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RELATIVISTIC QUANTUM SUPERGRAVITY. ADS/SCFT SUPERSYMMETRIES IN WEYL–DE SITTER AND ANTI–DE SITTER SUPERSPACES. VOLUME II

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SUPERGRAVEDAD CUÁNTICA RELATIVISTA. SUPERSIMETRÍAS AdS/SCFT EN SUPERESPACIOS DE WEYL –DE SITTER Y ANTI – DE SITTER. VOLUMEN II

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RESUMEN

En recientes trabajos, este autor ha esbozado planteamientos generales a propósito de la supergravedad cuántica relativista, la misma que, en sentido estricto, se tiene como una teoría del todo. La supergravedad cuántica relativista, es un pilar esencial en la Teoría Cuántica de Campos Relativistas o Curvos, planteada por este investigador, en la medida en que, unifica la relatividad general y la relatividad especial con la mecánica cuántica. Sin embargo, a diferencia de los postulados que son propios de la supergravedad, la supergravedad cuántica relativista, usa la supersimetría en grupos de gauge infinitos (supersimetrías de gauge) con la finalidad de integrar la gravedad a escala subatómica, a partir de la existencia de superpartículas, capaces de deformar el espacio – tiempo cuántico, en dimensiones más altas (supersimetrías de gauge en dimensiones D), lo que ocurre, por interacción de la propia superpartícula a razón de su masa y energía extremadamente densas, o por interacción de la superpartícula con el supergravitón, incluyendo, aquellas interacciones con regiones de antimateria, lo que supone, la permeabilización del campo cuántico repercutido por el campo supergravitónico. En este trabajo, se robustecerá este concepto esencial que explica la Teoría Cuántica de Campos Relativistas o Curvos.

Palabras clave: Supersimetría, Supergravedad en D dimensiones, relatividad general, grupos de gauge, superpartículas, supergravitón.

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RELATIVISTIC QUANTUM SUPERGRAVITY. ADS/SCFT SUPERSYMMETRIES IN WEYL–DE SITTER AND ANTI–DE SITTER SUPERSPACES. VOLUME II

ABSTRACT

In recent works, this author has outlined general approaches to relativistic quantum supergravity, which, strictly speaking, is considered a theory of everything. Relativistic quantum supergravity is an essential pillar in the Quantum Theory of Relativistic or Curved Fields, proposed by this researcher, insofar as it unifies general relativity and special relativity with quantum mechanics. However, unlike the postulates that are typical of supergravity, relativistic quantum supergravity uses supersymmetry in infinite gauge groups (gauge supersymmetries) in order to integrate gravity at the subatomic scale, based on the existence of superparticles, capable of deforming quantum space-time, in higher dimensions (gauge supersymmetries in D dimensions). which occurs by interaction of the superparticle itself due to its extremely dense mass and energy, or by interaction of the superparticle with the supergraviton, including those interactions with regions of antimatter, which supposes the permeabilization of the supergravitonic field, which supposes the permeabilization of the quantum field impacted by the supergravitonic field. In this work, this essential concept that explains the Quantum Theory of Relativistic or Curved Fields will be strengthened.

Keywords: Supersymmetry, Supergravity in D dimensions, general relativity, gauge groups, superparticles, supergraviton.

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INTRODUCCIÓN.

A diferencia de otras líneas de investigación relacionadas a la supergravedad, en este trabajo, abordaré la supergravedad a escala cuántica y en dimensión relativista, en D dimensiones, sin regulación por campos de gauge finitos, sino en contrario, suponiendo que, a escala cuántica, la supersimetría de gauge, está dada en infinitas dimensiones, a propósito de la distorsión del espacio – tiempo cuántico, en dimensiones más altas, a causa de la interacción de la superpartícula, por su propio centro de masa o energía (en el caso de las partículas estrella o blancas), o por interacción de la superpartícula con el supergravitón, incluyendo, aquellas interacciones con regiones de antimateria.

En este trabajo, esbozaré un modelo matemático unitario, a propósito de la supergravedad cuántica relativista, concebida así, en la Teoría Cuántica de Campos Relativistas o Curvos, esto es, la deformación del espacio – tiempo cuántico, y su desdoblamiento en dimensiones infinitas, por efectos de la gravedad causada por la interacción de la superpartícula, con el supergravitón, lo que supone, la permeabilización del campo cuántico repercutido por el campo supergravitónico, o en su propio campo cuántico, en condiciones relativistas, a razón de su masa y energía, extremadamente densas, como ocurre con las partículas blancas.

RESULTADOS Y DISCUSIÓN.

Los cálculos aquí contenidos, complementan con el desarrollo matemático desplegado en el volumen I de este trabajo.

Supersimetrías compactas en planos cuánticos relativistas.

$$\mathrm{SO}_0(2,2) \cong (\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R}))/\mathbb{Z}_2$$

$$\left\{\mathcal{D}_\alpha^{i\bar{\imath}}, \mathcal{D}_\beta^{j\bar{\jmath}}\right\} = 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\overline{\imath\jmath}}\mathcal{D}_{\alpha\beta} + 2\mathrm{i}\varepsilon_{\alpha\beta}X\left(\varepsilon^{\overline{\imath\jmath}}\mathbf{L}^{ij} - \varepsilon^{ij}\mathbf{R}^{\overline{\imath\jmath}}\right)$$

$$\left[\mathcal{D}_\alpha^{i\bar{\imath}}, \mathcal{D}_b\right] = 0, [\mathcal{D}_a, \mathcal{D}_b] = 0$$

$$\mathcal{D}_A = E_A{}^M\partial_M + \frac{1}{2}\Omega_A{}^{\beta\gamma}\mathcal{M}_{\beta\gamma} + \Phi_A{}^{jk}\mathbf{L}_{jk} + \Phi_A{}^{\overline{jk}}\mathbf{R}_{\overline{jk}}$$

$$\left[\mathbf{L}^{kl}, \mathcal{D}_\alpha^{i\bar{\imath}}\right] = \varepsilon^{i(k}\mathcal{D}_\alpha^{l)\bar{\imath}}, \left[\mathbf{R}^{\bar{k}\bar{l}}, \mathcal{D}_\alpha^{i\bar{\imath}}\right] = \varepsilon^{\bar{\imath}(\bar{k}}\mathcal{D}_\alpha^{i\bar{l})}$$

$$\begin{aligned}\left\{\mathcal{D}_\alpha^{i\bar{\imath}}, \mathcal{D}_\beta^{j\bar{\jmath}}\right\} =& 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{\imath}\bar{\jmath}}(\gamma^c)_{\alpha\beta}\mathcal{D}_c + 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{\imath}\bar{\jmath}}(2\mathcal{S} + X)\mathbf{L}^{ij} - 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{ij}\mathcal{S}^{kl\bar{\imath}\bar{\jmath}}\mathbf{L}_{kl} + 4\mathrm{i}C_{\alpha\beta}^{\bar{\imath}\bar{\jmath}}\mathbf{L}^{ij} \\ &+ 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{ij}(2\mathcal{S} - X)\mathbf{R}^{\bar{\imath}\bar{\jmath}} - 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{\imath}\bar{\jmath}}\mathcal{S}^{ij\bar{k}\bar{l}}\mathbf{R}_{\bar{k}\bar{l}} + 4\mathrm{i}B_{\alpha\beta}^{ij}\mathbf{R}^{\bar{\imath}\bar{\jmath}} \\ &+ 2\mathrm{i}\varepsilon_{\alpha\beta}\left(\varepsilon^{\bar{\imath}\bar{\jmath}}B^{\gamma\delta ij} + \varepsilon^{ij}C^{\gamma\delta\bar{\imath}\bar{\jmath}}\right)\mathcal{M}_{\gamma\delta} - 4\mathrm{i}\left(\mathcal{S}^{i\bar{\imath}\bar{\jmath}} + \varepsilon^{ij}\varepsilon^{\bar{\imath}\bar{\jmath}}\mathcal{S}\right)\mathcal{M}_{\alpha\beta}\end{aligned}$$

$$\mathfrak{M}: \mathrm{SU}(2)_{\mathrm{L}} \leftrightarrow \mathrm{SU}(2)_{\mathrm{R}}$$

$$\mathfrak{M} \cdot \mathcal{S} = \mathcal{S}, \quad \mathfrak{M} \cdot \mathcal{S}^{ij\bar{\imath}\bar{\jmath}} = \mathcal{S}^{ij\bar{\imath}\bar{\jmath}}, \quad \mathfrak{M} \cdot X = -X,$$
$$\mathfrak{M} \cdot C_a^{\bar{\imath}\bar{\jmath}} = B_a^{ij}, \quad \mathfrak{M} \cdot B_a^{ij} = C_a^{\bar{\imath}\bar{\jmath}}.$$

$$\mathcal{D}_\alpha^{i\bar{\imath}}\mathcal{S} = 0, \mathcal{D}_\alpha^{i\bar{\imath}}\mathcal{S}^{jkj\bar{k}} = 0,$$
$$\mathcal{D}_\alpha^{i\bar{\imath}}B_{\beta\gamma}^{jk} = 0, \mathcal{D}_\alpha^{i\bar{\imath}}C_{\beta\gamma}^{\bar{\jmath}\bar{k}} = 0.$$

$$\begin{aligned}
[\mathcal{D}_\alpha^{i\bar{\imath}}, \mathcal{D}_{\beta\gamma}] = &\Big\{\varepsilon_{\alpha(\beta}\Big[2\delta_{\gamma)}^{\delta}\left(\mathcal{S}^{ij\bar{\imath}\bar{\jmath}} + \varepsilon^{ij}\varepsilon^{\bar{\imath}\bar{\jmath}}\mathcal{S}\right) + \frac{4}{3}\left(\varepsilon^{\bar{\imath}\bar{\jmath}}B_\gamma{}^{\delta ij} + \varepsilon^{ij}C_\gamma{}^{\delta\bar{\imath}}\right)\Big] \\
&+\delta_{(\alpha}^{\delta}\left(\varepsilon^{\bar{\imath}\bar{\jmath}}B_{\beta\gamma)}^{ij} + \varepsilon^{ij}C_{\beta\gamma)}^{\bar{\imath}\bar{\jmath}}\right)\Big\}\mathcal{D}_{\delta j\bar{\jmath}} \\
[\mathcal{D}_{\alpha\beta}, \mathcal{D}_{\gamma\delta}] = &\varepsilon_{\alpha(\gamma}B_{\delta)\beta}^{ij}(2\mathcal{S} + X)\mathbf{L}_{ij} + \varepsilon_{\beta(\gamma}B_{\delta)\alpha}^{ij}(2\mathcal{S} + X)\mathbf{L}_{ij} \\
&+\varepsilon_{\alpha(\gamma}C_{\delta)\beta}^{\bar{\imath}\bar{\jmath}}(2\mathcal{S} - X)\mathbf{R}_{\bar{\imath}\bar{\jmath}} + \varepsilon_{\beta(\gamma}C_{\delta)\alpha}^{\bar{\imath}\bar{\jmath}}(2\mathcal{S} - X)\mathbf{R}_{\bar{\imath}\bar{\jmath}} \\
&+\varepsilon_{\alpha(\gamma}\left(B_{\delta)\beta}^{ij}B_{ij}^{\lambda\rho} + C_{\delta)\beta}^{\bar{\imath}\bar{\jmath}}C_{\bar{\imath}\bar{\jmath}}^{\lambda\rho}\right)\mathcal{M}_{\lambda\rho} + \varepsilon_{\beta(\gamma}\left(B_{\delta)\alpha}^{ij}B_{ij}^{\lambda\rho} + C_{\delta)\alpha}^{\bar{\imath}\bar{\jmath}}C_{\bar{\imath}\bar{\jmath}}^{\lambda\rho}\right)\mathcal{M}_{\lambda\rho} \\
&-\left(\mathcal{S}^{ij\bar{\imath}\bar{\jmath}}\mathcal{S}_{ij\bar{\imath}\bar{\jmath}} + 4\mathcal{S}^2\right)\left(\varepsilon_{\alpha(\gamma}\mathcal{M}_{\delta)\beta} + \varepsilon_{\beta(\gamma}\mathcal{M}_{\delta)\alpha}\right)
\end{aligned}$$

$$\mathcal{D}_{\alpha\beta}\mathcal{S} = 0, \mathcal{D}_{\alpha\beta}\mathcal{S}^{i\bar{\imath}\bar{\imath}\bar{\jmath}} = 0, \mathcal{D}_{\alpha\beta}X = 0$$
$$(\mathcal{D}_{\alpha\beta} - 2\mathcal{S}\mathcal{M}_{\alpha\beta})B_{\gamma\delta}^{ij} = 0, (\mathcal{D}_{\alpha\beta} - 2\mathcal{S}\mathcal{M}_{\alpha\beta})C_{\gamma\delta}^{\bar{\imath}\bar{\jmath}} = 0$$

$$\tilde{\mathcal{D}}_A = (\tilde{\mathcal{D}}_\alpha^{i\bar{\imath}}, \tilde{\mathcal{D}}_a), \tilde{\mathcal{D}}_\alpha^{i\bar{\imath}} = \mathcal{D}_\alpha^{i\bar{\imath}}, \tilde{\mathcal{D}}_a = \mathcal{D}_a - 2\mathcal{S}\mathcal{M}_a$$

$$B_{\alpha\beta ij}\mathcal{S}^{ij\bar{\imath}\bar{\jmath}} = 0, C_{\alpha\beta\bar{\imath}\bar{\jmath}}\mathcal{S}^{ij\bar{\imath}\bar{\jmath}} = 0, B_{\alpha\beta}{}^{(i}{}_k\mathcal{S}^{j)k\bar{\imath}\bar{\jmath}} + C_{\alpha\beta}{}^{(\bar{\imath}}{}_{\bar{k}}\mathcal{S}^{ij\bar{\jmath})\bar{k}}$$
$$B_{\alpha\beta}{}^{(ij}\mathcal{S}^{kl)\bar{\imath}\bar{\jmath}} = 0, C_{\alpha\beta}(\bar{\imath}\bar{\jmath}\mathcal{S}^{ij\bar{k}\bar{l}}) = 0, B_{(\alpha}{}^{\gamma ij}C_{\beta)\gamma}{}^{\bar{\imath}\bar{\jmath}} + \mathcal{S}^{ij\bar{k}(\bar{\imath}}C_{\alpha\beta}{}^{\bar{\jmath})}{}_{\bar{k}} = 0,$$
$$B^{\alpha\beta ij}C_{\alpha\beta}^{i\bar{\jmath}} = 0, B_{(\alpha\beta}^{ij}C_{\gamma\delta)}^{\bar{\imath}\bar{\jmath}} = 0, B_{(\alpha}{}^{\gamma ij}C_{\beta)\gamma}{}^{\bar{\imath}\bar{\jmath}} - \mathcal{S}^{k(i\bar{\imath}\bar{\jmath}}B_{\alpha\beta}{}^{j)}{}_k$$
$$B_{(\alpha}{}^{\gamma k(i}B_{\beta)\gamma k}{}^{j)} + (2\mathcal{S} + X)B_{\alpha\beta}^{ij} = 0, C_{(\alpha}{}^{\gamma\bar{k}(\bar{\imath}}C_{\beta)\gamma\bar{k}}{}^{\bar{\jmath})} + (2\mathcal{S} - X)C_{\alpha\beta}^{\bar{\imath}\bar{\jmath}} = 0,$$
$$\mathcal{S}^{k(i\bar{k}(\bar{\imath}}\mathcal{S}^{j)}{}_k{}^{\bar{\jmath}}{}_{\bar{k}} + 2\mathcal{S}\mathcal{S}^{ij\bar{\imath}\bar{\bar{\imath}}} = 0, X\mathcal{S}^{ij\bar{\imath}\bar{\bar{\imath}}} = 0$$

$$B_{\alpha\beta}^{ij}C_{\gamma\delta}^{\bar{\imath}\bar{\jmath}} = 0, B_{(\alpha}{}^{\gamma k(i}B_{\beta)\gamma k}{}^{j)} + 2\mathcal{S}B_{\alpha\beta}^{ij} = 0, C_{(\alpha}{}^{\gamma\bar{k}(\bar{\imath}}C_{\beta)\gamma\bar{k}}^{\bar{\jmath})} + 2\mathcal{S}C_{\alpha\beta}^{\bar{\imath}\bar{\jmath}} = 0$$

$$(i): \mathcal{S} = 0 \quad \Longrightarrow \quad B_{\alpha\beta}^{ij} = B_{\alpha\beta}B^{ij}, B^{ij}B_{ij} = 2$$
$$(ii): \mathcal{S} \neq 0 \quad \Longrightarrow \quad B_{\alpha\beta}^{ij} = -2\mathcal{S}\Lambda_{\alpha\beta},$$

$$\begin{aligned}
[\tilde{\mathcal{D}}_{\alpha\beta}, \tilde{\mathcal{D}}_{\gamma\delta}] = &4\mathcal{S}\varepsilon_{\alpha(\gamma}\tilde{\mathcal{D}}_{\delta)\beta} + 4\mathcal{S}\varepsilon_{\beta(\gamma}\tilde{\mathcal{D}}_{\delta)\alpha} + 2\mathcal{S}\left(\varepsilon_{\alpha(\gamma}B_{\delta)\beta}^{ij} + \varepsilon_{\beta(\gamma}B_{\delta)\alpha}^{ij}\right)\mathbf{L}_{ij} \\
&+\left(\varepsilon_{\alpha(\gamma}B_{\delta)\beta}^{ij}B_{ij}^{\lambda\rho} + \varepsilon_{\beta(\gamma}B_{\delta)\alpha}^{ij}B_{ij}^{\lambda\rho}\right)\mathcal{M}_{\lambda\rho} + 2\mathcal{S}^2\left(\varepsilon_{\alpha(\gamma}\mathcal{M}_{\delta)\beta} + \varepsilon_{\beta(\gamma}\mathcal{M}_{\delta)\beta}\right)
\end{aligned}$$

$$[\tilde{\mathcal{D}}_{\alpha\beta}, \tilde{\mathcal{D}}_{\gamma\delta}]B_{\lambda\rho}^{ij} = 0 \iff \mathcal{S} = 0$$

$$B_{\alpha\beta}^{ij}C_{\gamma\delta}^{\bar{\imath}\bar{\jmath}} = 0, B_{(\alpha}{}^{\gamma k(i}B_{\beta)\gamma k}{}^{j)} + (2\mathcal{S} + X)B_{\alpha\beta}^{ij} = 0, C_{(\alpha}{}^{\gamma\bar{k}(\bar{\imath}}C_{\beta)\gamma\bar{k}}{}^{\bar{\jmath})} + (2\mathcal{S} - X)C_{\alpha\beta}^{\bar{\imath}\bar{\jmath}} = 0,$$

$$(i): 2\mathcal{S} + X = 0 \implies B_{\alpha\beta}^{ij} = B_{\alpha\beta}B^{ij}, B^{ij}B_{ij} = 2$$
$$(ii): 2\mathcal{S} + X \neq 0 \implies B_{\alpha\beta}^{ij} = -(2\mathcal{S} + X)\Lambda_{\alpha\beta,}{}^{\gamma\delta}\delta_\gamma^k\delta_\delta^l(R^T)_{kl,}{}^{ij}$$

$$\left[\tilde{\mathcal{D}}_{\alpha\beta},\tilde{\mathcal{D}}_{\gamma\delta}\right]=4\mathcal{S}\varepsilon_{\alpha(\gamma}\tilde{\mathcal{D}}_{\delta)\beta}+4\mathcal{S}\varepsilon_{\beta(\gamma}\tilde{\mathcal{D}}_{\delta)\alpha}-\left(\varepsilon_{\alpha(\gamma}B_{\delta)\beta}B^{\lambda\rho}+\varepsilon_{\beta(\gamma}B_{\delta)\alpha}B^{\lambda\rho}\right)\mathcal{M}_{\lambda\rho}$$

$$\begin{aligned}\left[\tilde{\mathcal{D}}_{\alpha\beta},\tilde{\mathcal{D}}_{\gamma\delta}\right]=&4\mathcal{S}\varepsilon_{\alpha(\gamma}\tilde{\mathcal{D}}_{\delta)\beta}+4\mathcal{S}\varepsilon_{\beta(\gamma}\tilde{\mathcal{D}}_{\delta)\alpha}+(2\mathcal{S}+X)\left(\varepsilon_{\alpha(\gamma}B^{ij}_{\delta)\beta}+\varepsilon_{\beta(\gamma}B^{ij}_{\delta)\alpha}\right)\mathbf{L}_{ij}\\&+\left(\varepsilon_{\alpha(\gamma}B^{ij}_{\delta)\beta}B^{\lambda\rho}_{ij}+\varepsilon_{\beta(\gamma}B^{ij}_{\delta)\alpha}B^{\lambda\rho}_{ij}\right)\mathcal{M}_{\lambda\rho}+2\mathcal{S}^2\left(\varepsilon_{\alpha(\gamma}\mathcal{M}_{\delta)\beta}+\varepsilon_{\beta(\gamma}\mathcal{M}_{\delta)\beta}\right)\end{aligned}$$

$$\left[\tilde{\mathcal{D}}_{\alpha\beta},\tilde{\mathcal{D}}_{\gamma\delta}\right]B^{ij}_{\lambda\rho}=0\Leftrightarrow\mathcal{S}=0,$$

$$[\mathcal{D}_a,\mathcal{D}_b]=-X\varepsilon_{abc}B^{cij}\mathbf{L}_{ij}+X^2\mathcal{M}_{ab}$$

$$0=(-1)^{\varepsilon_A\varepsilon_C}[\mathcal{D}_A,[\mathcal{D}_B,\mathcal{D}_C\}]+(\text{ two cycles }).$$

$$B^{ij}_{\alpha\beta}=0,C^{\bar{i}\bar{j}}_{\alpha\beta}=0,\mathcal{S}^{k(i\bar{k}(\bar{\imath}}\mathcal{S}^{j)}{}_{k}{}^{\bar{j})}{}_{\bar{k}}+2\mathcal{S}\mathcal{S}^{ij\bar{i}\bar{j}}=0,$$

$$\begin{gathered}(i):\mathcal{S}=0\implies\mathcal{S}^{ij\bar{\imath}\bar{\jmath}}=\hat{\mathcal{S}}l^{ij}r^{\bar{\imath}\bar{\jmath}},l^{ik}l_{kj}=\delta^i_j,r^{\bar{\imath}\bar{k}}r_{\bar{k}\bar{\jmath}}=\delta^{\bar{\imath}}_{\bar{\jmath}},\hat{\mathcal{S}}\in\mathbb{R},\\(ii):\mathcal{S}\neq0\implies\mathcal{S}^{ij\bar{\imath}\bar{\jmath}}=-\mathcal{S}\left(w^{i\bar{\imath}}w^{j\bar{\jmath}}+w^{j\bar{\imath}}w^{i\bar{\jmath}}\right),w^{i\bar{k}}w_{j\bar{k}}=\delta^i_j,w^{k\bar{\imath}}w_{\kappa\bar{\jmath}}=\delta^{\bar{\imath}}_{\bar{\jmath}}\end{gathered}$$

$$\begin{gathered}\left\{\mathcal{D}_\alpha,\overline{\mathcal{D}}_\beta\right\}=-2\mathrm{i}\mathcal{D}_{\alpha\beta}-2\mathrm{i}\varepsilon_{\alpha\beta}X\hat{\mathcal{Z}},\left\{\mathcal{D}_\alpha,\mathcal{D}_\beta\right\}=0\\\left[\mathcal{D}_\alpha,\mathcal{D}_b\right]=0,\left[\mathcal{D}_a,\mathcal{D}_b\right]=0\end{gathered}$$

$$\hat{\mathcal{Z}}:=\mathbf{L}^{12}-\mathbf{R}^{\overline{12}},\left[\hat{\mathcal{Z}},\mathcal{D}^{1\bar{1}}_\alpha\right]=\left[\hat{\mathcal{Z}},\left(-\mathcal{D}^{2\bar{2}}_\alpha\right)\right]=0\Longrightarrow\left[\hat{\mathcal{Z}},\mathcal{D}_\alpha\right]=0$$

$$0=\left[\xi+\frac{1}{2}\Lambda^{\gamma\delta}\mathcal{M}_{\gamma\delta}+\Lambda^{kl}\mathbf{L}_{kl}+\Lambda^{\bar{k}\bar{l}}\mathbf{R}_{\bar{k}\bar{l}},\mathcal{D}_A\right]$$

$$\begin{aligned}\mathcal{D}^{i\bar{\imath}}_\alpha\xi_{\beta\gamma}&=4\mathrm{i}\varepsilon_{\alpha(\beta}\xi^{i\bar{\imath}}_{\gamma)}\\\mathcal{D}^{i\bar{\imath}}_\alpha\xi^{j\bar{\jmath}}_\beta&=\frac{1}{2}\Lambda_{\alpha\beta}\varepsilon^{ij}\varepsilon^{\bar{\imath}\bar{\jmath}}+\Lambda^{ij}\varepsilon^{\bar{\imath}\bar{\jmath}}\varepsilon_{\alpha\beta}+\Lambda^{\bar{\imath}\bar{\jmath}}\varepsilon^{ij}\varepsilon_{\alpha\beta}\\\mathcal{D}^{i\bar{\imath}}_\alpha\Lambda_{\beta\gamma}&=0\\\mathcal{D}^{i\bar{\imath}}_\alpha\Lambda^{kl}&=-2\mathrm{i}\varepsilon^{i(k}\xi^{l)\bar{\imath}}_\alpha X\\\mathcal{D}^{i\bar{\imath}}_\alpha\Lambda^{\bar{k}\bar{l}}&=2\mathrm{i}\varepsilon^{\bar{\imath}(\bar{k}}\xi_\alpha{}^{i\bar{l})}X\end{aligned}$$

$$\begin{aligned}\mathcal{D}_a\xi_b&=\Lambda_{ab}\\\mathcal{D}_a\xi^\beta_{j\bar{\jmath}}&=0,\mathcal{D}_a\Lambda^{\beta\gamma}=0,\mathcal{D}_a\Lambda^{kl}=0,\mathcal{D}_a\Lambda^{\bar{k}\bar{l}}=0\end{aligned}$$

$$\delta_\xi\mathfrak{T}=\left(\xi+\frac{1}{2}\Lambda^{\gamma\delta}\mathcal{M}_{\gamma\delta}+\Lambda^{kl}\mathbf{L}_{kl}+\Lambda^{\bar{k}\bar{l}}\mathbf{R}_{\bar{k}\bar{l}}\right)\mathfrak{T}$$

$$\mathfrak{T}|:=\mathfrak{T}\left(x,\theta_{\iota\bar{\jmath}}\right)\big|_{\theta_{1\bar{2}}=\theta_{2\bar{1}}=0}$$

$$\begin{gathered}\tau^a:=\xi^a|,\tau^\alpha:=\xi^\alpha_{\underline{1\bar{1}}}|,\bar{\tau}^\alpha=\xi^\alpha_{\underline{2\bar{2}}}\Big|,t:=\mathrm{i}\left(\Lambda^{12}+\Lambda^{\overline{12}}\right)\Big|=\bar{t},t^{ab}:=\Lambda^{ab}\,|\\\varepsilon^\alpha:=-\xi^\alpha_{1\bar{2}}|,\bar{\varepsilon}^\alpha=\xi^\alpha_{2\bar{1}}|,\sigma:=\mathrm{i}\left(\Lambda^{12}-\Lambda^{\overline{12}}\right)|=\bar{\sigma}\\\bar{\Lambda}_{\mathrm{L}}:=\Lambda^{11}|,\Lambda_{\mathrm{L}}=\Lambda^{22}|,\bar{\Lambda}_{\mathrm{R}}=\Lambda^{\overline{11}}\Big|,\Lambda_{\mathrm{R}}=\Lambda^{\overline{22}}\Big|\end{gathered}$$

$$2\Lambda^{12}\Big|\mathbf{L}_{12}+2\Lambda^{\overline{12}}\Big|\mathbf{R}_{\overline{12}}=\mathrm{i}t\hat{\mathcal{J}}+\mathrm{i}\sigma\hat{\mathcal{Z}},\hat{\mathcal{J}}:=\mathbf{L}^{12}+\mathbf{R}^{\overline{12}}$$

$$[\hat{\mathcal{J}}, \mathcal{D}_\alpha] = \mathcal{D}_\alpha, [\hat{\mathcal{J}}, \bar{\mathcal{D}}_\alpha] = -\bar{\mathcal{D}}_\alpha$$

$$\left[\tau + \frac{1}{2} t^{bc} \mathcal{M}_{bc} + \mathrm{i} t \hat{\mathcal{J}} + \mathrm{i} \sigma \hat{\mathcal{Z}}, \mathcal{D}_A\right] = 0$$

$$\begin{aligned}
\mathcal{D}_\alpha t &= 0 \\
\mathcal{D}_\alpha \sigma &= 2X \tau_\alpha \\
\mathcal{D}_\alpha \tau_b &= t_{ab} = -t_{ba} \\
\mathcal{D}_\alpha t_{bc} &= 0 \\
\bar{\mathcal{D}}_\alpha \tau^\beta &= 0 \\
\mathcal{D}_\alpha \tau^\beta &= \frac{1}{2} t_\alpha{}^\beta + \mathrm{i} \delta_\alpha{}^\beta t \\
\mathcal{D}_\alpha \tau^{\beta\gamma} &= -2\mathrm{i}\left(\delta_\alpha{}^\beta \bar{\tau}^\gamma + \delta_\alpha{}^\gamma \bar{\tau}^\beta\right)
\end{aligned}$$

$$\begin{aligned}
&\bar{\mathcal{D}}_\alpha \Lambda_{\mathrm{L}} = 0, && \mathcal{D}_\alpha \Lambda_{\mathrm{L}} = -2\mathrm{i} X \varepsilon_\alpha \\
&\bar{\mathcal{D}}_\alpha \Lambda_{\mathrm{R}} = 0, && \mathcal{D}_\alpha \Lambda_{\mathrm{R}} = -2\mathrm{i} X \bar{\varepsilon}_\alpha \\
&\mathcal{D}_\alpha \varepsilon_\beta = \varepsilon_{\alpha\beta} \bar{\Lambda}_{\mathrm{R}}, && \mathcal{D}_\alpha \bar{\varepsilon}_\beta = -\varepsilon_{\alpha\beta} \bar{\Lambda}_{\mathrm{L}} \\
&\bar{\mathcal{D}}_\alpha \varepsilon_\beta = \varepsilon_{\alpha\beta} \Lambda_{\mathrm{L}}, && \bar{\mathcal{D}}_\alpha \bar{\varepsilon}_\beta = -\varepsilon_{\alpha\beta} \Lambda_{\mathrm{R}}
\end{aligned}$$

$$\hat{\mathcal{Z}} \Lambda_{\mathrm{L}} = -\Lambda_{\mathrm{L}}, \hat{\mathcal{Z}} \varepsilon^\alpha = -\varepsilon^\alpha$$

$$\hat{\mathcal{Z}} \Lambda_{\mathrm{R}} = \Lambda_{\mathrm{R}}, \hat{\mathcal{Z}} \bar{\varepsilon}^\alpha = \bar{\varepsilon}^\alpha$$

$$\varepsilon_\alpha = \bar{\mathcal{D}}_\alpha \bar{\rho}_{\mathrm{L}}, \hat{\mathcal{Z}} \bar{\rho}_{\mathrm{L}} = -\bar{\rho}_{\mathrm{L}}, \mathcal{D}_\alpha \bar{\rho}_{\mathrm{L}} = 0$$

$$\bar{\rho}_{\mathrm{L}} = -\frac{\mathrm{i}}{2X} \bar{\Lambda}_{\mathrm{R}}, \rho_{\mathrm{L}} = \frac{\mathrm{i}}{2X} \Lambda_{\mathrm{R}}$$

$$\Lambda_{\mathrm{L}} = -\frac{1}{2} \bar{\mathcal{D}}^2 \bar{\rho}_{\mathrm{L}}, \bar{\Lambda}_{\mathrm{L}} = -\frac{1}{2} \mathcal{D}^2 \rho_{\mathrm{L}}$$

$$\varepsilon_\alpha = -\mathcal{D}_\alpha \rho_{\mathrm{R}}, \hat{\mathcal{Z}} \rho_{\mathrm{R}} = -\rho_{\mathrm{R}}, \bar{\mathcal{D}}_\alpha \rho_{\mathrm{R}} = 0$$

$$\rho_{\mathrm{R}} = -\frac{\mathrm{i}}{2X} \Lambda_{\mathrm{L}}, \bar{\rho}_{\mathrm{R}} = \frac{\mathrm{i}}{2X} \bar{\Lambda}_{\mathrm{L}}$$

$$\begin{aligned}
&\rho_{\mathrm{R}} = \frac{\mathrm{i}}{4X} \bar{\mathcal{D}}^2 \bar{\rho}_{\mathrm{L}}, && \rho_{\mathrm{L}} = -\frac{\mathrm{i}}{4X} \bar{\mathcal{D}}^2 \bar{\rho}_{\mathrm{R}} \\
&\Lambda_{\mathrm{R}} = -\frac{1}{2} \bar{\mathcal{D}}^2 \bar{\rho}_{\mathrm{R}}, && \bar{\Lambda}_{\mathrm{R}} = -\frac{1}{2} \mathcal{D}^2 \rho_{\mathrm{R}}
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_\alpha^{(1)\bar{\imath}} &:= v_i \mathcal{D}_\alpha^{i\bar{\imath}} \\
\mathcal{D}_\alpha^{(\bar{1})i} &:= v_{\bar{\imath}} \mathcal{D}_\alpha^{i\bar{\imath}}
\end{aligned}$$

$$\begin{aligned}
\left\{\mathcal{D}_\alpha^{(1)\bar{\imath}}, \mathcal{D}_\beta^{(1)\bar{\jmath}}\right\} &= 2\mathrm{i} \varepsilon_{\alpha\beta} \varepsilon^{\bar{\imath}\bar{\jmath}} X \mathbf{L}^{(2)}, \mathbf{L}^{(2)} := v_i v_j \mathbf{L}^{ij} \\
\left\{\mathcal{D}_\alpha^{(\bar{1})i}, \mathcal{D}_\beta^{(\bar{1})j}\right\} &= -2\mathrm{i} \varepsilon_{\alpha\beta} \varepsilon^{ij} X \mathbf{R}^{(\bar{2})}, \mathbf{R}^{(\bar{2})} := v_{\bar{\imath}} v_{\bar{\jmath}} \mathbf{R}^{\bar{\imath}\bar{\jmath}}
\end{aligned}$$

$$Q_{\mathrm{L}}^{(n)}(z, c v_{\mathrm{L}}) = c^n Q_{\mathrm{L}}^{(n)}(z, v_{\mathrm{L}}), c \in \mathbb{C}^* \equiv \mathbb{C} \setminus \{0\}$$

$$\delta_\xi Q_{\rm L}^{(n)} = (\xi + \Lambda^{ij}\mathbf{L}_{ij})Q_{\rm L}^{(n)}$$

$$\Lambda^{ij}\mathbf{L}_{ij}Q_{\rm L}^{(n)} = -\left(\Lambda^{(2)}\boldsymbol{\partial}_{\rm L}^{(-2)} - n\Lambda^{(0)}\right)Q_{\rm L}^{(n)}, \boldsymbol{\partial}_{\rm L}^{(-2)} := \frac{1}{(v_{\rm L}, u_{\rm L})}u^i\frac{\partial}{\partial v^i}$$

$$\Lambda^{(2)} := \Lambda^{ij}v_i v_j, \Lambda^{(0)} := \Lambda^{ij}\frac{v_i u_j}{(v_{\rm L}, u_{\rm L})}, (v_{\rm L}, u_{\rm L}) := v^i u_i$$

$$\mathcal{D}_\alpha^{(1)\bar{\imath}}Q_{\rm L}^{(n)} = 0$$

$$\boldsymbol{\partial}_{\rm L}^{(2)}Q_{\rm L}^{(n)} = 0 \Longrightarrow \boldsymbol{\partial}_{\rm L}^{(2)}\delta_\xi Q_{\rm L}^{(n)} = 0, \boldsymbol{\partial}_{\rm L}^{(2)} := \frac{1}{(v_{\rm L}, u_{\rm L})}v^i\frac{\partial}{\partial u^i}$$

$$\mathbf{L}^{(2)}Q_{\rm L}^{(n)} = 0 \Longrightarrow \left\{\mathcal{D}_\alpha^{(1)\bar{\imath}}, \mathcal{D}_\beta^{(1)\bar{\jmath}}\right\}Q_{\rm L}^{(n)} = 0$$

$$Q_{\rm L}^{(n)}(v^i) \longrightarrow \bar{Q}_{\rm L}^{(n)}(\bar{v}_i) \longrightarrow \bar{Q}_{\rm L}^{(n)}(\bar{v}_i \to -v_i) =: \breve{Q}_{\rm L}^{(n)}(v^i),$$

$$\breve{Q}_{\rm L}^{(n)}(v_{\rm L}) = (-1)^n Q_{\rm L}^{(n)}(v_{\rm L})$$

$$v^i = v^1(1, \zeta_{\rm L}), v_i = v^1(-\zeta_{\rm L}, 1)$$

$$Q_{\rm L}^{(n)}(z, v_{\rm L}) \longrightarrow Q_{\rm L}^{[n]}(z, \zeta_{\rm L}) \propto Q_{\rm L}^{(n)}(z, v_{\rm L}), \frac{\partial}{\partial\bar{\zeta}_{\rm L}}Q_{\rm L}^{[n]} = 0$$

$$\mathcal{D}_\alpha^{1\bar{2}}Q_{\rm L}^{[n]} = \frac{1}{\zeta_{\rm L}}\mathcal{D}_\alpha^{2\bar{2}}Q_{\rm L}^{[n]}, \mathcal{D}_\alpha^{2\bar{1}}Q_{\rm L}^{[n]} = \zeta_{\rm L}\mathcal{D}_\alpha^{1\bar{1}}Q_{\rm L}^{[n]}$$

$$Q_{\rm L}^{[n]}(z, \zeta_{\rm L}) = \sum_q^p Q_k(z)\zeta_{\rm L}^k, -\infty \le q < p \le +\infty$$

$$\Upsilon_{\rm L}^{(n)}(v_{\rm L}) = (v^1)^n\Upsilon_{\rm L}^{[n]}(\zeta_{\rm L}), \Upsilon_{\rm L}^{[n]}(\zeta_{\rm L}) = \sum_{k=0}^{\infty}\Upsilon_k\zeta_{\rm L}^k$$

$$\breve{\Upsilon}_{\rm L}^{(n)}(v_{\rm L}) = (v^2)^n\breve{\Upsilon}_{\rm L}^{[n]}(\zeta_{\rm L}) = (v^1)^n\zeta_{\rm L}^n\breve{\Upsilon}_{\rm L}^{[n]}(\zeta_{\rm L}), \breve{\Upsilon}_{\rm L}^{[n]}(\zeta_{\rm L}) = \sum_{k=0}^{\infty}\bar{\Upsilon}_k\frac{(-1)^k}{\zeta_{\rm L}^k}$$

$$G_{\rm L}^{(2n)}(v_{\rm L}) = ({\rm i}v^1v^2)^n G_{\rm L}^{[2n]}(\zeta_{\rm L}) = (v^1)^{2n}({\rm i}\zeta)^n G_{\rm L}^{[2n]}(\zeta_{\rm L})$$

$$G_{\rm L}^{[2n]}(\zeta_{\rm L}) = \sum_{k=-p}^{p} G_k\zeta_{\rm L}^k, \bar{G}_k = (-1)^k G_{-k}$$

$$G_{\rm L}^{(2n)}(v_{\rm L}) = G^{i_1\dots i_{2n}}v_{i_1}\dots v_{i_{2n}} = \breve{G}_{\rm L}^{(2n)}(v_{\rm L}).$$

$$\mathcal{D}_\alpha^{(j\bar{\jmath}}G^{i_1\dots i_{2n})} = 0$$

$$\overline{G^{i_1\dots i_{2n}}} = G_{i_1\dots i_{2n}} = \varepsilon_{i_1j_1}\cdots\varepsilon_{i_{2n}j_{2n}}G^{j_1\dots j_{2n}}$$

$$\mathcal{D}_\alpha^{(j\bar{\jmath}}Q^{i_1\dots i_n)} = 0$$

$$Q_{\mathrm{L}}^{(n)}(v_{\mathrm{L}}) = Q^{i_1 \dots i_n} v_{i_1} \dots v_{i_n}$$

$$\mathcal{D}_{\alpha}^{(\bar{1})i} Q_{\mathrm{R}}^{(n)} = 0$$

$$v^{\bar{\imath}} = v^{\bar{1}}(1, \zeta_{\mathrm{R}}), v_{\bar{\imath}} = v^{\bar{1}}(-\zeta_{\mathrm{R}}, 1)$$

$$Q_{\mathrm{R}}^{(n)}(z, v_{\mathrm{R}}) \longrightarrow Q_{\mathrm{R}}^{[n]}(z, \zeta_{\mathrm{R}}) \propto Q_{\mathrm{R}}^{(n)}(z, v_{\mathrm{R}}), \frac{\partial}{\partial \bar{\zeta}_{\mathrm{R}}} Q_{\mathrm{R}}^{[n]} = 0$$

$$\mathcal{D}_{\alpha}^{1\bar{2}} Q_{\mathrm{R}}^{[n]} = \zeta_{\mathrm{R}} \mathcal{D}_{\alpha}^{1\bar{1}} Q_{\mathrm{R}}^{[n]}, \mathcal{D}_{\alpha}^{2\bar{1}} Q_{\mathrm{R}}^{[n]} = \frac{1}{\zeta_{\mathrm{R}}} \mathcal{D}_{\alpha}^{2\bar{2}} Q_{\mathrm{R}}^{[n]}$$

$$U| := U(x, \theta_{i\bar{\jmath}})\big|_{\theta_{1\bar{2}} = \theta_{2\bar{1}} = 0},$$

$$u_i = (1,0) \implies (v_{\mathrm{L}}, u_{\mathrm{L}}) = v^1$$

$$\delta_{\xi} \Upsilon_{\mathrm{L}}^{[n]} \mid = [\tau + \mathrm{i}t\left(\zeta \frac{\partial}{\partial \zeta} - \frac{n}{2}\right) + \zeta \bar{\varepsilon}^{\alpha} \mathcal{D}_{\alpha} - \frac{1}{\zeta} \varepsilon_{\alpha} \overline{\mathcal{D}}^{\alpha} + \mathrm{i}\sigma\left(\zeta \frac{\partial}{\partial \zeta} - \frac{n}{2}\right)$$
$$+ \left(\zeta \bar{\Lambda}_{\mathrm{L}} + \frac{1}{\zeta} \Lambda_{\mathrm{L}}\right) \zeta \frac{\partial}{\partial \zeta} - n \zeta \bar{\Lambda}_{\mathrm{L}}\Big] \Upsilon_{\mathrm{L}}^{[n]} \mid$$

$$\delta_{\xi} \check{\Upsilon}_{\mathrm{L}}^{[n]} \mid = [\tau + \mathrm{i}t\left(\zeta \frac{\partial}{\partial \zeta} + \frac{n}{2}\right) + \zeta \bar{\varepsilon}^{\alpha} \mathcal{D}_{\alpha} - \frac{1}{\zeta} \varepsilon_{\alpha} \overline{\mathcal{D}}^{\alpha} + \mathrm{i}\sigma\left(\zeta \frac{\partial}{\partial \zeta} + \frac{n}{2}\right)$$
$$+ \left(\zeta \bar{\Lambda}_{\mathrm{L}} + \frac{1}{\zeta} \Lambda_{\mathrm{L}}\right) \zeta \frac{\partial}{\partial \zeta} + n \frac{1}{\zeta} \Lambda_{\mathrm{L}}\Big] \check{\Upsilon}_{\mathrm{L}}^{[n]} \mid$$

$$\Upsilon_{\mathrm{L}}^{[1]}(\zeta) \mid = \sum_{n=0}^{\infty} \zeta^n \Upsilon_n = \Phi + \zeta \Sigma + \cdots, \overline{\mathcal{D}}_{\alpha} \Phi = 0, \overline{\mathcal{D}}^2 \Sigma = 0$$
$$\check{\Upsilon}_{\mathrm{L}}^{[1]}(\zeta) \mid = \sum_{n=0}^{\infty} (-\zeta)^{-n} \bar{\Upsilon}_n$$

$$\hat{\mathcal{Z}} \Phi = -\frac{1}{2} \Phi, \hat{\mathcal{Z}} \Sigma = \frac{1}{2} \Sigma$$

$$\delta \Phi = \varepsilon^{\alpha} \overline{\mathcal{D}}_{\alpha} \Sigma + \Lambda_{\mathrm{L}} \Sigma = -\frac{1}{2} \overline{\mathcal{D}}^2 (\bar{\rho}_{\mathrm{L}} \Sigma)$$
$$\delta \Sigma = (\bar{\varepsilon}^{\alpha} \mathcal{D}_{\alpha} - \bar{\Lambda}_{\mathrm{L}}) \Phi - \overline{\mathcal{D}}_{\alpha} (\varepsilon^{\alpha} \Upsilon_2) = \overline{\mathcal{D}}_{\alpha} \left\{ \frac{\mathrm{i}}{2X} \mathcal{D}^{\alpha} (\bar{\Lambda}_{\mathrm{L}} \Phi) - \varepsilon^{\alpha} \Upsilon_2 \right\}$$

$$\delta_{\xi} G_{\mathrm{L}}^{[2n]} \mid = [\tau + \mathrm{i}t\zeta \frac{\partial}{\partial \zeta} + \zeta \bar{\varepsilon}^{\alpha} \mathcal{D}_{\alpha} - \frac{1}{\zeta} \varepsilon_{\alpha} \overline{\mathcal{D}}^{\alpha} + \mathrm{i}\sigma \zeta \frac{\partial}{\partial \zeta}$$
$$+ \left(\bar{\Lambda}_{\mathrm{L}} \zeta + \Lambda_{\mathrm{L}} \frac{1}{\zeta}\right) \zeta \frac{\partial}{\partial \zeta} + n \left(\Lambda_{\mathrm{L}} \frac{1}{\zeta} - \bar{\Lambda}_{\mathrm{L}} \zeta\right)\Big] G_{\mathrm{L}}^{[2n]} \mid$$

$$\mathcal{D}_{\alpha}^{(i\bar{\imath}} W^{kl)} = 0$$

$$W_{\mathrm{L}}^{(2)}(v) = \mathrm{i}(v^1)^2 \zeta W_{\mathrm{L}}^{[2]}(\zeta)$$
$$W_{\mathrm{L}}^{[2]}(\zeta) = \frac{1}{\zeta} \varphi + G - \zeta \bar{\varphi}, \overline{\mathcal{D}}_{\alpha} \varphi = 0, \overline{\mathcal{D}}^2 G = 0$$

$$\varphi := -\mathrm{i}W^{22}|, G := 2\mathrm{i}W^{12}|, \bar{\varphi} = \mathrm{i}W^{11}\,|.$$

$$\hat{\mathcal{Z}}\varphi = -\varphi, \hat{\mathcal{Z}}G = 0$$

$$\begin{aligned}
\delta\varphi &= \left(\varepsilon^{\alpha}\overline{\mathcal{D}}_{\alpha} + \Lambda_{\mathrm{L}}\right)G = -\frac{1}{2}\overline{\mathcal{D}}^{2}(\bar{\rho}_{\mathrm{L}}G) \\
\delta G &= (\bar{\varepsilon}^{\alpha}\mathcal{D}_{\alpha} - 2\bar{\Lambda}_{\mathrm{L}})\varphi - \left(\varepsilon^{\alpha}\overline{\mathcal{D}}_{\alpha} + 2\Lambda_{\mathrm{L}}\right)\bar{\varphi} \\
&= \mathcal{D}^{\alpha}(\bar{\varepsilon}_{\alpha}\varphi) + \overline{\mathcal{D}}_{\alpha}(\varepsilon^{\alpha}\bar{\varphi}) = \mathcal{D}^{\alpha}(\varphi\mathcal{D}_{\alpha}\rho_{\mathrm{L}}) + \overline{\mathcal{D}}_{\alpha}\left(\bar{\varphi}\overline{\mathcal{D}}^{\alpha}\bar{\rho}_{\mathrm{L}}\right) \\
&= -\mathcal{D}^{\alpha}\overline{\mathcal{D}}_{\alpha}(\bar{\rho}_{\mathrm{R}}\varphi - \rho_{\mathrm{R}}\bar{\varphi})
\end{aligned}$$

$$\mathcal{D}_{\alpha}^{(i\bar{\imath}}q^{j)} = 0$$

$$Q_{+} := q^{2}\big|, \overline{\mathcal{D}}_{\alpha}Q_{+} = 0;\ \bar{Q}_{-} := q^{1}\big|, \mathcal{D}_{\alpha}\bar{Q}_{-} = 0$$

$$\hat{\mathcal{Z}}Q_{+} = -\frac{1}{2}Q_{+}, \hat{\mathcal{Z}}\bar{Q}_{-} = \frac{1}{2}\bar{Q}_{-}$$

$$\delta Q_{+} = \frac{1}{2}\overline{\mathcal{D}}^{2}(\bar{\rho}_{\mathrm{L}}\bar{Q}_{-}), \delta\bar{Q}_{-} = -\frac{1}{2}\mathcal{D}^{2}(\rho_{\mathrm{L}}Q_{+})$$

$$\delta Q_{\pm} = \pm\frac{1}{2}\overline{\mathcal{D}}^{2}(\bar{\rho}_{\mathrm{L}}\bar{Q}_{\mp})$$

$$\hat{\mathcal{Z}}Q_{\pm} = -\frac{1}{2}Q_{\pm}$$

$$\delta_{\xi}Q_{\mathrm{R}}^{(n)} = \left(\xi + \Lambda^{\bar{\imath}\bar{\jmath}}\mathbf{R}_{\bar{\imath}\bar{\jmath}}\right)Q_{\mathrm{R}}^{(n)}$$

$$\begin{aligned}
\delta_{\xi}\Upsilon_{\mathrm{R}}^{[n]}\,| &= [\tau + \mathrm{i}t\left(\zeta\frac{\partial}{\partial\zeta} - \frac{n}{2}\right) - \zeta\varepsilon^{\alpha}\mathcal{D}_{\alpha} + \frac{1}{\zeta}\bar{\varepsilon}_{\alpha}\overline{\mathcal{D}}^{\alpha} - \mathrm{i}\sigma\left(\zeta\frac{\partial}{\partial\zeta} - \frac{n}{2}\right) \\
&+ \left(\zeta\bar{\Lambda}_{\mathrm{R}} + \frac{1}{\zeta}\Lambda_{\mathrm{R}}\right)\zeta\frac{\partial}{\partial\zeta} - n\zeta\bar{\Lambda}_{\mathrm{R}}\Big]\Upsilon_{\mathrm{L}}^{[n]}\,| \\
\delta_{\xi}\breve{\Upsilon}_{\mathrm{R}}^{[n]}\,| &= [\tau + \mathrm{i}t\left(\zeta\frac{\partial}{\partial\zeta} + \frac{n}{2}\right) - \zeta\varepsilon^{\alpha}\mathcal{D}_{\alpha} + \frac{1}{\zeta}\bar{\varepsilon}_{\alpha}\overline{\mathcal{D}}^{\alpha} - \mathrm{i}\sigma\left(\zeta\frac{\partial}{\partial\zeta} + \frac{n}{2}\right) \\
&+ \left(\zeta\bar{\Lambda}_{\mathrm{R}} + \frac{1}{\zeta}\Lambda_{\mathrm{R}}\right)\zeta\frac{\partial}{\partial\zeta} + n\frac{1}{\zeta}\Lambda_{\mathrm{R}}\Big]\breve{\Upsilon}_{\mathrm{R}}^{[n]}\,|
\end{aligned}$$

$$\begin{aligned}
\Upsilon_{\mathrm{R}}^{[1]}(\zeta)\,| &= \sum_{n=0}^{\infty}\zeta^{n}\Upsilon_{n} = \Phi + \zeta\Sigma + \cdots, \overline{\mathcal{D}}_{\alpha}\Phi = 0, \overline{\mathcal{D}}^{2}\Sigma = 0 \\
\breve{\Upsilon}_{\mathrm{R}}^{[1]}(\zeta)\,| &= \sum_{n=0}^{\infty}(-\zeta)^{-n}\bar{\Upsilon}_{n}
\end{aligned}$$

$$\hat{\mathcal{Z}}\Phi = \frac{1}{2}\Phi, \hat{\mathcal{Z}}\Sigma = -\frac{1}{2}\Sigma$$

$$\begin{aligned}
\delta\Phi &= \bar{\varepsilon}_{\alpha}\overline{\mathcal{D}}^{\alpha}\Sigma + \Lambda_{\mathrm{R}}\Sigma = -\frac{1}{2}\overline{\mathcal{D}}^{2}(\bar{\rho}_{\mathrm{R}}\Sigma) \\
\delta\Sigma &= -(\varepsilon^{\alpha}\mathcal{D}_{\alpha} + \bar{\Lambda}_{\mathrm{R}})\Phi - \overline{\mathcal{D}}_{\alpha}(\varepsilon^{\alpha}\Upsilon_{2})
\end{aligned}$$

$$G_{\mathrm{R}}^{(2n)}(v_{\mathrm{R}}) = \left(\mathrm{i}v^{\bar{1}}v^{\bar{2}}\right)^{n} G_{\mathrm{R}}^{[2n]}(\zeta) = \left(v^{\bar{1}}\right)^{2n} (\mathrm{i}\zeta)^{n} G_{\mathrm{R}}^{[2n]}(\zeta)$$

$$G_{\mathrm{R}}^{[2n]}(\zeta) = \sum_{k=-p}^{p} G_k \zeta^k, \bar{G}_k = (-1)^k G_{-k}$$

$$\delta_{\xi} G_{\mathrm{R}}^{[2n]} | = [\tau + \mathrm{i}t\zeta \frac{\partial}{\partial\zeta} - \zeta\varepsilon^{\alpha}\mathcal{D}_{\alpha} + \frac{1}{\zeta}\bar{\varepsilon}_{\alpha}\overline{\mathcal{D}}^{\alpha} - \mathrm{i}\sigma\zeta\frac{\partial}{\partial\zeta} + \left(\bar{\Lambda}_{\mathrm{R}}\zeta + \Lambda_{\mathrm{R}}\frac{1}{\zeta}\right)\zeta\frac{\partial}{\partial\zeta} + n\left(\Lambda_{\mathrm{R}}\frac{1}{\zeta} - \bar{\Lambda}_{\mathrm{R}}\zeta\right)\Big] G_{\mathrm{R}}^{[2n]} |$$

$$\mathcal{D}_{\alpha}^{i(\bar{\imath}} W^{\bar{k}\bar{l})} = 0$$

$$W_{\mathrm{R}}^{(2)}(v) = \mathrm{i}\left(v^{\bar{1}}\right)^{2} \zeta W_{\mathrm{R}}^{[2]}(\zeta)$$

$$W_{\mathrm{R}}^{[2]}(\zeta) = \frac{1}{\zeta}\varphi + G - \zeta\bar{\varphi}, \overline{\mathcal{D}}_{\alpha}\varphi = 0, \overline{\mathcal{D}}^{2} G = 0$$

$$\varphi := -\mathrm{i}W^{\overline{22}}\Big|, G := 2\mathrm{i}W^{\overline{12}}\Big|, \bar{\varphi} = \mathrm{i}W^{\overline{11}} |.$$

$$\hat{\mathcal{Z}}\varphi = \varphi, \hat{\mathcal{Z}}G = 0$$

$$\delta\varphi = -\frac{1}{2}\overline{\mathcal{D}}^{2}(\bar{\rho}_{\mathrm{R}}G), \delta G = -\mathcal{D}^{\alpha}(\varepsilon_{\alpha}\varphi) - \overline{\mathcal{D}}_{\alpha}(\bar{\varepsilon}^{\alpha}\bar{\varphi})$$

$$\delta\varphi = \frac{\mathrm{i}}{8X}\overline{\mathcal{D}}^{2}(G\mathcal{D}^{2}\rho_{\mathrm{L}}), \delta G = \mathcal{D}^{\alpha}\overline{\mathcal{D}}_{\alpha}(\rho_{\mathrm{L}}\bar{\varphi} - \bar{\rho}_{\mathrm{L}}\varphi)$$

$$\mathcal{D}_{\alpha}^{i(\bar{\imath}} q^{\bar{\jmath})} = 0$$

$$Q_{+} = q^{\bar{2}}\Big|, \overline{\mathcal{D}}_{\alpha}Q_{+} = 0;\ \bar{Q}_{-} = q^{\bar{1}}\Big|, \mathcal{D}_{\alpha}\bar{Q}_{-} = 0$$

$$\hat{\mathcal{Z}}Q_{\pm} = \frac{1}{2}Q_{\pm}, \hat{\mathcal{Z}}\bar{Q}_{\pm} = -\frac{1}{2}\bar{Q}_{\pm}$$

$$\delta Q_{\pm} = \pm\frac{1}{2}\overline{\mathcal{D}}^{2}(\bar{\rho}_{\mathrm{R}}\bar{Q}_{\mp}), \delta\bar{Q}_{\pm} = \pm\frac{1}{2}\mathcal{D}^{2}(\rho_{\mathrm{R}}Q_{\mp}).$$

$$S = S_{\mathrm{left}} + S_{\mathrm{right}}$$

$$S\left(\mathcal{L}_{\mathrm{L}}^{(2)}\right) = \int \mathrm{d}^{3|4}z \oint_{C} \frac{\mathrm{d}\zeta}{2\pi\mathrm{i}\zeta}\mathcal{L}_{\mathrm{L}}^{[2]}, \mathcal{L}_{\mathrm{L}}^{(2n)}(v_{\mathrm{L}}) = \mathrm{i}\zeta(v^{1})^{2}\mathcal{L}_{\mathrm{L}}^{[2n]}(\zeta_{\mathrm{L}})$$

$$\delta_{\xi}\mathcal{L}_{\mathrm{L}}^{[2]} = [\tau + \mathrm{i}t\zeta\frac{\partial}{\partial\zeta} + \zeta\bar{\varepsilon}^{\alpha}\mathcal{D}_{\alpha} - \frac{1}{\zeta}\varepsilon_{\alpha}\overline{\mathcal{D}}^{\alpha} + \mathrm{i}\sigma\zeta\frac{\partial}{\partial\zeta} + \left(\bar{\Lambda}_{\mathrm{L}}\zeta + \Lambda_{\mathrm{L}}\frac{1}{\zeta}\right)\zeta\frac{\partial}{\partial\zeta} + \left(\Lambda_{\mathrm{L}}\frac{1}{\zeta} - \bar{\Lambda}_{\mathrm{L}}\zeta\right)\Big]\mathcal{L}_{\mathrm{L}}^{[2]}$$

$$\mathcal{L}_{\mathrm{L}}^{(2)} = \mathrm{i}\mathfrak{K}_{\mathrm{L}}\left(\Upsilon_{\mathrm{L}}^{(1)}, \breve{\Upsilon}_{\mathrm{L}}^{(1)}\right) \Longrightarrow \mathcal{L}_{\mathrm{L}}^{[2]}(\Upsilon_{\mathrm{L}}, \breve{\Upsilon}_{\mathrm{L}}; \zeta) = \frac{1}{\zeta}\mathfrak{K}_{\mathrm{L}}(\Upsilon_{\mathrm{L}}, \zeta\breve{\Upsilon}_{\mathrm{L}})$$

$$\left(\Phi^{I}\frac{\partial}{\partial\Phi^{I}}+\bar{\Omega}^{\bar{I}}\frac{\partial}{\partial\bar{\Omega}^{\bar{I}}}\right)\mathfrak{K}(\Phi,\bar{\Omega})=2\mathfrak{K}(\Phi,\bar{\Omega})$$

$$\bar{\mathfrak{K}}(\bar{\Phi},-\Omega)=-\mathfrak{K}(\Omega,\bar{\Phi})$$

$$\Upsilon_{\mathrm{L}}^{I}(\zeta)=\sum_{n=0}^{\infty}\zeta^{n}\Upsilon_{\mathrm{L}n}^{I}=\Phi_{\mathrm{L}}^{I}+\zeta\Sigma_{\mathrm{L}}^{I}+\cdots,\breve{\Upsilon}_{\mathrm{L}}^{\bar{I}}(\zeta)=\sum_{n=0}^{\infty}(-\zeta)^{-n}\bar{\Upsilon}_{\mathrm{L}n}^{\bar{I}}$$

$$\frac{\partial\mathbb{L}_{\mathrm{L}}}{\partial\Upsilon_{\mathrm{L}n}^{I}}=0,n=2,3,\ldots,\mathbb{L}_{\mathrm{L}}:=\oint_{C}\frac{\mathrm{d}\zeta}{2\pi\mathrm{i}\zeta}\mathcal{L}_{\mathrm{L}}^{[2]}$$

$$\left(\Phi_{\mathrm{L}}^{I}\frac{\partial}{\partial\Phi_{\mathrm{L}}^{I}}-\Sigma_{\mathrm{L}}^{I}\frac{\partial}{\partial\Sigma_{\mathrm{L}}^{I}}\right)\mathbb{L}_{\mathrm{L}}=\left(\bar{\Phi}_{\mathrm{L}}^{\bar{I}}\frac{\partial}{\partial\bar{\Phi}_{\mathrm{L}}^{\bar{I}}}-\bar{\Sigma}_{\mathrm{L}}^{\bar{I}}\frac{\partial}{\partial\bar{\Sigma}_{\mathrm{L}}^{\bar{I}}}\right)\mathbb{L}_{\mathrm{L}}.$$

$$\left(\Phi^{I}\frac{\partial}{\partial\Phi^{I}}+\Sigma^{I}\frac{\partial}{\partial\Sigma^{I}}+\bar{\Phi}^{\bar{I}}\frac{\partial}{\partial\bar{\Phi}^{\bar{I}}}+\bar{\Sigma}^{\bar{I}}\frac{\partial}{\partial\bar{\Sigma}^{\bar{I}}}\right)\mathbb{L}_{\mathrm{L}}=2\mathbb{L}_{\mathrm{L}}.$$

$$\left(\Phi_{\mathrm{L}}^{I}\frac{\partial}{\partial\Phi_{\mathrm{L}}^{I}}+\bar{\Sigma}_{\mathrm{L}}^{\bar{I}}\frac{\partial}{\partial\bar{\Sigma}_{\mathrm{L}}^{\bar{I}}}\right)\mathbb{L}_{\mathrm{L}}=\mathbb{L}_{\mathrm{L}}$$

$$S_{\text{dual}}=\int\mathrm{d}^{3|4}z\mathbb{K}_{\mathrm{L}}(\Phi_{\mathrm{L}},\Psi_{\mathrm{L}},\bar{\Phi}_{\mathrm{L}},\bar{\Psi}_{\mathrm{L}}),\mathbb{K}_{\mathrm{L}}=\mathbb{L}_{\mathrm{L}}+\Sigma_{\mathrm{L}}^{I}\Psi_{\mathrm{L}I}+\bar{\Sigma}_{\mathrm{L}}^{\bar{J}}\bar{\Psi}_{\mathrm{L}\bar{J}}$$

$$\phi_{\mathrm{L}}^{\mathrm{a}}\frac{\partial}{\partial\phi_{\mathrm{L}}^{\mathrm{a}}}\mathbb{K}_{\mathrm{L}}=\mathbb{K}_{\mathrm{L}},\phi_{\mathrm{L}}^{\mathrm{a}}=\left(\Phi_{\mathrm{L}}^{I},\Psi_{\mathrm{L}I}\right)$$

$$\hat{\mathcal{Z}}\phi_{\mathrm{L}}^{\mathrm{a}}=-\frac{1}{2}\phi_{\mathrm{L}}^{\mathrm{a}}$$

$$\omega_{\mathrm{ab}}=\begin{pmatrix}0&\delta_{I}^{J}\\-\delta_{J}^{I}&0\end{pmatrix}$$

$$\delta\Phi_{\mathrm{L}}^{I}=-\frac{1}{2}\bar{\mathcal{D}}^{2}\left(\bar{\rho}_{\mathrm{L}}\frac{\partial\mathbb{K}_{\mathrm{L}}}{\partial\Psi_{\mathrm{L}I}}\right),\delta\Psi_{\mathrm{L}I}=\frac{1}{2}\bar{\mathcal{D}}^{2}\left(\bar{\rho}_{\mathrm{L}}\frac{\partial\mathbb{K}_{\mathrm{L}}}{\partial\Phi_{\mathrm{L}}^{I}}\right)$$

$$\delta\phi_{\mathrm{L}}^{\mathrm{a}}=-\frac{1}{2}\bar{\mathcal{D}}^{2}\left(\bar{\rho}_{\mathrm{L}}\omega^{\mathrm{ab}}\partial_{\mathrm{b}}\mathbb{K}_{\mathrm{L}}\right)$$

$$\Phi^{I}\frac{\partial}{\partial\Phi^{I}}K(\Phi,\bar{\Phi})=K(\Phi,\bar{\Phi}),\bar{\Phi}^{\bar{I}}\frac{\partial}{\partial\bar{\Phi}^{\bar{I}}}K(\Phi,\bar{\Phi})=K(\Phi,\bar{\Phi}).$$

$$\mathcal{L}_{\mathrm{L}}^{[2]}(\Upsilon_{\mathrm{L}},\breve{\Upsilon}_{\mathrm{L}};\zeta)=\frac{1}{\zeta}\mathfrak{K}_{\mathrm{L}}\left(\Upsilon_{\mathrm{L}},\zeta\breve{\Upsilon}_{\mathrm{L}}\right)=K\left(\Upsilon_{\mathrm{L}},\breve{\Upsilon}_{\mathrm{L}}\right)$$

$$\omega_{\mathrm{L}}^{\mathrm{ab}}\omega_{\mathrm{Rbc}}=0,\omega_{\mathrm{L}}^{\mathrm{ab}}\omega_{\mathrm{Lbc}}+\omega_{\mathrm{R}}^{\mathrm{ab}}\omega_{\mathrm{Rbc}}=-\delta_{\mathrm{c}}^{\mathrm{a}}$$

$$(P_{\mathrm{L}})^{\mathrm{a}}{}_{\mathrm{b}}=-\omega_{\mathrm{L}}^{\mathrm{ac}}\omega_{\mathrm{Lcb}},(P_{\mathrm{R}})^{\mathrm{a}}{}_{\mathrm{b}}=-\omega_{\mathrm{R}}^{\mathrm{ac}}\omega_{\mathrm{Rcb}}$$
$$P_{\mathrm{L}}P_{\mathrm{R}}=0,P_{\mathrm{L}}+P_{\mathrm{R}}=\mathbb{1}.$$

$$\chi=\chi^{\mathrm{a}}(\phi)\frac{\partial}{\partial\phi^{\mathrm{a}}}+\bar{\chi}^{\bar{\mathrm{a}}}(\bar{\phi})\frac{\partial}{\partial\bar{\phi}^{\bar{\mathrm{a}}}}.$$

$$\chi_{\mathrm{L}}^{\mathrm{a}}=(P_{\mathrm{L}})^{\mathrm{a}}{}_{\mathrm{b}}\chi^{\mathrm{b}},\chi_{\mathrm{R}}^{\mathrm{a}}=(P_{\mathrm{R}})^{\mathrm{a}}{}_{\mathrm{b}}\chi^{\mathrm{b}},$$

$$K=\chi^{\mathrm{a}}\chi_{\mathrm{a}}=\chi_{\mathrm{L}}^{\mathrm{a}}\chi_{\mathrm{La}}+\chi_{\mathrm{R}}^{\mathrm{a}}\chi_{\mathrm{Ra}}=K_{\mathrm{L}}+K_{\mathrm{R}}$$

$$\Delta=\Delta^{\mathrm{a}}(\phi)\frac{\partial}{\partial\phi^{\mathrm{a}}}+\bar{\Delta}^{\bar{\mathrm{a}}}(\bar{\phi})\frac{\partial}{\partial\bar{\phi}^{\bar{\mathrm{a}}}}=\mathrm{i}\hat{\mathcal{Z}}$$

$$\Delta K=\Delta^{\mathrm{a}}K_{\mathrm{a}}+\bar{\Delta}^{\bar{\mathrm{a}}}K_{\bar{\mathrm{a}}}=0$$

$$\Delta^{\mathrm{a}}=-\frac{\mathrm{i}}{2}\chi_{\mathrm{L}}^{\mathrm{a}}+\frac{\mathrm{i}}{2}\chi_{\mathrm{R}}^{\mathrm{a}}$$

$$\delta\phi^{\mathrm{a}}=-\frac{1}{2}\bar{\mathcal{D}}^{2}(\bar{\rho}_{\mathrm{L}}\Omega_{\mathrm{L}}^{\mathrm{a}})-\frac{1}{2}\bar{\mathcal{D}}^{2}(\bar{\rho}_{\mathrm{R}}\Omega_{\mathrm{R}}^{\mathrm{a}}),$$

$$K(\phi,\bar{\phi})=K_{\mathrm{L}}(\phi,\bar{\phi})+K_{\mathrm{R}}(\phi,\bar{\phi})$$

$$\varphi^{\mathrm{a}}:=\phi^{\mathrm{a}}\Big|,\psi_{\alpha}^{\mathrm{a}}:=\frac{1}{\sqrt{2}}\mathcal{D}_{\alpha}\phi^{\mathrm{a}}\Big|$$

$$F^{\mathrm{a}}:=-\frac{1}{4}g^{\mathrm{a}\bar{b}}\mathcal{D}^{2}K_{\bar{b}}\Big|=-\frac{1}{4}\left(\mathcal{D}^{2}\phi^{\mathrm{a}}+\Gamma_{\mathrm{bc}}^{\mathrm{a}}\mathcal{D}^{\alpha}\phi^{\mathrm{b}}\mathcal{D}_{\alpha}\phi^{\mathrm{c}}\right)\Big|$$

$$\hat{\mathcal{D}}_{m}\psi_{\alpha}^{\mathrm{a}}:=\mathcal{D}_{m}\psi_{\alpha}^{\mathrm{a}}+\Gamma^{\mathrm{a}}{}_{\mathrm{bc}}\mathcal{D}_{m}\varphi^{\mathrm{b}}\psi_{\alpha}^{\mathrm{c}}$$

$$\Delta\varphi^{\mathrm{a}}=\Delta^{\mathrm{a}}(\varphi),\Delta\psi_{\alpha}^{\mathrm{a}}=\psi_{\alpha}^{\mathrm{b}}\partial_{\mathrm{b}}\Delta^{\mathrm{a}}(\varphi)$$

$$L=-g_{\mathrm{a}\bar{\mathrm{a}}}\mathcal{D}_{m}\varphi^{\mathrm{a}}\mathcal{D}^{m}\bar{\varphi}^{\bar{\mathrm{a}}}-\mathrm{i}g_{\mathrm{a}\bar{\mathrm{a}}}\bar{\psi}_{\dot{\alpha}}^{\bar{\mathrm{a}}}\hat{\mathcal{D}}^{\alpha\beta}\psi_{\beta}^{\mathrm{a}}+\frac{1}{4}R_{\mathrm{a}\bar{\mathrm{a}}\,\mathrm{b}\,\bar{\mathrm{b}}}(\psi^{\mathrm{a}}\psi^{\mathrm{b}})\left(\bar{\psi}^{\bar{\mathrm{a}}}\bar{\psi}^{\bar{\mathrm{b}}}\right)$$
$$-\frac{\mathrm{i}}{2}X\left(\psi^{\mathrm{a}}\bar{\psi}^{\bar{\mathrm{b}}}\right)(P_{\mathrm{L}}-P_{\mathrm{R}})_{\mathrm{a}\,\bar{\mathrm{b}}}-\frac{1}{4}X^{2}(K_{\mathrm{L}}+K_{\mathrm{R}})$$

$$V=\frac{1}{4}X^{2}(K_{\mathrm{L}}+K_{\mathrm{R}})$$

$$K(\phi,\bar{\phi}):=g_{\mathrm{aa}}(\phi,\bar{\phi})\chi^{\mathrm{a}}(\phi)\bar{\chi}^{\bar{\mathrm{b}}}(\bar{\phi})$$

$$\delta V_{\mathrm{L}}=\mathrm{i}(\breve{\lambda}_{\mathrm{L}}-\lambda_{\mathrm{L}})$$

$$W^{\bar{\imath}\bar{\jmath}}=\frac{\mathrm{i}}{4}\mathcal{D}^{ij\bar{\imath}\bar{\jmath}}\oint_{\gamma}\frac{(v_{\mathrm{L}},\mathrm{d}v_{\mathrm{L}})}{2\pi}\frac{u_{i}u_{j}}{(v_{\mathrm{L}},u_{\mathrm{L}})^{2}}V_{\mathrm{L}}(v_{\mathrm{L}})=W^{\bar{\jmath}\bar{\imath}}=\overline{W_{\bar{\imath}\bar{\jmath}}},$$

$$\mathcal{D}^{ij\bar{\imath}\bar{\jmath}}:=\mathcal{D}^{\alpha(i(\bar{\imath}}\mathcal{D}_{\alpha}^{j)\bar{\jmath})}$$

$$\mathcal{D}_{\alpha}^{i(\bar{\imath}}W^{\bar{k}\bar{l})}=0$$

$$\delta V_{\mathrm{R}}=\mathrm{i}(\breve{\lambda}_{\mathrm{R}}-\lambda_{\mathrm{R}})$$

$$W^{ij}=\frac{\mathrm{i}}{4}\mathcal{D}^{ij\bar{\imath}\bar{\jmath}}\oint_{\gamma}\frac{(v_{\mathrm{R}},\mathrm{d}v_{\mathrm{R}})}{2\pi}\frac{u_{\bar{\imath}}u_{\bar{\jmath}}}{(v_{\mathrm{R}},u_{\mathrm{R}})^{2}}V_{\mathrm{R}}(v_{\mathrm{R}})=W^{ji}=\overline{W_{ij}}$$

$$\mathcal{D}_{\alpha}^{(i\bar{\imath}}W^{kl)}=0$$

$$\mathcal{L}_{\mathrm{L}}^{(2)} = \mathcal{F}_{\mathrm{L}}\left(W_I^{(2)}\right), W_I^{(2)} \frac{\partial}{\partial W_I^{(2)}} \mathcal{F}_{\mathrm{L}} = \mathcal{F}_{\mathrm{L}}$$

$$\mathcal{L}_{\mathrm{L}}^{(2)} = -W_{\mathrm{L}}^{(2)} \ln \frac{W_{\mathrm{L}}^{(2)}}{\mathrm{i}\boldsymbol{\Upsilon}_{\mathrm{L}}^{(1)} \breve{\boldsymbol{\Upsilon}}_{\mathrm{L}}^{(1)}}$$

$$\mathcal{L}_{\mathrm{L}}^{(2)} = \mathrm{i}\breve{\Upsilon}_{\mathrm{L}}^{(1)} \mathrm{e}^{eV_{\mathrm{L}}} \Upsilon_{\mathrm{L}}^{(1)} - W_{\mathrm{L}}^{(2)} \ln \frac{W_{\mathrm{L}}^{(2)}}{\mathrm{i}\Upsilon_{\mathrm{L}}^{(1)} \breve{\Upsilon}_{\mathrm{L}}^{(1)}}$$

$$\delta \Upsilon_{\mathrm{L}}^{(1)} = e\lambda_{\mathrm{L}} \Upsilon_{\mathrm{L}}^{(1)}$$

$$\mathbb{L}_{\mathrm{L}} = \oint_C \frac{\mathrm{d}\zeta}{2\pi \mathrm{i}\zeta} \mathcal{L}_{\mathrm{L}}^{[2]}$$

$$\left(G_I \frac{\partial}{\partial G_I} + \varphi_I \frac{\partial}{\partial \varphi_I} + \bar{\varphi}_I \frac{\partial}{\partial \bar{\varphi}_I}\right) L = L$$

$$\varphi_I \frac{\partial L}{\partial \varphi_I} - \bar{\varphi}_I \frac{\partial L}{\partial \bar{\varphi}_I} = 0$$

$$\frac{\partial^2 L}{\partial G_I \partial \varphi_J} - \frac{\partial^2 L}{\partial G_J \partial \varphi_I} = 0$$

$$L(G, \varphi, \bar{\varphi}) = \sqrt{G^2 + 4\varphi\bar{\varphi}} - G \ln\left(\frac{G + \sqrt{G^2 + 4\varphi\bar{\varphi}}}{\sqrt{4\varphi\bar{\varphi}}}\right)$$

$$\boldsymbol{W}^{ij} := -\frac{\mathrm{i}}{12} \mathcal{D}^{ij\bar{\imath}\bar{\jmath}} W_{\bar{\imath}\bar{\jmath}}$$

$$\mathcal{D}_\alpha^{(i\bar{\imath}} \boldsymbol{W}^{kl)} = 0$$

$$\mathcal{L}_{\mathrm{L}}^{(2)} = \mathrm{i}\breve{\Upsilon}_{\mathrm{L}}^{(1)} \mathrm{e}^{eV_{\mathrm{L}}} \Upsilon_{\mathrm{L}}^{(1)} + \frac{1}{2g^2} V_{\mathrm{L}} \boldsymbol{W}_{\mathrm{L}}^{(2)}$$

$$\mathcal{L}_{\mathrm{L}}^{(2)} = V_{\mathrm{L}} W_{\mathrm{L}}^{(2)} \Leftrightarrow \mathcal{L}_{\mathrm{R}}^{(2)} = V_{\mathrm{R}} W_{\mathrm{R}}^{(2)},$$

$$G^{ij} = \frac{\mathrm{i}}{4}\left(\mathcal{D}^{ij\bar{\imath}\bar{\jmath}} + 8\mathrm{i}\mathcal{S}^{ij\bar{\imath}\bar{\jmath}}\right)\left(\frac{W_{\bar{\imath}\bar{\jmath}}}{W_{\mathrm{R}}}\right), W_{\mathrm{R}} := \sqrt{W^{\bar{\imath}\bar{\jmath}} W_{\bar{\imath}\bar{\jmath}}}$$

$$W_{\bar{\imath}\bar{\jmath}} = \frac{\mathrm{i}}{\sqrt{2}\kappa} (\sigma_1)_{\bar{\imath}\bar{\jmath}} + \Delta W_{\bar{\imath}\bar{\jmath}},$$

$$D_\alpha^{i(\bar{\imath}} \Delta W^{\bar{k}\bar{l})} = 0$$

$$G^{ij} = -\frac{\mathrm{i}}{6\kappa} D^{ij\bar{\imath}\bar{\jmath}} \Delta W_{\bar{\imath}\bar{\jmath}}.$$

$$\mathcal{D}_\alpha = e^{-\hat{\mathcal{V}}} D_\alpha e^{\hat{\mathcal{V}}} = D_\alpha - 2\mathrm{i}X\bar{\theta}_\alpha \hat{\mathcal{Z}}, \bar{\mathcal{D}}_\alpha = \bar{D}_\alpha, \hat{\mathcal{V}} = -2\mathrm{i}X\bar{\theta}^\alpha \theta_\alpha \hat{\mathcal{Z}}$$

$$\left\{D_\alpha, \bar{D}_\beta\right\} = -2\mathrm{i}\partial_{\alpha\beta} = -2\mathrm{i}\mathcal{D}_{\alpha\beta}, \left\{D_\alpha, D_\beta\right\} = 0$$

$$\{\nabla_\alpha, \bar{\nabla}_\beta\} = -2\mathrm{i}\nabla_{\alpha\beta} - 2\mathrm{i}\varepsilon_{\alpha\beta}X\hat{\mathcal{Z}} + \mathrm{i}\varepsilon_{\alpha\beta}G$$
$$[\nabla_\alpha, \nabla_a] = -\frac{1}{2}(\gamma_a)_\alpha{}^\beta\nabla_\beta G, [\bar{\nabla}_\alpha, \nabla_a] = \frac{1}{2}(\gamma_a)_\alpha{}^\beta\bar{\nabla}_\beta G$$
$$[\nabla_a, \nabla_b] = -\frac{\mathrm{i}}{8}\varepsilon_{abc}(\gamma^c)^{\alpha\beta}[\nabla_\alpha, \bar{\nabla}_\beta]G$$

$$\nabla^2 G = \bar{\nabla}^2 G = 0$$

$$\mathrm{e}^V \to \mathrm{e}^{\mathrm{i}\lambda^\dagger}\mathrm{e}^V\mathrm{e}^{-\mathrm{i}\lambda}$$

$$\nabla_\alpha = \mathrm{e}^{-V}D_\alpha\mathrm{e}^V, \bar{\nabla}_\alpha = \bar{D}_\alpha$$

$$G = \frac{\mathrm{i}}{2}\bar{D}^\alpha(\mathrm{e}^{-V}D_\alpha\mathrm{e}^V)$$

$$S_{\mathrm{YM}}^{\mathcal{N}=2} = -\frac{1}{2g^2}\mathrm{tr}\int \mathrm{d}^{3|4}z\left[G^2 + \mathrm{i}X\int_0^1 \mathrm{d}t\bar{D}^\alpha(\mathrm{e}^{-tV}D_\alpha\mathrm{e}^{tV})\mathrm{e}^{-tV}\partial_t\mathrm{e}^{tV}\right]$$

$$\mathfrak{D}_A = \mathcal{D}_A + \mathrm{i}\mathfrak{V}_A,$$

$$\left\{\mathfrak{D}_\alpha^{i\bar{\imath}}, \mathfrak{D}_\beta^{j\bar{\jmath}}\right\} = 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{\imath}\bar{\jmath}}\mathfrak{D}_{\alpha\beta} + 2\mathrm{i}\varepsilon_{\alpha\beta}X\left(\varepsilon^{\bar{\imath}\bar{\jmath}}\mathbf{L}^{ij} - \varepsilon^{ij}\mathbf{R}^{\bar{\imath}\bar{\jmath}}\right) + 2\varepsilon_{\alpha\beta}\varepsilon^{ij}\mathfrak{W}^{\bar{\imath}\bar{\jmath}}$$

$$\mathfrak{D}_\gamma^{i(\bar{\imath}}\mathfrak{W}^{\bar{\jmath}\bar{k})} = 0$$

$$\nabla_\alpha\bar{\Phi} = 0, \bar{\nabla}_\alpha\Phi = 0$$

$$\bar{\Phi} = \mathrm{e}^{-V}\bar{\varphi}\mathrm{e}^V, \Phi = \varphi$$

$$\hat{\mathcal{Z}}\varphi = \varphi, \hat{\mathcal{Z}}\bar{\varphi} = -\bar{\varphi}$$

$$S_{\mathrm{YMCS}}^{\mathcal{N}=4} = \frac{1}{g^2}\mathrm{tr}\int \mathrm{d}^{3|4}z\left[\bar{\Phi}\Phi - \frac{1}{2}G^2 - \frac{\mathrm{i}X}{2}\int_0^1 \mathrm{d}t\bar{D}^\alpha(\mathrm{e}^{-tV}D_\alpha\mathrm{e}^{tV})\mathrm{e}^{-tV}\partial_t\mathrm{e}^{tV}\right]$$

$$\Delta V := \mathrm{e}^{-V}\delta\mathrm{e}^V = 2\mathrm{i}(\bar{\rho}\Phi - \rho\bar{\Phi})$$
$$\Delta\Phi := \delta\varphi = \frac{\mathrm{i}}{8X}\bar{\nabla}^2(G\mathcal{D}^2\rho)$$
$$\Delta\bar{\Phi} := \mathrm{e}^{-V}\delta\bar{\varphi}\mathrm{e}^V = -\frac{\mathrm{i}}{8X}\nabla^2\left(G\bar{\mathcal{D}}^2\bar{\rho}\right)$$

$$\mathcal{D}_\alpha\rho = 0, \bar{\mathcal{D}}_\alpha\rho = 0, \hat{\mathcal{Z}}\rho = \rho$$

$$\bar{\mathcal{D}}^\alpha\mathcal{D}^2\rho = -4\mathrm{i}X\mathcal{D}^\alpha\rho, \bar{\mathcal{D}}^2\mathcal{D}^2\rho = -16X^2\rho$$

$$\varphi \to \mathrm{e}^{\mathrm{i}\lambda}\varphi\mathrm{e}^{-\mathrm{i}\lambda}, \bar{\varphi} \to \mathrm{e}^{\mathrm{i}\lambda^\dagger}\bar{\varphi}\mathrm{e}^{-\mathrm{i}\lambda^\dagger}, \mathrm{e}^V \to \mathrm{e}^{\mathrm{i}\lambda^\dagger}\mathrm{e}^V\mathrm{e}^{-\mathrm{i}\lambda}$$

$$S_{\mathrm{VM}}^{\mathcal{N}=4} = \int \mathrm{d}^{3|4}z\left(\bar{\varphi}\varphi - \frac{1}{2}G^2 - \frac{X}{2}VG\right)$$

$$\bar{\mathcal{D}}_\alpha Q_\pm = 0, \hat{\mathcal{Z}}Q_\pm = -\frac{1}{2}Q_\pm$$

$$Q_+ \rightarrow \mathrm{e}^{\mathrm{i}\lambda} Q_+, Q_- \rightarrow Q_- \mathrm{e}^{-\mathrm{i}\lambda}$$

$$\mathcal{Q}_\pm = Q_\pm, \overline{\mathcal{Q}}_+ = \bar{Q}_+ \mathrm{e}^V, \overline{\mathcal{Q}}_- = \mathrm{e}^{-V} \bar{Q}_-$$

$$\bar{\nabla}_\alpha \mathcal{Q}_\pm = 0, \nabla_\alpha \overline{\mathcal{Q}}_\pm = 0$$

$$S_{\text{hyper}} = \int \mathrm{d}^{3|4} z [\overline{\mathcal{Q}}_+ \mathcal{Q}_+ + \mathcal{Q}_- \overline{\mathcal{Q}}_-] - \mathrm{i} \int \mathrm{d}^{3|2} z \mathcal{Q}_- \Phi \mathcal{Q}_+ + \mathrm{i} \int \mathrm{d}^{3|2} \bar{z} \overline{\mathcal{Q}}_+ \bar{\Phi} \overline{\mathcal{Q}}_-$$

$$\delta \mathcal{Q}_\pm = \pm \frac{1}{2} \bar{\nabla}^2 (\bar{\rho} \overline{\mathcal{Q}}_\mp), \delta \overline{\mathcal{Q}}_\pm = \pm \frac{1}{2} \nabla^2 (\rho \mathcal{Q}_\mp).$$

$$\mathcal{Q}_\pm | = f_\pm, D^\alpha \mathcal{Q}_\pm | = \chi_\pm^\alpha, -\frac{1}{4} D^2 \mathcal{Q}_\pm \Big| = F_\pm$$

$$S_{\text{hyper}} = -\frac{\mathrm{i}}{2} \int \mathrm{d}^3 x \left[\chi_+^\alpha ((\gamma^a)_{\alpha\beta} \nabla_a + \varepsilon_{\alpha\beta} X/2) \bar{\chi}_+^\beta + \chi_-^\alpha ((\gamma^a)_{\alpha\beta} \nabla_a + \varepsilon_{\alpha\beta} X/2) \bar{\chi}_-^\beta \right] + \cdots$$

$$\Gamma = \Gamma_{\text{odd}} [V] + \Gamma_{\text{even}} [V, \varphi]$$

$$\delta \Gamma = \int \mathrm{d}^{3|4} z \delta V \langle J \rangle$$

$$\langle J \rangle = \left\langle \frac{\delta S_{\text{hyper}}}{\delta V} \right\rangle = \langle \mathcal{Q}_+ \overline{\mathcal{Q}}_+ \rangle - \langle \mathcal{Q}_- \overline{\mathcal{Q}}_- \rangle = \sum_{e=\pm} e \langle \mathcal{Q}_e \overline{\mathcal{Q}}_e \rangle$$

$$\mathrm{i} \langle \mathcal{Q}_e(z) \overline{\mathcal{Q}}_e(z') \rangle = \frac{1}{16} \frac{1}{\hat{\square}_e} \bar{\nabla}^2 \nabla'^2 [\delta^{3|4}(z - z') \mathcal{I}(z, z')]$$

$$\hat{\square}_e \mathcal{Q}_e \equiv \frac{1}{16} \bar{\nabla}^2 \nabla^2 \mathcal{Q}_e$$

$$\hat{\square}_e = \nabla^a \nabla_a - \frac{\mathrm{i}e}{2} W^\alpha \nabla_\alpha - \frac{\mathrm{i}e}{4} (\nabla^\alpha W_\alpha) - \left(-X \hat{\mathcal{Z}} + \frac{e}{2} G \right)^2,$$

$$\langle \mathcal{Q}_e(z) \overline{\mathcal{Q}}_e(z) \rangle = \sqrt{-\mathrm{i}} \int_0^\infty \frac{\mathrm{d}s}{(4\pi s)^{3/2}} \mathrm{e}^{-\mathrm{i} \frac{s}{4} (X + eG)^2} = -\frac{1}{8\pi} |X + eG|$$

$$\langle J \rangle_{\text{odd}} = -\frac{1}{8\pi} [(X + G) - (X - G)] = -\frac{G}{4\pi}.$$

$$\Gamma_{\text{odd}} = -\frac{1}{8\pi} \int \mathrm{d}^{3|4} z V G$$

$$\Gamma = \frac{1}{4\pi X} \int \mathrm{d}^{3|4} z \left(\bar{\varphi} \varphi - \frac{1}{2} G^2 - \frac{X}{2} V G \right)$$

$$\frac{1}{8\pi} \frac{m}{|m|} \int \mathrm{d}^3 x \varepsilon^{abc} A_a \partial_b A_c$$

$$(\gamma_0)_{\alpha\beta} = \mathbb{1}, (\gamma_1)_{\alpha\beta} = \sigma_1, (\gamma_2)_{\alpha\beta} = \sigma_3$$
$$(\gamma_m)^{\alpha\beta} = (\gamma_m)^{\beta\alpha} = \varepsilon^{\alpha\gamma} \varepsilon^{\beta\delta} (\gamma_m)_{\gamma\delta}$$

$$\varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \varepsilon^{\alpha\gamma}\varepsilon_{\gamma\beta} = \delta^{\alpha}_{\beta}$$

$$\psi^{\alpha} = \varepsilon^{\alpha\beta}\psi_{\beta}, \psi_{\alpha} = \varepsilon_{\alpha\beta}\psi^{\beta}$$

$$\gamma_m := (\gamma_m)_{\alpha}{}^{\beta} = \varepsilon^{\beta\gamma}(\gamma_m)_{\alpha\gamma}$$

$$\{\gamma_m, \gamma_n\} = 2\eta_{mn}\mathbb{1}$$
$$\gamma_m\gamma_n = \eta_{mn}\mathbb{1} + \varepsilon_{mnp}\gamma^{p}$$

$$(\gamma^{a})_{\alpha\beta}(\gamma_a)_{\gamma\delta} = 2\varepsilon_{\alpha(\gamma}\varepsilon_{\delta)\beta}$$
$$\varepsilon_{abc}(\gamma^{b})_{\alpha\beta}(\gamma^{c})_{\gamma\delta} = \varepsilon_{\gamma(\alpha}(\gamma_a)_{\beta)\delta} + \varepsilon_{\delta(\alpha}(\gamma_a)_{\beta)\gamma}$$
$$\mathrm{tr}[\gamma_a\gamma_b\gamma_c\gamma_d] = 2\eta_{ab}\eta_{cd} - 2\eta_{ac}\eta_{db} + 2\eta_{ad}\eta_{bc}$$

$$V_{\alpha\beta} := (\gamma^{a})_{\alpha\beta}V_a = V_{\beta\alpha}, V_a = -\frac{1}{2}(\gamma_a)^{\alpha\beta}V_{\alpha\beta}$$

$$F_a = \frac{1}{2}\varepsilon_{abc}F^{bc}, F_{ab} = -\varepsilon_{abc}F^{c}$$

$$F_{\alpha\beta} := (\gamma^{a})_{\alpha\beta}F_a = \frac{1}{2}(\gamma^{a})_{\alpha\beta}\varepsilon_{abc}F^{bc}$$

$$-F^{a}G_a = \frac{1}{2}F^{ab}G_{ab} = \frac{1}{2}F^{\alpha\beta}G_{\alpha\beta}$$

$$\mathcal{M}_{ab}V_c = 2\eta_{c[a}V_{b]}$$

$$\mathcal{M}_{ab}\psi_{\alpha} = \frac{1}{2}\varepsilon_{abc}(\gamma^{c})_{\alpha}{}^{\beta}\psi_{\beta}$$

$$\mathcal{M}_{a}\psi_{\alpha} = -\frac{1}{2}(\gamma_a)_{\alpha}{}^{\beta}\psi_{\beta}, \mathcal{M}_{\alpha\beta}\psi_{\gamma} = \varepsilon_{\gamma(\alpha}\psi_{\beta)}$$

$$(\psi^{\alpha})^{*} = \psi^{\alpha}, (\psi_{\alpha})^{*} = \psi_{\alpha}$$

$$(\Sigma_I)_{i\bar{\imath}} = (\mathbb{1}, \mathrm{i}\sigma_1, \mathrm{i}\sigma_2, \mathrm{i}\sigma_3), I = \mathbf{1}, \cdots, \mathbf{4}, i = 1,2, \bar{\imath} = \bar{1}, \bar{2}$$

$$((\Sigma_I)_{i\bar{\imath}})^{*} = (\Sigma_I)^{i\bar{\imath}} = \varepsilon^{ij}\varepsilon^{\bar{\imath}\bar{\jmath}}(\Sigma_I)_{j\bar{\jmath}}$$

$$\psi^{i} = \varepsilon^{ij}\psi_j, \psi_i = \varepsilon_{ij}\psi^{j}, \chi^{\bar{\imath}} = \varepsilon^{\bar{\imath}\bar{\jmath}}\chi_{\bar{\jmath}}, \chi_{\bar{\imath}} = \varepsilon_{\bar{\imath}\bar{\jmath}}\chi^{\bar{\jmath}}$$

$$(\tau_I)_{i\bar{\imath}} := \frac{1}{\sqrt{2}}(\Sigma_I)_{i\bar{\imath}}$$

$$\left(\tau_{(I}\right)_{i\bar{\jmath}}\left(\tau_{J)}\right)^{j\bar{\jmath}} = \frac{1}{2}\delta_{IJ}\delta^{j}_{i}, \left(\tau_{(I}\right)_{j\bar{\imath}}\left(\tau_{J)}\right)^{j\bar{\jmath}} = \frac{1}{2}\delta_{IJ}\delta^{\bar{\jmath}}_{\bar{\imath}}$$
$$(\tau_I)_{i\bar{\imath}}(\tau^{I})_{j\bar{\jmath}} = \varepsilon_{ij}\varepsilon_{\bar{\imath}\bar{\jmath}}, (\tau_I)_{i\bar{\imath}}(\tau_J)^{i\bar{\imath}} = \delta_{IJ}$$

$$A_{i\bar{\imath}} := (\tau_I)_{i\bar{\imath}}A^{I} \leftrightarrow A_I = (\tau_I)^{i\bar{\imath}}A_{i\bar{\imath}}$$

$$\delta_I^J \to \delta_i^j \delta_{\bar{\imath}}^{\bar{\jmath}}, A_I B^I = A_{i\bar{\imath}} B^{i\bar{\imath}}$$

$$A_{i\bar{\imath}j\bar{\jmath}} = \varepsilon_{ij} A_{\bar{\imath}\bar{\jmath}} + \varepsilon_{\bar{\imath}\bar{\jmath}} A_{ij} \longrightarrow A^{i\bar{\imath}\bar{\jmath}} = -\varepsilon^{ij} A^{\bar{\imath}\bar{\jmath}} - \varepsilon^{\bar{\imath}\bar{\jmath}} A^{ij}, A_{ij} = A_{ji}, A_{\bar{\imath}\bar{\jmath}} = A_{\bar{\jmath}\bar{\imath}}$$

$$\frac{1}{2} A^{IJ} B_{IJ} = A^{ij} B_{ij} + A^{\overline{\imath}\overline{\jmath}} B_{\bar{\imath}\bar{\jmath}}$$

$$\varepsilon_{i\bar{\imath}j\bar{\jmath}k\bar{k}l\bar{l}} := \varepsilon_{IJKL} (\tau^I)_{i\bar{\imath}} (\tau^J)_{j\bar{\jmath}} (\tau^K)_{k\bar{k}} (\tau^L)_{l\bar{l}} = \left(\varepsilon_{ij} \varepsilon_{kl} \varepsilon_{\bar{\imath}\bar{l}} \varepsilon_{\bar{\jmath}\bar{k}} - \varepsilon_{il} \varepsilon_{jk} \varepsilon_{\bar{\imath}\bar{\jmath}} \varepsilon_{\bar{k}\bar{l}}\right).$$

$$\chi = \chi^{\mathrm{a}} \frac{\partial}{\partial \phi^{\mathrm{a}}} + \bar{\chi}^{\bar{\mathrm{a}}} \frac{\partial}{\partial \bar{\phi}^{\bar{\mathrm{a}}}} \equiv \chi^{\mu} \frac{\partial}{\partial \varphi^{\mu}}$$

$$\nabla_\nu \chi^\mu = \delta_\nu{}^\mu \iff \nabla_{\mathrm{b}} \chi^{\mathrm{a}} = \delta_{\mathrm{b}}{}^{\mathrm{a}}, \nabla_{\bar{\mathrm{b}}} \chi^{\mathrm{a}} = \partial_{\bar{\mathrm{b}}} \chi^{\mathrm{a}} = 0$$

$$\chi_{\mathrm{a}} := g_{\mathrm{a}\,\mathrm{b}} \bar{\chi}^{\bar{\mathrm{b}}} = \partial_{\mathrm{a}} K \implies \chi^{\mathrm{a}} K_{\mathrm{a}} = K$$

$$K := g_{\mathrm{a}\bar{\mathrm{b}}} \chi^{\mathrm{a}} \bar{\chi}^{\bar{\mathrm{b}}}$$

$$V^\mu = -\frac{1}{2} J_\nu^\mu \chi^\nu, \nabla_\mu V_\nu + \nabla_\nu V_\mu = 0$$

$$\chi = \phi^{\mathrm{a}} \frac{\partial}{\partial \phi^{\mathrm{a}}} + \bar{\phi}^{\bar{\mathrm{a}}} \frac{\partial}{\partial \bar{\phi}^{\bar{\mathrm{a}}}}$$

$$\phi^{\mathrm{a}} K_{\mathrm{a}}(\phi, \bar{\phi}) = K(\phi, \bar{\phi})$$

$$V_A^\mu := -\frac{1}{2} (J_A)^\mu{}_\nu \chi^\nu$$

$$[V_A, \chi] = 0, [V_A, V_B] = \epsilon_{ABC} V_C$$

$$\mathcal{L}_{V_A} J_B = \epsilon_{ABC} J_C$$

$$V| = 0, \mathcal{D}_\alpha V| = \overline{\mathcal{D}}_\alpha V| = 0, \mathcal{D}^2 V| = \overline{\mathcal{D}}^2 V| = 0,$$

$$\begin{aligned} \frac{1}{2} [D_\alpha, \bar{D}_\beta] V \Big| &= (\gamma^m)_{\alpha\beta} A_m + \mathrm{i} \varepsilon_{\alpha\beta} \sigma \\ \frac{1}{4} D^2 \bar{D}_\alpha V \Big| &= \mathrm{i} \bar{\lambda}_\alpha, \frac{1}{4} \bar{D}^2 D_\alpha V \Big| = -\mathrm{i} \lambda_\alpha \\ -\frac{1}{8} \{D^2, \bar{D}^2\} V \Big| &= D \end{aligned}$$

$$\varphi \Big| = \frac{\mathrm{i}}{\sqrt{2}} f, D_\alpha \varphi \Big| = \frac{\mathrm{i}}{\sqrt{2}} \psi_\alpha, -\frac{1}{4} D^2 \varphi \Big| = \frac{\mathrm{i}}{\sqrt{2}} F.$$

$$\phi_{\mathrm{I}} = (\phi_1, \phi_2, \sigma) = \left(\frac{f + \bar{f}}{\sqrt{2}}, \mathrm{i} \frac{f - \bar{f}}{\sqrt{2}}, \sigma \right), \mathrm{I} = 1,2,3.$$

$$\bar{\psi}^{1\alpha} = \bar{\lambda}^\alpha, \psi_{1\alpha} = \lambda_\alpha, \psi_{2\alpha} = \frac{1}{\sqrt{2}} \bar{\psi}_\alpha, \bar{\psi}^{2\alpha} = \frac{1}{\sqrt{2}} \psi^\alpha$$

$$
\begin{aligned}
S_{\mathrm{YM}}^{\mathcal{N}=4} &= \frac{1}{g^2}\mathrm{tr}\int \mathrm{d}^3x\left(\mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{spinor}}\right)\\
\mathcal{L}_{\text{vector}} &= -\frac{1}{4}F^{mn}F_{mn} + \frac{X}{2}\varepsilon^{mnp}\left(A_m\partial_n A_p + \frac{2}{3}A_m A_n A_p\right)\\
\mathcal{L}_{\text{scalar}} &= \frac{1}{2}\phi^{\mathrm{I}}(\nabla^m\nabla_m - X^2)\phi_{\mathrm{I}} - \frac{\mathrm{i}}{2}X\varepsilon^{\mathrm{IJK}}\phi_{\mathrm{I}}[\phi_{\mathrm{J}},\phi_{\mathrm{K}}] + \frac{1}{4}[\phi^{\mathrm{I}},\phi^{\mathrm{J}}][\phi_{\mathrm{I}},\phi_{\mathrm{J}}]\\
\mathcal{L}_{\text{spinor}} &= \mathrm{i}\bar{\psi}^{i\alpha}\left((\gamma^m)_\alpha{}^\beta\nabla_m + X\delta_\alpha^\beta\right)\psi_{i\beta} + \mathrm{i}(\sigma^{\mathrm{I}})_i{}^j\bar{\psi}^{i\alpha}[\phi_{\mathrm{I}},\psi_{j\alpha}]
\end{aligned}
$$

$$
F_{mn} = \partial_m A_n - \partial_n A_m + \mathrm{i}[A_m, A_n] \propto \varepsilon_{mnp}(\gamma^p)^{\alpha\beta}[\nabla_\alpha,\bar{\nabla}_\beta]G\big|_{\theta=0}
$$

$$
(\sigma^{\mathrm{I}})_j{}^i(\sigma^{\mathrm{J}})_i{}^k = \mathrm{i}\varepsilon^{\mathrm{IJK}}(\sigma_{\mathrm{K}})_j{}^k + \delta^{\mathrm{IJ}}\delta_i^k, (\sigma^{\mathrm{I}})_{ij}(\sigma_{\mathrm{I}})^{kl} = -(\delta_i^k\delta_j^l + \delta_i^l\delta_j^k)
$$

$$
\begin{aligned}
\delta A_m &= \mathrm{i}(\gamma_m)_\alpha{}^\beta(\bar{\epsilon}^{i\alpha}\psi_{i\beta} + \epsilon_{i\beta}\bar{\psi}^{i\alpha})\\
\delta\phi^{\mathrm{I}} &= (\sigma^{\mathrm{I}})^i{}_j(\bar{\epsilon}^{j\alpha}\psi_{i\alpha} + \epsilon_{i\alpha}\bar{\psi}^{j\alpha})\\
\delta\bar{\psi}^{i\alpha} &= -\frac{1}{2}\varepsilon^{mnp}(\gamma_p)^\alpha{}_\beta\bar{\epsilon}^{i\beta}F_{mn} - \mathrm{i}(\gamma^m)^\alpha{}_\beta(\sigma^{\mathrm{I}})^i{}_j\nabla_m\phi_{\mathrm{I}}\bar{\epsilon}^{j\beta}\\
&\quad -\mathrm{i}X\phi^{\mathrm{I}}(\sigma_{\mathrm{I}})^i{}_j\bar{\epsilon}^{j\alpha} + \frac{1}{2}\varepsilon^{\mathrm{IJK}}(\sigma_{\mathrm{K}})^i{}_j[\phi_{\mathrm{I}},\phi_{\mathrm{J}}]\bar{\epsilon}^{j\alpha}\\
\delta\psi_{i\alpha} &= \frac{1}{2}\varepsilon^{mnp}(\gamma_p)_\alpha{}^\beta\epsilon_{i\beta}F_{mn} - \mathrm{i}(\gamma^m)_\alpha{}^\beta(\sigma^{\mathrm{I}})_i{}^j\nabla_m\phi^{\mathrm{I}}\epsilon_{j\beta}\\
&\quad +\mathrm{i}X\phi^{\mathrm{I}}(\sigma_{\mathrm{I}})_i{}^j\epsilon_{j\alpha} - \frac{1}{2}\varepsilon^{\mathrm{IJK}}(\sigma_{\mathrm{K}})_i{}^j[\phi_{\mathrm{I}},\phi_{\mathrm{J}}]\epsilon_{j\alpha}
\end{aligned}
$$

Estructuras Holográficas AdS/SCFT.

$$
S_{\mathrm{EE}}^{(\mathrm{univ})} = -\frac{(\varpi,\varpi)}{5},
$$

$$
\mathrm{d}s^2 = 4kQ_{\mathrm{M5}}H_{\mathrm{M}'}^{-1/3}\left(\mathrm{d}s^2_{\mathrm{AdS}_3} + \mathrm{d}s^2_{\mathbb{S}^3/\mathbb{Z}_k}\right) + H_{\mathrm{M}'}^{2/3}\left(\mathrm{d}z^2 + \mathrm{d}\rho^2 + \rho^2\,\mathrm{d}s^2_{\tilde{\mathbb{S}}^3/\mathbb{Z}_{k'}}\right)
$$

$$
\nabla^2_{\mathbb{R}^3_{\hat{\rho}}}H_{\mathrm{M5}'}(z,\hat{\rho}) + \frac{k'\partial_z^2 H_{\mathrm{M5}'}(z,\hat{\rho})}{\hat{\rho}} = 0
$$

$$
H_{\mathrm{M}'}(z,\rho) = \frac{4\sqrt{2}}{g^3}\frac{1}{P_+P_-}\frac{\sqrt{P_+^2 + P_-^2 - 4\alpha^2 + 2P_+P_-}}{P_+^2 + P_-^2 + 2P_+P_-}
$$

$$
P_\pm = \sqrt{z^2 + (\rho \pm \alpha)^2}
$$

$$
\rho = \alpha\frac{\cos\,\xi}{\sqrt{1-\mu^5}}
$$
$$
z = \alpha\mu^{\frac{5}{2}}\frac{\sin\,\xi}{\sqrt{1-\mu^5}}
$$

$$
H_{\mathrm{M5}'}(\mu,\xi) = 2^{27/4}\left(\sqrt{g}kQ_{\mathrm{M5}}\right)^3\frac{\mu^{5/2}(1-\mu^5)^{3/2}}{\mu^5\cos^2\,\xi + \sin^2\,\xi}
$$

$$\mathcal{R} = H_{\mathrm{M5}'}^{-2/3}\left[\frac{1}{6}\frac{(\partial_z H_{\mathrm{M5}'})^2 + \left(\partial_\rho H_{\mathrm{M}'}\right)^2}{H_{\mathrm{M5}'}^2} - \frac{2}{3}\frac{\partial_z^2 H_{\mathrm{M}'} + \partial_\rho^2 H_{\mathrm{M}'}}{H_{\mathrm{M}'}} - 2\frac{\partial_\rho H_{\mathrm{M5}'}}{\rho H_{\mathrm{M5}'}}\right]$$

$$\mathcal{R} = \frac{g^2(2\alpha^2 - 3\rho^2)^2}{12\rho^{2/3}(\rho^2 - \alpha^2)^{5/3}} + \mathcal{O}(z^2)$$

$$\mathcal{R} = \frac{g^2\alpha^{2/3}(\alpha^2 - \rho^2)}{12z^{8/3}} + \mathcal{O}\left(\frac{1}{z^{2/3}}\right)$$

$$\left[T_i^{(a)}, T_j^{(b)}\right] = i\delta^{ab}\varepsilon_{ijk}\eta_a^{kl}T_l^{(a)} \quad \text{for } i,j \in \{1,2,3\}$$

$$\begin{aligned}\{F_{A_1A_2A_3}, F_{B_1B_2B_3}\} = {} & \beta_1 C_{A_2B_2}C_{A_3B_3}\left(C\sigma^i\right)_{A_1B_1}T_i^{(1)} \\ & +\beta_2 C_{A_1B_1}C_{A_3B_3}\left(C\sigma^i\right)_{A_2B_2}T_i^{(2)} \\ & +\beta_3 C_{A_1B_1}C_{A_2B_2}\left(C\sigma^i\right)_{A_3B_3}T_i^{(3)}\end{aligned}$$

$$\{F_{A_1A_2A_3}, F_{B_1B_2B_3}\} = \beta_1\left(C_{A_2B_2}C_{A_3B_3}\left(C\sigma^i\right)_{A_1B_1}T_i^{(1)} - C_{A_1B_1}C_{A_3B_3}\left(C\sigma^i\right)_{A_2B_2}T_i^{(2)}\right)$$

$$\begin{aligned}&\mathfrak{d}(2,1;\gamma;0) = \mathfrak{osp}(4^* \mid 2) \qquad \gamma \in \{-2, -1/2\} \\ &\mathfrak{d}(2,1;\gamma;0) = \mathfrak{osp}(4 \mid 2;\mathbb{R}) \qquad \gamma = 1\end{aligned}$$

$$\mathfrak{so}(2,2) \oplus \mathfrak{so}(4) \oplus \mathfrak{so}(4)$$

$$\left(\mathrm{AdS}_3 \times \mathbb{S}^3 \times \tilde{\mathbb{S}}^3\right) \ltimes \Sigma_2,$$

$$\partial_w \partial_{\bar{w}} h = 0$$

$$\partial_w G = \frac{1}{2}(G + \bar{G})\partial_w \ln\, h$$

$$\mathrm{d}s^2 = f_{\mathrm{AdS}_3}^2\, \mathrm{d}s_{\mathrm{AdS}_3}^2 + f_{\mathbb{S}^3}^2\, \mathrm{d}s_{\mathbb{S}^3}^2 + f_{\tilde{\mathbb{S}}^3}^2\, \mathrm{d}s_{\tilde{\mathbb{S}}^3}^2 + f_{\Sigma_2}^2\, \mathrm{d}s_{\Sigma_2}^2$$

$$W_\pm := |G \pm i|^2 + \gamma^{\pm 1}(G\bar{G} - 1)$$

$$\begin{aligned}f_{\mathrm{AdS}_3}^6 &= \frac{h^2 W_+ W_-}{\beta_1^6(G\bar{G} - 1)^2} \\ f_{\mathbb{S}^3}^6 &= \frac{h^2(G\bar{G} - 1)W_-}{\beta_2^3\beta_3^3 W_+^2} \\ f_{\tilde{\mathbb{S}}^3}^6 &= \frac{h^2(G\bar{G} - 1)W_+}{\beta_2^3\beta_3^3 W_-^2} \\ f_{\Sigma_2}^6 &= \frac{|\partial_w h|^6}{\beta_2^3\beta_3^3 h^4}(G\bar{G} - 1)W_+W_-\end{aligned}$$

$$\mathcal{C}_{(3)} = \sum_{i=1}^{3} b_{\mathcal{M}_i} \mathrm{vol}_{\mathcal{M}_i}$$

$$b_{\mathrm{AdS}_3} := \frac{\tau_1}{\beta_1^3}\left[-\frac{h(G+\bar{G})}{1-G\bar{G}} + (2+\gamma+\gamma^{-1})\Phi - (\gamma-\gamma^{-1})\tilde{h} + b_1^0\right]$$
$$b_{\mathbb{S}^3} := \frac{\tau_2}{\beta_2^3}\left[-\frac{\gamma h(G+\bar{G})}{W_+} + \gamma(\Phi-\tilde{h}) + b_2^0\right]$$
$$b_{\tilde{\mathbb{S}}^3} := \frac{\tau_3}{\beta_3^3}\left[\frac{h(G+\bar{G})}{\gamma W_-} - \frac{\Phi+\tilde{h}}{\gamma} + b_3^0\right]$$

$$\partial_w \tilde{h} = -i\partial_w h$$
$$\partial_w \Phi = \bar{G}\partial_w h$$

$$\prod_{i=1}^{3} \beta_i f_{\mathcal{M}_i} + h \prod_{i=1}^{3} \tau_i = 0$$

$$h = -ih_0 w + \text{ c.c. } = \frac{2h_0 \sin \vartheta}{\varrho}$$

$$G = -i + a_1 \varrho e^{i\vartheta} \sin \vartheta + \mathcal{O}(\varrho^2)$$

$$\mathrm{d}s^2 = L^2 \frac{\mathrm{d}\varrho^2}{\varrho^2} - \frac{2\gamma L^2}{a_1(1+\gamma)^2 \varrho}\left(\mathrm{d}s^2_{\mathrm{AdS}_3} + \frac{(1+\gamma)^2}{\gamma^2}\,\mathrm{d}s^2_{\mathbb{S}^3}\right) + L^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}s^2_{\tilde{\mathbb{S}}^3}\right) + \cdots$$

$$L^6 = \frac{a_1^2 h_0^2 (1+\gamma)^6}{\beta_1^6 \gamma^2}$$

$$G = -i\left(1 + \sum_{j=1}^{2n+2} (-1)^j \frac{w-\xi_j}{|w-\xi_j|}\right)$$

$$G = -i(1+\gamma^{-1}F)$$

$$\mathrm{d}s^2 = \left[\frac{4h^2}{\hat{\beta}^6(F+\bar{F})}\right]^{\frac{1}{3}}\left(\mathrm{d}s^2_{\mathrm{AdS}_3} + \mathrm{d}s^2_{\mathbb{S}^3}\right) + \left[\frac{h^2(F+\bar{F})^2}{16\hat{\beta}^6}\right]^{\frac{1}{3}}\left(\mathrm{d}s^2_{\tilde{\mathbb{S}}^3} + \frac{4|\partial_w h|^2}{h^2}\,\mathrm{d}w\,\mathrm{d}\bar{w}\right)$$

$$b_{\mathrm{AdS}_3} = b_{\mathbb{S}^3} = -\frac{2\tilde{h}}{\hat{\beta}^3}, b_{\tilde{\Phi}^3} = \frac{h}{2\hat{\beta}^3}\frac{-i(F-\bar{F})}{2} - \hat{\Phi}$$

$$\partial_w \hat{\Phi} = \bar{F}\frac{\partial_w \tilde{h}}{\hat{\beta}^3}$$

$$F = \frac{2\rho^2}{h_1} H + iF_I$$

$$\mathrm{d}s^2 = h_1 H^{-\frac{1}{3}}\left(\mathrm{d}s^2_{\mathrm{AdS}_3} + \mathrm{d}s^2_{\mathbb{S}^3}\right) + H^{\frac{2}{3}}\left(\mathrm{d}z^2 + \mathrm{d}\rho^2 + \rho^2\,\mathrm{d}s^2_{\tilde{\mathbb{S}}^3}\right)$$

$$\partial_w F = \frac{1}{2}(F - \bar{F})\partial_w \ln\, h$$

$$\partial_\rho^2 H + \frac{3}{\rho}\partial_\rho H + \partial_z^2 H = 0$$

$$b_{\mathrm{AdS}_3} = b_{\mathbb{S}^3} = 2h_1 z, b_{\tilde{\mathbb{S}}^3} = \frac{h_1\rho}{2}F_I + \hat{\Phi}$$

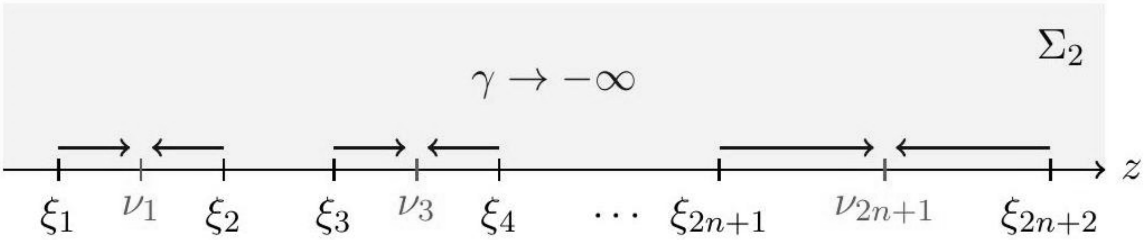
$$\partial_\rho b_{\tilde{\mathbb{S}}^3} = \rho^3\partial_z H, \partial_z b_{\tilde{\mathbb{S}}^3} = -\rho^3\partial_\rho H$$

$$z \to \lambda z, \rho \to \lambda\rho, H \to \lambda^{-3}H, h_1 \to \lambda^{-1}h_1$$

$$\begin{aligned} \mathrm{d}s^2_{\mathbb{S}^3} \to \ \mathrm{d}s^2_{\mathbb{S}^3/\mathbb{Z}_k} &= \frac{1}{4}\left[\left(\frac{\mathrm{d}\chi}{k} + \omega\right)^2 + \mathrm{d}s^2_{\mathbb{S}^2}\right] \\ \mathrm{d}s^2_{\tilde{\mathbb{S}}^3} \to \ \mathrm{d}s^2_{\tilde{\mathbb{S}}^3/\mathbb{Z}_{k'}} &= \frac{1}{4}\left[\left(\frac{\mathrm{d}\phi}{k'} + \eta\right)^2 + \mathrm{d}s^2_{\tilde{\mathbb{S}}^2}\right] \end{aligned}$$

$$\xi_j = \nu_j - \gamma^{-1}\hat{\xi}_j \in \partial\Sigma_2 \quad j \in \{1,2,\dots,2n+2\}$$

$$\nu_k \equiv \nu_{k+1} \equiv \cdots \equiv \nu_{k+2m+1}$$



$$\nu_{2i-1} \equiv \nu_{2i} \ \text{ for } \ i \in \{1,2,\dots,n+1\}$$

$$F(w,\bar{w}) = \sum_{j=1}^{2n+2} (-1)^j \hat{\xi}_j \frac{\bar{w} - w}{2(\bar{w} - \nu_j)|w - \nu_j|}$$

$$H(z,\rho) = \frac{h_1}{2}\sum_{j=1}^{2n+2} \frac{(-1)^j\hat{\xi}_j}{\left(\rho^2 + (z - \nu_j)^2\right)^{3/2}}$$

$$\begin{aligned} F_{1\mathrm{PV}}(w,\bar{w}) &= \hat{\xi}\frac{\bar{w} - w}{\bar{w}|w|} \\ H_{1\mathrm{PV}}(z,\rho) &= \frac{h_1\hat{\xi}}{(z^2+\rho^2)^{\frac{3}{2}}} \end{aligned}$$

$$\mathcal{R}|_{(z,\rho)=(\nu_{2j},0)} = \frac{3}{2L^2_{\mathbb{S}^4}}\left(\frac{\hat{\xi}_{2j} - \hat{\xi}_{2j-1}}{\hat{m}_1}\right)^{-2/3},$$

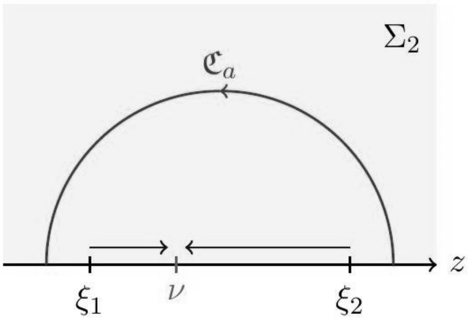
$$\mathcal{R} = \frac{3}{2L^2_{\mathbb{S}^4}},$$

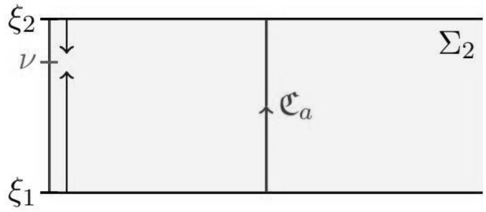
$$ds^2 = \frac{4L_{\mathbb{S}^4}^2}{v^2}\left[\, dv^2 + \alpha_1\left(ds_{\mathrm{AdS}_3}^2 + ds_{\mathbb{S}^3}^2\right)\right] + L_{\mathbb{S}^4}^2\left[\alpha_3\, d\phi^2 + \alpha_4 \sin^2\,\phi\, ds_{\tilde{\mathbb{S}}^3}^2\right]$$

$$\hat{m}_k := \sum_{j=1}^{2n+2} (-1)^j \hat{\xi}_j^k$$

$$L_{\mathbb{S}^4}^3 = \frac{h_1 \hat{m}_1}{2}$$

$$ds_{\mathrm{AdS}_7}^2 = 4L_{\mathbb{S}^4}^2\left[\, dx^2 + \cosh^2\, x\, ds_{\mathrm{AdS}_3}^2 + \sinh^2\, x\, ds_{\mathbb{S}^3}^2\right],$$





$$\Phi = -\tilde{h} = \frac{4\hat{m}_1 \cos\,\phi}{v^2} + \frac{2\hat{n}_1}{\hat{m}_1} + \frac{\hat{m}_1\hat{n}_2 - \hat{n}_1^2}{\hat{m}_1^3} \cos\,\phi v^2 + \mathcal{O}(v^4),$$

$$\frac{\mathcal{F}_{(4)}}{L_{\mathbb{S}^4}^3} = -\frac{16\cos\,\phi}{v^3}\, dv \wedge \left(\mathrm{vol}_{\mathbb{S}^3} + \mathrm{vol}_{\mathrm{AdS}_3}\right) - \frac{8\sin\,\phi}{v^2}\, d\phi \wedge \left(\mathrm{vol}_{\mathbb{S}^3} + \mathrm{vol}_{\mathrm{AdS}_3}\right)$$
$$+3\sin^3\,\phi\, d\phi \wedge \mathrm{vol}_{\tilde{\mathbb{S}}^3} + 4\cos\,\phi \frac{\hat{m}_1\hat{n}_2 - \hat{n}_1^2}{\hat{m}_1^4} v\, dv \wedge \left(\mathrm{vol}_{\mathbb{S}^3} + \mathrm{vol}_{\mathrm{AdS}_3}\right) + \mathcal{O}(v^2)$$

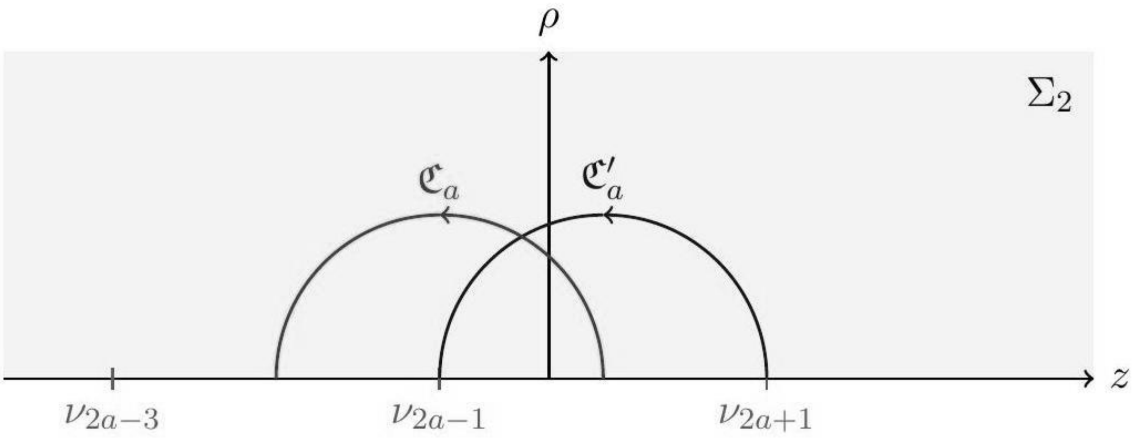
$$\frac{ds_{1\mathrm{PV}}^2(\gamma)}{L_{\mathbb{S}^4}^2} = \frac{4}{v^2}\left[dv^2 + \left(1 + \frac{2\gamma + 3 - (2\gamma+1)c_{2\phi}}{16(\gamma+1)^2} v^2\right) ds_{\mathrm{AdS}_3}^2\right.$$
$$\left. + \left(\frac{(\gamma+1)^2}{\gamma^2} + \frac{2\gamma - 1 - (2\gamma+1)c_{2\phi}}{16\gamma^2} v^2\right) ds_{\mathbb{S}^3}^2 + \mathcal{O}(v^4)\right]$$
$$+\left[\left(1 + \frac{(2\gamma+1)\left(2c_{2\phi}+1\right)}{12(\gamma+1)^2} v^2\right) s_\phi^2\, ds_{\tilde{\mathbb{S}}^3}^2 + \left(1 + \frac{(2\gamma+1)c_\phi^2}{4(\gamma+1)^2} v^2\right) d\phi^2 + \mathcal{O}(v^4)\right]$$

$$\cos\, x \equiv c_x,\ \text{and}\ \sin\, x \equiv s_x$$

$$ds_{1\mathrm{PV}}^2(\gamma \to -\infty) = \frac{4L_{\mathbb{S}^4}}{v^2}\left[\, dv^2 + ds_{\mathrm{AdS}_3}^2 + ds_{\mathbb{S}^3}^2 + \mathcal{O}(v^4)\right] + L_{\mathbb{S}^4}^2\left[s_\phi^2\, ds_{\tilde{\mathbb{S}}^3}^2 + d\phi^2 + \mathcal{O}(v^4)\right]$$

$$\mathfrak{C}_a \equiv \left\{Re^{i\theta} - \nu_{2a-1} \mid 0 \le \theta \le \pi\right\} \times \tilde{\mathbb{S}}^3,$$

$$\mathfrak{C}_a' \equiv \left\{\frac{1}{2}(\nu_{2a+1} - \nu_{2a-1})e^{i\theta} + \frac{1}{2}(\nu_{2a+1} + \nu_{2a-1}) \,\middle|\, 0 \le \theta \le \pi\right\} \times \mathbb{S}^3,$$



$$\mathcal{F}_{(4)} = 2h_1 \mathrm{vol}_{\mathrm{AdS}_3} \wedge \mathrm{d}z + 2h_1 \mathrm{vol}_{\mathbb{S}^3} \wedge \mathrm{d}z + \partial_z H \rho^3 \, \mathrm{d}\rho \wedge \mathrm{vol}_{\tilde{\mathbb{S}}^3} - \partial_\rho H \rho^3 \, \mathrm{d}z \wedge \mathrm{vol}_{\tilde{\mathbb{S}}^3}$$

$$\int_{\mathfrak{C}_a} P_{\mathfrak{C}_a}[\mathcal{F}_{(4)}] = 2h_1 \mathrm{Vol}(\tilde{\mathbb{S}}^3)(\hat{\xi}_{2a} - \hat{\xi}_{2a-1})$$

$$M_a = \frac{1}{2(4\pi^2 G_N)^{1/3}} \int_{\mathfrak{C}_a} P_{\mathfrak{C}_a}[\mathcal{F}_{(4)}]$$

$$\int_{\mathfrak{C}_{bc}} P_{\mathfrak{C}_{bc}}[\mathcal{F}_{(4)}] = 2h_1 \mathrm{Vol}(\tilde{\mathbb{S}}^3) \sum_{a=b}^{c} (\hat{\xi}_{2a} - \hat{\xi}_{2a-1})$$

$$\int_{\mathfrak{C}'_a} P_{\mathfrak{C}'_a}[\mathcal{F}_{(4)}] = 2h_1 \mathrm{Vol}(\mathbb{S}^3)(\nu_{2a+1} - \nu_{2a-1})$$

$$M'_a = \frac{1}{2(4\pi^2 G_N)^{1/3}} \int_{\mathfrak{C}'_a} P_{\mathfrak{C}'_a}[\mathcal{F}_{(4)}],$$

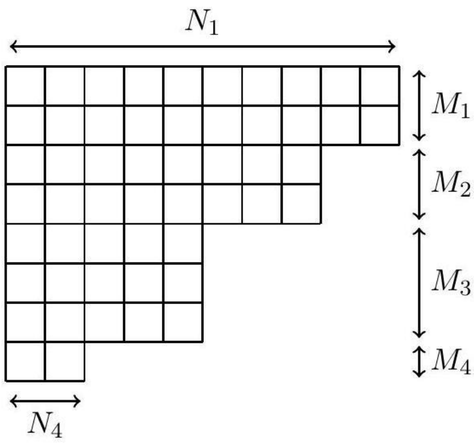
$$\int_{\mathfrak{C}'} P_{\mathfrak{C}'}[\mathcal{F}_{(4)}] = 2h_1 \mathrm{Vol}(\mathbb{S}^3) \sum_{a} (\nu_{2a+1} - \nu_{2a-1}).$$

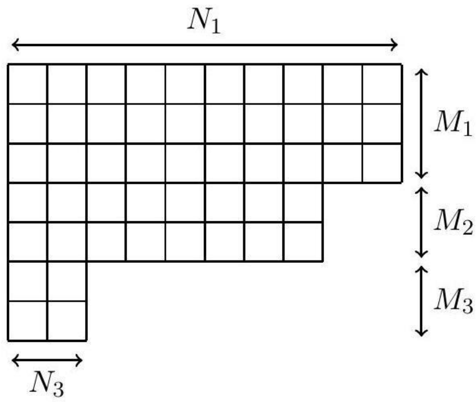
$$N_a = \sum_{b=a}^{n} M'_b = \frac{h_1 \mathrm{Vol}(\mathbb{S}^3)}{(4\pi^2 G_N)^{1/3}} \sum_{b=a}^{n} (\nu_{2b+1} - \nu_{2b-1}).$$

$$\hat{m}_1 = \sum_{j=1}^{2n+2} (-1)^j \hat{\xi}_j = \frac{(4\pi^2 G_N)^{1/3}}{h_1 \mathrm{Vol}(\tilde{\mathbb{S}}^3)} M$$

$$L^3_{\mathbb{S}^4} = \frac{h_1 \hat{m}_1}{2} = \frac{(G_N)^{1/3}}{(2\pi)^{4/3}} M$$

$$\sum_{k=0}^{j} (-1)^{j+k} \frac{j!}{k!\,(j-k)!} \nu^k_{2n+1} \hat{n}_{j-k} = \left(\frac{G_N^{1/3}}{h_1 \pi (2\pi)^{1/3}} \right)^{j+1} \sum_{a=1}^{n+1} M_a N_a^j,$$





$$\hat{n}_1^2 - \hat{m}_1 \hat{n}_2 = \frac{G_N^{4/3}}{h_1^4 \pi^4 (2\pi)^{4/3}} \left[\left(\sum_{a=1}^{n+1} M_a N_a \right)^2 - M \sum_{a=1}^{n+1} M_a N_a^2 \right],$$

$$\hat{\xi}_{2a} - \hat{\xi}_{2a-1} \longrightarrow \sum_{j=I_a}^{K_a} (-1)^j \hat{\xi}_j$$

$$S_{\mathrm{EE}} = \min_{\zeta} \frac{\mathcal{A}[\zeta]}{4G_{\mathrm{N}}},$$

$$S_{\mathrm{EE}}^{(\mathrm{univ})} = R \frac{\mathrm{d}}{\mathrm{d}R} \left(S_{\mathrm{EE}}[\Upsilon_2] - S_{\mathrm{EE}}[\emptyset] \right) \Big|_{R \to 0}$$

$$S_{\mathrm{EE}}^{(\mathrm{univ})} = \frac{1}{3} \left(a_{\Upsilon} - \frac{3}{5} d_2 \right)$$

$$\mathrm{d}s_{\mathrm{AdS}_3}^2 = \frac{1}{u^2} \left(\mathrm{d}u^2 - \mathrm{d}t^2 + \mathrm{d}x_{\parallel}^2 \right)$$

$$A[\zeta] = \mathrm{Vol}(\mathbb{S}^3) \mathrm{Vol}(\tilde{\mathbb{S}}^3) \int \mathrm{d}u \int_{\Sigma_2} \mathrm{d}\rho \, \mathrm{d}z \mathcal{L}$$

$$\mathcal{L} = \frac{h_1^2 \rho^3}{u^2} \left[h_1 \left(\left(\partial_\rho x_{\parallel} \right)^2 + \left(\partial_z x_{\parallel} \right)^2 \right) H(z,\rho) + u^2 \left(1 + \left(\partial_u x_{\parallel} \right)^2 \right) H(z,\rho)^2 \right]^{1/2}$$

$$x_{\parallel}^2 + u^2 = R^2$$

$$A[\zeta_{\mathrm{RT}}] = h_1^3 \mathrm{Vol}(\mathbb{S}^3) \mathrm{Vol}(\tilde{\mathbb{S}}^3) \log \left(\frac{2R}{\epsilon_u} \right) \int_{\Sigma_2} \mathrm{d}\rho \, \mathrm{d}z \rho^3 \sum_{j=1}^{2n+2} \frac{(-1)^j \hat{\xi}_j}{\left(\rho^2 + \left(z - \nu_j \right)^2 \right)^{3/2}} + \mathcal{O}(\epsilon_u^2)$$

$$S_{\mathrm{EE}}[\Upsilon_2] = \frac{\pi^4 L_{\mathbb{S}^4}^9}{G_{\mathrm{N}}} \log \left(\frac{2R}{\epsilon_u} \right) \left[\frac{64}{3} \frac{1}{\epsilon_v^4} + \frac{16}{5} \frac{\hat{n}_1^2}{\hat{m}_1^4} - \frac{16}{5} \frac{\hat{n}_2}{\hat{m}_1^3} + \mathcal{O}(\epsilon_v^2) \right] + \mathcal{O}(\epsilon_u^2)$$

$$S_{\mathrm{EE}}^{1\mathrm{PV}} = \frac{\pi^4 L_{\mathbb{S}^4}^9}{G_{\mathrm{N}}} \log \left(\frac{2R}{\epsilon_u} \right) \left[\frac{64}{3} \frac{1}{\epsilon_v^4} + \mathcal{O}(\epsilon_v^2) \right] + \mathcal{O}(\epsilon_u^2)$$

$$S_{\mathrm{EE}}^{(\mathrm{univ})} = \frac{16}{5}\frac{\pi^4 L_{\mathbb{S}^4}^9}{G_{\mathrm{N}}}\frac{\hat{n}_1^2 - \hat{m}_1\hat{n}_2}{\hat{m}_1^4}$$

$$= \frac{1}{5M}\left[\left(\sum_{a=1}^{n+1} M_a N_a\right)^2 - M\sum_{a=1}^{n+1} M_a N_a^2\right]$$

$$S_{\mathrm{EE}}^{(\mathrm{univ})} = -\frac{(\varpi,\varpi)}{5}$$

$$S_{\mathrm{EE}}^{(\mathrm{univ,bulk\text{-}def.})} = \frac{8}{3}\frac{\pi^4 L_{\mathbb{S}^4}^9}{G_{\mathrm{N}}} = \frac{M^3}{6}$$

$$S_{\mathrm{EE}}^{(\mathrm{univ})} \longrightarrow \frac{\mathrm{Vol}(\mathbb{S}^3/\mathbb{Z}_k)\mathrm{Vol}(\tilde{\mathbb{S}}^3/\mathbb{Z}_{k'})}{\mathrm{Vol}(\mathbb{S}^3)\mathrm{Vol}(\tilde{\mathbb{S}}^3)} S_{\mathrm{EE}}^{(\mathrm{univ})} = \frac{S_{\mathrm{EE}}^{(\mathrm{univ})}}{kk'}$$

$$\mathrm{d}s^2 = \frac{4L_{\mathbb{S}^4}^2}{v^2}\left[\,\mathrm{d}v^2 + \alpha_1\,\mathrm{d}s_{\mathrm{AdS}_3}^2 + \alpha_2\,\mathrm{d}s_{\mathbb{S}^3}^2\right] + L_{\mathbb{S}^4}^2\left[\alpha_3\,\mathrm{d}\phi^2 + \alpha_4 s_\phi^2\,\mathrm{d}s_{\tilde{\mathbb{S}}^3}^2\right]$$

$$r = \sqrt{z^2+\rho^2} \quad \text{and} \quad \theta = \arctan(\rho/z)$$

$$r(v,\phi) = -\frac{2(\gamma+1)^2 m_1}{\gamma v^2} + \frac{(2\gamma+1)m_1(c_{2\phi}-3)}{24\gamma} + \frac{m_2 c_\phi}{2m_1} + \left[\frac{3\gamma^2 m_2^2(7c_{2\phi}+1)}{4m_1^3} - \frac{\gamma^2 m_3(5c_{2\phi}+3)}{m_1^2} - \frac{2\gamma(2\gamma+1)m_2 c_\phi s_\phi^2}{m_1} - \frac{1}{16}(8\gamma(\gamma+1)+3)m_1 c_{2\phi} + \frac{37}{48}\gamma(\gamma+1)m_1 + \frac{73m_1}{192} + \frac{19}{192}(2\gamma+1)^2 m_1 c_{4\phi}\right]\frac{v^2}{48\gamma(\gamma+1)^2} + \mathcal{O}(v^4)$$

$$\theta(v,\phi) = \phi + \frac{(2\gamma+1)m_1^2 c_\phi + 3\gamma m_2}{12(\gamma+1)^2 m_1^2} s_\phi v^2 + \left[\frac{9\gamma^2 m_2^2 s_{2\phi}}{8m_1^4} - \frac{5\gamma^2 m_3 s_{2\phi}}{6m_1^3} + \frac{c_\phi s_\phi(5(2\gamma+1)^2 c_{2\phi} - 24\gamma(\gamma+1) - 7)}{48} + \frac{\gamma(2\gamma+1)m_2(3c_{2\phi}-1)s_\phi}{12m_1^2}\right]\frac{v^4}{16(\gamma+1)^4} + \mathcal{O}(v^6)$$

$$m_k := \sum_{j=1}^{2n+2} (-1)^j \xi_j^k$$

$$\alpha_1(\gamma) = 1 + \frac{2\gamma + 3 - (1+2\gamma)c_{2\phi}}{16(\gamma+1)^2}v^2 + [9(4(3\gamma - 13)\gamma + 16\gamma^2\kappa - 17)$$
$$-67(2\gamma+1)^2 c_{4\phi} + 12(8(3\gamma+1)\gamma + 20\gamma^2\kappa + 3)c_{2\phi}]\frac{v^4}{18432(\gamma+1)^4} + \mathcal{O}(v^6)$$
$$\alpha_2(\gamma) = \frac{(\gamma+1)^2}{\gamma^2} + \frac{2\gamma - 1 - (1+2\gamma)c_{2\phi}}{16\gamma^2}v^2 + [9(4(3\gamma+19)\gamma + 16\gamma^2\kappa + 47)$$
$$-67(2\gamma+1)^2 c_{4\phi} + 12(8(3\gamma+5)\gamma + 20\gamma^2\kappa + 19)c_{2\phi}]\frac{v^4}{18432\gamma^2(\gamma+1)^2} + \mathcal{O}(v^6)$$
$$\alpha_3(\gamma) = 1 + \frac{(2\gamma+1)c_\phi^2}{4(\gamma+1)^2}v^2 + [-12(\gamma+1)\gamma - 24\gamma^2\kappa - 9 + 6(2\gamma+1)^2 c_{2\phi}$$
$$+7(2\gamma+1)^2 c_{4\phi}]\frac{v^4}{768(\gamma+1)^4} + \mathcal{O}(v^6)$$
$$\alpha_4(\gamma) = 1 + \frac{(2\gamma+1)(2c_{2\phi}+1)}{12(\gamma+1)^2}v^2 + [-52(\gamma+1)\gamma - 24\gamma^2\kappa - 19$$
$$+79(2\gamma+1)^2 c_{4\phi} + 12(-2(\gamma+1)\gamma - 10\gamma^2\kappa - 3)c_{2\phi}]\frac{v^4}{4608(\gamma+1)^4} + \mathcal{O}(v^6)$$

$$\kappa \equiv \frac{3m_2^2 - 4m_1 m_3}{m_1^4}$$

$$L_{\mathbb{S}^4}^3 = \frac{|1+\gamma|^3}{\gamma^2}\frac{h_1 m_1}{2}.$$

$$\frac{\kappa}{\gamma^2} \to \frac{12(\hat{n}_1^2 - \hat{m}_1\hat{n}_2)}{\hat{m}_1^4}$$

$$\hat{n}_k := \sum_{j=1}^{2n+2}(-1)^j v_j^k \hat{\xi}_j.$$

$$\alpha_1(\gamma \to -\infty) = 1 + \frac{(3+5c_{2\phi})(\hat{n}_1^2 - \hat{m}_1\hat{n}_2)}{32\hat{m}_1^4}v^4$$
$$-\frac{5(1+7c_{2\phi})(2\hat{n}_1^3 - 3\hat{m}_1\hat{n}_1\hat{n}_2 + \hat{m}_1^2\hat{n}_3)c_\phi}{144\hat{m}_1^6}v^6 + \mathcal{O}(v^8)$$
$$\alpha_3(\gamma \to -\infty) = 1 + \frac{3(\hat{m}_1\hat{n}_2 - \hat{n}_1^2)}{8\hat{m}_1^4}v^4 + \frac{5(2\hat{n}_1^3 - 3\hat{m}_1\hat{n}_1\hat{n}_2 + \hat{m}_1^2\hat{n}_3)c_\phi}{12\hat{m}_1^6}v^6 + \mathcal{O}(v^8)$$
$$\alpha_4(\gamma \to -\infty) = 1 + \frac{(1+5c_{2\phi})(\hat{m}_1\hat{n}_2 - \hat{n}_1^2)}{16\hat{m}_1^4}v^4$$
$$+\frac{5(5c_\phi + 7c_{3\phi})(2\hat{n}_1^3 - 3\hat{m}_1\hat{n}_1\hat{n}_2 + \hat{m}_1^2\hat{n}_3)}{144\hat{m}_1^6}v^6 + \mathcal{O}(v^8)$$

$$r(v,\phi) = \frac{2\hat{m}_1}{v^2}$$
$$\theta(v,\phi) = \phi$$

$$H(r,\theta)=\begin{cases}\dfrac{h_1}{2}\displaystyle\sum_{j=1}^{2n+2}(-1)^j\hat{\xi}_j|\nu_j|^{-3}\left(\sum_{k=0}^{\infty}r^k\nu_j^{-k}P_k(c_\theta)\right)^3, r\in[0,|\nu_j|),\\ \dfrac{h_1}{2}\displaystyle\sum_{j=1}^{2n+2}(-1)^j\hat{\xi}_j r^{-3}\left(\sum_{k=0}^{\infty}r^{-k}\nu_j^{k}P_k(c_\theta)\right)^3, r\in(|\nu_j|,\Lambda_r(\epsilon_v,0)],\end{cases}$$

$$\Lambda_r(\epsilon_v,\theta)=\frac{2\hat{m}_1}{\epsilon_v^2}+\frac{\hat{n}_1c_\theta}{\hat{m}_1}+\frac{(3+5c_{2\theta})\hat{m}_1\hat{n}_2-(5+3c_{2\theta})\hat{n}_1^2}{16\hat{m}_1^3}\epsilon_v^2+\mathcal{O}(\epsilon_v^4)$$

$$A[\zeta_{\mathrm{RT}}]=\frac{32\pi^4L_{\mathbb{S}^4}^9}{\hat{m}_1^3}\log\left(\frac{2R}{\epsilon_u}\right)\sum_{j=1}^{2n+2}(-1)^j\hat{\xi}_j\left(\mathcal{I}_j^{(1)}+\mathcal{I}_j^{(2)}\right)+\mathcal{O}(\epsilon_u^2)$$

$$\mathcal{I}_j^{(1)}=\int_0^\pi \mathrm{d}\theta\int_0^{|\nu_j|}\mathrm{d}r r^4 s_\theta^3\left(\frac{1}{|\nu_j|}\sum_{k=0}^{\infty}\left(\frac{r}{\nu_j}\right)^kP_k(c_\theta)\right)^3$$

$$\mathcal{I}_j^{(2)}=\int_0^\pi \mathrm{d}\theta\int_{|\nu_j|}^{\Lambda_r(\epsilon_v,\theta)}\mathrm{d}r r s_\theta^3\left(\sum_{k=0}^{\infty}\left(\frac{\nu_j}{r}\right)^kP_k(c_\theta)\right)^3$$

$$\int_0^\pi \mathrm{d}\theta s_\theta P_{\ell_1}(c_\theta)P_{\ell_2}(c_\theta)P_{\ell_3}(c_\theta)=2\begin{pmatrix}\ell_1&\ell_2&\ell_3\\0&0&0\end{pmatrix}^2$$

$$\begin{pmatrix}\ell_1&\ell_2&\ell_3\\0&0&0\end{pmatrix}=\frac{(-1)^{\ell/2}(\ell/2)!}{\sqrt{(\ell+1)!}}\prod_{i=1}^{3}\frac{\sqrt{(\ell-2\ell_i)!}}{(\ell/2-\ell_i)!}$$

$$P_{\ell_1}P_{\ell_2}=\sum_{\ell_3=|\ell_1-\ell_2|}^{\ell_1+\ell_2}\begin{pmatrix}\ell_1&\ell_2&\ell_3\\0&0&0\end{pmatrix}^2(2\ell_3+1)P_{\ell_3}$$

$$\int_0^\pi \mathrm{d}\theta s_\theta^3P_{\ell_1}P_{\ell_2}P_{\ell_3}=\frac{4}{3}\begin{pmatrix}\ell_1&\ell_2&\ell_3\\0&0&0\end{pmatrix}^2-\frac{4}{3}\sum_{k=|2-\ell_1|}^{2+\ell_1}(2k+1)\begin{pmatrix}2&\ell_1&k\\0&0&0\end{pmatrix}^2\begin{pmatrix}k&\ell_2&\ell_3\\0&0&0\end{pmatrix}^2$$

$$\mathcal{I}_j^{(1)}=\sum_{\ell_1,\ell_2,\ell_3}\frac{1}{5+\ell}\frac{\nu_j^{\ell+2}}{|\nu_j|^\ell}\int_0^\pi \mathrm{d}\theta s_\theta^3P_{\ell_1}P_{\ell_2}P_{\ell_3}$$

$$\mathcal{I}_j^{(2)}=\sum_{\substack{\ell_1,\ell_2,\ell_3\\ \ell\neq2}}\frac{\nu_j^\ell}{2-\ell}\int_0^\pi \mathrm{d}\theta s_\theta^3P_{\ell_1}P_{\ell_2}P_{\ell_3}\left[\Lambda_r^{2-\ell}(\epsilon_v,\theta)-\frac{\nu_j^2}{|\nu_j|^\ell}\right]+\sum_{\substack{\ell_1,\ell_2,\ell_3\\ \ell=2}}\nu_j^2\int_0^\pi \mathrm{d}\theta s_\theta^3P_{\ell_1}P_{\ell_2}P_{\ell_3}\ln\left(\frac{\Lambda_r(\epsilon_v,\theta)}{|\nu_j|}\right)$$

$$\sum_{j=1}^{2n+2}(-1)^j\hat{\xi}_j\sum_{\substack{\ell_1,\ell_2,\ell_3\\ \ell=2}}\nu_j^2\int_0^\pi \mathrm{d}\theta s_\theta^3P_{\ell_1}P_{\ell_2}P_{\ell_3}\ln\left(\frac{\Lambda_r(\epsilon_v,\theta)}{|\nu_j|}\right)=\mathcal{O}(\epsilon_v^4)$$

$$\sum_{j=1}^{2n+2} (-1)^j \hat{\xi}_j \sum_{\substack{\ell_1,\ell_2,\ell_3 \\ \ell\neq 2}} \frac{v_j^{\ell}}{2-\ell} \int_0^{\pi} \mathrm{d}\theta s_\theta^3 P_{\ell_1} P_{\ell_2} P_{\ell_3} \Lambda_r^{2-\ell}(\epsilon_v, \theta) = \frac{8\hat{m}_1^3}{3\epsilon_v^4} + \frac{2\hat{n}_1^2}{5\hat{m}_1} + \mathcal{O}(\epsilon_v^2)$$

$$\sum_{\substack{\ell_1,\ell_2,\ell_3 \\ \ell\neq 2}} \left(\frac{1}{5+\ell} - \frac{1}{2-\ell}\right) \int_0^{\pi} \mathrm{d}\theta s_\theta^3 P_{\ell_1} P_{\ell_2} P_{\ell_3} = \sum_{\substack{a=0 \\ a\neq 2}}^{\infty} \left(\frac{1}{5+a} - \frac{1}{2-a}\right) \sum_{\substack{\ell_1,\ell_2,\ell_3 \\ \ell=a}} \int_0^{\pi} \mathrm{d}\theta s_\theta^3 P_{\ell_1} P_{\ell_2} P_{\ell_3}$$

$$= -\frac{2}{5} - \frac{5}{6} \sum_{\substack{\ell_1,\ell_2,\ell_3 \\ \ell=1}} \int_0^{\pi} \mathrm{d}\theta s_\theta^3 P_{\ell_1} P_{\ell_2} P_{\ell_3} + \cdots$$

$$\sum_{j=1}^{2n+2} (-1)^j \hat{\xi}_j \left(\mathcal{J}_j^{(1)} + \mathcal{J}_j^{(2)}\right) = \frac{8\hat{m}_1^3}{3\epsilon_v^4} + \frac{2}{5} \frac{\hat{n}_1^2 - \hat{n}_2\hat{m}_1}{\hat{m}_1} + \mathcal{O}(\epsilon_v^2)$$

$$A[\zeta_{\mathrm{RT}}] = \pi^4 L_{\mathbb{S}^4}^9 \log\left(\frac{2R}{\epsilon_u}\right) \left[\frac{256}{3} \frac{1}{\epsilon_v^4} + \frac{64}{5} \frac{\hat{n}_1^2}{\hat{m}_1^4} - \frac{64}{5} \frac{\hat{n}_2}{\hat{m}_1^3} + \mathcal{O}(\epsilon_v^2)\right] + \mathcal{O}(\epsilon_u^2)$$

Supercampos cuánticos relativistas bajo supergravedad.

$$\mathrm{d}\vec{F} = \vec{P}(\vec{F}) := \left(P^i(\vec{F})\right)_{i\in I}$$

$$\vec{F} := \left(F^i \in \Omega_{\mathrm{dR}}^{\deg_i}(X)\right)_{i\in I}$$

$$\star \vec{F} = \vec{\mu}(\vec{F})$$

$$\vec{F} = \begin{pmatrix} G_4 \in \Omega_{\mathrm{dR}}^4(X) \\ G_7 \in \Omega_{\mathrm{dR}}^7(X) \end{pmatrix}, \quad \begin{aligned} \mathrm{d}G_4 &= 0 \\ \mathrm{d}G_7 &= \frac{1}{2} G_4 \wedge G_4, \end{aligned}$$

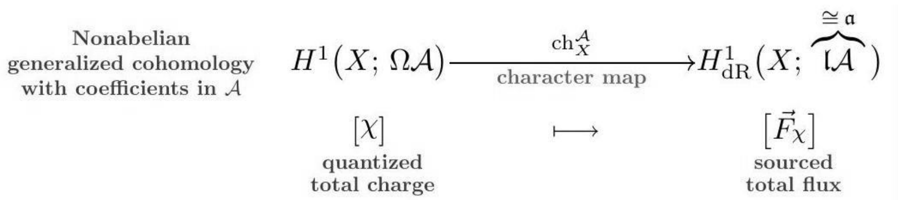
$$\begin{aligned} \star G_4 &= G_7, \\ \star G_7 &= -G_4. \end{aligned}$$

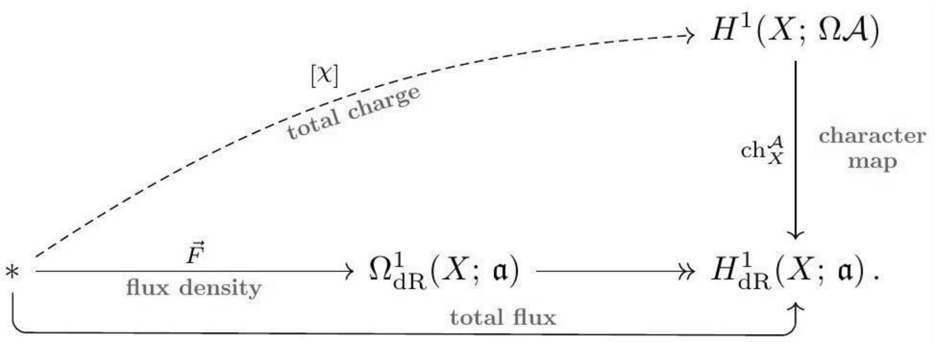
$$\mathrm{d}\vec{F} = \vec{P}(\vec{F}) \Leftrightarrow \vec{F} \in \Omega_{\mathrm{dR}}^1(X; \mathfrak{a})_{\mathrm{clsd}}$$

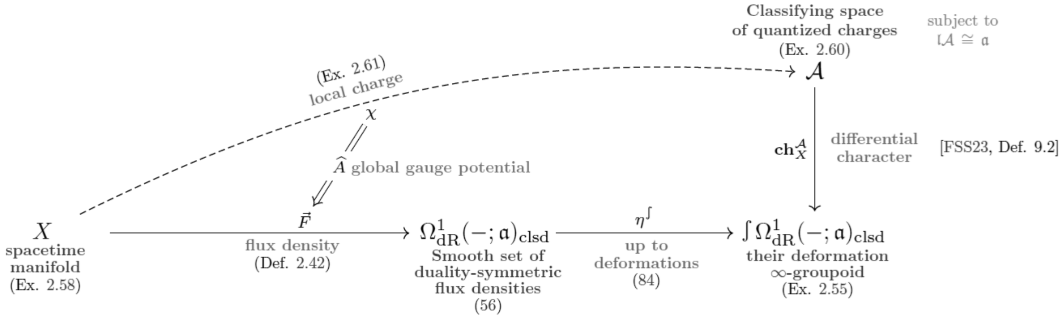
$$\vec{v} := \left(v_i \in \mathfrak{a}_{\deg_i - 1}\right)_{i\in I}$$

$$[-, \cdots, -] : \mathfrak{a}^{\otimes n} \longrightarrow \mathfrak{a}$$

$$\left[v_{j_1}, \cdots, v_{j_n}\right] = P_{j_1\cdots j_n}^i v_i, \text{ where } P^i\left(\left(F^j\right)_{j\in I}\right) = \sum_{n\in\mathbb{N}} P_{j_1\cdots j_n}^i F^{j_1} \cdots F^{j_n}$$



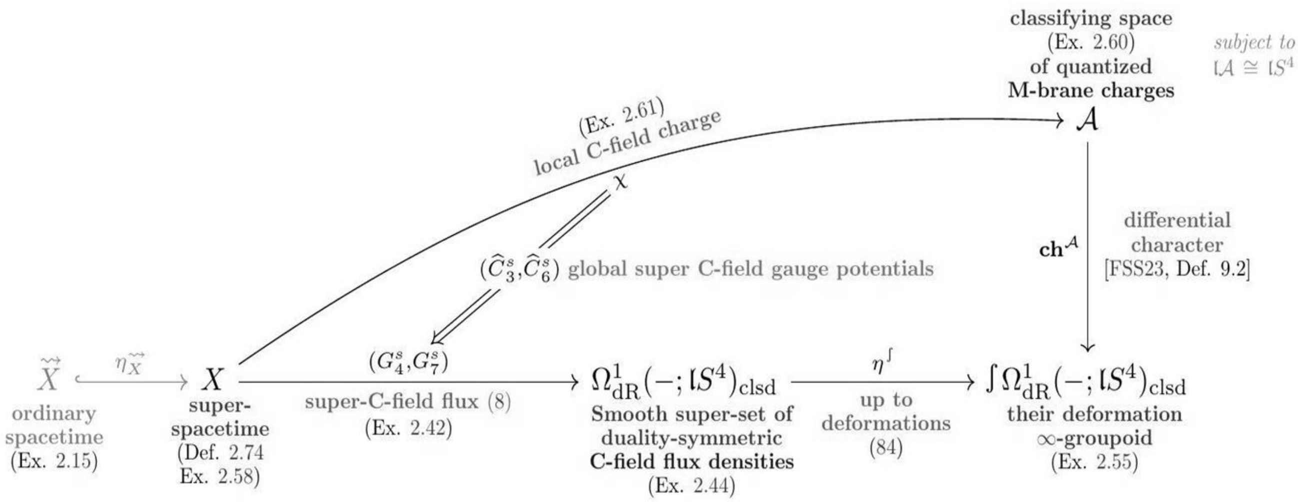


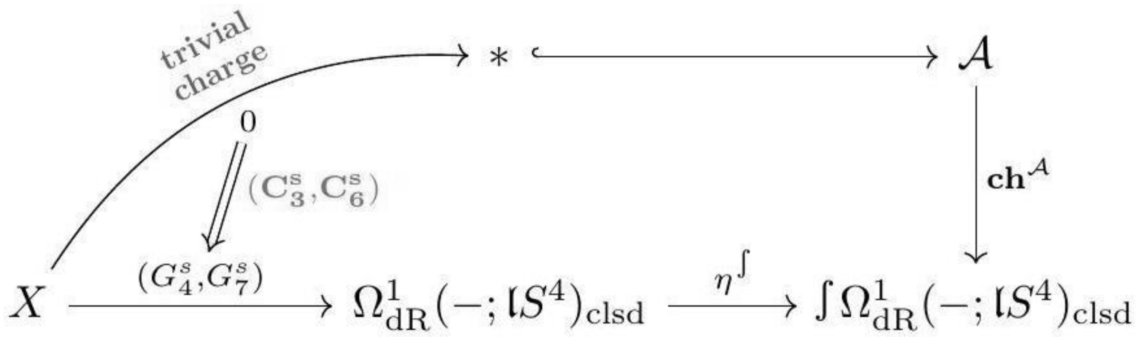


$$G_4^s := \frac{1}{4!}(G_4)_{a_1\cdots a_4} e^{a_1}\cdots e^{a_4} + \frac{1}{2}\left(\bar{\psi}\Gamma_{a_1 a_2}\psi\right)e^{a_1}e^{a_2}$$
$$G_7^s := \frac{1}{7!}(G_7)_{a_1\cdots a_7} e^{a_1}\cdots e^{a_7} + \frac{1}{5!}\left(\bar{\psi}\Gamma_{a_1\cdots a_5}\psi\right)e^{a_1}\cdots e^{a_5}$$

$$\mathrm{d}G_4^s = 0$$
$$\mathrm{d}G_7^s = \frac{1}{2}G_4^s \wedge G_4^s$$

$$\mathrm{d}G_7^s = \frac{1}{2}G_4^s \wedge G_4^s \;\Rightarrow\; (G_7)_{a_1\cdots a_7} = \frac{1}{4!}\epsilon_{a_1\cdots a_7 b_1\cdots b_4}(G_4)^{b_1\cdots b_4}$$





$$\left.\begin{matrix} C_3^s \in \Omega^3_{\mathrm{dR}}(X) \\ C_6^s \in \Omega^6_{\mathrm{dR}}(X) \end{matrix}\right\} \text{ s.t. } \begin{cases} \mathrm{d}C_3^s = G_4^s \\ \mathrm{d}C_6^s = G_7^s - \frac{1}{2} C_3^s G_4^s \end{cases}$$

$$(\eta_{\tilde{U}})^* G_4^s = \frac{1}{4!} (G_4)_{r_1 \cdots r_4}\Big|_{\theta^\rho = 0} \cdot \mathrm{d}x^{r_1} \cdots \mathrm{d}x^{r_4},$$
$$(\eta_{\tilde{U}})^* G_7^s = \frac{1}{7!} (G_7)_{r_1 \cdots r_7}\Big|_{\theta^\rho = 0} \cdot \mathrm{d}x^{r_1} \cdots \mathrm{d}x^{r_7},$$

$$\left(\frac{1}{4!} (G_4)_{r_1 \cdots r_4}\Big|_{\theta^\rho = 0} + \frac{1}{2} (\bar{\psi}_{r_1} \Gamma_{r_2 r_3} \psi_{r_4})\Big|_{\theta^\rho = 0}\right) \mathrm{d}x^{r_1} \cdots \mathrm{d}x^{r_4}$$
$$\left(\frac{1}{7!} (G_7)_{r_1 \cdots r_7}\Big|_{\theta^\rho = 0} + \frac{1}{5!} (\bar{\psi}_{r_1} \Gamma_{r_2 \cdots r_6} \psi_{r_7})\Big|_{\theta^\rho = 0}\right) \mathrm{d}x^{r_1} \cdots \mathrm{d}x^{r_7}$$

$$\mathrm{d}\left(\frac{1}{4!} (G_4)_{a_1 \cdots a_4} e^{a_1} \cdots e^{a_4}\right) = 0$$
$$\mathrm{d}\left(\frac{1}{7!} (G_7)_{a_1 \cdots a_7} e^{a_1} \cdots e^{a_7}\right) = \frac{1}{2}\left(\frac{1}{4!} (G_4)_{a_1 \cdots a_4} e^{a_1} \cdots e^{a_4}\right)^2$$

$$\mathrm{d}\left(\frac{1}{2} (\bar{\psi} \Gamma_{a_1 a_2}) e^{a_1} e^{a_2}\right) = 0$$
$$\mathrm{d}\left(\frac{1}{5!} (\bar{\psi} \Gamma_{a_1 \cdots a_5}) e^{a_1} \cdots e^{a_5}\right) = \frac{1}{2}\left(\frac{1}{2} (\bar{\psi} \Gamma_{a_1 a_2}) e^{a_1} e^{a_2}\right)^2$$

$$\mathrm{d}\left(\frac{1}{4!} (G_4)_{a_1 \cdots a_4} e^{a_1} \cdots e^{a_4} + \frac{1}{2} (\bar{\psi} \Gamma_{a_1 a_2}) e^{a_1} e^{a_2}\right) = 0$$
$$\mathrm{d}\left(\frac{1}{7!} (G_7)_{a_1 \cdots a_7} e^{a_1} \cdots e^{a_7} + \frac{1}{5!} (\bar{\psi} \Gamma_{a_1 \cdots a_5}) e^{a_1} \cdots e^{a_5}\right) = \frac{1}{2}\left(\frac{1}{4!} (G_4)_{a_1 \cdots a_4} e^{a_1} \cdots e^{a_4} + \frac{1}{2} (\bar{\psi} \Gamma_{a_1 a_2}) e^{a_1} e^{a_2}\right)^2$$

Flux densities on bosonic but curved manifolds, satisfying:

$$\mathrm{d}\Big(\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}\, e^{a_1}\cdots e^{a_4}\Big) = 0$$

$$\mathrm{d}\Big(\tfrac{1}{7!}(G_7)_{a_1\cdots a_7}\, e^{a_1}\cdots e^{a_7}\Big) = \tfrac{1}{2}\Big(\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}\, e^{a_1}\cdots e^{a_4}\Big)^2$$

Supersymmetric forms on flat but super spacetime, satisfying:

$$\mathrm{d}\Big(\tfrac{1}{2}\big(\overline{\psi}\,\Gamma_{a_1a_2}\big)e^{a_1}e^{a_2}\Big) = 0$$

$$\mathrm{d}\Big(\tfrac{1}{5!}\big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\big)e^{a_1}\cdots e^{a_5}\Big) = \tfrac{1}{2}\Big(\tfrac{1}{2}\big(\overline{\psi}\,\Gamma_{a_1a_2}\big)e^{a_1}e^{a_2}\Big)^2$$

Locally supersymmetric flux densities on curved supermanifolds, satisfying:

$$\mathrm{d}\Big(\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4} + \tfrac{1}{2}\big(\overline{\psi}\,\Gamma_{a_1a_2}\big)e^{a_1}e^{a_2}\Big) = 0$$

$$\mathrm{d}\Big(\tfrac{1}{7!}(G_7)_{a_1\cdots a_7}\, e^{a_1}\cdots e^{a_7} + \tfrac{1}{5!}\big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\big)e^{a_1}\cdots e^{a_5}\Big) = \tfrac{1}{2}\Big(\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4} + \tfrac{1}{2}\big(\overline{\psi}\,\Gamma_{a_1a_2}\big)e^{a_1}e^{a_2}\Big)^2$$

thereby enforcing the equations of motion of 11d supergravity.

$$\mathbb{R}^{1,10|32}_{\mathrm{ex},s} \overset{\phi^0}{\rightsquigarrow} \mathbb{R}^{1,10|32}$$

$$\overbrace{\mathbb{R}^{1,10|\mathbf{10}}_{\mathrm{ex},s}} \cong \mathbb{R}^{1,10} \times \Lambda^2\big(\mathbb{R}^{1,10|\mathbf{32}}\big)^* \times \Lambda^5\big(\mathbb{R}^{1,10|\mathbf{32}}\big)^*$$

$$\mathrm{d}(\underbrace{\alpha_0(s)e_{a_1a_2}e^{a_1}e^{a_2} + \cdots}_{H_3^0}) = (\phi^0)^*(\underbrace{\frac{1}{2}(\bar{\psi}\Gamma_{a_1a_2}\psi)e^{a_1}e^{a_2}}_{G_4^0}).$$

$$\mathrm{d}\big((H_3)_{a_1a_2a_3}e^{a_1}e^{a_2}e^{a_3}\big) = \phi^*\big((G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4}\big)$$

$$\mathrm{d}\Big(\frac{1}{3!}(H_3)_{a_1a_2a_3}e^{a_1}e^{a_2}e^{a_3} + \alpha_0(s)e_{a_1a_2}e^{a_1}e^{a_2} + \cdots\Big) = (\phi^s)^*\Big(\frac{1}{4!}(G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4} + \frac{1}{2!}(\bar{\psi}\Gamma_{a_1a_2}\psi)e^{a_1}e^{a_2}\Big)$$

	Even	Odd
Frame	$a \in \{0,\cdots,10\}$	$\alpha \in \{1,\cdots,32\}$
Coord	$r \in \{0,\cdots,10\}$	$\rho \in \{1,\cdots,32\}$

so that

frame-		**coord-differentials**		
e^a	$=$	$e^a_r\,\mathrm{d}x^r$	$+$	$e^a_\rho\,\mathrm{d}\theta^\rho$
ψ^α	$=$	$\psi^\alpha_r\,\mathrm{d}x^r$	$+$	$\psi^\alpha_\rho\,\mathrm{d}\theta^\rho$

$$(\eta_{ab})^D_{a,b=0} = \big(\eta^{ab}\big)^D_{a,b=0} := (\mathrm{diag}(-1,+1,+1,\cdots,+1))^D_{a,b=0}$$

$$V^a := V_b\eta^{ab}, V_a = V^b\eta_{ab}$$

$$V_{[a_1\cdots a_p]} := \frac{1}{p!}\sum_{\sigma\in\mathrm{Sym}(n)}(-1)^{|\sigma|}V_{a_{\sigma(1)}\cdots a_{\sigma(p)}}$$

$$\epsilon_{012\cdots} := +1 \text{ hence } \epsilon^{012\cdots} := -1.$$

$$\delta^{a_1\cdots a_p}_{b_1\cdots b_p} := \delta^{[a_1}_{[b_1}\cdots\delta^{a_p]}_{b_p]} = \delta^{a_1}_{[b_1}\cdots\delta^{a_p}_{b_p]} = \delta^{[a_1}_{b_1}\cdots\delta^{a_p]}_{b_p}$$

$$V_{a_1\cdots a_p}\delta^{a_1\cdots a_p}_{b_1\cdots b_p} = V_{[b_1\cdots b_p]} \text{ and } \epsilon^{c_1\cdots c_p a_1\cdots a_q}\epsilon_{c_1\cdots c_p b_1\cdots b_q} = -p!\cdot q!\,\delta^{a_1\cdots a_q}_{b_1\cdots b_q}$$

$$\Gamma_a\Gamma_b + \Gamma_b\Gamma_a = +2\eta_{ab}.$$

$$\Gamma_{a_1\cdots a_p} := \Gamma_{[a_1}\cdots\Gamma_{a_p]} := \frac{1}{p!}\sum_{\sigma} (-1)^{|\sigma|}\Gamma_{a_{\sigma(1)}}\cdots\Gamma_{a_{\sigma(p)}},$$

$$\Gamma_{a_1\cdots a_p} = \begin{cases}\Gamma_{a_1}\cdots\Gamma_{a_p}\\ 0\end{cases}$$

$$\eta = -\eta^{\mathrm{CDF}}, \Gamma_a = \mathrm{i}\Gamma_a^{\mathrm{CDF}}$$

$$(V\otimes V')_\sigma := V_0\otimes V'_{0+\sigma}\oplus V_1\otimes V'_{1+\sigma}$$

$$V\otimes V' \underset{\sim}{\overset{\mathrm{br}_{V,V'}}{\leftrightsquigarrow}} V'\otimes V$$

$$v\in V_\sigma, v'\in V'_{\sigma'} \Rightarrow \mathrm{brd}_{V,V'}(v\otimes v') := (-1)^{\sigma\cdot\sigma'}v'\otimes v$$

$$V\in \mathrm{Mod} \Rightarrow \begin{cases}V_{\mathrm{odd}}\in \mathrm{sMod}\\ (V_{\mathrm{odd}})_0 = 0\\ (V_{\mathrm{odd}})_1 = V\end{cases}$$

$$(V^*)_\sigma \cong (V_\sigma)^*$$

$$(-)\cdot(-): A\otimes A\longrightarrow A$$

$$a\in A_\sigma, a'\in A_{\sigma'} \Rightarrow \begin{cases}a\cdot a'\in A_{\sigma+\sigma'}\\ a\cdot a' = (-1)^{\sigma\cdot\sigma'}a'\cdot a\end{cases}$$

$$(-)\otimes(-): \mathrm{sCAlg}\times\mathrm{sCAlg}\longrightarrow \mathrm{sCAlg}$$

$$(A\otimes A')_\sigma := A_0\otimes A'_\sigma\oplus A_1\otimes A'_{1+\sigma}$$

$$\begin{array}{rcl}\mathrm{SmthMfd} & \hookrightarrow & \mathrm{sCAlg}^{\mathrm{op}}\\ X & \rightsquigarrow & C^\infty(X)\end{array}$$

$$\theta^{\rho_1}\theta^{\rho_2} = -\theta^{\rho_2}\theta^{\rho_1}, \theta^\rho\theta^\rho = 0$$

$$a + \sum_{\rho=1}^{q} a_\rho\theta^\rho + \sum_{\rho_1,\rho_2=1}^{q}\frac{1}{2}a_{\rho_1\rho_2}\theta^{\rho_1}\theta^{\rho_2} + \cdots + a_{1\cdots q}\theta^1\cdots\theta^q, a_{\dots}\in\mathbb{R}$$

$$\begin{array}{ccc}\mathrm{sCartSp} & \xhookrightarrow{C^\infty(-)} & \mathrm{sCAlg}^{\mathrm{op}}\\ \mathbb{R}^{n|q} & \longmapsto & C^\infty(\mathbb{R}^n)\otimes\wedge^\bullet(\mathbb{R}^q)^*.\end{array}$$

$$\left\{U_i\cong\mathbb{R}^{n|q}\overset{\iota_i}{\leftrightsquigarrow}\mathbb{R}^{n|q}\right\}_{i\in I}$$

$$\left\{ \overset{\rightsquigarrow}{U}_i \xrightarrow{\overset{\rightsquigarrow}{\iota}_i} \mathbb{R}^n \right\}_{i \in I}$$

$$\text{sSmthSet} := L^{\text{liso}}\ \text{Func}(\text{sCartSp}^{\text{op}}, \text{Set}).$$

$$\begin{array}{ccc} \text{sCartSp}^{\text{op}} & \longrightarrow & \text{Set} \\ \mathbb{R}^{n|q} & \longmapsto & \text{Plt}(\mathbb{R}^{n|q}, X) \end{array}$$

$$X \underset{\text{liso}}{\overset{p}{\longleftarrow}} \widehat{X} \overset{f}{\longrightarrow} Y$$

$$\widehat{X} \xrightarrow{\text{liso}} X \quad \Leftrightarrow \quad \underset{n,q \in \mathbb{N}}{\forall} \quad \text{PltGrm}(\mathbb{R}^{n|q}, \widehat{X}) \xrightarrow{\sim} \text{PltGrm}(\mathbb{R}^{n|q}, X)$$

$$\text{PltGrm}(\mathbb{R}^{n|q}, X) := \text{PltGrm}(\mathbb{R}^{n|q}, X) / \sim$$

$$X \in \text{sSmthMfd} \Rightarrow \text{Plt}(\mathbb{R}^{n|q}, X) := \text{Hom}_{\text{sSmthMfd}}(\mathbb{R}^{n|q}, X).$$

$$\text{sSmthMfd} \leftrightsquigarrow \text{sSmthSet}.$$

$$C^\infty(\overset{\rightsquigarrow}{X}|V_{\text{odd}}) := \wedge^{\bullet}_{C^\infty(\overset{\rightsquigarrow}{X})} \Gamma_X(V^*) = \Gamma_X(\wedge^{\bullet} V^*)$$

$$\overset{\rightsquigarrow}{X}|V_{\text{odd}} \in \text{sSmthSet}, \quad \text{with} \quad \text{Plt}(\mathbb{R}^{n|q}, \overset{\rightsquigarrow}{X}|V_{\text{odd}}) := \text{Hom}_{\text{sCAlg}}\left(C^\infty(\overset{\rightsquigarrow}{X}|V_{\text{odd}}), C^\infty(\mathbb{R}^{n|q})\right)$$

$$\begin{array}{cccc} \text{Plt}(-; \widehat{X}) & : \text{sCartSp}^{\text{op}} & \longrightarrow & \text{Set} \\ & \mathbb{R}^{n|q} & \longmapsto & \text{Hom}_{\text{sSmthMfd}}\left(\mathbb{R}^{n|q}, \coprod_{i \in I} U_i\right) / \sim \end{array}$$

$$\begin{array}{ccc} \text{Plt}(\mathbb{R}^{n|q}, \widehat{X}) & \longrightarrow & \text{Plt}(\mathbb{R}^{n|q}, X) \\ \phi_i & \longmapsto & \iota_i \circ \phi_i \end{array}$$

$$\eta_X : \overset{\rightsquigarrow}{X} \hookrightarrow X$$

$$f(x) + \sum_{\rho=1}^{q} f_\rho(x)\theta^\rho + \sum_{\rho_1,\rho_2=1}^{q} \frac{1}{2} f(x)_{\rho_1\rho_2}\theta^{\rho_1}\theta^{\rho_2} + \cdots + f(x)_{1\cdots q}\theta^1 \cdots \theta^q \longmapsto f(x)$$

$$\eta: \text{sSmthMfd} \longrightarrow \text{sSmthMfd}$$

$$V = \bigoplus_{\substack{n\in\mathbb{Z}\\ \sigma\in\mathbb{Z}_2}} V_{n,\sigma}$$

$$(V \otimes V')_{n,\sigma} := \bigoplus_{k\in\mathbb{Z},\rho\in\mathbb{Z}_2} V_{k,\sigma} \otimes V_{n-k,\sigma-\rho},$$

$$v \in V_{n,\sigma}, v' \in V_{n',\sigma'} \;\Rightarrow\; \mathrm{brd}_{V,V'}(v \otimes v') = (-1)^{n\cdot n'}(-1)^{\sigma\cdot\sigma'} v' \otimes v$$

$$\mathrm{sgMod}^{\,\mathrm{ft}} \hookrightarrow \mathrm{sgMod}$$

$$(V^*)_{n,\sigma} = \left(V_{-n,\sigma}\right)^*.$$

$$(V^\vee)_{n,\sigma} := \left(V_{n,\sigma}\right)^*$$

$$(bV)_{n,\sigma} := V_{n-1,\sigma}$$

$$A \equiv \bigoplus_{n\in\mathbb{Z}} \left(A_{n,0} \oplus A_{n,1}\right)$$

$$(-)\cdot(-): A \otimes A \longrightarrow A$$

$$a \in A_{n,\sigma}, a' \in A'_{n',\sigma'}, \Rightarrow \begin{cases} a \cdot a' \in A_{n+n',\sigma+\sigma'} \\ a \cdot a' = (-1)^{n\cdot n' + \sigma\cdot\sigma'} a' \cdot a \end{cases}$$

$$\mathbb{R}[V] := \mathrm{Sym}(V) \in \mathrm{sgCAlg}$$

$$v_i \cdot v'_{i'} = (-1)^{n_i\cdot n'_{i'} + \sigma_i\cdot\sigma'_{i'}} v'_{i'} \cdot v_i$$

$$\mathrm{d}: A \longrightarrow A$$

$$a \in A_{n,\sigma}, a' \in A'_{n',\sigma'} \Rightarrow \begin{cases} \mathrm{d}a \in A_{n+1,\sigma} \\ \mathrm{d}(a \cdot a') = (\mathrm{d}a) \cdot a' + (-1)^n a \cdot \mathrm{d}a' \\ \mathrm{dd}a = 0 \end{cases}$$

$$(A \otimes A')_{n,\sigma} := \bigoplus_{k\in\mathbb{Z},\rho\in\mathbb{Z}_2} A_{k,\sigma} \otimes A_{n-k,\sigma-\rho}.$$

$$\begin{array}{ccc} \mathrm{shLAlg}^{\mathrm{ft}} & \xhookrightarrow{\mathrm{CE}(-)} & \mathrm{sdgcAlg}^{\mathrm{op}} \\ \left(\mathfrak{a}, [-], [-,-], [-,-,-], \cdots\right) & \longmapsto & \left(\mathbb{R}\left[b\mathfrak{a}^\vee\right], \mathrm{d}_{|b\mathfrak{a}^\vee} = [-]^* + [-,-]^* + [-,-,-]^* + \cdots\right) \end{array}$$

$$\left[v^i, v^j\right] = f_k^{ij} v^k$$

$$\omega_i\omega_j = -(-1)^{\sigma_i\cdot\sigma_j}\omega_j\omega_i$$

$$\mathrm{d}\omega_k = \frac{1}{2} f_k^{ij}\omega_i\omega_j$$

$$\mathfrak{iso}\left(\mathbb{R}^{1,10|\mathbf{32}}\right) \in \mathrm{shLAlg}^{\mathrm{fr}}$$

$$\mathrm{CE}\left(\mathfrak{iso}\left(\mathbb{R}^{1,D-1|\mathbf{32}}\right)\right) = \mathbb{R}\begin{bmatrix} (e^a)_{a=0}^{10}, & \deg(e^a) = (1{,}0) \\ (\omega^{ab} = -\omega^{ba})_{a,b=0}^{10}, & \deg(\omega^{ab}) = (1{,}0) \\ (\psi)_{\alpha=1}^{32}, & \deg(\psi^\alpha) = (1{,}1) \end{bmatrix} \Big/ \begin{pmatrix} \mathrm{d}e^a = -\omega^a{}_b e^b + (\bar{\psi}\Gamma^a\psi) \\ \mathrm{d}\omega^{ab} = -\omega^a{}_c\omega^{cb} \\ \mathrm{d}\psi^\alpha = 0 \end{pmatrix}$$

$$\begin{array}{ccc} \mathfrak{so}(1,10) = \mathfrak{so}\left(\mathbb{R}^{1,10}\right) & \hookrightarrow & \mathfrak{iso}\left(\mathbb{R}^{1,10|\mathbf{32}}\right) \\ \mathrm{CE}(\mathfrak{so}(1,10)) & \twoheadleftarrow & \mathrm{CE}\left(\mathfrak{iso}\left(\mathbb{R}^{1,10|\mathbf{32}}\right)\right) \\ 0 & \longleftarrow & e^a \\ 0 & \longleftarrow & \psi^\alpha \\ \omega^a{}_b & \longleftarrow & \omega^a{}_b\,, \end{array}$$

$$\mathbb{R}^{1,10|\mathbf{32}} := \mathfrak{iso}\left(\mathbb{R}^{1,10|\mathbf{32}}\right)/\mathfrak{so}\left(\mathbb{R}^{1,10|\mathbf{32}}\right)$$

$$\mathrm{CE}\left(\mathbb{R}^{1,10|\mathbf{32}}\right) = \mathbb{R}\begin{bmatrix} (e^a)_{a=0}^{10}, & \deg(e^a) = (1{,}0) \\ (\psi)_{\alpha=1}^{32}, & \deg(\psi^\alpha) = (1{,}1) \end{bmatrix} \Big/ \begin{pmatrix} \mathrm{d}e^a = +(\bar{\psi}\Gamma^a\psi) \\ \mathrm{d}\psi^\alpha = 0 \end{pmatrix}$$

$$\left[\bar{Q}_\alpha, Q_\beta\right] = \Gamma^a_{\alpha\beta}P_a$$

$$\mathrm{CE}(\mathfrak{l}S^4) \cong \mathbb{R}[G_4, G_7] \Big/ \begin{pmatrix} \mathrm{d}G_4 = 0 \\ \mathrm{d}G_7 = \frac{1}{2}G_4G_4 \end{pmatrix}$$

$\mathfrak{l}S^4 \cong \mathbb{R}\langle v_3,\, v_6\rangle$ with only non-vanishing bracket of generators being $[v_3, v_3] = v_6$

$$\mathrm{d}G_7 = \frac{1}{2}G_4G_4$$
$$v_6 = [v_3, v_3]$$

$$\begin{array}{ccc} \mathbb{R}^{1,10|\mathbf{32}} & \xrightarrow{(G_4^0,\, G_7^0)} & \mathfrak{l}S^4 \\ \mathrm{CE}\left(\mathbb{R}^{1,10|\mathbf{32}}\right) & \longleftarrow & \mathrm{CE}\left(\mathfrak{l}S^4\right) \\ \frac{1}{2}\left(\overline{\psi}\,\Gamma_{a_1a_2}\,\psi\right)e^{a_1}\,e^{a_2} & \longleftarrow & G_4 \\ \frac{1}{5!}\left(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\psi\right)e^{a_1}\cdots e^{a_5} & \longleftarrow & G_7\,. \end{array}$$

$$\left.\begin{array}{l} \mathrm{d}\left(\frac{1}{2}(\bar{\psi}\Gamma_{a_1a_2}\psi)e^{a_1}e^{a_2}\right) = 0 \\ \mathrm{d}\left(\frac{1}{5!}(\bar{\psi}\Gamma_{a_1\cdots a_5}\psi)e^{a_1}\cdots e^{a_5}\right) = \frac{1}{2}\left(\frac{1}{2}(\bar{\psi}\Gamma_{a_1a_2}\psi)e^{a_1}e^{a_2}\right)\left(\frac{1}{2}(\bar{\psi}\Gamma_{a_1a_2}\psi)e^{a_1}e^{a_2}\right) \end{array}\right\} \in \mathrm{CE}\left(\mathbb{R}^{1,10|\mathbf{32}}\right)$$

$$\begin{array}{ccc} TX & \xrightarrow{(G_4^s,\, G_7^s)} & \mathfrak{l}S^4 \\ \Omega^\bullet_{\mathrm{dR}}(X) & \longleftarrow & \mathrm{CE}(\mathfrak{l}S^4) \\ (G_4)_{a_1\cdots a_4}\, e^{a_1}\cdots e^{a_4} + \tfrac{1}{2}\big(\overline{\psi}\,\Gamma_{a_1 a_2}\,\psi\big)e^{a_1}\,e^{a_2} & \longleftarrow\!\shortmid & G_4 \\ (G_7)_{a_1\cdots a_7}\, e^{a_1}\cdots e^{a_7} + \tfrac{1}{5!}\big(\overline{\psi}\,\Gamma_{a_1\cdots a_7}\,\psi\big)e^{a_1}\cdots e^{a_7} & \longleftarrow\!\shortmid & G_7 \end{array}$$

$$\mathbb{R}[G_4,\, G_7]\Big/\left(\begin{array}{rcl} \mathrm{d}\,G_4 & = & 0 \\ \mathrm{d}\,G_7 & = & \tfrac{1}{2}\,G_4\,G_4 \end{array}\right)$$

$$\begin{array}{ccc} \mathfrak{l}S^4 & \xrightarrow{\sim} & \mathfrak{l}'S^4 \\ \mathrm{CE}(\mathfrak{l}S^4) & \xleftarrow{\sim} & \mathrm{CE}(\mathfrak{l}'S^4) \\ G_4 & \longleftarrow\!\shortmid & G_4 \\ 2\,G_7 & \longleftarrow\!\shortmid & G_7 \end{array}$$

$$\Omega^\bullet_{\mathrm{dR}}\big(\mathbb{R}^{0\vert q}\big) \;\in\; \mathrm{sdgCAlg}$$

$$\Omega^\bullet_{\mathrm{dR}}\big(\mathbb{R}^{0\vert q}\big) \;\cong\; \bigoplus_{p=0}^{q}\; \bigoplus_{1\leq\rho_1<\cdots<\rho_p\leq q} C^\infty\big(\mathbb{R}^{0\vert q}\big)\big\langle \mathrm{d}\theta^{\rho_1}\cdots\mathrm{d}\theta^{\rho_p}\big\rangle$$

$$\Omega^\bullet_{\mathrm{dR}}\big(\mathbb{R}^{n\vert q}\big) \;\in\; \mathrm{sdgCAlg}$$

$$\Omega^\bullet_{\mathrm{dR}}\big(\mathbb{R}^{n\vert q}\big) \;:=\; \Omega^\bullet_{\mathrm{dR}}\big(\mathbb{R}^{n}\big)\otimes\Omega^\bullet_{\mathrm{dR}}\big(\mathbb{R}^{0\vert q}\big) \;\in\; \mathrm{sdgCAlg}\,.$$

$$\Omega^1_{\mathrm{dR}}\big(\mathbb{R}^{n\vert q}\big) \cong \mathrm{Hom}^{\mathrm{fib.lin.}}_{\mathrm{sSmthMfd}}\big(T(\mathbb{R}^{n\vert q}), \mathbb{R}\times\mathbb{R}_{\mathrm{odd}}\big)$$

$$T^{\times k}\big(\mathbb{R}^{n\vert q}\big) \longrightarrow \mathbb{R}\times\mathbb{R}_{\mathrm{odd}}$$

$$\Omega^1_{\mathrm{dR}}\big(\mathbb{R}^{n\vert q};\, V\big) \;:=\; \mathrm{Hom}_{\mathrm{sgCAlg}}\big(\mathrm{Sym}(bV^\vee),\, \Omega^\bullet_{\mathrm{dR}}(\mathbb{R}^{n\vert q})\big)$$

$$\Omega^1_{\mathrm{dR}}\big(\mathbb{R}^{n\vert q}; V\big) \;\cong\; \Big(\Omega^\bullet_{\mathrm{dR}}\big(\mathbb{R}^{n\vert q}\big)\otimes\big(V^\vee\big)^*\Big)_{(1,0)}$$

$$\Omega^1_{\mathrm{dR}}\big(\mathbb{R}^{n\vert q};\, V\big) \cong \mathrm{Hom}^{\mathrm{fib.lin.}}_{\mathrm{sSmthMfd}}\big(T(\mathbb{R}^{n\vert q}),\, V\big)$$

$$
\begin{aligned}
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{n|q};\mathbb{R}) &\cong \Omega^1_{\mathrm{dR}}(\mathbb{R}^{n|q})_0\\
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{n|q};b^k\mathbb{R}) &\cong \Omega^{1+k}_{\mathrm{dR}}(\mathbb{R}^{n|q})_0\\
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{n|0};b^k\mathbb{R}) &\cong \Omega^{1+k}_{\mathrm{dR}}(\mathbb{R})\\
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{n|0};b^k\mathbb{R}_{\mathrm{odd}}) &\cong 0\\
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{0|1};\mathbb{R}) &\cong \mathbb{R}\langle\theta\mathrm{d}\theta\rangle\\
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{0|1};\mathbb{R}_{\mathrm{odd}}) &\cong \mathbb{R}\langle\mathrm{d}\theta\rangle\\
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{1|1};\mathbb{R}) &\cong C^\infty(\mathbb{R})\langle\mathrm{dx}\rangle\oplus C^\infty(\mathbb{R})\langle\theta\mathrm{d}\theta\rangle\\
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{1|1};\mathbb{R}_{\mathrm{odd}}) &\cong C^\infty(\mathbb{R})\langle\theta\mathrm{dx}\rangle\oplus C^\infty(\mathbb{R})\langle\mathrm{d}\theta\rangle\\
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{1|2};\mathbb{R}) &\cong C^\infty(\mathbb{R})\langle\mathrm{dx},\theta^1\theta^2\mathrm{dx}\rangle\oplus C^\infty(\mathbb{R})\langle\theta^1\,\mathrm{d}\theta^1,\theta^2\,\mathrm{d}\theta^1,\theta^1\,\mathrm{d}\theta^2,\theta^2\,\mathrm{d}\theta^2\rangle\\
\Omega^1_{\mathrm{dR}}(\mathbb{R}^{1|2};\mathbb{R}_{\mathrm{odd}}) &\cong C^\infty(\mathbb{R})\langle\theta^1\,\mathrm{dx},\theta^2\,\mathrm{dx}\rangle\oplus C^\infty(\mathbb{R})\langle\mathrm{d}\theta^1,\mathrm{d}\theta^2\rangle
\end{aligned}
$$

$$
\begin{array}{ccccc}
\Omega^1_{\mathrm{dR}}(-;V) & : & \mathrm{sCartSp}^{\mathrm{op}} & \longrightarrow & \mathrm{Set}\\
 & & \mathbb{R}^{n|q} & \longmapsto & \Omega^1_{\mathrm{dR}}(\mathbb{R}^{n|q};V)
\end{array}
$$

$$F\colon X\longrightarrow \Omega^1_{\mathrm{dR}}(-;V).$$

$$\Omega^1_{\mathrm{dR}}(X;V) := \mathrm{Hom}_{\mathrm{sSmthSet}}\big(X,\Omega^1_{\mathrm{dR}}(-;V)\big).$$

$$
\begin{array}{ccc}
\Omega^1_{\mathrm{dR}}(Y;V) & \xrightarrow{\;f^*\;} & \Omega^1_{\mathrm{dR}}(X;V)\\
\| & & \|\\
\mathrm{Hom}(Y;\Omega^1_{\mathrm{dR}}(-;V)) & \longrightarrow & \mathrm{Hom}(X;\Omega^1_{\mathrm{dR}}(-;V))\\
\phi & \longmapsto & \phi\circ f\,.
\end{array}
$$

$$\Omega^1_{\mathrm{dR}}\big(\overset{\rightsquigarrow}{X},\,\mathbb{R}_{\mathrm{odd}}\big)\;\cong\;0$$

$$\Omega^1_{\mathrm{dR}}(\mathbb{R}^{n|q};\,\mathfrak{a})_{\mathrm{clsd}}\;:=\;\mathrm{Hom}_{\mathrm{sdgcAlg}}\big(\mathrm{CE}(\mathfrak{a}),\,\Omega^\bullet_{\mathrm{dR}}(\mathbb{R}^{n|q})\big)$$

$$\Omega^1_{\mathrm{dR}}(\mathbb{R}^{n|q};\mathfrak{a})_{\mathrm{clsd}}\;\cong\;\mathrm{MC}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathbb{R}^{n|q})\otimes(\mathfrak{a}^\vee)^*\Big)_0$$

$$T(\mathbb{R}^{n|q})\longrightarrow\mathfrak{a}$$

$$\Omega^1_{\mathrm{dR}}(-;\mathfrak{a})_{\mathrm{clsd}}\;\rightsquigarrow\;\Omega^1_{\mathrm{dR}}(-;\mathfrak{a})\in\;\mathrm{sSmthSet}.$$

$$\Omega^1_{\mathrm{dR}}(X;\mathfrak{a})_{\mathrm{clsd}} := \mathrm{Hom}_{\mathrm{sSmthSet}}\big(X;\Omega^1_{\mathrm{dR}}(-;\mathfrak{a})_{\mathrm{clsd}}\big)$$

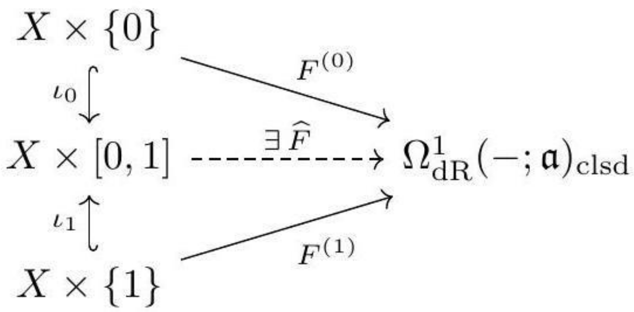
$$\Omega^1_{\mathrm{dR}}(X;b^k\mathbb{R})_{\mathrm{clsd}}\cong\{F\in\Omega^{1+k}_{\mathrm{dR}}(X)\mid \mathrm{d}F=0\}$$

$$\Omega^1_{\mathrm{dR}}\big(X;\, \mathfrak{l}S^4\big)_{\mathrm{clsd}} \;\cong\; \left\{ \begin{array}{lcl|lcl} G_4 & \in & \Omega^1_{\mathrm{dR}}(X;\, b^3\mathbb{R}) & \mathrm{d}\,G_4 & = & 0 \\ G_7 & \in & \Omega^1_{\mathrm{dR}}(X;\, b^6\mathbb{R}) & \mathrm{d}\,G_7 & = & \tfrac{1}{2} G_4 \wedge G_4 \end{array} \right\}$$

$$\mathrm{Hom}_{\mathrm{sSmthSet}}\Big(X, \Omega^1_{\mathrm{dR}}(-; \mathfrak{l}S^4)\Big) \cong \Omega^1_{\mathrm{dR}}(X; \mathfrak{l}S^4).$$

$$F^{(0)}, F^{(1)} \in \Omega^1_{\mathrm{dR}}(X; \mathfrak{a})_{\mathrm{clsd}}$$

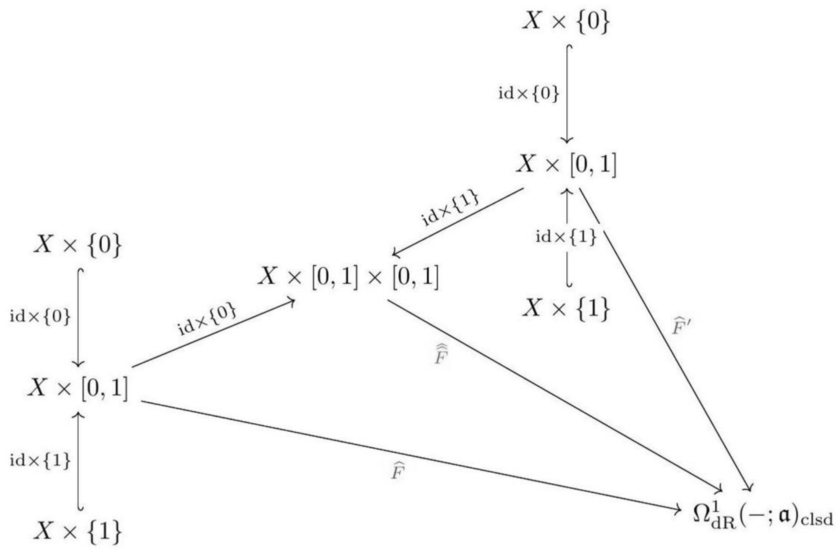
$$F^{(0)} \sim F^{(1)} \qquad \Leftrightarrow$$



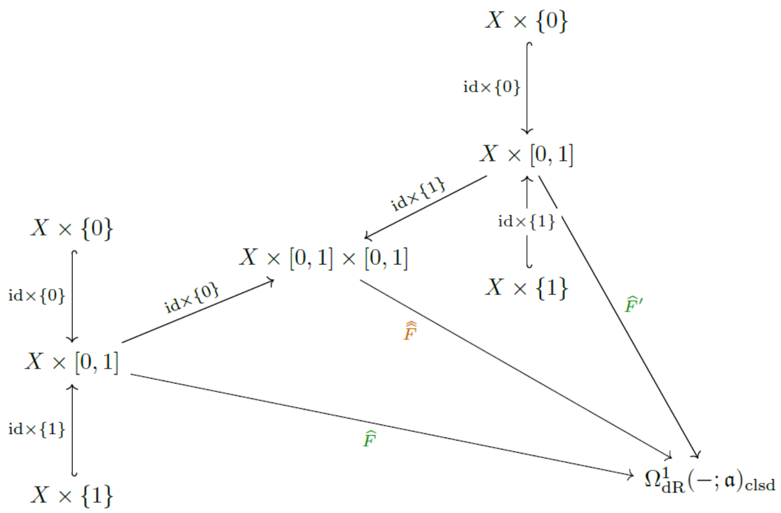
$$H^1_{\mathrm{dR}}(X; \mathfrak{a}) := \Omega^1_{\mathrm{dR}}(X; \mathfrak{a})_{\mathrm{clsd}} /\!\sim.$$

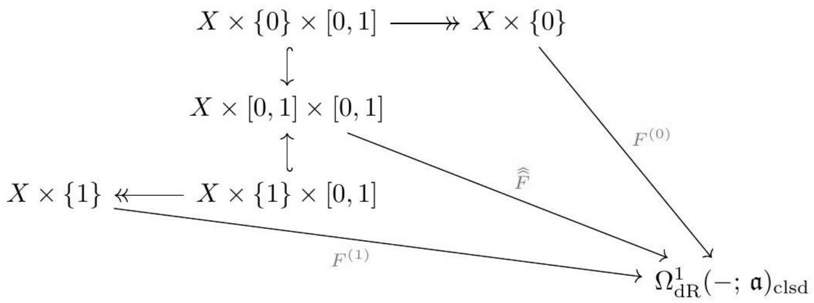
$$H^1_{\mathrm{dR}}(X; b^n\mathbb{R}) \cong H^{n+1}_{\mathrm{dR}}(X),$$

$$\begin{aligned} (\mathrm{id} \times \{0\})^* \widehat{\widehat{F}} &= \widehat{F} \\ (\mathrm{id} \times \{1\})^* \widehat{\widehat{F}} &= \widehat{F}' \end{aligned}$$

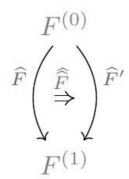


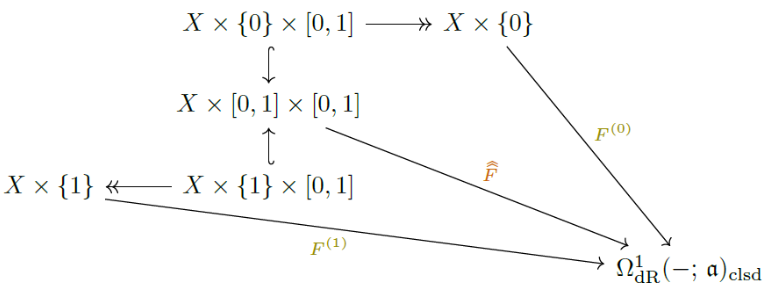
$$(\mathrm{id}\times\{0\})^*\widehat{\widehat{F}} \;=\; \widehat{F}$$
$$(\mathrm{id}\times\{1\})^*\widehat{\widehat{F}} \;=\; \widehat{F}'$$



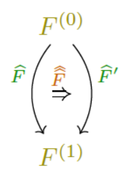


hence schematically of this form:





hence schematically of this form:



$$\iota_0^*\hat{G}_4 = 0$$

$$\iota_{\partial_t}\hat{C}_3 = 0 \;\text{ and }\; \underset{t\in[0,1]}{\forall}\hat{C}_3(-,t) = \int_{[0,t]}\hat{G}_4$$

$$\hat{G}_4 = A_4 + \mathrm{d}tB_3,\;\text{ with }\; \iota_{\partial_t}A_4 = 0 \;\text{ and }\; \iota_{\partial_t}B_3 = 0,$$

$$\mathrm{d}_XA_4 = 0 \;\text{ and }\; \mathrm{d}_{[0,1]}A_4 = \mathrm{d}t\;\mathrm{d}_XB_3$$

$$
\begin{aligned}
\mathrm{d}\int_{[0,-]} \hat{G}_4 &= \left(\mathrm{d}_{[0,1]} + \mathrm{d}_X\right)\int_{[0,-]} \mathrm{d}t' B_3(t') \\
&= \mathrm{d}t B_3 + \int_{[0,-]} \mathrm{d}t' \mathrm{d}_X B_3(t') \\
&= \mathrm{d}t B_3 + \int_{[0,-]} \mathrm{d}_{[0,1]} A_4 \\
&= \mathrm{d}t B_3 + A_4 \\
&= \hat{G}_4
\end{aligned}
$$

$$
\left.\begin{matrix} C_3 \in \Omega^3_{\mathrm{dR}}(X) \\ C_6 \in \Omega^6_{\mathrm{dR}}(X) \end{matrix}\right\} \text{ such that } \left\{\begin{aligned} \mathrm{d}C_3 &= G_4 \\ \mathrm{d}C_6 &= G_7 - \frac{1}{2} C_3 G_4 \end{aligned}\right.
$$

$$
(C_3, C_6) \sim (C_3', C_6') \;\Leftrightarrow\; \exists \left.\begin{matrix} B_2 \in \Omega^2_{\mathrm{dR}}(X) \\ B_5 \in \Omega^5_{\mathrm{dR}}(X) \end{matrix}\right\} \text{ such that } \left\{\begin{aligned} \mathrm{d}B_2 &= C_3' - C_3 \\ \mathrm{d}B_5 &= C_6' - C_6 - \frac{1}{2} C_3' C_3 \end{aligned}\right.
$$

$$
\begin{array}{ccc}
(C_3, C_6) & & \left(\widehat{G}_4 := t\,G_4 + \mathrm{d}t\,C_3, \quad \widehat{G}_7 := t^2\,G_7 + 2\,t\mathrm{d}t\,C_6\right) \\
\Big| & & \Big| \\
(B_2, B_5) & \longmapsto & \left(\begin{matrix} \widehat{\widehat{G}}_4 := t\,G_4 + \mathrm{d}t\,C_3 + s\,\mathrm{d}t(C_3' - C_3) - \mathrm{d}s\,\mathrm{d}t\,B_2 \\ \widehat{\widehat{G}}_7 := t^2\,G_7 + 2\,t\mathrm{d}t\,C_6 + 2\,s\,t\mathrm{d}t(C_6' - C_6) - 2\,\mathrm{d}s\,t\mathrm{d}t(B_5 + \tfrac{1}{2} B_2\,C_3) \end{matrix}\right) \\
\downarrow & & \downarrow \\
(C_3', C_6') & & \left(\widehat{G}_4' := t\,G_4 + \mathrm{d}t\,C_3', \quad \widehat{G}_7' := t^2\,G_7 + 2\,t\mathrm{d}t\,C_6'\right),
\end{array}
$$

$$
\begin{array}{ccc}
(C_3, C_6) & & \left(\widehat{G}_4 := t\,G_4 + \mathrm{d}t\,C_3, \quad \widehat{G}_7 := t^2\,G_7 + 2\,t\mathrm{d}t\,C_6\right) \\
\Big| & & \Big| \\
(B_2, B_5) & \longmapsto & \left(\begin{matrix} \widehat{\widehat{G}}_4 := t\,G_4 + \mathrm{d}t\,C_3 + s\,\mathrm{d}t(C_3' - C_3) - \mathrm{d}s\,\mathrm{d}t\,B_2 \\ \widehat{\widehat{G}}_7 := t^2\,G_7 + 2\,t\mathrm{d}t\,C_6 + 2\,s\,t\mathrm{d}t(C_6' - C_6) - 2\,\mathrm{d}s\,t\mathrm{d}t(B_5 + \tfrac{1}{2} B_2\,C_3) \end{matrix}\right) \\
\downarrow & & \downarrow \\
(C_3', C_6') & & \left(\widehat{G}_4' := t\,G_4 + \mathrm{d}t\,C_3', \quad \widehat{G}_7' := t^2\,G_7 + 2\,t\mathrm{d}t\,C_6'\right),
\end{array}
$$

$$
X \times [0,1]_t \times [0,1]_s \xrightarrow{p_{X \times [0,1]_t}} X \times [0,1]_t \xrightarrow{p_X} X
$$

$$
X \underset{\iota_1}{\overset{\iota_0}{\rightrightarrows}} X \times [0,1]
$$

$$
\mathrm{d}\int_{[0,1]} \hat{F} = \iota_1^* \hat{F} - \iota_0^* \hat{F} - \int_{[0,1]} \mathrm{d}\hat{F}
$$

$$\left(\hat{G}_4,\hat{G}_7\right)\in\Omega^1_{\mathrm{dR}}(X\times[0,1];\mathfrak{l}S^4)_{\mathrm{clsd}}\ \text{ with }\ \begin{cases}\iota_1^*\left(\hat{G}_4,\hat{G}_7\right)=(G_4,G_7)\\ \iota_0^*\left(\hat{G}_4,\hat{G}_7\right)=0\end{cases}$$

$$\left.\begin{aligned}C_3 &:= \int_{[0,1]}\widehat{G}_4\\ C_6 &:= \int_{[0,1]}\Big(\widehat{G}_7-\tfrac{1}{2}\underbrace{\Big(\int_{[0,-]}\widehat{G}_4\Big)}_{\widehat{C}_3}\widehat{G}_4\Big)\end{aligned}\right\}\quad \text{which indeed satisfies}\quad \begin{cases}\mathrm{d}\,C_3 = G_4\\ \mathrm{d}\,C_6 = G_7-\tfrac{1}{2}C_3\,G_4\end{cases}$$

$$\left.\begin{aligned}C_3 &:= \int_{[0,1]}\widehat{G}_4\\ C_6 &:= \int_{[0,1]}\Big(\widehat{G}_7-\tfrac{1}{2}\underbrace{\Big(\int_{[0,-]}\widehat{G}_4\Big)}_{\widehat{C}_3}\widehat{G}_4\Big)\end{aligned}\right\}\quad \text{which indeed satisfies}\quad \begin{cases}\mathrm{d}\,C_3 = G_4\\ \mathrm{d}\,C_6 = G_7-\tfrac{1}{2}C_3\,G_4\end{cases}$$

$$\hat{C}_3\in\Omega^3_{\mathrm{dR}}(X\times[0,1]),\hat{C}_3(-,t):=\int_{[0,t]}\hat{G}_4$$

$$\mathrm{d}\hat{C}_3=\hat{G}_4\ \text{ and }\ \iota_0^*\hat{C}_3=\int_{[0,0]}\hat{G}_4=0,\iota_1^*\hat{C}_3=\int_{[0,1]}\hat{G}_4=C_3$$

$$\mathrm{d}\int_{[0,1]}\left(\hat{G}_7-\frac{1}{2}\left(\int_{[0,-]}\hat{G}_4\right)\hat{G}_4\right)=\iota_1^*\left(\hat{G}_7-\frac{1}{2}\left(\int_{[0,-]}\hat{G}_4\right)\hat{G}_4\right)-\int_{[0,1]}\underbrace{\left(\hat{G}_7-\frac{1}{2}\left(\int_{[0,-]}\hat{G}_4\right)\hat{G}_4\right)}_{=0}$$

$$=G_7-\frac{1}{2}C_3G_4$$

$$\left.\begin{aligned}\hat{G}_4&:=tG_4+\mathrm{d}tC_3\\ \hat{G}_7&:=t^2G_7+2t\,\mathrm{d}tC_6\end{aligned}\right\}\overset{\mathrm{def}}{\Leftrightarrow}\begin{cases}\mathrm{d}(tG_4+\mathrm{d}tC_3)=0\\ \mathrm{d}(t^2G_7+2t\,\mathrm{d}tC_6)=\dfrac{1}{2}(tG_4+\mathrm{d}tC_3)(tG_4+\mathrm{d}tC_3)\end{cases}$$

$$\begin{aligned}&\int_{[0,t']}(tG_4+\mathrm{d}tC_3)=t'C_3\\ &\int_{[0,1]}(\underbrace{t^2G_7+2t\,\mathrm{d}tC_6}_{\hat{G}_7}-\frac{1}{2}\underbrace{tC_3}_{\hat{C}_3}\underbrace{(tG_4+\mathrm{d}tC_3)}_{\hat{G}_4})=2C_6\int_{[0,1]}t\,\mathrm{d}t=C_6\end{aligned}$$

$$\begin{aligned}&\int_{[0,t']}(t\,G_4+\mathrm{d}t\,C_3) = t'\,C_3\\ &\int_{[0,1]}\Big(\underbrace{t^2\,G_7+2t\mathrm{d}t\,C_6}_{\widehat{G}_7}-\tfrac{1}{2}\underbrace{t\,C_3}_{\widehat{C}_3}\underbrace{(t\,G_4+\mathrm{d}t\,C_3)}_{\widehat{G}_4}\Big) = 2\,C_6\int_{[0,1]}t\mathrm{d}t = C_6\end{aligned}$$

$$\left(\hat{\hat{G}}_4,\hat{\hat{G}}_7\right)\in\Omega^1_{\mathrm{dR}}(X\times[0,1]_t\times[0,1]_s;\mathfrak{l}S^4)_{\mathrm{clsd}},\ \text{such that:}\ \begin{cases}\iota_{s=1}^*\left(\hat{\hat{G}}_4,\hat{\hat{G}}_7\right)=\left(\hat{G}_4',\hat{G}_7'\right)\\ \iota_{s=0}^*\left(\hat{\hat{G}}_4,\hat{\hat{G}}_7\right)=\left(\hat{G}_4,\hat{G}_7\right)\end{cases}$$

$$B_2 := \int_{s\in[0,1]}\int_{t\in[0,1]} \hat{\hat{G}}_4$$

$$B_5 := \int_{s\in[0,1]}\int_{t\in[0,1]} \left(\hat{\hat{G}}_7 - \frac{1}{2}\left(\int_{t'\in[0,-]} \hat{\hat{G}}_4\right)\hat{\hat{G}}_4\right) - \frac{1}{2}B_2C_3$$

$$\begin{aligned}
&\mathrm{d}\int_{s\in[0,1]}\int_{t\in[0,1]} \hat{\hat{G}}_4 \\
=&\iota^*_{s=1}\int_{t\in[0,1]} \hat{\hat{G}}_4 - \iota^*_{s=0}\int_{t\in[0,1]} \hat{\hat{G}}_4 - \int_{s\in[0,1]} \mathrm{d}\int_{t\in[0,1]} \hat{\hat{G}}_4 \\
=&\int_{t\in[0,1]} \iota^*_{s=1}\hat{\hat{G}}_4 - \int_{t\in[0,1]} \iota^*_{s=0}\hat{\hat{G}}_4 - \int_{s\in[0,1]} \underbrace{\left(\iota^*_{t=1}\hat{\hat{G}}_4 - \iota^*_{t=0}\hat{\hat{G}}_4\right)}_{=0} + \int_{s\in[0,1]}\int_{t\in[0,1]} \underbrace{\mathrm{d}\hat{\hat{G}}_4}_{=0} \\
=&\int_{t\in[0,1]} \hat{G}'_4 - \int_{t\in[0,1]} \hat{G}_4 \\
=&C'_3 - C_3
\end{aligned}$$

$$\begin{aligned}
&\mathrm{d}\int_{s\in[0,1]}\int_{t\in[0,1]} \left(\hat{\hat{G}}_7 - \frac{1}{2}\left(\int_{t'\in[0,-]} \hat{\hat{G}}_4\right)\hat{\hat{G}}_4\right) \\
=&\int_{t\in[0,1]} \left(\hat{G}'_7 - \frac{1}{2}\hat{C}'_3\hat{G}'_4\right) - \int_{t\in[0,1]} \left(\hat{G}_7 - \frac{1}{2}\hat{C}_3\hat{G}_4\right) - \int_{s\in[0,1]} \mathrm{d}\int_{t\in[0,1]} \left(\hat{\hat{G}}_7 - \frac{1}{2}\left(\int_{t'\in[0,-]} \hat{\hat{G}}_4\right)\hat{\hat{G}}_4\right) \\
=&C'_6 - C_6 + \frac{1}{2}\left(\int_{s\in[0,1]}\int_{t\in[0,1]} \hat{\hat{G}}_4\right)G_4 \\
=&C'_6 - C_6 + \frac{1}{2}B_2G_4
\end{aligned}$$

$$\begin{aligned}
\mathrm{d}B_5 &= \mathrm{d}\int_{s\in[0,1]}\int_{t\in[0,1]} \left(\hat{\hat{G}}_7 - \frac{1}{2}\left(\int_{t'\in[0,-]} \hat{\hat{G}}_4\right)\hat{\hat{G}}_4\right) - \mathrm{d}\frac{1}{2}B_2C_3 \\
&= C'_6 - C_6 + \frac{1}{2}B_2G_4 \underbrace{-\frac{1}{2}(C'_3 - C_3)C_3 - \frac{1}{2}B_2G_4}_{\mathrm{d}\left(-\frac{1}{2}B_2C_3\right)} \\
&= C'_6 - C_6 - \frac{1}{2}C'_3C_3
\end{aligned}$$

$$\begin{array}{lcl}
\mathrm{d}\big(t\,G_4 + \mathrm{d}t\,C_3\big) & = & 0 \\
\mathrm{d}\left(s\,\mathrm{d}t(C'_3 - C_3)\right) & = & \mathrm{d}s\,\mathrm{d}t\big(C'_3 - C_3\big) \\
\mathrm{d}\big(-\mathrm{d}s\,\mathrm{d}t\,B_2\big) & = & -\mathrm{d}s\,\mathrm{d}t\big(C'_3 - C_3\big) \\
\hline
\mathrm{d}\big(\hat{\hat{G}}_4\big) & = & 0
\end{array}$$

$$\mathrm{d}(C'_6 - C_6) = -\frac{1}{2}(C'_3 - C_3)G_4,$$

$$\begin{array}{lcl}
\mathrm{d}(t^2 G_7 + 2t\mathrm{d}t\, C_6) & = & \tfrac{1}{2}(t\, G_4 + \mathrm{d}t\, C_3)^2 \\
\mathrm{d}(2s\, t\mathrm{d}t(C_6' - C_6)) & = & s\, t\mathrm{d}t(C_3' - C_3)G_4 \;+\; 2\,\mathrm{d}s\, t\mathrm{d}t(C_6' - C_6) \\
\mathrm{d}(-2\,\mathrm{d}s\, t\mathrm{d}t B_5) & = & -2\,\mathrm{d}s\, t\mathrm{d}t(C_6' - C_6) \;+\; \mathrm{d}s\, t\mathrm{d}t\, C_3' C_3 \\
\mathrm{d}(-\mathrm{d}s\, t\mathrm{d}t\, B_2 C_3) & = & -\mathrm{d}s\, t\mathrm{d}t\, B_2 G_4 \;-\; \mathrm{d}s\, t\mathrm{d}t\, C_3' C_3 \\
\hline
\mathrm{d}(\widehat{\widehat{G}}_7) & = & \tfrac{1}{2}\widehat{\widehat{G}}_4\, \widehat{\widehat{G}}_4
\end{array}$$

$$\psi \in \Omega^1_{\mathrm{dR}}(X; \mathbf{N}_{\mathrm{odd}}),$$

$$[X, Y](\mathbb{R}^{n|q}) := \mathrm{Hom}_{\mathrm{sSmthSet}}(X \times \mathbb{R}^{n|q}, F),$$

$$[X, Y](\mathbb{R}^{n|q}) \cong \mathrm{Hom}_{\mathrm{sSmthMfd}}(X \times \mathbb{R}^{n|q}, Y)$$

$$\mathrm{Hom}_{\mathrm{sSmthMfd}}(\overset{\rightsquigarrow}{X}, V_{\mathrm{odd}}) \cong \mathrm{Hom}_{\mathrm{sCAlg}}(C^\infty(V_{\mathrm{odd}}), C^\infty(\overset{\rightsquigarrow}{X})) \cong 0$$

$$[\overset{\rightsquigarrow}{X}, V_{\mathrm{odd}}](\mathbb{R}^{0|1}) \cong \mathrm{Hom}_{\mathrm{sSmthMfd}}(\overset{\rightsquigarrow}{X} \times \mathbb{R}^{0|1}, V_{\mathrm{odd}}) \cong (C^\infty(\overset{\rightsquigarrow}{X}) \otimes \mathbb{R}[\theta] \otimes V_{\mathrm{odd}})_0$$

$$C^\infty(\overset{\rightsquigarrow}{X} \times \mathbb{R}^{n|0}) \cong C^\infty(\overset{\rightsquigarrow}{X}) \,\hat{\otimes}\, C^\infty(\mathbb{R}^{n|0})$$

$$[X, \Omega^1_{\mathrm{dR}}(-; V)] \in \mathrm{sSmthSet}$$

$$\mathrm{Hom}_{\mathrm{sSmthSet}}(X \times \mathbb{R}^{n|q}, \Omega^1_{\mathrm{dR}}(-; V)) \cong \Omega^1_{\mathrm{dR}}(X \times \mathbb{R}^{n|q}; V),$$

$$[TX, V]^{\mathrm{fib.lin.}} \hookrightarrow [TX, V]$$

$$[TX, V]^{\mathrm{fib.lin.}}(\mathbb{R}^{n|q}) := \mathrm{Hom}^{\mathrm{fib.lin.}}_{\mathrm{sSmthMfd}}(TX \times \mathbb{R}^{n|q}, V).$$

$$[TX, V]^{\mathrm{fib.lin.}}(\mathbb{R}^{n|q}) \cong (\Omega^1_{\mathrm{dR}}(X) \,\hat{\otimes}\, C^\infty(\mathbb{R}^{n|q}) \otimes (V^\vee)^*)_{(1,0)}$$

