac{1}{4c^2}\left[-\frac{3\sqrt{N}}{2^4}E\left(\frac{3}{2};\tau,\bar{\tau}\right)+\frac{45}{2^8\sqrt{N}}E\left(\frac{5}{2};\tau,\bar{\tau}\right)+\frac{1}{N^{\frac{3}{2}}}\left[-\frac{39}{2^{13}}E\left(\frac{3}{2};\tau,\bar{\tau}\right)+\frac{4725}{2^{15}}E\left(\frac{7}{2};\tau,\bar{\tau}\right)\right]\right.\\ &\left.+\frac{1}{N^{\frac{5}{2}}}\left[-\frac{1125}{2^{16}}E\left(\frac{5}{2};\tau,\bar{\tau}\right)+\frac{99225}{2^{18}}E\left(\frac{9}{2};\tau,\bar{\tau}\right)\right]\right]_{k\neq 0}\end{aligned}$$

$$\begin{aligned}E(s,\tau,\bar{\tau})=&2\zeta(2s)\tau_2^s+2\sqrt{\pi}\tau_2^{1-s}\frac{\Gamma\left(s-\frac{1}{2}\right)}{\Gamma(s)}\zeta(2s-1)\\ &+\frac{2\pi^s\sqrt{\tau_2}}{\Gamma(s)}\sum_{k\neq 0}|k|^{s-\frac{1}{2}}\sigma_{1-2s}(|k|)K_{r-\frac{1}{2}}(2\pi\tau_2|k|)e^{2\pi ik\tau_1}\end{aligned}$$

$$\sum_{\mathcal{O}_{\Delta,\ell}}\lambda_{\Delta,\ell}^2\left(V^4G_{\Delta+4,\ell}(U,V)-U^4G_{\Delta+4,\ell}(U,V)\right)+U^2V^4\mathcal{T}_{\text{short}}(U,V)-U^4V^2\mathcal{T}_{\text{short}}(V,U)=0$$

$$\sum_{\mathcal{O}_{\Delta,\ell}}\lambda_{\Delta,\ell}^2 I_2\left[\frac{G_{\Delta+4,\ell}(U,V)}{U^2}\right]+I_2[\mathcal{T}_{\text{short}}]-\mathcal{F}_2(\tau,\bar{\tau})=0$$
$$\sum_{\mathcal{O}_{\Delta,\ell}}\lambda_{\Delta,\ell}^2 I_4\left[\frac{G_{\Delta+4,\ell}(U,V)}{U^2}\right]+I_4[\mathcal{T}_{\text{short}}]-\mathcal{F}_4(\tau,\bar{\tau})=0$$

$$\Delta_{2,0} = 2 + \frac{3\lambda}{4\pi^2} - \frac{3\lambda^2}{16\pi^4} + \frac{21\lambda^3}{256\pi^6} + \frac{\lambda^4\left(-1440\left(\frac{12}{N^2}+1\right)\zeta(5) + 576\zeta(3) - 2496\right)}{65536\pi^8} + O(\lambda^5)$$

$$\lambda_{2,0}^2 = \frac{1}{c}\left[\frac{1}{3} - \frac{\lambda}{4\pi^2} + \frac{\lambda^2(3\zeta(3)+7)}{32\pi^4} - \frac{\lambda^3(8\zeta(3)+25\zeta(5)+48)}{256\pi^6} + \frac{\lambda^4\left(2488 + 328\zeta(3) + 72\zeta(3)^2 + 980\zeta(5) + 1470\zeta(7) + \frac{45}{N^2}(8\zeta(5)+7\zeta(7))\right)}{16384\pi^8} + O(\lambda^5)\right]$$

$$\Delta_{2,0} = 2\lambda^{1/4} - 2 + \frac{2}{\lambda^{1/4}} + \frac{1/2 - 3\zeta(3)}{\lambda^{3/4}} + \frac{1/2 + 6\zeta(3) + 15\zeta(5)/2}{\lambda^{5/4}} + O(\lambda^{-3/2}).$$

$$\Delta_{4,0} = 4 - \frac{4}{c} + \frac{135}{7\sqrt{2}\pi^{3/2}c^{7/4}}E\left(\frac{3}{2},\tau\right) + \frac{1199}{42c^2} - \frac{3825}{32\sqrt{2}\pi^{5/2}c^{9/4}}E\left(\frac{5}{2},\tau\right) + O(c^{-5/2})$$

$$\lambda_{4,0}^2 = \frac{1}{10} + \frac{19}{300c} - \frac{4059}{1960\sqrt{2}\pi^{3/2}c^{7/4}}E\left(\frac{3}{2},\tau\right) + \frac{1}{c^2}\left[a - \frac{4059}{1960}\right] - \frac{40025}{1792\sqrt{2}\pi^{5/2}c^{9/4}}E\left(\frac{5}{2},\tau\right) + O(c^{-5/2})$$

$$a \approx 3.5897946432786394668.$$

$$F_{\Delta,\ell}(U,V) \equiv V^4 G_{\Delta+4,\ell}(U,V) - U^4 G_{\Delta+4,\ell}(V,U)$$
$$F_{\text{short}}(U,V) \equiv U^2V^4\mathcal{T}_{\text{short}}(U,V) - U^4V^2\mathcal{T}_{\text{short}}(V,U)$$

$$\sum_{O_{\Delta,\ell}} \lambda_{\Delta,\ell}^2 F_{\Delta,\ell}(U,V) + F_{\text{short}}(U,V) = 0.$$

$$\sum_{m,n} \alpha_{m,n}\left(\partial_z^m\partial_{\bar z}^n F_{\Delta,\ell}\big|_{z=\bar z=1/2}\right) \geq 0 \ \forall \ell = 0,2,\dots, \Delta \geq \begin{cases}\Delta_* & \ell = 0\\ \ell+2 & \ell > 0\end{cases}$$

$$\sum_{m,n} \alpha_{m,n}\left(\partial_z^m\partial_{\bar z}^n F_{\text{short}}\big|_{z=\bar z=1/2}\right) = 1$$

$$\alpha_2 I_2\left[\frac{G_{\Delta+4,\ell}(U,V)}{U^2}\right] + \alpha_4 I_4\left[\frac{G_{\Delta+4,\ell}(U,V)}{U^2}\right] + \sum_{m,n}\alpha_{m,n}\left(\partial_z^m\partial_{\bar z}^n F_{\Delta,\ell}\big|_{z=\bar z=1/2}\right) \geq 0,$$

$$\alpha_2\left(I_2[\mathcal{T}_{\text{short}}] - \mathcal{F}_2(\tau,\bar\tau)\right) + \alpha_4\left(I_4[\mathcal{T}_{\text{short}}] - \mathcal{F}_4(\tau,\bar\tau)\right) + \sum_{m,n}\alpha_{m,n}\left(\partial_z^m\partial_{\bar z}^n F_{\text{short}}\big|_{z=\bar z=1/2}\right) = 1.$$

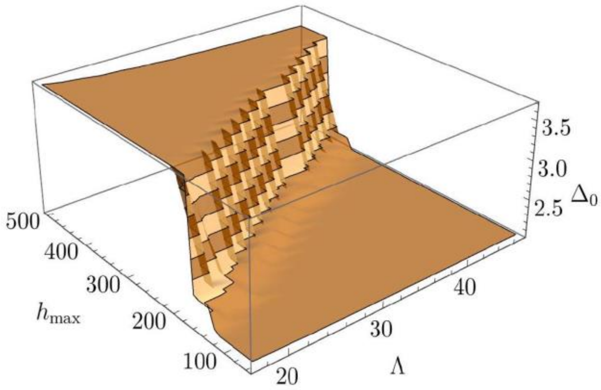
$$U = \frac{16r^2}{(r^2+2\eta r+1)^2}, V = \frac{(r^2-2\eta r+1)^2}{(r^2+2\eta r+1)^2}$$

$$D_1: r \leq -\sqrt{4|\eta| + \eta^2 + 3} + |\eta| + 2, |\eta| \leq 1$$

$$D_2: \quad r \geq -\sqrt{4|\eta| + \eta^2 + 3} + |\eta| + 2, 0 \leq \eta \leq 1$$
$$D_3: \quad r \geq -\sqrt{4|\eta| + \eta^2 + 3} + |\eta| + 2, -1 \leq \eta \leq 0$$

$$I_2\left[\frac{G_{\Delta+4,\ell}}{U^2}\right] = -3\int_{D_1} dr d\eta \frac{\sqrt{1-\eta^2}(r^2-1)^2\left((2-4\eta^2)r^2 + r^4 + 1\right)}{128\pi r^5} G_{\Delta+4,\ell}(r,\eta)$$

$$I_4\left[\frac{G_{\Delta+4,\ell}}{U^2}\right] = 3\int_{D_1} dr d\eta \left[\frac{\sqrt{1-\eta^2}(r^2-1)^2(2\eta r - r^2 - 1)\left((4\eta^2+10)r^2 + r^4 + 1\right)}{4\pi r^5(r^2+2\eta r+1)} \times \bar D_{1,1,1,1} G_{\Delta+4,\ell}(r,\eta)\right]$$



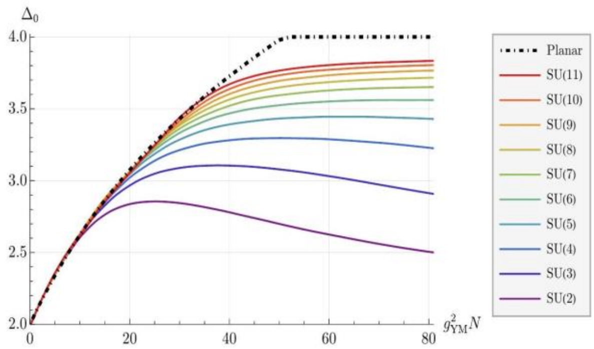
$$\partial_z^m \partial_{\bar{z}}^n G_{\ell+h,\ell}\big|_{z=\bar{z}=1/2} \underset{h\to\infty}{\sim} h^{m+n}(4(3-2\sqrt{2}))^h.$$

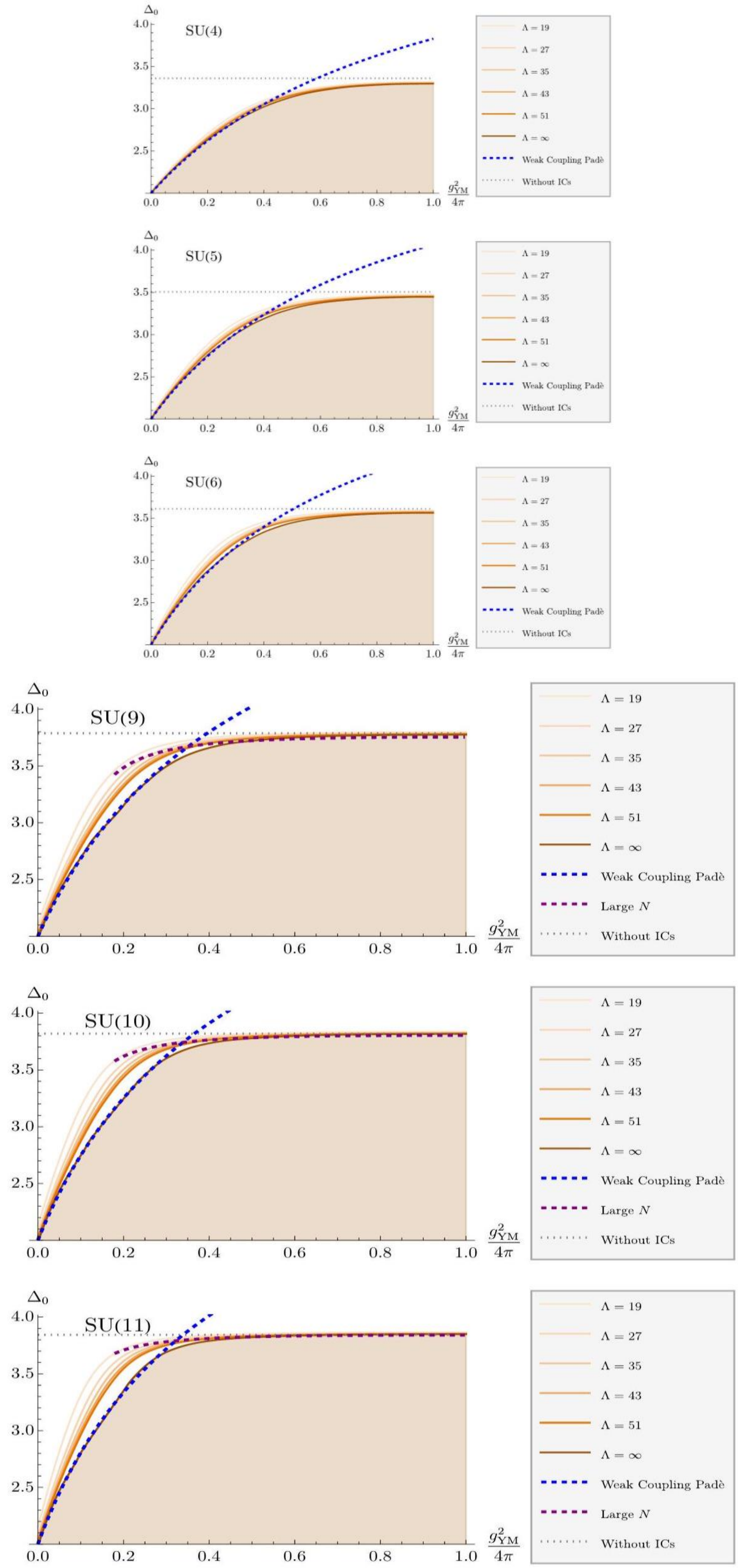
$$h^{\Lambda}(4(3-2\sqrt{2}))^h \approx (4(2-\sqrt{3}))^h$$

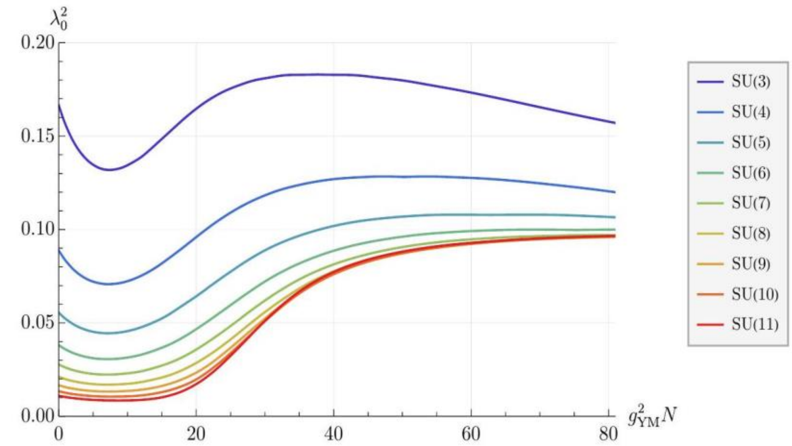
$$h_{\max} \approx -\frac{\Lambda}{C} W_{-1}\left(-\frac{C}{\Lambda}\right)$$

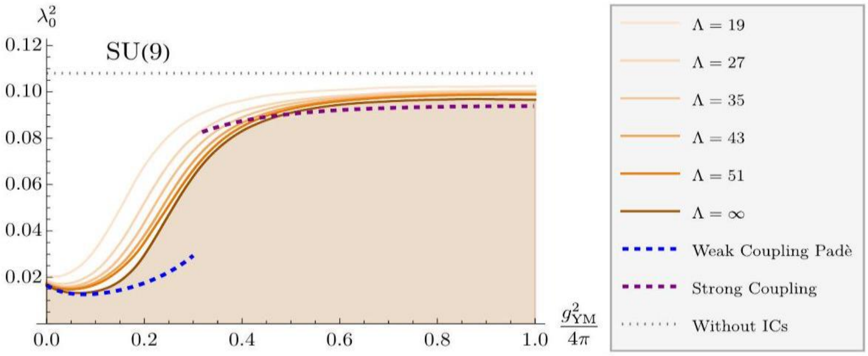
$$C = \log\frac{2-\sqrt{3}}{3-2\sqrt{2}} \approx 0.446$$

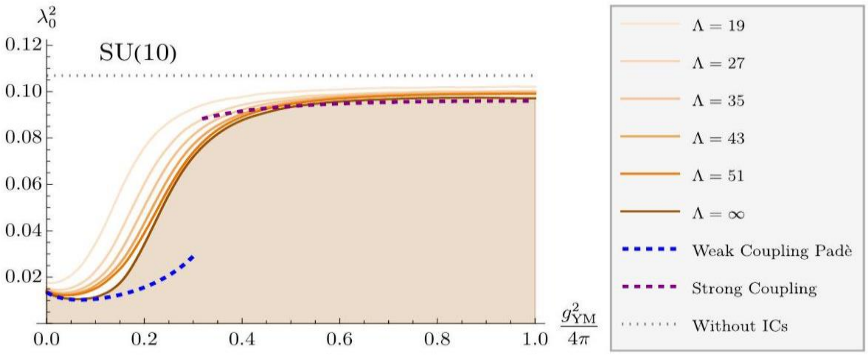
$$h_{\max} \approx \frac{\Lambda}{C}\left(\log\frac{\Lambda}{C} + \log\left(\log\frac{\Lambda}{C}\right)\right).$$

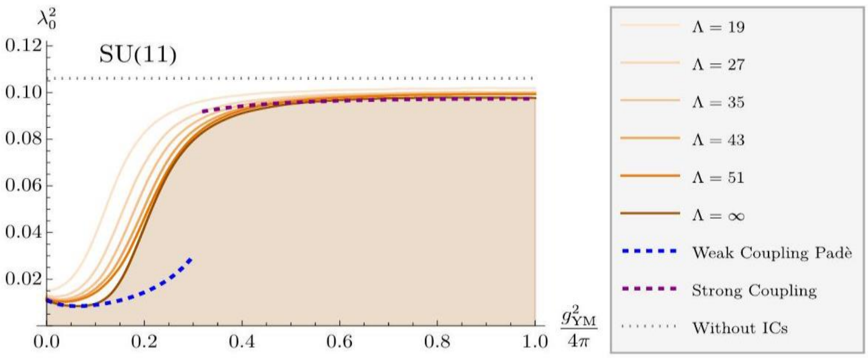



SU(4)
SU(5)
SU(6)
SU(9)
SU(10)
SU(11)
Δ0
g²YM / 4π
Λ = 19
Λ = 27
Λ = 35
Λ = 43
Λ = 51
Λ = ∞
Weak Coupling Padè
Large N
Without ICs


λ_0^2
0.20
0.15
0.10
0.05
0.00
0
20
40
60
80
$g_{\text{YM}}^2 N$
SU(3)
SU(4)
SU(5)
SU(6)
SU(7)
SU(8)
SU(9)
SU(10)
SU(11)


λ_0^2
SU(9)
0.12
0.10
0.08
0.06
0.04
0.02
0.0
0.2
0.4
0.6
0.8
1.0
$\frac{g_{\text{YM}}^2}{4\pi}$
$\Lambda = 19$
$\Lambda = 27$
$\Lambda = 35$
$\Lambda = 43$
$\Lambda = 51$
$\Lambda = \infty$
Weak Coupling Padè
Strong Coupling
Without ICs


λ_0^2
SU(10)
0.12
0.10
0.08
0.06
0.04
0.02
0.0
0.2
0.4
0.6
0.8
1.0
$\frac{g_{\text{YM}}^2}{4\pi}$
$\Lambda = 19$
$\Lambda = 27$
$\Lambda = 35$
$\Lambda = 43$
$\Lambda = 51$
$\Lambda = \infty$
Weak Coupling Padè
Strong Coupling
Without ICs


λ_0^2
SU(11)
0.12
0.10
0.08
0.06
0.04
0.02
0.0
0.2
0.4
0.6
0.8
1.0
$\frac{g_{\text{YM}}^2}{4\pi}$
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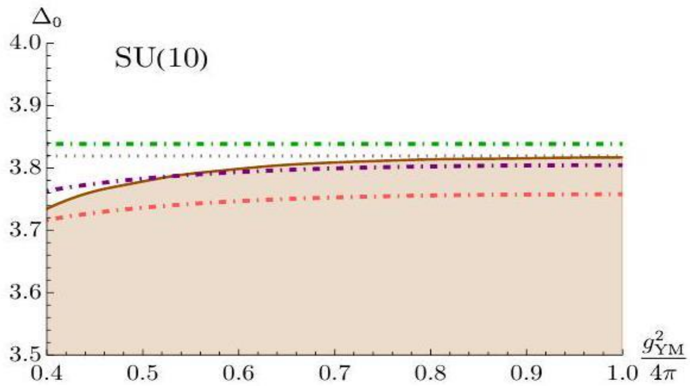
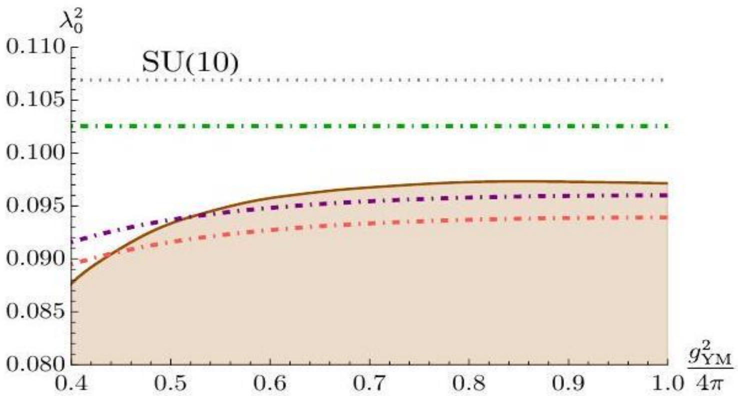




$$
\begin{aligned}
\mathcal{T}^{(0)}_{\text{short}} =& -1 - \frac{1}{(1-z)^2(1-\bar{z})^2} + \frac{24\log(1-z)\log(1-\bar{z})}{z^2\bar{z}^2} \\
&+ \frac{6(-2z\bar{z}(z^2+z\bar{z}+\bar{z}^2-4) + (z+\bar{z})(z^2\bar{z}^2+z^2+\bar{z}^2-6)+4)}{(1-z)^2 z(1-\bar{z})^2\bar{z}} \\
&+ \frac{2\big(z(2\bar{z}^4-\bar{z}^3+4\bar{z}^2-18\bar{z}+12) - 3(\bar{z}^4-6\bar{z}^2+4\bar{z})\big)\log(1-z)}{z^2(1-\bar{z})^2\bar{z}(z-\bar{z})} \\
&+ \frac{2(3(z^4-6z^2+4z) - (2z^4-z^3+4z^2-18z+12)\bar{z})\log(1-\bar{z})}{(1-z)^2 z\bar{z}^2(z-\bar{z})}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}^{(1)}_{\text{short}} =& -\frac{1}{(1-z)(1-\bar{z})} + \frac{36\log(1-z)\log(1-\bar{z})}{z^2\bar{z}^2} - \frac{2\left(\frac{9\bar{z}-18}{z^2\bar{z}} + \frac{4}{z-\bar{z}} - \frac{4}{z}\right)\log(1-z)}{1-\bar{z}} \\
&- \frac{2\left(\frac{9z-18}{z\bar{z}^2} - \frac{4}{z-\bar{z}} - \frac{4}{\bar{z}}\right)\log(1-\bar{z})}{1-z} + \frac{18\left(\frac{1}{(1-z)(1-\bar{z})}+1\right)}{z\bar{z}}
\end{aligned}
$$

$$
I_2[\mathcal{T}_{\text{short}}] \approx 0.0462845727 + \frac{0.3895281312}{c}
$$

$$
I_4[\mathcal{T}_{\text{short}}] \approx 5.60637758 + \frac{50.86596767}{c}
$$

$$
\mathcal{T}^{R|R}(U,V) = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}-2} \Gamma\left[2-\frac{s}{2}\right]^2 \Gamma\left[2-\frac{t}{2}\right]^2 \Gamma\left[2-\frac{u}{2}\right]^2 M^{R|R}(s,t)
$$

$$
M^{R|R}(s,t) = \sum_{m,n=2}^{\infty} \left[\frac{c_{mn}}{(s-2m)(t-2n)} + \frac{c_{mn}}{(t-2m)(u-2n)} + \frac{c_{mn}}{(u-2m)(s-2n)} - b_{mn}\right] + C
$$

$$c_{mn}^{SU(N)}=\frac{(m-1)^2m^2}{5(m+n-1)}+\frac{2(m-1)^2(3m^2-6m+8)}{5(m+n-2)}-\frac{9m^4-54m^3+123m^2-126m+44}{5(m+n-3)}$$
$$-\frac{4(m^2-4m+9)(m-2)^2}{5(m+n-4)}+\frac{6(m-3)^2(m-2)^2}{5(m+n-5)}$$

$$b_{mn}=\frac{9mn}{2(m+n)^3}, C=-\frac{39}{16}-\frac{13}{8}\pi^2+9\zeta(3)$$

$$\Phi(s,t)=\sum_{m,n=2}^{\infty}\left[\frac{c_{mn}}{(s-2m)(t-2n)}-\frac{3mn}{2(m+n)^3}+\frac{3mt-4m+3ns-4n}{4(m+n)^3}\right]$$

$$\Phi(s,t)=R_0(s,t)\left(\psi^{(1)}\left(2-\frac{s}{2}\right)+\psi^{(1)}\left(2-\frac{t}{2}\right)-\left(\psi^{(0)}\left(2-\frac{s}{2}\right)-\psi^{(0)}\left(2-\frac{t}{2}\right)\right)^2\right)$$
$$+R_1(s,t)\psi^{(0)}\left(2-\frac{s}{2}\right)+R_1(t,s)\psi^{(0)}\left(2-\frac{t}{2}\right)+R_2(s,t)$$

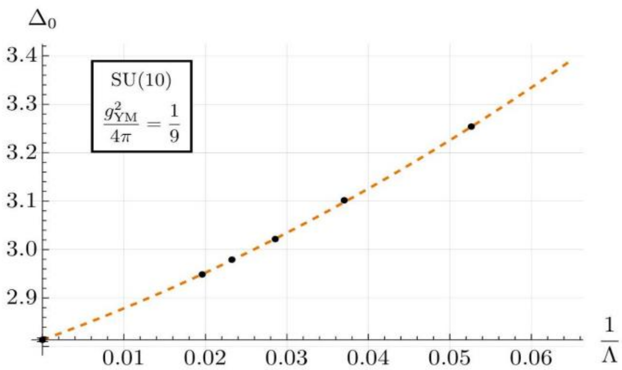
$$R_0(s,t)=\frac{P_0(s,t)}{40(s+t-10)(s+t-8)(s+t-6)(s+t-4)(s+t-2)}$$
$$R_1(s,t)=\frac{P_1(s,t)}{40(s+t-10)(s+t-8)(s+t-4)(s+t-2)}$$
$$R_2(s,t)=\frac{P_2(s,t)}{240(s+t-10)(s+t-8)(s+t-6)(s+t-4)(s+t-2)}-3\zeta(3)$$

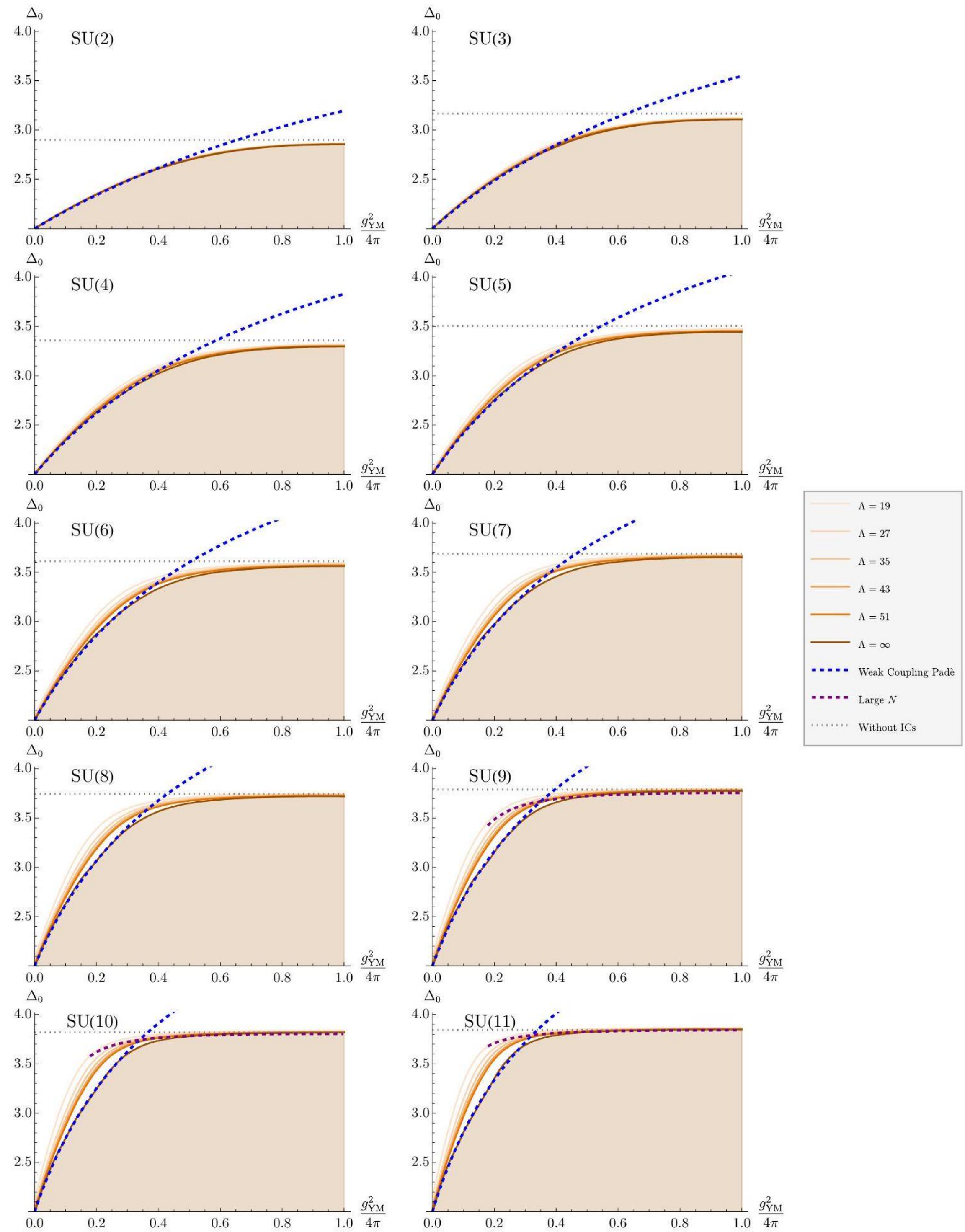
$$P_0(s,t)=15s^4t^2+30s^3t^3-360s^3t^2+15s^2t^4-360s^2t^3+2304s^2t^2-70s^4t+1096s^3t$$
$$-5048s^2t+88s^4-1024s^3+3552s^2-70st^4+1096st^3-5048st^2+8640st$$
$$-4736s+88t^4-1024t^3+3552t^2-4736t+2048$$
$$P_1(s,t)=-105s^3t^2-45s^2t^3+1200s^2t^2-75s^4t+1350s^3t-7720s^2t-15s^5+400s^4$$
$$-3532s^3+13008s^2+250st^3-3980st^2+17200st-21248s-368t^3$$
$$+4192t^2-13312t+12800$$
$$P_2(s,t)=-45s^4t^2-630s^3t^2-45s^2t^4-630s^2t^3+19404s^2t^2-72s^5t+585s^4t$$
$$+8640s^3t-125604s^2t-27s^6+477s^5-1062s^4-28908s^3+239688s^2-72st^5+585st^4$$
$$+8640st^3-125604st^2+520848st-683424s-27t^6+477t^5-1062t^4-28908t^3$$
$$+239688t^2-683424t+642816+\pi^2(45s^4t^2+520s^3t^2+45s^2t^4+520s^2t^3-14544s^2t^2$$
$$+54s^5t-400s^4t-7056s^3t+89808s^2t+9s^6-164s^5-648s^4+25984s^3-172656s^2$$
$$+54st^5-400st^4-7056st^3+89808st^2-354528st+458560s+9t^6-164t^5-648t^4$$
$$+25984t^3-172656t^2+458560t-419328)$$

$$\mathcal{T}^{R|R}(U,1)=\lambda_{4,0}^2U^2+\cdots$$

$$a=-\int_{-i\infty}^{i\infty}\frac{dt}{8\pi i}\Gamma\left[2-\frac{t}{2}\right]^2\operatorname{Res}\left[\Gamma\left[\frac{s+t}{2}\right]^2\Gamma\left[2-\frac{s}{2}\right]^2M^{R|R}(s,t)\right]_{s=4}$$

$$\{\ell+2,\ell+2.04,\dots,\ell+4,\ell+4.1,\dots,\ell+6,\ell+6.2,\dots,\ell+10,\ell+11,\dots,\ell+h_{\max}\}.$$





SU(2)
SU(3)
SU(4)
SU(5)
SU(6)
SU(7)
SU(8)
SU(9)
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Δ_0
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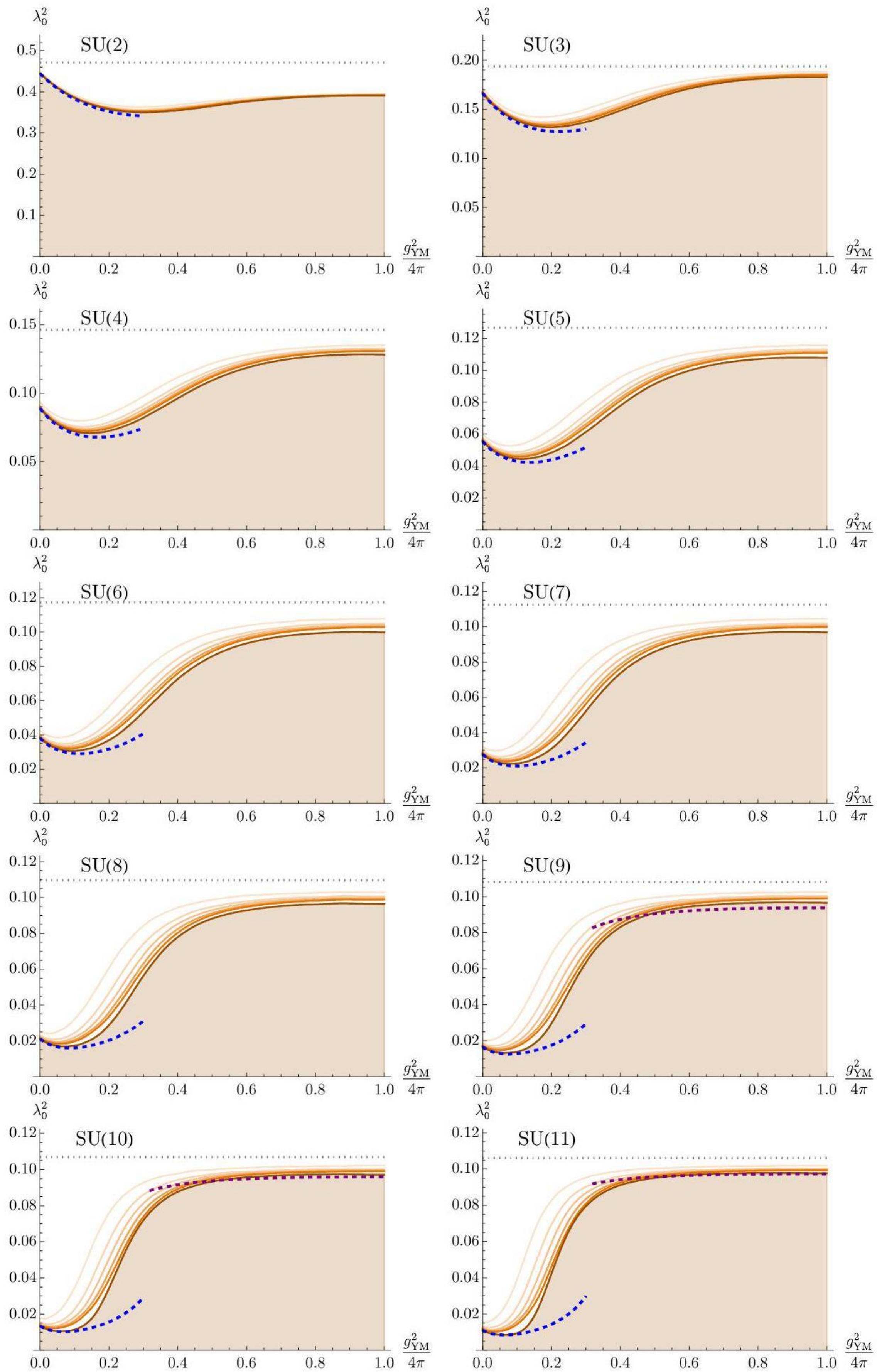
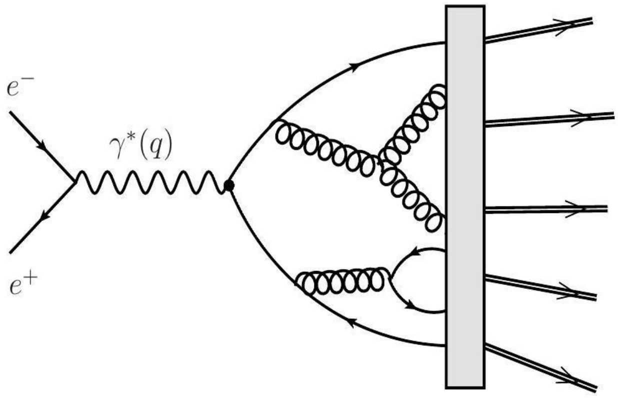


SU(2)
SU(3)
SU(4)
SU(5)
SU(6)
SU(7)
SU(8)
SU(9)
SU(10)
SU(11)
λ_0^2
$\frac{g_{YM}^2}{4\pi}$



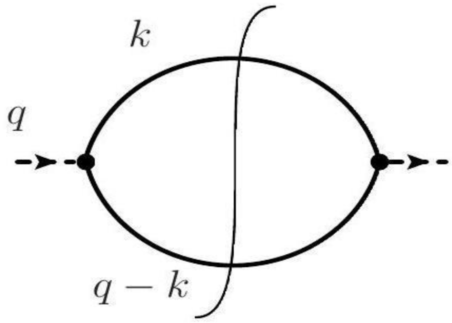
$$O_{\mathbf{20}'}^{IJ}(x) = \mathrm{tr}\left[\Phi^I\Phi^J - \frac{1}{6}\delta^{IJ}\Phi^K\Phi^K\right]$$

$$O(x,Y) = Y^I Y^J O_{\mathbf{20}}^{IJ}(x) = Y^I Y^J \mathrm{tr}[\Phi^I(x)\Phi^J(x)]$$

$$\langle X| \int d^4x \mathrm{e}^{iqx} O(x,Y)|0\rangle = (2\pi)^4 \delta^{(4)}(q-k_X)\mathcal{M}_{O_{20'}\to X}$$

$$\mathcal{M}_{O_{20'}\to X} = \langle X|O(0,Y)|0\rangle$$

$$\sigma_{\mathrm{tot}}(q) = \sum_X (2\pi)^4\delta^{(4)}(q-k_X)\left|\mathcal{M}_{O_{20'}\to X}\right|^2$$



$$\sigma_{\mathrm{tot}}(q) = \frac{1}{2}(N_c^2-1)(Y\bar{Y})^2 \int \frac{d^4k}{(2\pi)^4}(2\pi)^2\delta_+(k^2)\delta_+((q-k)^2) + \cdots$$

$$\sigma_{\mathrm{tot}}(q) = \int d^4x \mathrm{e}^{iqx} \sum_X \langle 0|O(0,\bar{Y})|X\rangle \mathrm{e}^{-ixk_X}\langle X|O(0,Y)|0\rangle$$
$$= \int d^4x \mathrm{e}^{iqx}\langle 0|O(x,\bar{Y})O(0,Y)|0\rangle$$

$$\sigma_{\mathrm{tot}}(q) = \mathrm{Im}\left[2i\int d^4x \mathrm{e}^{iqx}\langle 0|TO(x,\bar{Y})O(0,Y)|0\rangle\right]$$

$$\langle 0|TO(x,\bar{Y})O(0,Y)|0\rangle = \frac{1}{2}(N_c^2-1)(Y\bar{Y})^2[D_F(x)]^2$$

$$\sigma_{\mathrm{tot}}(q) = \frac{1}{16\pi}(N_c^2-1)(Y\bar{Y})^2\theta(q^0)\theta(q^2)$$

$$\sigma_W(q) = \sigma_{\mathrm{tot}}^{-1}\sum_X (2\pi)^4\delta^{(4)}(q-k_X)w(X)\left|\mathcal{M}_{O_{20'}\to X}\right|^2$$
$$= \sigma_{\mathrm{tot}}^{-1}\int d^4x \mathrm{e}^{iqx}\sum_X \langle 0|O(x,\bar{Y})|X\rangle w(X)\langle X|O(0,Y)|0\rangle$$

$$\left|\mathcal{M}_{O_{20'}\to X}\right|^2 \sim \mathrm{e}^{-f(g^2)/\epsilon^2} \to 0$$

$$w_{\mathcal{E}}(k_1, \ldots, k_\ell) = \sum_{i=1}^{\ell} k_i^0 \delta^{(2)}\left(\Omega_{\vec{k}_i} - \Omega_{\vec{n}}\right)$$

$$\Omega_{\vec{k}_i} = \vec{k}_i / |\vec{k}_i|$$

$$\mathcal{E}(\vec{n})|X\rangle = w_{\mathcal{E}}(X)|X\rangle$$

$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\vec{n})$$

$$\mathcal{E}(\vec{n}) = \int \frac{d^4k}{(2\pi)^4} 2\pi\delta_+(k^2) k_0 \delta^{(2)}(\Omega_{\vec{n}} - \Omega_{\vec{k}}) \sum_{p=\mathrm{s},\lambda,\,\mathrm{g}} a_p^{b\dagger}(k) a_p^b(k)$$

$$[\mathcal{E}(\vec{n}), \mathcal{E}(\vec{n}')] = 0,\ \text{for } \vec{n} \neq \vec{n}'.$$

$$\mathcal{E}(\vec{n}_1) \ldots \mathcal{E}(\vec{n}_\ell)|X\rangle = w_{\mathcal{E}(\vec{n}_1)}(X) \ldots w_{\mathcal{E}(\vec{n}_\ell)}(X)|X\rangle \equiv w(X)|X\rangle.$$

$$\langle \mathcal{E}(\vec{n}_1) \ldots \mathcal{E}(\vec{n}_\ell)\rangle \equiv \sigma_{\mathcal{E}}(q; \vec{n}_1, \ldots, \vec{n}_\ell)$$
$$= \sigma_{\mathrm{tot}}^{-1} \int d^4x \mathrm{e}^{iqx} \langle 0|O(x, \bar{Y})\mathcal{E}(\vec{n}_1) \ldots \mathcal{E}(\vec{n}_\ell) O(0, Y)|0\rangle$$

$$\mathcal{Q}_A^B(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 (J_0)_A^B(t, r\vec{n})$$

$$\mathcal{Q}_A^B(\vec{n}) = \int \frac{d^4k}{(2\pi)^4} 2\pi\delta_+(k^2)\delta^{(2)}(\Omega_{\vec{n}} - \Omega_{\vec{k}})$$
$$\times \left[a_{AC}^\dagger(k) a^{CB}(k) + a_{A,1/2}^\dagger(k) a_{-1/2}^B(k) - a_{-1/2}^{B,\dagger}(k) a_{A,1/2}(k)\right] - (\aleph)$$

$$|k\rangle_E \equiv a_{E,1/2}^\dagger(k) a_{AC}^\dagger(k) |\delta_B^A \mathcal{Q}_A^B(\vec{n}) a_{A,1/2}^\dagger a_{-1/2}^{B\dagger}\rangle$$

$$\mathcal{Q}_A^B(\vec{n})|k\rangle_E = \delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}}) \left[\delta_E^B |k\rangle_A - \frac{1}{4}\delta_A^B |k\rangle_E\right]$$

$$\mathcal{Q}(\vec{n}; Q) = Q_B^A \mathcal{Q}_A^B(\vec{n}).$$

$$Q_B^A = \sum_{\alpha=1}^4 Q_\alpha \bar{u}_\alpha^A u_B^\alpha, \sum_{A=1}^4 \bar{u}_\alpha^A u_A^\beta = \delta_\beta^\alpha, \sum_{\alpha=1}^4 \bar{u}_\alpha^A u_B^\alpha = \delta_B^A.$$

$$\sum_{\alpha=1}^4 Q_\alpha = |k\rangle_\alpha = \bar{u}_\alpha^A |k\rangle_A$$

$$\mathcal{Q}(\vec{n}; Q)|k\rangle_\alpha = Q_\alpha \delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}})|k\rangle_\alpha$$

$$\sum_A |k\rangle_A \langle k|^A = \sum_\alpha |k\rangle_A \bar{u}_\alpha^A u_B^\alpha \left\langle k\right|^B = \sum_\alpha \left| k\right\rangle_\alpha \langle k|^\alpha$$

$$\mathcal{Q}(\vec{n}; Q) \sum_{A=1}^4 |k\rangle_A \langle k|^A = \left(\delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}}) Q_A^B |k\rangle_B\right)\langle k|^A$$
$$= \mathcal{Q}(\vec{n}; Q) \sum_\alpha |k\rangle_\alpha \left\langle k\right|^\alpha = \sum_{\alpha=1}^4 \left| k\right\rangle_\alpha \left(Q_\alpha \delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}})\right)\langle k|^\alpha.$$

$$[\mathcal{Q}(\vec{n}; Q), \mathcal{Q}(\vec{n}'; Q')] = [\mathcal{Q}(\vec{n}; Q), \mathcal{E}(\vec{n}')] = 0.$$

$$\langle \mathcal{Q}(\vec{n}_1)\ldots\mathcal{Q}(\vec{n}_\ell)\rangle \equiv \sigma_{\mathcal{Q}}(q;\vec{n}_1,\ldots,\vec{n}_\ell;Q_1,\ldots,Q_\ell;Y)$$
$$= \sigma_{\text{tot}}^{-1}\int d^4x e^{iqx}\langle 0|O(x,\bar{Y})\mathcal{Q}(\vec{n}_1,Q_1)\ldots\mathcal{Q}(\vec{n}_\ell,Q_\ell)O(0,Y)|0\rangle$$

$$\mathcal{O}^{IJ}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 O_{20'}^{IJ}(t,r\vec{n})$$

$$\Delta_{\mathcal{O}} = -1, \Delta_{\mathcal{Q}} = 0, \Delta_{\mathcal{E}} = 1$$

$$\mathcal{O}^{IJ}(\vec{n}) = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} 2\pi\delta_+(k^2)k_0^{-1}\delta^{(2)}(\Omega_{\vec{n}} - \Omega_{\vec{k}})a^{\dagger\{I}(k)a^{J\}}(k),$$

$$\mathcal{O}(\vec{n};S) = S_{IJ}\mathcal{O}^{IJ}(\vec{n})$$

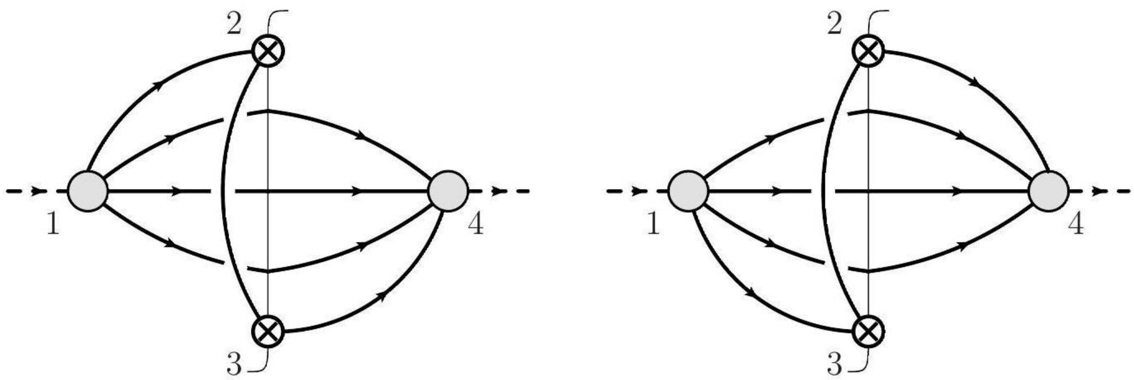
$$S_{IJ} = \sum_{i=1}^{6} S_i\phi_I^i\phi_J^i, \sum_{I=1}^{6}\phi_I^i\phi_I^j = \delta^{ij}, \sum_{i=1}^{6}\phi_I^i\phi_J^i = \delta_{IJ}$$

$$\mathcal{O}(\vec{n};S) = \frac{1}{2}\sum_{i=1}^{6} S_i \int \frac{d^4k}{(2\pi)^4} 2\pi\delta_+(k^2)k_0^{-1}\delta^{(2)}(\Omega_{\vec{n}} - \Omega_{\vec{k}})\big(\phi^i a^\dagger(k)\big)\big(\phi^i a(k)\big),$$

$$\mathcal{O}(\vec{n};S)\sum_{I=1}^{6}|k\rangle^I \left\langle k|^I = (2k_0)^{-1}\delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}})\sum_{I,J=1}^{6} S_{IJ} \mid k\right\rangle^J \langle k|^I$$

$$= \mathcal{O}(\vec{n};S)\sum_{i=1}^{6}|k\rangle^i \left\langle k|^i = \sum_{i=1}^{6} \mid k\right\rangle^i \left(\frac{S_i}{2k_0}\delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}})\right)\langle k|^i.$$

$$\mathcal{O}(\vec{n};S)\mathcal{O}(\vec{n}';S')|k\rangle \sim \delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}})\delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}'})|k\rangle = 0$$



$$[\mathcal{O}(\vec{n};S),\mathcal{O}(\vec{n}';S')] \sim \frac{a^{\dagger I}(0)(SS' - S'S)_{IJ}a^J(0)}{1 - (\vec{n}\vec{n}')}.$$

$$[\mathcal{O}(\vec{n};S),\mathcal{E}(\vec{n}')] = [\mathcal{O}(\vec{n};S),\mathcal{Q}(\vec{n}';Q)] = 0.$$

$$[S,S'] = 0.$$

$$\langle \mathcal{O}(\vec{n}_1)\ldots\mathcal{O}(\vec{n}_\ell)\rangle \equiv \sigma_S(q;\vec{n}_1,\ldots,\vec{n}_\ell;S_1,\ldots,S_\ell;Y)$$
$$= \sigma_{\text{tot}}^{-1}\int d^4x e^{iqx}\langle 0|O(x,\bar{Y})\mathcal{O}(\vec{n}_1,S_1)\ldots\mathcal{O}(\vec{n}_\ell,S_\ell)O(0,Y)|0\rangle$$

$$\sigma_0 = \left|\mathcal{M}_{O_{20'}\to ss}\right|^2 = \left|\langle s(k_1)s(k_2)|O(0,Y)|0\rangle\right|^2 = \frac{1}{2}(N_c^2 - 1)(Y\bar{Y})^2.$$

$$\left|\mathcal{M}_{O_{20'}\to\mathrm{ssg}}\right|^2 = |\langle \mathrm{s}(k_1)\mathrm{s}(k_2)\mathrm{g}(k_3)|O(0,Y)|0\rangle|^2 = g^2\sigma_0 \frac{4s_{12}}{s_{13}s_{23}}$$
$$\left|\mathcal{M}_{O_{20'}\to\mathrm{s}\lambda\lambda}\right|^2 = |\langle \lambda(k_1)\lambda(k_2)\mathrm{s}(k_3)|O(0,Y)|0\rangle|^2 = g^2\sigma_0 \frac{8}{s_{12}}$$

$$\sigma_{\mathrm{tot}}(q) = \int \mathrm{dPS}_2 \left|\mathcal{M}_{O_{20'}\to\mathrm{ss}}\right|^2 + \int \mathrm{dPS}_3 \left(\left|\mathcal{M}_{O_{20'}\to\mathrm{ssg}}\right|^2 + \left|\mathcal{M}_{O_{20'}\to\mathrm{s}\lambda\lambda}\right|^2\right) + O(g^4)$$
$$= \frac{\sigma_0}{8\pi}[1 + g^2 F_{\mathrm{virt}}(q^2)] + 4g^2\sigma_0 \int \mathrm{dPS}_3 \frac{s_{12}^2 + 2s_{13}s_{23}}{s_{12}s_{13}s_{23}} + O(g^4)$$

$$\int \mathrm{dPS}_\ell = \int \left(\prod_{i=1}^{\ell} \frac{d^4k_i}{(2\pi)^4} 2\pi\delta_+(k_i^2)\right)(2\pi)^4\delta^{(4)}\left(q - \sum_{i=1}^{\ell} k_i\right)$$

$$\sigma_{\mathrm{tot}}(q) = \frac{\sigma_0}{8\pi} + g^2\frac{\sigma_0}{8\pi}\left[F_{\mathrm{virt}}(q^2) + \int \mathrm{dPS}_3 \frac{32\pi(q^2)^2}{3s_{12}s_{13}s_{23}}\right] + O(g^4)$$

$$\sigma_{\mathcal{E}}(q;\vec{n}) = \sigma_{\mathrm{tot}}^{-1} \int \mathrm{dPS}_2 w_{\mathcal{E}}(k_1,k_2) \left|\mathcal{M}_{O_{20'}\to\mathrm{ss}}\right|^2$$
$$+\sigma_{\mathrm{tot}}^{-1} \int \mathrm{dPS}_3 w_{\mathcal{E}}(k_1,k_2,k_3)\left(\left|\mathcal{M}_{O_{20'}\to\mathrm{ssg}}\right|^2 + \left|\mathcal{M}_{O_{20'}\to\mathrm{s}\lambda\lambda}\right|^2\right) + O(g^4)$$
$$\langle\mathcal{E}(\vec{n})\rangle \equiv \sigma_{\mathcal{E}}(q;\vec{n}) = \frac{1}{8\pi}\int \mathrm{dPS}_2 \sum_{i=1,2} k_i^0\delta^{(2)}\left(\Omega_{\vec{k}_i} - \Omega_{\vec{n}}\right) = \frac{1}{4\pi}q^0$$

$$\left|\mathcal{M}_{O_{20'}\to\mathrm{ss}}\right|^2 \sim y_{A_1B_1}y_{A_2B_2}\bar{y}^{A_1B_1}\bar{y}^{A_2B_2} = \mathrm{tr}(y\bar{y})^2$$

$$y_{A_1B_1}\left[2Q_{A_1'}^{A_1}\delta^{(2)}\left(\Omega_{\vec{k}_1} - \Omega_{\vec{n}}\right)\right]\bar{y}^{A_1'B_1}y_{A_2B_2}\bar{y}^{A_2B_2} + y_{A_1B_1}\bar{y}^{A_1B_1}y_{A_2B_2}\left[2Q_{A_2'}^{A_2}\delta^{(2)}\left(\Omega_{\vec{k}_2} - \Omega_{\vec{n}}\right)\right]\bar{y}^{A_2'B_2}$$
$$= 2\mathrm{tr}(y\bar{y})\mathrm{tr}(yQ\bar{y})\left[\delta^{(2)}\left(\Omega_{\vec{k}_1} - \Omega_{\vec{n}}\right) + \delta^{(2)}\left(\Omega_{\vec{k}_2} - \Omega_{\vec{n}}\right)\right],$$

$$\int \mathrm{dPS}_2 \left|\mathcal{M}_{O_{20'}\to ss}\right|^2$$

$$\langle Q(\vec{n})\rangle \equiv \sigma_Q(q;(\vec{n},Q);y) = \frac{\langle Q\rangle}{4\pi}\int \mathrm{dPS}_2 \sum_{i=1,2} \delta^{(2)}\left(\Omega_{\vec{k}_i} - \Omega_{\vec{n}}\right) = \frac{1}{\pi}\langle Q\rangle$$

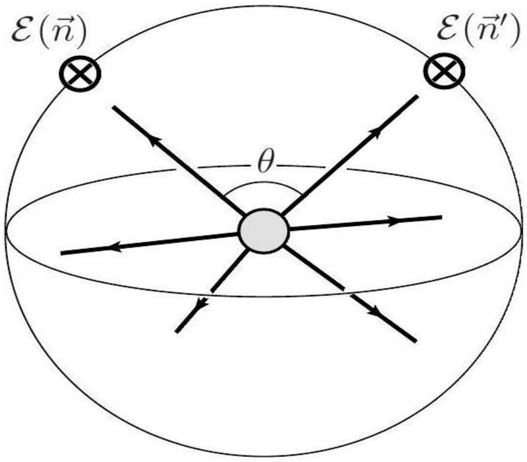
$$\langle Q\rangle = \frac{\mathrm{tr}(yQ\bar{y})}{\mathrm{tr}(y\bar{y})}$$

$$Y^I\delta^{IJ}\bar{Y}^J \to Y^I\left(\left(k_i^0\right)^{-1}\delta^{(2)}\left(\Omega_{\vec{k}_i} - \Omega_{\vec{n}}\right)\frac{1}{2}S^{IJ}\right)\bar{Y}^J$$

$$\langle\mathcal{O}(\vec{n})\rangle \equiv \sigma_{\mathcal{O}}(q;(\vec{n},S);Y) = \frac{\langle S\rangle}{16\pi}\int \mathrm{dPS}_2 \sum_{i=1,2} \left(k_i^0\right)^{-1}\delta^{(2)}\left(\Omega_{\vec{k}_i} - \Omega_{\vec{n}}\right) = \frac{1}{2\pi}\frac{\langle S\rangle}{q^0},$$

$$\langle S\rangle = \frac{(YS\bar{Y})}{(Y\bar{Y})},$$

$$\langle\mathcal{E}(\vec{n})\mathcal{E}(\vec{n}')\rangle^{(0)} = \frac{q_0^2}{8\pi}[\delta(\theta) + \delta(\pi - \theta)]$$



$$\langle\mathcal{E}(\vec{n})\mathcal{E}(\vec{n}')\rangle = \sigma_{\rm tot}^{-1}\int \mathrm{dPS}_3 \sum_{i,j=1}^{3} k_i^0 k_j^0 \delta^{(2)}\left(\Omega_{\vec{k}_i}-\Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_j}-\Omega_{\vec{n}'}\right)$$
$$\times\left(\left|\mathcal{M}_{O_{20'}\to s(k_1)s(k_2)g(k_3)}\right|^2+\left|\mathcal{M}_{O_{20'}\to s(k_1)\lambda(k_2)\lambda(k_3)}\right|^2\right)$$

$$\langle\mathcal{E}(\vec{n})\mathcal{E}(\vec{n}')\rangle = \frac{g^2}{2(2\pi)^4}\frac{q_0^2}{\sin^2\theta}\int_0^1\frac{d\tau_1}{1-\tau_1(1-\cos\theta)/2}+O(g^4)$$
$$=\frac{g^2}{(2\pi)^4}q_0^2\frac{1+\cos\theta}{\sin^4\theta}\ln\frac{2}{1+\cos\theta}+O(g^4)$$

$$z=(1-\cos\theta)/2,$$

$$\langle\mathcal{E}(\vec{n})\mathcal{E}(\vec{n}')\rangle = \frac{a}{4\pi^2}\frac{q_0^2}{8z^3}\left(-\frac{z\ln(1-z)}{1-z}\right)+O(a^2),$$

$$\langle\mathcal{E}(\vec{n})\mathcal{E}(\vec{n}')\rangle_{\rm QCD} = \frac{a_{\rm QCD}}{4\pi^2}\frac{q_0^2}{8z^3}\left[\left(-\frac{z}{1-z}+\frac{9}{z^2}-\frac{15}{z}+3\right)\ln(1-z)\right.$$
$$\left.+\left(\frac{9}{z}-\frac{3}{2(1-z)}-9\right)\right]+O\left(a_{\rm QCD}^2\right)$$

$$a_{\rm QCD}=g_{\rm QCD}^2C_2/(4\pi^2)$$

$$\langle\mathcal{O}(\vec{n})\mathcal{O}(\vec{n}')\rangle = \sigma_{\rm tot}^{-1}\int \mathrm{dPS}_3(k_1^0k_2^0)^{-1}2\delta^{(2)}\left(\Omega_{\vec{k}_1}-\Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_2}-\Omega_{\vec{n}'}\right)$$
$$\times(YS\bar{Y})(YS'\bar{Y})\left|\mathcal{M}_{O_{2'}\to s(k_1)s(k_2)g(k_3)}\right|^2$$

$$\langle\mathcal{O}(\vec{n})\mathcal{O}(\vec{n}')\rangle = \frac{a}{4\pi^2}\frac{\langle S\rangle\langle S'\rangle}{2q_0^2z}\left(-\frac{z\ln(1-z)}{1-z}\right)+O(a^2),$$

$$\langle\mathcal{Q}(\vec{n})\mathcal{Q}(\vec{n}')\rangle = 2\sigma_{\rm tot}^{-1}\int \mathrm{dPS}\Big[4\mathrm{tr}(yQ\bar{y})\mathrm{tr}(yQ'\bar{y})\delta^{(2)}\left(\Omega_{\vec{k}_1}-\Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_2}-\Omega_{\vec{n}'}\right)$$
$$\times\left(\left|\mathcal{M}_{O_{20'}\to s(k_1)s(k_2)g(k_3)}\right|^2+\left|\mathcal{M}_{O_{20'}\to s(k_1)\lambda(k_2)\lambda(k_3)}\right|^2\right)$$
$$+\mathrm{tr}[y\bar{y}]\mathrm{tr}[yQ\bar{y}Q']\delta^{(2)}\left(\Omega_{\vec{k}_2}-\Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_3}-\Omega_{\vec{n}'}\right)\left|\mathcal{M}_{O_{20'}\to s(k_1)\lambda(k_2)\lambda(k_3)}\right|^2\Big]$$

$$\langle\mathcal{Q}(\vec{n})\mathcal{Q}(\vec{n}')\rangle = -\frac{a}{\pi^2}\frac{\ln(1-z)}{4z^2}\left(\frac{2z}{1-z}\langle Q\rangle\langle Q'\rangle+\langle Q,Q'\rangle\right)+O(a^2),$$

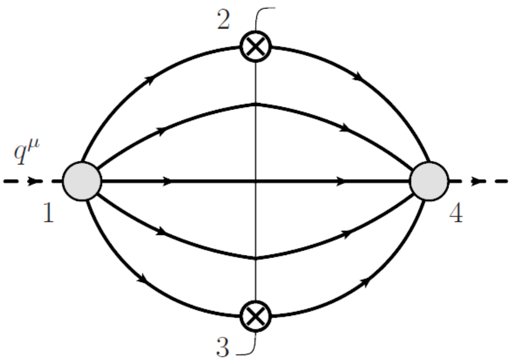
$$\langle Q,Q'\rangle\equiv\frac{\mathrm{tr}[yQ\bar{y}Q'+\tilde{\bar{y}}Q\tilde{y}Q']}{2\mathrm{tr}[y\bar{y}]}=\frac{\mathrm{tr}[yQ'\bar{y}Q+\tilde{\bar{y}}Q'\tilde{y}Q]}{2\mathrm{tr}[y\bar{y}]},$$

$$\tilde{\bar{y}}_{AB} = \frac{1}{2}\epsilon_{ABCD}\bar{y}^{CD} \text{ and } \tilde{y}^{AB} = \frac{1}{2}\epsilon^{ABCD}y_{CD}$$

$$\langle \mathcal{Q}(\vec{n})\mathcal{E}(\vec{n}')\rangle = \frac{a}{4\pi^2}\langle Q\rangle q^0\left(-\frac{\ln(1-z)}{z^2(1-z)}\right) + O(a^2)$$

$$\langle \mathcal{O}(\vec{n})\mathcal{E}(\vec{n}')\rangle = \frac{a}{16\pi^2}\langle S\rangle\left(-\frac{\ln(1-z)}{z^2(1-z)}\right) + O(a^2)$$

$$\langle \mathcal{Q}(\vec{n})\mathcal{O}(\vec{n}')\rangle = \frac{a}{4\pi^2}\langle Q\rangle\langle S'\rangle(q^0)^{-1}\left(-\frac{\ln(1-z)}{z(1-z)}\right) + O(a^2)$$



$$\Longrightarrow \quad \text{Limit}_{2,3}\ \langle 0|O(1)O(2)O(3)O(4)|0\rangle$$

$$x^\mu = x_+ n^\mu + x_- \bar{n}^\mu, n^\mu = (1,\vec{n}), \bar{n}^\mu = (1,-\vec{n})$$

$$n^\mu \to \rho n^\mu, \bar{n}^\mu \to \rho' \bar{n}^\mu,$$

$$x_+ = \frac{(x\bar{n})}{(n\bar{n})}, x_- = \frac{(xn)}{(n\bar{n})}.$$

$$\mathcal{E}(n) = (n\bar{n})\int_{-\infty}^{\infty} dx_- \lim_{x_+\to\infty} x_+^2 T_{++}(x_+ n + x_- \bar{n})$$

$$T_{++} \equiv \bar{n}^\mu \bar{n}^\nu T_{\mu\nu}(x)/(n\bar{n})^2$$

$$\mathcal{Q}_A^B(n) = (n\bar{n})\int_{-\infty}^{\infty} dx_- \lim_{x_+\to\infty} x_+^2 (J_+)_A^B(x_+ n + x_- \bar{n})$$

$$\mathcal{O}^{IJ}(n) = (n\bar{n})\int_{-\infty}^{\infty} dx_- \lim_{x_+\to\infty} x_+^2 O_{\mathbf{20}'}^{IJ}(x_+ n + x_- \bar{n})$$

$$J_+(x) \equiv \bar{n}^\mu J_\mu(x)/(n\bar{n})$$

$$\mathcal{E}(n) = \frac{1}{2(2\pi)^3}\int_0^\infty d\tau\tau^2 \sum_{i=s,q,g} a_i^\dagger(n\tau)a_i(n\tau)$$

$$\mathcal{E}(\rho n) = \rho^{-3}\mathcal{E}(n), \mathcal{Q}_A^B(\rho n) = \rho^{-2}\mathcal{Q}_A^B(n), \mathcal{O}^{IJ}(\rho n) = \rho^{-1}\mathcal{O}^{IJ}(n)$$

$$\langle \mathcal{D}_1(n_1)\ldots\mathcal{D}_\ell(n_\ell)\rangle_q = \sigma_{\text{tot}}^{-1}\int d^4x e^{iqx}\langle 0|O(x,\bar{Y})\mathcal{D}_1(n_1)\ldots\mathcal{D}_\ell(n_\ell)O(0,Y)|0\rangle$$

$$\langle \mathcal{D}_1(\Lambda n_1)\ldots\mathcal{D}_\ell(\Lambda n_\ell)\rangle_{\Lambda q} = \langle \mathcal{D}_1(n_1)\ldots\mathcal{D}_\ell(n_\ell)\rangle_q$$

$$\langle \mathcal{D}_1(\rho_1 n_1)\ldots\mathcal{D}_\ell(\rho_\ell n_\ell)\rangle_q = \rho_1^{-1-s_1}\ldots\rho_\ell^{-1-s_\ell}\langle \mathcal{D}_1(n_1)\ldots\mathcal{D}_\ell(n_\ell)\rangle_q$$

$$\langle \mathcal{D}_1(n_1)\ldots\mathcal{D}_\ell(n_\ell)\rangle_{\lambda q} = \lambda^{\Delta_1+\cdots+\Delta_\ell}\langle \mathcal{D}_1(n_1)\ldots\mathcal{D}_\ell(n_\ell)\rangle_q$$

$$\mathcal{O}(n) = \mathcal{D}(n;0), \mathcal{Q}(n) = \mathcal{D}(n;1), \mathcal{E}(n) = \mathcal{D}(n;2)$$

$$\langle \mathcal{D}(n;s)\rangle = c_{\mathcal{D}}\frac{(q^2)^s}{(qn)^{s+1}}$$

$$\int d\Omega_{\vec{n}}\langle \mathcal{Q}(\vec{n})\rangle = 4\langle Q\rangle$$

$$c_{\mathcal{E}} = 1/\int d\Omega_{\vec{n}} = \frac{1}{4\pi}, c_{\mathcal{Q}} = \frac{\langle Q\rangle}{\pi}$$

$$z = \frac{q^2(nn')}{2(qn)(qn')}$$

$$\langle \mathcal{D}(n;s)\mathcal{D}(n';s')\rangle = \frac{(q^2)^{s'-1}(qn')^{s-s'}}{(nn')^{s+1}}\frac{\mathcal{F}_{ss'}(z)}{4\pi^2}$$

$$\begin{aligned}
\langle \mathcal{E}(n)\mathcal{E}(n')\rangle &= \frac{q^2\mathcal{F}_{\mathcal{E}\mathcal{E}}(z)}{4\pi^2(nn')^3}, \quad \langle \mathcal{E}(n)\mathcal{Q}(n')\rangle = \frac{(qn')\mathcal{F}_{\mathcal{E}\mathcal{Q}}(z)}{4\pi^2(nn')^3}\\
\langle \mathcal{Q}(n)\mathcal{Q}(n')\rangle &= \frac{\mathcal{F}_{\mathcal{Q}\mathcal{Q}}(z)}{4\pi^2(nn')^2}, \quad \langle \mathcal{E}(n)\mathcal{O}(n')\rangle = \frac{(qn')^2\mathcal{F}_{\mathcal{E}\mathcal{O}}(z)}{4\pi^2q^2(nn')^3}\\
\langle \mathcal{O}(n)\mathcal{O}(n')\rangle &= \frac{\mathcal{F}_{\mathcal{O}\mathcal{O}}(z)}{4\pi^2q^2(nn')}, \quad \langle \mathcal{Q}(n)\mathcal{O}(n')\rangle = \frac{(qn')\mathcal{F}_{\mathcal{Q}\mathcal{O}}(z)}{4\pi^2q^2(nn')^2}
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{\mathcal{E}\mathcal{E}}(z) &= -a\frac{z\ln(1-z)}{1-z} + O(a^2)\\
\mathcal{F}_{\mathcal{Q}\mathcal{Q}}(z) &= -4a\left(\frac{2z\ln(1-z)}{1-z}\langle Q\rangle\langle Q'\rangle + \ln(1-z)\langle Q,Q'\rangle\right) + O(a^2)\\
\mathcal{F}_{\mathcal{O}\mathcal{O}}(z) &= -a\frac{z\ln(1-z)}{1-z}\langle S\rangle\langle S'\rangle + O(a^2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{\mathcal{E}\mathcal{Q}}(z) &= -8a\frac{z\ln(1-z)}{1-z}\langle Q'\rangle + O(a^2)\\
\mathcal{F}_{\mathcal{E}\mathcal{O}}(z) &= -2a\frac{z\ln(1-z)}{1-z}\langle S'\rangle + O(a^2)\\
\mathcal{F}_{\mathcal{Q}\mathcal{O}}(z) &= -4a\frac{z\ln(1-z)}{1-z}\langle Q\rangle\langle S'\rangle + O(a^2)
\end{aligned}$$

$$\langle \mathcal{O}(n)\rangle = \sigma_{\text{tot}}^{-1}\int d^4x_1 \mathrm{e}^{iqx_1}\langle 0|O(x_1,\bar{Y})S^{IJ}\mathcal{O}^{IJ}(n)O(0,Y)|0\rangle$$

$$\begin{aligned}
\langle \mathcal{O}(n)\rangle =& \frac{S^{IJ}}{\sigma_{\text{tot}}}\int d^4x_1 \mathrm{e}^{iqx_1}\int_{-\infty}^{\infty} dx_{2-}(n\bar{n})\\
&\times \lim_{x_{2+}\to\infty} x_{2+}^2\langle 0|O(x_1,\bar{Y})O^{IJ}_{\mathbf{20'}}(x_{2+}n + x_{2-}\bar{n})O(0,Y)|0\rangle_W
\end{aligned}$$

$$G_E(1,2,3) = \langle 0|O(x_1,Y_1)O(x_2,Y_2)O(x_3,Y_3)|0\rangle_E$$

$$G_E(1,2,3) = (N_c^2-1)(Y_1Y_2)(Y_2Y_3)(Y_1Y_3)D_E(x_{12})D_E(x_{23})D_E(x_{13})$$

$$G_W(1,2,3) = G_E(1,2,3)|_{D_E(x)\to D_W(x)}$$

$$\langle 0|\Phi^I(x_i)\Phi^J(x_j)|0\rangle = \delta^{IJ}D_W(x_{ij}) = -\frac{\delta^{IJ}}{4\pi^2}\frac{1}{x_{ij}^2 - i\epsilon x_{ij}^0}$$

$$(Y_1Y_2)(Y_2Y_3)(Y_1Y_3) \to (Y\bar{Y})(YS\bar{Y}) \equiv (Y\bar{Y})^2\langle S\rangle$$

$$x_{12}^2 - i0x_{12}^0 \to 2x_{2+}(x_{2-}(n\bar{n}) - (x_1n) + i0)$$

$$x_2^2 - i\epsilon x_2^0 \to 2x_{2+}(x_{2-}(n\bar{n}) - i\epsilon)$$

$$\lim_{x_{2+}\to\infty} x_{2+}^2 G_W(1,2,3) \sim \frac{(Y\bar{Y})^2\langle S\rangle}{(x_{2-}(n\bar{n}) - (x_1n) + i\epsilon)(x_{2-}(n\bar{n}) - i\epsilon)(x_1^2 - i\epsilon x_1^0)}.$$

$$(n\bar{n})\int_{-\infty}^{\infty} dx_{2-}\lim_{x_{2+}\to\infty} x_{2+}^2 G_W(1,2,3) \sim \frac{(Y\bar{Y})^2\langle S\rangle}{\left((x_1n) - i\epsilon\right)(x_1^2 - i\epsilon x_1^0)}$$

$$\langle\mathcal{O}(n)\rangle=\sigma_{\text{tot}}^{-1}\int d^4x\mathrm{e}^{iqx}\int_{-\infty}^{\infty}dx_{2-}(n\bar{n})\lim_{x_{2+}\to\infty}x_{2+}^2G_W(1,2,3)=\frac{1}{2\pi}\frac{\langle S\rangle}{(qn)}$$

$$\langle\mathcal{Q}(n)\rangle=\frac{Q_A^B}{\sigma_{\text{tot}}}\int d^4x\mathrm{e}^{iqx}\langle 0|O(x,\bar{Y})\mathcal{Q}_B^A(n)O(0,Y)|0\rangle$$

$$Q_A^B\bar{n}_\mu\langle 0|O(x_1,\bar{Y})(J^\mu)_B^A(x_2)O(x_3,Y)|0\rangle_W,$$

$$\begin{aligned}G_E^\mu &\equiv\langle 0|O^{I_1J_1}(x_1)(J^\mu)_B^A(x_2)O^{I_3J_3}(x_3)|0\rangle_E\\ &=-i\frac{N_c^2-1}{16\pi^6}\frac{\left(\Gamma^{\{I_1\{I_3}\right)_B^A\delta^{J_1\}J_3\}}}{x_{12}^2x_{23}^2x_{13}^2}\left(\frac{x_{12}^\mu}{x_{12}^2}+\frac{x_{23}^\mu}{x_{23}^2}\right)\end{aligned}$$

$$(Y\bar{Y})\bar{Y}^I(\Gamma^{IJ})_B^AQ_A^BY^J=\frac{1}{4}\mathrm{tr}(y\bar{y})\mathrm{tr}(yQ\bar{y})$$

$$\lim_{x_{2+}\to\infty}x_{2+}^2G_W{}^\mu\sim\frac{n^\mu(x_1n)}{\left((x_1n)-x_{2-}(n\bar{n})-i\epsilon\right)^2(x_{2-}(n\bar{n})-i\epsilon)^2(x_1^2-i\epsilon x_1^0)}.$$

$$(n\bar{n})\int_{-\infty}^{\infty}dx_{2-}\lim_{x_{2+}\to\infty}x_{2+}^2\bar{n}_\mu G_W^\mu\sim\frac{1}{\left((x_1n)-i\epsilon\right)^2(x_1^2-i\epsilon x_1^0)}$$

$$\langle\mathcal{Q}(n)\rangle=\sigma_{\text{tot}}^{-1}\int d^4x_1\mathrm{e}^{iqx_1}\int_{-\infty}^{\infty}dx_{2-}(n\bar{n})\lim_{x_{2+}\to\infty}x_{2+}^2\bar{n}_\mu G_W{}^\mu=\frac{\langle Q\rangle}{\pi}\frac{q^2}{(qn)^2}$$

$$\langle\mathcal{E}(n)\rangle=\sigma_{\text{tot}}^{-1}\int d^4x\mathrm{e}^{iqx}\langle 0|O(x,\bar{Y})\mathcal{E}(n)O(0,Y)|0\rangle$$

$$\begin{aligned}(G_E)^{\mu\nu}(1,2,3)&=\langle 0|O(x_1,\bar{Y})T^{\mu\nu}(x_2)O(x_3,Y)|0\rangle_E\\ &=-\frac{N_c^2-1}{16\pi^6}\frac{(Y\bar{Y})^2}{x_{12}^2x_{23}^2x_{13}^2}\left(\frac{x_{12}^\mu}{x_{12}^2}+\frac{x_{23}^\mu}{x_{23}^2}\right)\left(\frac{x_{12}^\nu}{x_{12}^2}+\frac{x_{23}^\nu}{x_{23}^2}\right)\end{aligned}$$

$$\langle\mathcal{E}(n)\rangle=\frac{1}{4\pi}\frac{(q^2)^2}{(qn)^3}$$

$$\mathcal{T}(x,Y,\theta,\bar{\theta})=O(x,Y)+\cdots+(\theta\sigma^\mu\bar{\theta})J_\mu(x,Y)+\cdots+(\theta\sigma^\mu\bar{\theta})(\theta\sigma^\nu\bar{\theta})T_{\mu\nu}(x)+\cdots,$$

$$\mathcal{G}_E(1,2,3,4)=\langle 0|\mathcal{T}(x_1,Y_1,\theta_1,\bar{\theta}_1)\ldots\mathcal{T}(x_4,Y_4,\theta_4,\bar{\theta}_4)|0\rangle.$$

$$G_E(1,2,3,4)=\langle 0|O(x_1,Y_1)O(x_2,Y_2)O(x_3,Y_3)O(x_4,Y_4)|0\rangle.$$

$$\begin{aligned}G_E^{(\text{Born})}(1,2,3,4)&=\frac{N_c^2-1}{(4\pi^2)^4}\left(\frac{y_{12}^2}{x_{12}^2}\frac{y_{23}^2}{x_{23}^2}\frac{y_{34}^2}{x_{34}^2}\frac{y_{14}^2}{x_{14}^2}+\frac{y_{13}^2}{x_{13}^2}\frac{y_{23}^2}{x_{23}^2}\frac{y_{24}^2}{x_{24}^2}\frac{y_{14}^2}{x_{14}^2}+\frac{y_{12}^2}{x_{12}^2}\frac{y_{24}^2}{x_{24}^2}\frac{y_{34}^2}{x_{34}^2}\frac{y_{13}^2}{x_{13}^2}\right)\\ &+\frac{(N_c^2-1)^2}{4(4\pi^2)^4}\left(\frac{y_{12}^4}{x_{12}^4}\frac{y_{34}^4}{x_{34}^4}+\frac{y_{13}^4}{x_{13}^4}\frac{y_{24}^4}{x_{24}^4}+\frac{y_{14}^4}{x_{14}^4}\frac{y_{23}^4}{x_{23}^4}\right)\end{aligned}$$

$$\begin{aligned}G_E^{(\text{loop})}(1,2,3,4)&=\frac{2(N_c^2-1)}{(4\pi^2)^4}\left[\frac{y_{12}^2y_{23}^2y_{34}^2y_{41}^2}{x_{12}^2x_{23}^2x_{34}^2x_{41}^2}(1-u-v)+\frac{y_{12}^2y_{13}^2y_{24}^2y_{34}^2}{x_{12}^2x_{13}^2x_{24}^2x_{34}^2}(v-u-1)\right.\\ &\left.+\frac{y_{13}^2y_{14}^2y_{23}^2y_{24}^2}{x_{13}^2x_{14}^2x_{23}^2x_{24}^2}(u-v-1)+\frac{y_{12}^4y_{34}^4}{x_{12}^4x_{34}^4}u+\frac{y_{13}^4y_{24}^4}{x_{13}^4x_{24}^4}+\frac{y_{14}^4y_{23}^4}{x_{14}^4x_{23}^4}v\right]\Phi_E(u,v)\end{aligned}$$

$$\Phi_E(u,v)=\sum_{\ell=1}^{\infty}a^\ell\Phi_\ell(u,v),u=\frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2},v=\frac{x_{23}^2x_{14}^2}{x_{13}^2x_{24}^2}$$

$$\Phi_E(u,v)=a\Phi^{(1)}(u,v)+O(a^2)=-\frac{a}{4\pi^2}\int\frac{d^4x_0x_{13}^2x_{24}^2}{x_{10}^2x_{20}^2x_{30}^2x_{40}^2}+O(a^2)$$

$$\hat{G}_E^{(\text{Born})}=\frac{N_c^2-1}{(4\pi^2)^4}\frac{y_{12}^2}{x_{12}^2}\frac{y_{24}^2}{x_{24}^2}\frac{y_{13}^2}{x_{13}^2}\frac{y_{34}^2}{x_{34}^2}+\frac{(N_c^2-1)^2}{4(4\pi^2)^4}\left(\frac{y_{12}^4}{x_{12}^4}\frac{y_{34}^4}{x_{34}^4}+\frac{y_{13}^4}{x_{13}^4}\frac{y_{24}^4}{x_{24}^4}\right),$$

$$\hat{G}_E^{(\text{loop})} = \frac{2(N_c^2-1)}{(4\pi^2)^4}[y_{12}^2y_{13}^2y_{24}^2y_{34}^2(v-u-1)+y_{12}^4y_{34}^4+y_{13}^4y_{24}^4u]\frac{\Phi_E(u,v)}{x_{12}^2x_{13}^2x_{24}^2x_{34}^2}$$

$$y_{12}^2y_{13}^2y_{24}^2y_{34}^2 \to (YS\bar{Y})(YS'\bar{Y})$$
$$(y_{12}^2y_{34}^2)^2 \to (YSY)(\bar{Y}S'\bar{Y})$$
$$(y_{13}^2y_{24}^2)^2 \to (YS'Y)(\bar{Y}S\bar{Y})$$

$$\langle \mathcal{O}(n)\mathcal{O}(n')\rangle^{(\text{Born})} = \sigma_{\text{tot}}^{-1}\int d^4x_1 e^{iqx_1}\int_{-\infty}^{\infty} dx_{2-}dx_{3-}\lim_{x_{2+},x_{3+}\to\infty}(x_{2+}x_{3+})^2\hat{G}_W^{(\text{Born})}$$
$$\sim \int d^4x_1 e^{iqx_1}\int_{-\infty}^{\infty}\frac{dx_{2-}dx_{3-}\langle S\rangle\langle S'\rangle}{(x_{12-}-i\epsilon)(x_{2-}-i\epsilon)(x_{13-}-i\epsilon)(x_{3-}-i\epsilon)}$$
$$\sim \int d^4x_1 e^{iqx_1}\frac{\langle S\rangle\langle S'\rangle}{\big((x_1n)-i\epsilon\big)\big((x_1n')-i\epsilon\big)}$$

$$x_{12-} = (x_1n) - x_{2-}, x_{13-} = (x_1n') - x_{3-}$$
$$\int_0^{\infty} dtdt'\delta^{(4)}(q-nt-n't')$$

$$\Phi_E(u,v) = \int_{-\delta-i\infty}^{-\delta+i\infty}\frac{dj_1dj_2}{(2\pi i)^2}M(j_1,j_2)u^{j_1}v^{j_2}$$

$$M(j_1,j_2;a) = \sum_{\ell\geq 1} a^{\ell}M^{(\ell)}(j_1,j_2)$$

$$M^{(1)}(j_1,j_2) = -\frac{1}{4}[\Gamma(-j_1)\Gamma(-j_2)\Gamma(j_1+j_2+1)]^2$$

$$M^{(\infty)}(j_1,j_2;a) = -[\Gamma(1-j_1)\Gamma(1-j_2)\Gamma(j_1+j_2+1)]^2\frac{1+j_1+j_2}{2j_1j_2}$$

$$\Phi_E(u,v) = \Phi_E(v,u) = \frac{1}{v}\Phi_E\left(\frac{u}{v},\frac{1}{v}\right)$$

$$M(j_1,j_2) = M(j_2,j_1) = M(j_1,-1-j_1-j_2).$$

$$\Phi_W(1,2,3,4) = \int_{-\delta-i\infty}^{-\delta+i\infty}\frac{dj_1dj_2}{(2\pi i)^2}M(j_1,j_2;a)(-x_{13}^2+i\epsilon x_{13}^0)^{-j_1-j_2}(-x_{24}^2+i\epsilon x_{24}^0)^{-j_1-j_2}$$
$$\times(-x_{12}^2+i\epsilon x_{12}^0)^{j_1}(-x_{34}^2+i\epsilon x_{34}^0)^{j_1}(-x_{23}^2+i\epsilon x_{23}^0)^{j_2}(-x_{14}^2+i\epsilon x_{14}^0)^{j_2}$$

$$\lim_{x_{2+},x_{3+}\to\infty}\Phi_W(1,2,3,4) = \int_{-\delta-i\infty}^{-\delta+i\infty}\frac{dj_1dj_2}{(2\pi i)^2}M(j_1,j_2;a)f(j_1,j_2+1)$$

$$f(j_1,j_2) = \big((nn')/2\big)^{-j_1-j_2}(-x_1^2+i\epsilon x_1^0)^{-j_1-j_2}$$
$$\times\big((x_1n)-x_{2-}(n\bar{n})-i\epsilon\big)^{j_1}(-x_{2-}(n\bar{n})+i\epsilon)^{j_2}$$
$$\times\big((x_1n')-x_{3-}(n'\bar{n}')-i\epsilon\big)^{j_2}(-x_{3-}(n'\bar{n}')+i\epsilon)^{j_1}$$

$$\lim_{x_{2+},x_{3+}\to\infty}(x_{2+}x_{3+})^2\hat{G}_W^{(\text{loop})} = \frac{4\sigma_{\text{tot}}}{(2\pi)^7(nn')^2}(-x_1^2+i\epsilon x_1^0)^{-2}\int_{-\delta-i\infty}^{-\delta+i\infty}\frac{dj_1dj_2}{(2\pi i)^2}M(j_1,j_2;a)$$
$$\times\left[\langle S\rangle\langle S'\rangle f(j_1-1,j_2-1)+\langle S,S'\rangle f(j_1-1,j_2)+\overline{\langle S,S'\rangle}f(j_1,j_2-1)\right]$$

$$\langle S,S'\rangle = \frac{(YSY)(\bar{Y}S'\bar{Y})-(YS\bar{Y})(YS'\bar{Y})}{(Y\bar{Y})^2}$$
$$\overline{\langle S,S'\rangle} = \langle S,S'\rangle^*, \langle S\rangle = \frac{(YS\bar{Y})}{(Y\bar{Y})}$$

$$\langle \mathcal{O}(n)\mathcal{O}(n')\rangle = \frac{1}{4\pi^2 q^2 (nn')}\int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} M(j_1,j_2;a)[\langle S\rangle\langle S'\rangle K(j_1,j_2;z) + \langle S,S'\rangle K(j_1,j_2+1;z) + \overline{\langle S,S'\rangle} K(j_1+1,j_2;z)\Big]$$

$$K(j_1,j_2;z) = \frac{1}{8\pi^5}\frac{q^2}{(nn')}\int \frac{d^4x_1 e^{iqx_1}}{(-x_1^2+i\epsilon x_1^0)^2} \times \int_{-\infty}^{\infty} dx_{2-}(n\bar{n})\int_{-\infty}^{\infty} dx_{3-}(n'\bar{n}') f(j_1-1,j_2-1;x_1,x_{2-},x_{3-})$$

$$K(j_1,j_2;z) = \left(\frac{z}{1-z}\right)^{1-j_1-j_2} \frac{2\pi}{\sin\left(\pi(j_1+j_2)\right)[\Gamma(j_1+j_2)\Gamma(1-j_1)\Gamma(1-j_2)]^2}$$

$$\mathcal{F}_{\mathcal{O}\mathcal{O}}(z) = \langle S\rangle\langle S'\rangle \mathcal{F}^{+}_{\mathcal{O}\mathcal{O}}(z) + \left(\langle S,S'\rangle + \overline{\langle S,S'\rangle}\right)\mathcal{F}^{-}_{\mathcal{O}\mathcal{O}}(z)$$

$$\mathcal{F}^{+}_{\mathcal{O}\mathcal{O}}(z) = \int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} M(j_1,j_2;a) K(j_1,j_2;z)$$

$$\mathcal{F}^{-}_{\mathcal{O}\mathcal{O}}(z) = \int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} M(j_1,j_2;a) K(j_1,j_2+1;z)$$

$$M(j_1,j_2;a)K(j_1,j_2;z) = -\frac{a}{2}\left(\frac{z}{1-z}\right)^{1-j_1-j_2} \frac{\pi(j_1+j_2)^2}{(j_1 j_2)^2 \sin\left(\pi(j_1+j_2)\right)} + O(a^2)$$

$$M(j_1,j_2;a)K(j_1,j_2+1;z) = \frac{a}{2}\left(\frac{z}{1-z}\right)^{-j_1-j_2} \frac{\pi}{j_1^2 \sin\left(\pi(j_1+j_2)\right)} + O(a^2)$$

$$\mathcal{F}^{+}_{\mathcal{O}\mathcal{O}}(z) = -\frac{a}{2}\int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} \frac{\pi j_2^2}{\left(j_1(j_2-j_1)\right)^2 \sin(\pi j_2)}\left(\frac{z}{1-z}\right)^{1-j_2} + O(a^2),$$

$$\mathcal{F}^{+}_{\mathcal{O}\mathcal{O}}(z) = -a\sum_{k=1}^{\infty}\frac{(-1)^k}{k}\left(\frac{z}{1-z}\right)^{1+k} + O(a^2) = -a\frac{z}{1-z}\ln(1-z) + O(a^2).$$

$$\mathcal{F}^{-}_{\mathcal{O}\mathcal{O}}(z) = \frac{a}{2}\int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} \frac{\pi}{j_1^2 \sin(\pi j_2)}\left(\frac{z}{1-z}\right)^{-j_2} + O(a^2)$$

$$\mathcal{F}^{-}_{\mathcal{O}\mathcal{O}}(z) = O(a^2)$$

$$\mathcal{F}_{\mathcal{A}\mathcal{B}}(z) = \sum_{R} \omega_R \mathcal{F}_{\mathcal{A}\mathcal{B};R}(z)$$

$$\mathcal{F}_{\mathcal{A}\mathcal{B};R}(z) = \int \frac{dj_1 dj_2}{(2\pi i)^2} M(j_1,j_2;a) K_{\mathcal{A}\mathcal{B};R}(j_1,j_2;z)$$

$$K_{\mathcal{E}\mathcal{E};\mathbf{1}} = K_{\mathcal{E}\mathcal{Q};\mathbf{15}} = K_{\mathcal{E}\mathcal{O};\mathbf{20}} = K(j_1,j_2;z).$$

$$20' \times 20' = 1 + 15 + 20' + 84 + 105 + 175,$$

$$K_{\mathcal{O}\mathcal{O};R}(j_1,j_2;z) = P_R^{\mathcal{O}\mathcal{O}}(t_1,t_2) K(j_1,j_2;z)$$

$$\left(P_1^{\mathcal{O}\mathcal{O}}, P_{15}^{\mathcal{O}\mathcal{O}}, P_{20'}^{\mathcal{O}}, P_{84}^{\mathcal{O}\mathcal{O}}, P_{175}^{\mathcal{O}\mathcal{O}}, P_{105}^{\mathcal{O}\mathcal{O}}\right) = (\mathcal{Y}_{105}, \mathcal{Y}_{175}, \mathcal{Y}_{84}, \mathcal{Y}_{20'}, \mathcal{Y}_{15}, \mathcal{Y}_{1})$$

$$K_{\mathcal{O}\mathcal{O};\mathbf{105}} = K(j_1,j_2;z)$$

$\langle \mathcal{O}\mathcal{O} \rangle$	$P_R^{\mathcal{O}\mathcal{O}}(t_1,t_2)$	$\mathcal{F}^{(1)}_{\mathcal{O}\mathcal{O};R}$	$\mathcal{F}^{(\infty)}_{\mathcal{O}\mathcal{O};R}$
1	$t_1^2+t_2^2+4t_1t_2-\frac{4}{5}(t_1+t_2)+\frac{1}{10}$	∞	∞
15	$t_1^2-t_2^2-\frac{1}{2}(t_1-t_2)$	0	0
20′	$(t_1-t_2)^2-\frac{1}{2}(t_1+t_2)+\frac{1}{10}$	$-\frac{1}{10}\frac{z\ln(1-z)+10(1-z)\ln z}{1-z}$	∞
84	$3(t_1+t_2)-1$	$\frac{z\ln(1-z)}{1-z}$	∞
175	t_1-t_2	0	0
105	1	$-\frac{z\ln(1-z)}{1-z}$	$2z^3$

$$15\times 20' = 15 + 20' + 45 + \overline{45} + 175$$

$\langle \mathcal{Q}\mathcal{O} \rangle$	$P_R^{\mathcal{Q}\mathcal{O}}(t_1,t_2)$	$\mathcal{F}^{(1)}_{\mathcal{Q}\mathcal{O};R}$	$\mathcal{F}^{(\infty)}_{\mathcal{Q}\mathcal{O};R}$
15	$\frac{1}{2}(t_1+t_2-\frac{1}{4})$	$\frac{z\ln(1-z)}{8(1-z)}$	∞
20′	$\frac{1}{2}(t_1-t_2)$	0	0
175	1	$-\frac{z\ln(1-z)}{1-z}$	$2z^3$

$$K_{\mathcal{Q}\mathcal{O},R}(j_1,j_2;z) = P_R^{\mathcal{Q}\mathcal{O}}(t_1,t_2)K(j_1,j_2;z),$$

$$15\times 15 = 1 + 15_s + 15_a + 20' + 45 + \overline{45} + 84,$$

$$K_{\mathcal{Q}\mathcal{Q},R}(j_1,j_2;z) = P_R^{\mathcal{Q}\mathcal{Q}}(t_1,t_2,z)K(j_1,j_2;z),$$

$$K_{\mathcal{E}\mathcal{E};\mathbf{1}} = K_{\mathcal{E}\mathcal{Q};\mathbf{15}} = K_{\mathcal{E}\mathcal{O};\mathbf{20}} = K_{\mathcal{Q}\mathcal{Q};\mathbf{84}} = K_{\mathcal{Q}\mathcal{O};\mathbf{175}} = K_{\mathcal{O}\mathcal{O};\mathbf{105}}.$$

$\langle QQ \rangle$	$P_R^{QQ}(t_1, t_2, z)$	$\mathcal{F}^{(1)}_{QQ;R}$	$\mathcal{F}^{(\infty)}_{QQ;R}$
1	$\frac{2}{3}\left(1 + 15(t_1+t_2) + 25\frac{1-z}{z}\right)$	$-\frac{2(25-24z)\ln(1-z)}{3(1-z)}$	∞
$\mathbf{15_a}$	$t_1 - t_2$	0	0
$\mathbf{15_s}$	0	0	0
$\mathbf{20'}$	$\frac{5}{3}\left(3 - \frac{4}{z}\right)$	$-\frac{5(3z-4)\ln(1-z)}{3(1-z)}$	$\frac{10(3z-4)z^2}{3}$
84	1	$-\frac{z\ln(1-z)}{1-z}$	$2z^3$

$$\mathcal{F}_{QQ}(z) = \sum_{R=\mathbf{1},\mathbf{15}_s,\mathbf{15}_a,\mathbf{20}',\mathbf{84}} \omega_R^{QQ}\mathcal{F}_{QQ;R}(z),$$

$$\int_0^\infty d\tau\tau^\ell\delta^{(4)}(k-n\tau) = 2k_0^{\ell-1}\delta_+(k^2)\delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}})$$

$$\mathcal{O}(\vec{n};S) = \frac{1}{2(2\pi)^3}\sum_{i=1}^{6} S_i\int_0^\infty d\tau\big(\phi^i a^\dagger(n\tau)\big)\big(\phi^i a(n\tau)\big)$$

$$\big(\phi^i a(n\tau)\big) \equiv \sum_{I=1}^{6}\phi_I^i a^I(n\tau)$$

$$\left[a^I(k), a^{\dagger J}(p)\right] = (2\pi)^3 2k_0\delta^{(3)}(\vec{k}-\vec{p})\delta^{IJ}$$

$$\begin{aligned}\left[\mathcal{O}(\vec{n};S), \mathcal{O}(\vec{n}';S')\right] = \frac{1}{2(2\pi)^3}\sum_{i,j=1}^{6} S_iS_j'\big(\phi^i\phi'^j\big)\int_0^\infty d\tau_1 d\tau_2\tau_2\delta^{(3)}(\tau_1\vec{n} - \tau_2\vec{n}')\\ \times\Big[\big(\phi^i a^\dagger(n\tau_1)\big)\big(\phi'^j a(n'\tau_2)\big) - \big(\phi'^j a^\dagger(n'\tau_2)\big)\big(\phi^i a(n\tau_1)\big)\Big]\end{aligned}$$

$$(\phi^i\phi'^j) = \Sigma_1^6\ \phi_I^i\phi_I'^j$$

$$\begin{aligned}&\int_0^\infty d\tau_1 d\tau_2\tau_2\delta^{(3)}(\tau_1\vec{n} - \tau_2\vec{n}')f(\tau_1,\tau_2)\\ &= \lim_{\epsilon\to 0}(\pi\epsilon)^{-3/2}\int_0^\infty d\tau_1 d\tau_2\tau_2 e^{-(\tau_1\vec{n}-\tau_2\vec{n}')^2/\epsilon}f(\tau_1,\tau_2)\\ &= \pi^{-3/2}\int_0^\infty d\tau_1 d\tau_2\tau_2 e^{-(\tau_1\vec{n}-\tau_2\vec{n}')^2}\lim_{\epsilon\to 0}f\big(\epsilon^{1/2}\tau_1, \epsilon^{1/2}\tau_2\big) = \frac{1}{4\pi}\frac{f(0,0)}{1-(\vec{n}\vec{n}')}\end{aligned}$$

$$\begin{aligned}\left[\mathcal{O}(\vec{n};S), \mathcal{O}(\vec{n}';S')\right] &= \frac{1}{4(2\pi)^4}\sum_{i,j=1}^{6}\frac{S_iS_j'(\phi^i\phi'^j)}{1-(\vec{n}\vec{n}')}\left[(\phi^i a^\dagger(0))(\phi'^j a(0)) - (\phi'^j a^\dagger(0))(\phi^i a(0))\right]\\ &= \frac{1}{4(2\pi)^4}\frac{a^{\dagger I}(0)(SS' - S'S)_{IJ}a^J(0)}{1-(\vec{n}\vec{n}')}\end{aligned}$$

$$y_{BA} = -y_{AB}, \tilde{y}^{AB} = \frac{1}{2}\epsilon^{ABCD}y_{CD}$$

$$Y_I = \frac{1}{\sqrt{2}}(\Sigma_I)^{AB}y_{AB}, y_{AB} = \frac{1}{\sqrt{2}}\epsilon_{ABCD}(\Sigma_I)^{CD}Y_I$$

$$\sum_{I=1}^{6}(\Sigma_I)^{AB}(\Sigma_I)^{CD}=\frac{1}{2}\epsilon^{ABCD},\frac{1}{2}\epsilon_{ABCD}(\Sigma_I)^{AB}(\Sigma_J)^{CD}=\delta_{IJ}$$

$$(\Gamma_{IJ})_C^{\ A}=-(\Gamma_{JI})_C^{\ A}=\frac{1}{2}(\Sigma_I)^{AB}(\bar{\Sigma}_J)_{BC}-(I\leftrightarrow J),(\Gamma_{IJ})_A^{\ A}=0$$

$$(Y_1Y_2)\equiv\sum_I Y_1^IY_2^I=\frac{1}{4}\epsilon^{ABCD}(y_1)_{AB}(y_2)_{CD}=\frac{1}{2}(y_1)_{AB}(\tilde{y}_2)^{AB}=\frac{1}{2}(y_2)_{AB}(\tilde{y}_1)^{AB}$$

$$\frac{1}{2}\epsilon^{ABCD}(y_1)_{AB}(y_1)_{CD}=(y_1)_{AB}(\tilde{y}_1)^{AB}=0$$

$$(Y\Phi)=\sum_I Y^I\Phi^I=y_{AB}\phi^{AB}=\tilde{y}^{AB}\tilde{\phi}_{AB}$$

$$\phi^{AB}=\frac{1}{\sqrt{2}}(\Sigma_I)^{AB}\Phi^I,\tilde{\phi}_{AB}=\frac{1}{2}\epsilon_{ABCD}\phi^{CD}$$

$$\bar{y}^{AB}=(y_{AB})^*\neq\tilde{y}^{AB}$$

$$[O(x,Y)]^\dagger=\mathrm{tr}[(\bar{Y}\Phi)^2]=O(x,\bar{Y})$$

$$O(x,Y)=\mathrm{tr}[(y_{AB}\phi^{AB})^2],O(x,\bar{Y})=\mathrm{tr}\left[\left(\bar{y}^{AB}\tilde{\phi}_{AB}\right)^2\right]$$

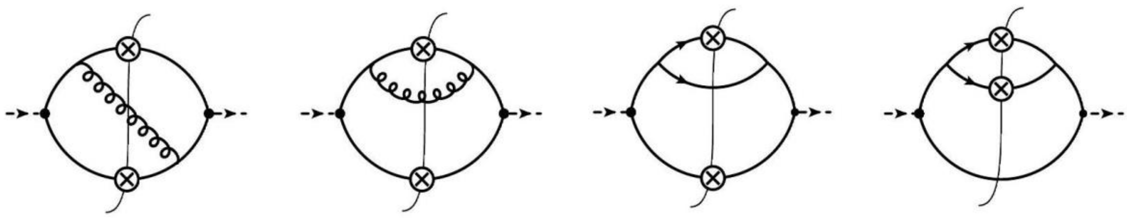
$$\phi^{AB}=\phi^{AB,a}T^a,\tilde{\phi}_{AB}=\tilde{\phi}_{AB}^aT^a$$

$$\mathrm{tr}[T^aT^b]=\frac{1}{2}\delta^{ab}$$

$$\langle O(x_1,\bar{Y})O(x_2,Y)\rangle=\frac{N_c^2-1}{2}\left[\frac{1}{2}y_{A_1B_1}\bar{y}^{A_1B_1}D(x_1-x_2)\right]^2=\frac{N_c^2-1}{2}[(Y\bar{Y})D(x_1-x_2)]^2,$$

$$\left\langle\Phi^{I,a}(x_1)\Phi^{J,b}(x_2)\right\rangle=\delta^{ab}\delta^{IJ}D(x_1-x_2)$$
$$\left\langle\phi^{A_1B_1,a_1}(x_1)\tilde{\phi}^{a_2}_{A_2B_2}(x_2)\right\rangle=\frac{1}{4}\delta^{a_1a_2}(\delta^{A_1}_{A_2}\delta^{B_1}_{B_2}-\delta^{A_1}_{B_2}\delta^{B_1}_{A_2})D(x_1-x_2)$$

$$D_F(x)=-\frac{1}{4\pi^2}\frac{1}{x^2-i\epsilon},D_W(x)=-\frac{1}{4\pi^2}\frac{1}{x^2-i\epsilon x_0}$$



(a) (b) (c) (d)

$$2\langle(\bar{Y}\Phi)\mathcal{O}(\vec{n},S)(Y\Phi)\rangle\langle(\bar{Y}\Phi)\mathcal{O}(\vec{n}',S')(Y\Phi)\rangle\sim 2\left(Y^IS_{IJ}\bar{Y}^J\right)\left(Y^{I'}S'_{I'J'}\bar{Y}^{J'}\right)=2(Y\bar{Y})^2\langle S\rangle\langle S'\rangle,$$

$\langle S\rangle=\left(Y^IS_{IJ}\bar{Y}^J\right)/(Y\bar{Y})$ and $(Y\bar{Y})=Y^I\bar{Y}^I$.

$$\langle\mathcal{O}(\vec{n})\mathcal{O}(\vec{n}')\rangle=\sigma_{\mathrm{tot}}^{-1}\int \mathrm{dPS}_3(k_1^0k_2^0)^{-1}(YS\bar{Y})(YS'\bar{Y})\left|\mathcal{M}_{O_{2'}\to s(k_1)s(k_2)g(k_3)}\right|^2$$
$$\times\left[\delta^{(2)}\left(\Omega_{\vec{k}_1}-\Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_2}-\Omega_{\vec{n}'}\right)+\delta^{(2)}\left(\Omega_{\vec{k}_1}-\Omega_{\vec{n}'}\right)\delta^{(2)}\left(\Omega_{\vec{k}_2}-\Omega_{\vec{n}}\right)\right]$$

$$s_{12}=q_0^2(1-\tau_3),s_{23}=q_0^2(1-\tau_1),s_{13}=q_0^2(1-\tau_2),$$

$$\int \mathrm{dPS}_3 \delta^{(2)}\left(\Omega_{\vec{k}_1} - \Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_2} - \Omega_{\vec{n}'}\right) = \frac{q_0^2}{64(2\pi)^5}\int_0^1 d\tau_1 d\tau_2 \tau_1\tau_2 \delta(1-\tau_1-\tau_2+\tau_1\tau_2 z)$$

$$\langle \mathcal{O}(\vec{n})\mathcal{O}(\vec{n}')\rangle = \frac{g^2}{32\pi^4}\frac{(YS\bar{Y})(YS'\bar{Y})}{q_0^2(Y\bar{Y})^2}\int_0^1 d\tau_1 d\tau_2 \frac{\tau_1+\tau_2-1}{(1-\tau_1)(1-\tau_2)}\delta(1-\tau_1-\tau_2+\tau_1\tau_2 z)$$

$$\mathcal{Q}(\vec{n}) \sim Q_B^A a_{AC}^{\dagger} a^{CB}$$

$$\begin{aligned} Q_{B_3}^{A_3}\langle y_{A_1B_1}\phi^{A_1B_1}a_{A_3C_3}^{\dagger}\rangle\langle a^{C_3B_3}\bar{y}^{A_2B_2}\bar{\phi}_{A_2B_2}\rangle(Q')_{B_4}^{A_4}\langle y_{A_1'B_1'}\phi^{A_1'B_1'}a_{A_4C_4}^{\dagger}\rangle\langle a^{C_4B_4}\bar{y}^{A_2'B_2'}\bar{\phi}_{A_2'B_2'}\rangle \\ \sim Q_{B_3}^{A_3}(Q')_{B_4}^{A_4}y_{A_1B_1}\bar{y}^{A_2B_2}y_{1,A_1'B_1'}\bar{y}^{A_2'B_2'}\delta_{A_3}^{A_1}\delta_{C_3}^{B_1}\delta_{A_2}^{C_3}\delta_{B_2}^{B_3}\delta_{A_4}^{A_1'}\delta_{C_4}^{B_1'}\delta_{A_2'}^{C_4}\delta_{B_2'}^{B_4} \\ \sim Q_{B_3}^{A_3}y_{A_3B_1}\bar{y}^{B_1B_3}(Q')_{B_4}^{A_4}y_{A_4B_1'}\bar{y}^{B_1'B_4} \equiv \mathrm{tr}(\bar{y}Qy)\mathrm{tr}(\bar{y}Q'y) \end{aligned}$$

$$\mathcal{Q}(\vec{n}) \sim Q_B^A a_{A,1/2}^{\dagger} a_{-1/2}^{B}$$

$$Q_{B_3}^{A_3}(Q')_{B_4}^{A_4}y_{A_3A_4}\bar{y}^{B_3B_4} \equiv \mathrm{tr}(\bar{y}QyQ')\mathrm{tr}(\bar{y}y) = \mathrm{tr}(\bar{y}Q'yQ)\mathrm{tr}(\bar{y}y) = \mathrm{tr}(yQ\bar{y}Q')\mathrm{tr}(\bar{y}y)$$

$$y_{AB} \to \tilde{\bar{y}}_{AB} \equiv= \tfrac{1}{2}\epsilon_{ABCD}\bar{y}^{CD} \text{ and } \bar{y}^{AB} \to \tilde{y}^{AB} \equiv \tfrac{1}{2}\epsilon^{ABCD}y_{CD}$$

$$\langle \mathcal{Q}(\vec{n})\mathcal{Q}(\vec{n}')\rangle = 4\frac{\mathrm{tr}(yQ\bar{y})}{\mathrm{tr}(y\bar{y})}\frac{\mathrm{tr}(yQ'\bar{y})}{\mathrm{tr}(y\bar{y})}(I_{a+b}+I_c) + \frac{\mathrm{tr}[yQ'\bar{y}Q + \tilde{\bar{y}}Q'\tilde{y}Q]}{2\mathrm{tr}(y\bar{y})}I_d,$$

$$I_{a+b} = \frac{2[\mathrm{tr}(\bar{y}y)]^2}{\sigma_{\mathrm{tot}}}\int \mathrm{dPS}_3\delta^{(2)}\left(\Omega_{\vec{k}_1}-\Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_2}-\Omega_{\vec{n}'}\right)\left|\mathcal{M}_{O_{20'}\to s(k_1)s(k_2)g(k_3)}\right|^2$$

$$I_c = \frac{2[\mathrm{tr}(\bar{y}y)]^2}{\sigma_{\mathrm{tot}}}\int \mathrm{dPS}_3\delta^{(2)}\left(\Omega_{\vec{k}_1}-\Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_2}-\Omega_{\vec{n}'}\right)\left|\mathcal{M}_{O_{20'}\to s(k_1)\lambda(k_2)\lambda(k_3)}\right|^2$$

$$I_d = \frac{2[\mathrm{tr}(\bar{y}y)]^2}{\sigma_{\mathrm{tot}}}\int \mathrm{dPS}_3\delta^{(2)}\left(\Omega_{\vec{k}_2}-\Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_3}-\Omega_{\vec{n}'}\right)\left|\mathcal{M}_{O_{20'}\to s(k_1)\lambda(k_2)\lambda(k_3)}\right|^2.$$

$$\begin{aligned} I_{a+b} &= \frac{a}{8\pi^2}\frac{(z-2)\ln(1-z)-2z}{z^2(1-z)} \\ I_c &= \frac{a}{4\pi^2}\frac{(1-z)\ln(1-z)+z}{z^2(1-z)} \\ I_d &= -\frac{a}{4\pi^2}\frac{\ln(1-z)}{z^2} \end{aligned}$$

$$\langle \mathcal{E}(n)\mathcal{E}(n')\rangle = 16\pi g^2 \int \mathrm{dPSS}_3 k_1^0 k_2^0 \delta^{(2)}\left(\Omega_{\vec{k}_1}-\Omega_{\vec{n}}\right)\delta^{(2)}\left(\Omega_{\vec{k}_2}-\Omega_{\vec{n}'}\right)\frac{4(q^2)^2}{s_{12}s_{13}s_{23}},$$

$$\begin{aligned} \langle \mathcal{E}(n)\mathcal{E}(n')\rangle &= \frac{g^2 q_0^2}{8(2\pi)^4}\int_0^1 \frac{d\tau_1 d\tau_2(\tau_1\tau_2)^2}{(1-\tau_1)(1-\tau_2)(\tau_1+\tau_2-1)}\delta(1-\tau_1-\tau_2+\tau_1\tau_2 z) \\ &= \frac{g^2 q_0^2}{8(2\pi)^4}\frac{1}{z(1-z)}\int_0^1 \frac{d\tau_1}{1-z\tau_1} \end{aligned}$$

$$\langle S\rangle = \frac{(YS\bar{Y})}{(Y\bar{Y})}, \qquad [S] = \frac{(YSY)}{(Y\bar{Y})}, \qquad \overline{[S]} = \frac{(\bar{Y}S\bar{Y})}{(Y\bar{Y})}$$

$$\langle SS'\rangle = \frac{(YSS'\bar{Y})}{(Y\bar{Y})}, \quad \overline{\langle SS'\rangle} = \frac{(YS'S\bar{Y})}{(Y\bar{Y})}, \quad (SS') = \mathrm{tr}(SS')$$

$$\omega_1^{OO} = \frac{1}{6}(SS')$$
$$\omega_{15}^{OO} = \frac{1}{6}\left\{\langle SS'\rangle - \overline{\langle SS'\rangle}\right\}$$
$$\omega_{20'}^{OO} = \frac{1}{6}\left\{\langle SS'\rangle + \overline{\langle SS'\rangle} - \frac{1}{3}(SS')\right\}$$
$$\omega_{84}^{OO} = \frac{1}{6}\left\{[S]\overline{[S']} + \overline{[S]}[S'] - 2\langle S\rangle\langle S'\rangle - \frac{1}{2}\left(\langle SS'\rangle + \overline{\langle SS'\rangle}\right) + \frac{1}{10}(SS')\right\}$$
$$\omega_{175}^{OO} = \frac{1}{6}\left\{[S]\overline{[S']} - \overline{[S]}[S'] - \frac{1}{2}\left(\langle SS'\rangle - \overline{\langle SS'\rangle}\right)\right\}$$
$$\omega_{105}^{OO} = \frac{1}{6}\left\{[S]\overline{[S']} + \overline{[S]}[S'] + 4\langle S\rangle\langle S'\rangle - \frac{4}{5}\left(\langle SS'\rangle + \overline{\langle SS'\rangle}\right) + \frac{1}{10}(SS')\right\}$$

$$S^{IJ} \to Y_2^I Y_2^J, (S')^{IJ} \to Y_3^I Y_3^J, Y \to Y_4, \bar{Y} \to Y_1$$

$$\omega_R^{OO} \to (Y_2Y_3)^2 \mathcal{Y}_R(t_1, t_2)$$

$$t_1 = (Y_1Y_2)(Y_3Y_4)/\left((Y_1Y_4)(Y_2Y_3)\right), t_2 = (Y_1Y_3)(Y_2Y_4)/\left((Y_1Y_4)(Y_2Y_3)\right)$$

$$\mathcal{Y}_{\mathbf{1}} = 1$$
$$\mathcal{Y}_{\mathbf{15}} = t_1 - t_2$$
$$\mathcal{Y}_{\mathbf{20}'} = t_1 + t_2 - \frac{1}{3}$$
$$\mathcal{Y}_{\mathbf{84}} = (t_1 - t_2)^2 - \frac{1}{2}(t_1 + t_2) + \frac{1}{10}$$
$$\mathcal{Y}_{\mathbf{175}} = t_1^2 - t_2^2 - \frac{1}{2}(t_1 - t_2)$$
$$\mathcal{Y}_{\mathbf{105}} = t_1^2 + t_2^2 + 4t_1t_2 - \frac{4}{5}(t_1 + t_2) + \frac{1}{10}$$

$$(Y_2Y_3) = 0 \to (SS') = \langle SS'\rangle = \langle S'S\rangle = 0 \to \omega_{\mathbf{1}}^{OO} = \omega_{\mathbf{15}}^{OO} = \omega_{\mathbf{20}}^{OO} = 0$$

$$Y_2 = Y_3 \to [S]\overline{[S']} = \overline{[S]}[S'] = \langle S\rangle\langle S'\rangle \to \omega_{\mathbf{84}}^{OO} = \omega_{\mathbf{175}}^{OO} = 0$$

$$[S]\overline{[S']} = \overline{[S]}[S'] = \langle S\rangle\langle S'\rangle$$

$$Y = (1,0,1,0,i,i), S = \mathrm{diag}(1,-1,0,0,0,0), S' = \mathrm{diag}(0,0,1,-1,0,0)$$

$$\omega_{15}^{QO} = -4[\langle QS\rangle + \overline{\langle QS\rangle}]$$
$$\omega_{20'}^{QO} = 4[\langle QS\rangle - \overline{\langle QS\rangle}]$$
$$\omega_{175}^{QO} = 4\left[\langle S\rangle\langle Q\rangle - \frac{1}{8}\{\langle QS\rangle + \overline{\langle QS\rangle}\}\right]$$

$$\langle QS\rangle = Y^I Q^{IJ} S^{KL} \bar{Y}^L$$

$$Q^{IJ} = -Q^{JI} = (\Gamma^{IJ})_A^B Q_B^A$$

$$\mathcal{Z}_1 = \mathrm{tr}(QQ'), \quad \mathcal{Z}_2 = \frac{\mathrm{tr}(yQ\bar{y})\mathrm{tr}(yQ'\bar{y})}{[\mathrm{tr}(y\bar{y})]^2}, \quad \mathcal{Z}_5 = \frac{\mathrm{tr}(yQ\bar{y}Q')}{\mathrm{tr}(y\bar{y})}.$$
$$\mathcal{Z}_3 = \frac{\mathrm{tr}(yQQ'\bar{y})}{\mathrm{tr}(y\bar{y})}, \quad \mathcal{Z}_4 = \frac{\mathrm{tr}(yQ'Q\bar{y})}{\mathrm{tr}(y\bar{y})},$$

$$\frac{\mathrm{tr}(\tilde{\bar{y}}Q\bar{y}Q')}{\mathrm{tr}(y\bar{y})} = \mathcal{Z}_5 - \frac{1}{2}\mathcal{Z}_1 - (\mathcal{Z}_3 + \mathcal{Z}_4) \;\to\; \langle Q, Q'\rangle = 2\mathcal{Z}_5 - \frac{1}{2}\mathcal{Z}_1 - (\mathcal{Z}_3 + \mathcal{Z}_4).$$

$$\omega_1^{QQ} = -\frac{1}{5}Z_1$$
$$\omega_{15_a}^{Q} = -\frac{1}{5}[Z_3 - Z_4]$$
$$\omega_{15_s}^{Q} = -\frac{1}{5}[Z_1 - 4(Z_3 + Z_4)]$$
$$\omega_{20'}^{Q} = -\frac{1}{5}[Z_1 - 3(Z_3 + Z_4) + 6Z_5]$$
$$\omega_{84}^{Q} = -\frac{1}{5}[Z_1 - 40Z_2 - 5(Z_3 + Z_4) + 10Z_5]$$

$$\omega_{\mathbf{1}}^{\mathcal{E}\mathcal{E}} = 1, \omega_{\mathbf{15}}^{\mathcal{E}Q} = 8\langle Q\rangle, \omega_{\mathbf{20}}^{\mathcal{E}Q} = 2\langle S\rangle.$$

$$K(j_1, j_2; z) = \frac{1}{16\pi^5} q^2\big((nn')/2\big)^{-j_1-j_2+1} \int \frac{d^4x_1 e^{iqx_1}}{(-x_1^2 + i\epsilon x_1^0)^{j_1+j_2}}$$
$$\times \int_{-\infty}^{\infty} dx_{2-}\big((x_1n) - x_{2-} - i\epsilon\big)^{j_1-1}(-x_{2-} + i\epsilon)^{j_2-1}$$
$$\times \int_{-\infty}^{\infty} dx_{3-}\big((x_1n') - x_{3-} - i\epsilon\big)^{j_2-1}(-x_{3-} + i\epsilon)^{j_1-1}$$

$$\int_{-\infty}^{\infty} dx_{2-}\big((x_1n) - x_{2-} - i0)^{j_1-1}(-x_{2-} + i0)^{j_2-1}$$
$$= \frac{2\pi i^{j_2-j_1}}{\Gamma(1-j_1)\Gamma(1-j_2)} \int_0^{\infty} d\omega_1 \omega_1^{-j_1-j_2} e^{-i\omega_1(x_1n)}$$

$$K(j_1, j_2; z) = \frac{q^2\big((nn')/2\big)^{-j_1-j_2+1}}{4\pi^3[\Gamma(1-j_1)\Gamma(1-j_2)]^2} \int_0^{\infty} d\omega_1 d\omega_2 (\omega_1\omega_2)^{-j_1-j_2} D_{j_1+j_2}(q - \omega_1 n - \omega_2 n')$$

$$D_j(q) = \int \frac{d^4x_1 e^{iqx_1}}{(-x_1^2 + i\epsilon x_1^0)^j} = 2\pi^3 \frac{(q^2/4)^{j-2}\theta(q^0)\theta(q^2)}{\Gamma(j)\Gamma(j-1)}.$$

$$K(j_1, j_2; z) = \left(\frac{z}{1-z}\right)^{1-j_1-j_2} \frac{2\pi}{\sin\big(\pi(j_1+j_2)\big)[\Gamma(j_1+j_2)\Gamma(1-j_1)\Gamma(1-j_2)]^2}.$$

$$\left\{Q_\alpha^i, \bar{Q}_{j\dot{\alpha}}\right\} = \delta_j^i (\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$$
$$\left\{Q_\alpha^i, Q_\beta^j\right\} = \left\{\bar{Q}_{i\dot{\alpha}}, \bar{Q}_{j\dot{\beta}}\right\} = 0, (2.1)$$

$$S_{YM} = \frac{1}{g^2}\mathrm{Tr}\int d^4x \left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - 4\lambda^{i\alpha}\sigma_{\alpha\dot{\alpha}}^{\mu} D_\mu \bar{\lambda}_i^{\dot{\alpha}} - \frac{1}{2}\bar{\phi}D_\mu D^\mu \phi\right.$$
$$\left. - \bar{\phi}\{\bar{\lambda}^{i\dot{\alpha}}, \bar{\lambda}_{i\dot{\alpha}}\} + \phi\{\lambda^{i\alpha}, \lambda_{i\alpha}\} - \frac{1}{32}[\phi, \bar{\phi}][\phi, \bar{\phi}]\right),$$

$$H = SU(2)_L \times SU(2)_R \times SU(2)_I \times U(1)$$

$$A_\mu: \left(\frac{1}{2}, \frac{1}{2}, 0\right)^0$$
$$\lambda^{i\alpha}: \left(\frac{1}{2}, 0, \frac{1}{2}\right)^{-1}$$
$$\bar{\lambda}_{i\alpha}: \left(0, \frac{1}{2}, \frac{1}{2}\right)^{+1}$$
$$\phi: (0,0,0)^{+2}$$
$$\bar{\phi}: (0,0,0)^{-2}$$

$$(SU(2)_L, SU(2)_R, SU(2)_I)^{U(1)}$$

$$Q_{i\alpha} = \left(\frac{1}{2}, 0, \frac{1}{2}\right)^{+1}; \ \bar{Q}_{i\dot{\alpha}} = \left(0, \frac{1}{2}, \frac{1}{2}\right)^{-1}.$$

$$\delta A_\mu = -\sqrt{2}\xi^{\alpha j}(\sigma_\mu)_{\alpha\dot{\alpha}}\bar{\lambda}_j^{\dot{\alpha}} - \sqrt{2}\bar{\xi}^{\dot{\alpha} j}(\sigma_\mu)_{\alpha\dot{\alpha}}\lambda_i^{\alpha}$$
$$\delta\phi = -4\sqrt{2}\bar{\xi}^{\dot{\alpha} j}\bar{\lambda}_{j\dot{\alpha}}$$
$$\delta\bar{\phi} = -4\sqrt{2}\xi^{\alpha j}\lambda_{j\alpha}$$
$$\delta\lambda_{i\kappa} = \frac{\sqrt{2}}{8}\xi^{\alpha j}\varepsilon_{\alpha\kappa}\varepsilon_{ij}[\phi,\bar{\phi}] + \frac{1}{\sqrt{2}}\xi^{\alpha j}\varepsilon_{ij}(\sigma^{\mu\nu})_{\kappa\alpha}F_{\mu\nu} + \frac{1}{\sqrt{2}}\bar{\xi}^{\dot{\alpha} j}\varepsilon_{ij}(\sigma_\mu)_{\kappa\dot{\alpha}}D^\mu\bar{\phi}$$
$$\delta\bar{\lambda}_{i\dot{\kappa}} = \frac{1}{\sqrt{2}}\xi^{\alpha j}\varepsilon_{ij}(\sigma_\mu)_{\alpha\dot{\kappa}}D^\mu\phi + \frac{\sqrt{2}}{8}\bar{\xi}^{\dot{\alpha} j}\varepsilon_{\dot{\kappa}\dot{\alpha}}\varepsilon_{ji}[\phi,\bar{\phi}] + \frac{1}{\sqrt{2}}\bar{\xi}^{\dot{\alpha} j}\varepsilon_{ji}(\bar{\sigma}^{\mu\nu})_{\dot{\kappa}\dot{\alpha}}F_{\mu\nu}$$

$$\delta S_{YM} = 0$$

$$q_i: \left(0,0,\frac{1}{2}\right)^0$$
$$\tilde{q}_i: \left(0,0,\frac{1}{2}\right)^0$$
$$(\psi_q)_\alpha: \left(\frac{1}{2},0,0\right)^{+1}$$
$$(\bar{\psi}_q)_{\dot{\alpha}}: \left(0,\frac{1}{2},0\right)^{-1}$$
$$(\psi_{\tilde{q}})_\alpha: \left(\frac{1}{2},0,0\right)^{+1}$$
$$(\bar{\psi}_{\tilde{q}})_{\dot{\alpha}}: \left(0,\frac{1}{2},0\right)^{-1}$$

$$S = S_{YM} + S_{matter}$$

$$S_{\text{matter}} = \frac{1}{g^2}\mathrm{Tr}_m\int d^4x\Big(\frac{1}{2}\tilde{q}^i D_\mu D^\mu q_i + 2\tilde{q}^i\bar{\lambda}_{i\dot{\alpha}}(\bar{\psi}_q)^{\dot{\alpha}} - 2q^i\bar{\lambda}_{i\dot{\alpha}}(\bar{\psi}_{\tilde{q}})^{\dot{\alpha}}$$
$$-\frac{1}{2}\tilde{q}^i\lambda_{i\alpha}(\psi_q)^\alpha - \frac{1}{2}q^i\lambda_{i\alpha}(\psi_{\tilde{q}})^\alpha + (\psi_{\tilde{q}})^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}D_\mu(\bar{\psi}_q)^{\dot{\alpha}}$$
$$-(\bar{\psi}_{\tilde{q}})_{\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}D_\mu(\psi_q)_\alpha + \frac{1}{8}(\psi_{\tilde{q}})^\alpha\bar{\phi}(\psi_q)_\alpha - 2(\bar{\psi}_{\tilde{q}})^{\dot{\alpha}}\phi(\bar{\psi}_q)_{\dot{\alpha}}$$
$$+\frac{1}{16}\tilde{q}^i\{\phi,\bar{\phi}\}q_i - \frac{1}{32}\tilde{q}^i q_i\tilde{q}^j q_j\Big)$$

$$\delta q_i = \sqrt{2}\varepsilon_{ji}\xi^{\alpha j}(\psi_q)_\alpha + \sqrt{2}\varepsilon_{ji}\bar{\xi}^{\dot{\alpha} j}(\bar{\psi}_q)_{\dot{\alpha}}$$
$$\delta\tilde{q}_i = \sqrt{2}\varepsilon_{ji}\xi^{\alpha j}(\psi_{\tilde{q}})_\alpha + \sqrt{2}\varepsilon_{ji}\bar{\xi}^{\dot{\alpha} j}(\bar{\psi}_{\tilde{q}})_{\dot{\alpha}}$$
$$\delta(\psi_q)_\gamma = \sqrt{2}\varepsilon_{\gamma\alpha}\xi^{\alpha j}\phi q_j + \frac{1}{\sqrt{2}}\bar{\xi}^{\dot{\alpha} j}(\sigma^\nu)_{\gamma\dot{\alpha}}D_\nu q_j$$
$$\delta(\bar{\psi}_q)_{\dot{\gamma}} = -\frac{1}{\sqrt{2}}\xi^{\alpha j}(\sigma^\nu)_{\alpha\dot{\gamma}}D_\nu q_j - \frac{\sqrt{2}}{16}\varepsilon_{\dot{\gamma}\dot{\alpha}}\bar{\xi}^{\dot{\alpha} j}\bar{\phi}q_j$$
$$\delta(\psi_{\tilde{q}})_\gamma = -\sqrt{2}\varepsilon_{\gamma\alpha}\xi^{\alpha j}\phi\tilde{q}_j + \frac{1}{\sqrt{2}}\bar{\xi}^{\dot{\alpha} j}(\sigma^\nu)_{\gamma\dot{\alpha}}D_\nu\tilde{q}_j$$
$$\delta(\bar{\psi}_{\tilde{q}})_{\dot{\gamma}} = -\frac{1}{\sqrt{2}}\xi^{\alpha j}(\sigma^\nu)_{\alpha\dot{\gamma}}D_\nu\tilde{q}_j + \frac{\sqrt{2}}{16}\varepsilon_{\dot{\gamma}\dot{\alpha}}\bar{\xi}^{\dot{\alpha} j}\bar{\phi}\tilde{q}_j$$

$$\delta S_{matter} = 0,$$

$$\delta S = \delta(S_{YM} + S_{matter}) = 0$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\mu\sigma}M_{\mu\rho})$$
$$[M_{\mu\nu}, P_{\rho}] = i(\eta_{\nu\rho}P_{\mu} - \eta_{\mu\rho}P_{\nu})$$
$$[M_{\mu\nu}, Q^i_{\alpha}] = -(\sigma_{\mu\nu})_{\alpha}^{\ \beta}Q^i_{\beta}$$
$$\{Q^i_{\alpha}, \bar{Q}^j_{\dot{\beta}}\} = 2\sigma^{\mu}_{\alpha\dot{\beta}}P_{\mu}\delta^{ij}$$
$$[T_i^{\ j}, Q_{k\alpha}] = -\frac{1}{2}\left(\delta_k^j Q_{i\alpha} - \frac{1}{2}\delta_i^j Q_{k\alpha}\right)$$
$$[T_i^{\ j}, T_k^{\ l}] = \frac{1}{2}(\delta_i^l T_k^{\ j} - \delta_k^j T_i^{\ l})$$
$$[\mathcal{R}, Q_{i\alpha}] = Q_{i\alpha}$$

$$J_{\alpha\beta} := \frac{1}{2}(\sigma^{\mu\nu})_{\alpha\beta}M_{\mu\nu}\ ;\ \bar{J}_{\dot{\alpha}\dot{\beta}} := \frac{1}{2}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}}M_{\mu\nu}\ ;\ P_{\alpha\dot{\beta}} := (\sigma^{\mu})_{\alpha\dot{\beta}}P_{\mu}$$

$$\rho: M_4 \to H\ , \rho(x_{\mu}) = x_{\mu}\sigma^{\mu}$$
$$\rho^{-1}: H \to M_4\ , \rho^{-1}(h) = \frac{1}{2}\mathrm{Tr}[h\bar{\sigma}^{\mu}]$$

$$i \overset{\text{twist}}{\to} \alpha$$

$$J'_{\alpha\beta} := J_{\alpha\beta} + kT_{\alpha\beta}$$

$$H \overset{twist}{\to} H' = SU(2)'_L \times SU(2)_R \times U(1).$$

$$Q_{i\alpha} \overset{\text{twist}}{\to} Q_{\beta\alpha} \text{ and } \bar{Q}_{i\dot{\alpha}} \overset{\text{twist}}{\to} \bar{Q}_{\beta\dot{\alpha}}.$$

$$Q_{\beta\alpha} = (0,0)^{+1} \oplus (1,0)^{+1} \text{ and } \bar{Q}_{\beta\dot{\alpha}} = \left(\tfrac{1}{2}, \tfrac{1}{2}\right)^{-1}$$

$$Q_{\beta\alpha} \overset{\text{twist}}{\to} \delta_W := \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}Q_{\beta\alpha} \oplus \delta_{\mu\nu} := \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta}Q_{\beta\alpha}$$
$$\bar{Q}_{\beta\dot{\alpha}} \overset{\text{twist}}{\to} \delta_{\mu} := \frac{1}{\sqrt{2}}\bar{Q}_{\beta\dot{\alpha}}(\bar{\sigma}_{\mu})^{\dot{\alpha}}_{\ \beta},$$

$$\delta_{\mu\nu} = \tilde{\delta}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\delta^{\rho\sigma}$$

$$\{\delta_W, \delta_W\} = 2\delta_W^2 = 0$$
$$\{\delta_W, \delta_{\mu}\} = \partial_{\mu}$$

$$\{\delta_{\mu}, \delta_{\nu}\} = 0$$
$$\{\delta_{\mu}, \delta_{\rho\sigma}\} = -(\varepsilon_{\mu\rho\sigma\nu}\partial^{\nu} + g_{\mu\rho}\partial_{\sigma} - g_{\mu\sigma}\partial_{\rho})$$
$$\{\delta_W, \delta_{\mu\nu}\} = 0$$
$$[\mathcal{R}, \delta_W] = +\delta_W$$
$$[\mathcal{R}, \delta_{\mu}] = -\delta_{\mu}$$
$$[\mathcal{R}, \delta_{\mu\nu}] = +\delta_{\mu\nu},$$

$$\delta_W^2 = 0$$

$$\delta^2 = 0\ , \{\delta, \delta_{\mu}\} = \partial_{\mu}\ , \{\delta_{\mu}, \delta_{\nu}\} = 0$$

$$\lambda^{i\alpha} \to \lambda_{\beta\alpha}\left(\frac{1}{2}, 0, \frac{1}{2}\right)^{-1} \to \eta(0,0)^{-1} \oplus \chi_{\mu\nu}(1,0)^{-1}$$
$$\bar{\lambda}_{i\dot{\alpha}} \to \bar{\lambda}_{\beta\dot{\alpha}}\left(0, \frac{1}{2}, \frac{1}{2}\right)^{+1} \to \psi_{\mu}\left(\frac{1}{2}, \frac{1}{2}\right)^{+1}$$

$$\lambda_{\beta\alpha} \to \eta := \varepsilon^{\alpha\beta}\lambda_{[\beta\alpha]} \oplus \chi_{\mu\nu} := \frac{1}{4}(\sigma_{\mu\nu})^{\alpha\beta}\lambda_{(\beta\alpha)}$$
$$\bar{\lambda}_{\beta\dot{\alpha}} \to \psi_{\mu} := \bar{\lambda}_{\beta\dot{\alpha}}(\bar{\sigma}_{\mu})^{\dot{\alpha}}_{\ \beta}$$

$$\chi_{\mu\nu} = \tilde{\chi}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\chi^{\rho\sigma}$$

$$\lambda_{\beta\alpha} = \frac{1}{2}\left(\lambda_{[\beta\alpha]} + \lambda_{(\beta\alpha)}\right)$$

$$\left(A_\mu, \lambda_{i\alpha}, \bar{\lambda}_{i\dot{\alpha}}, \phi, \bar{\phi}\right) \overset{twist}{\rightarrow} \left(A_\mu, \psi_\mu, \chi_{\mu\nu}, \eta, \phi, \bar{\phi}\right),$$

$$S_{YM} \overset{\text{twist}}{\rightarrow} S_{TYM} = \frac{1}{g^2}\text{Tr}\int \mathrm{d}^4x \left(\frac{1}{2}F^+_{\mu\nu}F^{+\mu\nu} - \chi^{\mu\nu}\left(D_\mu\psi_\nu - D_\nu\psi_\mu\right)^+\right.$$

$$+\eta D_\mu\psi^\mu - \frac{1}{2}\bar{\phi}D_\mu D^\mu\phi + \frac{1}{2}\bar{\phi}\{\psi^\mu, \psi_\mu\}$$
$$\left. -\frac{1}{2}\phi\{\chi^{\mu\nu}, \chi_{\mu\nu}\} - \frac{1}{8}[\phi, \eta]\eta - \frac{1}{32}[\phi, \bar{\phi}][\phi, \bar{\phi}]\right)$$

$$\delta^g_\epsilon A_\mu = -D_\mu\epsilon$$
$$\delta^g_\epsilon \lambda = [\epsilon, \lambda], \lambda = \chi, \psi, \eta, \phi, \bar{\phi},$$

$$\delta^g_\epsilon S_{TYM} = 0.$$

$$\delta_W A_\mu = \psi_\mu$$
$$\delta_W \psi_\mu = -D_\mu\phi$$
$$\delta_W \phi = 0$$
$$\delta_W \chi_{\mu\nu} = F^+_{\mu\nu}$$
$$\delta_W \bar{\phi} = 2\eta$$
$$\delta_W \eta = \frac{1}{2}[\phi, \bar{\phi}]$$

$$\delta_\mu A_\nu = \frac{1}{2}\chi_{\mu\nu} + \frac{1}{8}g_{\mu\nu}\eta$$
$$\delta_\mu \psi_\nu = F_{\mu\nu} - \frac{1}{2}F^+_{\mu\nu} - \frac{1}{16}g_{\mu\nu}[\phi, \bar{\phi}]$$
$$\delta_\mu \eta = \frac{1}{2}D_\mu\bar{\phi}$$

$$\delta_\mu \chi_{\sigma\tau} = \frac{1}{8}\left(\varepsilon_{\mu\sigma\tau\nu}D^\nu\bar{\phi} + g_{\mu\sigma}D_\tau\bar{\phi} - g_{\mu\tau}D_\sigma\bar{\phi}\right)$$
$$\delta_\mu \phi = -\psi_\mu$$
$$\delta_\mu \bar{\phi} = 0$$

$$\delta_{\mu\nu} A_\sigma = -\left(\varepsilon_{\mu\nu\sigma\tau}\psi^\tau + g_{\mu\sigma}\psi_\nu - g_{\nu\sigma}\psi_\mu\right)$$
$$\delta_{\mu\nu} \psi_\sigma = -\left(\varepsilon_{\mu\nu\sigma\tau}D^\tau\phi + g_{\mu\sigma}D_\nu\phi - g_{\nu\sigma}D_\mu\phi\right)$$
$$\delta_{\mu\nu} \phi = 0$$
$$\delta_{\mu\nu} \bar{\phi} = 8\chi_{\mu\nu}$$
$$\delta_{\mu\nu} \eta = -4F^+_{\mu\nu}$$
$$\delta_{\mu\nu} \chi_{\sigma\tau} = \frac{1}{8}\left(\varepsilon_{\mu\nu\sigma\tau} + g_{\mu\sigma}g_{\nu\tau} - g_{\mu\tau}g_{\nu\sigma}\right)[\phi, \bar{\phi}]$$
$$+\left(F^+_{\mu\sigma}g_{\nu\tau} - F^+_{\nu\sigma}g_{\mu\tau} - F^+_{\mu\tau}g_{\nu\sigma} + F^+_{\nu\tau}g_{\mu\sigma}\right)$$
$$+\left(\varepsilon_{\mu\nu\sigma}{}^\alpha F^+_{\tau\alpha} - \varepsilon_{\mu\nu\tau}{}^\alpha F^+_{\sigma\alpha} + \varepsilon_{\sigma\tau\mu}{}^\alpha F^+_{\nu\alpha} - \varepsilon_{\sigma\tau\nu}{}^\alpha F^+_{\mu\alpha}\right)$$

$$\delta_W S_{TYM} = \delta_\mu S_{TYM} = \delta_{\mu\nu} S_{TYM} = 0$$

$$\delta^2_W = \delta^g_\phi + (\mathfrak{G})$$

$$\{\delta_\mu, \delta_\nu\} = -\frac{1}{8}g_{\mu\nu}\delta\frac{g}{\phi} + (\mathfrak{G})$$

$$\{\delta_W, \delta_\mu\} = \partial_\mu + \delta^g_{A_\mu} + (\mathfrak{G})$$

$$\{\delta_W, \delta_{\mu\nu}\} = (\mathcal{g}_t) + (\mathfrak{G})$$

$$\{\delta_{\mu\nu}, \delta_{\rho\sigma}\} = (\mathcal{g}_t) + (\mathfrak{G})$$

$$\{\delta_\mu, \delta_{\rho\sigma}\} = -(\varepsilon_{\mu\rho\sigma\nu}\partial^\nu + g_{\mu\rho}\partial_\sigma - g_{\mu\sigma}\partial_\rho) + (\mathcal{g}_t) + (\mathfrak{G}).$$

$$q_i \overset{twist}{\to} q_\alpha\left(0,0,\frac{1}{2}\right)^0 \to \mathrm{H}_\alpha\left(\frac{1}{2},0\right)^0$$
$$\tilde{q}_i \overset{twist}{\to} \tilde{q}_\alpha\left(0,0,\frac{1}{2}\right)^0 \to \overline{\mathrm{H}}_\alpha\left(\frac{1}{2},0\right)^0$$
$$(\psi_q)_\alpha\left(\frac{1}{2},0,0\right)^{+1} \to u_\alpha\left(\frac{1}{2},0\right)^{+1}$$
$$(\bar{\psi}_q)_{\dot{\alpha}}\left(0,\frac{1}{2},0\right)^{-1} \to v_{\dot{\alpha}}\left(0,\frac{1}{2}\right)^{-1}$$
$$(\psi_{\tilde{q}})_\alpha\left(\frac{1}{2},0,0\right)^{+1} \to \bar{u}_\alpha\left(\frac{1}{2},0\right)^{+1}$$
$$(\bar{\psi}_{\tilde{q}})_{\dot{\alpha}}\left(0,\frac{1}{2},0\right)^{-1} \to \bar{v}_{\dot{\alpha}}\left(0,\frac{1}{2}\right)^{-1}$$

$$S_{\text{matter}} \overset{\text{twist}}{\to} S_{\text{Tmatter}},$$

$$S_{\text{Tmatter}} = \frac{1}{g^2}\mathrm{Tr_m}\int d^4x\left(\frac{1}{2}\overline{\mathrm{H}}^\gamma D_\mu D^\mu \mathrm{H}_\gamma + \overline{\mathrm{H}}^\gamma(\sigma^\mu)_{\gamma\dot{\gamma}}\psi_\mu v^{\dot{\gamma}}\right.$$
$$-\bar{v}_{\dot{\gamma}}(\bar{\sigma}^\mu)^{\dot{\gamma}\gamma}\psi_\mu \mathrm{H}_\gamma + \frac{1}{8}\overline{\mathrm{H}}^\gamma\eta u_\gamma + \frac{1}{8}\overline{\mathrm{H}}^\gamma(\sigma^{\mu\nu})_{\gamma\beta}\chi_{\mu\nu}u^\beta + \frac{1}{8}\bar{u}^\gamma\eta\mathrm{H}_\gamma$$
$$-\frac{1}{8}\bar{u}^\gamma(\sigma^{\mu\nu})_{\gamma\beta}\chi_{\mu\nu}\mathrm{H}^\beta + \bar{u}^\gamma(\sigma^\mu)_{\gamma\dot{\gamma}}D_\mu v^{\dot{\gamma}} - \bar{v}_{\dot{\gamma}}(\bar{\sigma}^\mu)^{\dot{\gamma}\gamma}D_\mu u_\gamma$$
$$\left.+\frac{1}{8}\bar{u}^\gamma\bar{\phi}u_\gamma - 2\bar{v}^{\dot{\gamma}}\phi v_{\dot{\gamma}} + \frac{1}{16}\overline{\mathrm{H}}^\gamma\{\phi,\bar{\phi}\}\mathrm{H}_\gamma - \frac{1}{32}\overline{\mathrm{H}}^\gamma\mathrm{H}_\gamma\overline{\mathrm{H}}^\delta\mathrm{H}_\delta\right)$$

$$\delta_W\mathrm{H}_\gamma = \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}Q_{\beta\alpha}\mathrm{H}_\gamma = \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}(\sqrt{2}u_\alpha\varepsilon_{\beta\gamma}) = u_\gamma$$
$$\delta_W\overline{\mathrm{H}}_\gamma = \bar{u}_\gamma$$
$$\delta_W u_\gamma = \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}(\sqrt{2}\varepsilon_{\gamma\alpha}\phi\mathrm{H}_\beta) = +\phi\mathrm{H}_\gamma$$
$$\delta_W\bar{u}_\gamma = -\phi\overline{\mathrm{H}}_\gamma$$
$$\delta_W v_{\dot{\gamma}} = \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}\left(-\frac{1}{\sqrt{2}}(\sigma^\nu)_{\alpha\dot{\gamma}}D_\nu\mathrm{H}_\beta\right) = -\frac{1}{2}(\sigma^\nu)_{\alpha\dot{\gamma}}D_\nu\mathrm{H}^\alpha$$
$$\delta_W\bar{v}_{\dot{\gamma}} = -\frac{1}{2}(\sigma^\nu)_{\alpha\dot{\gamma}}D_\nu\overline{\mathrm{H}}^\alpha$$

$$\delta_\mu\mathrm{H}_\gamma = \frac{1}{\sqrt{2}}(\bar{\sigma}_\mu)^{\dot{\alpha}\beta}(\sqrt{2}v_{\dot{\alpha}}\varepsilon_{\beta\alpha}) = (\sigma_\mu)_{\gamma\dot{\alpha}}v^{\dot{\alpha}}$$
$$\delta_\mu\overline{\mathrm{H}}_\gamma = (\sigma_\mu)_{\gamma\dot{\alpha}}\bar{v}^{\dot{\alpha}}$$
$$\delta_\mu u_\gamma = \frac{1}{\sqrt{2}}(\bar{\sigma}_\mu)^{\dot{\alpha}\beta}\left(\frac{1}{\sqrt{2}}(\sigma_\nu)_{\gamma\dot{\alpha}}D^\nu\mathrm{H}_\beta\right) = \frac{1}{2}D_\mu\mathrm{H}_\gamma - \frac{1}{2}(\sigma_{\mu\nu})_\gamma^{\ \beta}D^\nu\mathrm{H}_\beta$$

$$\delta_\mu\bar{u}_\gamma = \frac{1}{2}D_\mu\overline{\mathrm{H}}_\gamma - \frac{1}{2}(\sigma_{\mu\nu})_\gamma^{\ \beta}D^\nu\overline{\mathrm{H}}_\beta$$
$$\delta_\mu v_{\dot{\gamma}} = \frac{1}{\sqrt{2}}(\bar{\sigma}_\mu)^{\dot{\alpha}\beta}\left(\frac{\sqrt{2}}{16}\varepsilon_{\dot{\gamma}\dot{\alpha}}\bar{\phi}\mathrm{H}_\beta\right) = -\frac{1}{16}(\bar{\sigma}_\mu)_{\dot{\gamma}\beta}\bar{\phi}\mathrm{H}^\beta$$
$$\delta_\mu\bar{v}_{\dot{\gamma}} = \frac{1}{16}(\bar{\sigma}_\mu)_{\dot{\gamma}\beta}\overline{\phi H}^\beta$$

$$\delta_{\mu\nu}\mathrm{H}_\gamma = \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta}(\sqrt{2}u_\alpha\varepsilon_{\beta\gamma}) = -(\sigma_{\mu\nu})_\gamma^{\alpha}u_\alpha$$
$$\delta_{\mu\nu}\overline{\mathrm{H}}_\gamma = -(\sigma_{\mu\nu})_\gamma^{\alpha}\bar{u}_\alpha$$
$$\delta_{\mu\nu}u_\gamma = \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta}(\sqrt{2}\varepsilon_{\gamma\alpha}\phi\mathrm{H}_\beta) = (\sigma_{\mu\nu})_\gamma^{\beta}\phi\mathrm{H}_\beta$$
$$\delta_{\mu\nu}\bar{u}_\gamma = -(\sigma_{\mu\nu})_\gamma^{\beta}\phi\overline{\mathrm{H}}_\beta$$
$$\begin{aligned}\delta_{\mu\nu}v_{\dot\gamma} &= \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta}\left(-\frac{1}{\sqrt{2}}(\sigma^\lambda)_{\alpha\dot\gamma}D_\lambda\mathrm{H}_\beta\right) = \frac{1}{2}(\sigma_{\mu\nu})_\beta^{\alpha}(\sigma_\lambda)_{\alpha\dot\gamma}D^\lambda\mathrm{H}^\beta \\ &= \frac{1}{2}(\sigma_\mu)_{\beta\dot\gamma}D_\nu\mathrm{H}^\beta - \frac{1}{2}(\sigma_\nu)_{\beta\dot\gamma}D_\mu\mathrm{H}^\beta - \frac{1}{2}\varepsilon_{\mu\nu\lambda\tau}(\sigma^\tau)_{\beta\dot\gamma}D^\lambda\mathrm{H}^\beta \\ &= \frac{1}{2}\left[(\sigma_\mu)_{\beta\dot\gamma}D_\nu\mathrm{H}^\beta - (\sigma_\nu)_{\beta\dot\gamma}D_\mu\mathrm{H}^\beta\right]^+\end{aligned}$$
$$\delta_{\mu\nu}\bar{v}_{\dot\gamma} = \frac{1}{2}\left[(\sigma_\mu)_{\beta\dot\gamma}D_\nu\overline{\mathrm{H}}^\beta - (\sigma_\nu)_{\beta\dot\gamma}D_\mu\overline{\mathrm{H}}^\beta\right]^+.$$

$$\begin{aligned}\delta_W^2\chi_{\mu\nu} &= [\phi,\chi_{\mu\nu}] - g^2\frac{\delta S_{TYM}}{\delta\chi^{\mu\nu}} \\ &= [\phi,\chi_{\mu\nu}] - [\phi,\chi_{\mu\nu}] + (D_\mu\psi_\nu - D_\nu\psi_\mu)^+ \\ &\quad -\frac{1}{8}\overline{\mathrm{H}}^\gamma(\sigma^{\mu\nu})_{\gamma\beta}u^\beta - \frac{1}{8}\bar{u}^\gamma(\sigma^{\mu\nu})_{\gamma\beta}\mathrm{H}^\beta,\end{aligned}$$

$$\delta_W\chi_{\mu\nu} = F_{\mu\nu}^+ - \frac{1}{8}\overline{\mathrm{H}}^\gamma(\sigma_{\mu\nu})_{\gamma\beta}\mathrm{H}_\gamma$$

$$\begin{aligned}\delta_W\chi_{\mu\nu} &= F_{\mu\nu}^+ - \frac{1}{8}\overline{\mathrm{H}}^\gamma(\sigma_{\mu\nu})_{\gamma\beta}\mathrm{H}_\gamma \\ \delta_\mu\psi_\nu &= F_{\mu\nu}^- - \frac{1}{16}g_{\mu\nu}[\phi,\bar\phi] + \frac{1}{16}\overline{\mathrm{H}}^\alpha(\sigma_{\mu\nu})_{\alpha\beta}\mathrm{H}^\beta \\ \delta_\mu\eta &= -4F_{\mu\nu}^+ + \frac{1}{2}\overline{\mathrm{H}}^\gamma(\sigma_{\mu\nu})_{\gamma\beta}\mathrm{H}^\beta \\ \delta_{\mu\nu}\chi_{\rho\sigma} &= \frac{1}{8}(\varepsilon_{\mu\nu\sigma\tau} + g_{\mu\sigma}g_{\nu\tau} - g_{\mu\tau}g_{\nu\sigma})[\phi,\bar\phi] \\ &\quad + (F_{\mu\sigma}^+g_{\nu\tau} - F_{\nu\sigma}^+g_{\mu\tau} - F_{\mu\tau}^+g_{\nu\sigma} + F_{\nu\tau}^+g_{\mu\sigma}) \\ &\quad + (\varepsilon_{\mu\nu\sigma}{}^\alpha F_{\tau\alpha}^+ - \varepsilon_{\mu\nu\tau}{}^\alpha F_{\sigma\alpha}^+ + \varepsilon_{\sigma\tau\mu}{}^\alpha F_{\nu\alpha}^+ - \varepsilon_{\sigma\tau\nu}{}^\alpha F_{\mu\alpha}^+) \\ &\quad - \frac{1}{16}\Big[g_{\mu\sigma}(\sigma_{\rho\nu})_\gamma{}^\beta - g_{\mu\rho}(\sigma_{\sigma\nu})_\gamma{}^\beta \\ &\quad + g_{\rho\nu}(\sigma_{\sigma\nu})_\gamma{}^\beta + g_{\sigma\nu}(\sigma_{\mu\rho})_\gamma{}^\beta\Big](\overline{\mathrm{H}}_\beta\mathrm{H}^\gamma + \overline{\mathrm{H}}^\gamma\mathrm{H}_\beta)\end{aligned}$$

$$S_T = S_{TYM} + S_{\mathrm{Tmatter}}$$

$$\delta_W S_T = \delta_\mu S_T = \delta_{\mu\nu}S_T = 0$$

$$S_T = \delta_W\Delta + \widehat{\Delta}$$

$$\begin{aligned}\Delta &= \mathrm{Tr}\int d^4x\left(\frac{1}{2}\chi^{\mu\nu}F_{\mu\nu}^+ - \frac{1}{2}\bar\phi D^\mu\psi_\mu + \frac{1}{16}\eta[\phi,\bar\phi]\right. \\ &\quad \left. + \frac{1}{16}\overline{\mathrm{H}}^g(\sigma_{\mu\nu})_\gamma^{\beta}\chi^{\mu\nu}\mathrm{H}_\beta + \frac{1}{16}\bar{u}^\gamma\bar\phi\mathrm{H}_\gamma + \frac{1}{16}u_\gamma\overline{\phi\mathrm{H}}^\gamma\right),\end{aligned}$$

$$\widehat{\Delta} = \frac{1}{2}\chi^{\mu\nu}\frac{\delta S_T}{\delta\chi^{\mu\nu}} + v^{\dot\gamma}\frac{\delta S_T}{\delta v^{\dot\gamma}} + \bar{v}^{\dot\gamma}\frac{\delta S_T}{\delta\bar{v}^{\dot\gamma}}$$

$$\begin{aligned}[\delta_W,\delta_\epsilon^g] &= [\delta_\mu,\delta_\epsilon^g] = [\delta_{\mu\nu},\delta_\epsilon^g] = 0 \\ \delta_W^2 &= \delta_\phi^g + (\mathcal{G}) \\ \{\delta_\mu,\delta_\nu\} &= -\frac{1}{8}g_{\mu\nu}\delta_\phi^g + (\mathcal{G}) \\ \{\delta_W,\delta_\mu\} &= \partial_\mu + \delta_{A_\mu}^g + (\mathcal{G})\end{aligned}$$

$$\{\delta_W, \delta_{\mu\nu}\} = (\mathbb{G}_t) + (\mathfrak{g})$$
$$\{\delta_{\mu\nu}, \delta_{\rho\sigma}\} = (\mathbb{G}_t) + (\mathfrak{g})$$
$$\{\delta_\mu, \delta_{\rho\sigma}\} = -(\varepsilon_{\mu\rho\sigma\nu}\partial^\nu + g_{\mu\rho}\partial_\sigma - g_{\mu\sigma}\partial_\rho) + (\mathbb{G}_t) + (\mathfrak{g})$$

$$\epsilon^a(x) \to c^a(x)\,, \delta_\epsilon^g \to s$$

$$sA_\mu = -D_\mu c$$
$$s\psi_\mu = \{c, \psi_\mu\}$$
$$s\chi_{\mu\nu} = \{c, \chi_{\mu\nu}\}$$
$$s\eta = \{c, \eta\}$$
$$s\phi = [c, \phi]$$
$$s\bar{\phi} = [c, \bar{\phi}]$$
$$sc = c^2 = \frac{1}{2} f^{abc} c^b c^c$$
$$s\bar{c} = b$$
$$sb = 0$$
$$s\mathrm{H} = [c, \mathrm{H}]$$

$$s\overline{\mathrm{H}} = [c, \overline{\mathrm{H}}]$$
$$su = \{c, u\}$$
$$s\bar{u} = \{c, \bar{u}\}$$
$$sv = \{c, v\}$$
$$s\bar{v} = \{c, \bar{v}\}$$

$$\omega \leftrightarrow \delta_W, \varepsilon^\mu \leftrightarrow \delta_\mu, v^\mu \leftrightarrow \partial_\mu,$$

$$Q = s + \omega\delta_W + \varepsilon^\mu \delta_\mu + v^\mu \partial_\mu - \omega\varepsilon^\mu \frac{\partial}{\partial v^\mu}.$$

$$Qc = c^2 - \omega^2\phi - \omega\varepsilon^\mu A_\mu + \frac{\varepsilon^2}{16}\bar{\phi} + v^\mu \partial_\mu c$$
$$Q\omega = 0$$
$$Q\varepsilon^\mu = 0$$
$$Qv^\mu = -\omega\varepsilon^\mu$$

$$Q\bar{c} = b + v^\mu \partial_\mu \bar{c}$$
$$Qb = \omega\varepsilon^\mu \partial_\mu \bar{c} + v^\mu \partial_\mu b$$

$$Q^2\bar{c} = Q^2 b = 0$$

$$S_{gf} = Q\mathrm{Tr}\int d^4x\bar{c}\partial A$$
$$= \mathrm{Tr}\int d^4x\left(b\partial^\mu A_\mu + \bar{c}\partial^\mu D_\mu c - \omega\bar{c}\partial^\mu\psi_\mu - \frac{\varepsilon^\nu}{2}\bar{c}\partial^\mu\chi_{\nu\mu} - \frac{\varepsilon^\mu}{8}\bar{c}\partial_\mu\eta\right)$$

$$Q(S) = Q(S_T + S_{gf}) = 0$$

$$
\begin{aligned}
QA_\mu &= -D_\mu c + \omega\psi_\mu + \frac{\varepsilon^\nu}{2}\chi_{\nu\mu} + \frac{\varepsilon_\mu}{8}\eta + v^\nu\partial_\nu A_\mu \\
Q\psi_\mu &= \{c,\psi_\mu\} - \omega D_\mu\phi + \varepsilon^\nu\left(F_{\nu\mu} - \frac{1}{2}F^+_{\nu\mu}\right) - \frac{\varepsilon_\mu}{16}[\phi,\bar\phi] \\
&\quad + v^\nu\partial_\nu\psi_\mu + \frac{1}{16}\overline{\mathrm{H}}^\gamma(\sigma_{\nu\mu})_{\gamma\beta}\mathrm{H}^\beta\varepsilon^\nu \\
Q\chi_{\sigma\tau} &= \{c,\chi_{\sigma\tau}\} + \omega F^+_{\sigma\tau} + \frac{\varepsilon^\mu}{8}(\varepsilon_{\mu\sigma\tau\nu} + g_{\mu\sigma}g_{\nu\tau} - g_{\mu\tau}g_{\nu\sigma})D^\nu\bar\phi \\
&\quad + v^\nu\partial_\nu\chi_{\sigma\tau} - \frac{\omega}{8}\overline{\mathrm{H}}^\gamma(\sigma_{\sigma\tau})_{\gamma\beta}\mathrm{H}^\beta \\
Q\eta &= \{c,\eta\} + \frac{\omega}{2}[\phi,\bar\phi] + \frac{\varepsilon^\mu}{2}D_\mu\bar\phi + v^\nu\partial_\nu\eta \\
Q\phi &= [c,\phi] - \varepsilon^\mu\psi_\mu + v^\nu\partial_\nu\phi \\
Q\bar\phi &= [c,\bar\phi] + 2\omega\eta + v^\nu\partial_\nu\bar\phi \\
Qc &= c^2 - \omega^2\phi - \omega\varepsilon^\mu A_\mu + \frac{\varepsilon^2}{16}\bar\phi + v^\nu\partial_\nu c \\
Q\omega &= 0 \\
Q\varepsilon^\mu &= 0 \\
Qv^\mu &= -\omega\varepsilon^\mu
\end{aligned}
$$

$$
\begin{aligned}
Q\bar c &= b + v^\mu\partial_\mu\bar c \\
Qb &= \omega\varepsilon^\mu\partial_\mu\bar c + v^\mu\partial_\mu b \\
Q\mathrm{H}_\gamma &= [c,\mathrm{H}_\gamma] + \omega u_\gamma + \varepsilon^\mu(\sigma_\mu)_{\gamma\dot\alpha}v^{\dot\alpha} + v^\mu\partial_\mu\mathrm{H}_\gamma \\
Q\overline{\mathrm{H}}_\gamma &= [c,\overline{\mathrm{H}}_\gamma] + \omega\bar u_\gamma + \varepsilon^\mu(\sigma_\mu)_{\gamma\dot\alpha}\bar v^{\dot\alpha} + v^\mu\partial_\mu\overline{\mathrm{H}}_\gamma \\
Qu_\gamma &= [c,u_\gamma] + \omega\phi\mathrm{H}_\gamma + \varepsilon^\mu\left(\frac{1}{2}D_\mu\mathrm{H}_\gamma - \frac{1}{2}(\sigma_{\mu\nu})_\gamma^{\ \beta}D^\nu\mathrm{H}_\beta\right) + v^\mu\partial_\mu u_\gamma \\
Q\bar u_\gamma &= [c,\bar u_\gamma] + \omega\phi\overline{\mathrm{H}}_\gamma + \varepsilon^\mu\left(\frac{1}{2}D_\mu\overline{\mathrm{H}}_\gamma - \frac{1}{2}(\sigma_{\mu\nu})_\gamma^{\ \beta}D^\nu\overline{\mathrm{H}}_\beta\right) + v^\mu\partial_\mu\bar u_\gamma \\
Qv_{\dot\gamma} &= [c,v_{\dot\gamma}] - \frac{1}{2}\omega(\sigma^\nu)_{\alpha\dot\gamma}D_\nu\mathrm{H}^\alpha - \frac{1}{16}\varepsilon^\mu(\bar\sigma_\mu)_{\dot\gamma\beta}\bar\phi\mathrm{H}^\beta + v^\mu\partial_\mu v_{\dot\gamma} \\
Q\bar v_{\dot\gamma} &= [c,\bar v_{\dot\gamma}] - \frac{1}{2}\omega(\sigma^\nu)_{\alpha\dot\gamma}D_\nu\overline{\mathrm{H}}^\alpha - \frac{1}{16}\varepsilon^\mu(\bar\sigma_\mu)_{\dot\gamma\beta}\bar\phi\overline{\mathrm{H}}^\beta + v^\mu\partial_\mu\bar v_{\dot\gamma}
\end{aligned}
$$

$$Q^2 = 0 \ \text{on} \ (A,\phi,\bar\phi,\eta,\mathrm{H},\overline{\mathrm{H}},c,\omega,\varepsilon,v,\bar c,b)$$

$$
\begin{aligned}
Q^2\psi_\sigma &= \frac{g^2}{4}\omega\varepsilon^\mu\frac{\delta S}{\delta\chi^{\mu\sigma}} \\
&\quad + \frac{g^2}{32}\varepsilon^\mu\varepsilon^\nu\left(g_{\mu\sigma}\frac{\delta S}{\delta\psi^\nu} + g_{\nu\sigma}\frac{\delta S}{\delta\psi^\mu} - 2g_{\mu\nu}\frac{\delta S}{\delta\psi^\sigma}\right) \\
Q^2\chi_{\sigma\tau} &= -\frac{g^2}{2}\omega^2\frac{\delta S}{\delta\chi^{\sigma\tau}} \\
&\quad + \frac{g^2}{8}\omega\varepsilon^\mu\left(\varepsilon_{\mu\sigma\tau\nu}\frac{\delta S}{\delta\psi_\nu} + g_{\mu\sigma}\frac{\delta S}{\delta\psi^\tau} - g_{\mu\tau}\frac{\delta S}{\delta\psi^\sigma}\right) \\
Q^2u_\gamma &= \frac{g^2}{2}\left(\omega\varepsilon^\mu(\sigma_\mu)_{\gamma\dot\gamma}\frac{\delta S}{\delta\bar v^{\dot\gamma}} + \varepsilon^2\frac{\delta S}{\delta\bar u^\gamma}\right) \\
Q^2\bar u_\gamma &= \frac{g^2}{2}\left(\omega\varepsilon^\mu(\sigma_\mu)_{\gamma\dot\gamma}\frac{\delta S}{\delta v^{\dot\gamma}} + \varepsilon^2\frac{\delta S}{\delta u^\gamma}\right) \\
Q^2v_{\dot\gamma} &= \frac{g^2}{2}\left(\omega^2\frac{\delta S}{\delta\bar v_{\dot\gamma}} - \omega\varepsilon^\mu(\sigma_\mu)_{\beta\dot\gamma}\frac{\delta S}{\delta\bar u_\beta}\right) \\
Q^2\bar v_{\dot\gamma} &= \frac{g^2}{2}\left(\omega^2\frac{\delta S}{\delta v_{\dot\gamma}} - \omega\varepsilon^\mu(\sigma_\mu)_{\beta\dot\gamma}\frac{\delta S}{\delta u_\beta}\right)
\end{aligned}
$$

$$
\begin{array}{lcl}
L \to c & , & X^{\gamma} \to \mathrm{H}_{\gamma} \\
D \to \phi & , & \bar{X}^{\gamma} \to \overline{\mathrm{H}}_{\dot{\gamma}} \\
\Omega^{\mu} \to A_{\mu} & , & U^{\gamma} \to u_{\gamma} \\
\xi^{\mu} \to \psi_{\mu} & , & \bar{U}^{\gamma} \to \bar{u}_{\gamma} \\
\rho \to \bar{\phi} & , & V^{\dot{\gamma}} \to v_{\dot{\gamma}} \\
\tau \to \eta & , & \bar{V}^{\dot{\gamma}} \to \bar{v}_{\dot{\gamma}} \\
B^{\mu\nu} \to \chi_{\mu\nu} & & ,
\end{array}
$$

$$S_{ext} = \mathrm{Tr}\int d^4x \Phi^{\star i} Q \Phi^{i}$$

$$S_{\text{quad}} = \mathrm{Tr}\int \mathrm{d}^4x\left(\Omega_{ij}\Phi^{*i}\Phi^{*j}\right)$$

$$S_{\text{quad}} = g^2\mathrm{Tr}\int d^4x\Big(\frac{1}{8}\omega^2 B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}\omega B^{\mu\nu}\varepsilon_{\mu}\xi_{\nu} - \frac{1}{32}\varepsilon^{\mu}\varepsilon^{\nu}\xi_{\mu}\xi_{\nu} + \frac{1}{32}\varepsilon^2\xi^2$$
$$-\frac{1}{2}\varepsilon^2 U_{\gamma}\bar{U}^{\gamma} + \frac{1}{2}\omega^2 V_{\dot{\gamma}}\bar{V}^{\dot{\gamma}} - \frac{1}{2}\omega\varepsilon^{\mu}\bar{U}^{\alpha}\left(\sigma_{\mu}\right)_{\alpha\dot{\gamma}}V^{\dot{\gamma}}\Big)$$

$$\Sigma = S_T + S_{gf} + S_{ext} + S_{quad}$$

$$\mathcal{S}(\Sigma) = 0$$

$$\mathcal{S}(\Sigma) = \mathrm{Tr}\int d^4x\left(\frac{\delta\Sigma}{\delta\Phi^{\star i}}\frac{\delta\Sigma}{\delta\Phi^{i}} + \left(b + v^{\mu}\partial_{\mu}\bar{c}\right)\frac{\delta\Sigma}{\delta\bar{c}} + \left(\omega\varepsilon^{\mu}\partial_{\mu}\bar{c} + v^{\mu}\partial_{\mu}b\right)\frac{\delta\Sigma}{\delta b}\right)$$
$$-\omega\varepsilon^{\mu}\frac{\partial\Sigma}{\partial v^{\mu}}$$

$$\mathcal{P}_{\mu}\Sigma = \mathrm{Tr}\int d^4x\left(\partial_{\mu}\Phi^{i}\frac{\delta\Sigma}{\delta\Phi^{i}} + \partial_{\mu}\Phi^{*i}\frac{\delta\Sigma}{\delta\Phi^{*i}}\right) = 0$$

$$\frac{\partial\Sigma}{\partial v^{\mu}} = \Delta_{\mu}^{cl}$$

$$\Delta_{\mu}^{cl} = \mathrm{Tr}\int d^4x\big(L\partial_{\mu}c - D\partial_{\mu}\phi - \Omega^{\nu}\partial_{\mu}A_{\nu} + \xi^{\nu}\partial_{\mu}\psi_{\nu}$$
$$-\rho\partial_{\mu}\bar{\phi} + \tau\partial_{\mu}\eta + B^{\nu\sigma}\partial_{\mu}\chi_{\nu\sigma} - X^{\gamma}\partial_{\mu}\mathrm{H}_{\gamma} - \bar{X}^{\gamma}\partial_{\mu}\overline{\mathrm{H}}_{\gamma}$$
$$+U^{\gamma}\partial_{\mu}u_{\gamma} + \bar{U}^{\gamma}\partial_{\mu}\bar{u}_{\gamma} + V^{\dot{\gamma}}\partial_{\mu}v_{\dot{\gamma}} + \bar{V}^{\dot{\gamma}}\partial_{\mu}\bar{v}_{\dot{\gamma}}\big)$$

$$\Sigma = \hat{\Sigma} + v^{\mu}\Delta_{\mu}^{cl}$$

$$\frac{\partial\hat{\Sigma}}{\partial v^{\mu}} = 0$$

$$\mathcal{S}(\hat{\Sigma}) = \omega\varepsilon^{\mu}\Delta_{\mu}^{cl}$$

$$\mathcal{S}(\hat{\Sigma}) = \mathrm{Tr}\int d^4x\left(\frac{\delta\hat{\Sigma}}{\delta\Phi^{\star i}}\frac{\delta\hat{\Sigma}}{\delta\Phi^{i}} + b\frac{\delta\hat{\Sigma}}{\delta\bar{c}} + \omega\varepsilon^{\mu}\partial_{\mu}\bar{c}\frac{\delta\hat{\Sigma}}{\delta b}\right)$$

$$\mathcal{B}_{\hat{\Sigma}} = \mathrm{Tr}\int d^4x\left(\frac{\delta\hat{\Sigma}}{\delta\Phi^{i}}\frac{\delta}{\delta\Phi^{\star i}} + \frac{\delta\hat{\Sigma}}{\delta\Phi^{\star i}}\frac{\delta}{\delta\Phi^{i}} + b\frac{\delta\hat{\Sigma}}{\delta\bar{c}} + \omega\varepsilon^{\mu}\partial_{\mu}\bar{c}\frac{\delta\hat{\Sigma}}{\delta b}\right)$$

$$\mathcal{B}_{\hat{\Sigma}}\mathcal{B}_{\hat{\Sigma}} = \omega\varepsilon^{\mu}\mathcal{P}_{\mu}$$

$$\frac{\delta\hat{\Sigma}}{\delta b} = \partial^{\mu}A_{\mu}$$

$$\frac{\delta\hat{\Sigma}}{\delta\bar{c}} + \partial_{\mu}\frac{\delta\hat{\Sigma}}{\delta\Omega_{\mu}} = 0$$

$$\mathrm{Tr}\int d^4x\left(\frac{\delta\hat{\Sigma}}{\delta c}+\left[\bar{c},\frac{\delta\hat{\Sigma}}{\delta b}\right]\right)=\Delta_c^{cl}$$

$$\Delta_c^{cl}=\mathrm{Tr}\int d^4x([c,L]-[A,\Omega]-[\phi,D]+[\psi,\xi]-[\bar{\phi},\rho]+[\eta,\tau]$$
$$+[\chi,B]-[\mathrm{H},X]-[\overline{\mathrm{H}},\bar{X}]+[u,U]+[\bar{u},\bar{U}]+[v,V]+[\bar{v},\bar{V}])$$

$$[P_\mu,P_\nu]=0$$
$$[P_\lambda,M_{\mu\nu}]=(\eta_{\lambda\mu}P_\nu-\eta_{\lambda\nu}P_\mu)$$
$$[M_{\mu\nu},M_{\rho\sigma}]=-(\eta_{\mu\rho}M_{\nu\sigma}+\eta_{\nu\sigma}M_{\mu\rho}-\eta_{\mu\sigma}M_{\nu\rho}-\eta_{\nu\rho}M_{\mu\sigma})$$

$$[T^a,T^b]=if^{ab}{}_cT^c.$$

$$\{Q_\alpha^i,\bar{Q}_\beta^j\}=2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu\delta^{ij}$$
$$[P_\mu,Q_\alpha^i]=[P_\mu,\bar{Q}_{\dot{\alpha}j}]=0$$
$$[Q_\alpha^i,M_{\mu\nu}]=(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta^i$$

$$[\bar{Q}_{\dot{\alpha}j},M_{\mu\nu}]=(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}}\bar{Q}_j^{\dot{\beta}}$$
$$\{Q_\alpha^i,Q_\beta^j\}=\varepsilon_{\alpha\beta}Z^{ij}$$
$$\{\bar{Q}_{\dot{\alpha}i},\bar{Q}_{\dot{\beta}j}\}=\varepsilon_{\dot{\alpha}\dot{\beta}}Z_{ij}^\dagger$$
$$[Q_\alpha^i,T^a]=B^{ai}{}_jQ_\alpha^j$$
$$[T^a,\bar{Q}_{\dot{\alpha}i}]=B_j^{\dagger ai}\bar{Q}_{\dot{\alpha}i}$$
$$[Z^{ij},X]=[Z_{lk}^\dagger,X]=0,$$

$$Z^{ij}=a_b^{ij}T^b$$

$$Q_\alpha^i\longrightarrow U^i{}_kQ_\alpha^k\ \bar{Q}_{\dot{\alpha}j}\longrightarrow\bar{Q}_{\dot{\alpha}k}U_j^{\dagger k}$$

$$[Q_\alpha,R]=Q_\alpha,[\bar{Q}_{\dot{\alpha}},R]=-\bar{Q}_{\dot{\alpha}}$$

$$0\leq\sum_i\left(\{Q_1^i,(Q_{1i})^\dagger\}+\{Q_2^i,(Q_{2i})^\dagger\}\right)=-4\mathcal{N}P_0=4\mathcal{N}H$$

$$\langle\psi|H|\psi\rangle\geq0$$

$$W_\mu=\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}$$

$$C^2=C_{\mu\nu}C^{\mu\nu}$$
$$C_{\mu\nu}=B_\mu P_\nu-B_\nu P_\mu$$
$$B_\mu=W_\mu-\frac{1}{4}\bar{Q}_{\dot{\alpha}}\bar{\sigma}_\mu^{\dot{\alpha}\alpha}Q_\alpha.$$

$$(-)^{N_f}|B\rangle=+|B\rangle,(-)^{N_f}|F\rangle=-|F\rangle$$

$$(-)^{N_f}Q_\alpha=-Q_\alpha(-)^{N_f},(-)^{N_f}\bar{Q}_{\dot{\alpha}}=-\bar{Q}_{\dot{\alpha}}(-)^{N_f}$$

$$0=\mathrm{tr}[(-)^{N_f}\{Q_\alpha,\bar{Q}_{\dot{\alpha}}\}]=2\sigma_{\alpha\dot{\alpha}}^\mu\delta^{ij}P_\mu\mathrm{tr}(-)^{N_f},$$

$$\mathrm{tr}(-)^{N_f}=0$$

$$\{Q_\alpha^i,\bar{Q}_{\dot{\beta}j}\}=2M\delta_{\alpha\dot{\alpha}}\delta_j^i$$
$$\{Q_\alpha,Q_\beta\}=\{\bar{Q}_{\dot{\alpha}i}\bar{Q}_{\dot{\beta}j}\}=0$$

$$a^i_\alpha = \frac{1}{\sqrt{2M}} Q^i_\alpha$$
$$\left(a^i_\alpha\right)^\dagger = \frac{1}{\sqrt{2M}} \bar{Q}_{\dot{\alpha} i}$$

$$\left\{a^i_\alpha, \left(a^j_\beta\right)^\dagger\right\} = \delta^\beta_\alpha \delta^i_j$$
$$\left\{a^i_\alpha, a^j_\beta\right\} = \left\{\left(a^i_\alpha\right)^\dagger, \left(a^j_\beta\right)^\dagger\right\} = 0$$

$$a^i_\alpha |\Omega\rangle = 0 \ \forall i, \alpha$$

$$\left|\Omega^{(n)\alpha_1 \ldots \alpha_n}_{i_1 \ldots i_n}\right\rangle = \frac{1}{\sqrt{n!}} \left(a^{i_n}_{\alpha_n}\right)^\dagger \ldots \left(a^{i_1}_{\alpha_1}\right)^\dagger |\Omega\rangle$$

$$d = \sum_{n=0}^{2\mathcal{N}} \binom{2\mathcal{N}}{n} = 2^{2\mathcal{N}}$$

$$\{Q^i_\alpha, \bar{Q}_{\dot{\beta} j}\} = 2 \begin{pmatrix} 2E & 0 \\ 0 & 0 \end{pmatrix} \delta^i_j$$
$$\left\{Q^i_\alpha, Q^j_\beta\right\} = \{\bar{Q}_{\dot{\alpha} i}, \bar{Q}_{\dot{\beta} j}\} = 0$$

$$a^i = \frac{1}{2\sqrt{E}} Q^i_1$$
$$a^\dagger_i = \left(a^i\right)^\dagger = \frac{1}{2\sqrt{E}} \bar{Q}^i_{\dot{1}}.$$

$$\{a^i, a^\dagger_j\} = \delta^i_j$$
$$\{a^i, a^j\} = \{a^\dagger_i, a^\dagger_j\} = 0$$

$$a^i |\Omega_\lambda\rangle = 0$$

$$\left|\Omega^{(n)}_{\lambda + \frac{n}{2}; i_1, \ldots, i_n}\right\rangle = \frac{1}{\sqrt{n!}} a^\dagger_{i_n} \ldots a^\dagger_{i_1} |\Omega_\lambda\rangle.$$

$$d = \sum_{i=0}^{\mathcal{N}} \binom{\mathcal{N}}{n} = 2^{\mathcal{N}}$$

$$\{Q^i_\alpha, \bar{Q}_{\dot{\alpha} j}\} = 2M \delta^i_j \delta_{\alpha\dot{\alpha}}$$
$$\left\{Q^i_\alpha, Q^j_\beta\right\} = \varepsilon_{\alpha\beta} Z^{ij}$$
$$\{\bar{Q}_{\dot{\alpha} i}, \bar{Q}_{\dot{\beta} j}\} = -\varepsilon_{\dot{\alpha}\dot{\beta}} Z^\dagger_{ij},$$

$$Z^{ij} = U^i{}_k \tilde{Z}^{kl} (U^T)_l{}^j,$$

$$\tilde{Z} = \varepsilon \otimes D \qquad \text{(for } \mathcal{N} \text{ even)}$$
$$\tilde{Z} = \begin{pmatrix} \varepsilon \otimes D & 0 \\ 0 & 0 \end{pmatrix} \qquad \text{(for } \mathcal{N} \text{ odd)}$$

$$i = (a, R), j = (b, S),$$

$$\left\{Q^{aR}_\alpha, \bar{Q}^{bS}_{\dot{\beta}}\right\} = 2M \delta_{\alpha\dot{\alpha}} \delta^{ab} \delta^{RS}$$
$$\left\{Q^{aR}_\alpha, Q^{bS}_\beta\right\} = \varepsilon_{\alpha\beta} \varepsilon^{ab} \delta^{RS} Z_S$$
$$\left\{\bar{Q}^{aR}_{\dot{\alpha}}, \bar{Q}^{bS}_{\dot{\beta}}\right\} = \varepsilon_{\dot{\alpha}\dot{\beta}} \varepsilon^{ab} \delta^{RS} Z_S$$

$$a_\alpha^R = \frac{1}{\sqrt{2}}\left[Q_\alpha^{1R} + \varepsilon_{\alpha\beta}\left(Q_\beta^{2R}\right)^\dagger\right], \qquad (a_\alpha^R)^\dagger = a_\alpha^{\dagger R}$$
$$b_\alpha^R = \frac{1}{\sqrt{2}}\left[Q_\alpha^{1R} - \varepsilon_{\alpha\beta}\left(Q_\beta^{2R}\right)^\dagger\right], \qquad (b_\alpha^R)^\dagger = b_\alpha^{\dagger R}$$

$$\{a_\alpha^R, a_\beta^S\} = \{b_\alpha^R, b_\beta^S\} = \{a_\alpha^R, b_\beta^S\} = 0$$
$$\left\{a_\alpha^R, \left(a_\beta^S\right)^\dagger\right\} = \delta_{\alpha\beta}\delta^{RS}(2M + Z_S)$$
$$\left\{b_\alpha^R, \left(b_\beta^S\right)^\dagger\right\} = \delta_{\alpha\beta}\delta^{RS}(2M - Z_S).$$

$$|\Omega\rangle \qquad (\text{spin } 0)$$
$$(a_\alpha)^\dagger|\Omega\rangle \qquad \left(\text{spin } \frac{1}{2}\right)$$
$$\frac{1}{\sqrt{2}}(a_\alpha)^\dagger\left(a_\beta\right)^\dagger|\Omega\rangle = -\frac{1}{2\sqrt{2}}\varepsilon_{\alpha\beta}(a^\gamma)^\dagger\left(a_\gamma\right)^\dagger|\Omega\rangle \qquad (\text{spin } 0)$$

$$R(g)f(x) = f(\tau(g)x),$$

$$F = F\left(x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}\right)$$

$$g(x, \theta, \bar{\theta}) = e^{i\left(x^\mu P_\mu + \theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right)}$$

$$z_m = \left(x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}\right)$$

$$g = e^{i\xi^m L_m} e^{i\zeta^k H_k}$$

$$g = e^{i\left(a^\mu P_\mu + \eta^\alpha Q_\alpha + \bar{\eta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right)} e^{\frac{1}{2}w^{\mu\nu}M_{\mu\nu}}$$

$$k = e^{i\left(x^\mu P_\mu + \theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right)} = e^{z^m K_m}$$

$$g_0 \circ e^{z^m K_m} = e^{z'^m K_m} e^{\frac{1}{2}w'^{\mu\nu}M_{\mu\nu}} \equiv g'$$

$$x'^\mu = x^\mu + a^\mu + i\eta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}} + w^{\mu\nu}x_\nu$$
$$\theta'^\alpha = \theta^\alpha + \eta^\alpha + \frac{1}{4}w_{\mu\nu}\sigma^{\mu\nu\alpha}{}_\beta\theta_\beta$$
$$\bar{\theta}'^{\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}} + \bar{\eta}^{\dot{\alpha}} - \frac{1}{4}\bar{\theta}^{\dot{\beta}}\bar{\sigma}^{\mu\nu\dot{\alpha}}_{\dot{\beta}}w_{\mu\nu}.$$

$$(g_1 \circ g_2) \circ e^{z^m K_m}$$

$$g_1 \circ \left(g_2 \circ e^{z^m K_m}\right)$$

$$F(z) = F(x, \theta, \bar{\theta}).$$

$$\delta_g F = F(z + \delta z) - F(z) = \delta g_m X^m F(z),$$

$$\ell_\mu = \partial_\mu$$
$$\ell_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$$
$$\bar{\ell}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$
$$\ell_{\mu\nu} = -\left(x_\mu\partial_\nu - x_\nu\partial_\mu\right) - \frac{1}{2}\theta^\beta\sigma^{\mu\nu\alpha}_\beta\frac{\partial}{\partial\theta^\alpha} + \frac{1}{2}\bar{\theta}^{\dot{\beta}}\bar{\sigma}^{\mu\nu\dot{\alpha}}_{\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}.$$

$$\left\{\ell_\alpha, \ell_\beta\right\} = 0, \quad \left\{\ell_\alpha, \bar{\ell}_{\dot{\alpha}}\right\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\ell_\mu,$$
$$\left[\ell_\mu, \ell_\alpha\right] = 0, \quad \left[\ell_\alpha, \ell_{\mu\nu}\right] = \frac{1}{2}\sigma^\beta_{\mu\nu\alpha}\ell_\beta.$$

$$R(g)f_p(z) = D_p^q\left(e^{-\frac{1}{2}w^{\mu\nu}M_{\mu\nu}}\right)F_q(\tau(g)z)$$

$$F(x,\theta,\bar{\theta}) = f(x) + \theta^\alpha \chi_\alpha(x) + \bar{\theta}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}(x) + \theta^\alpha \theta_\alpha g(x) + \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} h(x)$$
$$+\theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} r_\mu(x) + \theta^\alpha \theta_\alpha \bar{\theta}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}(x) + \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \theta^\alpha \xi_\alpha(x) + \theta^\alpha \theta_\alpha \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} s(x)$$

$$\delta_\eta F(x,\theta,\bar{\theta}) = \delta_\eta f(x) + \theta^\alpha \delta_\eta \chi_\alpha(x) + \bar{\theta}_{\dot{\alpha}} \delta_\eta \bar{\psi}^{\dot{\alpha}}(x) + \theta^\alpha \theta_\alpha \delta_\eta g(x) + \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \delta_\eta h(x)$$
$$+\theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \delta_\eta r^\mu(x) + \theta^\alpha \theta_\alpha \bar{\theta}_{\dot{\alpha}} \delta_\eta \bar{\lambda}^{\dot{\alpha}}(x) + \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \theta^\alpha \delta_\eta \xi_\alpha(x) + \theta^\alpha \theta_\alpha \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \delta_\eta s(x)$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu, \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu$$

