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TEACHING ELECTROSTATIC FIELD MODELLING THROUGH MATHEMATICAL AND COMPUTATIONAL REPRESENTATIONS

ENSEÑANZA DE LA MODELIZACIÓN ELECTROSTÁTICA MEDIANTE REPRESENTACIONES MATEMÁTICAS Y COMPUTACIONALES

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Teaching Electrostatic Field Modelling Through Mathematical and Computational Representations

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ABSTRACT

This article examines how relationships among charges, potential, vector fields and computational representations can inform university electrostatics teaching. A narrative documentary review focuses on the correspondence between physical assumptions, mathematical operations, representational resources and limits of interpretation. Disciplinary literature is connected with educational work on electric-field teaching, multiple representations and pedagogical content knowledge. The analysis distinguishes prescribed-charge superposition from induced-charge problems, scalar potential from vector field, and numerical accuracy from physical and educational validation. These distinctions support proposed prediction, comparison and justification tasks connecting analytical expressions, potential maps, vector diagrams and simulation outputs. Computational applications serve as cases for discussing what each observable permits one to conclude, without equating indicators of different kinds. The contribution is an organisation of criteria for selecting and discussing representations in university physics education. No classroom intervention, learning measurements or validated teaching sequence is reported. Educational implications are proposals derived from documentary analysis and require empirical assessment before effectiveness can be attributed to them. The review is selective rather than exhaustive, and its conclusions are limited by the available documentary evidence.

Keywords: physics education, mathematical models, simulation models, higher education

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Enseñanza de la Modelización Electrostática Mediante Representaciones Matemáticas y Computacionales

RESUMEN

Este artículo examina cómo las relaciones entre cargas, potencial, campo vectorial y representaciones computacionales pueden orientar la enseñanza universitaria de la electrostática. Se desarrolla una revisión documental narrativa centrada en la correspondencia entre supuestos físicos, operaciones matemáticas, recursos de representación y límites de interpretación. La literatura disciplinar se articula con antecedentes sobre enseñanza del campo eléctrico, representaciones múltiples y conocimiento pedagógico del contenido. El análisis distingue la superposición de cargas prescritas de los problemas con carga inducida, el potencial escalar del campo vectorial y la precisión numérica de la validación física y educativa. A partir de estas distinciones se proponen tareas de predicción, comparación y justificación que relacionan expresiones analíticas, mapas de potencial, diagramas vectoriales y resultados de simulación. Las aplicaciones computacionales se utilizan como casos para discutir qué permite concluir cada observable, sin equiparar indicadores de naturaleza diferente. La contribución consiste en organizar criterios para seleccionar y discutir representaciones en la formación universitaria en física. No se presentan intervenciones de aula, mediciones de aprendizaje ni una secuencia didáctica validada. Las implicaciones educativas constituyen propuestas derivadas del análisis documental y requieren evaluación empírica antes de atribuirles eficacia.

***Palabras clave*:** enseñanza de la física, modelo matemático, modelo de simulación, enseñanza superior

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Ensino da Modelagem Eletrostática Mediante Representações Matemáticas e Computacionais

RESUMO

Este artigo examina como as relações entre cargas, potencial, campo vetorial e representações computacionais podem orientar o ensino universitário de eletrostática. Desenvolve-se uma revisão documental narrativa centrada na correspondência entre pressupostos físicos, operações matemáticas, recursos de representação e limites de interpretação. A literatura disciplinar é articulada com estudos sobre ensino do campo elétrico, representações múltiplas e conhecimento pedagógico do conteúdo. A análise distingue a superposição de cargas prescritas dos problemas com carga induzida, o potencial escalar do campo vetorial e a precisão numérica da validação física e educacional. Com base nessas distinções, propõem-se tarefas de previsão, comparação e justificação que relacionam expressões analíticas, mapas de potencial, diagramas vetoriais e resultados de simulação. As aplicações computacionais são utilizadas como casos para discutir o que cada observável permite concluir, sem equiparar indicadores de naturezas diferentes. A contribuição consiste em organizar critérios para selecionar e discutir representações na formação universitária em física. Não são apresentadas intervenções em sala de aula, medidas de aprendizagem ou uma sequência didática validada. As implicações educacionais são propostas derivadas da análise documental e requerem avaliação empírica antes que se possa atribuir eficácia a elas.