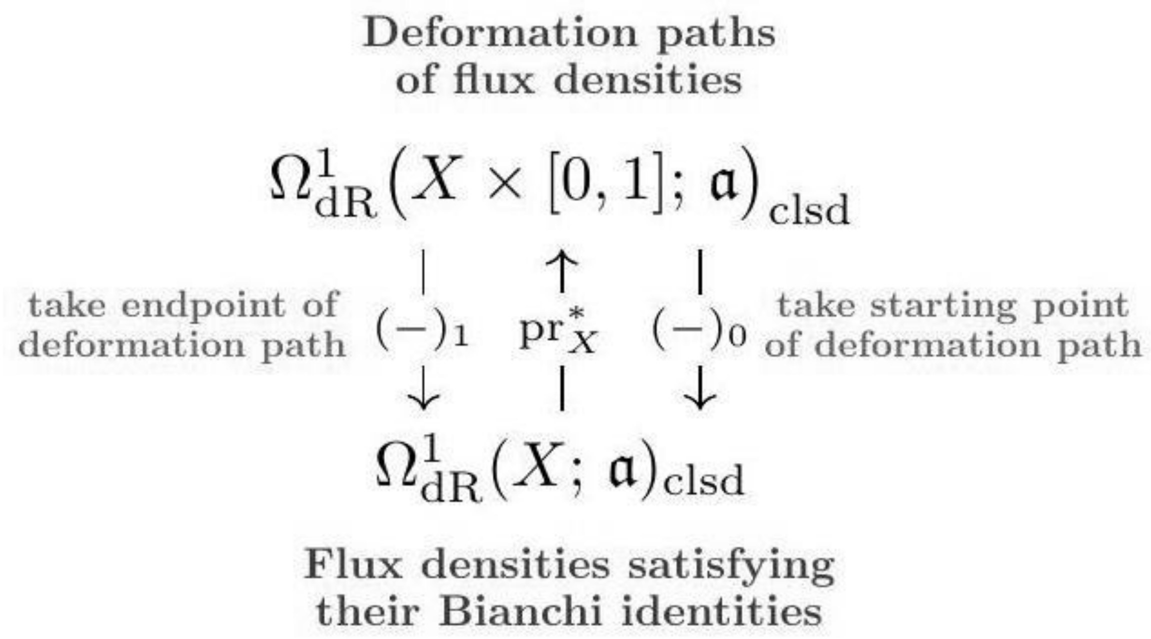
$$\mathbf{\Omega}^1_{\mathrm{dR}}(X;\, V)(\mathbb{R}^{n|q}) \;:=\; \left(\Omega^\bullet_{\mathrm{dR}}(X) \,\hat{\otimes}\, C^\infty(\mathbb{R}^{n|q}) \otimes (V)^*\right)_{(1,0)}$$

$$\mathbf{\Omega}^1_{\mathrm{dR}}(\overset{\rightsquigarrow}{X};\, \mathbf{32}_{\mathrm{odd}})(\mathbb{R}^{n|q}) \;=\; \left(\Omega^1_{\mathrm{dR}}(\overset{\rightsquigarrow}{X}) \,\hat{\otimes}\, C^\infty(\mathbb{R}^{n|q}) \otimes \mathbf{32}_{\mathrm{odd}}\right)_0$$

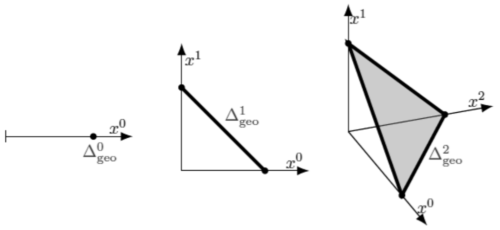
$$\psi^\alpha = \psi^\alpha{}_r \cdot \mathrm{d}x^r$$

$$\begin{array}{rccc}
\eta^*_X \;: & \mathbf{\Omega}^1_{\mathrm{dR}}(X;\, \mathbf{32}_{\mathrm{odd}}) & \longrightarrow & \mathbf{\Omega}^1_{\mathrm{dR}}(\overset{\rightsquigarrow}{X};\, \mathbf{32}_{\mathrm{odd}}) \\
& \psi^\alpha{}_r \cdot \mathrm{d}x^r + \psi^\alpha{}_\rho \cdot \mathrm{d}\theta^\rho & \longmapsto & \psi^\alpha{}_r|_{\theta^\rho = 0} \cdot \mathrm{d}x^r
\end{array}$$

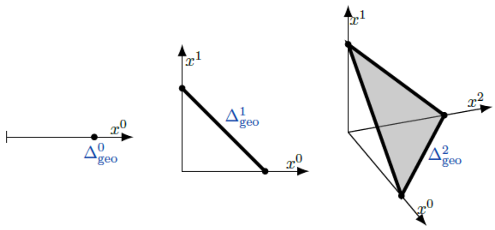
$$\mathbf{\Omega}^1_{\mathrm{dR}}\big(X;\,\mathfrak{a}\big)_{\mathrm{clsd}}\big(\mathbb{R}^{n|q}\big) \;:=\; \mathrm{MC}\Big(\Omega^\bullet_{\mathrm{dR}}(X)\,\hat{\otimes}\,C^\infty\big(\mathbb{R}^{n|q}\big)\otimes\big(\mathfrak{a}^\vee\big)^*\Big)_0$$

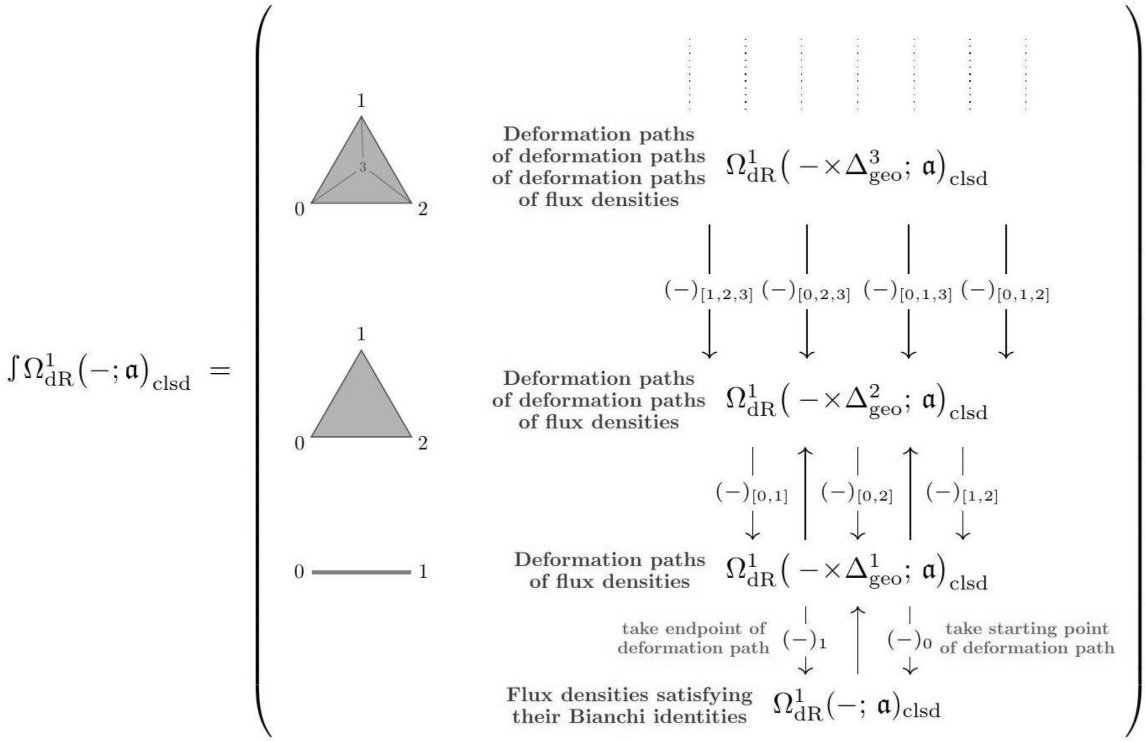


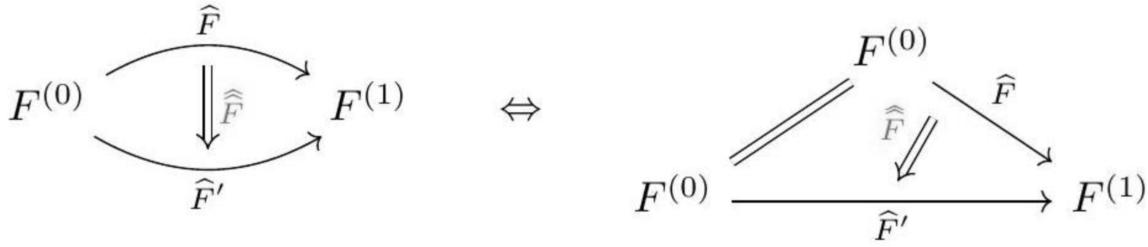
$$\Delta^n_{\mathrm{geo}} \;:=\; \Big\{ (x^0, x^1, \cdots, x^n) \in (\mathbb{R}_{\geq 0})^n \;\Big\vert\; \textstyle\sum_{i=0}^n x^i \;=\; 1 \Big\}$$

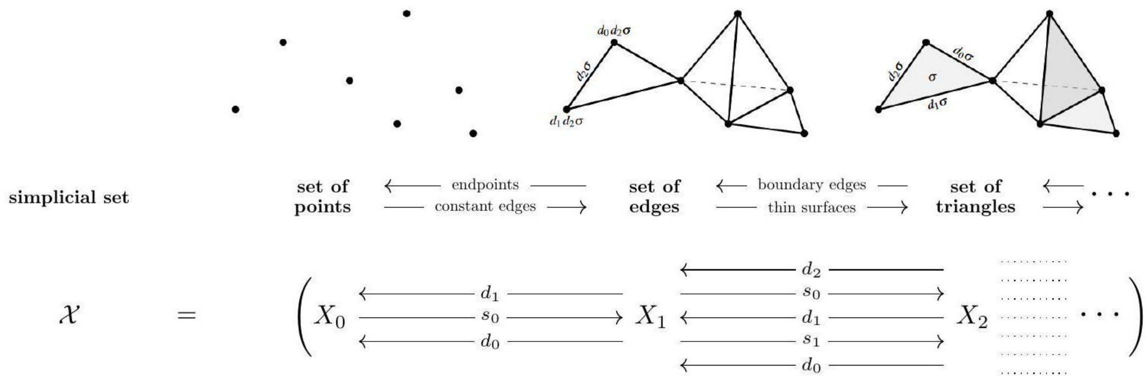


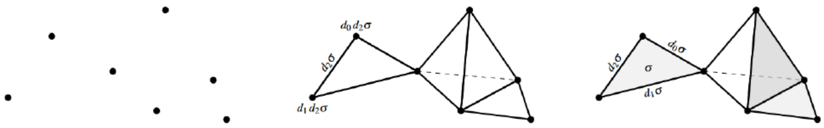
$$\Delta^n_{\mathrm{geo}} \;:=\; \Big\{ (x^0, x^1, \cdots, x^n) \in (\mathbb{R}_{\geq 0})^n \;\Big\vert\; \textstyle\sum_{i=0}^n x^i \;=\; 1 \Big\}$$




Deformation paths
of deformation paths
of deformation paths
of flux densities
Deformation paths
of deformation paths
of flux densities
Deformation paths
of flux densities
take endpoint of
deformation path
take starting point
of deformation path
Flux densities satisfying
their Bianchi identities

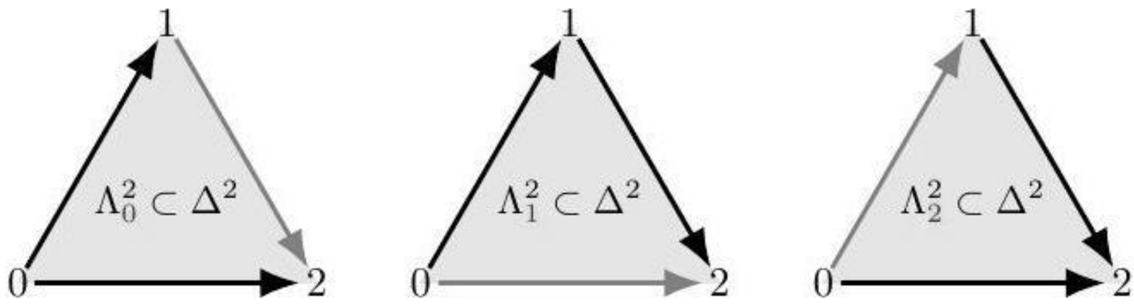



simplicial set
set of
points
endpoints
constant edges
set of
edges
boundary edges
thin surfaces
set of
triangles



simplicial set	set of points	$\leftarrow$ endpoints — — constant edges $\rightarrow$	set of edges	$\leftarrow$ boundary edges — — thin surfaces $\rightarrow$	set of triangles	$\leftarrow$ $\rightarrow$ $\cdots$

$$\mathcal{X} \quad = \quad \left(X_0 \underset{d_0}{\overset{d_1}{\underset{\overset{s_0}{\longrightarrow}}{\longleftarrow}}} X_1 \;\; \begin{matrix} \xleftarrow{d_2} \\ \xrightarrow{s_0} \\ \xleftarrow{d_1} \\ \xrightarrow{s_1} \\ \xleftarrow{d_0} \end{matrix} \;\; X_2 \;\; \vdots \;\; \cdots \right)$$



$$\int \Omega^1_{\mathrm{dR}}(-;\mathfrak{a})_{\mathrm{clsd}} : \mathrm{sCartSp}^{\mathrm{op}} \longrightarrow \mathrm{SimpSet}_{\mathrm{Kan}} .$$

$$\begin{array}{ccc} \Omega^1_{\mathrm{dR}}(-;\,\mathfrak{a})_{\mathrm{clsd}} & \xhookrightarrow{\;\eta^{\int}\;} & \int \Omega^1_{\mathrm{dR}}(-;\,\mathfrak{a})_{\mathrm{clsd}} \\ \vec{F} & \longmapsto & (p^\bullet)^* \vec{F} \end{array}$$

A *homotopy* $\mathcal{X} \underset{g}{\overset{f}{\Downarrow \eta}} \mathcal{Y}$ is a diagram of the form

$$\begin{array}{ccc} \mathcal{X} & & \\ \downarrow{\scriptstyle (\mathrm{id},0)} & \searrow^{f} & \\ \mathcal{X}\times\Delta^1 & \overset{\eta}{\dashrightarrow} & \mathcal{Y} \, . \\ \uparrow{\scriptstyle (\mathrm{id},1)} & \nearrow_{g} & \\ \mathcal{X} & & \end{array}$$

A *homotopy equivalence* is maps $\mathcal{X} \underset{\bar f}{\overset{f}{\rightleftarrows}} \mathcal{Y}$ with homotopies $\mathcal{X} \underset{\mathrm{id}}{\overset{\bar f \circ f}{\Downarrow}} \mathcal{X}$ and $\mathcal{Y} \underset{\mathrm{id}}{\overset{f \circ \bar f}{\Downarrow}} \mathcal{Y}$

$$\mathrm{sSmthGrpd}_{\infty} := L^{\mathrm{lhe}} \; \mathrm{Func}(\mathrm{sCartSp}^{\mathrm{op}},\; \mathrm{SimpSet}_{\mathrm{Kan}}),$$

$$\begin{array}{ccc} \mathrm{sCartSp}^{\mathrm{op}} & \longrightarrow & \mathrm{SimpSet}_{\mathrm{Kan}} \\ \mathbb{R}^{n|q} & \longmapsto & \mathrm{Plt}(\mathbb{R}^{n|q}, \mathcal{X}) \end{array}$$

$$\mathcal{X} \xleftarrow[\text{lheq}]{p} \widehat{\mathcal{X}} \xrightarrow{f} \mathcal{Y}$$

$$\mathrm{Plt}\left(\mathbb{R}^{n|q},\, \widehat{X}\right) \;:=\; \left(\begin{array}{c} \vdots \\ \mathrm{Hom}_{\mathrm{sSmthMfd}}\left(\mathbb{R}^{n|q},\ \coprod_{i_1,i_2\in I} U_{i_1}\cap U_{i_2}\right) \\ \downarrow\ \uparrow\ \downarrow \\ \mathrm{Hom}_{\mathrm{sSmthMfd}}\left(\mathbb{R}^{n|q},\ \coprod_{i\in I} U_i\right) \end{array}\right)$$

$$N_\bullet(\Delta^n) \;\in\; \mathrm{SimpAb}$$

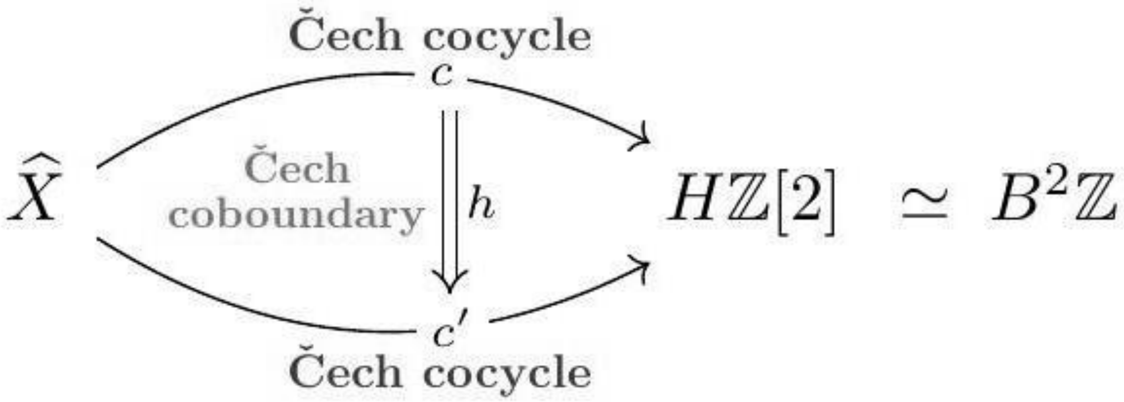
$$A_\bullet \;\in\; \mathrm{Ch}_{\geq 0}\Big(\mathrm{Ab}\big(\mathrm{Sh}(\mathrm{sCartSp})\big)\Big)$$

$$HA_\bullet \;\in\; \mathrm{sSmthGrpd}_\infty$$

$$\mathrm{Plt}\left(\mathbb{R}^{n|q},\, HA_\bullet\right)_k \;:=\; \mathrm{Hom}_{\mathrm{sAb}}\left(N_\bullet(\Delta^k),\, A_\bullet(\mathbb{R}^{n|q})\right)$$

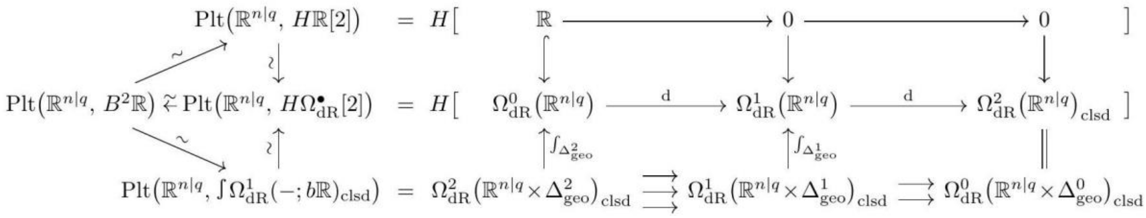
$$\mathrm{Plt}\left(\mathbb{R}^{n|q};\, \mathcal{A}\right) \;:=\; \left(\begin{array}{c} \vdots \\ \mathrm{Hom}_{\mathrm{TopSp}}\left(\Delta^2_{\mathrm{geo}},\, \mathcal{A}\right) \\ \downarrow\ \uparrow\ \downarrow\ \uparrow\ \downarrow \\ \mathrm{Hom}_{\mathrm{TopSp}}\left(\Delta^1_{\mathrm{geo}},\, A\right) \\ \downarrow\ \uparrow\ \downarrow \\ \mathrm{Hom}_{\mathrm{TopSp}}\left(\Delta^0_{\mathrm{geo}},\, A\right) \end{array}\right).$$

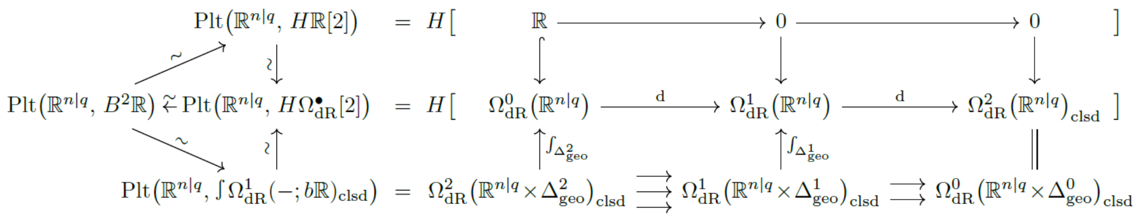
$$\mathbb{Z}[2] := [\ \cdots\to 0 \longrightarrow 0 \longrightarrow \mathbb{Z} \longrightarrow 0 \longrightarrow 0\ ]$$

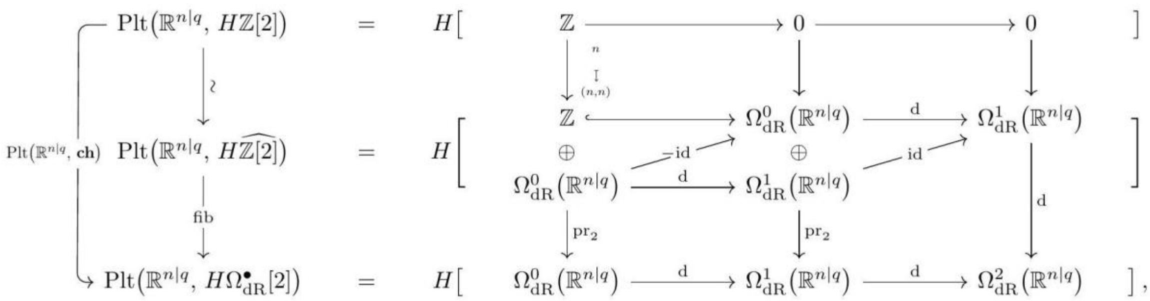


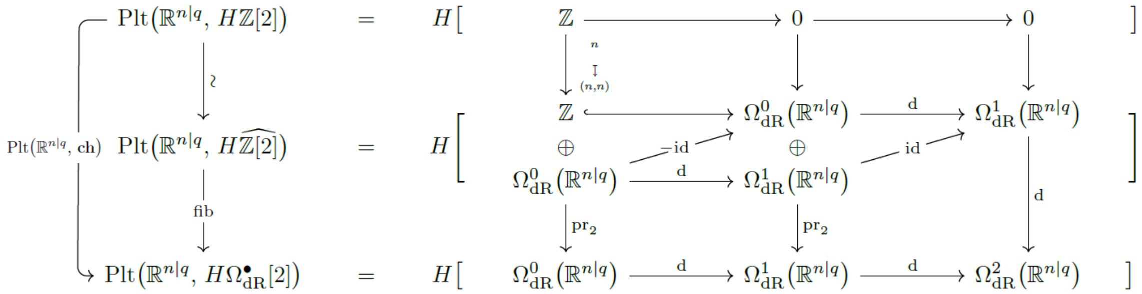
$$B^2\mathbb{Z} := K(\mathbb{Z},2) = H\mathbb{Z}[2]$$

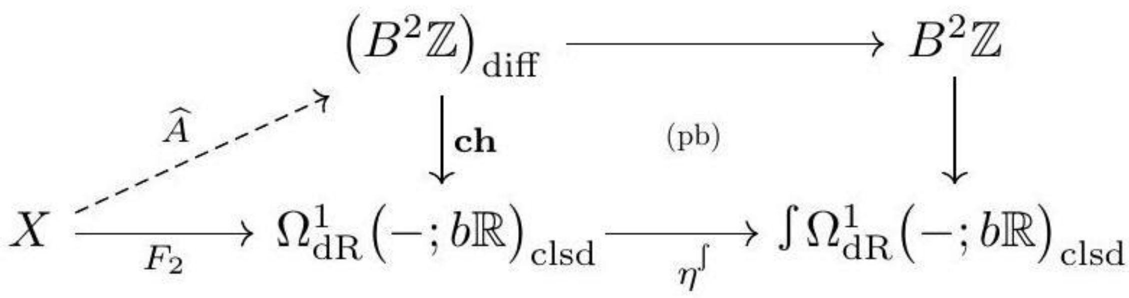
$$H^2(X;\mathbb{Z}) := H^1(X;B\mathbb{Z}) \simeq \pi_0 \mathrm{Map}(\hat{X}, H\mathbb{Z}[2])$$

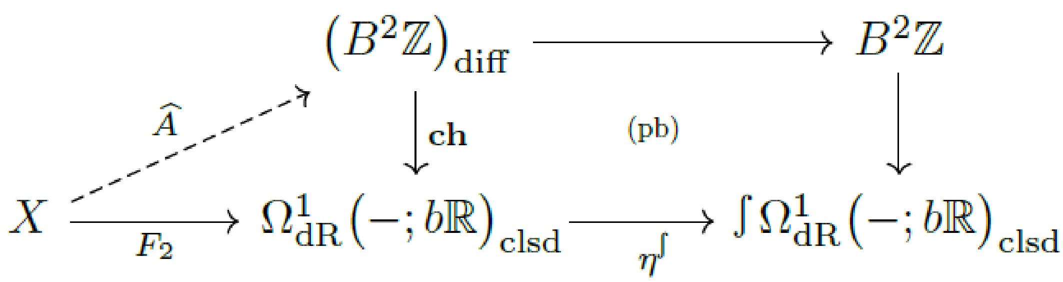








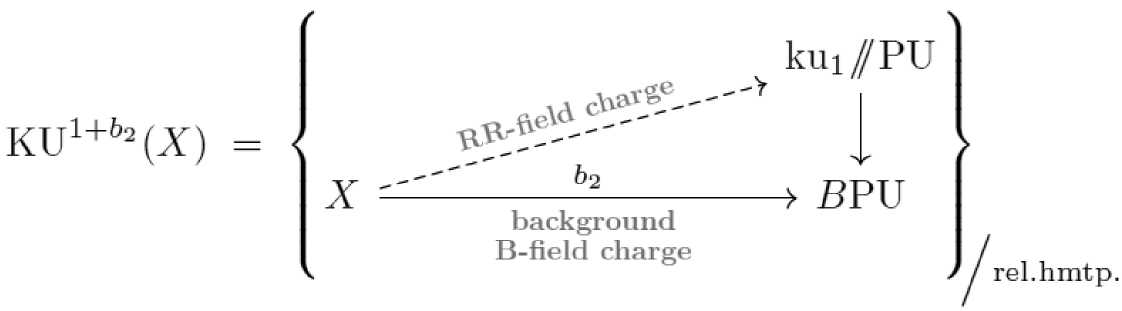




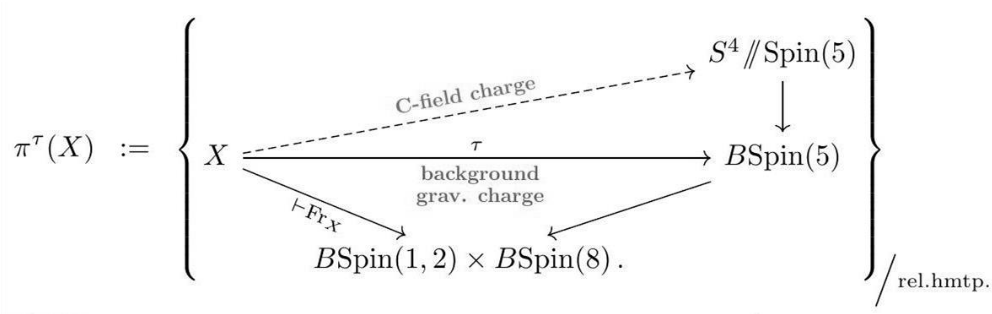
$$(B^2\mathbb{Z})_{\mathrm{diff}} \simeq \Omega^2_{\mathrm{dR}}(-)_{\mathrm{clsd}} \underset{H\Omega^{\bullet}_{\mathrm{dR}}[2]}{\times} H\widehat{\mathbb{Z}[2]} = H\big[\underbrace{\mathbb{Z} \hookrightarrow \Omega^0_{\mathrm{dR}}(-) \xrightarrow{\mathrm{d}} \Omega^1_{\mathrm{dR}}(-)}_{\text{Deligne complex}}\big]$$

$$\bigoplus_{k\in\mathbb{N}} \Omega^{2k+1}_{\mathrm{dR}}(-)_{\mathrm{clsd}} = \Omega^1_{\mathrm{dR}}\Big(-;\bigoplus_{k\in\mathbb{N}} b^{2k}\mathbb{R}\Big)_{\mathrm{clsd}}$$

$$\mathrm{d}H_3 = 0,\ \mathrm{d}F_{2k+1} = H_3 F_{2k-1}$$



$$\Omega^1_{\mathrm{dR}}(-;\, \mathfrak{l}S^4) \;\simeq\; \left\{ \begin{array}{lcl} G_4 & \in & \Omega^4_{\mathrm{dR}}(-) \\ G_7 & \in & \Omega^7_{\mathrm{dR}}(-) \end{array} \;\middle|\; \begin{array}{lcl} \mathrm{d}\, G_4 & = & 0 \\ \mathrm{d}\, G_7 & = & \tfrac{1}{2} G_4 G_4 \end{array} \right\}$$

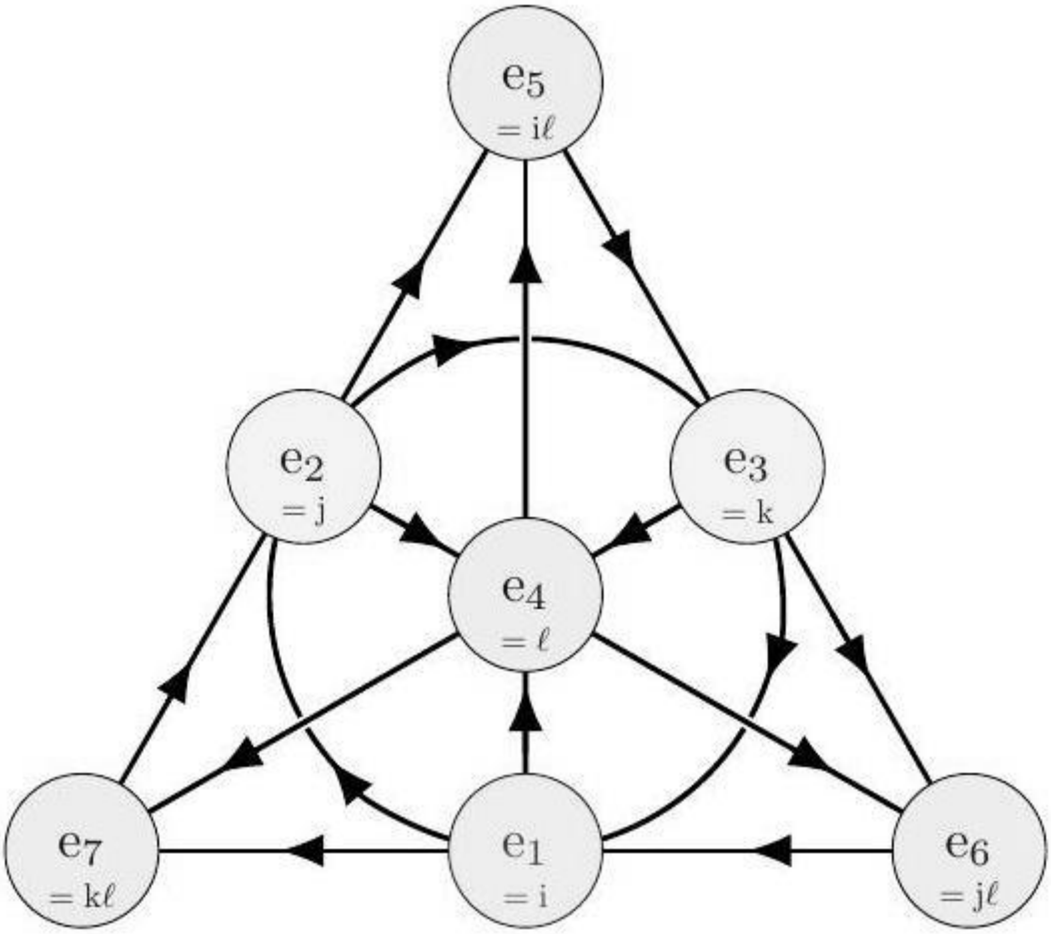


$$[G_4, G_7] \in \mathrm{im}\left(\pi^\tau(X) \xrightarrow{\mathrm{ch}} H^\tau_{\mathrm{dR}}(X; \mathfrak{l}S^4)\right) \Rightarrow \left[G_4 + \frac{1}{4}p_1\right] \in H^4(X; \mathbb{Z}) \to H^4_{\mathrm{dR}}(X)$$

$$\psi^\alpha, \psi^\beta \in \Omega^1_{\mathrm{dR}}(-; \mathbf{32}_{\mathrm{odd}}) \Rightarrow \psi^\alpha \psi^\beta = \psi^\beta \psi^\alpha \in \Omega^2_{\mathrm{dR}}(-; \mathbf{32}_{\mathrm{odd}} \otimes \mathbf{32}_{\mathrm{odd}})$$

$$a \cdot b = c, c \cdot a = b, b \cdot c = a, b \cdot a = -c$$

$$L_v : \mathbb{O} \to \mathbb{O}$$



$$L_v \circ L_v \ = \ -|v|^2 \, \mathrm{id}_{\mathbb{O}}$$

$$L_{e_7} L_{e_6} L_{e_5} L_{e_4} L_{e_3} L_{e_2} L_{e_1} \ = \ \mathrm{id}_{\mathbb{O}}$$

$$\begin{array}{lccccc}
\Gamma_a & \in \mathrm{End}(\mathbb{R}^2) & \otimes \mathrm{End}(\mathbb{R}^2) & \otimes \mathrm{End}(\mathbb{O}) & & \\
\Gamma_0 = & J & \otimes & 1 & \otimes & 1 \\
\Gamma_1 = & \epsilon & \otimes & \tau & \otimes & 1 \\
\Gamma_2 = & \epsilon & \otimes & \epsilon & \otimes & 1 \\
\Gamma_{2+i} = & \epsilon & \otimes & J & \otimes & L_{e_i} \\
\Gamma_{10} = & \tau & \otimes & 1 & \otimes & 1 \, ,
\end{array}$$

$$\tau := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \epsilon := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, J := \tau \cdot \epsilon := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

$$\mathbf{32} := \mathbb{O}^4 \cong_{\mathbb{R}} \mathbb{R}^{32}.$$

$$\Gamma^{a_1 \cdots a_p} = \frac{(-1)^{(p+1)(p-2)/2}}{(11-p)!} \epsilon^{a_1 \cdots a_p b_1 \cdots a_{11-p}} \Gamma_{b_1 \cdots b_{11-p}}.$$

$$\Gamma^{a_1 \cdots a_{11}} = \epsilon^{a_1 \cdots a_{11}} \mathrm{Id}_{\mathbf{32}}, \quad \Gamma^{a_1 \cdots a_6} = +\frac{1}{5!} \epsilon^{a_1 \cdots a_6 b_1 \cdots b_5} \Gamma_{b_1 \cdots b_5},$$
$$\Gamma^{a_1 \cdots a_{10}} = \epsilon^{a_1 \cdots a_{10} b} \Gamma_b, \quad \Gamma^{a_1 \cdots a_5} = -\frac{1}{6!} \epsilon^{a_1 \cdots a_5 b_1 \cdots b_6} \Gamma_{b_1 \cdots b_6}.$$

$$\Gamma_{10} \cdot \Gamma_9 \cdot \Gamma_8 \cdot \Gamma_7 \cdot \Gamma_6 \cdot \Gamma_5 \cdot \Gamma_4 \cdot \Gamma_3 \cdot \Gamma_2 \cdot \Gamma_1 = \tau \epsilon^3 J \otimes J \epsilon \tau \otimes 1 = -\mathrm{Id}_{\mathbf{32}} \cdot$$

$$\Gamma_0 \cdot \Gamma_1 \cdot \Gamma_2 \cdots \Gamma_{10} = +1,$$

$$\Gamma_{a_1 \cdots a_{11}} = \epsilon_{a_1 \cdots a_{11}} \mathrm{Id}_{\mathbf{32}}.$$

$$\begin{aligned} \Gamma^{a_1 \cdots a_p} &= \frac{-1}{(11-p)!} \epsilon^{b_1 \cdots b_{11-p} a_p a_{p-1} \cdots a_1} \Gamma_{b_1 \cdots b_{11-p}} \underbrace{\Gamma_{a_p a_{p-1} \cdots a_1}}_{\text{no sum}} \Gamma^{a_1 a_2 \cdots a_p} \\ &= \frac{-1}{(11-p)!} \epsilon^{b_1 \cdots b_{11-p} a_p \cdots a_1} \Gamma_{b_1 \cdots b_{11-p}} \qquad \Gamma_{a_{\sigma(i)}} \Gamma^{a_{\sigma(i)}} = 1 \\ &= \frac{-(-1)^{p(p-1)/2}}{(11-p)!} \epsilon^{b_1 \cdots b_{11-p} a_1 \cdots a_p} \Gamma_{b_1 \cdots b_{11-p}} \\ &= \frac{-(-1)^{p(p-1)/2 + p(11-p)}}{(11-p)!} \epsilon^{a_1 \cdots a_p b_1 \cdots b_{11-p}} \Gamma_{b_1 \cdots b_{11-p}} \\ &= \frac{(-1)^{(p+1)(p-2)/2}}{(11-p)!} \epsilon^{a_1 \cdots a_p b_1 \cdots b_{11-p}} \Gamma_{b_1 \cdots b_{11-p}}. \end{aligned}$$

$$\Gamma^{a_j \cdots a_1} \Gamma_{b_1 \cdots b_k} = \sum_{l=0}^{\min(j,k)} \pm\, l! \binom{j}{l} \binom{k}{l} \delta^{[a_1 \cdots a_l}_{[b_1 \cdots b_l} \Gamma^{a_j \cdots a_{l+1}]} {}_{b_{l+1} \cdots b_k]}$$

$$\Gamma^{a_j \cdots a_1} \Gamma_{b_1 \cdots b_k} = \frac{jk}{l} \Gamma^{[a_j \cdots a_2} \delta^{a_1]}_{[b_1} \Gamma_{b_2 \cdots b_k]}$$

$$\Gamma^{a_j \cdots a_1} \Gamma_{b_1 \cdots b_k} = (-1)^{j-1} \frac{jk}{l} \delta^{[a_1}_{[b_1} \Gamma^{a_j \cdots a_2]} \Gamma_{b_2 \cdots b_k]}$$

$$\Gamma^{a_j \cdots a_1} \Gamma_{b_1 \cdots b_k} = (-1)^{(j-1) \cdots (j-l)} \underbrace{\frac{j \cdots (j-l)\, k \cdots (k-l)}{l!}}_{l! \binom{k}{l} \binom{j}{l}} \delta^{[a_1 \cdots a_l}_{[b_1 \cdots b_l} \underbrace{\Gamma^{a_j \cdots a_{l+1}]} \Gamma_{b_{l+1} \cdots b_k]}}_{\Gamma^{a_j \cdots a_{l+1}]}{}_{b_{j+l} \cdots b_k]}}$$

$$\Gamma^{a_j \cdots a_1} \Gamma_{b_1 \cdots b_k} = (-1)^{(j-1) \cdots (j-l)} \underbrace{\frac{j \cdots (j-l)\, k \cdots (k-l)}{l!}}_{l! \binom{k}{l} \binom{j}{l}} \delta^{[a_1 \cdots a_l}_{[b_1 \cdots b_l} \underbrace{\Gamma^{a_j \cdots a_{l+1}]} \Gamma_{b_{l+1} \cdots b_k]}}_{\Gamma^{a_j \cdots a_{l+1}]}{}_{b_{j+l} \cdots b_k]}}$$

$$\mathrm{Tr}\left(\Gamma_{a_1 \cdots a_p}\right) = 0.$$

$$\phi = \frac{1}{32}\sum_{p=0}^{5} \frac{(-1)^{p(p-1)/2}}{p!} \mathrm{Tr}\left(\phi \circ \Gamma_{a_1\cdots a_p}\right) \Gamma^{a_1\cdots a_p}.$$

$$\begin{aligned} &\mathbf{32} \times \mathbf{32} \longrightarrow \mathbb{R} \\ &(\psi, \phi) \mapsto (\bar{\psi}\phi) := \mathrm{Re}\left(\psi^\dagger \cdot \Gamma_0 \cdot \phi\right) \end{aligned}$$

$$(\bar{\psi}\phi) = -(\bar{\phi}\psi).$$

$$(\Gamma_a)^\dagger = \begin{Bmatrix} -\Gamma_a & | \; a = 0 \\ +\Gamma_a & | \; a \neq 0 \end{Bmatrix} = \Gamma_0\Gamma_a\Gamma_0,$$

$$\begin{aligned} \left(\overline{\Gamma_a\psi}\phi\right) &= \mathrm{Re}\left((\Gamma_a\psi)^\dagger\Gamma_0\phi\right) \\ &= \mathrm{Re}(\psi^\dagger \underbrace{\Gamma_0\Gamma_a\Gamma_0}_{(\Gamma_a)^\dagger} \Gamma_0\phi) \\ &= -\mathrm{Re}\left(\psi^\dagger\Gamma_0\Gamma_a\phi\right) \\ &= -(\bar{\psi}\Gamma_a\phi) \end{aligned}$$

$$\overline{\Gamma_{a_1\cdots a_p}} = (-1)^{p+p(p-1)/2}\Gamma_{a_1\cdots a_p}.$$

$$(\bar{\phi}_1\psi)(\bar{\psi}\phi_2) = \frac{1}{32}\left((\bar{\psi}\Gamma^a\psi)(\bar{\phi}_1\Gamma_a\phi_2) - \frac{1}{2}(\bar{\psi}\Gamma^{a_1a_2}\psi)(\bar{\phi}_1\Gamma_{a_1a_2}\phi_2) + \frac{1}{5!}(\bar{\psi}\Gamma^{a_1\cdots a_5}\psi)(\bar{\phi}_1\Gamma_{a_1\cdots a_5}\phi_2)\right).$$

$$\psi\left(\bar{\psi}\Gamma_{a_1\cdots a_p}\psi\right) \in \mathbf{32}.$$

$$\left\langle \psi^\alpha\left(\bar{\psi}\Gamma_{a_1\cdots a_p}\psi\right)\right\rangle_{a_i\in\{0,\cdots,10\},\alpha\in\{1,\cdots,32\}} \in \mathrm{Rep}_{\mathbb{R}}(\mathrm{Spin}(1,10)).$$

$$\begin{aligned} (32 \otimes 32)_{\mathrm{sym}} &\cong 11 \oplus 55 \oplus 462 \\ (32 \otimes 32 \otimes 32)_{\mathrm{sym}} &\cong 32 \oplus 320 \oplus 1408 \oplus 4424 \\ (32 \otimes 32 \otimes 32 \otimes 32)_{\mathrm{sym}} &\cong 1 \oplus 165 \oplus 330 \oplus 462 \oplus 65 \oplus 429 \oplus 1144 \oplus 17160 \oplus 32604. \end{aligned}$$

$$\left\langle \Xi^\alpha_{a_1\cdots a_p} = \Xi^\alpha_{[a_1\cdots a_p]}\right\rangle_{a_i\in\{0,\cdots,10\},\alpha\in\{1,\cdots 32\}} \in \mathrm{Rep}_{\mathbb{R}}(\mathrm{Spin}(1,10))$$
$$\Gamma^{a_1}\Xi_{a_1a_2\cdots a_p} = 0$$

$$\begin{aligned} \psi(\overline{\psi}\,\Gamma_a\,\psi) &= \tfrac{1}{11}\,\Gamma_a\,\Xi^{(32)} + \Xi^{(320)}_a\,, \\ \psi(\overline{\psi}\,\Gamma_{a_1a_2}\,\psi) &= \tfrac{1}{11}\,\Gamma_{a_1a_2}\,\Xi^{(32)} - \tfrac{2}{9}\,\Gamma_{[a_1}\,\Xi^{(320)}_{a_2]} + \Xi^{(1408)}_{a_1a_2}\,, \\ \psi(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\psi) &= -\tfrac{1}{77}\Gamma_{a_1\cdots a_5}\Xi^{(32)} + \tfrac{5}{9}\Gamma_{[a_1\cdots a_4}\Xi^{(320)}_{a_5]} + 2\,\Gamma_{[a_1a_2a_3}\,\Xi^{(1408)}_{a_4a_5]} + \Xi^{(4224)}_{a_1\cdots a_5} \end{aligned}$$

$$\begin{aligned} \psi(\overline{\psi}\,\Gamma_a\,\psi) &= \tfrac{1}{11}\,\Gamma_a\,\Xi^{(32)} + \Xi^{(320)}_a\,, \\ \psi(\overline{\psi}\,\Gamma_{a_1a_2}\,\psi) &= \tfrac{1}{11}\,\Gamma_{a_1a_2}\,\Xi^{(32)} - \tfrac{2}{9}\,\Gamma_{[a_1}\,\Xi^{(320)}_{a_2]} + \Xi^{(1408)}_{a_1a_2}\,, \\ \psi(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\psi) &= -\tfrac{1}{77}\Gamma_{a_1\cdots a_5}\Xi^{(32)} + \tfrac{5}{9}\Gamma_{[a_1\cdots a_4}\Xi^{(320)}_{a_5]} + 2\,\Gamma_{[a_1a_2a_3}\,\Xi^{(1408)}_{a_4a_5]} + \Xi^{(4224)}_{a_1\cdots a_5} \end{aligned}$$

$$\bar{\psi}\psi = 0, \bar{\psi}\Gamma_{[a_1a_2a_3]}\psi = 0, \bar{\psi}\Gamma_{[a_1\cdots a_4]}\psi = 0, \bar{\psi}\Gamma_{[a_1\cdots a_7]}\psi = 0, \bar{\psi}\Gamma_{[a_1\cdots a_8]}\psi = 0, \bar{\psi}\Gamma_{[a_1\cdots a_{11}]}\psi = 0,$$

$$\begin{array}{ll} (\bar{\psi}\Gamma_a\psi) & \binom{11}{1} = 11 \\ (\bar{\psi}\Gamma_{ab}\psi) & \binom{11}{2} = 55 \\ (\bar{\psi}\Gamma_{a_1\cdots a_5}\psi) & \binom{11}{5} = 462. \end{array}$$

$$\begin{aligned} \left(\bar{\psi}\Gamma_{[a_1\cdots a_p]}\phi\right) &\equiv \mathrm{Re}\left(\psi^\dagger\Gamma_0\Gamma_{[a_1\cdots a_p]}\phi\right) \\ &= -\mathrm{Re}\left(\phi^\dagger\left(\Gamma_{[a_1\cdots a_p]}\right)^\dagger\Gamma_0\psi\right) \\ &= -\mathrm{Re}\left(\phi^\dagger\Gamma_0\Gamma_0^{-1}\left(\Gamma_{[a_1\cdots a_p]}\right)^\dagger\Gamma_0\psi\right) \\ &= -(-1)^{p+p(p-1)/2}\mathrm{Re}\left(\phi^\dagger\Gamma_0\Gamma_{[a_1\cdots a_p]}\psi\right) \\ &= -(-1)^{p(p+1)/2}\left(\bar{\phi}\Gamma_{[a_1\cdots a_p]}\psi\right). \end{aligned}$$

$$\left(\bar{\psi}\Gamma_{a_1}\xi\right) = 0, \text{ and } \left(\bar{\psi}\Gamma_{a_1a_2}\xi\right) = 0 \text{ and } \left(\bar{\psi}\Gamma_{a_1\cdots a_5}\xi\right) = 0 \text{ for all } a_1, a_2\cdots a_5,$$

$$\begin{aligned} (\bar{\psi}\Gamma_{ab}\psi)(\bar{\psi}\Gamma^a\psi) &= 0, \\ (\bar{\psi}\Gamma_{ab_1\cdots b_4}\psi)(\bar{\psi}\Gamma^a\psi) &= 3(\bar{\psi}\Gamma_{[b_1b_2}\psi)(\bar{\psi}\Gamma_{b_3b_4]}\psi) \\ &= -\frac{1}{6}(\bar{\psi}\Gamma^{a_1a_2}\psi)(\bar{\psi}\Gamma_{a_1a_2b_1\cdots b_4}\psi). \end{aligned}$$

$$(32 \otimes 32 \otimes 32 \otimes 32)_{\mathrm{sym}} \longrightarrow 11.$$

$$\begin{aligned} & (\bar{\psi}\Gamma^{a_1a_2}\psi)(\bar{\psi}\Gamma_{a_1a_2b_1\cdots b_4}\psi) \\ = & \frac{1}{5!}(\bar{\psi}\Gamma^{a_1a_2}\psi)(\bar{\psi}\Gamma^{c_1\cdots c_5}\psi)\epsilon_{a_1a_2b_1\cdots b_4c_1\cdots c_5} \\ = & \frac{1}{5!}(\bar{\psi}\Gamma^{a_1}\psi)(\bar{\psi}\Gamma^{a_2c_1\cdots c_5}\psi)\epsilon_{a_1a_2b_1\cdots b_4c_1\cdots c_5} \\ = & \frac{1}{5!\cdot 5!}(\bar{\psi}\Gamma^{a_1}\psi)(\bar{\psi}\Gamma_{d_1\cdots d_5}\psi)\epsilon^{a_2c_1\cdots c_5d_1\cdots d_5}\epsilon_{a_1a_2b_1\cdots b_4c_1\cdots c_5} \\ = & -\frac{5!\cdot 6!}{5!\cdot 5!}(\bar{\psi}\Gamma^{a_1}\psi)(\bar{\psi}\Gamma_{d_1\cdots d_5}\psi)\delta^{d_1\cdots d_5}_{a_1b_1\cdots b_4} \\ = & -6(\bar{\psi}\Gamma^{a_1}\psi)(\bar{\psi}\Gamma_{a_1b_1\cdots b_4}\psi) \\ = & -3\cdot 6(\bar{\psi}\Gamma_{[b_1b_2}\psi)(\bar{\psi}\Gamma_{b_1b_2]}\psi) \end{aligned}$$

$$\left\{\mathbb{R}^{1,D-1|\mathbf{N}} \xleftarrow{\sim} U_i \xrightarrow{\text{ét}} X\right\}_{i\in I}$$

$$((e_i^a)_{a=0}^{D-1}, (\psi_i^\alpha)_{\alpha=1}^N) \in \Omega^1_{\mathrm{dR}}\left(U_i; \mathbb{R}^{1,D-1|\mathbf{N}}\right)$$

$$\phi_i := (e_i, \psi_i) \ : \ TU_i \xrightarrow[\sim]{} T\mathbb{R}^{1,D-1|\mathbf{N}}$$

$$\gamma_{ij} \; : \; T(U_i \cap U_j) \xrightarrow{\phi_i} T\mathbb{R}^{1,D-1|\mathbf{N}} \xrightarrow{\phi_j^{-1}} T(U_i \cap U_j)$$

$$\left((\gamma_{ij})^a{}_b\right)_{a,b=0}^{D-1} \in \Omega^0_{\mathrm{dR}}\left(U_i \cap U_j; \mathrm{Spin}(1, D-1)\right)$$

$$(\omega^a{}_b)^d_{a,b=0} \in \Omega^1_{\mathrm{dR}}(U_i; \mathfrak{so}(1, D-1))$$

$$(\omega_i)^a{}_b = (\gamma_{ij})^a{}_{a'} (\omega_j)^{a'}{}_{b'} \left(\gamma_{ij}^{-1}\right)^{b'}{}_b + (\gamma_{ij})^a{}_c \, \mathrm{d} \left(\gamma_{ij}^{-1}\right)^c{}_b$$

$$\begin{aligned} &\left((e^a)^{D-1}_{a=0}, (\omega^{ab})^{D-1}_{a,b=0}\right) && \in \Omega^1_{\mathrm{dR}}\left(U; \mathfrak{iso}(\mathbb{R}^{1,D-1})\right) \\ &(\psi^\alpha)^N_{\alpha=1} && \in \Omega^1_{\mathrm{dR}}(U; \mathbf{N}_{\mathrm{odd}}) \end{aligned}$$

$$T_i^a := \mathrm{d}e_i^a + (\omega_i)^a{}_b e_i^b - (\bar{\psi}_i \Gamma^a \psi_i) = 0$$

$$e^a = e^a{}_r \, \mathrm{d}x^\mu + e^a{}_\rho \mathrm{d}\theta^\rho, \psi^\alpha = \psi^\alpha{}_r \, \mathrm{d}x^r + \psi^\alpha{}_\rho \mathrm{d}\theta^\rho.$$

$$\partial_a \; := \; e_a{}^r \frac{\partial}{\partial x^r} + e_a{}^\rho \frac{\partial}{\partial \theta^\rho}, \qquad \Gamma_{r_1 \cdots r_p} \; := \; \Gamma_{a_1 \cdots a_p} e^{a_1}{}_{[r_1} \cdots e^{a_p}{}_{r_p]}$$

$$\begin{aligned} e^a \quad & := \delta^a_r \, \mathrm{d}x^r + (\bar{\theta}\Gamma^a \, \mathrm{d}\theta) \\ \psi^\alpha \quad & := \delta^\alpha_\rho \theta^\rho \\ \omega^a{}_b \quad & := 0 \end{aligned}$$

$$X \; := \; \widetilde{\overrightarrow{X}} \Big| \underset{\mathrm{Spin}(1,10)}{(\mathbf{32} \times P)}$$

$$\begin{aligned} &\big(T^a && := && \mathrm{d}\, e^a && + \omega^a{}_b \, e^b - (\overline{\psi} \, \Gamma^a \, \psi)\big)_{a=0}^{D-1} && \in \Omega^2_{\mathrm{dR}}\left(U; \, \mathbb{R}^{1,D-1}\right), \\ &\big(R^{ab} && := && \mathrm{d}\, \omega^{ab} && + \omega^a{}_c \, \omega^{cb}\big)_{a,b=0}^{D-1} && \in \Omega^2_{\mathrm{dR}}\left(U; \, \mathfrak{so}(1, D-1)\right), \\ &\big(\rho && := && \mathrm{d}\, \psi && + \tfrac{1}{4}\omega^{ab} \, \Gamma_{ab} \psi\big)_{\alpha=1}^{N} && \in \Omega^2_{\mathrm{dR}}\left(U; \, \mathbf{N}_{\mathrm{odd}}\right). \end{aligned}$$

$$\begin{aligned} \overbrace{\mathrm{d}T^a + \omega^a_b T^b}^{0} \quad & = +R^{ab} e_b + 2(\bar{\psi}\Gamma^a \rho), \\ \mathrm{d}\rho + \frac{1}{4}\omega^{ab}\Gamma_{ab}\rho \quad & = +\frac{1}{4} R^{ab}\Gamma_{ab}\psi, \\ \mathrm{d}R^{ab} + \omega^a{}_{a'} R^{a'b} - R^{ab'} \omega^b{}_{b'} \quad & = 0, \end{aligned}$$

$$\rho =: \frac{1}{2}\rho_{ab} e^a e^b + H_a \psi e^a + (\bar{\psi}\kappa\psi)$$

$$R^{a_1 a_2} \; =: \; \tfrac{1}{2} R^{a_1 a_2}{}_{b_1 b_2} \, e^{b_1} \, e^{b_2} \; + \; \left(\overline{J}^{a_1 a_2}{}_b \, \psi\right) e^b \; + \; \left(\overline{\psi} K^{a_1 a_2} \psi\right)$$

$$\mathrm{d} = \mathrm{d}x^r \frac{\partial}{\partial x^r} + \mathrm{d}\theta^\rho \frac{\partial}{\partial \theta^\rho} = e^a \partial_a + \psi^\alpha \partial_\alpha$$

$$\omega := \frac{1}{p!} \omega_{a_1 \cdots a_p} e^{a_1} \cdots e^{a_p}$$

$$\mathrm{d}\left(\frac{1}{p_1} \omega_{a_1 \cdots a_p} e^{a_1} \cdots e^{a_p}\right) = \frac{1}{p!}\left(\nabla_{a_0} \omega_{a_1 \cdots a_p}\right) e^{a_0} \cdots e^{a_p} + \frac{1}{(p-1)!} \omega_{a_1 a_2 \cdots a_p} (\bar{\psi} \Gamma^{a_1} \psi) e^{a_2} \cdots e^{a_p}$$
$$+ \frac{1}{p!}\left(\nabla_\alpha \omega_{a_1 \cdots a_p}\right) \psi^\alpha e^{a_1} \cdots e^{a_p}$$

$$\mathrm{d}(\kappa_\alpha \psi^\alpha) = (\nabla_a \kappa_\alpha) e^a \psi^\alpha + \kappa_\alpha \rho^\alpha$$
$$+ (\nabla_\beta \kappa_\alpha) \psi^\beta \psi^\alpha$$

$$\mathrm{d}\left(\frac{1}{p!}\left(\bar{\psi} \Gamma_{a_1 \cdots a_p} \psi\right) e^{a_1} \cdots e^{a_p}\right)$$

$$= \frac{1}{(p-1)!}\left(\bar{\psi} \Gamma_{a_1 \cdots a_p} \psi\right) (\bar{\psi} \Gamma^{a_1} \psi) e^{a_2} \cdots e^{a_p} - \frac{2}{p!}\left(\bar{\psi} \Gamma_{a_1 \cdots a_p} \rho\right) e^{a_1} \cdots e^{a_p}.$$

$$\begin{aligned}
\nabla_r \Gamma_a^\alpha{}_\beta &= \underbrace{\partial_r \Gamma_a^\alpha{}_\beta}_{=0} + \omega_{ra}^{a'} \Gamma_{a'}^\alpha{}_\beta + \frac{1}{4} \omega_r^{b_1 b_2} \left(\left(\Gamma_{b_1 b_2}\right)^\alpha{}_{\alpha'} \Gamma_a^{\alpha'}{}_\beta - \Gamma_a^\alpha{}_{\beta'} \left(\Gamma_{b_1 b_2}\right)^{\beta'}{}_\beta \right) \\
&= \omega_{ra}{}^{a'} \Gamma_{a'}{}^\alpha{}_\beta + \frac{1}{4} \underbrace{\omega_r b_2 [\Gamma_{b_1 b_2}, \Gamma_a]^\alpha}_{= -4\omega_{ra}{}^{a'} \Gamma a'}{}_\beta \\
&= \omega_{ra}{}^{a'} \Gamma_{a'}{}^\alpha{}_\beta - \omega_{ra}{}^{a'} \Gamma_{a'}{}^\alpha{}_\beta = 0
\end{aligned}$$

$$\nabla_A \eta_{a_1 a_2} = \underbrace{\partial_A \eta_{a_1 a_2}}_{0} + \underbrace{\omega_{A a_1}{}^{a_1'} \eta_{a_1' a_2}}_{\omega_{A a_1 a_2}} + \underbrace{\omega_{A a_2}{}^{a_2'} \eta_{a_1 a_2'}}_{\omega_{A a_2 a_1}} = 0$$

$$0 = \nabla_A\left(\epsilon_{a_1 \cdots a_{11}} \epsilon^{a_1 \cdots a_{11}}\right) = 2\epsilon_{a_1 \cdots a_{11}} \nabla_A \epsilon^{a_1 \cdots a_{11}} \;\Rightarrow\; \nabla_A \epsilon_{a_1 \cdots a_{11}} = 0$$

$$(\bar{\psi} \phi) = \psi^\alpha \eta_{\alpha\beta} \phi^\beta$$

$$\nabla_A \psi_\beta = \nabla_A\left(\eta_{\beta\beta'} \psi^{\beta'}\right) = \eta_{\beta\beta'} \nabla_A\left(\psi^{\beta'}\right)$$

$$(G_4^s, G_7^s) \in \Omega^1_{\mathrm{dR}}(X; \mathrm{l}S^4)_{\mathrm{clsd}}$$

$$\nabla_{[a} (G_4)_{a_1 \cdots a_4]} = 0$$

$$H_a = \frac{1}{6}\frac{1}{3!} (G_4)_{a b_1 b_2 b_3} \Gamma^{b_1 b_2 b_3} - \frac{1}{12}\frac{1}{4!} (G_4)^{b_1 \cdots b_4} \Gamma_{a b_1 \cdots b_4}$$

$$\psi^\alpha \nabla_\alpha (G_4)_{a_1 \cdots a_4} = 12\left(\bar{\psi} \Gamma_{[a_1 a_2} \rho_{a_3 a_4]}\right)$$

$$\left(\bar{\psi} \Gamma_{a_1 a_2} (\bar{\psi} \kappa \psi)\right) = 0.$$

$$\mathrm{d}\left(\frac{1}{4!}(G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4}+\frac{1}{2}\left(\bar{\psi}\Gamma_{a_1a_2}\psi\right)e^{a_1}e^{a_2}\right)=0$$

$$\Leftrightarrow\begin{cases}(\psi^0) & \left(\nabla_{[a}(G_4)_{a_1\cdots a_4]}\right)e^{a}e^{a_1}\cdots e^{a_4}=0\\ (\psi^1) & \left(\frac{1}{4!}\psi^{\alpha}\nabla_{\alpha}(G_4)_{a_1\cdots a_4}-\frac{1}{2}\left(\bar{\psi}\Gamma_{[a_1a_2}\rho_{a_3a_4]}\right)\right)e^{a_1}\cdots e^{a_4}=0\\ (\psi^2) & \frac{1}{3!}(G_4)_{ab_1b_2b_3}(\bar{\psi}\Gamma^{a}\psi)e^{b_1b_2b_3}-\left(\bar{\psi}\Gamma_{[a_1a_2}H_{b]}\psi\right)e^{a_1}e^{a_2}e^{b}=0\\ (\psi^3) & \left(\bar{\psi}\Gamma_{a_1a_2}(\bar{\psi}\kappa\psi)\right)e^{a_1}e^{a_2}=0\end{cases}$$

$$H_a=\aleph_1\frac{1}{3!}(G_4)_{ab_1b_2b_3}\Gamma^{b_1b_2b_3}+\mathrm{const}_2\frac{1}{4!}(G_4)^{b_1\cdots b_4}\Gamma_{ab_1\cdots b_4}$$

$$\begin{aligned}\left(\bar{\psi}\Gamma_{a_1a_2}H_{a_3}\psi\right)e^{a_1}e^{a_2}e^{a_3} &=\beth_1\frac{1}{3!}(G_4)_{a_3b_1b_2b_3}\left(\bar{\psi}\Gamma_{a_1a_2}\Gamma^{b_1b_2b_3}\psi\right)e^{a_1}e^{a_2}e^{a_3}\\ &\quad+\gimel_2\frac{1}{4!}(G_4)^{b_1\cdots b_4}\left(\bar{\psi}\Gamma_{a_1a_2}\Gamma_{a_3b_1\cdots b_4}\psi\right)e^{a_1}e^{a_2}e^{a_3}\\ &=1\gimel_1\frac{1}{3!}(G_4)_{a_3b_1b_2b_3}\left(\bar{\psi}\Gamma^{b_1b_2b_3}_{a_1a_2}\psi\right)e^{a_1}e^{a_2}e^{a_3}\\ &\quad+6\daleth_1\frac{1}{3!}(G_4)_{b_3a_1a_2a_3}\left(\bar{\psi}\Gamma^{b_3}\psi\right)e^{a_1}e^{a_2}e^{a_3}\\ &\quad+8\Im_2\frac{1}{4!}(G_4)^{b_1\cdots b_3}{}_{a_3}\left(\bar{\psi}\Gamma^{a_1a_2}{}_{b_1\cdots b_3}\psi\right)e^{a_1}e^{a_2}e^{a_3},\end{aligned}$$

$$1=1!\binom{2}{0}\binom{3}{0},6=2!\binom{2}{2}\binom{3}{2},8=1!\binom{2}{1}\binom{4}{1}.$$

$$\left(\nabla_{a_1}\frac{1}{7!}(G_7)_{a_2\cdots a_8}\right)e^{a_1}\cdots e^{a_8}=\frac{1}{2}\left(\frac{1}{4!}(G_4)_{a_1\cdots a_4}\frac{1}{4!}(G_4)_{a_5\cdots a_8}\right)e^{a_1}\cdots e^{a_8}$$

$$\psi^{\alpha}\nabla_{\alpha}(G_7)_{a_1\cdots a_7}=\frac{7!}{5!}\left(\bar{\psi}\Gamma_{[a_1\cdots a_5}\rho_{a_6a_7]}\right)$$

$$(G_7)_{a_1\cdots a_7}=\epsilon_{a_1\cdots a_7b_1\cdots b_4}\frac{1}{4!}(G_4)^{b_1\cdots b_4},(G_4)_{a_1\cdots a_4}=-\epsilon_{a_1\cdots a_4b_1\cdots b_7}\frac{1}{7!}(G_7)^{b_1\cdots b_7}$$

$$\left(\bar{\psi}\Gamma_{a_1\cdots a_5}(\bar{\psi}\kappa\psi)\right)=0$$

$$
\mathrm{d}\,G_4^s = 0
$$

$$
\Rightarrow
\begin{cases}
\mathrm{d}\Big(\tfrac{1}{7!}(G_7)_{a_1\cdots a_7}\, e^{a_1}\cdots e^{a_7} + \tfrac{1}{5!}\big(\overline{\psi}\Gamma_{a_1\cdots a_5}\psi\big)e^{a_1}\cdots e^{a_5}\Big) \\
\quad = \tfrac{1}{2}\Big(\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4} + \tfrac{1}{2}\big(\overline{\psi}\Gamma_{a_1a_2}\psi\big)\Big)\Big(\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4} + \tfrac{1}{2}\big(\overline{\psi}\Gamma_{a_1a_2}\psi\big)\Big) \\
\Leftrightarrow
\begin{cases}
(\psi^0) & \Big(\nabla_{a_1}\tfrac{1}{7!}(G_7)_{a_2\cdots a_8} = \tfrac{1}{2}\,\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}\,\tfrac{1}{4!}(G_4)_{a_5\cdots a_8}\Big)e^{a_1}\cdots e^{a_8} \\
(\psi^1) & \Big(\psi^\alpha\nabla_\alpha\tfrac{1}{7!}(G_7)_{a_1\cdots a_7} - \tfrac{1}{5!}\big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\rho_{a_6a_7}\big)\Big)e^{a_1}\cdots e^{a_7} = 0 \\
(\psi^2) & \left.\begin{aligned}
&\tfrac{1}{6!}(G_7)_{a_1\cdots a_6 b}\big(\overline{\psi}\,\Gamma^b\,\psi\big)e^{a_1}\cdots e^{a_6} \\
&- \tfrac{2}{6}\tfrac{1}{5!}\tfrac{1}{3!}(G_4)_{ab_1b_2b_3}\big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\Gamma^{b_1b_2b_3}\psi\big)e^a\,e^{a_1}\cdots e^{a_5} \\
&+ \tfrac{2}{12}\,\tfrac{1}{5!}\,\tfrac{1}{4!}\,(G_4)^{b_1\cdots b_4}\big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\Gamma_{ab_1\cdots b_4}\,\psi\big)e^a\,e^{a_1}\cdots e^{a_5} \\
&+ \Big(\tfrac{1}{2}\big(\overline{\psi}\Gamma_{a_1a_2}\psi\big)e^{a_1}\,e^{a_2}\Big)\tfrac{1}{4!}(G_4)_{b_1\cdots b_4}\,e^{b_1}\cdots e^{b_4} = 0
\end{aligned}\right\} \Leftrightarrow \begin{aligned}&(G_7)_{a_1\cdots a_6 b} \\ &= \tfrac{1}{4!}\epsilon_{a_1\cdots a_6 b b_1\cdots b_4}(G_4)\end{aligned} \\
(\psi^3) & \Big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\big(\overline{\psi}\,\kappa\,\psi\big)\Big)e^{a_1}\cdots e^{a_5} = 0
\end{cases}
\end{cases}
$$

$$
(G_7)_{a_1\cdots a_6 b} = \frac{1}{4!}\epsilon_{a_1\cdots a_6 b b_1\cdots b_4}(G_4)^{b_1\cdots b_4}
$$

$$
\mathrm{d}\,G_4^s = 0
$$

$$
\Rightarrow
\begin{cases}
\mathrm{d}\Big(\tfrac{1}{7!}(G_7)_{a_1\cdots a_7}\, e^{a_1}\cdots e^{a_7} + \tfrac{1}{5!}\big(\overline{\psi}\Gamma_{a_1\cdots a_5}\psi\big)e^{a_1}\cdots e^{a_5}\Big) \\
\quad = \tfrac{1}{2}\Big(\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4} + \tfrac{1}{2}\big(\overline{\psi}\Gamma_{a_1a_2}\psi\big)\Big)\Big(\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4} + \tfrac{1}{2}\big(\overline{\psi}\Gamma_{a_1a_2}\psi\big)\Big) \\
\Leftrightarrow
\begin{cases}
(\psi^0) & \Big(\nabla_{a_1}\tfrac{1}{7!}(G_7)_{a_2\cdots a_8} = \tfrac{1}{2}\,\tfrac{1}{4!}(G_4)_{a_1\cdots a_4}\,\tfrac{1}{4!}(G_4)_{a_5\cdots a_8}\Big)e^{a_1}\cdots e^{a_8} \\
(\psi^1) & \Big(\psi^\alpha\nabla_\alpha\tfrac{1}{7!}(G_7)_{a_1\cdots a_7} - \tfrac{1}{5!}\big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\rho_{a_6a_7}\big)\Big)e^{a_1}\cdots e^{a_7} = 0 \\
(\psi^2) & \left.\begin{aligned}
&\tfrac{1}{6!}(G_7)_{a_1\cdots a_6 b}\big(\overline{\psi}\,\Gamma^b\,\psi\big)e^{a_1}\cdots e^{a_6} \\
&- \tfrac{2}{6}\tfrac{1}{5!}\tfrac{1}{3!}(G_4)_{ab_1b_2b_3}\big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\Gamma^{b_1b_2b_3}\psi\big)e^a\,e^{a_1}\cdots e^{a_5} \\
&+ \tfrac{2}{12}\,\tfrac{1}{5!}\,\tfrac{1}{4!}\,(G_4)^{b_1\cdots b_4}\big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\Gamma_{ab_1\cdots b_4}\,\psi\big)e^a\,e^{a_1}\cdots e^{a_5} \\
&+ \Big(\tfrac{1}{2}\big(\overline{\psi}\Gamma_{a_1a_2}\psi\big)e^{a_1}\,e^{a_2}\Big)\tfrac{1}{4!}(G_4)_{b_1\cdots b_4}\,e^{b_1}\cdots e^{b_4} = 0
\end{aligned}\right\} \Leftrightarrow \begin{aligned}&(G_7)_{a_1\cdots a_6 b} \\ &= \tfrac{1}{4!}\epsilon_{a_1\cdots a_6 b b_1\cdots b_4}\end{aligned} \\
(\psi^3) & \Big(\overline{\psi}\,\Gamma_{a_1\cdots a_5}\,\big(\overline{\psi}\,\kappa\,\psi\big)\Big)e^{a_1}\cdots e^{a_5} = 0
\end{cases}
\end{cases}
$$

$$
(G_7)_{a_1\cdots a_6 b} = \frac{1}{4!}\epsilon_{a_1\cdots a_6 b b_1\cdots b_4}(G_4)^{b_1\cdots b_4}
$$

$$\overbrace{\left(-\frac{2}{6}\frac{1}{5!}\frac{1}{3!}3!\binom{5}{3}\binom{3}{3}+\frac{2}{12}\frac{1}{5!}\frac{1}{4!}4!\binom{5}{4}\binom{4}{4}+\frac{1}{2}\frac{1}{4!}\right)}^{=0}(G_4)_{a_2\cdots a_5}(\bar{\psi}\Gamma_{aa_1}\psi)e^{a}e^{a_1}\cdots e^{a_6},$$

$$\underbrace{\left(-\frac{2}{6}\frac{1}{5!}\frac{1}{3!}1\binom{5}{1}\binom{3}{1}+\frac{2}{12}\frac{1}{5!}\frac{1}{4!}2\binom{5}{2}\binom{4}{2}\right)}_{=0}(G_4)_{a_1a_2b_1b_2}(\bar{\psi}\Gamma_{a_3\cdots a_6}{}^{b_1b_2}\psi)e^{a_1}\cdots e^{a_6}.$$

$$(\bar{\psi}\Gamma_{a_1\cdots a_5ab_1\cdots b_4}\psi)=+\epsilon_{a_1\cdots a_5ab_1\cdots b_4b}(\bar{\psi}\Gamma^{b}\psi)$$

$$\left(\frac{1}{6!}(G_7)_{a_1\cdots a_6b}-\frac{2}{12}\frac{1}{5!}\frac{1}{4!}(G_4)^{b_1\cdots b_4}\epsilon_{a_1\cdots a_6bb_1\cdots b_4}\right)(\bar{\psi}\Gamma^{b}\psi)e^{a_1}\cdots e^{a_6}=0$$

$$\begin{aligned}(G_4)_{a_1\cdots a_4}&=\delta^{a_1\cdots a_4}_{b_1\cdots b_4}(G_4)_{a_1\cdots a_4}=-\frac{1}{4!\cdot 7!}\epsilon^{c_1\cdots c_7a_1\cdots a_4}\epsilon_{c_1\cdots c_7b_1\cdots b_4}(G_4)_{a_1\cdots a_4}\\&=-\frac{1}{7!}\epsilon_{a_1\cdots a_4c_1\cdots c_7}(G_7)^{c_1\cdots c_7}\end{aligned}$$

$$\frac{5}{5!}(\bar{\psi}\Gamma_{a_1\cdots a_5}\psi)(\bar{\psi}\Gamma^{a_1})e^{a_2}\cdots e^{a_5}=\frac{1}{2^3}\left((\bar{\psi}\Gamma_{a_1a_2}\psi)e^{a_1}e^{a_2}\right)\left((\bar{\psi}\Gamma_{a_1a_2}\psi)e^{a_1}e^{a_2}\right)$$

$$\begin{aligned}\nabla_b(G_4)^{ba_1a_2a_3}&=-\frac{1}{7!}\epsilon^{ba_1a_2a_3c_1\cdots c_7}\nabla_b(G_7)_{c_1\cdots c_7}\\&=\frac{1}{2}\frac{1}{7!}\epsilon^{a_1a_2a_3bc_1\cdots c_7}\left(\frac{1}{4!}(G_4)_{bc_1\cdots c_3}\frac{1}{4!}(G_4)_{c_4\cdots c_7}\right)\end{aligned}$$

$$\begin{aligned}H_a&=\frac{1}{6}\frac{1}{3!}(G_4)_{ab_1b_2b_3}\Gamma^{b_1b_2b_3}-\frac{1}{12}\frac{1}{4!}(G_4)^{b_1\cdots b_4}\Gamma_{ab_1\cdots b_4}\\&=\frac{1}{6}\frac{1}{3!}(G_4)_{ab_1b_2b_3}\Gamma^{b_1b_2b_3}+\frac{1}{12}\frac{1}{4!}\frac{1}{6!}(G_4)^{b_1\cdots b_4}\epsilon_{ab_1\cdots b_4c_1\cdots c_6}\Gamma^{c_1\cdots c_6}\\&=\frac{1}{6}\frac{1}{3!}(G_4)_{ab_1b_2b_3}\Gamma^{b_1b_2b_3}+\frac{1}{12}\frac{1}{6!}(G_7)_{ac_1\cdots c_6}\Gamma^{c_1\cdots c_6}\end{aligned}$$

$$\Gamma^{a\,b_1b_2}\,\rho_{b_1b_2}\;=\;0\qquad\Rightarrow\qquad\left\{\begin{array}{rcll}\Gamma^{b_1b_2}\,\rho_{b_1b_2}&=&0\,,&\\\Gamma^{b_2}\,\rho_{b_1b_2}&=&0\,,&\text{irreducibility}\\\Gamma^{ab_1}\,\rho_{b_1b_2}&=&-\rho^{a}{}_{b_2}\,,&\\\Gamma^{a_1a_2\,b_1b_2}\,\rho_{b_1b_2}&=&-2\,\rho^{a_1a_2}\,,&\\\Gamma_{[a_1\cdots a_5}\,\rho_{a_6a_7]}&=&\frac{1}{84}\epsilon_{a_1\cdots a_7b_1\cdots b_4}\Gamma^{b_1b_2}\rho^{b_3b_4}\,.&\end{array}\right.$$

$$\cdots\qquad\Rightarrow\qquad\overline{\rho_{a_1a_2}}{}_{\alpha}\;=\;+6\,\Gamma_{b_1b_2}{}^{\beta}{}_{\alpha}\,\nabla_{\beta}(G_4)^{a_1a_2b_1b_2}$$

$$\underbrace{\Gamma_a\Gamma^{ab_1b_2}\rho_{b_1b_2}}_{=0}=9\Gamma^{b_1b_2}\rho_{b_1b_2},\underbrace{\Gamma_{ca}\Gamma^{ab_1b_2}\rho_{b_1b_2}}_{=0}=\underbrace{8\Gamma^{cb_1b_2}\rho_{b_1b_2}}_{=0}+18\Gamma^{b}\rho_{cb},$$

$$\begin{aligned}\Gamma^{ac}\rho_{cb}&=\overbrace{\frac{1}{2}\Gamma^{a}\Gamma^{c}\rho_{cb}}^{=0}-\frac{1}{2}\Gamma^{c}\Gamma^{a}\rho_{cb}\\&=\underbrace{\frac{1}{2}\Gamma^{a}\Gamma^{c}\rho_{cb}}_{=0}-\eta^{ac}\rho_{cb}=-\rho^{a}{}_{v},\end{aligned}$$

$$
\begin{aligned}
0 &= \Gamma_{c_1c_2a}\Gamma^{ab_1b_2}\rho_{b_1b_2} \\
&= 7\Gamma_{c_1c_2b_1b_2}\rho^{b_1b_2} + 32\Gamma_{[c_1}{}^{b}\rho_{c_2]b} - 18\rho_{c_1c_2} \\
&= 7\Gamma_{c_1c_2b_1b_2}\rho^{b_1b_2} + 14\rho_{c_1c_2}
\end{aligned}
$$

$$
\begin{aligned}
\overline{\rho^{a_1a_2}}_{\alpha} &= -\tfrac{1}{2}\,\overline{\Gamma^{a_1a_2b_1b_2}\ \rho_{b^1b^2}}\ {}_{\alpha} \\
&= -\tfrac{1}{2}\,\overline{\Gamma_{b_1b_2}\Gamma^{[a_1a_2}\rho^{b_1b_2]}}\ {}_{\alpha} \\
&= +\tfrac{1}{2}\Gamma_{b_1b_2}{}^{\beta}{}_{\alpha}\ \overline{\Gamma^{[a_1a_2}\rho^{b_1b_2]}}_{\beta} \\
&= +6\,\Gamma_{b_1b_2}{}^{\beta}{}_{\alpha}\nabla_{\beta}(G_4)^{a_1a_2b_1b_2}
\end{aligned}
$$

$$
\nabla_{[a_1}\rho_{a_2a_3]} = \frac{1}{3}\underbrace{\Gamma_{b[a_1}\nabla^{b}\rho_{a_2a_3]}}_{\blacksquare^2} - \frac{1}{15}\Gamma_{[a_1a_2}\underbrace{\nabla_b\rho^{b}{}_{a_3]}}_{\blacksquare^2} - \frac{1}{3}\underbrace{\Gamma^{b_1b_2}\Gamma_{[b_1b_2}\nabla_{a_1}\rho_{a_2a_3]}}_{\blacksquare^2}
$$

$$
\Gamma_{[a_1a_2}\nabla_{a_3}\rho_{a_4a_5]} = \bar{H}_{[a_1}\Gamma_{a_2a_3}\rho_{a_4a_5]} - \frac{1}{3}(G_4)_b\left[a_1a_2a_3\Gamma^{b}\rho_{a_4a_5}\right]
$$

$$
\nabla_b\rho^{b}{}_{a} = \frac{5}{84}\Gamma^{b_1\cdots b_4}\underbrace{\Gamma_{[b_1b_2}\nabla_{b_3}\rho_{b_4a]}}_{\blacksquare^2}
$$

$$
\Gamma^{a[c_1}\nabla_a\rho^{c_2c_3]} = -\Gamma^{[c_1c_2}\overbrace{\nabla_b\rho^{|b|c_3]}}^{\blacksquare^2} + 2\bar{H}_b\Gamma^{[bc_1}\rho^{c_2c_3]}
$$
$$
+2\,\frac{5!\cdot 84}{7!\cdot 4!}\epsilon^{c_1c_2c_3a_1\cdots a_8}\left(\frac{12}{4!\cdot 4!}(G_4)_{a_1\cdots a_4}\Gamma_{a_5a_6}\rho_{a_7a_8} - \frac{1}{6!}(G_7)_{ba_1\cdots a_6}\Gamma^{b}\rho_{a_7a_8}\right)
$$

$$
\Gamma^{b_1\cdots b_4}\Gamma_{[b_1b_2}\nabla_{b_3}\rho_{b_4a]} = \frac{84}{5}\nabla_b\rho^{b}_{a}
$$

$$
\Gamma^{b_1b_2}\Gamma_{[b_1b_2}\nabla_{a_1}\rho_{a_2a_3]} = \Gamma_{b[a_1}\nabla^{b}\rho_{a_2a_3]} - \frac{1}{5}\Gamma_{[a_1a_2}\nabla^{b}\rho_{|b|a_3]} - 3\nabla_{[a_1}\rho_{a_2a_3]}
$$

$$
\bar{H}_a = \frac{1}{6}\frac{1}{3!}(G_4)_{ab_1b_2b_3}\Gamma^{b_1b_2b_3} + \frac{1}{12}\frac{1}{4!}(G_4)^{b_1\cdots b_4}\Gamma_{ab_1\cdots b_4}
$$

$$
\begin{aligned}
0 =& \,\mathrm{dd}\frac{1}{4!}(G_4)_{a_1\cdots a_4}e^{a_1}\cdots e^{a_4} \\
=& \,\mathrm{d}\Bigg(\frac{1}{4!}\left(\nabla_{[a_1}(G_4)_{a_2\cdots a_5]}\right)e^{a_1}\cdots e^{a_5} + \frac{1}{4!}\psi^{\beta}\left(\nabla_{\beta}(G_4)_{a_1\cdots a_4}\right)e^{a_1}\cdots e^{a_4} \\
&+\frac{1}{3!}(G_4)_{ba_1a_2a_3}(\bar{\psi}\Gamma^{b}\psi)e^{a_1}e^{a_2}\,e^{a_3}\Bigg) \\
=& \frac{1}{4!}\left(H_{[a_1}\psi\right)^{\beta}\left(\nabla_{|\beta|}(G_4)_{a_2\cdots a_5]}\right)e^{a_1}\cdots e^{a_5} - \frac{1}{4!}\psi^{\beta}\left(\nabla_{[a_1}\nabla_{|\beta|}(G_4)_{a_2\cdots a_5]}\right)e^{a_1}\cdots e^{a_5} \\
&-\frac{1}{3!}(G_4)_{ba_1a_2a_3}\left(\bar{\psi}\Gamma^{b}\rho_{a_4a_5}\right)e^{a_2}\cdots e^{a_5} \\
&+\mathcal{O}(\psi^{\neq 1}) \\
=& \bar{\psi}\left(\frac{1}{2}\bar{H}_{[a_1}\Gamma_{a_2a_3}\rho_{a_4a_5]} - \frac{1}{2}\nabla_{[a_1}\Gamma_{a_2a_3}\rho_{a_4a_5]} - \frac{1}{3!}(G_4)_{b[a_1a_2a_3}\Gamma^{b}\rho_{a_4a_5]}\right)e^{a_1}\cdots e^{a_5} \\
&+\mathcal{O}(\psi^{\neq 1})
\end{aligned}
$$

$$\nabla^b \Gamma_{[bc_1}\rho_{c_2c_3]} = \frac{1}{2}\underbrace{\Gamma_{b[c_1}\nabla^b\rho_{c_2c_3]}}_{\blacksquare^2} + \frac{1}{2}\Gamma_{[c_1c_2}\underbrace{\nabla^b\rho_{|b|c_3]}}_{\blacksquare^2}$$

$$\begin{aligned}
0 =& \mathrm{dd}\frac{1}{7!}(G_7)_{a_1\cdots a_7}e^{a_1}\cdots e^{a_7}\\
=& \mathrm{d}\Big(\frac{1}{7!}(\nabla_{a_1}(G_7)_{a_2\cdots a_8})e^{a_1}\cdots e^{a_8} + \frac{1}{7!}\psi^\beta\nabla_\beta(G_7)_{a_1\cdots a_7}e^{a_1}\cdots e^{a_7}\\
&+\frac{1}{6!}(G_7)_{ba_1\cdots a_6}(\bar{\psi}\Gamma^b\psi)e^{a_1}\cdots e^{a_6}\Big)\\
=&\Big(\frac{1}{7!}\psi^\beta\nabla_\beta\nabla_{[a_1}(G_7)_{a_2\cdots a_8]}\\
&+\frac{1}{7!}\psi^\beta(\bar{H}_{[a_1}-\nabla_{[a_1})\nabla_{|\beta|}(G_7)_{a_2\cdots a_8]} - \frac{1}{6!}(G_7)_{ba_1\cdots a_6}(\bar{\psi}\Gamma^b\rho_{a_7a_8})\Big)e^{a_1}\cdots e^{a_8}\\
&+\mathcal{O}(\psi^{\neq 1})\\
=& \bar{\psi}\Big(\frac{12}{4!\cdot 4!}(G_4)_{[a_1\cdots a_4}\Gamma_{a_5a_6}\rho_{a_7a_8]}\\
&+\frac{1}{5!\cdot 84}(\bar{H}_{[a_1}-\nabla_{[a_1})\epsilon_{a_2\cdots a_8]b_1\cdots b_4}\Gamma^{[b_1b_2}\rho^{b_3b_4]} - \frac{1}{6!}(G_7)_{ba_1\cdots a_6}(\bar{\psi}\Gamma^b\rho_{a_7a_8})\Big)e^{a_1}\cdots e^{a_8}\\
&+\mathcal{O}(\psi^{\neq 1})
\end{aligned}$$

$$\begin{aligned}
&\frac{7!\cdot 4!}{5!\cdot 84}(\bar{H}_{a_1}-\nabla_{a_1})\Gamma^{[a_1c_1}\rho^{c_2c_3]}\\
&+\epsilon^{c_1c_2c_3a_1\cdots a_8}\big(\frac{12}{4!\cdot 4!}(G_4)_{a_1\cdots a_4}\Gamma_{a_5a_6}\rho_{a_7a_8} - \frac{1}{6!}\underbrace{(G_7)_{ba_1\cdots a_6}}_{\frac{1}{4!}\epsilon_{ba_1\cdots a_6d_1\cdots d_4}(G_4)^{d_1\cdots d_4}}\Gamma^b\rho_{a_7a_8}\big) = 0
\end{aligned}$$

$$\Gamma^{ab_1b_2}\rho_{b_1b_2} = 0$$

$$\begin{aligned}
J_{abc} &= +\Gamma_a\rho_{bc} - \Gamma_c\rho_{ab} + \Gamma_b\rho_{ca}\\
K^{a_1a_2} &= +\frac{1}{6}\Big((G_4)^{a_1a_2b_1b_2}\Gamma_{b_1b_2} + \frac{1}{4!}(G_4)_{b_1\cdots b_4}\Gamma^{a_1a_2b_1\cdots b_4}\Big)\\
&= +\frac{1}{6}\Big((G_4)^{a_1a_2b_1b_2}\Gamma_{b_1b_2} + \frac{1}{5!}(G_7)^{a_1a_2b_1\cdots b_5}\Gamma_{b_1\cdots b_5}\Big),
\end{aligned}$$

$$(\bar{\psi}\kappa\psi) = 0$$

$$\begin{aligned}
&R^{ab}e_b = -2(\bar{\psi}\Gamma^a\rho)\\
\Leftrightarrow &\begin{cases}
(\psi^0) & R^a{}_{[b_1b_2b_3]}e^{b_1}e^{b_2}e^{b_3} = 0\\
(\psi^1) & (\bar{\psi}J^a{}_{b_1b_2})e^{b_1}e^{b_2} = +(\bar{\psi}\Gamma^a\rho_{b_1b_2})e^{b_1}e^{b_2}\\
(\psi^2) & (\bar{\psi}K^{ab}\psi)e_b = -2(\bar{\psi}\Gamma^a H_b\psi)e^b\\
(\psi^3) & 2(\bar{\psi}\Gamma^a(\bar{\psi}\kappa\psi)) = 0
\end{cases}
\end{aligned}$$

$$\frac{1}{2}(J_{ab_1b_2} - J_{ab_2b_1}) = +\Gamma_a\rho_{b_1b_2}$$

$$
\frac{\begin{array}{r} \frac{1}{2}\left(J_{ab_1b_2} - J_{ab_2b_1}\right) \\ -\frac{1}{2}\left(J_{b_2ab_1} - J_{b_2b_1a}\right) \\ +\frac{1}{2}\left(J_{b_1b_2a} - J_{b_1ab_2}\right) \end{array}}{J_{ab_1b_2}\,,} = \begin{array}{l} +\Gamma_a\rho_{b_1b_2} \\ -\Gamma_{b_2}\rho_{ab_1} \\ +\Gamma_{b_1}\rho_{b_2a} \end{array}
$$

$$
\begin{aligned}
&+\Gamma_a\rho_{b_1b_2} - \Gamma_{b_2}\rho_{ab_1} + \Gamma_{b_1}\rho_{b_2a} \\
&-\Gamma_a\rho_{b_2b_1} + \Gamma_{b_1}\rho_{ab_2} - \Gamma_{b_2}\rho_{b_1a} \\
&= +2\Gamma_a\rho_{b_1b_2}
\end{aligned}
$$

$$
\begin{aligned}
(\bar{\psi}\Gamma^a H^b\psi) &= \frac{1}{6}\frac{1}{3!}(G_4)^b{}_{b_1b_2b_3}\left(\bar{\psi}\Gamma^a\Gamma^{b_1b_2b_3}\psi\right) - \frac{1}{12}\frac{1}{4!}(G_4)_{b_1\cdots b_4}\left(\bar{\psi}\Gamma^a\Gamma^{bb_1\cdots b_4}\psi\right) \\
&= -\frac{1}{6}\frac{1}{2!}(G_4)^{abb_2b_3}\left(\bar{\psi}\Gamma_{b_2b_3}\psi\right) - \frac{1}{12}\frac{1}{4!}(G_4)_{b_1\cdots b_4}\left(\bar{\psi}\Gamma^{abb_1\cdots b_4}\psi\right)
\end{aligned}
$$

$$
\begin{aligned}
K^{ab} &= +\frac{1}{6}\left((G_4)^{abb_1b_2}\Gamma_{b_1b_2} + \frac{1}{4!}(G_4)_{b_1\cdots b_4}\Gamma^{abb_1\cdots b_4}\right) \\
&= +\frac{1}{6}\left((G_4)^{abb_1b_2}\Gamma_{b_1b_2} + \frac{1}{4!\cdot 5!}(G_4)_{b_1\cdots b_4}\epsilon^{abb_1\cdots b_4c_1\cdots c_5}\Gamma_{c_1\cdots c_5}\right) \\
&= +\frac{1}{6}\left((G_4)^{abb_1b_2}\Gamma_{b_1b_2} + \frac{1}{5!}(G_7)^{abc_1\cdots c_5}\Gamma_{c_1\cdots c_5}\right)
\end{aligned}
$$

$$
(\bar{\psi}\Gamma_a(\bar{\psi}\kappa\psi)) = 0, \left(\bar{\psi}\Gamma_{a_1a_2}(\bar{\psi}\kappa\psi)\right) = 0, \left(\bar{\psi}\Gamma_{a_1\cdots a_5}(\bar{\psi}\kappa\psi)\right) = 0
$$

$$
\mathrm{d}\rho + \frac{1}{4}\omega^{ab}\Gamma_{ab}\rho = +\frac{1}{4}R^{ab}\Gamma_{ab}\psi
$$

$$
\Leftrightarrow \begin{cases}
(\psi^0) & \left(\nabla_{[a_1}\rho_{a_2a_3]} + H_{[a_1}\rho_{a_2a_3]}\right)e^{a_1}e^{a_2}e^{a_3} = 0 \\
(\psi^1) & \left(\psi^\alpha\frac{1}{2}\left(\nabla_\alpha\rho_{a_1a_2}\right) + \left(\nabla_{[a_1}H_{a_2]}\right)\psi - H_{a_1}H_{a_2}\psi - \frac{1}{4}\frac{1}{2}R^{ab}{}_{a_1a_2}\Gamma_{ab}\psi\right)e^{a_1}e^{a_2} = 0 \\
(\psi^2) & \rho_{ab}(\bar{\psi}\Gamma^a\psi)e^b + \left(\frac{1}{6}\frac{1}{3!}\psi^\alpha\left(\nabla_\alpha(G_4)_{ab_1b_2b_3}\right)\Gamma^{b_1b_2b_3} - \frac{1}{12}\frac{1}{4!}\psi^\alpha\left(\nabla_\alpha(G_4)^{b_1\cdots b_4}\right)\Gamma_{ab_1\cdots b_4}\right)\psi e^a \\
& +\left(\bar{\psi}J^{b_1b_2}{}_a\right)\frac{1}{4}\Gamma_{b_1b_2}\psi e^a = 0 \\
(\psi^3) & H_a\psi(\bar{\psi}\Gamma^a\psi) - \frac{1}{2}\Gamma_{ab}\psi\left(\bar{\psi}\Gamma^{[a}H^{b]}\psi\right) = 0
\end{cases}
$$

$$
\begin{aligned}
&\rho_{ca}(\bar{\psi}\Gamma^c\psi)e^a + \Big(\frac{1}{6}\frac{1}{3!}\underbrace{\psi^\alpha\left(\nabla_\alpha(G_4)_{ab_1b_2b_3}\right)}_{\frac{4!}{2}\left(\bar{\psi}\Gamma_{[ab_1}\rho^\alpha_{b_2b_3]}\right)}\Gamma^{b_1b_2b_3} + \frac{1}{12}\frac{1}{4!}\underbrace{\psi^\alpha(\nabla_\alpha(G_4)^{b_1\cdots b_4})}_{\frac{4!}{2}(\bar{\psi}\Gamma^{[b_1b_2}\rho^{b_3b_4]})}\Gamma_{ab_1\cdots b_4}\Big)\psi e^a \qquad \underbrace{+\left(\bar{\psi}J_{b_1b_2a}\right)}_{+\left(\bar{\psi}\left(\Gamma_{b_1}\rho_{b_2a} - \Gamma_a\rho_{b_1b_2} + \Gamma_{b_2}\rho_{ab_1}\right)\right)} \quad \frac{1}{4}\Gamma^{b_1b_2}\psi e^a \\
&= \rho_{ca}(\bar{\psi}\Gamma^c\psi)e^a - \frac{1}{3}\Gamma^{b_1b_2b_3}\psi\left(\bar{\psi}\Gamma_{[ab_1}\rho_{b_2b_3]}\right)e^a + \frac{1}{24}\Gamma_{ab_1\cdots b_4}\psi\left(\bar{\psi}\Gamma^{[b_1b_2}\rho^{b_3b_4]}\psi\right)e^a \\
&\quad - \frac{1}{4}\Gamma^{b_1b_2}\psi\left(\left(\bar{\psi}\Gamma_{b_1}\rho_{b_2a}\right) - \left(\bar{\psi}\Gamma_a\rho_{b_1b_2}\right) + \left(\bar{\psi}\Gamma_{b_2}\rho_{ab_1}\right)\right)e^a \\
&=: Q_{ca}(\bar{\psi}\Gamma^c\psi) + \frac{1}{2}Q_{c_1c_2a}(\bar{\psi}\Gamma^{c_1c_2}\psi) + \frac{1}{5!}Q_{c_1\cdots c_5a}(\bar{\psi}\Gamma^{c_1\cdots c_5}\psi)
\end{aligned}
$$

$$32Q_{ca}=32\cdot\rho_{ca}-\frac{1}{3}\Gamma^{b_1b_2b_3}\Gamma_c\Gamma_{[ab_1}\rho_{b_2b_3]}+\frac{1}{24}\Gamma_{ab_1\cdots b_4}\Gamma_c\Gamma^{[b_1b_2}\rho^{b_3b_4]}-\frac{1}{4}\Gamma^{b_1b_2}\Gamma_c\left(\Gamma_{b_1}\rho_{b_2a}-\Gamma_a\rho_{b_1b_2}+\Gamma_{b_2}\rho_{ab_1}\right),$$

$$32Q_{c_1c_2a}=+\frac{1}{3}\Gamma^{b_1b_2b_3}\Gamma_{c_1c_2}\Gamma_{[ab_1}\rho_{b_2b_3]}-\frac{1}{24}\Gamma_{ab_1\cdots b_4}\Gamma_{c_1c_2}\Gamma^{[b_1b_2}\rho^{b_3b_4]}+\frac{1}{4}\Gamma^{b_1b_2}\Gamma_{c_1c_2}\left(\Gamma_{b_1}\rho_{b_2a}-\Gamma_a\rho_{b_1b_2}+\Gamma_{b_2}\rho_{ab_1}\right),$$

$$32Q_{c_1\cdots c_5a}=-\frac{1}{3}\Gamma^{b_1b_2b_3}\Gamma_{c_1\cdots c_5}\Gamma_{[ab_1}\rho_{b_2b_3]}+\frac{1}{24}\Gamma_{ab_1\cdots b_4}\Gamma_{c_1\cdots c_5}\Gamma^{[b_1b_2}\rho^{b_3b_4]}-\frac{1}{4}\Gamma^{b_1b_2}\Gamma_{c_1\cdots c_5}\left(\Gamma_{b_1}\rho_{b_2a}-\Gamma_a\rho_{b_1b_2}+\Gamma_{b_2}\rho_{ab_1}\right).$$

$$\begin{aligned}\Gamma^cQ_{ca}&=-\frac{261}{2}\Gamma^b\rho_{ab}-\frac{31}{12}\Gamma^{ab_1b_2}\rho_{b_1b_2}\\ \Gamma^aQ_{ca}&=\frac{43}{2}\Gamma^b\rho_{cb}+\frac{53}{12}\Gamma^{cb_1b_2}\rho_{b_1b_2}\end{aligned}$$

$$Q_{ca}=0\ \Rightarrow\ \Gamma^{ab_1b_2}\rho_{b_1b_2}=0\ \Rightarrow\ \Gamma^b\rho_{ab}=0$$

$$\Gamma^{b'}\rho_{bb'}=0\ \Rightarrow\ \begin{cases}Q_{ca}=0\\ Q_{c_1c_2a}=0\\ Q_{c_1\cdots c_5a}=0\end{cases}$$

$$\begin{aligned}&\Big(-\frac{1}{6}\frac{1}{3!}\Gamma_{[a_1a_2a_3}\psi(\bar\psi\Gamma_{a_4]}\psi)-\frac{1}{12}\frac{1}{4!}\Gamma_{ba_1\cdots a_4}\psi(\bar\psi\Gamma^b\psi)\\ &+\frac{1}{4\cdot 6}\Gamma_{[a_1a_2}\psi(\bar\psi\Gamma_{a_3a_4]}\psi)+\frac{1}{4\cdot 6}\frac{1}{24}\Gamma^{b_1b_2}\psi\underbrace{(\bar\psi\Gamma_{b_1b_2a_1\cdots a_4}\psi)}_{\epsilon_{b_1b_2a_1\cdots a_4c_1\cdots c_5}(\bar\psi\Gamma^{c_1\cdots c_5}\psi)}\Big)(G_4)^{a_1\cdots a_4}\end{aligned}$$

$$\begin{aligned}&-\frac{1}{12}\frac{1}{4!}(\bar\psi\Gamma_{ba_1\cdots a_4}\psi)(\bar\psi\Gamma^b\psi)+\frac{1}{4\cdot 6}(\bar\psi\Gamma_{[a_1a_2}\psi)(\bar\psi\Gamma_{a_3a_4]}\psi)+\frac{1}{4\cdot 6}\frac{1}{24}(\bar\psi\Gamma^{b_1b_2}\psi)(\bar\psi\Gamma_{b_1b_2a_1\cdots a_4}\psi)\\ &=\left(-\frac{1}{12}\frac{1}{4!}+\frac{1}{3\cdot 4\cdot 6}-\frac{1}{4}\frac{1}{24}\right)(\bar\psi\Gamma_{ba_1\cdots a_4}\psi)(\bar\psi\Gamma^b\psi)\\ &=\frac{1}{12}\left(-\frac{2}{48}+\frac{8}{48}-\frac{6}{48}\right)(\bar\psi\Gamma_{ba_1\cdots a_4}\psi)(\bar\psi\Gamma^b\psi)=0\end{aligned}$$

$$\begin{aligned}\left(-\frac{1}{6}\frac{1}{3!}\frac{1}{11}\Gamma_{[a_1a_2a_3}\Gamma_{a_4]}-\frac{1}{12}\frac{1}{4!}\frac{1}{11}\Gamma_{ba_1\cdots a_4}\Gamma^b+\frac{1}{4\cdot 6}\frac{1}{11}\Gamma_{[a_1a_2}\Gamma_{a_3a_4]}-\frac{1}{4\cdot 6}\frac{1}{24}\frac{1}{77}\frac{1}{5!}\Gamma^{b_1b_2}\epsilon_{b_1b_2a_1\cdots a_4c_1\cdots c_5}\Gamma^{c_1\cdots c_5}\right)\Xi^{\blacksquare}&=0\\ -\frac{1}{6}\frac{1}{3!}\Gamma_{[a_1a_2a_3}\Xi^{\daleth}_{a_4]}-\frac{1}{12}\frac{1}{4!}\Gamma^{b}_{a_1\cdots a_4}\Xi^{\gimel}_{b}-\frac{1}{4\cdot 6}\frac{2}{9}\Gamma_{[a_1a_2}\Gamma_{a_3}\Xi^{\beth}_{a_4]}+\frac{1}{4\cdot 6}\frac{1}{24}\frac{5}{9}\frac{1}{5!}\Gamma_{b_1b_2}\epsilon^{b_1b_2a_1\cdots a_4c_1\cdots c_5}\Gamma_{[c_1\cdots c_4}\Xi^{\aleph}_{c_5]}&=0\\ \frac{1}{4\cdot 6}\Gamma_{[a_1a_2}\Xi^{\daleth}_{a_3a_4]}+\frac{1}{4\cdot 6}\frac{1}{24}2\frac{1}{5!}\Gamma_{b_1b_2}\epsilon^{b_1b_2a_1\cdots a_4c_1\cdots c_5}\Gamma_{[c_1c_2c_3}\Xi^{\mathfrak{H}}_{c_4c_5]}&=0\\ \frac{1}{4\cdot 6}\frac{1}{24}\epsilon^{b_1b_2a_1\cdots a_4c_1\cdots c_5}\Gamma_{b_1b_2}\Xi^{\blacksquare}_{c_1\cdots c_5}&=0\end{aligned}$$

$$\mathrm{d}R^{a_1a_2}+\omega^{a_1}{a'_1}R^{a'_1a_2}-R^{a_1a'_2}\omega^{a_2}{a'_2}=0$$

$$\Leftrightarrow\begin{cases}(\psi^0)\quad\left(\left(\nabla_{b_1}R^{a_1a_2}{}_{b_2b_3}\right)-\left(\bar J^{a_1a_2}{}_{b_1}\rho_{b_2b_3}\right)\right)e^{b_1}e^{b_2}e^{b_3}=0\\ (\psi^1)\quad\left(\psi^\alpha(\nabla_\alpha R^{a_1a_2}b_1b_2)-\left(\bar\psi\nabla_{b_1}J^{a_1a_2}b_2\right)+\left(\bar J^{a_1a_2}b_1H_{b_2}\psi\right)-\left(\bar\psi K^{a_1a_2}\rho_{b_1b_2}\right)\right)e^{b_1}e^{b_2}=0\\ (\psi^2)\quad\left(2R^{a_1a_2}{}_{bc}(\bar\psi\Gamma^b\psi)+\psi^\alpha\left(\overline{\left(\nabla_\alpha J^{a_1a_2}c\right)}\psi\right)+(\bar\psi\nabla_cK^{a_1a_2}\psi)-2(\bar\psi K^{a_1a_2}H_c\psi)\right)e^c=0\\ (\psi^3)\quad(\bar J^{a_1a_2}{}_b\psi)(\bar\psi\Gamma^b\psi)-\psi^\alpha(\bar\psi\nabla_\alpha K^{a_1a_2}\psi)=0\end{cases}$$

$$\begin{aligned}R_a{}^c{}_{bc}-\frac{1}{2}R^{c_1c_2}{}_{c_1c_2}\eta_{ab}&=+\frac{1}{12}\left((G_4)_{ac_1c_2c_3}(G_4)_b{}^{c_1c_2c_3}-\frac{1}{8}(G_4)_{c_1\cdots c_4}(G_4)^{c_1\cdots c_4}\eta_{ab}\right)\\ \Leftrightarrow\ R_a{}^c{}_{bc}&=+\frac{1}{12}\left((G_4)_{ac_1c_2c_3}(G_4)_b{}^{c_1c_2c_3}-\frac{1}{12}(G_4)_{c_1\cdots c_4}(G_4)^{c_1\cdots c_4}\eta_{ab}\right).\end{aligned}$$

$$0 = -\frac{1}{2}\left(\bar{\psi}\Gamma_a^{b_1b_2}\psi^\alpha\nabla_\alpha\rho_{b_1b_2}\right)$$
$$= \underbrace{\left(\bar{\psi}\Gamma_a^{b_1b_2}\nabla_{[b_1}H_{b_2]}\psi\right)}_{(C)} - \underbrace{\left(\bar{\psi}\Gamma_a^{b_1b_2}H_{b_1}H_{b_2}\psi\right)}_{(\mathbf{B})} - \underbrace{\frac{1}{4}\frac{1}{2}R_{b_1b_2a_1a_2}\left(\bar{\psi}\Gamma_a^{b_1b_2}\Gamma^{a_1a_2}\psi\right)}_{(A)}$$

$$\frac{1}{4}R_{b_1b_2a_1a_2}(\bar{\psi}\Gamma_a{}^{b_1b_2}\Gamma^{a_1a_2}\psi) = \frac{1}{4}\overbrace{R^{b_1[b_2a_1a_2]}}^{=0(\bar{\psi}\mathfrak{R})}(\bar{\psi}\Gamma_{ab_1b_2a_1a_2}\psi) - \frac{1}{2}R^{b_1b_2}{}_{a_1a_2}(\delta^{a_1a_2}_{b_1b_2}\eta_{ac} - \delta^{a_1a_2}_{ab_2}\eta_{b_1c} + \delta^{a_1a_2}_{ab_1}\eta_{b_2c})(\bar{\psi}\Gamma^c\psi)$$
$$= -\frac{1}{2}(R^{b_1b_2}{}_{b_1b_2}\eta_{ac} - R_a{}^b{}_{cb} - R_a{}^b{}_{cb})(\bar{\psi}\Gamma^c\psi)$$
$$= \left(R_a{}^b{}_{cb} - \frac{1}{2}R^{b_1b_2}{}_{b_1b_2}\eta_{ac}\right)(\bar{\psi}\Gamma^c\psi)$$

$$(\bar{\psi}\Gamma_a^{b_1b_2}H_{b_1}H_{b_2}\psi)$$
$$= \left(\bar{\psi}\Gamma_a^{b_1b_2}\left(\frac{1}{6}\frac{1}{3!}(G_4)_{b_1c_1c_2c_3}\Gamma^{c_1c_2c_3} - \frac{1}{12}\frac{1}{4!}(G_4)^{c_1\cdots c_4}\Gamma_{b_1c_1\cdots c_4}\right)\left(\frac{1}{6}\frac{1}{3!}(G_4)_{b_2c_1c_2c_3}\Gamma^{c_1c_2c_3} - \frac{1}{12}\frac{1}{4!}(G_4)^{c_1\cdots c_4}\Gamma_{b_2c_1\cdots c_4}\right)\psi\right)$$
$$= -\frac{1}{24}\left((G_4)_ab_1b_2b_3(G_4)_c{}^{b_1b_2b_3} - \frac{1}{8}(G_4)_{b_1\cdots b_4}(G_4)^{b_1\cdots b_4}\eta_{ac}\right)(\bar{\psi}\Gamma^c\psi) + Q_{aa_1a_2}(\bar{\psi}\Gamma^{a_1a_2}\psi) + Q_{aa_1\cdots a_5}(\bar{\psi}\Gamma^{a_1\cdots a_5}\psi)$$

$$(\bar{\psi}\Gamma_a{}^{b_1b_2}\nabla_{[b_1}H_{b_2]}\psi) = \left(\bar{\psi}\Gamma_a{}^{b_1b_2}\nabla_{[b_1}\left(\frac{1}{6}\frac{1}{3!}(G_4)_{b2]c_1c_2c_3}\Gamma^{c_1c_2c_3} - \frac{1}{12}\frac{1}{4!}(G_4)^{c_1\cdots c_4}\Gamma_{b_2]c_1\cdots c_4}\right)\psi\right)$$
$$= Q'_{aa_1a_2}(\bar{\psi}\Gamma^{a_1a_2}\psi) + Q'_{aa_1\cdots a_5}(\bar{\psi}\Gamma^{a_1\cdots a_5}\psi)$$

$$R^{c_1c_2}{}_{c_1c_2} = +\frac{1}{24}\frac{1}{12}(G_4)_{c_1\cdots c_4}(G_4)^{c_1\cdots c_4}$$

$$(G_4^s, G_7^s): X \longrightarrow \Omega^1_{\mathrm{dR}}(-; \mathfrak{l}S^4)$$

$$(\eta_{\widetilde{X}})^*(G_4^s, G_7^s) \quad : \quad \widetilde{X} \xrightarrow{\eta_{\widetilde{X}}} X \xrightarrow{(G_4^s, G_7^s)} \Omega^1_{\mathrm{dR}}(-; \mathfrak{l}S^4)$$

$$\theta^\rho e^a_\rho = 0$$
$$\theta^\rho \psi^\alpha_\rho = 0$$
$$\theta^\rho \omega_\rho{}^a{}_b = 0$$

$$(G_4)_{a_1\cdots a_4}(x_0, \{\theta^\rho\}_{\rho=1}^{32}) = \underbrace{(\eta_{\widetilde{X}})^*(G_4)_{a_1\cdots a_4}(x_0)}_{\text{ordinary flux density}} + \underbrace{12\left(\bar{\theta}\Gamma_{[a_1a_2}\rho_{a_3a_4]}(x_0, \{\theta^\rho\}_{\rho=1}^{32})\right)}_{\text{its higher superfield components}}$$

$$\left(\nabla_{a_0}\left(\bar{\theta}\Gamma_{[a_1a_2}\rho_{a_3a_4]}\right)\right)e^{a_0}\cdots e^{a_4} = (\bar{\theta}\Gamma_{[a_1a_2}\nabla_{a_0}\rho_{a_3a_4]})e^{a_0}\cdots e^{a_4} \quad (\psi^0)$$
$$= 0$$

$$\psi^\alpha\nabla_\alpha\frac{1}{7!}(G_7)_{a_1\cdots a_7} = \frac{1}{7!\cdot 4!}\psi^\alpha\nabla_\alpha\epsilon_{a_1\cdots a_7b_1\cdot b_4}(G_4)^{b_1\cdots b_4}$$
$$= f\frac{1}{2\cdot 7!}\epsilon_{a_1\cdots a_7b_1\cdots b_4}\left(\bar{\psi}\Gamma^{[b_1b_2}\rho^{b_3b_4]}\right)$$
$$= \frac{84}{\underbrace{2\cdot 7!}_{1/5!}}\left(\bar{\psi}\Gamma_{[a_1\cdots a_5}\rho_{a_6a_7]}\right)$$

Modelo Unificado de Supergravedad Cuántica Relativista AdS/SCFT en supersimetría de calibre y superespacios compactificados.

$$\langle O_1 \cdots O_n \rangle \sim \int \mathcal{D}\phi O_1(\phi) \cdots O_n(\phi) e^{\frac{i}{\hbar}S(\phi)}$$

$$S(\phi) = \int d^D x \left(\frac{1}{2}\phi K \phi - V(\phi, \partial\phi) \right)$$

$$S_{EH}(g_{\mu\nu}) \sim \frac{1}{\kappa^2} \int d^4 x \sqrt{-g}(R(g) - \Lambda)$$

$$l_{\text{Planck}} = \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-35} \text{ m}$$

$$m_{\text{Planck}} = \sqrt{\frac{\hbar c}{G}} \sim 10^{19} Gev \sim 10^{-8} \text{Kg}$$

$$t_{\text{Planck}} = \sqrt{\frac{G\hbar}{c^5}} \sim 10^{-44} \text{ s}$$

$$\lambda_c(m) = \frac{2\pi\hbar}{c}\frac{1}{m}$$

$$r_s(m) = \frac{2G}{c^2} m$$

$$r_s(m_{\text{Planck}}) = \frac{1}{\pi} \lambda_c(m_{\text{Planck}}).$$

$$\alpha' \sim l_{\text{Planck}}^2$$

$$T_{\text{string}} = \frac{1}{2\pi\alpha'}$$

$$\alpha' m^2 \sim N - 1$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}.$$

$$\mathcal{A} \sim g \int d^D p \frac{1}{p^2 + m^2} = \text{ divergent}$$

$$V(\phi) \to V(\tilde{\phi}),$$

$$\tilde{\phi} = e^{\alpha' \Box} \phi \to e^{-\alpha' p^2} \phi(p),$$

$$\mathcal{A} \sim g \int d^D p \frac{e^{-2\alpha' p^2}}{p^2 + m^2} = \text{ convergent}$$

$$S' = -m\int d\tau\sqrt{-\dot{X}^2} = -m\int d\tau\sqrt{-\eta_{\mu\nu}\frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\tau}},$$

$$\tilde{X}^\mu(\tilde{\tau}) = X^\mu(\tau) \Rightarrow \frac{dX^\mu}{d\tau} = \frac{d\tilde{\tau}}{d\tau}\frac{d\tilde{X}^\mu}{d\tilde{\tau}}$$

$$S[\tilde{X}^\mu(\tilde{\tau})] = -m\int d\tilde{\tau}\sqrt{-\dot{\tilde{X}}^2} = -m\int d\tau\frac{d\tilde{\tau}}{d\tau}\sqrt{-\left(\frac{d\tau}{d\tilde{\tau}}\right)^2\dot{X}^2} = S[X^\mu(\tau)]$$

$$P_\mu = \frac{\partial\mathcal{L}}{\partial\dot{X}^\mu} = \frac{m\dot{X}^\mu}{\sqrt{-\dot{X}^2}}$$

$$P_\mu P^\mu + m^2 = 0$$

$$H = P_\mu\dot{X}^\mu - \mathcal{L} = 0$$

$$S[X,e] = \frac{1}{2}\int d\tau\left(\frac{\dot{X}^2}{e} - em^2\right)$$

$$S[X^\mu, g_{\tau\tau}] = -\frac{1}{2}\int d\tau\sqrt{-g}\left(g^{\tau\tau}\dot{X}^\mu\dot{X}_\mu + m^2\right)$$

$$g^{\tau\tau} = \frac{1}{g_{\tau\tau}}, g = \det g = g_{\tau\tau}$$

$$\tilde{\phi}(\tilde{\tau}) = \phi(\tau) + \delta\phi(\tau) + \dot{\phi}(\tau)\delta\tau \equiv \tilde{\phi}(\tau) + \dot{\phi}(\tau)\theta(\tau),$$

$$\tilde{e}(\tilde{\tau})d\tilde{\tau} = e(\tau)d\tau,$$

$$\begin{aligned}\delta e &= -\frac{d(e\theta)}{d\tau},\\ \delta X^\mu &= -\theta\dot{X}(\tau),\\ \delta\tau &= \theta(\tau).\end{aligned}$$

$$\begin{aligned}\frac{\delta S}{\delta e} &= 0 \Rightarrow \dot{X}^2 + e^2m^2 = 0,\\ \frac{\delta S}{\delta X^\mu} &= 0 \Rightarrow \frac{d}{d\tau}\frac{\dot{X}^\mu}{e} = 0,\end{aligned}$$

$$e = \hat{e} = 1 \text{ (fiducial value)}.$$

$$\delta e = -\partial_\tau(\theta e) = 1 - e.$$

$$\theta(\tau) = -\frac{1}{e}\left((\tau - \tau_0) - \int_{\tau_0}^{\tau} d\tau' e(\tau')\right)$$

$$S^{\text{gauge-fixed}}(X) = S(X,e)|_{e=1} = \frac{1}{2}\int d\tau(\dot{X}^2 - m^2)$$

$$\ddot{X}^{\mu} = 0.$$

$$P_{\mu} = \dot{X}_{\mu},$$

$$\dot{X}^2 + m^2 = 0 \leftrightarrow P^2 + m^2 = 0.$$

$$X^{\mu}(\tau) = x^{\mu} + p^{\mu}\tau$$
$$P^{\mu}(\tau) = \dot{X}^{\mu} = p^{\mu}$$

$$\left[\hat{X}^{\mu}(\tau), \hat{P}^{\nu}(\tau)\right] = i(\hbar)\eta^{\mu\nu},$$

$$[\hat{x}^{\mu}, \hat{p}^{\nu}] = i\eta^{\mu\nu}$$

$$\hat{p}^{\mu}|0,p\rangle = p^{\mu}|0,p\rangle$$
$$\langle 0,p \mid 0,p'\rangle = \delta^{d}(p-p')$$

$$|\Psi\rangle = \int d^{d}p\psi(p)|0,p\rangle$$

$$^{(\text{phys})}\langle 0,p'|(\hat{p}^2 + m^2)|0,p\rangle^{(\text{phys})} = 0,$$

$$(\hat{p}^2 + m^2) \mid \text{phys} \rangle = 0$$

$$\psi(p) = \int d^{d}x\tilde{\psi}(x)e^{-ip\cdot x}$$

$$(\Box - m^2)\tilde{\psi}(x) = 0,$$

$$\delta\phi_i = \xi^{a}\delta_{a}\phi_i$$

$$[\delta_a, \delta_b] = f_{ab}^{c}\delta_c$$

$$Z = \int \frac{\mathcal{D}\phi_i}{\text{Vol}(\mathcal{G})} e^{iS[\phi_i]}$$

$$\int \mathcal{D}\xi^{a}\delta(\xi^{a}) = 1$$

$$\int \mathcal{D}\xi^{a}\delta(\xi^{a}) = \int \mathcal{D}\xi^{a}\det\left[\frac{\delta F^{A}\left(\phi_i^{\xi}\right)}{\delta\xi^{a}}\right]_{\xi=0} \delta\left(F^{A}\left(\phi_i^{\xi}\right)\right) = 1$$

$$Z = \int \frac{\mathcal{D}\phi_i}{\text{Vol}(\mathcal{G})} e^{iS[\phi_i]} \int \mathcal{D}\xi^{a}\det\left[\frac{\delta F^{A}\left(\phi_i^{\xi}\right)}{\delta\xi^{a}}\right]_{\xi=0} \delta\left(F^{A}\left(\phi_i^{\xi}\right)\right)$$

$$Z = \int \mathcal{D}\xi^{a} \int \frac{\mathcal{D}\phi_i}{\text{Vol}(\mathcal{G})} e^{iS[\phi_i]}\det\left[\frac{\delta F^{A}\left(\phi_i^{\xi}\right)}{\delta\xi^{a}}\right]_{\xi=0} \delta\left(F^{A}(\phi_i)\right)$$

$$\delta\left(F^A(\phi_i)\right) = \int \mathcal{D}B_A e^{iB_A F^A(\phi)}$$

$$\int db_1 dc_1 \dots db_m dc_m e^{-b^\alpha M_{\alpha\beta} c^\beta} = \det M$$

$$\int d\theta\theta^n = \delta_{1,n}$$

$$\int d\theta_1 d\theta_2 \theta_2 \theta_1 = 1$$

$$Z = \int \mathcal{D}\phi_i \mathcal{D}B_A \mathcal{D}b_A \mathcal{D}c^a e^{i\left[S[\phi_i] + B_A F^A + ib_A \frac{\delta F^A}{\delta \xi^a} c^a\right]}$$

$$S_{\mathrm{FP}}[\phi_i, B_A, c_a, b_A] = S[\phi_i] + B_A F^A + ib_A \frac{\delta F^A}{\delta \xi^a} c^a$$

$$\begin{aligned} \delta_B \phi_i &= c^a \delta_a \phi \\ \delta_B B_A &= 0 \\ \delta_B b_A &= iB_A \\ \delta_B c^a &= \frac{1}{2} f^a_{bc} c^b c^c \end{aligned}$$

$$\delta_B^2 = 0$$

$$\begin{aligned} \delta_B^2 F[\varphi_I] &= \delta_B^2 \varphi_I \frac{\delta F}{\delta \varphi_I} + (-1)^{|\varphi_I|+1} \delta_B \varphi_I \delta_B \varphi_J \frac{\delta^2 F}{\delta \varphi_J \delta \varphi_I} \\ &= \frac{1}{2}\left[(-1)^{|\varphi_I|+1} \delta_B \varphi_I \delta_B \varphi_J \frac{\delta^2 F}{\delta \varphi_J \delta \varphi_I} + (-1)^{|\varphi_J|+1} \delta_B \varphi_J \delta_B \varphi_I \frac{\delta^2 F}{\delta \varphi_I \delta \varphi_J}\right] + \delta_B^2 \varphi_I \frac{\delta F}{\delta \varphi_I} \\ &= \frac{1}{2}\left[(-1)^{|\varphi_I|+1} \delta_B \varphi_I \delta_B \varphi_J \frac{\delta^2 F}{\delta \varphi_J \delta \varphi_I} + (-1)^{|\varphi_I|} \delta_B \varphi_I \delta_B \varphi_J \frac{\delta^2 F}{\delta \varphi_J \delta \varphi_I}\right] + \delta_B^2 \varphi_I \frac{\delta F}{\delta \varphi_I} \\ &\Rightarrow \delta_B^2 F[\varphi_I] = \delta_B^2 \varphi_I \frac{\delta F}{\delta \varphi_I} \end{aligned}$$

$$\begin{aligned} \delta_B^2 B_A &= \delta_B 0 = 0, \\ \delta_B^2 b_A &= i\delta_B B_A = 0, \\ \delta_B^2 \phi_i = \delta_B(c^a \delta_a \phi_i) &= \delta_B c^a \delta_a \phi_i - c^a \delta_B(\delta_a \phi_i) = \frac{1}{2} f^a_{bc} c^b c^c \delta_a \phi_i - c^a c^b \delta_a \delta_b \phi_i \\ &= \frac{1}{2} f^a_{bc} c^b c^c \delta_a \phi_i - \frac{1}{2}[c^a c^b \delta_a \delta_b \phi_i + c^b c^a \delta_b \delta_a \phi_i] \\ &= \frac{1}{2} f^a_{bc} c^b c^c \delta_a \phi_i - \frac{1}{2} c^a c^b [\delta_a, \delta_b] \phi_i = \frac{1}{2} f^a_{bc} c^b c^c \delta_a \phi_i - \frac{1}{2} f^c_{ab} c^a c^b \delta_c \phi_i \\ &= 0, \\ \delta_B^2 c^a = \frac{1}{2} f^a_{bc} \delta_B(c^b c^c) &= \frac{1}{2} f^a_{bc} \delta_B(c^b) c^c - \frac{1}{2} f^a_{bc} c^b \delta_B(c^c) \\ &= \frac{1}{4} f^a_{bc} f^b_{de} c^d c^e c^c - \frac{1}{4} f^a_{bc} f^c_{de} c^b c^d c^e = \frac{1}{4} f^a_{bc} f^b_{de} c^d c^e c^c - \frac{1}{4} f^a_{cb} f^b_{de} c^c c^d c^e \\ &= \frac{1}{2} f^a_{bc} f^b_{de} c^c c^d c^e = \frac{1}{6} f^a_{b[c} f^b_{de]} c^c c^d c^e \\ &= 0, \end{aligned}$$

$$\delta_B(-ib_AF^A) = B_AF^A + ib_A\delta_BF^A = B_AF^A + ib_A\delta_B\phi_i\frac{\delta F^A}{\delta\phi_i}$$
$$= B_AF^A + ib_A\delta_a\phi_i\frac{\delta F^A}{\delta\phi_i}c^a = B_AF^A + ib_A\frac{\delta F^A}{\delta\xi^a}c^a$$

$$\delta_BS_{FP} = \delta_B[S[\phi_i] + \delta_B(-ib_AF^A)] = \delta_BS[\phi_i] + \delta_B^2(-ib_AF^A) = c^a\delta_aS[\phi_i] = 0,$$

$$\delta_B\varphi_i = i[\varphi_i, Q_B] = i(-1)^{|\varphi|+1}[Q_B, \varphi_i],$$

$$\begin{aligned}
0 &= \delta_B^2\varphi_i = \left[Q_B, [Q_B, \varphi_i]\right] \\
&= Q_B\left(Q_B\varphi_i - (-1)^{|\varphi_i|}\varphi_iQ_B\right) - (-1)^{|\varphi_i|+1}\left(Q_B\varphi_i - (-1)^{|\varphi_i|}\varphi_iQ_B\right)Q_B \\
&= Q_B^2\varphi_i - \varphi_iQ_B^2 = [Q_B^2, \varphi_i]\ \forall\varphi_i \\
&\Longrightarrow Q_B^2 = 0
\end{aligned}$$

$$|\Psi\rangle \sim |\Psi\rangle + Q_B|\Phi\rangle, |\Psi\rangle \in \mathrm{Ker}Q_B,$$

$$\mathrm{H(Q_B)} = \mathrm{Ker}Q_B/\mathrm{Im}Q_B.$$

$$\langle f \mid i\rangle = \int \mathcal{D}\varphi\varphi_f\varphi_ie^{iS_{GF}}$$

$$\begin{aligned}
\delta_{GF}\langle f \mid i\rangle &= \int \mathcal{D}\varphi\varphi_f\varphi_ie^{i\left(S_{GF}+\delta_B(-ib_A\delta F^A)\right)} - \int \mathcal{D}\varphi\varphi_f\varphi_ie^{iS_{GF}} \\
&= \int \mathcal{D}\varphi\varphi_f\delta_B(b_A\delta F^A)\varphi_ie^{-S_{GF}} = \langle f|\delta_B(b_A\delta F^A)|i\rangle \\
&= i\langle f|[b_A\delta F^A, Q_B]|i\rangle = 0
\end{aligned}$$

$$Q_B \mid \mathrm{phys}\ \rangle = 0$$

$$\langle E \mid \phi\rangle = \langle\Lambda|Q_B|\phi\rangle = 0, Q_B|\phi\rangle = 0$$

$$S[X^\mu, e] = \frac{1}{2}\int d\tau\left(\frac{\dot{X}^2}{e} - m^2e\right)$$

$$\begin{aligned}
\delta X^\mu &= -\theta(\tau)\partial_\tau X^\mu \\
\delta e &= -\partial_\tau(e\theta(\tau))
\end{aligned}$$

$$\begin{aligned}
\phi_i &\to \{X^\mu(\tau), e(\tau)\} \\
\xi^a &\to \theta(\tau')
\end{aligned}$$

$$\begin{aligned}
\xi^a\delta_aX^\mu(\tau) &\to \int d\tau'\theta(\tau')\delta_{\tau'}X^\mu(\tau) = -\theta(\tau)\partial_\tau X^\mu(\tau) \\
&\Longrightarrow \delta_{\tau'}X^\mu(\tau) = -\delta(\tau-\tau')\partial_\tau X^\mu(\tau) \\
\xi^a\delta_ae(\tau) &\to \int d\tau'\theta(\tau')\delta_{\tau'}e(\tau) = -\frac{d(\theta e)}{d\tau}(\tau) \\
&\Longrightarrow \delta_{\tau'}e(\tau) = \delta'(\tau'-\tau)e(\tau')
\end{aligned}$$

$$[\delta_{\tau_1}, \delta_{\tau_2}]X^\mu(\tau) = [\delta(\tau-\tau_1)\partial_\tau\delta(\tau-\tau_2) - \delta(\tau-\tau_2)\partial_\tau\delta(\tau-\tau_1)]\partial_\tau X^\mu(\tau)$$
$$= \int d\tau_3 f^{\tau_3}_{\tau_1\tau_2}\delta_{\tau_3}X^\mu(\tau)$$
$$\Longrightarrow f^{\tau_3}_{\tau_1\tau_2} = \delta(\tau_3-\tau_1)\delta'(\tau_3-\tau_2) - \delta(\tau_3-\tau_2)\delta'(\tau_3-\tau_1)$$

$$f^{\tau_3}_{\tau_1\tau_2} = \delta'(\tau_1-\tau_2)\big(\delta(\tau_1-\tau_3)+\delta(\tau_2-\tau_3)\big)$$

$$\delta_B X^\mu = [c^a\delta_a X^\mu] = \int d\tau_1 c(\tau_1)\delta_{\tau_1}X^\mu(\tau) = -c\dot{X}^\mu$$
$$\delta_B e = [c^a\delta_a e] = \int d\tau_1 c(\tau_1)\delta_{\tau_1}e(\tau) = -\frac{d(ce)}{d\tau}$$
$$\delta_B b = iB$$
$$\delta_B c = \left[\frac{1}{2}f^a{}_{bc}c^b c^c\right] = \frac{1}{2}\int d\tau_1 d\tau_2 f^\tau{}_{\tau_1\tau_2}c(\tau_1)c(\tau_2)$$

$$= \frac{1}{2}\int d\tau_1 d\tau_2[\delta(\tau-\tau_1)\delta'(\tau-\tau_2) - \delta(\tau-\tau_2)\delta'(\tau-\tau_1)]c(\tau_1)c(\tau_2)$$
$$= \frac{1}{2}[-c\dot{c}+\dot{c}c] = -c\dot{c}$$

$$S_{GF} = S[X^\mu, e] + B_A F^A + i b_A \delta_a F^A c^a$$
$$= S[X^\mu, e] + \int d\tau B(\tau)(e(\tau)-1) + i\int d\tau b(\tau)c^a\delta_a(e(\tau)-1)$$
$$= \int d\tau\left[\frac{1}{2}\left(\frac{\dot{X}^2}{e} - em^2\right) + B(e-1) - ib\frac{d(ce)}{d\tau}\right]$$
$$= \int d\tau\left[\frac{1}{2}\left(\frac{\dot{X}^2}{e} - em^2\right) + B(e-1) + i\dot{b}ce\right].$$

$$Z = \int \mathcal{D}X^\mu\mathcal{D}e\mathcal{D}B\mathcal{D}b\mathcal{D}c e^{i\int d\tau\left[\frac{1}{2}\left(\frac{\dot{X}^2}{e}-em^2\right)+B(e-1)+i\dot{b}ce\right]}$$
$$= \int \mathcal{D}X^\mu\mathcal{D}e\mathcal{D}b\mathcal{D}c e^{i\int d\tau\left[\frac{1}{2}\left(\frac{\dot{X}^2}{e}-em^2\right)+i\dot{b}ce\right]}\delta(e-1)$$
$$= \int \mathcal{D}X^\mu\mathcal{D}b\mathcal{D}c e^{i\int d\tau\left[\frac{1}{2}(\dot{X}^2-m^2)+i\dot{b}c\right]}$$

$$S_R[X,b,c] = \int d\tau\left[\frac{1}{2}(\dot{X}^2-m^2) + i\dot{b}c\right]$$

$$\left.\frac{\delta S_{GF}}{\delta e}\right|_{e=1} = \left[B + i\dot{b}c - \frac{1}{2}\left(\frac{\dot{X}^2}{e^2}+m^2\right)\right]_{e=1}$$
$$= B + i\dot{b}c - \frac{1}{2}(\dot{X}^2+m^2) = 0$$
$$\Longrightarrow B = \frac{1}{2}(\dot{X}^2+m^2) - i\dot{b}c$$

$$\delta_B X^\mu = -c\dot{X}^\mu$$
$$\delta_B c = -c\dot{c}$$
$$\delta_B b = -iB = i\left(\frac{1}{2}(\dot{X}^2+m^2) - i\dot{b}c\right)$$

$$\frac{\delta S}{\delta b} = \dot{c} = 0 \implies c(\tau) = c_0,$$
$$\frac{\delta S}{\delta c} = \dot{b} = 0 \implies b(\tau) = b_0,$$
$$\frac{\delta S}{\delta X^\mu} = \ddot{X}^\mu = 0 \implies X^\mu(\tau) = x^\mu + p^\mu \tau.$$

$$P_\mu(\tau) = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = \dot{X}_\mu = p_\mu$$
$$P_c(\tau) = \frac{\partial \mathcal{L}}{\partial \dot{c}} = -ib = -ib_0$$
$$P_b(\tau) = \frac{\partial \mathcal{L}}{\partial \dot{b}} = -ic = -ic_0$$

$$[X^\mu(\tau), P_\nu(\tau)] = [x^\mu, p_\nu] = i\delta^\mu_\nu$$
$$[c, b] = [c_0, b_0] = 1$$

$$x^\dagger = x, P^\dagger = P, c_0^\dagger = c_0, b_0^\dagger = b_0.$$

$$c_0|\uparrow\rangle = c_0^2|\downarrow\rangle = 0$$

$$\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 0, \langle\downarrow|\uparrow\rangle = \langle\uparrow|\downarrow\rangle = 1.$$

$$\langle\downarrow|c_0|\downarrow\rangle = \langle\uparrow|b_0|\uparrow\rangle = 1.$$

$$[Q_B, c_0] = [Q_B, p_\mu] = 0, [Q_B, b_0] = \frac{1}{2}(p^2 + m^2)$$

$$0 = 2Q_B|\downarrow, p\rangle = (p^2 + m^2)|\uparrow, p\rangle \to p^2 + m^2 = 0,$$
$$0 = 2Q_B|\uparrow, p\rangle = c_0^2(p^2 + m^2)|\downarrow, p\rangle \to \text{ identically zero.}$$

$$\text{Ker}Q_B = \text{Span}\{|\uparrow, p\rangle\} \cup \text{Span}\{|\downarrow, p\rangle|_{p^2+m^2=0}\}$$

$$\#\,|\downarrow, p\rangle = 0\,, \quad \#\,|\uparrow, p\rangle = 1$$

$$\#\,|\downarrow, p\rangle = \#\,(Q_B\,|\Lambda\rangle) = \#Q_B + \#\,|\Lambda\rangle \;\to\; 0 = 1 + \#\,|\Lambda\rangle \;\to\; \#\,|\Lambda\rangle = -1$$

$$2Q|\downarrow, p\rangle = (p^2 + m^2)|\uparrow, p\rangle,$$

$$|\uparrow, p\rangle = Q\frac{2}{p^2 + m^2}|\downarrow, p\rangle, (p^2 + m^2 \neq 0).$$

$$\text{H}(Q_B) = \{|\downarrow, p\rangle \oplus |\uparrow, p\rangle\}_{p^2+m^2=0}$$

$$|\psi\rangle = \int d^D p \psi(p)|\downarrow, p\rangle,$$

$$Q|\psi\rangle = 0 \to (p^2 + m^2)\psi(p) = 0.$$

$$(\Box - m^2)\tilde{\psi}(x) = 0.$$

$$S[\psi] = \frac{1}{2}\langle\psi|Q|\psi\rangle$$

$$\begin{aligned} S[\psi] &= \frac{1}{2}\int d^d p_1 d^d p_2 \langle\downarrow, p_1|Q|\downarrow, p_2\rangle \psi^\dagger(p_1)\psi(p_2) \\ &= \frac{1}{2}\int d^d p_1 d^d p_2 \psi^\dagger(p_1)\langle\downarrow, p_1 \,|\uparrow, p_2\rangle(p_2^2 + m^2)\psi(p_2) \\ &= \frac{1}{2}\int d^d p \psi^\dagger(p)(p^2 + m^2)\psi(p) \\ &= \frac{1}{2}\int d^d x \psi^\dagger(x)(-\Box + m^2)\psi(x) \end{aligned}$$

$$Q_B \sim c \cdot (\text{ constraint }) = c_0(p^2 + m^2)$$

$$[\mathcal{F}_j, \mathcal{F}_k] = \gamma^i{}_{jk}\mathcal{F}_i$$

$$[b_i, c^j] = \delta_i^j$$

$$\tilde{\mathcal{F}}_i = -\gamma_{ij}{}^k c^j b_k,$$

$$\begin{gathered} [\tilde{\mathcal{F}}_i, c^l] = -\gamma_{ij}{}^k[c^j b_k, c^l] = -\gamma_{ij}{}^k c^j[b_k, c^l] = -\gamma_{ij}{}^k c^j \delta_k^l = -\gamma_{ij}{}^l c^j, \\ [\tilde{\mathcal{F}}_i, b_l] = -\gamma_{ij}{}^k[c^j b_k, b_l] = \gamma_{il}{}^k b_k. \end{gathered}$$

$$[\tilde{\mathcal{F}}_i, \tilde{\mathcal{F}}_j] = \gamma_{ij}{}^k\tilde{\mathcal{F}}_k.$$

$$\begin{aligned} [\tilde{\mathcal{F}}_i, \tilde{\mathcal{F}}_j] &= [\tilde{\mathcal{F}}_i, -\gamma_{jk}{}^l c^k b_l] = -\gamma_{jk}{}^l([\tilde{\mathcal{F}}_i, c^k]b_l + c^k[\tilde{\mathcal{F}}_i, b_l]) \\ &= -\gamma_{jk}{}^l(-\gamma_{im}{}^k c^m b_l + \gamma_{il}{}^m c^k b_m) \\ &= \gamma_{jk}{}^l\gamma_{im}{}^k c^m b_l - \gamma_{jk}{}^l\gamma_{il}{}^m c^k b_m \\ &= (\gamma_{jk}{}^l\gamma_{im}{}^k - \gamma_{jk}{}^l\gamma_{jm}{}^k)c^m b_l \\ &= -\gamma_{ij}{}^k\gamma_{km}{}^l c^m b_l = \gamma_{ij}{}^k\tilde{\mathcal{F}}_k \end{aligned}$$

$$\gamma_{jk}^l\gamma_{im}^k + \gamma_{ik}^l\gamma_{mj}^k + \gamma_{mk}^l\gamma_{ji}^k = 0.$$

$$Q = c^i\mathcal{F}_i + \frac{1}{2}c^i\tilde{\mathcal{F}}_i = c^i\mathcal{F}_i - \frac{1}{2}\gamma_{ij}{}^k c^i c^j b_k$$

$$\begin{aligned} [Q, c^l] &= \frac{1}{2}c^i[\tilde{\mathcal{F}}_i, c^l] = -\frac{1}{2}\gamma_{ij}{}^l c^i c^j, \\ [Q, b_l] &= \mathcal{F}_i[c^i, b_l] + \frac{1}{2}c^i[\tilde{\mathcal{F}}_i, b_l] + \frac{1}{2}[c^i, b_l]\tilde{\mathcal{F}}_i \\ &= \mathcal{F}_l + \frac{1}{2}\gamma_{li}{}^k c^i b_k + \frac{1}{2}\tilde{\mathcal{F}}_l = \mathcal{F}_l + \tilde{\mathcal{F}}_l = \mathcal{F}_l^{\text{tot}}. \end{aligned}$$

$$
\begin{aligned}
Q^2 =& \frac{1}{2}[Q,Q] = \frac{1}{2}\left[c^i\mathcal{F}_i + \frac{1}{2}c^n\tilde{\mathcal{F}}_n, c^j\mathcal{F}_j + \frac{1}{2}c^m\tilde{\mathcal{F}}_m\right] \\
=& \frac{1}{2}\left[c^i\mathcal{F}_i, c^j\mathcal{F}_j\right] + \frac{1}{4}\left[c^i\mathcal{F}_i, c^m\tilde{\mathcal{F}}_m\right] + \frac{1}{4}\left[c^n\tilde{\mathcal{F}}_n, c^j\mathcal{F}_j\right] + \frac{1}{8}\left[c^n\tilde{\mathcal{F}}_n, c^m\tilde{\mathcal{F}}_m\right] \\
=& \frac{1}{2}c^ic^j\left[\mathcal{F}_i, \mathcal{F}_j\right] + \frac{1}{2}c^n\left[\tilde{\mathcal{F}}_n, c^i\right]\mathcal{F}_i + \frac{1}{8}c^nc^m\left[\tilde{\mathcal{F}}_n, \tilde{\mathcal{F}}_m\right] \\
&+ \frac{1}{8}c^n\left[\tilde{\mathcal{F}}_n, c^m\right]\tilde{\mathcal{F}}_m + \frac{1}{8}c^m\left[c^n, \tilde{\mathcal{F}}_m\right]\tilde{\mathcal{F}}_n \\
=& \frac{1}{2}c^ic^j\gamma_{ij}{}^k\mathcal{F}_k - \frac{1}{2}c^n\gamma_{nj}{}^ic^j\mathcal{F}_i + \frac{1}{8}c^nc^m\gamma_{nm}{}^k\tilde{\mathcal{F}}_k \\
=& \frac{1}{8}\gamma_{nm}{}^k\gamma_{ki}{}^jc^nc^mc^ib_j = -\frac{1}{8}\gamma_{[nm}{}^k\gamma_{i]k}{}^jc^nc^mc^ib_j = 0,
\end{aligned}
$$

$$X^\mu : (\tau,\sigma) \longrightarrow X^\mu(\tau,\sigma) \in \mathbb{R}^{1,D-1}.$$

$$S_{\mathrm{NG}}[X] = -T\int_{\mathrm{WS}} dA,$$

$$T = \frac{1}{2\pi\alpha'},$$

$$
\begin{aligned}
G_{\alpha\beta} &= \eta_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta} = \begin{pmatrix} G_{\tau\tau} & G_{\tau\sigma} \\ G_{\sigma\tau} & G_{\sigma\sigma} \end{pmatrix} \\
G &= \det\left(G_{\alpha\beta}\right) = G_{\tau\tau}G_{\sigma\sigma} - (G_{\tau\sigma})^2
\end{aligned}
$$

$$dA = \sqrt{-G}\,d^2\sigma.$$

$$S_{\mathrm{NG}}[X] = -T\int_{\mathrm{WS}} d^2\sigma\sqrt{-G} = -T\int_{\mathrm{WS}} d^2\sigma\sqrt{-\dot{X}^2(X')^2 + (\dot{X}\cdot X')^2},$$

$$\cdot \equiv \frac{\partial}{\partial\tau},\ ' \equiv \frac{\partial}{\partial\sigma}$$

$$S_{\mathrm{P}}[X,h] = -\frac{1}{4\pi\alpha'}\int_{\mathrm{WS}} d^2\sigma\sqrt{-h}h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu}$$

$$
\begin{aligned}
\delta_h S_{\mathrm{P}} &= -\frac{1}{4\pi\alpha'}\int_{\mathrm{WS}} d^2\sigma\left[\delta_h(\sqrt{-h})h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu} + \sqrt{-h}\delta_h\left(h^{\alpha\beta}\right)\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu}\right] \\
&= -\frac{1}{4\pi\alpha'}\int_{\mathrm{WS}} d^2\sigma\sqrt{-h}\delta_h h^{\alpha\beta}\left[-\frac{1}{2}h_{\alpha\beta}\partial_\gamma X^\mu\partial^\gamma X_\mu + \partial_\alpha X^\mu\partial_\beta X_\mu\right] \\
&= \frac{1}{4\pi}\int_{\mathrm{WS}} d^2\sigma\sqrt{-h}\delta_h h^{\alpha\beta}T_{\alpha\beta}
\end{aligned}
$$

$$h^{\alpha\beta}h_{\alpha\beta}=2$$
$$\delta_h(h^{\alpha\beta}h_{\alpha\beta})=0=\delta_h(h^{\alpha\beta})h_{\alpha\beta}+h^{\alpha\beta}\delta_h(h_{\alpha\beta})$$
$$\Longrightarrow\delta_h(h^{\alpha\beta})h_{\alpha\beta}=-h^{\alpha\beta}\delta_h(h_{\alpha\beta})$$
$$\delta_h h=\delta_h\left(\det(h_{\alpha\beta})\right)=\delta_h\left(e^{\mathrm{Tr}\{\log(h_{\alpha\beta})\}}\right)=\det(h_{\alpha\beta})\mathrm{Tr}\left\{\delta_h\left(\log\left(h_{\alpha\beta}\right)\right)\right\}$$
$$=\det(h_{\alpha\beta})\mathrm{Tr}\left\{\frac{\delta_h(h_{\alpha\beta})}{\det(h_{\alpha\beta})}\right\}=hh^{\alpha\beta}\delta_h(h_{\alpha\beta})$$
$$=-hh_{\alpha\beta}\delta_h(h^{\alpha\beta})$$

$$\alpha' T_{\alpha\beta}=\frac{1}{2}h_{\alpha\beta}\partial_\gamma X^\mu\partial^\gamma X_\mu-\partial_\alpha X^\mu\partial_\beta X_\mu$$

$$=\frac{1}{2}h_{\alpha\beta}h^{\gamma\delta}G_{\gamma\delta}-G_{\alpha\beta}.$$

$$T_{\alpha\beta}=0$$

$$G_{\alpha\beta}=\left(\frac{1}{2}h^{\gamma\delta}G_{\gamma\delta}\right)h_{\alpha\beta},$$

$$h_{\alpha\beta}=\left(\frac{1}{2}h^{\gamma\delta}G_{\gamma\delta}\right)^{-1}G_{\alpha\beta}$$
$$\Longrightarrow h=\left(\frac{1}{2}h^{\gamma\delta}G_{\gamma\delta}\right)^{-2}\det(G_{\alpha\beta})=\left(\frac{1}{2}h^{\gamma\delta}G_{\gamma\delta}\right)^{-2}G$$
$$\Longrightarrow\sqrt{-h}=\left(\frac{1}{2}h^{\alpha\beta}G_{\alpha\beta}\right)^{-1}\sqrt{-G}$$

$$S_{\mathrm{P}}=-\frac{1}{4\pi\alpha'}\int_{\mathrm{WS}}d^2\sigma\sqrt{-G}\left(\frac{1}{2}h^{\gamma\delta}G_{\gamma\delta}\right)^{-1}h^{\alpha\beta}G_{\alpha\beta}$$
$$=-\frac{1}{2\pi\alpha'}\int_{\mathrm{WS}}d^2\sigma\sqrt{-G}$$
$$=S_{\mathrm{NG}}$$

$$\alpha' T^\alpha_\alpha=-(\partial X\cdot\partial X)+\frac{1}{2}h^\alpha_\alpha(\partial X\cdot\partial X)$$
$$=-(\partial X\cdot\partial X)+\frac{1}{2}2(\partial X\cdot\partial X)=0$$

$$T^\alpha{}_\alpha=0$$

$$\delta_X S_{\mathrm{P}}=-\frac{1}{2\pi\alpha'}\int_{\mathrm{WS}}d^2\sigma\sqrt{-h}h^{\alpha\beta}\partial_\alpha\delta_X X^\mu\partial_\beta X_\mu$$
$$\overset{\mathrm{IBP}}{=}\frac{1}{2\pi\alpha'}\int_{\mathrm{WS}}d^2\sigma\left[\delta_X X^\mu\partial_\alpha\left(\sqrt{-h}h^{\alpha\beta}\partial_\beta X_\mu\right)-\partial_\alpha\left(\sqrt{-h}h^{\alpha\beta}\delta_X X^\mu\partial_\beta X_\mu\right)\right]$$

$$\begin{cases}\partial_\alpha\left(\sqrt{-h}h^{\alpha\beta}\partial_\beta X_\mu\right)=0\\ \left[\sqrt{-h}h^{\alpha\beta}\delta_X X_\mu\partial_\beta X_\mu\right]_{\partial(\mathrm{WS})}=0\end{cases}$$

$$\int_{\mathrm{WS}} d^2\sigma = \int_0^{\pi} d\sigma \int_{-\infty}^{+\infty} d\tau$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^{\pi} d\sigma \partial_\sigma\left[\delta X^\mu \sqrt{-h} h^{\sigma\beta} \partial_\beta X_\mu\right] =$$
$$= \int_{-\infty}^{+\infty} d\tau \sqrt{-h} h^{\sigma\beta}\left[\delta X^\mu \partial_\beta X_\mu\right]_{\sigma=0}^{\sigma=\pi} = 0$$

$$\begin{aligned} n^\beta &= h^{\sigma\beta} \\ t_\beta &= h_{\tau\beta} \\ t_\beta n^\beta &= \delta_\tau^\sigma = 0 \end{aligned}$$

$$\begin{cases} \partial_\alpha\left(\sqrt{-h} h^{\alpha\beta} \partial_\beta X_\mu\right) & = 0 \quad \text{(WS bulk eq.)} \\ \delta X^\mu\left(n^\alpha \partial_\alpha X_\mu\right)\big|_{\partial\,(\mathrm{WS})} & = 0 \quad \text{(WS boundary eq.)} \end{cases},$$

$$\begin{cases} X'(\sigma') = X(\sigma) \\ \dfrac{\partial \sigma'^\gamma}{\partial \sigma^\alpha} \dfrac{\partial \sigma'^\delta}{\partial \sigma^\beta} h'_{\gamma\delta}(\sigma') = h_{\alpha\beta}(\sigma) \end{cases};$$

$$h'_{\alpha\beta}(\sigma) = e^{2\omega(\sigma)} h_{\alpha\beta}(\sigma).$$

$$\left(h^{\alpha\beta}\right)' = e^{-2\omega(\sigma)} h^{\alpha\beta}(\sigma),$$
$$h' = \det\left(h'_{\alpha\beta}\right) = e^{4\omega(\sigma)} h \implies \sqrt{-h'} = e^{2\omega(\sigma)}\sqrt{-h},$$

$$\sqrt{-h'} h'^{\alpha\beta} = \sqrt{-h} h^{\alpha\beta}.$$

$$h'_{\alpha\beta} = e^{2\omega} h_{\alpha\beta} = \left(1 + 2\omega + \mathcal{O}(\omega^2)\right) h_{\alpha\beta},$$

$$\delta_\omega h_{\alpha\beta} = 2\omega h_{\alpha\beta}$$

$$\delta_\omega S_{\mathrm{P}} \propto \int_{\mathrm{WS}} d^2\sigma \sqrt{-h}\, \delta_\omega h^{\alpha\beta} T_{\alpha\beta} = 0$$
$$\implies h^{\alpha\beta} T_{\alpha\beta} = T_\alpha^\alpha = 0$$

$$\begin{cases} X'\mu = \Lambda_\nu^\mu X^\nu + a^\nu \\ h'_{\alpha\beta} = h_{\alpha\beta} \end{cases}.$$

$$\hat{h}_{\alpha\beta} = e^{-\phi} \eta_{\alpha\beta},$$
$$\hat{h}^{\alpha\beta} = e^{\phi} \eta^{\alpha\beta},$$

$$\sqrt{-\hat{h}} = e^{-\phi}\sqrt{-\eta} = e^{-\phi}$$

$$\hat{h}^{\alpha\beta}\sqrt{-\hat{h}} = \eta^{\alpha\beta}.$$

$$S[\hat{h}, \hat{X}] = -\frac{1}{4\pi\alpha'} \int_{\mathrm{WS}} d^2\sigma \eta^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu$$

$$\sigma^\pm = \tau \pm \sigma.$$

$$d^2\sigma = \frac{1}{2} d\sigma^+ d\sigma^-$$

$$\tau = \frac{\sigma^+ + \sigma^-}{2}, \sigma = \frac{\sigma^+ - \sigma^-}{2}.$$

$$\eta_{++} = \eta_{--} = 0, \qquad \eta^{++} = \eta^{--} = 0$$
$$\eta_{+-} = \eta_{-+} = -\frac{1}{2}, \quad \eta^{+-} = \eta^{-+} = -2$$

$$\eta = \begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}, \eta^{-1} = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}.$$

$$S = \frac{1}{2\pi\alpha'} \int_{\mathrm{WS}} d\sigma^+ d\sigma^- \partial_+ X^\mu \partial_- X_\mu$$

$$\begin{aligned} \sigma'^+ &= f(\sigma^+) = f^+ \\ \sigma'^- &= g(\sigma^-) = g^- \end{aligned}$$

$$-d\sigma'^- d\sigma'^+ = -(\partial_+ f_+ \partial_- g_-) df^+ dg^-$$

$$\begin{cases} \partial_+ \partial_- X^\mu(\sigma^+, \sigma^-) & = 0 \quad \text{(WS bulk eq.)} \\ \delta X_\mu \partial_\sigma X^\mu \big|_{\sigma=0,\pi} & = 0 \quad \text{(WS boundary eq.)} \end{cases}.$$

$$T_{\alpha\beta} = \frac{4\pi}{\sqrt{-h}} \frac{\delta S}{\delta h^{\alpha\beta}} = \frac{1}{2\alpha'} h_{\alpha\beta} \partial_\gamma X^\mu \partial^\gamma X_\mu - \frac{1}{\alpha'} \partial_\alpha X^\mu \partial_\beta X_\mu$$

$$\begin{aligned} T_{++} &= -\frac{1}{\alpha'} (\partial_+ X^\mu \partial_+ X_\mu), \\ T_{--} &= -\frac{1}{\alpha'} (\partial_- X^\mu \partial_- X_\mu), \\ \alpha' T_{+-} &= \alpha' T_{-+} = -\partial_+ X^\mu \partial_- X^\mu + \frac{1}{2} 2(\partial_+ X^\mu \partial_- X^\mu) = 0 \end{aligned}$$

$$\partial^\alpha T_{\alpha\beta} = 0 \Leftrightarrow \begin{cases} \partial_+ T_{--} = 0 \\ \partial_- T_{++} = 0 \end{cases}$$

$$\begin{cases} T_{++} = T_{++}(\sigma^+) \\ T_{--} = T_{--}(\sigma^-) \end{cases}$$

$$\partial_+ \partial_- X^\mu(\sigma^+, \sigma^-) = 0$$

$$X^\mu(\sigma^+, \sigma^-) = X_L^\mu(\sigma^+) + X_R^\mu(\sigma^-)$$

$$X_L^\mu(\sigma^+) = \frac{1}{2}(X_0^\mu + c^\mu) + \frac{\alpha'}{2} P^\mu \sigma^+ + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{\alpha_n^\mu}{n} e^{-in\sigma^+}$$

$$X_R^\mu(\sigma^-) = \frac{1}{2}\left(X_0^\mu - c^\mu\right) + \frac{\alpha'}{2}P^\mu\sigma^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n^\mu}{n}e^{-in\sigma^-}$$

$$X^\mu(\sigma^+,\sigma^-) = X_0^\mu + \alpha' P^\mu\tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^\mu e^{-in\sigma^+} + \tilde{\alpha}_n^\mu e^{-in\sigma^-}\right)$$

$$X^\mu = (X^\mu)^* \Leftrightarrow \overset{(\sim)}{\alpha}_{-n} = \left(\overset{(\sim)}{\alpha}_n\right)^*$$

$$\mathcal{P}^\mu(\tau,\sigma) = \frac{\partial\mathcal{L}}{\partial\dot{X}_\mu} = \frac{1}{2\pi\alpha'}\dot{X}^\mu$$
$$= \frac{P^\mu}{2\pi} + \frac{1}{2\pi\sqrt{2\alpha'}}\sum_{n\neq 0}\left(\alpha_n^\mu e^{-in(\tau+\sigma)} + \tilde{\alpha}_n^\mu e^{-in(\tau-\sigma)}\right)$$

$$\int_0^{2\pi} d\sigma\mathcal{P}^\mu = P^\mu$$

$$[X^\mu(\tau,\sigma),\mathcal{P}^\nu(\tau,\sigma')] = i\eta^{\mu\nu}\delta(\sigma-\sigma')$$

$$\left[\alpha_n^\mu,\alpha_m^\nu\right] = \left[\tilde{\alpha}_n^\mu,\tilde{\alpha}_m^\nu\right] = n\eta^{\mu\nu}\delta_{n+m,0}$$
$$\left[\alpha_n^\mu,\tilde{\alpha}_m^\nu\right] = 0$$
$$\left[X_0^\mu,P^\nu\right] = i\eta^{\mu\nu}$$

$$a_n^\mu = \frac{1}{\sqrt{n}}\alpha_n^\mu \text{ for } n > 0$$
$$a_n^{\mu\dagger} = \frac{1}{\sqrt{n}}\alpha_{-n}^\mu \text{ for } n > 0$$

$$\left[a_n^\mu,a_m^{\nu\dagger}\right] = \eta^{\mu\nu}\delta_{m,n}$$

$$\alpha_n|0\rangle = 0, \langle 0|\alpha_{-n} = 0, \forall n > 0$$

$$\left(\alpha_n^\mu\right)^\dagger = \alpha_{-n}^\mu$$

$$\alpha_{-n_1}^{\mu_1}\ldots\alpha_{-n_k}^{\mu_k}\tilde{\alpha}_{-m_1}^{\nu_1}\ldots\tilde{\alpha}_{-m_k}^{\nu_k}|0,P\rangle$$

$$T_{++} = -\frac{1}{\alpha'}\partial_+X^\mu\partial_+X_\mu = T_{++}(\sigma^+)$$

$$T_{--} = -\frac{1}{\alpha'}\partial_-X^\mu\partial_-X_\mu = T_{--}(\sigma^-)$$

$$\begin{cases} T_{++}(\sigma^+) = -\sum_{n\in\mathbb{Z}} L_n e^{-in\sigma^+} \\ T_{--}(\sigma^-) = -\sum_{n\in\mathbb{Z}} \tilde{L}_n e^{-in\sigma^-} \end{cases}$$

$$L_n = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{n-k}^{\mu}\alpha_k^{\nu}\eta_{\mu\nu}, \tilde{L}_n = \frac{1}{2}\sum_{k\in\mathbb{Z}} \tilde{\alpha}_{n-k}^{\mu}\tilde{\alpha}_k^{\nu}\eta_{\mu\nu}$$

$$\alpha_0^{\mu} = \tilde{\alpha}_0^{\mu} = \sqrt{\frac{\alpha'}{2}}P^{\mu}$$

$$L_{n\neq 0} = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{n-k}\cdot\alpha_k,$$
$$L_0 \sim \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{-k}\cdot\alpha_k,$$

$$[\alpha_{n-k}^{\mu}, \alpha_k^{\nu}] = 0$$
$$[\alpha_{-k}^{\mu}, \alpha_k^{\nu}] \neq 0$$

$$:L_0: = \frac{1}{2}\sum_{k\in\mathbb{Z}} :\alpha_{-k}\cdot\alpha_k: = \frac{1}{2}\alpha_0^2 + \sum_{k\geq 0} \alpha_{-k}\cdot\alpha_k \equiv \hat{L}_0.$$

$$\begin{cases}(\hat{L}_0 - a)|\text{phys}\rangle = 0\\ \left(\hat{\tilde{L}}_0 - \tilde{a}\right)|\text{phys}\rangle = 0\end{cases}$$

$$[L_n, \alpha_m^{\mu}] = -m\alpha_{n+m}^{\mu}$$

$$\begin{aligned}[L_n, L_m] &= \frac{1}{2}\sum_{k\in\mathbb{Z}} [L_n, :\alpha_{m-k}\cdot\alpha_k:]\\
&= \frac{1}{2}\left(\sum_{k\geq 0} [L_n, \alpha_{m-k}\cdot\alpha_k] + \sum_{k<0} [L_n, \alpha_k\cdot\alpha_{m-k}]\right)\\
&= \frac{1}{2}\left(\sum_{k\geq 0} \{(k-m)\alpha_{n+m-k}\cdot\alpha_k - k\alpha_{m-k}\cdot\alpha_{n+k}\}\right.\\
&\left.\quad + \sum_{k<0} \{-k\alpha_{n+k}\cdot\alpha_{m-k} + (k-m)\alpha_k\cdot\alpha_{n+m-k}\}\right)\end{aligned}$$

$$[L_n, L_m] = \frac{1}{2}\left(\sum_{k\geq 0} (k-m)\alpha_{n+m-k}\cdot\alpha_k + \sum_{q\geq n} (n-q)\alpha_{n+m-q}\cdot\alpha_q\right.$$
$$\left. + \sum_{q\leq n-1} (n-q)\alpha_q\cdot\alpha_{n+m-q} + \sum_{k<0} (k-m)\alpha_k\cdot\alpha_{n+m-k}\right)$$

$$= \frac{1}{2}\left(\sum_{k\geq 0} (n-m)\alpha_{n+m-k}\cdot\alpha_k + \sum_{k=0}^{n-1} (k-n)\alpha_{n+m-k}\cdot\alpha_k\right.$$
$$\left. + \sum_{k\leq -1} (n-m)\alpha_k\cdot\alpha_{n+m-k} + \sum_{k=0}^{n-1} (n-k)\alpha_k\cdot\alpha_{n+m-k}\right)$$

$$\sum_{k=0}^{n-1} (n-k)\alpha_k \cdot \alpha_{n+m-k} = \sum_{k=0}^{n-1} (n-k)\left(\alpha_{n+m-k} \cdot \alpha_k - [\alpha^{\mu}_{n+m-k}, \alpha^{\nu}_k]\eta_{\mu\nu}\right)$$
$$= \sum_{k=0}^{n-1} (n-k)\left(\alpha_{n+m-k} \cdot \alpha_k - (-k)\eta^{\mu\nu}\delta_{n+m,0}\eta_{\mu\nu}\right)$$
$$= \sum_{k=0}^{n-1} (n-k)\left(\alpha_{n+m-k} \cdot \alpha_k + kd\delta_{n+m,0}\right)$$

$$[L_n, L_m] = \frac{1}{2}\left(\sum_{k\in\mathbb{Z}} (n-m)\alpha_{n+m-k} \cdot \alpha_k + \sum_{k=0}^{n-1} (n-k)kD\delta_{n+m,0}\right)$$
$$= (n-m)L_{n+m} + \frac{1}{2}D\sum_{k=0}^{n-1} (n-k)k\delta_{n+m,0}$$

$$\sum_{k=0}^{n-1} (n-k)k = \frac{1}{6}n(n^2-1)$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{D}{12}n(n^2-1)\delta_{n+m,0}$$

$$\frac{c}{12}n(n^2-1)\delta_{n+m,0}$$

$$[L_0, L_1] = -L_1, [L_0, L_{-1}] = +L_{-1}, [L_1, L_{-1}] = 2L_0$$

$$f(z) = \frac{\alpha z + \beta}{\gamma z + \delta},$$

$$\det\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = 1.$$

$$|0, P\rangle|_{P=0} = |0\rangle.$$

$$P^{\mu}|0\rangle = 0$$
$$\alpha^{\mu}_0|0\rangle = 0$$
$$\tilde{\alpha}^{\mu}_0|0\rangle = 0$$

$$L_{+1}|0\rangle = 0$$
$$L_0|0\rangle = 0$$
$$L_{-1}|0\rangle = 0$$

$$L_{+1}|0\rangle = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{1-k}\cdot\alpha_k|0\rangle = \begin{cases} 0 & \text{for } k>0 \text{ since } \alpha_k \text{ annihilates } |0\rangle \\ 0 & \text{for } k<0 \text{ since } \alpha_{1-k} \text{ annihilates } |0\rangle \\ 0 & \text{for } k=0 \text{ since } \alpha_0|_{P=0}=0 \end{cases}$$

$$L_0|0\rangle = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{-k}\cdot\alpha_k = \left(\frac{1}{2}(\alpha_0)^2 + \frac{1}{2}\sum_{k\neq 0} \alpha_{-k}\cdot\alpha_k\right)|0\rangle$$
$$= \left(\frac{\alpha' P^2}{4}\Big|_{P=0} + \sum_{k>0} \alpha_{-k}\cdot\alpha_k\right)|0\rangle$$
$$= 0$$

$$L_{-1}|0\rangle = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{-1-k}\cdot\alpha_k|0\rangle = \begin{cases} 0 & \text{for } k>0 \text{ since } \alpha_{-1-k} \text{ annihilates } |0\rangle \\ 0 & \text{for } k<0 \text{ since } \alpha_k \text{ annihilates } |0\rangle \\ 0 & \text{for } k=0 \text{ since } \alpha_0|_{P=0}=0 \end{cases}.$$

$$e^{iP\cdot\hat{X}_0}|0\rangle = |0,P\rangle$$

$$\hat{P}^\mu e^{iP\cdot\hat{X}_0}|0\rangle = 0 + \left[\hat{P}^\mu, e^{iP\cdot\hat{X}_0}\right]|0\rangle$$
$$= iP_\nu\left[\hat{P}^\mu, \hat{X}_0^\nu\right]e^{iP\cdot\hat{X}_0}|0\rangle$$
$$= iP_\nu(-i\eta^{\mu\nu})e^{iP\cdot\hat{X}_0}|0\rangle$$
$$= P^\mu e^{iP\cdot\hat{X}_0}|0\rangle$$

$$L_n \mid \text{phys}\rangle \stackrel{?}{=} 0\ \forall n\neq 0$$

$$\langle \text{phys}|[L_n, L_m] \mid \text{phys'}\rangle =$$
$$= 0$$
$$= \langle \text{phys}|\left((n-m)L_{n+m} + \frac{D}{12}n(n^2-1)\delta_{n+m,0}\right)\Big| \text{phys'}\Big\rangle$$

$$\langle \text{phys} \mid \text{phys'}\rangle = 0$$

$$\langle \text{phys}|L_n \mid \text{phys'}\rangle = 0\ \forall n\neq 0$$

$$L_n \mid \text{phys}\rangle = 0\ \langle \text{phys}|L_{-n} = 0\ \forall n>0$$

$$(L_0 - a) \mid \text{phys}\rangle = 0$$

$$Q_B^2 = 0 \ \leftrightarrow\ D = 26$$

$$Q_B|\Psi\rangle = 0 \Rightarrow \begin{cases} L_n|\psi\rangle^{\text{matter}} = 0\ \forall n>0 \\ (L_0-1)|\psi\rangle^{\text{matter}} = 0 \end{cases}$$

$$L_n \mid \text{phys}\rangle = 0\ \forall n>0$$
$$(L_0 - a) \mid \text{phys}\rangle = 0$$

$$|\chi\rangle = \sum_{n>0} L_{-n}|\chi_n\rangle$$

$$\langle\chi| \text{phys}\rangle = 0$$

$$
\begin{aligned}
|\text{ null }\rangle &= \sum_{n>0} L_{-n}|\chi_n\rangle \\
L_n|\text{ null }\rangle &= 0\ \forall n>0 \\
(L_0-a)|\text{ null }\rangle &= 0
\end{aligned}
$$

$$\langle \text{null} \mid \text{null}'\rangle = 0, \langle \text{null} \mid \text{phys}\rangle = 0$$

$$|\text{ phys }\rangle \sim |\text{ phys }\rangle + |\text{ null }\rangle.$$

$$\overset{(\sim)}{N} = \sum_{k>0} \overset{(\sim)}{\alpha}_{-k} \cdot \overset{(\sim)}{\alpha}_{k}.$$

$$
\begin{aligned}
N\left(\alpha_{-1}^{\mu}\right)^2(\alpha_{-2}^{\nu})^3|0,P\rangle &= 8\left(\alpha_{-1}^{\mu}\right)^2(\alpha_{-2}^{\nu})^3|0,P\rangle \\
N\left(\alpha_{-3}^{\mu}\right)(\alpha_{-2}^{\nu})|0,P\rangle &= 5\left(\alpha_{-3}^{\mu}\right)(\alpha_{-2}^{\nu})|0,P\rangle
\end{aligned}
$$

$$\overset{(\sim)}{L}_0 = \frac{\alpha' P^2}{4} + \overset{(\sim)}{N}$$

$$
\begin{aligned}
\left(L_0+\tilde{L}_0\right)|0,P\rangle &= \left(\frac{\alpha' P^2}{2} + N + \tilde{N}\right)|0,P\rangle \\
&= \left(-\frac{\alpha' m^2}{2} + N + \tilde{N}\right)|0,P\rangle = 2a|0,P\rangle \\
\left(L_0-\tilde{L}_0\right)|0,P\rangle &= (N-\tilde{N})|0,P\rangle = 0
\end{aligned}
$$

$$N = \tilde{N}$$

$$-\frac{\alpha' m^2}{2} + 2N = 2a$$

$$\alpha' m^2 = 4(N-a).$$

$$
\begin{aligned}
\overset{(\sim)}{L}_n|0,P\rangle &= 0\ \forall n>0 \\
\left(\tilde{L}_0|0,P\rangle\right. &= \left(\frac{\alpha' P^2}{4} + 0\right)|0,P\rangle = a|0,P\rangle
\end{aligned}
$$

$$-\frac{\alpha' m^2}{4} = a \Longrightarrow m^2 = -\frac{4a}{\alpha'}$$

$$\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0,P\rangle$$

$$G_{\mu\nu}(P)\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0,P\rangle$$

$$G_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu} + \Phi\eta_{\mu\nu},$$

$$
\begin{aligned}
h_{\mu\nu} &= G_{(\mu\nu)} \underbrace{-\frac{1}{D}\eta^{\mu\nu}G_\rho^\rho}_{\text{remove the trace}} \text{ symmetric traceless} \\
B_{\mu\nu} &= G_{[\mu\nu]} \text{ anti-symmetric} \\
\Phi &= \underbrace{\frac{1}{d}G_\rho^\rho}_{\text{trace}} \text{ scalar}
\end{aligned}
$$

$$
|\psi\rangle = \int d^d P G_{\mu\nu}(P)\alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |0,P\rangle
$$

$$
\begin{aligned}
L_1 G_{\mu\nu}\alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |0,P\rangle &= 0 \\
&\propto G_{\mu\nu}\left(\alpha_0^\rho \alpha_1^\sigma\right)\eta_{\rho\sigma}\alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |0,P\rangle \\
&= G_{\rho\nu}P^\rho \tilde{\alpha}_{-1}^\nu |0,P\rangle
\end{aligned}
$$

$$
\begin{cases}
P_\mu G^{\mu\nu} = 0 \\
G^{\mu\nu}P_\nu = 0
\end{cases}
$$

$$
\alpha' m^2 = 4(1-a),
$$

$$
|\text{ state }\rangle = \xi_\nu \alpha_{-1}^0 \tilde{\alpha}_{-1}^\nu |0,P\rangle
$$

$$
\frac{\alpha' P^2}{4} = a - 1 > 0
$$

$$
\|\text{ state }\rangle\|^2 = \eta^{00}(\xi \cdot \xi)\delta(P+P') < 0
$$

$$
P^2 = 0
$$

$$
G_*^{\mu\nu} = P^\mu \xi^\nu + v^\mu P^\nu
$$

$$
\|G_*^{\mu\nu}\alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |0,P\rangle\|^2 \sim P^2 = 0
$$

$$
\begin{aligned}
P^\mu \xi^\nu \alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |0,P\rangle &\propto L_{-1}(\xi \cdot \tilde{\alpha}_{-1})|0,P\rangle \\
v^\mu P^\nu \alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |0,P\rangle &\propto (v \cdot \alpha_{-1})\tilde{L}_{-1}|0,P\rangle
\end{aligned}
$$

$$
\begin{aligned}
|\text{ phys }\rangle &\sim |\text{ phys }\rangle + |\text{ null }\rangle \\
G_{\mu\nu} &\sim G_{\mu\nu} + P^\mu \xi^\nu + v^\mu P^\nu \\
&(\text{ with } P \cdot \xi = 0, v \cdot P = 0, P^2 = 0)
\end{aligned}
$$

$$
\begin{aligned}
h_{\mu\nu} &\sim h_{\mu\nu} + P_{(\mu}\theta_{\nu)} \\
B_{\mu\nu} &\sim B_{\mu\nu} + P_{[\mu}\Lambda_{\nu]} \\
\Phi &\sim \Phi
\end{aligned}
$$

$$
\theta_\mu = \frac{\xi_\mu + v_\mu}{2}, \Lambda_\mu = \frac{\xi_\mu - v_\mu}{2}
$$

$$
\begin{cases}
P^2 h_{\mu\nu} = 0 \\
P_\mu h^{\mu\nu} = 0 \\
h_\mu{}^\mu = 0
\end{cases}
$$

$$P^2 h_{\mu\nu} - P^\rho P_{(\mu} h_{\nu)\rho} - P_\mu P_\nu h_\rho^\rho = 0.$$

$$B = B_{\mu\nu} dx^\mu \wedge dx^\nu.$$

$$\begin{cases} P^2 B_{\mu\nu} = 0 \\ P_\mu B^{\mu\nu} = 0' \end{cases}$$

$$\begin{aligned} H = dB = {} & \partial_{[\mu} B_{\nu\rho]} dx^\mu \wedge dx^\nu \wedge dx^\rho \\ & = H_{\mu\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho \end{aligned}$$

$$S \sim \int d^D x H_{\mu\nu\rho} H^{\mu\nu\rho} \sim \int H \wedge * H$$

$$\Box A_\mu - \partial_\mu (\partial \cdot A) = 0 \leftrightarrow P^2 A_\mu - P_\mu (P \cdot A) = 0.$$

$$\begin{cases} P^2 = 0 \\ P \cdot A = 0 \end{cases}$$

$$\begin{cases} * d * H = 0 & \rightarrow \Box B_{\mu\nu} + \partial^\rho \partial_{[\mu} B_{\nu]\rho} = 0 \\ dH = 0 & (\mathcal{B}_{bianchi\ identities}) \end{cases}$$

$$P^2 \Phi(P) = 0$$

$$q \int_{WL} A = q \int d\tau \dot{X}^\mu(\tau) A_\mu(X)$$

$$q \sim \int_{S^{D-2}} * F^{(2)}$$

$$-\frac{1}{4\pi\alpha'} \int_{WS} B = -\frac{1}{4\pi\alpha'} \int_{WS} d^2\sigma \epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu B_\mu(X)$$

$$\text{Particle charge } \sim \int_{S^{d-3}} * H^{(3)}$$

$$|\chi_2\rangle = \underbrace{(2L_{-2} + 3L_{-1}^2)}_{\substack{\text{left} \\ \text{excitation}}} \underbrace{(\tilde{\theta}_{-2})}_{\substack{\text{right} \\ \text{excitation}}} |0, P\rangle,$$

$$\begin{aligned} (\tilde{L}_0 - 1)\tilde{\theta}_{-2}|0, P\rangle &= 0 \\ \tilde{L}_1 \tilde{\theta}_{-2}|0, P\rangle &= 0 \\ \tilde{L}_2 \tilde{\theta}_{-2}|0, P\rangle &= 0 \end{aligned}$$

$$\begin{aligned} (L_0 - 1)|\chi_2\rangle &= \left(\frac{\alpha' P^2}{4} + 1\right)|\chi_2\rangle = 0 \rightarrow m^2 = \frac{4}{\alpha'} \\ L_1|\chi_2\rangle &= 0 \\ L_2|\chi_2\rangle &\propto (D - 26)\tilde{\theta}_{-2}|0, P\rangle \end{aligned}$$

$$\| |\chi\rangle \|^2 \propto (26 - D)$$

$$|\Psi\rangle = (\alpha_{-1} \cdot \alpha_{-1} + c_2 \alpha_0 \cdot \alpha_{-2} + 2c_3(\alpha_0 \cdot \alpha_{-1})^2)|0, P\rangle.$$

$$\langle \Psi \mid \Psi \rangle = \frac{2(D-1)[2(a-2)(8a-21)+(2a-3)D]}{(9-4a)^2} \langle 0,P \mid 0,P \rangle.$$

$$\langle \Psi \mid \Psi \rangle \geq 0 \;\rightarrow\; D \leq \frac{2(2-a)(21-8a)}{3-2a}$$

$$\begin{cases} \partial_+ \partial_- X^\mu(\sigma^+,\sigma^-) & = 0 \quad \text{(WS bulk eq.)} \\ \delta X_\mu \partial_\sigma X^\mu \big|_{\sigma=0,\pi} & = 0 \quad \text{(WS boundary eq.)} \end{cases}$$

$$\partial_\sigma X^\mu |_{\text{endpoint}} = 0$$

$$\delta X_\mu \big|_{\text{endpoint}} = 0$$

$$X^\mu(\sigma^+,\sigma^-) = X_L^\mu(\sigma^+) + X_R^\mu(\sigma^-)$$

$$\hat{X}^\mu(\tau,\sigma) = \begin{cases} X^\mu(\tau,\sigma) & \text{if } \sigma \in (0,\pi) \\ X^\mu(\tau,2\pi-\sigma) & \text{if } \sigma \in (\pi,2\pi) \end{cases}$$

$$\hat{X}^\mu(\tau,2\pi+\sigma) = \hat{X}^\mu(\tau,\sigma)$$

$$\hat{X}(\tau,\sigma) = \hat{X}(\tau,-\sigma)$$

$$\alpha_n^\mu = \tilde{\alpha}_n^\mu$$

$$X_{NN}^\mu(\sigma^+,\sigma^-) = X_0^\mu + \alpha' P^\mu \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu \left(e^{-in\sigma^+} + e^{-in\sigma^-}\right).$$

$$[\alpha_n^\mu, \alpha_m^\nu] = n\eta^{\mu\nu}\delta_{n+m,0},$$

$$[X_0^\mu, P^\nu] = i\eta^{\mu\nu}.$$

$$\alpha_{-n_1}^{\mu_1} \dots \alpha_{-n_k}^{\mu_k} |0,P\rangle$$

$$L_{n\neq 0} = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{n-k}\cdot\alpha_k$$

$$L_0 = \frac{1}{2}\sum_{k\in\mathbb{Z}} :\alpha_{-k}\cdot\alpha_k: = \frac{1}{2}(\alpha_0)^2 + N = \alpha' P^2 + N$$

$$\alpha_0^\mu = \sqrt{2\alpha'} P^\mu$$

$$X^i(\sigma=0) = x^i, X^i(\sigma=\pi) = x^i$$

$$X^i(\tau,\sigma) = x^i + Y^i(\tau,\sigma)$$

$$Y^i(\tau,\sigma)\big|_{\sigma=0,\pi} = 0$$

$$\hat{Y}^i(\tau,\sigma) = \begin{cases} Y^i(\tau,\sigma) & \text{if } \sigma \in (0,\pi) \\ -Y^i(\tau,2\pi-\sigma) & \text{if } \sigma \in (\pi,2\pi) \end{cases}$$

$$\hat{Y}^i(\tau,-\sigma) = \hat{Y}^i(\tau, 2\pi - \sigma) = -\hat{Y}^i(\tau,\sigma).$$

$$Y^i(\sigma^+,\sigma^-) = i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^i\left(e^{-in\sigma^+} - e^{-in\sigma^-}\right)$$

$$X_{DD}^i(\tau,\sigma) = x^i + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^i\left(e^{-in\sigma^+} - e^{-in\sigma^-}\right)$$

$$\partial_\sigma X^\mu|_{\sigma=0} = 0,\ X^\mu(\tau,\sigma)|_{\sigma=\pi} = x^\mu$$

$$X_{ND}^\mu(\tau,\sigma) = x^\mu + i\sqrt{\frac{\alpha'}{2}}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\frac{1}{r}\alpha_r^\mu\left(e^{-ir\sigma^+} + e^{-ir\sigma^-}\right)$$

$$X_{DN}^\mu(\tau,\sigma) = X_0^\mu + i\sqrt{\frac{\alpha'}{2}}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\frac{1}{r}\alpha_r^\mu\left(e^{-ir\sigma^+} - e^{-ir\sigma^-}\right)$$

$$L_n = \frac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}} :\alpha_{n-r}\cdot\alpha_r:$$

$$[\alpha_r^\mu, \alpha_s^\nu] = r\eta^{\mu\nu}\delta_{r+s,0}$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{1}{12}n(n^2-1)\delta_{n+m,0}$$

$$\alpha_r|0\rangle_{ND} = 0, \forall r \in \mathbb{Z} + \frac{1}{2}, r \geq \frac{1}{2}.$$

$$T_{\pm\pm}(\sigma^\pm) = -\sum_{n\in\mathbb{Z}} L_n e^{-in\sigma^\pm}$$

$$\partial_+ X_{NN}(\sigma^+) = \partial_- X_{NN}(\sigma^-)|_{\sigma=0,\pi}$$

$$\partial_+ X_{DD}(\sigma^+) = -\partial_- X_{DD}(\sigma^-)|_{\sigma=0,\pi}$$

$$T_{++}(\sigma^+) = T_{--}(\sigma^-)|_{\sigma=0,\pi}$$

$$\partial_+ X_{ND}(\sigma^+) = \partial_- X_{ND}(\sigma^-)|_{\sigma=0}$$
$$\partial_+ X_{ND}(\sigma^+) = -\partial_- X_{ND}(\sigma^-)|_{\sigma=\pi}$$

$$L_0 = \alpha' P^2 + N^{(NN)} + N^{(DD)} = \alpha' P^2 + N^{(\mathrm{tot})}.$$

$$\alpha' m^2 = N^{(\mathrm{tot})} - a.$$

$$t(P)|0,P\rangle,$$

$$\alpha' m^2 = -a.$$

$$a = 1,$$

$$m^2 = -\frac{1}{\alpha'}.$$

$$\left(A_\mu(P)\alpha_{-1}^\mu + \phi_i(P)\alpha_{-1}^i\right)|0, P\rangle,$$

$$SO(1, d-1) \longrightarrow \underbrace{SO(1,p)}_{\substack{\text{Lorentz on} \\ \text{the } D(p)\text{-brane}}} \times \underbrace{SO(d-p-1)}_{\substack{\text{rotations in the} \\ \text{Dirichlet directions}}}.$$

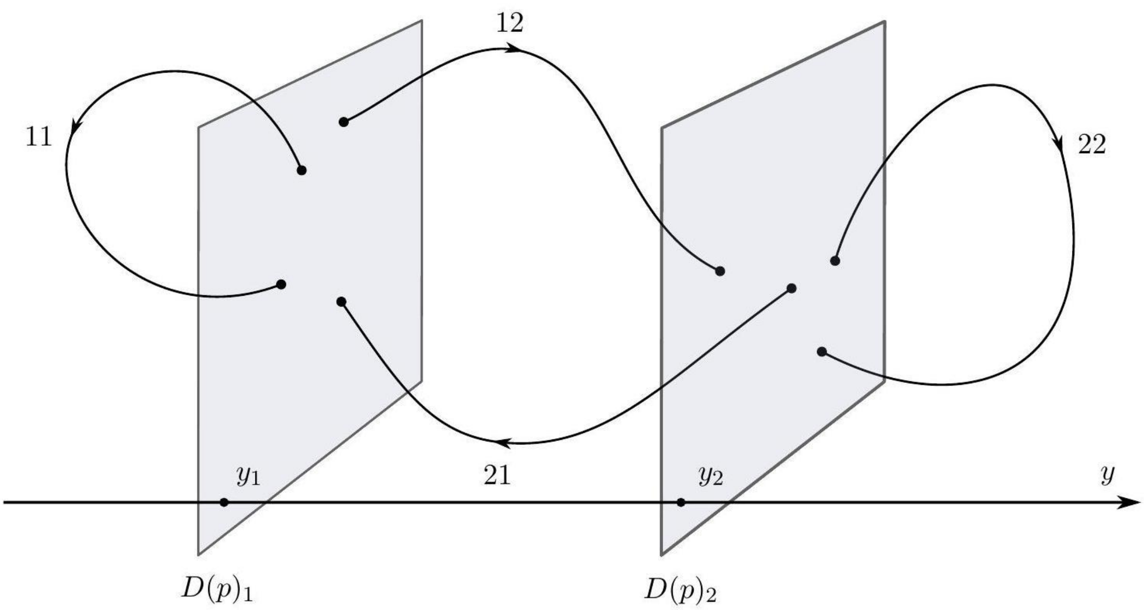
$$m = 0.$$

$$\begin{cases} P_\mu A^\mu(P) = 0 \\ P^2 A^\mu(P) = 0 \end{cases}$$
$$P^2 \phi^i(P) = 0$$

$$A_\mu(P) \sim A_\mu(P) + \lambda(P)P_\mu \longrightarrow A_\mu(x) \sim A_\mu(x) + i\partial_\mu \lambda(x),$$

$$\left(\xi_\mu \alpha_{-2}^\mu + \theta_{\mu\nu}\alpha_{-1}^\mu \alpha_{-1}^\nu\right)|0, P\rangle$$

$$\partial_+ \partial_- Y^{(12)}(\tau, \sigma) = 0 \text{ (wave equation)}$$
$$Y^{(12)}(\tau, \sigma = 0) = y_1 \text{ (BC)}$$



$$Y^{(12)}(\tau, \sigma = \pi) = y_2 \text{ (BC)}.$$

$$Y^{(12)}(\sigma^+, \sigma^-) = \left(1 - \frac{\sigma}{\pi}\right) y_1 + \frac{\sigma}{\pi} y_2 + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \alpha_n^{(12)} \left(e^{-in\sigma^+} - e^{-in\sigma^-}\right)$$

$$\left(1 - \frac{\sigma}{\pi}\right) y_1 + \frac{\sigma}{\pi} y_2 = y_1 + \frac{\Delta y}{2\pi}(\sigma^+ - \sigma^-).$$

$$\alpha_0^{(12)} = \frac{\Delta y}{2\pi}\sqrt{\frac{2}{\alpha'}},$$

$$\partial_+ Y^{(12)}(\sigma^+,\sigma^-) = \frac{\Delta y}{2\pi} + \sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0} \alpha_n^{(12)} e^{-in\sigma^+} = \sqrt{\frac{\alpha'}{2}}\sum_{n\in\mathbb{Z}} \alpha_n^{(12)} e^{-in\sigma^+},$$

$$\partial_- Y^{(12)}(\sigma^+,\sigma^-) = -\frac{\Delta y}{2\pi} - \sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0} \alpha_n^{(12)} e^{-in\sigma^-} = \sqrt{\frac{\alpha'}{2}}\sum_{n\in\mathbb{Z}} \alpha_n^{(12)} e^{-in\sigma^+}.$$

$$T_{\pm\pm}^{(y)} = -\frac{1}{\alpha'}\partial_\pm Y \partial_\pm Y.$$

$$\alpha' m^2 = -1 + N^{(\mathrm{tot})} + \frac{1}{2}\left(\frac{\Delta y}{2\pi}\right)^2 \frac{2}{\alpha'} = -1 + N^{(\mathrm{tot})} + \left(\frac{\Delta y}{2\pi\sqrt{\alpha'}}\right)^2.$$

$$t^{(12)}(P)|0,P\rangle,$$

$$\alpha' m^2 = -1 + \left(\frac{\Delta y}{2\pi\sqrt{\alpha'}}\right)^2.$$

$$-1 + \left(\frac{\Delta y}{2\pi\sqrt{\alpha'}}\right)^2 \geq 0$$

$$\left(W_\mu^{(12)}(P)\alpha_{-1}^\mu + \phi_i^{(12)}\alpha_{-1}^i\right)|0,P\rangle$$

$$\alpha' m^2 = \left(\frac{\Delta y}{2\pi\sqrt{\alpha'}}\right)^2.$$

$$\begin{cases} P^\mu W_\mu^{(12)}(P) = 0 \\ \left(\alpha' P^2 + \left(\frac{\Delta y}{2\pi\sqrt{\alpha'}}\right)^2\right) W_\mu^{(12)} = 0 \end{cases}$$

$$\left(\alpha' P^2 + \left(\frac{\Delta y}{2\pi\sqrt{\alpha'}}\right)^2\right)\phi_i^{(12)}(P) = 0$$

$$L_1 \phi_y^{(12)}(P)\alpha_{-1}^y|0,P\rangle = \phi_y^{(12)}(\alpha_0^y\alpha_1^y)\alpha_{-1}^y|0,P\rangle \sim (\Delta y)\phi_y^{(12)}(P)|0,P\rangle \neq 0$$

$$\begin{pmatrix} \phi_{i\neq y}^{(11)} & \phi_{i\neq y}^{(12)} \\ \phi_{i\neq y}^{(21)} & \phi_{i\neq y}^{(22)} \end{pmatrix}, \begin{pmatrix} \phi_y^{(11)} & 0 \\ 0 & \phi_y^{(22)} \end{pmatrix}, (\Delta y \neq 0)$$

$$\begin{pmatrix} A_\mu^{(11)} & W_\mu^{(12)} \\ W_\mu^{(21)} & A_\mu^{(22)} \end{pmatrix}$$

$$\underbrace{U(1)\times U(1)}_{\substack{A_\mu \text{ massless} \\ W_\mu \text{ massive}}} \xrightarrow{\Delta y = 0} \underbrace{U(2)}_{\substack{A_\mu \text{ massless} \\ W_\mu \text{ massless}}}$$

$$Z = \frac{1}{\mathrm{Vol}\mathcal{G}} \int \mathcal{D}X^\mu \mathcal{D}h_{\alpha\beta} e^{iS[X,h]}$$

$$S[h,X] = -\frac{1}{4\pi\alpha'} \int_{\mathrm{WS}} d^2\sigma \sqrt{-h} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu$$

$$\delta_D X^\mu = \xi^\rho \partial_\rho X^\mu;\ \delta_D h_{\alpha\beta} = \nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha$$

$$\delta_{\mathrm{W}} h_{\alpha\beta} = \omega h_{\alpha\beta}$$

$$\nabla_\alpha \xi_\beta = \partial_\alpha - \Gamma^\gamma_{\alpha\beta} \xi_\gamma$$

$$\Gamma^\gamma_{\alpha\beta} = \frac{1}{2} h^{\gamma\sigma} (\partial_\alpha h_{\sigma\beta} + \partial_\beta h_{\sigma\alpha} - \partial_\sigma h_{\alpha\beta}).$$

$$h = \hat{h}^g$$

$$F(h) = h - \hat{h} = 0 \rightarrow h = \hat{h}.$$

$$1 = \int \mathcal{D}h \delta(h - \hat{h}) = \int \mathcal{D}\hat{h}^g \delta(\hat{h}^g - \hat{h}) = \int \mathcal{D}g \delta(\hat{h}^g - \hat{h}) \det \frac{\delta \hat{h}^g}{\delta g}\bigg|_{g=1}$$

$$\begin{aligned}
Z &= \frac{1}{\mathrm{Vol}\mathcal{G}} \int \mathcal{D}X\mathcal{D}h\mathcal{D}g \delta(\hat{h}^g - \hat{h}) \det \frac{\delta \hat{h}^g}{\delta g}\bigg|_{g=1} e^{iS[X,h]} \\
&= \frac{1}{\mathrm{Vol}\mathcal{G}} \int \mathcal{D}X\mathcal{D}\hat{h}^g\mathcal{D}g \delta(\hat{h}^g - \hat{h}) \det \frac{\delta \hat{h}^g}{\delta g}\bigg|_{g=1} e^{iS[X,\hat{h}^g]} \\
&= \frac{1}{\mathrm{Vol}\mathcal{G}} \int \mathcal{D}X^g\mathcal{D}\hat{h}^g\mathcal{D}g \delta(\hat{h}^g - \hat{h}) \det \frac{\delta \hat{h}^g}{\delta g}\bigg|_{g=1} e^{iS[X^g,\hat{h}^g]} \\
&= \frac{1}{\mathrm{Vol}\mathcal{G}} \int \mathcal{D}X^g\mathcal{D}\hat{h}^g\mathcal{D}g \delta(\hat{h}^g - \hat{h}) \det \frac{\delta \hat{h}^g}{\delta g}\bigg|_{g=1} e^{iS[X,\hat{h}]} \\
&= \frac{1}{\mathrm{Vol}\mathcal{G}} \int \mathcal{D}X\mathcal{D}g \det \frac{\delta \hat{h}^g}{\delta g}\bigg|_{g=1} e^{iS[X,\hat{h}]} \\
&= \int \mathcal{D}X \det \frac{\delta \hat{h}^g}{\delta g}\bigg|_{g=1} e^{iS[X,\hat{h}]}
\end{aligned}$$

$$\det \frac{\delta h^g}{\delta g}\bigg|_{g=1} = \int \mathcal{D}b_{\alpha\beta}\mathcal{D}\lambda\mathcal{D}c^\rho \exp\left[\frac{1}{4\pi}\int d^2\sigma \sqrt{-\hat{h}} b_{\alpha\beta} \frac{\delta h^{\alpha\beta}}{\delta g^x}\bigg|_{g=1} c^x\right],$$

$$\delta_{\mathrm{tot}} h_{\alpha\beta} = \delta_{\mathrm{D}} h_{\alpha\beta} + \delta_{\mathrm{W}} h_{\alpha\beta} = \omega h_{\alpha\beta} + \nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha = \Omega h_{\alpha\beta} + 2(P_1 \xi)_{\alpha\beta},$$

$$\Omega \equiv \omega + \nabla \cdot \xi$$

$$(P_1 \xi)_{\alpha\beta} \equiv \frac{1}{2}(\nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha - h_{\alpha\beta} \nabla \cdot \xi)$$

$$\int d^2\sigma\sqrt{-\hat{h}}b_{\alpha\beta}\frac{\delta h^{\alpha\beta}}{\delta g^x}\bigg|_{g=1} c^x = \int d^2\sigma\sqrt{-\hat{h}}b_{\alpha\beta}\left[\hat{h}^{\alpha\beta}\left(\lambda+\frac{1}{2}\nabla\cdot c\right)+(2P_1c)^{\alpha\beta}\right].$$

$$Z=\int \mathcal{D}X^\mu \mathcal{D}b_{\alpha\beta}\mathcal{D}c^\rho e^{i\left(S[X,\hat{h}]-\frac{i}{4\pi}\int d^2\sigma\sqrt{-\hat{h}}b_{\alpha\beta}(2P_1c)^{\alpha\beta}\right)}$$

$$Z_{Pol}\left[X^\mu,b_{\alpha\beta},c^\rho\right]=\int \mathcal{D}X^\mu \mathcal{D}b_{\alpha\beta}\mathcal{D}c^\rho e^{i\left(S[X,\hat{h}]-\frac{i}{4\pi}\int d^2\sigma\sqrt{-\hat{h}}b_{\alpha\beta}(\nabla^\alpha c^\beta+\nabla^\beta c^\alpha)\right)}$$

$$S_{\mathrm{GH}}=-\frac{i}{4\pi}\int d^2\sigma\sqrt{-\hat{h}}b_{\alpha\beta}(\nabla^\alpha c^\beta+\nabla^\beta c^\alpha)=-\frac{i}{2\pi}\int D^2\sigma\sqrt{-\hat{h}}b_{\alpha\beta}\nabla^\alpha c^\beta$$

$$\begin{aligned}iS'_{\mathrm{GH}}&=\frac{1}{2\pi}\int d^2\sigma\sqrt{-h}e^{-\omega}e^{\omega}h^{\alpha\beta}b_{\beta\delta}\nabla'_\alpha c^\delta\\&=\frac{1}{2\pi}\int d^2\sigma\sqrt{-h}h^{\alpha\beta}b_{\beta\delta}\left(\partial_\alpha c^\delta+\Gamma^\delta_{\alpha\sigma}{}'c^\sigma\right)\\&=-S_{\mathrm{GH}}-\frac{1}{4\pi}\int d^2\sigma\sqrt{-h}h^{\alpha\beta}b_{\beta\delta}\left(h^\delta_\alpha\partial_\sigma\omega+h^\delta_\sigma\partial_\alpha\omega-h_{\alpha\sigma}\partial^\delta\omega\right)c^\sigma\end{aligned}$$

$$=iS_{\mathrm{GH}}-\underbrace{\frac{1}{4\pi}\int d^2\sigma\sqrt{-h}b^\alpha{}_\alpha\partial_\sigma\omega c^\sigma}_{=0\text{ because }b^\alpha_\alpha=0}-\underbrace{\frac{1}{4\pi}\int d^2\sigma\sqrt{-h}\left(b^\alpha_\delta c_\alpha\partial^\delta\omega-b^\alpha_\delta c^\delta\partial_\alpha\omega\right)}_{=0\text{ after index redefinition}},$$

$$S=-\frac{1}{4\pi\alpha'}\int d^2\sigma\partial_\alpha X^\mu\partial^\alpha X_\mu-\frac{i}{2\pi}\int d^2\sigma b_{\alpha\beta}\partial^\alpha c^\beta$$

$$\begin{aligned}S_{\mathrm{BRST}}=&-\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu-\frac{i}{2\pi}\int d^2\sigma\sqrt{-h}b^{\alpha\beta}\nabla_\alpha c_\beta\\&-\frac{1}{4\pi}\int d^2\sigma\sqrt{-h}B_{\alpha\beta}\left(h^{\alpha\beta}-\hat{h}^{\alpha\beta}\right)\end{aligned}$$

$$\frac{4\pi}{\sqrt{-h}}\frac{\delta S_{\mathrm{BRST}}}{\delta h^{\alpha\beta}}=-B_{\alpha\beta}+T^M_{\alpha\beta}+T^{GH}_{\alpha\beta}$$

$$\begin{aligned}\delta_h S_{GH}=-\frac{i}{2\pi}\int d^2\sigma\Big[&-\frac{1}{2}\sqrt{-h}h_{\rho\sigma}\delta h^{\rho\sigma}h^{\mu\alpha}b_{\mu\beta}\nabla_\alpha c^\beta+\sqrt{-h}\delta h^{\mu\alpha}b_{\mu\beta}\nabla_\alpha c^\beta\\&+\sqrt{-h}\delta h^{\mu\alpha}b_{\mu\beta}\delta\Gamma^\beta_{\alpha\gamma}c^\gamma\Big]\end{aligned}$$

$$\delta\Gamma^\beta_{\alpha\gamma}=\frac{1}{2}h^{\beta\lambda}\left(\nabla_\gamma\delta h_{\lambda\alpha}+\nabla_\alpha\delta h_{\lambda\gamma}-\nabla_\lambda\delta h_{\alpha\gamma}\right)$$

$$h_{\alpha\beta}h^{\beta\gamma}=\delta^\gamma_\alpha\to\delta h_{\alpha\beta}h^{\beta\gamma}+h_{\alpha\beta}\delta h^{\beta\gamma}=0\Longrightarrow\delta h_{\alpha\beta}=-h_{\alpha\delta}h_{\beta\gamma}\delta h^{\delta\gamma},$$

$$\delta\Gamma^\beta_{\alpha\gamma}=-\frac{1}{2}h^{\beta\gamma}\left(h_{\lambda\mu}h_{\alpha\nu}\nabla_\gamma\delta h^{\mu\nu}+h_{\lambda\mu}h_{\gamma\nu}\nabla_\alpha\delta h^{\mu\nu}+h_{\alpha\mu}h_{\gamma\nu}\nabla_\lambda\delta h^{\mu\nu}\right).$$

$$\delta h^{\mu\alpha} b_{\mu\beta} \delta\Gamma^{\beta}_{\alpha\gamma} c^{\gamma} = -\frac{1}{2} h^{\rho\alpha} h^{\beta\lambda} h_{\lambda\mu} h_{\alpha\nu} b_{\rho\beta} c^{\gamma} \nabla_{\gamma} \delta h^{\mu\nu} +$$
$$-\frac{1}{2} h^{\rho\alpha} h^{\beta\lambda} h_{\lambda\mu} h_{\gamma\nu} b_{\rho\beta} c^{\gamma} \nabla_{\alpha} \delta h^{\mu\nu} +$$
$$+\frac{1}{2} h^{\rho\alpha} h^{\beta\lambda} h_{\alpha\mu} h_{\gamma\nu} b_{\rho\beta} c^{\gamma} \nabla_{\lambda} \delta h^{\mu\nu}$$

$$\delta h^{\mu\alpha} b_{\mu\beta} \delta\Gamma^{\beta}_{\alpha\gamma} c^{\gamma} = \frac{1}{2} \delta h^{\mu\nu} \left[\nabla_{\gamma}\left(b_{\mu\nu} c^{\gamma} + \nabla_{\alpha}\left(b^{\alpha}_{\mu} c_{\nu}\right) - \nabla_{\lambda}\left(b^{\lambda}_{\mu} c_{\nu}\right)\right] + \text{boundary terms}$$
$$= \frac{1}{2} \delta h^{\mu\nu} \nabla_{\gamma}\left(b_{\mu\nu} c^{\gamma}\right) + \text{ boundary terms}$$

$$\delta_h S_{GH} = -\frac{i}{2\pi} \int d^2\sigma \sqrt{-h}\, \delta h^{\mu\nu} \left[\frac{1}{2} \nabla_{\gamma}\left(b_{\mu\nu} c^{\gamma}\right) + b_{\mu\beta} \nabla_{\nu} c^{\beta} - \frac{1}{2} h_{\mu\nu} b^{\alpha}_{\beta} \nabla_{\alpha} c^{\beta}\right]$$
$$= -\frac{i}{4\pi} \int d^2\sigma \sqrt{-h}\, \delta h^{\mu\nu} \left[\nabla_{\gamma}\left(b_{\mu\nu} c^{\gamma}\right) + b_{\mu\beta} \nabla_{\nu} + b_{\nu\beta} \nabla_{\mu} c^{\beta} - h_{\mu\nu} b^{\alpha}_{\beta} \nabla_{\alpha} c^{\beta}\right]$$
$$= -\frac{i}{4\pi} \int d^2\sigma \sqrt{-h} \left[\delta h^{\mu\nu} b_{\mu\nu} \nabla_{\gamma} c^{\gamma} + \delta h^{\mu\nu} \nabla_{\gamma} b_{\mu\nu} c^{\gamma}\right]$$
$$-\frac{i}{4\pi} \int d^2\sigma \sqrt{-h}\, \delta h^{\mu\nu} \left[b_{\mu\beta} \nabla_{\nu} c^{\beta} + b_{\nu\beta} \nabla_{\mu} c^{\beta} - h_{\mu\nu} b^{\alpha}_{\beta} \nabla_{\alpha} c^{\beta}\right].$$

$$\delta_h S_{GH} = -\frac{i}{4\pi} \int d^2\sigma \sqrt{-h}\, \delta h^{\mu\nu} \left[\nabla_{\gamma} b_{\mu\nu} c^{\gamma} + b_{\mu\beta} \nabla_{\nu} c^{\beta} + b_{\nu\beta} \nabla_{\mu} c^{\beta} - h_{\mu\nu} b^{\alpha}_{\beta} \nabla_{\alpha} c^{\beta}\right]$$

$$T^{GH}_{\alpha\beta} = \frac{4\pi}{\sqrt{-h}} \frac{\delta S}{\delta h^{\alpha\beta}} = -i\left(\nabla_{\gamma} b_{\alpha\beta} c^{\gamma} + b_{\alpha\gamma} \nabla_{\beta} c^{\gamma} + b_{\beta\gamma} \nabla_{\alpha} c^{\gamma} - h_{\alpha\beta} b^{\delta}_{\gamma} \nabla_{\delta} c^{\gamma}\right).$$

$$B_{\alpha\beta} = \left[T^{M}_{\alpha\beta} - i\left(\partial_{\gamma} b_{\alpha\beta} c^{\gamma} + b_{\alpha\gamma} \partial_{\beta} c^{\gamma} + b_{\beta\gamma} \partial_{\alpha} c^{\gamma} - \eta_{\alpha\beta} b^{\gamma}_{\delta} \partial_{\gamma} c^{\delta}\right)\right].$$

$$b^{\alpha}{}_{\alpha} = \eta^{\alpha\beta} b_{\alpha\beta} = \eta^{+-} b_{+-} + \eta^{-+} b_{-+} = -4 b_{-+} = -4 b_{+-} = 0,$$

$$\delta_B X^{\mu} = c^{+} \partial_{+} X^{\mu} + c^{-} \partial_{-} X^{\mu}$$
$$\delta_B c^{\pm} = c^{\pm} \partial_{\pm} c^{\pm}$$
$$\delta_B b_{\pm\pm} = i\left(-\frac{1}{\alpha'} \partial_{\pm} X^{\mu} \partial_{\pm} X_{\mu} - i(2 b_{\pm\pm} \partial_{\pm} c^{\pm} + \partial_{\pm} b_{\pm\pm} c^{\pm})\right)$$

$$S = \frac{1}{2\pi\alpha'} \int d\sigma^{+} d\sigma^{-} \partial_{+} X^{\mu} \partial_{-} X_{\mu} - \frac{i}{2\pi} \int d\sigma^{+} d\sigma^{-} (b_{++} \partial_{-} c^{+} + b_{--} \partial_{+} c^{-})$$

$$\partial_{+} \partial_{-} X^{\mu} = 0$$
$$\partial_{+} c^{-} = \partial_{-} c^{+} = \partial_{+} b_{-} = \partial_{-} b_{+} = 0$$

$$c^{+} = c(\sigma^{+}), \quad c^{-} = \tilde{c}(\sigma^{-})$$
$$b^{+} = b(\sigma^{+}), \quad b^{-} = \tilde{b}(\sigma^{-})$$

$$c(\sigma^{+}) = \sum_{n\in\mathbb{Z}} c_n e^{-in\sigma^{+}}, \tilde{c}(\sigma^{-}) = \sum_{n\in\mathbb{Z}} \tilde{c}_n e^{-in\sigma^{-}}$$
$$b(\sigma^{+}) = \sum_{n\in\mathbb{Z}} b_n e^{-in\sigma^{+}}, \tilde{b}(\sigma^{-}) = \sum_{n\in\mathbb{Z}} \tilde{b}_n e^{-in\sigma^{-}}$$

$$[b(\tau,\sigma), c(\tau,\sigma')] = \left[\tilde{b}(\tau,\sigma), \tilde{c}(\tau,\sigma')\right] = 2\pi\delta(\sigma - \sigma')$$

$$[b_n, c_m] = \delta_{n+m}, [\tilde{b}_n, \tilde{c}_m] = \delta_{n+m}$$
$$[c_n, c_m] = 0, [\tilde{b}_n, \tilde{b}_m] = 0$$

$$[b(\tau,\sigma), c(\tau,\sigma')] = \sum_{n,m} [b_n, c_m] e^{-i(n\sigma+m\sigma')} = \sum_{n\in\mathbb{Z}} e^{-in(\sigma-\sigma')} = 2\pi\delta(\sigma-\sigma').$$