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu$$
$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0.$$

$$\bar{D}_{\dot{\alpha}} \Phi(x,\theta,\bar{\theta}) = 0,$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + 2i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \frac{\partial}{\partial y^\mu}$$
$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}}$$

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y).$$

$$\Phi(x,\theta,\bar{\theta}) = \phi(x) + \sqrt{2}\theta\psi(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) +$$
$$+\theta\theta F(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \frac{1}{4}\theta\theta\overline{\theta\theta} \,\Box\, \phi(x).$$

$$D_\alpha \Phi^\dagger(x,\theta,\bar{\theta}) = 0.$$

$$\Phi^\dagger = \Phi^\dagger(y^\dagger,\bar{\theta}) = \phi^*(y^\dagger) + \sqrt{2}\bar{\theta}\bar{\psi}(y^\dagger) + \overline{\theta\theta}F^*(y^\dagger).$$

$$\Phi_i\Phi_j = \phi_i(y)\phi_j(y) + \sqrt{2}\theta[\psi_i(y)\phi_j(y) + \phi_i(y)\psi_j(y)] +$$
$$+\theta\theta[\phi_i(y)F_j(y) + \phi_j(y)F_i(y) - \psi_i(y)\psi_j(y)]$$

$\bar{D}^2 F = \Phi$ is chiral
$D^2 F = \Phi^\dagger$ is antichiral

$$V^\dagger(x,\theta,\bar{\theta}) = V(x,\theta,\bar{\theta})$$

$$V(x,\theta,\bar{\theta}) = C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + \frac{i}{\sqrt{2}}\theta\theta S(x)$$
$$-\frac{i}{\sqrt{2}}\overline{\theta\theta}S^\dagger(x) - \theta\sigma^\mu\bar{\theta}A_\mu(x) + i\theta\theta\bar{\theta}\left[\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right]$$
$$-i\overline{\theta\theta}\theta\left[\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\right] + \frac{1}{2}\theta\theta\overline{\theta\theta}\left[D(x) + \frac{1}{2}\Box\, C(x)\right].$$

$$V \longrightarrow V + \Phi + \Phi^\dagger$$

$$\Phi + \Phi^\dagger = \phi + \phi^* + \sqrt{2}(\theta\psi + \overline{\theta\psi}) + \theta\theta F + \overline{\theta\theta}F^* + i\theta\sigma^\mu\bar{\theta}\partial_\mu(\phi - \phi^*) +$$
$$+\frac{1}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi + \frac{1}{\sqrt{2}}\overline{\theta\theta}\sigma^\mu\theta\partial_\mu\bar{\psi} + \frac{1}{4}\theta\theta\overline{\theta\theta}\,\Box\,(\phi + \phi^*).$$

$$C \longrightarrow C + \phi + \phi^*$$
$$\chi \longrightarrow \chi - i\sqrt{2}\psi$$
$$S \longrightarrow S - i\sqrt{2}F$$
$$A_\mu \longrightarrow A_\mu - i\partial_\mu(\phi - \phi^*)$$
$$\lambda \longrightarrow \lambda$$
$$D \longrightarrow D.$$

$$V(x,\theta,\bar{\theta}) = -\theta\sigma^{\mu}\bar{\theta}A_{\mu}(x) + i\theta\theta\overline{\theta\lambda}(x) - i\overline{\theta\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x)$$
$$V^{2}(x,\theta,\bar{\theta}) = -\frac{1}{2}\theta\theta\overline{\theta\theta}A_{\mu}(x)A^{\mu}(x)$$
$$V^{n}(x,\theta,\bar{\theta}) = 0, \forall n \geq 3.$$

$$W_{\alpha}(x,\theta,\bar{\theta}) = -\frac{1}{4}\overline{DD}D_{\alpha}V(x,\theta,\bar{\theta})$$

$$\bar{W}_{\alpha}(x,\theta,\bar{\theta}) = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V(x,\theta,\bar{\theta}).$$

$$\bar{D}_{\dot{\alpha}}W_{\alpha} = 0, D_{\alpha}\bar{W}_{\dot{\alpha}} = 0$$

$$W[V + \Phi + \Phi^{\dagger}] = W[V].$$

$$W_{\alpha}(y,\theta) = -i\lambda(y) + \left[\delta_{\alpha}{}^{\beta}D - \frac{i}{2}(\sigma^{\mu}\sigma^{\nu})_{\alpha}{}^{\beta}\left(\partial_{\mu}A_{\nu}(y) - \partial_{\nu}A_{\mu}(y)\right)\right]\theta_{\beta}$$
$$+\theta\theta\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}\bar{\lambda}^{\dot{\alpha}}(y) =$$
$$= -i\lambda(y) + \left[\delta_{\alpha}{}^{\beta}D - \frac{i}{2}(\sigma^{\mu}\sigma^{\nu})_{\alpha}{}^{\beta}F_{\mu\nu}(y)\right]\theta_{\beta} + \theta\theta\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}\bar{\lambda}^{\dot{\alpha}}(y)$$

$$S = \int d^4x\left[\int d^2\theta\Phi + \int d^2\bar{\theta}\Phi^{\dagger}\right]$$

$$S = \int d^4x \int d^4\theta V$$

$$S = \int d^4x\Big\{d^4\theta\Phi_i^{\dagger}\Phi_i + \Big[\int d^2\theta\left(\frac{1}{2}m_{ij}\Phi_i\Phi_j + \right.$$
$$\left. + \frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k + \lambda_i\Phi_i\right) + \text{h.c.}\Big]\Big\}$$

$$S = \int d^4x\Big\{-\partial_{\mu}\phi_i^{*}\partial^{\mu}\phi_i - i\bar{\psi}_i\bar{\sigma}^{\mu}\partial_{\mu}\psi_i + F_i^{*}F_i + \Big[m_{ij}\left(\phi_iF_j - \frac{1}{2}\psi_i\psi_j\right) +$$
$$+g_{ijk}\left(\phi_i\phi_jF_k - \psi_i\psi_j\phi_k\right) + \lambda_iF_i + \text{h.c.}\Big]\Big\}$$

$$\delta_{\eta}\phi_i = \sqrt{2}\eta\psi_i$$
$$\delta_{\eta}\psi_i = i\sqrt{2}\sigma^{\mu}\bar{\eta}\partial_{\mu}\phi_i + \sqrt{2}\eta F_i$$
$$\delta_{\eta}F_i = i\sqrt{2}\overline{\eta\sigma}^{\mu}\partial_{\mu}\psi_i$$

$$\frac{\partial\mathcal{L}}{\partial F_k^{*}} = F_k + \lambda_k^{*} + m_{ik}^{*}\phi_i^{*} + g_{ijk}^{*}\phi_i^{*}\phi_j^{*} = 0$$
$$\frac{\partial\mathcal{L}}{\partial F_k} = F_k^{*} + \lambda_k + m_{ik}\phi_i + g_{ijk}\phi_i\phi_j = 0$$

$$\mathcal{L} = -i\bar{\psi}_i\bar{\sigma}^{\mu}\partial_{\mu}\psi_i - \partial^{\mu}\phi^{*}\partial_{\mu}\phi - \frac{1}{2}m_{ij}\psi_i\psi_j - \frac{1}{2}m_{ij}^{*}\bar{\psi}_i\bar{\psi}_j +$$
$$-g_{ijk}\psi_i\psi_j\phi_k - g_{ijk}^{*}\bar{\psi}_i\bar{\psi}_j\phi_k^{*} - \mathcal{V}(\phi_i,\phi_i^{*}),$$

$$\mathcal{V} = (F_k^{*}F_k) = \left(m_{ik}\phi_i + g_{ijk}\phi_i\phi_j\right)(m_{mk}^{*}\phi_m^{*} + g_{mnk}^{*}\phi_m^{*}\phi_n^{*}).$$

$$S = \int d^4x\left\{\int d^4\theta\Phi_i^{\dagger}\Phi_i + \left[\int d^2\theta\, \mathrm{W}(\Phi) + \text{h.c.}\right]\right\}$$

$$\mathcal{V}(\Phi) = \left|\frac{\partial \mathrm{W}(\Phi)}{\partial\Phi}\right|^2$$

$$\Phi = \bar{D}^2\Psi\ \Phi^{\dagger} = D^2\Psi^{\dagger}$$

$$S=\int d^4xd^4\theta\{(D^2\Psi^\dagger)(\bar{D}^2\Psi)-2m(\Psi\bar{D}^2\Psi+\Psi^\dagger D^2\Psi+$$
$$-\frac{4g}{3!}\left[\Psi(\bar{D}^2\Psi)^2+\Psi^\dagger\left(D^2\Psi^\dagger\right)^2\right]\}$$

$$\int d^4x\int d^2\theta\Psi=\int d^4x\int d^4\theta\left(-\frac{1}{4}\bar{D}^2\right)\Psi$$

$$\Phi_k'=e^{-iq_k\Lambda}\Phi_k,\quad \text{with}\quad \bar{D}_{\dot\alpha}\Lambda=0$$
$$\Phi_k^{\dagger\prime}=e^{iq_k\Lambda^\dagger}\Phi_k^\dagger,\quad \text{with}\quad D_\alpha\Lambda^\dagger=0$$

$$V'=V+i(\Lambda-\Lambda^\dagger),$$

$$S=\int d^4x\left\{\int d^2\theta\frac{1}{4}W^\alpha W_\alpha+\int d^2\bar\theta\frac{1}{4}\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}+\int d^4\theta\Phi_i^\dagger e^{q_iV}\Phi_i+\right.$$
$$\left.+\left[\int d^2\theta\left(\frac{1}{2}m_{ij}\Phi_i\Phi_j+\frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k\right)+\text{ h.c.}\right]\right\}$$

$$(\Phi)_{ij}=T^a_{ij}\Phi_a,\left(\Phi^\dagger\right)_{ij}=T^a_{ij}\Phi^\dagger_a,V_{ij}=T^a_{ij}V_a$$

$$\text{tr}(T^aT^b)=C(\mathbf{r})\delta^{ab}\ [T^a,T^b]=if^{ab}{}_cT^c$$

$$\Phi'=e^{-i\Lambda}\Phi,\Phi^{\dagger\prime}=\Phi^\dagger e^{i\Lambda^\dagger}$$

$$V\longrightarrow V':e^{V'}=e^{-i\Lambda^\dagger}e^Ve^{i\Lambda}.$$

$$\delta V=V'-V=i\mathcal{L}_{V/2}\left[(\Lambda+\Lambda^\dagger+\coth\left(\mathcal{L}_{V/2}\right)(\Lambda-\Lambda^\dagger)\right]$$

$$W_\alpha=-\frac{1}{4}\overline{DD}e^{-V}D_\alpha e^V$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+ig\left[A_\mu,A_\nu\right]$$

$$W_\alpha\longrightarrow W_\alpha'=e^{-i\Lambda}W_\alpha e^{i\Lambda}$$

$$S=\int d^4x\frac{1}{d_{\mathbf{r}}}\text{tr}\left\{\frac{1}{16g^2}\left[\int d^2\theta W^\alpha W_\alpha+\int d^2\bar\theta\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\right]+\int d^4\theta e^{-V}\Phi^\dagger e^V\Phi+\right.$$
$$\left.+\left[\int d^2\theta\left(\frac{1}{2}m_{ij}\Phi_i\Phi_j+\frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k\right)+\text{ h.c.}\right]\right\}$$

$$S=\int d^4x\left\{-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-i\bar\lambda^a\bar\sigma^\mu\mathcal{D}_\mu\lambda^a+\frac{1}{2}D^aD^a-\mathcal{D}_\mu\phi^\dagger\mathcal{D}^\mu\phi\right.$$
$$\left.-i\bar\psi\bar\sigma^\mu\mathcal{D}_\mu\psi+F^\dagger F+i\sqrt{2}g\left(\phi^\dagger T^a\psi\lambda^a-\bar\lambda^aT^a\phi\bar\psi\right)+gD^a\phi^\dagger T^a\phi\right\}$$

$$\mathcal{D}_\mu\phi=\partial_\mu\phi+igA^a_\mu T^a\phi$$
$$\mathcal{D}_\mu\psi=\partial_\mu\psi+igA^a_\mu T^a\psi$$
$$\mathcal{D}_\mu\lambda^a=\partial_\mu\lambda^a+igf^a{}_{bc}A^b_\mu\lambda^c$$

$$\mathcal{V}_D=\frac{1}{8}g^2\left([\phi^\dagger,\phi]\right)^2.$$

$$\delta_\xi\phi=\sqrt{2}\xi\psi$$
$$\delta_\xi\psi=i\sqrt{2}\sigma^\mu\bar\xi\mathcal{D}_\mu\phi+\sqrt{2}\xi F$$
$$\delta_\xi F=i\sqrt{2}\sigma^\mu\bar\xi\mathcal{D}_\mu\psi+2igT^a\phi\overline{\xi\lambda}{}^a$$
$$\delta_\xi A^a_\mu=-i\bar\lambda^a\bar\sigma_\mu\xi+i\bar\xi\bar\sigma_\mu\lambda^a$$
$$\delta_\xi\lambda^a=\sigma^{\mu\nu}\xi F^a_{\mu\nu}+i\xi D^a$$
$$\delta_\xi D^a=-\xi\sigma^\mu\mathcal{D}_\mu\bar\lambda^a-\mathcal{D}_\mu\lambda^a\sigma^\mu\bar\xi.$$

$$\tau=\frac{\theta}{2\pi}+\frac{4\pi i}{g^2}$$

$$S = \frac{1}{8\pi}\mathrm{Im}\left[\tau\int d^4xd^2\theta\mathrm{tr}W^\alpha W_\alpha\right] =$$
$$= \frac{1}{g^2}\int d^4x\mathrm{tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu\mathcal{D}_\mu\bar{\lambda} + \frac{1}{2}D^2\right] +$$
$$- \frac{\theta}{32\pi^2}\int d^4x\mathrm{tr}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$g = e^{i\left(a^\mu P_\mu + \eta_i^\alpha Q_\alpha^i + \bar{\eta}_{\dot{\alpha}}^i \bar{Q}_i^{\dot{\alpha}} + b^r Z_r + \frac{1}{2}w_{\mu\nu}M^{\mu\nu}\right)}$$

$$k = e^{i\left(x^\mu P_\mu + \theta_i^\alpha Q_\alpha^i + \bar{\theta}_{\dot{\alpha}}^i \bar{Q}_i^{\dot{\alpha}} + \zeta^r Z_r\right)}$$

$$z_m = \left(x_\mu, \theta_\alpha^i, \bar{\theta}_{\dot{\alpha}}^j, \zeta_r\right)$$

$$x'^\mu = x^\mu + i\eta_i^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}i} - i\theta_i^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\eta}^{\dot{\alpha}i}$$
$$\theta_i'^\alpha = \theta_i^\alpha + \eta_i^\alpha$$
$$\bar{\theta}^{\dot{\alpha}j} = \bar{\theta}^{\dot{\alpha}j} + \bar{\eta}^{\dot{\alpha}j}$$
$$\zeta'^r = \zeta^r + i\eta_i^\alpha \theta_{\alpha j}(\Omega^r)^{ij} + i\bar{\eta}_{\dot{\alpha}}^i \bar{\theta}^{\dot{\alpha}j}(\Omega^r)_{ij}.$$

$$\zeta'^r = \zeta^r + b^r.$$

$$F = F\left(x_\mu, \theta_\alpha^i, \bar{\theta}_{\dot{\alpha}}^j, \zeta_r\right).$$

$$R(g)F(z) = F(z')$$

$$\ell_\mu = \partial_\mu$$
$$\ell_\alpha^i = \frac{\partial}{\partial\theta_i^\alpha} + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}i}\partial_\mu + i(\Omega^r)^{ij}\theta_{\alpha j}\frac{\partial}{\partial\zeta^r}$$
$$\bar{\ell}_{\dot{\alpha}i} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}i}} + i\theta_i^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu + i\bar{\theta}_\alpha^j (\Omega^r)_{ij}\frac{\partial}{\partial\zeta^r}$$
$$\ell^r = \frac{\partial}{\partial\zeta_r}$$
$$\ell_{\mu\nu} = -\left(x_\mu\partial_\nu - x_\nu\partial_\mu\right) - \frac{1}{2}\theta^\beta \sigma_\beta^{\mu\nu\alpha}\frac{\partial}{\partial\theta^\alpha} + \frac{1}{2}\bar{\theta}^{\dot{\beta}}\bar{\sigma}_{\dot{\beta}}^{\mu\nu\dot{\alpha}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$$
$$\ell_a = -\theta_i^\alpha B_{aj}^i \frac{\partial}{\partial\theta_j^\alpha} - \bar{\theta}^{\dot{\alpha}i}\left(B_{aj}^i\right)^* \frac{\partial}{\partial\bar{\theta}_j^{\dot{\alpha}}}.$$

$$F(x,\theta,\bar{\theta},\zeta) = F^{(0)}(x,\theta,\bar{\theta}) + F_{(r)}^{(1)}(x,\theta,\bar{\theta})\zeta^{(r)} + \cdots,$$

$$F^{(0)}(x,\theta,\bar{\theta}) = F(x,\theta,\bar{\theta},0)$$
$$F_{(r)}^{(1)}(x,\theta,\bar{\theta}) = \frac{\partial}{\partial\zeta^r}F(x,\theta,\bar{\theta},\zeta)\Big|_{\zeta=0}$$

$$D_\mu = \partial_\mu$$
$$D_\alpha^i = \frac{\partial}{\partial\theta_i^\alpha} + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}i}\partial_\mu - i(\Omega^r)^{ij}\theta_{\alpha j}\frac{\partial}{\partial\zeta^r}$$
$$\bar{D}_{\dot{\alpha}i} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}i}} - i\theta_i^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu - i(\Omega^r)_{ij}\bar{\theta}_{\dot{\alpha}}^j \frac{\partial}{\partial\zeta^r}$$
$$D_r = \frac{\partial}{\partial\zeta^r}$$

$$D_r V(x,\theta,\bar{\theta},\zeta) = \frac{\partial}{\partial\zeta^r}V(x,\theta,\bar{\theta},\zeta) = 0$$

$$\left\{D_\alpha^i, D_\beta^j\right\} = 2\varepsilon_{\alpha\beta}\varepsilon^{ij}D_\zeta$$

$$\bar{D}_{\dot{\alpha}}^i W = 0, D_\zeta W = 0$$

$$W, D_\alpha^i W, D^{\alpha i} D_\alpha^j W, D_\alpha^i D_{\beta i} W,$$
$$D_\alpha^i D_\beta^j D_\gamma^k W, D_\alpha^i D_\beta^j D_\gamma^k D_\delta^l W.$$

$$D^{\alpha i} D_\alpha^j W = \bar{D}_{\dot{\alpha}}^i \bar{D}^{\dot{\alpha} j} \bar{W}.$$

$$D, \chi_\alpha^i, C^{ij}, F_{\mu\nu},$$

$$D_\alpha^i \Phi_j = \frac{1}{2} \delta_j^i D_\alpha^k \Phi_k$$
$$\bar{D}_{\dot{\alpha} i} \Phi_j + \bar{D}_{\dot{\alpha} j} \Phi_i = 0.$$

$$\Phi, D_\alpha^k \Phi_k, \bar{D}_{\dot{\alpha}}^k \Phi_k, D_\zeta \Phi$$

$$\psi_\alpha, \chi_\alpha, \phi_i, F_i.$$

$$z_m = (x_\mu, \theta_\alpha^i, \bar{\theta}_{\dot{\alpha} j}) = \left(x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}, \tilde{\theta}_\alpha, \bar{\tilde{\theta}}_{\dot{\alpha}}\right)$$

$$\bar{D}_{\dot{\alpha}} \Psi = 0, \bar{\tilde{D}}_{\dot{\alpha}} \Psi = 0$$

$$\tilde{y}_\mu = x_\mu + i\theta\sigma_\mu \bar{\theta} + i\tilde{\theta}\sigma_\mu \bar{\tilde{\theta}}$$

$$\Psi(\tilde{y}, \theta) = \Phi(\tilde{y}, \theta) + i\sqrt{2}\tilde{\theta}^\alpha W_\alpha(\tilde{y}, \theta) + \tilde{\theta}\tilde{\theta} G(\tilde{y}, \theta),$$

$$\Psi_{ij} = T_{ij}^a \Psi_a.$$

$$S = \frac{1}{4\pi} \mathrm{Im} \left\{ \tau \int d^4x \int d^2\theta d^2\tilde{\theta} \mathrm{tr}\left(\frac{1}{2}\Psi^2\right) \right\},$$

$$\Psi^2|_{\theta^2\tilde{\theta}^2} = W^\alpha W_\alpha|_{\theta^2} + 2G\Phi|_{\theta^2}.$$

$$D^{\alpha i} D_\alpha^j \Psi = \bar{D}_{\dot{\alpha}}^i \bar{D}^{\dot{\alpha} j} \Psi^\dagger.$$

$$G = \int d^2\theta \Phi^\dagger(\tilde{y} - i\theta\sigma\bar{\theta}, \bar{\theta}) e^{V(\tilde{y} - i\theta\sigma\bar{\theta}, \bar{\theta})}$$

$$S = \frac{1}{4\pi} \mathrm{Im} \left\{ \int d^4x \int d^2\theta d^2\tilde{\theta} \mathcal{F}(\Psi) \right\},$$

$$S_K = \int d^4x D^{\alpha i} D_\alpha^j [\Phi^{i\dagger} D^{\beta k} D_\beta^k \Phi^j]$$

$$S = \int d^4x \left\{ \int d^4\theta [Q^\dagger e^{2V} Q + \tilde{Q}^\dagger e^{2V} \tilde{Q} + \Phi^\dagger e^{2V} \Phi] \right.$$
$$\left. + \frac{1}{4\pi} \mathrm{Im} \left[\tau \int d^2\theta W^\alpha W_\alpha \right] + \left[\int d^2\theta \left(\sqrt{2}\tilde{Q}\Phi Q + m_i \tilde{Q}_i Q_i\right) + \text{ h.c.} \right] \right\}$$

$$Z[J] = N \int [\mathcal{D}\phi] e^{iS[\phi] + i\int dx \phi_i(x) J_i(x)},$$

$$S = \int d^4x d^4\theta K(\Phi, \Phi^\dagger, V) + \left[\int d^4x d^2\theta \, \mathrm{W}[\Phi] + \text{ h.c.} \right].$$

$$\frac{\delta F(x', \theta', \bar{\theta}')}{\delta F(x, \theta, \bar{\theta})} = \delta_4(x - x') \delta_4(\theta - \theta')$$

$$\delta_4(\theta - \theta') = \delta_2(\theta - \theta') \delta_2(\bar{\theta} - \bar{\theta}') = (\theta - \theta')^2 (\bar{\theta} - \bar{\theta}')^2$$

$$\int d^4\theta \delta_4(\theta - \theta') F(x, \theta, \bar{\theta}) = F(x, \theta', \bar{\theta}')$$

$$\frac{\delta}{\delta F(x,\theta,\bar{\theta})}\int d^4x'd^4\theta' F(x',\theta',\bar{\theta}')G(x',\theta',\bar{\theta}') = G(x,\theta,\bar{\theta})$$

$$\frac{\delta\Phi(x',\theta',\bar{\theta}')}{\delta\Phi(x,\theta,\bar{\theta})} = -\frac{1}{4}\bar{D}^2\delta_4(x-x')\delta_4(\theta-\theta')$$

$$\begin{aligned}&\frac{\delta}{\delta\Phi(x,\theta,\bar{\theta})}\int d^4x'd^2\theta'\Phi(x',\theta',\bar{\theta}')\tilde{\Phi}(x',\theta',\bar{\theta}') = \\ &= \int d^4x'd^2\theta'\left(-\frac{1}{4}\bar{D}^2\right)\delta_4(x-x')\delta_4(\theta-\theta')\tilde{\Phi}(x',\theta',\bar{\theta}') = \\ &= \int d^4x'd^4\theta'\delta_4(x-x')\delta_4(\theta-\theta')\tilde{\Phi}(x',\theta',\bar{\theta}') = \tilde{\Phi}(x,\theta,\bar{\theta})\end{aligned}$$

$$Z[J,J^\dagger,J_V] = N\int [\mathcal{D}\Phi\mathcal{D}\Phi^\dagger\mathcal{D}V]e^{iS[\Phi,\Phi^\dagger,V]+i(\Phi,J)+i(\Phi^\dagger,J^\dagger)+i(V,J_V)}$$

$$\begin{aligned}(\Phi,J) &= \int d^4xd^2\theta\Phi(x,\theta,\bar{\theta})J(x,\theta,\bar{\theta}) \\ (\Phi^\dagger,J^\dagger) &= \int d^4xd^2\bar{\theta}\Phi^\dagger(x,\theta,\bar{\theta})J^\dagger(x,\theta,\bar{\theta}) \\ (V,J_V) &= \int d^4xd^4\theta V(x,\theta,\bar{\theta})J_V(x,\theta,\bar{\theta})\end{aligned}$$

$$\begin{aligned}&\mathcal{G}_n(z_1,\dots,z_i,z_{i+1},\dots,z_j,z_{j+1},\dots,z_n) \\ &= \langle 0|T\{\Phi(z_1)\dots\Phi(z_i)\Phi^\dagger(z_{i+1})\dots\Phi^\dagger(z_j)V(z_{j+1})\dots V(z_n)\}|0\rangle \\ &= (-i)^n\frac{\delta^n Z[J,J^\dagger,J_V]}{\delta J(z_1)\dots\delta J(z_i)\delta J^\dagger(z_{i+1})\dots\delta J^\dagger(z_j)\delta J_V(z_{j+1})\dots\delta J_V(z_n)}\Bigg|_{J=J^\dagger=J_V=0}\end{aligned}$$

$$W[J,J^\dagger,J_V] = (-i)\log\left(Z[J,J^\dagger,J_V]\right).$$

$$\tilde{\Phi} = \frac{\delta W}{\delta J},\tilde{\Phi}^\dagger = \frac{\delta W}{\delta J^\dagger},\tilde{V} = \frac{\delta W}{\delta J_V},$$

$$\Gamma[\tilde{\Phi},\tilde{\Phi}^\dagger,\tilde{V}] = \left(W[J,J^\dagger,J_V] - [(J,\Phi)+(J^\dagger,\Phi^\dagger)+(J_V,V)]\right)\Big|_{(\tilde{\Phi},\tilde{\Phi}^\dagger,\tilde{J}_V)}$$

$$J = J[\tilde{\Phi},\tilde{\Phi}^\dagger,\tilde{V}], J^\dagger = J^\dagger[\tilde{\Phi},\tilde{\Phi}^\dagger,\tilde{V}] \text{ and } J_V = J_V[\tilde{\Phi},\tilde{\Phi}^\dagger,\tilde{V}]$$

$$S[\Phi,\Phi^\dagger,V] = S_0[\Phi,\Phi^\dagger,V] + S_{\text{int}}[\Phi,\Phi^\dagger,V]$$

$$Z[J,J^\dagger,J_V] = e^{iS_{\text{int}}\left[\frac{\delta}{\delta J},\frac{\delta}{\delta J^\dagger},\frac{\delta}{\delta J_V}\right]}Z_0[J,J^\dagger,J_V]$$

$$Z_0[J,J^\dagger J_V] = N\int [\mathcal{D}\Phi\mathcal{D}\Phi^\dagger\mathcal{D}V]e^{iS_0[\Phi,\Phi^\dagger,V]+i(\Phi,J)+i(\Phi^\dagger,J^\dagger)+i(V,J_V)}$$

$$e^{iS_{\text{int}}\left[\frac{\delta}{\delta J},\frac{\delta}{\delta J^\dagger},\frac{\delta}{\delta J_V}\right]}$$

$$\begin{aligned}Z_0[J,J^\dagger,J_V] = \exp\Bigg\{&-\frac{i}{2}\int d^4xd^4\theta(\Phi,\Phi^\dagger)\mathcal{M}_\Phi\begin{pmatrix}\Phi\\ \Phi^\dagger\end{pmatrix} \\ &+\frac{i}{2}\int d^4xd^4\theta V\mathcal{M}_V V + i(\Phi,J) + i(\Phi^\dagger,J^\dagger) + i(V,J_V)\Bigg\}\end{aligned}$$

$$\begin{aligned}Z_0[J,J^\dagger,J_V] = \exp\Bigg\{&\frac{i}{2}\int d^8zd^8z'(J(z),J^\dagger(z))\Delta_\Phi(z,z')\begin{pmatrix}J(z')\\ J^\dagger(z')\end{pmatrix} \\ &+\frac{i}{2}\int d^8zd^8z'J_V(z)\Delta_V(z,z')J_V(z')\Bigg\}\end{aligned}$$

$$S_0 = \int d^4x \left\{ \int d^4\theta \Phi_i^\dagger \Phi_i + \left[\int d^2\theta \frac{1}{2} m_{ij} \Phi_i \Phi_j + \int d^2\bar{\theta} \frac{1}{2} m_{ij} \Phi_i^\dagger \Phi_j^\dagger \right] \right\}$$
$$= \int d^4x d^4\theta \left[\Phi_i^\dagger \Phi_i - \frac{1}{8} m_{ij} \left(\Phi_i \frac{DD}{\Box} \Phi_j + \Phi_i^\dagger \frac{\overline{DD}}{\Box} \Phi_j^\dagger \right) \right]$$

$$P_1 \quad = \frac{1}{16} \frac{D^2 \bar{D}^2}{\Box}, \quad P_1 \Phi^\dagger \quad = \Phi^\dagger, P_1 \Phi = 0$$

$$S_0 = \frac{1}{2} \int d^4x d^4\theta (\Phi_i, \Phi_i^\dagger) \mathcal{M}_{ij} \begin{pmatrix} \Phi_j \\ \Phi_j^\dagger \end{pmatrix}$$

$$\mathcal{M}_{ij} = \begin{pmatrix} -\frac{1}{4} \frac{m_{ij}}{\Box} DD & \delta_{ij} \\ \delta_{ij} & -\frac{1}{4} \frac{m_{ij}}{\Box} \overline{DD} \end{pmatrix}.$$

$$i \int d^4x d^4\theta \left[\frac{1}{2} (\Phi, \Phi^\dagger) \mathcal{M} \begin{pmatrix} \Phi \\ \Phi^\dagger \end{pmatrix} + (\Phi, \Phi^\dagger) \begin{pmatrix} -\frac{1}{4} \frac{D^2}{\Box} & 0 \\ 0 & -\frac{1}{4} \frac{\bar{D}^2}{\Box} \end{pmatrix} \begin{pmatrix} J \\ J^\dagger \end{pmatrix} \right]$$

$$\Delta_\Phi(x, \theta, \bar{\theta}; x', \theta', \bar{\theta}') = \frac{1}{\Box - m^2} \begin{pmatrix} \frac{m}{4} \frac{D^2}{\Box} & 1 \\ 1 & \frac{m}{4} \frac{\bar{D}^2}{\Box} \end{pmatrix} \delta_8(z - z')$$

$$S_0 = \int d^4x \left\{ \int d^2\theta \frac{1}{4} W^\alpha W_\alpha + \int d^2\bar{\theta} \frac{1}{4} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} + \int d^4\theta m^2 V^2 \right\}$$

$$\begin{aligned} S &= S_0 + S_{\mathrm{GF}} + S_{\mathrm{FP}} = \\ &= \int d^4x d^4\theta \left[\frac{1}{4} W^\alpha W_\alpha \delta_2(\bar{\theta}) + \frac{1}{4} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \delta_2(\theta) \right] \\ &+ \int d^4x d^4\theta \left[-\frac{\xi}{8} (\bar{D}^2 V)(D^2 V) \right] \\ &+ i \int d^4x d^4\theta (C' + \bar{C}') \mathcal{L}_{\frac{V}{2}} \left[(C + \bar{C}) + \coth \mathcal{L}_{\frac{V}{2}} (C - \bar{C}) \right] \end{aligned}$$

$$\begin{aligned} S &= S_0 + S_{\mathrm{GF}} = \\ &= \int d^4x d^4\theta \{ V[-\Box P_T - \xi(P_1 + P_2)\Box] V \} = \\ &= \int d^4x d^4\theta V \mathcal{M}_V V \end{aligned}$$

$$Z[J_V] = \int [\mathcal{D}V] e^{i \int V \mathcal{M}_V V + i(V, J_V)}$$

$$\begin{aligned} \Delta_V(x, \theta, \bar{\theta}; x', \theta', \bar{\theta}') &= \left[-\frac{1}{\Box} P_T - \frac{\alpha}{\Box} (P_1 + P_2) \right] \delta_8(z - z') \\ &= -\frac{1}{\Box} [1 + (\alpha - 1)(P_1 + P_2)] \delta_8(z - z') (1.69) \end{aligned}$$

$$\begin{aligned} &\mathcal{G}_n(z_1, \ldots, z_i, z_{i+1}, \ldots, z_j, z_{j+1}, \ldots, z_n) \\ &= \langle 0 | T\{ \Phi(z_1) \ldots \Phi(z_i) \Phi^\dagger(z_{i+1}) \ldots \Phi^\dagger(z_j) V(z_{j+1}) \ldots V(z_n) \} | 0 \rangle \\ &= \frac{\delta}{\delta J(z_1)} \cdots \frac{\delta}{\delta J(z_i)} \frac{\delta}{\delta J^\dagger(z_{i+1})} \cdots \frac{\delta}{\delta J^\dagger(z_j)} \frac{\delta}{\delta J_V(z_{j+1})} \cdots \frac{\delta}{\delta J_V(z_n)} \\ &\cdot \sum_{k=0}^{\infty} \frac{(-i)^k}{k!} \left(S_{\mathrm{int}} \left[\frac{\delta}{\delta J}, \frac{\delta}{\delta J^\dagger}, \frac{\delta}{\delta J_V} \right] \right)^k Z_0[J, J^\dagger, J_V] \Big|_{J = J^\dagger = J_V = 0} \end{aligned}$$

$$d_s = 4L - 2P + 2V - C - E - 2L$$

$$d_s = 2 - C - E$$

$$d_s = -C$$

$$d_s = 1 - C - n,$$

$$\Gamma = \sum_n \int\int d^4x_1 \dots d^4x_n d^4\theta G(x_1, \dots, x_n) \cdot f_n[\Phi, \Phi^\dagger, V, D_\alpha\Phi, \bar{D}_{\dot\alpha}\Phi^\dagger, D_\alpha V, \bar{D}_{\dot\alpha}V, \dots]$$

$$\int d^4x d^2\theta d^2\bar\theta \frac{D^2}{16\,\square}\Phi^n = \int d^4x d^2\theta \frac{\bar{D}^2 D^2}{16\,\square}\Phi^n = \int d^4x d^2\theta\Phi^n$$

$$S = \int d^{10}x \mathrm{tr}\left\{-\frac{1}{4}F_{\Gamma\Lambda}F^{\Gamma\Lambda} + \frac{i}{2}\bar\lambda\Gamma^\Lambda D_\Lambda\lambda\right\}$$

$$\lambda = C_{(10)}\bar\lambda^T, \lambda = \pm\Gamma_5\lambda$$

$$\delta_\eta A_\Lambda = i\bar\eta\Gamma_\Lambda\lambda$$
$$\delta_\eta\lambda = \Sigma_{\Gamma\Lambda}F^{\Gamma\Lambda}\eta.$$

$$A^a_\mu = A^a_\mu \qquad \mu = 0,1,2,3$$
$$\varphi^a_{B4} = \frac{1}{\sqrt{2}}(A^a_{B+3} + iA^a_{B+6}) \qquad B = 1,2,3$$
$$\varphi^{aAB} = \frac{1}{2}\varepsilon^{ABC4}\varphi^a_{C4} = (\varphi^{aAB})^* \quad A, B, C = 1,2,3.$$

$$\lambda = \begin{pmatrix} L\chi^1 \\ \vdots \\ L\chi^4 \\ R\tilde\chi_1 \\ \vdots \\ R\tilde\chi_4 \end{pmatrix}, \qquad \tilde\chi_A = C(\bar\chi^A)^T$$

$$\lambda^A = \begin{pmatrix} L\chi^A \\ R\tilde\chi_A \end{pmatrix} \; A = 1,2,3,4$$

$$\varphi^i = \frac{1}{2}\bar t^i_{AB}\varphi^{AB}, i = 1,2, \dots, 6$$

$$S = \int d^4x \,\mathrm{tr}\Big\{(D_\mu\varphi^{AB})(D^\mu\overline{\varphi}_{AB}) - \frac{1}{2}i(\lambda^{\alpha A}\overleftrightarrow{\not{D}}_{\alpha\dot\alpha}\overline{\lambda}^{\dot\alpha}{}_A) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} +$$
$$-g\lambda^{\alpha A}[\lambda_\alpha{}^B, \overline{\varphi}_{AB}] - g\overline{\lambda}_{\dot\alpha A}[\overline{\lambda}^{\dot\alpha}{}_B, \varphi^{AB}] + 2g^2[\varphi^{AB}, \varphi^{CD}][\overline{\varphi}_{AB}, \overline{\varphi}_{CD}]\Big\},$$

$$\delta\varphi^{AB} = \frac{1}{2}(\lambda^{\alpha A}\eta_\alpha{}^B - \lambda^{\alpha B}\eta_\alpha{}^A) + \frac{1}{2}\varepsilon^{ABCD}\bar\eta_{\dot\alpha C}\bar\lambda^{\dot\alpha}_D$$
$$\delta\lambda_\alpha{}^A = -\frac{1}{2}F^-_{\mu\nu}\sigma^{\mu\nu}{}_\alpha{}^\beta\eta_\beta{}^A + 4i(\not{D}_{\alpha\dot\alpha}\varphi^{AB})\bar\eta^{\dot\alpha}_B - 8g[\bar\varphi_{BC}, \varphi^{CA}]\eta_\alpha{}^B$$
$$\delta A^\mu = -i\lambda^{\alpha A}\sigma^\mu{}_{\alpha\dot\alpha}\bar\eta^{\dot\alpha}{}_A - i\eta^{\alpha A}\sigma^\mu{}_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}{}_A,$$

$$\bar W_{AB} = \frac{1}{2}\varepsilon_{ABCD}W^{CD}$$

$$\mathcal{D}^A_\alpha W^{BC} = \mathcal{D}^{[A}_\alpha W^{BC]}$$

$$S = \frac{1}{d_r}\mathrm{tr}\Big\{\int d^4x\Big[d^4\theta e^{-gV}\Phi^\dagger_I e^{gV}\Phi^I + \frac{1}{4g^2}\Big(\int d^2\theta\frac{1}{4}W^\alpha W_\alpha + \text{h.c.}\Big)$$
$$+ig\frac{\sqrt{2}}{3!}\Big(\int d^2\theta\varepsilon_{IJK}\Phi^I[\Phi^J, \Phi^K] + \int d^2\bar\theta\varepsilon^{IJK}\Phi^\dagger_I[\Phi^\dagger_J, \Phi^\dagger_K]\Big)\Big]\Big\}$$

$$\int d^4x \left\{ \int d^2\theta \, \mathrm{W}(x,\theta,\bar\theta) + \text{h.c.} \right\}$$
$$= \int d^4x \left\{ \int d^2\theta \left[-i\frac{\sqrt{2}}{3!}\varepsilon_{IJK} f^{abc} \left(\Phi_a^I \Phi_b^J \Phi_c^K \right)(x,\theta,\bar\theta) \right] + \text{h.c.} \right\}$$

$$S = \int d^4x d^4\theta \{ V^a(x,\theta,\bar\theta)[-\Box P_T - \xi(P_1+P_2)\Box] V_a(x,\theta,\bar\theta)$$
$$+\Phi^{\dagger a}(x,\theta,\bar\theta)\Phi_a^I(x,\theta,\bar\theta) + igf_{abc}\Phi^{\dagger a}(x,\theta,\bar\theta)V^b(x,\theta,\bar\theta)\Phi^{Ic}(x,\theta,\bar\theta)$$
$$-\frac{1}{2}g^2 f_{ab}{}^e f_{ecd}\Phi^{\dagger a}(x,\theta,\bar\theta)V^b(x,\theta,\bar\theta)V^c(x,\theta,\bar\theta)\Phi^{Id}(x,\theta,\bar\theta) + \cdots$$
$$-\frac{i}{4}gf_{abc}[\bar D^2(D^\alpha V^a(x,\theta,\bar\theta))]V^b(x,\theta,\bar\theta)(D_\alpha V^c(x,\theta,\bar\theta))$$
$$-\frac{1}{8}g^2 f_{ab}{}^e f_{ecd}V^a(x,\theta,\bar\theta)(D^\alpha V^b(x,\theta,\bar\theta))[(\bar D^2 V^c(x,\theta,\bar\theta))(D_\alpha V^d(x,\theta,\bar\theta))]$$
$$+\cdots - \frac{\sqrt{2}}{3!}gf^{abc}\left[\varepsilon_{IJK}\Phi_a^I(x,\theta,\bar\theta)\Phi_b^J(x,\theta,\bar\theta)\Phi_c^K(x,\theta,\bar\theta)\delta(\bar\theta) + \right.$$
$$\left. +\varepsilon^{IJK}\Phi^\dagger{}_{Ia}(x,\theta,\bar\theta)\Phi^\dagger{}_{Jb}(x,\theta,\bar\theta)\Phi^\dagger{}_{Kc}(x,\theta,\bar\theta)\delta(\theta)\right] + (\bar C'{}_a(x,\theta,\bar\theta)C^a(x,\theta,\bar\theta)$$
$$-C'{}_a(x,\theta,\bar\theta)\bar C^a(x,\theta,\bar\theta)) + \frac{i}{2\sqrt{2}}gf_{abc}(C'^a(x,\theta,\bar\theta) + \bar C'^a(x,\theta,\bar\theta))$$
$$\cdot V^b(x,\theta,\bar\theta)(C^c(x,\theta,\bar\theta) + \bar C^c(x,\theta,\bar\theta)) - \frac{1}{8}g^2 f_{ab}{}^e f_{ecd}(C'^a(x,\theta,\bar\theta) + \bar C'^a(x,\theta,\bar\theta))$$
$$\cdot V^b(x,\theta,\bar\theta)V^c(x,\theta,\bar\theta)(C^d(x,\theta,\bar\theta) + \bar C^d(x,\theta,\bar\theta)) + \cdots \}$$

$$S = \int d^4x \left\{ -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - D_\mu\varphi_I^\dagger D^\mu\varphi^I + \frac{i}{2}\bar\lambda^a \not{D}\lambda^a + \frac{i}{2}\bar\psi_I^a \not{D}\psi^{aI} + \right.$$
$$+ig\sqrt{2}f_{abc}\left(\bar\lambda^a\bar\varphi_I^b\psi^{cI} - \bar\psi_I^a\varphi^{bI}\lambda^c\right) - \frac{i\sqrt{2}}{2}f_{abc}\left(\varepsilon_{JK}^I\bar\psi_I^a\varphi^{bJ}\psi^{cK} + \right.$$
$$\left. \left. -\varepsilon_K^{IJ}\bar\psi_I^a\varphi_J^{\dagger b}\psi^{cK}\right) - \frac{1}{2}g^2\left(f_{abc}\varphi_I^{\dagger a}\varphi_I^b\right)^2 + \frac{1}{2}g^2 f_{abc}f_{de}^a\varepsilon_{IJK}\varepsilon^{LMK}\varphi^{bI}\varphi^{cJ}\varphi_L^{\dagger d}\varphi_M^{\dagger e}\right\}$$

$$\mathcal{V}\to \lambda^u \in (\mathbf{2},\mathbf{1})_{+1}, \varphi \in (\mathbf{1},\mathbf{1})_{+2}, A_\mu \in (\mathbf{1},\mathbf{1})_0$$
$$\mathcal{H} \to \psi^{\dot u} \in (\mathbf{1},\mathbf{2})_{-1}, q_{u\dot u} \in (\mathbf{2},\mathbf{2})_0,$$

$$\mathcal{L}_Y^{(\mathcal{N}=2)} = \frac{\sqrt{2}}{2}gf_{abc}\{q_S^a(\lambda^{b\alpha u}\sigma^S{}_{u\dot u}\psi_\alpha{}^{c\dot u} + \bar\psi_{\dot\alpha\dot u}^b\bar\sigma^{S\dot u}\bar\lambda_u^{c\dot\alpha})$$
$$+\varphi^a(\varepsilon^{uv}\bar\lambda_{\dot\alpha u}^b\bar\lambda_v^{c\dot\alpha} + \varepsilon_{\dot u\dot v}\psi^{b\alpha\dot u}\psi_\alpha^{c\dot v})$$
$$+\bar\varphi^a(\varepsilon_{uv}\lambda^{b\alpha u}\lambda_\alpha^{cv} + \varepsilon^{\dot u\dot v}\bar\psi_{\dot\alpha\dot u}^b\bar\psi_{\dot v}^{c\dot\alpha})\}$$

$$\mathrm{tr}([\varphi^{AB},\varphi^{CD}][\bar\varphi_{AB},\bar\varphi_{CD}]) = 0 \Leftrightarrow \mathrm{tr}\left([\varphi^i,\varphi^j][\varphi_i,\varphi_j]\right) = 0.$$

$$\sum_{i,j}^{1,6}\sum_{a=1}^{\dim G}\varphi_{ia}^2\varphi_{jb}^2\,\mathrm{tr}([T^a,T^b])^2 = 0$$

$$\mathcal{M} = \mathbb{R}^{6r}/\mathcal{S}_r,$$

$$[M_{\mu\nu}, K_\lambda] = \eta_{\nu\lambda}K_\mu - \eta_{\mu\lambda}K_\nu$$
$$[D, P_\mu] = -P_\mu \qquad [D, K_\mu] = K_\mu$$
$$[P_\mu, K_\nu] = -2M_{\mu\nu} + 2\eta_{\mu\nu}D \qquad [K_\mu, K_\nu] = 0$$
$$[S_{\alpha i}, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta S_{\beta i} \qquad [\bar{S}^i_{\dot\alpha}, M^{\mu\nu}] = (\bar\sigma)^{\dot\alpha}{}_{\dot\beta}\bar{S}^{\dot\beta i}$$
$$\{S_{\alpha i}, \bar{S}^j_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}K_\mu\delta^j_i$$
$$\{S_{\alpha i}, S_{\beta j}\} = 0 \qquad \{\bar{S}^i_{\dot\alpha}, \bar{S}^j_{\dot\beta}\} = 0$$
$$[Q^i_\alpha, D] = \frac{1}{2}Q^i_\alpha \qquad [\bar{Q}_{\dot\alpha i}, D] = \frac{1}{2}\bar{Q}_{\dot\alpha i}$$
$$[S_{\alpha i}, D] = -\frac{1}{2}S_{\alpha i} \qquad [\bar{S}^i_{\dot\alpha}, D] = \frac{1}{2}\bar{S}^i_{\dot\alpha}$$
$$[Q^i_\alpha, K^\mu] = i\sigma^\mu_{\alpha\dot\alpha}\bar{S}^{\dot\alpha i} \qquad [\bar{Q}^{\dot\alpha}_i, K^\mu] = -i\bar\sigma^{\mu\dot\alpha\alpha}S_{\alpha i}$$
$$[S_{\alpha i}, K^\mu] = 0 \qquad [\bar{S}^{\dot\alpha i}, K^\mu] = 0$$
$$[S_{\alpha i}, P^\mu] = -i\sigma^\mu_{\alpha\dot\alpha}\bar{Q}^{\dot\alpha}_i \qquad [\bar{S}^{\dot\alpha i}, P^\mu] = i\bar\sigma^{\mu\dot\alpha\alpha}Q^i_\alpha$$
$$[S_{\alpha i}, T^a] = -B^{aj}_i S_{\alpha j} \qquad [T^a, \bar{S}^i_{\dot\alpha}] = -B^{\dagger ai}{}_j\bar{S}^j_{\dot\alpha}$$
$$\{Q^i_\alpha, S_{\beta j}\} = 2\varepsilon_{\alpha\beta}\delta^i_j D - i(\sigma^{\mu\nu})_\alpha{}^\gamma\varepsilon_{\gamma\beta}M_{\mu\nu}\delta^i_j - 4i\varepsilon_{\alpha\beta}\delta^i_j A + B^{ai}{}_j T_a$$
$$\{\bar{Q}_{\dot\alpha i}, \bar{S}^j_{\dot\beta}\} = 2\varepsilon_{\dot\alpha\dot\beta}\delta^j_i D - i\varepsilon_{\dot\alpha\dot\gamma}(\bar\sigma^{\mu\nu})^{\dot\gamma}{}_{\dot\beta}M_{\mu\nu}\delta^j_i + 4i\varepsilon_{\dot\alpha\dot\beta}\delta^j_i A + B^{\dagger aj}_i T_a$$
$$[Q^i_\alpha, A] = -i\left(\frac{4-\mathcal{N}}{4\mathcal{N}}\right)Q^i_\alpha \qquad [\bar{Q}_{\dot\alpha i}, A] = i\left(\frac{4-\mathcal{N}}{4\mathcal{N}}\right)\bar{Q}_{\dot\alpha i}$$
$$[S_{\alpha i}, A] = i\left(\frac{4-\mathcal{N}}{4\mathcal{N}}\right)S_{\alpha i} \qquad [\bar{S}^i_{\dot\alpha}, A] = -i\left(\frac{4-\mathcal{N}}{4\mathcal{N}}\right)\bar{S}^i_{\dot\alpha}.$$

$$[Q^i_\alpha, A] = [\bar{Q}_{\dot\alpha i}, A] = 0.$$

$$\begin{aligned}\mathcal{T}^{\mu\nu} = &\frac{1}{2}\left[\delta^{\mu\nu}\left(F^-_{\rho\sigma}\right)^2 - 4F^{-\mu}{}_\rho F^{-\nu\rho} + \text{ h.c.}\right] - \frac{1}{2}\lambda^{\alpha A}\sigma^{(\mu}{}_{\alpha\dot\alpha}\overleftrightarrow{\partial}^{\nu)}\bar\lambda^{\dot\alpha}_A \\ &+\delta^{\mu\nu}\left(\partial_\rho\bar\varphi_{AB}\right)\left(\partial^\rho\varphi^{AB}\right) - 2\left(\partial^\mu\bar\varphi_{AB}\right)\left(\partial^\nu\varphi^{AB}\right) \\ &-\frac{1}{3}\left(\delta^{\mu\nu}\,\square - \partial^\mu\partial^\nu\right)\left(\bar\varphi_{AB}\varphi^{AB}\right)\end{aligned}$$
$$\Sigma^\mu{}_{\alpha A} = -\sigma^{\kappa\nu}F^-_{\kappa\nu}\sigma^\mu{}_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}{}_A + 2i\bar\varphi_{AB}\overleftrightarrow{\partial}^\mu\lambda_\alpha{}^B + \frac{4}{3}i\sigma^{\mu\nu}{}_\alpha{}^\beta\partial_\nu\left(\bar\varphi_{AB}\lambda_\beta{}^B\right)$$
$$\mathcal{J}^\mu{}_A{}^B = \bar\varphi_{AC}\overleftrightarrow{\partial}^\mu\varphi^{CB} + \bar\lambda_{\dot\alpha A}\bar\sigma^{\mu\dot\alpha\alpha}\lambda_\alpha{}^B - \frac{1}{4}\delta_A{}^B\lambda^{\alpha C}\sigma^\mu{}_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}{}_C.$$

$$\begin{aligned}\mathcal{C} &= \left(F^-{}_{\mu\nu}\right)^2 \\ \hat\Lambda^A_\alpha &= -\sigma^{\mu\nu}{}_\alpha{}^\beta F^-{}_{\mu\nu}\lambda_\beta{}^A \\ \mathcal{E}^{AB} &= \lambda^{\alpha A}\lambda_\alpha{}^B \\ \mathcal{B}_{\mu\nu}{}^{AB} &= \lambda^{\alpha A}\sigma_{\mu\nu\alpha}{}^\beta\lambda_\beta{}^B + 2i\varphi^{AB}F^-{}_{\mu\nu} \\ \hat\chi^C_{\alpha AB} &= \frac{1}{2}\varepsilon_{ABDE}\left(\varphi^{DE}\lambda_\alpha{}^C + \varphi^{CE}\lambda_\alpha{}^D\right) \\ \mathcal{Q}^{AB}{}_{CD} &= \varphi^{AB}\bar\varphi_{CD} - \frac{1}{12}\delta^A{}_{[C}\delta^B{}_{D]}\varphi^{EF}\bar\varphi_{EF}.\end{aligned}$$

$$\mathcal{W}^{ij}_{(2)} = \text{tr}\left(W^iW^j - \frac{\delta^{ij}}{6}W_kW^k\right).$$

$$\mathcal{E}^{(AB)} = \text{tr}(\lambda^{\alpha A}\lambda_\alpha{}^B) + gt^{(AB)}_{[ijk]} + \text{tr}\left(\varphi^i\varphi^j\varphi^k\right),$$

$$\beta = -\frac{g^2}{16\pi^2}(3-n)N,$$

$$\beta(g) = \frac{3}{2}g^2\frac{\partial Z^{(1)}_g}{\partial g}$$

$$\frac{\beta(g)}{g}F^a_{\mu\nu}F^{\mu\nu}_a = \mathcal{T}^\mu_\mu$$

$$Z(K) = \sum_{\{\sigma\}} \exp\left(K \sum_{(i,j)} \sigma_i \sigma_j\right),$$

$$\sinh\ 2K^* = \frac{1}{\sinh\ 2K}$$

$$S_{\mathrm{SG}} = \int\ d^2x \left[\frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{\alpha}{\beta^2}(\cos\ \beta\phi - 1)\right]$$

$$S_{\mathrm{T}} = \int\ d^2x \left[\bar{\psi} i \gamma_\mu \partial^\mu \psi + m\bar{\psi}\psi - \frac{g}{2}\bar{\psi}\gamma_\mu \psi \bar{\psi}\gamma^\mu \psi\right]$$

$$\frac{\beta^2}{4\pi} = \frac{1}{1 + \frac{g}{\pi}},$$

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}D^\mu \Phi^a D_\mu \Phi_a - \frac{\lambda}{4}(\Phi^a \Phi_a - v^2)^2,$$

$$Q_m = \frac{1}{v}\int\ d^3x D_i \Phi^a B^i_a$$
$$Q_e = \frac{1}{v}\int\ d^3x D_i \Phi^a E^i_a$$

$$m \geq v\sqrt{Q_e^2 + Q_m^2}$$

$$m_W = ev \ll v, m_M = gv = \frac{4\pi}{e}v \gg v.$$

$$e \longrightarrow g = \frac{4\pi}{e},$$

$$\mathcal{L} = -\frac{1}{32\pi}\mathrm{Im}\left[\tau(F^{\mu\nu} + i * F^{\mu\nu})(F_{\mu\nu} + i * F_{\mu\nu})\right] - \frac{1}{2}D^\mu \Phi D_\mu \Phi,$$

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{e^2}$$

$$\tau \longrightarrow \tau + b, b \in \mathbb{Z},$$

$$\begin{pmatrix} n_e \\ n_m \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} n_e \\ n_m \end{pmatrix}.$$

$$\tau \longrightarrow -\frac{1}{\tau}$$

$$\begin{pmatrix} n_e \\ n_m \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} n_e \\ n_m \end{pmatrix}$$

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}, a, b, c, d \in \mathbb{Z}, ad - bc = 1$$

$$\begin{pmatrix} n_e \\ n_m \end{pmatrix} \longrightarrow \begin{pmatrix} -a & b \\ c & -d \end{pmatrix}\begin{pmatrix} n_e \\ n_m \end{pmatrix}.$$

$$m^2 \geq 4\pi v^2 (n_e, n_m)\frac{1}{\mathrm{Im}\tau}\begin{pmatrix} 1 & -\mathrm{Re}\tau \\ -\mathrm{Re}\tau & |\tau|^2 \end{pmatrix}\begin{pmatrix} n_e \\ n_m \end{pmatrix}$$

$$\left\{Q^i_\alpha, Q^j_\beta\right\} = \delta^{ij}\gamma^\mu_{\alpha\beta}P_\mu + \delta_{\alpha\beta}U^{ij} + (\gamma_5)_{\alpha\beta}V^{ij}, i, j = 1{,}2,$$

State	Spin (S_z)
$\|\Omega\rangle$	0
$a_{\pm}^{i\dagger}\|\Omega\rangle$	$\pm\frac{1}{2}$
$a_{-}^{i\dagger}a_{+}^{j\dagger}\|\Omega\rangle$	0
$a_{+}^{1\dagger}a_{+}^{2\dagger}\|\Omega\rangle$	1
$a_{-}^{1\dagger}a_{-}^{2\dagger}\|\Omega\rangle$	-1
$a_{\mp}^{1\dagger}a_{\mp}^{2\dagger}a_{\pm}^{i\dagger}\|\Omega\rangle$	$\mp\frac{1}{2}$
$a_{+}^{1\dagger}a_{+}^{2\dagger}a_{-}^{1\dagger}a_{-}^{2\dagger}\|\Omega\rangle$	0

$$U^{ij} = \varepsilon^{ij} v Q_e, V^{ij} = \varepsilon^{ij} v Q_m$$

$$Q_e^i = \frac{1}{v}\int_{S_\infty^2} d\vec{\sigma}\cdot \mathrm{tr}(\vec{E}\varphi^i)$$

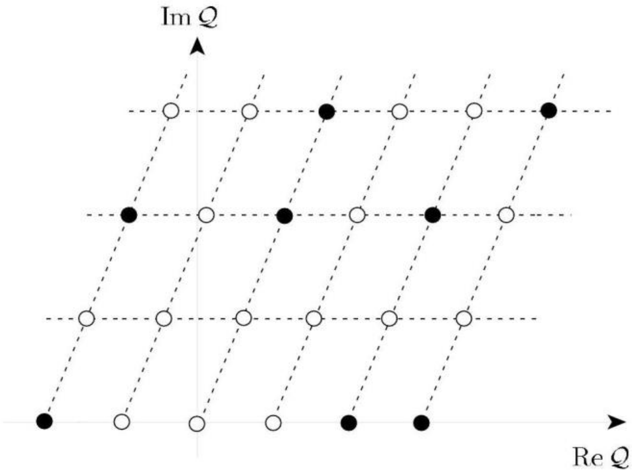
$$Q_m^i = \frac{1}{v}\int_{S_\infty^2} d\vec{\sigma}\cdot \mathrm{tr}(\vec{B}\varphi^i)$$

$$v^2 = \sum_{i=1}^{6}\left\langle \mathrm{tr}(\varphi^i)^2\right\rangle$$

$$m^2 \geq v^2\left[(Q_e^i)^2 + (Q_m^i)^2\right]$$

$$Q = Q_e + iQ_m = \left(n_e e + n_m \frac{e\theta}{2\pi}\right) + in_m\frac{4\pi}{e}.$$

$$m \geq m_1 + m_2$$



$$A_i^{\mathrm{cla}}(\vec{x}+\vec{X}), \Phi^{\mathrm{cla}}(\vec{x}+\vec{X})$$

$$\mathcal{C} = \mathcal{A}/\mathcal{G}$$

$$H = T + V = \frac{1}{2}\int d^3x, [\dot{A}_i^a\dot{A}_a^i + \dot{\Phi}^a\dot{\Phi}_a] + \frac{1}{2}\int d^3x, [B_i^a B_a^i + D_i\Phi^a D^i\Phi_a]$$

$$\delta A_i^a(\vec{x},t), \delta\Phi^a(\vec{x},t)$$

$$\varepsilon_{ijk}D^j\delta A^k - D_i\delta\Phi + e\left[\delta A_i, \Phi^{\rm cl}\right] = 0$$

$$D_i\delta A^i + e\left[\Phi^{\rm cl}, \delta\Phi\right] = 0.$$

$$\delta A_i = D_i\left(\chi(t)\Phi^{\rm cl}\right)$$
$$\delta\Phi = 0$$
$$\delta A_0 = D_0\left(\chi(t)\Phi^{\rm cl}\right) - \dot{\chi}(t)\Phi^{\rm cl}$$

$$\mathcal{G}_{\alpha\beta} = -\int d^3x {\rm tr}\left(\delta_\alpha A_i \delta_\beta A^i + \delta_\alpha\Phi\delta_\beta\Phi\right)$$

$$\delta_\alpha A_i = \frac{\partial A_i}{\partial z^\alpha} - D_i\epsilon_\alpha$$
$$\delta_\alpha\Phi = \frac{\partial\Phi}{\partial z^\alpha} - e\left[\Phi^{\rm cl}, \epsilon_\alpha\right]$$

$$A_i(\vec{x}, t) \longrightarrow A_i^{\rm cl}(\vec{x}, z^\alpha(t))$$
$$A_0 \longrightarrow \dot{z}^\alpha(t)\epsilon_\alpha$$
$$\Phi(\vec{x}, t) \longrightarrow \Phi^{\rm cl}(\vec{x}, z^\alpha(t))$$

$$S = -\frac{1}{2}\int d^3x dt {\rm tr}\left(F_{0i}^{\rm cl}F_{\rm cl}^{0i}\right) = \int dt \mathcal{G}_{\alpha\beta}\dot{z}^\alpha\dot{z}^\beta - \frac{4\pi v}{e}k$$

$$S = \frac{1}{2}\int dt\left[\frac{4\pi v}{e}\dot{\vec{X}}^2 + \frac{4\pi}{ve^2}\dot{\chi}^2\right] - \frac{4\pi v}{e}$$

$$\psi = e^{i\vec{P}\cdot\vec{X}}e^{in_e\chi}$$

$$m = \frac{n_e^2 ve^3}{8\pi} + \frac{4\pi v}{e} \approx v[Q_e^2 + Q_m^2]$$

$$\langle\varphi^2\rangle = \langle\varphi^3\rangle = \cdots = \langle\varphi^6\rangle = 0$$
$$\langle\varphi^1\rangle = \Phi, \langle{\rm tr}\Phi^2\rangle = v^2.$$

$$\lambda(\vec{x}, t) \sim \psi(t)\lambda^{\rm cl}(\vec{x}, z_\alpha(t))$$

$$S = \frac{1}{2}\int dt\left[\mathcal{G}_{\alpha\beta}\left(\dot{z}^\alpha\dot{z}^\beta + i\bar{\psi}^\alpha\gamma^0 D_t\psi^\beta\right) + \frac{1}{6}R_{\alpha\beta\gamma\delta}\bar{\psi}^\alpha\psi^\gamma\bar{\psi}^\beta\psi^\delta\right] - \frac{4\pi v}{e^2}k$$

$$D_t\psi^\alpha = \dot{\psi}^\alpha + \Gamma^\alpha_{\beta\gamma}\dot{z}^\beta\psi^\gamma$$

$$Q_e = n_e + \frac{e\theta}{2\pi}$$

$$\mathcal{M}_k = \mathbb{R}^3 \times \left(S^1 \times \tilde{\mathcal{M}}_k^0\right)/\mathbb{Z}_k,$$

$$\tilde{\mathcal{M}}_k^0, |s\rangle = |\omega, n_e\rangle \otimes |\alpha\rangle^2$$

$$Q_e = n_e e + n_m\frac{e\theta}{2\pi}$$

$$W_\alpha = -\frac{1}{4}\overline{DD}e^{-V}D_\alpha e^V = \sum_{k=1}^{\infty} W_\alpha^{(k)}$$

$$W_\alpha^{(1)} = -\frac{1}{4}\overline{DD}D_\alpha V$$
$$W_\alpha^{(2)} = \frac{1}{8}\overline{DD}[V, D_\alpha V]$$

$$S^{(\mathrm{E})}=\frac{1}{d_{\mathbf{r}}}\mathrm{tr}\int d^4xd^4\theta\Big\{-\Phi_I^\dagger\Phi^I-\frac{1}{4g^2}\Big[\frac{1}{4}W^{(1)\alpha}W_\alpha^{(1)}\delta(\bar\theta)+\frac{1}{4}\bar W_{\dot\alpha}^{(1)}\bar W^{(1)\dot\alpha}\delta(\theta)$$
$$-\frac{1}{8\alpha}\bar D^2VD^2V\Big]-g[\Phi_I^\dagger V]\Phi^I-\Big[\Big(\frac{1}{8g^2}W^{(1)\alpha}W_\alpha^{(2)}+\frac{1}{16g^2}W^{(2)\alpha}W_\alpha^{(2)}\Big)\delta(\bar\theta)$$
$$+\Big(\frac{1}{8g^2}\bar W_{\dot\alpha}^{(1)}\bar W^{(2)\dot\alpha}+\frac{1}{16g^2}\bar W_{\dot\alpha}^{(2)}\bar W^{(2)\dot\alpha}\Big)\delta(\theta)\Big]-\frac{1}{2}g^2\Big[V,[V,\Phi_I^\dagger]\Big]\Phi^I+\cdots\Big\}$$

$$S=\frac{1}{d_{\mathbf{r}}}\mathrm{tr}\int d^4xd^4\theta\Big\{\big[V\,\Box\,V-\Phi_I^\dagger\Phi^I\big]-\Big[\Big(\frac{1}{8g^2}W^{(1)\alpha}W_\alpha^{(2)}+\frac{1}{16g^2}W^{(2)\alpha}W_\alpha^{(2)}\Big)\delta(\bar\theta)+$$
$$+\Big(\frac{1}{8g^2}\bar W_{\dot\alpha}^{(1)}\bar W^{(2)\dot\alpha}+\frac{1}{16g^2}\bar W_{\dot\alpha}^{(2)}\bar W^{(2)\dot\alpha}\Big)\delta(\theta)\Big]-\Big[-g[\Phi_I^\dagger V]\Phi^I+\frac{1}{2}g^2\Big[V,[V,\Phi_I^\dagger]\Big]\Phi^I\Big]\Big\}$$

$$W_\alpha^{(1)}=-i\lambda_\alpha+\Big[\delta_\alpha^\beta D-\frac{1}{2}\delta_\alpha^\beta\,\Box\,C-\frac{i}{2}(\sigma^\mu\bar\sigma^\nu)_\alpha{}^\beta\big(\partial_\mu A_\nu-\partial_\nu A_\mu\big)\Big]\theta_\beta$$
$$+\theta\theta\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar\lambda^{\dot\alpha}.$$

$$S=S_0+S_{\mathrm{int}}$$

$$S_0=\int d^4x\{[(\partial_\mu\varphi_I^{a\dagger})(\partial^\mu\varphi_a^I)+\bar\psi_I^a\bar\sigma^\mu(\partial_\mu\psi_a^I)+F_I^{a\dagger}F_a^I]+[S_a^\dagger\,\Box\,S^a$$
$$-\frac{1}{2}C^a\,\Box\,D_a-\frac{1}{2}D^a\,\Box\,C_a-\frac{1}{2}C^a\,\Box^2\,C_a+\frac{1}{2}\chi^a\,\Box\,\lambda_a+\frac{1}{2}\bar\chi^a\,\Box\,\bar\lambda_a+\frac{1}{2}\lambda^a\,\Box\,\chi_a$$
$$+\frac{1}{2}\bar\lambda^a\,\Box\,\bar\chi_a+\frac{1}{2}\chi^a\,\Box\,\sigma^\mu(\partial_\mu\bar\chi_a)+\frac{1}{2}\bar\chi^a\,\Box\,\bar\sigma^\mu(\partial_\mu\chi_a)+\frac{1}{2}(\partial_\mu A_\nu^a)(\partial_\nu A_{a\mu})\Big]\Big\}.$$

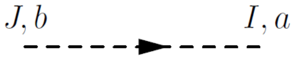
$$S_{\mathrm{int}}=\int d^4x\Big\{igf_{abc}\Big[\frac{1}{2}\phi_I^{a\dagger}D^b\phi^{cI}+\frac{1}{2}\phi_I^{a\dagger}(\Box\,C^b)\phi^{cI}-\frac{1}{2}(\partial_\mu\phi_I^{a\dagger})C^b(\partial^\mu\phi^{cI})$$
$$+\frac{i}{2}(\phi_I^{a\dagger}A_\mu^b(\partial^\mu\phi^{cI})-(\partial^\mu\phi_I^{a\dagger})A_\mu^b\phi^{cI})+\frac{i}{\sqrt2}(\phi_I^{a\dagger}S^{b\dagger}F^{cI}-F_I^{a\dagger}S^b\phi^{cI})$$
$$-F_I^{a\dagger}C^bF^{cI}+\frac{i}{\sqrt2}(\bar\psi_I^a\bar\lambda^b\phi^{cI}-\phi_I^{a\dagger}\lambda^b\psi^{cI})+\frac{i}{\sqrt2}(F_I^{a\dagger}\chi^b\psi^{cI}-\bar\psi_I^a\bar\chi^bF^{cI})$$
$$-\frac{i}{\sqrt2}(\phi^{a\dagger}(\partial_\mu\bar\chi^b)\sigma^\mu\psi^{cI}+\bar\psi_I^a\bar\sigma^\mu(\partial_\mu\chi^b)\phi^{cI})+\frac{1}{2}(C^b\bar\psi_I^a\bar\sigma^\mu(\partial_\mu\psi^{cI})-C^b(\partial_\mu\bar\psi_I^a)\bar\sigma^\mu\psi^{cI})$$
$$-\frac{i}{2}\bar\psi_I^a\bar\sigma^\mu\psi^{cI}A_\mu^b\Big]-\frac{\sqrt2}{3!}gf_{abc}[\varepsilon^{IJK}(\phi_I^{a\dagger}\phi_J^{b\dagger}F_K^{c\dagger}-\phi_I^{a\dagger}\bar\psi_J^b\bar\psi_K^c)+\varepsilon_{IJK}(\phi^{aI}\phi^{bJ}F^{cK}$$
$$-\phi^{aI}\psi^{bJ}\psi^{cK})]-\frac{g^2}{2}f_{abe}f_{cd}^e([-C^b\psi^{dI}\sigma^\mu\bar\psi_I^aA_\mu^c$$
$$+(\bar\chi^c\bar\psi_I^d)(\chi^b\psi^{aI})+\frac{i}{2}(\partial_\mu\psi^{dI}\sigma^\mu\bar\psi_I^a-\psi^{dI}\sigma^\mu\partial_\mu\bar\psi_I^a)C^bC^c\Big]+\varphi_I^{a\dagger}[C^b(D^c$$
$$+\frac{1}{2}\,\Box\,C^c\Big)-\chi^b\Big(\lambda^c+\frac{i}{2}\sigma^\mu\partial_\mu\bar\chi^c\Big)-\bar\chi^b\Big(\bar\lambda^c+\frac{i}{2}\sigma^\mu\partial_\mu\chi^c\Big)+S^{b\dagger}S^c$$
$$-\frac{1}{2}A_\mu^{bc}A^{c\mu}\Big]\varphi^{dI}+\frac{i}{2}C^bA^{c\mu}(\varphi_I^{a\dagger}(\partial_\mu\varphi^{dI})-(\partial_\mu\varphi_I^{a\dagger})\varphi^{dI})+\frac{1}{4}C^bC^c(\varphi_I^{a\dagger}(\Box\,\varphi^{dI})$$
$$+(\Box\,\varphi_I^{a\dagger})\varphi^{dI})+\frac{i}{2}\chi^b\sigma^\mu\bar\chi^c(\varphi_I^{a\dagger}(\partial_\mu\varphi^{dI})-(\partial_\mu\varphi_I^{a\dagger})\varphi^{dI})\Big)\Big\}$$

$$\mathcal{B}^a(x)=\begin{pmatrix}C^a(x)\\D^a(x)\end{pmatrix},\mathcal{F}^a(x)=\begin{pmatrix}\chi^a(x)\\\bar\chi^a(x)\\\lambda^a(x)\\\bar\lambda^a(x)\end{pmatrix}$$

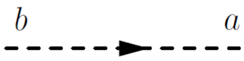
$$S_0=\int d^4x\{[(\partial_\mu\varphi_I^{a\dagger})(\partial^\mu\varphi_a^I)+\bar\psi_I^a\bar\sigma^\mu(\partial_\mu\psi_a^I)+F_I^{a\dagger}F_a^I]+$$
$$+\Big[S_a^\dagger\,\Box\,S^a-\frac{1}{2}A_\mu^aA_a^\mu+\mathcal{B}_a^TM\mathcal{B}^a+\mathcal{F}_a^TN\mathcal{F}^a\Big]\Big\},$$

$$M=\frac{1}{2}\begin{pmatrix}-\Box^2&-\Box\\-\Box&0\end{pmatrix},N=\frac{1}{2}\begin{pmatrix}0&\Box\,\sigma^\mu\partial_\mu&-\Box&0\\\Box\,\bar\sigma^\mu\partial_\mu&0&0&-\Box\\-\Box&0&0&0\\0&-\Box&0&0\end{pmatrix}$$

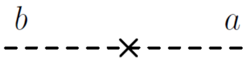
$$M^{-1}=\begin{pmatrix}0 & \frac{1}{\Box}\\ \frac{1}{\Box} & -1\end{pmatrix},N^{-1}=\begin{pmatrix}0 & 0 & -\frac{1}{\Box} & 0\\ 0 & 0 & 0 & -\frac{1}{\Box}\\ -\frac{1}{\Box} & 0 & 0 & \frac{\sigma^{\mu}\partial_{\mu}}{\Box}\\ 0 & -\frac{1}{\Box} & \frac{\bar{\sigma}^{\mu}\partial_{\mu}}{\Box} & 0\end{pmatrix},$$



$$\longrightarrow \quad \langle\varphi_J^{b\dagger}(x)\varphi_a^I(y)\rangle_{\text{free}}=-\frac{\delta_a^b\delta_J^I}{\Box}\delta(x-y)=\Delta_{aJ}^{bI}(x-y)$$

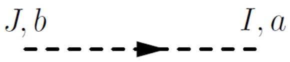


$$\longrightarrow \quad \langle S^{b\dagger}(x)S_a(y)\rangle_{\text{free}}=\frac{\delta_a^b}{\Box}\delta(x-y)=2\Delta_a^b(x-y)$$



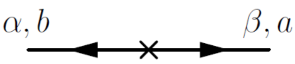
$$\longrightarrow \quad \langle C^b(x)D_a(y)\rangle_{\text{free}}=\frac{\delta_a^b}{\Box}\delta(x-y)=\Delta_a^b(x-y)$$

$$\longrightarrow \quad \langle D^b(x)D_a(y)\rangle_{\text{free}}=-\delta_a^b\delta(x-y)$$

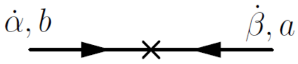


$$\longrightarrow \quad \langle F_J^{b\dagger}(x)F_a^I(y)\rangle_{\text{free}}=\delta_J^I\delta_a^b\delta(x-y)$$

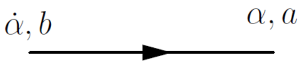
$$\longrightarrow \quad \langle A_\mu^b(x)A_{a\nu}(y)\rangle_{\text{free}}=-\frac{\delta_{\mu\nu}\delta_a^b}{\Box}\delta(x-y)=\Delta_{a\mu\nu}^b(x-y)$$



$$\longrightarrow \quad \langle\chi^{b\alpha}(x)\lambda_a^\beta(y)\rangle_{\text{free}}=-\frac{\varepsilon^{\alpha\beta}\delta_a^b}{\Box}\delta(x-y)=R_a^{b\alpha\beta}(x-y)$$

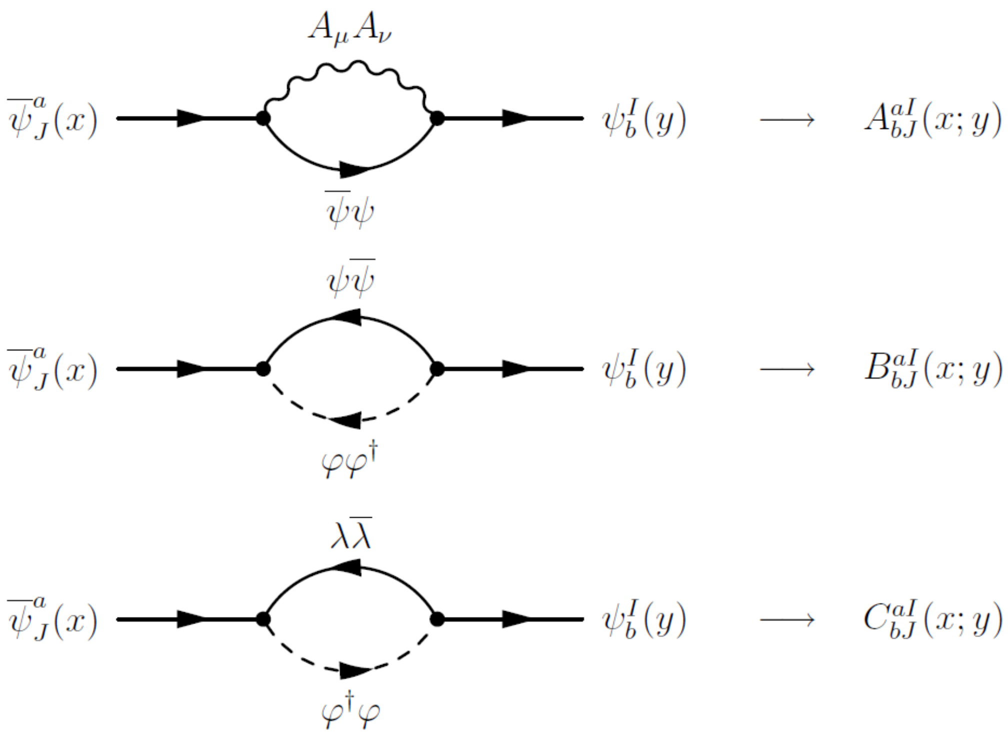


$$\longrightarrow \quad \langle\overline{\chi}^{b\dot{\alpha}}(x)\overline{\lambda}_a^{\dot{\beta}}(y)\rangle_{\text{free}}=-\frac{\varepsilon^{\dot{\alpha}\dot{\beta}}\delta_a^b}{\Box}\delta(x-y)=\overline{R}_a^{b\dot{\alpha}\dot{\beta}}(x-y)$$



$$\longrightarrow \quad \langle\overline{\lambda}^{b\dot{\alpha}}(x)\lambda_a^\alpha(y)\rangle_{\text{free}}=\frac{\delta_a^b\overline{\sigma}^\mu_{\dot{\alpha}\alpha}\partial_\mu}{\Box}\delta(x-y)=$$
$$=\overline{S}^b_{a\dot{\alpha}\alpha}(x-y)$$

$$\longrightarrow \quad \langle\overline{\psi}_I^{b\dot{\alpha}}(x)\psi_a^{\alpha J}(y)\rangle_{\text{free}}=\frac{\delta_a^b\delta_I^J\overline{\sigma}^\mu_{\dot{\alpha}\alpha}\partial_\mu}{\Box}\delta(x-y)=$$
$$=\overline{S}^{bJ}_{aI\dot{\alpha}\alpha}(x-y)$$



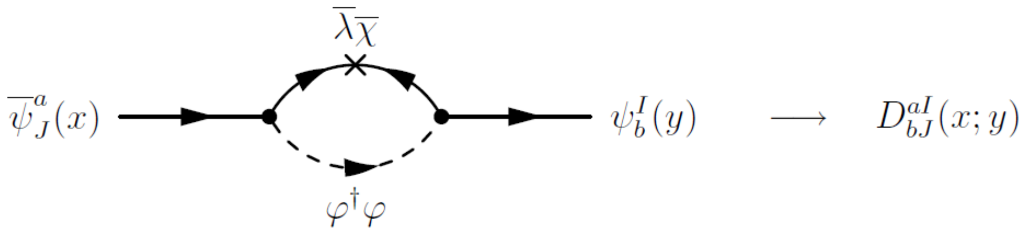
$$A^{\dot\alpha\alpha aI}_{bJ}(x;y) = -\frac{1}{2}g^2 f^d{}_{ef} f^l{}_{mn}\int d^4x_1 d^4x_2 \left\{\Delta^{me}_{\nu\mu}(x_2-x_1)\left[\bar{S}^{\dot\alpha\beta aI}_{lL}(x-x_2)\sigma^\nu_{\beta\dot\gamma}\right.\right.$$
$$\left.\left.\cdot\,\bar{S}^{\dot\gamma\gamma nL}_{dK}(x_2-x_1)\sigma^\mu_{\gamma\dot\beta}\bar{S}^{\dot\beta\alpha fK}_{bJ}(x_1-y)\right]\right\}$$

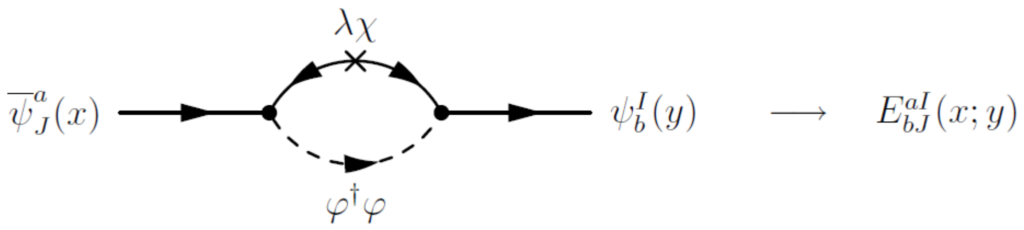
$$B^{\dot\alpha\alpha aI}_{bJ}(x;y) =-\frac{1}{2}g^2\varepsilon^{LMN}\varepsilon^{PQ}{}_R f^f_{de} f_{lmn}\int d^4x_1 d^4x_2\{\Delta^{ld}_{PL}(x_2-x_1)$$
$$\cdot\left[\bar{S}^{\dot\alpha\beta aI}_{fN}(x-x_2)S^{em}_{\beta\dot\beta MQ}(x_2-x_1)\bar{S}^{\dot\beta\alpha nR}_{bJ}(x_1-y)\right]\}$$
$$C^{\dot\alpha\alpha aI}_{bJ}(x;y) = -g^2 f^f_{de} f^n_{lm}\int d^4x_1 d^4x_2\{\Delta^{lK}_{fL}(x_2-x_1)$$
$$\cdot\left[\bar{S}^{\dot\alpha\beta aI}_{nK}(x-x_2)S^{em}_{\beta\dot\beta}(x_2-x_1)\bar{S}^{\dot\beta\alpha dL}_{bJ}(x_1-y)\right]\}.$$

$$\tilde{A}^{\dot\alpha\alpha aI}_{bJ}(p) = \frac{i}{2}g^2 N(N^2-1)\delta^a_b\delta^I_J\overline{\bar{S}}^{\dot\alpha\beta}(p)\sigma^\lambda_{\beta\dot\beta}p_\lambda\tilde{\bar{\bar{S}}}^{\dot\beta\alpha}(p)\left[\int\frac{d^4k}{(2\pi)^4}\frac{1}{\left(k+\frac{p}{2}\right)^2\left(k-\frac{p}{2}\right)^2}\right]$$
$$\tilde{B}^{\dot\alpha\alpha aI}_{bJ}(p) = -\tilde{C}^{\dot\alpha\alpha aI}_{bJ}(p) = \tilde{A}^{\dot\alpha\alpha aI}_{bJ}(p)$$

$$\tilde{\bar{\bar{S}}}^{\dot\alpha\alpha}(p) = -i\frac{\bar\sigma^{\dot\alpha\alpha}_\mu p^\mu}{p^2}$$

$$\left\langle(\bar\psi\psi)^{\dot\alpha\alpha I}_{bJ}\right\rangle^{1-\text{loop, WZ}}_{\text{FT}} = \tilde{A}^{\dot\alpha\alpha aI}_{bJ}(p) + \tilde{B}^{\dot\alpha\alpha aI}_{bJ}(p) + \tilde{C}^{\dot\alpha\alpha aI}_{bJ}(p)$$
$$= \frac{i}{2}g^2 N(N^2-1)\delta^a_b\delta^I_J\tilde{\bar{\bar{S}}}^{\dot\alpha\beta}(p)\sigma^\lambda_{\beta\dot\beta}p_\lambda\tilde{\bar{\bar{S}}}^{\dot\beta\alpha}(p)\left[\int\frac{d^4k}{(2\pi)^4}\frac{1}{\left(k+\frac{p}{2}\right)^2\left(k-\frac{p}{2}\right)^2}\right].$$

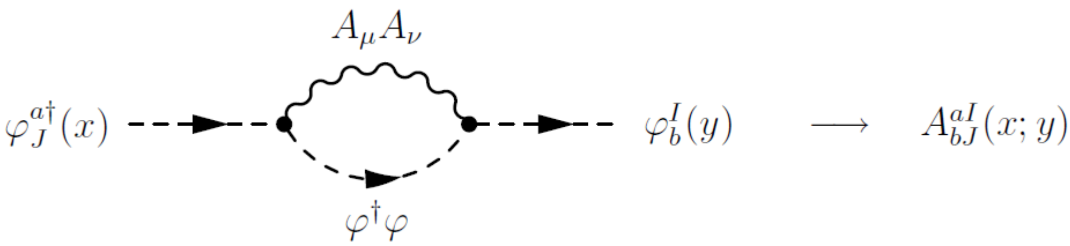


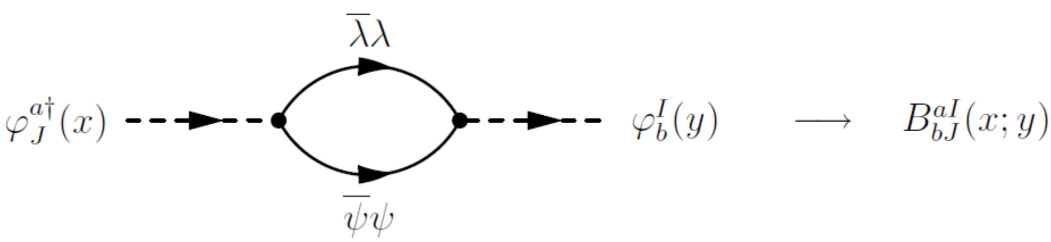


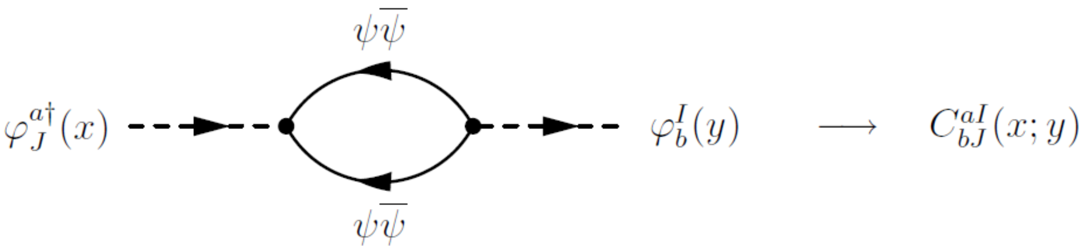
$$D^{\dot{\alpha}\alpha aI}_{bJ}(x;y) = -\frac{1}{2}g^2 f^f_{de} f^n_{lm} \int d^4x_1 d^4x_2 \{\Delta^{lK}_{fL}(x_2 - x_1)$$
$$\cdot \left[\bar{S}^{\dot{\alpha}\gamma aI}_{mK}(x - x_2)\sigma^\mu_{\gamma\dot{\gamma}}\left(\partial^{(2)}_\mu R^{\dot{\gamma}ne}_{\dot{\beta}}(x_2 - x_1)\right)\bar{S}^{\dot{\beta}\alpha dL}_{bJ}(x_1 - y)\right]\}$$
$$E^{\dot{\alpha}\alpha aI}_{bJ}(x;y) = D^{\dot{\alpha}\alpha aI}_{bJ}(x;y)$$

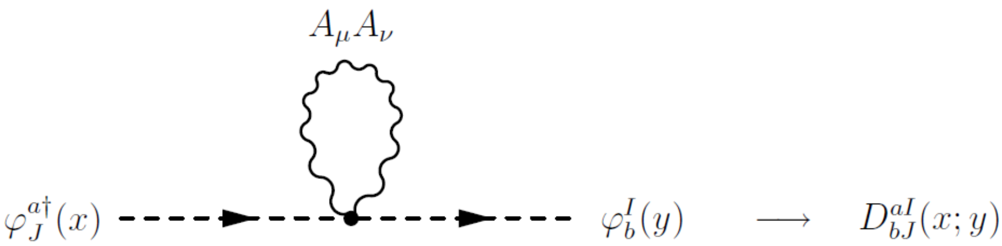
$$\tilde{D}^{\dot{\alpha}\alpha aI}_{bJ}(p) = -\frac{i}{4}g^2 N(N^2 - 1)\delta^a_b\delta^I_J \tilde{\bar{S}}^{\dot{\alpha}\beta}(p)\sigma^\lambda_{\beta\dot{\beta}} p_\lambda \tilde{\bar{S}}^{\dot{\beta}\alpha}(p)\left[\int \frac{d^4k}{(2\pi)^4}\frac{1}{\left(k + \frac{p}{2}\right)^2\left(k - \frac{p}{2}\right)^2}\right],$$
$$\tilde{E}^{\dot{\alpha}\alpha aI}_{bJ}(p) = \tilde{D}^{\dot{\alpha}\alpha aI}_{bJ}(p).$$

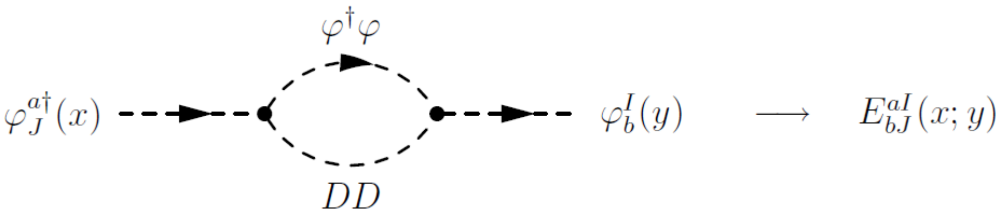
$$\left\langle(\bar{\psi}\psi)^{\dot{\alpha}\alpha aI}_{bJ}\right\rangle^{1-\text{loop}}_{\text{FT}} =$$
$$= \tilde{A}^{\dot{\alpha}\alpha aI}_{bJ}(p) + \tilde{B}^{\dot{\alpha}\alpha aI}_{bJ}(p) + \tilde{C}^{\dot{\alpha}\alpha aI}_{bJ}(p) + \tilde{D}^{\dot{\alpha}\alpha aI}_{bJ}(p) + \tilde{E}^{\dot{\alpha}\alpha aI}_{bJ}(p) = 0,$$

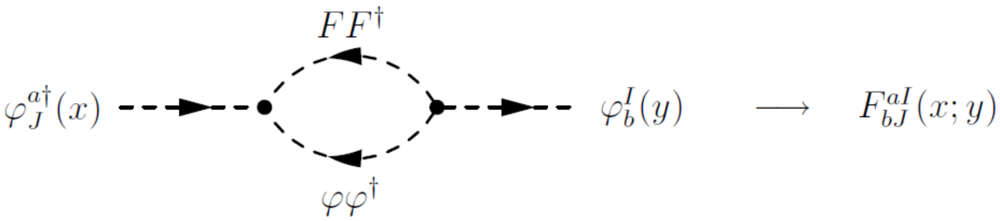




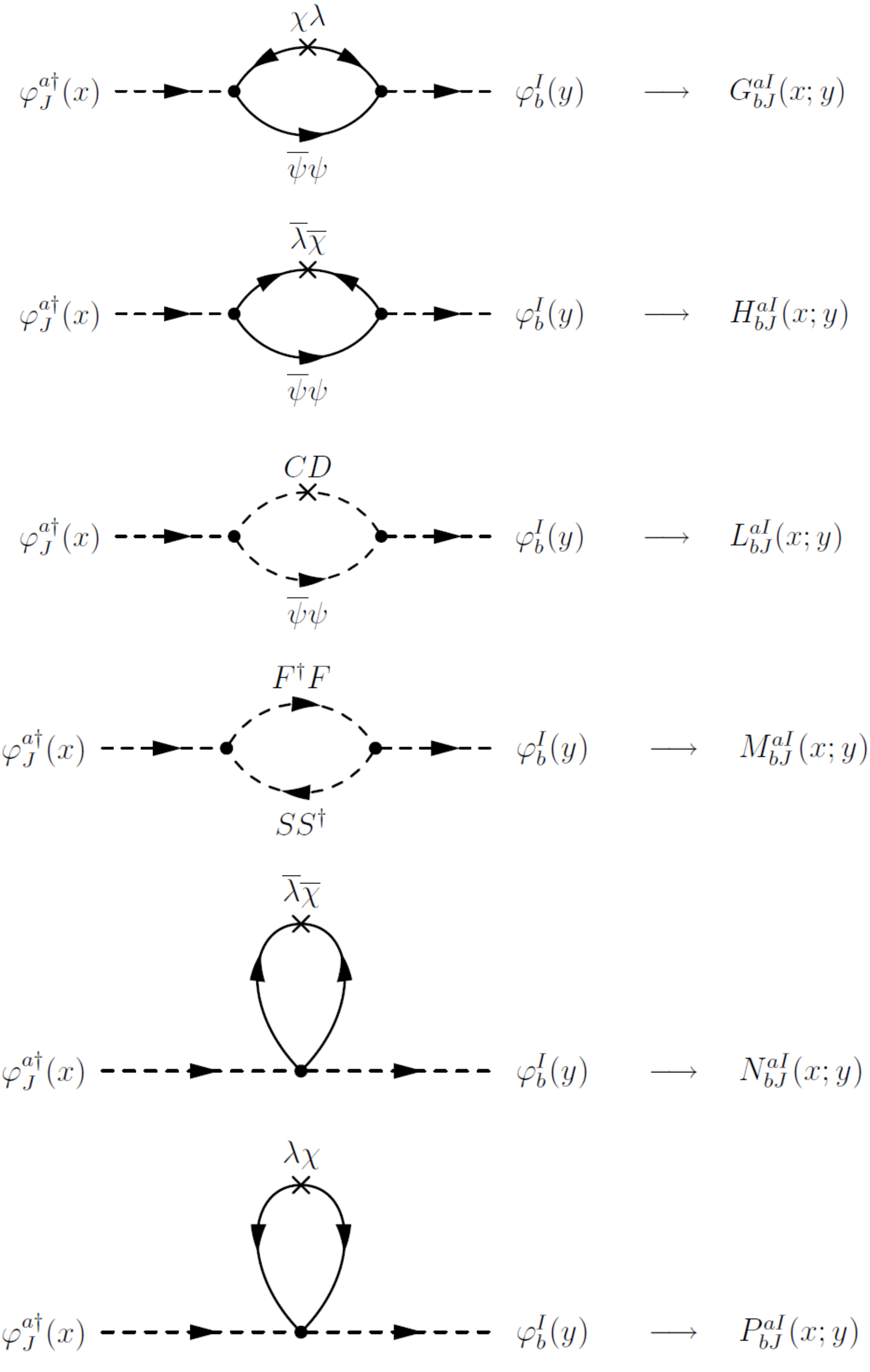


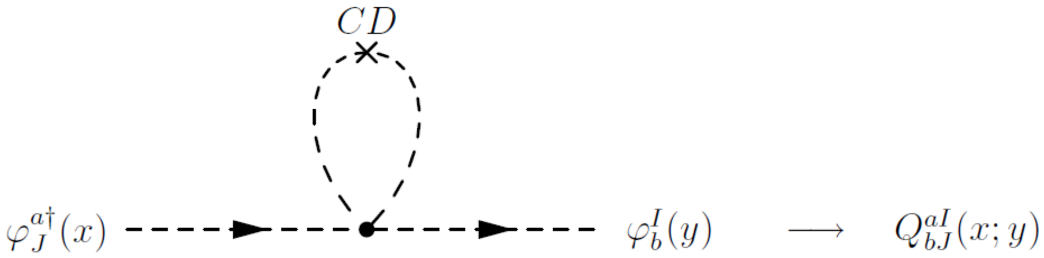


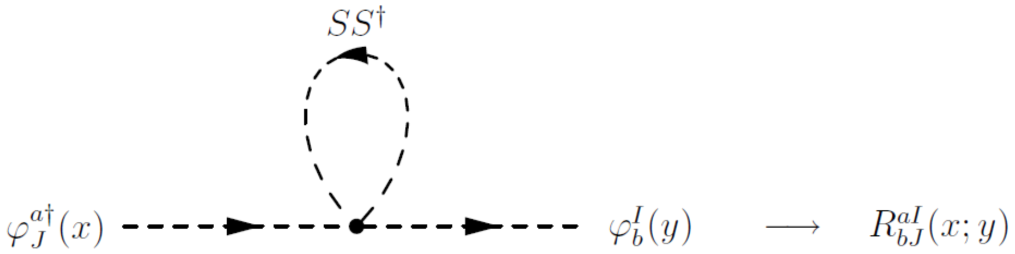


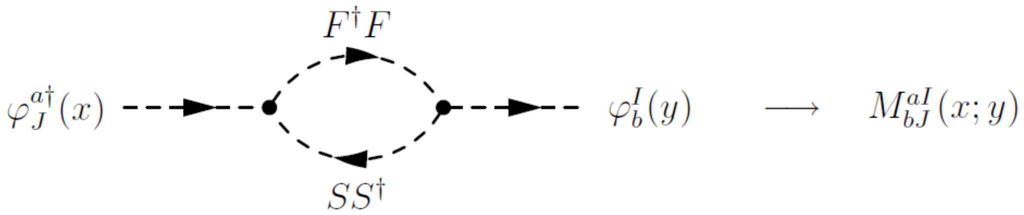


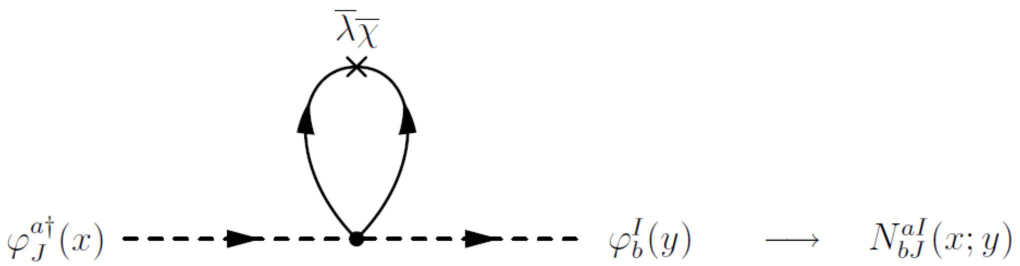
$$\left\langle (\varphi^\dagger\varphi)_{bJ}^{aI} \right\rangle_{\text{FT}}^{1-\text{loop},\text{WZ}} \sim g^2 \delta_J^I \delta_b^a p^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left(k+\frac{p}{2}\right)^2 \left(k-\frac{p}{2}\right)^2}.$$

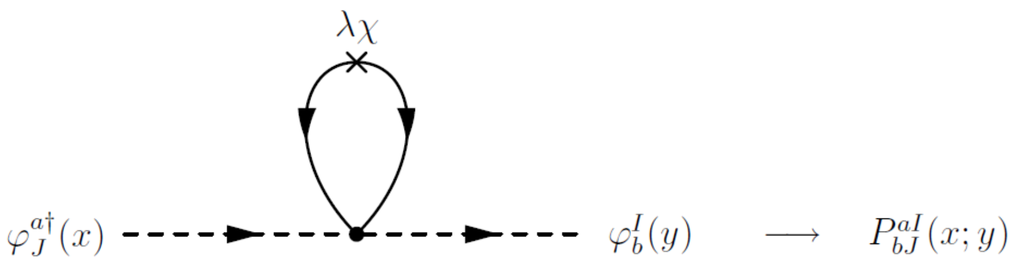

$\chi\lambda$
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow$ $G_{bJ}^{aI}(x;y)$
$\overline{\psi}\psi$
$\overline{\lambda}\overline{\chi}$
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow$ $H_{bJ}^{aI}(x;y)$
$\overline{\psi}\psi$
CD
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow$ $L_{bJ}^{aI}(x;y)$
$\overline{\psi}\psi$
$F^\dagger F$
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow$ $M_{bJ}^{aI}(x;y)$
$SS^\dagger$
$\overline{\lambda}\overline{\chi}$
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow$ $N_{bJ}^{aI}(x;y)$
$\lambda\chi$
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow$ $P_{bJ}^{aI}(x;y)$


CD
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow \quad Q_{bJ}^{aI}(x;y)$


$SS^\dagger$
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow \quad R_{bJ}^{aI}(x;y)$


$F^\dagger F$
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$SS^\dagger$
$\longrightarrow \quad M_{bJ}^{aI}(x;y)$


$\overline{\lambda}\overline{\chi}$
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow \quad N_{bJ}^{aI}(x;y)$

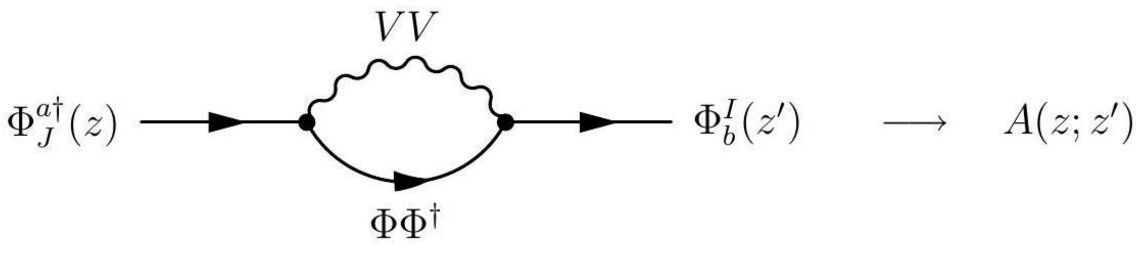

$\lambda\chi$
$\varphi_J^{a\dagger}(x)$
$\varphi_b^I(y)$
$\longrightarrow \quad P_{bJ}^{aI}(x;y)$

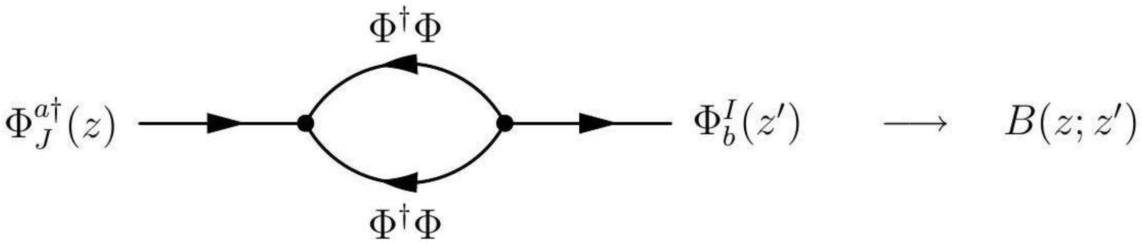
$$
\begin{aligned}
W_\alpha^{a(2)} =& -\frac{i}{2} f^a{}_{bc} \Big\{ -iC^b \lambda_\alpha^c - \frac{1}{2} \sigma^\mu_{\alpha\dot{\alpha}} \bar{\chi}^{\dot{\alpha}b} \partial_\mu C^c - \frac{i}{2} \sigma^\mu_{\alpha\dot{\alpha}} \bar{\chi}^{\dot{\alpha}b} A_\mu^c + \frac{1}{\sqrt{2}} S^{b\dagger} \chi_\alpha^c \\
& + \Big[S^{b\dagger} S^c \delta_\alpha{}^\beta + \delta_\alpha{}^\beta C^b D^c - \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu)_\alpha{}^\beta C^b \big(\partial_\mu A_\nu^c - \partial_\nu A_\mu^c \big) \\
& - \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu)_\alpha{}^\beta \big(A_\nu^b \partial_\mu C^c + A_\mu^b \partial_\nu C^c \big) - \frac{1}{2} \delta_\alpha{}^\beta C^b \, \square \, C^c - \delta_\alpha{}^\beta \bar{\chi}^b \bar{\lambda}^c \\
& + \frac{1}{2} (\sigma^\mu \sigma^\nu)_\alpha{}^\beta \partial_\mu C^b \partial_\nu C^c - \chi^{b\beta} \lambda_\alpha^c - \lambda^{b\beta} \chi_\alpha^c - i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\chi}^{b\dot{\alpha}} \partial_\mu \chi^{c\beta} \\
& + i\varepsilon^{\gamma\beta} \sigma^\mu_{\gamma\dot{\beta}} \partial_\mu \bar{\chi}^b \chi_\alpha^c + \frac{1}{2} (\sigma^\mu \sigma^\nu)_\alpha{}^\beta A_\nu^b A_\mu^c \Big] \theta_\beta + \big[C^b \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \bar{\lambda}^{c\dot{\alpha}} \\
& - i\chi_\alpha^b D^c - \frac{1}{4} (\sigma^\mu \bar{\sigma}^\nu)_\alpha{}^\beta \chi_\beta^b \big(\partial_\mu A_\nu^c - \partial_\nu A_\mu^c \big) - \frac{1}{2} A_\nu^b (\sigma^\mu \bar{\sigma}^\nu)_\alpha{}^\beta \partial_\mu \chi_\beta^c \\
& + \frac{1}{2} \partial_\mu A^{b\mu} \chi_\alpha^c - \frac{i}{2} \partial_\nu C^b (\sigma^\mu \bar{\sigma}^\nu)_\alpha{}^\beta \partial_\mu \chi_\beta^c - \frac{i}{\sqrt{2}} \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \bar{\chi}^{\dot{\alpha}b} S^c + \sqrt{2} S^b \lambda_\alpha^c \\
& - iA_\mu^b \sigma^\mu_{\alpha\dot{\alpha}} \bar{\lambda}^{c\dot{\alpha}} \big] \theta\theta \Big\}
\end{aligned}
$$

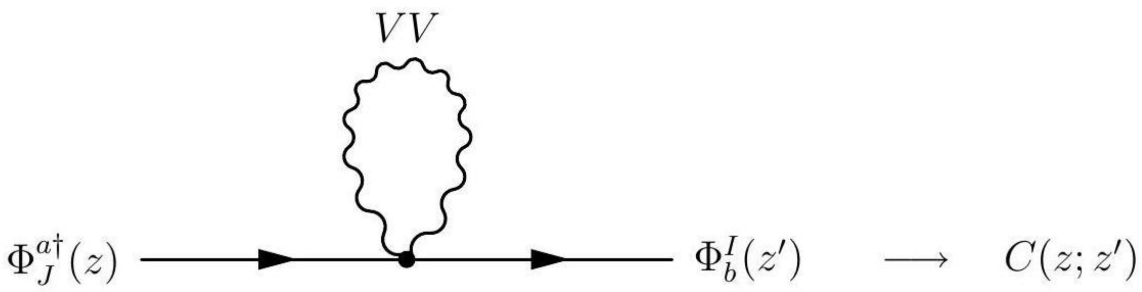
$$
\begin{aligned}
S_{\mathrm{m}} = -\int \ d^4x d^4\theta \Big[& \frac{1}{2} m \delta_{IJ} \Phi_a^I(x,\theta,\bar{\theta}) \Phi^{Ja}(x,\theta,\bar{\theta}) \delta(\bar{\theta}) \\
& + \frac{1}{2} m^* \delta^{IJ} \Phi_I^{\dagger a}(x,\theta,\bar{\theta}) \Phi_{Ja}^\dagger(x,\theta,\bar{\theta}) \delta(\theta) \Big]
\end{aligned}
$$

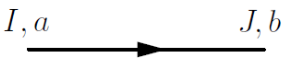
$$
\begin{aligned}
S = \int \ & d^4x d^4\theta \{ V^a(x,\theta,\bar{\theta}) [- \square P_T - \xi (P_1 + P_2) \square] V_a(x,\theta,\bar{\theta}) \\
& + \Phi_I^{\dagger a}(x,\theta,\bar{\theta}) \Phi_a^I(x,\theta,\bar{\theta}) - \frac{1}{2} m \delta_{IJ} \Phi_a^I(x,\theta,\bar{\theta}) \Phi^{Ja}(x,\theta,\bar{\theta}) \delta(\bar{\theta}) \\
& - \frac{1}{2} m \delta^{IJ} \Phi_I^{\dagger a}(x,\theta,\bar{\theta}) \Phi^\dagger{}_{Ja}(x,\theta,\bar{\theta}) \delta(\theta) + igf_{abc} \Phi_I^{\dagger a}(x,\theta,\bar{\theta}) V^b(x,\theta,\bar{\theta})
\end{aligned}
$$

$$
\begin{aligned}
& \cdot \Phi^{Ic}(x,\theta,\bar{\theta}) - \frac{1}{2} g^2 f_{ab}^e f_{ecd} \Phi_I^{\dagger a}(x,\theta,\bar{\theta}) V^b(x,\theta,\bar{\theta}) V^c(x,\theta,\bar{\theta}) \Phi^{Id}(x,\theta,\bar{\theta}) \\
& - \frac{i}{4} g f_{abc} [\bar{D}^2 (D^\alpha V^a(x,\theta,\bar{\theta}))] V^b(x,\theta,\bar{\theta}) (D_\alpha V^c(x,\theta,\bar{\theta})) \\
& - \frac{1}{8} g^2 f_{ab}^e f_{ecd} V^a(x,\theta,\bar{\theta}) (D^\alpha V^b(x,\theta,\bar{\theta})) [(\bar{D}^2 V^c(x,\theta,\bar{\theta})) (D_\alpha V^d(x,\theta,\bar{\theta}))] \\
& + \cdots - \frac{\sqrt{2}}{3!} g f^{abc} \big[\varepsilon_{IJK} \Phi_a^I(x,\theta,\bar{\theta}) \Phi_b^J(x,\theta,\bar{\theta}) \Phi_c^K(x,\theta,\bar{\theta}) \delta(\bar{\theta}) \\
& + \varepsilon^{IJK} \Phi_{Ia}^\dagger(x,\theta,\bar{\theta}) \Phi_{Jb}^\dagger(x,\theta,\bar{\theta}) \Phi_{Kc}^\dagger(x,\theta,\bar{\theta}) \delta(\theta) \big] + (\bar{C}_a'(x,\theta,\bar{\theta}) C^a(x,\theta,\bar{\theta}) \\
& - C_a'(x,\theta,\bar{\theta}) \bar{C}^a(x,\theta,\bar{\theta})) + \frac{i}{2\sqrt{2}} g f_{abc} (C'^a(x,\theta,\bar{\theta}) + \bar{C}'^a(x,\theta,\bar{\theta})) \\
& \cdot V^b(x,\theta,\bar{\theta}) (C^c(x,\theta,\bar{\theta}) + \bar{C}^c(x,\theta,\bar{\theta})) - \frac{1}{8} g^2 f_{ab}^e f_{ecd} (C'^a(x,\theta,\bar{\theta}) + \bar{C}'^a(x,\theta,\bar{\theta})) \\
& \cdot V^b(x,\theta,\bar{\theta}) V^c(x,\theta,\bar{\theta}) (C^d(x,\theta,\bar{\theta}) + \bar{C}^d(x,\theta,\bar{\theta})) + \cdots \}
\end{aligned}
$$



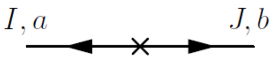




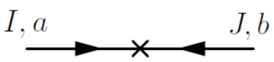


$$\longrightarrow \quad \langle \Phi^{aI}(z)\Phi^{b\dagger}_J(z')\rangle_{\text{free}} = \delta^I_J\delta^a_b \frac{1}{\Box + m^2}\delta_8(z-z')$$

$$\longrightarrow \quad \langle V^a(z)V_b(z')\rangle_{\text{free}} = -\frac{\delta^a_b}{\Box}[1+(\alpha-1)(P_1+P_2)]\delta_8(z-z')$$



$$\longrightarrow \quad \langle \Phi^{aI}(z)\Phi^J_b(z')\rangle_{\text{free}} = -\delta^{IJ}\delta^a_b \frac{m}{4}\frac{D^2}{\Box(\Box+m^2)}\delta_8(z-z')$$



$$\longrightarrow \quad \langle \Phi^{a\dagger}_I(z)\Phi^\dagger_{bJ}(z')\rangle_{\text{free}} = -\delta_{IJ}\delta^a_b \frac{m}{4}\frac{\overline{D}^2}{\Box(\Box+m^2)}\delta_8(z-z')$$

$$\tilde{A}(p) = \int \frac{d^4k}{(2\pi)^4} d^4\theta_1 d^4\theta_2 \left\{ ig f_{acd}\Phi^{a\dagger}_I(p,\theta_1,\bar\theta_1)\left[\left(-\frac{1}{4}\bar{D}_1^2\right)\frac{\delta^I_J\delta^{cf}\delta(1,2)}{(p-k)^2+m^2}\right.\right.$$
$$\left.\left.\cdot\left(-\frac{1}{4}\overleftarrow{\mathfrak{D}}_1^2\right)\right]\left[\left(1+\gamma(D_1^2\bar{D}_1^2+\bar{D}_1^2D_1^2)\right)\left(-\frac{\delta^{de}\delta(1,2)}{k^2}\right)\right] ig f_{efb}\Phi^{bJ}(-p,\theta_2,\bar\theta_2)\right\}$$

$$D_{1\alpha}\delta(1,2) = -\delta(1,2)\overleftarrow{D}_{2\alpha}\,, \qquad \overline{D}_{1\dot\alpha}\delta(1,2) = -\delta(1,2)\overleftarrow{\overline{D}}_{2\dot\alpha}\,,$$
$$\overline{D}_{1\dot\alpha}D_{1\alpha}\delta(1,2) = \delta(1,2)\overleftarrow{\overline{D}}_{2\dot\alpha}\overleftarrow{D}_{2\alpha}\,, \qquad \overline{D}_1^2 D_1^2\delta(1,2) = \delta(1,2)\overleftarrow{\overline{D}}{}_2^2\overleftarrow{D}{}_2^2\,,$$

$$\tilde{A}(p) = -g^2N(N^2-1)\delta^a_b\delta^I_J\left(\frac{1}{4}\right)^2\int\frac{d^4k}{(2\pi)^4}d^4\theta_1d^4\theta_2\frac{1}{k^2[(p-k)^2+m^2]}\{\Phi^\dagger_{aI}(p,1)$$
$$\cdot\,\Phi^{bJ}(-p,2)[\bar{D}_1^2D_1^2\delta(1,2)][1+\gamma(D_1^2\bar{D}_1^2+\bar{D}_1^2D_1^2)\delta(1,2)]\} = \tilde{A}_1(p)+\tilde{A}_2(p)$$

$$\int d^4\theta[\bar{D}^2D^2\delta(\theta-\theta')]\delta(\theta-\theta')=16$$
$$\int d^4\theta[\bar{D}^mD^n\delta(\theta-\theta')]\delta(\theta-\theta')=0 \text{ if } (m,n)\neq \mathfrak{X}$$

$$\tilde{A}_1(p)=-g^2N(N^2-1)\delta_b^a\delta_J^I\int \frac{d^4k}{(2\pi)^4}d^4\theta\frac{1}{k^2[(p-k)^2+m^2]}\{\Phi_{aI}^{\dagger}(p,\theta,\bar{\theta})\Phi^{bJ}(-p,\theta,\bar{\theta})\}$$

$$\bar{D}^2D^2\bar{D}^2=16\,\Box\,\bar{D}^2\; D^2\bar{D}^2D^2=16\,\Box\,D^2$$

$$\tilde{A}_2(p)=\gamma g^2N(N^2-1)\delta_b^a\delta_J^I\int \frac{d^4k}{(2\pi)^4}d^4\theta\frac{[(p-k)^2+p^2]}{k^4[(p-k)^2+m^2]}\{\Phi_{aI}^{\dagger}(p,\theta,\bar{\theta})\Phi^{bJ}(-p,\theta,\bar{\theta})\}.$$

$$\tilde{B}(p)=\int \frac{d^4k}{(2\pi)^4}d^4\theta_1d^4\theta_2\left\{\left(-\frac{\sqrt{2}}{3!}\right)g\varepsilon_{KL}^{I}f_{acd}\Phi_I^{\dagger a}(p,1)\right.$$
$$\cdot\left[\left(-\frac{1}{4}D_1^2\right)\frac{\delta^{ce}\delta_M^K\delta(1,2)}{(p-k)^2+m^2}\left(-\frac{1}{4}\bar{D}_2^2\right)\right]\left[\frac{\delta_f^d\delta_N^L\delta(1,2)}{k^2+m^2}\right]$$
$$\left.\cdot\left(-\frac{\sqrt{2}}{3!}\right)\varepsilon_J{}^{MN}f_{be}{}^{f}\Phi^{bJ}(-p,2)\right\}$$

$$\tilde{B}(p)=g^2N(N^2-1)\delta_b^a\delta_J^I\int \frac{d^4k}{(2\pi)^4}d^4\theta\left\{\frac{\Phi_{aI}^{\dagger}(p,\theta,\bar{\theta})\Phi^{bJ}(-p,\theta,\bar{\theta})}{(k^2+m^2)[(p-k)^2+m^2]}\right\}.$$

$$\tilde{C}(p)=\int \frac{d^4k}{(2\pi)^4}d^4\theta_1\left\{\left(-\frac{g^2}{2}\right)f_{ac}{}^{d}f_{deb}\Phi_I^{a\dagger}(p,1)\right.$$
$$\left.\cdot\left[-\left(1+\gamma(\bar{D}_1^2D_1^2+D_1^2\bar{D}_1^2)\right)\frac{\delta^{ce}\delta(1,1)}{k^2}\right]\Phi^{bJ}(-p,1)\right\}.$$

$$\tilde{C}(p)=-\gamma g^2N(N^2-1)\delta_a^b\delta_J^I\int \frac{d^4k}{(2\pi)^4}d^4\theta\frac{1}{k^4}\{\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(-p,\theta,\bar{\theta})\},$$

$$\tilde{A}_1(p)+\tilde{B}(p)=g^2N(N^2-1)\delta_a^b\delta_J^I\int \frac{d^4k}{(2\pi)^4}d^4\theta[\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(-p,\theta,\bar{\theta})]$$
$$\cdot\left\{\frac{1}{k^2[(p-k)^2+m^2]}-\frac{1}{(k^2+m^2)[(p-k)^2+m^2]}\right\}.$$

$$\tilde{A}_2(p)+\tilde{C}(p)=\gamma g^2N(N^2-1)\delta_a^b\delta_J^I\int \frac{d^4k}{(2\pi)^4}d^4\theta[\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(-p,\theta,\bar{\theta})]$$
$$\cdot\left\{\left(\frac{(p-k)^2}{k^4[(p-k)^2+m^2]}-\frac{1}{k^4}\right)+\left(\frac{p^2}{k^4[(p-k)^2+m^2]}\right)\right\}$$

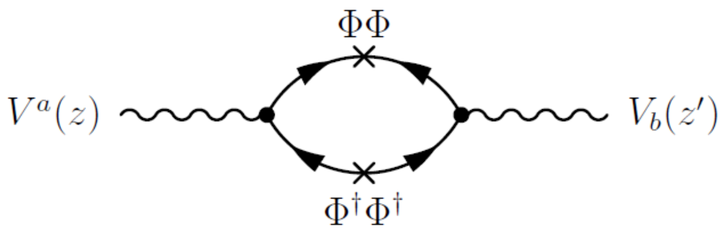
$$\tilde{A}_2(p)+\tilde{C}(p)=\gamma g^2N(N^2-1)\delta_a^b\delta_J^I[\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(-p,\theta,\bar{\theta})][I_1(p)+I_2(p)]$$

$$I_1(p)=\int \frac{d^4k}{(2\pi)^4}\left\{\frac{(p-k)^2}{k^4[(p-k)^2+m^2]}-\frac{1}{k^4}\right\}$$
$$=\int \frac{d^4k}{(2\pi)^4}\left\{\frac{m^2}{k^4[(p-k)^2+m^2]}\right\}$$
$$=2m^2\int_0^1 d\zeta\zeta\int \frac{d^4k}{(2\pi)^4}\frac{1}{[k^2+(p^2\zeta+m^2)(1-\zeta)]^3}$$
$$=\frac{2m^2}{2(4\pi)^2}\int_0^1 d\zeta\zeta\frac{1}{(p^2\zeta+m^2)(1-\zeta)}$$
$$=-\frac{m^2}{(4\pi)^2}\left[\frac{1}{(p^2+m^2)}\log\epsilon+\frac{m^2}{p^2(p^2+m^2)}\log\left(\frac{p^2+m^2}{m^2}\right)\right]$$

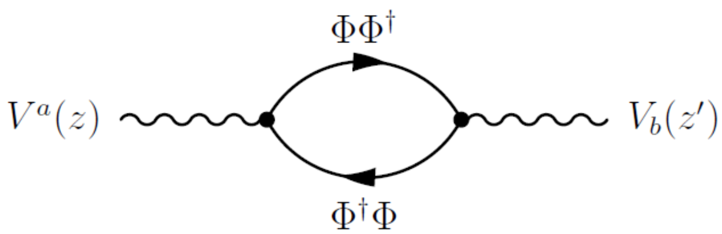
$$
\begin{aligned}
I_2(p) &= \int \frac{d^4k}{(2\pi)^4}\left\{\frac{p^2}{k^4[(p-k)^2+m^2]}\right\} \\
&= 2p^2\int_0^1 d\zeta\zeta \int \frac{d^4k}{(2\pi)^4}\frac{1}{[k^2+(p^2\zeta+m^2)(1-\zeta)]^3} \\
&= \frac{p^2}{(4\pi)^2}\int_0^1 d\zeta\zeta \frac{1}{(p^2\zeta+m^2)(1-\zeta)} \\
&= -\frac{1}{(4\pi)^2}\left[\frac{p^2}{(p^2+m^2)}\log\epsilon + \frac{m^2}{(p^2+m^2)}\log\left(\frac{p^2+m^2}{m^2}\right)\right]
\end{aligned}
$$

$$\longrightarrow \quad \langle \overline{C}^{a\prime}(z)C^{b}(z')\rangle_{\text{free}} = \delta^a_b\frac{1}{\Box}\delta_8(z-z')$$

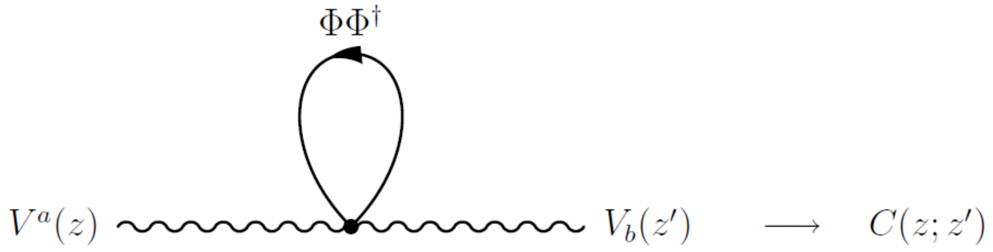
$$\longrightarrow \quad \langle \overline{C}^{a}(z)C^{b\prime}(z')\rangle_{\text{free}} = \delta^a_b\frac{1}{\Box}\delta_8(z-z')$$

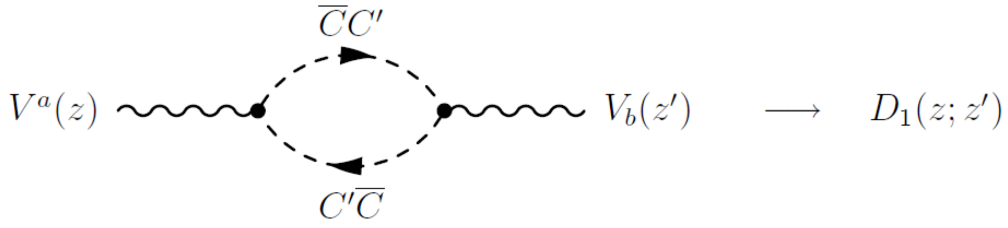


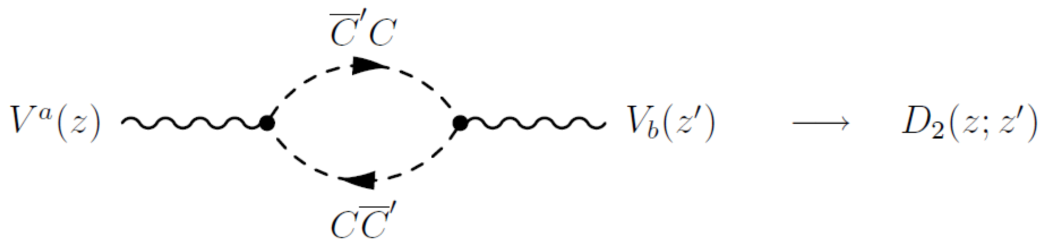
$$\longrightarrow \quad A(z;z')$$

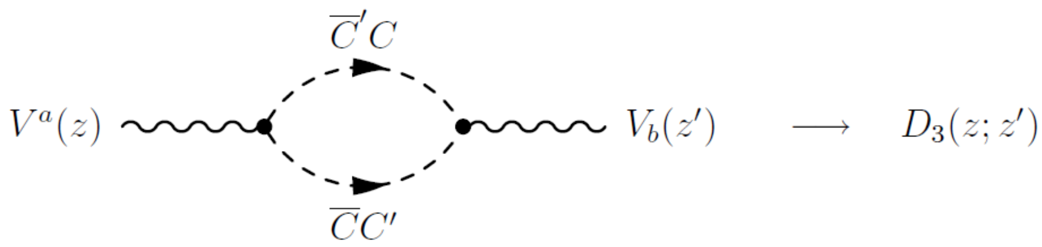


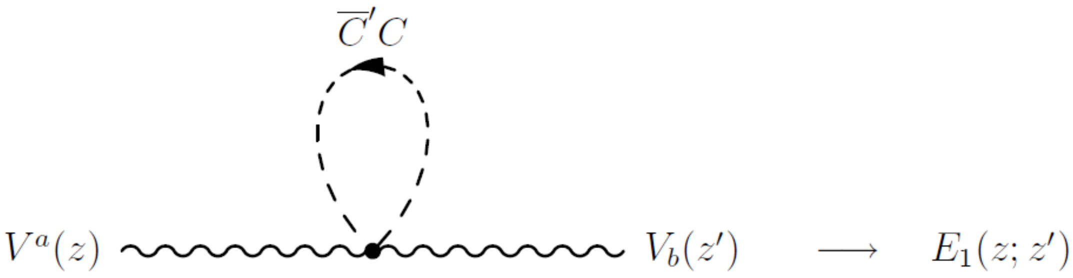
$$\longrightarrow \quad B(z;z')$$

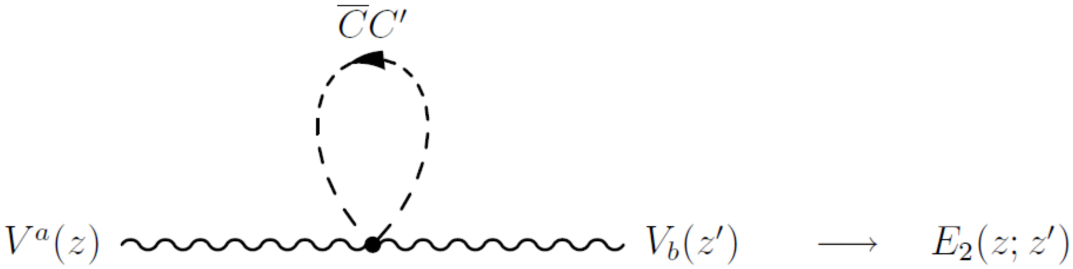

$\Phi\Phi^\dagger$
$V^a(z)$
$V_b(z')$
$\longrightarrow$
$C(z;z')$

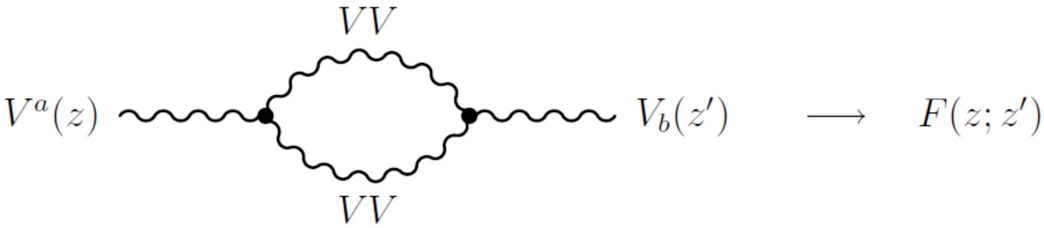

$\overline{C}C'$
$V^a(z)$
$V_b(z')$
$\longrightarrow$
$D_1(z;z')$
$C'\overline{C}$

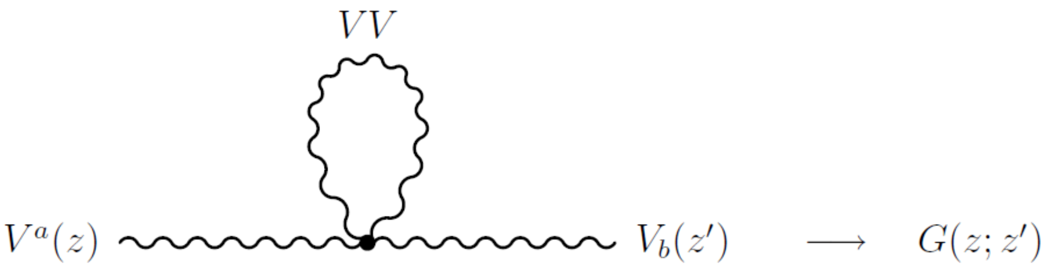

$\overline{C}'C$
$V^a(z)$
$V_b(z')$
$\longrightarrow$
$D_2(z;z')$
$C\overline{C}'$


$\overline{C}'C$
$V^a(z)$
$V_b(z')$
$\longrightarrow$
$D_3(z;z')$
$\overline{C}C'$









$$-\frac{i}{16\sqrt{2}} g f_{abc} [\bar{D}^2 (D^\alpha V^a)] V^b (D_\alpha V^c)$$
$$-\frac{1}{128} g^2 f_{ab}{}^{e} f_{ecd} V^a (D^\alpha V^b) [(\bar{D}^2 V^c)(D_\alpha V^d)]$$

$$\tilde{A}(p) = 3g^2 N(N^2-1)\delta_{ab} \int \frac{d^4k}{(2\pi)^4} d^4\theta \left\{ \frac{m^2 [V^a(p,\theta,\bar{\theta}) V^b(-p,\theta,\bar{\theta})]}{(k^2+m^2)[(p-k)^2+m^2]} \right\}$$

$$\tilde{B}(p) = 3g^2N(N^2-1)\delta_{ab}\int \frac{d^4k}{(2\pi)^4}d^4\theta \frac{1}{(k^2+m^2)[(p-k)^2+m^2]}$$
$$\cdot\Big\{k^2V^a(p,\theta,\bar{\theta})V^b(-p,\theta,\bar{\theta}) - \frac{i}{4}p_\mu\sigma^\mu_{\alpha\dot{\alpha}}V^a(p,\theta,\bar{\theta})\big[\big(\bar{D}^{\dot{\alpha}}D^\alpha\big)V^b(-p,\theta,\bar{\theta})\big]$$
$$+\frac{1}{4}V^a(p,\theta,\bar{\theta})[(\bar{D}^2D^2)V^b(-p,\theta,\bar{\theta})]\Big\}$$

$$\tilde{C} = -3g^2N(N^2-1)\delta^{ab}\int \frac{d^4k}{(2\pi)^4}d^4\theta\frac{1}{(k^2+m^2)}\{V^a(p,\theta,\bar{\theta})V^b(-p,\theta,\bar{\theta})\}$$

$$\tilde{A}(p)+\tilde{B}(p)+\tilde{C}(p) = -3g^2N(N^2-1)\delta_{ab}\int \frac{d^4k}{(2\pi)^4}d^4\theta\frac{1}{(k^2+m^2)[(p-k)^2+m^2]}$$
$$\cdot\Big\{\frac{i}{4}p_\mu\sigma^\mu_{\alpha\dot{\alpha}}V^a(p,\theta,\bar{\theta})\big[\big(\bar{D}^{\dot{\alpha}}D^\alpha\big)V^b(-p,\theta,\bar{\theta})\big]$$
$$+\frac{1}{16}V^a(p,\theta,\bar{\theta})[(\bar{D}^2D^2)V^b(-p,\theta,\bar{\theta})]\Big\}$$

$$\tilde{D}_1(p)+\tilde{D}_2(p)+\tilde{D}_3(p)+\tilde{E}_1(p)+\tilde{E}_2(p) = \frac{1}{8}g^2N(N^2-1)\delta_{ab}$$
$$\int\frac{d^4k}{(2\pi)^4}d^4\theta\frac{1}{k^2(p-k)^2}\Big\{ip_\mu\sigma^\mu_{\alpha\dot{\alpha}}V^a(p,\theta,\bar{\theta})\big[\big(\bar{D}^{\dot{\alpha}}D^\alpha\big)V^b(-p,\theta,\bar{\theta})\big]$$
$$+\frac{1}{8}V^a(p,\theta,\bar{\theta})[(\bar{D}^2D^2)V^b(-p,\theta,\bar{\theta})]\Big\}$$

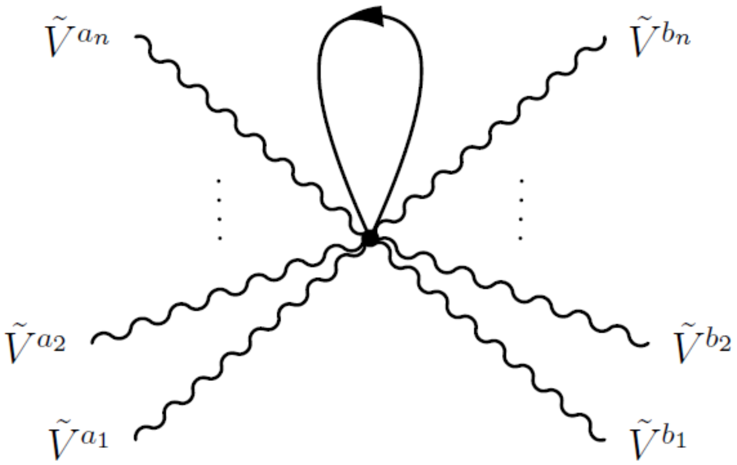
$$\bar{D}^2D_\alpha\bar{D}^2D^2 = 0,$$

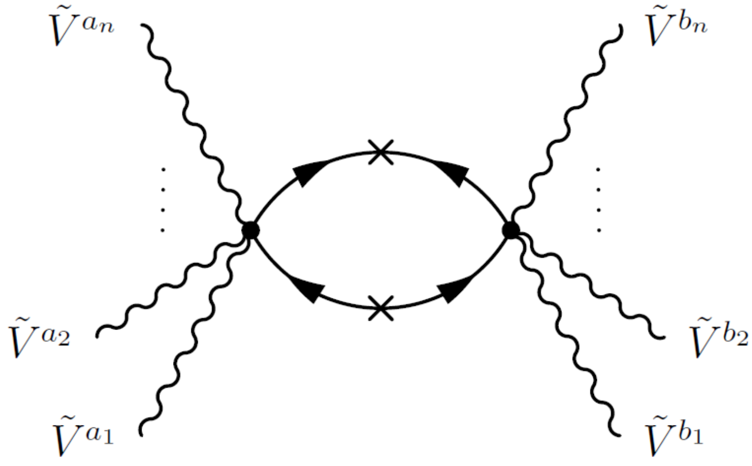
$$J^{(1)}(p) = c_1\gamma g^2\delta^a_b\int\frac{d^4k}{(2\pi)^4}d^4\theta\frac{\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}p_\mu p_\nu}{k^4(p-k)^2}\{V_a(p,\theta,\bar{\theta})\big[D^\alpha\bar{D}^{\dot{\alpha}}\bar{D}^{\dot{\beta}}D^\beta V^b(-p,\theta,\bar{\theta})\big]\}$$

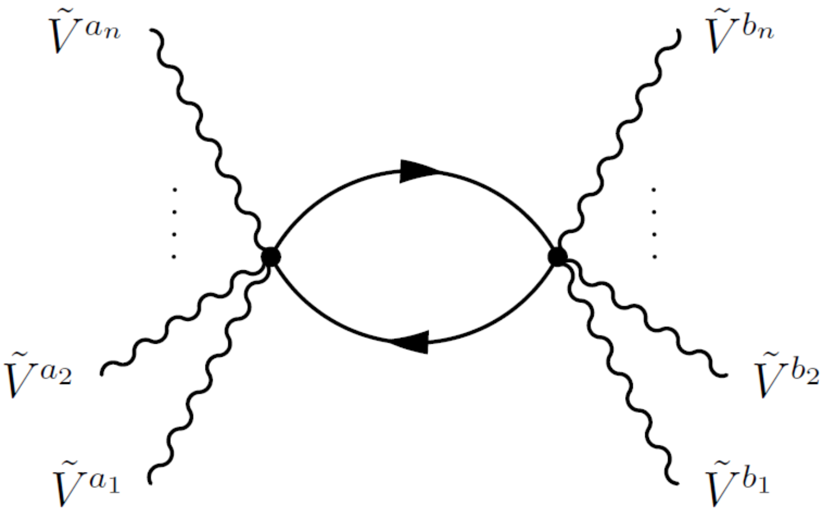
$$J^{(2)}(p) = c_2\gamma^2g^2\delta^a_b\int\frac{d^4k}{(2\pi)^4}d^4\theta\frac{\big(\sigma^\nu\bar{\sigma}^\mu\sigma^\lambda\big)_{\alpha\dot{\alpha}}(p-k)_\mu k_\nu p_\lambda}{k^4(p-k)^4}\cdot\{[(\bar{D}^2D^\alpha)V_a(p,\theta,\bar{\theta})]\big[\big(D^2\bar{D}^{\dot{\alpha}}\big)V^b(-p,\theta,\bar{\theta})\big]\}$$

$$d_s = 2-E-C$$

$$d_s = -C$$







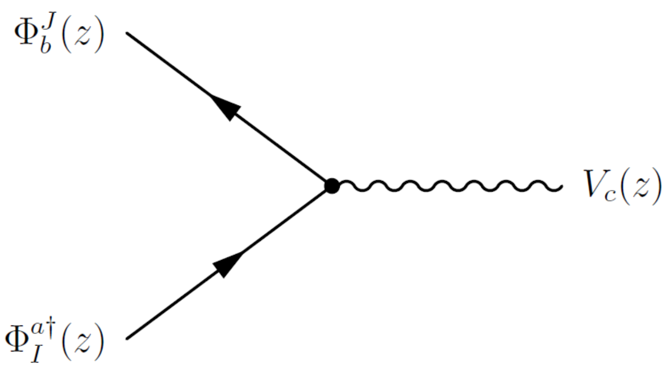
$$\langle (CC)(k) \rangle \sim \frac{1}{k^4}.$$

$$-\frac{1}{k^4}(\bar{D}^2 D^2 + D^2 \bar{D}^2)\delta_4(\theta - \theta')$$

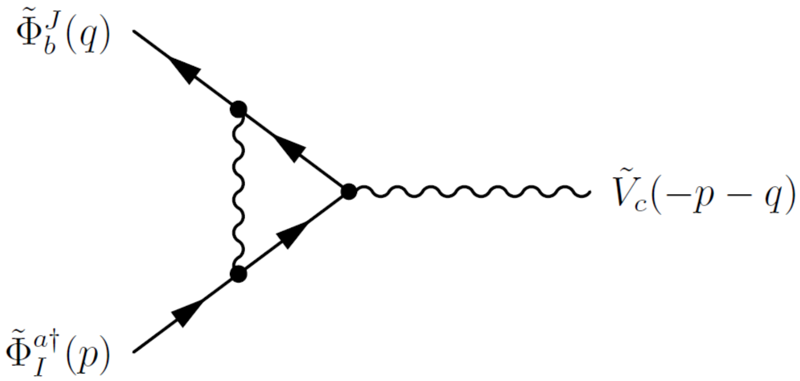
$$D_\alpha \left[-\frac{1}{k^4}(\bar{D}^2 D^2 + D^2 \bar{D}^2)\delta_4(\theta - \theta') \right] = -4i \frac{k_\mu \sigma^\mu_{\alpha\dot{\alpha}}}{k^4} \bar{D}^{\dot{\alpha}} D^2$$

$$\bar{D}^{\dot{\alpha}} \left[-\frac{1}{k^4}(\bar{D}^2 D^2 + D^2 \bar{D}^2)\delta_4(\theta - \theta') \right] = 4i \frac{k_\mu \bar{\sigma}^{\mu\dot{\alpha}\alpha}}{k^4} D_\alpha \bar{D}^2.$$

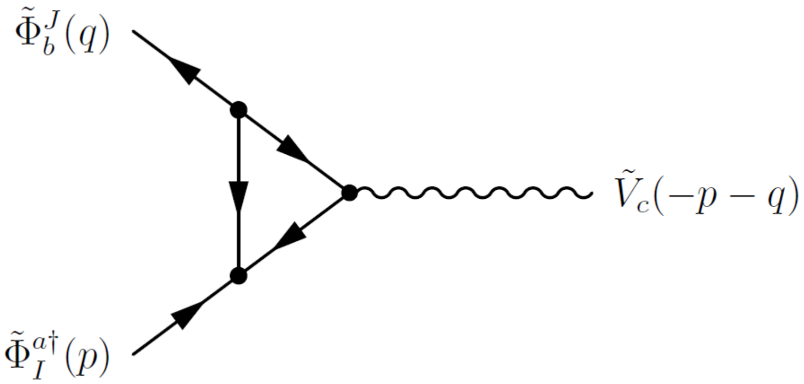
$$V_R = f(V).$$



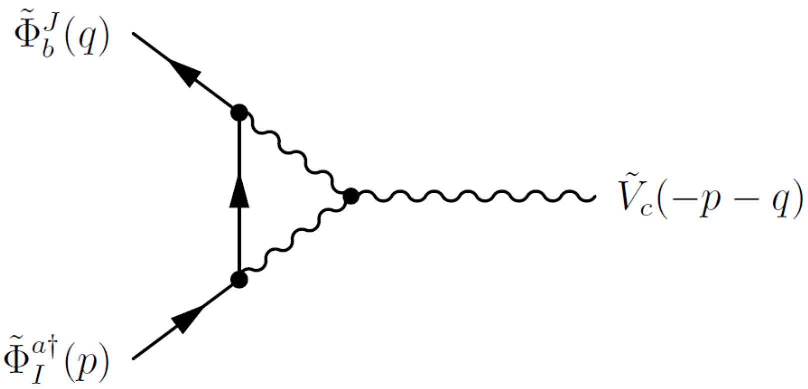
$$\longrightarrow \quad \left(\Phi_I^{a\dagger} V^c \Phi_b^I\right)(z)$$



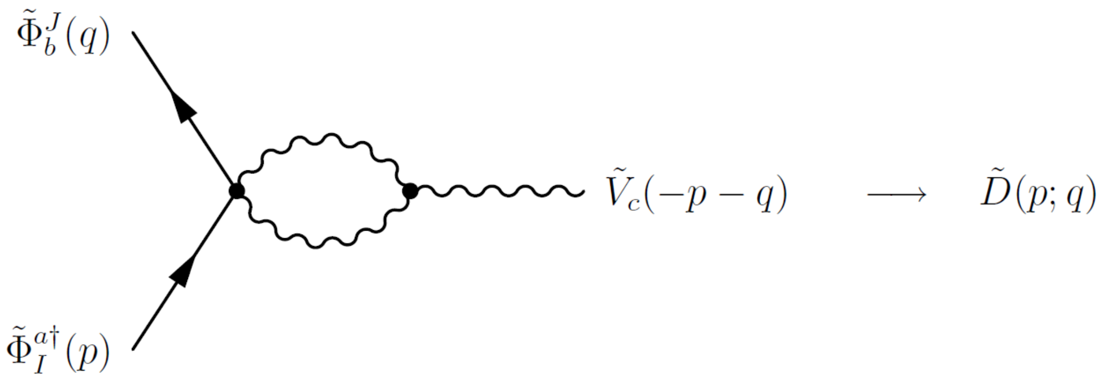
$$\longrightarrow \quad \tilde{A}(p;q)$$

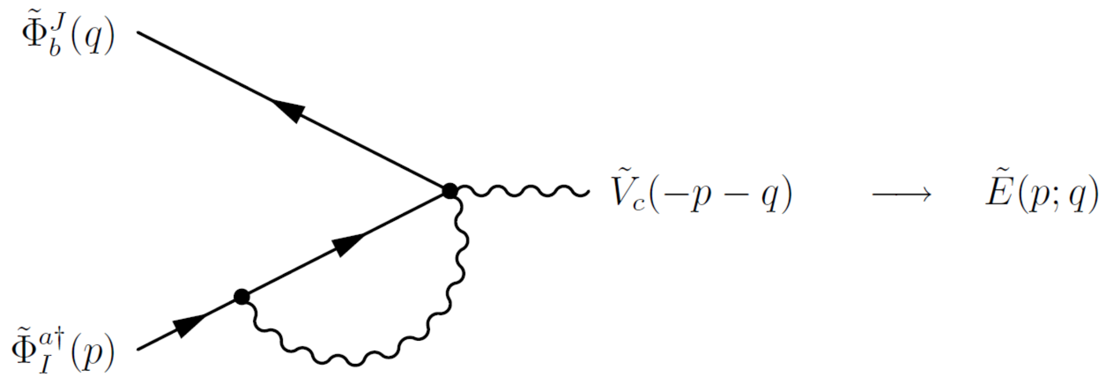


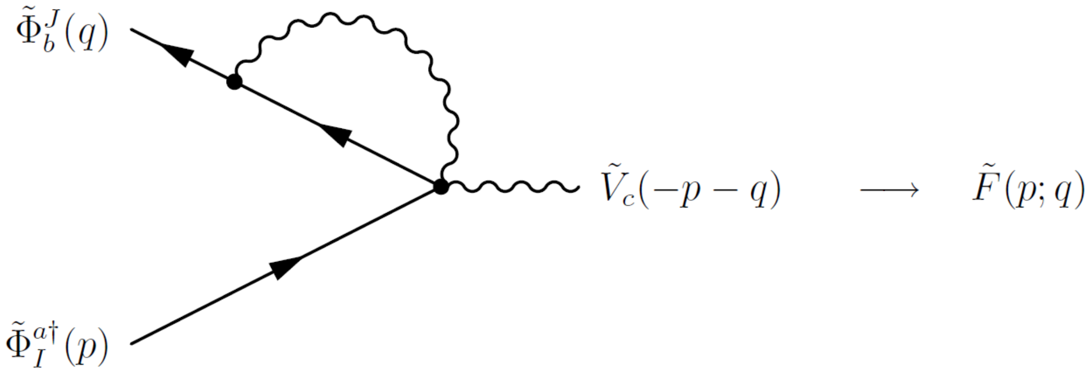
$$\longrightarrow \quad \tilde{B}(p;q)$$

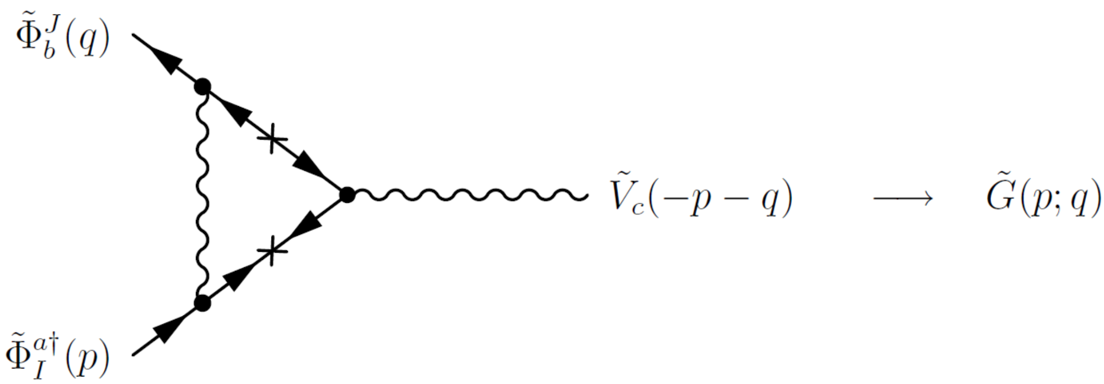


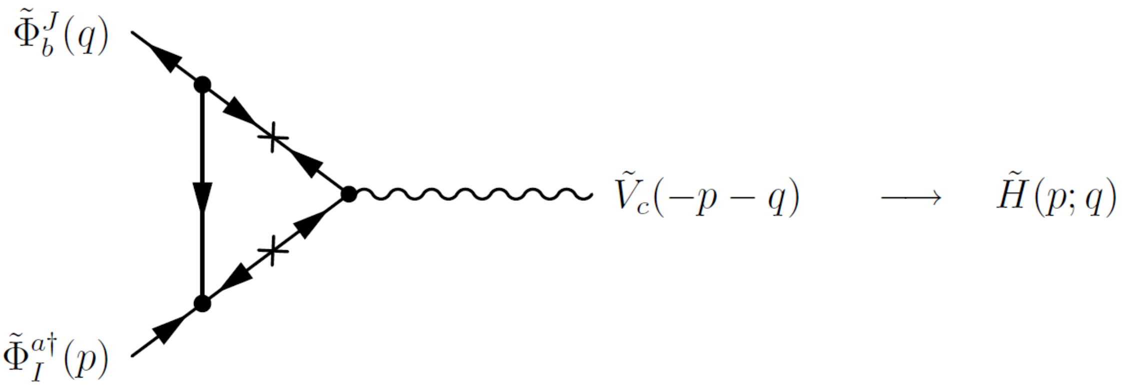
$$\longrightarrow \quad \tilde{C}(p;q)$$


$\tilde{\Phi}_b^J(q)$
$\tilde{\Phi}_I^{a\dagger}(p)$
$\tilde{V}_c(-p-q) \longrightarrow \tilde{D}(p;q)$


$\tilde{\Phi}_b^J(q)$
$\tilde{\Phi}_I^{a\dagger}(p)$
$\tilde{V}_c(-p-q) \longrightarrow \tilde{E}(p;q)$