Palavras-chave: ensino de física, modelo matemático, modelo de simulação, ensino superior

INTRODUCTION

University electrostatics teaching requires students to coordinate quantities that refer to the same configuration but encode different information. A charge distribution specifies sources; potential assigns a scalar value to position; the electric field assigns a vector; and a computational display selects how some of those properties become visible. An equation, a colour map and a set of arrows can therefore be mutually consistent without being interchangeable. The educational problem addressed here is how to make their relationships and limitations explicit when teaching field modelling, especially when three-dimensional configurations exceed the convenience of elementary symmetry arguments.

This problem cannot be reduced to providing more elaborate visualisations. A learner may reproduce a potential surface without explaining the direction of its gradient, or calculate a field magnitude while adding contributions as scalars. Conversely, a correct explanation may use a reduced representation whose assumptions are appropriate. These are analytically constructed examples of reasoning demands, not misconceptions measured in the present study. Research on conceptual knowledge in electricity and magnetism provides a reason to assess understanding separately from successful equation manipulation (Maloney et al., 2001).

The guiding question is: which distinctions among physical models, mathematical operations and computational representations can inform the design and interpretation of university learning tasks about electrostatic fields? The objective is to articulate disciplinary and educational literature into criteria for selecting representations, asking explanatory questions and delimiting claims based on simulations. The contribution concerns the structure of the knowledge to be taught and the evidence that students might be asked to produce. It does not establish that a particular interface, activity or instructional approach improves achievement.

Three-dimensional modelling is treated as an object of critical learning rather than an automatic indicator of sophistication. Students need to distinguish a three-dimensional physical domain from a surface plot of a scalar function of two coordinates. They also need to determine when symmetry permits a reduced solution and when interfaces or finite electrodes demand a fuller description. Accordingly, the article connects electrostatic equations to representational decisions and then uses published computational

applications to examine the scope of evidence. This educational organisation changes the purpose of comparison: the issue is what can be explained and justified, rather than which solver is universally best.

Background

The educational and disciplinary sources make different contributions. Villamizar (2025) develops a conceptual progression from mechanical work and potential energy towards electric potential and field, supported by geometrical representations and a numerical algorithm. Nascimento Neto and Siqueira (2025) present visual treatments of Laplace problems that connect boundary conditions with potential and field representations. These works offer material for examining how a mathematical operation is linked to an interpretation. Their presentation of instructional resources does not, by itself, supply a transferable estimate of learning gains.

Other contributions emphasise computational implementation. Tan et al. (2024) describe a MATLAB graphical interface addressing Coulomb's and Gauss's laws. Montilla López et al. (2025) discuss a computational treatment of electrostatic potential using Excel and Maple in an engineering-teaching context. Such resources differ in the degree to which a learner can inspect the equations, modify the source configuration or follow an iterative calculation. In this review, those features are considered opportunities for designing questions, not evidence that one software environment produces better learning than another.

The study by Reyes Roncancio and Martínez (2013), published in TED, examines pedagogical content knowledge in electric-field teaching through a qualitative case involving a preservice physics teacher. Its use of a questionnaire, interview and lesson-plan analysis provides an educational point of reference: disciplinary knowledge becomes a teaching object through decisions about explanation and representation. The present review does not reproduce that investigation or extend its findings to all physics teachers; it uses the study to locate the instructional decisions that a review of electrostatic models must address.

A more recent TED case study by Viafara-Ortiz et al. (2025) examines pedagogical content knowledge in a novice physics teacher using interviews and classroom observations. It situates teaching choices within a professional history rather than treating them as properties of a technological tool. Together, these educational antecedents motivate a distinction between the availability of a representation and the

teacher's decisions about its use. The present contribution concentrates on that distinction for electrostatic models, while recognising that documentary analysis cannot replace observations of how teachers and students actually work.

Conceptual framework

Physical models and representational choices

A physical model specifies a domain, sources, constitutive relations and conditions at boundaries or infinity. Its mathematical formulation identifies relations among quantities within those assumptions. A numerical method approximates a formulation, and a visual representation displays selected outputs using conventions such as colour, arrow length or line placement. These levels should be connected in teaching but should not be collapsed. Changing a colour scale alters a display; changing permittivity alters a material model; changing a boundary value can alter the physical problem itself.

The distinction matters when evaluating an explanation. An arrow may correctly indicate field direction while conveying no field magnitude if all arrows have been normalised. A potential map may be accurate but insufficient to determine a derivative without spatial information. A converged linear solve may satisfy an algebraic tolerance without adequately resolving a singularity or interface. The analysis therefore treats representation as an explicit selection of information. Its educational value depends on which relation learners are expected to infer and which omitted information they must recover from another representation.