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{matter}} \otimes \mathcal{H}_{\text{ghost}} = \mathcal{H}_m \otimes \mathcal{H}_{gh}.$$

$$b_n |\downarrow\rangle = c_n |\downarrow\rangle = 0, \langle\downarrow| b_{-n} = \langle\downarrow| c_{-n} = 0, \text{ for } n > 0.$$

$$\langle\downarrow|\uparrow\rangle = \langle\downarrow| c_0 |\downarrow\rangle = 1,$$
$$\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 0.$$

$$b_{-n_1} \dots b_{-n_N} c_{-m_1} \dots c_{-m_M} |\downarrow\rangle \; n_i > 0, m_j \geq 0.$$

$$c_n|0\rangle = 0 \; \forall n \geq 2$$
$$b_m|0\rangle = 0 \; \forall m \geq -1$$

$$T_{++}^{(\mathrm{gh})} = i(2\partial_+ c^+ b_{++} + c^+ \partial_+ b_{++})$$

$$= i \sum_{n,m\in\mathbb{Z}} 2(-in) c_n b_m e^{-i(n+m)\sigma^+} + i \sum_{n,m\in\mathbb{Z}} (-im) c_n b_m e^{-i(n+m)\sigma^+}$$
$$= \sum_{n,m\in\mathbb{Z}} (2n+m) c_n b_m e^{-i(n+m)\sigma^+} = -\sum_{n,k\in\mathbb{Z}} (n+k) b_{k-n} c_n e^{-ik\sigma^+}$$
$$= -\sum_{k\in\mathbb{Z}} \left[\sum_{n\in\mathbb{Z}} (k-n) b_{k+n} c_{-n}\right] e^{-ik\sigma^+} = \sum_{k\in\mathbb{Z}} L_k^{(\mathrm{gh})} e^{-ik\sigma^+}.$$

$$L_k^{(\mathrm{gh})} = \sum_{n\in\mathbb{Z}} (n-k) b_{k+n} c_{-n}, \text{(classical)}$$

$$L_k^{(\mathrm{gh})} = \sum_{n\in\mathbb{Z}} (n-k) : b_{k+n} c_{-n} :, \text{(quantum)}$$

$$\left[L_n^{(\mathrm{gh})}, b_m\right] = \sum_{k\leq -2} (n-k)[b_{n+k} c_{-k}, b_m] - \sum_{k\geq -1} (n-k)[c_{-k} b_{n+k}, b_m]$$
$$= \sum_{k\leq -2} (n-k) b_{n+k} [c_{-k}, b_m] + \sum_{k\geq -1} (n-k)[c_{-k}, b_m] b_{n+k} = (n-m) b_{n+m}$$
$$\left[L_n^{(\mathrm{gh})}, c_m\right] = \sum_{k\leq -2} (n-k)[b_{n+k} c_{-k}, c_m] - \sum_{k\geq -1} (n-k)[c_{-k} b_{n+k}, c_m]$$
$$= -\sum_{k\leq -2} (n-k) c_{-k} \delta_{n+m+k,0} = -(2n+m) c_{n+m}$$

$$\left[L_n^{(\text{gh})}, L_m^{(\text{gh})}\right] =$$

$$= \sum_{k\le -2} (m-k)\left[L_n^{(\text{gh})}, b_{m+k}c_{-k}\right] - \sum_{k\ge -1} (m-k)\left[L_n^{(\text{gh})}, c_{-k}b_{m+k}\right]$$

$$= \sum_{k\le -2} (m-k)(n-m-k)b_{n+m+k}c_{-k} - \sum_{k\le -2} (m-k)(2n-k)b_{m+k}c_{n-k}$$

$$+ \sum_{k\ge -1} (m-k)(2n-k)c_{n-k}b_{m+k} - \sum_{k\ge -1} (m-k)(n-m-k)c_{-k}b_{n+m+k}$$

$$= \sum_{k\le -2} (m-k)(n-m-k)b_{n+m+k}c_{-k} - \sum_{q\le -n-2} (m-n-q)(n-q)b_{m+n+q}c_{-q}$$

$$+ \sum_{q\ge -n-1} (n-q)(m-n-q)c_{-q}b_{n+m+k} - \sum_{k\ge -1} (m-k)(n-m-k)c_{-k}b_{n+m+k}$$

$$= \sum_{q\le -n-2} \underbrace{[(m-q)(n-m-q)-(n-q)(m-n-q)]}_{(n-m)(n+m-q)} b_{q+m+n}c_{-q}$$

$$- \sum_{q\ge -1} [(m-q)(n-m-q)-(n-q)(m-n-q)]c_{-q}b_{q+m+n}$$

$$+ \sum_{q=-n-1}^{-2} (m-q)(n-m-q)b_{q+m+n}c_{-q} + \sum_{q=-n-1}^{-2} \underbrace{(m-q)(n-m-q)c_{-q}b_{q+m+n}}_{\text{In this range this is not normal ordered}}$$

$$= \sum_{-1\le q\le -n-2} (n-m)(n+m-q):b_{q+m+n}c_{-q}: + \sum_{q\le -n-1}^{-2} (n-q)(m-n-q)\delta_{n+m}$$

$$+ \sum_{q=-n-1}^{-2} [(m-q)(n-m-q)-(n-q)(m-n-q)]b_{q+m+n}c_{-q}$$

$$= (n-m)\sum_{q\in\mathbb{Z}} (n+m-q):b_{q+m+n}c_{-q}: -\delta_{m+n}\sum_{q=-n-1}^{-2} (n-q)(2n+q)$$

$$= (n-m)L_{m+n}^{(\text{gh})} - \frac{13}{6}(n^3-n)\delta_{n+m}$$

$$\left[L_n^{(\text{gh})}, L_m^{(\text{gh})}\right] = (n-m)L_{m+n}^{(\text{gh})} + \frac{-26}{12}(n^3-n)\delta_{n+m}.$$

$$c^{\text{ghost}} = -26$$

$$|\downarrow\rangle = c_1|0\rangle$$

$$c_n|\downarrow\rangle = c_nc_1|0\rangle = 0 \;\forall n>0,$$
$$c_0|\downarrow\rangle = c_0c_1|0\rangle \ne 0 \implies |\uparrow\rangle = c_0c_1|0\rangle,$$
$$b_n|\downarrow\rangle = b_nc_1|0\rangle = 0 \;\forall n\ge 0,$$
$$\langle\uparrow|\uparrow\rangle = \langle\downarrow|c_0^2|\downarrow\rangle = 0,$$
$$\langle\downarrow|\downarrow\rangle = \langle\downarrow|[b_0,c_0]|\downarrow\rangle = 0,$$
$$\langle\downarrow|\uparrow\rangle = \langle 0|c_{-1}c_0c_1|0\rangle = 1.$$

$$|0\rangle, |\downarrow\rangle = c_1|0\rangle, |\uparrow\rangle = c_0c_1|0\rangle, |\hat{0}\rangle = c_{-1}c_0c_1|0\rangle.$$

$$J_{gh} = -\sum_{k\ge 2} b_{-k}c_k + \sum_{k\ge -1} c_{-k}b_k,$$

$$J_{gh}|0\rangle = 0, J_{gh}|\downarrow\rangle = |\downarrow\rangle, J_{gh}|\uparrow\rangle = 2|\uparrow\rangle, J_{gh}|\hat{0}\rangle = 3|\hat{0}\rangle.$$

$$\begin{aligned} \left|\{n_i\},\{m_j\}\right\rangle &= c_{-n_1} \dots c_{-n_k} b_{-m_1} \dots b_{-m_q}|0\rangle, \\ J_{gh}\left|\{n_i\},\{m_j\}\right\rangle &= (k-q)\left|\{n_i\},\{m_j\}\right\rangle, \text{ for } n_i \geq -1, m_j \geq 2. \end{aligned}$$

$$L_n \sim 0, \tilde{L}_m \sim 0.$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{D}{12}n(n^2-1)\delta_{n+m}.$$

$$f_{nm}^k = \delta_{k,n+m}(n-m).$$

$$\tilde{\mathcal{F}}_i = -f_{ij}^k c^j b_k = -\sum_{j,k} (i-j)\delta_{k,i+j} c^j b_k$$

$$\tilde{\mathcal{F}}_i = -\sum_{j,k} (i-j)\delta_{k,i+j} c_{-j} b_k = -\sum_{j,k} (i-j) c_{-j} b_{i+j} = \sum_{j} (i-j) b_{i+j} c_{-j} = L_i^{gh}.$$

$$Q = c^i \mathcal{F}_i + \frac{1}{2} c^i \tilde{\mathcal{F}}_i = \sum_{n\in\mathbb{Z}} \left[c_{-n} L_n^{(\mathrm{m})} + \frac{1}{2} : c_{-n} L_n^{(\mathrm{gh})} : \right],$$

$$\left[L_n^{(\mathrm{tot})}, L_m^{(\mathrm{tot})}\right] = (n-m) L_{n+m}^{(\mathrm{tot})} + \frac{D-26}{12} n(n^2-1)\delta_{n+m}$$

$$Q^2 = \frac{1}{2}[Q,Q] = \frac{1}{2}\sum_{n,m} \left[\left[L_n^{(\mathrm{tot})}, L_m^{(\mathrm{tot})}\right] - (n-m) L_{n+m}^{(\mathrm{tot})}\right] c_{-n} c_{-m} = 0$$

$$[Q, b_n] = \mathcal{F}_n^{tot} = L_n^{(\mathrm{tot})},$$

$$\left[Q, L_n^{(\mathrm{tot})}\right] = [Q,[Q,b_n]] = [[Q,Q],b_n] - [Q,[Q,b_n]] \to 2\left[Q, L_n^{(\mathrm{tot})}\right] = [Q^2, b_n] = 0.$$

$$\begin{aligned} \left[L_n^{(\mathrm{tot})}, L_m^{(\mathrm{tot})}\right] &= \left[L_n^{(\mathrm{tot})}, [Q, b_m]\right] = \left[\left[L_n^{(\mathrm{tot})}, Q\right], b_m\right] + \left[Q, \left[L_n^{(\mathrm{tot})}, b_m\right]\right] \\ &= \left[Q, \left[L_n^{(\mathrm{gh})}, b_m\right]\right] = (n-m)[Q, b_{n+m}] = (n-m) L_{n+m}^{(\mathrm{tot})} \end{aligned}$$

- $\mathcal{H}^m \to \left[\alpha_n^\mu, \alpha_m^\nu\right] = \eta^{\mu\nu} n \delta_{n+m}; \left[\hat{X}^\mu, \hat{P}^\nu\right] = i\eta^{\mu\nu}$
- $\mathcal{H}^{gh} \to [b_n, c_m] = \delta_{n+m}$

$$\begin{aligned} |0, P\rangle &= e^{i\hat{X}\cdot P}|0\rangle \\ |\downarrow, P\rangle &= e^{i\hat{X}\cdot P}|\downarrow\rangle = e^{i\hat{X}\cdot P} c_1|0\rangle \\ |\uparrow, P\rangle &= e^{i\hat{X}\cdot P}|\uparrow\rangle = e^{i\hat{X}\cdot P} c_0 c_1|0\rangle \\ |\hat{0}, P\rangle &= e^{i\hat{X}\cdot P}|\hat{0}\rangle = e^{i\hat{X}\cdot P} c_{-1} c_0 c_1|0\rangle \end{aligned}$$

$$\langle 0, P'|c_{-1} c_0 c_1|0, P\rangle = \delta(P - P')$$

$$Q = \sum_{n\in\mathbb{Z}} \left(c_n L_{-n} + \frac{1}{2} : c_n L_{-n}^{(\mathrm{gh})} : \right),$$

$$L_n = \frac{1}{2}\sum_{k\in\mathbb{Z}} :\alpha_{n-k}\alpha_k:, L_n^{(\text{gh})} = \sum_{k\in\mathbb{Z}} (n-k): b_{n+k}c_{-k}:$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{D}{12}(n^3-n)\delta_{n+m}$$

$$\left[L_n^{(\text{gh})}, L_m^{(\text{gh})}\right] = (n-m)L_{n+m}^{(\text{gh})} + \frac{-26}{12}(n^3-n)\delta_{n+m}$$

$$\left[Q, N^{(\text{tot})}\right] = \left[Q, L_0^{(\text{tot})}\right] - \frac{1}{2}[Q, \alpha_0^2] = 0$$

$$Q|\psi\rangle = Qc_1|\phi_m\rangle = [Q, c_1]|\phi_m\rangle - c_1Q|\phi_m\rangle$$

$$[Q, c_n] = -\sum_{m\in\mathbb{Z}} (m+2n)c_{-m}c_{n+m}$$

$$\begin{aligned}[Q, c_1]|\phi_m\rangle &= -|\phi_m\rangle \otimes \sum_{m\in\mathbb{Z}} (m+2)c_{-m}c_{1+m}|0\rangle_{gh} \\ &= -|\phi_m\rangle \otimes (\cdots - c_3c_{-2} + c_1c_0 + 2c_0c_1 + 3c_{-1}c_2 + \cdots)|0\rangle_{gh} \\ &= -|\phi_m\rangle \otimes (-c_1c_0)|0\rangle_{gh} = c_1c_0|\phi_m\rangle\end{aligned}$$

$$\begin{aligned}Q|\psi\rangle &= c_1c_0|\phi_m\rangle - c_1\sum_{n\in\mathbb{Z}} c_nL_{-n}|\phi_m\rangle \\ &= c_1(c_0 + \underbrace{\cdots - c_2L_{-2}}_{\text{All zero over } |\phi_m\rangle} - c_1L_{-1} - c_0L_0 - \sum_{n\geq 1} c_{-n}L_n)|\phi_m\rangle \\ &= (L_0-1)c_0c_1|\phi_m\rangle + \sum_{n\geq 1} c_{-n}L_nc_1|\phi_m\rangle = 0,\end{aligned}$$

$$(L_0-1)|\phi_m\rangle = 0$$

$$L_n|\phi_m\rangle = 0, \forall n \geq 1,$$

$$(L_0-1)\mid \text{null}\rangle = 0, L_{n\geq 1}\mid \text{null}\rangle = 0, \mid \text{null}\rangle = \sum_{n\geq 1} L_{-n}|\chi_n\rangle$$

$$|\psi\rangle = A_\mu(P)\alpha_{-1}^\mu c_1|0, P\rangle.$$

$$\begin{aligned}Q\lambda(P)|0, P\rangle &= \sum_{n\in\mathbb{Z}} c_{-n}L_n|0, P\rangle = \lambda(P)(\underbrace{\cdots + c_2L_{-2}}_{\text{Identically zero}} + c_1L_{-1} + \underbrace{c_0L_0 + \cdots}_{\text{zero when } P^2=0})|0, P\rangle \\ &= \lambda(P)c_1L_{-1}|0, P\rangle \sim P_\mu\lambda(P)\alpha_{-1}^\mu c_1|0, P\rangle.\end{aligned}$$

$$c_1\mid \text{null}\rangle = Q|\Lambda\rangle.$$

$$\begin{aligned}|\phi_m\rangle &\longrightarrow c_1|\phi_m\rangle = |\psi\rangle \\ \mid \text{phys}\rangle &\longrightarrow Q|\psi\rangle = 0 \\ \mid \text{null}\rangle &\longrightarrow c_1\mid \text{null}\rangle = Q|\Lambda\rangle\end{aligned}$$

$$|\hat{\psi}\rangle = c_0c_1|\phi_m\rangle = |\phi_m\rangle \otimes |\uparrow\rangle$$

$$Q|\hat{\psi}\rangle = \underbrace{[Q, c_0 c_1]}_{=0} |\phi_m\rangle + c_0 c_1 Q|\phi_m\rangle$$

$$= c_0 c_1 \left(\cdots + c_1 L_{-1} + c_0 L_0 + \sum_{n\geq 1} c_{-n} L_n \right) |\phi_m\rangle = 0$$

$$\Longrightarrow L_n|\phi_m\rangle = 0 \forall n \geq 1$$

$$\ker(Q)|_{\mathrm{gh}=2} = \{|\hat{\psi}^*\rangle = c_0 c_1 |\phi_m^*\rangle\}_{L_{n\geq 1}|\phi_m^*\rangle = 0}$$

$$b_0|\hat{\psi}^*\rangle = c_1|\phi_m^*\rangle = |\psi^*\rangle$$

$$Q|\psi^*\rangle = Q b_0|\hat{\psi}^*\rangle = [Q, b_0]|\hat{\psi}^*\rangle - \underbrace{b_0 Q|\hat{\psi}^*\rangle}_{=0} = L_0^{(\mathrm{tot})}|\hat{\psi}^*\rangle.$$

$$|\hat{\psi}^*\rangle = \frac{1}{L_0^{(\mathrm{tot})}} Q|\psi^*\rangle = Q\left[\frac{1}{L_0^{(\mathrm{tot})}}|\psi^*\rangle\right] = Q\left[\frac{b_0}{L_0^{(\mathrm{tot})}}|\hat{\psi}^*\rangle\right]$$

$$0 = L_0^{(\mathrm{tot})}|\hat{\psi}^*\rangle = L_0^{(\mathrm{tot})} c_0 c_1 |\phi_m^*\rangle = \left(L_0 + L_0^{(\mathrm{gh})}\right) c_0 c_1 |\phi_m^*\rangle$$
$$= c_0 c_1 (L_0 - 1)|\phi_m^*\rangle$$

$$Q|\hat{0}\rangle = \underbrace{\sum_{k\in\mathbb{Z}} (k-2) c_{-k} c_{k-1} c_0 c_1 |0\rangle}_{=0 \text{ for the properties of } c_l|0\rangle} - c_{-1} Q c_0 c_1 |0\rangle = c_{-1} Q^2 c_1 |0\rangle = 0$$

$$\langle 0 \mid \hat{0} \rangle = \langle 0|c_{-1} c_0 c_1|0\rangle = 1$$

$$|\phi_i^{OCQ}\rangle \otimes |\downarrow\rangle = |\psi_i\rangle \text{ at } \#_{gh} = 1,$$
$$|\phi_i^{OCQ}\rangle \otimes |\uparrow\rangle = |\hat{\psi}_i\rangle \text{ at } \#_{gh} = 2,$$

$$\langle \psi_i \mid \hat{\psi}_j \rangle = G_{ij}$$

$$|\Psi\rangle = \int d^{26}P\left(t(P) c_1 e^{i\hat{X}\cdot P}|0\rangle + A_\mu(P) c_1 \alpha_{-1}^\mu e^{i\hat{X}\cdot P}|0\rangle + iB(P) c_0 e^{i\hat{X}\cdot P}|0\rangle + \cdots\right).$$

$$Q|\Psi\rangle = 0.$$

$$S[\Psi] = \frac{1}{2}\langle\Psi|Q|\Psi\rangle.$$

$$|\Psi\rangle \sim |\Psi\rangle + Q|\Lambda\rangle,$$

$$|\Psi\rangle = \int d^{26}P t(P) c_1 |0, P\rangle, t \in \mathbb{R}$$

$$S[t] = \frac{\alpha'}{2}\int d^{26}P t(P^2 + m^2) t \text{ with } m^2 = -\frac{1}{\alpha'}$$

$$|\Psi\rangle = \int d^{26}P\left(A_\mu(P) c_1 \alpha_{-1}^\mu + iB(P) c_0\right)|0, P\rangle, A_\mu, B \in \mathbb{R}$$

$$S[\Psi]=\frac{1}{2}\langle\Psi,Q\Psi\rangle+\frac{1}{3}\langle\Psi,\Psi,\Psi\rangle$$

$$\begin{aligned}(b_0-\bar{b}_0)|\Psi_c\rangle &=0\\(L_0-\bar{L}_0)|\Psi_c\rangle &=0\end{aligned}$$

$$\langle\Psi_1,\Psi_2\rangle\equiv\frac{1}{2}\langle\Psi_1|(c_0-\bar{c}_0)|\Psi_2\rangle$$

$$S[\Psi_c]=\frac{1}{2}\langle\Psi_c,Q_{\text{tot}}\Psi_c\rangle$$

$$S[X]=\frac{1}{2\pi\alpha'}\int_{\text{WS}}d^2\sigma\partial_+X^\mu\partial_-X_\mu$$

$$\begin{aligned}\sigma^+ &\longrightarrow f_+(\sigma^+)\\\sigma^- &\longrightarrow f_-(\sigma^-)\end{aligned}$$

$$\begin{aligned}X_L^\mu(\sigma^+) &\longrightarrow X_L^\mu\big(f_+(\sigma^+)\big)\\X_R^\mu(\sigma^-) &\longrightarrow X_R^\mu\big(f_-(\sigma^-)\big)\end{aligned}$$

$$X^\pm=\frac{1}{\sqrt{2}}(X^0\pm X^{D-1})$$

$$\mu=(\underbrace{+,-}_{\substack{0,D-1\\(\text{ lightcone })}},\underbrace{i}_{1,\dots,D-2}),$$

$$X_L^+(\sigma^+)=\frac{x_0^++c_0^+}{2}+\frac{\alpha'}{2}P^+\sigma^+,X_R^+(\sigma^-)=\frac{x_0^+-c_0^+}{2}+\frac{\alpha'}{2}P^+\sigma^-,$$

$$X^+=x_0^++\alpha'P^+\tau$$

$$\alpha_{n\neq0}^+=0$$

$$L_n=\frac{1}{2}\sum_{k\in\mathbb{Z}}\left(-2\alpha_{n-k}^-\alpha_k^++\alpha_{n-k}^i\alpha_k^i\right)=-\alpha_n^-\alpha_0^++\frac{1}{2}\sum_{k\in\mathbb{Z}}\alpha_{n-k}^i\alpha_k^i$$

$$\alpha_n^-=\frac{1}{2\alpha_0^+}\sum_{k\in\mathbb{Z}}\alpha_{n-k}^i\alpha_k^i$$

$$[\alpha_n^i,\alpha_m^j]=n\delta^{ij}\delta_{n+m,0},i,j=1,\cdots,D-2.$$

$$\alpha_{-n_1}^{i_1}\dots\alpha_{-n_k}^{i_k}|0,P\rangle.$$

$$L_0=\frac{1}{2}(\alpha_0)^2+\frac{1}{2}\sum_{k\neq0}\alpha_{-k}^i\alpha_k^i$$

$$L_0=\frac{1}{2}(\alpha_0)^2+\sum_{k\geq1}\alpha_{-k}^i\alpha_k^i,\ \text{(quantum)}$$

$$N^{(\mathrm{lc})} = \sum_{k\geq 1} \alpha_{-k}^{i}\alpha_{k}^{i}$$

$$L_0 - a = \frac{1}{2}(\alpha_0)^2 + N^{(\mathrm{lc})} - a = 0$$

$$t(P)|0,P\rangle$$

$$\alpha' m^2 = -a.$$

$$A_i(P)\alpha_{-1}^{i}|0,P\rangle,$$

$$\alpha' m^2 = 1 - a.$$

$$\alpha' m^2 = 0$$

$$a = 1,$$

$$\left(\xi_i\alpha_{-2}^{i} + \xi_{ij}\alpha_{-1}^{i}\alpha_{-1}^{j}\right)|0,P\rangle,$$

$$\alpha' m^2 = 1.$$

$$\left(\Box + \Box^{\text{traceful}}\right)_{SO(D-2)} = \left(\Box^{\text{traceless}}\right)_{SO(D-1)},$$

$$(D-2) + \frac{1}{2}(D-2)(D-1) = \frac{1}{2}D(D-1) - 1$$

$$\begin{aligned}\hat{H} &= \frac{1}{2}\left(\hat{P}^2 + \omega^2\hat{x}^2\right)\\ &= \left(a^\dagger a + a a^\dagger\right) = a^\dagger a + \frac{1}{2}\end{aligned}$$

$$\begin{aligned}L_0 &= \frac{1}{2}(\alpha_0)^2 + \frac{1}{2}\sum_{k\neq 0} \alpha_{-k}^{i}\alpha_{k}^{i}\\ &= \frac{1}{2}(\alpha_0)^2 + \sum_{k\geq 1} \alpha_{-k}^{i}\alpha_{k}^{i} + \frac{1}{2}\sum_{k\geq 1} \left[\alpha_{-k}^{i}, \alpha_{k}^{i}\right]\\ &= \frac{1}{2}(\alpha_0)^2 + \sum_{k\geq 1} \alpha_{-k}^{i}\alpha_{k}^{i} + \frac{1}{2}(D-2)\sum_{k\geq 1} k = L_0^{\text{quantum}} - a\end{aligned}$$

$$a = -\frac{1}{2}(D-2)\sum_{k\geq 1} k = \infty.$$

$$\begin{aligned}\sum_{k=1}^{\infty} k \longrightarrow \sum_{k=1}^{\infty} k e^{-\epsilon k} &= -\frac{1}{2}\frac{1}{1-\cosh\,\epsilon} = \frac{1}{\epsilon^2} - \frac{1}{12} + \frac{\epsilon^2}{240} + \mathcal{O}(\epsilon^4)\\ &\overset{\epsilon\to 0}{=} \frac{1}{\epsilon^2} - \frac{1}{12}\end{aligned}$$

$$S = \frac{1}{2\pi\alpha'}\int_{\mathrm{WS}} d^2\sigma\left(\partial_+ X^i\partial_- X_i + \frac{r}{\epsilon^2}\right),$$

$$\left(\sum_{k=1}^{\infty} k\right)_{\text{renorm}} = -\frac{1}{12}.$$

$$\sum_{k=1}^{\infty} k \rightarrow \zeta(-1) = -\frac{1}{12}$$

$$a^{(\text{renorm})} = -\frac{D-2}{2}\left(-\frac{1}{12}\right) = \frac{D-2}{24}.$$

$$\alpha_{-k_1}^{i_1} \ldots \alpha_{-k_n}^{i_n} |0\rangle$$

$$\mathrm{Tr}\left\{q^{L_0^{D=24}}\right\} = \left(\sum_s \langle s| q^{L_0^{D=1}} |s\rangle\right)^{24}$$

$$\langle s \mid s' \rangle = \delta_{s,s'}.$$

$$\begin{aligned}
\mathrm{Tr}\left\{q^{L_0^{D=1}}\right\} &= \sum_{n_1,\ldots,n_\infty=0}^{\infty} \langle\{n_i\}| q^{L_0} |\{n_i\}\rangle \\
&= \sum_{n_1,\ldots,n_\infty=0}^{\infty} q^{n_1+2n_2+\cdots+kn_k+\cdots} \\
&= \sum_{n_1=0}^{\infty} q^{n_1} \sum_{n_2=0}^{\infty} (q^2)^{n_2} \cdots \sum_{n_k=0}^{\infty} \left(q^k\right)^{n_k} \cdots \\
&= \frac{1}{1-q}\frac{1}{1-q^2}\cdots\frac{1}{1-q^k}\cdots \\
&= \prod_{k=1}^{\infty} \frac{1}{1-q^k}
\end{aligned}$$

$$\mathrm{Tr}\{q^{L_0}\} = \left(\prod_{k=1}^{\infty} \frac{1}{1-q^k}\right)^{D-2} = \mathcal{P}(q)$$

$$\mathcal{P}(q) \overset{q\to 0}{=} 1 + 24q + 324q^2 + \cdots$$

$$\#_{\text{states}}(N) \sim \frac{1}{\sqrt{2}} N^{-\frac{27}{4}} \exp\left(4\pi\sqrt{N}\right)$$

$$\log \#_{\text{states}} \sim 4\pi\sqrt{N} \sim 4\pi\alpha' E$$

$$E \sim m = \sqrt{\frac{N-1}{\alpha'}} \sim \sqrt{\frac{N}{\alpha'}}.$$

$$S_{\text{micro}}(E) = K \log \#_{\text{states}}(E) = K 4\pi\alpha' E,$$

$$\frac{1}{KT} = \frac{1}{K}\frac{\partial S}{\partial E} = 4\pi\alpha'$$

$$KT_H \equiv \frac{1}{4\pi\alpha'}$$

$$\begin{cases} a = 1 \\ D = 26 \end{cases}$$

$$L_0^{\text{tot}} = L_0^{(\text{matter})} + L_0^{(\text{ghost})} = \frac{1}{2}(\alpha_0)^2 + N^{(\text{matter})} + N^{(\text{ghost})},$$

$$N^{(\text{matter})} = \frac{1}{2}\sum_{k\neq 0} \alpha_{-k}\cdot\alpha_k = N^{(\pm)} + N^{\perp}$$

$$N^{(\text{ghost})} = \sum_{k\in\mathbb{Z}} kb_{-k}c_k$$

$$N^{(\text{ghost})} = \sum_{k\in\mathbb{Z}} kb_{-k}c_k = \sum_{k\geq 1}(b_{-k}c_k - b_kc_{-k}) = \sum_{k\geq 1}(b_{-k}c_k + c_{-k}b_k) - \sum_{k\geq 1} k$$

$$N^{(\pm)} = \frac{1}{2}\sum_{k\neq 0} \alpha_{-k}^{\bar{\mu}}\alpha_k^{\bar{\nu}}\eta_{\bar{\mu}\bar{\nu}} = \sum_{k\geq 1}\alpha_{-k}^{\bar{\mu}}\alpha_k^{\bar{\nu}}\eta_{\bar{\mu}\bar{\nu}} + \frac{1}{2}\eta_{\bar{\mu}}^{\bar{\mu}}\sum_{k\geq 1} k$$

$$= \sum_{k\geq 1}\alpha_{-k}^{\bar{\mu}}\alpha_k^{\bar{\nu}}\eta_{\bar{\mu}\bar{\nu}} + \sum_{k\geq 1} k$$

$$\text{BRST} = \text{matter} + \text{ghosts} = (\text{Le} + \text{transverse}) + \text{ghosts}.$$

$$\int \mathcal{D}b\mathcal{D}c\mathcal{D}X^+\mathcal{D}X^-\mathcal{D}X^i(\ldots)e^{S_{\text{BRST}}[X^\pm,X^i,b,c]} = \int \mathcal{D}X^i dx_0^+ dx_0^-(\ldots)e^{S_{\text{LC}}[x_0^\pm,X^i]}$$

$$h_{\alpha\beta} = \hat{h}_{\alpha\beta}.$$

$$\delta_{\text{conf-diff}}\hat{h}_{\alpha\beta} = \nabla_\alpha\xi_\beta + \nabla_\beta\xi_\alpha = -2\omega(\sigma)\hat{h}_{\alpha\beta}$$

$$\delta_{\text{Weyl}}\hat{h}_{\alpha\beta} = 2\omega(\sigma)\hat{h}_{\alpha\beta},$$

$$\left(\delta_{\text{conf diff}} + \delta_{\text{Weyl}}\right)\hat{h}_{\alpha\beta} = 0.$$

$$S = \int d\tau \frac{1}{2}(\dot{X}^2 - m^2)$$

$$\tau \longrightarrow \tau + \delta\tau \text{ with } \delta\tau = \mathcal{G}$$

$$\hat{H} \mid \text{state}\rangle = \left(\hat{P}^2 + m^2\right) \mid \text{state}\rangle = 0$$

$$\hat{h}_{\alpha\beta} = \begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix},$$

$$\partial_+\xi_+ = \partial_+\xi^- = 0$$
$$\partial_-\xi_- = \partial_-\xi^+ = 0$$

$$S[X,b,c] = \frac{1}{2\pi\alpha'}\int_{\text{WS}} d^2\sigma_\pm\partial_+X^\mu\partial_-X_\mu - \frac{i}{2\pi}\int_{\text{WS}} d^2\sigma_\pm(b_{++}\partial_-c^+ + b_{--}\partial_+c^-)$$

$$\tau \to -it.$$

$$\tau + \sigma \to -it + \sigma = -i(t + i\sigma) \equiv -iw,$$
$$\tau - \sigma \to -it - \sigma = -i(t - i\sigma) \equiv -i\bar{w},$$

$$S[X,b,c] = \frac{1}{2\pi\alpha'}\int_{\mathrm{WS}} d^2w\partial X^\mu \bar{\partial} X_\mu + \frac{1}{2\pi}\int_{\mathrm{WS}} d^2w(b\bar{\partial}c + \bar{b}\partial\bar{c}),$$

$$\partial \ = \partial_w, \bar{\partial} = \partial_{\bar{w}}$$
$$b \ = b_{ww}, \bar{b} = b_{\bar{w}}\bar{w}$$
$$c \ = c^w, \bar{c} = c^{\bar{w}}$$

$$w \longrightarrow f(w) \ \text{ such that } \bar{\partial} f = 0$$
$$\bar{w} \longrightarrow \bar{f}(\bar{w}) \ \text{ such that } \partial \bar{f} = 0$$

$$X(w) \xrightarrow{f} X(f(w)).$$

$$b_{ww}(dw)^2 = b_{zz}(dz)^2 \Longrightarrow b(w) = \left(\frac{dz}{dw}\right)^2 b(z).$$

$$b(w) \xrightarrow{f} [f'(w)]^2 b(f(w)).$$

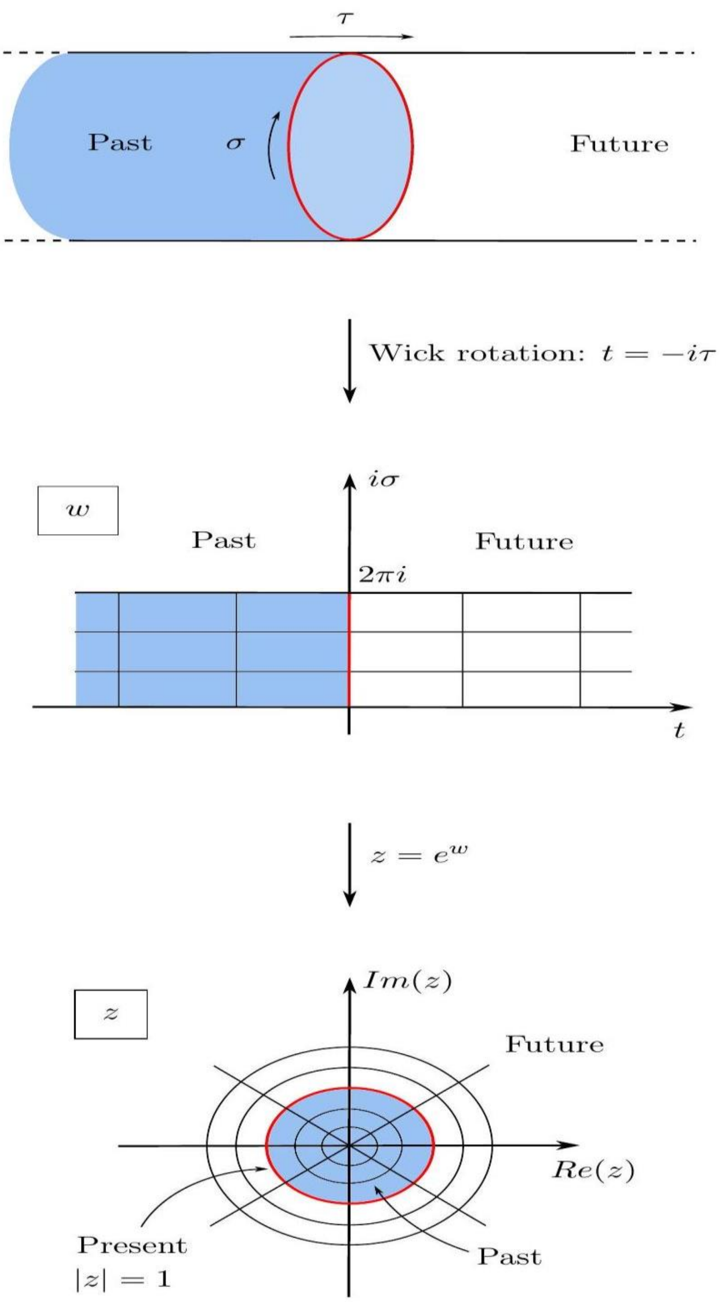
$$c^w\partial_w = c^z\partial_z \Longrightarrow c(w) = \left(\frac{dz}{dw}\right)^{-1} c(z)$$

$$c(w) \xrightarrow{f} [f'(w)]^{-1} c(f(w)),$$

$$z = e^w$$

$$z = 0 \ \longrightarrow t = -\infty,$$
$$z = \infty \ \longrightarrow t = +\infty,$$
$$|z| = 1 \ \longrightarrow t = 0.$$

$$\phi_{\mathrm{cyl}}(w,\bar{w}) = \left(\frac{dz}{dw}\right)^h \left(\frac{d\bar{z}}{d\bar{w}}\right)^{\bar{h}} \phi_{\mathbb{C}}(z,\bar{z})$$



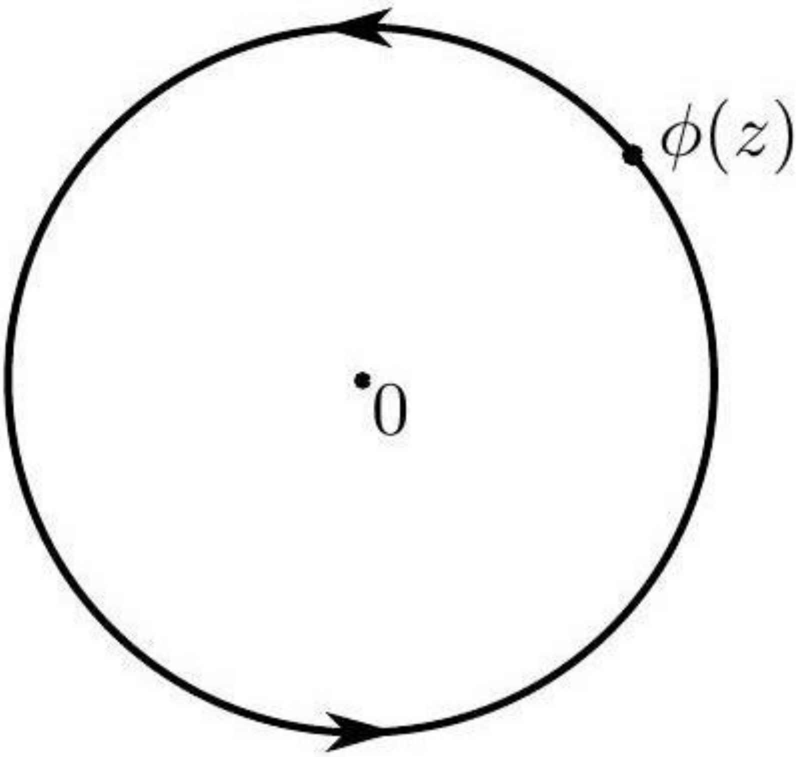
$$\phi_{\text{cyl}}^{(h)}(w) = \sum_{n\in\mathbb{Z}} \phi_n e^{-nw} = \sum_{n\in\mathbb{Z}} \phi_n (z(w))^{-n} = \sum_{n\in\mathbb{Z}} \phi_n z^{-n}$$

$$\phi_{\mathbb{C}}^{(h)}(z) = \left(\frac{dw}{dz}\right)^h \phi_{\text{cyl}}^{(h)}(w) = \left(\frac{\text{dlog } z}{dz}\right)^h \phi_{\text{cyl}}^{(h)}(w) = z^{-h} \sum_{n\in\mathbb{Z}} \phi_n z^{-n}$$

$$\phi_{\mathbb{C}}^{(h)}(z) = \sum_{n\in\mathbb{Z}} \phi_n z^{-n-h}$$

$$\phi_n = \oint_0 \frac{dz}{2\pi i} z^{n+h-1} \phi^{(h)}(z)$$

$$
\begin{aligned}
j^{\mu}(z) &\equiv i\sqrt{\frac{2}{\alpha'}}\partial X^{\mu} = \sum_{n\in\mathbb{Z}} \alpha_n^{\mu} z^{-n-1} \\
b(z) &= \sum_{n\in\mathbb{Z}} b_n z^{-n-2} \\
c(z) &= \sum_{n\in\mathbb{Z}} c_n z^{-n+1}
\end{aligned}
$$



$$z'(z) = z + \epsilon(z)$$

$$\phi^{(h)}(z) \longrightarrow \phi'(h)(z') = \left(\frac{dz}{dz'}\right)^{h} \phi^{(h)}(z)$$

$$
\begin{aligned}
\phi'(h)(z') &= \phi'(h) \\
&= (1 - h\partial\epsilon(z))\phi^{(h)}(z)
\end{aligned}
$$

$$
\begin{aligned}
\phi'^{(h)}(z) &= (1 - h\partial\epsilon(z))\phi(z - \epsilon(z)) \\
&= (1 - h\partial\epsilon(z))(\phi(z) - \epsilon(z)\partial\phi(z)) \\
&= \phi(z) - h\partial\epsilon(z)\phi(z) - \epsilon(z)\partial\phi(z) + \mathcal{O}(\epsilon^2) \\
&= \phi(z) - (\epsilon\partial\phi(z) + h(\partial\epsilon)\phi(z)) + \mathcal{O}(\epsilon^2) \\
&= \phi(z) + \delta_{\epsilon}\phi(z) + \mathcal{O}(\epsilon^2)
\end{aligned}
$$

$$\delta_{\epsilon}\phi(z) = -(\epsilon\partial\phi(z) + h(\partial\epsilon)\phi(z)).$$

$$\epsilon(z) = \sum_{n\in\mathbb{Z}} \epsilon_n z^{-n+1}$$

$$\delta_\epsilon(z) = \epsilon(z)\partial_z = \sum_{n\in\mathbb{Z}} \epsilon_n z^{-n+1}\partial_z = -\sum_{n\in\mathbb{Z}} \epsilon_n l_{-n},$$

$$l_n = -z^{n+1}\partial_z$$

$$[l_n, l_m] = (n-m)l_{n+m}.$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{\epsilon_1}\overleftrightarrow{\partial}\epsilon_2.$$

$$f(z) = e^{\epsilon(z)\partial_z}z.$$

$$T[\phi_1(w_1)\phi_2(w_2)] = \begin{cases}\phi_1(w_1)\phi_2(w_2) & \text{if Re}(w_1) > \text{Re}(w_2)\\ \phi_2(w_2)\phi_1(w_1) & \text{if Re}(w_1) < \text{Re}(w_2)\end{cases}.$$

$$R[\phi_1(z_1)\phi_2(z_2)] = \begin{cases}\phi_1(z_1)\phi_2(z_2) & \text{if } |z_1| > |z_2|\\ \phi_2(z_2)\phi_1(z_1) & \text{if } |z_1| < |z_2|\end{cases}.$$

$$[\phi_1(z_1), \phi_2(z_2)]\big|_{|z_1|=|z_2|} = \lim_{\varepsilon\to 0^+}\{\phi_1(z_1)\phi_2(z_2)|_{|z_1|=|z_2|+\varepsilon} - \phi_2(z_2)\phi_1(z_1)|_{|z_1|=|z_2|-\varepsilon}\}.$$

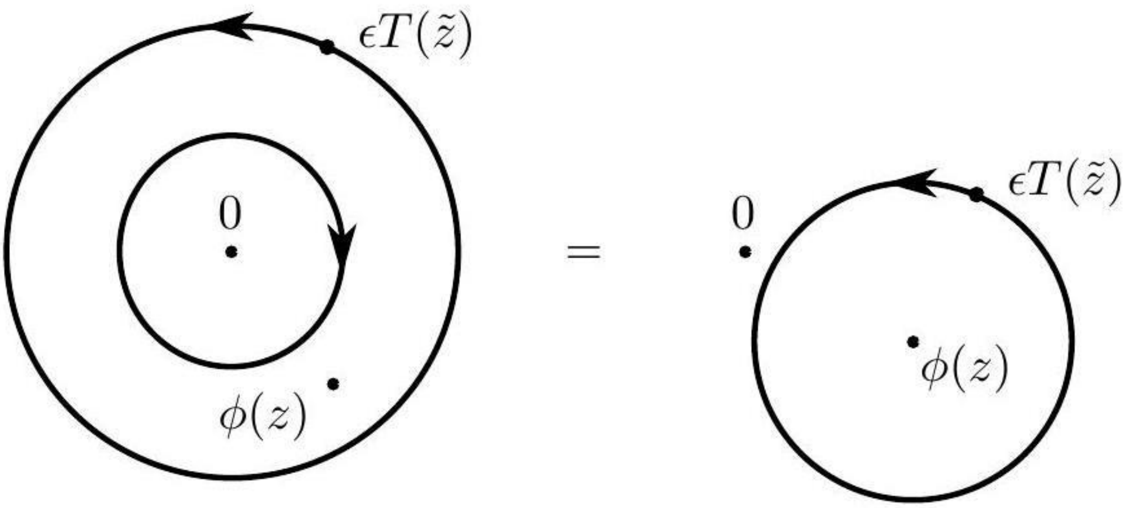
$$\begin{cases}T_{zz} = T(z)\\ T_{\bar{z}\bar{z}} = \bar{T}(\bar{z})\end{cases}.$$

$$T(z) = \sum_{n\in\mathbb{Z}} L_n z^{-n-2}$$

$$T_\epsilon = \oint_0 \frac{dz}{2\pi i}\epsilon(z)T(z)$$

$$\delta_\epsilon\phi(z) = -[T_\epsilon, \phi(z)].$$

$$\begin{aligned}\delta_\epsilon\phi(z) &= -\left[\oint_0 \frac{d\tilde{z}}{2\pi i}\epsilon(\tilde{z})T(\tilde{z}), \phi(z)\right]\\ &= -\left[\left(\oint_{|\tilde{z}|>|z|}\frac{d\tilde{z}}{2\pi i} - \oint_{|\tilde{z}|<|z|}\frac{d\tilde{z}}{2\pi i}\right)\epsilon(\tilde{z})T(\tilde{z})\phi(z)\right]\\ &= -\oint_z \frac{d\tilde{z}}{2\pi i}\epsilon(\tilde{z})T(\tilde{z})\phi(z)\end{aligned}$$



$$\phi_i(z_i)\phi_j(z_j) = \sum_k C_{ij}^k(z_i - z_j)\phi_k(z_j),$$

$$\phi_i^{(h_i)}(z_i)\phi_j^{(h_j)}(z_j) = \sum_k C_{ij}^k \phi_k^{(h_k)}(z_j)\frac{1}{(z_i - z_j)^{h_i+h_j-h_k}},$$

$$T(\tilde{z})\phi^{(h)}(z) = \sum_{k\geq 1}\frac{[T\phi]_k(z)}{(\tilde{z}-z)^k} + \varrho\,(k<1),$$

$$\delta_\epsilon\phi(z) = -\oint_z \frac{d\tilde{z}}{2\pi i}\epsilon(\tilde{z})T(\tilde{z})\phi(z) \overset{\text{must}}{=} -(\epsilon\partial\phi(z) + h(\partial\epsilon)\phi(z)).$$

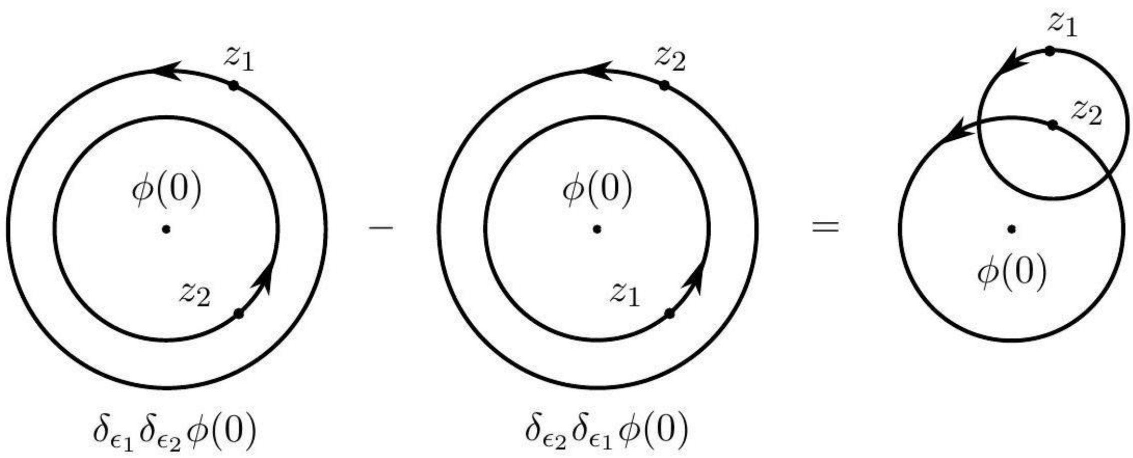
$$k = 1 \Longrightarrow [T\phi]_1(z) = \partial\phi(z)$$

$$k = 2 \Longrightarrow [T\phi]_2(z) = h\phi(z)$$

$$k > 2 \Longrightarrow [T\phi]_{k>2}(z) = 0.$$

$$T(\tilde{z})\phi^{(h)}(z) = \frac{h\phi(z)}{(\tilde{z}-z)^2} + \frac{\partial\phi(z)}{(\tilde{z}-z)} + (\,\varsigma\,).$$

$$\begin{aligned}
[\delta_{\epsilon_1},\delta_{\epsilon_2}]\phi(0) &= \delta_{\epsilon_1}\delta_{\epsilon_2}\phi(0) - \delta_{\epsilon_2}\delta_{\epsilon_1}\phi(0)\\
&= \left[T_{\epsilon_1},\left[T_{\epsilon_2},\phi(0)\right]\right] - \left[T_{\epsilon_2},\left[T_{\epsilon_1},\phi(0)\right]\right]\\
&= \oint_0 \frac{dz_2}{2\pi i}\oint_{z_2}\frac{dz_1}{2\pi i}\epsilon_1(z_1)\epsilon_2(z_2)T(z_1)T(z_2)\phi(0)\\
&= \delta_{\epsilon_1}\overset{\leftrightarrow}{\partial}\epsilon_2\phi(0)\\
&= -\oint_0 \frac{dz_2}{2\pi i}\left(\epsilon_1\partial\epsilon_2(z_2) - \epsilon_2\partial\epsilon_1(z_2)\right)T(z_2)\phi(0)
\end{aligned}$$



$$T(z_1)T(z_2) = \sum_{k\geq 1} \frac{[TT]_k(z_2)}{(z_1 - z_2)^k} + \tau\ (k < 1),$$

$$\sum_{k\geq 1} \oint_0 \frac{dz_2}{2\pi i} \epsilon_2(z_2) \frac{\partial^{k-1}\epsilon_1(z_2)}{(k-1)!} [TT]_k(z_2)\phi(0) \overset{\text{must}}{=} -\oint_0 \frac{dz_2}{2\pi i} \big(\epsilon_1 \partial\epsilon_2(z_2) - \epsilon_2 \partial\epsilon_1(z_2)\big) T(z_2)\phi(0)$$

$$[TT]_1 = \alpha\partial T.$$

$$\oint_0 \frac{dz_2}{2\pi i} \epsilon_2\epsilon_1 \alpha \partial T(z_2)\phi(0) \overset{\text{IBP}}{=} -\alpha \oint_0 \frac{dz_2}{2\pi i} \partial(\epsilon_2\epsilon_1) T(z_2)\phi(0),$$

$$[TT]_2 = \beta T$$

$$\beta \oint_0 \frac{dz_2}{2\pi i} \epsilon_2 \partial(\epsilon_1) T(z_2)\phi(0)$$

$$[TT]_3 = 0$$

$$[TT]_4 = \hbar \equiv \frac{c}{2}$$

$$\frac{c}{2} \oint_0 \frac{dz_2}{2\pi i} \frac{1}{6} \epsilon_2 \partial^3 \epsilon_1(z_2)\phi(0) = 0$$

$$T(z_1)T(z_2) = \frac{c}{2} \frac{1}{(z_1 - z_2)^4} + \frac{2T(z_2)}{(z_1 - z_2)^2} + \frac{\partial T(z_2)}{(z_1 - z_2)} + (\ \hbar\ )$$

$$L_n = \oint_0 \frac{dz}{2\pi i} z^{n+1} T(z)$$

$$[L_n, L_m] = \oint_0 \frac{d\tilde{z}}{2\pi i} \oint_{\tilde{z}} \frac{dz}{2\pi i} \tilde{z}^{m+1} z^{n+1} T(\tilde{z}) T(z)$$
$$= (n-m) L_{n+m} + \frac{c}{12} n(n^2-1) \delta_{n+m,0}$$

$$\left[L_n, \phi^{(h)}(z)\right] = z^n (z\partial + (n+1)h) \phi^{(n)}(z)$$
$$[L_n, \phi_m] = (n(h-1)-m) \phi_{n+m}$$

$$\delta_\epsilon T(z) = -[T_\epsilon, T(z)]$$
$$= -\frac{c}{12} \partial^3 \epsilon(z) - 2(\partial\epsilon) T(z) - \epsilon \partial T(z)$$

$$\epsilon_*(z) = \alpha + \beta z + \gamma z^2$$

$$L_{-1} \longrightarrow -\frac{d}{dz}$$

$$L_0 \longrightarrow -z\frac{d}{dz}$$
$$L_1 \longrightarrow -z^2\frac{d}{dz}$$

$$e^{-AL_{-1}} : z \longrightarrow z + A$$
$$e^{-BL_0} : z \longrightarrow e^B z$$
$$e^{-CL_1} : z \longrightarrow \frac{z}{1+Cz}$$

$$f(z) = \frac{Az+B}{Cz+D}$$

$$z \xrightarrow{\mathcal{I}} \frac{1}{z} \xrightarrow{\mathcal{T}} \frac{1}{z} + C \xrightarrow{\mathcal{I}} \frac{1}{\frac{1}{z}+C} = \frac{z}{1+Cz}.$$

$$\lim_{z\to 0} T(z)|0\rangle = \text{ך}$$

$$\lim_{z\to 0} T(z)|0\rangle = \lim_{z\to 0} \sum_{n\in\mathbb{Z}} L_n z^{-n-2} |0\rangle = \text{ב}$$

$$L_n|0\rangle = 0 \ \forall n \geq -1$$

$$\langle 0| L_n = 0 \ \forall n \leq 1$$

$$L_{-1}, L_0, L_1 |0\rangle = 0$$
$$\langle 0| L_{-1}, L_0, L_1 = 0$$

$$\lim_{z\to 0} \phi^{(h)}(z)|0\rangle = \lim_{z\to 0} \sum_{n\in\mathbb{Z}} \phi_n z^{-n-h} |0\rangle$$

$$\phi_n|0\rangle = 0 \ \forall n \geq -h+1,$$
$$\langle 0|\phi_n = 0 \ \forall n \leq h-1.$$

$$\phi(z) \leftrightarrow |\phi\rangle = \phi(0)|0\rangle.$$

$$\phi^{(h)}(z) \leftrightarrow \left|\phi^{(h)}\right\rangle = \phi^{(h)}(0)|0\rangle = \phi_{-h}|0\rangle.$$

$$\begin{aligned}
\phi^{(h)}(0)|0\rangle &= \phi_{-h}|0\rangle \\
\partial\phi^{(h)}(0)|0\rangle &= \phi_{-h-1}|0\rangle \\
\frac{1}{2}\partial^2\phi^{(h)}(0)|0\rangle &= \phi_{-h-2}|0\rangle \\
&\vdots \\
\frac{1}{n!}\partial^n\phi^{(h)}(0)|0\rangle &= \phi_{-h-n}|0\rangle
\end{aligned}$$

$$L_0\phi_n|0\rangle = -n\phi_n|0\rangle$$

$$\phi(z) = e^{zL_{-1}}\phi(0)e^{-zL_{-1}}$$

$$\begin{aligned}
e^{zL_{-1}}|\phi\rangle &= e^{zL_{-1}}\phi(0)|0\rangle = e^{zL_{-1}}\phi(0)e^{-zL_{-1}}e^{zL_{-1}}|0\rangle \\
&= \phi(z)e^{zL_{-1}}|0\rangle = \phi(z)|0\rangle
\end{aligned}$$

$$\text{BPZ: } \phi^{(h)}(z) \longrightarrow \mathcal{I} \circ \phi^{(h)}(z)$$

$$\mathcal{I}(z) = -\frac{1}{z}$$

$$\text{BPZ}(|\phi\rangle) = \langle 0|\mathcal{I} \circ \phi(0) = \langle 0|\phi\left(-\frac{1}{z}\right)\frac{1}{z^{2h}}\bigg|_{z=0}$$

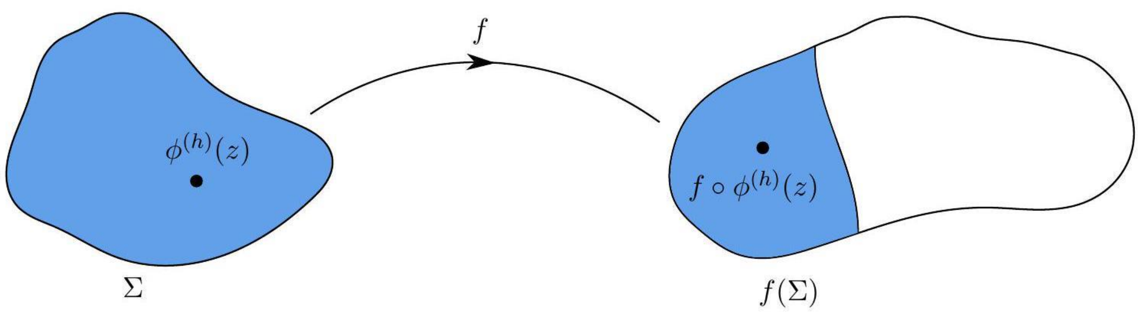
$$\left|\phi^{(h)}\right\rangle = \phi_{-h}|0\rangle \Longrightarrow \text{BPZ}\left(\left\langle\phi^{(h)}\right|\right) = \langle 0|\phi_h(-1)^{2h}$$

$$\text{BPZ}(\phi_n) = (-1)^{n-h}\phi_{-n}.$$

$$\langle\phi_1, \phi_2\rangle = \langle\phi_1 \mid \phi_2\rangle = \langle\phi_1(\infty)\phi_2(0)\rangle = \langle\mathcal{I} \circ \phi_1(0)\phi_2(0)\rangle$$

$$\begin{aligned}
(|\phi\rangle)^\dagger &= \langle\phi| = \text{BPZ}(|\phi\rangle) \Leftrightarrow |\phi\rangle \\
(|\phi\rangle)^\dagger &= -\langle\phi| = -\text{BPZ}(|\phi\rangle) \Leftrightarrow |\phi\rangle
\end{aligned}$$

$$f \circ \phi^{(h)}(z) = [f'(z)]^h\phi^{(h)}(f(z)).$$



$$f \circ \phi(z) = U_f\phi(z)U_f^{-1},$$

$$U_f = e^{\sum_{n\in\mathbb{Z}} v_n L_{-n}},$$

$$v(z) = \sum_{n\in\mathbb{Z}} v_n z^{n+1}$$

$$e^{v(z)\partial_z} z = f(z)$$

$$L_0 |\phi^{(h)}\rangle = h|\phi^{(h)}\rangle$$
$$L_{n>0}|\phi^{(h)}\rangle = 0$$

$$L_{-k_1} \dots L_{-k_{n-1}} L_{-k_n}|\phi^{(h)}\rangle \; k_i > 0 \forall i.$$

$$L_0\left(L_{-k_1} \dots L_{-k_n}|\phi^{(h)}\rangle\right) = \left(h + \sum_i k_i\right)\left(L_{-k_1} \dots L_{-k_n}|\phi^{(h)}\rangle\right).$$

$$\langle \phi_1(z_1) \dots \phi_n(z_n)\rangle = \langle 0|R[\phi_1(z_1) \dots \phi_n(z_n)]|0\rangle,$$

$$\begin{aligned}\langle \phi_1(z_1) \dots \phi_n(z_n)\rangle &= \langle 0|\phi_1(z_1) \dots \phi_n(z_n)|0\rangle \\ &= \langle 0|U_f^{-1}U_f\phi_1(z_1)U_f^{-1}U_f \dots U_f^{-1}U_f\phi_n(z_n)U_f^{-1}U_f|0\rangle \\ &= \langle 0|U_f\phi_1(z_1)U_f^{-1}U_f \dots U_f^{-1}U_f\phi_n(z_n)U_f^{-1}|0\rangle \\ &= \langle 0|f\circ\phi_1(z_1) \dots f\circ\phi_n(z_n)|0\rangle.\end{aligned}$$

$$\langle \phi^{(h)}(z)\rangle = \langle \phi^{(h)}(0)\rangle = \langle 0 \mid \phi^{(h)}\rangle,$$

$$\langle \phi^{(h)}(z)\rangle = A\delta_{h,0}$$

$$f(\xi) = \frac{\Lambda}{(z_i - z_j)}(\xi - z_j)$$

$$\begin{aligned}\langle f\circ\phi_i(z_i) f\circ\phi_j(z_j)\rangle &= \left(\frac{\Lambda}{(z_i - z_j)}\right)^{h_i+h_j} \langle \phi_i(\Lambda)\phi_j(0)\rangle \\ &= \left(\frac{\Lambda}{(z_i - z_j)}\right)^{h_i+h_j} \underbrace{\langle \phi_i(\Lambda)\phi_j(0)\rangle \cdot \frac{\Lambda^{-2h_i}}{\Lambda^{-2h_i}}}_{\text{for } \Lambda\to\infty \text{ it is the BPZ scalar product}} \\ &\overset{\Lambda\to\infty}{=} \langle \phi_i \mid \phi_j\rangle \left(\frac{1}{(z_i - z_j)}\right)^{h_i+h_j} \lim_{\Lambda\to\infty} \Lambda^{h_j-h_i}.\end{aligned}$$

$$\langle \phi_i(z_i)\phi_j(z_j)\rangle = \frac{G_{ij}}{(z_i - z_j)^{2h}}\delta_{h_i,h_j}$$

$$G_{ij} = \langle \phi_i \mid \phi_j\rangle.$$

$$f(\xi) = \frac{(z_j - z_i)}{(z_j - z_k)}\frac{(\xi - z_k)}{(\xi - z_i)}$$

$$\langle \phi_i^{(h_i)}(z_i)\phi_j^{(h_j)}(z_j)\phi_k^{(h_k)}(z_k)\rangle = \frac{C_{ijk}}{(z_i - z_j)^{h_i+h_j-h_k}(z_j - z_k)^{-h_i+h_j+h_k}(z_i - z_k)^{h_i-h_j+h_k}}$$

$$C_{ijk} = \langle \phi_i | \phi_j(1) | \phi_k \rangle = \langle \mathcal{I} \circ \phi_i(0) \phi_j(1) \phi_k(0) \rangle.$$

$$S_{\text{matter}} = \frac{1}{2\pi\alpha'} \int_{\text{WS}} d^2 z \partial X(z, \bar{z}) \cdot \bar{\partial} X(z, \bar{z})$$

$$\partial \bar{\partial} X^\mu = 0.$$

$$X^\mu(z, \bar{z}) = X^\mu(z) + \bar{X}^\mu(\bar{z})$$

$$X^\mu(z) = \frac{X_0^\mu + c^\mu}{2} - i \frac{\alpha'}{2} P^\mu \log z + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu z^{-n}$$
$$\bar{X}^\mu(\bar{z}) = \frac{X_0^\mu - c^\mu}{2} - i \frac{\alpha'}{2} P^\mu \log \bar{z} + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu \bar{z}^{-n}$$

$$j^\mu(z) = i \sqrt{\frac{2}{\alpha'}} \partial X^\mu(z) = \sum_{n \in \mathbb{Z}} \alpha_n^\mu z^{-n-1}$$
$$\bar{j}^\mu(\bar{z}) = i \sqrt{\frac{2}{\alpha'}} \bar{\partial} \bar{X}^\mu(\bar{z}) = \sum_{n \in \mathbb{Z}} \tilde{\alpha}_n^\mu \bar{z}^{-n-1}$$

$$\langle j(z_1) j(z_2) \rangle = \sum_{n,m \in \mathbb{Z}} z_1^{-n-1} z_2^{-m-1} \langle 0 | \alpha_n \alpha_m | 0 \rangle = \sum_{n \geq 0} n z_1^{-n-1} z_2^{n-1}$$
$$= \frac{1}{z_1 z_2} \sum_{n \geq 0} n \left(\frac{z_2}{z_1} \right)^n = \frac{1}{z_1 z_2} \frac{\frac{z_2}{z_1}}{\left(1 - \frac{z_2}{z_1}\right)^2}$$
$$= \frac{1}{(z_1 - z_2)^2}$$

$$\langle j^\mu(z_1) j^\nu(z_2) \rangle = \frac{\eta^{\mu\nu}}{(z_1 - z_2)^2}$$

$$\alpha_n = \oint_0 \frac{dz}{2\pi i} z^n j(z)$$

$$[\alpha_n, \alpha_m] = n \delta_{n+m,0} = \oint_0 \frac{dz_1}{2\pi i} \oint_{z_1} \frac{dz_2}{2\pi i} z_1^n z_2^m j(z_1) j(z_2).$$

$$j(z_1) j(z_2) = \frac{A}{(z_1 - z_2)^2} + \frac{B}{(z_1 - z_2)} + (\,7\,).$$

$$j(z_1) j(z_2) = \frac{1}{(z_1 - z_2)^2} + (\,\star\,)$$

$$A(z_1) B(z_2),$$

$$: A(z_1) B(z_2) :$$

$$j(z_1)j(z_2) = \frac{1}{(z_1 - z_2)^2}$$

$$\begin{aligned} \partial X(z_1)\partial X(z_2) &= -\frac{\alpha'}{2}\frac{1}{(z_1 - z_2)^2} \\ \partial X(z_1)X(z_2) &= -\frac{\alpha'}{2}\frac{1}{(z_1 - z_2)} \\ X(z_1)X(z_2) &= -\frac{\alpha'}{2}\log{(z_1 - z_2)} \end{aligned}$$

$$T^{(\mathrm{m})}(z) = -\frac{1}{\alpha'}:\partial X\partial X:(z) = \frac{1}{2}:jj:(z),$$

$$\begin{aligned} :AB:(z_1) &= \lim_{z_2 \to z_1}\left(A(z_1)B(z_2) - A(z_1)B(z_2)\right) \\ &= \oint_{z_1} \frac{dz_2}{2\pi i}\frac{A(z_1)B(z_2)}{(z_1 - z_2)} \end{aligned}$$

$$\begin{aligned} T^{(\mathrm{m})}(z_1)T^{(\mathrm{m})}(z_2) &= \frac{1}{4}\left(:jj:(z_1):jj:(z_2)\right) \\ &= \frac{1}{4}\left(2\frac{1}{(z_1 - z_2)^2}\frac{1}{(z_1 - z_2)^2} + 4\frac{1}{(z_1 - z_2)^2}:j(z_1)j(z_2):\right) \\ &= \frac{1}{2}\frac{1}{(z_1 - z_2)^4} + \frac{1}{(z_1 - z_2)^2}:(j(z_2) + (z_1 - z_2)\partial j(z_2) + \cdots)j(z_2) \\ &:= \frac{1}{2}\frac{1}{(z_1 - z_2)^4} + \frac{2T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)^2} + \frac{:(\partial j)j:(z_2)}{(z_1 - z_2)} \\ &= \frac{1}{2}\frac{1}{(z_1 - z_2)^4} + \frac{2T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)^2} + \frac{\partial T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)} \end{aligned}$$

$$c = 1.$$

$$T^{(\mathrm{m})}(z_1)T^{(\mathrm{m})}(z_2) = \frac{1}{2}\frac{1}{(z_1 - z_2)^4} + \frac{2T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)^2} + \frac{\partial T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)} + (\gimel).$$

$$\begin{aligned} \underbrace{T^{(\mathrm{m})}(z_1)j(z_2)} &= \frac{1}{2}\underbrace{:jj:(z_1)j(z_2)} \\ &= \frac{1}{2}2j(z_1)\frac{1}{(z_1 - z_2)^2} \\ &= \frac{j(z_2) + (z_1 - z_2)\partial j(z_2) + \ldots}{(z_1 - z_2)^2} \end{aligned}$$

$$= \frac{j(z_2)}{(z_1 - z_2)^2} + \frac{\partial j(z_2)}{(z_1 - z_2)}$$

$$T^{(\mathrm{m})}(z_1)j(z_2) = \frac{j(z_2)}{(z_1 - z_2)^2} + \frac{\partial j(z_2)}{(z_1 - z_2)} + (\dagger)$$

$$\mathcal{V}_P(z,\bar{z}) =: e^{iP\cdot X}:(z,\bar{z})$$
$$=: e^{iP\cdot X_L}:(z): e^{iP\cdot X_R}:(\bar{z})$$
$$= \mathcal{V}_P(z)\overline{\mathcal{V}}_P(\bar{z})$$

$$:e^{iP\cdot X}:(z) = \sum_{n=0}^{\infty} \frac{(iP)^n}{n!} :X^n:(z)$$

$$:X^n:(z) = X^n - \overbrace{\underbrace{XX\cdots X}}^{n}$$

$$j(z_1)X(z_2) = i\sqrt{\frac{2}{\alpha'}}\partial X(z_1)X(z_2)$$
$$= i\sqrt{\frac{2}{\alpha'}}\left(-\frac{\alpha'}{2}\right)\frac{1}{z_1 - z_2}$$
$$= -i\sqrt{\frac{\alpha'}{2}}\frac{1}{z_1 - z_2}$$

$$j(z_1):X^n:(z_2) = nj(z_1)X(z_2):X^{n-1}:(z_2)$$
$$= -in\sqrt{\frac{\alpha'}{2}}\frac{1}{z_1 - z_2}:X^{n-1}:(z_2)$$

$$j(z_1):e^{iP\cdot X}:(z_2) = \sum_{n=0}^{\infty} \frac{(iP)^n}{n!}\left(-in\sqrt{\frac{\alpha'}{2}}\frac{1}{z_1 - z_2}\right):X^{n-1}:(z_2)$$
$$= \frac{P\sqrt{\frac{\alpha'}{2}}}{z_1 - z_2}\sum_{n=1}^{\infty} \frac{(iP)^{n-1}}{(n-1)!}:X^{n-1}:(z_2)$$
$$= \frac{P\sqrt{\frac{\alpha'}{2}}}{z_1 - z_2}:e^{iP\cdot X}:(z_2)$$

$$j(z_1)X(z_2) = -i\sqrt{\frac{\alpha'}{2}}\frac{1}{z_1 - z_2} + (\boxdot)$$

$$j(z_1):X^n:(z_2) = -in\sqrt{\frac{\alpha'}{2}}\frac{1}{z_1 - z_2}:X^{n-1}:(z_2) + (\boxplus)$$

$$j(z_1):e^{iP\cdot X}:(z_2) = \frac{P\sqrt{\frac{\alpha'}{2}}}{z_1 - z_2}:e^{iP\cdot X}:(z_2) + (\boxtimes)$$

$$|0,P\rangle =: e^{iP\cdot X}:(z,\bar{z})|_{z=\bar{z}=0}|0\rangle$$

$$|0,P\rangle = e^{iP\cdot \hat{X}_0}|0\rangle$$

$$X(z,\bar{z}) = \hat{X}_0 + i\hat{P}\log|z|^2 + \mathcal{O}_{oscillators}$$

$$T^{(\mathrm{m})}(z_1){:}\,e^{iP\cdot X}{:}\,(z_2) = \frac{\frac{\alpha' P^2}{4}{:}\,e^{iP\cdot X}{:}\,(z_2)}{(z_1-z_2)^2} + \frac{\partial{:}\,e^{iP\cdot X}{:}\,(z_2)}{(z_1-z_2)} + \ (\mathfrak{K})$$

$$t(P){:}\,e^{iP\cdot X}{:}\,(z,\bar{z})$$

$$h = \frac{\alpha' P^2}{4} = 1$$

$$G_{\mu\nu}(P)\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0,P\rangle$$

$$G_{\mu\nu}(P){:}\,j^{\mu}\bar{j}^{\nu}e^{iP\cdot X}{:}\,(z,\bar{z})$$

$$\alpha' P^2 = 0$$

$$\begin{aligned} T^{(\mathrm{m})}(z_1)&{:}\,j^{\mu}e^{iP\cdot X}{:}\,(z,\bar{z})G_{\mu\nu}(P) = \\ &= \frac{\sqrt{\frac{\alpha'}{2}}P^{\mu}G_{\mu\nu}}{(z_1-z_2)^3}{:}\,e^{iP\cdot X}{:}\,(z_2) \\ &+ \left(\frac{\frac{\alpha' P^2}{4}+1}{(z_1-z_2)^2} + \frac{\partial_{z_2}}{(z_1-z_2)}\right){:}\,j^{\mu}e^{iP\cdot X}{:}\,(z_2)G_{\mu\nu} \\ &+(\mathfrak{L}) \end{aligned}$$

$$\begin{cases} \frac{\alpha' P^2}{4} + 1 = 1 \iff P^2 = 0 \\ P^{\mu}G_{\mu\nu} = 0 \end{cases}$$

$$G_{\mu\nu}P^{\nu} = 0$$

$$\begin{aligned} \langle j^{\mu}(z_1)\rangle &= 0 \\ \langle j^{\mu}(z_1)j^{\nu}(z_2)\rangle &= \frac{\eta^{\mu\nu}}{(z_1-z_2)^2}, \\ \langle j^{\mu}(z_1)j^{\nu}(z_2)j^{\rho}(z_3)\rangle &= 0 \\ \langle j^{\mu}(z_1)j^{\nu}(z_2)j^{\rho}(z_3)j^{\sigma}(z_4)\rangle &= \frac{\eta^{\mu\nu}\eta^{\rho\sigma}}{(z_1-z_2)^2(z_3-z_4)^2} \\ &+\frac{\eta^{\mu\rho}\eta^{\nu\sigma}}{(z_1-z_3)^2(z_2-z_4)^2} \\ &+\frac{\eta^{\mu\sigma}\eta^{\nu\rho}}{(z_1-z_4)^2(z_2-z_3)^2} \end{aligned}$$

$$\langle \mathcal{V}_{P_1}(z_1,\overline{z_1}) \ldots \mathcal{V}_{P_n}(z_n,\overline{z_n})\rangle = \langle \mathcal{V}_{P_1}(z_1) \ldots \mathcal{V}_{P_n}(z_n)\rangle \cdot \langle \overline{\mathcal{V}}_{P_1}(\overline{z_1}) \ldots \overline{\mathcal{V}}_{P_n}(\overline{z_n})\rangle$$

$$\begin{aligned} \langle \mathcal{V}_{P_1}(z_1) \ldots \mathcal{V}_{P_n}(z_n)\rangle &= \left\langle \prod_{k=1}^{n} {:}\,e^{iP_k\cdot X(z_k)}{:}\right\rangle \\ &= \prod_{k<l} (z_k - z_l)^{\frac{\alpha'}{2}P_k\cdot P_l} \cdot \delta\left(\sum_{k=1}^{n} P_k\right) \end{aligned}$$

$$:e^{iP_k\cdot X}:(z_k):e^{iP_l\cdot X}:(z_l)= e^{\,iP_k\cdot X(z_k) iP_l\cdot X(z_l)}:e^{iP_k\cdot X}(z_k)e^{iP_l\cdot X}(z_l)$$

$$:= e^{\frac{\alpha'}{2}P_k\cdot P_l \log(z_k - z_l)}:e^{iP_k\cdot X}(z_k)e^{iP_l\cdot X}(z_l)$$

$$:= (z_k - z_l)^{\frac{\alpha'}{2}P_k\cdot P_l}:e^{iP_k\cdot X}(z_k)e^{iP_l\cdot X}(z_l):$$

$$j_0^\mu = \oint_0 \frac{dz}{2\pi i} j^\mu(z) = \alpha_0^\mu = \sqrt{\frac{2}{\alpha'}}P^\mu$$

$$\left\langle j_0\left(\mathcal{V}_{P_1}(z_1)\dots\mathcal{V}_{P_n}(z_n)\right)\right\rangle = \langle 0|j_0\left(\mathcal{V}_{P_1}(z_1)\dots\mathcal{V}_{P_n}(z_n)\right)|0\rangle = 0$$

$$= \left\langle \oint_{z_1\dots z_n} \frac{dz}{2\pi i} j^\mu(z)\left(\mathcal{V}_{P_1}(z_1)\dots\mathcal{V}_{P_n}(z_n)\right)\right\rangle$$

$$= \left(\sum_{k=1}^{n} P_k\right)\left(\sqrt{\frac{\alpha'}{2}}\right)^n \langle \mathcal{V}_{P_1}(z_1)\dots\mathcal{V}_{P_n}(z_n)\rangle$$

$$j_0^\mu \mathcal{V}_{P_k}(z_k) = \oint_{z_k} \frac{dz}{2\pi i} j^\mu(z) e^{iP_k\cdot X}(z_k)$$

$$= \oint_{z_k} \frac{dz}{2\pi i} j^\mu(z) e^{iP_k\cdot X}(z_k)$$

$$= \oint_{z_k} \frac{dz}{2\pi i} \sqrt{\frac{\alpha'}{2}} \frac{P^\mu}{(z - z_k)} \mathcal{V}_{P_k}(z_k)$$

$$= \sqrt{\frac{\alpha'}{2}} P^\mu \mathcal{V}_{P_k}(z_k)$$

$$\sum_{k=1}^{n} P_k = 0$$

$$S_{\text{ghost}} = \frac{1}{2\pi}\int_{\text{WS}} d^2 z (b\bar{\partial}c + \bar{b}\partial\bar{c})$$

$$\begin{cases} b(z) = \sum_{n\in\mathbb{Z}} b_n z^{-n-2} \\ b_n = \oint_0 \frac{dz}{2\pi i} z^{n+1} b(z) \end{cases}$$

$$\begin{cases} c(z) = \sum_{n\in\mathbb{Z}} c_n z^{-n+1} \\ c_n = \oint_0 \frac{dz}{2\pi i} z^{n-2} c(z) \end{cases}$$

$$[b_n, c_m] = \delta_{n+m,0}$$

$$b(z_1)c(z_2) = \frac{1}{(z_1 - z_2)} + (\mathfrak{M})$$

$$T^{(\mathrm{gh})}(z) = -(2{:}\, b(\partial c){:} + {:}(\partial b)c{:})(z)$$

$$T^{(\mathrm{gh})}(z_1)b(z_2) = \frac{2b(z_2)}{(z_1 - z_2)^2} + \frac{\partial b(z_2)}{(z_1 - z_2)} + (\mathfrak{N})$$
$$T^{(\mathrm{gh})}(z_1)c(z_2) = -\frac{c(z_2)}{(z_1 - z_2)^2} + \frac{\partial c(z_2)}{(z_1 - z_2)} + (\mathfrak{O})$$

$$b(z_1)b(z_2) = 0,$$
$$c(z_1)c(z_2) = 0.$$

$$T^{(\mathrm{gh})}(z_1)T^{(\mathrm{gh})}(z_2) = \frac{-\frac{26}{2}}{(z_1 - z_2)^4} + \frac{2T^{(\mathrm{gh})}(z_2)}{(z_1 - z_2)^2} + \frac{\partial T^{(\mathrm{gh})}(z_2)}{(z_1 - z_2)} + (\mathfrak{P})$$

$$j^{(\mathrm{gh})}(z) = -{:}\, bc{:}\,(z) = \sum_{n\in\mathbb{Z}} j_n^{(\mathrm{gh})} z^{-n-1}$$

$$T^{(\mathrm{gh})}(z_1)j^{(\mathrm{gh})}(z_2) = \frac{-3}{(z_1 - z_2)^3} + \frac{j^{(\mathrm{gh})}(z_2)}{(z_1 - z_2)^2} + \frac{\partial j^{(\mathrm{gh})}(z_2)}{(z_1 - z_2)} + (\mathfrak{Q})$$

$$N_{(\mathrm{gh})} = \oint_0 \frac{dz}{2\pi i} j^{(\mathrm{gh})}(z)$$

$$\left[N_{(\mathrm{gh})}, c(z)\right] = c(z)$$
$$\left[N_{(\mathrm{gh})}, b(z)\right] = -b(z)$$

$$c(z), b(z)$$
$$c(\partial c)(z), b(\partial b)(z)$$
$$\frac{1}{2}c(\partial c)(\partial^2 c)(z), \frac{1}{2}b(\partial b)(\partial^2 b)(z)$$

$$|0\rangle \leftrightarrow \mathbb{1}\ (h = 0)$$
$$c_1|0\rangle \leftrightarrow c(z)\ (h = -1)$$
$$c_0c_1|0\rangle \leftrightarrow (\partial c)c(z)\ (h = -1)$$
$$c_{-1}c_0c_1|0\rangle \leftrightarrow \frac{1}{2}(\partial^2 c)(\partial c)c(z)\ (h = 0)$$

$$\langle 0 \mid \hat{0}\rangle = \langle 0|c_{-1}c_0c_1|0\rangle = 1 \leftrightarrow \frac{1}{2}\langle c(\partial c)(\partial^2 c)(z)\rangle = 1$$

$$c(z) = c_{-1}z^2 + c_0 z + c_1$$

$$\langle c(z_1)c(z_2)c(z_3)\rangle = -\det\begin{pmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ z_1^2 & z_2^2 & z_3^2 \end{pmatrix} = -(z_1 - z_2)(z_2 - z_3)(z_1 - z_3)$$

$$Q_B = \sum_{n\in\mathbb{Z}} c_{-n}L_n^{(\mathrm{m})} + \frac{1}{2}{:}c_{-n}L_n^{(\mathrm{gh})}{:}$$

$$= \oint_0 \frac{dz}{2\pi i}\left(cT^{(\mathrm{m})}(z) + \frac{1}{2}{:}cT^{(\mathrm{gh})}{:}(z)\right) = \oint_0 \frac{dz}{2\pi i} j_B(z)$$

$$j_B(z) = cT^{(\mathrm{m})}(z) + \frac{1}{2}{:}cT^{(\mathrm{gh})}{:}(z) + \frac{3}{2}\partial^2 c(z),$$

$$T^{(\mathrm{tot})}(z) = T^{(\mathrm{m})}(z) + T^{(\mathrm{gh})}(z)$$

$$[Q_B, X^\mu(z)] = \oint_z \frac{d\tilde{z}}{2\pi i} j_B(\tilde{z})X^\mu(z) = c\partial X^\mu(z)$$

$$[Q_B, c(z)] = c\partial c(z)$$

$$Q_B c_1|0\rangle = c_1 c_0|0\rangle.$$

$$Q_B^2 = 0$$

$$\left[Q_B, j_{\mathrm{gh}}(z)\right] = -j_B(z)$$

$$\left[N_{\mathrm{gh}}, Q_B\right] = Q_B$$

$$Q_B^2 = \oint_0 \frac{d\tilde{z}}{2\pi i}\oint_{\tilde{z}} \frac{dz}{2\pi i} j_B(z) j_B(\tilde{z})$$

$$j_B(z)j_B(\tilde{z}) = \cdots + \frac{c^{(\mathrm{m})} - 26}{12}\frac{1}{(z-\tilde{z})}(\partial^3 c\cdot c)(\tilde{z}) + \cdots$$

$$|0\rangle \leftrightarrow \mathbb{1}\ (gh = 0)$$
$$c_1\mathcal{V}_{h=1}^{(\mathrm{m})}|0\rangle \leftrightarrow c\mathcal{V}_{h=1}^{(\mathrm{m})}(z)\ (gh = 1)$$
$$c_0c_1\mathcal{V}_{h=1}^{(\mathrm{m})}|0\rangle \leftrightarrow c(\partial c)\mathcal{V}_{h=1}^{(\mathrm{m})}(z)\ (gh = 2)$$
$$c_{-1}c_0c_1|0\rangle \leftrightarrow \frac{1}{2}c(\partial c)(\partial^2 c)(z)\ (gh = 3)$$

$$\left[Q_B, \mathcal{V}_{h=1}^{(\mathrm{m})}(z)\right] = \partial\left(c\mathcal{V}_{h=1}^{(\mathrm{m})}\right)(z)$$

$$c\bar{c}\mathcal{V}(z,\bar{z}) = \int d^dP\,\Phi_{ij}(P){:}c\mathcal{V}_i\bar{c}\bar{\mathcal{V}}_j e^{iP\cdot X}{:}(z,\bar{z})$$

$$\alpha' P^2/4 + h_i - 1 = \alpha' P^2/4 + h_j - 1 = 0$$

$$\int dz d\bar{z}\mathcal{V}(z,\bar{z}) = \int dz d\bar{z}\int d^dP\,\Phi_{ij}(P){:}\mathcal{V}_i\bar{\mathcal{V}}_j e^{iP\cdot X}{:}(z,\bar{z})$$

$$z \longrightarrow f(z),$$
$$\bar{z} \longrightarrow \bar{f}(\bar{z}),$$

$$f(z) = \bar{f}(\bar{z}) \ \text{ for } z = \bar{z}$$

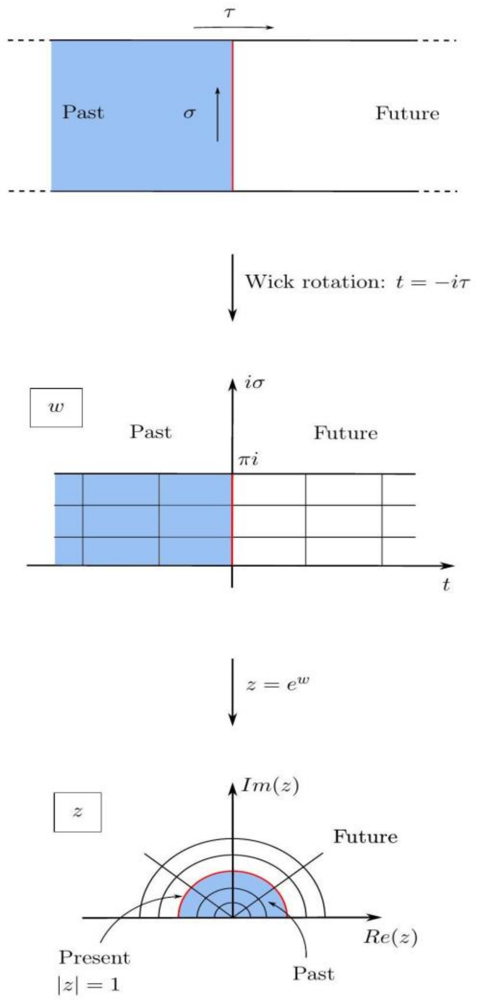
$$f(z) = \sum_n f_n z^n \text{ with } f_n = \bar{f}_n \Leftrightarrow f_n \in \mathbb{R}.$$

$$T(z) = \bar{T}(\bar{z}) \text{ for } z = \bar{z}.$$

$$T(z) = \sum_{n \in \mathbb{Z}} L_n z^{-n-2}$$
$$\bar{T}(\bar{z}) = \sum_{n \in \mathbb{Z}} \tilde{L}_n \bar{z}^{-n-2}$$

$$L_n = \tilde{L}_n,$$

$$f(z) = \frac{Az + B}{Cz + D} \text{ with } A, B, C, D \in \mathbb{R}, (AD - BC) \neq 0.$$

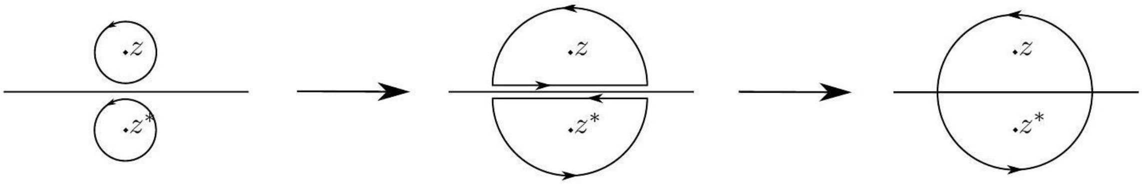


$$\delta_{(\epsilon,\bar{\epsilon})}\phi(z,\bar{z}) = -([T_\epsilon, \phi(z,\bar{z})] + [\bar{T}_{\bar{\epsilon}}, \phi(z,\bar{z})])$$
$$= -\oint_z \frac{d\tilde{z}}{2\pi i}\epsilon(\tilde{z})T(\tilde{z})\phi(z,\bar{z}) - \oint_{\bar{z}} \frac{d\tilde{\bar{z}}}{2\pi i}\epsilon(\tilde{\bar{z}})\bar{T}(\tilde{\bar{z}})\phi(z,\bar{z})$$
$$= -\oint_z \frac{d\tilde{z}}{2\pi i}\epsilon(\tilde{z})T(\tilde{z})\phi(z,\bar{z}) - \oint_{\bar{z}} \frac{d\tilde{\bar{z}}}{2\pi i}\epsilon(\tilde{\bar{z}})\bar{T}(\tilde{\bar{z}})\phi(z,\bar{z})$$

$$\delta_{(\epsilon,\bar{\epsilon})}\phi(z,\bar{z}) = -\oint_{z} \frac{d\tilde{z}}{2\pi i}\epsilon(\tilde{z})T(\tilde{z})\phi(z,\bar{z}) + \oint_{\bar{z}} \frac{d\bar{\tilde{z}}}{2\pi i}\epsilon(\bar{\tilde{z}})\bar{T}(\bar{\tilde{z}})\phi(z,\bar{z})$$

$$= -\oint_{z} \frac{d\tilde{z}}{2\pi i}\epsilon(\tilde{z})T(\tilde{z})\phi(z,z^*) - \oint_{z^*} \frac{d\tilde{z}}{2\pi i}\epsilon(\tilde{z})T(\tilde{z})\phi(z,z^*)$$

$$= -\oint_{z,z^*} \frac{d\tilde{z}}{2\pi i}\epsilon(\tilde{z})T_{\mathbb{C}}(\tilde{z})\phi(z,z^*)$$



$$T_{\mathbb{C}}(\tilde{z}) = \begin{cases} T_{\mathrm{UHP}}(\tilde{z}) & \text{for } \mathrm{Im}(\tilde{z}) > 0 \\ \bar{T}_{\mathrm{UHP}}(\bar{\tilde{z}}) & \text{for } \mathrm{Im}(\tilde{z}) < 0 \end{cases}.$$

$$S_{\mathrm{matter}} = \frac{1}{2\pi\alpha'}\int_{\mathrm{WS}} d^2z\partial X(z,\bar{z})\cdot\bar{\partial}X(z,\bar{z}),$$

$$X^\mu(z,\bar{z}) = X_L^\mu(z) + X_R^\mu(\bar{z})$$

$$X_L^\mu(z) = \frac{X_0^\mu}{2} + c^\mu/2 - i\alpha'\frac{P^\mu}{2}\log z + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu z^{-n},$$

$$X_R^\mu(\bar{z}) = \frac{X_0^\mu}{2} - c^\mu/2 - i\alpha'\frac{P^\mu}{2}\log \bar{z} + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu \bar{z}^{-n},$$

$$T(z) = -\frac{1}{\alpha'}:\partial X\cdot\partial X:.$$

$$j^\mu(z) = \Omega_{\mathrm{A}}^{(j)}\bar{j}^\mu(\bar{z}) \text{ for } z = \bar{z},$$

$$\Omega_{\mathrm{A}} = \begin{cases} +1 \text{ for A} = \mathcal{N}_{\mathrm{Neumann}} \\ -1 \text{ for A} = \mathcal{D}_{\mathrm{Dirichlet}} \end{cases}$$

$$X_L^\mu(z) = \Omega_{\mathrm{A}}^{(X)}X_R^\mu(\bar{z}) \text{ for } z = \bar{z}$$

$$X_L^{(\mathrm{N})}(z) \longrightarrow X(z), \quad X_R^{(\mathrm{N})}(\bar{z}) \longrightarrow X(z^*),$$
$$X_L^{(\mathrm{D})}(z) \longrightarrow X(z), \quad X_R^{(\mathrm{D})}(\bar{z}) \longrightarrow -X(z^*).$$

$$j^\mu(z_1)j_\mu(z_2) = \frac{1}{(z_1-z_2)^2} \overset{\mathrm{osc}}{\rightarrow} [\alpha_n^\mu, \alpha_m^\mu] = n\delta_{n+m,0}$$
$$\bar{j}^\mu(\bar{z}_1)\bar{j}_\mu(\bar{z}_2) = \frac{1}{(\bar{z}_1-\bar{z}_2)^2} \overset{\mathrm{osc}}{\rightarrow} [\tilde{\alpha}_n^\mu, \tilde{\alpha}_m^\mu] = n\delta_{n+m,0}$$

$$\Omega_{\mathrm{A}}^{(j)} = \pm 1 \Longrightarrow \left(\Omega_{\mathrm{A}}^{(j)}\right)^2 = 1$$

$$\begin{cases} T = \frac{1}{2} :j \cdot j: \\ \bar{T} = \frac{1}{2} :\bar{j} \cdot \bar{j}: \end{cases} \Longrightarrow \left(\Omega_{\mathrm{A}}^{(j)}\right)^2 = 1.$$

$$j_{\mathbb{C}}^{\mu}(z) = \begin{cases} j_{\mathrm{UHP}}^{\mu}(z) & \text{for } \mathrm{Im}(z) > 0 \\ \Omega_{\mathrm{A}}^{(j)} \bar{j}_{\mathrm{UHP}}^{\mu}(\bar{z}) & \text{for } \mathrm{Im}(z) < 0 \end{cases}$$

$$\mathcal{V}_{i,\mathbb{C}}^{(h)}(z) = \begin{cases} \mathcal{V}_{i,\mathrm{UHP}}^{(h)}(z) & \text{for } \mathrm{Im}(z) > 0 \\ \left(\Omega_{\mathrm{A}}^{(\mathcal{V})}\right)_i^{\ j} \overline{\mathcal{V}}_{j,\mathrm{UHP}}^{(\bar{h})}(\bar{z}) & \text{for } \mathrm{Im}(z) < 0 \end{cases}$$

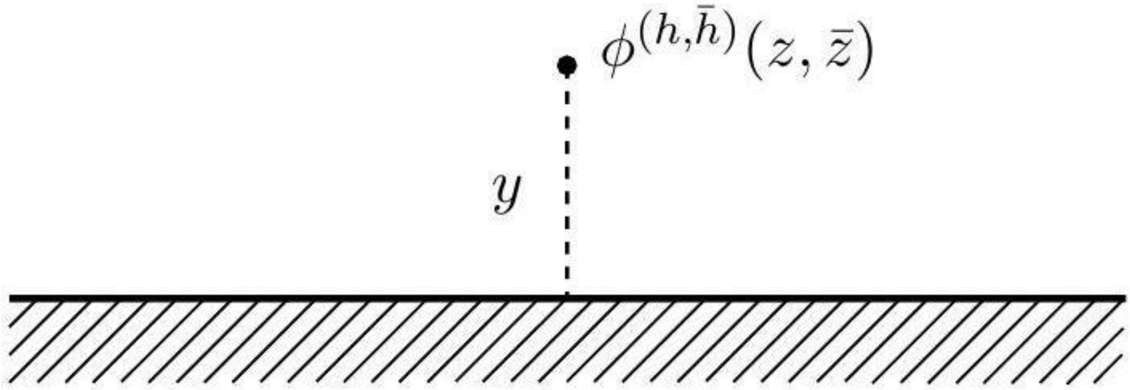
$$\phi(z,\bar{z}) = \phi_{ij} \mathcal{V}^i(z) \overline{\mathcal{V}}^j(\bar{z})$$

$$\phi(z,z^*) = \phi_{ij} \left(\Omega_{\mathrm{A}}^{(\mathcal{V})}\right)_k^{\ j} \mathcal{V}^i(z) \mathcal{V}^k(z^*).$$

$$\Phi(z,\bar{z}) = j(z) \cdot \bar{j}(\bar{z}) \longrightarrow \alpha_{-1} \cdot \tilde{\alpha}_{-1} |0\rangle$$

$$\Phi^{(\mathrm{N})}(z,z^*) = j(z) \Omega_{\mathrm{N}}^{(j)} j(z^*) = j(z) j(z^*)$$
$$\Phi^{(\mathrm{D})}(z,z^*) = j(z) \Omega_{\mathrm{D}}^{(j)} j(z^*) = -j(z) j(z^*)$$

$$\left\langle \phi^{(h,\bar{h})}(z,\bar{z}) \right\rangle^{(a)} = \frac{C_{\phi}^{(a)}}{|z-\bar{z}|^{2h}} \delta_{h,\bar{h}}.$$



$$\begin{aligned} \langle X(z_1,\bar{z}_1) \cdot X(z_2,\bar{z}_2) \rangle &= \langle X_L(z_1) \cdot X_L(z_2) \rangle + \langle X_R(\overline{z_1}) \cdot X_R(\overline{z_2}) \rangle \\ &= -\frac{\alpha'}{2} \left(\log(z_1 - z_2) + \log(\overline{z_1} - \overline{z_2}) \right) \\ &= -\frac{\alpha'}{2} \log(|z_1 - z_2|^2) \end{aligned}$$

$$\begin{aligned}\langle X(z_1,\bar{z}_1)\cdot X(z_2,\bar{z}_2)\rangle^{(\mathrm{N})}_{\mathrm{UHP}} =& \big\langle\big(X(z_1)+X(z_1^*)\big)\cdot\big(X(z_2)+X(z_2^*)\big)\big\rangle\\ =&\langle X(z_1)\cdot X(z_2)\rangle+\langle X(z_1)\cdot X(z_2^*)\rangle\\ &+\langle X(z_1^*)\cdot X(z_2)\rangle+\langle X(z_1^*)\cdot X(z_2^*)\rangle\\ =&-\frac{\alpha'}{2}\log\left(|z_1-z_2|^2\right)-\frac{\alpha'}{2}\big(\log\left(z_1-z_2^*\right)+\log\left(z_1^*-z_2\right)\big)\\ =&-\frac{\alpha'}{2}\big(\log\left(|z_1-z_2|^2\right)+\log\left(|z_1-\bar{z}_2|^2\right)\big)\end{aligned}$$

$$\langle X(z_1,\bar{z}_1)\cdot X(z_2,\bar{z}_2)\rangle^{(\mathrm{D})}_{\mathrm{UHP}}=-\frac{\alpha'}{2}\big(\log\left(|z_1-z_2|^2\right)-\log\left(|z_1-\bar{z}_2|^2\right)\big).$$

$$\begin{aligned}\langle\partial X\cdot\bar{\partial}X(z,\bar{z})\rangle^{(\mathrm{N})}_{\mathrm{UHP}}&=+\frac{\alpha'}{2}\frac{1}{|z-\bar{z}|^2}\\ \langle\partial X\cdot\bar{\partial}X(z,\bar{z})\rangle^{(\mathrm{D})}_{\mathrm{UHP}}&=-\frac{\alpha'}{2}\frac{1}{|z-\bar{z}|^2}\end{aligned}$$

$$:e^{iP\cdot X}:(z,\bar{z})=:e^{iP\cdot X_L}:(z):e^{iP\cdot X_R}:(\bar{z})$$

$$e^{iP\cdot X}:(z,\bar{z})=e^{iP\cdot X_L}(z)e^{iP\cdot X_L}(z^*)$$

$$e^{iP\cdot X}:(z,\bar{z})=e^{iP\cdot X_L}(z)e^{-iP\cdot X_L}(z^*)$$

$$\big\langle:e^{iP\cdot X}:(z,\bar{z})\big\rangle^{(\mathrm{N})}_{\mathrm{UHP}}=\big\langle e^{iP\cdot X_L}(z)e^{iP\cdot X_L}(z^*)\big\rangle=0$$

$$\begin{aligned}\big\langle:e^{iP\cdot X}:(z,\bar{z})\big\rangle^{(\mathrm{D})}_{\mathrm{UHP}}=&\big\langle e^{iP\cdot X_L}(z)e^{-iP\cdot X_L}(\bar{z})\big\rangle\\ &=(z-\bar{z})^{-\alpha'P^2}\delta(0)\\ &\propto|z-\bar{z}|^{-\alpha'P^2}=e^{-\alpha'\Delta P^2}\end{aligned}$$

$$\big\langle:e^{iP\cdot X}:(z,\bar{z})\big\rangle^{(\mathrm{D}),X_0}_{\mathrm{UHP}}\propto e^{iP\cdot X_0}e^{-\alpha'\Delta P^2}$$

$$\int dP e^{iP\cdot X_0}e^{-\alpha'\Delta P^2}\propto e^{-\frac{(X-X_0)^2}{\alpha'\Delta}}$$

$$\phi^{(h)}(z,\bar{z})=\sum_{i,j}\phi_{i,j}\mathcal{V}_i^{(h)}(z)\overline{\mathcal{V}}_j^{(h)}(\bar{z})=\sum_{i,j}\phi_{i,j}\frac{\mathcal{V}_i^{(h)}(z)}{\Omega_j^k\dot{\mathcal{V}}_k^{(h)}(z^*)}.$$

$$\mathcal{V}_i^{(h)}(z)\mathcal{V}_k^{(h)}(z^*)\propto\sum_p(z-z^*)^k[\mathcal{V}_i\mathcal{V}_k]_p(0)$$

$$\phi_{\mathrm{A}}^{(h)}(x)=\lim_{z\to x}\phi^{(h)}(z)$$

$$\begin{aligned}\partial_x&=\partial_{\parallel}\\ \partial_y&=\partial_{\perp}\end{aligned}$$

$$\begin{aligned}\partial&=\partial_x-i\partial_y\\ \bar{\partial}&=\partial_x+i\partial_y\end{aligned}$$

$$\partial X^\mu(z)=\bar{\partial}X^\mu(\bar{z})\ \text{ for } z=\bar{z}.$$

$$(\partial-\bar{\partial})X^{\mu}(z)\big|_{z=\bar{z}}=-2i\partial_{y}X^{\mu}(z)=0$$

$$\partial_{\perp}X^{\mu}(z)=0$$

$$\begin{aligned}\partial X^{\mu}(x)&=\left(\partial_{\perp}+\partial_{\|}\right)X^{\mu}(z)\big|_{z\to x}\\&=\partial_{\|}X^{\mu}(x)\\&=\partial_{x}X^{\mu}(x)\end{aligned}$$

$$\partial X^{\mu}(z)=-\bar{\partial}X^{\mu}(\bar{z})\big|_{z=\bar{z}}$$

$$\partial_{\|}X^{\mu}(z)=0$$

$$\begin{aligned}\partial X^{\mu}(x)&=\left(\partial_{\perp}+\partial_{\|}\right)X^{\mu}(z)\big|_{z\to x}\\&=\partial_{\perp}X^{\mu}(x)\\&=\partial_{y}X^{\mu}(x)\end{aligned}$$

$$:e^{iP\cdot X}:(z,\bar{z})=:e^{iP\cdot X_{L}}:(z):e^{iP\cdot X_{R}}:(\bar{z})$$

$$:e^{2iP\cdot X_{L}}:(z)|_{z\to x}=:e^{2iP\cdot X_{L}}:(x)$$

$$X^{\mu(\text{ open })}(x)=2X_{L}^{\mu}(z)\big|_{z\to x}$$

$$\alpha_{0}^{(\text{open })}=2\alpha_{0}^{(\text{closed })}=\alpha_{0}^{(\text{closed })}+\tilde{\alpha}_{0}^{(\text{closed })}.$$

$$X_{(\text{open})}^{\mu}(x)=X_{0}^{\mu}-i\alpha'P^{\mu}\log(x)+i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_{n}^{\mu}x^{-n}.$$

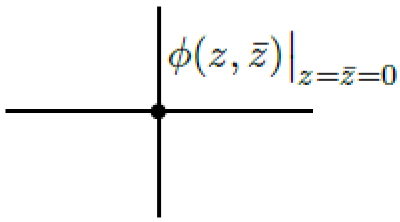
$$:e^{iP\cdot X_{(\text{open })}}:(x)=:e^{2iP\cdot X_{L}}:(z)|_{z\to x}$$

$$h=4\frac{\alpha'P^{2}}{4}=\alpha'P^{2}$$

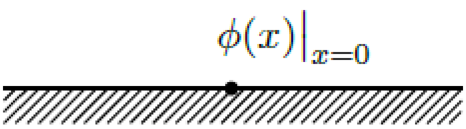
$$L_{0}^{(\text{open })}=L_{0}^{(\text{closed, L sector })}=\alpha'P^{2}$$

$$\langle X_{L}(z_{1})\cdot X_{L}(z_{2})\rangle=-\frac{\alpha'}{2}\log(z_{1}-z_{2}).$$

$$\phi(z,\bar{z})\Big|_{z=\bar{z}=0}|0\rangle_{SL(2,\mathbb{C})}=$$



$$\phi(x)\Big|_{x=0}|0\rangle_{SL(2,\mathbb{R})}=$$



$$\left\langle \phi^{(h)}(x) \right\rangle_{\text{UHP}} = A\delta_{h,0}$$

$$\left\langle \phi_1^{(h_1)}(x_1)\phi_2^{(h_2)}(x_2) \right\rangle_{\text{UHP}} = \frac{C}{(x_1 - x_2)^{2h_1}}\delta_{h1,h2},$$

$$\left\langle \phi_1^{(h_1)}(x_1)\phi_2^{(h_2)}(x_2)\phi_3^{(h_3)}(x_3) \right\rangle_{\text{UHP}} =$$
$$= \frac{C_{123}}{(x_1 - x_2)^{h_1+h_2-h_3}(x_2 - x_3)^{-h_1+h_2+h_3}(x_1 - x_3)^{h_1-h_2+h_3}}$$

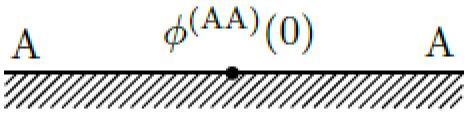
$$\begin{cases} x_1 & \longrightarrow & \Lambda \\ x_2 & \longrightarrow & 1 \\ x_3 & \longrightarrow & 0 \end{cases}$$

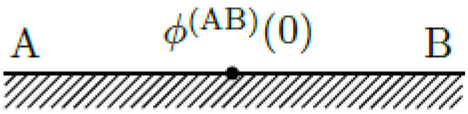
$$\left\langle \phi_1^{(h_1)}(x_1)\phi_2^{(h_2)}(x_2)\phi_3^{(h_3)}(x_3) \right\rangle_{\text{UHP}} =$$
$$= \left\langle f \circ \phi_1^{(h_1)}(x_1) f \circ \phi_2^{(h_2)}(x_2) f \circ \phi_3^{(h_3)}(x_3) \right\rangle_{\text{UHP}}$$
$$= [f'(x_1)]^{h_1}[f'(x_2)]^{h_2}[f'(x_3)]^{h_3} \left\langle \phi_1^{(h_1)}(\Lambda)\phi_2^{(h_2)}(1)\phi_3^{(h_3)}(0) \right\rangle_{\text{UHP}}$$
$$\overset{\Lambda^{2h_1}}{=} \frac{1}{\Lambda^{2h_1}}[f'(x_1)]^{h_1}[f'(x_2)]^{h_2}[f'(x_3)]^{h_3} \left\langle \mathcal{J} \circ \phi_1^{(h_1)}\left(-\frac{1}{\Lambda}\right)\phi_2^{(h_2)}(1)\phi_3^{(h_3)}(0) \right\rangle_{\text{UHP}}$$
$$\overset{\Lambda\to\infty}{=} \frac{C_{123}}{(x_1 - x_2)^{h_1+h_2-h_3}(x_2 - x_3)^{-h_1+h_2+h_3}(x_1 - x_3)^{h_1-h_2+h_3}}$$

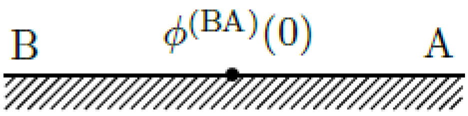
$$C_{123} = \left\langle \mathcal{J} \circ \phi_1^{(h_1)}\left(-\frac{1}{\Lambda}\right)\phi_2^{(h_2)}(1)\phi_3^{(h_3)}(0) \right\rangle_{\text{UHP}}.$$

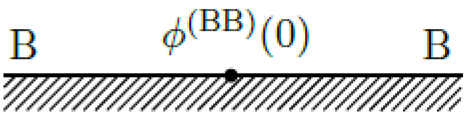
$$\left\langle e^{iP_1\cdot X}(x_1) \cdots e^{iP_n\cdot X}(x_n) \right\rangle_{\text{UHP}}^{(\text{N})} =$$
$$= \left\langle e^{2iP_1\cdot X_L}(z_1) \cdots e^{2iP_n\cdot X_L}(z_n) \right\rangle\Big|_{z_i\to x_i}$$
$$= \prod_{k<l} e^{\langle 2iP_k\cdot X_L(z_k) 2iP_l\cdot X_L(z_l)\rangle}\Bigg|_{z_i\to x_i} \cdot \delta\left(\sum_{k=1}^{n} P_k\right)$$
$$= \prod_{k<l} e^{-4P_k\cdot P_l\langle X_L(z_k) X_L(z_l)\rangle}\Bigg|_{z_i\to x_i} \cdot \delta\left(\sum_{k=1}^{n} P_k\right)$$
$$= \prod_{k<l} e^{4P_k\cdot P_l\frac{\alpha'}{2}\log(z_k - z_l)}\Bigg|_{z_i\to x_i} \cdot \delta\left(\sum_{k=1}^{n} P_k\right)$$
$$= \prod_{k<l} (x_k - x_l)e^{2\alpha' P_k\cdot P_l} \cdot \delta\left(\sum_{k=1}^{n} P_k\right)$$

$$\begin{pmatrix} |\text{AA}\rangle & |\text{AB}\rangle \\ |\text{BA}\rangle & |\text{BB}\rangle \end{pmatrix},$$

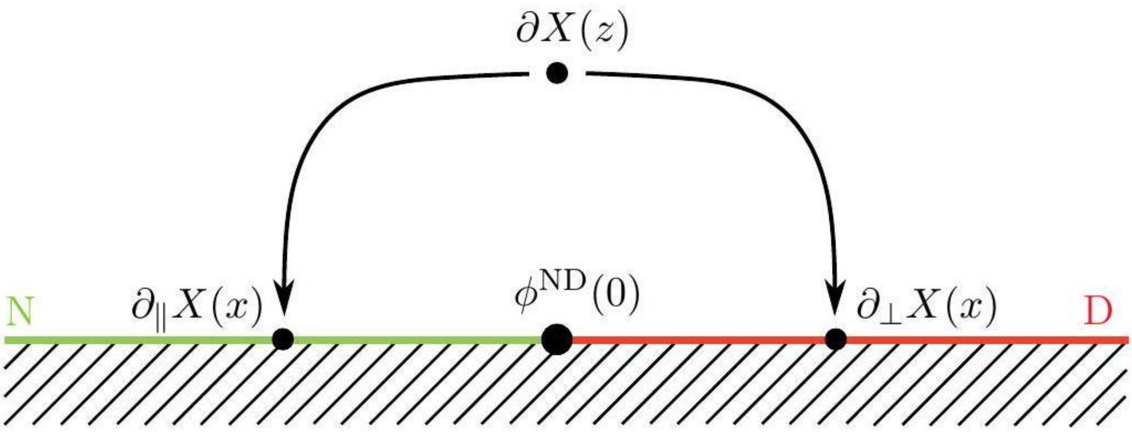
$$|\mathrm{AA}\rangle = \phi^{(\mathrm{AA})}(x=0)\,|0\rangle =$$  ,

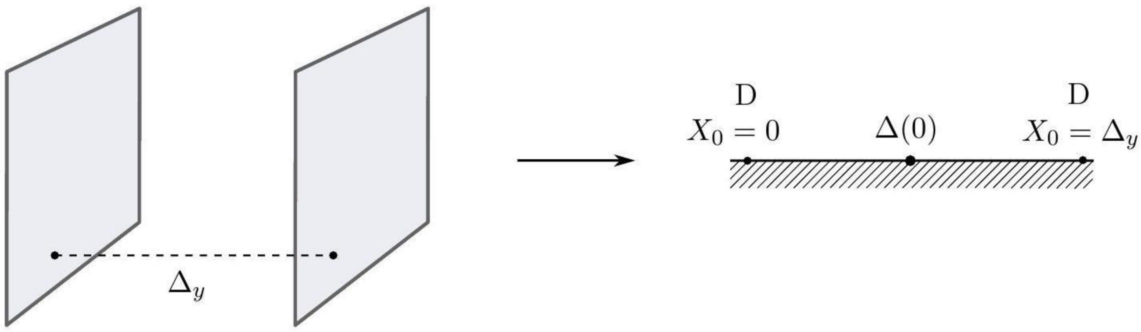
$$|\mathrm{AB}\rangle = \phi^{(\mathrm{AB})}(x=0)\,|0\rangle =$$  ,

$$|\mathrm{BA}\rangle = \phi^{(\mathrm{BA})}(x=0)\,|0\rangle =$$  ,

$$|\mathrm{BB}\rangle = \phi^{(\mathrm{BB})}(x=0)\,|0\rangle =$$  .

$$\phi_i^{(\mathrm{AB})}(x)\phi_j^{(\mathrm{CD})}(y) \sim \delta^{\mathrm{BC}} \sum_k C_{ijk}^{\mathrm{ABD}} \phi_k^{(\mathrm{AD})}(y),$$





$$\begin{pmatrix} |\phi^{11}\rangle & |\phi^{12}\rangle \\ |\phi^{21}\rangle & |\phi^{22}\rangle \end{pmatrix}$$

$$|\phi^{11}\rangle = t(P)e^{iP\cdot X}(0)|0\rangle = t(P)|0,P\rangle$$

$$|\phi^{12}\rangle = t(P)e^{iP\cdot X}\Delta(0)|0\rangle = t(P)|0,P\rangle^{1,2}$$

$$\Delta(x) = e^{i\frac{\Delta_y}{\pi\alpha'}X_L^y}(x).$$

$$T(z)\Delta(x) = \left(\frac{\Delta_y}{2\pi\sqrt{\alpha'}}\right)^2 \frac{\Delta(x)}{(z-x)^2} + \frac{\partial\Delta(x)}{(z-x)} + (\mathfrak{R}).$$

$$b(z) = \bar{b}(\bar{z}) \ \text{ for } z = \bar{z}.$$

$$b(z_1)c(z_2) = \frac{1}{(z_1 - z_2)} + (\mathfrak{S})$$
$$\bar{b}(\bar{z}_1)\bar{c}(\bar{z}_2) = \frac{1}{(\bar{z}_1 - \bar{z}_2)} + (\mathfrak{T})$$

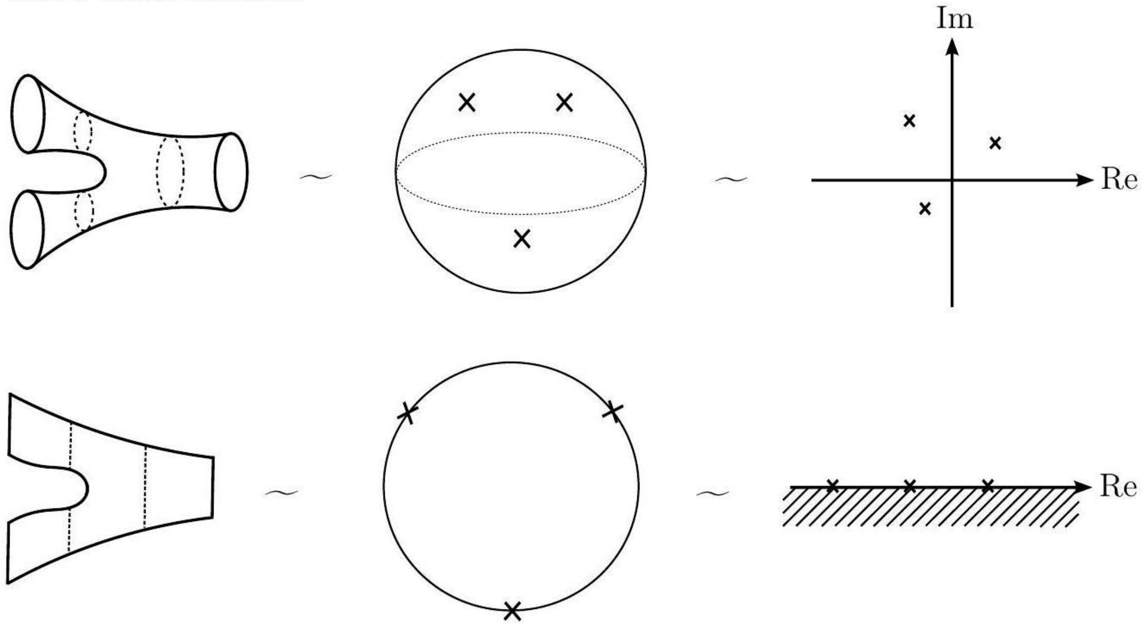
$$c(z) = \bar{c}(\bar{z}) \ \text{ for } z = \bar{z}.$$

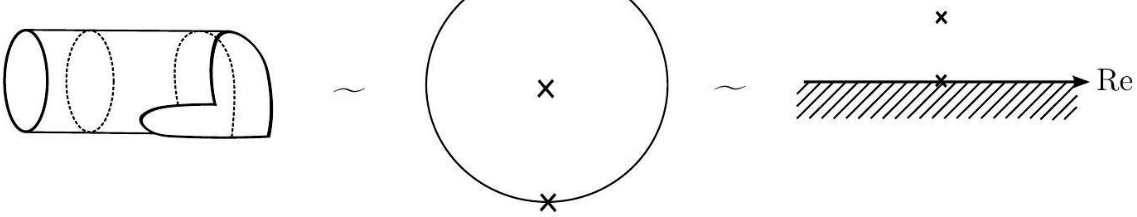
$$Q_B = \sum_{n\in\mathbb{Z}} c_n L_{-n}^{(\mathrm{m})} + \frac{1}{2} : c_n L_{-n}^{(\mathrm{gh})} :,$$

$$Q_B = \bar{Q}_B.$$

$$c\mathcal{V}(x),$$

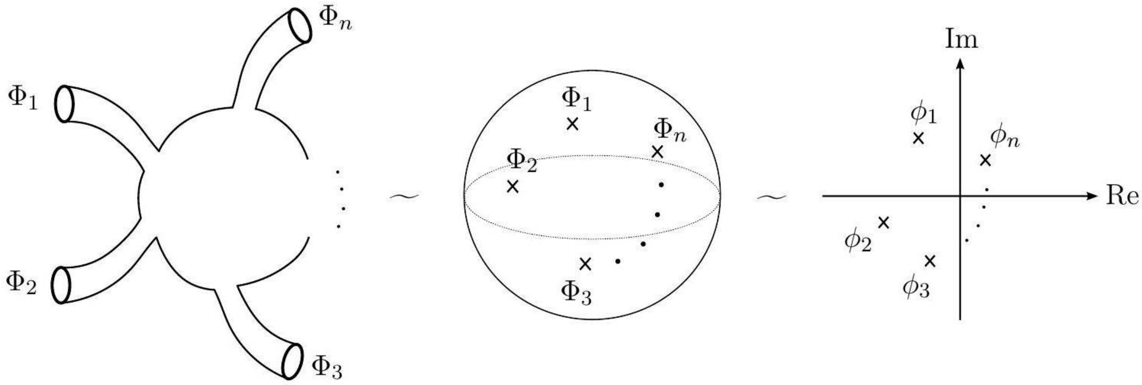
$$\int_{\mathbb{R}} \mathcal{V}(x),$$





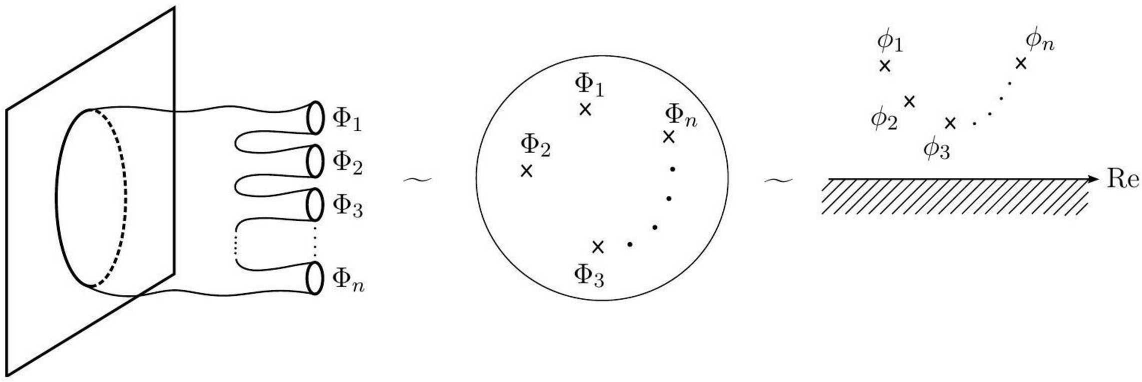
$$\phi_i(z_i,\bar{z}_i) = \sum_{k_i,l_i} (\Phi_i)_{k_i l_i} \mathcal{V}_{k_i}(z_i) \overline{\mathcal{V}}_{l_i}(\bar{z}_i)$$

$$\langle \phi_1(z_1,\bar{z}_1) \cdots \phi_n(z_n,\bar{z}_n) \rangle = \prod_{i=1}^{n} \left(\sum_{k_i,l_i} (\Phi_i)_{k_i l_i} \right) \left\langle \prod_{i=1}^{n} \mathcal{V}_{k_i}(z_i) \right\rangle \cdot \left\langle \prod_{i=1}^{n} \overline{\mathcal{V}}_{l_i}(\bar{z}_i) \right\rangle.$$



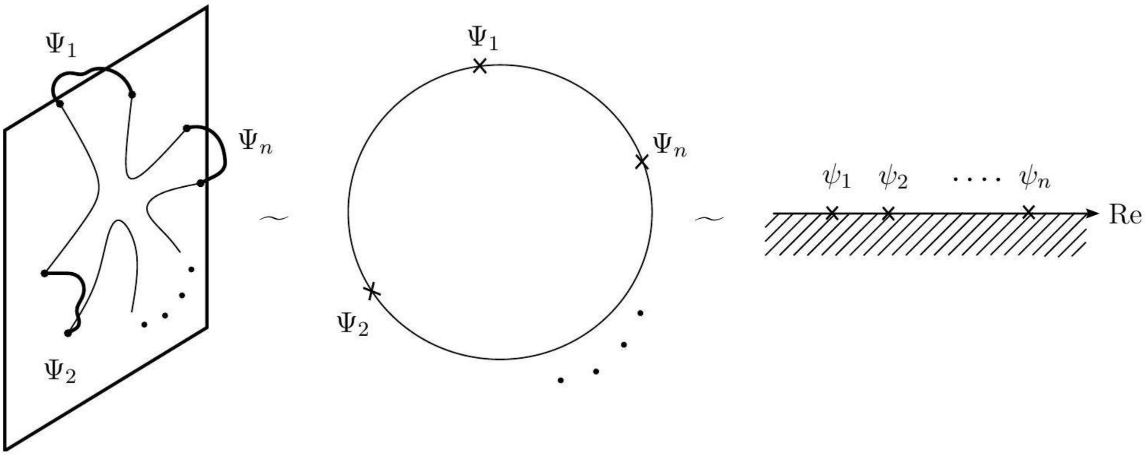
$$\phi_i(z_i,\overline{z_i}) = \sum_{k_i,l_i} (\Phi_i)_{k_i l_i} \Omega_i \mathcal{V}_{k_i}(z_i) \mathcal{V}_{l_i}(z_i^*)$$

$$\langle \phi_1(z_1,\bar{z}_1) \cdots \phi_n(z_n,\bar{z}_n) \rangle_{\mathrm{UHP}}^{(\mathrm{A})} = \prod_{i=1}^{n} \left(\sum_{k_i,l_i} (\Phi_i)_{k_i l_i} \Omega_i \right) \left\langle \prod_{i=1}^{n} \mathcal{V}_{k_i}(z_i) \mathcal{V}_{l_i}(z_i^*) \right\rangle,$$

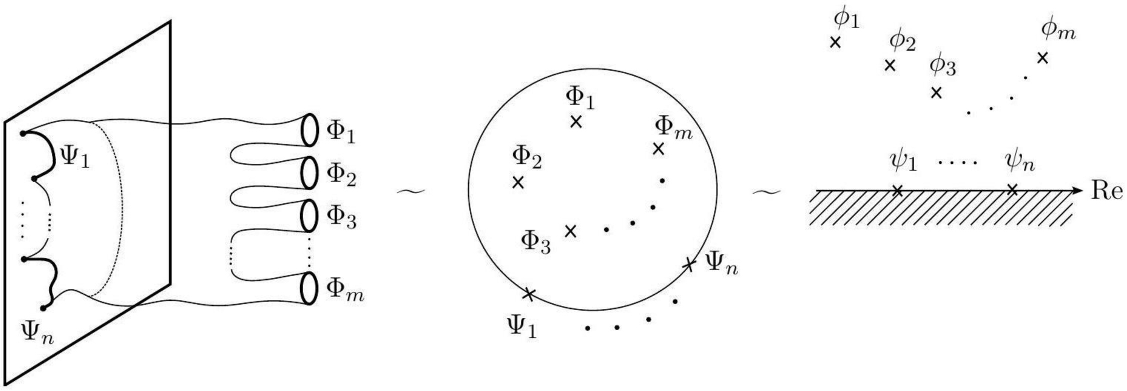


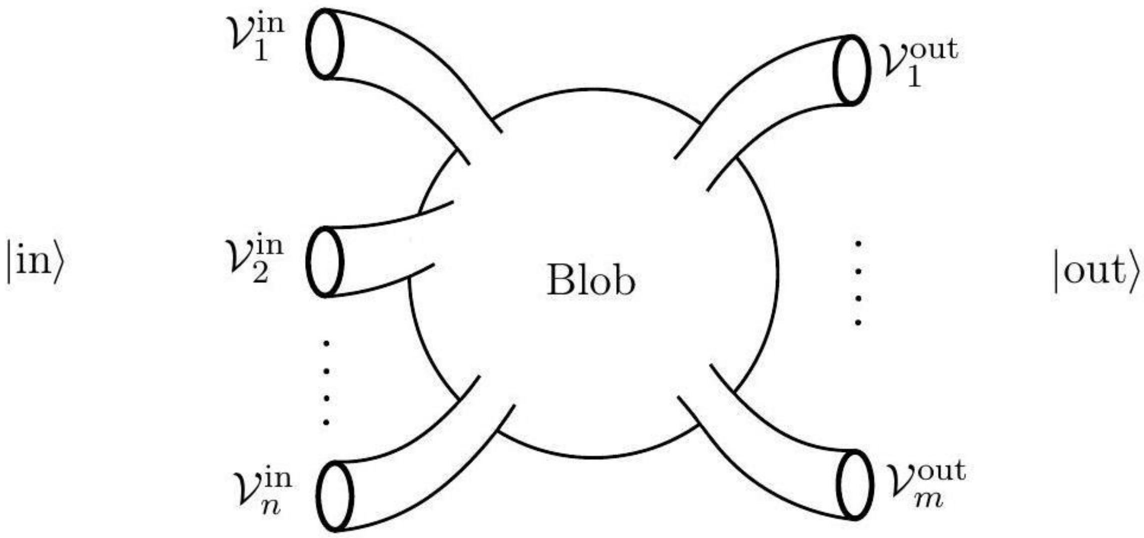
$$\psi_i(x_i) = \Psi_i \mathcal{V}_i(x_i),$$

$$\langle \psi_1(x_1) \cdots \psi_n(x_n) \rangle_{\mathrm{UHP}}^{(\mathrm{A})} = \mathrm{Tr} \left\{ \prod_{i=1}^{n} \Psi_i \right\} \left\langle \prod_{i=1}^{n} \mathcal{V}_i(x_i) \right\rangle.$$



$$\langle \psi_1(x_1)\cdots\psi_n(x_n)\phi_1(z_1,\bar{z}_1)\cdots\phi_n(z_n,\bar{z}_n)\rangle$$
$$= \mathrm{Tr}\left\{\prod_{i=1}^{n}\Psi_i\right\}\prod_{i=1}^{n}\left(\sum_{k_i,l_i}(\Phi_i)_{k_i l_i}\Omega_i\right)\left\langle\prod_{i=1}^{n}\mathcal{V}_i(x_i)\prod_{i=1}^{n}\mathcal{V}_{k_i}(z_i)\mathcal{V}_{l_i}(z_i^*)\right\rangle.$$

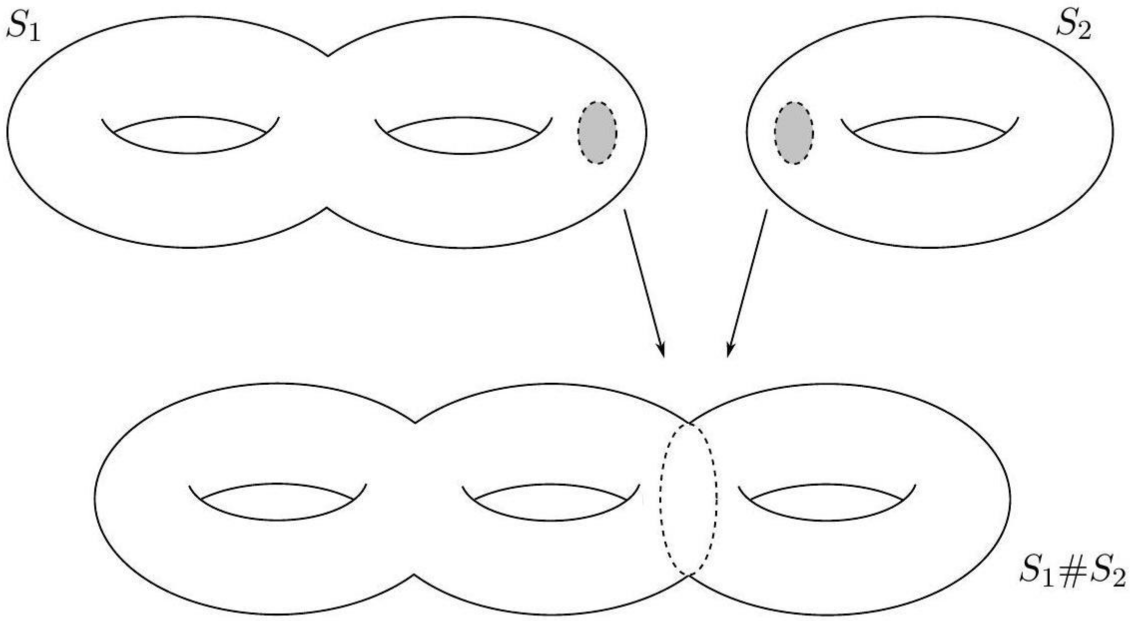




$$\mathcal{A}(1,\dots,n) = \int \frac{\mathcal{D}h_{\alpha\beta}\mathcal{D}X^{\mu}}{\mathrm{Vol}(\mathcal{G})}\mathcal{V}_1 \dots \mathcal{V}_n e^{-S[h,x]}$$

$$\mathcal{V}_i = \int d^2\sigma\sqrt{h}\mathcal{V}_i(\sigma)$$

$$\int \mathcal{D}h_{\alpha\beta} = \sum_{g\in \mathrm{Top}} \int \mathcal{D}h_{\alpha\beta}^{(g)}$$

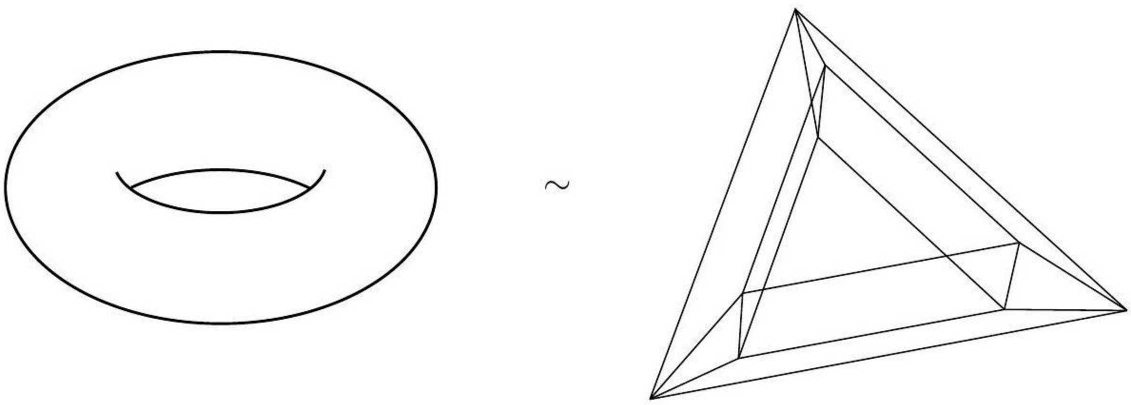


$$S_{EH} = \frac{1}{2^d\pi}\int d^dx\sqrt{h}R^{(d)}$$

$$\#\text{d.o.f.} = \frac{d(d-3)}{2} \quad d \geq 3\,.$$

$$S_{EH} = \frac{1}{4\pi}\int d^2z\sqrt{h}R^{(2)} = \chi$$

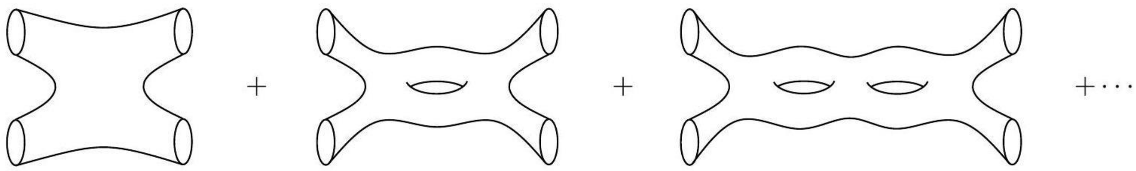
$$\chi = 2 - 2g$$

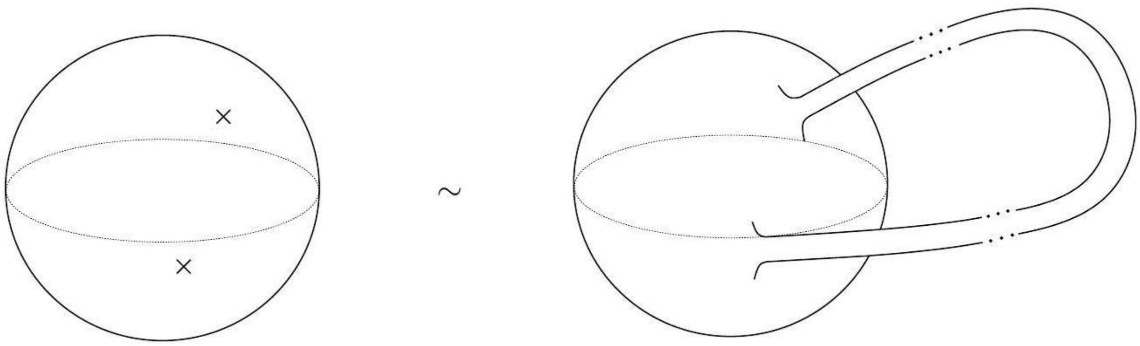


$$S_T[h,X] = -\frac{1}{4\pi\alpha}\int d^2z\sqrt{-h}h^{\alpha\beta}\partial_\alpha X_\mu \partial_\beta X^\mu + \frac{\lambda}{4\pi}\underbrace{\int d^2z\sqrt{-h}R^{(2)}}_{=8\pi(1-g)}$$

$$\mathcal{Z} = \sum_g e^{-2\lambda(1-g)} \int \frac{\mathcal{D}h_{\alpha\beta}^{(g)}\mathcal{D}X^\mu}{\mathrm{Vol}(\mathcal{G})} e^{-S_{\mathrm{Pol}}[h,X]}$$
$$= \sum_g \left(e^\lambda\right)^{2(g-1)} \int \frac{\mathcal{D}h_{\alpha\beta}^{(g)}\mathcal{D}X^\mu}{\mathrm{Vol}(\mathcal{G})} e^{-S_{\mathrm{Pol}}[h,X]}$$

$$g_s = e^\lambda$$

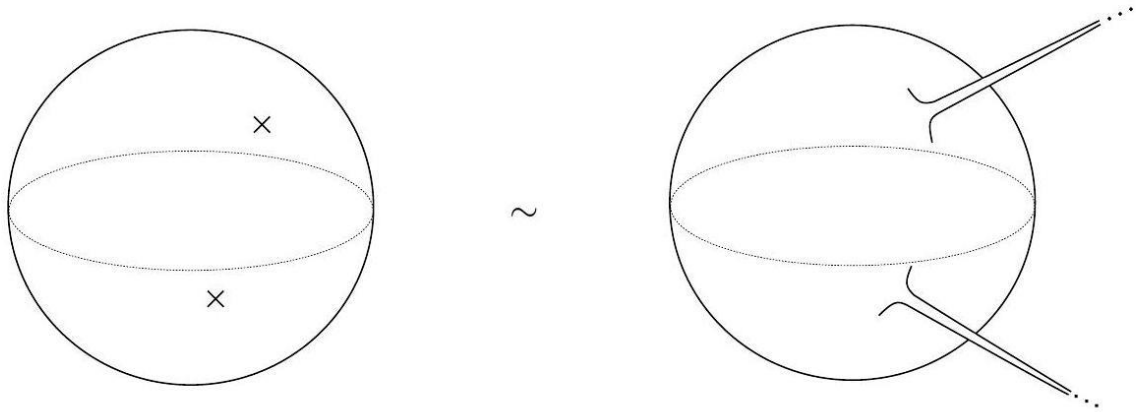




$$\chi(\Sigma_{g,n}) = 2(1-g) - n$$

$$\chi_0 \longrightarrow \chi = \chi_0 - \int d^2z\sqrt{-h}\delta^{(2)}(z-z^*) = \chi_0 - 1,$$

$$\mathcal{A}(1,\ldots,n) = g_s^n \sum_{g=0}^{\infty} g_s^{2(g-1)} \int \frac{\mathcal{D}h^g \mathcal{D}X}{\mathrm{Vol}(\mathcal{G})} \mathcal{V}_1 \ldots \mathcal{V}_n e^{-S_{\mathrm{Pol}}[h,X]}$$
$$= g_s^n \sum_{g=0}^{\infty} g_s^{2(g-1)} \langle\langle \mathcal{V}_1 \ldots \mathcal{V}_n \rangle\rangle^{(g)}$$



$$\langle\langle \mathcal{V}_1 \dots \mathcal{V}_n \rangle\rangle^{(0)} = \int \frac{\mathcal{D}h^g \mathcal{D}X}{\mathrm{Vol}(\mathcal{G})} \int d^2 z_1 \sqrt{h} \dots d^2 z_n \sqrt{h} \mathcal{V}_1 \dots \mathcal{V}_n e^{-S_{\mathrm{Pol}}[h,X]}$$
$$= \int \mathcal{D}X \mathcal{D}c \mathcal{D}b d^2 z_1 \dots d^2 z_n \mathcal{V}_1(z_1, \bar{z}_1) \dots \mathcal{V}_n(z_n, \bar{z}_n), e^{-S[X,b,c]}$$
$$= \int d^2 z_1 \dots d^2 z_n \underbrace{\langle \mathcal{V}_1(z_1, \bar{z}_1) \dots \mathcal{V}_n(z_n, \bar{z}_n) \rangle}_{\text{CFT correlator on } \mathbb{C}} = \mathcal{A}_0$$

$$\langle 0|1|0\rangle = \langle 0 \mid 0\rangle = 0.$$

$$\mathcal{A}_0 = 0 \times \infty,$$

$$\int \mathcal{D}b \mathcal{D}\bar{b} \mathcal{D}c \mathcal{D}\bar{c} (1) e^{-\frac{1}{2\pi}\int d^2 z (b\bar{\partial} c + \bar{b} \partial \bar{c})}$$

$$c^{(0)}(z) = \sum_{n \leq 1} c_n z^{-n+1}$$

$$\mathcal{I} \circ c^{(0)}(z) = \left(\frac{1}{z^2}\right)^{-1} \sum_{n \leq 1} c_n (-1)^n z^{n-1} = \sum_{n \leq 1} c_n (-1)^n z^{n+1}$$

$$c^{(0)}(z) = c_1 + c_0 z + c_{-1} z^2$$

$$\mathcal{A}_0 = \underbrace{\int d^2 z_1 \dots d^2 z_n}_{=\infty} \underbrace{\int \prod_{i=0,\pm 1} dc_i d\bar{c}_i}_{=0} \underbrace{\langle \mathcal{V}_1 \dots \mathcal{V}_n \rangle'}_{\text{no zero modes}}$$

$$\mathcal{A}_0 = \int d^2 z_4 \dots d^2 z_n \langle c\bar{c}\mathcal{V}_1(z_1, \bar{z}_1) \dots c\bar{c}\mathcal{V}_3(z_3, \bar{z}_3) \mathcal{V}_4(z_4, \bar{z}_4) \dots \mathcal{V}_n(z_n, \bar{z}_n) \rangle$$

$$\langle c\bar{c}(z_1) c\bar{c}(z_2) c\bar{c}(z_3) \rangle = \underbrace{\langle 0 | c_{-1}\bar{c}_{-1} c_0 \bar{c}_0 c_1 \bar{c}_1 | 0 \rangle}_{=1} \begin{vmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ z_1^2 & z_2^2 & z_3^2 \end{vmatrix}^2 = |z_{12} z_{13} z_{23}|^2$$

$$\mathcal{A}_0 = \int \frac{d^2 z_1 \dots d^2 z_n}{\mathrm{Vol}(SL(2, \mathbb{C}))} \langle \mathcal{V}_1(z_1, \bar{z}_1) \dots \mathcal{V}_n(z_n, \bar{z}_n) \rangle$$

$$\delta_\epsilon z = \epsilon_1 + \epsilon_2 z + \epsilon_3 z^2, \delta_\epsilon \bar{z} = \bar{\epsilon}_1 + \bar{\epsilon}_2 \bar{z} + \bar{\epsilon}_3 \bar{z}^2$$

$$1=\int |d\epsilon_1 d\epsilon_2 d\epsilon_3|^2 |\delta(\epsilon_1)\delta(\epsilon_2)\delta(\epsilon_3)|^2 = \int [d\epsilon]\prod_{i=1}^{3} |\delta(z_i-\hat{z}_i)|^2 \left|\frac{\delta z_i}{\delta \epsilon_j}\right|$$

$$= \int [d\epsilon]\prod_{i=1}^{3} |\delta(z_i-\hat{z}_i)|^2 \begin{vmatrix} 1 & 1 & 1 \\ \hat{z}_1 & \hat{z}_2 & \hat{z}_3 \\ \hat{z}_1^2 & \hat{z}_2^2 & \hat{z}_3^2 \end{vmatrix}^2 = \int [d\epsilon]\prod_{i=1}^{3} |\delta(z_i-\hat{z}_i)|^2 |z_{12}z_{13}z_{23}|^2$$

$$\mathcal{A}_0 = \underbrace{\left(\int [d\epsilon]\frac{1}{\mathrm{Vol}(SL(2,\mathbb{C}))}\right)}_{=1}\int d^2z_1 \dots d^2z_n \prod_{i=1}^{3} |\delta(z_i-\hat{z}_i)|^2 |z_{12}z_{13}z_{23}|^2 \langle \mathcal{V}_1 \dots \mathcal{V}_n\rangle$$

$$= \int d^2z_4 \dots d^2z_n |\hat{z}_{12}\hat{z}_{13}\hat{z}_{23}|^2 \langle \mathcal{V}_1(\hat{z}_1,\bar{\hat{z}}_1)\dots\mathcal{V}_3(\hat{z}_3,\bar{\hat{z}}_3)\mathcal{V}_4(z_4,\bar{z}_4)\dots\mathcal{V}_n(z_n,\bar{z}_n)\rangle$$

$$\mathcal{A}_0 - \mathcal{A}_0' \to [c\bar{c}\mathcal{V}(\hat{z}_1,\bar{\hat{z}}_1) - c\bar{c}\mathcal{V}(\hat{w}_1,\bar{\hat{w}}_1)].$$

$$c\bar{c}\mathcal{V}(\hat{z}_1,\bar{\hat{z}}_1) - c\bar{c}\mathcal{V}(\hat{w}_1,\bar{\hat{w}}_1) = \sum_{ij}\Phi_{ij}[c\mathcal{V}_i(\hat{z}_1)\bar{c}\bar{\mathcal{V}}_j(\bar{\hat{z}}_1) - c\mathcal{V}_i(\hat{w}_1)\bar{c}\bar{\mathcal{V}}_j(\bar{\hat{w}}_1)].$$

$$c\mathcal{V}_i(\hat{z}) = c\mathcal{V}_i(\hat{w}) - \int_{\hat{z}}^{\hat{w}} d\xi[Q,\mathcal{V}_i(\xi)]$$

$$\bar{c}\bar{\mathcal{V}}_i(\bar{\hat{z}}) = \bar{c}\bar{\mathcal{V}}_i(\bar{\hat{w}}) - \int_{\bar{\hat{z}}}^{\bar{\hat{w}}} d\bar{\xi}[\bar{Q},\bar{\mathcal{V}}_i(\bar{\xi})]$$

$$[c\bar{c}\mathcal{V}(\hat{z}_1,\bar{\hat{z}}_1) - c\bar{c}\mathcal{V}(\hat{w}_1,\bar{\hat{w}}_1)] =$$
$$= \sum_{ij}\Phi_{ij}\left\{-\left[Q,\int_{\hat{z}}^{\hat{w}} d\xi\mathcal{V}_i(\xi)\right]\bar{c}\bar{\mathcal{V}}_j(\bar{\hat{w}}) - c\mathcal{V}_i(\bar{w})\left[\bar{Q},\int_{\bar{\hat{z}}}^{\bar{\hat{w}}} d\bar{\xi}\bar{\mathcal{V}}_j(\bar{\xi})\right]\right.$$
$$\left.+\left[Q\bar{Q},\int_{\hat{z}}^{\hat{w}} d\xi\int_{\bar{\hat{z}}}^{\bar{\hat{w}}} d\bar{\xi}\mathcal{V}_i(\xi)\bar{\mathcal{V}}_j(\bar{\xi})\right]\right\}$$
$$= \sum_{ij}\{[Q,A_{ij}] + [\bar{Q},B_{ij}] + [Q\bar{Q},C_{ij}]\}$$

$$\mathcal{A}_0 - \mathcal{A}_0' = \int d^2z_4 \dots d^2z_n \langle [Q_B,\dots]\mathcal{V}_2(\hat{z}_2,\bar{\hat{z}}_2)\mathcal{V}_3(\hat{z}_3,\bar{\hat{z}}_3)\mathcal{V}_4\dots\mathcal{V}_n\rangle = 0$$

$$Q_B c\bar{c}\mathcal{V}(z,\bar{z}) = 0$$
$$Q_B\int dz d\bar{z}\mathcal{V}(z,\bar{z}) = 0$$

$$\mathcal{A}(p_1,p_2,p_3) = g_s t_1(p_1)t_2(p_2)t_3(p_3)\langle c\bar{c}e^{ip_1\cdot X}(z_1,\bar{z}_1)c\bar{c}e^{ip_2\cdot X}(z_2,\bar{z}_2)c\bar{c}e^{ip_3\cdot X}(z_3,\bar{z}_3)\rangle$$
$$= g_s t_1(p_1)t_2(p_2)t_3(p_3)\delta(p_1+p_2+p_3)$$

$$\mathcal{A}(p_1,p_2,p_3,p_4) = g_s^2 t_1(p_1)t_2(p_2)t_3(p_3)t_4(p_4)$$
$$\times\int d^2z\langle c\bar{c}e^{ip_1\cdot X}(z_1,\bar{z}_1)c\bar{c}e^{ip_2\cdot X}(z_2,\bar{z}_2)c\bar{c}e^{ip_3\cdot X}(z_3,\bar{z}_3)e^{ip_4\cdot X}(z,\bar{z})\rangle.$$

$$\langle c\bar{c}(\Lambda,\bar{\Lambda})c\bar{c}(1,\bar{1})c\bar{c}(0,\bar{0})\rangle = |(\Lambda-1)\Lambda|^2 \sim |\Lambda|^4$$

$$\langle e^{ip_1\cdot X}(\Lambda,\bar{\Lambda})e^{ip_2\cdot X}(1,\bar{1})e^{ip_3\cdot X}(0,\bar{0})e^{ip_4\cdot X}(z,\bar{z})\rangle$$
$$= |\Lambda-1|^{\alpha' p_1\cdot p_2}|\Lambda|^{\alpha' p_1\cdot p_3}|\Lambda-z|^{\alpha' p_1\cdot p_4}|1-z|^{\alpha' p_2\cdot p_4}|z|^{\alpha' p_3\cdot p_4}\delta(P)$$
$$\sim |\Lambda|^{\alpha' p_1\cdot(p_2+p_3+p_4)}|1-z|^{\alpha' p_2\cdot p_4}|z|^{\alpha' p_3\cdot p_4}\delta(P)$$
$$= |\Lambda|^{-\alpha' p_1\cdot p_1}|1-z|^{\alpha' p_2\cdot p_4}|z|^{\alpha' p_3\cdot p_4} = |\Lambda|^{-4}|1-z|^{\alpha' p_2\cdot p_4}|z|^{\alpha' p_3\cdot p_4}\delta(P),$$

$$\mathcal{A}(p_1,p_2,p_3,p_4) = g_s^2\prod_{i=1}^{4} t_i(p_i)\delta(P)\int d^2z|1-z|^{\alpha' p_2\cdot p_4}|z|^{\alpha' p_3\cdot p_4}$$
$$= g_s^2\prod_{i=1}^{4} t_i(p_i)\delta(P)\int d^2z|1-z|^{-\alpha'\frac{t}{2}-4}|z|^{-\alpha'\frac{s}{2}-4}$$

$$s \equiv -(p_1+p_2)^2 = -(p_3+p_4)^2$$
$$t \equiv -(p_1+p_3)^2 = -(p_2+p_4)^2$$
$$u \equiv -(p_1+p_4)^2 = -(p_3+p_2)^2$$

$$s+t+u = 4\times\left(-\frac{4}{\alpha'}\right)$$

$$\int d^2z|z|^{2(a-1)}|1-z|^{2(b-1)} = 2\pi\frac{\Gamma(a)\Gamma(b)\Gamma(1-a-b)}{\Gamma(1-a)\Gamma(1-b)\Gamma(a+b)}$$

$$\mathcal{A}(p_1,p_2,p_3,p_4) = 2\pi g_s^2\prod_{i=1}^{4} t_i(p_i)\delta(P)\frac{\Gamma\left(-\frac{\alpha' s}{4}-1\right)\Gamma\left(-\frac{\alpha' t}{4}-1\right)\Gamma\left(-\frac{\alpha' u}{4}-1\right)}{\Gamma\left(\frac{\alpha' s}{4}+2\right)\Gamma\left(\frac{\alpha' t}{4}+2\right)\Gamma\left(\frac{\alpha' u}{4}+2\right)}$$

$$\alpha' s = \alpha' m^2 = 4(n-1), n = 0,1,2,3,\cdots$$

$$\lambda\chi = \lambda\left(\frac{1}{4\pi}\int_M d^2z\sqrt{h}R^{(2)} + \frac{1}{2\pi}\int_{\partial M} dsk\right)$$

$$k = \frac{1}{2}t^\alpha t^\beta\nabla_\alpha n_\beta$$

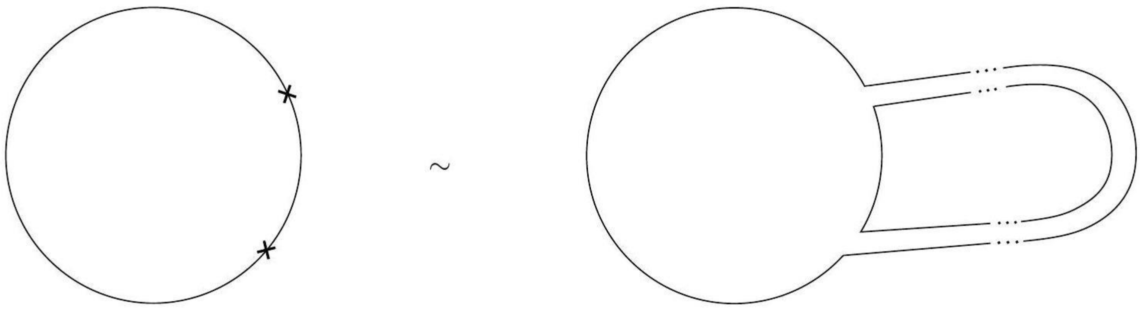
$$\gamma(\theta) = R(\cos\theta,\sin\theta), t(\theta) = (-\sin\theta,\cos\theta), n(\theta) = (\cos\theta,\sin\theta),$$

$$k = \frac{1}{R}$$

$$\chi = \frac{1}{2\pi}\int_{\partial M} dsk = \frac{1}{2\pi}\int_{\partial M}\frac{ds}{R} = \int_0^{2\pi}\frac{d\theta}{2\pi} = 1$$

$$\chi = 2(1-g)-b$$

$$\chi = 2(1-g)-b-n_c-\frac{n_o}{2}$$



$$\mathcal{A}(n_c,n_o) = g_s^{n_c+\frac{n_o}{2}} \sum_{n=0}^{\infty} g_s^{2(g-1)} \sum_{b=1}^{\infty} g_s^b \left\langle \langle \mathcal{V}_1^c \dots \mathcal{V}_{n_c}^c \mathcal{V}_1^o \dots \mathcal{V}_{n_o}^o \rangle \right\rangle_{g,b},$$

$$g_{\text{open}} = \sqrt{g_{\text{closed}}}$$

$$\left\langle \langle \mathcal{V}_1^c \dots \mathcal{V}_{n_c}^c \mathcal{V}_1^o \dots \mathcal{V}_{n_o}^o \rangle \right\rangle_{g=0,b=1} = \int \frac{\mathcal{D}X\mathcal{D}h}{\text{Vol}(\mathcal{G})} e^{-S_{\text{Pol}}[X,h]}$$
$$\times \int_{\mathbb{R}} dx_1 \cdots \int_{\mathbb{R}} dx_{n_o} \text{Tr}\mathcal{P}[\mathcal{V}_1^o(x_1) \dots \mathcal{V}_{n_o}^o(x_{n_o})]$$
$$\times \int_{\text{UHP}} d^2z_1 \cdots \int_{\text{UHP}} d^2z_{n_c} \mathcal{V}_1^c(z_1,\bar{z}_1) \dots \mathcal{V}_{n_c}^c(z_{n_c},\bar{z}_{n_c}).$$

$$\mathcal{P}[\dots \mathcal{V}_i(x_i) \dots \mathcal{V}_j(x_j) \dots] = \begin{cases} [\dots \mathcal{V}_i(x_i) \dots \mathcal{V}_j(x_j) \dots] & x_i > x_j \\ (-1)^{\epsilon}[\dots \mathcal{V}_j(x_j) \dots \mathcal{V}_i(x_i) \dots] & x_j < x_i \end{cases},$$

$$\left\langle \langle \mathcal{V}_1^o \dots \mathcal{V}_{n_o}^o \rangle \right\rangle_{g=0,b=1} = \text{Tr} \int \mathcal{D}X\mathcal{D}c\mathcal{D}b \int_{\mathbb{R}} dx_1 \dots dx_{n_o} \mathcal{P}[\mathcal{V}_1^o(x_1) \dots \mathcal{V}_{n_o}^o(x_{n_o})] e^{-S[X,b,c]}$$
$$= \text{Tr} \int_{\mathbb{R}} dx_1 \dots dx_{n_o} \mathcal{P}\langle \mathcal{V}_1^o(x_1) \dots \mathcal{V}_{n_o}^o(x_{n_o}) \rangle_{\text{UHP}}$$

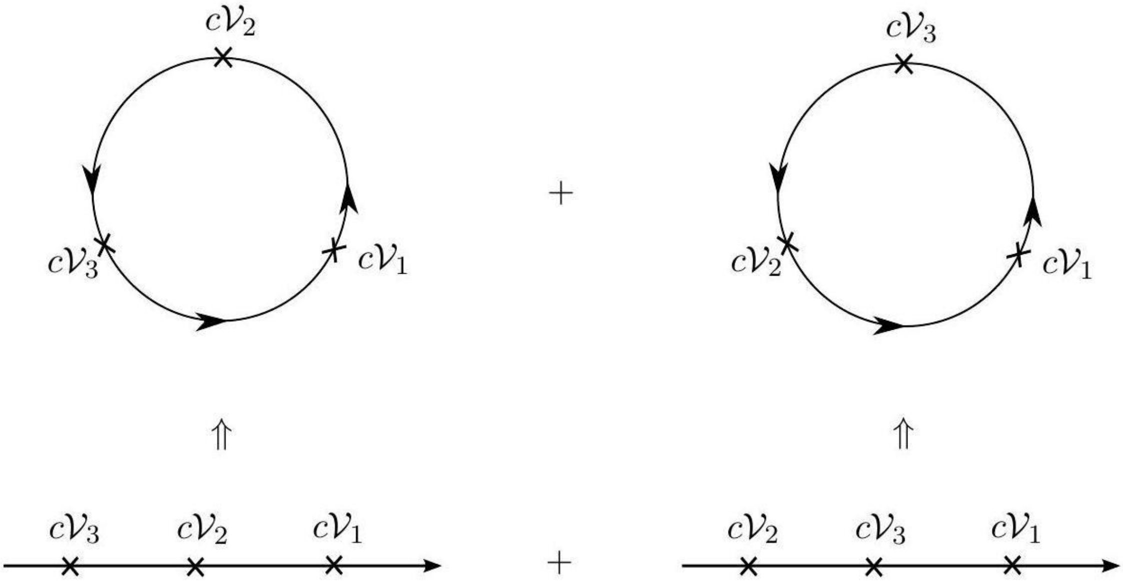
$$\int dx_i \mathcal{V}_i^o(x_i) \to c\mathcal{V}_i^o(x_i), i = 1,2,3.$$

$$\left\langle \langle \mathcal{V}_1^o \dots \mathcal{V}_{n_o}^o \rangle_{g=0,b=1} = \text{Tr} \int_{\mathbb{R}} dx_4 \dots dx_{n_o} \mathcal{P}\langle c\mathcal{V}_1(x_1) c\mathcal{V}_2(x_2) c\mathcal{V}_3(x_3) \mathcal{V}(x_4) \dots \mathcal{V}(x_{n_o}) \rangle$$
$$+ \text{Tr} \int_{\mathbb{R}} dx_4 \dots dx_{n_o} \mathcal{P}\langle c\mathcal{V}_1(x_1) c\mathcal{V}_3(x_2) c\mathcal{V}_2(x_3) \mathcal{V}(x_4) \dots \mathcal{V}(x_{n_o}) \rangle.$$

$$P_1 + P_2 + P_3 = 0, \alpha' P_i^2 = 1, i = 1, \dots, 3,$$

$$\mathcal{V}_i(x) = t(P_i) : e^{iP_i \cdot X} : (x).$$

$$\mathcal{A}(1,2,3) = g_s^{\frac{1}{2}} (\langle c\mathcal{V}_1(x_1) c\mathcal{V}_2(x_2) c\mathcal{V}_3(x_3) \rangle + \langle c\mathcal{V}_1(x_1) c\mathcal{V}_3(x_2) c\mathcal{V}_2(x_3) \rangle),$$



$$\langle c(x_1)c(x_2)c(x_3)\rangle = x_{12}x_{13}x_{23}$$
$$\left\langle e^{iP_1\cdot X}(x_1)e^{iP_2\cdot X}(x_2)e^{iP_3\cdot X}(x_3)\right\rangle = x_{12}^{2\alpha' P_1\cdot P_2}x_{13}^{2\alpha' P_1\cdot P_3}x_{23}^{2\alpha' P_2\cdot P_3},$$

$$P_1^2 + P_2^2 + 2P_1\cdot P_2 = P_3^2 \Longrightarrow P_1\cdot P_2 = -\frac{1}{2\alpha'} = P_i\cdot P_j \text{ for } i \neq j$$

$$\mathcal{A}(1,2,3) = 2g_s^{\frac{1}{2}}t(P_1)t(P_2)t(P_3)\delta(P_1+P_2+P_3)$$

$$\mathcal{A}(1,\ldots,4) = g_s\int_{-\infty}^{+\infty} dx\mathcal{P}\langle c\mathcal{V}_1(x_1)c\mathcal{V}_2(x_2)c\mathcal{V}_3(x_3)\mathcal{V}_4(x)\rangle$$
$$+g_s\int_{-\infty}^{+\infty} dx\mathcal{P}\langle c\mathcal{V}_1(x_1)c\mathcal{V}_3(x_3)c\mathcal{V}_2(x_2)\mathcal{V}_4(x)\rangle$$

$$\langle c(\Lambda)c(1)c(0)\rangle = \Lambda(\Lambda-1)$$
$$\left\langle e^{iP_1\cdot X}(\Lambda)e^{iP_2\cdot X}(1)e^{iP_3\cdot X}(0)e^{iP_4\cdot X}(x)\right\rangle = (\Lambda-1)^{2\alpha' P_1\cdot P_2}\Lambda^{2\alpha' P_1\cdot P_3}(\Lambda-x)^{2\alpha' P_1\cdot P_4}\times$$
$$\times |x|^{2\alpha' P_3\cdot P_4}|1-x|^{2\alpha' P_2\cdot P_4}$$

$$\Lambda^{2\alpha'(P_2+P_3+P_4)\cdot P_1} = \Lambda^{-2\alpha' P_1^2} = \Lambda^{-2}$$

$$\langle c\mathcal{V}_1(\Lambda)c\mathcal{V}_2(1)c\mathcal{V}_3(0)\mathcal{V}_4(x)\rangle|_{\Lambda\to\infty} = \left.\frac{\Lambda(\Lambda-1)}{\Lambda^2}\right|_{\Lambda\to\infty}|x|^{2\alpha' P_3\cdot P_4}|1-x|^{2\alpha' P_2\cdot P_4}$$
$$= |x|^{2\alpha' P_3\cdot P_4}|1-x|^{2\alpha' P_2\cdot P_4}$$

$$\int_{-\infty}^{+\infty} dx|x|^{2\alpha' P_3\cdot P_4}|1-x|^{2\alpha' P_2\cdot P_4} = I_1 + I_2 + I_3$$

$$I_1 = \int_0^1 dx x^{2\alpha' P_3 \cdot P_4} (1-x)^{2\alpha' P_2 \cdot P_4}$$
$$I_2 = \int_1^\infty dx x^{2\alpha' P_3 \cdot P_4} (x-1)^{2\alpha' P_2 \cdot P_4}$$
$$I_3 = \int_{-\infty}^0 dx (-x)^{2\alpha' P_3 \cdot P_4} (1-x)^{2\alpha' P_2 \cdot P_4}$$

$$I_2 = -\int_1^0 \frac{dy}{y^2} y^{-2\alpha' P_3 \cdot P_4} \left(\frac{1-y}{y}\right)^{2\alpha' P_2 \cdot P_4}$$
$$= \int_0^1 dy y^{-2-2\alpha'(P_2+P_3)\cdot P_4} (1-y)^{2\alpha' P_2 \cdot P_4}$$
$$= \int_0^1 dx x^{2\alpha' P_1 \cdot P_4} (1-y)^{2\alpha' P_2 \cdot P_4}$$

$$I_3 = -\int_1^0 \frac{dy}{(1-y)^2} \left(\frac{y}{1-y}\right)^{2\alpha' P_3 \cdot P_4} (1-y)^{-2\alpha' P_2 \cdot P_4}$$
$$= \int_0^1 dy y^{2\alpha' P_3 \cdot P_4} (1-y)^{-2-2\alpha' P_4 \cdot (P_3+P_2)}$$
$$= \int_0^1 dx x^{2\alpha' P_3 \cdot P_4} (1-x)^{2\alpha' P_1 \cdot P_4}$$

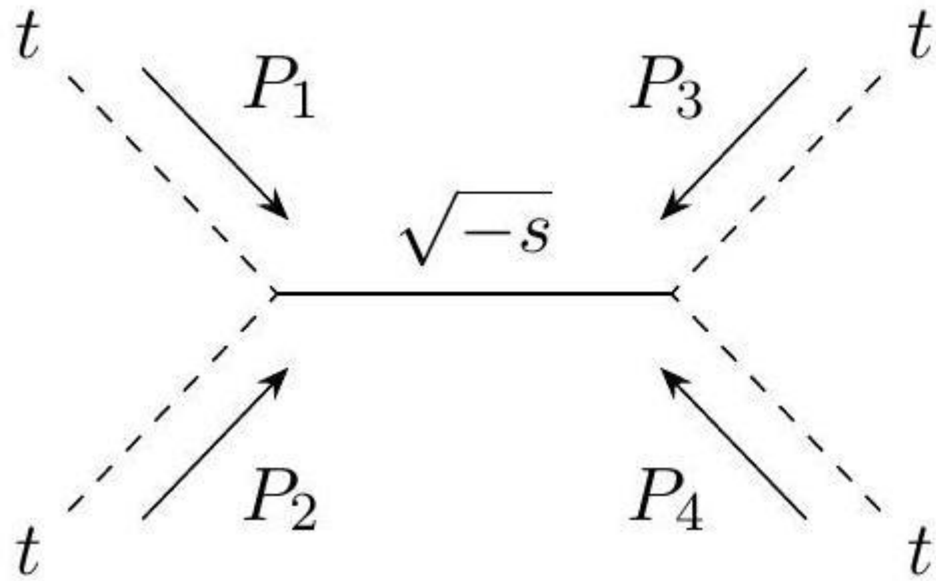
$$s = -(P_1+P_2)^2, t = -(P_1+P_3)^2, u = -(P_1+P_4)^2,$$

$$s + t + u = 4 \times \left(-\frac{1}{\alpha'}\right), \alpha' P_i \cdot P_j = -2 - \alpha' s_{ij}$$

$$s_{12} = s, s_{13} = t, s_{14} = u$$

$$\mathcal{A}(1, \dots, 4) = 2 g_o^2 \delta(P_1 + P_2 + P_3 + P_4)[I(s,t) + I(t,u) + I(u,s)]$$

$$I(a,b) = \int_0^1 dx x^{-2-\alpha' a} (1-x)^{-2-\alpha' b} = B(-1-\alpha' a, -1-\alpha' b)$$



$$I(s,t) = \frac{\Gamma(-\alpha' s - 1)\Gamma(-\alpha' t - 1)}{\Gamma(-\alpha'(s+t) - 2)}$$

$$\Gamma(z) = \frac{(-1)^n}{n!}\frac{1}{z+n} \quad \text{for } z \sim -n \in -\mathbb{N}$$

$$\begin{aligned} I(s,t) &= \frac{(-1)^n}{n!}\frac{1}{n-1-\alpha' s}\frac{\Gamma(-1-\alpha' t)}{\Gamma(-1-n-\alpha' t)} \\ &= \frac{1}{n!}\frac{1}{n-1-\alpha' s}(\alpha' t + 1)\ldots(\alpha' t + n + 1) \\ t &\to \infty \frac{(\alpha' t)^n}{s + m_n^2} \end{aligned}$$

$$\mathcal{A} \sim \frac{t^J}{s+m^2}, s \sim -m^2$$

$$\alpha' m^2 = J - 1$$

$$\mathcal{V}(x) = \mathbb{A}_\mu(P) : j^\mu e^{iP\cdot X} : (x) \quad \text{with} \quad P^2 = 0$$

$$\mathcal{A} = g_o \underbrace{\mathrm{Tr}\langle c\mathcal{V}_1(x_1) c\mathcal{V}_2(x_2) c\mathcal{V}_3(x_3)\rangle}_{\mathcal{A}_{123}} + g_o \underbrace{\mathrm{Tr}\langle c\mathcal{V}_1(x_1) c\mathcal{V}_3(x_2) c\mathcal{V}_2(x_3)\rangle}_{\mathcal{A}_{132}}$$

$$\begin{aligned} \mathcal{A}_{123} &= \mathrm{Tr}\left\langle c\mathbb{A}_\mu(P_1) : j^\mu e^{iP_1\cdot X} : (\Lambda) c\mathbb{A}_\nu(P_2) : j^\nu e^{iP_2\cdot X} : (1) c\mathbb{A}_\rho(P_3) : j^\rho e^{iP_3\cdot X} : (0)\right\rangle \\ &= \Lambda(\Lambda - 1)\mathrm{Tr}\left[\mathbb{A}_\mu(P_1)\mathbb{A}_\nu(P_2)\mathbb{A}_\rho(P_3)\right] \\ &\times \left\langle : j^\mu e^{iP_1\cdot X} : (\Lambda) : j^\nu e^{iP_2\cdot X} : (1) : j^\rho e^{iP_3\cdot X} : (0)\right\rangle \end{aligned}$$

$$\begin{aligned} \mathcal{A}_{123} &= \mathrm{Tr}\left[\mathbb{A}_\mu^{(1)}\mathbb{A}_\nu^{(2)}\mathbb{A}_\rho^{(3)}\right]\left(P_1^\rho \eta^{\mu\nu} + P_2^\mu \eta^{\rho\nu} + P_3^\nu \eta^{\rho\mu} + 2\alpha' P_1^\rho P_2^\mu P_3^\nu\right)\delta(P), \\ \mathcal{A}_{132} &= \mathrm{Tr}\left[\mathbb{A}_\mu^{(1)}\mathbb{A}_\rho^{(3)}\mathbb{A}_\nu^{(2)}\right]\underbrace{\left(P_1^\nu \eta^{\mu\rho} + P_3^\mu \eta^{\nu\rho} + P_2^\rho \eta^{\mu\nu} + 2\alpha' P_1^\nu P_3^\mu P_2^\rho\right)}_{t_{132}^{\mu\rho\nu}}\delta(P), \end{aligned}$$

$$\mathcal{A} = g_o\delta(P)\mathrm{Tr}\left[\mathbb{A}_\mu^{(1)}\mathbb{A}_\rho^{(3)}\mathbb{A}_\nu^{(2)}\right]\left(t_{123}^{\mu\nu\rho} + t_{132}^{\mu\rho\nu}\right) + g_o\delta(P)t_{123}^{\mu\nu\rho}\mathrm{Tr}\left[\mathbb{A}_\mu^{(1)}\left[\mathbb{A}_\nu^{(2)}, \mathbb{A}_\rho^{(3)}\right]\right],$$

$$t_{123}^{\mu\nu\rho} + t_{132}^{\mu\rho\nu} = -P_1^\mu\left(\eta^{\nu\rho} - 2\alpha' P_3^\nu P_2^\rho\right) - P_2^\nu\left(\eta^{\mu\rho} - 2\alpha' P_3^\mu P_2^\rho\right) - P_3^\rho\left(\eta^{\mu\nu} - 2\alpha' P_2^\mu P_3^\nu\right) = 0$$

$$\mathcal{A} = g_o\delta(P)t_{123}^{\mu\nu\rho}\mathrm{Tr}\left[\mathbb{A}_\mu^{(1)}\left[\mathbb{A}_\nu^{(2)}, \mathbb{A}_\rho^{(3)}\right]\right]$$

$$\mathcal{L}_{\mathrm{eff}} \supset -\frac{1}{4}\mathrm{Tr}\left[\partial_\mu A_\nu - \partial_\nu A_\mu\right]^2 - \frac{1}{2}g_o\mathrm{Tr}\left[\partial_\mu A_\nu[A^\mu, A^\nu]\right]$$

$$\mathcal{L}_{\mathrm{eff}} \supset -\frac{1}{4}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right]$$

$$\mathcal{L}_{\mathrm{eff}} \supset -\frac{1}{4}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu} + \alpha' F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\right] + O(\alpha'^2)$$

$$S[X] = \frac{1}{2\pi\alpha'}\int d^2z\,\partial X^\mu\bar{\partial}X^\nu G_{\mu\nu}(X)$$

$$G_{\mu\nu}(X) = \eta_{\mu\nu} + h_{\mu\nu}(X)$$

$$\begin{aligned} S_{\eta+h} &= \frac{1}{2\pi\alpha'}\int d^2z\,\partial X^\mu\bar{\partial}X^\nu\eta_{\mu\nu} + \frac{1}{2\pi\alpha'}\int d^2z\,\partial X^\mu\bar{\partial}X^\nu h_{\mu\nu}(X) \\ &= S_\eta + \frac{1}{2\pi\alpha'}\int d^2z\,\mathcal{V}_h(z,\bar{z}) \end{aligned}$$

$$\mathcal{V}_h(z,\bar{z}) = \int dp\,h_{\mu\nu}(p)\partial X^\mu\bar{\partial}X^\nu e^{ip\cdot X}(z,\bar{z})$$

$$h_{\mu\nu}(X) = \int dp\,e^{ip\cdot X}h_{\mu\nu}(p)$$

$$\begin{cases} P^2 = 0 \\ P_\mu h^{\mu\nu} = h^{\mu\nu}P_\nu = 0 \end{cases}$$

$$Z = \int \mathcal{D}X e^{-S[X] - \int d^2z\mathcal{V}_h(z,\bar{z})} = \int \mathcal{D}X e^{-S[X]}e^{-\int d^2z\mathcal{V}_h(z,\bar{z})}$$

$$e^{-\int d^2z\mathcal{V}_h(z,\bar{z})} = 1 - \int d^2z\mathcal{V}_h(z,\bar{z}) - \frac{1}{2}\int d^2z'\int d^2z\mathcal{V}_h(z,\bar{z})\mathcal{V}_h'(z',\bar{z}') + \cdots$$

$$G_{\mu\nu}(X) = G_{\mu\nu}(X_0) + G_{\mu\nu,\rho}(X_0)Y^\rho + G_{\mu\nu,\rho\sigma}(X_0)Y^\rho Y^\sigma + \mathcal{O}(Y^3)$$

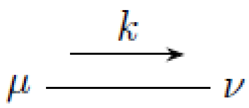
$$S[Y] = \frac{1}{2\pi\alpha'}\int d^2z\left(\partial Y^\mu\bar{\partial}Y^\nu G_{\mu\nu}(X_0) + \partial Y^\mu\bar{\partial}Y^\nu\cdot Y^\rho G_{\mu\nu,\rho}(X_0) + \cdots\right)$$

$$G_{\mu\nu}(X) = \underbrace{G_{\mu\nu}(X_0)}_{\eta_{\mu\nu}} - \frac{1}{3}R_{\mu\rho\nu\sigma}(X - X_0)^\rho(X - X_0)^\sigma + \mathcal{O}(X^3)$$

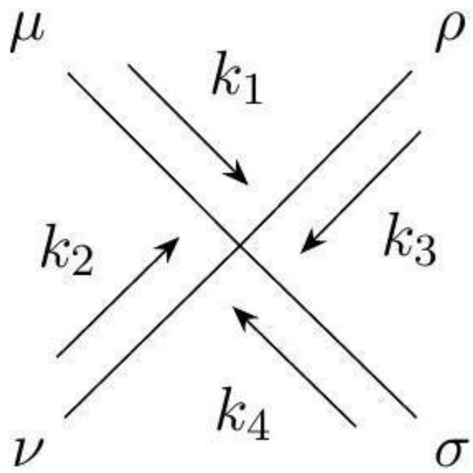
$$X^\mu = X_0^\mu + \sqrt{\alpha'}Y^\mu$$

$$G_{\mu\nu}(X) = G_{\mu\nu}(X_0) - \frac{1}{3}\alpha' R_{\mu\rho\nu\sigma} Y^\rho Y^\sigma + \mathcal{O}(X^3)$$

$$S[Y] = \frac{1}{2\pi\alpha'}\int \ d^2 z \alpha' \partial Y^\mu \bar{\partial} Y^\nu \left(\eta_{\mu\nu} - \frac{1}{3}\alpha' R_{\mu\rho\nu\sigma} Y^\rho Y^\sigma + \cdots\right)$$

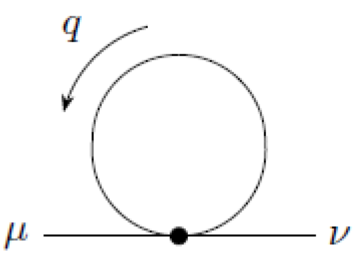


$$\xrightarrow{\text{FT}} \frac{\eta_{\mu\nu}}{k^2},$$



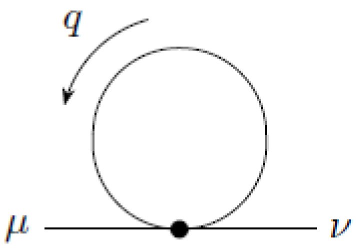
$$\xrightarrow{\text{FT}} \frac{\alpha'}{3} R_{\mu\rho\nu\sigma}(k_1 \cdot k_2),$$

$$\alpha' R \ll 1,$$



$$\propto (k_1 \cdot k_2)\, \delta(k_1 + k_2)\, R_{\mu\rho\nu}{}^{\rho} \int d^2 q \, \frac{1}{q^2}.$$

$$\int \ d^2 q \frac{1}{q^2} \to \mu^{-\epsilon} \int \ \frac{d^{2+\epsilon} q}{q^2} \sim \mu^{-\epsilon} \int^{\infty} d\, dq q^{\epsilon-1} \sim \mu^{-\epsilon} \frac{1}{\epsilon},$$



$$\propto \mu^{-\epsilon}\frac{R_{\mu\nu}}{\epsilon}(k_1\cdot k_2)\delta(k_1+k_2)\,.$$

$$S_{\text{ren}} = S_{\text{bare}} + \int d^2z\partial Y^\mu\bar{\partial}Y^\nu\left(-\frac{\alpha'}{3}\right)\mu^{-\epsilon}\frac{R_{\mu\nu}}{\epsilon} = \int d^2z\partial Y^\mu\bar{\partial}Y^\nu G^{\text{ren}}_{\mu\nu},$$

$$G^{\text{ren}}_{\mu\nu} = G^{\text{bare}}_{\mu\nu} - \mu^{-\epsilon}\frac{\alpha'}{3}R_{\mu\nu}\frac{1}{\epsilon}$$

$$\beta_{\mu\nu} = \mu\frac{\partial}{\partial\mu}G^{\text{ren}}_{\mu\nu}\Big|_{\epsilon\to 0} \propto \alpha' R_{\mu\nu},$$

$$\beta_{\mu\nu} = 0$$

$$R_{\mu\nu} = 0,$$

$$G_{\mu\nu}(X) = G_{\mu\nu}(X_0) - \frac{1}{3}\alpha' R_{\mu\nu\rho\sigma}Y^\rho Y^\sigma + \frac{8}{180}(\alpha')^2 R_{\mu\alpha\beta\lambda}R_{\nu\gamma\delta}{}^{\lambda}Y^\alpha Y^\beta Y^\gamma Y^\delta + \mathcal{O}(Y^5).$$

$$\beta_{\mu\nu} \propto \alpha' R_{\mu\nu} + \sharp(\alpha')^2 R_{\mu\rho}R_\nu{}^\rho.$$

$$S[X] = \int d^{2+\epsilon}z\sqrt{\gamma}\gamma^{ab}\partial_a X^\mu\bar{\partial}_b X^\nu G_{\mu\nu}(X),$$

$$\gamma^{ab} = e^{+\phi}\delta^{ab}$$
$$\gamma_{ab} = e^{-\phi}\delta_{ab}$$

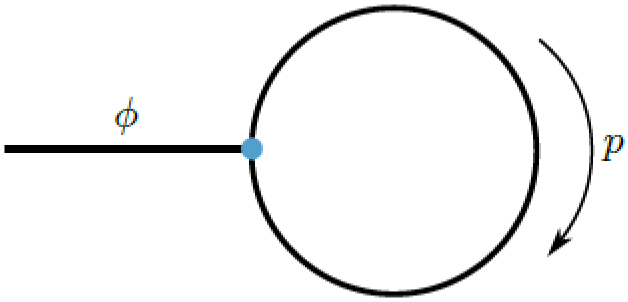
$$\gamma = \det(\gamma_{ab}) = e^{\text{Tr}\{\log\gamma_{ab}\}}$$
$$= e^{-\phi\text{Tr}\{\delta_{ab}\}} = e^{-\phi(2+2\epsilon)}$$
$$\sqrt{\gamma}\gamma^{ab} = e^{-\epsilon\phi}\delta^{ab} = (1-\epsilon\phi)\delta^{ab}$$

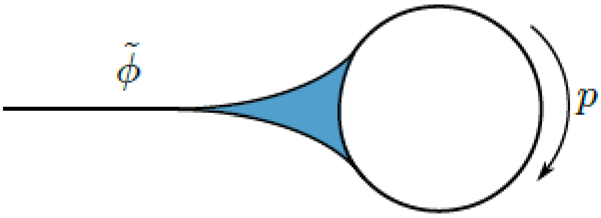
$$S[X] = \int d^{2+\epsilon}z(1-\epsilon\phi)\partial X^\mu\bar{\partial}X^\nu G_{\mu\nu}(X)$$

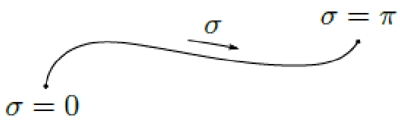
$$S[X]= \int d^{2+\epsilon}z(1-\epsilon\phi)\partial X^\mu\bar{\partial}X^\nu\left(G_{\mu\nu} - \frac{\alpha'}{3\epsilon}R_{\mu\nu}\right)$$
$$\overset{\epsilon\to 0}{=} \int d^2z\left(\partial X^\mu\bar{\partial}X^\nu G_{\mu\nu} + \alpha'\phi R_{\mu\nu}\partial X^\mu\bar{\partial}X^\nu - \frac{\alpha'}{3\epsilon}R_{\mu\nu}\partial X^\mu\bar{\partial}X^\nu\right)$$

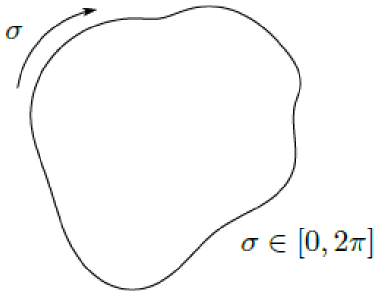
$$\delta_w S = \frac{\delta S}{\delta\gamma^{ab}}\delta_w\gamma^{ab} = T_{ab}\left(-w\gamma^{ab}\right) = -wT^a_a$$

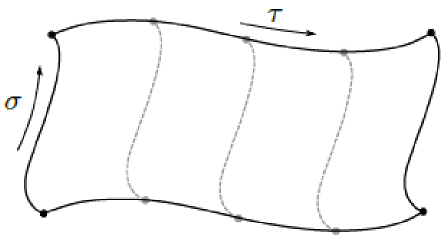
$$\delta_w S = 0 \iff T_a^a = 0$$

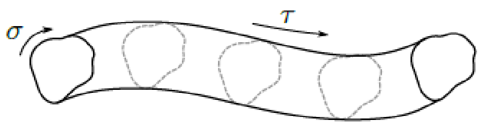


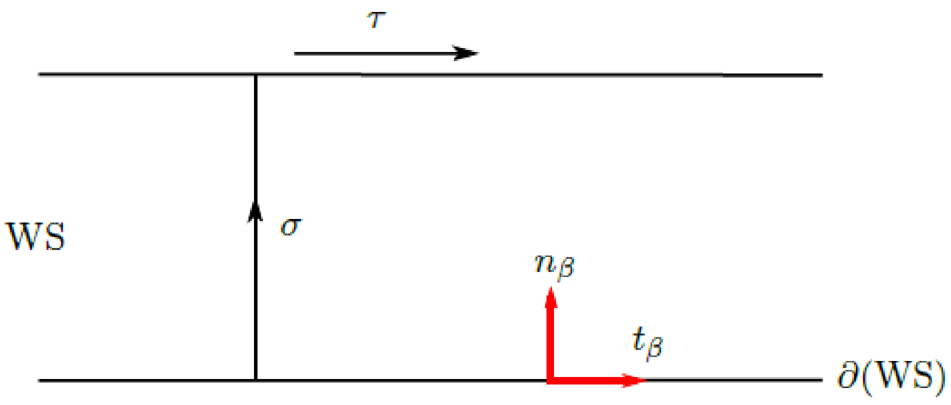


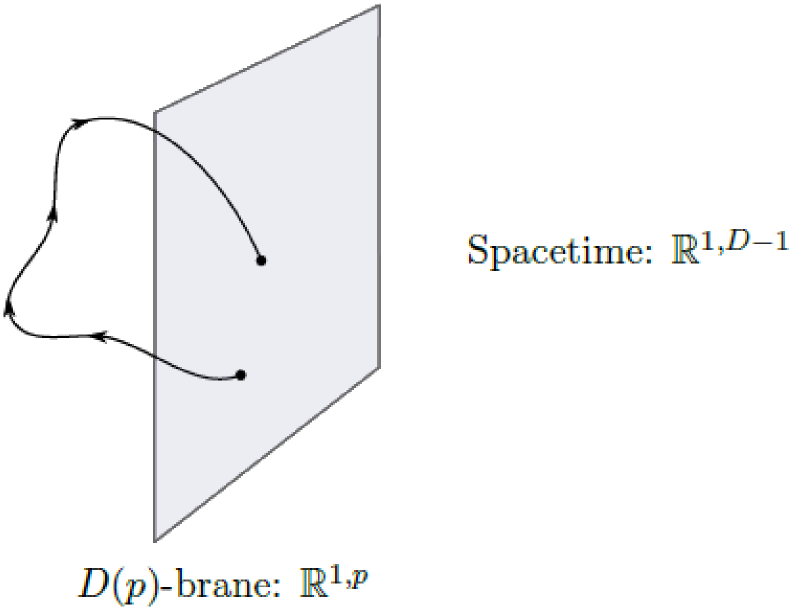


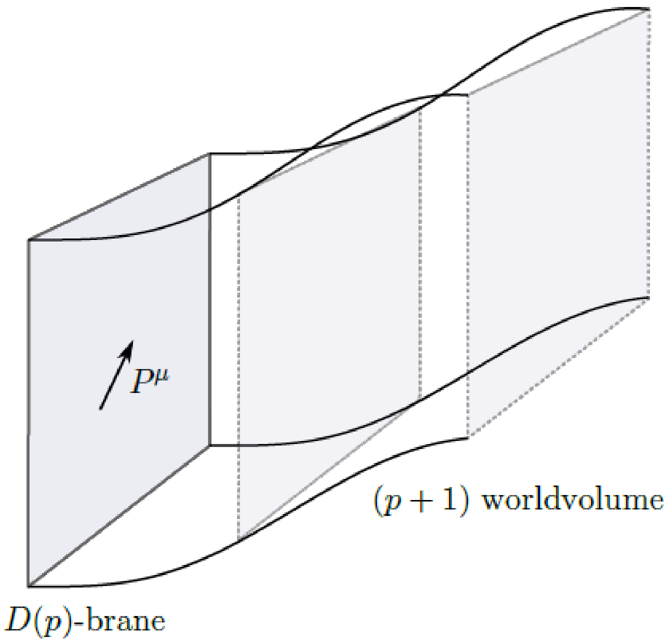


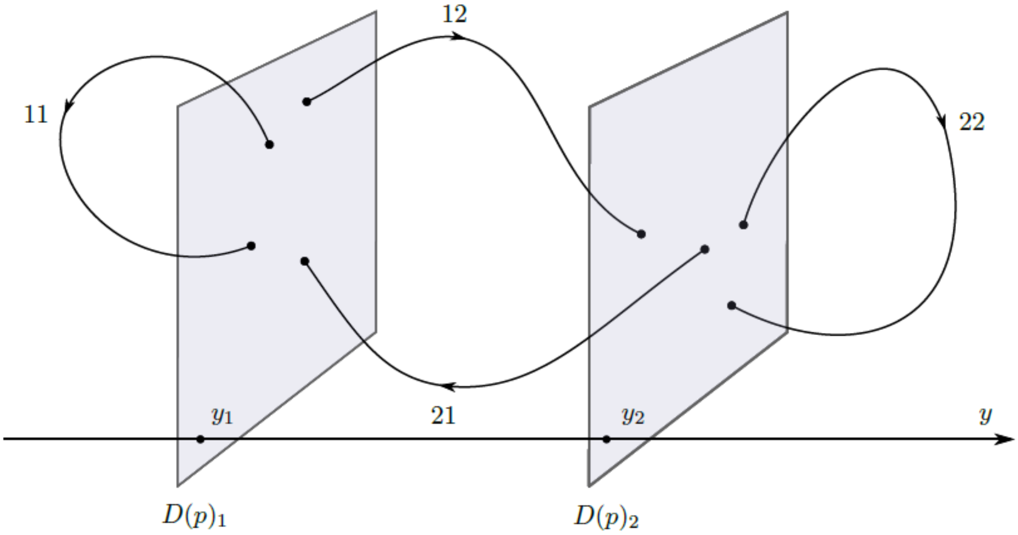


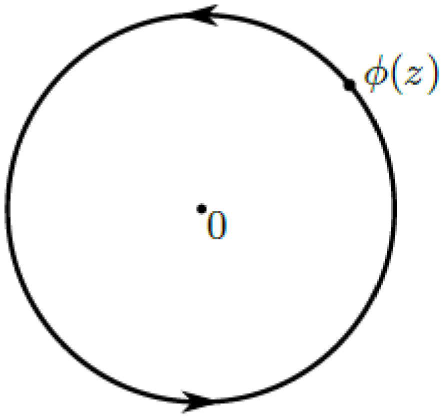



τ
WS
σ
n_β
t_β
$\partial(\mathrm{WS})$

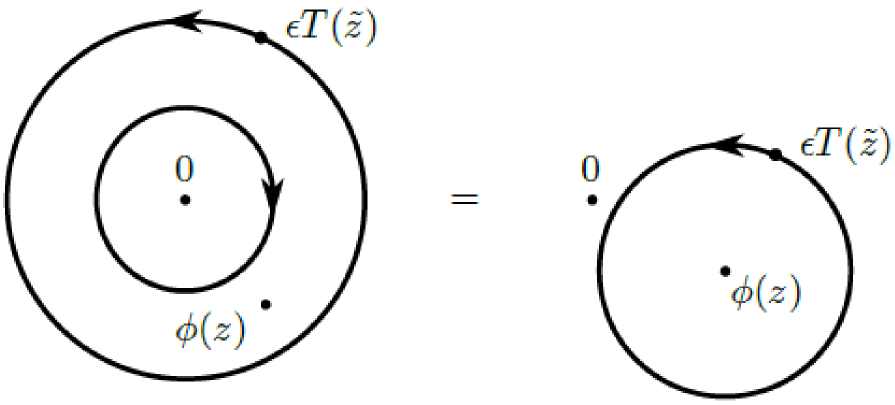

Spacetime: $\mathbb{R}^{1,D-1}$
$D(p)$-brane: $\mathbb{R}^{1,p}$


P^μ
$(p+1)$ worldvolume
$D(p)$-brane

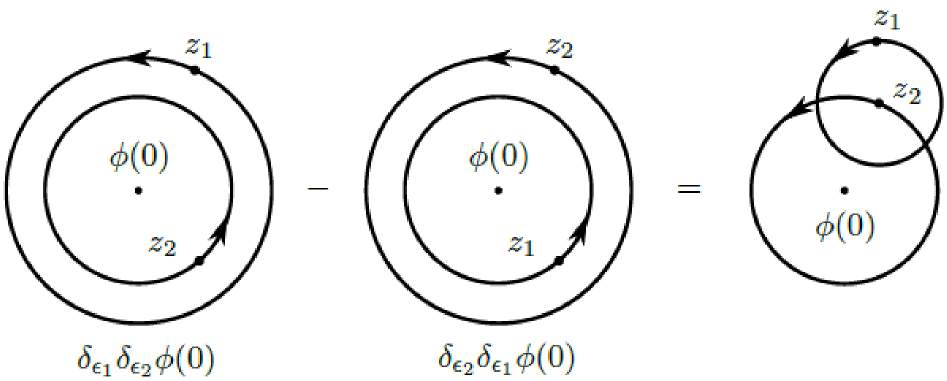

12
11
22
21
y_1
y_2
y
$D(p)_1$
$D(p)_2$



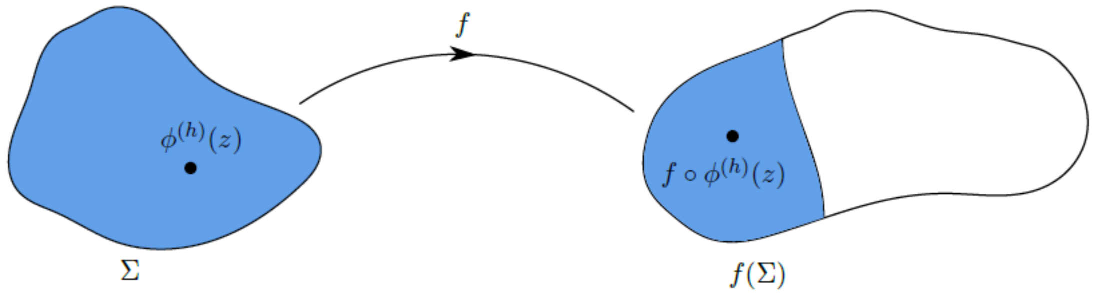
$\phi(z)$
0



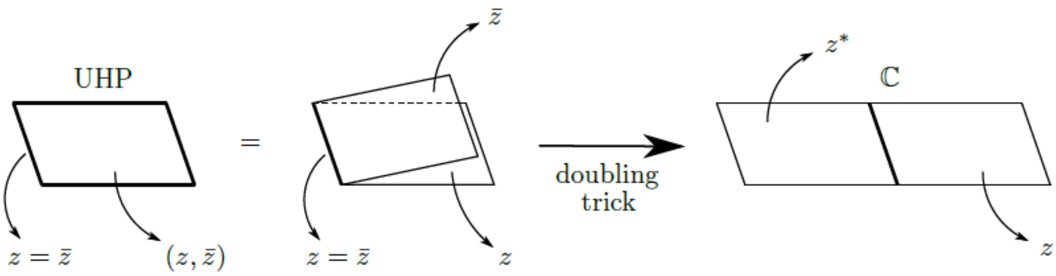
$\epsilon T(\tilde{z})$
0
$\phi(z)$
=
0
$\epsilon T(\tilde{z})$
$\phi(z)$



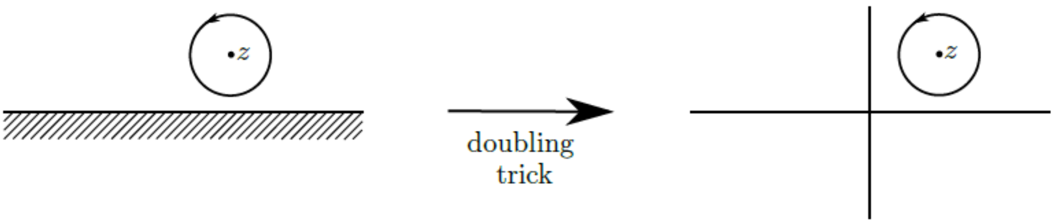
z_1
$\phi(0)$
z_2
$\delta_{\epsilon_1}\delta_{\epsilon_2}\phi(0)$
−
z_2
$\phi(0)$
z_1
$\delta_{\epsilon_2}\delta_{\epsilon_1}\phi(0)$
=
z_1
z_2
$\phi(0)$



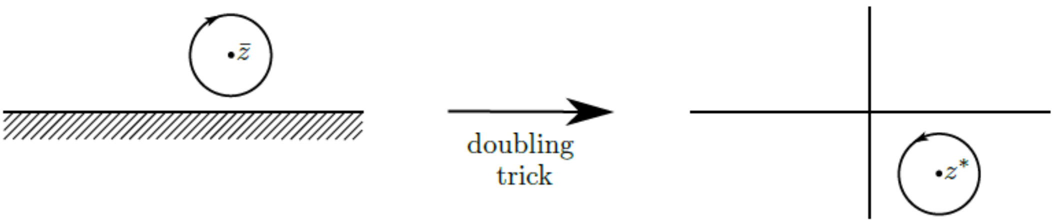
f
$\phi^{(h)}(z)$
Σ
$f \circ \phi^{(h)}(z)$
$f(\Sigma)$



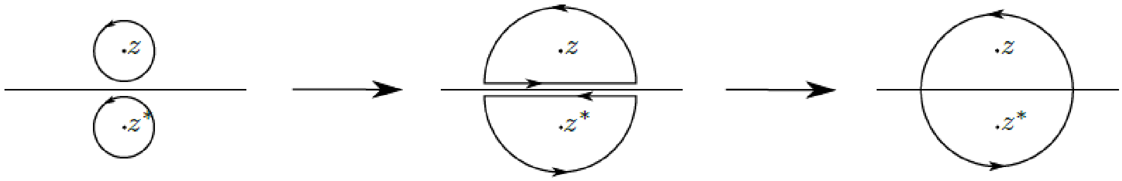
UHP
=
$\bar{z}$
z^*
$\mathbb{C}$
doubling trick
$z = \bar{z}$
$(z, \bar{z})$
$z = \bar{z}$
z
z



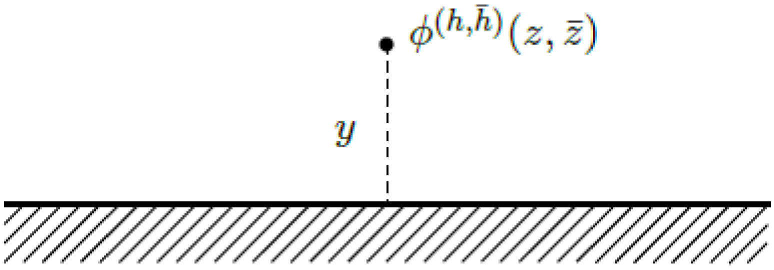
z
doubling trick
z

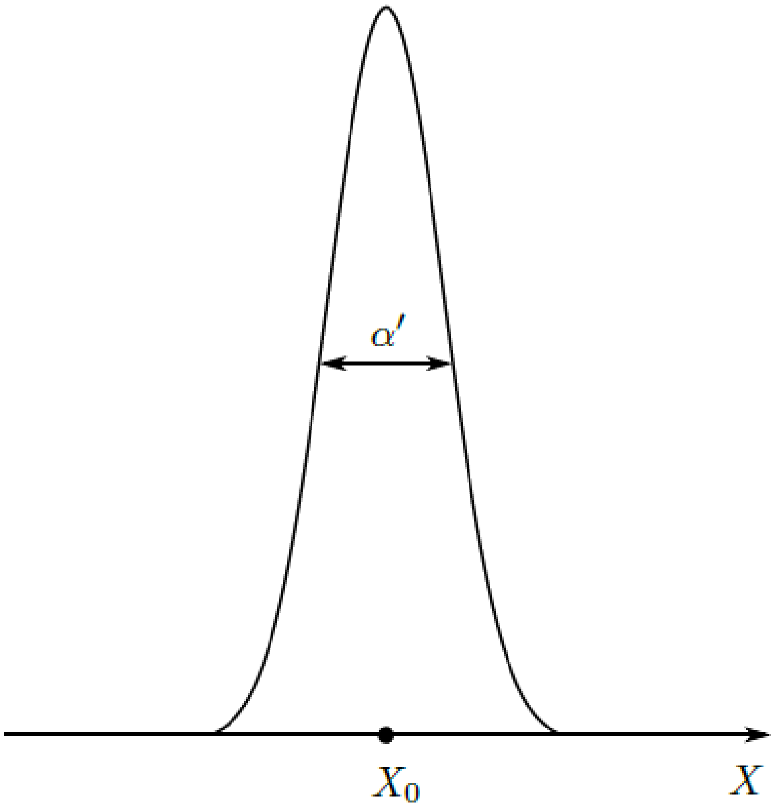


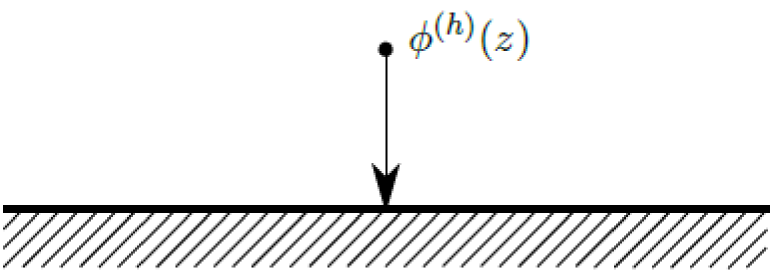
$\bar{z}$
doubling trick
z^*

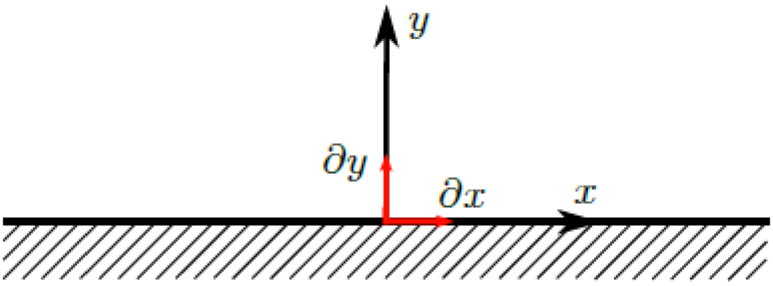


z
z^*
z
z^*
z
z^*

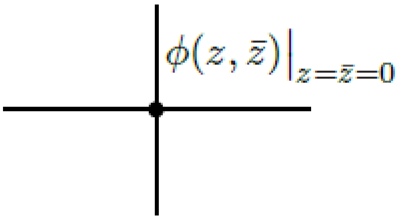

$\phi^{(h,\bar{h})}(z,\bar{z})$
y


α'
X_0
X


$\phi^{(h)}(z)$

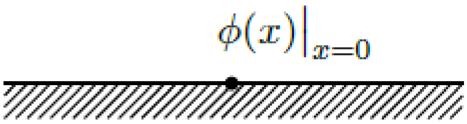


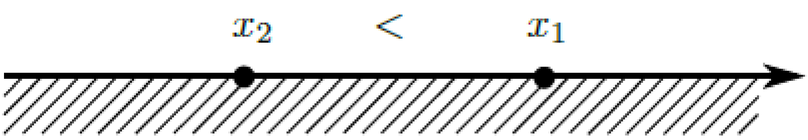
$$\phi(z,\bar{z})\Big|_{z=\bar{z}=0}\,|0\rangle_{SL(2,\mathbb{C})} =$$



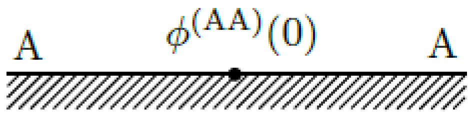
;

$$\phi(x)\Big|_{x=0}\,|0\rangle_{SL(2,\mathbb{R})} =$$



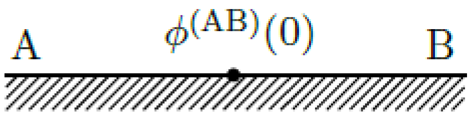


$$|AA\rangle = \phi^{(AA)}(x=0)\,|0\rangle =$$



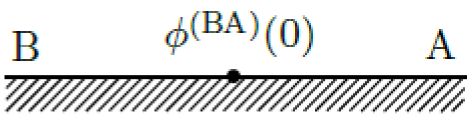
,

$$|AB\rangle = \phi^{(AB)}(x=0)\,|0\rangle =$$



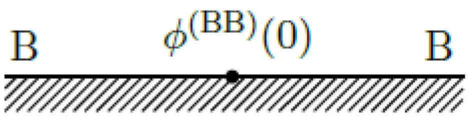
,

$$|BA\rangle = \phi^{(BA)}(x=0)\,|0\rangle =$$

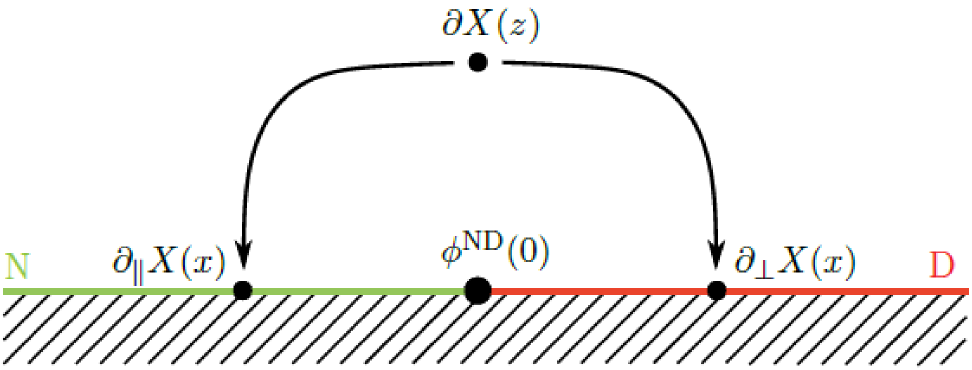


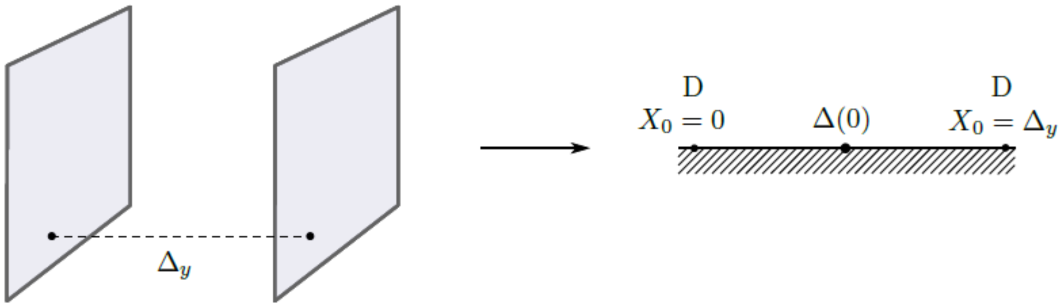
,

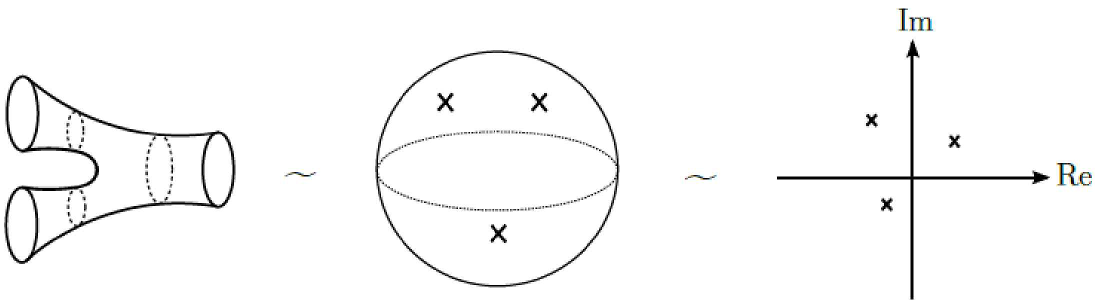
$$|BB\rangle = \phi^{(BB)}(x=0)\,|0\rangle =$$

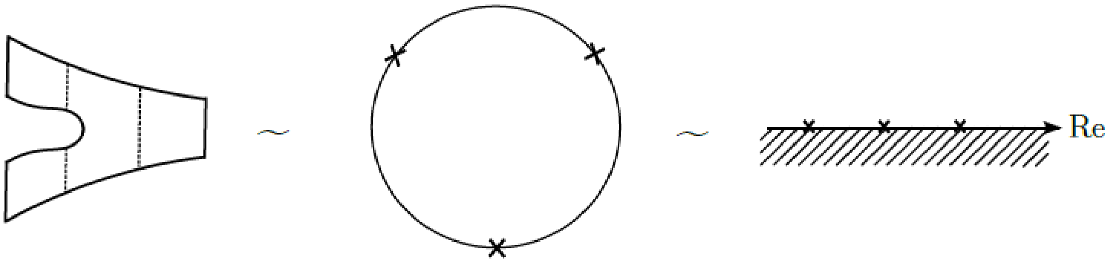


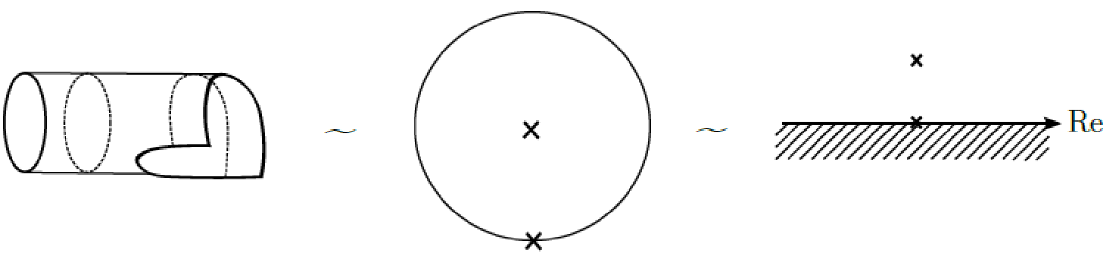
.