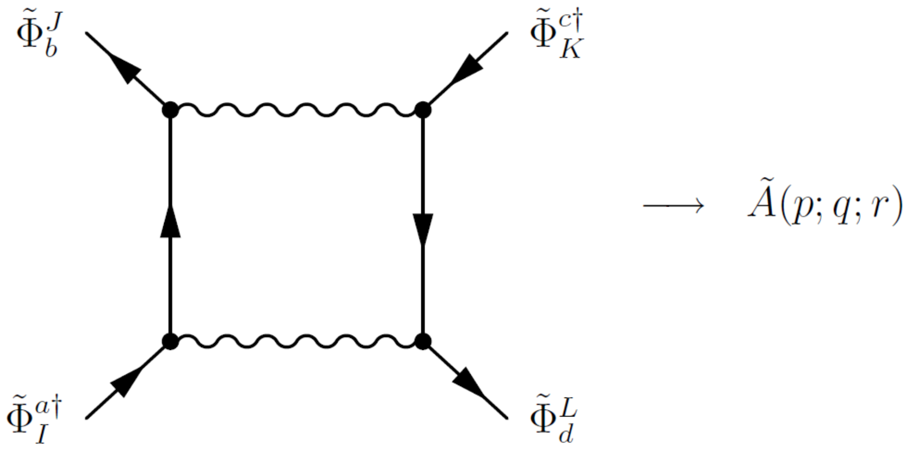

$\tilde{\Phi}_b^J(q)$
$\tilde{\Phi}_I^{a\dagger}(p)$
$\tilde{V}_c(-p-q) \longrightarrow \tilde{F}(p;q)$


$\tilde{\Phi}_b^J(q)$
$\tilde{\Phi}_I^{a\dagger}(p)$
$\tilde{V}_c(-p-q) \longrightarrow \tilde{G}(p;q)$



$$I_{\log} = c\delta_J^I f^{abc} \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2} \{\Phi_{aI}^{\dagger}(p,\theta,\bar{\theta}) V_b(-p-q,\theta,\bar{\theta}) \Phi_c^J(q,\theta,\bar{\theta})\}$$

$$\begin{aligned} I_{\infty} &= \delta_J^I f f^{abc} \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(q+k)^2(p-k)^2} \\ &\Phi_{aI}^{\dagger}(p,\theta,\bar{\theta}) \{c_1 [D^{\alpha}\bar{D}^2 D_{\alpha} V_b(-p-q,\theta,\bar{\theta})] \Phi_c^J(q,\theta,\bar{\theta}) \\ &+ c_2 \sigma_{\alpha\dot{\alpha}}^{\mu} (q+k)_{\mu} [D^{\alpha}\bar{D}^{\dot{\alpha}} V_b(-p-q,\theta,\bar{\theta})] \Phi_c^J(q,\theta,\bar{\theta})\} \end{aligned}$$



$$\tilde{A}_1(p,q,r) = \left(\frac{1}{4}\right)^2 g^4 \kappa N^2(N^2-1)(\delta_a^b \delta_c^d + \delta_{ac}\delta^{bd}) \delta_J^I \delta_L^K I_{1bdIK}^{(A)acJL}(p,q,r)$$

$$\begin{aligned} I_{1bdIK}^{(A)acJL}(p,q,r) &= \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(q+k)^2(q+k-r)^2} \\ &\cdot \{[(D^2\bar{D}^2)\Phi_I^{a\dagger}(p,\theta,\bar{\theta})] \Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta}) \Phi_d^L(r,\theta,\bar{\theta}) \Phi_b^J(q,\theta,\bar{\theta}) + \Phi_I^{a\dagger}(p,\theta,\bar{\theta}) \\ &\cdot [(D^2\bar{D}^2)\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})] \Phi_d^L(r,\theta,\bar{\theta}) \Phi_b^J(q,\theta,\bar{\theta}) + 2[(D^2\bar{D}_{\dot{\alpha}})\Phi_I^{a\dagger}(p,\theta,\bar{\theta})] \\ &\cdot [\bar{D}^{\dot{\alpha}}\Phi^{c\dagger}(p+q-r,\theta,\bar{\theta})] \Phi_d^L(r,\theta,\bar{\theta}) \Phi_b^J(q,\theta,\bar{\theta}) + 2[\bar{D}_{\dot{\alpha}}\Phi_I^{a\dagger}(p,\theta,\bar{\theta})] \\ &\cdot [(D^2\bar{D}^{\dot{\alpha}})\Phi^{c\dagger}(p+q-r,\theta,\bar{\theta})] \Phi_d^L(r,\theta,\bar{\theta}) \Phi_b^J(q,\theta,\bar{\theta}) + 4[(D^{\alpha}\bar{D}_{\dot{\alpha}})\Phi_I^{a\dagger}(p,\theta,\bar{\theta})] \\ &\cdot [D_{\alpha}\bar{D}^{\dot{\alpha}}\Phi^{c\dagger}(p+q-r,\theta,\bar{\theta})] \Phi_d^L(r,\theta,\bar{\theta}) \Phi_b^J(q,\theta,\bar{\theta})\} \\ &\equiv \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(q+k)^2(q+k-r)^2} K_{bdIK}^{acJL}(p,q,r;\theta;\bar{\theta}) \end{aligned}$$

$$f_{aef} f^{bfg} f_{cgh} f^{dhe} = \kappa N^2(N^2-1)(\delta_a^b \delta_c^d + \delta_a^d \delta_c^b);$$

$$\tilde{A}_2(p,q,r) = \left(\frac{1}{4}\right)^2 g^4 \kappa N^2(N^2-1)(\delta_a^d \delta_c^b + \delta_{ac}\delta^{bd}) \delta_L^I \delta_J^K I_{2bdIK}^{(A)acJL}(p,q,r),$$

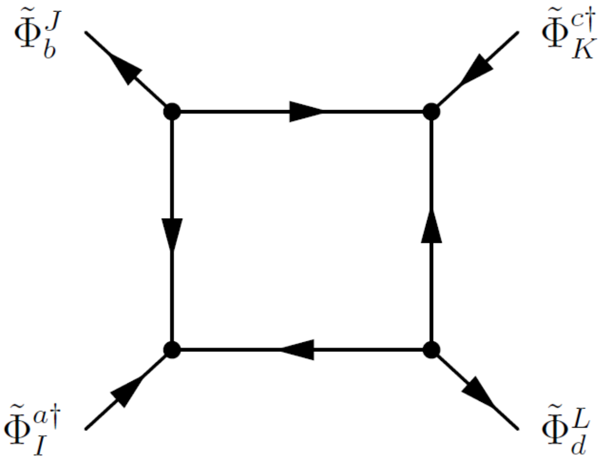
$$I_{2bdIK}^{(A)acJL}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(p-k-r)^2(q+k)^2(p+q-k-r)^2} \cdot K_{bdIK}^{acJL}(p,q,r;\theta;\bar{\theta})$$

$$\tilde{A}_3(p,q,r) = \left(\frac{1}{4}\right)^2 g^4 \kappa N^2(N^2-1)(\delta_a^b\delta_c^d + \delta_a^d\delta_c^b)\delta_J^I\delta_L^K I_{3bdIK}^{(A)acJL}(p,q,r),$$

$$I_{3bdIK}^{(A)acJL}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(q+k)^2(q+k-r)^2} \cdot \{[\bar{D}^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})][D^2\Phi_b^J(q,\theta,\bar{\theta})]\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})\Phi_d^L(r,\theta,\bar{\theta}) + 8i\sigma_{\alpha\dot{\alpha}}^{\mu}(p-k-r)_\mu[\bar{D}^{\dot{\alpha}}\Phi_I^{a\dagger}(p,\theta,\bar{\theta})][D^\alpha\Phi_b^J(q,\theta,\bar{\theta})]\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta}) \cdot \Phi_d^L(r,\theta,\bar{\theta}) - 16(p-k-r)^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta}) \cdot \Phi_d^L(r,\theta,\bar{\theta})\}$$

$$\tilde{A}_4(p,q,r) = \left(\frac{1}{4}\right)^2 g^4 \kappa N^2(N^2-1)(\delta_a^b\delta_c^d + \delta_a^d\delta_c^b)\delta_J^I\delta_L^K I_{4bdIK}^{(A)acJL}(p,q,r),$$

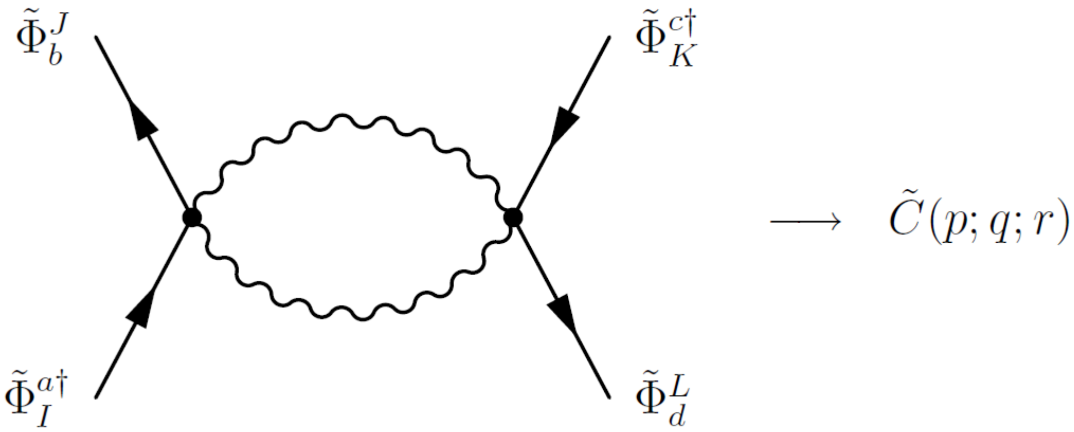
$$I_{4bdIK}^{(A)acJL}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(p-k-r)^2(q+k)^2} \cdot \{[\bar{D}^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})]\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})[D^2\Phi_d^L(r,\theta,\bar{\theta})] + 8i\sigma_{\alpha\dot{\alpha}}^{\mu}(q+k)_\mu[\bar{D}^{\dot{\alpha}}\Phi_I^{a\dagger}(p,\theta,\bar{\theta})]\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta}) \cdot [D^\alpha\Phi_d^L(r,\theta,\bar{\theta})] - 16(q+k)^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta}) \cdot \Phi_d^L(r,\theta,\bar{\theta})\}$$



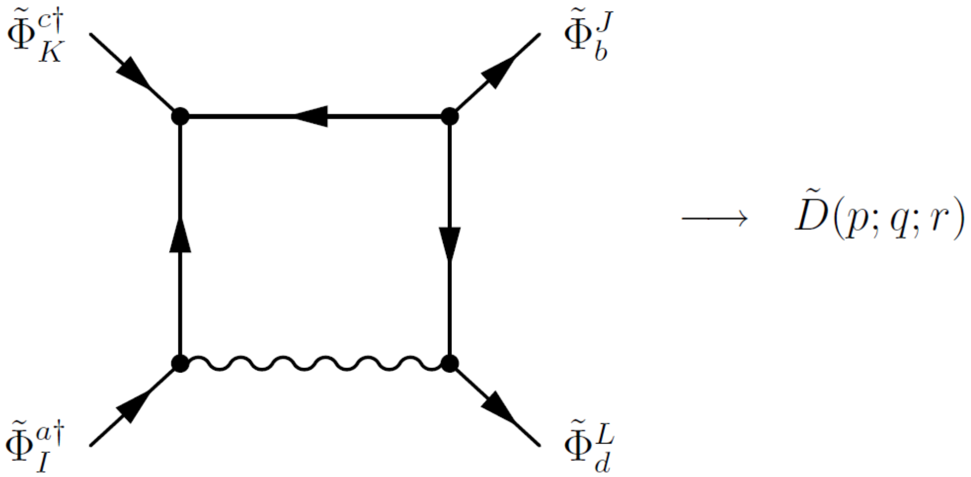
$\longrightarrow \quad \tilde{B}(p;q;r)$

$$\tilde{B}(p,q,r) = \frac{g^4}{6}\kappa N^2(N^2-1)(\delta_a^b\delta_c^d + \delta_a^d\delta_c^b)(\delta_J^I\delta_L^K + \delta_L^I\delta_J^K) I_{bdIK}^{(B)acJL}(p,q,r),$$

$$I_{bdIK}^{(B)acJL}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(p+q-r)^2(q+k)^2} \cdot \{[\bar{D}^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})]\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})[D^2\Phi_d^L(r,\theta,\bar{\theta})] + 8i\sigma_{\alpha\dot{\alpha}}^{\mu}(p-k)_\mu[\bar{D}^{\dot{\alpha}}\Phi_I^{a\dagger}(p,\theta,\bar{\theta})]\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta}) \cdot [D^\alpha\Phi_d^L(r,\theta,\bar{\theta})] - 16(p-k)^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta}) \cdot \Phi_d^L(r,\theta,\bar{\theta})\}$$



$$\delta_4(\theta_1-\theta_2)\delta_4(\theta_1-\theta_2)\equiv 0.$$



$$\tilde{D}_1(p,q,r)=\left(\frac{1}{4}\right)^2 g^4\kappa N^2(N^2-1)(\delta_{ac}\delta^{bd}+\delta_a^d\delta_c^b)(\delta_J^I\delta_L^K-\delta_L^I\delta_J^K)I_{1bdIK}^{(D)acJL}(p,q,r),$$

$$I_{1bdIK}^{(D)acJL}(p,q,r)=\int\frac{d^4k}{(2\pi)^4}d^4\theta\frac{1}{k^2(p-k)^2(k+r-p-q)^2(p-k-r)^2}$$
$$\cdot\{[\bar{D}^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})][D^2\Phi_b^J(q,\theta,\bar{\theta})]\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})\Phi_d^L(r,\theta,\bar{\theta})$$
$$+8i\sigma_{\alpha\dot{\alpha}}^{\mu}(k+r-p)_\mu[\bar{D}^{\dot{\alpha}}\Phi_I^{a\dagger}(p,\theta,\bar{\theta})][D^\alpha\Phi_b^J(q,\theta,\bar{\theta})]\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})$$
$$\cdot\Phi_d^L(r,\theta,\bar{\theta})-16(k+r-p)^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})$$
$$\cdot\Phi_d^L(r,\theta,\bar{\theta})\}$$

$$\tilde{D}_2(p,q,r)=\left(\frac{1}{4}\right)^2 g^4\kappa N^2(N^2-1)(\delta_{ac}\delta^{bd}+\delta_a^b\delta_c^d)(\delta_L^I\delta_J^K-\delta_J^I\delta_L^K)I_{2bdIK}^{(D)acJL}(p,q,r)$$

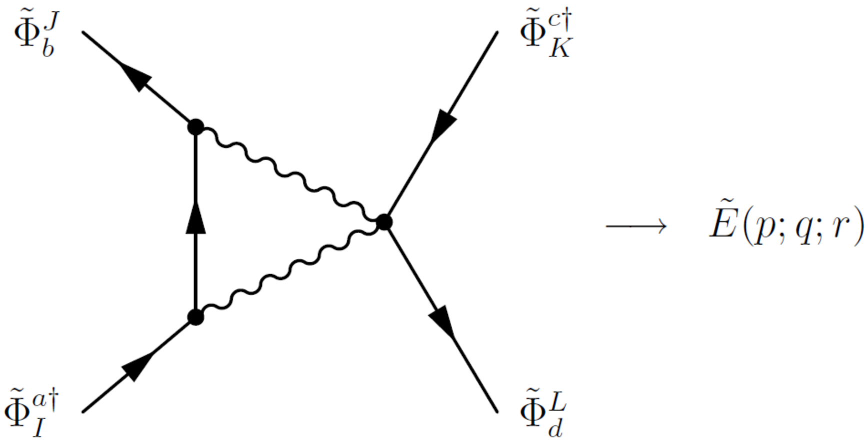
$$I_{2bdIK}^{(D)acJL}(p,q,r)=\int\frac{d^4k}{(2\pi)^4}d^4\theta\frac{1}{k^2(p-k)^2(k+r-p-q)^2(p+q-k)^2}$$
$$\cdot\{[\bar{D}^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})]\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})[D^2\Phi_d^L(r,\theta,\bar{\theta})]+$$
$$+8i\sigma_{\alpha\dot{\alpha}}^{\mu}(k-p-q)_\mu[\overline{D^\alpha}\Phi_I^{a\dagger}(p,\theta,\bar{\theta})]\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})$$
$$\cdot[D^\alpha\Phi_d^L(r,\theta,\bar{\theta})]-16(k-p-q)^2\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})$$
$$\cdot\Phi_d^L(r,\theta,\bar{\theta})\}$$

$$\tilde{D}_3(p,q,r)=\left(\frac{1}{4}\right)^2 g^4\kappa N^2(N^2-1)(\delta_a^d\delta_c^b+\delta_{ac}\delta^{bd})(\delta_J^I\delta_L^K-\delta_L^I\delta_J^K)I_{3bdIK}^{(D)acJL}(p,q,r)$$

$$I^{(D)acJL}_{3bdIK}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(k+r-p-q)^2(p+q-k)^2}$$
$$\cdot \{\Phi_I^{a\dagger}(p,\theta,\bar\theta)\Phi_b^J(q,\theta,\bar\theta)[\bar D^2\Phi_K^{c\dagger}(p+q-r,\theta,\bar\theta)][D^2\Phi_d^L(r,\theta,\bar\theta)]$$
$$-8i\sigma^\mu_{\alpha\dot\alpha}(p-k-r)_\mu\Phi_I^{a\dagger}(p,\theta,\bar\theta)\Phi_b^J(q,\theta,\bar\theta)[\bar D^{\dot\alpha}\Phi_K^{c\dagger}(p+q-r,\theta,\bar\theta)]$$
$$\cdot [D^\alpha\Phi_d^L(r,\theta,\bar\theta)] - 16(p-k-r)^2\Phi_I^{a\dagger}(p,\theta,\bar\theta)\Phi_b^J(q,\theta,\bar\theta)\Phi_K^{c\dagger}(p+q-r,\theta,\bar\theta)$$
$$\cdot \Phi_d^L(r,\theta,\bar\theta)\}$$

$$\tilde D_4(p,q,r) = \left(\frac{1}{4}\right)^2 g^4\kappa N^2(N^2-1)(\delta_a^b\delta_c^d + \delta_{ac}\delta^{bd})(\delta_L^I\delta_J^K - \delta_J^I\delta_L^K)I^{(D)acJL}_{4bdIK}(p,q,r),$$

$$I^{(D)acJL}_{4bdIK}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(k+r-p-q)^2(p+q-k)^2}$$
$$\cdot \{\Phi_I^{a\dagger}(p,\theta,\bar\theta)[D^2\Phi_b^J(q,\theta,\bar\theta)][\bar D^2\Phi_K^{c\dagger}(p+q-r,\theta,\bar\theta)]\Phi_d^L(r,\theta,\bar\theta)$$
$$-8i\sigma^\mu_{\alpha\dot\alpha}(p+q-k)_\mu\Phi_I^{a\dagger}(p,\theta,\bar\theta)[D^\alpha\Phi_b^J(q,\theta,\bar\theta)][\bar D^{\dot\alpha}\Phi_K^{c\dagger}(p+q-r,\theta,\bar\theta)]$$
$$\cdot \Phi_d^L(r,\theta,\bar\theta) - 16(p+q-k)^2\Phi_I^{a\dagger}(p,\theta,\bar\theta)\Phi_b^J(q,\theta,\bar\theta)\Phi_K^{c\dagger}(p+q-r,\theta,\bar\theta)$$
$$\cdot \Phi_d^L(r,\theta,\bar\theta)\}$$



$$\tilde E_1(p,q,r) = \left(\frac{1}{4}\right)^2 \frac{g^4}{2}\kappa N^2(N^2-1)\delta_a^b\delta_c^d + \delta_{ac}\delta^{bd}\Big)\delta_J^I\delta_L^K I^{(E)acJL}_{1bdIK}(p,q,r),$$

$$I^{(E)acJL}_{1bdIK}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(p+q-k)^2}$$
$$\cdot \{\Phi_I^{a\dagger}(p,\theta,\bar\theta)\Phi_b^J(q,\theta,\bar\theta)\Phi_K^{c\dagger}(p+q-r,\theta,\bar\theta)\Phi_d^L(r,\theta,\bar\theta)\}$$
$$\tilde E_2(p,q,r) = \left(\frac{1}{4}\right)^2 \frac{g^4}{2}\kappa N^2(N^2-1)(\delta_a^d\delta_c^b + \delta_{ac}\delta^{bd})\delta_L^I\delta_J^K I^{(E)acJL}_{2bdIK}(p,q,r),$$

$$I^{(E)acJL}_{2bdIK}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p-k)^2(r-k)^2}$$
$$\cdot \{\Phi_I^{a\dagger}(p,\theta,\bar\theta)\Phi_b^J(q,\theta,\bar\theta)\Phi_K^{c\dagger}(p+q-r,\theta,\bar\theta)\Phi_d^L(r,\theta,\bar\theta)\};$$

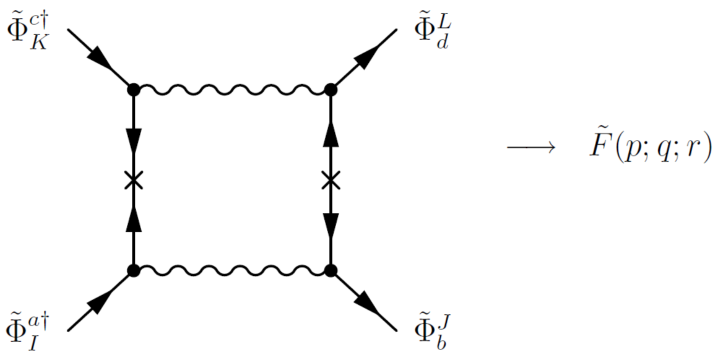
$$\tilde E_3(p,q,r) = \left(\frac{1}{4}\right)^2 \frac{g^4}{2}\kappa N^2(N^2-1)(\delta_a^b\delta_c^d + \delta_{ac}\delta^{bd})\delta_L^I\delta_J^K I^{(E)acJL}_{3bdIK}(p,q,r)$$

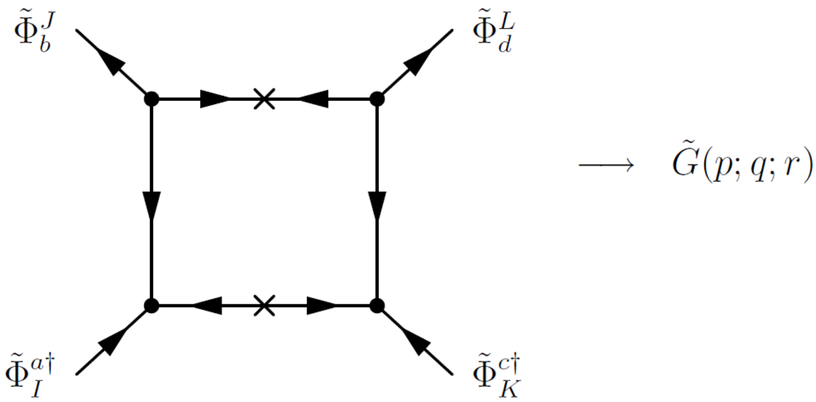
$$I^{(E)acJL}_{3bdIK}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(p+q-k)^2(k-r)^2}$$
$$\cdot \{\Phi_I^{a\dagger}(p,\theta,\bar\theta)\Phi_b^J(q,\theta,\bar\theta)\Phi_K^{c\dagger}(p+q-r,\theta,\bar\theta)\Phi_d^L(r,\theta,\bar\theta)\}$$
$$\tilde E_4(p,q,r) = \left(\frac{1}{4}\right)^2 \frac{g^4}{2}\kappa N^2(N^2-1)(\delta_a^d\delta_c^b + \delta_{ac}\delta^{bd})\delta_L^I\delta_J^K I^{(E)acJL}_{4bdIK}(p,q,r),$$

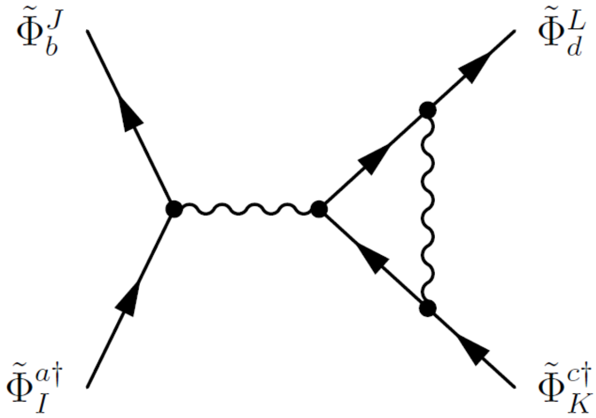
$$I_{4bdIK}^{(E)acJL}(p,q,r) = \int \frac{d^4k}{(2\pi)^4} d^4\theta \frac{1}{k^2(q+k)^2(k+r-p)^2}$$
$$\cdot \{\Phi_I^{a\dagger}(p,\theta,\bar{\theta})\Phi_b^J(q,\theta,\bar{\theta})\Phi_K^{c\dagger}(p+q-r,\theta,\bar{\theta})\Phi_d^L(r,\theta,\bar{\theta})\}$$

$$\langle \Phi^\dagger \Phi \Phi^\dagger \Phi \rangle = \kappa \left(\frac{1}{4}\right)^2 g^4 N^2 (N^2-1) \sum_{i=1}^{6} G^{(i)}$$

$$G^{(1)} = \delta_a^b \delta_c^d \delta_J^I \delta_L^K \Big(I_{1bdIK}^{(A)acJL} + I_{3bdIK}^{(A)acJL} + \frac{2}{3} I_{bdIK}^{(B)acJL} - I_{2bdIK}^{(D)acJL}$$
$$- I_{4bdIK}^{(D)acJL} + I_{1bdIK}^{(E)acJL} + I_{3bdIK}^{(E)acJL} \Big)$$
$$G^{(2)} = \delta_{ac} \delta^{bd} \delta_J^I \delta_L^K \Big(I_{1bdIK}^{(A)acJL} + I_{1bdIK}^{(D)acJL} - I_{2bdIK}^{(D)acJL} + I_{3bdIK}^{(D)acJL}$$
$$- I_{4bdIK}^{(D)acJL} + I_{1bdIK}^{(E)acJL} + I_{3bdIK}^{(E)acJL} \Big)$$
$$G^{(3)} = \delta_a^b \delta_c^d \delta_L^I \delta_J^K \left(I_{2bdIK}^{(A)acJL} + \frac{2}{3} I_{bdIK}^{(B)acJL} + I_{2bdIK}^{(D)acJL} + I_{4bdIK}^{(D)acJL} \right)$$
$$G^{(4)} = \delta_a^d \delta_c^b \delta_L^I \delta_J^K \Big(I_{2bdIK}^{(A)acJL} + I_{4bdIK}^{(A)acJL} + \frac{2}{3} I_{bdIK}^{(B)acJL} - I_{1bdIK}^{(D)acJL}$$
$$- I_{3bdIK}^{(D)acJL} + I_{2bdIK}^{(E)acJL} + I_{4bdIK}^{(E)acJL} \Big)$$
$$G^{(5)} = \delta_a^d \delta_c^b \delta_J^I \delta_L^K \left(I_{3bdIK}^{(A)acJL} + \frac{2}{3} I_{bdIK}^{(B)acJL} + I_{1bdIK}^{(D)acJL} + I_{3bdIK}^{(E)acJL} \right)$$
$$G^{(6)} = \delta_{ac} \delta^{bd} \delta_L^I \delta_J^K \Big(I_{4bdIK}^{(A)acJL} - I_{1bdIK}^{(D)acJL} + I_{2bdIK}^{(D)acJL} - I_{3bdIK}^{(D)acJL}$$
$$+ I_{4bdIK}^{(D)acJL} + I_{2bdIK}^{(E)acJL} + I_{4bdIK}^{(E)acJL} \Big)$$







$$\langle \mathcal{O}(x_1,\ldots,x_n)\rangle = \int [\mathcal{D}\phi] e^{-\frac{S[\phi]}{\hbar}} \mathcal{O}\big(\phi(x_1),\ldots,\phi(x_n)\big),$$

$$\phi(x) = \bar{\phi}(x) + \delta\phi(x) = \bar{\phi}(x) + \hbar^{1/2}\eta(x)$$

$$S[\phi] = S[\bar{\phi}] + \frac{\hbar}{2}\int d^4x d^4y \left(\frac{\delta^2 S}{\delta\phi(x)\delta\phi(y)}\right)\bigg|_{\phi=\bar{\phi}} \eta(x)\eta(y) + O\big(\hbar^{3/2}\big),$$

$$\langle \mathcal{O}(x_1,\ldots,x_n)\rangle = \int [\mathcal{D}\eta] e^{-\frac{S[\bar{\phi}+\eta]}{\hbar}} \mathcal{O}[(\bar{\phi}+\eta)(x_i)]$$

$$= \mathcal{O}(\bar{\phi}) e^{-\frac{\bar{S}}{\hbar}} \left[\det\left(\frac{\delta^2 S}{\delta\phi(x)\delta\phi(y)}\right)\bigg|_{\phi=\bar{\phi}}\right]^{-\frac{1}{2}} (1 + O(\hbar))$$

$$S_{\mathrm{YM}}[A] = -\frac{1}{4d_{\mathbf{r}}g^2}\int d^4x \operatorname{tr}\big(F_{\mu\nu}F^{\mu\nu}\big)$$

$$A_\mu = A_\mu^a T_a$$
$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + [A^\mu, A^\nu] = \big(\partial_\mu A_\mu^a - \partial_\nu A_\mu^a + i f_{bc}^a A_\mu^b A_\nu^c\big) T_a$$

$$A_\mu(x) \longrightarrow A_\mu^\Omega(x) = \Omega(x)\big[A_\mu(x) + i\partial_\mu\big]\Omega^\dagger(x)$$

$$K[A] = \frac{1}{32\pi^2}\int d^4x F_{\mu\nu}^a \tilde{F}_a^{\mu\nu}$$

$$S_{\mathrm{YM}}[A] = \pm\frac{8\pi^2}{g^2}K[A] + \frac{1}{8\pi^2}\int d^4x \big(F_{\mu\nu}^a \mp \tilde{F}_{\mu\nu}^a\big)\big(F_a^{\mu\nu} \mp \tilde{F}_a^{\mu\nu}\big) \geq \frac{8\pi^2}{g^2}|K[A]|$$

$$F_{\mu\nu}^a = \pm\tilde{F}_{\mu\nu}^a$$

$$\bar{A}_\mu = i\frac{\bar{\sigma}_{\mu\nu}}{2}\partial^\nu \log\Phi$$

$$\frac{\square\,\Phi}{\Phi} = 0.$$

$$\Phi = 1 + \frac{\rho^2}{(x-x_0)^2}$$

$$\bar{A}_\mu = -2i\rho^2\bar{\eta}_{\mu\nu}^a \frac{y^\nu}{y^2(y^2+\rho^2)}$$

$$\bar{\eta}_{\mu\nu}^a = -\bar{\eta}_{\nu\mu}^a = \begin{cases}\varepsilon^{a\mu\nu} & \text{for } \mu,\nu = 1,2,3\\ -\delta^{a\mu} & \text{for } \nu = 4\end{cases}$$

$$\bar{A}_\mu = -2ie^{i\theta_a T^a}\eta^a_{\mu\nu}\frac{y^\nu}{(y^2+\rho^2)}e^{-i\theta_a T^a}$$

$$\eta^a_{\mu\nu} = -\eta^a_{\nu\mu} = \begin{cases}\varepsilon^{a\mu\nu} & \text{for } \mu,\nu = 1,2,3\\ \delta^{a\mu} & \text{for } \nu = 4\end{cases}$$

$$\sigma_{\mu\nu} = \eta^a_{\mu\nu}\sigma_a$$

$$\bar{F}_{\mu\nu} = F_{\mu\nu}[\bar{A}] = -2ie^{i\theta_a T^a}\sigma_{\mu\nu}\frac{\rho^2}{(y^2+\rho^2)^2}e^{-i\theta_a T^a}$$

$$\Phi(x) = 1 + \sum_{i=1}^{n}\frac{\rho_i^2}{\left(x - x_0^{(i)}\right)^2}$$

$$S[\bar{A}^{(n)}] = nS[\bar{A}^{(1)}]$$

$$M^{ab}_{\mu\nu} = \left(\frac{\delta^2 S[A]}{\delta A^a_\mu \delta A^b_\nu}\right)\Bigg|_{\bar{A}} = [-D^2(\bar{A})\delta_{\mu\nu} + D_\mu(\bar{A})D_\nu(\bar{A}) - 2\bar{F}_{\mu\nu}]\delta^{ab},$$

$$\int d^4y\left(\frac{\delta^2 S[A]}{\delta A_\mu \delta A_\nu}\right)\Bigg|_{\bar{A}_\beta}\frac{\partial \bar{A}_\nu}{\partial \beta_i}(y,\beta) = 0$$

$$\bar{A}^\Omega_\mu(x,\beta) = \Omega^\dagger \bar{A}_\mu \Omega + i\Omega^\dagger\partial_\mu\Omega,$$

$$Z = \int [\mathcal{D}A]e^{-S[A]}$$

$$1 = \Delta_{\text{FP}}\int_G \prod_{a,x}[\mathcal{D}h^a(x)]\int_{\mathcal{M}}\prod_i d\beta_i \delta\left(\left(A_\mu - \bar{A}^\Omega_\mu, \frac{\delta \bar{A}^\Omega_\mu}{\delta h^a}\right)\right)\delta\left(\left(A_\mu - \bar{A}^\Omega_\mu, \frac{\partial \bar{A}^\Omega_\mu}{\partial \beta_i}\right)\right),$$

$$(f_\mu, g^\mu) = \frac{1}{2}\int d^4x \operatorname{tr}(f_\mu(x)g^\mu(x))$$

$$Z = \int [\mathcal{D}A_\mu]\int \prod_i d\beta_i e^{-S[A]}\Delta_{\text{FP}}(A,\beta)$$
$$\cdot\, \delta(\operatorname{tr}[T^a D_\mu(\bar{A})(A^\mu - \bar{A})(x)])\delta\left(\left(A_\mu - \bar{A}_\mu, \frac{\partial \bar{A}_\mu}{\partial \beta_i}\right)\right)$$

$$\Delta_{\text{FP}}(A,\beta)|_{A=\bar{A}} = \det_{\substack{x,y\\a,b}}\left(D^2_{ab}(\bar{A})\delta(x-y)\right)\det_{i,j}\left(d_{ij}(A,\beta)\right)$$

$$d_{ij}(A,\beta) = \left(\hat{a}^{(i)}_\mu(\beta), \hat{a}^{\mu(j)}(\beta)\right)$$

$$\hat{a}^{(i)a}_\mu(x,\beta) = \frac{\partial \bar{A}^a_\mu}{\partial \beta_i} + \xi^{(i)a}_\mu(x)$$

$$D^{ab}_\mu(\bar{A})\hat{a}^{(i)}_{\mu b}(x,\beta) = 0.$$

$$\langle \mathcal{O}(x_1,\dots,x_n)\rangle_{\bar{A}} = e^{-\frac{8\pi^2}{g^2}K[\bar{A}]}$$
$$\cdot\frac{\int [\mathcal{D}Q_\mu \mathcal{D}c\mathcal{D}\bar{c}]\prod_i d\beta_i \frac{\|\hat{a}\|}{\sqrt{2\pi}}e^{-\frac{1}{2}\int Q^\mu M^{\text{g.f.}}_{\mu\nu}Q^\nu - \int \bar{c}D^2(\bar{A})c}\mathcal{O}[\bar{A}]}{\int [\mathcal{D}Q_\mu \mathcal{D}c\mathcal{D}\bar{c}]e^{-\frac{1}{2}\int Q^\mu M^{(0)\,\text{g.f.}}_{\mu\nu}Q^\nu - \int \bar{c}\Box c}}$$
$$= e^{-\frac{8\pi^2}{g^2}K[\bar{A}]}\int \prod_i d\beta_i \frac{\|\hat{a}\|}{\sqrt{2\pi}}\mathcal{O}[\bar{A}]\frac{\left(\det' M^{\text{g.f.}}_{\mu\nu}\right)^{-\frac{1}{2}}\det D^2(\bar{A})}{\left(\det' M^{(0)\,\text{g.f.}}_{\mu\nu}\right)^{-\frac{1}{2}}\det \Box}$$

$$M_{\mu\nu}^{\text{g.f.}} = -D^2(\bar{A})\delta_{\mu\nu} - 2F_{\mu\nu}[\bar{A}], M_{\mu\nu}^{(0)\,\text{g.f.}} = -\,\Box\,\delta_{\mu\nu}.$$

$$\hat{a}_\mu^{(i)}(x) = \frac{\partial \bar{A}_\mu}{\partial \beta_i}(x,\beta) + D_\mu(\bar{A})\Lambda^{(i)}(x),$$

$$\hat{a}_\mu^{(i)}(x) \in \text{Ker}\left[M_{\mu\nu}^{\text{g.f.}}\right].$$

$$\bar{A}_\mu(x,\beta) = -\frac{i}{g}e^{i\theta_a T^a}\left[\bar{\sigma}_{\mu\nu}\frac{\rho^2(x-x_0)_\mu}{(x-x_0)^2[(x-x_0)^2+\rho^2]}\right]e^{-i\theta_a T^a},$$

$$\hat{a}_{\mu(\nu)} = \frac{\partial}{\partial x_0^\nu}\bar{A}_\mu + D_\mu(\bar{A})\bar{A}_\nu = F_{\mu\nu}[\bar{A}]$$

$$\left\|\hat{a}_{\mu(\nu)}\right\| = \frac{2\sqrt{2}\pi}{g}$$

$$\hat{a}_\mu^{(\text{dil})} = \frac{\partial \bar{A}_\mu}{\partial \rho} = -\frac{2i}{g}\bar{\sigma}_{\mu\nu}\frac{\rho y^\nu}{(y^2+\rho^2)^2}$$

$$\left\|\hat{a}_\mu^{(\text{dil})}\right\| = \frac{4\pi}{g}$$

$$\hat{a}_\mu^{(a)} = \frac{\partial \bar{A}_\mu}{\partial \theta_a} + D_\mu(\bar{A})\Lambda^{(a)} = D_\mu(\bar{A})\left[-\frac{i}{g}T^a + \Lambda^{(a)}\right] \equiv D_\mu(\bar{A})\phi^{(a)}$$

$$D^2(\bar{A})\phi^{(a)} = 0, \lim_{|x|\to\infty}\phi^{(a)} = -\frac{i}{g}T^{(a)}$$

$$\phi^{(a)} = \left(-\frac{i}{g}T^{(a)}\right)\frac{r^2}{r^2+\rho^2} \text{ with } r = |x|$$

$$\left\|\hat{a}_\mu^{(a)}\right\| = \frac{2\pi\rho}{g}$$

$$d\mu_{\text{B}} = \prod_i d\beta_i\frac{\|\hat{a}\|}{\sqrt{2\pi}} = \frac{1}{(2\pi)^4}\left(\frac{2\sqrt{2}\pi}{g}\right)^4\frac{4\pi}{g}\left(\frac{2\pi\rho}{g}\right)^3 d^4x_0 d\rho d^3\theta$$
$$= \frac{2^7\pi^4}{g^8}\rho^3 d^4x_0 d\rho d^3\theta.$$

$$\bar{A}_\mu = \frac{2i}{g}\frac{\bar{\eta}_{\mu\nu}^a x^\nu}{x^2(x^2+\rho^2)}\frac{\lambda_k}{2},$$

$$\hat{a}_\mu^r = D_\mu(\bar{A})\left[\frac{\lambda^r}{g}\left(\frac{x^2}{x^2+\rho^2}\right)^{\frac{1}{2}}\right].$$

$$S[\psi,\bar{\psi},A] = S_{\text{YM}}[A] + S_A[\psi,\bar{\psi}] = S_{\text{YM}}[A] + \int d^4x\bar{\psi}[iD_\mu(\mathbf{r})]\bar{\sigma}^\mu\psi$$

$$S_A[\Psi,\bar{\Psi}] = \int d^4x\bar{\Psi}[i\not{D}_L(\mathbf{r})]\Psi$$

$$\not{D}_L(\mathbf{r}) = \begin{pmatrix} 0 & \sigma^\mu\partial_\mu \\ \bar{\sigma}^\mu(\partial_\mu + A_\mu) & 0 \end{pmatrix}$$

$$\langle\mathcal{O}[\Psi,\bar{\Psi}]\rangle_A = \int [\mathcal{D}\Psi\mathcal{D}\bar{\Psi}]e^{-S_A[\Psi,\bar{\Psi}]}\mathcal{O}[\Psi,\bar{\Psi}]$$

$$i\not{D}_L(\mathbf{r})f^{(n)} = \lambda_n f^{(n)}, i\not{D}_R(\mathbf{r})g^{(n)} = \lambda_n^* g^{(n)}$$

$$\Psi = \sum_{i=1}^{m} b_i^{(0)} f_i^{(0)} + \sum_n b_n f^{(n)}$$
$$\bar{\Psi} = \sum_{j=1}^{\bar{m}} \bar{c}_j^{(0)} g_j^{(0)\dagger} + \sum_n \bar{c}_n g^{(n)\dagger}$$

$$S_A[\Psi,\bar{\Psi}] = \sum_{n\neq 0} \lambda_n \bar{c}_n b_n$$

$$\langle \mathcal{O} \rangle_A = \int \prod_{i=1}^{m} \left[db_i^{(0)}\right] \prod_{j=1}^{\bar{m}} \left[d\bar{c}_j^{(0)}\right] \prod_{n\neq 0} [db_n d\bar{c}_n] e^{-\sum_{n\neq 0} \lambda_n \bar{c}_n b_n} \mathcal{O}(b,\bar{c}).$$

$$n_L - n_R = 2d_{\mathbf{r}} K[\bar{A}].$$

$$n_L = 2d_{\mathbf{r}} K[\bar{A}], n_R = 0 \text{ when } F_{\mu\nu}[A] = \tilde{F}_{\mu\nu}[A]$$

$$\psi = a_0 f^{(0)} + \sum_{n\neq 0} a_n f^{(n)}$$
$$\chi = b_0 f^{(0)} + \sum_{n\neq 0} b_n f^{(n)}$$

$$\langle \mathcal{O} \rangle_A = \int [da_0 db_0] \prod_{n\neq 0} [da_n db_n d\bar{c}_n d\bar{d}_n] e^{-\sum_{n\neq 0} \lambda_n (\bar{c}_n a_n + \bar{d}_n b_n)}$$
$$\cdot \left(\sum_{n=0}^{\infty} a_n f^{(n)}\right)\left(\sum_{n=0}^{\infty} b_n f^{(n)}\right) = [\det'(i\not{D}_L(\mathbf{2}))]^2 f^{(0)}(x) f^{(0)}(y)$$

$$iD_\mu^{(\mathrm{f})}(\bar{A}) \bar{\sigma}^{\mu\dot{\alpha}\alpha} \psi_\alpha^{(0)} = 0$$

$$\psi_{\alpha,s}^{(0)}(x) = \frac{\varepsilon_{\alpha s}}{[(x-x_0)^2 \rho^2]^{3/2}}$$

$$\|\psi^{(0)}\| = \frac{\pi}{\rho}$$

$$\left[iD_\mu^{(\mathrm{adj})}(\bar{A})\right]_a {}^b \bar{\sigma}^{\mu\dot{\alpha}\alpha} \lambda_{\alpha b}^{(0)} = 0.$$

$$\lambda = \bar{\lambda} = 0, A_\mu = \bar{A}_\mu,$$

$$\delta_1 \lambda = \frac{\sqrt{\rho}}{2} \bar{F}_{\mu\nu} \sigma^{\mu\nu} \eta$$

$$\delta_2 \lambda = \frac{1}{2\sqrt{\rho}} \bar{F}_{\mu\nu} \sigma^{\mu\nu} \left[(x-x_0)_\kappa \sigma^\kappa \bar{\xi}\right]$$

$$\lambda_i^{(0)} = \frac{1}{2} \bar{F}_{\mu\nu} \sigma^{\mu\nu} \zeta_i(x),$$

$$\zeta_i^{(0)}(x) = \frac{1}{\sqrt{\rho}} \left[\rho \eta_i + (x-x_0)^\mu \sigma_\mu \bar{\xi}_{i-2}\right], i = 1,2,3,4$$

$$\eta_1 = \bar{\xi}_1 = \begin{pmatrix}1\\0\end{pmatrix}, \eta_2 = \bar{\xi}_2 = \begin{pmatrix}0\\1\end{pmatrix}, \eta_{3,4} = \bar{\xi}_{-1,-2} = 0$$

$$\left\|\lambda_j^{(0)}\right\| = \frac{4\sqrt{2}\pi\sqrt{\rho}}{g}, j = 1,2, \left\|\lambda_k^{(0)}\right\| = \frac{8\pi\sqrt{\rho}}{g}, k = 3,4.$$

$$|\det(i\not{D}_L(\mathbf{r}))|^2 = \det[(i\not{D}_L(\mathbf{r}) i\not{D}_R(\mathbf{r}))] = \det(i \not\supset (\mathbf{r}))\det(-\Box),$$

$$D^2(\bar{A})\varphi^{(n)} = -\lambda_n^2\varphi^{(n)}\ \lambda_n \neq 0$$

$$i\not{D}(\bar{A})\Psi^{(n)} = \mu_n\Psi^{(n)}$$
$$\left(-D^2(\bar{A})\delta_{\mu\nu} - 2F_{\mu\nu}[\bar{A}]\right)Q_\nu^{(n)} = \rho_n^2 Q_\mu^{(n)}$$

$$\mu_n = \lambda_n \ \leftrightarrow\ \Psi^{(n)}(x) = \begin{pmatrix}\xi^{(n)}\\ \bar{\chi}^{(n)}\end{pmatrix}$$

$$\xi^{(n)} = \frac{i}{\lambda_n}D_\mu(\bar{A})\varphi^{(n)}\sigma^\mu\bar{\epsilon}, \bar{\chi}^{(n)} = \bar{\epsilon}\varphi^{(n)}$$

$$\rho_n = \lambda_n \ \leftrightarrow\ Q_\mu^{(n)} = \overline{\eta\sigma}_\mu\xi^{(n)},$$

$$\left[\frac{\det\left(M_{\mu\nu}^{\text{g.f.}} + \mu^2\right)}{\det' M_{\mu\nu}^{\text{g.f.}}}\right]^{\frac{1}{2}}\frac{\det D^2(\bar{A})}{\det(D^2(\bar{A}) + \mu^2)}\frac{\det'\left(i\not{D}_L^{(\text{adj})}\right)}{\det\left(i\not{D}_L^{(\text{adj})} + i\mu\right)},$$

$$\mu^{n_B - \frac{1}{2}n_F},$$

$$Q^{ij} = \varphi^i\varphi^j - \frac{1}{6}\delta^{ij}\varphi_k\varphi^k, i, j, k = 1,2, \dots, 6,$$

$$\mathcal{W}_{(2)}^{ij} = \operatorname{tr}\left(W^iW^j - \frac{\delta^{ij}}{6}W_kW^k\right)$$

$$Q^{ij}(x) \sim \mathcal{W}_{(2)}^{ij}(\Upsilon)\Big|_{\theta=\bar{\theta}=0}$$

$$\left\langle Q^{i_1j_1}(x_1)Q^{i_2j_2}(x_2)Q^{i_3j_3}(x_3)Q^{i_4j_4}(x_4)\right\rangle.$$

$$\left\langle\varphi^{ia}(x)\varphi^{jb}(y)\right\rangle_{\text{free}} = \frac{1}{(2\pi)^2}\frac{\delta^{ij}\delta^{ab}}{(x-y)^2}.$$

$$\left\langle Q^{i_1j_1}(x_1)Q^{i_2j_2}(x_2)Q^{i_3j_3}(x_3)Q^{i_4j_4}(x_4)\right\rangle_{\text{free}}$$
$$= \frac{1}{(4\pi^2)^4}\left[N^4\frac{\delta^{i_1i_3}\delta^{j_1j_3}\delta^{i_2i_4}\delta^{j_2j_4}}{x_{13}^4x_{24}^4} + N^2\frac{\delta^{j_4i_1}\delta^{j_1i_3}\delta^{j_3i_2}\delta^{j_2i_4}}{x_{41}^2x_{13}^2x_{32}^2x_{24}^2} + \text{permutations}\right]$$

$$x_{ij} = x_i - x_j.$$

$$\left\langle Q^{i_1j_1}(x_1)Q^{i_2j_2}(x_2)Q^{i_3j_3}(x_3)Q^{i_4j_4}(x_4)\right\rangle_{K=1}$$

$$\Big\langle Q^{i_1j_1}(x_1)Q^{i_2j_2}(x_2)Q^{i_3j_3}(x_3)Q^{i_4j_4}(x_4)\left(\int d^4z_1\left[\frac{i}{4}gf^{abc}\bar{t}_{AB}^i\left(\lambda_a^{\alpha A}\lambda_{\alpha b}^B\varphi_{ic}\right)(z_1)\right]\right)$$
$$\dots\cdot\left(\int d^4z_8\left[\frac{i}{4}gf^{abc}\bar{t}_{AB}^i\left(\lambda_a^{\alpha A}\lambda_{\alpha b}^B\varphi_{ic}\right)(z_8)\right]\right)\Big\rangle_K$$

$$\left\langle\left(\int d^4z_1[g\Delta(x_1 - z_1)(\lambda\lambda)(z_1)]\right)\dots\left(\int d^4z_8[g\Delta(x_4 - z_8)(\lambda\lambda)(z_8)]\right)\right\rangle$$

$$\left[D^2(\bar{A})\varphi^i\right](x) = J^i(x),$$

$$Q^{S0} = q^S\varphi \quad Q^{S0^\dagger} = q^S\bar{\varphi} \quad Q^{ST} = q^Sq^T - \gimel$$
$$\mathcal{Q}_{(+)} = \varphi^2 \quad \mathcal{Q}_{(-)} = \bar{\varphi}^2 \quad \mathcal{Q}_{(0)} = \bar{\varphi}\varphi - \aleph$$

$$\langle\varphi^2(x_1)\varphi^2(x_2)\bar{\varphi}^2(x_3)\bar{\varphi}^2(x_4)\rangle_{\text{free}}$$
$$= \frac{1}{(4\pi^2)^4}\left(\frac{4N^4}{x_{13}^4x_{24}^4} + \frac{4N^4}{x_{14}^4x_{23}^4} + \frac{16N^2}{x_{41}^2x_{13}^2x_{32}^2x_{24}^2}\right)$$

$$\lambda^u_{(0)\alpha} = \frac{1}{2}\bar{F}_{\mu\nu}\sigma^{\mu\nu\beta}_{\alpha}\frac{1}{\sqrt{\rho_0}}\left(\rho_0\eta^u_\beta + (x-x_0)_\kappa\sigma^\kappa_{\beta\dot{\beta}}\bar{\xi}^{\dot{\beta}u}\right).$$

$$\zeta^u_{\pm\alpha}(\rho_0, x-x_0) = \frac{1}{\sqrt{\rho_0}}\left(\rho_0\eta^u_{\pm\alpha} + \sigma^\mu_{\alpha\dot{\alpha}}(x_\mu - x_{0\mu})\bar{\xi}^{\dot{\alpha}u}_\pm\right)$$

$$G_{Q^4}(x_p) = \langle g^2\varphi^2(x_1)g^2\varphi^2(x_2)g^2\bar{\varphi}^2(x_3)g^2\bar{\varphi}^2(x_4)\rangle_{K=1} = \frac{g^8}{2^{32}\pi^{10}}e^{-\frac{8\pi^2}{g^2}+i\theta}$$
$$\int \frac{d\rho_0 d^4x_0}{\rho_0^5}d^4\eta_+ d^4\bar{\xi}_+ d^4\eta_- d^4\bar{\xi}_- \varphi^2_{(0)}(x_1)\varphi^2_{(0)}(x_2)\bar{\varphi}^2_{(0)}(x_3)\bar{\varphi}^2_{(0)}(x_4)$$

$$\varphi(x) \to \varphi_{(0)}(x) = \frac{1}{2\sqrt{2}}\varepsilon_{uv}\zeta^u_+\sigma^{\mu\nu}\zeta^v_+\bar{F}_{\mu\nu}$$
$$\bar{\varphi}(x) \to \bar{\varphi}_{(0)}(x) = \frac{1}{2\sqrt{2}}\varepsilon_{\dot{u}\dot{v}}\zeta^{\dot{u}}_-\sigma^{\mu\nu}\zeta^{\dot{v}}_-\bar{F}_{\mu\nu},$$

$$G_{Q^4}(x_p) = \frac{3^4}{4\pi^{10}}g^8 e^{-\frac{8\pi^2}{g^2}+i\theta}\int \frac{d\rho_0 d^4x_0}{\rho_0^5}x^4_{12}x^4_{34}\prod_{p=1}^{4}\left[\frac{\rho_0}{\rho_0^2 + (x_p - x_0)^2}\right]^4.$$

$$G_{Q^4}(x_p) = \frac{3^4\Gamma(16)}{4\pi^{10}(\Gamma(4))^4}g^8 e^{-\frac{8\pi^2}{g^2}+i\theta}\int\prod_p \alpha_p^3 d\alpha_p\,\delta\left(1-\sum_q \alpha_q\right)$$
$$\cdot\int\frac{d\rho_0 d^4x_0}{\rho_0^5}\frac{x^4_{12}x^4_{34}\rho_0^{16}}{\left(\rho_0^2 + x_0^2 - 2x_0\cdot\sum_p \alpha_p x_p + \sum_p x_p^2\right)^{16}}$$

$$G_{Q^4}(x_p) = \frac{3^3\Gamma(11)}{2^7(\pi^3\Gamma(4))^4}g^8 e^{-\frac{8\pi^2}{g^2}+i\theta}$$
$$\cdot\int\prod_p \alpha_p^3 d\alpha_p\,\delta\left(1-\sum_q\alpha_q\right)\frac{x^4_{12}x^4_{34}}{\left(\sum_p \alpha_p\alpha_q x^2_{pq}\right)^8}$$

$$G_{Q^4}(x_p) = \frac{3^3\Gamma(11)}{2^7(\pi^3\Gamma(4))^4}g^8 e^{-\frac{8\pi^2}{g^2}+i\theta}x^4_{12}x^4_{34}\prod_{p<q}\frac{\partial}{\partial x^2_{pq}}B(x_{pq})$$

$$B(x_{pq}) = \int\prod_p d\alpha_p\,\delta\left(1-\sum_q\alpha_q\right)\frac{1}{\left(\sum_p \alpha_p\alpha_q x^2_{pq}\right)^2}$$

$$B(x_{pq}) = \frac{4}{\sqrt{\Delta(x_{pq})}}\left[\frac{1}{2}\log\left(\frac{u_+u_-}{(1-u_+)^2(1-u_-)^2}\right)\log\left(\frac{u_+}{u_-}\right)\right.$$
$$\left.-\mathrm{Li}_2(1-u_+) + \mathrm{Li}_2(1-u_-) - \mathrm{Li}_2\left(1-\frac{1}{u_-}\right) + \mathrm{Li}_2\left(1-\frac{1}{u_+}\right)\right]$$

$$\Delta = \det_{4\times 4}\left(\left(x^2_{pq}\right)\right) = X^2 + Y^2 + Z^2 - 2XY - 2YZ - 2ZX$$

$$u_\pm = \frac{Y + X - Z \pm \sqrt{\Delta}}{2Y}$$

$$\mathrm{Li}_2(z) + \mathrm{Li}_2(1-z) = \frac{\pi^2}{6} - \log(z)\log(1-z)$$
$$\mathrm{Li}_2(z) + \mathrm{Li}_2\left(\frac{1}{z}\right) = -\frac{\pi^2}{6} - \frac{1}{2}[\log(-z)]^2$$

$$\prod_{p<q}\frac{\partial}{\partial x^2_{pq}} = \mathcal{D}_Z\mathcal{D}_Y\mathcal{D}_X$$

$$\mathcal{D}_T = \left(\frac{\partial}{\partial T} + T\frac{\partial^2}{\partial T^2}\right), T = X, Y, Z$$

$$Q(x_{pq}) = \prod_{p<q} \frac{\partial}{\partial x_{pq}^2} B(x_{pq})$$

$$Q(x_{pq}) = \frac{2}{\Delta^5}(193X^6 - 114X^5Y - 1281X^4Y^2 + 2404X^3Y^3 - 1281X^2Y^4 +$$
$$-114XY^5 + 193Y^6 - 114X^5Z + 4734X^4YZ - 4620X^3Y^2Z - 4620X^2Y^3Z +$$
$$+4734XY^4Z - 114Y^5Z - 1281X^4Z^2 - 4620X^3YZ^2 + 15402X^2Y^2Z^2 +$$
$$-4620XY^3Z^2 - 1281Y^4Z^2 + 2404X^3Z^3 - 4620X^2YZ^3 - 4620XY^2Z^3 +$$
$$+2404Y^3Z^3 - 1281X^2Z^4 + 4734XYZ^4 - 1281Y^2Z^4 - 114XZ^5 - 114YZ^5 +$$
$$+193Z^6) + \frac{6}{\Delta^6}\{(-33X^8 - 96X^7Y + 714X^6Y^2 - 1008X^5Y^3 + 1008X^3Y^5 +$$
$$-714X^2Y^6 + 96XY^7 + 33Y^8 - 6X^7Z - 1770X^6YZ + 198X^5Y^2Z + 8298X^4Y^3Z +$$
$$-8298X^3Y^4Z - 198X^2Y^5Z + 1770XY^6Z + 6Y^7Z + 435X^6Z^2 + 3312X^5YZ^2 +$$
$$-13473X^4Y^2Z^2 + 13473X^2Y^4Z^2 - 3312XY^5Z^2 - 435Y^6Z^2 - 930X^5Z^3 +$$
$$+1914X^4YZ^3 + 15396X^3Y^2Z^3 - 15396X^2Y^3Z^3 - 1914XY^4Z^3 + 930Y^5Z^3 +$$
$$+645X^4Z^4 - 6384X^3YZ^4 + 6384XY^3Z^4 - 645Y^4Z^4 + 78X^3Z^5 + 3114X^2YZ^5 +$$
$$-3114XY^2Z^5 - 78Y^3Z^5 - 279X^2Z^6 + 279Y^2Z^6 + 90XZ^7 - 90YZ^7)\log\left[\frac{X}{Y}\right] +$$

$$-(11X^8 - 28X^7Y - 52X^6Y^2 + 284X^5Y^3 - 430X^4Y^4 + 284X^3Y^5 - 52X^2Y^6 +$$
$$-28XY^7 + 11Y^8 + 62X^7Z + 590X^6YZ - 2142X^5Y^2Z + 1490X^4Y^3Z +$$
$$+1490X^3Y^4Z - 2142X^2Y^5Z + 590XY^6Z + 62Y^7Z - 331X^6Z^2 + 972X^5YZ^2 +$$
$$+4491X^4Y^2Z^2 - 10264X^3Y^3Z^2 + 4491X^2Y^4Z^2 + 972XY^5Z^2 - 331Y^6Z^2 +$$
$$+362X^5Z^3 - 4894X^4YZ^3 + 5132X^3Y^2Z^3 + 5132X^2Y^3Z^3 - 4894XY^4Z^3 +$$
$$+362Y^5Z^3 + 215X^4Z^4 + 3404X^3YZ^4 - 8982X^2Y^2Z^4 + 3404XY^3Z^4 +$$
$$+215Y^4Z^4 - 646X^3Z^5 + 1170X^2YZ^5 + 1170XY^2Z^5 - 646Y^3Z^5 + 383X^2Z^6 +$$
$$-1180XYZ^6 + 383Y^2Z^6 - 34XZ^7 - 34YZ^7 - 22Z^8)\log\left[\frac{XY}{Z^2}\right] +$$
$$+\frac{36}{\Delta^{13/2}}\{(X^9 + 3X^8(Y+Z) - 6X^7(5Y^2 - 19YZ + 5Z^2) + (Y-Z)^6(Y^3 + 9Y^2Z +$$
$$+9YZ^2 + Z^3) + X^6(62Y^3 - 144Y^2Z - 144YZ^2 + 62Z^3) + 3X(Y-Z)^4 \cdot$$
$$\cdot (Y^4 + 42Y^3Z + 114Y^2Z^2 + 42YZ^3 + Z^4) - 6X^5(6Y^4 + 83Y^3Z - 252Y^2Z^2 +$$
$$+83YZ^3 + 6Z^4) - 6X^2(Y-Z)^2(5Y^5 + 34Y^4Z - 189Y^3Z^2 - 189Y^2Z^3 + 34YZ^4 +$$
$$+5Z^5) - 6X^4(6Y^5 - 175Y^4Z + 223Y^3Z^2 + 223Y^2Z^3 - 175YZ^4 + 6Z^5) +$$
$$+X^3(62Y^6 - 498Y^5Z - 1338Y^4Z^2 + 3948Y^3Z^3 - 1338Y^2Z^4 - 498YZ^5 + 62Z^6)\Big) \cdot$$
$$\cdot \left[\frac{1}{2}\log\left(\frac{XY}{Z^2}\right)\log\left(1 + \frac{(X+Y-Z)\sqrt{\Delta} + \Delta}{2XY}\right) - \mathrm{Li}_2\left(\frac{X - Y + Z - \sqrt{\Delta}}{2X}\right) +\right.$$
$$\left.-\mathrm{Li}_2\left(\frac{-X + Y + Z - \sqrt{\Delta}}{2Y}\right) + \mathrm{Li}_2\left(\frac{X - Y + Z + \sqrt{\Delta}}{2X}\right) + \mathrm{Li}_2\left(\frac{-X + Y + Z + \sqrt{\Delta}}{2Y}\right)\right]\Bigg\}.$$

$$G_{Q^4}(x_p) = \frac{3^3\Gamma(11)}{2^5(\pi^3\Gamma(4))^4} g^8 e^{-\frac{8\pi^2}{g^2} + i\theta} x_{12}^4 x_{34}^4 Q(x_{pq})$$

$$\Delta = \Delta(X, Y, Z) = X^2 + Y^2 + Z^2 - 2XY - 2XZ - 2YZ \leq 0.$$

$$r = \frac{X}{Y} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, s = \frac{Z}{Y} = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2},$$

$$\delta(r, s) = \frac{\Delta}{Y^2} = 1 + r^2 + s^2 - 2r - 2s - 2rs \leq 0$$

$$\sqrt{r} + \sqrt{s} = 1.$$

$$r = \eta^2, s = (1-\eta)^2$$

$$\tilde{Q} = \tilde{Q}(\eta) = 3\frac{\log(\eta^2)}{(-1+\eta)^7}(100\eta^6 + 429 + 2431\eta^2 - 2717\eta^3 + 1794\eta^4$$
$$-650\eta^5 - 1287\eta) - 3\frac{\log[(-1+\eta)^2]}{\eta^7}(100\eta^6 + 50\eta^5 + 44\eta^4 + 41\eta^3$$
$$+44\eta^2 + 50\eta + 100) - 2\frac{(\eta^2-\eta+1)^2}{\eta^6(-1+\eta)^6}(300 - 900\eta + 307\eta^2 + 886\eta^3$$
$$+307\eta^4 - 900\eta^5 + 300\eta^6)$$

$$G_{\hat{\Lambda}^{16}}(x_p) = \left\langle \prod_{p=1}^{16} g^2 \hat{\Lambda}_{\alpha_p}^{A_p}(x_p) \right\rangle_{K=1},$$

$$\lambda_{(0)\alpha}^{A} = \frac{1}{2}\bar{F}_{\mu\nu}\sigma_{\alpha}^{\mu\nu\beta}\frac{1}{\sqrt{\rho_0}}\left(\rho_0\eta_\beta^A + (x-x_0)_\mu \sigma_{\beta\dot{\beta}}^{\mu}\bar{\xi}^{\dot{\beta}A}\right).$$

$$G_{\hat{\Lambda}^{16}}(x_p) = \frac{2^6 3^{16}}{\pi^{10}} g^8 e^{-\frac{8\pi^2}{g^2}+i\theta} \int \frac{d^4x_0 d\rho_0}{\rho_0^5} \int d^8\eta d^8\bar{\xi}$$
$$\cdot \prod_{p=1}^{16}\left[\frac{\rho_0^4}{\left[\rho_0^2 + (x_p - x_0)^2\right]^4}\frac{1}{\sqrt{\rho_0}}\left(\rho_0\eta_{\alpha_p}^{A_p} + (x_p - x_0)_\mu \sigma_{\alpha_p\dot{\alpha}_p}^{\mu}\bar{\xi}^{\dot{\alpha}_p A_p}\right)\right]$$

$$G_{\mathcal{E}^8}(x_p) = \langle g^2\mathcal{E}^{A_1B_1}(x_1)\ldots g^2\mathcal{E}^{A_8B_8}(x_8)\rangle_{K=1}$$

$$\mathcal{E}^{AB} = \lambda^{\alpha a A}\lambda_{\alpha a}^{B} + g f_{abc} t_{ijk}^{(AB)+}\phi^{ia}\phi^{jb}\phi^{kc},$$

$$\lambda_{(0)}^{\alpha a A}\lambda_{(0)\alpha a}{}^{B} = \frac{3\cdot 2^6}{g^2}\frac{\rho_0^4}{(\rho_0^2 + (x-x_0)^2)^4}\zeta^{\alpha A}\zeta_\alpha{}^B$$

$$G_{\mathcal{E}^8}(x_p) = \frac{3^8 2^{14}}{\pi^{10}} g^8 e^{-\frac{8\pi^2}{g^2}+i\theta} \int \frac{d^4x_0 d\rho_0}{\rho_0^5} \int d^8\eta d^8\bar{\xi}$$
$$\cdot \prod_{p=1}^{8}\left[\frac{\rho_0^4}{\left(\rho_0^2 + (x_p - x_0)^2\right)^4}\frac{1}{\sqrt{\rho_0}}\left(\rho_0\eta_{\alpha_p}^{A_p} + (x_p - x_0)_\mu \sigma_{\alpha_p\dot{\alpha}_p}^{\mu}\bar{\xi}^{A_p\dot{\alpha}_p}\right)\right.$$
$$\left.\cdot\, \varepsilon^{\alpha_p\beta_p}\frac{1}{\sqrt{\rho_0}}\left(\rho_0\eta_{\beta_p}^{B_p} + (x_p - x_0)_\nu \sigma_{\beta_p\dot{\beta}_p}^{\nu}\bar{\xi}^{B_p\dot{\beta}_p}\right)\right]$$

$$S_{\text{inst}} = \frac{8\pi^2 K}{g^2} + S_{4\,\mathrm{F}}$$

$$S_{4\,\mathrm{F}} = \frac{\pi^2}{16\rho^2 g^2}\varepsilon_{ABCD}\left[\sum_{i=1}^{N-2}\bar{\mu}_i^A\mu_i^B\right]\left[\sum_{j=1}^{N-2}\bar{\mu}_j^C\mu_j^D\right].$$

$$G_{\hat{\Lambda}^{16}}^{N}(x_p) = C_N g^8 e^{-\frac{8\pi^2}{g^2}+i\theta} \int \frac{d^4x_0 d\rho_0}{\rho_0^5} \int d^8\eta d^8\bar{\xi} \int \prod_{A=1}^{4}\prod_{i=1}^{N-2} d\mu_i^A d\bar{\mu}_i^A e^{-S_{4\,\mathrm{F}}}$$
$$\cdot \prod_{p=1}^{16}\left[\frac{\rho_0^4}{\left[\rho_0^2 + (x_p - x_0)^2\right]^4}\frac{1}{\sqrt{\rho_0}}\left(\rho_0\eta_{\alpha_p}^{A_p} + (x_p - x_0)_\mu \sigma_{\alpha_p\dot{\alpha}_p}^{\mu}\bar{\xi}^{\dot{\alpha}_p A_p}\right)\right]$$

$$I_N = \int \prod_{A=1}^{4}\prod_{i=1}^{N-2} d\mu_i^A d\bar{\mu}_i^A e^{-S_{4\,\mathrm{F}}}$$

$$G_{\hat{\Lambda}^{16}}^{N}(x_p) \sim \sqrt{N} G_{\hat{\Lambda}^{16}}(x_p)$$

$$G_n^{(K)}(x_p) = g^8\sqrt{N}K^{n-\frac{7}{2}}e^{-\frac{8\pi^2 K}{g^2}}\sum_{d|K}\frac{1}{d^2}F_n(x_p),$$

$$S_{\rm P} = \frac{1}{4\pi\alpha'}\int_\Sigma d\sigma d\tau\sqrt{\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu$$

$$S = \frac{1}{4\pi\alpha'}\int_\Sigma d\sigma d\tau\{\sqrt{\gamma}\gamma^{ab}(\partial_a X^\mu\partial_b X_\mu + i\bar{\Psi}^\mu\rho_a\partial_b\Psi_\mu) + \\ +\bar{\chi}_a\rho^b\rho^a\left(X_\mu\partial_b X^\mu + \frac{1}{2}\bar{\Psi}^\mu\Psi_\mu\chi_b\right)\}$$

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma[\partial_+X^\mu\partial_-X_\mu - i\psi_L\cdot\partial_+\psi_L - i\psi_R\cdot\partial_-\psi_R]$$

$$\partial X_L^\mu = \sum_n \alpha_{-n}^\mu e^{in(\tau+\sigma)}$$
$$\psi_L^\mu = \sum_n \psi_{-n}^\mu e^{in(\tau+\sigma)}$$

$$\alpha_{-n_1}^{L\mu_1}\ldots\psi_{-n_k}^{L\mu_k}\ldots|P_L,a\rangle\otimes\alpha_{-m_1}^{R\nu_1}\ldots\psi_{-m_h}^{R\nu_h}\ldots|P_R,b\rangle,$$

$$NS\otimes NS \leftrightarrow \mathbf{v}\otimes\mathbf{v}$$

$$R\otimes R \leftrightarrow \mathbf{s}\otimes\mathbf{s},$$

$$NS\otimes NS \leftrightarrow (g_{\mu\nu}, B_{\mu\nu}, \phi)$$
$$R\otimes R \leftrightarrow (\chi = C^{(0)}, B'_{\mu\nu}, C^{(4)})$$

$$\tau = \tau_1 + i\tau_2 = \chi + ie^{-\phi},$$

$$S_{\text{supercurvature}} = \frac{1}{2\kappa_0^2}\int d^{10}X\sqrt{g}\left\{R - \frac{1}{2\tau_2^2}\partial_\Lambda\tau\partial^\Lambda\bar{\tau} + \left(F^{(5)}\right)^2 - \\ -\frac{1}{12\tau_2}[\tau H_1 + H_2]_{\Lambda\Gamma\Pi}[\bar{\tau}H_1 + H_2]^{\Lambda\Gamma\Pi}\right\} + \cdots,$$

$$g_{\mu\nu}^{(s)} = e^{\phi/2}g_{\mu\nu}^{(E)}$$

$$(\alpha')^{-4}\int d^{10}X\sqrt{g}[e^{-2\phi}R + \kappa(\alpha')^3 f_4(\tau,\bar{\tau})e^{-\phi/2}\mathcal{R}^4]$$

$$\mathcal{R}^4 \equiv \int d^{16}\Theta(R_{\Theta^4})^4$$

$$R_{\Theta^4} = \bar{\Theta}\Gamma^{\Lambda_1\Lambda_2\Lambda}\Theta\bar{\Theta}\Gamma^{\Lambda_3\Lambda_4}{}_\Lambda\Theta R_{\Lambda_1\Lambda_2\Lambda_3\Lambda_4}$$

$$(\alpha')^{-1}\int d^{10}X\sqrt{g}e^{-\phi/2}f_{16}(\tau,\bar{\tau})\Lambda^{16} + \text{c.c.}$$

$$\int_{V_{p+1}} C^{(p+1)},$$

$$Q_e = \int_{S_\infty^{d-p-2}} * dC^{(p+1)}$$

$$Q_m = \int_{S_\infty^{p+2}} dC^{(p+1)}$$

$$ds^2 = e^{2A(r)}d\vec{x}^2 + e^{2B(r)}d\vec{y}^2$$
$$\phi = \phi(r)$$
$$C_{01\ldots p}^{(p+1)} = -e^{C(r)},$$

$$H_p(r) = 1 + \frac{a_p}{r^{7-p}}$$

$$\begin{aligned} ds^2 &= [H_p(r)]^{\frac{p-7}{8}} d\vec{x}^2 + [H_p(r)]^{\frac{p+1}{8}} d\vec{y}^2 \\ C^{(p+1)}_{01\ldots p}(r) &= [H_p(r)]^{-1} - 1 \\ e^{2\phi(r)} &= [H_p(r)]^{\frac{3-p}{2}} \end{aligned}$$

$$ds^2 = [H_p(r)]^{-\frac{1}{2}} d\vec{x}^2 + [H_p(r)]^{\frac{1}{2}} d\vec{y}^2$$

$$ds^2 = \left[1 + \frac{L^4}{r^4}\right]^{-\frac{1}{2}} d\vec{x}^2 + \left[1 + \frac{L^4}{r^4}\right]^{\frac{1}{2}} d\vec{y}^2$$

$$g_s = e^{\phi_0}$$

$$L^4 = 4\pi g_s N \alpha'^2$$

$$ds^2 = \frac{L^2}{\rho^2}(dx \cdot dx + d\rho^2) + d\omega_5^2$$

$$g_s = \frac{g_{\text{YM}}^2}{4\pi}, \chi_0 = \frac{\theta_{\text{YM}}}{2\pi},$$

$$\begin{aligned} F_{MNPQR} &= \frac{1}{L}\varepsilon_{MNPQR} \qquad R_{MNPQ} = -\frac{1}{L^2}(g_{MP}g_{NQ} - g_{MQ}g_{NP}) \\ F_{mnpqr} &= \frac{1}{L}\varepsilon_{mnpqr} \qquad R_{mnpq} = +\frac{1}{L^2}(g_{mp}g_{nq} - g_{mq}g_{np}) \end{aligned}$$

$$R_{MN} = -\frac{4}{L^2} g_{MN} \; R_{mn} = +\frac{4}{L^2} g_{mn}$$

$$\begin{aligned} C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \frac{1}{d-2}[R_{\mu\nu} g_{\nu\sigma} + (3 \text{ terms })] + \\ + \frac{1}{(d-1)(d-2)} R(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}), \end{aligned}$$

$$\begin{aligned} D_\Lambda \epsilon - \frac{1}{2L}(\sigma_1 \otimes \mathbb{I} \otimes \mathbb{I})\Gamma_\Lambda \epsilon = 0 \\ \epsilon_\pm = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \zeta_\pm \otimes \kappa_\pm \end{aligned}$$

$$\begin{aligned} D_M \zeta_\pm \mp \frac{1}{2L}\gamma_M \zeta_\pm = 0 \\ D_m \kappa_\pm \mp i\frac{1}{2L}\gamma_m \kappa_\pm = 0 \end{aligned}$$

$$Z_{\text{supercurvature}}[J] = \int [DA] \exp\left(-S_{\text{YM}}[A] + \mathcal{O}[A]J\right)$$

$$Z_{\text{supercurvature}}[J] = \int [D\Phi]_J \exp\left(-S_{\text{IIB}}[\Phi]\right)$$

$$(mL)^2 = \Delta(\Delta - 4)$$

$$\delta g_{MN} = \frac{16}{15} f \delta_{MN} \; \delta g_{mn} = f \delta_{mn}$$

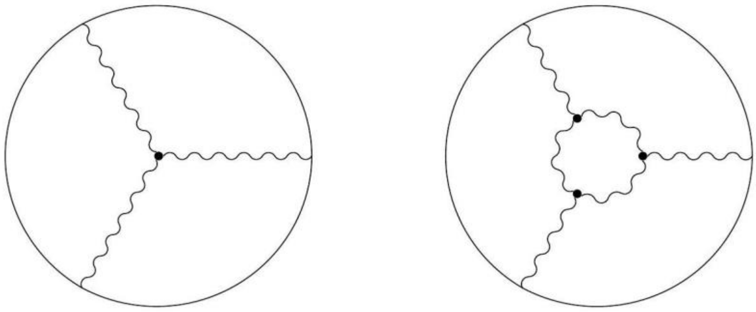
$$A_{MNPQ} = -\frac{L}{4}\varepsilon_{MNPQR}\nabla^R f \; A_{mnpq} = -\frac{L}{4}\varepsilon_{mnpqr}\nabla^r f.$$

$$f(z, \omega) = Q_{ij}(z) Y^{ij}_{\ell=2}(\omega)$$

$$W^{(i_1 \dots i_\ell)}\big|_{\theta=0} = \mathrm{tr}\left(\phi^{(i_1} \dots \phi^{i_\ell)}\right) - ב,$$

$$K_\Delta(x^\mu, \rho; x'^\mu, 0) = c_\Delta \frac{\rho^\Delta}{(\rho^2 + (x - x')^2)^\Delta}$$

$$\Phi_m(z; J) = \int d^4x' K_\Delta(x, \rho; x', 0) J_\Delta(x')$$



$$S = \frac{1}{2}\int d^5z\sqrt{g}[g^{MN}(\partial_M\Phi)(\partial_N\Phi) + m^2\Phi^2]$$

$$\frac{1}{\sqrt{g}}\partial_M\left(\sqrt{g}g^{MN}\partial_N\Phi\right) - m^2\Phi = 0$$

$$\langle\mathcal{O}(x)\mathcal{O}(y)\rangle = -\int \frac{d^4x'd\rho}{\rho^5}[\partial_M K_\Delta(x', \rho; x, 0)\rho^2\partial^M K_\Delta(x', \rho; y, 0)$$
$$+m^2 K_\Delta(x', \rho; x, 0) K_\Delta(x', \rho; y, 0)]$$

$$\langle\mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{\Gamma(\Delta + 1)}{\pi^2\Gamma(\Delta - 2)}\frac{1}{(x - y)^{2\Delta}}$$

$$\Delta = 2 \pm \sqrt{4 + (mL)^2 + p(p - 4)}$$

$$S = \int d^5z\sqrt{g}[\bar{\Psi}(\not{D} - m)\Psi]$$

$$\not{D}\Psi = e_{\hat{L}}^M \gamma^{\hat{L}}\left(\partial_M + \frac{1}{4}\omega_M^{\hat{M}\hat{N}}\gamma_{\hat{M}\hat{N}}\right)\Psi = \left(\rho\gamma^{\hat{5}}\partial_5 + \rho\gamma^{\hat{\mu}}\partial_\mu - 2\gamma^{\hat{5}}\right)\Psi$$

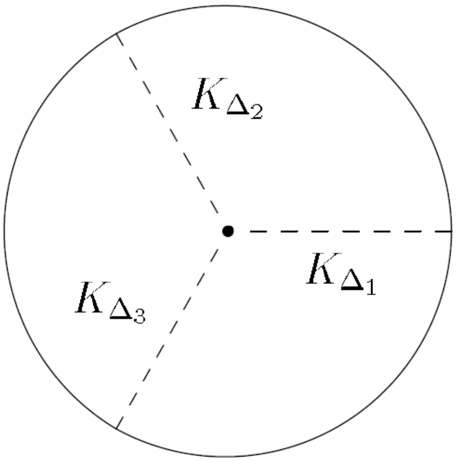
$$S_b = \lim_{\epsilon\to 0}\int_{M^\epsilon} d^4x\sqrt{g^\epsilon}\bar{\Psi}\Psi$$

$$\Delta = 2 - mL$$

$$\mathcal{L}^{(A)} = \Phi_1\Phi_2\Phi_3$$
$$\mathcal{L}^{(B)} = g^{MN}\Phi_1(\partial_M\Phi_2)(\partial_N\Phi_3)$$

$$\langle\mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle^{(A)} = -\int \frac{d^4x'd\rho}{\rho^5}K_{\Delta_1}(x', \rho; x, 0)K_{\Delta_2}(x', \rho; y, 0)$$
$$\cdot K_{\Delta_3}(x', \rho; z, 0)$$

$$\langle\mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle^{(B)} = -\int \frac{d^4x'd\rho}{\rho^5}K_{\Delta_1}(x', \rho; x, 0)\partial_M K_{\Delta_2}(x', \rho; y, 0)$$
$$\cdot \rho^2\partial^M K_{\Delta_3}(x', \rho; z, 0)$$



$$\langle \mathcal{J}_\mu^a(x)\mathcal{J}_\nu^b(y)\rangle = c\frac{\delta^{ab}}{(2\pi)^4}\left(\delta_{\mu\nu}\,\square - \partial_\mu\partial_\nu\right)\frac{1}{(x-y)^4},$$

$$c = \frac{1}{2}(N^2-1).$$

$$\begin{aligned}\langle \mathcal{J}_\mu^a(x)\mathcal{J}_\nu^b(y)\mathcal{J}_\lambda^c(z)\rangle &= \langle \mathcal{J}_\mu^a(x)\mathcal{J}_\nu^b(y)\mathcal{J}_\lambda^c(z)\rangle_+ + \langle \mathcal{J}_\mu^a(x)\mathcal{J}_\nu^b(y)\mathcal{J}_\lambda^c(z)\rangle_- \\ &= f^{abc}\left[k_1^{(+)}D_{\mu\nu\lambda}(x,y,z) + k_2^{(+)}C_{\mu\nu\lambda}(x,y,z)\right] + k^{(-)}d^{abc}M_{\mu\nu\lambda}(x,y,z)\end{aligned}$$

$$\mathcal{O}_\ell^I = g_{\mathrm{YM}}^\ell t_{i_1\ldots i_\ell}^I \operatorname{tr}\left(\varphi^{i_1}(x)\ldots\varphi^{i_\ell}(x)\right)$$

$$\langle \varphi_a^i \varphi_b^j\rangle = \frac{\delta^{ij}\delta_{ab}}{(2\pi)^2}\frac{1}{(x-y)^2}$$

$$\Delta \geq q = \ell$$

$$\mathcal{W}_{(\ell)} = \operatorname{tr}\left(W^{(i_1}\ldots W^{i_\ell)}\right) - \mathfrak{T}$$

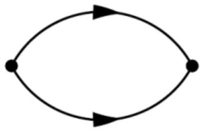
$$\Sigma = \Phi^I e^V \Phi_I^\dagger$$

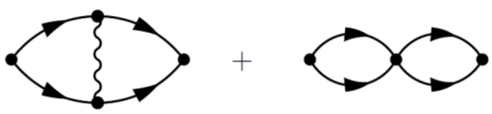
$$\langle \mathcal{O}_\ell^{I_1}(x)\mathcal{O}_\ell^{I_2}(y)\rangle = \hat{g}^{2\ell}\frac{\ell}{(2\pi)^{2\ell}}\delta^{I_1I_2}\frac{1}{(x-y)^{2\ell}}$$

$$\left\langle \mathcal{O}_{\ell_1}^{I_1}(x)\mathcal{O}_{\ell_2}^{I_2}(y)\mathcal{O}_{\ell_3}^{I_3}(z)\right\rangle = \frac{\hat{g}^\Sigma}{N}\frac{\ell_1\ell_2\ell_3}{(2\pi)^\Sigma}t^{I_1I_2I_3}\frac{1}{(x-y)^{2\alpha_3}(y-z)^{2\alpha_1}(x-z)^{2\alpha_2}},$$

$$\mathcal{O}_\ell^I \longrightarrow \tilde{\mathcal{O}}_\ell^I = \frac{(2\pi)^\ell}{\hat{g}^\ell\sqrt{\ell}}\mathcal{O}_\ell^I,$$

$$\begin{aligned}\operatorname{tr}\left(X^\ell\right) &= \operatorname{tr}\left(\varphi^{I_1}\ldots\varphi^{I_\ell}\right)\\ \operatorname{tr}\left(X^{\dagger\ell}\right) &= \operatorname{tr}\left(\varphi_{I_1}^\dagger\ldots\varphi_{I_\ell}^\dagger\right)\end{aligned}$$

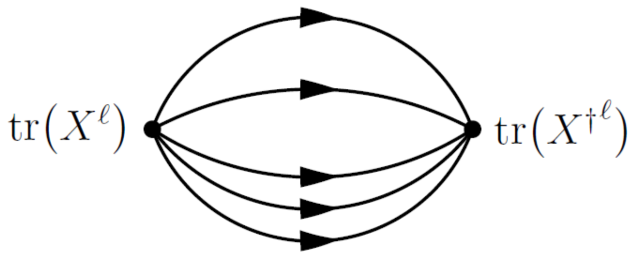
$\langle \operatorname{tr}(X^2)\operatorname{tr}(X^{\dagger 2})\rangle =$ $\operatorname{tr}(X^2)$  $\operatorname{tr}(X^{\dagger 2})$

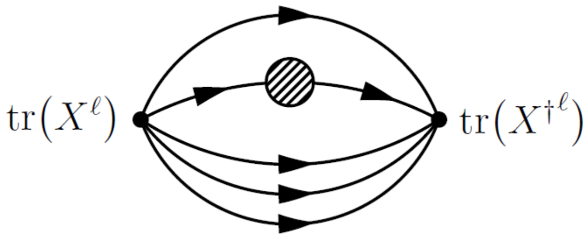


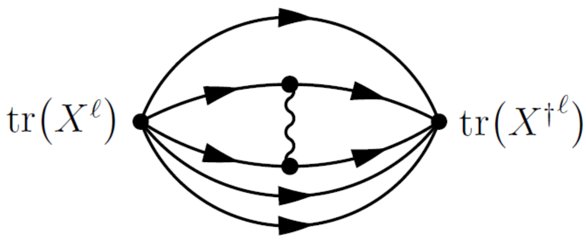
$= c\dfrac{B(x,y)}{(x-y)^4}$

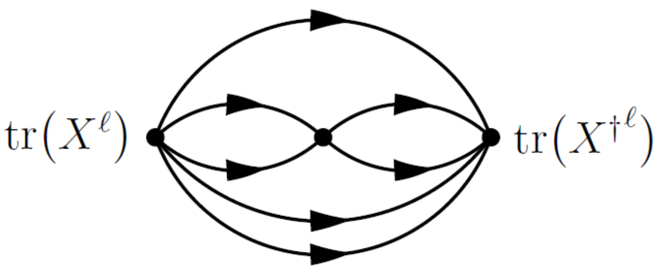
$$= 2c\frac{A(x,y)}{(x-y)^4}$$

$$B(x,y) + 2A(x,y) = 0.$$

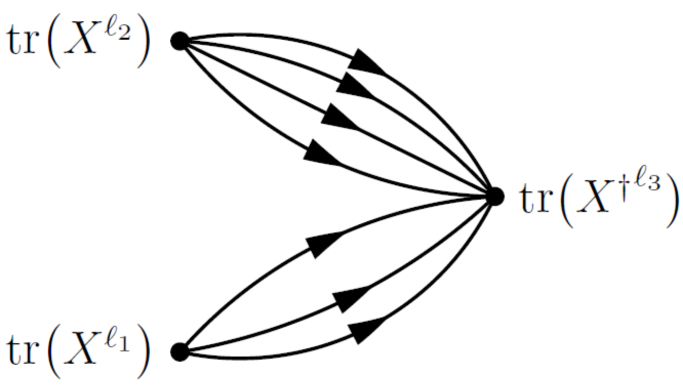




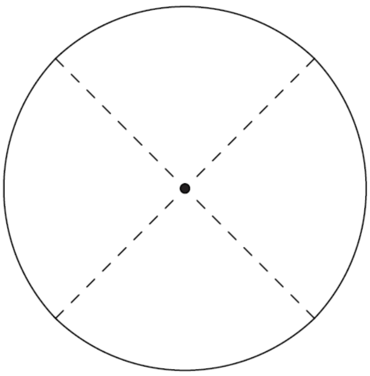


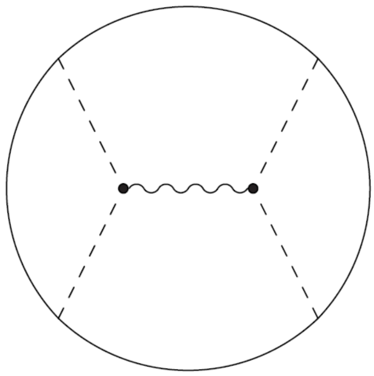


$$\left\langle \mathrm{tr}\left(X^k X^{\dagger k}\right) \right\| \sim kN \frac{[B(x,y) + 2A(x,y)]}{(x-y)^{2k}} \right\rangle,$$



$$r = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, s = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}.$$





$$A(x_1, x_2, x_3, x_4) = \int \frac{d^4x' d\rho'}{\rho'^5} \frac{d^4x'' d\rho''}{\rho''^5} [K(x', \rho'; x_1, 0)$$
$$\cdot K(x', \rho'; x_2, 0) G(x', \rho'; x'', \rho'') K(x'', \rho''; x_3, 0) K(x'', \rho''; x_4, 0)]$$

$$A_{\phi\phi\phi\phi}(x_1, x_2, x_3, x_4)$$
$$A_{\phi\chi\phi\chi}(x_1, x_2, x_3, x_4)$$
$$A_{\chi\chi\chi\chi}(x_1, x_2, x_3, x_4)$$

$$\langle \mathcal{G}(x_1)\mathcal{G}(x_2)\mathcal{G}(x_3)\mathcal{G}(x_4) \rangle$$
$$\langle \mathcal{G}(x_1)\mathcal{C}(x_2)\mathcal{G}(x_3)\mathcal{C}(x_4) \rangle$$
$$\langle \mathcal{C}(x_1)\mathcal{C}(x_2)\mathcal{C}(x_3)\mathcal{C}(x_4) \rangle$$

$$S = \frac{1}{2\kappa_0^2} \int d^5z \sqrt{g} \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2} e^{2\phi} (\partial\chi)^2 \right]$$

$$\Delta_z G(z; z') = \delta(z - z')$$

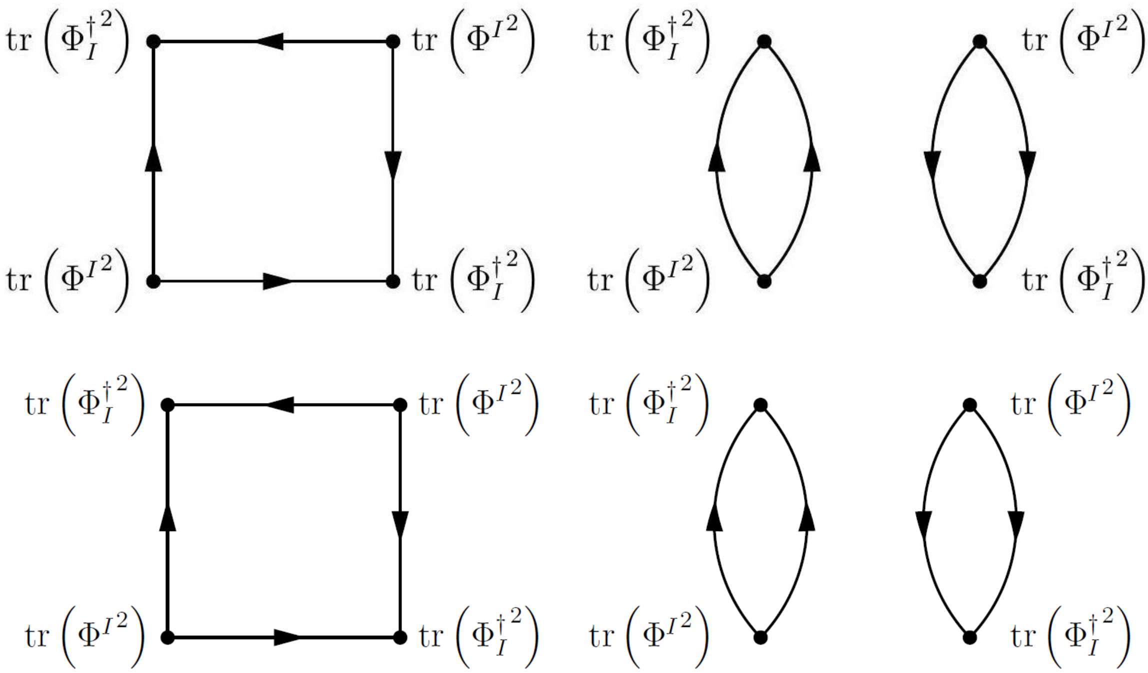
$$\int \frac{d^4x d\rho}{\rho^5} K_{\Delta_1} K_{\Delta_2} \dots K_{\Delta_n}$$

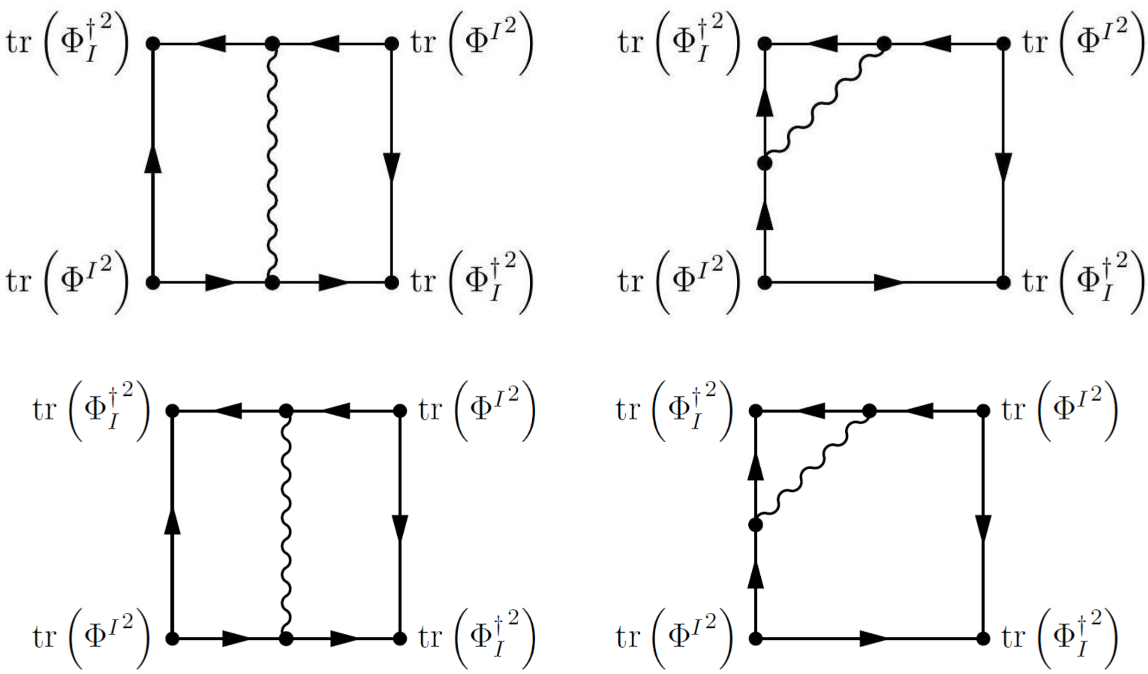
$$\mathrm{tr}(\Phi^{I^\ell}), \mathrm{tr}(\Phi_I^\dagger), I = 1,2,3$$

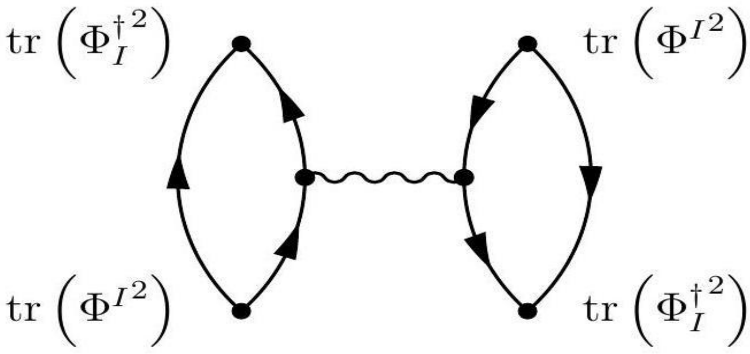
$$\mathbf{20}_{\mathbb{R}} \supset \overline{\mathbf{6}}_{-\frac{4}{3}} + \mathbf{6}_{\frac{4}{3}} + \mathbf{8}_0$$

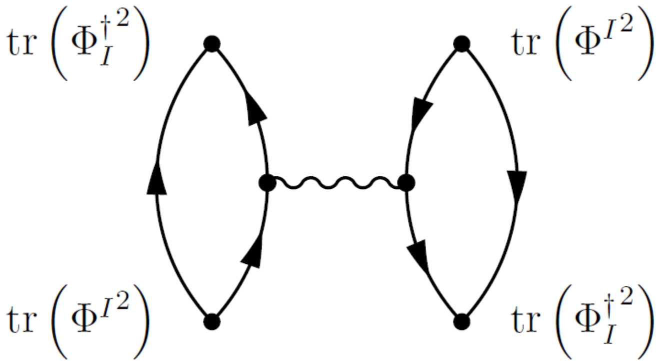
$$\mathcal{O}_{\mathbf{6}}^{IJ} = \mathrm{tr}(\Phi^I \Phi^J) \in \mathbf{6}, \mathcal{O}_{\overline{\mathbf{6}}IJ} = \mathrm{tr}(\Phi_I^\dagger \Phi_J^\dagger) \in \overline{\mathbf{6}}$$
$$\mathcal{O}_{\mathbf{8}J}^{I} = \mathrm{tr}\left(\Phi^I e^V \Phi_J^\dagger - \frac{\delta^I{}_J}{3} \Phi^K e^V \Phi_K^\dagger \right) \in \mathbf{8}$$

$$G_I^{(a)} = \left\langle \mathrm{tr}\left(\Phi_I^{\dagger 2}\right)\mathrm{tr}(\Phi^{I2})\mathrm{tr}\left(\Phi_I^{\dagger 2}\right)\mathrm{tr}(\Phi^{I2})\right\rangle,$$

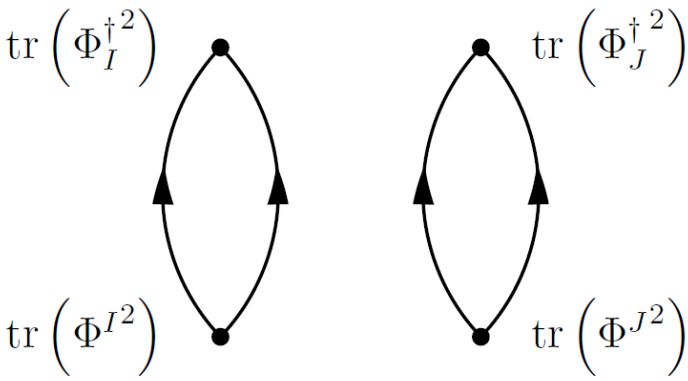


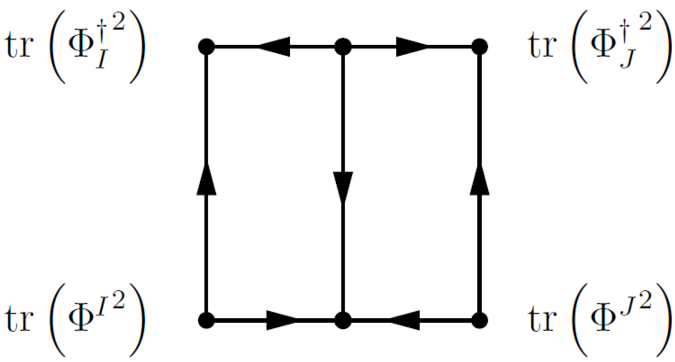






$$G_{IJ}^{(b)} = \left\langle \mathrm{tr}\left(\Phi_I^{\dagger 2}\right) \mathrm{tr}\left(\Phi^{I^2}\right) \mathrm{tr}\left(\Phi_J^{\dagger 2}\right) \mathrm{tr}\left(\Phi^{J^2}\right) \right\rangle,$$





$$S=\frac{1}{4}\int \left(d\phi \pm e^{-\phi} * dC^{(8)}\right) \wedge * \left(d\phi \pm e^{-\phi} * dC^{(8)}\right) \pm \frac{1}{2}\int e^{-\phi} d\phi \wedge dC^{(8)}$$
$$=\frac{1}{4}\int \left(d\phi \pm e^{-\phi} * dC^{(8)}\right) \wedge * \left(d\phi \pm e^{-\phi} * dC^{(8)}\right) \mp \frac{1}{2}\int_{\partial M} e^{-\phi} dC^{(8)}$$

$$S \geq \frac{1}{2}\left|\int_{\partial M} e^{-\phi} dC^{(8)}\right|$$

$$\partial_\Sigma \chi = \pm i \partial_\Sigma e^{-\phi}$$

$$\chi = \tilde{\chi} + i f(r),$$

$$e^{-\phi} g^{\Lambda\Sigma} \nabla_\Lambda \nabla_\Sigma e^{\phi} = 0$$

$$e^{\hat{\phi}^{(10)}} = g_s + \frac{3K\alpha'^4}{\pi^4 |X - X_0|^8}$$

$$\chi = \tilde{\chi} + i\left(A - \frac{1}{g_s} + e^{-\hat{\phi}}\right)$$

$$S_K = \frac{2\pi |K|}{g_s}.$$

$$f_{16}(\tau, \bar{\tau}) = (\tau_2 \mathcal{D})^{12} f_4(\tau, \bar{\tau}),$$

$$e^{-\phi/2} f_n = a_n \zeta(3) e^{-2\phi} + b_n +$$
$$+ \sum_{K=1}^{\infty} \mu_K \left(K e^{-\phi}\right)^{n-7/2} e^{2\pi i K \tau} \left(1 + \sum_{k=1}^{\infty} c_{k,n}^{K} \left(K e^{-\phi}\right)^{-k}\right).$$

$$\mathcal{Z}_K = \mu_K \left(K e^{-\phi}\right)^{-7/2} e^{2\pi i K \tau}$$

$$d\Omega_K^{(s)} = (\alpha')^{-1} d^{10}X d^{16}\Theta \mathcal{Z}_K$$

$$K_{7/2}^{\mathrm{F}}(\rho_0, x_0; x) = K_4(\rho_0, x_0; x) \frac{1}{\sqrt{\rho_0}} \left(\rho_0 \gamma_{\hat{5}} + (x_0 - x)^\mu \gamma_{\hat{\mu}}\right).$$

$$\Lambda_J(x_0, \rho_0) = \int d^4x K_{7/2}^{\mathrm{F}}(\rho_0, x_0; x) J_\Lambda(x)$$

$$S_\Lambda[J] = e^{-2\pi\left(\frac{1}{g_s} + i\chi\right)} g_s^{-12} V_{S^5} \int \frac{d^4x_0 d\rho_0}{\rho_0^5}$$
$$t_{16} \prod_{p=1}^{16} \left[K_4(\rho_0, x_0; x_p) \frac{1}{\sqrt{\rho_0}} \left(\rho_0 \gamma^{\hat{5}} + \left(x_0 - x_p\right)^\mu \gamma_{\hat{\mu}}\right) J_\Lambda(x_p)\right],$$

$$\frac{1}{\alpha'}\left(\frac{1}{g_s}\right)^{\frac{25}{2}} \sim \left(\frac{1}{g_s}\right)^{12},$$

$$g^{\Lambda\Sigma} \nabla_\Lambda \nabla_\Sigma e^{\phi} = 0$$

$$e^{\hat{\phi}} = g_s + \frac{\rho_0^4 \rho^4}{L^8}\left(e^{\hat{\phi}^{(10)}} - g_s\right)$$

$$\rho^{-4}\left(e^{\hat{\phi}} - g_s\right) = \frac{3K(\alpha')^4}{L^8 \pi^4} \frac{\rho_0^4}{((x - x_0)^2 + \rho_0^2)^4}$$

$$\hat{\chi} = \tilde{\chi} + i f(x, y),$$

$$f = A - \frac{1}{g_s} + e^{-\hat{\phi}}$$

$$S_K = -\frac{L^{10}}{(\alpha')^4}\int \frac{d\rho d^4 x d^5\omega}{\rho^5} g^{\Lambda\Sigma}\nabla_\Lambda\left(e^{2\hat{\phi}} f \partial_\Sigma f\right)$$

$$S_K = \frac{2\pi|K|}{g_s}$$

$$\mathcal{D}_M\zeta \equiv \left(D_M - \frac{i}{2}Q_M\right)\zeta = \frac{1}{2L}\gamma_M\zeta$$

$$Q_M = \frac{i}{2}e^{-\hat{\phi}}\partial_M e^{\hat{\phi}}$$

$$\zeta_\pm = e^{-\frac{\hat{\phi}}{4}}\frac{z_M\gamma^{\hat{M}}}{\sqrt{\rho_0}}\zeta_\pm^{(0)}$$

$$\Lambda_{(0)} = \delta\Lambda = (\gamma^M \hat{P}_M)\zeta_-$$

$$\hat{P}_M = e^{-\hat{\phi}}\partial_M e^{\hat{\phi}}$$

$$\gamma^M\mathcal{D}_M\Lambda_{(0)} = -\frac{3}{2L}\Lambda_{(0)}$$

$$P_M \sim \frac{1}{g_s}\partial_M e^{\hat{\phi}}$$

$$\Lambda_{(0)} \sim \frac{4}{g_s}\left(e^{\hat{\phi}} - g_s\right)\zeta_-$$

$$\left\langle \prod_{p=1}^{16}\Lambda_{\alpha_p}^{A_p}(x_p, 0)\right\rangle_J = g_s^{-12} e^{-2\pi K\left(\frac{1}{g_s}+iC^{(0)}\right)} V_{S^5}\int \frac{d^4x_0 d\rho_0}{\rho_0^5}\int d^{16}\zeta_-^{(0)}$$
$$\cdot\prod_{p=1}^{16}\left[K_4(x_0,\rho_0;x_p)\frac{1}{\sqrt{\rho_0}}\left(\rho_0\eta_{\alpha_p}^{A_p} + (x_p - x_0)_\mu \sigma^\mu_{\alpha_p\dot{\alpha}_p}\bar{\xi}^{\dot{\alpha}_p A_p}\right)J_\Lambda(x_p)\right],$$

$$\zeta(\rho_0, x - x_0) = \left(\frac{1-\gamma^5}{2}\right)\frac{z^M\gamma_M\zeta^{(0)}}{\sqrt{\rho_0}}$$

$$\zeta^{(0)} = \begin{pmatrix}\eta_\alpha \\ \bar{\xi}^{\dot{\alpha}}\end{pmatrix}, z^M = (x^\mu - x_0^\mu, \rho_0)$$

$$\mathcal{O}_{(q_1)}(x)\mathcal{O}_{(q_2)}(y) \text{ for } |x-y| \to 0,$$

$$\mathcal{O}_{(q_1)}(x)\mathcal{O}_{(q_2)}(y) \sim c\mathcal{O}^\Delta_{(q_1+q_2)}\left(\frac{x+y}{2}\right) + |x-y| \to 0,$$

$$\mathcal{O}_{(q_1)}(x)\mathcal{O}_{(q_2)}(y) \sim \mathcal{V}_q^\Delta\left(\frac{x+y}{2}\right) + \cdots,$$

$$\langle \mathcal{O}_{(q_1)}(x)\mathcal{O}_{(q_2)}(y)\rangle \sim \frac{1}{(x-y)^{2\Delta}}[1 - \gamma\log(\mu^2(x-y)^2) +$$
$$+\frac{1}{2}\gamma^2\left(\log(\mu^2(x-y)^2)\right)^2 + \cdots]$$

$$\eta_{\mu\nu} = \begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}.$$

$$\sigma^\mu = (-\mathbb{1}_2, \sigma^i), \bar{\sigma}^\mu = (-\mathbb{1}_2, -\sigma^i), i = 1,2,3,$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\psi_\alpha, \bar{\psi}^{\dot{\alpha}}, \text{ with } \alpha, \dot{\alpha} = 1{,}2.$$

$$\varepsilon^{\alpha\beta}(\varepsilon^{12} = -\varepsilon^{21} = -\varepsilon_{12} = \varepsilon_{21} = 1)$$

$$\varepsilon^{\dot{\alpha}\dot{\beta}}(\varepsilon^{\dot{1}\dot{2}} = -\varepsilon^{\dot{2}\dot{1}} = -\varepsilon_{\dot{1}\dot{2}} = \varepsilon_{\dot{2}\dot{1}} = 1)$$

$$\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta$$
$$\bar{\psi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}}, \quad \bar{\psi}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}}$$

$$\sigma^\mu = \sigma^\mu{}_{\alpha\dot{\alpha}}.$$

$$\bar{\sigma}^{\mu\dot{\alpha}\alpha} = \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}\sigma^\mu{}_{\beta\dot{\beta}}.$$

$$(\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu)_\alpha{}^\beta = -2\eta^{\mu\nu}\delta_\alpha{}^\beta$$
$$(\bar{\sigma}^\mu\sigma^\nu + \bar{\sigma}^\nu\sigma^\mu)^{\dot{\alpha}}{}_{\dot{\beta}} = -2\eta^{\mu\nu}\delta^{\dot{\alpha}}{}_{\dot{\beta}},$$

$$\mathrm{tr}(\sigma^\mu\bar{\sigma}^\nu) = -2\eta^{\mu\nu}$$
$$\sigma^\mu_{\alpha\dot{\alpha}}\bar{\sigma}_\mu^{\dot{\beta}\beta} = -2\delta_\alpha{}^\beta\delta_{\dot{\alpha}}{}^{\dot{\beta}}.$$

$$\sigma_\alpha^{\mu\nu}{}^\beta_\alpha = \frac{1}{4}(\sigma^\mu_{\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}\beta} - \sigma^\nu_{\alpha\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta})$$
$$\bar{\sigma}^{\mu\nu\dot{\alpha}}{}_{\dot{\beta}} = \frac{1}{4}\left(\bar{\sigma}^{\mu\dot{\alpha}\alpha}\sigma^\nu_{\alpha\dot{\beta}} - \bar{\sigma}^{\nu\dot{\alpha}\alpha}\sigma^\mu_{\alpha\dot{\beta}}\right).$$

$$\sigma^\mu_{\mathrm{E}} = (\mathbb{1}_2, i\sigma^i), \bar{\sigma}^\mu_{\mathrm{E}} = (\mathbb{1}_2, -i\sigma^i), i = 1{,}2{,}3.$$

$$\left(\sigma^\mu_{\mathrm{E}}\bar{\sigma}^\nu_{\mathrm{E}} + \sigma^\nu_{\mathrm{E}}\bar{\sigma}^\mu_{\mathrm{E}}\right)_\alpha{}^\beta = 2\delta^{\mu\nu}\delta_\alpha{}^\beta$$
$$\left(\bar{\sigma}^\mu_{\mathrm{E}}\sigma^\nu_{\mathrm{E}} + \bar{\sigma}^\nu_{\mathrm{E}}\sigma^\mu_{\mathrm{E}}\right)^{\dot{\alpha}}{}_{\dot{\beta}} = 2\delta^{\mu\nu}\delta^{\dot{\alpha}}{}_{\dot{\beta}},$$
$$\mathrm{tr}\left(\sigma^\mu_{\mathrm{E}}\bar{\sigma}^\nu_{\mathrm{E}}\right) = 2\delta^{\mu\nu}$$
$$\sigma^\mu_{\mathrm{E}\alpha\dot{\alpha}}\bar{\sigma}^{\dot{\beta}\beta}_{\mathrm{E}\mu} = 2\delta_\alpha{}^\beta\delta_{\dot{\alpha}}{}^{\dot{\beta}}.$$

$$\sigma^{\mu\nu}_{\mathrm{E}\alpha}{}^\beta = \frac{1}{4}\left(\sigma^\mu_{\mathrm{E}\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}\beta}_{\mathrm{E}} - \sigma^\nu_{\mathrm{E}\alpha\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta}_{\mathrm{E}}\right)$$
$$\bar{\sigma}^{\mu\nu\dot{\alpha}}_{\mathrm{E}}{}_{\dot{\beta}} = \frac{1}{4}\left(\bar{\sigma}^{\mu\dot{\alpha}\alpha}_{\mathrm{E}}\sigma^\nu_{\mathrm{E}\alpha\dot{\beta}} - \bar{\sigma}^{\nu\dot{\alpha}\alpha}_{\mathrm{E}}\sigma^\mu_{\mathrm{E}\alpha\dot{\beta}}\right)$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\psi\chi = \psi^\alpha\chi_\alpha = -\psi_\alpha\chi^\alpha = \chi\psi$$
$$\bar{\psi}\bar{\chi} = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = -\bar{\psi}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} = \bar{\chi}\bar{\psi}$$

$$\psi^\alpha\psi^\beta = -\frac{1}{2}\varepsilon^{\alpha\beta}\psi\psi, \qquad \psi_\alpha\psi_\beta = \frac{1}{2}\varepsilon_{\alpha\beta}\psi\psi$$
$$\bar{\psi}^{\dot{\alpha}}\bar{\psi}^{\dot{\beta}} = \frac{1}{2}\varepsilon^{\dot{\alpha}\dot{\beta}}\overline{\psi\psi}, \qquad \bar{\psi}_{\dot{\alpha}}\bar{\psi}_{\dot{\beta}} = -\frac{1}{2}\varepsilon_{\dot{\alpha}\dot{\beta}}\overline{\psi\psi}$$

$$\theta\sigma^\mu\bar{\theta}\theta\sigma^\nu\bar{\theta} = -\frac{1}{2}\theta\theta\overline{\theta\theta}\eta^{\mu\nu}$$
$$(\theta\psi)(\theta\chi) = -\frac{1}{2}(\theta\theta)(\psi\chi), \quad (\overline{\theta\psi})(\bar{\theta}\bar{\chi}) = -\frac{1}{2}(\overline{\theta\theta})(\bar{\psi}\bar{\chi})$$
$$\chi\sigma^\mu\bar{\psi} = -\psi\sigma^\mu\bar{\chi}, \quad (\chi\sigma^\mu\bar{\psi})^\dagger = \psi\sigma^\mu\bar{\chi}$$
$$\chi\sigma^\mu\bar{\sigma}^\nu\bar{\psi} = \psi\sigma^\nu\bar{\sigma}^\mu\bar{\chi}, \quad (\chi\sigma^\mu\bar{\sigma}^\nu\bar{\psi})^\dagger = \bar{\psi}\bar{\sigma}^\nu\sigma^\mu\bar{\chi}$$
$$(\psi\chi)\bar{\lambda}_{\dot{\alpha}} = -\frac{1}{2}(\chi\sigma^\mu\bar{\lambda})(\psi\sigma_\mu)_{\dot{\alpha}}$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu, \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$\{D_\alpha, \bar D_{\dot\alpha}\} = -2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$
$$\{D_\alpha, D_\beta\} = \{\bar D_{\dot\alpha}, \bar D_{\dot\beta}\} = 0.$$

$$\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha} = -\frac{\partial}{\partial\theta_\alpha}, \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = \varepsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\overline{\theta\theta} = 4.$$

$$[\bar D_{\dot\alpha}, \{\bar D_{\dot\beta}, D_\gamma\}] = 0, [D_\alpha, \bar D_{\dot\beta}\bar D^{\dot\beta}] = -4i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar D^{\dot\alpha}$$
$$[D^2, \bar D^2] = -8iD^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar D^{\dot\alpha}\partial_\mu - 16\,\square = 8i\bar D^{\dot\alpha}\sigma^\mu_{\alpha\dot\alpha}D^\alpha\partial_\mu + 16\,\square$$

$$D^{\mathrm{E}}_\alpha = \frac{\partial}{\partial\theta^\alpha} + \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu, \bar D^{\mathrm{E}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$\{D^{\mathrm{E}}_\alpha, \bar D^{\mathrm{E}}_{\dot\alpha}\} = -2\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$
$$\{D^{\mathrm{E}}_\alpha, D^{\mathrm{E}}_\beta\} = \{\bar D^{\mathrm{E}}_{\dot\alpha}, \bar D^{\mathrm{E}}_{\dot\beta}\} = 0.$$

$$[\bar D^{\mathrm{E}}_{\dot\alpha}, \{\bar D^{\mathrm{E}}_{\dot\beta}, D^{\mathrm{E}}_\gamma\}] = 0$$
$$[D^{\mathrm{E}}_\alpha, \bar D^{\mathrm{E}}_{\dot\beta}\bar D^{\mathrm{E}\dot\beta}] = -4\sigma^\mu_{\mathrm{E}\alpha\dot\alpha}\partial_\mu\bar D^{\mathrm{E}\dot\alpha},$$
$$[D^2_{\mathrm{E}}, \bar D^2_{\mathrm{E}}] = -8D^\alpha_{\mathrm{E}}\sigma^\mu_{\mathrm{E}\alpha\dot\alpha}\bar D^{\dot\alpha}_{\mathrm{E}}\partial_\mu - 16\,\square = 8\bar D^{\dot\alpha}_{\mathrm{E}}\sigma^\mu_{\mathrm{E}\alpha\dot\alpha}D^\alpha_{\mathrm{E}}\partial_\mu + 16\,\square.$$

$$D^2_{\mathrm{E}}\bar D^2_{\mathrm{E}}D^2_{\mathrm{E}} = 16\,\square\, D^2_{\mathrm{E}}, \bar D^2_{\mathrm{E}}D^2_{\mathrm{E}}\bar D^2_{\mathrm{E}} = 16\,\square\,\bar D^2_{\mathrm{E}}$$
$$\bar D^2_{\mathrm{E}}D^{\mathrm{E}}_\alpha D^{\mathrm{E}}_\beta\bar D^2_{\mathrm{E}} = -16\sigma^\mu_{\mathrm{E}\alpha\dot\alpha}\sigma^\nu_{\mathrm{E}\beta\dot\beta}\partial_\mu\partial_\nu\bar D^{\dot\alpha}_{\mathrm{E}}\bar D^{\dot\beta}_{\mathrm{E}}$$
$$D^2_{\mathrm{E}}\bar D^{\mathrm{E}}_{\dot\alpha}\bar D^{\mathrm{E}}_{\dot\beta}D^2_{\mathrm{E}} = -16\sigma^\mu_{\mathrm{E}\alpha\dot\alpha}\sigma^\nu_{\mathrm{E}\beta\dot\beta}\partial_\mu\partial_\nu D^\alpha_{\mathrm{E}}D^\beta_{\mathrm{E}}.$$

$$\int d\eta = 0, \int d\eta\eta = 1.$$

$$f(\eta) = a + b\eta$$

$$\int d\eta f(\eta) = b, \int d\eta\eta f(\eta) = a$$

$$\int d\eta\frac{\partial}{\partial\eta}f(\eta) = 0$$

$$\int d\eta f(\eta)\delta(\eta) = f(0)$$

$$d^4\theta = d^2\theta d^2\bar\theta$$
$$d^2\theta = \frac{1}{4}\varepsilon_{\alpha\beta}d\theta^\alpha d\theta^\beta, d^2\bar\theta = \frac{1}{4}\varepsilon^{\dot\alpha\dot\beta}d\bar\theta_{\dot\alpha}d\bar\theta_{\dot\beta}$$

$$\int d^2\theta\theta\theta = \int d^2\bar\theta\overline{\theta\theta} = 1$$

$$\vec\nabla\cdot\vec E = 0 \qquad \vec\nabla\cdot\vec B = 0$$
$$\vec\nabla\times\vec B - \frac{\partial E}{\partial t} = 0 \qquad \vec\nabla\times\vec E + \frac{\partial B}{\partial t} = 0$$

$$D: \vec E \longrightarrow \vec B, \vec B \longrightarrow -\vec E.$$

$$\begin{pmatrix}\vec E\\ \vec B\end{pmatrix} \longrightarrow \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\vec E\\ \vec B\end{pmatrix}$$

$$dF = 0, *^{-1}d * F = 0$$

$$dF = 0, *^{-1}d * F = j$$

$$dF = j_m, *^{-1} d * F = j_e$$

$$q_i \cdot g_j = 2\pi\hbar n_{ij}$$

$$i\hbar \frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\vec{\nabla} - \frac{iq}{\hbar}\vec{A}\right)^2 \psi + V\psi.$$

$$\psi \longrightarrow e^{-i\frac{q}{\hbar}\chi(\theta)}\psi, \vec{A} \longrightarrow \vec{A} - \vec{\nabla}\chi(\theta).$$

$$\vec{A}_N\big|_{\text{eq}} = \vec{A}_S\big|_{\text{eq}} + \vec{\nabla}\chi$$

$$\psi_N|_{\text{eq}} = e^{-i\frac{q}{\hbar}\chi}\psi_S\big|_{\text{eq}}.$$

$$g = \int_{S^3} d\vec{\sigma} \cdot \vec{B} = \int_{\text{eq}} d\vec{\ell} \cdot \vec{A}_N - \int_{\text{eq}} d\vec{\ell} \cdot \vec{A}_S = \chi(2\pi) - \chi(0).$$

$$q_i g_j - q_j g_i = 2\pi\hbar n_{ij}.$$

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}D^\mu\Phi^a D_\mu\Phi_a - V(\Phi),$$

$$V(\Phi) = \frac{\lambda}{4}(\Phi^a\Phi_a - v^2)^2$$

$$D_\mu F^{a\mu\nu} = \varepsilon^{abc}\Phi_b D^\nu\Phi_c$$
$$\left(D^\mu D_\mu\Phi\right)^a = -\lambda\Phi^a(\Phi^b\Phi_b - v^2)$$

$$E = \int d^3x\Theta_{00} = \int d^3x\left\{\left[\frac{1}{2}(B^a_i)^2 + (E^a_i)^2 + (\Pi^a)^2 + [(D_i\Phi)^a]^2\right] + V(\Phi)\right\}$$

$$F^a_{0i} = E^a_i, F^a_{ij} = -\varepsilon_{ijk}B^{ak}$$

$$\mathcal{M}_H = \left\{\Phi : D_\mu\Phi^a = 0, V(\Phi) = 0\right\}$$

$$A^a_\mu = \frac{1}{v^2 e}\varepsilon^{abc}\Phi_b\partial_\mu\Phi_c + \frac{1}{v}\Phi^a K_\mu,$$

$$F^a_{\mu\nu} = \frac{1}{v}\Phi^a F_{\mu\nu},$$

$$F_{\mu\nu} = \frac{1}{v^3 e}\varepsilon_{abc}\Phi^a\partial_\mu\Phi^b\partial_\nu\Phi^c + \partial_\mu K_\nu - \partial_\nu K_\mu,$$

$$g = \int_{S^2_\infty} d\vec{\sigma} \cdot \vec{B} = \frac{1}{2ev^3}\int_{S^2_\infty} d^2\sigma^i\varepsilon_{ijk}\varepsilon^{abc}\Phi_a\partial^j\Phi_b\partial^k\Phi_c = \frac{4\pi n}{e},$$
$$B^a_i = \frac{1}{2}\varepsilon_{ijk}F^{ajk} = \frac{1}{2v^4 e}\Phi^a\varepsilon_{ijk}\varepsilon^{bcd}\Phi_b\partial^j\Phi_c\partial^k\Phi_d = \frac{1}{v}\Phi^a B_i$$
$$g = \int_{S^2_\infty} d\vec{\sigma} \cdot \frac{1}{v}\Phi^a\vec{B}_a = \frac{1}{v^2}\int_{S^2_\infty} d\vec{\sigma} \cdot \Phi_a\Phi^a\vec{B} = \int_{S^2_\infty} d\vec{\sigma} \cdot \vec{B} = \frac{4\pi n}{e}.$$
$$g = \frac{1}{v}\int_{S^2_\infty} d\vec{\sigma} \cdot \vec{B}^a\Phi_a = \frac{1}{v}\int d^3x\vec{B}^a(\vec{D}\Phi)^a$$

$$m_M \geq v|g|$$

$$B^a_i = D_i\Phi^a$$

$$\Phi^a = \frac{x^a}{er^2}H(\xi), A^a_0 = 0, A^a_i = -\varepsilon^a{}_{ij}\frac{x^j}{er^2}[1 - K(\xi)]$$

$$m_D \geq v\sqrt{e^2+g^2}$$

$$\mathcal{L}_\theta = -\frac{\theta e^2}{32\pi^2} F_{\mu\nu}^a * F_a^{\mu\nu}$$

$$\tau = \frac{\theta}{2\pi} + \frac{4i\pi}{e^2}$$

$$\mathcal{L} = -\frac{1}{32\pi}\mathrm{Im}\tau(F^{\mu\nu} + i * F^{\mu\nu})(F_{\mu\nu} + i * F_{\mu\nu}) - \frac{1}{2}D^\mu \Phi D_\mu \Phi.$$

$$Q_m = \frac{1}{v}\int d^3x D_i \Phi^a B_a^i$$
$$Q_e = \frac{1}{v}\int d^3x D_i \Phi^a E_a^i$$

$$q = n_e e - \frac{e\theta n_m}{2\pi},$$

$$\mathcal{L}_\mathrm{F} = \sum_{k=1}^{N_f} i\bar{\psi}^k \gamma^\mu D_\mu \psi^k - if\bar{\psi}^k \Phi \psi^k.$$

$$(i\gamma^\mu D_\mu - \Phi)\psi^k = 0$$

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \gamma^i = \begin{pmatrix} -i\sigma^i & 0 \\ 0 & i\sigma^i \end{pmatrix}$$

$$\psi^k(x,t) = e^{iEt}\psi^k(x)$$

$$\psi^k(x) = \begin{pmatrix} \chi_+^k(x) \\ \chi_-^k(x) \end{pmatrix}$$

$$\not{D}\chi_+^k = (i\sigma^i D_i + \Phi)\chi_-^k = E\chi_+^k$$
$$\not{D}\chi_-^k = (i\sigma^i D_i - \Phi)\chi_+^k = E\chi_-^k$$

$$\psi^k = \sum_{i=1}^{n} a_0^i \psi_0^k i + \sum_{j=1}^{\infty} a^j \psi_j^k$$

$$\mathcal{L} = \mathrm{Tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\gamma_\mu D^\mu \Psi\right)$$

$$Q^- = 2^{3/4} g \int dx^-(i[\phi, \partial_- \phi] + 2\psi\psi)\frac{1}{\partial_-}\psi$$

$$\phi_{ij}(0,x^-) = \frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{dk^+}{\sqrt{2k^+}}\left[a_{ij}(k^+)e^{-\mathrm{i}k^+x^-} + a_{ji}^\dagger(k^+)e^{\mathrm{i}k^+x^-}\right]$$
$$\psi_{ij}(0,x^-) = \frac{1}{2\sqrt{\pi}}\int_0^\infty dk^+\left[b_{ij}(k^+)e^{-\mathrm{i}k^+x^-} + b_{ji}^\dagger(k^+)e^{\mathrm{i}k^+x^-}\right]$$

$$2P^+P^-|\varphi\rangle = M^2|\varphi\rangle$$

$$S: a_{ij}(k^+) \to -a_{ji}(k^+), b_{ij}(k^+) \to -b_{ji}(k^+).$$

$$P: a_{ij}(k^+) \to -a_{ij}(k^+), b_{ij}(k^+) \to b_{ij}(k^+).$$

$$M^2_{K_0,S+}(K_0) = -0.000316166 + 88.8448\frac{1}{K_0} - 9.84279\frac{1}{K_0^2} \pm \cdots,$$
$$M^2_{K_0,S-}(K_0) = 0.000125688 + 88.8097\frac{1}{K_0} - 8.68279\frac{1}{K_0^2} \pm \cdots,$$
$$M^2_{K_0,\mathrm{all}}(K_0) = -0.000353165 + 88.8513\frac{1}{K_0} - 10.1359\frac{1}{K_0^2} \pm \cdots,$$

$$N(M^2,K) = 2^{\left(K-\frac{84}{M^2}\right)}\Theta(14-M^2)\Theta(K-6).$$

$$N(K) = 2^{(K-6)}\Theta(K-6).$$

$$\rho(M^2,K) = \frac{dN(M^2,K)}{dM^2} = N(K)\tilde{\rho}(M^2),$$

$$\tilde{\rho}(M^2) = 2^{\left(6-\frac{84}{M^2}\right)}\frac{84}{M^4}\ln(2)\Theta(14-M^2).$$

$$s^\dagger = a^\dagger(3)b^\dagger(1)b^\dagger(1)b^\dagger(3)a^\dagger(1)a^\dagger(4),$$

$$s^\dagger = a++bbb++aa+++,$$

$$s_1^\dagger = b^\dagger(1)b^\dagger(1)b^\dagger(3)a^\dagger(1)a^\dagger(4)a^\dagger(3)$$
$$s_2^\dagger = b^\dagger(1)b^\dagger(3)a^\dagger(1)a^\dagger(4)a^\dagger(3)b^\dagger(1)$$
$$s_3^\dagger = b^\dagger(3)a^\dagger(1)a^\dagger(4)a^\dagger(3)b^\dagger(1)b^\dagger(1)$$

$$B(K,n,j) = n^j\binom{K-1}{j-1}.$$

$$A(K,n,j) = \frac{n^j}{j}\binom{K-1}{j-1} = \frac{n^j}{K}\binom{K}{j},$$

$$A(K,n) = \sum_{j=2}^{K}\frac{n^j}{K}\binom{K}{j} = \frac{1}{K}[(1+n)^K - 1 - nK].$$

$$A(K,n)_{\mathrm{prime}} = \frac{1}{K}[(1+n)^K - (1+n)].$$

$$C(K,n,1) = n = 2C_f(K,n,1) = 2C_b(K,n,1),$$
$$C(K,n,j) = B(K,n,j) - \sum_{q>1,\,\mathrm{odd}}^{K} C\left(\frac{K}{q},n,\frac{j}{q}\right),$$
$$C_f(K,n,j) = B_f(K,n,j) - \sum_{q>1,\,\mathrm{odd}}^{K} C_f\left(\frac{K}{q},n,\frac{j}{q}\right),$$
$$C_b(K,n,j) = B_b(K,n,j) - \sum_{q>1,\,\mathrm{odd}}^{K} C_b\left(\frac{K}{q},n,\frac{j}{q}\right).$$

$$A_f(K,n,j) = \sum_{q,\,\mathrm{odd}}\frac{q}{j}C_f\left(\frac{K}{q},n,\frac{j}{q}\right).$$

$$D(K,n,j) = C(K,n,j) - C\left(\frac{K}{2},n,\frac{j}{2}\right).$$

$$D_b(K,n,j) = C_b(K,n,j) - C\left(\frac{K}{2},n,\frac{j}{2}\right) = C_b(K,n,j) - 2C_b\left(\frac{K}{2},n,\frac{j}{2}\right).$$

$$A_b(K,n,j) = \sum_q \frac{q}{j} D_b\left(\frac{K}{q},n,\frac{j}{q}\right)$$

$$= \sum_q \frac{q}{j}\left[C_b\left(\frac{K}{q},n,\frac{j}{q}\right) - 2C_b\left(\frac{K}{2q},n,\frac{j}{2q}\right)\right] = \sum_{q,\,\text{odd}}^{K} \frac{q}{j} C_b\left(\frac{K}{q},n,\frac{j}{q}\right).$$

$$A_f(K,n) = \sum_j \sum_{q,\,\text{odd}} \frac{q}{j} C_f\left(\frac{K}{q},n,\frac{j}{q}\right) - A_f(K,n,1) = \sum_{q,\,\text{odd}} A_f'\left(\frac{K}{q},n\right) - \frac{n}{2},$$

$$A_f'(K,n) \equiv \sum_j \frac{1}{j} C_f(K,n,j) = \sum_j \frac{1}{j} B_f(K,n,j) - \sum_{q>1,\,\text{odd}} \sum_j \frac{1}{j} C_f\left(\frac{K}{q},n,\frac{j}{q}\right)$$
$$= \sum_j \frac{1}{j} B_f(K,n,j) - \sum_{q>1,\,\text{odd}} \frac{1}{q} A_f'\left(\frac{K}{q},n\right)$$
$$= \frac{(1+n)^K - 1}{2K} - \sum_{q>1,\,\text{odd}} \frac{1}{q} A_f'\left(\frac{K}{q},n\right)$$

$$Y: a_1(k^+) \leftrightarrow a_2(k^+), b_1(k^+) \to b_1(k^+), b_2(k^+) \to -b_2(k^+).$$

$$A_{fo}(K,n) = A_{bo}(K,n) \text{ and } A_{fe}(K,n) = A_{be}(K,n)$$

$$A_{be}(K,n) = A_{fe}(K,n) = \frac{1}{2}\left[A_f(K,n) + A_f(K,-n)\right],$$
$$A_{bo}(K,n) = A_{fo}(K,n) = \frac{1}{2}\left[A_f(K,n) - A_f(K,-n)\right].$$

$$\delta Y_{1+}(K,j) = \sum_{q,\,\text{even}} \frac{q}{j} C_f\left(\frac{K}{q},2,\frac{j}{q}\right).$$

$$\delta Y_{1-}(K,j) = \sum_{\substack{j/q,\,\text{odd}\\ q,\,\text{even}}} \frac{q}{j} D\left(\frac{K}{q},2,\frac{j}{q}\right) = \sum_{\substack{j/q,\,\text{odd}\\ q,\,\text{even}}} \frac{q}{j}\left[C\left(\frac{K}{q},2,\frac{j}{q}\right) - C\left(\frac{K}{2q},2,\frac{j}{2q}\right)\right].$$

$$\delta Y_{1+}(K,j) - \delta Y_{1-}(K,j) = \sum_{q,\,\text{even}} \frac{q}{j} C_f\left(\frac{K}{q},2,\frac{j}{q}\right) - \sum_{\substack{j/q,\,\text{odd}\\ q,\,\text{even}}} \frac{q}{j} 2C_f\left(\frac{K}{q},2,\frac{j}{q}\right)$$
$$= \sum_{q,\,\text{even}} (-1)^{j/q} \frac{q}{j} C_f\left(\frac{K}{q},2,\frac{j}{q}\right)$$

$$\mathrm{Tr}\left[\left(s^\dagger \bar{s}^\dagger\right)^q\right]|0\rangle\left(s'^\dagger \bar{s}'^\dagger\right)^{(2r+1)}$$

$$\frac{1}{2}\frac{2q}{j} C\left(\frac{K}{2q},n,\frac{j}{2q}\right)$$

$$\delta Y_{2+}(K,j) - \delta Y_{2-}(K,j) = -\sum_{q\geq 1} (-1)^{j/2q} \frac{q}{j} C\left(\frac{K}{2q},2,\frac{j}{2q}\right).$$

$$(-1)^j(-1)^{\frac{n_f(n_f-1)}{2}}$$

State	SY	number of states K even	K odd
$\mathrm{Tr}\left[a_1^\dagger(K/2)a_2^\dagger(K/2)\right]\|0\rangle$	1	1	0
$\mathrm{Tr}\left[b_1^\dagger(k)b_2^\dagger(K-k)\right]\|0\rangle$	-1	$K-1$	$K-1$
$\mathrm{Tr}\left[b_1^\dagger(k)b_1^\dagger(K-k)\right]\|0\rangle$	1	$K/2-1$	$(K-1)/2$
$\mathrm{Tr}\left[b_2^\dagger(k)b_2^\dagger(K-k)\right]\|0\rangle$	1	$K/2-1$	$(K-1)/2$

$$A_{fes^+y^+} = A_{fes^+y^-} \text{ and } A_{fes^-y^+} = A_{fes^-y^-},$$
$$A_{bos^+y^+} = A_{bos^+y^-} \text{ and } A_{bos^-y^+} = A_{bos^-y^-}.$$

$$A_{fos^+y^+} = A_{fos^+y^-} \text{ and } A_{fos^-y^+} = A_{fos^-y^-}$$

$$A_{bey^+} = A_{bey^-}$$

$$A_{bes^+y^+} + A_{bes^-y^-} = A_{bes^+y^-} + A_{bes^-y^+}$$

$$A_{bes^+y^+} = A_{bes^+y^-} \text{ and } A_{bes^-y^+} = A_{bes^-y^-}$$

$$A_{bes^+} - A_{bes^-} = A_{fes^+} - A_{fes^-} = \frac{n^2}{4}(K-1)$$
$$A_{bos^+} - A_{bos^-} = A_{fos^+} - A_{fos^-} = 0$$

$$\begin{aligned} A_{bes^+}(K,n) = A_{fes^+}(K,n) &= \frac{1}{4}\left\{A_f(K,n) + A_f(K,-n) + \frac{n^2}{2}(K-1)\right\} \\ A_{bes^-}(K,n) = A_{fes^-}(K,n) &= \frac{1}{4}\left\{A_f(K,n) + A_f(K,-n) - \frac{n^2}{2}(K-1)\right\} \\ A_{bos^+}(K,n) = A_{fos^+}(K,n) &= \frac{1}{4}\{A_f(K,n) - A_f(K,-n)\} \\ &= A_{bos^-}(K,n) = A_{fos^-}(K,n) \end{aligned}$$

$$A_{bes^\pm} = A_{fes^\pm} \text{and } A_{bos^+} = A_{fos^+} = A_{bos^-} = A_{fos^-}$$

$$A'_f(2K+1,-2) = \frac{1}{4K+2}\{[1+(-2)]^{2K+1} - 1 - 2A'_f(1,-2)\} = 0.$$

$$\begin{aligned} A_f(2K+1,-2) &= A'_f(2K+1,-2) + A'_f(1,-2) - (-1) = 0, \\ A_f(2K,-2) &= A'_f(2K,-2) - (-1) = 1. \end{aligned}$$

$$\mathcal{C} = \frac{1}{2}Y_I Z^I = 0$$

$$(\mu_\alpha^\dagger)_r, (\mu^{\dot\alpha})_r, (\psi_a^\dagger)_r\ (a=1,2), \psi_r^b\ (b=3,4), \left(-\frac{w-1}{2} \le r \le \frac{w-1}{2}\right)$$

$$Z_r^I \text{ and } (Y_J)_r \text{ with } r \le -\frac{w+1}{2}$$

$$\hat{\Phi}_j = \frac{1}{\sqrt{w}} \sum_{r=-(w-1)/2}^{(w-1)/2} \Phi_r e^{-2\pi i \frac{rj}{w}}$$

$$(L_0 + pw)|\Psi_{\text{phy}}\rangle = 0\ (p \in \mathbb{Z})$$

$$\mathcal{C}_n|\Psi_{\text{phy}}\rangle = 0\ (n = 0,1,\dots,w-1)$$

$$\left[\lambda_r^\alpha,\left(\mu_\beta^\dagger\right)_s\right]=\delta_\beta^\alpha\delta_{r,-s},\left[\mu_r^{\dot\alpha},\left(\lambda_{\dot\beta}^\dagger\right)_s\right]=\delta_{\dot\beta}^{\dot\alpha}\delta_{r,-s},\left\{\psi_r^a,\left(\psi_b^\dagger\right)_s\right\}=\delta_b^a\delta_{r,-s}$$

$$J^I{}_J=Y_JZ^I$$

$$\mathcal{B}_0|0\rangle=\mathcal{C}_0|0\rangle=\mathcal{D}_0|0\rangle=0$$

$$\begin{aligned}
\left(\tilde\lambda^\alpha\right)_r &=(\lambda^\alpha)_{r-w/2}, & \left(\tilde\lambda_{\dot\alpha}^\dagger\right)_r&=\left(\lambda_{\dot\alpha}^\dagger\right)_{r-w/2},\\
\left(\tilde\mu^{\dot\alpha}\right)_r &=\left(\mu^{\dot\alpha}\right)_{r+w/2}, & \left(\tilde\mu_\alpha^\dagger\right)_r&=\left(\mu_\alpha^\dagger\right)_{r+w/2},\\
\left(\tilde\psi_r^a\right) &=\psi_{r-w/2}^a, & \left(\tilde\psi_a^\dagger\right)_r&=\left(\psi_a^\dagger\right)_{r+w/2} & (a=1,2),\\
\left(\tilde\psi_r^b\right) &=\psi_{r+w/2}^b, & \left(\tilde\psi_b^\dagger\right)_r&=\left(\psi_b^\dagger\right)_{r-w/2} & (b=3,4).
\end{aligned}$$

$$\begin{aligned}
\mu_r^{\dot\alpha}|0\rangle_w &=\left(\mu_\alpha^\dagger\right)_r|0\rangle_w=\left(\psi_{1,2}^\dagger\right)_r|0\rangle_w=\psi_r^{3,4}|0\rangle_w=0, & \left(r\geq\frac{w+1}{2}\right)\\
\lambda_r^\alpha|0\rangle_w &=\left(\lambda_{\dot\alpha}^\dagger\right)_r|0\rangle_w=(\psi^{1,2})_r|0\rangle_w=\left(\psi_{3,4}^\dagger\right)_r|0\rangle_w=0, & \left(r\geq-\frac{w-1}{2}\right)\\
&\mu_r^{\dot\alpha},\left(\mu_\alpha^\dagger\right)_r,\left(\psi_{1,2}^\dagger\right)_r,\psi_r^{3,4}, & \left(r\leq\frac{w-1}{2}\right)\\
&\lambda_r^\alpha,\left(\lambda_{\dot\alpha}^\dagger\right)_r,(\psi^{1,2})_r,\left(\psi_{3,4}^\dagger\right)_r, & \left(r\leq-\frac{w+1}{2}\right)
\end{aligned}$$

$$\left(\tilde{\mathcal{P}}^{\dot\alpha}{}_\beta\right)_r=\left(\mathcal{P}^{\dot\alpha}{}_\beta\right)_{r+w},$$

$$\tilde U_n=U_n-w\delta_{n,0},\tilde{\dot U}_n=\dot U_n+w\delta_{n,0},\tilde V_n=V_n$$

$$\tilde{\mathcal{B}}_n=\mathcal{B}_n,\tilde{\mathcal{C}}_n=\mathcal{C}_n$$

$$\tilde{\mathcal{D}}_n=\mathcal{D}_n-w\delta_{n,0}$$

$$\left(\tilde{\mathcal{R}}^a{}_a\right)_n=\begin{cases}(\mathcal{R}^a{}_a)_n+\dfrac{w}{2}\delta_{n,0} & \text{if } a=1,2\\ (\mathcal{R}^a{}_a)_n-\dfrac{w}{2}\delta_{n,0} & \text{if } a=3,4,\end{cases}$$

$$\left(\tilde{\mathcal{R}}^a{}_b\right)_n=\begin{cases}(\mathcal{R}^a{}_b)_{n+w} & \text{if } b\in\{1,2\}\text{ and } a\in\{3,4\}\\ (\mathcal{R}^a{}_b)_{n-w} & \text{if } a\in\{1,2\}\text{ and } b\in\{3,4\}\\ (\mathcal{R}^a{}_b)_n & \text{if } a,b\in\{1,2\}\text{ or } a,b\in\{3,4\}\end{cases}$$

$$\left(\tilde H_1\right)_n=(H_1)_n,\left(\tilde H_2\right)_n=(H_2)_n-w\delta_{n,0},\left(\tilde H_3\right)_n=(H_3)_n$$

$$\tilde L_n=L_n-w(\mathcal{D}_n-\mathcal{R}_n)$$

$$\mathcal{R}_n=\frac{1}{2}[-(\mathcal{R}^1{}_1)_n-(\mathcal{R}^2{}_2)_n+(\mathcal{R}^3{}_3)_n+(\mathcal{R}^4{}_4)_n]$$

$$\tilde{\mathcal{R}}_n=\mathcal{R}_n-w\delta_{n,0}$$

$$m_1,m_2\in\frac{1}{2}\mathbb{Z},\dot m_1,\dot m_2\in\frac{1}{2}\mathbb{Z},m_1,\dot m_2\in\frac{1}{2}\mathbb{N}_0$$

$$m_1,\dot m_1,m_2,\dot m_2\in\frac{1}{2}\mathbb{N}_0$$

$$[0,1,0]\oplus[1,0,0]\oplus[0,0,1]\oplus 2\cdot[0,0,0]$$

$$\mathcal{R}_0=\bigoplus_{s=0}^{\infty}\left[\left(\frac{s}{2},\frac{s}{2};[0,1,0]\right)_{1+s}\oplus\left(\frac{s}{2}+1,\frac{s}{2};[0,0,0]\right)_{2+s}\oplus\left(\frac{s}{2},\frac{s}{2}+1;[0,0,0]\right)_{2+s}\right.$$
$$\left.\oplus\left(\frac{s+1}{2},\frac{s}{2};[0,0,1]\right)_{\frac{3}{2}+s}\oplus\left(\frac{s}{2},\frac{s+1}{2};[1,0,0]\right)_{\frac{3}{2}+s}\right]$$

$$\mathcal{R}_0 \equiv [0,0;[0,1,0]]_1$$

$$|0\rangle_{w=1} = (\psi_3^\dagger)_0 (\psi_4^\dagger)_0 |0,0;0,0\rangle$$

$$(\mu_\alpha^\dagger)_r, (\mu^{\dot{\alpha}})_r, (\psi_a^\dagger)_r \ (a = 1,2), \psi_r^b \ (b = 3,4), \text{ with } -\frac{w-1}{2} \leq r \leq \frac{w-1}{2},$$

$$Z_r^I \text{ and } (Y_J)_r \text{ with } r \leq -\frac{w+1}{2}.$$

$$2N + 2 = 4 + \sum_{i=1}^{2} (\hat{w}_i - 1) = \sum_{i=1}^{2} w_i$$

$$\mathcal{C}_n \phi = 0 \ (n \geq 0), (L_0 + pw)\phi = 0 \ (p \in \mathbb{Z}).$$

$$Z_r^I \text{ and } (Y_J)_r \text{ with } -\frac{w-1}{2} \leq r \leq \frac{w-1}{2}$$

$$\Phi_r = \frac{1}{\sqrt{w}} \sum_{j=1}^{w} \hat{\Phi}_j e^{2\pi i \frac{rj}{w}}, \hat{\Phi}_j = \frac{1}{\sqrt{w}} \sum_{r=-(w-1)/2}^{(w-1)/2} \Phi_r e^{-2\pi i \frac{rj}{w}}$$

$$\left[Z_r^I, (Y_J^\dagger)_s\right]_\pm = \delta_J^I \delta_{r,-s}$$

$$\left[\hat{Z}_{j_1}^I, (\hat{Y}_J^\dagger)_{j_2}\right]_\pm = \delta_J^I \delta_{j_1,j_2}$$

$$\left[e^{\frac{2\pi i}{w} L_0}, \hat{\Phi}_j\right] = \hat{\Phi}_{j+1}$$

$$\hat{\mathcal{C}}_j = \frac{1}{2} \sum_I :(\hat{Y}_I)_j \hat{Z}_j^I:,$$

$$[\hat{\mathcal{C}}_{j_1}, \hat{\Phi}_{j_2}] = \pm \frac{1}{2} \hat{\Phi}_{j_2} \delta_{j_1,j_2}$$

$$\begin{aligned} \sum_{j=1}^{w} e^{2\pi i \frac{jn}{w}} \hat{\mathcal{C}}_j &= \frac{1}{2w} \sum_{j=1}^{w} \sum_{r,s=-(w-1)/2}^{(w-1)/2} \sum_I :(Y_I)_r Z_s^I: e^{-2\pi i \frac{j(r+s-n)}{w}} \\ &= \frac{1}{2} \sum_{r=n-(w-1)/2}^{(w-1)/2} \sum_I :(Y_I)_r Z_{n-r}^I: \end{aligned}$$

$$S_m^{\mathbf{a}} \equiv (S_I^J)_m = \sum_{r=m-\frac{w-1}{2}}^{\frac{w-1}{2}} (Y_I)_r (Z^J)_{m-r}$$

$$[S_m^{\mathbf{a}}, S_n^{\mathbf{b}}]_\pm = \begin{cases} f^{\mathbf{ab}}{}_{\mathbf{c}} S_{m+n}^{\mathbf{c}} & m+n \leq w-1 \\ 0 & m+n \geq w \end{cases}$$

$$\begin{aligned} &\left[S_1^{\mathbf{a}}, [S_1^{\mathbf{b}}, S_0^{\mathbf{c}}]\right] + \left[S_1^{\mathbf{b}}, [S_1^{\mathbf{c}}, S_0^{\mathbf{a}}]\right] + \left[S_1^{\mathbf{c}}, [S_1^{\mathbf{a}}, S_0^{\mathbf{b}}]\right] \\ &= \frac{1}{24} f^{\mathbf{adk}} f^{\mathbf{bel}} f^{\mathbf{cfm}} f_{\mathbf{klm}} \{(S_{\mathbf{d}})_0, (S_{\mathbf{e}})_0, (S_{\mathbf{f}})_0\} \end{aligned}$$

$$(S_{I_1}{}^{J_1})_{w-1} (S_{I_2}{}^{J_2})_{w-1} = \pm (S_{I_1}{}^{J_2})_{w-1} (S_{I_2}{}^{J_1})_{w-1}$$

$$\mathcal{C}_n |0\rangle_w = 0 \ n \geq 0$$

$$[0,0;[0,w,0]]_w$$

$$L_0 = -2p \colon [p-1, p-1; [0,0,0]]_{2p}$$

$$\bigoplus_{r=0}^{3m} \left\{ \mathcal{V}_{0,3m+r} \oplus \bigoplus_{k=1}^{\infty} (\mathcal{V}_{2k,3m+r} \oplus \mathcal{V}_{-2k,3m+r}) \right\}$$

$$\bigoplus_{r=0}^{3m-2} \left\{ \mathcal{V}_{-1,3m+r-1} \oplus \mathcal{V}_{1,3m+r-1} \oplus \bigoplus_{k=1}^{\infty} (\mathcal{V}_{2k+1,3m+r-1} \oplus \mathcal{V}_{-2k-1,3m+r-1}) \right\}$$

$$\bigoplus_{r=0}^{3m+1} \left\{ \mathcal{V}_{0,3m+r+2} \oplus \bigoplus_{k=1}^{\infty} (\mathcal{V}_{2k,3m+r+2} \oplus \mathcal{V}_{-2k,3m+r+2}) \right\}$$

$$\bigoplus_{r=0}^{3m} \left\{ \mathcal{V}_{-1,3m+r} \oplus \mathcal{V}_{1,3m+r} \oplus \bigoplus_{k=1}^{\infty} (\mathcal{V}_{2k+1,3m+r} \oplus \mathcal{V}_{-2k-1,3m+r}) \right\}$$

$$m\frac{w-1}{2} \approx m\frac{w}{2} = pw,$$

$$\xi_r^+, \eta_r^+, \psi_r^-, \chi_r^-, -\frac{(w-1)}{2} \leq r \leq \frac{(w-1)}{2},$$

$$[Z_n, \eta_r^+] = \frac{1}{2}\eta_{r+n}^+, [Z_n, \chi_r^-] = \frac{1}{2}\chi_{r+n}^-$$
$$[Z_n, \xi_r^+] = -\frac{1}{2}\xi_{r+n}^+, [Z_n, \psi_r^-] = -\frac{1}{2}\psi_{r+n}^-$$

$$Z_n\phi = 0\ (n \geq 0), (L_0 + pw)\phi = 0\ (p \in \mathbb{Z})$$

$$\tilde{J}_0^3 = J_0^3 - \frac{w}{2}, \tilde{K}_0^3 = K_0^3 - \frac{w}{2}$$

$$Z^{(1)}(t,y) = \frac{1}{(1-t)}\left[t^{\frac{1}{2}}\chi_{\frac{1}{2}}(y) + 2t\chi_0(y)\right]$$

$$Z^{(2)}(t,y) = \frac{1}{2}\left(\left(Z^{(1)}(t,y)\right)^2 + \tilde{Z}^{(1)}(t^2,y^2)\right)$$

$$\tilde{Z}^{(1)}(t,y) = \frac{1}{(1-t)}\left[t^{\frac{1}{2}}\chi_{\frac{1}{2}}(y) - 2t\chi_0(y)\right]$$

$$Z^{(2)}(t,y) = \chi_0^{(\mathrm{BPS})}(t,y) + \sum_{n=1}^{\infty} \chi_{h=2n}^{\mathcal{N}=4}(t,y)$$

$$\chi_0^{(\mathrm{BPS})}(t,y) = \frac{1}{(1-t)}\left[t\chi_1(y) + 2t^{3/2}\chi_{\frac{1}{2}}(y) + t^2\right]$$

$$\chi_h^{\mathcal{N}=4}(t,y) = \frac{t^h}{(1-t)}\left[1 + 2t^{1/2}\chi_{1/2}(y) + t(\chi_1(y) + 3) + 2t^{3/2}\chi_{1/2}(y) + t^2\right]$$

$$w = \hat{w} + 1$$

$$\bar{\psi}_{-r}^1 \psi_{-s}^1 |0\rangle^{(2)}, \bar{\psi}_{-r}^1 \bar{\psi}_{-s}^2 |0\rangle^{(2)}, \text{ etc.}$$

$$\bar{\psi}_{-\frac{1}{2}}^1 \bar{\psi}_{-\frac{1}{2}}^2 |0\rangle^{(2)} = J_{-1}^+ |0\rangle^{(2)}$$

$$(\mathcal{H}_{\min})^{\otimes w} \cong (\mathcal{H}_{\min})^{\otimes (w-2)} \otimes \mathcal{H}_{\min} \otimes \mathcal{H}_{\min}.$$

$$\bar{\psi}_{-\frac{1}{2}}^1 \bar{\psi}_{-\frac{1}{2}}^2 |\mathrm{BPS_{lower}}\rangle^{(w-1)}$$

$$(\mathcal{H}_{\min})^{\otimes w} \cong \underbrace{(\mathcal{H}_{\min})^{\otimes(w-2)}}_{\in(w-1)-\text{ cycle}} \otimes \underbrace{\mathcal{H}_{\min} \otimes \mathcal{H}_{\min}}_{\bar{\psi}^1_{-\frac{1}{2}}\bar{\psi}^2_{-\frac{1}{2}},\dots}$$

$$\left(\xi^+_{\frac{1}{2}}\eta^+_{\frac{1}{2}}\right)^{2p-1}\psi^-_{\frac{1}{2}}\chi^-_{\frac{1}{2}}|0\rangle_2 : J^3_0 = 2p, K^3_0 = 0$$

$$L_0 = -2p : \chi^{\mathcal{N}=4}_{h=2p}(t, y)$$

$$\begin{aligned}\mathcal{L}^{\alpha}{}_{\beta} &= \mu^{\dagger}_{\beta}\lambda^{\alpha} - \frac{1}{2}\delta^{\alpha}_{\beta}U\\ \dot{\mathcal{L}}^{\dot{\alpha}}{}_{\dot{\beta}} &= \lambda^{\dagger}_{\dot{\beta}}\mu^{\dot{\alpha}} - \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\dot{U}\\ \mathcal{R}^{a}{}_{b} &= \psi^{\dagger}_{b}\psi^{a} - \frac{1}{4}\delta^{a}_{b}V\end{aligned}$$

$$U = \mu^{\dagger}_{\gamma}\lambda^{\gamma}, \dot{U} = \lambda^{\dagger}_{\dot{\gamma}}\mu^{\dot{\gamma}}, V = \psi^{\dagger}_{c}\psi^{c},$$

$$\mathcal{B}_n = \frac{1}{2}(U_n + \dot{U}_n), \mathcal{C}_n = \frac{1}{2}(U_n + \dot{U}_n) + \frac{1}{2}V_n, \mathcal{D}_n = \frac{1}{2}(U_n - \dot{U}_n).$$

$$\begin{aligned}\mathcal{Q}^{a}{}_{\alpha} &= \psi^{a}\mu^{\dagger}_{\alpha}, & \mathcal{S}^{\alpha}{}_{a} &= \lambda^{\alpha}\psi^{\dagger}_{a}\\ \dot{\mathcal{Q}}^{\dot{\alpha}}{}_{a} &= \mu^{\dot{\alpha}}\psi^{\dagger}_{a}, & \dot{\mathcal{S}}^{a}{}_{\dot{\alpha}} &= \psi^{a}\lambda^{\dagger}_{\dot{\alpha}}\\ \mathcal{P}^{\dot{\alpha}}{}_{\beta} &= \mu^{\dot{\alpha}}\mu^{\dagger}_{\beta}, & \mathcal{K}^{\alpha}{}_{\dot{\beta}} &= \lambda^{\alpha}\lambda^{\dagger}_{\dot{\beta}}\end{aligned}$$

$$H_1 = (\mathcal{R}^2_2)_0 - (\mathcal{R}^1_1)_0, H_2 = (\mathcal{R}^3_3)_0 - (\mathcal{R}^2_2)_0, H_3 = (\mathcal{R}^4_4)_0 - (\mathcal{R}^3_3)_0$$

$$\left[(\mathcal{L}^{\alpha}{}_{\beta})_m, (\mathcal{L}^{\gamma}{}_{\delta})_n\right] = \delta^{\alpha}_{\delta}(\mathcal{L}^{\gamma}{}_{\beta})_{m+n} - \delta^{\gamma}_{\beta}(\mathcal{L}^{\alpha}{}_{\delta})_{m+n} + m\left(-\delta^{\gamma}_{\beta}\delta^{\alpha}_{\delta} + \frac{1}{2}\delta^{\alpha}_{\beta}\delta^{\gamma}_{\delta}\right)\delta_{m,-n}$$

$$J^+_m = (\mathcal{L}^1{}_2)_m, J^-_m = (\mathcal{L}^2{}_1)_m, J^3_m = \frac{1}{2}((\mathcal{L}^2{}_2)_m - (\mathcal{L}^1{}_1)_m)$$

$$\begin{aligned}[J^3_m, J^3_n] &= +\frac{1}{2}km\delta_{m,-n}\\ [J^3_m, J^{\pm}_n] &= \pm J^{\pm}_{m+n}\\ [J^+_m, J^-_n] &= +2J^3_{m+n} + km\delta_{m,-n}\end{aligned}$$

$$[(\mathcal{R}^a{}_b)_m, (\mathcal{R}^c{}_d)_m] = \delta^a_d(\mathcal{R}^c)_{m+n} - \delta^c_b(\mathcal{R}^a{}_d)_{m+n} + m\left(\delta^a_d\delta^c_b - \frac{1}{4}\delta^a_b\delta^c_d\right)\delta_{m,-n}$$

$$\begin{aligned}[\mathcal{B}_m, \mathcal{B}_n] &= -m\delta_{m,-n}\\ [\mathcal{B}_m, \mathcal{C}_n] &= -m\delta_{m,-n}\\ [\mathcal{C}_m, \mathcal{C}_n] &= 0\\ [\mathcal{D}_m, \mathcal{D}_n] &= -m\delta_{m,-n}\end{aligned}$$

$$[U_m, U_n] = [\dot{U}_m, \dot{U}_n] = -2m\delta_{m,-n}, [V_m, V_n] = 4m\delta_{m,-n}$$

$$\begin{aligned}[\mathcal{C}_m, (\mathcal{Q}^a{}_\alpha)_n] &= [\mathcal{C}_m, (\mathcal{S}^\alpha{}_a)_n] = \left[\mathcal{C}_m, (\dot{\mathcal{Q}}_{\dot{\alpha}a})_n\right] = \left[\mathcal{C}_m, (\dot{\mathcal{S}}^{\dot{\alpha}a})_n\right]\\ &= \left[\mathcal{C}_m, (\mathcal{P}_{\alpha\dot{\beta}})_n\right] = \left[\mathcal{C}_m, (\mathcal{K}^{\alpha\dot{\beta}})_n\right] = 0\end{aligned}$$

$$\begin{aligned}[\mathcal{B}_m, (\mathcal{Q}^a{}_\alpha)_n] &= \frac{1}{2}(\mathcal{Q}^a{}_\alpha)_{m+n}, & [\mathcal{B}_m, (\mathcal{S}^\alpha{}_a)_n] &= -\frac{1}{2}(\mathcal{S}^\alpha{}_a)_{m+n}\\ \left[\mathcal{B}_m, (\dot{\mathcal{Q}}_{\dot{\alpha}a})_n\right] &= -\frac{1}{2}(\dot{\mathcal{Q}}_{\dot{\alpha}a})_{m+n}, & \left[\mathcal{B}_m, (\dot{\mathcal{S}}^{\dot{\alpha}a})_n\right] &= \frac{1}{2}(\dot{\mathcal{S}}^{\dot{\alpha}a})_{m+n}\\ \left[\mathcal{B}_m, (\mathcal{P}_{\alpha\dot{\beta}})_n\right] &= 0, & \left[\mathcal{B}_m, (\mathcal{K}^{\alpha\dot{\beta}})_n\right] &= 0\end{aligned}$$

$$[\mathcal{D}_m, (\mathcal{Q}^a{}_\alpha)_n] = \frac{1}{2}(\mathcal{Q}^a{}_\alpha)_{m+n}, \quad [\mathcal{D}_m, (\mathcal{S}^\alpha{}_a)_n] = -\frac{1}{2}(\mathcal{S}^\alpha{}_a)_{m+n}$$
$$\left[\mathcal{D}_m, (\dot{\mathcal{Q}}_{\dot\alpha a})_n\right] = \frac{1}{2}(\dot{\mathcal{Q}}_{\dot\alpha a})_{m+n}, \quad \left[\mathcal{D}_m, (\dot{\mathcal{S}}^{\dot\alpha a})_n\right] = -\frac{1}{2}(\dot{\mathcal{S}}^{\dot\alpha a})_{m+n}$$
$$\left[\mathcal{D}_m, (\mathcal{P}_{\alpha\dot\beta})_n\right] = (\mathcal{P}_{\alpha\dot\beta})_{m+n}, \quad \left[\mathcal{D}_m, (\mathcal{K}^{\alpha\dot\beta})_n\right] = -(\mathcal{K}^{\alpha\dot\beta})_{m+n}$$

$$\left[(\mathcal{K}^\alpha{}_{\dot\beta})_m, (\mathcal{P}^{\dot\gamma}{}_\delta)_n\right] = -\delta^{\dot\gamma}_{\dot\beta}(\mathcal{L}^\alpha{}_\delta)_{m+n} + \delta^\alpha_\delta(\dot{\mathcal{L}}^{\dot\gamma}{}_{\dot\beta})_{m+n} - \delta^\alpha_\delta\delta^{\dot\gamma}_{\dot\beta}(\mathcal{D}_{m+n} + m\delta_{m,-n})$$
$$\left\{(\mathcal{S}^\alpha{}_a)_m, (\mathcal{Q}^b{}_\beta)_n\right\} = \delta^b_a(\mathcal{L}^\alpha{}_\beta)_{m+n} + \delta^\alpha_\beta(\mathcal{R}^b{}_a)_{m+n} + \frac{1}{2}\delta^b_a\delta^\alpha_\beta(\mathcal{D}_{m+n} + \mathcal{C}_{m+n} + 2m\delta_{m,-n})$$
$$\left\{(\dot{\mathcal{S}}^a{}_{\dot\alpha})_m, (\dot{\mathcal{Q}}^{\dot\beta}{}_b)_n\right\} = \delta^a_b(\dot{\mathcal{L}}^{\dot\beta}{}_{\dot\alpha})_{m+n} + \delta^{\dot\beta}_{\dot\alpha}(\mathcal{R}^a{}_b)_{m+n} - \frac{1}{2}\delta^a_b\delta^{\dot\alpha}_{\dot\beta}(\mathcal{D}_{m+n} - \mathcal{C}_{m+n} + 2m\delta_{m,-n})$$

$$\lambda^1_0|m_1, m_2; \dot m_1, \dot m_2\rangle = 2m_1\left|m_1 - \frac{1}{2}, m_2; \dot m_1, \dot m_2\right\rangle$$
$$\lambda^2_0|m_1, m_2; \dot m_1, \dot m_2\rangle = 2m_2\left|m_1, m_2 - \frac{1}{2}; \dot m_1, \dot m_2\right\rangle$$
$$(\mu^\dagger_1)_0|m_1, m_2; \dot m_1, \dot m_2\rangle = \left|m_1 + \frac{1}{2}, m_2; \dot m_1, \dot m_2\right\rangle$$
$$(\mu^\dagger_2)_0|m_1, m_2; \dot m_1, \dot m_2\rangle = \left|m_1, m_2 + \frac{1}{2}; \dot m_1, \dot m_2\right\rangle,$$

$$\mu^1_0|m_1, m_2; \dot m_1, \dot m_2\rangle = \left|m_1, m_2; \dot m_1 + \frac{1}{2}, \dot m_2\right\rangle$$
$$\mu^2_0|m_1, m_2; \dot m_1, \dot m_2\rangle = \left|m_1, m_2; \dot m_1, \dot m_2 + \frac{1}{2}\right\rangle$$
$$(\lambda^\dagger_1)_0|m_1, m_2; \dot m_1, \dot m_2\rangle = -2\dot m_1\left|m_1, m_2; \dot m_1 - \frac{1}{2}, \dot m_2\right\rangle$$
$$(\lambda^\dagger_2)_0|m_1, m_2; \dot m_1, \dot m_2\rangle = -2\dot m_2\left|m_1, m_2; \dot m_1, \dot m_2 - \frac{1}{2}\right\rangle$$

$$\psi^a_0|m_1, m_2; \dot m_1, \dot m_2\rangle = 0, a = 1,2,3,4$$

$$J^3_0|m_1, m_2; \dot m_1, \dot m_2\rangle = (m_2 - m_1)|m_1, m_2; \dot m_1, \dot m_2\rangle$$
$$U_0|m_1, m_2; \dot m_1, \dot m_2\rangle = 2\left(m_1 + m_2 + \frac{1}{2}\right)|m_1, m_2; \dot m_1, \dot m_2\rangle$$
$$\hat J^3_0|m_1, m_2; \dot m_1, \dot m_2\rangle = (\dot m_1 - \dot m_2)|m_1, m_2; \dot m_1, \dot m_2\rangle$$
$$\dot U_0|m_1, m_2; \dot m_1, \dot m_2\rangle = -2\left(\dot m_1 + \dot m_2 + \frac{1}{2}\right)|m_1, m_2; \dot m_1, \dot m_2\rangle$$

$$J^+_0|m_1, m_2; \dot m_1, \dot m_2\rangle = 2m_1\left|m_1 - \frac{1}{2}, m_2 + \frac{1}{2}; \dot m_1, \dot m_2\right\rangle$$
$$J^-_0|m_1, m_2; \dot m_1, \dot m_2\rangle = 2m_2\left|m_1 + \frac{1}{2}, m_2 - \frac{1}{2}; \dot m_1, \dot m_2\right\rangle$$
$$\hat J^+_0|m_1, m_2; \dot m_1, \dot m_2\rangle = -2\dot m_2\left|m_1, m_2; \dot m_1 + \frac{1}{2}, \dot m_2 - \frac{1}{2}\right\rangle$$
$$\hat J^-_0|m_1, m_2; \dot m_1, \dot m_2\rangle = -2\dot m_1\left|m_1, m_2; \dot m_1 - \frac{1}{2}, \dot m_2 + \frac{1}{2}\right\rangle.$$

$$C^{\mathrm{su}(2)} = J^3_0J^3_0 + \frac{1}{2}(J^+_0J^-_0 + J^-_0J^+_0) = j(j+1)$$

$$C^{\mathrm{su}(2)} = j(j+1) = (m_1 + m_2)(m_1 + m_2 + 1),$$
$$\hat C^{\mathrm{su}(2)} = \hat\jmath(\hat\jmath + 1) = (\dot m_1 + \dot m_2)(\dot m_1 + \dot m_2 + 1).$$

$$(\psi^\dagger_b\psi^a)_0|m_1, m_2; \dot m_1, \dot m_2\rangle = 0, V_0|m_1, m_2; \dot m_1, \dot m_2\rangle = -2|m_1, m_2; \dot m_1, \dot m_2\rangle.$$

$$\mathcal{B}_0|m_1, m_2; \dot m_1, \dot m_2\rangle = (m_1 + m_2 - \dot m_1 - \dot m_2)|m_1, m_2; \dot m_1, \dot m_2\rangle$$
$$\mathcal{C}_0|m_1, m_2; \dot m_1, \dot m_2\rangle = (m_1 + m_2 - \dot m_1 - \dot m_2 - 1)|m_1, m_2; \dot m_1, \dot m_2\rangle$$
$$\mathcal{D}_0|m_1, m_2; \dot m_1, \dot m_2\rangle = (m_1 + m_2 + \dot m_1 + \dot m_2 + 1)|m_1, m_2; \dot m_1, \dot m_2\rangle.$$

$$\mathcal{J}^2 = \frac{1}{2}\mathcal{D}_0^2 + \frac{1}{2}\mathcal{L}^{\gamma}{}_{\delta}\mathcal{L}^{\delta}{}_{\gamma} + \frac{1}{2}\dot{\mathcal{L}}^{\dot\gamma}{}_{\dot\delta}\dot{\mathcal{L}}^{\dot\delta}{}_{\dot\gamma} - \frac{1}{2}\mathcal{R}^{c}{}_{d}\mathcal{R}^{d}{}_{c}$$
$$-\frac{1}{2}[\mathcal{Q}^{c}{}_{\gamma}, \mathcal{S}^{\gamma}{}_{c}] + \frac{1}{2}[\dot{\mathcal{Q}}^{\dot\gamma}{}_{c}, \dot{\mathcal{S}}^{c}{}_{\dot\gamma}] + \frac{1}{2}\{\mathcal{P}^{\dot\gamma}{}_{\delta}, \mathcal{K}^{\delta}{}_{\dot\gamma}\} + \mathcal{B}_0\mathcal{C}_0,$$

$$\mathcal{J}^2|m_1, m_2; \dot m_1, \dot m_2\rangle = \frac{5}{2}\mathcal{C}_0^2|m_1, m_2; \dot m_1, \dot m_2\rangle$$

$$\text{□□}: t^2\left((0,0;[0,2,0])^{(0)} \oplus (0,0;[0,0,0])^{(-2)}\right)$$
$$+t^{\frac{5}{2}}\left(\left(\frac{1}{2},0;[0,1,1]\right)^{(0)} \oplus \left(\frac{1}{2},0;[1,0,0]\right)^{(-2)} \oplus \left(0,\frac{1}{2};[1,1,0]\right)^{(0)} \oplus \left(0,\frac{1}{2};[0,0,1]\right)^{(-2)}\right)$$
$$+t^3\left(\left(\frac{1}{2},\frac{1}{2};[0,2,0]\right)^{(0)} \oplus 2\cdot(1,0;[0,1,0])^{(0/-2)} \oplus 2\cdot(0,1;[0,1,0])^{(0/-2)}\right.$$
$$\oplus (0,0;[0,0,2])^{(0)} \oplus (0,0;[2,0,0])^{(0)}$$
$$\left.\oplus 2\cdot\left(\frac{1}{2},\frac{1}{2};[1,0,1]\right)^{(0/-2)} \oplus 2\cdot\left(\frac{1}{2},\frac{1}{2};[0,0,0]\right)\right) + \mathcal{O}(t^{7/2})$$

$$\text{□□}:\ [0,0;[0,2,0]]_2^{(0)} \oplus \bigoplus_{p=1}^{\infty} [p-1,p-1;[0,0,0]]_{2p}^{(-2p)}.$$

$$\text{□□□}: t^3\left((0,0;[0,3,0])^{(0)} \oplus (0,0;[0,1,0])^{(-3)}\right)$$
$$+t^{\frac{7}{2}}\left(\left(\frac{1}{2},0;[0,2,1]\right)^{(0)} \oplus \left(0,\frac{1}{2};[1,2,0]\right)^{(0)} \oplus \left(\frac{1}{2},0;[1,1,0]\right) \oplus \left(0,\frac{1}{2};[0,1,1]\right)\right.$$
$$\left.\oplus \left(\frac{1}{2},0;[0,0,1]\right) \oplus \left(0,\frac{1}{2};[1,0,0]\right)\right)$$
$$+t^4\left(\left(\frac{1}{2},\frac{1}{2};[0,3,0]\right)^{(0)} \oplus 2\cdot(1,0;[0,2,0])^{(0/-3)} \oplus 2\cdot(0,1;[0,2,0])^{(0/-3)}\right.$$
$$\oplus (0,0;[2,1,0])^{(0)} \oplus (0,0;[0,1,2])^{(0)} \oplus 2\cdot\left(\frac{1}{2},\frac{1}{2};[1,1,1]\right)$$
$$\oplus (1,0;[1,0,1]) \oplus (0,1;[1,0,1])$$
$$\oplus \left(\frac{1}{2},\frac{1}{2};[2,0,0]\right) \oplus \left(\frac{1}{2},\frac{1}{2};[0,0,2]\right) \oplus 4\cdot\left(\frac{1}{2},\frac{1}{2};[0,1,0]\right) \oplus 2\cdot(0,0;[1,0,1])$$
$$\left.\oplus 2\cdot(1,0;[0,0,0]) \oplus 2\cdot(0,1;[0,0,0])\right) + \mathcal{O}(t^{9/2})$$