Ainsworth's (2006) DeFT framework distinguishes the design, functions and learning tasks associated with multiple representations. It provides a conceptual reason to avoid assuming that displaying several formats automatically supports understanding. Here, that perspective informs the organisation of explanatory tasks: representations are paired around a specified relation, such as superposition or differentiation, and learners are asked to explain the connection. DeFT is used as an interpretive reference, not as an intervention whose effectiveness is tested in this article.

Kinds of evidence and educational inference

Three questions organise the interpretation of evidence. First, does the mathematical or computational result follow from the stated assumptions? Second, does the physical model reproduce an appropriate measured observable under specified conditions? Third, do learners develop the intended understanding

when working with the model? These questions require different evidence. A residual or convergence study addresses numerical behaviour; a comparison with measurements addresses a physical prediction; and an educational assessment requires information about learners, tasks and the interpretation of their responses.

The categories used below are analytical distinctions rather than a validated scale of competence. Source specification, scalar vector coordination, boundary interpretation, dimensional reasoning, numerical verification and evidential judgement describe aspects of an explanation that a teacher might examine. Their use does not establish a developmental sequence or a universal hierarchy of difficulty. The proposed teaching implications remain conditional on students' mathematical preparation, available time and the purpose of the course. An introductory class may prioritise superposition and potential differences, whereas an advanced class may examine anisotropic media or weak formulations.

METHODOLOGY

The article retains a narrative documentary-review approach. Its starting material is a bounded set of 50 bibliographic records assembled for the study of electrostatic-field formulations and computational applications. That number describes the initial documentary working set, not 50 eligible educational studies or 50 references cited in the present article. The educational reorientation selects material relevant to the relationship between representations and teaching, and adds explicitly identified conceptual and educational antecedents. It does not reconstruct a systematic database search or claim exhaustive coverage of physics education research.

Four complementary references support the educational framing: Ainsworth (2006), Maloney et al. (2001), Reyes Roncancio and Martínez (2013), and Viafara-Ortiz et al. (2025). They were located through targeted searches by topic and title and consultation of publisher, journal or institutional records. Their inclusion is purposive: they address multiple representations, conceptual assessment or pedagogical content knowledge. They are not counted retrospectively as members of the initial 50-record working set. No search-flow totals, duplicate counts or screening agreement coefficients are asserted.

The unit discussed is a documentary claim about a formulation, representation, computational output or educational resource. Bibliographic identity, the physical regime and the reported observable are

distinguished from the interpretation proposed in this review. Available article text and publisher descriptions support the descriptions of the selected works; access was uneven, and no uniform full-text appraisal of every initial record is claimed. Consequently, an inaccessible or unreported educational evaluation cannot be treated as evidence that a resource has no educational value. It limits what this review can conclude about its effectiveness.

The exposition compares source representation, boundary assumptions, mathematical transformations, visual conventions and the type of evidence used to support a claim. Equations provide disciplinary constraints for that comparison. Tables distinguish reported properties of published work from teaching questions proposed here. The resulting synthesis is interpretive, without statistical aggregation of learning outcomes, computational errors or execution times. There are no participants, classroom observations, assessment scores, new experiments or newly executed simulations. The activities described in the results are instructional proposals for later examination; they are not reports of implemented teaching.

RESULTS AND ANALYSIS

From charge configurations to vector superposition

In an infinite, homogeneous, linear and isotropic medium, the Coulomb representation for prescribed point charges is given by equations (1) and (2). The observation position is $\mathbf{r}$, charge i is located at $\mathbf{r}_i$, its value is q_i, and ε denotes absolute permittivity. Evaluation points exclude the singular charge positions. The potential is expressed relative to a reference compatible with behaviour at infinity (Carrillo Avila, 2025; Polito, 2025).

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\frac{q_i}{\| \mathbf{r}-\mathbf{r}_i \|}. \qquad (1)$$

$$\mathbf{E}(\mathbf{r}) = -\nabla\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N} q_i \frac{\mathbf{r}-\mathbf{r}_i}{\| \mathbf{r}-\mathbf{r}_i \|^3}. \qquad (2)$$

Vector components must be summed before the norm is calculated. Adding individual field magnitudes removes directional cancellation and gives a different result from the physical field. For a continuous distribution, the sum is replaced by an integral involving the volume charge density ρ. Surface and line

distributions require the corresponding density and measure. Polito examines why improper integrals and asymptotic conditions require explicit treatment when sources extend indefinitely (Polito, 2025).