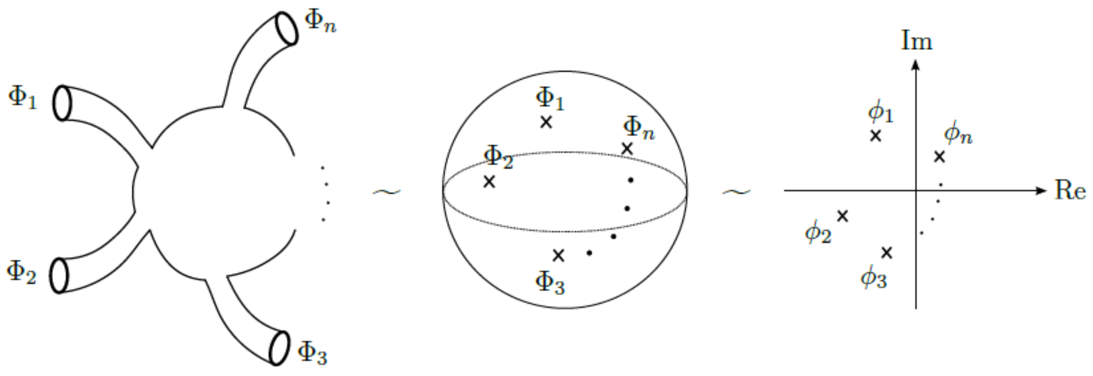

$\partial X(z)$
N
$\partial_{\parallel} X(x)$
$\phi^{\mathrm{ND}}(0)$
$\partial_{\perp} X(x)$
D

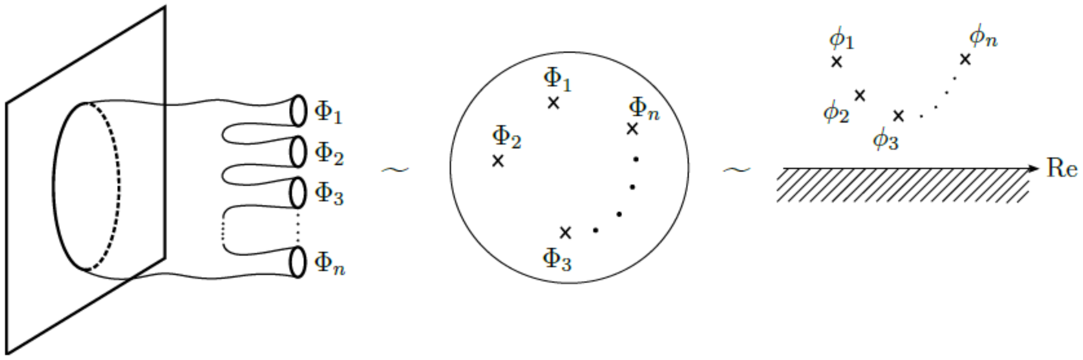

Δ_y
D
$X_0 = 0$
$\Delta(0)$
D
$X_0 = \Delta_y$

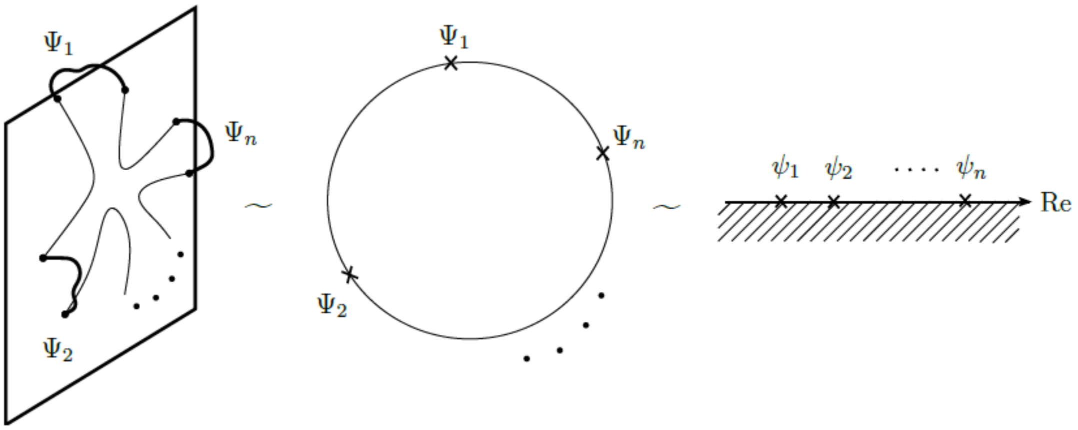

Im
Re

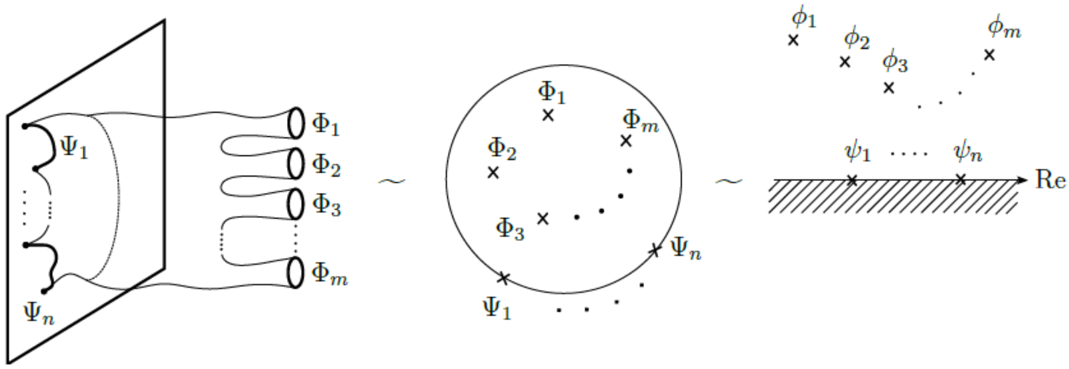

Re

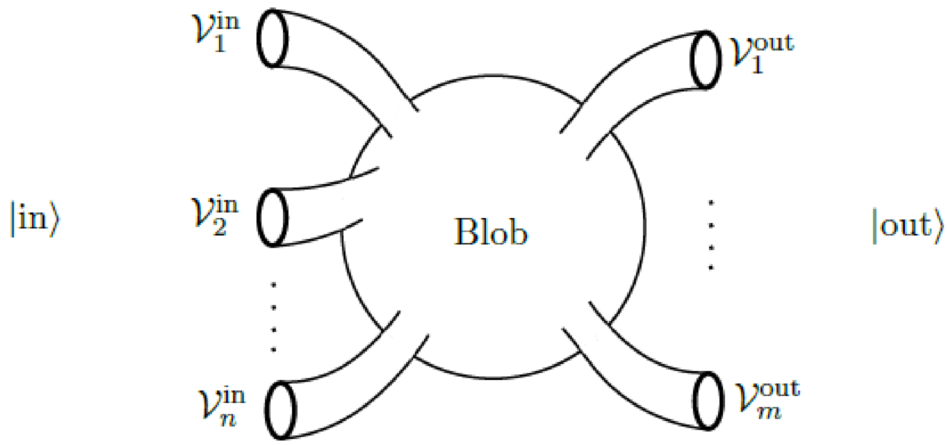

Re


Φ_n
Φ_1
Φ_2
Φ_3
$\sim$
Φ_1
Φ_n
Φ_2
Φ_3
$\sim$
Im
ϕ_1
ϕ_n
Re
ϕ_2
ϕ_3


Φ_1
Φ_2
Φ_3
Φ_n
$\sim$
Φ_1
Φ_n
Φ_2
Φ_3
$\sim$
ϕ_1
ϕ_n
ϕ_2
ϕ_3
Re


Ψ_1
Ψ_n
Ψ_2
$\sim$
Ψ_1
Ψ_n
Ψ_2
$\sim$
ψ_1 ψ_2 .... ψ_n
Re





$$T^{ab} = \frac{\delta S}{\delta \gamma^{ab}} = \frac{\delta S}{\delta(e^{-\phi}\delta_{ab})} = \frac{\delta S}{\delta((1-\phi)\delta_{ab})} = -\frac{\delta S}{\delta \phi}\delta^{ab}$$
$$= -\alpha' R_{\mu\nu}\partial X^\mu \cdot \bar{\partial} X^\nu \delta^{ab}$$
$$\delta_w S = -wT_a^a = \alpha' R_{\mu\nu}\partial X^\mu \cdot \bar{\partial} X^\nu w \delta_a^a$$
$$\propto w\alpha' R_{\mu\nu}\partial X^\mu \cdot \bar{\partial} X^\nu$$

$$T_a^a = -\alpha' R_{\mu\nu}\partial X^\mu \cdot \bar{\partial} X^\nu$$

$$T_a^a = 0 \iff R_{\mu\nu} = 0$$

$$S[X] = \frac{1}{2\pi\alpha'}\int d^2\sigma\left[\sqrt{\gamma}\gamma^{ab}\partial_a X^\mu \bar{\partial}_b X^\nu G_{\mu\nu}(X) + i\epsilon^{ab}\partial_a X^\mu \bar{\partial}_b X^\nu B_{\mu\nu}(X) + \alpha' R^{(2)}\Phi(X)\right]$$

$$\beta_{\mu\nu}^G = \alpha'\left(R_{\mu\nu} + \nabla_\mu\nabla_\nu\Phi - \frac{1}{4}H_{\mu\rho\sigma}H_\nu^{\rho\sigma}\right) + \mathcal{O}((\alpha')^2)$$
$$\beta_{\mu\nu}^B = \alpha'\left(-\frac{1}{2}\Phi\nabla_\rho H_{\mu\nu}^\rho + \nabla_\rho\Phi H_{\mu\nu}^\rho\right) + \mathcal{O}((\alpha')^2)$$
$$\beta^\Phi = \frac{D-26}{6} + \alpha'\left(-\frac{1}{2}\nabla^2\Phi + \nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{24}H_{\mu\nu\rho}H^{\mu\nu\rho}\right) + \mathcal{O}((\alpha')^2)$$

$$\begin{cases} G_{\mu\nu}(X) = \eta_{\mu\nu} \\ B_{\mu\nu}(X) = 0 \\ \Phi(X) = \Phi_0 \in \mathbb{R} \\ D = 26 \end{cases}$$

$$\begin{aligned} S &= \frac{1}{2\pi\alpha'} \int d^2\sigma \left(\sqrt{\gamma}\gamma^{ab}\partial_a X^\mu \bar{\partial}_b X_\mu + \alpha' \Phi_0 R^{(2)}\right) \\ &= \left(\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{\gamma}\gamma^{ab}\partial_a X^\mu \bar{\partial}_b X_\mu\right) + \Phi_0 \chi \end{aligned}$$

$$g_s = e^{\lambda}$$

$$g_s = e^{\langle \Phi \rangle}$$

$$\begin{cases} G_{\mu\nu}(X) = \eta_{\mu\nu} \\ B_{\mu\nu}(X) = 0 \\ \Phi(X) = \Phi_0 + v_\mu X^\mu \\ D \neq 26 \end{cases}$$

$$v_\mu v^\mu = \frac{26 - D}{6\alpha'}.$$

$$\mathcal{A}(1, \dots, n) = \sum_{g,b} g_s^{n_c + n_o/2} g_s^{2(g-1)+b} \left\langle \left\langle \mathcal{V}_1 \dots \mathcal{V}_{n_c} V_1 \dots V_{n_o} \right\rangle \right\rangle_{\Sigma_{g,b}},$$

$$\left\langle \left\langle \mathcal{V}_1 \dots \mathcal{V}_{n_c} V_1 \dots V_{n_o} \right\rangle \right\rangle_{\Sigma_{g,b}} = \int \frac{\mathcal{D}h^{(g,b)} \mathcal{D}X}{\mathrm{Vol}(\mathcal{G})} \mathcal{V}_1 \dots \mathcal{V}_{n_c} V_1 \dots V_{n_o} e^{-S_{\mathrm{Pol}}[X, h^{(g,b)}]}.$$

$$\hat{h}_{\alpha\beta} = \hat{h}_{\alpha\beta}(t^i)$$

$$h_{\alpha\beta}(t^i) = \left(\hat{h}_{\alpha\beta}(t^i)\right)^{\zeta}$$

$$\begin{aligned} \int \frac{\mathcal{D}h}{\mathrm{Vol}(\mathcal{G})} &= \int \frac{\mathcal{D}^{\mathrm{gauge}} h \mathcal{D}^{\mathrm{mod}} h}{\mathrm{Vol}(\mathcal{G})} = \int \frac{\mathcal{D}\zeta}{\mathrm{Vol}(\mathcal{G})} d^n t^i \det\left(\frac{\mathcal{D}\hat{h}}{\mathcal{D}\zeta}, \frac{\partial \hat{h}}{\partial t}\right) \\ &= \int_{\mathcal{M}_{g,b}} d^n t^i \det\left(\frac{\mathcal{D}\hat{h}}{\mathcal{D}\zeta}, \frac{\partial \hat{h}}{\partial t}\right) \end{aligned}$$

$$\det\left(\frac{\mathcal{D}\hat{h}}{\mathcal{D}\zeta}, \frac{\partial \hat{h}}{\partial t}\right) = \int \mathcal{D}b\mathcal{D}c \prod_{i=1}^{\dim \mathcal{M}} \left(b, \partial_{t^i} \hat{h}(t)\right) e^{-\frac{1}{4\pi}(b, 2P_1 c)}$$

$$(\psi, \chi) \equiv \int d^2\sigma \sqrt{\hat{h}} \psi_{\alpha\beta} \hat{h}^{\alpha\gamma} \hat{h}^{\beta\delta} \chi_{\gamma\delta}$$

$$(P_1 c)_{\alpha\beta} \equiv \frac{1}{2}\left(\hat{\nabla}_\alpha c_\beta + \hat{\nabla}_\beta c_\alpha\right) - \hat{h}_{\alpha\beta} \hat{\nabla} \cdot c.$$

$$(b, P_1 c) = \int d^2\sigma \sqrt{\hat{h}} b_{\alpha\beta} (P_1 c)^{\alpha\beta} = \int d^2\sigma \sqrt{\hat{h}} (P_1^T b)_\alpha c^\alpha$$

$$(P_1^T \delta h)_\alpha = -2\hat{\nabla}^\beta \delta h_{\alpha\beta}$$

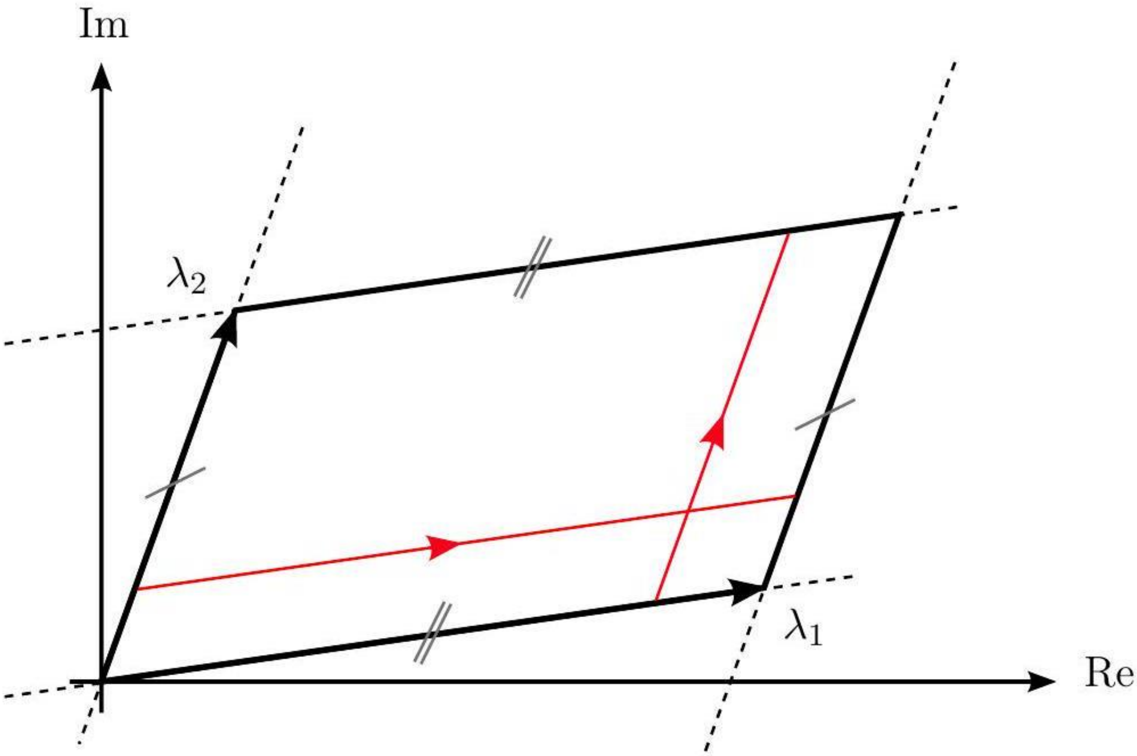
$$P_1^T \delta h = 0$$

$$P_1^T b = 0$$

$$\dim \mathcal{M}_{g,b} - \dim \mathrm{CKG}_{g,b} = \dim \mathrm{Ker} P_1^T - \dim \mathrm{Ker} P_1 = -3\chi = 6g + 3b - 6.$$

$$z \sim z + n\lambda_1 + m\lambda_2,$$

$$\Lambda = \{\xi \in \mathbb{C} \mid \xi = n\lambda_1 + m\lambda_2, n, m \in \mathbb{Z}\}$$



$$\tau = \lambda_2/\lambda_1 \in \mathbb{C}$$

$$z \to \alpha z \;\leftrightarrow\; \lambda_{1,2} \to \alpha\lambda_{1,2}$$

$$\Lambda' = \begin{pmatrix} \lambda_1' \\ \lambda_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = A\Lambda$$

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \in \mathbb{Z}$$

$$\det(A) = ad - bc = 1$$

$$\tau' = \frac{\lambda_2'}{\lambda_1'} = \frac{c\lambda_1 + d\lambda_2}{a\lambda_1 + b\lambda_2} = \frac{d\tau + c}{b\tau + a}$$

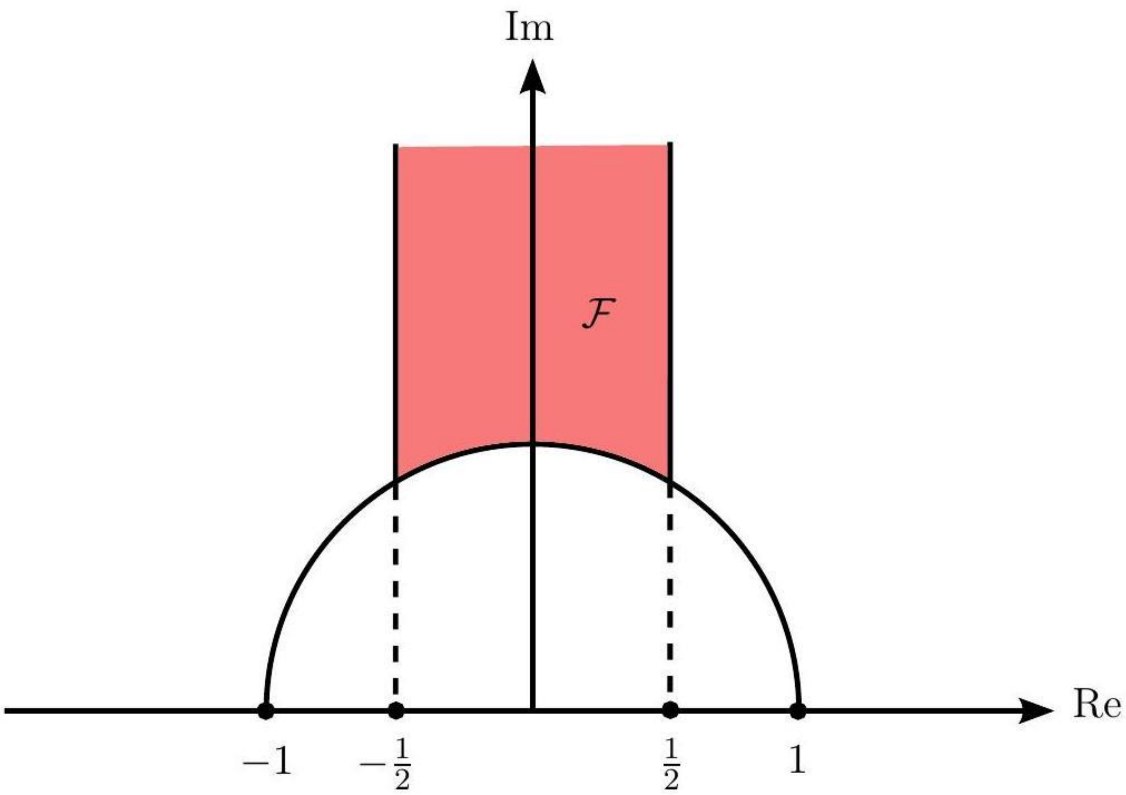
$$\mathcal{F} = \mathrm{UHP}/SL(2, \mathbb{Z})$$

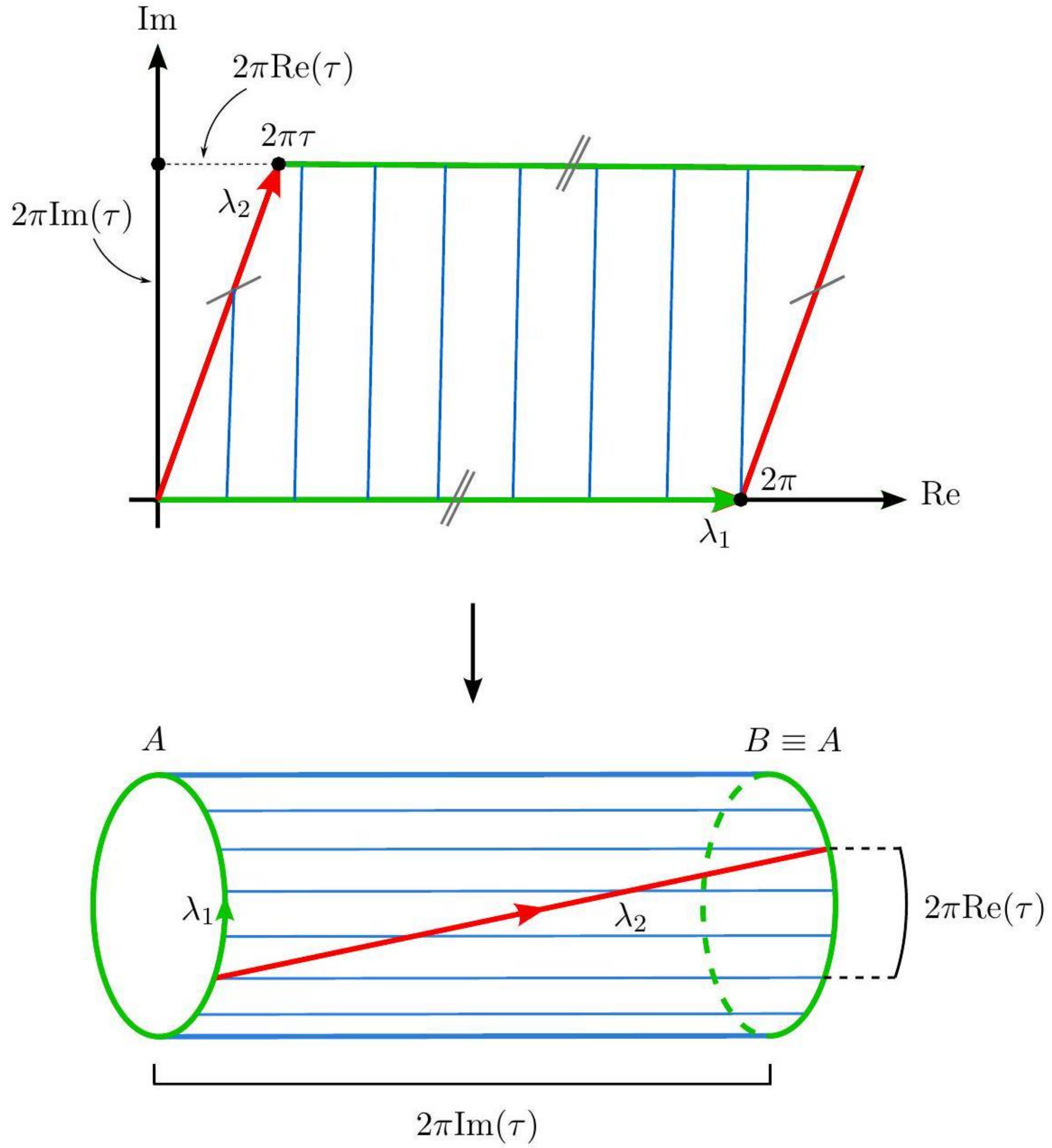
$$T: \tau \to \tau + 1 \implies T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$

$$S: \tau \to -\frac{1}{\tau} \implies S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

$$z = \sigma_1 + \tau\sigma_2$$
$$\bar{z} = \sigma_1 + \bar{\tau}\sigma_2$$

$$\sigma_{1,2} \in (0, 2\pi]$$





$$dz \otimes_{\text{sym}} d\bar{z} = (d\sigma_1 + \tau d\sigma_2) \otimes_{\text{sym}} (d\sigma_1 + \bar{\tau} d\sigma_2)$$

$$z = \sigma_1 + \tau\sigma_2 \longrightarrow \sigma_1 + \frac{a\tau + b}{c\tau + d}\sigma_2 = \frac{1}{c\tau + d}[(d\sigma_1 + b\sigma_2) + \tau(c\sigma_1 + a\sigma_2)]$$
$$= \frac{1}{c\tau + d}(\sigma_1' + \tau\sigma_2')$$

$$\begin{pmatrix} \sigma_1' \\ \sigma_2' \end{pmatrix} \simeq \begin{pmatrix} d & b \\ c & a \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}$$

$$\int \frac{\mathcal{D}h}{Vol(G)} \to \int_{\mathcal{F}} d^2\tau [\text{ ghosts }]$$

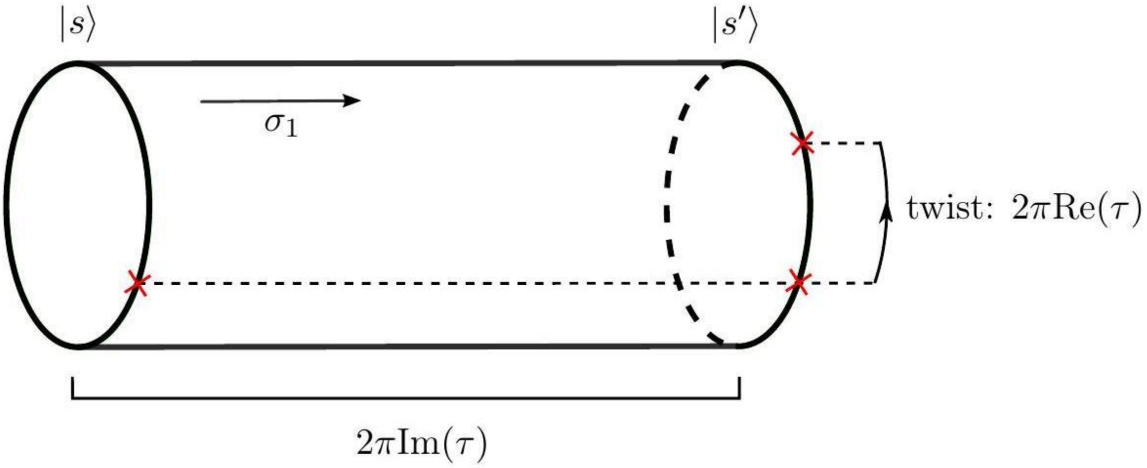
$$\text{CKG}_{T^2} \sim U(1) \times U(1).$$

$$\mathcal{A} = \int_{\mathcal{F}} \frac{d^2\tau}{4\pi^2 \mathrm{Im}(\tau)} f(\tau,\bar{\tau}),$$

$$f(\tau,\bar{\tau}) = \int \mathcal{D}X\mathcal{D}c\mathcal{D}b[B_0\bar{B}_0][C_0\bar{C}_0]\mathcal{V}_1 \dots \mathcal{V}_n e^{-S[X,b,c]}$$

$$f(\tau,\bar{\tau}) = \int \mathcal{D}b\mathcal{D}c\mathcal{D}X^+\mathcal{D}X^-\mathcal{D}X^i[B_0\bar{B}_0][C_0\bar{C}_0]e^{-S[X,b,c]} \sim \int \mathcal{D}x_0^{\pm}\mathcal{D}X^i e^{-S_{lc}[x_0^{\pm},X^i]},$$

$$f(\tau,\bar{\tau}) = \langle [B_0\bar{B}_0][C_0\bar{C}_0]\rangle_{\text{Torus }\tau}^{(\text{matter }^2\text{ hosts})} = \langle 1\rangle_{\text{Torus }\tau}^{(\text{light cone})},$$



$$f(\tau,\bar{\tau}) = \sum_s \langle s|e^{-2\pi i \mathrm{Re}\tau P}e^{-2\pi \mathrm{Im}(\tau)H}|s\rangle = \mathrm{Tr}\left[e^{-2\pi i \mathrm{Re}\tau P}e^{-2\pi \mathrm{Im}(\tau)H}\right]$$

$$H = \int \frac{d\sigma}{2\pi}T_{tt}(t,\sigma) = \int \frac{dw}{2\pi i}T(w) + \int \frac{d\bar{w}}{2\pi i}\bar{T}(\bar{w})$$

$$T(w) = T(z)z^2 - \frac{c}{24}, \bar{T}(\bar{w}) = \bar{T}(\bar{z})\bar{z}^2 - \frac{c}{24}$$

$$H = \oint \frac{dz}{2\pi i}\frac{1}{z}\left(T(z)z^2 - \frac{c}{24}\right) + \oint \frac{d\bar{z}}{2\pi i}\frac{1}{\bar{z}}\left(\bar{T}(\bar{z})\bar{z}^2 - \frac{c}{24}\right)$$
$$= L_0 + \bar{L}_0 - \frac{c}{12}$$

$$P = \int \frac{d\sigma}{2\pi}T_{t\sigma} = \int \frac{dw}{2\pi i}T(w) - \int \frac{d\bar{w}}{2\pi i}\bar{T}(\bar{w})$$
$$= \oint \frac{dz}{2\pi i}\frac{1}{z}\left(T(z)z^2 - \frac{c}{24}\right) - \oint \frac{d\bar{z}}{2\pi i}\frac{1}{\bar{z}}\left(\bar{T}(\bar{z})\bar{z}^2 - \frac{c}{24}\right)$$
$$= L_0 - \bar{L}_0$$

$$f(\tau,\bar{\tau}) = \mathrm{Tr}\left[q^{L_0 - \frac{c}{24}}\bar{q}^{\bar{L}_0 - \frac{c}{24}}\right]$$

$$q = e^{2\pi i\tau}$$

$$f(\tau,\bar{\tau}) = |q|^{-\frac{c}{12}} \int \frac{d^{26}P}{(2\pi\sqrt{\alpha'})^{26}} \sum_s \langle s,P|\, |q|^{\frac{\alpha' P^2}{2}} q^N \bar{q}^{\bar{N}} \,|s,P\rangle$$

$$= |q|^{-\frac{c}{12}} \int \frac{d^{26}P}{(2\pi\sqrt{\alpha'})^{26}} |q|^{\frac{\alpha' P^2}{2}} \underbrace{\sum_s \langle s|\, q^N \bar{q}^{\bar{N}} \,|s\rangle}_{\text{oscillators sum}}$$

$$= |q|^{-\frac{c}{12}} \int \frac{d^{26}P}{(2\pi\sqrt{\alpha'})^{26}} |q|^{\frac{\alpha' P^2}{2}} \left[\prod_{k=1,\bar{k}=1}^{\infty} \sum_{n_k,\bar{n}_{\bar{k}}=0}^{\infty} q^{k n_k}\, \bar{q}^{\bar{k}\bar{n}_{\bar{k}}}\right]^{24}$$

$$= |q|^{-\frac{c}{12}} \int \frac{d^{26}P}{(2\pi\sqrt{\alpha'})^{26}} |q|^{\frac{\alpha' P^2}{2}} \left[\prod_{k=1}^{\infty} \frac{1}{1-q^k} \prod_{\bar{k}=1}^{\infty} \frac{1}{1-\bar{q}^{\bar{k}}}\right]^{24}$$

$$\stackrel{c=24}{=} \left[\prod_{k=1}^{\infty} \frac{q^{-\frac{1}{24}}}{1-q^k} \prod_{\bar{k}=1}^{\infty} \frac{\bar{q}^{-\frac{1}{24}}}{1-\bar{q}^{\bar{k}}}\right]^{24} \int \frac{d^{26}P}{(2\pi\sqrt{\alpha'})^{26}} |q|^{\frac{\alpha' P^2}{2}}$$

$$= \left[\prod_{k=1}^{\infty} \frac{q^{-\frac{1}{24}}}{1-q^k} \prod_{\bar{k}=1}^{\infty} \frac{\bar{q}^{-\frac{1}{24}}}{1-\bar{q}^{\bar{k}}}\right]^{24} (\mathrm{Im}(\tau))^{-\frac{26}{2}}$$

$$= \frac{1}{\mathrm{Im}(\tau)} \left[\frac{1}{\sqrt{\mathrm{Im}(\tau)}} \frac{1}{|\eta(\tau)|^2}\right]^{24},$$

$$\mathcal{A}^{1-\text{loop}}_{\text{Vacuum}} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}(\tau))^2} \left[\frac{1}{\sqrt{\mathrm{Im}(\tau)}} \frac{1}{|\eta(\tau)|^2}\right]^{24}$$

$$\tau' = \frac{a\tau+b}{c\tau+d}, \bar{\tau}' = \frac{a\bar{\tau}+b}{c\bar{\tau}+d}, A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

$$\begin{aligned}
\mathrm{Im}(\tau') &= \frac{1}{|c\tau+d|^2} \mathrm{Im}((a\tau+b)(c\bar{\tau}+d)) \\
&= \frac{1}{|c\tau+d|^2} \mathrm{Im}(bc\bar{\tau}+ad\tau) \\
&= \frac{\det(A)}{|c\tau+d|^2} \mathrm{Im}(\tau) = \frac{\mathrm{Im}(\tau)}{|c\tau+d|^2} \\
d\tau' &= \frac{d\tau a}{c\tau+d} - \frac{a\tau+b}{(c\tau+d)^2} d\tau c
\end{aligned}$$

$$= \frac{d\tau\tau ac + d\tau ad - d\tau\tau ac - d\tau bc}{(c\tau + d)^2}$$

$$= \frac{\det(A)}{(c\tau + d)^2} d\tau = \frac{d\tau}{(c\tau + d)^2}$$

$$\frac{d\tau' d\bar{\tau}'}{\left(\mathrm{Im}(\tau')\right)^2} = \frac{|c\tau + d|^4}{(\mathrm{Im}(\tau))^2} \frac{d\tau}{(c\tau + d)^2} \frac{d\bar{\tau}}{(c\bar{\tau} + d)^2} = \frac{d\tau d\bar{\tau}}{(\mathrm{Im}(\tau))^2}$$

$$\tilde{f}(\tau, \bar{\tau}) = \left[\frac{1}{\sqrt{\mathrm{Im}(\tau)}} \frac{1}{|\eta(\tau)|^2}\right]^{24}$$

$$\eta(\tau + 1) = e^{\frac{i\pi}{12}}\eta(\tau), \eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\eta(\tau).$$

$$\tilde{f}(T(\tau)) = \tilde{f}(\tau + 1) = \left[\frac{1}{\sqrt{\mathrm{Im}(\tau)}} \frac{1}{\left|e^{\frac{i\pi}{12}}\eta(\tau)\right|^2}\right]^{24} = \tilde{f}(\tau).$$

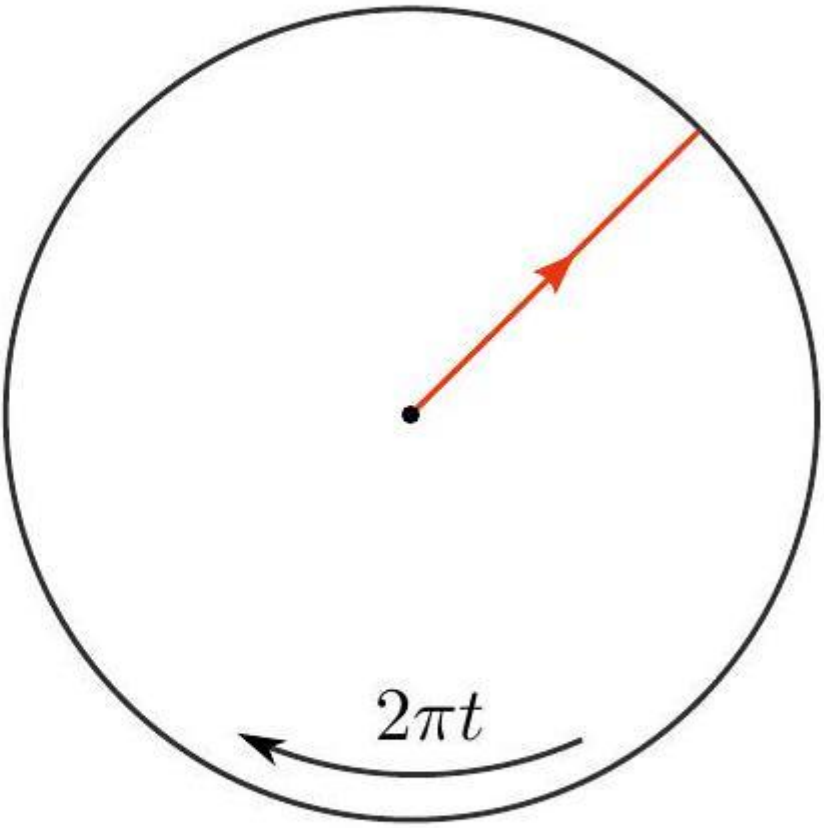
$$\mathrm{Im}\left(-\frac{1}{\tau}\right) = -\mathrm{Im}\left(\frac{\bar{\tau}}{|\tau|^2}\right) = \frac{\mathrm{Im}(\tau)}{|\tau|^2},$$

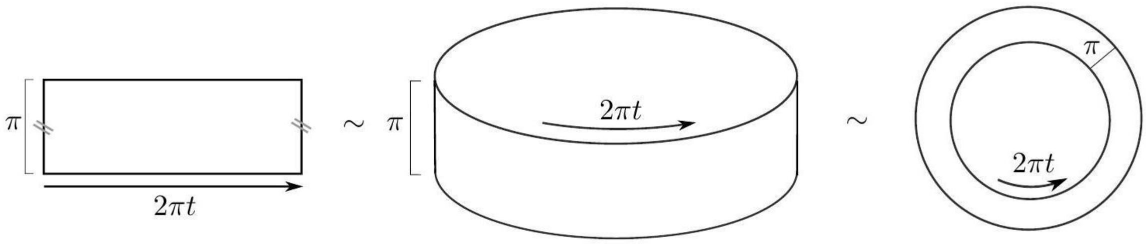
$$\tilde{f}(S(\tau)) = \tilde{f}\left(-\frac{1}{\tau}\right) = \left[\frac{|\tau|}{\sqrt{\mathrm{Im}(\tau)}} \frac{1}{|\sqrt{-i\tau}\eta(\tau)|^2}\right]^{24} = \tilde{f}(\tau)$$

$$\mathcal{A}_{pp} = \int_0^\infty \frac{dt}{\mathrm{Vol}(CKG)} \sum_s \langle s|e^{-2\pi tH}|s\rangle = \int_0^\infty \frac{dt}{\mathrm{Vol}(CKG)} \mathrm{Tr}(e^{-2\pi tH})$$

$$= \int_0^\infty \frac{dt}{2\pi t} \int \frac{d^d p}{(2\pi)^d} e^{-2\pi t(p^2 + m^2)} = \int_0^\infty \frac{dt}{2\pi t} \left(\frac{1}{8\pi^2 t}\right)^{\frac{d}{2}} e^{-2\pi t m^2}$$

$$\mathcal{A}_{g=1}(1, \ldots, n) \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}(\tau))^2} f_n(\tau, \bar{\tau})$$





$$\mathcal{A}^{1-\text{ loop}}_{\text{Vacuum}} = \int_0^\infty \frac{dt}{\text{Vol}(CKG)} \text{Tr}[e^{-2\pi t H}]^{lc} = \int_0^\infty \frac{dt}{2\pi t} \text{Tr}\left[e^{-2\pi t\left(L_0 - \frac{c}{24}\right)}\right]^{lc}$$

$$L_0 = \alpha' P^2 + \hat{N}_N + \hat{N}_D$$

$$\mathcal{A}^{1-\text{ loop}}_{\text{Vacuum}} = \int_0^\infty \frac{dt}{2\pi t} \text{Tr}\left[e^{-2\pi t \alpha' p^2} e^{-2\pi t(\hat{N}_N + \hat{N}_D)} e^{2\pi t \frac{D-2}{24}}\right]$$

$$= \int_0^\infty \frac{dt}{2\pi t} \int \frac{d^{p+1}P}{(2\pi)^{p+1}} e^{-2\pi t \alpha' P^2} q^{-\frac{D-2}{24}} \left[\prod_{n=1}^{\infty} \frac{1}{1-q^n}\right]^{D-2}$$

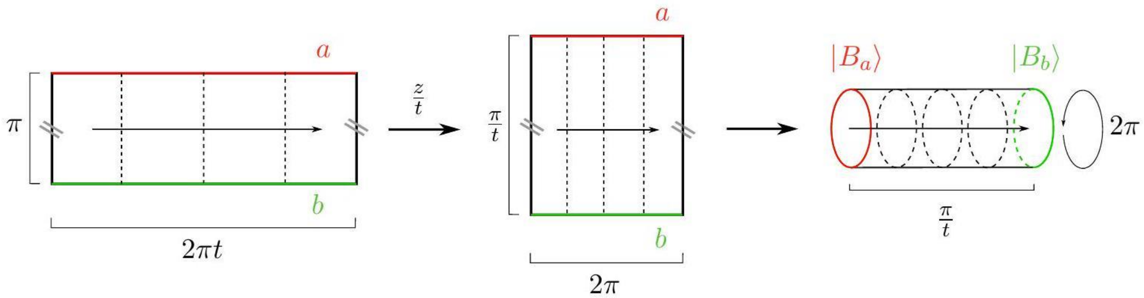
$$\propto \int_0^\infty \frac{dt}{2t} \left(\frac{1}{2t}\right)^{\frac{p+1}{2}} \left[\frac{1}{\eta(it)}\right]^{D-2}$$

$$L_0 = \alpha' P^2 + \left(\frac{\Delta Y}{2\pi\sqrt{\alpha'}}\right)^2 + \hat{N}_N + \hat{N}_D$$

$$\mathcal{A}^{1-\text{ loop}}_{\text{Vacuum}} = \int_0^\infty \frac{dt}{(2t)^2}\left(\frac{1}{2t}\right)^{\frac{p-1}{2}} e^{-2\pi t\left[\frac{\Delta Y}{2\pi\sqrt{\alpha'}}\right]^2}\left[\frac{1}{\eta(it)}\right]^{D-2}$$

$$= \int_0^\infty \frac{dt}{(2t)^2}\mathrm{Tr}_{\mathcal{H}^{ab}_{\text{open}}}\left[e^{-2\pi t\left(L_0-\frac{c}{24}\right)}\right]$$

$$\mathrm{Tr}_{\mathcal{H}^{ab}_{\text{open}}}\left[e^{-2\pi t\left(L_0-\frac{c}{24}\right)}\right] = \int_{\substack{X(\sigma,\tau+2\pi t)=X(\sigma,\tau)\\ F_a(X(0,\tau))=0, F_b(X(\pi,\tau))=0}} \frac{\mathcal{D}X\mathcal{D}h(t)}{\mathrm{Vol}(\mathcal{G})} e^{-S[X,h(t)]}$$

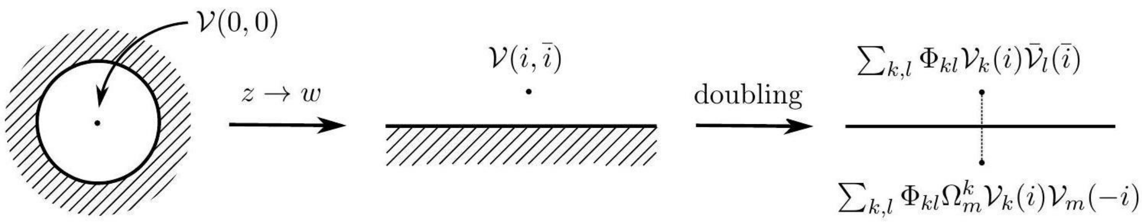


$$\mathrm{Tr}_{\mathcal{H}^{ab}_{\text{open}}}\left[e^{-2\pi t\left(L_0-\frac{c}{24}\right)}\right] = \langle B_b| e^{-\frac{\pi}{t}\left(L_0+\bar{L}_0-\frac{c}{12}\right)}|B_a\rangle.$$

$$\langle B_a \mid \mathcal{V}\rangle \equiv \langle \mathcal{V}(0,0)\rangle^{(a)}_{\text{Disk}}, \forall \mathcal{V} \in \mathcal{H}_{\text{closed}}$$

$$z \to w = \frac{1+iz}{1-iz}$$
$$\bar{z} \to \bar{w} = \frac{1-i\bar{z}}{1+i\bar{z}}$$

$$\mathcal{V}(w,\bar{w}) = \left(\frac{\partial w}{\partial z}\right)^h \left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^{\bar{h}} \mathcal{V}(z(w),\bar{z}(\bar{w}))$$
$$\Longrightarrow \langle \mathcal{V}(0,0)\rangle^{(a)}_{\text{Disk}} = (2i)^h(-2i)^{\bar{h}}\delta_{h,\bar{h}}\langle \mathcal{V}(i,-i)\rangle^{(a)}_{\text{UHP}}.$$

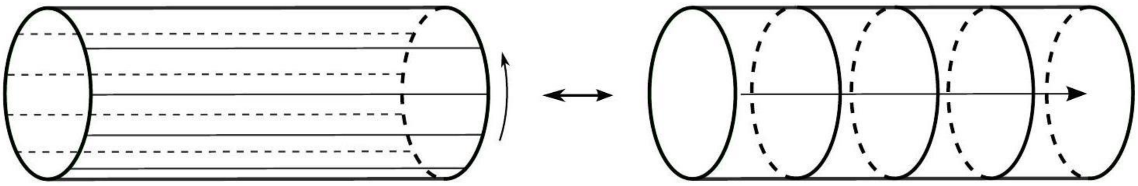


$$\mathcal{V}(z,\bar{z}) = j(z)\bar{j}(\bar{z}) = -\frac{2}{\alpha'}\partial X\bar{\partial}X.$$

$$\langle B_{D,N} \mid \mathcal{V}\rangle = -\frac{2}{\alpha'}\langle \partial X(0)\bar{\partial}X(0)\rangle^{D,N}_{\text{Disk}}$$
$$= -\frac{8}{\alpha'}\langle \partial X(i)\bar{\partial}X(-i)\rangle^{D,N}_{\text{UHP}}$$
$$= -\frac{8}{\alpha'}\left(\pm\frac{\alpha'}{2}\frac{1}{(2i)^2}\right)$$
$$= \pm 1$$

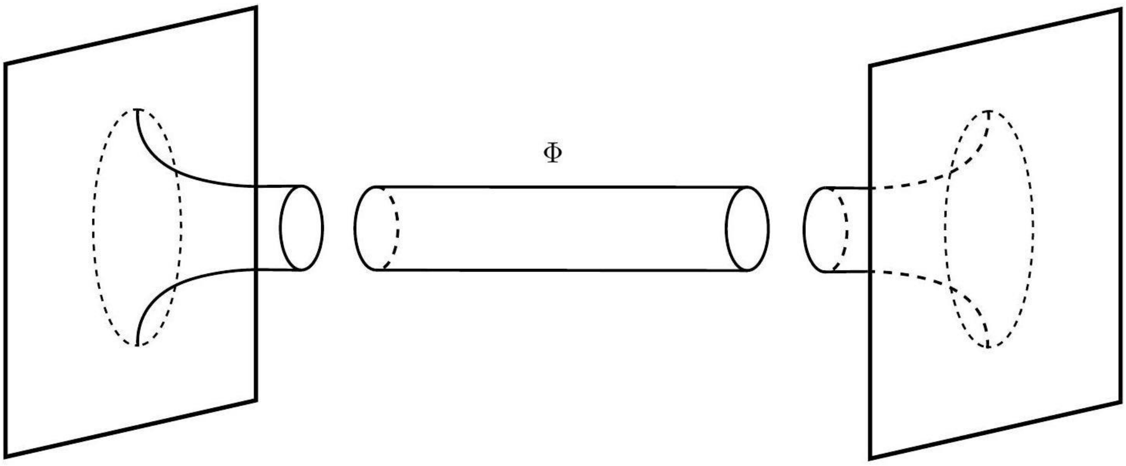
$$\mathrm{Tr}_{\mathcal{H}^{ab}_{\text{open}}}\left[e^{-2\pi t\left(L_0-\frac{c}{24}\right)}\right] = \langle B_b|e^{-\frac{\pi}{t}\left(L_0+\bar{L}_0-\frac{c}{12}\right)}|B_a\rangle.$$

$$\int_0^\infty \frac{dt}{(2t)^2}\mathrm{Tr}_{\mathcal{H}^{ab}_{\text{open}}}\left[e^{-2\pi t\left(L_0-\frac{c}{24}\right)}\right] = \frac{1}{4\pi}\int_0^\infty ds\langle B_b|e^{-s\left(L_0+\bar{L}_0-\frac{c}{12}\right)}|B_a\rangle$$



$$S \sim \left[\frac{1}{2}\langle\Phi|K|\Phi\rangle + \langle\Phi \mid B_a\rangle\right],$$

$$S_{EM} = \int d^d x\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\mu J^\mu\right]$$



$$\mathcal{A}^{1-\text{ loop}}_{\text{Vacuum}} = \int_0^\infty \frac{dt}{(2t)^2}\mathrm{Tr}_{\text{BCFT}}\left[e^{-2\pi t(L_0-1)}\right]$$
$$= \frac{1}{4\pi}\int_0^\infty ds\langle B_a|e^{-s(L_0+\bar{L}_0-2)}|B_b\rangle$$

$$\mathcal{A}^{1-\text{ loop}}_{\text{Vacuum}} = \int_0^\infty \frac{dt}{(2t)^2}\frac{1}{(2t)^{\frac{p-1}{2}}}\frac{1}{\eta(it)^{24}} = \frac{1}{4\pi}\int_0^\infty ds\left(\frac{s}{2\pi}\right)^{\frac{p-1}{2}}\eta\left(-\frac{\pi}{is}\right)^{-24}$$
$$= \frac{1}{4\pi}\int_0^\infty ds\left(\frac{s}{2\pi}\right)^{\frac{p-1}{2}}\left(\frac{\pi}{s}\right)^{-12}\eta\left(\frac{is}{\pi}\right)^{-24}$$
$$= \frac{1}{4\pi}\int_0^\infty ds\left(\frac{s}{2\pi}\right)^{\frac{p-25}{2}}\left[\sqrt{2}\eta\left(\frac{is}{\pi}\right)\right]^{-24}$$

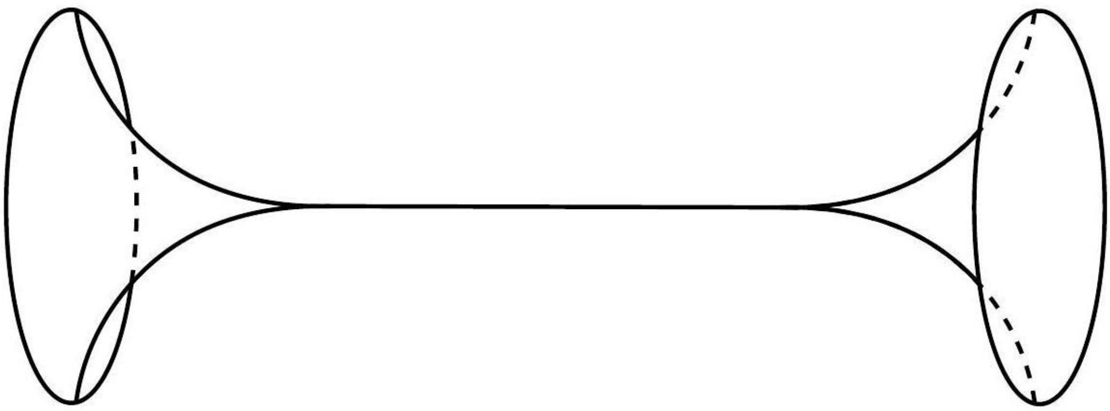
$$\left\langle B_p \right| e^{-s(L_0+\bar{L}_0-2)} \left| B_p \right\rangle = \left(\frac{s}{2\pi}\right)^{\frac{p-25}{2}} \left[\sqrt{2}\eta\left(\frac{is}{\pi}\right)\right]^{-24}$$

$$\left\langle B_p \right| e^{-s(L_0+\bar{L}_0-2)} \left| B_p^{\Delta Y} \right\rangle = \left(\frac{s}{2\pi}\right)^{\frac{p-25}{2}} e^{-\frac{\Delta Y^2}{2s\alpha'}} \left[\sqrt{2}\eta\left(\frac{is}{\pi}\right)\right]^{-24}.$$

$$\begin{pmatrix} \phi_{aa} & \phi_{ab} \\ \phi_{ba} & \phi_{bb} \end{pmatrix}$$

$$\begin{aligned}
\mathcal{A}_{\text{Vacuum}}^{1-\text{ loop}} &= \mathrm{Tr}_{CP}\left[e^{-2\pi t\left(L_0-\frac{c}{24}\right)}\right] = \mathrm{Tr}_{CP}\left[\begin{pmatrix} \langle aa| & \langle ab| \\ \langle ba| & \langle bb| \end{pmatrix} q^{L_0-\frac{c}{24}} \begin{pmatrix} |aa\rangle & |ab\rangle \\ |ba\rangle & |bb\rangle \end{pmatrix}\right] \\
&= \mathrm{Tr}_{aa}\left[q^{L_0-\frac{c}{24}}\right] + \mathrm{Tr}_{bb}\left[q^{L_0-\frac{c}{24}}\right] + 2\mathrm{Tr}_{ab}\left[q^{L_0-\frac{c}{24}}\right] \\
&= \langle B_a| e^{-s\left(L_0+\bar{L}_0-\frac{c}{12}\right)} |B_a\rangle + \langle B_b| e^{-s\left(L_0+\bar{L}_0-\frac{c}{12}\right)} |B_b\rangle + \\
&\quad +2\langle B_a| e^{-s\left(L_0+\bar{L}_0-\frac{c}{12}\right)} |B_b\rangle
\end{aligned}$$

$$\mathcal{A} = \frac{1}{4\pi}\int_0^\infty ds \left(\frac{2\pi}{s}\right)^{\frac{d_\perp}{2}} e^{-\frac{\Delta Y^2}{2\alpha' s}} \left(\sqrt{2}\eta\left(\frac{is}{\pi}\right)\right)^{-24}$$



$$\eta\left(\frac{is}{\pi}\right) \sim e^{2s} + 24 + 324e^{-2s} + \cdots$$

$$\mathcal{A} \sim \frac{1}{4\pi}\int_0^\infty ds \left(\frac{2\pi}{s}\right)^{\frac{d_\perp}{2}} e^{-\frac{\Delta Y^2}{2\alpha' s}} (24) = G(\Delta Y)$$

$$\begin{aligned}
G(\Delta Y) &= \frac{6}{\pi}\int_0^\infty \frac{dt}{t^2} (2\pi t)^{\frac{d_\perp}{2}} e^{-t\frac{\Delta Y^2}{2\alpha'}} \\
&= 12(2\pi)^{\frac{d_\perp}{2}-1} \int_0^\infty dt\, t^{\frac{d_\perp}{2}-2} e^{-t\frac{\Delta Y^2}{2\alpha'}}
\end{aligned}$$

$$G(\Delta Y) = 12(2\pi)^{\frac{d_\perp}{2}-1}\left(\frac{2\alpha'}{\Delta Y^2}\right)^{\frac{d_\perp}{2}-1}\int_0^\infty dt x^{\frac{d_\perp}{2}-2}e^{-x}$$

$$= 12\left(\frac{4\pi\alpha'}{\Delta Y^2}\right)^{\frac{d_\perp}{2}-1}\Gamma\left(\frac{d_\perp}{2}-1\right)\propto\frac{1}{\Delta Y^{d_\perp-2}}$$

$$S = \int d^d x\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu\right]$$

$$j_\mu(x) = j_\mu^{(1)}(x) + j_\mu^{(2)}(x)$$

$$j_\mu^{(i)}(x) = q_i\delta_{\mu,0}\delta^{d_\perp}(\vec{x}-\vec{y}_i) = q_i\delta_{\mu,0}\int d^{d_\perp}\vec{k}e^{i\vec{k}\cdot(\vec{x}-\vec{y}_i)}, i = 1,2$$

$$G(\Delta x) = \langle j^{(1)}(\vec{y}_1)j^{(2)}(\vec{y}_2)\rangle \sim q_1q_2\int d^{d_\perp}\vec{k}e^{i\vec{k}\cdot\overrightarrow{\Delta y}}\frac{1}{\vec{k}^2},$$

$$\frac{1}{\vec{k}^2} = \int_0^\infty dse^{-s\vec{k}^2}$$

$$G(\Delta y) \sim q_1q_2\int_0^\infty ds\frac{1}{s^{\frac{d}{2}}}e^{-\frac{\Delta y^2}{4s}}$$

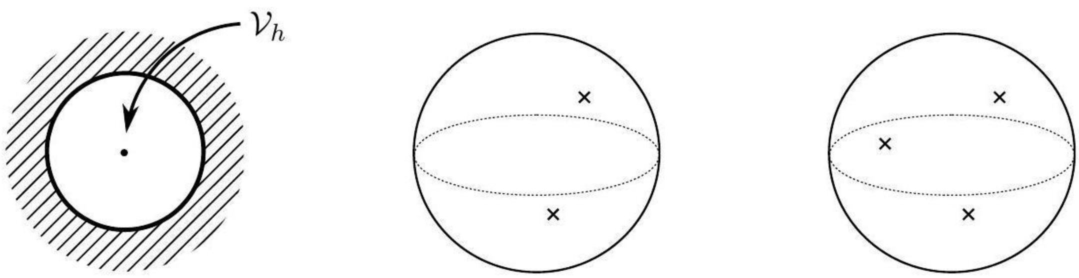
$$G(\Delta y) \sim \frac{q_1q_2}{|\Delta y|^{d_\perp-2}}$$

$$S = \int d^d x\left[-\frac{1}{2}h_{\mu\nu}\nabla_{\mu\nu\rho\lambda}h^{\rho\lambda} - T_{\mu\nu}h^{\mu\nu}\right]$$

$$\nabla^{\mu\nu}_{\rho\lambda} = \left(\delta_\rho^{(\mu}\delta_\lambda^{\nu)} - \eta^{\mu\nu}\eta_{\rho\lambda}\right)\Box - 2\delta^{(\mu}_{(\rho}\partial^{\nu)}\partial_{\lambda)} + \eta_{\rho\lambda}\partial^\mu\partial^\nu + \eta^{\mu\nu}\partial_\rho\partial_\lambda$$

$$T^{\mu\nu}(x) = m_1\eta^{\mu 0}\eta^{\nu 0}\delta(\vec{x}-\vec{y}_1) + m_2\eta^{\mu 0}\eta^{\nu 0}\delta(\vec{x}-\vec{y}_2)$$

$$V(\Delta y) = m_1m_2\int d^{d_\perp}\vec{p}\frac{e^{i\vec{p}\cdot\vec{\Delta}y}}{|\vec{p}|^2}\propto\frac{m_1m_2}{|\Delta y|^{d_\perp-2}}$$



(a) *D-brane interaction, disk insertion.*

(b) *Kinetic term, 2-vertex insertion on the sphere.*

(c) *Typical interaction term, 3-vertex insertion on the sphere.*

$$\mathcal{L}_{eff}(h) = \tilde{T}_{\mu\nu}h^{\mu\nu} + hKh + g_sh^3 + g_s^2h^4 + \cdots.$$

$$\mathcal{L}_{EH} \sim \frac{R}{g_s^2} + T_{\mu\nu}\hat{h}^{\mu\nu} = \frac{1}{g_s^2}\left(\hat{h}K\hat{h} + \hat{h}^3 + \hat{h}^4 + \cdots\right) + T_{\mu\nu}\hat{h}^{\mu\nu},$$

$$\mathcal{L}_{eff}(\hat{h}) = \frac{1}{g_s}\tilde{T}_{\mu\nu}\hat{h}^{\mu\nu} + \frac{1}{g_s^2}\left[\hat{h}K\hat{h} + \hat{h}^3 + \hat{h}^4 + \cdots\right].$$

$$\tau_p \propto \frac{1}{g_s}.$$

$$\begin{aligned} S_{oc}(\Psi_o, \Psi_c) = & \frac{1}{g_s}\left(\frac{1}{2}\langle \Psi_o | K_o | \Psi_o \rangle + \text{ (open particle interactions)}\right) \\ & + \frac{1}{g_s^2}\left(\frac{1}{2}\langle \Psi_c | K_c | \Psi_c \rangle + \text{ (closed particle interactions)}\right) \\ & + \frac{1}{g_s}\left(\langle B \mid \Psi_c \rangle + \text{ (open-closed particle interactions)}\right), \end{aligned}$$

$$S_{oc}(\Psi_o^*, \Psi_c \equiv 0) \sim \frac{1}{g_s}$$

$$\Psi_o \to \Psi_o^* + \varphi_o, \longrightarrow\ |B\rangle \to |B_*\rangle = |B\rangle + |\ \text{interactions with } \Psi_o^*\rangle.$$

$$S = \frac{1}{4\pi}\int_{\text{WS}} d^2z\left(\frac{2}{\alpha'}\partial X^\mu \bar{\partial} X_\mu + \psi^\mu \bar{\partial}\psi_\mu + \bar{\psi}^\mu \partial \bar{\psi}_\mu\right)$$

$$\begin{aligned} X^\mu(z,\bar{z})X^\nu(0,0) \ &= -\frac{\alpha'}{2}\eta^{\mu\nu}\log|z| + (\ \mathrm{r}\mathfrak{U}\ ) \\ \psi^\mu(z)\psi^\nu(0) \ &= \eta^{\mu\nu}\frac{1}{z} + (\ \mathfrak{V}\ ) \\ \bar{\psi}^\mu(\bar{z})\bar{\psi}^\nu(0) \ &= \eta^{\mu\nu}\frac{1}{\bar{z}} + (\ \mathfrak{W}\ ) \end{aligned}$$

$$\begin{aligned} T^{(\mathrm{m})}(z) &= -\frac{1}{\alpha'}{:}\partial X \cdot \partial X{:} - \frac{1}{2}{:}\psi \cdot \partial\psi{:} \\ \bar{T}^{(\mathrm{m})}(\bar{z}) &= -\frac{1}{\alpha'}{:}\bar{\partial} X \cdot \bar{\partial} X{:} - \frac{1}{2}{:}\bar{\psi} \cdot \bar{\partial}\bar{\psi}{:} \end{aligned}$$

$$T^{(\mathrm{m})}(z_1)T^{(\mathrm{m})}(z_2) = \frac{c}{2}\frac{1}{(z_1 - z_2)^4} + \frac{2T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)^2} + \frac{\partial T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)} + (\ \mathfrak{X}\ ),$$

$$c = c_X + c_\psi = d + \frac{1}{2}d = \frac{3}{2}d$$

$$\begin{cases} \delta_\epsilon^{\mathrm{c}} X^\mu(z,\bar{z}) & = -\left[\epsilon(z)\partial X^\mu(z,\bar{z}) + \bar{\epsilon}(\bar{z})\bar{\partial} X^\mu(z,\bar{z})\right] \\ \delta_\epsilon^{\mathrm{c}} \psi^\mu(z) & = -\left[\epsilon\partial\psi^\mu(z) + \frac{1}{2}(\partial\epsilon)\psi^\mu(z)\right] \\ \delta_\epsilon^{\mathrm{c}} \bar{\psi}^\mu(\bar{z}) & = -\left[\bar{\epsilon}\bar{\partial}\bar{\psi}^\mu(\bar{z}) + \frac{1}{2}(\bar{\partial}\bar{\epsilon})\bar{\psi}^\mu(\bar{z})\right] \end{cases}$$

$$\epsilon(z) = \sum_n \epsilon_n z^{-n+1},$$

$$\left\langle \begin{aligned} \delta_\eta^{\rm sc} X^\mu(z,\bar z) &= \sqrt{\frac{\alpha'}{2}}[\eta(z)\psi^\mu(z)+\bar\eta(\bar z)\bar\psi^\mu(\bar z)] \\ \delta_\eta^{\rm sc}\psi^\mu(z) &= \sqrt{\frac{2}{\alpha'}}[-\eta(z)\partial X^\mu(z)] \\ \delta_\eta^{\rm sc}\bar\psi^\mu(\bar z) &= \sqrt{\frac{2}{\alpha'}}[-\bar\eta(\bar z)\bar\partial \bar X^\mu(\bar z)] \end{aligned} \right\rangle$$

$$\eta(z)=\sum_r \eta_r z^{-r+\frac{1}{2}}$$

$$\begin{aligned} \delta_\eta^{\rm sc} S = \frac{1}{2\pi}\int_{\rm WS} d^2z\Big[\frac{1}{\sqrt2}\partial(\eta\psi)\cdot\bar\partial X+\frac{1}{\sqrt2}\partial X\cdot\bar\partial(\eta\psi) \\ +\frac{1}{2}(-\sqrt2\eta\partial X)+\frac{1}{2}\psi\cdot\partial(-\sqrt2\eta\partial X)\Big] \\ =\frac{1}{2\pi}\int_{\rm WS} d^2z\Big[\frac{2}{\sqrt2}\partial(\eta\psi)\cdot\bar\partial X-\sqrt2(\eta\partial X)\cdot\bar\partial\psi\Big] \\ \overset{\rm IBP}{=}\frac{1}{2\pi}\int_{\rm WS} d^2z\sqrt2[\partial(\eta\psi)\cdot\bar\partial X+\eta(\bar\partial\partial X)\cdot\psi] \\ \overset{\rm IBP}{=}\frac{1}{2\pi}\int_{\rm WS} d^2z\sqrt2[\partial(\eta\psi)\cdot\bar\partial X-\bar\partial X\cdot\partial(\eta\psi)]=0 \end{aligned}$$

$$\begin{aligned} [\delta_{\eta_1}^{\rm sc},\delta_{\eta_2}^{\rm sc}]X^\mu &= (\delta_{\eta_1}^{\rm sc}\delta_{\eta_2}^{\rm sc}-\delta_{\eta_2}^{\rm sc}\delta_{\eta_1}^{\rm sc})X^\mu \\ &=\delta_{\eta_1}^{\rm sc}\left(\frac{1}{\sqrt2}\eta_2\psi^\mu\right)-\delta_{\eta_2}^{\rm sc}\left(\frac{1}{\sqrt2}\eta_1\psi^\mu\right) \\ &=\frac{1}{\sqrt2}\eta_2(-\sqrt2\eta_1\partial X^\mu)+\frac{1}{\sqrt2}\eta_1(-\sqrt2\eta_2\partial X^\mu) \\ &=-\eta_2\eta_1\partial X^\mu+\eta_1\eta_2\partial X^\mu \end{aligned}$$

$$\begin{aligned} &= 2\eta_1\eta_2\partial X^\mu \\ &=\delta^{\rm c}_{2\eta_1\eta_2}X^\mu=\delta^{\rm c}_{2\eta_2\eta_1}X^\mu \\ [\delta_{\eta_1}^{\rm sc},\delta_{\eta_2}^{\rm sc}]\psi^\mu &= (\delta_{\eta_1}^{\rm sc}\delta_{\eta_2}^{\rm sc}-\delta_{\eta_2}^{\rm sc}\delta_{\eta_1}^{\rm sc})\psi^\mu \\ &=\delta_{\eta_1}^{\rm sc}(-\sqrt2\eta_2\partial X^\mu)-\delta_{\eta_2}^{\rm sc}(-\sqrt2\eta_1\partial X^\mu) \\ &=-\sqrt2\eta_2\partial\left(\sqrt{\frac{1}{2}}\eta_1\psi^\mu\right)+\sqrt2\eta_1\partial\left(\sqrt{\frac{1}{2}}\eta_2\psi^\mu\right) \\ &=-\eta_2\partial(\eta_1\psi^\mu)+\eta_1\partial(\eta_2\psi^\mu) \\ &=[-\eta_2\partial\eta_1-(\partial\eta_2)\eta_1]\psi^\mu-2\eta_2\eta_1\partial\psi^\mu \\ &=-\left[\frac{1}{2}\partial(2\eta_2\eta_1)\psi^\mu+2\eta_2\eta_1(\partial\psi^\mu)\right] \\ &=\delta^{\rm c}_{2\eta_2\eta_1}\psi^\mu \end{aligned}$$

$$[\delta_{\eta_1}^{\rm sc},\delta_{\eta_2}^{\rm sc}]=\delta^{\rm c}_{2\eta_2\eta_1}$$

$$T_{\epsilon(z)}=\oint_0\frac{dz}{2\pi i}\epsilon(z)T^{(\rm m)}(z)$$

$$\delta_{\epsilon}^{\mathrm{c}}\phi(z) = -[T_{\epsilon}, \phi(z)]$$

$$G_{\eta(z)} = \oint_{0} \frac{dz}{2\pi i}\eta(z)G^{(\mathrm{m})}(z)$$

$$\delta_{\eta}^{\mathrm{sc}}\phi(z) = -\left[G_{\eta}, \phi(z)\right]$$

$$G^{(\mathrm{m})}(z) = i\sqrt{\frac{2}{\alpha'}}\psi^{\mu}\partial X_{\mu} = \psi \cdot j$$

$$G^{(\mathrm{m})}(z_1)G^{(\mathrm{m})}(z_2) = \frac{d}{(z_1 - z_2)^3} + \frac{2T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)}$$

$$T^{(\mathrm{m})}(z_1)G^{(\mathrm{m})}(z_2) = \frac{\frac{3}{2}G^{(\mathrm{m})}(z_2)}{(z_1 - z_2)^2} + \frac{\partial G^{(\mathrm{m})}(z_2)}{(z_1 - z_2)}$$

$$T^{(\mathrm{m})}(z_1)T^{(\mathrm{m})}(z_2) = \frac{\frac{c}{2}}{(z_1 - z_2)^4} + \frac{2T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)^2} + \frac{\partial T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)} + (\text{ד})$$

$$G^{(\mathrm{m})}(z_1)G^{(\mathrm{m})}(z_2) = \frac{\frac{2}{3}c}{(z_1 - z_2)^3} + \frac{2T^{(\mathrm{m})}(z_2)}{(z_1 - z_2)} + (\text{ג})$$

$$T^{(\mathrm{m})}(z_1)G^{(\mathrm{m})}(z_2) = \frac{\frac{3}{2}G^{(\mathrm{m})}(z_2)}{(z_1 - z_2)^2} + \frac{\partial G^{(\mathrm{m})}(z_2)}{(z_1 - z_2)} + (\text{א})$$

$$T^{(\mathrm{m})}(z_1)\phi^{(h)}(z_2) = \frac{h\phi^{(h)}(z_2)}{(z_1 - z_2)^2} + \frac{\partial \phi^{(h)}(z_2)}{(z_1 - z_2)} + (\text{א})$$

$$G^{(\mathrm{m})}(z_1)\phi^{(h)}(z_2) = \frac{\psi^{\left(h+\frac{1}{2}\right)}(z_2)}{(z_1 - z_2)} + (\text{א})$$

$$G^{(\mathrm{m})}(z_1)\psi^{\mu}(z_2) = j \cdot \psi(z_1)\psi^{\mu}(z_2) = \frac{j^{\mu}(z_1)}{(z_1 - z_2)} \overset{\text{Taylor}}{=} \frac{j^{\mu}(z_2)}{(z_1 - z_2)} + O(1)$$

$$G^{(\mathrm{m})}(z_1)j^{\mu}(z_2) = \psi \cdot j(z_1)j^{\mu}(z_2) = \frac{\psi^{\mu}(z_1)}{(z_1 - z_2)^2} \overset{\text{Taylor}}{=} \frac{\psi^{\mu}(z_2)}{(z_1 - z_2)^2} + \frac{\partial \psi^{\mu}(z_2)}{(z_1 - z_2)},$$

$$G^{(\mathrm{m})}(z_1)i\sqrt{\frac{2}{\alpha'}}X^{\mu}(z_2) = \frac{\psi^{\mu}(z_2)}{(z_1 - z_2)}$$

$$\begin{aligned} G^{(\mathrm{m})}(z_1){:}e^{iP\cdot X_L}{:}(z_2) &= \psi \cdot j(z_1){:}e^{iP\cdot X_L}{:}(z_2) \\ &= \psi_{\mu}(z_1)iP_{\nu}j^{\mu}(z_1)X_L^{\nu}(z_2){:}e^{iP\cdot X_L}{:}(z_2) \\ &= \sqrt{\frac{\alpha'}{2}}P_{\mu}\psi^{\mu}{:}e^{iP\cdot X_L}{:}(z_2)\frac{1}{(z_1 - z_2)} \end{aligned}$$

$$[L_{-1},\cdot] = \oint_{0} \frac{dz}{2\pi i}T^{(\mathrm{m})}(z)$$

$$[L_{-1}, \phi(z_2)] = \oint_0 \frac{dz_1}{2\pi i} T^{(\mathrm{m})}(z_1)\phi(z_2) = \partial\phi(z_2)$$

$$\delta_{\mathrm{SUSY}} = \oint_0 \frac{dz}{2\pi i} G^{(\mathrm{m})}(z)$$

$$G^{(\mathrm{m})}(z) = \sum_{r \in \mathbb{Z}+\frac{1}{2}} G_r z^{-r-\frac{3}{2}}$$

$$\delta_{\mathrm{SUSY}} = \left[G_{-\frac{1}{2}}, \cdot\right]$$

$$\delta_{\mathrm{SUSY}}\left(i\sqrt{\frac{2}{\alpha'}}X^\mu(z)\right) = \psi^\mu(z)$$

$$\delta_{\mathrm{SUSY}}(\psi^\mu(z)) = j^\mu(z)$$

$$\delta_{\mathrm{SUSY}}(j^\mu(z)) = \partial\psi^\mu(z)$$

$$[\delta_{\mathrm{SUSY}}, \delta_{\mathrm{SUSY}}] = \partial_z$$

$$S_{\mathrm{gh}} = \frac{1}{2\pi}\int_{\mathrm{WS}} d^2z(b\bar{\partial}c + \bar{b}\partial\bar{c} + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma})$$

$$b(z_1)c(z_2) = \frac{1}{(z_1 - z_2)} + (\aleph)$$

$$c(z_1)b(z_2) = \frac{1}{(z_1 - z_2)} + (\wp)$$

$$\beta(z_1)\gamma(z_2) = -\frac{1}{(z_1 - z_2)} + (\beth)$$

$$\gamma(z_1)\beta(z_2) = \frac{1}{(z_1 - z_2)} + (\varkappa)$$

$$T^{(\mathrm{gh})}(z) = T^{(b,c)}(z) + T^{(\beta,\gamma)}(z)$$

$$T^{(b,c)}(z) =: (\partial b)c : (z) - 2 : \partial(bc) : (z)$$

$$T^{(\beta,\gamma)}(z) =: (\partial\beta)\gamma : (z) - \frac{3}{2} : \partial(\beta\gamma) : (z)$$

$$T^{(b,c)}(z_1)T^{(b,c)}(z_2) \approx \frac{\frac{1}{2}(-26)}{(z_1 - z_2)^4}$$

$$T^{(\beta,\gamma)}(z_1)T^{(\beta,\gamma)}(z_2) \approx \frac{\frac{1}{2}(11)}{(z_1 - z_2)^4}$$

$$c^{(b,c)} = -26$$

$$c^{(\beta,\gamma)} = 11$$

$$c^{(\mathrm{gh})} = c^{(b,c)} + c^{(\beta,\gamma)} = -15$$

$$G^{(\mathrm{gh})}(z) = -\frac{1}{2}(\partial\beta)c + \frac{3}{2}\partial(\beta c) - 2b\gamma$$

$$c^{(\mathrm{tot})} = c^{(\mathrm{m})} + c^{(\mathrm{gh})} = \frac{3}{2}d - 15$$

$$d = 10$$

$$j_{\mathrm{B}}(z) = cT^{(\mathrm{m})}(z) + \frac{1}{2}:cT^{(\mathrm{gh})}:(z).$$

$$Q_{\mathrm{B}} = \oint_0 \frac{dz}{2\pi i} j_{\mathrm{B}}(z)$$

$$Q_{\mathrm{B}}^2 = 0 \Leftrightarrow c^{(\mathrm{m})} + c^{(\mathrm{gh})} = 0.$$

$$\begin{aligned} T^{(\mathrm{m})} &= 0, \\ G^{(\mathrm{m})} &= 0, \end{aligned}$$

$$j_{\mathrm{B}}(z) = cT^{(\mathrm{m})}(z) + \gamma G^{(\mathrm{m})}(z) + \frac{1}{2}\left(:cT^{(\mathrm{gh})}:(z) + :\gamma G^{(\mathrm{gh})}:(z)\right)$$

$$Q_{\mathrm{B}} = \oint_0 \frac{dz}{2\pi i} j_{\mathrm{B}}(z).$$

$$Q_{\mathrm{B}}^2 = 0 \Leftrightarrow c^{(\mathrm{m})} + c^{(\mathrm{gh})} = 0,$$

$$\begin{aligned} [Q_{\mathrm{B}}, b(z)] &= T^{(\mathrm{tot})}(z) = T^{(\mathrm{m})}(z) + T^{(\mathrm{gh})}(z), \\ [Q_{\mathrm{B}}, \beta(z)] &= G^{(\mathrm{tot})}(z) = G^{(\mathrm{m})}(z) + G^{(\mathrm{gh})}(z). \end{aligned}$$

$$\begin{aligned} \psi^{\mu}\left(h = \frac{1}{2}\right) &\leftrightarrow X^{\mu}(h = 0) \\ \gamma\left(h = -\frac{1}{2}\right) &\leftrightarrow c(h = -1) \\ \beta\left(h = \frac{3}{2}\right) &\leftrightarrow b(h = 2). \end{aligned}$$

$$\chi(w + 2\pi i) = \begin{cases} +\chi(w) & \text{Ramond (R) sector} \\ -\chi(w) & \text{Neveu-Schwarz (NS) sector} \end{cases}.$$

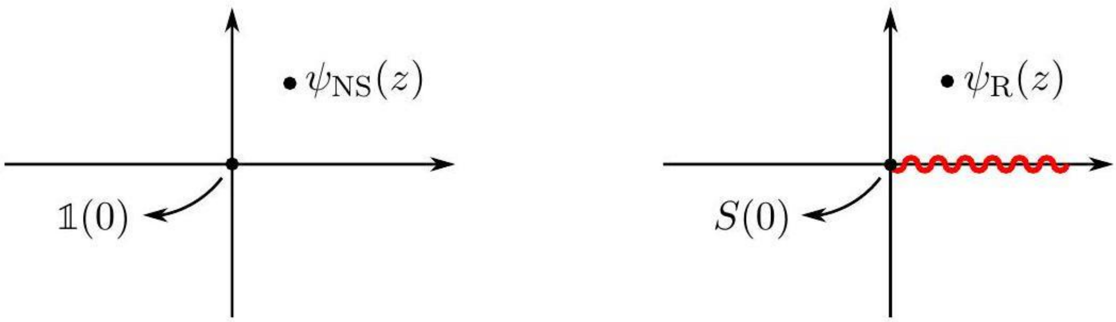
$$G(w + 2\pi i) = \begin{cases} +G(w) & \text{(R)} \\ -G(w) & \text{(NS)} \end{cases}$$

$$\psi(z)(dz)^{\frac{1}{2}} = \psi(w)(dw)^{\frac{1}{2}}$$

$$\psi(z)= \left(\frac{dz}{dw}\right)^{\frac{1}{2}} \psi(w(z))$$
$$= \left(\frac{d(e^{w})}{dw}\right)^{\frac{1}{2}} \psi(\log(z))$$
$$= \frac{1}{\sqrt{z}} \psi(\log(z))$$

$$\begin{cases} \psi^{\mu}_{\mathrm{NS}}(z) = \displaystyle\sum_{r\in\mathbb{Z}+\frac{1}{2}} \psi^{\mu}_{r} z^{-r-\frac{1}{2}} \\ \psi^{\mu}_{\mathrm{R}}(z) = \displaystyle\sum_{n\in\mathbb{Z}} \psi^{\mu}_{n} z^{-n-\frac{1}{2}} \end{cases}$$

$$\psi(z_1)S(z_2) = \frac{(\cdots)}{(z_1 - z_2)^{\pm\frac{1}{2}}}$$



(a) *NS case* (b) *R case*

$$\psi(z_1)\psi(z_2) = \frac{1}{(z_1 - z_2)} + (\,ð\,),$$

$$\text{NS)}\ \left[\psi^{\mu}_{r}, \psi^{\nu}_{s}\right] = \eta^{\mu\nu}\delta_{r+s,0}, \quad \text{with } r, s \in \mathbb{Z} + \frac{1}{2}$$
$$\text{R)}\ \left[\psi^{\mu}_{n}, \psi^{\nu}_{m}\right] = \eta^{\mu\nu}\delta_{n+m,0}, \quad \text{with } n, m \in \mathbb{Z}$$

$$\begin{cases} \beta_{\mathrm{NS}}(z) = \displaystyle\sum_{r\in\mathbb{Z}+\frac{1}{2}} \beta_{r} z^{-r-\frac{3}{2}} \\ \beta_{\mathrm{R}}(z) = \displaystyle\sum_{n\in\mathbb{Z}} \beta_{n} z^{-n-\frac{3}{2}} \end{cases}$$

$$\begin{cases} \gamma_{\mathrm{NS}}(z) = \displaystyle\sum_{r\in\mathbb{Z}+\frac{1}{2}} \gamma_{r} z^{-r+\frac{1}{2}} \\ \gamma_{\mathrm{R}}(z) = \displaystyle\sum_{n\in\mathbb{Z}} \gamma_{n} z^{-n+\frac{1}{2}} \end{cases}$$

$$\text{NS)}\ \ [\beta_r, \gamma_s] = \delta_{r+s,0}, \qquad \text{with } r, s \in \mathbb{Z} + \frac{1}{2}$$
$$\text{R)}\ \ [\beta_n, \gamma_m] = \delta_{n+m,0}, \qquad \text{with } n, m \in \mathbb{Z}.$$

$$\begin{cases} G_{\text{NS}}(z) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} G_r z^{-r-\frac{3}{2}} \\ G_{\text{R}}(z) = \sum_{n\in\mathbb{Z}} G_n z^{-n-\frac{3}{2}} \end{cases},$$

$$\text{NS)}\ \ [G_r, G_s] = 2L_{r+s} + \frac{c}{12}(4r^2 - 1)\delta_{r+s,0}, \qquad \text{with } r, s \in \mathbb{Z} + \frac{1}{2},$$
$$\text{R)}\ \ [G_n, G_m] = 2L_{n+m} + \frac{c}{12}(4n^2 - 1)\delta_{n+m,0}, \qquad \text{with } n, m \in \mathbb{Z}.$$

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}, \\ [L_n, G_r] &= \frac{n-2r}{2}G_{n+r}, \\ \text{NS)}\ [G_r, G_s] &= 2L_{r+s} + \frac{c}{12}(4r^2-1)\delta_{r+s,0}, \ \text{with } r, s \in \mathbb{Z} + \frac{1}{2}, \\ \text{R)}\ [G_r, G_s] &= 2L_{r+s} + \frac{c}{12}(4r^2-1)\delta_{r+s,0}, \ \text{with } r, s \in \mathbb{Z}, \end{aligned}$$

$$\alpha_n^\mu |0, P\rangle_{\text{NS}} = 0 \qquad \text{for } n \geq 1$$
$$\psi_r^\mu |0, P\rangle_{\text{NS}} = 0 \qquad \text{for } r \geq \frac{1}{2}$$

$$\alpha_n^\mu |0, P\rangle_{\text{R}} = 0 \qquad \text{for } n \geq 1,$$
$$\psi_n^\mu |0, P\rangle_{\text{R}} = 0 \qquad \text{for } n \geq 1.$$

$$[\psi_0^\mu, \psi_0^\nu] = \eta^{\mu\nu}$$

$$\psi_0^\mu |\alpha, P\rangle_{\text{R}} = (\psi^\mu)_{\alpha\beta} |\beta, P\rangle_{\text{R}}$$

$$\Gamma^\mu = \sqrt{2}\psi_0^\mu$$

$$[\Gamma^\mu, \Gamma^\nu] = 2\eta^{\mu\nu}$$

$$|\alpha\rangle_{\text{R}} = \underbrace{|\alpha\rangle}_{16_c} \oplus \underbrace{|\dot{\alpha}\rangle}_{16_s},$$

$$\begin{aligned} \Gamma|\alpha\rangle &= +|\alpha\rangle \\ \Gamma|\dot{\alpha}\rangle &= -|\dot{\alpha}\rangle \end{aligned}$$

$$|0, P\rangle_{\text{NS}}^{\text{matter}} = \underbrace{|0, P\rangle}_{X \text{ sector}} \otimes \underbrace{|0\rangle_{\text{NS}}}_{\psi \text{ sector}} = |0, P\rangle \otimes |0\rangle,$$

$$|\alpha, P\rangle_{\text{R}}^{\text{matter}} = \underbrace{|0, P\rangle}_{X \text{ sector}} \otimes \underbrace{|\alpha\rangle_{\text{R}}}_{\psi \text{ sector}} = |0, P\rangle \otimes (|\alpha\rangle \oplus |\dot{\alpha}\rangle).$$

$$|\overset{(\cdot)}{\alpha}\rangle = S_{(\cdot)}(0)|0\rangle.$$

$$G_{\{\mu_1 \dots \mu_k\}\{\nu_1 \dots \nu_q\}}(P)\left(\alpha_{-n_1}^{\mu_1} \dots \alpha_{-n_k}^{\mu_k}\right)\left(\psi_{-r_1}^{\nu_1} \dots \psi_{-r_q}^{\nu_q}\right)|0,P\rangle_{\mathrm{NS}}$$

$$\chi_{\alpha\{\mu_1 \dots \mu_k\}\{\nu_1 \dots \nu_q\}}(P)\left(\alpha_{-n_1}^{\mu_1} \dots \alpha_{-n_k}^{\mu_k}\right)\left(\psi_{-r_1}^{\nu_1} \dots \psi_{-r_q}^{\nu_q}\right)|\alpha,P\rangle_{\mathrm{R}},$$

$$\begin{aligned}(L_0 - a_{\mathrm{NS}})|\mathrm{phys}\rangle_{\mathrm{NS}} &= 0\\ L_{n>0}|\mathrm{phys}\rangle_{\mathrm{NS}} &= 0\\ G_{r\geq\frac{1}{2}}|\mathrm{phys}\rangle_{\mathrm{NS}} &= 0\end{aligned}$$

$$\begin{aligned}(L_0 - a_{\mathrm{R}})|\mathrm{phys}\rangle_{\mathrm{R}} &= 0\\ L_{n>0}|\mathrm{phys}\rangle_{\mathrm{R}} &= 0\\ G_{n\geq 0}|\mathrm{phys}\rangle_{\mathrm{R}} &= 0\end{aligned}$$

$$a_{\mathrm{NS}} = \frac{1}{2}, a_{\mathrm{R}} = \frac{5}{8}$$

$$\begin{aligned}L_0^{(\mathrm{tot})} &= L_0^{(\mathrm{m})} + L_0^{(\mathrm{gh})}\\ &= L_0^{(\mathrm{m})} - a\\ &= L_0^{(\mathrm{m})} - 1\\ &= 0\end{aligned}$$

$$c_1|0\rangle_{\mathrm{NS}} = c\mathcal{V}_{\frac{1}{2}}^{(\beta,\gamma)}(0)|0\rangle,$$

$$\begin{aligned}L_0^{(\mathrm{tot})} &= L_0^{(\mathrm{m})} + L_0^{(\mathrm{gh})}\\ &= L_0^{(\mathrm{m})} - a_{\mathrm{NS}}\\ &= L_0^{(\mathrm{m})} - \frac{1}{2}\\ &= 0.\end{aligned}$$

$$c_1|\overset{(\cdot)}{\alpha}\rangle_{\mathrm{R}} = S_{(\dot{\alpha})}c\mathcal{V}_{\frac{3}{8}}^{(\beta,\gamma)}(0)|0\rangle,$$

$$\begin{aligned}L_0^{(\mathrm{tot})} &= L_0^{(\mathrm{m})} + L_0^{(\mathrm{gh})}\\ &= L_0^{(\mathrm{m})} - a_{\mathrm{R}}\\ &= L_0^{(\mathrm{m})} + \left(-1 + \frac{3}{8}\right)\\ &= L_0^{(\mathrm{m})} - \frac{5}{8}\\ &= 0 \quad (6.87)\end{aligned}$$

$$\mathrm{NS)}\ L_n = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{n-k}\cdot\alpha_k + \frac{1}{4}\sum_{r\in\mathbb{Z}+\frac{1}{2}} (2r-n):\psi_{n-r}\cdot\psi_r:,$$

$$\mathrm{R)}\ L_n = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{n-k}\cdot\alpha_k + \frac{1}{4}\sum_{k\in\mathbb{Z}} (2k-n):\psi_{n-k}\cdot\psi_k: + \frac{d}{16}\delta_{n,0}.$$

$$G_r = \sum_{n\in\mathbb{Z}} \alpha_n \cdot \psi_{n-r},$$

$$L_0 = \frac{1}{2}(\alpha_0)^2 + N,$$

$$\text{NS)}\ N_{\text{NS}} = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{-k}\cdot\alpha_k + \frac{1}{4}\sum_{r\in\mathbb{Z}+\frac{1}{2}} (2r){:}\psi_{-r}\cdot\psi_r{:}$$

$$\text{R)}\ N_{\text{R}} = \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha_{-k}\cdot\alpha_k + \frac{1}{4}\sum_{k\in\mathbb{Z}} (2k){:}\psi_{-k}\cdot\psi_k{:} + \frac{5}{8}$$

$$t(P)|0,P\rangle_{\text{NS}}$$

$$L_0^{(\text{m})} - a_{\text{NS}} = L_0^{(\text{m})} - \frac{1}{2} = 0 \implies \frac{\alpha' P^2}{4} = \frac{1}{2} = a_{\text{NS}} \implies m^2 = -\frac{2}{\alpha'}$$

$$\xi_\mu(P)\psi^\mu_{\frac{1}{2}}|0,P\rangle_{\text{NS}}$$

$$m^2 = \frac{4}{\alpha'}\left(\frac{1}{2} - a_{\text{NS}}\right) = 0$$

$$G_{\frac{1}{2}} = \sum_n \alpha_n \cdot \psi_{\frac{1}{2}-n}$$

$$G_{\frac{1}{2}} = 0 \implies \alpha_0\cdot\xi = 0 \implies P\cdot\xi = 0$$

$$u_\alpha(P)|\alpha,P\rangle_{\text{R}} + v_{\dot\alpha}(P)|\dot\alpha,P\rangle_{\text{R}},$$

$$G_0 = \sum_n \alpha_n\cdot\psi_{-n},$$

$$G_0 = \alpha_0\cdot\psi_0 = \sqrt{\frac{\alpha'}{2}}P\cdot\Gamma\frac{1}{\sqrt{2}} = \frac{\sqrt{\alpha'}}{2}\not{P},$$

$$\not{P}u(P) = 0 \implies m^2 = 0,$$
$$\not{P}v(P) = 0 \implies m^2 = 0.$$

$$SO(8) \longrightarrow \underbrace{8_C}_{\text{left}} \oplus \underbrace{8_S}_{\text{right}}\ .$$

NS		R
$N_{\text{NS}} = 0$:	tachyon	$N_{\text{R}} = 0$:
$N_{\text{NS}} = \frac{1}{2}$:	8_V massless	$8_C \oplus 8_S$ massless

$$X_L^+(z) = \frac{X_0^+}{2} - i\alpha' P^+ \log\, z \;\; (\text{no } \mathcal{O}_{\text{oscillators}})$$

$$j^+(z) = i\sqrt{\frac{2}{\alpha'}}\partial X^+(z) = \sqrt{\frac{\alpha'}{2}}P^+\frac{1}{z}$$

$$\delta_\eta^{\text{sc}}\psi^+(z) = -i\eta j^+(z)$$

$$\psi^+(z) = 0$$

$$\begin{cases} X_L^+(z) = \dfrac{X_0^+}{2} - i\alpha' P^+ \log\, z \\ \psi^+(z) = 0 \end{cases}$$

$$\left\| \begin{aligned} \partial X_L^-(z) &= \frac{z}{2P^+}\left(\frac{2}{\alpha'}\partial X^j \partial X_j + i\psi^j\partial\psi_j\right) \\ \psi^-(z) &= \frac{2z}{\alpha' P^+}\psi^j \partial X_{L,j} \end{aligned} \right\|$$

$$\text{NS)}\; \left(\alpha_{-n_1}^{i_1} \ldots \alpha_{-n_k}^{i_k}\right)\left(\psi_{-r_1}^{j_1} \ldots \psi_{-r_q}^{j_q}\right)|0,P\rangle_{\text{NS}}$$

$$\text{R)}\; \left(\alpha_{-n_1}^{i_1} \ldots \alpha_{-n_k}^{i_k}\right)\left(\psi_{-r_1}^{j_1} \ldots \psi_{-r_q}^{j_q}\right)|\overset{(\cdot)}{\alpha}, P\rangle_{\text{R}}$$

$$\text{NS)}\; L_0^{(\text{lc})} - a_{\text{NS}}^{(\text{lc})} = 0$$

$$\text{R)}\; L_0^{(\text{lc})} - a_{\text{R}}^{(\text{lc})} = 0.$$

$$t(P)|0,P\rangle_{\text{NS}}$$

$$m^2 = -a_{\text{NS}}^{(\text{lc})}\frac{4}{\alpha'}$$

$$\xi_i(P)\psi_{-\frac{1}{2}}^i|0,P\rangle_{\text{NS}}$$

$$\frac{\alpha' m^2}{4} = \frac{1}{2} - a_{\text{NS}}^{(\text{lc})}$$

$$a_{\text{NS}}^{(\text{lc})} = \frac{1}{2},$$

$$
\begin{aligned}
L_0^{(\mathrm{lc})} &= \frac{\alpha' P^2}{4} + N_\perp \\
&= \frac{\alpha' P^2}{4} + \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha^i_{-k}\alpha_{k,i} + \frac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}} r\psi^i_{-r}\psi_{r,i} \\
&= \frac{\alpha' P^2}{4} + \sum_{k\geq 1}\left(\alpha^i_{-k}\alpha_{k,i} + \frac{1}{2}k(d-2)\right) + \sum_{r\geq\frac{1}{2}}\left(r\psi^i_{-r}\psi_{r,i} - \frac{1}{2}r(d-2)\right) \\
&= \frac{\alpha' P^2}{4} + \hat{N}_\perp + \underbrace{\frac{1}{2}(d-2)\sum_{k\geq 1} k - \frac{1}{2}(d-2)\sum_{r\geq\frac{1}{2}} r}_{\text{infinite zero point energies}},
\end{aligned}
$$

$$
\sum_{k\geq\nu} k \longrightarrow \sum_{k\geq\nu} k e^{-\epsilon k} = \frac{e^{\epsilon\nu}\left(\nu + e^{\epsilon(1-\nu)}\right)}{(1-e^{\epsilon})^2} \overset{\epsilon\to 0}{=} \frac{1}{\epsilon^2} - \frac{1}{12}(1 - 6\nu + 6\nu^2) + O(\epsilon)
$$

$$
\zeta(s,\nu) = \sum_{n=0}^{\infty} (n+\nu)^{-s}, \zeta(-1,\nu) = -\frac{1}{12}(1-6\nu+6\nu^2)
$$

$$
\sum_{k\geq 1} k = -\frac{1}{12}, \sum_{r\geq\frac{1}{2}} r = \frac{1}{24}
$$

$$
L_0^{(\mathrm{lc})} = \frac{\alpha' P^2}{4} + \hat{N}_\perp - \frac{d-2}{16}
$$

$$
a_{\mathrm{NS}}^{(\mathrm{lc})} = \frac{d-2}{16} = \frac{1}{2} \Longrightarrow d = 10
$$

$$
u_\alpha(P)|\alpha, P\rangle_{\mathrm{R}} + v_{\dot{\alpha}}(P)|\dot{\alpha}, P\rangle_{\mathrm{R}}
$$

$$
\begin{aligned}
L_0^{(\mathrm{lc})} &= \frac{\alpha' P^2}{4} + \frac{1}{2}\sum_{k\in\mathbb{Z}} \alpha^i_{-k}\alpha_{k,i} + \frac{1}{2}\sum_{n\in\mathbb{Z}} n\psi^i_{-n}\psi_{n,i} \\
&= \frac{\alpha' P^2}{4} + \hat{N}_\perp + \frac{1}{2}(d-2)\sum_{k\geq 1} k - \frac{1}{2}(d-2)\sum_{n\geq 1} n
\end{aligned}
$$

$$
a_{\mathrm{R}}^{(\mathrm{lc})} = 0.
$$

	$\mathrm{NS}-\overline{\mathrm{NS}}$	$\mathrm{R}-\overline{\mathrm{NS}}$	$\mathrm{NS}-\overline{\mathrm{R}}$	$\mathrm{R}-\overline{\mathrm{R}}$
$L_0-a=0$ $\bar{L}_0-a=0$	$m^2=\frac{4}{\alpha'}\left(N_{\mathrm{NS}}-\frac{1}{2}\right)$ $N_{\mathrm{NS}}=\bar{N}_{\mathrm{NS}}$	$m^2=\frac{4}{\alpha'}N_{\mathrm{R}}$ $N_{\mathrm{R}}=\bar{N}_{\mathrm{NS}}-\frac{1}{2}$	$m^2=\frac{4}{\alpha'}\bar{N}_{\mathrm{R}}$ $\bar{N}_{\mathrm{R}}=N_{\mathrm{NS}}-\frac{1}{2}$	$m^2=\frac{4}{\alpha'}\bar{N}_{\mathrm{R}}$ $N_{\mathrm{R}}=\bar{N}_{\mathrm{R}}$
$N=0$	tachyon: $t(P)\,\lvert 0,P\rangle$ $m^2=-\frac{2}{\alpha'}$			
$N_{NS}=\frac{1}{2}$ $N_R=0$	massless boson: $\underbrace{8_V}_{\text{bos}}\otimes\underbrace{8_V}_{\text{bos}}$ $m^2=0$	massless fermion: $\underbrace{(8_C\oplus 8_S)}_{\text{ferm}}\otimes\underbrace{8_V}_{\text{bos}}$ $m^2=0$	massless fermion: $\underbrace{8_V}_{\text{bos}}\otimes\underbrace{(8_C\oplus 8_S)}_{\text{ferm}}$ $m^2=0$	massless boson: $\underbrace{(8_C\oplus 8_S)}_{\text{ferm}}\otimes\underbrace{(8_C\oplus 8_S)}_{\text{ferm}}$ $m^2=0$

$$Z_{1\text{-loop}}=\int_{\mathcal{F}}\frac{d\tau d\bar{\tau}}{(\mathrm{Im}(\tau))^2}\mathrm{Tr}_{\perp}\left[(-1)^{\mathbb{F}}q^{L_0-\frac{c}{24}}\bar{q}^{\bar{L}_0-\frac{c}{24}}\right]$$

$$L_0-\frac{c}{24}=\left(L_0^X-\frac{8}{24}\right)+\left(L_0^\psi-\frac{4}{24}\right).$$

$$\text{NS)}\quad L_{0\mathrm{NS}}^{\psi}=N_{\mathrm{NS}}$$

$$\text{R)}\quad L_{0\mathrm{R}}^{\psi}=N_{\mathrm{R}}+\frac{8}{16}=N_{\mathrm{R}}+\frac{1}{2}$$

$$L_0|a\rangle_R=\frac{1}{2}|a\rangle_R$$

$$\begin{aligned}f(\tau,\bar{\tau})&=\mathrm{Tr}_{\perp}\left[(-1)^{\mathbb{F}}q^{L_0-\frac{c}{24}}\bar{q}^{\bar{L}_0-\frac{c}{24}}\right]\\
&=\mathrm{Tr}_X\left[q^{L_0-\frac{1}{3}}\bar{q}^{\bar{L}_0-\frac{1}{3}}\right]\left(\mathrm{Tr}_{\mathrm{NS}}\left[q^{N-\frac{1}{6}}\right]-\mathrm{Tr}_{\mathrm{R}}\left[q^{N+\frac{1}{3}}\right]\right)\left(\mathrm{Tr}_{\overline{\mathrm{NS}}}\left[\bar{q}^{\bar{N}-\frac{1}{6}}\right]-\mathrm{Tr}_{\overline{\mathrm{R}}}\left[\bar{q}^{\bar{N}+\frac{1}{3}}\right]\right)\\
&=\left(\frac{1}{\sqrt{\mathrm{Im}\tau}\,\eta(\tau)\eta(\bar{\tau})}\right)^8\left(\mathrm{Tr}_{\mathrm{NS}}\left[q^{N-\frac{1}{6}}\right]-\mathrm{Tr}_{\mathrm{R}}\left[q^{N+\frac{1}{3}}\right]\right)\left(\mathrm{Tr}_{\overline{\mathrm{NS}}}\left[\bar{q}^{\bar{N}-\frac{1}{6}}\right]-\mathrm{Tr}_{\overline{\mathrm{R}}}\left[\bar{q}^{\bar{N}+\frac{1}{3}}\right]\right)\end{aligned}$$

$$\eta(\tau)=q^{\frac{1}{24}}\prod_{n=1}^{\infty}(1-q^n),\ q=e^{2\pi i\tau}$$

$$\mathrm{Tr}_{\mathrm{NS}}\left[q^{N-\frac{1}{6}}\right]=q^{-\frac{1}{6}}\left(\prod_{r\geq\frac{1}{2}}(1+q^r)\right)^8=\left(\frac{\prod_{n=1}^{\infty}(1-q^n)\left(1+q^{n-\frac{1}{2}}\right)^2}{\eta(\tau)}\right)^4$$

$$\equiv\frac{\theta_3(\tau)^4}{\eta(\tau)^4}$$

$$\mathrm{Tr_R}\left[q^{N+\frac{1}{3}}\right] = q^{-\frac{1}{6}+\frac{1}{2}}\left(\prod_{n\geq 0}(1+q^n)\right)^8 = \left(\frac{q^{\frac{1}{8}}\prod_{n=1}^{\infty}(1-q^n)(1+q^n)^2(1+q^0)^2}{\eta(\tau)}\right)^4 \equiv \frac{\theta_2(\tau)^4}{\eta(\tau)^4}$$

$$\begin{aligned} \mathrm{NS})\ (-1)^F &= -e^{i\pi\sum_{r\geq\frac{1}{2}}\psi^i_{-r}\psi^i_r} \\ \mathrm{R})\ (-1)^F &= e^{i\pi\sum_{n\geq 0}\psi^i_{-n}\psi^i_n} \to \Gamma_9 e^{i\pi\sum_{n\geq 1}\psi^i_{-n}\psi^i_n} \end{aligned}$$

$$(-1)^F|0,P\rangle_{\mathrm{NS}} = -|0,P\rangle_{\mathrm{NS}}$$

$$(-1)^F\psi^i_{-\frac{1}{2}}|0,P\rangle_{\mathrm{NS}} = +\psi^i_{-\frac{1}{2}}|0,P\rangle_{\mathrm{NS}}$$

$$\begin{aligned} (-1)^F|\alpha,P\rangle_{\mathrm{R}} &= +|\alpha,P\rangle_{\mathrm{R}} \\ (-1)^F|\dot{\alpha},P\rangle_{\mathrm{R}} &= -|\dot{\alpha},P\rangle_{\mathrm{R}} \end{aligned}$$

$$\begin{aligned} \mathrm{Tr_{NS}}\left[(-1)^F q^{N-\frac{1}{6}}\right] &= -q^{-\frac{1}{6}}\left(\prod_{r\geq\frac{1}{2}}(1-q^r)\right)^8 = -\left(\frac{\prod_{n=1}^{\infty}(1-q^n)\left(1-q^{n-\frac{1}{2}}\right)^2}{\eta(\tau)}\right)^4 \\ &\equiv -\frac{\theta_4(\tau)^4}{\eta(\tau)^4} \end{aligned}$$

$$\begin{aligned} \mathrm{Tr_R}\left[(-1)^F q^{N+\frac{1}{3}}\right] &= q^{-\frac{1}{6}+\frac{1}{2}}\left(\prod_{n\geq 0}(1-q^n)\right)^8 = \left(\frac{q^{\frac{1}{8}}\prod_{n=1}^{\infty}(1-q^n)(1+q^n)^2(1-q^0)^2}{\eta(\tau)}\right)^4 \\ &\equiv \frac{\theta_1(\tau)^4}{\eta(\tau)^4} \equiv 0 \end{aligned}$$

$$\theta_1(\tau) \equiv 0$$

$$\begin{aligned} \theta_2(\tau+1) &= e^{i\frac{\pi}{4}}\theta_2(\tau) \\ \theta_3(\tau+1) &= \theta_4(\tau) \\ \theta_4(\tau+1) &= \theta_3(\tau) \\ \eta(\tau+1) &= e^{i\frac{\pi}{12}}\eta(\tau) \end{aligned}$$

$$\begin{aligned} \theta_2\left(-\frac{1}{\tau}\right) &= (-i\tau)^{\frac{1}{2}}\theta_4(\tau) \\ \theta_3\left(-\frac{1}{\tau}\right) &= (-i\tau)^{\frac{1}{2}}\theta_3(\tau) \\ \theta_4\left(-\frac{1}{\tau}\right) &= (-i\tau)^{\frac{1}{2}}\theta_2(\tau) \\ \eta\left(-\frac{1}{\tau}\right) &= (-i\tau)^{\frac{1}{2}}\eta(\tau) \end{aligned}$$

$$\mathrm{Tr}_{\psi}\left[(-1)^{\mathbb{F}} q^{L_0-\frac{c}{24}} \bar{q}^{\bar{L}_0-\frac{c}{24}}\right] = \left|\frac{\theta_3(\tau)^4 - \theta_2(\tau)^4}{\eta(\tau)^4}\right|^2$$

$$\mathcal{P}_{\pm}^{(\mathrm{R,NS})} = \frac{1}{2}\left(1 \pm (-1)^{F_{(\mathrm{R,NS})}}\right)$$

$$\mathcal{P}_{IIA} \equiv \left(\mathcal{P}_{+}^{(\mathrm{NS})} + \mathcal{P}_{+}^{(\mathrm{R})}\right)\overline{\left(\mathcal{P}_{+}^{(\mathrm{NS})} + \mathcal{P}_{-}^{(\mathrm{R})}\right)}$$
$$\mathcal{P}_{IIB} \equiv \left(\mathcal{P}_{+}^{(\mathrm{NS})} + \mathcal{P}_{+}^{(\mathrm{R})}\right)\overline{\left(\mathcal{P}_{+}^{(\mathrm{NS})} + \mathcal{P}_{+}^{(\mathrm{R})}\right)}$$

$$\mathrm{Tr}_{\psi}\left[\mathcal{P}_{IIA}(-1)^{\mathbb{F}} q^{L_0-\frac{c}{24}} \bar{q}^{\bar{L}_0-\frac{c}{24}}\right] = \mathrm{Tr}_{\psi}\left[\mathcal{P}_{IIB}(-1)^{\mathbb{F}} q^{L_0-\frac{c}{24}} \bar{q}^{\bar{L}_0-\frac{c}{24}}\right]$$
$$= \left|\frac{\theta_3(\tau)^4 - \theta_4(\tau)^4 - \theta_2(\tau)^4}{\eta(\tau)^4}\right|^2$$

$$\theta_3(\tau)^4 - \theta_4(\tau)^4 - \theta_2(\tau)^4 = 0$$

$$Z_{1-\mathrm{loop}}^{IIA,B} = \int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{(\mathrm{Im}(\tau))^2} \mathrm{Tr}_{\perp}\left[\mathcal{P}_{IIA,B}(-1)^{\mathbb{F}} q^{L_0-\frac{1}{2}} \bar{q}^{\bar{L}_0-\frac{1}{2}}\right]$$
$$= \int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{(\mathrm{Im}(\tau))^2}\left(\frac{1}{\sqrt{\mathrm{Im}\tau}\,\eta(\tau)\eta(\bar{\tau})}\right)^8 \left|\frac{\theta_3(\tau)^4 - \theta_4(\tau)^4 - \theta_2(\tau)^4}{\eta(\tau)^4}\right|^2 = 0$$

$$\mathcal{P}_{0A} \equiv \mathcal{P}_{+}^{(\mathrm{NS})}\overline{\mathcal{P}_{+}^{(\mathrm{NS})}} + \mathcal{P}_{-}^{(\mathrm{NS})}\overline{\mathcal{P}_{-}^{(\mathrm{NS})}} + \mathcal{P}_{+}^{(\mathrm{R})}\overline{\mathcal{P}_{-}^{(\mathrm{R})}} + \mathcal{P}_{-}^{(\mathrm{R})}\overline{\mathcal{P}_{+}^{(\mathrm{R})}}$$
$$\mathcal{P}_{0B} \equiv \mathcal{P}_{+}^{(\mathrm{NS})}\overline{\mathcal{P}_{+}^{(\mathrm{NS})}} + \mathcal{P}_{-}^{(\mathrm{NS})}\overline{\mathcal{P}_{-}^{(\mathrm{NS})}} + \mathcal{P}_{+}^{(\mathrm{R})}\overline{\mathcal{P}_{+}^{(\mathrm{R})}} + \mathcal{P}_{-}^{(\mathrm{R})}\overline{\mathcal{P}_{-}^{(\mathrm{R})}}$$

$$\mathrm{Tr}_{\psi}\left[\mathcal{P}_{0A} q^{L_0-\frac{c}{24}} \bar{q}^{\bar{L}_0-\frac{c}{24}}\right] = \mathrm{Tr}_{\psi}\left[\mathcal{P}_{0B} q^{L_0-\frac{c}{24}} \bar{q}^{\bar{L}_0-\frac{c}{24}}\right]$$
$$= \frac{|\theta_3(\tau)|^8 + |\theta_4(\tau)|^8 + |\theta_2(\tau)|^8}{|\eta(\tau)|^8}$$

$$Z_{1-\mathrm{loop}}^{0A,B} = \int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{(\mathrm{Im}(\tau))^2} \mathrm{Tr}_{\perp}\left[\mathcal{P}_{0A,B} q^{L_0-\frac{1}{2}} \bar{q}^{\bar{L}_0-\frac{1}{2}}\right]$$
$$= \int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{(\mathrm{Im}(\tau))^2}\left(\frac{1}{\sqrt{\mathrm{Im}\tau}\,\eta(\tau)\eta(\bar{\tau})}\right)^8 \frac{|\theta_3(\tau)|^8 + |\theta_4(\tau)|^8 + |\theta_2(\tau)|^8}{|\eta(\tau)|^8}$$

$$t^{ij} = \delta^{ij}\Phi + a^{[ij]} + s^{(ij)}$$

$$8_V \otimes 8_V = 1 \oplus 28_a \oplus 35_s,$$

$$8_V \otimes 8_S = 8_C \oplus 56_S,$$

$$\eta_\beta = \Gamma^i_{\beta\dot{\alpha}}\chi^i_{\dot{\alpha}};$$

$$\psi^i_{\dot{\beta}} = \chi^i_{\dot{\beta}} - \frac{1}{D-2}\Gamma^i_{\dot{\beta}\alpha}\eta_\alpha.$$

$$8_C \otimes 8_V = 8_S \oplus 56_C,$$

$$\eta_{\dot{\beta}} = \Gamma^i_{\dot{\beta}\alpha}\chi^i_{\alpha};$$

$$\psi^i_{\beta} = \chi^i_{\beta} - \frac{1}{D-2}\Gamma^i_{\beta\dot{\alpha}}\eta_{\dot{\alpha}}.$$

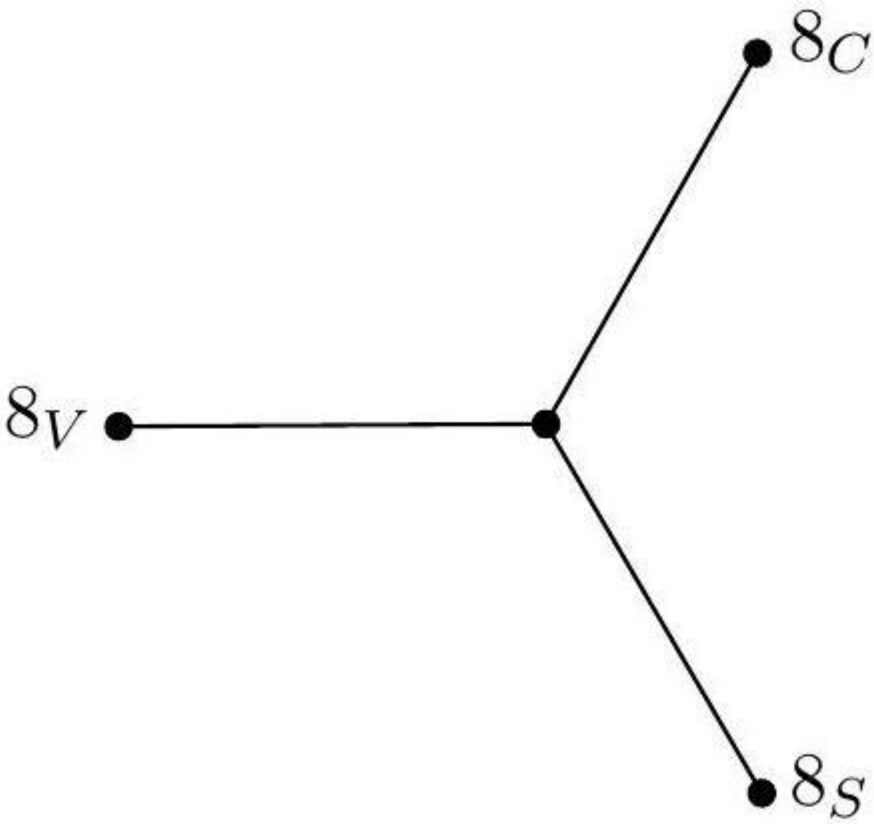
$$H_{\alpha\dot{\alpha}} = C_i\Gamma^i_{\alpha\dot{\alpha}} + C_{ijk}\Gamma^{ijk}_{\alpha\dot{\alpha}},$$

$$\Gamma^{i_1 \ldots i_n} = \frac{1}{n!}\Gamma^{[i_1} \cdots \Gamma^{i_n]}.$$

$$8_C \otimes 8_S = 8_V \oplus 56_a,$$

$$F_{\mu\nu} = \partial_{[\mu}C_{\nu]};$$

$$F_{\mu\nu\rho\sigma} = \partial_{[\mu}C_{\nu\rho\sigma]}.$$



$$S_{\text{IIA}} = \frac{1}{2K_{10}^2}\int d^{10}x\sqrt{-G}\left\{e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - \frac{1}{2}\left|H_{(3)}\right|^2\right) - \frac{1}{2}\left|F_{(2)}\right|^2 - \frac{1}{2}\left|F_{(4)}\right|^2\right\}$$
$$+ \frac{1}{4K_{10}^2}\int B_{(2)} \wedge dC_{(3)} \wedge dC_{(3)}$$
$$+(\mathfrak{F}_{\text{fermions couplings}})$$
$$+(\alpha'_{\text{corrections}})$$

$$S_{\text{IIB}} = \frac{1}{2K_{10}^2}\int d^{10}x\sqrt{-G}\Big\{e^{-2\Phi}\Big(R + 4(\partial\Phi)^2 - \frac{1}{2}|H_{(3)}|^2\Big)$$
$$-\frac{1}{2}|F_{(1)}|^2 - \frac{1}{2}|F_{(3)}|^2 - \frac{1}{2}|F_{(5)}|^2\Big\}$$
$$+\frac{1}{4K_{10}^2}\int C_{(4)} \wedge H_{(3)} \wedge F_{(3)}$$
$$+(\mathfrak{F}_{\text{fermions couplings}})$$
$$+(\alpha'_{\text{corrections}})$$

$$F_{(1)} = dC$$
$$F_{(3)} = dC_{(2)} - CdB_{(2)}$$
$$F_{(5)} = dC_{(4)} - \frac{1}{2}dC_{(2)} \wedge B_{(2)} + \frac{1}{2}B_{(2)} \wedge dC_{(2)}$$

$$H_{\alpha\dot{\alpha}} = C_i\Gamma^i_{\alpha\dot{\alpha}} + C_{i_1\dots i_3}\Gamma^{i_1\dots i_3}_{\alpha\dot{\alpha}} + C_{i_1\dots i_5}\Gamma^{i_1\dots i_5}_{\alpha\dot{\alpha}} + C_{i_1\dots i_7}\Gamma^{i_1\dots i_7}_{\alpha\dot{\alpha}}.$$

$$\Gamma^{i_1\dots i_n} \propto \varepsilon^{i_1\dots i_n j_1\dots j_{8-n}}\Gamma_{j_1\dots j_{8-n}}$$

$$C_{i_1\dots i_7} \propto \varepsilon_{i_1\dots i_7}{}^{j}C_j \implies C_{(7)} = \star_8 C_{(1)},$$
$$C_{i_1\dots i_3} \propto \varepsilon^{j_1\dots j_5}_{i_1\dots i_3}C_{j_1\dots j_5} \implies C_{(3)} = \star_8 C_{(5)},$$

$$\begin{array}{lll} \text{R)} & S_\alpha(0)|0\rangle & \text{with } \alpha \in 16_C, \\ \overline{\text{R}}) & S_{\dot{\alpha}}(0)|0\rangle & \text{with } \dot{\alpha} \in 16_S. \end{array}$$

$$\mathcal{C}^{-1}(\Gamma^\mu)^T\mathcal{C} = \Gamma^\mu$$

$$S_\alpha^\dagger\Gamma^0 = S_\alpha^T\mathcal{C}$$

$$F_{\mu_1\dots\mu_p}(P)\big((S(z))^T\mathcal{C}\Gamma^{\mu_1\dots\mu_p}\tilde{S}(\bar{z})\big).$$

$$(\Gamma^{11}S)_\alpha = +S_\alpha$$
$$(\Gamma^{11}\tilde{S})_{\dot{\alpha}} = -\tilde{S}_{\dot{\alpha}}$$

$$(S)^T\mathcal{C}\Gamma^{\mu_1\dots\mu_p}\tilde{S} = (S)^T(\Gamma^{11})^T\mathcal{C}\Gamma^{\mu_1\dots\mu_p}\tilde{S}$$

$$(S_\alpha)^T\mathcal{C}\Gamma^{\mu_1\dots\mu_p}\tilde{S}_{\dot{\beta}} = (S_\alpha)^T(\Gamma^{11})^T\mathcal{C}\Gamma^{\mu_1\dots\mu_p}\tilde{S}_{\dot{\beta}}$$
$$= (-1)^{p+1}(S_\alpha)^T\mathcal{C}\Gamma^{\mu_1\dots\mu_p}\Gamma^{11}\tilde{S}_{\dot{\beta}}$$
$$= (-1)^{p+1}(S_\alpha)^T\mathcal{C}\Gamma^{\mu_1\dots\mu_p}(-\tilde{S}_{\dot{\beta}})$$
$$= (-1)^p(S_\alpha)^T\mathcal{C}\Gamma^{\mu_1\dots\mu_p}\tilde{S}_{\dot{\beta}}$$

$$|\text{ state }\rangle = F_{\mu_1\dots\mu_p}(S_\alpha)^T\mathcal{C}\Gamma^{\mu_1\dots\mu_p}\tilde{S}_{\dot{\beta}}e^{iP\cdot X}(0,0)|0\rangle$$

$$G_0 |\text{ state }\rangle = 0 \implies P^\mu\psi_{0\mu} = 0,$$
$$\bar{G}_0 |\text{ state }\rangle = 0 \implies P^\mu\bar{\psi}_{0\mu} = 0,$$

$$
\begin{aligned}
G_0 \mid \text{ state } \rangle &= P_\mu F_{\mu_1 \ldots \mu_p} (S_\alpha)^T (\Gamma^\mu)^T \mathcal{C} \Gamma^{\mu_1 \ldots \mu_p} \tilde{S}_{\dot{\beta}}(0,0)|0\rangle \\
&= P_\mu F_{\mu_1 \ldots \mu_p} (S_\alpha)^T \mathcal{C} \Gamma^\mu \Gamma^{\mu_1 \ldots \mu_p} \tilde{S}_{\dot{\beta}}(0,0)|0\rangle \\
&= P_\mu F_{\mu_1 \ldots \mu_p} (S_\alpha)^T \mathcal{C} \left(\Gamma_\mu^{\mu_1 \ldots \mu_p} + p \delta_\mu^{[\mu_1} \Gamma^{\mu_2 \ldots \mu_p]} \right) \tilde{S}_{\dot{\beta}}(0,0)|0\rangle \\
&= 0
\end{aligned}
$$

$$
P_{[\mu} F_{\mu_1 \ldots \mu_p]} = 0 \Longrightarrow dF^{(p)} = 0
$$

$$
P_\mu F^\mu_{\mu_1 \ldots \mu_p} = 0 \Longrightarrow d \star_{10} F^{(p)} = 0
$$

$$
\begin{aligned}
F^{(6)} &= \star_{10} F^{(4)}, \\
F^{(8)} &= \star_{10} F^{(2)},
\end{aligned}
$$

$$
\left(\star_{10} F^{(p)} \right)_{\mu_1 \ldots \mu_{d-p}} = \sqrt{-g} \varepsilon_{\mu_1 \ldots \mu_{d-p} \nu_1 \ldots \nu_p} F^{\nu_1 \ldots \nu_p}.
$$

$$
\begin{aligned}
S &= \int d^d x \sqrt{-g} F_{\mu_1 \ldots \mu_p} F^{\mu_1 \ldots \mu_p} \\
&= \int d^d x \sqrt{-g} \left| F^{(p)} \right|^2 \\
&= \int_{\mathcal{M}_d} F^{(p)} \wedge \star F^{(p)}
\end{aligned}
$$

$$
F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}.
$$

$$
(\star F)_{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = \tilde{F}^{\mu\nu} = \begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & -E_3 & E_2 \\ -B_2 & E_3 & 0 & E_1 \\ -B_3 & -E_2 & E_1 & 0 \end{pmatrix}.
$$

$$
\begin{aligned}
\vec{E} &\longrightarrow \vec{B} \\
\vec{B} &\longrightarrow -\vec{E}
\end{aligned}
$$

$$
\partial_\mu F^{\mu\nu} = 0;
$$

$$
\partial_\mu \tilde{F}^{\mu\nu} = 0
$$

$$
\begin{aligned}
\partial_\mu F^{\mu\nu} &= j^\nu_{\text{electr.}} \\
\partial_\mu \tilde{F}^{\mu\nu} &= j^\nu_{\text{magn.}}
\end{aligned}
$$

$$
\begin{aligned}
F &\leftrightarrow \tilde{F} = \star F \\
j_{\text{electr.}} &\leftrightarrow j_{\text{magn.}} .
\end{aligned}
$$

$$
\int_{\mathcal{S}_2} \star F^{(2)} = \int_{\mathcal{S}_2} \vec{E} \cdot d\vec{s} = q_e
$$

$$
\int_{\mathcal{S}_2} F^{(2)} = \int_{\mathcal{S}_2} \vec{B} \cdot d\vec{s} = q_m
$$

$$\int_\gamma A^{(1)} = \int d\tau A_\mu \dot{x}^\mu$$

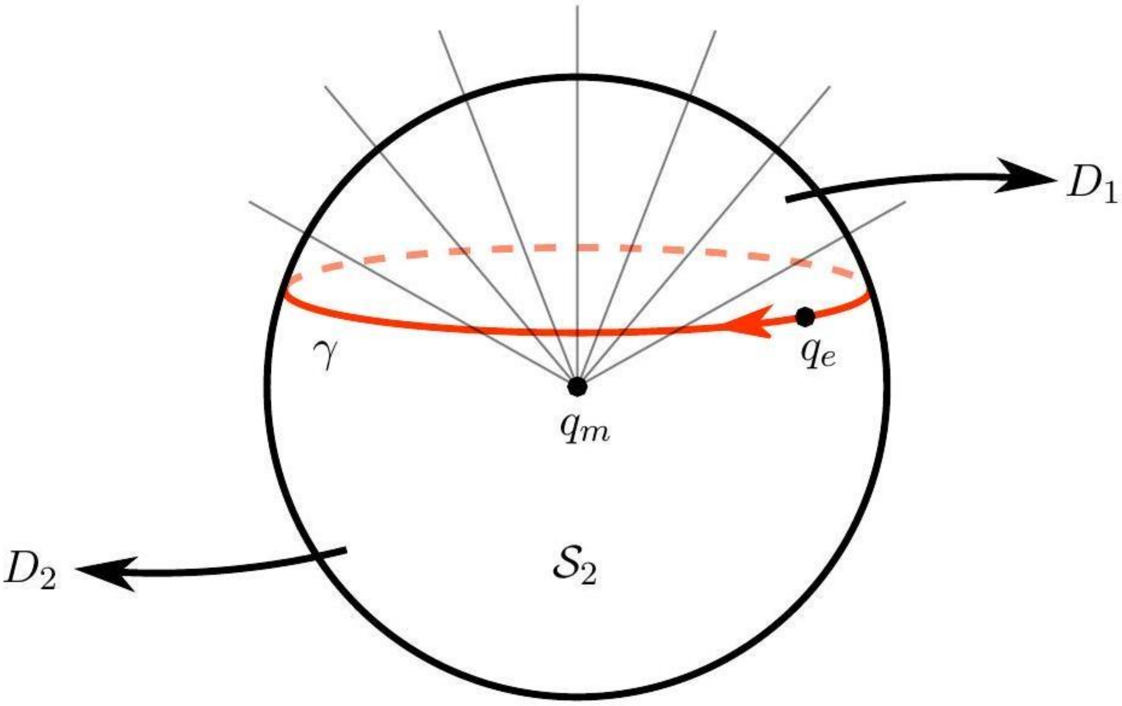
$$\Psi_{q_e}(\vec{x}) \longrightarrow e^{iq_e \int_\gamma A^{(1)}} \Psi_{q_e}(\vec{x}).$$

$$\gamma = \partial D_1 = -\partial D_2.$$

$$\int_\gamma A^{(1)} = \int_{\partial D_1} A^{(1)} = \int_{D_1} F^{(2)},$$

$$\int_\gamma A^{(1)} = \int_{-\partial D_2} A^{(1)} = -\int_{D_2} F^{(2)}$$

$$-\int_{D_2} F^{(2)} = -\int_{\mathcal{S}_2} F^{(2)} + \int_{D_1} F^{(2)}$$



$$e^{-iq_e \int_{S^2} F^{(2)}} = e^{-iq_e q_m} = 1,$$

$$q_e q_m = 2\pi n, \text{ with } n \in \mathbb{Z},$$

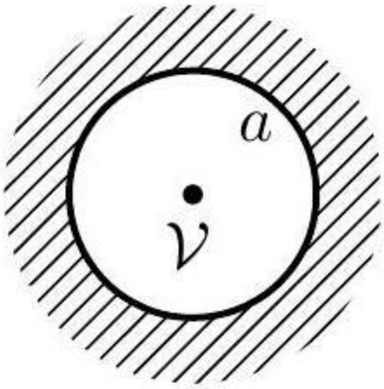
$$\hat{q}_e \hat{q}_m = 2\pi.$$

$$q \int_\gamma A^{(1)} = q \int d\tau A_\mu \dot{x}^\mu$$

$$\frac{1}{2\pi\alpha'} \int_{\text{WS}} B^{(2)} = \frac{1}{2\pi\alpha'} \int_{\text{WS}} d^2\sigma B_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu$$

$$\langle D(p) \mid \mathcal{V} \rangle = \langle \mathcal{V}(0,0) \rangle_{\text{disk}}^{D(p)\text{BC}}$$

$$\langle B_a|\mathcal{V}\rangle =$$

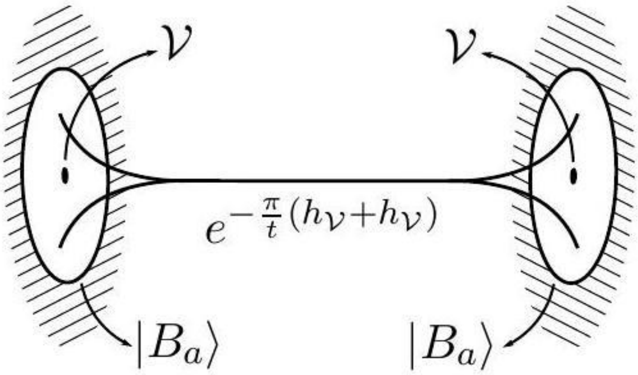


$$= \langle \mathcal{V}(0,\bar{0})\rangle_{\text{disk}}^{(a)}$$

$$\mathrm{Tr}\{e^{-2\pi t H}\} \overset{\text{Cardy}}{=} \langle B_a| e^{-2\frac{\pi}{t}(H+\bar{H})}|B_a\rangle = \langle B_a|\mathbb{1}e^{-2\frac{\pi}{t}(H+\bar{H})}\mathbb{1}|B_a\rangle$$

$$= \sum_{\mathcal{V},\mathcal{V}'} \langle B_a \mid \mathcal{V}\rangle\langle \mathcal{V}|e^{-2\frac{\pi}{t}(H+\bar{H})}|\mathcal{V}'\rangle\langle \mathcal{V}' \mid B_a\rangle = \sum_{\mathcal{V}} |\langle \mathcal{V} \mid B_a\rangle|^2 e^{-\frac{\pi}{t}(h_{\mathcal{V}}+h_{\mathcal{V}})}$$

$$\mathrm{Tr}\left\{e^{-2\pi t H}\right\} = \sum_{\mathcal{V}}$$



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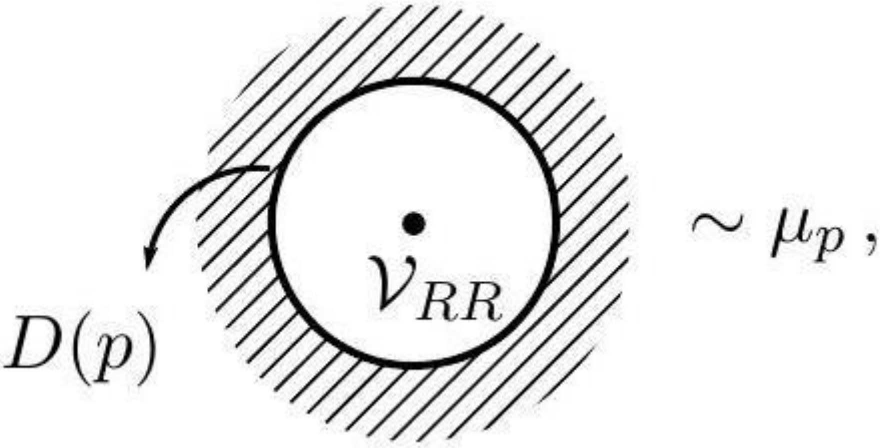
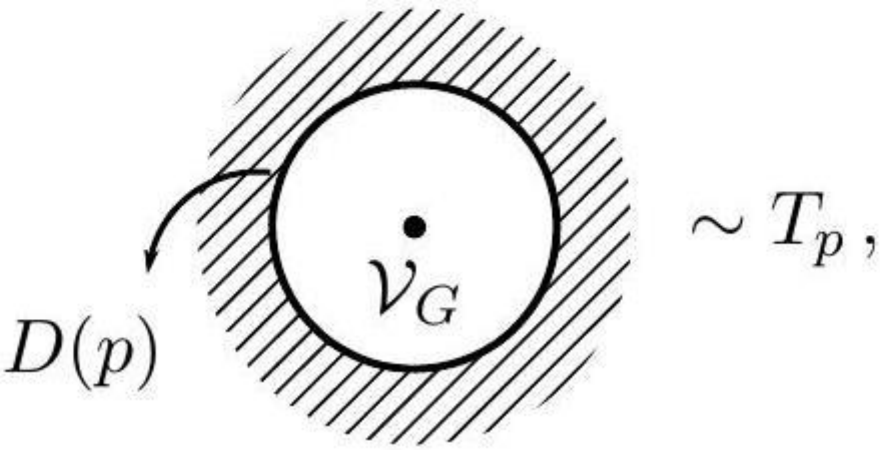
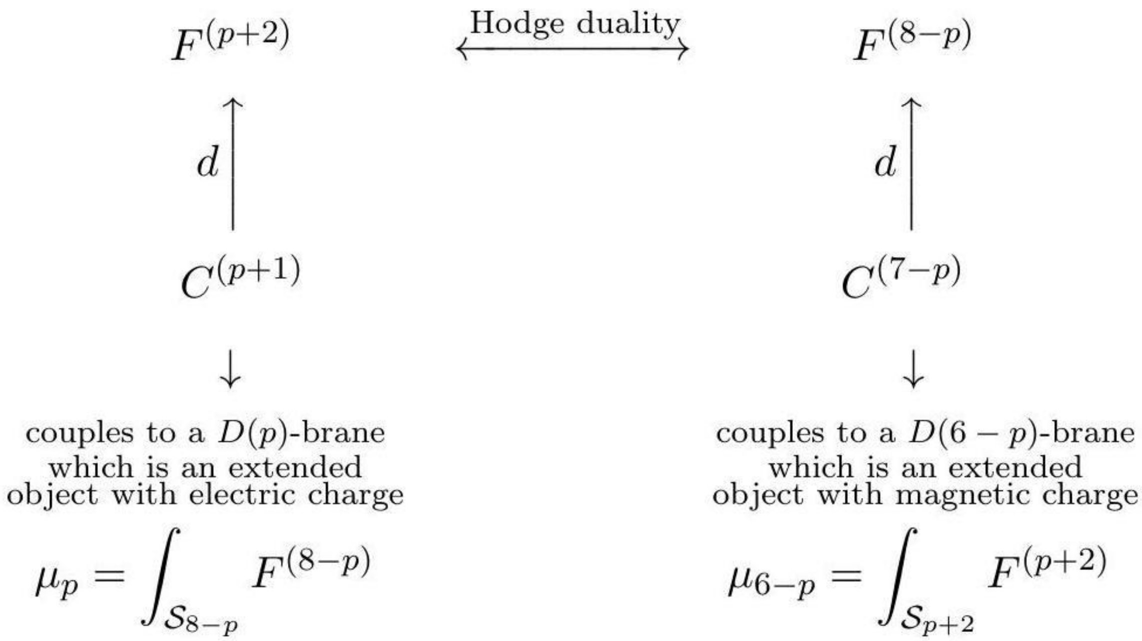




$$\langle D(p)| = T_p \langle \mathrm{NS}\overline{\mathrm{NS}}| + \mu_p \langle \mathrm{R}\overline{\mathrm{R}}|,$$

$$\mu_p = \int_{\mathcal{S}_{8-p}} \star F^{(p+2)} = \int_{\mathcal{S}_{8-p}} F^{(8-p)}$$



$$\mu_p \mu_{6-p} = 2\pi.$$

$$\begin{cases} D(0)\text{-brane:} & \langle D(0)| = T_0\langle \mathrm{NS}\overline{\mathrm{NS}}| + \mu_0 \langle \mathrm{R}\overline{\mathrm{R}}| \\ \text{anti- } D(0)\text{-brane:} & \langle \overline{D(0)}| = T_0\langle \mathrm{NS}\overline{\mathrm{NS}}| - \mu_0 \langle \mathrm{R}\overline{\mathrm{R}}| \end{cases}$$
$$\begin{cases} D(2)\text{-brane:} & \langle D(2)| = T_2\langle \mathrm{NS}\overline{\mathrm{NS}}| + \mu_2 \langle \mathrm{R}\overline{\mathrm{R}}| \\ \text{anti- } D(2)\text{-brane:} & \langle \overline{D(2)}| = T_2\langle \mathrm{NS}\overline{\mathrm{NS}}| - \mu_2 \langle \mathrm{R}\overline{\mathrm{R}}| \\ \vdots \end{cases}$$

$$\langle D(p) - \overline{D(p)}| = 2T_p \langle \mathrm{NS}\overline{\mathrm{NS}}|.$$

$$\langle D(1)| = T_1 \langle \mathrm{NS}\overline{\mathrm{NS}}|.$$

$$\begin{aligned} T(z) &= \bar{T}(\bar{z}) \\ \text{NS) } G(z) &= \bar{G}(\bar{z}) \\ \text{R) } G(z) &= \begin{cases} +\bar{G}(\bar{z}) & \text{for } z = \bar{z} < 0 \\ -\bar{G}(\bar{z}) & \text{for } z = \bar{z} > 0 \end{cases} \end{aligned}$$

$$T_{\mathbb{C}} = \begin{cases} T_{\mathrm{UHP}}(z) & \text{for } \mathrm{Im}(z) > 0 \\ \bar{T}_{\mathrm{UHP}}(z^*) & \text{for } \mathrm{Im}(z) < 0 \end{cases}$$

$$\epsilon = \begin{cases} +1 & \text{for NS} \\ -1 & \text{for R} \end{cases}.$$

$$G_{\mathbb{C}}^{(\epsilon)}(z) = \begin{cases} G_{\mathrm{UHP}}(z) & \text{for } \mathrm{Im}(z) > 0 \\ & \begin{cases} \bar{G}_{\mathrm{UHP}}(z^*) & \text{for } \mathrm{Im}(z) > 0, \mathrm{Re}(z) < 0 \\ \epsilon \bar{G}_{\mathrm{UHP}}(z^*) & \text{for } \mathrm{Im}(z) > 0, \mathrm{Re}(z) > 0 \end{cases} \end{cases}$$

$$G_{\mathbb{C}}^{(\epsilon)}\left(z e^{2i\pi}\right) = \epsilon G_{\mathbb{C}}^{(\epsilon)}(z),$$

$$j^{\mu}(z) = \Omega_{\mathrm{A}}^{(j)} \bar{j}^{\mu}(\bar{z}) \ \text{ for } z = \bar{z}$$

$$\Omega_{\mathrm{A}}^{(j)} = \begin{cases} +1 \text{ for A} = \mathfrak{N} \\ -1 \text{ for A} = \mathfrak{D} \end{cases}$$

$$\delta_{\mathrm{SUSY}}^{(\epsilon)} \psi^{\mu(\epsilon)}(z) = j^{\mu}(z)$$

$$\psi^{\mu(\epsilon)}(z) = \begin{cases} \Omega_{\mathrm{A}}^{(\psi)} \bar{\psi}^{\mu(\epsilon)}(\bar{z}) & \text{for } z = \bar{z} < 0 \\ \epsilon \Omega_{\mathrm{A}}^{(\psi)} \bar{\psi}^{\mu(\epsilon)}(\bar{z}) & \text{for } z = \bar{z} > 0 \end{cases}$$

$$\langle D(p_1)| e^{-\frac{\pi}{t}\left(L_0 + \bar{L}_0 - \frac{c}{12}\right)} |D(p_2)\rangle = \mathrm{Tr}_{\mathcal{H}_{12}} \left\{ e^{-2\pi t\left(L_0 - \frac{c}{24}\right)} \right\}$$

$$\begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$$

$$\begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \longrightarrow \begin{pmatrix} \mathrm{GSO}^{\pm}(+) & \mathrm{GSO}^{\pm}(+) \\ \mathrm{GSO}^{\pm}(+) & \mathrm{GSO}^{\pm}(+) \end{pmatrix}$$

$$\begin{aligned} &\int_0^{\infty} \frac{dt}{(2t)^2} \mathrm{Tr}^{\perp} \left[(-1)^{\mathbb{F}} \frac{1}{2} (1 + (-1)^F) e^{-2\pi t\left(L_0 - \frac{1}{2}\right)} \right] = \\ &= \int_0^{\infty} \frac{dt}{(2t)^2} \left(\frac{1}{2t}\right)^{\frac{p-1}{2}} e^{-\frac{t(\Delta y)^2}{2\pi\alpha'}} \left(\frac{1}{\eta(it)}\right)^8 \left(\frac{{\theta_3}^4 - {\theta_4}^4 - {\theta_2}^4}{\eta^4}\right)(it) \\ &= \frac{1}{4\pi} \int_0^{\infty} ds \left(\frac{\pi}{s}\right)^{\frac{d_{\perp}}{2}} e^{-\frac{(\Delta y)^2}{2\alpha' s}} \left(\frac{1}{\sqrt{2}\eta\left(\frac{is}{\pi}\right)}\right)^8 \left(\frac{{\theta_3}^4 - {\theta_4}^4 - {\theta_2}^4}{\eta^4}\right)\left(\frac{is}{\pi}\right) \\ &= \frac{1}{4\pi} \int_0^{\infty} ds \langle D(p)| e^{-s\left(L_0 + \bar{L}_0 - \frac{c}{12}\right)} |D(p)\rangle = 0 \end{aligned}$$

$$\left(\frac{1}{\eta\left(\frac{is}{\pi}\right)}\right)^8 \left(\frac{[{\theta_3}^4 - {\theta_4}^4]_{\mathrm{NSNS}} - [{\theta_2}^4]_{\mathrm{RR}}}{\eta^4}\right)\left(\frac{is}{\pi}\right) \underset{s\to\infty}{\sim} (16_{\mathrm{NSNS}} - 16_{\mathrm{RR}}) + (0)e^{-s} + \cdots$$

$$\Big(\underbrace{\overbrace{16}^{\mathrm{NS}} - \overbrace{16}^{\mathrm{R}}}_{\substack{=0 \\ \text{matching of bosonic} \\ \text{and fermionic d.o.f.}}} \Big) G(\Delta y) = 0,$$

$$\underbrace{\text{tension}}_{\substack{\text{produces gravitational attraction} \\ \text{between the two } D(p)\text{-branes}}} = \underbrace{\text{R-R charge}}_{\substack{\text{produces electric repulsion} \\ \text{between the two } D(p)\text{-branes}}} .$$

$$\begin{aligned} X^{\mu}, \psi^{\mu} &\longrightarrow \begin{cases} (\mathrm{NS}) & \mathrm{GSO}_{\mathrm{NS}}^{+} \\ (\mathrm{R}) & \mathrm{GSO}_{\mathrm{R}}^{+} \end{cases} \\ X^{i}, \psi^{i} &\longrightarrow \begin{cases} (\mathrm{NS}) & \mathrm{GSO}_{\mathrm{NS}}^{+} \\ (\mathrm{R}) & \mathrm{GSO}_{\mathrm{R}}^{+} \end{cases} \end{aligned}$$

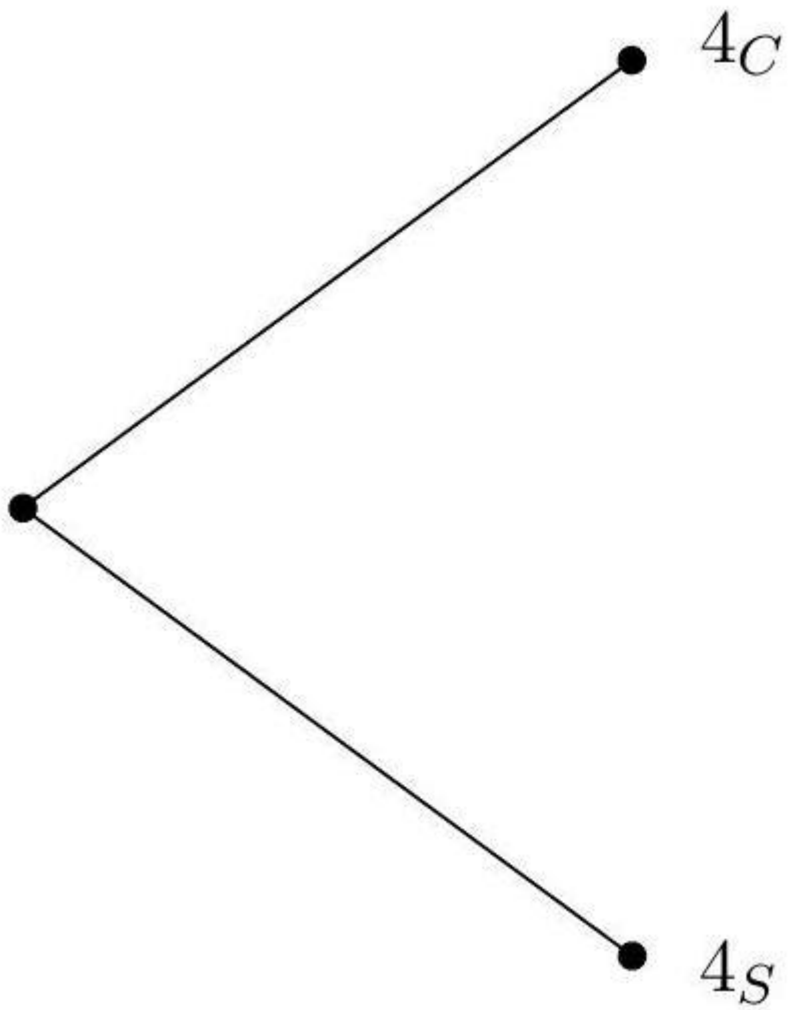
$$A_{\mu}(P) \psi_{\frac{1}{2}}^{\mu} |0, P\rangle_{\mathrm{NS}} + \lambda_i(P) \psi_{-\frac{1}{2}}^{\mu} |0, P\rangle_{\mathrm{NS}}$$

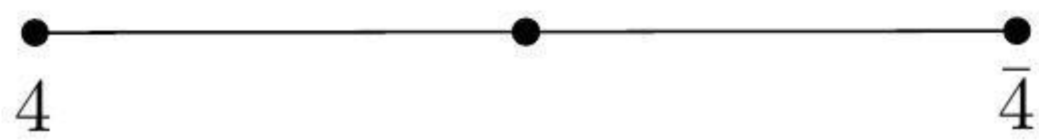
$$G_{\frac{1}{2}} \Longrightarrow P \cdot A = 0, L_0 \Longrightarrow P^2 = 0$$

$$\chi_\alpha(P)|\alpha, P\rangle_{\mathrm{R}} + \chi_{\dot\alpha}(P)|\dot\alpha, P\rangle_{\mathrm{R}},$$

$$\alpha = \left(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\right),$$

$$\alpha = (\underbrace{\pm\frac{1}{2}, \pm\frac{1}{2}}_{\alpha,\dot\alpha}, \underbrace{\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}}_{I,\bar{I}}),$$





$$S_{\mathcal{N}=4}^{\mathrm{SYM}} = \frac{1}{g_s}\int d^4x \mathrm{Tr}\{F_{\mu\nu}F^{\mu\nu} + D_\mu\lambda^i D^\mu\lambda_i + [\lambda_i, \lambda_j][\lambda^i, \lambda^j]$$
$$+\bar\chi^{\bar I}\not{\phi}^I + \bar\chi^I[\chi^{\bar I}, \lambda_i]\gamma_i^{I,\bar I}\} + (\alpha' \text{ corrections })$$

$$\frac{1}{4\pi}\int_0^\infty ds\langle\bar{D}(p)|e^{-s\left(L_0+\bar{L}_0-\frac{c}{12}\right)}|D(p)\rangle =$$

$$= \frac{1}{4\pi}\int_0^\infty ds\left(\frac{\pi}{s}\right)^{\frac{d_\perp}{2}} e^{-\frac{(\Delta y)^2}{2\alpha' s}}\left(\frac{1}{\sqrt{2}\eta\left(\frac{is}{\pi}\right)}\right)^8\left(\frac{\theta_3{}^4-\theta_4{}^4+\theta_2{}^4}{\eta^4}\right)\left(\frac{is}{\pi}\right)$$

$$= \int_0^\infty \frac{dt}{(2t)^2}\left(\frac{1}{2t}\right)^{\frac{p-1}{2}} e^{-\frac{t(\Delta y)^2}{2\pi\alpha'}}\left(\frac{1}{\eta(it)}\right)^8\left(\frac{\theta_3{}^4+\theta_4{}^4-\theta_2{}^4}{\eta^4}\right)(it)$$

$$\theta_2/\eta \leftrightarrow \theta_4/\eta.$$

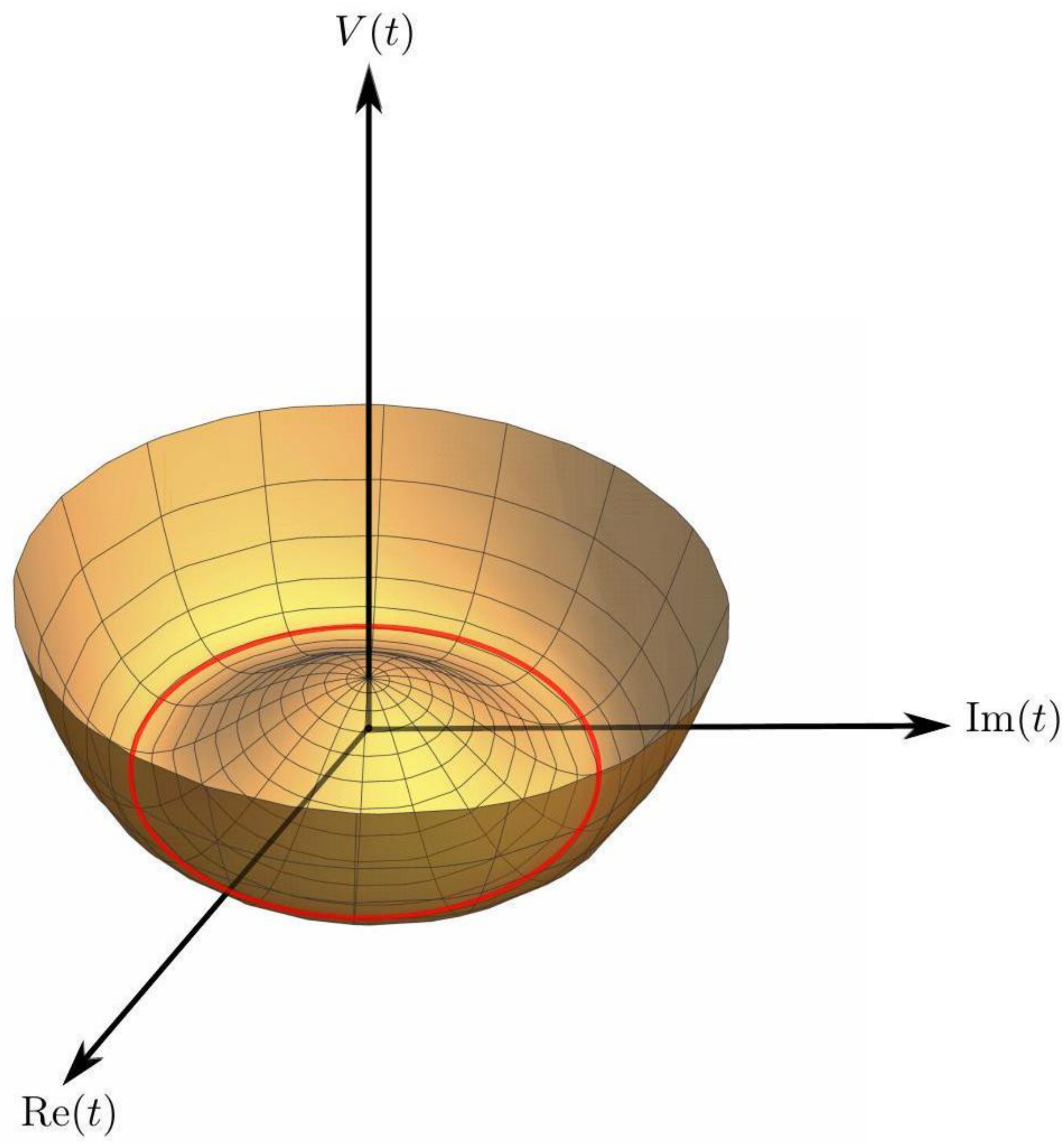
$$\begin{pmatrix}\phi_{11} & \phi_{12}\\ \phi_{21} & \phi_{22}\end{pmatrix} \longrightarrow \begin{pmatrix}\mathrm{GSO}^+(+) & \mathrm{GSO}^+(-)\\ \mathrm{GSO}^-(-) & \mathrm{GSO}^-(+)\end{pmatrix}$$

$$|D(p)-\overline{D(p)}\rangle = 2T_p|\mathrm{NS}-\overline{\mathrm{NS}}\rangle$$

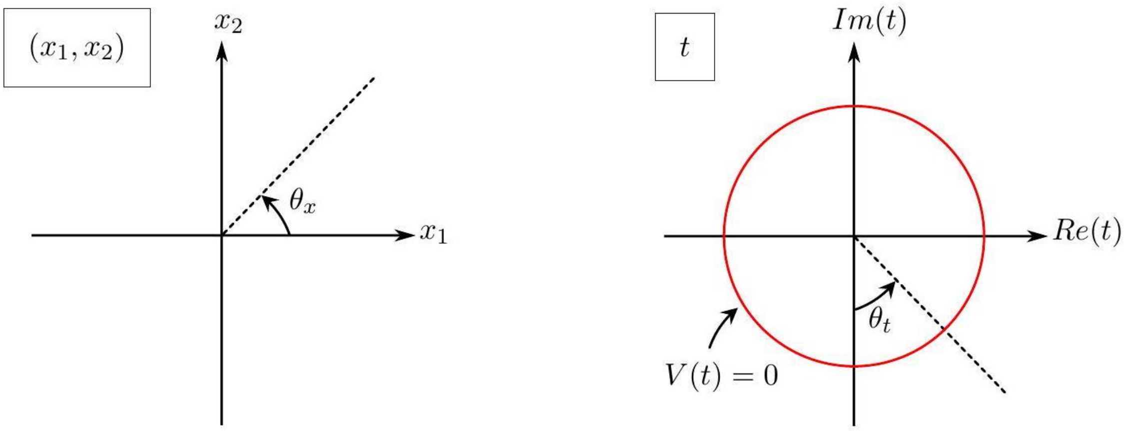
$$\Box\, t - V'(t) = 0$$

$$t = 0,$$

$$t = t_* e^{i\theta}$$



$$\lim_{\vec{x}\to\infty} t(\vec{x}) = \lim_{|x|\to\infty} t\left(|x|e^{i\theta_x}\right) = t_* e^{i\theta_t},$$



$$m: \mathcal{S}_1^{(x)} \longrightarrow \mathcal{S}_1^{(t)},$$

$$|\alpha\rangle_{\mathrm{R}} = S_\alpha(0)|0\rangle$$
$$|\dot{\alpha}\rangle_{\mathrm{R}} = S_{\dot{\alpha}}(0)|0\rangle$$

$$\psi_I^+(z_1)\psi_J^-(z_2) = \frac{\delta_{IJ}}{(z_1 - z_2)}$$
$$\psi_I^\pm(z_1)\psi_J^\pm(z_2) = 0$$

$$H_I(z_1)H_J(z_2) = -\delta_{IJ}\log(z_1 - z_2)$$

$$T_H(z) = \frac{1}{2} : \partial H \cdot \partial H : (z).$$

$$e^{+iH_I}(z_1)e^{-iH_J}(z_2) = \frac{\delta_{IJ}}{(z_1 - z_2)},$$
$$e^{\pm iH_I}(z_1)e^{\pm iH_J}(z_2) = 0.$$

$$\psi_I^\pm(z) = e^{\pm iH_I}(z)$$

$$S_\alpha(z) = e^{i\sum_I s_I H_I}(z)$$

$$s_I = \left\{\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\right\}$$

$$h(S_\alpha) = h(S_{\dot{\alpha}}) = \frac{5}{8}$$

$$(\eta, \xi) \oplus \varphi$$

$$\beta(z) = e^{-\varphi}\partial\xi(z)$$
$$\gamma(z) = \eta e^{\varphi}(z)$$

$$h(\eta) = 1, h(\xi) = 0$$

$$\xi(z_1)\eta(z_2) = \frac{1}{(z_1 - z_2)}.$$

$$T^{(\eta,\xi)}(z) =: (\partial\eta)\xi : (z) - \partial(:\eta\xi:)(z)$$

$$\langle 0|\xi_0|0\rangle = 1$$

$$\varphi(z_1)\varphi(z_2) = -\log(z_1 - z_2).$$

$$T^{\varphi}(z) = -\frac{1}{2}(\partial\varphi)(\partial\varphi) - \partial^2\varphi$$

$$h(e^{q\varphi}) = -\frac{q}{2}(q + 2)$$

$$e^{q\varphi} = \begin{cases} \mathbb{1} \longrightarrow q = 0 \\ e^{-2\varphi} \longrightarrow q = -2 \end{cases}$$

$$\langle \mathbb{1} \rangle = 0, \langle e^{-2\varphi} \rangle = 1 \neq 0$$

$$Q_\varphi = \oint_0 \frac{dz}{2\pi i}(-\partial\varphi) = \oint_0 \frac{dz}{2\pi i} j_\varphi(z)$$

$$[Q_\varphi, e^{q\varphi}] = q e^{q\varphi}$$

$$T_\varphi(z_1) j_\varphi(z_2) = -\frac{2}{(z_1 - z_2)^3} + \frac{j_\varphi(z_2)}{(z_1 - z_2)^2} + \frac{\partial j_\varphi(z_2)}{(z_1 - z_2)} + (\mathfrak{S})$$

$$Q_\varphi |0\rangle = 0 \overset{\mathcal{I}(z) = -\frac{1}{z}}{\rightarrow} \langle 0 | (Q_\varphi + 2) = 0,$$

$$\left\langle \prod_i e^{q_i\varphi}(z_1) \right\rangle \neq 0 \Leftrightarrow \sum_i q_i = -2.$$

$$|0\rangle_{\text{NS}} = e^{-\varphi}(0)|0\rangle, |\overset{(\cdot)}{\alpha}\rangle_{\text{R}} = S_{(\cdot)} e^{-\frac{\varphi}{2}}(0)|0\rangle$$

$$j_p(z) =: \xi\eta : (z) - \partial\varphi(z) =: \xi\eta : (z) + j_\varphi(z)$$

Field	p
β	0
γ	0
η	-1
ξ	1
$e^{-q\varphi}$	$-q$
$\|0\rangle_{\text{NS}}$	-1
$\|\alpha\rangle_{\text{R}}$	$-\frac{1}{2}$

$$\left\langle \xi e^{-2\varphi} \frac{1}{2} (\partial^2 c)(\partial c) c(z) \right\rangle_{\text{LHS}}^{\text{chiral}} = 1$$

$$\xi(z) = \sum_{n=0}^{\infty} \xi_n z^{-n} = \xi_0 + \sum_{n=1}^{\infty} \xi_n z^{-n}$$

$$\langle \ldots \rangle_{\mathrm{SHS}} = \langle \xi_0(\ldots) \rangle_{\mathrm{LHS}}$$

$$\left\langle e^{-2\varphi} \frac{1}{2} (\partial^2 c)(\partial c) c(z) \right\rangle_{\mathrm{SHS}}^{\mathrm{chiral}} = 1$$

$$\xi_0 = \oint{}_0 \frac{dz}{2\pi i} \frac{1}{z} \xi(z), \eta_0 = \oint{}_0 \frac{dz}{2\pi i} \eta(z)$$

$$[\eta_0, \xi_0] = 1$$

$$\frac{\partial}{\partial \xi_0} = \eta_0 = 0$$

$$\mathrm{SHS} = \mathcal{H}_S = \ker(\eta_0).$$

$$Q_B = \oint{}_0 \frac{dz}{2\pi i} j_{\mathrm{B}}(z),$$

$$Q_B^2 = 0, \eta_0^2 = 0$$

$$[Q_B, \eta_0] = 0.$$

$$[\eta_0, \xi(z)] = 1, [Q_B, -c\xi\partial\xi e^{-2\varphi}(z)] = 1.$$

$$\mathcal{X}(z) = [Q_B, \xi(z)]$$
$$\mathcal{Y}(z) = [\eta_0, -c\xi\partial\xi e^{-2\varphi}(z)] = c\partial\xi e^{-2\varphi}(z)$$

$$\begin{cases} [Q_B, \mathcal{X}(z)] = 0 \\ [\eta_0, \mathcal{X}(z)] = 0 \end{cases}, \begin{cases} [Q_B, \mathcal{Y}(z)] = 0 \\ [\eta_0, \mathcal{Y}(z)] = 0 \end{cases},$$

$$\lim_{z \to 0} \mathcal{X}(z)\mathcal{Y}(0) = 1$$

$$\mathcal{X}_0 = \oint{}_0 \frac{dz}{2\pi i} \frac{1}{z} \mathcal{X}$$

$$\mathcal{X}_0 \phi_p = \phi_{p+1}, Q_B \phi_{p+1} = 0$$

$$\ldots \underset{\mathcal{Y}}{\overset{\mathcal{X}}{\rightleftarrows}} \phi_{p-1} \underset{\mathcal{Y}}{\overset{\mathcal{X}}{\rightleftarrows}} \phi_p \underset{\mathcal{Y}}{\overset{\mathcal{X}}{\rightleftarrows}} \phi_{p+1} \underset{\mathcal{Y}}{\overset{\mathcal{X}}{\rightleftarrows}} \ldots.$$

$$\psi_I^{\pm}(z) = e^{\pm i H_I}(z)$$
$$\beta(z) = e^{-\varphi} \partial\xi(z)$$
$$\gamma(z) = \eta e^{\varphi}(z)$$

$$|\ \text{NS state}\ \rangle = e^{i v_I H_I + v_6 \varphi}, \text{ with } v_I, v_6 \in \mathbb{Z},$$
$$|\ \text{R state}\ \rangle = e^{i s_I H_I + s_6 \varphi}, \text{ with } s_I, s_6 \in \mathbb{Z} + \frac{1}{2}.$$

integer + integer = integer
integer + half-integer = half-integer
half-integer + half-integer = integer

$$| \text{ NS state } \rangle \cdot | \text{ NS state } \rangle \approx | \text{ NS state } \rangle$$
$$| \text{ NS state } \rangle \cdot | \text{ R state } \rangle \approx | \text{ R state } \rangle$$
$$| \text{ R state } \rangle \cdot | \text{ R state } \rangle \approx | \text{ NS state } \rangle$$

$$\text{NS} \longrightarrow \begin{cases} \text{GSO}_{\text{NS}}^{+} \quad \text{where} \sum_{I} v_I + v_6 \text{ is even} \\ \text{GSO}_{\text{NS}}^{-} \quad \text{where} \sum_{I} v_I + v_6 \text{ is odd} \end{cases}$$
$$\text{R} \longrightarrow \begin{cases} \text{GSO}_{\text{R}}^{+} \quad \text{where} \sum_{I} s_I + s_6 \text{ is even} \\ \text{GSO}_{\text{R}}^{-} \quad \text{where} \sum_{I} s_I + s_6 \text{ is odd} \end{cases}$$

$$\text{GSO}_{\text{NS}}^{+} \cdot \text{GSO}_{\text{NS}}^{+} \approx \text{GSO}_{\text{NS}}^{+}$$

$$e^{ip_I H_I + p_6 \varphi}(z_1) e^{iq_I H_I + q_6 \varphi}(z_2) = (z_1 - z_2)^{-p_I q_I + p_6 q_6} e^{i(p_I + q_I)H_I + (p_6 + q_6)\varphi}(z_2).$$

$$-p_I q_I + p_6 q_6 \in \mathbb{Z}$$

$$\text{GSO}_{\text{NS}}^{-}(z_1) \cdot \text{GSO}_{\text{R}}^{\pm}(z_2) \approx \frac{1}{\sqrt{z_1 - z_2}}$$
$$\text{GSO}_{\text{R}}^{-}(z_1) \cdot \text{GSO}_{\text{R}}^{+}(z_2) \approx \frac{1}{\sqrt{z_1 - z_2}}$$

$$\text{GSO}_{\text{NS}}^{+} \oplus \text{GSO}_{\text{R}}^{+}, \text{GSO}_{\text{NS}}^{+} \oplus \text{GSO}_{\text{R}}^{-}.$$

$$Q_\alpha(z) = S_\alpha e^{-\frac{\varphi}{2}}(z)$$
$$\bar{Q}_{\dot{\alpha}}(z) = \bar{S}_{\dot{\alpha}} e^{-\frac{\bar{\varphi}}{2}}(\bar{z})$$

$$S_\alpha(z_1) S_\beta(z_2) = \frac{1}{\sqrt{2}} \frac{(\Gamma_\mu)_{\alpha\beta}}{(z_1 - z_2)^{\frac{3}{4}}} \psi^\mu(z_2)$$
$$S_\alpha(z_1) S_{\dot{\beta}}(z_2) = \frac{C_{\alpha\dot{\beta}}}{(z_1 - z_2)^{\frac{5}{4}}} + \frac{1}{2} \frac{\Gamma_\mu \Gamma_\nu}{(z_1 - z_2)^{\frac{1}{4}}} : \psi^\mu \psi^\nu : (z_2)$$
$$e^{-\frac{\varphi}{2}}(z_1) e^{-\frac{\varphi}{2}}(z_2) = \frac{1}{(z_1 - z_2)^{\frac{1}{4}}} e^{-\varphi}(z_2)$$

$$\underbrace{S_\alpha(z_1)S_\beta(z_2)} = \frac{1}{\sqrt{2}}\frac{(\Gamma_\mu)_{\alpha\beta}}{(z_1-z_2)^{\frac{3}{4}}}\psi^\mu(z_2)\,,$$

$$\underbrace{S_\alpha(z_1)S_{\dot\beta}(z_2)} = \frac{C_{\alpha\dot\beta}}{(z_1-z_2)^{\frac{5}{4}}} + \frac{1}{2}\frac{\Gamma_\mu\Gamma_\nu}{(z_1-z_2)^{\frac{1}{4}}}:\psi^\mu\psi^\nu:(z_2)$$

$$\underbrace{e^{-\frac{\varphi}{2}}(z_1)e^{-\frac{\varphi}{2}}(z_2)} = \frac{1}{(z_1-z_2)^{\frac{1}{4}}}e^{-\varphi}(z_2)\,,$$

$$\underbrace{Q_\alpha(z_1)Q_\beta(z_2)} = \frac{1}{\sqrt{2}}\frac{(\Gamma_\mu)_{\alpha\beta}}{(z_1-z_2)}\psi^\mu e^{-\varphi}(z_2)$$

$$\underbrace{Q_\alpha(z_1)Q_{\dot\beta}(z_2)} = \frac{C_{\alpha\dot\beta}}{(z_1-z_2)^{\frac{3}{2}}}e^{-\varphi}(z_2) + \frac{1}{2}\frac{\Gamma_\mu\Gamma_\nu}{(z_1-z_2)^{\frac{1}{2}}}:\psi^\mu\psi^\nu:(z_2)$$

$$\underbrace{Q_\alpha(z_1)Q_\beta(z_2)} = \frac{1}{\sqrt{2}}\frac{(\Gamma_M)_{\alpha\beta}}{(z_1-z_2)}\psi^M e^{-\varphi}(z_2)$$

$$\underbrace{\mathscr{P}_\alpha{}^\gamma\bar{Q}_\gamma(\bar{z}_1)\mathscr{P}_\beta{}^\delta\bar{Q}_\delta(\bar{z}_2)} = \frac{1}{\sqrt{2}}\frac{(\Gamma_M)_{\alpha\beta}}{(z_1-z_2)}\Omega^{(\psi)}{}^M{}_N\,\bar{\psi}^N e^{-\bar{\varphi}}$$

$$Q_\alpha(z_1)Q_\beta(z_2) = \frac{1}{\sqrt{2}}\frac{(\Gamma_\mu)_{\alpha\beta}}{(z_1-z_2)}\psi^\mu e^{-\varphi}(z_2),$$

$$\mathcal{X}_0(\psi^\mu e^{-\varphi}) = j^\mu.$$

$$Q_\alpha(z_1)Q_{\dot\beta}(z_2) = \frac{C_{\alpha\dot\beta}}{(z_1-z_2)^{\frac{3}{2}}}e^{-\varphi}(z_2) + \frac{1}{2}\frac{\Gamma_\mu\Gamma_\nu}{(z_1-z_2)^{\frac{1}{2}}}:\psi^\mu\psi^\nu:(z_2).$$

$$Q_\alpha(z) = \mathcal{P}_\alpha^\beta\bar{Q}_\beta(\bar{z}) \text{ for } z=\bar{z}$$

$$Q_\alpha(z_1)Q_\beta(z_2) = \frac{1}{\sqrt{2}}\frac{(\Gamma_M)_{\alpha\beta}}{(z_1-z_2)}\psi^M e^{-\varphi}(z_2).$$

$$\mathcal{P}_\alpha^\gamma \bar{Q}_\gamma(\bar{z}_1)\mathcal{P}_\beta^\delta \bar{Q}_\delta(\bar{z}_2) = \frac{1}{\sqrt{2}}\frac{(\Gamma_M)_{\alpha\beta}}{(z_1 - z_2)}\Omega_N^{(\psi)}\,{}_N^M\bar{\psi}^N e^{-\bar{\varphi}}.$$

$$\Omega_N^M = \begin{pmatrix} \delta_\nu^\mu & 0 \\ 0 & -\delta_j^i \end{pmatrix} \leftrightarrow \Omega_A = \begin{cases} +1 \text{ for A} = \mathcal{N} \\ -1 \text{ for A} = \mathcal{D} \end{cases}$$

$$\begin{cases} j^M(z) = \Omega_N^M \bar{J}^N(\bar{z}) & \text{for } z = \bar{z} \\ \psi^M(z) = \Omega_N^M \bar{\psi}^N(\bar{z}) & \text{for } z = \bar{z} \end{cases}$$

$$\mathcal{P}_\alpha{}^\gamma \mathcal{P}_\beta{}^\delta (\Gamma_N)_{\gamma\delta} = (\Gamma_M)_{\alpha\beta}\Omega_M^N,$$

$$\mathcal{P}\Gamma_N \mathcal{C}^{-1}\mathcal{P}^T = \Omega_N^M \Gamma_M \mathcal{C}^{-1},$$

$$\mathcal{P}_p^\pm = \pm \prod_{i=p+2}^{9} (\Gamma_i \Gamma),$$

$$Q_\alpha(z) = \mathcal{P}_\alpha^{\dot{\beta}} Q_{\dot{\beta}}(\bar{z}) \;\text{ for } z = \bar{z}$$

$$Q(z) = \mathcal{P}\bar{Q}(\bar{z}) \;\text{ for } z = \bar{z}$$

$$\mathcal{P}_p^\pm = \pm \prod_{i=p+2}^{9} (\Gamma_i \Gamma)$$

$$\int_{\mathcal{M}_{p+1}} C^{(p+1)} \longrightarrow \begin{cases} > 0 & \Rightarrow \quad \text{positive orientation} \\ < 0 & \Rightarrow \quad \text{negative orientation} \end{cases} \Rightarrow \overline{D(p)\text{-brane}}$$

$$Q + \mathcal{P}_q \bar{Q} = Q + (\mathcal{P}_p \mathcal{P}_p^{-1})\mathcal{P}_q \bar{Q} = Q + \mathcal{P}_p \left(\prod_{i \in \mathrm{ND}} \Gamma_i \right) \bar{Q}.$$

$$\mathcal{P}_p^{-1}\mathcal{P}_q \bar{Q} = \bar{Q},$$

$$p - q = 4n, \;\text{ with } n \in \mathbb{N}.$$

$$g_s^{\text{heterotic}} = \frac{1}{g_s^{\text{type I}}}$$

$$\mathcal{M}^{1,d} = \mathbb{R}^{1,d-1} \times \mathcal{S}_R^1,$$

$$y \sim y + 2\pi R.$$

$$\Box\, \phi(x^\mu, y) = 0,$$

$$\phi(x^\mu, y) = \sum_{n \in \mathbb{Z}} \phi_n(x^\mu) e^{in\frac{y}{R}},$$

$$\phi_{-n}^*(x^\mu) = \phi_n(x^\mu).$$

$$\Box\,\phi(x^\mu,y)=\sum_{n\in\mathbb{Z}}\left(\Box_x-\frac{n^2}{R^2}\right)\phi_n(x^\mu)e^{in\frac{y}{R}}=0.$$

$$\left(\Box_x-\frac{n^2}{R^2}\right)\phi_n(x^\mu)=0.$$

$$m_n^2=\frac{n^2}{R^2}$$

$$A_\mu(x,y)=A_\mu^0(x)+\sum_{n\neq 0}A_\mu^n(x)e^{iny/R}$$

$$A_y(x,y)=A_y^0(x)+\sum_{n\neq 0}A_y^n(x)e^{iny/R}$$

$$-\frac{1}{4}\int d^dxdyF_{MN}F^{MN}=-\frac{1}{4}\int d^dx\left(F_{\mu\nu}F^{\mu\nu}+\partial_\mu\phi\partial^\mu\phi+\mathcal{K}_{\text{fields}}\right)$$

$$g_{MN}=\begin{pmatrix} g_{\mu\nu}-e^{2\phi}A_\mu A_\nu & e^{2\phi}A_\mu \\ e^{2\phi}A_\mu & e^{2\phi}. \end{pmatrix}$$

$$\int d^dxdyR^{(d+1)}=\int d^dx\left(R^{(d)}-\frac{1}{4}e^{2\phi}F_{\mu\nu}F^{\mu\nu}-2e^{-\phi}\,\Box\,e^{\phi}+\mathcal{K}_{\text{fields}}\right).$$

$$Y^{(R)}(w,\bar{w})=Y(w,\bar{w})+A\sigma,$$

$$\sigma\to\sigma+2\pi\implies y\to y+2\pi R\omega,$$

$$A=R\omega$$

$$z=e^w, w=t+i\sigma$$

$$Y(z,\bar{z})=Y_0-\frac{i}{2}\alpha'(P_y+R\omega/\alpha')\log z-\frac{i}{2}\alpha'(P_y-R\omega/\alpha')\log\bar{z}+(\mathcal{O}_{\text{oscillators}})$$

$$P_y=\frac{n}{R},\ \text{with } n\in\mathbb{Z}$$

$$Y(z,\bar{z})=Y_0-\frac{i}{2}\alpha'\left(\frac{n}{R}+\frac{R\omega}{\alpha'}\right)\log z-\frac{i}{2}\alpha'\left(\frac{n}{R}-\frac{R\omega}{\alpha'}\right)\log\bar{z}+(\mathcal{O}_{\text{oscillators}})$$

$$Y(z,\bar{z})=Y_L(z)+Y_R(\bar{z})$$

$$\begin{cases} Y_L(z)=\dfrac{Y_0-c}{2}-\dfrac{i}{2}\left(\alpha'\dfrac{n}{R}+R\omega\right)\log z+i\sqrt{\dfrac{\alpha'}{2}}\displaystyle\sum_{m\neq 0}\dfrac{\alpha_m^y}{m}z^{-m} \\ Y_R(\bar{z})=\dfrac{Y_0+c}{2}-\dfrac{i}{2}\left(\alpha'\dfrac{n}{R}-R\omega\right)\log\bar{z}+i\sqrt{\dfrac{\alpha'}{2}}\displaystyle\sum_{m\neq 0}\dfrac{\tilde{\alpha}_m^y}{m}\bar{z}^{-m} \end{cases}$$

$$\begin{cases} j(z) = i\sqrt{\frac{2}{\alpha'}}\partial Y_L(z) = \sum_{m\in\mathbb{Z}} \alpha_m^y z^{-m-1} \\ \bar{J}(\bar{z}) = i\sqrt{\frac{2}{\alpha'}}\bar{\partial} Y_R(\bar{z}) = \sum_{m\in\mathbb{Z}} \tilde{\alpha}_m^y \bar{z}^{-m-1} \end{cases}$$

$$\begin{cases} \alpha_0^y = \oint_0 \frac{dz}{2\pi i} j(z) = \sqrt{\frac{\alpha'}{2}}\left(\frac{n}{R} + \frac{R\omega}{\alpha'}\right) \equiv \sqrt{\frac{\alpha'}{2}} p_L \\ \tilde{\alpha}_0^y = \oint_0 \frac{d\bar{z}}{2\pi i} \bar{J}(\bar{z}) = \sqrt{\frac{\alpha'}{2}}\left(\frac{n}{R} - \frac{R\omega}{\alpha'}\right) \equiv \sqrt{\frac{\alpha'}{2}} p_R \end{cases}$$

$$\hat{\mathcal{P}} = \frac{1}{\sqrt{2\alpha'}}(\alpha_0^y + \tilde{\alpha}_0^y),$$
$$\hat{\mathcal{W}} = \frac{1}{\sqrt{2\alpha'}}(\alpha_0^y - \tilde{\alpha}_0^y),$$

$$\left|\frac{n}{R}\right\rangle = e^{i\frac{n}{R}(Y_L+Y_R)}(z,\bar{z})|0\rangle\Big|_{z=\bar{z}=0}$$

$$\hat{\mathcal{P}}\left|\frac{n}{R}\right\rangle = \frac{n}{R}\left|\frac{n}{R}\right\rangle$$

$$\left|\frac{\omega R}{\alpha'}\right\rangle = e^{i\frac{\omega R}{\alpha'}(Y_L-Y_R)}(z,\bar{z})|0\rangle\Big|_{z=\bar{z}=0}$$

$$\hat{\mathcal{W}}\left|\frac{\omega R}{\alpha'}\right\rangle = \frac{\omega R}{\alpha'}\left|\frac{\omega R}{\alpha'}\right\rangle$$

$$\left[\begin{array}{c} Y(z,\bar{z}) = Y_L(z) + Y_R(\bar{z}) \\ = Y_0 - i\frac{\alpha'}{2}\hat{\mathcal{P}}\log z\bar{z} - i\frac{\alpha'}{2}\hat{\mathcal{W}}\log\frac{z}{\bar{z}} + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\left(\frac{\alpha_m^y}{m}z^{-m} + \frac{\tilde{\alpha}_m^y}{m}\bar{z}^{-m}\right) \\ \tilde{Y}(z,\bar{z}) = Y_L(z) - Y_R(\bar{z}z) \\ = C - i\frac{\alpha'}{2}\hat{\mathcal{W}}\log z\bar{z} - i\frac{\alpha'}{2}\hat{\mathcal{P}}\log\frac{z}{\bar{z}} + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\left(\frac{\alpha_m^y}{m}z^{-m} - \frac{\tilde{\alpha}_m^y}{m}\bar{z}^{-m}\right) \end{array}\right]$$

$$\begin{cases} Y(z,\bar{z}) & = Y_L(z) + Y_R(\bar{z}) \\ & = Y_0 - i\frac{\alpha'}{2}\hat{\mathcal{P}}\log z\bar{z} - i\frac{\alpha'}{2}\hat{\mathcal{W}}\log\frac{z}{\bar{z}} + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\left(\frac{\alpha_m^y}{m}z^{-m} + \frac{\tilde{\alpha}_m^y}{m}\bar{z}^{-m}\right) \\ \widetilde{Y}(z,\bar{z}) & = Y_L(z) - Y_R(\bar{z}z) \\ & = C - i\frac{\alpha'}{2}\hat{\mathcal{W}}\log z\bar{z} - i\frac{\alpha'}{2}\hat{\mathcal{P}}\log\frac{z}{\bar{z}} + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\left(\frac{\alpha_m^y}{m}z^{-m} - \frac{\tilde{\alpha}_m^y}{m}\bar{z}^{-m}\right) \end{cases}$$

$$\mathcal{V}_{n,\omega}(z,\bar{z}) = e^{i\frac{n}{R}Y}(z,\bar{z}) \cdot e^{i\frac{R\omega}{\alpha'}\tilde{Y}}(z,\bar{z})$$
$$= e^{i\left(\frac{n}{R}+\frac{R\omega}{\alpha'}\right)Y_L}(z) \cdot e^{i\left(\frac{n}{R}-\frac{R\omega}{\alpha'}\right)Y_R}(\bar{z})$$

$$\left[L_0, \mathcal{V}_{n,\omega}(z,\bar{z})\right] = \frac{\alpha'}{4}\left(\frac{n}{R}+\frac{R\omega}{\alpha'}\right)^2 \mathcal{V}_{n,\omega}(z,\bar{z})$$
$$\left[\bar{L}_0, \mathcal{V}_{n,\omega}(z,\bar{z})\right] = \frac{\alpha'}{4}\left(\frac{n}{R}-\frac{R\omega}{\alpha'}\right)^2 \mathcal{V}_{n,\omega}(z,\bar{z})$$

$$L_0 + \bar{L}_0 = \frac{1}{2}(\alpha_0^2 + \tilde{\alpha}_0^2) + N + \tilde{N} = 2$$

$$= \left(\frac{\alpha' P^2}{2} + \frac{\alpha'}{2}\left(\frac{n}{R}\right)^2 + \frac{\alpha'}{2}\left(\frac{R\omega}{\alpha'}\right)^2\right) + N + \tilde{\mathfrak{N}}$$

$$\frac{\alpha' P^2}{2} = -\frac{\alpha' m^2}{2}$$

$$m^2 = \underbrace{\left(\frac{n}{R}\right)^2}_{\text{Kaluza-Klein momentum}} + \underbrace{\left(\frac{R\omega}{\alpha'}\right)^2}_{\text{winding}} + \underbrace{\frac{2(N+\tilde{N})}{\alpha'}}_{\text{oscillators}} - \underbrace{\frac{4}{\alpha'}}_{\text{zero point energy}},$$

$$L_0 - \bar{L}_0 = \frac{1}{2}(\alpha_0^2 - \tilde{\alpha}_0^2) + N - \tilde{\mathfrak{N}} = 0$$
$$= \frac{1}{2}\left((\alpha_0^y)^2 - (\tilde{\alpha}_0^y)^2\right) + N - \tilde{\mathfrak{N}}$$
$$= \frac{1}{2}\left(4\frac{\alpha'}{2}\frac{n}{R}\frac{R\omega}{\alpha'}\right) + N - \tilde{\mathfrak{N}}$$
$$= \frac{1}{2}(2n\omega) + N - \tilde{\mathfrak{N}},$$

$$n\omega = \tilde{N} - N$$

$$G_{MN}(P)\alpha_{-1}^M \tilde{\alpha}_{-1}^N |0, P\rangle.$$

$$SO(1,d) \longrightarrow \underbrace{SO(1,d-1)}_{\mu}$$

$$G_{MN} \longrightarrow \begin{cases} G_{\mu\nu} \\ G_{\mu y}, G_{y\nu} \\ G_{yy} \end{cases}$$

$$h_{MN} \longrightarrow h_{\mu\nu}, A_\mu = h_{\mu y}(\text{ and } A_\nu = h_{y\nu}), \varphi = h_{yy}$$
$$B_{MN} \longrightarrow B_{\mu\nu}, B_\mu = B_{\mu y}(\text{ and } B_\nu = B_{y\nu})$$
$$\Phi \longrightarrow \Phi$$

$$\frac{1}{R} = \frac{R}{\alpha'}.$$

$$R = \sqrt{\alpha'}.$$

$$\alpha' m^2 = n^2 + \omega^2 + 2(N + \tilde{N}) - 4.$$

$$\begin{cases} n = \pm 2, \omega = 0 \\ n = 0, \omega = \pm 2 \end{cases}$$

$$\begin{cases} n = \pm 1, \omega = \pm 1 \\ n = \pm 1, \omega = \mp 1 \end{cases}$$

$$\begin{cases} 1 = N - \tilde{N} \\ -1 = N - \tilde{N} \end{cases} \Rightarrow \begin{cases} N = 1, \tilde{N} = 0 \\ N = 0, \tilde{N} = 1 \end{cases}$$

$$\begin{cases} | \pm 1, \pm 1\rangle = e^{\pm\frac{2i}{\sqrt{\alpha'}}Y_L}(z)|0\rangle \Big|_{z=0} \\ | \pm 1, \mp 1\rangle = e^{\pm\frac{2i}{\sqrt{\alpha'}}Y_R}(\bar{z})|0\rangle \Big|_{\bar{z}=0} \end{cases}$$

$$A^{(0)}_{L,\mu} j^{\mu} j^{y} e^{iP\cdot X}(z,\bar{z})$$
$$A^{(+)}_{L,\mu} j^{\mu} e^{\frac{2i}{\sqrt{\alpha'}}Y_R} e^{iP\cdot X}(z,\bar{z})$$
$$A^{(-)}_{L,\mu} j^{\mu} e^{\frac{-2i}{\sqrt{\alpha'}}Y_R} e^{iP\cdot X}(z,\bar{z})$$

$$A^{(0)}_{R,\mu} j^{y^{-}j^{\mu}} e^{iP\cdot X}(z,\bar{z})$$
$$A^{(+)}_{R,\mu} e^{\frac{2i}{\sqrt{\alpha'}}Y_L} \bar{j}^{\mu} e^{iP\cdot X}(z,\bar{z})$$
$$A^{(-)}_{R,\mu} e^{\frac{-2i}{\sqrt{\alpha'}}Y_L} \bar{j}^{\mu} e^{iP\cdot X}(z,\bar{z})$$

$$j^{0} = j^{y}$$
$$j^{+} = e^{\frac{2i}{\sqrt{\alpha'}}Y_L}$$
$$j^{-} = e^{-\frac{2i}{\sqrt{\alpha'}}Y_L}$$

$$j^{1} = \frac{1}{\sqrt{2}}(j^{+} + j^{-}) = \sqrt{2}\cos(2Y)$$
$$j^{2} = \frac{1}{\sqrt{2}}(j^{+} - j^{-}) = \sqrt{2}\sin(2Y)$$
$$j^{3} = j^{0}$$

$$j^{a}(z_1)j^{b}(z_2) = \frac{\delta^{ab}}{(z_1 - z_2)^2} + i\varepsilon^{ab}_{c}\frac{j^{c}(z_2)}{(z_1 - z_2)}$$

$$\underbrace{j^{a}(z_1)j^{b}(z_2)} = \frac{\delta^{ab}}{(z_1 - z_2)^2} + i\varepsilon^{ab}{}_{c}\frac{j^{c}(z_2)}{(z_1 - z_2)}$$

$$\begin{cases} m^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{R\omega}{\alpha'}\right)^2 + \frac{2(N+\tilde{N})}{\alpha'} - \frac{4}{\alpha'} \\ n\omega = N - \tilde{N} \end{cases}$$

$$\begin{cases} R \longrightarrow \frac{\alpha'}{R} \\ n \longrightarrow \end{cases}$$

$$Y \to \tilde{Y} \Longrightarrow \begin{cases} Y_L \to Y_L \\ Y_R \to -Y_R \end{cases}$$

$$j^y(z) = \Omega^{(j^y)}\bar{J}(\bar{z}), \text{ for } z = \bar{z}$$

$$j^y(z) = -\Omega^{(j^y)}\bar{J}(\bar{z}), \text{ for } z = \bar{z}$$

$$\mathfrak{N} \overset{T\text{-duality}}{\longleftrightarrow} \mathfrak{D}.$$

$$\begin{matrix} D(p)\text{-brane wrapping around } S^1_R \\ \text{(i.e. } S^1_R \text{ with Neumann BC)} \end{matrix} \overset{T\text{-duality}}{\longleftrightarrow} D(p-1)\text{-brane transverse to } S^1_{\alpha'/R}\,,$$

$$\begin{matrix} D(p)\text{-brane transverse to } S^1_R \\ \text{(i.e. } S^1_R \text{ with Dirichlet BC)} \end{matrix} \overset{T\text{-duality}}{\longleftrightarrow} D(p+1)\text{-brane wrapping around } S^1_{\alpha'/R}\,.$$

$$\begin{cases} \delta_{\mathrm{SUSY}}\sqrt{2/\alpha'}Y = \psi^y \\ \delta_{\mathrm{SUSY}}\psi^y = j^y \end{cases}$$

$$Y_R \overset{T\text{-duality}}{\rightarrow} -Y_R \Longrightarrow \bar{\psi}^y \overset{T\text{-duality}}{\rightarrow} -\bar{\psi}^y,$$

$$\bar{\psi}^y_0 \overset{T\text{-duality}}{\rightarrow} -\bar{\psi}^y_0$$

$$(-1)^{\bar{F}} = \bar{\Gamma}_{11}e^{-i\pi\sum_{n\geq 1}\bar{\psi}_{-n}\cdot\bar{\psi}_n}$$

$$\bar{\Gamma}_{11}e^{-i\pi\sum_{n\geq 1}\bar{\psi}_{-n}\cdot\bar{\psi}_n} \overset{T\text{-duality}}{\rightarrow} -\bar{\Gamma}_{11}(-1)^{-i\pi\sum_{n\geq 1}\bar{\psi}_{-n}\cdot\bar{\psi}_n}.$$

$$\begin{matrix} \text{type IIA} \\ \text{compactified on } S^1_R \end{matrix} \overset{T\text{-duality}}{\longleftrightarrow} \begin{matrix} \text{type IIB} \\ \text{compactified on } S^1_{\alpha'/R} \end{matrix}$$

$$X_R{}^9 \overset{T\text{-duality}}{\rightarrow} -X_R{}^9,$$

$$\tilde{S}_\alpha \overset{T\text{-duality}}{\rightarrow} \left(\Gamma_{11}\Gamma^9\tilde{S}\right)_\alpha$$

$$S^T\mathcal{C}\Gamma^j\tilde{S} \overset{T\text{-duality}}{\rightarrow} \begin{cases} S^T\mathcal{C}\Gamma^j\Gamma^9\tilde{S} & \text{for } j \neq 9 \\ S^T\mathcal{C}\tilde{S} & \text{for } j = 9 \end{cases}$$

$$F^{(p)}S^T\mathcal{C}\Gamma^{\mu_1\dots\mu_9}\tilde{S} \overset{T\text{-duality}}{\rightarrow} \begin{cases} F^{(p-1)} & \text{for } 9 \in \{\mu_1,\dots,\mu_9\} \\ F^{(p+1)} & \text{for } 9 \notin \{\mu_1,\dots,\mu_9\} \end{cases}$$

$$F^{(p+1)} \ (\text{i.e. } F^{\mu_1 \dots \mu_p 9}) \xrightarrow{T\text{-duality}} F^{(p)} \ (\text{i.e. } F^{\mu_1 \dots \mu_p})$$
$$F^{(p-1)} \ (\text{i.e. } F^{\mu_1 \dots \mu_{p-1}}) \xrightarrow{T\text{-duality}} F^{(p+1)} \ (\text{i.e. } F^{\mu_1 \dots \mu_{p-1} 9}) \, .$$

$$D(2p)\text{-brane} \overset{T\text{-duality}}{\rightarrow} \begin{cases} D(2p-1)\text{-brane} & \begin{array}{c}\text{if the } D(2p)\text{-brane was} \\ \text{wrapped around } S^1\end{array} \\ D(2p+1)\text{-brane} & \begin{array}{c}\text{if the } D(2p)\text{-brane was not} \\ \text{wrapped around } S^1\end{array} \end{cases}.$$

$$e^{-\Phi} \overset{T\text{-duality}}{\rightarrow} e^{-\Phi'} = e^{-\Phi} \frac{R}{\sqrt{\alpha'}},$$

$$g_s = e^{\langle\Phi\rangle} \overset{T\text{-duality}}{\rightarrow} g_s' = g_s \frac{\sqrt{\alpha'}}{R},$$

$$S_{\text{IIB}} = \frac{1}{2K_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\Phi} \left(R + 4(\partial\Phi)^2 - \frac{1}{2} \left| H_{(3)} \right|^2 \right) + \cdots \right\}$$

$$G_{MN}^{(\mathrm{E})} = e^{-\frac{1}{2}\Phi} G_{MN}^{(\mathrm{s})}$$

$$S_{\text{IIB}} = \frac{1}{2K_{10}^2} \int d^{10}x \sqrt{-G^{(E)}} \left\{ R - \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{2(\mathrm{Im}(\tau))^2} - \frac{1}{2} \frac{\left| G_{(3)} \right|^2}{\mathrm{Im}(\tau)} - \frac{1}{4} \left| F_{(5)} \right| \right\}$$
$$+ \frac{1}{8iK_{10}^2} \int \frac{1}{\mathrm{Im}(\tau)} G_{(4)} \wedge G_{(3)} \wedge \bar{G}_{(3)}$$

$$\tau = C_{(0)} + ie^{-\Phi}$$

$$G_{(3)} = F_{(3)} - ie^{-\Phi} H_{(3)}$$

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} C_{(2)} \\ B_{(2)} \end{pmatrix} \longrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_{(2)} \\ B_{(2)} \end{pmatrix}$$

$$S\colon \tau \longrightarrow -\frac{1}{\tau}$$

$$S\colon g_s \longrightarrow \frac{1}{g_s}.$$

$$B_2 \rightarrow -C_2$$
$$C_2 \rightarrow B_2.$$

$$T_p = \frac{1}{g_s} \left(\frac{1}{2\pi} \right)^p (\alpha')^{-\frac{p+1}{2}}.$$

$$T_{D(1)} = \frac{1}{2\pi g_s \alpha'} \overset{S}{\longleftrightarrow} \frac{1}{2\pi\alpha'},$$

$$S: \alpha' \longrightarrow g_s\alpha'.$$

$$D(3) \overset{S}{\longleftrightarrow} D(3)$$

$$T_{D(3)} = \frac{1}{(2\pi)^3 g_s (\alpha')^2} \overset{S}{\longleftrightarrow} \frac{g_s}{(2\pi)^3 (g_s\alpha')^2} = \frac{1}{(2\pi)^3 g_s (\alpha')^2} = T_{D(3)}.$$

$$F_{(5)} \overset{S}{\longleftrightarrow} F_{(5)}.$$

$$D(5) \overset{S}{\longleftrightarrow} NS(5)\text{-brane}$$

$$T_{D(5)} = \frac{1}{(2\pi)^5 g_s (\alpha')^3} \overset{S}{\longleftrightarrow} \frac{1}{(2\pi)^5 g_s^2 (\alpha')^3}$$

$$T_{D(0)} = \frac{1}{g_s\sqrt{\alpha'}}$$

$$T_{nD(0)} = nT_{D(0)}$$

$$\left(m_{nD(0)}\right)^2 = \left(\frac{n}{g_s\sqrt{\alpha'}}\right)^2$$

$$m_n^2 = \left(\frac{n}{R}\right)^2$$

$$R = g_s\sqrt{\alpha'}.$$

$$S_{d=11} = \frac{1}{2K_{11}^2}\int d^{11}x\sqrt{-G}\left(R - \frac{1}{2}\left|dA_{(3)}\right|^2\right) + \frac{1}{2K_{11}^2}\left(-\frac{1}{6}\right)\int A_{(3)} \wedge F_{(4)} \wedge F_{(4)}$$

$$\mathcal{M}^{11} = \mathbb{R}^{1,9} \times \mathcal{S}_R^1$$

$$K_{11}^2 = 2\pi R K_{10}^2$$

$$\int d^d X$$

$$\downarrow$$

$$\text{compactification:} \quad \underbrace{\mathcal{M}^d}_{X} = \underbrace{\mathbb{R}^{1,d-2}}_{x} \times \underbrace{\mathcal{S}^1_R}_{y}$$

$$\downarrow$$

$$\int d^d X = \int d^{d-1}x \int dy = 2\pi R \int d^{d-1}x$$

$$\frac{1}{K_d^2} \int d^d X \, (\cdots)$$

$$\downarrow$$

$$\frac{2\pi R}{K_{d-1}^2} \int d^{d-1}x(\cdots)\,.$$

$$G_{MN} \longrightarrow \begin{cases} G_{\mu\nu}(\mu,\nu = 0,1,\dots,9) \\ G_{\mu 10} \to C_\mu \\ G_{1010} \to e^{2\Phi} \end{cases}$$

$$A_{MNP} \longrightarrow \begin{cases} A_{\mu\nu\rho}(\mu,\nu,\rho = 0,1,\dots,9) \\ A_{\mu\nu 10} \to B_{\mu\nu} \end{cases}$$

$$T_{M(2)} = \frac{(m_{11})^3}{(2\pi)^2}$$

$$K_{10} = \frac{(4\pi\alpha')^2}{2\sqrt{\pi}} g_s$$

$$K_{11}^2 = 2\pi R K_{10}^2 = \frac{1}{4\pi}\left(\frac{2\pi}{m_{11}}\right)^2.$$

$$R_{11} = g_s\sqrt{\alpha'}$$

$$m_{11} = g_s^{-\frac{1}{3}} \frac{1}{\sqrt{\alpha'}}$$

$$\alpha' = \frac{l_{11}^3}{R_{11}}, g_s = \left(\frac{R_{11}}{l_{11}}\right)^{\frac{3}{2}},$$

$$T_{M(2)} = \frac{1}{(2\pi)^2}\frac{1}{g_s}\frac{1}{(\alpha')^{\frac{3}{2}}} = T_{D(2)}$$

$$T_{M(2)/S_1} = 2\pi R T_{M(2)} = \frac{1}{2\pi\alpha'}$$

$$F^{(7)} = \star_{11} F^{(4)} = dA^{(6)}$$

$$T_{M(5)} = \frac{m_{11}^6}{(2\pi)^5}$$

$$T_{M(2)}T_{M(5)} = \frac{2\pi}{2K_{11}^2}$$

$$T_{M(5)} = \frac{m_{11}^6}{(2\pi)^5} = \frac{1}{(2\pi)^5}\frac{1}{g_s^2}\frac{1}{(\alpha')^3} = T_{NS(5)}$$

$$T_{M(5)/S_1} = 2\pi R T_{M(5)} = \frac{1}{(2\pi)^4}\frac{1}{g_s}\frac{1}{(\alpha')^{\frac{5}{2}}} = T_{D(4)}$$

$$(0,1),(2,3),\dots,(d-2,d-1)$$

$$I = 0,1,\dots,\frac{d}{2}-1 = (0,j) \text{ with } j = 1,\dots,\frac{d}{2}-1$$

$$[\psi^\mu, \psi^\nu] = \eta^{\mu\nu}$$

$$\psi_0^{\pm} \equiv \frac{i}{\sqrt{2}}(\psi_0^0 \pm \psi_0^1)$$
$$\psi_j^{\pm} \equiv \frac{1}{\sqrt{2}}(\psi_0^{2j} \pm i\psi_0^{2j+1})$$

$$[\psi_I^+, \psi_J^-] = \delta_{I,J},$$
$$[\psi_I^{\pm}, \psi_J^{\pm}] = 0.$$

$$\psi^+|+1/2\rangle = 0$$
$$\psi^-|+1/2\rangle = |-1/2\rangle$$
$$\psi^-|-1/2\rangle = 0$$

$$|0\rangle = \underbrace{|+1/2\rangle \otimes |+1/2\rangle \otimes \cdots \otimes |+1/2\rangle}_{\frac{d}{2}\text{ times}} = |\underbrace{+1/2, +1/2, \dots, +1/2}_{\frac{d}{2}\text{ times}}\rangle,$$

$$|\alpha\rangle = \underbrace{|\pm 1/2\rangle \otimes |\pm 1/2\rangle \otimes \cdots \otimes |\pm 1/2\rangle}_{\frac{d}{2}\text{ times}} = |\underbrace{\pm 1/2, \pm 1/2, \dots, \pm 1/2}_{\frac{d}{2}\text{ times}}\rangle.$$

$$\alpha = 1, \cdots, 2^{\frac{d}{2}}.$$

$$[\psi_0^+, \psi_0^-] = \psi_0^+\psi_0^- + \psi_0^-\psi_0^+ = \delta_{0,0} = 1.$$

$$\begin{aligned} -\psi_0^0\psi_0^1 &= \frac{1}{2}(\psi_0^+ + \psi_0^-)(\psi_0^+ - \psi_0^-) \\ &= -\frac{1}{2}(\psi_0^-\psi_0^+ - \psi_0^+\psi_0^-) \\ &= \psi_0^+\psi_0^- - \frac{1}{2}, \\ -i\psi_0^{2j}\psi_0^{2j+1} &= \psi_j^+\psi_j^- - \frac{1}{2}. \end{aligned}$$

$$\chi_I = \begin{cases} -\psi_0^0\psi_0^1 & \text{if } I = 0 \\ -i\psi_0^{2j}\psi_0^{2j+1} & \text{if } I = j \end{cases} \Rightarrow \chi_I = \psi_I^+\psi_I^- - \frac{1}{2},$$

$$\chi_I|+1/2\rangle = \psi_I^+\psi_I^- - \frac{1}{2}|+1/2\rangle = \left(1 - \frac{1}{2}\right)|+1/2\rangle = \frac{1}{2}|+1/2\rangle$$

$$\chi_I|-1/2\rangle = \psi_I^+\psi_I^- - \frac{1}{2}|-1/2\rangle = \left(0 - \frac{1}{2}\right)|+1/2\rangle = -\frac{1}{2}|+1/2\rangle$$

$$\Gamma = 2^n \prod_{I=0}^{n-1} \chi_I = -(-i)^{n-1}\Gamma^0\Gamma^1 \ldots \Gamma^{(2n-2)}\Gamma^{(2n-1)}$$

$$|\alpha\rangle = |(\text{even } \# \text{ of } -1/2)\rangle$$

$$|\dot{\alpha}\rangle = |(\text{odd } \# \text{ of } -1/2)\rangle$$

$$\begin{aligned} \Gamma|\alpha\rangle &= +|\alpha\rangle \\ \Gamma|\dot{\alpha}\rangle &= -|\dot{\alpha}\rangle \end{aligned}$$

$$\Gamma^{\mu\nu} = [\Gamma^\mu, \Gamma^\nu]$$

$$[\Gamma^{\mu\nu}, \Gamma] = 0$$

$$\tilde{\eta}(\tau) = q^{\frac{1}{24}} \prod_{l=1}^{\infty} (1 - q^l), \text{ with } q = e^{2\pi i\tau}, |q| < 1$$

$$\eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\,\eta(\tau)$$

$$\frac{1}{12}i\pi\tau - \log\,\eta(\tau) = -\sum_{l=1}^{\infty} \log\left(1 - q^l\right) = \sum_{k,l=1}^{\infty} \frac{1}{k}q^{lk} = \sum_{k=1}^{\infty} \frac{1}{k}\frac{1}{q^{-k} - 1}.$$

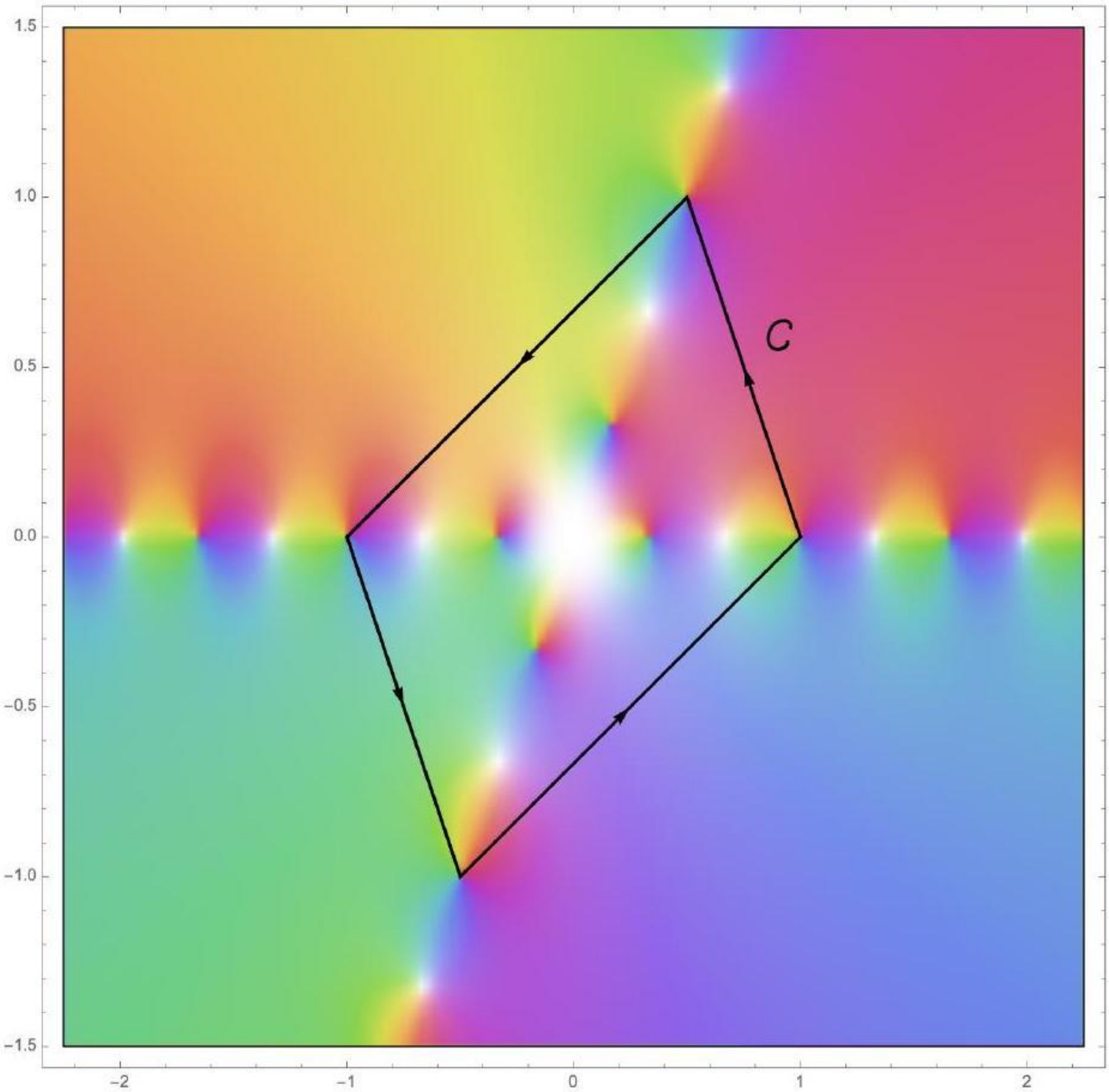
$$-\frac{i\pi}{12\tau} - \log\,\eta\left(-\frac{1}{\tau}\right) = -\frac{i\pi}{12\tau} - \frac{1}{2}\log\frac{\tau}{i} - \log\,\eta(\tau) = \sum_{k=1}^{\infty} \frac{1}{k}\frac{1}{e^{\frac{2\pi ik}{\tau}} - 1}.$$

$$\frac{\pi i}{12}\left(\tau+\frac{1}{\tau}\right)+\frac{1}{2}\log\frac{\tau}{i}=\sum_{k=1}^{\infty}\frac{1}{k}\left[\frac{1}{e^{-2\pi ik\tau}-1}-\frac{1}{e^{2\pi ik/\tau}-1}\right].$$

$$f_\nu(z)=\frac{1}{z}g(\nu z),\nu=\left(n+\frac{1}{2}\right)\pi,(n=0,1,\dots)$$

$$\mathrm{Res}_{z_k}(f_\nu)=\frac{1}{\pi k}\cot\frac{\pi k}{\tau},\mathrm{Res}_{z_k^\tau}(f_\nu)=\frac{1}{\pi k}\cot\pi k\tau$$

$$\mathrm{Res}_0(f_\nu)=-\frac{1}{3}\left(\tau+\frac{1}{\tau}\right)$$



$$\frac{\pi i}{12}\left(\tau+\frac{1}{\tau}\right)+\frac{1}{8}\int_C f_\nu(z)dz=\frac{i}{2}\sum_{k=1}^{n}\frac{1}{k}(\cot\pi k\tau+\cot\pi k/\tau)$$
$$=\sum_{k=1}^{n}\frac{1}{k}\left[\frac{1}{e^{-2\pi ik\tau}-1}-\frac{1}{e^{2\pi ik/\tau}-1}\right]$$

$$\lim_{n\to\infty}\int_C f_\nu(z)dz = \left(\int_1^\tau - \int_\tau^{-1} + \int_{-1}^{-\tau} - \int_{-\tau}^{1}\right)\frac{dz}{z} = 4\log\frac{\tau}{i}$$

$$\phi_i\varphi^i = \sum_i \phi_i\varphi_i$$

$$\phi_a\varphi^a = \int da\phi(a)\varphi(a)$$

$$\int dx\partial_x\delta(x-x_0)f(x) = \lim_{h\to 0}\int dx\frac{\delta(x+h-x_0)-\delta(x-x_0)}{h}f(x) = \lim_{h\to 0}\frac{f(x_0-h)-f(x_0)}{h} = -f'(x_0)$$

$$\int dx\partial_{x_0}\delta(x-x_0)f(x) = \lim_{h\to 0}\int dx\frac{\delta(x-h-x_0)-\delta(x-x_0)}{h}f(x) = \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} = f'(x_0).$$

$$\delta_B\phi = i[\phi, Q_B], \text{ for } \phi = (X^\mu, c, b).$$

$$A_p = A_{\mu_1\cdots\mu_p}dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_p}, (D-p)\text{-form } *A_p = A^{\mu_1\cdots\mu_p}\epsilon_{\mu_1\cdots\mu_d}dx^{\mu_{p+1}}\wedge\cdots\wedge dx^{\mu_D}$$

$$L_0^{\text{tot}}\ b_0|\psi\rangle = Q|\psi\rangle = 0\ [Q, b_0]|\psi\rangle = L_0^{\text{tot}}|\psi\rangle = 0.$$

$$\partial_\mu A^\mu = 0, \delta A_\mu = \partial_\mu\lambda\ \square\ \lambda = 0.$$

$$\phi'(z') = \phi(z) + \delta\phi(z) + \partial\phi(z)\delta z, z' = z + \delta z.$$

$$\int_{-\infty}^{\infty} dx_1 dx_2 dx_3.$$

$$(h, \bar{h}), s = h - \bar{h}.$$

$$\sim ce^{-\varphi/2}S_\alpha|0\rangle_{SL(2,\mathbb{C})}.$$

$$\begin{aligned} 8_i\otimes 8_i &= 1\oplus 28\oplus 35, && \text{with } i = V, C, S,\\ 8_i\otimes 8_j &= 8_k\oplus 56, && \text{with } i\neq j\neq k.\end{aligned}$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu.$$

$$\delta v^\alpha = -\Gamma^\alpha{}_{\mu\nu}v^\mu dx^\nu, \nabla_\mu v_\nu = \partial_\mu v_\nu - \Gamma^\lambda_{\mu\nu}v_\lambda.$$

$$\Gamma^\alpha_{\{\mu\nu\}} = \frac{1}{2}g^{\alpha\lambda}(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}), \Gamma^\alpha_{[\mu\nu]} = 0.$$

$$R^\rho_{\sigma\mu\nu}(g) = \partial_\mu\Gamma^\rho_{\nu\sigma}(g) - \partial_\nu\Gamma^\rho_{\mu\sigma}(g) + \Gamma^\rho_{\mu\lambda}(g)\Gamma^\lambda_{\nu\sigma}(g) - \Gamma^\rho_{\nu\lambda}(g)\Gamma^\lambda_{\mu\sigma}(g),$$