□
$$\text{□}: t^3\left((0,0;[2,0,0])^{(-3)} \oplus (0,0;[0,0,2])^{(-3)}\right)$$
□
$$+t^{\frac{7}{2}}\left(\left(\frac{1}{2},0;[1,1,0]\right) \oplus \left(0,\frac{1}{2};[0,1,1]\right) \oplus \left(\frac{1}{2},0;[1,0,2]\right) \oplus \left(0,\frac{1}{2};[2,0,1]\right)\right.$$
$$\left.+\left(\frac{1}{2},0;[0,0,1]\right) \oplus \left(0,\frac{1}{2};[1,0,0]\right)\right)$$
$$+t^4\left(2\cdot(0,0;[0,2,0])^{(-3)} \oplus (1,0;[0,1,2])^{(-3)} \oplus (0,1;[2,1,0])^{(-3)}\right.$$
$$\oplus 2\cdot\left(\frac{1}{2},\frac{1}{2};[1,1,1]\right) \oplus 2\cdot(1,0;[1,0,1]) \oplus 2\cdot(0,1;[1,0,1])$$
$$\oplus 2\cdot\left(\frac{1}{2},\frac{1}{2};[2,0,0]\right) \oplus 2\cdot\left(\frac{1}{2},\frac{1}{2};[0,0,2]\right) \oplus 3\cdot\left(\frac{1}{2},\frac{1}{2};[0,1,0]\right)$$
$$\left.\oplus 2\cdot(0,0;[1,0,1]) \oplus 2\cdot(0,0;[0,0,0])\right) + \mathcal{O}(t^{9/2})$$

$$\text{□□□}:\ \underbrace{[0,0;[0,3,0]]_3^{(0)}}_{\mathcal{V}_{0,0}} \oplus \underbrace{[0,0;[0,1,0]]_3^{(-3)}}_{\mathcal{V}_{0,2}} \oplus \cdots$$

□
$$\text{□}:\ \underbrace{[0,0;[2,0,0]]_3^{(-3)}}_{\mathcal{V}_{-1,0}} \oplus \underbrace{[0,0;[0,0,2]]_3^{(-3)}}_{\mathcal{V}_{1,0}} \oplus \cdots,$$
□

$$
\begin{aligned}
w=2: &\quad \ydiagram{2}: && [0,2,0]^{(0)} \oplus [0,0,0]^{(-2)} \\
w=3: &\quad \ydiagram{3}: && [0,3,0]^{(0)} \oplus [0,1,0]^{(-3)} \\
&\quad \ydiagram{1,1,1}: && [2,0,0]^{(-3)} \oplus [0,0,2]^{(-3)} \\
w=4: &\quad \ydiagram{4}: && [0,4,0]^{(0)} \oplus [0,2,0]^{(-4)} \oplus [0,0,0]^{(-8)} \\
&\quad \ydiagram{2,2}: && [2,0,2]^{(-4)} \oplus [0,2,0]^{(-4)} \oplus [0,0,0]^{(-8)} \\
&\quad \ydiagram{2,1,1}: && [2,1,0]^{(-4)} \oplus [0,1,2]^{(-4)} \oplus [1,0,1]^{(-4)} \\
w=5: &\quad \ydiagram{5}: && [0,5,0]^{(0)} \oplus [0,3,0]^{(-5)} \oplus [0,1,0] \\
&\quad \ydiagram{3,2}: && [0,3,0]^{(-5)} \oplus [2,1,2]^{(-5)} \oplus [1,1,1]^{(-5/10)} \oplus [0,1,0] \\
&\quad 2\cdot\ydiagram{3,1,1}: && [2,2,0]^{(-5)} \oplus [0,2,2]^{(-5)} \oplus [1,1,1]^{(-5/10)} \oplus [2,0,0] \oplus [0,0,2] \\
&\quad \ydiagram{2,2,1}: && [1,1,1]^{(-5/10)} \oplus [3,0,1]^{(-5)} \oplus [1,0,3]^{(-5)} \oplus [0,1,0] \\
&\quad \ydiagram{1,1,1,1,1}: && [0,1,0]\ ,
\end{aligned}
$$

$$
\begin{aligned}
\square\square: t^2(0,0)^{(0)} &+ t^3\left(\frac{1}{2},\frac{1}{2}\right)^{(0)} + t^4\left(2\cdot(1,1)^{(0/-2)} \oplus (0,0)^{(0)}\right) \\
&+ t^5\left(2\cdot\left(\frac{3}{2},\frac{3}{2}\right)^{(0/-2)} \oplus \left(\frac{1}{2},\frac{3}{2}\right)^{(-2)} \oplus \left(\frac{3}{2},\frac{1}{2}\right)^{(-2)} \oplus \left(\frac{1}{2},\frac{1}{2}\right)^{(0)}\right) + \mathcal{O}(t^6)
\end{aligned}
$$

$$
\begin{aligned}
\ydiagram{3}: &\quad t^3(0,0)^{(0)} + t^4(\tfrac{1}{2},\tfrac{1}{2})^{(0)} + t^5\left(2\cdot(1,1)^{(0/-3)} \oplus (0,0)^{(0)}\right) \\
&\quad + t^6\left(3\cdot(\tfrac{3}{2},\tfrac{3}{2}) \oplus (\tfrac{1}{2},\tfrac{3}{2})^{(-3)} \oplus (\tfrac{3}{2},\tfrac{1}{2})^{(-3)} \oplus 2\cdot(\tfrac{1}{2},\tfrac{1}{2})^{(0/-3)}\right) + \mathcal{O}(t^7) \\
\ydiagram{1,1,1}: &\quad t^5\left((1,0)^{(-3)} \oplus (0,1)^{(-3)}\right) + t^6\left((\tfrac{3}{2},\tfrac{3}{2}) \oplus (\tfrac{1}{2},\tfrac{3}{2})^{(-3)} \oplus (\tfrac{3}{2},\tfrac{1}{2})^{(-3)} \oplus 2\cdot(\tfrac{1}{2},\tfrac{1}{2})^{(-3)}\right) \\
&\quad + \mathcal{O}(t^7)\ ,
\end{aligned}
$$

$$
\begin{aligned}
\ydiagram{4}: &\quad t^4(0,0)^{(0)} + t^5(\tfrac{1}{2},\tfrac{1}{2})^{(0)} + t^6\left(2\cdot(1,1)^{(0/-4)} \oplus (0,0)^{(0)}\right) \\
&\quad + t^7\left(3\cdot(\tfrac{3}{2},\tfrac{3}{2}) \oplus (\tfrac{1}{2},\tfrac{3}{2}) \oplus (\tfrac{3}{2},\tfrac{1}{2}) \oplus 2\cdot(\tfrac{1}{2},\tfrac{1}{2})^{(0/-4)}\right) + \mathcal{O}(t^8) \\
\ydiagram{2,2}: &\quad t^6\left((1,1)^{(-4)} \oplus (0,0)^{(-4)}\right) + t^7\left((\tfrac{3}{2},\tfrac{3}{2}) \oplus 2\cdot(\tfrac{1}{2},\tfrac{3}{2}) \oplus 2\cdot(\tfrac{3}{2},\tfrac{1}{2}) \oplus 2\cdot(\tfrac{1}{2},\tfrac{1}{2})^{(-4)}\right) \\
&\quad + \mathcal{O}(t^8) \\
\ydiagram{2,1,1}: &\quad t^6\left((1,0)^{(-4)} \oplus (0,1)^{(-4)}\right) + t^7\left((\tfrac{3}{2},\tfrac{3}{2}) \oplus 2\cdot(\tfrac{1}{2},\tfrac{3}{2}) \oplus 2\cdot(\tfrac{3}{2},\tfrac{1}{2}) \oplus 3\cdot(\tfrac{1}{2},\tfrac{1}{2})^{(-4)}\right) \\
&\quad + \mathcal{O}(t^8)\ ,
\end{aligned}
$$

$$\left\{\psi_r^\alpha, \chi_s^\beta\right\} = \epsilon^{\alpha\beta}\delta_{r,-s}, \left[\xi_r^\alpha, \eta_s^\beta\right] = \epsilon^{\alpha\beta}\delta_{r,-s}$$

$$
\begin{aligned}
J_m^3 &= -\frac{1}{2}(\eta^+\xi^-)_m - \frac{1}{2}(\eta^-\xi^+)_m, K_m^3 = -\frac{1}{2}(\chi^+\psi^-)_m - \frac{1}{2}(\chi^-\psi^+)_m \\
J_m^\pm &= (\eta^\pm\xi^\pm)_m, K_m^\pm = \pm(\chi^\pm\psi^\pm)_m \\
S_m^{\alpha\beta+} &= \left(\chi^\beta\xi^\alpha\right)_m, S_m^{\alpha\beta-} = -\left(\eta^\alpha\psi^\beta\right)_m \\
U_m &= -\frac{1}{2}(\eta^+\xi^-)_m + \frac{1}{2}(\eta^-\xi^+)_m, V_m = -\frac{1}{2}(\chi^+\psi^-)_m + \frac{1}{2}(\chi^-\psi^+)_m
\end{aligned}
$$

$$\tilde{\xi}_r^\pm = \xi_{r\pm\frac{w}{2}}^\pm, \tilde{\eta}_r^\pm = \eta_{r\pm\frac{w}{2}}^\pm, \tilde{\psi}_r^\pm = \psi_{r\mp\frac{w}{2}}^\pm, \tilde{\chi}_r^\pm = \chi_{r\mp\frac{w}{2}}^\pm$$

$$\tilde{J}_n^3 = J_n^3 - \frac{w}{2}\delta_{n,0} \qquad \tilde{K}_n^3 = K_n^3 - \frac{w}{2}\delta_{n,0}$$
$$\tilde{U}_m = U_n \qquad \tilde{V}_n = V_n$$
$$\tilde{L}_n = L_n - w(K_n^3 - J_n^3), \qquad \tilde{Z}_n = Z_n$$

$$\eta_r^{\pm}|0\rangle = \xi_r^{\pm}|0\rangle = \chi_r^{\pm}|0\rangle = \psi_r^{\pm}|0\rangle = 0 \; r \geq \frac{1}{2}$$

$$U_0|0\rangle = V_0|0\rangle = Z_0|0\rangle = 0$$

$$|0\rangle_1 = \left|\frac{1}{2}, 0\right\rangle$$

$$J_0^3|m_1, m_2\rangle = (m_1 + m_2)|m_1, m_2\rangle$$
$$U_0|m_1, m_2\rangle = \left(m_1 - m_2 - \frac{1}{2}\right)|m_1, m_2\rangle$$

$$\chi_0^{+}|m_1, m_2\rangle = \psi_0^{+}|m_1, m_2\rangle = 0, K_0^3|m_1, m_2\rangle = +\frac{1}{2}|m_1, m_2\rangle$$

$$Z^{(3)}_{\square\square\square}(t, y) = \frac{1}{6}\left(\left(Z^{(1)}(t, y)\right)^3 + 3\,\tilde{Z}^{(1)}(t^2, y^2)Z^{(1)}(t, y) + 2Z^{(1)}(t^3, y^3)\right)$$
$$Z^{(3)}_{\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}}(t, y) = \frac{1}{6}\left(\left(Z^{(1)}(t, y)\right)^3 - 3\,\tilde{Z}^{(1)}(t^2, y^2)Z^{(1)}(t, y) + 2Z^{(1)}(t^3, y^3)\right)$$

$$\chi_h^{(\mathrm{BPS})}(t, y) = \frac{t^{h+1}}{1-t}\left(\chi_{h+1}(y) + 2t^{1/2}\chi_{h+\frac{1}{2}}(y) + t\chi_h(y)\right)$$

$$Z^{(3)}_{\square\square\square}(t, y) = \chi^{(\mathrm{BPS})}_{\frac{1}{2}}(t, y) + \sum_{n=2}^{\infty} c_n\,\chi_{\frac{1}{2}}(y)\,\chi^{\mathcal{N}=4}_{\frac{1}{2}+n}(t, y) + 2\sum_{n=0}^{\infty} c_n\,\chi^{\mathcal{N}=4}_{5+n}(t, y)$$
$$Z^{(3)}_{\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}}(t, y) = 2\sum_{n=0}^{\infty} c_n\,\chi^{\mathcal{N}=4}_{2+n}(t, y) + \sum_{n=0}^{\infty} c_n\,\chi_{\frac{1}{2}}(y)\,\chi^{\mathcal{N}=4}_{\frac{7}{2}+n}(t, y)\ ,$$

$$\sum_{n=0}^{\infty} c_n x^n = \frac{1}{(1-x^2)(1-x^3)}$$

$$c_n = 1 + \left\lfloor\frac{n}{6}\right\rfloor - \delta_{n,1}^{(6)}, \delta_{n,1}^{(6)} = \begin{cases}1 & \text{if } n = 1 \bmod 6\\ 0 & \text{otherwise}\end{cases}$$

$$S_2^{\eta\xi} = \eta_1^{+}\xi_1^{+}, S_1^{\eta\xi} = \eta_1^{+}\xi_0^{+} + \eta_0^{+}\xi_1^{+}$$

$$S_2^{\eta\xi}S_2^{\chi\psi} = S_2^{\chi\xi}S_2^{\eta\psi}$$
$$S_2^{\eta\xi}S_1^{\chi\psi} + S_1^{\eta\xi}S_2^{\chi\psi} = S_2^{\chi\xi}S_1^{\eta\psi} + S_1^{\chi\xi}S_2^{\eta\psi}$$

$$\left(S_2^{\eta\xi}\right)^{3m-1} S_2^{\chi\psi}|0\rangle_3 \leftrightarrow \chi_{\frac{1}{2}}(y)\chi^{\mathcal{N}=4}_{3m+\frac{1}{2}}(t, y), \quad (C.20)$$
$$\left(S_2^{\eta\xi}\right)^{3m-r}\left(S_1^{\eta\xi}\right)^{2r-1} S_1^{\chi\psi}|0\rangle_3 \leftrightarrow \chi_{\frac{1}{2}}(y)\chi^{\mathcal{N}=4}_{3m+\frac{1}{2}+r}(t, y), (r = 1, \ldots, 3m), (r = 0, \ldots, 3m-2),$$
$$\left(S_2^{\eta\xi}\right)^{3m-2-r}\left(S_1^{\eta\xi}\right)^{2r+1} S_1^{\chi\xi}S_2^{\chi\psi}|0\rangle_3 \leftrightarrow \chi^{\mathcal{N}=4}_{3m+r+1}(t, y), (r = 0, \ldots, 3m-2),$$
$$\left(S_2^{\eta\xi}\right)^{3m-2-r}\left(S_1^{\eta\xi}\right)^{2r+1} S_1^{\eta\psi}S_2^{\chi\psi}|0\rangle_3 \leftrightarrow \chi^{\mathcal{N}=4}_{3m+r+1}(t, y), (r = 0, \ldots, 3m-2),$$

$$\left(S_2^{\eta\xi}\right)^{3m+1-r}\left(S_1^{\eta\xi}\right)^{2r}S_1^{\chi\psi}|0\rangle_3\leftrightarrow\chi_{\frac{1}{2}}(y)\chi^{\mathcal{N}=4}_{3m+\frac{5}{2}+r}(t,y),(r=0,\dots,3m+1)$$
$$\left(S_2^{\eta\xi}\right)^{3m-r}\left(S_1^{\eta\xi}\right)^{2r}S_1^{\chi\xi}S_2^{\chi\psi}|0\rangle_3\leftrightarrow\chi^{\mathcal{N}=4}_{3m+r+2}(t,y),(r=0,\dots,3m)$$
$$\left(S_2^{\eta\xi}\right)^{3m-r}\left(S_1^{\eta\xi}\right)^{2r}S_1^{\eta\psi}S_2^{\chi\psi}|0\rangle_3\leftrightarrow\chi^{\mathcal{N}=4}_{3m+r+2}(t,y),(r=0,\dots,3m)$$

$$Z^{(3)}_{L_0=-6m}(t,y)=\sum_{r=0}^{3m}\chi_{\frac{1}{2}}(y)\chi^{\mathcal{N}=4}_{3m+\frac{1}{2}+r}(t,y)+2\sum_{r=0}^{3m-2}\chi^{\mathcal{N}=4}_{3m+1+r}(t,y)$$
$$Z^{(3)}_{L_0=-6m-3}(t,y)=\sum_{r=0}^{3m+1}\chi_{\frac{1}{2}}(y)\chi^{\mathcal{N}=4}_{3m+\frac{5}{2}+r}(t,y)+2\sum_{r=0}^{3m}\chi^{\mathcal{N}=4}_{3m+2+r}(t,y)$$

$$Z^{(3)}_{\square\square\square}(t,y)+Z^{(3)}_{\substack{\square\\\square\\\square}}(t,y)=\chi^{(\mathrm{BPS})}_{\frac{1}{2}}(t,y)+\sum_{m=0}^{\infty}\left[Z^{(3)}_{L_0=-6m-6}(t,y)+Z^{(3)}_{L_0=-6m-3}(t,y)\right]$$

$$(\partial\phi^1,\partial\phi^2),(\partial\bar\phi^1,\partial\bar\phi^2),(\psi^1,\psi^2),(\bar\psi^1,\bar\psi^2)$$

$$\partial\phi^i(z)\partial\bar\phi^j(w)\sim\frac{\delta^{ij}}{(z-w)^2},\psi^i(z)\bar\psi^j(w)\sim\frac{\delta^{ij}}{z-w}$$

$$\begin{aligned}J&=\bar\psi^1\psi^1+\bar\psi^2\psi^2\\G^+&=\sqrt{2}(\partial\phi^1\bar\psi^1+\partial\phi^2\bar\psi^2)\\G^-&=\sqrt{2}(\partial\bar\phi^1\psi^1+\partial\bar\phi^2\psi^2)\\L&=\frac{1}{2}(\partial\bar\psi^1\psi^1-\bar\psi^1\partial\psi^1+\partial\bar\psi^2\psi^2-\bar\psi^2\partial\psi^2)+\partial\bar\phi^1\partial\phi^1+\partial\bar\phi^2\partial\phi^2\end{aligned}$$

$$J^+=\bar\psi^1\bar\psi^2,J^-=\psi^2\psi^1$$

$$G'^+=\sqrt{2}(\bar\psi^1\partial\bar\phi^2-\bar\psi^2\partial\bar\phi^1),G'^-=\sqrt{2}(\psi^1\partial\phi^2-\psi^2\partial\phi^1)$$

$$\partial^lS^i\partial^m\bar S^i,i=1,2,\partial^lS^1\partial^mS^2,\partial^l\bar S^1\partial^m\bar S^2$$

$$W^{(1)}=\bar\psi^1\psi^1-\bar\psi^2\psi^2.$$

$$S^1\mapsto S^2\mapsto -S^1,\bar S^1\mapsto\bar S^2\mapsto-\bar S^1.$$

$$\bar\psi^1_{-r}|0\rangle_{\mathrm{NS}},\psi^2_{-r}|0\rangle_{\mathrm{NS}},\bar\alpha^1_{-n}|0\rangle_{\mathrm{NS}},\alpha^2_{-n}|0\rangle_{\mathrm{NS}},$$

$$\psi^1_{-r}|0\rangle_{\mathrm{NS}},\bar\psi^2_{-r}|0\rangle_{\mathrm{NS}},\alpha^1_{-n}|0\rangle_{\mathrm{NS}},\bar\alpha^2_{-n}|0\rangle_{\mathrm{NS}}.$$

$$\alpha_0^i|0\rangle^{(2)}=\bar\alpha_0^i|0\rangle^{(2)}=0,\bar\psi_0^1|0\rangle^{(2)}=\bar\psi_0^2|0\rangle^{(2)}=0$$

$$J_0^-|0\rangle^{(2)}=\psi_0^2\psi_0^1|0\rangle^{(2)}$$

$$G^-_{-\frac{1}{2}}|0\rangle^{(2)}=\left(\bar\alpha^1_{-\frac{1}{2}}\psi_0^1+\bar\alpha^2_{-\frac{1}{2}}\psi_0^2\right)|0\rangle^{(2)}$$
$$G'^-_{-\frac{1}{2}}|0\rangle^{(2)}=\left(\alpha^2_{-\frac{1}{2}}\psi_0^1-\alpha^1_{-\frac{1}{2}}\psi_0^2\right)|0\rangle^{(2)}$$

$$|0\rangle_w=[|0\rangle]^{(w)}$$

$$\lambda_r^\alpha[|0\rangle]^{(w)}=[\sigma^w(\lambda_r^\alpha)|0\rangle]^{(w)}$$

$$\sigma^w(\lambda_r^\alpha)=\lambda^\alpha_{r+\frac{w}{2}}$$

$$S=\frac{1}{g^2}\mathrm{Tr}_{U(\hat N)}\left(\frac{1}{4}[\hat A_\mu,\hat A_\nu]^2\right)$$

$$S=\int d^dx\frac{1}{\tilde{g}^2}\mathrm{tr}\left(-\frac{1}{4}F_{\mu\nu}^2\right)_*$$

$$S=\frac{\hat{N}}{g_m}2\pi i\tau\mathrm{Str}\left(\left\{\hat{A}_\alpha,\overline{\hat{A}}_{\dot{\alpha}}\right\}^2\right)$$

$$S=\frac{\hat{N}}{\tilde{g}_m}\int d^2\theta 2\pi i\tau\mathrm{Tr}_{U(\hat{N})}\left(-\frac{1}{4}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}e^{-\hat{V}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{V}}\right)_\star^2+\text{c.c.}$$

$$S=\int d^4xd^2\theta 2\pi i\tau\mathrm{Tr}_{U(n)}\left(-\frac{1}{4}\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}e^{-V}D_\alpha e^V\right)_*^2+\text{c.c..}$$

$$S=\frac{\hat{N}}{g_m}2\pi i\tau\mathrm{Str}\left(\epsilon^{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\left\{\hat{A}_\alpha,\overline{\hat{A}}^{\dot{\alpha}}\right\}\left\{\hat{A}_\beta,\overline{\hat{A}}^{\dot{\beta}}\right\}\right),$$

$$\hat{A}_\alpha=A_{i\alpha}^aT^a\otimes t^i$$

$$\mathrm{Str}(\hat{O})\equiv\mathrm{Tr}\left(\begin{pmatrix}1_{2\hat{N}} & \\ & -1_{2\hat{N}}\end{pmatrix}\hat{O}\right)$$

$$\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}\to\mathrm{Tr}\left(\epsilon^{\mu\nu\rho\sigma}[\hat{A}_\mu,\hat{A}_\nu][\hat{A}_\rho,\hat{A}_\sigma]\right)=0$$

$$\hat{A}'_\alpha=e^{-i\hat{K}}\hat{A}_\alpha e^{i\hat{K}},\overline{\hat{A}}'_{\dot{\alpha}}=e^{-i\hat{K}}\overline{\hat{A}}_{\dot{\alpha}}e^{i\hat{K}}$$

$$\hat{K}=K_i^aT^a\otimes t^i$$

$$\{\hat{A}_\alpha,\hat{A}_\beta\}=\left\{\overline{\hat{A}}_{\dot{\alpha}},\overline{\hat{A}}_{\dot{\beta}}\right\}=0$$

$$\left\{\hat{\theta}^\alpha,\overline{\hat{\theta}}^{\dot{\alpha}}\right\}=\gamma^{\alpha\dot{\alpha}}$$

$$\{\hat{\theta}^\alpha,\hat{\theta}^\beta\}=\left\{\overline{\hat{\theta}}^{\dot{\alpha}},\overline{\hat{\theta}}^{\dot{\beta}}\right\}=0$$

$$\hat{\theta}^1=\sqrt{\gamma}\begin{pmatrix}0&0&0&1\\0&0&0&0\\0&-1&0&0\\0&0&0&0\end{pmatrix},\hat{\theta}^2=\sqrt{\gamma}\begin{pmatrix}0&0&1&0\\0&0&0&0\\0&0&0&0\\0&1&0&0\end{pmatrix},$$

$$\overline{\hat{\theta}}^{\mathrm{i}}=\sqrt{\gamma}\begin{pmatrix}0&0&0&0\\0&0&-1&0\\0&0&0&0\\1&0&0&0\end{pmatrix},\overline{\hat{\theta}}^2=\sqrt{\gamma}\begin{pmatrix}0&0&0&0\\0&0&0&1\\1&0&0&0\\0&0&0&0\end{pmatrix}.$$

$$\hat{O}=\int 2^4d^2\kappa d^2\bar{\kappa}\tilde{O}(\kappa,\bar{\kappa})e^{\kappa^\alpha\hat{\theta}_\alpha+\bar{\kappa}_{\dot{\alpha}}\overline{\hat{\theta}}^{\dot{\alpha}}}\leftrightarrow O(\theta,\bar{\theta})=\int 2^4d^2\kappa d^2\bar{\kappa}\tilde{O}(\kappa,\bar{\kappa})e^{\kappa^\alpha\theta_\alpha+\bar{\kappa}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}}$$

$$O_1\star O_2(\theta,\bar{\theta})=\exp\left(-\frac{1}{2}\gamma^{\alpha\dot{\alpha}}\left(\frac{\partial}{\partial\theta_1^\alpha}\frac{\partial}{\partial\bar{\theta}_2^{\dot{\alpha}}}+\frac{\partial}{\partial\bar{\theta}_1^{\dot{\alpha}}}\frac{\partial}{\partial\theta_2^\alpha}\right)\right)O_1(\theta_1,\bar{\theta}_1)O_2(\theta_2,\bar{\theta}_2)\Big|_{\theta=\theta_1=\theta_2}$$

$$\hat{\pi}_\alpha\equiv\beta_{\alpha\dot{\alpha}}\overline{\hat{\theta}}^{\dot{\alpha}},\overline{\hat{\pi}}_{\dot{\alpha}}\equiv\beta_{\alpha\dot{\alpha}}\hat{\theta}^\alpha,\hat{\pi}_\alpha\equiv\beta_{\alpha\dot{\alpha}}\overline{\hat{\theta}}^{\dot{\alpha}},\overline{\hat{\pi}}_{\dot{\alpha}}\equiv\beta_{\alpha\dot{\alpha}}\hat{\theta}^\alpha$$

$$\gamma^{\alpha\dot{\alpha}}\beta_{\dot{\alpha}\beta}=\delta_\beta^\alpha,\beta_{\dot{\alpha}\alpha}\gamma^{\alpha\dot{\beta}}=\delta_{\dot{\alpha}}^{\dot{\beta}}$$

$$\{\hat{\pi}_\alpha,\hat{\theta}^\beta\}=\delta_\alpha^\beta,\left\{\overline{\hat{\pi}}_{\dot{\alpha}},\overline{\hat{\theta}}^{\dot{\beta}}\right\}=\delta_{\dot{\alpha}}^{\dot{\beta}},\{\hat{\pi}_\alpha,\overline{\hat{\pi}}_{\dot{\alpha}}\}=\beta_{\alpha\dot{\alpha}}$$

$$\left\{\hat{\pi}_\alpha,\overline{\hat{\theta}}^{\dot{\alpha}}\right\}=\{\overline{\hat{\pi}}_{\dot{\alpha}},\hat{\theta}^\alpha\}=\{\hat{\pi}_\alpha,\hat{\pi}_\beta\}=\{\overline{\hat{\pi}}_{\dot{\alpha}},\overline{\hat{\pi}}_{\dot{\beta}}\}=0$$

$$\int d^2\theta d^2\bar{\theta} O = \frac{1}{4\gamma^2}\mathrm{Tr}_{GL(4,R)}\left(\begin{pmatrix} 1_2 & \\ & -1_2 \end{pmatrix}\hat{O}\right) \equiv \frac{1}{4\det\gamma}\mathrm{Str}\hat{O}$$

$$S = \frac{\hat{N}}{g_m}2\pi i\tau \mathrm{Str}\left(\epsilon^{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\left\{\hat{A}_\alpha, \bar{\hat{A}}^{\dot{\alpha}}\right\}\left\{\hat{A}_\beta, \bar{\hat{A}}^{\dot{\beta}}\right\} + \hat{\Lambda}^{\alpha\beta}\{\hat{A}_\alpha, \hat{A}_\beta\} + \bar{\hat{\Lambda}}_{\dot{\alpha}\dot{\beta}}\left\{\bar{\hat{A}}^{\dot{\alpha}}, \bar{\hat{A}}^{\dot{\beta}}\right\}\right),$$

$$\left[\bar{\hat{A}}_{\dot{\alpha}}, \left\{\bar{\hat{A}}^{\dot{\alpha}}, \hat{A}^\alpha\right\}\right] + [\hat{\Lambda}^{\alpha\beta}, \hat{A}_\beta] = 0$$

$$\left[\hat{A}^\alpha, \left\{\hat{A}_\alpha, \bar{\hat{A}}_{\dot{\alpha}}\right\}\right] + \left[\bar{\hat{\Lambda}}_{\dot{\alpha}\dot{\beta}}, \bar{\hat{A}}^{\dot{\beta}}\right] = 0$$

$$\left\{\hat{A}_\alpha, \hat{A}_\beta\right\} = \left\{\bar{\hat{A}}_{\dot{\alpha}}, \bar{\hat{A}}_{\dot{\beta}}\right\} = 0$$

$$\hat{A}_\alpha = \hat{\pi}_\alpha \otimes 1_{\hat{N}}, \bar{\hat{A}}_{\dot{\alpha}} = \bar{\hat{\pi}}_{\dot{\alpha}} \otimes 1_{\hat{N}}, \hat{\Lambda}_{\alpha\beta} = \bar{\hat{\Lambda}}_{\dot{\alpha}\dot{\beta}} = 0$$

$$\hat{A}_\alpha = e^{-\hat{\omega}}\hat{\pi}_\alpha e^{\hat{\omega}}, \bar{\hat{A}}_{\dot{\alpha}} = e^{\bar{\hat{\omega}}}\bar{\hat{\pi}}_{\dot{\alpha}} e^{-\bar{\hat{\omega}}}$$

$$S = \frac{\hat{N}}{g_m}2\pi i\tau \mathrm{Str}\left(\left\{e^{-\hat{\omega}}\hat{\pi}_\alpha e^{\hat{\omega}}, e^{\bar{\hat{\omega}}}\bar{\hat{\pi}}_{\dot{\alpha}} e^{-\bar{\hat{\omega}}}\right\}^2\right)$$

$$S = \frac{\hat{N}}{g_m}4\det\gamma\int d^2\theta d^2\bar{\theta}\, 2\pi i\tau \mathrm{Tr}_{U(\hat{N})}\left(\left(\left\{e^{-\hat{\Omega}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{\Omega}}, e^{\bar{\hat{\Omega}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\bar{\hat{\Omega}}}\right\} + \beta_{\alpha\dot{\alpha}}\right)^2\right)_\star$$

$$\begin{aligned} S =& \frac{\hat{N}}{g_m}4\det\gamma\int d^2\theta d^2\bar{\theta}\, 2\pi i\tau \mathrm{Tr}\left(\left\{e^{-\hat{\Omega}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{\Omega}}, e^{\bar{\hat{\Omega}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\bar{\hat{\Omega}}}\right\}^2\right. \\ &\left. +2\beta^{\alpha\dot{\alpha}}\left\{e^{-\hat{\Omega}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{\Omega}}, e^{\bar{\hat{\Omega}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\bar{\hat{\Omega}}}\right\} + (\beta^{\alpha\dot{\alpha}})^2\right)_\star \end{aligned}$$

$$\begin{aligned} S =& \frac{\hat{N}}{g_m}4\det\gamma\int d^2\theta d^2\bar{\theta}\, 2\pi i\tau \mathrm{Tr}\left(\left\{e^{-\hat{\Omega}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{\Omega}}, e^{\bar{\hat{\Omega}}}\frac{\partial}{\partial\bar{\theta}\dot{\alpha}}e^{-\bar{\hat{\Omega}}}\right\}^2\right)_\star \\ =& 2\frac{\hat{N}}{g_m}\det\gamma\int d^2\theta d^2\bar{\theta}\, 2\pi i\tau \mathrm{Tr}\left(e^{-\bar{\hat{\Omega}}}\left\{e^{-\hat{\Omega}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{\Omega}}, e^{\bar{\hat{\Omega}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\bar{\hat{\Omega}}}\right\}e^{\bar{\hat{\Omega}}}\right)^2_\star \\ &+2\frac{\hat{N}}{g_m}\det\gamma\int d^2\theta d^2\bar{\theta}\, 2\pi i\tau \mathrm{Tr}\left(e^{\hat{\Omega}}\left\{e^{-\hat{\Omega}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{\Omega}}, e^{\bar{\hat{\Omega}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\bar{\hat{\Omega}}}\right\}e^{-\hat{\Omega}}\right)^2_\star \end{aligned}$$

$$e^{\hat{\mathcal{V}}} \equiv e^{\hat{\Omega}}e^{\bar{\hat{\Omega}}}$$

$$\begin{aligned} S =& \quad 2\frac{\hat{N}}{g_m}\det\gamma\int d^2\theta d^2\bar{\theta}\, 2\pi i\tau \mathrm{Tr}\left(\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\hat{\mathcal{V}}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{\mathcal{V}}}\right)^2_\star \\ &+2\frac{\hat{N}}{g_m}\det\gamma\int d^2\theta d^2\bar{\theta}\, 2\pi i\tau \mathrm{Tr}\left(\frac{\partial}{\partial\theta^\alpha}e^{\hat{\mathcal{V}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\hat{\mathcal{V}}}\right)^2_\star \end{aligned}$$

$$\int d^2\bar{\theta} = -\frac{1}{4}\epsilon^{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}$$

$$S = 8\frac{\hat{N}}{g_m}\det\gamma\int d^2\theta\, 2\pi i\tau \mathrm{Tr}\left(-\frac{1}{4}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}e^{-\hat{\mathcal{V}}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{\mathcal{V}}}\right)^2_\star$$

$$+8\frac{\hat{N}}{g_m}\det\gamma\int d^2\bar{\theta}\, 2\pi i\tau \mathrm{Tr}\left(-\frac{1}{4}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\alpha}e^{\hat{\mathcal{V}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\hat{\mathcal{V}}}\right)^2_\star$$

$$S = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[\hat{A}_\mu, \hat{A}_\nu]^2 + \frac{1}{2}\bar{\hat{\Psi}}\Gamma^\mu[\hat{A}_\mu, \hat{\Psi}]\right),$$

$$(\delta e^{\hat{\mathcal{V}}})e^{-\hat{\mathcal{V}}}\frac{\partial}{\partial\theta_\alpha}\left(e^{\hat{\mathcal{V}}}\left(-\frac{1}{4}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}e^{-\hat{\mathcal{V}}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{\mathcal{V}}}\right)e^{-\hat{\mathcal{V}}}\right) = 0$$

$$e^{\hat{\mathcal{V}}} = e^{-2\theta\sigma^\mu\bar{\theta}\hat{p}_\mu\otimes 1_n}$$

$$\hat{A}_\mu = \hat{p}_\mu \otimes 1_n$$

$$\hat{A}_\alpha = e^{\hat{\theta}\sigma^\mu\bar{\hat{\theta}}\hat{p}_\mu}\hat{\pi}_\alpha e^{-\hat{\theta}\sigma^\mu\bar{\hat{\theta}}\hat{p}_\mu}$$

$$e^{\hat{\mathcal{V}}} = e^{-\theta\sigma^\mu\bar{\theta}\hat{p}_\mu}e^{\hat{V}}e^{-\theta\sigma^\mu\bar{\theta}\hat{p}_\mu} = e^{\hat{A}}e^{\hat{V}}e^{\hat{A}}$$

$$\begin{aligned} S &= \frac{8\hat{N}\det\gamma}{g_m}\int d^2\theta 2\pi i\tau \mathrm{Tr}\left(-\frac{1}{4}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}e^{-\hat{A}}e^{-\hat{V}}e^{-\hat{A}}\frac{\partial}{\partial\theta^\alpha}e^{\hat{A}}e^{\hat{V}}e^{\hat{A}}\right)^2 + \text{c.c.} \\ &= \frac{8\hat{N}\det\gamma}{g_m}\int d^2\theta 2\pi i\tau \mathrm{Tr}\left(-\frac{1}{4}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}e^{-\hat{A}}e^{-\hat{V}}e^{\hat{A}}e^{-2\hat{A}}\frac{\partial}{\partial\theta^\alpha}e^{2\hat{A}}e^{-\hat{A}}e^{\hat{V}}e^{\hat{A}}\right)^2 + \text{c.c.} \end{aligned}$$

$$\begin{aligned} e^{-\hat{A}}f(\hat{x})e^{\hat{A}} &= e^{\theta\sigma^\mu\bar{\theta}\hat{p}_\mu}f(\hat{x})e^{-\theta\sigma^\mu\bar{\theta}\hat{p}_\mu} = f(\hat{x} - i\theta\sigma^\mu\bar{\theta}), \\ e^{-2\hat{A}}\frac{\partial}{\partial\theta^\alpha}e^{2\hat{A}} &= \frac{\partial}{\partial\theta^\alpha} - 2\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\hat{p}_\mu - 2(\theta\sigma^\mu\bar{\theta})\sigma^\nu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}iB_{\mu\nu}, \\ \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\hat{V}}\hat{p}_\mu e^{\hat{V}} &= \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\hat{V}}[\hat{p}_\mu, e^{\hat{V}}] + \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\hat{p}_\mu = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}e^{-\hat{V}}[\hat{p}_\mu, e^{\hat{V}}], \end{aligned}$$

$$\begin{aligned} S =& \frac{8\hat{N}\det\gamma}{g_m(2\pi)^2\sqrt{\det C}}2\pi i\tau \times \\ & \int d^4x d^2\theta \mathrm{Tr}_{U(n)}\left(-\frac{1}{4}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}e^{-V(x-i\theta\sigma\bar{\theta},\theta,\bar{\theta})}\left(\frac{\partial}{\partial\theta^\alpha} + 2i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\right)e^{V(x-i\theta\sigma\bar{\theta},\theta,\bar{\theta})}\right)^2_* + \text{c.c.} \end{aligned}$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial y^\mu}, \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$$

$$\frac{\hat{N}}{g_m} = \frac{(2\pi)^2\sqrt{\det C}}{8\det\gamma}$$

$$S = \int d^4y d^2\theta 2\pi i\tau \mathrm{Tr}\left(-\frac{1}{4}\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}e^{-V}D_\alpha e^V\right)^2_* + \text{c.c.}$$

$$\hat{A}_\alpha \leftrightarrow \nabla_\alpha, \bar{\hat{A}}_{\dot{\alpha}} \leftrightarrow \bar{\nabla}_{\dot{\alpha}}$$

$$\left[\bar{\hat{A}}_{\dot{\alpha}}, \hat{\Phi}\right] = 0, \left[\hat{A}_\alpha, \bar{\hat{\Phi}}\right] = 0$$

$$S = \frac{\hat{N}}{g_m}\mathrm{Str}\left(2\pi i\tau\left\{\hat{A}_\alpha, \bar{\hat{A}}_{\dot{\alpha}}\right\}^2 + 2\bar{\hat{\Phi}}\hat{\Phi}\right) + \frac{\hat{N}}{g_m}2\mathrm{Str}'(W(\hat{\Phi})) + \text{c.c.}$$

$$\{\hat{A}_\alpha, \hat{\Theta}^\beta\} = \delta^\beta_\alpha, \left\{\bar{\hat{A}}_{\dot{\alpha}}, \bar{\hat{\Theta}}^{\dot{\beta}}\right\} = \delta^{\dot{\beta}}_{\dot{\alpha}}$$

$$\mathrm{Str}'\hat{O} \equiv \mathrm{Str}\left(\bar{\hat{\Theta}}^2\hat{O}\right)$$

$$\left[\bar{\hat{\pi}}_{\dot{\alpha}}, \hat{\Phi}_0\right] = 0, \left[\hat{\pi}_\alpha, \bar{\hat{\Phi}}_0\right] = 0$$

$$\begin{aligned} \mathrm{Str}'W(\hat{\Phi}) = \mathrm{Str}\left(\bar{\hat{\theta}}^2 W(\hat{\Phi}_0)\right) &= 4\det\gamma\int d^2\theta d^2\bar{\theta}\mathrm{Tr}_{U(\hat{N})}\left(\bar{\theta}^2 W(\hat{\Phi}_0(\theta))\right)_\star \\ &= 4\det\gamma\int d^2\theta \mathrm{Tr}_{U(\hat{N})}\left(W(\hat{\Phi}_0(\theta))\right)_\star \end{aligned}$$

$$\begin{aligned} S =& \frac{\hat{N}}{g_m}2\mathrm{Str}\left(e^{-\hat{\mathcal{V}}}\bar{\hat{\Phi}}_0 e^{\hat{\mathcal{V}}}\hat{\Phi}_0\right) \\ &+ \frac{\hat{N}}{g_m}2\mathrm{Str}\bar{\hat{\theta}}^2\left(2\pi i\tau\left(-\frac{1}{4}\left[\bar{\hat{\pi}}_{\dot{\alpha}}, \left\{\bar{\hat{\pi}}^{\dot{\alpha}}, e^{-\hat{\mathcal{V}}}\left[\hat{\pi}_\alpha, e^{\hat{\mathcal{V}}}\right]\right\}\right]\right)^2 + W(\hat{\Phi}_0)\right) + \text{c.c.} \end{aligned}$$

$$S=\frac{\hat{N}}{g_m}2\pi i\tau\mathrm{Str}\left(\left\{\hat{A}_\alpha,\overline{\hat{A}}_{\dot\alpha}\right\}^2+2\overline{\hat{\Phi}}\hat{\Phi}\right)$$

$$\begin{aligned}\delta_\epsilon\hat{A}_\alpha&=\epsilon_\alpha\overline{\hat{\Phi}}\\ \delta_\epsilon\hat{\Phi}&=-\epsilon^\alpha\left[\overline{\hat{A}}_{\dot\alpha},\left\{\overline{\hat{A}}^{\dot\alpha},\hat{A}_\alpha\right\}\right]\\ \delta_{\bar\epsilon}\overline{\hat{A}}_{\dot\alpha}&=\bar\epsilon_{\dot\alpha}\hat{\Phi}\\ \delta_{\bar\epsilon}\overline{\hat{\Phi}}&=-\bar\epsilon_{\dot\alpha}\left[\hat{A}^\alpha,\left\{\hat{A}_\alpha,\overline{\hat{A}}^{\dot\alpha}\right\}\right]\end{aligned}$$

$$(\delta_{\bar\epsilon}\delta_\epsilon-\delta_\epsilon\delta_{\bar\epsilon})\hat{\Phi}=2\epsilon^\alpha\bar\epsilon_{\dot\alpha}\left[\left\{\overline{\hat{A}}^{\dot\alpha},\hat{A}_\alpha\right\},\hat{\Phi}\right]$$

$$\begin{aligned}S=&\frac{\hat{N}}{g_m}\mathrm{Str}\left(\left\{\hat{A}_\alpha,\overline{\hat{A}}_{\dot\alpha}\right\}^2+2\overline{\hat{\Phi}}_1\hat{\Phi}_1+2\overline{\hat{\Phi}}_2\hat{\Phi}_2+2\overline{\hat{\Phi}}_3\hat{\Phi}_3\right)\\ &+\frac{\hat{N}}{g_m}2\mathrm{Str}'(\hat{\Phi}_1[\hat{\Phi}_2,\hat{\Phi}_3])+\text{ c.c.}\end{aligned}$$

$$\left\{\hat\theta^\alpha,\overline{\hat\theta}^{\dot\alpha}\right\}=\gamma^{\alpha\dot\alpha},\{\hat\theta^\alpha,\hat\theta^\beta\}=\{\overline{\hat\theta}\dot\alpha,\overline{\hat\theta}\dot\beta\}=0$$

$$\{\hat\theta^\alpha,\hat\theta^\beta\}=\gamma^{\alpha\beta},\left\{\overline{\hat\theta}^{\dot\alpha},\overline{\hat\theta}^{\dot\beta}\right\}=\gamma^{*\dot\alpha\dot\beta},\left\{\hat\theta^\alpha,\overline{\hat\theta}^{\dot\alpha}\right\}=0$$

$$\hat\pi_\alpha=\beta_{\alpha\beta}\hat\theta^\beta,\overline{\hat\pi}_{\dot\alpha}=\beta^*_{\dot\alpha\dot\beta}\overline{\hat\theta}^{\dot\beta}$$

$$\beta_{\alpha\beta}\gamma^{\beta\gamma}=\delta_\alpha^\gamma,\beta^*_{\dot\alpha\dot\beta}\gamma^{\dot\beta\dot\gamma}=\delta_{\dot\alpha}^{\dot\gamma}$$

$$\bar{D}_{\dot\alpha}\Phi=0,D_\alpha\bar\Phi=0$$

$$\begin{aligned}D_\alpha&=\frac{\partial}{\partial\theta^\alpha}+i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu=e^{-A}\frac{\partial}{\partial\theta^\alpha}e^A\\ \bar{D}_{\dot\alpha}&=-\frac{\partial}{\partial\bar\theta^{\dot\alpha}}-i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu=e^A\left(-\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\right)e^{-A}\end{aligned}$$

$$\Phi=\Phi(e^Axe^{-A},e^A\theta e^{-A}),\bar\Phi=\bar\Phi(e^{-A}xe^A,e^{-A}\bar\theta e^A)$$

$$\begin{aligned}e^Ax^\mu e^{-A}&=x^\mu+i\theta\sigma^\mu\bar\theta=y^\mu,e^A\theta^\alpha e^{-A}=\theta^\alpha\\ e^{-A}x^\mu e^A&=x^\mu-i\theta\sigma^\mu\bar\theta=\bar y^\mu,e^{-A}\bar\theta^{\dot\alpha}e^A=\bar\theta^{\dot\alpha}\end{aligned}$$

$$\begin{aligned}D_\alpha&\equiv e^{-A}\star\frac{\partial}{\partial\theta^\alpha}\star e^A=\frac{\partial}{\partial\theta^\alpha}+i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu+\cdots\\ \bar D_{\dot\alpha}&\equiv e^A\star\left(-\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\right)\star e^{-A}=-\frac{\partial}{\partial\bar\theta^{\dot\alpha}}-i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu+\cdots\end{aligned}$$

$$\begin{aligned}\Phi&=\Phi(e^A\star x\star e^{-A},e^A\star\theta\star e^{-A})=e^A\star\Phi(x,\theta)\star e^{-A}\\ \bar\Phi&=\bar\Phi(e^{-A}\star x\star e^A,e^{-A}\star\bar\theta\star e^A)=e^{-A}\star\bar\Phi(x,\bar\theta)\star e^A\end{aligned}$$

$$O(\theta,\bar\theta)=\int 2^4d^2\kappa d^2\bar\kappa\tilde{O}(\kappa,\bar\kappa)e^{\kappa^\alpha\theta_\alpha+\bar\kappa_{\dot\alpha}\bar\theta^{\dot\alpha}}$$

$$O_1\star O_2(\theta,\bar\theta)=\exp\left(-\frac{1}{2}\gamma^{\alpha\beta}\frac{\partial}{\partial\theta_1^\alpha}\frac{\partial}{\partial\theta_2^\beta}-\frac{1}{2}\gamma^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_1^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_2^{\dot\beta}}\right)O_1(\theta_1,\bar\theta_1)O_2(\theta_2,\bar\theta_2)\Big|_{\theta=\theta_1=\theta_2}$$

$$\begin{aligned}S=&\int d^4xd^2\theta d^2\bar\theta\mathrm{Tr}\left(e^{-A}\bar\Phi(x,\bar\theta)e^Ae^{V(x,\theta,\bar\theta)}e^A\Phi(x,\theta)e^{-A}e^{-V(x,\theta,\bar\theta)}\right)_\star\\ &+\int d^4xd^2\theta\mathrm{Tr}(2\pi i\tau W^\alpha W_\alpha+W(\Phi))_\star+\text{ c.c.}\end{aligned}$$

$$\mathrm{Str}\left(\prod_i \exp\left(\epsilon_i^{\alpha}\hat{A}_{\alpha}+\epsilon_i^{\dot{\alpha}}\overline{\hat{A}}_{\dot{\alpha}}\right)\right)$$

$$[\hat{x}^{\mu},\hat{x}^{\nu}]=-iC^{\mu\nu},$$

$$\hat{p}_{\mu}=B_{\mu\nu}\hat{x}^{\nu}$$

$$[\hat{p}_{\mu},\hat{x}^{\nu}]=-i\delta_{\mu}^{\nu},[\hat{p}_{\mu},\hat{p}_{\nu}]=iB_{\mu\nu}$$

$$S=\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[\hat{A}_{\mu},\hat{A}_{\nu}]^2\right),$$

$$A_{\mu}=\hat{p}_{\mu}\otimes 1_n$$

$$S=\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[\hat{p}_{\mu}+\hat{a}_{\mu},\hat{p}_{\nu}+\hat{a}_{\nu}]^2\right)$$

$$\hat{O}=\int\frac{d^dk}{(2\pi)^d}e^{ik_{\mu}\hat{x}^{\mu}}\tilde{O}(k)\leftrightarrow O(x)=\int\frac{d^dk}{(2\pi)^d}e^{ik_{\mu}x^{\mu}}\tilde{O}(k)$$

$$\hat{O}_1\hat{O}_2\leftrightarrow O_1*O_2(x)\equiv\exp\left(-\frac{i}{2}C^{\mu\nu}\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial y^{\mu}}\right)O_1(x)O_2(y)\Big|_{y=x}$$

$$(2\pi)^{\frac{d}{2}}\sqrt{\mathrm{det}C}\,\mathrm{Tr}_{U(\hat{N})}(\hat{O})=\int d^4x\,\mathrm{tr}_{U(n)}O(x)$$

$$S=\int d^dx\frac{1}{\tilde{g}^2}\mathrm{tr}_{U(n)}\left(-\frac{1}{4}F_{\mu\nu}^2\right)_*$$

$$\frac{1}{\tilde{g}^2}\equiv\frac{1}{(2\pi)^{\frac{d}{2}}\sqrt{\mathrm{det}C}}\frac{1}{g^2}$$

$$\hat{A}'_{\mu}=e^{i\hat{\Lambda}}\hat{A}_{\mu}e^{-i\hat{\Lambda}}$$

$$S=\int d^4xd^2\theta d^2\bar{\theta}\,\mathrm{Tr}(\bar{\Phi}_0e^V\Phi_0e^{-V})+\int d^4xd^2\theta\,\mathrm{Tr}\big(2\pi i\tau W_0^{\alpha}W_{0\alpha}+W(\Phi_0)\big)+\text{ c.c.}$$

$$W_{0\alpha}=-\frac{1}{4}\bar{D}^2e^{-V}D_{\alpha}e^V$$

$$\Phi'_0=e^{-i\Lambda}\Phi_0e^{i\Lambda}$$
$$\bar{\Phi}'_0=e^{-i\bar{\Lambda}}\bar{\Phi}_0e^{i\bar{\Lambda}}$$
$$e^{V'}=e^{-i\bar{\Lambda}}e^Ve^{i\Lambda}$$

$$\nabla_{c\alpha}=e^{-V}D_{\alpha}e^V\big(=D_{\alpha}+e^{-V}(D_{\alpha}e^V)\big)$$
$$\bar{\nabla}_{c\dot{\alpha}}=\bar{D}_{\dot{\alpha}}$$
$$\nabla_{c\alpha\dot{\alpha}}=-i\{\nabla_{c\alpha},\bar{\nabla}_{c\dot{\alpha}}\}$$

$$\nabla'_{c\alpha}=e^{-i\Lambda}e^{-V}e^{i\bar{\Lambda}}D_{\alpha}e^{-i\bar{\Lambda}}e^Ve^{i\Lambda}=e^{-i\Lambda}e^{-V}D_{\alpha}e^Ve^{i\Lambda}=e^{-i\Lambda}\nabla_{c\alpha}e^{i\Lambda}$$
$$\bar{\nabla}'_{c\dot{\alpha}}=\bar{D}_{\dot{\alpha}}=e^{-i\Lambda}\bar{D}_{\dot{\alpha}}e^{i\Lambda}=e^{-i\Lambda}\bar{\nabla}_{c\dot{\alpha}}e^{i\Lambda}$$
$$\nabla'_{c\alpha\dot{\alpha}}=-i\{\nabla'_{c\alpha},\bar{\nabla}'_{c\dot{\alpha}}\}=e^{-i\Lambda}\nabla_{c\alpha\dot{\alpha}}e^{i\Lambda}$$

$$\nabla_{a\alpha}=D_{\alpha},\bar{\nabla}_{a\dot{\alpha}}=e^V\bar{D}_{\dot{\alpha}}e^{-V},\nabla_{a\alpha\dot{\alpha}}=-i\{\nabla_{a\alpha},\bar{\nabla}_{a\dot{\alpha}}\}$$

$$W_{0\alpha}=-\frac{1}{4}[\bar{\nabla}_{c\dot{\alpha}},\{\bar{\nabla}_c^{\dot{\alpha}},\nabla_{c\alpha}\}]$$
$$\bar{W}_{0\dot{\alpha}}=-\frac{1}{4}[\nabla_a^{\alpha},\{\nabla_{a\alpha},\bar{\nabla}_{a\dot{\alpha}}\}]$$

$$\overline{(\nabla_{\alpha\dot{\alpha}})}=-\nabla_{\alpha\dot{\alpha}},\overline{(\nabla_{\alpha})}=\bar{\nabla}_{\dot{\alpha}}.$$

$$\nabla'_A = e^{-iK}\nabla_A e^{iK}$$

$$\nabla_A = D_A - i\Gamma_A$$
$$[\nabla_A, \nabla_B\} = T_{AB}^C \nabla_C - iF_{AB}$$

$$D_{\alpha\dot{\alpha}} = -2\sigma^{\mu}_{\alpha\dot{\alpha}} D_\mu = -2\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_\mu, D_\alpha$$

$$\nabla_A = (\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}, -i\{\nabla_\alpha, \nabla_{\dot{\alpha}}\}).$$

$$\bar{\nabla}_{\dot{\alpha}}\Phi = 0, \nabla_\alpha\bar{\Phi} = 0$$

$$\Phi' = e^{-iK}\Phi e^{iK}, \bar{\Phi}' = e^{-iK}\bar{\Phi} e^{iK}$$

$$-i\{\bar{\nabla}_{\dot{\alpha}}, \bar{\nabla}_{\dot{\beta}}\}\Phi = F_{\dot{\alpha}\dot{\beta}}\Phi = 0, F_{\alpha\beta}\bar{\Phi} = 0.$$

$$F_{\dot{\alpha}\dot{\beta}} = -i\{\bar{\nabla}_{\dot{\alpha}}, \bar{\nabla}_{\dot{\beta}}\} = 0,$$
$$F_{\alpha\beta} = -i\{\nabla_\alpha, \nabla_\beta\} = 0.$$

$$\nabla_\alpha = e^{-\Omega} D_\alpha e^{\Omega}\left(= D_\alpha + e^{-\Omega}(D_\alpha e^{\Omega})\right)$$
$$\bar{\nabla}_{\dot{\alpha}} = e^{\bar{\Omega}}\bar{D}_{\dot{\alpha}} e^{-\bar{\Omega}}$$

$$\{e^{-\Omega}D_\alpha e^{\Omega}, e^{-\Omega}D_\beta e^{\Omega}\} = e^{-\Omega}\{D_\alpha, D_\beta\}e^{\Omega} = 0$$

$$W_\alpha = -\frac{1}{4}[\bar{\nabla}_{\dot{\alpha}}, \{\bar{\nabla}^{\dot{\alpha}}, \nabla_\alpha\}]$$
$$\bar{W}_{\dot{\alpha}} = -\frac{1}{4}[\nabla^{\alpha}, \{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\}]$$

$$\bar{\nabla}_{\dot{\alpha}} W_\alpha = \nabla_\alpha \bar{W}_{\dot{\alpha}} = 0$$
$$\nabla^\alpha W_\alpha = \bar{\nabla}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}$$

$$S = \int d^4x d^2\theta d^2\bar{\theta}\,\mathrm{Tr}\bar{\Phi}\Phi + \int d^4x d^2\theta\,\mathrm{Tr}(2\pi i\tau W^\alpha W_\alpha + W(\Phi)) + \text{ c.c..}$$

$$e^{\Omega'} = e^{\Omega}e^{iK}, e^{\bar{\Omega}'} = e^{-iK}e^{\bar{\Omega}}$$

$$e^{\Omega'} = e^{-i\bar{\Lambda}}e^{\Omega}, e^{\bar{\Omega}'} = e^{\bar{\Omega}}e^{i\Lambda}$$

∇_A	$\nabla_{cA} = e^{-\bar{\Omega}}\nabla_A e^{\bar{\Omega}}$
$e^{-\Omega}D_\alpha e^{\Omega}$	$e^{-V}D_\alpha e^{V}$
$e^{\bar{\Omega}}\bar{D}_{\dot{\alpha}}e^{-\bar{\Omega}}$	$\bar{D}_{\dot{\alpha}}$
$-i\{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\}$	$-i\{\nabla_{c\alpha}, \bar{\nabla}_{c\dot{\alpha}}\}$
	$\nabla_{aA} = e^{\Omega}\nabla_A e^{-\Omega}$
	D_α
	$e^{V}\bar{D}_{\dot{\alpha}}e^{-V}$
	$-i\{\nabla_{a\alpha}, \bar{\nabla}_{a\dot{\alpha}}\}$
W_α	$W_{0\alpha} = e^{-\bar{\Omega}}W_\alpha e^{\bar{\Omega}}$
$\bar{W}_{\dot{\alpha}}$	$\bar{W}_{0\dot{\alpha}} = e^{\Omega}\bar{W}_{\dot{\alpha}}e^{-\Omega}$
Φ	$\Phi_0 = e^{-\bar{\Omega}}\Phi e^{\bar{\Omega}}$
$(\bar{\nabla}_{\dot{\alpha}}\Phi = 0)$	$(\bar{\nabla}_{c\dot{\alpha}}\Phi_0 = \bar{D}_{\dot{\alpha}}\Phi_0 = 0)$
$\bar{\Phi}$	$\bar{\Phi}_0 = e^{\Omega}\bar{\Phi}e^{-\Omega}$

$$
\begin{array}{cc}
K & \Lambda,\ \bar{\Lambda} \\
(\bar{K} = K) & (\bar{D}_{\dot{\alpha}}\Lambda = 0,\ D_{\alpha}\bar{\Lambda} = 0) \\
\nabla'_A = e^{-iK}\nabla_A e^{iK} & \nabla'_{cA} = e^{-i\Lambda}\nabla_{cA} e^{i\Lambda} \\
 & \nabla'_{aA} = e^{-i\bar{\Lambda}}\nabla_{aA} e^{i\bar{\Lambda}} \\
\Phi' = e^{-iK}\Phi e^{iK} & \Phi'_0 = e^{-i\Lambda}\Phi_0 e^{i\Lambda} \\
\bar{\Phi}' = e^{-iK}\bar{\Phi} e^{iK} & \bar{\Phi}'_0 = e^{-i\bar{\Lambda}}\bar{\Phi}_0 e^{i\bar{\Lambda}} \\
e^{\Omega'} = e^{-i\bar{\Lambda}} e^{\Omega} e^{iK} & e^{V'} = e^{-i\bar{\Lambda}} e^{V} e^{i\Lambda} \\
e^{\bar{\Omega}'} = e^{-iK} e^{\Omega} e^{i\Lambda} &
\end{array}
$$

$$\int d^2\theta d^2\bar{\theta}$$

$$\hat{\theta}^1 = \sqrt{\gamma}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \hat{\theta}^2 = \sqrt{\gamma}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\overline{\hat{\theta}^1} = \sqrt{\gamma}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \overline{\hat{\theta}}^{\dot{2}} = \sqrt{\gamma}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\cdot,$$

$$\hat{\theta}^1\hat{\theta}^2 - \hat{\theta}^2\hat{\theta}^1 = 2\gamma\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \overline{\hat{\theta}}^{\dot{1}}\overline{\hat{\theta}}^{\dot{2}} - \overline{\hat{\theta}}^{2}\overline{\hat{\theta}}^{\dot{1}} = 2\gamma\begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{\theta}^1\overline{\hat{\theta}}^{\dot{1}} - \overline{\hat{\theta}}^{\dot{1}}\hat{\theta}^1 = \gamma\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \hat{\theta}^1\overline{\hat{\theta}}^{\dot{2}} - \overline{\hat{\theta}}^{2}\hat{\theta}^1 = 2\gamma\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{\theta}^2\overline{\hat{\theta}}^{\dot{2}} - \overline{\hat{\theta}}^{2}\hat{\theta}^2 = \gamma\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \hat{\theta}^2\overline{\hat{\theta}}^{\dot{1}} - \overline{\hat{\theta}}^{\dot{1}}\hat{\theta}^2 = 2\gamma\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$$

$$\left\{[\hat{\theta}^1, \hat{\theta}^2], \hat{\hat{\theta}}^{\dot{1}}\right\} = 2\gamma^{\frac{3}{2}}\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & & 0 \end{pmatrix}, \left\{[\hat{\theta}^1, \hat{\theta}^2], \overline{\hat{\theta}}^{2}\right\} = 2\gamma^{\frac{3}{2}}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\left\{\left[\overline{\hat{\theta}}^{\dot{1}}, \overline{\hat{\theta}}^{\dot{2}}\right], \hat{\theta}^1\right\} = 2\gamma^{\frac{3}{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \left\{\left[\overline{\hat{\theta}}^{\dot{1}}, \overline{\hat{\theta}}^{\dot{2}}\right], \hat{\theta}^2\right\} = 2\gamma^{\frac{3}{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}.$$

$$(C \cdot 4)\left[\hat{\theta}^1, \left\{\hat{\theta}^2, \left[\bar{\theta}^{\dot{1}}, \overline{\hat{\theta}}^{\dot{2}}\right]\right\}\right] = 2\gamma^2\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$u_k = \mathrm{Tr}X^k$$
$$v_k = -\frac{1}{16\pi^2}\mathrm{Tr}W^{\alpha}W_{\alpha}X^k$$

$$R(z) = \sum_{k\geq 0}\frac{\langle u_k\rangle}{z^{k+1}}$$
$$S(z) = \sum_{k\geq 0}\frac{\langle v_k\rangle}{z^{k+1}}$$

$$L = \frac{1}{4\pi}\mathrm{Im}\int \ d^2\theta \mathrm{Tr}(\tau(X)W^{\alpha}W_{\alpha}) + 2N\mathrm{Re}\int \ d^2\theta \mathrm{Tr}W(X)$$
$$NV(z) = 2i\pi\tau(z)$$

$$L = 2N\text{Re}\int \mathrm{d}^2\theta \mathcal{W}$$

$$\mathcal{W} = -\frac{1}{16\pi^2}\text{Tr}V(X)W^\alpha W_\alpha + \text{Tr}W(X)$$

$$V(z) = \lambda_{-1} + \sum_{k=0}^{d_V}\frac{\lambda_k}{k+1}z^{k+1}$$

$$W(z) = \sum_{k=0}^{d_W}\frac{g_k}{k+1}z^{k+1}$$

$$\mathcal{W} = \lambda_{-1}v_0 + \sum_{k\geq 0}\frac{\lambda_k}{k+1}v_{k+1} + \sum_{k\geq 0}\frac{g_k}{k+1}u_{k+1}$$

$$t(z) = \sum_{k\geq 1}\frac{t_k}{k+1}z^{k+1}$$

$$t''(z) = NV(z) = 2i\pi\tau(z)$$

$$t_1 = N\lambda_{-1}, t_k = \frac{N\lambda_{k-2}}{k(k-1)} \text{ for } k \geq 2$$

$$q = e^{t_1} = e^{N\lambda_{-1}}$$

	θ	W^α	X	u_k	v_k	g_k	$\lambda_k,\ k\geq 0$	q	w
$\text{U}(1)_\text{A}$	0	0	1	k	k	$-k-1$	$-k-1$	$2N$	0
$\text{U}(1)_\text{R}$	1	1	0	0	2	2	0	0	2 .

$$\mathcal{F}_{\mathcal{N}=2}(X) = t(X).$$

$$v_{k,\text{mac}}(\boldsymbol{s},\boldsymbol{g},\varepsilon) = N\varepsilon\left\langle\langle\boldsymbol{s}|\text{Tr}X^k \mid \boldsymbol{s}\rangle\right\rangle_\varepsilon = \frac{N\varepsilon}{Z_\text{mac}}\int \mathrm{d}\mu_\text{mac}^M \text{Tr}M^k$$

$$Z_\text{mac}(\boldsymbol{s},\boldsymbol{g},\varepsilon) = \int \mathrm{d}\mu_\text{mac}^M = \exp\frac{\mathcal{F}_\text{mac}(\boldsymbol{s},\boldsymbol{g},\varepsilon)}{\varepsilon^2}$$

$$\mathrm{d}\mu_\text{mac}^M = \prod_{I=1}^{n}\mathrm{d}M_{II}\prod_{1\leq I<J\leq n}\mathrm{dRe}M_{IJ}\,\mathrm{dIm}M_{IJ}\exp\left(-\frac{1}{\varepsilon}\text{Tr}W(M)\right).$$

$$\varepsilon = \frac{s}{n},$$

$$s = \sum_i s_i$$

$$v_{k,\text{mac}}(\boldsymbol{s},\boldsymbol{g},\varepsilon) = -Nk\frac{\partial\mathcal{F}_\text{mac}}{\partial g_{k-1}}, k \geq 1.$$

$$S_\text{mac}(z;\boldsymbol{s},\boldsymbol{g},\varepsilon) = \sum_{k\geq 0}\frac{v_{k,\text{mac}}(\boldsymbol{s},\boldsymbol{g},\varepsilon)}{z^{k+1}}$$

$$v_{k,\text{mac}}(\boldsymbol{s},\boldsymbol{g}) = \lim_{\varepsilon\to 0}v_{k,\text{mac}}(\boldsymbol{s},\boldsymbol{g},\varepsilon)$$
$$S_\text{mac}(z;\boldsymbol{s},\boldsymbol{g}) = \lim_{\varepsilon\to 0}S_\text{mac}(z;\boldsymbol{s},\boldsymbol{g},\varepsilon)$$

$$\lambda_\text{mac} = S_\text{mac}(z;\boldsymbol{s},\boldsymbol{g})\mathrm{d}z,$$

$$W_{\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \frac{1}{2i\pi}\oint\ {}_{\alpha}V\lambda_{\mathrm{mac}} - \sum_i N_i \frac{\partial \mathcal{F}_{\mathrm{mac}}}{\partial s_i},$$

$$u_{k,\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = k\frac{\partial W_{\mathrm{mac}}}{g_{k-1}},$$

$$R_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \sum_{k\geq 0} \frac{u_{k,\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g},\boldsymbol{t})}{z^{k+1}}.$$

$$\frac{\partial W_{\mathrm{mac}}}{\partial s_i}(s = s^*) = 0$$

$$S^*_{\mathrm{mac}}(z;\boldsymbol{g},\boldsymbol{t}) = S_{\mathrm{mac}}(z;\boldsymbol{s}^*,\boldsymbol{g})$$
$$R^*_{\mathrm{mac}}(z;\boldsymbol{g},\boldsymbol{t}) = R_{\mathrm{mac}}(z;\boldsymbol{s}^*,\boldsymbol{g},\boldsymbol{t})$$

$$S(z;\boldsymbol{g},\boldsymbol{t}) = S^*_{\mathrm{mac}}(z;\boldsymbol{g},\boldsymbol{t}),$$
$$R(z;\boldsymbol{g},\boldsymbol{t}) = R^*_{\mathrm{mac}}(z;\boldsymbol{g},\boldsymbol{t}).$$

$$X_\infty = \mathrm{diag}(a_1,\dots,a_N) = \mathrm{diag}\boldsymbol{a},$$

$$u_{k,\,\mathrm{mic}}\,(\boldsymbol{a},\boldsymbol{t},\epsilon) = \langle \boldsymbol{a}|\mathrm{Tr}X^k|\boldsymbol{a}\rangle_\epsilon = \frac{1}{Z_{\mathrm{mic}}}\sum_{\vec{p}} \mu^{\vec{p}}_{\mathrm{mic}}\, u_{k,\vec{p}}$$

$$Z_{\mathrm{mic}}\,(\boldsymbol{a},\boldsymbol{t},\epsilon) = \sum_{\vec{p}} \mu^{\vec{p}}_{\mathrm{mic}} = \exp\frac{\mathcal{F}_{\mathrm{mic}}\,(\boldsymbol{a},\boldsymbol{t},\epsilon)}{\epsilon^2}$$

$$p_{i,1} \geq p_{i,2} \geq \cdots \geq p_{i,\tilde{p}_{i,1}} > p_{i,\tilde{p}_{i,1}+1} = 0$$
$$\sum_{\alpha=1}^{\tilde{p}_{i,1}} p_{i,\alpha} = |\mathrm{p}_i|$$

$$|\vec{\mathrm{p}}| = \sum_{i=1}^{N} |\mathrm{p}_i|$$

$$\tilde{p}_{i,1} \geq \tilde{p}_{i,2} \geq \cdots \geq \tilde{p}_{i,p_{i,1}} > \tilde{p}_{i,p_{i,1}+1} = 0$$
$$\sum_{\beta=1}^{p_{i,1}} \tilde{p}_{i,\beta} = |\mathrm{p}_i|$$

$$\mu^{\vec{p}}_{\mathrm{mic}} = \left(\nu^{\vec{p}}_{\mathrm{mic}}\right)^2$$

$$\nu^{\vec{\mathrm{p}}}_{\mathrm{mic}} = \frac{1}{\epsilon|\vec{\mathrm{p}}|}\prod_{i=1}^{N}\left[\prod_{\square(\alpha,\beta)\in Y_{\mathrm{p}_i}} \frac{1}{p_{i,\alpha}-\beta+\tilde{p}_{i,\beta}-\alpha+1}\prod_{j\neq i}\frac{1}{a_i-a_j+\epsilon(\beta-\alpha)}\right]\times$$
$$\prod_{i<j}\prod_{\alpha=1}^{\tilde{p}_{i,1}}\prod_{\beta=1}^{p_{j,1}} \frac{\left(a_i-a_j+\epsilon\left(\tilde{p}_{j,\beta}-\alpha-\beta+1\right)\right)\left(a_i-a_j+\epsilon\left(p_{i,\alpha}-\beta-\alpha+1\right)\right)}{\left(a_i-a_j+\epsilon(1-\alpha-\beta)\right)\left(a_i-a_j+\epsilon\left(\tilde{p}_{j,\beta}-\alpha+p_{i,\alpha}-\beta+1\right)\right)}\times$$
$$\exp\left(\frac{1}{2\epsilon^2}\sum_{k\geq 1}\frac{t_k}{k+1}u_{k+1,\vec{\mathrm{p}}}\right)$$

$$u_{k,\vec{\mathrm{p}}} = \sum_{i=1}^{N}\left[a_i^k + \sum_{\alpha=1}^{\tilde{p}_{i,1}}\left(\left(a_i+\epsilon\left(p_{i,\alpha}-\alpha+1\right)\right)^k - \left(a_i+\epsilon\left(p_{i,\alpha}-\alpha\right)\right)^k\right.\right.$$
$$\left.\left. +(a_i-\epsilon\alpha)^k - (a_i-\epsilon(\alpha-1))^k\right)\right].$$

$$u_{2,\vec{\mathrm{p}}} = \sum_{i=1}^{N} a_i^2 + 2\epsilon^2|\vec{\mathrm{p}}|$$

$$u_{k,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{t},\epsilon)=2k\frac{\partial\mathcal{F}_{\mathrm{mic}}}{\partial t_{k-1}},k\geq 2.$$

$$R_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{t},\epsilon)=\sum_{k\geq 0}\frac{u_{k,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{t},\epsilon)}{z^{k+1}}$$

$$\begin{aligned}u_{k,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{t})&=\langle\boldsymbol{a}|\mathrm{Tr}X^k|\boldsymbol{a}\rangle=\lim_{\epsilon\to 0}u_{k,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{t},\epsilon)\\ R_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{t})&=\lim_{\epsilon\to 0}R_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{t},\epsilon)\end{aligned}$$

$$\lambda_{\mathrm{mic}}=zR_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{t})\mathrm{d}z,$$

$$W_{\mathrm{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})=\langle\boldsymbol{a}|\mathrm{Tr}W(X)|\boldsymbol{a}\rangle=\frac{1}{2i\pi}\oint_{\alpha}\frac{W\lambda_{\mathrm{mic}}}{z}$$

$$\begin{aligned}v_{0,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})&=\frac{\partial W_{\mathrm{mic}}}{\partial\lambda_{-1}}\\ v_{k,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})&=k\frac{\partial W_{\mathrm{mic}}}{\partial\lambda_{k-1}}\quad\text{for } k\geq 1\end{aligned}$$

$$v_{k,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})=\frac{N}{k+1}\frac{\partial W_{\mathrm{mic}}}{\partial t_{k+1}}.$$

$$S_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})=\sum_{k\geq 0}\frac{v_{k,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})}{z^{k+1}}$$

$$\frac{\partial W_{\mathrm{mic}}}{\partial a_i}(\boldsymbol{a}=\boldsymbol{a}^*)=0$$

$$\begin{aligned}R^*_{\mathrm{mic}}(z;\boldsymbol{g},\boldsymbol{t})&=R_{\mathrm{mic}}(z;\boldsymbol{a}^*,\boldsymbol{t})\\ S^*_{\mathrm{mic}}(z;\boldsymbol{g},\boldsymbol{t})&=S_{\mathrm{mic}}(z;\boldsymbol{a}^*,\boldsymbol{g},\boldsymbol{t})\end{aligned}$$

$$\begin{aligned}R(z;\boldsymbol{g},\boldsymbol{t})&=R^*_{\mathrm{mic}}(z;\boldsymbol{g},\boldsymbol{t}),\\ S(z;\boldsymbol{g},\boldsymbol{t})&=S^*_{\mathrm{mic}}(z;\boldsymbol{g},\boldsymbol{t}).\end{aligned}$$

$$\left|\begin{aligned}R^*_{\mathrm{mac}}(z;\boldsymbol{g},\boldsymbol{t})&=R^*_{\mathrm{mic}}(z;\boldsymbol{g},\boldsymbol{t})\\ S^*_{\mathrm{mac}}(z;\boldsymbol{g},\boldsymbol{t})&=S^*_{\mathrm{mic}}(z;\boldsymbol{g},\boldsymbol{t})\end{aligned}\right|$$

$$\begin{aligned}\delta_{L_n}X&=-\zeta X^{n+1}\\ \delta_{J_n}X&=\frac{\zeta}{16\pi^2}W^{\alpha}W_{\alpha}X^{n+1}\end{aligned}$$

$$L_n=-X^{n+1}\frac{\delta}{\delta X},J_n=\frac{1}{16\pi^2}W^{\alpha}W_{\alpha}X^{n+1}\frac{\delta}{\delta X}$$

$$L_n\cdot u_m=-mu_{n+m},J_n\cdot u_m=-mv_{n+m},L_n\cdot v_m=-mv_{n+m},J_n\cdot v_m=0$$

$$[L_n,L_m]=(n-m)L_{n+m},[L_n,J_m]=(n-m)J_{n+m},[J_n,J_m]=0$$

$$\begin{aligned}-NW'(z)R_{\mathrm{mac}}(z)-NV'(z)S_{\mathrm{mac}}(z)+2R_{\mathrm{mac}}(z)S_{\mathrm{mac}}(z)+N^2\Delta_R(z)&=0\\ -NW'(z)S_{\mathrm{mac}}(z)+S_{\mathrm{mac}}(z)^2+N^2\Delta_S(z)&=0\end{aligned}$$

$$S_{\mathrm{mac}}(z)\underset{z\to\infty}{\sim}\frac{v_{0,\mathrm{mac}}}{z},R_{\mathrm{mac}}(z)\underset{z\to\infty}{\sim}\frac{N}{z}$$

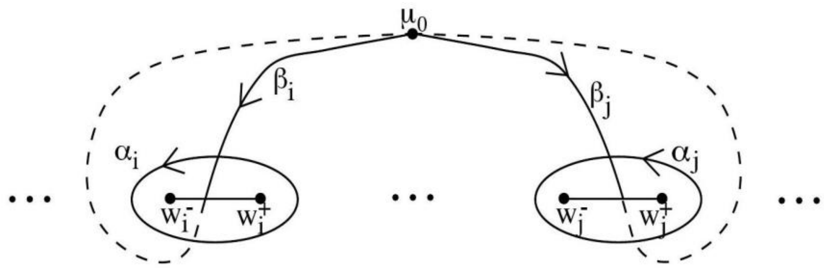
$$u_{N+p,\mathrm{mac}}=\mathcal{P}_{\mathrm{pert},p}\left(u_{1,\mathrm{mac}},\dots,u_{N,\mathrm{mac}}\right),p\geq 1.$$

$$u_{N+p,\mathrm{mac}}=\mathcal{P}_p\left(u_{1,\mathrm{mac}},\dots,u_{N,\mathrm{mac}};q,\lambda_0,\dots,\lambda_{d_V}\right)$$

$$S_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g})=\frac{N}{2}\left(W'(z)-\sqrt{W'(z)^2-4\Delta_S(z)}\right)$$

$$W'(z)^2 - 4\Delta_S(z) = N_{d_W-r}(z)^2 y_{\text{mac},r}^2$$

$$\mathcal{C}_{\text{mac},r}: y_{\text{mac},r}^2 = \prod_{i=1}^{r} (z - w_i^-)(z - w_i^+)$$



$$s_i = \frac{1}{2i\pi N} \oint_{\alpha_i} \lambda_{\text{mac}}$$

$$s = \sum_i s_i = \frac{1}{2i\pi N} \oint_{\alpha} \lambda_{\text{mac}} = \frac{v_{0,\text{mac}}}{N}$$

$$v_{0,\text{mac}}(\boldsymbol{s}, \boldsymbol{g}) = v_{0,\text{mac}}(\boldsymbol{s}) = N \sum_i s_i$$

$$\oint_{\alpha_i} S'_{\text{mac}} \text{dz} = 0$$

$$\int_{\beta_i} S'_{\text{mac}} \text{dz} = N W'(\mu_0)$$

$$v_{0,\text{mac}}(\boldsymbol{s}, \boldsymbol{g}) = \frac{\partial W_{\text{mac}}}{\partial \lambda_{-1}}$$

$$v_{k,\text{mac}}(\boldsymbol{s}, \boldsymbol{g}) = k \frac{\partial W_{\text{mac}}}{\partial \lambda_{k-1}} \text{ for } k \geq 1$$

$$\mathcal{L}_{\text{mac}}(z) = -\sum_{k \geq 1} \frac{k}{z^{k+1}} \frac{\partial}{\partial g_{k-1}}.$$

$$S_{\text{mac}}(z) = \frac{v_{0,\text{mac}}}{z} + N \mathcal{L}_{\text{mac}}(z) \cdot \mathcal{F}_{\text{mac}}$$

$$R_{\text{mac}}(z) = \frac{N}{z} - \mathcal{L}_{\text{mac}}(z) \cdot W_{\text{mac}}$$

$$R_{\text{mac}}(z) = \frac{1}{N} \sum_{i=1}^{r} N_i \frac{\partial S_{\text{mac}}(z)}{\partial s_i} - \frac{1}{2i\pi} \oint_{\alpha} \mathcal{L}_{\text{mac}}(z) \cdot S_{\text{mac}}(z') V(z') \text{dz}'$$

$$\frac{1}{2i\pi} \oint_{\alpha_j} h_i = \delta_{ij},$$

$$h_i = \psi_i(z) \text{dz} = \frac{p_i}{y_{\text{mac},r}} \text{dz}$$

$$\frac{\partial S_{\text{mac}}(z)}{\partial s_i} = \frac{N \partial \Delta_S(z) / \partial s_i}{\sqrt{W'(z)^2 - 4\Delta_S(z)}}$$

$$\frac{\partial S_{\text{mac}}(z)}{\partial s_i} = N \psi_i(z).$$

$$\frac{\partial \lambda_{\mathrm{mac}}}{\partial s_i} = N h_i$$

$$\mathcal{L}_{\mathrm{mac}}(z)\cdot \mathcal{S}_{\mathrm{mac}}(z') = \frac{N}{\varepsilon^2}\left(\left\langle \langle s|\varepsilon \mathrm{Tr}\frac{1}{z-X}\varepsilon \mathrm{Tr}\frac{1}{z'-X}\right| s\right\rangle_{\varepsilon} - \left\langle \langle s|\varepsilon \mathrm{Tr}\frac{1}{z-X}\right| s\right\rangle_{\varepsilon}\left\langle \langle s|\varepsilon \mathrm{Tr}\frac{1}{z'-X}\right| s\right\rangle_{\varepsilon}\Bigg).$$

$$R_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \sum_{i=1}^{r} N_i\psi_i(z) - \frac{N}{\varepsilon^2}\left(\left\langle \langle \boldsymbol{s}|\varepsilon \mathrm{Tr}\frac{1}{z-X}\varepsilon \mathrm{Tr}V(X)\right| \boldsymbol{s}\right\rangle_{\varepsilon} - \left\langle \langle \boldsymbol{s}|\varepsilon \mathrm{Tr}\frac{1}{z-X}\right| \boldsymbol{s}\right\rangle_{\varepsilon} \langle\langle \boldsymbol{s}|\varepsilon \mathrm{Tr}V(X) \mid \boldsymbol{s}\rangle_{\varepsilon}\Bigg)$$

$$\mathcal{L}_{\mathrm{mac}}(z)\cdot \mathcal{S}_{\mathrm{mac}}(z') = \mathcal{L}_{\mathrm{mac}}(z')\cdot \mathcal{S}_{\mathrm{mac}}(z)$$

$$\frac{1}{2i\pi}\oint_{\alpha} \mathcal{L}_{\mathrm{mac}}(z)\cdot \mathcal{S}_{\mathrm{mac}}(z')V(z')\mathrm{d}z' = \frac{1}{2i\pi}\oint_{\alpha} \mathcal{L}_{\mathrm{mac}}(z')V(z')\mathrm{d}z'\cdot \mathcal{S}_{\mathrm{mac}}(z) = -\sum_{k\geq 0}\lambda_k\frac{\partial \mathcal{S}_{\mathrm{mac}}(z)}{\partial g_k}$$

$$R_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \sum_{i=1}^{r} N_i\psi_i(z) + \sum_{k\geq 0}\lambda_k\frac{\partial \mathcal{S}_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g})}{\partial g_k}$$

$$R_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \frac{N}{2}\left(V'(z) + \frac{D_R(z)}{y_{\mathrm{mac},r}}\right)$$

$$\frac{1}{2i\pi}\oint_{\alpha_i} R_{\mathrm{mac}}\mathrm{d}z = N_i$$

$$R_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \frac{N}{2}\left(V'(z) + \frac{2\Delta_R(z) - W'(z)V'(z)}{\sqrt{W'(z)^2 - 4\Delta_S(z)}}\right)$$

$$W_{\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \sum_{k\geq 0} g_k\frac{\partial W_{\mathrm{mac}}}{\partial g_k} + \sum_{i=1}^{r} s_i\frac{\partial W_{\mathrm{mac}}}{\partial s_i} = \frac{1}{2i\pi}\oint_{\alpha} R_{\mathrm{mac}}W\,\mathrm{d}z + \sum_{i=1}^{r} s_i\frac{\partial W_{\mathrm{mac}}}{\partial s_i}$$

$$N\varepsilon\langle\langle s|\mathrm{Tr}V(X)\mid s\rangle\rangle_{\varepsilon} = \frac{1}{2i\pi}\oint_{\alpha} V\lambda_{\mathrm{mac}} = \frac{N\varepsilon}{Z_{\mathrm{mac}}}\int \mathrm{d}\mu_{\mathrm{mac}}^{M}\mathrm{Tr}V(M),$$

$$\delta s_i = \zeta s_i, \delta\varepsilon = \zeta\varepsilon$$

$$\frac{1}{2i\pi}\oint_{\alpha}\sum_i s_i V\frac{\partial \lambda_{\mathrm{mac}}}{\partial s_i} = \frac{N}{2i\pi}\oint_{\alpha}\sum_i s_i V h_i = N\varepsilon\langle\langle \boldsymbol{s}|\mathrm{Tr}V(X)|\boldsymbol{s}\rangle_{\varepsilon} + \frac{N}{\varepsilon^2}\left(\langle\langle \boldsymbol{s}|\varepsilon\mathrm{Tr}W(X)\varepsilon\mathrm{Tr}V(X)|\boldsymbol{s}\rangle\rangle - \langle\langle \boldsymbol{s}|\varepsilon\mathrm{Tr}W(X)|\boldsymbol{s}\rangle\langle\langle \boldsymbol{s}|\varepsilon\mathrm{Tr}V(X)|\boldsymbol{s}\rangle\right).$$

$$\sum_i s_i\frac{\partial \mathcal{F}_{\mathrm{mac}}}{\partial s_i} - 2\mathcal{F}_{\mathrm{mac}} = \frac{1}{2i\pi N}\oint_{\alpha} W\lambda_{\mathrm{mac}}$$

$$-\frac{\partial\mathcal{F}_{\text{mac}}}{\partial s_i}+\sum_j s_j\frac{\partial^2\mathcal{F}_{\text{mac}}}{\partial s_i\partial s_j}=\frac{1}{2i\pi}\oint_\alpha Wh_i$$

$$\begin{aligned}\sum_i s_i\frac{\partial W_{\text{mac}}}{\partial s_i}&=\frac{N}{2i\pi}\oint_\alpha\sum_i s_iVh_i-\sum_{i,j}N_js_i\frac{\partial^2\mathcal{F}_{\text{mac}}}{\partial s_i\partial s_j}\\&=\frac{1}{2i\pi}\oint_\alpha V\lambda_{\text{mac}}-\sum_i N_i\frac{\partial\mathcal{F}_{\text{mac}}}{\partial s_i}-\frac{1}{2i\pi}\oint_\alpha W\sum_i N_ih_i\\&+\frac{N}{\varepsilon^2}\left(\langle\langle\boldsymbol{s}|\varepsilon\text{Tr}W(X)\varepsilon\text{Tr}V(x)|\boldsymbol{s}\rangle\rangle-\langle\langle\boldsymbol{s}|\varepsilon\text{Tr}W(X)|\boldsymbol{s}\rangle\right\rangle\langle\langle\boldsymbol{s}|\varepsilon\text{Tr}V(X)|\boldsymbol{s}\rangle\Big)\Big)\\&=W_{\text{mac}}-\frac{1}{2i\pi}\oint_\alpha R_{\text{mac}}W\,\text{dz}\end{aligned}$$

$$\frac{1}{2i\pi}\oint_{\beta_i-\beta_j}R_{\text{mac}}\text{dz}\in\mathbb{Z}$$

$$F^*_{\text{mac}}(z;\boldsymbol{g},\boldsymbol{t})=\langle\det(z-X)\rangle,$$

$$\frac{F^{*\prime}_{\text{mac}}(z;\boldsymbol{g},\boldsymbol{t})}{F^*_{\text{mac}}(z;\boldsymbol{g},\boldsymbol{t})}=R^*_{\text{mac}}(z;\boldsymbol{g},\boldsymbol{t}),$$

$$\frac{\partial\mathcal{F}_{\text{mac}}}{\partial s_i}=\frac{1}{N}\int_{\beta_i}\lambda_{\text{mac}}+2s\ln\,\mu_0-W(\mu_0).$$

$$\frac{\partial W_{\text{mac}}}{\partial s_j}=\frac{N}{2i\pi}\oint_\alpha Vh_j-\frac{1}{2i\pi}\sum_{i=1}^{r}\oint_{\alpha_i}R_{\text{mac}}\,\text{dz}\int_{\beta_i}h_j-2N\ln\,\mu_0.$$

$$H_j(P)=\int_{P_0}^{P}h_j+\ln\,\mu_0$$

$$H_j(z)+\hat{H}_j(z)=\int_{\beta_r}h_j+2\ln\,\mu_0$$

$$R_{\text{mac}}(z)+\hat{R}_{\text{mac}}(z)=NV'(z)$$

$$\begin{aligned}&\int_{\beta_j}R_{\text{mac}}\text{dz}-\frac{1}{2i\pi}\sum_{i=1}^{r}\oint_{\alpha_i}R_{\text{mac}}\text{dz}\int_{\beta_i}h_j+\frac{1}{2i\pi}\oint_\alpha R_{\text{mac}}\text{dz}\int_{\beta_r}h_j=\\&-\frac{1}{2i\pi}\oint_\alpha\left[R_{\text{mac}}H_j+(NV'-R_{\text{mac}})\left(\int_{\beta_r}h_j+2\ln\,\mu_0-H_j\right)\right]\end{aligned}$$

$$\int_{\beta_j}R_{\text{mac}}\text{dz}=\frac{1}{2i\pi}\sum_{i=1}^{r}\oint_{\alpha_i}R_{\text{mac}}\text{dz}\int_{\beta_i}h_j+2N\ln\,\mu_0+\frac{1}{2i\pi}\oint_\alpha\left(NV'H_j-2R_{\text{mac}}H_j\right)\text{dz}$$

$$\frac{1}{2i\pi}\oint_\alpha V'H_j\,\text{dz}=-\frac{1}{2i\pi}\oint_\alpha Vh_j+V(\mu_0)$$

$$\frac{1}{2i\pi}\oint_\alpha R_{\text{mac}}H_j\,\text{dz}=N\ln\,\mu_0$$

$$\frac{\partial W_{\text{mac}}}{\partial s_i} = -\int_{\beta_i} R_{\text{mac}}\,\mathrm{d}z + NV(\mu_0) - 2N\ln\,\mu_0$$

$$\int_{\beta_i} R^*_{\text{mac}}\,\mathrm{d}z = NV(\mu_0) - 2N\ln\,\mu_0 + 2i\pi\mathbb{Z}$$

$$\sum_{i=1}^{r}\left[\oint_{\alpha_i} R_{\text{mac}}\mathrm{d}z \int_{\beta_i} S_{\text{mac}}\mathrm{d}z - \oint_{\alpha_i} S_{\text{mac}}\mathrm{d}z \int_{\beta_i} R_{\text{mac}}\mathrm{d}z\right] =$$

$$N\oint_{\alpha}(S_{\text{mac}}V - R_{\text{mac}}W)\mathrm{d}z + 2i\pi N^2\big(W(\mu_0) - sV(\mu_0)\big)$$

$$\rho(P) = \int_{P_0}^{P} R_{\text{mac}}\mathrm{d}z + N\ln\,\mu_0$$

$$S_{\text{mac}}(z) + \hat{S}_{\text{mac}}(z) = NW'(z)$$

$$\rho(z) + \hat{\rho}(z) = NV(z) + \int_{\beta_r} R_{\text{mac}}\,\mathrm{d}z - NV(\mu_0) + 2N\ln\,\mu_0$$

$$\frac{1}{2i\pi}\oint_{\alpha} S_{\text{mac}}\rho\mathrm{d}z = N^2 s\ln\,\mu_0$$

$$-NW'(z)R^*_{\text{mac}}(z) - NV'(z)S^*_{\text{mac}}(z) + 2R^*_{\text{mac}}(z)S^*_{\text{mac}}(z) + N^2\Delta_R(z) = 0\,,$$
$$-NW'(z)S^*_{\text{mac}}(z) + S^*_{\text{mac}}(z)^2 + N^2\Delta_S(z) = 0$$
$$S^*_{\text{mac}}(z) \underset{z\to\infty}{\sim} \frac{Ns}{z}\,, \quad R^*_{\text{mac}}(z) \underset{z\to\infty}{\sim} \frac{N}{z}\,,$$
$$\oint_{\alpha_i} R^*_{\text{mac}}\,\mathrm{d}z \in 2i\pi\mathbb{Z}\,, \quad \int_{\beta_i} R^*_{\text{mac}}\,\mathrm{d}z - NV(\mu_0) + 2N\ln\mu_0 \in 2i\pi\mathbb{Z}\,.$$

$$F^*_{\text{mac}}(z) \underset{z\to\infty}{\sim} z^N\,, \quad \hat{F}^*_{\text{mac}}(z) \underset{z\to\infty}{\sim} \frac{e^{NV(z)}}{z^N}$$

$$R_{\text{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t}) = R_{\text{mic}}(z;\boldsymbol{a},\boldsymbol{t})$$

$$\mathcal{C}_{\text{mic}}: y^2 = \prod_{i=1}^{N}(x - x_i^-)(x - x_i^+)$$

$$R_{\text{mic}}(z;\boldsymbol{a},\boldsymbol{t}) = \frac{N}{2}\left(V'(z) + \frac{E_R(z)}{y}\right)$$

$$R_{\text{mic}}(z) \underset{z\to\infty}{\sim} \frac{N}{z}\,,$$
$$\frac{1}{2i\pi}\oint_{\alpha_i} R_{\text{mic}}\,\mathrm{d}z = 1\,,$$
$$\int_{\beta_i} R_{\text{mic}}\,\mathrm{d}z = NV(\mu_0) - 2N\ln\mu_0 + 2i\pi\mathbb{Z}$$
$$a_i = \frac{1}{2i\pi}\oint_{\alpha_i}\lambda_{\text{mic}}\,.$$

$$v_{k,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t},\epsilon)=\frac{N}{(k+1)(k+2)}\frac{1}{\epsilon^2}(\langle\boldsymbol{a}|\mathrm{Tr}W(X)\mathrm{Tr}X^{k+2}|\boldsymbol{a}\rangle_\epsilon$$
$$-\langle\boldsymbol{a}|\mathrm{Tr}W(X)|\boldsymbol{a}\rangle_\epsilon\langle\boldsymbol{a}|\mathrm{Tr}X^{k+2}|\boldsymbol{a}\rangle_\epsilon)$$

$$S''_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})=\frac{N}{\epsilon^2}\Big(\langle\boldsymbol{a}|\mathrm{Tr}\frac{1}{z-X}\mathrm{Tr}W(X)|\boldsymbol{a}\rangle_\epsilon$$
$$-\langle\boldsymbol{a}|\mathrm{Tr}\frac{1}{z-X}|\boldsymbol{a}\rangle_\epsilon\langle\boldsymbol{a}|\mathrm{Tr}W(X)|\boldsymbol{a}\rangle_\epsilon\Big)$$

$$u_{1,\vec{\mathrm{p}}}=\sum_{i=1}^{N}a_i$$

$$u_{1,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{t})=u_{1,\mathrm{mic}}(\boldsymbol{a})=\sum_{i=1}^{N}a_i=a$$

$$\langle\boldsymbol{a}|\mathrm{Tr}X\mathrm{Tr}X^k|\boldsymbol{a}\rangle_\epsilon=\langle\boldsymbol{a}|\mathrm{Tr}X|\boldsymbol{a}\rangle_\epsilon\langle\boldsymbol{a}|\mathrm{Tr}X^k|\boldsymbol{a}\rangle_\epsilon$$

$$\mathcal{L}_{\mathrm{mic}}(z)=-\frac{1}{z}\frac{\partial}{\partial\lambda_{-1}}-\sum_{k\geq1}\frac{k}{z^{k+1}}\frac{\partial}{\partial\lambda_{k-1}},$$

$$\mathcal{L}''_{\mathrm{mic}}(z)=-N\sum_{k\geq2}\frac{k}{z^{k+1}}\frac{\partial}{\partial t_{k-1}}.$$

$$R_{\mathrm{mic}}(z)=\frac{N}{z}+\frac{a}{z^2}-\frac{2}{N}\mathcal{L}''_{\mathrm{mic}}(z)\cdot\mathcal{F}_{\mathrm{mic}}$$
$$S_{\mathrm{mic}}(z)=-\mathcal{L}_{\mathrm{mic}}(z)\cdot W_{\mathrm{mic}}$$

$$S_{\mathrm{mic}}(z)=-\frac{1}{2i\pi}\oint_\alpha\mathcal{L}_{\mathrm{mic}}(z)\cdot R_{\mathrm{mic}}(z')W(z')\mathrm{d}z'$$

$$\mathcal{L}''_{\mathrm{mic}}(z)\cdot R_{\mathrm{mic}}(z')=-\frac{N}{\epsilon^2}(\langle\boldsymbol{a}|\mathrm{Tr}\frac{1}{z-X}\mathrm{Tr}\frac{1}{z'-X}|\boldsymbol{a}\rangle_\epsilon$$
$$-\langle\boldsymbol{a}|\mathrm{Tr}\frac{1}{z-X}|\boldsymbol{a}\rangle_\epsilon\langle\boldsymbol{a}|\mathrm{Tr}\frac{1}{z'-X}|\boldsymbol{a}\rangle_\epsilon\Big)$$

$$\mathcal{L}''_{\mathrm{mic}}(z)\cdot R_{\mathrm{mic}}(z')=\mathcal{L}''_{\mathrm{mic}}(z')\cdot R_{\mathrm{mic}}(z)$$

$$\frac{1}{2i\pi}\oint_\alpha\mathcal{L}''_{\mathrm{mic}}(z)\cdot R_{\mathrm{mic}}(z')W(z')\mathrm{d}z'=\frac{1}{2i\pi}\oint_\alpha\mathcal{L}''_{\mathrm{mic}}(z')W(z')\mathrm{d}z'\cdot R_{\mathrm{mic}}(z)$$
$$=-N\sum_{k\geq1}g_k\frac{\partial R_{\mathrm{mic}}(z)}{\partial t_k}$$

$$S''_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})=N\sum_{k\geq1}g_k\frac{\partial R_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{t})}{\partial t_k}$$

$$\oint_{\alpha_i}S''_{\mathrm{mic}}\,\mathrm{d}z=0,\ \oint_{\delta_i}S''_{\mathrm{mic}}\,\mathrm{d}z=0$$

$$S'_{\mathrm{mic}}(P)=\int_{P_0}^{P}S''_{\mathrm{mic}}\mathrm{d}z$$

$$F_{\mathrm{mic}}(P)=\langle\boldsymbol{a}|\det(z-X)|\boldsymbol{a}\rangle=\mu_0^N\exp\int_{P_0}^{P}R_{\mathrm{mic}}\,\mathrm{d}z$$

$$F_{\mathrm{mic}}(z)\underset{z\to\infty}{\sim}z^N,\hat{F}_{\mathrm{mic}}(z)\underset{z\to\infty}{\sim}\frac{e^{NV(z)}}{z^N}$$

$$F_{\text{mic}}(z) = \phi_1(z) + \phi_2(z)y$$

$$\frac{1}{F_{\text{mic}}(z)} = \varphi_1(z) + \varphi_2(z)y$$

$$f_\delta(z) = \frac{\delta F_{\text{mic}}}{F_{\text{mic}}} = \delta \ln\, F_{\text{mic}}$$

$$\delta y = \frac{\rho_\delta}{y}$$

$$f_\delta(z) = p_\delta(z) + \frac{q_\delta(z)}{y}$$

$$S'_{\text{mic}}(z; \boldsymbol{a}, \boldsymbol{g}, \boldsymbol{t}) = N \sum_{k\geq 1} g_k \frac{\partial \ln\, F_{\text{mic}}(z; \boldsymbol{a}, \boldsymbol{t})}{\partial t_k}$$

$$S'_{\text{mic}}(z) = \frac{N}{2}\left(p(z) + \frac{E_S(z)}{y}\right)$$

$$R_{\text{mic}}(z) + \hat{R}_{\text{mic}}(z) = NV'(z)$$

$$S''_{\text{mic}}(z) + \hat{S}''_{\text{mic}}(z) = NW'''(z)$$

$$S'_{\text{mic}}(z) + \hat{S}'_{\text{mic}}(z) = NW''(z) + c = Np(z)$$

$$\int_{\beta_i} S''_{\text{mic}} \mathrm{dz} = NW''(\mu_0)$$

$$S'_{\text{mic}}(z) = \frac{N}{2}\left(W''(z) + \frac{E_S(z)}{y}\right)$$

$$S'_{\text{mic}}\,(z) \underset{z\to\infty}{\sim} -\frac{v_{0,\,\text{mic}}}{z^2}$$

$$\oint_{\alpha_i} S'_{\text{mic}}\, \mathrm{dz} = 0$$

$$\oint_{\alpha_i} S'_{\text{mic}}\, \mathrm{dz} = -\oint_{\alpha_i} zS''_{\text{mic}}\, \mathrm{dz} = -N \sum_{k\geq 1} g_k \frac{\partial}{\partial t_k} \oint_{\alpha_i} \lambda_{\text{mic}}$$

$$\frac{1}{2i\pi} \oint_{\alpha_j} h_i = \delta_{ij}$$

$$h_i = \psi_i\, \mathrm{dz} = \frac{p_i}{y}\, \mathrm{dz}$$

$$\frac{\partial \lambda_{\text{mic}}}{\partial a_i} = h_i - \mathrm{d}(z\psi_i)$$

$$\lambda_{\text{mic}} = zR_{\text{mic}}\, \mathrm{dz} = -\ln\, F_{\text{mic}}\, \mathrm{dz} + \mathrm{d}(z\ln\, F_{\text{mic}})$$

$$\frac{\partial \ln\, F_{\text{mic}}}{\partial a_i} = p(z) + \frac{q(z)}{y}$$

$$\frac{1}{F_{\text{mic}}} \frac{\partial F_{\text{mic}}}{\partial a_i} = \mathcal{O}(1/z)$$

$$\frac{\partial \ln\ F_{\text{mic}}}{\partial a_i} = -\psi_i$$

$$\frac{\partial W_{\text{mic}}}{\partial a_i} = \frac{1}{2i\pi} \oint_{\alpha} \frac{W}{z} \frac{\partial \lambda_{\text{mic}}}{\partial a_i}$$

$$\frac{\partial W_{\text{mic}}}{\partial a_i} = \frac{1}{2i\pi} \oint_{\alpha} W' h_i$$

$$\int_{\beta_i} S'_{\text{mic}}\, \mathrm{dz} \ = -\frac{1}{2i\pi} \oint_{\alpha} \left[S'_{\text{mic}} H_i + (NW'' - S'_{\text{mic}}) \left(\int_{\beta_r} h_i + 2\ln\ \mu_0 - H_i \right) \right]$$

$$= NW'(\mu_0) - \frac{N}{2i\pi} \oint_{\alpha} W' h_i$$

$$\frac{\partial W_{\text{mic}}}{\partial a_i} = -\frac{1}{N} \int_{\beta_i} S'_{\text{mic}} \mathrm{dz} + W'(\mu_0)$$

$$\int_{\beta_i} S^{*'}_{\text{mic}} \mathrm{dz} = NW'(\mu_0)$$

$$S^{*}_{\text{mic}}(z) + \hat{S}^{*}_{\text{mic}}(z) = NW'(z) + \tilde{c}$$

$$\int_{\beta_i} S^{*'}_{\text{mic}} \mathrm{dz} = NW'(\mu_0) + \tilde{c}$$

$$S^{*}_{\text{mic}}(z)(NW'(z) - S^{*}_{\text{mic}}(z)) = N^2 \Delta^{*}_{S,\text{mic}}(z)$$

$$S^{*}_{\text{mic}}(z; \boldsymbol{g}, \boldsymbol{t}) = \frac{N}{2} \left(W'(z) - \sqrt{W'(z)^2 - 4\Delta^{*}_{S,\text{mic}}(z)} \right)$$

$$y^2 = M_{N-r}(z)^2 y^2_{\text{mic},r}$$
$$W'(z)^2 - 4\Delta^{*}_{S,\text{mic}}(z) = N_{d_W - r}(z)^2 y^2_{\text{mic},r}$$

$$\mathcal{C}_{\text{mic},r}\colon y^2_{\text{mic},r} = \prod_{i=1}^{r} (z - v_i^-)(z - v_i^+)$$

$$\oint_{\alpha_i} R^{*}_{\text{mic}}\, \mathrm{dz} \in 2i\pi\mathbb{Z}$$

$$\int_{\beta_i} R^{*}_{\text{mic}}\, \mathrm{dz} - NV(\mu_0) + 2N\ln\ \mu_0 \in 2i\pi\mathbb{Z}$$

$$L_n \ = -\frac{1}{2i\pi} \sum_{i=1}^{N} \oint_{\alpha_i} z^{n+1} R_{\text{mic}}\, (z; \boldsymbol{a}, \boldsymbol{t}) \mathrm{dz} \frac{\partial}{\partial a_i}$$

$$J_n \ = -\frac{1}{2i\pi} \sum_{i=1}^{N} \oint_{\alpha_i} z^{n+1} S_{\text{mic}}\, (z; \boldsymbol{a}, \boldsymbol{g}, \boldsymbol{t}) \mathrm{dz} \frac{\partial}{\partial a_i}$$

$$NL_n \cdot W_{\text{mic}}(\boldsymbol{a}, \boldsymbol{g}, \boldsymbol{t}) = \mathcal{A}_n(\boldsymbol{a}, \boldsymbol{g}, \boldsymbol{t}),$$
$$NJ_n \cdot W_{\text{mic}}(\boldsymbol{a}, \boldsymbol{g}, \boldsymbol{t}) = \mathcal{B}_n(\boldsymbol{a}, \boldsymbol{g}, \boldsymbol{t}),$$

$$\mathcal{A}_n = -N \sum_{k \geq 0} g_k u_{n+k+1,\text{mic}} - N \sum_{k \geq 0} \lambda_k v_{n+k+1,\text{mic}} + 2 \sum_{k_1 + k_2 = n} u_{k_1,\text{mic}} v_{k_2,\text{mic}}$$
$$\mathcal{B}_n = -N \sum_{k \geq 0} g_k v_{n+k+1,\text{mic}} + \sum_{k_1 + k_2 = n} v_{k_1,\text{mic}} v_{k_2,\text{mic}}$$

$$\mathcal{A}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t}) = \sum_{n\geq -1} \frac{\mathcal{A}_n(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})}{z^{n+2}} = NL(z)\cdot W_{\text{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})$$
$$= -NW'(z;\boldsymbol{g})R_{\text{mic}}(z;\boldsymbol{a},\boldsymbol{t}) - NV'(z;\boldsymbol{t})S_{\text{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})$$
$$+2R_{\text{mic}}(z;\boldsymbol{a},\boldsymbol{t})S_{\text{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t}) + N^2\Delta_{R,\text{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})$$
$$\mathcal{B}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t}) = \sum_{n\geq -1} \frac{\mathcal{B}_n(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})}{z^{n+2}} = NJ(z)\cdot W_{\text{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})$$
$$= -NW'(z;\boldsymbol{g})S_{\text{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})$$
$$+S_{\text{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})^2 + N^2\Delta_{S,\text{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t})$$

$$NL_n\cdot W_{\text{mic}} = \frac{1}{2i\pi}\sum_i \oint_{\alpha_i} z^{n+1}R_{\text{mic}}\,\mathrm{d}z \int_{\beta_i} S'_{\text{mic}}\,\mathrm{d}z - NW'(\mu_0)u_{n+1,\,\text{mic}}$$

$$S_{\text{mic}}(z) + \hat{S}_{\text{mic}}(z) = NW'(z) + \int_{\beta_r} S'_{\text{mic}}\mathrm{d}z - NW'(\mu_0)$$

$$NL_n\cdot W_{\text{mic}} = -NW'(\mu_0)u_{n+1,\,\text{mic}} + u_{n+1,\,\text{mic}}\int_{\beta_r} S'_{\text{mic}}\,\mathrm{d}z$$
$$+\frac{1}{2i\pi}\oint_{\alpha} z^{n+1}\left[R_{\text{mic}}S_{\text{mic}} + (NV' - R_{\text{mic}})\left(NW' - S_{\text{mic}} - NW'(\mu_0) + \int_{\beta_r} S'_{\text{mic}}\,\mathrm{d}z\right)\right]$$
$$= \frac{1}{2i\pi}\oint_{\alpha} z^{n+1}(-NW'R_{\text{mic}} - NV'S_{\text{mic}} + 2R_{\text{mic}}S_{\text{mic}})\mathrm{d}z = \mathcal{A}_n$$

$$NJ_n\cdot W_{\text{mic}} = \frac{1}{2i\pi}\sum_i \oint_{\alpha_i} z^{n+1}S_{\text{mic}}\,\mathrm{d}z \int_{\beta_i} S'_{\text{mic}}\,\mathrm{d}z - NW'(\mu_0)v_{n+1,\,\text{mic}}$$

$$2\times\frac{1}{2i\pi}\sum_i \oint_{\alpha_i} z^{n+1}S_{\text{mic}}\,\mathrm{d}z \int_{\beta_i} S'_{\text{mic}}\,\mathrm{d}z = 2\times v_{n+1,\,\text{mic}}\int_{\beta_r} S'_{\text{mic}}\,\mathrm{d}z$$
$$+\frac{1}{2i\pi}\oint_{\alpha} z^{n+1}\left[S_{\text{mic}}^2 + \left(NW' - S_{\text{mic}} - NW'(\mu_0) + \int_{\beta_r} S'_{\text{mic}}\,\mathrm{d}z\right)^2\right]$$
$$= \frac{1}{i\pi}\oint_{\alpha} z^{n+1}(-NW'S_{\text{mic}} + S_{\text{mic}}^2)\mathrm{d}z + 2NW'(\mu_0)v_{n+1,\,\text{mic}}$$

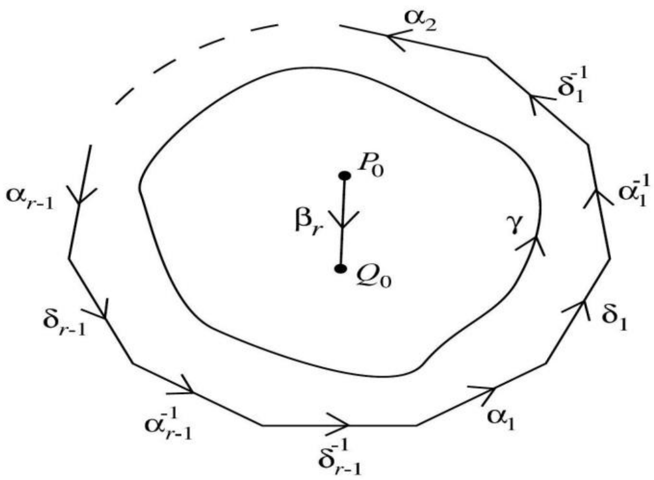
$$L_n\cdot W_{\text{mic}} = 0, J_n\cdot W_{\text{mic}} = 0$$

$$-NW'(z)R^*_{\text{mic}}(z) - NV'(z)S^*_{\text{mic}}(z) + 2R^*_{\text{mic}}(z)S^*_{\text{mic}}(z) + N^2\Delta_{R,\,\text{mic}}(z) = 0$$
$$-NW'(z)S^*_{\text{mic}}(z) + S^*_{\text{mic}}(z)^2 + N^2\Delta_{S,\,\text{mic}}(z) = 0$$
$$R^*_{\text{mic}}(z) \underset{z\to\infty}{\sim} \frac{N}{z}, \qquad S^*_{\text{mic}}(z) \underset{z\to\infty}{\sim} \frac{v_{0,\,\text{mic}}}{z},$$
$$\oint_{\alpha_i} R^*_{\text{mic}}\,\mathrm{d}z \in 2i\pi\mathbb{Z}, \qquad \int_{\beta_i} R^*_{\text{mic}}\,\mathrm{d}z - NV(\mu_0) + 2N\ln\mu_0 \in 2i\pi\mathbb{Z}.$$

$v_{k,\,\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g}) = N\varepsilon\langle\!\langle \boldsymbol{s}\vert\,\mathrm{Tr}\,X^k\vert\boldsymbol{s}\rangle\!\rangle$	$u_{k,\,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{t}) = \langle\boldsymbol{a}\vert\,\mathrm{Tr}\,X^k\vert\boldsymbol{a}\rangle$
$\lambda_{\mathrm{mac}} = S_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g})\,\mathrm{d}z$	$\lambda_{\mathrm{mic}} = zR_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{t})\,\mathrm{d}z$
$\boldsymbol{s} = \frac{1}{2i\pi N}\oint_\alpha \lambda_{\mathrm{mac}}$	$\boldsymbol{a} = \frac{1}{2i\pi}\oint_\alpha \lambda_{\mathrm{mic}}$
$\frac{\partial\lambda_{\mathrm{mac}}}{\partial s_i} = Nh_i$	$\frac{\partial\lambda_{\mathrm{mic}}}{\partial a_i} = h_i - \mathrm{d}(z\psi_i)$
$W_{\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \frac{1}{2i\pi}\oint_\alpha S_{\mathrm{mac}}V\,\mathrm{d}z - \sum_i N_i\frac{\partial\mathcal{F}_{\mathrm{mac}}}{\partial s_i}$	$W_{\mathrm{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t}) = \frac{1}{2i\pi}\oint_\alpha R_{\mathrm{mic}}W\,\mathrm{d}z$
$\sum_i s_i\frac{\partial W_{\mathrm{mac}}}{\partial s_i} = -\frac{1}{2i\pi}\oint_\alpha R_{\mathrm{mac}}W\,\mathrm{d}z + W_{\mathrm{mac}}$	$\sum_i a_i\frac{\partial W_{\mathrm{mac}}}{\partial a_i} = -2v_{0,\,\mathrm{mic}} + \frac{1}{2i\pi}\oint_\alpha (zW'R_{\mathrm{mic}} + zV'S_{\mathrm{mic}})\mathrm{d}z$
$u_{k,\,\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = k\frac{\partial W_{\mathrm{mac}}}{\partial g_{k-1}}$	$v_{k,\,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t}) = \frac{N}{k+1}\frac{\partial W_{\mathrm{mic}}}{\partial t_{k+1}}$
$v_{k,\,\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g}) = \frac{N}{k+1}\frac{\partial W_{\mathrm{mac}}}{\partial t_{k+1}}$	$u_{k,\,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{t}) = k\frac{\partial W_{\mathrm{mic}}}{\partial g_{k-1}}$
$v_{k,\,\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g}) = -Nk\frac{\partial\mathcal{F}_{\mathrm{mac}}}{\partial g_{k-1}}$, $k \geq 1$	$u_{k,\,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{t}) = 2k\frac{\partial\mathcal{F}_{\mathrm{mic}}}{\partial t_{k-1}}$, $k \geq 2$
$u_{k,\,\mathrm{mac}}(\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \frac{1}{2i\pi}\oint_\alpha z^k\sum_i N_ih_i - \frac{N}{\varepsilon^2}\Big[\langle\!\langle\boldsymbol{s}\vert\varepsilon\mathrm{Tr}\,X^k\,\varepsilon\mathrm{Tr}\,V(X)\vert\boldsymbol{s}\rangle\!\rangle_\varepsilon - \langle\!\langle\boldsymbol{s}\vert\varepsilon\mathrm{Tr}\,X^k\vert\boldsymbol{s}\rangle\!\rangle_\varepsilon\langle\!\langle\boldsymbol{s}\vert\varepsilon\mathrm{Tr}\,V(X)\vert\boldsymbol{s}\rangle\!\rangle_\varepsilon\Big]$	$v_{k,\,\mathrm{mic}}(\boldsymbol{a},\boldsymbol{g},\boldsymbol{t}) = \frac{N}{(k+1)(k+2)}\times \frac{1}{\epsilon^2}\Big[\langle\boldsymbol{a}\vert\,\mathrm{Tr}\,X^{k+2}\,\mathrm{Tr}\,W(X)\vert\boldsymbol{a}\rangle_\epsilon - \langle\boldsymbol{a}\vert\,\mathrm{Tr}\,X^{k+2}\vert\boldsymbol{a}\rangle_\epsilon\langle\boldsymbol{a}\vert\,\mathrm{Tr}\,W(X)\vert\boldsymbol{a}\rangle_\epsilon\Big]$
$R_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g},\boldsymbol{t}) = \sum_i N_i\psi_i(z) + \sum_{k\geq 0}\lambda_k\frac{\partial S_{\mathrm{mac}}(z;\boldsymbol{s},\boldsymbol{g})}{\partial g_k}$	$S''_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{g},\boldsymbol{t}) = N\sum_{k\geq 1} g_k\frac{\partial R_{\mathrm{mic}}(z;\boldsymbol{a},\boldsymbol{t})}{\partial t_k}$
$\frac{1}{2i\pi}\oint_{\alpha_i} S'_{\mathrm{mac}}\,\mathrm{d}z = 0$	$\frac{1}{2i\pi}\oint_{\alpha_i} R_{\mathrm{mic}}\,\mathrm{d}z \in \mathbb{Z}$
$\int_{\beta_i} S'_{\mathrm{mac}}\,\mathrm{d}z = NW'(\mu_0)$	$\int_{\beta_i} R_{\mathrm{mic}}\,\mathrm{d}z = NV(\mu_0) - 2N\ln\mu_0$

$\frac{1}{2i\pi}\oint_{\alpha_i} R_{\text{mac}}\,\mathrm{d}z \in \mathbb{Z}$	$\frac{1}{2i\pi}\oint_{\alpha_i} S'_{\text{mic}}\,\mathrm{d}z = 0$
$\frac{\partial W_{\text{mac}}}{\partial s_i} = -\int_{\beta_i} R_{\text{mac}}\,\mathrm{d}z - 2N\ln\mu_0 + NV(\mu_0)$	$\frac{\partial W_{\text{mic}}}{\partial a_i} = -\frac{1}{N}\int_{\beta_i} S'_{\text{mic}}\,\mathrm{d}z + W'(\mu_0)$
$-NW'R_{\text{mac}} - NV'S_{\text{mac}} + 2R_{\text{mac}}S_{\text{mac}} + N^2\Delta_R = 0$	$NL(z)\cdot W_{\text{mic}} = -NW'R_{\text{mic}} - NV'S_{\text{mic}} + 2R_{\text{mic}}S_{\text{mic}} + N^2\Delta_{R,\text{mic}}$
$-NW'S_{\text{mac}} + S_{\text{mac}}^2 + N^2\Delta_S = 0$	$NJ(z)\cdot W_{\text{mic}} = -NW'S_{\text{mic}} + S_{\text{mic}}^2 + N^2\Delta_{S,\text{mic}}$
$\boldsymbol{R}^*_{\text{mac}} = \boldsymbol{R},\quad \boldsymbol{S}^*_{\text{mac}} = \boldsymbol{S}$	$\boldsymbol{R}^*_{\text{mic}} = \boldsymbol{R},\quad \boldsymbol{S}^*_{\text{mic}} = \boldsymbol{S}$

$$\mathcal{C}: y^2 = \prod_{i=1}^{r} (z - z_i^-)(z - z_i^+)$$



$$\oint_{\alpha_i} f\,\mathrm{d}z = 0,\ \oint_{\delta_i} f\,\mathrm{d}z = 0$$

$$F(P) = \int_O^P f\,\mathrm{d}z$$

$$G(P) = \int_O^P g\,\mathrm{d}z$$

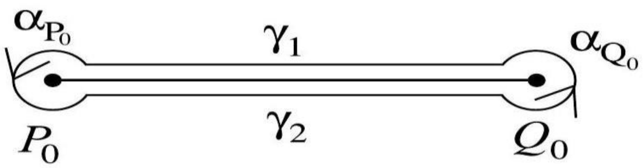
$$\mathcal{I} = \oint_{\gamma} pFG\,\mathrm{d}z,$$

$$\mathcal{I}=\sum_{i=1}^{r-1}\left[\oint_{\alpha_i}\left(pFG-p\left(F+\oint_{\delta_i}f\,\mathrm{du}\right)\left(G+\oint_{\delta_i}g\,\mathrm{du}\right)\right)\mathrm{dz}\right.$$
$$\left.+\oint_{\delta_i}\left(pFG-p\left(F-\oint_{\alpha_i}f\,\mathrm{du}\right)\left(G-\oint_{\alpha_i}g\,\mathrm{du}\right)\right)\mathrm{dz}\right]$$
$$=\sum_{i=1}^{r-1}\left[\oint_{\delta_i}pF\,\mathrm{dz}\oint_{\alpha_i}g\,\mathrm{dz}+\oint_{\delta_i}pG\,\mathrm{dz}\oint_{\alpha_i}f\,\mathrm{dz}\right.$$
$$\left.-\oint_{\delta_i}g\,\mathrm{dz}\oint_{\alpha_i}pF\,\mathrm{dz}-\oint_{\delta_i}f\,\mathrm{dz}\oint_{\alpha_i}pG\,\mathrm{dz}\right]$$

$$\mathcal{I}=\sum_{i=1}^{r}\left[\int_{\beta_i}pF\,\mathrm{dz}\oint_{\alpha_i}g\,\mathrm{dz}+\int_{\beta_i}pG\,\mathrm{dz}\oint_{\alpha_i}f\,\mathrm{dz}-\int_{\beta_i}g\,\mathrm{dz}\oint_{\alpha_i}pF\,\mathrm{dz}-\int_{\beta_i}f\,\mathrm{dz}\oint_{\alpha_i}pG\,\mathrm{dz}\right]$$
$$-\int_{\beta_r}pF\,\mathrm{dz}\oint_{\alpha}g\,\mathrm{dz}-\int_{\beta_r}pG\,\mathrm{dz}\oint_{\alpha}f\,\mathrm{dz}+\int_{\beta_r}g\,\mathrm{dz}\oint_{\alpha}pF\,\mathrm{dz}+\int_{\beta_r}f\,\mathrm{dz}\oint_{\alpha}pG\,\mathrm{dz}$$

$$\int_{\gamma_1+\gamma_2}pFG\,\mathrm{dz}=\int_{\beta_r}p\big(FG-(F-2i\pi f_0)(G-2i\pi g_0)\big)\mathrm{dz}$$

$$\mathcal{I}=-\oint_{\alpha}g\,\mathrm{dz}\int_{\beta_r}pF\,\mathrm{dz}-\oint_{\alpha}f\,\mathrm{dz}\int_{\beta_r}pG\,\mathrm{dz}+\oint_{\alpha_{P_0}+\beta_{P_0}}pFG\,\mathrm{dz}$$



$$\sum_{i=1}^{r}\left[\int_{\beta_i}pF\,\mathrm{dz}\oint_{\alpha_i}g\,\mathrm{dz}+\int_{\beta_i}pG\,\mathrm{dz}\oint_{\alpha_i}f\,\mathrm{dz}-\int_{\beta_i}g\,\mathrm{dz}\oint_{\alpha_i}pF\,\mathrm{dz}-\int_{\beta_i}f\,\mathrm{dz}\oint_{\alpha_i}pG\,\mathrm{dz}\right]$$
$$+\int_{\beta_r}g\,\mathrm{dz}\oint_{\alpha}pF\,\mathrm{dz}+\int_{\beta_r}f\,\mathrm{dz}\oint_{\alpha}pG\,\mathrm{dz}=\oint_{\alpha_{P_0}+\beta_{P_0}}pFG\,\mathrm{dz}$$

$$\oint_{\alpha_{P_0}+\beta_{P_0}}pFG\,\mathrm{dz}=-\oint_{\alpha}p(FG+\hat{F}\hat{G})\mathrm{dz}$$

$$\sum_{i=1}^{r}\left[\int_{\beta_i}pF\,\mathrm{dz}\oint_{\alpha_i}g\,\mathrm{dz}-\int_{\beta_i}g\,\mathrm{dz}\oint_{\alpha_i}pF\,\mathrm{dz}\right]+\int_{\beta_r}g\,\mathrm{dz}\oint_{\alpha}pF\,\mathrm{dz}=-\oint_{\alpha}p(FG+\hat{F}\hat{G})\mathrm{dz}$$

$$W(z)=\frac{1}{2}mz^2,V(z)=\lambda_{-1}+\lambda_0z+\frac{1}{2}\lambda_1z^2$$

$$R(z)\ =\frac{N}{2}\left[\lambda_0+\lambda_1z+\frac{2+2s\lambda_1/m-\lambda_0z-\lambda_1z^2}{\sqrt{z^2-4s/m}}\right]$$
$$S(z)\ =\frac{Nm}{2}\left[z-\sqrt{z^2-4s/m}\right]$$

$$\int_{\beta_1}R\,\mathrm{dz}=NV(\mu_0)-2N\ln\,\mu_0+2i\pi k=N\int_{2\sqrt{s/m}}^{\mu_0}\frac{\lambda_1z^2+\lambda_0z-2s\lambda_1/m-2}{\sqrt{z^2-4s/m}}$$

$$q=\left(\frac{s}{m}\right)^{N}e^{-N\lambda_1 s/m}$$

$$s=me^{2i\pi k/N}q^{1/N}\left(1+\lambda_1 e^{2i\pi k/N}q^{1/N}+\mathcal{O}(\lambda_1^2)\right),0\leq k\leq N-1$$

$$s\simeq-\frac{m}{N\lambda_1}(\ln(\lambda_1^N q)+2i\pi k),k\in\mathbb{Z}$$

$$s\simeq\frac{m}{N\lambda_1}\left(\frac{8\pi^2}{g_{\mathrm{YM}}^2}-i\vartheta-\ln\,\lambda_1^N\right)$$

$$(\partial V)\frac{\sinh\,L_V-L_V}{L_V^2}(\partial V)=(\partial V)\left(\frac{L_V}{3!}+\frac{L_V^3}{5!}+\cdots\right)(\partial V)$$

$$\begin{aligned}\delta A_M&=\mathrm{i}\bar{\varepsilon}^I\gamma_M\lambda_I\\ \delta B&=\mathrm{i}\bar{\varepsilon}^I\lambda_I\\ \delta\lambda_I&=\gamma^{MN}F_{MN}\varepsilon_I+\gamma^M D_M B\varepsilon_I+\mathrm{i}X_i(\sigma^i)_I^{\ J}\varepsilon_J\\ \delta X^i&=\bar{\varepsilon}^I(\sigma^i)_I^{\ J}\gamma^M D_M\lambda_J+\mathrm{i}[B,\lambda_J](\sigma^i)_I^{\ J}\bar{\varepsilon}^I\end{aligned}$$

$$\begin{aligned}V&=-\theta\sigma^\mu\bar{\theta}A_\mu+\mathrm{i}\theta^2\bar{\theta}\bar{\lambda}-\mathrm{i}\bar{\theta}^2\theta\lambda+\frac{1}{2}\theta^4(X^3-D_5B)\\ \Phi&=A_5+\mathrm{i}B+2\theta\psi+\theta^2(X^1+\mathrm{i}X^2)\end{aligned}$$

$$e^{2V}\longrightarrow e^{-\mathrm{i}\bar{\Lambda}}e^{2V}e^{\mathrm{i}\Lambda},\Phi\longrightarrow e^{-\mathrm{i}\Lambda}(\Phi-\mathrm{i}\partial_5)e^{\mathrm{i}\Lambda}$$

$$\nabla=\partial_5+\mathrm{i}\Phi$$

$$Z=e^{-2V}\nabla e^{2V},Z\longrightarrow e^{-\mathrm{i}\Lambda}Ze^{\mathrm{i}\Lambda}$$

$$\begin{aligned}\mathcal{L}_5&=\frac{1}{4}\int\ \mathrm{d}^4\theta\mathrm{tr}Z^2+\frac{1}{4}\left(\int\ \mathrm{d}^2\theta\mathrm{tr}W^\alpha W_\alpha+\text{ H.c.}\right)\\ &=-\frac{1}{4}F_{MN}F^{MN}-\frac{1}{2}D_MBD^MB-\frac{\mathrm{i}}{2}\bar{\lambda}^I\Gamma^M D_M\lambda_I+\frac{1}{2}X^iX_i+\frac{1}{2}\bar{\lambda}^I[B,\lambda_I]\end{aligned}$$

$$\mathcal{L}_{10}=-\frac{1}{4}F_{\widehat{M}\widehat{N}}F^{\widehat{M}\widehat{N}}-\frac{\mathrm{i}}{2}\bar{\Xi}\Gamma^{\widehat{M}}D_{\widehat{M}}\Xi$$

$$\delta A_{\widehat{M}}=\frac{\mathrm{i}}{2}\bar{\epsilon}\Gamma_{\widehat{M}}\Xi,\delta\Xi=-\frac{1}{4}F_{\widehat{M}\widehat{N}}\Gamma^{\widehat{M}\widehat{N}}\epsilon$$

$$\Psi_I=\varepsilon_{IJ}C(\bar{\Psi}^J)^T$$

$$\begin{aligned}\mathcal{L}_7=&-\frac{1}{4}\mathrm{tr}F_{MN}F^{MN}-\frac{1}{2}\mathrm{tr}D_MB_iD^MB^i+\frac{1}{4}\mathrm{tr}[B_i,B_j][B^i,B^j]\\ &-\frac{\mathrm{i}}{2}\mathrm{tr}\bar{\Psi}^I\Gamma^M D_M\Psi_I-\frac{\mathrm{i}}{2}\mathrm{tr}\bar{\Psi}^I\left[B_i(\sigma^i)_I^{\ J},\Psi_J\right].\end{aligned}$$

$$\begin{aligned}\delta A_M&=\frac{\mathrm{i}}{2}\bar{\varepsilon}^I\Gamma_M\Psi_I\\ \delta B_i&=\frac{1}{2}\bar{\varepsilon}^I(\sigma_i)_I^{\ J}\Psi_J\\ \delta\Psi_I&=-\frac{1}{4}F_{MN}\Gamma^{MN}\varepsilon_I+\frac{\mathrm{i}}{2}\Gamma^M D_M(B_i\sigma^i)_I^{\ J}\varepsilon_J+\frac{1}{4}\varepsilon^{ijk}[B_i,B_j](\sigma_k)_I^{\ J}\varepsilon_J\end{aligned}$$

$$\Gamma_*=\mathrm{i}\Gamma^0\Gamma^1\Gamma^2\Gamma^3=\begin{pmatrix}-\mathbb{1}&&&\\&\mathbb{1}&&\\&&-\mathbb{1}&\\&&&\mathbb{1}\end{pmatrix}$$

$$B=\Gamma^2\Gamma^5=\begin{pmatrix} 0 & 0 & & -\epsilon_{\alpha\beta} \\ 0 & \epsilon_{\alpha\beta} & -\epsilon^{\dot{\alpha}\dot{\beta}} & 0 \\ \epsilon^{\dot{\alpha}\dot{\beta}} & 0 & 0 & \end{pmatrix}.$$

$$\Psi_1=\begin{pmatrix} \lambda_{1\alpha} \\ \bar{\lambda}_2^{\dot{\alpha}} \\ \lambda_{3\alpha} \\ \bar{\lambda}_4^{\dot{\alpha}} \end{pmatrix}, \Psi_2=\begin{pmatrix} -\lambda_{4\alpha} \\ -\bar{\lambda}_3^{\dot{\alpha}} \\ \lambda_{2\alpha} \\ \bar{\lambda}_1^{\dot{\alpha}} \end{pmatrix}$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_4 \end{pmatrix}, \begin{pmatrix} \lambda_2 \\ \lambda_3 \end{pmatrix}$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_3 \end{pmatrix}, \begin{pmatrix} \lambda_2 \\ \lambda_4 \end{pmatrix}$$

$$\begin{aligned} \phi_1 &= A_5+\mathrm{i}B_1, \quad \psi_1=\mathrm{i}(\lambda_1-\lambda_2), \\ \phi_2 &= A_6+\mathrm{i}B_2, \quad \psi_2=-(\lambda_1+\lambda_2), \\ \phi_3 &= A_7+\mathrm{i}B_3, \quad \psi_3=\mathrm{i}(\lambda_4-\lambda_3). \end{aligned}$$

$$\delta\phi_i=\sqrt{2}\epsilon\psi_i, \delta\psi_i=-\sqrt{2}\mathrm{i}\left(\partial_\mu\phi_i-\partial_i A_\mu+\mathrm{i}\left[A_\mu,\phi_i\right]\right)\sigma^\mu\bar{\epsilon}-\sqrt{2}F_i\epsilon.$$

$$F_i=-\frac{1}{2}\varepsilon_{ijk}\left(F_{jk}+2\mathrm{i}D_jB_k-\mathrm{i}\left[B_j,B_k\right]\right).$$

$$\delta A_\mu=-\frac{\mathrm{i}}{\sqrt{2}}\left(\epsilon\sigma_\mu\bar{\chi}-\chi\sigma_\mu\bar{\epsilon}\right), \delta\chi=-\sqrt{2}F_{\mu\nu}\sigma^{\mu\nu}\epsilon+\sqrt{2}\mathrm{i}D\epsilon,$$

$$\left[\delta_\epsilon,\delta_\eta\right]A_\mu=-2\mathrm{i}(\epsilon\sigma^\nu\bar{\eta}-\eta\sigma^\nu\epsilon)\partial_\nu A_\mu-\delta_{\text{gauge}}$$

$$\mathrm{i}(\epsilon\sigma^\mu\bar{\eta}-\eta\sigma^\mu\epsilon)A_\mu$$

$$\begin{aligned} V &= -\theta\sigma^\mu\bar{\theta}A_\mu+\frac{1}{\sqrt{2}}\theta^2\bar{\theta}\bar{\chi}+\frac{1}{\sqrt{2}}\bar{\theta}^2\theta\chi+\frac{1}{2}\theta^4 D \\ \Phi_i &= \phi_i+\sqrt{2}\mathrm{i}\theta\psi_i+\theta^2F_i \end{aligned}$$

$$\Phi_i \longrightarrow e^{-\mathrm{i}\Lambda}(\Phi_i-\mathrm{i}\partial_i)e^{\mathrm{i}\Lambda}, e^{2V}\longrightarrow e^{-\mathrm{i}\bar{\Lambda}}e^{2V}e^{\mathrm{i}\Lambda}$$

$$\nabla_i=\partial_i+\mathrm{i}\Phi_i,$$

$$\nabla_i e^{2V}=\partial_i e^{2V}+\mathrm{i}\bar{\Phi}_i e^{2V}-\mathrm{i}e^{2V}\Phi_i$$

$$\frac{1}{16}\int \mathrm{d}^2\theta W^\alpha W_\alpha+\text{ H.c. }=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\mathrm{i}}{2}\chi\sigma^\mu D_\mu\bar{\chi}+\frac{1}{2}D^2.$$

$$Z_i=e^{-2V}\nabla_i e^{2V}.$$

$$\Lambda=\lambda+\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu\lambda+\frac{1}{4}\theta^4\,\Box\,\lambda$$

$$\delta Z_i=\mathrm{i}[Z_i,\Lambda]=\mathrm{i}[Z_i,\lambda]+\mathrm{i}\left[Z_i,\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu\lambda+\frac{1}{4}\theta^4\,\Box\,\lambda\right]$$

$$\begin{aligned} \frac{1}{4}\int \mathrm{d}^4\theta\mathrm{tr}Z_iZ_i =& -\frac{1}{2}F_{\mu i}F^{\mu i}-\frac{1}{2}D_\mu B_i D^\mu B_i-DD_iB_i+2F_i\bar{F}_i \\ & -\frac{\mathrm{i}}{2}\psi_i\sigma^\mu D_\mu\bar{\psi}_i-\frac{1}{2}\chi D_i\psi_i-\frac{1}{2}\bar{\chi}D_i\bar{\psi}_i \\ & +\frac{1}{2}\chi[B_i,\psi_i]-\frac{1}{2}\bar{\chi}[B_i,\bar{\psi}_i] \end{aligned}$$

$$W=\varepsilon_{ijk}\Phi_i\left(\partial_j\Phi_k+\frac{\mathrm{i}}{3}[\Phi_j,\Phi_k]\right)$$

$$\frac{1}{4}\int \mathrm{d}^2\theta W + \mathrm{H.c.} = \frac{1}{4}\varepsilon_{ijk}F_i\left(F_{jk} + 2\mathrm{i}D_jB_k - \mathrm{i}[B_j, B_k]\right)$$
$$+\frac{1}{4}\varepsilon_{ijk}\psi_iD_j\psi_k - \frac{1}{4}\varepsilon_{ijk}\psi_i[B_j, \psi_k] + \mathrm{H.c.}$$

$$\delta W = \frac{1}{3}\varepsilon_{ijk}\left(e^{-\mathrm{i}\Lambda}\partial_i e^{\mathrm{i}\Lambda}\right)\left(e^{-\mathrm{i}\Lambda}\partial_j e^{\mathrm{i}\Lambda}\right)\left(e^{-\mathrm{i}\Lambda}\partial_k e^{\mathrm{i}\Lambda}\right)$$

$$\int \mathrm{d}^3 y\delta W = \blacksquare \cdot n$$

$$\int \mathrm{d}^3 y\delta W = z + \theta\zeta + \theta^2 F_Z$$

$$\int \mathrm{d}^2\theta\delta W = F_Z$$

$$\mathcal{L}_{\mathrm{SF}} = \frac{1}{4}\int \mathrm{d}^4\theta \operatorname{tr} Z_i Z_i$$
$$+\left[\frac{1}{16}\int \mathrm{d}^2\theta W^\alpha W_\alpha + \frac{1}{4}\int \mathrm{d}^2\theta\varepsilon_{ijk}\Phi_i\left(\partial_j\Phi_k + \frac{\mathrm{i}}{3}[\Phi_j, \Phi_k]\right) + \mathrm{H.c.}\right]$$
$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}F_{\mu i}F^{\mu i} - \frac{1}{2}D_\mu B_i D^\mu B_i + \frac{1}{2}D^2 - DD_iB_i$$
$$+\frac{1}{2}F_i\bar{F}_i + \left[\frac{1}{4}F_i\varepsilon_{ijk}\left(F_{jk} + 2\mathrm{i}D_jB_k - \mathrm{i}[B_j, B_k]\right) + \mathrm{H.c.}\right]$$
$$-\frac{\mathrm{i}}{2}\chi\sigma^\mu D_\mu\bar{\chi} - \frac{\mathrm{i}}{2}\psi_i\sigma^\mu D_\mu\bar{\psi}_i$$
$$-\frac{1}{2}\left[\chi(D_i\psi_i - [B_i, \psi_i]) - \frac{1}{2}\varepsilon_{ijk}\psi_i\left(D_j\psi_k - [B_j, \psi_k]\right) + \mathrm{H.c.}\right].$$

$$F_i = -\frac{1}{2}\varepsilon_{ijk}\left(F_{jk} + 2\mathrm{i}D_jB_k - \mathrm{i}[B_j, B_k]\right)$$
$$D = D_iB_i$$

$$\mathcal{L}_{\mathrm{SF}} = -\frac{1}{4}F_{MN}F^{MN} - \frac{1}{2}D_M B_i D^M B_i + \frac{1}{4}[B_i, B_j][B_i, B_j]$$
$$-\frac{\mathrm{i}}{2}\chi\sigma^\mu D_\mu\bar{\chi} - \frac{\mathrm{i}}{2}\psi_i\sigma^\mu D_\mu\bar{\psi}_i$$
$$-\frac{1}{2}\left[\chi(D_i\psi_i - [B_i, \psi_i]) - \frac{1}{2}\varepsilon_{ijk}\psi_i\left(D_j\psi_k - [B_j, \psi_k]\right) + \mathrm{H.c.}\right]$$

$$\Gamma^\mu = \mathbb{1} \otimes \gamma^\mu, \Gamma^{3+i} = \sigma^i \otimes \gamma^5$$

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

$$\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$

$$C\Gamma_M C^{-1} = -\Gamma_M^T, B\Gamma_M B^{-1} = \Gamma_M^*$$

$$\Psi_I = \varepsilon_{IJ}C(\bar{\Psi}^T)^J, \bar{\Psi}^I = -\varepsilon^{IJ}\Psi_J^T C$$
$$\Psi_I = \varepsilon_{IJ}B(\Psi^*)^J, (\Psi^*)^J = \varepsilon^{IJ}B^*\Psi_J$$

$$C = \Gamma^0\Gamma^2\Gamma^5 = \begin{pmatrix} 0 & \varepsilon & 0 \\ -\varepsilon & 0 & 0 \\ 0 & -\varepsilon & 0 \end{pmatrix}, B = \Gamma^2\Gamma^5 = \begin{pmatrix} 0 & 0 & -\varepsilon \\ 0 & \varepsilon & 0 \\ -\varepsilon & 0 & 0 \end{pmatrix}.$$

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \psi^\beta\epsilon_{\beta\alpha}$$
$$\bar{\chi}^{\dot{\alpha}} = \bar{\chi}_{\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}}, \quad \bar{\chi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\chi}^{\dot{\beta}}$$

$$\epsilon_{12} = \epsilon^{12} = 1, \epsilon_{\dot{1}\dot{2}} = \epsilon^{\dot{1}\dot{2}} = -1$$

$$\psi\chi = \psi^\alpha\chi_\alpha = \chi\psi, \bar{\psi}\bar{\chi} = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = \bar{\chi}\bar{\psi}$$

$$(\psi_\alpha)^\dagger = \bar{\psi}_{\dot{\alpha}}, (\psi^\alpha)^\dagger = \bar{\psi}^{\dot{\alpha}}, \Rightarrow \ (\psi^\alpha \chi_\alpha)^\dagger = \bar{\chi}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} = \bar{\chi}\bar{\psi}$$

$$\epsilon^{\alpha\beta} \sigma^\mu_{\beta\dot{\beta}} \epsilon^{\dot{\beta}\dot{\alpha}} = -(\tilde{\sigma}^\mu)^{\dot{\alpha}\alpha}, \epsilon_{\dot{\alpha}\dot{\beta}} (\tilde{\sigma}^\mu)^{\dot{\beta}\beta} \epsilon_{\beta\alpha} = -\sigma^\mu_{\alpha\dot{\alpha}}$$

$$\Gamma^{M \cdots P} = \begin{pmatrix} A_\alpha{}^\beta & B_{\alpha\dot{\beta}} & * \\ C^{\dot{\alpha}\beta} & D^{\dot{\alpha}}{}_{\dot{\beta}} & \\ & * & \end{pmatrix}$$

$$B = \Gamma^2 \Gamma^5 = \begin{pmatrix} 0 & 0 & -\epsilon_{\alpha\beta} \\ 0 & \epsilon_{\alpha\beta} & 0 \\ \epsilon^{\dot{\alpha}\dot{\beta}} & 0 & 0 \end{pmatrix}$$

$$\Gamma_* = i \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 = \begin{pmatrix} -\mathbb{1} & & & \\ & \mathbb{1} & & \\ & & -\mathbb{1} & \\ & & & \mathbb{1} \end{pmatrix}$$

$$\Psi_1 = \begin{pmatrix} \lambda_{1\alpha} \\ \bar{\lambda}_2^{\dot{\alpha}} \\ \lambda_{3\alpha} \\ \bar{\lambda}_4^{\dot{\alpha}} \end{pmatrix}, \Psi_2 = \begin{pmatrix} -\lambda_{4\alpha} \\ -\bar{\lambda}_3^{\dot{\alpha}} \\ \lambda_{2\alpha} \\ \bar{\lambda}_1^{\dot{\alpha}} \end{pmatrix}$$

$$\bar{\Psi}_1 = \left(-\lambda_2^\alpha, \bar{\lambda}_{1\dot{\alpha}}, -\lambda_4^\alpha, \bar{\lambda}_{3\dot{\alpha}}\right), \bar{\Psi}_2 = \left(\lambda_3^\alpha, -\bar{\lambda}_{4\dot{\alpha}}, -\lambda_1^\alpha, \bar{\lambda}_{2\dot{\alpha}}\right)$$

$$\hat{\Gamma}^M = \sigma_3 \otimes \sigma_3 \otimes \Gamma^M, \hat{\Gamma}^7 = \sigma_2 \otimes \mathbb{1} \otimes \mathbb{1}_8$$
$$\hat{\Gamma}^8 = -\sigma_1 \otimes \mathbb{1} \otimes \mathbb{1}_8, \hat{\Gamma}^9 = \sigma_3 \otimes \sigma_1 \otimes \mathbb{1}_8$$

$$\hat{\Gamma}_* = \hat{\Gamma}^0 \cdots \hat{\Gamma}^9 = \sigma_3 \otimes \sigma_2 \otimes \mathbb{1}_8$$

$$\Xi = \begin{pmatrix} \mathrm{i}\xi_1 \\ \xi_1 \\ \mathrm{i}\xi_2 \\ -\xi_2 \end{pmatrix}$$

$$\hat{C} = \sigma_1 \otimes \sigma_3 \otimes C$$

$$\hat{C} \hat{\Gamma}_{\hat{M}} \hat{C}^{-1} = \hat{\Gamma}^T_{\hat{M}}$$

$$\Xi = \begin{pmatrix} \mathrm{i}\xi_1 \\ \xi_1 \\ \mathrm{i}\xi_2 \\ -\xi_2 \end{pmatrix} = \Xi^c = \hat{C} \bar{\Xi}^T = \begin{pmatrix} \mathrm{i}C\bar{\xi}_2^T \\ C\bar{\xi}_2^T \\ -\mathrm{i}C\bar{\xi}_1^T \\ C\bar{\xi}_1^T \end{pmatrix},$$

$$\xi_1 = C\bar{\xi}_2^T, \xi_2 = -C\bar{\xi}_1^T$$

$$\Sigma^{89}: \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \longrightarrow -\frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}, \Sigma^{97}: \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \longrightarrow -\frac{1}{2} \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix},$$
$$\Sigma^{78}: \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \longrightarrow -\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}.$$

$$\bar{\Xi} \hat{\Gamma}^{7,8,9} \Xi = -2\mathrm{i} \bar{\xi}_I \sigma^{1,2,3}_{IJ} \xi_J$$

$$A_M = \hat{A}_M, B_i = \hat{A}_{6+i}, \Psi_I = \sqrt{2}\xi_I$$

$$\delta A_\mu = \frac{\mathrm{i}}{2}[\bar{\epsilon}_1\tilde{\sigma}_\mu\lambda_1 - \epsilon_1\sigma_\mu\bar{\lambda}_1 + \bar{\epsilon}_2\tilde{\sigma}_\mu\lambda_2 - \epsilon_2\sigma_\mu\bar{\lambda}_2$$
$$+\bar{\epsilon}_3\tilde{\sigma}_\mu\lambda_3 - \epsilon_3\sigma_\mu\bar{\lambda}_3 + \bar{\epsilon}_4\tilde{\sigma}_\mu\lambda_4 - \epsilon_4\sigma_\mu\bar{\lambda}_4]$$
$$\delta A_4 = \frac{\mathrm{i}}{2}[\bar{\epsilon}_1\bar{\lambda}_4 - \epsilon_1\lambda_4 - \bar{\epsilon}_2\bar{\lambda}_3 + \epsilon_2\lambda_3 + \bar{\epsilon}_3\bar{\lambda}_2 - \epsilon_3\lambda_2 - \bar{\epsilon}_4\bar{\lambda}_1 + \epsilon_4\lambda_1]$$
$$\delta A_5 = \frac{1}{2}[\bar{\epsilon}_1\bar{\lambda}_4 + \epsilon_1\lambda_4 + \bar{\epsilon}_2\bar{\lambda}_3 + \epsilon_2\lambda_3 - \bar{\epsilon}_3\bar{\lambda}_2 - \epsilon_3\lambda_2 - \bar{\epsilon}_4\bar{\lambda}_1 - \epsilon_4\lambda_1]$$
$$\delta A_6 = \frac{\mathrm{i}}{2}[\bar{\epsilon}_1\bar{\lambda}_2 - \epsilon_1\lambda_2 - \bar{\epsilon}_2\bar{\lambda}_1 + \epsilon_2\lambda_1 - \bar{\epsilon}_3\bar{\lambda}_4 + \epsilon_3\lambda_4 + \bar{\epsilon}_4\bar{\lambda}_3 - \epsilon_4\lambda_3]$$
$$\delta B_1 = \frac{1}{2}[-\bar{\epsilon}_1\bar{\lambda}_3 - \epsilon_1\lambda_3 + \bar{\epsilon}_2\bar{\lambda}_4 + \epsilon_2\lambda_4 + \bar{\epsilon}_3\bar{\lambda}_1 + \epsilon_3\lambda_1 - \bar{\epsilon}_4\bar{\lambda}_2 - \epsilon_4\lambda_2]$$
$$\delta B_2 = \frac{\mathrm{i}}{2}[\bar{\epsilon}_1\bar{\lambda}_3 - \epsilon_1\lambda_3 + \bar{\epsilon}_2\bar{\lambda}_4 - \epsilon_2\lambda_4 - \bar{\epsilon}_3\bar{\lambda}_1 + \epsilon_3\lambda_1 - \bar{\epsilon}_4\bar{\lambda}_2 + \epsilon_4\lambda_2]$$
$$\delta B_3 = \frac{1}{2}[\bar{\epsilon}_1\bar{\lambda}_2 + \epsilon_1\lambda_2 - \bar{\epsilon}_2\bar{\lambda}_1 - \epsilon_2\lambda_1 + \bar{\epsilon}_3\bar{\lambda}_4 + \epsilon_3\lambda_4 - \bar{\epsilon}_4\bar{\lambda}_3 - \epsilon_4\lambda_3]$$

$$\delta\lambda_1 = -\frac{1}{2}(F_{\mu\nu}\sigma^{\mu\nu} + \mathrm{i}F_{45})\epsilon_1 - \frac{1}{2}F_{\mu 6}\sigma^\mu\bar{\epsilon}_2 - \frac{1}{2}(F_{64} - \mathrm{i}F_{65})\epsilon_3 - \frac{1}{2}(F_{\mu 4} - \mathrm{i}F_{\mu 5})\sigma^\mu\bar{\epsilon}_4$$
$$-\frac{\mathrm{i}}{2}D_\mu(B_1 - \mathrm{i}B_2)\sigma^\mu\bar{\epsilon}_3 + \frac{\mathrm{i}}{2}D_\mu B_3\sigma^\mu\bar{\epsilon}_2 - \frac{\mathrm{i}}{2}(D_4 - \mathrm{i}D_5)(B_1 - \mathrm{i}B_2)\epsilon_2$$
$$-\frac{\mathrm{i}}{2}(D_4 - \mathrm{i}D_5)B_3\epsilon_3 + \frac{\mathrm{i}}{2}D_6(B_1 - \mathrm{i}B_2)\epsilon_4 - \frac{\mathrm{i}}{2}D_6B_3\epsilon_1$$
$$+\frac{1}{2}[B_1, B_2]\epsilon_1 - \frac{\mathrm{i}}{2}[B_1 - \mathrm{i}B_2, B_3]\epsilon_4,$$
$$\delta\lambda_2 = -\frac{1}{2}(F_{\mu\nu}\sigma^{\mu\nu} - \mathrm{i}F_{45})\epsilon_2 + \frac{1}{2}F_{\mu 6}\sigma^\mu\bar{\epsilon}_1 - \frac{1}{2}(F_{64} + \mathrm{i}F_{65})\epsilon_4 + \frac{1}{2}(F_{\mu 4} + \mathrm{i}F_{\mu 5})\sigma^\mu\bar{\epsilon}_3$$
$$+\frac{\mathrm{i}}{2}D_\mu(B_1 + \mathrm{i}B_2)\sigma^\mu\bar{\epsilon}_4 - \frac{\mathrm{i}}{2}D_\mu B_3\sigma^\mu\bar{\epsilon}_1 - \frac{\mathrm{i}}{2}(D_4 + \mathrm{i}D_5)(B_1 + \mathrm{i}B_2)\epsilon_1$$
$$-\frac{\mathrm{i}}{2}(D_4 + \mathrm{i}D_5)B_3\epsilon_4 + \frac{\mathrm{i}}{2}D_6(B_1 + \mathrm{i}B_2)\epsilon_3 - \frac{\mathrm{i}}{2}D_6B_3\epsilon_2$$
$$-\frac{1}{2}[B_1, B_2]\epsilon_2 - \frac{\mathrm{i}}{2}[B_1 + \mathrm{i}B_2, B_3]\epsilon_3,$$
$$\delta\lambda_3 = -\frac{1}{2}(F_{\mu\nu}\sigma^{\mu\nu} - \mathrm{i}F_{45})\epsilon_3 + \frac{1}{2}F_{\mu 6}\sigma^\mu\bar{\epsilon}_4 + \frac{1}{2}(F_{64} + \mathrm{i}F_{65})\epsilon_1 - \frac{1}{2}(F_{\mu 4} + \mathrm{i}F_{\mu 5})\sigma^\mu\bar{\epsilon}_2$$
$$+\frac{\mathrm{i}}{2}D_\mu(B_1 - \mathrm{i}B_2)\sigma^\mu\bar{\epsilon}_1 + \frac{\mathrm{i}}{2}D_\mu B_3\sigma^\mu\bar{\epsilon}_4 + \frac{\mathrm{i}}{2}(D_4 + \mathrm{i}D_5)(B_1 - \mathrm{i}B_2)\epsilon_4$$
$$-\frac{\mathrm{i}}{2}(D_4 + \mathrm{i}D_5)B_3\epsilon_1 + \frac{\mathrm{i}}{2}D_6(B_1 - \mathrm{i}B_2)\epsilon_2 + \frac{\mathrm{i}}{2}D_6B_3\epsilon_3$$
$$+\frac{1}{2}[B_1, B_2]\epsilon_3 + \frac{\mathrm{i}}{2}[B_1 - \mathrm{i}B_2, B_3]\epsilon_2,$$
$$\delta\lambda_4 = -\frac{1}{2}(F_{\mu\nu}\sigma^{\mu\nu} + \mathrm{i}F_{45})\epsilon_4 - \frac{1}{2}F_{\mu 6}\sigma^\mu\bar{\epsilon}_3 + \frac{1}{2}(F_{64} - \mathrm{i}F_{65})\epsilon_2 + \frac{1}{2}(F_{\mu 4} - \mathrm{i}F_{\mu 5})\sigma^\mu\bar{\epsilon}_1$$
$$-\frac{\mathrm{i}}{2}D_\mu(B_1 + \mathrm{i}B_2)\sigma^\mu\bar{\epsilon}_2 - \frac{\mathrm{i}}{2}D_\mu B_3\sigma^\mu\bar{\epsilon}_3 + \frac{\mathrm{i}}{2}(D_4 - \mathrm{i}D_5)(B_1 + \mathrm{i}B_2)\epsilon_3$$
$$-\frac{\mathrm{i}}{2}(D_4 - \mathrm{i}D_5)B_3\epsilon_2 + \frac{\mathrm{i}}{2}D_6(B_1 + \mathrm{i}B_2)\epsilon_1 + \frac{\mathrm{i}}{2}D_6B_3\epsilon_4$$
$$-\frac{1}{2}[B_1, B_2]\epsilon_4 + \frac{\mathrm{i}}{2}[B_1 + \mathrm{i}B_2, B_3]\epsilon_1.$$

$$F_{\alpha\beta} = \frac{1}{5!}\Gamma_{\alpha\beta}^{a_1\ldots a_5}J_{a_1\ldots a_5}$$

$$J^A_{abcde} = -\frac{1}{2}\alpha'^2 M^A{}_{BCD}(\lambda^B\Gamma^f\Gamma_{abcde}\Gamma^g\lambda^C)F^D_{fg}$$

$$\begin{aligned}
\mathcal{L}' &= \mathcal{L}'^{(0)} + \alpha'^2 \mathcal{L}'^{(2)} \\
&= -\frac{1}{4} G^{Aij} G^A_{ij} + \frac{1}{2}\chi^A \not{D} \chi^A \\
&\quad -6\alpha'^2 M_{ABCD}\Big[\mathrm{tr} G^A G^B G^C G^D - \frac{1}{4}(\mathrm{tr} G^A G^B)(\mathrm{tr} G^C G^D) \\
&\quad -2 G^{Ai}{}_k G^{Bjk}(\chi^C \Gamma_i D_j \chi^D) + \frac{1}{2} G^{Ail} D_l G^{Bjk}(\chi^C \Gamma_{ijk}\chi^D) \\
&\quad + \frac{1}{180}(\chi^A \Gamma^{ijk}\chi^B)(D_l \chi^C \Gamma_{ijk} D^l \chi^D) + \frac{3}{10}(\chi^A \Gamma^{ijk}\chi^B)(D_i \chi^C \Gamma_j D_k \chi^D) \\
&\quad + \frac{7}{60} f^D{}_{EF} G^{Aij}(\chi^B \Gamma_{ijk}\chi^C)(\chi^E \Gamma^k \chi^F) \\
&\quad - \frac{1}{360} f^D{}_{EF} G^{Aij}(\chi^B \Gamma^{klm}\chi^C)(\chi^E \Gamma_{ijklm}\chi^F)\Big] + O(\alpha'^3)
\end{aligned}$$

$$J_{abcde} = 10(\lambda\Gamma_{[abc}\lambda)F_{de]} + \frac{1}{2}(\lambda\Gamma_{abcde}{}^{fg}\lambda)F_{fg} = -\frac{1}{2}(\lambda\Gamma^f\Gamma_{abcde}\Gamma^g\lambda)F_{fg}$$

$$\begin{aligned}
0 =& D^b F_{ab} - \lambda\Gamma_a\lambda - 8D^b K_{ab} + 36 w_a - \frac{4}{3}\{\lambda, \tilde{J}_a\} - 2\tilde{J}_b \Gamma_a \tilde{J}^b + \frac{1}{140\cdot 3!}\tilde{J}_{bcd}\Gamma_a \tilde{J}^{bcd} \\
&+ \frac{1}{42}[K_{bcde}, J_a{}^{bcde}] + \frac{1}{42\cdot 4!}[D^f J_{fbcde}, J_a{}^{bcde}]
\end{aligned}$$

$$0 = \not{D}\lambda - 30\psi + \frac{4}{3}D^a \tilde{J}_a + \frac{5}{126\cdot 5!}\Gamma^{abcde}[\lambda, J_{abcde}]$$

$$\begin{aligned}
\tilde{J}_a &= \frac{1}{1680}\Gamma^{bcde} D J_{bcdea} \\
\tilde{J}_{abc} &= -\frac{1}{12}\Gamma^{de} D J_{deabc} - \frac{1}{224}\Gamma_{[ab}\Gamma^{defg} D J_{|defg|c]} \\
\tilde{J}_{abcde} &= D J_{abcde} + \frac{5}{6}\Gamma_{[ab}\Gamma^{fg} D J_{|fg|cde]} + \frac{1}{24}\Gamma_{[abcd}\Gamma^{fghi} D J_{|fghi|e]} \\
K_{ab} &= \frac{1}{5376}(D\Gamma^{cde} D) J_{cdeab} \\
K_{abcd} &= \frac{1}{480}\left(D\Gamma^{fg}_{[a} D\right) J_{|fg|bcd]}
\end{aligned}$$

$$\psi_\alpha = -\frac{1}{840\cdot 3!\cdot 5!}\Gamma_{abc}{}^{\beta\gamma}\Gamma_{dea}{}^{\delta} D_{[\beta} D_\gamma D_{\delta]} J^{abcde}$$

$$w_a = \frac{1}{4032\cdot 4!\cdot 5!}\Gamma^{[\alpha\beta}_{abc}\Gamma^{\gamma\delta]}_{def} D_\alpha D_\beta D_\gamma D_\delta J^{bcdef}$$

$$D J_{abcde} = \tilde{J}_{abcde} + 10\Gamma_{[ab}\tilde{J}_{cde]} + 5\Gamma_{[abcd}\tilde{J}_{e]}$$

$$\Gamma^{ab} D K_{ab} = -\frac{225}{2}\psi + \frac{5}{2}D^a \tilde{J}_a + \frac{1}{2016}\Gamma^{abcde}[\lambda, J_{abcde}]$$

$$0 = \not{D}\lambda + \frac{2}{3}D^a\tilde{J}_a + \frac{4}{15}\Gamma^{ab} D K_{ab} + \frac{1}{42\cdot 5!}\Gamma^{abcde}[\lambda, J_{abcde}]$$

$$D_\alpha \lambda^\beta = \Lambda\delta_\alpha{}^\beta + \frac{1}{2}(\Gamma^{ij})_\alpha{}^\beta \Lambda_{ij} + \frac{1}{24}(\Gamma^{ijkl})_\alpha{}^\beta \Lambda_{ijkl}$$

$$\tilde{J}_a = -\frac{3}{70}F^{ij}F^{kl}\Gamma_{aijkl}\lambda + \frac{9}{35}F_a{}^{i}F^{jk}\Gamma_{ijk}\lambda$$
$$-\frac{1}{5}F_{ij}F^{ij}\Gamma_a\lambda + 2F_{aj}F^{ij}\Gamma_i\lambda$$
$$+\frac{1}{42}(\lambda\Gamma^{ijk}\lambda)\Gamma_{ijk}D_a\lambda - \frac{1}{35}(\lambda\Gamma^{ijk}\lambda)\Gamma_{aij}D_k\lambda + \frac{3}{35}(\lambda\Gamma_a{}^{ij}\lambda)\Gamma_i D_j\lambda$$
$$K_{ab} = \frac{4}{7}F_{ij}F^{ij}F_{ab} - \frac{22}{7}F_a{}^{i}F_b{}^{j}F_{ij}$$
$$+\frac{13}{28}F^{ij}(\lambda\Gamma_{abi}D_j\lambda) + \frac{25}{56}F_{ij}(\lambda\Gamma_a{}^{ij}D_b\lambda)$$
$$+\frac{12}{7}F_a{}^{i}(\lambda\Gamma_b D_i\lambda) + \frac{43}{28}F_a{}^{i}(\lambda\Gamma_i D_b\lambda) + \frac{3}{28}D_a F^{ij}(\lambda\Gamma_{bij}\lambda)$$
$$+\frac{11}{2688}(\lambda\Gamma^{ijk}\lambda)\{\lambda,\Gamma_{abijk}\lambda\} - \frac{3}{448}(\lambda\Gamma_{ab}{}^{i}\lambda)\{\lambda,\Gamma_i\lambda\}$$

$$0 = \not{D}\lambda + \frac{28}{5}F^{ij}F^{kl}\Gamma_{ijk}D_l\lambda + 24F_k^i F^{jk}\Gamma_i D_j\lambda$$
$$-\frac{16}{5}F^{il}D_l F^{jk}\Gamma_{ijk}\lambda + \frac{4}{5}F^{jk}D_i F_{jk}\Gamma^i\lambda$$
$$-\frac{17}{60}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda - \frac{9}{5}(D_i\lambda\Gamma_j D_k\lambda)\Gamma^{ijk}\lambda$$
$$+\frac{1}{5}(\lambda\Gamma^{ijk}\lambda)[F_i^l,\Gamma_{jkl}\lambda] + \frac{1}{2}(\lambda\Gamma^{ijk}\lambda)[F_{ij},\Gamma_k\lambda]$$
$$+\frac{5}{48}F_{ij}\{\lambda,\Gamma^{ijklm}\lambda\}\Gamma_{klm}\lambda - \frac{19}{40}F^{ij}\{\lambda,\Gamma^k\lambda\}\Gamma_{ijk}\lambda + F^{ij}\{\lambda,\Gamma_i\lambda\}\Gamma_j\lambda$$

$$E_1 = F^{ij}F^{kl}\Gamma_{ijk}D_l\lambda$$
$$E_2 = F_k^i F^{jk}\Gamma_i D_j\lambda$$
$$E_3 = F^{il}D_l F^{jk}\Gamma_{ijk}\lambda$$
$$E_4 = F^{jk}D_i F_{jk}\Gamma^i\lambda$$
$$E_5 = (D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda$$
$$E_6 = (D_i\lambda\Gamma_j D_k\lambda)\Gamma^{ijk}\lambda$$
$$E_7 = (\lambda\Gamma^{ijk}\lambda)[F^{lm},\Gamma_{ijklm}\lambda]$$
$$E_8 = (\lambda\Gamma^{ijk}\lambda)[F_i^l,\Gamma_{jkl}\lambda]$$
$$E_9 = (\lambda\Gamma^{ijk}\lambda)[F_{ij},\Gamma_k\lambda]$$
$$E_{10} = F_{ij}\{\lambda,\Gamma^{ijklm}\lambda\}\Gamma_{klm}\lambda$$
$$E_{11} = F^{ij}\{\lambda,\Gamma^k\lambda\}\Gamma_{ijk}\lambda$$
$$E_{12} = F^{ij}\{\lambda,\Gamma_i\lambda\}\Gamma_j\lambda$$

$$0 = \not{D}\lambda + \sum_{n=1}^{12} x_n E_n$$

$$(x_1,\ldots,x_{12}) = \left(\frac{28}{5},24,-\frac{16}{5},\frac{4}{5},-\frac{17}{60},-\frac{9}{5},0,\frac{1}{5},\frac{1}{2},\frac{5}{48},-\frac{19}{40},1\right)$$

$$\mathcal{L} = \mathfrak{F} + \frac{1}{2}\lambda\not{D}\lambda + \sum_{m=1}^{10} a_m\mathcal{L}_m$$

$$\mathcal{L}_1 = F^{ij}F^{kl}(\lambda\Gamma_{ijklm}D^m\lambda)$$
$$\mathcal{L}_2 = F^i{}_k F^{jk}(\lambda\Gamma_i D_j\lambda)$$
$$\mathcal{L}_3 = F^{il}D_l F^{jk}(\lambda\Gamma_{ijk}\lambda)$$
$$\mathcal{L}_4 = F^{ij}F_{ij}(\lambda \not{D}\lambda)$$
$$\mathcal{L}_5 = F^{ij}D_l F^{kl}(\lambda\Gamma_{ijk}\lambda)$$
$$\mathcal{L}_6 = (\lambda\Gamma^{ijk}\lambda)(D_l\lambda\Gamma_{ijk}D^l\lambda)$$
$$\mathcal{L}_7 = (\lambda\Gamma^{ijk}\lambda)(D_i\lambda\Gamma_j D_k\lambda)$$
$$\mathcal{L}_8 = (\lambda\Gamma^{ijk}\lambda)(D^l\lambda\Gamma_{lijkm}D^m\lambda)$$
$$\mathcal{L}_9 = F^{ij}(\lambda\Gamma_{ijk}\lambda)\{\lambda, \Gamma^k\lambda\}$$
$$\mathcal{L}_{10} = F^{ij}(\lambda\Gamma^{klm}\lambda)\{\lambda, \Gamma_{ijklm}\lambda\}$$

$$(a_1, \dots, a_{10}) = \left(-\frac{7}{10}, 12, -\frac{8}{5}, -\frac{13}{5}, -\frac{11}{10}, -\frac{1}{30}, -\frac{9}{5}, 0, \frac{11}{20}, \frac{1}{60}\right).$$