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon}\int_{\Omega} \rho\,(\mathbf{r}')\,\frac{\mathbf{r}-\mathbf{r}'}{\|\,\mathbf{r}-\mathbf{r}'\,\|^3}\,\mathrm{d}^3 r'. \qquad (3)$$

Becerra Becerra et al. represent finite-size bodies through sets of charge sub-elements and examine spherical and non-spherical geometries. Their procedure retains Coulomb superposition and explores the approximation of an extended distribution by a discrete collection (Becerra Becerra et al., 2026). Discretising a prescribed charge distribution, however, does not itself solve induced charge redistribution on an equipotential conductor. In that problem, surface charge is an unknown linked to boundary conditions rather than an input that can be distributed uniformly for convenience.

Dimensional consistency requires $[\phi] = \mathrm{V}$, $[\mathbf{E}] = \mathrm{V\,m^{-1}} = \mathrm{N\,C^{-1}}$, $[\rho] = \mathrm{C\,m^{-3}}$ and $[\varepsilon] = \mathrm{F\,m^{-1}}$. Coordinates expressed in millimetres must be converted before they are combined with International System constants. Potential and field-strength plots do not share the same physical scale even when both use colour maps. Carrillo Avila's discussion of permittivity provides a conceptual and dimensional check rather than a performance evaluation of a 3D algorithm (Carrillo Avila, 2025).

For teaching, the relevant operation is the transition from a physical configuration to a vector contribution at an observation point. A task can ask students to mark source and observation positions before computing any magnitude. They then identify the displacement vector, determine the sign of each contribution and predict whether components reinforce or cancel. The calculation becomes evidence for a prior geometrical argument rather than a replacement for one. This task is a proposal derived from the structure of the equations; no classroom outcome is inferred from its mathematical validity.

Two equal positive charges at positions symmetric about the origin illustrate a useful distinction. Their fields cancel at the midpoint, whereas their potentials add when the reference is at infinity. Replacing one charge by an equal negative charge makes the midpoint potential zero while the fields reinforce along the line joining the charges.

These idealised examples show why a zero value of potential does not generally imply a zero field. They also separate cancellation of vector components from cancellation of scalar contributions without requiring a large numerical model.

A computational representation can make this comparison inspectable by presenting the signed components and the scalar potential at the same locations. Normalised arrows should be labelled as such, and a table of component values can accompany a colour map. The explanatory requirement is to justify why the representations agree, including at points where a magnitude or potential vanishes. An isolated image is insufficient evidence of that coordination.

Potential, boundaries and the meaning of spatial variation

The macroscopic formulation for a linear dielectric distinguishes the electric field from electric displacement. The density ρ_f denotes free charge, and $\boldsymbol{\varepsilon}$ allows anisotropy. Equations (4) and (5) establish this distinction; inserting a position-dependent permittivity into Coulomb's formula is not an automatic replacement for solving a heterogeneous boundary-value problem (Ballas, 2025; Carrillo Avila, 2025; Han et al., 2025).

$$\nabla \times \mathbf{E} = \mathbf{0}, \qquad \nabla \cdot \mathbf{D} = \rho_f, \qquad \mathbf{D} = \boldsymbol{\varepsilon}\mathbf{E}. \qquad (4)$$

$$-\nabla \cdot (\boldsymbol{\varepsilon}\nabla\phi) = \rho_f. \qquad (5)$$

For constant scalar permittivity, the equation becomes $\nabla^2\phi = -\rho_f/\varepsilon$; in a free-charge-free region under the same constitutive assumption, it reduces to Laplace's equation. Dirichlet conditions prescribe potential, whereas Neumann conditions prescribe its normal derivative or the associated flux. With $g = \mathbf{n} \cdot \mathbf{D}$, the flux condition is $-\mathbf{n} \cdot \boldsymbol{\varepsilon}\nabla\phi = g$. A pure Neumann problem requires compatibility between total flux and charge, together with a reference removing the additive indeterminacy. Cho's study concerns mixed boundary conditions and does not remove the need to specify their physical meaning (Cho, 2022).