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R(g) - 2\Lambda) + S_{matter}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$T^\alpha_{\mu\nu} - g^\alpha{}_\mu T^\rho_{\nu\rho} - g^\alpha{}_\nu T^\rho_{\rho\mu} = 8\pi G S^\alpha_{\mu\nu}$$

$$dx^a = e^a_\mu dx^\mu$$

$$\eta_{ab} = g_{\mu\nu}(x)e_a^\mu(x)e_b^\nu(x) \text{ and } g_{\mu\nu}(x) = \eta_{ab}e_\mu^a(x)e_\nu^b(x)$$

$$D_\mu^\omega X^a = \partial_\mu X^a + \omega_{\mu b}^a X^b, D_\mu^\omega Y^{ab} = \partial_\mu Y^{ab} + \omega_{\mu c}^a Y^{cb} + \omega_{\mu c}^b Y^{ac}$$

$$e^a = e_\mu^a dx^\mu, \omega^{ab} = \omega_\mu^{ab} dx^\mu$$

$$R_{\mu\nu}{}^{ab}(\omega) = \partial_\mu \omega_\nu{}^{ab} - \partial_\nu \omega_\mu{}^{ab} + \omega_\mu{}^a{}_c \omega_\nu{}^{cb} - \omega_\nu{}^a{}_c \omega_\mu{}^{cb},$$

$$R^{ab}(\omega) = d\omega^{ab} + \omega_c^a \wedge \omega^{cb} = \frac{1}{2} R_{\mu\nu}^{ab}(\omega) dx^\mu \wedge dx^\nu$$

$$T_{\mu\nu}^a = T_{\mu\nu}^\rho e_\rho^a \; T_{\mu\nu}^a = D_\mu^\omega e_\nu^a - D_\nu^\omega e_\mu^a$$
$$T^a = D^\omega e^a = de^a + \omega_b^a \wedge e^b$$

$$D^\omega R^{ab} = 0, D^\omega T^a = R^{ab} \wedge e_b$$

$$S = \frac{1}{64\pi G} \int d^4x \epsilon^{abcd} \left(R_{\mu\nu ab}(\omega) e_{\rho k} e_{\sigma d} - \frac{\Lambda}{3} e_{\mu a} e_{\nu b} e_{\rho c} e_{\sigma d} \right) \epsilon^{\mu\nu\rho\sigma}$$

$$S(\omega, e) = \frac{1}{32\pi G} \int \left(R^{ab}(\omega) \wedge e^c \wedge e^d - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d \right) \epsilon_{abcd}$$

$$D\left(e^a \wedge e^b \epsilon_{abcd}\right) = 0, \left(R^{ab}(\omega) \wedge e^c - \frac{\Lambda}{3} e^a \wedge e^b \wedge e^c \right) \epsilon_{abcd} = 0$$

$$0 = D^\omega e^a = de^a + \omega_b^a \wedge e^b$$

$$\omega_\mu^{ab}(e) = e^{\nu a} \nabla_\mu e_\nu^b = e^{\nu a} \left(\partial_\mu e_\nu^b - \Gamma_{\mu\nu}^\lambda(g) e_\lambda^b \right)$$

$$R_{\sigma\mu\nu}^\rho(g) = R_{\mu\nu}^{ab}(\omega(e)) e_a^\rho e_{\sigma b}$$

$$S = \frac{1}{32\pi G} \int R^{ab} \wedge e^c \wedge e^d \epsilon_{abcd}$$

$$A_\mu^{ab} = \omega_\mu^{ab}, A_\mu^{a4} = \frac{1}{\ell} e_\mu^a$$

$$\frac{\Lambda}{3} = -\frac{1}{\ell^2}$$

$$F^{IJ}(A) = \frac{1}{2} F_{\mu\nu}^{IJ} dx^\mu \wedge dx^\nu$$

$$F^{IJ}(A) = dA^{IJ} + A^{IK} \wedge A_K^J, F_{\mu\nu}^{IJ} = \partial_\mu A_\nu^{IJ} - \partial_\nu A_\mu^{IJ} + A_{\mu K}^I A_\nu^{KJ} - A_{\nu K}^I A_\mu^{KJ}$$

$$F_{\mu\nu}^{a4} = \frac{1}{\ell}\left(\partial_\mu e_\nu^a + \omega_\mu{}^a{}_b e_\nu^b - \partial_\nu e_\mu^a - \omega_\nu{}^a{}_b e_\mu^b \right) = \frac{1}{\ell}\left(D_\mu^\omega e_\nu^a - D_\nu^\omega e_\mu^a \right) = \frac{1}{\ell} T_{\mu\nu}^a$$

$$F_{\mu\nu}^{ab} = R_{\mu\nu}^{ab} + \frac{1}{\ell^2}\left(e_\mu^a e_\nu^b - e_\nu^a e_\mu^b \right)$$

$$R_{\mu\nu}^{ab} = \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} + \omega_{\mu c}^{a} \omega_\nu^{cb} - \omega_{\nu c}^{a} \omega_\mu^{cb}$$

$$F^{IJ} \rightarrow \hat{F}^{IJ} = F^{ab} \text{ where } F^{ab} = R^{ab} + \frac{1}{\ell^2} e^a \wedge e^b$$

$$S_{MM}(A) = \frac{\ell^2}{64\pi G} \int \operatorname{tr}(\hat{F} \wedge \star \hat{F})$$

$$S_{MM}(A) = \frac{\ell^2}{64\pi G} \int \left(R^{ab} + \frac{1}{\ell^2} e^a \wedge e^b\right) \wedge \left(R^{cd} + \frac{1}{\ell^2} e^c \wedge e^d\right) \epsilon_{abcd}$$

$$32\pi G S_{MM} = \int \mathfrak{E}_{\text{Einstein}} / \mathfrak{C}_{\text{Cartan}} + \frac{1}{2\ell^2} \int \Lambda_{\text{cosmological}} + \frac{\ell^2}{2} \int \zeta_{\text{Euler}}$$

$$\frac{1}{2} \epsilon_{abcd} \left(R^{ab} + \frac{1}{\ell^2} e^a \wedge e^b\right) \wedge e^c = 16\pi G t_d$$

$$\epsilon_{abcd} T^c \wedge e^d = 16\pi G s_{ab}$$

$$t_d = \frac{1}{3!} T_{df} \epsilon^{abcf} e_a \wedge e_b \wedge e_c, s_{mn} = \frac{1}{3!} S_{mn}^{f} \epsilon^{abcf} e_a \wedge e_b \wedge e_c$$

$$\left(\epsilon_{IJKLa} F^{KL} \wedge A^{a4} - 16\pi G s_{IJ}\right) \wedge \delta A^{IJ} = 0$$

$$\delta\left(\epsilon_{abcd4} F^{ab} \wedge F^{cd}\right) - 16\pi G s_{IJ} \wedge \delta A^{IJ} = 0$$

$$s_{IJ} = \begin{cases} s_{ab} \\ s_{a4} = t_a \end{cases}$$

$$16\pi S_{BF}(A, B) = \int \operatorname{tr}\left(B \wedge F - \frac{\alpha}{4} \hat{B} \wedge \star \hat{B}\right),$$

$$\delta B_{a5}: F^{a5} = \frac{1}{\ell} T^a = 0, \delta B_{ab}: F^{ab} = \frac{\alpha}{2} \epsilon^{abcd} B_{cd}$$

$$S_{BF}(A, B) \equiv S_{MM}(A)$$

$$\omega_i^a = \omega_i^{0a} + \frac{i}{2} \epsilon^{0abc} \omega_{ibc}, \mathcal{P}_a^i = \frac{4}{16\pi G} \epsilon_{abc} \epsilon^{ijk} e_j^b e_k^c$$

$${}^{\gamma}\omega_i^a = \omega_i^{0a} + \frac{\gamma}{2} \epsilon^{0abc} \omega_{ibc}, \text{ and } \mathcal{P}_a^i = \frac{4}{16\pi G} \epsilon_{abc} \epsilon^{ijk} e_j^b e_k^c,$$

$$\left\{ {}^{\gamma}\omega_i^a(x), \mathcal{P}_b^j(y) \right\} = \gamma \delta(x - y) \delta_i^j \delta_b^a$$

$$\frac{2}{64\pi G \gamma} \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu}^{ab} e_{\rho a} e_{\sigma b}$$

$$\mathcal{L}_{EC} = R^{ab} \wedge e^c \wedge e^d \epsilon_{abcd}$$

$$\mathcal{L}_\Lambda = e^a \wedge e^b \wedge e^c \wedge e^d \epsilon_{abcd}$$

$$H_4 = R^{ab} \wedge e_a \wedge e_b$$

$$\mathcal{E}_4 = R^{ab} \wedge R^{cd} \epsilon_{abcd}$$
$$\mathcal{P}_4 = R^{ab} \wedge R_{ab}$$
$$\mathcal{NY}_4 = T^a \wedge T_a - R^{ab} \wedge e_a \wedge e_b$$

$$F^{IJ}(A) \wedge F_{IJ}(A) = R^{ab}(\omega) \wedge R_{ab}(\omega) - \frac{2}{\ell^2}\left(T^a \wedge T_a - R_{ab} \wedge e^a \wedge e^b\right)$$
$$\mathcal{P}_5(A) = \mathcal{P}_4(\omega) - \frac{2}{\ell^2}\mathcal{NY}_4$$

$$P_4 = \frac{1}{4}\int d^4x R_{\mu\nu ab} R^{ab}_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$
$$E_4 = \frac{1}{4}\int d^4x R_{\mu\nu ab} R_{\rho\sigma cd} \epsilon^{abcd} \epsilon^{\mu\nu\rho\sigma}$$
$$NY_4 = \frac{1}{4}\int d^4x \left(T_{\mu\nu a} T^a_{\rho\sigma} - 2R_{\mu\nu ab} e^a_\nu e^b_\rho\right)\epsilon^{\mu\nu\rho\sigma}$$

$$NY_4 = 2\int \partial_\mu\left(e_{\nu\alpha} T^\alpha_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma} = 4\int \partial_\mu\left(e_{\nu\alpha} \mathcal{D}^\omega_\rho e^\alpha_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$
$$P_4 = \int R_{\mu\nu ab} R^{ab}_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} = 4\int \partial_\mu C^\mu(\omega)$$

$$C^\mu(\omega) = \left(\omega_{\nu ab}\partial_\rho \omega^{ab}_\sigma + \frac{2}{3}\omega_{\nu ab}\omega^a_{\rho\ c}\omega^{cb}_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$

$$C^\mu(A) = C^\mu(\omega) - \frac{2}{\ell^2}\left(e_{a\nu} D^\omega_\rho e^a_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$

$${}^{\pm}\omega^{ab}_\mu = \frac{1}{2}\left(\omega^{ab}_\mu \mp \frac{i}{2}\epsilon^{ab}_{cd}\omega^{cd}_\mu\right)$$
$${}^{\pm}R^{ab}_{\mu\nu} = \frac{1}{2}\left(\frac{1}{2}\delta^{ab}_{cd} \mp \frac{i}{2}\epsilon^{ab}_{cd}\right)R^{cd}_{\mu\nu},\ {}^{\pm}R^{ab}_{\mu\nu} = \frac{1}{2}\left(R^{ab}_{\mu\nu} \mp \frac{i}{2}\epsilon^{ab}_{cd}R^{cd}_{\mu\nu}\right),$$

$$\epsilon^{\mu\nu\sigma\rho\pm}R^{ab\pm}_{\mu\nu}R_{\rho\sigma ab} = \frac{1}{4}\epsilon^{\mu\nu\sigma\rho}\left(2R^{ab}_{\mu\nu}R_{\rho\sigma ab} \mp iR^{ab}_{\mu\nu}R^{cd}_{\rho\sigma}\epsilon_{abcd}\right).$$
$$4\partial_\mu \mathcal{C}^\mu({}^{\pm}\omega) = \frac{1}{4}(2\mathcal{P}_4(\omega) \mp i\mathcal{E}_4(\omega)).$$

$$P_4 = 4\int \left(\partial_\mu \mathcal{C}^\mu({}^{+}\omega) + \partial_\mu \mathcal{C}^\mu({}^{-}\omega)\right)$$
$$E_4 = 8i\int \left(\partial_\mu \mathcal{C}^\mu({}^{+}\omega) - \partial_\mu \mathcal{C}^\mu({}^{-}\omega)\right)$$

$$16\pi S(A,B) = \int F^{IJ} \wedge B_{IJ} - \frac{\beta}{2}B^{IJ} \wedge B_{IJ} - \frac{\alpha}{4}\epsilon^{abcd4}B_{ab} \wedge B_{cd}$$

$$\alpha = \frac{G\Lambda}{3(1+\gamma^2)}, \beta = \frac{\gamma G\Lambda}{3(1+\gamma^2)} \quad \text{with} \ \Lambda = -\frac{3}{\ell^2}$$

$$F^{a4} = \beta B^{a4}$$
$$F^{ab} = \beta B^{ab} + \frac{\alpha}{2}\epsilon^{abcd}B_{cd}$$

$$B^{ab}=\frac{1}{\alpha^2+\beta^2}\left(\beta F^{ab}-\frac{\alpha}{2}\epsilon^{abcd}F_{cd}\right).$$

$$S(\omega,e)=\frac{1}{16\pi}\int\left(\frac{1}{4}M^{abcd}F_{ab}\wedge F_{cd}-\frac{1}{\beta\ell^2}T^a\wedge T_a\right)$$

$$M^{ab}{}_{cd}=\frac{\alpha}{(\alpha^2+\beta^2)}\left(\gamma\delta^{ab}_{cd}-\epsilon^{ab}_{cd}\right)\equiv-\frac{\ell^2}{G}\left(\gamma\delta^{ab}_{cd}-\epsilon^{ab}_{cd}\right).$$

$$\begin{aligned}32\pi GS=&\int R^{ab}\wedge e^c\wedge e^d\epsilon_{abcd}+\frac{1}{2\ell^2}\int e^a\wedge e^b\wedge e^c\wedge e^d\epsilon_{abcd}+\frac{2}{\gamma}\int R^{ab}\wedge e_a\wedge e_b\\&+\frac{\ell^2}{2}\int R^{ab}\wedge R^{cd}\epsilon_{abcd}-\ell^2\gamma\int R^{ab}\wedge R_{ab}\\&+\frac{\gamma^2+1}{\gamma}\int 2\left(T^a\wedge T_a-R^{ab}\wedge e_a\wedge e_b\right)\end{aligned}$$

$$\begin{aligned}64\pi GS=&\int\epsilon^{abcd}\left(R_{\mu\nu ab}e_{\rho c}e_{\sigma d}-\frac{\Lambda}{3}e_{\mu a}e_{\nu b}e_{\rho c}e_{\sigma d}\right)\epsilon^{\mu\nu\rho\sigma}+\frac{2}{\gamma}\int R_{\mu\nu ab}e^a_\nu e^b_\rho\epsilon^{\mu\nu\rho\sigma}\\&+\frac{\gamma^2+1}{\gamma}NY_4+\frac{3\gamma}{2\Lambda}P_4-\frac{3}{4\Lambda}E_4\end{aligned}$$

$$(D^AB)^{IJ}=0$$
$$F^{IJ}-\beta B^{IJ}-\frac{\alpha}{2}\epsilon^{IJKL4}B_{KL}=0$$

$$(D^AB)^{IJ}=dB^{IJ}+A^I{}_K\wedge B^{KJ}+A^J{}_K\wedge B^{IK}.$$

$$D^\omega B^{ab}+\frac{1}{\ell}e^a\wedge B^{b4}-\frac{1}{\ell}e^b\wedge B^{a4}=0$$
$$D^\omega B^{a4}-\frac{1}{\ell}e_b\wedge B^{ab}=0$$

$$\left(\frac{1}{\gamma}\delta^{ab}_{cd}+\epsilon^{ab}_{cd}\right)F_{ab}\wedge e^c=0,D^\omega\left(\left(\frac{1}{\gamma}\delta^{ab}_{cd}+\epsilon^{ab}_{cd}\right)e_a\wedge e_b\right)=0$$

$$\left(R^{ab}\wedge e^c+\frac{1}{\ell^2}e^a\wedge e^b\wedge e^c\right)\epsilon_{abcd}=0$$

$$[\mathcal{M}_{IJ},\mathcal{M}_{KL}]=-i(\eta_{IK}\mathcal{M}_{JL}+\eta_{JL}\mathcal{M}_{IK}-\eta_{IL}\mathcal{M}_{JK}-\eta_{JK}\mathcal{M}_{IL}),$$

$$[\mathcal{M}_{ab},\mathcal{M}_{cd}]=-i(\eta_{ac}\mathcal{M}_{bd}+\eta_{bd}\mathcal{M}_{ac}-\eta_{ad}\mathcal{M}_{bc}-\eta_{bc}\mathcal{M}_{ad})$$
$$[\mathcal{M}_{ab},\mathcal{P}_c]=-i(\eta_{ac}\mathcal{P}_b-\eta_{bc}\mathcal{P}_a),[\mathcal{P}_a,\mathcal{P}_b]=-i\eta_{44}\mathcal{M}_{ab}=i\mathcal{M}_{ab}$$

$$\mathbb{A}_\mu=\frac{1}{2}A^{IJ}_\mu\mathcal{M}_{IJ}=\frac{1}{2}\omega^{ab}_\mu\mathcal{M}_{ab}+\frac{1}{\ell}e^a_\mu\mathcal{P}_a$$

$$\mathbb{F}_{\mu\nu}(\mathbb{A})=\partial_\mu\mathbb{A}_\nu-\partial_\nu\mathbb{A}_\mu-i[\mathbb{A}_\mu,\mathbb{A}_\nu]$$

$$\{\gamma^a,\gamma^b\}=2\eta^{ab},\eta^{ab}=\mathrm{diag}(-,+,+,+)$$

$$\gamma^5 = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} \text{ and } \gamma^0 = \begin{pmatrix} 0 & -i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} \gamma^2 = \begin{pmatrix} 0 & i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix} \gamma^3 = \begin{pmatrix} -\sigma^1 & 0 \\ 0 & -\sigma^1 \end{pmatrix}.$$

$$m_{a4} = \frac{1}{2}\gamma_a, m^{a4} = -\frac{1}{2}\gamma^a, m_{ab} = m^{ab} = \frac{1}{4}[\gamma_a, \gamma_b] = \frac{1}{2}\gamma_{ab}$$

$$[M_{IJ}, Q_\alpha] = -i(m_{IJ})_\alpha^\beta Q_\beta, \text{ i.e. } [M_{ab}, Q] = -\frac{i}{2}\gamma_{ab}Q, [P_a, Q] = -\frac{i}{2}\gamma_a Q$$

$$\{Q_\alpha, Q_\beta\} = -im_{\alpha\beta}^{IJ}\mathcal{M}_{IJ}, \text{ which can be split to } \{Q_\alpha, Q_\beta\} = -\frac{i}{2}(\gamma^{ab})_{\alpha\beta}\mathcal{M}_{ab} + i\gamma^a\mathcal{P}_a$$

$$[b_1, \{f_2, f_3\}] + \{f_2, [f_3, b_1]\} - \{f_3, [b_1, f_2]\} = 0$$

$$\kappa^2 = \frac{4\pi G}{\ell}$$

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}e_\mu^a\mathcal{P}_a + \kappa\bar{\psi}_\mu^\alpha Q_\alpha$$

$$\mathbb{F}_{\mu\nu} = \frac{1}{2}F_{\mu\nu}^{(s)IJ}\mathcal{M}_{IJ} + \overline{\mathcal{F}}_{\mu\nu}^\alpha Q_\alpha = \frac{1}{2}F_{\mu\nu}^{(s)ab}\mathcal{M}_{ab} + F_{\mu\nu}^{(s)a}\mathcal{P}_a + \overline{\mathcal{F}}_{\mu\nu}^\alpha Q_\alpha$$

$$\begin{aligned} F_{\mu\nu}^{(s)ab} &= F_{\mu\nu}^{ab} - \kappa^2\bar{\psi}_\mu\gamma^{ab}\psi_\nu \\ F_{\mu\nu}^{(s)a} &= F_{\mu\nu}^a + \kappa^2\bar{\psi}_\mu\gamma^a\psi_\nu \end{aligned}$$

$$F_{\mu\nu}^{ab} = R_{\mu\nu}^{ab} + \tfrac{1}{\ell^2}\left(e_\mu^a e_\nu^b - e_\nu^a e_\mu^b\right) \ell F_{\mu\nu}^a = D_\mu^\omega e_\nu^a - D_\nu^\omega e_\mu^a.$$

$$\begin{aligned} \mathcal{D}_\mu\psi_\nu &= \partial_\mu\psi_\nu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\psi_\nu + \frac{1}{2\ell}e_\mu^a\gamma_a\psi_\nu = \mathcal{D}_\mu^\omega\psi_\nu + \frac{1}{2\ell}e_\mu^a\gamma_a\psi_\nu \\ \mathcal{D}_\mu\bar{\psi}_\nu &= \partial_\mu\bar{\psi}_\nu - \frac{1}{4}\omega_\mu^{ab}\bar{\psi}_\nu\gamma_{ab} - \frac{1}{2\ell}e_\mu^a\bar{\psi}_\nu\gamma_a = \mathcal{D}_\mu^\omega\bar{\psi}_\nu - \frac{1}{2\ell}e_\mu^a\gamma_a\bar{\psi}_\nu \end{aligned}$$

$$\mathcal{F}_{\mu\nu} = \kappa(D_\mu\psi_\nu - D_\nu\psi_\mu) = \kappa\left(\mathcal{D}_\mu^\omega\psi_\nu - \mathcal{D}_\nu^\omega\psi_\mu + \frac{1}{2\ell}(e_\mu^a\gamma_a\psi_\nu - e_\nu^a\gamma_a\psi_\mu)\right)$$

$$\delta_\Theta\mathbb{A}_\mu = \partial_\mu\Theta - i[\mathbb{A}_\mu, \Theta] \equiv D_\mu^{\mathbb{A}}\Theta$$

$$\Theta = \frac{1}{2}\lambda^{ab}\mathcal{M}_{ab} + \xi^a\mathcal{P}_a + \bar{\epsilon}^\alpha Q_\alpha$$

$$\delta_\epsilon e_\mu^a = -\ell\kappa\bar{\epsilon}\gamma^a\psi, \delta_\epsilon\omega_\mu^{ab} = \kappa\bar{\epsilon}\gamma^{ab}\psi\mu, \delta_\epsilon\bar{\psi}_\mu = \frac{1}{\kappa}\left(\mathcal{D}_\mu^\omega\bar{\epsilon} - \frac{1}{2l}e_\mu^a\bar{\epsilon}\gamma_a\right),$$

$$D_\mu^\omega\bar{\epsilon} = \partial_\mu\bar{\epsilon} - \frac{1}{4}\omega_\mu^{ab}\bar{\epsilon}\gamma_{ab}$$

$$\delta\mathbb{F}_{\mu\nu} = D_\mu\delta\mathbb{A}_\nu - D_\nu\delta\mathbb{A}_\mu = [D_\mu, D_\nu]\Theta = i[\Theta, \mathbb{F}_{\mu\nu}],$$

$$\delta_\epsilon F^{(s)a4} = -\bar{\epsilon}\gamma^a \mathcal{F}\ \delta_\epsilon F^{(s)ab} = \bar{\epsilon}\gamma^{ab}\mathcal{F}$$

$$\delta_\epsilon \overline{\mathcal{F}} = -\frac{1}{4}\bar{\epsilon}\gamma^{ab}F^{(s)}_{ab} - \frac{1}{2}\bar{\epsilon}\gamma_a F^{(s)a}$$

$$16\pi\mathcal{L}^{(\text{sugra-topological })} = 16\pi\left(\mathcal{L}^{(\text{sugra-topological, },b)} - 4\mathcal{L}^{(\text{sugra-topopological, },f)}\right)$$

$$=\epsilon^{\mu\nu\rho\sigma}\left(B^{(s)IJ}_{\mu\nu}F^{(s)}_{\rho\sigma}IJ - \frac{\beta}{2}B^{(s)IJ}_{\mu\nu}B^{(s)}_{\rho\sigma IJ}\right) - 4\epsilon^{\mu\nu\rho\sigma}\left(\overline{\mathcal{B}}_{\mu\nu}\mathcal{F}_{\rho\sigma} - \frac{\beta}{2}\overline{\mathcal{B}}_{\mu\nu}\mathcal{B}_{\rho\sigma}\right)$$

$$\delta_\epsilon B^{(s)a4} = -\bar{\epsilon}\gamma^a\mathcal{B}\ \delta_\epsilon B^{(s)ab} = \bar{\epsilon}\gamma^{ab}\mathcal{B}, \delta_\epsilon\overline{\mathcal{B}} = -\frac{1}{4}\bar{\epsilon}\gamma^{ab}B^{(s)}_{ab} - \frac{1}{2}\bar{\epsilon}\gamma_a B^{(s)a}$$

$$16\pi\mathcal{L}^{\text{sugra-gb}} = -\frac{\alpha}{4}\epsilon^{\mu\nu\rho\sigma}\left(\epsilon_{abcd}B^{(s)ab}_{\mu\nu}B^{(s)cd}_{\rho\sigma} - 8\overline{\mathcal{B}}_{\mu\nu}\gamma^5\mathcal{B}_{\rho\sigma}\right)$$

$$2\alpha\epsilon^{\mu\nu\rho\sigma}B^{(s)a}_{\mu\nu}\bar{\epsilon}\gamma_a\gamma^5\mathcal{B}_{\rho\sigma}$$

$$B^{(s)a}_{\rho\sigma} = \frac{1}{\beta}\left(F^a_{\rho\sigma} + \kappa^2\bar{\psi}_\rho\gamma^a\psi_\sigma\right)$$

$$B^{(s)ab}_{\rho\sigma} = \frac{\beta}{\alpha^2+\beta^2}\left(F^{ab}_{\rho\sigma} - \kappa^2\bar{\psi}_\rho\gamma^{ab}\psi_\sigma\right) - \frac{\alpha}{2(\alpha^2+\beta^2)}\left(F^{cd}_{\rho\sigma} - \kappa^2\bar{\psi}_\rho\gamma^{cd}\psi_\sigma\right)\epsilon^{ab}_{cd}$$

$$\mathcal{B} = \frac{1}{\alpha^2+\beta^2}(\beta\mathbb{1} - \alpha\gamma^5)\mathcal{F}$$

$$16\pi\mathcal{L}^f = \epsilon^{\mu\nu\rho\sigma}\frac{\alpha}{(\alpha^2+\beta^2)}\overline{\mathcal{F}}_{\mu\nu}\left(\frac{\beta\mathbb{1} - \alpha\gamma^5}{2\alpha}\right)\mathcal{F}_{\rho\sigma}$$

$$16\pi\mathcal{L}^b = \epsilon^{\mu\nu\rho\sigma}\left(\frac{1}{\beta}F^{(s)a4}{}_{\mu\nu}F^{(s)}_{a4}{}_{\rho\sigma} + \frac{1}{4}M^{abcd}F^{(s)}_{ab}{}_{\mu\nu}F^{(s)}_{cd}{}_{\rho\sigma}\right)$$

$$\overline{\mathcal{F}}_{\mu\nu}\left(\frac{\mathbb{1}\beta - \gamma^5\alpha}{2\alpha}\right)\mathcal{F}_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 4\frac{\kappa^2}{2}(D_\mu\bar{\psi}_\nu)(\gamma\mathbb{1} - \gamma^5)(D_\rho\psi_\sigma)\epsilon^{\mu\nu\rho\sigma}$$

$$\overline{\mathcal{F}}_{\mu\nu}\left(\frac{\mathbb{1}\beta - \gamma^5\alpha}{2\alpha}\right)\mathcal{F}_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = \frac{\kappa^2}{4}\bar{\psi}_\mu(\gamma\mathbb{1} - \gamma^5)\left(\gamma_{ab}F^{ab}_{\nu\rho} + \gamma_a\frac{2}{\ell}T^a_{\nu\rho}\right)\psi_\sigma\epsilon^{\mu\nu\rho\sigma}$$
$$+\kappa^2\bar{\psi}_\mu\left(\frac{1}{\ell^2}\gamma^5\gamma_{ab}e^a_\nu e^b_\rho + \frac{2}{\ell}\gamma^5\gamma_a e^a_\nu\mathcal{D}^\omega_\rho\right)\psi_\sigma\epsilon^{\mu\nu\rho\sigma}$$

$$16\pi\mathcal{L} = -\left(\frac{\kappa^2}{G}\bar{\psi}_\mu\gamma^5\gamma_{ab}e^a_\nu e^b_\rho + \frac{2\kappa^2\ell}{G}\bar{\psi}_\mu\gamma^5\gamma_a e^a_\nu\mathcal{D}^\omega_\rho\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$
$$-\bar{\psi}_\mu\left(\frac{1}{4\beta}\frac{2\kappa^2}{\ell}\gamma_a T^a_{\nu\rho} + \frac{2\kappa^2\ell}{4G}(\gamma\mathbb{1} - \gamma^5)\gamma_a T^a_{\nu\rho}\right)\psi_\sigma\epsilon^{\mu\nu\rho\sigma}$$

$$-\frac{1}{4\beta}\left(\frac{1}{\ell^2}T^a_{\mu\nu}T_{\rho\sigma a} + \kappa^4\bar{\psi}_\mu\gamma^a\psi_\nu\bar{\psi}_\rho\gamma_a\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+\frac{1}{16}M_{abcd}\left(F^{ab}_{\mu\nu}F^{cd}_{\rho\sigma} + \kappa^4\bar{\psi}_\mu\gamma^{ab}\psi_\nu\bar{\psi}_\rho\gamma^{cd}\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+\xi.$$

$$\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\Gamma^A\psi_\nu = 0,\ \text{for}\ \Gamma^A = (1, \gamma^5, \gamma^5\gamma^a)$$

$$16\pi\mathcal{L} = \left(\frac{1}{16}M_{abcd}F^{ab}_{\mu\nu}F^{cd}_{\rho\sigma} - \frac{1}{4\beta\ell^2}T^{a}_{\mu\nu}T_{\rho\sigma a}\right)\epsilon^{\mu\nu\rho\sigma}$$
$$-\left(\frac{\kappa^2}{G}\bar{\psi}_\mu\gamma^5\gamma_{ab}e^a_\nu e^b_\rho + \frac{2\kappa^2\ell}{G}\bar{\psi}_\mu\gamma^5\gamma_a e^a_\nu \mathcal{D}^\omega_\rho\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+\frac{\kappa^2\ell}{2\gamma G}\bar{\psi}_\mu\gamma_a\psi_\nu T^a_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} + \xi$$

$$\mathcal{L}^{\text{sugra}} = \frac{1}{64\pi G}\left(R^{ab}_{\mu\nu}e^c_\rho e^d_\sigma + \frac{1}{\ell^2}e^a_\mu e^a_\nu e^c_\rho e^d_\sigma\right)\epsilon_{abcd}\epsilon^{\mu\nu\rho\sigma}$$
$$+\left(\frac{1}{2}\bar{\psi}_\mu\gamma_5\gamma_a e^a_\nu D^\omega_\rho\psi_\sigma + \frac{1}{4\ell}\bar{\psi}_\mu\gamma_5\gamma_{ab}e^a_\nu e^b_\rho\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$

$$\mathcal{L}^{add} = \frac{1}{\gamma}\left(\frac{2}{64\pi G}R^{ab}_{\mu\nu}e_{\rho a}e_{\sigma b} + \frac{1}{4}\bar{\psi}_\mu\gamma_a\psi_\nu D^\omega_\rho e^a_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$

$$\frac{32\pi G}{\ell^2}\mathcal{L}^{\text{boundary}} = \partial_\mu\left[\left(\frac{1}{\gamma}+i\right)\mathcal{SC}^\mu({}^+\omega) + \left(\frac{1}{\gamma}-i\right)\mathcal{SC}^\mu({}^-\omega) - \left(\frac{1}{\gamma}+\gamma\right)\mathcal{SC}^\mu(A)\right]$$
$$+4\kappa^2\partial_\mu\left[\bar{\psi}_\nu\frac{1}{2}\left(\frac{1}{\gamma}\mathbb{I}-\gamma^5\right)\frac{1}{2}A^{IJ}_\rho m_{IJ}\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}\Big]$$

$$\mathcal{SC}^\mu(A) = \left(A_{\nu IJ}\partial_\rho A^{IJ}_\sigma + \frac{2}{3}A_{\nu IJ}A^I_{\rho K}A^{KJ}_\sigma\right)\epsilon^{\mu\nu\rho\sigma} + 4\kappa^2(\bar{\psi}_\nu D^A_\rho\psi_\sigma)\epsilon^{\mu\nu\rho\sigma}$$
$$= \mathcal{C}^\mu(A) + 4\kappa^2(\bar{\psi}_\nu D^A_\rho\psi_\sigma)\epsilon^{\mu\nu\rho\sigma}$$
$$\mathcal{SC}^\mu({}^\pm\omega) = \mathcal{C}^\mu({}^\pm\omega) + 4\kappa^2({}^\pm\bar{\psi}_\nu D^\pm_\rho\omega \pm \psi_\sigma)\epsilon^{\mu\nu\rho\sigma}$$

$$\delta I = \delta e\left.\frac{\delta I}{\delta e}\right|_{\psi,\omega(e,\psi)} + \delta\psi\left.\frac{\delta I}{\delta\psi}\right|_{e,\omega(e,\psi)} + \left.\frac{\delta I}{\delta\omega}\right|_{e,\psi}\left(\delta e\frac{\delta\omega(e,\psi)}{\delta e} + \delta\psi\frac{\delta\omega(e,\psi)}{\delta\psi}\right)$$

$$\left(\frac{1}{\ell}T^a_{\mu\nu} + \kappa^2\bar{\psi}_\mu\gamma^a\psi_\nu\right)e^b_\rho\left(\frac{1}{\gamma}\delta_{abcd} + \epsilon_{abcd}\right)\epsilon^{\mu\nu\rho\sigma}\delta\omega^{cd}_\sigma = 0$$

$$T^a_{\mu\nu} = -4\pi G\bar{\psi}_\mu\gamma^a\psi_\nu$$

$$\partial_\mu e^a_\nu - \partial_\nu e^a_\mu + \omega_\mu{}^a{}_b e^b_\nu - \omega_\nu{}^a{}_b e^b_\mu + 4\pi G\bar{\psi}_\mu\gamma^a\psi_\nu = 0$$

$$\frac{1}{4\gamma}\epsilon^{\mu\nu\rho\sigma}\delta\bar{\psi}_\mu\gamma_a\psi_\nu D^\omega_\rho e^a_\sigma$$

$$\epsilon^{\mu\nu\rho\sigma}\gamma_a\psi_\nu\bar{\psi}_\rho\gamma^a\psi_\sigma = 0$$

$$\frac{1}{\gamma}\left(\frac{2}{8\pi G}R^{ab}_{\mu\nu}e_{\rho a}\delta e_{\sigma b} + \bar{\psi}_\mu\gamma_a\psi_\nu D^\omega_\rho\delta e^a_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$

$$\epsilon^{\mu\nu\rho\sigma}R^{ab}_{\mu\nu}e_{\rho a} = -2\epsilon^{\mu\nu\rho\sigma}D^\omega_\mu D^\omega_\nu e^b_\rho$$

$$\frac{1}{\gamma}\left(\frac{1}{4\pi G}T^b_{\nu\rho}D^\omega_\mu\delta e_{\sigma b} + \bar{\psi}_\mu\gamma_a\psi_\nu D^\omega_\rho\delta e_\sigma{}^a\right)\epsilon^{\mu\nu\rho\sigma}$$

$$dM = \frac{\kappa}{8\pi G} dA + \Omega dJ \text{ (black hole dynamics)}$$
$$dE = TdS + dW \text{ (thermodynamics),}$$

$$\text{Entropy } = \frac{\text{Area}}{4l_p^2}, \text{ where } l_p = \sqrt{G\hbar/c^3},$$

$$\text{Temperature } = \hbar c \frac{\kappa}{2\pi}$$

$$S_{LQG} = \frac{\gamma_M}{\gamma} \frac{\text{Area}}{4G}$$

$$\delta\mathcal{L}(\varphi, \partial\varphi) = (\text{ f.e. }) \cdot \delta\varphi + d\Theta$$

$$J[\xi] = \Theta[\varphi, L_\xi \varphi] - I_\xi \mathcal{L}$$

$$I_\xi \alpha_p = \frac{1}{(p-1)!} \xi^\mu \alpha_{\mu v^1 \dots v^{p-1}} dx^{v^1} \wedge \dots \wedge dx^{v^{p-1}}$$

$$Q[\xi_t + \Omega \xi_\varphi]_H = \frac{\kappa}{2\pi} \text{ Entropy}$$

$$\text{Mass } = Q[\xi_t]_\infty = \frac{1}{2} M + \lim_{r\to\infty} \frac{r^3}{2G\ell^2}.$$

$$32\pi GS = \int R^{ab} \wedge e^c \wedge e^d \epsilon_{abcd} + \frac{1}{2\ell^2} \int e^a \wedge e^b \wedge e^c \wedge e^d \epsilon_{abcd} + \rho \int R^{ab} \wedge R^{cd} \epsilon_{abcd}$$

$$\text{Mass } = Q[\partial_t]_\infty = \frac{M}{2}\left(1 + \frac{2}{\ell^2}\rho\right) + \lim_{r\to\infty} \frac{r^3}{2G\ell^2}\left(1 - \frac{2}{\ell^2}\rho\right).$$

$$\left(R^{ab}(\omega) + \frac{1}{\ell^2} e^a \wedge e^b\right)\Big|_\infty = 0$$

$$\delta S_{\left(\text{Einstein / Cartan } + \Lambda + \frac{\ell^2}{2} \text{ Euler}\right)} = \int_{\mathcal{M}} (\text{ f.e. })_a \delta e^a + \int_{\mathcal{M}} (\text{ f.e. })_{ab} \delta\omega^{ab} + \int_{\mathcal{M}} d\Theta = 0$$

$$\Theta|_{\partial\mathcal{M}} = \epsilon_{abcd} \delta\omega^{ab} \wedge \left(R^{cd}(\omega) + \frac{1}{\ell^2} e^c \wedge e^d\right)\Big|_{\partial\mathcal{M}} = 0$$

$$\int_{\partial M} \delta A^{IJ} \wedge B_{IJ} \to \int_{\partial M} \delta\omega^{ab} \wedge B_{ab} = \int_{\partial M} \delta\omega^{ab} \wedge M_{abcd} F^{ab} = 0$$

$$\left[\mathfrak{E}_{\text{Einstein}}/\mathfrak{C}_{\text{Cartan}} \text{ with } \Lambda + \frac{\ell^2}{2} \mathbb{E}_{\text{Euler}}\right] - \gamma\left[\mathfrak{H}_{\text{Holst}} + \frac{\ell^2}{2} \mathfrak{P}_{\text{Pontryagin}}\right],$$

$$16\pi\delta S = \int \left(\delta B^{IJ} \wedge \left(F_{IJ} - \beta B_{IJ} - \frac{\alpha}{2} B^{KL} \epsilon_{IJKL4}\right) + \right.$$
$$\left. + \delta A_{IJ} \wedge (D^A B^{IJ}) + d(B^{IJ} \wedge \delta A_{IJ})\right)$$

$$\Theta = B^{IJ} \wedge \delta A_{IJ}$$

$$J[\xi] = \Theta[\phi, L_\xi \phi] - I_\xi \mathcal{L}, J[\xi] = B^{IJ} \wedge L_\xi A_{IJ} - I_\xi \mathcal{L}$$

$$I_\xi \alpha_p = \frac{1}{(p-1)!} \xi^\mu \alpha_{\mu\nu^1 \ldots \nu^{p-1}} dx^{\nu^1} \wedge \ldots \wedge dx^{\nu^{p-1}}$$

$$16\pi J[\xi] = \left(F_{IJ} - \beta B_{IJ} - \frac{\alpha}{2} B^{KL} \epsilon_{IJKL4}\right) \wedge I_\xi B_{IJ}$$
$$+ I_\xi A_{IJ} \wedge (D^A B^{IJ}) + d\left(B^{IJ} \wedge I_\xi A_{IJ}\right)$$

$$Q[\xi] = \frac{1}{16\pi} \int_{\partial\Sigma} B^{IJ} I_\xi A_{IJ}$$

$$Q = \frac{1}{16\pi} \int_{\partial\Sigma} \left(\frac{1}{2} M^{ab}_{cd} F_{ab} I_\xi \omega^{cd} - \frac{2}{\beta \ell^2} T_a I_\xi e^a\right)$$

$$Q[\xi] = \frac{1}{16\pi} \int_{\partial\Sigma} \frac{\delta \mathcal{L}}{\delta F^{IJ}} I_\xi A^{IJ} + (\text{ f.e. })_{IJ} I_\xi A^{IJ}$$

$$Q[\xi] = \frac{1}{16\pi} \int_{\partial\Sigma} I_\xi \omega^{cd} \left(\frac{1}{2} M^{ab}_{cd} F_{ab}\right)$$

$$Q[\xi] = \frac{\ell^2}{32\pi G} \int_{\partial\Sigma} I_\xi \omega_{ab} \left(\epsilon^{ab}{}_{cd} F^{cd}_{\theta\varphi} - 2\gamma F^{ab}_{\theta\varphi}\right) d\theta d\varphi$$

$$ds^2 = -f(r)^2 dt^2 + f(r)^{-2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), f(r)^2 = \left(1 - \frac{2GM}{r} + \frac{r^2}{\ell^2}\right)$$

$$Q[\partial_t] = \frac{4\ell^2}{32\pi G} \int_{\partial\Sigma} \omega^{01}_t \left(\epsilon_{0123} F^{23}_{\theta\varphi} - \gamma F_{\theta\varphi 01}\right) d\theta d\varphi$$

$$Q[\partial_t] = \frac{4\ell^2}{32\pi G} \int_{\partial\Sigma} \left(\frac{1}{2} \frac{\partial f(r)^2}{\partial r}\right) \left(1 - f(r)^2 + \frac{r^2}{\ell^2}\right) \sin \theta d\theta d\varphi$$

$$Q[\xi]_\infty = \lim_{r \to \infty} \frac{1}{4\pi} \int_{\partial\Sigma_\infty} \left(M + \frac{\ell^2 G M^2}{r^3}\right) \sin \theta d\theta d\varphi = M$$

$$I_\xi \omega^a_b \xi^b = \kappa \xi^a$$

$$\kappa = \omega^{01}_t \Big|_{r_H} = \left(\frac{1}{2} \frac{\partial f(r)^2}{\partial r}\right)\Bigg|_{r_H} \quad T = \frac{\kappa}{2\pi},$$

$$f(r_H)^2 = 0, \frac{r_H^3}{\ell^2} + r_H - 2GM = 0$$

$$Q[\xi_t]_H = \frac{\kappa \ell^2}{8\pi G} \left(1 + \frac{r_H^2}{\ell^2}\right) \int_{\partial\Sigma_H} \sin \theta d\theta d\varphi = \frac{\kappa}{2\pi} \frac{4\pi (r_H^2 + \ell^2)}{4G},$$

$$\text{Entropy} = \frac{\text{Area}}{4G} + \frac{4\pi \ell^2}{4G}.$$

$$f(r)^2 = \left(k - \frac{2GM}{r}\frac{4\pi}{\Sigma_k} + \frac{r^2}{\ell^2}\right)$$

$$d\Sigma_k = \begin{cases} \sinh\ \theta d\theta d\phi & \text{for} \quad k = -1 \\ d\theta d\phi & \text{for} \quad k = 0 \\ \sin\ \theta d\theta d\phi & \text{for} \quad k = 1 \end{cases}$$

$$Q[\xi] = \frac{4\ell^2}{32\pi G}\int_{\partial\Sigma}\left(\frac{1}{2}\frac{\partial f(r)^2}{\partial r}\right)\left(k - f(r)^2 + \frac{r^2}{\ell^2}\right)d\Sigma_k$$

$$Q[\xi_t]_\infty = \frac{M}{\Sigma_k}\int_{\partial\Sigma_\infty} d\Sigma_k = M, Q[\xi]_H = \frac{\kappa}{2\pi}\frac{(\ell^2 k + r_H^2)}{4G}\int_{\partial\Sigma_H} d\Sigma_k$$

$$\text{Entropy } = \frac{\text{Area}}{4G} + \frac{4\pi\ell^2 k}{4G},$$

$$e^0 = \frac{\sqrt{\Delta_r}}{\rho}\left(dt - \frac{a}{\Xi}\sin^2\ \theta d\varphi\right), e^1 = \rho\frac{dr}{\sqrt{\Delta_r}}$$

$$e^2 = \rho\frac{d\theta}{\sqrt{\Delta_\theta}}, e^3 = \frac{\sqrt{\Delta_\theta}}{\rho}\sin\ \theta\left(\frac{(r^2 + a^2)}{\Xi}d\varphi - adt\right)$$

$$\rho^2 = r^2 + a^2\cos^2\ \theta, \Delta_r = (r^2 + a^2)\left(1 + \frac{r^2}{l^2}\right) - 2MGr$$

$$\Delta_\theta = 1 - \frac{a^2}{l^2}\cos^2\ \theta, \Xi = 1 - \frac{a^2}{l^2}$$

$$\tilde{\Omega} = -\frac{g_{t\varphi}}{g_{\varphi\varphi}} = \frac{a\Xi(\Delta_\theta(r^2 + a^2) - \Delta_r)}{(r^2 + a^2)^2\Delta_\theta - a^2\Delta_r\sin^2\ \theta}$$

$$\Omega_H = \frac{a\left(1 - \frac{a^2}{\ell^2}\right)}{r_H^2 + a^2}$$

$$Q\left[\frac{\partial}{\partial t}\right] = \frac{M}{\Xi}, Q\left[\frac{\partial}{\partial\varphi}\right] = \frac{Ma}{\Xi^2}$$

$$\text{Mass } = Q\left[\partial_t - \frac{a}{l^2}\partial_\varphi\right] = \frac{M}{\Xi^2}$$

$$\Omega = \Omega_H - \Omega_\infty = \frac{a\left(1 + \frac{r_H^2}{\ell^2}\right)}{r_H^2 + a^2}$$

$$\kappa = \frac{r_H\left(\frac{a^2}{l^2} - \frac{a^2}{r_H^2} + \frac{3r_H^2}{l^2} + 1\right)}{2(a^2 + r_H^2)}, \text{ Entropy } = \frac{4\pi(r_H^2 + a^2)}{4G\left(1 - \frac{a^2}{l^2}\right)} + \frac{4\pi l^2}{4G},$$

$$\omega_\chi^{ab} = e^{\nu a}\nabla_\chi e_\nu^b = e^{\nu a}\left(\partial_\chi e_\nu^b - \Gamma_{\chi\nu}^\lambda e_\lambda^b\right)$$

$$Q[\partial_\chi]=\frac{2}{32\pi G}\int_{\partial\Sigma}\left(\epsilon^{\mu\nu}{}_{\theta\varphi}\Gamma_{\mu\chi\nu}-\gamma\left(\Gamma_{\theta\chi\varphi}-\Gamma_{\varphi\chi\theta}\right)\right)$$
$$+\frac{\ell^2}{32\pi G}\int_{\partial\Sigma}\left(\epsilon^{\mu\nu\rho\sigma}R_{\rho\sigma\theta\varphi}\Gamma_{\mu\chi\nu}-2\gamma R^{\mu\nu}_{\theta\varphi}\Gamma_{\mu\chi\nu}\right)$$

$$-\gamma\int_{\partial\Sigma}\left(\Gamma_{\theta t\varphi}-\Gamma_{\varphi t\theta}\right)=\gamma\int_{\partial\Sigma}\partial_\theta g_{t\varphi}$$

$$e^0=f(r)dt+2nf(r)\cos\,\theta d\varphi\; e^1=\frac{1}{f(r)}dr,$$
$$e^2=\sqrt{n^2+r^2}d\theta, e^3=\sqrt{n^2+r^2}\sin\,\theta d\varphi,$$

$$ds^2=-f(r)^2(dt+2n\cos\,\theta d\varphi)^2+\frac{dr^2}{f(r)^2}+(n^2+r^2)d\Omega^2$$

$$f(r)^2=\frac{r^2-2GMr-n^2+(r^4+6n^2r^2-3n^4)\ell^{-2}}{n^2+r^2},$$

$$Q[\partial_t]=\frac{f(r)f'(r)(n^2+r^2)}{2G}+\gamma\frac{nf(r)^2}{2G}$$
$$+\frac{\ell^2}{2G}\left(f(r)f'(r)\left(1+\frac{5n^2-r^2}{n^2+r^2}f(r)^2\right)-\frac{2rn^2f(r)^4}{(r^2+n^2)^2}\right)$$
$$+\gamma\frac{\ell^2n}{2G}\left(-2(f(r)f'(r))^2+\frac{2rf(r)^2}{r^2+n^2}f(r)f'(r)+\frac{f(r)^2}{r^2+n^2}\left(1+\frac{3n^2-r^2}{r^2+n^2}f(r)^2\right)\right)$$

$$Mass=M+\gamma\frac{n(\ell^2+4n^2)}{G\ell^2}$$

$$Q[\partial_t]_{\text{Einstein + Holst}}=\frac{f(r)f'(r)(n^2+r^2)}{2G}+\frac{\gamma nf(r)^2}{2G}$$

$$\kappa=\left(\frac{1}{2}\frac{\partial f(r)^2}{\partial r}\right)\Bigg|_{r_H}=\frac{1}{2}\left(\frac{1}{r_H}+\frac{3(n^2+r_H^2)}{\ell^2r_H}\right)$$

$$Q[\partial_t]_H=\frac{\kappa}{2\pi}\frac{4\pi(r_H^2+n^2+\ell^2(1-2n\gamma\kappa))}{4G}$$

$$\text{Entropy}=\frac{\text{Area}}{4G}+\frac{4\pi\ell^2}{4G}-\gamma\frac{2\pi n\ell^2}{G}\kappa$$

$$\text{Entropy}=\frac{\text{Area}}{4G}\left(1-\gamma\frac{3n}{r_H}\right)+\frac{4\pi\ell^2}{4G}\left(1-\gamma\frac{n}{r_H}\right)$$

$$[\mathcal{P}_a,\mathcal{P}_b]=-i\mathcal{Z}_{ab}$$

$$[\mathcal{M}_{ab},\mathcal{Z}_{cd}]=-i(\eta_{ac}\mathcal{Z}_{bd}+\eta_{bd}\mathcal{Z}_{ac}-\eta_{ad}\mathcal{Z}_{bc}-\eta_{bc}\mathcal{Z}_{ad}).$$

$$
\begin{aligned}
[\mathcal{P}_a, \mathcal{P}_b] &= i(\mathcal{M}_{ab} - \mathcal{Z}_{ab}),\\
[\mathcal{M}_{ab}, \mathcal{M}_{cd}] &= -i(\eta_{ac}\mathcal{M}_{bd} + \eta_{bd}\mathcal{M}_{ac} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad}),\\
[\mathcal{M}_{ab}, \mathcal{Z}_{cd}] &= -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}),\\
[\mathcal{Z}_{ab}, \mathcal{Z}_{cd}] &= -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}),\\
[\mathcal{M}_{ab}, \mathcal{P}_c] &= -i(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a),\\
[\mathcal{Z}_{ab}, \mathcal{P}_c] &= 0.
\end{aligned}
$$

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}e_\mu^a\mathcal{P}_a + \frac{1}{2}h_\mu^{ab}\mathcal{Z}_{ab}$$

$$\mathbb{F}_{\mu\nu} = \partial_\mu\mathbb{A}_\nu - \partial_\nu\mathbb{A}_\mu - i[\mathbb{A}_\mu, \mathbb{A}_\nu]$$

$$\mathbb{F}_{\mu\nu} = \frac{1}{2}F_{\mu\nu}^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}T_{\mu\nu}^a\mathcal{P}_a + \frac{1}{2}G_{\mu\nu}^{ab}\mathcal{Z}_{ab}$$

$$G_{\mu\nu}^{ab} = D_\mu^\omega h_\nu^{ab} - D_\nu^\omega h_\mu^{ab} - \frac{1}{\ell^2}\left(e_\mu^a e_\nu^b - e_\nu^a e_\mu^b\right) + \left(h_\mu^{ac}h_{\nu c}{}^b - h_\nu^{ac}h_{\mu c}{}^b\right),$$

$$G^{ab} = \frac{1}{2}G_{\mu\nu}^{ab}dx^\mu \wedge dx^\nu = dh^{ab} + \omega^{ac}\wedge h_c{}^b + \omega^{bc}\wedge h^a{}_c - \frac{1}{\ell^2}e^a \wedge e^b + h^{ac}\wedge h_c{}^b.$$

$$\epsilon^{\mu\nu\rho\sigma}D_\mu^{\mathbb{A}}\mathbb{F}_{\nu\rho}(\mathbb{A}) = 0$$

$$
\begin{aligned}
\epsilon^{\mu\nu\rho\sigma}D_\mu^\omega R_{\nu\rho}^{ab} &= 0\\
\epsilon^{\mu\nu\rho\sigma}\left(D_\mu^\omega T_{\nu\rho}^a - R_{\mu\nu}^{ab}e_{\rho b}\right) &= 0\\
\epsilon^{\mu\nu\rho\sigma}\left(D_\mu^{(\omega+h)}G_{\nu\rho}^{ab} + 2h_\mu^{ac}F_{\nu\rho c}{}^b - \frac{2}{\ell^2}e_\mu^a T_{\mu\nu}^b\right) &= 0
\end{aligned}
$$

$$\epsilon^{\mu\nu\rho\sigma}D_\mu^{(\omega+h)}\left(G_{\nu\rho}^{ab} + F_{\nu\rho}^{ab}\right) = \epsilon^{\mu\nu\rho\sigma}D_\mu^{(\omega+h)}R_{\nu\rho}^{ab}(\omega+h) = 0$$

$$\delta_\Theta\mathbb{A}_\mu = \partial_\mu\Theta - i[\mathbb{A}_\mu, \Theta] \equiv D_\mu^{\mathbb{A}}\Theta, \delta_\Theta\mathbb{F}_{\mu\nu} = i[\Theta, \mathbb{F}_{\mu\nu}]$$

$$\Theta = \frac{1}{2}\lambda^{ab}\mathcal{M}_{ab} + \xi^a\mathcal{P}_a + \frac{1}{2}\tau^{ab}\mathcal{Z}_{ab}$$

$$
\begin{aligned}
&\delta_\Theta h_\mu^{ab} = D_\mu^\omega\tau^{ab} - \frac{1}{\ell}\left(e_\mu^a\xi^b - e_\mu^b\xi^a\right) + h_\mu^{ac}\left(\lambda_c^b + \tau_c^b\right) + h_\mu^{bc}(\lambda_c^a + \tau_c^a)\\
&\delta_\Theta\omega_\mu^{ab} = D_\mu^\omega\lambda^{ab} + \frac{1}{\ell}\left(e_\mu^a\xi^b - e_\mu^b\xi^a\right)\\
&\frac{1}{\ell}\delta_\Theta e_\mu^a = D_\mu^\omega\xi^a - \frac{1}{\ell}\lambda_b^a e_\mu^b
\end{aligned}
$$

$$
\begin{aligned}
\delta_\Theta G_{\mu\nu}^{ab} &= \frac{1}{\ell}[\xi, T_{\mu\nu}]^{ab} - [\tau, F_{\mu\nu}]^{ab} - [(\lambda+\tau), G_{\mu\nu}]^{ab}\\
\delta_\Theta F_{\mu\nu}^{ab} &= -\frac{1}{\ell}[\xi, T_{\mu\nu}]^{ab} - [\lambda, F_{\mu\nu}]^{ab}
\end{aligned}
$$

$$\frac{1}{\ell}\delta_\Theta T_{\mu\nu}^a = -\frac{1}{\ell}\lambda_b^a T_{\mu\nu}^b + \xi_b F_{\mu\nu}^{ab}$$

$$2B_a \wedge T^a + B_{ab}\wedge F^{ab} + C_{ab}\wedge G^{ab}, \text{ with } B^a = B^{a4}.$$

$$\delta_\xi B^{ab} = \left(B^a\xi^b - B^b\xi^a\right), \delta_\xi C^{ab} = 0, \delta_\xi B^a = \left(B^{ab} - C^{ab}\right)\xi_b$$
$$\delta_\lambda B^{ab} = -[\lambda, B]^{ab}, \delta_\lambda C^{ab} = -[\lambda, C]^{ab}, \delta_\lambda B^a = -\lambda^a_b B^b$$
$$\delta_\tau B^{ab} = -[\tau, C]^{ab}, \delta_\tau C^{ab} = -[\tau, C]^{ab}, \delta_\tau B^a = 0$$

$$C^{ab} \wedge C_{ab} \text{ and } 2B^a \wedge B_a + B^{ab} \wedge B_{ab} - 2C^{ab} \wedge B_{ab}.$$

$$\epsilon^{abcd} C_{ab} \wedge C_{cd} \text{ and } \epsilon^{abcd}(B_{ab} \wedge B_{cd} - 2C_{ab} \wedge B_{cd}).$$

$$\begin{aligned} 16\pi S(A,B) = \int \; & 2\left(B^{a4} \wedge F_{a4} - \frac{\beta}{2} B^{a4} \wedge B_{a4}\right) \\ & + B^{ab} \wedge F_{ab} - \frac{\beta}{2} B^{ab} \wedge B_{ab} - \frac{\alpha}{4}\epsilon^{abcd} B_{ab} \wedge B_{cd} \\ & + C^{ab} \wedge G_{ab} - \frac{\rho}{2} C^{ab} \wedge C_{ab} - \frac{\sigma}{4}\epsilon^{abcd} C_{ab} \wedge C_{cd} \\ & + \beta C^{ab} \wedge B_{ab} + \frac{\alpha}{2}\epsilon^{abcd} C_{ab} \wedge B_{cd} \end{aligned}$$

$$\begin{aligned} \frac{1}{\ell} T^a &= \beta B^a \\ G^{ab} &= \rho C^{ab} + \frac{\sigma}{2}\epsilon^{abcd} C_{cd} - \beta B^{ab} - \frac{\alpha}{2}\epsilon^{abcd} B_{cd} \\ F^{ab} &= \beta B^{ab} + \frac{\alpha}{2}\epsilon^{abcd} B_{cd} - \beta C^{ab} - \frac{\alpha}{2}\epsilon^{abcd} C_{cd} \end{aligned}$$

$$16\pi S(A,B) = \frac{1}{2}\int \left(B^{ab} \wedge F_{ab} + C^{ab} \wedge G_{ab} + \frac{2}{\beta} B^{a4} \wedge F_{a4}\right),$$

$$\begin{aligned} 16\pi S(\omega,h,e) = & \int \left(\frac{1}{4} M^{abcd} F_{ab} \wedge F_{cd} - \frac{1}{\beta\ell^2} T^a \wedge T_a\right) \\ & + \int \frac{1}{4} N^{abcd}(F_{ab} + G_{ab}) \wedge (F_{cd} + G_{cd}) \end{aligned}$$

$$N^{abcd} = \frac{(\sigma - \alpha)}{(\sigma - \alpha)^2 + (\rho - \beta)^2}\left(\frac{\rho - \beta}{\sigma - \alpha}\delta^{abcd} - \epsilon^{abcd}\right)$$

$$F^{ab}(\omega, e) + G^{ab}(h, e) = R^{ab}(\omega + h) \equiv d(\omega + h)^{ab} + (\omega + h)^a{}_c \wedge (\omega + h)^{cb}$$

$$[\mathcal{M}_{ab}, Q_\alpha] = -\frac{i}{2}(\gamma_{ab}Q)_\alpha$$
$$[\mathcal{M}_{ab}, \Sigma_\alpha] = -\frac{i}{2}(\gamma_{ab}\Sigma)_\alpha$$
$$[\mathcal{Z}_{ab}, Q_\alpha] = -\frac{i}{2}(\gamma_{ab}\Sigma)_\alpha$$
$$[\mathcal{Z}_{ab}, \Sigma_\alpha] = -\frac{i}{2}(\gamma_{ab}\Sigma)_\alpha$$
$$[\mathcal{P}_a, Q_\alpha] = -\frac{i}{2}\gamma_a(Q_\alpha - \Sigma_\alpha)$$
$$[\mathcal{P}_a, \Sigma_\alpha] = 0$$
$$\{Q_\alpha, Q_\beta\} = -\frac{i}{2}\left(\gamma^{ab}\right)_{\alpha\beta}\mathcal{M}_{ab} + i(\gamma^a)_{\alpha\beta}\mathcal{P}_a$$
$$\{Q_\alpha, \Sigma_\beta\} = -\frac{i}{2}\left(\gamma^{ab}\right)_{\alpha\beta}\mathcal{Z}_{ab}$$
$$\{\Sigma_\alpha, \Sigma_\beta\} = -\frac{i}{2}\left(\gamma^{ab}\right)_{\alpha\beta}\mathcal{Z}_{ab}$$

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}e_\mu^a\mathcal{P}_a + \frac{1}{2}h_\mu^{ab}\mathcal{Z}_{ab} + \kappa\bar{\psi}_\mu^\alpha Q_\alpha + \tilde{\kappa}\bar{\chi}_\mu^\alpha\Sigma_\alpha$$

$$\mathbb{F}_{\mu\nu} = \frac{1}{2}F_{\mu\nu}^{(s)ab}\mathcal{M}_{ab} + F_{\mu\nu}^{(s)a}\mathcal{M}_a + \frac{1}{2}G_{\mu\nu}^{(s)ab}\mathcal{Z}_{ab} + \overline{\mathcal{F}}_{\mu\nu}^\alpha Q_\alpha + \overline{\mathcal{G}}_{\mu\nu}^\alpha\Sigma_\alpha$$

$$G_{\mu\nu}^{(s)ab} = G_{\mu\nu}^{ab} - \tilde{\kappa}\kappa\left(\bar{\psi}_\mu\gamma^{ab}\chi_\nu + \bar{\chi}_\mu\gamma^{ab}\psi_\nu\right) - \tilde{\kappa}^2\bar{\chi}_\mu\gamma^{ab}\chi_\nu$$

$$G_{\mu\nu}^{ab} = D_\mu^\omega h_\nu^{ab} - D_\nu^\omega h_\mu^{ab} - \frac{1}{\ell^2}\left(e_\mu^a e_\nu^b - e_\nu^a e_\mu^b\right) + \left(h_\mu^{ac}h_{\nu c}^b - h_\nu^{ac}h_{\mu c}^b\right)$$

$$\mathcal{G}_{\mu\nu} = \tilde{\kappa}\Bigg(\left(\mathcal{D}_\mu^\omega\chi_\nu - \mathcal{D}_\nu^\omega\chi_\mu\right) + \frac{1}{4}\left(h_\mu^{ab}\gamma_{ab}\chi_\nu - h_\nu^{ab}\gamma_{ab}\chi_\mu\right)$$
$$+\frac{\kappa}{4\tilde{\kappa}}\left(h_\mu^{ab}\gamma_{ab}\psi_\nu - h_\nu^{ab}\gamma_{ab}\psi_\mu\right) - \frac{1}{2\ell}\frac{\kappa}{\tilde{\kappa}}\left(e_\mu^a\gamma_a\psi_\nu - e_\nu^a\gamma_a\psi_\mu\right)\Bigg)$$

$$64\pi\mathcal{L} = \left(B_{\mu\nu}^{IJ}F_{\rho\sigma IJ}^{(s)} - \frac{\beta}{2}B_{\mu\nu}^{IJ}B_{\rho\sigma IJ} - \frac{\alpha}{4}\epsilon_{abcd}B_{\mu\nu}^{ab}B_{\rho\sigma}^{cd}\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+\left(C_{\mu\nu}^{ab}G_{\rho\sigma ab}^{(s)} - \frac{\rho}{2}C_{\mu\nu}^{ab}C_{\rho\sigma ab} - \frac{\sigma}{4}\epsilon_{abcd}C_{\mu\nu}^{ab}C_{\rho\sigma}^{cd}\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+\left(\beta C_{\mu\nu}^{ab}B_{\rho\sigma ab} + \frac{\alpha}{2}\epsilon_{abcd}C_{\mu\nu}^{ab}B_{\rho\sigma}^{cd}\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+4\left(\overline{\mathcal{B}}_{\mu\nu}\mathcal{F}_{\rho\sigma} - \frac{\beta}{2}\overline{\mathcal{B}}_{\mu\nu}\mathcal{B}_{\rho\sigma} - \frac{\alpha}{2}\overline{\mathcal{B}}_{\mu\nu}\gamma^5\mathcal{B}_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+4\left(\overline{\mathcal{C}}_{\mu\nu}\mathcal{G}_{\rho\sigma} - \frac{\rho}{2}\overline{\mathcal{C}}_{\mu\nu}\mathcal{C}_{\rho\sigma} - \frac{\sigma}{2}\overline{\mathcal{C}}_{\mu\nu}\gamma^5\mathcal{C}_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+4\left(\frac{\beta}{2}\overline{\mathcal{C}}_{\mu\nu}\mathcal{B}_{\rho\sigma} + \frac{\beta}{2}\overline{\mathcal{B}}_{\mu\nu}\mathcal{C}_{\rho\sigma} + \frac{\alpha}{2}\overline{\mathcal{C}}_{\mu\nu}\gamma^5\mathcal{B}_{\rho\sigma} + \frac{\alpha}{2}\overline{\mathcal{B}}_{\mu\nu}\gamma^5\mathcal{C}_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma}$$

$$\mathcal{B} - \mathcal{C} = \frac{1}{\alpha^2 + \beta^2}(\beta\mathbb{1} - \alpha\gamma^5)\mathcal{F}, \text{ and } \mathcal{C} = \frac{(\rho - \beta)\mathbb{1} - (\sigma - \alpha)\gamma^5}{(\sigma - \alpha)^2 + (\rho - \beta)^2}(\mathcal{G} + \mathcal{F})$$

$$16\pi\mathcal{L}^f = \epsilon^{\mu\nu\rho\sigma}\frac{\alpha}{(\alpha^2+\beta^2)}\overline{\mathcal{F}}_{\mu\nu}\left(\frac{\beta\mathbb{1}-\alpha\gamma^5}{2\alpha}\right)\mathcal{F}_{\rho\sigma}$$
$$+\epsilon^{\mu\nu\rho\sigma}\frac{(\sigma-\alpha)}{(\sigma-\alpha)^2+(\rho-\beta)^2}\left(\overline{\mathcal{G}}_{\mu\nu}+\overline{\mathcal{F}}_{\mu\nu}\right)\left(\frac{(\rho-\beta)\mathbb{1}-(\sigma-\alpha)\gamma^5}{2(\sigma-\alpha)}\right)\left(\mathcal{G}_{\rho\sigma}+\mathcal{F}_{\rho\sigma}\right)$$

$$16\pi\mathcal{L}^b = \epsilon^{\mu\nu\rho\sigma}\left(\frac{1}{\beta}F^{(s)a4}{}_{\mu\nu}F^{(s)}_{a4}{}_{\rho\sigma}+\frac{1}{4}M^{abcd}F^{(s)}_{ab}{}_{\mu\nu}F^{(s)}_{cd}{}_{\rho\sigma}\right)$$
$$+\epsilon^{\mu\nu\rho\sigma}\frac{1}{4}N^{abcd}\left(G^{(s)}_{ab}{}_{\mu\nu}+F^{(s)}_{ab}{}_{\mu\nu}\right)\left(G^{(s)}_{cd}{}_{\rho\sigma}+F^{(s)}_{cd}{}_{\rho\sigma}\right)$$

$$M^{abcd} = \frac{\alpha}{(\alpha^2+\beta^2)}\left(\gamma\delta^{abcd}-\epsilon^{abcd}\right)$$
$$N^{abcd} = \frac{(\sigma-\alpha)}{(\sigma-\alpha)^2+(\rho-\beta)^2}\left(\frac{\rho-\beta}{\sigma-\alpha}\delta^{abcd}-\epsilon^{abcd}\right)$$

$$G^{(s)ab}_{\mu\nu}+F^{(s)ab}_{\mu\nu} = R^{ab}_{\mu\nu}(\omega+h)-\left(\kappa\bar{\psi}_\mu+\tilde{\kappa}\bar{\chi}_\mu\right)\gamma^{ab}\left(\kappa\psi_\nu+\tilde{\kappa}\chi_\nu\right)$$
$$\mathcal{G}_{\mu\nu}+\mathcal{F}_{\mu\nu} = \mathcal{D}^{(\omega+h)}_\mu(\kappa\psi_\nu+\tilde{\kappa}\chi_\nu)-\mathcal{D}^{(\omega+h)}_\nu\left(\kappa\psi_\mu+\tilde{\kappa}\chi_\mu\right).$$

$$16\pi\mathcal{L} = -\left(\frac{\kappa^2}{G}\bar{\psi}_\mu\gamma^5\gamma_{ab}e^a_\nu e^b_\rho+\frac{2\kappa^2\ell}{G}\bar{\psi}_\mu\gamma^5\gamma_a e^a_\nu\mathcal{D}^\omega_\rho\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$
$$-\bar{\psi}_\mu\left(\frac{1}{4\beta}\frac{2\kappa^2}{\ell}\gamma_a T^a_{\nu\rho}+\frac{2\kappa^2\ell}{4G}(\gamma\mathbb{1}-\gamma^5)\gamma_a T^a_{\nu\rho}\right)\psi_\sigma\epsilon^{\mu\nu\rho\sigma}$$
$$-\frac{1}{4\beta}\left(\frac{1}{\ell^2}T^a_{\mu\nu}T_{\rho\sigma a}+\kappa^4\bar{\psi}_\mu\gamma^a\psi_\nu\bar{\psi}_\rho\gamma_a\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+\frac{1}{16}M_{abcd}\left(F^{ab}_{\mu\nu}F^{cd}_{\rho\sigma}+\kappa^4\bar{\psi}_\mu\gamma^{ab}\psi_\nu\bar{\psi}_\rho\gamma^{cd}\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+\frac{1}{16}N_{abcd}\left(\kappa\bar{\psi}_\mu+\bar{\kappa}\bar{\chi}_\mu\right)\gamma^{ab}\left(\kappa\psi_\nu+\bar{\kappa}\chi_\nu\right)\left(\kappa\bar{\psi}_\rho+\bar{\kappa}\bar{\chi}_\rho\right)\gamma^{cd}\left(\kappa\psi_\sigma+\bar{\kappa}\chi_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$
$$+\xi$$

$$F^{IJ}_{0i} = \dot{A}^{IJ}_i-\partial_i A^{IJ}_0+A^I_0{}_K A_i{}^{KJ}-A_i{}^I{}_K A_0{}^{KJ} = \dot{A}^{IJ}_i-\mathcal{D}_i A^{IJ}_0$$
$$F^{IJ}_{ij} = \partial_i A^{IJ}_j+A^I_i{}_K A_j{}^{KJ}-i\leftrightarrow j$$

$$B^{IJ}_{\mu\nu}\to\left(B^{IJ}_{0i}\equiv B^{IJ}_i,\mathcal{P}^{iIJ}\equiv 2\epsilon^{ijk}B^{IJ}_{jk}\right).$$

$$S = \int dt\mathcal{L},\mathcal{L} = \int d^3x\left(\mathcal{P}^i{}_{IJ}\dot{A}_i{}^{IJ}+B_i{}^{IJ}\Pi^i{}_{IJ}+A_0{}^{IJ}\Pi_{IJ}\right)$$

$$\Pi_{IJ}(x) = \left(\mathcal{D}_i\mathcal{P}^i\right)_{IJ}(x) = \left(\partial_i\mathcal{P}^i_{IJ}+A_{iI}{}^K\mathcal{P}^i{}_{KJ}+A_{iJ}{}^K\mathcal{P}^i{}_{IK}\right)(x)\approx 0$$
$$\Pi^i{}_{IJ}(x) = \left(2\epsilon^{ijk}F_{jkIJ}-\beta\mathcal{P}^i_{IJ}-\frac{\alpha}{2}\epsilon_{IJKL4}\mathcal{P}^{iKL}\right)(x)\approx 0$$

$$\left\{A^{IJ}_i(x),\mathcal{P}^j{}_{KL}(y)\right\} = \frac{1}{2}\delta(x-y)\delta^j_i\delta^{IJ}_{KL}$$

$$\Phi^i_\alpha \;= \mathcal{P}^i_\alpha - \frac{4}{\ell\beta}\epsilon^{ijk}\mathcal{D}^\omega_j e_{k\alpha} \approx 0$$
$$\Phi^i_{\alpha\beta} \;= \mathcal{P}^i_{\alpha\beta} - M^{\gamma\delta}_{\alpha\beta}F_{jk\gamma\delta}\epsilon^{ijk} \approx 0$$
$$\Pi_{\alpha\beta} \;= \frac{2}{\ell^2}\epsilon^{ijk}\mathcal{D}^\omega_i\left(K^{\gamma\delta}_{\alpha\beta}e_{j\gamma}e_{k\delta}\right) \approx 0$$
$$\Pi_\alpha \;= \frac{1}{\ell}\epsilon^{ijk}K^{\gamma\delta}_{\alpha\beta}e^\beta_i R_{jk\gamma\delta} - \frac{2\alpha}{(\alpha^2+\beta^2)\ell^3}\epsilon^{ijk}\epsilon_{\alpha\beta\gamma\delta}e^\beta_i e^\gamma_j e^\delta_k \approx 0$$

$$M^{\alpha\beta}{}_{\gamma\delta} \equiv \frac{\alpha}{(\alpha^2+\beta^2)}\left(\gamma\delta^{\alpha\beta}_{\gamma\delta} - \epsilon^{\alpha\beta}{}_{\gamma\delta}\right), K^{\alpha\beta}{}_{\gamma\delta} \equiv \frac{\alpha}{(\alpha^2+\beta^2)}\left(\frac{1}{\gamma}\delta^{\alpha\beta}_{\gamma\delta} + \epsilon^{\alpha\beta}{}_{\gamma\delta}\right),$$

$$H = -2A^\alpha\Pi_\alpha - A^{\alpha\beta}\Pi_{\alpha\beta} - 2B^\alpha_i\Phi^i_\alpha - B^{\alpha\beta}_i\Phi^i_{\alpha\beta}$$

$$S_T = \frac{2\alpha}{(\alpha^2+\beta^2)}\frac{\beta}{\alpha}\int \partial_\mu\left(\mathcal{C}^\mu({}^+\omega) + \mathcal{C}^\mu({}^-\omega)\right) - i\frac{2\alpha}{(\alpha^2+\beta^2)}\int \partial_\mu\left(\mathcal{C}^\mu({}^+\omega) - \mathcal{C}^\mu({}^-\omega)\right)$$
$$+ \frac{4}{\beta\ell^2}\int \partial_\mu\left(e_{\nu\alpha}\mathcal{D}^\omega_\rho e^\alpha_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$

$$W(e,\omega) = \frac{4}{\beta\ell^2}\int_\Sigma \epsilon^{ijk}\left(e_{i\alpha}\mathcal{D}^\omega_j e^\alpha_k\right) + \frac{2\alpha}{(\alpha^2+\beta^2)}\int_\Sigma \left((\gamma - i)\mathcal{L}_{CS}({}^+\omega) + (\gamma + i)\mathcal{L}_{CS}({}^-\omega)\right)$$

$$\mathcal{P}^i_\alpha = \mathcal{P}^i_\alpha + \{\mathcal{P}^i_\alpha, W(\omega,e)\}, \mathcal{P}^i_{\alpha\beta} = \mathcal{P}^i_{\alpha\beta} + \{\mathcal{P}^i_{\alpha\beta}, W(\omega,e)\}$$

$$\left\{e^\alpha_i, \mathcal{P}^j_\beta\right\} = \frac{1}{2}\ell\delta^j_i\delta^\alpha_\beta \text{ and } \left\{\omega^{\alpha\beta}_i, \mathcal{P}^j_{\gamma\delta}\right\} = \frac{1}{2}\delta^j_i\delta^{\gamma\delta}_{\alpha\beta}$$

$$\frac{1}{2}\frac{\delta W}{\delta\omega^{\alpha\beta}_i} \;= M^{\gamma\delta}_{\alpha\beta}R_{jk\gamma\delta}\epsilon^{ijk} - \frac{4}{\beta\ell^2}e_{j\alpha}e_{k\beta}\epsilon^{ijk}$$
$$\frac{1}{2}\frac{\delta W}{\delta e^\alpha_i} \;= \frac{4}{\ell\beta}\epsilon^{ijk}\mathcal{D}^\omega_j e_{k\alpha}$$

$$\Phi^i_\alpha \;= \mathcal{P}^i_\alpha \approx 0$$
$$\Phi^i_{\alpha\beta} \;= \mathcal{P}^i_{\alpha\beta} - \frac{2}{\ell^2}K_{\alpha\beta}{}^{\gamma\delta}e_{j\gamma}e_{k\delta}\epsilon^{ijk} \approx 0$$
$$\Pi_{\alpha\beta} \;= \frac{2}{\ell^2}\epsilon^{ijk}K_{\alpha\beta}{}^{\gamma\delta}\mathcal{D}^\omega_i\left(e_{j\gamma}e_{k\delta}\right) \approx 0$$
$$\Pi_\alpha \;= \frac{1}{\ell}\epsilon^{ijk}K_{\alpha\beta}{}^{\gamma\delta}e^\beta_i F_{jk\gamma\delta} \approx 0$$

$$e^0_i \approx 0$$

$$-w^a_i = \omega^{0a}_i - \frac{1}{2\gamma}\epsilon^{abc}\omega_{ibc}$$

$$-\frac{4\alpha}{(\alpha^2+\beta^2)\ell^2}\epsilon^{ijk}\epsilon_{abc}e^b_j e^c_k$$

$$A \wedge B = (-1)^{pq}B \wedge A$$
$$d(A \wedge B) = dA \wedge B + (-1)^p A \wedge dB$$

$$dA = d\left(A_{k_1k_2\dots k_p}dx^{k_1}\wedge dx^{k_2}\wedge\dots\wedge dx^{k_p}\right) = \frac{\partial A_{k_1k_2\dots k_p}}{\partial x^k}dx^k\wedge dx^{k_1}\wedge dx^{k_2}\wedge\dots\wedge dx^{k_p}$$

$$\epsilon^{abmn}\epsilon_{mncd} = -(4-2)!\,\delta^{ab}_{cd} = -2\left(\delta^a_c\delta^b_d - \delta^a_d\delta^b_c\right)$$

$$\left(\det e^i_\mu\right)^2 = -\det g_{\mu\nu} = -g, \qquad e = \sqrt{-g}$$

$$e = \frac{1}{4!}\epsilon_{abcd}e^a_\mu e^b_\nu e^c_\rho e^d_\sigma\epsilon^{\mu\nu\rho\sigma}, \quad \epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho\sigma} = 4!\sqrt{-g}$$

$$\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3\ \gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$$

$$\gamma^5 = -\frac{1}{4!}\epsilon_{abcd}\gamma^a\gamma^b\gamma^c\gamma^d\ \gamma_5 = \frac{1}{4!}\epsilon^{abcd}\gamma_a\gamma_b\gamma_c\gamma_d$$

$$\gamma^{ab} = \frac{1}{2}\epsilon^{abcd}\gamma_{cd}\gamma_5, \gamma_{ab} = -\frac{1}{2}\epsilon_{abcd}\gamma^{cd}\gamma^5$$

$$\gamma_c\gamma_{ab} = \eta_{ca}\gamma_b - \eta_{cb}\gamma_a - \epsilon_{abcd}\gamma^d\gamma^5$$

$$\gamma_{ab}\gamma_c = \eta_{cb}\gamma_a - \eta_{ca}\gamma_b - \epsilon_{abcd}\gamma^d\gamma^5$$

$$\bar\psi_\mu\Gamma^A\psi_\nu\epsilon^{\mu\nu\rho\sigma} = 0 \text{ where } \Gamma^A = \{1,\gamma^5,\gamma^5\gamma^a\}$$

$$\bar\psi\chi = \bar\chi\psi\ \bar\psi\gamma_5\chi = \bar\chi\gamma_5\psi\ \bar\psi\gamma_5\gamma_i\chi = \bar\chi\gamma_5\gamma_i\psi$$

$$\left(\bar\psi_\mu\Gamma\psi_\nu\right)\left(\bar\psi_\rho\Gamma\psi_\sigma\right)\epsilon^{\mu\nu\rho\sigma} = 0$$

$$\mathcal{D}_\mu\bar\psi_\nu = \partial_\mu\bar\psi_\nu - \frac{1}{4}\omega^{ab}_\mu\bar\psi_\nu\gamma_{ab} - \frac{1}{2\ell}e^a_\mu\bar\psi_\nu\gamma_a$$

$$\mathcal{D}_\mu\psi_\nu = \partial_\mu\psi_\nu + \frac{1}{4}\omega^{ab}_\mu\gamma_{ab}\psi_\nu + \frac{1}{2\ell}e^a_\mu\gamma_a\psi_\nu$$

$$\mathcal{R}^{ab} \equiv \mathrm{d}\omega^{ab} + \omega^{ac}\wedge\omega_c^b$$

$$T^a \equiv \mathcal{D}V^a - \frac{i}{2}\bar\Psi_A\Gamma^a\wedge\Psi_A$$

$$\rho_A \equiv \mathcal{D}\Psi_A - \frac{1}{2\ell}A\epsilon_{AB}\wedge\Psi_B$$

$$F \equiv \mathrm{d}A - \bar\Psi_A\wedge\Psi_B\epsilon_{AB} = \mathcal{F} - \bar\Psi_A\wedge\Psi_B\epsilon_{AB}$$

$$\begin{aligned}\mathcal{L}_{\text{bulk}} = &\frac{1}{4}\mathcal{R}^{ab}\wedge V^c\wedge V^d\epsilon_{abcd} + \bar\Psi_A\Gamma_a\Gamma_5\wedge\rho_A\wedge V^a\\ &+\frac{1}{4}\epsilon_{abcd}\tilde F^{cd}V^a\wedge V^b\wedge F - \frac{1}{48}\tilde F_{\ell m}\tilde F^{\ell m}V^a\wedge V^b\wedge V^c\wedge V^d\epsilon_{abcd}\\ &+\frac{i}{2}\left(F + \frac{1}{2}\bar\Psi_A\wedge\Psi_B\epsilon_{AB}\right)\wedge\bar\Psi_C\Gamma_5\wedge\Psi_D\epsilon_{CD}\\ &-\frac{i}{2\ell}\bar\Psi_A\Gamma_{ab}\Gamma_5\wedge\Psi_A\wedge V^a\wedge V^b - \frac{1}{8\ell^2}V^a\wedge V^b\wedge V^c\wedge V^d\epsilon_{abcd}\end{aligned}$$

$$\mathcal{A} = \int_{\mathcal{M}_4\subset\mathcal{M}^{4|8}}\mathcal{L}$$

$$T^a \equiv \mathcal{D}V^a - \frac{i}{2}\bar{\Psi}_A\Gamma^a\Psi_A = 0$$

$$\delta_\epsilon\Phi = \ell_\epsilon\Phi = \mathrm{d}\iota_\epsilon(\Phi) + \iota_\epsilon(\mathrm{d}\Phi),$$

$$\delta_\epsilon\Phi = \ell_\epsilon\Phi = \nabla\iota_\epsilon(\Phi) + \iota_\epsilon(\nabla\Phi).$$

$$\iota_\epsilon(\Psi_A) = \epsilon_A, \iota_\epsilon(V^a) = 0, \iota_\epsilon\left(\omega^{ab}\right) = 0, \iota_\epsilon(A) = 0$$

$$\begin{aligned}\mathcal{R}^{ab} =& \tilde{\mathcal{R}}^{ab}{}_{cd}V^cV^d + \bar{\Theta}_A^{ab|c}\Psi_AV_c + \frac{1}{2\ell}\bar{\Psi}_A\Gamma^{ab}\Psi_B\delta_{AB} \\ & -\frac{1}{2}\epsilon_{AB}\left(\tilde{F}^{ab}\bar{\Psi}_A \wedge \Psi_B + \frac{i}{2}\epsilon^{abcd}\tilde{F}_{cd}\bar{\Psi}_A\Gamma_5 \wedge \Psi_B\right), \\ T^a =& 0,\end{aligned}$$

$$\begin{aligned}\rho_A =& \tilde{\rho}_{ab}V^aV^b - \frac{i}{2}\epsilon_{AB}\left(\tilde{F}_{ab} - \frac{i}{2}\epsilon_{abcd}\tilde{F}^{cd}\Gamma_5\right)\Gamma^a\Psi_BV^b \\ & + \frac{i}{2\ell}\Gamma_a\Psi_AV^a, \\ F =& \tilde{F}_{ab}V^aV^b,\end{aligned}$$

$$\delta_\epsilon\mathcal{A} = \int_{\mathcal{M}_4\subset\mathcal{M}^{4|8}} \ell_\epsilon\mathcal{L} = \int_{\mathcal{M}_4} (d\iota_\epsilon(\mathcal{L}) + \iota_\epsilon(d\mathcal{L})).$$

$$\iota_\epsilon(d\mathcal{L}) = 0$$

$$\delta_\epsilon\mathcal{A} = \int_{\mathcal{M}_4} d\iota_\epsilon(\mathcal{L}) = \int_{\partial\mathcal{M}_4} \iota_\epsilon(\mathcal{L}) = 0.$$

$$\mathcal{L}_{\text{bulk}} \to \mathcal{L}_{\text{full}} \equiv \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\text{bdy}}$$

$$\iota_\epsilon(\mathcal{L}_{\text{full}})|_{\partial\mathcal{M}_4} = 0$$

$$\begin{aligned}\mathcal{L}_{\text{bdy}} = -\frac{\ell^2}{8}(&\mathcal{R}^{ab}\mathcal{R}^{cd}\epsilon_{abcd} + \frac{8i}{\ell}\bar{\rho}_A\Gamma_5\rho_A \\ & - \frac{2i}{\ell}\mathcal{R}^{ab}\bar{\Psi}_A\Gamma_{ab}\Gamma_5\Psi_A + \frac{4i}{\ell^2}dA\bar{\Psi}_A\Gamma_5\Psi_B\epsilon_{AB}\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{R}^{ab}\big|_{\partial\mathcal{M}_4} &= \left[\frac{1}{\ell^2}V^aV^b + \frac{1}{2\ell}\delta^{AB}\bar{\Psi}_A\Gamma^{ab}\Psi_B\right]_{\partial\mathcal{M}_4} \\ \mathcal{D}V^a|_{\partial\mathcal{M}_4} &= \left[\frac{i}{2}\bar{\Psi}_A\Gamma^a\Psi_A\right]_{\partial\mathcal{M}_4} \\ \rho_A|_{\partial\mathcal{M}_4} &= \left[\frac{i}{2\ell}\delta_{AB}\Gamma_a\Psi_BV^a\right]_{\partial\mathcal{M}_4} \\ F|_{\partial\mathcal{M}_4} &= 0\end{aligned}$$

$$R^{ab} \equiv d\omega^{ab} + \omega^{ac}\omega_c^b - \frac{1}{\ell^2}V^aV^b - \frac{1}{2\ell}\delta^{AB}\bar{\Psi}_A\Gamma^{ab}\Psi_B \to 0$$

$$T^a \equiv dV^a + \omega^{ab}V_b - \frac{i}{2}\bar{\Psi}_A\Gamma^a\Psi_A = 0$$

$$\hat{\rho}_A \equiv d\Psi_A + \frac{1}{4}\omega^{ab}\Gamma_{ab}\Psi_A - \frac{1}{2\ell}A\epsilon_{AB}\Psi_B - \frac{i}{2\ell}\delta_{AB}\Gamma_a\Psi_BV^a \to 0$$

$$F \equiv dA - \bar{\Psi}_A\Psi_B\epsilon_{AB} \to 0$$

$$E_\pm^i \equiv \pm\frac{1}{2}\left(V^i \mp \ell h^i\right)$$

$$\Psi_A = \Psi_{+A} + \Psi_{-A}, \Gamma^3\Psi_{\pm A} = \pm i\Psi_{\pm A}$$

$$\mathcal{R}^{ij} \equiv d\omega^{ij} + \omega_k^i \wedge \omega^{kj} = -\frac{4}{\ell^2}E_+^{[i} \wedge E_-^{j]} + \frac{1}{\ell}\bar{\Psi}_{+A}\Gamma^{ij}\Psi_{-A}$$

$$dV^3 = -\omega_i^3 \wedge V^i + \bar{\Psi}_{+A}\Psi_{-A}$$

$$\mathcal{D}E_\pm^i \equiv dE_\pm^i + \omega_j^i \wedge E_\pm^j = \pm\frac{i}{2}\bar{\Psi}_{\pm A}\Gamma^i\Psi_{\pm A} \pm \frac{1}{\ell}E_\pm^i \wedge V^3$$

$$\mathcal{D}\Psi_{\pm A} \equiv d\Psi_{\pm A} + \frac{1}{4}\omega_{ij}\Gamma^{ij} \wedge \Psi_{\pm A}$$

$$= \mp\frac{i}{\ell}E_{\pm i} \wedge \Gamma^i\Psi_{\mp A} - \epsilon_{AB}A^{(4)} \wedge \Psi_{\pm|B} \pm \frac{1}{2\ell}\Psi_{\pm A} \wedge V^3$$

$$dA^{(4)} = \epsilon_{AB}\bar{\Psi}_A \wedge \Psi_B = 2\epsilon_{AB}\bar{\Psi}_{+A} \wedge \Psi_{-B}$$

$$\omega_i^3\big|_{\partial\mathcal{M}_4} = \left(h_{i|j}V^j\right)_{\partial\mathcal{M}_4} \text{ with } h_{i|j} = h_{j|i};\ \bar{\Psi}_{+A}\Psi_{-A} = 0$$

$$E_+^i(x,r) = \frac{r}{2\ell}\left[E^i(x) + \mathcal{O}\left(\frac{\ell^2}{r^2}\right)\right];\ E_-^i(x,r) = -\frac{\ell}{2r}E^i(x) + \mathcal{O}\left(\frac{\ell^2}{r^2}\right)$$

$$\omega^{ij}(x,r) = \omega^{ij}(x) + \mathcal{O}\left(\frac{\ell}{r}\right);\ A^{(4)}(x,r) = 2\ell\varepsilon A_\mu(x)dx^\mu + \mathcal{O}\left(\frac{\ell}{r}\right)$$

$$\Psi_{+A\mu}(x,r) = \sqrt{\frac{r}{2\ell}}\left[\begin{pmatrix}\psi_{A\mu}\\ \mathbf{0}\end{pmatrix} + \mathcal{O}\left(\frac{\ell}{r}\right)\right];\Psi_{-A\mu}(x,r) = \sqrt{\frac{\ell}{2r}}\left[\begin{pmatrix}\mathbf{0}\\ \varepsilon\psi_{A\mu}\end{pmatrix} + \mathcal{O}\left(\frac{\ell}{r}\right)\right]$$

$$V^3(r) = \frac{\ell}{r}\left[dr + \mathcal{O}\left(\frac{\ell^2}{r^2}\right)\right]$$

$$\mathcal{R}^{ij} = \frac{1}{\ell^2}E^iE^j + \frac{\varepsilon}{2\ell}\bar{\psi}_A\gamma^{ij}\psi_A,$$

$$\mathcal{D}E^i = \frac{i}{2}\bar{\psi}_A\gamma^i\psi_A,$$

$$\mathcal{D}\psi_A = -\varepsilon\frac{i}{2\ell}E_i\gamma^i\psi_A - \epsilon_{AB}A\psi_B,$$

$$dA = -\frac{\varepsilon}{\ell}\epsilon_{AB}\bar{\psi}_A\psi_B,$$

$$\mathcal{L}^{(3)} = \left(\mathcal{R}^{ij} - \frac{1}{3\ell^2}E^iE^j - \frac{\varepsilon}{2\ell}\bar{\psi}_A\gamma^{ij}\psi_A\right)E^k\epsilon_{ijk} - \frac{\varepsilon}{2\ell}AdA$$

$$+2\bar{\psi}_A\left(\mathcal{D}\psi_A - \frac{\varepsilon}{2\ell}\epsilon_{AB}A\psi_B\right)$$

$$\omega^{ij}_{(\pm\varepsilon)} = \omega^{ij} \pm \frac{\varepsilon}{\ell} E_k \epsilon^{ijk} \equiv \epsilon^{ijk} \omega_{(\pm\varepsilon)k},$$

$$\mathcal{R}^i_{(\varepsilon)} = i\frac{\varepsilon}{\ell} \bar{\psi}_A \gamma^i \psi_A, \mathcal{R}^i_{(-\varepsilon)} = 0, dA = -\frac{\varepsilon}{\ell} \epsilon_{AB} \bar{\psi}_A \psi_B$$
$$\mathcal{D}_{(\varepsilon)} \psi_A = -\epsilon_{AB} A \psi_B$$

$$\mathcal{R}_{(\pm)i} \equiv \frac{1}{2} \epsilon_{ijk} \mathcal{R}^{jk}_{(\pm)} = d\omega_{(\pm)i} - \frac{1}{2} \omega^j_{(\pm)} \omega^k_{(\pm)} \varepsilon_{ijk}\ \mathcal{D}_{(\varepsilon)}\ \omega^{ij}_{(+\varepsilon)} \mathfrak{osp}(2 \mid 2)_{(\varepsilon)} \times \mathfrak{so}(1,2)_{(-\varepsilon)}$$

$$\mathcal{L}^{(3)} = \varepsilon\big(\mathcal{L}_{(\varepsilon)} - \mathcal{L}_{(-\varepsilon)}\big) + d\mathcal{B} \equiv \mathcal{L}^{(3)}_+ - \mathcal{L}^{(3)}_- + d\mathcal{B}$$

$$\mathcal{L}_{(\varepsilon)} = \frac{\ell}{2}\left(\omega^i_{(\varepsilon)} d\omega_{(\varepsilon)|i} - \frac{1}{3} \omega^i_{(\varepsilon)} \omega^j_{(\varepsilon)} \omega^k_{(\varepsilon)} \varepsilon_{ijk}\right) + 2\varepsilon \bar{\psi}_A \nabla^{(\varepsilon)} \psi_A - \frac{\varepsilon}{2\ell} A dA$$
$$\nabla^{(\varepsilon)} \psi_A \equiv \left(d + \frac{1}{4} \omega^{ij}_{(\varepsilon)} \gamma_{ij}\right) \psi_A + A \psi_B \epsilon_{AB}$$
$$\mathcal{L}_{(-\varepsilon)} = \frac{\ell}{2}\left(\omega^i_{(-\varepsilon)} d\omega_{(-\varepsilon)|i} - \frac{1}{3} \omega^i_{(-\varepsilon)} \omega^j_{(-\varepsilon)} \omega^k_{(-\varepsilon)} \varepsilon_{ijk}\right)$$
$$\mathcal{B} = -\frac{\ell}{2} \omega^i_{(+)} \omega_{(-)i}$$

$$\delta\omega^i_{(-\varepsilon)} = 0, \delta\omega^i_{(\varepsilon)} = \varepsilon \frac{2i}{\ell} \bar{\epsilon}_A \gamma^i \psi_A, \delta A = 2\epsilon_{AB} \bar{\epsilon}_A \psi_B$$
$$\delta\psi_A = \mathcal{D}_{(\varepsilon)} \epsilon_A - \frac{\varepsilon}{2\ell} \epsilon_{AB} A \epsilon_B \equiv \nabla^{(\varepsilon)} \epsilon_A$$

$$\mathbb{A} = \frac{1}{2} \omega^{ij} \mathbb{J}_{ij} + A \cdot \mathbb{T} + \bar{\psi}_A \mathbb{Q}^A + \overline{\mathbb{Q}}_A \psi^A,$$

$$\mathcal{A} = \int_{\mathcal{M}_3} \mathcal{L}(\mathbb{A}) = \int_{\mathcal{M}_3} \left\langle \mathbb{A} \wedge d\mathbb{A} + \frac{2}{3} \mathbb{A} \wedge \mathbb{A} \wedge \mathbb{A} \right\rangle$$

$$\psi_{\mu A} = i e^i_\mu \gamma_i \chi_A$$

$$\mathcal{D}^{(\varepsilon)} e^i = \tau_0 \epsilon^i{}_{jk} e^j e^k,$$

$$\mathcal{L}_{(\varepsilon)} = \frac{\ell}{2}\left(\omega^i_{(\varepsilon)} d\omega_{(\varepsilon)|i} - \frac{1}{3} \omega^i_{(\varepsilon)} \omega^j_{(\varepsilon)} \omega^k_{(\varepsilon)} \varepsilon_{ijk}\right) + 2\varepsilon\, e_i\, \mathcal{D}^{(\varepsilon)} e^i\, \bar{\chi}_A \chi_A + - 4\, i\, \varepsilon\, \bar{\chi}_A \nabla\!\!\!\!/^{(\varepsilon)} \chi_A\, e\, d^3x - \frac{\varepsilon}{2\ell} A\, dA\,,$$

$$\nabla\!\!\!\!/^{(\varepsilon)} \chi_A \equiv \gamma^i\, \nabla^{(\varepsilon)}_i \chi_A = \mathcal{D}\!\!\!\!/^{(\varepsilon)} \chi_A - \frac{\varepsilon}{2\ell} A_i \epsilon_{AB} \gamma^i \chi_B\,.$$

$$\nabla\!\!\!\!/^{(\varepsilon)} \chi_A + 3i\, \tau_0 \chi_A = 0\,.$$

$$\nabla\!\!\!/^{(\varepsilon)}\chi + 3i\,\tau_0\chi = 0\,,$$

$$\nabla_i^{(\varepsilon)}\chi_A = i\frac{\varepsilon}{\ell}\gamma_i\chi_A$$

$$d(\bar{\chi}\chi) = 0$$

$$\mathcal{R}^i_{(\varepsilon)} = -\frac{\varepsilon}{\ell}\bar{\chi}_A\chi_A\epsilon^{ijk}e_je_k, dA = i\epsilon_{AB}\bar{\chi}_A\gamma^k\chi_Be^ie^j\epsilon_{ijk}$$

$$e^i \to \lambda(x)e^i, \chi_A \to \frac{1}{\lambda(x)}\chi_A, \lambda \neq 0$$

$$\mathcal{D}^{(\varepsilon)}e^i = \hat{T}^i_{jk}e^j \wedge e^k = \beta \wedge e^i + \tau e^j \wedge e^k \epsilon^i_{jk} + \mathring{T}^{i\ell}\epsilon_{jk\ell}e^j \wedge e^k$$

$$\mathcal{R}^i_{(\varepsilon)} = \frac{\varepsilon}{\ell}\bar{\chi}_A\chi_A e^j \wedge e^k \epsilon_{ijk} = \frac{1}{2}\epsilon^i_{jk}\mathcal{R}^{jk}_{(\varepsilon)}$$

$$\left(\mathcal{D}^{(\varepsilon)}\right)^2 e^i = 0$$

$$\beta = -\frac{d\tau}{\tau}$$

$$e^i \to \lambda(x)e^i \;\Rightarrow\; \mathcal{D}^{(\varepsilon)}e^i \to \lambda\left(\mathcal{D}^{(\varepsilon)}e^i + \frac{d\lambda}{\lambda} \wedge e^i\right)$$

$$\beta \to \beta + \frac{d\lambda}{\lambda}, \tau \to \frac{\tau}{\lambda},.$$

$$\omega'^i = \omega^i_{(\varepsilon)} + \tau_0 e^i$$

$$\mathcal{R}^{ij}[\omega'] = \left(\tau_0^2 + \frac{\varepsilon}{\ell}\bar{\chi}_A\chi_A\right)e^i \wedge e^j$$

$$^*d\beta = -2\tau\left(\beta + \frac{d\tau}{\tau}\right)$$

$$\omega^{ij}_{(\pm\varepsilon)}\;\omega^i = \tfrac{1}{2}\left(\omega^i_{(\varepsilon)} + \omega^i_{(-\varepsilon)}\right)\tau_0 = -\tfrac{\varepsilon}{\ell}$$

$$\mathcal{D}_{(\varepsilon)}e^i = -\frac{\varepsilon}{\ell}\epsilon^i{}_{jk}e^je^k,$$

$$\mathcal{D}_{(\varepsilon)}E^i = -\frac{\varepsilon}{\ell}\epsilon^{ijk}E_jE_k + \frac{1}{2}\bar{\chi}_A\chi_A\epsilon^{ijk}e_je_k$$

$$E^i = M(\bar{\chi}\chi)e^i,\ \text{with: } M(\bar{\chi}\chi) = \left(1 + \varepsilon\frac{\ell}{2}\bar{\chi}\chi - \frac{\ell^2}{4}(\bar{\chi}\chi)^2\right)$$

$$\chi_A = -\frac{i}{3}\gamma_i e^{i|\mu}\psi_{A\mu}$$

$$\Gamma_a V^{a|\hat{\mu}} \Psi_{A\hat{\mu}} = 0$$

$$\Psi_{+A3} = \frac{3i\varepsilon}{2M(\bar{\chi}\chi)} \left(\frac{\ell}{r}\right)^{\frac{5}{2}} \begin{pmatrix} \chi_A \\ \mathbf{0} \end{pmatrix}, \Psi_{-A3} = \frac{3i}{M(\bar{\chi}\chi)} \left(\frac{\ell}{r}\right)^{\frac{3}{2}} \begin{pmatrix} \mathbf{0} \\ \chi_A \end{pmatrix}.$$

$$d\omega^{ab} + \omega_c^a \wedge \omega^{cb} - \ell^{-2} V^a \wedge V^b - \frac{1}{2\ell} (\bar{\Psi}_A \wedge \Gamma^{ab} \Psi_A) = 0$$
$$dV^a + \omega_b^a \wedge V^b - \frac{i}{2} (\bar{\Psi}_A \wedge \Gamma^a \Psi_A) = 0$$
$$dA^{CD} + A_B^C \wedge A^{BD} - (\bar{\Psi}^C \wedge \Psi^D) = 0$$
$$d\Psi^A + \frac{1}{4} \omega^{ab} \wedge \Gamma_{ab} \Psi^A + \frac{i}{2\ell} V^a \wedge \Gamma_a \Psi^A - \frac{1}{2\ell} A^{AB} \wedge \Psi^B = 0$$

$$E_\pm^i \equiv \pm \frac{1}{2} (V^i \mp \ell \omega^{3i})$$

$$\Gamma^3 : \Psi^A = \Psi_+^A + \Psi_-^A, \Gamma^3 \Psi_\pm^A = \pm i \Psi_\pm^A$$

$$\left[d\omega^{ij} + \omega_k^i \wedge \omega^{kj} + \frac{4}{\ell^2} E_+^{[i} \wedge E_-^{j]} - \frac{1}{\ell} (\bar{\Psi}_+^A \wedge \Gamma^{ij} \Psi_{A-}) \right]_{\partial\mathcal{M}} = 0$$
$$\left[dE_\pm^i + \omega_j^i \wedge E_\pm^j \mp \frac{1}{\ell} E_\pm^i \wedge V^3 \mp \frac{i}{2} (\bar{\Psi}_\pm^A \wedge \Gamma^i \Psi_{A\pm}) \right]_{\partial\mathcal{M}} = 0$$
$$\left[dV^3 - \frac{1}{\ell} (E_+^i + E_-^i) \wedge V_i + \bar{\Psi}_-^A \wedge \Psi_{A+} \right]_{\partial\mathcal{M}} = 0$$
$$\left[dA^{CD} + A^C{}_M \wedge A^{MD} - 2 \left(\bar{\Psi}_+^{[C} \wedge \Psi_-^{D]} \right) \right]_{\partial\mathcal{M}} = 0$$
$$\left[d\Psi_\pm^{M\beta} + \frac{1}{4} \omega^{ij} \wedge \left(\Gamma_{ij} \Psi_\pm^M \right)^\beta \pm \frac{i}{\ell} E_\pm^i \wedge \left(\Gamma_i \Psi_\mp^M \right)^\beta \pm \frac{1}{2\ell} V^3 \wedge \Psi_\pm^{M\beta} + \right.$$
$$\left. - \frac{1}{2\ell} \delta^M{}_{[C} \delta_{D]B} A^{CD} \wedge \Psi_\pm^{B\beta} \right]_{\partial\mathcal{M}} = 0$$

$$A^{AB}(r,x) = -2\ell A_\mu^{AB}(x) dx^\mu + O\left(\frac{\ell}{r}\right)$$

$$\Psi_{+\mu}^A(r,x) dx^\mu = \sqrt{\frac{r}{2\ell}} \begin{pmatrix} \psi^A(x) \\ \mathbf{0} \end{pmatrix} + O\left(\frac{\ell}{r}\right)$$
$$\Psi_{-\mu}^A(r,x) dx^\mu = \sqrt{\frac{\ell}{2r}} \begin{pmatrix} \mathbf{0} \\ \eta^{AB} \psi^B(x) \end{pmatrix} + O\left(\frac{\ell}{r}\right)$$

$$\mathrm{O}(\mathcal{N}) \to \mathrm{O}(p) \times \mathrm{O}(q), p + q = \mathcal{N}$$

$$\eta_{AB} = \begin{pmatrix} \delta_{a_1 b_1} & \mathbf{0} \\ \mathbf{0} & -\delta_{a_2 b_2} \end{pmatrix}$$

$$R^i_\pm \equiv d\Omega^i_\pm - \frac{1}{2}\epsilon^{ijk}\Omega_{\pm j}\wedge\Omega_{\pm k} = \pm\frac{i}{\ell}(\bar{\psi}_\pm \wedge \gamma^i \psi_\pm)$$
$$\mathcal{D}[\Omega_\pm, A_\pm]\psi_\pm = 0$$
$$\mathcal{F}^{a_1b_1} \equiv dA^{a_1b_1} + A^{a_1}{}_{c_1}\wedge A^{c_1b_1} = -\frac{1}{\ell}(\bar{\psi}^{a_1}\wedge\psi^{b_1})$$
$$\mathcal{F}^{a_2b_2} \equiv dA^{a_2b_2} + A^{a_2}{}_{c_2}\wedge A^{c_2b_2} = \frac{1}{\ell}(\bar{\psi}^{a_2}\wedge\psi^{b_2})$$

$$\Omega^i_{(\pm)} \equiv \omega^i \pm \frac{E^i}{\ell}, A_+ \equiv \left(A^{a_1b_1}\right), A_- \equiv \left(A^{a_2b_2}\right)$$
$$\psi_+ \equiv (\psi^{a_1}), \mathcal{D}[\Omega_+, A_+]\psi_+ \equiv \left(d\psi^{a_1} + \frac{i}{2}\Omega^i_+ \wedge \gamma_i\psi^{a_1} + A^{a_1b_1}\wedge\psi^{\beta}_{b_1}\right)$$
$$\psi_- \equiv (\psi^{a_2}), \mathcal{D}[\Omega_-, A_-]\psi_- \equiv \left(d\psi^{a_2} + \frac{i}{2}\Omega^i_- \wedge \gamma_i\psi^{a_2} + A^{a_2b_2}\wedge\psi_{b_2}\right)$$

$$\mathcal{L} = \mathcal{L}_{(+)} - \mathcal{L}_{(-)} - \frac{1}{2}d\left(\Omega_{+k}\wedge\Omega^k_-\right)$$

$$\mathcal{L}_{(\pm)} \equiv \frac{1}{2}\left(\Omega_{\pm i}d\Omega^i_\pm - \frac{1}{3}\epsilon_{ijk}\Omega^i_\pm\wedge\Omega^j_\pm\wedge\Omega^k_\pm\right) \pm \frac{2}{\ell}\bar{\psi}_\pm \wedge \mathcal{D}[\Omega_\pm, A_\pm]\psi_\pm + $$
$$+\mathrm{Tr}\left(A_\pm\wedge dA_\pm + \frac{2}{3}A_\pm\wedge A_\pm\wedge A_\pm\right)$$

$$E^i \to \mathcal{O}_Y{}^i{}_j E^j, \omega^i \to -\mathcal{O}_Y{}^i{}_j\omega^j \Rightarrow \Omega^i_\pm \to -\mathcal{O}_Y{}^i{}_j\Omega^j_\mp.$$

$$\mathrm{SO}(1,2)_+ \times \mathrm{SO}(1,2)_- \longrightarrow \mathrm{SO}(1,2)_- \times \mathrm{SO}(1,2)_+$$

$$\psi_\pm \to \tilde{\psi}_\pm = \sigma^1\psi_\mp, A_\pm \to \tilde{A}_\pm = A_\mp$$

$$\psi_{\mu A} = i e^i_\mu \gamma_i \chi_A$$

$$R^i_\pm = \pm\frac{1}{\ell}\bar{\chi}_\pm\chi_\pm\epsilon^{ijk}e_j\wedge e_k$$
$$\mathcal{D}[\Omega_\pm]E^i = \mp\frac{1}{\ell}\epsilon^{ijk}E_j\wedge E_k + \frac{1}{2}(\bar{\chi}_+\chi_+ + \bar{\chi}_-\chi_-)\epsilon^{ijk}e_j\wedge e_k$$
$$\mathcal{F}^{a_1b_1} = -\frac{i}{\ell}(\bar{\chi}^{a_1}\gamma^i\chi^{b_1})\epsilon_{ijk}e^j\wedge e^k$$
$$\mathcal{F}^{a_2b_2} = \frac{i}{\ell}(\bar{\chi}^{a_2}\gamma^i\chi^{b_2})\epsilon_{ijk}e^j\wedge e^k$$

$$E^i = f e^i$$

$$T^i_\pm = \mathcal{D}[\Omega_\pm]e^i = \beta_\pm e^i + \tau_\pm\epsilon^{ijk}e_j\wedge e_k$$

$$\beta_\pm \to \beta_\pm + \frac{d\lambda}{\lambda}, \tau_\pm \to \frac{1}{\lambda}\tau_\pm$$

$$\mathcal{D}[\Omega_\pm]^2 e^i = -\epsilon^{ijk}R_{\pm j}e_k = 0$$

$$\beta_\pm = -\frac{d\tau_\pm}{\tau_\pm} = -d\ln\left(|\tau_\pm|\right)$$

$$\Omega^i_\pm = \omega^i \pm E^i/\ell$$

$$\mathcal{D}[\omega]e^i = \beta \wedge e^i + \tau\epsilon^{ijk} e_j \wedge e_k$$

$$\mathcal{D}[\Omega_\pm]e^i = \mathcal{D}[\omega]e^i \mp (f/\ell)\epsilon^{ijk} e_j \wedge e_k$$

$$(\beta_+ - \beta_-)e^i + (\tau_+ - \tau_- + 2f/\ell)\epsilon^{ijk} e_j \wedge e_k = 0$$

$$\beta_+ = \beta_- = \beta, \tau_+ + \frac{f}{\ell} = \tau_- - \frac{f}{\ell} = \tau$$

$$df + \beta f \ = 0$$

$$f\tau \ = \frac{1}{2}(\bar{\chi}_+\chi_+ + \bar{\chi}_-\chi_-)$$

$$\mathcal{D}[\Omega_\pm]R^i_\pm = 0 \Rightarrow d(\bar{\chi}_\pm\chi_\pm) = -2\beta\bar{\chi}_\pm\chi_\pm$$

$$\not{D}[\Omega_\pm, A_\pm]\chi_\pm = -3i\tau_\pm\chi_\pm$$

$$\mathcal{D}[\Omega_+, A_+]\chi_+^{a_1} \equiv d\chi^{a_1} + \frac{i}{2}\Omega^i_+\gamma_i\chi^{a_1} + A^{a_1b_1}\chi_{b_1}$$

$$\mathcal{D}[\Omega_-, A_-]\chi_-$$

$$\omega'^i = \Omega^i_+ + \tau_+ e^i = \Omega^i_- + \tau_- e^i.$$

$$\not{D}[\omega', A_\pm]\chi_\pm = -\frac{3}{2}i\,\tau_\pm\,\chi_\pm\,,$$

$$m_\pm = \frac{3}{2}\tau_\pm.$$

$$R^i[\omega'] = \frac{1}{2}\left(\frac{f^2}{\ell^2} + \tau^2 + \frac{\eta_{AB}\bar{\chi}^A\chi^B}{\ell}\right)\epsilon^{ijk} e_j \wedge e_k.$$

$$\Omega^i_{(\lambda)} \equiv \omega^i + \frac{\lambda}{\ell}E^i$$

$$\Omega'^i_{(\lambda)} \equiv \omega'^i + \frac{\lambda}{\ell}e^i$$

$$\tau = \frac{\lambda}{\ell}(f-1),$$

$$\tau_\pm \ = \frac{1}{\ell}[\lambda(f-1) \mp f].$$

$$\lambda f(f-1) = \frac{\ell}{2}(\bar{\chi}_+\chi_+ + \bar{\chi}_-\chi_-)$$

$$\chi_{(+)} \equiv \chi_{a_1=1} + i\chi_{a_1=2}, \chi_{(-)} \equiv \chi_{a_2=1} + i\chi_{a_2=2}$$

$$\zeta = \begin{pmatrix} \sqrt{n_A} e^{i\alpha_A} \\ \sqrt{n_B} e^{i\alpha_B} \end{pmatrix}$$

$$n \equiv n_A + n_B = \zeta^\dagger \zeta, \Delta n \equiv n_B - n_A = \bar{\zeta}\zeta \equiv \zeta^\dagger \gamma^0 \zeta$$

$$\chi = \sqrt{\frac{\ell}{2}} U \zeta$$

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$$

$$\bar{\chi}\chi = \frac{\ell}{2} \bar{\zeta}\zeta = \frac{\ell}{2}(n_B - n_A)$$

$$\lambda = 0: \bar{\chi}\chi = 0 \Rightarrow n_A = n_B,$$

$$n_A = n_A^{(+)} + n_A^{(-)}; n_B = n_B^{(+)} + n_B^{(-)}, \text{ and } n_A^{(\pm)}, n_B^{(\pm)}$$

$$\bar{\chi}_+ \chi_+ = -\bar{\chi}_- \chi_-, \text{ and } n_A = n_B$$

$$\chi_{(\pm)} \to \tilde{\chi}_{(\pm)} = -\sigma^1 \chi_{(\mp)}$$

$$e^i \to \tilde{e}^i = \mathcal{O}_Y{}^i{}_j e^j$$

$$\begin{aligned} \mathbf{K}: \quad & E_{\mathbf{q}} \chi_{\mathbf{K}}(\mathbf{q}) = [\gamma^0(\gamma^1 q^1 + \gamma^2 q^2 + m_{\mathbf{K}})] \chi_{\mathbf{K}}(\mathbf{q}) \\ \mathbf{K}': \quad & E_{\mathbf{q}} \chi_{\mathbf{K}'}(\mathbf{q}) = [\gamma^0(\gamma^1 q^1 - \gamma^2 q^2 + m_{\mathbf{K}'})] \chi_{\mathbf{K}'}(\mathbf{q}) \end{aligned}$$

$$\begin{aligned} \mathbf{K}: \quad & i\partial_t \chi_{\mathbf{K}}(x) = \left[\gamma^0\left(-i\gamma^1 \partial_x - i\gamma^2 \partial_y + m_{\mathbf{K}}\right)\right] \chi_{\mathbf{K}}(x) \\ \mathbf{K}': \quad & i\partial_t \chi_{\mathbf{K}'}(x) = \left[\gamma^0\left(-i\gamma^1 \partial_x + i\gamma^2 \partial_y + m'_{\mathbf{K}}\right)\right] \chi_{\mathbf{K}'}(x) \end{aligned}$$

$$\left(\nabla_\mu^{\mathbf{K}} \equiv \mathcal{D}_\mu[\omega', A_{\mathbf{K}}], \nabla_\mu^{\mathbf{K}'} \equiv \mathcal{D}_\mu[\omega', A_{\mathbf{K}'}]\right)$$

$$\begin{aligned} \mathbf{K}: \quad & i\nabla_t^{\mathbf{K}} \chi_{\mathbf{K}}(x) = \left[-i\gamma^0\left(\gamma^1 \nabla_x^{\mathbf{K}} + \gamma^2 \nabla_y^{\mathbf{K}} + i m_{\mathbf{K}}\right)\right] \chi_{\mathbf{K}}(x), \\ \mathbf{K}': \quad & i\nabla_t^{\mathbf{K}'} \chi'_{\mathbf{K}}(x) = \left[-i\gamma^0\left(\gamma^1 \nabla_x^{\mathbf{K}'} - \gamma^2 \nabla_y^{\mathbf{K}'} + i m'_{\mathbf{K}}\right)\right] \chi'_{\mathbf{K}}(x), \end{aligned}$$

$$\chi_{\mathbf{K}}(x^\mu) = \chi_+(x^\mu), \chi_{\mathbf{K}'}(x^0, x^1, x^2) = \sigma^1 \chi_-(-x^0, x^1, x^2)$$

$$A_{\mathrm{K}} = A_+, A_{\mathrm{K}'} = A_-,$$

$$m_{\mathbf{K}} = m_+ = \frac{3}{2}\tau_+, m_{\mathbf{K}'} = m_- = \frac{3}{2}\tau_-.$$

$$m_{\mathbf{K}} = M - 3\sqrt{3} t_2 \sin \varphi, m_{\mathbf{K}'} = M + 3\sqrt{3} t_2 \sin \varphi$$

$$\tau_\pm = \tau \mp 2\frac{f}{\ell}$$

$$M = \frac{3}{2}\tau, \sqrt{3}t_2 \sin(\varphi) = \frac{f}{\ell}$$

$$S = \int_{\mathbb{R}^{1,4}} \left(\frac{M_P^3}{2} \star R - \frac{M_P^3}{4} g_{ij} \, d\varphi^i \wedge \star \, d\varphi^j - \frac{M_P}{4} f_{IJ} F^I \wedge \star F^J - \frac{1}{12} \mathcal{F}_{IJK} A^I \wedge F^J \wedge F^K \right)$$

$$\mathcal{F}[\mathbf{X}] = \frac{1}{3!} \mathcal{F}_{IJK} X^I X^J X^K,$$

$$\mathcal{F}[\mathbf{X}] = 1.$$

$$f_{IJ} = \mathcal{F}_I \mathcal{F}_J - \mathcal{F}_{IJ},$$

$$\mathcal{F}_I := \frac{\partial \mathcal{F}}{\partial X^I} = \frac{1}{2} \mathcal{F}_{IJK} X^J X^K$$

$$\mathcal{F}_{IJ} := \frac{\partial^2 \mathcal{F}}{\partial X^I \partial X^J} = \mathcal{F}_{IJK} X^K$$

$$f^{IJ} = g^{ij} \partial_i X^I \partial_j X^J + \frac{1}{3} X^I X^J,$$

$$g_{ij} = f_{IJ} \partial_i X^I \partial_j X^J$$

$$\mathbf{X} = X^I K_I.$$

$$\mathcal{F}_{IJK} =: K_I \cdot K_J \cdot K_K$$

$$\mathcal{F}[\mathbf{X}] = \frac{1}{3!} \mathbf{X} \cdot \mathbf{X} \cdot \mathbf{X}$$

$$S_{\text{part}} = -\int_{\mathcal{L}} d\tau \sqrt{-\gamma} M_{\text{part}} + q_I \int_{\mathcal{L}} A^I$$

$$M_{\text{part}} = M_P q_I X^I$$

$$\mathrm{q}_{\text{part}}^2 = \frac{2}{M_P} q_I f^{IJ} q_J$$

$$M_P^3 \mathrm{q}_{\text{part}}^2 = \frac{2}{3} M_{\text{part}}^2 + 2 g^{ij} \partial_i M_{\text{part}} \partial_j M_{\text{part}}$$

$$S_{\text{str}} = -\int_{\mathcal{W}} d^2\xi \sqrt{-h} \mathcal{T}_{\text{str}} + p^I \int_{\mathcal{W}} B_I$$

$$\mathcal{T}_{\text{str}} = \frac{M_P^2}{2} p^I \mathcal{F}_I$$

$$\mathcal{Q}_{\text{str}}^2 = \frac{M_P}{2} p^I f_{IJ} p^J$$

$$M_P^3 \mathcal{Q}_{\text{str}}^2 = \frac{2}{3} \mathcal{T}_{\text{str}}^2 + 2 g^{ij} \partial_i \mathcal{T}_{\text{str}} \partial_j \mathcal{T}_{\text{str}}$$

$$M_{\mathrm{P}} \equiv 1$$

$$\mathcal{C}_{\text{part}} = \{\mathbf{q} : \mathbf{q} \text{ superparticles }\},$$

$$\Delta = \left\{\mathbf{X} \in \mathbb{R}^{n+1} : q_I X^I \geq 0 \ \forall \mathbf{q} \in \mathcal{C}_{\text{part}}\right\}$$

$$\mathcal{C}_{\text{part}} = \Delta^{\vee} \cap N_{\mathbb{Z}},$$

$$\mathbb{R}^{n+1}, \mathbf{e}^1 = (1,0,\ldots,0)^T, \mathbf{e}^2 = (0,1,0,\ldots,0)^T, \ldots, \mathbf{e}^{n+1} = (0,\ldots,0,1)^T \cdot$$

$$\mathcal{C}_{\text{part}} = \{\mathbf{q} \in \mathbb{N}^{n+1}\}$$

$$\Delta = \{\mathbf{X} \in \mathbb{R}^{n+1}_{\geq 0}\}$$

$$\Gamma = \left\{\mathcal{F} \in \mathbb{R}^{n+1} : \mathcal{F}_I = \left.\frac{\partial \mathcal{F}}{\partial X^I}\right|_{\mathbf{X} \in \Delta}\right\}$$

$$\mathcal{C}_{\text{str}} = \Gamma^{\vee} \cap N_{\mathbb{Z}},$$

$$\mathcal{C}_{\text{SUGRA}} = \Delta \cap N_{\mathbb{Z}}$$

$$\mathcal{C}_{\text{SUGRA}} \subseteq \mathcal{C}_{\text{str}}$$

$$k^{(\mathbf{p})} : \mathcal{A} \times \mathcal{A} \longrightarrow \mathbb{R}$$

$$k_{IJ}^{(\mathbf{p})} := k(K_I, K_J) = \mathcal{F}_{IJK} p^K$$

$$\operatorname{sgn}\left(k_{IJ}^{(\mathbf{p})}\right) = (1, r-1) \ \forall \mathbf{p} \in \mathcal{C}_{\text{SUGRA}} \quad \text{with } r \leq n := n_V + 1.$$

$$\tilde{k}_{IJ}^{(\mathbf{p})} := k(\tilde{K}_I, \tilde{K}_J) = \operatorname{diag}(1, -1, \ldots, -1, 0, \ldots, 0).$$

$$\mathcal{F}_{IJK} \geq 0 \ \forall I, J, K \in \mathcal{J}$$

$$\mathcal{T}_{\mathbf{p}^{(I)}} \longrightarrow \frac{1}{4} \mathcal{F}_{III} (X^I)^2$$

$$\mathcal{F}_{III} \geq 0,$$

$$\mathcal{T}_{\mathbf{p}^{(I)}} \longrightarrow \frac{1}{4} \mathcal{F}_{IJJ} (X^J)^2.$$

$$\mathcal{F}_{IJJ} \geq 0.$$

$$k(K_I, K_I) \geq 0, k(K_J, K_J) \geq 0, k(K_K, K_K) \geq 0$$

$$k(K_I, K_J) \geq 0, k(K_I, K_K) \geq 0.$$

$$\mathrm{q}^2 \sim \mathcal{Q} \longrightarrow 0$$

$$\mathrm{d}\left(\varphi_0^i, \varphi_\infty^i\right) = \int_1^\infty \mathrm{d}\lambda \sqrt{\frac{1}{2} g_{ij}(\varphi) \frac{\mathrm{d}\varphi^i}{\mathrm{d}\lambda} \frac{\mathrm{d}\varphi^j}{\mathrm{d}\lambda}}$$

$$\mathcal{P} = \{\mathbf{X}(\lambda) \in \Delta : \mathcal{F}[\mathbf{X}] = 1, \lambda \in [1, \infty[\}$$

$$\mathrm{d}(X_0^I, X_\infty^I) = \int_1^\infty \mathrm{d}\lambda \sqrt{\frac{1}{2} f_{IJ} \frac{\mathrm{d}X^I}{\mathrm{d}\lambda} \frac{\mathrm{d}X^J}{\mathrm{d}\lambda}}\Bigg|_{\mathcal{F}[\mathbf{X}]=1}$$

$$X^0 \sim \lambda, X^I \lesssim \lambda \; \forall I \in \mathcal{J} \setminus \{0\}.$$

$$\mathcal{J}_\lambda = \{I \in \mathcal{J} \mid X^I \sim \lambda\}.$$

$$\mathcal{F}_{abc} = 0 \; \forall a, b, c \in \mathcal{J}_\lambda,$$

$$\mathcal{J} = \bigsqcup_{k=0}^{3} \mathcal{J}_k$$

$$\mathcal{J}_1 = \{i \in \mathcal{J} : \mathcal{F}_{00i} \neq 0\},$$
$$\mathcal{J}_2 = \left\{\mu \in \mathcal{J} : \mathcal{F}_{00\mu} = 0 \text{ and } \exists \nu \in \mathcal{J}_2 : \mathcal{F}_{0\mu\nu} \neq 0\right\},$$
$$\mathcal{J}_3 = \{r \in \mathcal{J} : \mathcal{F}_{00r} = 0 \text{ and } \mathcal{F}_{0rs} = 0 \forall s \in \mathcal{J}_2 \sqcup \mathcal{J}_3\}.$$

$$A_{\min} = \sum_{i \in \mathcal{J}_1} c_i A^i,$$

$$\mathcal{F} = \frac{1}{2}\mathcal{F}_{00i}(X^0)^2 X^i + \frac{1}{2}\mathcal{F}_{0ij} X^0 X^i X^j + \frac{1}{6}\mathcal{F}_{0ir} X^0 X^i X^r + \frac{1}{6}\mathcal{F}_{ijk} X^i X^j X^k + \frac{1}{2}\mathcal{F}_{ijr} X^i X^j X^r + \frac{1}{2}\mathcal{F}_{irs} X^i X^r X^s \quad (4.1)$$

$$\mathcal{F}_0 \supset \mathcal{F}_{00i} X^0 X^i \sim \lambda^{-1}, \mathcal{F}_i \supset \mathcal{F}_{00i}(X^0)^2 \sim \lambda^2, \mathcal{F}_r \supset \mathcal{F}_{0ir} X^0 X^i \sim \lambda^{-1},$$

$$f_{ij} = \mathcal{F}_i \mathcal{F}_j - \mathcal{F}_{ij} \sim \lambda^4,$$

$$\mathrm{q}_{\min}^2 \sim \lambda^{-4}.$$

$$\mathcal{Q}_{\mathbf{p}}^2 = \frac{1}{2}(p^0)^2 f_{00} + p^0 f_{0i} p^i + p^0 f_{0r} p^r + \frac{1}{2} p^i f_{ij} p^j + p^i f_{ir} p^r + \frac{1}{2} p^r f_{rs} p^s.$$

$$f_{00} \sim \lambda^{-2}, f_{rr} \sim \lambda^{-2},$$

$$\mathcal{Q}_{\mathbf{p}}^2 \gtrsim \lambda^{-2} \; \forall \mathbf{p} \in \mathcal{C}_{\text{SUGRA}}$$

$$f_{00}|_{\mathcal{O}(\lambda^{-2})} = \mathcal{F}_{00i} X^i \left(\mathcal{F}_{00j}(X^0)^2 X^j + 2\mathcal{F}_{0jr} X^0 X^j X^r - 1\right) + \mathcal{F}_{00i} \beta_s(i) \mathcal{F}_{0jr} X^i X^j X^r X^s$$

$$\beta_r(i) = \frac{\mathcal{F}_{0ir}}{\mathcal{F}_{00i}}$$

$$\lim_{\lambda\to\infty}\mathcal{F}[\mathbf{X}]=\frac{1}{2}\mathcal{F}_{00i}(X^0)^2X^i+\mathcal{F}_{0ir}X^0X^iX^r+\frac{1}{2}\mathcal{F}_{irs}X^iX^rX^s=1$$

$$\frac{f_{00}|_{\mathcal{O}(\lambda^{-2})}}{\mathcal{F}_{00i}X^i}=\frac{1}{2}\mathcal{F}_{00j}(X^0)^2X^j+\mathcal{F}_{0jr}X^0X^jX^r+\left(\mathcal{F}_{0jr}\beta_s-\frac{1}{2}\mathcal{F}_{jrs}\right)X^jX^rX^s$$

$$\frac{k(K_0,K_r)k(K_0,K_s)}{k(K_0,K_0)}-\frac{1}{2}k(K_r,K_s)\geq 0$$

$$\frac{f_{rr}|_{\mathcal{O}(\lambda^{-2})}}{\mathcal{F}_{irr}X^i}=\left(\beta^2\mathcal{F}_{jrr}-\frac{1}{2}\mathcal{F}_{00j}\right)(X^0)^2X^j+(2\beta\gamma_s\mathcal{F}_{jrr}-\mathcal{F}_{0js})X^0X^jX^s$$
$$+\left(\gamma_s\gamma_t\mathcal{F}_{jrr}-\frac{1}{2}\mathcal{F}_{jst}\right)X^jX^sX^t$$

$$\beta=\frac{\mathcal{F}_{0jr}}{\mathcal{F}_{jrr}}\ \text{ and }\ \gamma_s=\frac{\mathcal{F}_{jrs}}{\mathcal{F}_{jrr}}.$$

$$f_{ij}\big|_{\mathcal{O}(\lambda^4)}=\left(\frac{1}{2}\mathcal{F}_{00i}(X^0)^2+\mathcal{F}_{0ir}X^0X^r+\frac{1}{2}\mathcal{F}_{irs}X^rX^s\right)$$
$$\times\left(\frac{1}{2}\mathcal{F}_{00j}(X^0)^2+\mathcal{F}_{0jt}X^0X^t+\frac{1}{2}\mathcal{F}_{itu}X^tX^u\right)$$

$$(\,f_{ij}\big|_{\mathcal{O}(\lambda^4)}\,)\ \mathrm{rk}\left(f_{ij}\big|_{\mathcal{O}(\lambda^4)}\right)=1$$

$$c_if_{jk}\big|_{\mathcal{O}(\lambda^4)}=c_jf_{ik}\big|_{\mathcal{O}(\lambda^4)}\ \forall k\in\mathcal{J}_1$$

$$c_i\mathcal{F}_{jab}=c_j\mathcal{F}_{iab}\ \text{ for all } a,b\in\{0\}\sqcup\mathcal{J}_3,$$

$$A_{\min}=\sum_{i\in\mathcal{J}_1}c_iA^i$$

$$\mathcal{F}_{00I}=0\ \forall I\in\mathcal{J},$$

$$\mathcal{F}=\frac{1}{2}\mathcal{F}_{0\mu\nu}X^0X^\mu X^\nu+\frac{1}{6}\mathcal{F}_{\mu\nu\rho}X^\mu X^\nu X^\rho$$

$$\mathcal{F}_0\sim\lambda^{-1},\mathcal{F}_\mu\succ\lambda^{-1}$$

$$X^\nu\lesssim\lambda^{-1/2}\ \forall\nu\in\mathcal{J}_2.$$

$$\mathcal{Q}^2_{\min}=\mathcal{Q}^2_{\delta^0}=\frac{1}{2}f_{00}\sim\lambda^{-2}$$

$$f'=(f_{\mu\nu})\big|_{\mathcal{O}(\lambda)}=\left(\mathcal{F}_{0\mu\rho}\mathcal{F}_{0\nu\sigma}(X^0)^2X^\rho X^\sigma\big|_{\mathcal{O}(\lambda)}-\mathcal{F}_{0\mu\nu}X^0\right),$$

$$\mathcal{C}=\langle K_\mu\mid\mu\in\mathcal{J}_2\rangle\subset\mathcal{A}$$

$$k^{(\delta^0)}_{\mu\nu}=k_{\mu\nu}=\mathcal{F}_{0\mu\nu},f'$$

$$f'_{\mu\nu} = X^0\left(k_{\mu\rho}k_{\nu\sigma}X^0X^\rho X^\sigma - k_{\mu\nu}\right)$$

$$\tilde{f}'_{\mu\nu} = X^0\left(\tilde{k}_{\mu\rho}\tilde{k}_{\nu\sigma}X^0\tilde{X}^\rho\tilde{X}^\sigma - \tilde{k}_{\mu\nu}\right),$$

$$\tilde{f}' = X^0\begin{pmatrix} X^0\left(\delta_{0\rho}\tilde{X}^\rho\right)^2 - 1 & 0 & 0 \\ 0 & \left(X^0\delta_{\alpha\gamma}\tilde{X}^\gamma\delta_{\beta\tau}\tilde{X}^\tau + \delta_{\alpha\beta}\right) & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A^I \longmapsto \Lambda^I_J A^J, X^I \longmapsto \Lambda^I_J X^J$$

$$k_{IJ} \longmapsto \Sigma^L_I\Sigma^M_J k_{LM}, \mathcal{F}_{IJK} \longmapsto \Sigma^L_I\Sigma^M_J\Sigma^N_K\mathcal{F}_{LMN}, f_{IJ} \longmapsto \Sigma^L_I\Sigma^M_J f_{LM},$$

$$f_{\mu_0\rho}\Big|_{\mathcal{O}(\lambda^{-1/2})} = c f_{\nu_0\rho}\Big|_{\mathcal{O}(\lambda^{-1/2})}$$

$$f'_{\mu_0\rho} = c f'_{\nu_0\rho},\ f_{\mu_0\rho}\Big|_{\mathcal{O}(\lambda^{-1/2})} = c f_{\nu_0\rho}\Big|_{\mathcal{O}(\lambda^{-1/2})}$$

$$\left(\mathcal{F}_{\mu_0} - c\mathcal{F}_{\nu_0}\right)\left(1 - \mathcal{F}_{0\rho\sigma}X^0X^\rho X^\sigma\right) = \frac{1}{2}\mathcal{F}_{\rho\gamma\delta}X^\rho X^\gamma X^\delta\left(\mathcal{F}_{0\mu_0\nu} - c\mathcal{F}_{\nu_0\nu}\right)X^0X^\nu$$

$$\frac{1}{2}\left(\mathcal{F}_{\mu_0\alpha\beta} - c\mathcal{F}_{\nu_0\alpha\beta}\right)X^\alpha X^\beta\left(1 - \mathcal{F}_{0\rho\sigma}X^0X^\rho X^\sigma\right) = 0$$

$$\mathfrak{q}^2_{\min} \sim \lambda^{-1}$$

$$N_{\min} = \mathrm{rk}(f') \geq 1$$

$$X^{\mu_0} \sim \lambda^{-1/2+x},\ \text{for some } x \in \left(0, \frac{3}{2}\right).$$

$$\mathcal{Q}^2_{\max} \supset \frac{1}{2} f_{\nu\nu} \sim \lambda^{1+2x} \sim \mathfrak{q}^{-2}_{\min},$$

$$\mathcal{Q}^2_{\min} \geq \frac{2}{3}\mathcal{T}^2_{\min} \succ \lambda^{-2}$$

$$\mathcal{Q}^2_{\min} \succ \lambda^{-2}$$

$$\emptyset \neq \mathcal{J}^{\mu_0}_2 \subseteq \mathcal{J}_2\left(f_{\mu\nu}\Big|_{\mathcal{O}(\lambda^{1+2x})}\right)\mu,\nu \in \mathcal{J}^{\mu_0}_2$$

$$c_\mu\mathcal{F}_{0\mu_0\nu} = c_\nu\mathcal{F}_{0\mu_0\mu},$$

$$A_{\min} = \sum_{\mu\in\mathcal{J}^{\mu_0}_2} c_\mu A^\mu.$$

$$\mathcal{J}'_2 = \left\{\mu' \in \mathcal{J}_2 : \mathcal{F}_{ab\mu'} = 0\ \forall a, b \in \mathcal{J}_\lambda\right\}$$
$$\mathcal{J}''_2 = \left\{\mu'' \in \mathcal{J}_2 : \mathcal{F}_{ab\mu''} \neq 0\ \forall a \neq b \in \mathcal{J}_\lambda\right\}$$

$$\mathcal{J} = \mathcal{J}_\lambda \sqcup \mathcal{J}'_2 \sqcup \mathcal{J}''_2$$

$$X^{\mu''} \precsim \lambda^{-2}\ \forall\mu'' \in \mathcal{J}''_2, X^{\mu'} \prec \lambda\ \forall\mu' \in \mathcal{J}'_2$$

$$X^a = \lambda \hat{X}^a \; a = 0, \alpha, \alpha \in \mathcal{J}'_\lambda$$

$$K_D := \sum_{a \in \mathcal{J}_\lambda} \hat{X}^a K_a$$

$$\mathbf{X} = \lambda K_D + X^{\mu'} K_{\mu'} + X^{\mu''} K_{\mu''}, X^{\mu'}, X^{\mu''} \prec \lambda.$$

$$\mathcal{F}_{DDD} = 0, \mathcal{F}_{DD\alpha} = 0, \mathcal{F}_{DD\mu'} = 0, \mathcal{F}_{DD\mu''} \neq 0$$

$$\{D\} \leftrightarrow \{0\}, \mathcal{J}'_\lambda \cup \mathcal{J}'_2 \leftrightarrow \mathcal{J}_3, \mathcal{J}''_2 \leftrightarrow \mathcal{J}_1$$

$$\mathfrak{q}_{\min}^{-2} \sim \mathcal{Q}_{\max}^2 \sim \lambda^4 \sim f_{\mu''\nu''}$$

$$\left(f_{\mu''\nu''}\big|_{\mathcal{O}(\lambda^4)} \right) \mathfrak{q}_{\min}^2 \sim \lambda^{-4}$$

$$\operatorname{rk}\left(f_{\mu''\nu''}\big|_{\mathcal{O}(\lambda^4)} \right) = 1.$$

$$\mathcal{Q}_{\min}^2 \sim \lambda^{-2}, \mathcal{Q}_{\max}^2 \sim \mathfrak{q}_{\min}^{-2} \sim \lambda^4.$$

$$\frac{\mathrm{d}X^0}{\mathrm{d}\lambda} = \hat{X}^0, \frac{\mathrm{d}X^i}{\mathrm{d}\lambda} = -2\lambda^{-3}\hat{X}^i, \frac{\mathrm{d}X^0}{\mathrm{d}\lambda} = \hat{X}^r,$$

$$\lim_{\lambda\to\infty} \mathcal{F}[\mathbf{X}] = \frac{1}{2}\mathcal{F}_{00i}(X^0)^2 X^i + \mathcal{F}_{0ir} X^0 X^i X^r + \frac{1}{2}\mathcal{F}_{irs} X^i X^r X^s = 1$$

$$f_{IJ} \frac{\mathrm{d}X^I}{\mathrm{d}\lambda}\frac{\mathrm{d}X^J}{\mathrm{d}\lambda} = 6\lambda^{-2} + \mathcal{O}(\lambda^{-3})$$

$$\mathfrak{q}_{\min} \sim \lambda^{-2} = \exp\left(-\alpha \mathrm{d}(X_0^I, X_\lambda^I)\right), \alpha = \frac{2}{\sqrt{3}} = \sqrt{\frac{d-1}{d-2}}\Bigg|_{d=5}.$$

$$\alpha = \sqrt{\frac{d-1}{d-2}}\Bigg|_{d=5}$$

$$\frac{m_{\mathrm{KK}}}{M_{\mathrm{Pl}}} \sim \exp\left(-\alpha \mathrm{d}(X_0^I, X_\lambda^I)\right)$$

$$\alpha = \sqrt{\frac{d-1}{d-2}}\Bigg|_{d=5}$$

$$\mathcal{Q}_{\min}^2 \sim \lambda^{-2} \sim \mathfrak{q}_{\min}^4$$

$$\lim_{\lambda\to\infty} \mathcal{F}[\mathbf{X}] = \frac{1}{2}\mathcal{F}_{0\mu\nu} X^0 X^\mu X^\nu = 1,$$

$$\frac{\mathrm{d}X^0}{\mathrm{d}\lambda} = \hat{X}^0, \frac{\mathrm{d}X^\mu}{\mathrm{d}\lambda} = -\frac{1}{2}\lambda^{-3/2}\hat{X}^\mu$$

$$f_{IJ}\frac{\mathrm{d}X^I}{\mathrm{d}\lambda}\frac{\mathrm{d}X^J}{\mathrm{d}\lambda}=\frac{3}{2}\lambda^{-2}+\mathcal{O}(\lambda^{-3}),$$

$$\mathcal{Q}_{\min}^{1/2}\sim \mathfrak{q}_{\min}\sim \exp\left(-\alpha \mathrm{d}(X_0,X_\lambda)\right),\alpha=\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{d-2}}\bigg|_{d=5}.$$

$$\frac{M_{\mathrm{str}}}{M_{\mathrm{P}}}\sim \exp\left(-\alpha \mathrm{d}(X_0,X_\lambda)\right),$$

$$S_{ARR}=\frac{1}{192}\int_{\mathbb{R}^{1,4}}C_I A^I\wedge \mathrm{tr}(\mathcal{R}\wedge\mathcal{R})$$

$$c_L=C_0,c_R=\frac{1}{2}C_0$$

$$C_0\in 12\mathbb{N}_0$$

$$\mathcal{F}_{00I}=0\ \forall I\in\mathcal{J}\overset{\mathrm{ESC}}{\Longrightarrow}C_0\in\{0,24\}.$$

$$\mathcal{F}_{IJK}\geq 0\ \forall I,J,K\in\mathcal{J}$$

$$k(K_I,K_J)=\mathcal{F}_{IIJ}=0,k(K_J,K_J)=\mathcal{F}_{IJJ}=0$$

$$K_I=a_0\tilde{K}_0+a_\alpha\tilde{K}_\alpha+a_\zeta\tilde{K}_\zeta$$
$$K_J=b_0\tilde{K}_0+b_\alpha\tilde{K}_\alpha+b_\zeta\tilde{K}_\zeta$$

$$a_0b_0=a_\alpha b_\alpha,b_0^2=b_\alpha b_\alpha$$

$$a_0^2=\frac{(a_\alpha b_\alpha)^2}{b_0^2}=\frac{(a_\alpha b_\alpha)^2}{b_\alpha b_\alpha}\leq a_\alpha a_\alpha$$

$$b_0^2>b_\alpha b_\alpha,a_0b_0=a_\alpha b_\alpha$$

$$a_0^2=\frac{(a_\alpha b_\alpha)^2}{b_0^2}<\frac{(a_\alpha b_\alpha)^2}{b_\alpha b_\alpha}\leq a_\alpha a_\alpha$$

$$\begin{aligned}\mathcal{F}[X]=&\frac{1}{2}\mathcal{F}_{IIK}(X^I)^2X^K+\mathcal{F}_{IJK}X^IX^JX^K+\frac{1}{2}\mathcal{F}_{IKK}X^I(X^K)^2\\&+\frac{1}{2}\mathcal{F}_{JJK}(X^J)^2X^K+\frac{1}{2}\mathcal{F}_{JKK}X^J(X^K)^2+\frac{1}{6}\mathcal{F}_{KKK}(X^K)^3\end{aligned}$$

$$\mathcal{Q}_{\delta^I}^2=\mathcal{F}_I^2-\mathcal{F}_{II}=\left(\mathcal{F}_{IIK}X^IX^K+\mathcal{F}_{IJK}X^JX^K+\mathcal{O}\left(\frac{1}{\lambda^4}\right)\right)^2-\mathcal{F}_{IIK}X^K$$

$$X^I=-\frac{\mathcal{F}_{\mathrm{IJK}}}{\mathcal{F}_{\mathrm{IIK}}}X^J$$

$$\mathcal{F}_{IJK}\geq 0\ \forall I,J,K\in\mathcal{J}.$$

$$K_0=a_0\tilde{K}_0+a_\alpha\tilde{K}_\alpha+a_\zeta\tilde{K}_\zeta,K_\mu=b_0^\mu\tilde{K}_0+b_\alpha^\mu\tilde{K}_\alpha+b_\zeta^\mu\tilde{K}_\zeta$$

$$0 = \mathcal{F}_{000} = k(K_0, K_0) = a_0^2 - a_\alpha a_\alpha, 0 = \mathcal{F}_{00\mu} = k(K_0, K_\mu) = a_0 b_0^\mu - a_\alpha b_\alpha^\mu$$

$$a_0^2 = a_\alpha a_\alpha, b_0^\mu = \frac{a_\alpha b_\alpha^\mu}{a_0}$$

$$\mathcal{F}_{0\mu\mu} = k(K_\mu, K_\mu) = \left(b_0^\mu\right)^2 - b_\alpha^\mu b_\alpha^\mu = \frac{\left(a_\alpha b_\alpha^\mu\right)^2}{a_0^2} - b_\alpha^\mu b_\alpha^\mu$$

$$\mathcal{F}_{0\mu\mu} \leq \frac{(a_\alpha a_\alpha)\left(b_\beta^\mu b_\beta^\mu\right)}{a_0^2} - b_\alpha^\mu b_\alpha^\mu = 0$$

$$K_0|_{\mathcal{B}^0} = \beta_{(\mu)} K_\mu|_{\mathcal{B}^0}$$

$$\begin{aligned} 0 &= \mathcal{F}_{00\mu} = k(K_0, K_\mu) = a_0 b_0^\mu - a_\alpha b_\alpha^\mu \\ &= \beta_{(\nu)}\left(b_0^\nu b_0^\mu - b_\alpha^\nu b_\alpha^\mu\right) = \beta_{(\nu)} k(K_\nu, K_\mu) = \beta_{(\nu)} \mathcal{F}_{0\mu\nu} \end{aligned}$$

$$K_0|_{\mathcal{B}^0} = \beta_{(\mu)} K_\mu|_{\mathcal{B}^0}$$

$$a_0 = \beta_{(\mu)} b_0^\mu, a_\alpha = \beta_{(\mu)} b_\alpha^\mu$$

$$\mathcal{J}_2 = \emptyset.$$

$$K_0 = a_0 \tilde{K}_0 + a_\alpha \tilde{K}_\alpha + a_\zeta \tilde{K}_\zeta, K_r = c_0 \tilde{K}_0 + c_\alpha \tilde{K}_\alpha + c_\zeta \tilde{K}_\zeta$$

$$\begin{aligned} \mathcal{F}_{00i} &= k(K_0, K_0) = a_0^2 - a_\alpha a_\alpha > 0 \\ \mathcal{F}_{irr} &= k(K_r, K_r) = c_0^2 - c_\alpha c_\alpha \geq 0 \end{aligned}$$

$$|a_0| > \sqrt{a_\alpha a_\alpha} \text{ and } |c_0| \geq \sqrt{c_\alpha c_\alpha}$$

$$k(K_0, K_r) = \mathcal{F}_{0ir} = a_0 c_0 - a_\alpha c_\alpha$$

$$|c_0| = \frac{|a_\alpha c_\alpha|}{|a_0|} < \frac{|a_\alpha c_\alpha|}{\sqrt{a_\alpha a_\alpha}} \leq \sqrt{c_\alpha c_\alpha},$$

$$\mathcal{F}_{0ir} \neq 0 \text{ for all } i \in \mathcal{J}_1 \text{ and } r \in \mathcal{J}_3.$$

$$K_0 = a_0 \tilde{K}_0 + a_\alpha \tilde{K}_\alpha + a_\zeta \tilde{K}_\zeta, K_s = b_0^s \tilde{K}_0 + b_\alpha^s \tilde{K}_\alpha + b_\zeta^s \tilde{K}_\zeta$$

$$a_0^2 - a_\alpha a_\alpha = 0, a_0 b_0^s - a_\alpha b_\alpha^s = 0$$

$$k(K_s, K_s) = \mathcal{F}_{rss} \geq 0$$

$$|b_0^s| = \frac{|a_\alpha b_\alpha^s|}{|a_0|} = \frac{|a_\alpha b_\alpha^s|}{\sqrt{a_\alpha a_\alpha}}, |b_0^s| \geq \sqrt{b_\alpha^s b_\alpha^s}$$

$$|b_0^s| \leq \sqrt{b_\alpha^s b_\alpha^s}$$

$$|b_0^s| = \sqrt{b_\alpha^s b_\alpha^s}$$

$$a_\alpha = \beta_{(s)} b_\alpha^s$$

$$a_0 = \beta_{(s)} b_0$$

$$K_0 \big| \mathcal{B}^r = \beta_{(s)} K_s \big|_{\mathcal{B}^r}$$

$$K_0|_{\mathcal{B}} = \beta_{(t)} K_t \big|_{\mathcal{B}^r}$$

$$k(K_I, K_0) = \mathcal{F}_{rI0} = 0$$

$$\mathcal{F}_{rst} = k(K_s, K_t) = \frac{1}{\beta_{(s)}\beta_{(t)}} k(K_0, K_0) = \frac{1}{\beta_{(s)}\beta_{(t)}} \mathcal{F}_{00r} = 0$$

$$\mathcal{F}_{rst} = 0 \text{ for all } r, s, t \in \mathcal{J}_3.$$

$$\beta_{(r)} K_0 \big|_{\mathcal{B}^0} = K_r|_{\mathcal{B}^0}$$

$$\mathcal{F}_{0ir} = k(K_i, K_r) = \beta_{(r)} k(K_0, K_i) = \frac{k(K_0, K_i)}{k(K_0, K_j)} k(K_j, K_r),$$

$$c_i \mathcal{F}_{0jr} = c_j \mathcal{F}_{0ir} \text{ for all } i, j \in \mathcal{J}_1 \text{ and } r \in \mathcal{J}_3,$$

$$c_i = \mathcal{F}_{00i}$$

$$\beta'_{(s)} K_0 \big|_{\mathcal{B}^r} = K_s|_{\mathcal{B}^r}$$

$$c_i \mathcal{F}_{jab} = c_j \mathcal{F}_{iab} \text{ for all } a, b \in \{0\} \sqcup \mathcal{J}_3.$$

$$B = k(K_r, A)K_0 - k(K_0, A)K_r \, \{\tilde{K}_0, \tilde{K}_\alpha, \tilde{K}_\zeta\}$$

$$K_s = a_0 \tilde{K}_0 + a_\alpha \tilde{K}_\alpha + a_\zeta \tilde{K}_\zeta, B = b_0 \tilde{K}_0 + b_\alpha \tilde{K}_\alpha + b_\zeta \tilde{K}_\zeta$$

$$k(K_s, K_s) = a_0^2 - a_\alpha a_\alpha \geq 0, k(K_s, B) = a_0 b_0 - a_\alpha b_\alpha = 0$$

$$k(K_r, K_s)^2 k(K_0, K_0) + k(K_0, K_s)^2 k(K_r, K_r) - 2k(K_r, K_s)k(K_0, K_s)k(K_r, K_0) \leq 0.$$

$$k(K_0, K_r)k(K_0, K_s) \geq \frac{k(K_0, K_0)k(K_r, K_s)}{2}$$

$$k(K_0, K_r)^2 \geq \frac{k(K_0, K_0)k(K_r, K_r)}{2}, k(K_0, K_r)k(K_r, K_s) \geq \frac{k(K_0, K_s)k(K_r, K_r)}{2}$$

$$k(K_r, K_s)k(K_r, K_t) \geq \frac{k(K_r, K_r)k(K_s, K_t)}{2}$$

$$k_{IJ}^{(\mathbf{p})} = k_{IJ} = \sum_{L} \mathcal{F}_{LIJ}$$

$$K_0 = a_0\tilde{K}_0 + a_\alpha\tilde{K}_\alpha, K_r = b_0^r\tilde{K}_0 + b_\alpha^r\tilde{K}_\alpha$$

$$k(K_0, K_0) = \sum_{L} k_{L00} = a_0a_0 - a_\alpha a_\alpha = 0,$$
$$k(K_r, K_r) = \sum_{L} k_{Lrr} = b_0^rb_0^r - b_\alpha^rb_\alpha^r \geq 0,$$

$$|b_0^r| \geq \sqrt{b_\alpha^rb_\alpha^r}$$

$$k(K_0, K_r) = \sum_{L} k_{L0r} = a_0b_0^r - a_\alpha b_\alpha^r$$

$$|b_0^r| = \frac{|a_\alpha b_\alpha^r|}{\sqrt{a_\alpha a_\alpha}} \leq \sqrt{b_\alpha^rb_\alpha^r}.$$

$$|b_0^r| = \sqrt{b_\alpha^rb_\alpha^r}$$

$$k(K_0, K_r) = \sum_{L} k_{L0r} = \sum_{i} k_{I0r}$$

$$\mathcal{J}_3 = \emptyset$$

$$k^{(\delta^a)}(K_b, K_{\mu'}) = k(K_b, K_{\mu'}) = \mathcal{F}_{ab\mu'} = 0$$

$$k(K_{\mu'}, K_{\mu'}) = K_{\mu'}\big|_{\mathcal{B}^a} \propto K_b\big|_{\mathcal{B}^a}$$

$$\mathcal{F}_{a\mu'\nu'} = k(K_{\mu'}, K_{\nu'}) \propto k(K_b, K_{\nu'}) = \mathcal{F}_{ab\nu'}$$

$$\mathcal{F}_{a\mu'\nu'} = 0$$

$$k(K_b, K_b) = k(K_b, K_{\mu'}) = 0$$

$$K_b|_{\mathcal{B}^a} = \beta K_{\mu'}\big|_{\mathcal{B}^a}$$

$$\mathcal{F}_{ab\nu''} = k(K_b, K_{\nu''}) = \beta k(K_{\mu'}, K_{\nu''}) = \mathcal{F}_{a\mu'\nu''}$$

$$\mathcal{F}_{a\mu'\nu''} \neq 0$$

$$k_{IJ}^{(\delta^{\mu'})} = k_{IJ} = \mathcal{F}_{\mu'IJ}$$

$$K_a = a_0\tilde{K}_0 + a_\alpha\tilde{K}_\alpha + a_\zeta\tilde{K}_\zeta, K_{\nu'} = b_0\tilde{K}_0 + b_\alpha\tilde{K}_\alpha + b_\zeta\tilde{K}_\zeta$$

$$K_a\big|_{\mathcal{B}^{\mu'}} = \beta^a_{\nu'}K_{\nu'}\big|_{\mathcal{B}^{\mu'}},$$

$$\mathcal{F}_{\mu'\nu'\rho'} = \beta^a_{\nu'}\mathcal{F}_{a\mu'\rho'} = \beta^a_{\nu'}\beta^b_{\rho'}\mathcal{F}_{ab\mu'}$$

$$\mathcal{F}_{\mu'\nu'\rho} = 0.$$

$$\mathcal{F} = X^0\mathcal{F}_0 + \frac{1}{6}\mathcal{F}_{\mu\nu\rho}X^\mu X^\nu X^\rho,\ \text{with}\ \mathcal{F}_0 = \frac{1}{2}\mathcal{F}_{0\mu\nu}X^\mu X^\nu$$

$$\mathcal{F}_{0\mu\nu}X^\mu X^\nu \lesssim \lambda^{-1}, \mathcal{F}_{0\nu\rho}X^\nu X^\rho \lesssim \lambda^{-1}, \mathcal{F}_{0\rho\mu}X^\rho X^\mu \lesssim \lambda^{-1}$$

$$K_0 = a_0\tilde{K}_0 + a_\alpha\tilde{K}_\alpha + a_\zeta\tilde{K}_\zeta, K_\mu = b_0\tilde{K}_0 + b_\alpha\tilde{K}_\alpha + b_\zeta\tilde{K}_\zeta$$

$$k(K_0, K_0) = a_0^2 - a_\alpha a_\alpha = 0, k\big(K_\mu, K_\mu\big) = b_0^2 - b_\alpha b_\alpha \geq 0$$

$$|b_0| = \frac{|a_\alpha b_\alpha|}{\sqrt{a_\alpha a_\alpha}} \leq \sqrt{b_\alpha b_\alpha}$$

$$|b_0| = \sqrt{b_\alpha b_\alpha}$$

$$K_\mu\big|_{\mathcal{B}^\nu} = \beta K_0|_{\mathcal{B}^\nu}$$

$$\beta\mathcal{F}_{0\rho\nu} = \beta k\big(K_0, K_\rho\big) = k\big(K_\mu, K_\rho\big) = \mathcal{F}_{\mu\nu\rho} > 0$$

$$\mathcal{F}_0 \sim \lambda^{-1}.$$

$$\mathcal{F}_\mu = \mathcal{F}_{0\mu\nu}X^0X^\nu + \frac{1}{2}\mathcal{F}_{\mu\rho\sigma}X^\rho X^\sigma \prec \lambda^{-1}$$

$$X^{\rho_0}X^{\sigma_0} \sim \lambda^{-1}\ \text{and}\ \mathcal{F}_{0\rho_0\sigma_0} \neq 0$$

$$k\big(K_\mu, K_\mu\big) = \mathcal{F}_{\mu\mu\sigma_0} \geq 0, k\big(K_{\rho_0}, K_{\rho_0}\big) = \mathcal{F}_{\rho_0\rho_0\sigma_0} \geq 0$$

$$K_\mu = a_0\tilde{K}_0 + a_\alpha\tilde{K}_\alpha + a_\zeta\tilde{K}_\zeta, K_{\rho_0} = b_0\tilde{K}_0 + b_\alpha\tilde{K}_\alpha + b_\zeta\tilde{K}_\zeta$$

$$|b_0| \geq \sqrt{b_\alpha b_\alpha}, \mathcal{F}_{0\mu\sigma_0} = k\big(K_0, K_\mu\big) = 0$$

$$|b_0| = \frac{|a_\alpha b_\alpha|}{|a_0|} \leq \frac{|a_\alpha b_\alpha|}{\sqrt{a_\alpha a_\alpha}} \leq \sqrt{b_\alpha b_\alpha}$$

$$K_{\rho_0}\big|_{\mathcal{B}^{\sigma_0}} = \beta K_\mu\big|_{\mathcal{B}^{\sigma_0}}$$

$$0 \neq \mathcal{F}_{0\rho_0\sigma_0} = k\big(K_0, K_{\rho_0}\big) = \beta k\big(K_0, K_\mu\big) = \beta\mathcal{F}_{0\mu\sigma_0}$$

$$K_\mu = a_0\tilde{K}_0 + a_\alpha\tilde{K}_\alpha + a_\zeta\tilde{K}_\zeta$$
$$K_{\rho_0} = b_0\tilde{K}_0 + b_\alpha\tilde{K}_\alpha + b_\zeta\tilde{K}_\zeta, K_{\sigma_0} = c_0\tilde{K}_0 + c_\alpha\tilde{K}_\alpha + c_\zeta\tilde{K}_\zeta$$

$$k(K_\mu, K_{\rho_0}) = 0, k(K_\mu, K_{\sigma_0}) = 0$$

$$a_0 b_0 - a_\alpha b_\alpha = 0, a_0 c_0 - a_\alpha c_\alpha = 0$$

$$k(K_\mu, K_\mu) = a_0^2 - a_\alpha a_\alpha \geq 0$$

$$|b_0| = \frac{\sqrt{a_\alpha b_\alpha}}{|a_0|} \leq \frac{\sqrt{a_\alpha b_\alpha}}{\sqrt{a_\alpha a_\alpha}} \leq \sqrt{b_\alpha b_\alpha}$$

$$|c_0| \leq \sqrt{c_\alpha c_\alpha}, |a_0| \leq \sqrt{a_\alpha a_\alpha}$$

$$k(K_{\rho_0}, K_{\rho_0}) \leq 0, k(K_{\sigma_0}, K_{\sigma_0}) \leq 0$$

$$k(K_{\rho_0}, K_{\sigma_0}) = b_0 c_0 - b_\alpha c_\alpha = \beta k(K_{\sigma_0}, K_{\sigma_0}) = 0$$

$$\mathcal{F}_\mu \succ \lambda^{-1}$$

$$\mathcal{J} = \mathcal{J}_\lambda \sqcup \mathcal{J}_2' \sqcup \mathcal{J}_2''$$

$$\mathcal{J}_2' = \{\mu' \in \mathcal{J}_2 : \mathcal{F}_{ab\mu'} = 0 \forall a, b \in \mathcal{J}_\lambda\},$$
$$\mathcal{J}_2'' = \{\mu'' \in \mathcal{J}_2 : \mathcal{F}_{ab\mu''} \neq 0 \forall a \neq b \in \mathcal{J}_\lambda\}.$$

$$k_{a_0 I}^{(\delta^{a_0})} = \mathcal{F}_{a_0 a_0 I}\ I \in \mathcal{J}_\lambda k^{(\delta^{a_0})}\ k^{(\delta^{a_0})}(K_{b_0}, K_{b_0}) = \mathcal{F}_{a_0 b_0 b_0}\ k^{(\delta^{a_0})}(K_\mu, K_\mu) = \mathcal{F}_{a_0 \mu\mu}\ k^{(\delta^{a}0)}(K_{b_0}, K_\mu) =$$

$$\mathcal{F}_{a_0 b_0 \mu}\ k^{(\delta^{a_0})}(K_\mu, K_\mu) K_\mu \mid \mathcal{B}^{a_0} = \beta K_{b_0 | \mathcal{B}^{a_0}} \beta$$

$$\mathcal{F}_{a_0 b\mu} = k^{(\delta^{a_0})}(K_b, K_\mu) = \beta k^{(\delta^{a_0})}(K_b, K_{b_0}) = \beta \mathcal{F}_{a_0 b_0 b} = 0$$

$$k^{(\delta^b)}(K_{a_0}, K_{a_0}) = k^{(\delta^b)}(K_\mu, K_\mu) k^{(\delta^b)}(K_{a_0}, K_\mu) = \mathcal{F}_{a_0 b\mu}$$

$$k^{(\delta^b)}(K_\mu, K_\mu) = K_\mu\big|_{\mathcal{B}^b} = \beta' K_{a_0}\big|_{\mathcal{B}^b} \mathcal{A} \setminus \mathcal{B}^b$$

$$\mathcal{F}_{ab\mu} = k^{(\delta^b)}(K_a, K_\mu) = \beta' k^{(\delta^b)}(K_a, K_{a_0}) = \beta' \mathcal{F}_{a a_0 b} = 0$$

$$\mathcal{J} = \mathcal{J}_\lambda \sqcup \mathcal{J}_2' \sqcup \mathcal{J}_2''.$$

$$X^{\mu_0} \sim \lambda^{-1/2+x} \sim X^{\nu_0} \mathcal{F}_{0\mu_0\mu_0} = \mathcal{F}_{0\mu_0\nu_0} = \mathcal{F}_{0\nu_0\nu_0} K_{\mu_0}\big|_{\mathcal{B}^0} \propto K_{\nu_0}\big|_{\mathcal{B}^0}$$

$$G_{MN} = 8\pi G_D T_{MN}$$

$$G_{MN} = R_{MN} - \frac{1}{2} g_{MN} R$$

$$S_{\text{EH}} = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} R + S_{\text{matter}}$$

$$T_{MN} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\text{matter}}}{\delta g^{MN}}$$

$$ds^2 = -\left[1-\left(\frac{r_0}{r}\right)^{D-3}\right]dt^2 + \left[1-\left(\frac{r_0}{r}\right)^{D-3}\right]^{-1} dr^2 + r^2 d\Omega_{D-2}^2$$

$$M = \frac{(D-2)\Omega_{D-2} r_0^{D-3}}{16\pi G_D}, T = \frac{\kappa}{2\pi} = \frac{D-3}{4\pi r_0}, A = \Omega_{D-2} r_0^{D-2}$$

is $S = A/(4G_D)$

$$dM = TdS$$

$$(D-3)M = (D-2)TS$$

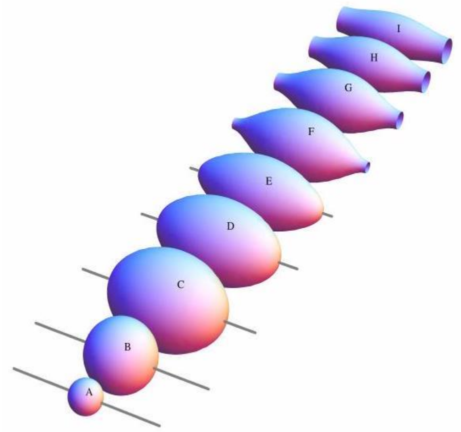
$$ds_{5\text{D}}^2 = g_{MN}dX^M dX^N = e^{\phi/\sqrt{3}} g_{\mu\nu} dx^\mu dx^\nu + e^{-2\phi/\sqrt{3}}\left(dy + A_\mu dx^\mu\right)^2$$

$$\begin{aligned} S &= \frac{1}{16\pi G_5}\int d^5X\sqrt{-g_5}R_{5\text{D}} \\ &= \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R_{4\text{D}} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{-\sqrt{3}\phi}F_{\mu\nu}F^{\mu\nu}\right) \end{aligned}$$

$$G_4 = G_5/(2\pi R).$$

$$\phi(x,y) = \sum_n \phi_n(x)e^{iny/R}$$

$$ds_{5\text{D}}^2 = -\left(1-\frac{2G_4M}{r}\right)dt^2 + \left(1-\frac{2G_4M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2 + dy^2$$

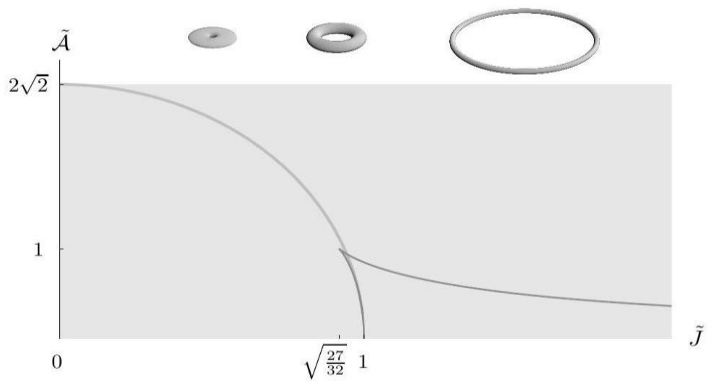


$$\begin{aligned} ds^2 = & -dt^2 + \frac{\mu r^2}{\Delta}(dt + a_1\cos^2\theta d\phi_1 + a_2\sin^2\theta d\phi_2)^2 \\ & + \frac{\Delta}{(r^2+a_1^2)(r^2+a_2^2) - \mu r^2}dr^2 \\ & + (r^2+a_1^2)(\sin^2\theta d\theta^2 + \cos^2\theta d\phi_1^2) + (r^2+a_2^2)(\cos^2\theta d\theta^2 + \sin^2\theta d\phi_2^2) \end{aligned}$$

$$\Delta = (r^2 + a_1^2)(r^2 + a_2^2)\left(1 - \frac{a_1^2 \cos^2\ \theta}{r^2 + a_1^2} - \frac{a_2^2 \sin^2\ \theta}{r^2 + a_2^2}\right).$$

$$M = \frac{3\pi\mu}{8G_5}, J_i = \frac{\pi\mu}{4G_5} a_i$$

$$dM = TdS + \Omega_1 dJ_1 + \Omega_2 dJ_2$$



$$\tilde{\mathcal{A}} = \sqrt{27/(256\pi G_5^3)}\mathcal{A}$$

$$\tilde{J} = \sqrt{27\pi/(32G_5)}J$$

$$M^3 \geq (\tilde{J}_1^2 + \tilde{J}_2^2 + 2|\tilde{J}_1 \tilde{J}_2|), \text{ where } \tilde{J}_i = \sqrt{27\pi/32G_5}J_i \tilde{J}_1^2 < M^3$$

$$ds^2 = -dt^2 + \frac{\mu}{r^{D-5}\rho^2}(dt + a\sin^2\ \theta d\phi)^2 + (r^2 + a^2)\sin^2\ \theta d\phi^2$$

$$+\rho^2 d\theta^2 + r^2 \cos^2\ \theta d\Omega_{D-4}^2 + \frac{\rho^2}{\Delta} dr^2$$

$$\rho^2 = r^2 + a^2 \cos^2\ \theta \ \text{ and } \ \Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$

$$M = (D-2)\Omega_{D-2}\mu/(16\pi G_D)$$

$$J = 2Ma/(D-2)$$

$$\mathcal{L} = i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi - \partial_\mu \bar{\phi} \partial^\mu \phi + \frac{1}{2} g\phi\chi\chi + \frac{1}{2} g^* \bar{\phi} \chi^\dagger \chi^\dagger - \frac{1}{4}|g|^2 |\phi|^4$$

$$\delta_\epsilon \phi = \epsilon^\alpha \chi_\alpha, \qquad \delta_\epsilon \bar{\phi} = \epsilon_{\dot{\alpha}}^\dagger \chi^{\dagger\dot{\alpha}}$$

$$\delta_\epsilon \chi_\alpha = -i\sigma_{\alpha\dot{\beta}}^\mu \epsilon^{\dagger\dot{\beta}} \partial_\mu \phi + \frac{1}{2} g^* \bar{\phi}^2 \epsilon_\alpha \qquad \delta_\epsilon \chi_{\dot{\alpha}}^\dagger = i\partial_\mu \bar{\phi} \epsilon^\beta \sigma_{\beta\dot{\alpha}}^\mu + \frac{1}{2} g\phi^2 \epsilon_{\dot{\alpha}}^\dagger.$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \sim (\epsilon_1^\dagger \sigma^\mu \epsilon_2)\partial_\mu$$

$$\{Q^\dagger, Q\} \sim P^\mu, \{Q, Q\} = 0, \{Q^\dagger, Q^\dagger\} = 0$$

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} [R - \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho]$$

$$D_\nu \psi_\rho = \partial_\nu \psi_\rho + \frac{1}{4} \omega_{\nu ab} \gamma^{ab} \psi_\rho$$

$$\omega_\mu^{ab} = 2e^{\nu[a} \partial_{[\mu} e_{\nu]}^{b]} - e^{\nu[a} e^{b]\rho} e_{\mu c} \partial_\nu e_\rho^c$$

$$\delta_\epsilon e_\mu^a = \frac{1}{2} \bar{\epsilon} \gamma^a \psi_\mu, \delta_\epsilon \psi_\mu = D_\mu \epsilon$$

$$\delta_\epsilon (\sqrt{-g} R) \to \sqrt{-g} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) (-\bar{\epsilon} \gamma^\mu \psi^\nu).$$

$$\delta_\epsilon \left(-\sqrt{-g} \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho \right) \Big|_{\text{lin. } \psi} \to \frac{1}{4} \sqrt{-g} \bar{\epsilon} \gamma^{\mu\nu\rho} \gamma^{ab} R_{\mu\nu ab} \psi_\rho$$

$$\delta_\epsilon \left(-\sqrt{-g} \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho \right) \Big|_{\text{lin. } \psi} \to \sqrt{-g} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) (\bar{\epsilon} \gamma^\mu \psi^\nu)$$

$$S = \frac{1}{16\pi G_{11}} \int d^{11} x \left[\sqrt{-g} \left(R - \frac{1}{4!} F_{\mu\nu\rho\sigma}^{(4)} F^{(4)\mu\nu\rho\sigma} \right) - \frac{\sqrt{2}}{3} A^{(3)} \wedge F^{(4)} \wedge F^{(4)} \right]$$

$$Q_{\text{E}} \propto \int_{S^7} \star F^{(4)}$$

$$Q_{\text{M}} \propto \int_{S^4} F^{(4)}$$

$$Q_{\text{E}} \propto \int_{S^7} \star H$$

$$\delta_\epsilon B = \bar{\epsilon} f(B) F + O(F^3), \text{ and } \delta_\epsilon F = g(B) \epsilon + O(F^2)$$

$$0 = \delta_\epsilon \psi_\mu = D_\mu \epsilon - \frac{1}{2L} \gamma_\mu \epsilon$$

$$\hat{D}_\mu \epsilon \equiv D_\mu \epsilon - \frac{1}{4} F_{\nu\rho} \gamma^\nu \gamma^\rho \gamma_\mu \epsilon = 0$$

$$M \geq (Q^2 + P^2)^{1/2}$$

$$M \geq \frac{\sqrt{3}}{2} Q$$

$$S_{\text{EH}} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} R$$
$$= \int d^D x [h \partial^2 h + \kappa h^2 \partial^2 h + \kappa^2 h^3 \partial^2 h + \kappa^3 h^4 \partial^2 h + \cdots]$$

$$\partial^\mu h_{\mu\nu} = \frac{1}{2} \partial_\nu h_\mu{}^\mu$$

$$h\partial^2 h \to -\frac{1}{2}h_{\mu\nu} \Box h^{\mu\nu} + \frac{1}{4}h_\mu{}^\mu \Box h_\nu{}^\nu.$$

$$P_{\mu_1\nu_1,\mu_2\nu_2} = -\frac{i}{2}\left(\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{D-2}\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\right)\frac{1}{k^2}.$$

$$e_-^{\mu\nu}(p_i) = \epsilon_-^\mu(p_i)\epsilon_-^\nu(p_i), e_+^{\mu\nu}(p_i) = \epsilon_+^\mu(p_i)\epsilon_+^\nu(p_i)$$

$$M_4^{\text{tree}}(1234) = -sA_4^{\text{tree}}[1234]A_4^{\text{tree}}[1243],$$

$$\text{graviton}^{\pm 2}(p_i) = \text{gluon}^{\pm 1}(p_i) \otimes \text{gluon}^{\pm 1}(p_i),$$

$$\text{gravity 1-loop diagram} \sim \int^\Lambda d^4\, d^4k \frac{(k^2)^m}{(k^2)^m} \sim \Lambda^4$$

$$S[X^\mu, \gamma_{ab}] = \frac{1}{4\pi\ell_s^2}\int d\tau d\sigma\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

$$\int DX^\mu D\psi^\mu D\gamma_{ab} e^{-S[X^\mu,\psi^\mu,\gamma_{ab}]}$$

$$\frac{1}{4\pi}\int \phi\mathcal{R}[\gamma]$$

$$S_{\text{BH}} \sim Mr_0 \sim M\ell_s \sim S_{\text{superparticle}}$$

$$r_0 = 2G_4M \sim \ell_s\, g_s \sim (M\ell_s)^{-1/2}$$

$$S = \frac{1}{16\pi G_{10}}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}[R + 4(\nabla\phi)^2] - \frac{1}{12}F_3^2\right]$$

$$Q_1 = \frac{1}{4\pi^2 g_s}\int e^{2\phi} * F_3, Q_5 = \frac{1}{4\pi^2 g_s}\int F_3$$

$$ds^2 = f(r)^{-1}\left[-dt^2 + dx_5^2 + \frac{r_0^2}{r^2}(\cosh\,\sigma dt + \sinh\,\sigma dx_5)^2 + \left(1 + \frac{g_sQ_1}{r^2}\right)dx_i dx^i\right]$$
$$+f(r)\left[\left(1 - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2\right]$$

$$f(r) = \left(1 + \frac{g_sQ_1}{r^2}\right)^{1/2}\left(1 + \frac{g_sQ_5}{r^2}\right)^{1/2}$$

$$e^{-2\phi} = \left(1 + \frac{g_sQ_5}{r^2}\right)\left(1 + \frac{g_sQ_1}{r^2}\right)^{-1}$$
$$F_3 = 2g_sQ_5\epsilon_3 + 2g_sQ_1e^{-2\phi} * \epsilon_3$$

$$n = \frac{r_0^2R^2\sinh\,2\sigma}{2g_s^2}$$

$$E = \frac{RQ_1}{g_s} + \frac{RQ_5}{g_s} + \frac{n}{R} + \frac{Rr_0^2 e^{-2\sigma}}{2g_s^2}.$$

$$S_{\mathrm{BH}} = \frac{A}{4G_{10}} = 2\pi\sqrt{Q_1 Q_5}\frac{r_0 R\cosh\,\sigma}{g_s}.$$

$$S_{\mathrm{BH}} = 2\pi\sqrt{Q_1 Q_5 n}.$$

$$S = 2\pi\sqrt{Q_1 Q_5 n},$$

$$r_0^2 \ll g_s Q_1, g_s Q_5 \text{ and } g_s^2 n/R^2 \ll g_s Q_1, g_s Q_5$$

$$S = 2\pi\sqrt{Q_1 Q_5}\left(\sqrt{n_R} + \sqrt{n_L}\right),$$

$$P = (n_R - n_L)/R(n_R + n_L)/R$$

$$n_R = \frac{r_0^2 R^2 e^{2\sigma}}{4g_s^2}, n_L = \frac{r_0^2 R^2 e^{-2\sigma}}{4g_s^2}.$$

$$\frac{d}{dt}\mathcal{O}(t) = i[H, \mathcal{O}(t)]$$

$$ds^2 = H^{-1/2}[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2] + H^{1/2}[dr^2 + r^2 d\Omega_5^2]$$

$$H(r) = 1 + \frac{L^4}{r^4}, L^4 = 4\pi g_s N \ell_s^4$$

$$ds^2 = \frac{r^2}{L^2}[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2] + \frac{L^2 dr^2}{r^2}$$

$$(\,g_s, L/\ell_s\,)(g_{\mathrm{YM}}, N)4\pi g_s = g_{\mathrm{YM}}^2 (L/\ell_s)^4 = g_{\mathrm{YM}}^2 N G_{10} \sim g_s^2 \ell_s^8 L^4 \sim N \ell_p^4 \ell_p$$

$$N \to \infty, g_{\mathrm{YM}} \to 0 \text{ with } \lambda \equiv g_{\mathrm{YM}}^2 N$$

$$G_{10} \sim g_s^2 \ell_s^8 \sim L^8/N^2\ E < O(N^2) O(N^2) E \sim O(N^2)$$

$$\mathcal{N} = 4\ \mathrm{SYM} r \to ar, (t, x_i) \to (t, x_i)/a$$

$$S_{\mathrm{SYM}} \sim N^2 T^3 V_3,$$

$$ds^2 = \frac{r^2}{L^2}\left[\left(1 - \frac{r_0^4}{r^4}\right)dt^2 + dx_i dx^i\right] + \left(1 - \frac{r_0^4}{r^4}\right)^{-1}\frac{L^2 dr^2}{r^2} + L^2 d\Omega_5^2$$

$$T = 3r_0/4\pi L^2$$

$$S_{\mathrm{BH}} = \frac{A}{4G_{10}} \sim \frac{L^8 T^3 V_3}{G_{10}} \sim N^2 T^3 V_3$$

$$S[\Phi_0] = S_{\mathrm{SYM}} + \int \Phi_0 \mathcal{O}$$

$$Z_{\text{quantum supergravity}}(\Phi \rightarrow \Phi_0) = \int DAD\phi e^{iS[\Phi_0]} \equiv Z_{\text{SYM}}[\Phi_0]$$

$$R_{\mu\nu} = F_{\mu\alpha\beta\gamma\delta} F_{\nu}^{\alpha\beta\gamma\delta}$$

$$J/N \leq 1$$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

$$S = \int d^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\nabla\Psi - iqA\Psi|^2 - m^2|\Psi|^2\right)$$

$$\Lambda = -3/L^2\Psi\ m_{\text{eff}}^2 = m^2 + q^2 g^{tt} A_t^2 m_{\text{eff}}^2$$

$$|\sigma(\omega)| = B/\omega^{2/3} + C$$

$$S_{\text{EE}} = -\text{Tr}\rho_A \ln \rho_A$$

$$S_{\text{EE}} = A_\Sigma/4G$$

$$\ell_s = 1$$

$$\Phi \rightarrow \Phi_0/r^\Delta$$

Morfología y fenomenología de una partícula blanca o estrella en supergravedad cuántica.

$$\frac{dL_r}{dr} = 4\pi r^2 \varrho \epsilon$$

$$\frac{dT}{dr} = -\frac{3\kappa\varrho}{16\sigma T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT}{dr} = -\left(1 - \frac{1}{\gamma}\right)\frac{G_{\text{N}} M_r \varrho T}{r^2 P}$$

$$\frac{dM_r}{dr} = 4\pi r^2 \varrho$$

$$\frac{dP}{dr} = -\frac{G_{\text{N}} M_r}{r^2}\varrho$$

Layer	Region	Temperature T	Energy density ρ	Matter

Empty	$R_\star \leq r < \infty$	$T_\star \sqrt{\frac{1-\frac{r_H}{R_\star}}{1-\frac{r_H}{r}}}$	0	(γ)
Atmosphere	$R_* \leq r \leq R_*$	$T_\star \left(1 - \frac{G_N \nu_*}{2c^2} + \cdots\right)$	$\left(\bar{m}c^2 + \frac{3}{2} k_B T\right) \hat{n}_g$	NR($+\gamma$)
Interior	$R_c \leq r \leq R_*$	$T(R_*) \left(\frac{R_*}{r}\right)^{2\left(1-\frac{1}{\gamma_b}\right)}$	$\rho(R_*) \left(\frac{R_*}{r}\right)^2$	NR $+$ B($+\gamma$)
Core	$0 < r \leq R_c$	$T(R_c) \frac{2\sqrt{r_c^2 - R_c^2}}{3\sqrt{r_c^2 - R_c^2} - \sqrt{r_c^2 - r^2}}$	ρ_c	D

Layer	Region	Temperature T	Energy density ρ	Matter
Empty	$R_\star \leq r < \infty$	$T_\star \sqrt{\frac{1-\frac{r_H}{R_\star}}{1-\frac{r_H}{r}}}$	0	(γ)
Atmosphere	$R_* \leq r \leq R_*$	$T_\star \left(1 - \frac{G_N \nu_*}{2c^2} + \cdots\right)$	$(\bar{m}c^2 + \frac{3}{2} k_B T)\hat{n}_g$	NR$(+\gamma)$
Interior	$R_c \leq r \leq R_*$	$T(R_*) \left(\frac{R_*}{r}\right)^{2(1-\frac{1}{\gamma_b})}$	$\rho(R_*) \left(\frac{R_*}{r}\right)^2$	NR+B$(+\gamma)$
Core	$0 < r \leq R_c$	$T(R_c) \frac{2\sqrt{r_c^2 - R_c^2}}{3\sqrt{r_c^2 - R_c^2} - \sqrt{r_c^2 - r^2}}$	ρ_c	D

$$r_H = \frac{2G_N M_\star}{c^2}, r_c = \sqrt{\frac{3c^2}{8\pi G_N \rho_c}}, M_\star$$

$$g_{\mu\nu} dx^\mu dx^\nu = -e^\nu (cdt)^2 + e^\lambda dr^2 + r^2 (\, d\theta^2 + (\cos\ \theta)^2\ d\phi^2),$$

$$T_{\mu\nu} = (\rho + P) \frac{u_\mu}{c} \frac{u_\nu}{c} + P g_{\mu\nu}$$

$$\begin{aligned} P' &= -\frac{(P+\rho)}{2} \nu' \\ \lambda' &= r e^\lambda 8\pi G \frac{\rho}{c^2} - \frac{e^\lambda - 1}{r} \\ \nu' &= r e^\lambda 8\pi G \frac{P}{c^2} + \frac{e^\lambda - 1}{r} \end{aligned}$$

$$P' = -\frac{(P+\rho)}{c^2}\frac{G\left(\check{E}_r + 4\pi r^3 P\right)}{r^2\left(1 - 2\frac{G\check{E}_r}{rc^2}\right)}$$

$$\check{E}_r' = 4\pi r^2 \rho$$

$$Ts = u + Pv, Ts' = u' + Pv'$$

$$v = r^2 \cos\,\theta\left(1 - \frac{2G\check{E}_r}{rc^2}\right)^{-\frac{1}{2}}, u = \rho v$$

$$T' = \frac{P'}{P+\rho}T$$

$$T' = -\frac{G\left(\check{E}_r + 4\pi r^3 P\right)}{c^2 r^2\left(1 - 2\frac{G\check{E}_r}{rc^2}\right)}T$$

$$\bar{T}\left(\frac{\partial \bar{P}}{\partial \bar{T}}\right)_{\bar{V},\bar{N}} = \left(\frac{\partial \bar{U}}{\partial \bar{V}}\right)_{\bar{T},\bar{N}} + \bar{P}$$

$$\left(\frac{\partial \bar{P}}{\partial T}\right)_{\bar{V},\bar{N}} = \frac{\bar{T}}{P+\bar{\rho}}, \text{ where } \bar{\rho} = \left(\frac{\partial \bar{U}}{\partial V}\right)_{\bar{T},\bar{N}}$$

$$P \ll \rho, P \ll \frac{\check{E}_r}{4\pi r^3}, \frac{2G_N \check{E}_r}{c^2} \ll r \text{ so that } \rho \to \varrho c^2, \check{E}_r \to M_r c^2$$

$$\check{E}_r = \frac{\frac{c^2 r^2 v'}{G} - 8\pi r^3 P}{2(1 + rv')},$$

$$v'' + \frac{2v'}{r} + v'^2 - \frac{4\pi G}{c^2}\left((rv'+1)(rv'+2)\rho - (2r^2 v'' + rv'(rv'-3) - 6)P\right) = 0.$$

$$\nabla^2 v = \frac{8\pi G}{c^2}(g^{\omega\mu} + 2u^\omega u^\mu)T_{\omega\mu}$$

$$\nabla^2 v = e^{-\lambda}\left(v'' + \left(\frac{2}{r} + \frac{v' - \lambda'}{2}\right)v'\right)$$

$$= e^{-\lambda}v'' + \frac{v' + 4\pi Gr/c^2(P(rv'+3) - (rv'+1)\rho) + 2/r}{rv'+1}v'$$

$$\nabla^2 v = \frac{1}{\sqrt{|g|}}\partial_\omega \sqrt{|g|}g^{\omega\mu}\partial_\mu v$$

$$e^{-\lambda}\frac{v' - \lambda'}{2} = 4\pi Gr\frac{P-\rho}{c^2} + \frac{2G\check{E}_r}{r^2 c^2}$$

$$e^{-\lambda} = \frac{1 + 8\pi Gr^2 P/c^2}{rv'+1} 1 + 8\pi Gr^2 P/c^2 = e^{-\lambda}(rv'+1).$$

$$\nabla^2 v = 8\pi G(\rho + 3P)/c^2.$$

$$\nu'' + \frac{2\nu'}{r} + \nu'^2 = 0.$$

$$\nu' = \frac{1}{r(-1 + C_0 r)} =: \nu'_\star$$

$$\check{E}_r = \frac{c^2 - 8\pi G r^2(-1 + C_0 r)P}{2C_0 G}.$$

$$r = R_\star \; \check{E}_r \approx \frac{c^2}{2C_0 G} r \geq R_\star \; C_0 = \frac{1}{2M_\star G} \; \nu_\star = \log\left(\frac{1}{r} - \frac{1}{2M_\star G}\right) + \tilde{C}_0, \nu(R_\star) = 0.$$

$$\nu_\star = \log\left(\frac{1 - \frac{2M_\star G}{R_\star}}{1 - \frac{2M_\star G}{r}}\right).$$

$$\nu_\star = 2M_\star G\left(-\frac{1}{r} + \frac{1}{R_\star}\right) + \cdots.$$

$$c^2 \gg 4\pi R_\star^3 \frac{P(R_\star)}{M_\star},$$

$$T = T(R_\star)e^{-\frac{\nu}{2}} = T(R_\star)\sqrt{\frac{1 - \frac{r_\mathrm{H}}{R_\star}}{1 - \frac{r_\mathrm{H}}{r}}} =: T_\mathrm{vac}$$

$$T_\mathrm{vac}\left(T''_\mathrm{vac} + \frac{2}{r}T'_\mathrm{vac}\right) - 3T'^2_\mathrm{vac} = 0$$

$$T\left(T'' + \frac{2}{r}T'\right) - 3T'^2$$
$$= \frac{4\pi G}{c^2}\left(\rho(-2r^2T'^2 + 3rTT' - T^2) + P(4r^2T'^2 - rT(2rT'' - 3T') - 3T^2)\right)$$

$$\nu''_1 + \frac{2}{r}\nu'_1 = \frac{8\pi}{c^2}(\rho_0 + 3P_0),$$

$$T''_1 + \frac{2}{r}T'_1 = -\frac{4\pi T_0}{c^2}(\rho_0 + 3P_0)$$

$$P = \hat{n} k_\mathrm{B} T$$

$$\nabla_\mu J^\mu = 0, \text{ where } J^\mu = \hat{n} u^\mu$$

$$\nabla_\mu J^\mu = u^r\left(\hat{n}' + \hat{n}\left(\log\left(\sqrt{|g|}u^r\right)\right)'\right)$$

$$\nabla_\mu J^\mu = u^r\left(\hat{n}' - \hat{n}\frac{\rho'}{\rho + P}\right)$$

$$\hat{n}' = \hat{n}\frac{\rho'}{\rho + P}$$

$$P = w\rho$$

$$w = \frac{k_{\mathrm{B}}}{C_V} = \gamma - 1$$

$$\frac{T'}{T} = \left(1 - \frac{1}{\gamma}\right)\frac{P'}{P}$$

$$P \propto T^{\frac{\gamma}{\gamma-1}}, \rho = aT^{\frac{\gamma}{\gamma-1}}$$

$$\gamma \sim 1.$$

$$T\left(T'' + \frac{2}{r}T'\right) - 3T'^2$$
$$= \frac{4\pi aG}{c^2} T^{\frac{\gamma}{\gamma-1}}\left(-6r^2T'^2 + 2r^2TT'' + 2T^2 + \gamma\left(4r^2T'^2 - rT(2rT'' - 3T') - 3T^2\right)\right).$$

$$A^2(2n-1)r^{2n-2}\left(4\pi aGA^{\frac{\gamma}{\gamma-1}}(n(\gamma-2) + 3\gamma - 2)r^{\frac{n}{\gamma-1}+n+2} + c^2 n\right) = 0$$

$$\frac{n}{\gamma-1} + n + 2 = 4\pi aGA^{\frac{\gamma}{\gamma-1}}(n(\gamma-2) + 3\gamma - 2)\, r^{\frac{n}{\gamma-1}+n+2} + c^2 n = 0$$

$$n = -2(\gamma-1)/\gamma, A = \left(\frac{c^2(\gamma-1)}{2\pi aG(\gamma^2+4\gamma-4)}\right)^{\frac{\gamma-1}{\gamma}}.$$

$$T = \left(\frac{c^2(\gamma-1)}{2\pi aG(\gamma^2+4\gamma-4)}\right)^{1-\frac{1}{\gamma}} \frac{1}{r^{2\left(1-\frac{1}{\gamma}\right)}}$$

$$\nu = 4\left(1 - \frac{1}{\gamma}\right)\log\left(r/R_\star\right)$$

$$\hat{n} = \frac{P}{k_{\mathrm{B}}T} = \frac{a(\gamma-1)}{k_{\mathrm{B}}}\left(\frac{c^2(\gamma-1)}{2\pi aG(\gamma^2+4\gamma-4)}\right)^{\frac{1}{\gamma}} \frac{1}{r^{\frac{2}{\gamma}}},$$

$$w = \tfrac{1}{3}, \text{ so } P_{\mathrm{rad}} = \tfrac{1}{3}\rho_{\mathrm{rad}} \propto T^4, \rho_{\mathrm{rad}} = aT^4$$

$$a = 4\sigma/c$$

$$\rho_{\mathrm{rad}} = \frac{4\sigma}{c}T^4$$

$$P_{\mathrm{g}} = \hat{n}_{\mathrm{g}}k_{\mathrm{B}}T, \rho_{\mathrm{g}} = \left(\bar{m}c^2 + \frac{3}{2}k_{\mathrm{B}}T\right)\hat{n}_{\mathrm{g}},$$

$$\hat{n}_{\mathrm{g}} = C_{\mathrm{g}}T^{3/2}e^{-\frac{\bar{m}c^2}{k_{\mathrm{B}}T}}$$

$$C_{\mathrm{g}} = \bar{g}\left(\frac{2\pi\bar{m}k_{\mathrm{B}}}{h^2}\right)^{\frac{3}{2}}$$

$$P_0 = \hat{n}_{g0} k_B T_0, \rho_0 = \left(\bar{m}c^2 + \frac{3}{2} k_B T_0\right) \hat{n}_{g0}, \hat{n}_{g0} = \bar{g} \left(\frac{2\pi \bar{m} k_B}{h^2}\right)^{\frac{3}{2}} T_0^{3/2} e^{-\frac{\bar{m}c^2}{k_B T_0}}.$$

$$\nu_1 = \frac{4\pi r^2 (\rho_0 + 3P_0)}{3c^2} - \frac{C_1}{r} + \tilde{C}_1$$

$$\check{E}_r = \frac{8\pi c^2 r^4 \rho_0 - 3c^4 C_1 r}{16\pi G r^3 (\rho_0 + 3P_0) + 6c^2 (r + C_1 G)}.$$

$$c^2 M_* = \frac{8\pi c^2 R_*^4 \rho_0 - 3c^4 C_1 R_*}{16\pi G R_*^3 (\rho_0 + 3P_0) + 6c^2 (R_* + C_1 G)}$$

$$C_1 = \frac{2R_* M_* (8\pi G R_*^2 (\rho_0 + 3P_0)/c^2 + 3) - 8\pi R_*^4 \rho_0 / c^2}{3R_* - 6GM_*}$$
$$\approx 2M_* - \frac{8}{3}\pi R_*^3 \rho_0 / c^2$$

$$\tilde{C}_1 = \frac{2M_*}{R_\star} - \frac{4\pi\left(3P_0 R_\star^3 + \rho_0 (R_\star^3 + 2R_*^3)\right)}{3c^2 R_\star}$$

$$\nu_* = 2\left(\frac{1}{R_\star} - \frac{1}{r}\right)\left(M_* + \frac{6\pi P_0 r R_\star (r + R_\star) + 2\pi \rho_0 (r^2 R_\star + r R_\star^2 - 2R_*^3)}{3c^2}\right).$$

$$M_\star = M_* + 4\pi R_\star^3 \frac{P_0}{c^2} + \frac{4}{3}\pi (R_\star^3 - R_*^3) \frac{\rho_0}{c^2},$$

$$T = T(R_\star)(1 - G\nu_*/2 + \cdots),$$

$$\frac{T(R_*)}{T(R_\star)} = 1 + G\left(\frac{1}{R_*} - \frac{1}{R_\star}\right)\left(M_* + \frac{6\pi P_0 R_* R_\star (R_* + R_\star) + 2\pi \rho_0 (R_*^2 R_\star + R_* R_\star^2 - 2R_*^3)}{3c^2}\right)$$

$$\rho = \rho_b + \rho_g, P = P_b + P_g$$

$$P_b = \hat{n}_b k_B T$$

$$\hat{n}_b' = \hat{n}_b \frac{\rho'}{\rho + P}$$

$$P_b'(P_g + \rho) = P_b (P_g + \rho)'$$

$$P_b = w_b (P_g + \rho)$$

$$T_g := \frac{\bar{m}c^2}{k_B}, \rho_g \approx \bar{m}c^2 \hat{n}_g, P_g \ll \rho_g, P = (1 + w_b) P_g + w_b \rho \approx w_b \rho, \gamma_b \text{ as } w_b = \gamma_b - 1, \rho \approx a_b T^{\frac{\gamma_b}{\gamma_b - 1}}$$

$$\rho_b(R_*) = 0$$

$$a_b \approx \rho(R_*)/T(R_*)^{\frac{\gamma_b}{\gamma_b - 1}}, \rho(R_*) = \rho_g(R_*) \approx \bar{m}c^2 \hat{n}_g(R_*)\, T(R_*), T \approx T(R_*)\left(\frac{R_*}{r}\right)^{2\left(1 - \frac{1}{\gamma_b}\right)}$$

$$T(R_*) = \left(\frac{c^2(\gamma_b - 1)}{2\pi a_b G(\gamma_b^2 + 4\gamma_b - 4)}\right)^{1-\frac{1}{\gamma_b}} \frac{1}{R_*^{2\left(1-\frac{1}{\gamma_b}\right)}}.$$

$$R_* \approx \sqrt{\frac{c^2(\gamma_b - 1)}{2\pi\rho(R_*)G(\gamma_b^2 + 4\gamma_b - 4)}}$$

$$\gamma_b \approx 1 + 2\pi \frac{\rho(R_*)}{c^2} G R_*^2$$

$$\rho \approx \rho(R_*) \left(\frac{R_*}{r}\right)^2, \rho_b \approx \rho \times \left(1 - \left(\frac{R_*}{r}\right)^{1-\frac{3}{\gamma_b}} e^{T_g\left(\frac{1}{T(R_*)} - \frac{1}{T}\right)}\right).$$

$$P = \frac{\sqrt{r_c^2 - r^2} - \sqrt{r_c^2 - R_c^2}}{3\sqrt{r_c^2 - R_c^2} - \sqrt{r_c^2 - r^2}} \rho_c, T = \frac{2\sqrt{r_c^2 - R_c^2}}{3\sqrt{r_c^2 - R_c^2} - \sqrt{r_c^2 - r^2}} T(R_c),$$

$$r_c = \sqrt{\frac{3}{8\pi G \rho_c}}$$

$$M_* = \frac{1}{c^2} \int_0^{R_*} dr 4\pi r^2 \rho, M_\star = M_* + \frac{1}{c^2} \int_{R_*}^{R_\star} dr 4\pi r^2 \rho$$

$$M_* \approx \frac{\rho(R_*)}{c^2} 4\pi R_*^3$$

$$\rho(R_*)/c^2 = \bar{m}\hat{n}_g(R_*) = \mu_e m_u / \left(\frac{4}{3}\pi a_B\right)^3, \hat{n}_g(R_*) \approx 1/\left(\frac{4}{3}\pi a_B\right)^3$$

$$M_\star \approx \frac{\mu_e m_u}{\frac{4}{3}\pi a_B^3} \times 4\pi \left(\frac{R_\odot}{\eta}\right)^3 \approx \frac{1}{\eta^3} \times 10^{34} \text{ g.}$$

$$M_\odot = 2.0 \times 10^{33} \text{ g}$$

$$L_r = 4\pi r^2 e^{\lambda/2} F$$

$$F = \int d\nu F_\nu, P_\nu \text{ by } \frac{dP_\nu}{d\tau_\nu} = \frac{F_\nu}{c}, \alpha_\nu \text{ as } d\tau_\nu = -\alpha_\nu dr, \frac{dP_\nu}{dr} = -\alpha_\nu \frac{F_\nu}{c}$$

$$P'_{\text{rad}} = -\alpha_R \frac{F}{c},$$

$$P_{\text{rad}} = \frac{4\sigma}{3c} T^4, \text{ so } T' = \frac{3c}{16\sigma T^3} P'_{\text{rad}}.$$

$$T' = -\frac{3\alpha_R}{16\sigma T^3} F$$

$$T' = -\frac{3\alpha_{\rm R}}{16\sigma T^3}\frac{L_r}{4\pi r^2\left(1-\frac{2G\check{E}_r}{rc^2}\right)^{-\frac{1}{2}}}$$

$$L_r = \frac{64\pi\sigma}{3c^2\alpha_{\rm R}}\frac{G\left(\check{E}_r + 4\pi r^3 P\right)}{\left(1-2\frac{G\check{E}_r}{rc^2}\right)^{3/2}}T^4$$

$$L_r \approx \frac{64\pi\sigma}{3c^2\alpha_{\rm R}}G\check{E}_r T^4.$$

$$L_r \approx \frac{64\pi\sigma}{3\alpha_{{\rm R}_\star}}GM_\star T(R_\star)^4$$

$$L_\star = 4\pi R_\star^2\sigma T_\star^4$$

$$R_\star^2 = \frac{16GM_\star}{3\alpha_{{\rm R}\star}}.$$

$$R_\star := R_\odot, R_* = \frac{1}{2}R_*$$

$$T(R_*) \approx T_\odot, T\left(\tilde{R}_*\right) = T(R_*)\left(\frac{R_*}{\tilde{R}_*}\right)^{1/2} \approx 7\times 10^3 \text{ K}, T(R_{\rm c}) = T(R_*)\left(\frac{R_*}{R_{\rm c}}\right)^{5/4} \approx 2\times 10^7 \text{ K}$$

$$\rho(R_*)/c^2 \approx m_{\rm u}/\left(\frac{4}{3}\pi a_{\rm B}^3\right) \approx 3\times 10^3 \text{ kg/m}^3$$

$$\rho\left(\tilde{R}_*\right)/c^2 = \rho(R_*)/c^2\left(\frac{R_*}{\tilde{R}_*}\right)^2 \approx 5\times 10^3 \text{ kg/m}^3$$

$$\rho_{\rm c}/c^2 = \rho(R_*)/c^2\left(\frac{R_*}{R_{\rm c}}\right)^2 \approx 1\times 10^{12} \text{ kg/m}^3$$

$$1.5\times 10^5 \text{ kg/m}^3$$

$$5\times 10^{14} \text{ kg/m}^3$$

$$\left|\frac{dP_{\rm rad}}{dr}\right| \ll \left|\frac{dP}{dr}\right|.$$

$$|P'_{\rm rad}| = \frac{\alpha_{\rm R}L_r}{4\pi r^2\left(1-2\frac{G\tilde{E}_r}{rc^2}\right)^{-\frac{1}{2}}c}$$

$$L_r \ll (1+w_{\rm b})\rho\frac{4\pi G\left(\check{E}_r + 4\pi r^3 P\right)}{\alpha_{\rm R}c\left(1-2\frac{G\check{E}_r}{rc^2}\right)^{\frac{3}{2}}},$$

$$L_r \ll \gamma_{\rm b}\frac{4\pi cG\left(\check{E}_r+4\pi r^3P\right)}{\kappa\left(1-2\frac{G\check{E}_r}{rc^2}\right)^{\frac{3}{2}}}.$$

$$L_{R_\star} \ll \frac{4\pi cG_{\rm N}M_\star}{\kappa}.$$

$$T\propto r^{\frac{-2(\gamma-1)}{\gamma}}, \rho\propto p\propto r^{-2}$$

$$\frac{dL_r}{dr}=\frac{4\pi r^2}{\left(1-2\frac{G\check{E}_r}{rc^2}\right)^{\frac{1}{2}}}\rho\epsilon$$

$$\check{E}_r\approx\frac{c^2r^2\nu'}{2G}, P\propto\varrho^{1+1/n}\,\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right)=-\theta^n\xi\propto r \text{ as } m\propto\xi_1^2|\theta(\xi_1)'|, \text{where } \theta(\xi_1)=0$$

$$P\propto\rho^\gamma$$

$$\bar{\tau}=\frac{2}{3}, \bar{\tau}=\int_R^\infty\kappa\varrho dr$$

$$\gamma_{\rm b}-1\approx1\times10^{-6}, \rho(R_*)/c^2\to m_{\rm u}/\left(\frac{4}{3}\pi a_{\rm B}^3\right)\approx2.7\times10^3\ {\rm kg/m^3}, R_*\to\frac{R_\star}{2}=\frac{R_\odot}{2}=$$

$$3.5\times10^8\ {\rm m}, a_{\rm B}=5.3\times10^{-11}\ {\rm m}$$

$$\alpha_{{\rm R}_\star}=\frac{16G\left(M_\star+4\pi R_\star^3P(R_\star)/c^2\right)}{3R_\star^2}$$

$$f(\Box^{-1}R)=\Box^{-1}R+\alpha(\Box^{-1}R)^2$$

$$\frac{P}{P_c}=a\left(1-e^{-b\frac{\rho}{\rho_c}}\right)$$

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R\left(1+f(\Box^{-1}R)\right)+\mathcal{L}_{\rm matter}\right\}$$

$$\Box=\nabla_\mu\nabla^\mu=\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right)$$

$$\nabla_\mu V_\nu=\partial_\mu V_\nu-\Gamma^\lambda_{\mu\nu}V_\lambda$$
$$R^\sigma_{\mu\nu\rho}=\partial_\nu\Gamma^\sigma_{\mu\rho}-\partial_\rho\Gamma^\sigma_{\mu\nu}+\Gamma^\omega_{\mu\rho}\Gamma^\sigma_{\omega\nu}-\Gamma^\omega_{\mu\nu}\Gamma^\sigma_{\omega\rho}$$

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\{R(1+f(\psi))+\xi(\Box\psi-R)\}+\mathcal{L}_{\rm matter}\right]$$
$$=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\{R(1+f(\psi))-\partial_\mu\xi\partial^\mu\psi-\xi R\}+\mathcal{L}_{\rm matter}\right].$$

$$\frac{\delta S}{\delta \xi} = 0, \Box\, \psi = R$$

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0$$

$$0 = \frac{1}{2} g_{\mu\nu} \{R(1 + f(\psi) - \xi) - \partial_\rho \xi \partial^\rho \psi\} - R_{\mu\nu}(1 + f(\psi) - \xi)$$
$$+ \frac{1}{2}(\partial_\mu \xi \partial_\nu \psi + \partial_\mu \psi \partial_\nu \xi) - (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu)(f(\psi) - \xi) + \kappa^2 T_{\mu\nu}.$$

$$0 = \Box\, \xi + f'(\psi) R.$$

$$ds^2 = c^2 e^{2\phi} dt^2 - e^{2\lambda} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$T_\mu^\nu = \mathrm{diag}(\rho c^2, -p, -p, -p)$$

$$\xi \equiv \xi(r), \psi \equiv \psi(r)$$

$$-\frac{1}{2}\xi'\phi' + (1 + f(\phi) - \xi)\left(-\frac{2\lambda'}{r} + \frac{1 - e^{2\lambda}}{r^2}\right) - (\psi'' f_\psi + \psi'^2 f_{\psi\psi} - \xi'')$$
$$-\left(\frac{2}{r} - \lambda'\right)(\psi' f_\psi - \xi') = \kappa^2 \rho c^2 e^{2\lambda}$$

$$(1 + f(\phi) - \xi)\left(\frac{2\phi'}{r} + \frac{1 - e^{2\lambda}}{r^2}\right) + \frac{\xi'\phi'}{2} - \left(\frac{2}{r} + \phi'\right)(\psi' f_\psi - \xi') = -\kappa^2 e^{2\lambda} p($$

$$\psi'' + \left(\frac{2}{r} + \phi' - \lambda'\right)\psi' - \phi'' - \phi'^2 + \phi'\lambda' - \frac{2}{r}(\phi' - \lambda') - \frac{1 - e^{2\lambda}}{r^2}$$

$$\xi'' + \left(\frac{2}{r} + \phi' - \lambda'\right)\xi' + 2f_\psi\left(\phi'' + \phi'^2 - \phi'\lambda' + 2\frac{\phi' - \lambda'}{r} + \frac{1 - e^{2\lambda}}{r^2}\right)$$

$$R(1 + f(\psi) - \xi) - \partial_\mu \xi \partial^\mu \psi - 3\, \Box\, (f(\psi) - \xi) = -\kappa^2(\rho c^2 - p)$$

$$2\left(\phi'' + \phi'^2 - \phi'\lambda' + \frac{2}{r}(\phi' - \lambda') + \frac{1 - e^{2\lambda}}{r^2}\right)(1 + f(\psi) - \xi) - \psi'\xi'$$
$$-3\left[(\psi'' f_\psi + \psi'^2 f_{\psi\psi} - \xi'') + \left(\frac{2}{r} + \phi' - \lambda'\right)(\psi' f_\psi - \xi')\right] = \kappa^2 e^{2\lambda}(\rho c^2 - p)$$

$$\nabla_\mu T_\nu^\mu = 0$$

$$\frac{dp}{dr} = -(p + \rho c^2)\phi'$$

$$e^{-2\lambda} = 1 - \frac{2GM}{c^2 r} \Rightarrow \frac{GdM}{c^2 dr} = \frac{1}{2}\left[1 - e^{-2\lambda}(1 - 2r\lambda')\right]$$

$$\left\{\frac{dp}{dr}, \frac{dM}{dr}, \rho\right\}$$

$$M \to mM_{\odot}, r \to r_g r, \rho \to \frac{\rho M_{\odot}}{r_g^3}, p \to \frac{p M_{\odot} c^2}{r_g^3}$$

$$r_g = \frac{GM_{\odot}}{c^2} = 1.47473\text{nm and } M_{\odot}$$

$$\phi' = -\frac{p'}{(p+\rho)}$$

$$\lambda' = \frac{m}{r^2}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1}$$

$$e^{2\lambda} = \left(1-\frac{2m}{r}\right)^{-1}, 1-e^{2\lambda} = \left(1-\frac{r}{2m}\right)^{-1}.$$

$$\frac{p'\xi'}{2(p+\rho)} + (1+f(\phi)-\xi)\left(-\frac{2m}{r^3}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1} + \frac{1}{r^2\left(1-\frac{r}{2m}\right)}\right)$$

$$-(\psi'' f_\psi + \psi'^2 f_{\psi\psi} - \xi'') - \left(\frac{2}{r} - \frac{m}{r^2}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1}\right)(\psi' f_\psi - \xi') = \frac{8\pi\rho}{1-\frac{2m}{r}}$$

$$(1+f(\phi)-\xi)\left(\frac{2p'}{r(p+\rho)} - \frac{1}{r^2\left(1-\frac{r}{2m}\right)}\right) + \frac{\xi' p'}{2(p+\rho)}$$

$$+\left(\frac{2}{r} - \frac{p'}{p+\rho}\right)(\psi' f_\psi - \xi') = \frac{8\pi p}{1-\frac{2m}{r}}$$

$$\psi'' + \left(\frac{2}{r} - \frac{p'}{p+\rho} - \frac{m}{r^2}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1}\right)\psi' + \left(\frac{p'}{p+\rho}\right)' - \left(\frac{p'}{p+\rho}\right)^2$$

$$-\frac{m}{r^2}\frac{p'}{p+\rho}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1} + \frac{2}{r}\left(\frac{p'}{p+\rho} + \frac{m}{r^2}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1}\right) - \frac{1}{r^2\left(1-\frac{r}{2m}\right)}$$

$$\xi'' + \left(\frac{2}{r} - \frac{p'}{p+\rho} - \frac{m}{r^2}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1}\right)\xi' + 2f_\psi\left(-\left(\frac{p'}{p+\rho}\right)'\right.$$

$$\left. + \left(\frac{p'}{p+\rho}\right)^2 + \frac{m}{r^2}\frac{p'}{p+\rho}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1} - \frac{2}{r}\left(\frac{p'}{p+\rho} + \frac{m}{r^2}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1}\right) + \frac{1}{r^2\left(1-\frac{r}{2m}\right)}\right) = 0$$

$$\left(-\left(\frac{p'}{p+\rho}\right)' + \left(\frac{p'}{p+\rho}\right)^2 + \frac{p'}{p+\rho}\frac{m}{r^2}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1}\right.$$

$$\left. - \frac{2}{r}\left(\frac{p'}{p+\rho} + \frac{m}{r^2}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1}\right) + \frac{1}{r^2\left(1-\frac{r}{2m}\right)}\right)(1+f(\psi)-\xi) - \psi'\xi'$$

$$-3\left[\left(\psi''f_\psi + \psi'^2 f_{\psi\psi} - \xi''\right) + \left(\frac{2}{r} - \frac{p'}{p+\rho} - \frac{m}{r^2}\frac{1-\frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r}-1}\right)\left(\psi' f_\psi - \xi'\right)\right] = \frac{8\pi(\rho-p)}{\left(1-\frac{2m}{r}\right)}$$

$$2\lambda = Ar^2, 2\phi = Br^2 + C$$

$$f(\Box^{-1}R) = \Box^{-1}R + \alpha(\Box^{-1}R)^2$$

$$\psi(r) = C_1 \int \frac{g_{rr}dr}{\sqrt{-g}} + \int_0^r \sqrt{-g(y)}R(y)(r-y)dy + C_2$$

$$\xi(r) = C_3 \int \frac{g_{rr}dr}{\sqrt{-g}} - \int_0^r \sqrt{-g(y)}(r-y)f_\psi(y)\,\Box\,\psi(y)dy + C_4$$

$$V_{rv}^2 \equiv \frac{dp}{d\rho}.$$

$$z = e^{-\phi} - 1$$

$$\frac{dP/P_c}{d\rho/\rho_c}$$

$$\begin{aligned}
\text{NEC} &\iff \rho_{\text{eff}} + p_{\text{eff}} \geq 0\\
\text{WEC} &\iff \rho_{\text{eff}} \geq 0 \text{ and } \rho_{\text{eff}} + p_{\text{eff}} \geq 0\\
\text{SEC} &\iff \rho_{\text{eff}} + 3p_{\text{eff}} \geq 0 \text{ and } \rho_{\text{eff}} + p_{\text{eff}} \geq 0\\
\text{DEC} &\iff \rho_{\text{eff}} \geq 0 \text{ and } \rho_{\text{eff}} \pm p_{\text{eff}} \geq 0
\end{aligned}$$

$$\frac{P}{P_c} = a\left(1 - e^{-b\frac{\rho}{\rho_c}}\right)$$

$$\frac{P}{P_c} + \frac{\rho}{\rho_c}$$

$$\frac{3P}{P_c} + \frac{\rho}{\rho_c}$$

$$\frac{P}{P_c} - \frac{\rho}{\rho_c}$$

$$\rho \ll \rho_c$$

$$\frac{p}{\rho k_B T} = \exp\left(\Sigma_{m=2}^{\infty} K_m \rho^{m-1}\right)$$

$$\frac{P}{P_c} = a\left(1 - e^{-b\frac{\rho}{\rho_c}}\right)$$

$$f(\Box^{-1} R) = \Box^{-1} R + \alpha(\Box^{-1} R)^2$$

$$\frac{P}{P_c} = a\left(1 - e^{-b\frac{\rho}{\rho_c}}\right)$$

$$\Gamma_{12}^{1} = \phi', \Gamma_{11}^{2} = \phi' e^{2\phi - 2\lambda}, \Gamma_{22}^{2} = \lambda', \Gamma_{23}^{2} = -re^{-2\lambda},$$
$$\Gamma_{44}^{2} = -r\sin^2\theta e^{-2\lambda}, \Gamma_{23}^{3} = \frac{1}{r}, \Gamma_{44}^{3} = -\sin\theta\cos\theta, \Gamma_{24}^{4} = \frac{1}{r}, \Gamma_{34}^{4} = \cot\theta$$

$$R_{tt} = e^{2\phi - 2\lambda}\left(-\phi'' - \phi'^2 + \phi'\lambda' - \frac{2\phi'}{r}\right)$$
$$R_{rr} = \phi'' + \phi'^2 - \phi'\lambda' - \frac{2\lambda'}{r}$$

$$R = -2e^{-2\lambda}\left(\phi'' + \phi'^2 - \phi'\lambda' + \frac{2(\phi' - \lambda')}{r} + \frac{1 - e^{2\lambda}}{r^2}\right)$$

$$\Box A(r) = -e^{-2\lambda}\left(A'' + \left(\frac{2}{r} + \phi' - \lambda'\right)A'\right).$$

$$\rho_c = \frac{M_\odot}{r_g^3}, p_c = \frac{M_\odot c^2}{r_g^3}$$

$$\frac{\partial \log(r/\tilde{r})}{\partial \tilde{r}} = \frac{\tilde{r}^{1/2} - (\tilde{r} - 2m)^{1/2}}{\tilde{r}(\tilde{r} - 2m)^{1/2}}$$

$$d\tilde{r}/dr = (\tilde{r}/r)\sqrt{1 - 2m(\tilde{r})/\tilde{r}}$$

$$ds^2 = A(r)dt^2 + B(r)dr^2 + 2C(r)drdt + D(r)d\Omega^2$$

$$d\Omega^2 = \sin^2\theta d\phi^2 + d\theta^2$$

$$A(\tilde{r}) = e^{2\tilde{\nu}(\tilde{r})}, B(\tilde{r}) = e^{2\tilde{\lambda}(\tilde{r})} \text{ or } A(r) = e^{2\nu(r)}, B(r) = e^{2\mu(r)},$$

$$ds^2 = -e^{2\tilde{\nu}}d\tilde{t}^2 + e^{2\tilde{\lambda}}d\tilde{r}^2 + \tilde{r}^2\left(\sin^2\tilde{\theta}d\tilde{\phi}^2 + d\tilde{\theta}^2\right)$$

$$ds^2 = -e^{2\nu}dt^2 + e^{2\mu}[dr^2 + r^2(\sin^2\theta d\phi^2 + d\theta^2)]$$

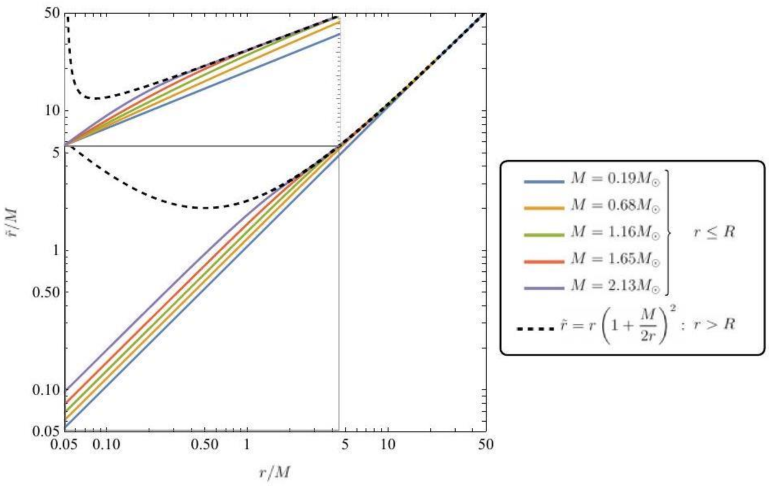
$$\tilde{r}^2 \equiv e^{2\mu}r^2, e^{2\mu}dr^2 \equiv e^{2\tilde{\lambda}}d\tilde{r}^2$$

$$r = \exp\left(\int e^{\tilde{\lambda}}\tilde{r}^{-1}d\tilde{r}\right) = \exp[-\text{Ei}(-\tilde{\lambda}\tilde{r})]$$

$$d\tilde{r} = e^{\mu}(r\mu' + 1)dr$$

$$e^{2\tilde{\lambda}} = (r\mu' + 1)^{-2}$$

$$\tilde{r} = \sqrt{\tilde{A}/4\pi}$$



$$g|_{t=t_0,r=r_0} = e^{2\mu(r_0)}r_0^2 g_\Omega,$$

$$g_{\mu\nu} = \mathrm{diag}(-e^{2\nu}, e^{2\mu}, e^{2\mu}r^2, e^{2\mu}r^2\sin^2\ \theta).$$

$$\Gamma^t_{tr} = \nu', \Gamma^r_{tt} = e^{2\nu-2\mu}\nu', \Gamma^r_{rr} = \mu', \Gamma^\theta_{r\theta} = \mu' + \frac{1}{r}$$
$$\Gamma^r_{\theta\theta} = -r(r\mu' + 1), \Gamma^r_{\phi\phi} = -r\sin^2\ \theta(r\mu' + 1)$$
$$\Gamma^\theta_{\phi\phi} = -\sin\ \theta\cos\ \theta, \Gamma^\phi_{r\phi} = \mu' + \frac{1}{r}, \Gamma^\theta_{\theta\phi} = \cot\ \theta$$

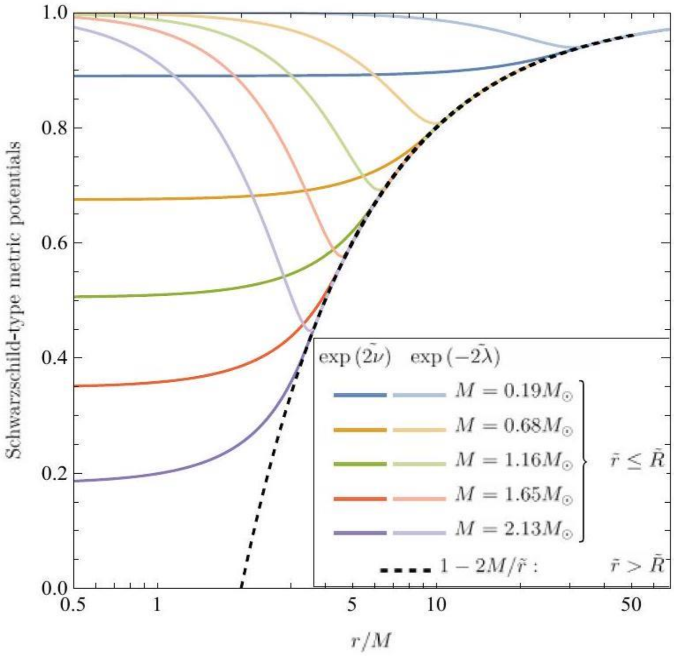
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$8\pi T^t_t = -e^{2\nu-2\mu}\frac{2r\mu'' + \mu'(r\mu' + 4)}{r},$$
$$8\pi T^r_r = \frac{\mu'(r\mu' + 2r\nu' + 2) + 2\nu'}{r},$$
$$8\pi T^\theta_\theta = 8\pi T^\phi_\phi = r[r(\mu'' + \nu'') + \mu' + r\nu'^2 + \nu'],$$

$$T_{\mu\nu} = 0$$

$$ds^2 = -\left(\frac{1 - M/2r}{1 + M/2r}\right)^2 dt^2 + \left(1 + \frac{M}{2r}\right)^4 [dr^2 + r^2(\sin^2\ \theta d\phi^2 + d\theta^2)]$$

$$ds^2 = -\left(1 - \frac{2M}{\tilde{r}}\right)dt^2 + \left(1 - \frac{2M}{\tilde{r}}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2\left(\sin^2\ \tilde{\theta}d\tilde{\phi}^2 + d\tilde{\theta}^2\right)$$



$$\tilde{r} = r\left(1 + \frac{M}{2r}\right)^2 \text{ and } r = \frac{\tilde{r}}{2}\left[1 + \left(1 - \frac{2M}{\tilde{r}}\right)^{1/2} - \frac{M}{\tilde{r}}\right],$$

$$\tilde{t} = t, \tilde{\theta} = \theta, \tilde{\phi} = \phi$$

$$R = \frac{\tilde{R}}{2}\left[1 + \left(1 - \frac{2M}{\tilde{R}}\right)^{1/2} - \frac{M}{\tilde{R}}\right]$$

$$T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu + p g_{\mu\nu},$$

$$8\pi\varepsilon = -\frac{e^{-2\mu}}{r}[2r\mu'' + \mu'(r\mu' + 4)],$$

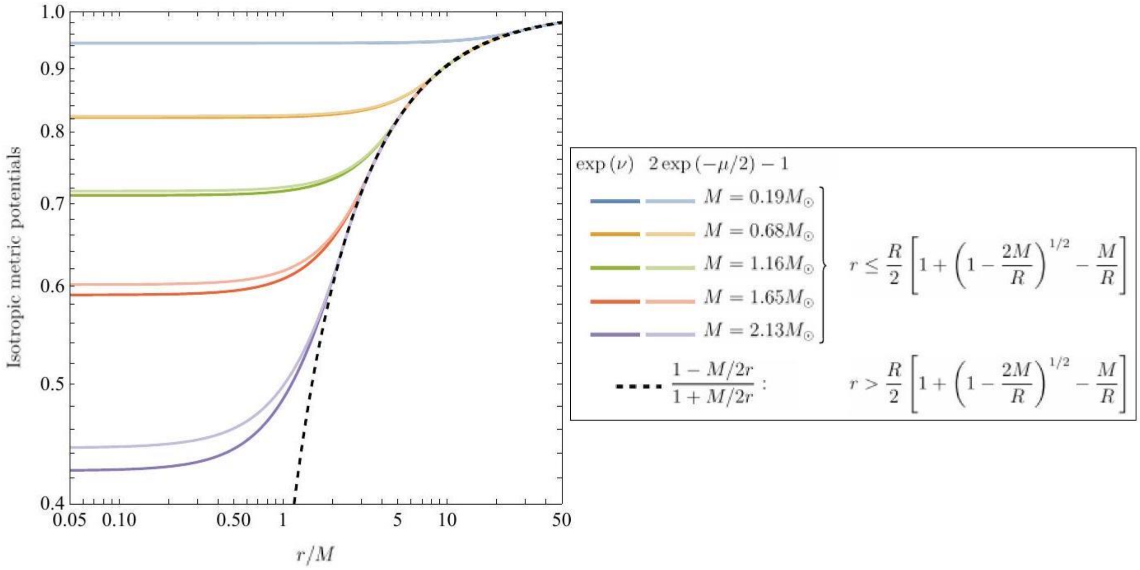
$$8\pi p = \frac{e^{-2\mu}}{r}[\mu'(r\mu' + 2r\nu' + 2) + 2\nu'],$$

$$8\pi p = \frac{e^{-2\mu}}{r}[r(\mu'' + \nu'') + \mu' + r\nu'^2 + \nu'].$$

$$\mu'' = -4\pi e^{2\mu}\varepsilon - \frac{2\mu'}{r} - \frac{\mu'^2}{2}$$

$$\nu'' = 4\pi e^{2\mu}(\varepsilon + 2p) + \frac{\mu'(r\mu' + 2) + 2\nu'(r\nu' + 1)}{2r}.$$

$$\nu' = \frac{8\pi e^{2\mu} r p + \mu'(r\mu' + 2)}{2(r\mu' + 1)},$$



$$p' = -\frac{1}{2}(p+\varepsilon)\frac{8\pi e^{2\mu}rp + \mu'(r\mu'+2)}{r\mu'+1},$$

$$8\pi\varepsilon = e^{-2\tilde{\lambda}}\left(\tilde{r}^{-1}\tilde{\lambda}' - \tilde{r}^{-2}\right) + \tilde{r}^{-2}\tilde{\lambda}\Big|_{\tilde{r}=0} = 0.$$

$$e^{-2\tilde{\lambda}} = 1 - \frac{8\pi}{\tilde{r}}\int_0^{\tilde{r}_1} \varepsilon\tilde{r}^2 d\tilde{r}$$

$$e^{-2\tilde{\lambda}} = 1 - 2M/\tilde{R},$$

$$M = \int_0^{\tilde{R}} dm = 4\pi\int_0^{\tilde{R}} \varepsilon\tilde{r}^2 d\tilde{r}$$

$$dm = 4\pi\varepsilon\tilde{r}^2 d\tilde{r} = 4\pi\varepsilon(e^{2\mu}r^2)(e^{\mu}[r\mu'+1]\mathrm{dr})$$

$$\frac{dm}{dr} = \frac{4\pi\varepsilon}{3}\frac{d}{dr}(e^{\mu}r)^3$$

$$d\tilde{A} \equiv d(e^{\mu}r)^3/dr$$

$$d\tilde{V} \equiv d(e^{\mu}r)^3$$

$$\mu'(r\mu'+2) = \frac{2m}{e^{\mu}r^2}$$

$$r\mu'+1 = \left(1 - \frac{2m}{e^{\mu}r}\right)^{1/2}$$

$$\nu' = \left(4\pi e^{2\mu}rp + \frac{m}{e^{\mu}r^2}\right)\left(1 - \frac{2m}{e^{\mu}r}\right)^{-1/2}$$

$$p' = -\frac{Gm}{e^{\mu}r^2}(p+\varepsilon)\left(\frac{4\pi}{c^4}\frac{(e^{\mu}r)^3 p}{m} + 1\right)\left(1 - \frac{2G}{c^2}\frac{m}{e^{\mu}r^2}\right)^{-1/2}$$

$$p' = -\frac{Gm}{\tilde{r}^2}(p+\varepsilon)\left(\frac{4\tilde{r}^3 p}{mc^2}+1\right)\left(1-\frac{2Gm}{\tilde{r}c^2}\right)^{-1}$$

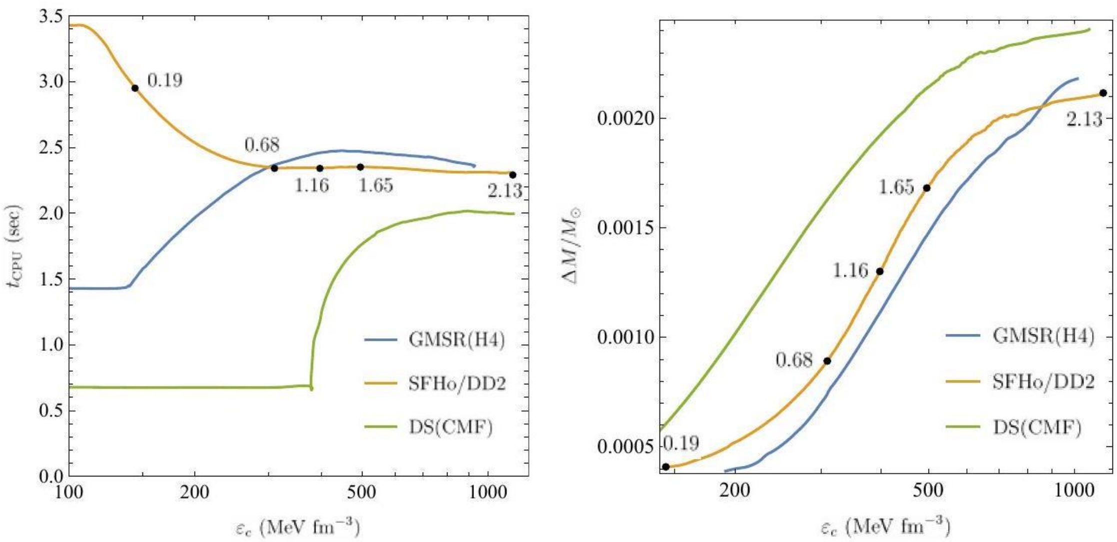
$$\frac{dm}{dr} = \frac{4\pi\varepsilon}{3}\frac{d}{dr}(e^{\mu}r)^3$$

$$\frac{d\mu}{dr} = \frac{1}{r^2}\left[\left(1-\frac{2m}{e^{\mu}r}\right)^{1/2}-1\right]$$

$$\frac{d\nu}{dr} = \left(4\pi e^{2\mu}rp-\frac{m}{e^{\mu}r^2}\right)\left(1-\frac{2m}{e^{\mu}r}\right)^{-1/2}$$

$$\frac{dp}{dr} = -\frac{m}{e^{\mu}r^2}(p+\varepsilon)\left(\frac{4\pi(e^{\mu}r)^3 p}{m}+1\right)\left(1-\frac{2m}{e^{\mu}r^2}\right)^{-1/2}$$

Name	Particles	$\boldsymbol{M_{\max}}$	$\mathbf{min}(\boldsymbol{n_b})$	$\mathbf{max}(\boldsymbol{n_b})$
GMSR(H4)	Nucleonic	$2.33M_{\odot}$	10^{-7}fm^{-3}	2.00fm^{-3}
SFHo/DD2	Nucleons	$2.13M_{\odot}$	10^{-9}fm^{-3}	1.86fm^{-3}
DS(CMF)-2 Hybrid	Hybrid	$1.96M_{\odot}$	0.03fm^{-3}	1.6fm^{-3}



(a) The computation time for evaluating equilibrium configurations, measured as the average CPU execution time.