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F^{ij}F_{ij} + \frac{1}{2}\lambda\not{D}\lambda \\
& +\alpha'^2\Big[-6\left(\mathrm{tr}F^4 - \frac{1}{4}(\mathrm{tr}F^2)^2\right) \\
& -\frac{7}{10}F^{ij}F^{kl}(\lambda\Gamma_{ijklm}D^m\lambda) + 12F^i{}_k F^{jk}(\lambda\Gamma_i D_j\lambda) \\
& -\frac{8}{5}F^{il}D_l F^{jk}(\lambda\Gamma_{ijk}\lambda) - \frac{13}{5}F^{ij}F_{ij}(\lambda\not{D}\lambda) - \frac{11}{10}F^{ij}D_l F^{kl}(\lambda\Gamma_{ijk}\lambda) \\
& -\frac{1}{30}(\lambda\Gamma^{ijk}\lambda)(D_l\lambda\Gamma_{ijk}D^l\lambda) - \frac{9}{5}(\lambda\Gamma^{ijk}\lambda)(D_i\lambda\Gamma_j D_k\lambda) \\
& +\frac{11}{20}F^{ij}(\lambda\Gamma_{ijk}\lambda)\{\lambda, \Gamma^k\lambda\} + \frac{1}{60}F^{ij}(\lambda\Gamma^{klm}\lambda)\{\lambda, \Gamma_{ijklm}\lambda\}\Big] + O(\alpha'^3)
\end{aligned}$$

$$\begin{aligned}
\lambda &= \chi + \alpha'^2[\alpha G^{ij}G_{ij}\chi + \beta G^{ij}G^{kl}\Gamma_{ijkl}\chi], \\
A_a &= B_a + \gamma\alpha'^2 G^{ij}(\chi\Gamma_{aij}\chi),
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}' &= \mathcal{L}'^{(0)} + \alpha'^2\mathcal{L}'^{(2)} \\
&= -\frac{1}{4}G^{ij}G_{ij} + \frac{1}{2}\chi\not{D}\chi \\
& -6\alpha'^2\Big[\mathrm{tr}G^4 - \frac{1}{4}(\mathrm{tr}G^2)^2 \\
& -2G^i{}_k G^{jk}(\chi\Gamma_i D_j\chi) + \frac{1}{2}G^{il}D_l G^{jk}(\chi\Gamma_{ijk}\chi) \\
& +\frac{1}{180}(\chi\Gamma^{ijk}\chi)(D_l\chi\Gamma_{ijk}D^l\chi) + \frac{3}{10}(\chi\Gamma^{ijk}\chi)(D_i\chi\Gamma_j D_k\chi) \\
& +\frac{7}{60}G^{ij}(\chi\Gamma_{ijk}\chi)\{\chi, \Gamma^k\chi\} - \frac{1}{360}G^{ij}(\chi\Gamma^{klm}\chi)\{\chi, \Gamma_{ijklm}\chi\}\Big] + O(\alpha'^3)
\end{aligned}$$

$$\delta_\varepsilon(\phi)_{\theta=0} = -(\varepsilon^\alpha\hat{D}_\alpha\phi)_{\theta=0}$$

$$\{\hat{D}_\alpha, \hat{D}_\beta\} = -T_{\alpha\beta}{}^c\partial_c = -2\Gamma^c_{\alpha\beta}\partial_c$$

$$D_a = \partial_a - A_a, D_\alpha = \hat{D}_\alpha - A_\alpha$$

$$[D_A, D_B\} = -T^C_{AB}D_C - F_{AB}$$

$$\Lambda = \varepsilon^\alpha A_\alpha$$

$$Q_\varepsilon = \delta_\varepsilon + T[\varepsilon^\alpha A_\alpha]$$

$$Q_\varepsilon \cdot A_a = \varepsilon^\alpha F_{a\alpha} = (\varepsilon\Gamma_a\lambda) - 7(\varepsilon\tilde{J}_a)$$

$$Q_\varepsilon \cdot \lambda = -\varepsilon^\alpha D_\alpha\lambda = \frac{1}{2}\left(F_{ij} - \frac{28}{5}K_{ij}\right)\Gamma^{ij}\varepsilon - \frac{1}{24}\left(2K_{ijkl} + \frac{7}{30}D^m J_{mijkl}\right)\Gamma^{ijkl}\varepsilon$$

$$Q_\varepsilon \cdot (\varepsilon'^\alpha F_{a\alpha}) - (\varepsilon \leftrightarrow \varepsilon') = -2\varepsilon^\alpha \varepsilon'^\beta D_{(\alpha} F_{|a|\beta)}$$
$$= -\varepsilon^\alpha \varepsilon'^\beta \left(2\Gamma^i_{\alpha\beta} F_{ia} + D_a F_{\alpha\beta}\right) = -2\left(\varepsilon\Gamma^i\varepsilon'\right)\partial_i A_a + D_a\left(2\left(\varepsilon\Gamma^i\varepsilon'\right)A_i - \varepsilon^\alpha\varepsilon'^\beta F_{\alpha\beta}\right)$$

$$[Q_\varepsilon, Q_{\varepsilon'}] = -2\left(\varepsilon\Gamma^i\varepsilon'\right)\partial_i + T\left[2\left(\varepsilon\Gamma^i\varepsilon'\right)A_i - \varepsilon^\alpha\varepsilon'^\beta F_{\alpha\beta}\right].$$

$$Q_\varepsilon = -\varepsilon^\alpha \hat{D}_\alpha + T[\varepsilon^\alpha A_\alpha] = -\varepsilon^\alpha D_\alpha$$

$$\left.\begin{array}{r} \delta_\eta A_a = 0 \\ \delta_\eta \lambda = \eta \end{array}\right\} \Rightarrow [\delta_\eta, \delta_{\eta'}] = 0$$

$$\delta_\eta^{(2)} \mathcal{L}'^{(0)} = \left(D_j G^{ji} + \frac{1}{2}\{\chi, \Gamma^i\chi\}\right)\delta_\eta^{(2)} B_i + \delta_\eta^{(2)}\chi \not{D}\chi$$

$$\delta_\eta B_a = -6\alpha'^2\left[2G_{ai}\left(\eta\Gamma^i\chi\right) - G^{ij}\left(\eta\Gamma_{aij}\chi\right)\right],$$
$$\delta_\eta \chi = \eta - 6\alpha'^2\left[\frac{1}{2}G^{ij}G_{ij}\eta + \frac{1}{4}G^{ij}G^{kl}\Gamma_{ijkl}\eta\right.$$
$$-2\left(\eta\Gamma^i\chi\right)D_i\chi + \frac{1}{15}\left(\eta\Gamma^{ijk}\chi\right)\Gamma_{ij}D_k\chi$$
$$\left.+\frac{19}{25}\left(\eta\Gamma^i\chi\right)\Gamma_i\not{D}\chi - \frac{1}{90}\left(\eta\Gamma^{ijk}\chi\right)\Gamma_{ijk}\not{D}\chi\right]$$

$$\delta_\eta A_a = -6\alpha'^2\left[2F_{ai}\left(\eta\Gamma^i\lambda\right) - \frac{1}{6}F^{ij}\left(\eta\Gamma_{aij}\lambda\right)\right]$$
$$\delta_\eta \lambda = \eta - 6\alpha'^2\left[\frac{1}{15}F^{ij}F_{ij}\eta + \frac{2}{15}F^{ij}F^{kl}\Gamma_{ijkl}\eta\right.$$
$$-2\left(\eta\Gamma^i\lambda\right)D_i\lambda + \frac{1}{15}\left(\eta\Gamma^{ijk}\lambda\right)\Gamma_{ij}D_k\lambda$$
$$\left.+\frac{19}{25}\left(\eta\Gamma^i\lambda\right)\Gamma_i\not{D}\lambda - \frac{1}{90}\left(\eta\Gamma^{ijk}\lambda\right)\Gamma_{ijk}\not{D}\lambda\right]$$

$$[\delta_\eta, \delta_{\eta'}]A_a = -24\alpha'^2\left(\eta\Gamma^i\eta'\right)F_{ia}$$
$$[\delta_\eta, \delta_{\eta'}]\lambda = -24\alpha'^2\left(\eta\Gamma^i\eta'\right)D_i\lambda$$

$$[S_\varrho, S_{\varrho'}] = -2\left(\varrho\Gamma^i\varrho'\right)\partial_i + T\left[2\left(\varrho\Gamma^i\varrho'\right)A_i\right],$$

$$[Q_\varepsilon, S_\varrho] = T\left[-\frac{1}{2\sqrt{3\mu\alpha'}}(\varepsilon\Gamma_i\varrho)x^i + \frac{\alpha'}{6\sqrt{3\mu}}\left(\varepsilon\Gamma^{ijk}\varrho\right)\left(\lambda\Gamma_{ijk}\lambda\right)\right]$$

$$\mathfrak{g} = \mathfrak{u}(N) \simeq \mathfrak{su}(N) \oplus \mathfrak{u}(1)$$

$$M_{A'B'C'D'} = a d_{(A'B'}{}^{E'} d_{C'D')E'} + b\delta_{(A'B'}\delta_{C'D')}$$
$$M_{0A'B'C'} = c d_{A'B'C'}$$
$$M_{00A'B'} = d\delta_{A'B'}$$
$$M_{000A'} = 0$$
$$M_{0000} = \mu$$

$$M_{ABCD} \propto \mathrm{Tr}\left(T_{(A}T_B T_C T_{D)}\right)$$

$$T_{A'}T_{B'} = \frac{1}{N}\delta_{A'B'} + \frac{1}{2}\left(d^{C'}_{A'B'} + f^{C'}_{A'B'}\right)T_{C'}$$

$$a = 1, b = \frac{4}{N}$$

$$c = \frac{2}{\sqrt{N}}, d = \mu = \frac{4}{N}$$

$$(\lambda A\{\lambda), B\lambda\} = \frac{1}{16}\{\lambda, \Gamma^i\lambda\}B\Gamma_i A^t\lambda + \frac{1}{32\cdot 5!}\{\lambda, \Gamma^{ijklm}\lambda\}B\Gamma_{ijklm}A^t\lambda$$

$$(\lambda\Gamma_{[a}\{\lambda),\Gamma_{b]}\lambda\}=\frac{1}{16}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda+\frac{1}{96}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda,$$

$$(\lambda\Gamma^i\{\lambda),\Gamma_{abi}\lambda\}=\{\lambda,\Gamma_{[a}\lambda\}\Gamma_{b]}\lambda-\frac{3}{8}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda+\frac{1}{48}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda,$$

$$(\lambda\Gamma_{abi}\{\lambda),\Gamma^i\lambda\}=\{\lambda,\Gamma_{[a}\lambda\}\Gamma_{b]}\lambda+\frac{3}{8}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda-\frac{1}{48}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda,$$

$$(\lambda\Gamma_{ij[a}\{\lambda),\Gamma^{ij}{}_{b]}\lambda\}=\frac{7}{4}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda-\frac{1}{24}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda,$$

$$(\lambda\Gamma^{ijk}\{\lambda),\Gamma_{abijk}\lambda\}=42\{\lambda,\Gamma_{[a}\lambda\}\Gamma_{b]}\lambda-\frac{21}{4}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda-\frac{3}{8}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda,$$

$$(\lambda\Gamma_{abijk}\{\lambda),\Gamma^{ijk}\lambda\}=42\{\lambda,\Gamma_{[a}\lambda\}\Gamma_{b]}\lambda+\frac{21}{4}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda+\frac{3}{8}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda.$$

$$(\lambda A\{\lambda),B\lambda\}=(\lambda^A A\lambda^B)(B\lambda^C)_\alpha-(\lambda^A A\lambda^C)(B\lambda^B)_\alpha$$

$$\lambda_\alpha^{[B}\lambda_\beta^{C]}=\frac{1}{16}(\Gamma^a)_{\alpha\beta}(\lambda^B\Gamma_a\lambda^C)+\frac{1}{32\cdot 5!}(\Gamma^{abcde})_{\alpha\beta}(\lambda^B\Gamma_{abcde}\lambda^C)$$

$$(\Gamma^a)^{\alpha\beta}(\Gamma^b)_{\alpha\beta}=16\eta^{ab}$$

$$(\Gamma^{a_1\ldots a_5})^{\alpha\beta}\left(\Gamma_{b_1\ldots b_5}\right)_{\alpha\beta}=\mathrm{tr}\left(\frac{1}{2}(\hat{1}+\hat{\Gamma}^{11})\hat{\Gamma}^{a_1\ldots a_5}\hat{\Gamma}^{b_1\ldots b_5}\right)$$
$$=16\cdot 5!\,\delta^{a_1\ldots a_5}_{b_1\ldots b_5}+16\epsilon^{a_1\ldots a_5}{}_{b_1\ldots b_5}$$
$$(\Gamma^{a_1\ldots a_5})_{\alpha\beta}\left(\Gamma_{b_1\ldots b_5}\right)^{\alpha\beta}=\mathrm{tr}\left(\frac{1}{2}(\hat{1}-\hat{\Gamma}^{11})\hat{\Gamma}^{a_1\ldots a_5}\hat{\Gamma}^{b_1\ldots b_5}\right)$$
$$=16\cdot 5!\,\delta^{a_1\ldots a_5}_{b_1\ldots b_5}-16\epsilon^{a_1\ldots a_5}{}_{b_1\ldots b_5}$$

$$(\lambda\Gamma^i D^j\lambda)\Gamma_i D_j\lambda=\frac{1}{24}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda$$
$$(\lambda\Gamma^i D^j\lambda)\Gamma_j D_i\lambda=\frac{1}{48}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda+\frac{1}{4}(D_i\lambda\Gamma_j D_k\lambda)\Gamma^{ijk}\lambda$$
$$(\lambda\Gamma^{ijk}D^l\lambda)\Gamma_{ijk}D_l\lambda=-\frac{1}{2}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda$$
$$(\lambda\Gamma^{ijk}D^l\lambda)\Gamma_{ijl}D_k\lambda=-\frac{1}{24}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda+\frac{5}{2}(D_i\lambda\Gamma_j D_k\lambda)\Gamma^{ijk}\lambda$$

$$S=\int d^3x\,\mathrm{tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{i}{2}\bar{\Lambda}\Gamma^\mu D_\mu\Lambda+D_\mu\xi^\dagger D^\mu\xi+i\bar{\Psi}D_\mu\Gamma^\mu\Psi\right.$$
$$\left.-g\left[\bar{\Psi}\Lambda\xi+\xi^\dagger\bar{\Lambda}\Psi\right]\right)$$

$$D_\mu\Lambda=\partial_\mu\Lambda+ig\left[A_\mu,\Lambda\right],D_\mu\xi=\partial_\mu\xi+igA_\mu\xi,D_\mu\Psi=\partial_\mu\Psi+igA_\mu\Psi$$

$$\delta A_\mu=\frac{i}{2}\bar{\varepsilon}\Gamma_\mu\Lambda,\delta\Lambda=\frac{1}{4}F_{\mu\nu}\Gamma^{\mu\nu}\varepsilon,$$
$$\delta\xi=\frac{i}{2}\bar{\varepsilon}\Psi,\delta\Psi=-\frac{1}{2}\Gamma^\mu\varepsilon D_\mu\xi.$$

$$\bar{\varepsilon}q^\mu=\frac{i}{4}\bar{\varepsilon}\Gamma^{\alpha\beta}\Gamma^\mu\mathrm{tr}\left(\Lambda F_{\alpha\beta}\right)+\frac{i}{2}D^\mu\xi^\dagger\bar{\varepsilon}\Psi+\frac{i}{2}\xi^\dagger\bar{\varepsilon}\Gamma^{\mu\nu}D_\nu\Psi$$
$$-\frac{i}{2}\bar{\Psi}\varepsilon D^\mu\xi+\frac{i}{2}D_\nu\bar{\Psi}\Gamma^{\mu\nu}\varepsilon\xi$$

$$\bar{\varepsilon}Q=\int dx^-dx^2\left(\frac{i}{4}\bar{\varepsilon}\Gamma^{\alpha\beta}\Gamma^+\mathrm{tr}\left(\Lambda F_{\alpha\beta}\right)+\frac{i}{2}D_-\xi^\dagger\bar{\varepsilon}\Psi+\frac{i}{2}\xi^\dagger\bar{\varepsilon}\Gamma^{+\nu}D_\nu\Psi\right.$$
$$\left.-\frac{i}{2}\bar{\Psi}\varepsilon D^+\xi+\frac{i}{2}D_\nu\bar{\Psi}\Gamma^{+\nu}\varepsilon\xi\right)$$

$$\Gamma^0=\sigma_2,\Gamma^1=i\sigma_1,\Gamma^2=i\sigma_3$$

$$\Lambda=(\lambda,\tilde{\lambda})^T,\Psi=(\psi,\tilde{\psi})^T,Q=(Q^+,Q^-)^T$$

$$\{Q^+,Q^+\}=2\sqrt{2}P^+,\{Q^-,Q^-\}=2\sqrt{2}P^-,\{Q^+,Q^-\}=0$$

$$Q^{+} = 2\int dx^{-}\left(\lambda\partial_{-}A^{2}+\frac{i}{2}\partial_{-}\xi^{\dagger}\psi-\frac{i}{2}\psi^{\dagger}\partial_{-}\xi-\frac{i}{2}\xi^{\dagger}\partial_{-}\psi+\frac{i}{2}\partial_{-}\psi^{\dagger}\xi\right)$$
$$Q^{-} = -2\int dx^{-}\left(-\lambda\partial_{-}A^{-}+i\xi^{\dagger}D_{2}\psi-iD_{2}\psi^{\dagger}\xi+\frac{i}{\sqrt{2}}\partial_{-}\left(\tilde{\psi}^{\dagger}\xi-\xi^{\dagger}\tilde{\psi}\right)\right)$$

$$\partial_{-}\tilde{\lambda} = -\frac{ig}{\sqrt{2}}\left([A^{2},\lambda]+i\xi\psi^{\dagger}-i\psi\xi^{\dagger}\right)$$
$$\partial_{-}\tilde{\psi} = -\frac{ig}{\sqrt{2}}A^{2}\psi+\frac{g}{\sqrt{2}}\lambda\xi$$

$$\partial_{-}^{2}A^{-} = gJ$$

$$J \equiv i[A^{2},\partial_{-}A^{2}]+\frac{1}{\sqrt{2}}\{\lambda,\lambda\}-ih\partial_{-}\xi\xi^{\dagger}+i\xi\partial_{-}\xi^{\dagger}+\sqrt{2}\psi\psi^{\dagger}$$

$$A_{ij}^{2}(0,x^{-}) = \frac{1}{\sqrt{4\pi}}\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}\left(a_{ij}(k)e^{-ik\pi x^{-}/L}+a_{ji}^{\dagger}(k)e^{ik\pi x^{-}/L}\right)$$
$$\lambda_{ij}(0,x^{-}) = \frac{1}{2^{\frac{1}{4}}\sqrt{2L}}\sum_{k=1}^{\infty}\left(b_{ij}(k)e^{-ik\pi x^{-}/L}+b_{ji}^{\dagger}(k)e^{ik\pi x^{-}/L}\right)$$
$$\xi_{i}(0,x^{-}) = \frac{1}{\sqrt{4\pi}}\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}\left(c_{i}(k)e^{-ik\pi x^{-}/L}+\tilde{c}_{i}^{\dagger}(k)e^{ik\pi x^{-}/L}\right)$$
$$\psi_{i}(0,x^{-}) = \frac{1}{2^{\frac{1}{4}}\sqrt{2L}}\sum_{k=1}^{\infty}\left(d_{i}(k)e^{-ik\pi x^{-}/L}+\tilde{d}_{i}^{\dagger}(k)e^{ik\pi x^{-}/L}\right)$$

$$\left[A_{ij}^{2}(0,x^{-}),\partial_{-}A_{kl}^{2}(0,y^{-})\right] = i\left(\delta_{il}\delta_{kj}-\frac{1}{N}\delta_{ij}\delta_{kl}\right)\delta(x^{-}-y^{-})$$
$$\{\lambda_{ij}(0,x^{-}),\lambda_{kl}(0,y^{-})\} = \sqrt{2}\left(\delta_{il}\delta_{kj}-\frac{1}{N}\delta_{ij}\delta_{kl}\right)\delta(x^{-}-y^{-})$$
$$\left[\xi_{i}(0,x^{-}),\partial_{-}\xi_{j}(0,y^{-})\right] = i\delta_{ij}\delta(x^{-}-y^{-})$$
$$\{\psi_{i}(0,x^{-}),\psi_{j}(0,y^{-})\} = \sqrt{2}\delta_{ij}\delta(x^{-}-y^{-})$$

$$\left[a_{ij},a_{kl}^{\dagger}\right] = \left(\delta_{il}\delta_{kj}-\frac{1}{N}\delta_{ij}\delta_{kl}\right),\{b_{ij},b_{kl}^{\dagger}\}=\left(\delta_{il}\delta_{kj}-\frac{1}{N}\delta_{ij}\delta_{kl}\right),$$
$$\left[c_{i},c_{j}^{\dagger}\right] = \delta_{ij},\left[\tilde{c}_{i},\tilde{c}_{j}^{\dagger}\right]=\delta_{ij},\{d_{i},d_{j}^{\dagger}\}=\delta_{ij}\ \{\tilde{d}_{i},\tilde{d}_{j}^{\dagger}\}=\delta_{ij}$$

$$Q^{-} = Q_{s}^{-}+Q_{1}^{-}+Q_{2}^{-}+Q_{3}^{-}$$

$$Q_{1}^{-} = -\frac{g}{\sqrt{2}}\int dx^{-}\left(i\sqrt{2}\xi\partial_{-}\xi^{\dagger}-i\sqrt{2}\partial_{-}\xi\xi^{\dagger}\right)\frac{1}{\partial_{-}}\lambda$$
$$Q_{2}^{-} = -\frac{g}{\sqrt{2}}\int dx^{-}\left(2\psi\psi^{\dagger}\right)\frac{1}{\partial_{-}}\lambda$$
$$Q_{3}^{-} = -2g\int dx^{-}\left(\xi^{\dagger}A^{2}\psi+\psi^{\dagger}A^{2}\xi\right)$$

$$Q_{\alpha}^{-} = \frac{i2^{-1/4}g\sqrt{L}}{\pi}\sum_{k_{1},k_{2},k_{3}=1}^{\infty}q_{\alpha}^{-}(k_{1},k_{2},k_{3})$$

$$
\begin{aligned}
q_1^- =&\frac{(k_2+k_3)}{2k_1\sqrt{k_2k_3}}\left[\tilde{c}_i^\dagger(k_2)\tilde{c}_j(k_3)\tilde{b}_{ji}(k_1)-\tilde{c}_i^\dagger(k_2)b_{ij}^\dagger(k_1)\tilde{c}_j(k_3)\right.\\
&\left.+b_{ji}^\dagger(k_1)c_i^\dagger(k_2)c_j(k_3)-c_i^\dagger(k_2)b_{ij}(k_1)c_j(k_3)\right]\delta_{k_3,k_1+k_2},\\
q_2^- =&\frac{1}{k_1}\left[\tilde{d}_i^\dagger(k_2)b_{ij}^\dagger(k_1)\tilde{d}_j(k_3)+\tilde{d}_j^\dagger(k_3)\tilde{d}_i(k_2)b_{ij}(k_1)+\right.\\
&\left.d_i^\dagger(k_2)b_{ij}^\dagger(k_1)d_j(k_3)+d_j^\dagger(k_3)d_i(k_2)b_{ij}(k_1)\right]\delta_{k_3,k_1+k_2},\\
q_3^- =&\frac{-i}{2\sqrt{k_2k_3}}\left[\left(d_j^\dagger(k_1)\tilde{c}_i(k_3)a_{ij}(k_2)+\tilde{c}_i^\dagger(k_3)a_{ij}^\dagger(k_2)d_j(k_1)+\right.\right.\\
&\left(\tilde{d}_i^\dagger(k_1)a_{ij}(k_2)c_j(k_3)+a_{ij}^\dagger(k_2)c_j^\dagger(k_3)\tilde{d}_i(k_1)\right)\delta_{k_1,k_3+k_2}\\
&\left(\tilde{c}_j^\dagger(k_3)d_i(k_1)a_{ij}(k_2)+d_i^\dagger(k_1)a_{ij}^\dagger(k_2)\tilde{c}_j(k_3)+\right.\\
&\left.\left(c_i^\dagger(k_3)a_{ij}(k_2)\tilde{d}_j(k_1)+a_{ij}^\dagger(k_2)\tilde{d}_j^\dagger(k_1)c_i(k_3)\right)\delta_{k_3,k_1+k_2}\right].
\end{aligned}
$$

$$
\bar{f}_{i_1}^\dagger(k_1)a_{i_1i_2}^\dagger(k_2)\ldots b_{i_ni_{n+1}}^\dagger(k_{n-1})\ldots f_{i_p}^\dagger(k_n)|0\rangle.
$$

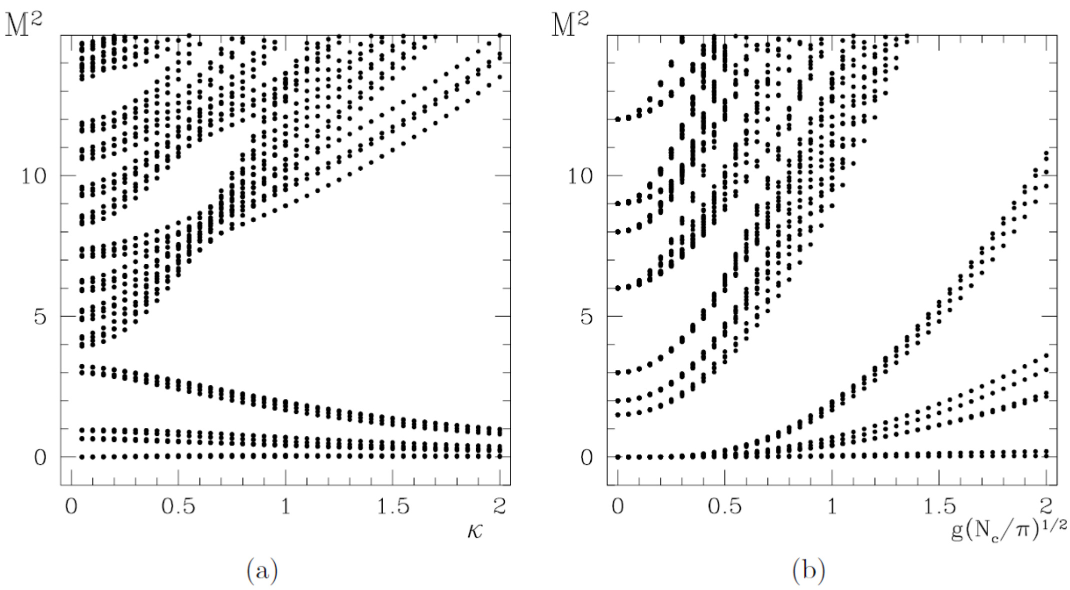
$$
\mathcal{L}=\mathcal{L}_{\mathrm{SQCD}}+\frac{\kappa}{2}\mathcal{L}_{\mathrm{CS}}
$$

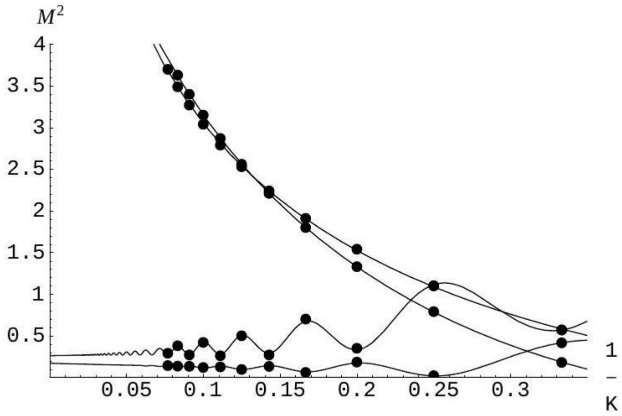
$$
\mathcal{L}_{\mathrm{CS}}=\epsilon^{\mu\nu\lambda}\left(A_\mu\partial_\nu A_\lambda+\frac{2i}{3}gA_\mu A_\nu A_\lambda\right)+2\bar{\Psi}\Psi
$$

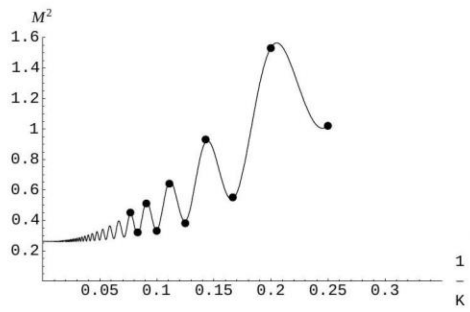
$$
Q_{CS}^-=\left(\frac{2^{-1/4}\sqrt{L}}{\sqrt{\pi}}\right)\sum_n\frac{\kappa}{\sqrt{n}}\left(a_{ij}^\dagger(n)b_{ij}(n)+b_{ij}^\dagger(n)a_{ij}(n)\right).
$$

$$
Q_\perp^-=i\left(\frac{2^{-1/4}\sqrt{L}}{\sqrt{\pi}}\right)\sum_{n,n_\perp}\frac{k_\perp}{\sqrt{n}}\left(a_{ij}^\dagger(n,n_\perp)b_{ij}(n,n_\perp)-b_{ij}^\dagger(n,n_\perp)a_{ij}(n,n_\perp)\right),
$$

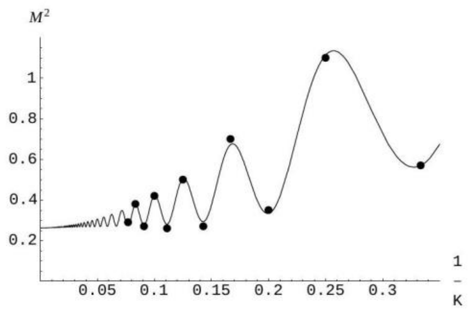
$$
M^2=m^2K\left[\frac{1}{K-n}+\frac{1}{n}\right],
$$



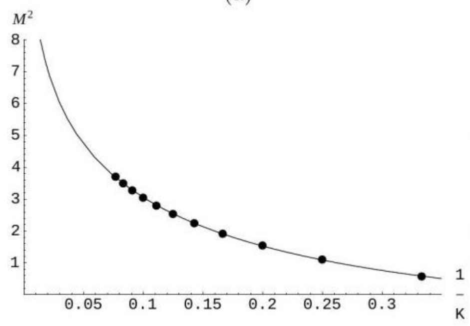

M²
4
3.5
3
2.5
2
1.5
1
0.5
0.05 0.1 0.15 0.2 0.25 0.3
1/K


M²
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0.05 0.1 0.15 0.2 0.25 0.3
1/K

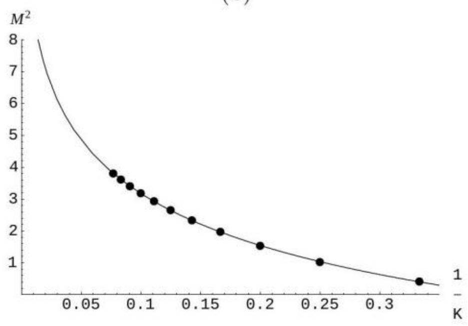
(a)


M²
1
0.8
0.6
0.4
0.2
0.05 0.1 0.15 0.2 0.25 0.3
1/K

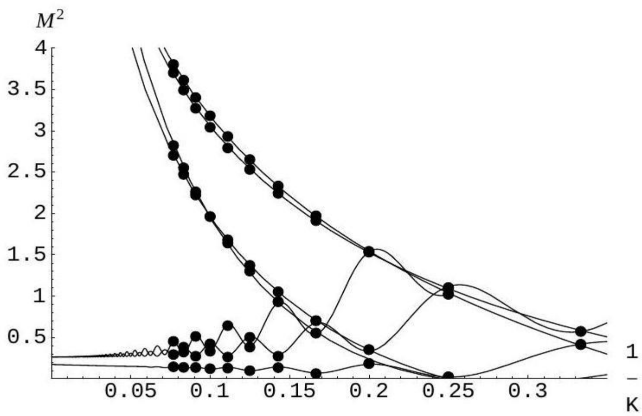
(b)

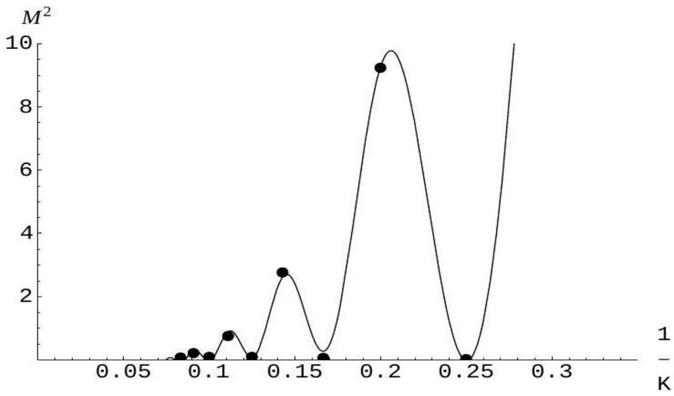

M²
8
7
6
5
4
3
2
1
0.05 0.1 0.15 0.2 0.25 0.3
1/K

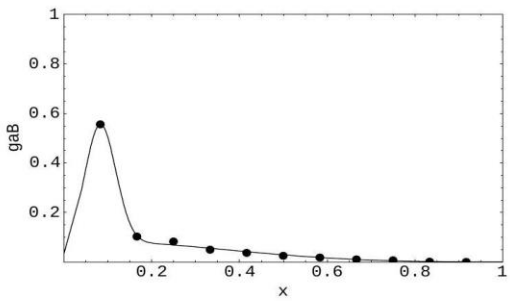
(c)


M²
8
7
6
5
4
3
2
1
0.05 0.1 0.15 0.2 0.25 0.3
1/K

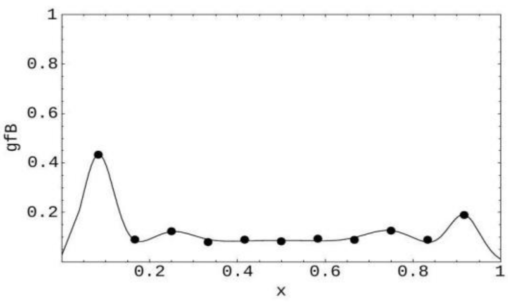
(d)


M²
4
3.5
3
2.5
2
1.5
1
0.5
0.05 0.1 0.15 0.2 0.25 0.3
1/K

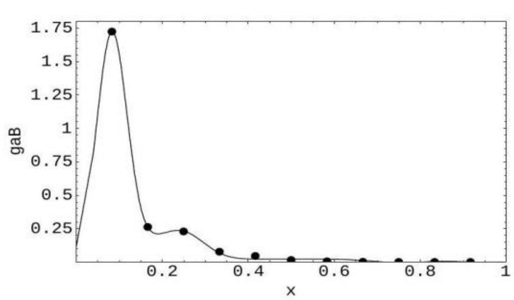




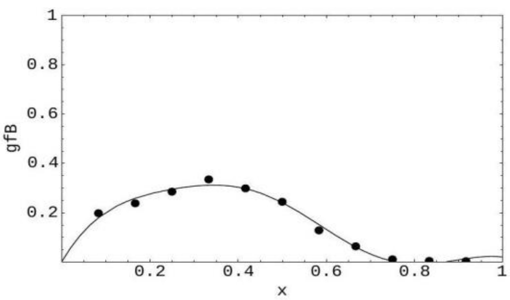
(a)



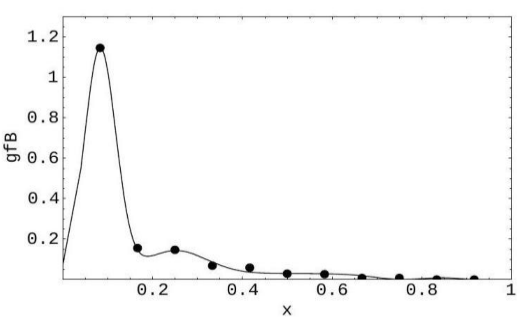
(b)



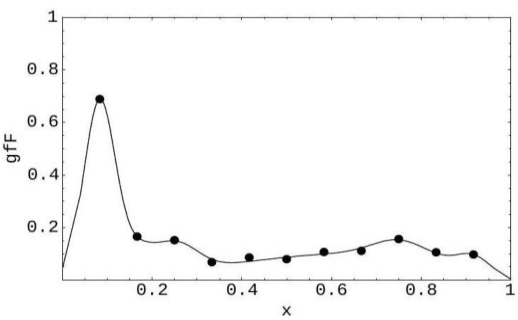
(a)



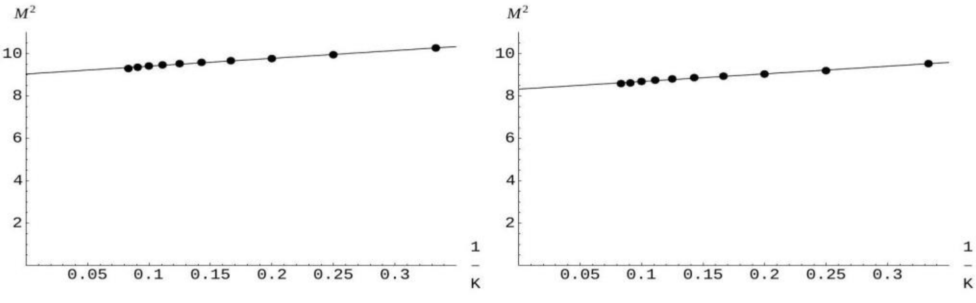
(b)



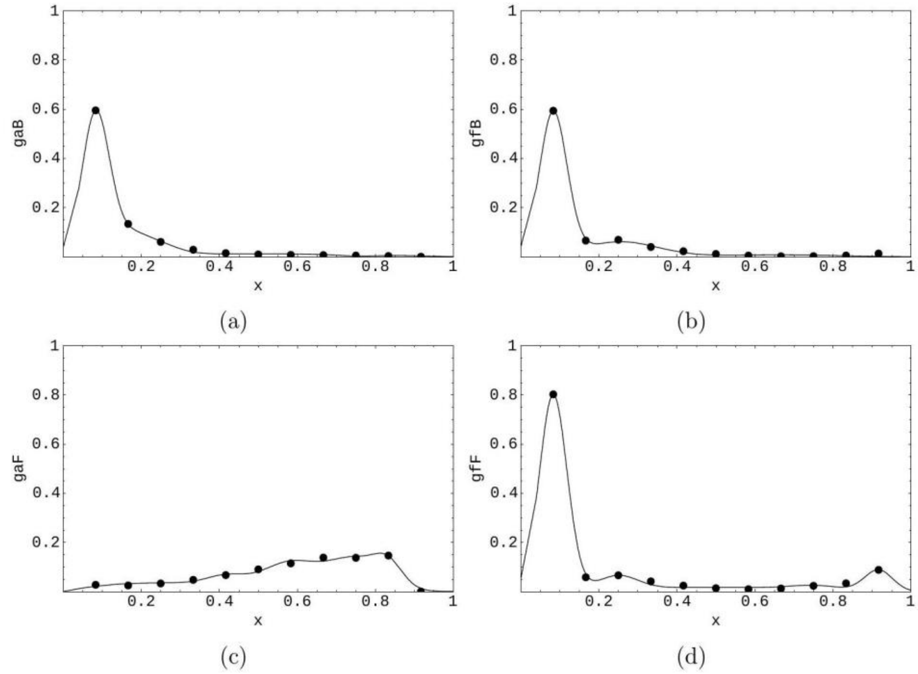
(a)

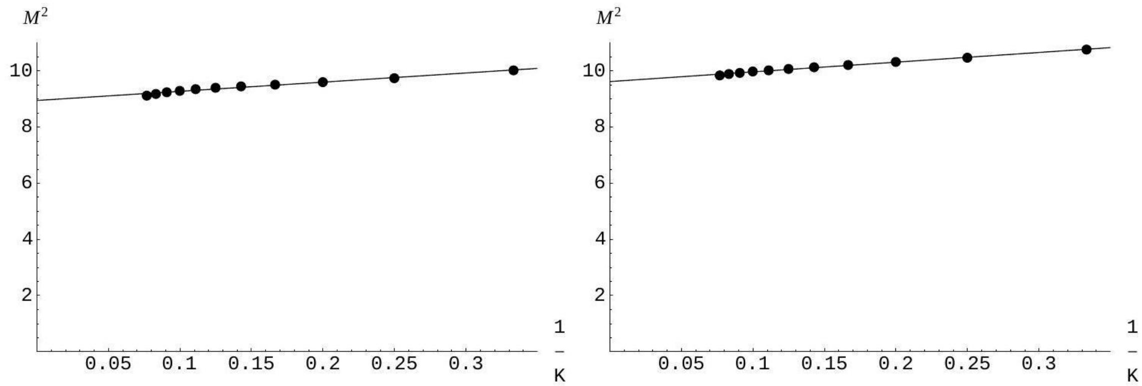


(b)

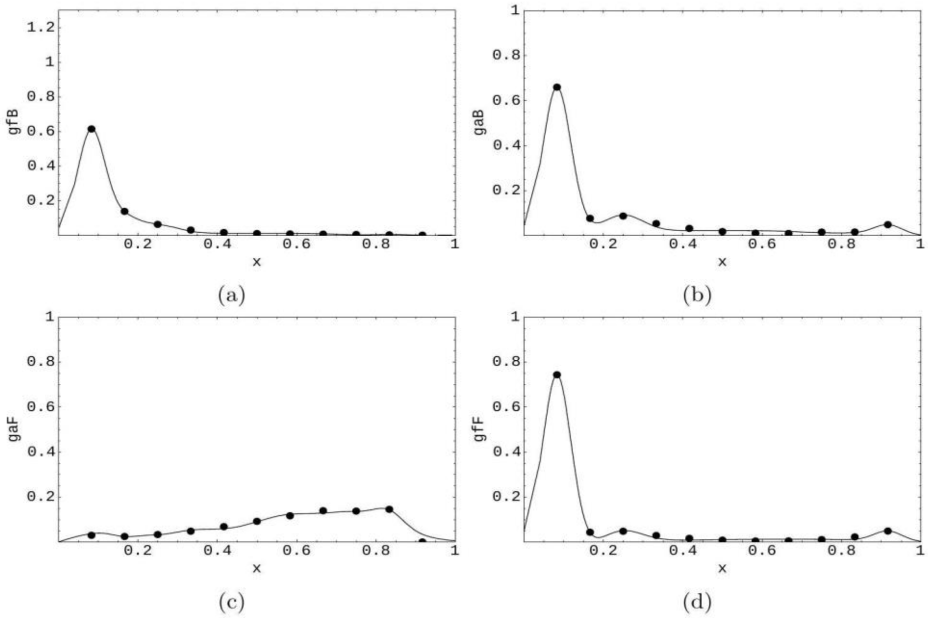

M^2
10
8
6
4
2
0.05 0.1 0.15 0.2 0.25 0.3
1/K

(a) (b)


gaB
gfB
gaF
gfF
x
(a)
(b)
(c)
(d)


M^2
10
8
6
4
2
0.05 0.1 0.15 0.2 0.25 0.3
1/K

(a) (b)



$$S = S_{SYM} + S_{\text{Gravity}} + \cdots$$

$$S_{SYM} = K \int d^{d+2}X\sqrt{-G}\,\delta(W(X))\left\{-\frac{1}{4g_{YM}^2}\Omega^{2\frac{d-4}{d-2}}F_{MN}^aF_a^{MN} + \frac{i}{2}\left[\bar{\lambda}^a V\bar{D}\lambda^a + \bar{\lambda}^a\overleftarrow{\bar{D}}\bar{V}\lambda^a\right]\right\}$$

$$S_{\text{Gravity}} = K \int d^{d+2}X\sqrt{G}\begin{bmatrix} \delta(W)\left\{a_d\Omega^2R(G) + \frac{1}{2}\partial\Omega\cdot\partial\Omega - V(\Omega)\right\} \\ +\delta'(W)\{a_d\Omega^2(4-\nabla^2W) + a_d\partial W\cdot\partial\Omega^2\}\end{bmatrix}$$

$$V \equiv \Gamma^M V_M = \Gamma^i V_i$$

$$\bar{V} \equiv \bar{\Gamma}^M V_M = \bar{\Gamma}^i V_i$$

$$\Gamma^M = \Gamma^i E_i^M$$

$$V_M(X) \equiv \frac{1}{2}\partial_M W$$

$$F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + f^{abc}A_M^bA_N^c - \frac{1}{4g^2}\Omega^{2\frac{d-4}{d-2}}F_{MN}^aF_{PQ}^aG^{MP}G^{NQ}$$

$$\bar{D}\lambda^a \equiv \bar{\Gamma}^M D_M\lambda^a = \bar{\Gamma}^k E_k^M\left(\partial_M\lambda^a + \frac{1}{4}\omega_M^{ij}\Gamma_{ij}\lambda^a + f^{abc}A_M^b\lambda^c\right)$$

$$V_M = \frac{1}{2}\partial_M W, V^M = G^{MN}V_N, V^i = V^M E_M^i$$

$$W = V^iV_i = G^{MN}V_MV_N = \frac{1}{2}V^M\partial_M W$$

$$G_{MN} = \nabla_M V_N = \frac{1}{2}(\partial_M\partial_N W - \Gamma_{MN}^P\partial_P W)$$

$$E_M^i = D_M V^i = \partial_M V^i + \omega_M^{ij}V_j$$

$$\left(V^M\partial_M + \frac{d-2}{2}\right)\Omega = 0$$

$$\mathrm{SO}(d,2), \Gamma^{ij} = \frac{1}{2}(\Gamma^i\Gamma^j - \Gamma^j\Gamma^i)$$

$$(\Gamma^{ik})_{(AB}(\Gamma_k^j)_{C)D} + (\Gamma^{jk})_{(AB}(\Gamma_k^i)_{C)D} = \frac{2\eta^{ij}}{d+2}(\Gamma^{kl})_{(AB}(\Gamma_{lk})_{C)D}$$

$$\left\{-\frac{d-4}{d-2}(\bar{\Gamma}^{PQN}\Gamma^M\varepsilon)_A V_N\partial_M \ln\,\Omega + (\bar{\Gamma}^M\Gamma^{PQN}D_M\varepsilon)_A V_N = V^PU_A^Q - V^QU_A^Q\right\}_{W=0}$$

$$\delta_\varepsilon \lambda_A^a = \frac{i}{g_{YM}} \Omega^{\frac{d-4}{d-2}} F_{MN}^a (\Gamma^{MN}\varepsilon)_A, \delta_\varepsilon A_M^a = \Omega^{-\frac{d-4}{d-2}} [-2\bar{\varepsilon}\Gamma_M \bar{V}\lambda^a + W\bar{\varepsilon}\Gamma_{MN} D^N \lambda^a] + h.c..$$

$$\bar{\varepsilon} J^M = \delta(W)\sqrt{G}\Omega^{\frac{d-4}{d-2}} F_{PQ}^a V_N \bar{\varepsilon}(\Gamma^{PQN}\bar{\Gamma}^M)\lambda^a$$

$$V^M F_{MN}^a = 0, \left(V \cdot D + \frac{d}{2}\right)\lambda_A^a = 0$$

$$(V\bar{D}\lambda^a)_A = 0, \hat{D}_N \left(\Omega^{\frac{2(d-4)}{d-2}} F_a^{NM}\right) = f_{abc}\left(\overline{\lambda^b}\Gamma^{MN}\lambda^c\right) V_N.$$

$$V \cdot D\varepsilon_A \equiv V^M \left(\partial_M \varepsilon_A + \frac{1}{4}\omega_M^{ij} (\Gamma_{ij}\varepsilon)_A\right) = 0$$

$$\delta S \sim \int \delta\Phi[\alpha(X)\delta(W) + \beta(X)\delta'(W)] = 0$$

$$\Delta J_A^M = \delta(W)\sqrt{G} V^M \xi_A$$

$$\Delta J_A^M = \delta'(W)\sqrt{G} V^M \tilde{\xi}_A$$

$$\Delta J_A^M = \delta(W)\sqrt{G} V^M \Omega^{\frac{d-4}{d-2}} F_{PQ}^a (\Gamma^{PQ}\lambda^a)_A$$

$$\partial_M\left(\sqrt{G} V^M\right) = \sqrt{G}\nabla_M V^M = \sqrt{G}\delta_M^M = (d+2)\sqrt{G}$$

$$V \cdot \partial\delta(W) = \delta'(W) V \cdot \partial W = 2W\delta'(W) = -2\delta(W)$$

$$V \cdot \partial\delta'(W) = -4\delta'(W)$$

$$\partial_M(\bar{\varepsilon} J^M) = \sqrt{G}\delta(W) \left\{ \begin{matrix} 2\Omega^{-\frac{d-4}{d-2}} f_{abc} V_N V^P (\bar{\varepsilon}\Gamma^{QN}\lambda^a)\left(\bar{\lambda}^b \Gamma_{QP}\lambda^c\right) \\ +F_{PQ} V_N \left[\begin{matrix} -\partial_M \Omega^{\frac{d-4}{d-2}} (\bar{\varepsilon}\Gamma^M \bar{\Gamma}^{PQN}\lambda) \\ +(D_M\bar{\varepsilon})\Gamma^{PQN}\bar{\Gamma}^M \lambda \end{matrix} \right] \end{matrix} \right\}$$

$$f_{abc} V_N V^P (\bar{\varepsilon}\Gamma^{QN}\lambda^a)\left(\bar{\lambda}^b \Gamma_{QP}\lambda^c\right)\delta(W) = \frac{2}{d+2} f_{abc}\left(\bar{\lambda}^b \Gamma_{kl}\lambda^a\right)(\bar{\varepsilon}\Gamma^{kl}\lambda^c) W\delta(W) = 0$$

$$[D_M\varepsilon]_{W=0} \equiv \left[\partial_M \varepsilon + \frac{1}{4}\omega_M^{ij}\Gamma_{ij}\varepsilon\right]_{W=0} \boxtimes \left[(\Gamma^M\varepsilon)_A \left(\partial_M \ln\, \Omega^{\frac{d-4}{d-2}}\right)\right]_{W=0}$$

$$A_M^a(X) = \begin{cases} A_\mu^a = A_\mu^a(x), \quad F_{MN}^a(X) = \begin{cases} F_{\mu\nu}^a = F_{\mu\nu}^a(x), \\ A_w = A_u = 0, \end{cases} \\ \qquad F_{w\mu}^a = F_{u\mu}^a = F_{wu}^a = 0, \end{cases}$$

$$\lambda_A^a(X) = \begin{pmatrix} \lambda_\alpha(x) \\ 0 \end{pmatrix} e^{(d-1)u}, G_{MN}(X) = e^{(d-2)u}\phi(x),$$
$$G_{\mu\nu} = e^{-4u} g_{\mu\nu}(x), G_{wu} = -1,$$
$$G_{ww} = G_{w\mu} = G_{u\mu} = 0.$$

$$S = S_{SYM} + \int d^d x \sqrt{-g}\left(\frac{d-2}{8(d-1)}\phi^2 R(g) + \frac{1}{2}\partial\phi \cdot \partial\phi\right)$$
$$S_{SYM} = \int d^d x \sqrt{-g}\left(-\frac{1}{4g_{YM}^2}\phi^{2\frac{d-4}{d-2}} F_{\mu\nu}^a F_a^{\mu\nu} + i\bar{\lambda}^a \gamma^\mu D_\mu \lambda_a\right)$$

$$\hat{g}_{YM} = g_{YM}\phi_0^{-\frac{d-4}{d-2}}$$

$$\delta_\varepsilon \lambda^a = \frac{i}{g_{YM}} \phi^{\frac{d-4}{d-2}} F_{\mu\nu}^a \gamma^{\mu\nu}\varepsilon, \delta_\varepsilon A_\mu^a = -2\phi^{-\frac{d-4}{d-2}} \bar{\varepsilon}\gamma_\mu \lambda^a + h.c.$$

$$D_\mu \varepsilon = \frac{1}{d}\gamma_\mu (\bar{\gamma} \cdot D\varepsilon) \text{ and } (d-4)\bar{\gamma}^\mu D_\mu \left(\phi^{\frac{d}{d-2}}\varepsilon\right) = 0$$

$$D_\mu \varepsilon_\alpha(x) = \partial_\mu \varepsilon_\alpha(x) + \frac{1}{4}\omega_\mu^{ab}(x)(\gamma_{ab}\varepsilon(x))_\alpha$$

$$W = V^M V_M = w, V_M = \left(\frac{1}{2},0,0\right)_M, V^M = \left(2w,-\frac{1}{2},0\right)^M$$

$$V_i = E_i^M V_M = \left(\frac{1}{2},-w,0\right)_i, V = V_i\Gamma^i = \left(\frac{1}{2}\Gamma^{-'} - w\Gamma^{+'}\right) = \begin{pmatrix} 0 & -i\sqrt{2}w \\ \frac{i}{\sqrt{2}} & 0 \end{pmatrix},$$

$$d^{d+2}X\sqrt{G}\delta(W) = (d^d x du dw)e^{-2du}\sqrt{-g}\delta(w)$$

$$G_{MN} = \begin{array}{c} M\backslash N \\ w \\ u \\ \mu \end{array} \begin{array}{c} \begin{array}{ccc} w & u & \nu \end{array} \\ \begin{pmatrix} 0 & -1 & 0 \\ -1 & -4w & 0 \\ 0 & 0 & e^{-4u}g_{\mu\nu} \end{pmatrix} \end{array}, \quad G^{MN} = \begin{array}{c} M\backslash N \\ w \\ u \\ \mu \end{array} \begin{array}{c} \begin{array}{ccc} w & u & \nu \end{array} \\ \begin{pmatrix} 4w & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & e^{4u}g^{\mu\nu} \end{pmatrix} \end{array}$$

$$E_M{}^i = \begin{array}{c} M\backslash i \\ w \\ u \\ \mu \end{array} \begin{array}{c} \begin{array}{ccc} -' & +' & a \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 2w & 1 & 0 \\ 0 & 0 & e^{-2u}e_\mu{}^a \end{pmatrix} \end{array}, \quad E_i{}^M = \begin{array}{c} i\backslash M \\ -' \\ +' \\ a \end{array} \begin{array}{c} \begin{array}{ccc} w & u & \nu \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ -2w & 1 & 0 \\ 0 & 0 & e^{2u}e_a{}^\mu \end{pmatrix} \end{array}$$

$$\Gamma_{wN}^P = \begin{array}{c} N\backslash P \\ w \\ u \\ \nu \end{array} \begin{array}{c} \begin{array}{ccc} w & u & \lambda \end{array} \\ \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & \frac{1}{2}g^{\lambda\sigma}\partial_w g_{\sigma\nu} \end{pmatrix} \end{array}, \quad \Gamma_{uN}^P = \begin{array}{c} N\backslash P \\ w \\ u \\ \nu \end{array} \begin{array}{c} \begin{array}{ccc} w & u & \lambda \end{array} \\ \begin{pmatrix} 2 & 0 & 0 \\ 8w & -2 & 0 \\ 0 & 0 & -2\delta_\nu^\lambda + 2wg^{\lambda\sigma}\partial_w g_{\sigma\nu} \end{pmatrix} \end{array}$$

$$\omega_w^{ij} = \begin{array}{c} i\backslash j \\ -' \\ +' \\ a \end{array} \begin{array}{c} \begin{array}{ccc} -' & +' & b \end{array} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}e^{\mu[a}\partial_w e_\mu^{b]} \end{pmatrix} \end{array}, \quad \omega_u^{ab} = \begin{array}{c} i\backslash j \\ -' \\ +' \\ a \end{array} \begin{array}{c} \begin{array}{ccc} -' & +' & b \end{array} \\ \begin{pmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 2we^{\mu[a}\partial_w e_\mu^{b]} \end{pmatrix} \end{array}$$

$$\omega_\mu^{ij} = \begin{array}{c} i\backslash j \\ -' \\ +' \\ a \end{array} \begin{array}{c} \begin{array}{ccc} -' & +' & b \end{array} \\ \begin{pmatrix} 0 & 0 & e^{-2u}\left(-2e_\mu^b + we^{\ b\sigma}\partial_w g_{\mu\sigma}\right) \\ 0 & 0 & \frac{e^{-2u}}{2}e^{\ b\nu}\partial_w g_{\mu\nu} \\ e^{-2u}\left(2e_\mu{}^a - we^{\ a\sigma}\partial_w g_{\lambda\sigma}\right) & -\frac{e^{-2u}}{2}e^{\ a\sigma}\partial_w g_{\mu\sigma} & \omega_\mu^{ab}(e) \end{pmatrix} \end{array}$$

$$V^M\Gamma_{MN}^P = 2w\Gamma_{wN}^P - \frac{1}{2}\Gamma_{uN}^P = \delta_M^P - 2\delta_w^P\delta_M^w, V^M\omega_M^{ij} = 2w\omega_w^{ij} - \frac{1}{2}\omega_u^{ij} = -\delta_{+'}^{[i}\delta_{-'}^{j]}$$

$$\left(2w\partial_w - \frac{1}{2}\partial_u + \frac{d-2}{2}\right)\Omega(X) = 0$$

$$\left(2wD_w - \frac{1}{2}D_u\right)\varepsilon(X) = \left(2w\partial_w - \frac{1}{2}\partial_u + \frac{1}{2}\Gamma^{+'-'}\right)\varepsilon(X) = 0$$

$$\Omega(X) = e^{(d-2)u}\hat{\Omega}(x, we^{4u}), \varepsilon(X) = \exp\left(u\Gamma^{+'-'}\right)\hat{\varepsilon}(x, we^{4u})$$

$$0 = V\cdot\delta_\Lambda A^a = V\cdot D\Lambda^a = V\cdot\partial\Lambda^a = \left(2w\partial_w - \frac{1}{2}\partial_u\right)\Lambda^a$$

$$\Lambda^a(X) = \hat{\Lambda}^a(x, we^{4u})$$

$$0 = V^M F^a_{MN} = (V \cdot \nabla + 1) A^a_N$$

$$\nabla_M V_N = G_{MN}$$

$$0 = (V \cdot \nabla + 1) A^a_N = \left(2w\partial_w - \frac{1}{2}\partial_u + 1\right) A^a_N - \left(2w\Gamma^P_{wN} - \frac{1}{2}\Gamma^P_{uN}\right) A^a_P.$$

$$A^a_w(X) = e^{4u} \hat{A}^a_w(x, we^{4u}), A^a_u(X) = \hat{A}^a_u(x, we^{4u}), A^a_\mu(X) = \hat{A}^a_\mu(x, we^{4u}).$$

$$0 \ = \left(V \cdot D + \frac{d}{2}\right) \lambda^a = \left(2wD_w - \frac{1}{2}D_u + \frac{d}{2}\right) \lambda^a$$
$$\Rightarrow \lambda^a = \exp\left(ud + u\Gamma^{+'-'}\right) \hat{\lambda}^a(x, we^{4u})$$

[8] $\delta_s A^a_M = W s^a_M(X)$ and $\delta_\kappa \lambda^a = V \kappa^a_1(X) + W \kappa^a_2(X)$

$$\delta_s \hat{A}^a_M = we^{4u} \hat{s}^a_M(x, we^{4u})$$
$$\delta_\kappa \hat{\lambda}^a = \left(\frac{1}{2}\Gamma^{-'} - we^{4u}\Gamma^{+'}\right) \hat{\kappa}^a_1(x, we^{4u}) + we^{4u} \hat{\kappa}^a_2(x, we^{4u})$$

$$\hat{A}^a_M(x, we^{4u}) \to A_M(x), \hat{\lambda}^a_A(x, we^{4u}) \to \begin{pmatrix} \lambda_\alpha(x) \\ 0 \end{pmatrix}, \overline{\hat{\lambda}^a}(x, we^{4u}) \to i\left(0 \overline{\lambda^a}(x)\right)$$

$$X^M = (x^m, y^I), \begin{cases} x^m \mathrm{SO}(4{,}2) \\ y^I \mathrm{SO}(6) \end{cases}$$

$$V^M(x,y) = \frac{1}{2} G^{MN} \partial_M W = \begin{pmatrix} m & I \\ x^m & , 0 \end{pmatrix} \text{ and } V^i = V^M E^i_M = \begin{pmatrix} \alpha \\ \delta^\alpha_m x^m, & 0 \end{pmatrix}.$$

$$\pounds_V G_{MN} = 2G_{MN}, \pounds_V E^i_M = E^i_M.$$

$$x \cdot \partial \ln\ a = 1, \text{or } a(tx, y) = ta(x, y).$$

$$\left(x \cdot \partial + \frac{d-2}{2}\right) \Omega = 0$$

$$\Omega(tx, y) = t^{-4} \Omega(x, y) \text{ for } d + 2 = 12.$$

$\omega^{ij}_M(x, y) =$

$M \setminus ij$	$\alpha\beta$	βa	ab
$\omega^{ij}_m =$	0	0	0
$\omega^{ij}_I =$	0	$\omega^{\beta a}_I = \delta^a_I \delta^{\beta m} \partial_m \ln a$	$\omega^{ab}_I = \delta^{[a}_I \delta^{b]J} (\partial_J \ln a)$

$$T^i_{MN} = D_{[M} E^i_{N]} = \partial_{[M} E^i_{N]} + \omega^{ij}_{[M} E_{N]j} = R^{ij}_{MN} V_j = 0$$

$$F_{mI} = D_m A_I = \partial_m A_I + A_m \times A_I;\ F_{IJ} = A_I \times A_J,$$

$$(A_m \times A_I)_a \equiv f_{abc} A^b_m A^c_I, \text{etc.}$$

$$-\frac{1}{4g^2_{YM}} \delta(W) \Omega^{3/2} \sqrt{G} G^{MP} G^{NQ} F_{MN} F_{NQ}$$
$$= -\frac{1}{4g^2_{YM}} \delta(x^2) \Omega^{3/2} a^6 \left((F_{mn})^2 + 2a^{-2}(D_m A_I)^2 + a^{-4}(A_I \times A_J)^2\right)$$
$$= -\frac{1}{4g^2_{YM}} \delta(x^2) \Omega^{3/2} a^6 \left((F_{mn})^2 + 2\left(D_m \frac{A_I}{a} + \frac{A_I}{a}\partial_m \ln\ a\right)^2 + \left(\frac{A_I}{a} \times \frac{A_J}{a}\right)^2\right)$$

$$\phi_I(x) = \frac{1}{g_{YM}} \frac{A_I(x)}{a(x, y)} \text{SO }(6).$$

$$D_m\left(\frac{A_I}{ag_{YM}}\right)+\left(\frac{A_I}{ag_{YM}}\right)\partial_m \ln\ a=(D_m\phi_I+\phi_I\partial_m \ln\ a)$$

$$-\frac{1}{2}\delta(x^2)(D_m\phi_I+\phi_I\partial_m \ln\ a)^2$$
$$=-\frac{1}{2}\delta(x^2)((D_m\phi_I)^2+\partial_m \ln\ a\partial_m\phi^2+\phi^2(\partial_m \ln\ a)^2)$$
$$=\left\{\begin{matrix}+\frac{1}{2}\delta(x^2)\phi_I D_m^2\phi_I+\frac{1}{2}\phi^2\left\{\begin{matrix}\delta'(x^2)(-2+2x\cdot\partial \ln\ a)\\+\delta(x^2)(\partial_m^2 \ln\ a-(\partial_m \ln\ a)^2)\end{matrix}\right\}\\+\frac{1}{2}\partial^m[-\delta(x^2)\phi_I D_m\phi_I+\delta'(x^2)x_m\phi^2-\delta(x^2)(\partial_m \ln\ a)\phi^2]\end{matrix}\right\}$$

$$\delta(x^2)\Omega^{3/2}a^6\left\{\frac{1}{2}\phi_I D^2\phi_I+\frac{1}{2}\phi_I^2(\partial_m^2 \ln\ a-(\partial_m \ln\ a)^2)\right\}.$$

$$\partial^2 \ln\ a-(\partial_m \ln\ a)^2=0\ \rightarrow\ a(x)=x\cdot b \text{ and } b^m b_m=0.$$

$$a(x)=\Omega^{-\frac{1}{4}}(x)=x\cdot b$$

$$\frac{1}{2}\delta(x^2)\phi_I D_m^2\phi_I$$

$$-\frac{1}{2}\delta(x^2)\eta^{mn}D_m\phi_I D_n\phi_I$$

$$-\frac{1}{2}\delta(x^2)(D_m\phi_I+\phi_I\partial_m \ln\ a)^2=\frac{1}{2}\delta(x^2)\phi_I D^2\phi_I+\text{ total derivative}$$

$$\lambda_A(x,y)=a^{-3}(x)\psi_A(x), A=1,2,\cdots,32.$$

$$\frac{i}{2}\delta(W)\sqrt{G}\bar{\lambda}VD\lambda+h.c$$
$$=\frac{i}{2}\delta(x^2)\sqrt{G}a^{-3}\bar{\psi}x\left(\Gamma^m D_m+\Gamma^a E_a^I(\partial_I+\omega_I+A_I\times)\right)(\psi a^{-3})+h.c$$
$$=\frac{i}{2}\delta(x^2)(a^6a^{-6})\bar{\psi}x\left(\begin{matrix}\Gamma^m D_m+\Gamma^m\partial_m \ln\ a^{-3}\\+\frac{1}{a}\Gamma^I\left(\begin{matrix}\partial_I \ln\ a^{-3}-\frac{1}{2}\Gamma_\beta\Gamma_a\omega_I^{\beta a}\\+\frac{1}{4}\Gamma_{cd}\omega_I^{cd}+A_I\times\end{matrix}\right)\end{matrix}\right)\psi(x)+h.c$$

$$\omega_I^{cd}=\delta_I^{[c}\delta^{d]J}(\partial_J \ln\ a)=0$$

$$\omega_I^{\beta a}=\delta_I^a\delta^{\beta m}\partial_m \ln\ a$$

$$-3\Gamma^m\partial_m \ln\ a+\frac{1}{2}\Gamma^I\Gamma^I\Gamma^m\partial_m \ln\ a=0.$$

$$\frac{i}{2}\delta(W)\sqrt{G}\bar{\lambda}VD\lambda=\frac{i}{2}\delta(x^2)\bar{\psi}[x(\Gamma^m D_m+g\Gamma^I\phi_I\times)]\psi(x)$$

$$L_{SYM}(x,y)=\delta(x^2)\left(\begin{matrix}-\frac{1}{4g_{YM}^2}(F_{mn})^2+\frac{1}{2}\phi_I D^2\phi_I-\frac{g_{YM}^2}{4}(\phi_I\times\phi_J)^2\\+\frac{i}{2}\bar{\psi}[x(\bar{\Gamma}^m D_m+g_{YM}\bar{\Gamma}^I\phi_I\times)]\psi(x)+\text{ h.c}\end{matrix}\right)$$

$$L^{\mathcal{N}=4}(x)=\delta(x^2)\left\{\begin{matrix}-\frac{1}{4g_{YM}^2}F_{mn}^aF_a^{mn}+\frac{1}{2}\phi_I^aD^mD_m\phi_I^a-\frac{g_{YM}^2}{4}\sum\left(f_{abc}\phi_I^b\phi_J^c\right)^2\\+\frac{i}{2}[\bar{\psi}^{ar}x\bar{D}\psi_r^a+g_{YM}f_{abc}(\psi_r^aC\bar{x}\psi_s^b)(\bar{\gamma}^I)^{rs}\phi_I^c]+h.c.\end{matrix}\right\}$$

$$\varphi_{rs}=(\gamma^I)_{rs}\phi_I^c=\frac{1}{2}\varepsilon_{rstu}\bar{\varphi}^{tu}$$

$$L_{YM} = -\frac{1}{4g_{YM}^2}\sqrt{G}\delta(W)\Omega^{\frac{3}{2}}\frac{1}{2}\mathrm{Tr}\big(F_{MN}F_{NQ}\big)G^{MP}G^{NQ}$$

$$X^M = (\sigma^m, x^\mu)$$

$$F_{mn} = \partial_m A_n - \partial_n A_m - i[A_m, A_n]$$
$$F_{m\mu} = D_m A_\mu = \partial_m A_\mu - i\big[A_m, A_\mu\big], F_{\mu\nu} = -i\big[A_\mu, A_\nu\big]$$

$$G_{MN} = \begin{pmatrix} g_{mn}(\sigma) & 0 \\ 0 & a^2(\sigma)\eta_{\mu\nu} \end{pmatrix}, \sqrt{G} = \sqrt{g}a^{10}.$$

$$V_M = (v_m(\sigma),0)_M, V^M = (v^m, 0)^M, v_m = \frac{1}{2}\partial_m W, v^m = g^{mn}v_n$$
$$0 = (v\cdot\partial - 1)a(\sigma), (v\cdot\partial + 4)\Omega(\sigma), \pounds_v g_{mn} = 2g_{mn}$$

$$a(\sigma) = \Omega^{-1/4}(\sigma)$$

$$\sqrt{G}\Omega^{3/2} = \sqrt{g}\Omega^{3/2}a^{10} = \sqrt{g}a^4$$

$$L_{YM} = -\frac{1}{4g_{YM}^2}\delta(W)\sqrt{G}\Omega^{\frac{3}{2}}\frac{1}{2}\mathrm{Tr}\big(F_{MN}F_{PQ}\big)G^{MP}G^{NQ}$$
$$= -\frac{1}{4g_{YM}^2}\delta(W)\sqrt{g}\frac{1}{2}\mathrm{Tr}\begin{Bmatrix} a^4 F_{mn}F^{mn} + 2a^2\big(D_m A_\mu\big)\big(D^m A^\mu\big) \\ -\big[A_\mu, A_\nu\big][A^\mu, A^\nu] \end{Bmatrix}$$

$$L_{YM}^{\text{shadow}} = \frac{1}{4g_{YM}^2}\frac{1}{2}\mathrm{Tr}\big(\big[A_\mu, A_\nu\big][A^\mu, A^\nu]\big)$$

$$\big[A_\mu, A_\nu\big]_i^j \leftrightarrow \{X_\mu, X_\nu\}(\xi^\alpha) = \frac{\partial X_\mu}{\partial\tau}\frac{\partial X_\nu}{\partial s} - \frac{\partial X_\mu}{\partial\sigma}\frac{\partial X_\nu}{\partial\tau}$$

$$\mathrm{Tr} \leftrightarrow \int d^2\xi$$

$$\int d^2\xi \det(g)$$

$$g_{\alpha\beta} = \frac{\partial X^\mu}{\partial\xi^\alpha}\frac{\partial X^\nu}{\partial\xi^\beta}\eta_{\mu\nu}, \det(g) = \{X_\mu, X_\nu\}\{X^\mu, X^\nu\}$$

$$\int d^2\xi \det(-g) \leftrightarrow \int d^2\xi \det\sqrt{-g}$$

$$L_{SYM}^{fermi} = \frac{i}{2}\sqrt{-G}\delta(W(X))\frac{1}{2}\mathrm{Tr}[\bar\lambda V\bar D\lambda + \bar\lambda \bar D V\lambda]$$

$$\sqrt{-G}\delta(W(X)) = \sqrt{-g}a^{10}\delta(W(\sigma))$$

$$\delta(W(X))\sqrt{-G}\bar\lambda V\bar D\lambda$$
$$= \delta(W(\sigma))\sqrt{-g}a^{10}\bar\lambda(v_n\Gamma^n)\left\{\bar\Gamma^m D_m\lambda + \bar\Gamma^\mu\left(\frac{1}{4}\omega_\mu^{ij}\Gamma_{ij}\lambda - i\big[A_\mu, \lambda\big]\right)\right\}$$

$\omega_M^{ij}(x,y) =$

$M \setminus ij$	$\hat m\hat n$	$\hat m\hat\mu$	$\hat\mu\hat\nu$
$\omega_m^{ij} =$	$\omega_m^{\hat m\hat n}$	0	0
$\omega_\mu^{ij} =$	0	$\omega_\mu^{\hat m\hat\mu} = \delta_\mu^{\hat\mu}\delta^{\hat m n}\partial_n \ln a$	0

$$\frac{2}{4}\omega_\mu^{\hat m\hat\mu}\bar\Gamma^\mu\Gamma_{\hat m}\bar\Gamma_{\hat\mu} = \frac{1}{2}\delta_\mu^{\hat\mu}\delta^{\hat m n}\partial_n \ln\, a\big(-\Gamma_{\hat m}\Gamma^\mu\bar\Gamma_{\hat\mu}\big) = -\frac{10}{2}\bar\Gamma^n\partial_n \ln\, a$$

$$(a^5\bar{\lambda})(v_n\Gamma^n)\{\bar{\Gamma}^m D_m(\lambda a^5) + \bar{\Gamma}^\mu(-i[A_\mu,(\lambda a^5)])\}$$

$$\psi \equiv (\lambda a^5)\sqrt{4g_{YM}}$$

$$L_{SYM}^{fermi} = \frac{1}{4g_{YM}^2}\delta(W(\sigma))\sqrt{-g}\frac{1}{2}\mathrm{Tr}\left\{\frac{1}{2}(i\bar{\psi}v\bar{\Gamma}^m D_m\psi + \text{h.c.}) + \bar{\psi}v\bar{\Gamma}^\mu[A_\mu,\psi]\right\}$$

$$S_{SYM}^{\text{reduced}} = \frac{1}{8g_{YM}^2}\int d^2\sigma\delta(W(\sigma))\sqrt{-g}Tr\begin{Bmatrix} -a^4F_{mn}F^{mn} - 2a^2(D_mA_\mu)(D^mA^\mu) \\ +\frac{1}{2}(i\bar{\psi}v\bar{\Gamma}^m D_m\psi + h.c.) \\ +[A_\mu,A_\nu][A^\mu,A^\nu] + \bar{\psi}v\bar{\Gamma}^\mu[A_\mu,\psi] \end{Bmatrix}$$

$$L_{SYM}^{\text{shadow}} = \frac{1}{8g_{YM}^2}\mathrm{Tr}\{[A_\mu,A_\nu][A^\mu,A^\nu] + \bar{\psi}_+\bar{\gamma}^\mu[A_\mu,\psi_+]\}$$

$$\begin{aligned} ds_{d+2}^2 &= dX^i dX^j\eta_{ij} = -\left(dX^{0'}\right)^2 + \left(dX^{1'}\right)^2 + dX^\alpha dX^a\eta_{ab} \\ &= -2dX^{+'}dX^{-'} + dX^\alpha dX^a\eta_{ab}, \end{aligned}$$

$$\begin{aligned} X^{+'} &= \frac{X^{0'} + X^{1'}}{\sqrt{2}} = \pm e^{-2\Sigma}, X^a = e^{-2\Sigma}q^a, \\ X^{-'} &= \frac{X^{0'} - X^{1'}}{\sqrt{2}} = \pm e^{-2\Sigma}\frac{q^2}{2} \mp e^{2\Sigma}\frac{w}{2}, \end{aligned}$$

$$ds_{d+2}^2 = -2dw(d\Sigma) - 4w(d\Sigma)^2 + e^{-4\Sigma}(dq)^2.$$

$$\Sigma(w,u,x) = u + \frac{1}{2}\sigma(x,we^{4u}), q^a(w,u,x) = q^a(x,we^{4u}).$$

$$ds_{d+2}^2 = \begin{Bmatrix} -(dw)^2[\sigma'(1+we^{4u}\sigma') - e^{-2\sigma}(q')^2]e^{4u} \\ +(du)^2[-4w + 16(we^{4u})^2[\sigma'(1+we^{4u}\sigma') - e^{-2\sigma}(q')^2]] \\ +dx^\mu dx^\nu[-(we^{4u})\partial_\mu\sigma\partial_\nu\sigma + e^{-2\sigma}\partial_\mu q\cdot\partial_\nu q]e^{-4u} \\ +2dwdu[-1 - 4z(\sigma'(1+we^{4u}\sigma') - e^{-2\sigma}(q')^2)] \\ +2dwdx^\mu\left[-\left(\frac{1}{2} + we^{4u}\sigma'\right)\partial_\mu\sigma + e^{-2\sigma}\partial_\mu q\cdot q'\right] \\ +2dudx^\mu\left[-\left(\frac{1}{2} + we^{4u}\sigma'\right)\partial_\mu\sigma + e^{-2\sigma}\partial_\mu q\cdot q'\right]4w \end{Bmatrix}$$

$$q_a' \equiv \frac{dq^a(x,z)}{dz}, \sigma' \equiv \frac{d\sigma(q^a(x,z),z)}{dz} = \frac{\partial\sigma}{\partial z} + \frac{\partial\sigma}{\partial q^a}q_a'$$

$$ds^2 = -2dwdu - 4w(du)^2 + e^{-4u}g_{\mu\nu}(x,we^{4u})dx^\mu dx^\nu$$

$$\begin{aligned} e^{-2\sigma}(q')^2 &= \sigma'(1+we^{4u}\sigma') \\ 2\partial_\mu q\cdot q' &= (1+2we^{4u}\sigma')e^{2\sigma}\partial_\mu\sigma \end{aligned}$$

$$\partial_\mu\sigma = \frac{\partial q^a}{\partial x^\mu}\frac{\partial\sigma}{\partial q^a}$$

$$\begin{aligned} q_a' &= \frac{e^{2\sigma}\partial_a\sigma}{2\sqrt{1 - ze^{2\sigma}(\partial_a\sigma)^2}} \\ \sigma' &= \frac{e^{2\sigma}(\partial_a\sigma)^2}{2\sqrt{1 - ze^{2\sigma}(\partial_a\sigma)^2}\left(1 + \sqrt{1 - ze^{2\sigma}(\partial_a\sigma)^2}\right)} \end{aligned}$$

$$\partial_a\sigma \equiv \frac{\partial\sigma}{\partial q^a}$$

$$(\partial_a\sigma)^2 \equiv \eta^{ab}\partial_a\sigma\partial_b\sigma$$

$$q^a(x,z) = q_0^a(x) + \int_0^z dz' \frac{e^{2\sigma}\partial_b\sigma\eta^{ab}}{2\sqrt{1 - z'e^{2\sigma}(\partial_a\sigma)^2}}$$

$$q^a(x,z) \ = q_0^a(x) + zq_1^a(x) + \frac{z^2}{2}q_2^a(x) + \cdots$$

$$\sigma(q^a(x,z),z) \ = \sigma_0(q_0(x)) + z\sigma_1(q_0(x)) + \frac{z^2}{2}\sigma_2(q_0(x)) + \cdots$$

$$\partial_a\sigma = \frac{\partial\sigma}{\partial q^a} = \frac{\partial\sigma}{\partial q_0^a}$$

$$q^a(x,z) = q_0^a(x) + ze^{2\sigma_0}\frac{\partial\sigma_0(q_0)}{2\partial q_0^a} + \cdots$$

$$\sigma(x,z) = \sigma_0(q_0) + ze^{2\sigma_0}\left(\frac{\partial\sigma_0(q_0)}{2\partial q_0^a}\right)^2 + \cdots$$

$$q_0^a(x), \sigma_0(q_0(x))$$

$$ds^2 = -2dwdu - 4w(du)^2 + e^{-4u}g_{\mu\nu}(x, we^{4u})dx^\mu dx^\nu$$

$$g_{\mu\nu}(x,z) \ = e^{-2\sigma}\partial_\mu q^a\partial_\nu q^b\eta_{ab} - z\partial_\mu\sigma\partial_\nu\sigma$$

$$= g_{\mu\nu}^{(0)}(x) + zg_{\mu\nu}^{(1)}(x) + \frac{z^2}{2}g_{\mu\nu}^{(2)}(x) + \cdots$$

$$g_{\mu\nu}^{(0)}(x) \ = e_\mu^a(x)e_\nu^b(x)\eta_{ab}, \text{where } e_\mu^a(x) = e^{-\sigma_0(x)}\frac{\partial q_0^a(x)}{\partial x^\mu}$$

$$g_{\mu\nu}^{(1)}(x) \ = -\frac{1}{2}(\partial_a\sigma_0)^2\partial_\mu q_0 \cdot \partial_\nu q_0 + \partial_\mu\sigma_0\partial_\nu\sigma_0 + \left(\partial_\mu q_0^b\partial_\nu q_0^a\right)\partial_b\partial_a\sigma_0$$

$$g_{\mu\nu}^{(2)}(x) \ = \cdots$$

$$\partial_a\sigma_0\big(x(q_0)\big) = \frac{\partial x^\mu}{\partial q_0^a}\partial_\mu\sigma_0(x) = e^{-\sigma_0(x)}e_a^\mu(x)\partial_\mu\sigma_0(x),$$

$$\partial_a(e^{\sigma_0}) = e_a^\mu(x)\partial_\mu\sigma_0(x)$$

$$g_{\mu\nu}^{(0)}(x) = e_\mu^a(x)e_\nu^b(x)\eta_{ab}$$

$$e_\mu^a(x) = e^{-\sigma_0(x)}\frac{\partial q_0^a(x)}{\partial x^\mu}$$

$$\left[-\frac{d-4}{d-2}(\bar{\Gamma}^{PQN}\Gamma^M\varepsilon)_A V_N\partial_M \ln\,\Omega + (\bar{\Gamma}^M\Gamma^{PQN}D_M\varepsilon)_A V_N\right]_{W=0} = \left[V^P U_A^Q - V^Q U_A^P\right]_{W=0}$$

$$V^N = \left(2w, -\frac{1}{2}, 0\right)^N$$

$$\left[-\frac{d-4}{d-2}(\bar{\Gamma}^{PQw}\Gamma^M\varepsilon)_A\partial_M \ln\,\Omega + (\bar{\Gamma}^M\Gamma^{PQw}D_M\varepsilon)_A = -(\delta_u^P U_A^Q - \delta_u^Q U_A^P)\right]_{w=0}$$

$$\left[-\frac{d-4}{d-2}(\bar{\Gamma}^{wuw}\Gamma^M\varepsilon)_A\partial_M \ln\,\Omega + (\bar{\Gamma}^M\Gamma^{wuw}D_M\varepsilon)_A = -(\delta_u^w V^w U_A^u - \delta_u^u U_A^w)\right]_{w=0}$$

$$U_A^w = 0.$$

$$\left[-\frac{d-4}{d-2}\left(\bar{\Gamma}^{w\lambda w}\Gamma^M\varepsilon\right)_A\partial_M \ln\,\Omega + \left(\bar{\Gamma}^M\Gamma^{w\lambda w}D_M\varepsilon\right)_A = -\left(\delta_u^w U_A^\lambda - \delta_u^\lambda U_A^w\right)\right]_{w=0}$$

$$\left[-\frac{d-4}{d-2}\left(\bar{\Gamma}^{u\lambda w}\Gamma^M\varepsilon\right)_A\partial_M \ln\,\Omega + \left(\bar{\Gamma}^M\Gamma^{u\lambda w}D_M\varepsilon\right)_A = -\left(\delta_u^u U_A^\lambda - \delta_u^\lambda U_A^u\right)\right]_{w=0}$$

$$U^\lambda = -\bar{\Gamma}^{wu}\left[-\frac{d-4}{d-2}\bar{\Gamma}^\lambda\Gamma^M\varepsilon\partial_M \ln\,\Omega + \left(\tilde{\Gamma}^M\Gamma^\lambda D_M\varepsilon\right)\right]_{w=0}$$

$$\tilde{\Gamma}^M = \bar{\Gamma}^{wu}\bar{\Gamma}^M\bar{\Gamma}^{wu} = (-\Gamma^w, -\Gamma^u, \Gamma^\mu)^M$$

$$\Omega(X) = e^{(d-2)u}\hat{\Omega}(x, we^{4u})$$

$$\varepsilon(X) = \exp\left(u\Gamma^{+'-'}\right)\hat{\varepsilon}(x, we^{4u})$$

$$\left[-\frac{d-4}{d-2}\left(\bar{\Gamma}^{\nu\lambda w}\Gamma^M\varepsilon\right)\partial_M \ln\,\Omega + \left(\bar{\Gamma}^M\Gamma^{\nu\lambda w}D_M\varepsilon\right) = -\left(\delta_u^\nu U_A^\lambda - \delta_u^\lambda U_A^\nu\right)\right]_{w=0}$$

$$\left[-\frac{d-4}{d-2}\left(\bar{\Gamma}^{\nu\lambda}\bar{\Gamma}^w\Gamma^M\varepsilon\right)\partial_M \ln\,\Omega + \left(\bar{\Gamma}^M\Gamma^{\nu\lambda}\Gamma^w D_M\varepsilon\right)\right]_{w=0} = 0$$

$$0 = \left\{\bar{\Gamma}^w\Gamma^{\nu\lambda}\left[-(d-4)\Gamma^u\varepsilon - \frac{d-4}{d-2}\left(\partial_\mu \ln\,\Omega\right)\Gamma^\mu\varepsilon\right] - \bar{\Gamma}^w\Gamma^\mu\bar{\Gamma}^{\nu\lambda}D_\mu\varepsilon\right\}_{w=0}$$

$$[\varepsilon_A(X)]_{w=0} = \begin{pmatrix} e^{-u}\varepsilon_1(x) \\ e^{+u}\varepsilon_2(x) \end{pmatrix}$$

$$0 = -(d-4)(-i\sqrt{2})\bar{\gamma}^{\nu\lambda}\varepsilon_2 - \partial_\mu \ln\,\phi^{\frac{d-4}{d-2}}\bar{\gamma}^{\nu\lambda}\bar{\gamma}^\mu\varepsilon_1 - \bar{\gamma}^\mu\gamma^{\nu\lambda}\left(D_\mu\varepsilon_1 + i\sqrt{2}\gamma_\mu\varepsilon_2\right)$$

$$\bar{\gamma}^\mu\gamma^{\nu\lambda}\gamma_\mu = (d-4)\gamma^{\nu\lambda}$$

$$-\bar{\gamma}^{\nu\lambda}\bar{\gamma}^\mu\varepsilon_1\left(\partial_\mu \ln\,\phi^{\frac{d-4}{d-2}}\right) - \bar{\gamma}^\mu\gamma^{\nu\lambda}D_\mu\varepsilon_1 = 0, \varepsilon_2(x) = \mathfrak{A}_{arbitrary}$$

$$\bar{\gamma}_{\nu\lambda}\bar{\gamma}^{\nu\lambda} = -d(d-1), \bar{\gamma}_{\nu\lambda}\bar{\gamma}^\mu\gamma^{\nu\lambda} = -(d-1)(d-4)\bar{\gamma}^\mu$$

$$\left(\partial_\mu \ln\,\phi^{\frac{d}{d-2}}\bar{\gamma}^\mu\varepsilon_1 + \bar{\gamma}^\mu D_\mu\varepsilon_1\right)(d-1)(d-4) = 0$$

$$D_\mu\varepsilon_1 = \frac{1}{d}\gamma_\mu(\bar{\gamma}\cdot D\varepsilon_1)\text{ and }(d-4)\bar{\gamma}^\mu D_\mu\left(\phi^{\frac{d}{d-2}}\varepsilon_1\right) = 0, \varepsilon_2(x) = \mathfrak{A}_{arbitrary}$$

$$\bar{\varepsilon}J^M = \delta(W)\sqrt{G}\,\Omega^{\frac{d-4}{d-2}}F_{PQ}^a V_N\bar{\varepsilon}(\Gamma^{PQN}\bar{\Gamma}^M)\lambda^a$$

$$\lambda = \begin{pmatrix}\lambda_1 \\ 0\end{pmatrix}e^{(d-1)u}, F_{wu} = F_{w\mu} = F_{u\mu} = 0, \Omega = \phi e^{(d-2)u}$$
$$\sqrt{G} = e^{-2du}\sqrt{-g}, \Gamma^w \sim \begin{pmatrix}0 & 0 \\ 1 & 0\end{pmatrix}, \Gamma^u \sim \begin{pmatrix}0 & 1 \\ 0 & 0\end{pmatrix}$$

$$\begin{aligned}
\bar{\varepsilon}J^w &= \frac{1}{2}\delta(w)e^{-5u}\sqrt{-g}\phi^{\frac{d-4}{d-2}}F_{Pq}^a\bar{\varepsilon}(\Gamma^{Pq}\Gamma^w\bar{\Gamma}^w)\begin{pmatrix}\lambda_1 \\ 0\end{pmatrix} = 0 \\
\bar{\varepsilon}J^u &= \frac{1}{2}\delta(w)e^{-5u}\sqrt{-g}\phi^{\frac{d-4}{d-2}}F_{Pq}^a\bar{\varepsilon}(\Gamma^{Pq}\Gamma^w\bar{\Gamma}^u)\begin{pmatrix}\lambda_1 \\ 0\end{pmatrix} = 0 \\
\bar{\varepsilon}J^\mu &= \frac{1}{2}\delta(w)e^{-5u}\sqrt{-g}\phi^{\frac{d-4}{d-2}}F_{Pq}^a\bar{\varepsilon}(\Gamma^{Pq}\Gamma^w\bar{\Gamma}^\mu)\begin{pmatrix}\lambda_1 \\ 0\end{pmatrix} \\
&\sim -\frac{1}{2}\delta(w)e^{-5u}\sqrt{-g}\phi^{\frac{d-4}{d-2}}F_{Pq}^a(\bar{\varepsilon}_2\bar{\varepsilon}_1)(\Gamma^{Pq}\bar{\Gamma}^\mu)\begin{pmatrix}0 \\ \lambda_1\end{pmatrix} \\
&= -\frac{1}{2}\delta(w)e^{-5u}\sqrt{-g}\phi^{\frac{d-4}{d-2}}F_{Pq}^a\bar{\varepsilon}_1(-\bar{\gamma}^{Pq}\gamma^\mu)\lambda_1
\end{aligned}$$

$$\bar{\psi} = \psi^\dagger\eta; \bar{\psi}_r^\rho = \left(\psi^\dagger\right)_{r\dot{\sigma}}\eta^{\dot{\sigma}\rho}$$

$$\psi^c = C\bar{\psi}^T;\ (\psi^c)_{\dot{\rho}r} = C_{\dot{\rho}\sigma}(\bar{\psi}^T)_r^\sigma = C_{\dot{\rho}\sigma}(\eta^T)^{\sigma\dot{\kappa}}(\psi^*)_{\dot{\kappa}r} = \tilde{C}_{\dot{\rho}\dot{\kappa}}(\psi^*)_{\dot{\kappa}r}$$

$$\psi_A = \frac{1}{\sqrt{2}}\begin{pmatrix}\psi_\rho^r \\ (\psi^c)_{\dot{\rho}r}\end{pmatrix} \sim 32 = \begin{pmatrix}(4, \bar{4}) \\ (\bar{4}, 4)\end{pmatrix}$$

$$\delta_\omega\psi_A = -\frac{1}{4}\omega^{MN}(\Gamma_{MN})_A^B\psi_B$$

$$(\delta_\omega\psi)^r_\rho = -\frac{1}{4}\omega^{mn}(\gamma_{mn}\psi)^r_\rho + \frac{1}{4}\omega^{IJ}\left(\psi\gamma_{IJ}\right)^r_\rho + \frac{1}{2}\omega^{mI}(\gamma_m\psi^c\gamma_I)^r_\rho$$
$$= -\frac{1}{4}\omega^{mn}(\gamma_{mn})^\sigma_\rho\psi^r_\sigma + \frac{1}{4}\omega^{IJ}\psi^s_\rho\left(\gamma_{IJ}\right)^r_s + \frac{1}{2}\omega^{mI}(\gamma_m)^{\dot\sigma}_\rho(\psi^c)_{\dot\sigma s}(\bar\gamma_I)^{sr}$$

$$(\delta_\omega\psi^c)_{\dot\rho r} = -\frac{1}{4}\omega^{mn}(\bar\gamma_{mn}\psi^c)_{\dot\rho r} + \frac{1}{4}\omega^{IJ}\left(\psi^c\bar\gamma_{IJ}\right)_{\dot\rho r} + \frac{1}{2}\omega^{mI}(\bar\gamma_m\psi\gamma_I)_{\dot\rho r}$$
$$= -\frac{1}{4}\omega^{mn}(\bar\gamma_{mn})^{\dot\sigma}_{\dot\rho}(\psi^c)_{\dot\sigma r} + \frac{1}{4}\omega^{IJ}(\psi^c)_{\dot\rho s}\left(\bar\gamma_{IJ}\right)^s_r + \frac{1}{2}\omega^{mI}(\bar\gamma_m)^\sigma_{\dot\rho}\psi^s_\sigma(\gamma_I)_{sr}$$

$$\delta_\omega\psi^c = C\overline{(\delta_\omega\psi)}^T = C\eta^T(\delta_\omega\psi)^*$$

$$C\eta^T(\gamma_{mn})^*(\eta^T)^{-1}C^{-1} = \bar\gamma_{mn}, C\eta^T(\gamma_m)^*(\eta^T)^{-1}C^{-1} = -\bar\gamma_m$$
$$\left(\gamma_{IJ}\right)^* = \bar\gamma_{IJ}, (\bar\gamma_I)^* = -\gamma_I;\text{ also } (\gamma_I)_{rs}, (\bar\gamma_I)^{rs}$$

$$(\bar\gamma_I)^\dagger = \gamma_I, \text{and } \left(\gamma_{IJ}\right)^\dagger = -\gamma_{IJ}\,, \left(\bar\gamma_{IJ}\right)^\dagger = -\bar\gamma_{IJ}$$

$$(\gamma_I)_{rs} = \left((\sigma_2\times i\sigma_2\vec\sigma), (\sigma_2\vec\sigma\times\sigma_2)\right), \left(\text{ note } i\sigma_2\vec\sigma = (\sigma_1, i, -\sigma_3)\right)$$
$$(\bar\gamma_I)^{rs} = \left((\sigma_2\times i\sigma_2\vec\sigma^*), (-\sigma_2\vec\sigma^*\times\sigma_2)\right), \left(\text{ note } (-\sigma_2\vec\sigma^*) = (i\sigma_3, 1, -i\sigma_1)\right)$$

$$\left(\gamma_I\bar\gamma_J + \gamma_J\bar\gamma_I\right)^s_r = 2\delta_{IJ}\delta^s_r$$

$$(\bar\gamma_I)^{rs} = \frac{1}{2}\varepsilon^{rsuv}(\gamma_I)_{uv}$$

$$\delta_\omega\psi_A = -\frac{1}{4}\omega^{mn}(\gamma_{mn})^B_A\psi_B$$

$$\delta_\omega\begin{pmatrix}\psi\\ \psi^c\end{pmatrix} = -\frac{1}{4}\omega^{MN}(\Gamma_{MN})\begin{pmatrix}\psi\\ \psi^c\end{pmatrix}$$
$$\omega^{MN}(\Gamma_{MN}) = \begin{pmatrix}\omega^{mn}(\gamma_{mn}\otimes 1_4) + \omega^{IJ}\left(1_4\otimes\gamma_{IJ}\right) & 2\omega^{mI}(\gamma_m\otimes\bar\gamma_I)\\ 2\omega^{mI}(\bar\gamma_m\otimes\gamma_I) & \omega^{mn}(\bar\gamma_{mn}\otimes 1_4) + \omega^{IJ}\left(1_4\otimes\bar\gamma_{IJ}\right)\end{pmatrix}$$

$$\overline{32} = \begin{pmatrix}(4,4)\\ (\bar 4,\bar 4)\end{pmatrix}, \text{versus } 32 = \begin{pmatrix}(4,\bar 4)\\ (\bar 4, 4)\end{pmatrix} = \begin{pmatrix}\psi\\ \psi^c\end{pmatrix}$$

$$\bar\Gamma_m\begin{pmatrix}(\psi)^r_\rho\\ (\psi^c)_{\dot\rho r}\end{pmatrix} = \begin{pmatrix}(\gamma_m\psi^c)_{\rho r}\\ (\bar\gamma_m\psi)^r_{\dot\rho}\end{pmatrix}\sim\overline{32}, \text{and } \bar\Gamma_I\begin{pmatrix}(\psi)^r_\rho\\ (\psi^c)_{\dot\rho r}\end{pmatrix} = \begin{pmatrix}(\psi\gamma_I)_{\rho r}\\ -(\psi^c\bar\gamma_I)^r_{\dot\rho}\end{pmatrix}\sim\overline{32}$$

$$\overline{32}\times 32, (\bar\Gamma_M)^{\dot A B}:\quad \bar\Gamma_m = \begin{pmatrix}0 & \gamma_m\otimes 1_4\\ \bar\gamma_m\otimes 1_4 & 0\end{pmatrix}, \bar\Gamma_I = \begin{pmatrix}1_4\otimes\gamma_I & 0\\ 0 & -1_4\otimes\bar\gamma_I\end{pmatrix}$$
$$32\times\overline{32}, (\Gamma_M)_{A\dot B}:\quad \Gamma_m = \begin{pmatrix}0 & \gamma_M\otimes 1_4\\ \bar\gamma_m\otimes 1_4 & 0\end{pmatrix}, \Gamma_I = \begin{pmatrix}1_4\otimes\bar\gamma_I & 0\\ 0 & -1_4\otimes\gamma_I\end{pmatrix}$$

$$\Gamma_M\bar\Gamma_N + \Gamma_N\bar\Gamma_M = 2\eta_{MN}, \text{and } \bar\Gamma_M\Gamma_N + \bar\Gamma_N\Gamma_N = 2\eta_{MN}$$

$$\Gamma_m\bar\Gamma_n + \Gamma_n\bar\Gamma_m = \begin{pmatrix}0 & \gamma_m\otimes 1_4\\ \bar\gamma_m\otimes 1_4 & 0\end{pmatrix}\begin{pmatrix}0 & \gamma_n\otimes 1_4\\ \bar\gamma_n\otimes 1_4 & 0\end{pmatrix} + (m\leftrightarrow n)$$
$$= \begin{pmatrix}(\gamma_m\bar\gamma_n + (m\leftrightarrow n))\otimes 1_4 & 0\\ 0 & (\bar\gamma_m\gamma_n + (m\leftrightarrow n))\otimes 1_4\end{pmatrix} = 2\eta_{mn}$$

$$\Gamma_I\bar\Gamma_J + \Gamma_J\bar\Gamma_I = \begin{pmatrix}1_4\otimes\bar\gamma_I & 0\\ 0 & -1_4\otimes\gamma_I\end{pmatrix}\begin{pmatrix}1_4\otimes\gamma_J & 0\\ 0 & -1_4\otimes\bar\gamma_J\end{pmatrix} + (I\leftrightarrow J)$$
$$= \begin{pmatrix}1_4\otimes(\gamma_J\bar\gamma_I + (I\leftrightarrow J)) & 0\\ 0 & 1_4\otimes(\bar\gamma_J\gamma_I + (I\leftrightarrow J))\end{pmatrix} = 2\delta_{IJ}$$

$$\Gamma_m\bar{\Gamma}_I+\Gamma_I\bar{\Gamma}_m=\begin{pmatrix}0 & \gamma_m\otimes 1_4\\ \bar{\gamma}_m\otimes 1_4 & 0\end{pmatrix}\begin{pmatrix}1_4\otimes\gamma_I & 0\\ 0 & -1_4\otimes\bar{\gamma}_I\end{pmatrix}+\begin{pmatrix}1_4\otimes\bar{\gamma}_I & 0\\ 0 & -1_4\otimes\gamma_I\end{pmatrix}\begin{pmatrix}0 & \gamma_m\otimes 1_4\\ \bar{\gamma}_m\otimes 1_4 & 0\end{pmatrix}$$

$$=\begin{pmatrix}0 & -(\gamma_m\otimes\bar{\gamma}_I)+(\gamma_m\otimes\bar{\gamma}_I)\\ (\bar{\gamma}_m\otimes\gamma_I)-(\bar{\gamma}_m\otimes\gamma_I) & 0\end{pmatrix}=0$$

$$(\Gamma_{MN})_A^{\ B}=\frac{1}{2}(\Gamma_M\bar{\Gamma}_N-\Gamma_N\bar{\Gamma}_M)_A^{\ B}$$

$$\Gamma_{mn}=\begin{pmatrix}\gamma_{mn}\otimes 1_4 & 0\\ 0 & \bar{\gamma}_{mn}\otimes 1_4\end{pmatrix},\Gamma_{IJ}=\begin{pmatrix}-1_4\otimes\gamma_{IJ} & 0\\ 0 & -1_4\otimes\bar{\gamma}_{IJ}\end{pmatrix}$$

$$\Gamma_{mI}=\begin{pmatrix}0 & -\gamma_m\otimes\bar{\gamma}_I\\ \bar{\gamma}_m\otimes\gamma_I & 0\end{pmatrix}$$

$$a_{AB}=\begin{pmatrix}0 & C\otimes 1_4\\ \bar{C}\otimes 1_4 & 0\end{pmatrix}.$$

$$(\Gamma^M\bar{C})^T=-(\Gamma^M\bar{C}),(\Gamma^{MN}C)^T=(\bar{\Gamma}^{MN}C)$$
$$(\Gamma^I)^T=-(\Gamma^I),(\Gamma^{IJ})^T=(\bar{\Gamma}^{IJ})$$

$W_{\text{curvature}}=X\cdot X=\eta_{MN}X^MX^N,V_M^{\text{curvature}}=X_M,G_{MN}^{\text{curvature}}=\eta_{MN},(\Gamma_{MN}^p)_{\text{curvature}}=0=(\omega_M^{ij})_{\text{curvature}}$ and $\Omega_{\text{curvature}}=(c\cdot X)^{1-d/2}$

$$G_{MN}=\eta_{MN}+h_{MN}(X)$$

$$W=\eta_{MN}X^MX^N,V^M=X^M,V_M=\eta_{MN}X^N$$

$$s_w^a(X)=\hat{s}_w^a(x,we^{4u}),s_u^a(X)=e^{4u}\hat{s}_u^a(x,we^{4u})$$

$$s_\mu^a(X)=e^{4u}\hat{s}_\mu^a(x,we^{4u})$$

$$\kappa_1^a(X)=\exp\left(u\Gamma^{+'-'}+u(d+2)\right)\hat{\kappa}_1^a(x,we^{4u})$$

$$\kappa_2^a(X)=\exp\left(u\Gamma^{+'-'}+u(d+4)\right)\hat{\kappa}_2^a(x,we^{4u})$$

$\Gamma^i\bar{\Gamma}^j+\Gamma^j\bar{\Gamma}^i=2\eta^{ij}:\Gamma^{+'}=\begin{pmatrix}0 & -i\sqrt{2}\\ 0 & 0\end{pmatrix},\Gamma^{-'}=\begin{pmatrix}0 & 0\\ -i\sqrt{2} & 0\end{pmatrix},\Gamma^\mu=\begin{pmatrix}\bar{\gamma}^\mu & 0\\ 0 & -\gamma^\mu\end{pmatrix}$ with $\gamma^\mu=(-1,\gamma^i),\bar{\gamma}^\mu=(1,\gamma^i)$ and $\bar{\Gamma}^{+'}=\begin{pmatrix}0 & -i\sqrt{2}\\ 0 & 0\end{pmatrix},\bar{\Gamma}^{-'}=\begin{pmatrix}0 & 0\\ -i\sqrt{2} & 0\end{pmatrix},\bar{\Gamma}^\mu=\begin{pmatrix}\gamma^\mu & 0\\ 0 & -\bar{\gamma}^\mu\end{pmatrix}$

$$\Psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\ \bar{\Psi}=i(\bar{\psi}_2,\bar{\psi}_1)$$

$$[A_M,A_N]=it^a\left(f_{abc}A_M^bA_N^c\right)$$

$$\text{Tr}(t_at_b)=2\delta_{ab}$$

$$\left(D_\mu\varepsilon\right)_A=\left(\partial_\mu+\frac{1}{4}\omega_\mu^{ab}\bar{\Gamma}_{ab}+\frac{1}{2}\omega_\mu^{+'b}\bar{\Gamma}_b\Gamma^{-'}+\frac{1}{2}\omega_\mu^{-'b}\bar{\Gamma}_b\Gamma^{+'}\right)\varepsilon_A$$

$$
\begin{aligned}
\{Q_{i\alpha}, Q_{j\beta}\} &= C_{\alpha\beta} Z_{ij}, & \{\bar{Q}^{i}{}_{\dot{\alpha}}, \bar{Q}^{j}{}_{\dot{\beta}}\} &= C_{\dot{\alpha}\dot{\beta}} \bar{Z}^{ij}, \\
\{Q_{i\alpha}, \bar{Q}^{j}{}_{\dot{\beta}}\} &= 2\delta_i{}^{j} (\sigma^{\mu})_{\alpha\dot{\beta}} P_{\mu}, & [P_{\mu}, P_{\nu}] &= 0, \\
[Q_{i\alpha}, P_{\mu}] &= 0, & [\bar{Q}^{i}{}_{\dot{\alpha}}, P_{\mu}] &= 0, \\
[J_{\mu\nu}, Q_{i\alpha}] &= \frac{i}{2} (\sigma_{\mu\nu})_{\alpha}{}^{\beta} Q_{i\beta}, & [J_{\mu\nu}, \bar{Q}^{i\dot{\alpha}}] &= \frac{i}{2} (\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} \bar{Q}^{i\dot{\beta}}, \\
[J_{\mu\nu}, P_{\rho}] &= i(\eta_{\mu\rho} P_{\nu} - \eta_{\nu\rho} P_{\mu}), & [J_{\mu\nu}, J_{\rho\sigma}] &= i(\eta_{\nu\rho} J_{\mu\sigma} - \eta_{\nu\sigma} J_{\mu\rho} \\
& & &\quad - \eta_{\mu\rho} J_{\nu\sigma} + \eta_{\mu\sigma} J_{\nu\rho}), \\
[R^{a}, Q_{i\alpha}] &= (X^{a})_{i}{}^{j} Q_{j\alpha}, & [R^{a}, \bar{Q}^{i}{}_{\dot{\alpha}}] &= -(X^{a})_{j}{}^{i} \bar{Q}^{j}{}_{\dot{\alpha}}, \\
[R^{a}, P_{\mu}] &= 0, & [R^{a}, J_{\mu\nu}] &= 0, \\
[Z_{ij}, \text{any}] &= 0, & [\bar{Z}^{ij}, \text{any}] &= 0,
\end{aligned}
$$

$$\bar{Q}^{i}{}_{\dot{\alpha}} = (Q_{i\alpha})^{\dagger},$$

$$Q_{i\alpha} \in (\mathbf{2}, \mathbf{0}, \boldsymbol{N}), \bar{Q}^{i}{}_{\dot{\alpha}} \in (\mathbf{0}, \mathbf{2}, \overline{\boldsymbol{N}}) \text{ under } SL(2, \mathbb{C}) \times SU(N);$$

$$Z_{ij} + Z_{ji} = 0, \bar{Z}^{ij} + \bar{Z}^{ji} = 0$$
$$Z_{ij} \in \frac{N(N-1)}{2}, \bar{Z}^{ij} \in \overline{\frac{N(N-1)}{2}} \text{ under } SU(N),$$

$$\bar{Z}^{ij} = (Z_{ij})^{*}, \bar{Z} + Z^{\dagger} = 0$$

$$\eta^{\mu\rho} \eta_{\rho\nu} = \eta^{\mu}_{\nu} := \delta^{\mu}_{\nu}$$
$$P_{\mu} \in (\mathbf{2}, \mathbf{2}), J_{\mu\nu} \in (\mathbf{3}, \mathbf{0}) \oplus (\mathbf{0}, \mathbf{3}) \text{ under } SL(2, \mathbb{C})$$
$$\exp\left(-\frac{i}{2} \omega^{\mu\nu} J_{\mu\nu}\right) \in SO(1,3)$$

$$R^{a} \in \mathfrak{su}(N)$$

$$\exp\left(i \sum_{a} t^{a} R^{a}\right) \in SU(N)$$
$$(X^{a})_{i}^{j} \in (\boldsymbol{N}^{2} - \mathbf{1}) \text{ under } SU(N)$$

$$(X^{a*})_{j}^{i} := ((X^{a})_{i}^{j})^{*} = (X^{a})_{j}^{i}$$

$$C_{\alpha\beta} + C_{\beta\alpha} = 0, C_{\dot{\alpha}\dot{\beta}} + C_{\dot{\beta}\dot{\alpha}} = 0$$

$$C_{\alpha\beta} \in (\mathbf{1}, \mathbf{0}), C_{\dot{\alpha}\dot{\beta}} \in (\mathbf{0}, \mathbf{1}) \text{ under } SL(2, \mathbb{C})$$

$$C_{\dot{\alpha}\dot{\beta}} = (C_{\alpha\beta})^{*}$$

$$C_{\alpha\gamma} C^{\beta\gamma} = \delta_{\alpha}{}^{\beta}, C_{\dot{\alpha}\dot{\gamma}} C^{\dot{\beta}\dot{\gamma}} = \delta_{\dot{\alpha}}^{\dot{\beta}};$$
$$\sigma^{\mu} = (\mathbf{1}, \tau^{i}), \bar{\sigma}^{\mu} = (\mathbf{1}, -\tau^{i})$$
$$(\sigma^{\mu})_{\alpha\dot{\beta}} \in (\mathbf{2}, \mathbf{2}), (\bar{\sigma}^{\mu})^{\dot{\alpha}\beta} \in (\mathbf{2}, \mathbf{2}) \text{ under } SL(2, \mathbb{C})$$

$$(\sigma^{\mu} \bar{\sigma}^{\nu} + \sigma^{\nu} \bar{\sigma}^{\mu})_{\alpha}{}^{\beta} = 2\eta^{\mu\nu} \delta_{\alpha}{}^{\beta}, (\bar{\sigma}^{\mu} \sigma^{\nu} + \bar{\sigma}^{\nu} \sigma^{\mu})^{\dot{\alpha}}_{\dot{\beta}} = 2\eta^{\mu\nu} \delta^{\dot{\alpha}}_{\dot{\beta}};$$

$$(\sigma^{\mu\nu})^{\beta}_{\alpha} := \frac{1}{2} (\sigma^{\mu} \bar{\sigma}^{\nu} - \sigma^{\nu} \bar{\sigma}^{\mu})^{\beta}_{\alpha}, (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} := \frac{1}{2} (\bar{\sigma}^{\mu} \sigma^{\nu} - \bar{\sigma}^{\nu} \sigma^{\mu})^{\dot{\alpha}}_{\dot{\beta}}$$
$$\sigma^{\mu\nu} \in (\mathbf{3}, \mathbf{0}), \bar{\sigma}^{\mu\nu} \in (\mathbf{0}, \mathbf{3}) \text{ under } SL(2, \mathbb{C}).$$

$$\mathcal{A} = \mathfrak{su}(N) \oplus \mathcal{A}_1 \oplus \mathcal{A}_2$$

$$[B^{l}, Q_{I\alpha}] = (S^{l})_{I}^{J} Q_{J\alpha}, [B^{l}, \bar{Q}^{I}_{\dot{\alpha}}] = -(S^{l})_{J}^{I} \bar{Q}^{J}_{\dot{\alpha}}$$
$$[B^{l}, P_{\mu}] = 0, [B^{l}, J_{\mu\nu}] = 0.$$

$$\{Q_{I\alpha}, Q_{J\beta}\} = C_{\alpha\beta}Z_{IJ} + \frac{1}{2}(\sigma^{\mu\nu})_{\alpha\beta}(Y_{\mu\nu})_{IJ}$$

$$Z_{IJ} = \sum_{l} (a^l)_{IJ}B^l \in \mathcal{A}, (Y_{\mu\nu})_{IJ} = Y_{IJ}J_{\mu\nu}$$

$$[Z_{IJ}, B^l] = 0, \{Q_{I\alpha}, Q_{J\beta}\} = C_{\alpha\beta}Z_{IJ}$$

$$(X^a)_i^k Z_{kj} + Z_{ik}(X^a)_j^k = 0$$

$$Z_{ij} = \sum_{B^l \in \mathcal{A}} (a^l)_{ij}B^l$$

$$(X^a)_i{}^k (a^l)_{kj} = -(a^l)_{ik}(X^{a*})^k{}_j.$$

$$(X^a)_i^j = (a^l)_{ik}(X^{a*})^k{}_l (a^l)^{lj}$$

$$(a^l)^{ij} := (((a^l)^T)^{-1})^{ij}$$

$$0 = (N^2 - N - 2)(a^l)_{im} = (N-2)(N+1)(a^l)_{im},$$

$$(X^a)_i^j = \Omega_{ik}(X^{a*})^k{}_l \Omega^{lj}, \Omega^{ij} := ((\Omega^T)^{-1})^{ij}$$

$$2m \geqslant z^l$$

$$\{Q_{i\alpha}, \bar{Q}^j{}_{\dot\beta}\} = 2m\delta_{\alpha\dot\beta}\delta_i{}^j$$
$$\{Q_{i\alpha}, Q_{j\beta}\} = 0, \{\bar{Q}^i{}_{\dot\alpha}, \bar{Q}^j{}_{\dot\beta}\} = 0$$

$$a_{i\alpha} := \frac{1}{\sqrt{2m}}Q_{i\alpha}, (a_{i\alpha})^\dagger := \frac{1}{\sqrt{2m}}\bar{Q}^i\dot\alpha$$

$$\{a_{i\alpha}, (a_{j\beta})^\dagger\} = \delta_\alpha^\beta \delta_i^j$$
$$\{a_{i\alpha}, a_{j\beta}\} = 0, \{(a_{i\alpha})^\dagger, (a_{j\beta})^\dagger\} = 0$$

$$a_i := \frac{1}{2\sqrt{P^0}}Q_{1i}, (a_i)^\dagger := \frac{1}{2\sqrt{P^0}}\bar{Q}^i$$
$$b_i := \frac{1}{2\sqrt{P^0}}Q_{2i}, (b_i)^\dagger := \frac{1}{2\sqrt{P^0}}\bar{Q}^i$$

$$\{a_i, (a_j)^\dagger\} = \delta_i^j$$

$$\{a_i, a_j\} = 0, \{(a_i)^\dagger, (a_j)^\dagger\} = 0$$

$$\{b_i, (b_j)^\dagger\} = 0, \{b_i, b_j\} = 0, \{(b_i)^\dagger, (b_j)^\dagger\} = 0$$
$$\{a_i, b_j\} = 0, \{(a_i)^\dagger, (b_j)^\dagger\} = 0, \{a_i, (b_j)^\dagger\} = 0, \{(a_i)^\dagger, b_j\} = 0$$

$$\{Q_{i\alpha}, \bar{Q}^j{}_{\dot\beta}\} = 2m\delta_{\alpha\dot\beta}\delta_i{}^j$$
$$\{Q_{i\alpha}, Q_{j\beta}\} = C_{\alpha\beta}Z_{ij}, \{\bar{Q}^i{}_{\dot\alpha}, \bar{Q}^j{}_{\dot\beta}\} = C_{\dot\alpha\dot\beta}(Z^*)^{ij}$$

$$Z_{ij} = U_i{}^k (Z^{\text{std}})_{kl} U_j^l$$
$$(Z^{\text{std}})_{ij} := iC \otimes Z^{\text{d}} \ (N \text{ even}) \ (Z^{\text{std}})_{ij} := \begin{pmatrix} iC \otimes Z^{\text{d}} & 0 \\ 0 & 0 \end{pmatrix} \ (N \text{ odd}),$$

$$Z^{\text{d}} = \text{diag}(z^1, \cdots, z^{\text{rank}(Z)}) \text{ with } z^l \geqslant 0$$

$$(A_i{}^k) \otimes (B_j{}^l) = (A_i{}^k B_j{}^l)$$

$$(i,j) = (1,1), \cdots, (n,1), \cdots, (1,n), \cdots, (n,n),$$

$$\tilde{Q}_{i\alpha} = (U^{-1})_i{}^j Q_{j\alpha}, (\tilde{Q}_{i\alpha})^\dagger = \bar{\tilde{Q}}^i_{\dot{\alpha}} = U_j{}^i \bar{Q}^j{}_{\dot{\alpha}}$$

$$\left\{\tilde{Q}_{am\alpha}, (\tilde{Q}_{bn\beta})^\dagger\right\} = 2m\delta_\alpha{}^\beta \delta_a{}^b \delta_m{}^n$$

$$\{\tilde{Q}_{am\alpha}, \tilde{Q}_{bn\beta}\} = iC_{\alpha\beta}C_{ab}(Z^{\mathrm{d}})_{mn}, \left\{(\tilde{Q}_{am\alpha})^\dagger, (\tilde{Q}_{bn\beta})^\dagger\right\} = iC^{\alpha\beta}C^{ab}(Z^{\mathrm{d}})^{mn}$$

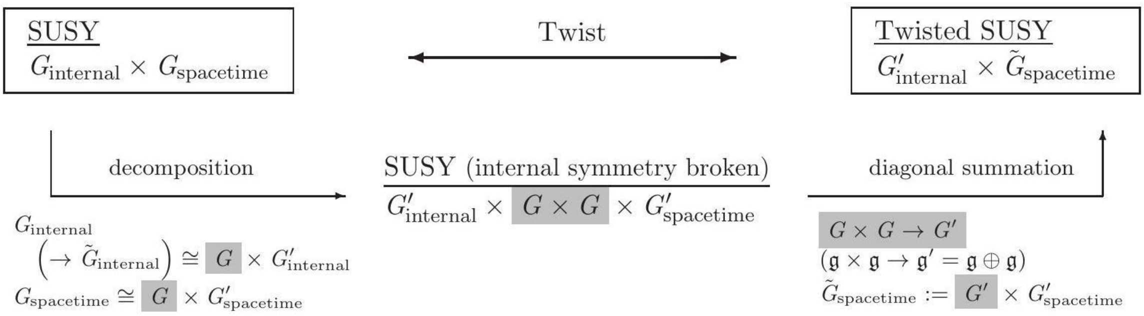
$$Z^{\mathrm{d}} = \mathrm{diag}(z^1, \cdots, z^{\mathrm{rank}Z}, 0, \cdots, 0)$$

$$a_{m\alpha} := \frac{1}{\sqrt{2}}\left(\tilde{Q}_{1m\alpha} + iC_{\alpha\gamma}(\tilde{Q}_2{}^m{}_\gamma)^\dagger\right), \quad (a_{m\alpha})^\dagger := \frac{1}{\sqrt{2}}\left((\tilde{Q}_{1m\alpha})^\dagger - iC^{\alpha\gamma}\tilde{Q}_2{}^m{}_\gamma\right),$$
$$b_{m\alpha} := \frac{1}{\sqrt{2}}\left(\tilde{Q}_{1m\alpha} - iC_{\alpha\gamma}(\tilde{Q}_2{}^m{}_\gamma)^\dagger\right), \quad (b_{m\alpha})^\dagger := \frac{1}{\sqrt{2}}\left((\tilde{Q}_{1m\alpha})^\dagger + iC^{\alpha\gamma}\tilde{Q}_2{}^m{}_\gamma\right),$$

$$\left\{a_{m\alpha}, (a_{n\beta})^\dagger\right\} = \delta_{\alpha\beta}\left(2m\delta_{mn} + (Z^{\mathrm{d}})_{mn}\right), \left\{b_{m\alpha}, (b_{n\beta})^\dagger\right\} = \delta_{\alpha\beta}\left(2m\delta_{mn} - (Z^{\mathrm{d}})_{mn}\right),$$
$$\{a_{m\alpha}, a_{n\beta}\} = 0, \{b_{m\alpha}, b_{n\beta}\} = 0, \left\{(a_{m\alpha})^\dagger, (a_{n\beta})^\dagger\right\} = 0, \left\{(b_{m\alpha})^\dagger, (b_{n\beta})^\dagger\right\} = 0,$$
$$\{a_{m\alpha}, b_{n\beta}\} = 0, \left\{(a_{m\alpha})^\dagger, (b_{n\beta})^\dagger\right\} = 0, \left\{a_{m\alpha}, (b_{n\beta})^\dagger\right\} = 0, \{(a_{m\alpha})^\dagger, b_{n\beta}\} = 0.$$

$$2m \geqslant z^l, \text{ for } l = 1, \cdots, \mathrm{rank}Z.$$

$$z^i = 2m \text{ for } i = 1, \cdots, r \leqslant \mathrm{rank}Z$$



$$J_{\alpha\beta} := \frac{1}{4}(\sigma^{\mu\nu})_{\alpha\beta}J_{\mu\nu}, J_{\dot{\alpha}\dot{\beta}} := \frac{1}{4}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}}J_{\mu\nu}$$

$$\{Q_{i\alpha}, Q_{j\beta}\} = C_{ij}C_{\alpha\beta}Z, \{\bar{Q}^i{}_{\dot{\alpha}}, \bar{Q}^j{}_{\dot{\beta}}\} = C^{ij}C_{\dot{\alpha}\dot{\beta}}Z,$$
$$\{Q_{i\alpha}, \bar{Q}^j{}_{\dot{\beta}}\} = \delta_i{}^j P_{\alpha\dot{\beta}}, [P_{\alpha\dot{\beta}}, P_{\gamma\dot{\delta}}] = 0,$$
$$[Q_{i\alpha}, P_{\gamma\dot{\delta}}] = 0, [\bar{Q}^i{}_{\dot{\alpha}}, P_{\gamma\dot{\delta}}] = 0,$$
$$[J_{\alpha\beta}, Q_{i\gamma}] = \frac{i}{2}C_{\gamma(\alpha}Q_{i\beta)}, [J_{\alpha\beta}, \bar{Q}^i{}_{\dot{\gamma}}] = 0,$$
$$[J_{\dot{\alpha}\dot{\beta}}, Q_{i\gamma}] = 0, [J_{\dot{\alpha}\dot{\beta}}, \bar{Q}^i{}_{\dot{\gamma}}] = \frac{i}{2}C_{\dot{\gamma}(\dot{\alpha}}\bar{Q}^i{}_{\dot{\beta})},$$
$$[J_{\alpha\beta}, P_{\gamma\dot{\delta}}] = \frac{i}{2}C_{\gamma(\alpha}P_{\beta)\dot{\delta}}, [J_{\dot{\alpha}\dot{\beta}}, P_{\gamma\dot{\delta}}] = \frac{i}{2}C_{\dot{\delta}(\dot{\alpha}}P_{\gamma\dot{\beta})},$$

$$[J_{\alpha\beta}, J_{\gamma\delta}] = \frac{i}{2}C_{(\alpha|(\gamma}J_{\delta)|\beta)}, [J_{\dot{\alpha}\dot{\beta}}, J_{\dot{\gamma}\dot{\delta}}] = \frac{i}{2}C_{(\dot{\alpha}|(\dot{\gamma}}J_{\dot{\delta})|\dot{\beta})}$$
$$[J_{\alpha\beta}, J_{\dot{\gamma}\dot{\delta}}] = 0, [Z, \mathrm{any}] = 0$$
$$[R_{ij}, Q_{k\gamma}] = \frac{i}{2}C_{k(i}Q_{j)\gamma}, [R_{ij}, \bar{Q}_{k\gamma}] = \frac{i}{2}C_{k(i}\bar{Q}_{j)\gamma}$$
$$[R_{ij}, R_{kl}] = \frac{i}{2}C_{(i|(k}R_{l)|j)}, [R_{ij}, P_{\alpha\dot{\beta}}] = 0$$
$$[R_{ij}, J_{\alpha\beta}] = 0, [R_{ij}, J_{\dot{\alpha}\dot{\beta}}] = 0$$

$$J'_{\alpha\beta} := J_{\alpha\beta} + R_{\alpha\beta}, \text{ or } J'_{\dot{\alpha}\dot{\beta}} := J_{\dot{\alpha}\dot{\beta}} + R_{\dot{\alpha}\dot{\beta}}.$$

$$\begin{array}{ccc} \mathbf{2} & \to & (\mathbf{2},\mathbf{1}) \\ i & \to & \alpha \end{array}, \text{ or } \begin{array}{ccc} \mathbf{2} & \to & (\mathbf{1},\mathbf{2}) \\ i & \to & \dot{\alpha} \end{array}$$

$$\begin{array}{ccccc} \mathbf{2}\otimes\mathbf{2} & = & \mathbf{1} & \oplus & \mathbf{3} \\ Q_{\alpha\beta} & \to & Q_{[\alpha\beta]}, & & Q_{(\alpha\beta)} \end{array}$$

$$Q := \frac{1}{2} C^{\alpha\beta} Q_{[\alpha\beta]}, H_{\alpha\beta} := Q_{(\alpha\beta)}, G_{\alpha\dot{\beta}} := \bar{Q}_{\alpha\dot{\beta}}$$

$$\begin{gathered} \{Q,Q\} = Z, \{H_{\alpha\beta}, H_{\gamma\delta}\} = C_{(\alpha|(\gamma}C_{\delta)|\beta)}Z, \\ \{G_{\alpha\dot{\beta}}, G_{\gamma\dot{\delta}}\} = C_{\alpha\beta}C_{\dot{\beta}\dot{\delta}}Z, \{Q, H_{\alpha\beta}\} = 0, \\ \{Q, G_{\alpha\dot{\beta}}\} = P_{\alpha\dot{\beta}}, \{H_{\alpha\beta}, G_{\gamma\dot{\delta}}\} = C_{\alpha(\gamma}P_{\beta)\dot{\delta}} \\ [Q, P_{\gamma\dot{\delta}}] = 0, [H_{\alpha\beta}, P_{\gamma\dot{\delta}}] = 0, \\ [G_{\alpha\dot{\beta}}, P_{\gamma\dot{\delta}}] = 0, [P_{\alpha\dot{\beta}}, P_{\gamma\dot{\delta}}] = 0, \\ [J_{\alpha\beta}, Q] = 0, [J_{\dot{\alpha}\dot{\beta}}, Q] = 0, \\ [J_{\alpha\beta}, H_{\gamma\delta}] = \frac{i}{2} C_{(\alpha|(\gamma}H_{\delta)|\beta)}, [J_{\dot{\alpha}\dot{\beta}}, H_{\gamma\delta}] = 0, \end{gathered}$$

$$\begin{gathered} [J_{\alpha\beta}, G_{\gamma\dot{\delta}}] = \frac{i}{2} C_{(\alpha|\gamma}G_{\beta)\dot{\delta}}, [J_{\dot{\alpha}\dot{\beta}}, G_{\gamma\dot{\delta}}] = \frac{i}{2} C_{(\dot{\alpha}|\dot{\delta}}G_{\gamma\dot{\beta})} \\ [J_{\alpha\beta}, P_{\gamma\dot{\delta}}] = \frac{i}{2} C_{(\alpha|\gamma}P_{\beta)\dot{\delta}}, [J_{\dot{\alpha}\dot{\beta}}, P_{\gamma\dot{\delta}}] = \frac{i}{2} C_{(\dot{\alpha}|\dot{\delta}}P_{\gamma\dot{\beta})} \\ [J_{\alpha\beta}, J_{\gamma\delta}] = \frac{i}{2} C_{(\alpha|(\gamma}J_{\delta)|\beta)}, [J_{\dot{\alpha}\dot{\beta}}, J_{\dot{\gamma}\dot{\delta}}] = \frac{i}{2} C_{(\dot{\alpha}|(\dot{\gamma}}J_{\dot{\delta})|\dot{\beta})} \\ [J_{\alpha\beta}, J_{\dot{\gamma}\dot{\delta}}] = 0, [Z, \text{any}] = 0 \end{gathered}$$

$$\begin{gathered} A_{\alpha\dot{\beta}} \to A_{\alpha\dot{\beta}} \\ \lambda_{i\beta} \to \lambda_{\alpha\beta} \to \eta, \chi_{\alpha\beta} \\ \bar{\lambda}_{i\dot{\beta}} \to \psi_{\alpha\dot{\beta}} \\ \phi \to \phi \\ \bar{\phi} \to \rho \\ G_{ij} \to G_{\alpha\beta} \end{gathered}$$

$$\eta = C^{\alpha\beta}\psi_{\alpha\beta}, \chi_{\alpha\beta} = \psi_{(\alpha\beta)}$$

$$\begin{cases} SU(4) \to SU(2) \times SU(2), \\ \mathfrak{su}(4) \to \mathfrak{su}(2) \oplus \mathfrak{su}(2) = \{\sqrt{1/2}(X^{12}_{\text{s,a}} + X^{34}_{\text{s,a}}), \sqrt{1/2}X^1 + \sqrt{1/6}(\sqrt{2}X^3 - X^2)\} \\ \qquad \oplus \{\sqrt{1/2}(X^{13}_{\text{s,a}} + X^{24}_{\text{s,a}}), \sqrt{1/3}(\sqrt{2}X^2 + X^3)\} \end{cases}$$

$$\begin{cases} SU(4) \to SU(2) \times SU(2) \times U(1), \\ \mathfrak{su}(4) \to \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \tilde{\mathfrak{u}}(1) = \{X^{12}_{\text{s,a}}, X^1\} \oplus \{X^{34}_{\text{s,a}}, \sqrt{1/3}(\sqrt{2}X^3 - X^2)\} \\ \qquad \oplus \{\sqrt{1/3}(\sqrt{2}X^2 + X^3)\} \end{cases}$$

$$SU(2)_{\text{L}} \times SU(2)_{\text{I}}, \text{ or equivalently}, SU(2)_{\text{R}} \times SU(2)_{\text{I}},$$

$$\begin{cases} \mathbf{4} \to (\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{1}), \\ i \to a\alpha, \end{cases} \text{ or equivalently, } \begin{cases} \mathbf{4} \to (\mathbf{1},\mathbf{2}) \oplus (\mathbf{1},\mathbf{2}) \\ i \to a\dot{\alpha} \end{cases}$$

$$SU(2)_{\text{L}} \times SU(2)_{\text{I}} \times U, \text{ or equivalently}, SU(2)_{\text{R}} \times SU(2)_{\text{I}} \times U,$$

$$SU(2)_{\text{L}} \times SU(2)_{\text{R}} \times U,$$

$$\begin{cases} \mathbf{4} \to (\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1}), \\ i \to (\alpha, a) \equiv a \oplus \alpha, \end{cases} \text{ or equivalently, } \begin{cases} \mathbf{4} \to (\mathbf{1},\mathbf{2}) \oplus (\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1}) \\ i \to (\dot{\alpha}, a) \equiv \dot{\alpha} \oplus a \end{cases}$$

$$\left| \begin{array}{c} \mathbf{4} \to (\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2}) \\ i \to (\alpha, \dot{\alpha}) \equiv \alpha \oplus \dot{\alpha} \end{array} \right\rangle$$

$$SU(4) \to SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{I}}, \begin{cases} \mathbf{4} \to (\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{1}) \\ i \to a\alpha \end{cases}$$

$$\begin{cases} Q_{i\beta} \to Q_{a\alpha,\beta} \to Q_a, Q_{a\alpha\beta} \\ \bar{Q}^i_{\dot{\beta}} \to Q^a_{\alpha\dot{\beta}} \end{cases}$$

$$\begin{aligned} A_{\alpha\dot{\beta}} &\to A_{\alpha\dot{\beta}} \\ \lambda_{i\beta} &\to \lambda_{a\alpha,\beta} \to \eta_a, \chi_{a\alpha\beta} \\ \bar{\lambda}^i_{\dot{\beta}} &\to \psi^a_{\alpha\dot{\beta}} \\ \phi_{ij} &\to \phi_{a\alpha,b\beta} \to \varphi_{ab}, G_{\alpha\beta} \end{aligned}$$

$$SU(4) \to SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{I}} \times U(1), \quad \begin{cases} \mathbf{4} \to (\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1}), \\ i \to (a,\alpha) \equiv a \oplus \alpha. \end{cases}$$

$$\begin{cases} Q_{i\beta} \to Q_{a\oplus\alpha,\beta} \to Q^{(-1)}_{a\beta}, Q^{(+1)}, Q^{(+1)}_{\alpha\beta} \\ \bar{Q}^i{}_{\dot{\beta}} \to Q^{a\oplus\alpha}{}_{\dot{\beta}} \to Q^{(+1)}_{a\dot{\beta}}, Q^{(-1)}_{\alpha\dot{\beta}} \end{cases}$$

$$\begin{aligned} A_{\alpha\dot{\beta}} &\to A^{(0)}_{\alpha\dot{\beta}} \\ \lambda_{i\beta} &\to \lambda_{a\oplus\alpha\beta} \to \lambda^{(+1)}_{a\beta}, \eta^{(-1)}, \chi^{(-1)}_{\alpha\beta} \\ \bar{\lambda}^i_{\dot{\beta}} &\to \bar{\lambda}^{a\oplus\alpha}{}_{\dot{\beta}} \to \psi^{(+1)}_{\alpha\dot{\beta}}, \zeta^{(-1)a}_{\dot{\beta}} \\ \phi_{ij} &\to \phi_{a\oplus\alpha,b\oplus\beta} \to B^{(-2)}, C^{(+2)}, G^{(0)}_{a\alpha} (= \phi_{a\beta}, \phi_{b\alpha}). \end{aligned}$$

$$SU(4) \to SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times U(1), \begin{cases} \mathbf{4} \to (\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2}) \\ i \to (\alpha,\dot{\beta}) \equiv \alpha \oplus \dot{\beta} \end{cases}$$

$$\begin{cases} Q_{i\gamma} \to Q_{\alpha\oplus\dot{\beta},\gamma} \to Q^{(+1)}, Q^{(-1)}_{\gamma\dot{\beta}}, Q^{(+1)}_{\alpha\gamma} \\ \bar{Q}^i_{\dot{\gamma}} \to \bar{Q}^{\alpha\oplus\dot{\beta}}_{\dot{\gamma}} \to \tilde{Q}^{(+1)}, \tilde{Q}^{(-1)}_{\alpha\dot{\gamma}}, \tilde{Q}^{(+1)}_{\dot{\beta}\dot{\gamma}} \end{cases}$$

$$\begin{aligned} A_{\alpha\dot{\beta}} &\to A^{(0)}_{\alpha\dot{\beta}} \\ \lambda_{i\gamma} &\to \lambda_{\alpha\oplus\dot{\beta}\gamma} \to \tilde{\psi}^{(+1)}_{\gamma\dot{\beta}}, \eta^{(-1)}, \chi^{(-1)}_{\alpha\gamma} \\ \bar{\lambda}^i_{\dot{\gamma}} &\to \bar{\lambda}^{\alpha\oplus\dot{\beta}}{}_{\dot{\gamma}} \to \psi^{(+1)}_{\alpha\dot{\gamma}}, \tilde{\eta}^{(-1)}, \tilde{\chi}^{(-1)}_{\dot{\beta}\dot{\gamma}} \\ \phi_{ij} &\to \phi_{\alpha\oplus\dot{\beta},\gamma\oplus\dot{\delta}} \to B^{(-2)}, C^{(+2)}, V^{(0)}_{\alpha\dot{\beta}} \end{aligned}$$

$$\bar{Q}^{A\alpha} = \sum_{B\beta} (Q_{B\beta})^{\dagger} (\Gamma^0)^A_B (\Gamma^0)^{\alpha}_{\beta}$$

$$Q_{A\alpha} = (Q_{\mathrm{c}})_{A\alpha} := C_{AB} C_{\alpha\beta} (\bar{Q}^T)^{B\beta},$$

$$C^T = \varepsilon' C, C^{\dagger} C = 1, C\gamma^{\mu} C^{-1} = \eta' (\gamma^{\mu})^T, C\gamma^i C^{-1} = \eta' (\gamma^i)^T, \varepsilon', \eta' = \pm 1.$$

$$G_{\mathrm{I}} \to Spin(D), \quad \begin{cases} \mathbf{2}^{D/2} \to \mathbf{2}^{D/2}, \\ A \to \alpha, \\ Q_{A\beta} \to Q_{\alpha\beta}. \end{cases}$$

$$Q_{\alpha\beta} = \sum_{p=0}^{D} \frac{1}{p!} Q_{\mu_1 \cdots \mu_p} (\gamma^{\mu_1 \cdots \mu_p} C^{-1})_{\alpha\beta}, Q_{\mu_1 \cdots \mu_p} := (-1)^{p(p-1)/2} \frac{1}{2^{D/2}} \left(C\gamma_{\mu_1 \cdots \mu_p} \right)^{\alpha\beta} Q_{\alpha\beta}$$

$$Q := \frac{1}{2^{D/2}} C^{\alpha\beta} Q_{\alpha\beta}$$

$$\tilde{Q} := \frac{1}{2^{D/2}}(C\Gamma^5)^{\alpha\beta}Q_{\alpha\beta}$$

$$Q^{\pm}_{\mu_1\cdots\mu_p} := \frac{1}{2}\left(Q_{\mu_1\cdots\mu_p} \pm \frac{1}{(D-p)!}(-1)^{D/4+p(p+1)/2}\varepsilon_{\mu_1\cdots\mu_p}^{\mu_{p+1}\cdots\mu_D}Q_{\mu_{p+1}\cdots\mu_D}\right)$$
$$= \frac{1}{2^{D/2}}(-1)^{p(p-1)/2}\left(C\gamma_{\mu_1\cdots\mu_p}\frac{1}{2}(1\pm\Gamma^5)\right)^{\alpha\beta}Q_{\alpha\beta}$$

$$\{Q_{A\alpha}, Q_{B\beta}\} = 2C^{-1}_{AB}(\gamma^\mu C^{-1})_{\alpha\beta}P_\mu + \Gamma_{\alpha\beta}Z_{AB}$$
$$[J_{\mu\nu}, Q_{A\alpha}] = \frac{i}{2}(\gamma_{\mu\nu})_\alpha{}^\beta Q_{A\beta}, [R_{ij}, Q_{A\alpha}] = \frac{i}{2}(\gamma_{ij})_A{}^B Q_{B\alpha}$$

$$\{Q_{\mu_1\cdots\mu_p}, Q_{\nu_1\cdots\nu_{p-1}}\} = \frac{2}{2^{D/2}}\eta_{\mu_1\cdots\mu_p,\nu_1\cdots\nu_p}P^{\nu_p}, \{Q_{\mu_1\cdots\mu_p}, Q_{\nu_1\cdots\nu_p}\} = \frac{1}{2^{D/2}}\eta_{\mu_1\cdots\mu_p,\nu_1\cdots\nu_p}Z$$
$$[J_{\mu\nu}, Q_{\rho_1\cdots\rho_p}] = \frac{i}{2}\varepsilon'(-1)^{p(p-1)/2}\left(Q_{\mu\nu\rho_1\cdots\rho_p} + \sum_{i=1}^{p}\eta_{[\mu|\rho_i}Q_{\rho_1\cdots\nu]\cdots\rho_p}\right.$$
$$\left. + \sum_{1\leqslant i<j\leqslant p}(-1)^{i+j}\left(\eta_{[\mu|\rho_i}\eta_{\nu]\rho_j}\right)Q_{\rho_1\cdots\check{\rho}_i\cdots\check{\rho}_j\cdots\rho_p}\right)$$
$$[R_{\mu\nu}, Q_{\rho_1\cdots\rho_p}] = \frac{i}{2}\varepsilon'(-1)^{p(p-1)/2}\left(-Q_{\mu\nu\rho_1\cdots\rho_p} + \sum_{i=1}^{p}\eta_{[\mu|\rho_i}Q_{\rho_1\cdots\nu]\cdots\rho_p}\right.$$
$$\left. - \sum_{1\leqslant i<j\leqslant p}(-1)^{i+j}\left(\eta_{[\mu|\rho_i}\eta_{\nu]\rho_j}\right)Q_{\rho_1\cdots\check{\rho}_i\cdots\check{\rho}_j\cdots\rho_p}\right)$$

$$[J'_{\mu\nu}, Q_{\rho_1\cdots\rho_p}] = i\varepsilon'(-1)^{p(p-1)/2}\sum_{i=1}^{p}\eta_{[\mu|\rho_i}Q_{\rho_1\cdots\nu]\cdots\rho_p}$$

$$Q := C^{\alpha\beta}Q_{\alpha\beta}, \quad Q_\mu := (C\gamma_\mu)^{\alpha\beta}Q_{\alpha\beta}, \quad Q_{\mu\nu} := (C\gamma_{\mu\nu})^{\alpha\beta}Q_{\alpha\beta},$$
$$\tilde{Q} := (C\gamma^5)^{\alpha\beta}Q_{\alpha\beta}, \quad \tilde{Q}_\mu := (C\gamma^5\gamma_\mu)^{\alpha\beta}Q_{\alpha\beta}, \quad \tilde{Q}_{\mu\nu} := (C\gamma^5\gamma_{\mu\nu})^{\alpha\beta}Q_{\alpha\beta}.$$

$$\gamma^\mu := \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \sigma^\mu = (\mathbf{1}, i\tau^i), \bar{\sigma}^\mu = (\mathbf{1}, -i\tau^i)$$

$$C := i\gamma^1\gamma^3 = \begin{pmatrix} -\tau^2 & 0 \\ 0 & -\tau^2 \end{pmatrix}, \gamma^5 := \gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\psi_\alpha \to \begin{pmatrix} \psi_\alpha \\ \psi^{\dot{\alpha}} \end{pmatrix}, Q_{\alpha\beta} \to \begin{pmatrix} Q_{\alpha\beta} & Q_\alpha^{\dot{\beta}} \\ Q^{\dot{\alpha}}_\beta & Q^{\dot{\alpha}\dot{\beta}} \end{pmatrix}$$

$$Q = C^{\alpha\beta}_{(2)}Q_{\alpha\beta} + C^{(2)}_{\dot{\alpha}\dot{\beta}}Q^{\dot{\alpha}\dot{\beta}}, \tilde{Q} = C^{\alpha\beta}_{(2)}Q_{\alpha\beta} - C^{(2)}_{\dot{\alpha}\dot{\beta}}Q^{\dot{\alpha}\dot{\beta}}$$
$$Q_\mu = -(\sigma_\mu)_{\alpha\dot{\beta}}Q^{\alpha\dot{\beta}} + (\bar{\sigma}_\mu)^{\dot{\alpha}\beta}Q_{\dot{\alpha}\beta}, \tilde{Q}_\mu = (\sigma_\mu)_{\alpha\dot{\beta}}Q^{\alpha\dot{\beta}} + (\bar{\sigma}_\mu)^{\dot{\alpha}\beta}Q_{\dot{\alpha}\beta}$$
$$Q_{\mu\nu} = (\sigma_{\mu\nu})^{\alpha\beta}Q_{\alpha\beta} + (\bar{\sigma}_{\mu\nu})_{\dot{\alpha}\dot{\beta}}Q^{\dot{\alpha}\dot{\beta}}, \tilde{Q}_{\mu\nu} = (\sigma_{\mu\nu})^{\alpha\beta}Q_{\alpha\beta} - (\bar{\sigma}_{\mu\nu})_{\dot{\alpha}\dot{\beta}}Q^{\dot{\alpha}\dot{\beta}}$$

$$Q_{[\alpha\beta]} = \frac{1}{2}(Q+\tilde{Q})C^{(2)}_{\alpha\beta}, Q^{[\dot{\alpha}\dot{\beta}]} = \frac{1}{2}(Q-\tilde{Q})C^{\dot{\alpha}\dot{\beta}}_{(2)}$$
$$Q_{\alpha\dot{\beta}} = \frac{1}{4}(Q_\mu + \tilde{Q}_\mu)(\sigma^\mu)_{\alpha\dot{\beta}}, Q^{\dot{\alpha}\beta} = -\frac{1}{4}(Q_\mu - \tilde{Q}_\mu)(\bar{\sigma}^\mu)^{\dot{\alpha}\beta}$$
$$Q_{(\alpha\beta)} = \frac{1}{4}(Q_{\mu\nu} + \tilde{Q}_{\mu\nu})(\sigma^{\mu\nu})_{\alpha\beta}, Q^{(\dot{\alpha}\dot{\beta})} = \frac{1}{4}(Q_{\mu\nu} - \tilde{Q}_{\mu\nu})(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}}$$

$$
\begin{aligned}
\{Q_{i\alpha}, Q_{j\beta}\} &= C_{\alpha\beta}\Omega_{ij}Z, & \{\bar{Q}_{i\dot{\alpha}}, \bar{Q}_{j\dot{\beta}}\} &= C_{\dot{\alpha}\dot{\beta}}\Omega_{ij}Z,\\
\{Q_{i\alpha}, \bar{Q}_{j\dot{\beta}}\} &= 2\Omega_{ij}(\sigma^{\mu})_{\alpha\dot{\beta}}P_{\mu}, & [P_{\mu}, P_{\nu}] &= 0,\\
[Q_{i\alpha}, P_{\mu}] &= 0, & [\bar{Q}_{i\dot{\alpha}}, P_{\mu}] &= 0,\\
[J_{\mu\nu}, Q_{i\alpha}] &= \frac{i}{2}(\sigma_{\mu\nu})_{\alpha}{}^{\beta}Q_{i\beta}, & [J_{\mu\nu}, \bar{Q}_{i}{}^{\dot{\alpha}}] &= \frac{i}{2}(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}}\bar{Q}_{i}{}^{\dot{\beta}},\\
[J_{\mu\nu}, P_{\rho}] &= i(\eta_{\nu\rho}P_{\mu} - \eta_{\mu\rho}P_{\nu}), & [J_{\mu\nu}, J_{\rho\sigma}] &= i(\eta_{\nu\rho}J_{\mu\sigma} - \eta_{\nu\sigma}J_{\mu\rho}\\
& & &\quad -\eta_{\mu\rho}J_{\nu\sigma} + \eta_{\mu\sigma}J_{\nu\rho}),\\
[R^{a}, Q_{i\alpha}] &= (X^{a})_{i}{}^{j}Q_{j\alpha}, & [R^{a}, \bar{Q}_{i\dot{\alpha}}] &= (X^{a})_{i}{}^{j}\bar{Q}_{j\dot{\alpha}},\\
[R^{a}, P_{\mu}] &= 0, & [R^{a}, J_{\mu\nu}] &= 0,\\
[Z, \text{any}] &= 0,
\end{aligned}
$$

$$\bar{Q}^{i}{}_{\dot{\alpha}} := (Q_{i\alpha})^{\dagger},$$

$$Q^{i} = \Omega^{ij}Q_{j}, Q_{i} = Q^{j}\Omega_{ji}, \Omega_{ij} + \Omega_{ji} = 0, \Omega^{ik}\Omega_{jk} = \delta_{j}^{i}$$

$$(X^{a})_{ij} = (X^{a})_{ji}, (X^{a})_{ij} := (X^{a})_{i}{}^{k}\Omega_{kj}$$

$$Z^{\dagger} = Z$$

$$
\begin{aligned}
S = \mathrm{tr}\int d^{4}x\Big(&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}V_{\mu}V^{\mu} + \frac{1}{2}D_{\mu}\phi_{ij}D^{\mu}\phi^{ij} + \frac{1}{2}H_{ij}H^{ij}\\
&-\frac{i}{4}\bar{\lambda}^{i}\overleftrightarrow{\not{D}}\lambda_{i} - \bar{\lambda}^{i}[\lambda^{j}, \phi_{ij}] + \frac{1}{4}[\phi_{ij}, \phi_{kl}][\phi^{ij}, \phi^{kl}]\Big)
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{i\alpha} &= \bar{\lambda}^{j\beta}(C^{-1})_{\beta\alpha}\Omega_{ji},\\
\delta A_{\mu} &= i\bar{\zeta}^{i}(\gamma_{\mu})\lambda_{i}\\
\delta\phi_{ij} &= -i\left(\bar{\zeta}_{i}\lambda_{j} - \bar{\zeta}_{j}\lambda_{i} + \frac{1}{2}\Omega_{ij}\bar{\zeta}^{k}\lambda_{k}\right)
\end{aligned}
$$

$$\delta\lambda_{i} = -\frac{1}{2}\gamma_{\mu\nu}F^{\mu\nu}\zeta_{i} + 2\gamma^{\mu}D_{\mu}\phi_{i}^{j}\zeta_{j} + \gamma^{5}\gamma^{\mu}V_{\mu}\zeta_{i} + 2\gamma^{5}H_{i}^{j}\zeta_{j} - 2i[\phi_{ik}, \phi^{kj}]\zeta_{j}$$

$$
\begin{aligned}
\delta H_{ij} = i\Big(&\bar{\zeta}_{i}\gamma^{5}\gamma^{\mu}D_{\mu}\lambda_{j} - \bar{\zeta}_{j}\gamma^{5}\gamma^{\mu}D_{\mu}\lambda_{i} + \frac{1}{2}\Omega_{ij}\bar{\zeta}^{k}\gamma^{5}\gamma^{\mu}D_{\mu}\lambda_{k}\Big)\\
&-2\left(\bar{\zeta}_{i}\gamma^{5}[\lambda^{l}, \phi_{jl}] - \bar{\zeta}_{j}\gamma^{5}[\lambda^{l}, \phi_{il}] + \frac{1}{2}\Omega_{ij}\bar{\zeta}^{k}\gamma^{5}[\lambda^{l}, \phi_{kl}]\right) + \bar{\zeta}^{k}\gamma^{5}[\lambda_{k}, \phi_{ij}]\\
\delta V_{\mu} &= i\bar{\zeta}^{i}\gamma^{5}\gamma_{\mu\nu}D^{\nu}\lambda_{i} + 2\bar{\zeta}^{i}\gamma^{5}\gamma_{\mu}[\lambda^{j}, \phi_{ij}]
\end{aligned}
$$

$$
\begin{aligned}
\delta_{z}A_{\mu} &= \omega V_{\mu}\\
\delta_{z}\phi_{ij} &= -\omega H_{ij}\\
\delta_{z}\lambda_{i} &= -\omega(\gamma^{5}\gamma^{\mu}D_{\mu}\lambda_{i} - 2i\gamma^{5}[\lambda^{j}, \phi_{ij}])\\
\delta_{z}H_{ij} &= \omega\left(-D^{\mu}D_{\mu}\phi_{ij} + i\left(\frac{1}{4}\Omega_{ij}\{\bar{\lambda}_{k}, \lambda^{k}\}\right) - [[\phi_{kl}, \phi^{kl}], \phi_{ij}]\right)\\
\delta_{z}V_{\mu} &= \omega\left(D_{\nu}F_{\mu}^{\nu} - \frac{1}{2}\{\bar{\lambda}, \gamma_{\mu}\lambda\} + i[\phi^{ij}, D_{\mu}\phi_{ij}]\right)\\
0 &= D^{\mu}V_{\mu} + \frac{1}{2}\{\bar{\lambda}^{i}, \gamma^{5}\lambda_{i}\} - i[\phi^{ij}, H_{ij}]
\end{aligned}
$$

$$P_{\mu} = i\partial_{\mu}$$

$$
\begin{aligned}
Q_{i\alpha} &= \frac{\partial}{\partial\theta^{i\alpha}} - i(\sigma^{\mu})_{\alpha\dot{\beta}}\bar{\theta}_{i}{}^{\dot{\beta}}\partial_{\mu} - \frac{i}{2}\theta_{i\alpha}\partial_{z}\\
\bar{Q}_{i\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}^{i\dot{\alpha}}} - i\theta_{i}{}^{\beta}(\sigma^{\mu})_{\beta\dot{\alpha}}\partial_{\mu} - \frac{i}{2}\bar{\theta}_{i\dot{\alpha}}\partial_{z}\\
Z &= i\partial_{z}
\end{aligned}
$$

$$
\begin{aligned}
\{Q_{i\alpha}, Q_{j\beta}\} &= -C_{\alpha\beta}\Omega_{ij}Z, & \{\bar{Q}_{i\dot{\alpha}}, \bar{Q}_{j\dot{\beta}}\} &= -C_{\dot{\alpha}\dot{\beta}}\Omega_{ij}Z,\\
\{Q_{i\alpha}, \bar{Q}_{j\dot{\beta}}\} &= -2\Omega_{ij}(\sigma^{\mu})_{\alpha\dot{\beta}}P_{\mu}, & [P_{\mu}, P_{\nu}] &= 0,\\
[Q_{i\alpha}, P_{\mu}] &= 0, & [\bar{Q}_{i\dot{\alpha}}, P_{\mu}] &= 0,\\
[Z, \text{any}] &= 0.
\end{aligned}
$$

$$\begin{aligned} D_\mu &= \partial_\mu \\ D_{i\alpha} &= \frac{\partial}{\partial\theta^{i\alpha}} + i(\sigma^\mu)_{\alpha\dot\beta}\bar\theta_i{}^{\dot\beta}\partial_\mu + \frac{i}{2}\theta_{i\alpha}\partial_z \\ \bar D_{i\dot\alpha} &= \frac{\partial}{\partial\bar\theta^{i\dot\alpha}} + i\theta_i{}^{\beta}(\sigma^\mu)_{\beta\dot\alpha}\partial_\mu + \frac{i}{2}\bar\theta_{i\dot\alpha}\partial_z \\ D_z &= \partial_z \end{aligned}$$

$$\begin{aligned} \{D_{i\alpha}, D_{j\beta}\} &= iC_{\alpha\beta}\Omega_{ij}D_z, \{\bar D_{i\dot\alpha}, \bar D_{j\dot\beta}\} = iC_{\dot\alpha\dot\beta}\Omega_{ij}D_z, \\ \{D_{i\alpha}, \bar D_{j\dot\beta}\} &= 2i\Omega_{ij}(\sigma^\mu)_{\alpha\dot\beta}D_\mu, [D_\mu, D_\nu] = 0, \\ [D_{i\alpha}, D_\mu] &= 0, [\bar D_{i\dot\alpha}, D_\mu] = 0, \\ [D_z, \text{any}\,] &= 0, \{\bar Q_{i\dot\alpha}, \text{any}\, D\} = 0, \\ \{Q_{i\alpha}, \text{any}\, D\} &= 0, [Z, \text{any}\, D] = 0, \end{aligned}$$

$$\nabla_I := D_I - i\Gamma_I$$

$$\nabla_I' = e^K \nabla e^{-K}, \text{ or } \delta_K\nabla_I = [\nabla_I, K], \delta_K\Gamma_I = i[\nabla_I, K],$$

$$K = \kappa(x,z) + \theta^{i\alpha}(i\Gamma_{i\alpha} - i[\kappa(x,z), \Gamma_{i\alpha}]) + \bar\theta^{i\dot\alpha}(i\bar\Gamma_{i\dot\alpha} - i[\kappa(x,z), \bar\Gamma_{i\dot\alpha}]) + K'(x,z,\theta,\bar\theta)$$

$$\partial_z\kappa(x,z) = i[\Gamma_z(x,\theta=0,\bar\theta=0,z), \kappa(x,z)] + i\Gamma_z(x,\theta=0,\bar\theta=0,z).$$

$$\Gamma'_{i\alpha}| = \Gamma_{i\alpha} + \delta\Gamma_{i\alpha}| = 0, \bar\Gamma'_{i\dot\alpha}| = \bar\Gamma_{i\dot\alpha} + \delta\bar\Gamma_{i\dot\alpha}| = 0, \Gamma'_z| = \Gamma_z + \delta\Gamma_z| = 0,$$

$$B\,| := B(x,\theta=0,\bar\theta=0,z)$$

$$\Gamma_{i\alpha}| = \bar\Gamma_{i\dot\alpha}|\Gamma_z\,| = 0$$

$$[\nabla_I, \nabla_J]_\pm = -iF_{IJ}$$

$$\begin{aligned} \delta_K F_{IJ} &= i[\delta_K\nabla_I, \nabla_J]_\pm + i[\nabla_I, \delta_K\nabla_J]_\pm = i\left[[\nabla_I, K], \nabla_J\right]_\pm + i\left[\nabla_I, [\nabla_J, K]\right]_\pm \\ &= i\left[[\nabla_I, \nabla_J]_\pm, K\right] = [F_{IJ}, K] \end{aligned}$$

$$\{\nabla_{i\alpha}, \bar\nabla_{j\dot\beta}\}_\pm = 2\Omega_{ij}(\sigma^\mu)_{\alpha\dot\beta}\nabla_\mu - i\{D_{i\alpha}, \bar\Gamma_{j\dot\beta}\} - i\{\bar D_{j\dot\beta}, \Gamma_{i\alpha}\} - \{\Gamma_{i\alpha}, \bar\Gamma_{j\dot\beta}\} + 2i\Omega_{ij}(\sigma^\mu)_{\alpha\dot\beta}\Gamma_\mu.$$

$$\begin{aligned} \{\nabla_{i\alpha}, \nabla_{j\beta}\} &= i\Omega_{ij}C_{\alpha\beta}\nabla_z - iF_{ij\alpha\beta}, \\ \{\nabla_{i\alpha}, \bar\nabla_{j\dot\beta}\} &= i\Omega_{ij}(\sigma^\mu)_{\alpha\dot\beta}\nabla_\mu - iF_{ij\alpha\dot\beta}, \end{aligned}$$

$$F_{ij\alpha\beta} = F_{ji\beta\alpha}$$

$$F_{ij\alpha\beta} = W^{\mathrm{a}}_{ij\alpha\beta} + W^{\mathrm{s}}_{ij\alpha\beta},$$

$$\begin{aligned} W^{\mathrm{a}}_{ij\alpha\beta} + W^{\mathrm{a}}_{ji\alpha\beta} &= W^{\mathrm{a}}_{ij\alpha\beta} + W^{\mathrm{a}}_{ij\beta\alpha} = 0 \\ W^{\mathrm{s}}_{ij\alpha\beta} &= W^{\mathrm{s}}_{ji\alpha\beta} = W^{\mathrm{s}}_{ij\beta\alpha} \end{aligned}$$

$$W^{\mathrm{a}}_{ij\alpha\beta} = C_{\alpha\beta}W^{\mathrm{a}}_{ij}, W^{\mathrm{a}}_{ij} + W^{\mathrm{a}}_{ji} = 0.$$

$$W^{\mathrm{a}}_{ij} = W_{ij} + \Omega_{ij}W, \text{ where } \Omega^{ij}W_{ij} = 0\ (\aleph).$$

$$\begin{aligned} W^{\mathrm{s}}_{ij\alpha\beta} &= \frac{1}{2}\sum_a (\sigma_{\mu\nu})_{\alpha\beta}(X^a)_{ij}W^{\mu\nu a}, \\ &(X^a)_{ij} \in \mathfrak{usp}(4), \end{aligned}$$

$$(\sigma_{\mu\nu})_{\alpha\beta} = (\sigma_{\mu\nu})_{\beta\alpha}$$

$$[S_i^\pm, S_j^\pm] = i\epsilon_{ijk}S^{\pm k}, [S_i^\pm, S_j^\mp] = 0, S_i^\pm := \frac{1}{2}\left(-\frac{1}{2}\epsilon_{ijk}J^{jk} \mp J_i^4\right) = -\frac{1}{2}\epsilon_{ijk}(J^\pm)^{jk}$$

$$J := (S^+)^3 + (S^-)^3 = J^{12}$$

$$[J_{\mu\nu}, Q_{i\alpha}] = \frac{i}{2}(\sigma_{\mu\nu})_\alpha{}^\beta Q_{i\beta}, [J_{\mu\nu}, \bar{Q}_{i\dot{\alpha}}] = \frac{i}{2}(\bar{\sigma}_{\mu\nu})^{\dot{\beta}\dot{\alpha}} \bar{Q}_{i\dot{\beta}}$$

$$[J, F_{ij1\mathrm{i}}] = +F_{ij1\mathrm{i}}, [J, F_{ij2\dot{2}}] = -F_{ij2\dot{2}}$$
$$[J, F_{ij1\dot{2}}] = [J, F_{ij2\mathrm{i}}] = 0$$

$$[J, W^{\mathrm{s}}_{ij11}] = +W^{\mathrm{s}}_{ij11}, [J, W^{\mathrm{s}}_{ij22}] = -W^{\mathrm{s}}_{ij22}$$
$$[J, W^{\mathrm{s}}_{ij12}] = [J, W^{\mathrm{s}}_{ij21}] = 0$$

$$\nabla_{I_1} \cdots \nabla_{I_r} F,$$

$$F_{ij\alpha\dot{\beta}} = W^{\mathrm{s}} = 0$$

$$\{\nabla_{i\alpha}, \nabla_{j\beta}\} = i\Omega_{ij} C_{\alpha\beta} \nabla_z - i\Omega_{ij} C_{\alpha\beta} W - iC_{\alpha\beta} W_{ij},$$
$$\{\nabla_{i\alpha}, \bar{\nabla}_{j\dot{\beta}}\} = i\Omega_{ij} (\sigma^\mu)_{\alpha\dot{\beta}} \nabla_\mu,$$

$$(W_{ij})^* \equiv \bar{W}^{ij} = W^{ij}, W^* = W$$

$$W^{ij} := \Omega^{ik} \Omega^{jl} W_{kl}$$

$$\{\nabla_{i\alpha}, \nabla_{j\beta}\} = i\Omega_{ij} C_{\alpha\beta} \nabla_z - iC_{\alpha\beta} W_{ij}, \{\bar{\nabla}_{i\dot{\alpha}}, \bar{\nabla}_{j\dot{\beta}}\} = i\Omega_{ij} C_{\dot{\alpha}\dot{\beta}} \nabla_z + iC_{\dot{\alpha}\dot{\beta}} W_{ij}$$
$$\{\nabla_{i\alpha}, \bar{\nabla}_{j\dot{\beta}}\} = i\Omega_{ij} (\sigma^\mu)_{\alpha\dot{\beta}} \nabla_\mu,$$
$$[\nabla_{i\alpha}, \nabla_\mu] = -iF_{i\alpha\mu}, [\bar{\nabla}_{i\dot{\alpha}}, \nabla_\mu] = +i\bar{F}_{i\dot{\alpha}\mu}$$
$$[\nabla_{i\alpha}, \nabla_z] = -iG_{i\alpha}, [\bar{\nabla}_{i\dot{\alpha}}, \nabla_z] = +i\bar{G}_{i\dot{\alpha}}$$
$$[\nabla_\mu, \nabla_z] = -ig_\mu [\nabla_\mu, \nabla_\nu] = -iF_{\mu\nu}$$

$$\Omega^{ij} W_{ij} = 0, (W_{ij})^* = W^{ij}$$

$$[\nabla_A, [\nabla_B, \nabla_C\}\} \pm [\nabla_B, [\nabla_C, \nabla_A\}\} \pm [\nabla_C, [\nabla_A, \nabla_B\}\} = 0$$

$$[\nabla_{j\alpha}, W_i^j] = 5iG_{i\alpha}, [\bar{\nabla}_{j\dot{\alpha}}, W_i^j] = 5i\bar{G}_{i\dot{\alpha}}$$
$$G_{i\alpha} = -\frac{1}{4}(\sigma^\mu C)_\alpha{}^{\dot{\beta}} \bar{F}_{i\dot{\beta}\mu}, \bar{G}_{i\dot{\alpha}} = -\frac{1}{4}(C\sigma^\mu)^\beta_{\dot{\alpha}} F_{i\beta\mu}$$
$$[\nabla_{i\alpha}, W_{jk}] = 2i\Omega_{i[j} G_{k]\alpha} + i\Omega_{jk} G_{i\alpha}, [\bar{\nabla}_{i\dot{\alpha}}, W_{jk}] = 2i\Omega_{i[j} \bar{G}_{k]\dot{\alpha}} + i\Omega_{jk} \bar{G}_{i\dot{\alpha}}$$
$$F_{i\alpha\mu} = (\bar{\sigma}_\mu C)^{\dot{\gamma}}{}_\alpha \bar{G}_{i\dot{\gamma}}, \bar{F}_{i\dot{\alpha}} = (C\bar{\sigma}_\mu)_{\dot{\alpha}}{}^\beta G_{i\beta}$$

$$\{\nabla_{i\alpha}, G_{j\beta}\} = -\frac{i}{4}\Omega_{ij} (\sigma^{\mu\nu})_{\alpha\beta} F_{\mu\nu} - \frac{1}{2} C_{\alpha\beta} [\nabla_z, W_{ij}] - \frac{1}{4} C_{\alpha\beta} [W_{ik}, W_j^k],$$
$$\{\bar{\nabla}_{i\dot{\alpha}}, \bar{G}_{j\dot{\beta}}\} = -\frac{i}{4}\Omega_{ij} (\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}} F_{\mu\nu} - \frac{1}{2} C_{\dot{\alpha}\dot{\beta}} [\nabla_z, W_{ij}] + \frac{1}{4} C_{\dot{\alpha}\dot{\beta}} [W_{ik}, W_j^k],$$
$$\{\bar{\nabla}_{i\dot{\alpha}}, G_{j\beta}\} = -\frac{1}{2}(\sigma^\mu)_{\beta\dot{\alpha}} (i\Omega_{ij} g_\mu - [\nabla_\mu, W_{ij}]),$$
$$\{\nabla_{i\alpha}, \bar{G}_{j\dot{\beta}}\} = -\frac{1}{2}(\sigma^\mu)_{\alpha\dot{\beta}} (i\Omega_{ij} g_\mu + [\nabla_\mu, W_{ij}]).$$