At an ideal interface without a dipole layer, potential is continuous. If $\mathbf{n}$ points from medium 1 to medium 2 and σ_f denotes free surface charge density, the jumps in equation (6) apply. The normal component of $\mathbf{E}$ can be discontinuous even without free surface charge when permittivity changes. Interpolation that imposes continuity on every field component may therefore smooth a physical discontinuity that the model must preserve.

$$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = \mathbf{0}, \qquad \mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \sigma_f. \qquad (6)$$

Lima studies the field on a uniformly charged cylindrical shell using a representation by rings and elliptic integrals (Lima, 2025). This problem requires distinguishing the two one-sided limits from a value assigned on the ideal surface. A regularised mean value does not invalidate Gauss's law, nor does it imply that every surface field equals half the normal jump. Geometry, contributions from external sources and the definition of the limit enter that identification. The review retains the result within its geometric context rather than treating it as a universal interface condition.

The instructional connection is between a scalar distribution and its spatial rate of change. Equipotential contours encode equal values, while the gradient depends on how those values vary with position. Learners can compare two plots that use different vertical scales but represent the same potential data, and explain why changing the display does not change the physical field. They can also shift every potential value by the same constant and identify which predicted quantities remain invariant. These comparisons make the reference dependence of potential visible without confusing it with an ambiguity in the electric field.

Boundary conditions provide a second explanatory focus. Specifying a conductor potential is not equivalent to prescribing a uniform surface charge on an arbitrarily shaped conductor. A classroom comparison can keep the geometry fixed while changing the physical quantity assigned at the boundary, then ask which unknown must be solved for. At a dielectric interface, students can annotate which field components must satisfy continuity or jump relations and from which side a numerical value is sampled. This connects the calculation to the physical meaning of its input rather than treating boundary settings as software options without interpretation.

The distinction between geometric dimension and display dimension should be explicit. A graph of potential above a two-coordinate domain has a third plotting axis for the value of the potential; that axis is not automatically a third spatial coordinate. Conversely, a planar section of a three-dimensional solution retains only part of its spatial information. A teaching task can request the domain coordinates, the plotted quantity, units and any symmetry assumption before interpreting the figure. The task assesses representational description, not an ability to produce the most visually elaborate image.

Alternative formulations and their domains of validity

Salazar et al. develop a representation of surface-electrode fields through an electric vector potential and an integral analogous to Biot–Savart (Salazar et al., 2022). The analogy is mathematical: it does not transform electrostatic charges into magnetic currents. Its scope includes charge-free regions and specific conditions in a planar electrode system. Conditions on the conductors must be distinguished from those imposed in the gap; assigning Neumann conditions indiscriminately to every surface changes the physical problem.

$$\mathbf{E} = \nabla \times \boldsymbol{\Theta} \quad \Rightarrow \quad \nabla \cdot \mathbf{E} = 0. \qquad (7)$$

Identity (7) identifies a restriction that must accompany this representation. In a homogeneous medium with nonzero volume charge, the field cannot globally be expressed solely as the curl of a regular potential. In heterogeneous media, the absence of free charge implies $\nabla \cdot \mathbf{D} = 0$ but not necessarily $\nabla \cdot \mathbf{E} = 0$. Beyond this local restriction, global existence depends on the domain and its fluxes. Equivalence between formulations must therefore be checked for the same sources, boundaries and topological conditions; the corpus does not establish general superiority of the electric vector potential.

The educational purpose of introducing this advanced formulation is to examine the conditions attached to an equivalence claim. Students can be asked to identify which divergence statement is required, where it holds and whether the domain introduces global restrictions. This does not require presenting the electric vector potential as a replacement for the scalar potential in an introductory course. It offers an advanced comparison in which a familiar vector-calculus identity constrains the admissible physical representation. The mathematical analogy with magnetostatics must be explained as an analogy between integral structures, not as an identity between their sources.

Numerical methods as objects of explanation

The finite-element method approximates a weak formulation of the potential problem. For a test function v vanishing on the Dirichlet boundary, and using the preceding flux convention, the structure in equation (8) follows. The discrete matrix incorporates geometry and material constitution. Solving it provides potential coefficients; the field subsequently requires differentiation or a compatible reconstruction. The studies by Han et al. and Liao et al. illustrate applications where interfaces or three-dimensional geometries enter explicitly (Han et al., 2025; Liao et al., 2023).

$$\int_{\Omega} \nabla v \cdot \boldsymbol{\varepsilon} \nabla \phi \, \mathrm{d}\Omega = \int_{\Omega} v \, \rho_f \, \mathrm{d}\Omega - \int_{\Gamma_N} v \, g \, \mathrm{d}\Gamma. \qquad (8)$$

Arun et al. investigate an algebraic topological method for three-dimensional Laplace problems (Arun et al., 2023). Its interest lies in the discrete representation of the problem's relationships, which must be distinguished from metric and constitutive choices. Exact topological identities in a discretisation do not, by themselves, imply exactness of the approximated field. Without consistent information on system size, tolerance, hardware and error, individual comparisons cannot establish a general ranking against finite elements.