(b) The absolute difference in the gravitational mass between the one computed in the proposed scheme and in `LORENE`.

$$\frac{dm}{dr} = 4\pi e^{\mu} r^2 \left(1 - \frac{2m}{e^{\mu} r}\right)^{1/2}$$

$$\frac{d\mu}{dr} = \left[\left(1 - \frac{2m}{e^{\mu} r}\right)^{1/2} - 1\right]$$

$$\frac{d\nu}{dr} = \left(4\pi e^{2\mu} rp - \frac{m}{e^{\mu} r^2}\right)\left(1 - \frac{2m}{e^{\mu} r}\right)^{-1/2}$$

$$\frac{dp}{dr} = -\frac{m}{e^{\mu} r}(p + \varepsilon)\left(\frac{4\pi (e^{\mu} r)^3 p}{m} + 1\right)\left(1 - \frac{2m}{e^{\mu} r}\right)^{-1/2}$$

$$(r, \theta, \phi) \mapsto (x^1, x^2, x^3): \begin{cases} x^1 = r\sin\,\theta\cos\,\phi \\ x^2 = r\sin\,\theta\sin\,\phi \\ x^3 = r\cos\,\theta \end{cases}$$

$$ds^2 = -\left(\frac{1 - M/2r}{1 + M/2r}\right)^2 dt^2 + \left(1 - \frac{M}{2r}\right)^4 [(x^1)^2 + (x^2)^2 + (x^3)^2]$$

$$r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$$

$$\left(\frac{dx^1}{dt}\right)^2 + \left(\frac{dx^2}{dt}\right)^2 + \left(\frac{dx^3}{dt}\right)^2 = \frac{(1 - M/2r)^2}{(1 + M/2r)^6}$$

$$u^{\mu} = dx^{\mu}/dt$$

$$d\Sigma^2 = dr^2 + r^2(\sin^2\,\theta d\phi^2 + d\theta^2)$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + e^{2\mu}[dr^2 + r^2(\sin^2\,\theta d\phi^2 + d\theta^2)]$$

$$\frac{dr^2}{r^2} = \frac{d\tilde{r}^2}{\tilde{r}^2 - 2M\tilde{r}}$$

$$\frac{d\tilde{r}}{dr} = \frac{\tilde{r}}{r}\sqrt{1 - \frac{2m(\tilde{r})}{\tilde{r}}},$$

$$(r\mu' + 1)^2 = 1 - \frac{8\pi}{e^{\mu} r}\int \varepsilon \cdot e^{3\mu} r^2 (r\mu' + 1) dr$$

$$e^{\mu} r^2 \mu' (2r\mu'' + r\mu'^2 + 3\mu') = e^{\mu}(r\mu' + 1) - 8\pi\varepsilon \cdot e^{3\mu} r^2 (r\mu' + 1)$$

$$r^3\mu'^3 + 4r^2\mu'^2 + r\mu' + 1 - 8\pi\varepsilon e^{2\mu} r^2 = 0$$

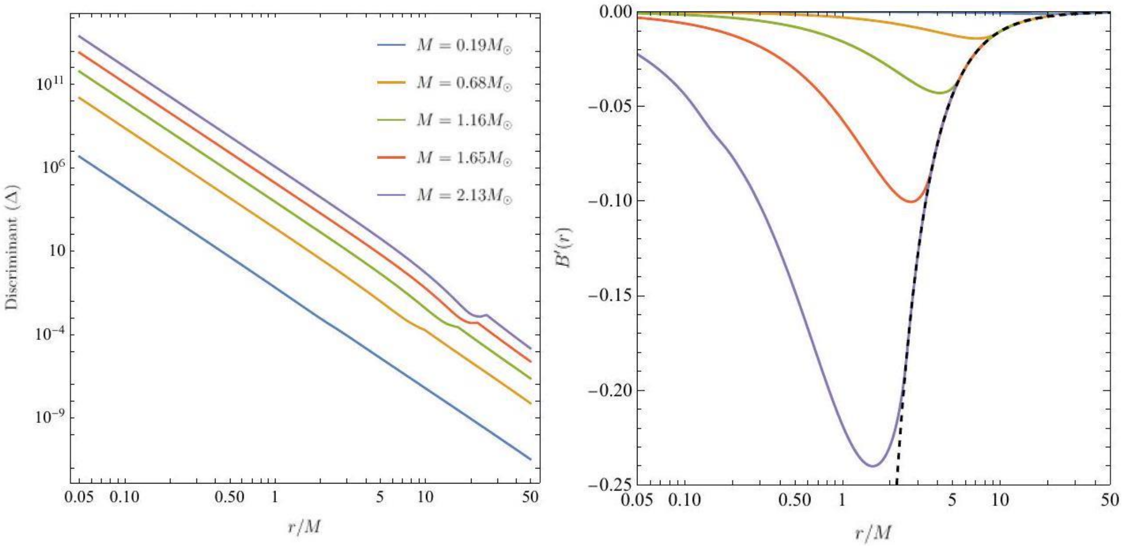
$$\mu(r) = \frac{1}{2}\log\,B(r), \mu'(r) = \frac{B'(r)}{2B(r)}$$

$$aB'^3 + bB'^2 + cB' + d = 0 \;\text{ with }\; \begin{cases} a = r^3 \\ b = 8r^2 B \\ c = 4rB^2 \\ d = 8B(1 - 8\pi\varepsilon r^2 B) \end{cases}$$

$$t = B' + \frac{b}{3a}$$

$$t^3 + pt + q = 0$$

$$p = \frac{3ac - b^2}{3a^2}, q = \frac{2b^3 - 9abc + 27a^2 d}{27a^3}$$



$$B'(r \geq R) = (M/r)(1 + M/2r), r = R, t \equiv w - \frac{p}{3w}$$

$$(w^3)^2 + q(w^3) - \frac{p^3}{27} = 0.$$

$$u_{\pm} = -\frac{q}{2} \pm \sqrt{\Delta},$$

$$\Delta = q^2/4 + p^3/27 > 0$$

$$w_1 = \sqrt[3]{u_+} + \sqrt[3]{u_-}.$$

$$w_2 = -\frac{1}{2}\left(\sqrt[3]{u_+} + \sqrt[3]{u_-}\right) + \frac{i\sqrt{3}}{2}\left(\sqrt[3]{u_+} - \sqrt[3]{u_-}\right)$$

$$w_3 = -\frac{1}{2}\left(\sqrt[3]{u_+} + \sqrt[3]{u_-}\right) + \frac{i\sqrt{3}}{2}\left(-\sqrt[3]{u_+} + \sqrt[3]{u_-}\right)$$

$$\left(\frac{237276}{\beta^3} + \frac{6084\sqrt[3]{2}}{\beta} + \beta^3 + 78\sqrt[3]{4}\beta + 184\right)e^{4\mu} - 432\pi r^2 \varepsilon e^{2\mu} + 54 = 0$$

$$\alpha^3 = -92e^{4\mu} + 216\pi r^2 \varepsilon e^{2\mu} - 27 - \sqrt{(92e^{4\mu} - 216\pi r^2 \varepsilon e^{2\mu} + 27)^2 - 8788e^{8\mu}},$$

$$\beta = \alpha + \sqrt[3]{\alpha^3 - 432\pi r^2 \varepsilon e^{2\mu}}$$

$$\nabla^{\mu} T_{\mu\nu} = 0$$

$$\partial_t \varepsilon = \partial_t p = 0$$

$$\partial_\theta p = \partial_\psi p = 0$$

$$\nabla_\mu T_t^\mu = -p' - \tfrac{1}{2}(p + \varepsilon)\nu' = 0$$

$$\delta^3 = 1$$

$$\delta^3 - 1 = (\delta - 1)(\delta^2 + \delta + 1)$$

$$\delta_0 = 1, \delta_1 = \tfrac{-1+i\sqrt{3}}{2}, \delta_2 = \tfrac{-1-i\sqrt{3}}{2}$$

$$\mathcal{L}_{QHD} = \sum_B \bar{\psi}_B \left[\gamma^\mu \left(\mathrm{i}\partial_\mu - g_{B\omega}\omega_\mu - g_{B\rho}\frac{1}{2}\vec{\tau}_B \cdot \vec{\rho}_\mu\right) - (M_B - g_{B\sigma}\sigma)\right]\psi_B - U(\sigma)$$
$$+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4}\vec{\mathbf{P}}^{\mu\nu} \cdot \vec{\mathbf{P}}_{\mu\nu}$$

$$U(\sigma) = \frac{\kappa M_N (g_\sigma\sigma)^3}{3} + \frac{\lambda(g_\sigma\sigma)^4}{4}$$

$$\mathcal{L}_\phi = -g_{Y\phi}\bar{\psi}_Y(\gamma^\mu\phi_\mu)\psi_Y + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu - \frac{1}{4}\Phi^{\mu\nu}\Phi_{\mu\nu}$$

$$\mathcal{L}_{\omega\rho} = \Lambda_{\omega\rho}\left(g_\rho^2\overrightarrow{\rho^\mu} \cdot \overrightarrow{\rho_\mu}\right)\left(g_\omega^2\omega^\mu\omega_\mu\right)$$

$$\varepsilon = \langle T_{00}\rangle \text{ and } P = \frac{1}{3}\langle T_{jj}\rangle$$

$$\frac{g_{\Lambda\omega}}{g_{N\omega}} = \frac{4 + 2\alpha_v}{5 + 4\alpha_v}, \frac{g_{\Sigma\omega}}{g_{N\omega}} = \frac{8 - 2\alpha_v}{5 + 4\alpha_v}, \text{ and } \frac{g_{\Xi\omega}}{g_{N\omega}} = \frac{5 - 2\alpha_v}{5 + 4\alpha_v}$$

$$\frac{g_{\Lambda\phi}}{g_{N\omega}} = \sqrt{2}\left(\frac{2\alpha_v - 5}{5 + 4\alpha_v}\right), \frac{g_{\Sigma\phi}}{g_{N\omega}} = \sqrt{2}\left(\frac{-2\alpha_v - 1}{5 + 4\alpha_v}\right), \text{ and } \frac{g_{\Xi\phi}}{g_{N\omega}} = \sqrt{2}\left(\frac{-2\alpha_v - 4}{5 + 4\alpha_v}\right)$$

$$\frac{g_{\Lambda\rho}}{g_{N\rho}} = 0, \frac{g_{\Sigma\rho}}{g_{N\rho}} = 2\alpha_v, \text{ and } \frac{g_{\Xi\rho}}{g_{N\rho}} = -(1 - 2\alpha_v)$$

Parameters	Values	Parameters	Values	Meson masses [MeV]
N13$\omega\rho$ parameterization				
$g_{N\sigma}$	10.094	$n_0[\mathrm{fm}^{-3}]$	0.150	$m_\sigma = 508.194$
$g_{N\omega}$	12.807	M^*/M	0.594	$m_\omega = 782.501$

$g_{N\rho}$	14.441	K[MeV]	258	$m_\rho = 763$
λ	-0.002904	S_0[MeV]	30.7	$m_\phi = 1020$
κ	-0.002208	L[MeV]	42	-
$\Lambda_{\omega\rho}$	0.045	B/A[MeV]	16.31	-
El3 $\omega\rho$ parameterization				
$(g_{N\sigma}/m_\sigma)^2$	12.108[fm^2]	n_0[fm^{-3}]	0.156	$m_\sigma = 512$
$(g_{N\omega}/m_\omega)^2$	7.132[fm^2]	M^*/M	0.69	$m_\omega = 783$
$(g_{N\rho}/m_\rho)^2$	5.85[fm^2]	K[MeV]	256	$m_\rho = 770$
κ	0.004138	S_0[MeV]	32.1	$m_\phi = 1020$
λ	-0.00390	L[MeV]	66	-
$\Lambda_{\omega\rho}$	0.0185	B/A[MeV]	16.2	-

Parameters	Values	Parameters	Values	Meson masses [MeV]
Nl3$\omega\rho$ parameterization				
$g_{N\sigma}$	10.094	n_0 [fm^{-3}]	0.150	$m_\sigma = 508.194$
$g_{N\omega}$	12.807	M^*/M	0.594	$m_\omega = 782.501$
$g_{N\rho}$	14.441	K [MeV]	258	$m_\rho = 763$
λ	-0.002904	S_0[MeV]	30.7	$m_\phi = 1020$
κ	-0.002208	L [MeV]	42	–
$\Lambda_{\omega\rho}$	0.045	B/A [MeV]	16.31	–
El3$\omega\rho$ parameterization				
$(g_{N\sigma}/m_\sigma)^2$	12.108 [fm^2]	n_0 [fm^{-3}]	0.156	$m_\sigma = 512$
$(g_{N\omega}/m_\omega)^2$	7.132 [fm^2]	M^*/M	0.69	$m_\omega = 783$
$(g_{N\rho}/m_\rho)^2$	5.85 [fm^2]	K [MeV]	256	$m_\rho = 770$
κ	0.004138	S_0[MeV]	32.1	$m_\phi = 1020$
λ	-0.00390	L [MeV]	66	–
$\Lambda_{\omega\rho}$	0.0185	B/A [MeV]	16.2	–

Model	$g_{\Lambda\sigma}$ $/g_{N\sigma}$	$g_{\Sigma\sigma}$ $/g_{N\sigma}$	$g_{\Xi\sigma}$ $/g_{N\sigma}$
NL3 $\omega\rho$	0.613	0.461	0.279
EL3 $\omega\rho$	0.610	0.406	0.269

Model	$g_{\Lambda\sigma}/g_{N\sigma}$	$g_{\Sigma\sigma}/g_{N\sigma}$	$g_{\Xi\sigma}/g_{N\sigma}$
NL3$\omega\rho$	0.613	0.461	0.279
EL3$\omega\rho$	0.610	0.406	0.269

$$\frac{dP}{dr} = -\frac{(1+\alpha)(\varepsilon+\beta P)(M+4\chi\pi r^3 P)}{r(r-2M)}$$

$$\frac{dM}{dr} = 4\pi r^2(\varepsilon+\theta P)+\Gamma\sqrt{\varepsilon}M$$

$$\gamma(\Psi^i)(G_{\mu\nu}-W_{\mu\nu}) = kT_{\mu\nu},$$

$$G_{\mu\nu} = kT_{\mu\nu}^{\text{eff}} = \frac{k}{\gamma}T_{\mu\nu}+W_{\mu\nu}$$

$$T_{\mu\nu} = Fg_{\mu\nu}+(F+\bar{\rho})U_\mu U_\nu+(P-F)N_\mu N_\nu$$

$$U^\mu = \left(\frac{1}{\sqrt{B}},0,0,0\right)$$

$$N^\mu = \left(0,\frac{1}{\sqrt{A}},0,0\right)$$

$$U_\nu U^\nu = -1, N_\nu N^\nu = 1 \text{ and } U_\nu N^\nu = 0$$

$$ds^2 = Bdt^2-Adr^2-r^2(d\theta^2+\sin\,\theta^2 d\phi^2),$$

$$-\frac{B}{r^2A}+\frac{B}{r^2}+\frac{A'B}{rA^2} = 8\pi B\bar{\rho}$$

$$-\frac{A}{r^2}+\frac{B'}{rB}+\frac{1}{r^2} = 8\pi AP$$

$$-\frac{B'^2r^2}{4AB^2}-\frac{A'B'r^2}{4A^2B}+\frac{B''r^2}{2AB}-\frac{A'r}{2A^2}+\frac{B'r}{2AB} = 8\pi r^2 F$$

$$P' = -(P+\bar{\rho})\frac{M+4\pi r^3P}{r(r-2M)}-\frac{2}{r}\sigma$$

$$M' = 4\pi\bar{\rho}r^2$$

$$\sigma \equiv P-F,$$

$$\bar{\rho} = \rho+\theta P+\frac{\Gamma M\sqrt{\rho}}{4\pi r^2}$$

$$\sigma = \frac{(M+4\pi r^3P)(P(1+\theta)+\rho)\left[(\alpha+1)\left(\frac{(\chi-1)4\pi r^3P}{M+4\pi r^3P}+1\right)\left(\frac{P(\beta-\theta-1)}{P(1+\theta)+\rho}+1\right)-1\right]}{2(r-2M)} - \frac{\Gamma M\sqrt{\rho}(M+4\pi r^3P)}{8\pi r^2(r-2M)},$$

$$P' = -(P+\rho)\frac{M+4\pi r^3P}{r(r-2M)}-\frac{2}{r}\sigma,\ M' = 4\pi\rho r^2,\ \sigma\equiv F-P.$$

$$\frac{d\rho}{dr}<0,\frac{dP}{dr}<0$$

a) null energy ($\rho > 0$), b) dominant energy ($\rho + P > 0, \rho + F > 0$), and c) strong energy ($\rho + P + 2F > 0$)

$0 < \frac{\partial P}{\partial \rho} < 1$ and $0 < \frac{\partial F}{\partial \rho} < 1$, where $c_s^2(\text{ radial }) = \frac{\partial P}{\partial \rho}$ and $c_s^2(\text{ transverse }) = \frac{\partial F}{\partial \rho}$

$$P(0) = F(0).$$

$$Q_{ij} = -\lambda \epsilon_{ij}$$

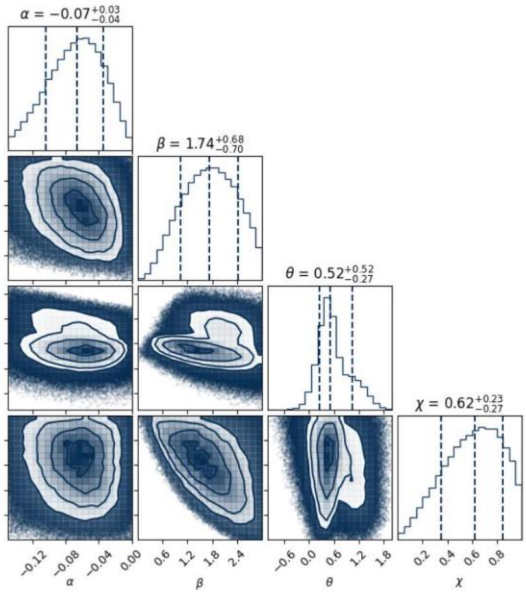
$$\Lambda \equiv \frac{\lambda}{M^5} \equiv \frac{2k_2}{3C^5}$$

$$\begin{aligned} k_2 = & \frac{8C^5}{5}(1-2C)^2[2-y+2C(y-1)] \\ & \times \{2C[6-3y+3C(5y-8)] \\ & +4C^3[13-11y+C(3y-2)+2C^2(1+y)] \\ & +3(1-2C)^2[2-y+2C(y-1)]\log(1-2C)\}^{-1} \end{aligned}$$

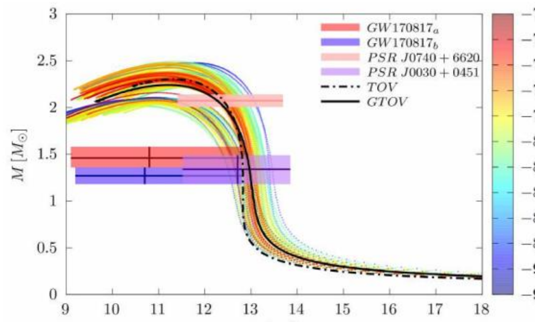
$$r\frac{dy}{dr} + y^2 + yB_1 + r^2 B_2 = 0$$

$$B_1 = \frac{r - 4\pi r^3(\bar{\rho} - P)}{r - 2M}$$

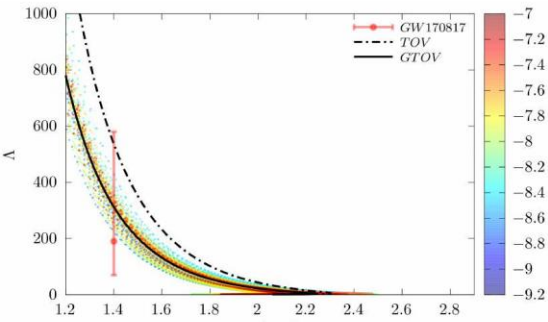
$$B_2 = \frac{4\pi r\left[4\bar{\rho} + 8P + \frac{(\bar{\rho}+P)(1+d\bar{\rho}/dP)}{1-d\sigma/dP} - \frac{6}{4\pi r^2} + 4\sigma\right]}{r-2M} - 4\left[\frac{M+4\pi r^3 P}{r^2(1-2M/r)}\right]^2$$



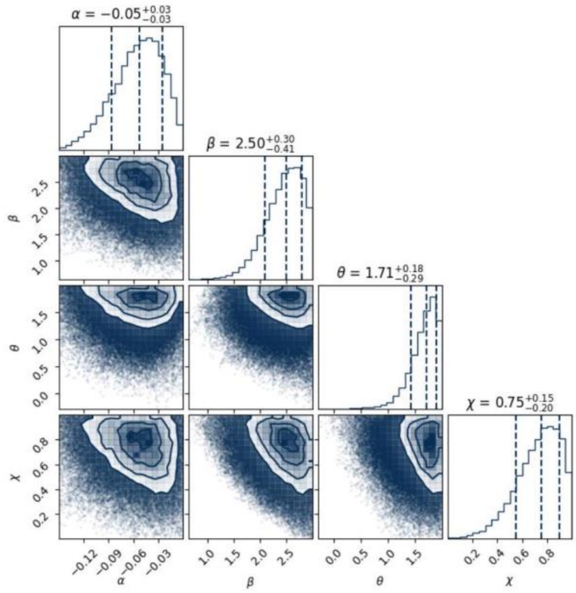
$\alpha = -0.07^{+0.03}_{-0.04}$
$\beta = 1.74^{+0.68}_{-0.70}$
$\theta = 0.52^{+0.52}_{-0.27}$
$\chi = 0.62^{+0.23}_{-0.27}$
α
β
θ
χ

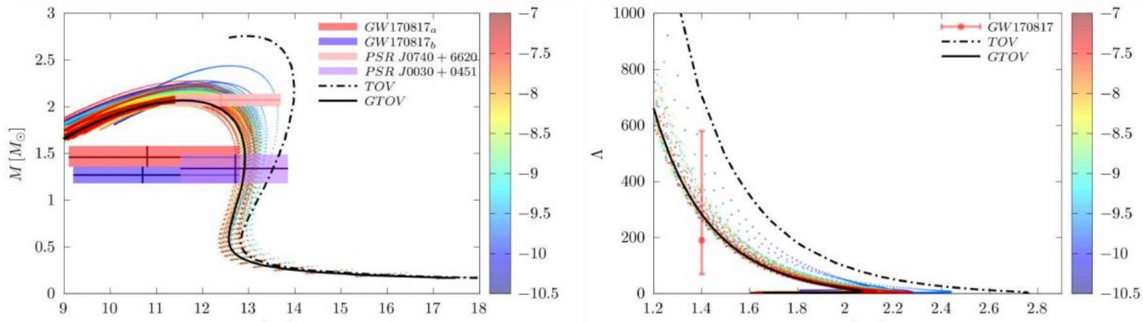


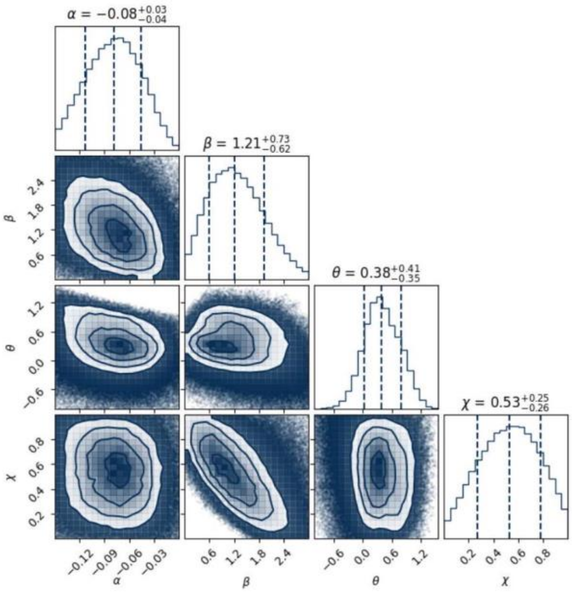
$GW170817_a$
$GW170817_b$
$PSR\ J0740+6620$
$PSR\ J0030+0451$
TOV
$GTOV$
$M\ [M_\odot]$



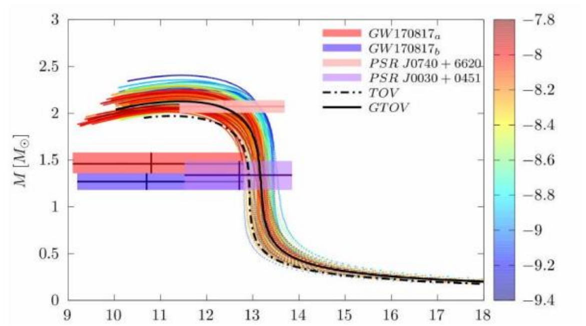
$GW170817$
TOV
$GTOV$
Λ


$\alpha = -0.05^{+0.03}_{-0.03}$
$\beta = 2.50^{+0.30}_{-0.41}$
$\theta = 1.71^{+0.18}_{-0.29}$
$\chi = 0.75^{+0.15}_{-0.20}$
α
β
θ
χ

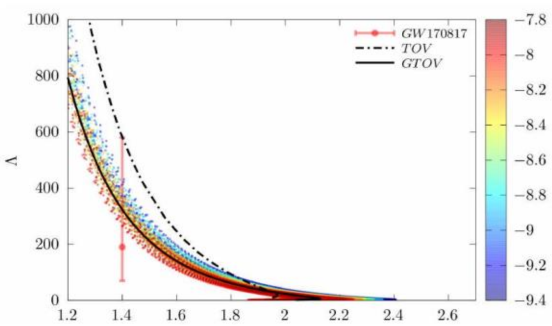

$M\,[M_\odot]$
$GW170817_a$
$GW170817_b$
$PSR\ J0740+6620$
$PSR\ J0030+0451$
TOV
GTOV
Λ
GW170817
TOV
GTOV



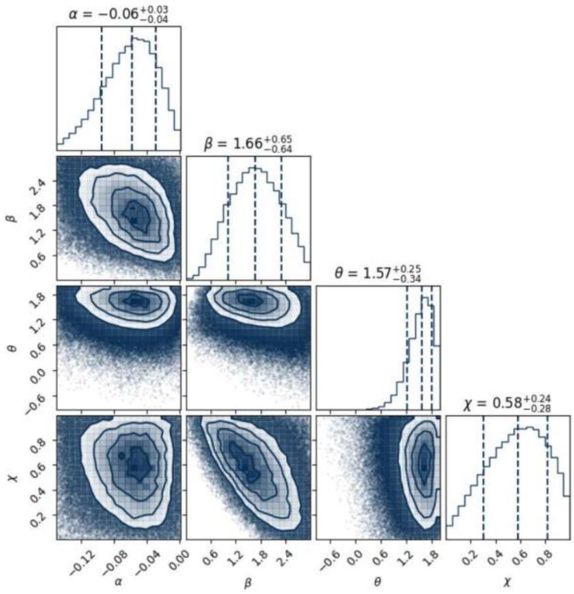
$\alpha = -0.08^{+0.03}_{-0.04}$
$\beta = 1.21^{+0.73}_{-0.62}$
$\theta = 0.38^{+0.41}_{-0.35}$
$\chi = 0.53^{+0.25}_{-0.26}$

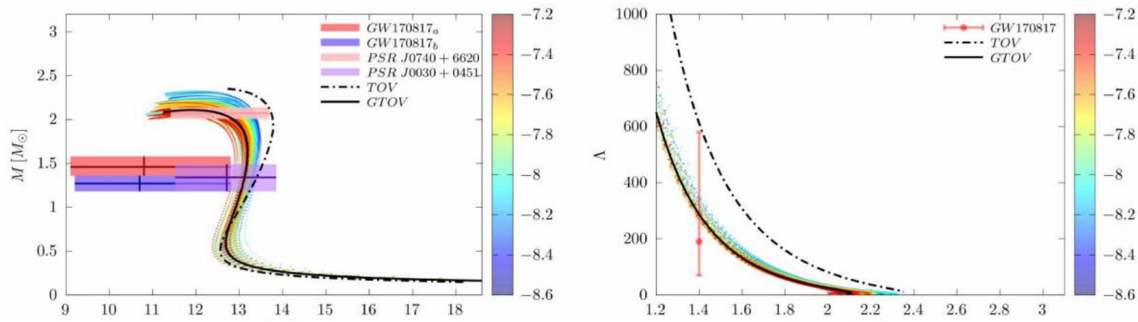


$GW170817_a$
$GW170817_b$
$PSR\ J0740+6620$
$PSR\ J0030+0451$
TOV
$GTOV$
$M\ [M_\odot]$



$GW170817$
TOV
$GTOV$
Λ





Figuras complementarias 1, 2, 3, 4, 5, 6, 7 y 8. Distribución GTOV respecto de una partícula estrella o blanca.

$$M_1 = 1.46^{+0.12}_{-0.10} M_\odot$$

$$R_1 = 10.8^{+2.0}_{-1.7} \text{ nm}$$

$$M_2 = 1.27^{+0.09}_{-0.09} M_\odot$$

$$R_2 = 10.7^{+2.1}_{-1.5} \text{ nm}$$

$$2.072^{+0.067}_{-0.066} M_\odot$$

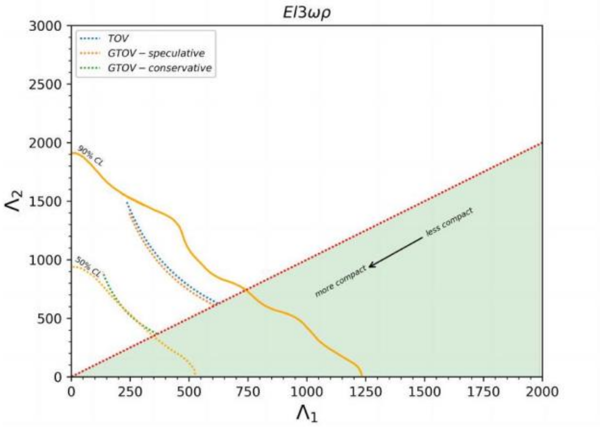
$$12.39^{+1.30}_{-0.98} \text{ nm}$$

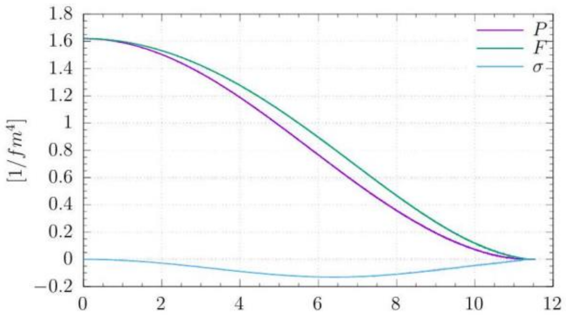
$$1.34^{+0.15}_{-0.16} M_\odot$$

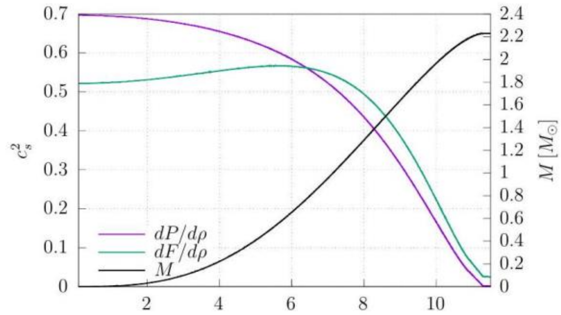
$$12.71^{+1.14}_{-1.19} \text{ nm}$$

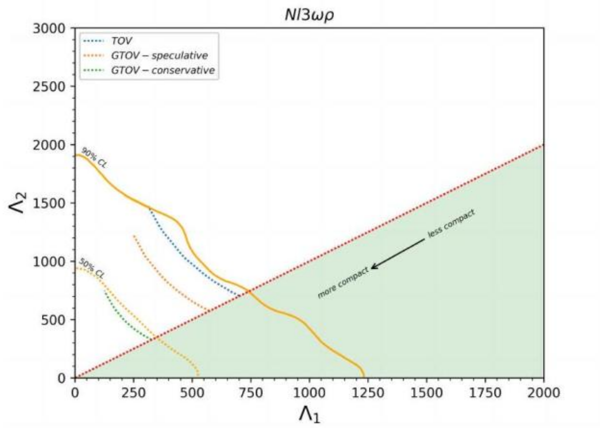
$$\Lambda \times M[M_\odot]$$

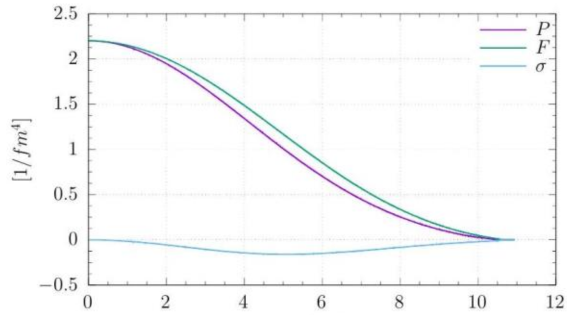
$$\Lambda_{1.4} = 190^{+390}_{-120}$$

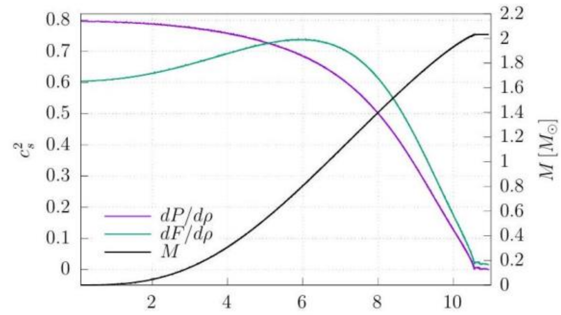

El3ωρ
TOV
GTOV – speculative
GTOV – conservative
90% CL
50% CL
less compact
more compact
Λ2
Λ1

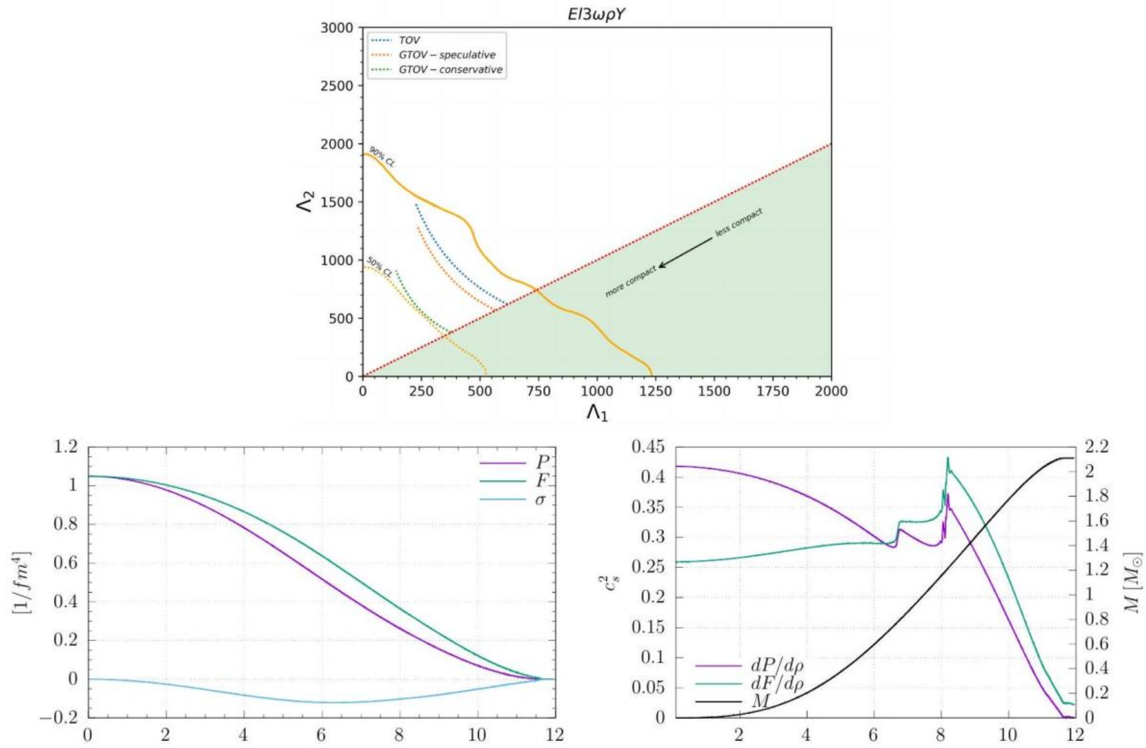

P
F
σ
[1/fm^4]

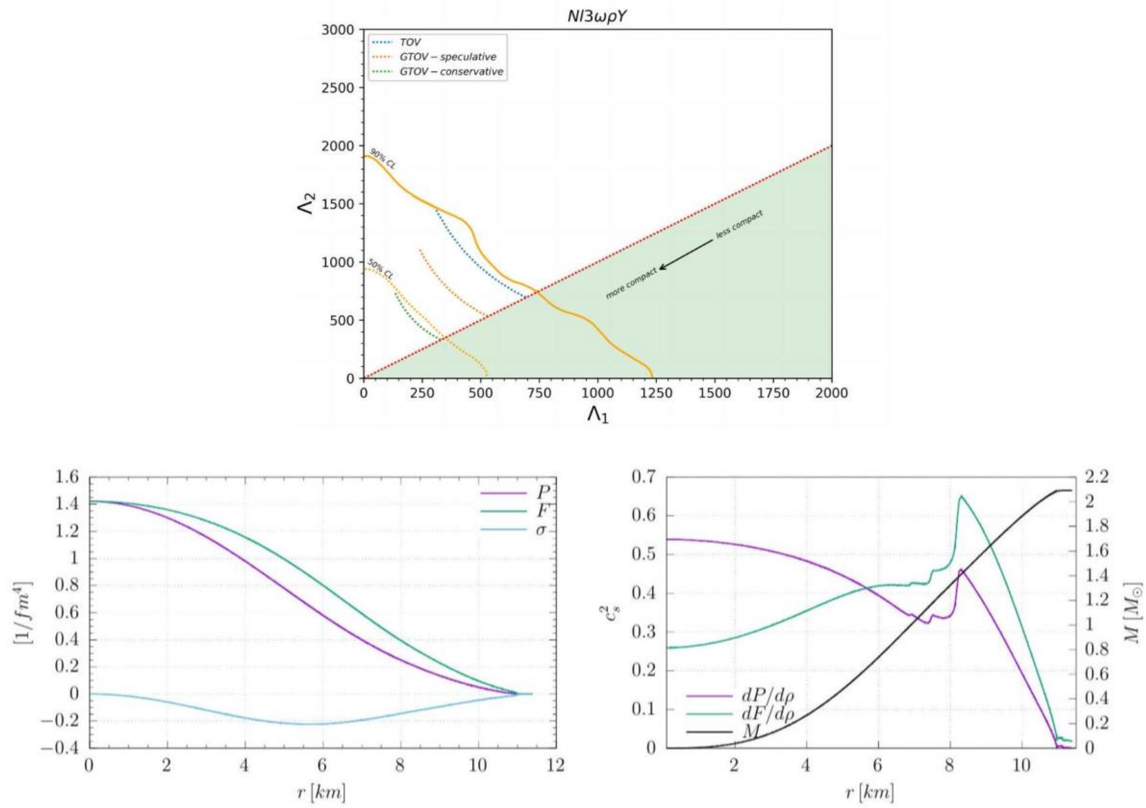

dP/dρ
dF/dρ
M
c_s^2
M [M_⊙]


Nl3ωρ
TOV
GTOV – speculative
GTOV – conservative
90% CL
50% CL
less compact
more compact
Λ2
Λ1


P
F
σ
[1/fm^4]

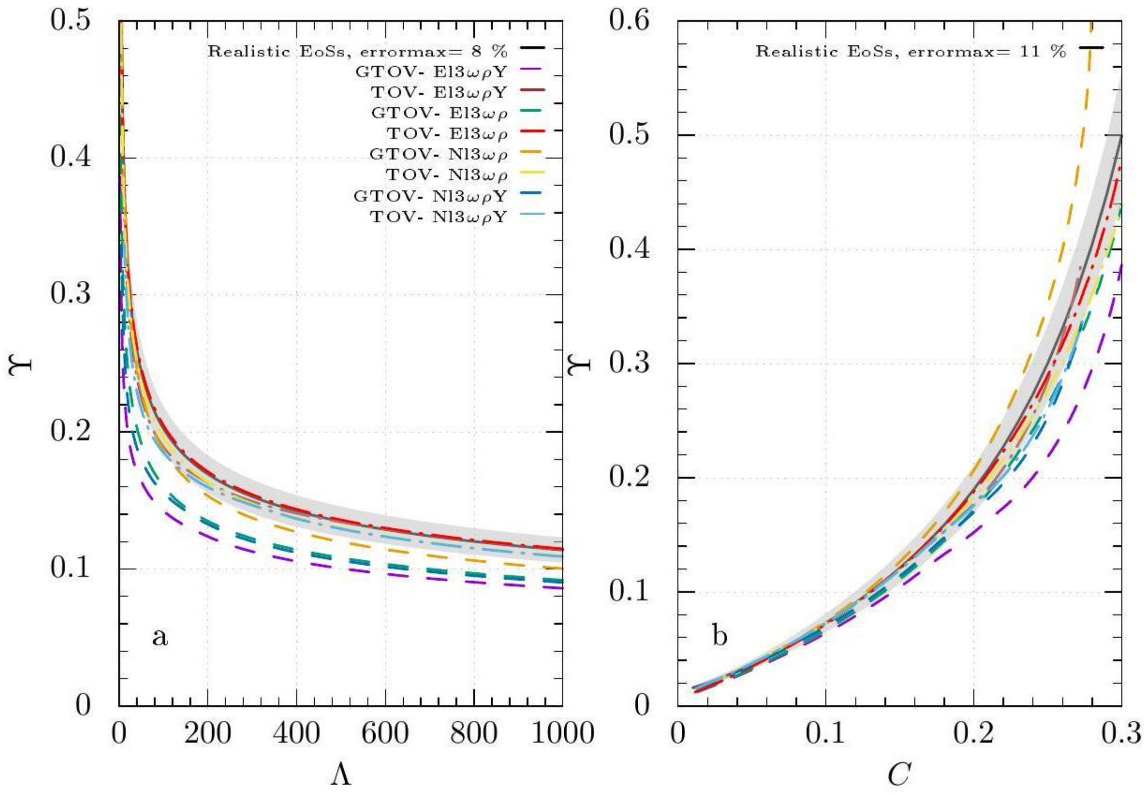

dP/dρ
dF/dρ
M
c_s^2
M [M_⊙]





Figuras complementarias 9, 10, 11, 12, 13, 14, 15 y 16. Fluctaciones de deformación por interacción de una partícula blanca o estrella.

$$\mathcal{M}_{\odot} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}},$$



EoS	($\alpha, \beta, \theta, \chi$)	$M_{\max}$ $[M_{\odot}]$	$R_{\max}$ [nm]	$\Lambda_{1.4}$	$R_{1.4}$[nm]
El3$\omega\rho$Y	(-0.03, 0.77,-0.78, 0.36)	2.57✓	11.72	490✓✓	13.44✓✓
El3$\omega\rho$	(-0.02, 1.85, -0.44, 0.57)	2.53✓	11.87	509✓✓	13.10✓✓
Nl3$\omega\rho$Y	(-0.02, 1.13, -0.08, 0.44)	2.63✓	13.18	467✓✓	13.62✓✓
$N13\omega\rho$	(-0.01, 1.85, 0.24, 0.63)	2.62✓	12.49	504✓✓	13.40✓✓
El3$\omega\rho$Y	(-0.08, 1.21, 0.38, 0.53)	2.12✓	11.37✓	323✓	13.14✓✓
El3$\omega\rho$	(-0.07, 1.74, 0.52, 0.62)	2.24✓	11.31✓	312✓	12.94✓✓
Nl3$\omega\rho$Y	(-0.06, 1.66, 1.57, 0.58)	2.19✓	11.74✓	287✓	13.13✓✓

$N13\omega\rho$	(-0.05, 2.5, 1.71, 0.75)	2.06✓	11.71✓	282✓	12.91✓✓
El3$\omega\rho$Y	(0, 1, 0, 1)	1.96	11.43	530✓✓	12.81✓✓
El3$\omega\rho$	(0, 1, 0, 1)	2.30✓	11.22✓	536✓✓	12.81✓✓
N13$\omega\rho$Y	(0, 1, 0, 1)	2.35✓	12.73✓	611✓	13.45✓✓
$N13\omega\rho$	(0, 1, 0, 1)	2.74✓	12.64	619✓	13.45✓✓

$$\Lambda_{1.4}(GW170817) = 190^{+390}_{-120}$$

and $\Lambda_{1.4}(GW190814) = 616^{+273}_{-158}$.

$$(\text{GW190814}) = 2.5^{+0.08}_{-0.09}\text{M}_{\odot},$$

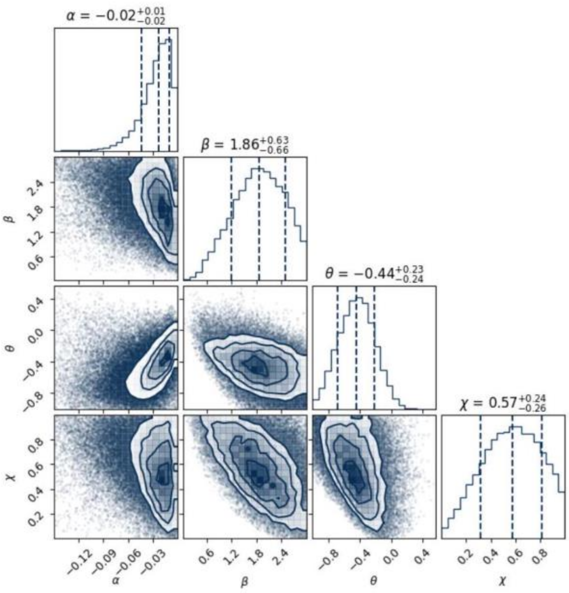
$$(\text{PSR J07740+6620}) = 2.072^{+0.067}_{-0.066}\text{M}_{\odot}$$

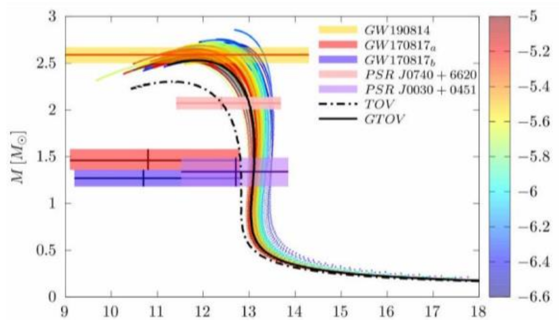
$$12.39^{+1.30}_{-0.98}\text{ nm},$$

$$(\text{ PSR J0030} + 0451) = 1.34^{+0.15}_{-0.16}\text{M}_{\odot}$$

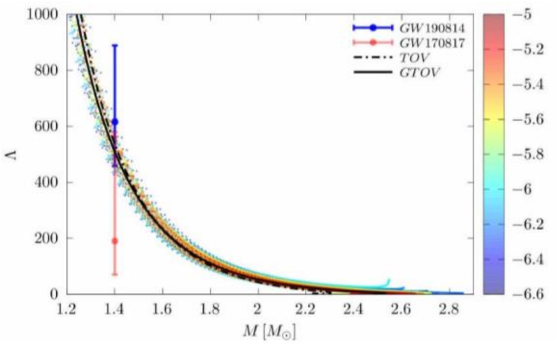
$$12.71^{+1.14}_{-1.19}\text{ nm}$$

$$\alpha = -0.05 \pm 0.03, \beta = 2.50^{+0.30}_{-0.41}, \theta = 1.71^{+0.18}_{-0.29} \text{ and } \chi = 0.75^{+0.15}_{-0.20}$$

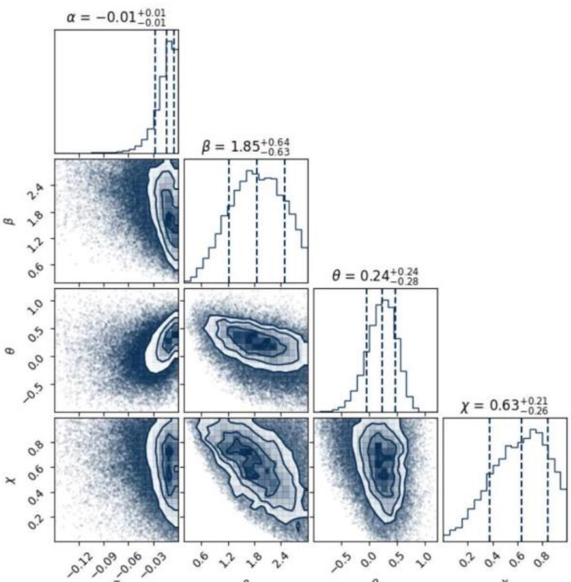




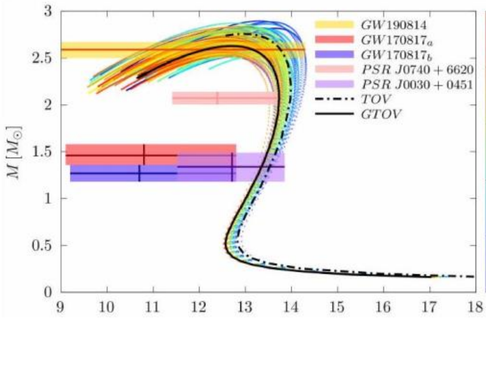
$GW190814$
$GW170817_a$
$GW170817_b$
$PSR\ J0740+6620$
$PSR\ J0030+0451$
TOV
$GTOV$
$M\,[M_\odot]$



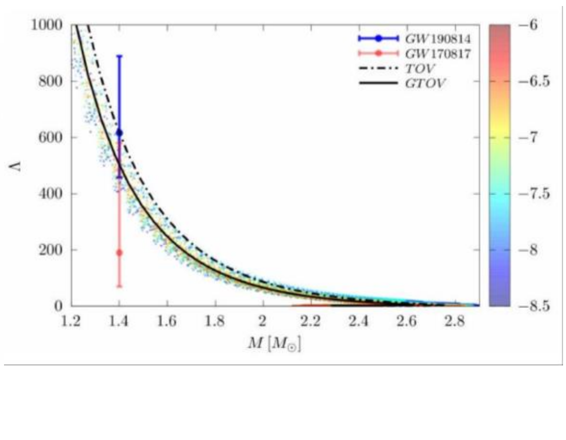
$GW190814$
$GW170817$
TOV
$GTOV$
Λ
$M\,[M_\odot]$



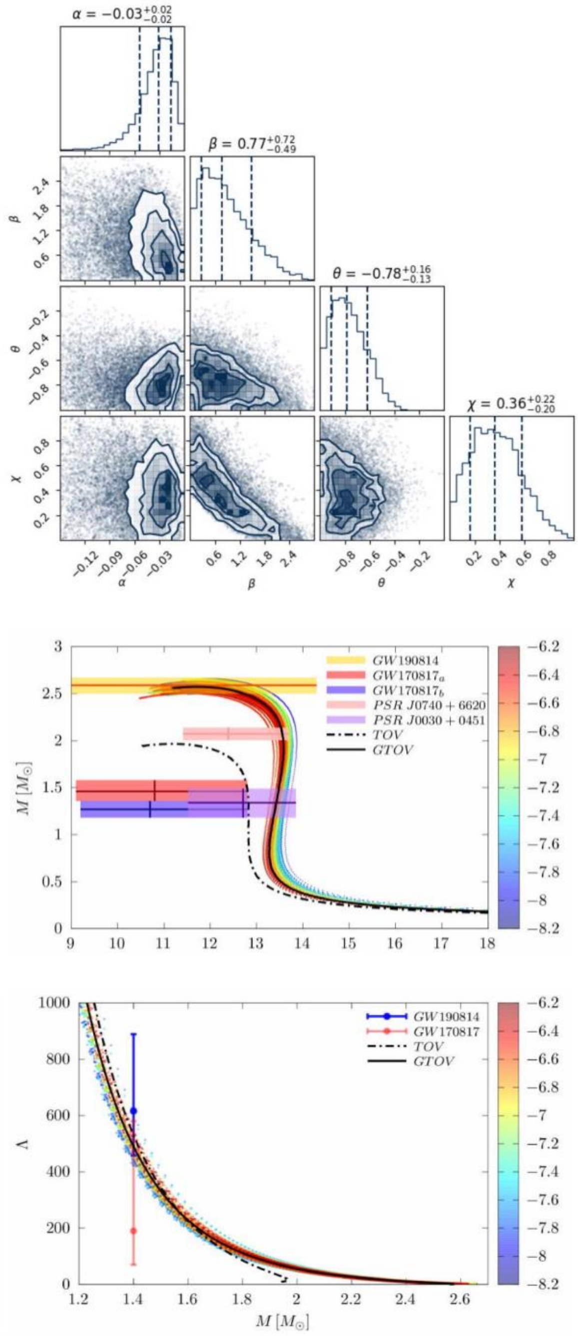
$\alpha = -0.01^{+0.01}_{-0.01}$
$\beta = 1.85^{+0.64}_{-0.63}$
$\theta = 0.24^{+0.24}_{-0.28}$
$\chi = 0.63^{+0.21}_{-0.26}$
α
β
θ
χ

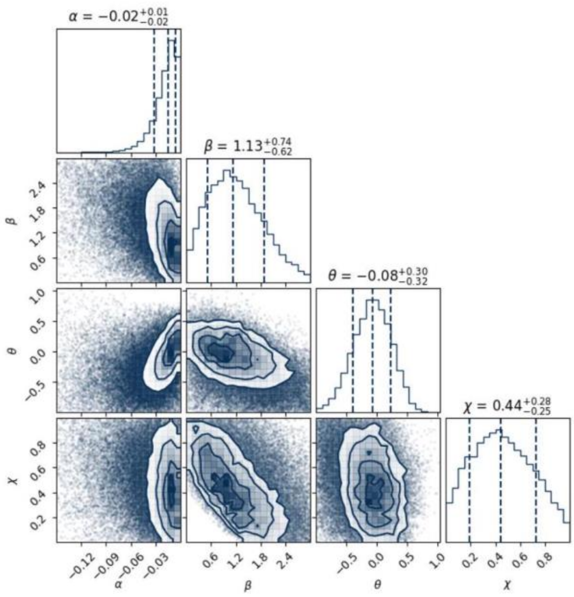


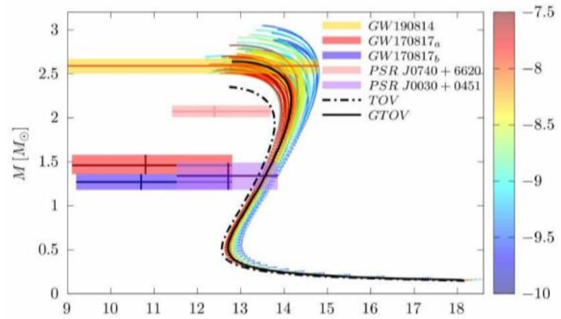
$GW190814$
$GW170817_a$
$GW170817_b$
$PSR\ J0740+6620$
$PSR\ J0030+0451$
TOV
$GTOV$
$M\,[M_\odot]$

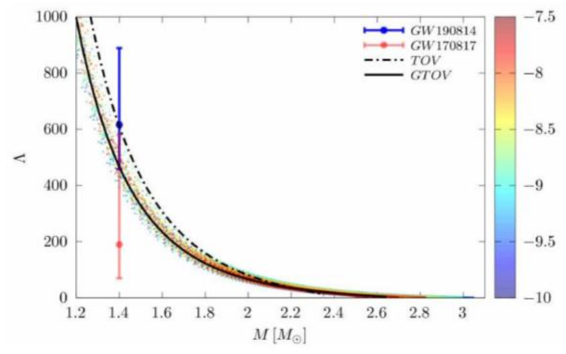


$GW190814$
$GW170817$
TOV
$GTOV$
Λ
$M\,[M_\odot]$

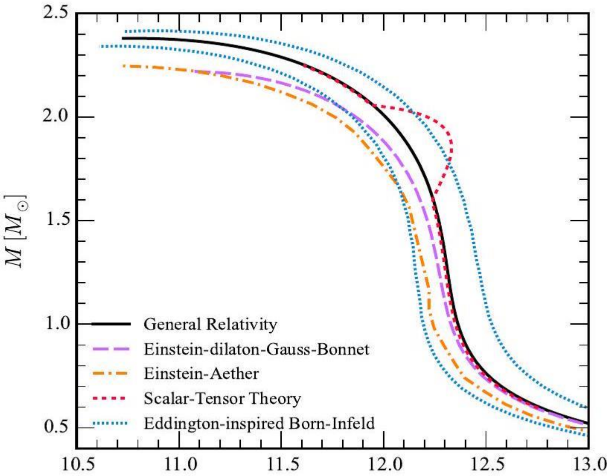

$\alpha = -0.03^{+0.02}_{-0.02}$
$\beta = 0.77^{+0.72}_{-0.49}$
$\theta = -0.78^{+0.16}_{-0.13}$
$\chi = 0.36^{+0.22}_{-0.20}$
$GW190814$
$GW170817_a$
$GW170817_b$
$PSR\ J0740+6620$
$PSR\ J0030+0451$
TOV
$GTOV$
$M\ [M_\odot]$
Λ
$GW170817$

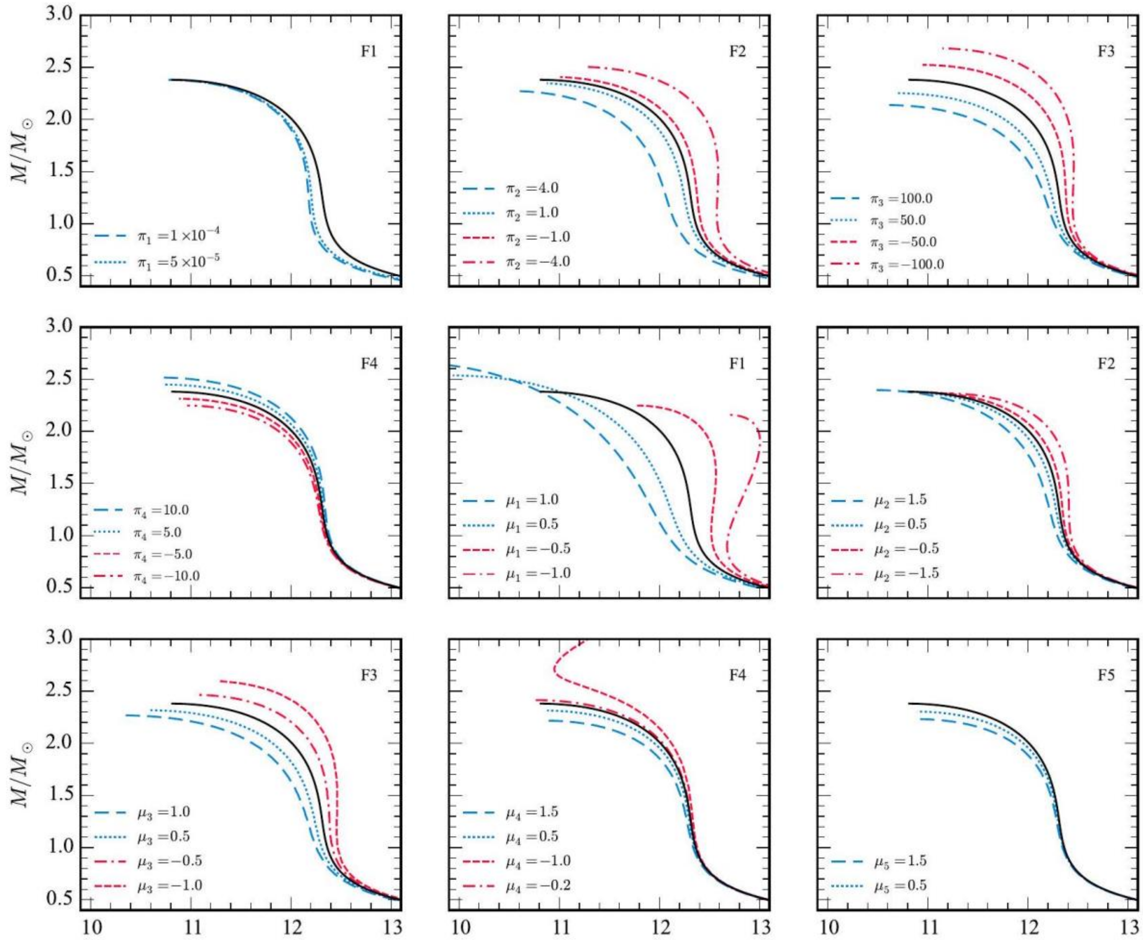






$(1 - d\sigma/dP) = dF/dP = c_s^2$ (transversal) $/c_s^2$ (radial)





Figuras complementarias 17 y 18. Fluctuaciones de masa de una partícula estrella o blanca.

$$\frac{dp}{dr} = \left(\frac{dp}{dr}\right)_{\mathrm{GR}} - \frac{\rho m}{r^2}(\mathcal{P}_1 + \mathcal{P}_2)$$

$$\frac{dm}{dr} = \left(\frac{dm}{dr}\right)_{\mathrm{GR}} + 4\pi r^2 \rho(\mathcal{M}_1 + \mathcal{M}_2)$$

$$\mathcal{P}_1 \equiv \delta_1 \frac{m}{r} + 4\pi\delta_2 \frac{r^3 p}{m}$$
$$\mathcal{M}_1 \equiv \delta_3 \frac{m}{r} + \delta_4 \Pi$$
$$\mathcal{P}_2 \equiv \pi_1 \frac{m^3}{r^5 \rho} + \pi_2 \frac{m^2}{r^2} + \pi_3 r^2 p + \pi_4 \frac{\Pi p}{\rho}$$
$$\mathcal{M}_2 \equiv \mu_1 \frac{m^3}{r^5 \rho} + \mu_2 \frac{m^2}{r^2} + \mu_3 r^2 p + \mu_4 \frac{\Pi p}{\rho} + \mu_5 \Pi^3 \frac{r}{m}$$

$$\Pi \equiv (\epsilon - \rho)/\rho$$

$$\nabla_\nu T^{\mu\nu} = 0, T^{\mu\nu} = (\epsilon_{\text{eff}} + p) u^\mu u^\nu + p g^{\mu\nu},$$

$$\epsilon_{\text{eff}} = \epsilon + \rho \mathcal{M}_2$$

$$g_{\mu\nu} = \text{diag}\left[e^{\nu(r)}, (1 - 2m(r)/r)^{-1}, r^2, r^2 \sin^2 \theta\right]$$

$$\frac{d\nu}{dr} = \frac{2}{r^2}\left[(1 - \mathcal{M}_2)\frac{m + 4\pi r^3 p}{1 - 2m/r} + m\mathcal{P}_2\right]$$

$$\left(\frac{dp}{dr}\right)_{\text{GR}} = -\frac{(\epsilon + p)}{r^2}\frac{(m_{\text{T}} + 4\pi r^3 p)}{(1 - 2m_{\text{T}}/r)}$$
$$\left(\frac{dm_{\text{T}}}{dr}\right)_{\text{GR}} = 4\pi r^2 \epsilon$$

$$\frac{dp}{dr} = -\frac{m_{\text{T}}\rho}{r^2}\left(1 + \Pi + \frac{p}{\rho} + \frac{2m_{\text{T}}}{r} + 4\pi \frac{r^3 p}{m_{\text{T}}}\right) + \mathcal{O}(2\text{PN})$$
$$\frac{dm_{\text{T}}}{dr} = 4\pi r^2 \rho (1 + \Pi)$$

$$\frac{dp}{dr} = -\frac{\epsilon\bar{m}}{r^2}\left[1 + (5 + 3\gamma - 6\beta + \zeta_2)\frac{\bar{m}}{r} + \frac{p}{\epsilon} + \zeta_3 \frac{E}{\bar{m}}\right.$$
$$\left. + (\gamma + \zeta_4)\frac{4\pi r^3 p}{\bar{m}} + \frac{1}{2}(11 + \gamma - 12\beta + \zeta_2 - 2\zeta_4)\frac{\Omega}{\bar{m}}\right]$$
$$\frac{d\bar{m}}{dr} = 4\pi r^2 \epsilon$$

$$\frac{d\Omega}{dr} = -4\pi r \rho \bar{m}, \frac{dE}{dr} = 4\pi r^2 \rho \Pi$$

$$m(r) = \bar{m} + AE + B\Omega + C\frac{\bar{m}^2}{r} + D(4\pi r^3 p),$$

$$A = \zeta_3, B = \frac{1}{2}(11 + \gamma - 12\beta + \zeta_2 - 2\zeta_4)$$

$$\frac{dp}{dr} = -\frac{\rho m}{r^2}\left[1+\Pi+\frac{p}{\rho}+(5+3\gamma-6\beta+\zeta_2-C)\frac{m}{r} + (\gamma+\zeta_4-D)4\pi\frac{r^3 p}{m}\right]$$

$$\frac{dm}{dr} = 4\pi r^2\rho\left[1+(1+\zeta_3)\Pi+3D\frac{p}{\rho}-\frac{C}{4\pi}\frac{m^2}{\rho r^4} - \frac{1}{2}(11+\gamma-12\beta+\zeta_2-2\zeta_4-4C+2D)\frac{m}{r}\right]$$

$$D=\gamma+\zeta_4, C=\frac{1}{2}(7+3\gamma-8\beta+\zeta_2)$$

$$\frac{dp}{dr} = -\frac{\rho\tilde{m}}{r^2}\left(1+\Pi+\frac{p}{\rho}+a\frac{\tilde{m}}{r}\right)$$

$$\frac{d\tilde{m}}{dr} = 4\pi r^2\rho\left[1+(1+\zeta_3)\Pi+a\frac{\tilde{m}}{r}+3(\gamma+\zeta_4)\frac{p}{\rho} - \frac{b}{4\pi}\frac{\tilde{m}^2}{\rho r^4}\right]$$

$$a \equiv (3+3\gamma-4\beta+\zeta_2)/2$$

$$b=(7+3\gamma-8\beta+\zeta_2)/2$$

$$\tilde{m} = m_{\mathrm{T}} + \frac{m_{\mathrm{T}}^2}{r} + 4\pi r^3 p,$$

$$C=D=0.$$

$$\frac{dp}{dr} = -\frac{\rho m}{r^2}\left[1+\Pi+\frac{p}{\rho}+(5+3\gamma-6\beta+\zeta_2)\frac{m}{r} + (\gamma+\zeta_4)4\pi\frac{r^3 p}{m}\right]$$

$$\frac{dm}{dr} = 4\pi r^2\rho[1+(1+\zeta_3)\Pi - \frac{1}{2}(11+\gamma-12\beta+\zeta_2-2\zeta_4)\frac{m}{r}]$$

$$M_{\mathrm{in}} = \bar{m}(\bar{R}) + \left(\frac{17}{2}+\frac{3}{2}\gamma-10\beta+\frac{5}{2}\zeta_2\right)\Omega(\bar{R})$$

$$M_{\mathrm{a}} = M_{\mathrm{in}} + \left(4\beta-\gamma-3-\frac{1}{2}\alpha_3-\frac{1}{3}\zeta_1-2\zeta_2\right)\Omega(\bar{R}) + \zeta_3 E(\bar{R}) - \left(\frac{3}{2}\alpha_3-3\zeta_4+\zeta_1\right)P$$

$$M_{\mathrm{p}} = M_{\mathrm{in}} + \left(4\beta-\gamma-3-\alpha_1+\frac{2}{3}\alpha_2-\frac{2}{3}\zeta_1-\frac{1}{3}\zeta_2\right) \times \Omega(\bar{R})$$

$$P = 4\pi\int_0^{\bar{R}} dr r^2 p$$

$$M_{\mathrm{in}} = M_{\mathrm{a}} = M_{\mathrm{p}}$$

$$M_{\mathrm{a}} = M_{\mathrm{p}}$$

$$\zeta_3 = 0$$

$$\zeta_1 - 3\zeta_4 + \frac{3}{2}\alpha_3 = 0$$

$$\zeta_1 + 3\alpha_1 - 2\alpha_2 - 5\zeta_2 - \frac{3}{2}\alpha_3 = 0$$

$$M_{\mathrm{g}} = M_{\mathrm{a}} = M_{\mathrm{p}} = \bar{m}(\bar{R}) + F\Omega(\bar{R})$$

$$F = \frac{1}{2}\left(11 + \gamma - 12\beta - \alpha_3 + \zeta_2 - \frac{2}{3}\zeta_1\right)$$

$$m(r) = \bar{m}(r) + \frac{1}{2}(11 + \gamma - 12\beta + \zeta_2 - 2\zeta_4)\Omega(r)$$

$$M_{\mathrm{g}} = m(\bar{R}) + \left(\zeta_4 - \frac{1}{2}\alpha_3 - \frac{1}{3}\zeta_1\right)\Omega(\bar{R}) = m(\bar{R})$$

$$m(\bar{R}) \approx m(R) - \frac{dm}{dr}(R)\delta R$$

$$M_{\mathrm{g}} = m(R)$$

$$\begin{aligned}\frac{dp}{dr} = &-\frac{\rho m}{r^2}\left(1 + \Pi + \frac{p}{\rho} + \frac{2m}{r} + 4\pi\frac{r^3 p}{m}\right)\\ &-\frac{\rho m}{r^2}\left[(3 + 3\gamma - 6\beta + \zeta_2)\frac{m}{r} + (\gamma - 1 + \zeta_4)4\pi\frac{r^3 p}{m}\right]\end{aligned}$$

$$\begin{aligned}\frac{dm}{dr} &= 4\pi r^2\rho(1 + \Pi)\\ &+4\pi r^2\rho\left[\zeta_3\Pi - \frac{1}{2}(11 + \gamma - 12\beta + \zeta_2 - 2\zeta_4)\frac{m}{r}\right]\end{aligned}$$

$$\frac{dp}{dr} = -\frac{(\epsilon + p)}{r^2}\left(\frac{m + 4\pi r^3 p}{1 - 2m/r}\right) - \frac{\rho m}{r^2}\left(\delta_1\frac{m}{r} + \delta_2 4\pi\frac{r^3 p}{m}\right),$$

$$\frac{dm}{dr} = 4\pi r^2\left[\epsilon + \rho\left(\delta_3\frac{m}{r} + \delta_4\Pi\right)\right]$$

$$\delta_1 \equiv 3(1 + \gamma) - 6\beta + \zeta_2, \delta_2 \equiv \gamma - 1 + \zeta_4$$
$$\delta_3 \equiv -\frac{1}{2}(11 + \gamma - 12\beta + \zeta_2 - 2\zeta_4), \delta_4 \equiv \zeta_3$$

$$\begin{aligned}\frac{dp}{dr} &= \left(\frac{dp}{dr}\right)_{\mathrm{GR}} - \frac{\rho m}{r^2}\left(\delta_1\frac{m}{r} + \delta_2 4\pi\frac{r^3 p}{m}\right)\\ \frac{dm}{dr} &= \left(\frac{dm}{dr}\right)_{\mathrm{GR}} + 4\pi r^2\rho\left(\delta_3\frac{m}{r} + \delta_4\Pi\right)\end{aligned}$$

$$\frac{dp}{dr} = -\frac{\rho m}{r^2}\left[\left(1+\Pi+\frac{p}{\rho}\right)\left(1+\frac{2m}{r}+4\pi\frac{r^3 p}{m}\right) + \frac{4m^2}{r^2} + 8\pi r^2 p\right] + \mathcal{O}(3\text{PN})$$

$$\frac{m^2}{r^2}, \Pi\frac{m}{r}, r^2 p, \frac{mp}{r\rho}, \Pi\frac{r^3 p}{m}, \frac{r^3 p^2}{\rho m}$$

$$1\text{PN}: \Pi, \frac{p}{\rho}, \frac{m}{r}, \frac{r^3 p}{m}, \frac{m^2}{\rho r^4}$$

$$\frac{r^6 p^2}{m^2}, \Pi\frac{m^2}{\rho r^4}, \frac{m^3}{\rho r^5}, \Pi^2, \Pi\frac{p}{\rho}$$
$$\underbrace{\frac{p^2}{\rho^2}, \frac{m^4}{\rho^2 r^8}, \frac{m^2 p}{\rho^2 r^4}}$$

$$\{\blacksquare_{\text{geometry}}\} = 8\pi T^{\mu\nu}$$
$$\nabla_\nu T^{\mu\nu} = 0 \rightarrow \frac{dp}{dr} = (\epsilon + p)\{\blacksquare_{\text{geometry}}\}$$

$$\{\blacksquare_{\text{geometry}}\} \sim (\epsilon + \tau p)^n \sim \rho^n \left(1+\Pi+\tau\frac{p}{\rho}\right)^n$$

$$\text{PN term} \sim (r^2\rho)^{n-1}\left(\frac{p}{\rho}\right)^k, k = n, n-1, \ldots$$

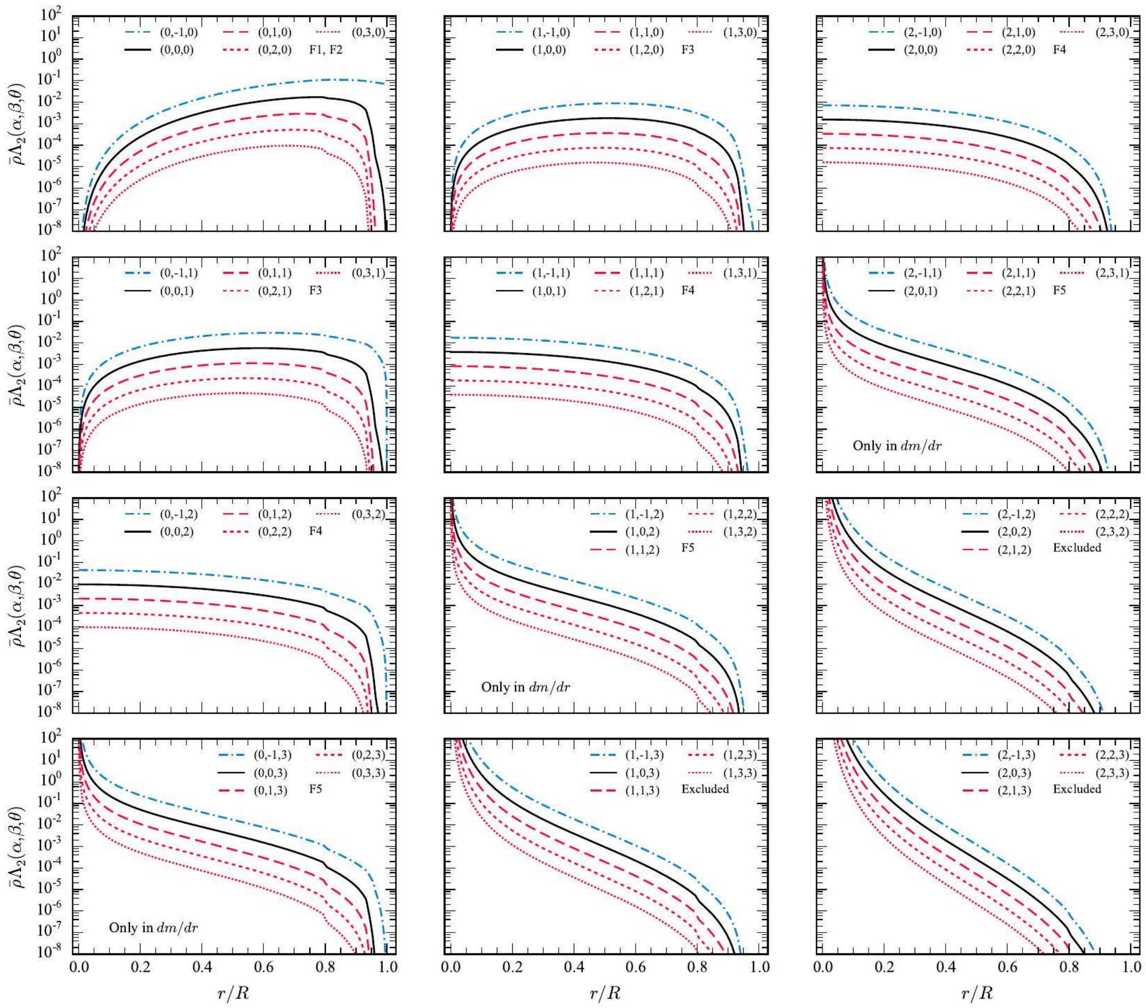
$$\text{PN term} \sim \rho^\beta, \beta \geq -1$$

$$\Lambda_2 \sim \Pi^\theta (r^2 p)^\alpha (r^2\rho)^\beta \left(\frac{m}{r}\right)^{2-2\alpha-\beta-\theta}$$

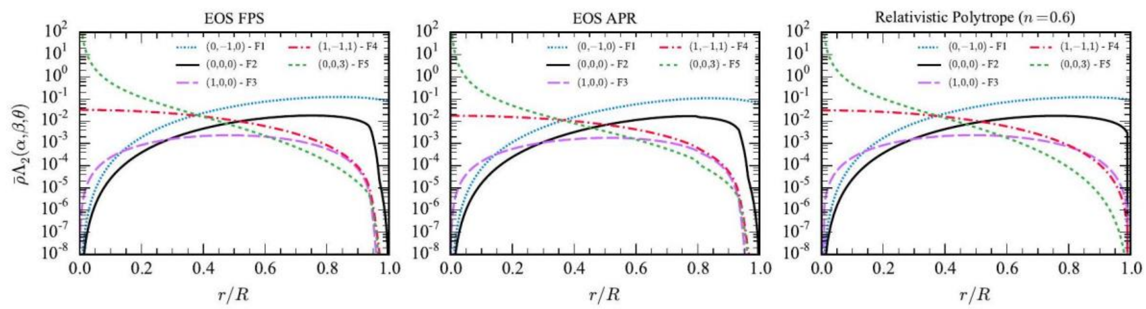
$$\beta \geq -1$$

$$\frac{dp}{dr}: \quad 0 \leq \theta \leq 2, 0 \leq \alpha \leq 2-\theta$$
$$\frac{dm}{dr}: \quad 0 \leq \theta \leq 3, 0 \leq \alpha \leq 3-\theta$$

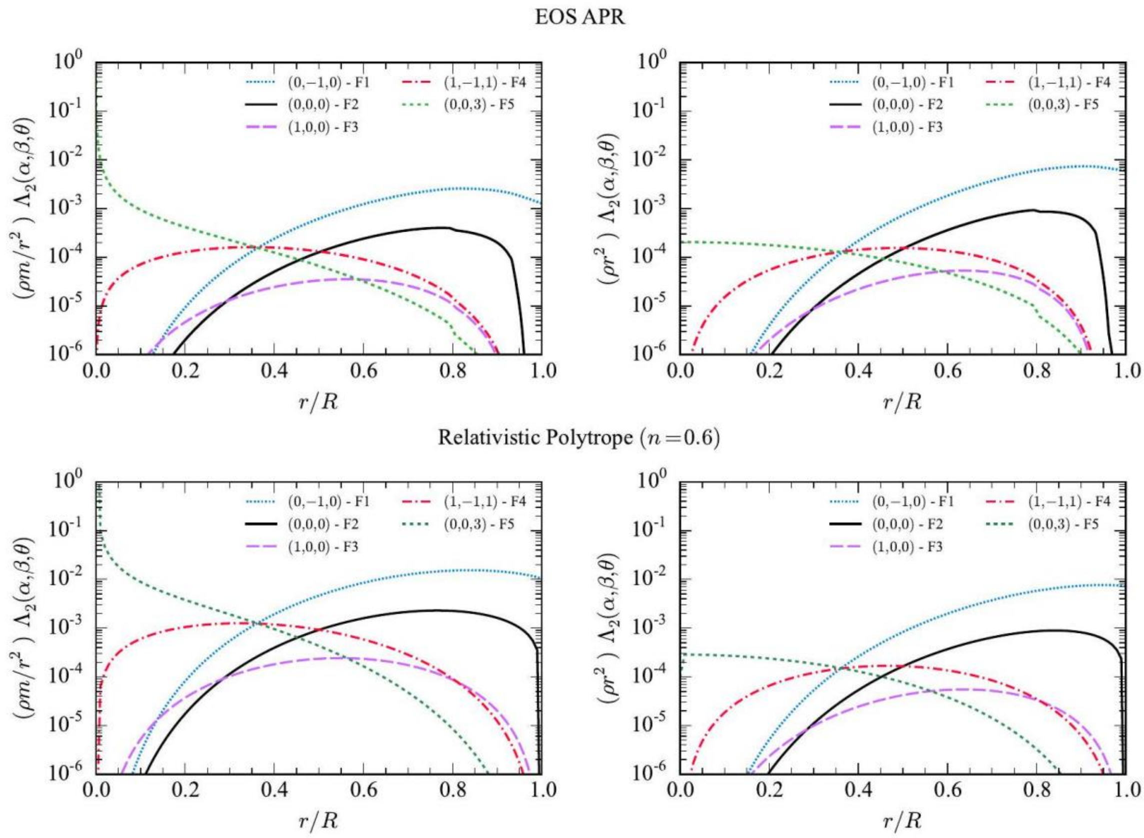
$$2\text{PN}: \frac{m^3}{r^5\rho}, \frac{m^2}{r^2}, r\rho m, \frac{mp}{r\rho}, r^2 p, \frac{r^3 p^2}{\rho m}, \frac{r^6 p^2}{m^2}$$
$$\frac{r^7 p^3}{\rho m^3}\frac{r^{10} p^3}{m^4}, \Pi\frac{m^2}{r^4\rho}, \Pi\frac{m}{r}, \Pi r^2\rho, \Pi\frac{p}{\rho}$$
$$\Pi\frac{r^3 p}{m}, \Pi\frac{r^4 p^2}{\rho m^2}, \Pi\frac{r^7 p^2}{m^3}, \Pi^2\frac{m}{\rho r^3}, \Pi^2, \Pi^2\frac{rp}{m\rho}$$
$$\Pi^2\frac{r^4 p}{m^2}, \frac{\Pi^3}{r^2\rho}, \Pi^3\frac{r}{m}$$

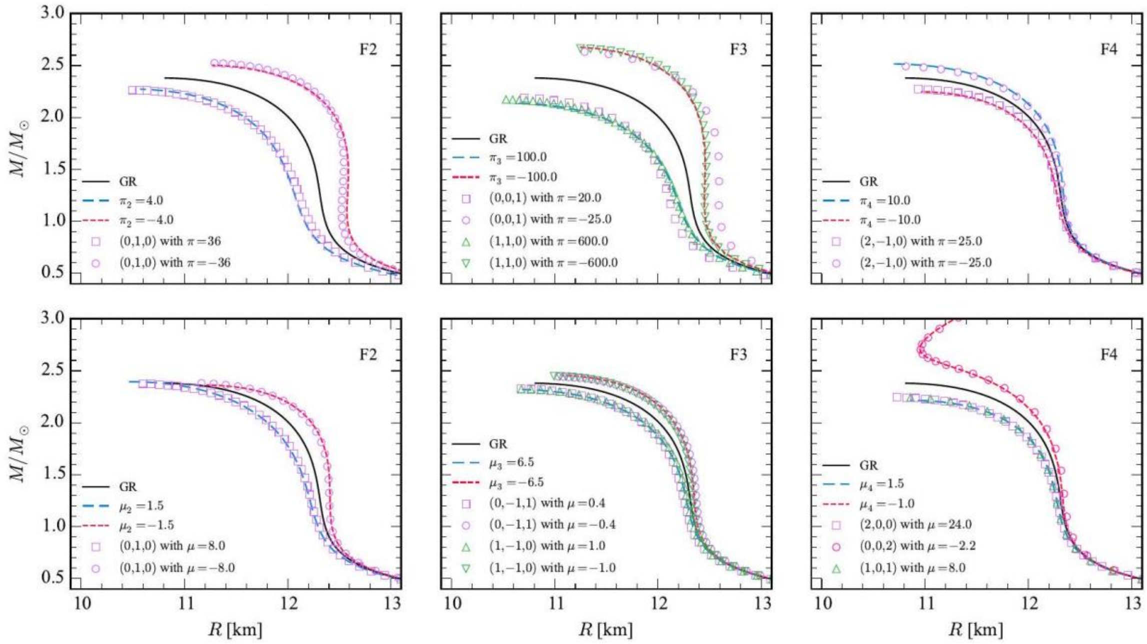


$\bar{\rho}\Lambda_2(\alpha,\beta,\theta)$
(0,-1,0) (0,1,0) (0,3,0) (0,0,0) (0,2,0) F1, F2
(1,-1,0) (1,1,0) (1,3,0) (1,0,0) (1,2,0) F3
(2,-1,0) (2,1,0) (2,3,0) (2,0,0) (2,2,0) F4
(0,-1,1) (0,1,1) (0,3,1) (0,0,1) (0,2,1) F3
(1,-1,1) (1,1,1) (1,3,1) (1,0,1) (1,2,1) F4
(2,-1,1) (2,1,1) (2,3,1) (2,0,1) (2,2,1) F5
Only in dm/dr
(0,-1,2) (0,1,2) (0,3,2) (0,0,2) (0,2,2) F4
(1,-1,2) (1,2,2) (1,0,2) (1,3,2) (1,1,2) F5
Only in dm/dr
(2,-1,2) (2,2,2) (2,0,2) (2,3,2) (2,1,2) Excluded
(0,-1,3) (0,2,3) (0,0,3) (0,3,3) (0,1,3) F5
Only in dm/dr
(1,-1,3) (1,2,3) (1,0,3) (1,3,3) (1,1,3) Excluded
(2,-1,3) (2,2,3) (2,0,3) (2,3,3) (2,1,3) Excluded
r/R



EOS FPS
EOS APR
Relativistic Polytrope ($n = 0.6$)
(0,−1,0) - F1
(1,−1,1) - F4
(0,0,0) - F2
(0,0,3) - F5
(1,0,0) - F3
$\bar{\rho}\Lambda_2(\alpha,\beta,\theta)$
r/R





Figuras complementarias 19, 20, 21 y 22. Radio de una partícula estrella o blanca.

$\epsilon_c = 0.861 \times 10^{15}$ g/nm^3($\lambda \equiv p_c/\epsilon_c = 0.165$), $M = 1.51 M_\odot$ and $R = 12.3$ nm; $\epsilon_c = 1.450 \times 10^{15}$ g/nm^3($\lambda = 0.198$), $M = 1.50 M_\odot$ and $R = 10.7$ nm; $\lambda = 0.165$, $M = 1.50 M_\odot$, $R = 11.75$ nm.

$$\bar{\rho}\Lambda_2(\alpha, \beta, \theta), \bar{\rho} = \rho/\rho_c$$

$$\epsilon_c/c^2 = 0.86 \times 10^{15}\ \mathrm{g/nm}^3$$

$$M = 1.51 M_\odot$$

$$R = 12.3\ \mathrm{nm}$$

$$\Lambda_2(0, \beta, 1) \sim r^{-1+3\beta} \frac{\Pi \rho^\beta}{m^{\beta-1}}.$$

$$\Lambda_2(0, \beta, 1) \sim r^{2+3\beta} p \left(\frac{\rho}{m}\right)^\beta \sim \Lambda_2(1, \beta, 0).$$

$$(\rho m/r^2)\Lambda_2(\alpha, \beta, \theta)\ \rho r^2 \Lambda_2(\alpha, \beta, \theta)$$

$$\mathfrak{F} : \frac{m^3}{r^5 \rho}, \frac{m^2}{r^2}, r^2 p, \Pi \frac{p}{\rho}, \Pi^3 \frac{r}{m}.$$

$$\bar{\rho}\Lambda_2(\alpha, \beta, \theta)$$

$$m\rho\Lambda_2/r^2 \text{ and } r^2 \rho \Lambda_2$$

2PN term	(α, β, θ)
$m^3/(r^5\rho)$	(0, -1, 0)
$(m/r)^2$	(0, 0, 0)
$rm\rho$	(0, 1, 0)
$mp/(r\rho)$	(1, -1, 0)
$r^2 p$	(1, 0, 0)
$\Pi m^2/(r^4\rho)$	(0, -1, 1)
$\Pi m/r$	(0, 0, 1)
$r^2 \Pi \rho$	(0, 1, 1)
$r^3 p^2/(\rho m)$	(2, -1, 0)

$r^6p^2/(m^2)$	(2, 0, 0)
$\Pi p/\rho$	(1, -1, 1)
$\Pi r^3p/m$	(1, 0, 1)
$\Pi^2m/(r^3\rho)$	(0, -1, 2)
Π^2	(0, 0, 2)
$\Pi r^4p^2/(\rho m^2)$	(2, -1, 1)
$\Pi r^7p^2/m^3$	(2, 0, 1)
$\Pi^2rp/m\rho$	(1, -1, 2)
Π^2r^4p/m^2	(1, 0, 2)
$\Pi^3/(r^2\rho)$	(0, -1, 3)
Π^3r/m	(0, 0, 3)

2PN term	(α, β, θ)
$m^3/(r^5\rho)$	$(0, -1, 0)$
$(m/r)^2$	$(0, 0, 0)$
$rm\rho$	$(0, 1, 0)$
$mp/(r\rho)$	$(1, -1, 0)$
r^2p	$(1, 0, 0)$
$\Pi m^2/(r^4\rho)$	$(0, -1, 1)$
$\Pi m/r$	$(0, 0, 1)$
$r^2\Pi\rho$	$(0, 1, 1)$
$r^3p^2/(\rho m)$	$(2, -1, 0)$
$r^6p^2/(m^2)$	$(2, 0, 0)$
$\Pi p/\rho$	$(1, -1, 1)$
$\Pi r^3p/m$	$(1, 0, 1)$
$\Pi^2 m/(r^3\rho)$	$(0, -1, 2)$
Π^2	$(0, 0, 2)$
$\Pi r^4p^2/(\rho m^2)$	$(2, -1, 1)$
$\Pi r^7p^2/m^3$	$(2, 0, 1)$
$\Pi^2 rp/m\rho$	$(1, -1, 2)$
$\Pi^2 r^4p/m^2$	$(1, 0, 2)$
$\Pi^3/(r^2\rho)$	$(0, -1, 3)$
$\Pi^3 r/m$	$(0, 0, 3)$

$$\frac{dp}{dr} = \left(\frac{dp}{dr}\right)_{\text{GR}} - \frac{\rho m}{r^2}\left(\pi_1 \frac{m^3}{r^5\rho} + \pi_2 \frac{m^2}{r^2} + \pi_3 r^2 p + \pi_4 \Pi \frac{p}{\rho}\right)$$

$$\frac{dm}{dr} = \left(\frac{dm}{dr}\right)_{\text{GR}} + 4\pi r^2\rho\left(\mu_1 \frac{m^3}{r^5\rho} + \mu_2 \frac{m^2}{r^2} + \mu_3 r^2 p + \mu_4 \Pi \frac{p}{\rho} + \mu_5 \Pi^3 \frac{r}{m}\right)$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^{\nu(r)} dt^2 + \left(1 - \frac{2\mathcal{M}(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$T^{\mu\nu} = (\mathcal{E} + \mathcal{P})u^{\mu}u^{\nu} + \mathcal{P}g^{\mu\nu}$$

$$\nabla_{\nu}T^{\mu\nu} = 0$$

$$\frac{d\mathcal{P}}{dr} = -(\mathcal{E} + \mathcal{P})\Gamma^{t}_{rt} = -\frac{1}{2}(\mathcal{E} + \mathcal{P})\frac{d\nu}{dr}$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \mathcal{E}[1 + Z(r)]$$

$$\frac{dp}{dr} = -\frac{(\epsilon + p)}{r^2}\Gamma(r) - \frac{\rho m}{r^2}\left[\left(1 + \Pi + \frac{p}{\rho}\right)\mathcal{P}_1 + \tilde{\mathcal{P}}_2\right]$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon + 4\pi r^2 \rho[\mathcal{M}_1 + \mathcal{M}_2]$$

$$\tilde{\mathcal{P}}_2 \equiv \mathcal{P}_2 - \left(\Pi + \frac{p}{\rho}\right)\mathcal{P}_1$$

$$\epsilon_{\text{eff}} \equiv \epsilon + \rho(\mathcal{M}_1 + \mathcal{M}_2)$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon_{\text{eff}}$$

$$\frac{dp}{dr} = -[\epsilon_{\text{eff}} + p - \rho(\mathcal{M}_1 + \mathcal{M}_2)]\frac{\Gamma}{r^2} - \frac{m}{r^2}\left[(\epsilon + p)\mathcal{P}_1 + (\epsilon_{\text{eff}} + p)\tilde{\mathcal{P}}_2\right]$$

$$\frac{dp}{dr} = -\frac{(\epsilon_{\text{eff}} + p)}{r^2}\left[(1 - \mathcal{M}_2)\Gamma + m\left(\mathcal{P}_1 + \tilde{\mathcal{P}}_2 - \mathcal{M}_1\mathcal{P}_1\right)\right] + \frac{\rho}{r^2}\mathcal{M}_1\Gamma$$

$$\frac{dp}{dr} \approx -\frac{(\epsilon_{\text{eff}} + p)}{r^2}\left[(1 - \mathcal{M}_2)\Gamma + m\left(\mathcal{P}_1 + \tilde{\mathcal{P}}_2 - \mathcal{M}_1\mathcal{P}_1\right)\right] \approx -\frac{(\epsilon_{\text{eff}} + p)}{r^2}[(1 - \mathcal{M}_2)\Gamma + m\mathcal{P}_2]$$

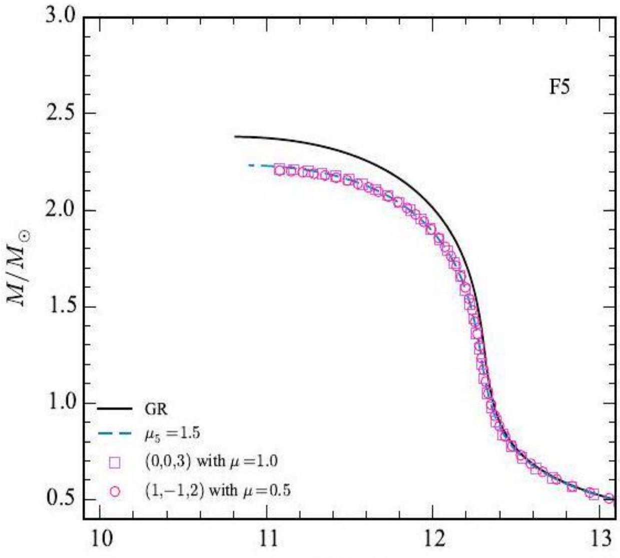
$$\mathcal{P} = p, \mathcal{M} = m, \mathcal{E} = \epsilon_{\text{eff}}$$

$$g_{\mu\nu} = \text{diag}\left[-e^{\nu(r)}, (1 - 2m(r)/r)^{-1}, r^2, r^2\sin^2\ \theta\right],$$

$$\frac{d\nu}{dr} \approx \frac{2}{r^2}\left[(1 - \mathcal{M}_2)\Gamma + m\left(\mathcal{P}_1 + \tilde{\mathcal{P}}_2 - \mathcal{M}_1\mathcal{P}_1\right)\right] \approx \frac{2}{r^2}[(1 - \mathcal{M}_2)\Gamma + m\mathcal{P}_2]$$

$$T^{\mu\nu} = (\epsilon_{\text{eff}} + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$$

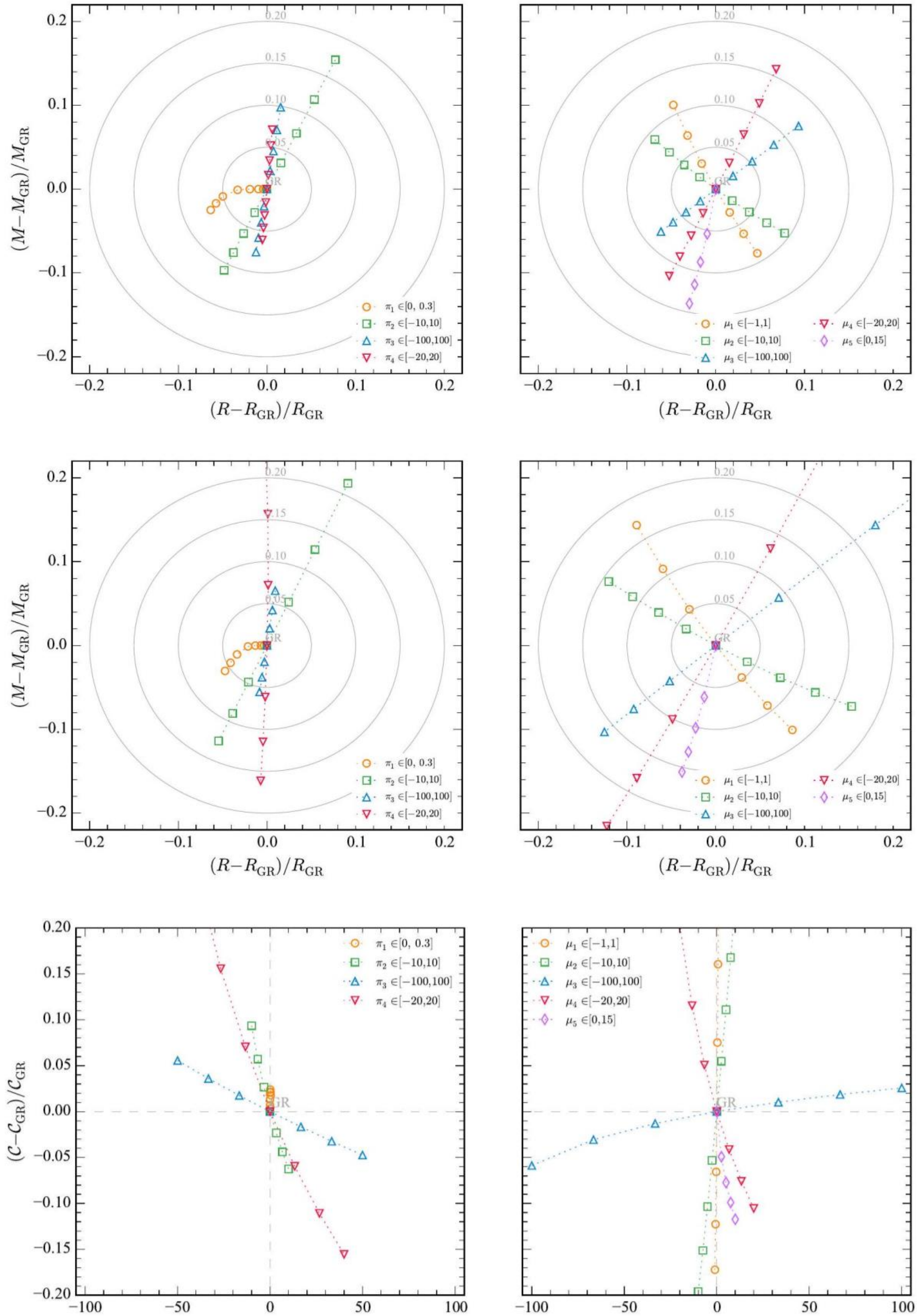
$$p(\epsilon) \to p(\epsilon_{\text{eff}}),$$
$$\epsilon_{\text{eff}} \approx \epsilon + \rho\mathcal{M}_2.$$



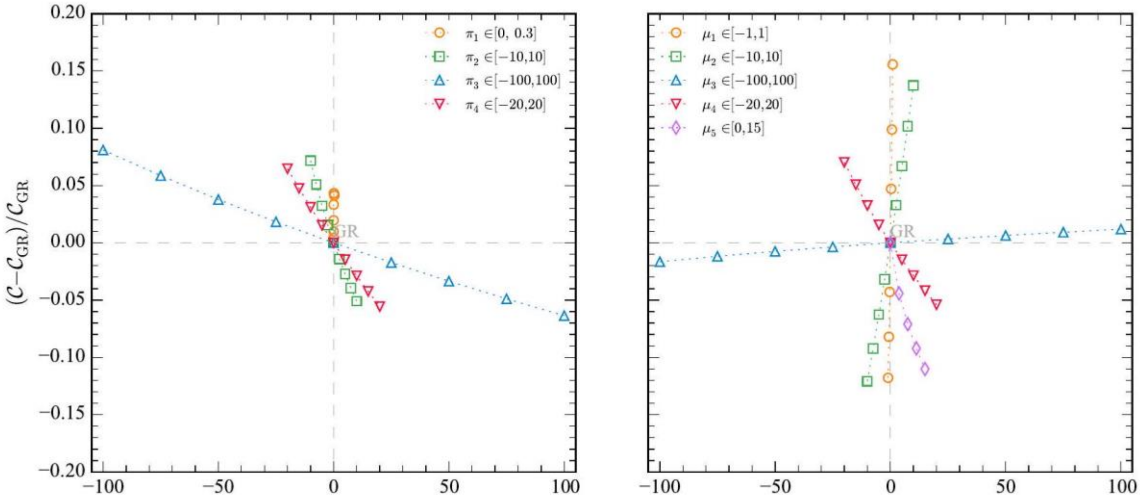
$p_c = p(\epsilon_c)$, densidad de masa central $\rho_c = m_b n_b(\epsilon_c)$ energía central interna $\Pi_c = (\epsilon_c - \rho_c)/\rho_c$,

donde $m_b = 1.66 \times 10^{-24}$ g

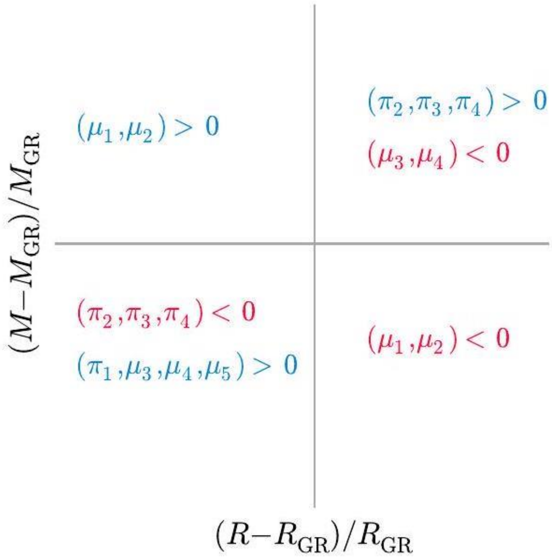
$\lambda = p_c/\epsilon_c$



$(M - M_{GR})/M_{GR}$
$(R - R_{GR})/R_{GR}$
0.05
0.10
0.15
0.20
GR
$\pi_1 \in [0, 0.3]$
$\pi_2 \in [-10,10]$
$\pi_3 \in [-100,100]$
$\pi_4 \in [-20,20]$
$\mu_1 \in [-1,1]$
$\mu_2 \in [-10,10]$
$\mu_3 \in [-100,100]$
$\mu_4 \in [-20,20]$
$\mu_5 \in [0,15]$
$(\mathcal{C} - \mathcal{C}_{GR})/\mathcal{C}_{GR}$



Top row: $\epsilon_c/c^2 = 8.61 \times 10^{14}$ g/nm^3, $M_{\rm GR} = 1.51 M_\odot$ and $R_{\rm GR} = 12.3$ nm. Bottom row: $\epsilon_c/c^2 = 1.20 \times 10^{15}$ g/nm^3, $M_{\rm GR} = 2.04 M_\odot$ and $R_{\rm GR} = 11.9$ nm.



$$\delta M/M_{\rm GR} \equiv (M - M_{\rm GR})/M_{\rm GR},\ \delta R/R_{\rm GR} \equiv (R - R_{\rm GR})/R_{\rm GR}$$

$$\sigma_i = \{\pi_i \neq \pi_1, \mu_i\}$$

$$\frac{\delta M}{M_{\rm GR}} \approx \sigma_i K_M, \frac{\delta R}{R_{\rm GR}} \approx \sigma_i K_R$$

$$\mathcal{C} \equiv M/R$$

$$\frac{\delta \mathcal{C}}{\mathcal{C}_{\rm GR}} \approx \sigma_i (K_M - K_R).$$

$$\Lambda = p^\alpha \rho^\beta m^\gamma r^\delta \Pi^\theta G^\kappa c^\lambda$$

$$p \sim G\frac{m\rho}{r}, m \sim \rho r^3$$

$$\Lambda \sim [M]^{\alpha+\beta+\gamma-\kappa}[L]^{-\alpha+\delta-3\beta+\lambda+3\kappa}[T]^{-2\alpha-\lambda-2\kappa}$$

$$\lambda = -2(\alpha+\kappa), \kappa = \alpha+\beta+\gamma$$

$$-\alpha+\delta-3\beta+\lambda+3\kappa = 0$$

$$\gamma+\delta = 2(\alpha+\beta)$$

$$\gamma+\delta = 2\beta$$

$$\Lambda \sim (r^2\rho)^\beta \left(\frac{m}{r}\right)^\gamma$$

$$\Lambda_N(\beta) \sim (r^2\rho)^\beta \left(\frac{m}{r}\right)^{N-\beta}$$

$$\Lambda_1(-1) \sim \frac{m^2}{r^4\rho}, \Lambda_1(0) \sim \frac{m}{r}, \Lambda_1(1) \sim r^2\rho$$
$$\Lambda_2(-1) \sim \frac{m^3}{r^5\rho}, \Lambda_2(0) \sim \frac{m^2}{r^2}, \Lambda_2(1) \sim r\rho m$$

$$\gamma+\delta = 2(1+\beta)$$

$$\Lambda \sim r^2p(r^2\rho)^\beta \left(\frac{m}{r}\right)^\gamma$$

$$\Lambda_N(\beta) \sim r^2p(r^2\rho)^\beta \left(\frac{m}{r}\right)^{N-2-\beta}$$

$$\Lambda_1(-1) \sim \frac{p}{\rho}, \Lambda_1(0) \sim \frac{r^3p}{m}, \Lambda_1(1) \sim \frac{r^6\rho p}{m^2}$$
$$\Lambda_2(-1) \sim \frac{pm}{\rho r}, \Lambda_2(0) \sim r^2p, \Lambda_2(1) \sim \frac{r^5\rho p}{m}$$

$$\Lambda_N(\beta) = (r^2p)^2(r^2\rho)^\beta \left(\frac{m}{r}\right)^{N-4-\beta}$$

$$\Lambda_1(-1) \sim \frac{r^4p^2}{m^2\rho}, \quad \Lambda_1(0) \sim \frac{r^7p^2}{m^3}$$
$$\Lambda_2(-1) \sim \frac{r^3p^2}{\rho m}, \quad \Lambda_2(0) \sim \frac{r^6p^2}{m^2}$$

$$\Lambda_N(\alpha,\beta,\theta) \sim \Pi^\theta(r^2p)^\alpha(r^2\rho)^\beta \left(\frac{m}{r}\right)^{N-2\alpha-\beta-\theta}$$

$$\rho\Lambda_N(0,\beta,0) \sim \rho^{1+\beta}$$

$$\sim (\epsilon+\tau p)^n = \rho^n(1+\Pi+\tau p/\rho)^n \sim \rho^{n-1}(p/\rho)^k, k = n, n-1, \dots \rho^{-1}\,\Lambda_N(\alpha,\beta,\theta)$$

$$\Lambda_N(\alpha,\beta,\theta)$$

$$\Lambda_N(r\to 0)\sim r^{2(N-\alpha-\theta)}$$

$$\frac{dp}{dr}\sim\frac{\rho m}{r^2}\Lambda_N\sim r^{2(N-\alpha-\theta)+1}$$
$$\frac{dm}{dr}\sim r^2\rho\Lambda_N\sim r^{2(N-\alpha-\theta+1)}$$

$$\frac{dp}{dr}:\quad 0\le\alpha\le N-\theta$$
$$\frac{dm}{dr}:\quad 0\le\alpha\le N+1-\theta$$

$$\frac{dp}{dr}:0\le\theta\le N,\frac{dm}{dr}:0\le\theta\le N+1$$

$$\frac{dp}{dr}:0\le\theta\le 2,0\le\alpha\le 2-\theta$$
$$\frac{dm}{dr}:0\le\theta\le 3,0\le\alpha\le 3-\theta$$

$$\theta^n\equiv\frac{\rho}{\rho_c},p=K\rho_c^{1+1/n}\theta^{n+1}$$

$$r=\alpha\xi,\alpha\equiv\left[\frac{(n+1)K}{4\pi G}\rho_c^{-1+1/n}\right]^{1/2}$$

$$\frac{dp}{dr}=-\frac{Gm_N}{r^2}\rho$$
$$\frac{dm_N}{dr}=4\pi r^2\rho$$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right)=-\theta^n$$

$$\Gamma=1+\frac{1}{n}=\frac{\rho}{p}\frac{dp}{d\rho}=\frac{\epsilon+p}{p}\frac{dp}{d\epsilon}$$

$$\epsilon=\rho+np$$

$$\Pi=n\frac{p}{\rho}$$

$$\rho=\rho_c\theta^n,r=\alpha\xi,p=K\rho_c^{1+1/n}\theta^{n+1}$$

$$\lambda\equiv\frac{p_c}{\epsilon_c}=\frac{K\rho_c^{1+1/n}}{\rho_c+nK\rho_c^{1+1/n}}$$

$$\begin{aligned}\epsilon&=\rho_c\theta^n+nK\rho_c^{1+1/n}\theta^{n+1}\\&=\epsilon_c[1+n\lambda(\theta-1)]\theta^n\end{aligned}$$

$$\frac{dm_{\mathrm{T}}}{d\xi} = 4\pi\epsilon_{\mathrm{c}}a^3[1 + n\lambda(\theta - 1)]\theta^n\xi^2$$

$$\bar{m} \equiv \frac{m_{\mathrm{T}}}{a^3\epsilon_c},$$

$$\frac{d\bar{m}}{d\xi} = 4\pi[1 + n\lambda(\theta - 1)]\theta^n\xi^2$$

$$\frac{d\theta}{d\xi} = -\frac{\bar{m}}{\xi^2}(1 - n\lambda)\left[1 + (n + 1)\frac{\lambda}{1 - n\lambda}\theta\right] \times \left(1 + \lambda\frac{4\pi\xi^3\theta^{n+1}}{\bar{m}}\right)\left[1 - 2(n + 1)\lambda\frac{\bar{m}}{\xi}\right]^{-1}$$

$$a = \left[(n + 1)K\rho_{\mathrm{c}}^{-1+1/n}(1 - n\lambda)^2\right]^{1/2}$$

$$\rho_{\mathrm{c}} = K^{-n}\ell^n, \ell \equiv \frac{\lambda}{1 - n\lambda}$$

$$\bar{\rho} \equiv \rho K^n = \ell^n\theta^n$$

$$a = K^{n/2}\sqrt{(n + 1)\ell^{1-n}}(1 - n\lambda),$$

$$\bar{r} \equiv rK^{-n/2} = \sqrt{(n + 1)\ell^{1-n}}(1 - n\lambda)\xi.$$

$$\bar{\epsilon} \equiv \epsilon K^n = \left(\frac{\ell^n}{1 - n\lambda}\right)[1 + n\lambda(\theta - 1)]\theta^n$$

$$\bar{\mu} \equiv m_{\mathrm{T}}K^{-n/2} = \left[\sqrt{(n + 1)\ell^{1-n}}(1 - n\lambda)\right]^3\left(\frac{\ell^n}{1 - n\lambda}\right)\bar{m}$$

$$\bar{p} \equiv pK^n = \ell^{n+1}\theta^{n+1}$$

$$\Pi = n\frac{\bar{p}}{\bar{\rho}} = n\ell\theta$$

$$U(r) = -\int_0^r dr'\frac{m_{\mathrm{N}}}{r'^2} + U(0)$$

$$E(r) = 4\pi\int_0^r dr'r'^2\rho\Pi$$

$$\Omega(r) = -4\pi\int_0^r dr'r'\rho m_{\mathrm{N}}$$

$$m_{\mathrm{N}}(r) = 4\pi\int_0^r dr'\rho r'^2 = 4\pi m_{\mathrm{b}}\int_0^r dr'n_{\mathrm{b}}r'^2$$

$$U = \frac{m_{\mathrm{N}}}{r} + 4\pi \int_r^R dr'r'\rho$$
$$\frac{E}{m_{\mathrm{N}}} = \Pi - \frac{1}{m_{\mathrm{N}}} \int_0^r dr' m_{\mathrm{N}} \frac{d\Pi}{dr'}$$
$$\frac{\Omega}{m_{\mathrm{N}}} = -\frac{m_{\mathrm{N}}}{2r} - \frac{1}{2m_{\mathrm{N}}} \int_0^r dr' \left(\frac{m_{\mathrm{N}}}{r'}\right)^2$$
$$= 4\pi \frac{r^3 p}{m_{\mathrm{N}}} - \frac{12\pi}{m_{\mathrm{N}}} \int_0^r dr' r'^2 p$$

$$U(0) = 4\pi \int_0^R dr r\rho, \frac{\Omega}{m_{\mathrm{N}}}(0) = 0, \frac{E}{m_{\mathrm{N}}}(0) = \Pi_{\mathrm{c}}$$

$$\frac{m_{\mathrm{N}}}{r}, \Pi, \frac{r^3 p}{m_{\mathrm{N}}}$$

$$U = (n+1)\left(\frac{p}{\rho} - \frac{p_{\mathrm{c}}}{\rho_{\mathrm{c}}}\right) + U(0).$$

$$U = \frac{(n+1)}{n}(\Pi - \Pi_{\mathrm{c}}) + U(0)$$

$$|\delta_1| \lesssim 6 \times 10^{-4}, |\delta_2| \lesssim 7 \times 10^{-3}, |\delta_3| \lesssim 7 \times 10^{-3}, |\delta_4| \lesssim 10^{-8}$$

$dp/dr = (dp/dr)_{\mathrm{GR}} - \rho m \pi_i f_i(r)/r^2$, with $f_i(r) > 0$, $dp_r/dr = (dp_r/dr)_{\mathrm{GR}} - 2\sigma/r$, with $\sigma =$

$p_r - p_q$, $2r\sigma = \rho m \pi_i f_i(r)$.

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$$

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} + f(G)\right] + S_m$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + 8\big[R_{\mu\rho\nu\sigma} + R_{\rho\nu}g_{\sigma\mu} - R_{\rho\sigma}g_{\nu\mu} - R_{\mu\nu}g_{\sigma\rho} + R_{\mu\sigma}g_{\nu\rho} + \frac{R}{2}(g_{\mu\nu}g_{\sigma\rho} - g_{\mu\sigma}g_{\nu\rho})\big]\nabla^\rho\nabla^\sigma f_G + (Gf_G - f)g_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

$$f_{GG\ldots} = \frac{d^n f(G)}{dG^n}.$$

$$\nabla_\mu V_\nu = \partial_\mu V_\nu - \Gamma^\lambda_{\mu\nu}V_\lambda, R^\sigma_{\mu\nu\rho} = \partial_\nu\Gamma^\sigma_{\mu\rho} - \partial_\rho\Gamma^\sigma_{\mu\nu} + \Gamma^\omega_{\mu\rho}\Gamma^\sigma_{\omega\nu} - \Gamma^\omega_{\mu\nu}\Gamma^\sigma_{\omega\rho}$$

$$\nabla^\mu T_{\mu\nu} = 0.$$

$$ds^2 = c^2 e^{2\phi}dt^2 - e^{2\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$T^\nu_\mu = \mathrm{diag}(\rho c^2, -p, -p, -p)$$

$$-\frac{1}{r^2}\left(2r\lambda' + e^{2\lambda} - 1\right) + 8e^{-2\lambda}\left(f_{GG}(G'' - 2\lambda' G') + f_{GGG}(G')^2\right)\langle\frac{1-e^{2\lambda}}{r^2} - 2(\phi'' + \phi'^2)\rangle$$
$$+(Gf_G - f)e^{2\lambda} = \kappa^2 \rho c^2 e^{2\lambda}$$
$$-\frac{1}{r^2}\left(2r\phi' - e^{2\lambda} + 1\right) - (Gf_G - f)e^{2\lambda} = \kappa^2 p e^{2\lambda}$$

$$R + 8G_{\rho\sigma}\nabla^\rho\nabla^\sigma f_G - 4(Gf_G - f) = -\kappa^2(\rho c^2 - p)$$

$$2\left(\phi'' + \phi'^2 - \phi'\lambda' + \frac{2}{r}(\phi' - \lambda') + \frac{1-e^{2\lambda}}{r^2}\right)$$
$$+8e^{-2\lambda}\left(\frac{2\phi'}{r} + \frac{1-e^{2\lambda}}{r^2}\right)\left(f_{GG}(G'' - 2\lambda' G') + f_{GGG}(G')^2\right) + 4(Gf_G - f)e^{2\lambda} = \kappa^2 e^{2\lambda}(\rho c^2 - 3p)$$

$$\frac{dp}{dr} = -(p + \rho c^2)\phi'$$

$$e^{-2\lambda} = 1 - \frac{2GM}{c^2 r} \Longrightarrow \frac{GdM}{c^2 dr} = \frac{1}{2}\left[1 - e^{-2\lambda}(1 - 2r\lambda')\right]$$

$$\left\{\frac{dp}{dr}, \frac{dM}{dr}, \rho\right\}$$

$$M \to mM_\star, r \to r_g r, \rho \to \frac{\rho M_\star}{r_g^3}, p \to \frac{pM_\star c^2}{r_g^3}, G \to \frac{G}{r_g^4}.$$

$$r_g = \frac{GM_\star}{c^2} = 1.47473(nm)$$

$$\frac{dp}{dr} = -(p + \rho)\phi'$$

$$\frac{d\lambda}{dr} = \frac{m}{r^3}\frac{1 - \frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r} - 1}$$

$$\frac{2}{r}\frac{1 - \frac{2m}{r}}{p + \rho c^2} + \frac{2m}{r^3} - r_g^2(Gf_G - f) = 8\pi p$$

$$-\frac{2}{r^2}\frac{dm}{dr} + 8r_g^2\left(1 - \frac{2m}{r}\right)^2\left[f_{GG}\left(r_g^2 G'' - r_g^3\frac{2m}{r^2}\frac{1 - r\frac{dm}{dr}}{\frac{2m}{r} - 1}G'\right) + r_g^2 f_{GGG}G^2\right]$$
$$\times\left[-\frac{2m/r^3}{1 - \frac{2m}{r}} + 2\frac{d}{dr}\left(\frac{\frac{dp}{dr}}{p + \rho}\right) - 2\left(\frac{\frac{dp}{dr}}{p + \rho}\right)^2\right] + (Gf_G - f) = 8\pi\rho$$

$$2\left(1 - \frac{2m}{r}\right)\left(-\frac{d}{dr}\left(\frac{\frac{dp}{dr}}{p + \rho}\right) + \left(\frac{\frac{dp}{dr}}{p + \rho}\right)^2 + \frac{m}{r^3}\frac{1 - \frac{r}{m}\frac{dm}{dr}}{\frac{2m}{r} - 1}\left(\frac{\frac{dp}{dr}}{p + \rho}\right) - \frac{2m/r^3}{1 - \frac{2m}{r}}\right)$$
$$+8\left[-\frac{2\frac{dp}{dr}}{r(p + \rho)} - \frac{2m/r^3}{1 - \frac{2m}{r}}\right]\left[f_{GG}\left(r_g^2 G'' - r_g^3\frac{2m}{r^2}\frac{1 - r\frac{dm}{dr}}{\frac{2m}{r} - 1}G'\right) + r_g^2 f_{GGG}G^2\right]$$
$$4(Gf_G - f) = 8\pi(\rho - 3p)$$

$$\Gamma^1_{12} = \phi', \Gamma^2_{11} = \phi' e^{2\phi - 2\lambda}, \Gamma^2_{22} = \lambda', \Gamma^2_{23} = -re^{-2\lambda},$$
$$\Gamma^2_{44} = -r\sin^2\theta e^{-2\lambda}, \Gamma^3_{23} = \frac{1}{r}, \Gamma^3_{44} = -\sin\theta\cos\theta, \Gamma^4_{24} = \frac{1}{r}, \Gamma^4_{34} = \cot\theta.$$

$$G_{11} = -e^{2\phi-2\lambda}\left(\frac{2\lambda'}{r} + \frac{e^{2\lambda}-1}{r^2}\right)$$
$$G_{22} = -\frac{2\phi'}{r} + \frac{e^{2\lambda}-1}{r^2}$$
$$G_{33} = -re^{-2\lambda}(\phi' - \lambda' + r(\phi'' + \phi'^2) - r\phi'\lambda')$$
$$G_{44} = -\sin^2\ \theta G_{33}$$

$$R_{1212} = e^{2\phi}(\phi'\lambda' - \phi'' - \phi'^2), R_{1313} = -re^{2\phi-2\lambda}\phi', R_{1414} = \sin^2\ \theta R_{1313}$$
$$R_{2323} = -r\lambda', R_{2424} = \sin^2\ \theta R_{2323}, R_{3434} = \sin^2\ \theta r^2 e^{-2\lambda}(1 - e^{2\lambda})$$

$$R = -2e^{-2\lambda}\left(\phi'' + \phi'^2 - \phi'\lambda' + 2\frac{\phi'-\lambda'}{r} + \frac{1-e^{2\lambda}}{r^2}\right)$$

$$\frac{-e^{4\lambda}G}{8} = \frac{1}{r^2}\left[(\phi'' + \phi'^2 - \lambda'\phi')(e^{2\lambda} - 1) + 2\phi'\lambda'\right]$$

$$\nabla_\mu T_2^\mu = \partial_\mu T_2^\mu + \Gamma_{\mu\sigma}^\mu T_2^\sigma - \Gamma_{\mu 2}^\sigma T_\sigma^\mu = 0, \partial_\mu(-p\delta^{\mu,2}) - p\Gamma_{\mu 2}^\mu + p\Gamma_{a2}^a - \rho c^2\Gamma_{12}^1 = 0$$
$$-\frac{dp}{dr} - (p + \rho c^2)\Gamma_{12}^1 = 0 \Longrightarrow \frac{dp}{dr} = -(p + \rho c^2)\phi'$$

$$M(r) = 4\pi\int_0^r r'^2\rho(r')dr'$$

$$\frac{dp}{dr}\left[\frac{1}{\kappa_G} - \frac{M(r)c^2}{4\pi r}\right] = -\frac{p+\rho c^2}{2}\left[pr + \frac{M(r)c^2}{4\pi r^2}\right].$$

$$\kappa_G = 8\pi G/c^4$$

$$H_{jkl} = \kappa_G T_{jkl}$$
$$H_{jkl} = \nabla_j R_{kl} + \nabla_k R_{lj} + \nabla_l R_{jk}$$
$$-\frac{1}{3}(g_{kl}\nabla_j R + g_{lj}\nabla_k R + g_{jk}\nabla_l R)$$
$$T_{jkl} = \nabla_j T_{kl} + \nabla_k T_{lj} + \nabla_l T_{jk}$$
$$-\frac{1}{6}(g_{kl}\nabla_j T + g_{lj}\nabla_k T + g_{jk}\nabla_l T)$$

$$\nabla_j R^j{}_k = \frac{1}{2}\nabla_k R$$

$$R_{kl} - \frac{1}{2}Rg_{kl} = \kappa_G(T_{kl} + K_{kl})$$
$$\nabla_j K_{kl} + \nabla_k K_{jl} + \nabla_l K_{jk}$$
$$= \frac{1}{6}(g_{kl}\nabla_j K + g_{jl}\nabla_k K + g_{jk}\nabla_l K).$$

$$ds^2 = -b^2(r,t)dt^2 + f_1^2(r,t)dr^2 + r^2 d\Omega_2^2$$

$$ds^2 = -y(r)dt^2 + \frac{h(r)}{y(r)}dr^2 + r^2 d\Omega_2^2$$

$$\nabla_i u_j = -u_i\dot{u}_j, \nabla_i\dot{u}_j = \nabla_j\dot{u}_i$$

$$u_0 = -\sqrt{y}, u_\mu = 0.$$

$$\dot{u}_0 = 0, \dot{u}_r = y'/(2y), \dot{u}_\theta = \dot{u}_\phi = 0$$

$$\eta = \dot{u}^k \dot{u}_k = y'^2/(4hy).$$

$$\chi_k = \dot{u}_k/\sqrt{\eta}, \chi_r = \sqrt{h/y}$$

$$R_{kl} = \frac{R+4\nabla_p \dot{u}^p}{3} u_k u_l + \frac{R+\nabla_p \dot{u}^p}{3} g_{kl} + \Sigma(r)\left[\chi_k \chi_l - \frac{u_k u_l + g_{kl}}{3}\right]$$

$$R = R^\star - 2\nabla_p \dot{u}^p$$
$$\frac{R^\star}{2} = \frac{1}{r^2} + \frac{y}{r}\frac{h'}{h^2} - \frac{y'}{rh} - \frac{y}{r^2 h}$$
$$\nabla_p \dot{u}^p = \frac{y''}{2h} - \frac{y'}{4}\frac{h'}{h^2} + \frac{y'}{rh}$$
$$\Sigma = -\frac{y''}{2h} + \frac{y'}{4}\frac{h'}{h^2} + \frac{y}{r^2 h} + \frac{y}{2r}\frac{h'}{h^2} - \frac{1}{r^2}$$

$$\frac{R^\star}{2} + \nabla_p \dot{u}^p + \Sigma = \frac{3y}{2r}\frac{h'}{h^2}$$

$$K_{kl} = \mathrm{A}(r) u_k u_l + \mathrm{B}(r) g_{kl} + \mathrm{C}(r) \chi_k \chi_l$$

$$\mathrm{A}(r) = \kappa_2 r^2 - 2\kappa_3 y(r)$$
$$\mathrm{B}(r) = \kappa_1 + 2\kappa_2 r^2 + \kappa_3 y(r)$$
$$\mathrm{C}(r) = -\kappa_2 r^2$$

$$T_{kl} = (\mu_m + P_m) u_k u_l + P_m g_{kl} + (p_{mr} - p_{m\perp})\left[\chi_k \chi_l - \frac{g_{kl} + u_k u_l}{3}\right]$$

$$P_m = \frac{1}{3}(p_{mr} + 2p_{m\perp})$$

$$K_{kl} = (\mu_d + P_d) u_k u_l + P_d g_{kl} + (p_{dr} - p_{d\perp})\left[\chi_k \chi_l - \frac{g_{kl} + u_k u_l}{3}\right]$$

$$P_d = \frac{1}{3}(p_{dr} + 2p_{d\perp})$$

$$\mu_d = \mathrm{A} - \mathrm{B} = -\kappa_1 - \kappa_2 r^2 - 3\kappa_3 y(r)$$
$$p_{dr} = \mathrm{B} + \mathrm{C} = \kappa_1 + \kappa_2 r^2 + \kappa_3 y(r)$$
$$p_{d\perp} = \mathrm{B} = \kappa_1 + 2\kappa_2 r^2 + \kappa_3 y(r)$$

$$\mu_d(r) + 5p_{dr}(r) - 2p_{d\perp}(r) = 2\kappa_1$$

$$R_{kl} - \frac{1}{2} R g_{kl} = T_{kl} + K_{kl}$$

$$\frac{1}{2} R^\star = \mu_m + \mu_d$$
$$\nabla_p \dot{u}^p = \frac{3}{2}(P_m + P_d) + \frac{1}{2}(\mu_m + \mu_d)$$
$$\Sigma = (p_{mr} - p_{m\perp}) + (p_{dr} - p_{d\perp})$$

$$\frac{1}{2} R^\star + \nabla_p \dot{u}^p + \Sigma = \frac{3}{2}(\mu_m + \mu_d + p_{mr} + p_{dr}), \mu_d + p_{dr} = -2\kappa_3 r$$

$$\mu_m + p_{mr} = \frac{y(r)}{r}\frac{d}{dr}\left[\kappa_3 r^2 - \frac{1}{h(r)}\right]$$

$$\frac{dp_{mr}}{dr} = -\frac{y'}{2y}(\mu_m + p_{mr}) - \frac{2}{r}(p_{mr} - p_{m\perp}).$$

$$p_{mr} = p_{m\perp} \equiv p_m$$

$$\frac{1}{2}R^\star = -\kappa_1 - \kappa_2 r^2 - 3\kappa_3 y(r)$$
$$\nabla_p \dot{u}^p = \kappa_1 + 2\kappa_2 r^2$$
$$\Sigma = -\kappa_2 r^2$$

$$h(r) = \frac{1}{\kappa_3 r^2 + \kappa_4}$$

$$\frac{y'}{r}(\kappa_3 r^2 + \kappa_4) + \frac{y}{r^2}\kappa_4 = \kappa_1 + \kappa_2 r^2 + \frac{1}{r^2}$$
$$y''(\kappa_3 r^2 + \kappa_4) + \frac{y'}{r}(3\kappa_3 r^2 + 2\kappa_4) = 4\kappa_2 r^2 + 2\kappa_1$$
$$y''(\kappa_3 r^2 + \kappa_4) + y'\kappa_3 r - 2\frac{y}{r^2}\kappa_4 = 2\kappa_2 r^2 - \frac{2}{r^2}$$

$$\frac{y'}{r} + \frac{y}{r^2} = \kappa_1 + \kappa_2 r^2 + \frac{1}{r^2}$$

$$C = -2\bar{M}, \kappa_1 = -\Lambda, \kappa_2 = -\lambda$$

$$y(r) = 1 - \frac{2\bar{M}}{r} - \frac{\Lambda}{3}r^2 - \frac{\lambda}{5}r^4$$

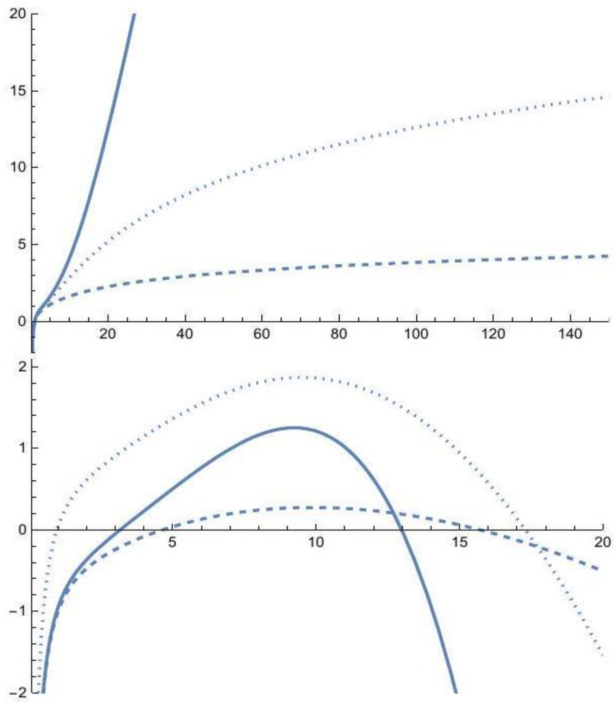
$$y' = \frac{\kappa_1}{\kappa_3}\frac{1}{r} + \frac{\kappa_2}{\kappa_3}r + \frac{1}{\kappa_3}\frac{1}{r^3}$$

$$y(r) = \frac{\kappa_1}{\kappa_3}\log\left(r\sqrt{\kappa_1}\right) + \frac{\kappa_2}{2\kappa_3}r^2 - \frac{1}{2\kappa_3}\frac{1}{r^2} + \aleph$$

$$y(r) = C\frac{\sqrt{\kappa_3 r^2 + \kappa_4}}{r} + \frac{1}{\kappa_4} + \frac{\kappa_2}{2\kappa_3}r^2$$
$$+\left[\frac{\kappa_1}{\kappa_3} - \frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right]\left[\frac{\sqrt{\kappa_3 r^2 + \kappa_4}}{r\sqrt{|\kappa_3|}}F\left(r\sqrt{\frac{|\kappa_3|}{|\kappa_4|}}\right) - 1\right]$$
$$F(x) = \begin{cases} \mathrm{ArcSinh}x & \kappa_3 > 0, \kappa_4 > 0 \\ \mathrm{ArcCosh}x & \kappa_3 > 0, \kappa_4 < 0 \\ \mathrm{ArcSin}x & \kappa_3 < 0, \kappa_4 > 0 \end{cases}$$

$$r^2 \gg \frac{|\kappa_4|}{\kappa_3}$$

$$y(r) \approx \frac{\kappa_2}{2\kappa_3}r^2 + \left[\frac{\kappa_1}{\kappa_3} - \frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right]\log\left(2r\sqrt{\frac{\kappa_3}{|\kappa_4|}}\right)$$
$$+\left[\frac{1}{\kappa_4} - \frac{\kappa_1}{\kappa_3} + \frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right] + \cdots$$



$$C = -1, \kappa_1 = 0.1, \kappa_2 = 0, \kappa_2 = -0.001, \kappa_4 = 1, \kappa_3 = \frac{1}{1000}, \kappa_3 = \frac{1}{50}, \kappa_3 = \frac{1}{10}$$

$$ds^2 = -b^2(r)dt^2 + \left[1 - \frac{2M(r)}{r}\right]^{-1} dr^2 + r^2 d\Omega_2^2$$

$$y/h = 1 - 2M(r)/r$$

$$R^\star = \frac{4M'}{r^2}$$

$$\Sigma = -\left[\frac{b''}{b} - \frac{b'}{br}\right]\left[1 - \frac{2M(r)}{r}\right] + \frac{b'}{b}\left[\frac{M'r - M}{r^2}\right] - \frac{3M - rM'}{r^3}$$

$$\nabla_p \dot{u}^p = \left[\frac{b''}{b} + \frac{2b'}{br}\right]\left[1 - \frac{2M(r)}{r}\right] - \frac{b'}{b}\left[\frac{M'r - M}{r^2}\right]$$

$$M(r) = \frac{1}{2}\int_0^r dr' r^2 [\mu_m(r') + \mu_d(r')]$$

$$\left[p'_{mr} + 2\frac{p_{mr} - p_{m\perp}}{r}\right]\left[1 - \frac{2M(r)}{r}\right] = -\frac{\mu_m + p_{mr}}{2}\left[r(p_m + p_{dr}) + \frac{2M(r)}{r^2}\right]$$

$$\nabla_p \dot{u}^p + \Sigma - \frac{R^\star}{4} = \frac{3b'}{br}\left[1 - \frac{2M(r)}{r}\right] - \frac{3M(r)}{r^3}$$

$$\nabla_p \dot{u}^p + \Sigma - \frac{R^\star}{4} = \frac{3}{2}(p_{mr} + p_{dr})$$

$$b'/b = y'/2y$$

$$\mu_d = p_{dr} = 0$$

$$p_{mr} = p_{m\perp} = p_m$$

$$ds_-^2 = -b_-^2(r)dt^2 + \frac{dr^2}{1-\frac{2M(r)}{r}} + r^2 d\Omega_2^2 \qquad r < R$$
$$ds_+^2 = -b_+^2(r)dt^2 + f_{1+}^2(r)dr^2 + r^2 d\Omega_2^2 \quad r > R$$

$$b_-^2(R) = b_+^2(R)$$
$$1 - \frac{2M(R)}{R} = \frac{1}{f_{1+}^2(R)}$$
$$p_{mr}(R) = p_{dr}^+(R) - p_{dr}^-(R)$$

$$u_0^\pm(r) = -b_\pm(r)$$

$$u_k^-(R) = u_k^+(R) \equiv u_k(R)$$

$$\chi_r^\pm(r) = f_1^\pm(r)$$

$$\chi_k^-(R) = \chi_k^+(R) \equiv \chi_k(R)$$

$$K_{kl}^\pm(r) = \mathrm{A}^\pm(r)u_k^\pm(r)u_l^\pm(r) + \mathrm{B}^\pm(r)g_{kl}^\pm(r) + \mathrm{C}^\pm(r)\chi_k^\pm(r)\chi_l^\pm(r)$$

$$\delta K_{kl}(R) = K_{kl}^+(R) - K_{kl}^-(R) = \delta\mathrm{A}(R)u_k(R)u_l(R) + \delta\mathrm{B}(R)g_{kl}(R) + \delta\mathrm{C}(R)\chi_k(R)\chi_l(R)$$

$$\delta\mathrm{A}(R) = (\kappa_2^+ - \kappa_2^-)R^2 - 2(\kappa_3^+ - \kappa_3^-)b^2(R)$$
$$\delta\mathrm{B}(R) = (\kappa_1^+ - \kappa_1^-) + 2(\kappa_2^+ - \kappa_2^-)R^2 + (\kappa_3^+ - \kappa_3^-)b^2(R)$$
$$\delta\mathrm{C}(R) = -(\kappa_2^+ - \kappa_2^-)R^2$$

$$b^2(R) = b_\pm^2(R)$$

$$\nabla_k^- \chi_l^-(R) = \nabla_k^+ \chi_l^+(R)$$

$$\delta R_{jklm} = \chi_j\chi_l B_{km} - \chi_j\chi_m B_{kl} + \chi_k\chi_m B_{jl} - \chi_k\chi_l B_{jm}$$

$$\delta R_{kl} = B_{kl} + \chi_k\chi_l B,\ B = g^{kl}B_{kl},\ \delta R = 2B$$

$$\delta G_{kl} = B_{kl} + B(g_{kl} - \chi_k\chi_l)$$

$$0 = \delta G_{kl}(R)\chi^k = G_{kl}^+(R)\chi^k - G_{kl}^-(R)\chi^k$$

$$G_{kl}^- = T_{kl} + K_{kl}^-,\ G_{kl}^+ = K_{kl}^+,\ T_{kl}\chi^k(R) = \delta K_{kl}(R)\chi^k(R)$$

$$p_{mr}(R) = \delta\mathrm{B}(R) + \delta\mathrm{C}(R) = (\kappa_1^+ - \kappa_1^-) + (\kappa_2^+ - \kappa_2^-)R^2 + (\kappa_3^+ - \kappa_3^-)b^2(R)$$

$$b_+^2(r) = 1/f_{1+}^2(r) = 1 - 2\bar{M}/r,\ \bar{M} = M(R),\ p_{mr}(R) = 0$$

$$\kappa_1^+ = \kappa_1^-, \kappa_2^+ = \kappa_2^-\ \kappa_3^+ = \kappa_3^-$$
$$p_{mr}(R) = 0$$

$$\delta\mathrm{A}(R)u_k(R)u_l(R) + \delta\mathrm{B}(R)g_{kl}(R) + \delta\mathrm{C}(R)\chi_k(R)\chi_l(R) = 0$$

$$\delta \mathrm{A}(R)b^2(R) - \delta \mathrm{B}(R)b^2(R) = 0, \delta \mathrm{A}(R) = 0$$

$$\delta \mathrm{B}(R)f_1^2(R) + \delta \mathrm{C}(R)f_1^2(R) = 0, \delta \mathrm{C}(R) = 0$$

$$\kappa_2^+ = \kappa_2^-.$$

$$\delta \mathrm{A}(R) = 0$$

$$\kappa_3^+ = \kappa_3^-$$

$$\delta \mathrm{B}(R) = 0 \text{ it is } \kappa_1^+ = \kappa_1^-$$

$$p_{mr}(R) = 0.$$

$$1 - \frac{2M(R)}{R} = 1 - \frac{2\bar{M}}{R} - \frac{\Lambda}{3}R^2 - \frac{\lambda}{5}R^4$$

$$\frac{1}{2}\int_0^R dr'[\mu_m(r') + \mu_d(r')]r'^2 = \bar{M} + \frac{\Lambda}{6}R^3 + \frac{\lambda}{10}R^5$$

$$p_{mr}(R) = 0\text{: } \kappa_j^+ = \kappa_j^- \equiv \kappa_j (j = 1,2,3)$$

$$K_{kl}^+\text{: } \kappa_1 = -\Lambda, \kappa_2 = -\lambda, \kappa_3 = 0$$

$$\mu_d(r) = \lambda r^2 + \Lambda$$
$$p_{dr}(r) = -\lambda r^2 - \Lambda$$
$$p_{d\perp}(r) = -2\lambda r^2 - \Lambda$$

$$\frac{1}{2}\int_0^R drr^2\mu_m(r) = \bar{M}$$

$$\Sigma(r) = p_{dr} - p_{d\perp} = \lambda r^2$$

$$\left[1 - \frac{2M(r)}{r}\right]b_-'' - \left[\frac{1}{r} + \frac{rM'(r) - 3M(r)}{r^2}\right]b_-' + \left[\lambda r^2 - \frac{rM'(r) - 3M(r)}{r^3}\right]b_- = 0$$

$$\nabla_p \dot{u}^p = \frac{3}{2}p_m + \frac{1}{2}\mu_m - 2\lambda r^2 - \Lambda$$

$$\left[1 - \frac{2M(r)}{r}\right]b_-'' + \left[\frac{2}{r} - \frac{rM'(r) + 3M(r)}{r^2}\right]b_-' - \left[\frac{3}{2}p_m + \frac{1}{2}\mu_m - 2\lambda r^2 - \Lambda\right]b_- = 0$$

$$\frac{p_m'}{p_m + \mu_m} = -\frac{b'}{b}$$

$$M(r) = \frac{\mu_m + \Lambda}{6}r^3 + \frac{\lambda}{10}r^5$$

$$b_-(r) = \gamma\frac{\mu_m}{p_m(r) + \mu_m}$$

$$b_-(R) = b_+(R), p_m(R) = 0$$

$$\gamma = \sqrt{1 - \frac{2M(R)}{R}} = \sqrt{1 - \frac{\mu_m+\Lambda}{3}R^2 - \frac{\lambda}{5}R^4}$$

$$y(r) = p_m(r) + \mu_m$$

$$y'\left[1 - \frac{\mu_m+\Lambda}{3}r^2 - \frac{\lambda}{5}r^4\right] + \frac{y^2}{2}r - y\left[\frac{\mu_m+\Lambda}{3}r + 2\frac{\lambda}{5}r^3\right] = 0$$

$$\frac{1}{p_m(r)+\mu_m} = \frac{1}{4}\frac{\frac{\mu_m+\Lambda}{6}+\frac{\lambda}{5}r^2}{\left(\frac{\mu_m+\Lambda}{6}\right)^2+\frac{\lambda}{5}}$$

$$+\sqrt{\frac{1-\frac{2M(r)}{r}}{1-\frac{2M(R)}{R}}}\left[\frac{1}{\mu_m} - \frac{1}{4}\frac{\frac{\mu_m+\Lambda}{6}+\frac{\lambda}{5}R^2}{\left(\frac{\mu_m+\Lambda}{6}\right)^2+\frac{\lambda}{5}}\right]$$

$$M(r) = \frac{\mu_m}{6}r^3$$

$$p_m(r) = \mu_m\frac{\sqrt{1-2M(R)/R}-\sqrt{1-2M(r)/r}}{\sqrt{1-2M(r)/r}-3\sqrt{1-2M(R)/R}}$$

$$M(R) = \frac{4}{9}R$$

$$b_-(r) = \sqrt{1 - \frac{2M(r)}{r}}\left[1 - \frac{\mu_m}{4}\frac{\frac{\mu_m+\Lambda}{6}+\frac{\lambda}{5}R^2}{\left(\frac{\mu_m+\Lambda}{6}\right)^2+\frac{\lambda}{5}}\right]$$

$$+\frac{\mu_m}{4}\frac{\frac{\mu_m+\Lambda}{6}+\frac{\lambda}{5}r^2}{\left(\frac{\mu_m+\Lambda}{6}\right)^2+\frac{\lambda}{5}}\sqrt{1 - \frac{2M(R)}{R}}$$

$$\sqrt{1 - 2M(r)/r}, \frac{\mu_m+\Lambda}{6} + \frac{\lambda}{5}r^2.$$

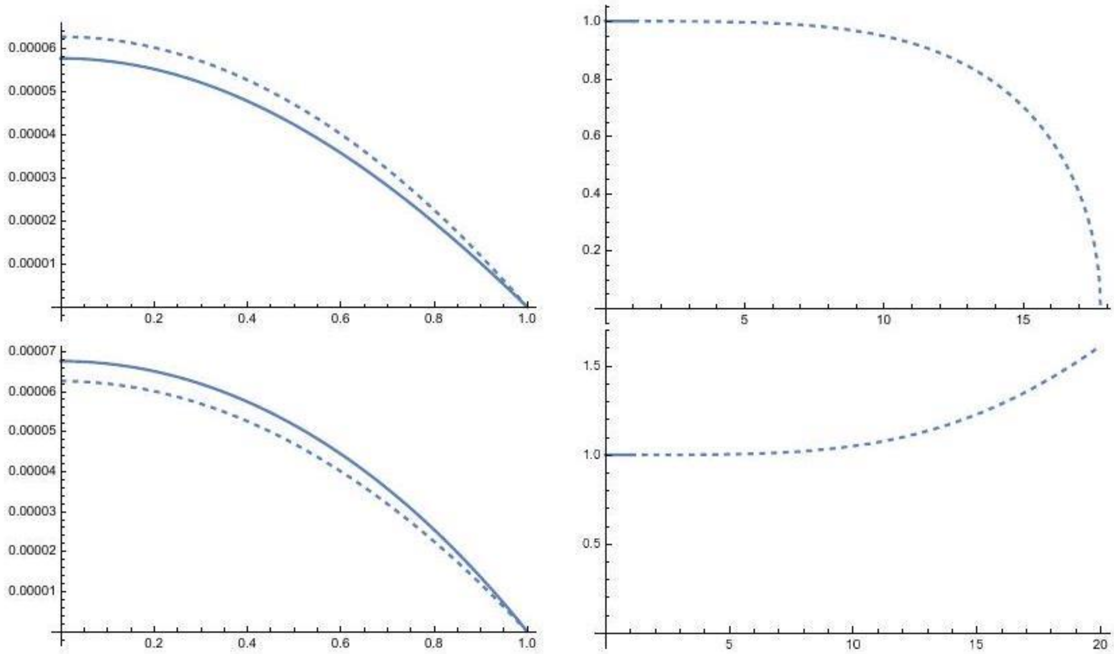
$$\left[1 - \frac{\Lambda + \mu_m}{3}r^2 - \frac{\lambda}{5}r^4\right]b_-'' - \left[\frac{1}{r} + \frac{\lambda}{5}r^3\right]b_-' + \frac{4\lambda}{5}r^2 b_- = 0$$

$$b(p_m + \mu_m) = \gamma\mu_m$$

$$\left[1 - \frac{2M(r)}{r}\right]b_-' + 2\left[\frac{\mu_m+\Lambda}{6}r + \frac{\lambda}{5}r^2\right]b_- = \frac{r}{2}\gamma\mu_m$$

$$b_-(r) = \gamma\mu_m/y(r)$$

$$\bar{M} = \frac{1}{2}\int_0^R drr^2\mu_m = \frac{1}{6}\mu_m R^3$$



$p_m(r)/\mu_m$, $\Lambda = 0, X = \frac{1}{3}\mu_m R^2 = \frac{1}{4}10^{-3}$, $Y = \frac{1}{5}\lambda R^4 = 10^{-5}$, (in GR: $X = \frac{1}{4}10^{-3}, Y = 0$)

$$\Lambda = 0, X = \frac{1}{4}10^{-3}, Y = -10^{-5}$$

$$y(r) = 1 - \frac{2\bar{M}}{r} - \frac{\lambda}{5}r^4$$

$$0 = \frac{1}{\mu_m}\left[\frac{\mu_m^2}{36} + \frac{\lambda}{5}\right] - \frac{1}{4}\left[\frac{\mu_m}{6} + \frac{\lambda}{5}R^2\right] + \frac{\mu_m}{24}\sqrt{1 - \frac{2M(R)}{R}}$$

$$X = \frac{1}{3}\mu_m R^2, Y = \frac{\lambda}{5}R^4$$

$$0 = -\frac{1}{6} + \frac{4}{3}\frac{Y}{X^2} - \frac{Y}{X} + \frac{1}{2}\sqrt{1 - X - Y}$$

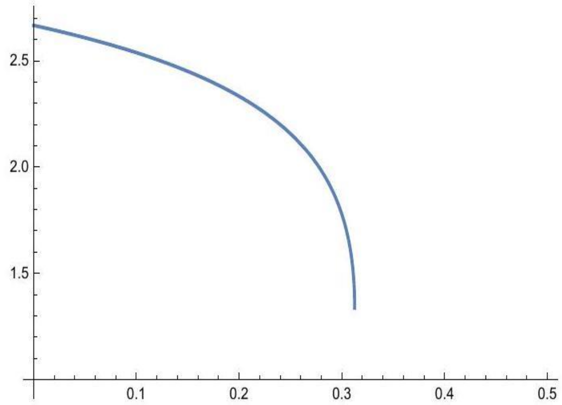
$$Y = \frac{\frac{4}{9} - \frac{X}{3} - \frac{X^2}{4} \pm \left(\frac{X^2}{4} - \frac{5X}{3} + \frac{4}{3}\right)}{2\left(\frac{4}{3X} - 1\right)^2}$$

$$\left(\frac{4}{3} - X\right)^2 Y = X^2\left(\frac{8}{9} - X\right)$$

$$\frac{\lambda}{5}(\mu_m R^2)^2 - \left(\frac{8}{5}\lambda - \frac{\mu_m^2}{3}\right)(\mu_m R^2) + \frac{16}{5}\lambda - \frac{8}{9}\mu_m^2 = 0$$

$$\mu_m R^2 = 4 - \frac{5}{6}\frac{\mu_m^2}{\lambda}\left[1 - \sqrt{1 - \frac{16}{5}\frac{\lambda}{\mu_m^2}}\right]$$

$$\mu_m R^2 = 8/3, M(R) = \frac{1}{6}\mu_m R^3, M_{cr} = \frac{4}{9}R$$



El radio crítico $\mu_m R^2$ de una partícula blanca, es una función de λ/μ_m^2. Para $\lambda = 0$, el valor es $8/3$. El valor más bajo es $\mu_m R^2 = 4/3$ for $\lambda/\mu_m^2 = 5/16$.

$$\kappa_3 r^2 + \kappa_4 \geq 0 \text{ is } y(r) = Cy_0(r) + y_P(r)$$

$$y_P(r) = y_0(r)v(r)$$

$$y_0(r) = \frac{\sqrt{\kappa_3 r^2+\kappa_4}}{r}, v(r) = \int \; dr \frac{1+\kappa_1 r^2+\kappa_2 r^4}{(\kappa_3 r^2+\kappa_4)^{3/2}}$$

$$\kappa_3 > 0, \kappa_4 > 0$$

$$r = \sqrt{\frac{\kappa_4}{\kappa_3}}\text{sh}\theta, y_0 = \sqrt{\kappa_3}\frac{\text{ch}\theta}{\text{sh}\theta}$$

$$v(r) = \frac{1}{\sqrt{\kappa_3}}\int \; d\theta \frac{\frac{1}{\kappa_4}+\frac{\kappa_1}{\kappa_3}\text{sh}^2\theta+\frac{\kappa_2\kappa_4}{\kappa_3^2}\text{sh}^4\theta}{\text{ch}^2\theta}$$
$$= \frac{1}{\sqrt{\kappa_3}}\left[\left(\frac{\kappa_1}{\kappa_3} - \frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\theta + \left(\frac{1}{\kappa_4} - \frac{\kappa_1}{\kappa_3} + \frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\frac{\text{sh}\theta}{\text{ch}\theta} + \frac{\kappa_2\kappa_4}{4\kappa_3^2}\text{sh}(2\theta)\right]$$

$$y_P(r) = \left[\frac{\kappa_1}{\kappa_3} - \frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right]\theta\frac{\text{ch}\theta}{\text{sh}\theta} + \left[\frac{1}{\kappa_4} - \frac{\kappa_1}{\kappa_3} + \frac{\kappa_2\kappa_4}{\kappa_3^2}\right] + \frac{\kappa_2\kappa_4}{2\kappa_3^2}(\text{sh}^2\theta + 1)$$
$$= \left[\frac{\kappa_1}{\kappa_3} - \frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right]\left[\frac{\sqrt{\kappa_3 r^2+\kappa_4}}{r\sqrt{\kappa_3}}\text{Arsh}\sqrt{\frac{\kappa_3}{\kappa_4}}r - 1\right] + \frac{1}{\kappa_4} + \frac{\kappa_2}{2\kappa_3}r^2$$

$$\kappa_3 > 0, \;\; \kappa_4 < 0$$

$$r = \sqrt{\frac{|\kappa_4|}{\kappa_3}}\text{ch}\theta, y_0 = \sqrt{\kappa_3}\frac{\text{sh}\theta}{\text{ch}\theta}$$

$$v(r)=\frac{1}{\sqrt{\kappa_3}}\int d\theta\frac{\frac{1}{|\kappa_4|}+\frac{\kappa_1}{\kappa_3}\text{ch}^2\theta+\frac{\kappa_2|\kappa_4|}{\kappa_3^2}\text{ch}^4\theta}{\text{sh}^2\theta}$$
$$=\frac{1}{\sqrt{\kappa_3}}\left[\left(\frac{\kappa_1}{\kappa_3}-\frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\theta+\left(\frac{1}{\kappa_4}-\frac{\kappa_1}{\kappa_3}+\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\frac{\text{ch}\theta}{\text{sh}\theta}\right.$$
$$\left.-\frac{\kappa_2\kappa_4}{4\kappa_3^2}\text{sh}(2\theta)\right]$$

$$y_P(r)=\left(\frac{\kappa_1}{\kappa_3}-\frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\theta\frac{\text{sh}\theta}{\text{ch}\theta}+\left(\frac{1}{\kappa_4}-\frac{\kappa_1}{\kappa_3}+\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)$$
$$-\frac{\kappa_2\kappa_4}{2\kappa_3^2}(\text{ch}^2\theta-1)$$
$$=\left(\frac{\kappa_1}{\kappa_3}-\frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\left[\frac{\sqrt{\kappa_3r^2+\kappa_4}}{r\sqrt{\kappa_3}}\text{Arch}\sqrt{\frac{\kappa_3}{|\kappa_4|}}r-1\right]$$
$$+\frac{1}{\kappa_4}+\frac{\kappa_2}{2\kappa_3}r^2$$

$$\kappa_3<0,\kappa_4>0(r^2\leq\kappa_4/|\kappa_3|)$$

$$r=\sqrt{\frac{\kappa_4}{|\kappa_3|}}\sin\,\theta, y_0=\sqrt{|\kappa_3|}\frac{\cos\,\theta}{\sin\,\theta}$$

$$v(r)=\frac{1}{\sqrt{|\kappa_3|}}\int d\theta\frac{\frac{1}{\kappa_4}+\frac{\kappa_1}{|\kappa_3|}\sin^2\,\theta+\frac{\kappa_2\kappa_4}{\kappa_3^2}\sin^4\,\theta}{\cos^2\,\theta}$$
$$=\frac{1}{\sqrt{\kappa_3}}\left[\left(\frac{\kappa_1}{\kappa_3}-\frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\theta+\left(\frac{1}{\kappa_4}-\frac{\kappa_1}{\kappa_3}+\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\frac{\sin\,\theta}{\cos\,\theta}\right.$$
$$\left.+\frac{\kappa_2\kappa_4}{4\kappa_3^2}\sin\,(2\theta)\right]$$
$$y_P(r)=\left(\frac{\kappa_1}{\kappa_3}-\frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\theta\frac{\cos\,\theta}{\sin\,\theta}+\left(\frac{1}{\kappa_4}-\frac{\kappa_1}{\kappa_3}+\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)$$
$$+\frac{\kappa_2\kappa_4}{2\kappa_3^2}(1-\sin^2\,\theta)$$
$$=\left(\frac{\kappa_1}{\kappa_3}-\frac{3}{2}\frac{\kappa_2\kappa_4}{\kappa_3^2}\right)\left[\frac{\sqrt{\kappa_3r^2+\kappa_4}}{r\sqrt{|\kappa_3|}}\text{ArcSin}\sqrt{\frac{|\kappa_3|}{\kappa_4}}r-1\right]$$
$$+\frac{1}{\kappa_4}+\frac{\kappa_2}{2\kappa_3}r^2$$

$$F(r)=1-ar^2-br^4$$

$$y'F+\frac{1}{2}yF'+\frac{r}{2}y^2=0$$

$$\frac{d}{dr}\left(\frac{1}{y\sqrt{F}}\right)=\frac{1}{2}\frac{r}{F^{3/2}}$$

$$\frac{1}{y(r)}=\sqrt{F(r)}\left[C-\frac{1}{2}\int_r^R\frac{r'dr'}{F(r')^{3/2}}\right]$$
$$=\sqrt{F(r)}\left[C-\frac{1}{2(a^2+4b)}\left(\frac{a+2bR^2}{\sqrt{F(R)}}-\frac{a+2br^2}{\sqrt{F(r)}}\right)\right]$$

$$\frac{1}{y(r)}=\sqrt{\frac{F(r)}{F(R)}}\left[\frac{1}{\mu_m}-\frac{a+2bR^2}{2(a^2+4b)}\right]+\frac{a+2br^2}{2(a^2+4b)}$$

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu=e^{2F(r)}c^2dt^2-e^{2H(r)}dr^2-r^2(d\theta^2+\sin^2\,\theta d\phi^2)$$

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\Lambda g_{\mu\nu}=\frac{8\pi G}{c^4}T_{\mu\nu}$$

$$T^{\mu\nu} = (c^2\rho + P)U^\mu U^\mu - Pg^{\mu\nu}$$

$$g_{\mu\nu}dx^\mu dx^\nu, R = g^{\alpha\beta}R_{\alpha\beta}$$

$$\frac{dm}{dr} = 4\pi r^2\rho, \frac{dP}{dr} = -(\rho + P/c^2)\frac{G\left(m+\frac{4\pi r^3}{c^2}P\right)-\frac{c^2\Lambda}{3}r^3}{r^2\left(1-\frac{2Gm}{c^2r}-\frac{\Lambda}{3}r^2\right)}.$$

$$e^{2F(r)} = \kappa_+ e^{-2u(r)/c^2}$$

$$e^{-2H(r)} = 1 - \frac{2Gm(r)}{c^2r} - \frac{\Lambda}{3}r^2.$$

$$\frac{dm}{dr} = 4\pi r^2\rho, \frac{dP}{dr} = -(\rho + P/c^2)\frac{G\left(m+\frac{4\pi r^3}{c^2}P\right)}{r^2\left(1-\frac{2Gm}{c^2r}\right)}$$

$$P = A\rho^\gamma\Omega(A\rho^{\gamma-1}/c^2)$$

$$P = Kc^5\int_0^\zeta \frac{q^4dq}{\sqrt{1+q^2}}, \rho = 3Kc^3\int_0^\zeta \sqrt{1+q^2}q^2dq$$

$$\kappa(r,m) := 1 - \frac{2Gm}{c^2r} - \frac{\Lambda}{3}r^2,$$

$$Q(r,m,P) := G\left(m + \frac{4\pi r^3}{c^2}P\right) - \frac{c^2\Lambda}{3}r^3,$$

$$\frac{dP}{dr} = -(\rho + P/c^2)\frac{Q(r,m,P)}{r^2\kappa(r,m)}$$

$$\mathcal{D} = \{(r,m,P)|0 < r, |m| < +\infty, 0 < \rho, 0 < \kappa(r,m)\}.$$

$$m = \frac{4\pi}{3}\rho_c r^3 + O(r^5)$$
$$P = P_c - (\rho_c + P_c/c^2)(4\pi G(\rho_c + 3P_c/c^2) - c^2\Lambda)\frac{r^2}{6} + O(r^4)$$

$$\kappa_+ := \lim_{r\to r_+-0}\kappa(r,m(r)) = 1 - \frac{2Gm_+}{c^2r_+} - \frac{\Lambda}{3}r_+^2$$
$$Q_+ := \lim_{r\to r_+-0}Q(r,m(r),P(r)) = Gm_+ - \frac{c^2\Lambda}{3}r_+^3$$

$$m_+ := \lim_{r\to r_+-0}m(r)$$

$$\Lambda < \frac{4\pi G}{c^2}(\rho_c + 3P_c/c^2)$$

$$\Lambda = \frac{4\pi G}{c^2}(\rho_c + 3P_c/c^2)$$

$$ds^2 = c^2dt^2 - \left(1 - \frac{L}{3}r^2\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\xi^1 = r\sin\theta\cos\phi, \xi^2 = r\sin\theta\sin\phi, \xi^3 = r\cos\theta, \xi^4 = \sqrt{\frac{3}{L} - r^2}$$

$$\Lambda > \frac{4\pi G}{c^2}(\rho_c + 3P_c/c^2)$$

$$u := \int_0^\rho \frac{dP}{\rho + P/c^2}$$

$$u = \frac{\gamma A}{\gamma-1}\rho^{\gamma-1}\Omega_u(A\rho^{\gamma-1}/c^2)$$
$$\rho = A_1 u^{\frac{1}{\gamma-1}}\Omega_\rho(u/c^2)$$
$$P = AA_1^\gamma u^{\frac{\gamma}{\gamma-1}}\Omega_P(u/c^2)$$

$$\Omega_u(\zeta) = \frac{1}{\zeta}\int_0^\zeta \frac{\Omega(\zeta') + \frac{\gamma-1}{\gamma}\zeta' D\Omega(\zeta')}{1+\zeta'\Omega(\zeta')} d\zeta'$$
$$\zeta = \frac{\gamma-1}{\gamma}\eta\Omega_\rho(\eta) \Leftrightarrow \eta = \frac{\gamma}{\gamma-1}\zeta\Omega_u(\zeta)$$
$$\Omega_P(\eta) = \Omega(\zeta)\Omega_u(\zeta)^{-\frac{\gamma}{\gamma-1}} \text{ with } \zeta = \frac{\gamma-1}{\gamma}\eta\Omega_\rho(\eta)$$

$$u_c := \int_0^{\rho_c} \frac{dP}{\rho+P/c^2} = \frac{\gamma A}{\gamma-1}\rho_c^{\gamma-1}\Omega_u\left(A\rho_c^{\gamma-1}/c^2\right)$$

$$u_c \leq c^2\epsilon_0, \Lambda \leq \frac{4\pi}{c^2}G\left(\frac{\gamma-1}{\gamma A}\right)^{\frac{1}{\gamma-1}}(u_c)^{\frac{1}{\gamma-1}}\epsilon_0$$

$$\frac{dm}{dr} = 4\pi r^2 A_1(u_\sharp)^{\frac{1}{\gamma-1}}\Omega_\rho(u/c^2)$$
$$\frac{du}{dr} = -\frac{G\left(m + \frac{4\pi}{c^2}r^3 AA_1^\gamma(u_\sharp)^{\frac{\gamma}{\gamma-1}}\Omega_P(u/c^2)\right) - \frac{c^2\Lambda}{3}r^3}{r^2\left(1 - \frac{2Gm}{c^2 r} - \frac{\Lambda}{3}r^2\right)}.$$

$$\mathcal{D}_u = \{(r,m,u)|0 < r, |m| < \infty, -\delta_\Omega < u/c^2 < +\infty, \kappa > 0\}.$$

$$r = aR, m = a^3 b^{\frac{1}{\gamma-1}} \cdot 4\pi A_1 M, u = bU$$

$$4\pi G A_1 a^2 b^{\frac{2-\gamma}{\gamma-1}} = 1$$

$$\lambda := \frac{c^2}{4\pi G A_1}\Lambda, \alpha := b/c^2 = u_c/c^2, \beta := b^{-\frac{1}{\gamma-1}}\lambda = \frac{c^2}{4\pi G A_1}(u_c)^{-\frac{1}{\gamma-1}}\Lambda.$$

$$\frac{dM}{dR} = R^2(U_\sharp)^{\frac{1}{\gamma-1}}\Omega_\rho(\alpha U)$$
$$\frac{dU}{dR} = -\frac{1}{R^2}\frac{\left(M + \frac{\gamma-1}{\gamma}\alpha R^3(U_\sharp)^{\frac{\gamma}{\gamma-1}}\Omega_P(\alpha U) - \frac{1}{3}\beta R^3\right)}{\left(1 - 2\alpha\frac{M}{R} - \frac{1}{3}\alpha\beta R^2\right)}$$

$$\mathcal{D}_U = \{(R,M,U)|0 < R, |M| < \infty, -\delta_\Omega < U < 2, \kappa > 0\}$$

$$\kappa = 1 - 2\alpha\frac{M}{R} - \frac{1}{3}\alpha\beta R^2$$

$$M(R) = \Omega_\rho(\alpha)\frac{R^3}{3} + O(R^5)$$
$$U(R) = 1 - \left(\Omega_\rho(\alpha) + \frac{3\gamma}{\gamma-1}\alpha\Omega_P(\alpha) - \beta\right)\frac{R^2}{6} + O(R^4)$$

$$q(R) = \frac{3}{R^3}\int_0^R U(R')^{\frac{1}{\gamma-1}}\frac{\Omega_\rho\big(\alpha U(R')\big)}{\Omega_\rho(\alpha)}R'^2 dR'$$

$$U(R) = 1 - \int_0^R \frac{\frac{1}{3}\Omega_\rho(\alpha)q(R') + \frac{\gamma-1}{\gamma}\alpha U(R')^{\frac{\gamma}{\gamma-1}}\Omega_P\big(\alpha U(R')\big) - \frac{1}{3}\beta}{1 - 2\alpha\frac{1}{3}\Omega_\rho(\alpha)q(R')R'^2 - \frac{1}{3}\alpha\beta R'^2}R' dR'$$

$$C_q := \max\left\{U^{\frac{1}{\gamma-1}}\frac{\Omega_\rho(\alpha U)}{\Omega_\rho(\alpha)}\,\middle|\, \frac{1}{2} \le U \le 2, 0 \le \alpha \le 1\right\}.$$

$$\frac{dM}{dR} = R^2 (U_\sharp)^\mu, \frac{dU}{dR} = -\frac{M}{R^2}$$

$$-\frac{1}{R^2}\frac{d}{dR}\left(R^2\frac{dU}{dR}\right) = (U_\sharp)^\mu.$$

$$\bar{U}(R) = \left(R^2\frac{d\bar{U}}{dR}\right)_{R=\xi_1}\left(\frac{1}{\xi_1} - \frac{1}{R}\right)$$

$$\alpha \le \epsilon_0 \text{ and } \beta \le \epsilon_0,$$

$$-\frac{\delta_\Omega}{2} \le \bar{U}(\xi_1 + \delta_R) < 0$$

$$\rho + 3P/c^2 = \left(\frac{\gamma-1}{\gamma A}\right)^{\frac{1}{\gamma-1}} u^{\frac{1}{\gamma-1}}\Omega_{\rho+3P/c^2}(u/c^2),$$

$$\Omega_{\rho+3P/c^2}(\eta) := \Omega_\rho(\eta) + 3\frac{\gamma-1}{\gamma}\eta\Omega_P(\eta)$$

$$\Lambda \le 4\pi c^{\frac{2(2-\gamma)}{\gamma-1}} G\left(\frac{\gamma-1}{\gamma A}\right)^{\frac{1}{\gamma-1}}\epsilon_0^{\frac{\gamma}{\gamma-1}}$$

$$\frac{dm}{dr} = 4\pi r^2 A_1 (u_\sharp)^{\frac{1}{\gamma-1}}\Omega_\rho(u/c^2)$$

$$\frac{du}{dr} = -\frac{G\left(m + \frac{4\pi}{c^2}r^3 A A_1^\gamma (u_\sharp)^{\frac{\gamma}{\gamma-1}}\Omega_P(u/c^2)\right)}{r^2\left(1 - \frac{2Gm}{c^2 r}\right)}.$$

$$m^0(r) = m_+^0\left(:= m^0(r_+^0)\right)$$

$$u^0(r) = \frac{c^2}{2}\left(\log\left(1 - \frac{2Gm_+^0}{c^2 r_+^0}\right) - \log\left(1 - \frac{2Gm_+^0}{c^2 r}\right)\right)$$

$$r = aR, m = a^3 \cdot 4\pi A_1 M, 4\pi G A_1 a^2 = 1, u = U, \lambda = \frac{c^2}{4\pi G A_1}\Lambda$$

$$\frac{dM}{dR} = R^2 U^\mu \Omega_\rho(U/c^2)$$

$$\frac{dU}{dR} = -\frac{1}{R^2}\left(M + \frac{\gamma-1}{\gamma}\frac{R^3}{c^2}U^{\mu+1}\Omega_P(U/c^2) - \frac{\lambda}{3}R^3\right) \times\left(1 - \frac{2M}{c^2 R} - \frac{\lambda}{3c^2}R^2\right)^{-1}$$

$$\left(R^2 U^\mu, -\frac{1}{R^2}\left(M - \frac{\lambda}{3}R^3\right)\right)^T \; c \to \infty$$

$$-\frac{1}{R^2}\frac{d}{dR}\left(R^2\frac{dU}{dR}\right) = U^{\mu} - \lambda$$

$$\hat{U}(R) = \lambda + (1-\lambda)\frac{\sin\ R}{R}$$

$$\hat{U}(R) = 1 - \frac{1-\lambda}{6}R^2 + O(R^4)$$

$$\frac{d\hat{U}}{dR} = \frac{1-\lambda}{R}\left(\cos\ R - \frac{\sin\ R}{R}\right)$$

$$\frac{2Gm_+}{c^2 r_+} = 1 - \frac{\Lambda}{3}r_+^2$$

$$\kappa_+' := \frac{d\kappa}{dr}\bigg|_{r=r_+-0} = \lim_{r\to r_+-0} -\frac{2G}{c^2}4\pi\rho + \frac{2Gm}{c^2}\frac{1}{r^2} - \frac{2}{3}\Lambda r$$
$$= \frac{2Gm_+}{c^2}\frac{1}{r_+^2} - \frac{2}{3}\Lambda r_+ = \frac{1}{r_+}(1-\Lambda r_+^2) \leq 0$$

$$Q_+ = Gm_+ - \frac{c^2\Lambda}{3}r_+^3 = \frac{c^2 r_+}{2}(1-\Lambda r_+^2)$$
$$\geq 0$$

$$ds^2 = \kappa_+ e^{-2u/c^2}c^2dt^2 - \frac{1}{\kappa}dr^2 - r^2d\omega^2$$

$$d\omega^2 = d\theta^2 + \sin^2\ \theta d\phi^2$$

$$ds^2 = \left(1 - \frac{2Gm_+}{c^2 r} - \frac{\Lambda}{3}r^2\right)c^2dt^2 - \left(1 - \frac{2Gm_+}{c^2 r} - \frac{\Lambda}{3}r^2\right)^{-1}dr^2 - r^2d\omega^2$$

$$ds^2 = g_{00}c^2dt^2 - g_{11}dr^2 - r^2d\omega^2$$

$$g_{00} = \begin{cases}\kappa_+ e^{-2u(r)/c^2} & (0 \leq r < r_+)\\ 1 - \frac{2Gm_+}{c^2 r} - \frac{\Lambda}{3}r^2 & (r_+ \leq r < r_E)'\end{cases}$$
$$-g_{11} = \left(1 - \frac{2G\tilde{m}(r)}{c^2 r} - \frac{\Lambda}{3}r^2\right)^{-1} \quad (0 \leq r < r_E),$$

$$\tilde{m}(r) = \begin{cases} m(r) & (0 \leq r < r_+)\\ m_+ & (r_+ \leq r < r_E).\end{cases}$$

$$\kappa(r, m_+) = \frac{\Lambda}{3r}(r - r_I)(r_E - r)(r + r_I + r_E).$$

$$\sqrt{\Lambda} < \frac{c^2}{3Gm_+}.$$

$$u(r) = B(r_+ - r)\big(1 + O(r_+ - r)\big)$$

$$\rho(r) = \left(\frac{(\gamma-1)B}{\gamma A}\right)^{\frac{1}{\gamma-1}}(r_+ - r)^{\frac{1}{\gamma-1}}\big(1 + O(r_+ - r)\big).$$

$$\frac{du}{dr}\bigg|_{r=r_+-0} = -\frac{Q+}{r_+^2\kappa_+} = -B.$$

$$\frac{d}{dr}\tilde{m}(r) = \begin{cases}4\pi r^2\rho(r) & (r < r_+)\\ 0 & (r_+ \leq r < r_E)\end{cases}$$

$$\left.\frac{d}{dr}g_{00}\right|_{r=r_+-0}=-\frac{2\kappa_+}{c^2}\left.\frac{du}{dr}\right|_{r=r_+-0}=\frac{2Q_+}{c^2r_+^2}$$

$$\left.\frac{d}{dr}g_{00}\right|_{r=r_++0}=\left(\frac{2Gm_+}{c^2r^2}-\frac{2\Lambda}{3}r\right)_{r=r_+}=\frac{2Q_+}{c^2r_+^2}$$

$$\left.\frac{d^2}{dr^2}g_{00}\right|_{r=r_+-0}=\frac{4\kappa_+}{c^4}\left(\frac{du}{dr}\right)^2_{r=r_+-0}-\frac{2\kappa_+}{c^2}\left(\frac{d^2u}{dr^2}\right)_{r=r_+-0}.$$

$$\left.\frac{d^2u}{dr^2}\right|_{r=r_+-0}=\frac{c^2\Lambda}{\kappa_+}+\frac{2Q_+}{r_+^3\kappa_+}+\frac{2(Q_+)^2}{c^2r_+^4\kappa_+^2}.$$

$$\left.\frac{d^2}{dr^2}g_{00}\right|_{r=r_+-0}=\left.\frac{d^2}{dr^2}g_{00}\right|_{r=r_++0}=-\frac{4Q_+}{c^2r_+^3}-2\Lambda.$$

$$u(r)=B(r_+-r)\big(1+O(r_+-r)\big)$$

$$u(r)=B(r_+-r)\left(1+\left[r_+-r,(r_+-r)^{\frac{\gamma}{\gamma-1}}\right]_1\right),$$

$$\rho(r)=\left(\frac{(\gamma-1)B}{\gamma A}\right)^{\frac{1}{\gamma-1}}(r_+-r)^{\frac{1}{\gamma-1}}\left(1+\left[r_+-r,(r_+-r)^{\frac{\gamma}{\gamma-1}}\right]_1\right).$$

$$\sum_{k_1+k_2\geq 1} a_{k_1k_2}X_1^{k_1}X_2^{k_2}$$

$$\frac{dm}{dr}=4\pi r^2A_1u^{\mu}\Omega_\rho(u/c^2)$$

$$\frac{du}{dr}=-\frac{G\left(m+\frac{4\pi}{c^2}r^3AA_1^{\gamma}u^{\mu+1}\Omega_P(u/c^2)\right)-\frac{c^2\Lambda}{3}r^3}{r^2\left(1-\frac{2Gm}{c^2r}-\frac{\Lambda}{3}r^2\right)}$$

$$\frac{dm}{du}=-4\pi r^4\left(1-\frac{2Gm}{c^2r}-\frac{\Lambda}{3}r^2\right)\cdot Q^{-1}\cdot A_1u^{\mu}\Omega_\rho(u/c^2)$$

$$\frac{dr}{du}=-r^2\left(1-\frac{2Gm}{c^2r}-\frac{\Lambda}{3}r^2\right)\cdot Q^{-1}$$

$$Q=G\left(m+\frac{4\pi}{c^2}r^3AA_1^{\gamma}u^{\mu+1}\Omega_P(u/c^2)\right)-\frac{c^2\Lambda}{3}r^3$$

$$m(u)=m_++u[u,u^{\mu}]_0$$
$$r(u)=r_++u[u,u^{\mu}]_0$$

$$\frac{dy_\alpha}{dx}=f^{\alpha}(x,x^{\mu},y_1,y_2),\ y_\alpha|_{x=0}=0,\alpha=1,2$$

$$m=m_+-Cu^{\mu+1}+\sum_{n\geq 2} m_{1n}u^{\mu n+1}+\sum_{n\geq 0,l\geq 2} m_{ln}u^{\mu n+l}$$
$$r=r_+-\frac{1}{B}u+\sum_{n\geq 1} c_{1n}u^{\mu n+1}+\sum_{n\geq 0,l\geq 2} c_{ln}u^{\mu n+l}$$

$$m=m_+-Cu^{\mu+1}+\sum_{n\geq 0,l\geq 2} m_{ln}u^{\mu n+l}$$
$$r=r_+-\frac{1}{B}u+\sum_{n\geq 0,l\geq 2} c_{ln}u^{\mu n+l}$$

$$r=r_+-\frac{1}{B}u+\sum_{n\geq 0,l\geq n+1,l\geq 2} c_{ln}u^{(\mu+1)n+l-n}$$
$$=r_+-\frac{1}{B}u(1+[u,u^{\mu+1}]_1)$$

$$u = B(r_+ - r)(1 + [r_+ - r, (r_+ - r)^{\mu+1}]_1).$$

$$\varphi(x) = \sum \tilde{c}_{ij} x^i (x^\mu)^j, |\tilde{c}_{ij}| \leq \frac{\tilde{M}}{\tilde{\delta}^{i+j}}, (\tilde{\delta} < 1),$$

$$\varphi(x) = \sum_l \sum_{n=0}^{p-1} c_{ln} x^{\mu n + l}, |c_{ln}| \leq \frac{M}{\delta^l} \text{ for } 0 \leq n \leq p-1,$$

$$c_{ln} = \sum \{\tilde{c}_{ij} \mid i + qJ = l, j = pJ + n, \exists J \in \mathbb{N}\}$$

$$f^\alpha(x, x^\mu, y_1, y_2) = \sum \sum_{n=0}^{p-1} a^\alpha_{lnk_1k_2} x^{\mu n + l} y_1^{k_1} y_2^{k_2}$$

$$|a^\alpha_{lnk_1k_2}| \leq \frac{M}{\delta^{l+k_1+k_2}} \ (0 \leq n \leq p-1)$$

$$F(x, y_1, y_2) = \sum \frac{M}{\delta^{l+k_1+k_2}} \sum_{n=0}^{p-1} x^{\mu n + l} y_1^{k_1} y_2^{k_2}$$
$$= \frac{M}{1 - x/\delta} \frac{1 - x^{\mu p}}{1 - x^\mu} \frac{1}{1 - y_1/\delta} \frac{1}{1 - y_2/\delta}$$

$$\frac{dY}{dx} = F(x, Y, Y), \ Y|_{x=0} = 0$$

$$Y = Mx(1 + [x, x^\mu]_1)$$
$$= \sum_l \sum_{n=0}^{p-1} C_{ln} x^{\mu n + l}, 0 \leq C_{ln} \leq \frac{M'}{(\delta')^l}$$

$$y_\alpha = \sum_l \sum_{n=0}^{p-1} c^\alpha_{ln} x^{\mu n + l},$$

$$c^\alpha_{0n} = 0, c^\alpha_{l+1,n} = \frac{1}{l+1+\mu n} b^\alpha_{ln}$$
$$b^\alpha_{LR} = \sum a^\alpha_{lnk_1k_2} c^1_{l'(1)n'(1)} \cdots c^1_{l'(k_1)n'(k_1)} c^2_{l''(1)n''(1)} \cdots c^2_{l''(k_2)n''(k_2)}$$

$$L = qJ + l + l'(1) + \cdots + l'(k_1) + l''(1) + \cdots + l''(k_2)$$

$$pJ + R = n + n'(1) + \cdots + n'(k_1) + n''(1) + \cdots + n''(k_2)$$

$$\frac{dp(r)}{dr} = -\frac{[\rho(r) + p(r)][m(r) + 4\pi p(r) r^3]}{r^2\left[1 - \frac{2m(r)}{r}\right]}$$
$$\frac{dm(r)}{dr} = 4\pi\rho(r) r^2$$

$$\mathrm{d}\, m_0(r) = 4\pi \int \rho_0(r) r^2 dr$$

$$g_0 = \frac{m_0(r) + 4\pi p_0(r) r^3}{r^2[1 - 2m_0(r)/r]}$$

$$\delta p(r) = \frac{\delta p_c \sqrt{1 - 2m_0/r} \exp\left\{-2\int_0^r g_0 dr\right\}}{1 + 4\pi\delta p_c \int_0^r \frac{1}{\sqrt{1 - 2m_0/r}} \exp\left\{-2\int_0^r g_0 dr\right\} r dr}$$

$$g_0 = \frac{m_0(r) + 4\pi p_0(r) r^3}{r^2[1 - 2m_0(r)/r]} = \frac{m(r) + 4\pi p(r) r^3}{r^2[1 - 2m(r)/r]}$$

$$\delta m(r) = \frac{4\pi r^3 \delta\rho_c}{3[1 + r g_0]^2} \exp\left\{2\int g_0 \frac{1 - r g_0}{1 + r g_0} dr\right\}$$

$$\delta p(r) = -\frac{\delta m}{4\pi r^3}\frac{1+8\pi p_0 r^2}{1-2m_0/r}$$

$$\delta p(r) = \frac{\delta p_c}{[1+rg_0]^2}\frac{1+8\pi p_0 r^2}{1-\frac{2m_0}{r}} * \exp\left\{2\int_0^r g_0 \frac{1-rg_0}{1+rg_0}dr\right\}$$

$$\delta\rho(r) = -\frac{1}{r^2}\frac{d}{dr}\left(\frac{\delta p(r)r^3}{1+2rg_0(r)}\right)$$

$$T_{\hat{a}\hat{b}} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & p_r & 0 & 0 \\ 0 & 0 & p_t & 0 \\ 0 & 0 & 0 & p_t \end{bmatrix}$$

$$ds^2 = -\zeta_0(r)^2 dt^2 + \frac{dr^2}{B_0(r)} + r^2 d\Omega^2$$

$$G_{\hat{r}\hat{r}} - G_{\hat{\theta}\hat{\theta}} = 8\pi\Delta$$

$$T_f^{ab} = (\rho_f + p_f)V^a V^b + p_f g^{ab}$$

$$T_{em}^{ab} = F^{ac} g_{cd} F^{bd} - \frac{1}{4} g^{ab}(F_{cd}F^{cd})$$

$$T_s^{ab} = \phi^{;a}\phi^{;b} - \frac{1}{2} g^{ab}(g^{cd}\phi_{;c}\phi_{;d})$$

$$(\rho_f + p_f)V^a{}_{;b}V^b + g^{ab}\left([p_f]_{;b} + \sigma_s\phi_{;b}\right) - F^{ab}(\sigma_{em}V_b) = 0$$

$$\frac{dp_f}{dr} = -\frac{[\rho_f + p_f][m(r) + 4\pi p_f r^3]}{r^2\left[1-\frac{2m(r)}{r}\right]} - \frac{\sigma_{em}E}{\sqrt{1-\frac{2m(r)}{r}}} - \sigma_s\frac{d\phi}{dr}$$

$$\frac{dm}{dr} = 4\pi\rho r^2 = 4\pi(\rho_f + \rho_{em} + \rho_s)r^2$$

$$\frac{dp_f}{dr} = -\frac{[\rho_f + p_f][m(r) + 4\pi p_f r^3]}{r^2\left[1-\frac{2m(r)}{r}\right]} - \frac{\sigma_{em}E}{\sqrt{1-\frac{2m(r)}{r}}}$$

$$\frac{dm}{dr} = 4\pi\rho r^2 = 4\pi(\rho_f + \rho_{em})r^2$$

$$m(r) = \frac{4}{3}\pi\rho r^3$$

Charge density $(\mathbf{C/nm^3})$	Pressure $(\mathbf{N/nm^2})$
1	4.08×10^{28}

2×10^{11}	7.25×10^{29}
3×10^{11}	1.08×10^{30}
4×10^{11}	2.00×10^{30}
5×10^{11}	1.84×10^{30}

$$\sigma_{em} = 10^{11} \text{ ng/nm}^3$$

$$\sigma_{em} = 5 \times 10^{11} \text{ ng/nm}^3$$

Density ($\mathbf{kg/nm^3}$)	Pressure ($\mathbf{N/nm^2}$)
1×10^{12}	4.08×10^{28}
1.1×10^{12}	6.28×10^{28}
1.2×10^{12}	1.04×10^{29}
1.3×10^{12}	2.07×10^{29}
1.4×10^{12}	9.47×10^{29}

$$\rho_f = 10^{22} \text{ ng/nm}^3$$

$$\rho_f = 1.2 \times 10^{22} \text{ ng/nm}^3$$

$$ds^2 = \zeta_0(r)^2 dt^2 - \frac{dr^2}{B_0(r)} - r^2 d\Omega^2.$$

$$\frac{dC}{dr} = \left(\frac{\sqrt{B_0}}{r^2} \frac{d}{dr}\left(\frac{1}{\sqrt{B_0}}\right) - \frac{\frac{1}{B_0} - 1}{r^3} \right) + \sqrt{B_0} \frac{d}{dr}\left(\frac{1}{\sqrt{B_0}}\right) C - C^2 r + \frac{2q^2}{B_0 r^5},$$

$$C = \frac{\zeta_0{}'(r)}{r\zeta_0(r)}.$$

$$q(a) = Ka^n,$$

$$m = \frac{na^2(2-n) + 2q^2}{2(1+2n-n^2)a}.$$

$$\zeta_0^2 = 1 - \frac{2m}{a} + \frac{q^2}{a^2}.$$
$$B_0 = 1 - \frac{2m}{a} + \frac{q^2}{a^2} = 1 - \frac{2q^2}{a^2}.$$

$$ds^2 = \left(1 - \frac{2m}{a} + \frac{q^2}{a^2}\right)^2 dt^2 - \left(1 + \frac{q^2}{a^2}\right)^{-2} dr^2 - r^2 d\Omega^2.$$

$$T \geq \operatorname{sech}^2\left[\frac{1}{2\omega}\int_{-\infty}^{\infty} V_l(r) dr_*\right].$$

$$V(r) = \frac{l(l+1)}{r^2}\left(1 - \frac{2m}{a} + \frac{q^2}{a^2}\right)^2$$

$$\frac{dr_*}{dr} = \frac{1}{\left(1-\frac{2m}{a}+\frac{q^2}{a^2}\right)\left(1-\frac{2q^2}{a^2}\right)}.$$

$$T \geq \operatorname{sech}^2\left[\frac{1}{2\omega}\int_{-\infty}^{\infty} \frac{l(l+1)}{r^2}\left(1 - \frac{2m}{a} + \frac{q^2}{a^2}\right)^2 dr_*\right]$$
$$= \operatorname{sech}^2\left[\frac{1}{2\omega}\int_{r_0}^{\infty} \frac{l(l+1)}{r^2}\frac{1-\frac{2m}{a}+\frac{q^2}{a^2}}{1-\frac{2q^2}{a^2}} dr\right]$$

$$= \operatorname{sech}^2\left[\frac{1}{2\omega}\int_{r_0}^{\infty} \frac{l(l+1)}{r^2}\frac{a^2-2ma+q^2}{a^2-2q^2} dr\right]$$
$$= \operatorname{sech}^2\left[\frac{1}{2\omega} l(l+1)\frac{a^2-2ma+q^2}{a^2-2q^2}\left(-\frac{1}{r}\right)_{r_0}^{\infty}\right]$$
$$= \operatorname{sech}^2\left[\frac{1}{2\omega} l(l+1)\frac{a^2-2ma+q^2}{a^2-2q^2}\left(\frac{1}{r_0}\right)\right].$$

$$R + T = 1.$$

$$R = 1 - T = 1 - \operatorname{sech}^2\left[\frac{1}{2\omega} l(l+1)\frac{a^2-2ma+q^2}{a^2-2q^2}\left(\frac{1}{r_0}\right)\right].$$

$$L_{\text{ChS}}^{(5)} = \alpha_1 l^2 \varepsilon_{abcde} R^{ab} R^{cd} e^e + \alpha_3 \varepsilon_{abcde}\left(\frac{2}{3} R^{ab} e^c e^d e^e + 2l^2 k^{ab} R^{cd} T^e + l^2 R^{ab} R^{cd} h^e\right)$$

$$\varepsilon_{abcde} R^{cd} T^e = 0,$$
$$\alpha_3 l^2 \varepsilon_{abcde} R^{bc} R^{de} = -\frac{\delta L_{\text{M}}}{\delta h^a},$$
$$\varepsilon_{abcde}\left(2\alpha_3 R^{bc} e^d e^e + \alpha_1 l^2 R^{bc} R^{de} + 2\alpha_3 l^2 D_\omega k^{bc} R^{de}\right) = -\frac{\delta L_{\text{M}}}{\delta e^a},$$
$$2\varepsilon_{abcde}\left(\alpha_1 l^2 R^{cd} T^e + \alpha_3 l^2 D_\omega k^{cd} T^e + \alpha_3 e^c e^d T^e + \alpha_3 l^2 R^{cd} D_\omega h^e + \alpha_3 l^2 R^{cd} k_f^e e^f\right) = -\frac{\delta L_{\text{M}}}{\delta \omega^{ab}}.$$

$$de^a + \omega_b^a e^b = 0,$$
$$\varepsilon_{abcde} R^{cd} D_\omega h^e = 0,$$
$$\alpha_3 l^2 \star \left(\varepsilon_{abcde} R^{bc} R^{de}\right) = - \star \left(\frac{\delta L_{\text{M}}}{\delta h^a}\right),$$
$$\star \left(\varepsilon_{abcde} R^{bc} e^d e^e\right) + \frac{1}{2\alpha} l^2 \star \left(\varepsilon_{abcde} R^{bc} R^{de}\right) = \kappa_{\text{E}} T_{ab} e^b,$$

$$\alpha = \alpha_3/\alpha_a, \kappa_{\text{E}} = \kappa/2\alpha_3, T_{ab} = \star \left(\delta L_{\text{M}}/\delta e^a\right) \star$$

$$ds^2 = -e^{2f(r)}dt^2 + e^{2g(r)}dr^2 + r^2 d\Omega_3^2 = \eta_{ab}e^a e^b$$

$$e^T = e^{f(r)}dt, e^R = e^{g(r)}dr, e^1 = rd\theta_1, e^2 = r\sin\ \theta_1 d\theta_2, e^3 = r\sin\ \theta_1 \sin\ \theta_2 d\theta_3$$

$$\begin{aligned}
\frac{e^{-2g}}{r^2}(g'r + e^{2g} - 1) + \mathrm{sgn}(\alpha)l^2\frac{e^{-2g}}{r^3}g'(1-e^{-2g}) &= \frac{\kappa_{\mathrm{E}}}{12}\rho,\\
\frac{e^{-2g}}{r^2}(f'r - e^{2g} + 1) + \mathrm{sgn}(\alpha)l^2\frac{e^{-2g}}{r^3}f'(1-e^{-2g}) &= \frac{\kappa_{\mathrm{E}}}{12}p,\\
\frac{e^{-2g}}{r^2}\{(-f'g'r^2 + f''r^2 + (f')^2r^2 + 2f'r - 2g'r - e^{2g} + 1)&\\
+\mathrm{sgn}(\alpha)l^2(f'' + (f')^2 - f'g' - e^{-2g}f'' - e^{-2g}(f')^2 + 3e^{-2g}f'g')\} &= \frac{\kappa_{\mathrm{E}}}{4}p.
\end{aligned}$$

$$D_\omega(\star\, T_a) = 0$$

$$f'(r) = -\frac{p'(r)}{\rho(r) + p(r)}$$

$$e^{-2g(r)} = 1 + \mathrm{sgn}(\alpha)\frac{r^2}{l^2} - \mathrm{sgn}(\alpha)\sqrt{\frac{r^4}{l^4} + \mathrm{sgn}(\alpha)\frac{\kappa_{\mathrm{E}}}{6\pi^2 l^2}\mathcal{M}(r)},$$

$$\mathcal{M}(r) = 2\pi^2\int_0^r \rho(\bar{r})\bar{r}^3 d\bar{r}$$

$$\frac{df(r)}{dr} = f'(r) = \mathrm{sgn}(\alpha)\frac{\kappa_{\mathrm{E}}p(r)r^3 + 12r\left(1 - e^{-2g(r)}\right)}{12l^2 e^{-2g(r)}\left(1 - e^{-2g(r)} + \mathrm{sgn}(\alpha)\frac{r^2}{l^2}\right)}$$

$$\frac{dp(r)}{dr} = p'(r) = -\mathrm{sgn}(\alpha)\frac{(\rho(r) + p(r))\left(\kappa_{\mathrm{E}}p(r)r^3 + 12r\left(1 - e^{-2g(r)}\right)\right)}{12l^2 e^{-2g(r)}\left(1 - e^{-2g(r)} + \mathrm{sgn}(\alpha)\frac{r^2}{l^2}\right)}$$

$$\begin{aligned}
\frac{dp(r)}{dr} = -\frac{\kappa_{\mathrm{E}}\mathcal{M}(r)\rho(r)}{12\pi^2 r^3}&\left(1 + \frac{p(r)}{\rho(r)}\right)\left(1 + \mathrm{sgn}(\alpha)\frac{\kappa_{\mathrm{E}}}{6\pi^2 r^4}l^2\mathcal{M}(r)\right)^{-1/2}\\
&\times\left[\frac{\pi^2 r^4 p(r)}{\mathcal{M}(r)} - \frac{12\mathrm{sgn}(\alpha)\pi^2 r^4}{\kappa_{\mathrm{E}}l^2\mathcal{M}(r)}\left(1 - \sqrt{1 + \mathrm{sgn}(\alpha)\frac{\kappa_{\mathrm{E}}}{6\pi^2 r^4}l^2\mathcal{M}(r)}\right)\right]\\
&\times\left[1 + \mathrm{sgn}(\alpha)\frac{r^2}{l^2}\left(1 - \sqrt{1 + \mathrm{sgn}(\alpha)\frac{\kappa_{\mathrm{E}}}{6\pi^2 r^4}l^2\mathcal{M}(r)}\right)\right]^{-1}
\end{aligned}$$

$$\sqrt{1 + \mathrm{sgn}(\alpha)\frac{\kappa_{\mathrm{E}}}{6\pi^2 r^4}l^2\mathcal{M}(r)} = 1 + \mathrm{sgn}(\alpha)\frac{\kappa_{\mathrm{E}}}{12\pi^2 r^4}l^2\mathcal{M}(r) + \mathcal{O}(l^4)$$

$$\frac{dp(r)}{dr} \approx -\frac{\frac{\kappa_{\mathrm{E}}\mathcal{M}(r)\rho(r)}{12\pi^2 r^3}\left(1 + \frac{p(r)}{\rho(r)}\right)\left(1 + \frac{\pi^2 r^4 p(r)}{\mathcal{M}(r)}\right)}{\left(1 + \mathrm{sgn}(\alpha)\frac{\kappa_{\mathrm{F}}}{6\pi^2 r^4}l^2\mathcal{M}(r)\right)\left(1 - \frac{\kappa_{\mathrm{F}}}{12\pi^2 r^2}\mathcal{M}(r)\right)}$$

$$\frac{dp(r)}{dr} = p'(r) \approx -\frac{\kappa_E\mathcal{M}(r)}{12\pi^2 r^3}\left(1 + \frac{p(r)}{\rho(r)}\right)\left(1 + \frac{\pi^2 r^4 p(r)}{\mathcal{M}(r)}\right)\left(1 - \frac{\kappa}{12\pi^2 r^2}\mathcal{M}(r)\right)^{-1}$$

$$\mathcal{M}'(r) = 2\pi^2 r^3 \rho(r)$$

$$f(r) = -\int_r^\infty \frac{\kappa_E \mathcal{M}(\bar{r})}{12\pi^2\bar{r}^3}\left(1+\text{sgn}(\alpha)\frac{\kappa_E}{6\pi^2\bar{r}^4}l^2\mathcal{M}(\bar{r})\right)^{-1/2} \times\left[\frac{\pi^2\bar{r}^4 p(\bar{r})}{\mathcal{M}(\bar{r})}-\frac{12\text{sgn}(\alpha)\pi^2\bar{r}^4}{\kappa_E l^2\mathcal{M}(\bar{r})}\left(1-\sqrt{1+\text{sgn}(\alpha)\frac{\kappa_E}{6\pi^2\bar{r}^4}l^2\mathcal{M}(\bar{r})}\right)\right] \times\left[1+\text{sgn}(\alpha)\frac{\bar{r}^2}{l^2}\left(1-\sqrt{1+\text{sgn}(\alpha)\frac{\kappa_E}{6\pi^2\bar{r}^4}l^2\mathcal{M}(\bar{r})}\right)\right]^{-1} d\bar{r}$$

$$\mathcal{M}(r) = M\,, p(r) = \rho(r) = 0$$

$$f(r) = \frac{1}{2}\ln\left[1+\text{sgn}(\alpha)\frac{r^2}{l^2}\left(1-\sqrt{1+\text{sgn}(\alpha)\frac{\kappa_E}{6\pi^2 r^4}l^2 M}\right)\right],$$

$$e^{2f(r)} = e^{-2g(r)} = 1+\text{sgn}(\alpha)\frac{r^2}{l^2}-\text{sgn}(\alpha)\sqrt{\frac{r^4}{l^4}+\text{sgn}(\alpha)\frac{\kappa_E}{6\pi^2 l^2}M},$$

$$\rho_0 + p(r) = Ce^{-f(r)}$$

$$\mathcal{M}(r) = \frac{\pi^2}{2}\rho_0 r^4$$

$$e^{-2g(r)} = 1+\text{sgn}(\alpha)\frac{r^2}{l^2}-\text{sgn}(\alpha)\sqrt{\frac{r^4}{l^4}+\text{sgn}(\alpha)\frac{\kappa_E}{12l^2}\rho_0 r^4}$$

$$\frac{e^{-2g}}{r^3}(f'+g')[r^2+\text{sgn}(\alpha)l^2(1-e^{-2g})] = \frac{\kappa_E}{12}(\rho_0+p).$$

$$e^f = \frac{\kappa_E}{12}Ce^{-g}\int \frac{r^3 dr}{e^{-3g}[r^2+\text{sgn}(\alpha)l^2(1-e^{-2g})]} + C_0 e^{-g}$$

$$\int \frac{r^3 dr}{e^{-3g}[r^2+\text{sgn}(\alpha)l^2(1-e^{-2g})]} = \frac{-\text{sgn}(\alpha)l^2 e^{g(r)}}{\sqrt{1+\text{sgn}(\alpha)\frac{\kappa_E}{12}l^2\rho_0}\left(1-\sqrt{1+\text{sgn}(\alpha)\frac{\kappa_E}{12}l^2\rho_0}\right)}$$

$$e^f = C_1 + C_0 e^{-g}$$

$$C_1 := -\frac{\text{sgn}(\alpha)\kappa_E l^2 C}{12\sqrt{1+\text{sgn}(\alpha)\frac{\kappa_E}{12}l^2\rho_0}\left(1-\sqrt{1+\text{sgn}(\alpha)\frac{\kappa_E}{12}l^2\rho_0}\right)}$$

$$C = \rho_0 \sqrt{1 + \operatorname{sgn}(\alpha)\frac{R^2}{l^2}\left(1 - \sqrt{1 + \operatorname{sgn}(\alpha)\frac{\kappa_E}{12}l^2\rho_0}\right)},$$

$$C_1 = -\frac{\operatorname{sgn}(\alpha)\kappa_E l^2 \rho_0 \sqrt{1 + \operatorname{sgn}(\alpha)\frac{R^2}{l^2}\left(1 - \sqrt{1 + \operatorname{sgn}(\alpha)\frac{\kappa_E}{12}l^2\rho_0}\right)}}{12\sqrt{1 + \operatorname{sgn}(\alpha)\frac{\kappa_E}{12}l^2\rho_0}\left(1 - \sqrt{1 + \operatorname{sgn}(\alpha)\frac{\kappa_E}{12}l^2\rho_0}\right)},$$

$$C_0 = -\frac{1}{\sqrt{1 + \operatorname{sgn}(\alpha)\frac{\kappa_E}{12}l^2\rho_0}}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu T^{\mu r} = \frac{f'(r)(\rho(r) + p(r)) + p'(r)}{e^{2g(r)}}$$

$$f' = -\frac{p'}{\rho + p}$$

$$f'(r) = \frac{\kappa_E \mathcal{M}(r)}{12\pi^2 r^3}\left(1 + \frac{\pi^2 r^4 p(r)}{\mathcal{M}(r)}\right)\left(1 - \frac{\kappa_E}{12\pi^2 r^2}\mathcal{M}(r)\right)^{-1}$$

$$p'(r) = -\frac{\kappa_E \mathcal{M}(r)}{12\pi^2 r^3}\left(1 + \frac{p(r)}{\rho(r)}\right)\left(1 + \frac{\pi^2 r^4 p(r)}{\mathcal{M}(r)}\right)\left(1 - \frac{\kappa_E}{12\pi^2 r^2}\mathcal{M}(r)\right)^{-1}$$

$$\hat{T}_a := T_{\mu\nu} e_a^\mu dx^\nu$$

$$\nabla_\mu T_\nu^\mu = -e_\nu^a \star D_\omega(\star \hat{T}_a)$$

$$\star \hat{T}_a = \frac{\sqrt{-g}}{4!}\epsilon_{\mu\nu\rho\sigma\tau} T_a^\mu dx^\nu dx^\rho dx^\sigma dx^\tau.$$

$$-e_\nu^a \star D_\omega(\star \hat{T}_a) = \frac{1}{\sqrt{-g}}\partial_\lambda(\sqrt{-g})T_\nu{}^\lambda + \partial_\lambda T_\nu{}^\lambda - T_a{}^\lambda(\partial_\lambda e_\nu^a + \omega_{\lambda b}^a e_\nu^b),$$

$$\partial_\lambda e_\nu^a + \omega^a{}_{\lambda b} e_\nu^b - \Gamma_{\lambda\nu}{}^\rho e_\rho^a = 0,$$

$$-e_\nu^a \star D_\omega(\star \hat{T}_a) = \frac{1}{\sqrt{-g}}\partial_\lambda(\sqrt{-g})T_\nu{}^\rho + \partial_\lambda T_\nu{}^\lambda - \Gamma_{\lambda\nu}{}^\rho T_\rho{}^\lambda$$
$$-e_\nu^a \star D_\omega(\star \hat{T}_a) = \partial_\lambda T_\nu{}^\lambda + \Gamma_{\lambda\rho}{}^\lambda T_\nu{}^\rho - \Gamma_{\lambda\nu}{}^\rho T_\rho{}^\lambda = \nabla_\lambda T_\nu{}^\lambda$$

$$\star P = \frac{\sqrt{|g|}}{(d-p)!\,p!}\varepsilon_{\alpha_1 \cdots \alpha_d} g^{\alpha_1\beta_1} \cdots g^{\alpha_p\beta_p} P_{\beta_1 \cdots \beta_p} dx^{\alpha_{p+1}} \cdots dx^{\alpha_d}$$

$$\hat{T}_a = T_{ab} e^b$$

$$T_{TT} = \rho(r)\,, T_{RR} = T_{ii} = p(r)$$

$$D_\omega(\star\hat{T}_a)=0$$

$$D_\omega(\star\hat{T}_a)=D_\omega(T_{ab}\star e^b)=\frac{1}{4!}\epsilon_{fbcde}(D_\omega T_a^f)e^be^ce^de^e$$

$$D_\omega(\star\hat{T}_a)=\frac{1}{4!}\epsilon_{fbcde}(dT_a^f+\omega_a^g T_g^f+\omega_g^f T_a^g)e^be^ce^de^e$$

$$D_\omega(\star\hat{T}_R)=e^{-g}(p'+f'(\rho+p))e^Te^Re^1e^2e^3=0$$

$$p'+f'(\rho+p)=0$$

$$h_a=h_{\mu\nu}e_a^\mu dx^\nu$$

$$\begin{aligned}
\xi_0 &= \partial_t\\
\xi_1 &= \partial_{\theta_3}\\
\xi_2 &= \sin\theta_3\partial_{\theta_2}+\cot\theta_2\cos\theta_3\partial_{\theta_3}\\
\xi_3 &= \sin\theta_2\sin\theta_3\partial_{\theta_1}+\cot\theta_1\cos\theta_2\sin\theta_3\partial_{\theta_2}+\cot\theta_1\csc\theta_2\cos\theta_3\partial_{\theta_3}\\
\xi_4 &= \cos\theta_3\partial_{\theta_2}-\cot\theta_2\sin\theta_3\partial_{\theta_3}\\
\xi_5 &= \sin\theta_2\cos\theta_3\partial_{\theta_1}+\cot\theta_1\cos\theta_2\cos\theta_3\partial_{\theta_2}-\cot\theta_1\csc\theta_2\sin\theta_3\partial_{\theta_3}\\
\xi_6 &= \cos\theta_2\partial_{\theta_1}-\cot\theta_1\sin\theta_2\partial_{\theta_2}
\end{aligned}$$

$$\begin{aligned}
h^T &= h_{tt}(r)e^T+h_{tr}(r)e^R\\
h^R &= h_{rt}(r)e^T+h_{rr}(r)e^R\\
h^i &= h(r)e^i
\end{aligned}$$

$$\epsilon_{abcde}R^{cd}Dh^e=0$$

$$Dh^a=dh^a+\omega_b^a h^b$$

$$\begin{aligned}
Dh^T &= e^{-g}(-h_{tt}'-f'h_{tt}+f'h_{rr})e^Te^R\\
Dh^R &= e^{-g}(-h_{rt}'-f'h_{rt}+f'h_{tr})e^Te^R\\
Dh^i &= \frac{e^{-g}}{r}(rh'+h-h_{rr})e^Re^i-\frac{e^{-g}}{r}h_{rt}e^Te^i
\end{aligned}$$

$$\begin{aligned}
h_{tr} &= h_{rt}=0\\
h_r &= (rh)'\\
h_t' &= f'(h_r-h_t)
\end{aligned}$$

$$h_t(r)=h_t(f(r))$$

$$\frac{dh_t(f)}{df}f'(r)=f'(h_r-h_t)$$

$$\dot{h}_t+h_t=h_r$$

$$\dot{h}_t:=\frac{dh_t(f)}{df}$$

$$h_t^{\mathrm{h}}(f)=Ae^{-f(r)}$$

$$h_r(r) = h_r(f(r)) = \sum_{n=0}^{\infty} B_n e^{nf(r)} + \sum_{m=2}^{\infty} C_m e^{-mf(r)},$$

$$h_t^{\mathrm{p}}(f) = \sum_{n=0}^{\infty} \frac{B_n}{n+1} e^{nf(r)} - \sum_{m=2}^{\infty} \frac{C_m}{m-1} e^{-mf(r)}.$$

$$h_t(f(r)) = Ae^{-f(r)} + \sum_{n=0}^{\infty} \frac{B_n}{n+1} e^{nf(r)} - \sum_{m=2}^{\infty} \frac{C_m}{m-1} e^{-mf(r)}$$

$$h(r) = \frac{1}{r}\left(\int h_r(r)dr + D\right)$$

$$h(r) = \frac{1}{r}\sum_{n=0}^{\infty} \left(B_n \int e^{nf(r)}dr\right) + \frac{1}{r}\sum_{m=2}^{\infty} \left(C_m \int e^{-mf(r)}dr\right) + \frac{D}{r}$$

$$h_r(r) = h = \hbar$$

$$h(r) = h + \frac{D}{r}$$

$$h_t(r) = Ae^{-f(r)} + h.$$

$$e^{2f(r\to\infty)} = e^{-2g(r\to\infty)} = 1$$

$$h_r(r \to \infty) = h, h(r \to \infty) = h, h_t(r \to \infty) = A + h.$$

$$h_r(r) = h, h(r) = h$$

$$h_t(r) = \begin{cases} \dfrac{A}{C_0 + C_1 e^{-g(r)}} + h & \text{if } r < R \\ \dfrac{A}{e^{-g(r)}} + h & \text{if } r \geq R \end{cases}$$

$$e^{-g(r)} = \begin{cases} \sqrt{1 + \mathrm{sgn}(\alpha)\dfrac{r^2}{l^2} - \mathrm{sgn}(\alpha)\sqrt{\dfrac{r^4}{l^4} + \mathrm{sgn}(\alpha)\dfrac{\kappa_{\mathrm{E}}}{6\pi^2 l^2}\mathcal{M}(r)}} & \text{if } r < R \\ \sqrt{1 + \mathrm{sgn}(\alpha)\dfrac{r^2}{l^2} - \mathrm{sgn}(\alpha)\sqrt{\dfrac{r^4}{l^4} + \mathrm{sgn}(\alpha)\dfrac{\kappa_{\mathrm{E}}}{6\pi^2 l^2}M}} & \text{if } r \geq R \end{cases}$$