$$[\nabla_{i\alpha}, g_\mu] = -(\sigma^{\mu\nu})_\alpha{}^\beta[\nabla^\nu, G_{i\beta}] + (\bar{\sigma}_\mu C)^{\dot{\beta}}{}_\alpha[\bar{G}^j{}_{\dot{\beta}}, W_{ij}],$$
$$[\bar{\nabla}_{i\dot{\alpha}}, g_\mu] = -(\bar{\sigma}^{\mu\nu})^{\dot{\beta}}_{\dot{\alpha}}[\nabla^\nu, \bar{G}_{i\dot{\beta}}] + (C\bar{\sigma}_\mu)_{\dot{\alpha}}{}^\beta[G^j{}_\beta, W_{ij}],$$
$$[\nabla_z, G_{i\alpha}] = -(\sigma^\mu C)_\alpha{}^{\dot{\alpha}}[\nabla_\mu, \bar{G}_{i\dot{\alpha}}] + [G^j{}_\alpha, W_{ij}],$$
$$[\nabla_z, \bar{G}_{i\dot{\alpha}}] = -(C\sigma^\mu)^\beta{}_{\dot{\alpha}}[\nabla_\mu, G_{i\beta}] - [\bar{G}^j{}_{\dot{\alpha}}, W_{ij}],$$
$$\begin{aligned}[\nabla_{i\alpha}, [\nabla_z, W_{jk}]] = &-i(\sigma^\mu C)_\alpha{}^{\dot{\alpha}}(\Omega_{jk}[\nabla_\mu, \bar{G}_{i\dot{\alpha}}] + 2\Omega_{i[j}[\nabla_\mu, \bar{G}_{k]\dot{\alpha}}])\\ &+i(\Omega_{jk}[G^l_\alpha, W_{il}] + 2\Omega_{i[j}[G^l_\alpha, W_{k]l}])\\ &-i[G_{i\alpha}, W_{jk}],\end{aligned}$$
$$\begin{aligned}[\bar{\nabla}_{i\dot{\alpha}}, [\nabla_z, W_{jk}]] = &-i(C\sigma^\mu)^\beta{}_{\dot{\alpha}}(\Omega_{jk}[\nabla_\mu, G_{i\beta}] + 2\Omega_{i[j}[\nabla_\mu, G_{k]\beta}])\\ &-i(\Omega_{jk}[\bar{G}^l_{\dot{\alpha}}, W_{il}] + 2\Omega_{i[j}[\bar{G}^l_{\dot{\alpha}}, W_{k]l}])\\ &+i[\bar{G}_{i\dot{\alpha}}, W_{jk}].\end{aligned}$$

$$\begin{aligned}&[\nabla_z, [\nabla_z, W_{jk}]] = [\nabla_\mu, [\nabla^\mu, W_{jk}]]\\ &+i(\Omega_{jk}\{G_{i\alpha}, G^{i\alpha}\} - 4\{G_{j\alpha}, G_k{}^\alpha\})\\ &-i(\Omega_{jk}\{\bar{G}_{i\dot{\alpha}}, \bar{G}^{i\dot{\alpha}}\} - 4\{\bar{G}_{j\dot{\alpha}}, \bar{G}_k{}^{\dot{\alpha}}\})\\ &+\frac{1}{4}[W_{[j|l}, [W_{k]m}, W^{ml}]]\\ &[\nabla_z, g_\mu] = [\nabla^\nu, F_{\mu\nu}] - 2(\bar{\sigma}^{\dot{\alpha}\alpha})\{G_{i\alpha}, \bar{G}^i_{\dot{\alpha}}\} + \frac{i}{4}[W^{ij}, [\nabla_\mu, W_{ij}]]\\ &[\nabla^\mu, g_\mu] = -\{G_{i\alpha}, G^{i\alpha}\} - \{\bar{G}_{i\dot{\alpha}}, \bar{G}^{i\dot{\alpha}}\} + \frac{i}{4}[W^{ij}, [\nabla_z, W_{ij}]]\end{aligned}$$

$$\phi_{ij} := W_{ij}\,|,$$

$$\lambda_{i\alpha} := 2iG_{i\alpha}\Big| = \frac{2}{5}[\nabla_{j\alpha}, W_i^j]\Big| = -2[\nabla_{i\alpha}, \nabla_z]\Big|\,,$$
$$\bar{\lambda}_{i\dot{\alpha}} := -2i\bar{G}_{i\dot{\alpha}}\Big| = -\frac{2}{5}[\bar{\nabla}_{j\dot{\alpha}}, W_i^j]\Big| = -2[\bar{\nabla}_{i\dot{\alpha}}, \nabla_z]\Big|\,,$$
$$\begin{aligned}H_{ij} &:= -i[\nabla_z, W_{ij}]\,|\\ &= \frac{1}{10}\{\nabla_{[i\alpha}, [\nabla^\alpha_k, W^k_{j]}]\}\Big| = \frac{i}{2}\{\nabla_{[i\alpha}, G^\alpha_{j]}\}\Big|\\ &= \frac{1}{10}\{\bar{\nabla}_{[i\dot{\alpha}}, [\bar{\nabla}^{\dot{\alpha}}_k, W^k_{j]}]\}\Big| = \frac{i}{2}\{\bar{\nabla}_{[i\dot{\alpha}}, \bar{G}^{\dot{\alpha}}_{j]}\}\Big|,\end{aligned}$$
$$\begin{aligned}V_\mu &:= ig_\mu| = -[\nabla_\mu, \nabla_z]|\\ &= \frac{i}{20}(\bar{\sigma}_\mu)^{\dot{\beta}\beta}\{\nabla_{i\beta}, [\bar{\nabla}_{j\dot{\beta}}, W^{ji}]\}\Big|\\ &= \frac{i}{20}(\bar{\sigma}_\mu)^{\dot{\beta}\beta}\{\bar{\nabla}_{i\dot{\beta}}, [\nabla_{j\beta}, W^{ji}]\}\Big|\,,\end{aligned}$$
$$A_\mu := i\nabla_\mu|,$$
$$\begin{aligned}F_{\mu\nu} &:= i[\nabla_\mu, \nabla_\nu]\,|\\ &= \frac{i}{8}\Big((\sigma_{\mu\nu})^{\alpha\beta}\{\nabla_{i\alpha}, G^i{}_\beta\} + (\bar{\sigma}_{\mu\nu})^{\dot{\alpha}\dot{\beta}}\{\bar{\nabla}_{i\dot{\alpha}}, \bar{G}^i{}_\beta\}\Big)\Big|\\ &= \frac{1}{40}\Big((\sigma_{\mu\nu})^{\alpha\beta}\{\nabla_{i\alpha}, [\nabla_{j\beta}, W^{ji}]\} + (\bar{\sigma}_{\mu\nu})^{\dot{\alpha}\dot{\beta}}\{\bar{\nabla}_{i\dot{\alpha}}, [\bar{\nabla}_{j\dot{\beta}}, W^{ji}]\}\Big)\Big|\,\Big)\end{aligned}$$

$$\delta b = \delta(B\,|) := (\delta B)| = [\xi^{i\alpha}Q_{i\alpha} + \bar{\xi}^{i\alpha}\bar{Q}_{i\dot{\alpha}}, B]| = |\xi^{i\alpha}D_{i\alpha} + \bar{\xi}^{i\alpha}\bar{D}_{i\dot{\alpha}}, B|$$

$$\delta b = \xi^{i\alpha}[D_{i\alpha}, B]_\pm + \bar{\xi}^{i\dot{\alpha}}[\bar{D}_{i\dot{\alpha}}, B]_\pm| = \xi^{i\alpha}[\nabla_{i\alpha}, B]_\pm + \bar{\xi}^{i\dot{\alpha}}[\bar{\nabla}_{i\dot{\alpha}}, B]_\pm|$$

$$\delta_z b = \delta_z(B\,|) := (\delta_z B)| = [i\omega D_z, B]| = |i\omega[\nabla_z, B]|$$

$$
\begin{aligned}
\delta\phi_{ij} =& \delta W_{ij} \mid \\
=& \xi^{k\alpha}\left[\nabla_{k\alpha}, W_{ij}\right] + \bar{\xi}^{k\dot{\alpha}}\left[\bar{\nabla}_{k\dot{\alpha}}, W_{ij}\right] \mid \\
=& \left(\xi_{[i}{}^{\alpha}\lambda_{j]\alpha} + \frac{1}{2}\Omega_{ij}\xi^{k\alpha}\lambda_{k\alpha}\right) - \left(\bar{\xi}_{[i}{}^{\alpha}\bar{\lambda}_{j]\dot{\alpha}} + \frac{1}{2}\Omega_{ij}\bar{\xi}^{k\dot{\alpha}}\bar{\lambda}_{k\dot{\alpha}}\right) \\
\delta\lambda_{i\alpha} =& 2i\left(\xi^{k\beta}\{\nabla_{k\beta}, G_{i\alpha}\} + \bar{\xi}^{k\dot{\beta}}\{\bar{\nabla}_{k\dot{\beta}}, G_{i\alpha}\}\right) \mid \\
=& \frac{1}{2}\xi_i{}^{\beta}(\sigma^{\mu\nu})_{\beta\alpha}F_{\mu\nu} \\
&+ i(\sigma^{\mu})_{\alpha\dot{\beta}}\bar{\xi}^{k\dot{\beta}}\left[\nabla_\mu, \phi_{ki}\right] - i(\sigma^{\mu})_{\alpha\dot{\beta}}\bar{\xi}_i{}^{\dot{\beta}}V_\mu + i\xi^k{}_{\alpha}H_{ki} - \frac{i}{2}\xi^k{}_{\alpha}[\phi_{kl}, \phi^l{}_i] \\
\delta\bar{\lambda}_{i\dot{\alpha}} =& -2i\left(\xi^{k\beta}\{\nabla_{k\beta}, \bar{G}_{i\dot{\alpha}}\} + \bar{\xi}^{k\dot{\beta}}\{\bar{\nabla}_{k\dot{\beta}}, \bar{G}_{i\dot{\alpha}}\}\right) \mid \\
=& -\frac{1}{2}\bar{\xi}_i{}^{\dot{\beta}}(\bar{\sigma}^{\mu\nu})_{\beta\alpha}F_{\mu\nu} \\
&+ i(\sigma^{\mu})_{\beta\dot{\alpha}}\xi^{k\beta}\left[\nabla_\mu, \phi_{ki}\right] + i(\sigma^{\mu})_{\beta\dot{\alpha}}\xi_i{}^{\beta}V_\mu + i\bar{\xi}^k{}_{\dot{\alpha}}H_{ki} - \frac{i}{2}\bar{\xi}^k{}_{\dot{\alpha}}[\phi_{kl}, \phi^l{}_i] \\
\delta H_{ij} =& -i\left(\xi^{k\beta}\left[\nabla_{k\beta}, [\nabla_z, W_{ij}]\right] + \bar{\xi}^{k\dot{\beta}}\left[\bar{\nabla}_{k\dot{\beta}}, [\nabla_z, W_{ij}]\right]\right) \mid \\
=& -i\left(\xi_{[i}{}^{\beta}(\sigma^{\mu})_{\beta\dot{\beta}}[\nabla_\mu, \bar{\lambda}_{j]}{}^{\dot{\beta}}] + \frac{1}{2}\Omega_{ij}\xi^{k\beta}(\sigma_\mu)_{\beta\dot{\beta}}[\nabla_\mu, \bar{\lambda}_k{}^{\dot{\beta}}]\right) \\
&- i\left(\bar{\xi}_{[i\dot{\beta}}(\bar{\sigma}^{\mu})^{\dot{\beta}\beta}[\nabla_\mu, \lambda_{j]\beta}] + \frac{1}{2}\Omega_{ij}\bar{\xi}^k{}_{\dot{\beta}}(\bar{\sigma}_\mu)^{\dot{\beta}\beta}[\nabla_\mu, \lambda_{k\beta}]\right) \\
&- i\left(\xi_{[i}{}^{\beta}[\lambda^l{}_{\beta}, \phi_{j]l}] + \frac{1}{2}\Omega_{ij}\xi^{k\beta}[\lambda^l{}_{\beta}, \phi_{kl}]\right) \\
&- i\left(\bar{\xi}_{[i}{}^{\dot{\beta}}[\bar{\lambda}^l{}_{\dot{\beta}}, \phi_{j]l}] + \frac{1}{2}\Omega_{ij}\bar{\xi}^{k\dot{\beta}}[\bar{\lambda}^l{}_{\dot{\beta}}, \phi_{kl}]\right) \\
&+ \frac{i}{2}\xi^{k\beta}[\lambda_{k\beta}, \phi_{ij}] + \frac{i}{2}\bar{\xi}^{k\dot{\beta}}[\bar{\lambda}_{k\dot{\beta}}, \phi_{ij}] \\
\delta V_\mu =& i\xi^{i\alpha}[\nabla_{i\alpha}, g_\mu] + i\bar{\xi}^{i\dot{\alpha}}[\bar{\nabla}_{i\dot{\alpha}}, g_\mu] \mid \\
=& -\frac{1}{2}\xi^{i\alpha}(\sigma_{\mu\nu})_{\alpha}{}^{\beta}[\nabla^\nu, \lambda_{i\beta}] + \frac{1}{2}\bar{\xi}^i{}_{\dot{\alpha}}(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}}[\nabla^\nu, \bar{\nabla}_i{}^{\dot{\beta}}] \\
&+ \frac{1}{2}\xi^{i\alpha}(\sigma_\mu)_{\alpha\dot{\alpha}}[\bar{\lambda}^{j\dot{\alpha}}, \phi_{ij}] + \frac{1}{2}\bar{\xi}^i{}_{\dot{\alpha}}(\bar{\sigma}_\mu)^{\dot{\alpha}\alpha}[\lambda^j{}_{\alpha}, \phi_{ij}] \\
\delta A_\mu =& i\left(\xi^{i\alpha}[\nabla_{i\alpha}, \nabla_\mu] + \bar{\xi}^{i\dot{\alpha}}[\bar{\nabla}_{i\dot{\alpha}}, \nabla_\mu]\right) \mid \\
=& -\frac{i}{2}\xi^{i\alpha}(\sigma_\mu)_{\alpha\dot{\alpha}}\bar{\lambda}_i^{\dot{\alpha}} + \frac{i}{2}\bar{\xi}^i{}_{\dot{\alpha}}(\bar{\sigma}_\mu)^{\dot{\alpha}\alpha}\lambda_{i\alpha}
\end{aligned}
$$

$$
\begin{aligned}
\delta_z\phi_{ij} =& i\omega[\nabla_z, W_{ij}] \mid \\
=& -\omega H_{ij}
\end{aligned}
$$

$$
\begin{aligned}
\delta_z\lambda_{i\alpha} =& -2\omega[\nabla_z, G_{i\alpha}] \mid \\
=& i\omega\left((\sigma^{\mu})_{\alpha\dot{\alpha}}[\nabla_\mu, \bar{\lambda}_i^{\dot{\alpha}}] + [\lambda_\alpha^j, \phi_{ij}]\right) \\
\delta_z\bar{\lambda}_{i\dot{\alpha}} =& -2\omega[\nabla_z, G_{i\alpha}] \mid \\
=& -i\omega\left((C\bar{\sigma}^{\mu})_{\dot{\alpha}}^{\alpha}[\nabla_\mu, \lambda_{i\alpha}] + [\bar{\lambda}_{\dot{\alpha}}^j, \phi_{ij}]\right) \\
\delta_z H_{ij} =& \omega\left[\nabla_z, [\nabla_z, W_{ij}]\right] \mid \\
=& \omega\left(\left[\nabla_\mu, [\nabla^\mu, \phi_{ij}]\right] - \frac{1}{4}\left[\phi_{[i|k}, [\phi_{j]l}, \phi^{kl}]\right]\right. \\
&- i\left(\frac{1}{4}\Omega_{ij}\{\lambda_{k\gamma}, \lambda^{k\gamma}\} - \left\{\lambda_{i\gamma}, \lambda_j^\gamma\right\}\right) \\
&\left.+ i\left(\frac{1}{4}\Omega_{ij}\{\bar{\lambda}_{k\dot{\gamma}}, \bar{\lambda}^{k\dot{\gamma}}\} - \left\{\bar{\lambda}_{i\dot{\gamma}}, \bar{\lambda}_j^{\dot{\gamma}}\right\}\right)\right) \\
\delta_z V_\mu =& -\omega[\nabla_z, g_\mu] \mid \\
=& \omega\left([\nabla_\nu, F_\mu^\nu] - \frac{1}{2}(\bar{\sigma})^{\dot{\alpha}\alpha}\{\bar{\lambda}_{i\dot{\alpha}}, \lambda_\alpha^i\} + \frac{1}{4}\left[\phi^{ij}, [\nabla_\mu, \phi_{ij}]\right]\right) \\
\delta_z A_\mu =& -\omega[\nabla_z, \nabla_\mu] \mid \\
=& -\omega V_\mu.
\end{aligned}
$$

$$(\delta_1\delta_2-\delta_2\delta_1)b=-i\left((\xi_1)_{i\alpha}(\xi_2)^{i\alpha}+(\bar{\xi}_1)_{i\dot{\alpha}}(\bar{\xi}_2)^{i\dot{\alpha}}\right)[\partial_z,b]$$
$$-i\left((\xi_1)_i^{\alpha}(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\xi}_2)^{i\dot{\alpha}}-(\xi_2)_i^{\alpha}(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\xi}_1)^{i\dot{\alpha}}\right)[\partial_{\mu},b].$$

$$\delta b=\xi^{i\alpha}[\nabla_{i\alpha},B]_{\pm}+\bar{\xi}^{i\dot{\alpha}}[\bar{\nabla}_{i\dot{\alpha}},B]_{\pm}\mid$$
$$\delta_1\delta_2 b=-(\xi_1)^{j\beta}(\xi_2)^{i\alpha}\left[\nabla_{j\beta},[\nabla_{i\alpha},B]_{\pm}\right]_{\mp}-(\bar{\xi}_1)^{j\dot{\beta}}(\xi_2)^{i\alpha}\left[\bar{\nabla}_{j\dot{\beta}},[\nabla_{i\alpha},B]_{\pm}\right]_{\mp}$$
$$-(\xi_1)^{j\beta}(\bar{\xi}_2)^{i\dot{\alpha}}\left[\nabla_{j\beta},[\bar{\nabla}_{i\dot{\alpha}},B]_{\pm}\right]_{\mp}-(\bar{\xi}_1)^{j\dot{\beta}}(\bar{\xi}_2)^{i\dot{\alpha}}\left[\bar{\nabla}_{j\dot{\beta}},[\bar{\nabla}_{i\dot{\alpha}},B]_{\pm}\right]_{\mp}\mid,$$

$$(\delta_1\delta_2-\delta_2\delta_1)b=-i\left((\xi_1)_i^{\alpha}(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\xi}_2)^{i\dot{\alpha}}-(\xi_2)_i^{\alpha}(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\xi}_1)^{i\dot{\alpha}}\right)[\nabla_{\mu},B]$$
$$-i\left((\xi_1)_{i\alpha}(\xi_2)^{i\alpha}+(\xi_1)_{i\dot{\alpha}}(\xi_2)^{i\dot{\alpha}}\right)[\nabla_z,B]-i\left((\xi_1)_{\alpha}^{i}(\xi_2)^{j\alpha}-(\xi_1)_{\dot{\alpha}}^{i}(\xi_2)^{j\dot{\alpha}}\right)[W_{ij},B]\mid$$

$$(\delta_1\delta_2-\delta_2\delta_1)b=-i\left((\xi_1)_i^{\alpha}(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\xi}_2)^{i\dot{\alpha}}-(\xi_2)_i^{\alpha}(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\xi}_1)^{i\dot{\alpha}}\right)[\mathcal{D}_{\mu},b]$$

$$-i\left((\xi_1)_{i\alpha}(\xi_2)^{i\alpha}+(\xi_1)_{i\dot{\alpha}}(\xi_2)^{i\dot{\alpha}}\right)[\partial_z,b]+i\left[\left((\xi_1)_{\alpha}^{i}(\xi_2)^{j\alpha}-(\xi_1)_{\dot{\alpha}}^{i}(\xi_2)^{j\dot{\alpha}}\right)\phi_{ij},b\right]$$

$$(\delta_1\delta_2-\delta_2\delta_1)A_{\mu}=-\left[\mathcal{D}_{\mu},\left((\xi_1)_{\alpha}^{i}(\xi_2)^{j\alpha}-(\xi_1)_{\dot{\alpha}}^{i}(\xi_2)^{j\dot{\alpha}}\right)\phi_{ij}\right]$$
$$-i\left((\xi_1)_{i\alpha}(\xi_2)^{i\alpha}+(\xi_1)_{i\dot{\alpha}}(\xi_2)^{i\dot{\alpha}}\right)[\partial_z,A_{\mu}]$$
$$-i\left((\xi_1)_i^{\alpha}(\sigma^{\nu})_{\alpha\dot{\alpha}}(\bar{\xi}_2)^{i\dot{\alpha}}-(\xi_2)_i^{\alpha}(\sigma^{\nu})_{\alpha\dot{\alpha}}(\bar{\xi}_1)^{i\dot{\alpha}}\right)F_{\nu\mu}$$

$$[\nabla^{\mu},g_{\mu}]=-\{G_{i\alpha},G^{i\alpha}\}-\{\bar{G}_{i\dot{\alpha}},\bar{G}^{i\dot{\alpha}}\}+\frac{i}{4}\left[W^{ij},[\nabla_z,W_{ij}]\right]$$

$$[\nabla^{\mu},g_{\mu}]=0$$

$$g_{\mu}=[\nabla^{\nu},(*A)_{\mu\nu}],(*A)_{\mu\nu}+(*A)_{\nu\mu}=0$$

$$S\sim\mathrm{tr}\int d^4x(\{\nabla,[\nabla,GG]\}+\{\bar{\nabla},[\bar{\nabla},\overline{GG}]\})\mid$$

$$A^{(i_1\cdots i_p)}:=\sum_{\sigma\in\mathfrak{S}_p}A^{i_{\sigma(1)}\cdots i_{\sigma(p)}},A^{[i_1\cdots i_p]}:=\sum_{\sigma\in\mathfrak{S}_p}\mathrm{sgn}(\sigma)A^{i_{\sigma(1)}\cdots i_{\sigma(p)}}.$$

$$\tau^i\tau^j=\delta^{ij}\mathbf{1}+i\sum_{k=1}^{3}\epsilon^{ijk}\tau^k,\tau^2\tau^i\tau^2=-(\tau^i)^*$$

$$\sigma^{\mu}:=(\mathbf{1},\tau^i),\bar{\sigma}^{\mu}:=(\mathbf{1},-\tau^i)$$

$$C_{\downarrow}:=\tau^2,C^{\uparrow}:=(C_{\downarrow}^T)^{-1}\equiv-\tau^2,C_{\downarrow}^*:=(C_{\downarrow})^*\equiv-\tau^2,C^{\uparrow *}:=(C^{\uparrow})^*\equiv((C_{\downarrow}^*)^T)^{-1}\equiv\tau^2.$$

$$C^{\uparrow}=-(C_{\downarrow})^{-1},C^{\uparrow *}=-(C_{\downarrow}^*)^{-1}.$$

$$\sigma^{\mu}\bar{\sigma}^{\nu}+\sigma^{\nu}\bar{\sigma}^{\mu}=2\eta^{\mu\nu},\bar{\sigma}^{\mu}\sigma^{\nu}+\bar{\sigma}^{\nu}\sigma^{\mu}=2\eta^{\mu\nu}$$

$$(\bar{\sigma}^{\mu})^T=C^{\uparrow}\sigma^{\mu}(C^{\uparrow *})^T,\text{ or }(\sigma^{\mu})^T=C_{\downarrow}^*\bar{\sigma}^{\mu}(C_{\downarrow})^T$$

$$(\sigma^{\mu})^{\dagger}=\sigma^{\mu},(\bar{\sigma}^{\mu})^{\dagger}=\bar{\sigma}^{\mu}$$

$$(\sigma^{\mu})_{\alpha\dot{\beta}},(\bar{\sigma}^{\mu})^{\dot{\alpha}\beta}$$

$$(C_{\downarrow})_{\alpha\beta}=:C_{\alpha\beta},(C^{\uparrow})^{\alpha\beta}=:C^{\alpha\beta},(C_{\downarrow}^*)_{\dot{\alpha}\dot{\beta}}:=C_{\dot{\alpha}\dot{\beta}},(C^{\uparrow *})^{\dot{\alpha}\dot{\beta}}:=C^{\dot{\alpha}\dot{\beta}}$$

$$C_{\alpha\beta}C^{\gamma\beta}=\delta_{\alpha}^{\gamma},C_{\dot{\alpha}\dot{\beta}}C^{\dot{\gamma}\dot{\beta}}=\delta_{\dot{\alpha}}^{\dot{\beta}}$$
$$(\sigma^{\mu})_{\alpha\dot{\gamma}}(\bar{\sigma}^{\nu})^{\dot{\gamma}\beta}+(\sigma^{\nu})_{\alpha\dot{\gamma}}(\bar{\sigma}^{\mu})^{\dot{\gamma}\beta}=2\eta^{\mu\nu}\delta_{\alpha}^{\beta},(\bar{\sigma}^{\mu})^{\dot{\alpha}\gamma}(\sigma^{\nu})_{\gamma\dot{\beta}}+(\bar{\sigma}^{\nu})^{\dot{\alpha}\gamma}(\sigma^{\mu})_{\gamma\dot{\beta}}=2\eta^{\mu\nu}\delta_{\dot{\beta}}^{\dot{\alpha}};$$
$$(\bar{\sigma}^{\mu})^{\dot{\alpha}\beta}=C^{\beta\delta}C^{\dot{\alpha}\dot{\gamma}}(\sigma^{\mu})_{\delta\dot{\gamma}},(\sigma^{\mu})_{\alpha\dot{\beta}}=C_{\dot{\beta}\dot{\delta}}C_{\alpha\gamma}(\bar{\sigma}^{\mu})^{\dot{\delta}\gamma}$$

$$\mathrm{tr}\sigma^\mu\bar{\sigma}^\nu = 2\eta^{\mu\nu}$$

$$(\sigma^\mu)_{\alpha\dot{\gamma}}(\bar{\sigma}_\mu)^{\dot{\delta}\beta} = 2\delta_\alpha{}^\beta\delta^{\dot{\delta}}{}_{\dot{\gamma}}$$

$$(\sigma^\mu)_{\alpha\dot{\gamma}}(\sigma_\mu)_{\beta\dot{\delta}} = 2C_{\alpha\beta}C_{\dot{\gamma}\dot{\delta}}, (\bar{\sigma}^\mu)^{\dot{\alpha}\gamma}(\bar{\sigma}_\mu)^{\dot{\beta}\delta} = 2C^{\dot{\alpha}\dot{\beta}}C^{\gamma\delta}$$

$$\sigma^\mu\bar{\sigma}^\rho\sigma^\nu + \sigma^\nu\bar{\sigma}^\rho\sigma^\mu = 2(\eta^{\mu\rho}\sigma^\nu + \eta^{\nu\rho}\sigma^\mu - \eta^{\mu\nu}\sigma^\rho)$$
$$\bar{\sigma}^\mu\sigma^\rho\bar{\sigma}^\nu + \bar{\sigma}^\nu\sigma^\rho\bar{\sigma}^\mu = 2(\eta^{\mu\rho}\bar{\sigma}^\nu + \eta^{\nu\rho}\bar{\sigma}^\mu - \eta^{\mu\nu}\bar{\sigma}^\rho)$$

$$\mathrm{tr}(\sigma^\mu\bar{\sigma}^\rho\sigma^\nu\bar{\sigma}^\sigma - \sigma^\nu\bar{\sigma}^\rho\sigma^\mu\bar{\sigma}^\sigma), \mathrm{tr}(\bar{\sigma}^\mu\sigma^\rho\bar{\sigma}^\nu\sigma^\sigma - \bar{\sigma}^\nu\sigma^\rho\bar{\sigma}^\mu\sigma^\sigma)$$

$$\sigma^\mu\bar{\sigma}^\rho\sigma^\nu - \sigma^\nu\bar{\sigma}^\rho\sigma^\mu = 2i\varepsilon^{\mu\nu\rho\sigma}\sigma_\sigma$$
$$\bar{\sigma}^\mu\sigma^\rho\bar{\sigma}^\nu - \bar{\sigma}^\nu\sigma^\rho\bar{\sigma}^\mu = -2i\varepsilon^{\mu\nu\rho\sigma}\bar{\sigma}_\sigma$$

$$\sigma^{\mu\nu} := \frac{1}{2}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu), \bar{\sigma}^{\mu\nu} := \frac{1}{2}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu).$$

$$(\sigma^{\mu\nu})_\alpha^\beta, (\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}},$$

$$\mathrm{tr}\sigma^{\mu\nu} = (\sigma^{\mu\nu})_\alpha^\alpha = 0, \mathrm{tr}\bar{\sigma}^{\mu\nu} = (\bar{\sigma}^{\mu\nu})_{\dot{\alpha}}^{\dot{\alpha}} = 0.$$

$$(\sigma^{\mu\nu})_{\alpha\beta} := (\sigma^{\mu\nu})_\alpha^\gamma C_{\gamma\beta}, (\sigma^{\mu\nu})^{\alpha\beta} := C^{\alpha\gamma}(\sigma^{\mu\nu})_\gamma^\beta$$
$$(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}} := C^{\dot{\beta}\dot{\gamma}}(\bar{\sigma}^{\mu\nu})_{\dot{\gamma}}^{\dot{\alpha}}, (\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}} := (\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\gamma}}C_{\dot{\gamma}\dot{\alpha}},$$

$$\sigma^{\mu\nu}C_\downarrow = \frac{1}{2}\left(\sigma^\mu C^{\uparrow *}(\sigma^\nu)^T + \left(\sigma^\mu C^{\uparrow *}(\sigma^\nu)^T\right)^T\right),$$

$$(\sigma^{\mu\nu})^\dagger = -\bar{\sigma}^{\mu\nu}, (\bar{\sigma}^{\mu\nu})^\dagger = -\sigma^{\mu\nu}$$

$$\tilde{\sigma}^{\mu\nu} := \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma} = i\sigma^{\mu\nu}, \tilde{\bar{\sigma}}^{\mu\nu} := \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\sigma}_{\rho\sigma} = -i\bar{\sigma}^{\mu\nu}$$

$$\mathrm{tr}\sigma^{\mu\nu}\sigma^{\rho\sigma} = -4\wp^{+\mu\nu\rho\sigma}, \mathrm{tr}\bar{\sigma}^{\mu\nu}\bar{\sigma}^{\rho\sigma} = -4\wp^{-\mu\nu\rho\sigma}$$

$$\wp^{+\mu\nu\rho\sigma} := \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho} - i\varepsilon^{\mu\nu\rho\sigma}), \wp^{-\mu\nu\rho\sigma} := \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho} + i\varepsilon^{\mu\nu\rho\sigma})$$

$$\frac{1}{2}\wp^{\pm\mu\nu\rho\sigma}\wp^{\pm}{}_{\rho\sigma}{}^{\tau\lambda} = \wp^{\pm\mu\nu\tau\lambda}, \frac{1}{2}\wp^{\pm\mu\nu\rho\sigma}\wp^{\mp}{}_{\rho\sigma}{}^{\tau\lambda} = 0$$
$$\wp^{\pm\rho\sigma\mu\nu} = \wp^{\pm\mu\nu\rho\sigma}, \wp^{\pm\nu\mu\rho\sigma} = \wp^{\pm\mu\nu\sigma\rho} = -\wp^{\pm\mu\nu\rho\sigma}$$

$$\left(C_{\alpha\beta}, (\sigma^{\mu\nu})_{\alpha\beta}\right), \left(C_{\dot{\alpha}\dot{\beta}}, (\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}}\right),$$

$$\mathrm{tr}C_\downarrow C^\uparrow = -2, \mathrm{tr}\sigma^{\mu\nu}C_\downarrow = 0, \text{ etc.}$$

$$C_{\alpha\beta}C^{\gamma\delta} + \frac{1}{2}(\sigma^{\mu\nu})_{\alpha\beta}(\sigma_{\mu\nu})^{\gamma\delta} = 2\delta_\alpha^\gamma\delta_\beta^\delta$$
$$C_{\dot{\alpha}\dot{\beta}}C^{\dot{\gamma}\dot{\delta}} + \frac{1}{2}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}}(\bar{\sigma}_{\mu\nu})^{\dot{\gamma}\dot{\delta}} = 2\delta_{\dot{\alpha}}^{\dot{\gamma}}\delta_{\dot{\beta}}^{\dot{\delta}}$$

$$C_{\alpha\beta}C_{\gamma\delta} = C_{\alpha\gamma}C_{\beta\delta} - C_{\alpha\delta}C_{\beta\gamma}, \frac{1}{2}(\sigma^{\mu\nu})_{\alpha\beta}(\sigma_{\mu\nu})_{\gamma\delta} = C_{\alpha\gamma}C_{\beta\delta} + C_{\alpha\delta}C_{\beta\gamma},$$

$$\psi'_\alpha = \exp\left(+\frac{1}{4}\omega_{\mu\nu}\sigma^{\mu\nu}\right)_\alpha{}^\beta\psi_\beta, \bar{\psi}'_{\dot{\alpha}} = \exp\left(-\frac{1}{4}\omega_{\mu\nu}\bar{\sigma}^{\mu\nu}\right)^{\dot{\beta}}{}_{\dot{\alpha}}\bar{\psi}_{\dot{\beta}}$$

$$\psi'^\alpha = \exp\left(-\frac{1}{4}\omega_{\mu\nu}\sigma^{\mu\nu}\right)_\beta^\alpha\psi^\beta, \bar{\psi}'^{\dot{\alpha}} = \exp\left(+\frac{1}{4}\omega_{\mu\nu}\bar{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}{}_{\dot{\beta}}\bar{\psi}^{\dot{\beta}}$$

$$\psi^\alpha = C^{\alpha\beta}\psi_\beta, \psi_\alpha = \psi^\beta C_{\beta\alpha}$$

$$\psi_\alpha \chi^\alpha = -\psi^\alpha \chi_\alpha = +\chi_\alpha \psi^\alpha$$

$$\exp\left(-\frac{1}{4}\omega_{\mu\nu}\bar{\sigma}^{\mu\nu}\right)^{\dot{\beta}}_{\ \dot{\alpha}} = \left(\exp\left(+\frac{1}{4}\omega_{\mu\nu}\sigma^{\mu\nu}\right)_{\alpha}^{\ \beta}\right)^\dagger, \text{ i.e. } \bar{\psi}_{\dot{\alpha}} = (\psi_\alpha)^\dagger, \bar{\psi}^{\dot{\alpha}} = (\psi^\alpha)^\dagger$$

$$(C^*)_{\dot{\alpha}\dot{\beta}}, (\sigma^\mu)^\dagger{}_{\alpha\dot{\beta}} = (\sigma^\mu)_{\alpha\dot{\beta}}$$

$$(\psi_\alpha \chi^\alpha)^\dagger = \left(\psi_\alpha C^{\alpha\beta}\chi_\beta\right)^\dagger = \left(\chi_\beta\right)^\dagger \left(C^{\dot{\eta}\dot{\alpha}}\right)(\psi_\alpha)^\dagger = \bar{\chi}_{\dot{\beta}}\bar{\psi}^{\dot{\beta}}$$

$$\left\{\frac{\partial}{\partial\theta^\alpha}, \theta^\beta\right\} = \delta_\alpha{}^\beta, \left\{\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}, \bar{\theta}^{\dot{\beta}}\right\} = \delta^{\dot{\beta}}_{\dot{\alpha}}$$

$$\left(\frac{\partial}{\partial\theta^\alpha}\right)^\dagger = \frac{\partial}{\partial(\theta^\alpha)^\dagger} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$$

$$C^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha}, \text{ etc.}$$

$$\sigma^\mu := \left(\mathbf{1}, i\tau^i\right), \bar{\sigma}^\mu := \left(\mathbf{1}, -i\tau^i\right)$$

$$\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu = 2\eta^{\mu\nu}, \bar{\sigma}^\mu\sigma^\nu + \bar{\sigma}^\nu\sigma^\mu = 2\eta^{\mu\nu}$$

$$(\sigma^\mu)^\dagger = \bar{\sigma}^\mu, (\bar{\sigma}^\mu)^\dagger = \sigma^\mu$$

$$(\sigma^{\mu\nu})^\dagger = -(\sigma^{\mu\nu}), (\bar{\sigma}^{\mu\nu})^\dagger = -(\bar{\sigma}^{\mu\nu})$$

$$(\psi_\alpha)^\dagger = \psi^\alpha, (\bar{\psi}_{\dot{\alpha}})^\dagger = \bar{\psi}^{\dot{\alpha}}, \text{ etc.}$$

$$SU(N) = \left\{\left(M_i{}^j\right) \in GL(N, \mathbb{C}) \mid \left(M^\dagger\right)_i{}^k M_k{}^j = \delta_i{}^j, \det M = 1\right\}.$$

$$\mathfrak{su}(N) = \left\{\left(X_i{}^j\right) \in \mathfrak{gl}(N, \mathbb{C}) \mid \left(X^\dagger\right)_i{}^j = X_i{}^j, \operatorname{tr} X = 0\right\}.$$

$$\langle X, Y\rangle := \operatorname{tr}\left(X^\dagger Y\right), X, Y \in M(n, \mathbb{C}),$$

$$\left\langle X^a, X^b\right\rangle = \frac{1}{2}\delta^{ab}.$$

$$\sum_{a=0}^{N^2-1}(X^a)_i^j (X^a)_k^l = \frac{1}{2}\delta_i^l\delta_k^j,$$

$$\sum_{a=1}^{N^2-1}(X^a)_i^j (X^a)_k^l = \frac{1}{2N}\left(N\delta_i^l\delta_k^j - \delta_i^j\delta_k^l\right).$$

$$F^{abc} := 2\left\langle X^a, X^b X^c\right\rangle = \left\langle X^a, \{X^b, X^c\}\right\rangle + \left\langle X^a, [X^b, X^c]\right\rangle.$$

$$\left\langle X^a, \{X^b, X^c\}\right\rangle^* = \left\langle \{X^b, X^c\}, X^a\right\rangle = \left\langle X^a, \{X^b, X^c\}\right\rangle,$$
$$\left\langle X^a, [X^b, X^c]\right\rangle^* = \left\langle [X^b, X^c], X^a\right\rangle = -\left\langle X^a, [X^b, X^c]\right\rangle,$$

$$\left\langle X^a, \{X^b, X^c\}\right\rangle = d^{abc} := \Re F^{abc}, -i\left\langle X^a, [X^b, X^c]\right\rangle = f^{abc} := \Im F^{abc},$$

$$\{X^a, X^b\} = \frac{1}{\sqrt{2N}}\delta^{ab} + \sum_{c=1}^{N^2-1} d^{abc}X^c$$

$$[X^a, X^b] = i\sum_{c=1}^{N^2-1} f^{abc}X^c$$

$$(X_\mathrm{R})^T = X_\mathrm{R}, (X_\mathrm{I})^T + X_\mathrm{I} = 0, \text{ where } X_\mathrm{R} := \Re X, X_\mathrm{I} := \Im X.$$

$$(X^m)_{kl} := \frac{1}{\sqrt{2m(m+1)}}\left(\sum_{i=1}^{m} \delta^i{}_k\delta^i{}_l - m\delta^{m+1}{}_k\delta^{m+1}{}_l\right), (m = 1, \cdots, N-1),$$

$$\left(X_s^{ij}\right)_{kl} := \frac{1}{2}\left(\delta^i{}_k\delta^j{}_l + \delta^j{}_k\delta^i{}_l\right), \left(X_a^{ij}\right)_{kl} := \frac{1}{2}i\left(\delta^i{}_k\delta^j{}_l - \delta^j{}_k\delta^i{}_l\right), (1 \leqslant i < j \leqslant N),$$

$$v^m = \left(0, \cdots, 0, -\frac{m-1}{\sqrt{2(m-1)m}}, \frac{1}{\sqrt{2m(m+1)}}, \cdots, \frac{1}{\sqrt{2(N-1)N}}\right) \in \mathbb{R}^{N-1}, \quad (m = 1, \cdots, N-1),$$

$$v^N = \left(0, \cdots, 0, -\frac{N-1}{\sqrt{2(N-1)N}}\right) \in \mathbb{R}^{N-1},$$

$$\left\|v^i\right\|^2 = \frac{N-1}{2N}, v^i \cdot v^j = -\frac{1}{2N}, (i \neq j), \sum_{i=1}^{N} v^i = 0$$

$$\alpha^i := v^i - v^{i+1}, (i = 1, \cdots, N-1)$$

$$\text{with } \left\|\alpha^i\right\|^2 = 1, \alpha^i \cdot \alpha^j = \begin{cases} 0, & (j \neq i, i \pm 1) \\ -\frac{1}{2}, & (j = i \pm 1) \end{cases}$$

$$\mu^j := \sum_{i=1}^{j} v^i$$

$$\sum_{i=1}^{N} v^i = 0, \text{ i.e. } \sum_{i=1}^{j} v^i = -\sum_{i=j+1}^{N} v^i,$$

$$\overline{[j]} = [N-j], \qquad \bar{N} = [N-1]$$

$$N \otimes \bar{N} = \mathbf{1} \oplus \left(N^2 - \mathbf{1}\right)$$

$$^\forall v \in V, \tilde{v} := G(v) \text{ s.t. } ^\forall w \in V, \tilde{v}(w) = \omega(v, w).$$

$$^\forall \phi \in V^*, \tilde{\phi} := G^{-1}(\phi) \text{ s.t. } ^\forall \psi \in V^*, \psi(\tilde{\phi}) = \tilde{\omega}(\psi, \phi).$$

$$\omega^{ij} := \omega\left(e^i, e^j\right), \omega_{ij} := \tilde{\omega}\left(e_i, e_j\right), v_i := e_i(v), \phi^i := \phi\left(e^i\right), v \in V, \phi \in V^*,$$

$$\omega_{ik}\omega^{jk} = \delta_i^j, \tilde{v}^i = \omega^{ij}v_j, \tilde{\phi}_i = \phi^j\omega_{ji}$$

$$\omega(f(v), f(w)) = \omega(v, w), \text{for}^\forall v, w \in V$$

$$M_k{}^i\omega^{kl}M_l{}^j = \omega^{ij}, \text{ equivalently, } M_i{}^k\omega_{kl}M_j^l = \omega_{ij},$$

$$g\left(e^i, e^j\right) = \delta^{\bar{i}j}$$

$$\omega\left(e^i, e'^j\right) = \Omega^{ij} := \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}$$

$$\tilde{\omega}\left(e'^i, e'^j\right) = \Omega_{ij} := \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}$$

$$USp(2n) = \left\{\left(M_i^j\right) \in GL(2n, \mathbb{C}) \mid M_k^i\Omega^{kl}M_l^j = \Omega^{ij}, \left(M^\dagger\right)_i^k M_k^j = \delta_i^j\right\}$$

$$\omega_\downarrow + (\omega_\downarrow)^T = 0, \omega^\uparrow + \left(\omega^\uparrow\right)^T = 0, \omega_\downarrow = \left(\left(\omega^\uparrow\right)^T\right)^{-1}, (\omega_\downarrow)^\dagger = c(\omega_\downarrow)^{-1} = -c\omega^\uparrow,$$

$$M(\omega_\downarrow)^\dagger M^T = (\omega_\downarrow)^\dagger, \text{ and } M(\omega_\downarrow)^{-1}M^T = (\omega_\downarrow)^{-1}$$

$$(\Omega_\downarrow)^\dagger = (\Omega_\downarrow)^{-1} = -\Omega^\uparrow$$

$$\mathfrak{usp}(2n) = \left\{ \left(X_i{}^j\right) \in \mathfrak{gl}(2n,\mathbb{C}) \mid X_k{}^i\Omega^{kj} + \Omega^{ik}X_k{}^j = 0, \left(X^\dagger\right)_i{}^j = X_i{}^j \right\}$$

$$X = \begin{pmatrix} T & S \\ S' & T' \end{pmatrix}, T, T', S, S' \in \mathfrak{gl}(n,\mathbb{C})$$

$$T^\dagger = T, S^T = S, T' = -T^T, S' = S^\dagger$$

$$X^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, X^a = \begin{pmatrix} T^a & 0 \\ 0 & -(T^a)^* \end{pmatrix}, T^a \in \mathfrak{su}(n), (a = 1, \cdots, n^2 - 1)$$
$$X_s^a = \begin{pmatrix} 0 & S_\mathrm{R} \\ S_\mathrm{R} & 0 \end{pmatrix}, X_a^a = \begin{pmatrix} 0 & -iS_\mathrm{I} \\ iS_\mathrm{I} & 0 \end{pmatrix}, S_{\mathrm{R,I}} \in \mathfrak{gl}(n,\mathbb{R}), S_{\mathrm{R,I}}^T = S_{\mathrm{R,I}}$$
$$\left(a = 1, \cdots, \frac{1}{2}n(n+1)\right)$$

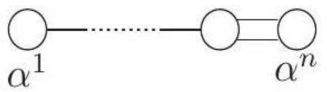
$$\tilde{X}^0 := \frac{1}{\sqrt{2n}} X^0, \tilde{X}^m := \begin{pmatrix} X^m & 0 \\ 0 & -X^m \end{pmatrix}, (m = 1, \cdots, n-1),$$

$$\pm\left(\nu^i + \frac{1}{\sqrt{2n}}\nu^{n+1}\right)$$

$$\nu^i - \nu^j, (i \neq j), \pm\left(\nu^i + \nu^j + \sqrt{\frac{2}{n}}\nu^{n+1}\right)$$

$$\alpha^i = \begin{cases} \nu^i - \nu^{i+1}, & (i = 1, \cdots, n-1), \\ 2\nu^n + \sqrt{\frac{2}{n}}\nu^{n+1}, & (i = n), \end{cases}$$

$$\|\alpha^i\|^2 = \begin{cases} 1, & (i = 1, \cdots, n-1), \\ 2, & (i = n) \end{cases} \quad \alpha^i \cdot \alpha^j = \begin{cases} -\frac{1}{2}, & (j = i+1, i = 1, \cdots, n-1, \text{or } i \leftrightarrow j) \\ -1, & (i = n-1, j = n, \text{or } i \leftrightarrow j) \\ 0, & (\text{otherwise}) \end{cases}$$



$$\eta_{\mu\nu} = (\underbrace{+, \cdots, +}_{t}, \underbrace{-, \cdots, -}_{s}).$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

$$\eta^{\mu\nu}\eta_{\nu\rho} = \eta^\mu{}_\rho \equiv \delta^\mu{}_\rho.$$

$$(\gamma^{\mu_1\cdots\mu_p})_{1 \leqslant \mu_1 < \cdots < \mu_p \leqslant D,\ 0 \leqslant p \leqslant D},$$
$$\gamma^{\mu_1\cdots\mu_p} := \begin{cases} 1 & (p = 0), \\ \frac{1}{p!} \sum_{\sigma \in \mathfrak{S}_p} \mathrm{sgn}(\sigma) \gamma^{\mu_{\sigma(1)}} \cdots \gamma^{\mu_{\sigma(p)}} & (p > 0), \end{cases}$$

$$\mathrm{tr}\gamma^{\mu_1\cdots\mu_p}\gamma^{\nu_1\cdots\nu_q} = (-1)^{p(p-1)/2} \dim V \delta_{pq} \eta_{\mu_1\cdots\mu_p,\nu_1\cdots\nu_p}$$

$$\eta_{\mu_1\cdots\mu_p,\nu_1\cdots\nu_p} := \sum_{\sigma\mathfrak{S}_p} \mathrm{sgn}(\sigma) \eta_{\mu_1\nu_{\sigma(1)}} \cdots \eta_{\mu_p\nu_{\sigma(p)}}$$

$$\mathrm{tr}\gamma^{\mu_1\cdots\mu_p} = 0, \left(1 \leqslant \mu_1 < \cdots < \mu_p \leqslant D, 1 \leqslant p \leqslant D\right)$$

$$\gamma = \sum_{p=0}^{D} \frac{1}{p!}(-1)^{p(p-1)/2}\frac{1}{\dim V}\gamma^{\mu_1\cdots\mu_p}\mathrm{tr}\gamma_{\mu_1\cdots\mu_p}\gamma,\ \text{for}\ {}^\forall\gamma \in \mathcal{C}(t,s)$$

$$\Gamma^5 := i^{(t-s)/2}(-1)^s\gamma^1\cdots\gamma^D$$

$$\{\Gamma^5,\gamma^\mu\} = 0, (\Gamma^5)^2 = 1$$

$$\gamma^{\mu_1\cdots\mu_p}\Gamma^5 = \frac{1}{(D-p)!}i^{(t-s)/2}(-1)^{p(p-1)/2}\varepsilon^{\mu_1\cdots\mu_p}{}_{\mu_{p+1}\cdots\mu_D}\gamma^{\mu_{p+1}\cdots\mu_D}$$

$$\varepsilon^{\mu_1\cdots\mu_D} = \frac{1}{|\det g|}\epsilon^{\mu_1\cdots\mu_D}, \epsilon^{1\cdots D} = 1$$

$$\{-(\gamma^\mu)^*, -(\gamma^\nu)^*\} = 2\eta^{\mu\nu}$$

$$B\gamma^\mu B^{-1} = \eta(\gamma^\mu)^*, B^\dagger B = 1, B^*B = \varepsilon, \eta = \pm 1, \varepsilon = \pm 1$$

$$\{(-\gamma^\mu)^T, (-\gamma^\nu)^T\} = 2\eta^{\mu\nu}$$

$$C\gamma^\mu C^{-1} = \eta'(\gamma^\mu)^T, C^\dagger C = 1, C^T = \varepsilon' C, \eta' = \pm 1, \varepsilon' = \pm 1$$

$$C^*B\gamma^\mu B^{-1}C^{*-1} = \eta(C\gamma^\mu C^{-1})^* = \eta\eta'(\gamma^\mu)^\dagger$$

$$\Gamma^0(\gamma^\mu)^\dagger(\Gamma^0)^{-1} = \kappa\gamma^\mu, (\Gamma^0)^\dagger\Gamma^0 = 1, \Gamma^0 := \varepsilon(-1)^{t(t-1)/2}B^{-1}C^{*-1}, \kappa := \eta\eta'$$

$$\begin{cases}(\gamma^{\mathrm{t}})^2 = +\mathbf{1}, \\ (\gamma^{\mathrm{s}})^2 = -\mathbf{1},\end{cases} \text{i.e.} \begin{cases}(\gamma^{\mathrm{t}})^\dagger(\gamma^{\mathrm{t}}) = +\kappa'_{\mathrm{t}}\mathbf{1} \\ (\gamma^{\mathrm{s}})^\dagger(\gamma^{\mathrm{s}}) = -\kappa'_{\mathrm{s}}\mathbf{1}\end{cases}$$

$$\begin{cases}(\gamma^{\mathrm{t}})^\dagger = +(\gamma^{\mathrm{t}}) \\ (\gamma^{\mathrm{s}})^\dagger = -(\gamma^{\mathrm{s}})\end{cases}$$

$$\begin{cases}\Gamma^0 = \gamma^1\cdots\gamma^t, \\ \kappa = (-1)^{t+1},\end{cases} \text{or} \begin{cases}\Gamma^0 = \gamma^{t+1}\cdots\gamma^D \\ \kappa = (-1)^s\end{cases}$$

$$\begin{cases}\eta' = (-1)^{t+1}\eta, \\ \varepsilon' = \varepsilon\eta^t(-1)^{t(t-1)/2},\end{cases} \text{or} \begin{cases}\eta' = (-1)^s\eta \\ \varepsilon' = \varepsilon\eta^s(-1)^{s(s-1)/2}\end{cases}$$

$$(C\gamma^{\mu_1\cdots\mu_p})^T = (-1)^{p(t+1)+p(p-1)/2+t(t-1)/2}\eta^{t+p}\varepsilon(C\gamma^{\mu_1\cdots\mu_p})$$

$$\#(\mathrm{AS}) = \frac{1}{2}2^{D/2}(2^{D/2}-1),$$

$$\#(\mathrm{AS}) = \sum_p \binom{D}{p}$$

$$(-1)^{p(t+1)+p(p-1)/2+t(t-1)/2}\eta^{t+p}\varepsilon = -1,\ \text{i.e.}\ (-1)^{p(t+1)+p(p-1)/2}\eta^p = -(-1)^{t(t-1)/2}\eta^t\varepsilon$$

$p \pmod 4$	$f(p)$
0	1
1	$(-1)^{t+1}\eta$
2	-1
3	$-(-1)^{t+1}\eta$

$(-1)^{t+1}\eta$	$f(p)$
1	$1, 1, -1, -1$
-1	$1, -1, -1, 1$

$$\#(\mathrm{AS}) = \sum_{p=0}^{D} \binom{D}{p} \frac{1}{2}\left(1 + \Re\sqrt{2}e^{-i\frac{\pi}{4}+i\frac{n\pi}{2}}\right)$$
$$= \frac{1}{2}2^{D/2}\left(2^{D/2} - \sqrt{2}\cos\frac{\pi}{4}(D+3)\right)$$

$$\#(\mathrm{AS}) = \sum_{p=0}^{D} \binom{D}{p} \frac{1}{2}\left(1 - \Re\sqrt{2}e^{-i\frac{\pi}{4}+i\frac{n\pi}{2}}\right)$$
$$= \frac{1}{2}2^{D/2}\left(2^{D/2} - (-1)\sqrt{2}\cos\frac{\pi}{4}(D+3)\right)$$

$$\#(\mathrm{AS}) = \sum_{p=0}^{D} \binom{D}{p} \frac{1}{2}\left(1 + \Re\sqrt{2}e^{i\frac{\pi}{4}+i\frac{n\pi}{2}}\right)$$
$$= \frac{1}{2}2^{D/2}\left(2^{D/2} - \sqrt{2}\cos\frac{\pi}{4}(-D+3)\right)$$

$$\#(\mathrm{AS}) = \sum_{p=0}^{D} \binom{D}{p} \frac{1}{2}\left(1 - \Re\sqrt{2}e^{i\frac{\pi}{4}+i\frac{n\pi}{2}}\right)$$
$$= \frac{1}{2}2^{D/2}\left(2^{D/2} - (-1)\sqrt{2}\cos\frac{\pi}{4}(-D+3)\right)$$

$$\#(\mathrm{AS}) = \frac{1}{2}2^{D/2}\left(2^{D/2} - \left(-(-1)^{t(t-1)/2}\eta^t\varepsilon\right)\sqrt{2}\cos\frac{\pi}{4}\left(\eta(-1)^{t+1}D+3\right)\right).$$

$$\varepsilon = -\sqrt{2}\eta^t(-1)^{t(t-1)/2}\cos\frac{1}{4}\pi(\eta(-1)^{t+1}D+3) = \cos\frac{1}{4}\pi(s-t) - \eta\sin\frac{1}{4}\pi(s-t)$$

$$B\Gamma^5B^{-1} = (-1)^{(t-s)/2}(\Gamma^5)^*, C\Gamma^5C^{-1} = (-1)^{D/2}(\Gamma^5)^T$$
$$\Gamma^0(\Gamma^5)^\dagger(\Gamma^0)^{-1} = (-1)^t\Gamma^5, (\Gamma^5)^\dagger = \Gamma^5$$

$$B\Gamma^0B^{-1} = \eta^t(\Gamma^0)^*, C\Gamma^0C^{-1} = \eta^t(-1)^{t(t-1)/2}(\Gamma^0)^T, (\Gamma^0)^2 = (-1)^{t(t-1)/2}\mathbf{1}$$

$$\Sigma^{\mu\nu} := \frac{i}{4}[\gamma^\mu,\gamma^\nu] \equiv \frac{i}{2}\gamma^{\mu\nu}$$

$$[\Sigma^{\mu\nu},\Sigma^{\rho\sigma}] = i(\eta^{\nu\rho}\Sigma^{\mu\sigma} - \eta^{\nu\sigma}\Sigma^{\mu\rho} - \eta^{\mu\rho}\Sigma^{\nu\sigma} + \eta^{\mu\sigma}\Sigma^{\nu\rho})$$

$$\mathrm{Spin}(t,s) := \left\{\gamma \in \mathcal{C}(t,s) \,\middle|\, \gamma = \exp\left(-i\frac{1}{2}\omega_{\mu\nu}\Sigma^{\mu\nu}\right)\right\}$$

$$\gamma\gamma^\mu\gamma^{-1} = \exp(\omega)^\mu{}_\nu\gamma^\nu$$

$$W = \langle\{\gamma^1,\cdots,\gamma^D\}\rangle$$

$$\Gamma \in \mathrm{Spin}(t,s), \gamma \in W, \mathrm{Ad}(\Gamma)(\gamma) = \Gamma\gamma\Gamma^{-1}$$

$$\gamma^p = \Gamma\gamma^p\Gamma^{-1},\ \text{i.e. } \gamma^p(\Gamma_0 + \gamma^p\Gamma') = (\Gamma_0 + \gamma^p\Gamma')\gamma^p,$$

$$(\gamma^p)^2\Gamma' = \gamma^p\Gamma'\gamma^p = -(\gamma^p)^2\Gamma',\ \text{i.e. } \Gamma' = 0$$

$$[\Gamma^5,\Sigma^{\mu\nu}] = 0$$

$$P^\pm = \frac{1 \pm \Gamma^5}{2}$$

$$P^\pm P^\pm = P^\pm, P^\pm P^\mp = 0, (P^\pm)^\dagger = P^\pm, P^+ + P^- = 1$$

$$[\Sigma^{2a-1,2a},\Sigma^{2b-1,2b}] = 0$$

$$S^a := (-i)^{\delta_{a,(t+1)/2}+\theta(a-(t+2)/2)}\Sigma^{2a-1,2a} = \Gamma^{a+}\Gamma^{a-} - \frac{1}{2}.$$

$$\psi^{\pm} = P^{\pm}\psi, P^{\pm}\psi^{\pm} = \pm\psi^{\pm}$$

$$\psi = (P^{+} + P^{-})\psi = \psi^{+} + \psi^{-}$$
$$\mathbf{2}^{D/2} = \mathbf{2}^{D/2-1} \oplus \mathbf{2}^{D/2-1}$$

$$\gamma_{\alpha}^{\beta}, (\gamma^{-1})_{\alpha}^{\beta}, (\gamma^{\dagger})_{\alpha}^{\beta}, (\gamma^{T})^{\alpha}{}_{\beta} = \gamma_{\beta}^{\alpha}, (\gamma^{*})^{\alpha}{}_{\beta} = \left(\gamma_{\alpha}^{\beta}\right)^{*},$$

$$\mathfrak{C}^{\alpha\beta}, (\mathfrak{C}^{-1})_{\alpha\beta}, (\mathfrak{C}^{T})^{\alpha\beta} = \mathfrak{C}^{\beta\alpha}, (\mathfrak{C}^{*})_{\alpha\beta} = \left(\mathfrak{C}^{\alpha\beta}\right)^{*}, \left(\mathfrak{C}^{\dagger}\right)_{\alpha\beta} = \left(\mathfrak{C}^{\beta\alpha}\right)^{*}$$

$$(\Gamma^0)_{\alpha}^{\beta} = (B^{-1})_{\alpha\gamma}(C^{*-1})^{\gamma\beta}.$$

$$\psi'_{\alpha} = \exp\left(-\frac{i}{2}\omega_{\mu\nu}\Sigma^{\mu\nu}\right)\alpha^{\beta}\psi_{\beta}, \text{ or } \delta\psi_{\alpha} = \frac{1}{4}\omega_{\mu\nu}(\gamma^{\mu\nu})_{\alpha}{}^{\beta}\psi_{\beta}$$

$$\psi'^{\alpha} = \exp\left(+\frac{i}{2}\omega_{\mu\nu}\Sigma^{\mu\nu}\right)\beta^{\alpha}\psi^{\beta}, \text{ or } \delta\psi^{\alpha} = -\frac{1}{4}\omega_{\mu\nu}(\gamma^{\mu\nu})_{\beta}{}^{\alpha}\psi^{\beta}.$$

$$\bar{\psi}^{\alpha} := \sum_{\beta}\left(\psi_{\beta}\right)^{\dagger}((\Gamma^0)^{-1})_{\beta}^{\alpha}$$

$$\Gamma^0(\Sigma^{\mu\nu})^{\dagger}(\Gamma^0)^{-1} = \Sigma^{\mu\nu}$$

$$\bar{\psi}'^{\alpha} = \left(\psi_{\gamma}\right)^{\dagger}\exp\left(+\frac{i}{2}\omega^{\mu\nu}\left(\Sigma_{\mu\nu}\right)^{\dagger}\right)\gamma^{\beta}(\Gamma^0)_{\beta}^{-1\alpha} = \bar{\psi}^{\beta}\exp\left(+\frac{i}{2}\omega^{\mu\nu}\Sigma_{\mu\nu}\right)_{\beta}^{\alpha}$$

$$\bar{\psi}_{\alpha} := (\Gamma^0)^{-1}{}_{\alpha}{}^{\beta}\left(\psi^{\beta}\right)^{\dagger},$$

$$\bar{\bar{\psi}} = (\Gamma^0)^{-1}(\bar{\psi})^{\dagger} = (\Gamma^0)^{-1}\Gamma^0\psi = \psi$$

$$\tilde{\psi}^{\alpha} := (\psi^{T})_{\beta}(C^{T})^{\beta\alpha} = C^{\alpha\beta}\psi_{\beta}$$

$$\tilde{\psi}_{\alpha} := (C^{-1})_{\alpha\beta}(\psi^{T})^{\beta}$$

$$C\Sigma^{\mu\nu}C^{-1} = -(\Sigma^{\mu\nu})^{T}$$

$$\tilde{\psi}'_{\alpha} = (C^{-1})_{\alpha\beta}\exp\left(+\frac{i}{2}\omega^{\mu\nu}\Sigma_{\mu\nu}\right)^{T}{}_{\gamma}(\psi^{T})^{\gamma} = \exp\left(-\frac{i}{2}\omega^{\mu\nu}\Sigma_{\mu\nu}\right)\alpha^{\beta}\tilde{\psi}_{\beta}$$

$$(\psi_c)_{\alpha} := (C^{-1})_{\alpha\beta}(\bar{\psi}^{T})^{\beta} = \tilde{\bar{\psi}}_{\alpha}, \text{ with } \psi_{cc} = \varepsilon\psi$$

$$(\psi_c)^{\alpha} := C^{\alpha\beta}\bar{\psi}_{\beta} = \tilde{\bar{\psi}}^{\alpha}, \text{ with } \psi_{cc} = \varepsilon\psi$$

$$(\psi_b)_{\alpha} := (B^{-1})_{\alpha\beta}(\psi^{*})^{\beta}, (\psi^{*})^{\alpha} := (\psi_{\alpha})^{*}$$
$$(\psi_b)^{\alpha} := \eta^{t}B^{\alpha\beta}(\psi^{*})_{\beta}, (\psi^{*})_{\alpha} := (\psi^{\alpha})^{*}$$

$$\psi_{i\alpha} = M_{ij}(\psi_c)_{\alpha}^{j}, M_{ij}M^{ik} = \delta_{j}^{k}$$

$$(\psi_c)^{i}{}_{\alpha} = (C^{-1})_{\alpha\beta}(\bar{\psi}^{T})^{i\beta} = (C^{-1})_{\alpha\beta}((\Gamma^0)^{-1T})^{\beta}{}_{\gamma}(\psi^{*})^{j\gamma}, (\psi^{*})^{i\alpha} := (\psi_{i\alpha}),$$
$$\psi_{i\alpha} = \varepsilon M_{ik}(M^{*})^{kj}\psi_{j\alpha}$$

$$M_{ik}(M^{*})^{kj} = \varepsilon, \text{ i.e., } M^{\dagger} = \varepsilon(M^{-1})^{T}$$
$$(\psi^{\pm})_{c} = \psi^{\pm}$$

$$P^{\pm(-1)^{t}} = CP^{\pm}C^{-1} = P^{\pm(-1)^{D/2}}, \text{ i.e. } P^{\pm\sigma} = \mathbb{1}, \sigma := (-1)^{(s-t)/2}$$

$$\{\nabla_{i\alpha},\nabla_{j\beta}\} = i\Omega_{ij}C_{\alpha\beta}\nabla_z - iC_{\alpha\beta}W_{ij}, \{\bar{\nabla}_{i\dot{\alpha}},\bar{\nabla}_{j\dot{\beta}}\} = i\Omega_{ij}C_{\dot{\alpha}\dot{\beta}}\nabla_z + iC_{\dot{\alpha}\dot{\beta}}W_{ij},$$

$$\begin{aligned}
&\{\nabla_{i\alpha},\bar{\nabla}_{j\dot{\beta}}\} = i\Omega_{ij}(\sigma^\mu)_{\alpha\dot{\beta}}\nabla_\mu, \\
&[\nabla_{i\alpha},\nabla_\mu] = -iF_{i\alpha\mu}, && [\bar{\nabla}_{i\dot{\alpha}},\nabla_\mu] = +i\bar{F}_{i\dot{\alpha}\mu}, \\
&[\nabla_{i\alpha},\nabla_z] = -iG_{i\alpha}, && [\bar{\nabla}_{i\dot{\alpha}},\nabla_z] = +i\bar{G}_{i\dot{\alpha}}, \\
&[\nabla_\mu,\nabla_z] = -ig_\mu, && [\nabla_\mu,\nabla_\nu] = -iF_{\mu\nu}, \\
&\Omega^{ij}W_{ij} = 0, && (W_{ij})^* = W^{ij},
\end{aligned}$$

$$\begin{aligned}
G_{i\alpha} &= i[\nabla_{i\alpha},\nabla_z] = i\left[\nabla_{i\alpha}, -\frac{i}{8}C^{\beta\gamma}\Omega^{jk}\{\nabla_{j\beta},\nabla_{k\gamma}\}\right] \\
&= -\frac{1}{8}C^{\beta\gamma}\Omega^{jk}([\nabla_{j\beta},\{\nabla_{k\gamma},\nabla_{i\alpha}\}] + [\nabla_{k\gamma},\{\nabla_{i\alpha},\nabla_{j\beta}\}]) \\
&= -\frac{1}{8}C^{\beta\gamma}\Omega^{jk}([\nabla_{j\beta}, i\Omega_{ki}C_{\gamma\alpha}\nabla_z - iC_{\gamma\alpha}W_{ki}] + [\nabla_{k\gamma}, i\Omega_{ij}C_{\alpha\beta}\nabla_z - iC_{\alpha\beta}W_{ij}]) \\
&= -\frac{i}{4}([\nabla_{i\alpha},\nabla_z] + [\nabla_{j\alpha},W_i^{\ j}]) = -\frac{i}{4}(-iG_{i\alpha} + [\nabla_{j\alpha},W_i^{\ j}]),
\end{aligned}$$

$$[\nabla_{j\alpha},W_i^{\ j}] = 5iG_{i\alpha}.$$

$$[\bar{\nabla}_{j\dot{\alpha}},W^j{}_i] = 5i\bar{G}_{i\dot{\alpha}}$$

$$\begin{aligned}
G_{i\alpha} &= i[\nabla_{i\alpha},\nabla_z] = i\left[\nabla_{i\alpha}, -\frac{i}{8}C^{\dot{\beta}\dot{\gamma}}\Omega^{jk}\{\bar{\nabla}_{j\dot{\beta}},\bar{\nabla}_{k\dot{\gamma}}\}\right] \\
&= -\frac{1}{8}C^{\dot{\beta}\dot{\gamma}}\Omega^{jk}([\bar{\nabla}_{j\dot{\beta}}, i\Omega_{ik}(\sigma^\mu)_{\alpha\dot{\gamma}}\nabla_\mu] + [\bar{\nabla}_{k\dot{\gamma}}, i\Omega_{ij}(\sigma^\mu)_{\alpha\dot{\beta}}\nabla_\mu]) \\
&= \frac{i}{4}(\sigma^\mu C)_\alpha{}^{\dot{\beta}}[\bar{\nabla}_{i\dot{\beta}},\nabla_\mu] = -\frac{1}{4}(\sigma^\mu C)_\alpha{}^{\dot{\beta}}\bar{F}_{i\dot{\beta}\mu},
\end{aligned}$$

$$G_{i\alpha} = -\frac{1}{4}(\sigma^\mu C)_\alpha{}^{\dot{\beta}}\bar{F}_{i\dot{\beta}\mu}$$

$$\bar{G}_{i\dot{\alpha}} = -\frac{1}{4}(C\sigma^\mu)^\beta{}_{\dot{\alpha}}F_{i\beta\mu},$$

$$\begin{aligned}
[\nabla_{i\alpha},W_{jk}] &= \left[\nabla_{i\alpha}, \frac{i}{2}C^{\beta\gamma}(\{\nabla_{j\beta},\nabla_{k\gamma}\} - iC\beta\gamma\Omega_{jk}\nabla_z)\right] \\
&= -\frac{i}{2}C^{\beta\gamma}([\nabla_{j\beta}, i\Omega_{ki}C_{\gamma\alpha}\nabla_z - iC_{\gamma\alpha}W_{ki}] + [\nabla_{k\gamma}, i\Omega_{ij}C_{\alpha\beta}\nabla_z - iC_{\alpha\beta}W_{ij}]) \\
&\quad + \Omega_{jk}[\nabla_{i\alpha},\nabla_z] \\
&= -\frac{1}{2}(\Omega_{ki}[\nabla_{j\alpha},\nabla_z] + \Omega_{ij}[\nabla_{k\alpha},\nabla_z]) + \Omega_{jk}[\nabla_{i\alpha},\nabla_z] \\
&\quad + \frac{1}{2}([\nabla_{j\alpha},W_{ki}] + [\nabla_{k\alpha},W_{ij}]),
\end{aligned}$$

$$[\nabla_{i\alpha},W_{ji}] = \frac{i}{2}\Omega_{i[j}G_{k]\alpha} - i\Omega_{jk}G_{i\alpha} + \frac{1}{2}[\nabla_{[j\alpha},W_{k]i}]$$

$$[\bar{\nabla}_{i\dot{\alpha}},W_{jk}] = \frac{i}{2}\Omega_{i[j}\bar{G}_{k]\dot{\alpha}} - i\Omega_{jk}\bar{G}_{i\dot{\alpha}} + \frac{1}{2}[\bar{\nabla}_{[j\dot{\alpha}},W_{k]i}]$$

$$\begin{aligned}
[\nabla_{i\alpha},W_{jk}] &= \left[\nabla_{i\alpha}, -\frac{i}{2}C^{\dot{\beta}\dot{\gamma}}(\{\bar{\nabla}_{j\dot{\beta}},\bar{\nabla}_{k\dot{\gamma}}\} - iC_{\dot{\beta}\dot{\gamma}}\Omega_{jk}\nabla_z)\right] \\
&= \frac{i}{2}C^{\dot{\beta}\dot{\gamma}}([\bar{\nabla}_{j\dot{\beta}}, i\Omega_{ik}(\sigma^\mu)_{\alpha\dot{\gamma}}\nabla_\mu] + [\bar{\nabla}_{k\dot{\gamma}}, i\Omega_{ij}(\sigma^\mu)_{\alpha\dot{\beta}}\nabla_\mu]) \\
&\quad - \Omega_{jk}[\nabla_{i\alpha},\nabla_z] \\
&= -\frac{i}{2}\Omega_{i[j}(\sigma^\mu C)_\alpha{}^{\dot{\beta}}\bar{F}_{k]\dot{\beta}\mu} + i\Omega_{jk}G_{i\alpha}
\end{aligned}$$

$$[\nabla_{i\alpha},W_{jk}] = 2i\Omega_{i[j}G_{k]\alpha} + i\Omega_{jk}G_{i\alpha}$$

$$[\bar{\nabla}_{i\dot{\alpha}},W_{jk}] = 2i\Omega_{i[j}\bar{G}_{k]\dot{\alpha}} + i\Omega_{jk}\bar{G}_{i\dot{\alpha}}$$

$$F_{i\alpha\mu} = i[\nabla_{i\alpha}, \nabla_\mu] = i\left[\nabla_{i\alpha}, -\frac{i}{8}\Omega^{jk}(\bar{\sigma}_\mu)^{\dot{\gamma}\beta}\{\nabla_{j\beta}, \bar{\nabla}_{k\dot{\gamma}}\}\right]$$
$$= -\frac{1}{8}\Omega^{jk}(\bar{\sigma}_\mu)^{\dot{\gamma}\beta}\left([\nabla_{j\beta}, i\Omega_{ik}(\sigma^\nu)_{\alpha\dot{\gamma}}\nabla_\nu] + [\bar{\nabla}_{k\dot{\gamma}}, i\Omega_{ij}C_{\alpha\beta}\nabla_z - iC_{\alpha\beta}W_{ij}]\right)$$
$$= -\frac{1}{8}(\sigma^\mu\bar{\sigma}_\mu)_\alpha{}^\beta F_{i\beta\nu} + \frac{3}{4}(\bar{\sigma}_\mu C)^{\dot{\gamma}}{}_\alpha \bar{G}_{i\dot{\gamma}}$$

$$= -\frac{1}{4}F_{i\alpha\mu} - \frac{1}{8}(\bar{\sigma}_\mu C)^{\dot{\gamma}}{}_\alpha (C\sigma^\nu)^\beta{}_{\dot{\gamma}} F_{i\beta\nu} + \frac{3}{4}(\bar{\sigma}_\mu C)^{\dot{\gamma}}{}_\alpha \bar{G}_{i\dot{\gamma}}$$

$$F_{i\alpha\mu} = (\bar{\sigma}_\mu C)^{\dot{\gamma}}{}_\alpha \bar{G}_{i\dot{\gamma}}$$

$$\bar{F}_{i\dot{\alpha}\mu} = (C\bar{\sigma}_\mu)_{\dot{\alpha}}{}^\beta G_{i\beta},$$

$$\{\nabla_{i\alpha}, G_{j\beta}\} = i\{\nabla_{i\alpha}, [\nabla_{j\beta}, \nabla_z]\}$$
$$= -\{\nabla_{j\beta}, G_{i\alpha}\} + [\nabla_z, \Omega_{ij}C_{\alpha\beta}\nabla_z - C_{\alpha\beta}W_{ij}]$$

$$\{\nabla_{i\alpha}, G_{j\beta}\} + \{\nabla_{j\beta}, G_{i\alpha}\} = -C_{\alpha\beta}[\nabla_z, W_{ij}].$$

$$\{\bar{\nabla}_{i\dot{\alpha}}, \bar{G}_{j\dot{\beta}}\} + \{\bar{\nabla}_{j\dot{\beta}}, \bar{G}_{i\dot{\alpha}}\} = -C_{\dot{\alpha}\dot{\beta}}[\nabla_z, W_{ij}].$$

$$\{\nabla_{i\alpha}, G_{j\beta}\} = -\frac{i}{5}\{\nabla_{i\alpha}, [\nabla_{k\beta}, W_j^k]\}$$
$$= \frac{1}{5}\left(\{\nabla_{j\beta}, G_{i\alpha}\} - 2\{\nabla_{i\beta}, G_{j\alpha}\} + 2\Omega_{ij}\{\nabla_{k\beta}, G_\alpha^k\}\right.$$
$$\left. - C_{\alpha\beta}[\nabla_z, W_{ij}] + C_{\alpha\beta}[W_j^k, W_{ik}]\right)$$

$$\{\nabla_{(i\alpha}, G_{j)\beta}\} = -\frac{1}{2}[W_{ik}, W_j^k],$$

$$\{\nabla_{[i\alpha}, G_{j]\beta}\} = -C_{\alpha\beta}[\nabla_z, W_{ij}] + \frac{1}{2}\Omega_{ij}\{\nabla_{k\alpha}, G_\beta^k\}.$$

$$\{\bar{\nabla}_{(i\dot{\alpha}}, \bar{G}_{j)\dot{\beta}}\} = \frac{1}{2}C_{\dot{\alpha}\dot{\beta}}[W_{ik}, W_j^k],$$
$$\{\bar{\nabla}_{[i\dot{\alpha}}, \bar{G}_{j]\dot{\beta}}\} = -C_{\dot{\alpha}\dot{\beta}}[\nabla_z, W_{ij}] + \frac{1}{2}\Omega_{ij}\left\{\bar{\nabla}_{k\dot{\alpha}}, \bar{G}_{\dot{\beta}}^k\right\}.$$

$$\{\nabla_{i\alpha}, G_{j\beta}\} = -\frac{1}{4}(\sigma^\mu C)_\beta{}^{\dot{\beta}}\{\nabla_{i\alpha}, \bar{F}_{j\dot{\beta}\mu}\} = \frac{i}{4}(\sigma^\mu C)_\beta{}^{\dot{\beta}}\{\nabla_{i\alpha}, [\bar{\nabla}_{j\dot{\beta}}, \nabla_\mu]\}$$

$$= -\frac{1}{2}C_{\alpha\beta}\{\bar{\nabla}_{j\dot{\alpha}}, \bar{G}_i^{\dot{\alpha}}\} - \frac{i}{4}\Omega_{ij}(\sigma^{\mu\nu})_{\alpha\beta}F_{\mu\nu}$$

$$\{\nabla_{i\alpha}, G_\beta^i\} = \frac{1}{2}C_{\alpha\beta}\{\bar{\nabla}_{i\dot{\alpha}}, \bar{G}^{i\dot{\alpha}}\} - i(\sigma^{\mu\nu})_{\alpha\beta}F_{\mu\nu} = -i(\sigma^{\mu\nu})_{\alpha\beta}F_{\mu\nu},$$

$$\{\nabla_{[i\alpha}, G_{j]\beta}\} = -C_{\alpha\beta}[\nabla_z, W_{ij}] - \frac{i}{2}\Omega_{ij}(\sigma^{\mu\nu})_{\alpha\beta}F_{\mu\nu}$$

$$\{\nabla_{i\alpha}, G_{j\beta}\} = -\frac{i}{4}\Omega_{ij}(\sigma^{\mu\nu})_{\alpha\beta}F_{\mu\nu} - \frac{1}{2}C_{\alpha\beta}[\nabla_z, W_{ij}] - \frac{1}{4}C_{\alpha\beta}[W_{ik}, W_j^k].$$

$$\{\bar{\nabla}_{[i\dot{\alpha}}, \bar{G}_{j]\dot{\beta}}\} = -C_{\dot{\alpha}\dot{\beta}}[\nabla_z, W_{ij}] - \frac{i}{2}\Omega_{ij}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}}F_{\mu\nu}$$
$$\{\bar{\nabla}_{i\dot{\alpha}}, \bar{G}_{j\dot{\beta}}\} = -\frac{i}{4}\Omega_{ij}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}}F_{\mu\nu} - \frac{1}{2}C_{\dot{\alpha}\dot{\beta}}[\nabla_z, W_{ij}] + \frac{1}{4}C_{\dot{\alpha}\dot{\beta}}[W_{ik}, W_j^k].$$

$$\{\bar{\nabla}_{i\dot{\alpha}}, G_{j\beta}\} = i\{\bar{\nabla}_{i\dot{\alpha}}, [\nabla_{j\beta}, \nabla_z]\}$$
$$= -i\left(i\{\nabla_{j\beta}, \bar{G}_{i\dot{\alpha}}\} + i\Omega_{ji}(\sigma^\mu)_{\beta\dot{\alpha}}[\nabla_z, \nabla_\mu]\right) = \{\nabla_{j\beta}, \bar{G}_{i\dot{\alpha}}\} - i\Omega_{ij}(\sigma^\mu)_{\beta\dot{\alpha}}g_\mu$$

$$\{\nabla_{i\alpha}, \bar{G}_{j\dot{\beta}}\} = \{\bar{\nabla}_{j\dot{\beta}}, G_{i\alpha}\} - i\Omega_{ij}(\sigma^\mu)_{\alpha\dot{\beta}}g_\mu.$$

$$\{\bar{\nabla}_{i\dot{\alpha}}, G_{j\beta}\} = -\frac{i}{5}\{\bar{\nabla}_{i\dot{\alpha}}, [\nabla_{k\beta}, W_j^k]\}$$
$$= \frac{1}{5}\left(\{\nabla_{j\beta}, \bar{G}_{i\dot{\alpha}}\} - 2\{\nabla_{i\beta}, \bar{G}_{j\dot{\alpha}}\} + 2\Omega_{ij}\{\nabla_{k\beta}, \bar{G}^k_{\dot{\alpha}}\}\right) + \frac{1}{5}(\sigma^\mu)_{\beta\dot{\alpha}}[\nabla_\mu, W_{ij}]$$

$$\{\bar{\nabla}_{(i\dot{\alpha}}, G_{j)\beta}\} = 0$$

$$\{\bar{\nabla}_{[i\dot{\alpha}}, G_{j]\beta}\} = -5i\Omega_{ij}(\sigma^\mu)_{\beta\dot{\alpha}} g_\mu - 2\Omega_{ij}\{\bar{\nabla}_{k\dot{\alpha}}, G^k{}_\beta\} + (\sigma^\mu)_{\beta\dot{\alpha}}[\nabla_\mu, W_{ij}].$$

$$\{\bar{\nabla}_{k\dot{\alpha}}, G^k{}_\beta\} = -2i(\sigma^\mu)_{\beta\dot{\alpha}} g_\mu$$

$$\{\bar{\nabla}_{[i\dot{\alpha}}, G_{j]\beta}\} = -i\Omega_{ij}(\sigma^\mu)_{\beta\dot{\alpha}} g_\mu + (\sigma^\mu)_{\beta\dot{\alpha}}[\nabla_\mu, W_{ij}].$$

$$\{\bar{\nabla}_{i\dot{\alpha}}, G_{j\beta}\} = -\frac{1}{2}(\sigma^\mu)_{\beta\dot{\alpha}}\left(i\Omega_{ij} g_\mu - [\nabla_\mu, W_{ij}]\right).$$

$$\{\nabla_{(i\alpha}, \bar{G}_{j)\dot{\beta}}\} = 0$$
$$\{\nabla_{[i\alpha}, \bar{G}_{j]\dot{\beta}}\} = -i\Omega_{ij}(\sigma^\mu)_{\alpha\dot{\beta}} g_\mu - (\sigma^\mu)_{\alpha\dot{\beta}}[\nabla_\mu, W_{ij}]$$
$$\{\nabla_{i\alpha}, \bar{G}_{j\dot{\beta}}\} = -\frac{1}{2}(\sigma^\mu)_{\alpha\dot{\beta}}\left(i\Omega_{ij} g_\mu + [\nabla_\mu, W_{ij}]\right)$$

$$g_\mu = i[\nabla_\mu, \nabla_z] = \frac{1}{8}(\bar{\sigma}^\mu)^{\dot{\beta}\alpha}\left[\left\{\nabla_{i\alpha}, \bar{\nabla}^i_{\dot{\beta}}\right\}, \nabla_z\right] = \frac{i}{8}(\bar{\sigma}^\mu)^{\dot{\beta}\alpha}\left(\left\{\nabla_{i\alpha}, \bar{G}^i_{\dot{\beta}}\right\} + \{\bar{\nabla}_{i\dot{\beta}}, G^i_\alpha\}\right)$$

$$g_\mu = i[\nabla_\mu, \nabla_z] = \frac{1}{8}\Omega^{ij} C^{\alpha\beta}[\nabla_\mu, \{\nabla_{i\alpha}, \nabla_{j\beta}\}]$$
$$= \frac{i}{4}(\bar{\sigma}_\mu)^{\dot{\beta}\alpha}\left\{\nabla_{i\alpha}, \bar{G}^i_{\dot{\beta}}\right\}$$

$$g_\mu = \frac{i}{4}(\bar{\sigma}_\mu)^{\dot{\beta}\alpha}\{\bar{\nabla}_{i\dot{\beta}}, G^i_\alpha\}$$

$$[\nabla_{i\alpha}, g_\mu] = i\left[\nabla_{i\alpha}, [\nabla_\mu, \nabla_z]\right] = [\nabla_\mu, G_{i\alpha}] - (\bar{\sigma}_\mu C)^{\dot{\alpha}}{}_\alpha[\nabla_z, \bar{G}_{i\dot{\alpha}}]$$

$$[\bar{\nabla}_{i\dot{\alpha}}, g_\mu] = -[\nabla_\mu, \bar{G}_{i\dot{\alpha}}] + (C\bar{\sigma}_\mu)^\alpha_{\dot{\alpha}}[\nabla_z, G_{i\alpha}].$$

$$[\nabla_{i\alpha}, g_\mu] = \frac{i}{4}(\bar{\sigma}^\mu)^{\dot{\beta}\beta}[\nabla_{i\alpha}, \{\nabla_{j\beta}, \bar{G}^j{}_{\dot{\beta}}\}],$$

$$\left[\nabla_{i\alpha}, \left\{\nabla_{j\beta}, \bar{G}^j_{\dot{\beta}}\right\}\right] = \left[\nabla^j{}_\beta, -\frac{1}{2}(\sigma^\mu)_{\alpha\dot{\beta}}\left(i\Omega_{ij} g_\mu + [\nabla_\mu, W_{ij}]\right)\right] - \left[\bar{G}^j_{\dot{\beta}}, i\Omega_{ij} C_{\alpha\beta}\nabla_z - iC_{\alpha\beta} W_{ij}\right]$$
$$= \frac{i}{2}(\sigma^\mu)_{\alpha\dot{\beta}}[\nabla_{i\beta}, g_\mu] - \frac{5}{2}i(\sigma^\mu)_{\alpha\dot{\beta}}[\nabla_\mu, G_{i\beta}]$$
$$+2iC_{\alpha\beta}[\bar{G}^j{}_{\dot{\beta}}, W_{ij}] - iC_{\alpha\beta}[\nabla_z, \bar{G}_{i\dot{\beta}}].$$

$$[\nabla_{i\alpha}, g_\mu] = -\frac{1}{8}(\sigma^\nu\bar{\sigma}_\mu)_\alpha{}^\beta[\nabla_{i\beta}, g_\nu] - \frac{5}{8}(\sigma^\nu\bar{\sigma}_\mu)_\alpha{}^\beta[\nabla_\nu, G_{i\beta}]$$
$$+\frac{1}{2}(\bar{\sigma}_\mu C)^{\dot{\beta}}{}_\alpha[\bar{G}^j{}_{\dot{\beta}}, W_{ij}] - \frac{1}{4}(\bar{\sigma}_\mu C)^{\dot{\beta}}{}_\alpha[\nabla_z, \bar{G}_{i\dot{\beta}}].$$

$$[\bar{\nabla}_{i\dot{\alpha}}, g_\mu] = -\frac{1}{8}(\bar{\sigma}_\mu\sigma^\nu)^{\dot{\beta}}_{\dot{\alpha}}[\bar{\nabla}_{i\dot{\beta}}, g_\nu] - \frac{5}{8}(\bar{\sigma}_\mu\sigma_\nu)^{\dot{\beta}}_{\dot{\alpha}}[\nabla^\nu, \bar{G}_{i\dot{\beta}}]$$
$$+\frac{1}{2}(C\bar{\sigma}_\mu)_{\dot{\alpha}}{}^\beta[G^j{}_\beta, W_{ij}] + \frac{1}{4}(C\bar{\sigma}_\mu)_{\dot{\alpha}}{}^\beta[\nabla_z, G_{i\beta}].$$

$$[\nabla_{i\alpha}, g_\mu] = \frac{i}{4}(\bar{\sigma})^{\dot{\beta}\beta}\left[\nabla_{i\alpha}, \left\{\bar{\nabla}_{j\dot{\beta}}, G^j_\beta\right\}\right]$$

$$[\nabla_{i\alpha},\{\overline{\nabla}_{j\dot\beta},G^j{}_\beta\}]=\Big[\overline{\nabla}^j{}_{\dot\beta},-\frac{i}{4}\Omega_{ij}(\sigma^{\mu\nu})_{\alpha\beta}F_{\mu\nu}-\frac{1}{2}C_{\alpha\beta}[\nabla_z,W_{ij}]-\frac{1}{4}C_{\alpha\beta}[W_{ik},W^k{}_j]\Big]$$
$$+i\Omega_{ij}(\sigma^\nu)_{\alpha\dot\beta}[\nabla_\nu,G^j{}_\beta],$$
$$\left(\begin{aligned}(\sigma^{\mu\nu})_{\alpha\beta}[\overline{\nabla}_{i\dot\beta},F_{\mu\nu}]&=i(\sigma^{\mu\nu})_{\alpha\beta}[\overline{\nabla}_{i\dot\beta},[\nabla_\mu,\nabla_\nu]]\\&=-2\big(C_{\alpha\beta}(C\bar\sigma^\mu)^{\dot\gamma\gamma}[\nabla_\mu,G_{i\gamma}]+2(\sigma^\mu)_{\beta\dot\beta}[\nabla_{\mu,G_{i\alpha}}]\big),\\ [\overline{\nabla}^j{}_{\dot\beta},[\nabla_z,W_ij]]&=5i[\nabla_z,\overline{G}_{i\dot\beta}]+i[\overline{G}^j{}_{\dot\beta},W_{ij}],\\ [\overline{\nabla}^j{}_{\dot\beta},[W_{ik},W^k{}_j]]&=-5i[\overline{G}^k{}_{\dot\beta},W_{ik}]-[i\Omega_{ik}\overline{G}_{j\dot\beta}+2i\Omega_{j[i}\overline{G}_{k]\dot\beta},W^{kj}]\\&=-8i[\overline{G}^k{}_{\dot\beta},W_{ik}],\end{aligned}\right)$$
$$=-\frac{i}{2}C_{\alpha\beta}(C\bar\sigma^\mu)_{\dot\beta}{}^\gamma[\nabla_\mu,G_{i\gamma}]-i(\sigma^\mu)_{\beta\dot\beta}[\nabla_\mu,G_{i\alpha}]-i(\sigma^\mu)_{\alpha\dot\beta}[\nabla_\mu,G_{i\beta}]$$
$$-\frac{5}{2}iC_{\alpha\beta}[\nabla_z,\overline{G}_{i\dot\beta}]+\frac{3}{2}iC_{\alpha\beta}[\overline{G}^j{}_{\dot\beta},W_{ij}],$$

$$[\nabla_{i\alpha},g_\mu]=\frac{5}{8}\Big([\nabla_\mu,G_{i\alpha}]-(\bar\sigma_\mu C)^{\dot\beta}_{\alpha}[\nabla_z,\bar G_{i\dot\beta}]\Big)$$
$$-\frac{3}{8}\Big((\sigma_{\mu\nu})^{\beta}_{\alpha}[\nabla^\nu,G_{i\beta}]-(\bar\sigma_\mu C)^{\dot\beta}_{\alpha}[\bar G^j_{\dot\beta},W_{ij}]\Big)$$

$$(\sigma^\nu\bar\sigma_\mu)^\beta_\alpha[\nabla_{i\beta},g_\nu]=-2(\sigma_{\mu\nu})^\beta_\alpha[\nabla^\nu,G_{i\beta}]+(\bar\sigma_\mu C)^{\dot\beta}_\alpha[\bar G^j_{\dot\beta},W_{ij}]+3(\bar\sigma_\mu C)^{\dot\beta}_\alpha[\nabla_z,\bar G_{i\dot\beta}]$$

$$[\nabla_{i\alpha},g_\mu]=\frac{1}{2}\Big((\sigma_{\mu\nu})^\beta_\alpha[\nabla^\nu,G_{i\beta}]-(\bar\sigma_\mu C)^{\dot\beta}_\alpha[\bar G^j_{\dot\beta},W_{ij}]\Big)$$
$$+\frac{3}{2}\Big([\nabla_\mu,G_{i\alpha}]-(\bar\sigma_\mu C)^{\dot\beta}_\alpha[\nabla_z,\bar G_{i\dot\beta}]\Big)$$

$$[\nabla_\mu,G_{i\alpha}]-(\bar\sigma_\mu C)^{\dot\beta}_\alpha[\nabla_z,\bar G_{i\dot\beta}]=-(\sigma_{\mu\nu})^\beta_\alpha[\nabla^\nu,G_{i\beta}]+(\bar\sigma_\mu C)^{\dot\beta}_\alpha[\bar G^j{}_{\dot\beta},W_{ij}]$$

$$[\nabla_{i\alpha},g_\mu]=[\nabla_\mu,G_{i\alpha}]-(\bar\sigma_\mu C)^{\dot\beta}_\alpha[\nabla_z,\bar G_{i\dot\beta}]$$
$$=-(\sigma_{\mu\nu})^\beta_\alpha[\nabla^\nu,G_{i\beta}]+(\bar\sigma_\mu C)^{\dot\beta}_\alpha[\bar G^j_{\dot\beta},W_{ij}]$$

$$[\nabla_z,\bar G_{i\dot\alpha}]=-(C\sigma^\mu)^\alpha_{\dot\alpha}[\nabla_\mu,G_{i\alpha}]-[\bar G^j_{\dot\alpha},W_{ij}].$$

$$[\bar\nabla_{i\dot\alpha},g_\mu]=-\frac{5}{8}\Big([\nabla_\mu,\bar G_{i\dot\alpha}]-(C\bar\sigma_\mu)^\beta_{\dot\alpha}[\nabla_z,G_{i\beta}]\Big)$$
$$-\frac{3}{8}\Big((\bar\sigma_{\mu\nu})^{\dot\beta}{}\dot\alpha[\nabla^\nu,\bar G_{i\dot\beta}]-(C\bar\sigma_\mu)^\beta_{\dot\alpha}[G^j{}_\beta,W_{ij}]\Big)$$
$$[\bar\nabla_{i\dot\alpha},g_\mu]=-[\nabla_\mu,\bar G_{i\dot\alpha}]+(C\bar\sigma_\mu)^\beta_{\dot\alpha}[\nabla_z,G_{i\beta}]$$
$$=-(\bar\sigma_{\mu\nu})^{\dot\beta}_{\dot\alpha}[\nabla^\nu,\bar G_{i\dot\beta}]+(C\bar\sigma_\mu)^\beta_{\dot\alpha}[G^j{}_\beta,W_{ij}]$$
$$[\nabla_z,G_{i\alpha}]=-(\sigma^\mu C)_\alpha{}^{\dot\beta}[\nabla_\mu,\bar G_{i\dot\beta}]+[G^j{}_\alpha,W_{ij}]$$

$$\big[\nabla_{i\alpha},[\nabla_z,W_{jk}]\big]=i[\nabla_z,\Omega_{jk}G_{i\alpha}+2\Omega_{i[j}G_{k]\alpha}]-i[G_{i\alpha},W_{jk}]$$
$$=-i(\sigma^\mu C)^{\dot\alpha}_\alpha\big(\Omega_{jk}[\nabla_\mu,\bar G_{i\dot\alpha}]+2\Omega_{i[j}[\nabla_\mu,\bar G_{k]\dot\alpha}]\big)$$
$$+i\big(\Omega_{jk}[G^l_\alpha,W_{il}]+2\Omega_{i[j}[G^l_\alpha,W_{k]l}]\big)$$
$$-i[G_{i\alpha},W_{jk}].$$

$$\big[\bar\nabla_{i\dot\alpha},[\nabla_z,W_{jk}]\big]=-i(C\sigma^\mu)^\alpha{}_{\dot\alpha}\big(\Omega_{jk}[\nabla_\mu,G_{i\alpha}]+2\Omega_{i[j}[\nabla_\mu,G_{k]\alpha}]\big)$$
$$-i\big(\Omega_{jk}[\bar G^l_{\dot\alpha},W_{il}]+2\Omega_{i[j}[\bar G^l_{\dot\alpha},W_{k]l}]\big)$$
$$+i[\bar G_{i\dot\alpha},W_{jk}].$$

$$
\begin{aligned}
[\nabla_z,[\nabla_z,W_{jk}]] &= -\frac{i}{8}\left[\{\nabla_{i\alpha},\nabla^{i\alpha}\},[\nabla_z,W_{jk}]\right] = -\frac{i}{4}\{\nabla_{i\alpha},[\nabla_{i\alpha},[\nabla_z,W_{jk}]]\}\\
&= -\frac{1}{4}\Big\{\nabla^{i\alpha},(\sigma^\mu C)_\alpha{}^{\dot\alpha}\Big(\Omega_{jk}[\nabla_\mu,\overline{G}_{i\dot\alpha}]+2\Omega_{i[j}[\nabla_\mu,\overline{G}_{k]\dot\alpha}]\Big)\\
&\qquad -\Big(\Omega_{jk}[G^j{}_\alpha,W_{il}]+2\Omega_{i[j}[G^l{}_\alpha,W_{k]l}]\Big)+[G_{i\alpha},W_{jk}]\Big\}
\end{aligned}
$$

$$
\left|\;\begin{pmatrix}
\{\nabla^{i\alpha},[\nabla_\mu,\overline{G}_{i\dot\alpha}]\} = 2i(C\sigma^\nu)^\alpha{}_{\dot\alpha}[\nabla_\mu,g_\nu]-i(\bar\sigma_\mu)^{\dot\beta\alpha}\{\overline{G}_{i\dot\alpha},\overline{G}^i{}_{\dot\alpha}\},\\
\{\nabla^{i\alpha},[\nabla_\mu,\overline{G}_{k]\dot\alpha}]\} = -\frac{1}{2}(C\sigma^\nu)^\alpha{}_{\dot\alpha}\left(-i\delta_{k]}{}^i[\nabla_\mu,g_\nu]+[\nabla_\mu,[\nabla_\nu,W^i{}_{k]}]]\right)\\
\qquad -i(\bar\sigma_\mu)^{\dot\beta\alpha}\{\overline{G}_{k]\dot\alpha},\overline{G}^i{}_{\dot\beta}\},\\
\{\nabla^{i\alpha},[G^l{}_\alpha,W_{il}]\} = 5i\{G^l{}_\alpha,G_l{}^\alpha\}-[W_{il},[\nabla_z,W^{il}]],\\
\{\nabla^{i\alpha},[G^l{}_\alpha,W_{k]l}]\} = 3i\{G_{k]\alpha},G^{i\alpha}\}-2i\delta^i{}_{k]}\{G_{l\alpha},G^{l\alpha}\}\\
\qquad -[W_{k]l},[\nabla_z,W^{il}]]-\frac{1}{2}[W_{k]l},[W^i{}_m,W^{ml}]],\\
\{\nabla^{i\alpha},[G_{i\alpha},W_{jk}]\} = -i\Omega_{jk}\{G_{i\alpha},G^{i\alpha}\}+4i\{G_{j\alpha},G_k{}^\alpha\},
\end{pmatrix}\right.
$$

$$
\begin{aligned}
&= [\nabla_\mu,[\nabla_\mu,W_{jk}]]+\frac{1}{4}[W_{[j|l},[W_{k]m},W^{ml}]]\\
&\quad + i\left(\Omega_{jk}\{G_{i\alpha},G^{i\alpha}\}-4\{G_{j\alpha},G_k{}^\alpha\}\right)-i\left(\Omega_{jk}\{\overline{G}_{i\dot\alpha},\overline{G}^{i\dot\alpha}\}-4\{\overline{G}_{j\dot\alpha},\overline{G}_k{}^{\dot\alpha}\}\right)\\
&\quad -\frac{1}{4}\Big(\Omega_{jk}[W_{il},[\nabla_z,W^{il}]]-2[W_{[j|l},[\nabla_z,W_{k]}{}^l]]\Big).
\end{aligned}
$$

$$
\begin{aligned}
\left[\nabla_z,[\nabla_z,W_{jk}]\right] &= -\frac{i}{8}\left[\{\bar\nabla_{i\dot\alpha},\bar\nabla^{i\dot\alpha}\},[\nabla_z,W_{jk}]\right] = -\frac{i}{4}\left\{\bar\nabla_{i\dot\alpha},\left[\bar\nabla_{i\dot\alpha},[\nabla_z,W_{jk}]\right]\right\}\\
&=\left[\nabla_\mu,[\nabla_\mu,W_{jk}]\right]+\frac{1}{4}\left[W_{[j|l},[W_{k]m},W^{ml}]\right]\\
&\quad +i\left(\Omega_{jk}\{G_{i\alpha},G^{i\alpha}\}-4\{G_{j\alpha},G_k{}^\alpha\}\right)-i\left(\Omega_{jk}\{\bar G_{i\dot\alpha},\bar G^{i\dot\alpha}\}-4\{\bar G_{j\dot\alpha},\bar G_k{}^{\dot\alpha}\}\right)\\
&\quad +\frac{1}{4}\left(\Omega_{jk}\left[W_{il},[\nabla_z,W^{il}]\right]-2\left[W_{[j|l},[\nabla_z,W^l_{k]}]\right]\right)
\end{aligned}
$$

$$
\Omega_{jk}\left[W_{il},[\nabla_z,W^{il}]\right]-2\left[W_{[j|l},[\nabla_z,W^l_{k]}]\right]=0
$$

$$
\begin{aligned}
\left[\nabla_z,[\nabla_z,W_{jk}]\right] &=\left[\nabla_\mu,[\nabla_\mu,W_{jk}]\right]+\frac{1}{4}\left[W_{[j|l},[W_{k]m},W^{ml}]\right]\\
&\quad +i\left(\Omega_{jk}\{G_{i\alpha},G^{i\alpha}\}-4\{G_{j\alpha},G_k{}^\alpha\}\right)-i\left(\Omega_{jk}\{\bar G_{i\dot\alpha},\bar G^{i\dot\alpha}\}-4\{\bar G_{j\dot\alpha},\bar G_k{}^{\dot\alpha}\}\right)
\end{aligned}
$$

$$
\begin{aligned}
[\nabla_z,g_\mu] &= \frac{i}{4}(\bar\sigma^\mu)^{\dot\alpha\alpha}[\nabla_z,\{\nabla_{i\alpha},\overline{G}^i{}_{\dot\alpha}\}]\\
&= \frac{i}{4}(\bar\sigma^\mu)^{\dot\alpha\alpha}\Big(\{\nabla_{i\alpha},-(C\sigma^\nu)^\beta{}_{\dot\beta}[\nabla_\nu,G^i{}_\beta]-[\overline{G}^j{}_{\dot\alpha},W^i{}_j]\}+i\{\overline{G}^j{}_{\dot\alpha},G_{i\alpha}\}\Big)
\end{aligned}
$$

$$
\left|\;\begin{pmatrix}
\{\nabla_{i\alpha},[\nabla_\nu,G^i{}_\beta]\} = -i(\sigma^{\rho\sigma})_{\alpha\beta}[\nabla_\nu,F_{\rho\sigma}]-i(\bar\sigma_\nu C)^{\dot\beta}{}_\alpha\{G^i{}_\beta,\overline{G}_{i\dot\beta}\},\\
\{\nabla_{i\alpha},[\overline{G}^j{}_{\dot\alpha},W^i{}_j]\} = -5i\{\overline{G}^j{}_{\dot\alpha},G_{j\alpha}\}+\frac{1}{2}(\sigma^\nu)_{\alpha\dot\alpha}[W^i{}_j,[\nabla_\nu,W_i{}^j]],
\end{pmatrix}\right.
$$

$$
= \wp^+{}_\mu{}^{\nu\rho\sigma}[\nabla_\nu,F_{\rho\sigma}]-2(\bar\sigma_\mu)^{\dot\alpha\alpha}\{G_{i\alpha},\overline{G}^i{}_{\dot\alpha}\}+\frac{i}{4}[W^{ij},[\nabla_\nu,W_{ij}]].
$$

$$
\varepsilon^{\mu\nu\rho\sigma}\left[\nabla_\nu,[\nabla_\rho,\nabla_\sigma]\right]=0
$$

$$
[\nabla_z,g_\mu]=[\nabla^\nu,F_{\mu\nu}]-2(\bar\sigma_\mu)^{\dot\alpha\alpha}\{G_{i\alpha},\bar G^i_{\dot\alpha}\}+\frac{i}{4}\left[W^{ij},[\nabla_\nu,W_{ij}]\right].
$$

$$[\nabla_z, g_\mu] = \wp_\mu^{-\ \nu\rho\sigma}[\nabla_\nu, F_{\rho\sigma}] - 2(\bar{\sigma}_\mu)^{\dot{\alpha}\alpha}\{G_{i\alpha}, \bar{G}^i_{\dot{\alpha}}\} + \frac{i}{4}\left[W^{ij}, [\nabla_\nu, W_{ij}]\right]$$

$$= [\nabla^\nu, F_{\mu\nu}] - 2(\bar{\sigma}_\mu)^{\dot{\alpha}\alpha}\{G_{i\alpha}, \bar{G}^i_{\dot{\alpha}}\} + \frac{i}{4}\left[W^{ij}, [\nabla_\nu, W_{ij}]\right].$$

$$[\nabla^\mu, g_\mu] = -\frac{i}{8}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}[\{\nabla_{i\alpha}, \bar{\nabla}^i_{\dot{\alpha}}\}, g_\mu]$$

$$= -\frac{i}{8}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}\left(\left\{-(\bar{\sigma}_{\mu\nu})^{\dot{\beta}}_{\dot{\alpha}}\left[\nabla^\nu, \bar{G}^i_{\dot{\beta}}\right] + (C\bar{\sigma}_\mu)^{\beta}_{\dot{\alpha}}\left[G^j_\beta, W^i_j\right], \nabla_{i\alpha}\right\}\right.$$

$$\left.\left\{-(\sigma_{\mu\nu})^{\beta}_{\alpha}[\nabla^\nu, G_{i\beta}] + (\bar{\sigma}_\mu C)^{\dot{\beta}}_{\alpha}\left[\bar{G}^j_{\dot{\beta}}, W_{ij}\right], \bar{\nabla}^i_{\dot{\alpha}}\right\}\right)$$

$$\left|\left(\begin{array}{c}
\left\{\left[\nabla^\nu, \bar{G}^i_{\dot{\beta}}\right], \nabla_{i\alpha}\right\} = -2i(\sigma^\rho)_{\alpha\dot{\beta}}[\nabla^\nu, g_\rho] - i(\bar{\sigma}^\nu C)^{\dot{\gamma}}_{\alpha\ \alpha}\left\{\bar{G}_{i\dot{\gamma}}, \bar{G}^i_{\dot{\beta}}\right\}, \\
\{[G^j{}_\beta, W^i{}_j], \nabla_{i\alpha}\} = -5i\{G_{j\alpha}, G^j{}_\beta\} + \frac{1}{2}C_{\alpha\beta}\left[[\nabla_z, W_{ij}], W^{ij}\right], \\
\{[\nabla^\nu, G_{i\beta}], \bar{\nabla}^i_{\dot{\alpha}}\} = 2i(\sigma^\rho)_{\beta\dot{\alpha}}[\nabla^\nu, g_\rho] + i(C\bar{\sigma}^\nu)^{\gamma}_{\dot{\alpha}}\{G^i{}_\gamma, G_{i\beta}\}, \\
\{[\bar{G}^j{}_{\dot{\beta}}, W_{ij}], \bar{\nabla}^i_{\dot{\alpha}}\} = 5i\{\bar{G}_{j\dot{\alpha}}, \bar{G}^j{}_{\dot{\beta}}\} - \frac{1}{2}C_{\dot{\alpha}\dot{\beta}}\left[[\nabla_z, W^{ij}], W_{ij}\right],
\end{array}\right)\right.$$

$$= -3[\nabla^\rho, g_\rho] - 4\{\bar{G}_{i\dot{\gamma}}, \bar{G}^i\dot{\gamma}\} - 4\{G_{i\gamma}, G^{i\gamma}\} + i\left[W^{ij}, [\nabla_z, W_{ij}]\right],$$

$$[\nabla^\mu, g_\mu] = -\{\bar{G}_{i\dot{\gamma}}, \bar{G}^i\dot{\gamma}\} - \{G_{i\gamma}, G^{i\gamma}\} + \frac{i}{4}\left[W^{ij}, [\nabla_z, W_{ij}]\right].$$

$$Z(\beta, g) = \mathrm{tr}\left[\hat{U}(g)e^{-\beta\hat{H}}\right]$$

$$Z(\beta, g) = \mathrm{tr}\left[\hat{U}(\eta)\hat{U}(g)\hat{U}(\eta)^{-1}e^{-\beta H}\right] = \mathrm{tr}\left[\hat{U}(\eta g\eta^{-1})e^{-\beta H}\right] = Z(\beta, \eta g\eta^{-1})$$

$$Z(\beta, \gamma_p) = \mathrm{tr}e^{-\beta\hat{H}}e^{i\gamma_p\hat{Q}_p}$$

$$Z(\beta, \mu_p) \equiv \mathrm{trexp}\left[-\beta(\hat{H} - \mu_p\hat{Q}_p)\right]$$

$$\phi^{ij} \equiv \frac{1}{2}\epsilon^{ijkl}\phi_{kl}.$$

$$\phi_{12} = \phi^*_{34}, \phi_{13} = \phi^*_{42}, \phi_{14} = \phi^*_{23},$$

$$\phi_1 \equiv \phi_{12}, \phi_2 \equiv \phi_{13}, \phi_3 \equiv \phi_{14}.$$

$$Q_1^4 = \frac{1}{2}\mathrm{diag}(1,1,-1,-1),$$
$$Q_2^4 = \frac{1}{2}\mathrm{diag}(1,-1,1,-1),$$
$$Q_3^4 = \frac{1}{2}\mathrm{diag}(1,-1,-1,1).$$

$$\phi^T \equiv (\phi_1, \phi_1^*, \phi_2, \phi_2^*, \phi_3, \phi_3^*)^T$$

$$Q_1^6 = \mathrm{diag}(1,-1,0,0,0,0),$$
$$Q_2^6 = \mathrm{diag}(0,0,1,-1,0,0),$$
$$Q_3^6 = \mathrm{diag}(0,0,0,0,1,-1).$$

$$\sum_{p=1}^{3}\mu_p Q_p^6 = \mathrm{diag}(\mu_1, -\mu_1, \mu_2, -\mu_2, \mu_3, -\mu_3).$$

$$\tilde{\mu}_1 \equiv \frac{1}{2}(\mu_1 + \mu_2 + \mu_3), \quad \tilde{\mu}_2 \equiv \frac{1}{2}(\mu_1 - \mu_2 - \mu_3),$$
$$\tilde{\mu}_3 \equiv \frac{1}{2}(-\mu_1 + \mu_2 - \mu_3), \quad \tilde{\mu}_4 \equiv \frac{1}{2}(-\mu_1 - \mu_2 + \mu_3),$$

$$D_\nu \to D_\nu - \mu_p Q_p \delta_{\nu 0}$$

$$\lambda^{\alpha}(\tau_{\nu})_{\alpha\dot{\beta}}\left(\overleftrightarrow{D}_{\nu}-\tilde{\mu}\delta_{\nu,0}\right)\bar{\lambda}^{\dot{\beta}}=\frac{1}{2}\bar{\psi}(\not{D}-\tilde{\mu}\gamma_0\gamma_5)\psi$$

$$\tau_{\nu}\equiv(\mathbf{1},i\vec{\sigma}),\qquad \bar{\tau}_{\nu}\equiv(\mathbf{1},-i\vec{\sigma})$$
$$\gamma_{\nu}\equiv\begin{pmatrix}0 & \tau_{\nu}\\ \bar{\tau}_{\nu} & 0\end{pmatrix},\quad \gamma_5\equiv\gamma_0\gamma_1\gamma_2\gamma_3=\begin{pmatrix}\mathbf{1} & 0\\ 0 & -\mathbf{1}\end{pmatrix}$$

$$\psi=C\bar{\psi},$$

$$[T^a,T^b]=if^{abc}T^c$$

$$\mathrm{tr}(T^aT^b)=\frac{1}{2}\delta^{ab}$$

$$\Phi\equiv(X_1,Y_1,X_2,Y_2,X_3,Y_3).$$

$$\mathcal{L}=\mathrm{tr}\left\{\frac{1}{2}\left(F_{\mu\nu}\right)^2+\left(D_{\nu}X_p-i\mu_p\delta_{\nu,0}Y_p\right)^2+\left(D_{\nu}Y_p+i\mu_p\delta_{\nu,0}X_p\right)^2+R^{-2}(\Phi_A)^2\right.$$
$$\left.+i\bar{\psi}_i(\not{D}-\tilde{\mu}_i\gamma_0\gamma_5)\psi_i+\frac{1}{2}g^2(i[\Phi_A,\Phi_B])^2-g\bar{\psi}_i\left[\left(\alpha_{ij}^pX_p+i\beta_{ij}^q\gamma_5Y_q\right),\psi_j\right]\right\}$$

$$\{\alpha^p,\alpha^q\}=-2\delta^{pq}\mathbf{1}_{4\times4},\{\beta^p,\beta^q\}=-2\delta^{pq}\mathbf{1}_{4\times4},[\alpha^p,\beta^q]=0$$

$$\alpha^1=\begin{pmatrix}0 & \sigma_1\\ -\sigma_1 & 0\end{pmatrix},\alpha^2=\begin{pmatrix}0 & -\sigma_3\\ \sigma_3 & 0\end{pmatrix},\alpha^3=\begin{pmatrix}i\sigma_2 & 0\\ 0 & i\sigma_2\end{pmatrix},$$
$$\beta^1=\begin{pmatrix}0 & i\sigma_2\\ i\sigma_2 & 0\end{pmatrix},\beta^2=\begin{pmatrix}0 & \sigma_0\\ -\sigma_0 & 0\end{pmatrix},\beta^3=\begin{pmatrix}-i\sigma_2 & 0\\ 0 & i\sigma_2\end{pmatrix}.$$

$$Z=\int\mathcal{D}A_{\mu}\mathcal{D}\psi_i\mathcal{D}\Phi_Ae^{-\int d^4x\mathcal{L}}$$

$$A_0(x)\longrightarrow\tilde{A}_0(x)+a/g$$

$$\left(\omega_k+q_m-q_n\pm i\mu_p\right)^2+(h+1)^2,$$

$$S_{\mathrm{eff}}(U)=-\sum_{n=1}^{\infty}\frac{1}{n}\{z_B(x^n)+(-1)^{n+1}z_F(x^n)\}\left[\mathrm{tr}(U^n)\mathrm{tr}\left(U^{\dagger n}\right)-1\right],$$

$$z_B(x)\equiv z_S(x)+z_V(x)$$
$$z_V(x)\equiv\frac{6x^2-2x^3}{(1-x)^3}$$
$$z_S(x)\equiv\frac{x+x^2}{(1-x)^3}(x^{\mu_1}+x^{-\mu_1}+x^{\mu_2}+x^{-\mu_2}+x^{\mu_3}+x^{-\mu_3})$$
$$z_F(x)\equiv\frac{2x^{3/2}}{(1-x)^3}\left(x^{\frac{1}{2}\mu_1}+x^{-\frac{1}{2}\mu_1}\right)\left(x^{\frac{1}{2}\mu_2}+x^{-\frac{1}{2}\mu_2}\right)\left(x^{\frac{1}{2}\mu_3}+x^{-\frac{1}{2}\mu_3}\right)$$

$$Z(x)=\int dU\exp\left[-S_{\mathrm{eff}}(U)\right]$$

$$S_{\mathrm{eff}}[\rho]=N_c^2\sum_{n=1}^{\infty}V_n\rho_n^2,$$

$$\rho_n\equiv\int_{-\pi}^{\pi}d\theta\rho(\theta)\cos(n\theta)$$

$$V_n\equiv\frac{1}{n}\{1-[z_B(x^n)+(-1)^{n+1}z_F(x^n)]\}.$$

$$F=\min_{\{\rho\}}\frac{S_{\mathrm{eff}}[\rho]}{\beta}+\mathcal{O}(N_c^0)$$

$$z_B(x^n)+(-1)^{n+1}z_F(x^n)<1$$

$$z_B(x) + z_F(x) = 1$$

$$\left\langle \frac{1}{N_c} \mathrm{tr}\mathcal{P}\left(e^{ig\int_0^\beta dx^0 A_0}\right)\right\rangle = \left\langle \frac{1}{N_c} \mathrm{tr} e^{i\beta a}\right\rangle = \left\langle \frac{1}{N_c} \mathrm{tr} U\right\rangle = \int_{-\pi}^{\pi} d\theta \rho(\theta) e^{i\theta} = \rho_1$$

$$e^{-\beta(1-\mu_1)} + e^{-\beta(1-\mu_2)} + e^{-\beta(1-\mu_3)} + 2e^{-\frac{1}{2}\beta(3-\mu_1-\mu_2-\mu_3)} = 1,$$

$$\lim_{N_c\to\infty} \frac{\beta F}{N_c^2} = -\frac{\epsilon^2}{4} + \mathcal{O}(\epsilon^3)$$

$$q_p \equiv \lim_{N_c\to\infty} \frac{Q_p}{N_c^2}$$

$$A_i \equiv \sqrt{Z_1 T}\tilde{A}_i, A_0 \equiv \sqrt{Z_2 T}\tilde{A}_0, \Phi_A \equiv \sqrt{Z_3 T}\tilde{\Phi}_A$$

$$\mathcal{L}_{\mathrm{ESYM}_3} = f + \mathrm{tr}\left[\frac{1}{2}\left(\tilde{F}_{ij}\right)^2 + \left(D_i\tilde{A}_0\right)^2 + M_D^2\tilde{A}_0^2 + \left(D_i\tilde{\Phi}_A\right)^2 + m_A^2\tilde{\Phi}_A^2\right] + V\left(\tilde{A}_0, \tilde{\Phi}_A\right)$$

$$f = -\frac{N_c^2 T^3}{12}\left\{2\pi^2 - 3\lambda - \frac{3}{T^2R^2} + \sum_{p=1}^{3}\frac{2\mu_p^2}{T^2} + \sum_{i=1}^{4}\frac{\tilde{\mu}_i^2}{T^2} + \mathcal{O}(\lambda^2)\right\}$$

$$m_A^2 = R^{-2} - \mu_A^2 + \delta m^2(T),$$

$$\mu_A \equiv \begin{cases}\mu_1, & A = 1 \text{ or } 2\\ \mu_2, & A = 3 \text{ or } 4\\ \mu_3, & A = 5 \text{ or } 6\end{cases}$$

$$\delta m^2(T) = T^2[\lambda + \mathcal{O}(\lambda^2)].$$

$$M_D^2 = T^2[2\lambda + \mathcal{O}(\lambda^2)].$$

$$V\left(\tilde{A}_0, \tilde{\Phi}_A\right) = V_{\mathrm{tree}}\left(\tilde{A}_0, \tilde{\Phi}_A\right) + \delta V\left(\tilde{A}_0, \tilde{\Phi}_A\right).$$

$$V_{\mathrm{tree}}\left(\tilde{A}_0, \tilde{\Phi}_A\right) = \mathrm{tr}\left\{2g_3\mu_p\left([\tilde{A}_0, \tilde{X}_p]\tilde{Y}_p\right) + g_3^2\left(i[\tilde{A}_0, \tilde{\Phi}_A]\right)^2 + \frac{1}{2}g_3^2\left(i[\tilde{\Phi}_A, \tilde{\Phi}_B]\right)^2\right\}.$$

$$\delta V_{\mathrm{quartic}}\left(\tilde{\Phi}_A\right) = \frac{\ln 2}{2\pi^2}\frac{g_3^4}{T}\mathrm{tr}\left(\tilde{\Phi}_B^{\mathrm{adj}}\tilde{\Phi}_C^{\mathrm{adj}}\tilde{\Phi}_B^{\mathrm{adj}}\tilde{\Phi}_C^{\mathrm{adj}}\right)$$

$$\delta V_{\mathrm{flat}}\left(\tilde{\Phi}_A\right) = \frac{1}{2}\pi^2 T^3 \mathrm{tr}\Bigg[(\ln 2)\left(\frac{g_3^2}{\pi^2T^2}\sum_A \tilde{\Phi}_A^{\mathrm{adj}}\tilde{\Phi}_A^{\mathrm{adj}}\right)^2 + \sum_{l=3}^{\infty} 8(1-4^{-l+2})\frac{(2l-5)!!}{(2l)!!}\zeta(2l-3)\left(-\frac{g_3^2}{\pi^2T^2}\sum_A \tilde{\Phi}_A^{\mathrm{adj}}\tilde{\Phi}_A^{\mathrm{adj}}\right)^l\Bigg]$$

$$\rho_A^m \equiv \frac{g_3}{\pi T}\tilde{\lambda}_A^m$$

$$\mathrm{tr}\left[\left(\frac{g_3^2}{\pi^2T^2}\sum_A \tilde{\Phi}_A^{\mathrm{adj}}\tilde{\Phi}_A^{\mathrm{adj}}\right)^l\right] = \sum_{m,n}\sum_A (\rho_A^m - \rho_A^n)^{2l}$$

$$\mu_A^2 < R^{-2} + \lambda T^2.$$

$$\tilde{\Phi}_A = 0, \tilde{A}_0 = 0$$

$$\frac{1}{2}N_c^2 T \int \frac{d^3p}{(2\pi)^3}\ln\left(p^2 + m^2\right) = -N_c^2 T\frac{m^3}{12\pi}$$

$$-\frac{N_c^2 T}{12\pi}\left(M_D^3+\sum_{A=1}^{6} m_A^3\right).$$

$$F_{\text{plasma}}=-\frac{N_c^2}{12}T^4\mathcal{V}\{2\pi^2-3\lambda-\frac{3}{T^2R^2}+\sum_{p=1}^{3}\frac{2\mu_p^2}{T^2}+\sum_{i=1}^{4}\frac{\tilde{\mu}_i^2}{T^2}$$

$$+\frac{(2\lambda)^{3/2}}{\pi}+\frac{2}{\pi}\sum_{p=1}^{3}\left(\lambda+\frac{1}{T^2R^2}-\frac{\mu_p^2}{T^2}\right)^{3/2}+\mathcal{O}(\lambda^2)\right\}$$

$$(\rho_A^m)^2\sim\frac{|m_A^2|}{\lambda T^2}.$$

$$-N_c^2T^3\left[\max_A\left(-\frac{m_A^2}{\lambda T^2}\right)\right]^2$$

$$\tilde{\Phi}_A=\langle\tilde{\Phi}_A\rangle+\delta\tilde{\Phi}_A,$$

$$\left\langle\left(\tilde{\Phi}_A\right)_{mn}\right\rangle=(\pi T/g_3)\rho_A^m\delta_{mn}$$

$$\delta V_{\text{tree}}\left(\tilde{\Phi}_A\right)=\sum_{m,n}M_{mn}^2\left\{\left|\left(\tilde{A}_0\right)_{mn}\right|^2+\sum_B\left|\left(\delta\tilde{\Phi}_B\right)_{mn}\right|^2\right\}+\mathcal{O}(\delta\tilde{\Phi}^3)+\mathcal{O}(\delta\tilde{\Phi}\tilde{A}_0^2)$$

$$M_{mn}^2\equiv\pi^2T^2\sum_A(\rho_A^m-\rho_A^n)^2$$

$$\mathcal{L}_{\text{HSYM}_3}=\text{tr}\left[\frac{1}{2}\left(\tilde{F}_{ij}\right)^2+\left(D_i\tilde{A}_0\right)^2+M_D^2\tilde{A}_0^2+\left(D_i\tilde{\Phi}_A\right)^2\right]+\bar{V}\left(\tilde{\Phi}_A\right)$$

$$\bar{V}\left(\tilde{\Phi}_A\right)=f+\text{tr}\left(m_A^2\tilde{\Phi}_A^2\right)+\delta V_{\text{flat}}\left(\tilde{\Phi}_A\right)+\delta V_{\text{off-diag}}\left(\tilde{\Phi}_A\right),$$

$$\delta V_{\text{off-diag}}\left(\tilde{\Phi}_A\right)=-\frac{2}{3}\pi^2T^3\text{tr}\left[\left(\frac{g_3^2}{\pi^2T^2}\sum_A\tilde{\Phi}_A^{\text{adj}}\tilde{\Phi}_A^{\text{adj}}\right)^{3/2}\right]$$

$$\bar{V}(\rho_A^m)=\frac{1}{2}\pi^2T^3\sum_{m,n}\left[h(\rho^{mn})+\sum_A\delta\hat{m}_A^2(\rho_A^m-\rho_A^n)^2\right]$$

$$\rho^{mn}\equiv\left[\sum_A(\rho_A^m-\rho_A^n)^2\right]^{1/2}$$

$$\delta\hat{m}_A^2\equiv\frac{m_A^2}{\lambda T^2}-1=\frac{R^{-2}-\mu_A^2}{\lambda T^2},$$

$$h(v)=-\frac{1}{3}+v^2-\frac{4}{3}v^3+(\ln 2)v^4+\sum_{l=3}^{\infty}8(1-4^{-l+2})\frac{(2l-5)!!}{(2l)!!}\zeta(2l-3)(-v^2)^l$$

$$h(v)=-\frac{1}{3}+v^2-\frac{4}{3}v^3+\frac{8}{3}\sum_{j=1}^{\infty}(-1)^{j-1}\left[(j^2+v^2)^{3/2}-j^3-\frac{3}{2}v^2j\right]$$

$$h(v)\sim-\frac{4}{\pi^2}(2v)^{3/2}e^{-\pi v}$$

$$\rho_A^m=v\delta_{A,1}\left(\delta_{m,1}-1/N_c\right)$$

$$\bar{V}(v)=-\frac{1}{6}\pi^2T^3N_c^2+\pi^2T^3N_c\left[h(v)+\frac{1}{3}+\delta\hat{m}_1^2v^2\right]+\mathcal{O}(N_c^0).$$

$$\Delta\bar{V}\equiv\max_v\bar{V}(v)-\bar{V}(0)=\mathcal{O}(N_cT^3).$$

$$e^{-\mathcal{V}\Delta\bar{V}} = e^{-\mathcal{O}(N_c(TR)^3)},$$

$$e^{-S_{\text{bounce}}} = e^{-\mathcal{O}(N_c\lambda^{-3/2})}$$

$$\mu_{\max}(T) = \sqrt{\lambda T^2 + R^{-2}} \times [1 + \mathcal{O}(\lambda)]$$

$$A_\nu(x) \longrightarrow \tilde{A}_\nu(x) + \delta_{\nu,0}a/g$$

$$\text{tr}\left\{\left(\partial_\nu\tilde{A}_\nu + i[a,\tilde{A}_0]\right)^2\right\}$$

$$\square^{mn} \equiv \left(\partial_\nu + iq^{mn}\delta_{\nu,0}\right)^2$$

$$\begin{aligned}\hat{\mathcal{L}}_{\text{quad}} = \sum_{m,n} & \Big\{(\tilde{A}_\nu)^*_{mn}(-\square^{mn})(\tilde{A}_\nu)_{mn} \\ & + \left((X_p)^*_{mn}, (Y_p)^*_{mn}\right)\begin{pmatrix} -\square^{mn} + 1 - \mu_p^2 & 2i\mu_p(\partial_0 + iq^{mn}) \\ -2i\mu_p(\partial_0 + iq^{mn}) & -\square^{mn} + 1 - \mu_p^2\end{pmatrix}\begin{pmatrix}(X_p)_{mn} \\ (Y_p)_{mn}\end{pmatrix} \\ & + (\bar{\psi}_i)^*_{mn}(i\,\partial\!\!\!/ - \gamma_0 q^{mn} - i\tilde{\mu}_i\gamma_0\gamma_5)(\psi_i)_{mn}\Big\}\end{aligned}$$

$$\ln Z_{\text{g}} = -\frac{1}{2}\left[\sum_k \sum_{m,n} \text{trln}\left\{(\omega_k + q^{mn})^2 + \Delta_g^2\right\} - \text{trln}\left(\omega_k^2 + \Delta_g^2\right)\right].$$

$$\begin{aligned}\ln Z_{\text{s}} = -\frac{1}{2}\sum_k \sum_p \Bigg[\sum_{m,n} & \text{trln}\left[\left\{\left(\omega_k + q^{mn} + i\mu_p\right)^2 + \Delta_s^2\right\}\left\{\left(\omega_k + q^{mn} - i\mu_p\right)^2 + \Delta_s^2\right\}\right] \\ & -\text{trln}\left[\left\{\left(\omega_k + i\mu_p\right)^2 + \Delta_s^2\right\}\left\{\left(\omega_k - i\mu_p\right)^2 + \Delta_s^2\right\}\right]\Bigg].\end{aligned}$$

$$\begin{aligned}\ln Z_{\text{f}} = \frac{1}{2}\sum_k \sum_i \Bigg[\sum_{m,n} & \text{trln}\left[\{(\omega_k + q^{mn} + i\tilde{\mu}_i)^2 + \Delta_f^2\}\{(\omega_k + q^{mn} - i\tilde{\mu}_i)^2 + \Delta_f^2\}\right] \\ & -\text{trln}\left[\{(\omega_k + i\tilde{\mu}_i)^2 + \Delta_f^2\}\{(\omega_k - i\tilde{\mu}_i)^2 + \Delta_f^2\}\right]\Bigg].\end{aligned}$$

$$\ln Z_{\text{g}} = -\frac{1}{2}\sum_{m,n} \text{trln}\left[\left\{1 - e^{-\beta(\Delta_g + iq^{mn})}\right\}\left\{1 - e^{-\beta(\Delta_g - iq^{mn})}\right\}\right] + \text{trln}\left(1 - e^{-\beta\Delta_g}\right)$$

$$\begin{aligned}\ln Z_{\text{s}} = -\sum_p \Bigg[\frac{1}{2}\sum_{m,n} & \text{trln}\left[\left\{1 - e^{-\beta(\Delta_s + \mu_p + iq^{mn})}\right\}\left\{1 - e^{-\beta(\Delta_s + \mu_p - iq^{mn})}\right\}\right. \\ & \left.\times\left\{1 - e^{-\beta(\Delta_s - \mu_p + iq^{mn})}\right\}\left\{1 - e^{-\beta(\Delta_s - \mu_p - iq^{mn})}\right\}\right] \\ & -\text{trln}\left[\left\{1 - e^{-\beta(\Delta_s + \mu_p)}\right\}\left\{1 - e^{-\beta(\Delta_s - \mu_p)}\right\}\right]\Bigg]\end{aligned}$$

$$\begin{aligned}\ln Z_{\text{f}} = \sum_i \Bigg[\frac{1}{2}\sum_{m,n} & \text{trln}\left[\left\{1 + e^{-\beta(\Delta_f + \tilde{\mu}_i + iq^{mn})}\right\}\left\{1 + e^{-\beta(\Delta_f + \tilde{\mu}_i - iq^{mn}}\right\}\right. \\ & \left.\times\left\{1 + e^{-\beta(\Delta_f - \tilde{\mu}_i + iq^{mn})}\right\}\left\{1 + e^{-\beta(\Delta_f - \tilde{\mu}_i - iq^{mn})}\right\}\right] \\ & -\text{trln}\left[\left\{1 + e^{-\beta(\Delta_f + \tilde{\mu}_i)}\right\}\left\{1 + e^{-\beta(\Delta_f - \tilde{\mu}_i)}\right\}\right]\Bigg]\end{aligned}$$

$$\begin{aligned}& -\sum_p \left[\frac{1}{2}\sum_{m,n} \text{trln}\left\{1 - e^{-\beta(\Delta_s + \mu_p + iq^{mn})}\right\} - \frac{1}{2}\text{trln}\left\{1 - e^{-\beta(\Delta_s + \mu_p)}\right\}\right] \\ & = \frac{1}{2}\sum_p \sum_{h=0}^{\infty} (h+1)^2 \sum_{l=1}^{\infty} \frac{1}{l} e^{-l\beta\Delta_s} e^{-l\beta\mu_p}\left(\sum_{m,n} e^{-il\beta q^m} e^{il\beta q^n} - 1\right) \\ & = \frac{1}{2}\sum_{l=1}^{\infty} \frac{1}{l}\left(\sum_{h=0}^{\infty} (h+1)^2 e^{-l\beta\Delta_s}\right)\left(\sum_p e^{-l\beta\mu_p}\right)\left(\sum_m e^{-il\beta q^m}\sum_n e^{il\beta q^n} - 1\right) \\ & = \frac{1}{2}\sum_{l=1}^{\infty} \frac{1}{l}\frac{x^l + x^{2l}}{(1 - x^l)^3}\left(\sum_p x^{l\mu_p}\right)\left(\text{tr}U^l \text{tr}U^{\dagger l} - 1\right)\end{aligned}$$

$$\ln Z_g = \sum_{l=1}^{\infty} \frac{1}{l} \frac{6x^{2l} - 2x^{3l}}{(1-x^l)^3} (\mathrm{tr}U^l \mathrm{tr}U^{\dagger l} - 1)$$

$$\ln Z_s = \sum_{l=1}^{\infty} \frac{1}{l} \frac{x^l + x^{2l}}{(1-x^l)^3} \left[\sum_{p=1}^{3} (x^{l\mu_p} + x^{-l\mu_p}) \right] (\mathrm{tr}U^l \mathrm{tr}U^{\dagger l} - 1)$$

$$\ln Z_f = \sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{l} \frac{2x^{\frac{3}{2}l}}{(1-x^l)^3} \left[\sum_{i=1}^{4} (x^{l\tilde{\mu}_i} + x^{-l\tilde{\mu}_i}) \right] (\mathrm{tr}U^l \mathrm{tr}U^{\dagger l} - 1)$$

$$= \sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{l} \frac{2x^{\frac{3}{2}l}}{(1-x^l)^3} \left[\prod_{p=1}^{3} \left(x^{\frac{1}{2}l\mu_p} + x^{-\frac{1}{2}l\mu_p} \right) \right] (\mathrm{tr}U^l \mathrm{tr}U^{\dagger l} - 1)$$

$$S_{\mathrm{eff}}(U) \equiv -\ln Z_g - \ln Z_s - \ln Z_f,$$

$$\frac{F}{\mathcal{V}} = -\frac{\pi^2}{45} T^4 + \frac{m^2 - 2\mu^2}{12} T^2 - \frac{(m^2 - \mu^2)^{3/2}}{6\pi} T + \cdots,$$

$$\frac{F^{n=0}}{\mathcal{V}} = T \int \frac{d^3p}{(2\pi)^3} \ln(p^2 + m^2 - \mu^2) = -\frac{(m^2 - \mu^2)^{3/2}}{6\pi} T.$$

$$\frac{F^{n\neq0}}{\mathcal{V}} = -T^4 \left\{ \frac{\pi^2}{45} - \frac{1}{12} \left(\frac{m^2 - 2\mu^2}{T^2} \right) + \mathcal{O}\left(\frac{\mu^2 m^2}{T^4}, \frac{m^4}{T^4}, \frac{\mu^4}{T^4} \right) \right\}.$$

$$\frac{F_B^{n\neq0}}{\mathcal{V}} = -N_c^2 T^4 \left\{ \frac{4\pi^2}{45} - \frac{1}{12} \left[\frac{3}{T^2 R^2} - \frac{2(\mu_1^2 + \mu_2^2 + \mu_3^2)}{T^2} \right] + \mathcal{O}(\lambda^2) \right\},$$

$$\frac{F}{\mathcal{V}} = -T^4 \left(\frac{7\pi^2}{360} + \frac{\mu^2}{12T^2} + \frac{\mu^4}{24T^4} \right)$$

$$\frac{F_F}{\mathcal{V}} = -N_c^2 T^4 \left\{ \frac{7\pi^2}{90} + \frac{1}{12} \left(\frac{\tilde{\mu}_1^2 + \tilde{\mu}_2^2 + \tilde{\mu}_3^2 + \tilde{\mu}_4^2}{T^2} \right) + \mathcal{O}(\lambda^2) \right\}$$

$$\frac{F_{1-\mathrm{loop}}}{\mathcal{V}} = -\frac{N_c^2 T^4}{12} \left\{ 2\pi^2 - \frac{3}{T^2 R^2} + \frac{2(\mu_1^2 + \mu_2^2 + \mu_3^2)}{T^2} + \frac{\tilde{\mu}_1^2 + \tilde{\mu}_2^2 + \tilde{\mu}_3^2 + \tilde{\mu}_4^2}{T^2} + \mathcal{O}(\lambda^2) \right\}.$$

$$\frac{F_{2-\mathrm{loop}}}{\mathcal{V}} = \frac{1}{4} N_c^2 T^4 \lambda + \mathcal{O}(\lambda^2)$$

$$f = -\frac{N_c^2 T^3}{12} \left\{ 2\pi^2 - 3\lambda - \frac{3}{T^2 R^2} + \frac{2(\mu_1^2 + \mu_2^2 + \mu_3^2)}{T^2} + \frac{\tilde{\mu}_1^2 + \tilde{\mu}_2^2 + \tilde{\mu}_3^2 + \tilde{\mu}_4^2}{T^2} + \mathcal{O}(\lambda^2) \right\}$$

$$\mathcal{Y}_p' \equiv \left(\frac{e^{\gamma_E} \Lambda^2}{4\pi} \right)^{\epsilon} T \sum_{n\neq0} \int \frac{d^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}}.$$

$$(S1) = -\frac{5}{12} \lambda T^2$$

$$(S2) = -\frac{1}{12} (3 + \alpha) \lambda T^2$$

$$(S3) = \frac{\alpha}{12} \lambda T^2 + \frac{\lambda}{16\pi^2} \left(\frac{1}{\epsilon} + L_b \right) (3 - \alpha) p^2 + \mathcal{O}\left(\frac{p^4}{T^2} \right)$$

$$(S4) = -\frac{1}{3} \lambda T^2 - \frac{\lambda}{4\pi^2} \left(\frac{1}{\epsilon} + L_f \right) p^2 + \mathcal{O}\left(\frac{p^4}{T^2} \right)$$

$$(S5) = -\delta Z^{(1)} p^2$$

$$L_b \equiv \ln \frac{\Lambda^2}{T^2} - 2\ln(4\pi) + 2\gamma_E, L_f \equiv \ln \frac{\Lambda^2}{T^2} - 2\ln(4\pi) + 2\gamma_E + 4\ln 2$$

$$-\Pi_{\text{hard}}(p) \equiv (S1)+(S2)+(S3)+(S4)+(S5)$$
$$= -\lambda T^2 + p^2 \frac{\lambda}{16\pi^2}\left\{(3-\alpha)L_b - 4L_f - \frac{1+\alpha}{\epsilon}\right\} - p^2 \delta Z^{(1)} + \mathcal{O}\left(\frac{p^4}{T^2}\right)$$

$$\delta Z^{(1)} = -(1+\alpha)\frac{\lambda}{16\pi^2\epsilon}$$

$$-\Pi_{\text{hard}}(p) = -\lambda T^2 + p^2 \frac{\lambda}{16\pi^2}\{(3-\alpha)L_b - 4L_f\} + \mathcal{O}\left(\frac{p^4}{T^2}\right)$$

$$\langle \Phi_A(x)\Phi_B(y)\rangle = \delta_{AB} T \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} G(p)$$

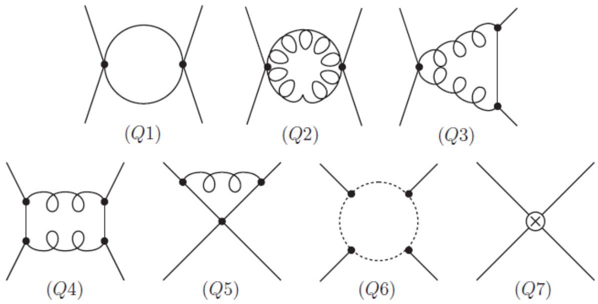
$$G^{-1}(p) = p^2 + \Pi_{\text{hard}}(p) + \Pi_{\text{soft}}(p)$$
$$= p^2(1+A) + \lambda T^2 + \Pi_{\text{soft}}(p) + \mathcal{O}(p^4/T^2) + \mathcal{O}(\lambda^2 T^2)$$

$$\langle \tilde{\Phi}_A(x)\tilde{\Phi}_B(y)\rangle = \delta_{AB} \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \tilde{G}(p)$$

$$\tilde{G}^{-1}(p) = p^2 + \delta m^2(T) + \Pi_{\text{soft}}(p),$$

$$Z_3 = (1+A)^{-1} = 1 + \frac{\lambda}{16\pi^2}\{(3-\alpha)L_b - 4L_f\} + \mathcal{O}(\lambda^2),$$

$$\delta m^2(T) = \lambda T^2 + \mathcal{O}(\lambda^2 T^2).$$



$$(\Gamma_1)^{abcd}_{ABCD} \equiv N_c^{-2}[\text{tr}(V^aV^bV^cV^d)(\delta_{AB}\delta_{CD} + \delta_{AD}\delta_{BC} - 2\delta_{AC}\delta_{BD})$$
$$+\text{tr}(V^aV^bV^dV^c)(\delta_{AC}\delta_{BD} + \delta_{AB}\delta_{CD} - 2\delta_{AD}\delta_{BC})$$
$$+\text{tr}(V^aV^cV^bV^d)(\delta_{AC}\delta_{BD} + \delta_{AD}\delta_{BC} - 2\delta_{AB}\delta_{CD})]$$
$$(\Gamma_2)^{abcd}_{ABCD} \equiv N_c^{-2}[\text{tr}(V^aV^bV^cV^d)(\delta_{AB}\delta_{CD} + \delta_{AD}\delta_{BC})$$
$$+\text{tr}(V^aV^bV^dV^c)(\delta_{AC}\delta_{BD} + \delta_{AB}\delta_{CD})$$
$$+\text{tr}(V^aV^cV^bV^d)(\delta_{AC}\delta_{BD} + \delta_{AD}\delta_{BC})]$$

$$(Q1) = \frac{\lambda^2}{16\pi^2}\left(\frac{1}{\epsilon} + L_b\right)[4(\Gamma_1)^{abcd}_{ABCD} + 5(\Gamma_2)^{abcd}_{ABCD}],$$
$$(Q2) = (3+\alpha^2)\frac{\lambda^2}{16\pi^2}\left(\frac{1}{\epsilon} + L_b\right)(\Gamma_2)^{abcd}_{ABCD},$$
$$(Q3) = -2\alpha^2\frac{\lambda^2}{16\pi^2}\left(\frac{1}{\epsilon} + L_b\right)(\Gamma_2)^{abcd}_{ABCD},$$
$$(Q4) = \alpha^2\frac{\lambda^2}{16\pi^2}\left(\frac{1}{\epsilon} + L_b\right)(\Gamma_2)^{abcd}_{ABCD},$$
$$(Q5) = -4\alpha\frac{\lambda^2}{16\pi^2}\left(\frac{1}{\epsilon} + L_b\right)(\Gamma_1)^{abcd}_{ABCD},$$
$$(Q6) = -8\frac{\lambda^2}{16\pi^2}\left(\frac{1}{\epsilon} + L_f\right)[(\Gamma_1)^{abcd}_{ABCD} + (\Gamma_2)^{abcd}_{ABCD}],$$
$$(Q7) = -4\lambda\delta Z^{(1)}(\Gamma_1)^{abcd}_{ABCD}.$$

$$(Q7) = 4\frac{\lambda^2}{16\pi^2\epsilon}(1+\alpha)(\Gamma_1)^{abcd}_{ABCD}$$

$$\langle \Phi_A^a \Phi_B^b \Phi_C^c \Phi_D^d \rangle_{1\text{PI}} = (\Gamma_1)_{ABCD}^{abcd} \left(-2\lambda + \frac{\lambda^2}{4\pi^2} \left[(1-\alpha) L_b - 2L_f \right] \right)$$

$$+ (\Gamma_2)_{ABCD}^{abcd} \frac{\lambda^2}{2\pi^2} \left[L_b - L_f \right] + \mathcal{O}(\lambda^3)$$

$$\langle \tilde{\Phi}_A^a \tilde{\Phi}_B^b \tilde{\Phi}_C^c \tilde{\Phi}_D^d \rangle_{1\text{PI}} = -2\lambda T^2 (\Gamma_1)_{ABCD}^{abcd} - (\ln\ 4) \frac{\lambda^2 T^2}{\pi^2} \left[(\Gamma_2)_{ABCD}^{abcd} - (\Gamma_1)_{ABCD}^{abcd} \right] + \mathcal{O}(\lambda^3)$$

$$\frac{1}{2} g_3^2 \operatorname{tr}\left(i [\tilde{\Phi}_A, \tilde{\Phi}_B] \right)^2 + \frac{\ln\ 2}{2\pi^2} \frac{g_3^4}{T} \operatorname{tr}\left(\tilde{\Phi}_A^a V_a \tilde{\Phi}_B^b V_b \right)^2$$

$$\frac{1}{\xi} \operatorname{tr} \left\{ \left(\partial_\mu A_\mu - i\xi g [\sigma_A, \bar{\Phi}_A] \right)^2 \right\}$$

$$M_{mn}^2 \equiv g^2 \sum_A (\lambda_A^m - \lambda_A^n)^2$$

$$\sum_{m,n} \left[(A_\mu)_{mn}^* (-\partial^2 + M_{mn}^2)(A_\mu)_{mn} + (\sigma_A)_{mn}^* (-\partial^2 + M_{mn}^2)(\sigma_A)_{mn} \right.$$

$$\left. + (\bar{\psi}_i)_{mn} \left[\delta_{ij} i \not{\partial} - g \left\{ \alpha_{ij}^p + i\gamma_5 \beta_{ij}^p \right\} (\lambda_p^m - \lambda_p^n) \right] (\psi_j)_{mn} \right]$$

$$I = \mathcal{Y}_p' \ln (p^2 + M_{mn}^2),$$

$$\frac{I_{\text{dark particle}}}{\pi^2 T^4} = -\frac{1}{45} + \frac{1}{3} \left(\frac{M_{mn}^2}{4\pi^2 T^2} \right) - \frac{1}{2} \left(\frac{1}{\epsilon} + L_b \right) \left(\frac{M_{mn}^2}{4\pi^2 T^2} \right)^2 - \sum_{l=3}^{\infty} 8 \frac{(2l-5)!!}{(2l)!!} \zeta(2l-3) \left(\frac{-M_{mn}^2}{4\pi^2 T^2} \right)^l,$$

$$\frac{I_{\text{white particle}}}{\pi^2 T^4} = \frac{7}{360} - \frac{1}{6} \left(\frac{M_{mn}^2}{4\pi^2 T^2} \right) - \frac{1}{2} \left(\frac{1}{\epsilon} + L_f \right) \left(\frac{M_{mn}^2}{4\pi^2 T^2} \right)^2 - \sum_{l=3}^{\infty} (4^l - 8) \frac{(2l-5)!!}{(2l)!!} \zeta(2l-3) \left(\frac{-M_{mn}^2}{4\pi^2 T^2} \right)^l,$$

$$\bar{V}(\bar{\Phi}_A) = \frac{1}{2} \pi^2 T^3 \sum_{m,n} \left[-\frac{1}{3} + \left(\frac{M_{mn}^2}{\pi^2 T^2} \right) + (\ln\ 2) \left(\frac{M_{mn}^2}{\pi^2 T^2} \right)^2 \right.$$

$$\left. + \sum_{l=3}^{\infty} 8(1 - 4^{-l+2}) \frac{(2l-5)!!}{(2l)!!} \zeta(2l-3) \left(-\frac{M_{mn}^2}{\pi^2 T^2} \right)^l \right].$$

$$\left(\bar{\Phi}_A^{\text{adj}} \right)_{ab} = 2 \operatorname{tr}(T_a [\bar{\Phi}_A, T_b])$$

$$\operatorname{tr}\left(\bar{\Phi}_A^{\text{adj}} \bar{\Phi}_B^{\text{adj}} \right) = 2 \sum_a \operatorname{tr}\left(T_a [\bar{\Phi}_A, [\bar{\Phi}_B, T_a]] \right)$$

$$= 2 \sum_{a,m,n} (T_a)_{nm} (\lambda_A^m - \lambda_A^n)(\lambda_B^m - \lambda_B^n)(T_a)_{mn}$$

$$= \sum_{m,n} (\lambda_A^m - \lambda_A^n)(\lambda_B^m - \lambda_B^n)$$

$$\sum_a (T_a)_{ij} (T_a)_{kl} = \frac{1}{2} \left(\delta_{il} \delta_{jk} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \right)$$

$$\operatorname{tr}\left[g^2 \sum_A \bar{\Phi}_A^{\text{adj}} \bar{\Phi}_A^{\text{adj}} \right] = g^2 \sum_{A,m,n} (\lambda_A^m - \lambda_A^n)^2 = \sum_{m,n} M_{mn}^2$$

$$\frac{1}{2} g^2 T^2 \operatorname{tr}\left(\bar{\Phi}_A^{\text{adj}} \bar{\Phi}_A^{\text{adj}} \right) = \frac{1}{2} T^2 \sum_{m,n} M_{mn}^2 = g^2 T^2 N_c \sum_{A,m} (\lambda_A^m)^2 = \lambda T^2 \operatorname{tr}(\bar{\Phi}_A)^2,$$

$$\operatorname{tr}\left[\left(g^2 \sum_A \bar{\Phi}_A^{\text{adj}} \bar{\Phi}_A^{\text{adj}} \right)^l \right] = \sum_{m,n} (M_{mn}^2)^l$$

$$\frac{\beta}{2\pi^2}(\ln\ 2)g^4\mathrm{tr}\left[\left(\bar{\Phi}_A^{\mathrm{adj}}\,\bar{\Phi}_A^{\mathrm{adj}}\right)^2\right]$$

$$-N_c^2\int\ d\theta d\theta'\rho(\theta)\rho(\theta')\ln\ \left|2\sin\ \left(\frac{\theta}{2}-\frac{\theta'}{2}\right)\right| S_{\mathrm{eff}}[\rho]$$

$$\ln\ \left|2\sin\ \frac{x}{2}\right| = -\sum_{n=1}^{\infty}\frac{1}{n}\cos\ nx$$

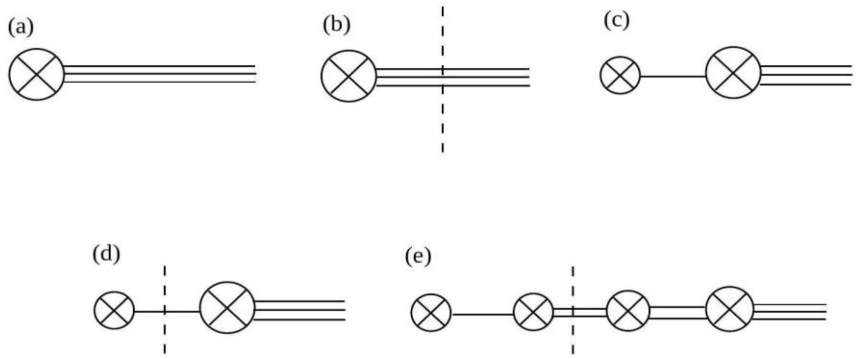
$$N_c^2\sum_{n=1}^{\infty}\frac{1}{n}\rho_n^2$$

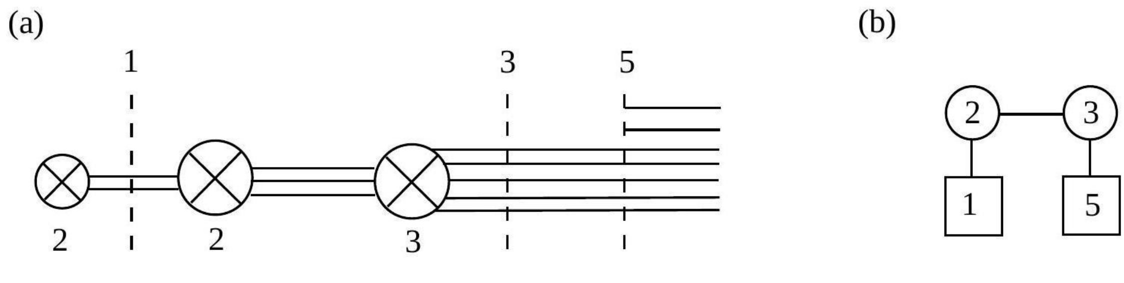
$$\langle\mathrm{tr}U\rangle = N_c\left[1+\left(1-\frac{1}{N_c^2}\right)\frac{\lambda^{3/2}}{8\sqrt{2}\pi}+\mathcal{O}(\lambda^2)\right]$$

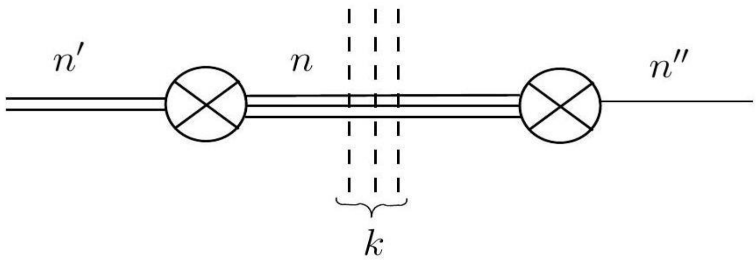
$$h(z) = -\frac{1}{3}+v^2+\frac{4}{3}\lim_{\epsilon\to 0}\int_C\frac{dz}{2i}\csc\,(\pi z)\left[(z^2+v^2)^{3/2}-(z^2+\epsilon^2)^{3/2}-\frac{3}{2}v^2(z^2+\epsilon^2)^{1/2}\right]$$

$$\int\ d^3y\left[\frac{1}{2}(\nabla\rho)^2+h(\rho)\right]$$

$$\begin{aligned}
\mathrm{tr}(V^aV^b) &= N_c\delta^{ab}\\
\mathrm{tr}(V^aV^bV^c) &= -\frac{i}{2}N_cf^{abc}\\
\mathrm{tr}(V^aV^bV^cV^d) &= N_c\mathrm{tr}(T^aT^bT^cT^d+T^bT^aT^dT^c)+\frac{1}{2}(\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc})\\
\mathrm{tr}(V^aV^bV^cV^d) &- \mathrm{tr}(V^bV^aV^cV^d) = -2N_cf^{abe}f^{cde}
\end{aligned}$$







$$n' + n'' + k \geq 2n.$$

$$n_f \geq 2n.$$

$$A = (M \quad 0), B = \binom{0}{N}$$

$$\ell_i = n_i - n_{i-1} + \#\{a|\widetilde{\ell}_a < i\}.$$

$$n_i = \sum_{j=1}^{i} \ell_j - \sum_{a|\tilde{\ell}_a<i} (i - \tilde{\ell}_a)$$

$$n_P = \sum_{j=1}^{P} \ell_j - \sum_{a|\tilde{\ell}_a<P} (P - \tilde{\ell}_a).$$

$$n = \sum_{j=1}^{P} \ell_j - \sum_{a=1}^{Q} (P - \tilde{\ell}_a) = \sum_{j=1}^{P} \ell_j + \sum_{a=1}^{Q} \tilde{\ell}_a - PQ.$$

$$\sum_{j=1}^{i} \ell_j > \sum_{a|\tilde{\ell}_a<i} (i - \tilde{\ell}_a)$$

$$\sum_{j=1}^{i} \ell_j + \sum_{a=1}^{b} \tilde{\ell}_a > bi$$

$$\tilde{n}_b = \sum_{a=1}^{b} \tilde{\ell}_a - \sum_{j|\ell_j<b} (b - \ell_j)$$

$$F = \frac{a}{2} \star \mathrm{d}\frac{1}{|\vec{x} - \vec{x}_0|}$$

$$q_R = \frac{1}{2}\sum_{i=1}^{s} |aq_i|,$$

$$F = \frac{\rho(1)}{2} \star \mathrm{d}\frac{1}{|\vec{x} - \vec{x}_0|}$$

$$q_R = \frac{1}{2}\left(\sum_i |h_i| - \sum_j |v_j|\right).$$

$$q_R = \frac{n_f}{2}\sum_{i=1}^{n_c} |a_i| - \sum_{1\leq i<j\leq n_c} |a_i - a_j|$$

$$q_R = \frac{n_f - 2n_c + 2}{2}\sum_{i=1}^{n_c} |a_i| + \sum_{1\leq i<j\leq n_c} (|a_i| + |a_j| - |a_i - a_j|).$$

$$e_i = m_i + n_{i-1} + n_{i+1} - 2n_i$$

$$\frac{m_i}{2}\sum_{k=1}^{n_i} |a_{i,k}|$$

$$-\frac{1}{2}\sum_{k=1}^{n_i}\sum_{t=1}^{n_i}|a_{i,k}-a_{i,t}|$$

$$\frac{1}{2}\sum_{k=1}^{n_i}\sum_{t=1}^{n_{i+1}}|a_{i,k}-a_{i+1,t}|$$

$$\Delta_i=\frac{e_i}{2}\sum_{k=1}^{n_i}|a_{i,k}|$$

$$A_i=\frac{1}{2}\sum_{k=1}^{n_i}\sum_{t=1}^{n_i}\left(|a_{i,k}|+|a_{i,t}|-|a_{i,k}-a_{i,t}|\right)$$
$$B_i=-\frac{1}{2}\sum_{k=1}^{n_i}\sum_{t=1}^{n_{i+1}}\left(|a_{i,k}|+|a_{i+1,t}|-|a_{i,k}-a_{i+1,t}|\right).$$

$$A_i=\frac{1}{2}\sum_{k=1}^{n_i}\sum_{t=1}^{n_i}2\min(a_{i,k},a_{i,t})=\sum_{k=1}^{n_i}a_{i,k}(2n_i-2k+1)$$

$$B_i=-\frac{1}{2}\sum_{k=1}^{n_i}\sum_{t=1}^{n_{i+1}}2\min(a_{i,k},a_{i+1,t})$$

$$B_i\geq-\sum_{k=1}^{n_i}a_{i,k}(n_i-k)-\sum_{t=1}^{n_{i+1}}a_{i+1,t}(n_{i+1}-t+1)$$

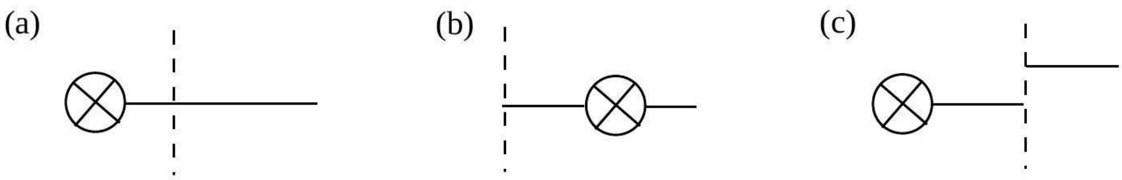
$$\sum_{i=1}^{P-1}(A_i+B_i)\geq\sum_{k=1}^{n_1}a_{1,k}(n_1-k+1)+\sum_{k=1}^{n_{P-1}}a_{P-1,k}(n_{P-1}-k).$$

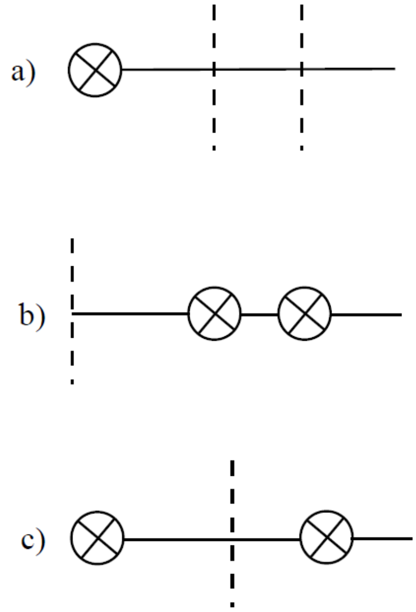
$$q_R\geq\sum_{i=1}^{P-1}\frac{e_i}{2}\sum_{k=1}^{n_i}a_{i,k}+\sum_{k=1}^{n_1}a_{1,k}(n_1-k+1)+\sum_{k=1}^{n_{P-1}}a_{P-1,k}(n_{P-1}-k).$$

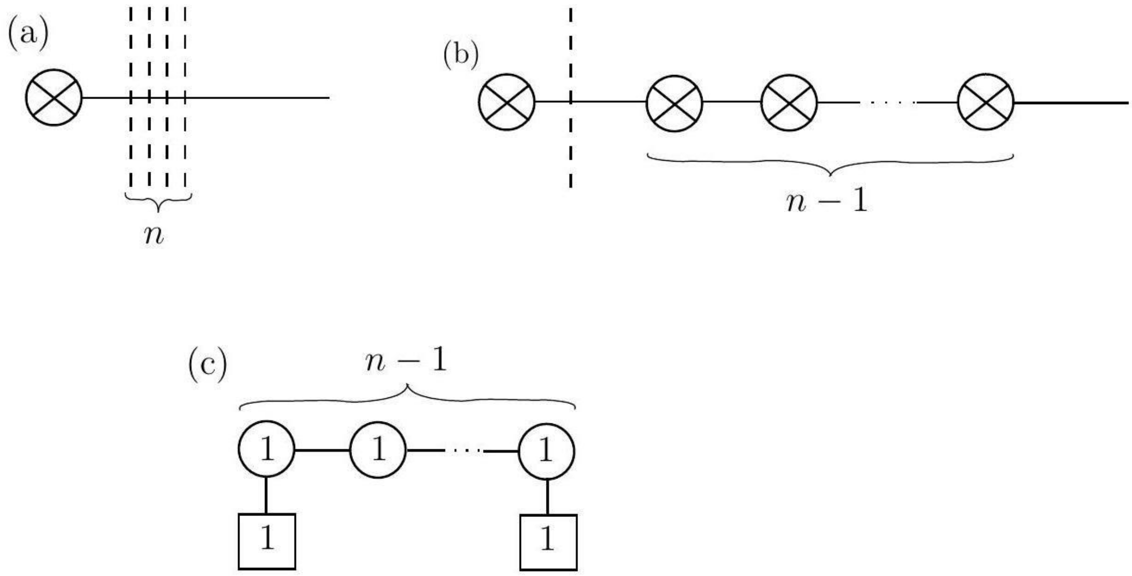
$$B_i\geq-\sum_{k=1}^{n_i}a_{i,k}(n_i-k+1)-\sum_{t=1}^{n_{i+1}}a_{i+1,t}(n_{i+1}-t)$$

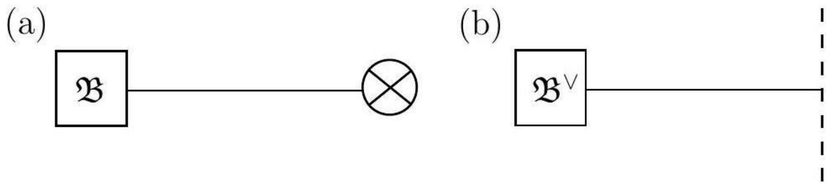
$$q_R\geq\sum_{i=1}^{P-1}\frac{e_i}{2}\sum_{k=1}^{n_i}a_{i,k}+\sum_{k=1}^{n_1}a_{1,k}(n_1-k)+\sum_{k=1}^{n_s}a_{s,k}+\sum_{k=1}^{n_{P-1}}a_{P-1,k}(n_{P-1}-k)$$

$$q_R\geq\sum_{i\neq s_0}\frac{e_i}{2}\sum_{k=1}^{n_i}a_{i,k}+\sum_{k=1}^{n_1}a_{1,k}(n_1-k)+\frac{1}{2}\sum_{k=1}^{n_{s_0}}a_{s_0,k}+\sum_{k=1}^{n_{P-1}}a_{P-1,k}(n_{P-1}-k).$$

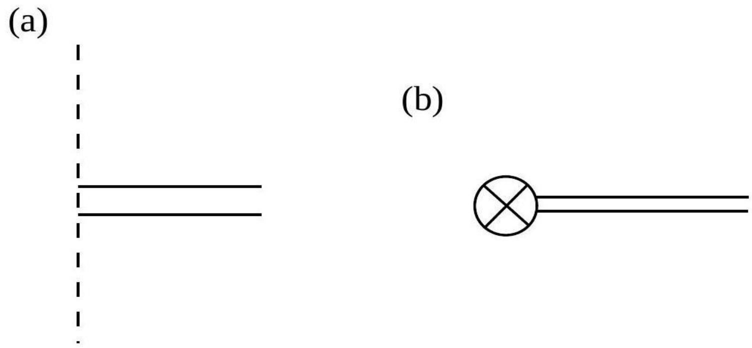


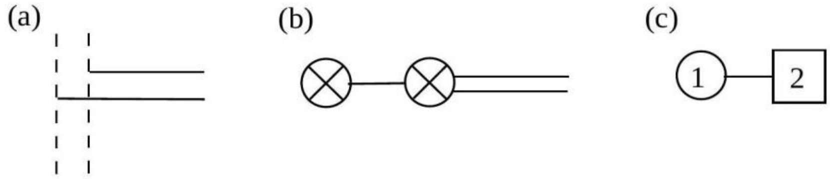


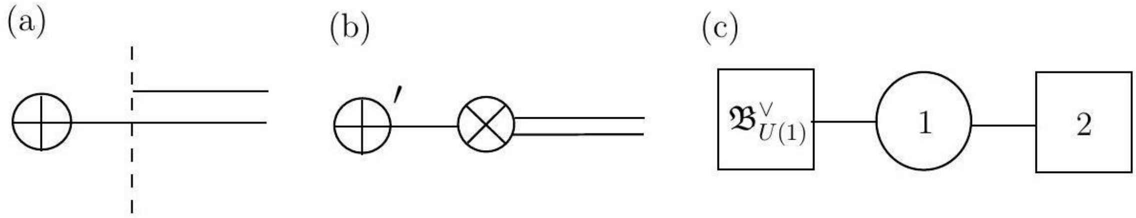


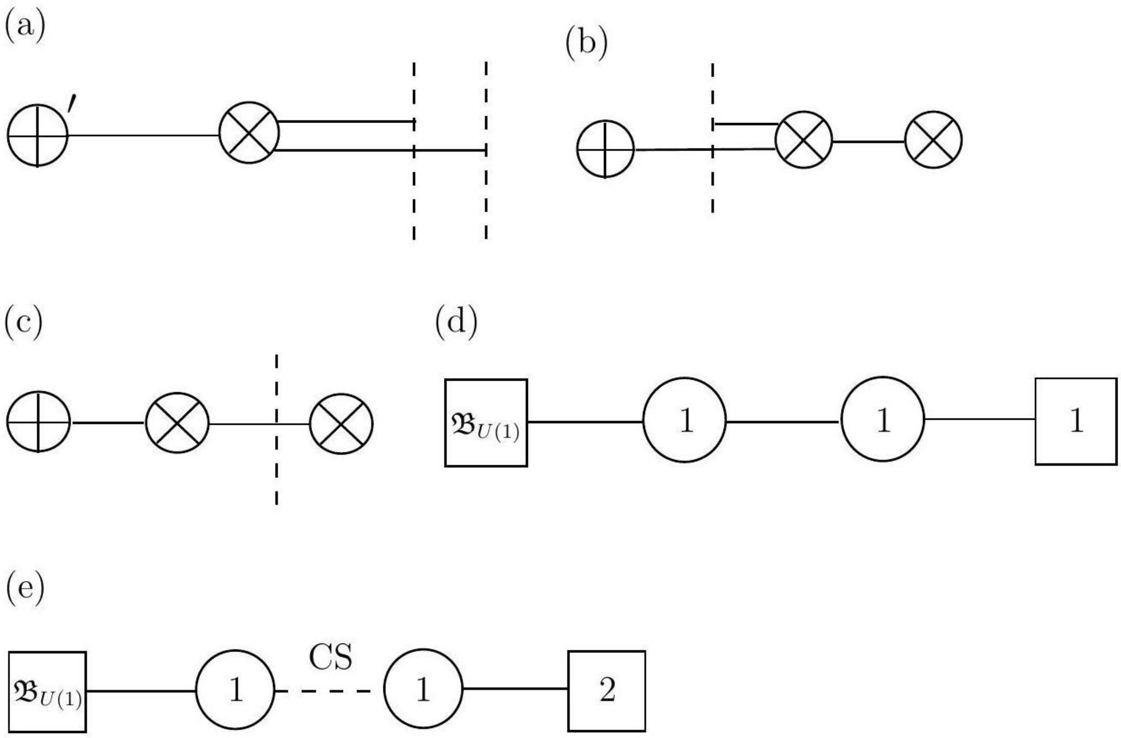


$$\frac{1}{2\pi}\int_{y=0} A \wedge dB$$

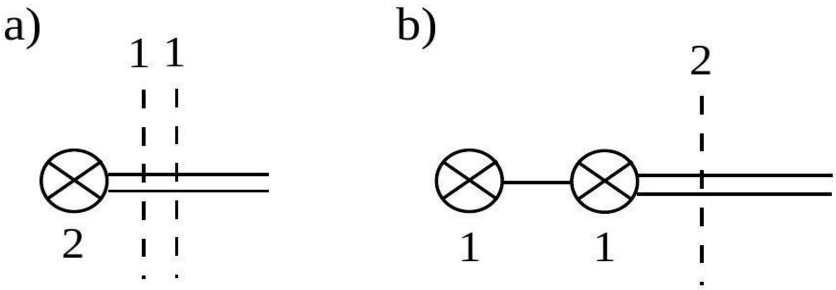

(a)
(b)