Finite differences offer a direct implementation on structured meshes. Montilla López et al. use the Liebmann method on a square domain and compare computational presentation in Excel and Maple (Montilla López et al., 2025). This work supports a teaching treatment of Poisson's equation, but a plotted potential surface does not make it a three-dimensional validation. Similarly, Wang et al.'s study using superLU is an explicitly two-dimensional conference contribution; superLU is the algebraic solver rather than an alternative physical formulation (Wang et al., 2022).

Han et al. combine a direct-implicit particle-in-cell method with trilinear anisotropic immersed finite elements (Han et al., 2025). Their model uses Cartesian meshes independent of the interface and allows larger spatial and temporal steps in the cases examined. Stability, temporal accuracy and spatial resolution must be distinguished: they are related but different properties. Avoiding redundant interpolation is an advantage of the computational arrangement studied; it does not establish that every interface or plasma regime can be resolved at the same cost and accuracy.

A learning task based on a solver should distinguish the physical model, the discretisation and the stopping rule. Refining a mesh while retaining the same boundary values tests a numerical choice; changing the size of a truncated domain can test sensitivity to an artificial boundary; altering a material law changes a physical assumption. These operations answer different questions even when all produce changes in a plotted field.

Students can document one change at a time and state which claim their comparison supports. No universal convergence rate should be assigned without the regularity and discretisation conditions required by the method.

Table 1 organises the formulations and approaches as possible objects of instruction. Its rows are not competing categories at the same conceptual level: a particle model may use a finite-element field solve, and a response surface may summarise outputs from that solve. The educational implication is to ask learners to identify the role of each component before comparing accuracy or efficiency. This avoids treating an algorithm name, a field equation and an optimisation strategy as interchangeable descriptions of a computational model.

Table 1 Formulations, computational approaches and limits of comparison

Formulation or approach	Represented quantity	Limit relevant to teaching
Discrete Coulomb superposition	Field from prescribed charges	Induced charge requires additional boundary constraints.
Poisson and Laplace formulations	Potential subject to source, material and boundary assumptions	Accurate potential values do not alone establish accurate gradients.
Immersed FEM with PIC	Field solution coupled to particle evolution	Stability and resolution are different properties.
Algebraic topological method	Discrete relations in a boundary-value problem	Exact identities do not imply zero field error.
Response surface fitted to FEM	Approximate design response	Agreement with simulations is not experimental or educational validation.

Note. Synthesis of the formulations discussed above and the studies by Arun et al. (2023), Han et al. (2025), and Nadjem and Rouagdia (2025). FEM = finite-element method; PIC = particle-in-cell. The teaching implications are interpretations of this review, not measured learning outcomes.

Published observables and critical reading of computational evidence

In electrostatic separation, Nadjem and Rouagdia combine finite elements and response surface methodology to explore electrode dimensions and inclination (Nadjem & Rouagdia, 2025). The study reports an approximately 65% improvement in field parameters relative to comparison designs and close agreement between simulations and statistical predictions. These values describe the selected design and responses. They do not represent a 65% reduction in numerical error or, by themselves, demonstrate an equivalent improvement in the purity or recovery of separated materials.

Tang et al. study multisource corona discharges through a three-dimensional fluid model and current validation (Tang et al., 2026). Increasing the number of electrodes can increase total current while reducing the mean current per needle through shielding. The comparison concerns the geometries and

conditions investigated. A maximum normalised current error of 10% is not a 10% error in every local field component: current integrates transport and ionisation processes and is a different observable.

Yu et al. present indirect electrostatic–fluid–structure coupling for a microelectromechanical comb resonator (Yu et al., 2022). Their experimental comparison reports errors of 15.47% in amplitude and 12.48% in resonant frequency. These values must retain their reference variables and denominators rather than becoming a generic measure of electromagnetic accuracy. Agreement in motion also depends on damping, structural behaviour and residual thermal stress; adjusting only the field may leave a dominant discrepancy unchanged.