SUPLEMENTO. ECUACIONES ADICIONALES.

$$\mathcal{L}_{\text{gravity}} = -\frac{2}{\kappa^2}\sqrt{g}R$$

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4g^2} F_{\mu\nu}^a F^{a\mu\nu}$$

$$G_{3\mu\alpha,\nu\beta,\rho\gamma}(k,p,q) = -\frac{i}{32}\kappa\big[V_{3\mu\nu\rho}(k,p,q) \times V_{3\alpha\beta\gamma}(k,p,q) + \{\mu \leftrightarrow \alpha\},\{\nu \leftrightarrow \beta\},\{\rho \leftrightarrow \gamma\}\big]$$

$$G_{3\mu\alpha,\nu\beta,\rho\gamma}(k_1,k_2,k_3) \sim k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\rho\gamma} + \cdots$$

$$V_{3\mu\nu\rho}(k,p,q) = \eta_{\nu\rho}(p-q)_\mu - 2\eta_{\mu\rho}k_\nu + 2\eta_{\mu\nu}k_\rho$$

$$V^{\text{closed}} = V^{\text{open}}_{\text{left}} \bar{V}^{\text{open}}_{\text{right}}$$

$$A_n \sim \int \frac{dx_1 \cdots dx_n}{\mathcal{V}_{abc}} \prod_{1\le i<j\le n} |x_i - x_j|^{k_i\cdot k_j} \exp\left[\sum_{i<j}\left(\frac{\epsilon_i\cdot\epsilon_j}{(x_i-x_j)^2} + \frac{k_i\cdot\epsilon_j - k_j\cdot\epsilon_i}{(x_i-x_j)}\right)\right]\Bigg|_{\text{supercurvature}}$$

$$\mathcal{V}_{abc} = \frac{dx_a dx_b dx_c}{|(x_a-x_b)(x_b-x_c)(x_c-x_a)|}$$

$$M_n \sim \int \frac{d^2z_1 \cdots d^2z_n}{\Delta_{abc}} \prod_{1\le i<j\le n} (z_i - z_j)^{k_i\cdot k_j} \exp\left[\sum_{i<j}\left(\frac{\epsilon_i\cdot\epsilon_j}{(z_i-z_j)^2} + \frac{k_i\cdot\epsilon_j - k_j\cdot\epsilon_i}{(z_i-z_j)}\right)\right]$$
$$\times \prod_{1\le i<j\le n} (\bar{z}_i - \bar{z}_j)^{k_i\cdot k_j} \exp\left[\sum_{i<j}\left(\frac{\bar{\epsilon}_i\cdot\bar{\epsilon}_j}{(\bar{z}_i-\bar{z}_j)^2} + \frac{k_i\cdot\bar{\epsilon}_j - k_j\cdot\bar{\epsilon}_i}{(\bar{z}_i-\bar{z}_j)}\right)\right]\Bigg|_{\text{supercurvature}}$$

$$\Delta_{abc} = \frac{d^2z_a d^2z_b d^2z_c}{|z_a-z_b|^2|z_b-z_c|^2|z_c-z_a|^2}$$

$$\epsilon_i^{\mu\nu} = \epsilon_i^\mu \bar{\epsilon}_i^\nu$$

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12}A_4^{\text{tree}}(1,2,3,4)A_4^{\text{tree}}(1,2,4,3)$$
$$M_5^{\text{tree}}(1,2,3,4,5) = is_{12}s_{34}A_5^{\text{tree}}(1,2,3,4,5)A_5^{\text{tree}}(2,1,4,3,5)$$
$$+is_{13}s_{24}A_5^{\text{tree}}(1,3,2,4,5)A_5^{\text{tree}}(3,1,4,2,5)$$

$$\mathcal{M}_n^{\text{tree}}(1,2,\ldots n) = \left(\frac{\kappa}{2}\right)^{(n-2)} M_n^{\text{tree}}(1,2,\ldots n)$$
$$\mathcal{A}_n^{\text{tree}}(1,2,\ldots n) = g^{(n-2)} \sum_{\sigma\in S_n/Z_n} \text{Tr}(T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}\cdots T^{a_{\sigma(n)}})A_n^{\text{tree}}(\sigma(1),\sigma(2),\ldots,\sigma(n)),$$

$$\mathcal{A}_4^{\text{tree}}(1,2,3,4) = g^2\big[\tilde{f}^{a_2a_3c}\tilde{f}^{ca_4a_1}A_4^{\text{tree}}(1,2,3,4) + \tilde{f}^{a_1a_3c}\tilde{f}^{ca_4a_2}A_4^{\text{tree}}(2,1,3,4)\big]$$

$$\tilde{f}^{abc} = i\sqrt{2}f^{abc} = \text{Tr}\big([T^a,T^b]T^c\big)$$

$$M_4^{\text{tree}}(1_h,2_h,3_{\tilde{h}},4_{\tilde{h}}) = -is_{12}A_4^{\text{tree}}\big(1_g,2_g,3_g,4_g\big)A_4^{\text{tree}}\big(1_g,2_g,4_{\tilde{g}},3_{\tilde{g}}\big)$$

$$A_4^{\text{tree}}(1,2,3,4) = \frac{-4i}{st}(t_8)_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}k_1^{\mu_1}k_2^{\mu_2}k_3^{\mu_3}k_4^{\mu_4}\epsilon_1^{\nu_1}\epsilon_2^{\nu_2}\epsilon_3^{\nu_3}\epsilon_4^{\nu_4} \equiv \frac{-4iK}{st}$$

$$M_4^{\text{tree}}(1,2,3,4) = \frac{16iK^2}{stu}$$

$$stuM_4^{\text{tree}}(1,2,3,4) = -i(st[A_4^{\text{tree}}(1,2,3,4)])^2.$$

$$A_4^{\text{tree}}\left(1^-,2^-,3^+,4^+\right) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle},$$

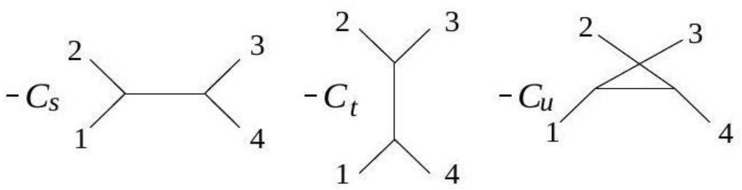
$$M_4^{\text{tree}}\left(1^-,2^-,3^+,4^+\right) = -is_{12}\frac{\langle 12\rangle^8}{\langle 12\rangle^2\langle 23\rangle\langle 24\rangle\langle 34\rangle^2\langle 31\rangle\langle 41\rangle}$$

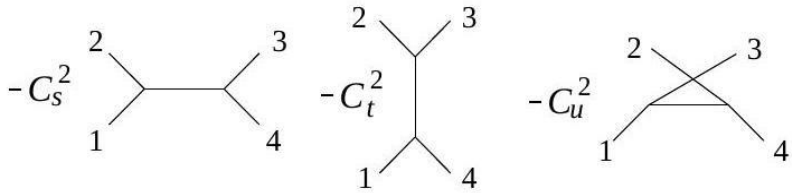
$$A_4^{\text{tree}}\left(1^-,2^-,3^+,4^+\right) = -\frac{i}{t}\times\left[s\frac{\langle 12\rangle}{[12]}\frac{[34]}{\langle 34\rangle}\right]$$
$$A_4^{\text{tree}}\left(1^-,2^-,4^+,3^+\right) = -\frac{i}{u}\times\left[s\frac{\langle 12\rangle}{[12]}\frac{[34]}{\langle 34\rangle}\right]$$

$$M_4^{\text{tree}}\left(1^-,2^-,3^+,4^+\right) = -i\left(\frac{1}{t}+\frac{1}{u}\right)\left[s\frac{\langle 12\rangle}{[12]}\frac{[34]}{\langle 34\rangle}\right]^2,$$

$$C_s = 0, C_t = C_u = s\frac{\langle 12\rangle}{[12]}\frac{[34]}{\langle 34\rangle}$$

$$C_s = 0, C_t = C_u = \left(\frac{\langle 42\rangle}{\langle 12\rangle}\right)^{2(1-\lambda)}\times\left[s\frac{\langle 12\rangle}{[12]}\frac{[34]}{\langle 34\rangle}\right]$$





$$\int\frac{d^Dp}{(2\pi)^D}\frac{\exp\left[\epsilon_i\cdot p_i\right]\times\left(\mathfrak{O}_{\text{oscillator contributions}}\right)\big|_{\text{supercurvature}}}{p_1^2p_2^2\cdots p_n^2}$$

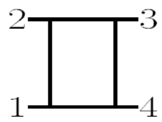
$$p_i = p - k_1 - k_2 - \cdots - k_{i-1} = p + k_i + \cdots + k_n$$

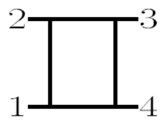
$$\int\frac{d^Dp}{(2\pi)^D}\frac{\exp\left[\epsilon_i\cdot p_i+\bar{\epsilon}_i\cdot p_i\right]\times\left(\mathbb{O}_{\text{left oscillator contrib.}}\right)\times\left(\mathcal{O}_{\text{right oscillator contrib.}}\right)\big|_{\text{supercurvature}}}{p_1^2p_2^2\cdots p_n^2}.$$

$$\mathcal{A}_4^{N=4,\text{1-loop}}(1,2,3,4) = ig^4s_{12}s_{23}A_4^{\text{tree}}(1,2,3,4)\Big(C_{1234}\mathcal{I}_4^{1-\text{ loop}}(s_{12},s_{23}) + C_{3124}\mathcal{I}_4^{1-\text{ loop}}(s_{12},s_{13})$$
$$+C_{2314}\mathcal{I}_4^{1-\text{ loop}}(s_{23},s_{13})\Big)$$

$$\mathcal{I}_4^{1-\text{loop}}(s_{12},s_{23}) = \int\frac{d^Dp}{(2\pi)^D}\frac{1}{p^2(p-k_1)^2(p-k_1-k_2)^2(p+k_4)^2}$$

$$\mathcal{M}_4^{N=8,\text{1-loop}}(1,2,3,4) = -i\left(\frac{\kappa}{2}\right)^4 s_{12}s_{23}s_{13}M_4^{\text{tree}}(1,2,3,4)\left(\mathcal{I}_4^{1-\text{loop}}(s_{12},s_{23}) + \mathcal{I}_4^{1-\text{loop}}(s_{12},s_{13}) + \mathcal{I}_4^{1-\text{loop}}(s_{23},s_{13})\right)$$

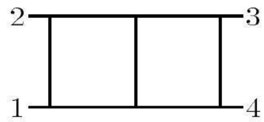
$ist\ A_4^{\text{tree}}$ 

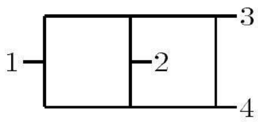
$\left[ist\ A_4^{\text{tree}}\right]^2$ 

$$\mathcal{A}_4^{2-\text{loop}}(1,2,3,4) = -g^6 s_{12}s_{23}A_4^{\text{tree}}(1,2,3,4)\left(C_{1234}^{\text{P}}s_{12}\mathcal{I}_4^{2-\text{loop},\text{P}}(s_{12},s_{23}) + C_{3421}^{\text{P}}s_{12}\mathcal{I}_4^{2-\text{loop},\text{P}}(s_{12},s_{24}) + C_{1234}^{\text{NP}}s_{12}\mathcal{I}_4^{2-\text{loop},\text{NP}}(s_{12},s_{23}) + C_{3421}^{\text{NP}}s_{12}\mathcal{I}_4^{2-\text{loop},\text{NP}}(s_{12},s_{24}) + \text{ cyclic }\right)$$

$$\mathcal{I}_4^{2-\text{loop},\text{P}}(s_{12},s_{23}) = \int \frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{1}{p^2(p-k_1)^2(p-k_1-k_2)^2(p+q)^2q^2(q-k_4)^2(q-k_3-k_4)^2}$$

$$\mathcal{I}_4^{2-\text{loop},\text{NP}}(s_{12},s_{23}) = \int \frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{1}{p^2(p-k_2)^2(p+q)^2(p+q+k_1)^2q^2(q-k_3)^2(q-k_3-k_4)^2}$$





$$t_8F^4 \equiv t_8^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}F_{\mu_1\nu_1}^aF_{\mu_2\nu_2}^bF_{\mu_3\nu_3}^cF_{\mu_4\nu_4}^dC_{abcd} = 4!\left(F_{\alpha\beta}^aF^{b\beta\gamma}F_{\gamma\delta}^cF^{d\delta\alpha} - \frac{1}{4}F_{\alpha\beta}^aF^{b\alpha\beta}F_{\gamma\delta}^cF^{d\gamma\delta}\right)C_{abcd}$$

$$t_8F^4 = \frac{3}{2}(F-\tilde{F})^2(F+\tilde{F})^2 = \frac{3}{2}\left(F_{\alpha\beta}-\tilde{F}_{\alpha\beta}\right)\left(F^{\alpha\beta}-\tilde{F}^{\alpha\beta}\right)\left(F_{\gamma\delta}+\tilde{F}_{\gamma\delta}\right)\left(F^{\gamma\delta}+\tilde{F}^{\gamma\delta}\right)$$

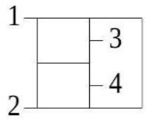
$$t_8^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}\partial_\alpha F_{\mu_1\nu_1}^a\partial^\alpha F_{\mu_2\nu_2}^bF_{\mu_3\nu_3}^cF_{\mu_4\nu_4}^d\tilde{C}_{abcd}$$

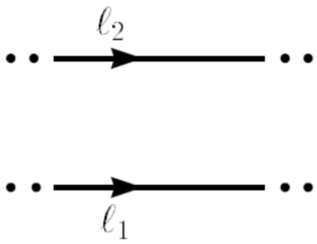
$$(F-\tilde{F})^2\partial^2(F+\tilde{F})^2$$

$$T_D\left(F_{\alpha\beta}F^{\beta\gamma}F_{\gamma\delta}F^{\delta\alpha} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta} + \cdots\right)$$

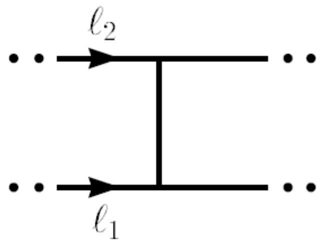
$$T_7 = -\frac{g^6\pi}{(4\pi)^7 2\epsilon}\left[s\left(\frac{1}{10}\left(C_{1234}^{\text{P}} + C_{1243}^{\text{P}}\right) + \frac{2}{15}C_{1234}^{\text{NP}}\right) + \text{ cyclic }\right]$$

$$T_9 = -\frac{g^6\pi s}{(4\pi)^9 4\epsilon}\left[\frac{1}{99792}(-45s^2+18st+2t^2)C_{1234}^{\text{P}} + \frac{1}{99792}(-45s^2+18su+2u^2)C_{1243}^{\text{P}} - \frac{2}{83160}(75s^2+2tu)C_{1234}^{\text{NP}}\right] + \text{ cyclic}$$



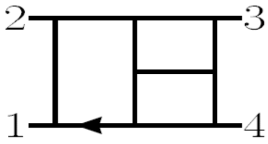


$$\longrightarrow\ i\,(\ell_1+\ell_2)^2\times$$



$$\int (d^D p)^3\,\frac{(p^2)}{(p^2)^{10}}$$

$$-ist\,A_4^{\text{tree}}\,s(\ell+k_4)^2$$



$$+\ \text{cyclic}$$

$$\int (d^D p)^L\,\frac{(p^2)^{(L-2)}}{(p^2)^{3L+1}}$$

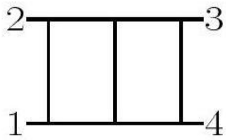
$$D<\frac{6}{L}+4,(L>1)$$

$$D<\frac{2(N-1)}{L}+4, N=1,2,4.$$

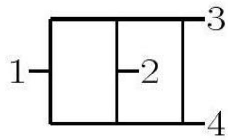
$$\mathcal{M}_4^{\text{2-loop}}(1,2,3,4)=-i\left(\frac{\kappa}{2}\right)^6[s_{12}s_{23}A_4^{\text{tree}}(1,2,3,4)]^2\Big(s_{12}^2\mathcal{I}_4^{2-\text{ loop ,P}}(s_{12},s_{23})+s_{12}^2\mathcal{I}_4^{2-\text{ loop ,P}}(s_{12},s_{24})$$
$$+s_{12}^2\mathcal{I}_4^{2-\text{ loop ,NP}}(s_{12},s_{23})+s_{12}^2\mathcal{I}_4^{2-\text{ loop ,NP}}(s_{12},s_{24})\ +\ \text{cyclic}\Big)$$

$$\mathcal{M}_4^{\text{2-loop}}(1,2,3,4)=\left(\frac{\kappa}{2}\right)^6 s_{12}s_{23}s_{13}M_4^{\text{tree}}(1,2,3,4)\Big(s_{12}^2\mathcal{I}_4^{2-\text{ loop ,P}}(s_{12},s_{23})+s_{12}^2\mathcal{I}_4^{2-\text{ loop, P}}(s_{12},s_{24})$$
$$+s_{12}^2\mathcal{I}_4^{2-\text{ loop, NP}}(s_{12},s_{23})+s_{12}^2\mathcal{I}_4^{2-\text{ loop, NP}}(s_{12},s_{24})\ +\ \text{cyclic}\Big)$$

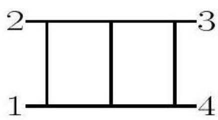
s



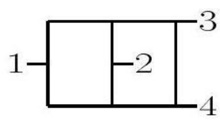
s



s^2



s^2



$$\mathcal{M}_4^{2-\text{ loop },D=7-2\epsilon}\Big|_{\text{pole}} = \frac{1}{2\epsilon(4\pi)^7}\frac{\pi}{3}(s^2+t^2+u^2)\times\left(\frac{\kappa}{2}\right)^6 \times stuM_4^{\text{tree}}$$

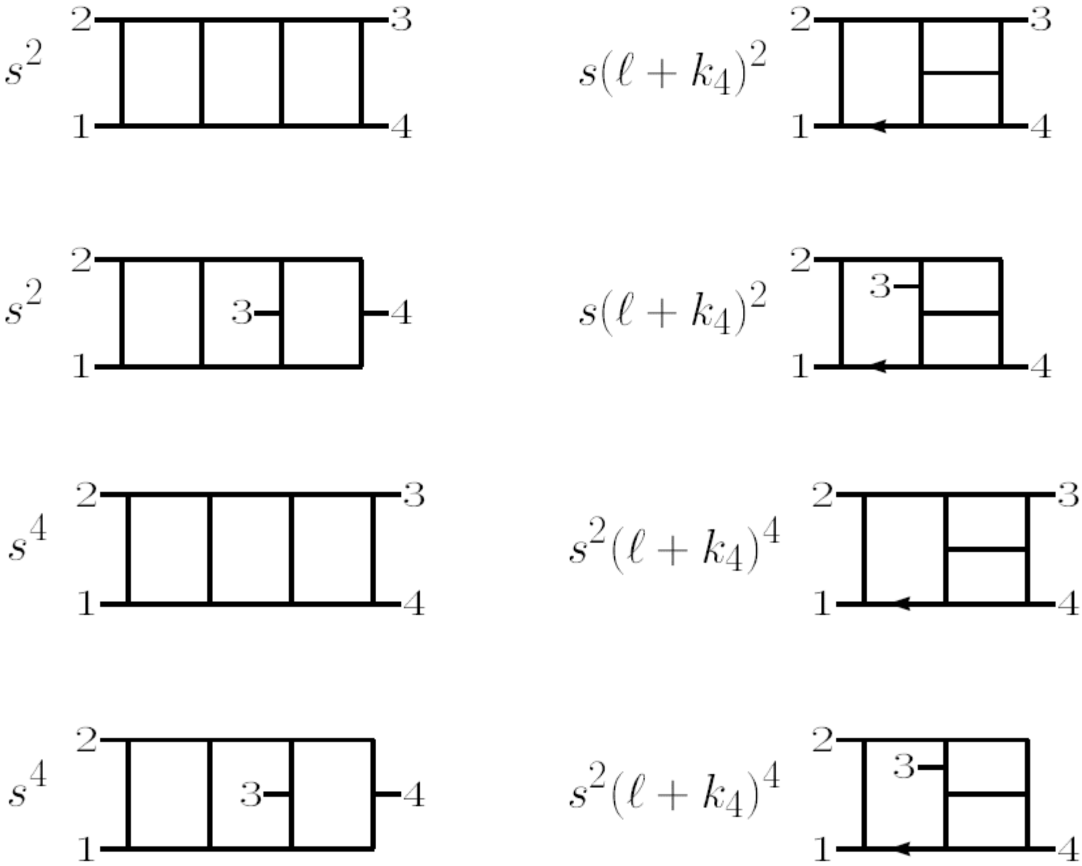
$$\mathcal{M}_4^{2-\text{ loop },D=9-2\epsilon}\Big|_{\text{pole}} = \frac{1}{4\epsilon(4\pi)^9}\frac{-13\pi}{9072}(s^2+t^2+u^2)^2\times\left(\frac{\kappa}{2}\right)^6 \times stuM_4^{\text{tree}}$$

$$\mathcal{M}_4^{2-\text{ loop },D=11-2\epsilon}\Big|_{\text{pole}} = \frac{1}{48\epsilon(4\pi)^{11}}\frac{\pi}{5791500}(438(s^6+t^6+u^6)-53s^2t^2u^2)\times\left(\frac{\kappa}{2}\right)^6 \times stuM_4^{\text{tree}}$$

$$\mathcal{M}_4^{2-\text{ loop },D=8-2\epsilon}\Big|_{\text{pole}} = \frac{1}{2(4\pi)^8}\left(-\frac{1}{24\epsilon^2}+\frac{1}{144\epsilon}\right)(s^3+t^3+u^3)\times\left(\frac{\kappa}{2}\right)^6 \times stuM_4^{\text{tree}}$$

$$\mathcal{M}_4^{2-\text{ loop },D=10-2\epsilon}\Big|_{\text{pole}} = \frac{1}{12\epsilon(4\pi)^{10}}\frac{-13}{25920}\,\text{stu}\,(s^2+t^2+u^2)\times\left(\frac{\kappa}{2}\right)^6 \times stuM_4^{\text{tree}}$$

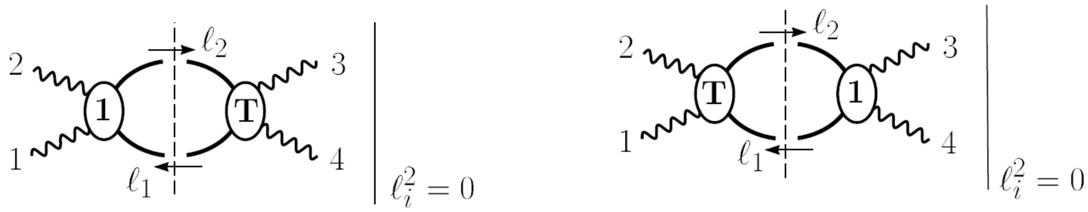
$$t_8t_8R^4 \equiv t_8^{\mu_1\mu_2\cdots\mu_8}t_8^{\nu_1\nu_2\cdots\nu_8}R_{\mu_1\mu_2\nu_1\nu_2}R_{\mu_3\mu_4\nu_3\nu_4}R_{\mu_5\mu_6\nu_5\nu_6}R_{\mu_7\mu_8\nu_7\nu_8}$$



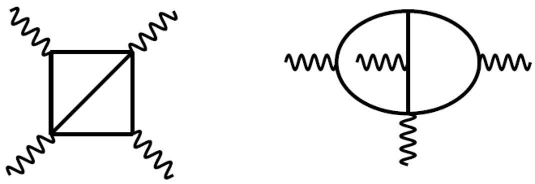
$$\int (d^Dp)^L \frac{(p^2)^{2(L-2)}}{(p^2)^{3L+1}}$$

$$D < \frac{10}{L}+2,\,(L>1)$$

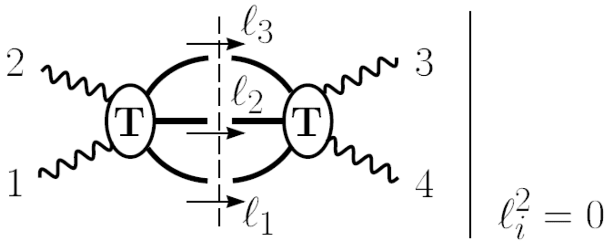
$$M_4^{2-\text{ loop}}(1,2,3,4)\Big|_{\text{cut(a)}} = \int \frac{d^D\ell_1}{(2\pi)^D}\sum_{S_1,S_2}\frac{i}{\ell_1^2}M_4^{\text{tree}}\left(-\ell_1^{S_1},1,2,\ell_2^{S_2}\right)\frac{i}{\ell_2^2}M_4^{1-\text{ loop}}\left(-\ell_2^{S_2},3,4,\ell_1^{S_1}\right)\Bigg|_{\ell_1^2=\ell_2^2=0}$$



(a) (b)



$$M_4^{2-\text{ loop}}(1,2,3,4)\Big|_{3-\text{ cut}}$$
$$= \int \frac{d^D \ell_1}{(2\pi)^D} \frac{d^D \ell_2}{(2\pi)^D} \sum_{S_1,S_2,S_3} M_5^{\text{tree}}\left(1,2,\ell_3^{S_3},\ell_2^{S_2},\ell_1^{S_1}\right) \frac{i}{\ell_1^2}\frac{i}{\ell_2^2}\frac{i}{\ell_3^2} M_5^{\text{tree}}\left(3,4,-\ell_1^{S_1},-\ell_2^{S_2},-\ell_3^{S_3}\right)\Bigg|_{\ell_i^2=0}$$



$$\sum_{S_1,S_2 \in \{N=4\}} A_4^{\text{tree}}\left(-\ell_1^{S_1},1,2,\ell_2^{S_2}\right) \times A_4^{\text{tree}}\left(-\ell_2^{S_2},3,4,\ell_1^{S_1}\right) = -istA_4^{\text{tree}}(1,2,3,4)\frac{1}{(\ell_1-k_1)^2}\frac{1}{(\ell_2-k_3)^2}$$

$$\sum_{N=8 \text{ states}} M_4^{\text{tree}}(-\ell_1,1,2,\ell_2) \times M_4^{\text{tree}}(-\ell_2,3,4,\ell_1)$$
$$= -s^2\left(\sum_{N=4 \text{ states}} A_4^{\text{tree}}(-\ell_1,1,2,\ell_2) \times A_4^{\text{tree}}(-\ell_2,3,4,\ell_1)\right)$$
$$\times\left(\sum_{N=4 \text{ states}} A_4^{\text{tree}}(\ell_2,1,2,-\ell_1) \times A_4^{\text{tree}}(\ell_1,3,4,-\ell_2)\right)$$
$$= s^2(st)^2[A_4^{\text{tree}}(1,2,3,4)]^2 \frac{1}{(\ell_1-k_1)^2(\ell_2-k_3)^2(\ell_2+k_1)^2(\ell_1+k_3)^2}$$
$$= is^2stuM_4^{\text{tree}}(1,2,3,4)\frac{1}{(\ell_1-k_1)^2(\ell_2-k_3)^2(\ell_1-k_2)^2(\ell_2-k_4)^2}$$

$$-\frac{s}{(\ell_1-k_1)^2(\ell_1-k_2)^2} = \frac{1}{(\ell_1-k_1)^2} + \frac{1}{(\ell_1-k_2)^2}$$
$$-\frac{s}{(\ell_2-k_3)^2(\ell_2-k_4)^2} = \frac{1}{(\ell_2-k_3)^2} + \frac{1}{(\ell_2-k_4)^2}$$

$$\sum_{N=8\text{ states}} M_4^{\text{tree}}(-\ell_1,1,2,\ell_2)\times M_4^{\text{tree}}(-\ell_2,3,4,\ell_1)$$
$$= istuM_4^{\text{tree}}(1,2,3,4)\left[\frac{1}{(\ell_1-k_1)^2}+\frac{1}{(\ell_1-k_2)^2}\right]\left[\frac{1}{(\ell_2-k_3)^2}+\frac{1}{(\ell_2-k_4)^2}\right]$$

$$M_4^{2-\text{loop}}(1,2,3,4)\Big|_{s-\text{cut}} = \int\frac{d^D\ell_1}{(2\pi)^D}\sum_{N=8\text{ states}}\frac{i}{\ell_1^2}M_4^{\text{tree}}(-\ell_1,1,2,\ell_2)\frac{i}{\ell_2^2}M_4^{1-\text{loop}}(-\ell_2,3,4,\ell_1)\Big|_{\ell_1^2=\ell_2^2=0}$$

$$M_4^{2-\text{loop}}(1,2,3,4)\Big|_{s-\text{cut}} = -stuM_4^{\text{tree}}\int\frac{d^D\ell_1}{(2\pi)^D}s(\ell_2-k_3)^2(\ell_2-k_4)^2$$
$$\times\left[\frac{1}{(\ell_1-k_1)^2}+\frac{1}{(\ell_1-k_2)^2}\right]\frac{i}{\ell_1^2}\left[\frac{s}{(\ell_2-k_3)^2(\ell_2-k_4)^2}\right]\frac{i}{\ell_2^2}$$
$$\times\left[\mathcal{I}_4^{1-\text{loop}}(s,(\ell_2-k_3)^2)+\mathcal{I}_4^{1-\text{loop}}((\ell_2-k_3)^2,(\ell_2-k_4)^2)+\mathcal{I}_4^{1-\text{loop}}((\ell_2-k_4)^2,s)\right]\Big|_{\ell_1^2=\ell_2^2=0}$$

$$M_4^{2-\text{loop}}(1,2,3,4)\Big|_{s-\text{cut}} = stuM_4^{\text{tree}}s^2\big(\mathcal{I}_4^{2-\text{loop, P}}(s,t)+\mathcal{I}_4^{2-\text{loop, P}}(s,u)$$
$$+\mathcal{I}_4^{2-\text{loop, NP}}(s,t)+\mathcal{I}_4^{2-\text{loop, NP}}(s,u)\big)\Big|_{s-\text{cut}}$$

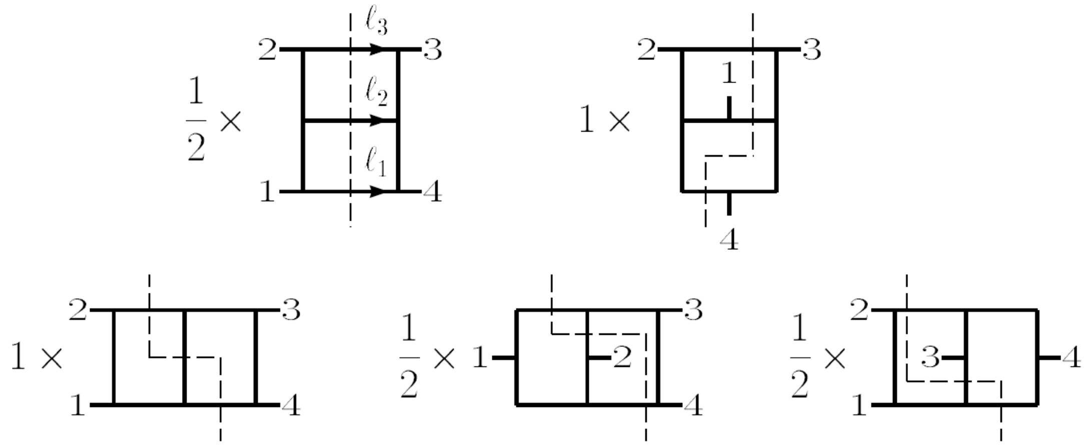
$$M_5^{\text{tree}}(\ell_1,1,2,\ell_3,\ell_2)=i(\ell_1+k_1)^2(\ell_3+k_2)^2A_5^{\text{tree}}(\ell_1,1,2,\ell_3,\ell_2)A_5^{\text{tree}}(1,\ell_1,\ell_3,2,\ell_2)+\{1\leftrightarrow 2\},$$

$$M_5^{\text{tree}}(-\ell_3,3,4,-\ell_1,-\ell_2)=i(\ell_3-k_3)^2(\ell_1-k_4)^2A_5^{\text{tree}}(-\ell_3,3,4,-\ell_1,-\ell_2)A_5^{\text{tree}}(3,-\ell_3,-\ell_1,4,-\ell_2)$$
$$+\{3\leftrightarrow 4\}$$

$$\sum_{N=8\text{ states}} M_5^{\text{tree}}(1,2,\ell_3,\ell_2,\ell_1)M_5^{\text{tree}}(3,4,-\ell_1,-\ell_2,-\ell_3)$$
$$=-(\ell_1+k_1)^2(\ell_3+k_2)^2(\ell_3-k_3)^2(\ell_1-k_4)^2$$
$$\times\left[\sum_{N=4\text{ states}}A_5^{\text{tree}}(\ell_1,1,2,\ell_3,\ell_2)A_5^{\text{tree}}(-\ell_3,3,4,-\ell_1,-\ell_2)\right]$$
$$\times\left[\sum_{N=4\text{ states}}A_5^{\text{tree}}(1,\ell_1,\ell_3,2,\ell_2)A_5^{\text{tree}}(3,-\ell_3,-\ell_1,4,-\ell_2)\right]$$
$$+\{1\leftrightarrow 2\}+\{3\leftrightarrow 4\}+\{1\leftrightarrow 2,3\leftrightarrow 4\}$$

$$\sum_{N=4\text{ states}} A_5^{\text{tree}}(\ell_1,1,2,\ell_3,\ell_2)A_5^{\text{tree}}(-\ell_3,3,4,-\ell_1,-\ell_2)=-istA_4^{\text{tree}}(1,2,3,4)$$
$$\times\left[\frac{s}{(\ell_1-k_4)^2(\ell_3+k_2)^2(\ell_1+\ell_2)^2(\ell_2+\ell_3)^2}+\frac{s}{(\ell_3-k_3)^2(\ell_1+k_1)^2(\ell_1+\ell_2)^2(\ell_2+\ell_3)^2}\right.$$
$$\left.+\frac{t}{(\ell_3-k_3)^2(\ell_1-k_4)^2(\ell_3+k_2)^2(\ell_1+k_1)^2}\right]$$

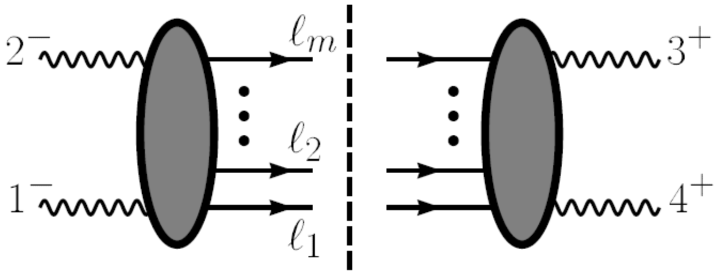
$$\sum_{N=4\text{ states}} A_5^{\text{tree}}(1,\ell_1,\ell_3,2,\ell_2)A_5^{\text{tree}}(3,-\ell_3,-\ell_1,4,-\ell_2) = -istA_4^{\text{tree}}(1{,}2{,}3{,}4)$$
$$\times\Bigg[-\frac{s}{(\ell_1+\ell_3)^2(\ell_3+k_2)^2(\ell_1+k_1)^2(\ell_2-k_3)^2}+\frac{t}{(\ell_1+k_1)^2(\ell_2+k_2)^2(\ell_2-k_3)^2(\ell_1-k_4)^2}$$
$$+\frac{t}{(\ell_1+k_1)^2(\ell_3+k_2)^2(\ell_1-k_4)^2(\ell_2-k_3)^2}-\frac{t}{(\ell_1+\ell_3)^2(\ell_2+k_2)^2(\ell_3-k_3)^2(\ell_1-k_4)^2}$$
$$+\frac{t}{(\ell_1+k_1)^2(\ell_2+k_2)^2(\ell_1-k_4)^2(\ell_3-k_3)^2}+\frac{t}{(\ell_1+k_1)^2(\ell_3+k_2)^2(\ell_1-k_4)^2(\ell_3-k_3)^2}$$
$$-\frac{u}{(\ell_2+k_1)^2(\ell_3+k_2)^2(\ell_1-k_4)^2(\ell_2-k_3)^2}-\frac{t}{(\ell_1+\ell_3)^2(\ell_2+k_1)^2(\ell_3-k_3)^2(\ell_1-k_4)^2}$$
$$+\frac{t}{(\ell_2+k_1)^2(\ell_3+k_2)^2(\ell_3-k_3)^2(\ell_1-k_4)^2}-\frac{u}{(\ell_1+\ell_3)^2(\ell_1+k_1)^2(\ell_3+k_2)^2(\ell_2-k_4)^2}$$
$$-\frac{t}{(\ell_1+k_1)^2(\ell_2+k_2)^2(\ell_3-k_3)^2(\ell_2-k_4)^2}+\frac{t}{(\ell_1+k_1)^2(\ell_3+k_2)^2(\ell_3-k_3)^2(\ell_2-k_4)^2}$$
$$+\frac{t}{(\ell_2+k_1)^2(\ell_3+k_2)^2(\ell_3-k_3)^2(\ell_2-k_4)^2}\Bigg]$$



$$\text{stuM } M_4^{\text{tree}}\int\frac{d^D\ell_1}{(2\pi)^D}\frac{d^D\ell_2}{(2\pi)^D}\frac{1}{\ell_1^2\ell_2^2\ell_3^2}$$
$$\times\Bigg\{\Bigg[\Bigg(\frac{1}{2}\frac{t^2}{(\ell_1+k_1)^2(\ell_3+k_2)^2(\ell_3-k_3)^2(\ell_1-k_4)^2}+\frac{t^2}{(\ell_2+k_1)^2(\ell_3+k_2)^2(\ell_3-k_3)^2(\ell_1-k_4)^2}$$
$$+\frac{s^2}{(\ell_1+\ell_2)^2(\ell_2+\ell_3)^2(\ell_3+k_2)^2(\ell_1-k_4)^2}+\frac{1}{2}\frac{s^2}{(\ell_2+\ell_3)^2(\ell_3+k_1)^2(\ell_2+k_2)^2(\ell_1-k_4)^2}$$
$$+\frac{1}{2}\frac{s^2}{(\ell_1+\ell_2)^2(\ell_3+k_2)^2(\ell_2-k_3)^2(\ell_1-k_4)^2}\Bigg)+\text{perms}(\ell_1,\ell_2,\ell_3)\Bigg]$$
$$+\{1\leftrightarrow 2\}+\{3\leftrightarrow 4\}+\{1\leftrightarrow 2{,}3\leftrightarrow 4\}\Bigg\}\Bigg|_{\ell_i^2=0}$$

$$\mathcal{A}_4^{L-\text{loop}}(1^-,2^-,3^+,4^+)\Big|_{m-\text{cut}}=\int\frac{d^D\ell_1}{(2\pi)^D}\frac{d^D\ell_2}{(2\pi)^D}\cdots\frac{d^D\ell_{m-1}}{(2\pi)^D}\sum_{N=4\text{ states}}\mathcal{A}_{m+2}^{r-\text{loop}}(1^-,2^-,\ell_m,\ell_{m-1},\dots,\ell_1)$$
$$\times\frac{i}{\ell_1^2}\frac{i}{\ell_2^2}\cdots\frac{i}{\ell_m^2}\mathcal{A}_{m+2}^{(L-r-m+1)-\text{loop}}(3^+,4^+,-\ell_1,-\ell_2,\dots,-\ell_m)\Big|_{\ell_i^2=0}$$

$$\frac{(M+3)!}{4!\,(M-1)!}+1$$



$$\mathcal{A}_{m+2}(1^-,2^-,\ell_1^+,\ldots\ell_m^+)=\frac{\langle 12\rangle^4}{\langle \ell_i\ell_j\rangle^4}\left[\mathcal{A}_{m+2}\left(1^-,2^-,\ell_1^-,\ldots,\ell_{i-1}^-,\ell_i^+,\ell_{i+1}^-,\ldots,\ell_{j-1}^-,\ell_j^+,\ell_{j+1}^-,\ldots,\ell_m^-\right)\right]^\dagger,$$

$$\left(\sum_{i\neq j} a_{ij}\right)^4 = s^4$$

$$\frac{(M+7)!}{8!\,(M-1)!}+1,$$

$$M_{m+2}^{\text{tree}}(1,2,\ell_1,\ldots,\ell_m)\sim\sum_{\{i\}\{j\}}\left(\prod_{m-1\text{ factors}}\left(\ell_p+\ell_q\right)^2\right)A_{m+2}^{\text{tree}}\left(1,\ell_{i_1},\ldots,\ell_{i_n},2,\ell_{i_{n+1}},\ldots,\ell_{i_m}\right)$$
$$\times A_{m+2}^{\text{tree}}\left(1,\ell_{j_1},\ldots,\ell_{j_k},2,\ell_{j_{k+1}},\ldots,\ell_{j_m}\right)$$

$$\left(\sum_{i\neq j} a_{ij}\right)^8 = s^8$$

$$\sum_{\lambda,\lambda'} M_4^{\text{tree}}\left(-\ell_1^{\lambda'},1^-,2^-,\ell_2^{\lambda}\right)\times M_4^{\text{tree}}\left(-\ell_2^{-\lambda},3^+,4^+,\ell_1^{-\lambda'}\right)$$

$$M_4^{\text{tree}}\left(-\ell_1^+,1^-,2^-,\ell_2^+\right)\times M_4^{\text{tree}}\left(-\ell_2^-,3^+,4^+,\ell_1^-\right)$$

$$M_4^{\text{tree}}\left(-\ell_1^+,1^-,2^-,\ell_2^+\right)=-i\left(s\frac{\langle 12\rangle\,[\ell_1\ell_2]}{[12]\,\langle \ell_1\ell_2\rangle}\right)^2\left[\frac{1}{(\ell_1-k_1)^2}+\frac{1}{(\ell_1-k_2)^2}\right]$$
$$M_4^{\text{tree}}\left(-\ell_2^-,3^+,4^+,\ell_1^-\right)=-i\left(s\frac{[34]\,\langle \ell_1\ell_2\rangle}{\langle 34\rangle\,[\ell_1\ell_2]}\right)^2\left[\frac{1}{(\ell_2-k_3)^2}+\frac{1}{(\ell_2-k_4)^2}\right]$$

$$M_4^{\text{tree}}\left(-\ell_1^+,1^-,2^-,\ell_2^+\right)\times M_4^{\text{tree}}\left(-\ell_2^-,3^+,4^+,\ell_1^-\right)$$
$$=-\left(s^2\frac{\langle 12\rangle\,[34]}{[12]\,\langle 34\rangle}\right)^2\left[\frac{1}{(\ell_1-k_1)^2}+\frac{1}{(\ell_1-k_2)^2}\right]\left[\frac{1}{(\ell_2-k_3)^2}+\frac{1}{(\ell_2-k_4)^2}\right]$$
$$=istuM_4^{\text{tree}}\left(1^-,2^-,3^+,4^+\right)\left[\frac{1}{(\ell_1-k_1)^2}+\frac{1}{(\ell_1-k_2)^2}\right]\left[\frac{1}{(\ell_2-k_3)^2}+\frac{1}{(\ell_2-k_4)^2}\right]$$

$$M_4^{2-\text{ loop}}\left(1^-,2^-,3^+,4^+\right)\big|_{s^-\text{channel }3-\text{ cut}}$$
$$=\int\frac{d^D\ell_1}{(2\pi)^D}\frac{d^D\ell_2}{(2\pi)^D}\sum_{N=8\text{ states}}M_5^{\text{tree}}\left(1^-,2^-,\ell_3,\ell_2,\ell_1\right)\frac{i}{\ell_1^2}\frac{i}{\ell_2^2}\frac{i}{\ell_3^2}M_5^{\text{tree}}\left(3^+,4^+,-\ell_1,-\ell_2,-\ell_3\right)\Bigg|_{\ell_i^2=0}$$

$$M_5^{\text{tree}}(\ell_1^s,1^-,2^-,\ell_3^+,\ell_2^+) = i(\ell_1+k_1)^2(\ell_3+k_2)^2 A_5^{\text{tree}}(\ell_1^+,1^-,2^-,\ell_3^+,\ell_2^+) A_5^{\text{tree}}(1^-,\ell_1^-,\ell_3^+,2^-,\ell_2^+) + \{1 \leftrightarrow 2\}$$

$$= i(\ell_1+k_1)^2(\ell_3+k_2)^2 \frac{\langle 12\rangle^4}{\langle \ell_1 1\rangle\langle 12\rangle\langle 2\ell_3\rangle\langle \ell_3\ell_2\rangle\langle \ell_2\ell_1\rangle} \frac{[\ell_3\ell_2]^4}{[1\ell_1][\ell_1\ell_3][\ell_3 2][2\ell_2][\ell_2 1]} + \{1 \leftrightarrow 2\}$$

$$= 0$$

$$A_5^{\text{tree}}(1^-,2^-,\ell_3^+,\ell_2^+,\ell_1^+) \times A_5^{\text{tree}}(3^+,4^+,-\ell_1^-,-\ell_2^-,-\ell_3^-),$$
$$A_5^{\text{tree}}(1^-,\ell_1^+,\ell_3^+,2^-,\ell_2^+) \times A_5^{\text{tree}}(3^+,-\ell_3^-,-\ell_1^-,4^+,-\ell_2^-).$$

$$A_5^{\text{tree}}(1^-,2^-,\ell_3^+,\ell_2^+,\ell_1^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 2\ell_3\rangle\langle \ell_3\ell_2\rangle\langle \ell_2\ell_1\rangle\langle \ell_1 1\rangle}$$

$$A_5^{\text{tree}}(3^+,4^+,-\ell_1^-,-\ell_2^-,-\ell_3^-) = -i\frac{[34]^4}{[34][4-\ell_1][-\ell_1-\ell_2][-\ell_2-\ell_3][-\ell_3 3]}$$

$$A_5^{\text{tree}}(1^-,2^-,\ell_3^+,\ell_2^+,\ell_1^+) \times A_5^{\text{tree}}(3^+,4^+,-\ell_1^-,-\ell_2^-,-\ell_3^-)$$
$$= -\langle 12\rangle^2[34]^2 \frac{\text{tr}_+[1\ell_1 43\ell_3 2]}{(\ell_1+\ell_2)^2(\ell_2+\ell_3)^2(\ell_3-k_3)^2(\ell_1-k_4)^2(\ell_3+k_2)^2(\ell_1+k_1)^2}$$

$$A_5^{\text{tree}}(1^-,\ell_1^+,\ell_3^+,2^-,\ell_2^+) \times A_5^{\text{tree}}(3^+,-\ell_3^-,-\ell_1^-,4^+,-\ell_2^-)$$
$$= -\frac{\langle 12\rangle^2[34]^2\text{tr}_+[1\ell_1 43\ell_2 2]\text{tr}_+[1\ell_2 43\ell_3 2]}{(\ell_1+\ell_3)^2(\ell_1+k_1)^2(\ell_3+k_2)^2(\ell_2+k_2)^2(\ell_2+k_1)^2(\ell_3-k_3)^2(\ell_1-k_4)^2(\ell_2-k_4)^2(\ell_2-k_3)^2}$$

$$M_4^{2-\text{loop}}(1^-,2^-,3^+,4^+)|_{\text{singlet 3-cut}} = stuM_4^{\text{tree}}(1^-,2^-,3^+,4^+)\int \frac{d^D\ell_1}{(2\pi)^D}\frac{d^D\ell_2}{(2\pi)^D}$$
$$\times\Bigg[\frac{\text{tr}_+[1\ell_1 43\ell_3 21\ell_1 43\ell_2 21\ell_2 43\ell_3 2]}{\ell_1^2\ell_2^2\ell_3^2(\ell_3-k_3)^2(\ell_1-k_4)^2(\ell_3+k_2)^2(\ell_1+k_1)^2(\ell_1+\ell_2)^2(\ell_2+\ell_3)^2}$$
$$\times\frac{1}{(\ell_1+\ell_3)^2(\ell_2+k_2)^2(\ell_2+k_1)^2(\ell_2-k_4)^2(\ell_2-k_3)^2}$$
$$+\{1\leftrightarrow 2\}+\{3\leftrightarrow 4\}+\{1\leftrightarrow 2, 3\leftrightarrow 4\}\Bigg]\Bigg|_{\ell_i^2=0}$$

$$\text{tr}_+[1\ell_1 43\ell_3 2]\text{tr}_+[1\ell_1 43\ell_2 2]\text{tr}_+[1\ell_2 43\ell_3 2] = \text{tr}_+[1\ell_1 43\ell_3 21\ell_1 43\ell_2 21\ell_2 43\ell_3 2]$$

$$A_5^{\text{tree}}(3^+,4^+,\ell_1^-,\ell_2^-,\ell_3^+) = i\frac{\langle \ell_1\ell_2\rangle^4}{\langle 34\rangle\langle 4\ell_1\rangle\langle \ell_1\ell_2\rangle\langle \ell_2\ell_3\rangle\langle \ell_3 3\rangle}$$

$$A_5^{\text{tree}}(3^+,4^+,\ell_1^-,\ell_{2\tilde{g}}^-,\ell_{3\tilde{g}}^+) = \frac{\langle \ell_1\ell_3\rangle}{\langle \ell_1\ell_2\rangle}A_5^{\text{tree}}(3^+,4^+,\ell_1^-,\ell_2^-,\ell_3^+) = i\frac{\langle \ell_1\ell_2\rangle^3\langle \ell_1\ell_3\rangle}{\langle 34\rangle\langle 4\ell_1\rangle\langle \ell_1\ell_2\rangle\langle \ell_2\ell_3\rangle\langle \ell_3 3\rangle},$$

$$h^+h^+v^-\tilde{v}^-\tilde{v}^- \to g^+g^+\tilde{g}^-\tilde{g}^-s^- \times g^+g^+\tilde{g}^-s^-\tilde{g}^-$$

Amplitude	Left YM	Left Factor	Right YM	Right Factor
$h^+h^+h^-h^-h^+$	$g^+g^+g^-g^-g^+$	$\langle \ell_1\, \ell_2\rangle^4$	$g^+g^+g^-g^-g^+$	$\langle \ell_1\, \ell_2\rangle^4$
$h^+h^+h^-\tilde{h}^-\tilde{h}^+$	$g^+g^+g^-\tilde{g}^-\tilde{g}^+$	$\langle \ell_1\, \ell_2\rangle^3 \langle \ell_1\, \ell_3\rangle$	$g^+g^+g^-g^-g^+$	$\langle \ell_1\, \ell_2\rangle^4$
$h^+h^+h^-v^-v^+$	$g^+g^+g^-s^-s^+$	$\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle^2$	$g^+g^+g^-g^-g^+$	$\langle \ell_1\, \ell_2\rangle^4$
$h^+h^+h^-\tilde{v}^-\tilde{v}^+$	$g^+g^+g^-s^-s^+$	$\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle^2$	$g^+g^+g^-\tilde{g}^-\tilde{g}^+$	$\langle \ell_1\, \ell_2\rangle^3 \langle \ell_1\, \ell_3\rangle$
$h^+h^+h^-s^-s^+$	$g^+g^+g^-s^-s^+$	$\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle^2$	$g^+g^+g^-s^-s^+$	$\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle^2$
$h^+h^+\tilde{h}^-\tilde{h}^-v^+$	$g^+g^+\tilde{g}^-\tilde{g}^-s^+$	$i\,\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle \langle \ell_2\, \ell_3\rangle$	$g^+g^+g^-g^-g^+$	$\langle \ell_1\, \ell_2\rangle^4$
$h^+h^+\tilde{h}^-v^-\tilde{v}^+$	$g^+g^+g^-s^-s^+$	$\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle^2$	$g^+g^+\tilde{g}^-g^-\tilde{g}^+$	$-\langle \ell_1\, \ell_2\rangle^3 \langle \ell_2\, \ell_3\rangle$
$h^+h^+\tilde{h}^-\tilde{v}^-s^+$	$g^+g^+g^-s^-s^+$	$\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle^2$	$g^+g^+\tilde{g}^-\tilde{g}^-s^+$	$i\,\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle \langle \ell_2\, \ell_3\rangle$
$h^+h^+v^-v^-s^+$	$g^+g^+\tilde{g}^-\tilde{g}^-s^+$	$i\,\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle \langle \ell_2\, \ell_3\rangle$	$g^+g^+\tilde{g}^-\tilde{g}^-s^+$	$i\,\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle \langle \ell_2\, \ell_3\rangle$
$h^+h^+v^-\tilde{v}^-\tilde{v}^-$	$g^+g^+g^-s^-s^-$	$-\langle \ell_1\, \ell_2\rangle^2 \langle \ell_1\, \ell_3\rangle^2$	$g^+g^+s^-\tilde{g}^-\tilde{g}^-$	$i\,\langle \ell_1\, \ell_2\rangle \langle \ell_1\, \ell_3\rangle \langle \ell_2\, \ell_3\rangle^2$

$$\text{(a) } A_5^{\text{tree}}(1^-,2^-,\ell_3^-,\ell_2^+,\ell_1^+) \times A_5^{\text{tree}}(3^+,4^+,-\ell_1^-,-\ell_2^-,-\ell_3^+),$$
$$\text{(b) } A_5^{\text{tree}}(1^-,\ell_1^+,\ell_3^-,2^-,\ell_2^+) \times A_5^{\text{tree}}(3^+,-\ell_3^+,-\ell_1^-,4^+,-\ell_2^-);$$

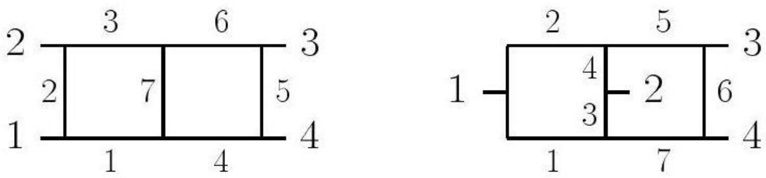
$$M_4^{2-\text{loop}}(1^-,2^-,3^+,4^+)\Big|_{\text{non-singlet 3-cut}} = stu M_4^{\text{tree}}(1^-,2^-,3^+,4^+)\int \frac{d^D\ell_1}{(2\pi)^D}\frac{d^D\ell_2}{(2\pi)^D}\frac{(\ell_1+\ell_2)^{16}}{s^8}$$
$$\times\Bigg[\frac{\text{tr}\_[1\ell_1 4 3\ell_3 2 1\ell_1 4 3\ell_2 2 1\ell_2 4 3\ell_3 2]}{\ell_1^2\ell_2^2\ell_3^2(\ell_3-k_3)^2(\ell_1-k_4)^2(\ell_3+k_2)^2(\ell_1+k_1)^2(\ell_1+\ell_2)^2(\ell_2+\ell_3)^2}$$
$$\times\frac{1}{(\ell_1+\ell_3)^2(\ell_2+k_2)^2(\ell_2+k_1)^2(\ell_2-k_4)^2(\ell_2-k_3)^2}$$
$$+\{1\leftrightarrow 2\}+\{3\leftrightarrow 4\}+\{1\leftrightarrow 2, 3\leftrightarrow 4\}\Bigg]_{\ell_i^2=0}.$$

$$\int \frac{d^4p}{(2\pi)^4}\frac{d^{-2\epsilon}\mu_p}{(2\pi)^{-2\epsilon}}\frac{d^4q}{(2\pi)^4}\frac{d^{-2\epsilon}\mu_q}{(2\pi)^{-2\epsilon}}\frac{f(p,q,k_i)\times\{\mu_p^2,\mu_q^2,\mu_p\cdot\mu_q,\dots\}}{(p^2-\mu_p^2)(q^2-\mu_q^2)\cdots}$$

$$\mathcal{I}_4^{2-\text{loop},X}(s,t) = \frac{\Gamma(7-D)}{(4\pi)^D}\int_0^1 d^7a\,\delta\left(1-\sum_{i=1}^{7}a_i\right)\left(-Q_X(s,t,a_i)\right)^{D-7}\left(\Delta_X(a_i)\right)^{7-3D/2}, X = \text{P}, \text{NP},$$

$$\Delta_{\text{P}}(a_i) = (a_1+a_2+a_3)(a_4+a_5+a_6)+a_7(1-a_7)$$
$$\Delta_{\text{NP}}(a_i) = (a_1+a_2)(a_3+a_4)+(a_1+a_2+a_3+a_4)(a_5+a_6+a_7)$$

$$Q_{\text{P}}(a_i) = s\big(a_1a_3(a_4+a_5+a_6)+a_4a_6(a_1+a_2+a_3)+a_7(a_1+a_4)(a_3+a_6)\big)+ta_2a_5a_7,$$
$$Q_{\text{NP}}(a_i) = s\big(a_1a_3a_5+a_2a_4a_7+a_5a_7(a_1+a_2+a_3+a_4)\big)+ta_2a_3a_6+ua_1a_4a_6.$$



$$\mathcal{I}_4^{2-\text{loop},X}(s,t)\Big|_{\text{pole}} = \frac{\Gamma(7-D)}{(4\pi)^D}\int_0^1 d^7a\,\delta\left(1-\sum_{i=1}^{7}a_i\right)\left(-Q_X(s,t,a_i)\right)^{n-7}\left(\Delta_X(a_i)\right)^{7-n-D/2}, X = \text{P}, \text{NP}.$$

$$\mathcal{J}_4^{2-\text{ loop },P,D=7-2\epsilon}\Big|_{\text{pole}} = \frac{1}{2\epsilon(4\pi)^7}\frac{1}{4}\int_0^1 dy y^2(1-y)^2\int_0^1 dx\frac{x^{3/2}}{[1-x(1-y(1-y))]^{7/2}} = \frac{1}{2\epsilon(4\pi)^7}\frac{\pi}{10}$$

$$\mathcal{J}_4^{2-\text{ loop },P,D=9-2\epsilon}\Big|_{\text{pole}} = \frac{1}{4\epsilon(4\pi)^9}\frac{1}{498960}\int_0^1 \frac{dy}{[y(1-y)]^{3/2}}[3s^2(16y^2(1-y)^2-77y(1-y)+132) +8\,\text{st}\,y(1-y)(2y(1-y)+11)+80t^2y^2(1-y)^2]$$

$$\mathcal{J}_4^{2-\text{ loop },P,D=9-2\epsilon}\Big|_{\text{pole, }s^2\text{ term}} = \frac{s^2}{4\epsilon(4\pi)^9}\int_0^1 dy\int_0^1 dx[C_1(x,y)+C_2(x,y)]$$

$$C_1(x,y) = \frac{[y(1-y)]^4}{480}\frac{x^{5/2+\epsilon}(-x^2y(1-y)+(1-x)(2-3x))}{[1-x(1-y(1-y))]^{13/2-\epsilon}}$$

$$C_2(x,y) = \frac{[y(1-y)]^2}{360}\frac{x^{5/2+\epsilon}}{[1-x(1-y(1-y))]^{9/2-\epsilon}}$$

$$\begin{aligned}I(p,q,\alpha) &\equiv \int_0^1 dy[y(1-y)]^p\int_0^1 dx\frac{x^{\alpha-q-2+\epsilon}(1-x)^q}{[1-x(1-y(1-y))]^{\alpha-\epsilon}}\\ &= \frac{\Gamma(\alpha-q-1+\epsilon)\Gamma(q+1)}{\Gamma(\alpha+\epsilon)}\int_0^1 dy[y(1-y)]^p\,{}_2F_1(\alpha-\epsilon,\alpha-q-1+\epsilon;\alpha+\epsilon;1-y(1-y))\\ &= \frac{\Gamma(\alpha-q-1+\epsilon)\Gamma(q+1)}{\Gamma(\alpha+\epsilon)}\int_0^1 dy[y(1-y)]^p\\ &\times\Big\{\frac{\Gamma(\alpha+\epsilon)\Gamma(-\alpha+q+1+\epsilon)}{\Gamma(2\epsilon)\Gamma(q+1)}\,{}_2F_1(\alpha-\epsilon,\alpha-q-1+\epsilon;\alpha-q-\epsilon;y(1-y))\\ &+[y(1-y)]^{-\alpha+q+1+\epsilon}\frac{\Gamma(\alpha+\epsilon)\Gamma(\alpha-q-1-\epsilon)}{\Gamma(\alpha-\epsilon)\Gamma(\alpha-q-1+\epsilon)}\,{}_2F_1(2\epsilon,q+1;-\alpha+q+2+\epsilon;y(1-y))\Big\}\end{aligned}$$

$$\begin{aligned}I(p,q,\alpha) &= \frac{\Gamma(\alpha-q-1-\epsilon)\Gamma(q+1)}{\Gamma(\alpha-\epsilon)}\int_0^1 dy[y(1-y)]^{-\alpha+p+q+1+\epsilon}\\ &= \frac{\Gamma(\alpha-q-1)\Gamma(q+1)\Gamma^2(-\alpha+p+q+2)}{\Gamma(\alpha)\Gamma(2(-\alpha+p+q+2))}\end{aligned}$$

$$\mathcal{J}_4^{2-\text{ loop },P,D=9-2\epsilon}\Big|_{\text{pole}} = \frac{1}{4\epsilon(4\pi)^9}\frac{\pi}{99792}(-45s^2+18st+2t^2)$$

$$\mathcal{J}_4^{2-\text{ loop },P,D=11-2\epsilon}\Big|_{\text{pole}} = \frac{1}{48\epsilon(4\pi)^{11}}\frac{\pi}{196911000}(2100s^4-880s^3t+215s^2t^2+30st^3+12t^4)$$

$$\mathcal{J}_4^{2-\text{ loop },NP,D=7-2\epsilon}\Big|_{\text{pole}} = \frac{1}{2\epsilon(4\pi)^7}\frac{\pi}{15}$$

$$\mathcal{J}_4^{2-\text{ loop },NP,D=9-2\epsilon}\Big|_{\text{pole}} = \frac{1}{4\epsilon(4\pi)^9}\frac{-\pi}{83160}(75s^2+2tu)$$

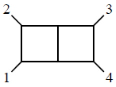
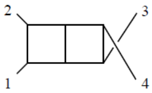
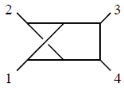
$$\mathcal{J}_4^{2-\text{ loop },NP,D=11-2\epsilon}\Big|_{\text{pole}} = \frac{1}{48\epsilon(4\pi)^{11}}\frac{\pi}{1654052400}(40383s^4-1138s^2tu+144t^2u^2)$$

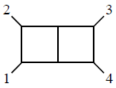
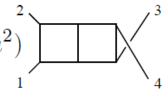
$$\mathcal{J}_4^{2-\text{ loop },P,D=8-2\epsilon}\Big|_{\text{subtracted, pole}} = \frac{1}{2(4\pi)^8}\left[-\frac{1}{72}\frac{s}{\epsilon^2} + \frac{1}{864}\frac{5s+2t}{\epsilon}\right]$$

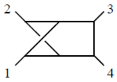
$$\mathcal{J}_4^{2-\text{ loop },P,D=10-2\epsilon}\Big|_{\text{subtracted, pole}} = \frac{1}{12(4\pi)^{10}}\left[-\frac{1}{25200}\frac{s^2(4s+t)}{\epsilon^2} + \frac{1}{21168000}\frac{-704s^3+55s^2t+252st^2+63t^3}{\epsilon}\right]$$

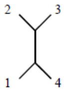
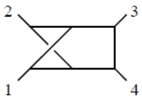
$$\mathcal{J}_4^{2-\text{ loop, NP},D=8-2\epsilon}\Big|_{\text{subtracted, pole}} = \frac{1}{2(4\pi)^8}\left[-\frac{1}{144}\frac{s}{\epsilon^2} - \frac{1}{864}\frac{s}{\epsilon}\right]$$

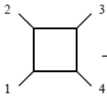
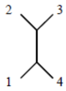
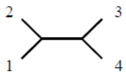
$$\mathcal{J}_4^{2-\text{ loop, NP},D=10-2\epsilon}\Big|_{\text{subtracted, pole}} = \frac{1}{12(4\pi)^{10}}\left[\frac{1}{7200}\frac{s^3}{\epsilon^2} - \frac{1}{4536000}\frac{s(301s^2+10tu)}{\epsilon}\right]$$

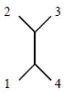
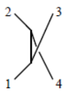
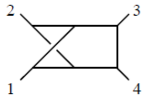
$T_7 = -\frac{8\,g^6\,\pi\,s}{(4\pi)^7\,2\epsilon}\Big[\frac{1}{10}\Big($  $+$  $\Big) + \frac{2}{15}$  $\Big]$ + cyclic,

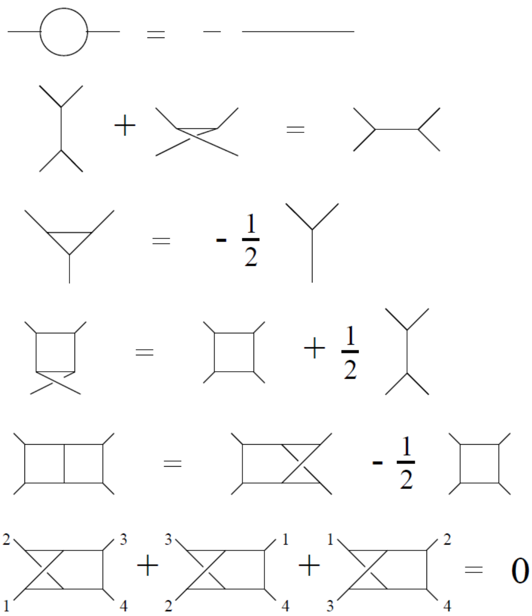
$T_9 = -\frac{8\,g^6\,\pi\,s}{(4\pi)^9\,4\epsilon}\Big[\frac{1}{99792}(-45s^2+18st+2t^2)$  $+ \frac{1}{99792}(-45s^2+18su+2u^2)$ 

$- \frac{2}{83160}(75s^2+2tu)$  $\Big]$ + cyclic

$T_7 = \frac{3}{2}(-4)\frac{g^6\,\pi}{(4\pi)^7\,2\epsilon}\Big\{s\Big[\frac{1}{90}\Big($  $+$  $\Big) + \frac{4}{9}$  $\Big]$ + cyclic$\Big\}$,

$T_9 = (-4)\frac{g^6\,\pi}{(4\pi)^9\,2\epsilon}\Big\{\frac{5}{3024}stu\Big[$  $+\frac{1}{6}\Big($  $+$  $\Big)\Big]$

$-\Big[s^3\Big(\frac{5}{133056}\Big($  $+$  $\Big) + \frac{13}{4536}$  $\Big)$ + cyclic$\Big]\Big\}$



$$[Q(p), g^{\pm}(k)] = \mp\Gamma^{\pm}(k,p)\tilde{g}^{\pm}(k), [Q(p), \tilde{g}^{\pm}(k)] = \mp\Gamma^{\mp}(k,p)g^{\pm}(p)$$

$$\Gamma^{+}(k,p) = \bar{\theta}\langle pk\rangle, \Gamma^{-}(k,p) = \theta\langle pk\rangle$$

$$\begin{aligned} 0 = \langle 0|[Q, g^{-}g^{-}\tilde{g}^{+}g^{+}g^{+}]|0\rangle = &\Gamma^{-}(k_1,p)A_5\left(1^{-}_{\tilde{g}}, 2^{-}_{g}, 3^{+}_{\tilde{g}}, 4^{+}_{g}, 5^{+}_{g}\right) + \Gamma^{-}(k_2,p)A_5\left(1^{-}_{g}, 2^{-}_{\tilde{g}}, 3^{+}_{\tilde{g}}, 4^{+}_{g}, 5^{+}_{g}\right) \\ &-\Gamma^{-}(k_3,p)A_5\left(1^{-}_{g}, 2^{-}_{g}, 3^{+}_{g}, 4^{+}_{g}, 5^{+}_{g}\right) - \Gamma^{+}(k_4,p)A_5\left(1^{-}_{g}, 2^{-}_{g}, 3^{+}_{\tilde{g}}, 4^{+}_{\tilde{g}}, 5^{+}_{g}\right) \\ &-\Gamma^{+}(k_5,p)A_5\left(1^{-}_{g}, 2^{-}_{g}, 3^{+}_{\tilde{g}}, 4^{+}_{g}, 5^{+}_{\tilde{g}}\right) \end{aligned}$$

$$A_5\left(1^{-}_{g}, 2^{-}_{\tilde{g}}, 3^{+}_{\tilde{g}}, 4^{+}_{g}, 5^{+}_{g}\right) = \frac{\langle 13\rangle}{\langle 12\rangle}A_5\left(1^{-}_{g}, 2^{-}_{g}, 3^{+}_{g}, 4^{+}_{g}, 5^{+}_{g}\right)$$

$$\begin{aligned} &[Q_a, g^{\pm}(k)] = \mp\Gamma^{\pm}(k,p)\tilde{g}^{\pm}_{a} \\ &\left[Q_a, \tilde{g}^{\pm}_{b}(k)\right] = \mp\Gamma^{\mp}(k,p)g^{\pm}\delta_{ab} \mp i\Gamma^{\pm}(k,p)s^{\pm}_{ab}\epsilon_{ab} \\ &\left[Q_a, s^{\pm}_{ab}(k)\right] = \pm i\Gamma^{\mp}(k,p)\epsilon_{ab}\tilde{g}^{\pm}_{b} \end{aligned}$$

$$\begin{aligned} 0 = \langle 0|[Q_1, g^{+}g^{+}g^{-}\tilde{g}^{-}_{2}s^{+}_{12}]|0\rangle = &\Gamma^{-}(k_3,p)A_5\left(1^{+}_{g}, 2^{+}_{g}, 3^{-}_{\tilde{g}_1}, 4^{-}_{\tilde{g}_2}, 5^{+}_{s_{12}}\right) + i\Gamma^{-}(k_4,p)A_5\left(1^{+}_{g}, 2^{+}_{g}, 3^{-}_{g}, 4^{-}_{s_{12}}, 5^{+}_{s_{12}}\right) \\ &+i\Gamma^{-}(k_5,p)A_5\left(1^{+}_{g}, 2^{+}_{g}, 3^{-}_{g}, 4^{-}_{\tilde{g}_2}, 5^{+}_{\tilde{g}_2}\right) \end{aligned}$$

$$A_5\left(1^{+}_{g}, 2^{+}_{g}, 3^{-}_{\tilde{g}_1}, 4^{-}_{\tilde{g}_2}, 5^{+}_{s_{12}}\right) = -i\frac{\langle 45\rangle}{\langle 43\rangle}A_5\left(1^{+}_{g}, 2^{+}_{g}, 3^{-}_{g}, 4^{-}_{\tilde{g}_2}, 5^{+}_{\tilde{g}_2}\right) = +i\frac{\langle 35\rangle\langle 45\rangle}{\langle 34\rangle^2}A_5\left(1^{+}_{g}, 2^{+}_{g}, 3^{-}_{g}, 4^{-}_{g}, 5^{+}_{g}\right)$$

Amplitude	Relative Factor	Amplitude	Relative Factor
$g^- g^- g^+ g^+$	$\langle 1\,2\rangle^4$	$\tilde g_1^- \tilde g_1^- \tilde g_1^+ \tilde g_1^+$	$-\langle 1\,2\rangle^3 \langle 3\,4\rangle$
$g^- \tilde g_1^- \tilde g_1^+ g^+$	$\langle 1\,2\rangle^3 \langle 1\,3\rangle$	$\tilde g_1^- s_{23}^- s_{23}^+ \tilde g_1^+$	$\langle 1\,2\rangle \langle 1\,3\rangle^2 \langle 2\,4\rangle$
$g^- s_{12}^- s_{12}^+ g^+$	$\langle 1\,2\rangle^2 \langle 1\,3\rangle^2$	$\tilde g_1^- s_{23}^- s_{12}^+ \tilde g_3^+$	$\langle 1\,2\rangle \langle 1\,3\rangle \langle 1\,4\rangle \langle 2\,3\rangle$
$\tilde g_1^- \tilde g_2^- s_{12}^+ g^+$	$i\,\langle 1\,2\rangle^2 \langle 1\,3\rangle \langle 2\,3\rangle$	$s_{12}^- s_{23}^- s_{12}^+ s_{23}^+$	$-\langle 1\,2\rangle \langle 1\,4\rangle \langle 2\,3\rangle \langle 3\,4\rangle$
$g^- s_{12}^- \tilde g_1^+ \tilde g_2^+$	$-i\,\langle 1\,2\rangle^2 \langle 1\,3\rangle \langle 1\,4\rangle$	$s_{12}^- s_{12}^- s_{12}^+ s_{12}^+$	$\langle 1\,2\rangle^2 \langle 3\,4\rangle^2$
$\tilde g_1^- \tilde g_2^- \tilde g_2^+ \tilde g_1^+$	$-\langle 1\,2\rangle^2 \langle 1\,3\rangle \langle 2\,4\rangle$		

$$\frac{1}{\langle ij\rangle^4}\mathcal{A}_n(1^+,2^+,\dots,i^-,\dots,j^-,\dots n^+) = \frac{1}{\langle ab\rangle^4}\mathcal{A}_n(1^+,2^+,\dots,a^-,\dots,b^-,\dots n^+),$$

Cut Legs	SWI Factor	Cut Legs	SWI Factor
$g^- g^- g^+ g^+ \cdots$	$\langle \ell_1\,\ell_2\rangle^4$	$s_{12}^- s_{23}^- s_{12}^+ s_{23}^+ \cdots$	$-\langle \ell_1\,\ell_2\rangle \langle \ell_1\,\ell_4\rangle \langle \ell_2\,\ell_3\rangle \langle \ell_3\,\ell_4\rangle$
$g^- \tilde g_1^- \tilde g_1^+ g^+ \cdots$	$\langle \ell_1\,\ell_2\rangle^3 \langle \ell_1\,\ell_3\rangle$	$s_{12}^- s_{12}^- s_{12}^+ s_{12}^+ \cdots$	$\langle \ell_1\,\ell_2\rangle^2 \langle \ell_3\,\ell_4\rangle^2$
$g^- s_{12}^- s_{12}^+ g^+ \cdots$	$\langle \ell_1\,\ell_2\rangle^2 \langle \ell_1\,\ell_3\rangle^2$	$s_{12}^- s_{12}^- s_{12}^+ \tilde g_1^+ \tilde g_2^+ \cdots$	$i\,\langle \ell_1\,\ell_2\rangle^2 \langle \ell_3\,\ell_4\rangle \langle \ell_3\,\ell_5\rangle$
$\tilde g_1^- \tilde g_2^- s_{12}^+ g^+ \cdots$	$i\,\langle \ell_1\,\ell_2\rangle^2 \langle \ell_1\,\ell_3\rangle \langle \ell_2\,\ell_3\rangle$	$s_{12}^- s_{23}^- s_{23}^+ \tilde g_1^+ \tilde g_2^+ \cdots$	$i\,\langle \ell_1\,\ell_2\rangle \langle \ell_1\,\ell_3\rangle \langle \ell_2\,\ell_4\rangle \langle \ell_3\,\ell_5\rangle$
$g^- s_{12}^- \tilde g_1^+ \tilde g_2^+ \cdots$	$-i\,\langle \ell_1\,\ell_2\rangle^2 \langle \ell_1\,\ell_3\rangle \langle \ell_1\,\ell_4\rangle$	$s_{12}^- s_{23}^- s_{13}^+ \tilde g_2^+ \tilde g_2^+ \cdots$	$-i\,\langle \ell_1\,\ell_2\rangle \langle \ell_1\,\ell_3\rangle \langle \ell_2\,\ell_3\rangle \langle \ell_4\,\ell_5\rangle$
$g^- \tilde g_1^+ \tilde g_2^+ \tilde g_3^+ \tilde g_4^+ \cdots$	$i\,\langle \ell_1\,\ell_2\rangle \langle \ell_1\,\ell_3\rangle \langle \ell_1\,\ell_4\rangle \langle \ell_1\,\ell_5\rangle$	$s_{12}^- s_{12}^- \tilde g_1^+ \tilde g_1^+ \tilde g_2^+ \tilde g_2^+ \cdots$	$\langle \ell_1\,\ell_2\rangle^2 \langle \ell_3\,\ell_4\rangle \langle \ell_5\,\ell_6\rangle$
$\tilde g_1^- \tilde g_2^- \tilde g_2^+ \tilde g_1^+ \cdots$	$-\langle \ell_1\,\ell_2\rangle^2 \langle \ell_1\,\ell_3\rangle \langle \ell_2\,\ell_4\rangle$	$s_{12}^- s_{23}^- \tilde g_1^+ \tilde g_2^+ \tilde g_2^+ \tilde g_3^+ \cdots$	$\langle \ell_1\,\ell_2\rangle \langle \ell_1\,\ell_5\rangle \langle \ell_2\,\ell_6\rangle \langle \ell_3\,\ell_4\rangle$
$\tilde g_1^- \tilde g_1^- \tilde g_1^+ \tilde g_1^+ \cdots$	$-\langle \ell_1\,\ell_2\rangle^3 \langle \ell_3\,\ell_4\rangle$	$s_{12}^- s_{34}^- \tilde g_1^+ \tilde g_2^+ \tilde g_3^+ \tilde g_4^+ \cdots$	$-\langle \ell_1\,\ell_5\rangle \langle \ell_1\,\ell_6\rangle \langle \ell_2\,\ell_3\rangle \langle \ell_2\,\ell_4\rangle$
$\tilde g_1^- s_{23}^- s_{23}^+ \tilde g_1^+ \cdots$	$\langle \ell_1\,\ell_2\rangle \langle \ell_1\,\ell_3\rangle^2 \langle \ell_2\,\ell_4\rangle$	$\tilde g_1^- \tilde g_1^+ \tilde g_1^+ \tilde g_2^+ \tilde g_3^+ \tilde g_4^+ \cdots$	$i\,\langle \ell_1\,\ell_4\rangle \langle \ell_1\,\ell_5\rangle \langle \ell_1\,\ell_6\rangle \langle \ell_2\,\ell_3\rangle$
$\tilde g_1^- s_{23}^- s_{12}^+ \tilde g_3^+ \cdots$	$\langle \ell_1\,\ell_2\rangle \langle \ell_1\,\ell_3\rangle \langle \ell_1\,\ell_4\rangle \langle \ell_2\,\ell_3\rangle$	$s_{12}^- \tilde g_1^+ \tilde g_2^+ \tilde g_1^+ \tilde g_2^+ \tilde g_3^+ \tilde g_4^+ \cdots$	$-\langle \ell_1\,\ell_6\rangle \langle \ell_1\,\ell_7\rangle \langle \ell_2\,\ell_4\rangle \langle \ell_3\,\ell_5\rangle$
$s_{12}^- \tilde g_3^- \tilde g_3^+ \tilde g_2^+ \tilde g_1^+ \cdots$	$i\,\langle \ell_1\,\ell_2\rangle \langle \ell_1\,\ell_3\rangle \langle \ell_2\,\ell_4\rangle \langle \ell_2\,\ell_5\rangle$	$\tilde g_1^+ \tilde g_2^+ \tilde g_3^+ \tilde g_4^+ \tilde g_1^+ \tilde g_2^+ \tilde g_3^+ \tilde g_4^+ \cdots$	$-\langle \ell_1\,\ell_5\rangle \langle \ell_2\,\ell_6\rangle \langle \ell_3\,\ell_7\rangle \langle \ell_4\,\ell_8\rangle$
$s_{12}^- \tilde g_2^- \tilde g_2^+ \tilde g_2^+ \tilde g_1^+ \cdots$	$-i\,\langle \ell_1\,\ell_2\rangle^2 \langle \ell_2\,\ell_5\rangle \langle \ell_3\,\ell_4\rangle$		

$$\begin{aligned}
[Q_a, h^\pm] &= \pm\Gamma^\pm(k,p)\tilde h_a^\pm \\
[Q_a, \tilde h_b^\pm] &= \pm\Gamma^\mp(k,p)\delta^{ab}h^\pm \pm i\Gamma^\pm(k,p)\epsilon^{ab}v_{ab}^\pm \\
[Q_a, v_{bc}^\pm] &= \mp i\Gamma^\mp(k,p)\delta^{ab}\tilde h_c^\pm \mp i\Gamma^\mp(k,p)\delta^{ac}\tilde h_b^\pm \mp \Gamma^\pm(k,p)\epsilon^{abc}\tilde v_{abc}^\pm \\
[Q_a, \tilde v_{bcd}^\pm] &= \mp\Gamma^\mp(k,p)\delta^{ab}v_{cd}^\pm \mp \Gamma^\mp(k,p)\delta^{ac}v_{bd}^\pm \mp \Gamma^\mp(k,p)\delta^{ad}v_{bc}^\pm \mp i\Gamma^\pm(k,p)\epsilon^{abcd}s_{abcd}^\pm \\
[Q_a, s_{bcde}^\pm] &= \pm i\Gamma^\mp(k,p)\delta^{ab}\tilde v_{cde}^\pm \pm i\Gamma^\mp(k,p)\delta^{ac}\tilde v_{bde}^\pm \pm i\Gamma^\mp(k,p)\delta^{ad}\tilde v_{bce}^\pm \pm i\Gamma^\mp(k,p)\delta^{ae}\tilde v_{bcd}^\pm \\
&\quad \pm\Gamma^\pm(k,p)\epsilon^{abcde}\tilde v_{abcde}^\mp
\end{aligned}$$

$$\begin{aligned}
0 = \langle 0|[Q_1, h^+h^+h^-\tilde v_{234}^- s_{1234}^+]|0\rangle &= -\Gamma^-(k_3,p)M_5\left(1_h^+, 2_h^+, 3_{\tilde h_1}^-, 4_{\tilde v_{234}}^-, 5_{s_{1234}}^+\right) \\
&\quad + i\Gamma^-(k_4,p)M_5\left(1_h^+, 2_h^+, 3_h^-, 4_{s_{1234}}^-, 5_{s_{1234}}^+\right) + i\Gamma^-(k_5,p)M_5\left(1_h^+, 2_h^+, 3_h^-, 4_{\tilde v_{234}}^-, 5_{\tilde v_{234}}^+\right)
\end{aligned}$$

$$M_5\left(1_h^+, 2_h^+, 3_{\tilde h}^-, 4_{\tilde v}^-, 5_s^+\right) = i\,\frac{\langle 54\rangle}{\langle 53\rangle}M_5\left(1_h^+, 2_h^+, 3_h^-, 4_s^-, 5_s^+\right) = i\,\frac{\langle 45\rangle\langle 35\rangle^3}{\langle 34\rangle^4}M_5\left(1_h^+, 2_h^+, 3_h^-, 4_h^-, 5_h^+\right).$$

Amplitude	Relative Factor	Amplitude	Relative Factor
$h^+h^+h^-h^-h^+$	$\langle 3\,4\rangle^8$	$h^+h^+h^-\tilde{h}^-\tilde{h}^+$	$\langle 3\,4\rangle^7\langle 3\,5\rangle$
$h^+h^+h^-v^-v^+$	$\langle 3\,4\rangle^6\langle 3\,5\rangle^2$	$h^+h^+h^-\tilde{v}^-\tilde{v}^+$	$\langle 3\,4\rangle^5\langle 3\,5\rangle^3$
$h^+h^+h^-s^-s^+$	$\langle 3\,4\rangle^4\langle 3\,5\rangle^4$	$h^+h^+\tilde{h}^-\tilde{h}^-v^+$	$i\langle 3\,4\rangle^6\langle 3\,5\rangle\langle 4\,5\rangle$
$h^+h^+\tilde{h}^-v^-\tilde{v}^+$	$\langle 3\,4\rangle^5\langle 4\,5\rangle\langle 3\,5\rangle^2$	$h^+h^+\tilde{h}^-\tilde{v}^-s^+$	$i\langle 3\,4\rangle^4\langle 4\,5\rangle\langle 3\,5\rangle^3$
$h^+h^+v^-v^-s^+$	$-\langle 3\,4\rangle^4\langle 4\,5\rangle^2\langle 3\,5\rangle^2$	$h^+h^+v^-\tilde{v}^-\tilde{v}^-$	$-i\langle 3\,4\rangle^3\langle 4\,5\rangle^2\langle 3\,5\rangle^3$

$$\frac{1}{\langle ij\rangle^8}\mathcal{M}_n(1^+,2^+,\ldots,i^-,\ldots,j^-,\ldots n^+)=\frac{1}{\langle ab\rangle^8}\mathcal{M}_n(1^+,2^+,\ldots,a^-,\ldots,b^-,\ldots n^+)$$

$$X^{AB}=-X^{BA},(X^{AB})^\dagger=\frac{1}{2}\varepsilon_{ABCD}X^{CD}=:X_{AB}$$

$$S=\frac{1}{g^2}\int dtr^3d^3\Omega\mathrm{Tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}D_\mu X_{AB}D^\mu X^{AB}+\frac{1}{4}[X_{AB},X_{CD}][X^{AB},X^{CD}]-\frac{1}{2r^2}X_{AB}X^{AB}$$
$$+i(\lambda_A)^\dagger\gamma^\mu D_\mu\lambda_A+\lambda_A[X^{AB},\lambda_B]+(\lambda_A)^\dagger[X_{AB},(\lambda_B)^\dagger]\Big]$$

$$D_\mu X^{AB}=\partial_\mu X^{AB}+i[A_\mu,X^{AB}]$$
$$D_\mu\lambda_A=\partial_\mu\lambda_A+\frac{1}{4}\omega_{\mu,\nu\rho}\gamma^{\nu\rho}\lambda_A+i[A_\mu,\lambda_A]$$

$$D_\mu F^{\mu\nu}-i[X_{AB},D^\nu X^{AB}]=0$$
$$D^\mu D_\mu X^{AB}+[X_{CD},[X^{AB},X^{CD}]]-\frac{1}{r^2}X^{AB}=0$$

$$R_A^C=i\int d^3\Omega\frac{1}{g^2}\mathrm{Tr}(-X^{CB}D^0X_{AB}+D^0X^{CB}X_{AB})$$

$$H=\int d^3\Omega\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{2}(F^{0i})^2+\frac{1}{4}(F^{ij})^2+\frac{1}{2}|D_0X_{AB}|^2+\frac{1}{2}|D_iX^{AB}|^2+\frac{1}{4}|[X_{AB},X_{CD}]|^2+\frac{1}{2r^2}|X_{AB}|^2\right)$$

$$J^i=\int d^3\Omega\frac{1}{g^2}\mathrm{Tr}\left(F^0{}_\mu F^{i\mu}+\frac{1}{2}(D^0X_{AB}D^iX^{AB}+D^iX_{AB}D^0X^{AB})\right)$$

$$\Delta_\epsilon A_\mu=i((\epsilon_A)^*\gamma_\mu\lambda_A+\epsilon_A\gamma_\mu(\lambda_A)^\dagger)$$
$$\Delta_\epsilon X^{AB}=i(-\varepsilon^{ABCD}\epsilon_C\lambda_D-(\epsilon_A)^*(\lambda_B)^\dagger+(\epsilon_B)^*(\lambda_A)^\dagger)$$
$$\Delta_\epsilon\lambda_A=\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon_A-2D_\mu X_{AB}\gamma^\mu(\epsilon_B)^*-2i[X_{AB},X^{BC}]\epsilon_C-X_{AB}\gamma^\mu\nabla_\mu(\epsilon_B)^*$$

$$\nabla_\mu\epsilon_A^{(\pm)}=\pm\frac{i}{2r}\gamma_\mu\gamma^t\epsilon_A^{(\pm)}$$

$$\partial_t\epsilon_A^{(+)}=\frac{i}{2r}\epsilon_A^{(+)},\partial_\psi\epsilon_A^{(+)}=\frac{i}{2}\gamma_3{}^t\epsilon_A^{(+)},\partial_\theta\epsilon_A^{(+)}=0,\partial_\phi\epsilon_A^{(+)}=0$$

$$\epsilon_A^{(+)}=e^{\frac{i}{2r}t}e^{-\frac{i}{2}\gamma_{30}\psi}\eta_A^{(+)}$$

$$\partial_t \epsilon_A^{(-)} = -\frac{i}{2r}\epsilon_A^{(-)}, \nabla_\theta^{S^2}\epsilon_A^{(-)} = -\frac{i}{r}\gamma_\theta{}^t\epsilon_A^{(-)}, \nabla_\phi^{S^2}\epsilon_A^{(-)} = -\frac{i}{r}\gamma_\phi{}^t\epsilon_A^{(-)}, \partial_\psi\epsilon_A^{(-)} = 0$$

$$\nabla_\theta\epsilon_A^{(-)} = \partial_\theta\epsilon_A^{(-)}, \nabla_\phi\epsilon_A^{(-)} = \partial_\phi\epsilon_A^{(-)} + \frac{1}{2}\cos\,\theta\gamma^{21}\epsilon_A^{(-)}$$

$$\epsilon_A^{(-)} = e^{-\frac{i}{2r}t}e^{-\frac{i}{2}\gamma_{01}\theta}e^{-\frac{1}{2}\gamma_{21}\phi}\eta_A^{(-)}$$

$$\Delta_{\epsilon^{(\pm)}}\lambda_A = \frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon_A^{(\pm)} - 2i[X_{AB}, X^{BC}]\epsilon_C^{(\pm)} + \left(-2D_0X_{AB} \pm X_{AB}\frac{2i}{r}\right)\gamma^t\left(\epsilon_B^{(\pm)}\right)^* - 2D_iX_{AB}\gamma^i\left(\epsilon_B^{(\pm)}\right)^*$$

$$F_{\mu\nu} = 0, [X_{AB}, X^{B_4}] = 0, -2D_0X_{A_4} + X_{A_4}\frac{2i}{r} = 0, D_iX_{A_4} = 0$$

$$F_{\mu\nu} = 0, 2\partial_0X_{A_4} = X_{A_4}\frac{2i}{r}, \partial_iX_{A_4} = 0$$

$$X_{A_4} = x_{A_4}^{(+)}e^{i\frac{t}{r}}$$

$$X_{12} = X_{34}^\dagger = x_{34}^{(+)\dagger}e^{-i\frac{t}{r}}, X_{13} = -X_{24}^\dagger = -x_{24}^{(+)\dagger}e^{-i\frac{t}{r}}, X_{23} = -X_{14}^\dagger = -x_{14}^{(+)\dagger}e^{-i\frac{t}{r}}$$

$$\Delta_\epsilon(\pm)\lambda_1 = \left(X_{12}\frac{2i}{r} \pm X_{12}\frac{2i}{r}\right)\gamma^t\left(\epsilon_2^{(\pm)}\right)^* + \left(X_{13}\frac{2i}{r} \pm X_{13}\frac{2i}{r}\right)\gamma^t\left(\epsilon_3^{(\pm)}\right)^*,$$

$$\Delta_\epsilon(\pm)\lambda_2 = \left(X_{21}\frac{2i}{r} \pm X_{21}\frac{2i}{r}\right)\gamma^t\left(\epsilon_1^{(\pm)}\right)^* + \left(X_{23}\frac{2i}{r} \pm X_{23}\frac{2i}{r}\right)\gamma^t\left(\epsilon_3^{(\pm)}\right)^*,$$

$$\Delta_\epsilon(\pm)\lambda_3 = \left(X_{31}\frac{2i}{r} \pm X_{31}\frac{2i}{r}\right)\gamma^t\left(\epsilon_1^{(\pm)}\right)^* + \left(X_{32}\frac{2i}{r} \pm X_{32}\frac{2i}{r}\right)\gamma^t\left(\epsilon_2^{(\pm)}\right)^*,$$

$$\Delta_\epsilon(\pm)\lambda_4 = \left(-X_{41}\frac{2i}{r} \pm X_{41}\frac{2i}{r}\right)\gamma^t\left(\epsilon_1^{(\pm)}\right)^* + \left(-X_{42}\frac{2i}{r} \pm X_{42}\frac{2i}{r}\right)\gamma^t\left(\epsilon_2^{(\pm)}\right)^* + \left(-X_{43}\frac{2i}{r} \pm X_{43}\frac{2i}{r}\right)\gamma^t\left(\epsilon_3^{(\pm)}\right)^*.$$

$$\left(\epsilon_1^{(+)}\right)^* = 0 \text{ or } \left(\epsilon_1^{(+)}\right)^* \neq 0, X_{34} = X_{24} = 0$$
$$\left(\epsilon_2^{(+)}\right)^* = 0 \text{ or } \left(\epsilon_2^{(+)}\right)^* \neq 0, X_{34} = X_{14} = 0$$
$$\left(\epsilon_3^{(+)}\right)^* = 0 \text{ or } \left(\epsilon_3^{(+)}\right)^* \neq 0, X_{24} = X_{14} = 0$$
$$\left(\epsilon_1^{(-)}\right)^* = 0 \text{ or } \left(\epsilon_1^{(-)}\right)^* \neq 0, X_{41} = 0$$
$$\left(\epsilon_2^{(-)}\right)^* = 0 \text{ or } \left(\epsilon_2^{(-)}\right)^* \neq 0, X_{42} = 0$$
$$\left(\epsilon_3^{(-)}\right)^* = 0 \text{ or } \left(\epsilon_3^{(-)}\right)^* \neq 0, X_{43} = 0$$

$$F_{\mu\nu} = 0, [X_{AB}, X^{B_4}] = 0, 2D_0X_{A_4} - X_{A_4}\frac{2i}{r} = 0, D_iX_{A_4} = 0$$

$$F_{\mu\nu} = 0, 2\partial_0X_{A_4} + X_{A_4}\frac{2i}{r} = 0, \partial_iX_{A_4} = 0.$$

$$X_{A_4} = x_{A_4}^{(-)} e^{-i\frac{t}{r}}$$

BPS solutions.	Preserved Killing spinors.	Number of SUSY.
$X_{A4} = x_{A4}^{(\pm)} e^{\pm i\frac{t}{r}}$ for $A = 1, 2, 3.$	$\epsilon_4^{(\pm)}.$	4 ($\frac{1}{8}$ BPS)
$X_{A4} = x_{A4}^{(\pm)} e^{\pm i\frac{t}{r}}$ for $A = 2, 3,$ $X_{14} = 0.$	$\epsilon_1^{(\mp)}, \epsilon_4^{(\pm)}.$	8 ($\frac{1}{4}$ BPS)
$X_{34} = x_{34}^{(\pm)} e^{\pm i\frac{t}{r}},$ $X_{14} = X_{24} = 0.$	$\epsilon_1^{(\mp)}, \epsilon_2^{(\mp)}, \epsilon_3^{(\pm)}, \epsilon_4^{(\pm)}.$	16 ($\frac{1}{2}$ BPS)
$X_{14} = X_{24} = X_{34} = 0.$	$\epsilon_1^{(\pm)}, \epsilon_2^{(\pm)}, \epsilon_3^{(\pm)}, \epsilon_4^{(\pm)}.$	32 (Unique vacuum)

$$H = \frac{1}{g^2}\int d^3\Omega \frac{4}{r^2} \mathrm{Tr}\left|X_{A_4}\right|^2$$

$$P_i = 0$$

$$R_4^4 = \int d^3\Omega \frac{2}{g^2}\frac{\mp 1}{r} \mathrm{Tr}\left|X_{A_4}\right|^2$$

$$rH = \mp 2R_4^4 = \pm 2\sum_{A=1}^{3} R_A^A.$$

$$\gamma_0 \eta_4^{(+)} = iw^{(+)}\eta_4^{(+)}$$

$$\epsilon_4^{(+)} = e^{\frac{i}{2r}t} e^{i\frac{w^{(+)}}{2}\psi}\eta_4^{(+)}.$$

$$rH = -2R_4^4 + 2w^{(+)}J_\psi.$$

$$\Delta_\epsilon(\pm)\lambda_C = \Delta_{(\pm)}\lambda_C + \bar{\Delta}_{(\pm)}\lambda_C$$

$$\Delta_{(\pm)}\lambda_A = \frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon_A^{(\pm)} - 2i[X_{AB}, X^{BC}]\epsilon_C^{(\pm)}$$

$$\bar{\Delta}_{(\pm)}\lambda_A = \left(-2D_0 X_{AB} \pm X_{AB}\frac{2i}{r}\right)\gamma^t\left(\epsilon_B^{(\pm)}\right)^* - 2D_i X_{AB}\gamma^i\left(\epsilon_B^{(\pm)}\right)^*$$

$$\Delta_{(+)}\lambda_A = e^{\frac{i\mu}{4}t} e^{i\frac{w^{(+)}}{2}\psi}\left[iw^{(+)}\delta_A^4(-F^{01} + iF^{23} + iw^{(+)}F^{02} - w^{(+)}F^{13})\gamma_1\eta_4^{(+)}\right.$$
$$\left. + \{iw^{(+)}\delta_A^4(iF^{03} + F^{12}) + (-2i[X_{AB}, X^{B4}])\}\eta_4^{(+)}\right] + \cdots$$

$$\bar{\Delta}_{(+)}\lambda_A = e^{-\frac{i\mu}{4}t} e^{i\frac{-w^{(+)}}{2}\psi}\left[\left(\left(-2D_0 X_{A_4} + X_{A_4}\frac{2i}{r}\right)(iw^{(+)}) - 2iD_3 X_{A_4}\right)\left(\eta_4^{(+)}\right)^*\right.$$
$$\left. + (-2D_1 X_{A_4} - 2iD_2 X_{A_4} w^{(+)})\gamma^1\left(\eta_4^{(+)}\right)^*\right] + \cdots$$

$$-F^{01} + iF^{23} + iw^{(+)}F^{02} - w^{(+)}F^{13} = 0$$
$$iw^{(+)}\delta_A^4(iF^{03} + F^{12}) - 2i[X_{AB}, X^{B4}] = 0$$
$$\left(-2D_0X_{A_4} + X_{A_4}\frac{2i}{r}\right)\left(iw^{(+)}\right) - 2iD_3X_{A_4} = 0$$
$$-2D_1X_{A_4} - 2iD_2X_{A_4}w^{(+)} = 0$$

$$F^{01} = -w^{(+)}F^{13}, F^{23} = -w^{(+)}F^{02}$$
$$w^{(+)}\delta_A^4(iF^{03} + F^{12}) - 2[X_{AB}, X^{B4}] = 0$$
$$\left(D_0 - \frac{i}{r} - w^{(+)}D_3\right)X_{A_4} = 0$$
$$(D_1 + iw^{(+)}D_2)X_{A_4} = 0$$

$$F^{12} = F^{03} = 0, F^{13} = -w^{(+)}F^{01}, F^{02} = -w^{(+)}F^{23}$$

$$\left(\partial_0 - \frac{i}{r} - w^{(+)}\partial_3\right)X_{A_4} = 0, \left(\partial_1 + iw^{(+)}\partial_2\right)X_{A_4} = 0$$

$$\hat{L}^2Y_{s,l_3,r_3} = \hat{R}^2Y_{s,l_3,r_3} = s(s+1)Y_{s,l_3,r_3}$$
$$\hat{L}_3Y_{s,l_3,r_3} = l_3Y_{s,l_3,r_3}, \hat{R}_3Y_{s,l_3,r_3} = r_3Y_{s,l_3,r_3}$$

$$\hat{R}_{w^{(+)}}X_{A_4} = 0$$

$$X_{A_4} = \sum_{s\geq 0,|l_3|\leq s} x_{A_4}^{s,l_3}(t)Y_{s,l_3,w^{(+)}s}(\theta,\phi,\psi)$$

$$\partial_0 x_{A_4}^{s,l_3}(t) = \frac{i}{r}x_{A_4}^{s,l_3}(t) - \frac{2}{r}w^{(+)}\left(-iw^{(+)}s\right)x_{A_4}^{s,l_3}(t)$$

$$x_{A_4}^{s,l_3}(t) = x_{A_4}^{s,l_3}e^{i\frac{2s+1}{r}t}$$

$$X_{A_4} = \sum_{s\geq 0,|l_3|\leq s} x_{A_4}^{s,l_3}e^{i\frac{2s+1}{r}t}Y_{s,l_3,w^{(+)}s}(\theta,\phi,\psi)$$

$$\partial_t F^{01} + w^{(+)}\partial_3 F^{01} = 0$$
$$\partial_t F^{23} + w^{(+)}\partial_3 F^{32} = 0$$
$$\partial_1\left(w^{(+)}F^{01}\right) - \partial_2 F^{23} + \frac{2}{r}\cot\,\theta\left(w^{(+)}F^{01}\right) = 0$$

$$\partial_1(\sin\,\theta F^{23}) + w^{(+)}\partial_2(\sin\,\theta F^{01}) = 0$$

$$\partial_1(\sin\,\theta F^{01}) - w^{(+)}\partial_2(\sin\,\theta F^{23}) = 0$$

$$\sin\,\theta F^{01} = \sum_{s\geq\frac{1}{2},|l_3|\leq s}\left(B^{s,l_3}(t)\mathrm{Re}\left[Y_{s,l_3,w^{(+)}s}(\theta,\phi,\psi)\right] - A^{s,l_3}(t)\mathrm{Im}\left[Y_{s,l_3,w^{(+)}s}(\theta,\phi,\psi)\right]\right)$$
$$\sin\,\theta F^{23} = \sum_{s\geq\frac{1}{2},|l_3|\leq s}\left(A^{s,l_3}(t)\mathrm{Re}\left[Y_{s,l_3,w^{(+)}s}(\theta,\phi,\psi)\right] + B^{s,l_3}(t)\mathrm{Im}\left[Y_{s,l_3,w^{(+)}s}(\theta,\phi,\psi)\right]\right)$$

$$\partial_1 u - w^{(+)}\partial_2 v = 0, \partial_1 v + w^{(+)}\partial_2 u = 0$$

$$\partial_1(Au + Bv) + w^{(+)}\partial_2(Bu - Av) = 0, \partial_1(Bu - Av) - w^{(+)}\partial_2(Au + Bv) = 0$$

$$\sin\,\theta F^{01} = \sum_{s\geq\frac{1}{2},|l_3|\leq s} a_{s,l_3}^{(+)}\mathrm{Re}\left[Y_{s,l_3,w^{(+)}s}\left(\theta,\phi,\psi - w^{(+)}\frac{2}{r}t + \alpha_{s,l_3}^{(+)}\right)\right]$$

$$\sin\,\theta F^{23} = \sum_{s\geq\frac{1}{2},|l_3|\leq s} a_{s,l_3}^{(+)}\mathrm{Im}\left[Y_{s,l_3,w^{(+)}s}\left(\theta,\phi,\psi - w^{(+)}\frac{2}{r}t + \alpha_{s,l_3}^{(+)}\right)\right]$$

$$F^{01} = \sum_{s\geq\frac{1}{2},|l_3|\leq s} a_{s,l_3}^{(+)}c_{s,l_3}\tan^{w^{(+)}l_3}\left(\frac{\theta}{2}\right)\sin^{s-1}\,\theta\cos\,\left(l_3\phi + s\left(\frac{2}{r}t - w^{(+)}\psi\right) + \alpha_{s,l_3}^{(+)}\right)$$

$$F^{23} = \sum_{s\geq\frac{1}{2},|l_3|\leq s} a_{s,l_3}^{(+)}c_{s,l_3}\tan^{w^{(+)}l_3}\left(\frac{\theta}{2}\right)\sin^{s-1}\,\theta\sin\,\left(l_3\phi + s\left(\frac{2}{r}t - w^{(+)}\psi\right) + \alpha_{s,l_3}^{(+)}\right)$$

$$x_{14}^{s,l_3}, x_{24}^{s,l_3}, x_{34}^{s,l_3} \in \mathbf{C}, a_{s,l_3}^{(+)} \geq 0, 0 \leq \alpha_{s,l_3}^{(+)} < 2\pi$$

$$H|_{F_{\mu\nu}=0} = \int d^3\Omega\frac{4}{g^2}\mathrm{Tr}\left(\frac{1}{r^2}\left|X_{A_4}\right|^2 - w^{(+)}\frac{i}{r}X_{A_4}\partial_3 X^{A_4} + \left|\partial_2 X_{A_4}\right|^2 + \left|\partial_3 X_{A_4}\right|^2\right),$$

$$J_\theta|_{F_{\mu\nu}=0} = 0,$$

$$J_\phi|_{F_{\mu\nu}=0} = \int d^3\Omega\frac{2}{g^2}\mathrm{Tr}\left(-\left(\frac{i}{r}X_{A_4} - w^{(+)}\partial_3 X_{A_4}\right)\partial_\phi X^{A_4} + (\text{ c.c. })\right),$$

$$J_\psi|_{F_{\mu\nu}=0} = \int d^3\Omega\frac{2}{g^2}\mathrm{Tr}\left(-\left(\frac{i}{r}X_{A_4} - w^{(+)}\partial_3 X_{A_4}\right)\partial_\psi X^{A_4} + (\text{ c.c. })\right),$$

$$R_4^4|_{F_{\mu\nu}=0} = i\int d^3\Omega\frac{2}{g^2}\mathrm{Tr}\left(\frac{i}{r}|X_{4B}|^2 - w^{(+)}X^{4B}\partial_3 X_{4B}\right),$$

$$H|_{X^{AB}=0} = \int d^3\Omega\frac{1}{g^2}\mathrm{Tr}((F^{01})^2 + (F^{23})^2)$$

$$J_\theta|_{X^{AB}=0} = 0$$

$$J_\phi|_{X^{AB}=0} = w^{(+)}\frac{r}{2g^2}\int d^3\Omega\cos\,\theta\mathrm{Tr}((F^{01})^2 + (F^{23})^2)$$

$$J_\psi|_{X^{AB}=0} = w^{(+)}\frac{r}{2g^2}\int d^3\Omega\mathrm{Tr}((F^{01})^2 + (F^{23})^2)$$

$$R_4^4|_{X^{AB}=0} = 0$$

$$H|_{F_{\mu\nu}=0} = \frac{8}{r^2 g^2}\sum_{s\geq 0,|l_3|\leq s}(2s+1)^2\mathrm{Tr}\left|x_{A_4}^{s,l_3}\right|^2,$$

$$J_\phi|_{F_{\mu\nu}=0} = \frac{-4}{rg^2}\sum_{s\geq 0,|l_3|\leq s}(2s+1)l_3\mathrm{Tr}\left|x_{A_4}^{s,l_3}\right|^2,$$

$$J_\psi|_{F_{\mu\nu}=0} = \frac{4}{rg^2}w^{(+)}\sum_{s\geq 0,|l_3|\leq s}(2s+1)s\mathrm{Tr}\left|x_{A_4}^{s,l_3}\right|^2,$$

$$R_4^4|_{F_{\mu\nu}=0} = -\frac{2}{rg^2}\sum_{s\geq 0,|l_3|\leq s}(2s+1)\mathrm{Tr}\left|x_{A_4}^{s,l_3}\right|^2.$$

$$H|_{X_{AB}=0} = w^{(+)}\frac{2}{r}J_\psi\Big|_{X_{AB}=0} = \frac{1}{g^2}\sum_{s\geq\frac{1}{2},|l_3|\leq s}\frac{s(2s+1)}{2(s^2-l_3^2)}\mathrm{Tr}\left(a_{s,l_3}^{(+)}\right)^2,$$

$$J_\phi|_{X_{AB}=0} = \frac{r}{2g^2}\sum_{s\geq\frac{1}{2},|l_3|\leq s}\frac{l_3(2s+1)}{2(l_3^2-s^2)}\mathrm{Tr}\left(a_{s,l_3}^{(+)}\right)^2.$$

$$\gamma_0\eta_1^{(-)} = iw^{(-)}\eta_1^{(-)}$$

$$\epsilon_1^{(-)} = e^{-\frac{i}{2r}t}e^{\frac{-iw^{(-)}\phi}{2}}(c_\theta - w^{(-)}\gamma_1 s_\theta)\eta_1^{(-)}$$

$$c_\theta = \cos\frac{\theta}{2}, s_\theta = \sin\frac{\theta}{2}$$

$$rH = 2R^1{}_1 + 2w^{(-)}J_\phi$$

$$\Delta_{(-)}\lambda_A$$
$$= e^{-\frac{i\mu}{4}t}e^{\frac{-iw^{(-)}\phi}{2}}\times$$
$$\Big[\{(-iw^{(-)}c_\theta)(F^{01}-iF^{23}) - c_\theta(F^{02}+iF^{13}) + s_\theta(F^{03}-iF^{12}) - 2i[X_{AB},X^{B1}](-w^{(-)}s_\theta)\}\gamma_1\eta_1^{(-)}$$
$$+\{-is_\theta(F^{01}-iF^{23}) + s_\theta w^{(-)}(F^{02}+iF^{13}) + c_\theta w^{(-)}(F^{03}-iF^{12}) - 2i[X_{AB},X^{B1}]c_\theta\}\eta_1^{(-)}\Big] + \cdots$$
$$\bar{\Delta}_{(-)}\lambda_A$$
$$= e^{\frac{i\mu}{4}t}e^{\frac{iw^{(-)}\phi}{2}}\times$$
$$\Big[\Big\{\Big(-2D_0X_{A1} - X_{A1}\frac{2i}{r}\Big)(iw^{(-)}c_\theta) - 2D_1X_{A1}(-w^{(-)}s_\theta) - 2D_2X_{A1}(is_\theta) - 2D_3X_{A1}(ic_\theta)\Big\}\left(\eta_1^{(-)}\right)^*$$
$$+\Big\{\Big(-2D_0X_{A1} - X_{A1}\frac{2i}{r}\Big)(is_\theta) - 2D_1X_{A1}(c_\theta) - 2D_2X_{A1}(iw^{(-)}c_\theta) - 2D_3X_{A1}(-iw^{(-)}s_\theta)\Big\}\gamma_1\left(\eta_1^{(-)}\right)^*\Big]$$
$$+\cdots$$

$$\{-is_\theta(F^{01}-iF^{23}) + s_\theta w^{(-)}(F^{02}+iF^{13}) + c_\theta w^{(-)}(F^{03}-iF^{12})\}\delta_A^1 - 2i[X_{AB},X^{B1}]c_\theta = 0,$$
$$\{(-iw^{(-)}c_\theta)(F^{01}-iF^{23}) - c_\theta(F^{02}+iF^{13}) + s_\theta(F^{03}-iF^{12})\}\delta_A^1 - 2i[X_{AB},X^{B1}](-w^{(-)}s_\theta) = 0,$$
$$\Big(-2D_0X_{A_1} - X_{A_1}\frac{2i}{r}\Big)(iw^{(-)}c_\theta) - 2D_1X_{A_1}(-w^{(-)}s_\theta) - 2D_2X_{A_1}(is_\theta) - 2D_3X_{A_1}(ic_\theta) = 0,$$
$$\Big(-2D_0X_{A_1} - X_{A_1}\frac{2i}{r}\Big)(is_\theta) - 2D_1X_{A_1}(c_\theta) - 2D_2X_{A_1}(iw^{(-)}c_\theta) - 2D_3X_{A_1}(-iw^{(-)}s_\theta) = 0,$$

$$F^{02} = -w^{(-)}F^{23}\sec\theta + F^{03}\tan\theta, F^{13} = -w^{(-)}F^{01}\sec\theta - F^{12}\tan\theta$$
$$[X_{AB},X^{B1}] = \frac{1}{2i}\{w^{(-)}\sec\theta(F^{03}-iF^{12}) - (F^{23}+iF^{01})\tan\theta\}\delta_A^1$$
$$D_0X_{A1} = -\frac{i}{r}X_{A1} - w^{(-)}\frac{2}{r}D_\phi X_{A1}$$
$$D_1X_{A1} = iw^{(-)}(-\cos\theta D_2X_{A1} + \sin\theta D_3X_{A1})$$

$$F^{02} = -w^{(-)}\cos\theta F^{23}, \quad F^{03} = w^{(-)}\sin\theta F^{23}$$
$$F^{12} = -w^{(-)}\sin\theta F^{01}, \quad F^{13} = -w^{(-)}\cos\theta F^{01}$$
$$\Big(\partial_0 + \frac{i}{r} + \frac{2}{r}w^{(-)}\partial_\phi\Big)X_{A_1} = 0$$
$$\Big(\partial_1 - iw^{(-)}(-\cos\theta\partial_2 + \sin\theta\partial_3)\Big)X_{A_1} = 0$$

$$X_{A_1} = \sum_{s\geq 0,|r_3|\leq s} x_{A_1}^{s,r_3}(t)Y_{s,w^{(-)}}^{s,r_3}(\theta,\phi,\psi)$$

$$\partial_0 x_{A_1}^{s,r_3}(t) = \frac{-i}{r}x_{A_1}^{s,r_3}(t) - \frac{2}{r}w^{(-)}\left(iw^{(-)}s\right)x_{A_1}^{s,r_3}(t)$$

$$x_{A_1}^{s,r_3}(t) = x_{A_1}^{s,r_3}e^{-i\frac{2s+1}{r}t}$$

$$X_{A1} = \sum_{s\geq 0,|r_3|\leq s} x_{A1}^{s,r_3}e^{-i\frac{2s+1}{r}t}Y_{s,w^{(-)}s,r_3}(\theta,\phi,\psi)$$

$$\partial_t F^{01} = -w^{(-)}\frac{2}{r}\partial_\phi F^{01}$$

$$\partial_t F^{23} = -w^{(-)}\frac{2}{r}\partial_\phi F^{23}$$

$$\partial_3 F^{23} = -w^{(-)}\left(\partial_t(\cos\ \theta F^{23}) + \partial_1(\sin\ \theta F^{01})\right)$$

$$\left(-\frac{1}{\sin\ \theta}\partial_\psi + \cot\ \theta\partial_\phi\right)(\sin\ \theta F^{01}) = -w^{(-)}\partial_\theta(\sin\ \theta F^{23})$$

$$G = \sum_{s\geq\frac{1}{2},|l_3|\leq s} C_{s,r_3}^{(-)}(t)Y_{s,w^{(-)}s_{s,r_3}}(\theta,\phi,\psi)$$

$$\partial_t C_{s,r_3}^{(-)}(t) = -i\frac{2}{r}sC_{s,r_3}^{(-)}(t)$$

$$C_{s,r_3}^{(-)}(t) = C_{s,r_3}^{(-)}e^{-i\frac{2}{r}st} = a_{s,r_3}^{(-)}e^{-i\frac{2}{r}st+i\alpha_{s,r_3}^{(-)}}$$

$$G = \sum_{s\geq\frac{1}{2},|l_3|\leq s} a_{s,r_3}^{(-)}e^{-i\frac{2}{r}st+i\alpha_{s,r_3}^{(-)}}Y_{s,w^{(-)}s,r_3}(\theta,\phi,\psi)$$

$$F^{01} = \sum_{s\geq\frac{1}{2},|r_3|\leq s} a_{s,r_3}^{(-)}c_{s,r_3}\tan^{w^{(-)}r_3}\left(\frac{\theta}{2}\right)\sin^{s-1}\ \theta\cos\left(-r_3\psi + s\left(w^{(-)}\phi - \frac{2}{r}t\right) + \alpha_{s,r_3}^{(-)}\right),$$

$$F^{23} = \sum_{s\geq\frac{1}{2},|r_3|\leq s} a_{s,r_3}^{(-)}c_{s,r_3}\tan^{w^{(-)}r_3}\left(\frac{\theta}{2}\right)\sin^{s-1}\ \theta\sin\left(-r_3\psi + s\left(w^{(-)}\phi - \frac{2}{r}t\right) + \alpha_{s,r_3}^{(-)},\right.$$

$$x_{12}^{s,r_3}, x_{12}^{s,r_3}, x_{13}^{s,r_3} \in \mathbf{C}, a_{s,r_3}^{(-)} \geq 0, 0 \leq \alpha_{s,r_3}^{(-)} < 2\pi$$

$$H|_{F_{\mu\nu}}=0\;=\frac{4}{g^2}\int\;d^3\Omega\mathrm{Tr}\left(\frac{1}{r^2}\left|X_{A_1}\right|^2+\left|\partial_2 X_{A_1}\right|^2+\left|\partial_3 X_{A_1}\right|^2+\frac{2i}{r^2}w^{(-)}X_{A_1}\partial_\phi X^{A_1}\right)$$

$$J_\theta|_{F_{\mu\nu}}=0\;=0$$

$$J_\phi\big|_{F_{\mu\nu}=0}=\int\;d^3\Omega\frac{1}{g^2}\mathrm{Tr}\left(-2\left(-\frac{i}{r}X_{A_1}-w^{(-)}\frac{2}{r}D_\phi X_{A_1}\right)\right)D_\phi X^{A_1}+(\,\mathrm{c.c.}\,)\Big)$$

$$J_\psi\big|_{F_{\mu\nu}=0}\;=\int\;d^3\Omega\frac{1}{g^2}\mathrm{Tr}\left(-2\left(-\frac{i}{r}X_{A_1}-w^{(-)}\frac{2}{r}D_\phi X_{A_1}\right)D_\psi X^{A_1}+(\,\mathrm{c.c.}\,)\right)$$

$$R_1^1\big|_{F_{\mu\nu}=0}\;=i\int\;d^3\Omega\frac{2}{g^2}\mathrm{Tr}\left(-\frac{i}{r}|X_{1A}|^2-w^{(-)}\frac{2}{r}X^{1A}D_\phi X_{A_1}\right)$$

$$H|_{X_{AB}=0}=\int\;d^3\Omega\frac{1}{g^2}\mathrm{Tr}((F^{01})^2+(F^{23})^2)$$

$$J_\theta|_{X_{AB}=0}=0$$

$$J_\phi\big|_{X_{AB}=0}=w^{(-)}\frac{r}{2g^2}\int\;d^3\Omega\mathrm{Tr}((F^{01})^2+(F^{23})^2)$$

$$J_\psi\big|_{X_{AB}=0}=w^{(-)}\frac{r}{2g^2}\int\;d^3\Omega\cos\,\theta\mathrm{Tr}((F^{01})^2+(F^{23})^2)$$

$$H|_{F_{\mu\nu}=0}\;=\frac{8}{r^2g^2}\sum_{s\geq 0,|r_3|\leq s}\;(2s+1)^2\mathrm{Tr}\left|x_{A_1}^{s,r_3}\right|^2$$

$$J_\phi\big|_{F_{\mu\nu}=0}\;=\frac{4}{rg^2}w^{(-)}\sum_{s\geq 0,|r_3|\leq s}\;(2s+1)s\mathrm{Tr}\left|x_{A_1}^{s,r_3}\right|^2$$

$$J_\psi\big|_{F_{\mu\nu}=0}\;=\frac{4}{rg^2}\sum_{s\geq 0,|r_3|\leq s}\;-(2s+1)r_3\mathrm{Tr}\left|x_{A_1}^{s,r_3}\right|^2$$

$$R_1^1\big|_{F_{\mu\nu}=0}\;=\frac{2}{rg^2}\sum_{s\geq 0,|r_3|\leq s}\;(2s+1)\mathrm{Tr}\left|x_{A_1}^{s,r_3}\right|^2$$

$$H|_{X_{AB}=0}=w^{(-)}\frac{2}{r}J_\phi\Big|_{X_{AB}=0}=\frac{1}{g^2}\sum_{s\geq\frac{1}{2},|r_3|\leq s}\frac{s(1+2s)}{2(s^2-r_3^2)}\mathrm{Tr}\left(a_{s,r_3}^{(-)}\right)^2$$

$$J_\psi\big|_{X_{AB}=0}=\frac{r}{2g^2}\sum_{s\geq\frac{1}{2},|r_3|\leq s}\frac{r_3(1+2s)}{2(r_3^2-s^2)}\mathrm{Tr}\left(a_{s,r_3}^{(-)}\right)^2$$

$$X_{A_4}\;=\sum_{s\geq 0,|l_3|\leq s}\;x_{A_4}^{s,l_3}e^{i\frac{2s+1}{r}t}Y_{s,l_3,w^{(+)}s}(\theta,\phi,\psi)$$

$$F^{01}\;=\sum_{s\geq\frac{1}{2},|l_3|\leq s}\;a_{s,l_3}^{(+)}c_{s,l_3}\tan^{w^{(+)}l_3}\left(\frac{\theta}{2}\right)\sin^{s-1}\;\theta\cos\left(l_3\phi+s\left(\frac{2}{r}t-w^{(+)}\psi\right)+\alpha_{s,l_3}^{(+)}\right)$$

$$F^{23}\;=\sum_{s\geq\frac{1}{2},|l_3|\leq s}\;a_{s,l_3}^{(+)}c_{s,l_3}\tan^{w^{(+)}l_3}\left(\frac{\theta}{2}\right)\sin^{s-1}\;\theta\sin\left(l_3\phi+s\left(\frac{2}{r}t-w^{(+)}\psi\right)+\alpha_{s,l_3}^{(+)}\right)$$

$$X_{A_3} = \sum_{s\geq 0,|l_3|\leq s} x_{A_3}^{s,l_3} e^{i\frac{2s+1}{r}t} Y_{s,l_3,w_3^{(+)}s}(\theta,\phi,\psi)$$

$$F^{01} = \sum_{s\geq\frac{1}{2},|l_3|\leq s} a_{s,l_3}^{(+)} c_{s,l_3} \tan^{w_3^{(+)}l_3}\left(\frac{\theta}{2}\right) \sin^{s-1}\theta \cos\left(l_3\phi + s\left(\frac{2}{r}t - w_3^{(+)}\psi\right) + \alpha_{s,l_3}^{(+)}\right)$$

$$F^{23} = \sum_{s\geq\frac{1}{2},|l_3|\leq s} a_{s,l_3}^{(+)} c_{s,l_3} \tan^{w_3^{(+)}l_3}\left(\frac{\theta}{2}\right) \sin^{s-1}\theta \sin\left(l_3\phi + s\left(\frac{2}{r}t - w_3^{(+)}\psi\right) + \alpha_{s,l_3}^{(+)}\right)$$

Parameter region.	Preserved Killing spinors.	Number of SUSY.
$x_{14}^{s,l_3}, x_{24}^{s,l_3}, x_{34}^{s,l_3}, a_{s,l_3}^{(+)}$: generic.	$\eta_4^{(+)}$ with $\gamma_0\eta_4^{(+)} = iw^{(+)}\eta_4^{(+)}$.	2 ($\frac{1}{16}$ BPS)
$x_{14}^{s,l_3} = x_{24}^{s,l_3} = 0$ for $\forall s, l_3$.	$\eta_3^{(+)}, \eta_4^{(+)}$ with $\gamma_0\eta_A^{(+)} = iw^{(+)}\eta_A^{(+)}$.	4 ($\frac{1}{8}$ BPS)
$x_{24}^{s,l_3}, x_{34}^{s,l_3}, a_{s,l_3}^{(+)} = 0$ for $\forall s > 0, l_3 \neq -w^{(-)}s$, $x_{14}^{s,l_3} = 0$ for $\forall s, l_3$.	$\eta_1^{(-)}, \eta_4^{(+)}$ with $\gamma_0\eta_A^{(\pm)} = iw^{(\pm)}\eta_A^{(\pm)}$.	4 ($\frac{1}{8}$ BPS)
$x_{34}^{s,l_3}, a_{s,l_3}^{(+)} = 0$ for $\forall s > 0, l_3 \neq -w^{(-)}s$, $x_{14}^{s,l_3} = x_{24}^{s,l_3} = 0$ for $\forall s, l_3$.	$\eta_1^{(-)}, \eta_2^{(-)}, \eta_3^{(+)}, \eta_4^{(+)}$ with $\gamma_0\eta_A^{(\pm)} = iw^{(\pm)}\eta_A^{(\pm)}$.	8 ($\frac{1}{4}$ BPS)
$x_{14}^{s,l_3} = x_{24}^{s,l_3} = x_{34}^{s,l_3} = 0$ for $\forall s, l_3$.	$\eta_1^{(+)}, \eta_2^{(+)}, \eta_3^{(+)}, \eta_4^{(+)}$ with $\gamma_0\eta_A^{(+)} = iw^{(+)}\eta_A^{(+)}$.	8 ($\frac{1}{4}$ BPS)
$a_{s,l_3}^{(+)} = 0$ for $\forall s > 0, l_3 \neq -w^{(-)}s$, $x_{14}^{s,l_3} = x_{24}^{s,l_3} = x_{34}^{s,l_3} = 0.$ for $\forall s, l_3$	$\eta_1^{(\pm)}, \eta_2^{(\pm)}, \eta_3^{(\pm)}, \eta_4^{(\pm)}$ with $\gamma_0\eta_A^{(\pm)} = iw^{(\pm)}\eta_A^{(\pm)}$.	16 ($\frac{1}{2}$ BPS)

Parameter region.	Preserved Killing spinors.	Number of SUSY.
$x_{14}^{s,l_3}, x_{24}^{s,l_3}, x_{34}^{s,l_3}, a_{s,l_3}^{(+)}$: generic.	$\eta_4^{(+)}$ with $\gamma_0\eta_4^{(+)} = iw^{(+)}\eta_4^{(+)}$.	2 ($\frac{1}{16}$ BPS)
$x_{14}^{s,l_3} = x_{24}^{s,l_3} = 0$ for $\forall s, l_3$.	$\eta_3^{(+)}, \eta_4^{(+)}$ with $\gamma_0\eta_A^{(+)} = iw^{(+)}\eta_A^{(+)}$.	4 ($\frac{1}{8}$ BPS)
$x_{24}^{s,l_3}, x_{34}^{s,l_3}, a_{s,l_3}^{(+)} = 0$ for $\forall s > 0, l_3 \neq -w^{(-)}s$, $x_{14}^{s,l_3} = 0$ for $\forall s, l_3$.	$\eta_1^{(-)}, \eta_4^{(+)}$ with $\gamma_0\eta_A^{(\pm)} = iw^{(\pm)}\eta_A^{(\pm)}$.	4 ($\frac{1}{8}$ BPS)
$x_{34}^{s,l_3} = 0$ for $\forall s > 0, l_3 \neq -w^{(-)}s$ $x_{24}^{s,l_3}, a_{s,l_3}^{(+)} = 0$ for $\forall s > 0, l_3$; $x_{14}^{s,l_3} = 0$ for $\forall s, l_3$.	$\eta_1^{(-)}, \eta_4^{(+)}$ with $\gamma_0\eta_4^{(+)} = iw^{(+)}\eta_4^{(+)}$.	6 ($\frac{3}{16}$ BPS)
$x_{34}^{s,l_3}, a_{s,l_3}^{(+)} = 0$ for $\forall s > 0, l_3 \neq -w^{(-)}s$, $x_{14}^{s,l_3} = x_{24}^{s,l_3} = 0$ for $\forall s, l_3$.	$\eta_1^{(-)}, \eta_2^{(-)}, \eta_3^{(+)}, \eta_4^{(+)}$ with $\gamma_0\eta_A^{(\pm)} = iw^{(\pm)}\eta_A^{(\pm)}$.	8 ($\frac{1}{4}$ BPS)

$$X_{A1} = \sum_{s\geq 0,|r_3|\leq s} x_{A1}^{s,r_3} e^{-i\frac{2s+1}{r}t} Y_{s,w_3^{(-)}}^{s,r_3}(\theta,\phi,\psi),$$

$$F^{01} = \sum_{s\geq\frac{1}{2},|r_3|\leq s} a_{s,r_3}^{(-)} c_{s,r_3} \tan^{w^{(-)}r_3}\left(\frac{\theta}{2}\right) \sin^{s-1}\theta \cos\left(-r_3\psi + s\left(w^{(-)}\phi - \frac{2}{r}t\right) + \alpha_{s,r_3}^{(-)}\right),$$

$$F^{23} = \sum_{s\geq\frac{1}{2},|r_3|\leq s} a_{s,r_3}^{(-)} c_{s,r_3} \tan^{w^{(-)}r_3}\left(\frac{\theta}{2}\right) \sin^{s-1}\theta \sin\left(-r_3\psi + s\left(w^{(-)}\phi - \frac{2}{r}t\right) + \alpha_{s,r_3}^{(-)}\right)$$

$$ds^2_{\mathbf{R}\times\mathbf{S}^2} = -dt^2 + \mu^{-2}(d\theta^2 + \sin^2\,\theta d\phi^2)$$

Parameter region.	Preserved Killing spinors.	Number of SUSY.
$x_{12}^{s,r_3}, x_{13}^{s,r_3}, x_{14}^{s,r_3}, a_{s,r_3}^{(-)}$: generic.	$\eta_1^{(-)}$ with $\gamma_0\eta_1^{(-)} = iw^{(-)}\eta_1^{(-)}$.	2 ($\frac{1}{16}$ BPS)
$x_{13}^{s,r_3} = x_{14}^{s,r_3} = 0$ for $\forall s, r_3$.	$\eta_1^{(-)}, \eta_2^{(-)}$ with $\gamma_0\eta_A^{(-)} = iw^{(-)}\eta_A^{(-)}$.	4 ($\frac{1}{8}$ BPS)
$x_{12}^{s,r_3}, x_{13}^{s,r_3}, a_{s,r_3}^{(-)} = 0$ for $\forall s, r_3 \neq -w^{(+)}s$, $x_{14}^{s,r_3} = 0$ for $\forall s, r_3$.	$\eta_1^{(-)}, \eta_4^{(+)}$ with $\gamma_0\eta_A^{(\pm)} = iw^{(\pm)}\eta_A^{(\pm)}$.	4 ($\frac{1}{8}$ BPS)
$x_{12}^{s,r_3}, x_{13}^{s,r_3}, x_{14}^{s,r_3} = 0$ for $\forall s > 0, r_3$, $a_{s,r_3}^{(-)} = 0$ for $\forall s, r_3$.	$\eta_1^{(-)}$.	4 ($\frac{1}{8}$ BPS)
$x_{12}^{s,r_3} = 0$ for $\forall s, r_3 \neq -w^{(+)}s$ $x_{13}^{s,r_3}, a_{s,r_3}^{(-)} = 0$ for $\forall s > 0, l_3$; $x_{14}^{s,r_3} = 0$ for $\forall s, l_3$.	$\eta_1^{(-)}, \eta_4^{(+)}$, with $\gamma_0\eta_1^{(-)} = iw^{(-)}\eta_1^{(-)}$.	6 ($\frac{3}{16}$ BPS)
$x_{12}^{s,r_3}, a_{s,r_3}^{(-)} = 0$ for $\forall s, r_3 \neq -w^{(+)}s$, $x_{13}^{s,r_3} = x_{14}^{s,r_3} = 0$ for $\forall s, r_3$.	$\eta_1^{(-)}, \eta_2^{(-)}, \eta_3^{(+)}, \eta_4^{(+)}$ with $\gamma_0\eta_A^{(\pm)} = iw^{(\pm)}\eta_A^{(\pm)}$.	8 ($\frac{1}{4}$ BPS)
$x_{12}^{s,r_3} = x_{13}^{s,r_3} = x_{14}^{s,r_3} = 0$ for $\forall s, r_3$.	$\eta_1^{(-)}, \eta_2^{(-)}, \eta_3^{(-)}, \eta_4^{(-)}$ with $\gamma_0\eta_A^{(-)} = iw^{(-)}\eta_A^{(-)}$.	8 ($\frac{1}{4}$ BPS)
$x_{12}^{s,r_3} = x_{13}^{s,r_3} = 0$ for $\forall s > 0, r_3$, $x_{14}^{s,r_3} = a_{s,r_3}^{(-)} = 0$ for $\forall s, r_3$.	$\eta_1^{(-)}, \eta_4^{(+)}$.	8 ($\frac{1}{4}$ BPS)
$a_{s,r_3}^{(-)} = 0$ for $\forall s, r_3 \neq -w^{(+)}s$, $x_{12}^{s,r_3} = x_{13}^{s,r_3} = x_{14}^{s,r_3} = 0$ for $\forall s, r_3$.	$\eta_1^{(\pm)}, \eta_2^{(\pm)}, \eta_3^{(\pm)}, \eta_4^{(\pm)}$ with $\gamma_0\eta_A^{(\pm)} = iw^{(\pm)}\eta_A^{(\pm)}$.	16 ($\frac{1}{2}$ BPS)
$x_{12}^{s,r_3} = 0$ for $\forall s > 0, r_3$, $x_{13}^{s,r_3} = x_{14}^{s,r_3} = a_{s,r_3}^{(-)} = 0$ for $\forall s, r_3$.	$\eta_1^{(-)}, \eta_2^{(-)}, \eta_3^{(+)}, \eta_4^{(+)}$.	16 ($\frac{1}{2}$ BPS)
$x_{12}^{s,r_3} = x_{13}^{s,r_3} = x_{14}^{s,r_3} = a_{s,r_3}^{(-)} = 0$ for $\forall s, r_3$.	$\eta_1^{(\pm)}, \eta_2^{(\pm)}, \eta_3^{(\pm)}, \eta_4^{(\pm)}$.	32 (Unique vacuum)

$$A_i = a_i, A_3 = \phi$$

$$F_{01} = f_{01}, F_{02} = f_{02}, F_{\mu 3} = D_\mu\phi, F_{12} = f_{12} - \mu\phi$$

$$\begin{aligned} S =& \frac{1}{g_3^2}\int dt \frac{d^2\Omega}{\mu^2}\mathrm{Tr}\Big[-\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}D_\mu\phi D^\mu\phi + \mu\phi f_{12} - \frac{1}{2}\mu^2\phi^2 \\ & -\frac{1}{2}D_\mu X_{AB}D^\mu X^{AB} - \frac{\mu^2}{8}X_{AB}X^{AB} + \frac{1}{2}[X_{AB},\phi][X^{AB},\phi] + \frac{1}{4}[X_{AB},X_{CD}][X^{AB},X^{CD}] \\ & +i(\psi_A)^\dagger\gamma^\mu D_\mu\psi_A + \frac{\mu}{4}(\psi_A)^\dagger\rho^t\psi_A + i(\psi_A)^\dagger[\phi,\psi_A] + (\psi_A)^\dagger[X_{AB},(\psi_B)^\dagger] + \psi_A[X^{AB},\psi_B]\Big] \end{aligned}$$

$$\begin{aligned} D_\mu f^{\mu\nu} - i[\phi, D^\nu\phi] - i[X_{AB}, D^\nu X^{AB}] + \mu\gamma^{0\mu\nu}D_\mu\phi = 0 \\ D^\mu D_\mu\phi + \mu f_{12} - \mu^2\phi^2 + [X_{AB},[\phi, X^{AB}]] = 0 \\ D^\mu D_\mu X^{AB} - \frac{\mu^2}{4}X^{AB} + [\phi,[X^{AB},\phi]] + [X_{CD},[X^{AB},X^{CD}]] = 0 \end{aligned}$$

$$R_A^C = i\int d^2\Omega\frac{1}{g_3^2}\mathrm{Tr}(-X^{CB}D^0X_{AB} + D^0X^{CB}X_{AB})$$

$$\begin{aligned} H =& \int d^2\Omega\frac{1}{g_3^2}\mathrm{Tr}\Big(\frac{1}{2}(f^{0i})^2 + \frac{1}{4}(f^{ij})^2 + \frac{1}{2}|D_0\phi|^2 + \frac{1}{2}|\vec{D}\phi|^2 + \frac{1}{2}|D_0X_{AB}|^2 + \frac{1}{2}|\vec{D}X^{AB}|^2 \\ & -\mu\phi f_{12} + \frac{1}{2}\mu^2\phi^2 + \frac{1}{2}|[X_{AB},\phi]|^2 + \frac{1}{4}|[X_{AB},X_{CD}]|^2 + \frac{\mu^2}{8}|X_{AB}|^2\Big) \end{aligned}$$

$$P^i = \int d^2\Omega \frac{1}{g_3^2}\mathrm{Tr}\left(f^0{}_\mu f^{i\mu} + D^0\phi D^i\phi + \frac{1}{2}\left(D^0X_{AB}D^iX^{AB} + D^iX_{AB}D^0X^{AB}\right)\right)$$

$$\partial_t\xi_A = -\frac{i\mu}{4}\xi_A, \nabla_\theta^{\mathbf{S}^2}\xi_A = -\frac{i\mu}{2}\rho_\theta{}^t\xi_A, \nabla_\phi^{\mathbf{S}^2}\xi_A = -\frac{i\mu}{2}\rho_\phi{}^t\xi_A$$

$$\xi_A = e^{-i\frac{\mu}{4}t}e^{-\frac{i}{2}\rho_{01}\theta}e^{-\frac{1}{2}\rho_{21}\phi}\eta_A$$

$$\begin{aligned}
\Delta_\xi A_\mu &= i\left((\xi_A)^*\rho_\mu\psi_A - (\psi_A)^\dagger\rho_\mu\xi_A\right)\\
\Delta_\xi\phi &= (\xi_A)^*\psi_A - (\psi_A)^\dagger\xi_A\\
\Delta_\xi X^{AB} &= i\left(-\varepsilon^{ABCD}\xi_C\psi_D - (\xi_A)^*(\psi_B)^\dagger + (\xi_B)^*(\psi_A)^\dagger\right)\\
\Delta_\xi\psi_A &= \left(-iD_\mu\phi\rho^\mu + \frac{1}{2}f_{\mu\nu}\rho^{\mu\nu} - \mu\phi\rho^{12}\right)\xi_A - 2\left(D_\mu X_{AB}\rho^\mu - [\phi, X_{AB}]\right)(\xi_B)^*\\
&\quad -2i[X_{AB}, X^{BC}]\xi_C - X_{AB}\left(\rho^\mu\nabla_\mu^{\mathbf{S}^2} - \frac{i\mu}{4}\rho^{12}\right)(\xi_B)^*
\end{aligned}$$

$$\gamma_0\eta_1 = iw\eta_1$$

$$\mu^{-1}H = R_1^1 + wJ_\phi$$

$$\begin{aligned}
&f^{02} = -wD^2\phi\sec\theta + D^0\phi\tan\theta, D^1\phi = -wf^{01}\sec\theta - (f_{12} - \mu\phi)\tan\theta\\
&[X_{AB}, X^{B1}] = \frac{1}{2i}\left\{w\sec\theta\left(D^0\phi - i(f_{12} - \mu\phi)\right) - (D^2\phi + if^{01})\tan\theta\right\}\delta_A^1\\
&D_0X_{A_1} = -\frac{i\mu}{2}X_{A_1} - w\mu D_\phi X_{A_1}\\
&D_1X_{A_1} = iw\left(-\cos\theta D_2X_{A_1} + \sin\theta\left[i\phi, X_{A_1}\right]\right)
\end{aligned}$$

$$\begin{aligned}
&f^{02} = -w\cos\theta\partial^2\phi, \partial_t\phi = -w\mu\partial_\phi\phi\\
&f_{12} = \mu\phi - w\sin\theta f^{01}, \partial^1\phi = -w\cos\theta f^{01}\\
&\left(\partial_0 + \frac{i\mu}{2} + \mu w\partial_\phi\right)X_{A_1} = 0, (\partial_1 + iw\cos\theta\partial_2)X_{A_1} = 0
\end{aligned}$$

$$\partial_t f^{01} = -w\mu\partial_\phi f^{01}, \mu\partial_\phi^2\phi = \tan\theta\partial_\theta(\sin\theta f^{01})$$

$$\begin{aligned}
X_{A1} &= \sum_{s\geq 0} x_{A_1}^s e^{-i\mu\left(s+\frac{1}{2}\right)t}Y_{s,ws}(\theta,\phi)\\
f^{01} &= \sum_{s\geq\frac{1}{2}} a_s c_s \sin^{s-1}\theta\cos\left(s(w\phi - \mu t) + \alpha_s\right)\\
\partial^2\phi &= \sum_{s\geq\frac{1}{2}} a_s c_s \sin^{s-1}\theta\sin\left(s(w\phi - \mu t) + \alpha_s\right)
\end{aligned}$$

$$\phi = \phi_0 - \frac{w}{\mu}\sum_{s\geq\frac{1}{2}}\frac{a_s}{s}c_s\sin^s\theta\cos\left(s(w\phi - \mu t) + \alpha_s\right)$$

$$H = \frac{1}{g_3^2}\left(2\mu^2\sum_{s\geq 0}(2s+1)^2\mathrm{Tr}\left|x_{A_1}^s\right|^2 + \sum_{s\geq\frac{1}{2}}\frac{(1+2s)}{2s}\mathrm{Tr}(a_s)^2\right),$$

$$J_\theta = 0$$

$$J_\phi = \frac{w}{g_3^2}\left(\sum_{s\geq 0}2\mu(2s+1)s\mathrm{Tr}\left|x_{A_1}^s\right|^2 + \mu^{-1}\sum_{s\geq\frac{1}{2}}\frac{(1+2s)}{2s}\mathrm{Tr}(a_s)^2\right),$$

$$R^1{}_1 = \frac{\mu}{g_3^2}\sum_{s\geq 0}(2s+1)\mathrm{Tr}\left|x_{A_1}^s\right|^2$$

Parameter region.	Preserved Killing spinors.	Number of SUSY.
$x_{12}^s, x_{13}^s, x_{14}^s, a_s, \phi_0$: generic.	η_1 with $\gamma_0\eta_1 = iw\eta_1$.	2 ($\frac{1}{8}$ BPS)
$x_{13}^s = x_{14}^s = 0$ for $\forall s$.	η_1, η_2 with $\gamma_0\eta_A = iw\eta_A$.	4 ($\frac{1}{4}$ BPS)
$x_{12}^s, x_{13}^s, x_{14}^s = 0$ for $\forall s > 0$, $a_s = 0$ for $\forall s, r_3$.	η_1.	4 ($\frac{1}{4}$ BPS)
$x_{12}^s = x_{13}^s = x_{14}^s = 0$ for $\forall s$.	$\eta_1, \eta_2, \eta_3, \eta_4$ with $\gamma_0\eta_A = iw\eta_A$.	8 ($\frac{1}{2}$ BPS)
$x_{12}^s = 0$ for $\forall s > 0$, $x_{13}^s = x_{14}^s = a_s = 0$ for $\forall s$.	η_1, η_2.	8 ($\frac{1}{2}$ BPS)
ϕ_0: generic, $x_{12}^s = x_{13}^s = x_{14}^s = a_s = 0$ for $\forall s$.	$\eta_1, \eta_2, \eta_3, \eta_4$.	16 (Vacua)

$$\sum_{n_1+n_2+n_3=E/R} c_{n_1n_2n_3}e^{-iEt}z_1^{n_1}z_2^{n_2}z_3^{n_3}$$

$$\gamma_\mu = \begin{pmatrix} 0 & \rho_\mu \\ \rho_\mu & 0\end{pmatrix}, \gamma_3 = \begin{pmatrix} 0 & i \\ -i & 0\end{pmatrix}$$

$$\psi^\dagger\gamma_\mu\chi = \psi^\dagger{}_{\dot\alpha}\gamma_\mu{}^{\dot\alpha}{}_\beta\chi^\beta = C_{\dot\alpha\dot\gamma}\psi^{\dagger\dot\gamma}\gamma_\mu{}^{\dot\alpha}{}_\beta\chi^\beta$$

$$\gamma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

$$g = e^{-i\frac{\phi}{2}\sigma_3}e^{-i\frac{\theta}{2}\sigma_2}e^{-i\frac{\psi}{2}\sigma_3} = \begin{pmatrix} \cos\frac{\theta}{2}e^{\frac{-i(\phi+\psi)}{2}} & -\sin\frac{\theta}{2}e^{\frac{i(-\phi+\psi)}{2}} \\ \sin\frac{\theta}{2}e^{\frac{-i(-\phi+\psi)}{2}} & \cos\frac{\theta}{2}e^{\frac{i(\phi+\psi)}{2}}\end{pmatrix}$$

$$z^i = rg^{i1} = r\begin{pmatrix} \cos\frac{\theta}{2}e^{\frac{-i(\phi+\psi)}{2}} \\ \sin\frac{\theta}{2}e^{\frac{-i(-\phi+\psi)}{2}}\end{pmatrix}$$

$$ds_{\mathbf{S}^3}^2 = |dz^1|^2 + |dz^2|^2 = \frac{r^2}{4}(d\theta^2 + \sin^2\theta d\phi^2 + (d\psi + \cos\theta d\phi)^2)$$

$$ds^2_{\mathbf{R}\times\mathbf{S}^3} = -dt^2 + ds^2_{\mathbf{S}^3}$$

$$e^0 = dt, e^1 = \frac{r}{2}d\theta, e^2 = \frac{r}{2}\sin\,\theta d\phi, e^3 = \frac{r}{2}(d\psi + \cos\,\theta d\phi)$$

$$e^0_t = 1, e^1_\theta = \frac{r}{2}, e^2_\phi = \frac{r}{2}\sin\,\theta, e^3_\phi = \frac{r}{2}\cos\,\theta, e^3_\psi = \frac{r}{2}$$

$$e^t_0 = 1, e^\theta_1 = \frac{2}{r}, e^\phi_2 = \frac{2}{r\sin\,\theta}, e^\psi_2 = \frac{2}{r}(-\cot\,\theta), e^\psi_3 = \frac{2}{r}.$$

$$\omega_{21} = \frac{2}{r}\left(\cot\,\theta e_2 - \frac{1}{2}e_3\right), \omega_{13} = \frac{1}{r}e_2, \omega_{23} = -\frac{1}{r}e_1$$

$$\omega_{2,21} = \frac{2}{r}\cot\,\theta, \omega_{3,21} = -\frac{1}{r}, \omega_{2,13} = \frac{1}{r}, \omega_{1,23} = -\frac{1}{r}.$$

$$\begin{aligned}\partial_1 &= \frac{2}{r}\partial_\theta, \quad \partial_2 = \frac{2}{r}\left(\frac{1}{\sin\theta}\partial_\phi - \cot\theta\partial_\psi\right), \quad \partial_3 = \frac{2}{r}\partial_\psi,\\ \partial_\theta &= \frac{r}{2}\partial_\theta, \quad \partial_\phi = \frac{r}{2}(\sin\theta\partial_2 + \cos\theta\partial_3), \quad \partial_\psi = \frac{r}{2}\partial_3.\end{aligned}$$

$$\hat{L}_a g = -\frac{1}{2}\sigma_a g, \hat{R}_a g = \frac{1}{2}g\sigma_a$$

$$[\hat{L}_a, \hat{L}_b] = i\epsilon_{abc}\hat{L}_c, [\hat{R}_a, \hat{R}_b] = i\epsilon_{abc}\hat{R}_c, [\hat{L}_a, \hat{R}_b] = 0$$

$$\begin{aligned}&\hat{L}_1 = i(\sin\,\phi\partial_\theta - \csc\,\theta\cos\,\phi\partial_\psi + \cot\,\theta\cos\,\phi\partial_\phi)\\ &\hat{L}_2 = i(-\cos\,\phi\partial_\theta - \csc\,\theta\sin\,\phi\partial_\psi + \cot\,\theta\sin\,\phi\partial_\phi)\\ &\hat{L}_3 = -i\partial_\phi\\ &\hat{R}_1 = i(\sin\,\psi\partial_\theta + \cot\,\theta\cos\,\psi\partial_\psi - \csc\,\theta\cos\,\psi\partial_\phi)\\ &\hat{R}_2 = i(\cos\,\psi\partial_\theta - \cot\,\theta\sin\,\psi\partial_\psi + \csc\,\theta\sin\,\psi\partial_\phi)\\ &\hat{R}_3 = i\partial_\psi\end{aligned}$$

$$\hat{L}_\pm = \hat{L}_1 \pm i\hat{L}_2, \hat{R}_\pm = \hat{R}_1 \pm i\hat{R}_2$$

$$\begin{aligned}[\hat{L}_3, \hat{L}_\pm] = \pm\hat{L}_\pm, [\hat{L}_+, \hat{L}_-] = 2\hat{L}_3\\ [\hat{R}_3, \hat{R}_\pm] = \pm\hat{R}_\pm, [\hat{R}_+, \hat{R}_-] = 2\hat{R}_3\end{aligned}$$

$$\begin{aligned}\hat{L}_\pm &= e^{\pm i\phi}\left(i\cot\,\theta\partial_\phi \pm \partial_\theta - i\frac{1}{\sin\,\theta}\partial_\psi\right)\\ \hat{R}_\pm &= e^{\mp i\psi}\left(i\cot\,\theta\partial_\psi \mp \partial_\theta - i\frac{1}{\sin\,\theta}\partial_\phi\right)\end{aligned}$$

$$\hat{L}^2 = \hat{R}^2 = \partial^2_\theta + \cot\,\theta\partial_\theta + \frac{1}{\sin^2\,\theta}(\partial^2_\phi + \partial^2_\psi - 2\cos\,\theta\partial_\phi\partial_\psi)$$

$$\begin{aligned}\hat{L}^2 Y_{s,l_3,r_3} &= \hat{R}^2 Y_{s,l_3,r_3} = s(s+1)Y_{s,l_3,r_3}\\ \hat{L}_3 Y_{s,l_3,r_3} &= l_3 Y_{s,l_3,r_3}, \hat{R}_3 Y_{s,l_3,r_3} = r_3 Y_{s,l_3,r_3}\end{aligned}$$

$$\left(\partial_\theta \mp \frac{1}{\sin\,\theta} l_3 - s\cot\,\theta\right) y_{s,l_3,\pm s}(\theta) = 0$$

$$Y_{s,l_3,\pm s} = y_{s,l_3,\pm s}(\theta) e^{i(l_3\phi \mp s\psi)}$$

$$Y_{s,l_3,\pm s} = c_{s,l_3} \tan^{\pm l_3}\left(\frac{\theta}{2}\right) \sin^s\,\theta e^{i(l_3\phi\mp s\psi)}$$

$$c_{s,l_3} = \frac{1}{\pi}\sqrt{\frac{\Gamma(2s+2)}{2^{2s+1}\Gamma(l_3+s+1)\Gamma(-l_3+s+1)}}$$

$$\int d^3\Omega \left|Y_{s,l_3,\pm s}\right|^2 = 1$$

$$\left(\partial_\theta \mp \frac{1}{\sin\,\theta} r_3 - s\cot\,\theta\right) y_{s,\pm s,r_3}(\theta) = 0$$

$$Y_{s,\pm s,r_3} = y_{s,\pm s,r_3}(\theta) e^{i(-r_3\psi \pm s\phi)}$$

$$Y_{s,\pm s,r_3} = c_{s,r_3} \tan^{\pm r_3}\left(\frac{\theta}{2}\right) \sin^s\,\theta e^{i(-r_3\psi\pm s\phi)}$$

$$\int d^3\Omega \left|Y_{s,\pm s,r_3}\right|^2 = 1$$

$$c_{s,r_3} = \frac{1}{\pi}\sqrt{\frac{\Gamma(2s+2)}{2^{2s+1}\Gamma(r_3+s+1)\Gamma(-r_3+s+1)}}$$

$$Y_{s,l_3,r_3} = \left(\prod_{n=l_3+1}^{s} \frac{\hat{L}_-}{\sqrt{s(s+1)-n(n-1)}}\right) Y_{s,s,r_3}$$

$$\int d^3\Omega \left(Y_{s',l_3',r_3'}\right)^* Y_{s,l_3,r_3} = \delta_{ss'}\delta_{l_3 l_3'}\delta_{r_3 r_3'}$$

$$ds^2_{\mathbf{R}\times\mathbf{S}^2} = -dt^2 + \frac{1}{\mu^2}(d\theta^2 + \sin^2\,\theta d\phi^2)$$

$$e^0 = dt, e^1 = \mu^{-1}d\theta, e^2 = \mu^{-1}\sin\,\theta d\phi$$

$$e^0_t = 1, \quad e^1_\theta = \mu^{-1}, e^2_\phi = \mu^{-1}\sin\,\theta,$$
$$e^t_0 = 1, \quad e^\theta_1 = \mu, e^\phi_2 = \frac{\mu}{\sin\,\theta}.$$

$$\omega^{\mathbf{S}^2}_{21} = \mu\cot\,\theta e^2.$$

$$\hat{L}^2 Y_{s,s_3} = s(s+1)Y_{s,s_3}, \hat{L}_3 Y_{s,s_3} = l_3 Y_{s,s_3}$$

$$Y_{s,s_3} = \sqrt{\frac{\pi}{2}} Y_{s,s_3,r_3=0}$$

$$Y_{s,\pm s} = c_s \sin^s \theta e^{\pm is\phi}$$

$$c_s = \sqrt{\frac{\pi}{2}} c_{s,0} = \sqrt{\frac{\Gamma(2s+2)}{\pi 2^{2s+2}\Gamma(s+1)^2}}$$

$$Y_{s,s_3} = \left(\prod_{n=s_3+1}^{s} \frac{\hat{L}_-}{\sqrt{s(s+1)-n(n-1)}} \right) Y_{s,s}.$$

$$\int d^2\Omega \left(Y_{s',s_3'}\right)^* Y_{s,s_3} = \delta_{ss'}\delta_{ss_3'}$$

$$d^2\Omega = d\theta \sin\theta d\phi$$

$$\int d^3\Omega \frac{1}{\sin^2\theta}\left|Y_{s,l_3,w^{(+)}s}\right|^2 = \frac{s(1+2s)}{2(-l_3^2+s^2)}$$
$$\int d^3\Omega \frac{\cot\theta}{\sin\theta}\left|Y_{s,l_3,w^{(+)}s}\right|^2 = \frac{w^{(+)}l_3(1+2s)}{2(l_3^2-s^2)}$$
$$\int d^3\Omega \cot^2\theta \left|Y_{s,l_3,w^{(+)}s}\right|^2 = -\frac{2l_3^2+s}{2(l_3^2-s^2)}$$

$$\int d^3\Omega \frac{1}{\sin^2\theta}\left|Y_{s,w^{(-)}s,r_3}\right|^2 = \frac{s(1+2s)}{2(-r_3^2+s^2)}$$
$$\int d^3\Omega \frac{\cot\theta}{\sin\theta}\left|Y_{s,w^{(-)}s,r_3}\right|^2 = \frac{w^{(-)}r_3(1+2s)}{2(r_3^2-s^2)}$$
$$\int d^3\Omega \cot^2\theta \left|Y_{s,w^{(-)}s,r_3}\right|^2 = -\frac{2r_3^2+s}{2(r_3^2-s^2)}$$

$$f^{01} = \frac{a_0'}{\sin\theta} + \sum_{s\geq\frac{1}{2}} a_s c_s \sin^{s-1}\theta \cos\left(s(w\phi-\mu t)+\alpha_s\right)$$

$$\phi = \phi_0 - \frac{w}{\mu}\left(a_0' \log\sin\theta + \sum_{s\geq\frac{1}{2}} \frac{a_s}{s} c_s \sin^s\theta \cos\left(s(w\phi-\mu t)+\alpha_s\right) \right).$$

$$\partial_0 A_1 - \partial_1 A_0 + [A_0, A_1] = 0$$

$$W(\gamma_t) = U(t)W(\gamma_0)U^{-1}(t)$$

$$W(\gamma) = Pe^{-\int_\gamma d\sigma A_\mu \frac{dx^\mu}{d\sigma}}$$

$$\mathcal{L}^{(d-1)} \equiv \{f: S^{d-1} \to M \mid \aleph \to x_R\}$$

$$\delta\mathcal{A} + \mathcal{A}\wedge\mathcal{A} = 0$$

$$P_2 e^{\int_{\partial\Omega}\mathcal{B}} = P_3 e^{\int_\Omega \mathcal{A}}$$

$$P_2 e^{\int_{\partial\Omega}\mathcal{B}} = P_3 e^{\int_{\Omega'}\mathcal{A}} \text{ and so } P_3 e^{\int_{\Omega}\mathcal{A}} = P_3 e^{\int_{\Omega'}\mathcal{A}}$$

$$P_3 e^{\int_{\mathbb{R}_t^3}\mathcal{A}} = U(t) P_3 e^{\int_{\mathbb{R}_0^3}\mathcal{A}} U^{-1}(t)$$

$$Q \equiv P_3 e^{\int_{\mathbb{R}_t^3}\mathcal{A}} = P_2 e^{\int_{S_\infty^2}\mathcal{B}}$$

$$q_a(\alpha,\beta) = \sum_{m,n=0}^{\infty} \alpha^m \beta^n q_a^{(m,n)}$$

$$Q_N \equiv \frac{1}{N}\mathrm{Tr}Q^N$$

$$\delta X \equiv \varepsilon\{X, Q_N\}_{PB}$$

$$\{H_{E/B/M/C}, Q_N\}_{PB} \cong 0$$

$$H_E = \frac{1}{2}\int d^3x \mathrm{Tr}(E_i)^2;\ H_B = \frac{1}{2}\int d^3x \mathrm{Tr}(B_i)^2;\ H_C = \int d^3x \mathrm{Tr}\big(A_0(eJ_0 - D_i E_i)\big)$$

$$\left\{\mathcal{A}^{\otimes}_{(\alpha_1,\beta_1,\zeta_1)}, \mathcal{A}_{(\alpha_2,\beta_2,\zeta_2)}\right\}_{PB} = \delta(\zeta_1 - \zeta_2)\{[\mathcal{R}, \mathcal{A}_{(\alpha_1,\beta_1,\zeta_1)} \otimes \mathbb{1} + \mathbb{1} \otimes \mathcal{A}_{(\alpha_2,\beta_2,\zeta_2)}]$$
$$+(\alpha_1 - \alpha_2)[\Xi_{\text{constr.}} + \Xi_{\text{anom.}}]\}$$

$$\mathcal{R} = -e^2\vartheta \frac{\beta_1\beta_2}{\beta_1 - \beta_2} T_a \otimes T_a$$

$$[T_a, T_b] = i f_{abc} T_c;\ \mathrm{Tr}(T_a T_b) = \delta_{ab};\ a,b,c = 1,2,\ldots \mathrm{dim}G$$

$$\left\{Q(\alpha_1,\beta_1)\right)^{\otimes} Q(\alpha_2,\beta_2)\Big\}_{PB} = [\mathcal{R}, Q(\alpha_1,\beta_1) \otimes Q(\alpha_2,\beta_2)]$$
$$+(\alpha_1 - \alpha_2)[\tilde{\Xi}_{\text{constr.}} + \tilde{\Xi}_{\text{anom.}}]$$

$$\{Q_{N_1}(\alpha_1,\beta_1), Q_{N_2}(\alpha_2,\beta_2)\}_{PB} \cong 0$$

$$[\delta_{N_1,\alpha_1,\beta_1}, \delta_{N_2,\alpha_2,\beta_2}]X = \varepsilon_1\varepsilon_2(\alpha_1 - \alpha_2)\{X, \mathrm{Tr}_{RL}(Q^{N_1}(\alpha_1,\beta_1) \otimes Q^{N_2}(\alpha_2,\beta_2)\Upsilon)\}_{PB}$$

$$\mathrm{Tr}_{RL}(Q^{N_1}(\alpha_1,\beta_1) \otimes Q^{N_2}(\alpha_2,\beta_2)\Upsilon)$$

$$\psi_r \to R(g)_{rs}\psi_s;\ g \in G;\ r,s = 1,2,\ldots \mathrm{dim}R$$

$$\delta\psi_r(x) = \varepsilon\{\psi_r(x), Q_N(\alpha,\beta)\}_{PB} = -\varepsilon\beta e^2\vartheta \sum_{a=1}^{\mathrm{dim}G} [R(T_a)\psi(x)]_r \mathrm{Tr}\big(Q^N(\zeta_f) S(x) T_a S^{-1}(x)\big)$$

$$S(x_R) = \mathbb{1}$$

$$\delta(\psi_r^\dagger \psi_r) = \varepsilon\{(\psi_r^\dagger\psi_r), Q_N\}_{PB} = 0$$

$$\delta A_\sigma \equiv \varepsilon\{A_\sigma, Q_N\}_{PB} = 0$$

$$\frac{d\omega_\xi}{d\xi} + A_i \frac{dx^i(\xi)}{d\xi}\omega_\xi = 0;\ \xi \equiv \sigma, \tau, \zeta$$

$$\delta\omega_\sigma \equiv \varepsilon\{\omega_\sigma, Q_N\}_{PB} = 0$$

$$\delta\omega_{\tau/\zeta} = \varepsilon\{\omega_{\tau/\zeta}, Q_N\}_{PB}$$
$$= \varepsilon e^2 \beta\vartheta \left[\omega_{\tau/\zeta} T_a \mathrm{Tr}\left[Q^N(\zeta_f) S_R^{-1} T_a S_R\right] - T_a \omega_{\tau/\zeta} \mathrm{Tr}\left[Q^N(\zeta_f) S_L^{-1} T_a S_L\right]\right]$$

$$\mathfrak{e}_{\tau/\zeta} = \int_{\sigma_i}^{\sigma_f} d\sigma \omega_\sigma^{-1} E_i \omega_\sigma \varepsilon_{ijk} \frac{dx^j}{d\sigma}\frac{dx^k}{d\tau/\zeta}\ \mathfrak{b}_{\tau/\zeta} = \int_{\sigma_i}^{\sigma_f} d\sigma \omega_\sigma^{-1} B_i \omega_\sigma \varepsilon_{ijk} \frac{dx^j}{d\sigma}\frac{dx^k}{d\tau/\zeta}$$

$$\delta\mathfrak{b}_{\tau/\zeta} = \varepsilon\{\mathfrak{b}_{\tau/\zeta}, Q_N\}_{PB} = 0$$
$$\delta\mathfrak{e}_{\tau/\zeta} = \varepsilon\{\mathfrak{e}_{\tau/\zeta}, Q_N\}_{PB} = -\varepsilon e^2 \beta\vartheta [T_a, \mathfrak{e}_{\tau/\zeta}] \mathrm{Tr}\left(Q(\zeta_f)^N S_\mathfrak{e}^{-1} T_a S_\mathfrak{e}\right)$$

$$\frac{d\hat{g}(\tau)}{d\tau} - \hat{g}(\tau)\mathfrak{a}(\tau) = 0 \text{ such that } \hat{g}(x_R) = \mathbb{1}$$

$$\hat{g}_3(\tau) = \hat{g}_1(\tau)\hat{g}_2(\tau)$$

$$\mathfrak{a}_3(\tau) = \hat{g}_2^{-1}(\tau)\mathfrak{a}_1(\tau)\hat{g}_2(\tau) + \mathfrak{a}_2(\tau)$$

$$\hat{g}_3(\tau_f) = \hat{g}_1(\tau_f)\hat{g}_2(\tau_f) \text{ or equivalently } \hat{g}_3(\partial\Omega) = \hat{g}_1(\partial\Omega)\hat{g}_2(\partial\Omega)$$

$$\hat{g}(\partial\Omega) = \hat{g}(\Omega)$$

$$P_2 e^{\int_{\partial\Omega}\mathcal{B}} \to \hat{g}_L(\partial\Omega) P_2 e^{\int_{\partial\Omega}\mathcal{B}} \text{ and } P_3 e^{\int_\Omega \mathcal{A}} \to \hat{g}_L(\Omega) P_3 e^{\int_\Omega \mathcal{A}}$$

$$P_2 e^{\int_{\partial\Omega}\mathcal{B}} \to P_2 e^{\int_{\partial\Omega}\mathcal{B}} \hat{g}_R(\partial\Omega) \text{ and } P_3 e^{\int_\Omega \mathcal{A}} \to P_3 e^{\int_\Omega \mathcal{A}} \hat{g}_R(\Omega)$$

$$\delta\mathcal{A} + \mathcal{A}\wedge\mathcal{A} = 0$$

$$\mathcal{A} = \delta\hat{g}(\partial\Omega)\hat{g}^{-1}(\partial\Omega)$$

$$\mathcal{A} \to \hat{g}_L(\partial\Omega)\mathcal{A}\hat{g}_L^{-1}(\partial\Omega) + \delta\hat{g}_L(\partial\Omega)\hat{g}_L^{-1}(\partial\Omega)$$

$$\mathcal{A} \to \mathcal{A} + \hat{g}(\partial\Omega)\delta\hat{g}_R(\partial\Omega)\hat{g}_R^{-1}(\partial\Omega)\hat{g}^{-1}(\partial\Omega)$$

$$P_2 e^{\int_{\partial\Omega}\mathcal{B}} = P_3 e^{\int_\Omega \mathcal{A}} = \mathbb{1}$$

$$P_2 e^{\int_{\partial\Omega}\mathcal{B}'} = \hat{g}_{L/R}(\partial\Omega) \text{ and } P_3 e^{\int_\Omega \mathcal{A}'} = \hat{g}_{L/R}(\Omega)$$

$$\mathcal{T} \equiv \int_{\sigma_i}^{\sigma_f} d\sigma W^{-1} B_{\mu\nu} W \frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}$$

$$\frac{dW}{d\sigma} + C_\mu \frac{dx^\mu}{d\sigma} W = 0$$

$$W(\sigma) = \left[\mathbb{1} - \int_{\sigma_i}^{\sigma} d\sigma' C(\sigma') + \int_{\sigma_i}^{\sigma} d\sigma' \int_{\sigma_i}^{\sigma'} d\sigma'' C(\sigma') C(\sigma'')\right.$$
$$\left. - \int_{\sigma_i}^{\sigma} d\sigma' \int_{\sigma_i}^{\sigma'} d\sigma'' \int_{\sigma_i}^{\sigma''} d\sigma''' C(\sigma') C(\sigma'') C(\sigma''') + \cdots \right] W_R \equiv P_1 e^{\int_{\sigma_i}^{\sigma} d\sigma' C(\sigma')} W_R$$

$$\frac{dV}{d\tau} - V\mathcal{T} = 0$$

$$V(\tau) = V_R \left[\mathbb{1} + \int_{\tau_i}^{\tau} d\tau' \mathcal{T}(\tau') + \int_{\tau_i}^{\tau} d\tau' \int_{\tau_i}^{\tau'} d\tau'' \mathcal{T}(\tau'') \mathcal{T}(\tau')\right.$$
$$\left. + \int_{\tau_i}^{\tau} d\tau' \int_{\tau_i}^{\tau'} d\tau'' \int_{\tau_i}^{\tau''} d\tau''' \mathcal{T}(\tau''') \mathcal{T}(\tau'') \mathcal{T}(\tau') + \cdots \right] \equiv V_R P_2 e^{\int_{\tau_i}^{\tau} d\tau' \mathcal{T}(\tau')}$$

$$\delta V(\tau) V^{-1}(\tau) = V(\tau) \mathcal{T}(\tau, \delta) V^{-1}(\tau) + \mathcal{K}(\tau, \delta)$$

$$\mathcal{T}(\tau, \delta) \equiv \int_{\sigma_i}^{\sigma_f} d\sigma W^{-1} B_{\mu\nu} W \frac{dx^{\mu}}{d\sigma} \delta x^{\nu}$$

$$\mathcal{K}(\tau, \delta) \equiv \int_{\tau_i}^{\tau} d\tau' \int_{\sigma_i}^{\sigma_f} d\sigma V(\tau') \left\{ W^{-1} \left[D_{\lambda} B_{\mu\nu} + D_{\mu} B_{\nu\lambda} + D_{\nu} B_{\lambda\mu} \right] W \frac{dx^{\mu}}{d\sigma} \frac{dx^{\nu}}{d\tau'} \delta x^{\lambda} \right.$$
$$- \int_{\sigma_i}^{\sigma} d\sigma' \left[B_{\kappa\rho}^{W}(\sigma') - H_{\kappa\rho}^{W}(\sigma'), B_{\mu\nu}^{W}(\sigma) \right] \frac{dx^{\kappa}}{d\sigma'} \frac{dx^{\mu}}{d\sigma}$$
$$\left. \times \left(\frac{dx^{\rho}(\sigma')}{d\tau'} \delta x^{\nu}(\sigma) - \delta x^{\rho}(\sigma') \frac{dx^{\nu}(\sigma)}{d\tau'} \right) \right\} V^{-1}(\tau')$$

$$D_{\mu} * \equiv \partial_{\mu} * + [C_{\mu}, *]$$

$$C_{\mu}, H_{\mu\nu} \equiv \partial_{\mu} C_{\nu} - \partial_{\nu} C_{\mu} + [C_{\mu}, C_{\nu}]$$

$$\delta x^{\mu} = \frac{dx^{\mu}}{d\zeta} d\zeta$$

$$\frac{dV(\tau)}{d\zeta} V^{-1}(\tau) = V(\tau) \mathcal{T}(\tau) V^{-1}(\tau) + \mathcal{K}(\tau)$$

$$\frac{dV}{d\zeta} - \mathcal{K} V = 0$$

$$V(\zeta) = \left[\mathbb{1} + \int_{\zeta_i}^{\zeta} d\zeta' \mathcal{K}(\zeta') + \int_{\zeta_i}^{\zeta} d\zeta' \int_{\zeta_i}^{\zeta'} d\zeta'' \mathcal{K}(\zeta') \mathcal{K}(\zeta'')\right.$$
$$\left. + \int_{\zeta_i}^{\zeta} d\zeta' \int_{\zeta_i}^{\zeta'} d\zeta'' \int_{\zeta_i}^{\zeta''} d\zeta''' \mathcal{K}(\zeta') \mathcal{K}(\zeta'') \mathcal{K}(\zeta''') + \cdots \right] \hat{V}_R \equiv P_3 e^{\int_{\zeta_i}^{\zeta} d\zeta' \mathcal{A}(\zeta')} \hat{V}_R$$

$$V_R P_2 e^{\int_{\tau_i}^{\tau_f} d\tau \mathcal{T}(\tau)} = P_3 e^{\int_{\zeta_i}^{\zeta_f} d\zeta \mathcal{K}(\zeta)} V_R$$

$$P_2 e^{\int_{\tau_i}^{\tau_f} d\tau \mathcal{T}(\tau)} \rightarrow \mathbb{1}$$

$$P_3 e^{\int_{\zeta_i}^{\zeta_f} d\zeta \mathcal{K}(\zeta)} \to \mathbb{1}$$

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi + \left(D_\mu\varphi\right)^\dagger D^\mu\varphi - V(|\varphi|)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie\left[A_\mu, A_\nu\right]$$

$$A_\mu \to gA_\mu g^{-1} + \frac{i}{e}\partial_\mu g g^{-1}; \qquad F_{\mu\nu} \to gF_{\mu\nu}g^{-1}$$
$$\psi \to R^\psi(g)\psi; \qquad \varphi \to R^\varphi(g)\varphi$$

$$D_\mu F^{\mu\nu} = eJ^\nu;\ D_\mu \tilde{F}^{\mu\nu} = 0$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\lambda}F^{\rho\lambda}$$

$$J^\mu = \left[\bar{\psi}\gamma^\mu R^\psi(T_a)\psi + \frac{i}{2}\left(\varphi^\dagger R^\varphi(T_a)D^\mu\varphi - (D^\mu\varphi)^\dagger R^\varphi(T_a)\varphi\right)\right]T_a$$

$$J^\mu \to gJ^\mu g^{-1}$$

$$\left(i\gamma^\mu D_\mu - m\right)\psi = 0;\ D_\mu D^\mu\varphi + \frac{\delta V}{\delta|\varphi|^2}\varphi = 0$$

$$C_\mu = ieA_\mu;\ \ B_{\mu\nu} = ie\left(\alpha F_{\mu\nu} + \beta\tilde{F}_{\mu\nu}\right)$$

$$D_\lambda B_{\mu\nu} + D_\mu B_{\nu\lambda} + D_\nu B_{\lambda\mu} = ie^2\beta\tilde{J}_{\lambda\mu\nu}$$

$$\mathcal{T} \equiv ie\int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}\left(\alpha F_{\mu\nu} + \beta\tilde{F}_{\mu\nu}\right)W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}$$

$$\mathcal{A} = ie^2\int_{\tau_i}^{\tau_f} d\tau V(\tau)\mathcal{J}V^{-1}(\tau)$$

$$\begin{aligned}\mathcal{J} \equiv &\int_{\sigma_i}^{\sigma_f} d\sigma\left\{\beta\tilde{J}^W_{\mu\nu\lambda}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\zeta}\right. \\ &-i\int_{\sigma_i}^{\sigma} d\sigma'\left[(\alpha-1)F^W_{\kappa\rho}(\sigma') + \beta\tilde{F}^W_{\kappa\rho}(\sigma'), \alpha F^W_{\mu\nu}(\sigma) + \beta\tilde{F}^W_{\mu\nu}(\sigma)\right]\frac{dx^\kappa}{d\sigma'}\frac{dx^\mu}{d\sigma} \\ &\times\left.\left(\frac{dx^\rho(\sigma')}{d\tau}\frac{dx^\nu(\sigma)}{d\zeta} - \frac{dx^\rho(\sigma')}{d\zeta}\frac{dx^\nu(\sigma)}{d\tau}\right)\right\}\end{aligned}$$

$$\frac{dV}{d\zeta} - \mathcal{A}V = 0$$

$$\mathcal{J} = \beta\mathcal{J}_M + \beta\mathcal{J}_G + i\alpha\left[\mathcal{F}_\tau, \mathcal{F}_\zeta\right] - i\left[\alpha\mathcal{F}_\tau + \beta\tilde{\mathcal{F}}_\tau, \alpha\mathcal{F}_\zeta + \beta\tilde{\mathcal{F}}_\zeta\right]$$

$$\mathcal{J}_M \equiv \int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}\tilde{J}_{\mu\nu\lambda}W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\zeta}$$

$$\mathcal{J}_G \equiv i\int_{\sigma_i}^{\sigma_f} d\sigma \int_{\sigma_i}^{\sigma} d\sigma' \left\{\left[F_{\kappa\rho}^W(\sigma')\frac{dx^\kappa}{d\sigma'}\frac{dx^\rho(\sigma')}{d\tau}, \tilde{F}_{\mu\nu}^W(\sigma)\frac{dx^\mu}{d\sigma}\frac{dx^\nu(\sigma)}{d\zeta}\right] - \left[F_{\kappa\rho}^W(\sigma')\frac{dx^\kappa}{d\sigma'}\frac{dx^\rho(\sigma')}{d\zeta}, \tilde{F}_{\mu\nu}^W(\sigma)\frac{dx^\mu}{d\sigma}\frac{dx^\nu(\sigma)}{d\tau}\right]\right\}$$

$$\tilde{\mathcal{F}}_{\tau/\zeta} \equiv \int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}\tilde{F}_{\mu\nu}W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau/\zeta}$$

$$\mathcal{F}_{\tau/\zeta} \equiv \int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}F_{\mu\nu}W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau/\zeta} = -\frac{i}{e}W_c^{-1}\frac{dW_c}{d\tau/\zeta}$$

$$\frac{dW}{d\sigma} + ieA_\mu\frac{dx^\mu}{d\sigma}W = 0$$

$$\mathcal{T} = ie[\alpha\mathcal{F}_\tau + \beta\tilde{\mathcal{F}}_\tau] = \alpha W_c^{-1}\frac{dW_c}{d\tau} + ie\beta\tilde{\mathcal{F}}_\tau$$

$$W \to g(x)WW_R^{-1}g^{-1}(x_R)W_R = g(x)W\tilde{g}^{-1}(x_R);\ \tilde{g}(x_R) \equiv W_R^{-1}g(x_R)W_R$$

$$X(x) \to g(x)X(x)g(x)^{-1};\ W^{-1}X(x)W \to g(x_R)W^{-1}X(x)Wg(x_R)^{-1}$$

$$\mathcal{T} \to g(x_R)\mathcal{T}g^{-1}(x_R);\ \mathcal{J} \to g(x_R)\mathcal{J}g^{-1}(x_R)$$

$$V(\tau) \to (V_Rg(x_R)V_R^{-1})V(\tau)g^{-1}(x_R)$$

$$\mathcal{A} \to (V_Rg(x_R)V_R^{-1})\mathcal{A}(V_Rg(x_R)V_R^{-1})^{-1}$$

$$V_RP_2e^{\int_{\tau_i}^{\tau_f} d\tau\mathcal{T}(\tau)} \to (V_Rg(x_R)V_R^{-1})\left(V_RP_2e^{\int_{\tau_i}^{\tau_f} d\tau\mathcal{T}(\tau)}\right)g^{-1}(x_R)$$

$$P_3e^{\int_{\zeta_i}^{\zeta_f} d\zeta\mathcal{A}(\zeta)}V_R \to (V_Rg(x_R)V_R^{-1})\left(P_3e^{\int_{\zeta_i}^{\zeta_f} d\zeta\mathcal{A}(\zeta)}V_R\right)g^{-1}(x_R)$$

$$V_RP_2e^{ie\int_{\partial\Omega} d\tau d\sigma W^{-1}(\alpha F_{\mu\nu}+\beta\tilde{F}_{\mu\nu})W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}} = P_3e^{ie^2\int_\Omega d\zeta d\tau V\mathcal{J}V^{-1}}V_R$$

$$V_Re\int_{\partial\Omega} d\tau d\sigma W^{-1}F_{\mu\nu}W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau} = ie^2\int_\Omega d\zeta d\tau[\mathcal{F}_\tau, \mathcal{F}_\zeta]V_R$$

$$V_Re\int_{\partial\Omega} d\tau d\sigma W^{-1}\tilde{F}_{\mu\nu}W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau} = e^2\int_\Omega d\zeta d\tau(\mathcal{J}_M + \mathcal{J}_G)V_R$$

$$V(\Omega) \equiv P_3e^{ie^2\int_\Omega d\zeta d\tau V\mathcal{J}V^{-1}}V_R = P_3e^{ie^2\int_{\Omega'} d\zeta d\tau V\mathcal{J}V^{-1}}V_R \equiv V(\Omega');\ \text{ if } \partial\Omega = \partial\Omega'$$

$$P_3e^{ie^2\int_\Omega d\zeta d\tau V\mathcal{J}V^{-1}}$$

$$V_{x_R^0}(I \times S_\infty^2)V_{x_R^0}(\mathbb{R}_0^3)$$

$$V_{x_R^0}(X) \equiv P_3e^{ie^2\int_X d\zeta d\tau V\mathcal{J}V^{-1}}$$

$$V_{x_R^0}(\mathbb{R}_t^3)V_{x_R^0}(I \times S_0^2)$$

$$F_{\mu\nu} \to \frac{1}{r^{\frac{3}{2}+\delta}}\, J_\mu \to \frac{1}{r^{2+\delta'}}\, r \to \infty$$

$$V_{x_R^0}(I \times S_\infty^2) \to \mathbb{1}\, V_{x_R^0}(I \times S_0^2) \to \mathbb{1}$$

$$V_{x_R^0}(\mathbb{R}_t^3) = W^{-1}(x_R^t, x_R^0) V_{x_R^t}(\mathbb{R}_t^3) W(x_R^t, x_R^0)$$

$$V_{x_R^t}(\mathbb{R}_t^3) = W(x_R^t, x_R^0) V_{x_R^0}(\mathbb{R}_0^3) W^{-1}(x_R^t, x_R^0)$$

$$Q(\alpha,\beta) \equiv V_{x_R^t}(\mathbb{R}_t^3)\; Q_N(\alpha,\beta) \equiv \frac{1}{N}\mathrm{Tr}[Q(\alpha,\beta)]^N$$

$$Q(\alpha,\beta) = V_R P_2 e^{ie\int_{S_\infty^2} d\tau d\sigma W^{-1}(\alpha F_{\mu\nu}+\beta\tilde{F}_{\mu\nu})W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}} = P_3 e^{ie^2\int_{\mathbb{R}_t^3} d\zeta d\tau V \mathcal{J} V^{-1}} V_R$$

$$Q(\alpha,\beta) = P_3 e^{ie^2\int_{\mathbb{R}^3} d\zeta d\tau V \mathcal{J} V^{-1}} V_R$$

$$Q(\alpha,\beta) \to (V_R g(x_R) V_R^{-1}) Q(\alpha,\beta) g^{-1}(x_R)$$

$$Q(\alpha,\beta) = \sum_{n,m=0}^{\infty} \alpha^m \beta^n Q(m,n)$$

$$Q(m,n,t) = W(x_R^t, x_R^0) Q(m,n,0) W^{-1}(x_R^t, x_R^0)$$

$$Q_N(m,n) = \frac{1}{N}\mathrm{Tr}[Q(m,n)]^N$$

$$V(\partial\Omega) = V_R P_2 e^{ie\int_{\partial\Omega} d\tau d\sigma W^{-1}(\alpha F_{\mu\nu}+\beta\tilde{F}_{\mu\nu})W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}} = \sum_{n,m=0}^{\infty} \alpha^m \beta^n V(\partial\Omega, m, n)$$

$$V(\Omega) = P_3 e^{ie^2\int_{\Omega} d\zeta d\tau V \mathcal{J} V^{-1}} V_R = \sum_{n,m=0}^{\infty} \alpha^m \beta^n V(\Omega, m, n)$$

$$V(\partial\Omega, m, n) = V(\Omega, m, n)$$

$$\delta Q_N(\alpha,\beta) = \mathrm{Tr}\left[Q^N(\alpha,\beta)\int_{\tau_i}^{\tau_f} d\tau V(\tau)\mathcal{N}V^{-1}(\tau)\right] = 0$$

$$\mathcal{N} = e^2\int_{\sigma_i}^{\sigma_f} d\sigma \int_{\sigma_i}^{\sigma} d\sigma' \left[(\alpha-1)F_{ij}^W(\sigma') + \beta\tilde{F}_{ij}^W(\sigma'), \alpha F_{kl}^W(\sigma) + \beta\tilde{F}_{kl}^W(\sigma)\right]$$
$$\times \frac{dx^i}{d\sigma'}\frac{dx^k}{d\sigma}\left(\frac{dx^j(\sigma')}{d\tau}\delta x^l(\sigma) - \delta x^j(\sigma')\frac{dx^l(\sigma)}{d\tau}\right) \text{ on } S_\infty^2$$

$$F_{ij} \sim \varepsilon_{ijk}\frac{\hat{r}_k}{r^2}G(\hat{r});\; \tilde{F}_{ij} \sim \varepsilon_{ijk}\frac{\hat{r}_k}{r^2}\tilde{G}(\hat{r})$$

$$D_k G(\hat{r}) = 0;\; D_k\tilde{G}(\hat{r}) = 0$$

$$\frac{d}{d\sigma}(W^{-1}G(\hat{r})W) = 0;\; \frac{d}{d\sigma}\left(W^{-1}\tilde{G}(\hat{r})W\right) = 0$$

$$W^{-1}F_{ij}W \sim \varepsilon_{ijk}\frac{\hat{r}_k}{r^2}c;\ W^{-1}\tilde{F}_{ij}W \sim \varepsilon_{ijk}\frac{\hat{r}_k}{r^2}\tilde{c}$$

$$\mathcal{L}= -\frac{1}{2}\mathrm{Tr}\left[\left(\partial_\mu A_\nu - \partial_\nu A_\mu + ie\left[A_\mu, A_\nu\right] - \frac{1}{2}F_{\mu\nu}\right)F^{\mu\nu}\right] + \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi$$
$$+\frac{1}{2}\left[\varphi_\mu^\dagger D^\mu\varphi + (D^\mu\varphi)^\dagger\varphi_\mu - V(|\varphi|)\right] - \frac{1}{2}\varphi_\mu^\dagger\varphi^\mu$$

$$E_i = F_{0i};\ B_i = -\frac{1}{2}\varepsilon_{ijk}F_{jk}$$

$$\pi_i^a = \frac{\delta\mathcal{L}}{\delta\partial_0 A_i^a} = E_i^a$$

$$\pi_\alpha^\psi = \frac{\delta\mathcal{L}}{\delta\partial_0\psi_\alpha} = i\psi_\alpha^\dagger;\ \alpha = 1,2,3,4$$

$$\pi_\varphi = \frac{\delta\mathcal{L}}{\delta\partial_0\varphi} = \frac{1}{2}\varphi_0^\dagger;\ \pi_{\varphi^\dagger} = \frac{\delta\mathcal{L}}{\delta\partial_0\varphi^\dagger} = \frac{1}{2}\varphi_0$$

$$\mathcal{L} = \pi_i^a\partial_0 A_i^a + \pi_\alpha^\psi\partial_0\psi_\alpha + \pi_\varphi\partial_0\varphi + \pi_{\varphi^\dagger}\partial_0\varphi^\dagger - \mathcal{H} + A_0^a\mathcal{C}_a$$

$$\mathcal{H}= \frac{1}{2}\left[(E_i^a)^2 + (B_i^a)^2\right] + i\bar{\psi}\gamma_i D_i\psi + m\bar{\psi}\psi + \frac{1}{2}\left[\varphi_i^\dagger D_i\varphi + (D_i\varphi)^\dagger\varphi_i\right]$$
$$+V(|\varphi|) + 2\pi_\varphi\pi_{\varphi^\dagger} - \frac{1}{2}\varphi_i^\dagger\varphi_i$$

$$\mathcal{C}_a = (D_iE_i)_a - e\psi^\dagger R^\psi(T_a)\psi + ie\left[\pi_\varphi R^\varphi(T_a)\varphi - \varphi^\dagger R^\varphi(T_a)\pi_{\varphi^\dagger}\right]$$

$$\varphi_\mu = D_\mu\varphi;\ \varphi_\mu^\dagger = \left(D_\mu\varphi\right)^\dagger;\ F_{ij} = \partial_i A_j - \partial_j A_i + ie\left[A_i, A_j\right]$$

$$H_T = \int d^3x(\mathcal{H} - A_0^a\mathcal{C}_a)$$

$$\left\{A_i^a(x), A_j^b(y)\right\}_{PB} = 0$$
$$\left\{\pi_i^a(x), \pi_j^b(y)\right\}_{PB} = 0$$
$$\left\{A_i^a(x), \pi_j^b(y)\right\}_{PB} = \delta^{ab}\delta_{ij}\delta^{(3)}(x-y)$$
$$\left\{\psi_\alpha(x), \pi_\beta^\psi(y)\right\}_{PB} = \delta^{\alpha\beta}\delta^{(3)}(x-y)$$
$$\left\{\varphi(x), \pi_\varphi(y)\right\}_{PB} = \delta^{(3)}(x-y)$$
$$\left\{\varphi^\dagger(x), \pi_{\varphi^\dagger}(y)\right\}_{PB} = \delta^{(3)}(x-y)$$

$$\left\{E_i^a(x), B_j^b(y)\right\}_{PB} = -\varepsilon_{ijk}\left[ef_{abc}A_k^c(x)\delta^{(3)}(x-y) - \delta^{ab}\frac{\partial\delta^{(3)}(x-y)}{\partial y^k}\right]$$

$$J_0 = J_0^a T_a = \left[\rho_a^\psi + \rho_a^\varphi\right]T_a$$

$$\rho_a^\psi = -i\pi^\psi R^\psi(T_a)\psi\ \rho_a^\varphi = -i\left[\pi_\varphi R^\varphi(T_a)\varphi - \varphi^\dagger R^\varphi(T_a)\pi_{\varphi^\dagger}\right]$$

$$\mathcal{C} = \mathcal{C}_a T_a = D_i E_i - eJ_0$$

$$\{\mathcal{C}_a(x), \psi(y)\}_{PB} = -ieR^{\psi}(T_a)\psi(x)\delta^{(3)}(x-y)$$
$$\left\{\mathcal{C}_a(x), \pi^{\psi}(y)\right\}_{PB} = ie\pi^{\psi}(x)R^{\psi}(T_a)\delta^{(3)}(x-y)$$
$$\{\mathcal{C}_a(x), \varphi(y)\}_{PB} = -ieR^{\varphi}(T_a)\varphi(x)\delta^{(3)}(x-y)$$
$$\left\{\mathcal{C}_a(x), \pi_{\varphi}(y)\right\}_{PB} = ie\pi_{\varphi}(x)R^{\varphi}(T_a)\delta^{(3)}(x-y)$$
$$\left\{\mathcal{C}_a(x), E_i^b(y)\right\}_{PB} = -ef_{abc}E_i^c(x)\delta^{(3)}(x-y)$$
$$\left\{\mathcal{C}_a(x), A_i^b(y)\right\}_{PB} = -ef_{abc}A_i^c(x)\delta^{(3)}(x-y) - \delta^{ab}\frac{\partial\delta^{(3)}(x-y)}{\partial x^i}$$