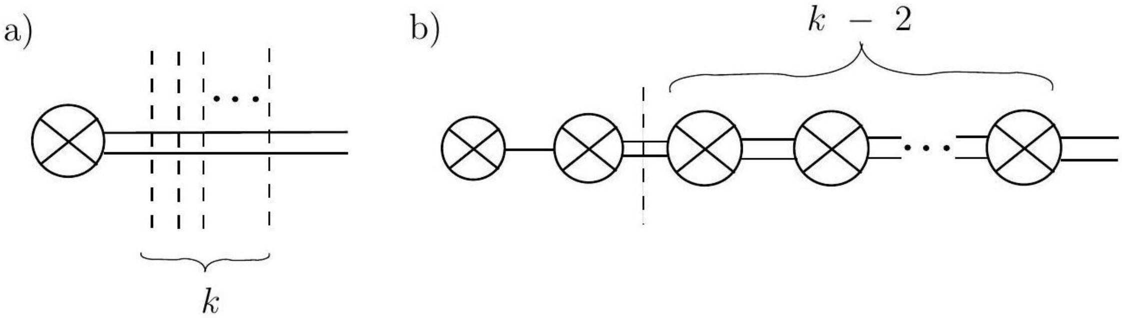

(a)
(b)
(c)
1
2

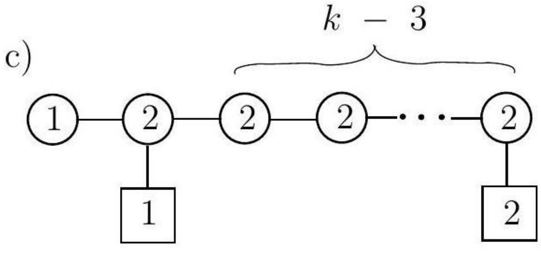

(a)
(b)
(c)
$\mathfrak{B}^{\vee}_{U(1)}$
1
2

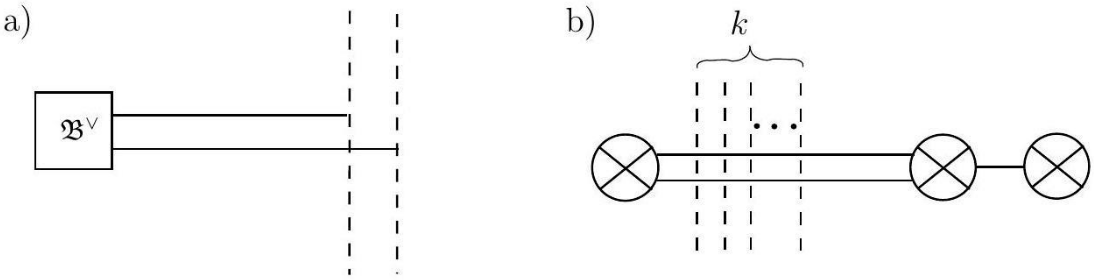

(a)
(b)
(c)
(d)
$\mathfrak{B}_{U(1)}$
1
1
1
(e)
$\mathfrak{B}_{U(1)}$
1
CS
1
2


(a)
(b)

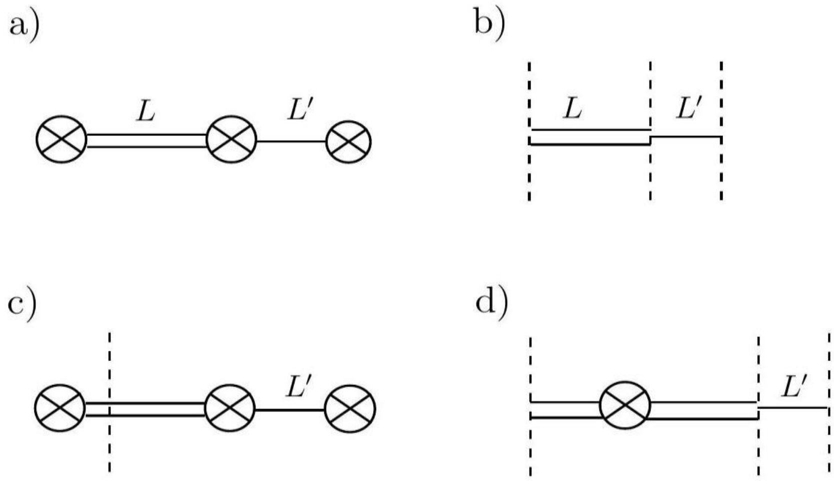


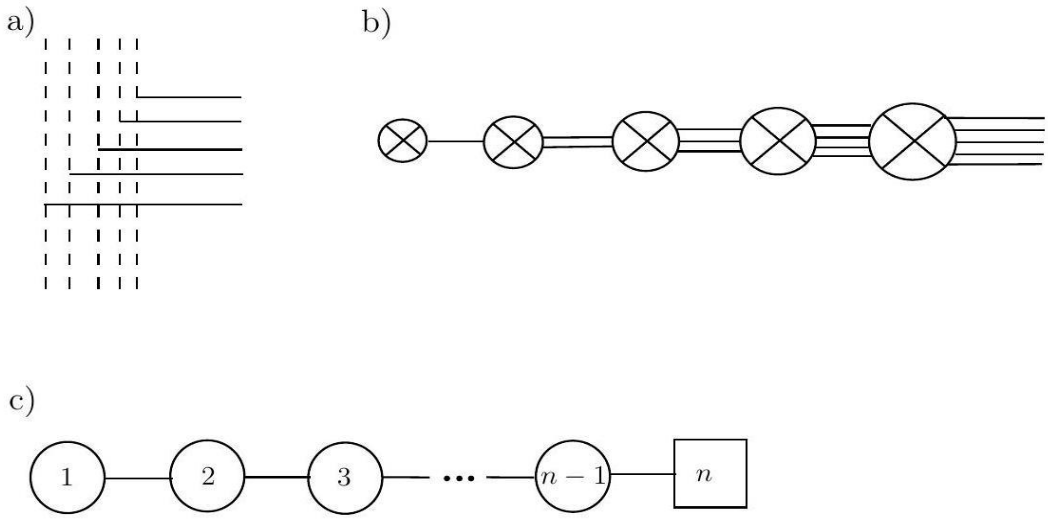


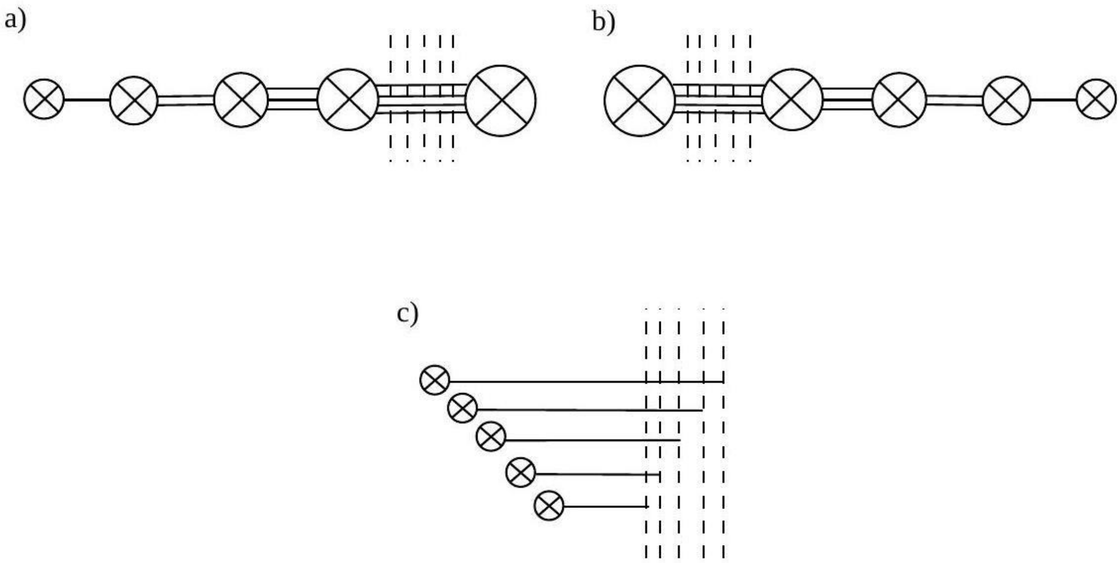


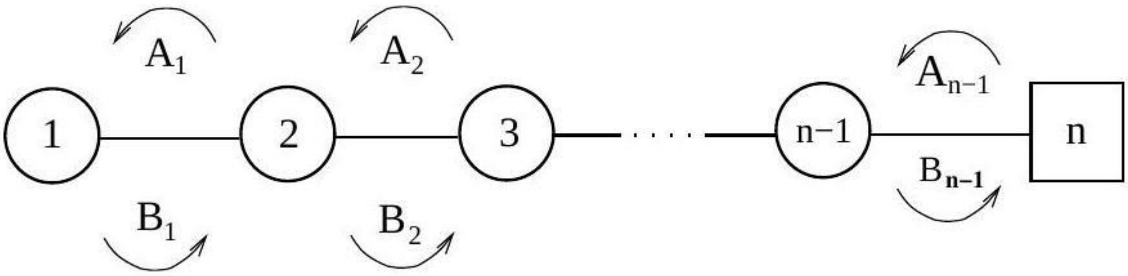


$$\widetilde{\mathfrak{B}}^{\vee} = \mathfrak{B} \times_{U(2)} T(SU(2)).$$


a)
b)
c)
d)
L
L'


a)
b)
c)
1
2
3
$n-1$
n


a)
b)
c)



$$A_{i+1}B_{i+1} = B_iA_i, i = 1, \dots, n-2$$
$$A_1B_1 = 0$$

$$M^2 = B_{n-1}A_{n-1}B_{n-1}A_{n-1} = B_{n-1}B_{n-2}A_{n-2}A_{n-1}$$

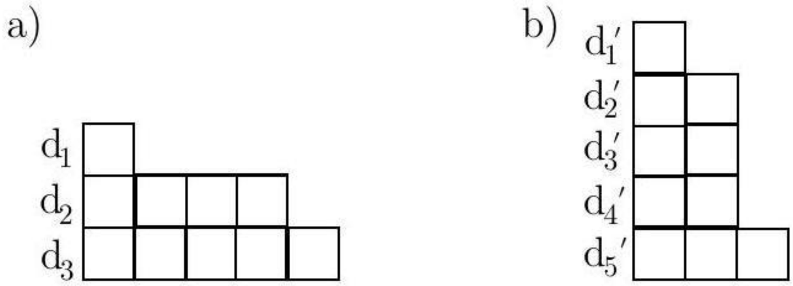
$$M^a = \prod_{i=n-1}^{n-a} B_i \prod_{j=n-a}^{n-1} A_j$$

$$A_{i+1}B_{i+1} = B_iA_i + t_{i+1}, i = 1, \dots, n-2$$
$$A_1B_1 = t_1$$

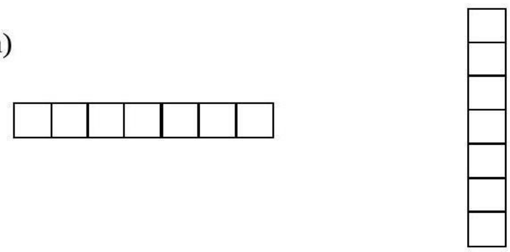
$$M^2 = B_{n-1}A_{n-1}B_{n-1}A_{n-1} = B_{n-1}B_{n-2}A_{n-2}A_{n-1} + t_{n-1}M$$

$$M(M - t_{n-1})(M - t_{n-1} - t_{n-2}) \cdots \left(M - \sum_{i=n-a}^{n-1} t_i\right) = \prod_{i=n-1}^{n-a} B_i \prod_{j=n-a}^{n-1} A_j$$

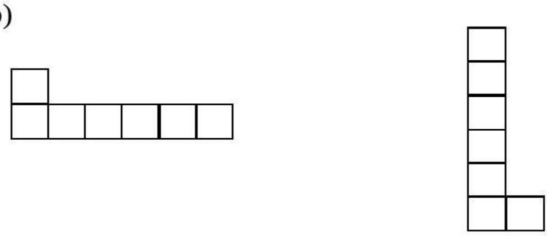
$$M(M - t_{n-1})(M - t_{n-1} - t_{n-2}) \cdots \left(M - \sum_{i=1}^{n-1} t_i\right) = 0$$

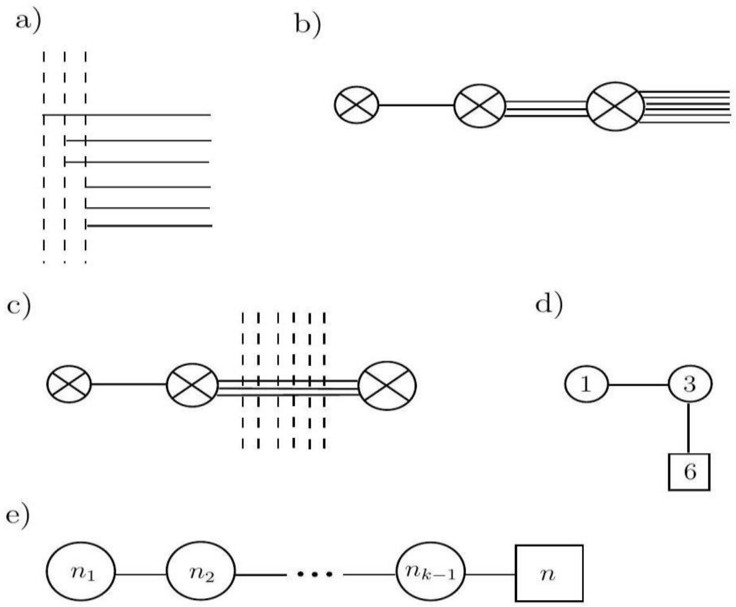


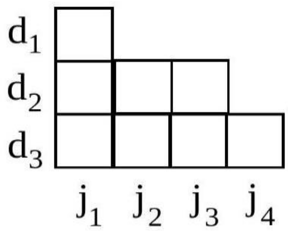
a)



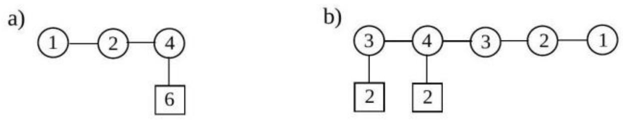
b)

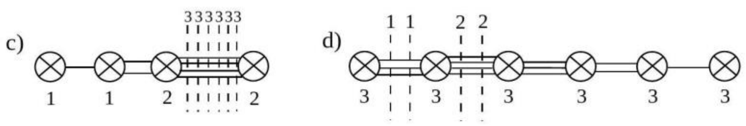


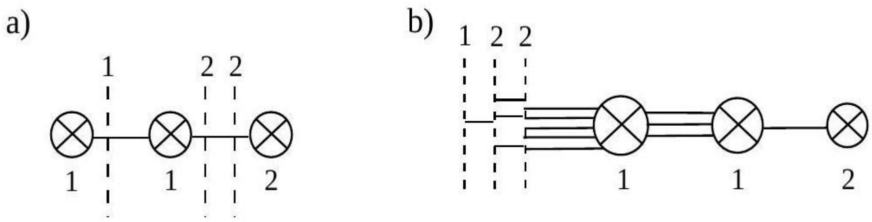


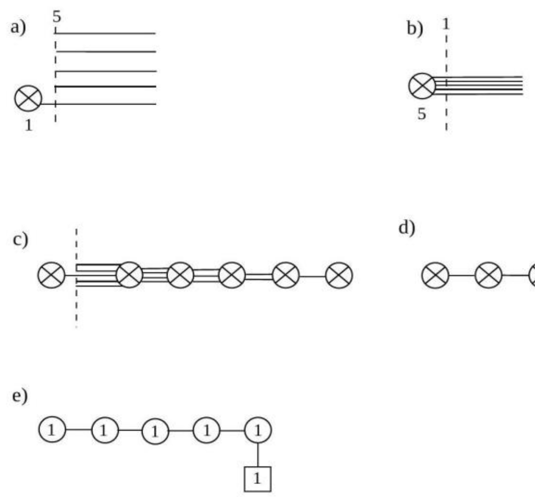


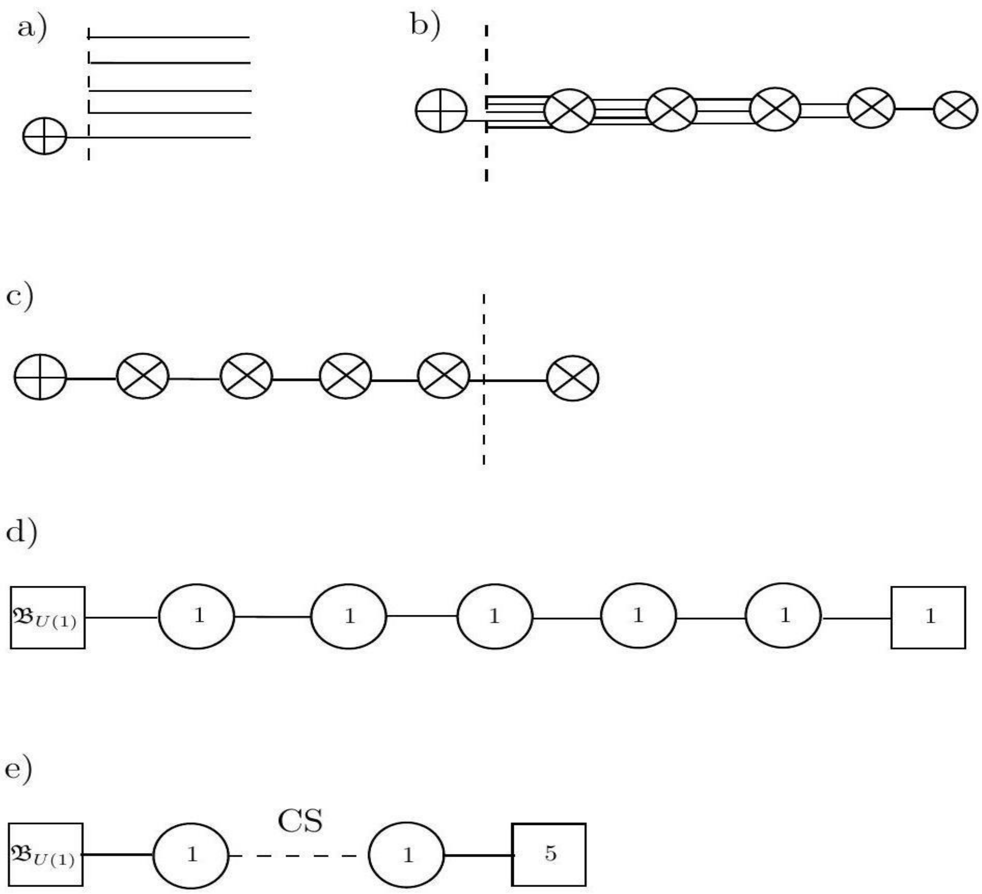
$$A_{i+1}B_{i+1} \ = B_iA_i + t_{i+1}, i = 1, \ldots, k-2$$
$$A_1B_1 \ = t_1$$



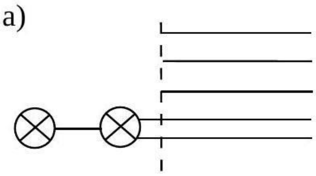




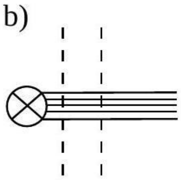

a)
5
1
b)
1
5
c)
d)
e)
1
1
1
1
1
1


a)
b)
c)
d)
$\mathfrak{B}_{U(1)}$
1
1
1
1
1
1
e)
$\mathfrak{B}_{U(1)}$
1
CS
1
5

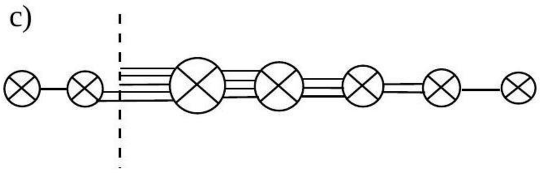
a)



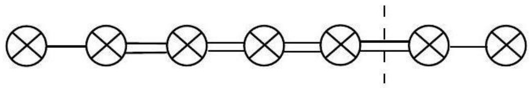
b)



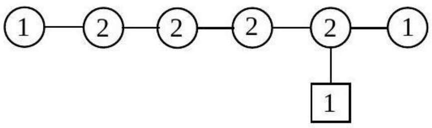
c)



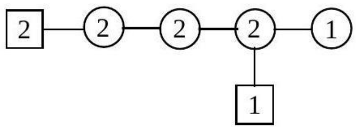
d)



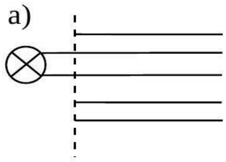
e)



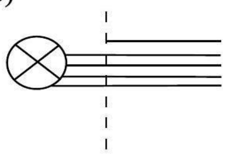
f)



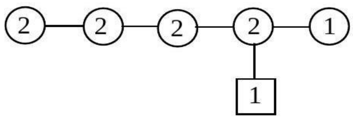
a)



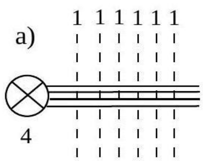
b)



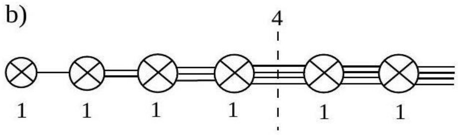
c)



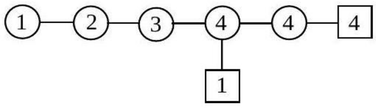
a)

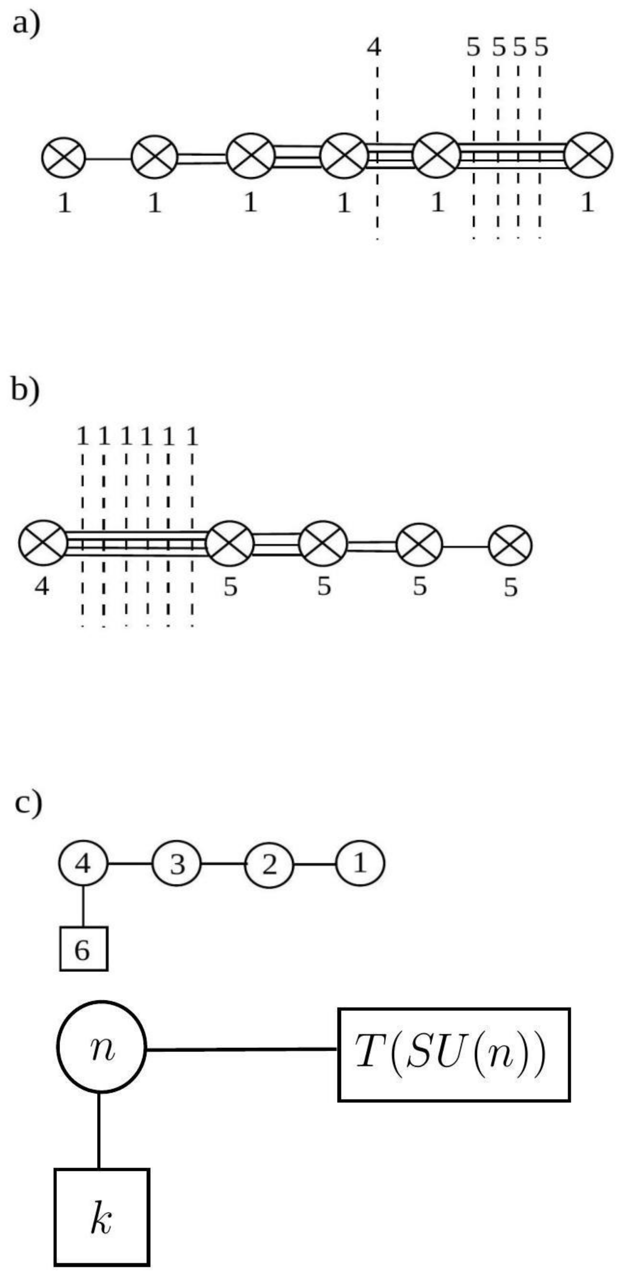


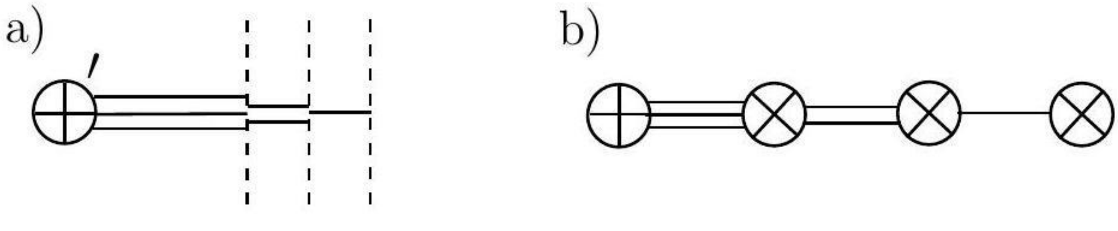
b)

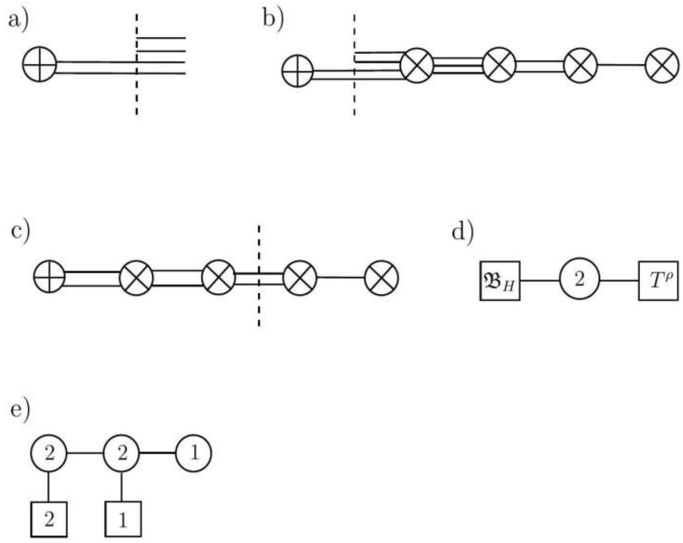


c)




a)
4
5 5 5 5
1 1 1 1 1 1
b)
1 1 1 1 1 1
4 5 5 5 5
c)
4 3 2 1
6
n
T(SU(n))
k

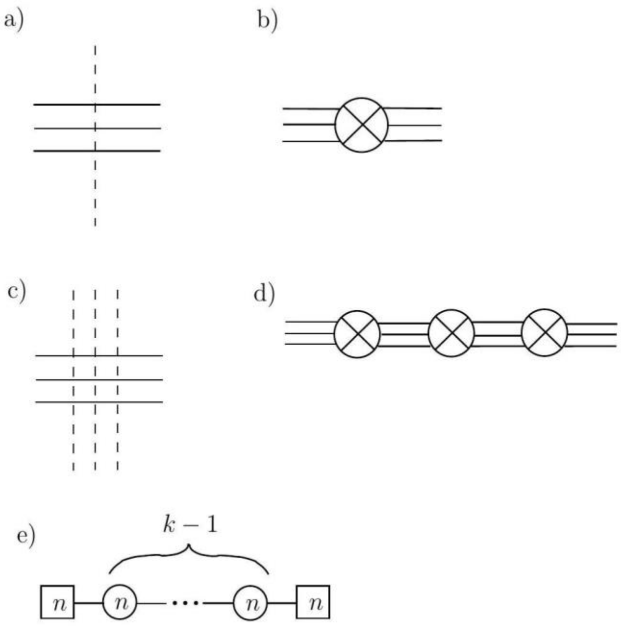

a)
b)

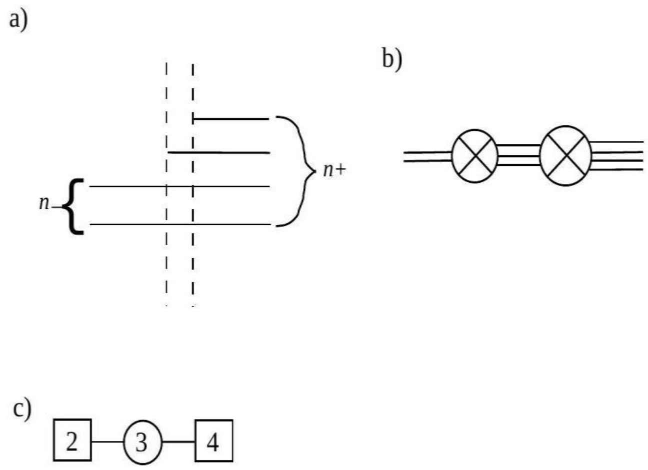


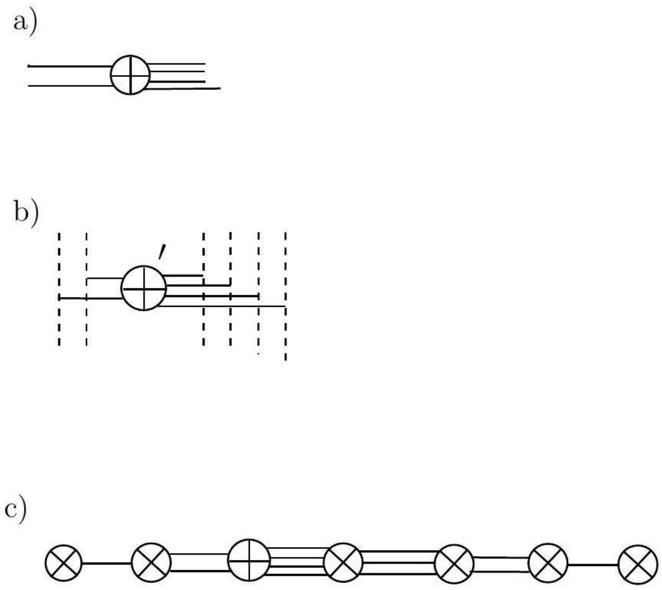
$$\widetilde{\mathfrak{B}}^\vee = \mathfrak{B}_H \times_H T^\rho(SU(n)).$$

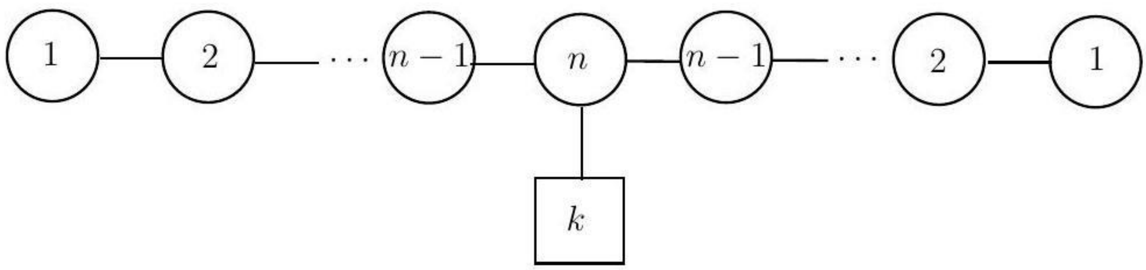
$$W(p) = P\exp \int_S \mathcal{A}$$

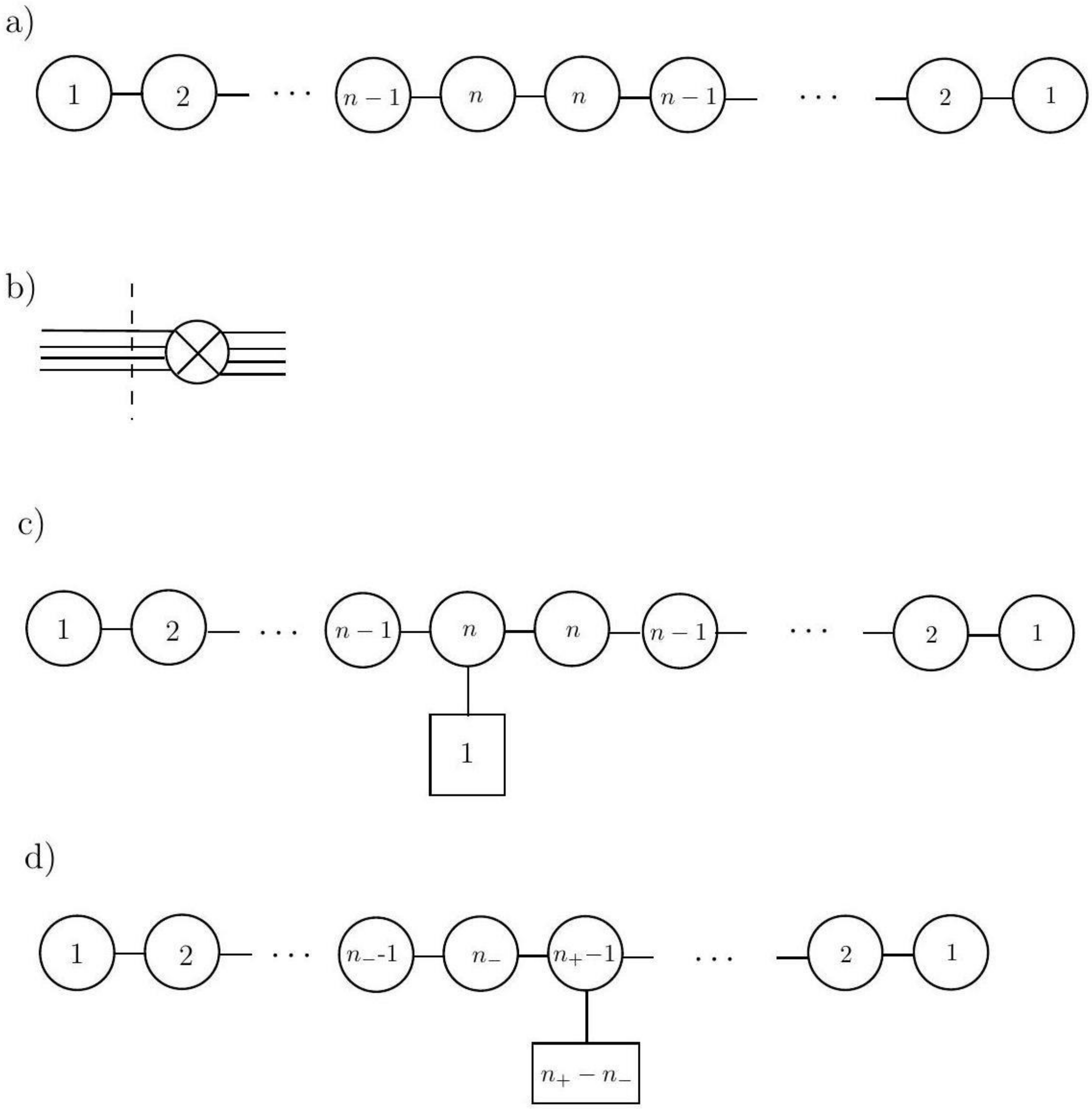
$$\widehat{W} = \lim_{\epsilon \to 0} \epsilon^{-it_3} W_\epsilon(p)$$




a)
n
$n+$
b)
c)
2
3
4

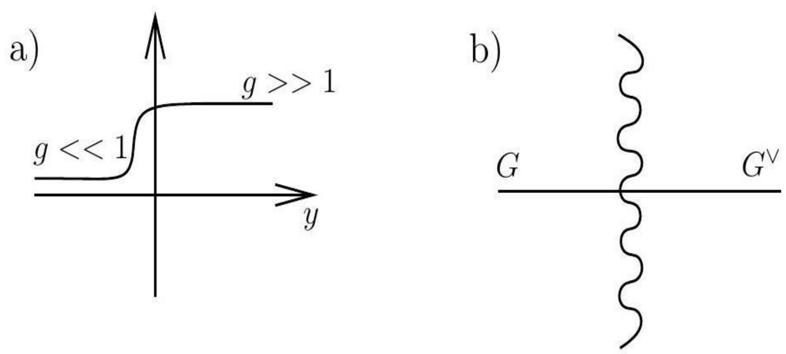

a)
b)
c)

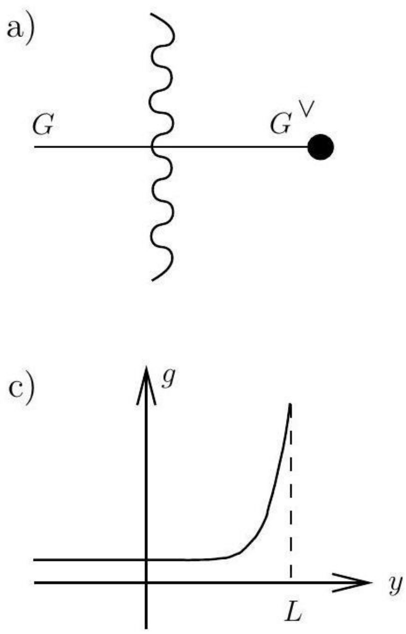

1
2
$n-1$
n
$n-1$
2
1
k



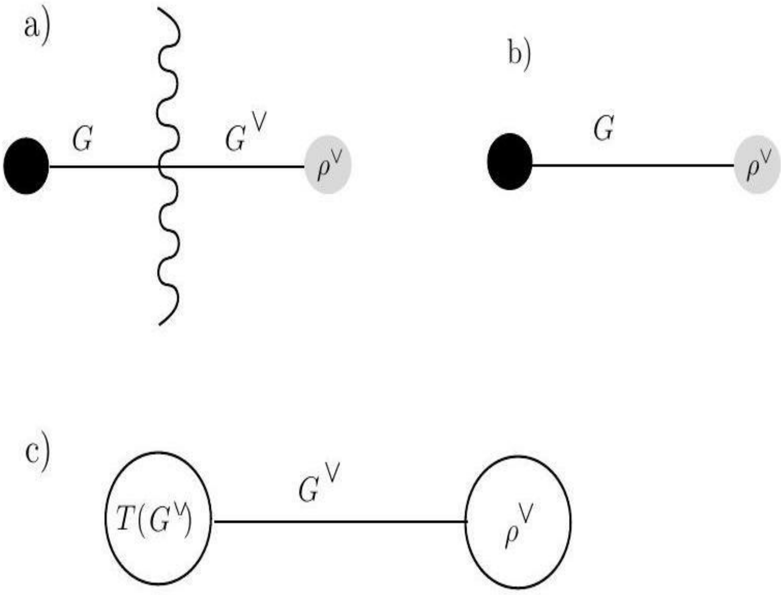
$$\mathcal{M}_{T(G)} = \bigcup_{\alpha \in S} \mathcal{C}_{\alpha} \times \mathcal{H}_{\alpha}$$

$$\mathcal{M}_{T(G)} = \bigcup_{\alpha \in S} \overline{\mathcal{O}}_{\rho_{\alpha}} \times \overline{\mathcal{O}}_{\rho_{\alpha}^{\vee}}$$

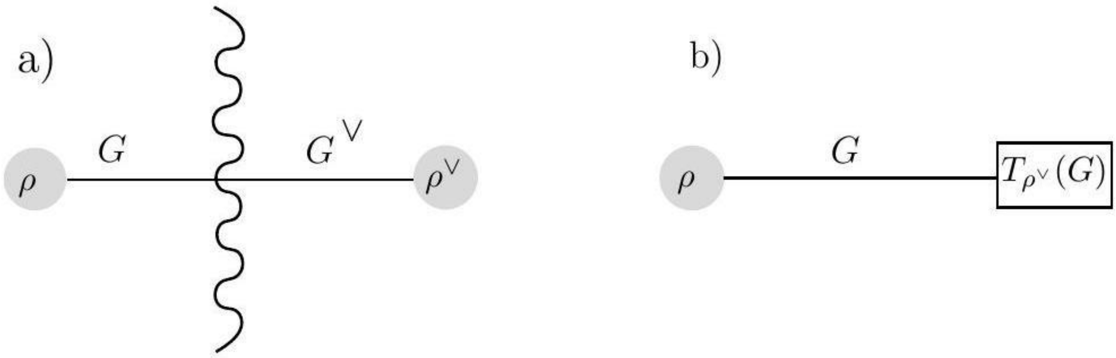




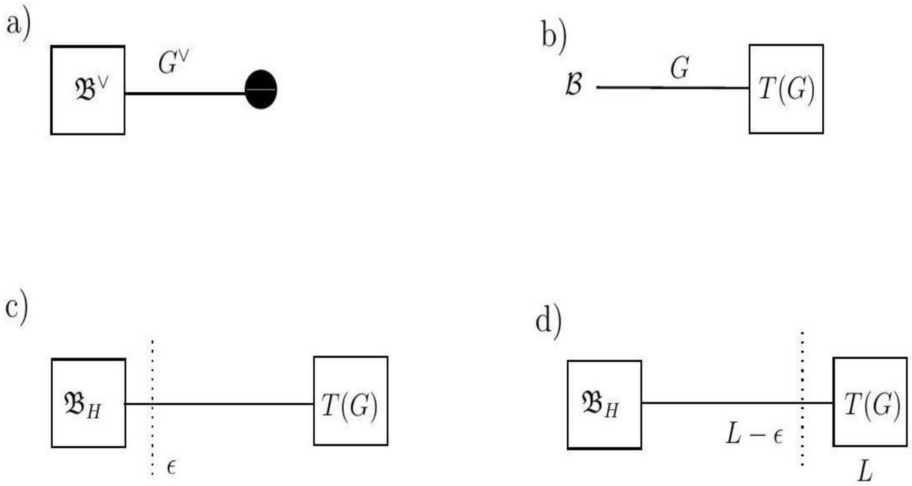
	$\mathcal{H}$	$\mathcal{C}$
$T(G)$	$\mathcal{N}$	$\mathcal{N}^\vee$
$T_{\rho^\vee}(G)$	$\cup_{\alpha\in C_{\rho^\vee}} \overline{\mathcal{O}}_{\rho_\alpha}$	$\mathcal{S}_{\rho^\vee} \cap \mathcal{N}^\vee$
$T^{\rho}(G)$	$\mathcal{S}_\rho \cap \mathcal{N}$	$\cup_{\alpha\in C_\rho} \overline{\mathcal{O}}_{\rho_\alpha^\vee}$
$T^{\rho}_{\rho^\vee}(G)$	$\mathcal{S}_\rho \cap \left(\cup_{\alpha\in C_{\rho^\vee}} \overline{\mathcal{O}}_{\rho_\alpha}\right)$	$\mathcal{S}_{\rho^\vee} \cap \left(\cup_{\alpha\in C_\rho} \overline{\mathcal{O}}_{\rho_\alpha^\vee}\right)$

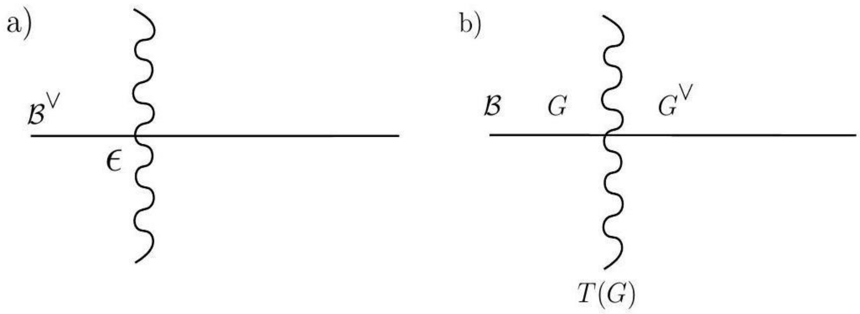


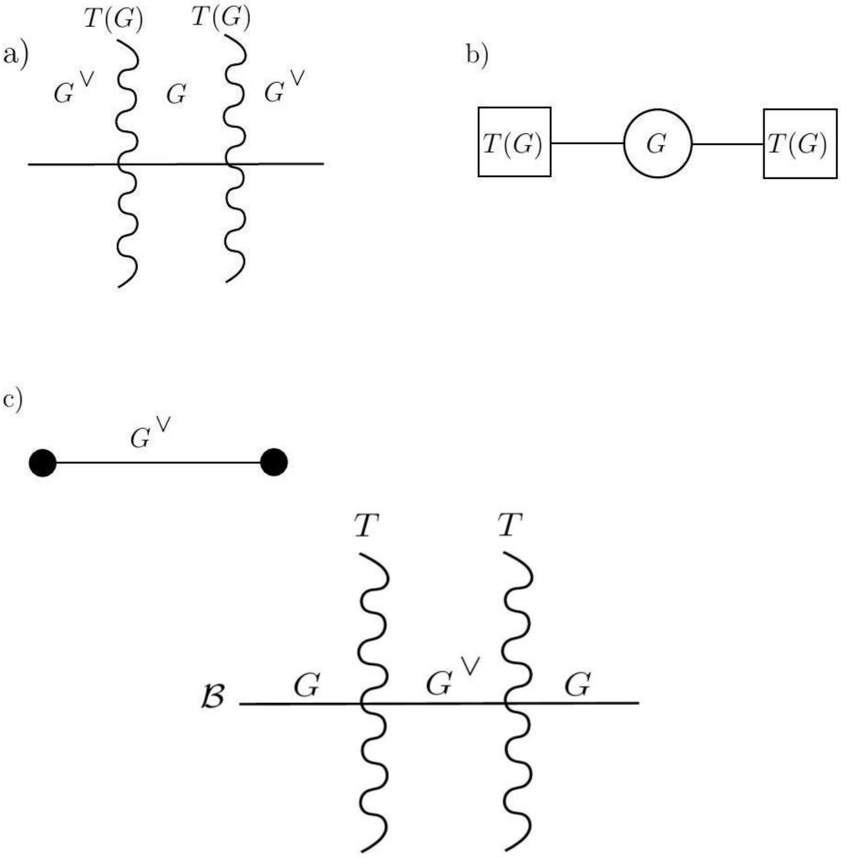
$$\mathcal{M}_{T_{\rho^\vee}(G)} = \bigcup_{\alpha\in S} \overline{\mathcal{O}}_{\rho_\alpha} \times (\mathcal{S}_{\rho^\vee} \cap \overline{\mathcal{O}}_{\rho_\alpha^\vee})$$



$$\mathcal{M}_{T'_{\rho^\vee}}(G) = \bigcup_{\alpha\in S} (\mathcal{S}_\rho \cap \overline{\mathcal{O}}_{\rho_\alpha}) \times (\mathcal{S}_{\rho^\vee} \cap \overline{\mathcal{O}}_{\rho_\alpha^\vee}).$$







$$\frac{1}{2\pi}\int C \wedge dB$$

$$J = \frac{\star F_B}{2\pi}$$

$$\boldsymbol{I} = \frac{1}{e^2}\int_M F_B \wedge \star F_B$$

$$B \rightarrow B + b$$
$$\boldsymbol{k} \rightarrow \boldsymbol{k} + db$$

$$\frac{1}{2\pi}\int_M C \wedge d\boldsymbol{k} = \frac{1}{2\pi}\int_M F_C \wedge \mathcal{F}$$

$$\hat{\boldsymbol{I}} = \frac{1}{2\pi}\int_M F_C \wedge \mathcal{F} + \frac{1}{e^2}\int_M \mathcal{F} \wedge \star \mathcal{F}$$

$$\boldsymbol{I}_C = \frac{e^2}{16\pi^2}\int_M F_C \wedge \star F_C$$

$$\frac{2}{e^2} \star F_C\Big|_{\partial M} = \frac{1}{2\pi}F_A.$$

$$\boldsymbol{I}_C = \frac{e^2}{16\pi^2}\int_M F_C \wedge \star F_C + \frac{1}{2\pi}\int_{\partial M} C \wedge dA$$

$$\frac{4\pi}{e^2} \star F_C\Big|_{\partial M} = F_A$$

$$B|_N = A|_N.$$

$$\frac{1}{2\pi}\int_N C \wedge dA$$

$$\mathcal{M} = \cup_{\alpha \in S} \mathcal{C}_\alpha \times \mathcal{H}_\alpha$$

$$\vec{Y}(0) = \vec{\zeta}$$
$$\vec{X}(L) = \vec{\mathbf{m}}$$

$$q_R = \frac{1}{2}\left(\sum_i |h_i| - \sum_j |v_j|\right).$$

$$\begin{pmatrix} 0 & \alpha \\ -\alpha & 0 \end{pmatrix}$$

$$q_R = \frac{n_f}{2}\sum_i |a_i| - \frac{1}{2}\left(\sum_{1\le i<j\le k} |a_i - a_j| - \sum_{1\le i\le k} |a_i|\right).$$

$$q_R = \frac{n_f + 2 - k}{2}\sum_i |a_i| + \frac{1}{2}\sum_{i<j} \left(|a_i| + |a_j| - |a_i - a_j|\right).$$

$$n_f \ge k - 1.$$

$$e = n_f - k + 1$$

$$q_R = \frac{n_f}{2}\sum_i |a_i| - \frac{1}{2}\left(\sum_{1\le i<j\le 2t} |a_i - a_j| + \sum_{1\le i\le 2t} |a_i|\right),$$

$$q_R = \frac{n_f - 2t}{2}\sum_i |a_i| + \frac{1}{2}\sum_{i<j} \left(|a_i| + |a_j| - |a_i - a_j|\right).$$

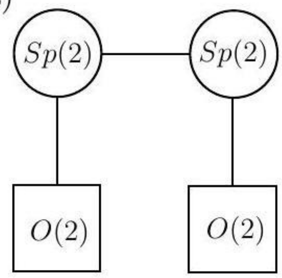
$$n_f \ge 2t + 1$$

$$e = n_f - 2t - 1$$

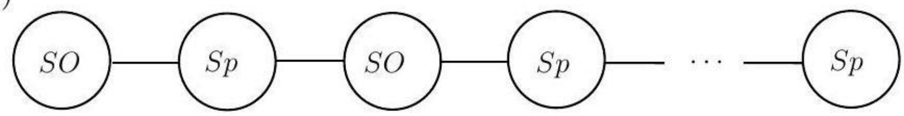
$$q_R = \sum_i \left(\Delta_i + \epsilon_i A_i + \epsilon_{i,i+1} B_i\right)$$

$$\Delta_i = \frac{e_i}{2}\sum_{k=1}^{n_i} |a_{i,k}|$$
$$A_i = \frac{1}{2}\sum_{k=1}^{n_i}\sum_{t=1}^{n_i} \left(|a_{i,k}| + |a_{i,t}| - |a_{i,k} - a_{i,t}|\right)$$
$$B_i = -\frac{1}{2}\sum_{k=1}^{n_i}\sum_{t=1}^{n_{i+1}} \left(|a_{i,k}| + |a_{i+1,t}| - |a_{i,k} - a_{i+1,t}|\right)$$

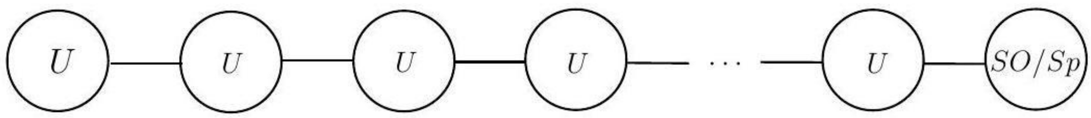
a)



b)



c)

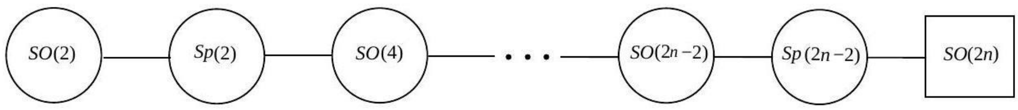


$$q_R \geq \sum_{i=1}^{P-1} \frac{e_i}{2} \sum_{m=1}^{n_i} |a_{i,m}| + \frac{1}{2} \sum_{m=1}^{n_1} |a_{1,m}|(n_1 - m + 1) + \frac{1}{2} \sum_{m=1}^{n_{P-1}} |a_{P-1,m}|(n_{P-1} - m)$$

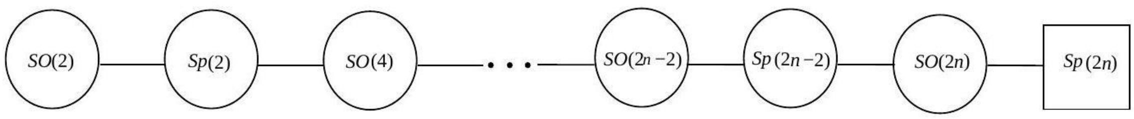
$$q_R \geq \sum_{i=1}^{P-1} \frac{e_i}{2} \sum_{m=1}^{n_i} |a_{i,m}| + \frac{1}{2} \sum_{m=1}^{n_1} |a_{1,m}|(n_1 - m) + \frac{1}{2} \sum_{m=1}^{n_s} |a_{s,m}| + \frac{1}{2} \sum_{m=1}^{n_{P-1}} |a_{P-1,m}|(n_{P-1} - m)$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

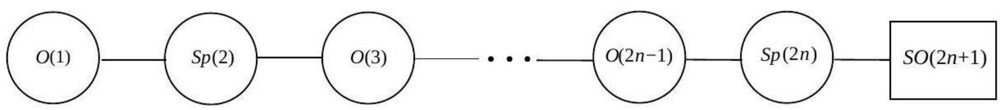
a)



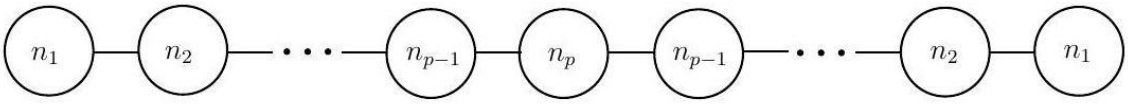
b)



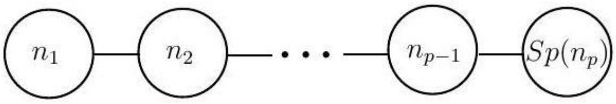
c)



a)



b)



$$q_R = q_R^+ + q_R^-$$

$$q_R^+ \geq \sum_{i=1}^{P} \frac{e_i}{2} \sum_{k|a_{i,k}\geq 0} a_{i,k} + \sum_{k|a_{1,k}\geq 0} a_{1,k}(n_1 - k) + \sum_{k|a_{s,k}\geq 0}^{n_s} a_{s,k} - \frac{1}{2} \sum_{k|a_{P,k}\geq 0} a_{P,k}$$

$$2n_i - \sum_{j|(i,j)\in E} n_j = m_i$$

$$\mathfrak{C}\mathfrak{n} = \mathfrak{m}.$$

$$\Delta_i = \frac{e_i}{2} \sum_{k=1}^{n_i} a_{i,k}$$

$$A_i = \sum_{k=1}^{n_i} a_{i,k}(2n_i - 2k + 1)$$

$$B_{ij} = -\frac{1}{2} \sum_{k=1}^{n_i} \sum_{t=1}^{n_j} 2\min(a_{i,k}, a_{j,t})$$

$$B_{ij} \geq -\sum_{k=1}^{n_i} a_{i,k}(n_i - k) - \sum_{t=1}^{n_j} a_{j,t}(n_j - t + 1).$$

$$B_{1c} = B_{12} + B_{13} = -\frac{1}{2} \sum_{k=1}^{n_1} \sum_{t=1}^{n_2+n_3} 2\min(a_{1,k}, a_{c,t})$$

$$B_{1c} \geq -\sum_{k=1}^{n_1} a_{1,k}(n_1 - k + 1) - \sum_{t=1}^{n_2+n_3} a_{c,t}(n_2 + n_3 - t)$$

$$-\sum_{t=1}^{n_2+n_3} a_{c,t}(n_2 + n_3 - t) =- \sum_{t=1}^{n_2-n_3} a_{2,t}(n_2 + n_3 - t) - \sum_{t=1}^{n_3} a_{3,t}\big(n_2 + n_3 - (n_2 - n_3 + 2t - 1)\big)$$
$$- \sum_{t=n_2-n_3+1}^{n_2} a_{2,t}\big(n_2 + n_3 - (2t - n_2 + n_3)\big)$$

$$B_{1c} \geq -\sum_{k=1}^{n_1} a_{1,k}(n_1 - k + 1) - \sum_{t=1}^{n_3} a_{3,t}(2n_3 - 2t + 1) - \sum_{t=1}^{n_2} a_{2,t}(2n_2 - 2t),$$

$$B_{1c} \geq -\sum_{k=1}^{n_1} a_{i,k}(n_i - k + 1) - \sum_{t=1}^{n_3} a_{3,t}(2n_3 - 2t) - \sum_{t=1}^{n_2} a_{2,t}(2n_2 - 2t + 1),$$

$$B_{1c} \geq -\sum_{k=1}^{n_1} a_{1,k}(n_1 - k) - \sum_{t=1}^{n_2+n_3} a_{c,t}(n_2 + n_3 - t + 1)$$

$$B_{1c} \geq -\sum_{k=1}^{n_1} a_{i,k}(n_i - k) - \sum_{t=1}^{n_3} a_{3,t}(2n_3 - 2t + 2) - \sum_{t=1}^{n_2} a_{2,t}(2n_2 - 2t + 1)$$

$$q_R \geq \sum_{j \in N} \Delta_j + \sum_{k=1}^{n_2} a_{2,k} + \sum_{k=1}^{n_0} a_{0,k}(n_0 - k)$$

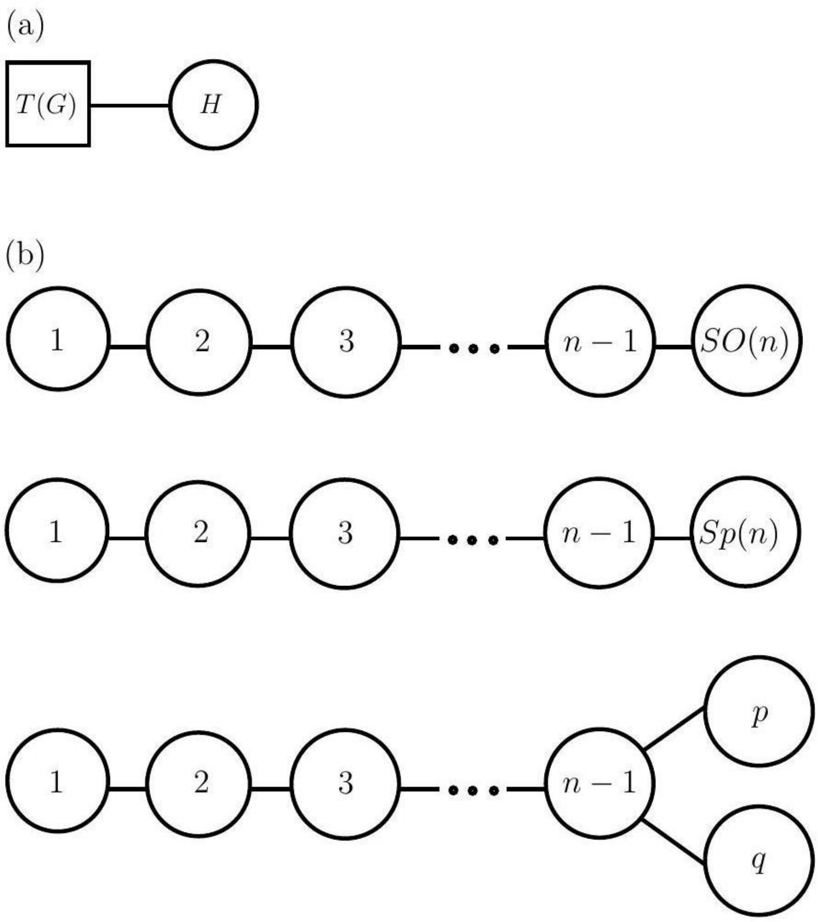
$$q_R \geq \sum \Delta_i + \sum_{k=1}^{n_3} a_{3,k} + \sum_{k=1}^{n_0} a_{0,k}(n_0 - k)$$

$$q_R \geq \sum \Delta_i - \sum_{k=1}^{n_3} a_{3,k} + \sum_{k=1}^{n_s} a_{s,k} + \sum_{k=1}^{n_0} a_{0,k}(n_0 - k)$$

$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{h}^{\perp}$$

$$\vec{X}^{+}(0) = 0 = \vec{Y}^{-}(0).$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$



$$\vec{X}^{+}(0)+\vec{\mu}_Z=0$$
$$\vec{Y}^{-}(0)=0$$

$$\vec{X}^{+}(0)+\vec{\mu}_Z=\vec{v}$$
$$\vec{Y}^{-}(0)=\vec{w}.$$

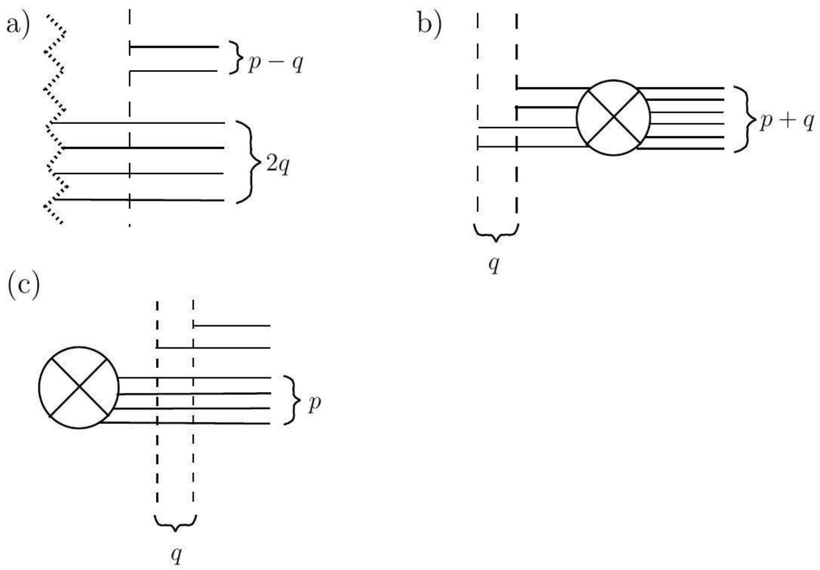
$$\begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix},$$

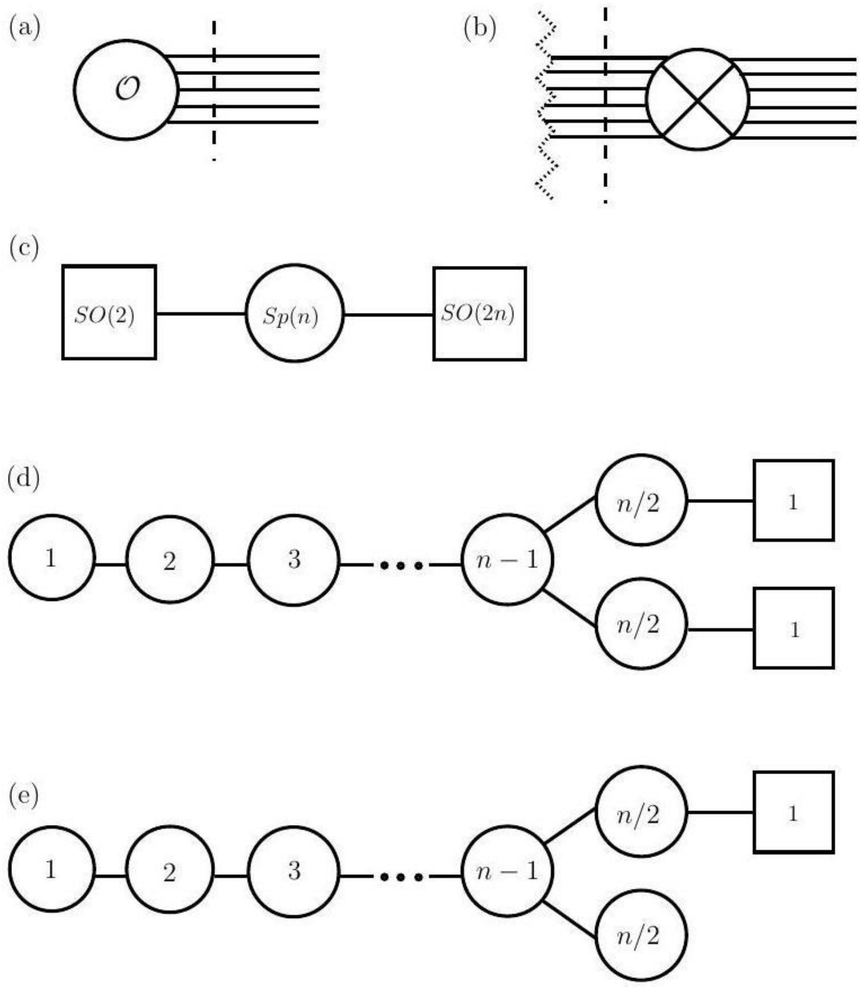
$$\vec{X}(0)=\begin{pmatrix} \vec{c}_1\cdot 1_{n/2} & * \\ * & \vec{c}_2\cdot 1_{n/2} \end{pmatrix}.$$

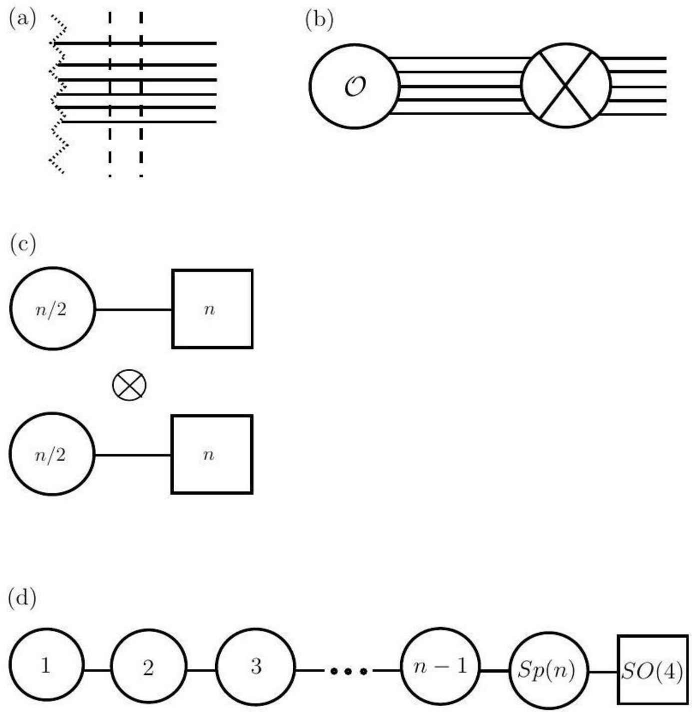
$$\vec{X}(0)=\begin{pmatrix} \vec{\mathbf{m}}\cdot 1_{n/2} & * \\ * & -\vec{\mathbf{m}}\cdot 1_{n/2} \end{pmatrix}.$$

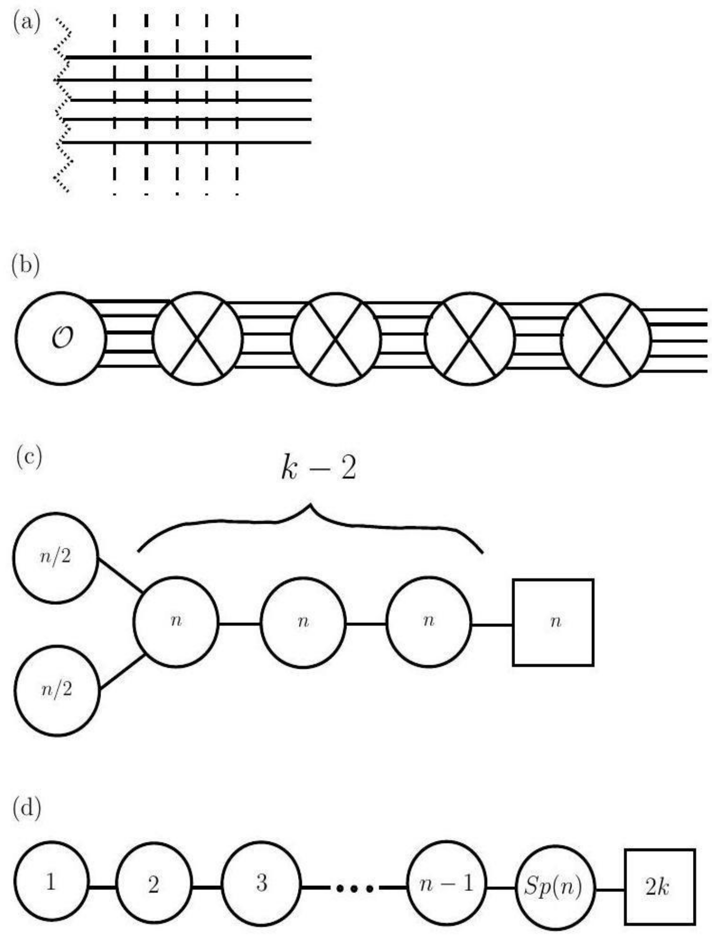
$$d\vec{X}/dy+\vec{X}\times\vec{X}=0$$

$$\vec{X}=f\frac{\vec{t}}{y+y_0}f^{-1},$$

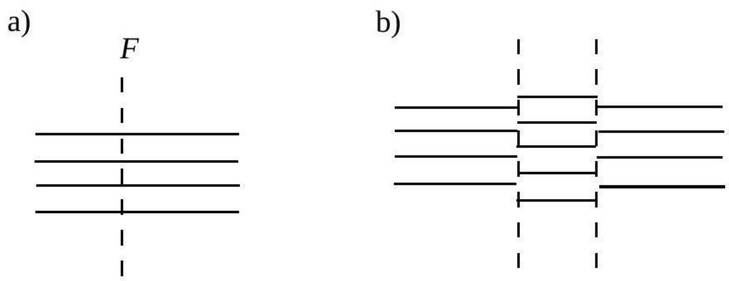



(a)
$\mathcal{O}$
(b)
(c)
$SO(2)$
$Sp(n)$
$SO(2n)$
(d)
1
2
3
$n-1$
$n/2$
$n/2$
1
1
(e)
1
2
3
$n-1$
$n/2$
$n/2$
1


(a)
(b)
$\mathcal{O}$
(c)
$n/2$
n
$n/2$
n
(d)
1
2
3
$n-1$
$Sp(n)$
$SO(4)$

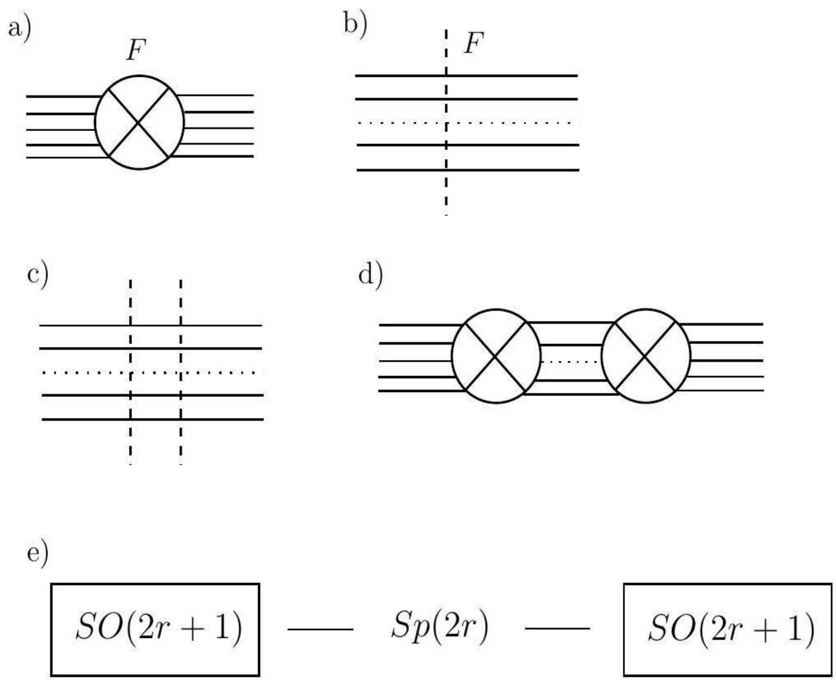


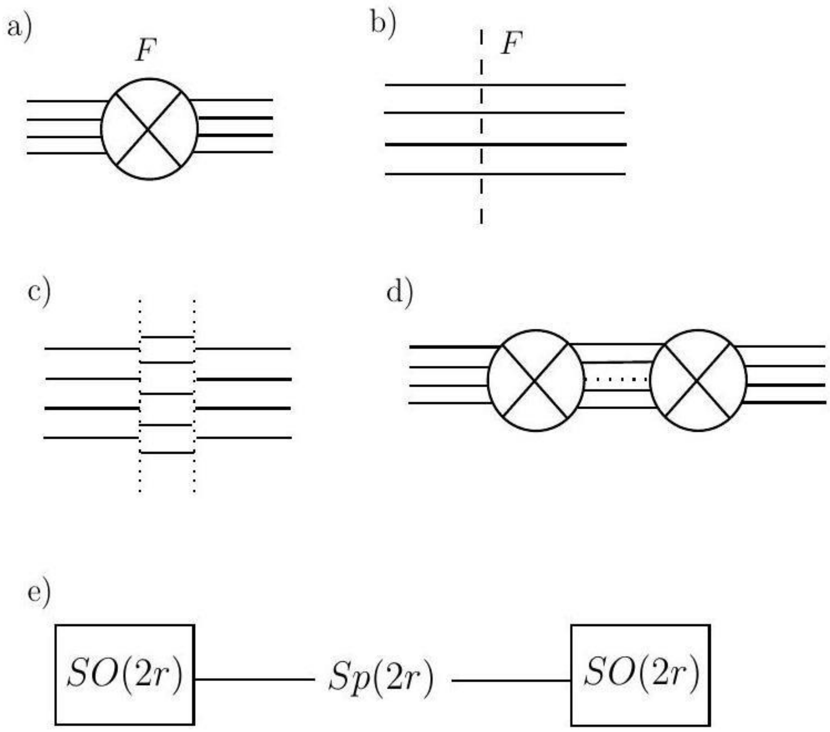
$$\Phi(-\vec{x}) = -\Phi(\vec{x}),$$



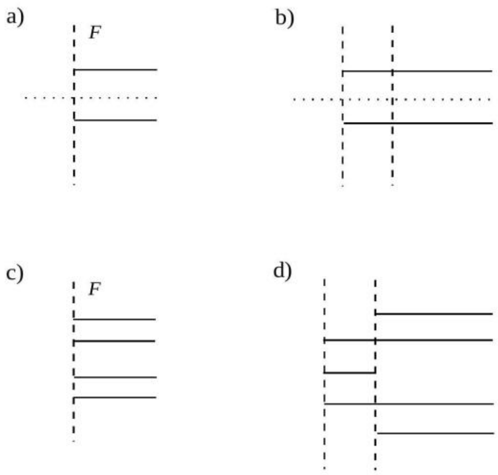
$$F = \star\, D\Phi.$$

$$\Phi = f(r)\vec{x} \cdot \vec{t},$$

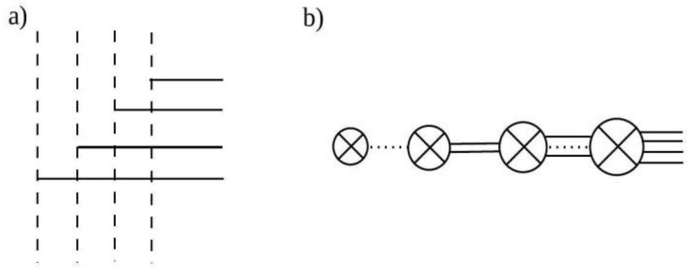


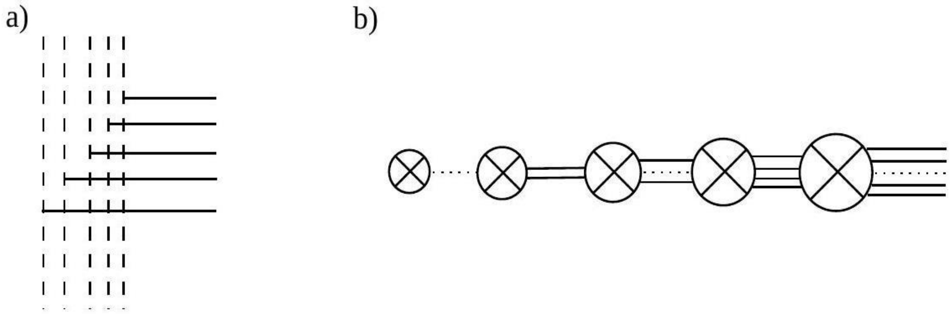


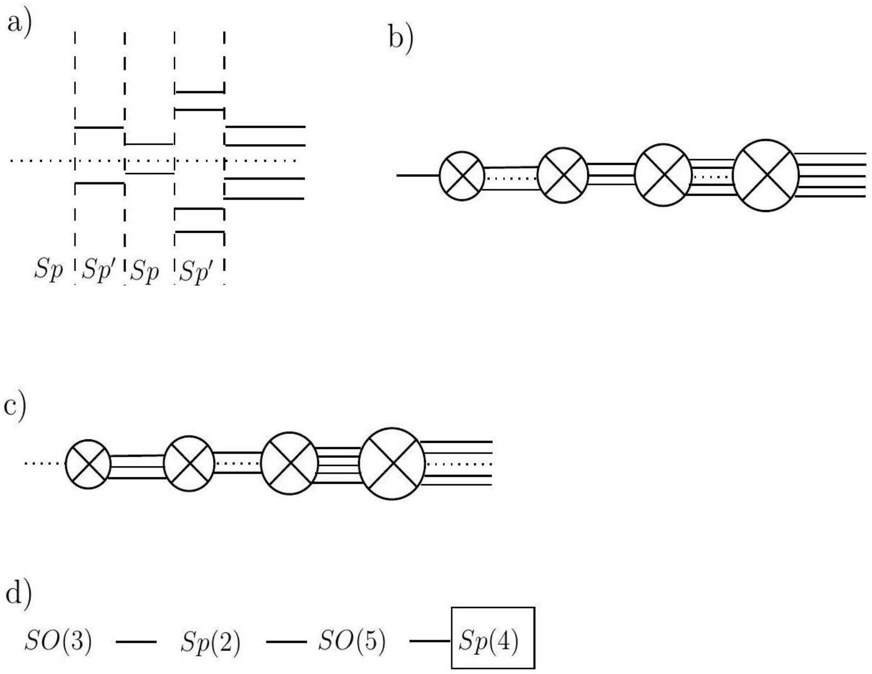
$$\tilde{\tau} = \begin{pmatrix} 0 & \phi \\ -\phi & 0 \end{pmatrix}$$

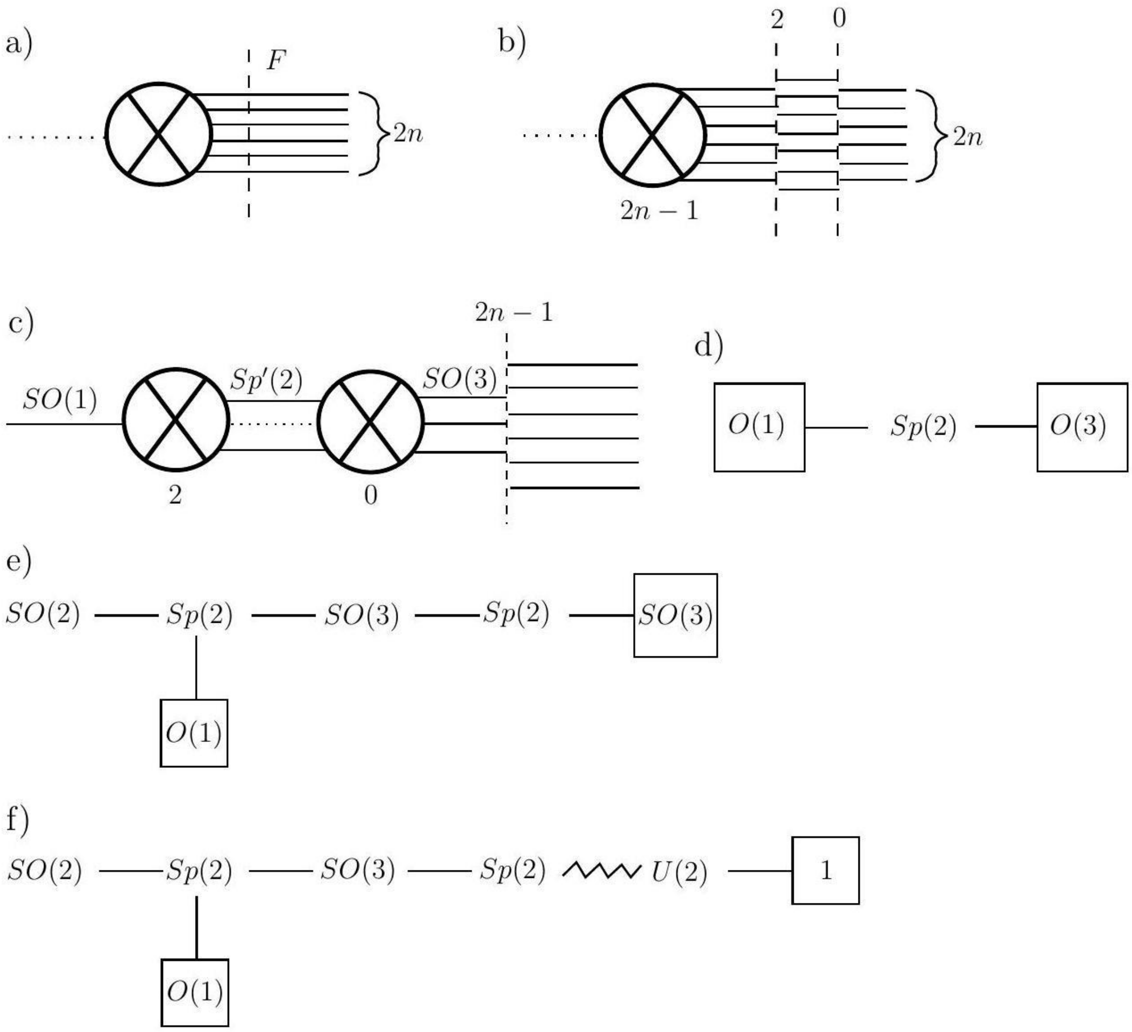


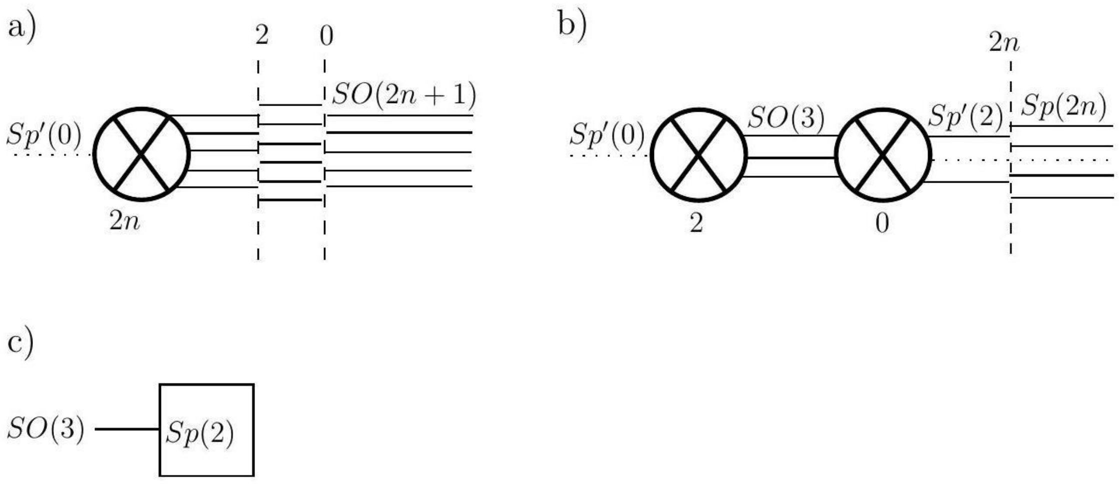
$$E = \mathcal{L} \oplus \mathcal{L}, c_1(\mathcal{L}) = 2s$$

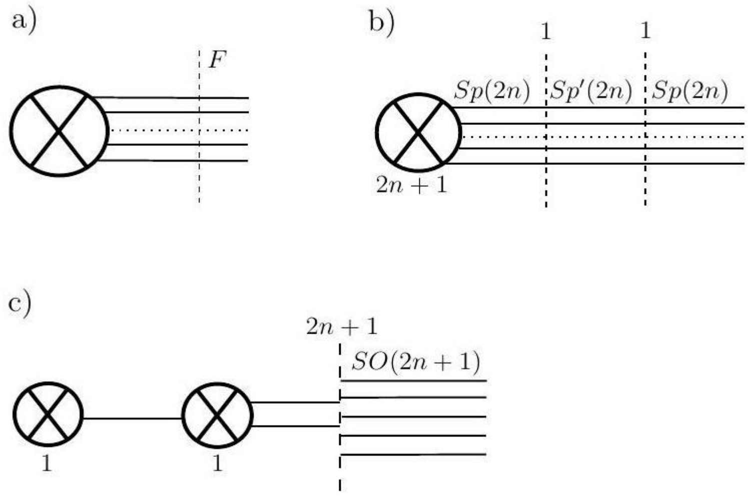


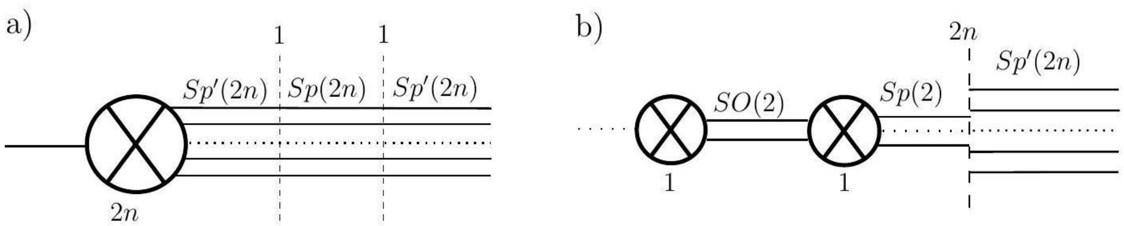


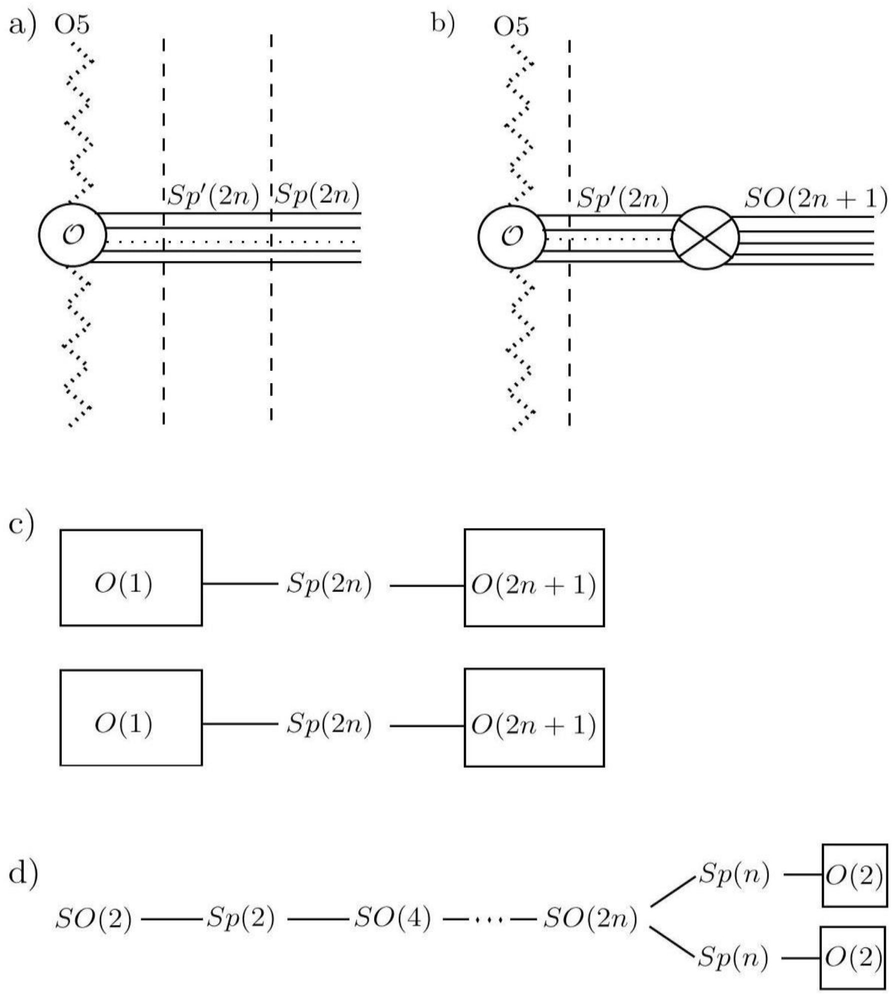

a)
b)
Sp
Sp'
Sp
Sp'
c)
d)
SO(3)
Sp(2)
SO(5)
Sp(4)

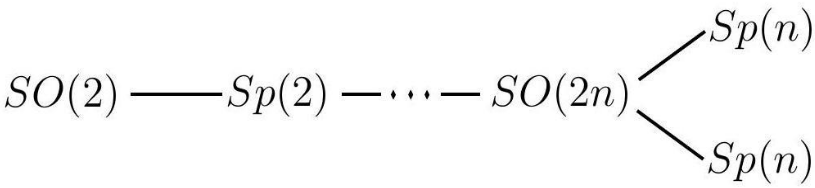

a)
F
2n
b)
2
0
2n
2n − 1
c)
2n − 1
SO(1)
Sp'(2)
SO(3)
2
0
d)
O(1)
Sp(2)
O(3)
e)
SO(2)
Sp(2)
SO(3)
Sp(2)
SO(3)
O(1)
f)
SO(2)
Sp(2)
SO(3)
Sp(2)
U(2)
1
O(1)

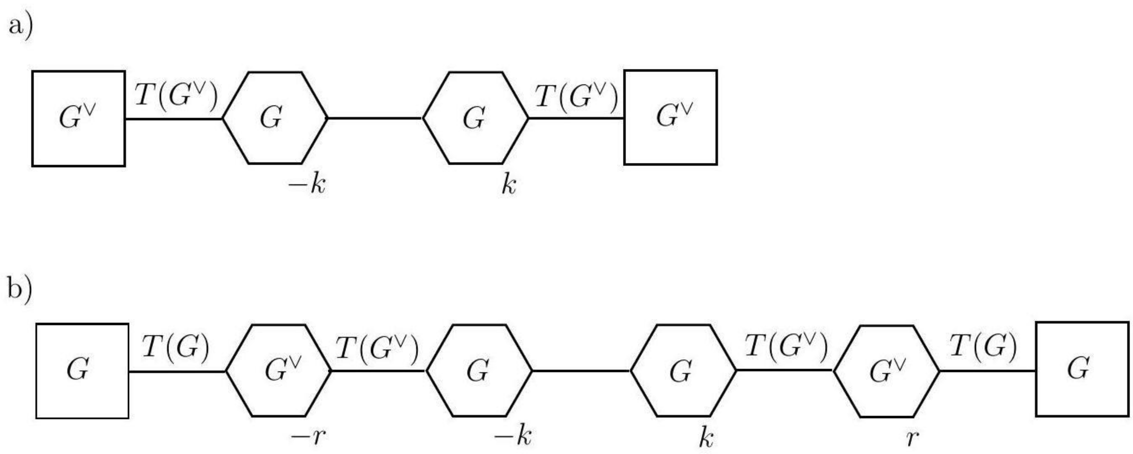

a)
2
0
$Sp'(0)$
$SO(2n+1)$
$2n$
b)
$2n$
$Sp'(0)$
$SO(3)$
$Sp'(2)$
$Sp(2n)$
2
0
c)
$SO(3)$
$Sp(2)$


a)
F
b)
1
1
$Sp(2n)$
$Sp'(2n)$
$Sp(2n)$
$2n+1$
c)
$2n+1$
$SO(2n+1)$
1
1


a)
1
1
$Sp'(2n)$
$Sp(2n)$
$Sp'(2n)$
$2n$
b)
$2n$
$Sp'(2n)$
$SO(2)$
$Sp(2)$
1
1







$$\frac{1}{2\pi}\int_{\partial M} A\wedge dB+\frac{k}{4\pi}\int_{\partial M} B\wedge dB$$

$$-\frac{1}{4\pi k}\int_{\partial M} A\wedge dA$$

$$e^{-1}\mathcal{L}=R-\frac{1}{2}(\partial\phi)^2+\frac{8}{L^2}e^{\frac{\phi}{\sqrt{6}}}+\frac{4}{L^2}e^{\frac{-2\phi}{\sqrt{6}}}-e^{\frac{-4\phi}{\sqrt{6}}}f_{\mu\nu}f^{\mu\nu}-2e^{\frac{2\phi}{\sqrt{6}}}F_{\mu\nu}F^{\mu\nu}-2\epsilon^{\mu\nu\rho\sigma\tau}f_{\mu\nu}F_{\rho\sigma}A_\tau$$

$$ds^2 = e^{2A(r)}(-h(r)dt^2+d\vec{x}^2)+\frac{e^{2B(r)}}{h(r)}dr^2$$
$$a_\mu dx^\mu = \Phi_1(r)dt, A_\mu dx^\mu=\Phi_2(r)dt, \phi=\phi(r)$$

$$A(r)=\log\frac{r}{L}+\frac{1}{6}\log\left(1+\frac{Q_1^2}{r^2}\right)+\frac{1}{3}\log\left(1+\frac{Q_2^2}{r^2}\right)$$
$$B(r)=-\log\frac{r}{L}-\frac{1}{3}\log\left(1+\frac{Q_1^2}{r^2}\right)-\frac{2}{3}\log\left(1+\frac{Q_2^2}{r^2}\right)$$
$$h(r)=1-\frac{r^2(r_H^2+Q_1^2)(r_H^2+Q_2^2)^2}{r_H^2(r^2+Q_1^2)(r^2+Q_2^2)^2},\phi(r)=-\sqrt{\frac{2}{3}}\log\left(1+\frac{Q_1^2}{r^2}\right)+\sqrt{\frac{2}{3}}\log\left(1+\frac{Q_2^2}{r^2}\right)$$
$$\Phi_1(r)=\frac{Q_1(r_H^2+Q_2^2)}{2Lr_H\sqrt{r_H^2+Q_1^2}}\left(1-\frac{r_H^2+Q_1^2}{r^2+Q_1^2}\right),\Phi_2(r)=\frac{Q_2\sqrt{r_H^2+Q_1^2}}{2Lr_H}\left(1-\frac{r_H^2+Q_2^2}{r^2+Q_2^2}\right)$$

$$T=\frac{2r_H^4+Q_1^2r_H^2-Q_1^2Q_2^2}{2\pi L^2r_H^2\sqrt{r_H^2+Q_1^2}},\mu_1=\frac{Q_1(r_H^2+Q_2^2)}{L^2r_H\sqrt{r_H^2+Q_1^2}},\mu_2=\frac{\sqrt{2}Q_2\sqrt{r_H^2+Q_1^2}}{L^2r_H},$$
$$s=\frac{1}{4GL^3}(r_H^2+Q_1^2)^{1/2}(r_H^2+Q_2^2),\rho_1=\frac{Q_1 s}{2\pi r_H},\rho_2=\frac{\sqrt{2}Q_2 s}{2\pi r_H},$$

$$2r_H^4+Q_1^2r_H^2-Q_1^2Q_2^2=0\ (\mathfrak{E}_{extremal}(2+1)\langle\text{QBH}\rangle),$$

$$e^{2A}\to\frac{r^2}{L^2},e^{2B}\to\frac{L^2}{r^2},h\to 1,\phi\to 0,\Phi_i\to\mathfrak{G}$$

$$A(r)=\log\frac{r}{L}+\frac{1}{3}\log\left(1+\frac{Q_2^2}{r^2}\right),B(r)=-\log\frac{r}{L}-\frac{2}{3}\log\left(1+\frac{Q_2^2}{r^2}\right)$$
$$h(r)=1-\frac{(r_H^2+Q_2^2)^2}{(r^2+Q_2^2)^2},\phi(r)=\sqrt{\frac{2}{3}}\log\left(1+\frac{Q_2^2}{r^2}\right),\Phi_2(r)=\frac{Q_2}{2L}\left(1-\frac{r_H^2+Q_2^2}{r^2+Q_2^2}\right)$$

$$T=\frac{r_H}{\pi L^2},\mu_2=\frac{\sqrt{2}Q_2}{L^2},s=\frac{r_H(r_H^2+Q_2^2)}{4GL^3},\rho_2=\frac{\sqrt{2}Q_2 s}{2\pi r_H}$$

$$r_H=0\ (\mathfrak{E}_{extremal}\langle 2\text{QBH}\rangle).$$

$$\frac{Q_1Q_2}{r_H^2}\to 0\ (\mathfrak{E}_{extremal}\langle 2\rangle)$$

$$\frac{Q_1Q_2}{r_H^2}\to\sqrt{2}\ (\mathfrak{E}_{extremal}\langle 2+1\rangle)$$

$$\Phi_1\to\frac{Q_2}{\sqrt{2}L}$$

$$\lim_{Q_1\to 0,T=0}\frac{\mu_1}{\mu_2}\to 1.$$

$$\frac{Q_1}{Q_2}=\frac{\sqrt{2}\rho_1}{\rho_2}$$

$$\left(i\gamma^\mu\nabla_\mu-m(\phi)+gq_1\gamma^\mu a_\mu+gq_2\gamma^\mu A_\mu+ip_1e^{\frac{-2\phi}{\sqrt{6}}}f_{\mu\nu}\gamma^{\mu\nu}+ip_2e^{\frac{\phi}{\sqrt{6}}}F_{\mu\nu}\gamma^{\mu\nu}\right)\chi=0$$

$$m(\phi) \equiv g\left(m_1 e^{-\frac{\phi}{\sqrt{6}}} + m_2 e^{\frac{2\phi}{\sqrt{6}}}\right)$$

$\chi^{q_a q_b q_c}$	Dual operator	m_1	m_2	q_1	q_2	p_1	p_2
$\chi^{(\frac{3}{2},\frac{1}{2},\frac{1}{2})}$	$\lambda_1 Z_1$	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$	1	$-\frac{1}{4}$	$\frac{1}{2}$
$\chi^{(\frac{3}{2},-\frac{1}{2},-\frac{1}{2})}$	$\lambda_2 Z_1$	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$	-1	$-\frac{1}{4}$	$-\frac{1}{2}$
$\bar{\chi}^{(\frac{3}{2},-\frac{1}{2},\frac{1}{2})}$, $\bar{\chi}^{(\frac{3}{2},\frac{1}{2},-\frac{1}{2})}$	$\bar{\lambda}_3 Z_1$, $\bar{\lambda}_4 Z_1$	$\frac{1}{2}$	$-\frac{3}{4}$	$\frac{3}{2}$	0	$\frac{1}{4}$	0
$\chi^{(\frac{1}{2},\frac{3}{2},\frac{1}{2})}$, $\chi^{(\frac{1}{2},\frac{1}{2},\frac{3}{2})}$	$\lambda_1 Z_2$, $\lambda_1 Z_3$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	2	$\frac{1}{4}$	0
$\bar{\chi}^{(-\frac{1}{2},\frac{3}{2},\frac{1}{2})}$, $\bar{\chi}^{(-\frac{1}{2},\frac{1}{2},\frac{3}{2})}$	$\bar{\lambda}_2 Z_2$, $\bar{\lambda}_2 Z_3$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{2}$	2	$\frac{1}{4}$	0
$\chi^{(-\frac{1}{2},\frac{3}{2},-\frac{1}{2})}$, $\chi^{(-\frac{1}{2},-\frac{1}{2},\frac{3}{2})}$	$\lambda_3 Z_2$, $\lambda_4 Z_3$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	1	$-\frac{1}{4}$	$-\frac{1}{2}$
$\bar{\chi}^{(\frac{1}{2},-\frac{1}{2},\frac{3}{2})}$, $\bar{\chi}^{(\frac{1}{2},\frac{3}{2},-\frac{1}{2})}$	$\bar{\lambda}_3 Z_3$, $\bar{\lambda}_4 Z_2$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$-\frac{1}{4}$	$\frac{1}{2}$

$$|p_1| = \frac{1}{4}$$

$$m_2 = -2q_1 p_1$$

$$\chi = e^{-2A} h^{-1/4} e^{-i\omega t + ikx} \Psi$$

$$(\partial_r + X\sigma_3 + Yi\sigma_2 + Z\sigma_1)\psi_\alpha = 0$$

$$X \equiv \frac{me^B}{\sqrt{h}}, Y \equiv -\frac{e^{B-A}}{\sqrt{h}}u, Z \equiv -\frac{e^{B-A}}{\sqrt{h}}((-1)^\alpha k - v)$$

$$u \equiv \frac{1}{\sqrt{h}}(\omega + gq_1\Phi_1 + gq_2\Phi_2), v \equiv 2e^{-B}\left(p_1 e^{-\frac{2\phi}{\sqrt{6}}}\partial_r\Phi_1 + p_2 e^{\frac{\phi}{\sqrt{6}}}\partial_r\Phi_2\right)$$

$$\psi_\alpha = \begin{pmatrix}\psi_{\alpha-}\\ \psi_{\alpha+}\end{pmatrix}$$

$$U_\pm \equiv \psi_- \pm i\psi_+$$

$$U'_- + iYU_- = (-X + iZ)U_+$$
$$U'_+ - iYU_+ = (-X - iZ)U_-$$

$$U''_- + pU'_- + (iY' - X^2 + Y^2 - Z^2 + iYp)U_- = 0$$
$$U''_+ + \bar{p}U'_+ + (-iY' - X^2 + Y^2 - Z^2 - iY\bar{p})U_+ = 0$$

$$p \equiv -\partial_r \log(-X + iZ)$$

$$\psi_+ \sim A(\omega,k)r^{mL} + B(\omega,k)r^{-mL-1}, \psi_- \sim C(\omega,k)r^{mL-1} + D(\omega,k)r^{-mL}$$
$$G_R = \frac{D}{A}$$

$$A(\omega = 0, k = k_F) \equiv 0$$

$$e^A \to k_0, e^B \to \frac{\tau_0 L_2}{k_0}, h \to \left(\frac{\tau_0}{k_0}\right)^2 (r - r_H)^2, \Phi_i \to \beta_i(r - r_H), \phi \to \phi_0,$$

$$ds^2 = -\tau_0^2(r - r_H)^2 dt^2 + \frac{L_2^2 dr^2}{(r - r_H)^2} + k_0^2 d\vec{x}^2$$

$$U'' + \left(\frac{1}{r - r_H} + \cdots\right)U' + \left(\frac{L^4(Q_2^4 - r_H^4)\omega^2}{16(2Q_2^2 - r_H^2)(r - r_H)^4} + \frac{\#\omega}{(r - r_H)^3} - \frac{v^2}{(r - r_H)^2} + \cdots\right)U = 0,$$

$$\nu^2 = \frac{(m_1(1-\mu_R)^2 + m_2\mu_R^2)^2}{(1-\mu_R^4)} + \frac{\tilde{k}^2}{\mu_2^2}\frac{1}{2(1+\mu_R^2)} - \frac{\left(\sqrt{2}q_1\mu_R^3 + q_2(1-\mu_R^2)\right)^2}{4(1-\mu_R^2)(1+\mu_R^2)^2},$$

$$\tilde{k} \equiv k - (-1)^\alpha\left(2p_1\mu_1 + \sqrt{2}p_2\mu_2\right).$$

$$U'' + \frac{1}{r-r_H}U' + \left(\frac{L^4(Q_2^4 - r_H^4)\omega^2}{16(2Q_2^2 - r_H^2)(r-r_H)^4} + \frac{\#\omega}{(r-r_H)^3} - \frac{\nu^2}{(r-r_H)^2}\right)U = 0.$$

$$U'' + \frac{1}{r-r_H}U' - \frac{\nu^2}{(r-r_H)^2}U = 0.$$

$$U \sim (r-r_H)^{-\frac{1}{2}\pm\nu}.$$

$$U \sim (r-r_H)^{-\frac{1}{2}+\nu} + \mathcal{G}(\omega)(r-r_H)^{-\frac{1}{2}-\nu},$$

$$\mathcal{G}(\omega) = |c(k)|e^{i\gamma_k}(2\omega)^{2\nu}$$

$$U'' + \frac{1}{r-r_H}U' - \frac{\nu^2}{(r-r_H)^2}U = 0$$

$$\eta_\pm^0 \to (r-r_H)^{-\frac{1}{2}\pm\nu},$$

$$U \sim \eta_+^0 + \mathcal{G}(\omega)\eta_-^0$$

$$\eta_\pm = \eta_\pm^0 + \omega\eta_\pm^1 + \cdots$$

$$U \sim \eta_+ + \mathcal{G}(\omega)\eta_-.$$

$$G = \frac{D}{A} = \frac{b_+^0 + \omega b_+^1 + \cdots + \mathcal{G}(\omega)(b_-^0 + \omega b_-^1 + \cdots)}{a_+^0 + \omega a_+^1 + \cdots + \mathcal{G}(\omega)(a_-^0 + \omega a_-^1 + \cdots)},$$

$$G_R(k,\omega) \sim \frac{h_1}{k_\perp - \frac{1}{v_F}\omega + \cdots - h_2 e^{i\gamma_{k_F}}(2\omega)^{2\nu_{k_F}}}$$

$$k_\perp = \frac{1}{v_F}\omega + \cdots + h_2\cos\,\gamma_{k_F}(2\omega)^{2\nu_{k_F}} + ih_2\sin\,\gamma_{k_F}(2\omega)^{2\nu_{k_F}}$$

$$\frac{\Gamma}{\omega_*} = \tan\,\frac{\gamma_{\nu_F}}{2\nu_F}$$

$$Z \sim (k_\perp)^{\frac{1}{2\nu_k}-1}$$

$$X = \frac{a}{r^2} + b, Y = \frac{c\omega}{r^2} + d\omega + f, Z = \frac{P}{r}$$

$$a = m_2\mu L^2, b = \frac{1}{\mu L^2}\left(2m_1 + \frac{3}{2}m_2\right), c = -\frac{L^2}{2}, d = -\frac{1}{2\mu^2L^2}, f = -\frac{q_2}{\sqrt{2}}\frac{1}{\mu L^2},$$

$$P = -(-1)^\alpha\frac{k}{\mu} + \sqrt{2}p_2$$

$$p = \frac{2}{r} + \frac{iP}{a} + \mathcal{O}(r)$$

$$iY' + iYp = -\frac{c\omega P}{ar^2} + \mathcal{O}\left(\frac{1}{r}\right)$$

$$\psi'' + \left(\frac{2}{r} + \cdots\right)\psi' + \left(\frac{c^2(\omega^2 - \Delta^2)}{r^4} + \frac{\frac{1}{4} - 4\nu^2 + \mathcal{O}(|\omega| - \Delta)}{r^2} + \cdots\right)\psi = 0$$

$$\Delta \equiv \left|\frac{a}{c}\right| = 2|m_2|\mu$$

$$\nu^2 \equiv \frac{1}{4}\left(P - \frac{1}{2}\mathrm{sgn}(\omega m_2)\right)^2 + \frac{1}{4}m_2^2 + m_1 m_2 - \frac{\mathrm{sgn}(\omega m_2)}{2\sqrt{2}} q_2 m_2$$

$$\psi'' + \frac{2}{r}\psi' + \frac{c^2(\omega^2 - \Delta^2)}{r^4}\psi = 0$$

$$\epsilon^2 \equiv \frac{\omega^2}{\Delta^2} - 1$$

$$\psi \sim \exp\left(\pm\frac{i\Delta L^2 \epsilon}{2r}\right)$$

$$\psi \sim \exp\left(\pm\frac{\Delta L^2 |\epsilon|}{2r}\right)$$

$$\psi'' + \frac{2}{r}\psi' + \frac{\frac{1}{4} - 4\nu^2}{r^2}\psi = 0$$

$$\psi \sim r^{-\frac{1}{2} \pm 2\nu}$$

$$\psi'' + \frac{2}{r}\psi' + \left(\frac{a^2\epsilon^2}{r^4} + \frac{\frac{1}{4} - 4\nu^2}{r^2}\right)\psi = 0$$

$$\psi \sim \frac{1}{\sqrt{r}} J_{2\nu}\left(\frac{\Delta L^2 \epsilon}{2r}\right), \frac{1}{\sqrt{r}} Y_{2\nu}\left(\frac{\Delta L^2 \epsilon}{2r}\right), \omega > \Delta.$$

$$\psi \sim \frac{1}{\sqrt{r}} K_{2\nu}\left(\frac{\Delta L^2 |\epsilon|}{2r}\right), \frac{1}{\sqrt{r}} I_{2\nu}\left(\frac{\Delta L^2 |\epsilon|}{2r}\right), \omega < \Delta.$$

$$\psi = \frac{1}{\sqrt{r}} H_{2\nu}^{(1)}\left(\frac{\Delta L^2 \epsilon}{2r}\right) \to \sqrt{\frac{2}{\epsilon\pi}} e^{-\frac{i\pi}{4} - \frac{2\pi i\nu}{2}} \exp\left(\frac{i\Delta L^2 \epsilon}{2r}\right)$$

$$H_{2\nu}^{(1)}(x) = -\frac{i\Gamma(2\nu)}{\pi}\left(\frac{x}{2}\right)^{-2\nu} + \cdots - \frac{i\Gamma(-2\nu)}{\pi} e^{-2\pi i\nu}\left(\frac{x}{2}\right)^{2\nu} + \cdots$$

$$\psi = -\frac{i}{\pi}\left(\Gamma(2\nu)\left(\frac{\Delta L^2 \epsilon}{4}\right)^{-2\nu} r^{2\nu - \frac{1}{2}} + \cdots + \Gamma(-2\nu) e^{-2\pi i\nu}\left(\frac{\Delta L^2 \epsilon}{4}\right)^{2\nu} r^{-2\nu - \frac{1}{2}}\right)$$

$$\begin{aligned} \mathcal{G}_+(\epsilon) &= e^{-2\pi i\nu}\frac{\Gamma(-2\nu)}{\Gamma(2\nu)}\left(\frac{\Delta L^2 \epsilon}{4}\right)^{4\nu} \\ &= e^{-2\pi i\nu}\frac{\Gamma(-2\nu)}{\Gamma(2\nu)}\left(\frac{L^4(\omega^2 - \Delta^2)}{16}\right)^{2\nu} \end{aligned}$$

$$\mathcal{G}_+(\epsilon) \approx e^{-2\pi i\nu}\frac{\Gamma(-2\nu)}{\Gamma(2\nu)}\left(\frac{L^4 \Delta(|\omega| - \Delta)}{8}\right)^{2\nu}$$

$$\gamma \equiv \arg\mathcal{G} = -2\pi\nu$$

$$|\epsilon| = \sqrt{1 - \omega^2/\Delta^2}$$

$$\psi = \frac{1}{\sqrt{r}} K_{2\nu}\left(\frac{\Delta L^2|\epsilon|}{2r}\right)$$

$$\psi = \frac{\Gamma(2\nu)}{2}\left(\frac{\Delta L^2|\epsilon|}{4}\right)^{-2\nu} r^{2\nu-\frac{1}{2}} + \cdots + \frac{\Gamma(-2\nu)}{2}\left(\frac{\Delta L^2|\epsilon|}{4}\right)^{2\nu} r^{-2\nu-\frac{1}{2}} + \cdots$$

$$\mathcal{G}_-(\epsilon) = \frac{\Gamma(-2\nu)}{\Gamma(2\nu)}\left(\frac{\Delta L^2|\epsilon|}{4}\right)^{4\nu} = \frac{\Gamma(-2\nu)}{\Gamma(2\nu)}\left(\frac{L^4(\Delta^2-\omega^2)}{16}\right)^{2\nu}$$

$$\mathcal{G}_-(\epsilon) \approx \frac{\Gamma(-2\nu)}{\Gamma(2\nu)}\left(\frac{L^4\Delta(\Delta-|\omega|)}{8}\right)^{2\nu}$$

$$K_{2\nu}(x) = \frac{\pi}{2} i^{2\nu+1} H_{2\nu}^{(1)}(ix)$$

$$\psi = \eta_+ + \mathcal{G}(\epsilon)\eta_-$$

$$\eta_\pm \sim r^{-\frac{1}{2}\pm 2\nu}$$

$$\epsilon^2 \approx \frac{2(|\omega|-\Delta)}{\Delta}$$

$$G_R = \frac{D}{A} = \frac{b_+^{(0)} + \epsilon^2 b_+^{(2)} + \cdots + \mathcal{G}(\epsilon)\left(b_-^{(0)} + \epsilon^2 b_-^{(2)} + \cdots\right)}{a_+^{(0)} + \epsilon^2 a_+^{(2)} + \cdots + \mathcal{G}(\epsilon)\left(a_-^{(0)} + \epsilon^2 a_-^{(2)} + \cdots\right)}$$

$$G_R \sim \frac{h_1}{(k-k_\Delta) - \frac{|\omega|-\Delta}{v_F} + \cdots - h_2 e^{-2\pi i \nu_{k_\Delta}}(|\omega|-\Delta)^{2\nu_{k_\Delta}}},$$

$$\frac{\Gamma}{\omega_*} = \tan\frac{-2\pi\nu_\Delta}{2\nu_\Delta} = 0$$

$$Z \sim (k-k_\Delta)^{\frac{1}{2\nu_\Delta}-1}$$

$$G_R \sim \frac{h_1}{(k-k_\Delta) + \frac{|\omega|-\Delta}{v_F} + \cdots - h_2(\Delta-|\omega|)^{2\nu_{k_\Delta}}}$$

$$\begin{aligned}\omega &= \omega_{2+1} + \frac{\sqrt{2}q_1 Q_2}{L^2} \\ &= \omega_{2+1} + q_1\mu_2\end{aligned}$$

$$\omega = \omega_{2+1} + q_1\mu_1$$

$$|q_1| = 2|m_2|$$

$$\omega = \omega_{2+1} + \operatorname{sgn}(q_1)\Delta$$

$$\frac{\tilde{k}}{\mu_2} \to \frac{k}{\mu_2} - (-1)^\alpha\left(2p_1 + \sqrt{2}p_2\right) = -(-1)^\alpha(P + 2p_1)$$

$$\nu^2 \to \frac{4m_2^2 - q_1^2}{16(1-\mu_R)} + \frac{1}{4}\left((P+2p_1)^2 + 4m_1m_2 + \frac{7q_1^2}{8} - \frac{5m_2^2}{2} - \frac{q_1q_2}{\sqrt{2}}\right) + \mathcal{O}(1-\mu_R)$$

$$\nu^2 \to \frac{1}{4}\left((P+2p_1)^2 + 4m_1m_2 + m_2^2 - \operatorname{sgn}(q_1m_2)\sqrt{2}q_2m_2\right)$$

$$\operatorname{sgn}(m_2) = -\operatorname{sgn}(q_1)\operatorname{sgn}(p_1)$$

$$\nu^2 \to \frac{1}{4}\left(\left(P - \frac{1}{2}\mathrm{sgn}(\omega m_2)\right)^2 + 4m_1m_2 + m_2^2 - \mathrm{sgn}(\omega m_2)\sqrt{2}q_2m_2\right),$$

$$\gamma_k \equiv \arg\left(\Gamma(-2\nu_k)\left(e^{-2\pi i\nu_k} - e^{-2\pi(qe)_{\mathrm{eff}}}\right)\right)$$

$$(qe)_{\mathrm{eff}}^2 = \frac{\left(\sqrt{2}q_1\mu_R^3 + q_2(1-\mu_R^2)\right)^2}{4(1-\mu_R^2)(1+\mu_R^2)^2}$$

$$\gamma_k \to -2\pi\nu_k$$

$$\psi'' + \frac{2}{r}\psi' + \left(\frac{q_1^2 - 4m_2^2}{2r^4} + \frac{\frac{1}{4} - 4\nu^2}{r^2}\right)\psi = 0$$

$\chi^{q_a q_b q_c}$	Dual operator	m_1	m_2	q_1	q_2	p_1	p_2
$\chi^{(\frac{1}{2},\frac{3}{2},\frac{1}{2})}$, $\chi^{(\frac{1}{2},\frac{1}{2},\frac{3}{2})}$	$\lambda_1 Z_2$, $\lambda_1 Z_3$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	2	$\frac{1}{4}$	0
$\overline{\chi}^{(-\frac{1}{2},\frac{3}{2},\frac{1}{2})}$, $\overline{\chi}^{(-\frac{1}{2},\frac{1}{2},\frac{3}{2})}$	$\overline{\lambda}_2 Z_2$, $\overline{\lambda}_2 Z_3$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	2	$-\frac{1}{4}$	0

$$\Delta \equiv 2|m_2|\mu = \frac{\mu}{2}$$

$$\frac{k_F}{\mu} \approx 0.83934.$$

$$\omega \approx v_F k_\perp, v_F \approx 0.724,$$

$$\frac{k_{-\Delta} - k_F}{\mu} \approx -0.92513,$$

$$\nu_{k_{-\Delta}} = 0.33211,$$

$$\frac{k_\Delta - k_F}{\mu} \approx 0.743052,$$

$$\frac{k_\Delta}{\mu} = 1.58239,$$

$$\nu_{k_\Delta} = 0.08211.$$

$$\frac{k_\Delta}{\mu} \approx -0.58239$$

$\chi^{q_a q_b q_c}$	Dual operator	m_1	m_2	q_1	q_2	p_1	p_2
$\chi^{(-\frac{1}{2},\frac{3}{2},-\frac{1}{2})}$, $\chi^{(-\frac{1}{2},-\frac{1}{2},\frac{3}{2})}$	$\lambda_3 Z_2$, $\lambda_4 Z_3$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	1	$-\frac{1}{4}$	$-\frac{1}{2}$
$\overline{\chi}^{(\frac{1}{2},-\frac{1}{2},\frac{3}{2})}$, $\overline{\chi}^{(\frac{1}{2},\frac{3}{2},-\frac{1}{2})}$	$\overline{\lambda}_3 Z_3$, $\overline{\lambda}_4 Z_2$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{1}{2}$

$$\frac{k_F}{\mu} \approx 0.05202$$

$$\frac{k_\Delta}{\mu} \approx 0.79289$$

$$\nu_{k_\Delta} = 0.22855$$

$\chi^{q_a q_b q_c}$	Dual operator	m_1	m_2	q_1	q_2	p_1	p_2
$\bar{\chi}^{(\frac{3}{2},-\frac{1}{2},\frac{1}{2})}$, $\bar{\chi}^{(\frac{3}{2},\frac{1}{2},-\frac{1}{2})}$	$\overline{\lambda}_3 Z_1$, $\overline{\lambda}_4 Z_1$	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$	0	$-\frac{1}{4}$	0

$\chi^{q_a q_b q_c}$	Dual operator	m_1	m_2	q_1	q_2	p_1	p_2
$\chi^{(\frac{3}{2},\frac{1}{2},\frac{1}{2})}$	$\lambda_1 Z_1$	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$	1	$-\frac{1}{4}$	$\frac{1}{2}$

$$ds^2 = -\frac{2r^{8/3}}{L^2 Q^{2/3}} dt^2 + \frac{r^{2/3} Q^{4/3}}{L^2} d\vec{x}^2 + \frac{L^2}{2r^{4/3} Q^{2/3}} dr^2$$

$$e^{\frac{\phi}{2\sqrt{6}}} = \left(\frac{Q}{r}\right)^{1/3}.$$

$$\begin{aligned} d\hat{s}^2 &= e^{2\alpha\phi} ds^2 + e^{2\beta\phi}(dz + \mathcal{A})^2 \\ &= e^{\frac{\phi}{\sqrt{6}}} ds^2 + e^{-\frac{3\phi}{\sqrt{6}}}(L d\varphi_3 + \mathcal{A})^2 \end{aligned}$$

$$\alpha = \frac{1}{2\sqrt{6}}, \beta = -3\alpha = -\frac{3}{2\sqrt{6}}$$

$$d\hat{s}^2 = -\frac{2r^2}{L^2} dt^2 + \frac{L^2}{2r^2} dr^2 + \frac{r^2 L^2}{Q^2} d\varphi_3^2 + \frac{Q^2}{L^2} d\vec{x}^2$$

$$\mathcal{L}_{EH} = \sqrt{-\hat{g}}\hat{R} = \sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4} e^{\frac{-4\phi}{\sqrt{6}}} \mathcal{F}^2\right)$$

$$a_\mu \equiv \frac{1}{2}\mathcal{A}_\mu$$

$$\mathcal{L}_\Lambda = \sqrt{-\hat{g}}\hat{\Lambda},$$

$$\mathcal{L}_\Lambda = \sqrt{-g} e^{\frac{\phi}{\sqrt{6}}} \hat{\Lambda}$$

$$\mathcal{L}_{\text{pot}} = \sqrt{-g}\left(\frac{8}{L^2} e^{\frac{\phi}{\sqrt{6}}} + \frac{4}{L^2} e^{\frac{-2\phi}{\sqrt{6}}}\right)$$

$$\hat{\Lambda} = \frac{8}{L^2}$$

$$\mathcal{L}_\Lambda = \sqrt{-\hat{g}} \frac{8}{L^2}$$

$$-\frac{1}{2}\hat{F}_2 \wedge \hat{*} \hat{F}_2 = -\frac{1}{2} e^{-2\alpha\phi} F_2 \wedge * F_2 - \frac{1}{2} e^{6\alpha\phi} F_1 \wedge * F_1$$

$$F_2 \equiv dA_1 - dA_0 \wedge \mathcal{A}_1, F_1 = dA_0$$

$$-\frac{1}{2}\hat{H}_3 \wedge \hat{*} \hat{H}_3 = -\frac{1}{2} e^{-4\alpha\phi} H_3 \wedge * H_3 - \frac{1}{2} e^{4\alpha\phi} H_2 \wedge * H_2,$$

$$H_3 \equiv dB_2 - dB_1 \wedge \mathcal{A}_1, H_2 = dB_1$$

$$B_\mu \propto A_\mu$$

$$S_\eta \equiv -\int d^6x\sqrt{-\hat{g}}\hat{g}^{MN}\partial_M\hat{\eta}\partial_N\hat{\eta}$$

$$\hat{\eta}(\hat{x}^M) = e^{in\varphi_3}\eta(x^\mu),$$

$$S_\eta \to -2\pi L\int d^5x\sqrt{-g}\left(g^{\mu\nu}\overline{D_\mu\eta}D_\nu\eta + e^{\frac{4\phi}{\sqrt{6}}}\frac{n^2}{L^2}|\eta|^2\right)$$

$$D_\mu \equiv \partial_\mu - i\frac{n}{L}\mathcal{A}_\mu = \partial_\mu - igna_\mu$$

$$m = \pm\frac{n}{L}e^{\frac{2\phi}{\sqrt{6}}}$$

$$\begin{aligned} k^M k_M &= \frac{Q^2}{r^2L^2}n^2 - \frac{L^2}{2r^2}\omega^2 \\ &= \frac{L^2}{2r^2}(\Delta^2 - \omega^2) \end{aligned}$$

$$\Gamma\lambda^a = \varepsilon\lambda^a, \lambda^a = B_6\Omega_{ab}(\lambda^b)^*$$

$$\mathcal{L} = \sqrt{-\hat{g}}\frac{i}{2}\bar{\lambda}^a\Gamma^{\underline{M}}\hat{e}^N_{\underline{M}}\hat{\nabla}_N\lambda_a$$

$$\hat{\nabla}_N = \partial_N - \frac{1}{4}\hat{\omega}_N{}^{\underline{PQ}}\Gamma_{\underline{PQ}}.$$

$$\lambda_1(z,x^\mu) \equiv e^{in\varphi_3}e^{\frac{\eta}{\sqrt{6}}\phi}\begin{pmatrix}\chi_1(x^\mu)\\0\end{pmatrix}, \lambda_2(z,x^\mu) \equiv e^{-in\varphi_3}e^{\frac{\eta}{\sqrt{6}}\phi}\begin{pmatrix}\chi_2(x^\mu)\\0\end{pmatrix},$$

$$e^{-1}\mathcal{L} = \frac{i}{2}\bar{\chi}\gamma^\mu\nabla_\mu\chi + \frac{1}{2}\bar{\chi}\left(\frac{n\varepsilon}{L}\right)e^{\frac{2\phi}{\sqrt{6}}}\tau_3\chi + \frac{1}{2}\bar{\chi}\left(\frac{2n}{L}\right)\gamma^\mu a_\mu\tau_3\chi + \frac{i}{2}\bar{\chi}\left(\frac{\varepsilon}{4}\right)e^{\frac{-2\phi}{\sqrt{6}}}\gamma^{\mu\nu}f_{\mu\nu}\chi,$$

$$\mathcal{L} = \frac{1}{2}(i\bar{\chi}\gamma^\mu\nabla_\mu\chi - m\bar{\chi}\tau_3\chi + q\bar{\chi}\gamma^\mu a_\mu\tau_3\chi + ip\bar{\chi}\gamma^{\mu\nu}f_{\mu\nu}\chi),$$

$$m(\phi) = g\left(m_1e^{\frac{-\phi}{\sqrt{6}}} + m_2e^{\frac{2\phi}{\sqrt{6}}}\right) \approx \frac{2m_2}{L}e^{\frac{2\phi}{\sqrt{6}}} + \cdots,$$

$$m_2 = -\frac{n\varepsilon}{2}, q_1 = n, p_1 = \frac{\varepsilon}{4}.$$

$$m_2 = -2q_1p_1$$

$\chi^{q_aq_bq_c}$	Dual operator	m_1	m_2	q_1	q_2	p_1	p_2	n	ε
$\chi^{(\frac{3}{2},\frac{1}{2},\frac{1}{2})}$	λ_1Z_1	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$	1	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{2}$	-1
$\chi^{(\frac{3}{2},-\frac{1}{2},-\frac{1}{2})}$	λ_2Z_1	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$	-1	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{3}{2}$	-1
$\bar{\chi}^{(\frac{3}{2},-\frac{1}{2},\frac{1}{2})}$, $\bar{\chi}^{(\frac{3}{2},\frac{1}{2},-\frac{1}{2})}$	$\bar{\lambda}_3Z_1$, $\bar{\lambda}_4Z_1$	$\frac{1}{2}$	$-\frac{3}{4}$	$\frac{3}{2}$	0	$\frac{1}{4}$	0	$\frac{3}{2}$	1
$\chi^{(\frac{1}{2},\frac{3}{2},\frac{1}{2})}$, $\chi^{(\frac{1}{2},\frac{1}{2},\frac{3}{2})}$	λ_1Z_2, λ_1Z_3	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	2	$\frac{1}{4}$	0	$\frac{1}{2}$	1
$\bar{\chi}^{(-\frac{1}{2},\frac{3}{2},\frac{1}{2})}$, $\bar{\chi}^{(-\frac{1}{2},\frac{1}{2},\frac{3}{2})}$	$\bar{\lambda}_2Z_2$, $\bar{\lambda}_2Z_3$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{2}$	2	$\frac{1}{4}$	0	$-\frac{1}{2}$	1
$\chi^{(-\frac{1}{2},\frac{3}{2},-\frac{1}{2})}$, $\chi^{(-\frac{1}{2},-\frac{1}{2},\frac{3}{2})}$	λ_3Z_2, λ_4Z_3	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	1	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
$\bar{\chi}^{(\frac{1}{2},-\frac{1}{2},\frac{3}{2})}$, $\bar{\chi}^{(\frac{1}{2},\frac{3}{2},-\frac{1}{2})}$	$\bar{\lambda}_3Z_3$, $\bar{\lambda}_4Z_2$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	-1

$$\mathcal{L}_{\text{Pauli}} = \frac{i}{2}\sqrt{-\hat{g}}\bar{\lambda}^a\Gamma^{MNP}H_{MNP}\lambda_a$$

$$\mathcal{L}_{\text{Pauli}} \sim -\frac{i\varepsilon}{2}e^{\frac{\phi}{\sqrt{6}}}\sqrt{-g}\bar{\chi}^a\gamma^{\mu\nu}F_{\mu\nu}\chi_a$$

$$\left(\partial_r + \left(\frac{a}{r^2} + b\right)\sigma_3 + \left(\frac{c\omega}{r^2} + d\omega + f\right) i\sigma_2 + \frac{P}{r}\sigma_1\right)\psi = 0$$

$$\nu(|P| \gg 1) \to |P| \mp \frac{1}{2} \approx |P|.$$

$$\psi''_\pm - F_\pm \psi'_\pm + (\mp X' - X^2 + Y^2 - Z^2 \pm X F_\pm)\psi_\pm$$

$$F_\pm \equiv \partial_r \log(\mp Y + Z)$$

$$F_\pm = -\frac{2}{r} \mp \frac{P}{c\omega} + \mathcal{O}(r)$$

$$\mp X' \pm X F_\pm = -\frac{aP}{c\omega r^2} + \mathcal{O}\left(\frac{1}{r}\right)$$

$$\frac{c\omega P}{a} \to \frac{aP}{c\omega}.$$

$$\{\gamma^{\underline{\mu}}, \gamma^{\underline{\nu}}\} = -2\eta^{\underline{\mu\nu}},$$

$$\gamma^{\underline{4}} = \gamma^{\underline{0}}\gamma^{\underline{1}}\gamma^{\underline{2}}\gamma^{\underline{3}}.$$

$$B\gamma^\mu B^{-1} = (\gamma^\mu)^*, C\gamma^\mu C^{-1} = (\gamma^\mu)^T$$

$$B^{-1} = B, B^T = B^* = -B, C^{-1} = C^T = -C, C^* = C.$$

$$\Gamma^{\underline{\mu}} = \gamma^{\underline{\mu}} \otimes \sigma_1, \Gamma^{\underline{5}} = \mathbb{I} \otimes i\sigma_2,$$

$$\Gamma \equiv \Gamma^0\Gamma^{\underline{1}}\Gamma^{\underline{2}}\Gamma^{\underline{3}}\Gamma^{\underline{4}}\Gamma^{\underline{5}} = \mathbb{I} \otimes \sigma_3$$

$$\Gamma^2 = 1, \{\Gamma, \Gamma^\mu\} = 0$$

$$B_6\Gamma^M B_6^{-1} = (\Gamma^M)^*, C_6\Gamma^M C_6^{-1} = (\Gamma^M)^T,$$

$$B_6 = B \otimes \mathbb{I}_{2\times 2}, C_6 = C \otimes \sigma_1.$$

$$B_6 \equiv -i\Gamma^4\Gamma^5\Gamma, C_6 = \Gamma^0 B_6 = -i\Gamma^0\Gamma^4\Gamma^5\Gamma$$

$$\chi_a = \Omega_{ab}\chi^b, \chi^a = \Omega^{ab}\chi_b,$$

$$\chi^a = B(\chi_a)^* = B\Omega_{ab}(\chi^b)^*$$

$$\bar{\chi}^a \equiv (\chi_a)^\dagger \gamma^0,$$

$$\bar{\chi}^a = (\chi^a)^T C$$

$$\bar{\chi}^a \gamma^{\mu_1} \cdots \gamma^{\mu_n} \psi^b = \bar{\psi}^b \gamma^{\mu_n} \cdots \gamma^{\mu_1} \chi^a$$

$$\mathcal{L} = \frac{1}{2}\left(i\bar{\chi}\gamma^\mu \nabla_\mu \chi - m\bar{\chi}\tau_3\chi + q\bar{\chi}\gamma^\mu A_\mu \tau_3 \chi + ip\bar{\chi}\gamma^{\mu\nu}F_{\mu\nu}\chi\right)$$

$$(i\gamma^\mu \nabla_\mu - m\tau_3 + q\gamma^\mu A_\mu \tau_3 + ip\gamma^{\mu\nu}F_{\mu\nu})\chi = 0$$

$$[B_6, \Gamma] = 0$$

$$\Gamma\lambda = \pm\lambda$$

$$d\hat{s}^2 = e^{2\alpha\phi}ds^2 + e^{2\beta\phi}(dz + \mathcal{A})^2$$

$$\hat{e}^{\underline{\mu}}{}_\nu = e^{\alpha\phi}e^{\underline{\mu}}{}_\nu, \hat{e}^{\underline{\mu}}{}_z = 0, \hat{e}^{\underline{z}}{}_z = e^{\beta\phi}, \hat{e}^{\underline{z}}{}_\nu = e^{\beta\phi}\mathcal{A}_\nu$$

$$\hat{e}_{\underline{\nu}}^{\mu} = e^{-\alpha\phi} e_{\underline{\nu}}^{\mu}, \hat{e}_{\underline{z}}^{\mu} = 0, \hat{e}_{\underline{z}}^{z} = e^{-\beta\phi}, \hat{e}_{\underline{\nu}}^{z} = -e^{-\alpha\phi} e_{\underline{\nu}}^{\mu} \mathcal{A}_{\mu}.$$

$$\hat{\omega}_{\mu}{}^{\underline{\nu\rho}} = \omega_{\mu}{}^{\underline{\nu\rho}} + \alpha\left(e^{\underline{\nu}}{}_{\mu}\partial^{\underline{\rho}}\phi - e^{\underline{\rho}}{}_{\mu}\partial^{\underline{\nu}}\phi\right) - \frac{1}{2} e^{2(\beta-\alpha)\phi} \mathcal{A}_{\mu} \mathcal{F}\underline{\nu}$$

$$\hat{\omega}_{\mu}{}^{\underline{\nu z}} = \frac{1}{2} e^{(\beta-\alpha)\phi} \mathcal{F}_{\mu}\underline{\nu} - \beta e^{(\beta-\alpha)\phi} \mathcal{A}_{\mu} \partial^{\underline{\nu}}\phi$$

$$\hat{\omega}_{z}{}^{\underline{\nu\rho}} = -\frac{1}{2} e^{2(\beta-\alpha)\phi} \mathcal{F}\underline{\nu\rho}$$

$$\hat{\omega}_{z}\underline{\nu z} = -\beta e^{(\beta-\alpha)\phi} \partial^{\underline{\nu}}\phi$$

$$\hat{\omega}_{\mu}{}^{\underline{\nu\rho}} = \omega_{\mu}{}^{\underline{\nu\rho}} + \alpha\left(e^{\underline{\nu}}{}_{\mu}\partial^{\underline{\rho}}\phi - e^{\underline{\rho}}{}_{\mu}\partial^{\underline{\nu}}\phi\right) + \mathcal{A}_{\mu}\hat{\omega}_{z}{}^{\underline{\nu\rho}}$$

$$\hat{\omega}_{\mu}{}^{\underline{\nu z}} = \frac{1}{2} e^{(\beta-\alpha)\phi} \mathcal{F}_{\mu}{}^{\underline{\nu}} + \mathcal{A}_{\mu}\hat{\omega}_{z}{}^{\underline{\nu z}}$$

$$\Gamma = \mathbb{I}_{4\times 4} \otimes \sigma_3$$

$$\mathcal{L}_{\partial} = \sqrt{-\hat{g}} \frac{i}{2} (\lambda_a)^{\dagger} \Gamma^{\underline{0}} \left(\Gamma^{\underline{\mu}} \hat{e}_{\underline{\mu}}^{\nu} \partial_{\nu} + \Gamma^{\underline{5}} \hat{e}_{\underline{z}}^{z} \partial_z + \Gamma^{\underline{\mu}} \hat{e}_{\underline{\mu}}^{z} \partial_z\right) \lambda_a,$$

$$\Gamma^{\underline{0}} \Gamma^{\underline{\mu}} = \gamma^{\underline{0}} \gamma^{\underline{\mu}} \otimes \mathbb{I}_{2\times 2}$$

$$\mathcal{L}_{\text{kin}} = e^{\frac{\phi}{\sqrt{6}}} \sqrt{-g} \frac{i}{2} e^{\frac{2\eta\phi}{\sqrt{6}}} (\chi_a)^{\dagger} \gamma^{\underline{0}} \gamma^{\underline{\mu}} e^{\frac{-\phi}{2\sqrt{6}}} e^{\nu}{}_{\underline{\mu}} \partial_{\nu} \chi_a + \cdots = \sqrt{-g} e^{\frac{(1+4\eta)\phi}{2\sqrt{6}}} \frac{i}{2} \bar{\chi}^a \gamma^{\mu} \partial_{\mu} \chi_a + \cdots$$

$$\mathcal{L}_{\text{kin}} = \sqrt{-g} \left(\frac{i}{2} \bar{\chi}^a \gamma^{\mu} \partial_{\mu} \chi_a - \frac{i}{8\sqrt{6}} \bar{\chi}^a \gamma^{\mu} (\partial_{\mu}\phi) \chi_a\right)$$

$$\partial_z \lambda_1 = \frac{in}{L} \lambda_1, \partial_z \lambda_2 = -\frac{in}{L} \lambda_2$$

$$\partial_z \lambda = \frac{in}{L} \tau_3 \lambda$$

$$\Gamma^0 \Gamma^{\underline{5}} = -\gamma^0 \sigma_3$$

$$\mathcal{L}_{\text{mass}} = e^{\frac{\phi}{\sqrt{6}}} \sqrt{-g} \frac{n}{2L} e^{-\frac{\phi}{2\sqrt{6}}} (\chi_a)^{\dagger} \sigma_3 \gamma^{\underline{0}} e^{\frac{3\phi}{2\sqrt{6}}} (\tau_3)_a{}^b \chi_b$$
$$= e^{\frac{2\phi}{\sqrt{6}}} \sqrt{-g} \frac{n\varepsilon}{2L} \bar{\chi}^a (\tau_3)_a{}^b \chi_b$$

$$\mathcal{L}_{\text{KKgauge}} = \sqrt{-\hat{g}} \frac{i}{2} (\lambda_a)^{\dagger} \Gamma^0 \Gamma^{\underline{\mu}} \hat{e}_{\underline{\underline{\mu}}}^{z} \partial_z \lambda_a$$
$$= \sqrt{-g} \frac{n}{2L} \bar{\chi}^a \gamma^{\mu} \mathcal{A}_{\mu} (\tau_3)_a{}^b \chi_b$$

$$\mathcal{L}_{\text{KKgauge}} = \sqrt{-g} \frac{1}{2} \bar{\chi} \left(\frac{2n}{L}\right) \gamma^{\mu} a_{\mu} \tau_3 \chi$$

$$\mathcal{L}_{\partial} = \sqrt{-g} \left(\frac{i}{2} \bar{\chi} \gamma^{\mu} \partial_{\mu} \chi + \frac{1}{2} \bar{\chi} \left(\frac{n\varepsilon}{L} e^{\frac{2\phi}{\sqrt{6}}}\right) \tau_3 \chi + \frac{1}{2} \bar{\chi} \left(\frac{2n}{L}\right) \gamma^{\mu} a_{\mu} \tau_3 \chi - \frac{i}{8\sqrt{6}} \bar{\chi} \gamma^{\mu} (\partial_{\mu}\phi) \chi\right)$$

$$\mathcal{L}_{\omega} = -\frac{i}{8} \sqrt{-\hat{g}} \bar{\lambda}^a \Gamma^{\underline{M}} \hat{e}_{\underline{M}}^{N} \hat{\omega}_N{}^{PQ} \Gamma_{\underline{PQ}} \lambda_a$$

$$\Gamma^{\underline{M}} \hat{e}_{\underline{M}}^{N} \hat{\omega}_N \underline{PQ} \Gamma_{\underline{PQ}} = \Gamma^{\underline{\mu}} \hat{e}_{\underline{\mu}}^{\nu} \hat{\omega}_{\nu} \underline{PQ} \Gamma_{\underline{PQ}} + \Gamma^{\underline{\mu}} \hat{e}_{\underline{\mu}}^{z} \hat{\omega}_z \underline{PQ} \Gamma_{\underline{PQ}} + \Gamma^{\underline{z}} \hat{e}_{\underline{z}}^{z} \hat{\omega}_z{}^{\underline{PQ}} \Gamma_{\underline{PQ}}$$
$$= \sigma_1 e^{-\alpha\phi} \gamma^{\mu} \left(\hat{\omega}_{\mu} \underline{PQ} - \mathcal{A}_{\mu} \hat{\omega}_z \underline{PQ}\right) \Gamma_{\underline{PQ}} + i\sigma_2 e^{-\beta\phi} \hat{\omega}_z \underline{PQ} \Gamma_{\underline{PQ}}$$

$$\sigma_1 e^{-\alpha\phi} \gamma^{\mu} \left(\hat{\omega}_{\mu} \underline{PQ} - \mathcal{A}_{\mu} \hat{\omega}_z \underline{PQ}\right) \Gamma_{PQ} = \sigma_1 e^{-\alpha\phi} \left(\gamma^{\mu} \omega_{\mu} \underline{\nu\rho} \gamma_{\underline{\nu\rho}} - 8\alpha \gamma^{\mu} \partial_{\mu}\phi - \sigma_3 e^{(\beta-\alpha)\phi} \gamma^{\mu\nu} \mathcal{F}_{\mu\nu}\right)$$

$$\gamma^{\mu} \gamma_{\mu\nu} = -4\gamma_{\nu}$$

$$i\sigma_2 e^{-\beta\phi}\hat{\omega}_z \overset{PQ}{\rightarrow} \Gamma_{PQ} = -2\beta\sigma_1 e^{-\alpha\phi}\gamma^\mu\partial_\mu\phi - \frac{i}{2}\sigma_2 e^{(\beta-2\alpha)\phi}\gamma^{\mu\nu}\mathcal{F}_{\mu\nu}$$

$$\Gamma^{\underline{M}}\hat{e}^N{}_{\underline{M}}\hat{\omega}_N{}^{PQ}\Gamma_{\underline{PQ}} = \sigma_1 e^{-\alpha\phi}\left(\gamma^\mu\omega_\mu{}^{\nu\rho}\gamma_{\underline{\nu\rho}} - 2\alpha\gamma^\mu\partial_\mu\phi - \frac{1}{2}\sigma_3 e^{(\beta-\alpha)\phi}\gamma^{\mu\nu}\mathcal{F}_{\mu\nu}\right)$$

$$\mathcal{L}_\omega = \sqrt{-g}\bar{\chi}^a\left(-\frac{i}{8}\gamma^\mu\omega_\mu{}^{\nu\rho}\gamma_{\underline{\nu\rho}} + \frac{i}{8\sqrt{6}}\gamma^\mu\partial_\mu\phi + \frac{i\varepsilon}{16}e^{(\beta-\alpha)\phi}\gamma^{\mu\nu}\mathcal{F}_{\mu\nu}\right)\chi_a$$

$$e^{-1}\mathcal{L} = \frac{i}{2}\bar{\chi}\gamma^\mu\nabla_\mu\chi + \frac{1}{2}\bar{\chi}\left(\frac{n\varepsilon}{L}\right)e^{\frac{2\phi}{\sqrt{6}}}\tau_3\chi + \frac{1}{2}\bar{\chi}\left(\frac{2n}{L}\right)\gamma^\mu a_\mu\tau_3\chi + \frac{i}{2}\bar{\chi}\left(\frac{\varepsilon}{4}\right)e^{\frac{-2\phi}{\sqrt{6}}}\gamma^{\mu\nu}f_{\mu\nu}\chi$$

$$\langle\lambda^2\rangle_k = \frac{16\pi^2}{g^2}M_{\mathrm{PV}}^3\exp\left(\frac{2\pi i(\tau+k)}{T_G}\right) = 3T_G\Lambda^3\exp\left(\frac{2\pi ik}{T_G}\right),$$

$$\mathcal{Z} = \frac{T_G}{8\pi^2}\int \mathrm{d}z\frac{\partial}{\partial z}\lambda^2$$

$$\mathcal{Z}_{kn} = \langle kn|\mathcal{Z}|kn\rangle = \frac{T_G}{8\pi^2}(\langle\lambda^2\rangle_{k+n} - \langle\lambda^2\rangle_n)$$

$$T_k = |\mathcal{Z}_{kn}| = \frac{3}{4\pi^2}T_G^2\Lambda^3\sin\frac{\pi k}{T_G},$$

$$\nu_k = \mathrm{Tr}_{kn}[F(-1)^F],$$

$$\nu_k = \mathrm{Tr}_{\mathrm{WV}}[(-1)^F].$$

$$S_L = \int dt\left[\frac{1}{2e^2}\dot{A}_i^2 - \frac{N}{8\pi}\epsilon_{ij}A_i\dot{A}_j\right]$$

$$\nu_k = \binom{N}{k} \equiv \frac{N!}{k!\,(N-k)!}$$

$$V^a = \phi^a + \tau\sigma^a$$

$$\mathcal{W} \propto \left[\sum_{a=1}^{N-1} e^{-V^a} + e^{2\pi i\tau}e^{\sum_a V^a}\right]$$

$$\mathcal{W} = \mathrm{Tr}(mM) + \frac{(\Lambda_{N-1})^{2N+1}}{\det M}$$

$$M_f^g = Q_f\bar{Q}^g$$

$$\det M = (\Lambda_N)^{2N},$$

$$\mathcal{W}_{\mathrm{tree}} = \mathrm{Tr}(mM),$$

$$\mathcal{W} = \mathrm{Tr}(mM) + \lambda[\det M - (\Lambda_N)^{2N}]$$

$$\langle M\rangle_k = m^{-1}\mu\Lambda_N^2\omega_N^k, k = 0, \dots N-1$$

$$\mu = (\det m)^{1/N}$$

$$m = \begin{pmatrix} m_N^N & 0\cdots 0 \\ 0 & \\ \vdots & m' \\ 0 & \end{pmatrix}, m_N^N \gg (m')_f^g, f, g = 1, \dots, N-1$$

$$\mathcal{W} = \mathrm{Tr}(m'M') + \frac{(\Lambda_{N-1})^{2N+1}}{\det M'}, (\Lambda_{N-1})^{2N+1} = m_N^N(\Lambda_N)^{2N}$$

$$X = mM(\mu\Lambda_N)^{-2}$$

$$\mathcal{W} = \mu\Lambda_N^2[\mathrm{Tr}X + \lambda(\det X - 1)]$$

$$\mathcal{K} \propto \mathrm{Tr}[\bar{X}\bar{m}^{-1}m^{-1}X]^{1/2}$$

$$\mathcal{Z} = \int \mathrm{d}z \frac{\partial}{\partial z}\{2\widehat{\mathcal{W}}\}_{\theta=0}, \widehat{\mathcal{W}} = \mathcal{W}_{\mathrm{tree}} - \frac{T_G - \sum_f T(R_f)}{16\pi^2}\mathrm{Tr}W^2$$

$$\mathcal{Z}_{kn} = 2[\mathcal{W}_{k+n} - \mathcal{W}_n]$$

$$X_k = \omega_N^k \cdot \mathbb{1}, \mathcal{W}_k = N\mu\Lambda_N^2\omega_N^k,$$

$$\mathcal{Z}_{kn} = |\mathcal{Z}_{kn}|e^{i\gamma_{kn}} = 4iN\mu\Lambda_N^2 \sin\frac{\pi k}{N}\exp\left(\frac{i\pi(2n+k)}{N}\right),$$

$$g_{\bar{a}b}\partial_z X^b = e^{i\gamma}\partial_{\bar{a}}\overline{\mathcal{W}}$$

$$g_{\bar{a}b} = \partial_{\bar{a}}\partial_b\mathcal{K}$$

$$\partial_z\mathcal{W} = e^{i\gamma}\|\partial_i X\|^2$$

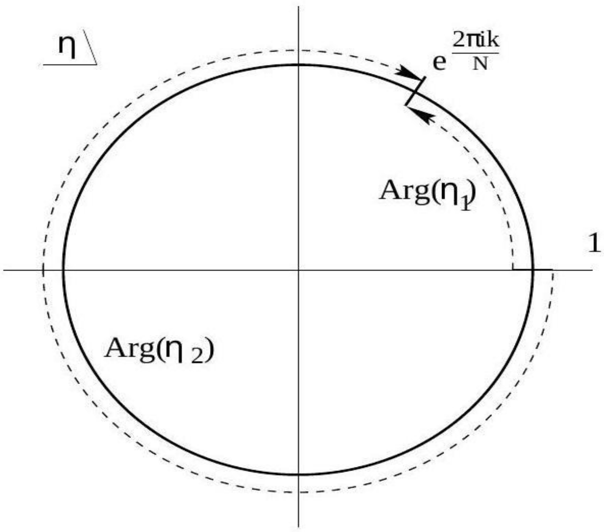
$$\nu_k = \Delta_n \circ \Delta_{n+k}$$

$$\mathcal{M} = \mathbb{R} \times \widetilde{\mathcal{M}}$$

$$\mathcal{W} = \mu\Lambda_N^2\left[\sum_{i=1}^{N} \eta_i + \lambda\left(\prod_{i=1}^{N} \eta_i - 1\right)\right]$$

$$w_i = \frac{1}{2\pi i}\int_\Gamma \frac{d\eta_i}{\eta_i}$$

$$w_1 = k/N, w_2 = (k/N) - 1$$



$$N(w_1) = k, N(w_2) = N - k.$$

$$\mathcal{W} = \mu\Lambda_N^2\{l\eta_1 + (N-l)\eta_2 + \lambda[\eta_1^l\eta_2^{N-l} - 1]\}.$$

$$\mathrm{SU}(k) \times \mathrm{SU}(N-k) \times \mathrm{U}(1).$$

$$\widetilde{\mathcal{M}}_k = G(k,N) \equiv \frac{\mathrm{U}(N)}{\mathrm{U}(k) \times \mathrm{U}(N-k)}$$

$$X = \sqrt{N}(X^0T_0 + iX^AT_A)$$

$$\langle X^0 \rangle = \omega_N^k, \langle X^A \rangle = 0$$

$$\mathcal{W} = \mu \Lambda_N^2 X^0$$

$$X^0(t) = f(t) \equiv (1-t) + t\omega_N^k,$$
$$\det\left[X^0(t)\mathbb{1} + i\sqrt{N} X^A T_A\right] = 1$$

$$\sum_{a=0}^{3} (X^a)^2 = 1,$$

$$r^2 = \mathrm{Tr}(\bar{X}X),$$

$$ds_C^2 = K'(r^2)\mathrm{Tr}(d\bar{X}dX) + K''(r^2)|\mathrm{Tr}(\bar{X}dX)|^2.$$

$$ds_{r\to 1}^2 = \Lambda_2^2 \left[d\rho^2 + d\Omega_3^2 + \frac{1}{2}\rho^2 d\Omega_2^2\right]$$

$$X^a = \cosh\left(\sqrt{p_b p_b}\right)x^a + i\frac{\sinh\left(\sqrt{p_b p_b}\right)}{\sqrt{p_c p_c}} p_a, a, b, c = 0,1,2,3$$

$$\sum_{a=0}^{3} (x^a)^2 = 1, \sum_{a=0}^{3} x^a p_a = 0$$

$$\mathcal{W} = \mu \Lambda_N^2 \cosh\left(\sqrt{p_A p_A}\right) x^0$$

$$E = \sum_{a=0}^{3} (p_a)^2/2$$

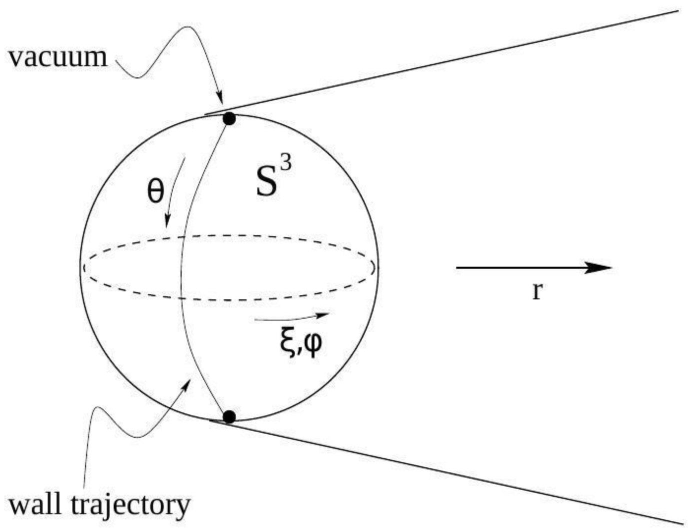
$$ds_{r=1}^2 = \Lambda_2^2 d\Omega_3^2, d\Omega_3^2 = d\theta^2 + \sin^2\theta(d\xi^2 + \sin^2\xi d\phi^2),$$

$$\mathcal{W}|_{\text{trajectory}} = 2\mu\Lambda_2^2 \cos\theta.$$

$$\partial_z\theta = 2\mu \sin\theta, \partial_z\xi = \partial_z\phi = 0,$$

$$\theta(z) = 2\arctan e^{2\mu(z-z_0)}.$$

$$\mathcal{M}^{\mathrm{N}=2} = \mathbb{R} \times \mathbb{CP}^1,$$



$$L_{\text{particle}} = -M + \frac{1}{2}M\dot{z}_0^2 + R_{\widetilde{\mathcal{M}}}\left(\dot{\xi}^2 + \sin^2\xi\dot{\phi}^2\right),$$

$$R_{\widetilde{\mathcal{M}}} \sim L^2 \frac{\Lambda_2^2}{\mu}$$

$$\mathcal{W}|_{\text{trajectory}} = N\mu\Lambda_N^2 e^{i\pi k/N}\left[\cos\frac{k\pi}{N} - i\sin\frac{k\pi}{N}\cos\,\theta\right].$$

$$\mathcal{K} = \Lambda_N^2\left[\theta^2 + \mathrm{Tr}f(\tilde{\tilde{X}}\tilde{X})\right]$$

$$\partial_z\theta = N\mu\sin\frac{k\pi}{N}\sin\,\theta,$$

$$\theta(z) = 2\arctan\,e^{2\tilde{\mu}(z-z_0)}, \tilde{\mu} = \frac{N}{2}\mu\sin\frac{k\pi}{N}.$$

$$\prod_{i=1}^{N}\eta_i(x(t)) = 1$$

$$X \to f(t)\mathbb{1} + x\sqrt{N}\Omega$$

$$\Omega = \mathrm{diag}\left\{-\sqrt{\frac{N-k}{Nk}}\mathbb{1}_k, \sqrt{\frac{k}{N(N-k)}}\mathbb{1}_{N-k}\right\},$$

$$\left(f(t) + \sqrt{\frac{N-k}{k}}x\right)^k\left(f(t) - \sqrt{\frac{k}{N-k}}x\right)^{N-k} = 1.$$

$$y^k = f(t) - \sqrt{\frac{k}{N-k}}x, y^{-(N-k)} = f(t) + \sqrt{\frac{N-k}{k}}x.$$

$$\frac{\mathcal{W}_{\text{ansatz}}}{\mu\Lambda_N^2} = Nf(t) = ky^{k-N} + (N-k)y^k,$$

$$\widetilde{\mathcal{M}}_k = G(k,N),$$

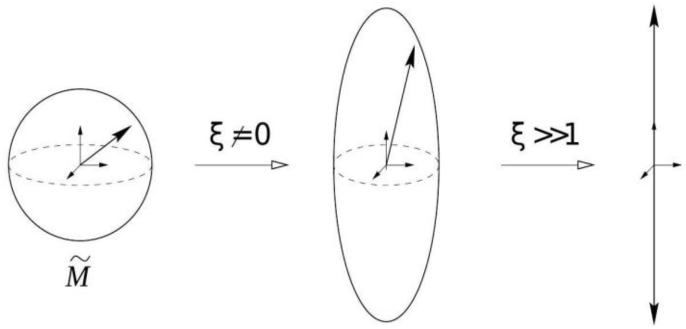
$$\nu_k = \chi(G(k,N)) = \binom{N}{k} \equiv \frac{N!}{k!\,(N-k)!},$$

$$\mathcal{W} = \mu\Lambda_N^2\left[\sum_{i=1}^{N}\eta_i + \lambda\left(\prod_{i=1}^{N}\eta_i - 1\right)\right]$$

$$M_{\widetilde{\mathcal{M}}} \propto \frac{\Lambda_2^2}{\mu} = \frac{3\Lambda^3}{8\pi^2\mu^2}$$

$$m = \mathrm{diag}\{m_1, m_2\}, m_1 \ll m_2 \ll \Lambda_2.$$

$$\xi \equiv \frac{m_2 - m_1}{\sqrt{m_1 m_2}}$$



$$T_2 - 2T_1 = -\frac{3\pi\Lambda^3}{4N} + \mathcal{O}(N^{-3})$$

$$2(\mathbb{R}\times\mathbb{CP}^{N-1}) \longrightarrow \mathbb{R}\times G(2,N)$$

$$\mathcal{S}_{TSYM} = \frac{1}{g^2}\mathrm{tr}\int d^4x\Big(\frac{1}{2}F^+_{\mu\nu}F^{+\mu\nu} + \frac{1}{2}\bar{\phi}\{\psi^\mu,\psi_\mu\}$$
$$-\chi^{\mu\nu}\left(D_\mu\psi_\nu - D_\nu\psi_\mu\right)^+ + \eta D_\mu\psi^\mu - \frac{1}{2}\bar{\phi}D_\mu D^\mu\phi$$
$$-\frac{1}{2}\phi\{\chi^{\mu\nu},\chi_{\mu\nu}\} - \frac{1}{8}[\phi,\eta]\eta - \frac{1}{32}[\phi,\bar{\phi}][\phi,\bar{\phi}]\Big)$$

$$\delta^g_\epsilon A_\mu = -D_\mu\epsilon$$
$$\delta^g_\epsilon\lambda = [\epsilon,\lambda], \lambda = \chi,\psi,\eta,\phi,\bar{\phi}$$

$$\left\{Q^i_\alpha,\overline{Q}_{j\dot{\alpha}}\right\} = \delta^i_j\partial_{\alpha\dot{\alpha}} + \mathbb{G}_{gauge\ transformations} + \mathfrak{M}_{equations\ of\ motion}$$
$$\left\{Q^i_\alpha,Q^j_\beta\right\} = \left\{\overline{Q}^i_{\dot{\alpha}},\overline{Q}^j_{\dot{\beta}}\right\} = \mathbb{G}_{gauge\ transformations} + \mathfrak{M}_{equations\ of\ motion}$$

$$\delta = \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}Q_{\beta\alpha}, \delta_\mu = \frac{1}{\sqrt{2}}\overline{Q}_{\alpha\dot{\alpha}}(\bar{\sigma}_\mu)^{\dot{\alpha}\alpha}$$
$$\delta_{\mu\nu} = \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta}Q_{\beta\alpha} = -\delta_{\nu\mu}$$

$$\delta^2 = \mathbb{G}_{gauge\ transformations} + \mathfrak{M}_{equations\ of\ motion}$$
$$\{\delta,\delta_\mu\} = \partial_\mu + \mathbb{G}_{gauge\ transformations} + \mathfrak{M}_{equations\ of\ motion}$$
$$\{\delta_\mu,\delta_\nu\} = \mathbb{G}_{gauge\ transformations} + \mathfrak{M}_{equations\ of\ motion}$$
$$\{\delta,\delta_{\mu\nu}\} = \{\delta_{\mu\nu},\delta_{\rho\sigma}\} = \mathbb{G}_{gauge\ transformations} + \mathfrak{M}_{equations\ of\ motion}$$
$$\{\delta_\mu,\delta_{\rho\sigma}\} = -\varepsilon_{\mu\rho\sigma\nu}\partial^\nu - g_{\mu[\rho}\partial_{\sigma]} + \mathbb{G}_{gauge\ transformations} + \mathfrak{M}_{equations\ of\ motion}$$

$$\bar{\psi}^i_{\dot{\alpha}} \overset{twist}{\to} \bar{\psi}_{\alpha\dot{\alpha}} \longrightarrow \psi_\mu = (\bar{\sigma}_\mu)^{\dot{\alpha}\alpha}\bar{\psi}_{\alpha\dot{\alpha}}$$

$$\psi_{[\alpha\beta]} \longrightarrow \eta = \varepsilon^{\alpha\beta}\psi_{[\alpha\beta]},$$
$$\psi_{(\alpha\beta)} \longrightarrow \chi_{\mu\nu} = \tilde{\chi}_{\mu\nu} = (\sigma_{\mu\nu})^{\alpha\beta}\psi_{(\alpha\beta)}.$$

$$\mathcal{S}^{N=2}_{YM}(A_\mu,\psi^i_\alpha,\bar{\psi}^i_{\dot{\alpha}},\phi,\bar{\phi}) \overset{\text{twist}}{\to} \mathcal{S}_{TSYM}(A_\mu,\psi_\mu,\chi_{\mu\nu},\eta,\phi,\bar{\phi}).$$

$$\delta_W A_\mu = \psi_\mu, \quad \delta_W\psi_\mu = -D_\mu\phi, \quad \delta_W\phi = 0$$
$$\delta_W\chi_{\mu\nu} = F^+_{\mu\nu}, \quad \delta_W\bar{\phi} = 2\eta, \quad \delta_W\eta = \frac{1}{2}[\phi,\bar{\phi}]$$

$$\delta_\mu A_\nu = \frac{1}{2}\chi_{\mu\nu} + \frac{1}{8}g_{\mu\nu}\eta$$
$$\delta_\mu \psi_\nu = F_{\mu\nu} - \frac{1}{2}F^+_{\mu\nu} - \frac{1}{16}g_{\mu\nu}[\phi,\bar{\phi}]$$
$$\delta_\mu \eta = \frac{1}{2}D_\mu\bar{\phi}$$
$$\delta_\mu \chi_{\sigma\tau} = \frac{1}{8}\left(\varepsilon_{\mu\sigma\tau\nu}D^\nu\bar{\phi} + g_{\mu\sigma}D_\tau\bar{\phi} - g_{\mu\tau}D_\sigma\bar{\phi}\right)$$
$$\delta_\mu \phi = -\psi_\mu$$
$$\delta_\mu \bar{\phi} = 0$$

$$\delta_{\mathcal{W}}\mathcal{S}_{TSYM} = \delta_\mu \mathcal{S}_{TSYM} = 0$$

$$sA_\mu = -D_\mu c, \qquad sc = c^2, \qquad s\phi = [c,\phi]$$
$$s\psi_\mu = \{c,\psi_\mu\}, \quad s\chi_{\mu\nu} = \{c,\chi_{\mu\nu}\}, \quad s\eta = \{c,\eta\}$$
$$s\bar{\phi} = [c,\bar{\phi}],$$
$$s\mathcal{S}_{TSYM} = 0, \qquad s^2 = 0$$

$$Q = s + \omega\delta_{\mathcal{W}} + \varepsilon^\mu\delta_\mu + v^\mu\partial_\mu - \omega\varepsilon^\mu\frac{\partial}{\partial v^\mu}$$

$$s\omega = 0, \delta_{\mathcal{W}}\omega = 0, \delta_{\mathcal{W}}c = -\omega\phi.$$

$$Qc = c^2 - \omega^2\phi - \omega\varepsilon^\mu A_\mu + \frac{\varepsilon^2}{16}\bar{\phi} + v^\mu\partial_\mu c$$
$$Q\omega = 0, Q\varepsilon^\mu = 0, Qv^\mu = -\omega\varepsilon^\mu$$

$$Q^2 = 0 \text{ on } (A,\phi,\bar{\phi},\eta,c,\omega,\varepsilon,v)$$

$$Q^2\psi_\sigma = \frac{g^2}{4}\omega\varepsilon^\mu\frac{\delta\mathcal{S}_{TYM}}{\delta\chi^{\mu\sigma}}$$
$$+\frac{g^2}{32}\varepsilon^\mu\varepsilon^\nu\left(g_{\mu\sigma}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\nu} + g_{\nu\sigma}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\mu} - 2g_{\mu\nu}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\sigma}\right)$$
$$Q^2\chi_{\sigma\tau} = -\frac{g^2}{2}\omega^2\frac{\delta\mathcal{S}_{TYM}}{\delta\chi^{\sigma\tau}}$$
$$+\frac{g^2}{8}\omega\varepsilon^\mu\left(\varepsilon_{\mu\sigma\tau\nu}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi_\nu} + g_{\mu\sigma}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\tau} - g_{\mu\tau}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\sigma}\right)$$

$$Q\bar{c} = b + v^\mu\partial_\mu\bar{c}, Qb = \omega\varepsilon^\mu\partial_\mu\bar{c} + v^\mu\partial_\mu b$$

$$\mathcal{S}_{gf} = Q\int d^4x\,\mathrm{tr}(\bar{c}\partial A)$$
$$= \mathrm{tr}\int d^4x\left(b\partial A + \bar{c}\partial Dc - \omega\bar{c}\partial\psi - \frac{\varepsilon^\nu}{2}\bar{c}\partial^\mu\chi_{\nu\mu} - \frac{\varepsilon^\mu}{8}\bar{c}\partial_\mu\eta\right)$$

$$\mathcal{S}_{ext} = \mathrm{tr}\int d^4x\left(\Phi^{*i}Q\Phi_i\right)$$

$$\mathcal{S}_{\text{quad}} = \mathrm{tr}\int d^4x\left(\frac{g^2}{8}\omega^2\chi^{*\mu\nu}\chi^*_{\mu\nu} - \frac{g^2}{4}\omega\chi^{*\mu\nu}\varepsilon_\mu\psi^*_\nu - \frac{g^2}{32}\varepsilon^\mu\varepsilon^\nu\psi^*_\mu\psi^*_\nu + \frac{g^2}{32}\varepsilon^2\psi^{*\mu}\psi^*_\mu\right)$$

$$\Sigma = \mathcal{S}_{TSYM} + \mathcal{S}_{gf} + \mathcal{S}_{ext} + \mathcal{S}_{quad}$$

$$\mathcal{S}(\Sigma) = \mathrm{tr}\int d^4x\left(\frac{\delta\Sigma}{\delta\Phi^{*i}}\frac{\delta\Sigma}{\delta\Phi_i} + \left(b + v^\mu\partial_\mu\bar{c}\right)\frac{\delta\Sigma}{\delta\bar{c}}\right.$$
$$\left.+\left(\omega\varepsilon^\mu\partial_\mu\bar{c} + v^\mu\partial_\mu b\right)\frac{\delta\Sigma}{\delta b}\right) - \omega\varepsilon^\mu\frac{\partial\Sigma}{\partial v^\mu} = 0$$

$$\frac{\partial\Sigma}{\partial v^{\mu}} = \Delta_{\mu}^{cl} = \mathrm{tr}\int d^4x\big(c^*\partial_\mu c - \phi^*\partial_\mu\phi - A^{*\nu}\partial_\mu A_\nu + \psi^{*\nu}\partial_\mu\psi_\nu - \bar{\phi}^*\partial_\mu\bar{\phi}$$
$$+\eta^*\partial_\mu\eta + \frac{1}{2}\chi^{*\nu\sigma}\partial_\mu\chi_{\nu\sigma}\big)$$

$$\Sigma = \hat{\Sigma} + v^\mu\Delta_\mu^{cl}, \frac{\partial\hat{\Sigma}}{\partial v^\mu} = 0$$

$$\mathcal{S}(\hat{\Sigma}) = \omega\varepsilon^\mu\Delta_\mu^{cl}.$$

$$\frac{\delta\hat{\Sigma}}{\delta b} = \partial A, \frac{\delta\hat{\Sigma}}{\delta\bar{c}} + \partial_\mu\frac{\delta\hat{\Sigma}}{\delta A_\mu^*} = 0$$
$$\int d^4x\left(\frac{\delta\hat{\Sigma}}{\delta c} + \left[\bar{c}, \frac{\delta\hat{\Sigma}}{\delta b}\right]\right) = \Delta_c^{cl}$$

$$\Delta_c^{cl} = \int d^4x\left([c,c^*] - [A,A^*] - [\phi,\phi^*] + [\psi,\psi^*] - [\bar{\phi},\bar{\phi}^*] + [\eta,\eta^*] + \frac{1}{2}[\chi,\chi^*]\right).$$

$$\hat{\Sigma} = \tilde{\mathcal{S}} + \mathrm{tr}\int d^4x b\partial A,$$

$$\mathcal{S}(\tilde{\mathcal{S}}) = \mathrm{tr}\int d^4x\left(\frac{\delta\tilde{\mathcal{S}}}{\delta\Phi^{*i}}\frac{\delta\tilde{\mathcal{S}}}{\delta\Phi_i}\right) = \omega\varepsilon^\mu\Delta_\mu^{cl}$$

$$\mathcal{B}_{\tilde{\mathcal{S}}} = \mathrm{tr}\int d^4x\left(\frac{\delta\tilde{\mathcal{S}}}{\delta\Phi^i}\frac{\delta}{\delta\Phi_i^*} + \frac{\delta\tilde{\mathcal{S}}}{\delta\Phi_i^*}\frac{\delta}{\delta\Phi^i}\right)$$

$$\mathcal{B}_{\tilde{\mathcal{S}}}\mathcal{B}_{\tilde{\mathcal{S}}} = \omega\varepsilon^\mu\mathcal{P}_\mu,$$

$$\mathcal{B}_{\tilde{\mathcal{S}}}\Delta^G = 0, G = 0,1.$$

$$\mathcal{B}_{\tilde{\mathcal{S}}} = b_{\tilde{\mathcal{S}}} + \varepsilon^\mu\mathcal{W}_\mu + \frac{1}{2}\varepsilon^\mu\varepsilon^\nu\mathcal{W}_{\mu\nu},$$

$$b_{\tilde{\mathcal{S}}}b_{\tilde{\mathcal{S}}} = 0, \{b_{\tilde{\mathcal{S}}}, \mathcal{W}_\mu\} = \omega\mathcal{P}_\mu$$
$$\{\mathcal{W}_\mu, \mathcal{W}_\nu\} + \{b_{\tilde{\mathcal{S}}}, \mathcal{W}_{\mu\nu}\} = 0$$
$$\{\mathcal{W}_\mu, \mathcal{W}_{\nu\rho}\} + \{\mathcal{W}_\nu, \mathcal{W}_{\rho\mu}\} + \{\mathcal{W}_\rho, \mathcal{W}_{\mu\nu}\} = 0$$
$$\{\mathcal{W}_{\mu\nu}, \mathcal{W}_{\rho\sigma}\} + \{\mathcal{W}_{\mu\rho}, \mathcal{W}_{\nu\sigma}\} + \{\mathcal{W}_{\mu\sigma}, \mathcal{W}_{\nu\rho}\} = 0$$

$$b_{\tilde{\mathcal{S}}} = s + \omega\delta_{\mathcal{W}}$$

$$b_{\tilde{\mathcal{S}}}\Omega_4^G + \omega\partial^\mu\Omega_{\frac{7}{2}\mu}^G = 0,$$
$$b_{\tilde{\mathcal{S}}}\Omega_{\frac{7}{2}\mu}^G + \omega\partial^\nu\Omega_{3[\mu\nu]}^G = 0,$$
$$b_{\tilde{\mathcal{S}}}\Omega_{3[\mu\nu]}^G + \omega\partial^\rho\Omega_{\frac{5}{2}[\mu\nu\rho]}^G = 0,$$
$$b_{\tilde{\mathcal{S}}}\Omega_{\frac{5}{2}[\mu\nu\rho]}^G + \omega\partial^\sigma\Omega_{2[\mu\nu\rho\sigma]}^G = 0,$$
$$b_{\tilde{\mathcal{S}}}\Omega_{2[\mu\nu\rho\sigma]}^G = 0,$$

$$\Delta = \frac{1}{2}\mathrm{tr}\phi^2$$

$$\Delta_{\mu\nu} = a\left(F_{\mu\nu}^+\phi + \frac{g^2\omega}{2}\chi_{\mu\nu}^*\phi\right),$$

$$\Delta = \frac{1}{2}b_{\tilde{\mathcal{S}}}\mathrm{tr}\left(-\frac{1}{\omega^2}c\phi + \frac{1}{3\omega^4}c^3\right)$$
$$\Delta_{\mu\nu} = ab_{\tilde{\mathcal{S}}}\mathrm{tr}\left(\frac{1}{\omega}\phi\chi_{\mu\nu}\right)$$

$$\Omega^0 = \varepsilon^{\mu\nu\rho\tau}\mathcal{W}_\mu\mathcal{W}_\nu\mathcal{W}_\rho\mathcal{W}_\tau\int d^4x\Delta + \mathcal{W}^\mu\mathcal{W}^\nu\int d^4x\Delta_{\mu\nu} + b_{\tilde{s}} - \mathfrak{V}$$
$$= \mathcal{S}_{TSYM} + a\Xi + b_{\tilde{s}}\widetilde{\Omega}^{-1}$$

$$\Xi = \int d^4x\Big(\frac{g^2\omega}{4}F^{+\mu\nu}\chi^*_{\mu\nu} + \frac{g^4\omega^2}{8}\chi^*_{\mu\nu}\chi^{*\mu\nu} - \frac{1}{4}\chi^{\mu\nu}\big(D_\mu\psi_\nu - D_\nu\psi_\mu\big)^+$$
$$-\frac{1}{4}\phi\{\chi^{\mu\nu},\chi_{\mu\nu}\} - \frac{3}{4}\psi^\mu D_\mu\eta - \frac{3}{4}\omega g^2\phi D^\mu\psi^*_\mu - \frac{3}{4}\omega g^2\psi^\mu A^*_\mu$$
$$+\frac{3}{16}\phi\{\eta,\eta\} - \frac{3}{2}\omega^2 g^2\phi c^* + \frac{3}{4}\omega g^2\phi[\bar{\phi},\eta^*]\Big)$$

$$\mathcal{W}_\mu\Omega^0 = \mathcal{W}_\mu\big(\mathcal{S}_{TSYM} + a\Xi + b_{\tilde{s}}\widetilde{\Omega}^{-1}\big) = 0$$

$$a\mathcal{W}_\mu\Xi = b_{\tilde{s}}\Lambda^{-1}_\mu,$$

$$\mathcal{W}_\mu\Xi = -\frac{1}{\omega}b_{\tilde{s}}\Big(\frac{3}{8}F^-_{\mu\nu}D^\nu\bar{\phi} + \frac{3}{64}[\phi,\bar{\phi}]D_\mu\bar{\phi} - \frac{3}{8}\psi^\nu[\chi_{\mu\nu},\bar{\phi}]$$
$$+\frac{3}{32}\psi_\mu[\eta,\bar{\phi}] - \frac{1}{16}\omega g^2\chi^*_{\mu\nu}D^\nu\bar{\phi} + \frac{1}{4}\omega g^2\chi_{\mu\nu}A^{*\nu}$$
$$-\frac{3}{8}\omega g^2 F^-_{\mu\nu}\psi^{*\nu} - \frac{3}{64}\omega g^2\psi^*_\mu[\phi,\bar{\phi}] - \frac{3}{4}\omega g^2\psi_\mu c^* + \frac{1}{16}\omega^2 g^4\chi^*_{\mu\nu}\psi^{*\nu}\Big)$$

$$\Omega^0 = \mathcal{S}_{TSYM} + b_{\tilde{s}}\widetilde{\Omega}^{-1} = \varepsilon^{\mu\nu\rho\tau}\mathcal{W}_\mu\mathcal{W}_\nu\mathcal{W}_\rho\mathcal{W}_\tau\int d^4x\frac{1}{2}\mathrm{tr}\phi^2 + b_{\tilde{s}} - \mathfrak{V}$$

$$\mathcal{S}_{TSYM} \approx \varepsilon^{\mu\nu\rho\tau}\mathcal{W}_\mu\mathcal{W}_\nu\mathcal{W}_\rho\mathcal{W}_\tau\int d^4x\frac{1}{2}\mathrm{tr}\phi^2,$$

$$\langle\hat{\mathcal{O}}_\alpha(x)\hat{\mathcal{O}}_\beta(0)\rangle = \frac{\delta_{\alpha\beta}}{|x|^{2\Delta_\alpha}}$$

$$D\hat{\mathcal{O}}_\alpha = \Delta_\alpha\hat{\mathcal{O}}_\alpha$$

$$D_0 = \mathrm{Tr}\Phi_m\check{\Phi}_m$$

$$\check{\Phi}_m = \frac{\delta}{\delta\Phi_m} = T^a\frac{\delta}{\delta\Phi^{(a)}_m}.$$

$$D = \sum_{k=0}^{\infty}\left(\frac{g^2_{\mathrm{YM}}}{16\pi^2}\right)^k D_{2k},$$

$$D_2 = -:\mathrm{Tr}[\Phi_m,\Phi_n][\check{\Phi}_m,\check{\Phi}_n]: - \frac{1}{2}:\mathrm{Tr}[\Phi_m,\check{\Phi}_n][\Phi_m,\check{\Phi}_n]:.$$

$$\mathcal{Q}_{nm} = \mathrm{Tr}\Phi_n\Phi_m - \frac{1}{6}\delta_{nm}\mathrm{Tr}\Phi_k\Phi_k \text{ and } \mathcal{K} = \mathrm{Tr}\Phi_k\Phi_k$$

$$\Delta_{\mathcal{Q}} = 2 \text{ and } \Delta_{\mathcal{K}} = 2 + \frac{3g^2_{\mathrm{YM}}N}{4\pi^2}$$

$$\mathcal{O} = \begin{pmatrix}\mathrm{Tr}\Phi_m\Phi_n\mathrm{Tr}\Phi_m\Phi_n\\ \mathrm{Tr}\Phi_m\Phi_m\mathrm{Tr}\Phi_n\Phi_n\\ \mathrm{Tr}\Phi_m\Phi_n\Phi_m\Phi_n\\ \mathrm{Tr}\Phi_m\Phi_m\Phi_n\Phi_n\end{pmatrix}$$

$$D_2 = N\begin{pmatrix}0 & 4 & -\frac{20}{N} & \frac{20}{N}\\ 0 & 24 & -\frac{24}{N} & \frac{24}{N}\\ -\frac{24}{N} & \frac{4}{N} & 8 & -4\\ -\frac{4}{N} & \frac{14}{N} & -4 & 18\end{pmatrix}$$

$$D_2\mathrm{Tr}\Phi_m\Phi_n\mathrm{Tr}\Phi_m\Phi_n = 0\mathrm{Tr}\Phi_m\Phi_n\mathrm{Tr}\Phi_m\Phi_n + 4N\mathrm{Tr}\Phi_m\Phi_m\mathrm{Tr}\Phi_n\Phi_n$$
$$-20\mathrm{Tr}\Phi_m\Phi_n\Phi_m\Phi_n + 20\mathrm{Tr}\Phi_m\Phi_m\Phi_n\Phi_n$$

$$\omega^4 - 25\omega^3 + \left(188 - \frac{160}{N^2}\right)\omega^2 - \left(384 - \frac{1760}{N^2}\right)\omega - \frac{7680}{N^2} = 0$$

$$\Delta = 4 + \frac{g_{\mathrm{YM}}^2 N}{8\pi^2}\omega$$

$$\begin{aligned} D_0 &= \mathrm{Tr}Z\check{Z} + \mathrm{Tr}\phi\check{\phi}, \\ D_2 &= -2{:}\,\mathrm{Tr}[\phi, Z][\check{\phi}, \check{Z}]{:}\,, \\ D_4 &= -2{:}\,\mathrm{Tr}[[\phi, Z], \check{Z}][[\check{\phi}, \check{Z}], Z]{:} \\ &\quad -2{:}\,\mathrm{Tr}[[\phi, Z], \check{\phi}][[\check{\phi}, \check{Z}], \phi]{:} \\ &\quad -2{:}\,\mathrm{Tr}[[\phi, Z], T^a]\big[[\check{\phi}, \check{Z}], T^a]{:}\,, \end{aligned}$$

$$\mathcal{K}' = \mathrm{Tr}[\phi, Z][\phi, Z].$$

$$\Delta_{\mathcal{K}'} = 4 + \frac{3g_{\mathrm{YM}}^2 N}{4\pi^2} - \frac{3g_{\mathrm{YM}}^4 N^2}{16\pi^4}$$

$$\Delta_{\mathcal{K}'} = 4 + \frac{3g_{\mathrm{YM}}^2 N}{4\pi^2} - \frac{3g_{\mathrm{YM}}^4 N^2}{16\pi^4} + \frac{21g_{\mathrm{YM}}^6 N^3}{256\pi^6}$$

$$\Phi_m^+ = \Phi_m(x), \Phi_m^- = \Phi_m(0)$$

$$\mathcal{O}_\alpha(\Phi) = \mathrm{Tr}\Phi_m\Phi_n\Phi_m \ldots \mathrm{Tr}\Phi_n \ldots, \mathcal{O}_\alpha^\pm = \mathcal{O}_\alpha(\Phi^\pm)$$

$$\langle \mathcal{O}_\alpha^+ \mathcal{O}_\beta^- \rangle_{\text{tree}} = \exp\left(W_0(x, \check{\Phi}^+, \check{\Phi}^-)\right)\mathcal{O}_\alpha^+ \mathcal{O}_\beta^-\Big|_{\Phi=0}$$

$$W_0(x, \check{\Phi}^+, \check{\Phi}^-) = I_{0x}\mathrm{Tr}\check{\Phi}_m^+ \check{\Phi}_m^-$$

$$\langle \mathcal{O}_\alpha^+ \mathcal{O}_\beta^- \rangle_{\text{one-loop}} = \exp\left(W_0(x, \check{\Phi}^+, \check{\Phi}^-)\right)\left(1 + g^2 W_2(x, \check{\Phi}^+, \check{\Phi}^-)\right)\mathcal{O}_\alpha^+ \mathcal{O}_\beta^-\Big|_{\Phi=0},$$

$$\langle \mathcal{O}_\alpha^+ \mathcal{O}_\beta^- \rangle_{\text{one-loop}} = \exp\left(W_0(x, \check{\Phi}^+, \check{\Phi}^-)\right)(1 + g^2 V_2^-(x))\mathcal{O}_\alpha^+ \mathcal{O}_\beta^-\Big|_{\Phi=0}$$

$$V_2(x) =: W_2(x, I_{0x}^{-1}\Phi, \check{\Phi}):.$$

$$\tilde{\mathcal{O}} = \left(1 - \frac{1}{2}g^2 V_2(x_0)\right)\mathcal{O},$$

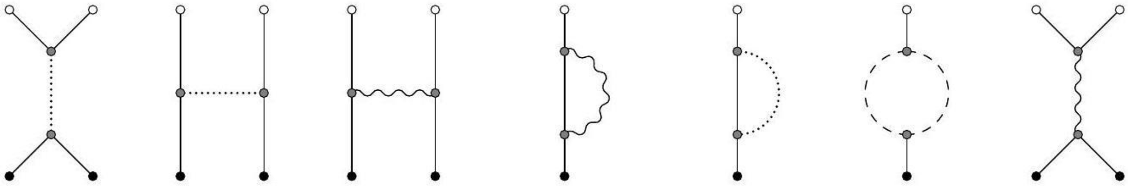
$$\langle \tilde{\mathcal{O}}_\alpha^+ \tilde{\mathcal{O}}_\beta^- \rangle_{\text{one-loop}} = \exp\left(W_0(x, \check{\Phi}^+, \check{\Phi}^-)\right)(1 + g^2 V_2^-(x) - g^2 V_2^-(x_0))\mathcal{O}_\alpha^+ \mathcal{O}_\beta^-\Big|_{\Phi=0}$$

$$V_2(x) = \frac{\Gamma(1-\epsilon)}{\left|\frac{1}{2}\mu^2 x^2\right|^{-\epsilon}} V_2$$

$$\lim_{\epsilon\to 0}\left(V_2(x) - V_2(x_0)\right) = \log(x_0^2/x^2)D_2,$$

$$D_2 = -\lim_{\epsilon\to 0}\epsilon V_2$$

$$\langle \tilde{\mathcal{O}}_\alpha^+ \tilde{\mathcal{O}}_\beta^- \rangle_{\text{one-loop}} = \exp(W_0)\exp(\log(x_0^2/x^2)g^2 D_2^-)\mathcal{O}_\alpha^+ \mathcal{O}_\beta^-\Big|_{\Phi=0},$$



a b c d1 d2 d3 e

$$
\begin{aligned}
W_{2,a} &= \frac{1}{4} X_{00xx} \mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_n^+][\check{\Phi}_m^-, \check{\Phi}_n^-] \\
W_{2,b} &= \frac{1}{4} X_{00xx} \left(\mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_n^-][\check{\Phi}_m^+, \check{\Phi}_n^-] + \mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_n^-][\check{\Phi}_m^-, \check{\Phi}_n^+]\right), \\
W_{2,c} &= \left(-\frac{1}{2} \tilde{H}_{0x,0x} - Y_{00x} I_{0x} + \frac{1}{4} X_{00xx}\right) \mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_m^-][\check{\Phi}_n^+, \check{\Phi}_n^-], \\
W_{2,d} &= -Y_{00x} \mathrm{Tr}[\check{\Phi}_m^+, T^a][T^a, \check{\Phi}_m^-].
\end{aligned}
$$

$$
\mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_n^-][\check{\Phi}_m^-, \check{\Phi}_n^+] = \mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_n^+][\check{\Phi}_m^-, \check{\Phi}_n^-] - \mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_m^-][\check{\Phi}_n^+, \check{\Phi}_n^-].
$$

$$
\begin{aligned}
W_{2,A} &= X_{00xx} \left(\frac{1}{2} \mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_n^+][\check{\Phi}_m^-, \check{\Phi}_n^-] + \frac{1}{4} \mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_n^-][\check{\Phi}_m^+, \check{\Phi}_n^-]\right), \\
W_{2,B} &= -\frac{1}{2} \tilde{H}_{0x,0x} \mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_m^-][\check{\Phi}_n^+, \check{\Phi}_n^-] \\
W_{2,C} &= -Y_{00x} \left(I_{0x} \mathrm{Tr}[\check{\Phi}_m^+, \check{\Phi}_m^-][\check{\Phi}_n^+, \check{\Phi}_n^-] + \mathrm{Tr}[\check{\Phi}_m^+, T^a][T^a, \check{\Phi}_m^-]\right).
\end{aligned}
$$

$$
\begin{aligned}
V_{2,A} &= X_{00xx} I_{0x}^{-2} \left(\frac{1}{2} : \mathrm{Tr}[\Phi_m, \Phi_n][\check{\Phi}_m, \check{\Phi}_n] : + \frac{1}{4} : \mathrm{Tr}[\Phi_m, \check{\Phi}_n][\Phi_m, \check{\Phi}_n] : \right), \\
V_{2,B} &= -\frac{1}{2} \tilde{H}_{0x,0x} I_{0x}^{-2} : \mathrm{Tr}[\Phi_m, \check{\Phi}_m][\Phi_n, \check{\Phi}_n] : , \\
V_{2,C} &= -Y_{00x} I_{0x}^{-1} \left(: \mathrm{Tr}[\Phi_m, \check{\Phi}_m][\Phi_n, \check{\Phi}_n] : + : \mathrm{Tr}[\Phi_m, T^a][T^a, \check{\Phi}_m] : \right).
\end{aligned}
$$