Tsuchiya presents a device for quantitative visualisation of a two-dimensional distribution of electrostatic vectors (Tsuchiya, 2021). It is a relevant measurement contribution, but its description is not equivalent to complete real-time volumetric reconstruction of three components. Nor does it establish that it is the only existing experimental method. This distinction matters because measurement of one component, a projection or a planar map cannot automatically validate a three-dimensional field. These cases can support an educational task in which students identify the numerator, denominator and reference of an error statement before comparing results. A percentage referring to current cannot be assigned to every field component, and an error in resonant motion does not isolate the error of a coupled electrostatic calculation. Likewise, an improvement in a design response is not necessarily a reduction in numerical error. Table 2 preserves these distinctions and can be used to discuss what evidence would be needed for a stronger conclusion. Its instructional use is proposed here; the cited application studies are not recast as educational experiments.

Table 2 Reported observables and limits of inference in selected studies

Study	Reported observable or result	What cannot be inferred
Nadjem and Rouagdia (2025)	Approximately 65% improvement in field parameters relative to conventional designs	An equivalent error reduction or improvement in material recovery.
Tang et al. (2026)	Maximum normalised current error of 10%	The same local error in every electric-field component.
Yu et al. (2022)	Amplitude error 15.47%; resonant-frequency error 12.48%	Accuracy of the electrostatic solver independently of the coupled model.
Tsuchiya (2021)	Imaging of a two-dimensional distribution of field vectors	Complete volumetric reconstruction from the reported planar map.

Note. Source values are attributed to the studies in the first column. The final column states limits of inference applied in this review. The quantities are not combined into a common performance indicator.

Teaching implications and evidence of explanation

The interface developed by Tan et al., the visualisations of Nascimento Neto and Siqueira, and Villamizar's didactic treatment make the relationship between potential, field and geometry explicit (Nascimento Neto & Siqueira, 2025; Tan et al., 2024; Villamizar, 2025). Their representational usefulness must be distinguished from metrological validation. Field-line density depends on graphical seeding, normalised arrows can hide magnitude differences, and a planar section does not represent the complete three-dimensional distribution. A scientific figure must identify the domain, units, scale, charge positions and representation convention.

The synthesis suggests designing tasks around a relation that learners must explain, rather than around a sequence of software commands. The intended product may be an annotated diagram, a comparison of signed components, a boundary-condition argument or an explanation of an error metric. Each product should expose the connection between a representation and a physical assumption. Table 3 translates the distinctions developed in the analysis into possible prompts and observable features of an explanation. These features are proposed assessment criteria, not validated indicators or measurements obtained from students.

A coherent classroom activity would make prediction, calculation and interpretation distinguishable. Before running a model, students state a directional or qualitative prediction with its assumptions. During calculation, they record the quantities varied and the conventions used to display the output.

Afterwards, they compare the result with the prediction and explain any discrepancy using physical or numerical reasons. This organisation is offered as a design option. The review does not prescribe a fixed number of sessions or claim that the order is optimal for every population.

Assessment should preserve the difference between computational execution and physical explanation. A functioning script can show that a procedure was implemented, but it does not by itself show that a learner understands why the model is appropriate. Conversely, a minor programming error need not invalidate a correct argument about superposition. A teacher can request both an interpretable output and an explanation of one limiting case, then examine them separately. This makes it possible to locate whether a discrepancy concerns vector reasoning, units, boundary assumptions, differentiation or the implementation of an otherwise appropriate model.

Table 3 Proposed teaching tasks derived from the documentary synthesis

Analytical focus	Possible task	Evidence to examine
Scalar–vector distinction	Compare the midpoint of equal-charge and opposite-charge pairs	An explanation of scalar addition and vector cancellation or reinforcement.
Potential and gradient	Shift the reference potential and compare field predictions	Recognition that a constant shift preserves the field.
Boundary assumptions	Contrast fixed potential with prescribed surface charge	Identification of the specified quantity and the induced unknown.
Spatial representation	Label domain coordinates, plotted values and units	Distinction between a potential surface and a spatial domain.
Numerical verification	Vary mesh resolution while retaining the physical problem	Separation of discretisation sensitivity from model changes.
Scope of evidence	Interpret the different quantities reported in Table 2	A conclusion limited to the observable and comparison actually reported.

Note. Author-proposed tasks derived from the preceding analysis. They have not been implemented or validated as a teaching sequence or assessment instrument.

DISCUSSION

Educational contribution and limits of transfer

The contribution of the review is an organisation of explanatory relations for teaching electrostatics, rather than a claim that visualisation alone improves conceptual understanding. The educational antecedents address representational resources and teacher knowledge, while the disciplinary sources impose constraints on what a representation may legitimately mean.

Bringing them into dialogue makes it possible to distinguish an attractive display from an accountable explanation. A task becomes relevant when it requires the learner to connect a physical assumption to an operation and to the information preserved in the resulting representation.