$$\frac{dQ(\alpha,\beta)}{d\zeta} - \mathcal{A}(\alpha,\beta)Q(\alpha,\beta) = 0$$

$$\mathfrak{e}_{\tau/\zeta}(\sigma) \equiv \int_{\sigma_i}^{\sigma} d\sigma' W^{-1}E_i W\varepsilon_{ijk}\frac{dx^j}{d\sigma'}\frac{dx^k}{d\tau/\zeta}$$
$$\mathfrak{b}_{\tau/\zeta}(\sigma) \equiv \int_{\sigma_i}^{\sigma} d\sigma' W^{-1}B_i W\varepsilon_{ijk}\frac{dx^j}{d\sigma'}\frac{dx^k}{d\tau/\zeta}$$

$$\mathfrak{e}_{\tau/\zeta}(\sigma) \to g(x_R)\mathfrak{e}_{\tau/\zeta}(\sigma)g^{-1}(x_R);\ \ \mathfrak{b}_{\tau/\zeta}(\sigma) \to g(x_R)\mathfrak{b}_{\tau/\zeta}(\sigma)g^{-1}(x_R)$$

$$\frac{dV}{d\tau} - V\mathcal{T}_\tau = 0;\ \mathcal{T}_\tau = -ie\left[\alpha\mathfrak{b}_\tau(\sigma_f) + \beta\mathfrak{e}_\tau(\sigma_f)\right]$$

$$F_{ij} = -\varepsilon_{ijk}B_k;\ \tilde{F}_{ij} = -\varepsilon_{ijk}E_k$$

$$\mathcal{J}_{\text{spatial}} = \beta(\rho_M + \rho_G) + \alpha\rho_{\text{mag.}} - i\left[\alpha\mathfrak{b}_\tau(\sigma_f) + \beta\mathfrak{e}_\tau(\sigma_f), \alpha\mathfrak{b}_\zeta(\sigma_f) + \beta\mathfrak{e}_\zeta(\sigma_f)\right]$$

$$\rho_M(\tau,\zeta) = -\int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}J_0 W\varepsilon_{ijk}\frac{dx^i}{d\sigma}\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}$$

$$\rho_G(\tau,\zeta) = i\int_{\sigma_i}^{\sigma_f} d\sigma\left\{\left[\mathfrak{b}_\tau(\sigma), \frac{d\mathfrak{e}_\zeta(\sigma)}{d\sigma}\right] - \left[\mathfrak{b}_\zeta(\sigma), \frac{d\mathfrak{e}_\tau(\sigma)}{d\sigma}\right]\right\}$$

$$\rho_{\text{mag.}} = i\left[\mathfrak{b}_\tau(\sigma_f), \mathfrak{b}_\zeta(\sigma_f)\right]$$

$$\int_{S^2_{\infty,t}} d\tau d\sigma W^{-1}B_i W\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau} = -e\int_{\mathbb{R}^3_t} d\zeta d\tau\rho_{\text{mag}}$$

$$\int_{S^2_{\infty,t}} d\tau d\sigma W^{-1}E_i W\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau} = -e\int_{\mathbb{R}^3_t} d\zeta d\tau(\rho_M + \rho_G)$$

$$Q(1,0) = ie^2\int_{\mathbb{R}^3_t} d\zeta d\tau\rho_{\text{mag.}}$$

$$Q(0,1) = ie^2\int_{\mathbb{R}^3_t} d\zeta d\tau(\rho_M + \rho_G)$$

$$\left\{\rho_G^a(\tau,\zeta),\rho_G^b(\tau',\zeta')\right\}_{PB} = -\vartheta f_{abc}\rho_G^c(\tau,\zeta)\delta(\zeta-\zeta')\delta(\tau-\tau')$$
$$\left\{\rho_M^a(\tau,\zeta),\rho_M^b(\tau',\zeta')\right\}_{PB} = -\vartheta f_{abc}\rho_M^c(\tau,\zeta)\delta(\zeta-\zeta')\delta(\tau-\tau')$$
$$\left\{\rho_{\text{mag.}}^a(\tau,\zeta),\rho_{\text{mag.}}^b(\tau',\zeta')\right\}_{PB} = 0$$

$$\left\{\rho_G^a(\tau,\zeta),\rho_M^b(\tau',\zeta')\right\}_{PB} = 0$$
$$\left\{\rho_G^a(\tau,\zeta),\rho_{\text{mag.}}^b(\tau',\zeta')\right\}_{PB} = 0$$
$$\left\{\rho_M^a(\tau,\zeta),\rho_{\text{mag.}}^b(\tau',\zeta')\right\}_{PB} = 0$$

$$\delta X \equiv \varepsilon\{X,Q_N(\alpha,\beta)\}_{PB} = \frac{\varepsilon}{N}\{X,\text{Tr}[Q(\alpha,\beta)]^N\}_{PB}$$

$$\frac{d\{X,Q(\alpha,\beta)\}_{PB}}{d\zeta} - \{X,\mathcal{A}(\alpha,\beta)\}_{PB}Q(\alpha,\beta) - \mathcal{A}(\alpha,\beta)\{X,Q(\alpha,\beta)\}_{PB} = 0$$

$$\frac{dQ^{-1}(\alpha,\beta)}{d\zeta} + Q^{-1}(\alpha,\beta)\mathcal{A}(\alpha,\beta) = 0$$

$$\frac{d[Q^{-1}(\alpha,\beta)\{X,Q(\alpha,\beta)\}_{PB}]}{d\zeta} - Q^{-1}(\alpha,\beta)\{X,\mathcal{A}(\alpha,\beta)\}_{PB}Q(\alpha,\beta) = 0$$

$$\{X,Q(\zeta)\}_{PB} = Q(\zeta)\int_{\zeta_i}^{\zeta} d\zeta' Q^{-1}(\zeta')\{X,\mathcal{A}(\zeta')\}_{PB}Q(\zeta')$$

$$\frac{d\{X,V\}_{PB}}{d\tau} - \{X,V\}_{PB}\mathcal{T}_\tau - V\{X,\mathcal{T}_\tau\}_{PB} = 0$$

$$\frac{dV^{-1}}{d\tau} + \mathcal{T}_\tau V^{-1} = 0$$

$$\frac{d[\{X,V\}_{PB}V^{-1}]}{d\tau} = V\{X,\mathcal{T}_\tau\}_{PB}V^{-1}$$

$$\{X,V(\tau)\}_{PB}V^{-1}(\tau) = \int_{\tau_i}^{\tau} d\tau' V(\tau')\{X,\mathcal{T}_\tau(\tau')\}_{PB}V^{-1}(\tau')$$

$$\{X,V(\tau)T_aV^{-1}(\tau)\}_{PB} = \left[\left[\int_{\tau_i}^{\tau} d\tau' V(\tau')\{X,\mathcal{T}_\tau(\tau')\}_{PB}V^{-1}(\tau'), V(\tau)T_aV^{-1}(\tau)\right]\right]$$

$$\{X,\mathcal{A}(\zeta)\}_{PB} = ie^2\int_{\tau_i}^{\tau_f} d\tau\left[V(\tau)\left\{X,\mathcal{J}_{\text{spatial}}(\tau)\right\}_{PB}V^{-1}(\tau)\right.$$
$$\left.+\left[\left[\int_{\tau_i}^{\tau} d\tau' V(\tau')\{X,\mathcal{T}_\tau(\tau')\}_{PB}V^{-1}(\tau'), V(\tau)\mathcal{J}_{\text{spatial}}(\tau)V^{-1}(\tau)\right]\right]\right]$$

$$V(\tau)\mathcal{J}_{\text{spatial}}(\tau)V^{-1}(\tau) = \frac{d}{d\tau}\int_{\tau_i}^{\tau} d\tau' V(\tau')\mathcal{J}_{\text{spatial}}(\tau')V^{-1}(\tau')$$

$$ie^2\int_{\tau_i}^{\tau_f} d\tau\left[\int_{\tau_i}^{\tau} d\tau' V(\tau')\{X,\mathcal{T}_\tau(\tau')\}_{PB}V^{-1}(\tau'),V(\tau)\mathcal{J}_{\text{spatial}}(\tau)V^{-1}(\tau)\right]$$
$$=\left[\int_{\tau_i}^{\tau_f} d\tau V(\tau)\{X,\mathcal{T}_\tau(\tau)\}_{PB}V^{-1}(\tau),\frac{dQ(\zeta)}{d\zeta}Q^{-1}(\zeta)\right]$$
$$-\int_{\tau_i}^{\tau_f} d\tau[V(\tau)\{X,\mathcal{T}_\tau(\tau)\}_{PB}V^{-1}(\tau),\mathcal{A}(\tau)]$$

$$\mathcal{A}(\tau)=ie^2\int_{\tau_i}^{\tau} d\tau' V(\tau')\mathcal{J}_{\text{spatial}}(\tau')V^{-1}(\tau')$$

$$\mathcal{A}(\tau_f)=\frac{dQ(\zeta)}{d\zeta}Q^{-1}(\zeta)$$

$$V(\tau)\mathcal{T}_\zeta(\tau)V^{-1}(\tau)=\frac{dV(\tau)}{d\zeta}V^{-1}(\tau)-\mathcal{K}(\tau)$$

$$\mathcal{T}_\zeta=-ie[\alpha\mathfrak{b}_\zeta(\sigma_f)+\beta\mathfrak{e}_\zeta(\sigma_f)]$$

$$\mathcal{K}(\tau)=\int_{\tau_i}^{\tau} d\tau' V(\tau')\left\{-ie\int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}[\alpha D_i B_i+\beta D_i E_i]W\varepsilon_{jkl}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau'}\frac{dx^l}{d\zeta}\right.$$
$$\left.+ie^2(\beta\rho_G+\alpha\rho_{\text{mag.}}-i[\alpha\mathfrak{b}_\tau(\sigma_f)+\beta\mathfrak{e}_\tau(\sigma_f),\alpha\mathfrak{b}_\zeta(\sigma_f)+\beta\mathfrak{e}_\zeta(\sigma_f)])\right\}V^{-1}(\tau')$$

$$\mathcal{K}(\tau_f)=\frac{dV(\tau_f)}{d\zeta}V^{-1}(\tau_f)$$

$$ie^2\int_{\tau_i}^{\tau_f} d\tau V\{X,\mathcal{J}_{\text{spatial}}\}_{PB}V^{-1}$$
$$=ie^2\int_{\tau_i}^{\tau_f} d\tau V\left[\{X,\beta(\rho_M+\rho_G)+\alpha\rho_{\text{mag.}}\}_{PB}+\frac{i}{e^2}\{X,[\mathcal{T}_\tau,\mathcal{T}_\zeta]\}_{PB}\right]V^{-1}$$
$$=\int_{\tau_i}^{\tau_f} d\tau V\left[\left\{X,ie^2[\beta(\rho_M+\rho_G)+\alpha\rho_{\text{mag.}}]+\frac{d\mathcal{T}_\zeta}{d\tau}-\frac{d\mathcal{T}_\tau}{d\zeta}\right\}_{PB}\right]V^{-1}$$
$$+\int_{\tau_i}^{\tau_f} d\tau\left[\frac{d}{d\zeta}(V\{X,\mathcal{T}_\tau\}_{PB}V^{-1})-\frac{d}{d\tau}\left(V\{X,\mathcal{T}_\zeta\}_{PB}V^{-1}\right)\right]$$
$$+\int_{\tau_i}^{\tau_f} d\tau[V(\tau)\{X,\mathcal{T}_\tau\}_{PB}V^{-1}(\tau),\mathcal{K}(\tau)]$$

$$\{X,\mathcal{A}(\zeta)\}_{PB}=\int_{\tau_i}^{\tau_f} d\tau\left\{V(\tau)\{X,\mathcal{M}\}_{PB}V^{-1}(\tau)-\frac{d}{d\tau}\left(V(\tau)\{X,\mathcal{T}_\zeta(\tau)\}_{PB}V^{-1}(\tau)\right)\right.$$
$$\left.+[V(\tau)\{X,\mathcal{T}_\tau(\tau)\}_{PB}V^{-1}(\tau),\mathcal{K}(\tau)-\mathcal{A}(\tau)]\right\}$$
$$+\frac{d}{d\zeta}\int_{\tau_i}^{\tau_f} d\tau(V(\tau)\{X,\mathcal{T}_\tau(\tau)\}_{PB}V^{-1}(\tau))$$
$$+\left[\int_{\tau_i}^{\tau_f} d\tau V(\tau)\{X,\mathcal{T}_\tau(\tau)\}_{PB}V^{-1}(\tau),\frac{dQ(\zeta)}{d\zeta}Q^{-1}(\zeta)\right]$$

$$\mathcal{M} \equiv ie^2\left[\beta(\rho_M + \rho_G) + \alpha\rho_{\text{mag.}}\right] + \frac{d\mathcal{T}_\zeta}{d\tau} - \frac{d\mathcal{T}_\tau}{d\zeta}$$

$$\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\zeta} = 0 \ \text{ at } \ \tau = \tau_i \ \text{ and } \ \tau = \tau_f$$

$$\mathfrak{e}_{\tau/\zeta} = \mathfrak{b}_{\tau/\zeta} = 0 \ \text{ at } \ \tau = \tau_i \ \text{ and } \ \tau = \tau_f$$

$$\int_{\tau_i}^{\tau_f} d\tau \frac{d}{d\tau}\left(V(\tau)\{X, \mathcal{T}_\zeta(\tau)\}_{PB} V^{-1}(\tau)\right) = 0$$

$$\mathcal{K}(\tau) - \mathcal{A}(\tau)$$
$$= -ie\int_{\tau_i}^{\tau} d\tau' V(\tau') \int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}[\alpha D_i B_i + \beta(D_i E_i - eJ_0)]W\varepsilon_{jkl}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau'}\frac{dx^l}{d\zeta}V^{-1}(\tau')$$

$$\mathcal{M} = ie\int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}[\alpha D_i B_i + \beta(D_i E_i - eJ_0)]W\varepsilon_{jkl}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\frac{dx^l}{d\zeta}$$

$$\{X, \mathcal{A}(\zeta)\}_{PB} = ie\beta\vartheta\int_{\tau_i}^{\tau_f} d\tau V(\tau)\int_{\sigma_i}^{\sigma_f} d\sigma\{X, \mathcal{C}_a\}_{PB} W^{-1}T_a W V^{-1}(\tau)\Delta(\sigma,\tau,\zeta)$$
$$+Q(\zeta)\frac{d}{d\zeta}\left[Q^{-1}(\zeta)\int_{\tau_i}^{\tau_f} d\tau(V(\tau)\{X, \mathcal{T}_\tau(\tau)\}_{PB}V^{-1}(\tau))Q(\zeta)\right]Q^{-1}(\zeta)$$
$$+ie\beta\vartheta\mathcal{X}$$

$$\mathcal{X} \equiv \int_{\tau_i}^{\tau_f} d\tau\left\{V(\tau)\int_{\sigma_i}^{\sigma_f} d\sigma\{X, W^{-1}T_a W\}_{PB}V^{-1}(\tau)\mathcal{C}_a\Delta(\sigma,\tau,\zeta)\right.$$
$$\left.-\left[V(\tau)\{X, \mathcal{T}_\tau(\tau)\}_{PB}V^{-1}(\tau), \int_{\tau_i}^{\tau} d\tau' V(\tau')\int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}\mathcal{C}WV^{-1}(\tau')\Delta(\sigma,\tau',\zeta)\right]\right\}$$

$$\Delta(\sigma,\tau,\zeta) \equiv \vartheta\varepsilon_{ijk}\frac{dx^i}{d\sigma}\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}; \ \vartheta = \pm 1$$

$$\{X, Q(\zeta)\}_{PB} = \left[\int_{\tau_i}^{\tau_f} d\tau(V(\tau)\{X, \mathcal{T}_\tau(\tau)\}_{PB}V^{-1}(\tau))\right]_{\zeta=\zeta} Q(\zeta)$$
$$+ie\beta\vartheta Q(\zeta)\int_{\zeta_i}^{\zeta} d\zeta' Q^{-1}(\zeta')\int_{\tau_i}^{\tau_f} d\tau V(\tau)\int_{\sigma_i}^{\sigma_f} d\sigma\{X, \mathcal{C}_a\}_{PB}W^{-1}T_a W$$
$$\times V^{-1}(\tau)Q(\zeta')\Delta(\sigma,\tau,\zeta')$$
$$+ie\beta\vartheta Q(\zeta)\int_{\zeta_i}^{\zeta} d\zeta' Q^{-1}(\zeta')\mathcal{X}Q(\zeta')$$

$$\frac{dx^\mu}{d\tau} = 0 \ \text{ at } \ \zeta = \zeta_i$$

$$\mathfrak{e}_\tau = \mathfrak{b}_\tau = 0 \ \text{ at } \ \zeta = \zeta_i$$

$$\delta X = \varepsilon\{X, Q_N(\alpha,\beta)\}_{PB}$$
$$= \varepsilon \mathrm{Tr}\left[Q^N(\zeta_f)\left\{\left[\left[\int_{\tau_i}^{\tau_f} d\tau(V(\tau)\{X,\mathcal{J}_\tau(\tau)\}_{PB}V^{-1}(\tau))\right]_{\zeta=\zeta_f}\right.\right.\right.$$
$$+ie\beta\vartheta\int_{\zeta_i}^{\zeta_f} d\zeta Q^{-1}(\zeta)\int_{\tau_i}^{\tau_f} d\tau V(\tau)\int_{\sigma_i}^{\sigma_f} d\sigma\{X,\mathcal{C}_a\}_{PB}W^{-1}T_aWV^{-1}(\tau)Q(\zeta)\Delta(\sigma,\tau,\zeta)$$
$$\left.\left.+ie\beta\vartheta\int_{\zeta_i}^{\zeta_f} d\zeta Q^{-1}(\zeta)\mathcal{X}Q(\zeta)\right\}\right]$$

$$\delta_{(\beta=0)}X = \varepsilon\{X, Q_N(\alpha,0)\}_{PB}$$
$$= -ie\alpha\varepsilon \mathrm{Tr}\left[Q^N(\zeta_f,\alpha,0)\left[\int_{\tau_i}^{\tau_f} d\tau\left(V_{\beta=0}(\tau)\{X,\mathfrak{b}_\tau(\sigma_f)\}_{PB}V_{\beta=0}^{-1}(\tau)\right)\right]_{\zeta=\zeta_f}\right]$$

$$\delta^{(N,1,0)}X = \varepsilon\{X, Q_N(1,0)\}_{PB}$$
$$= -ie\varepsilon \mathrm{Tr}\left[Q^{N-1}(1{,}0)\left[\int_{\tau_i}^{\tau_f} d\tau\{X,\mathfrak{b}_\tau(\sigma_f)\}_{PB}\right]_{\zeta=\zeta_f}\right]$$

$$\delta^{(N,0,1)}X = \varepsilon\{X, Q_N(0,1)\}_{PB}$$
$$= \varepsilon \mathrm{Tr}\left\{Q^{N-1}(0{,}1)\left[-ie\left[\int_{\tau_i}^{\tau_f} d\tau\{X,\mathfrak{e}_\tau(\sigma_f)\}_{PB}\right]_{\zeta=\zeta_f}\right.\right.$$
$$+ie\vartheta\int_{\zeta_i}^{\zeta_f} d\zeta\int_{\tau_i}^{\tau_f} d\tau\int_{\sigma_i}^{\sigma_f} d\sigma\{X,\mathcal{C}_a\}_{PB}W^{-1}T_aW\Delta(\sigma,\tau,\zeta)$$
$$\left.\left.+ie\vartheta\int_{\zeta_i}^{\zeta_f} d\zeta\int_{\tau_i}^{\tau_f} d\tau\int_{\sigma_i}^{\sigma_f} d\sigma\{X,W^{-1}T_aW\}_{PB}\Delta(\sigma,\tau,\zeta)\mathcal{C}_a\right]\right\}$$

$$\delta X - \delta' X = \varepsilon\{X, Q_N(\alpha,\beta) - Q'_N(\alpha,\beta)\}_{PB}$$

$$\mathcal{O}(X) \equiv \int_{\zeta_i}^{\zeta_f} d\zeta\int_{\tau_i}^{\tau_f} d\tau\int_{\sigma_i}^{\sigma_f} d\sigma\{X,\mathcal{C}_a\}_{PB}\otimes Q^{-1}(\zeta)V(\tau)W^{-1}T_aWV^{-1}(\tau)Q(\zeta)\Delta(\sigma,\tau,\zeta)$$
$$= \int d^3x d_{ba}(x)\{X,\mathcal{C}_a\}_{PB}\otimes T_b \equiv \mathcal{O}_b(X)\otimes T_b$$

$$gT_ag^{-1} = T_bd_{ba}(g)$$

$$Q^{-1}(\zeta)V(\tau)W^{-1}(\sigma)T_aW(\sigma)V^{-1}(\tau)Q(\zeta) = T_bd_{ba}(Q^{-1}(\zeta)V(\tau)W^{-1}(\sigma))$$
$$\equiv T_bd_{ba}(x)$$

$$\mathcal{O}_b(\psi(y)) = ied_{ba}(y)R^\psi(T_a)\psi(y);\ \mathcal{O}_b(\pi^\psi(y)) = -ied_{ba}(y)\pi^\psi(y)R^\psi(T_a)$$
$$\mathcal{O}_b(\varphi(y)) = ied_{ba}(y)R^\varphi(T_a)\varphi(y);\ \mathcal{O}_b(\pi^\varphi(y)) = -ied_{ba}(y)\pi^\varphi(y)R^\varphi(T_a)$$
$$\mathcal{O}_b(E_i(y)) = ied_{ba}(y)[T_a,E_i(y)]$$

$$\mathcal{O}_b(A_i(y)) = ied_{ba}(y)[T_a,A_i(y)] - \frac{\partial d_{ba}(y)}{\partial y^i}T_a + \int d^3x\frac{\partial}{\partial x^i}\left(d_{ba}(x)\delta^{(3)}(x-y)\right)T_a$$
$$= -D_i(d_{ba}(y)T_a) + \int d^3x\frac{\partial}{\partial x^i}\left(d_{ba}(x)\delta^{(3)}(x-y)\right)T_a$$

$$\mathcal{C}_a = 0 \text{ and } D_iB_i = 0 \rightarrow Q(\zeta_f) \equiv Q(\mathbb{R}_t^3) = V(S_\infty^2)$$

$$\left[\int_{\tau_i}^{\tau_f} d\tau (V(\tau)\{X, \mathcal{T}_\tau(\tau)\}_{PB} V^{-1}(\tau))\right]_{\zeta=\zeta_f} = \{X, V(S_\infty^2)\}_{PB} V^{-1}(S_\infty^2)$$

$$\begin{aligned}\delta X &= \varepsilon\{X, Q_N(\alpha,\beta)\}_{PB} \\ &\cong \frac{\varepsilon}{N}\{X, \mathrm{Tr}[V^N(S_\infty^2)]\}_{PB} + ie\varepsilon\beta\vartheta \mathrm{Tr}[Q^N(\zeta_f)\mathcal{O}(X)]\end{aligned}$$

$$\{X, \mathrm{Tr}[Q^N(\mathbb{R}_t^3)]\}_{PB} \neq \{X, \mathrm{Tr}[V^N(S_\infty^2)]\}_{PB}$$

$$H_T = H_E + H_B + H_\psi + H_\varphi - H_C$$

$$H_E = \frac{1}{2}\int d^3x (E_i^a)^2 \quad H_B = \frac{1}{2}\int d^3x (B_i^a)^2$$

$$H_C = \int d^3x A_0^a \mathcal{C}_a \qquad H_\psi = \int d^3x [i\bar{\psi}\gamma_i D_i\psi + m\bar{\psi}\psi]$$

$$H_\varphi = \int d^3x \left[2\pi_\varphi \pi_{\varphi^\dagger} + \frac{1}{2}(D_i\varphi)^\dagger D_i\varphi + V(|\varphi|)\right]$$

$$\{\bar{\psi}\psi(x), \mathcal{C}_a(y)\}_{PB} = \left\{\pi_\varphi\pi_{\varphi^\dagger}(x), \mathcal{C}_a(y)\right\}_{PB} = \{V(|\varphi|), \mathcal{C}_a(y)\}_{PB} = 0$$

$$\begin{aligned}\mathcal{O}_b(\bar{\psi}(y)\gamma_i D_i\psi(y)) &= ie\bar{\psi}(y)\left[\int d^3x \frac{\partial}{\partial x^i}\left(\gamma_i d_{ba}(x)\delta^{(3)}(x-y)\right)\right] R^\psi(T_a)\psi(y) \\ &= \left[\vartheta \int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma d_{ba}(x)\delta^{(3)}(x-y)\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\right]_{\zeta=\zeta_f} ie\bar{\psi}(y)\gamma_i R^\psi(T_a)\psi(y)\end{aligned}$$

$$\begin{aligned}\mathcal{O}(i\bar{\psi}(y)\gamma_i D_i\psi(y)) &= -e\vartheta Q^{-1}(\zeta_f) \\ \times &\left[\int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma V(\tau)W^{-1}(\sigma)J_i^\psi(y)W(\sigma)V^{-1}(\tau)\delta^{(3)}(x-y)\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\right]_{\zeta=\zeta_f} Q(\zeta_f)\end{aligned}$$

$$J_i^\psi = \bar{\psi}\gamma_i R^\psi(T_a)\psi T_a$$

$$\begin{aligned}\mathcal{O}\left(\frac{1}{2}(D_i\varphi(y))^\dagger D_i\varphi(y)\right) &= -e\vartheta Q^{-1}(\zeta_f) \\ \times &\left[\int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma V(\tau)W^{-1}(\sigma)J_i^\varphi(y)W(\sigma)V^{-1}(\tau)\delta^{(3)}(x-y)\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\right]_{\zeta=\zeta_f} Q(\zeta_f)\end{aligned}$$

$$J_i^\varphi = \frac{i}{2}[\varphi^\dagger R^\varphi(T_a)D_i\varphi - (D_i\varphi)^\dagger R^\varphi(T_a)\varphi]T_a$$

$$\left\{H_{\psi/\varphi}, \mathcal{T}_\tau\right\}_{PB} = ie^2\beta\int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}(\sigma)J_i^{\psi/\varphi}(y)W(\sigma)\varepsilon_{ijk}\frac{dy^j}{d\sigma}\frac{dy^k}{d\tau}$$

$$\delta H_{\psi/\varphi} = \varepsilon\left\{H_{\psi/\varphi}, Q_N(\alpha,\beta)\right\}_{PB} \cong 0$$

$$\left\{\left(E_i^b(x)\right)^2, \mathcal{C}_a(y)\right\}_{PB} = 0$$

$$\{H_E, \mathcal{T}_\tau\}_{PB} = e^2\beta \int_{\sigma_i}^{\sigma_f} d\sigma' \left[\frac{d\mathfrak{e}_\tau(\sigma')}{d\sigma'}, \int_{\sigma_i}^{\sigma'} d\sigma'' W^{-1}(\sigma'') E_i(z) W(\sigma'') \frac{dz^i}{d\sigma''}\right]$$
$$+e^2\alpha \left[\mathfrak{b}_\tau(\sigma_f), \int_{\sigma_i}^{\sigma_f} d\sigma' W^{-1}(\sigma') E_i(y) W(\sigma') \frac{dy^i}{d\sigma'}\right]$$
$$+ie\alpha \frac{d}{d\tau} \int_{\sigma_i}^{\sigma_f} d\sigma' W^{-1}(\sigma') E_i(y) W(\sigma') \frac{dy^i}{d\sigma'}$$

$$\frac{dy^j}{d\sigma/\tau} \to r \text{ as } r \to \infty$$

$$\mathfrak{e}_\tau \to \frac{1}{r^{\delta-1/2}};\ \mathfrak{b}_\tau \to \frac{1}{r^{\delta-1/2}} \text{ as } r \to \infty$$

$$E_i \frac{dy^i}{d\sigma} \to \frac{1}{r^{\delta+1/2}} \text{ as } r \to \infty$$

$$\left[\int_{\tau_i}^{\tau_f} d\tau \left(V(\tau)\{H_E, \mathcal{T}_\tau(\tau)\}_{PB} V^{-1}(\tau)\right)\right]_{\zeta=\zeta_f} \to \frac{s_1}{r^{2\delta}} + \frac{s_2}{r^{\delta+1/2}} \text{ as } r \to \infty$$

$$\delta H_E = \varepsilon\{H_E, Q_N(\alpha,\beta)\}_{PB} \cong 0$$

$$\mathcal{O}_b(\mathcal{C}(y)) = ied_{ba}(y)[T_a, \mathcal{C}(y)] + ie[T_a, E_i(y)] \int d^3x \frac{\partial}{\partial x^i}\left[d_{ba}(x)\delta^{(3)}(x-y)\right]$$
$$\cong ie\vartheta[T_a, E_i(y)] \int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma \varepsilon_{ijk} \frac{dx^j}{d\sigma}\frac{dx^k}{d\tau} d_{ba}(x)\delta^{(3)}(x-y)\bigg|_{\zeta=\zeta_f}$$

$$\mathcal{O}_b(H_C) \cong -ie\vartheta \int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma \varepsilon_{ijk} \frac{dx^j}{d\sigma}\frac{dx^k}{d\tau} d_{ba}(x) \mathrm{Tr}(T_a[A_0(x), E_i(x)])\bigg|_{\zeta=\zeta_f}$$

$$A_0 \to \frac{1}{r^{1/2+\delta}};\ A_i \to \frac{1}{r^{3/2+\delta}} \text{ as } r \to \infty \text{ for } i = 1,2,3$$

$$\mathcal{O}_b(H_C) \to \frac{1}{r^{2\delta}} \text{ as } r \to \infty$$

$$\{H_C, \mathcal{T}_\tau(\tau)\}_{PB} = ie[\mathcal{T}_\tau(\tau), W_R^{-1} A_0(x_R) W_R]$$

$$\left[\int_{\tau_i}^{\tau_f} d\tau \left(V(\tau)\{H_C, \mathcal{T}_\tau(\tau)\}_{PB} V^{-1}(\tau)\right)\right]_{\zeta=\zeta_f} \to \frac{1}{r^{2\delta}} \text{ as } r \to \infty$$

$$\delta H_C = \varepsilon\{H_C, Q_N(\alpha,\beta)\}_{PB} \cong 0$$

$$\mathcal{O}_b(B_i(y)) = -\varepsilon_{ijk} D_j \mathcal{O}_b(A_k(y))$$

$$\mathcal{O}_b(H_B) = \int d^3y \mathrm{Tr}[B_i(y)\mathcal{O}_b(B_i(y))]$$
$$= \varepsilon_{ijk}\int d^3y \mathrm{Tr}[B_i(y)D_jD_k(d_{ba}(y)T_a)]$$
$$-\int d^3y\left[\mathrm{Tr}(B_i(y)T_a)\frac{\partial \mathcal{S}_{ij}^{ba}(y)}{\partial y^j} - ie\mathrm{Tr}([A_j(y),B_i(y)]T_a)\mathcal{S}_{ij}^{ba}(y)\right]$$

$$\mathcal{S}_{ij}^{ba}(y) \equiv \varepsilon_{ijk}\int d^3x \frac{\partial}{\partial x^k}[d_{ba}(x)\delta^{(3)}(x-y)]$$
$$= \vartheta\int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma d_{ba}(x)\delta^{(3)}(x-y)\left(\frac{dx^i}{d\sigma}\frac{dx^j}{d\tau} - \frac{dx^j}{d\sigma}\frac{dx^i}{d\tau}\right)\Bigg|_{\zeta=\zeta_f}$$

$$\mathcal{O}_b(H_B) = \varepsilon_{ijk}\int d^3y \frac{\partial}{\partial y^j}\frac{\partial}{\partial y^k}[d_{ba}(y)\mathrm{Tr}(B_i(y)T_a)] + \int d^3y \mathrm{Tr}[D_jB_i(y)T_a]\mathcal{S}_{ij}^{ba}(y)$$
$$-\int d^3y \frac{\partial}{\partial y^j}[\mathrm{Tr}(B_i(y)T_a)\mathcal{S}_{ij}^{ba}(y)]$$

$$\mathcal{O}_b(H_B) \to \frac{1}{r^{\frac{1}{4}+\frac{3}{2}\delta}} \quad \text{as } r \to \infty$$

$$\{H_B, \mathfrak{b}_\tau(\sigma)\}_{PB} = 0$$

$$\{H_B, \mathfrak{e}_\tau(\sigma_f)\}_{PB} = \int_{\sigma_i}^{\sigma_f} d\sigma' \left(\frac{dy^i}{d\sigma'}\frac{dy^j}{d\tau} - \frac{dy^j}{d\sigma'}\frac{dy^i}{d\tau}\right) W^{-1}(\sigma')D_jB_iW(\sigma')$$

$$\left[\int_{\tau_i}^{\tau_f} d\tau(V(\tau)\{H_B, \mathcal{T}_\tau(\tau)\}_{PB}V^{-1}(\tau))\right]_{\zeta=\zeta_f} \to \frac{1}{r^{\frac{1}{4}+\frac{3}{2}\delta}} \quad \text{as } r \to \infty$$

$$\delta H_B = \varepsilon\{H_B, Q_N(\alpha,\beta)\}_{PB} \cong 0$$

$$\delta H_T = \varepsilon\{H_T, Q_N(\alpha,\beta)\}_{PB} \cong 0$$

$$\delta\psi(x,t) = -\varepsilon e^2\beta\vartheta R^\psi(T_a)\psi(x,t)$$
$$\times \mathrm{Tr}[Q^N(\zeta_f)Q^{-1}(\zeta_x)V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)Q(\zeta_x)]$$

$$\delta^{(3)}(x-y)\Delta(\sigma,\tau,\zeta) = \delta(\zeta-\zeta')\delta(\tau-\tau')\delta(\sigma-\sigma')$$

$$\delta\varphi(x,t) = -\varepsilon e^2\beta\vartheta R^\varphi(T_a)\varphi(x,t)$$
$$\times \mathrm{Tr}[Q^N(\zeta_f)Q^{-1}(\zeta_x)V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)Q(\zeta_x)]$$

$$\delta_{(\beta=0)}\psi = 0; \; \delta^{(N,1,0)}\psi = 0$$

$$\delta^{(N,0,1)}\psi(x,t) = -\varepsilon e^2\vartheta R^\psi(T_a)\psi(x,t)\mathrm{Tr}[Q^{N-1}(0,1)W^{-1}(\sigma_x)T_aW(\sigma_x)]$$
$$= -\varepsilon e^2\vartheta R^\psi(W_x)R^\psi(T_a)R^\psi(W_x^{-1})\psi(x,t)\mathrm{Tr}[Q^{N-1}(0,1)T_a]$$

$$T_a \otimes gT_ag^{-1} = T_a \otimes T_bd_{ba}(g) = T_ad_{ab}(g^{-1}) \otimes T_b = g^{-1}T_bg \otimes T_b$$

$$\delta^{(2,0,1)}\psi(x,t) = -i\varepsilon e^4\vartheta R^{\psi}\left(W_x\left[\int_{\mathbb{R}_t^3} d\zeta d\tau(\rho_M + \rho_G)\right]W_x^{-1}\right)\psi(x,t)$$

$$\delta_{(\beta=0)}A_i = 0;\ \delta_{(\beta=0)}B_i = 0$$

$$A = A_{\sigma_x}d\sigma_x + A_{\tau_x}d\tau_x + A_{\zeta_x}d\zeta_x$$

$$A_{\sigma_x} = A_i(x)\frac{dx^i}{d\sigma_x}, A_{\tau_x} = A_i(x)\frac{dx^i}{d\tau_x}$$

$$A_{\zeta_x} = A_i(x)\frac{dx^i}{d\zeta_x}$$

$$\left\{A_i^a(x)\frac{dx^i}{d\sigma_x}, Q_N(\alpha,\beta)\right\}_{PB} = ie\beta\vartheta\mathrm{Tr}\left[Q^N(\zeta_f)\left(I_{i,a,\sigma_x}^{(1)} + I_{i,a,\sigma_x}^{(2)}\right)\right]$$

$$I_{i,a,\sigma_x}^{(1)} = ie\int_{\zeta_i}^{\zeta_f} d\zeta Q^{-1}(\zeta)\int_{\tau_i}^{\tau_f} d\tau V(\tau)\int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}[A_i(x),T_a]WV^{-1}(\tau)Q(\zeta)$$
$$\times\,\delta^{(3)}(x-y)\Delta(\sigma,\tau,\zeta)\frac{dx^i}{d\sigma_x}$$

$$I_{i,a,\sigma_x}^{(2)} = \int_{\zeta_i}^{\zeta_f} d\zeta Q^{-1}(\zeta)\int_{\tau_i}^{\tau_f} d\tau V(\tau)\int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}T_aWV^{-1}(\tau)Q(\zeta)\frac{\partial\delta^{(3)}(x-y)}{\partial y^i}\frac{dx^i}{d\sigma_x}\Delta(\sigma,\tau,\zeta)$$

$$I_{i,a,\sigma_x}^{(2)} = \frac{dx^i}{d\sigma_x}\Bigg[-\frac{\partial}{\partial x^i}[Q^{-1}(\zeta_x)V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)Q(\zeta_x)]$$
$$+\,Q^{-1}(\zeta_f)\left[\left[\int_{S_\infty^2} d\tau d\sigma V(\tau)W^{-1}(\sigma)T_aW(\sigma)V^{-1}(\tau)\vartheta\varepsilon_{ijk}\frac{dy^j}{d\sigma}\frac{dy^k}{d\tau}\delta^{(3)}(x-y)\right]_{\zeta=\zeta_f} Q(\zeta_f)\right]$$

$$\varepsilon_{ijk}\frac{dx^i}{d\sigma_x}\frac{dy^j}{d\sigma}\frac{dy^k}{d\tau}\delta^{(3)}(x-y) = 0$$

$$I_{i,a,\sigma_x}^{(2)} = -Q^{-1}(\zeta_x)V(\tau_x)\frac{d}{d\sigma_x}[W^{-1}(\sigma_x)T_aW(\sigma_x)]V^{-1}(\tau_x)Q(\zeta_x)$$
$$= -ieQ^{-1}(\zeta_x)V(\tau_x)W^{-1}(\sigma_x)[A_i,T_a]W(\sigma_x)V^{-1}(\tau_x)Q(\zeta_x)\frac{dx^i}{d\sigma_x}$$

$$\delta\left(A_i^a(x)\frac{dx^i}{d\sigma_x}\right) = \varepsilon\left\{A_i^a(x)\frac{dx^i}{d\sigma_x}, Q_N(\alpha,\beta)\right\}_{PB} = 0$$

$$\left\{A_i^a(x)\frac{dx^i}{d\tau_x}, Q_N(\alpha,\beta)\right\}_{PB} = ie\beta\vartheta\mathrm{Tr}\left[Q^N(\zeta_f)\left(I_{i,a,\tau_x}^{(1)} + I_{i,a,\tau_x}^{(2)}\right)\right]$$

$$\varepsilon_{ijk}\frac{dx^i}{d\tau_x}\frac{dy^j}{d\sigma}\frac{dy^k}{d\tau}\delta^{(3)}(x-y) = 0$$

$$I^{(2)}_{i,a,\tau_x} = -Q^{-1}(\zeta_x)\frac{d}{d\tau_x}[V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)]Q(\zeta_x)$$
$$= -ieQ^{-1}(\zeta_x)V(\tau_x)W^{-1}(\sigma_x)[A_i,T_a]W(\sigma_x)V^{-1}(\tau_x)Q(\zeta_x)\frac{dx^i}{d\tau_x}$$
$$-Q^{-1}(\zeta_x)V(\tau_x)[\mathcal{T}_\tau(\tau_x)+ie\mathfrak{b}_\tau(\sigma_x),W^{-1}(\sigma_x)T_aW(\sigma_x)]V^{-1}(\tau_x)Q(\zeta_x)$$

$$\delta\left(A^a_i(x)\frac{dx^i}{d\tau_x}\right) = \varepsilon\left\{A^a_i(x)\frac{dx^i}{d\tau_x},Q_N(\alpha,\beta)\right\}_{PB} = -ie\beta\vartheta\varepsilon\mathrm{Tr}\big[Q^N(\zeta_f)$$
$$\times Q^{-1}(\zeta_x)V(\tau_x)[\mathcal{T}_\tau(\tau_x)+ie\mathfrak{b}_\tau(\sigma_x),W^{-1}(\sigma_x)T_aW(\sigma_x)]V^{-1}(\tau_x)Q(\zeta_x)]$$

$$\left\{A^a_i(x)\frac{dx^i}{d\zeta_x},Q_N(\alpha,\beta)\right\}_{PB} = ie\beta\vartheta\mathrm{Tr}\Big[Q^N(\zeta_f)\Big[I^{(1)}_{i,a,\zeta_x}+I^{(2)}_{i,a,\zeta_x}$$
$$-[V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)]_{\zeta=\zeta_f}$$
$$+ie\beta\vartheta Q^{-1}(\zeta_x)[V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)$$
$$,\int_{\tau_i}^{\tau_x}d\tau' V(\tau')\int_{\sigma_i}^{\sigma_f}d\sigma W^{-1}\mathcal{C}WV^{-1}(\tau')\Delta(\sigma,\tau',\zeta)\Big]Q(\zeta_x)\Big]\Big]$$

$$I^{(2)}_{i,a,\zeta_x} = -\frac{d}{d\zeta_x}[Q^{-1}(\zeta_x)V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)Q(\zeta_x)]$$
$$+Q^{-1}(\zeta_f)\left[\left[\iint_{S^2_\infty}d\tau d\sigma V(\tau)W^{-1}(\sigma)T_aW(\sigma)V^{-1}(\tau)\vartheta\varepsilon_{ijk}\frac{dy^j}{d\sigma}\frac{dy^k}{d\tau}\frac{dx^i}{d\zeta_x}\delta^{(3)}(x-y)\right]_{\zeta=\zeta_f}Q(\zeta_f)\right.$$

$$I^{(2)}_{i,a,\zeta_x} = Q^{-1}(\zeta_f)[V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)]_{\zeta=\zeta_f}Q(\zeta_f)$$
$$+Q^{-1}(\zeta_x)[\mathcal{A}(\tau_f,\zeta_x),V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)]Q(\zeta_x)$$
$$-Q^{-1}(\zeta_x)[V(\tau_x)\mathcal{T}_\zeta(\tau_x)V^{-1}(\tau_x)+\mathcal{K}(\tau_x),V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)]Q(\zeta_x)$$
$$-ieQ^{-1}(\zeta_x)V(\tau_x)\left[\mathfrak{b}_\zeta(\sigma_x)+W^{-1}(\sigma_x)A_i\frac{dx^i}{d\zeta_x}W(\sigma_x),W^{-1}(\sigma_x)T_aW(\sigma_x)\right]V^{-1}(\tau_x)Q(\zeta_x)$$

$$I^{(1)}_{i,a,\zeta_x}+I^{(2)}_{i,a,\zeta_x} = Q^{-1}(\zeta_f)[V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)]_{\zeta=\zeta_f}Q(\zeta_f)$$
$$-Q^{-1}(\zeta_x)V(\tau_x)[\mathcal{T}_\zeta(\tau_x)+ie\mathfrak{b}_\zeta(\sigma_x),W^{-1}(\sigma_x)T_aW(\sigma_x)]V^{-1}(\tau_x)Q(\zeta_x)$$
$$+Q^{-1}(\zeta_x)[\mathcal{A}(\tau_f,\zeta_x)-\mathcal{K}(\tau_x,\zeta_x),V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)]Q(\zeta_x)$$

$$\delta\left(A^a_i(x)\frac{dx^i}{d\zeta_x}\right) = \varepsilon\left\{A^a_i(x)\frac{dx^i}{d\zeta_x},Q_N(\alpha,\beta)\right\}_{PB} = -ie\beta\vartheta\varepsilon\mathrm{Tr}[Q^N(\zeta_f)Q^{-1}(\zeta_x)$$
$$\times\{V(\tau_x)[\mathcal{T}_\zeta(\tau_x)+ie\mathfrak{b}_\zeta(\sigma_x),W^{-1}(\sigma_x)T_aW(\sigma_x)]V^{-1}(\tau_x)$$
$$-[\mathcal{A}(\tau_f,\zeta_x)-\mathcal{A}(\tau_x,\zeta_x),V(\tau_x)W^{-1}(\sigma_x)T_aW(\sigma_x)V^{-1}(\tau_x)]\}Q(\zeta_x)]$$

$$\delta W(\sigma) = \varepsilon\{W(\sigma),Q_N(\alpha,\beta)\}_{PB} = 0$$

$$\delta\mathfrak{b}_{\tau/\zeta}(\sigma_f) = \varepsilon\{\mathfrak{b}_{\tau/\zeta}(\sigma_f),Q_N(\alpha,\beta)\}_{PB} = 0$$

$$\delta\mathfrak{b}_{\tau/\zeta}(\sigma) = \varepsilon\{\mathfrak{b}_{\tau/\zeta}(\sigma),Q_N(\alpha,\beta)\}_{PB}$$
$$= ie\beta\vartheta\varepsilon T_a\mathrm{Tr}[Q^N(\zeta_f)Q^{-1}(\zeta)V(\tau)[\mathcal{T}_{\tau/\zeta}(\tau)+ie\mathfrak{b}_{\tau/\zeta}(\sigma),T_a]V^{-1}(\tau)Q(\zeta)]$$

$$\left\{\mathrm{e}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}Q(\zeta_f)\right\}_{PB} = ie^2\mathbb{1}\otimes Q(\zeta_f)\int_{\zeta_i}^{\zeta_f}d\zeta'\int_{\tau_i}^{\tau_f}d\tau'\mathbb{1}\otimes\left(Q^{-1}(\zeta')V(\tau')\right)$$
$$\times\left\{\mathrm{e}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}\mathcal{J}_{\text{spatial}}(\tau')\right\}_{PB}\mathbb{1}\otimes\left(V^{-1}(\tau')Q(\zeta')\right)$$

$$\delta\mathrm{e}_{\tau/\zeta}(\sigma) = \varepsilon\left\{\mathrm{e}_{\tau/\zeta}(\sigma),Q_N(\alpha,\beta)\right\}_{PB}$$
$$= -\varepsilon e^2\beta\vartheta\left[T_a,\mathrm{e}_{\tau/\zeta}(\sigma)\right]\mathrm{Tr}\left(Q(\zeta_f)^N Q^{-1}(\zeta)V(\tau)T_aV^{-1}(\tau)Q(\zeta)\right)$$

$$\frac{d\omega_{\tau/\zeta}}{d\tau/\zeta}+ieA_i\frac{dx^i}{d\tau/\zeta}\omega_{\tau/\zeta}=0;\ \omega_\tau(\tau_i)=\omega_\zeta(\zeta_i)=W_R$$

$$\delta\omega_\xi(\xi) = \varepsilon\left\{\omega_\xi(\xi),Q_N(\alpha,\beta)\right\}_{PB}$$
$$= -ie\varepsilon\omega_\xi(\xi)\int_{\xi_i}^{\xi}d\xi_x\omega_\xi^{-1}(\xi_x)\{A_i(x),Q_N(\alpha,\beta)\}_{PB}\omega_\xi(\xi_x)\frac{dx^i}{d\xi_x}$$

$$ie\mathfrak{b}_\xi(\sigma) = \frac{d\left(W^{-1}(\sigma)\omega_\xi(\xi_x)\right)}{d\xi_x}\left(W^{-1}(\sigma)\omega_\xi(\xi_x)\right)^{-1}$$

$$\omega_\xi^{-1}(\xi_x)T_a\omega_\xi(\xi_x)\otimes W^{-1}(\sigma)T_aW(\sigma) = T_a\otimes W^{-1}(\sigma)\omega_\xi(\xi_x)T_a\omega_\xi^{-1}(\xi_x)W(\sigma)$$

$$\left[ie\mathfrak{b}_\xi(\sigma),W^{-1}(\sigma)\omega_\xi(\xi_x)T_a\omega_\xi^{-1}(\xi_x)W(\sigma)\right] = \frac{d}{d\xi_x}\left[W^{-1}(\sigma)\omega_\xi(\xi_x)T_a\omega_\xi^{-1}(\xi_x)W(\sigma_x)\right]$$

$$\delta\omega_\tau(\tau) = -\varepsilon e^2\beta\vartheta\omega_\tau(\tau)\int_{\tau_i}^{\tau}d\tau_x\omega_\tau^{-1}(\tau_x)T_a\omega_\tau(\tau_x)\times$$
$$\times\mathrm{Tr}\left[Q^N(\zeta_f)Q^{-1}(\zeta)V(\tau_x)[\mathcal{J}_\tau(\tau_x)+ie\mathfrak{b}_\tau(\sigma),W^{-1}(\sigma)T_aW(\sigma)]V^{-1}(\tau_x)Q(\zeta)\right]$$

$$\delta\omega_\tau(\tau) = -\varepsilon e^2\beta\vartheta\omega_\tau(\tau)T_a\int_{\tau_i}^{\tau}d\tau_x$$
$$\times\mathrm{Tr}\left[Q^N(\zeta_f)Q^{-1}(\zeta)\frac{d}{d\tau_x}[V(\tau_x)W^{-1}(\sigma)\omega_\tau(\tau_x)T_a\omega_\tau^{-1}(\tau_x)W(\sigma)V^{-1}(\tau_x)]Q(\zeta)\right]$$

$$\delta\omega_\tau(\tau) = -\varepsilon e^2\beta\vartheta\omega_\tau(\tau)T_a\times$$
$$\times\mathrm{Tr}\left[Q^N(\zeta_f)Q^{-1}(\zeta)[V(\tau)W^{-1}(\sigma)\omega_\tau(\tau)T_a\omega_\tau^{-1}(\tau)W(\sigma)V^{-1}(\tau)-T_a]Q(\zeta)\right]$$

$$\delta\omega_\tau(\tau) = \varepsilon e^2\beta\vartheta\left\{\omega_\tau(\tau)T_a\mathrm{Tr}\left[Q^N(\zeta_f)Q^{-1}(\zeta)T_aQ(\zeta)\right]\right.$$
$$\left.-T_a\omega_\tau(\tau)\mathrm{Tr}\left[Q^N(\zeta_f)Q^{-1}(\zeta)V(\tau)W^{-1}(\sigma)T_aW(\sigma)V^{-1}(\tau)Q(\zeta)\right]\right\}$$

$$\delta\omega_\zeta(\zeta) = -\varepsilon e^2\beta\vartheta\omega_\zeta(\zeta)\int_{\zeta_i}^{\zeta}d\zeta_x\omega_\zeta^{-1}(\zeta_x)T_a\omega_\zeta(\zeta_x)\mathrm{Tr}\left[Q^N(\zeta_f)Q^{-1}(\zeta_x)\right.$$
$$\times\left\{V(\tau)\left[\mathcal{J}_\zeta(\tau)+ie\mathfrak{b}_\zeta(\sigma),W^{-1}(\sigma)T_aW(\sigma)\right]V^{-1}(\tau)\right.$$
$$\left.\left.-\left[\mathcal{A}(\tau_f,\zeta_x)-\mathcal{A}(\tau,\zeta_x),V(\tau)W^{-1}(\sigma)T_aW(\sigma)V^{-1}(\tau)\right]\right\}Q(\zeta_x)\right]$$

$$\delta\omega_\zeta(\zeta) = -\varepsilon e^2\beta\vartheta\omega_\zeta(\zeta)T_a\int_{\zeta_i}^{\zeta} d\zeta_x \mathrm{Tr}\left[Q^N(\zeta_f)Q^{-1}(\zeta_x)\right.$$
$$\times\left\{\left[\mathcal{A}(\tau,\zeta_x) - \mathcal{K}(\tau,\zeta_x) - \mathcal{A}(\tau_f,\zeta_x), V(\tau)W^{-1}(\sigma)\omega_\zeta(\zeta_x)T_a\omega_\zeta^{-1}(\zeta_x)W(\sigma)V^{-1}(\tau)\right]\right.$$
$$\left.\left.+\frac{d}{d\zeta_x}\left[V(\tau)W^{-1}(\sigma)\omega_\zeta(\zeta_x)T_a\omega_\zeta^{-1}(\zeta_x)W(\sigma)V^{-1}(\tau)\right]\right\}Q(\zeta_x)\right]$$

$$\mathcal{A}(\tau_f,\zeta_x) = \frac{dQ(\zeta_x)}{d\zeta_x}Q^{-1}(\zeta_x)$$

$$\delta\omega_\zeta(\zeta) \cong -\varepsilon e^2\beta\vartheta\omega_\zeta(\zeta)T_a\int_{\zeta_i}^{\zeta} d\zeta_x \mathrm{Tr}\left[Q^N(\zeta_f)\times\right.$$
$$\left.\frac{d}{d\zeta_x}\left[Q^{-1}(\zeta_x)V(\tau)W^{-1}(\sigma)\omega_\zeta(\zeta_x)T_a\omega_\zeta^{-1}(\zeta_x)W(\sigma)V^{-1}(\tau)Q(\zeta_x)\right]\}\right]$$

$$\delta\omega_\zeta(\zeta) \cong \varepsilon e^2\beta\vartheta\left\{\omega_\zeta(\zeta)T_a\mathrm{Tr}\left[Q^N(\zeta_f)T_a\right]\right.$$
$$\left.-T_a\omega_\zeta(\zeta)\mathrm{Tr}\left[Q^N(\zeta_f)Q^{-1}(\zeta)V(\tau)W^{-1}(\sigma)T_aW(\sigma)V^{-1}(\tau)Q(\zeta)\right]\right\}$$

$$\left\{\mathcal{A}(\zeta_1,\alpha_1,\beta_1)\overset{\otimes}{\otimes}\mathcal{A}(\zeta_2,\alpha_2,\beta_2)\right\}_{PB} = ie\beta_1\vartheta T_b\otimes\mathbb{1}$$
$$\times\int_{\tau_i}^{\tau_f} d\tau_1\int_{\sigma_i}^{\sigma_f} d\sigma_1\Delta(\sigma_1,\tau_1,\zeta_1)d_{ba}\left(V_{(1)}(\tau_1)W^{-1}(\sigma_1)\right)\left\{\mathcal{C}_a(x)\overset{\otimes}{,}\mathcal{A}(\zeta_2,\alpha_2,\beta_2)\right\}_{PB}$$
$$+ie^3\beta_1\beta_2\vartheta\left[\frac{d}{d\zeta_1}\left[\mathcal{Y}(\zeta_1,\tau_f)\delta(\zeta_1-\zeta_2)\right] - \delta(\zeta_1-\zeta_2)\left[\mathcal{A}(\zeta_1,\alpha_1,\beta_1)\otimes\mathbb{1},\mathcal{Y}(\zeta_1,\tau_f)\right]\right]$$
$$+e^4\beta_1^2\beta_2\delta(\zeta_1-\zeta_2)\int_{\tau_i}^{\tau_f} d\tau_1\left[\frac{d\mathcal{Y}(\zeta_1,\tau_1)}{d\tau_1},T_b\otimes\mathbb{1}\right]\times$$
$$\times\int_{\tau_i}^{\tau_1} d\tau_1'\int_{\sigma_i}^{\sigma_f} d\sigma_1\Delta(\sigma_1,\tau_1',\zeta_1)d_{ba}\left(V_{(1)}(\tau_1')W^{-1}(\sigma_1)\right)\mathcal{C}_a(\sigma_1,\tau_1',\zeta_1)$$

$$\mathcal{Y}(\zeta_1,\tau) \equiv \int_{\tau_i}^{\tau} d\tau_1 V_{(1)}(\tau_1)\otimes V_{(2)}(\tau_1)\left[\mathbb{C},\mathfrak{e}_\tau(\sigma_f,\tau_1,\zeta_1)\otimes\mathbb{1}\right]V_{(1)}^{-1}(\tau_1)\otimes V_{(2)}^{-1}(\tau_1)$$
$$= -\int_{\tau_i}^{\tau} d\tau_1 V_{(1)}(\tau_1)\otimes V_{(2)}(\tau_1)\left[\mathbb{C},\mathbb{1}\otimes\mathfrak{e}_\tau(\sigma_f,\tau_1,\zeta_1)\right]V_{(1)}^{-1}(\tau_1)\otimes V_{(2)}^{-1}(\tau_1)$$

$$T_b\otimes\mathbb{1}\int_{\tau_i}^{\tau_f} d\tau_1\int_{\sigma_i}^{\sigma_f} d\sigma_1\Delta(\sigma_1,\tau_1,\zeta_1)d_{ba}(x)\left\{\mathcal{C}_a(x)\overset{\otimes}{A}(\zeta_2,\alpha_2,\beta_2)\right\}_{PB} =$$
$$= -e^2\beta_2\left\{\vartheta\delta(\zeta_1-\zeta_2)\int_{\tau_i}^{\tau_f} d\tau_2\int_{\sigma_i}^{\sigma_f} d\sigma_2\Delta(\sigma_2,\tau_2,\zeta_2)\times\right.$$
$$\times V_{(1)}(\tau_2)\otimes V_{(2)}(\tau_2)\left[\mathbb{C},W^{-1}(\sigma_2)\mathcal{C}W(\sigma_2)\otimes\mathbb{1}\right]V_{(1)}^{-1}(\tau_2)\otimes V_{(2)}^{-1}(\tau_2)$$
$$-\delta(\zeta_1-\zeta_2)\frac{d\mathcal{Y}(\zeta_2,\tau_f)}{d\zeta_2} + \frac{d\left(\mathcal{Y}(\zeta_1,\tau_f)\delta(\zeta_1-\zeta_2)\right)}{d\zeta_1}$$
$$+\delta(\zeta_1-\zeta_2)\left[\mathbb{1}\otimes\mathcal{A}(\zeta_2,\alpha_2,\beta_2),\mathcal{Y}(\zeta_2,\tau_f)\right]$$
$$+ie\beta_2\vartheta\delta(\zeta_1-\zeta_2)\int_{\tau_i}^{\tau_f} d\tau_2\left[\frac{d\mathcal{Y}(\zeta_2,\tau_2)}{d\tau_2},\mathbb{1}\otimes T_b\right]\times$$
$$\left.\times\int_{\tau_i}^{\tau_2} d\tau_2'\int_{\sigma_i}^{\sigma_f} d\sigma_2\Delta(\sigma_2,\tau_2',\zeta_2)d_{ba}\left(V_{(2)}(\tau_2')W^{-1}(\sigma_2)\right)\mathcal{C}_a(\sigma_2,\tau_2',\zeta_2)\right\}$$

$$\{\mathcal{A}(\zeta_1,\alpha_1,\beta_1)\otimes\mathcal{A}(\zeta_2,\alpha_2,\beta_2)\}_{PB}=-ie^3\beta_1\beta_2\vartheta\delta(\zeta_1-\zeta_2)\left\{\int_{\tau_i}^{\tau_f}d\tau_2\frac{(-i)}{e(\beta_1-\beta_2)}\right.$$

$$\times\left[V_{(1)}(\tau_2)\otimes V_{(2)}(\tau_2)\mathbb{C}V_{(1)}^{-1}(\tau_2)\otimes V_{(2)}^{-1}(\tau_2),\frac{d\widetilde{\mathcal{M}}_{(1)}(\tau_2)}{d\tau_2}\otimes\mathbb{1}+\mathbb{1}\otimes\frac{d\widetilde{\mathcal{M}}_{(2)}(\tau_2)}{d\tau_2}\right]$$

$$+\left[\mathcal{A}(\zeta_1,\alpha_1,\beta_1)\otimes\mathbb{1}+\mathbb{1}\otimes\mathcal{A}(\zeta_1,\alpha_2,\beta_2),\mathcal{Y}(\zeta_1,\tau_f)\right]-\frac{d\mathcal{Y}(\zeta_2,\tau_f)}{d\zeta_2}$$

$$\left.+\int_{\tau_i}^{\tau_f}d\tau_1\left[\frac{d\mathcal{Y}(\zeta_1,\tau_1)}{d\tau_1},\widetilde{\mathcal{M}}_{(1)}(\tau_1)\otimes\mathbb{1}+\mathbb{1}\otimes\widetilde{\mathcal{M}}_{(2)}(\tau_1)\right]\right\}$$

$$\widetilde{\mathcal{M}}_{(s)}(\tau_1)\equiv ie\beta_s\vartheta\int_{\tau_i}^{\tau_1}d\tau_1'\int_{\sigma_i}^{\sigma_f}d\sigma_1\Delta(z)V_{(s)}(\tau_1')W^{-1}(\sigma_1)\mathcal{C}(z)W(\sigma_1)V_{(s)}^{-1}(\tau_1')$$

$$-ie(\beta_1-\beta_2)\mathcal{Y}(\zeta_1,\tau)=\int_{\tau_i}^{\tau}d\tau_1V_{(1)}(\tau_1)\otimes V_{(2)}(\tau_1)$$

$$\times\left[\mathbb{C},V_{(1)}^{-1}(\tau_1,\zeta_1)\frac{dV_{(1)}(\tau_1,\zeta_1)}{d\tau_1}\otimes\mathbb{1}+\mathbb{1}\otimes V_{(2)}^{-1}(\tau_1,\zeta_1)\frac{dV_{(2)}(\tau_1,\zeta_1)}{d\tau_1}\right]V_{(1)}^{-1}(\tau_1)\otimes V_{(2)}^{-1}(\tau_1)$$

$$+(\alpha_1-\alpha_2)\tilde{\mathcal{Z}}(\tau,\zeta_1)$$

$$=\mathbb{C}-V_{(1)}(\tau)\otimes V_{(2)}(\tau)\mathbb{C}V_{(1)}^{-1}(\tau)\otimes V_{(2)}^{-1}(\tau)+(\alpha_1-\alpha_2)\tilde{\mathcal{Z}}(\tau,\zeta_1)$$

$$\tilde{\mathcal{Z}}(\tau,\zeta_1)\equiv ie\int_{\tau_i}^{\tau}d\tau_1V_{(1)}(\tau_1)\otimes V_{(2)}(\tau_1)\left[\mathbb{C},\mathfrak{b}_\tau(\sigma_f,\tau_1,\zeta_1)\otimes\mathbb{1}\right]V_{(1)}^{-1}(\tau_1)\otimes V_{(2)}^{-1}(\tau_1)$$

$$\widetilde{\mathcal{M}}_{(s)}(\tau_1)=\mathcal{A}(\zeta_1,\tau_1\alpha_s,\beta_s)-\mathcal{K}(\zeta_1,\tau_1,\alpha_s,\beta_s)$$

$$\{\mathcal{A}(\zeta_1,\alpha_1,\beta_1)\otimes\mathcal{A}(\zeta_2,\alpha_2,\beta_2)\}_{PB}=-ie^3\beta_1\beta_2\vartheta\delta(\zeta_1-\zeta_2)\left\{-\frac{d\mathcal{Y}(\zeta_1,\tau_f)}{d\zeta_1}\right.$$

$$+\frac{i}{e(\beta_1-\beta_2)}\left\{-\left[\mathbb{C}+(\alpha_1-\alpha_2)\tilde{\mathcal{Z}}(\tau_f,\zeta_1),\mathcal{A}(\zeta_1,\alpha_1,\beta_1)\otimes\mathbb{1}+\mathbb{1}\otimes\mathcal{A}(\zeta_1,\alpha_2,\beta_2)\right]\right.$$

$$+\left[V_{(1)}(\tau_f)\otimes V_{(2)}(\tau_f)\mathbb{C}V_{(1)}^{-1}(\tau_f)\otimes V_{(2)}^{-1}(\tau_f),\mathcal{K}(\zeta_1,\alpha_1,\beta_1)\otimes\mathbb{1}+\mathbb{1}\otimes\mathcal{K}(\zeta_1,\alpha_2,\beta_2)\right]$$

$$\left.\left.+(\alpha_1-\alpha_2)\int_{\tau_i}^{\tau_f}d\tau_1\left[\frac{d\tilde{\mathcal{Z}}(\zeta_1,\tau_1)}{d\tau_1},\widetilde{\mathcal{M}}_{(1)}(\tau_1)\otimes\mathbb{1}+\mathbb{1}\otimes\widetilde{\mathcal{M}}_{(2)}(\tau_1)\right]\right\}\right\}$$

$$\left\{\mathcal{A}_{(1)}(\zeta_1)\overset{\otimes}{\otimes}\mathcal{A}_{(2)}(\zeta_2)\right\}_{PB}=\delta(\zeta_1-\zeta_2)\left[\mathcal{R}(\beta_1,\beta_2),\mathcal{A}_{(1)}(\zeta_1)\otimes\mathbb{1}+\mathbb{1}\otimes\mathcal{A}_{(2)}(\zeta_1)\right]$$

$$-(\alpha_1-\alpha_2)\delta(\zeta_1-\zeta_2)\left\{\int_{\tau_i}^{\tau_f}d\tau_1\left[\frac{d\mathcal{Z}(\zeta_1,\tau_1)}{d\tau_1},\widetilde{\mathcal{M}}_{(1)}(\tau_1)\otimes\mathbb{1}+\mathbb{1}\otimes\widetilde{\mathcal{M}}_{(2)}(\tau_1)\right]\right.$$

$$\left.-Q_\otimes(\zeta_1)\frac{d}{d\zeta_1}\left(Q_\otimes^{-1}(\zeta_1)\mathcal{Z}(\zeta_1,\tau_f)Q_\otimes(\zeta_1)\right)Q_\otimes^{-1}(\zeta_1)\right\}$$

$$Q_\otimes(\zeta_1)\equiv Q(\zeta_1,\alpha_1,\beta_1)\otimes Q(\zeta_1,\alpha_2,\beta_2)$$

$$\mathcal{R}(\beta_1,\beta_2)\equiv-e^2\vartheta\frac{\beta_1\beta_2}{(\beta_1-\beta_2)}\mathbb{C}$$

$$\mathcal{Z}(\zeta_1,\tau) \equiv -e^2\vartheta \frac{\beta_1\beta_2}{(\beta_1-\beta_2)}\tilde{\mathcal{Z}}(\zeta_1,\tau) = ie\int_{\tau_i}^{\tau} d\tau_1 V_{(1)}(\zeta_1,\tau_1)\otimes V_{(2)}(\zeta_1,\tau_1)$$
$$\times\left[\mathcal{R}(\beta_1,\beta_2),\mathfrak{b}_\tau(\sigma_f,\tau_1,\zeta_1)\otimes\mathbb{1}\right]V_{(1)}^{-1}(\zeta_1,\tau_1)\otimes V_{(2)}^{-1}(\zeta_1,\tau_1)$$

$$\{Q(\zeta_f,\alpha_1,\beta_1)\otimes Q(\zeta_f,\alpha_2,\beta_2)\}_{PB} = Q_\otimes(\zeta_f)\int_{\zeta_i}^{\zeta_f} d\zeta Q_\otimes^{-1}(\zeta)$$
$$\times\left\{\left[\mathcal{R}(\beta_1,\beta_2),\mathcal{A}_{(1)}(\zeta)\otimes\mathbb{1}+\mathbb{1}\otimes\mathcal{A}_{(2)}(\zeta)\right]\right.$$
$$-(\alpha_1-\alpha_2)\left\{\int_{\tau_i}^{\tau_f} d\tau\left[\frac{d\mathcal{Z}(\zeta,\tau)}{d\tau},\widetilde{\mathcal{M}}_{(1)}(\tau)\otimes\mathbb{1}+\mathbb{1}\otimes\widetilde{\mathcal{M}}_{(2)}(\tau)\right]\right.$$
$$\left.\left.-Q_\otimes(\zeta)\frac{d}{d\zeta}\left(Q_\otimes^{-1}(\zeta)\mathcal{Z}(\zeta,\tau_f)Q_\otimes(\zeta)\right)Q_\otimes^{-1}(\zeta)\right\}Q_\otimes(\zeta)\right\}$$

$$\{Q(\zeta_f,\alpha_1,\beta_1)\overset{\otimes}{,}Q(\zeta_f,\alpha_2,\beta_2)\}_{PB} = Q_\otimes(\zeta_f)\int_{\zeta_i}^{\zeta_f} d\zeta$$
$$\times\left\{\frac{d}{d\zeta}\{Q_\otimes^{-1}(\zeta)[\mathcal{R}(\beta_1,\beta_2)+(\alpha_1-\alpha_2)\mathcal{Z}(\zeta,\tau_f)]Q_\otimes(\zeta)\}\right.$$
$$\left.-(\alpha_1-\alpha_2)Q_\otimes^{-1}(\zeta)\int_{\tau_i}^{\tau_f} d\tau\left[\frac{d\mathcal{Z}(\zeta,\tau)}{d\tau},\widetilde{\mathcal{M}}_{(1)}(\tau)\otimes\mathbb{1}+\mathbb{1}\otimes\widetilde{\mathcal{M}}_{(2)}(\tau)\right]Q_\otimes(\zeta)\right\}$$

$$\{Q(\zeta_f,\alpha_1,\beta_1)\otimes^{\otimes} Q(\zeta_f,\alpha_2,\beta_2)\}_{PB} = \left[\mathcal{R}(\beta_1,\beta_2),Q(\zeta_f,\alpha_1,\beta_1)\otimes Q(\zeta_f,\alpha_2,\beta_2)\right]$$
$$+(\alpha_1-\alpha_2)\mathcal{Z}(\zeta_f,\tau_f)Q(\zeta_f,\alpha_1,\beta_1)\otimes Q(\zeta_f,\alpha_2,\beta_2)$$
$$-(\alpha_1-\alpha_2)Q_\otimes(\zeta_f)\int_{\zeta_i}^{\zeta_f} d\zeta Q_\otimes^{-1}(\zeta)\int_{\tau_i}^{\tau_f} d\tau\left[\frac{d\mathcal{Z}(\zeta,\tau)}{d\tau},\widetilde{\mathcal{M}}_{(1)}(\tau)\otimes\mathbb{1}+\mathbb{1}\otimes\widetilde{\mathcal{M}}_{(2)}(\tau)\right]Q_\otimes(\zeta)$$

$$\{Q_N(\alpha_1,\beta_1),Q(\zeta_f,\alpha_2,\beta_2)\}_{PB} = \mathrm{Tr}_L\left(Q^{N-1}(\zeta_f,\alpha_1,\beta_1)\otimes\mathbb{1}[\mathcal{R},\mathbb{1}\otimes Q(\zeta_f,\alpha_2,\beta_2)]\right)$$
$$-\frac{ie^3\vartheta\beta_1\beta_2(\alpha_1-\alpha_2)}{(\beta_1-\beta_2)}\int_{\tau_i}^{\tau_f} d\tau\mathrm{Tr}\left[\left[\mathfrak{b}_\tau(\sigma_f,\tau,\zeta_f),V_{(1)}^{-1}(\zeta_f,\tau)Q^N(\zeta_f,\alpha_1,\beta_1)V_{(1)}(\zeta_f,\tau)\right]T_a\right]$$
$$\times V_{(2)}(\zeta_f,\tau)T_aV_{(2)}^{-1}(\zeta_f,\tau)Q(\zeta_f,\alpha_2,\beta_2)$$
$$-(\alpha_1-\alpha_2)\mathrm{Tr}_L\left(Q^N(\zeta_f,\alpha_1,\beta_1)\otimes Q(\zeta_f,\alpha_2,\beta_2)\Upsilon\right)$$

$$\Upsilon \equiv \int_{\zeta_i}^{\zeta_f} d\zeta Q_\otimes^{-1}(\zeta)\int_{\tau_i}^{\tau_f} d\tau\left[\frac{d\mathcal{Z}(\zeta,\tau)}{d\tau},\widetilde{\mathcal{M}}_{(1)}(\tau)\otimes\mathbb{1}+\mathbb{1}\otimes\widetilde{\mathcal{M}}_{(2)}(\tau)\right]Q_\otimes(\zeta)$$

$$\mathfrak{b}_\tau(\sigma_f,\tau,\zeta_f)\sim c\int_{\sigma_i}^{\sigma_f} d\sigma\varepsilon_{ijk}\frac{\hat{r}_i}{r^2}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau};\ \mathfrak{e}_\tau(\sigma_f,\tau,\zeta_f)\sim\tilde{c}\int_{\sigma_i}^{\sigma_f} d\sigma\varepsilon_{ijk}\frac{\hat{r}_i}{r^2}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}$$

$$V_{(i)}^{-1}(\tau)Q^N(\zeta_f,\alpha_1,\beta_1)V_{(i)}\equiv\mathbb{E}$$

$$\{Q_N(\alpha_1,\beta_1),Q(\zeta_f,\alpha_2,\beta_2)\}_{PB}\cong\mathrm{Tr}_L\left(Q^{N-1}(\zeta_f,\alpha_1,\beta_1)\otimes\mathbb{1}[\mathcal{R},\mathbb{1}\otimes Q(\zeta_f,\alpha_2,\beta_2)]\right)$$

$$\{Q_N(\alpha_1,\beta_1),Q_M(\alpha_2,\beta_2)\}_{PB}\cong 0$$

$$[\delta_{N_1,\alpha_1,\beta_1},\delta_{N_2,\alpha_2,\beta_2}]X = -\varepsilon_1\varepsilon_2\left\{X,\{Q_{N_1}(\alpha_1,\beta_1),Q_{N_2}(\alpha_2,\beta_2)\}_{PB}\right\}_{PB}$$

$$\{Q_{N_1}(\alpha_1,\beta_1),Q_{N_2}(\alpha_2,\beta_2)\}_{PB} = -(\alpha_1-\alpha_2)\Big\{\mathrm{Tr}_{RL}\left(Q_{(1)}^{N_1}(\zeta_f)\otimes Q_{(2)}^{N_2}(\zeta_f)\Upsilon\right)$$
$$+ie^3\vartheta\frac{\beta_1\beta_2}{(\beta_1-\beta_2)}\int_{\tau_i}^{\tau_f} d\tau \mathrm{Tr}\left(\left[\mathfrak{b}_\tau(\sigma_f,\tau,\zeta_f),V_{(1)}^{-1}(\zeta_f,\tau)Q_{(1)}^{N_1}(\zeta_f)V_{(1)}(\zeta_f,\tau)\right]T_a\right)$$
$$\times\mathrm{Tr}\left(T_a V_{(2)}^{-1}(\zeta_f,\tau)Q_{(2)}^{N_2}(\zeta_f)V_{(2)}(\zeta_f,\tau)\right)\Big\}$$

$$\int_{\tau_i}^{\tau_f} d\tau\mathrm{Tr}\left(\{X,\mathfrak{b}_\tau(\sigma_f,\tau,\zeta_f)\}_{PB}T_a\right)$$
$$\times\mathrm{Tr}\left(\left[V_{(1)}^{-1}(\zeta_f,\tau)Q_{(1)}^{N_1}(\zeta_f)V_{(1)}(\zeta_f,\tau),V_{(2)}^{-1}(\zeta_f,\tau)Q_{(2)}^{N_2}(\zeta_f)V_{(2)}(\zeta_f,\tau)\right]T_a\right)$$
$$-\int_{\tau_i}^{\tau_f} d\tau\mathrm{Tr}\left(\left\{X,V_{(1)}^{-1}(\zeta_f,\tau)Q_{(1)}^{N_1}(\zeta_f)V_{(1)}(\zeta_f,\tau)\right\}_{PB}T_a\right)$$
$$\times\mathrm{Tr}\left(\left[\mathfrak{b}_\tau(\sigma_f,\tau,\zeta_f),V_{(2)}^{-1}(\zeta_f,\tau)Q_{(2)}^{N_2}(\zeta_f)V_{(2)}(\zeta_f,\tau)\right]T_a\right)$$
$$+\int_{\tau_i}^{\tau_f} d\tau\mathrm{Tr}\left(\left[\mathfrak{b}_\tau(\sigma_f,\tau,\zeta_f),V_{(1)}^{-1}(\zeta_f,\tau)Q_{(1)}^{N_1}(\zeta_f)V_{(1)}(\zeta_f,\tau)\right]T_a\right)$$
$$\times\mathrm{Tr}\left(T_a\left\{X,V_{(2)}^{-1}(\zeta_f,\tau)Q_{(2)}^{N_2}(\zeta_f)V_{(2)}(\zeta_f,\tau)\right\}_{PB}\right)$$

$$[\delta_{N_1,\alpha_1,\beta_1},\delta_{N_2,\alpha_2,\beta_2}]X\cong\varepsilon_1\varepsilon_2(\alpha_1-\alpha_2)\left\{X,\mathrm{Tr}_{RL}\left(Q_{(1)}^{N_1}(\zeta_f)\otimes Q_{(2)}^{N_2}(\zeta_f)\Upsilon\right)\right\}_{PB}$$

$$\delta_\Upsilon H_T\equiv\varepsilon\left\{H_T,\mathrm{Tr}_{RL}\left(Q_{(1)}^{N_1}(\zeta_f)\otimes Q_{(2)}^{N_2}(\zeta_f)\Upsilon\right)\right\}_{PB}\cong 0$$

$$\mathrm{Tr}_{RL}\left(Q_{(1)}^{N_1}(\zeta_f)\otimes Q_{(2)}^{N_2}(\zeta_f)\Upsilon\right)$$

$$\mathcal{L}^{(2)}\equiv\{f:S^2\to M\mid\ \mathbb{N}\mathbb{p}\ \to x_R\}$$

$$\mathcal{L}^{(1)}\equiv\{f:S^1\to M\mid\ \mathbb{N}\mathbb{p}\to x_R\}$$

$$\frac{d\hat{g}(\tau)}{d\tau}-\hat{g}(\tau)\mathfrak{a}(\tau)=0$$

$$\frac{d\tilde{g}(\tau)}{d\tau}+\mathfrak{a}(\tau)\tilde{g}(\tau)=0$$

$$\hat{g}(x_R)=\mathbb{1};\ \tilde{g}(x_R)=\mathbb{1}$$

$$\frac{d(\hat{g}\tilde{g})}{d\tau}=\hat{g}\mathfrak{a}\tilde{g}-\hat{g}\mathfrak{a}\tilde{g}=0\ \to\ \hat{g}\tilde{g}=\daleth\ =\hat{g}(x_R)\tilde{g}(x_R)=\mathbb{1}$$

$$\frac{d(\tilde{g}\hat{g})}{d\tau}=-\mathfrak{a}\tilde{g}\hat{g}+\tilde{g}\hat{g}\mathfrak{a}=[\tilde{g}\hat{g},\mathfrak{a}]$$

$$\frac{d(\tilde{g}\hat{g})}{d\tau}=[\tilde{g}\hat{g},\tilde{g}\mathfrak{a}'\hat{g}]=\tilde{g}[\mathbb{1},\mathfrak{a}']\hat{g}=0\ \to\ \tilde{g}\hat{g}=\beth=\tilde{g}(x_R)\hat{g}(x_R)=\mathbb{1}$$

$$\frac{d(\hat{g}_1\hat{g}_2)}{d\tau}=\hat{g}_1\mathfrak{a}_1\hat{g}_2+\hat{g}_1\hat{g}_2\mathfrak{a}_2=\hat{g}_1\hat{g}_2[\mathfrak{a}_2+\hat{g}_2^{-1}\mathfrak{a}_1\hat{g}_2]$$

$$\mathfrak{a}_3(\tau) \equiv \mathfrak{a}_2(\tau) + \hat{g}_2^{-1}(\tau)\mathfrak{a}_1(\tau)\hat{g}_2(\tau)$$

$$\hat{g}_3(\tau) \equiv \hat{g}_1(\tau)\hat{g}_2(\tau)$$

$$\frac{d\hat{g}_3(\tau)}{d\tau} - \hat{g}_3(\tau)\mathfrak{a}_3(\tau) = 0$$

$$(\hat{g}_1\hat{g}_2)\hat{g}_3 = \hat{g}_1(\hat{g}_2\hat{g}_3)$$

$$\mathfrak{a}_{(12)3} \equiv \mathfrak{a}_3 + \hat{g}_3^{-1}[\mathfrak{a}_2 + \hat{g}_2^{-1}\mathfrak{a}_1\hat{g}_2]\hat{g}_3 = [\mathfrak{a}_3 + \hat{g}_3^{-1}\mathfrak{a}_2\hat{g}_3] + (\hat{g}_2\hat{g}_3)^{-1}\mathfrak{a}_1\hat{g}_2\hat{g}_3 \equiv \mathfrak{a}_{1(23)}$$

$$\frac{d\hat{g}_{\mathrm{id}}(\tau)}{d\tau} = 0 \;\rightarrow\; \hat{g}_{\mathrm{id}} = \mathbb{1}$$

$$\mathfrak{a}_{\hat{g}\hat{g}_{\mathrm{ig}}}(\tau) = \mathfrak{a}_{\hat{g}_{\mathrm{id}}}(\tau) + \hat{g}_{\mathrm{id}}^{-1}(\tau)\mathfrak{a}(\tau)\hat{g}_{\mathrm{id}}(\tau) = \mathfrak{a}(\tau) = \mathfrak{a}(\tau) + \hat{g}^{-1}(\tau)\mathfrak{a}_{\hat{g}_{\mathrm{id}}}(\tau)\hat{g}(\tau) = \mathfrak{a}_{\hat{g}_{\mathrm{id}}\hat{g}}(\tau)$$

$$\mathfrak{a}(\tau) = \int_{\sigma_i}^{\sigma_f} d\sigma\omega^{-1}b_{\mu\nu}\omega\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}$$

$$\frac{d\omega}{d\sigma} + a_\mu\frac{dx^\mu}{d\sigma}\omega = 0$$

$$\frac{d\delta\hat{g}(\tau)}{d\tau} - \delta\hat{g}(\tau)\mathfrak{a}(\tau) - \hat{g}(\tau)\delta\mathfrak{a}(\tau) = 0$$

$$\frac{d(\delta\hat{g}(\tau)\tilde{g}(\tau))}{d\tau} - \hat{g}(\tau)\delta\mathfrak{a}(\tau)\tilde{g}(\tau) = 0$$

$$\delta\hat{g}(\tau_f)\tilde{g}(\tau_f) = \int_{\tau_i}^{\tau_f} d\tau\hat{g}(\tau)\delta\mathfrak{a}(\tau)\tilde{g}(\tau)$$

$$\frac{d\hat{g}(\zeta,\tau_f)}{d\zeta} = \left[\int_{\tau_i}^{\tau_f} d\tau\hat{g}(\tau)\frac{d\mathfrak{a}(\tau)}{d\zeta}\tilde{g}(\tau)\right]\hat{g}(\zeta,\tau_f)$$

$$\hat{g}(\partial\Omega) = \hat{g}(\Omega)$$

$$b_{\mu\nu} = ie(\alpha F_{\mu\nu} + \beta\tilde{F}_{\mu\nu});\; a_\mu = ieA_\mu$$

$$V(\partial\Omega) \equiv P_2 e^{ie\int_{\partial\Omega} d\tau d\sigma W^{-1}(\alpha F_{\mu\nu}+\beta\tilde{F}_{\mu\nu})W\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}}$$
$$V(\Omega) \equiv P_3 e^{ie^2\int_\Omega d\zeta d\tau V\mathcal{J}V^{-1}}$$

$$V(\partial\Omega) = V(\Omega)$$

$$V(\partial\Omega) \rightarrow \hat{g}_L(\partial\Omega)V(\partial\Omega);\; V(\Omega) \rightarrow \hat{g}_L(\Omega)V(\Omega)$$

$$V(\partial\Omega) \rightarrow V(\partial\Omega)\hat{g}_R(\partial\Omega);\; V(\Omega) \rightarrow V(\Omega)\hat{g}_R(\Omega)$$

$$\mathcal{A} = \frac{dV}{d\zeta}V^{-1}$$

$$\mathcal{A}(\partial\Omega) = \delta V(\partial\Omega)V^{-1}(\partial\Omega)$$

$$\mathcal{A} \to \hat{g}_L(\partial\Omega)\mathcal{A}\hat{g}_L^{-1}(\partial\Omega) + \delta\hat{g}_L(\partial\Omega)\hat{g}_L^{-1}(\partial\Omega)$$

$$\mathcal{A} \to \mathcal{A} + V(\partial\Omega)\delta\hat{g}_R(\partial\Omega)\hat{g}_R^{-1}(\partial\Omega)V^{-1}(\partial\Omega)$$

$$\partial_t A_x - \partial_x A_t + [A_t, A_x] = 0$$

$$A_\mu \to \tilde{A}_\mu = gA_\mu g^{-1} - \partial_\mu g g^{-1} \; \mu = t, x$$

$$\begin{aligned} x^1 &= \zeta\cos^2\tau(1-\cos\sigma) - L; && 0 \le \sigma \le 2\pi; \\ x^2 &= -\zeta\cos\tau\sin\sigma; && -\frac{\pi}{2} \le \tau \le \frac{\pi}{2} \\ x^3 &= \zeta\cos\tau\sin\tau(1-\cos\sigma); && 0 \le \zeta < \infty \end{aligned}$$

$$\varepsilon_{ijk}\frac{dx^i}{d\sigma}\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta} = 4\zeta^2\cos^3\tau\sin^4\left(\frac{\sigma}{2}\right)$$

$$\begin{aligned} &\int_{\sigma_i}^{\sigma_f} d\sigma \int_{\sigma_i}^{\sigma} d\sigma' [M_{\kappa\rho}(\sigma'), M_{\mu\nu}(\sigma)] \frac{dx^\kappa}{d\sigma'}\frac{dx^\mu}{d\sigma}\left(\frac{dx^\rho(\sigma')}{d\tau}\frac{dx^\nu(\sigma)}{d\zeta} - \frac{dx^\rho(\sigma')}{d\zeta}\frac{dx^\nu(\sigma)}{d\tau}\right) \\ &= \int_{\sigma_i}^{\sigma_f} d\sigma \int_{\sigma_i}^{\sigma} d\sigma' \left[\left[M_{\kappa\rho}(\sigma')\frac{dx^\kappa}{d\sigma'}\frac{dx^\rho(\sigma')}{d\tau}, M_{\mu\nu}(\sigma)\frac{dx^\mu}{d\sigma}\frac{dx^\nu(\sigma)}{d\zeta}\right]\right. \\ &+ \left.\left[M_{\mu\nu}(\sigma)\frac{dx^\mu}{d\sigma}\frac{dx^\nu(\sigma)}{d\tau}, M_{\kappa\rho}(\sigma')\frac{dx^\kappa}{d\sigma'}\frac{dx^\rho(\sigma')}{d\zeta}\right]\right] \\ &= \left[\int_{\sigma_i}^{\sigma_f} d\sigma \int_{\sigma_i}^{\sigma} d\sigma' + \int_{\sigma_i}^{\sigma_f} d\sigma' \int_{\sigma_i}^{\sigma'} d\sigma\right]\left[M_{\kappa\rho}(\sigma')\frac{dx^\kappa}{d\sigma'}\frac{dx^\rho(\sigma')}{d\tau}, M_{\mu\nu}(\sigma)\frac{dx^\mu}{d\sigma}\frac{dx^\nu(\sigma)}{d\zeta}\right] \\ &= \left[\int_{\sigma_i}^{\sigma_f} d\sigma' M_{\kappa\rho}(\sigma')\frac{dx^\kappa}{d\sigma'}\frac{dx^\rho(\sigma')}{d\tau}, \int_{\sigma_i}^{\sigma_f} d\sigma M_{\mu\nu}(\sigma)\frac{dx^\mu}{d\sigma}\frac{dx^\nu(\sigma)}{d\zeta}\right] \end{aligned}$$

$$W^{-1}(\sigma)\delta W(\sigma) = -ieW^{-1}(\sigma)A_\mu(\sigma)W(\sigma)\delta x^\mu(\sigma) + ie\int_{\sigma_i}^{\sigma} d\sigma' W^{-1}F_{\mu\nu}W\frac{dx^\mu}{d\sigma'}\delta x^\nu$$

$$W^{-1}(\sigma)\frac{dW}{d\tau/\zeta}(\sigma) = -ieW^{-1}(\sigma)A_\mu(\sigma)W(\sigma)\frac{dx^\mu}{d\tau/\zeta}(\sigma) + ie\int_{\sigma_i}^{\sigma} d\sigma' W^{-1}F_{\mu\nu}W\frac{dx^\mu}{d\sigma'}\frac{dx^\nu}{d\tau/\zeta}$$

$$W^{-1}(\sigma_f)\frac{dW}{d\tau/\zeta}(\sigma_f) = ie\int_{\sigma_i}^{\sigma_f} d\sigma' W^{-1}F_{\mu\nu}W\frac{dx^\mu}{d\sigma'}\frac{dx^\nu}{d\tau/\zeta}$$

$$\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\zeta} = 0 \text{ at } \sigma = \sigma_i \text{ and } \sigma = \sigma_f$$

$$\mathfrak{b}_{\tau/\zeta}(\sigma) = -W^{-1}(\sigma)A_i W(\sigma)\frac{dx^i}{d\tau/\zeta} + \frac{i}{e}W^{-1}(\sigma)\frac{dW(\sigma)}{d\tau/\zeta}; \quad \text{for } \sigma < \sigma_f$$

$$\mathfrak{b}_{\tau/\zeta}(\sigma_f) = \frac{i}{e}W^{-1}(\sigma_f)\frac{dW(\sigma_f)}{d\tau/\zeta}$$

$$\begin{aligned} \frac{d}{d\tau/\zeta}(W^{-1}(\sigma)L(\sigma)W(\sigma)) &= W^{-1}(\sigma)D_i L(\sigma)W(\sigma)\frac{dx^i}{d\tau/\zeta} \\ &\quad -ie\left[W^{-1}(\sigma)L(\sigma)W(\sigma), \mathfrak{b}_{\tau/\zeta}(\sigma)\right] \end{aligned}$$

$$\frac{d}{d\sigma}(W^{-1}(\sigma)L(\sigma)W(\sigma)) = W^{-1}(\sigma)D_i L(\sigma)W(\sigma)\frac{dx^i}{d\sigma}$$

$$\frac{d}{d\tau}\left(\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\zeta}\right) - \frac{d}{d\zeta}\left(\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\right) = \frac{d}{d\sigma}\left(\varepsilon_{ijk}\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}\right)$$

$$\begin{aligned}\frac{d\mathcal{T}_\tau}{d\zeta} - \frac{d\mathcal{T}_\zeta}{d\tau} = -ie\int_{\sigma_i}^{\sigma_f} d\sigma\Bigg\{&-\frac{d}{d\sigma}\left(W^{-1}(\alpha B_i + \beta E_i)W\varepsilon_{ijk}\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}\right)\\ &+W^{-1}D_l(\alpha B_i + \beta E_i)W\varepsilon_{ijk}\left(\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\frac{dx^l}{d\zeta} - \frac{dx^j}{d\sigma}\frac{dx^k}{d\zeta}\frac{dx^l}{d\tau} + \frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}\frac{dx^l}{d\sigma}\right)\\ &+ie[W^{-1}(\alpha B_i + \beta E_i)W, \mathfrak{b}_\tau(\sigma)]\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\zeta}\\ &-ie[W^{-1}(\alpha B_i + \beta E_i)W, \mathfrak{b}_\zeta(\sigma)]\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\Bigg\}\end{aligned}$$

$$\varepsilon_{ijk}\left(\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\frac{dx^l}{d\zeta} - \frac{dx^j}{d\sigma}\frac{dx^k}{d\zeta}\frac{dx^l}{d\tau} + \frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}\frac{dx^l}{d\sigma}\right) = \delta_{il}\varepsilon_{jkm}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\frac{dx^m}{d\zeta}$$

$$\begin{aligned}&\int_{\sigma_i}^{\sigma_f} d\sigma\Bigg\{[W^{-1}(\alpha B_i + \beta E_i)W, \mathfrak{b}_\tau(\sigma)]\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\zeta}\\ &-[W^{-1}(\alpha B_i + \beta E_i)W, \mathfrak{b}_\zeta(\sigma)]\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\Bigg\}\\ &=\int_{\sigma_i}^{\sigma_f} d\sigma\left\{\left[\alpha\frac{d\mathfrak{b}_\zeta(\sigma)}{d\sigma} + \beta\frac{d\mathfrak{e}_\zeta(\sigma)}{d\sigma}, \mathfrak{b}_\tau(\sigma)\right] - \left[\alpha\frac{d\mathfrak{b}_\tau(\sigma)}{d\sigma} + \beta\frac{d\mathfrak{e}_\tau(\sigma)}{d\sigma}, \mathfrak{b}_\zeta(\sigma)\right]\right\}\\ &=i(\beta\rho_G + \alpha\rho_{\text{mag.}})\end{aligned}$$

$$\frac{d\mathcal{T}_\tau}{d\zeta} - \frac{d\mathcal{T}_\zeta}{d\tau} - ie^2(\beta\rho_G + \alpha\rho_{\text{mag.}}) = -ie\int_{\sigma_i}^{\sigma_f} d\sigma W^{-1}(\alpha D_l B_l + \beta D_l E_l)W\varepsilon_{ijk}\frac{dx^i}{d\sigma}\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}$$

$$\frac{dW}{d\sigma} + ieA_i\frac{dx^i}{d\sigma}W = 0;\ \frac{dW^{-1}}{d\sigma} - ieW^{-1}A_i\frac{dx^i}{d\sigma} = 0$$

$$\frac{d(W^{-1}\{X, W\}_{PB})}{d\sigma} + ieW^{-1}\{X, A_i\}_{PB}W\frac{dx^i}{d\sigma} = 0$$

$$\{X, W(\sigma)\}_{PB} = -ieW(\sigma)\int_{\sigma_i}^{\sigma} d\sigma' W^{-1}(\sigma')\{X, A_i(\sigma')\}_{PB}W(\sigma')\frac{dx^i}{d\sigma'}$$

$$\{X, W^{-1}T_a W\}_{PB} = [W^{-1}T_a W, W^{-1}\{X, W\}_{PB}]$$

$$\{A_i^a(x,t), \mathfrak{b}_{\tau/\zeta}(\sigma)\}_{PB} = 0$$

$$\{A_i^a(x,t), \mathfrak{e}_\tau(\sigma)\}_{PB} = \int_{\sigma_i}^{\sigma} d\sigma' \delta^{(3)}(x-y)\varepsilon_{ijk}\frac{dy^j}{d\sigma'}\frac{dy^k}{d\tau}W^{-1}(\sigma')T_a W(\sigma')$$

$$(x^1, x^2, x^3) \equiv (\zeta_x, \tau_x, \sigma_x);\ (y^1, y^2, y^3) \equiv (\zeta, \tau, \sigma')$$

$$\delta^{(3)}(x-y)\varepsilon_{ijk}\frac{dx^i}{d\sigma_x/\tau_x}\frac{dy^j}{d\sigma'}\frac{dy^k}{d\tau}=0$$

$$\left\{A_i^a(x,t)\frac{dx^i}{d\sigma_x},\mathfrak{e}_\tau(\sigma)\right\}_{PB}=0$$

$$\left\{A_i^a(x,t)\frac{dx^i}{d\tau_x},\mathfrak{e}_\tau(\sigma)\right\}_{PB}=0$$

$$\left\{A_i^a(x,t)\frac{dx^i}{d\zeta_x},\mathfrak{e}_\tau(\sigma)\right\}_{PB}=\vartheta\delta(\zeta-\zeta_x)\delta(\tau-\tau_x)\theta(\sigma-\sigma_x)W^{-1}(\sigma_x)T_aW(\sigma_x)$$

$$\{B_i^a(x,t),\mathfrak{b}_{\tau/\zeta}(\sigma)\}_{PB}=0$$

$$\{B_i^a(x,t),\mathfrak{e}_{\tau/\zeta}(\sigma)\}_{PB}=\int_{\sigma_i}^{\sigma}d\sigma'\left(\frac{dy^i}{d\sigma'}\frac{dy^j}{d\tau/\zeta}-\frac{dy^j}{d\sigma'}\frac{dy^i}{d\tau/\zeta}\right)$$
$$\times W^{-1}(\sigma')\left(-T_a\frac{\partial\delta^{(3)}(x-y)}{\partial x^j}+ie[A_j(x),T_a]\delta^{(3)}(x-y)\right)W(\sigma')$$

$$\{E_i^a(x,t),\mathfrak{e}_{\tau/\zeta}(\sigma)\}_{PB}=ie\int_{\sigma_i}^{\sigma}d\sigma'\left[\frac{d\mathfrak{e}_{\tau/\zeta}(\sigma')}{d\sigma'},\int_{\sigma_i}^{\sigma'}d\sigma''W^{-1}(\sigma'')T_aW(\sigma'')\frac{dz^i}{d\sigma''}\delta(x-z)\right]$$

$$(z^1,z^2,z^3)\equiv(\zeta,\tau,\sigma'')$$

$$\varepsilon_{ijk}\frac{dz^i}{d\sigma''}\frac{dx^j}{d\sigma_x}\frac{dx^k}{d\tau_x/\zeta_x}\delta(x-z)=0$$

$$\left\{E_i^a(x,t)\varepsilon_{ijk}\frac{dx^j}{d\sigma_x}\frac{dx^k}{d\tau_x/\zeta_x},\mathfrak{e}_{\tau/\zeta}(\sigma)\right\}_{PB}=0$$

$$\{E_i^a(x,t),\mathfrak{b}_{\tau/\zeta}(\sigma)\}_{PB}=W^{-1}(\sigma)T_aW(\sigma)\frac{dw^i}{d\tau/\zeta}\delta^{(3)}(x-w)$$
$$+ie\left[\mathfrak{b}_{\tau/\zeta}(\sigma),\int_{\sigma_i}^{\sigma}d\sigma'W^{-1}(\sigma')T_aW(\sigma')\frac{dy^i}{d\sigma'}\delta^{(3)}(x-y)\right]$$
$$-\frac{d}{d\tau/\zeta}\int_{\sigma_i}^{\sigma}d\sigma'W^{-1}(\sigma')T_aW(\sigma')\frac{dy^i}{d\sigma'}\delta^{(3)}(x-y)$$

$$(w^1,w^2,w^3)\equiv(\zeta,\tau,\sigma)$$

$$\{E_i^a(x,t),\mathfrak{b}_{\tau/\zeta}(\sigma_f)\}_{PB}=ie\left[\mathfrak{b}_{\tau/\zeta}(\sigma_f),\int_{\sigma_i}^{\sigma_f}d\sigma'W^{-1}(\sigma')T_aW(\sigma')\frac{dy^i}{d\sigma'}\delta^{(3)}(x-y)\right]$$
$$-\frac{d}{d\tau/\zeta}\int_{\sigma_i}^{\sigma_f}d\sigma'W^{-1}(\sigma')T_aW(\sigma')\frac{dy^i}{d\sigma'}\delta^{(3)}(x-y)$$

$$\left\{E_i^a(x,t)\varepsilon_{ijk}\frac{dx^j}{d\sigma_x}\frac{dx^k}{d\tau_x/\zeta_x},\mathfrak{b}_{\tau/\zeta}(\sigma_f)\right\}_{PB}=0$$

$$\{H_C, A_i(y)\}_{PB} = D_i A_0(y) - \int d^3x \frac{\partial}{\partial x^i}\left[A_0(x)\delta^{(3)}(x-y)\right]$$

$$\frac{d}{d\sigma}(W^{-1}(\sigma)XW(\sigma)) = W^{-1}(\sigma)(\partial_i X + ie[A_i, X])W(\sigma)\frac{dx^i}{d\sigma}$$

$$\{H_C, W(\sigma)\}_{PB} = -ieW(\sigma)\Bigg[\left[\int_{\sigma_i}^{\sigma} d\sigma' \frac{d}{d\sigma'}[W^{-1}(\sigma')A_0(\sigma')W(\sigma')]\right.$$

$$\left. - \int_{\sigma_i}^{\sigma} d\sigma' W^{-1}(\sigma')T_b W(\sigma')\frac{dy^i}{d\sigma'}\int d^3x \frac{\partial}{\partial x^i}\left[A_0^b(x)\delta^{(3)}(x-y)\right]\right]$$

$$\frac{dy^i}{d\sigma'}\int d^3x \frac{\partial}{\partial x^i}\left[A_0^b(x)\delta^{(3)}(x-y)\right]$$
$$= \vartheta \int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma A_0^b(x)\delta^{(3)}(x-y)\varepsilon_{ijk}\frac{dy^i}{d\sigma'}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\bigg|_{\zeta=\zeta_f} = 0$$

$$\{H_C, W(\sigma)\}_{PB} = -ie[A_0(x(\sigma))W(\sigma) - W(\sigma)W_R^{-1}A_0(x_R)W_R]$$

$$\{H_C, \mathfrak{e}_{\tau/\zeta}(\sigma_f)\}_{PB} = ie\left[\mathfrak{e}_{\tau/\zeta}(\sigma_f), W_R^{-1}A_0(x_R)W_R\right]$$

$$\{H_C, \mathfrak{b}_{\tau/\zeta}(\sigma_f)\}_{PB} = ie\left[\mathfrak{b}_{\tau/\zeta}(\sigma_f), W_R^{-1}A_0(x_R)W_R\right]$$

$$\varepsilon_{ijk}\int d^3y \frac{\partial}{\partial y^j}\frac{\partial}{\partial y^k}[d_{ba}(y)\mathrm{Tr}(B_i(y)T_a)]$$
$$= \vartheta \int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma \frac{\partial}{\partial y^k}[d_{ba}(y)\mathrm{Tr}(B_i(y)T_a)]\left(\frac{dy^k}{d\sigma}\frac{dy^i}{d\tau} - \frac{dy^i}{d\sigma}\frac{dy^k}{d\tau}\right)\bigg|_{\zeta=\zeta_f}$$
$$\to \frac{1}{r^{\frac{1}{2}+\delta}} \text{ as } r \to \infty$$

$$\int d^3y \mathrm{Tr}\left[D_j B_i(y) T_a\right] \mathcal{S}_{ij}^{ba}(y)$$
$$= \vartheta \int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma d_{ba}(x)\mathrm{Tr}\left[D_j B_i(x) T_a\right]\left(\frac{dx^i}{d\sigma}\frac{dx^j}{d\tau} - \frac{dx^j}{d\sigma}\frac{dx^i}{d\tau}\right)\bigg|_{\zeta=\zeta_f}$$

$$A_i \to \frac{1}{r^{1/2+\delta}} \text{ as } r \to \infty \text{ for } \delta \geq 1/2;\ i = 1,2,3$$

$$A_i \to \frac{1}{r^{3/4+\delta/2}} \text{ as } r \to \infty \text{ for } 0 < \delta \leq 1/2;\ i = 1,2,3$$

$$D_j B_i \to \frac{1}{r^{\frac{9}{4}+\frac{3}{2}\delta}} \text{ as } r \to \infty \text{ for } 0 < \delta \leq 1/2;\ i, j = 1,2,3$$

$$\int d^3y \mathrm{Tr}\left[D_j B_i(y) T_a\right] \mathcal{S}_{ij}^{ba}(y) = \frac{1}{r^{\frac{1}{4}+\frac{3}{2}\delta}} \text{ as } r \to \infty \text{ for } 0 < \delta \leq 1/2$$

$$\int d^3y \frac{\partial}{\partial y^j}\left[\mathrm{Tr}(B_i(y)T_a)\mathcal{S}_{ij}^{ba}(y)\right]$$
$$= \int_{\tau_i}^{\tau_f} d\tau' \int_{\sigma_i}^{\sigma_f} d\sigma' \varepsilon_{jmn}\frac{dy^m}{d\sigma'}\frac{dy^n}{d\tau'}\mathrm{Tr}(B_i(y)T_a)$$
$$\times \int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma d_{ba}(x)\delta^{(3)}(x-y)\left(\frac{dx^i}{d\sigma}\frac{dx^j}{d\tau} - \frac{dx^j}{d\sigma}\frac{dx^i}{d\tau}\right)\Bigg|_{\zeta=\zeta_f} = 0$$

$$\varepsilon_{jmn}\frac{dy^m}{d\sigma'}\frac{dy^n}{d\tau'}\frac{dx^j}{d\tau}\delta^{(3)}(x-y) = 0;\ \varepsilon_{jmn}\frac{dy^m}{d\sigma'}\frac{dy^n}{d\tau'}\frac{dx^j}{d\sigma}\delta^{(3)}(x-y) = 0$$

$$\left\{W(\sigma)\overset{\otimes}{,}W(\sigma')\right\}_{PB} = 0$$

$$\left\{\mathfrak{b}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}W(\sigma')\right\}_{PB} = 0$$

$$\left\{\mathfrak{b}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}\mathfrak{b}_{\tau'/\zeta'}(\sigma')\right\}_{PB} = 0$$

$$\left\{E_i^a\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau/\zeta}, W(\sigma')\right\}_{PB}$$
$$= ieW(\sigma')\int_{\sigma_i}^{\sigma'} d\sigma'' W^{-1}(\sigma'')T_aW(\sigma'')\varepsilon_{ijk}\frac{dy^i}{d\sigma''}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau/\zeta}\delta^{(3)}(x-y)$$

$$\left\{\mathfrak{e}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}W(\sigma')\right\}_{PB} = 0$$

$$\left\{\mathfrak{e}_{\tau/\zeta}(\sigma) \otimes_{\mathfrak{e}_{\tau'/\zeta'}} (\sigma')\right\}_{PB} = 0$$

$$\left\{\mathfrak{e}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}\mathfrak{b}_{\tau'/\zeta'}(\sigma_f)\right\}_{PB} = 0$$

$$\left\{\mathfrak{e}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}\mathfrak{b}_{\tau'/\zeta'}(\sigma')\right\}_{PB} = \int_{\sigma_i}^{\sigma} d\sigma'' W^{-1}(\sigma'')T_aW(\sigma'') \otimes W^{-1}(\sigma')T_aW(\sigma')$$
$$\times \varepsilon_{ijk}\frac{dx^j}{d\sigma''}\frac{dx^k}{d\tau/\zeta}\frac{dy^i}{d\tau'/\zeta'}\delta^{(3)}(x-y)$$

$$\left\{\mathfrak{e}_{\tau}(\sigma)\overset{\otimes}{,}\mathfrak{b}_{\tau'}(\sigma')\right\}_{PB} = 0$$
$$\left\{\mathfrak{e}_{\zeta}(\sigma)\overset{\otimes}{,}\mathfrak{b}_{\zeta'}(\sigma')\right\}_{PB} = 0$$
$$\left\{\mathfrak{e}_{\tau}(\sigma)\overset{\otimes}{,}\mathfrak{b}_{\zeta'}(\sigma')\right\}_{PB} = \vartheta\mathbb{C}\delta(\zeta-\zeta')\delta(\tau-\tau')\theta(\sigma-\sigma');\ \text{for } \sigma' < \sigma_f'$$
$$\left\{\mathfrak{e}_{\zeta}(\sigma)\overset{\otimes}{,}\mathfrak{b}_{\tau'}(\sigma')\right\}_{PB} = -\vartheta\mathbb{C}\delta(\zeta-\zeta')\delta(\tau-\tau')\theta(\sigma-\sigma');\ \text{for } \sigma' < \sigma_f'$$

$$gT_ag^{-1} \otimes gT_ag^{-1} = T_a \otimes T_a$$

$$\varepsilon_{ijk}\frac{dx^i}{d\sigma}\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}\delta^{(3)}(x-y) = \vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')\delta(\sigma-\sigma');\ \vartheta = \pm 1$$

$$\left\{\mathfrak{e}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}\mathcal{J}_{\tau}(\tau)\right\}_{PB} = 0$$

$$\left\{\mathfrak{b}_{\tau/\zeta}(\sigma_f)\overset{\otimes}{,}\mathcal{T}_\tau(\tau)\right\}_{PB} = 0$$

$$\left\{\mathcal{T}_\tau(\tau_1)\overset{\otimes}{,}\mathcal{T}_\tau(\tau_2)\right\}_{PB} = 0$$

$$\begin{aligned}
&\left\{\mathfrak{e}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}\mathcal{S}_{\text{spatial}}(\tau')\right\}_{PB} = \beta\left\{\mathfrak{e}_{\tau/\zeta}(\sigma)\overset{\otimes}{,}\rho_G(\tau')\right\}_{PB}\\
&= -i\beta\vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')\int_{\sigma_i}^{\sigma_f} d\sigma'\theta(\sigma-\sigma')\left[\mathbb{C},\mathbb{1}\otimes\frac{d\mathfrak{e}_{\tau/\zeta}(\sigma')}{d\sigma'}\right]\\
&= i\beta\vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')\left[\mathbb{C},\mathfrak{e}_{\tau/\zeta}(\sigma)\otimes\mathbb{1}\right]
\end{aligned}$$

$$\left\{\mathfrak{b}_{\tau/\zeta}(\sigma_f)\overset{\otimes}{,}\mathcal{J}_{\text{spatial}}(\tau)\right\}_{PB} = 0$$

$$\begin{aligned}
&\left\{\mathcal{T}_\tau(\tau_1,\zeta_1,\alpha_1,\beta_1)\overset{\oplus}{,}\mathcal{A}(\zeta_2,\alpha_2,\beta_2)\right\}_{PB} =\\
&= ie^3\beta_1\beta_2\vartheta\delta(\zeta_1-\zeta_2)\mathbb{1}\otimes V_{(2)}(\tau_1)\left[\mathbb{C},\mathfrak{e}_\tau(\sigma_f,\tau_1,\zeta_1)\otimes\mathbb{1}\right]\mathbb{1}\otimes V_{(2)}^{-1}(\tau_1)
\end{aligned}$$

$$\left\{W(\sigma_1,\tau_1,\zeta_1)\overset{\otimes}{,}\mathcal{A}(\zeta_2,\alpha_2,\beta_2)\right\}_{PB} = 0$$

$$\begin{aligned}
&\left\{\rho_a^\psi(x),\rho_b^\psi(y)\right\}_{PB} = f_{abc}\rho_c^\psi(x)\delta^{(3)}(x-y)\\
&\left\{\rho_a^\varphi(x),\rho_b^\varphi(y)\right\}_{PB} = f_{abc}\rho_c^\varphi(x)\delta^{(3)}(x-y)\\
&\left\{\rho_a^\psi(x),\rho_b^\varphi(y)\right\}_{PB} = 0
\end{aligned}$$

$$\left\{J_0(x)\overset{\otimes}{,}J_0(y)\right\}_{PB} = i\delta^{(3)}(x-y)[\mathbb{C},\mathbb{1}\otimes J_0(x)] = -i\delta^{(3)}(x-y)[\mathbb{C},J_0(x)\otimes\mathbb{1}]$$

$$\mathbb{C}\equiv T_a\otimes T_a$$

$$[\mathbb{C},\mathbb{1}\otimes L + L\otimes\mathbb{1}] = 0;\ \rightarrow\ [T_a,L]\otimes T_a = -T_a\otimes[T_a,L]$$

$$\begin{aligned}
&\left\{\rho_G(\tau,\zeta)\overset{\otimes}{,}\rho_G(\tau',\zeta')\right\}_{PB} = -\vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')\\
&\times\int_{\sigma_i}^{\sigma_f} d\sigma\left\{[T_a,\mathfrak{b}_\tau(\sigma)]\otimes\left[T_a,\frac{d\mathfrak{e}_\zeta(\sigma)}{d\sigma}\right]-\left[T_a,\frac{d\mathfrak{e}_\zeta(\sigma)}{d\sigma}\right]\otimes[T_a,\mathfrak{b}_\tau(\sigma)]\right.\\
&\left.-[T_a,\mathfrak{b}_\zeta(\sigma)]\otimes\left[T_a,\frac{d\mathfrak{e}_\tau(\sigma)}{d\sigma}\right]+\left[T_a,\frac{d\mathfrak{e}_\tau(\sigma)}{d\sigma}\right]\otimes[T_a,\mathfrak{b}_\zeta(\sigma)]\right\}
\end{aligned}$$

$$[T_a,X]\otimes[T_a,Y]-[T_a,Y]\otimes[T_a,X] = T_a\otimes[[X,Y],T_a]$$

$$\begin{aligned}
\left\{\rho_G(\tau,\zeta)\overset{\otimes}{,}\rho_G(\tau',\zeta')\right\}_{PB} &= -i\vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')[\mathbb{C},\mathbb{1}\otimes\rho_G(\tau,\zeta)]\\
&= i\vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')[\mathbb{C},\rho_G(\tau,\zeta)\otimes\mathbb{1}]
\end{aligned}$$

$$\begin{aligned}
\left\{\rho_M(\tau,\zeta)\overset{\otimes}{,}\rho_M(\tau',\zeta')\right\}_{PB} &= -i\vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')[\mathbb{C},\mathbb{1}\otimes\rho_M(\tau,\zeta)]\\
&= i\vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')[\mathbb{C},\rho_M(\tau,\zeta)\otimes\mathbb{1}]
\end{aligned}$$

$$\left\{\rho_G(\tau,\zeta)\overset{\otimes\otimes}{,}\rho_M(\tau',\zeta')\right\}_{PB} = 0$$

$$\left\{\rho_M(\tau,\zeta)\overset{\otimes}{,}\mathfrak{b}_{\tau'/\zeta'}(\sigma')\right\}_{PB} = \left\{\rho_M(\tau,\zeta)\overset{\otimes}{,}\mathfrak{e}_{\tau'/\zeta'}(\sigma')\right\}_{PB} = 0$$

$$\left\{\rho_M(\tau,\zeta)\overset{\otimes}{,}\rho_{\text{mag.}}(\zeta',\tau')\right\}_{PB} = 0$$

$$\left\{\rho_G(\tau,\zeta)\overset{\otimes}{,}\mathfrak{b}_{\tau'/\zeta'}(\sigma_f')\right\}_{PB} = 0$$

$$\left\{\rho_G(\tau,\zeta)\overset{\otimes}{,}\rho_{\text{mag.}}(\zeta',\tau')\right\}_{PB} = 0$$

$$\left\{\rho_{\text{mag.}}(\zeta,\tau)^{\otimes},\rho_{\text{mag.}}(\zeta',\tau')\right\}_{PB} = 0$$

$$\begin{aligned}\left\{\rho_G(\tau,\zeta)\overset{\otimes}{,}\mathfrak{e}_{\tau'/\zeta'}(\sigma_f')\right\}_{PB} &= i\vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')\left[\mathbb{C},\mathfrak{e}_{\tau'/\zeta'}(\sigma_f')\otimes\mathbb{1}\right]\\ &= -i\vartheta\delta(\zeta-\zeta')\delta(\tau-\tau')\left[\mathbb{C},\mathbb{1}\otimes\mathfrak{e}_{\tau'/\zeta'}(\sigma_f')\right]\end{aligned}$$

$$\left\{\rho_G^a(\tau,\zeta),\mathfrak{e}^b_{\tau'/\zeta'}(\sigma_f')\right\}_{PB} = -\vartheta f_{abc}\mathfrak{e}^c_{\tau'/\zeta'}(\sigma_f')\delta(\zeta-\zeta')\delta(\tau-\tau')$$

$$b_i \equiv \varepsilon_{ijk}B_{jk}$$

$$\frac{d\ln V}{d\tau} = \mathcal{T};\ \mathcal{T} = \frac{1}{2}\int_{\sigma_i}^{\sigma_f} d\sigma b_i\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}$$

$$\frac{d\ln V}{d\zeta} = \mathcal{K};\ \mathcal{K} = \frac{1}{2}\int_{\tau_i}^{\tau_f} d\tau\int_{\sigma_i}^{\sigma_f} d\sigma\frac{\partial b_l}{\partial x^l}\varepsilon_{ijk}\frac{dx^i}{d\sigma}\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}$$

$$\vartheta\int_{\tau_i}^{\tau_f} d\tau\int_{\sigma_i}^{\sigma_f} d\sigma b_i\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\bigg|_{\zeta=\zeta_f} = \int_{\mathbb{R}^3} d^3x\frac{\partial b_l}{\partial x^l}$$

$$I_{\zeta,i}(f) \equiv \int_{\tau_i}^{\tau_f} d\tau\int_{\sigma_i}^{\sigma_f} d\sigma\Delta(\sigma,\tau,\zeta)\frac{\partial}{\partial x^i}\left(f(x)\delta^{(3)}(x-y)\right)$$

$$\frac{d\sigma}{\delta x^i} = \frac{\vartheta}{\Delta}\varepsilon_{ijk}\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta};\ \frac{d\tau}{\delta x^i} = \frac{\vartheta}{\Delta}\varepsilon_{ijk}\frac{dx^j}{d\zeta}\frac{dx^k}{d\sigma};\ \frac{d\zeta}{\delta x^i} = \frac{\vartheta}{\Delta}\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}$$

$$\begin{aligned}I_{\zeta,i}(f) = \vartheta\int_{\tau_i}^{\tau_f} d\tau\int_{\sigma_i}^{\sigma_f} d\sigma\varepsilon_{ijk}&\left[\frac{d}{d\sigma}\left(\frac{dx^j}{d\tau}\frac{dx^k}{d\zeta}f(x)\delta^{(3)}(x-y)\right)\right.\\ &\left.+\frac{d}{d\tau}\left(\frac{dx^j}{d\zeta}\frac{dx^k}{d\sigma}f(x)\delta^{(3)}(x-y)\right)+\frac{d}{d\zeta}\left(\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}f(x)\delta^{(3)}(x-y)\right)\right]\end{aligned}$$

$$\frac{dx^i}{d\tau} = \frac{dx^i}{d\zeta} = 0 \text{ at } \sigma = \sigma_i,\sigma_f \text{ and } \tau = \tau_i,\tau_f$$

$$I_{\zeta,i}(f) = \vartheta\frac{d}{d\zeta}\int_{\tau_i}^{\tau_f} d\tau\int_{\sigma_i}^{\sigma_f} d\sigma\varepsilon_{ijk}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}f(x)\delta^{(3)}(x-y)$$

$$\varepsilon_{ijk}\frac{dy^i}{d\sigma'}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\delta^{(3)}(x-y) = 0;\ \varepsilon_{ijk}\frac{dy^i}{d\tau'}\frac{dx^j}{d\sigma}\frac{dx^k}{d\tau}\delta^{(3)}(x-y) = 0$$

$$\frac{dy^i}{d\sigma'}I_{\zeta,i}(f) = 0;\ \frac{dy^i}{d\tau'}I_{\zeta,i}(f) = 0$$

$$\mathcal{O}_\zeta(X) \equiv \int_{\tau_i}^{\tau_f} d\tau\int_{\sigma_i}^{\sigma_f} d\sigma\Delta(\sigma,\tau,\zeta)d_{ba}(x)T_b\otimes\{\mathcal{C}_a(x),X\}_{PB} \equiv T_b\otimes\mathcal{O}_\zeta^b(X)$$

$$\mathcal{O}^{b}_{\zeta_1}\big(W(\sigma_2)\big) = -ieW(\sigma_2)\int_{\tau_i}^{\tau_f} d\tau_1 \int_{\sigma_i}^{\sigma_f} d\sigma_1 \Delta(\sigma_1,\tau_1,\zeta_1) d^{(1)}_{ba}(x)$$

$$\times \int_{\sigma_i}^{\sigma_2} d\sigma_2' W^{-1}(\sigma_2')\left\{[ieA_i(z),T_a]\delta^{(3)}(x-z) - \frac{\partial\delta^{(3)}(x-z)}{\partial x^i}T_a\right\}\frac{dz^i}{d\sigma_2'}W(\sigma_2')$$

$$\frac{i}{e}W^{-1}(\sigma_2)\mathcal{O}^{b}_{\zeta_1}\big(W(\sigma_2)\big) = -\int_{\sigma_i}^{\sigma_2} d\sigma_2' W^{-1}(\sigma_2')T_a W(\sigma_2')\frac{dz^i}{d\sigma_2'} I_{\zeta_1,i}\left(d^{(1)}_{ba}\right)$$

$$+\delta(\zeta_1-\zeta_2)\int_{\sigma_i}^{\sigma_2} d\sigma_2' \frac{d}{d\sigma_2'}\left(d^{(1)}_{ba}(z)W^{-1}(\sigma_2')T_a W(\sigma_2')\right)$$

$$d_{ba}(z)T_a = W(\sigma_2')V^{-1}(\tau_2)T_b V^{-1}(\tau_2)W^{-1}(\sigma_2')$$

$$\frac{d}{d\sigma_2'}\big(d_{ba}(z)W^{-1}(\sigma_2')T_a W(\sigma_2')\big) = \frac{d}{d\sigma_2'}\big(V^{-1}(\tau_2)T_b V(\tau_2)\big) = 0$$

$$\mathcal{O}^{b}_{\zeta_1}\big(W(\sigma_2)\big) = 0$$

$$\mathcal{O}^{b}_{\zeta_1}\left(\mathfrak{b}_{\tau_2/\zeta_2}(\sigma_f)\right) = 0$$

$$\mathcal{O}^{b}_{\zeta_1}\left(\mathfrak{e}_{\tau_2/\zeta_2}(\sigma_f)\right) = ie\delta(\zeta_1-\zeta_2)\left[\mathfrak{e}_{\tau_2/\zeta_2}(\sigma_f), V^{-1}_{(1)}(\tau_2)T_b V_{(1)}(\tau_2)\right]$$

$$\mathcal{O}^{b}_{\zeta_1}\left(\mathcal{T}^{(2)}_{\tau_2}(\tau_2)\right) = e^2\beta_2\delta(\zeta_1-\zeta_2)\left[\mathfrak{e}_{\tau_2}(\sigma_f), V^{-1}_{(1)}(\tau_2)T_b V_{(1)}(\tau_2)\right]$$

$$\mathcal{O}^{c}_{\zeta_1}(\mathcal{C}_b(y)) = -ef_{abd}\int_{\tau_i}^{\tau_f} d\tau_1 \int_{\sigma_i}^{\sigma_f} d\sigma_1 \Delta(\sigma_1,\tau_1,\zeta_1) d^{(1)}_{ca}(x)\Big[\mathcal{C}_d(x)\delta^{(3)}(x-y)$$

$$+E^d_i(y)\frac{\partial\delta^{(3)}(x-y)}{\partial x^i} + E^d_i(x)\frac{\partial\delta^{(3)}(x-y)}{\partial y^i} - \frac{\partial E^d_i(x)}{\partial x^i}\delta^{(3)}(x-y)\Big]$$

$$\mathcal{O}^{c}_{\zeta_1}(\mathcal{C}_b(y)) = -ef_{abd}\left\{\delta(\zeta_1-\zeta_2)\left[d^{(1)}_{ca}(y)\mathcal{C}_d(y) - \frac{\partial\left(d^{(1)}_{ca}(y)E^d_i(y)\right)}{\partial y^i}\right] + E^d_i(y)I_{\zeta_1,i}\left(d^{(1)}_{ca}\right)\right.$$

$$\left.+\int_{\tau_i}^{\tau_f} d\tau_1 \int_{\sigma_i}^{\sigma_f} d\sigma_1 \Delta(\sigma_1,\tau_1,\zeta_1) d^{(1)}_{ca}(x)E^d_i(x)\frac{\partial\delta^{(3)}(x-y)}{\partial y^i}\right\}$$

$$
\begin{aligned}
\mathcal{I}_{cd} \equiv & \int_{\tau_i}^{\tau_f} d\tau_1 \int_{\sigma_i}^{\sigma_f} d\sigma_1 \Delta(\sigma_1, \tau_1, \zeta_1) d_{ca}^{(1)}(x) \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \Delta(\sigma_2, \tau_2, \zeta_2) d_{db}^{(2)}(y) \\
& \times \{\mathcal{C}_a(x), \mathcal{C}_b(y)\}_{PB} \\
= & \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \Delta(\sigma_2, \tau_2, \zeta_2) d_{db}^{(2)}(y) \mathcal{O}_{\zeta_1}^{c}(\mathcal{C}_b(y)) \\
= & -e f_{abe} \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \Delta(\sigma_2, \tau_2, \zeta_2) d_{db}^{(2)}(y) \\
& \times \left\{ \delta(\zeta_1 - \zeta_2) \left[d_{ca}^{(1)}(y) \mathcal{C}_e(y) - \frac{\partial \left(d_{ca}^{(1)}(y) E_i^e(y) \right)}{\partial y^i} \right] + E_i^e(y) I_{\zeta_1, i} \left(d_{ca}^{(1)} \right) \right\} \\
& - e f_{abe} \int_{\tau_i}^{\tau_f} d\tau_1 \int_{\sigma_i}^{\sigma_f} d\sigma_1 \Delta(\sigma_1, \tau_1, \zeta_1) d_{ca}^{(1)}(x) E_i^e(x) \left[I_{\zeta_2, i} \left(d_{db}^{(2)} \right) - \delta(\zeta_1 - \zeta_2) \frac{\partial d_{db}^{(2)}(x)}{\partial x^i} \right]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{I}_{cd} = & -e f_{abe} \delta(\zeta_1 - \zeta_2) \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \Delta(\sigma_2, \tau_2, \zeta_2) \\
& \times \left[d_{ca}^{(1)}(y) d_{db}^{(2)}(y) \mathcal{C}_e(y) - \frac{\partial \left(d_{ca}^{(1)}(y) d_{db}^{(2)}(y) E_i^e(y) \right)}{\partial y^i} \right] \\
& - e f_{abe} \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \Delta(\sigma_2, \tau_2, \zeta_2) d_{db}^{(2)}(y) E_i^e(y) I_{\zeta_1, i} \left(d_{ca}^{(1)} \right) \\
& - e f_{abe} \int_{\tau_i}^{\tau_f} d\tau_1 \int_{\sigma_i}^{\sigma_f} d\sigma_1 \Delta(\sigma_1, \tau_1, \zeta_1) d_{ca}^{(1)}(x) E_i^e(x) I_{\zeta_2, i} \left(d_{db}^{(2)} \right)
\end{aligned}
$$

$$
\begin{aligned}
& \int_{\tau_i}^{\tau_f} d\tau_1 \int_{\sigma_i}^{\sigma_f} d\sigma_1 \Delta(\sigma_1, \tau_1, \zeta_1) d_{ca}^{(1)}(x) E_i^e(x) I_{\zeta_2, i} \left(d_{db}^{(2)} \right) \\
& = \vartheta \frac{d}{d\zeta_2} \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \varepsilon_{ijk} \frac{dy^j}{d\sigma_2} \frac{dy^k}{d\tau_2} d_{db}^{(2)}(y) d_{ca}^{(1)}(y) E_i^e(y) \delta(\zeta_1 - \zeta_2)
\end{aligned}
$$

$$
\begin{aligned}
& \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \Delta(\sigma_2, \tau_2, \zeta_2) \frac{\partial \left(d_{ca}^{(1)}(y) d_{db}^{(2)}(y) E_i^e(y) \right)}{\partial y^i} \\
& = \vartheta \frac{d}{d\zeta_2} \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \varepsilon_{ijk} \frac{dy^j}{d\sigma_2} \frac{dy^k}{d\tau_2} d_{db}^{(2)}(y) d_{ca}^{(1)}(y) E_i^e(y)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{I}_{cd} = & -e f_{abe} \delta(\zeta_1 - \zeta_2) \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \Delta(\sigma_2, \tau_2, \zeta_2) d_{ca}^{(1)}(y) d_{db}^{(2)}(y) \mathcal{C}_e(y) \\
& + e \vartheta f_{abe} \delta(\zeta_1 - \zeta_2) \frac{d}{d\zeta_2} \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \varepsilon_{ijk} \frac{dy^j}{d\sigma_2} \frac{dy^k}{d\tau_2} d_{ca}^{(1)}(y) d_{db}^{(2)}(y) E_i^e(y) \\
& - e \vartheta f_{abe} \frac{d}{d\zeta_1} \int_{\tau_i}^{\tau_f} d\tau_1 \int_{\sigma_i}^{\sigma_f} d\sigma_1 \varepsilon_{ijk} \frac{dx^j}{d\sigma_1} \frac{dx^k}{d\tau_1} d_{ca}^{(1)}(x) d_{db}^{(2)}(x) E_i^e(x) \delta(\zeta_1 - \zeta_2) \\
& - e \vartheta f_{abe} \frac{d}{d\zeta_2} \int_{\tau_i}^{\tau_f} d\tau_2 \int_{\sigma_i}^{\sigma_f} d\sigma_2 \varepsilon_{ijk} \frac{dy^j}{d\sigma_2} \frac{dy^k}{d\tau_2} d_{ca}^{(1)}(y) d_{db}^{(2)}(y) E_i^e(y) \delta(\zeta_1 - \zeta_2)
\end{aligned}
$$

$$f_{abe}d_{ca}^{(1)}(y)d_{db}^{(2)}(y)T_c\otimes T_d =$$
$$= f_{abe}V_{(1)}(\tau_2)W^{-1}(\sigma_2)T_aW(\sigma_2)V_{(1)}^{-1}(\tau_2)\otimes V_{(2)}(\tau_2)W^{-1}(\sigma_2)T_bW(\sigma_2)V_{(2)}^{-1}(\tau_2)$$
$$= -iV_{(1)}(\tau_2)W^{-1}(\sigma_2)[T_b,T_e]W(\sigma_2)V_{(1)}^{-1}(\tau_2)\otimes V_{(2)}(\tau_2)W^{-1}(\sigma_2)T_bW(\sigma_2)V_{(2)}^{-1}(\tau_2)$$
$$= -iV_{(1)}(\tau_2)[T_b,W^{-1}(\sigma_2)T_eW(\sigma_2)]V_{(1)}^{-1}(\tau_2)\otimes V_{(2)}(\tau_2)T_bV_{(2)}^{-1}(\tau_2)$$
$$= -iV_{(1)}(\tau_2)\otimes V_{(2)}(\tau_2)[\mathbb{C},W^{-1}(\sigma_2)T_eW(\sigma_2)\otimes \mathbb{1}]V_{(1)}^{-1}(\tau_2)\otimes V_{(2)}^{-1}(\tau_2)$$

$$\mathcal{I}_{cd}T_c\otimes T_d = ie\delta(\zeta_1-\zeta_2)\int_{\tau_i}^{\tau_f}d\tau_2\int_{\sigma_i}^{\sigma_f}d\sigma_2\Delta(\sigma_2,\tau_2,\zeta_2)$$
$$\times V_{(1)}(\tau_2)\otimes V_{(2)}(\tau_2)[\mathbb{C},W^{-1}(\sigma_2)\mathcal{C}W(\sigma_2)\otimes\mathbb{1}]V_{(1)}^{-1}(\tau_2)\otimes V_{(2)}^{-1}(\tau_2)$$
$$+ie\vartheta\left[-\delta(\zeta_1-\zeta_2)\frac{d\mathcal{Y}(\zeta_2,\tau_f)}{d\zeta_2}+\frac{d\left(\mathcal{Y}(\zeta_1,\tau_f)\delta(\zeta_1-\zeta_2)\right)}{d\zeta_1}+\frac{d\left(\mathcal{Y}(\zeta_2,\tau_f)\delta(\zeta_1-\zeta_2)\right)}{d\zeta_2}\right]$$

$$\left[\delta_{N_1,\alpha_1,\beta_1},\delta_{N_2,\alpha_2,\beta_2}\right]H_T\cong 0$$

$$\left[\delta_{N_1,\alpha_1,\beta_1},\delta_{N_2,\alpha_2,\beta_2}\right]X\ \cong \varepsilon_1\varepsilon_2(\alpha_1-\alpha_2)\mathrm{Tr}_{RL}\left[Q_{(1)}^{N_1}(\zeta_f)\otimes Q_{(2)}^{N_2}(\zeta_f)\right.$$
$$\left.\times\int_{\zeta_i}^{\zeta_f}d\zeta\int_{\tau_i}^{\tau_f}d\tau\int_{\tau_i}^{\tau}d\tau'\int_{\sigma_i}^{\sigma_f}d\sigma\Delta(y)\Upsilon_a(\sigma,\tau,\tau',\zeta)\{X,\mathcal{C}_a(y)\}_{PB}\right]$$

$$\Upsilon_a(\sigma,\tau,\tau',\zeta)=ie\vartheta\left[Q_{\otimes}^{-1}(\zeta)\frac{d\mathcal{Z}(\zeta,\tau)}{d\tau}Q_{\otimes}(\zeta),\beta_1T_b\otimes\mathbb{1}d_{ba}^{(1)}(y)+\beta_2\mathbb{1}\otimes T_bd_{ba}^{(2)}(y)\right]$$

$$\left[\delta_{N_1,\alpha_1,\beta_1},\delta_{N_2,\alpha_2,\beta_2}\right]H_E=0$$

$$\{H_\psi+H_\varphi,\mathcal{C}_a(y)\}_{PB}\ = e\int\ d^3z\frac{\partial}{\partial z^i}\left(J_i^a(z)\delta^{(3)}(y-z)\right)$$
$$\{H_C,\mathcal{C}_a(y)\}_{PB}\ = ef_{abc}\left[A_0^b(y)\mathcal{C}_c(y)+\int\ d^3zE_i^c(y)\frac{\partial}{\partial z^i}\left(A_0^b(z)\delta^{(3)}(y-z)\right)\right]$$
$$\{H_B,\mathcal{C}_a(y)\}_{PB}\ = -\varepsilon_{ijk}\int\ d^3z\frac{\partial}{\partial z^i}\left[B_k^a(z)\frac{\partial\delta^{(3)}(y-z)}{\partial y^j}+ef_{abc}A_j^c(y)B_k^b(z)\delta^{(3)}(y-z)\right]$$

$$\mathcal{I}_a^{\Upsilon}\equiv\int_{\zeta_i}^{\zeta_f}d\zeta\int_{\tau_i}^{\tau_f}d\tau\int_{\tau_i}^{\tau}d\tau'\int_{\sigma_i}^{\sigma_f}d\sigma\Upsilon_a(\sigma,\tau,\tau',\zeta)\Delta(y)\delta^{(3)}\left(y-z(\sigma_z,\tau_z,\zeta_z)\right)$$
$$=\int_{\tau_z}^{\tau_f}d\tau\Upsilon_a(\sigma_z,\tau_z,\zeta_z,\tau)$$

$$\mathcal{I}_{a,i}^{\Upsilon}\equiv\int_{\zeta_i}^{\zeta_f}d\zeta\int_{\tau_i}^{\tau_f}d\tau\int_{\tau_i}^{\tau}d\tau'\int_{\sigma_i}^{\sigma_f}d\sigma\Upsilon_a(\sigma,\tau,\tau',\zeta)\Delta(y)\frac{\partial}{\partial y^i}\delta^{(3)}\left(y-z(\sigma_z,\tau_z,\zeta_z)\right)$$
$$=\int_{\zeta_i}^{\zeta_f}d\zeta\int_{\tau_i}^{\tau_f}d\tau\int_{\tau_i}^{\tau}d\tau'\int_{\sigma_i}^{\sigma_f}d\sigma\Delta(y)\left[\frac{\partial}{\partial y^i}\left(\Upsilon_a\delta^{(3)}(y-z)\right)-\delta^{(3)}(y-z)\frac{\partial\Upsilon_a}{\partial y^i}\right]$$

$$\int_{\tau_i}^{\tau} d\tau' \int_{\sigma_i}^{\sigma_f} d\sigma \Delta(y) \frac{\partial}{\partial y^i}\left(\Upsilon_a \delta^{(3)}(y-z)\right) =$$

$$= \varepsilon_{ijk}\vartheta \int_{\tau_i}^{\tau} d\tau' \int_{\sigma_i}^{\sigma_f} d\sigma \left[\frac{d}{d\tau'}\left(\frac{dy^j}{d\zeta}\frac{dy^k}{d\sigma}\Upsilon_a \delta^{(3)}(y-z)\right) + \frac{d}{d\zeta}\left(\frac{dy^j}{d\sigma}\frac{dy^k}{d\tau'}\Upsilon_a \delta^{(3)}(y-z)\right)\right]$$

$$= \varepsilon_{ijk}\vartheta \left\{\int_{\sigma_i}^{\sigma_f} d\sigma \left(\frac{dx^j}{d\zeta}\frac{dx^k}{d\sigma}\Upsilon_a \delta^{(3)}(x-z)\right) + \frac{d}{d\zeta}\left[\int_{\tau_i}^{\tau} d\tau' \int_{\sigma_i}^{\sigma_f} d\sigma \left(\frac{dy^j}{d\sigma}\frac{dy^k}{d\tau'}\Upsilon_a \delta^{(3)}(y-z)\right)\right]\right\}$$

$$\mathcal{J}_{a,i}^{\Upsilon} = \varepsilon_{ijk}\vartheta \left[\int_{\tau_i}^{\tau_f} d\tau \int_{\tau_i}^{\tau} d\tau' \int_{\sigma_i}^{\sigma_f} d\sigma \left(\frac{dy^j}{d\sigma}\frac{dy^k}{d\tau'}\Upsilon_a \delta^{(3)}(y-z)\right)\right]_{\zeta=\zeta_f}$$

$$+\varepsilon_{ijk}\vartheta \int_{\zeta_i}^{\zeta_f} d\zeta \int_{\tau_i}^{\tau_f} d\tau \int_{\sigma_i}^{\sigma_f} d\sigma \left(\frac{dx^j}{d\zeta}\frac{dx^k}{d\sigma}\Upsilon_a \delta^{(3)}(x-z)\right) - \int_{\tau_z}^{\tau_f} d\tau \frac{\partial}{\partial z^i}\Upsilon_a(\sigma_z,\tau_z,\zeta_z,\tau)$$

$$[\delta_{N_1,\alpha_1,\beta_1}, \delta_{N_2,\alpha_2,\beta_2}]H_M \cong \vartheta e \varepsilon_1 \varepsilon_2 (\alpha_1 - \alpha_2)$$

$$\times \mathrm{Tr}_{RL}\left[Q_{(1)}^{N_1}(\zeta_f) \otimes Q_{(2)}^{N_2}(\zeta_f)\left(\varepsilon_{imn}\int_{\tau_i}^{\tau_f} d\tau_z \int_{\sigma_i}^{\sigma_f} d\sigma_z \frac{dz^m}{d\sigma_z}\frac{dz^n}{d\tau_z} J_i^a(z)\mathcal{J}_a^{\Upsilon}(z)\right)_{\zeta_z=\zeta_f}\right]$$

$$\frac{dz^i}{d\sigma_z/\tau_z} \to r, \frac{dz^i}{d\zeta_z} \to s(\sigma,\tau), r \to \infty$$

$$\Delta(z) \to r^2, r \to \infty$$

$$J_i^a \to \frac{1}{r^{2+\delta'}}, r \to \infty$$

$$\Upsilon_a \to \frac{1}{r^{\delta-\frac{1}{2}}}, r \to \infty$$

$$\mathcal{J}_a^{\Upsilon} \to \frac{1}{r^{\delta-\frac{1}{2}}}\ r \to \infty$$

$$[\delta_{N_1,\alpha_1,\beta_1}, \delta_{N_2,\alpha_2,\beta_2}]H_M \to \frac{1}{r^{\delta+\delta'-1/2}}, r \to \infty$$

$$[\delta_{N_1,\alpha_1,\beta_1}, \delta_{N_2,\alpha_2,\beta_2}]H_M \cong 0;\ \text{ if } \delta+\delta' > \frac{1}{2}$$

$$[\delta_{N_1,\alpha_1,\beta_1}, \delta_{N_2,\alpha_2,\beta_2}]H_C \cong \vartheta e \varepsilon_1 \varepsilon_2 (\alpha_1 - \alpha_2) \times$$

$$\times \mathrm{Tr}_{RL}\left[Q_{(1)}^{N_1}(\zeta_f) \otimes Q_{(2)}^{N_2}(\zeta_f)\left(\varepsilon_{imn} f_{abc}\int_{\tau_i}^{\tau_f} d\tau_z \int_{\sigma_i}^{\sigma_f} d\sigma_z \frac{dz^m}{d\sigma_z}\frac{dz^n}{d\tau_z} A_0^b(z) E_i^c(z)\mathcal{J}_a^{\Upsilon}(z)\right)_{\zeta_z=\zeta_f}\right]$$

$$[\delta_{N_1,\alpha_1,\beta_1}, \delta_{N_2,\alpha_2,\beta_2}]H_C \cong 0;\ \text{ if } \delta > \frac{1}{6}$$

$$[\delta_{N_1,\alpha_1,\beta_1},\delta_{N_2,\alpha_2,\beta_2}]H_B \cong -\vartheta\varepsilon_1\varepsilon_2(\alpha_1-\alpha_2)\times$$

$$\times \mathrm{Tr}_{RL}\left[Q_{(1)}^{N_1}(\zeta_f)\otimes Q_{(2)}^{N_2}(\zeta_f)\left[\left(\varepsilon_{ijk}\int_{\tau_i}^{\tau_f}d\tau_z\int_{\sigma_i}^{\sigma_f}d\sigma_z\frac{dz^j}{d\sigma_z}\frac{dz^k}{d\tau_z}B_k^a(z)\mathcal{I}_{a,i}^{\Upsilon}(z)\right)_{\zeta_z=\zeta_f}\right.\right.$$

$$\left.\left.+\left(e\varepsilon_{ijk}f_{abc}\int_{\tau_i}^{\tau_f}d\tau_z\int_{\sigma_i}^{\sigma_f}d\sigma_z\frac{dz^j}{d\sigma_z}\frac{dz^k}{d\tau_z}A_j^c(z)B_k^b(z)\mathcal{I}_a^{\Upsilon}(z)\right)_{\zeta_z=\zeta_f}\right]\right]$$

$$[\delta_{N_1,\alpha_1,\beta_1},\delta_{N_2,\alpha_2,\beta_2}]H_B \cong -\vartheta\varepsilon_1\varepsilon_2(\alpha_1-\alpha_2)\mathrm{Tr}_{RL}\left[Q_{(1)}^{N_1}(\zeta_f)\otimes Q_{(2)}^{N_2}(\zeta_f)\right.$$

$$\times\left[\int_{\tau_i}^{\tau_f}d\tau_z\int_{\sigma_i}^{\sigma_f}d\sigma_z\int_{\tau_i}^{\tau_f}d\tau\int_{\tau_i}^{\tau}d\tau'\int_{\sigma_i}^{\sigma_f}d\sigma B_k^a(z)\Upsilon_a\delta^{(3)}(y-z)\right.$$

$$\left.\left.\times\left(\frac{dz^m}{d\sigma_z}\frac{dz^n}{d\tau_z}\frac{dy^m}{d\sigma}\frac{dy^n}{d\tau'}-\frac{dz^m}{d\sigma_z}\frac{dz^n}{d\tau_z}\frac{dy^n}{d\sigma}\frac{dy^m}{d\tau'}\right)\right]_{\zeta,\zeta_z=\zeta_f}\right]$$

$$[\delta_{N_1,\alpha_1,\beta_1},\delta_{N_2,\alpha_2,\beta_2}]H_B \cong 0$$

$$\delta n^a=\omega^{ab}n_b=-2An^a, A\in\mathbb{R}$$

$$\eta=\begin{pmatrix} & -1 & \\ -1 & & \\ & & \mathbf{1}_{D-2}\end{pmatrix},$$

$$\begin{aligned}\omega^{+b}n_b=An^+ &\quad\Rightarrow\quad \omega^{+-}=2A,\\ \omega^{-b}n_b=An^- &\quad\Rightarrow\quad 0=0,\\ \omega^{ab}n_b=An^a &\quad\Rightarrow\quad \omega^{a-}=0,\quad \text{for } a=2,\dots,D-1.\end{aligned}$$

$$\omega^{ab}=\begin{pmatrix}0 & \omega^{+-} & \omega^{+2}\\ \omega^{-+} & 0 & \omega^{-2}\\ \omega^{2+} & \omega^{2-} & 0\end{pmatrix}=\begin{pmatrix}0 & 2A & -\sqrt{2}B\\ -2A & 0 & 0\\ \sqrt{2}B & 0 & 0\end{pmatrix}$$

$$x^+\to e^{-2A}x^+-\sqrt{2}e^{-A}Bx^2+B^2x^-, x^-\to e^{2A}x^-, x^2\to x^2-\sqrt{2}e^{A}Bx^-.$$

$$x=\begin{pmatrix}x^0+x^1 & x^2\\ x^2 & x^0-x^1\end{pmatrix}=\begin{pmatrix}\sqrt{2}x^+ & x^2\\ x^2 & \sqrt{2}x^-\end{pmatrix}$$

$$x'=gxg^T, g\in SL(2,\mathbb{R})$$

$$g\xi=\pm e^{-A}\xi, A\in\mathbb{R}$$

$$g=\pm\begin{pmatrix}e^{-A} & -B\\ 0 & e^{A}\end{pmatrix}$$

$$\begin{pmatrix}z\\1\end{pmatrix}\to g\begin{pmatrix}z\\1\end{pmatrix}=\pm\begin{pmatrix}e^{-A}z-B\\ e^{A}\end{pmatrix}\sim\begin{pmatrix}e^{-2A}z-e^{-A}B\\ 1\end{pmatrix}=\begin{pmatrix}z'\\1\end{pmatrix}$$

$$Q_\alpha=\partial_\alpha-(\gamma^a\theta)_\alpha\partial_a=\partial_\alpha+\gamma_{\alpha\beta}^a\theta^\beta\partial_a$$

$$D_\alpha=\partial_\alpha+(\gamma^a\theta)_\alpha\partial_a=\partial_\alpha-\gamma_{\alpha\beta}^a\theta^\beta\partial_a$$

$$\{Q_\alpha, Q_\beta\} = 2\gamma^a_{\alpha\beta}\partial_a, \{Q_\alpha, D_\beta\} = 0, \{D_\alpha, D_\beta\} = -2\gamma^a_{\alpha\beta}\partial_a$$

$$\begin{pmatrix}\psi'^+\\ \psi'^-\end{pmatrix} = \begin{pmatrix}e^{-A} & -B\\ 0 & e^A\end{pmatrix}\begin{pmatrix}\psi^+\\ \psi^-\end{pmatrix} \Leftrightarrow \begin{pmatrix}\psi'_+\\ \psi'_-\end{pmatrix} = \begin{pmatrix}e^A & 0\\ B & e^{-A}\end{pmatrix}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}$$

$$\not{n}\psi = 0 \Rightarrow \psi = \begin{pmatrix}0\\ \psi_-\end{pmatrix}$$

$$\psi = \begin{pmatrix}\psi_+\\ 0\end{pmatrix}$$

$$\begin{pmatrix}0\\ \psi'_-\end{pmatrix} = e^{-A}\begin{pmatrix}0\\ \psi'_-\end{pmatrix}, \begin{pmatrix}\psi'_+\\ 0\end{pmatrix} = e^A\begin{pmatrix}\psi'_+\\ 0\end{pmatrix}$$

$$\psi = \frac{1}{2}\tilde{n}h\not{n}\psi + \frac{1}{2}h\tilde{n}\psi \Leftrightarrow \psi_\alpha = -\tilde{n}_{\alpha\beta}n^{\beta\gamma}\psi_\gamma - n_{\alpha\beta}\tilde{n}^{\beta\gamma}\psi_\gamma$$

$$\frac{1}{2}\tilde{n}h\left(\psi = \begin{pmatrix}\psi_+\\ 0\end{pmatrix}, \frac{1}{2}\not{n}\tilde{n}\psi = \begin{pmatrix}0\\ \psi_-\end{pmatrix}\right.$$

$$S = \frac{1}{2}\tilde{n}\not{n}Q, \zeta = \frac{1}{2}\not{n}\tilde{n}\theta, d = \frac{1}{2}\tilde{n}\not{n}D\Big|_{n\theta=0}$$

$$S_+ = \partial_+ + i\zeta_-\partial_{++}\ \zeta_- = \theta_-, d_+ = \partial_+ - i\zeta_-\partial_{++}$$

$$\{S_+, S_+\} = 2\partial_{++}, \{S_+, d_+\} = 0, \{d_+, d_+\} = -2\partial_{++}, \partial_+\zeta_- = -i$$

$$\phi = \Phi|_{\theta_+=0}, \tilde{\phi}_- = (D_-\Phi)|_{\theta_+=0}$$

$$\Phi = \phi - i\theta_+(\tilde{\phi}_- + i\theta_-\partial_{+-}\phi)$$

$$\phi'(x', \theta') = \phi(x, \theta), \tilde{\phi}'_-(x', \theta') = e^{-A}\tilde{\phi}_-(x, \theta) + B\phi(x, \theta)$$

$$\hat{q} = \frac{\eta\,\partial}{2n\cdot\partial}D$$

$$\hat{q}_- = D_- - \frac{\partial_{-+}}{\partial_{++}}D_+$$

$$\hat{\phi}_- = (\hat{q}_-\Phi)|_{\theta_+=0}$$

$$\hat{\phi}'_-(x', \theta') = e^{-A}\hat{\phi}_-(x, \theta),$$

$$\nabla_\alpha = D_\alpha - i\Gamma_\alpha, \nabla_{\alpha\beta} = \partial_{\alpha\beta} - i\Gamma_{\alpha\beta}$$

$$\{\nabla_\alpha, \nabla_\beta\} = -2\nabla_{\alpha\beta}$$
$$[\nabla_\alpha, \nabla_{\beta\gamma}] = C_{\alpha(\beta}W_{\gamma)}$$
$$[\nabla_{\alpha\beta}, \nabla_{\gamma\delta}] = -\frac{1}{2}C_{\alpha\gamma}F_{\beta\delta} - \frac{1}{2}C_{\alpha\delta}F_{\beta\gamma} - \frac{1}{2}C_{\beta\delta}F_{\alpha\gamma} - \frac{1}{2}C_{\beta\gamma}F_{\alpha\delta}$$

$$\Gamma_{\alpha\beta} = -\frac{1}{2}\left(D_{(\alpha}\Gamma_{\beta)} - i\{\Gamma_\alpha, \Gamma_\beta\}\right)$$
$$W_\alpha = -\frac{i}{2}D^\beta D_\alpha \Gamma_\beta - \frac{1}{2}\left[\Gamma^\beta, D_\beta \Gamma_\alpha\right] + \frac{i}{6}\left[\Gamma^\beta, \{\Gamma_\beta, \Gamma_\alpha\}\right], \nabla^\alpha W_\alpha = 0$$
$$F_{\alpha\beta} = \frac{1}{2}\nabla_{(\alpha} W_{\beta)}$$

$$q = n\nabla \frac{1}{2n \cdot \nabla}\nabla,$$

$$\phi_- = (q_- \Phi)|_{\theta_+=0} = \left(\left(\nabla_- - \nabla_{+-}\frac{\nabla_+}{\nabla_{++}}\right)\Phi\right)\Big|_{\theta_+=0}.$$

$$\left[\frac{1}{\partial_{++}}, \partial_{++}\right] f(x) = \left(\frac{1}{\partial_{++}}\partial_{++} - \partial_{++}\frac{1}{\partial_{++}}\right) f(x) = \frac{1}{\partial_{++}}\partial_{++} f(x) - f(x) = 0.$$

$$\frac{1}{\partial_{++}} f(x^{++}, x^{--}, x^{+-}) = \int_{-\infty}^{x^{++}} \mathrm{d}t^{++} f(t^{++}, x^{--}, x^{+-})$$

$$\phi_- = \left(\left(\nabla_- - \nabla_{-+}\frac{1}{\nabla_{++}}\nabla_+\right)\Phi\right)\Big|_{\theta_+=0, \bar{\theta}_+=0}$$

$$w_+ = W_+|_{\theta_+=0}, \tilde{w}_- = W_-|_{\theta_+=0},$$

$$w'_+(x', \theta') = e^A w_+(x, \theta), \tilde{w}'_-(x', \theta') = e^{-A}\tilde{w}_-(x, \theta) + B w_+(x, \theta)$$

$$\triangle = \frac{i}{2}\left(\frac{1}{2n \cdot \nabla}\not{n}\nabla + \not{n}\nabla\frac{1}{2n \cdot \nabla}\right)$$

$$\Delta_{-+} = 1, \Delta_{--} = \frac{1}{2}\nabla_{+-}\frac{1}{\nabla_{++}} + \frac{1}{2}\frac{1}{\nabla_{++}}\nabla_{+-}.$$

$$w_- = i(\triangle_-{}^\alpha W_\alpha)|_{\theta_+=0} = (W_- - \triangle_{--} W_+)|_{\theta_+=0}.$$

$$f_{++} = F_{++} \mid \theta_+ = 0$$
$$f_{+-} = i(\triangle_-{}^\alpha F_{+\alpha})|_{\theta_+=0} = (F_{+-} - \triangle_{--} F_{++})|_{\theta_+=0}$$
$$f_{--} = -\left(\triangle_-{}^\alpha \triangle_-{}^\beta F_{\alpha\beta}\right)\Big|_{\theta_+=0} = (F_{--} - 2\triangle_{--} F_{+-} + \triangle_{--}\triangle_{--} F_{++})|_{\theta_+=0}$$

$$S_m = \frac{1}{2}\int \mathrm{d}^3x \nabla^2\left[(\nabla^\alpha \Phi^\dagger)(\nabla_\alpha \Phi)\right]$$

$$S_g = \mathrm{tr}\int \mathrm{d}^3x \nabla^2[W^2]$$

$$S_m = \frac{1}{2}\int d^3x\nabla_+\Big[(\nabla_+\phi_-^\dagger)\phi_- + \phi_-^\dagger(\nabla_+\phi_-) - 2i\phi_-^\dagger\left(w_+\frac{\nabla_+}{\nabla_{++}}\phi\right) - 2i\left(w_+\frac{\nabla_+}{\nabla_{++}}\phi^\dagger\right)\phi_-$$

$$+(\Box_{\mathrm{cov}}\,\phi^\dagger)\left(\frac{\nabla_+}{\nabla_{++}}\phi\right) + \left(\frac{\nabla_+}{\nabla_{++}}\phi^\dagger\right)(\Box_{\mathrm{cov}}\,\phi)$$

$$-i\left(\frac{\nabla_+}{\nabla_{++}}\phi^\dagger\right)\left((\nabla_{++}w_-)\frac{\nabla_+}{\nabla_{++}}\phi\right) + \frac{1}{2}\left(\frac{\nabla_+}{\nabla_{++}}\phi^\dagger\right)\left(\left[f_{++},\frac{1}{\nabla_{++}}w_+\right]\frac{\nabla_+}{\nabla_{++}}\phi\right)\Big]$$

$$+\,\mathbb{St}$$

$$S_g = \mathrm{tr}\int d^3x\nabla_+(-\tilde{f}_{+-}\tilde{w}_- + w_+\tilde{f}_{--}) + \mathbb{St}$$

$$S_g' = S_g + (e^AB)\mathrm{tr}\int d^3x\nabla_+(w_+\tilde{f}_{+-} - \tilde{f}_{++}\tilde{w}_-)$$

$$+(e^{2A}B^2)\mathrm{tr}\int d^3x\nabla_+(w_+f_{++} - f_{++}w_+)$$

$$= S_g + (e^AB)\mathrm{tr}\int d^3x\nabla_+(-\nabla_+(w_+\tilde{w}_-)) = S_g + \mathbb{St}$$

$$S_g = \mathrm{tr}\int d^3x\nabla_+\Big[-f_{+-}w_- + w_+f_{--} - \frac{i}{2}\left\{w_+,\frac{1}{\nabla_{++}}w_+\right\}w_- - \frac{i}{2}\left(\frac{1}{\nabla_{++}}\{w_+,w_+\}\right)w_-$$

$$-\frac{1}{2}(\triangle_-{}^{\alpha}w_+)\left\{w_+,\frac{1}{\nabla_{++}}(\triangle_{-\alpha}\,w_+)\right\}\Big] + \mathbb{St}$$

$$-\frac{i}{2}(\triangle_{--}\,w_+)\left\{w_+,\frac{1}{\nabla_{++}}w_+\right\} + \frac{i}{2}w_+\left\{w_+,\frac{1}{\nabla_{++}}(\triangle_{--}\,w_+)\right\}$$

$$\delta\left(\mathrm{tr}\int d^3x\nabla_+\left[-\frac{i}{2}(\Delta_{--}w_+)\left\{w_+,\frac{1}{\nabla_{++}}w_+\right\} + \frac{i}{2}w_+\left\{w_+,\frac{1}{\nabla_{++}}(\Delta_{--}w_+)\right\}\right]\right)$$

$$= e^AB\mathrm{tr}\int d^3x\nabla_+\left[-\frac{i}{2}w_+\left\{w_+,\frac{1}{\nabla_{++}}w_+\right\} + \frac{i}{2}w_+\left\{w_+,\frac{1}{\nabla_{++}}w_+\right\}\right] = 0$$

$$S_m = \frac{1}{2}\int d^3x\nabla^\alpha\Big[-(\nabla^\beta q_\beta\Phi^\dagger)(q_\alpha\Phi) - (q_\alpha\Phi^\dagger)(\nabla^\beta q_\beta\Phi)$$

$$+2(q_\alpha\Phi^\dagger)\left(W^\beta\frac{n_\beta^\gamma}{\sqrt{2}n\cdot\nabla}\nabla_\gamma\Phi\right) + 2\left(W^\beta\frac{n_\beta^\gamma}{\sqrt{2}n\cdot\nabla}\nabla_\gamma\Phi^\dagger\right)(q_\alpha\Phi)$$

$$+(\Box_{\mathrm{cov}}\,\Phi^\dagger)\left(\frac{n_\alpha^\beta}{\sqrt{2}n\cdot\nabla}\nabla_\beta\Phi\right) + \left(\frac{n_\alpha^\beta}{\sqrt{2}n\cdot\nabla}\nabla_\beta\Phi^\dagger\right)(\Box_{\mathrm{cov}}\,\Phi)$$

$$-i\left(\frac{n_\alpha^\beta}{\sqrt{2}n\cdot\nabla}\nabla_\beta\Phi^\dagger\right)\left(((\sqrt{2}n\cdot\nabla)\Delta^{\gamma\delta}W_\delta)\frac{1}{\sqrt{2}n\cdot\nabla}\nabla_\gamma\Phi\right)$$

$$+\frac{1}{2}\left(\frac{n_\alpha^\beta}{\sqrt{2}n\cdot\nabla}\nabla_\beta\Phi^\dagger\right)\left(\left[n^{\gamma\delta}f_{\gamma\delta},\frac{1}{\sqrt{2}n\cdot\nabla}W_\sigma\right]\frac{n^{\sigma\epsilon}}{\sqrt{2}n\cdot\nabla}\nabla_\epsilon\Phi\right)\Big] + \mathbb{St}$$

$$S_g = \mathrm{tr}\int d^3x\nabla^\alpha\Big[(\triangle^{\gamma\delta}\,F_{\gamma\delta})(\triangle_\alpha{}^{\beta}W_\beta) + (\triangle_\alpha{}^{\gamma}\,\triangle_\beta{}^{\delta}F_{\gamma\delta})W^\beta$$

$$-\frac{i}{2}\left\{W_\gamma,\frac{n^{\gamma\delta}}{\sqrt{2}n\cdot\nabla}W_\delta\right\}(\triangle_\alpha{}^{\beta}W_\beta) - \frac{i}{2}\left(\frac{n^{\gamma\delta}}{\sqrt{2}n\cdot\nabla}\{W_\gamma,W_\delta\}\right)(\triangle_\alpha{}^{\beta}W_\beta)$$

$$-\frac{1}{2}(\triangle^{\gamma\beta}\,W_\gamma)\left\{W_\sigma,\frac{n_\alpha{}^{\sigma}}{\sqrt{2}n\cdot\nabla}(\triangle^{\delta}{}_{\beta}W_\delta)\right\}\Big] + \mathbb{St}$$

$$S_b = \int \, \mathrm{d}^3 x D_+ \mathcal{L}_- = \int \, \mathrm{d}^3 x d_+ (\mathcal{L}_-|_{\theta_+} = 0)$$

$$\mathcal{L}'_-(x',\theta') = e^{-A}\mathcal{L}_-(x,\theta)$$

$$\delta S_b = \int \, \mathrm{d}^3 x D_+(\delta\mathcal{L}_-) = \int \, \mathrm{d}^3 x D_+(-\epsilon^\alpha Q_\alpha \mathcal{L}_-)$$
$$= \epsilon^\alpha \int \, \mathrm{d}^3 x D_+ \left((D_\alpha + 2\theta^\beta \partial_{\beta\alpha})\mathcal{L}_- \right) = \epsilon^- \int \, \mathrm{d}^3 x D_+ D_- \mathcal{L}_-$$

$$S_b = -m^2 \int \, \mathrm{d}^3 x \nabla_+ \left(\phi^\dagger \frac{\nabla_+}{\nabla_{++}} \phi \right) = m^2 \int \, \mathrm{d}^3 x \nabla_\alpha \left(\Phi^\dagger \frac{n^{\alpha\beta}}{\sqrt{2} n \cdot \nabla} \nabla_\beta \Phi \right)$$

$$S_m + S_b = \int \, \mathrm{d}^3 x \Big(-A^\dagger (\Box_{\text{cov}} - m^2) A - \psi^{\dagger\alpha} \Big(\nabla_{\alpha\beta} - m^2 \frac{n_{\alpha\beta}}{\sqrt{2} n \cdot \nabla} \Big) \psi^\beta$$
$$-A^\dagger W^\alpha \psi_\alpha + \psi^{\dagger\alpha} W_\alpha A - F^\dagger F \Big)$$

$$\Phi|_{x^{--}=0} = 0$$

$$\delta\Phi|_{x^{--}=0} = -(\epsilon^\alpha Q_\alpha \Phi)|_{x^{--}=0}$$
$$= -[\epsilon^+(\partial_+ + \theta^+ \partial_{++} + \theta^- \partial_{+-})\Phi + \epsilon^-(\partial_- + \theta^+ \partial_{+-} + \theta^- \partial_{--})\Phi]|_{x^{--}=0}$$
$$= -\epsilon^-(\theta^- \partial_{--}\Phi)|_{x^{--}=0}$$

$$\theta^\alpha = \theta_\beta C^{\beta\alpha}, \theta_\alpha = \theta^\beta C_{\beta\alpha}$$

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, (\gamma^a)^* = \gamma^a$$

$$\theta \sim \theta_\alpha, \bar{\theta} = \theta^\dagger i\gamma^0 = \theta^T C \sim \theta^\alpha, C \sim C^{\alpha\beta}, C^{-1} \sim C_{\alpha\beta}, \gamma^a \sim (\gamma^a)_\alpha{}^\beta.$$

$$\theta^2 = \frac{1}{2}\bar{\theta}\theta = \frac{1}{2}\theta^\alpha\theta_\alpha, \qquad \psi = v_a \gamma^a$$
$$\gamma^a_{\alpha\beta} = (\gamma^a C^{-1})_{\alpha\beta} = (\gamma^a)_\alpha^\gamma C_{\gamma\beta}, \qquad \gamma_a^{\alpha\beta} = -(C\gamma_a)_{\alpha\beta} = (\gamma_a)_\gamma^\beta C^{\gamma\alpha}$$

$$(\theta_\alpha)^* = \theta_\alpha, \qquad (\theta^\alpha)^* = -\theta^\alpha$$
$$C_{\alpha\gamma}C^{\gamma\beta} = \delta_\alpha^\beta, \qquad C_{\alpha\beta} = -C_{\beta\alpha} = -C^*_{\alpha\beta}$$
$$\partial_\alpha \theta^\beta = \delta_\alpha^\beta, \qquad \theta_\alpha \theta_\beta = -C_{\alpha\beta}\theta^2$$
$$\gamma^a_{\alpha\beta} = \gamma^a_{\beta\alpha} = -(\gamma^a_{\alpha\beta})^*, \qquad \gamma_a^{\alpha\beta} = \gamma_a^{\beta\alpha} = -\left(\gamma_a^{\alpha\beta}\right)^*$$
$$\gamma^a_{\alpha\beta}\gamma_b^{\alpha\beta} = -2\delta^a_b, \qquad \gamma^a_{\alpha\beta}\gamma_a^{\gamma\delta} = -\delta^\gamma_{(\alpha}\delta^\delta_{\beta)}$$

$$C^{\alpha\beta} = \sigma_2 = C_{\alpha\beta}, (\gamma^a)_\alpha^\beta = (i\sigma_2, \sigma_1, -\sigma_3), \gamma_a^{\alpha\beta} = \gamma^a_{\alpha\beta} = (i\mathbf{1}, i\sigma_3, i\sigma_1)$$

$$x^{\alpha\beta} = \frac{1}{2}\gamma_a^{\alpha\beta} x^a, \partial_{\alpha\beta} = \gamma^a_{\alpha\beta}\partial_a, n^{\alpha\beta} = \frac{1}{\sqrt{2}}\gamma_a^{\alpha\beta} n^a$$

$$x^a = -\gamma^a_{\alpha\beta} x^{\alpha\beta}, \partial_a = -\frac{1}{2}\gamma_a^{\alpha\beta}\partial_{\alpha\beta}, n^a = -\frac{1}{\sqrt{2}}\gamma^a_{\alpha\beta} n^{\alpha\beta}$$

$$\partial_{\alpha\beta}x^{\gamma\delta} = -\frac{1}{2}\delta^{\gamma}_{(\alpha}\delta^{\delta}_{\beta)}, \qquad \partial^{\alpha\gamma}\partial_{\beta\gamma} = -\delta^{\alpha}_{\beta}\,\Box$$
$$D_{\alpha}\theta^{\beta} = \delta^{\beta}_{\alpha}, \qquad D^2\theta^2 = -1$$

$$\int d^3x\nabla_+\left((\triangle_{\alpha\beta} f)g\right) = \int d^3x\nabla_+\left(f(\triangle_{\alpha\beta}\, g)\right) + \mathbb{St}$$

$$\int d^4xD^2(\Phi^2) = \int d^4xD^2\bar{D}^2\left(\Phi\frac{D^2}{\Box}\Phi\right)$$

$$\tilde{\mathcal{D}}_0 = \mathcal{D}_0 + A + JAJ^{-1}, A = \sum a_i[\mathcal{D}, b_i], a_i, b_i \in \mathcal{A}.$$

$$\langle\psi\tilde{\mathcal{D}}_0\psi\rangle + \mathrm{Tr}(f(P)).$$

$$\mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_F.$$

$$\mathcal{H}_M = \mathcal{H}_+ \oplus \mathcal{H}_- = \{(\Psi_+, 0)^T\} + \{(0, \Psi_-)^T\}$$

$$\Psi_+(x_+) = \varphi_+(x_+) + \sqrt{2}\theta^{\alpha}\psi_{+\alpha}(x_+) + \theta\theta F_+(x_+)$$

$$\Psi_-(x_-) = \varphi_-(x_-)^* + \sqrt{2}\bar{\theta}_{\dot{\alpha}}\bar{\psi}_-^{*\dot{\alpha}}(x_-) + \overline{\theta\theta}F_-^*(x_-)$$

$$\gamma_M = \begin{cases} -i\mathbb{I}_+ & \text{in } \mathcal{H}_+ \\ i\mathbb{I}_- & \text{in } \mathcal{H}_- \end{cases}$$

$$Q^a = (q_L^a, q_R^a)^T$$

$$Q_c^a = ((q_c^a)_L, (q_c^a)_R)^T$$

$$\gamma_F(q_L^a) = -1, \gamma_F(q_R^a) = 1$$

$$\gamma = \gamma_M\gamma_F = \mathrm{i}$$

$$\Phi_L = q_L^a \otimes (\Psi_+(x_+), 0)^T = q_L^a \otimes \left(\varphi_+ + \sqrt{2}\theta^{\alpha}\psi_{+\alpha} + \theta\theta F_+, 0\right)^T$$

$$\varphi_L = q_L^a \otimes (\varphi_+, 0)^T, \psi_{L\alpha} = q_L^a \otimes (\psi_{+\alpha}, 0)^T, F_L = q_L^a \otimes (F_+, 0)^T.$$

$$\Phi_R = q_R^a \otimes \left(0, \Psi_-(x_-)\right)^T = q_R^a \otimes \left(0, \varphi_-^* + \sqrt{2}\bar{\theta}_{\dot{\alpha}}\bar{\psi}_-^{\dot{\alpha}} + \overline{\theta\theta}F_-^*\right)^T$$

$$\varphi_R = q_R^a \otimes (0, \varphi_-^*)^T, \psi_R^{\dot{\alpha}} = q_R^a \otimes \left(0, \bar{\psi}_-^{\dot{\alpha}}\right)^T, F_R = q_R^a \otimes (0, F_-^*)^T.$$

$$\mathcal{J}_M\Psi_+(x_+) = \Psi_+^{\dagger}(x_-) = \varphi_+^* + \sqrt{2}\bar{\theta}\bar{\psi} + \overline{\theta\theta}F_+^*,$$
$$\mathcal{J}_M\Psi_-(x_-) = \Psi_-^{\dagger}(x_+) = \varphi_- + \sqrt{2}\theta\psi_- + \theta\theta F_-,$$

$$\mathcal{J}_F q_L^a = (q_L^a)^c = (q_a^c)_R, \mathcal{J}_F q_R^a = (q_R^a)^c = (q_a^c)_L$$

$$J\Phi_L = \varphi_L^* + \sqrt{2}\bar{\theta}\bar{\psi}_L + \overline{\theta\theta}F_L^* = (\varphi^c)_R + \sqrt{2}\bar{\theta}(\psi^c)_R + \overline{\theta\theta}(F^c)_R = (\Phi^c)_R$$
$$J\Phi_R = \varphi_R^* + \sqrt{2}\theta\bar{\psi}_R + \theta\theta F_R^* = (\varphi^c)_L + \sqrt{2}\theta(\psi^c)_L + \theta\theta(F^c)_L = (\Phi^c)_L$$

$$\mathcal{A} = \mathcal{A}_M \otimes \mathcal{A}_F,$$
$$\mathcal{A}_M = \mathcal{A}_+ \oplus \mathcal{A}_-$$

$$u_a(x_+) = \frac{1}{m_0}\left(\varphi_a + \sqrt{2}\theta\psi_a + \theta\theta F_a\right)$$
$$\bar{u}_a(x_-) = \frac{1}{m_0}\left(\varphi_a^* + \sqrt{2}\bar{\theta}\bar{\psi}_a + \overline{\theta\theta} F_a^*\right)$$

$$\mathrm{i}\mathcal{D}_{tot} = \mathrm{i}\mathcal{D}_M + \gamma_M \otimes \mathcal{D}_F$$

$$\mathcal{D}(x_-) = -\frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta}\frac{\partial}{\partial\theta^\alpha} = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta},$$

$$\overline{\mathcal{D}}(x_+) = -\frac{1}{4}\varepsilon^{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}$$

$$\Psi_+(x_-) = \varphi_+ + \sqrt{2}\theta\psi_+ + \theta\theta F_+ + 2\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi_+ + \theta\theta\overline{\theta\theta}\,\square\,\varphi_+ - \sqrt{2}\mathrm{i}\theta\theta\partial_\mu\psi_+\sigma^\mu\bar{\theta}$$
$$\Psi_-(x_+) = \varphi_-^* + \sqrt{2}\bar{\theta}\bar{\psi}_- + \overline{\theta\theta}F_-^* - 2\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi_-^* + \theta\theta\overline{\theta\theta}\,\square\,\varphi_-^* + \sqrt{2}\mathrm{i}\overline{\theta\theta}\theta\sigma^\mu\partial_\mu\bar{\psi}_-$$

$$\Phi_L(x_-) = q_L^a \otimes (\Psi_+(x_-),0)^T, \Phi_R(x_+) = q_R^a \otimes \left(0,\Psi_-(x_+)\right)^T.$$

$$\mathcal{D}\Psi_+(x_-) = F_+ + \sqrt{2}\bar{\theta}\mathrm{i}\bar{\sigma}^\mu\partial_\mu\psi_+ + \overline{\theta\theta}\,\square\,\varphi_+, \mathcal{D}\Phi_L = q_L^a \otimes \left(0,\mathcal{D}\Psi_+(x_-)\right)^T,$$
$$\overline{\mathcal{D}}\Psi_-(x_+) = F_-^* + \sqrt{2}\theta\mathrm{i}\sigma^\mu\partial_\mu\bar{\psi}_- + \theta\theta\,\square\,\varphi_-^*, \overline{\mathcal{D}}\Phi_R = q_R^a \otimes \left(\overline{\mathcal{D}}\Psi_-(x_+),0\right)^T,$$

$$\mathcal{D}\mathcal{H}_+ \subset \mathcal{H}_-, \overline{\mathcal{D}}\mathcal{H}_- \subset \mathcal{H}_+.$$

$$\mathrm{i}\mathcal{D}_M = \begin{pmatrix} 0 & \overline{\mathcal{D}} \\ \mathcal{D} & 0 \end{pmatrix}$$

$$\mathcal{D}_F = \begin{pmatrix} m & 0 \\ 0 & m^\dagger \end{pmatrix}$$

$$(\Psi_-,\Psi_+')_s = \int_M d^4x d^2\theta\delta(\bar{\theta})\Psi_-^\dagger\Psi_+'$$

$$(\Psi_+,\Psi_-')_s = \int_M d^4x d^2\bar{\theta}\delta(\theta)\Psi_+^\dagger\Psi_-'$$

$$(\Phi_L,\mathcal{D}\Phi_L)_s = \int_M d^4x d^2\bar{\theta}\delta(\theta)q_L^a \otimes \left(0,\Psi_+^\dagger\right)\mathcal{D}\Phi_L = \int d^4x\left(\varphi_L^*\,\square\,\varphi_L - \mathrm{i}\bar{\psi}_L\bar{\sigma}^\mu\partial_\mu\psi + F_L^*F_L\right)$$
$$\left(\Phi_R,\overline{\mathcal{D}}\Phi_R\right)_s = \int_M d^4x d^2\theta\delta(\bar{\theta})q_R^a \otimes \left(\Psi_-^\dagger,0\right)\overline{\mathcal{D}}\Phi_R = \int d^4x\left(\varphi_R^*\,\square\,\varphi_R - \mathrm{i}\bar{\psi}_R\sigma^\mu\partial_\mu\psi_R + F_R^*F_R\right)$$

$$\mathrm{i}\mathcal{D}_{tot} \to \mathrm{i}\tilde{\mathcal{D}}_{tot} = \mathrm{i}\mathcal{D}_{tot} + V + JVJ^{-1}, V = \sum_a U_a'[\mathrm{i}\mathcal{D}_{tot},U_a], U_a \in A$$

$$\Pi_+ = \{u_a : a = 1,2,\cdots n\} \subset \mathcal{A}_+ \otimes \mathcal{A}_F,$$
$$\Pi_- = \{\bar{u}_a : a = 1,2,\cdots n\} \subset \mathcal{A}_- \otimes \mathcal{A}_F,$$

$$m_0^2 C = \sum_a c_a \varphi_a^* \varphi_a$$

$$m_0^2 \chi_\alpha = -\mathrm{i}\sqrt{2} \sum_a c_a \varphi_a^* \psi_{a\alpha}$$

$$m_0^2 (M + \mathrm{i}N) = -2\mathrm{i} \sum_a c_a \varphi_a^* F_a$$

$$m_0^2 A_\mu = -\mathrm{i} \sum_a c_a \left[\left(\varphi_a^* \partial_\mu \varphi_a - \partial_\mu \varphi_a^* \varphi_a \right) - \mathrm{i} \bar{\psi}_{a\dot{\alpha}} \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \psi_{a\alpha} \right]$$

$$m_0^2 \lambda_\alpha = \sqrt{2}\mathrm{i} \sum_a c_a \left(F_a^* \psi_{a\alpha} - \mathrm{i} \sigma_{\alpha\dot{\alpha}}^\mu \bar{\psi}_a^{\dot{\alpha}} \partial_\mu \varphi_a \right)$$

$$m_0^2 D = \sum_a c_a \left[2F_a^* F_a - 2\left(\partial^\mu \varphi_a^* \partial_\mu \varphi_a \right) \right.$$
$$\left. + \mathrm{i} \left\{ \partial_\mu \bar{\psi}_{a\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \psi_{a\alpha} - \bar{\psi}_{a\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial_\mu \psi_{a\alpha} \right\} \right]$$

$$\sum_a c_a \varphi_a^* \varphi_a = 0$$

$$\sum_a c_a \varphi_a^* \psi_a^\alpha = 0$$

$$\sum_a c_a \varphi_a^* F_a = 0$$

$$A_\mu(x) = \sum_{l=0}^{N^2-1} A_\mu^l(x) \frac{T_l}{2}$$

$$D(x) = \sum_{l=0}^{N^2-1} D^l(x) \frac{T_l}{2}$$

$$\lambda_\alpha(x) = \sum_{l=0}^{N^2-1} \lambda_\alpha^l(x) \frac{T_l}{2}$$

$$\mathrm{Tr}(T_a T_b) = 2\delta_{ab}$$

$$\tilde{\mathcal{D}}_M = -\mathrm{i} \begin{pmatrix} 0 & \tilde{\tilde{\mathcal{D}}} \\ \tilde{\mathcal{D}} & 0 \end{pmatrix}$$

$$U_a = \sqrt{-2c_a} \begin{pmatrix} u_a & 0 \\ 0 & 0 \end{pmatrix}, U_a' = \sqrt{-2c_a} \begin{pmatrix} 0 & 0 \\ 0 & \bar{u}_a \end{pmatrix}$$

$$V_{\mathcal{D}} = -2 \sum_a c_a \bar{u}_a [\mathrm{i}\mathcal{D}_M, u_a] = -2 \sum_a c_a \bar{u}_a \mathcal{D} u_a$$

$$U_a = \sqrt{2c_a} \begin{pmatrix} 0 & 0 \\ 0 & \bar{u}_a \end{pmatrix}, U_a' = \sqrt{2c_a} \begin{pmatrix} u_a & 0 \\ 0 & 0 \end{pmatrix}$$

$$V_{\overline{\mathcal{D}}} = 2 \sum_a c_a u_a [\mathrm{i}\mathcal{D}_M, \bar{u}_a] = 2 \sum_a c_a u_a \overline{\mathcal{D}} \bar{u}_a.$$

$$-\frac{V_{\mathcal{D}}}{2} = \sum_a c_a \bar{u}_a \mathcal{D} u_a = \overline{\theta\theta}\frac{1}{2}\left(D + \mathrm{i}\left(\partial^\mu A_\mu\right)\right) + \mathrm{i}\overline{\lambda\theta}$$

$$-\frac{1}{2}\left(D + \mathrm{i}\left(\partial^\mu A_\mu\right)\right)\overline{\theta\theta}\theta^\alpha \frac{\partial}{\partial\theta^\alpha} + \frac{\mathrm{i}}{2}\lambda^\alpha \overline{\theta\theta}\frac{\partial}{\partial\theta^\alpha} - \mathrm{i}\overline{\lambda\theta}\theta^\alpha \frac{\partial}{\partial\theta^\alpha} - \frac{1}{2}A^\mu \bar{\theta}_{\dot{\alpha}}\sigma_\mu^{\dot{\alpha}\alpha}\frac{\partial}{\partial\theta^\alpha}$$

$$+\left(\frac{1}{2}\left(D + \mathrm{i}\left(\partial^\mu A_\mu\right)\right)\theta\theta\overline{\theta\theta} - A_\mu \theta\sigma^\mu\bar{\theta} + \mathrm{i}\bar{\lambda}\bar{\theta}\theta\theta - \mathrm{i}\overline{\theta\theta}\theta\lambda\right)\mathcal{D}$$

$$\frac{V_{\overline{\mathcal{D}}}}{2} = \sum_a c_a u_a \overline{\mathcal{D}}\bar{u}_a = \theta\theta\frac{1}{2}\left(\overline{\mathcal{D}} - \mathrm{i}\left(\partial^\mu A_\mu\right)\right) - \mathrm{i}\theta\lambda$$

$$-\frac{1}{2}\left(\overline{\mathcal{D}} - \mathrm{i}\left(\partial^\mu A_\mu\right)\right)\theta\theta\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + \frac{\mathrm{i}}{2}\bar{\lambda}_{\dot{\alpha}}\theta\theta\varepsilon^{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}} + \mathrm{i}\lambda\theta\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - \frac{1}{2}A^\mu \bar{\sigma}_\mu^{\dot{\alpha}\alpha}\theta_\alpha \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$$

$$+\left(\frac{1}{2}\left(D - i\left(\partial_\mu A^\mu\right)\right)\theta\theta\overline{\theta\theta} - \mathrm{i}\theta\lambda\overline{\theta\theta} + \mathrm{i}\overline{\lambda\theta}\theta\theta - \theta\sigma^\mu\bar{\theta}A_\mu\right)\overline{\mathcal{D}}$$

$$\sum_{a,b} c_a c_b \bar{u}_{ab}\mathcal{D}u_{ab} = -\frac{1}{2}A^\mu A_\mu \overline{\theta\theta}\left(1 - \theta^\alpha \frac{\partial}{\partial\theta^\alpha} + \theta\theta\right)\mathcal{D}$$

$$\sum_{a,b} c_a c_b u_{ab}\overline{\mathcal{D}}\bar{u}_{ab} = -\frac{1}{2}A^\mu A_\mu \theta\theta\left(1 - \bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + \overline{\theta\theta}\right)\overline{\mathcal{D}}$$

$$\tilde{\mathcal{D}}(x_-) = \mathcal{D} - 2\bar{u}_a \mathcal{D}u_a + 2\bar{u}_{ab}\mathcal{D}u_{ab}$$

$$= \mathcal{D} - \left(D + \mathrm{i}\left(\partial_\mu A^\mu\right) + A^\mu A_\mu\right)\overline{\theta\theta} - 2\mathrm{i}\overline{\lambda\theta} + \left(D + \mathrm{i}\left(\partial_\mu A^\mu\right) + A^\mu A_\mu\right)\overline{\theta\theta}\theta^\alpha \frac{\partial}{\partial\theta^\alpha}$$

$$-\mathrm{i}\lambda^\alpha \overline{\theta\theta}\frac{\partial}{\partial\theta^\alpha} + 2\mathrm{i}\overline{\lambda\theta}\theta^\alpha \frac{\partial}{\partial\theta^\alpha} + A^\mu \bar{\theta}_{\dot{\alpha}}\bar{\sigma}_\mu^{\dot{\alpha}\alpha}\frac{\partial}{\partial\theta^\alpha}$$

$$-\left(D + \mathrm{i}\left(\partial_\mu A^\mu\right) + A^\mu A_\mu\right)\theta\theta\overline{\theta\theta}\mathcal{D} + 2A_\mu \theta\sigma^\mu\bar{\theta}\mathcal{D} - 2\mathrm{i}\overline{\lambda\theta}\theta\theta\mathcal{D} + 2\mathrm{i}\lambda\theta\overline{\theta\theta}\mathcal{D}$$

$$\tilde{\overline{\mathcal{D}}}(x_+) = \overline{\mathcal{D}} + 2u_a \overline{\mathcal{D}}\bar{u}_a + 2u_{ab}\overline{\mathcal{D}}\bar{u}_{ab}$$

$$= \overline{\mathcal{D}} + \left(D - \mathrm{i}\left(\partial^\mu A_\mu\right) - A^\mu A_\mu\right)\theta\theta - 2\mathrm{i}\theta\lambda - \left(D - \mathrm{i}\left(\partial^\mu A_\mu\right) - A^\mu A_\mu\right)\theta\theta\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$$

$$+\mathrm{i}\bar{\lambda}_{\dot{\alpha}}\theta\theta\varepsilon^{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}} + 2\mathrm{i}\lambda\theta\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - A_\mu \bar{\sigma}^{\dot{\alpha}\alpha}\theta_\alpha \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$$

$$+\left(D - \mathrm{i}\left(\partial^\mu A_\mu\right) - A^\mu A_\mu\right)\theta\theta\overline{\theta\theta}\overline{\mathcal{D}} - 2A_\mu \theta\sigma^\mu\bar{\theta}\overline{\mathcal{D}} - 2\mathrm{i}\theta\lambda\overline{\theta\theta}\overline{\mathcal{D}} + 2\mathrm{i}\overline{\lambda\theta}\theta\theta\overline{\mathcal{D}}$$

$$\mathrm{i}\tilde{\mathcal{D}}_{tot} = \mathrm{i}\tilde{\mathcal{D}}_M \otimes \mathbf{1_F} + \gamma_{\mathbf{M}} \otimes \mathcal{D}_{\mathbf{F}}$$

$$I_{\mathrm{matter}} = \left(\Phi_L + \Phi_R, \mathrm{i}\mathcal{D}_{tot}(\Phi_L + \Phi_R)\right)_s$$

$$= \left(\Phi_L + \Phi_R, \mathrm{i}\mathcal{D}_M(\Phi_L + \Phi_R)\right)_s + \left(\Phi_L + \Phi_R, \gamma_M \otimes \mathcal{D}_F(\Phi_L + \Phi_R)\right)_s$$

$$= \left(\Phi_L, \tilde{\mathcal{D}}\Phi_L\right)_s + \left(\Phi_R, \tilde{\overline{\mathcal{D}}}\Phi_R\right)_s + \left(\Phi_L, im^\dagger \Phi_R\right)_s - \left(\Phi_R, im\Phi_L\right)_s$$

$$I_L = \left(\Phi_L, \tilde{\mathcal{D}}\Phi_L\right)_s$$

$$= \int_M d^4x \left(\varphi_L^*\left(D^\mu D_\mu - D\right)\varphi_L - \mathrm{i}\bar{\psi}_L \bar{\sigma}^\mu D_\mu \psi_L + F_L^* F_L - \sqrt{2}\mathrm{i}\left(\varphi_l^* \lambda\psi_L - \bar{\psi}\bar{\lambda}\psi_L\right)\right)$$

$$I_R = \left(\Phi_R, \tilde{\tilde{\mathcal{D}}}\Phi_R\right)_s$$
$$= \int_M d^4x \left(\varphi_R^*(D^\mu D_\mu + D)\varphi_R - \mathrm{i}\bar{\psi}_R\sigma^\mu D_\mu\psi_R + F_R^*F_R - \sqrt{2}\mathrm{i}(\varphi_R^*\bar{\lambda}\psi_R - \bar{\psi}_R\lambda\varphi_R)\right)$$

$$I_{\text{mass}} = (\Phi_R, m\Phi_L) + \text{ h.c.}$$
$$= \int_M \mathrm{d}^4x[\varphi_R^* mF_L + F_R^* m\varphi_L - \bar{\psi}_R^\alpha m\psi_{L\alpha} + \text{ h.c.}]$$

$$\mathrm{Tr}_{L^2} f(P) \simeq \sum_{n\geq 0} c_n a_n(P)$$

$$P = (i\mathcal{D}_{tot})^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu + \mathbb{A}^\mu\partial_\mu + \mathbb{B}$$

$$a_0(P) = \frac{1}{16\pi^2}\int_M \mathrm{d}^4x\mathrm{Str}(\mathbb{I})$$
$$a_2(P) = \frac{1}{16\pi^2}\int_M \mathrm{d}^4x\mathrm{Str}(\mathbb{E})$$
$$a_4(P) = \frac{1}{32\pi^2}\int_M \mathrm{d}^4x\mathrm{Str}\left(\mathbb{E}^2 + \frac{1}{3}\mathbb{E}^\mu_{;\mu} + \frac{1}{6}\Omega^{\mu\nu}\Omega_{\mu\nu}\right)$$

$$\mathbb{E} = \mathbb{B} - \left(\partial_\mu\omega^\mu + \omega_\mu\omega^\mu\right)$$
$$\Omega^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu + [\omega^\mu, \omega^\nu]$$
$$\omega^\mu = \frac{1}{2}\mathbb{A}^\mu$$

$$f_+(x_+) = f_0 + \sqrt{2}\theta^\alpha f_{1\alpha} + \theta\theta f_2, f_-(x_-) = f_0^* + \sqrt{2}\bar{\theta}_{\dot{\alpha}}\bar{f}_1^{\dot{\alpha}} + \overline{\theta\theta} f_2^*$$

$$\mathcal{D}_+ = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}, \hat{f}_0 = \mathcal{D}_+\theta\theta, \hat{f}_{1\alpha} = -\sqrt{2}\mathcal{D}_+\theta_\alpha, \hat{f}_2 = \mathcal{D}_+$$

$$\mathcal{D}_- = -\frac{1}{4}\varepsilon^{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}, \hat{f}_0^* = \mathcal{D}_-\overline{\theta\theta}, \hat{f}_1^{\dot{\alpha}} = -\sqrt{2}\mathcal{D}_-\bar{\theta}^{\dot{\alpha}}, \hat{f}_2^* = \mathcal{D}_-$$

$$\hat{f}_0 f_+ = f_0, \hat{f}_{1\alpha} f_+ = f_{1\alpha}, \hat{f}_2 f_+ = f_2, \hat{f}_0^* f_- = f_0^*, \hat{f}_1^{\dot{\alpha}} f_- = \bar{f}_1^{\dot{\alpha}}, \hat{f}_2^* f_- = f_2^*$$

$$\mathbb{I}_+ = \hat{f}_0 + \sqrt{2}\theta^\alpha\hat{f}_{1\alpha} + \theta\theta\hat{f}_2, \mathbb{I}_- = \hat{f}_0^* + \sqrt{2}\bar{\theta}_{\dot{\alpha}}\hat{f}_1^{\dot{\alpha}} + \overline{\theta\theta}\hat{f}_2^*$$

$$\mathrm{Str}_+ = \hat{f}_0\frac{\delta}{\delta\hat{f}_0} + \hat{f}_{1\alpha}\frac{\delta}{\delta\hat{f}_{1\alpha}} + \hat{f}_2\frac{\delta}{\delta\hat{f}_2}$$

$$\mathrm{Str}_- = \hat{f}_0^*\frac{\delta}{\delta\hat{f}_0^*} + \hat{\bar{f}}_{1\alpha}\frac{\delta}{\delta\hat{\bar{f}}_{1\alpha}} + \hat{f}_2^*\frac{\delta}{\delta\hat{f}_2^*}$$

$$\begin{aligned}\hat{\mathcal{A}} =& \mathcal{A}_0\hat{f}_0 + (\cdots)^\alpha\hat{f}_{1\alpha} + (\cdots)\hat{f}_2 \\ &+\sqrt{2}\theta^\alpha(\cdots)_\alpha\hat{f}_0 + \sqrt{2}\theta^\beta\mathcal{A}_{1\beta}^\alpha\hat{f}_{1\alpha} + \sqrt{2}\theta^\alpha(\cdots)_\alpha\hat{f}_2 \\ &+\theta\theta(\cdots)\hat{f}_0 + \theta\theta(\cdots)^\alpha\hat{f}_{1\alpha} + \theta\theta\mathcal{A}_2\hat{f}_2\end{aligned}$$

$$\begin{aligned}\mathrm{Str}_+(\hat{\mathcal{A}}) =&\hat{f}_0\big(\mathcal{A}_0+\sqrt{2}\theta^\alpha(\cdots)_\alpha+\theta\theta(\cdots)\big)\\ &+\hat{f}_{1\alpha}\big(-(\cdots)^\alpha-\sqrt{2}\theta^\beta\mathcal{A}_{1\beta}^\alpha-\theta\theta(\cdots)^\alpha\big)\\ &+\hat{f}_2\big((\cdots)+\sqrt{2}\theta^\alpha(\cdots)_\alpha+\theta\theta\mathcal{A}_2\big)\\ =&\mathcal{A}_0-\mathcal{A}_{1\alpha}^\alpha+\mathcal{A}_2\end{aligned}$$

$$\begin{aligned}\hat{\bar{\mathcal{A}}} =&\mathcal{A}_0^*\hat{f}_0^*+(\cdots)^{\dot\alpha}\hat{\bar{f}}_{1\dot\alpha}+(\cdots)\hat{f}_2^*\\ &+\sqrt{2}\bar\theta_{\dot\alpha}(\cdots)^{\dot\alpha}\hat{f}_0^*+\sqrt{2}\bar\theta_{\dot\alpha}\mathcal{A}_1^{*\dot\alpha}{}_{\dot\beta}\hat{\bar{f}}_1^{\dot\beta}+\sqrt{2}\bar\theta_{\dot\alpha}(\cdots)^{\dot\alpha}\hat{f}_2\\ &+\overline{\theta\theta}(\cdots)\hat{f}_0^*+\overline{\theta\theta}(\cdots)_{\dot\alpha}\hat{f}_1^{\dot\alpha}+\theta\theta\mathcal{A}_2^*\hat{f}_2^*\end{aligned}$$

$$\mathrm{Str}_-(\hat{\bar{\mathcal{A}}})=\mathcal{A}_0^*-\mathcal{A}_1^{*\dot\alpha}{}_{\dot\alpha}+\mathcal{A}_2^*$$

$$\mathrm{Str}(\gamma_M^2)=-\mathrm{Str}(\mathbb{I}_+)-\mathrm{Str}(\mathbb{I}_-)=0$$

$$P=\begin{pmatrix}P_+&0\\0&P_-\end{pmatrix}=\big(\mathrm{i}\tilde{\mathcal{D}}_M\big)^2=\begin{pmatrix}\tilde{\mathcal{D}}\tilde{\mathcal{D}}&0\\0&\tilde{\mathcal{D}}\tilde{\mathcal{D}}\end{pmatrix}$$

$$\begin{aligned}P_+ =&\mathcal{D}^\mu\mathcal{D}_\mu\mathbb{I}_+-D\hat{f}_0-\mathrm{i}\sqrt{2}\lambda^\alpha\hat{f}_{1\alpha}\\ &+\sqrt{2}\theta^\alpha\left(\big(\sqrt{2}\sigma^\mu_{\alpha\dot\alpha}(\mathcal{D}_\mu\bar\lambda^{\dot\alpha})+\bar\lambda^{\dot\alpha}\mathcal{D}^\mu\big)\hat{f}_0+\mathrm{i}\sigma_\alpha^{\mu\nu}{}_\alpha F_{\mu\nu}\hat{f}_{1\beta}-\mathrm{i}\sqrt{2}\lambda_\alpha f_2\right)\\ &+\theta\theta\big(-2\bar\lambda_{\dot\alpha}\bar\lambda^\alpha\hat{f}_0+\sqrt{2}\bar\lambda_{\dot\alpha}\bar\sigma^{\mu\dot\alpha\beta}\mathcal{D}_\mu\hat{f}_{1\beta}+D\hat{f}_2\big),\end{aligned}$$

$$\begin{aligned}P_- =&\mathcal{D}^\mu\mathcal{D}_\mu\mathbb{I}_-+D\hat{f}_0^*-\mathrm{i}\sqrt{2}\bar\lambda_{\dot\alpha}\hat{f}_1^{\dot\alpha}\\ &+\sqrt{2}\bar\theta_{\dot\alpha}\left(\sqrt{2}\bar\sigma^{\mu\dot\alpha\alpha}\left((\mathcal{D}_\mu\lambda_\alpha)+\lambda_\alpha\mathcal{D}_\mu\right)\hat{f}_0^*+\mathrm{i}\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}F_{\mu\nu}\hat{f}_1^{\dot\beta}-\mathrm{i}\sqrt{2}\bar\lambda^{\dot\alpha}\hat{f}_2^*\right)\\ &+\overline{\theta\theta}\big(-2\lambda\lambda\hat{f}_0^*+\sqrt{2}\lambda^\alpha\sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu\hat{f}_1^{\dot\alpha}-D\hat{f}_2^*\big)\end{aligned}$$

$$P_\pm=\mathbb{I}_\pm\partial^\mu\partial_\mu+\mathbb{A}_\pm^\mu\partial_\mu+\mathbb{B}_\pm$$

$$\begin{aligned}\mathbb{E}_+(x_+) =&\mathbb{B}_+-\big(\partial_\mu\omega_+^\mu\big)-\omega_\mu^+\omega_+^\mu\\ =&-D\hat{f}_0-\mathrm{i}\sqrt{2}\lambda^\alpha\hat{f}_{1\alpha}+\sqrt{2}\theta^\alpha\left(\frac{1}{\sqrt{2}}\sigma^\mu_{\alpha\dot\alpha}(\mathcal{D}_\mu\bar\lambda^{\dot\alpha})\hat{f}_0+\mathrm{i}\sigma^{\mu\nu}{}_\alpha{}^\beta F_{\mu\nu}\hat{f}_{1\beta}-\mathrm{i}\sqrt{2}\lambda_\alpha\hat{f}_2\right)\\ &+\theta\theta\left(-\frac{1}{\sqrt{2}}(\mathcal{D}_\mu\bar\lambda_{\dot\alpha})\bar\sigma^{\mu\dot\alpha\alpha}\hat{f}_{1\alpha}+D\hat{f}_2\right)\end{aligned}$$

$$\begin{aligned}\mathbb{E}_-(x_-) =&\mathbb{B}_--\big(\partial_\mu\omega_-^\mu\big)-\omega_\mu^-\omega_-^\mu\\ =&D\hat{f}_0^*-\mathrm{i}\sqrt{2}\bar\lambda_{\dot\alpha}\hat{\bar{f}}_1^{\dot\alpha}+\sqrt{2}\bar\theta_{\dot\alpha}\left(\frac{1}{\sqrt{2}}\bar\sigma^{\mu\dot\alpha\alpha}(\mathcal{D}_\mu\lambda_\alpha)\hat{f}_0^*+\mathrm{i}\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}F_{\mu\nu}\hat{\bar{f}}_1^{\dot\beta}-\mathrm{i}\sqrt{2}\bar\lambda^{\dot\alpha}\hat{f}_2^*\right)\\ &+\overline{\theta\theta}\left(-\frac{1}{\sqrt{2}}(\mathcal{D}_\mu\lambda^\alpha)\sigma^\mu_{\alpha\dot\alpha}\hat{\bar{f}}_1^{\dot\alpha}-D\hat{f}_2^*\right)\end{aligned}$$

$$\begin{aligned}\Omega_+^{\mu\nu}(x_+) =&-\mathrm{i}F^{\mu\nu}\mathbb{I}_++\sqrt{2}\theta^\alpha\frac{1}{\sqrt{2}}\left(\sigma^\nu_{\alpha\dot\alpha}(\mathcal{D}^\mu\bar\lambda^{\dot\alpha})-\sigma^\mu_{\alpha\dot\alpha}(\mathcal{D}^\nu\bar\lambda^{\dot\alpha})\right)\hat{f}_0\\ &+\theta\theta\left((\mathcal{D}^\mu\bar\lambda_{\dot\alpha})\bar\sigma^{\nu\dot\alpha\alpha}-(\mathcal{D}^\nu\bar\lambda_{\dot\alpha})\bar\sigma^{\mu\dot\alpha\alpha}\right)\hat{f}_{1\alpha}\\ \Omega_-^{\mu\nu}(x_-) =&-\mathrm{i}F^{\mu\nu}\mathbb{I}_-+\sqrt{2}\bar\theta_{\dot\alpha}\frac{1}{\sqrt{2}}\left(\bar\sigma^{\nu\dot\alpha\alpha}(\mathcal{D}^\mu\lambda_\alpha)-\bar\sigma^{\mu\dot\alpha\alpha}(\mathcal{D}^\nu\lambda_\alpha)\right)\hat{f}_0^*\\ &+\theta\theta\left((\mathcal{D}^\mu\lambda^\alpha)\sigma^\nu_{\alpha\dot\alpha}-(\mathcal{D}^\nu\lambda^\alpha)\sigma^\mu_{\alpha\dot\alpha}\right)\hat{\bar{f}}_1^{\dot\alpha}\end{aligned}$$

$$\mathbb{E}_{+}^{2}(x_{+}) = \left(D^2 - \mathrm{i}\lambda^{\alpha}\sigma_{\alpha\dot{\alpha}}^{\mu}(\mathcal{D}_{\mu}\bar{\lambda}^{\dot{\alpha}})\right)\hat{f}_0 + (\cdots)^{\beta}\hat{f}_{1\beta} + (\cdots)\hat{f}_2 + \sqrt{2}\theta^{\alpha}(\cdots)_{\alpha}\hat{f}_0$$
$$+\sqrt{2}\theta^{\alpha}\left(-\mathrm{i}\sigma_{\alpha\dot{\alpha}}^{\mu}(\mathcal{D}_{\mu}\bar{\lambda}^{\dot{\alpha}})\lambda^{\beta} - \sigma_{\alpha}^{\mu\nu\ \gamma}\sigma_{\gamma}^{\lambda\kappa\ \beta}F_{\mu\nu}F_{\lambda\kappa} + \mathrm{i}\lambda_{\alpha}(\mathcal{D}_{\mu}\bar{\lambda}_{\dot{\alpha}})\bar{\sigma}^{\mu\dot{\alpha}\beta}\right)\hat{f}_{1\beta} + \sqrt{2}\theta^{\alpha}(\cdots)_{\alpha}\hat{f}_2$$
$$+\theta\theta(\cdots)\hat{f}_0 + \theta\theta(\cdots)^{\alpha}\hat{f}_{1\alpha} + \theta\theta\left(\mathrm{i}(\mathcal{D}_{\mu}\bar{\lambda}_{\dot{\alpha}})\bar{\sigma}^{\mu\dot{\alpha}\alpha}\lambda_{\alpha} + D^2\right)\hat{f}_2$$

$$\mathbb{E}_{-}^{2}(x_{-}) = \left(D^2 - \mathrm{i}\bar{\lambda}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}(\mathcal{D}_{\mu}\lambda_{\alpha})\right)\hat{f}_0^{*} + (\cdots)_{\dot{\beta}}\hat{\bar{f}}_1^{\dot{\beta}} + (\cdots)\hat{f}_2^{*} + \sqrt{2}\bar{\theta}_{\dot{\alpha}}(\cdots)^{\dot{\alpha}}\hat{f}_0^{*}$$
$$+\sqrt{2}\bar{\theta}_{\dot{\alpha}}\left(-\mathrm{i}\bar{\sigma}^{\mu\dot{\alpha}\alpha}(\mathcal{D}_{\mu}\lambda_{\alpha})\bar{\lambda}_{\dot{\beta}} - \bar{\sigma}^{\mu\nu\dot{\alpha}}{}_{\dot{\gamma}}\bar{\sigma}^{\lambda\kappa\dot{\gamma}}{}_{\dot{\beta}}F_{\mu\nu}F_{\lambda\kappa} + \mathrm{i}\bar{\lambda}^{\dot{\alpha}}(\mathcal{D}_{\mu}\lambda^{\alpha})\sigma_{\alpha\dot{\beta}}^{\mu}\ \hat{f}\right)\hat{\bar{f}}_1^{\dot{\beta}} + \sqrt{2}\bar{\theta}_{\dot{\alpha}}(\cdots)^{\dot{\alpha}}\hat{f}_2^{*}$$
$$+\overline{\theta\theta}(\cdots)\hat{f}_0^{*} + \overline{\theta\theta}(\cdots)_{\dot{\alpha}}\hat{\bar{f}}_1^{\dot{\alpha}} + \overline{\theta\theta}\left(\mathrm{i}(\mathcal{D}_{\mu}\lambda^{\alpha})\sigma_{\alpha\dot{\alpha}}^{\mu}\bar{\lambda}^{\dot{\alpha}} + D^2\right)\hat{f}_2^{*}$$

$$\Omega_{+}^{\mu\nu}\Omega_{\mu\nu}^{+}(x_{+}) = -F^{\mu\nu}F_{\mu\nu}\mathbb{I}_{+} + \sqrt{2}\theta^{\alpha}(\cdots)_{\alpha}\hat{f}_0 + \theta\theta(\cdots)^{\alpha}\hat{f}_{1\alpha}$$
$$\Omega_{-}^{\mu\nu}\Omega_{\mu\nu}^{-}(x_{-}) = -F^{\mu\nu}F_{\mu\nu}\mathbb{I}_{-} + \sqrt{2}\bar{\theta}_{\dot{\alpha}}(\cdots)^{\dot{\alpha}}\hat{f}_0^{*} + \overline{\theta\theta}(\cdots)_{\dot{\alpha}}\hat{\bar{f}}\,{}_1^{\dot{\alpha}}$$

$$\mathrm{Str}(\mathbb{E}_{+}) = -\mathrm{Tr}[D] + \mathrm{i}\sigma_{\alpha}^{\mu\nu\alpha}\mathrm{Tr}[F_{\mu\nu}] + \mathrm{Tr}[D] = 0, \mathrm{Str}(\mathbb{E}_{-}) = 0$$

$$\mathrm{Str}(\mathbb{E}_{+}^{2}) = \mathrm{Tr}\Big(\left(D^2 - \mathrm{i}\lambda^{\alpha}\sigma_{\alpha\dot{\alpha}}^{\mu}(\mathcal{D}_{\mu}\bar{\lambda}^{\dot{\alpha}})\right)$$
$$-\left(-\mathrm{i}\sigma_{\alpha\dot{\alpha}}^{\mu}(\mathcal{D}_{\mu}\bar{\lambda}^{\dot{\alpha}})\lambda^{\alpha} - \sigma_{\alpha}^{\mu\nu\ \gamma}\sigma_{\gamma}^{\lambda\kappa}{}_{\gamma}{}^{\alpha}F_{\mu\nu}F_{\lambda\kappa} + \mathrm{i}\lambda_{\alpha}(\mathcal{D}_{\mu}\bar{\lambda}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha})\right)$$
$$+\left(\mathrm{i}(\mathcal{D}_{\mu}\bar{\lambda}_{\dot{\alpha}})\bar{\sigma}^{\mu\dot{\alpha}\alpha}\lambda_{\alpha} + D^2\right)\Big)$$
$$= \mathrm{Tr}\left(2D^2 - 4\mathrm{i}\bar{\lambda}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}(\mathcal{D}_{\mu}\lambda_{\alpha}) - F_{\mu\nu}F_{\lambda\kappa} - \frac{\mathrm{i}}{2}\varepsilon^{\mu\nu\lambda\kappa}F_{\mu\nu}F_{\lambda\kappa}\right),$$
$$\mathrm{Str}(\mathbb{E}_{-}^{2}) = \mathrm{Tr}\left(2D^2 - 4\mathrm{i}\bar{\lambda}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}(\mathcal{D}_{\mu}\lambda_{\alpha}) - F_{\mu\nu}F_{\lambda\kappa} + \frac{\mathrm{i}}{2}\varepsilon^{\mu\nu\lambda\kappa}F_{\mu\nu}F_{\lambda\kappa}\right),$$

$$\mathrm{Str}(\Omega_{\pm}^{\mu\nu}\Omega_{\mu\nu}^{\pm}) = -\mathrm{Tr}[F^{\mu\nu}F_{\mu\nu}]\mathrm{Str}\mathbb{I}_{\pm} = 0$$

$$a_0(P) = a_2(P) = 0$$

$$a_4(P) = \frac{1}{16\pi^2}\int_M d^4x\mathrm{Tr}\left(2D^2 - 4\mathrm{i}\bar{\lambda}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}(\mathcal{D}_{\mu}\lambda_{\alpha}) - F_{\mu\nu}F^{\mu\nu}\right)$$

$$\frac{c_4}{8\pi^2} = \frac{1}{g^2}$$

$$I_{\mathrm{SYM}} = \int_M d^4x\mathrm{Tr}\left[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - 2\mathrm{i}\bar{\lambda}_{\dot{\beta}}\bar{\sigma}^{\mu\dot{\beta}\beta}(\mathcal{D}_{\mu}\lambda_{\beta}) + D^2\right]$$

CONCLUSIONES.

De los resultados obtenidos en el apartado anterior, se concluye: 1) que la supergravedad cuántica relativista consiste en la deformación extrema del espacio – tiempo cuántico; 2) que la deformación extrema del espacio – tiempo cuántico, se desarrolla en dimensiones altas; 3) que las dimensiones o supermembranas formadas por la referida distorsión, comportan otros puntos lejanos del espacio – tiempo cuántico, en los que, la dimensión temporal es diferente para cada caso; 4) la formación de dimensiones altas, supone la existencia de múltiples regiones que no convergen en un mismo espacio –

tiempo cuántico, sino en distintos sectores espaciales, por lo que, la transdimensionalidad y la multidimensionalidad ocurren a escalas infinitas y en ramificación, pues parten de una dimensión cero o de origen; 5) la supergravedad cuántica relativista, obedece a la interacción de una superpartícula, llamada también partícula blanca o estrella, es decir, aquella cuyo centro de masa y energía es extremadamente denso, capaz de deformar el espacio – tiempo cuántico en forma extrema, formando así, la supergravedad. Sin embargo, esta partícula también puede alcanzar la velocidad de la luz, por lo que, se la denomina suprapartícula, con las mismas propiedades que la aquí referida. Asimismo, la superpartícula o suprapartícula, según sea el caso, también produce supergravedad cuando interactúa con el supergravitón, esto es, con la supercompañera del gravitón, por lo que el espacio – tiempo cuántico distorsionado, se ve permeado y por ende, mutado por el campo supergravitónico vinculante; y, 6) la supersimetría de gauge y su modelamiento matemático, permite conciliar la relatividad general y especial y la mecánica cuántica, en la medida en que, estamos ante campos cuánticos en los que interfiere la dualidad holográfica y el principio cosmológico.

ACLARACIONES FINALES:

Algunas aclaraciones finales a tener en consideración y aplicar, a propósito de la Teoría Cuántica de Campos Relativistas o Curvos (TCCR) propuesta por este autor:

1. En todos los casos, este símbolo † será reemplazado por este símbolo†o por este símbolo †, equivaliendo lo mismo.

Símbolo a ser reemplazado.	**Símbolos de reemplazo.**
†	†
	†

2. En todos los casos, este símbolo ‡, será reemplazado por este símbolo‡o por este símbolo ‡.

Símbolo a ser reemplazado.	**Símbolos de reemplazo.**
‡	‡
	‡

3. En todos los casos, se añadirá y por ende, se calculará la magnitud§que equivale a un campo de Yang – Mills y por ende, a la teoría de Yang – Mills en sentido amplio, en relación a la Teoría Cuántica de Campos Relativistas propuesta por este autor.

4. Este símbolo • podrá usarse como exponente u operador, según sea el caso.

Las aclaraciones antes referidas aplican tanto a este trabajo como a todos los trabajos previos y posteriores publicados por este autor, según corresponda.

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