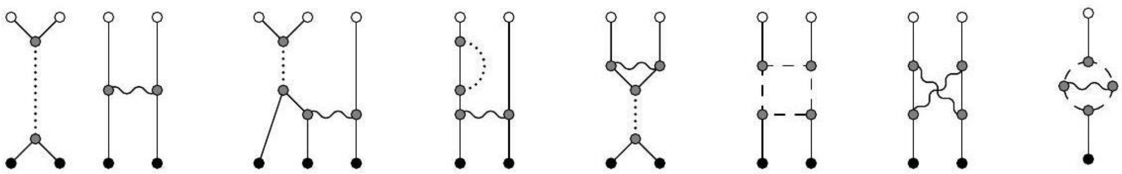
$$
\begin{aligned}
V_{2,C} &= -Y_{00x} I_{0x}^{-1} \mathrm{Tr}[\Phi_m, \check{\Phi}_m][\Phi_n, \check{\Phi}_n] \\
&= -Y_{00x} I_{0x}^{-1} \mathrm{Tr}\left([T^a, \Phi_m] \check{\Phi}_m\right) \mathrm{Tr}\left([T^a, \Phi_n] \check{\Phi}_n\right) \\
&= -Y_{00x} I_{0x}^{-1} G^a G^a.
\end{aligned}
$$

$$
\begin{aligned}
\frac{X_{00xx}}{I_{0x}^2} &= \left(\frac{2}{\epsilon} + 2 + \mathcal{O}(\epsilon^2)\right) \frac{\Gamma(1-\epsilon)}{\left|\frac{1}{2}\mu^2 x^2\right|^{-\epsilon}} \\
\frac{\tilde{H}_{0x,0x}}{I_{0x}^2} &= \left(-48\zeta(3)\epsilon + \mathcal{O}(\epsilon^2)\right) \frac{\Gamma(1-\epsilon)}{\left|\frac{1}{2}\mu^2 x^2\right|^{-\epsilon}}
\end{aligned}
$$

$$
D_2 = D_{2,A} = - : \mathrm{Tr}[\Phi_m, \Phi_n][\check{\Phi}_m, \check{\Phi}_n] : - \frac{1}{2} : \mathrm{Tr}[\Phi_m, \check{\Phi}_n][\Phi_m, \check{\Phi}_n] :
$$

$$
D_4 = -2 : \mathrm{Tr}\Phi_m \left[\Phi_n, \left[\check{\Phi}_m, \left[\Phi_p, [\check{\Phi}_n, \check{\Phi}_p]\right]\right]\right] : + : \mathrm{Tr}\Phi_m \left[\Phi_n, \left[T^a, \left[T^a, [\check{\Phi}_m, \check{\Phi}_n]\right]\right]\right] :
$$

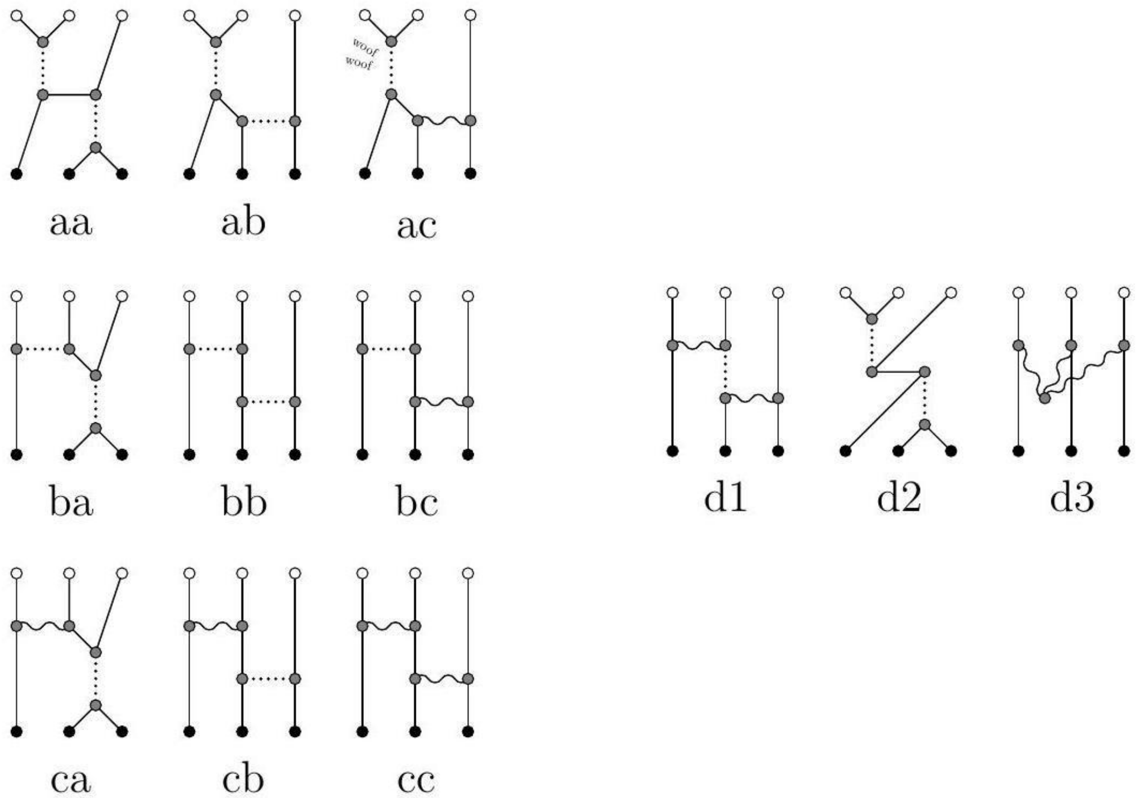
$$
(a, b, 0) \simeq [b, a-b, b] \text{ with } \Delta_0 = a + b
$$



a b c d e f g

$$
Z = \frac{1}{\sqrt{2}}(\Phi_5 + i\Phi_6), \phi = \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2).
$$

$$
\begin{aligned}
D_0 &= \mathrm{Tr} Z\check{Z} + \mathrm{Tr}\phi\check{\phi},\\
D_2 &= -2 : \mathrm{Tr}[\phi, Z][\check{\phi}, \check{Z}] : ,\\
D_4 &= -2 : \mathrm{Tr}[[\phi, Z], \check{Z}][[\check{\phi}, \check{Z}], Z] :\\
&\quad -2 : \mathrm{Tr}[[\phi, Z], \check{\phi}][[\check{\phi}, \check{Z}], \phi] :\\
&\quad -2 : \mathrm{Tr}[[\phi, Z], T^a][[\check{\phi}, \check{Z}], T^a] :
\end{aligned}
$$



$$: \mathrm{Tr}\Phi_m \left[\Phi_n, \left[\check{\Phi}_m, \left[\Phi_p, [\check{\Phi}_n, \check{\Phi}_p]\right]\right]\right] :$$

$$: \mathrm{Tr} \ldots [\ldots, [\ldots, [\ldots, [\ldots, \ldots]]]] :$$

$$[T^a, [T^a, [X, Y]]] = \left[X, [T^a, [T^a, Y]]\right] = 2(\mathrm{Tr}T^aT^a/\mathrm{Tr}1)[X, Y]$$

$$\left[T^a, \left[X, [T^a, Y]\right]\right] = (\mathrm{Tr}T^aT^a/\mathrm{Tr}1)[X, Y].$$

$$\mathrm{Tr}\Phi_m \left[\Phi_n, \left[T^a, \left[T^a, [\check{\Phi}_m, \check{\Phi}_n]\right]\right]\right] \sim N\mathrm{Tr}[\Phi_m, \Phi_n][\check{\Phi}_m, \check{\Phi}_n],$$

$$: \mathrm{Tr}\Phi_m \left[\check{\Phi}_m, \left[T^a, \left[T^a, [\Phi_n, \check{\Phi}_n]\right]\right]\right] : \sim N : \mathrm{Tr}[\Phi_m, \check{\Phi}_m][\Phi_n, \check{\Phi}_n] :$$

$$: \mathrm{Tr}T^a \left[X_1, \left[X_2, \left[X_3, [X_4, T^a]\right]\right]\right] :$$

$$: \mathrm{Tr}T^a \left[\Phi_m, \left[\check{\Phi}_m, \left[\Phi_n, [\check{\Phi}_n, T^a]\right]\right]\right] :$$

$$\mathrm{Tr}\Phi_m \left[T^a, \left[T^a, \left[T^b, [T^b, \check{\Phi}_m]\right]\right]\right] \sim N^2\mathrm{Tr}\Phi_m\check{\Phi}_m - N\mathrm{Tr}\Phi_m\mathrm{Tr}\check{\Phi}_m$$

$$2J\sum_{p=1}^{J-1}\left(N\mathrm{Tr}Z^{p}\mathrm{Tr}Z^{J-p}+\mathrm{Tr}Z^{2}\mathrm{Tr}Z^{p-1}\mathrm{Tr}Z^{J-1-p}\right)$$

$$D_4=\alpha{:}\,\mathrm{Tr}\Phi_m\left[\Phi_n,\left[\check{\Phi}_m,\left[\Phi_p,\left[\check{\Phi}_n,\check{\Phi}_p\right]\right]\right]\right]{:}+\beta\mathrm{Tr}\Phi_m\left[\Phi_n,\left[T^a,\left[T^a,\left[\check{\Phi}_m,\check{\Phi}_n\right]\right]\right]\right].$$

$$-\frac{(2\beta+\alpha)g_{\mathrm{YM}}^4N^2n^2}{16\pi^2J^2}+\frac{\alpha g_{\mathrm{YM}}^4N^2n^4}{8J^4}$$

$$-\frac{1}{4}\lambda'^2n^4=-\frac{g_{\mathrm{YM}}^4N^2n^4}{4J^4}$$

$$\alpha=-2,\beta=1$$

$$\begin{aligned}\mathcal{K}'=\mathcal{O}_{\mathrm{a}}&=\mathrm{Tr}[\phi,Z][\phi,Z],\\ \mathcal{O}_{\mathrm{b}}&=\mathrm{Tr}[\phi,Z][\phi,Z]Z,\\ \mathcal{O}_{\mathrm{c}}&=\mathrm{Tr}[\phi,Z][\phi,Z][\phi,Z],\end{aligned}$$

$$\begin{aligned}\mathcal{K}=\mathcal{O}_{\mathrm{a},0}&=\mathrm{Tr}\Phi_m\Phi_m\\ \mathcal{O}_{\mathrm{b},0}&=\mathrm{Tr}\Phi_m\Phi_m\Phi_n\\ \mathcal{O}_{\mathrm{c},0}&=\mathrm{Tr}\Phi_m\Phi_m[\Phi_n,\Phi_p]\end{aligned}$$

$$\begin{aligned}\Delta_{\mathcal{K}'}=\Delta_{\mathrm{a}}&=4+\frac{3g_{\mathrm{YM}}^2N}{4\pi^2}-\frac{3g_{\mathrm{YM}}^4N^2}{16\pi^4}\\ \Delta_{\mathrm{b}}&=5+\frac{g_{\mathrm{YM}}^2N}{2\pi^2}-\frac{3g_{\mathrm{YM}}^4N^2}{32\pi^4}\\ \Delta_{\mathrm{c}}&=6+\frac{3g_{\mathrm{YM}}^2N}{4\pi^2}-\frac{9g_{\mathrm{YM}}^4N^2}{64\pi^4}\end{aligned}$$

$$\mathcal{O}=\begin{pmatrix}\mathrm{Tr}[\phi,Z][\phi,Z]Z^2\\ \mathrm{Tr}[\phi,Z]Z[\phi,Z]Z\\ \hline \mathrm{Tr}Z^2\mathrm{Tr}[\phi,Z][\phi,Z]\end{pmatrix}$$

$$D_2=N\begin{pmatrix}+12 & -2 & +\frac{6}{N}\\ -8 & +8 & -\frac{4}{N}\\ +\frac{16}{N} & -\frac{16}{N} & +12\end{pmatrix}$$

$$D_4=N^2\begin{pmatrix}-52-\frac{24}{N^2} & +4+\frac{24}{N^2} & -\frac{52}{N}\\ +48+\frac{16}{N^2} & -16-\frac{16}{N^2} & +\frac{48}{N}\\ -\frac{144}{N} & +\frac{80}{N} & -48-\frac{64}{N^2}\end{pmatrix}$$

$$\mathcal{O}=\begin{pmatrix}\mathrm{Tr}[\phi,Z][\phi,Z]Z^3\\ \mathrm{Tr}[\phi,Z]Z[\phi,Z]Z^2\\ \hline \mathrm{Tr}Z^2\mathrm{Tr}[\phi,Z][\phi,Z]Z\\ \mathrm{Tr}Z^3\mathrm{Tr}[\phi,Z][\phi,Z]\end{pmatrix},$$

$$D_2=N\begin{pmatrix}+12 & 0 & 0 & +\frac{6}{N}\\ -4 & +4 & +\frac{4}{N} & -\frac{6}{N}\\ +\frac{8}{N} & -\frac{8}{N} & +8 & 0\\ +\frac{24}{N} & -\frac{24}{N} & 0 & +12\end{pmatrix}$$

$$D_4 = N^2 \begin{pmatrix} -48-\frac{36}{N^2} & -12+\frac{36}{N^2} & 0 & -\frac{48}{N} \\ +20+\frac{4}{N^2} & -\frac{4}{N^2} & -\frac{16}{N} & +\frac{20}{N} \\ -\frac{54}{N} & +\frac{16}{N} & -24+\frac{8}{N^2} & -\frac{24}{N^2} \\ -\frac{192}{N} & +\frac{72}{N} & +\frac{24}{N^2} & -48-\frac{72}{N^2} \end{pmatrix}$$

$$\omega^3 - 8\omega^2 + \left(20-\frac{10}{N^2}\right)\omega - \left(15-\frac{10}{N^2}\right)$$
$$+\frac{g_{\mathrm{YM}}^2 N}{16\pi^2}\left(\left(29+\frac{26}{N^2}\right)\omega^2 - \left(141-\frac{54}{N^2}\right)\omega + \left(150-\frac{100}{N^2}\right)\right)$$

$$\Delta = 6 + \frac{g_{\mathrm{YM}}^2 N}{4\pi^2}\omega$$

$$\mathcal{D} = \sum_{i,j} c_{i,j}\frac{(g^2N)^i}{N^{2j}},$$

$$\Delta_{\mathrm{single}} = 6 + \frac{g_{\mathrm{YM}}^2 N}{\pi^2}\left(\frac{5\pm\sqrt{5}}{8} + \frac{5\pm 2\sqrt{5}}{2N^2} - \frac{75\pm 34\sqrt{5}}{N^4}\right)$$
$$+\frac{g_{\mathrm{YM}}^4 N^2}{\pi^4}\left(-\frac{17\pm 5\sqrt{5}}{128} - \frac{131\pm 57\sqrt{5}}{64N^2} + \frac{1675\pm 751\sqrt{5}}{16N^4}\right)$$

$$\Delta_{\mathrm{double}} = 6 + \frac{g_{\mathrm{YM}}^2 N}{\pi^2}\left(\frac{3}{4} - \frac{5}{N^2} + \frac{150}{N^4}\right) + \frac{g_{\mathrm{YM}}^4 N^2}{\pi^4}\left(-\frac{3}{16} + \frac{59}{16N^2} - \frac{1675}{8N^4}\right).$$

$$\mathcal{D} = \sum_{i,j} c_{i,j}\frac{(g^2N)^i}{N^{2j}}$$

$$\Delta_{\mathrm{single}} = 7 + \frac{g_{\mathrm{YM}}^2 N}{\pi^2}\left(\frac{1}{4} + \frac{11}{16N^2} + \frac{253}{64N^4}\right) + \frac{g_{\mathrm{YM}}^4 N^2}{\pi^4}\left(-\frac{3}{128} - \frac{103}{1024N^2} - \frac{5541}{8192N^4}\right)$$
$$\Delta_{\mathrm{double}} = 7 + \frac{g_{\mathrm{YM}}^2 N}{\pi^2}\left(\frac{1}{2} - \frac{1}{2N^2} - \frac{17}{2N^4}\right) + \frac{g_{\mathrm{YM}}^4 N^2}{\pi^4}\left(-\frac{3}{32} + \frac{7}{32N^2} + \frac{129}{32N^4}\right)$$

$$\Delta_{\mathrm{remaining}} = 7 + \frac{g_{\mathrm{YM}}^2 N}{\pi^2}(\mathcal{M} \pm \sqrt{\mathcal{D}})$$
$$\mathcal{M} = \left(\frac{3}{4} - \frac{3}{32N^2} + \frac{291}{128N^4}\right) + \frac{g_{\mathrm{YM}}^2 N}{\pi^2}\left(-\frac{45}{256} - \frac{537}{2048N^2} - \frac{27483}{16384N^4}\right)$$
$$\mathcal{D} = \left(\frac{27}{32N^2} - \frac{531}{1024N^4}\right) + \frac{g_{\mathrm{YM}}^2 N}{\pi^2}\left(-\frac{3123}{4096N^2} + \frac{29295}{32768N^4}\right) + \frac{9g_{\mathrm{YM}}^4 N^2}{65536\pi^4}$$

$$\Delta_{\mathrm{single,curve}} = 7 + \frac{3g_{\mathrm{YM}}^2 N}{4\pi^2} - \frac{21 g_{\mathrm{YM}}^4 N^2}{128\pi^4}$$
$$\Delta_{\mathrm{double,curve}} = 7 + \frac{3g_{\mathrm{YM}}^2 N}{4\pi^2} - \frac{3 g_{\mathrm{YM}}^4 N^2}{16\pi^4}$$

$$\Delta_{\mathrm{one-loop}} = 7 + \frac{g_{\mathrm{YM}}^2 N}{\pi^2}\left(\frac{3}{4} \pm \frac{3\sqrt{6}}{8N} - \frac{3}{32N^2} \mp \frac{59\sqrt{6}}{512N^3}\right)$$

$$\mathcal{O}_p^{J_0;J_1,\ldots,J_k} = \mathrm{Tr}(\phi Z^p \phi Z^{J_0-p})\prod_{i=1}^{k}\mathrm{Tr}Z^{J_i}$$
$$\mathcal{Q}^{J_0,J_1;J_2,\ldots,J_k} = \mathrm{Tr}(\phi Z^{J_0})\mathrm{Tr}(\phi Z^{J_1})\prod_{i=2}^{k}\mathrm{Tr}Z^{J_i}$$

$$(g^2 D_2 + g^4 D_4)\begin{pmatrix}\mathcal{O}\\ \mathcal{Q}\end{pmatrix} = \begin{pmatrix} * & 0 \\ * & 0\end{pmatrix}\begin{pmatrix}\mathcal{O}\\ \mathcal{Q}\end{pmatrix}$$

$$D_2 =-2\colon \mathrm{Tr}[Z,\phi][\check{Z},\check{\phi}]\colon$$

$$\begin{aligned} D_4 =&2\colon \mathrm{Tr}[Z,\phi][\check{Z},[Z,[\check{Z},\check{\phi}]]]\colon \\ &+4N\colon \mathrm{Tr}[Z,\phi][\check{Z},\check{\phi}]\colon \\ &+2\colon \mathrm{Tr}[Z,\phi][\check{\phi},[\phi,[\check{Z},\check{\phi}]]]\colon \end{aligned}$$

$$\begin{aligned} D_4 &= -\frac{1}{4}(D_2)^2 + 2\colon \mathrm{Tr}[Z,\phi][\check{\phi},[\phi,[\check{Z},\check{\phi}]]]\colon \\ &\equiv -\frac{1}{4}(D_2)^2 + \delta D_4 \end{aligned}$$

$$D_2 \equiv N D_{2;0} + D_{2;+} + D_{2;-}$$

$$\begin{aligned} D_{2;0}\mathcal{O}_p^{J_0;J_1,\ldots,J_k} =& -4\big(\delta_{p\neq J_0}\mathcal{O}_{p+1}^{J_0;J_1,\ldots,J_k} - (\delta_{p\neq J_0}+\delta_{p\neq 0})\mathcal{O}_p^{J_0;J_1,\ldots,J_k} \\ &+\delta_{p\neq 0}\mathcal{O}_{p-1}^{J_0;J_1,\ldots,J_k}\big) \end{aligned}$$

$$\begin{aligned} D_{2;+}\mathcal{O}_p^{J_0;J_1,\ldots,J_k} =& 4\sum_{J_{k+1}=1}^{p-1}\left(\mathcal{O}_{p-J_{k+1}}^{J_0-J_{k+1};J_1,\ldots,J_{k+1}} - \mathcal{O}_{p-1-J_{k+1}}^{J_0-J_{k+1};J_1,\ldots,J_{k+1}}\right) \\ &-4\sum_{J_{k+1}=1}^{J_0-p-1}\left(\mathcal{O}_{p+1}^{J_0-J_{k+1};J_1,\ldots,J_{k+1}} - \mathcal{O}_{p}^{J_0-J_{k+1};J_1,\ldots,J_{k+1}}\right), \end{aligned}$$

$$\begin{aligned} D_{2;-}\mathcal{O}_p^{J_0;J_1,\ldots,J_k} =& 4\sum_{i=1}^{k} J_i\left(\mathcal{O}_{J_i+p}^{J_0+J_i;J_1,\ldots,\mathcal{F}_i,\ldots,J_k} - \mathcal{O}_{J_i+p-1}^{J_0+J_i;J_1,\ldots,\mathcal{F}_i,\ldots,J_k}\right) \\ &-4\sum_{i=1}^{k} J_i\left(\mathcal{O}_{p+1}^{J_0+J_i;J_1,\ldots,\mathcal{F}_i,\ldots,J_k} - \mathcal{O}_{p}^{J_0+J_i;J_1,\ldots,\mathcal{H}_i,\ldots,J_k}\right). \end{aligned}$$

$$\delta D_4 = N^2\delta D_{4;0} + N\delta D_{4;+} + N\delta D_{4;-} + \delta D_{4;++} + \delta D_{4;--} + \delta D_{4;+-}$$

$$\delta D_{4;0}\mathcal{O}_p^{J_0;J_1,\ldots,J_k} = 4\big(\delta_{p,0}+\delta_{p,J_0}-\delta_{p,1}-\delta_{p,J_0-1}\big)\big(\mathcal{O}_1^{J_0;J_1,\ldots,J_k} - \mathcal{O}_0^{J_0;J_1,\ldots,J_k}\big)$$

$$D_{2;0} = 4\cdot\begin{pmatrix} +1 & -1 & & & & \\ -1 & +2 & -1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & & & -1 & +2 & -1 \\ & & & & -1 & +1 \end{pmatrix}$$

$$\mathcal{O}_n^J = \frac{1}{J+1}\sum_{p=0}^{J} \cos\left(\frac{\pi n(2p+1)}{J+1}\right)\mathcal{O}_p^J$$

$$\mathcal{O}_p^J = \mathcal{O}_{n=0}^J + 2\sum_{n=1}^{[J/2]} \cos\left(\frac{\pi n(2p+1)}{J+1}\right)\mathcal{O}_n^J$$

$$D_{4;0} = -\frac{1}{4}\left(D_{2;0}\right)^2 + 4\cdot\begin{pmatrix} -1 & +1 & & & & \\ +1 & -1 & & & & \\ & & 0 & & & \\ & & \ddots & & & \\ & & & 0 & & \\ & & & & -1 & +1 \\ & & & & +1 & -1 \end{pmatrix}$$

$$\delta D_{4;0}\mathcal{O}_n^J = -\frac{256}{J+1}\sin^2\frac{\pi n}{J+1}\cos\frac{\pi n}{J+1}\sum_{m=1}^{[J/2]}\sin^2\frac{\pi m}{J+1}\cos\frac{\pi m}{J+1}\mathcal{O}_m^J$$

$$\Delta_n^J = J+2+\frac{g_{\mathrm{YM}}^2 N}{\pi^2}\sin^2\frac{\pi n}{J+1} - \frac{g_{\mathrm{YM}}^4 N^2}{\pi^4}\sin^4\frac{\pi n}{J+1}\left(\frac{1}{4}+\frac{\cos^2\frac{\pi n}{J+1}}{J+1}\right)$$

$$\mathcal{O}_n^J \to \mathcal{O}_n^J - \frac{g_{\mathrm{YM}}^2 N}{\pi^2}\frac{1}{J+1}\sum_{m=1}^{[J/2]} \delta_{m\neq n}\frac{\sin^2\frac{\pi n}{J+1}\cos\frac{\pi n}{J+1}\sin^2\frac{\pi m}{J+1}\cos\frac{\pi m}{J+1}}{\sin^2\frac{\pi n}{J+1}-\sin^2\frac{\pi m}{J+1}}\mathcal{O}_m^J$$

$$J\to\infty, N\to\infty, \lambda' = \frac{g_{\mathrm{YM}}^2 N}{J^2}, g_2 = \frac{J^2}{N}\ \text{fixed.}$$

$$\mathcal{O}_p^{J_0;J_1,\dots,J_k} \to |x;r_1,\dots,r_k\rangle = |r_0 - x;r_1,\dots,r_k\rangle,$$

$$x\in[0,r_0], r_0, r_i\in[0,1]\ \text{and}\ r_0 = 1-(r_1+\cdots+r_k),$$

$$|x;r_1,\dots,r_k\rangle = \left|x;r_{\pi(1)},\dots,r_{\pi(k)}\right\rangle$$

$$g^2 D_2 = \frac{\lambda'}{4\pi^2}d_2$$

$$d_{2;0}|x;r_1,\dots,r_k\rangle = -\partial_x^2|x;r_1,\dots,r_k\rangle$$

$$d_{2;+}|x;r_1,\dots,r_k\rangle = \int_0^x dr_{k+1}\partial_x|x-r_{k+1};r_1,\dots,r_{k+1}\rangle - \int_0^{r_0-x} dr_{k+1}\partial_x|x;r_1,\dots,r_{k+1}\rangle$$

$$d_{2;-}|x;r_1,\dots,r_k\rangle = \sum_{i=1}^{k} r_i\partial_x|x+r_i;r_1,\dots,\mathcal{R}_i,\dots,r_k\rangle - \sum_{i=1}^{k} r_i\partial_x|x;r_1,\dots,\mathcal{R}_i,\dots,r_k\rangle$$

$$|n;r_1,\dots,r_k\rangle = \frac{1}{r_0}\int_0^{r_0} dx\cos(2\pi n x/r_0)|x;r_1,\dots,r_k\rangle$$

$$|x;r_1,\dots,r_k\rangle = |0;r_1,\dots,r_k\rangle + 2\sum_{m=1}^{\infty}\cos(2\pi m x/r_0)|m;r_1,\dots,r_k\rangle.$$

$$d_{2;0}|n;r_1,\dots,r_k\rangle = \left(\frac{2\pi n}{r_0}\right)^2|n;r_1,\dots,r_k\rangle$$

$$d_{2;+}|n;r_1,\dots,r_k\rangle = \frac{8}{r_0}\int_0^{r_0} dr_{k+1}\sum_{m=1}^{\infty}\frac{\left(\frac{2\pi m}{r_0-r_{k+1}}\right)^2\sin^2\left(\pi n\frac{r_{k+1}}{r_0}\right)}{\left(\frac{2\pi m}{r_0-r_{k+1}}\right)^2-\left(\frac{2\pi n}{r_0}\right)^2}|m;r_1,\dots,r_{k+1}\rangle$$

$$d_{2;-}|n;r_1,\dots,r_k\rangle = 8\sum_{i=1}^{k}\frac{r_i}{r_0}\sum_{m=1}^{\infty}\frac{\left(\frac{2\pi m}{r_0+r_i}\right)^2\sin^2\left(\pi m\frac{r_i}{r_0+r_i}\right)}{\left(\frac{2\pi m}{r_0+r_i}\right)^2-\left(\frac{2\pi n}{r_0}\right)^2}|m;r_1,\dots,x_i,\dots,r_k\rangle$$

$$g^4 D_4 = \left(\frac{\lambda'}{4\pi^2}\right)^2 d_4$$

$$\mathcal{O}_1^{J_0;J_1,\dots,J_k} - \mathcal{O}_0^{J_0;J_1,\dots,J_k} \sim \frac{1}{J_0^2}$$

$$d_4 = -\frac{1}{4}(d_2)^2$$

$$D - J \longrightarrow 2 + \left(\frac{\lambda'}{4\pi^2}\right)d_2 - \frac{1}{4}\left(\frac{\lambda'}{4\pi^2}\right)^2(d_2)^2$$

$$\Delta(\lambda', g_2) - \Delta_0 = \sum_{k=1}^{\infty}\left(\frac{\lambda'}{4\pi^2}\right)^k\Delta_k(g_2)$$

$$\Delta_2(g_2) = -\frac{1}{4}\left(\Delta_1(g_2)\right)^2, \forall g_2$$

$$\Delta \longrightarrow J + 2\sqrt{1 + \lambda' n^2}$$

$$D - D_0 \longrightarrow 2\sqrt{1 + \left(\frac{\lambda'}{4\pi^2}\right) d_2}$$

$$(D_2)^l \sim 2^{l-1} \phi^{(a_0)} D_2^{a_0 a_1} D_2^{a_1 a_2} \dots D_2^{a_{l-1} a_l} \check{\phi}^{(a_l)}$$

$$D_2^{ab} = -2\!: \mathrm{Tr}[Z, T^a][\check{Z}, T^b]:$$

$$D_{2l} = (-2)^{1-l} \frac{(2l-2)!}{(l-1)!\, l!} \phi^{(a_0)} D_2^{a_0 a_1} D_2^{a_1 a_2} \dots D_2^{a_{l-1} a_l} \check{\phi}^{(a_l)} + \cdots,$$

$$P \mathrm{Tr} Z^3 \phi^2 Z \phi = \mathrm{Tr} \phi Z \phi^2 Z^3 = \mathrm{Tr} Z^3 \phi Z \phi^2.$$

$$PD = DP.$$

$$\mathcal{O}_- = \mathrm{Tr}[\phi, Z][\phi, Z][\phi, Z]Z,$$

$$\mathcal{O}_+ = \begin{pmatrix} 2\mathrm{Tr}Z^4\phi^3 + 2\mathrm{Tr}Z^2\phi Z^2\phi^2 + 2\mathrm{Tr}Z^2\phi Z\phi Z\phi - 3\mathrm{Tr}Z^3\phi\{\phi, Z\}\phi \\ \mathrm{Tr}Z\phi\mathrm{Tr}Z^2[\phi, Z]\phi - \mathrm{Tr}Z^2\mathrm{Tr}Z[\phi, Z]\phi^2 \end{pmatrix}.$$

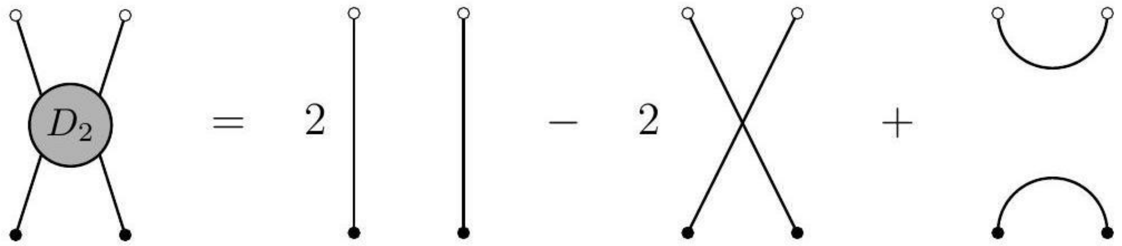
$$\Delta_- = 7 + \frac{5 g_{\mathrm{YM}}^2 N}{8\pi^2} - \frac{15 g_{\mathrm{YM}}^4 N^2}{128\pi^4}$$

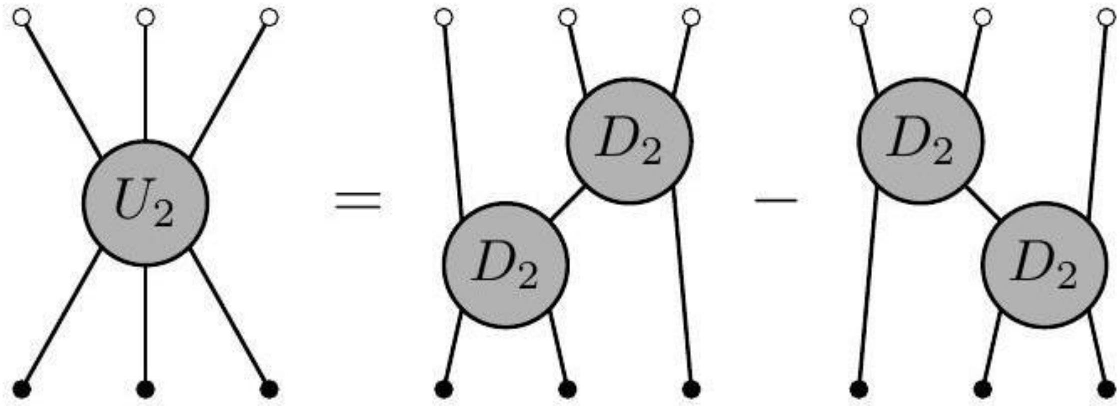
$$D_2 = N \begin{pmatrix} 10 & \frac{8}{N} \\ \frac{20}{N} & 8 \end{pmatrix}, D_4 = N^2 \begin{pmatrix} -30 - \frac{40}{N^2} & -\frac{40}{N} \\ -\frac{140}{N} & -24 - \frac{64}{N^2} \end{pmatrix}$$

$$\Delta_+ = 7 + \frac{g_{\mathrm{YM}}^2 N}{16\pi^2}\left(9 \pm \sqrt{1 + \frac{160}{N^2}}\right) - \frac{g_{\mathrm{YM}}^4 N^2}{256\pi^4}\left(27 + \frac{52}{N^2} \pm \frac{3 + \frac{948}{N^2}}{\sqrt{1 + \frac{160}{N^2}}}\right)$$

$$[D, U] = \{P, U\} = 0$$

$$D_2 = N \sum_{k=1}^{L} D_{2,k(k+1)},$$





$$D_{2,k(k+1)} = 2I_{k(k+1)} - 2P_{k(k+1)} + K_{k(k+1)}.$$

$$U_2 = \sum_{k=1}^{L} U_{2,k(k+1)(k+2)}, U_{2,k(k+1)(k+2)} = \left[D_{2,(k+1)(k+2)}, D_{2,k(k+1)}\right].$$

$$U_2\mathcal{O}_{+,1} = -60\mathcal{O}_-, U_2\mathcal{O}_- = +4\mathcal{O}_{+,1},$$

$$R_{0k}(u) = P_{0k}\left(\left(1-\frac{3}{2}u+\frac{1}{2}u^2\right)I_{0k} + u\left(1-\frac{3}{2}u\right)\frac{1}{2}D_{2,0k} + \frac{1}{2}u^2\left(\frac{1}{2}D_{2,0k}\right)^2\right),$$

$$R_{12}(u)R_{13}(u+v)R_{23}(v) = R_{23}(v)R_{13}(u+v)R_{12}(u),$$

$$t_1 = \frac{1}{2N}D_2 - \frac{3}{2}L,$$

$$t_2 = -\frac{1}{8}U_2 + \frac{1}{2}(t_1)^2 - \frac{5}{8}L$$

$$[D_2, U_2] = 0$$

$$\{n_1, n_2, \dots\} = \sum_{k=1}^{L} P_{k+n_1,k+n_1+1}P_{k+n_2,k+n_2+1}\cdots$$

$$D_2 = 2(\{\} - \{0\}), U_2 = 4(\{1,0\} - \{0,1\})$$

$$D_4 = 2(-4\{\} + 6\{0\} - (\{0,1\} + \{1,0\}))$$

$$[D_4, U_2] + [D_2, U_4] = 0, \text{ and } \{P, U_4\} = 0$$

$$U_4 = 8(\{2,1,0\} - \{0,1,2\})$$

$$Q_k = \sum_{l=0}^{\infty}\left(\frac{g_{\mathrm{YM}}^2}{16\pi^2}\right)^l Q_{k,2l},$$

$$D_6 = 4(15\{\} - 26\{0\} + 6(\{0,1\} + \{1,0\}) + \{0,2\} - (\{0,1,2\} + \{2,1,0\})).$$

$$\Delta_{\mathcal{K}'} = 4 + \frac{3g_{\mathrm{YM}}^2N}{4\pi^2} - \frac{3g_{\mathrm{YM}}^4N^2}{16\pi^4} + \frac{21g_{\mathrm{YM}}^6N^3}{256\pi^6}$$

$$D_6 = \frac{1}{8}(D_2)^3 + 4\cdot\begin{pmatrix} +9 & -10 & +1 & & \\ -10 & +10 & & & \\ +1 & & -1 & & \\ & & 0 & & \\ & & & & \ddots \end{pmatrix}$$

$$\delta\Delta_n^J = \frac{g_{\mathrm{YM}}^6 N^3}{\pi^6}\sin^6\frac{\pi n}{J+1}\left(\frac{1}{8}+\frac{\cos^2\frac{\pi n}{J+1}}{4(J+1)^2}\left(3J+2(J+6)\cos^2\frac{\pi n}{J+1}\right)\right),$$

$$D' = (1+\beta' g^6[D_2, D_4])^{-1} D (1+\beta' g^6[D_2, D_4]).$$

$$\beta' g^8\left[D_2,[D_2,D_4]\right]$$

$$\mathcal{O} = \sum_{k=1}^{\Delta_0-4}(-1)^k \mathrm{Tr}\phi Z^k \phi Z^{\Delta_0-3-k}\phi$$

$$\Delta = \Delta_0 + \frac{3g_{\mathrm{YM}}^2 N}{4\pi^2} - \frac{9g_{\mathrm{YM}}^4 N^2}{64\pi^4}$$

$$\Delta = \Delta_0 + \frac{3g_{\mathrm{YM}}^2 N}{4\pi^2} - \frac{9g_{\mathrm{YM}}^4 N^2}{64\pi^2} + \frac{66g_{\mathrm{YM}}^6 N^3}{1024\pi^6} - \frac{645g_{\mathrm{YM}}^8 N^4}{16384\pi^8}$$

$$\Delta = 8 + \frac{3g_{\mathrm{YM}}^2 N}{4\pi^2} - \frac{9g_{\mathrm{YM}}^4 N^2}{64\pi^2} + \frac{66g_{\mathrm{YM}}^6 N^3}{1024\pi^6} - \frac{648g_{\mathrm{YM}}^8 N^4}{16384\pi^8},$$

$$\Delta = 6 + \frac{3g_{\mathrm{YM}}^2 N}{4\pi^2} - \frac{9g_{\mathrm{YM}}^4 N^2}{64\pi^2} + \frac{63g_{\mathrm{YM}}^6 N^3}{1024\pi^6} - \frac{621g_{\mathrm{YM}}^8 N^4}{16384\pi^8}$$

$$\left\langle \mathcal{O}_\alpha(x_1)\mathcal{O}_\beta(x_2)\mathcal{O}_\gamma(x_3)\right\rangle = \frac{C_{\alpha\beta\gamma}}{|x_{12}|^{\Delta_\alpha+\Delta_\beta-\Delta_\gamma}|x_{23}|^{\Delta_\beta+\Delta_\gamma-\Delta_\alpha}|x_{31}|^{\Delta_\gamma+\Delta_\alpha-\Delta_\beta}}.$$

$$\begin{aligned}
S =& \frac{1}{2}\int\frac{d^{4-2\epsilon}x}{(2\pi)^{2-\epsilon}}\mathrm{Tr}\left(\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+\frac{1}{2}\mathcal{D}_\mu\Phi_m\mathcal{D}_\mu\Phi_m-\frac{1}{4}g^2\mu^{2\epsilon}[\Phi_m,\Phi_n][\Phi_m,\Phi_n]\right.\\
&\left.+\frac{1}{2}\Psi^\top\Sigma_\mu D_\mu\Psi-\frac{i}{2}g\mu^\epsilon\Psi^\top\Sigma_m[\Phi_m,\Psi]\right),\\
\mathcal{D}_\mu X =& \partial_\mu X - ig\mu^\epsilon\left[A_\mu, X\right],\\
F_{\mu\nu} =& \partial_\mu A_\nu - \partial_\nu A_\mu - ig\mu^\epsilon\left[A_\mu, A_\nu\right].
\end{aligned}$$

$$g^2 = \frac{g_{\mathrm{YM}}^2}{4(2\pi)^{2-\epsilon}} \to \frac{g_{\mathrm{YM}}^2}{16\pi^2}$$

$$\mathrm{Tr}T^aT^b = \delta^{ab}, \sum_a (T^a)^\alpha_\beta (T^a)^\gamma_\delta = \delta^\alpha_\delta\delta^\gamma_\beta$$

$$\mathrm{Tr}T^aA\mathrm{Tr}T^aB = \mathrm{Tr}AB, \mathrm{Tr}T^aAT^aB = \mathrm{Tr}A\mathrm{Tr}B$$

$$\check{\Phi}_m = \frac{\delta}{\delta\Phi_m} = T^a\frac{\delta}{\delta\Phi_m^{(a)}}.$$

$$I_{xy} = \frac{\Gamma(1-\epsilon)}{\left|\frac{1}{2}(x-y)^2\right|^{1-\epsilon}}$$

$$\begin{aligned}
Y_{x_1x_2x_3} =& \mu^{2\epsilon}\int\frac{d^{4-2\epsilon}z}{(2\pi)^{2-\epsilon}}I_{x_1z}I_{x_2z}I_{x_3z}\\
X_{x_1x_2x_3x_4} =& \mu^{2\epsilon}\int\frac{d^{4-2\epsilon}z}{(2\pi)^{2-\epsilon}}I_{x_1z}I_{x_2z}I_{x_3z}I_{x_4z}\\
\tilde{H}_{x_1x_2,x_3x_4} =& \frac{1}{2}\mu^{2\epsilon}\left(\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_3}\right)^2\int\frac{d^{4-2\epsilon}z_1 d^{4-2\epsilon}z_2}{(2\pi)^{4-2\epsilon}}I_{x_1z_1}I_{x_2z_1}I_{z_1z_2}I_{z_2x_3}I_{z_2x_4}
\end{aligned}$$

$$\frac{Y_{00x}}{I_{0x}} = \frac{1}{\epsilon(1-2\epsilon)}\xi$$
$$\frac{X_{00xx}}{I_{0x}^2} = \frac{2(1-3\epsilon)\gamma}{\epsilon(1-2\epsilon)^2}\xi$$
$$\frac{\tilde{H}_{0x,0x}}{I_{0x}^2} = -\frac{2(1-3\epsilon)(\gamma-1)}{\epsilon^2(1-2\epsilon)}\xi$$

$$\xi = \frac{\Gamma(1-\epsilon)}{\left|\frac{1}{2}\mu^2 x^2\right|^{-\epsilon}}, \gamma = \frac{\Gamma(1-\epsilon)\Gamma(1+\epsilon)^2\Gamma(1-3\epsilon)}{\Gamma(1-2\epsilon)^2\Gamma(1+2\epsilon)} = 1 + 6\zeta(3)\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$\langle \mathcal{O}_\alpha^+ \mathcal{O}_\beta^- \rangle = \exp(W_0)\exp\left(\sum_{l=1}^{\infty} g^{2l} W_{2l}(x, \check{\Phi}^+, \check{\Phi}^-)\right)\mathcal{O}_\alpha^+ \mathcal{O}_\beta^-\bigg|_{\Phi=0}$$

$$\langle \mathcal{O}_\alpha^+ \mathcal{O}_\beta^- \rangle = \exp(W_0):\exp\left(\sum_{l=1}^{\infty} g^{2l} W_{2l}(x, I_{0x}^{-1}\Phi^-, \check{\Phi}^-)\right):\mathcal{O}_\alpha^+ \mathcal{O}_\beta^-\big|_{\Phi=0}$$

$$\langle \mathcal{O}_\alpha^+ \mathcal{O}_\beta^- \rangle = \exp(W_0)\exp(V^-(x))\mathcal{O}_\alpha^+ \mathcal{O}_\beta^-\big|_{\Phi=0}$$

$$V(x) = \sum_{l=1}^{\infty} g^{2l} V_{2l}(x) - \frac{1}{48} g^8 [V_2(x), [V_2(x), V_4(x)]] + \cdots.$$

$$\exp(V(x)) =: \exp\left(\sum_{l=1}^{\infty} g^{2l} W_{2l}(x, I_{0x}^{-1}\Phi, \check{\Phi})\right):,$$

$$V_4(x) =: W_4(x, I_{0x}^{-1}\Phi, \check{\Phi}): -\frac{1}{2}(V_2(x)V_2(x) -: V_2(x)V_2(x):).$$

$$\exp(W_0)V_{2l}^-(x) = \exp(W_0)V_{2l}^+(x)$$

$$\exp(W_0)X^- = \exp(W_0)X^{\top +}$$

$$V_{2l}^\top(x) = V_{2l}(x)$$

$$\tilde{\mathcal{O}} = \exp\left(-\frac{1}{2}Z(x_0)\right)\mathcal{O}$$

$$Z(x_0) = \sum_{l=1}^{\infty} g^{2l} V_{2l}(x_0) - \frac{1}{12} g^6 [V_2(x_0), V_4(x_0)] + \cdots$$

$$\langle \mathcal{O}_\alpha^+ \mathcal{O}_\beta^- \rangle = \exp(W_0)\exp(V^-(x))\exp\left(-\frac{1}{2}Z^+(x_0)\right)\mathcal{O}_\alpha^+ \exp\left(-\frac{1}{2}Z^-(x_0)\right)\mathcal{O}_\beta^-\bigg|_{\Phi=0}.$$

$$\langle \mathcal{O}_\alpha^+ \mathcal{O}_\beta^- \rangle = \exp(W_0)\exp\left(-\frac{1}{2}Z^{\top -}(x_0)\right)\exp(V^-(x))\exp\left(-\frac{1}{2}Z^-(x_0)\right)\mathcal{O}_\alpha^+ \mathcal{O}_\beta^-\bigg|_{\Phi=0}$$

$$Z^\top(x_0) = \sum_{l=1}^{\infty} g^{2l} V_{2l}(x_0) + \frac{1}{12} g^6 [V_2(x_0), V_4(x_0)] + \cdots$$

$$V_{2l}(x) = \xi^l V_{2l}, \xi = \frac{\Gamma(1-\epsilon)}{\left|\frac{1}{2}\mu^2 x^2\right|^{-\epsilon}}$$

$$\sum_{l=1}^{\infty} (\xi^l - \xi_0^l) g^{2l} V_{2l}^- - \frac{1}{48} g^8 (\xi - \xi_0)^4 [V_2^-, [V_2^-, V_4^-]] + \cdots$$

$$\langle \tilde{\mathcal{O}}_{\alpha}^{+}\tilde{\mathcal{O}}_{\beta}^{-}\rangle = \exp{(W_0)}\exp{\left(\log{(x_0^2/x^2)}\sum_{l=1}^{\infty} g^{2l}D_{2l}^{-}\right)}\mathcal{O}_{\alpha}^{+}\mathcal{O}_{\beta}^{-}\Big|_{\Phi=0}$$

$$D_{2l} = -l\lim_{\epsilon\to 0}\epsilon V_{2l}$$

$$\Delta = 5 + \frac{5g_{\text{YM}}^2 N}{4\pi^2}$$

$$\Delta = 5 + \frac{g_{\text{YM}}^2 N}{4\pi^2}\omega$$

$$\omega^6 - 17\omega^5 + \left(110 - \frac{50}{N^2}\right)\omega^4 - \left(335 - \frac{565}{N^2}\right)\omega^3 + \left(475 - \frac{2440}{N^2} + \frac{400}{N^4}\right)\omega^2$$
$$-\left(250 - \frac{4850}{N^2} + \frac{1600}{N^4}\right)\omega - \left(\frac{3750}{N^2} - \frac{4000}{N^4}\right) = 0$$

$$\Delta = 6 + \frac{7g_{\text{YM}}^2 N}{8\pi^2}$$

$$\Delta = 6 + \frac{g_{\text{YM}}^2 N}{\pi^2}$$

$$\Delta = 6 + \frac{g_{\text{YM}}^2 N}{8\pi^2}\omega$$

$$\omega^6 - 43\omega^5 + 731\omega^4 - 6238\omega^3 + 27936\omega^2 - 61776\omega + 52272 = 0.$$

$$\Delta = 6 + \frac{g_{\text{YM}}^2 N}{\pi^2}$$

$$\Delta = 6 + \frac{5g_{\text{YM}}^2 N}{4\pi^2}$$

$$\Delta = 6 + \frac{g_{\text{YM}}^2 N}{8\pi^2}\omega$$

$$\omega^3 - 23\omega^2 + 158\omega - 308 = 0$$

$$\Delta = 6 + \frac{g_{\text{YM}}^2 N}{8\pi^2}\omega$$

$$\omega^5 - 43\omega^4 + 701\omega^3 - 5338\omega^2 + 18480\omega - 21960 = 0.$$

$$\Delta = 8 + \frac{g_{\text{YM}}^2 N}{2\pi^2} - \frac{5g_{\text{YM}}^4 N^2}{64\pi^4}$$

$$\Delta = 8 + \frac{3g_{\text{YM}}^2 N}{4\pi^2} - \frac{9g_{\text{YM}}^4 N^2}{64\pi^4}$$

$$\Delta = 9 + \frac{g_{\text{YM}}^2 N}{16\pi^2}\omega$$

$$\omega^3 - 34\omega^2 + 360\omega - 1176 + \frac{g_{\text{YM}}^2 N}{16\pi^2}(102\omega^2 - 2100\omega + 9912) = 0$$

$$\Delta = 10 + \frac{3g_{\text{YM}}^2 N}{4\pi^2} - \frac{9g_{\text{YM}}^4 N^2}{64\pi^4}$$

$$\Delta = 10 + \frac{g_{\text{YM}}^2 N}{16\pi^2}\omega$$

$$\omega^3 - 30\omega^2 + 276\omega - 768 + g_{\text{YM}}^2 N(86\omega^2 - 1516\omega + 5984) = 0$$

$$\Delta = 8 + \frac{g_{\mathrm{YM}}^2 N}{16\pi^2}\omega$$

$$\omega^3 - 40\omega^2 + 464\omega - 1600 + \frac{g_{\mathrm{YM}}^2 N}{16\pi^2}(128\omega^2 - 2720\omega + 12800) = 0$$

$$\Delta = 9 + \frac{5 g_{\mathrm{YM}}^2 N}{8\pi^2} - \frac{15 g_{\mathrm{YM}}^4 N^2}{128\pi^4}$$

$$\Delta = 9 + \frac{g_{\mathrm{YM}}^2 N(3 \pm \sqrt{3})}{4\pi^2} - \frac{g_{\mathrm{YM}}^4 N^2(18 \pm 9\sqrt{3})}{128\pi^4}$$

$$\Delta = 10 + \frac{g_{\mathrm{YM}}^2 N(11 \pm \sqrt{5})}{16\pi^2} - \frac{g_{\mathrm{YM}}^4 N^2(31 \pm 3\sqrt{5})}{256\pi^4}$$

$$\Delta = 10 + \frac{g_{\mathrm{YM}}^2 N}{4\pi^2}\omega$$

$$\begin{gathered}\omega^6 - 21\omega^5 + 173\omega^4 - 711\omega^3 + 1525\omega^2 - 1603\omega + 637 \\ + \frac{g_{\mathrm{YM}}^2 N}{16\pi^2}(67\omega^5 - 1074\omega^4 + 6409\omega^3 - 17623\omega^2 + 22078\omega - 9947) = 0\end{gathered}$$

$$\Delta = 10 + \frac{g_{\mathrm{YM}}^2 N}{4\pi^2}\omega$$

$$\begin{gathered}\omega^4 - 15\omega^3 + 78\omega^2 - 165\omega + 120 \\ + \frac{g_{\mathrm{YM}}^2 N}{16\pi^2}(47\omega^3 - 472\omega^2 + 1430\omega - 1305) = 0\end{gathered}$$

$$\mathcal{O}_{1-0}^{J_0;J_1,\dots,J_k} = \mathcal{O}_{1}^{J_0;J_1,\dots,J_k} - \mathcal{O}_{0}^{J_0;J_1,\dots,J_k}$$

$$\delta D_{4;0}\mathcal{O}_p^{J_0;J_1,\dots,J_k} = 4\left(\delta_{p,0} + \delta_{p,J_0} - \delta_{p,1} - \delta_{p,J_0-1}\right)\mathcal{O}_{1-0}^{J_0;J_1,\dots,J_k}$$

$$\begin{aligned}\delta D_{4;+}\mathcal{O}_p^{J_0;J_1,\dots,J_k} = {} & 4\delta_{p\neq 0,p\neq J_0}\left(\mathcal{O}_{1-0}^{p;J_1,\dots,J_k,J_0-p} + \mathcal{O}_{1-0}^{J_0-p;J_1,\dots,J_k,p}\right) \\ & -8\delta_{p>1}\mathcal{O}_{1-0}^{J_0-p+1;J_1,\dots,J_k,p-1} \\ & -8\delta_{p<J_0-1}\mathcal{O}_{1-0}^{p+1;J_1,\dots,J_k,J_0-p-1} \\ & +4\left(\delta_{p,0} + \delta_{p,J_0}\right)\sum_{J_{k+1}=1}^{J_0-1}\mathcal{O}_{1-0}^{J_0-J_{k+1};J_1,\dots,J_{k+1}}\end{aligned}$$

$$\begin{aligned}\delta D_{4;++}\mathcal{O}_p^{J_0;J_1,\dots,J_k} = {} & 4\delta_{p\neq 0}\sum_{J_{k+1}=1}^{J_0-p-1}\mathcal{O}_{1-0}^{J_0-p-J_{k+1};J_1,\dots,J_{k+1},p} \\ & +4\delta_{p\neq J_0}\sum_{J_{k+1}=1}^{p-1}\mathcal{O}_{1-0}^{p-J_{k+1};J_1,\dots,J_{k+1},J_0-p} \\ & -4\sum_{J_{k+1}=1}^{J_0-p-2}\mathcal{O}_{1-0}^{p+1;J_1,\dots,J_{k+1},J_0-p-J_{k+1}-1} \\ & -4\sum_{J_{k+1}=1}^{p-2}\mathcal{O}_{1-0}^{J_0-p+1;J_1,\dots,J_{k+1},p-J_{k+1}-1}\end{aligned}$$

$$\delta D_{4;-}\mathcal{O}_p^{J_0;J_1,\dots,J_k} = 4\left(\delta_{p,0} + \delta_{p,J_0}\right)\sum_{i=1}^{k} J_i\mathcal{O}_{1-0}^{J_0+J_i;J_1,\dots,\mathcal{Y}_i,\dots,J_k},$$

$$
\begin{aligned}
\delta D_{4;+-}\mathcal{O}_p^{J_0;J_1,\dots,J_k} =& 4\delta_{p\neq 0}\sum_{i=1}^{k} J_i \mathcal{O}_{1-0}^{J_0+J_i-p;J_1,\dots,\mathcal{X}_i,\dots,J_k,p} \\
&+4\delta_{p\neq J_0}\sum_{i=1}^{k} J_i \mathcal{O}_{1-0}^{J_i+p;J_1,\dots,\mathcal{X}_i,\dots,J_k,J_0-p} \\
&-4\sum_{i=1}^{k} J_i \mathcal{O}_{1-0}^{p+1;J_1,\dots,\mathcal{W}_i,\dots,J_k,J_0+J_i-p-1} \\
&-4\sum_{i=1}^{k} J_i \mathcal{O}_{1-0}^{J_0-p+1;J_1,\dots,\mathcal{K}_i,\dots,J_k,J_i+p-1}
\end{aligned}
$$

$$
\{n_1, n_2, \dots\} = \sum_{k=1}^{L} P_{k+n_1,k+n_1+1} P_{k+n_2,k+n_2+1} \cdots
$$

$$
\begin{aligned}
\{\dots, n, n\pm 1, n, \dots\} =& \{\dots,\dots\} - \{\dots, n, \dots\} - \{\dots, n\pm 1, \dots\} \\
&+ \{\dots, n, n\pm 1, \dots\} + \{\dots, n\pm 1, n, \dots\}
\end{aligned}
$$

$$
\begin{aligned}
D_0 &= \{\}, \\
D_2 &= 2\{\} - 2\{0\}, \\
D_4 &= -8\{\} + 12\{0\} - 2\left(\{0,1\} + \{1,0\}\right), \\
D_6 &= 60\{\} - 104\{0\} + 24\left(\{0,1\} + \{1,0\}\right) + 4\{0,2\} - 4\left(\{0,1,2\} + \{2,1,0\}\right) \\
D_8 &= +(-572 + 4\alpha)\{\} + (1072 - 12\alpha + 4\beta)\{0\} \\
&\quad + (-278 + 4\alpha - 4\beta)(\{0,1\} + \{1,0\}) + (-84 + 6\alpha - 2\beta)\{0,2\} - 4\{0,3\} \\
&\quad + 4(\{0,1,3\} + \{0,2,3\} + \{0,3,2\} + \{1,0,3\}) \\
&\quad + (78 + 2\beta)(\{0,1,2\} + \{2,1,0\}) + (-6 - 4\alpha + 2\beta)(\{0,2,1\} + \{1,0,2\}) \\
&\quad + (1 - \beta)(\{0,1,3,2\} + \{0,3,2,1\} + \{1,0,2,3\} + \{2,1,0,3\}) \\
&\quad + (2\alpha - 2\beta)\{1,0,2,1\} + 2\beta(\{0,2,1,3\} + \{1,0,3,2\}) \\
&\quad - 10(\{0,1,2,3\} + \{3,2,1,0\}).
\end{aligned}
$$

$$
\begin{aligned}
U_2 &= 4\left(\{1,0\} - \{0,1\}\right), \\
U_4 &= 8\left(\{2,1,0\} - \{0,1,2\}\right), \\
U_6 &= 8\left(\{0,1,3\} + \{0,2,3\} - \{0,3,2\} - \{1,0,3\}\right) \\
&\quad + 40\left(\{0,1,2\} - \{2,1,0\}\right) + 16\left(\{3,2,1,0\} - \{0,1,2,3\}\right)
\end{aligned}
$$

$$
\begin{aligned}
Q_{3,2} &= -2\{0\} + \left(\{0,1\} + \{1,0\}\right) \\
&\quad + \left(\{0,2,1\} + \{1,0,2\}\right) - \left(\{0,1,2\} + \{2,1,0\}\right), \\
Q_{3,4} &= -2\{0\} + \left(\{0,1\} + \{1,0\}\right) - 4\{0,2\} \\
&\quad - 3\left(\{0,1,2\} + \{2,1,0\}\right) + 5\left(\{0,2,1\} + \{1,0,2\}\right) + 2\{1,0,2,1\} \\
&\quad - 3\left(\{0,1,2,3\} + \{3,2,1,0\}\right) + \left(\{0,2,1,3\} + \{1,0,3,2\}\right) \\
&\quad + \left(\{0,1,3,2\} + \{2,1,0,3\} + \{0,3,2,1\} + \{1,0,2,3\}\right).
\end{aligned}
$$

$$
\begin{aligned}
Q_{4,2} &= -2\left(\{0,1,2\} - \{2,1,0\}\right) \\
&\quad - \left(\{0,2,1,3\} - \{1,0,3,2\}\right) + \left(\{0,1,2,3\} - \{3,2,1,0\}\right) \\
&\quad + \left(\{0,3,2,1\} - \{0,1,3,2\} - \{1,0,2,3\} + \{2,1,0,3\}\right).
\end{aligned}
$$

$$
D_2 \mathrm{Tr}\phi' Z^p \phi Z^{J-p} = P D_2 \mathrm{Tr}\phi' Z^p \phi Z^{J-p} = D_2 P \mathrm{Tr}\phi' Z^p \phi Z^{J-p} = D_2 \mathrm{Tr} Z^{J-p} \phi Z^p \phi' = D_2 \mathrm{Tr}\phi Z^p \phi' \mathrm{Tr} Z^{J-p}
$$

$$d\omega + \frac{1}{2}[\omega, \omega] = 0$$

$$L = \sum_k \Omega^{0,k} \otimes Mat_n = \Omega \otimes Mat_n$$

$$Q^a = Q^a_b x^b + Q^a_{bc} x^b x^c + \cdots$$

$$(u^\alpha, \theta^\alpha, z^l) :: \left(\lambda u^\alpha, (\theta^\alpha - \varepsilon u^\alpha), \left(z^l - \varepsilon \Gamma^l_{\alpha\beta} u^\alpha \theta^\beta\right)\right)$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - \Gamma^m_{\alpha\beta} \theta^\beta \frac{\partial}{\partial x^m}$$

$$S_{SYM}(A_m, \chi^\alpha) = -\frac{1}{4} \mathrm{Tr} F_{mn} F^{mn} + \frac{1}{2} \mathrm{Tr} \chi^\alpha \Gamma^m_{\alpha\beta} [\mathcal{D}_m, \chi^\beta]$$

$$\mathcal{D}_m = \frac{\partial}{\partial x_m} + A_m(x)$$

$$F_{nm} = \partial_m A_n - \partial_n A_m + [A_m, A_n]$$

$$\Gamma_m = \begin{pmatrix} 0 & \Gamma^{\alpha\beta}_m \\ (\Gamma_m)_{\alpha\beta} & 0 \end{pmatrix}$$

$$V_\phi(A_m) = [\mathcal{D}_m, \phi], V_\phi(\chi^\alpha) = [\chi^\alpha, \phi].$$

$$\delta_\epsilon(A_m) = \epsilon^\alpha (\Gamma_m)_{\alpha\beta} \chi^\beta$$
$$\delta_\epsilon(\chi^\alpha) = \frac{1}{2} (\Gamma^{mn})^\alpha{}_\beta \epsilon^\beta F_{mn}$$

$$\tilde{\delta}_\epsilon(A_m) = 0, \tilde{\delta}_\epsilon(\chi^\alpha) = \epsilon^\alpha$$

$$D_m = \partial_m + A_m(x, \theta), D_\alpha = D_\alpha + A_\alpha(x, \theta)$$

$$\partial_m = \frac{\partial}{\partial x^m}, D_\alpha = \frac{\partial}{\partial \theta^\alpha} - \Gamma_{\alpha\beta}{}^m \theta^\beta \partial_m, A_m$$

$$F_{\alpha\beta} = \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} + 2\Gamma^m_{\alpha\beta} \mathcal{D}_m$$
$$F_{\alpha m} = [\mathcal{D}_\alpha, \mathcal{D}_m], F_{mn} = [\mathcal{D}_m, \mathcal{D}_n]$$

$$F_{\alpha\beta} = 0$$

$$A_m \mapsto g^{-1} A_m g + g^{-1} \mathcal{D}_m g, A_\alpha \mapsto g^{-1} A_\alpha g + g^{-1} \mathcal{D}_\alpha g$$

$$\delta_\gamma(\mathcal{D}_m) = [\mathcal{D}_m, \tilde{D}_\gamma], \delta_\gamma(\mathcal{D}_\alpha) = \{\mathcal{D}_\alpha, \tilde{D}_\gamma\}$$

$$\tilde{D}_\alpha = \frac{\partial}{\partial \theta^\alpha} + \Gamma^m_{\alpha\beta} \theta^\beta \partial_m$$

$$\Gamma^m_{\alpha\beta} \mathcal{D}_m \chi^\beta = 0$$
$$\mathcal{D}_m F^{mn} = \frac{1}{2} \Gamma^n_{\alpha\beta} \{\chi^\alpha, \chi^\beta\}$$

$$\theta^\alpha A_\alpha = 0$$

$$\{D(u), D(u)\} = 0$$

$$\mathfrak{p} = (W \otimes W^*) \oplus \Lambda^2(W^*)$$

$$[m,m'] = [n,n'] = 0$$
$$[m,n]_a^b = m_{ac}n^{cb}$$
$$[m,k]_{ab} = m_{ac}k_b^c + m_{cb}k_a^c$$
$$[n,k]_{ab} = n^{ac}k_c^b + n^{cb}k_c^a$$

$$D = u^\alpha D_\alpha = u^\alpha \left(\frac{\partial}{\partial\theta^\alpha} + \Gamma_{\alpha\beta}{}^m\theta^\beta \frac{\partial}{\partial x^m}\right)$$

$$E = u^\alpha \frac{\partial}{\partial u^\alpha}$$

$$\Omega \otimes Mat_n$$

$$\bar{\partial}\omega + \frac{1}{2}\{\omega,\omega\} = 0$$

$$L_D\omega = L_{\bar{D}}\omega = i_D\omega = i_{\bar{D}}\omega = 0$$
$$L_E\omega = L_{\bar{E}}\omega = i_E\omega = i_{\bar{E}}\omega = 0$$

$$gDg^{-1} = D + u^\alpha A_\alpha = u^\alpha(D_\alpha + A_\alpha)$$

$$gDg^{-1} = D(u) = u^\alpha(D_\alpha + A_\alpha)$$

$$g^{-1}\cdot\bar{D}\cdot g = \bar{D}, g^{-1}\cdot E\cdot g = E, g^{-1}\cdot\bar{E}\cdot g = \bar{E}$$

$$\bar{D}g = 0, Eg = 0, \bar{E}g = 0.$$

$$E = u^\alpha \frac{\partial}{\partial u^\alpha} + \theta^\alpha \frac{\partial}{\partial\theta^\alpha}$$

$$d = u^\alpha \frac{\partial}{\partial\theta^\alpha}$$

$$(\Lambda^2(W) + \Lambda^4(W)) \otimes det^{-\frac{1}{2}}(W)$$

$$f(\omega) = \mathrm{Tr}\left(\frac{1}{2}\omega\bar{\partial}\omega + \frac{2}{3}\omega[\omega,\omega]\right)$$

$$\psi = C^{A_1\ldots A_k}(z)\rho(z)$$

$$\Omega_{int}^{-k} = \mathrm{Ber}\otimes\Lambda^k(T) = \bigoplus_{i+j=k} \mathrm{Ber}_{\mathbb{C}}\otimes\Lambda^i(T_{\mathbb{C}})\otimes\overline{\mathrm{Ber}}_{\mathbb{C}}\otimes\Lambda^j(\bar{T}_{\mathbb{C}})$$

$$\mathrm{Ber}_{\mathbb{C}}\otimes\overline{\mathrm{Ber}}_{\mathbb{C}}\otimes\Lambda^3(\bar{T}_{\mathbb{C}})$$

$$(\Lambda^3(W)\oplus\Lambda^1(W))\otimes\det^{-1}(W)\otimes\overline{\Lambda^3(W)\otimes\det^{-1}(W)}$$

$$\mathrm{tr}\in\Lambda^7(\Lambda^3(W)\oplus\Lambda^1(W))\otimes\det^{-7}(W)\otimes\overline{\Lambda^7(\Lambda^3(W))\otimes\det^{-7}(W)}$$

$$\mathcal{B} = A^\bullet \otimes \overline{A}^\bullet \otimes det^{-\frac{7}{2}}(W)$$

$$A^\bullet = \Lambda^\bullet[(\Lambda^3(W)\oplus\Lambda^1(W))\otimes det^{-\frac{1}{2}}(W)]$$

$$\Lambda^3(T) = \left(\bigoplus_{i=0}^{3}\Lambda^i(\Lambda^2(W))\otimes S^{3-i}(\Lambda^2(W)\oplus\Lambda^4(W))\otimes det^{-\frac{3-i}{2}}(W)\right)\otimes A^\bullet\otimes\overline{A}^\bullet$$

$$\mathcal{B} \underset{A^\bullet \otimes \overline{A}^\bullet}{\otimes} \overline{\mathcal{B} \underset{A^\bullet \otimes \overline{A}^\bullet}{\otimes} \Lambda^3(T_{\mathcal{R}})}.$$

$$\overline{\Lambda^3(\Lambda^2(W)) \otimes det^{-\frac{8}{2}}(W)} \subset$$

$$\subset \overline{\Lambda^3(\Lambda^2(W)) \otimes \Lambda^{15}[(\Lambda^3(W) \oplus \Lambda^1(W)) \otimes det^{-\frac{1}{2}}(W)] \otimes det^{-\frac{7}{2}}(W)} \subset$$

$$\subset \overline{\mathcal{B} \underset{A^\bullet \otimes \overline{A}^\bullet}{\otimes} \Lambda^3(T_{\mathcal{R}})}.$$

$$\mathrm{Tr}(a) = \int_{\mathcal{R}_0} < tr, a >$$

$$\sigma(a,b) = \mathrm{Tr}(a \wedge b)$$

$$d = u^\alpha \frac{\partial}{\partial \theta^\alpha}$$

$$d = u^\alpha \left(\frac{\partial}{\partial \theta^\alpha} + \Gamma^m_{\alpha\beta} \theta^\beta \frac{\partial}{\partial x^m} \right)$$

$$f(\omega) = \mathrm{Tr}\left(\frac{1}{2} \omega d\omega + \frac{2}{3} \omega[\omega, \omega] \right)$$

$$\mathrm{Tr}(\omega)\lambda = p(\omega)$$

$$W = \langle p_1, \dots, p_3 \rangle \oplus \langle u^1, \dots, u^3 \rangle \oplus \langle z_1, z_2 \rangle \oplus \langle w_1, w_2 \rangle$$

$$\tilde{A} = \mathbb{C}[p_1, \dots, p_3, u^1, \dots, u^3, z_1, z_2, w_1, w_2]/I$$

$$p_1 u^1 + p_2 u^2 + p_3 u^3$$

$$\tilde{A} \otimes \Lambda[\pi_{\alpha i}] \otimes \Lambda\left[\psi_j^\beta\right]$$

$$d(\pi_{\alpha i}) = p_\alpha z_i \; (1 \leq \alpha \leq 3, 1 \leq i \leq 2)$$
$$d\left(\psi_j^\beta\right) = u^\beta w_j \; (1 \leq \beta \leq 3, 1 \leq j \leq 2)$$
$$d(p_\alpha) = 0$$
$$d(u^\beta) = 0$$
$$d(z_i) = 0$$
$$d(w_j) = 0$$

$$R_{N=3} = A \otimes \Lambda[\pi_{\alpha i}] \otimes \Lambda\left[\psi_j^\beta\right]$$

$$S_{IKKT}(A, \chi) = \mathrm{tr}\left(-\frac{1}{4} \delta^{ij} \delta^{kl} [A_i, A_k][A_j, A_l] + \frac{1}{2} \Gamma^i_{\alpha\beta} [A_i, \chi^\alpha] \chi^\beta \right)$$

$$\delta A_i = [A_i, \varepsilon]; \delta\chi^\alpha = [\chi^\alpha, \varepsilon]$$

$$S = S_{IKKT} + \mathrm{tr} A^{*i} [A_i, c] + \mathrm{tr} \chi^*_\alpha [\chi^\alpha, c] + \frac{1}{2} \mathrm{tr}[c, c] c^*$$

$$QA^{*l} = \delta^{ij}\delta^{kl}\left[A_i,[A_j,A_k]\right] - \frac{1}{2}\Gamma^l_{\alpha\beta}\{\chi^\alpha,\chi^\beta\} - [A^{*l},c]$$
$$Q\chi^*_\alpha = -\Gamma^i_{\alpha\beta}[A_i,\chi^\beta] - [\chi^*_\alpha,c]$$
$$Qc^* = -[A^{*i},A_i] - [\chi^*_\alpha,\chi^\alpha] + [c,c^*]$$
$$Qc = \frac{1}{2}[c,c]$$
$$QA_i = [A_i,c]$$
$$Q\chi^\alpha = [\chi^\alpha,c]$$

$$m_2(\chi^\alpha,\chi^\beta) = \Gamma^{\alpha\beta}_k A^{*k}$$
$$m_2(\chi^\alpha,A_k) = m_2(A_k,\chi^\alpha) = \Gamma^{\alpha\beta}_k \chi^*_\beta$$
$$m_2(\chi^\alpha,\chi^*_\beta) = m_2(\chi^*_\beta,\chi^\alpha) = c^*$$
$$m_2(A_k,A^{*k}) = m_2(A^{*k},A_k) = c^*$$
$$m_3(A_k,A_l,A_m) = \delta_{kl}A^{*m} - \delta_{km}A^{*l}$$
$$m_2(c,\bullet) = m_2(\bullet,c) = \bullet$$

$$C_0 = \mathbb{C}, C_1 = C_2 = \{0\}, C_3 = V, C_4 = S, C_5 = \Lambda^2(V), C_6 = \Lambda^2(V), C_7 = S^*, C_8 = V, C_9 = C_{10} = \{0\}, C_{11} = \mathbb{C}$$

$$C_3 \otimes C_3 = V \otimes V \to \Lambda^2(V) = C_6$$

$$C_3 \otimes C_4 = V \otimes S \xrightarrow{\Gamma} S^* = C_7$$

$$C_5 \otimes C_4 = \Lambda^2(V) \otimes V \to V = C_8$$

$$S(a) = \mathrm{tr}\left(\frac{1}{2}a * d(a) + \frac{2}{3}a * a * a\right)$$

$$u^\alpha\Gamma^m_{\alpha\beta}u^\beta = 0, u^\alpha u^\beta = u^\beta u^\alpha$$

$$\Gamma^{\alpha\beta}_{m_1,m_2,m_3,m_4,m_5}(\lambda_\alpha\lambda_\beta + \lambda_\beta\lambda_\alpha) = 0$$

$$\Gamma^{\alpha\delta_1}_{m_1}\Gamma_{m_2\delta_1\delta_2}\Gamma^{\delta_2\delta_3}_{m_3}\Gamma_{m_4\delta_3\delta_4}\Gamma^{\delta_4\beta}_{m_5}$$

$$[\lambda_\alpha,\lambda_\beta]_+ = \lambda_\alpha\lambda_\beta + \lambda_\beta\lambda_\alpha$$

$$\Gamma_m \otimes \Gamma_n \to \Gamma_{m+n}$$

$$\Gamma = \bigoplus_{n\geq 0}\Gamma_n$$

$$\alpha_1^{\otimes n_1} \otimes \cdots \otimes \alpha_k^{\otimes n_k}(n_1,\ldots,n_k \geq 0)$$

$$u^\alpha\Gamma^m_{\alpha\beta}u^\beta = 0, u^\alpha u^\beta = u^\beta u^\alpha, \theta^\alpha\theta^\beta + \theta^\beta\theta^\alpha = 0, u^\alpha\theta^\beta = \theta^\beta u^\alpha$$

$$\Gamma^{\alpha\beta}_{m_1,\ldots m_5}(\lambda_\alpha\lambda_\beta + \lambda_\beta\lambda_\alpha) = 0, t_\alpha t_\beta - t_\beta t_\alpha = 0, \lambda_\alpha t_\beta - \lambda_\beta t_\alpha = 0$$

$$A_m = \Gamma^{\alpha\beta}_m\lambda_\alpha\lambda_\beta \text{ and } \chi^\alpha = \Gamma^{\alpha\beta}_m[\lambda_\beta, A^m]$$

$$[A_i,[A_i,A_k]] - \frac{1}{2}\Gamma^k_{\alpha\beta}\{\chi^\alpha,\chi^\beta\} = 0$$
$$-\Gamma^i_{\alpha\beta}[A_i,\chi^\beta] = 0$$

$$H^{0,k}\left(\hat{\beta}^{\otimes n}\right) n \geq 0$$

$$\left(\Omega(\hat{\mathcal{Q}}) \otimes \Lambda(S^*), \bar{\partial} + d\right)$$

$$\Omega(\hat{\mathcal{Q}}) \otimes \Lambda(S^*)$$

$$\left(\Omega(\hat{\mathcal{Q}}) \otimes \Lambda(S^*), \bar{\partial} + d\right)$$

$$\Omega(\hat{\mathcal{Q}} \times \Pi S) = \Omega(\hat{\mathcal{Q}}) \otimes \Lambda(S^*) \otimes \Lambda(\bar{S}^*) \otimes S(\bar{S}^*)$$

$$\lambda = (\det W)^{-1/2}, \left(\mathbb{C} + \Lambda^2(W) + \Lambda^4(W)\right) \otimes (\det W)^{-1/2}\right)$$

$$\left(\{0\}, \det W^{-1/2} + \{0\}\right) \subset \lambda$$

$$\varkappa = \left(\Lambda^2(W) + \Lambda^4(W)\right) \otimes (\det W)^{-1/2}$$

$$\mathcal{K}^{f_1,\dots,f_n} = \left(\mathcal{K}^{f_1,\dots,f_r}\right)^{f_{r+1},\dots,f_n}$$

$$V \otimes T_L \oplus V^* \otimes T_R \oplus T_L \oplus T_R$$

$$t_{\alpha 1} z_2 - t_{\alpha 2} z_1 = \det\left(s_j^\beta\right)$$
$$s_1^\alpha w_2 - s_2^\alpha w_1 = \det(t_{\alpha i})$$
$$s_i^\alpha t_{\alpha j} = 0$$

$$\mathcal{N} \otimes \mathbb{C}[w_i, z_j]$$

$$H^{\blacksquare}(V \otimes A^{x_1,\dots,x_r}) = V \otimes H^{\blacksquare}(A^{x_1,\dots,x_r})$$

$$H^{\blacksquare}(\mathcal{K}^{w_i,z_j}) = H^{\blacksquare}\left(V \otimes \mathbb{C}[w_i, z_j]^{w_i,z_j}\right) = V \otimes H^{\blacksquare}\left(\mathbb{C}[w_i, z_j]^{w_i,z_j}\right)$$

$$\rho(t_{\alpha i}) = p_\alpha z_i$$
$$\rho\left(s_j^\beta\right) = u^\beta w_j$$

$$Q^a(z) = \sum_{k=1}^{\infty} \sum_{i_1,\dots,i_k} {}^{(k)}m^a_{i_1,\dots,i_k} z^{i_1} \dots z^{i_k}$$

$$\sum_{\substack{k,l=0 \\ k+l=n+1}}^{n+1} \sum_{p=1}^{k} \sum_{\substack{perm \\ i_1,\dots,i_p,\dots,i_k \\ j_1,\dots,j_l}} \pm {}^{(k)}m^a_{i_1,\dots,i_p,\dots,i_k} \, {}^{(l)}m^{i_p}_{j_1,\dots,j_l} = 0$$

$$d^a_b = {}^{(1)}m^{a'}_{b'}$$

$$f^a_{bc} = \pm {}^{(2)}m^{a'}_{b'c'}$$

$$\pm = (-1)^{(\epsilon(a')+1)\epsilon(b')}$$

$$f^a_{bc} = (-1)^{\epsilon(b)\epsilon(c)+1} f^a_{cb}$$

$$d^m_b d^c_m = 0$$
$$f^r_{mb} d^m_c + (-1)^{\epsilon(b)\epsilon(c)} f^r_{mc} d^m_b + d^r_m f^m_{cb} = 0$$

$$f^r_{mb} f^m_{cd} + (-1)^{(\epsilon(b)+\epsilon(d))\epsilon(c)} f^r_{mc} f^m_{db} + (-1)^{(\epsilon(c)+\epsilon(d))\epsilon(b)} f^r_{md} f^m_{bc} = 0$$

$$\sum_{k=1}^{\infty} \frac{1}{k!} m_k(a, \dots, a) = 0$$

$$S(a)=\sum_{k=1}^{\infty}\frac{1}{k!}\mu_k(a,\dots,a)$$

$$Qx^i=\sum m^i_{i_1,\dots,i_k}x^{i_1}\dots x^{i_n}$$

$$m_k\left(e_{i_1},\dots,e_{i_k}\right)=\pm m^a_{i_1,\dots,i_k}e_a$$

$$\sum_{i+j=n+1}\sum_{0\le l\le i}\epsilon(l,j)m_i\left(a_0,\dots,a_{l-1},m_j\left(a_l,\dots,a_{l+j-1}\right),a_{l+j},\dots,a_n\right)=0$$

$$\epsilon(l,j)=(-1)^{j\sum_{0\le s\le l-1}\deg(a_s)+l(j-1)+j(i-1)}$$

$$T(V)=\bigoplus_{n\ge 0}V^{\otimes n}$$

$$m_k\colon A^{\otimes k}\to A$$

$$m_n^B\colon B^{\otimes n}\to \Pi^n B$$

$$m_1^B:=d^B=p\circ m_1\circ i$$
$$m_2^B=p\circ m_2\circ(i\otimes i);$$
$$m_n^B=\sum_T\pm m_{n,T},n\ge 3$$

$$m_k\left(\left(e_{i_k}\right)^{\beta_1}_{\alpha_1},\dots,\left(e_{i_k}\right)^{\beta_k}_{\alpha_k}\right)=m^a_{i_1,\dots,i_1}\delta^{\alpha_2}_{\beta_1}\delta^{\alpha_3}_{\beta_2}\dots\delta^{\alpha_n}_{\beta_{n-1}}\delta^{\alpha_1}_{\beta}\delta^{\alpha}_{\beta_n}(e_a)^{\beta}_{\alpha}$$

$$S=\frac{LT}{2}\int d\tau\int_0^{2\pi}d\sigma d\rho\left[\left(D_\tau X^i\right)^2-\frac{1}{2L^2}\left\{X^i,X^j\right\}^2\right]$$
$$D_\tau X^i=\partial_\tau X^i-\frac{1}{L}\left\{A,X^i\right\}$$
$$\{A,B\}\equiv\partial_\sigma A\partial_\rho B-\partial_\rho A\partial_\sigma B$$

$$X^i(\sigma,\rho)\ =\sum_{k_1,k_2=-\infty}^{\infty}X^i_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$
$$A(\sigma,\rho)\ =\sum_{k_1,k_2=-\infty}^{\infty}A_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$

$$X^9(\sigma,\rho)\ =w_1L_1\rho+\sum_{k_1,k_2=-\infty}^{\infty}Y^1_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}\equiv w_1L_1\rho+Y^1(\sigma,\rho)$$
$$X^k(\sigma,\rho)\ =\sum_{k_1,k_2=-\infty}^{\infty}X^k_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$
$$A(\sigma,\rho)\ =\sum_{k_1,k_2=-\infty}^{\infty}A_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$

$$X^9(\sigma,\rho)\ =w_1L_1\rho+\sum_{k_1,k_2=-\infty}^{\infty}Y^1_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}=w_1L_1\rho+Y^1(\sigma,\rho)$$
$$X^8(\sigma,\rho)\ =w_2L_2\sigma+\sum_{k_1,k_2=-\infty}^{\infty}Y^2_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}\equiv w_2L_2\sigma+Y^2(\sigma,\rho)$$
$$X^m(\sigma,\rho)\ =\sum_{k_1,k_2=-\infty}^{\infty}X^m_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$
$$A(\sigma,\rho)\ =\sum_{k_1,k_2=-\infty}^{\infty}A_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$

$$f * g = f\exp\left(i\frac{1}{2}\Theta\epsilon^{\alpha\beta}\overleftarrow{\partial_\alpha}\vec{\partial}_\beta\right)g.(\alpha,\beta=\sigma,\rho)$$

$$\left[e^{ik_1\sigma+ik_2\rho},e^{ik_1'\sigma+ik_2'\rho}\right]_* = -2i\sin\left(\frac{1}{2}\Theta k\times k'\right)e^{i(k_1+k_1')\sigma+i(k_2+k_2')\rho}.$$

$$\{f,g\} = -i\lim_{\Theta\to 0}\Theta^{-1}[f,g]_*$$

$$\begin{aligned} e^{i(k_1+pN)\sigma+ik_2\rho} &\approx (-1)^{pk_2}e^{ik_1\sigma+ik_2\rho}, \\ e^{ik_1\sigma+i(k_2+rN)\rho} &\approx (-1)^{rk_1}e^{ik_1\sigma+ik_2\rho}. \end{aligned}$$

$$\begin{aligned} X^i(\sigma,\rho) &= \sum_{k_1,k_2=-M}^{M} X^i_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho} \\ A(\sigma,\rho) &= \sum_{k_1,k_2=-M}^{M} A_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho} \end{aligned}$$

$$e^{ik_1\sigma+ik_2\rho}\to\lambda^{-k_1k_2/2}V^{k_2}U^{k_1},$$

$$U = \begin{pmatrix} 1 & & & & 0 \\ & \lambda & & & 0 \\ & & \lambda^2 & & \\ & & & \ddots & \\ & 0 & & & \lambda^{N-1} \end{pmatrix},$$

$$V = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ \vdots & & \ddots & \ddots & \\ 0 & & & 0 & 1 \\ 1 & 0 & \cdots & & 0 \end{pmatrix}.$$

$$\begin{aligned} U^N &= V^N = 1 \\ VU &= \lambda UV \end{aligned}$$

$$S = \frac{1}{\sqrt{N}}\begin{pmatrix} 1 & 1 & 1 & & 1 \\ 1 & \lambda & \lambda^2 & \cdots & \lambda^{(N-1)} \\ 1 & \lambda^2 & \lambda^4 & \cdots & \lambda^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \lambda^{N-1} & \lambda^{2(N-1)} & \cdots & \lambda^{(N-1)^2} \end{pmatrix}$$

$$\begin{aligned} X^i(\sigma,\rho) &\to X^i = \sum_{k_1,k_2=-M}^{M} X^i_{(k_1,k_2)}\lambda^{-k_1k_2/2}V^{k_2}U^{k_1} \\ A(\sigma,\rho) &\to A = \sum_{k_1,k_2=-M}^{M} A_{(k_1,k_2)}\lambda^{-k_1k_2/2}V^{k_2}U^{k_1} \end{aligned}$$

$$\begin{aligned} \{\cdot,\cdot\} &\to -i\frac{N}{2\pi}[\cdot,\cdot] \\ \int_0^{2\pi} d\sigma d\rho &\to \frac{(2\pi)^2}{N}\mathrm{Tr} \end{aligned}$$

$$S_{0+1} = (2\pi)^2 LT\int d\tau\mathrm{Tr}\left[\left(D_\tau X^i\right)^2+\frac{1}{2(2\pi L)^2}\left[X^i,X^j\right]^2\right]$$

$$D_\tau X^i = \partial_\tau X^i + i\frac{1}{2\pi L}\left[A,X^i\right]$$

$$\begin{aligned} \left[e^{ik_1\sigma+ik_2\rho},e^{ik_1'\sigma+ik_2'\rho}\right]_* &= -2i\sin\left(\frac{\pi}{N}k\times k'\right)e^{i(k_1+k_1')\sigma+i(k_2+k_2')\rho} \\ \left[\rho,e^{ik_1\sigma+ik_2\rho}\right]_* &= \frac{2\pi k_1}{N}e^{ik_1\sigma+ik_2\rho} \end{aligned}$$

$$X^9(\sigma,\rho) = w_1 L_1 \rho + \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-M}^{M} Y^1_{(k_1,k_2)} e^{ik_1\sigma + ik_2\rho}$$
$$= w_1 L_1 \rho + \sum_{p=-\infty}^{\infty} \sum_{q=-M}^{M} \sum_{k=-M}^{M} Y^1_{(pN+q,k)} e^{i(pN+q)\sigma + ik\rho},$$

$$X^k(\sigma,\rho) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-M}^{M} X^k_{(k_1,k_2)} e^{ik_1\sigma + ik_2\rho}$$
$$= \sum_{p=-\infty}^{\infty} \sum_{q=-M}^{M} \sum_{k=-M}^{M} X^k_{(pN+q,k)} e^{i(pN+q)\sigma + ik\rho}$$
$$A(\sigma,\rho) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-M}^{M} A_{(k_1,k_2)} e^{ik_1\sigma + ik_2\rho}$$
$$= \sum_{p=-\infty}^{\infty} \sum_{q=-M}^{M} \sum_{k=-M}^{M} A_{(pN+q,k)} e^{i(pN+q)\sigma + ik\rho}$$
$$e^{i(pN+q)\sigma + ik\rho} \rightarrow e^{i(pN+q)\theta_1/N} \lambda^{-kq/2} V^k U^q,$$
$$\rho \rightarrow -2\pi i \partial_{\theta_1} I,$$

$$X^9(\sigma,\rho) \rightarrow -2\pi i w_1 L_1 \partial_{\theta_1} I + Y^1(\theta_1)$$
$$= -2\pi i w_1 L_1 \partial_{\theta_1} I + \sum_{p=-\infty}^{\infty} \sum_{q=-M}^{M} \sum_{k=-M}^{M} Y^1_{(pN+q,k)} e^{i(pN+q)\theta_1/N} \lambda^{-kq/2} V^k U^q,$$
$$X^k(\sigma,\rho) \rightarrow X^k(\theta_1) = \sum_{p=-\infty}^{\infty} \sum_{q=-M}^{M} \sum_{k=-M}^{M} X^k_{(pN+q,k)} e^{i(pN+q)\theta_1/N} \lambda^{-kq/2} V^k U^q,$$
$$A(\sigma,\rho) \rightarrow A(\theta_1) = \sum_{p=-\infty}^{\infty} \sum_{q=-M}^{M} \sum_{k=-M}^{M} A_{(pN+q,k)} e^{i(pN+q)\theta_1/N} \lambda^{-kq/2} V^k U^q.$$

$$Y^1(\theta_1 + 2\pi) = V Y^1(\theta_1) V^\dagger,$$
$$X^k(\theta_1 + 2\pi) = V X^k(\theta_1) V^\dagger,$$
$$A(\theta_1 + 2\pi) = V A(\theta_1) V^\dagger,$$

$$\{\cdot,\cdot\} \rightarrow -i\frac{N}{2\pi}[\cdot,\cdot]$$
$$\int_0^{2\pi} d\sigma d\rho \rightarrow \frac{2\pi}{N}\int_0^{2\pi} d\theta_1 \mathrm{Tr}$$

$$S_{1+1} = \frac{2\pi LT}{2}\int d\tau \int_0^{2\pi} d\theta_1 \mathrm{Tr}\Big[\left(F_{\tau\theta_1}\right)^2 + (D_\tau X^k)^2$$
$$-\left(D_{\theta_1} X^k\right)^2 + \frac{1}{2(2\pi L)^2}[X^k, X^l]^2\Big]$$
$$F_{\tau\theta_1} = \partial_\tau Y^1 - \frac{L_1}{L}\partial_{\theta_1} A + i\frac{1}{2\pi L}[A, Y^1]$$
$$D_\tau X^k = \partial_\tau X^k + i\frac{1}{2\pi L}[A, X^k]$$
$$D_{\theta_1} X^k = \frac{L_1}{L}\partial_{\theta_1} X^k + i\frac{1}{2\pi L}[Y^1, X^k]$$

$$S_{YM} = -\frac{1}{4g_{YM}^2}\int d^D x \mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$$

$$Y^1(\theta_1) \to \alpha A_1(x^1),$$
$$X^k(\theta_1) \to \alpha \phi^k(x^1),$$
$$A(\theta_1) \to \alpha A_0(x^1),$$
$$\theta_1 \to \frac{x^1}{\Sigma_1},$$
$$\tau \to \frac{x^0}{\Sigma},$$

$$S_{1+1} = \frac{2\pi LT}{2}\frac{1}{\Sigma_1\Sigma}\int dx^0 \int_0^{2\pi\Sigma_1} dx^1 \mathrm{Tr}\left[\left(F_{\tau\theta_1}\right)^2 + (D_\tau X^k)^2\right.$$

$$\left. -\left(D_{\theta_1}X^k\right)^2 + \frac{\alpha^4}{2(2\pi L)^2}[\phi^k,\phi^l]^2\right],$$
$$F_{\tau\theta_1} = \Sigma\alpha\partial_0 A_1 - \frac{L_1}{L}\Sigma_1\alpha\partial_1 A_0 + i\frac{\alpha^2}{2\pi L}[A_0, A_1],$$
$$D_\tau X^k = \Sigma\alpha\partial_0\phi^k + i\frac{\alpha^2}{2\pi L}[A_0,\phi^k],$$
$$D_{\theta_1}X^k = \frac{L_1}{L}\Sigma_1\alpha\partial_1\phi^k + i\frac{\alpha^2}{2\pi L}[A_1,\phi^k].$$
$$\Sigma = \frac{\alpha}{2\pi L}$$
$$\Sigma_1 = \frac{\alpha}{2\pi L_1}$$

$$S_{1+1} = \frac{1}{2g_{YM}^2}\int dx^0 \int_0^{2\pi\Sigma_1} dx^1 \mathrm{Tr}\left[(F_{01})^2 + (D_0\phi^k)^2 - (D_1\phi^k)^2 + \frac{1}{2}[\phi^k,\phi^l]^2\right]$$
$$F_{01} = \partial_0 A_1 - \partial_1 A_0 + i[A_0, A_1]$$
$$D_0\phi^k = \partial_0\phi^k + i[A_0,\phi^k]$$
$$D_1\phi^k = \partial_1\phi^k + i[A_1,\phi^k]$$

$$A_0(x^1 + 2\pi\Sigma_1) = VA_0(x^1)V^\dagger,$$
$$A_1(x^1 + 2\pi\Sigma_1) = VA_1(x^1)V^\dagger,$$
$$\phi^k(x^1 + 2\pi\Sigma_1) = V\phi^k(x^1)V^\dagger,$$

$$g_{YM}^2 = (2\pi)^{-3}\Sigma_1^{-2}L_1^{-3}T^{-1}$$

$$\tilde{g}_{YM}^2 \equiv g_{YM}^2(2\pi\Sigma_1)^2 = (2\pi)^{-1}L_1^{-3}T^{-1} = 2\pi\frac{l_{11}^3}{L_1^3}$$

$$\frac{1}{2\pi\alpha'} = 2\pi L_1 T$$
$$g_s = \frac{L_1}{\sqrt{\alpha'}}$$

$$\tilde{g}_{YM}^2 = \frac{2\pi}{g_s^2}$$

$$\left[e^{ik_1\sigma+ik_2\rho}, e^{ik_1'\sigma+ik_2'\rho}\right]_* = -2i\sin\left(\frac{\pi}{N}k\times k'\right)e^{i(k_1+k_1')\sigma+i(k_2+k_2')\rho}$$
$$\left[\sigma, e^{ik_1\sigma+ik_2\rho}\right]_* = -\frac{2\pi k_2}{N}e^{ik_1\sigma+ik_2\rho}$$
$$\left[\rho, e^{ik_1\sigma+ik_2\rho}\right]_* = \frac{2\pi k_1}{N}e^{ik_1\sigma+ik_2\rho}$$
$$[\sigma,\rho]_* = i\frac{2\pi}{N}$$

$$X^9(\sigma,\rho)= w_1L_1\rho + \sum_{k_1,k_2=-\infty}^{\infty} Y^1_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$

$$= w_1L_1\rho + \sum_{p,r=-\infty}^{\infty}\sum_{q,s=-M}^{M} Y^1_{(pN+q,rN+s)}e^{i(pN+q)\sigma+i(rN+s)\rho}$$

$$X^8(\sigma,\rho) = w_2L_2\sigma + \sum_{k_1,k_2=-\infty}^{\infty} Y^2_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$

$$= w_2L_2\sigma + \sum_{p,r=-\infty}^{\infty}\sum_{q,s=-M}^{M} Y^2_{(pN+q,rN+s)}e^{i(pN+q)\sigma+i(rN+s)\rho}$$

$$X^m(\sigma,\rho) = \sum_{k_1,k_2=-\infty}^{\infty} X^m_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$

$$= \sum_{p,r=-\infty}^{\infty}\sum_{q,s=-M}^{M} X^m_{(pN+q,rN+s)}e^{i(pN+q)\sigma+i(rN+s)\rho}$$

$$A(\sigma,\rho) = \sum_{k_1,k_2=-\infty}^{\infty} A_{(k_1,k_2)}e^{ik_1\sigma+ik_2\rho}$$

$$= \sum_{p,r=-\infty}^{\infty}\sum_{q,s=-M}^{M} A_{(pN+q,rN+s)}e^{i(pN+q)\sigma+i(rN+s)\rho}$$

$$e^{i(pN+q)\sigma+i(rN+s)\rho} \rightarrow e^{i(pN+q)\theta_1/N}e^{-i(rN+s)\theta_2/N}\lambda^{-sq/2}V^sU^q$$
$$\rho \rightarrow -2\pi i\partial_{\theta_1}I$$
$$\sigma \rightarrow -2\pi i\partial_{\theta_2}I + \frac{\theta_1}{N}I$$

$$X^9(\sigma,\rho) \rightarrow -2\pi i w_1L_1\partial_{\theta_1}I + Y^1(\theta_1,\theta_2)$$
$$= -2\pi i w_1L_1\partial_{\theta_1}I$$
$$+ \sum_{p,r=-\infty}^{\infty}\sum_{q,s=-M}^{M} Y^1_{(pN+q,rN+s)}e^{i(pN+q)\theta_1/N}e^{-i(rN+s)\theta_2/N}\lambda^{-sq/2}V^sU^q,$$

$$X^8(\sigma,\rho) \rightarrow -2\pi i w_2L_2\partial_{\theta_2}I + \frac{w_2L_2}{N}\theta_1I + Y^2(\theta_1,\theta_2)$$
$$= -2\pi i w_2L_2\partial_{\theta_2}I + \frac{w_2L_2}{N}\theta_1I$$

$$+ \sum_{p,r=-\infty}^{\infty}\sum_{q,s=-M}^{M} Y^2_{(pN+q,rN+s)}e^{i(pN+q)\theta_1/N}e^{-i(rN+s)\theta_2/N}\lambda^{-sq/2}V^sU^q,$$

$$X^m(\sigma,\rho) \rightarrow X^m(\theta_1,\theta_2)$$
$$= \sum_{p,r=-\infty}^{\infty}\sum_{q,s=-M}^{M} X^m_{(pN+q,rN+s)}e^{i(pN+q)\theta_1/N}e^{-i(rN+s)\theta_2/N}\lambda^{-sq/2}V^sU^q,$$

$$A(\sigma,\rho) \rightarrow A(\theta_1,\theta_2)$$
$$= \sum_{p,r=-\infty}^{\infty}\sum_{q,s=-M}^{M} A_{(pN+q,rN+s)}e^{i(pN+q)\theta_1/N}e^{-i(rN+s)\theta_2/N}\lambda^{-sq/2}V^sU^q.$$

$$\begin{aligned}
Y^1(\theta_1+2\pi,\theta_2) &= VY^1(\theta_1,\theta_2)V^\dagger\\
Y^2(\theta_1+2\pi,\theta_2) &= VY^2(\theta_1,\theta_2)V^\dagger\\
X^m(\theta_1+2\pi,\theta_2) &= VX^m(\theta_1,\theta_2)V^\dagger\\
A(\theta_1+2\pi,\theta_2) &= VA(\theta_1,\theta_2)V^\dagger\\
Y^1(\theta_1,\theta_2+2\pi) &= UY^1(\theta_1,\theta_2)U^\dagger\\
Y^2(\theta_1,\theta_2+2\pi) &= UY^2(\theta_1,\theta_2)U^\dagger\\
X^m(\theta_1,\theta_2+2\pi) &= UX^m(\theta_1,\theta_2)U^\dagger\\
A(\theta_1,\theta_2+2\pi) &= UA(\theta_1,\theta_2)U^\dagger
\end{aligned}$$

$$VUV^\dagger = \lambda UUVU^\dagger = \lambda^{-1}V$$

$$\{\cdot,\cdot\} \to -i\frac{N}{2\pi}[\cdot,\cdot]$$

$$\int_0^{2\pi} d\sigma d\rho \to \frac{1}{N}\int_0^{2\pi} d\theta_1 d\theta_2 \mathrm{Tr}$$

$$S_{2+1} = \frac{LT}{2}\int d\tau \int_0^{2\pi} d\theta_1 d\theta_2 \mathrm{Tr}[\left(F_{\tau\theta_1}\right)^2 + \left(F_{\tau\theta_2}\right)^2 - \left(F_{\theta_1\theta_2}\right)^2 + (D_\tau X^m)^2 - \left(D_{\theta_1}X^m\right)^2 - \left(D_{\theta_2}X^m\right)^2 + \frac{1}{2(2\pi L)^2}[X^m, X^n]^2\Big]$$

$$F_{\tau\theta_1} = \partial_\tau Y^1 - \frac{L_1}{L}\partial_{\theta_1}A + i\frac{1}{2\pi L}[A, Y^1]$$

$$F_{\tau\theta_2} = \partial_\tau Y^2 - \frac{L_2}{L}\partial_{\theta_2}A + i\frac{1}{2\pi L}[A, Y^2]$$

$$F_{\theta_1\theta_2} = \frac{1}{NL}L_1 L_2 I + \frac{L_1}{L}\partial_{\theta_1}Y^2 - \frac{L_2}{L}\partial_{\theta_2}Y^1 + i\frac{1}{2\pi L}[Y^1, Y^2]$$

$$D_\tau X^m = \partial_\tau X^k + i\frac{1}{2\pi L}[A, X^m]$$

$$D_{\theta_1}X^m = \frac{L_1}{L}\partial_{\theta_1}X^m + i\frac{1}{2\pi L}[Y^1, X^m]$$

$$D_{\theta_2}X^m = \frac{L_2}{L}\partial_{\theta_2}X^m + i\frac{1}{2\pi L}[Y^2, X^m]$$

$$Y^1(\theta_1,\theta_2) \to \alpha A_1(x^1,x^2)$$

$$Y^2(\theta_1,\theta_2) \to \alpha A_2(x^1,x^2)$$

$$X^m(\theta_1,\theta_2) \to \alpha\phi^m(x^1,x^2)$$

$$A(\theta_1,\theta_2) \to \alpha A_0(x^1,x^2)$$

$$\theta_1 \to \frac{x^1}{\Sigma_1}$$

$$\theta_2 \to \frac{x^2}{\Sigma_2}$$

$$\tau \to \frac{x^0}{\Sigma}$$

$$S_{1+1} = \frac{LT}{2}\frac{1}{\Sigma_1\Sigma_2\Sigma}\int dx^0 \int_0^{2\pi\Sigma_1} dx^1 \int_0^{2\pi\Sigma_2} dx^2 \mathrm{Tr}\left[\left(F_{\tau\theta_1}\right)^2 + \left(F_{\tau\theta_2}\right)^2 - \left(F_{\theta_1\theta_2}\right)^2 + (D_\tau X^m)^2 - \left(D_{\theta_1}X^m\right)^2 - \left(D_{\theta_2}X^m\right)^2 + \frac{\alpha^4}{2(2\pi L)^2}[\phi^m,\phi^n]^2\right]$$

$$F_{\tau\theta_1} = \Sigma\alpha\partial_0 A_1 - \frac{L_1}{L}\Sigma_1\alpha\partial_1 A_0 + i\frac{\alpha^2}{2\pi L}[A_0, A_1]$$

$$F_{\tau\theta_2} = \Sigma\alpha\partial_0 A_2 - \frac{L_2}{L}\Sigma_2\alpha\partial_2 A_0 + i\frac{\alpha^2}{2\pi L}[A_0, A_2]$$

$$F_{\theta_1\theta_2} = \frac{1}{NL}L_1 L_2 I + \frac{L_1}{L}\Sigma_1\alpha\partial_1 A_2 - \frac{L_2}{L}\Sigma_2\alpha\partial_2 A_1 + i\frac{\alpha^2}{2\pi L}[A_1, A_2]$$

$$D_\tau X^m = \Sigma\alpha\partial_0\phi^m + i\frac{\alpha^2}{2\pi L}[A_0,\phi^m]$$

$$D_{\theta_1}X^m = \frac{L_1}{L}\Sigma_1\alpha\partial_1\phi^m + i\frac{\alpha^2}{2\pi L}[A_1,\phi^m]$$

$$D_{\theta_2}X^m = \frac{L_2}{L}\Sigma_2\alpha\partial_2\phi^m + i\frac{\alpha^2}{2\pi L}[A_2,\phi^m]$$

$$\Sigma = \frac{\alpha}{2\pi L}$$

$$\Sigma_1 = \frac{\alpha}{2\pi L_1}$$

$$\Sigma_2 = \frac{\alpha}{2\pi L_2}$$

$$S_{1+1} = \frac{1}{2g_{YM}^2}\int dx^0 \int_0^{2\pi\Sigma_1} dx^1 \int_0^{2\pi\Sigma_2} dx^2 \mathrm{Tr}[(F_{01})^2 + (F_{02})^2 - (F_{12})^2 + (D_0\phi^m)^2$$
$$-(D_1\phi^m)^2 - (D_2\phi^m)^2 + \frac{1}{2}[\phi^m, \phi^n]^2\Big]$$
$$F_{01} = \partial_0 A_1 - \partial_1 A_0 + i[A_0, A_1]$$
$$F_{02} = \partial_0 A_2 - \partial_2 A_0 + i[A_0, A_2]$$
$$F_{12} = \frac{1}{2\pi N \Sigma_1 \Sigma_2} I + \partial_1 A_2 - \partial_2 A_1 + i[A_1, A_2]$$
$$D_0\phi^m = \partial_0\phi^m + i[A_0, \phi^m]$$
$$D_1\phi^m = \partial_1\phi^m + i[A_1, \phi^m]$$
$$D_2\phi^m = \partial_2\phi^m + i[A_2, \phi^m]$$

$$\begin{aligned} A_0(x^1 + 2\pi\Sigma_1, x^2) &= VA_0(x^1, x^2)V^\dagger \\ A_1(x^1 + 2\pi\Sigma_1, x^2) &= VA_1(x^1, x^2)V^\dagger \\ A_2(x^1 + 2\pi\Sigma_1, x^2) &= VA_2(x^1, x^2)V^\dagger \\ \phi^m(x^1 + 2\pi\Sigma_1, x^2) &= V\phi^m(x^1, x^2)V^\dagger \\ A_0(x^1, x^2 + 2\pi\Sigma_2) &= UA_0(x^1, x^2)U^\dagger \\ A_1(x^1, x^2 + 2\pi\Sigma_2) &= UA_1(x^1, x^2)U^\dagger \\ A_2(x^1, x^2 + 2\pi\Sigma_2) &= UA_2(x^1, x^2)U^\dagger \\ \phi^m(x^1, x^2 + 2\pi\Sigma_2) &= U\phi^m(x^1, x^2)U^\dagger \end{aligned}$$

$$g_{YM}^2 = (2\pi)^{-2}(\Sigma_1\Sigma_2)^{-1/2}(L_1L_2)^{-3/2}T^{-1}$$

$$\tilde{g}_{YM}^2 \equiv g_{YM}^2(2\pi\Sigma_1 2\pi\Sigma_2)^{1/2} = (2\pi)^{-1}(L_1L_2)^{-3/2}T^{-1} = 2\pi\frac{l_{11}^3}{(L_1L_2)^{3/2}}$$

$$\mathrm{Tr}(\phi^I Z^{L-I}), \mathrm{Tr}(\phi^{I-1} Z \phi Z^{L-I-1}), \mathrm{Tr}(\phi^{I-2} Z \phi^2 Z^{L-I-1}), \ldots$$

$$H_{\mathrm{su}(2)} = I + \sum_n \left(\frac{\lambda}{8\pi^2}\right)^n H_{\mathrm{su}(2)}^{(2n)}$$

$$\{n_1, n_2, \ldots\} = \sum_{k=1}^{L} P_{k+n_1, k+n_1+1} P_{k+n_2, k+n_2+1} \cdots$$

$$\begin{aligned} H_{\mathrm{su}(2)}^{(2)} &= 2(\{\} - \{0\}) \\ H_{\mathrm{su}(2)}^{(4)} &= 2(-4\{\} + 6\{0\} - (\{0,1\} + \{1,0\})) \\ H_{\mathrm{su}(2)}^{(6)} &= 4(15\{\} - 26\{0\} + 6(\{0,1\} + \{1,0\}) + \{0,2\} - (\{0,1,2\} + \{2,1,0\})) \end{aligned}$$

$$\mathrm{Tr}(\phi^2 Z^6)\ \mathrm{Tr}(\phi Z \phi Z^5)\ \mathrm{Tr}(\phi Z^2 \phi Z^4)\ \mathrm{Tr}(\phi Z^3 \phi Z^3).$$

$$H_{\mathrm{su}(2)}^{(2)} = \begin{pmatrix} 2 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2\sqrt{2} \\ 0 & 0 & -2\sqrt{2} & 4 \end{pmatrix}$$

$$E_{\mathrm{su}(2)}^{(2)} = 8\sin^2\left(\frac{\pi n}{L-1}\right)\ n = 0, \ldots, n_{\max} = \begin{cases} (L-2)/2, & L \text{ even} \\ (L-3)/2, & L \text{ odd} \end{cases}.$$

$$E(\{n_i\}) = I + \frac{\lambda}{2\pi^2}\sum_{i=1}^{I} \sin^2\frac{n_i\pi}{L} + \sum_{N=2}^{2I} \frac{\lambda}{L^{2N-1}} V_{N-\mathrm{body}}(n_1, \ldots, n_I) + \cdots$$

$$V_N = \sum_{n_i, m_i} \delta_{n_1+\cdots+n_N, m_1+\cdots+m_N} f_N(\{n_i\}, \{m_i\}) \prod_{i=1}^{N} b_{n_i}^\dagger \prod_{i=1}^{N} b_{m_i},$$

$$S^\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)\ S^z = \frac{1}{2}\sigma_z$$

$$P_{i,i+1} = S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2S_i^z S_{i+1}^z + \frac{1}{2}$$

$$H_{\mathfrak{su}(2)}^{(2)} = -\sum_{j=1}^{L} \left(S_j^+ S_{j+1}^- + S_j^- S_{j+1}^+\right) - 2\sum_{j=1}^{L} S_j^z S_{j+1}^z + \frac{1}{2}.$$

$$S_j^+ = b_j^\dagger K(j) = K(j) b_j^\dagger$$
$$S_j^- = K(j) b_j = b_j K(j)$$
$$S_j^z = b_j^\dagger b_j - 1/2,$$

$$K(j) = \exp\left(i\pi \sum_{k=1}^{j-1} b_k^\dagger b_k\right)$$

$$[K(j), \mathbf{S}_k] = 0$$

$$b_j^\dagger = S_j^+ K(j)\ b_j = S_j^- K(j),$$

$$\{b_j, b_k^\dagger\} = \delta_{jk}\ \{b_j^\dagger, b_k^\dagger\} = \{b_j, b_k\} = 0.$$

$$b_{L+1} = (-1)^{I+1} b_1\ I \equiv \sum_{j=1}^{L} b_j^\dagger b_j$$

$$H_{\mathfrak{su}(2)}^{(2)} = \sum_{j=1}^{L} \left(b_j^\dagger b_j + b_{j+1}^\dagger b_{j+1} - b_{j+1}^\dagger b_j - b_j^\dagger b_{j+1} + 2 b_j^\dagger b_{j+1}^\dagger b_j b_{j+1}\right).$$

$$b_j = \frac{1}{\sqrt{L}} \sum_{p=0}^{L-1} e^{-\frac{2\pi i j}{L} p} \tilde{b}_p$$

$$H_{\mathfrak{su}(2)}^{(2)} = 4\sum_{p=0}^{L-1} \sin^2\left(\frac{\pi p}{L}\right) \tilde{b}_p^\dagger \tilde{b}_p + \frac{2}{L} \sum_{p,q,r,s=0}^{L-1} e^{\frac{2\pi i(q-s)}{L}} \tilde{b}_p^\dagger \tilde{b}_q^\dagger \tilde{b}_r \tilde{b}_s \delta_{p+q,r+s}.$$

$$\tilde{b}_{k_1}^\dagger \tilde{b}_{k_2}^\dagger \tilde{b}_{k_3}^\dagger |L\rangle\ k_1 + k_2 + k_3 = 0 \bmod L$$

$$\tilde{b}_0^\dagger \tilde{b}_1^\dagger \tilde{b}_5^\dagger |L\rangle\ \tilde{b}_0^\dagger \tilde{b}_2^\dagger \tilde{b}_4^\dagger |L\rangle\ \tilde{b}_1^\dagger \tilde{b}_2^\dagger \tilde{b}_3^\dagger |L\rangle\ \tilde{b}_3^\dagger \tilde{b}_4^\dagger \tilde{b}_5^\dagger |L\rangle,$$

$$H_{\mathfrak{su}(2)}^{(2)} = \begin{pmatrix} \frac{1}{3} & -1 & \frac{1}{3} & \frac{1}{3} \\ -1 & 3 & -1 & -1 \\ \frac{1}{3} & -1 & \frac{19}{3} & \frac{1}{3} \\ \frac{1}{3} & -1 & \frac{1}{3} & \frac{19}{3} \end{pmatrix}$$

$$E_L(\{k_i\}) = \frac{\lambda}{L^2} E^{(1,2)}(\{k_i\}) + \frac{\lambda}{L^3} E^{(1,3)}(\{k_i\}) + O(\lambda L^{-4}).$$

$$\begin{aligned} E_{\mathfrak{su}(2)}^{(1,2)} &= (k_1^2 + k_2^2 + k_3^2)/2 & \qquad k_1 + k_2 + k_3 = 0 \\ E_{\mathfrak{su}(2)}^{(1,3)} / E_{\mathfrak{su}(2)}^{(1,2)} &= 2 & \qquad (k_1 \neq k_2 \neq k_3) \\ E_{\mathfrak{su}(2)}^{(1,3)} / E_{\mathfrak{su}(2)}^{(1,2)} &= \frac{7}{3} & \qquad (k_1 = k_2,\ k_3 = -2k_1) \end{aligned}$$

$$\left(\frac{u_i+\frac{i}{2}V_{q_i}}{u_i-\frac{i}{2}V_{q_i}}\right)^L=\prod_{j\neq i}^{I}\left(\frac{u_i-u_j+\frac{i}{2}M_{q_i,q_j}}{u_i-u_j-\frac{i}{2}M_{q_i,q_j}}\right)$$

$$V_{q_i}=2\alpha^{(q_i)}\cdot\mu/\left(\alpha^{(q_i)}\right)^2$$

$$M_{q_i,q_j}=2\alpha^{(q_i)}\cdot\alpha^{(q_j)}/\left(\alpha^{(q_j)}\right)^2$$

$$1=\prod_{i}^{I}\left(\frac{u_i+\frac{i}{2}V_{q_i}}{u_i-\frac{i}{2}V_{q_i}}\right)$$

$$E=\sum_{j=1}^{I}\left(\frac{V_{q_j}}{u_j^2+V_{q_j}^2/4}\right)$$

$$u_i=\frac{1}{2\pi k_i}\left(L+A_i\sqrt{L}+B_i+\cdots\right)$$

$$\left(\frac{u_i+i/2}{u_i-i/2}\right)^L=\prod_{j\neq i}^{I}\left(\frac{u_i-u_j+i}{u_i-u_j-i}\right)$$

$$1=\prod_{i}^{I}\left(\frac{u_i+i/2}{u_i-i/2}\right)$$

$$\begin{aligned}u_1&=\frac{L-4}{2\pi k_1}+\frac{3k_1}{\pi(k_1-k_2)(2k_1+k_2)}+O(L^{-1})\\u_2&=\frac{(L-4)k_1^2+(L-4)k_1k_2-2(L-1)k_2^2}{2\pi k_2(k_1^2+k_1k_2-2k_2^2)}+O(L^{-1})\\u_3&=-\frac{(L-1)k_1^2-(8-5L)k_1k_2+2(L-1)k_2^2}{2\pi(k_1+k_2)(2k_1+k_2)(k_1+2k_2)}+O(L^{-1}).\end{aligned}$$

$$E_{\mathrm{su}(2)}^{(2)}(k_1,k_2)=\frac{8\pi^2}{L^3}(k_1^2+k_1k_2+k_2^2)(L+2)+O(L^{-4})\ (k_1\neq k_2\neq k_3).$$

$$\begin{aligned}u_1&=\frac{-7+3i\sqrt{L}+3L}{6\pi k_1}+O\left(L^{-1/2}\right)\\u_2&=-\frac{7+3i\sqrt{L}-3L}{6\pi k_1}+O\left(L^{-1/2}\right)\\u_3&=\frac{4-3L}{12\pi k_1}+O\left(L^{-1/2}\right).\end{aligned}$$

$$E_{\mathrm{su}(2)}^{(2)}(k_1)=\frac{8\pi^2}{L^3}k_1^2(3L+7)+O(L^{-4})\ (k_1=k_2,k_3=-2k_1)$$

$$\begin{aligned}H_{\mathrm{su}(2)}^{(4)}=&\sum_{j=1}^{L}\Big\{-\frac{1}{2}\left[b_{j+2}^\dagger b_j+b_j^\dagger b_{j+2}-4\left(b_{j+1}^\dagger b_j+b_j^\dagger b_{j+1}\right)\right]-3b_j^\dagger b_j-4b_j^\dagger b_{j+1}^\dagger b_j b_{j+1}\\&+b_{j+1}^\dagger b_{j+2}^\dagger b_j b_{j+1}+b_j^\dagger b_{j+1}^\dagger b_{j+1}b_{j+2}+b_j^\dagger b_{j+2}^\dagger b_j b_{j+2}\Big\}\end{aligned}$$

$$\begin{aligned}H_{\mathrm{su}(2)}^{(4)}=&-8\sum_{p=0}^{L-1}\sin^4\left(\frac{p\pi}{L}\right)\tilde{b}_p^\dagger\tilde{b}_p\\&+\frac{1}{L}\sum_{p,q,r,s=0}^{L-1}\left(e^{\frac{2\pi i(q+r)}{L}}+e^{\frac{-2\pi i(p+s)}{L}}+e^{\frac{4\pi i(q-s)}{L}}-4e^{\frac{2\pi i(q-s)}{L}}\right)\tilde{b}_p^\dagger\tilde{b}_q^\dagger\tilde{b}_r\tilde{b}_s\delta_{p+q,r+s}\end{aligned}$$

$$E_L^{(2)}(\{k_i\})=\frac{\lambda^2}{L^4}E^{(2,4)}(\{k_i\})+\frac{\lambda^2}{L^5}E^{(2,5)}(\{k_i\})+O(\lambda^2L^{-6}).$$

$$E_{\mathfrak{su}(2)}^{(2,4)} = -(k_1^2 + k_2^2 + k_3^2)^2/16 \quad k_1 + k_2 + k_3 = 0$$
$$E_{\mathfrak{su}(2)}^{(2,5)}/E_{\mathfrak{su}(2)}^{(2,3)} = 8 \quad (k_1 \neq k_2 \neq k_3)$$
$$E_{\mathfrak{su}(2)}^{(2,5)}/E_{\mathfrak{su}(2)}^{(2,3)} = \frac{76}{9} \quad (k_1 = k_2, k_3 = -2k_1)$$

$E_{\mathfrak{su}(2)}^{(2,4)}$	$E_{\mathfrak{su}(2)}^{(2,5)}$	$E_{\mathfrak{su}(2)}^{(2,5)}/E_{\mathfrak{su}(2)}^{(2,4)}$	(k_1, k_2, k_3)
$-0.25 - 4.6 \times 10^{-9}$	$-2 + 8.0 \times 10^{-7}$	$8 - 3.4 \times 10^{-6}$	$(1, 0, -1)$
$-2.25 - 1.4 \times 10^{-6}$	$-19 + 2.6 \times 10^{-4}$	$76/9 + 1.2 \times 10^{-4}$	$(1, 1, -2)$
$-2.25 - 1.4 \times 10^{-6}$	$-19 + 2.6 \times 10^{-4}$	$76/9 + 1.2 \times 10^{-4}$	$(-1, -1, 2)$
$-4 + 8.3 \times 10^{-7}$	$-32 - 1.1 \times 10^{-4}$	$8 + 3.0 \times 10^{-5}$	$(2, 0, -2)$
$-12.25 - 9.9 \times 10^{-6}$	$-98 + 2.3 \times 10^{-3}$	$8 - 2.0 \times 10^{-4}$	$(1, 2, -3)$
$-12.25 - 9.9 \times 10^{-6}$	$-98 + 2.3 \times 10^{-3}$	$8 - 2.0 \times 10^{-4}$	$(-1, -2, 3)$
$-20.25 + 3.2 \times 10^{-3}$	-161.4	7.97	$(3, 0, -3)$
$-36 - 2.8 \times 10^{-3}$	-304.6	8.46	$(2, 2, -4)$
$-36 - 2.8 \times 10^{-3}$	-304.6	8.46	$(-2, -2, 4)$
$-42.25 + 4.9 \times 10^{-3}$	-337.0	7.97	$(1, 3, -4)$
$-42.25 + 4.9 \times 10^{-3}$	-337.0	7.97	$(-1, -3, 4)$

$$\begin{aligned}
H_{\mathfrak{su}(2)}^{(6)} = &32 \sum_{p=0}^{L-1} \sin^6\left(\frac{p\pi}{L}\right) \tilde{b}_p^\dagger \tilde{b}_p + \frac{1}{2L} \sum_{p,q,r,s=0}^{L-1} \Big\{ -10 e^{\frac{2\pi i(q+r)}{L}} + e^{\frac{2\pi i(2q+r)}{L}} + e^{\frac{2\pi i(q+2r)}{L}} + e^{\frac{2\pi i(q-3s)}{L}} \\
&+ e^{\frac{2\pi i(2q-2r-3s)}{L}} + e^{\frac{2\pi i(3q-2r-3s)}{L}} + e^{\frac{2\pi i(q-r-3s)}{L}} + e^{\frac{2\pi i(2q-r-3s)}{L}} - e^{\frac{2\pi i(q-2s)}{L}} \\
&-10 e^{\frac{2\pi i(q-r-2s)}{L}} - e^{\frac{2\pi i(2q-r-2s)}{L}} - e^{\frac{2\pi i(3q-r-2s)}{L}} - e^{\frac{2\pi i(q+r-2s)}{L}} + 29 e^{\frac{2\pi i(q-s)}{L}} - 10 e^{\frac{4\pi i(q-s)}{L}} + e^{\frac{6\pi i(q-s)}{L}} \\
&- e^{\frac{2\pi i(2q-s)}{L}} + e^{\frac{2\pi i(3q-s)}{L}} - e^{\frac{2\pi i(q+r-s)}{L}} + e^{\frac{2\pi i(2q+r-s)}{L}} + e^{\frac{2\pi i(q+2r-s)}{L}} \Big\} \tilde{b}_p^\dagger \tilde{b}_q^\dagger \tilde{b}_r \tilde{b}_s \delta_{p+q,r+s} \\
&+ \frac{1}{L^2} \sum_{p,q,r,s,t,u=0}^{L-1} \Big\{ e^{\frac{2\pi i(q+3r-2t-3u)}{L}} + e^{\frac{2\pi i(q+2r-s-2t-3u)}{L}} \\
&+ e^{\frac{2\pi i(2q+3r-t-3u)}{L}} + e^{\frac{2\pi i(q+2r+s-u)}{L}} \Big\} \tilde{b}_p^\dagger \tilde{b}_q^\dagger \tilde{b}_r^\dagger \tilde{b}_s \tilde{b}_t \tilde{b}_u \delta_{p+q+r,s+t+u}
\end{aligned}$$

$E_{\mathfrak{su}(2)}^{(3,4)}$	$E_{\mathfrak{su}(2)}^{(3,7)}$	$E_{\mathfrak{su}(2)}^{(3,7)}/E_{\mathfrak{su}(2)}^{(3,6)}$	(k_1, k_2, k_3)
0.1250	2.0003	16.003	$(1, 0, -1)$
4.125	58.03	14.07	$(1, 1, -2)$
4.125	58.03	14.07	$(-1, -1, 2)$
7.999	128.2	16.03	$(2, 0, -2)$
49.62	713.3	14.37	$(1, 2, -3)$
49.62	713.3	14.37	$(-1, -2, 3)$
91.15	1,454	15.96	$(3, 0, -3)$
263.8	3,739	14.17	$(2, 2, -4)$
263.8	3,739	14.17	$(-2, -2, 4)$

$$e^{iLp_i} = \prod_{j \neq i}^{I} \frac{\varphi(p_i) - \varphi(p_j) + i}{\varphi(p_i) - \varphi(p_j) - i} \quad \sum_{i=1}^{I} p_i = 0.$$

$$\varphi(p_i) \equiv \frac{1}{2}\cot(p_i/2)\sqrt{1+\frac{\lambda}{\pi^2}\sin^2(p_i/2)}$$

$$E_{\mathfrak{su}(2)} = \sum_{i=1}^{I}\frac{8\pi^2}{\lambda}\left(\sqrt{1+\frac{\lambda}{\pi^2}\sin^2(p_i/2)}-1\right)$$

$$p_i = \frac{2\pi k_i}{L} + \sum_{n=1}\frac{p_i^{(n)}}{L^{\frac{n+2}{2}}},$$

$$E_{\mathfrak{su}(2)} = L - I + \sum_{i=1}^{I}\left(\sqrt{1+\lambda' k_i^2} - \frac{\lambda'}{L-I}\frac{I k_i^2}{\sqrt{1+\lambda' k_i^2}}\right) - \frac{\lambda'}{L-I}\sum_{\substack{i,j=1\\ i\neq j}}^{I}\frac{2k_i^2 k_j}{k_i^2-k_j^2}\left(k_j + k_i\sqrt{\frac{1+\lambda' k_j^2}{1+\lambda' k_i^2}}\right) + O(L^{-2}),$$

$$E_{\mathfrak{su}(2)}^{(1,2)} = k_1^2 + k_1k_2 + k_2^2 E_{\mathfrak{su}(2)}^{(1,3)} = 2(k_1^2 + k_1k_2 + k_2^2)$$

$$E_{\mathfrak{su}(2)}^{(2,4)} = -\frac{1}{4}(q^2+qr+r^2)^2 E_{\mathfrak{su}(2)}^{(2,5)} = -2(q^2+qr+r^2)^2.$$

$$E_{\mathfrak{su}(2)} = L - 3 + 2\sqrt{1+\lambda' n^2} + \sqrt{1+\lambda' 4n^2} - \frac{\lambda' n^2}{L-3}\left(\frac{1}{1+\lambda' n^2} + \frac{6}{\sqrt{1+\lambda' n^2}} + \frac{12}{\sqrt{1+\lambda' 4n^2}} - \frac{8}{\sqrt{1+\lambda' n^2}\sqrt{1+\lambda' 4n^2}}\right)$$

$$E_{\mathfrak{su}(2)}^{(1,2)} = 3n^2 \qquad E_{\mathfrak{su}(2)}^{(1,3)} = 7n^2$$

$$E_{\mathfrak{su}(2)}^{(2,4)} = -\frac{9}{4}n^4 \qquad E_{\mathfrak{su}(2)}^{(2,5)} = -19n^4$$

$$E_{\mathfrak{su}(2)}^{(3,6)} = \frac{1}{16}\left(2k_1^6 + 6k_1^5k_2 + 15k_1^4k_2^2 + 20k_1^3k_2^3 + 15k_1^2k_2^4 + 6k_1k_2^5 + 2k_2^6\right)$$

$$E_{\mathfrak{su}(2)}^{(3,7)} = \frac{1}{4}\left(8k_1^6 + 24k_1^5k_2 + 51k_1^4k_2^2 + 62k_1^3k_2^3 + 51k_1^2k_2^4 + 24k_1k_2^5 + 8k_2^6\right)$$

$$E_{\mathfrak{su}(2)}^{(3,6)} = \frac{33}{8}n^6 \; E_{\mathfrak{su}(2)}^{(3,7)} = 58n^6,$$

$E_{\mathfrak{su}(2)}^{(3,6)}$	$E_{\mathfrak{su}(2)}^{(3,7)}$	$E_{\mathfrak{su}(2)}^{(3,7)}/E_{\mathfrak{su}(2)}^{(3,6)}$	(k_1,k_2,k_3)
0.125	2	16	$(1,0,-1)$
4.125	58	14.06	$(1,1,-2)$
4.125	58	14.06	$(-1,-1,2)$
8	128	16	$(2,0,-2)$
49.625	713	14.37	$(1,2,-3)$
49.625	713	14.37	$(-1,-2,3)$
91.125	1,458	16	$(3,0,-3)$
264	3,712	14.06	$(2,2,-4)$
264	3,712	14.06	$(-2,-2,4)$

$$|L\rangle = \mathrm{Tr}(Z^L)\; b_j^{\dagger}|L\rangle = \mathrm{Tr}\left(Z_1 \cdots Z_{j-1}\psi Z_{j+1}\cdots Z_L\right).$$

$$\begin{Bmatrix} A_1 \ldots A_N \\ B_1 \ldots B_N \end{Bmatrix}$$

$$H^{(2)}_{\mathfrak{su}(|1)} = \begin{Bmatrix} Z\psi \\ Z\psi \end{Bmatrix} + \begin{Bmatrix} \psi Z \\ \psi Z \end{Bmatrix} - \begin{Bmatrix} Z\psi \\ \psi Z \end{Bmatrix} - \begin{Bmatrix} \psi Z \\ Z\psi \end{Bmatrix} + 2 \begin{Bmatrix} \psi\psi \\ \psi\psi \end{Bmatrix}.$$

$$H^{(2)}_{\mathfrak{su}(1|1)} = \sum_{j=1}^{L} \left(b_j^\dagger b_j + b_{j+1}^\dagger b_{j+1} - b_{j+1}^\dagger b_j - b_j^\dagger b_{j+1}\right).$$

$$\langle L| b_{i+1} b_i \left(H^{(2)}_{\mathfrak{su}(1|1)}\right) b_i^\dagger b_{i+1}^\dagger |L\rangle = 2$$

$$H^{(2)}_{\mathfrak{su}(1|1)} = 4 \sum_{p=0}^{L-1} \sin^2 \left(\frac{p\pi}{L}\right) \tilde{b}_p^\dagger \tilde{b}_p$$

$$H^{(4)}_{\mathfrak{su}(1|1)} = -8 \sum_{p=0}^{L-1} \sin^4 \left(\frac{p\pi}{L}\right) \tilde{b}_p^\dagger \tilde{b}_p + \frac{1}{4L} \sum_{p,q,r,s=0}^{L-1} \Big\{ e^{\frac{2\pi i(q-2r)}{L}} + e^{\frac{2\pi i(2q-r)}{L}} - 4e^{\frac{2\pi i(q-r)}{L}}$$
$$-2e^{\frac{2\pi i(q-2r-s)}{L}} - 2e^{\frac{2\pi i(q+s)}{L}} + e^{\frac{2\pi i(q-r+s)}{L}} + e^{\frac{2\pi i(2q-2r-s)}{L}} \Big\} \tilde{b}_p^\dagger \tilde{b}_q^\dagger \tilde{b}_r \tilde{b}_s \delta_{p+q,r+s}$$

$$H^{(6)}_{\mathfrak{su}(|1)} = 32 \sum_{p=0}^{L-1} \sin^6 \left(\frac{p\pi}{L}\right) \tilde{b}_p^\dagger \tilde{b}_p - \frac{1}{16} \sum_{p,q,r,s=0}^{L-1} e^{\frac{60\pi i(q-r)}{L}} \Big\{ 2e^{-\frac{2\pi i(27q-29r)}{L}} + 2e^{-\frac{2\pi i(28q-29r)}{L}}$$
$$-4e^{-\frac{2\pi i(27q-28r)}{L}} + 37e^{-\frac{2\pi i(29q-28r)}{L}} - 6e^{-\frac{2\pi i(29q-27r)}{L}} + 8e^{-\frac{56\pi i(q-r)}{L}} - 72e^{-\frac{58\pi i(q-r)}{L}}$$
$$-6e^{-\frac{2\pi i(29q-29r-2s)}{L}} - 40e^{-\frac{2\pi i(29q-30r-s)}{L}} + 37e^{-\frac{2\pi i(29q-29r-s)}{L}} - 8e^{-\frac{2\pi i(29q-28r-s)}{L}}$$
$$+8e^{-\frac{2\pi i(27q-28r+s)}{L}} + 2e^{-\frac{2\pi i(28q-28r+s)}{L}} - 40e^{-\frac{2\pi i(29q-28r+s)}{L}} - 4e^{-\frac{2\pi i(27q-27r+s)}{L}}$$
$$+8e^{-\frac{2\pi i(29q-27r+s)}{L}} + 2e^{-\frac{2\pi i(27q-27r+2s)}{L}} + 8e^{-\frac{2\pi i(29q-30r-2s)}{L}} \Big\} \tilde{b}_p^\dagger \tilde{b}_q^\dagger \tilde{b}_r \tilde{b}_s \delta_{p+q,r+s}$$
$$+\frac{1}{16} \sum_{p,q,r,s,t,u=0}^{L-1} \Big\{ 2e^{\frac{2\pi i(q+2r-3s-2t)}{L}} - e^{\frac{2\pi i(q+3r-3s-2t)}{L}} - 4e^{\frac{2\pi i(q+2r-3s-t)}{L}}$$
$$-e^{\frac{2\pi i(2q+3r-3s-t)}{L}} + 8e^{\frac{2\pi i(q+2r-2s-t)}{L}} + 2e^{\frac{2\pi i(2q+3r-2s-t)}{L}} - 4e^{\frac{2\pi i(q+2r-3s-2t-u)}{L}}$$
$$+2e^{\frac{2\pi i(q+3r-3s-2t-u)}{L}} + 2e^{\frac{2\pi i(q+2r-2s+u)}{L}}$$
$$-4e^{\frac{2\pi i(q+2r-s+u)}{L}} - 4e^{\frac{2\pi i(q+2r-2s-t+u)}{L}} \Big\} \tilde{b}_p^\dagger \tilde{b}_q^\dagger \tilde{b}_r^\dagger \tilde{b}_s \tilde{b}_t \tilde{b}_u \delta_{p+q+r,s+t+u}$$

$$E^{(1,2)}_{\mathfrak{su}(1|1)} = (k_1^2 + k_1 k_2 + k_2^2) E^{(1,3)}_{\mathfrak{su}(1|1)} = 0$$
$$E^{(2,4)}_{\mathfrak{su}(1|1)} - \frac{1}{4}(k_1^2 + k_1 k_2 + k_2^2)^2 E^{(2,5)}_{\mathfrak{su}(1|1)} = -(k_1^2 + k_1 k_2 + k_2^2)^2$$

$E^{(1,2)}_{\mathfrak{su}(1\|1)}$	$E^{(1,3)}_{\mathfrak{su}(1\|1)}$	$E^{(1,3)}_{\mathfrak{su}(1\|1)}/E^{(1,2)}_{\mathfrak{su}(1\|1)}$	(k_1, k_2, k_3)
$1 + 1.3 \times 10^{-10}$	-1.9×10^{-8}	-1.9×10^{-8}	$(1, 0, -1)$
$4 - 1.0 \times 10^{-7}$	1.8×10^{-5}	4.6×10^{-6}	$(2, 0, -2)$
$7 - 2.5 \times 10^{-7}$	4.4×10^{-5}	6.3×10^{-6}	$(1, 2, -3)$
$7 - 2.5 \times 10^{-7}$	4.4×10^{-5}	6.3×10^{-6}	$(-1, -2, 3)$
$9 - 3.9 \times 10^{-7}$	7.9×10^{-5}	8.7×10^{-6}	$(3, 0, -3)$
$13 - 4.0 \times 10^{-6}$	8.2×10^{-4}	6.3×10^{-5}	$(1, 3, -4)$
$13 - 4.0 \times 10^{-6}$	8.2×10^{-4}	6.3×10^{-5}	$(-1, -3, 4)$
$16 - 2.0 \times 10^{-5}$	4.1×10^{-3}	2.6×10^{-4}	$(4, 0, -4)$
$19 - 3.5 \times 10^{-5}$	7.3×10^{-3}	3.8×10^{-4}	$(2, 3, -5)$
$19 - 3.5 \times 10^{-5}$	7.3×10^{-3}	3.8×10^{-4}	$(-2, -3, 5)$

$E^{(2,4)}_{\mathfrak{su}(1\|1)}$	$E^{(2,5)}_{\mathfrak{su}(1\|1)}$	$E^{(2,5)}_{\mathfrak{su}(1\|1)}/E^{(2,4)}_{\mathfrak{su}(1\|1)}$	(k_1, k_2, k_3)
-0.25	-0.99999	3.99995	$(1, 0, -1)$
-4.00006	-15.990	3.998	$(2, 0, -2)$
-12.251	-48.899	3.992	$(1, 2, -3)$
-12.251	-48.899	3.992	$(-1, -2, 3)$
-20.25	-80.89	3.995	$(3, 0, -3)$
-42.25	-168.2	3.98	$(1, 3, -4)$
-42.25	-168.2	3.98	$(-1, -3, 4)$
-64.00	-254.6	3.98	$(4, 0, -4)$
-90.26	-359.3	3.98	$(2, 3, -5)$
-90.26	-359.8	3.99	$(-2, -3, 5)$

$E^{(3,7)}_{\mathfrak{su}(1\|1)}/E^{(3,6)}_{\mathfrak{su}(1\|1)}$	(k_1, k_2, k_3)
-86.41	$(1, 0, -1)$
-85.71	$(2, 0, -2)$
-83.74	$(1, 2, -3)$
-83.74	$(-1, -2, 3)$
-101.9	$(3, 0, -3)$
-96.01	$(1, 3, -4)$
-96.01	$(-1, -3, 4)$
-158.1	$(4, 0, -4)$

$$\left(\frac{u_i + \frac{i}{2}}{u_i - \frac{i}{2}}\right)^L = 1$$

$$u_i = \frac{1}{2}\cot\left(\frac{k_i \pi}{L}\right)$$

$$E_{\mathfrak{su}(1|1)} = 4\sum_{i=1}^{I} \sin^2\left(\frac{\pi k_i}{L}\right),$$

$$\mathrm{Tr}(\mathcal{D}^I Z Z^{L-1}), \mathrm{Tr}(\mathcal{D}^{I-1} Z \mathcal{D} Z Z^{L-2}), \mathrm{Tr}(\mathcal{D}^{I-1} Z Z \mathcal{D} Z Z^{L-3}), \ldots$$

$$\left(a_i^\dagger\right)^n |L\rangle \sim \mathrm{Tr}\left(Z^{i-1}\mathcal{D}^n Z Z^{L-i}\right), \ldots$$

$$H^{(2)}_{\mathfrak{sl}(2)} = \sum_{j=1}^{L} H^{\mathfrak{sl}(2)}_{j,j+1},$$

$$H^{\mathfrak{sl}(2)}_{1,2}\left(a_1^\dagger\right)^j\left(a_2^\dagger\right)^{n-j}|L\rangle = \sum_{j'=0}^{n}\left[\delta_{j=j'}(h(j) + h(n-j)) - \frac{\delta_{j\neq j'}}{|j-j'|}\right]\left(a_1^\dagger\right)^{j'}\left(a_2^\dagger\right)^{n-j'}|L\rangle$$

$$H^{(2)}_{\mathfrak{sl}(2)} = -\sum_{j=1}^{L}\left[\left(a^\dagger_{j+1} - 2a^\dagger_j + a^\dagger_{j-1}\right)\left(a_j - \frac{1}{2}a^\dagger_j a_j^2\right) + \frac{1}{4}\left(a^{\dagger 2}_{j+1} - 2a_j^{\dagger 2} + a^{\dagger 2}_{j-1}\right)a_j^2\right] + \cdots$$

$$H_{\mathrm{sl}(2)}^{(2)} = \sum_{p=0}^{L-1} 4\sin^2\frac{p\pi}{L}\tilde{a}_p^\dagger\tilde{a}_p$$
$$+\frac{1}{L}\sum_{p,q,r,s=0}^{L-1}\delta_{p+q,r+s}\left(-\sin^2\frac{p\pi}{L}-\sin^2\frac{q\pi}{L}+\sin^2\frac{(p+q)\pi}{L}\right)\tilde{a}_p^\dagger\tilde{a}_q^\dagger\tilde{a}_r\tilde{a}_s+\cdots$$

$$\tilde{a}_{k_1}^\dagger\tilde{a}_{k_2}^\dagger\tilde{a}_{k_3}^\dagger\cdots|L\rangle$$

$$E_{\mathrm{sl}(2)}^{(1,2)} = (k_1^2+k_1k_2+k_2^2)\ E_{\mathrm{sl}(2)}^{(1,3)}/E_{\mathrm{sl}(2)}^{(1,2)} = -2\ k_1 \neq k_2 \neq k_3$$
$$E_{\mathrm{sl}(2)}^{(1,2)} = 3n^2\ E_{\mathrm{sl}(2)}^{(1,3)}/E_{\mathrm{sl}(2)}^{(1,2)} = -7/3\ k_1 = k_2 = n, k_3 = -2n,$$

$$\left(\frac{u_i - i/2}{u_i + i/2}\right)^L = \prod_{j\neq i}^{n}\left(\frac{u_i-u_j+i}{u_i-u_j-i}\right)$$
$$1 = \prod_{i}^{n}\left(\frac{u_i-i/2}{u_i+i/2}\right)$$

$E_{\mathrm{sl}(2)}^{(1,2)}$	$E_{\mathrm{sl}(2)}^{(1,3)}$	$E_{\mathrm{sl}(2)}^{(1,3)}/E_{\mathrm{sl}(2)}^{(1,2)}$	(k_1,k_2,k_3)
$1+1.2\times10^{-9}$	$-2-3.1\times10^{-7}$	$-2-3.1\times10^{-7}$	$(1,0,-1)$
$3-7.6\times10^{-9}$	$-7+1.9\times10^{-6}$	$-7/3+6.3\times10^{-7}$	$(1,1,-2)$
$3-7.6\times10^{-9}$	$-7+1.9\times10^{-6}$	$-7/3+6.3\times10^{-7}$	$(-1,-1,2)$
$4-2.8\times10^{-7}$	$-8+6.9\times10^{-6}$	$-2+1.7\times10^{-6}$	$(2,0,-2)$
$7-2.9\times10^{-7}$	$-14+7.1\times10^{-5}$	$-2+1.0\times10^{-5}$	$(1,2,-3)$
$7-2.9\times10^{-7}$	$-14+7.1\times10^{-5}$	$-2+1.0\times10^{-5}$	$(-1,-2,3)$
$9-4.1\times10^{-7}$	$-18+1.0\times10^{-4}$	$-2+1.0\times10^{-5}$	$(3,0,-3)$
$12+8.4\times10^{-7}$	$-28-1.5\times10^{-4}$	$-7/3-1.2\times10^{-5}$	$(2,2,-4)$
$12+8.4\times10^{-7}$	$-28-1.5\times10^{-4}$	$-7/3-1.2\times10^{-5}$	$(-2,-2,4)$
$13-7.0\times10^{-6}$	$-26+1.7\times10^{-3}$	$-2+1.3\times10^{-4}$	$(1,3,-4)$
$13-7.0\times10^{-6}$	$-26+1.7\times10^{-3}$	$-2+1.3\times10^{-4}$	$(-1,-3,4)$
$16-1.4\times10^{-6}$	$-32+3.9\times10^{-4}$	$-2+2.4\times10^{-5}$	$(4,0,-4)$
$19-7.5\times10^{-6}$	$-38+2.2\times10^{-3}$	$-2+1.1\times10^{-4}$	$(2,3,-5)$
$19-7.5\times10^{-6}$	$-38+2.2\times10^{-3}$	$-2+1.1\times10^{-4}$	$(-2,-3,5)$
$21-3.4\times10^{-6}$	$-42+8.8\times10^{-4}$	$-2+4.2\times10^{-5}$	$(1,4,-5)$
$21-3.4\times10^{-6}$	$-42+8.8\times10^{-4}$	$-2+4.2\times10^{-5}$	$(-1,-4,5)$

$$u_1 = -\frac{2(1+L)k_1^2-(4+L)k_1k_2-(4+L)k_2^2}{2\pi k_1(k_2^2+k_1k_2-2k_1^2)}+O(L^{-1})$$
$$u_2 = -\frac{2(1+L)k_2^2-(4+L)k_1k_2-(4+L)k_1^2}{2\pi k_2(k_1^2+k_1k_2-2k_2^2)}+O(L^{-1})$$
$$u_3 = -\frac{2(1+L)k_1^2+(8+5L)k_1k_2+2(1+L)k_2^2}{2\pi(k_1+k_2)(2k_1+k_2)(k_1+2k_2)}+O(L^{-1}).$$

$$E_{\mathrm{sl}(2)}^{(2)}(k_1,k_2) = \frac{\lambda}{L^3}(k_1^2+k_1k_2+k_2^2)(L-2)+O(L^{-4})\ (k_1\neq k_2\neq k_3).$$

$$u_1 = \frac{7-3\sqrt{L}+3L}{6\pi n}+O\left(L^{-1/2}\right)$$
$$u_2 = \frac{7+3\sqrt{L}+3L}{6\pi n}+O\left(L^{-1/2}\right)$$
$$u_3 = -\frac{4+3L}{12\pi n}+O\left(L^{-1/2}\right),$$

$$E_{\mathrm{sl}(2)}^{(2)}(n) = \frac{\lambda n^2}{L^3}(3L-7)+O(L^{-4})\ (k_1=k_2=n, k_3=-2n).$$

CONCLUSIONES

Según los resultados antes referidos, se concluye que, en un campo cuántico – relativista, los quarks y gluones, para lograr el confinamiento de color, intermedia la gravedad o supergravedad cuánticas, según el caso, es decir, según la naturaleza de la partícula subatómica de origen (partícula oscura o blanca, según corresponda), lo que permite la formación de hadrones (mesones y bariones respectivamente), los cuales, al igual que las partículas que los originan, pueden ser partículas blancas u oscuras, según sea el caso. Téngase en cuenta, que si bien es cierto, que el quark top, es un candidato genuino para actuar como partícula oscura, debido a su enorme masa, sin embargo, no es susceptible de hadronización, debido a que se desintegra inmediatamente sin que le sea posible confinarse, más, como se ha dicho, esta partícula supermasiva, es capaz de producir gravedad, incluso en condiciones extremas y entrópicas.

Aclaraciones finales

Algunas aclaraciones finales a tener en consideración y aplicar, a propósito de la Teoría Cuántica de Campos Relativistas o Curvos (TCCR) propuesta por este autor:

1. En todos los casos, este símbolo † será reemplazado por este símbolo†o por este símbolo †, equivaliendo lo mismo.

Símbolo a ser reemplazado.	**Símbolos de reemplazo.**
†	†
	†

2. En todos los casos, este símbolo ‡, será reemplazado por este símbolo‡o por este símbolo ‡.

Símbolo a ser reemplazado.	**Símbolos de reemplazo.**
‡	‡
	‡

3. En todos los casos, se añadirá y por ende, se calculará la magnitud§que equivale a un campo de Yang – Mills y por ende, a la teoría de Yang – Mills en sentido amplio, en relación a la Teoría Cuántica de Campos Relativistas propuesta por este autor.

4. Este símbolo • podrá usarse como exponente u operador, según sea el caso.

Las aclaraciones antes referidas aplican tanto a este trabajo como a todos los trabajos previos y posteriores publicados por este autor, según corresponda.

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