This distinction also limits transfer from industrial or multiphysics applications. Their inclusion can support scientific reading and analysis of evidence, but their reported performance does not validate a teaching resource. A separator study, for example, may supply a question about design response; it does not establish how students understand the electric field. Similarly, a measurement device can illustrate the difference between a planar map and a volume, without proving that its use is practical or beneficial in a particular classroom. Educational relevance requires an explicit instructional question, not merely a sophisticated physical application.

For teacher education, the tables can serve as material for discussing why a representation is selected and what a learner is expected to infer from it. The two TED studies provide examples of researching teacher knowledge through educational evidence (Reyes Roncancio & Martínez, 2013; Viafara-Ortiz et al., 2025). The present review contributes a disciplinary set of questions to that discussion, but it does not characterise any teacher's pedagogical content knowledge. Such a claim would require data from actual teaching decisions and practice.

Physical consistency and the interpretation of learning evidence

Field recovery imposes an additional requirement beyond potential accuracy. Small spatial oscillations in potential can produce appreciable changes in its gradient; a smooth plot or a small algebraic residual therefore cannot establish local field accuracy. At interfaces, values must be evaluated from the appropriate material side. For point charges, self-field is excluded when calculating force on the same particle. If a singularity is regularised, the regularisation length and form must be stated because they modify the effective model.

Error indicators must identify the variable, norm, domain and reference. Relative error becomes misleading near a field null if the denominator is almost zero. Angular error is also undefined at such points. Timing comparisons require equivalent tolerances and a description of mesh generation, assembly, solution and post-processing costs. The examined literature contains useful results, but

heterogeneous observables preclude calculating an aggregate improvement representative of every methodological family.

These disciplinary checks can inform educational assessment without becoming a substitute for it. If students are asked to explain a field null, their response should be interpreted with respect to the selected configuration and representation. If they are asked about error, the reference and the observable must be specified. A low score on a task with undefined symbols or an ambiguous colour convention may reflect the task design rather than the targeted understanding. Any later investigation of the proposed activities would need to document the instructional context and justify how the assessment captures the intended reasoning.

The distinction between a conceptual survey and a locally designed modelling task is also relevant. Maloney et al. (2001) provide a published approach to assessing conceptual knowledge in electricity and magnetism. That work is an assessment antecedent, not evidence that the tasks in Table 3 are validated. The latter target explanations involving particular modelling decisions and would require their own examination. No reliability coefficient, achievement gain or causal effect can be assigned to them from the present documentary synthesis.

Scope of the documentary conclusions

The review is selective and cannot establish which instructional approach is most effective across courses or populations. Its initial working set favoured disciplinary and computational publications, and the educational supplementation is purposive rather than exhaustive. The analysis therefore offers a reasoned connection between physical constraints and possible teaching questions, not a complete state of research on learning electrostatics. The difference is consequential: a topic insufficiently documented in this selection cannot be described as absent from the literature.

The proposed tasks have not been applied to students. Their feasibility, difficulty and capacity to elicit the intended reasoning remain empirical questions. Mathematical soundness does not establish pedagogical effectiveness. The tables organise distinctions for instructional scrutiny; they are not validated educational instruments.

CONCLUSIONS

The educational reorientation of electrostatic modelling centres on the relationships learners must explain among sources, potential, vector fields and computational representations. The documentary analysis identifies distinctions that can guide those explanations: prescribed versus induced charge, scalar values versus vector components, boundary assumptions versus software settings, and numerical consistency versus physical or educational validation. Retaining these distinctions allows computational work to be discussed as a modelling practice with explicit assumptions.

The proposed tasks connect analytical expressions, graphical conventions and the interpretation of published results. They offer material for university physics teaching and teacher education, while remaining proposals rather than demonstrated instructional outcomes. The review supports the selection of explanatory questions and the delimitation of evidence; it does not establish learning gains, universal superiority of three-dimensional modelling or an optimal teaching sequence. Its practical contribution is a transparent basis for subsequent author-led design and empirical examination of teaching activities.

Author contribution

The four authors contributed to the conceptualisation of the study and the documentary analysis underlying its conceptual framework. All authors participated in the scientific review and interpretation of the literature, approved the final version of the manuscript for submission and accept responsibility for its content.

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ChatGPT was used for grammatical correction and linguistic editing of the manuscript. The authors are responsible for the study design, the preparation of the article and the interpretation of the literature.

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